System for Linear Logic

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1 Measure of derivations

• Case(s) rule 1

• Case(s) rule!

• Case(s) rule ?W

$$\frac{\mathbf{h}_1 :\vdash \Delta_2}{\bullet \mathbf{h}_1 :\vdash \Delta_2, ?_{\mathsf{F}_3}} \ ?_W \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1 :\vdash \Delta_2}{\bullet \mathbf{h}_1 :\vdash \Delta_2} \ ^{\mathsf{nx}}}{\bullet \bullet \mathbf{h}_1 :\vdash \Delta_2, ?_{\mathsf{F}_3}} \ ^{\mathsf{P}W}$$

• Case(s) rule ?C

$$\begin{array}{cccc} \underline{\mathbf{h}_1 :\vdash \Delta_2, ?\mathsf{F}_3, ?\mathsf{F}_3} & \mathsf{ac} & & & & & & & \\ \underline{\mathbf{h}_1 :\vdash \Delta_2, ?\mathsf{F}_3, ?\mathsf{F}_3} & \mathsf{ac} & & & & & \\ \underline{\mathbf{h}_1 :\vdash \Delta_2, ?\mathsf{F}_3, ?\mathsf{F}_3} & & \mathsf{IH} & & \\ \underline{\mathbf{e}} \underline{\mathbf{h}_1 :\vdash \Delta_2, ?\mathsf{F}_3, ?\mathsf{F}_3} & & \mathsf{IH} & & \\ & & \underline{\mathbf{e}} \underline{\mathbf{h}_1 :\vdash \Delta_2, ?\mathsf{F}_3, ?\mathsf{F}_3} & ?C & & & \\ \end{array} \right.$$

• Case(s) rule?

$$\begin{array}{c|c} \underline{h_1 :\vdash \Delta_2, F_3} \\ \bullet h_1 :\vdash \Delta_2, ?F_3 \end{array} ? \qquad \rightarrow \qquad \begin{array}{c|c} \hline \underline{h_1 :\vdash \Delta_2, F_3} \\ \bullet h_1 :\vdash \Delta_2, F_3 \end{array} \text{II} \\ \hline \bullet \bullet h_1 :\vdash \Delta_2, ?F_3 \end{array} ?$$

• Case(s) rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_3, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_3 \$ \mathbf{F}_4} \quad \$ \qquad \rightarrow \qquad \frac{\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_3, \mathbf{F}_4}{\bullet} \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \end{array} \overset{\mathbf{ax}}{} \\ \bullet \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_3 \$ \mathbf{F}_4} \quad \$ \end{array}$$

• Case(s) rule &

$$\frac{\mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_3 \quad \mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4} \quad \& \qquad \rightarrow \qquad \underbrace{\frac{\mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_3}}_{\bullet \bullet \ \mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_3} \overset{\mathtt{ax}}{=} \underbrace{\frac{\mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_4}}_{\bullet \mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_4} \overset{\mathtt{ax}}{=} \underbrace{\frac{\mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_4}}_{\bullet \&}$$

• Case(s) rule \oplus_B

$$\frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{f}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\frac{\overline{\mathbf{h}_1} : \vdash \Delta_2, \mathbf{f}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{f}_4} \ \text{IH}}{\bullet \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_B$$

• Case(s) rule \oplus_A

• Case(s) rule ⊥

 \bullet Case(s) rule \top

$$\overline{{}_{\bullet \mathbf{h}_1} :\vdash \top, \Delta_2} \ \top \qquad \rightarrow \qquad \overline{{}_{\bullet \bullet \mathbf{h}_1} :\vdash \Delta_2, \top} \ \top$$

 \bullet Case(s) rule I

 \bullet Case(s) rule \otimes

$$\frac{\mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_4 \quad \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \vdash \Delta_2, \Delta_3, \mathtt{F}_4 \otimes \mathtt{F}_5} \ \otimes \qquad \rightarrow \qquad \frac{\frac{\overline{\mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_4}}{\bullet \mathtt{h}_1 : \vdash \Delta_2, \mathtt{F}_4} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{ax}}{=} \ \frac{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_5} \ \overset{\mathtt{h}_2 : \bot}{=} \ \overset{\mathtt{h}_3 : \vdash \Delta_3, \mathtt{h}_3 : \bot}{\bullet \mathtt{h}_1 : \vdash \Delta_3, \mathtt{h}_3 : } \ \overset{\mathtt{h}_3 : \bot}{=} \ \overset{\mathtt{h}_3 : \vdash \Delta_3, \mathtt{h}_3 : \mathtt{h}_3 :$$

2 Weakening on bang: $\vdash \Gamma$, !F implies $\vdash \Gamma$, F.

- Case(s) rule 1
- Case(s) rule!

$$\frac{\mathtt{h}_1 : \vdash \mathtt{F}_3, ? \Upsilon 2}{\bullet \mathtt{h}_1 : \vdash ! \mathtt{F}_3, ? \Upsilon 2} \ ! \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_1 : \vdash ? \Upsilon 2, \mathtt{F}_3}}{\bullet \mathtt{h}_1 : \vdash ? \Upsilon 2, \mathtt{F}_3} \ ^{\mathtt{ax}}_{\mathtt{H}}$$

• Case(s) rule ?W

• Case(s) rule ?C

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbb{IF}_1, \mathbb{?F}_3, \mathbb{?F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbb{IF}_1, \Delta_4, \mathbb{?F}_3} \ ?C \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbb{IF}_1, \mathbb{?F}_3, \mathbb{?F}_3}{\mathbf{h}_2 : \vdash \Delta_4, \mathbb{F}_1, \mathbb{?F}_3, \mathbb{?F}_3}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbb{F}_1, \mathbb{?F}_3, \mathbb{?F}_3} \ \frac{\mathbf{ax}}{\mathbb{I}} \\ \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbb{F}_1, \mathbb{?F}_3, \mathbb{?F}_3}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbb{F}_1, \mathbb{?F}_3} \ ?C$$

• Case(s) rule?

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{lF_1}}{\bullet \mathbf{h}_2 : \vdash \mathbf{lF_1}, \Delta_4, ?\mathbf{F}_3} \ ? \\ & \rightarrow \\ & \frac{\frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{lF_1}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_1, \mathbf{F}_3}} \\ \bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_1, ?\mathbf{F}_3 \\ \end{array} \begin{array}{c} \mathbf{IH} \\ \mathbf{lF}_3 : \vdash \Delta_4, \mathbf{h}_1, \mathbf{h}_2 \\ \mathbf{h}_2 : \vdash \Delta_4, \mathbf{h}_1, ?\mathbf{h}_3 \\ \mathbf{h}_3 : \vdash \Delta_4, \mathbf{h}_1, ?\mathbf{h}_3 \\ \mathbf{h}_4 : \vdash \Delta_4, \mathbf{h}_1, ?\mathbf{h}_3 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_1, ?\mathbf{h}_3 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_1, \mathbf{h}_2 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_1, \mathbf{h}_3 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_1, \mathbf{h}_2 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_1, \mathbf{h}_3 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 \\ \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h}_5 : \vdash \Delta_4, \mathbf{h$$

• Case(s) rule \$

$$\begin{array}{c} \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, !\mathbf{F}_1 \\ \bullet \mathbf{h}_2 : \vdash !\mathbf{F}_1, \Delta_5, \mathbf{F}_3 \$ \mathbf{F}_4 \end{array} \ \mathbf{s} \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, !\mathbf{F}_1} \\ \overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3, \mathbf{F}_4} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \$ \mathbf{F}_4 \end{array} \ \begin{array}{c} \mathbf{n} \\ \mathbf{n} \\$$

• Case(s) rule &

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, !\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, !\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash !\mathbf{F}_1, \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \underbrace{\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{h}} \quad \underbrace{\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{h}} \quad \underbrace{\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{h}} \quad \underbrace{\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4}$$

• Case(s) rule \oplus_B

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_4, !\mathbf{f}_1}{\bullet \mathbf{h}_2 :\vdash !\mathbf{f}_1, \Delta_5, \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_4, !\mathbf{f}_1}}{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_1, \mathbf{f}_4} \overset{\mathrm{ax}}{}_{\mathrm{IH}}}{}_{\oplus_B}$$

• Case(s) rule \oplus_A

$$\begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{lF}_1 \\ \bullet \mathbf{h}_2 : \vdash \mathbf{lF}_1, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4 \end{array}}_{\bullet \mathbf{h}_2 : \vdash \mathbf{l}_5, \mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_{A} \qquad \rightarrow \qquad \begin{array}{c} \overline{\begin{array}{c} \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{lF}_1 \\ \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \end{array}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_{A} \end{array}$$

• Case(s) rule \perp

• Case(s) rule \top

- \bullet Case(s) rule I
- Case(s) rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, !\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash !\mathbf{F}_1, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \otimes \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, !\mathbf{F}_1}}{\underline{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4}} \quad \overset{ax}{\underset{\mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, !\mathbf{F}_1}} \quad \overset{ax}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, !\mathbf{F}_1}} \quad \overset{ax}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, !\mathbf{F}_1}} \quad \overset{ax}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, !\mathbf{F}_1}}} \end{array}$$

$$\begin{array}{c} \underline{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, \mathbf{!F_1}} \\ \bullet \mathbf{h}_2 : \vdash \mathbf{!F_1}, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \otimes \\ \end{array} \quad \rightarrow \quad \begin{array}{c} \underline{\underline{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4}} \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \end{array} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}{\mathbf{h}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_2 :$$

3 Invertibility of Rules

3.1 Status of 1: : Invertible

• Case rule 1

- Case rule!
- \bullet Case rule ?W
- Case rule ?C
- Case rule?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I
- Case rule \otimes

3.2 Status of !: : Invertible

- Case rule 1
- Case rule!

• Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, \mathsf{IF}_1}{\bullet \mathbf{h}_2 : \vdash (? \Upsilon 4, \mathsf{IF}_1), ? \mathbf{F}_3} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash ? \Upsilon 4, \mathbf{F}_1}}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \mathbf{F}_1, ? \mathbf{F}_3} \ ?W$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_1, ? \mathbf{F}_3, ? \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash (? \Upsilon 4, ! \mathbf{F}_1), ? \mathbf{F}_3} & ?C & \rightarrow & & \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, \mathbf{F}_1, ? \mathbf{F}_3, ? \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \mathbf{F}_1, ? \mathbf{F}_3} & \frac{\mathsf{ax/ind}}{?C} \end{array}$$

• Case rule?

$$\frac{\underset{\bullet_1:\vdash F_3, ?\Upsilon 4, \ !F_1}{\overset{\bullet_1}{\bullet}} ?}{\underset{\bullet_1:\vdash (?\Upsilon 4, \ !F_1), ?F_3}{\bullet}}? \qquad \rightarrow \qquad \frac{\frac{\underset{\bullet_1:\vdash ?\Upsilon 4, F_3, \ !F_1}{\overset{\bullet}{\bullet}}}{\overset{\bullet_1}{\bullet}} \underset{:\vdash ?\Upsilon 4, F_1, ?F_3}{\overset{\bullet_2}{\bullet}}?}{\underset{\bullet_1:\vdash ?\Upsilon 4, F_1, ?F_3}{\bullet}} \overset{ax}{\underset{!}{\bullet}} ?$$

- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I
- Case rule \otimes

3.3 Status of ?W: Non invertible

- Case rule 1
- Case rule!

$$\frac{h_2 : \vdash F_3, ?\Upsilon 4, ?F_1}{\bullet h_2 : \vdash (?\Upsilon 4, ?F_1), !F_3} \ ! \qquad \rightarrow \qquad \frac{\overline{h_2 : \vdash ?\Upsilon 4, F_3}}{\bullet h_2 : \vdash ?\Upsilon 4, !F_3} \overset{ax/ind}{!}$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 :\vdash \Delta_4, ?\mathbf{f}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_4, ?\mathbf{f}_1), ?\mathbf{f}_3} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_4}}{\bullet \mathbf{h}_2 :\vdash \Delta_4, ?\mathbf{f}_3} \ ?W$$

• Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, ? \mathbf{F}_1, ? \mathbf{F}_3, ? \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ? \mathbf{F}_1), ? \mathbf{F}_3} \ ?C \qquad \rightarrow \qquad \frac{\mathbf{h}_2 : \vdash \Delta_4, ? \mathbf{F}_3, ? \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \Delta_4, ? \mathbf{F}_3} \begin{array}{c} \mathbf{ax/ind} \\ ?C \end{array}$$

• Case rule?

• Case rule \$

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \$\mathbf{F}_4} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4}}{\bullet \mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3 \$\mathbf{F}_4} \quad \overset{\mathsf{ax/ind}}{\$}$$

• Case rule &

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3} \quad \text{ax/ind} \quad \overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4} \quad \frac{\text{ax/ind}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \quad \\$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_4}{\bullet \mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4}}{\bullet \mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \stackrel{\mathrm{ax/ind}}{\oplus_B}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_3, ?\mathbf{f}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{f}_1), \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_3}}{\bullet \mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_3 \oplus \mathbf{f}_4} \stackrel{\mathrm{ax/ind}}{\oplus_A}$$

 \bullet Case rule \bot

$$\frac{\mathtt{h}_2 : \vdash \Delta_3, ?_{\mathsf{F_1}}}{\bullet \mathtt{h}_2 : \vdash \bot, \Delta_3, ?_{\mathsf{F_1}}} \ \bot \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_2 : \vdash \Delta_3}}{\bullet \mathtt{h}_2 : \vdash \Delta_3, \bot} \ \bot$$

 \bullet Case rule \top

- ullet Case rule I
- Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, ?\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash (\Delta_6, ?\mathbf{F}_1), \Delta_3, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \otimes \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5}}{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \overset{ax}{\otimes} \\ \end{array}$$

$$\begin{array}{c} \underline{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, ?\mathbf{F}_1} \\ \bullet \underline{\mathbf{h}_2 : \vdash \Delta_3, (\Delta_6, ?\mathbf{F}_1), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \end{array} \rightarrow \\ \begin{array}{c} \overline{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4} \quad \overline{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5} \\ \bullet \underline{\mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \end{array} \right. \\ \times \\ \end{array}$$

3.4 Status of ?C: Non invertible

- ullet Case rule $oldsymbol{1}$
- Case rule!

$$\frac{\mathtt{h}_2 : \vdash \mathtt{F}_3, ?\Upsilon 4, ?\mathtt{F}_1}{\bullet \mathtt{h}_2 : \vdash (\Upsilon \Upsilon 4, ?\mathtt{F}_1), !\mathtt{F}_3} \ ! \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_2 : \vdash ?\Upsilon 4, \mathtt{F}_3, ?\mathtt{F}_1, ?\mathtt{F}_1}}{\bullet \mathtt{h}_2 : \vdash ?\Upsilon 4, !\mathtt{F}_3, ?\mathtt{F}_1, ?\mathtt{F}_1} \ !$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ?\mathbf{F}_1), ?\mathbf{F}_3} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_4, ?\mathbf{F}_1, ?\mathbf{F}_1}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_3} \ ?W$$

$$\frac{\mathbf{h}_1 :\vdash \Delta_2}{\bullet \mathbf{h}_1 :\vdash \Delta_2, ?_{\mathbf{F}_3}} \ ?_W \qquad \rightarrow \qquad \frac{\bullet \mathbf{h}_1 :\vdash \Delta_2, ?_{\mathbf{F}_3}, ?_{\mathbf{F}_3}}{\bullet \mathbf{h}_1 :\vdash \Delta_2, ?_{\mathbf{F}_3}, ?_{\mathbf{F}_3}} \ \text{fail}$$

 \bullet Case rule ?C

$$\frac{\mathbf{h}_2 :\vdash \Delta_4, ?\mathsf{F}_1, ?\mathsf{F}_3, ?\mathsf{F}_3}{\bullet \mathbf{h}_2 :\vdash (\Delta_4, ?\mathsf{F}_1), ?\mathsf{F}_3} \ ?C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_4, ?\mathsf{F}_1, ?\mathsf{F}_1, ?\mathsf{F}_3, ?\mathsf{F}_3}}{\bullet \mathbf{h}_2 :\vdash \Delta_4, ?\mathsf{F}_1, ?\mathsf{F}_1, ?\mathsf{F}_3} \ \frac{\mathsf{ax/ind}}{?C}$$

$$\frac{\mathbf{h}_1 : \vdash \Delta_2, ?_{\mathsf{F}_3}, ?_{\mathsf{F}_3}}{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?_{\mathsf{F}_3}} \ ?C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \Delta_2, ?_{\mathsf{F}_3}, ?_{\mathsf{F}_3}, ?_{\mathsf{F}_3}}}{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?_{\mathsf{F}_3}, ?_{\mathsf{F}_3}} \ \frac{\mathsf{ax/ind}}{?C}$$

• Case rule?

$$\frac{\mathbf{h}_2 :\vdash \Delta_4, \mathbf{F}_3, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_4, ?\mathbf{F}_1), ?\mathbf{F}_3} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_4, \mathbf{F}_3, ?\mathbf{F}_1, ?\mathbf{F}_1}}{\bullet \mathbf{h}_2 :\vdash \Delta_4, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_3} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \$\mathbf{F}_4} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}}{\bullet \mathbf{h}_2 :\vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \$\mathbf{F}_4} \quad \$$$

• Case rule &

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad \text{ax/ind}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{h}_3, ?\mathbf{F}_1, ?\mathbf{$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1} \ \ ^{\mathrm{ax/ind}}}{\bullet \mathbf{h}_2 :\vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1, ?\mathbf{F}_1}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A$$

 \bullet Case rule \bot

$$\frac{\mathbf{h}_2 : \vdash \Delta_3, ?\mathbf{F_1}}{\bullet \mathbf{h}_2 : \vdash \bot, \Delta_3, ?\mathbf{F_1}} \ \bot \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_3, ?\mathbf{F_1}, ?\mathbf{F_1}}}{\bullet \mathbf{h}_2 : \vdash \Delta_3, \bot, ?\mathbf{F_1}, ?\mathbf{F_1}} \overset{\mathsf{ax/ind}}{\bot}$$

 \bullet Case rule \top

- \bullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, ?\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash (\Delta_6, ?\mathbf{F}_1), \Delta_3, \mathbf{F}_4 \otimes \mathbf{F}_5} \ \otimes \\ \end{array} \Rightarrow \begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \overset{ax/ind}{\bullet} \frac{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \overset{ax}{\otimes} \\$$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_3, (\Delta_6, ?\mathbf{F}_1), \mathbf{F}_4 \otimes \mathbf{F}_5} \ \otimes \\ \end{array} \rightarrow \\ \begin{array}{c} \overline{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4} \quad \text{ax} \quad \overline{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad \overline{\mathbf{ax/ind}} \\ \underline{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \end{array}$$

3.5 Status of ?: : Non invertible

- Case rule 1
- Case rule!

$$\frac{\mathtt{h}_2 : \vdash \mathtt{F}_3, ? \Upsilon 4, ? \mathtt{F}_1}{\bullet \mathtt{h}_2 : \vdash (? \Upsilon 4, ? \mathtt{F}_1), ! \mathtt{F}_3} \ ! \qquad \rightarrow \qquad \overline{\bullet \mathtt{h}_2 : \vdash \mathtt{F}_1, ? \Upsilon 4, ! \mathtt{F}_3} \ \mathtt{fail}$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, ?_{\mathbf{f_1}}}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ?_{\mathbf{f_1}}), ?_{\mathbf{f_3}}} ?_W \qquad \rightarrow \qquad \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{f_1}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{f_1}, ?_{\mathbf{f_3}}} \overset{\mathrm{ax/ind}}{?W}$$

$$\frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_3} \ ?W \qquad \rightarrow \qquad \overline{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_3} \ \mathbf{fail}$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, ?_{\mathbf{F}_1}, ?_{\mathbf{F}_3}, ?_{\mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ?_{\mathbf{F}_1}), ?_{\mathbf{F}_3}} \ ?C \qquad \rightarrow \qquad \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{f}_1, ?_{\mathbf{F}_3}, ?_{\mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{f}_1, ?_{\mathbf{F}_3}} \begin{array}{c} \mathbf{ax/ind} \\ ?C \end{array}$$

$$\begin{array}{ccc} \underbrace{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{f}_3, ?\mathbf{f}_3}_{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{f}_3} & ?C & \rightarrow & & \underbrace{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{f}_3}_{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathsf{f}_3} & \mathtt{fail} \end{array}$$

• Case rule?

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_3, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ?\mathbf{F}_1), ?\mathbf{F}_3} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_1, \mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_1, ?\mathbf{F}_3} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

• Case rule &

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1 \quad \mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3} \quad \overset{\mathrm{ax/ind}}{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3} \quad \frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}{\bullet \mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & & & & & & & & \\ \hline$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A$$

 \bullet Case rule \bot

 \bullet Case rule \top

- ullet Case rule I
- Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_5, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_3, (\Delta_6, ?\mathbf{F}_1), \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \otimes \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4} \quad ^{\mathrm{ax}} \quad \overline{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}}{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \otimes \\ \end{array}$$

3.6 Status of \$: : Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1\$\mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_5, \mathbf{F}_1\$\mathbf{F}_2), ?\mathbf{F}_4} ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2, ?\mathbf{F}_4} ?W$$

 \bullet Case rule ?C

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, ?\mathsf{F}_4, ?\mathsf{F}_4, \mathsf{F}_1 \$\mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash (\Delta_5, \mathsf{F}_1 \$\mathsf{F}_2), ?\mathsf{F}_4} \ ?C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathsf{F}_1, \mathsf{F}_2, ?\mathsf{F}_4, ?\mathsf{F}_4}}{\bullet \mathsf{h}_3 : \vdash \Delta_5, \mathsf{F}_1, \mathsf{F}_2, ?\mathsf{F}_4} \ \frac{\mathsf{ax/ind}}{?C}$$

• Case rule?

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, \mathbf{F}_1 \$ \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \$ \mathbf{F}_2), ?\mathbf{F}_4} \quad ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2, ?\mathbf{F}_4} \quad ?$$

• Case rule \$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \$ \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_5, \mathbf{F}_1 \$ \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{F}_1 \$ \mathbf{F}_2), \mathbf{F}_4 \& \mathbf{F}_5} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3 : \vdash \Delta_6, \mathbf{F}_$$

• Case rule \oplus_B

$$\frac{\mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \$ \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash (\Delta_6, \mathtt{F}_1 \$ \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}}{\bullet \mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_B$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_4, \mathbf{f}_1\$\mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{f}_1\$\mathbf{f}_2), \mathbf{f}_4 \oplus \mathbf{f}_5} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_4}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_4 \oplus \mathbf{f}_5} \ \oplus_A$$

• Case rule ⊥

$$\frac{\mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_1\$\mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash \bot, \Delta_4, \mathbf{F}_1\$\mathbf{F}_2} \ \bot \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_1, \mathbf{F}_2, \bot} \overset{\mathsf{ax/ind}}{\bot}$$

• Case rule \top

- ullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \$ \mathbf{F}_2 \quad \mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_6} \\ \bullet \mathbf{h}_3 :\vdash (\Delta_7, \mathbf{F}_1 \$ \mathbf{F}_2), \Delta_4, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \ \otimes \qquad \rightarrow \qquad \begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5} \quad \mathbf{ax/ind} \quad \overline{\mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_6} \\ \bullet \mathbf{h}_3 :\vdash \Delta_4, \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \ \begin{array}{c} \mathbf{ax/ind} \\ \bullet \mathbf{h}_3 :\vdash \Delta_7, \bullet \mathbf{h}_3 :\vdash \Delta_4, \bullet \mathbf{h}_3 :\vdash \Delta_4, \bullet \mathbf{h}_3 :\vdash \Delta_4, \bullet \mathbf{h}_4 \end{array} \ \begin{array}{c} \mathbf{ax/ind} \\ \bullet \mathbf{h}_3 :\vdash \Delta_7, \bullet \mathbf{h}_3 :\vdash \Delta_7, \bullet \mathbf{h}_4 :$$

3.7 Status of &: (Left Premise): Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \& \mathbf{F}_2), ?\mathbf{F}_4} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4} \ ?W$$

 \bullet Case rule ?C

$$\begin{array}{c} \underline{\mathbf{h}_3} : \vdash \Delta_5, ?\mathbf{F}_4, ?\mathbf{F}_4, \mathbf{F}_1 \& \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \& \mathbf{F}_2), ?\mathbf{F}_4 \end{array} ? C \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3} : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4, ?\mathbf{F}_4 \\ \bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4 \end{array} \begin{array}{c} \mathbf{ax/ind} \\ ?C \end{array}$$

• Case rule?

• Case rule \$

• Case rule &

• Case rule \oplus_B

$$\frac{\mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{B} \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}}{\bullet \mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{B}$$

• Case rule \oplus_A

$$\frac{\mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{A} \qquad \rightarrow \qquad \frac{\mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}{\bullet \mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{A}$$

• Case rule \perp

• Case rule \top

$$\frac{}{\bullet \mathsf{h}_3 : \vdash \top, \Delta_4, \mathsf{F}_1 \& \mathsf{F}_2} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3 : \vdash \Delta_4, \mathsf{F}_1, \top} \ \top$$

- \bullet Case rule I
- Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \& \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_4, \mathbf{F}_6}{\bullet \mathbf{h}_3 : \vdash (\Delta_7, \mathbf{F}_1 \& \mathbf{F}_2), \Delta_4, \mathbf{F}_5 \otimes \mathbf{F}_6} \quad \otimes \qquad \rightarrow \qquad \\ \frac{\overline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3 : \vdash \Delta_4, \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6} \quad \overset{\mathbf{ax/ind}}{\otimes} \\ \frac{\mathbf{h}_3 : \vdash \Delta_4, \mathbf{F}_5 \quad \mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_6, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \Delta_4, (\Delta_7, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6} \quad \otimes \qquad \rightarrow \qquad \\ \frac{\overline{\mathbf{h}_3 : \vdash \Delta_4, \mathbf{F}_5} \quad \mathbf{ax}}{\bullet \mathbf{h}_3 : \vdash \Delta_4, \Delta_7, \mathbf{F}_1, \mathbf{F}_5} \quad \overset{\mathbf{ax/ind}}{\otimes} \\ \frac{\mathbf{ax/ind}}{\otimes} \\ \end{array}$$

3.8 Status of & (Right Premise): : Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_5, \mathbf{F}_1 \& \mathbf{F}_2), ?\mathbf{F}_4} \quad ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_2}}{\bullet \mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_2, ?\mathbf{F}_4} \overset{\mathsf{ax/ind}}{?W}$$

• Case rule ?C

$$\begin{array}{l} \underline{\mathbf{h}_3 :\vdash \Delta_5, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}, F_1 \& F_2}} \\ \bullet \underline{\mathbf{h}_3 :\vdash (\Delta_5, F_1 \& F_2), ?_{\mathbf{F}_4}} \end{array} ?C \qquad \rightarrow \qquad \\ \overline{\begin{array}{l} \underline{\mathbf{h}_3 :\vdash \Delta_5, F_2, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}} \\ \bullet \underline{\mathbf{h}_3 :\vdash \Delta_5, F_2, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}} \end{array}} \overset{\mathrm{ax/ind}}{?C} \end{array}$$

• Case rule?

$$\frac{\mathsf{h}_3 :\vdash \Delta_5, \mathsf{F}_4, \mathsf{F}_1 \& \mathsf{F}_2}{\bullet \mathsf{h}_3 :\vdash (\Delta_5, \mathsf{F}_1 \& \mathsf{F}_2), ?\mathsf{F}_4} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathsf{h}_3 :\vdash \Delta_5, \mathsf{F}_2, \mathsf{F}_4}}{\bullet \mathsf{h}_3 :\vdash \Delta_5, \mathsf{F}_2, ?\mathsf{F}_4} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4, \mathbf{F}_5}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$^{\mathsf{ax/ind}}$$

• Case rule &

• Case rule \oplus_B

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{B} \qquad \rightarrow \qquad \frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_5}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{B}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4} \ \ ^{\mathrm{ax/ind}}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_A$$

• Case rule \perp

 \bullet Case rule \top

- ullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \Delta_7, \mathsf{F}_5, \mathsf{F}_1 \& \mathsf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_4, \mathsf{F}_6}{\bullet \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_1 \& \mathsf{F}_2), \Delta_4, \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad \rightarrow \qquad \\ \overline{\frac{\mathbf{h}_3 : \vdash \Delta_7, \mathsf{F}_2, \mathsf{F}_5}{\bullet \mathsf{h}_3 : \vdash (\Delta_4, \Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\mathsf{h}_3 : \vdash (\Delta_4, \Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6}} \quad \frac{\mathsf{ax}/\mathsf{ind}}{\otimes} \\ \overline{\frac{\mathbf{h}_3 : \vdash \Delta_4, \mathsf{F}_5}{\bullet \mathsf{h}_3 : \vdash (\Delta_4, \mathsf{F}_5, \mathsf{F}_1 \& \mathsf{F}_2), \mathsf{F}_5 \otimes \mathsf{F}_6}} \quad \otimes \qquad \rightarrow \qquad \overline{\frac{\mathsf{h}_3 : \vdash (\Delta_4, \mathsf{F}_5)}{\bullet \mathsf{h}_3 : \vdash (\Delta_4, \Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}} \quad \overline{\frac{\mathsf{ax}/\mathsf{ind}}{\diamond \mathsf{h}_3 : \vdash (\Delta_7, \mathsf{F}_2, \mathsf{F}_5 \otimes \mathsf{F}_6)}}$$

3.9 Status of \oplus_B : Non invertible

- \bullet Case rule 1
- Case rule!
- ullet Case rule ?W

$$\frac{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2), ?\mathbf{F}_4} ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_2}}{\bullet \mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_2, ?\mathbf{F}_4} \overset{\mathsf{ax/ind}}{?W}$$

 \bullet Case rule ?C

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_5, ?\mathbf{F}_4, ?\mathbf{F}_4, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2), ?\mathbf{F}_4} & ?C & \rightarrow & & & & & \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_2, ?\mathbf{F}_4} & \frac{\mathsf{ax/ind}}{?C} \end{array}$$

• Case rule?

$$\frac{h_3:\vdash\Delta_5,F_4,F_1\oplus F_2}{\bullet h_3:\vdash(\Delta_5,F_1\oplus F_2),?F_4}\ ? \qquad \rightarrow \qquad \frac{h_3:\vdash\Delta_5,F_2,F_4}{\bullet h_3:\vdash\Delta_5,F_2,?F_4} \stackrel{ax/ind}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{f}_4, \mathbf{f}_5, \mathbf{f}_1 \oplus \mathbf{f}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{f}_1 \oplus \mathbf{f}_2), \mathbf{f}_4 \$ \mathbf{f}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{f}_2, \mathbf{f}_4, \mathbf{f}_5}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{f}_2, \mathbf{f}_4 \$ \mathbf{f}_5} \quad \overset{\text{ax/ind}}{\$}$$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_4 \& \mathbf{F}_5} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4} \quad \text{ax/ind} \quad \overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4 \& \mathbf{F}_5} \quad & & \text{ax/ind} \quad \overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4} \\ \bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4 \& \mathbf{F}_5 \\ \end{pmatrix}$$

• Case rule \oplus_B

• Case rule \oplus_A

 \bullet Case rule \bot

 \bullet Case rule \top

- ullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_4, \mathbf{F}_6}{\bullet \mathbf{h}_3 : \vdash (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \Delta_4, \mathbf{F}_5 \otimes \mathbf{F}_6} \end{array} \otimes \\ \end{array} \quad \rightarrow \quad \begin{array}{c} \frac{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash \Delta_4, \Delta_7, \mathbf{F}_2, \mathbf{F}_5} \overset{\text{ax/ind}}{\bullet} \frac{\mathbf{h}_3 : \vdash \Delta_4, \mathbf{F}_6}{\bullet} & \otimes \\ \\ \bullet \mathbf{h}_3 : \vdash \Delta_4, \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \otimes \mathbf{F}_6 & \otimes \\ \end{array}$$

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_5 \quad \mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_6, \mathbf{F}_1 \oplus \mathbf{F}_2} \\ \bullet \mathbf{h}_3 :\vdash \Delta_4, (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \otimes \\ \end{array} \quad \rightarrow \quad \begin{array}{c} \overline{\mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_5} \quad \overset{\mathrm{ax}}{} \quad \overline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_6} \\ \bullet \mathbf{h}_3 :\vdash \Delta_4, \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \overset{\mathrm{ax}/\mathrm{ind}}{\otimes} \end{array}$$

3.10 Status of \oplus_A : Non invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2), ?\mathbf{F}_4} ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1} \quad \text{ax/ind}}{\bullet \mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4} ?W$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \Delta_5, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2), ?_{\mathbf{F}_4}} \ ?C \\ \end{array} \rightarrow \\ \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}} \\ \bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?_{\mathbf{F}_4} \end{array} \begin{array}{c} \mathbf{ax/ind} \\ ?C \\ \end{array}$$

• Case rule?

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2), ?\mathbf{F}_4} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4, \mathbf{F}_5}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$'$$

• Case rule &

$$\frac{\mathbf{a}_3 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{a}_3 : \vdash \Delta_6, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{b}_3 : \vdash (\Delta_6, \mathbf{F}_1) \oplus \mathbf{F}_2), \mathbf{F}_4 \& \mathbf{F}_5} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{a}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4} \quad \text{ax/ind}}{\bullet \mathbf{b}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \& \mathbf{F}_5} \quad \frac{\text{ax/ind}}{\&} \quad \& \quad \Rightarrow \quad \frac{\overline{\mathbf{a}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4} \quad \mathbf{ax/ind}}{\bullet \mathbf{b}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \& \mathbf{F}_5} \quad \frac{\mathbf{ax/ind}}{\&} \quad \& \quad \Rightarrow \quad \frac{\mathbf{ax/ind}}{\bullet \mathbf{b}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \& \mathbf{F}_5} \quad \frac{\mathbf{ax/ind}}{\lozenge} \quad \frac{\mathbf{ax/ind}}$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_4 \oplus \mathbf{F}_5} \oplus_B \longrightarrow \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \oplus \mathbf{F}_5} \oplus_B$$

$$\frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B \qquad \rightarrow \qquad \overline{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_3} \ \text{fail}$$

• Case rule \oplus_A

• Case rule \perp

• Case rule \top

- \bullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \underline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_4, \mathbf{F}_6} \\ \bullet \mathbf{h}_3 : \vdash (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \Delta_4, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \otimes \\ \end{array} \quad \Rightarrow \quad \begin{array}{c} \underline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3 : \vdash \Delta_4, \mathbf{F}_6} \\ \bullet \mathbf{h}_3 : \vdash \Delta_4, \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \otimes \\ \end{array} \otimes \\$$

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_5 \quad \mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_6, \mathbf{F}_1 \oplus \mathbf{F}_2} \\ \bullet \mathbf{h}_3 :\vdash \Delta_4, (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \otimes \\ \end{array} \quad \rightarrow \quad \begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_5} \quad \text{ax} \quad \overline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_6} \\ \bullet \mathbf{h}_3 :\vdash \Delta_4, \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \otimes \\ \end{array} \quad \stackrel{\text{ax/ind}}{\otimes}$$

3.11 Status of \perp : Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_1 : \vdash \bot, \Delta_3}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_3), ?\mathsf{F}_2} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \Delta_3}}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathsf{F}_2} ?W$$

• Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \bot, \Delta_3, ?_{\mathsf{F}_2}, ?_{\mathsf{F}_2}}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_3), ?_{\mathsf{F}_2}} \ ?C \qquad \rightarrow \qquad \overline{\frac{\mathbf{h}_1 : \vdash \Delta_3, ?_{\mathsf{F}_2}, ?_{\mathsf{F}_2}}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?_{\mathsf{F}_2}}} \begin{array}{c} \mathbf{ax/ind} \\ ?C \end{array}$$

• Case rule?

$$\frac{\mathtt{h}_1 : \vdash \bot, \Delta_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \vdash (\bot, \Delta_3), ?\mathtt{F}_2} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_1 : \vdash \Delta_3, \mathtt{F}_2}}{\bullet \mathtt{h}_1 : \vdash \Delta_3, ?\mathtt{F}_2} \ \frac{\mathtt{ax/ind}}{?}$$

• Case rule \$

• Case rule &

$$\frac{\mathbf{h}_1 : \vdash \bot, \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \bot, \Delta_4, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_4), \mathbf{F}_2 \& \mathbf{F}_3} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2} \quad \text{ax/ind} \quad \overline{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3}}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3} \quad & \overset{\text{ax/ind}}{\&}$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_1 : \vdash \bot, \Delta_4, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_4), \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3}}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_B$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_1 : \vdash \bot, \Delta_4, \mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_4), \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2}}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_A$$

 \bullet Case rule \bot

 \bullet Case rule \top

$$\overline{{}_{\bullet \mathbf{h}_1} :\vdash \top, \bot, \Delta_2} \quad \top \qquad \rightarrow \qquad \overline{{}_{\bullet \mathbf{h}_1} :\vdash \Delta_2, \top} \quad \top$$

- $\bullet\,$ Case rule I
- \bullet Case rule \otimes

3.12 Status of \top : Invertible

- Case rule 1
- Case rule!
- $\bullet \;$ Case rule ?W

$$\frac{\mathbf{h}_1 : \vdash \top, \Delta_3}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_3), ?\mathbf{F}_2} \ ?W \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule ?C

$$\begin{array}{ll} \frac{\mathbf{h}_1 : \vdash \top, \Delta_3, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_3), ?\mathbf{F}_2} \ ?C & \rightarrow & \text{trivial} \end{array}$$

• Case rule?

$$\frac{\mathtt{h}_1 : \vdash \top, \Delta_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \vdash (\top, \Delta_3), ?\mathtt{F}_2} \ ? \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \$

$$\begin{array}{ccc} \frac{\mathbf{h}_1 : \vdash \top, \Delta_4, \mathbf{F}_2, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_4), \mathbf{F}_2\$\mathbf{F}_3} & \$ & \rightarrow & \text{trivial} \end{array}$$

• Case rule &

$$\frac{\mathbf{h}_1 : \vdash \top, \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \top, \Delta_4, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_4), \mathbf{F}_2 \& \mathbf{F}_3} \quad \& \qquad \rightarrow \qquad \text{trivial}$$

• Case rule \oplus_B

$$\frac{\mathtt{h}_1 : \vdash \top, \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash (\top, \Delta_4), \mathtt{F}_2 \oplus \mathtt{F}_3} \ \oplus_B \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_1 : \vdash \top, \Delta_4, \mathbf{f}_2}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_4), \mathbf{f}_2 \oplus \mathbf{f}_3} \ \oplus_A \qquad \rightarrow \qquad \mathtt{trivial}$$

 \bullet Case rule \bot

$$\frac{\mathtt{h}_1 : \vdash \top, \Delta_2}{\bullet \mathtt{h}_1 : \vdash \bot, \top, \Delta_2} \ \bot \qquad \to \qquad \mathtt{trivial}$$

 \bullet Case rule \top

$$\overline{\bullet \mathtt{h}_1 : \vdash \top, \Delta_2} \ \top \qquad \rightarrow \qquad \mathtt{trivial}$$

 \bullet Case rule I

• Case rule \otimes

$$\begin{array}{ccc} \frac{\textbf{h}_1 : \vdash \top, \Delta_5, \textbf{F}_3 & \textbf{h}_1 : \vdash \Delta_2, \textbf{F}_4}{\bullet \textbf{h}_1 : \vdash (\top, \Delta_5), \Delta_2, \textbf{F}_3 \otimes \textbf{F}_4} & \otimes & \rightarrow & \text{trivial} \end{array}$$

$$\begin{array}{ll} \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_3 \quad \mathbf{h}_1 : \vdash \top, \Delta_5, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_2, (\top, \Delta_5), \mathbf{F}_3 \otimes \mathbf{F}_4} \ \otimes & \rightarrow & \text{trivial} \end{array}$$

3.13 Status of I: Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W
- \bullet Case rule ?C
- Case rule?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I

$$\overline{\bullet \mathbf{h}_1 : \vdash p(\mathbf{n}_2), \, \hat{\ } (\mathbf{n}_2)} \quad I \qquad \rightarrow \qquad \mathsf{trivial}$$

 \bullet Case rule \otimes

3.14 Status of \otimes : (Left Premise): Non invertible

- \bullet Case rule ${\bf 1}$
- Case rule!
- $\bullet \;$ Case rule ?W

$$\frac{\mathbf{h}_4 :\vdash \Delta_1, \Delta_6, \mathbf{f}_2 \otimes \mathbf{f}_3}{\bullet \mathbf{h}_4 :\vdash (\Delta_1, \Delta_6, \mathbf{f}_2 \otimes \mathbf{f}_3), ?\mathbf{f}_5} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 :\vdash \Delta_6, \mathbf{f}_2}}{\bullet \mathbf{h}_4 :\vdash \Delta_6, \mathbf{f}_2, ?\mathbf{f}_5} \ ?W$$

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), ?\mathbf{F}_5} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2} \ \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

• Case rule ?C

$$\begin{array}{lll} \frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_6, ? \mathbf{F}_5, ? \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), ? \mathbf{F}_5} & ?C & \rightarrow & & \frac{\overline{\mathbf{h}_4 : \vdash \Delta_6, \mathbf{F}_2}}{\bullet \mathbf{h}_4 : \vdash \Delta_6, \mathbf{F}_2, ? \mathbf{F}_5} & ?W \end{array}$$

$$\begin{array}{lll} \frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_6, ? \mathbf{F}_5, ? \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), ? \mathbf{F}_5} & ?C & \rightarrow & & & & \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2} & \\ \end{array} \xrightarrow{\mathbf{ax/ind}} ^{\mathbf{ax/ind}}$$

• Case rule?

$$\frac{\mathbf{h}_4 :\vdash \Delta_1, \Delta_6, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 :\vdash (\Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), ?\mathbf{F}_5} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 :\vdash \Delta_6, \mathbf{F}_2}}{\bullet \mathbf{h}_4 :\vdash \Delta_6, \mathbf{F}_2, ?\mathbf{F}_5} \ ?W$$

• Case rule \$

$$\frac{\mathbf{h}_4 :\vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 :\vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \$ \mathbf{F}_6} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 :\vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6}}{\bullet \mathbf{h}_4 :\vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \$ \mathbf{F}_6} \quad \$$$

$$\frac{\mathbf{h}_4 :\vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 :\vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \$ \mathbf{F}_6} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 :\vdash \Delta_1, \mathbf{F}_2}}{\bullet \mathbf{h}_4 :\vdash \Delta_1, \mathbf{F}_2} \quad \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

• Case rule &

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3 \quad \mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \quad \rightarrow \quad \frac{\mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5}{\bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6} \quad \frac{\mathbf{ax/ind}}{\bullet} \quad & \quad & \quad & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6 & \quad & \quad & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6 & \quad & \quad & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6 & \quad & \quad & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6 & \quad & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \quad & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_2, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3 & \quad & \\ \bullet \mathbf{h}_4$$

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3 \quad \mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_6}}{\bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5} \stackrel{\mathrm{ax/ind}}{\oplus}_B$$

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2} \ ^{\mathrm{ax/ind}}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5} \stackrel{\mathrm{ax/ind}}{\oplus}_A$$

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_2} \ \overset{\mathrm{ax/ind}}{\vdash}$$

• Case rule \perp

• Case rule \top

$$\frac{}{\bullet \mathsf{h}_4 : \vdash \top, \Delta_1, \Delta_5, \mathsf{F}_2 \otimes \mathsf{F}_3} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_4 : \vdash \Delta_1, \mathsf{F}_2} \ \mathsf{fail}$$

- ullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4, \mathbf{f}_1 \otimes \mathbf{f}_2 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{f}_4 \otimes \mathbf{f}_2), (\Delta_8, \Delta_9), \mathbf{f}_4 \otimes \mathbf{f}_5} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_4 \otimes \mathbf{f}_5} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{f}_4, \mathbf{f}_1 \otimes \mathbf{f}_2 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4, \mathbf{f}_1 \otimes \mathbf{f}_2 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_4 \otimes \mathbf{f}_5} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_4 \otimes \mathbf{f}_5} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_4 \otimes \mathbf{f}_5} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_4 \otimes \mathbf{f}_5} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_4 \otimes \mathbf{f}_5} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_4 \otimes \mathbf{f}_5} \quad \otimes \\ \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \Delta_8, \mathbf{f}_1} \quad \mathbf{f}_1 \otimes \mathbf{f}_2} \quad \otimes \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_1 \otimes \mathbf{f}_2} \quad \otimes \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_5, \mathbf{f}_4 \otimes \mathbf{f}_5} \quad \otimes \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_8, \mathbf{f}_1}{\bullet \mathbf{h}_3} : \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{f}_1 \otimes \mathbf{f}_2} \quad \otimes \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_8, \mathbf$$

$$\begin{array}{c} \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_6, \Delta_7, \mathbf{F}_3 \\ \bullet \mathbf{h}_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), \mathbf{F}_2 \otimes \mathbf{F}_3 \end{array} \ \otimes \qquad \rightarrow \qquad \begin{array}{c} \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_2 \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_2 \end{array} \ \text{fail}$$

3.15 Status of \otimes (Right Premise): Non invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), ?\mathbf{F}_5} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_3}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_3} \ \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), ?\mathbf{F}_5} ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_6, \mathbf{F}_3} \quad \text{ax/ind}}{\bullet \mathbf{h}_4 : \vdash \Delta_6, \mathbf{F}_3, ?\mathbf{F}_5} ?W$$

• Case rule ?C

$$\begin{array}{lll} \frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_6, ?\mathsf{F}_5, ?\mathsf{F}_5, \mathsf{F}_2 \otimes \mathsf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_6, \mathsf{F}_2 \otimes \mathsf{F}_3), ?\mathsf{F}_5} & ?C & \rightarrow & & & & \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathsf{F}_3}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathsf{F}_3} & \text{ax/ind} \\ \end{array}$$

$$\begin{array}{lll} \frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_6, ? \mathbf{F}_5, ? \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), ? \mathbf{F}_5} & ?C & \rightarrow & & \frac{\overline{\mathbf{h}_4 : \vdash \Delta_6, \mathbf{F}_3}}{\bullet \mathbf{h}_4 : \vdash \Delta_6, \mathbf{F}_3, ? \mathbf{F}_5} & ?W \end{array}$$

• Case rule?

$$\frac{\mathtt{h}_4 : \vdash \Delta_1, \Delta_6, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : \vdash (\Delta_1, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_4 : \vdash \Delta_1, \mathtt{F}_3}}{\bullet \mathtt{h}_4 : \vdash \Delta_1, \mathtt{F}_3} \ \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_6, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), ?\mathbf{F}_5} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_6, \mathbf{F}_3}}{\bullet \mathbf{h}_4 : \vdash \Delta_6, \mathbf{F}_3, ?\mathbf{F}_5} \ ?W$$

• Case rule \$

$$\frac{\mathbf{h}_4 :\vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 :\vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \$ \mathbf{F}_6} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 :\vdash \Delta_1, \mathbf{F}_3}}{\bullet \mathbf{h}_4 :\vdash \Delta_1, \mathbf{F}_3} \quad \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathbf{h}_4 :\vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 :\vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \$ \mathbf{F}_6} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 :\vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5, \mathbf{F}_6}}{\bullet \mathbf{h}_4 :\vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5 \$ \mathbf{F}_6} \quad \$^{\mathsf{ax/ind}}$$

• Case rule &

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3 \quad \mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_3}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_3} \quad \overset{\mathrm{ax/ind}}{\mathsf{h}_4 : \vdash \Delta_1, \mathbf{F}_3} \quad \mathsf{H}$$

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3 \quad \mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5} \quad \overset{\mathrm{ax/ind}}{}{} \overline{\mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5}} \quad \overset{\mathrm{ax/ind}}{}{} & & & & & & \\ \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5 \& \mathbf{F}_6 & & & & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5 \& \mathbf{F}_6 & & & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5 \& \mathbf{F}_6 & & & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5 \& \mathbf{F}_6 & & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5 \& \mathbf{F}_6 & & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{h}_3, \mathbf{h}_5 \& \mathbf{h}_6 & & \\ \hline \bullet \mathbf{h}_$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_3}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_3} \ \underset{\mathbb{H}}{\text{ax/ind}}$$

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{f}_6, \mathbf{f}_2 \otimes \mathbf{f}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{f}_2 \otimes \mathbf{f}_3), \mathbf{f}_5 \oplus \mathbf{f}_6} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_7, \mathbf{f}_3, \mathbf{f}_6}}{\bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{f}_3, \mathbf{f}_5 \oplus \mathbf{f}_6} \ \oplus_B$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_3}}{\bullet \mathbf{h}_4 : \vdash \Delta_1, \mathbf{F}_3} \ _{\mathbf{H}}^{\mathbf{ax/ind}}$$

$$\frac{\mathbf{h}_4 : \vdash \Delta_1, \Delta_7, \mathbf{F}_5, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash (\Delta_1, \Delta_7, \mathbf{F}_2 \otimes \mathbf{F}_3), \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5}}{\bullet \mathbf{h}_4 : \vdash \Delta_7, \mathbf{F}_3, \mathbf{F}_5} \stackrel{\mathrm{ax/ind}}{\oplus_A}$$

• Case rule \perp

 \bullet Case rule \top

$$\frac{}{\bullet \mathsf{h}_4 : \vdash \top, \Delta_1, \Delta_5, \mathsf{F}_2 \otimes \mathsf{F}_3} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_4 : \vdash \Delta_5, \mathsf{F}_3, \top} \ \top$$

- ullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2 & \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), (\Delta_8, \Delta_9), \mathbf{F}_4 \otimes \mathbf{F}_5} & \otimes & & \rightarrow & & \hline{\bullet \mathbf{h}_3 : \vdash \Delta_7, \Delta_9, \mathbf{F}_2} & \mathbf{fail} \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \ \otimes \\ \end{array} \quad \rightarrow \quad \boxed{\bullet \mathbf{h}_3 : \vdash \Delta_7, \Delta_9, \mathbf{F}_2} \quad \mathbf{fail}$$

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2 & \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), (\Delta_8, \Delta_9), \mathbf{F}_4 \otimes \mathbf{F}_5} & \otimes & & \rightarrow & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), (\Delta_8, \Delta_9), \mathbf{F}_4 \otimes \mathbf{F}_5} & & & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{$$

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2} \\ \bullet \underline{\mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \phantom{\mathbf{h}_3 :\vdash \Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \mathbf{fail}$$

$$\begin{array}{c} \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_6, \Delta_7, \mathbf{F}_3 \\ \bullet \mathbf{h}_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), \mathbf{F}_2 \otimes \mathbf{F}_3 \end{array} \ \otimes \qquad \rightarrow \qquad \begin{array}{c} \bullet \mathbf{h}_1 : \vdash \Delta_5, \Delta_7, \mathbf{F}_3 \\ \bullet \mathbf{h}_1 : \vdash \Delta_5, \Delta_7, \mathbf{F}_3 \end{array} \ \text{fail} \end{array}$$

4 Identity-Expansion

5 Cut-Elimination

5.1 Status of 1: OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\underbrace{\bullet_{\mathbf{h}_1} : \vdash \mathbf{1}, *}_{\bullet \mathbf{h}_1} \ \mathbf{1} \ \frac{\mathbf{h}_2 : \vdash \mathbf{1}, \Delta_4}{\bullet_{\mathbf{h}_2} : \vdash dual(\mathbf{1}), \Delta_4, ?\mathsf{F}_3} }_{\bullet \mathbf{h}_2 : \vdash \mathbf{1}} \ \frac{?W}{\mathsf{cut}}$$

$$\frac{\bullet_{\mathbf{h}_1} : \vdash \mathbf{1}}{\bullet_{\mathbf{h}_1} : \vdash \mathbf{1}} \ \frac{\rightarrow}{\mathsf{h}_2 : \vdash \Delta_4, \bot} \ \underset{\mathsf{hCut}}{\bullet_{\mathbf{h}_1} : \vdash \Delta_4} \ \underset{\mathsf{hCut}}{\bullet_{\mathbf{h}_1} : \vdash \Delta_4, ?\mathsf{F}_3} ?W$$

 \bullet Case rule ?C

• Case rule?

• Case rule \$

• Case rule &

$$\frac{\underbrace{\bullet \mathbf{h}_1 : \vdash \mathbf{1}, *}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}, *} \mathbf{1} \quad \frac{\mathbf{h}_2 : \vdash \bot, \Delta_5, \mathsf{F}_3 \quad \mathbf{h}_2 : \vdash \bot, \Delta_5, \mathsf{F}_4}{\bullet \mathbf{h}_2 : \vdash dual(\mathbf{1}), \Delta_5, \mathsf{F}_3 \& \mathsf{F}_4} \underbrace{\mathsf{Cut}} \\ - : \vdash *, \Delta_5, \mathsf{F}_3 \& \mathsf{F}_4} \\ \underbrace{\bullet \mathbf{h}_1 : \vdash \mathbf{1}}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}} \mathbf{1} \quad \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathsf{F}_3, \bot}{\mathbf{h}_2 : \vdash \Delta_5, \mathsf{F}_3, \bot}}_{- : \vdash \Delta_5, \mathsf{F}_3} \underbrace{\mathsf{ax}}_{\mathsf{h}\mathsf{Cut}} \\ \underbrace{- : \vdash \Delta_5, \mathsf{F}_3}_{- : \vdash \Delta_5, \mathsf{F}_3 \& \mathsf{F}_4} \quad \&}_{\mathsf{h}\mathsf{Cut}}$$

• Case rule \oplus_B

• Case rule \oplus_A

• Case rule \perp

 \bullet Case rule \top

- \bullet Case rule I
- \bullet Case rule \otimes

5.2 Status of !: OK

- Case rule 1
- Case rule!

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \mathbf{F}_4, ?\Upsilon2 \\ \bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4, ?\Upsilon2 \end{array} ! \quad \begin{array}{c} \mathbf{h}_5 : \vdash \mathbf{F}_6, ?\Upsilon7, ?dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 : \vdash dual(!\mathbf{F}_4), ?\Upsilon7, ! \mathbf{F}_6 \end{array} }{ \begin{array}{c} \bullet \mathbf{h}_5 : \vdash ?\Upsilon2, ?\Upsilon7, ! \mathbf{F}_6 \\ \end{array} } \quad \begin{array}{c} \mathbf{Cut} \\ \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, ! \mathbf{F}_4 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, ! \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, ! \mathbf{F}_4 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, ! \mathbf{F}_6 \\ \hline \begin{array}{c} \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, ! \mathbf{F}_6 \end{array} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, ! \mathbf{F}_6 \\ \hline \begin{array}{c} \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, ?\Upsilon7, \mathbf{F}_6 \\ \hline \begin{array}{c} \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, ?\Upsilon7, ! \mathbf{F}_6 \end{array} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, ?\Upsilon7, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_2 : \vdash ?\Upsilon2, ?\Upsilon7, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_3 : \vdash ?\Upsilon2, ?\Upsilon7, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash ?\Upsilon2, ?\Upsilon3, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash ?\Upsilon3, ?T4, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash ?T4, ?T4, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash ?T4, ?T4, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash ?T4, ?T4, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash ?T4, ?T4, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash ?T4, ?T4, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash ?T4, ?T4, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash ?T4, ?T4, ?T6 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_4 : \vdash \mathbf{h}_4 : \vdash$$

 \bullet Case rule ?W

• Case rule ?C

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ? \Upsilon 4, 5}{\bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, ! \mathbf{F}_3} : \frac{\mathbf{h}_6 : \vdash \Delta_8, ? \mathbf{F}_7, ? \mathbf{F}_7, dual(5)}{\bullet \mathbf{h}_6 : \vdash dual(5), \Delta_8, ? \mathbf{F}_7} ? Cut \\ \hline \\ \bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, ! \mathbf{F}_3 & \xrightarrow{\rightarrow} \frac{\rightarrow}{\mathbf{h}_6 : \vdash \Delta_8, ? \mathbf{F}_7, ? \mathbf{F}_7, dual(5)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, ! \mathbf{F}_3 & \xrightarrow{\rightarrow} \frac{\rightarrow}{\mathbf{h}_6 : \vdash \Delta_8, ? \mathbf{F}_7, ? \mathbf{F}_7, dual(5)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, ! \mathbf{F}_3 & \xrightarrow{\rightarrow} \frac{\rightarrow}{\mathbf{h}_6 : \vdash \Delta_8, ? \mathbf{F}_7, ? \mathbf{F}_7, dual(5)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \Delta_8, ! \mathbf{F}_3, ? \mathbf{F}_7, ? \mathbf{F}_7, dual(5)} \\ \hline \\ \bullet \mathbf{h}_3 : \vdash ? \Upsilon 4, \Delta_8, ! \mathbf{F}_3, ? \mathbf{F}_7, ? \mathbf{F}_7, dual(\mathbf{F}_4) \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \mathbf{F}_4, ? \Upsilon 2} : \frac{\mathbf{h}_5 : \vdash \Delta_7, ? \mathbf{F}_6, ? \mathbf{f}_6, ? dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(! \mathbf{F}_4), \Delta_7, ? \mathbf{F}_6} Cut \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, ! \mathbf{F}_4 & \text{ax} \\ \hline \\ \bullet \mathbf{h}_5 : \vdash \Delta_7, ? \mathbf{F}_6, ? \mathbf{F}_6, ? dual(\mathbf{F}_4) \\ \hline \\ \bullet \mathbf{h}_5 : \vdash \Delta_7, ? \mathbf{F}_6, ? \mathbf{F}_6, ? dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, ! \mathbf{F}_4 & \text{ax} \\ \hline \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \Delta_7, ? \mathbf{F}_6, ? \mathbf{F}_6, ? dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \Delta_7, ? \mathbf{F}_6, ? \mathbf{F}_6, ? dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \Delta_7, ? \mathbf{F}_6, ? \mathbf{F}_6, ? dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \Delta_7, ? \mathbf{F}_6, ? \mathbf{F}_6, ? dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \Delta_7, ? \mathbf{F}_6, ? \mathbf{F}_6, ? dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \Delta_7, ? \mathbf{F}_6, ? \mathbf{F}_6, ? dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \Delta_7, ? \mathbf{F}_6, ? \mathbf{F}_6, ? \mathbf{h}_7, ? \mathbf{h}_7,$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \mathbf{F}_3,?\Upsilon 4,5}{\bullet \mathbf{h}_2 : \vdash 5,?\Upsilon 4,! \mathbf{F}_3} : \frac{\mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_7, dual(5)}{\bullet \mathbf{h}_6 : \vdash dual(5), \Delta_8,? \mathbf{F}_7} \\ \hline - : \vdash (?\Upsilon 4,! \mathbf{F}_3), \Delta_8,? \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_2 : \vdash 5,?\Upsilon 4,! \mathbf{F}_3 & \mathbf{ax} & \xrightarrow{h_6 : \vdash \Delta_8, \mathbf{F}_7, dual(5)} \\ \hline - : \vdash ?\Upsilon 4, \Delta_8,! \mathbf{F}_7,! \mathbf{F}_3 \\ \hline - : \vdash ?\Upsilon 4, \Delta_8,! \mathbf{F}_3,? \mathbf{F}_7 & ? \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 2}{\bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4,?\Upsilon 2} : & \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6,? dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(! \mathbf{F}_4), \Delta_7,? \mathbf{F}_6} \\ \hline - : \vdash ?\Upsilon 2, \Delta_7,? \mathbf{F}_6 \\ \hline & \xrightarrow{h_1 : \vdash ?\Upsilon 2,! \mathbf{F}_4} & \mathbf{ax} & \xrightarrow{h_5 : \vdash \Delta_7, \mathbf{F}_6,? dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 2} : & \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6,? dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6,? dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ \hline & \xrightarrow{-} : \vdash ?\Upsilon 2, \Delta_7,? \mathbf{F}_6} ? \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_4,? \Upsilon 2} : & \frac{\mathbf{h}_5 : \vdash \Delta_6, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(! \mathbf{F}_4), \Delta_6} ? \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ?\Upsilon 2, \Delta_6 & \xrightarrow{-} : \vdash ?\Upsilon 2, \Delta_6 \\ \hline \\ \hline - : \vdash ?\Upsilon 2, \mathbf{h}_6 & \mathbf{ax} & \xrightarrow{-} : \vdash \Delta_6, dual(\mathbf{F}_4)} \\ \hline \\ - : \vdash ?\Upsilon 2, \mathbf{h}_6 & \mathbf{ax} & \xrightarrow{-} : \vdash \Delta_6, dual(\mathbf{F}_4)} \\ \hline \\ - : \vdash ?\Upsilon 2, \Delta_6 & \mathbf{ax} & \xrightarrow{-} : \vdash \Delta_6, dual(\mathbf{F}_4)} \\ \hline \\ - : \vdash ?\Upsilon 2, \Delta_6 & \mathbf{ax} & \xrightarrow{-} : \vdash \Delta_6, dual(\mathbf{F}_4)} \\ \hline \\ - : \vdash ?\Upsilon 2, \Delta_6 & \mathbf{ax} & \xrightarrow{-} : \vdash \Delta_6, dual(\mathbf{F}_4)} \\ \hline \\ - : \vdash ?\Upsilon 2, \Delta_6 & \mathbf{ax} & \xrightarrow{-} : \vdash \Delta_6, dual(\mathbf{F}_4) \\ \hline \end{array}$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ? \Upsilon 4, 5}{\bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, ! \mathbf{F}_3} : \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8, dual(5)}{\bullet \mathbf{h}_6 : \vdash dual(5), \Delta_9, \mathbf{F}_7\$ \mathbf{F}_8} \\ \hline - : \vdash (? \Upsilon 4, ! \mathbf{F}_3), \Delta_9, \mathbf{F}_7\$ \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, ! \mathbf{F}_3 \\ \hline \bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, ! \mathbf{F}_3 \\ \hline - : \vdash ? \Upsilon 4, \Delta_9, \mathbf{F}_7, \mathbf{F}_8, ! \mathbf{F}_3 \\ \hline - : \vdash ? \Upsilon 4, \Delta_9, ! \mathbf{F}_3, \mathbf{F}_7\$ \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_4, ? \Upsilon 2 \\ \hline \bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4, ? \Upsilon 2 \\ \hline \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \Delta_8, \mathbf{F}_6\$ \mathbf{F}_7 \\ \hline - : \vdash ? \Upsilon 2, \Delta_8, \mathbf{F}_6\$ \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, ! \mathbf{F}_4 \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, ? dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}$$

• Case rule &

$$\frac{\frac{\mathbf{h}_2 : \vdash \mathbf{F}_3,?\Upsilon 4,5}{\bullet \mathbf{h}_2 : \vdash 5,?\Upsilon 4,!\mathbf{F}_3} ! \frac{\mathbf{h}_6 : \vdash \Delta_9,\mathbf{F}_7,dual(5) \quad \mathbf{h}_6 : \vdash \Delta_9,\mathbf{F}_8,dual(5)}{\bullet \mathbf{h}_6 : \vdash \Delta_9,\mathbf{F}_7\&\mathbf{F}_8} \underbrace{\mathbf{Cut}}_{} \\ - : \vdash (?\Upsilon 4,!\mathbf{F}_3),\Delta_9,\mathbf{F}_7\&\mathbf{F}_8} \\ \underline{\bullet}_{\mathbf{h}_2 : \vdash 5,?\Upsilon 4,!\mathbf{F}_3} \underbrace{\mathbf{ax}}_{\mathbf{h}_6 : \vdash \Delta_9,\mathbf{F}_7,dual(5)} \underbrace{\mathbf{ax}}_{\mathbf{h}_6 : \vdash \Delta_9,\mathbf{F}_7,dual(5)} \underbrace{\mathbf{ax}}_{\mathbf{h}_{\mathbf{Cut}}} \\ \underline{- : \vdash ?\Upsilon 4,\Delta_9,\mathbf{F}_7,!\mathbf{F}_3} \underbrace{- : \vdash ?\Upsilon 4,\Delta_9,!\mathbf{F}_3,\mathbf{F}_7\&\mathbf{F}_8}_{} \underbrace{\bullet}_{\mathbf{h}_{\mathbf{Cut}}} \underbrace{- : \vdash ?\Upsilon 4,\Delta_9,\mathbf{F}_8,!\mathbf{F}_3}_{} \underbrace{\&}_{\mathbf{h}_{\mathbf{Cut}}}$$

$$\frac{\frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon2}{\bullet \mathbf{h}_1 : \vdash !\mathbf{F}_4,?\Upsilon2} \; ! \; \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6,?dual(\mathbf{F}_4) \quad \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7,?dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(!\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7} \; \underbrace{- : \vdash ?\Upsilon2, \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7}_{\bullet \mathbf{h}_1 : \vdash ?\Upsilon2, !\mathbf{F}_4} \; \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6,?dual(\mathbf{F}_4)} \; \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6,?dual(\mathbf{F}_4)}^{\bullet \mathbf{x}} \; \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash ?\Upsilon2, \Delta_8, \mathbf{F}_6}^{\bullet \mathbf{h}_1 : \vdash ?\Upsilon2, !\mathbf{F}_4} \; \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7,?dual(\mathbf{F}_4)}^{\bullet \mathbf{x}} \; \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash 2}^{\bullet \mathbf{x}_6, \mathbf{x}_7,?dual(\mathbf{F}_4)}^{\bullet \mathbf{x}_7, \mathbf{x}$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ? \Upsilon 4, 5}{\bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, \mathsf{IF}_3} : \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathsf{F}_8, dual(5)}{\bullet \mathbf{h}_6 : \vdash dual(5), \Delta_9, \mathsf{F}_7 \oplus \mathsf{F}_8} \xrightarrow{\oplus \mathbf{h}_6} \frac{\oplus \mathcal{B}_8}{\mathsf{Cut}} \\ \\ \frac{\bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, \mathsf{IF}_3}{\bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, \mathsf{IF}_3} \overset{\mathrm{ax}}{\underset{\mathsf{h}_6 : \vdash \Delta_9, \mathsf{F}_8, dual(5)}{\bullet \mathbf{h}_6 : \vdash \Delta_9, \mathsf{F}_8, dual(5)}} \overset{\mathrm{ax}}{\underset{\mathsf{h}_6 : \vdash \Delta_9, \mathsf{F}_8, dual(5)}{\bullet \mathbf{h}_6 : \vdash \Delta_9, \mathsf{F}_8, dual(5)}} \overset{\mathrm{ax}}{\underset{\mathsf{h}_6 : \vdash 2}{\bullet \mathbf{h}_6 : \vdash 2}} \\ \\ \frac{\mathbf{h}_1 : \vdash \mathcal{F}_4, ? \Upsilon 2}{\bullet \mathbf{h}_1 : \vdash \mathsf{IF}_4, ? \Upsilon 2} : \overset{\mathbf{h}_5 : \vdash \Delta_8, \mathsf{F}_7, ? dual(\mathsf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(\mathsf{IF}_4), \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7} \overset{\oplus \mathcal{B}}{\underset{\mathsf{h}_5 : \vdash \Delta_8, \mathsf{F}_7, ? dual(\mathsf{F}_4)}{\bullet \mathbf{h}_5 : \vdash 2}} \overset{\mathrm{dx}}{\underset{\mathsf{h}_5 : \vdash 2}{\bullet \mathbf{h}_6, \mathsf{F}_7, ? dual(\mathsf{F}_4)}} \\ \\ \frac{\bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \mathsf{IF}_4}{\bullet \mathbf{h}_1 : \vdash ? \Upsilon 2, \mathsf{IF}_4} \overset{\mathrm{ax}}{\underset{\mathsf{h}_5 : \vdash \Delta_8, \mathsf{F}_7, ? dual(\mathsf{F}_4)}{\bullet \mathbf{h}_5 : \vdash 2}} \overset{\mathrm{ax}}{\underset{\mathsf{h}_5 : \vdash 2}{\bullet \mathbf{h}_6, \mathsf{F}_7, ? dual(\mathsf{F}_4)}} \overset{\mathrm{ax}}{\underset{\mathsf{h}_5 : \vdash 2}{\bullet \mathbf{h}_6, \mathsf{F}_7, ? dual(\mathsf{F}_4)}} \\ \\ \frac{- : \vdash ? \Upsilon 2, \Delta_8, \mathsf{F}_6}{\bullet \mathsf{IF}_7, ? \mathsf{I}_7, ?$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \mathbf{F}_3,?\Upsilon 4,5}{\bullet \mathbf{h}_2 : \vdash 5,?\Upsilon 4,! \mathbf{F}_3} : \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(5)}{\bullet \mathbf{h}_6 : \vdash dual(5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \\ \hline - : \vdash (?\Upsilon 4,! \mathbf{F}_3), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_2 : \vdash 5,?\Upsilon 4,! \mathbf{F}_3} \xrightarrow{\mathbf{ax}} \frac{\rightarrow}{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(5)} \\ \hline - : \vdash ?\Upsilon 4, \Delta_9, \mathbf{F}_7,! \mathbf{F}_3 \\ \hline - : \vdash ?\Upsilon 4, \Delta_9,! \mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{h}_{\mathbf{Cut}} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 2} : \xrightarrow{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6,?dual(\mathbf{F}_4)} \\ \bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4,?\Upsilon 2} : \xrightarrow{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6,?dual(\mathbf{F}_4)} \\ \hline - : \vdash ?\Upsilon 2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1 : \vdash ?\Upsilon 2,! \mathbf{F}_4} \xrightarrow{\mathbf{ax}} \xrightarrow{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6,?dual(\mathbf{F}_4)} \\ \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6,?dual(\mathbf{F}_4) \\ \hline - : \vdash ?\Upsilon 2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1 : \vdash ?\Upsilon 2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline - : \vdash ?\Upsilon 2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline - : \vdash ?\Upsilon 2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline - : \vdash ?\Upsilon 2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \end{array}$$

\bullet Case rule \bot

$$\begin{array}{c|c} \frac{\mathbf{h}_2 :\vdash \mathbf{F}_3,?\Upsilon 4,5}{\bullet \mathbf{h}_2 :\vdash 5,?\Upsilon 4,!\mathbf{F}_3} : & \frac{\mathbf{h}_6 :\vdash \Delta_7,dual(5)}{\bullet \mathbf{h}_6 :\vdash dual(5),\perp,\Delta_7} \\ \hline \\ \frac{-:\vdash (?\Upsilon 4,!\mathbf{F}_3),\perp,\Delta_7}{\bullet \mathbf{h}_2 :\vdash 5,?\Upsilon 4,!\mathbf{F}_3} & \frac{\rightarrow}{\mathbf{h}_6 :\vdash \Delta_7,dual(5)} \\ \hline \\ \frac{-:\vdash ?\Upsilon 4,\Delta_7,l\mathbf{F}_3}{-:\vdash ?\Upsilon 4,\Delta_7,\perp,l\mathbf{F}_3} \perp \\ \hline \\ \frac{\mathbf{h}_1 :\vdash \mathbf{F}_4,?\Upsilon 2}{\bullet \mathbf{h}_1 :\vdash !\mathbf{F}_4,?\Upsilon 2} : & \frac{\mathbf{h}_5 :\vdash \Delta_6,?dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 :\vdash dual(!\mathbf{F}_4),\perp,\Delta_6} \\ \hline \\ \frac{-:\vdash ?\Upsilon 2,\perp,\Delta_6}{\bullet \mathbf{h}_1 :\vdash ?\Upsilon 2,!\mathbf{F}_4} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 :\vdash \Delta_6,?dual(\mathbf{F}_4)} \\ \hline \\ \frac{-:\vdash ?\Upsilon 2,\Delta_6}{-:\vdash ?\Upsilon 2,\Delta_6,\perp} & \bot \\ \hline \end{array}$$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ? \Upsilon 4, 5}{\bullet \mathbf{h}_2 : \vdash 5, ? \Upsilon 4, ! \mathbf{F}_3} & ! & \frac{}{\bullet \mathbf{h}_6 : \vdash dual(5), \top, \Delta_7} & \top \\ \hline & - : \vdash (? \Upsilon 4, ! \mathbf{F}_3), \top, \Delta_7 \\ & \xrightarrow{} - : \vdash ? \Upsilon 4, \Delta_7, \top, ! \mathbf{F}_3 & \top \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \mathbf{F}_4, ? \Upsilon 2}{\bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4, ? \Upsilon 2} & ! & \frac{}{\bullet \mathbf{h}_5 : \vdash dual(! \mathbf{F}_4), \top, \Delta_6} & \top \\ \hline & - : \vdash ? \Upsilon 2, \top, \Delta_6 \\ & \xrightarrow{} - : \vdash ? \Upsilon 2, \Delta_6, \top & \top \\ \hline \end{array}$$

- \bullet Case rule I
- Case rule \otimes

5.3 Status of ?W: OK

- Case rule 1
- Case rule!

$$\frac{ \begin{array}{c|c} \mathbf{h}_1 : \vdash \Delta_2 \\ \hline \bullet \mathbf{h}_1 : \vdash ? \mathbf{F}_4, \Delta_2 \end{array} ?W & \begin{array}{c} \mathbf{h}_5 : \vdash ? \Upsilon \mathbf{G}, dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash dual(? \mathbf{F}_4), ? \Upsilon \mathbf{G} \end{array} \\ & \begin{array}{c} - : \vdash \Delta_2, ? \Upsilon \mathbf{G} \\ \hline \hline & - : \vdash \Delta_2 \end{array} \\ \hline & \begin{array}{c} - : \vdash \Delta_2 \\ \hline - : \vdash 2 \Upsilon \mathbf{G}, \Delta_2 \end{array} W \end{array} \end{aligned} \mathbf{Cut}$$

\bullet Case rule ?W

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} & ?W \\ \hline & - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_8, ?\mathbf{F}_7 & \rightarrow \\ \hline & \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} & \text{ax} \\ \hline & \frac{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_7}{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_7} & W \\ \hline & \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} & ?W \\ \hline & - : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6 & \rightarrow \\ \hline & \frac{\bullet}{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \text{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & \frac{- : \vdash \Delta_2, \Delta_7}{- : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6} & W \\ \hline \end{array}$$

\bullet Case rule ?C

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, ?\mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} & Cut \\ \hline & - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_8, ?\mathbf{F}_7 & \rightarrow \\ \hline & \frac{\rightarrow}{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5} & \text{ax} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} & \text{ax} \\ \hline & - : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5) & \text{hCut} \\ \hline & - : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_7 & ?W \\ \hline & \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} & ?C \\ \hline & - : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6 & \rightarrow \\ \hline & \frac{\rightarrow}{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \text{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & \text{ax} \\ \hline & - : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6, ?\mathbf{F}_6 & ?C \\ \hline & - : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6, ?\mathbf{F}_6 & ?C \\ \hline \end{array} \end{array}$$

• Case rule?

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} & ? \\ \hline \\ \frac{- : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_8, ?\mathbf{F}_7}{\bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} & \text{ax} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)}{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_7} & W \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} & ? \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4}{\bullet} & \text{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4}{\bullet} & \text{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)} \\ \hline \\ \frac{- : \vdash \Delta_2, \Delta_7, \mathbf{F}_6}{- : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6} & ? \\ \hline \end{array}$$

• Case rule \$

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8} & \mathbf{Cut} \\ \hline \\ - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8 & \rightarrow \\ \hline \\ \underline{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5} & \mathbf{ax} & \underbrace{- : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7\$\mathbf{F}_8}_{\bullet \mathbf{h}_6 : \vdash \Delta_9, f_7\$\mathbf{F}_8} & \mathbf{hCut} \\ \hline \\ \underline{- : \vdash \Delta_4, \Delta_9, \mathbf{F}_7\$\mathbf{F}_8} & W & \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} \\ \underline{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} & ?W & \underbrace{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6\$\mathbf{F}_7} & \mathbf{Cut} \\ \hline \\ \underline{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6\$\mathbf{F}_7} & \mathbf{ax} & \underbrace{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \mathbf{ax} & \mathbf{hCut} \\ \hline \\ \underline{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6\$\mathbf{F}_7} & \mathbf{s} & \mathbf{hCut} \\ \hline \end{array}$$

• Case rule &

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5) \quad \mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8} & \mathbf{Cut} \\ & - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8 \\ \hline & \rightarrow \\ \hline & \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5 & \mathbf{ax} & \rightarrow \\ \hline & \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \& \mathbf{F}_8} & \mathbf{ax} \\ \hline & - : \vdash \Delta_4, \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8 \\ \hline & - : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8 \\ \hline & - : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8 \\ \hline & - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) & \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_9, \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) \\ \hline & - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \mathbf{h}_7 \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) \\ \hline & - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \mathbf{h}_7 \\ \hline \end{array} \quad \mathbf{ax} \quad \mathbf{hCut} \quad \mathbf{hCut} \quad \mathbf{ax} \quad \mathbf{ax} \quad \mathbf{ax} \quad \mathbf{hCut} \quad \mathbf{ax} \quad \mathbf{ax}$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \frac{\oplus_B}{\mathsf{Cut}} \\ \hline & - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \rightarrow \\ \hline \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5} & \mathbf{ax} & \frac{\rightarrow_B}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathsf{ax} \\ \hline & - : \vdash \Delta_4, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & ?W \\ \hline & - : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8 & ?W \\ \hline \hline \bullet \mathbf{h}_1 : \vdash \Delta_2 & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_B \\ \hline & - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \rightarrow \\ \hline & \frac{\rightarrow_B}{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \mathbf{ax} & \rightarrow \\ \hline & \frac{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline & \frac{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_7}{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_B \\ \hline \end{array}$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{Cut} \\ \hline \\ - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & \rightarrow & \mathbf{ax} \\ \hline \underline{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5} & \mathbf{ax} & \xrightarrow{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{nCut} \\ \hline \\ - : \vdash \Delta_4, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & ?W & \bullet \mathbf{nCut} \\ \hline \underline{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8} & ?W & \bullet \mathbf{nCut} \\ \hline \\ \underline{\bullet \mathbf{h}_1 : \vdash \Delta_2} & ?W & \underbrace{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \mathbf{cut} \\ \hline \\ - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \underbrace{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \mathbf{ax} & \underbrace{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)}_{\bullet \mathbf{nCut}} & \mathbf{ax} & \mathbf{hCut} \\ \hline \\ - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus_A & \mathbf{hCut} \\ \hline \end{array}$$

• Case rule \perp

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \bot, \Delta_7} \\ \hline \\ & - : \vdash (\Delta_4, ?\mathbf{F}_3), \bot, \Delta_7 \\ \hline \\ & \xrightarrow{\bullet} \frac{}{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5} & \mathbf{ax} & \xrightarrow{\bullet} \frac{}{\bullet \mathbf{h}_6 : \vdash \Delta_7, \bot, dual(\mathbf{F}_5)} & \mathbf{ax} \\ \hline \\ & - : \vdash \Delta_4, \Delta_7, \bot \\ \hline \\ & - : \vdash \Delta_4, \Delta_7, \bot, ?\mathbf{F}_3 & W \\ \hline \\ & \bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2 & ?W & \underbrace{\bullet \mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \bot, \Delta_6} & \bot \\ \hline \\ & - : \vdash \Delta_2, \bot, \Delta_6 \\ \hline \\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \xrightarrow{\bullet} \frac{}{\mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \xrightarrow{\bullet} \frac{}{\mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ & - : \vdash \Delta_2, \Delta_6, \bot & \bot \\ \hline \end{array}$$

\bullet Case rule \top

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?W & \hline \bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \top, \Delta_7 \\ \hline - : \vdash (\Delta_4, ?\mathbf{F}_3), \top, \Delta_7 \\ \hline - : \vdash \Delta_4, \Delta_7, \top, ?\mathbf{F}_3 & \top \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} & ?W & \hline \bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \top, \Delta_6 \\ \hline - : \vdash \Delta_2, \top, \Delta_6 \\ \hline - : \vdash \Delta_2, \Delta_6, \top & \top \\ \hline \end{array}$$

- $\bullet\,$ Case rule I
- \bullet Case rule \otimes

$$\frac{ \begin{array}{c} \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5 \\ \bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3 \end{array} ?W \quad \begin{array}{c} \mathbf{h}_6 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \quad \mathbf{h}_6 : \vdash \Delta_7, \mathbf{F}_9 \\ \bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_7, \Delta_{10}, \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline \\ - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_7, \Delta_{10}, \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_6 : \vdash \Delta_{10}, \Delta_7, dual(\mathbf{F}_5), \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline \\ - : \vdash \Delta_{10}, \Delta_4, \Delta_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline \\ - : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?\mathbf{F}_3, \mathbf{F}_8 \otimes \mathbf{F}_9 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \mathbf{hCut} \\ \bullet \mathbf{hCut} \\ \end{array}$$

5.4 Status of ?C: OK

- \bullet Case rule 1
- Case rule!

$$\frac{ \begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ?C & \frac{\mathbf{h}_5 : \vdash ?\Upsilon\mathbf{6}, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), ?\Upsilon\mathbf{6}} \\ & \\ - : \vdash \Delta_2, ?\Upsilon\mathbf{6} & \\ & \\ \frac{\mathbf{h}_5 : \vdash ?\Upsilon\mathbf{6}, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash ?\Upsilon\mathbf{6}, ldual(\mathbf{F}_4)} \\ & \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_5 : \vdash ?\Upsilon\mathbf{6}, ldual(\mathbf{F}_4)} \\ & \\ - : \vdash ?\Upsilon\mathbf{6}, \Delta_2 & \\ \end{array} \\ \mathbf{mCut}$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ?C \qquad \frac{\mathbf{h}_6 : \vdash \Delta_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} \\ - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_8, ?\mathbf{F}_7 \\ \hline \underbrace{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3}_{\bullet \mathbf{h}_2 : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3} \text{ax} \qquad \frac{\rightarrow}{\mathbf{h}_6 : \vdash \Delta_8, dual(\mathbf{F}_5)}}_{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_7} W$$

$$\frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ?C \qquad \frac{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} ?W \\ \hline \underbrace{- : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6}_{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} \text{ax} \qquad \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)}}_{\bullet \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)} \text{ax} \\ \underbrace{- : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6}_{- : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6} ?W$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?C & \frac{\mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, ?\mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} & \mathbf{Cut} \\ \hline \\ \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{h}_5, ?\mathbf{F}_3, ?\mathbf{F}_3} & \mathbf{ax} & \xrightarrow{\bullet} \frac{\bullet}{\mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_3, ?\mathbf{F}_7}{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_7} & ?C & \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4 & ?C & \bullet \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2 & ?C & \bullet \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{h}_7, ?\mathbf{F}_6, ?\mathbf{F}_6 & ?C & \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{h}_7, ?\mathbf{h}_7, ?\mathbf{$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} &?C & \frac{\mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} &?\\ \hline \\ - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_8, ?\mathbf{F}_7 & \rightarrow \\ \hline \\ \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_3, ?\mathbf{F}_7}{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_7} &?C \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} &?C & \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} &?\\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4}{\bullet} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4}{\bullet} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6}{- : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6} &? & \mathbf{hCut} \\ \hline \end{array}$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} &?C & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8} & \frac{\$}{\mathsf{Cut}} \\ \hline \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3} & \text{ax} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7\$\mathbf{F}_8} \\ \hline \frac{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, ?\mathbf{F}_3, \mathbf{F}_7\$\mathbf{F}_8}{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, \mathbf{F}_7\$\mathbf{F}_8} &?C \\ \hline \frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} &?C & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6\$\mathbf{F}_7} & \$ \\ \hline \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \text{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline \frac{\rightarrow}{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \text{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4) & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4) & \text{ax} \\ \hline & & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4) & \text{ax} \\ \hline & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4) & \text{ax} \\ \hline & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4) & \text{ax} \\ \hline & \rightarrow \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4) & \text{ax} \\ \hline & \rightarrow \mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_8, \mathbf{F}_8, \mathbf{F}_$$

• Case rule &

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} &?C & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5) \quad \mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8} & \mathbf{Cut} \\ & & - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8 \\ & & \xrightarrow{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3} & \mathbf{ax} & \xrightarrow{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \& \mathbf{F}_8} & \mathbf{ax} \\ & & & \xrightarrow{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, ?\mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8} &?C \end{array}$$

$$\frac{\frac{\mathbf{h}_1 :\vdash \Delta_2, ?F_4, ?F_4}{\bullet \mathbf{h}_1 :\vdash ?F_4, \Delta_2}}{\underbrace{ ?C \quad \frac{\mathbf{h}_5 :\vdash \Delta_8, F_6, !dual(F_4) \quad \mathbf{h}_5 :\vdash \Delta_8, F_7, !dual(F_4)}{\bullet \mathbf{h}_5 :\vdash dual(?F_4), \Delta_8, F_6 \& F_7}}_{\text{Cut}} \& \\ \frac{- :\vdash \Delta_2, \Delta_8, F_6 \& F_7}{\bullet \mathbf{h}_5 :\vdash \Delta_8, F_6, !dual(F_4)} \quad \text{ax} \quad \frac{- :\vdash \Delta_2, \Delta_8, F_6, !dual(F_4)}{\bullet \mathbf{h}_5 :\vdash \Delta_8, !dual(F_4), F_6 \& F_7}}_{\bullet \mathbf{h}_5 :\vdash \Delta_8, F_6, !dual(F_4), F_6 \& F_7}} \quad \text{mCut} \\ \frac{\mathbf{h}_1 :\vdash \Delta_2, ?F_4, ?F_4}{\bullet \mathbf{h}_5 :\vdash \Delta_8, F_6, !dual(F_4), F_6 \& F_7}}_{\bullet \mathbf{h}_5 :\vdash \Delta_8, F_6 \& F_7} \quad \text{mCut}$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ?C & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \\ \underline{- : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{Cut} \\ \hline \\ \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, ?\mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{ax} \\ \underline{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8} & ?C \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ?C & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \mathbf{Cut} \\ \hline \\ \underline{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \mathbf{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)} \\ \underline{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \mathbf{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)} \\ \hline \\ \underline{\bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \mathbf{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)} \\ \underline{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ \underline{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_B \\ \end{array}$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ?C & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{Cut} \\ \hline & - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & \rightarrow \\ \hline \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{ax} \\ \hline & \frac{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, ?\mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8}{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8} ?C & \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ?C & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_A \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 & \bullet_A \\$$

\bullet Case rule \bot

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} & ?C & \frac{\mathbf{h}_6 : \vdash \Delta_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \bot, \Delta_7} & \mathsf{Cut} \\ \hline \\ \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3} & \mathbf{ax} & \xrightarrow{\bullet} \\ \frac{- : \vdash \Delta_4, \Delta_7, \bot, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_6 : \vdash \Delta_7, \bot, dual(\mathbf{F}_5)} & \mathsf{ax} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_7, \bot, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_4 : \vdash 2} & ?C & \frac{\mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)} & \bot \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} & ?C & \frac{\mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)} & \bot \\ \hline \\ \frac{- : \vdash \Delta_2, \bot, \Delta_6}{\bullet \mathbf{h}_5 : \vdash \Delta_6, \bot, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_5 : \vdash \Delta_6, \bot, !dual(\mathbf{F}_4)} & \bot \\ \hline \\ \mathbf{h}_5 : \vdash \Delta_6, \bot, !dual(\mathbf{F}_4)} & \mathbf{mCut} \\ \hline \end{array}$$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3, ?\mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ?C \quad & \frac{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \top, \Delta_7}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \top, \Delta_7} \quad & \mathsf{Cut} \\ \hline & - : \vdash (\Delta_4, ?\mathbf{F}_3), \top, \Delta_7 \\ & \xrightarrow{-} : \vdash \Delta_4, \Delta_7, \top, ?\mathbf{F}_3} \quad \top \\ \hline \frac{\mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ?C \quad & \frac{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \top, \Delta_6}{\bullet \mathbf{h}_5 : \vdash \Delta_2, \top, \Delta_6} \quad & \mathsf{Cut} \\ \hline & - : \vdash \Delta_2, \top, \Delta_6 \\ & \xrightarrow{-} : \vdash \Delta_2, \Delta_6, \top} \quad \top \end{array}$$

- \bullet Case rule I
- Case rule \otimes

5.5 Status of ?: OK

- Case rule 1
- Case rule!

\bullet Case rule ?W

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? & \frac{\mathbf{h}_6 : \vdash \Delta_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} ?W \\ \hline \\ - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_8, ?\mathbf{F}_7 \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3 & \xrightarrow{\bullet} \frac{\rightarrow}{\mathbf{h}_6 : \vdash \Delta_8, dual(\mathbf{F}_5)} \\ \hline \\ - : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3 \\ \hline \\ - : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_7 & W \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2 ? & \frac{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2 ? & \xrightarrow{\bullet} \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash 2, ?\mathbf{F}_4 & \xrightarrow{\bullet} \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \xrightarrow{\bullet} \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4) & \text{hCut} \\ \hline \\ \hline \\ - : \vdash \Delta_2, \Delta_7 \\ \hline - : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6 & W \\ \hline \end{array}$$

• Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? & \frac{\mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, ?\mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} & ?C \\ \hline & - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_8, ?\mathbf{F}_7 & \rightarrow \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, ?\mathbf{F}_3} & \mathbf{ax} & \xrightarrow{} \frac{\rightarrow}{\mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, ?\mathbf{F}_7, dual(\mathbf{F}_5)} & \mathbf{ax} \\ \hline & \frac{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_7, ?\mathbf{F}_7}{- : \vdash \Delta_4, \Delta_8, ?\mathbf{F}_3, ?\mathbf{F}_7, ?\mathbf{F}_7} & ?C \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ? & \xrightarrow{} \frac{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & ?C \\ \hline \\ \bullet \mathbf{h}_5 : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6 & \xrightarrow{} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \mathbf{ax} & \xrightarrow{} \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \mathbf{ax} & \xrightarrow{} \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \mathbf{ax} & \xrightarrow{} \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4} & \mathbf{ax} & \xrightarrow{} \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6, ?\mathbf{F}_6 & ?C & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_3 : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6, ?\mathbf{F}_6 & ?C & \mathbf{ax} \\ \hline \end{array}$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? \frac{\mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} \\ \hline \\ \frac{-}{\mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} \\ \frac{\rightarrow}{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_5} \text{ ax} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} \\ \hline \\ \frac{-}{-} : \vdash \Delta_4, \Delta_8, \mathbf{F}_3, ?\mathbf{F}_7} ? \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ? & \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} \\ \hline \\ \frac{-}{-} : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6} & \rightarrow\\ \hline \\ \frac{-}{-} : \vdash \Delta_2, \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)} \\ \hline \\ \frac{-}{-} : \vdash \Delta_2, \Delta_7, \mathbf{F}_6} ? \\ \hline \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \mathbf{hCut} \\ \mathbf{hCut} \\ \end{array}$$

• Case rule \$

• Case rule &

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{Cut} \\ \hline \\ - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & \rightarrow \\ \hline \\ \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_5}{\bullet} & \mathbf{ax} & \frac{- : \vdash \Delta_4, \Delta_9, \mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \oplus \mathbf{F}_8} ? \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_9, \mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8}{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8} ? \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ? & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_B \\ \hline \\ \frac{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7}{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7}{- : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_B \\ \hline \end{array}$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{Cut} \\ \hline - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & \rightarrow \\ \hline \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_5}{\bullet} & \mathbf{ax} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \oplus \mathbf{F}_8} \\ \hline - : \vdash \Delta_4, \Delta_9, \mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8 & ? & \mathbf{hCut} \\ \hline - : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_3, \mathbf{F}_7 \oplus \mathbf{F}_8 & ? & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2 & ? & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \mathbf{Cut} \\ \hline - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) & \mathbf{hCut} \\ \hline - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus \mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) & \mathbf{hCut} \\ \hline - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus \mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) & \mathbf{hCut} \\ \hline - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus \mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) & \mathbf{hCut} \\ \hline - : \vdash \Delta_2, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus \mathbf{h}_6 : \vdash \Delta_8, \mathbf{h}_6$$

• Case rule \perp

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? & \frac{\mathbf{h}_6 : \vdash \Delta_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \bot, \Delta_7} \\ \hline \\ - : \vdash (\Delta_4, ?\mathbf{F}_3), \bot, \Delta_7 \\ \hline \\ \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_5}{\bullet} & \mathbf{ax} & \frac{\bullet}{\bullet \mathbf{h}_6 : \vdash \Delta_7, \bot, dual(\mathbf{F}_5)} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_7, \mathbf{F}_3, \bot}{- : \vdash \Delta_4, \Delta_7, \bot, ?\mathbf{F}_3} ? \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ? & \frac{\mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \bot, \Delta_6} \\ \hline \\ - : \vdash \Delta_2, \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)} \\ \hline \\ \frac{-}{\bullet \mathbf{h}_1} : \vdash \Delta_2, ?\mathbf{F}_4 & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)} \\ \hline \\ - : \vdash \Delta_2, \Delta_6 \\ \hline \\ - : \vdash \Delta_2, \Delta_6, \bot & \mathbf{hCut} \\ \hline \end{array}$$

\bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? & \frac{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \top, \Delta_7}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \top, \Delta_7} & \mathsf{Cut} \\ & \xrightarrow{-} : \vdash (\Delta_4, ?\mathbf{F}_3), \top, \Delta_7 & \\ & \xrightarrow{-} : \vdash \Delta_4, \Delta_7, \top, ?\mathbf{F}_3} \top \\ \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ? & \xrightarrow{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \top, \Delta_6} & \top \\ & \xrightarrow{-} : \vdash \Delta_2, \top, \Delta_6 & \\ & \xrightarrow{-} : \vdash \Delta_2, \Delta_6, \top} \top \end{array}$$

- ullet Case rule I
- Case rule \otimes

$$\frac{\frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? \frac{\frac{\mathbf{h}_6 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \cdot \mathbf{h}_6 : \vdash \Delta_7, \mathbf{F}_9}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_7, \Delta_{10}, \mathbf{F}_8 \otimes \mathbf{F}_9}} \underbrace{ } } \underbrace{\phantom{ \underbrace{ \underbrace{ \underbrace{ \underbrace{ }} } \underbrace{ \underbrace{\phantom{$$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_5, \Delta_4, ?\mathbf{F}_3} ? & \frac{\mathbf{h}_6 : \vdash \Delta_7, \mathbf{F}_8 \quad \mathbf{h}_6 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_7, \Delta_{10}, \mathbf{F}_8 \otimes \mathbf{F}_9} \\ \hline - : \vdash (\Delta_4, ?\mathbf{F}_3), \Delta_7, \Delta_{10}, \mathbf{F}_8 \otimes \mathbf{F}_9} & \frac{\rightarrow}{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_5} & \mathbf{ax} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_{10}, \Delta_7, dual(\mathbf{F}_5), \mathbf{F}_8 \otimes \mathbf{F}_9} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \Delta_7, \mathbf{F}_3, \mathbf{F}_8 \otimes \mathbf{F}_9} ? & \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_2} ? & \frac{\mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8} & \mathbf{Cut} \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{cut} \\ \hline - : \vdash \Delta_2, \Delta_9, \mathbf{F}_7 & \mathbf{h}_2 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash 2, \mathbf{F}_4, \Delta_2 & ? & \frac{\mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline - : \vdash \Delta_2, \Delta_9, \mathbf{F}_7 & \mathbf{h}_2 & \mathbf{h}_2 & \mathbf{h}_2 & \mathbf{h}_3 \\ \hline \bullet \mathbf{h}_1 : \vdash 2, \mathbf{F}_4, \Delta_2 & ? & \frac{\mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4)} & \otimes \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{cut} \\ \hline \bullet \mathbf{h}_1 : \vdash 2, \mathbf{F}_4, \Delta_2 & ? & \frac{\mathbf{h}_5 : \vdash \Delta_6, \mathbf{F}_7 \quad \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{h}_7 \otimes \mathbf{F}_8} & \mathbf{cut} \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{cut} \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{cut} \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4) \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{cut} \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4) \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4) \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4) \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4) \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4) \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4) \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8, !dual(\mathbf{F}_4) \\ \hline - : \vdash \Delta_2, \Delta_6, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 & \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_8$$

5.6 Status of \$: OK

- ullet Case rule $oldsymbol{1}$
- Case rule!
- Case rule ?W

 \bullet Case rule ?C

$$\frac{\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \$ \mathbf{F}_4} \$ \frac{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \underbrace{?C}_{\mathbf{Cut}} \\ - : \vdash (\Delta_5, \mathbf{F}_3 \$ \mathbf{F}_4), \Delta_9, ?\mathbf{F}_8 \\ \xrightarrow{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \$ \mathbf{F}_4} \underbrace{\mathbf{ax}}_{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_6)} \\ - : \vdash \Delta_5, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, \mathbf{F}_3 \$ \mathbf{F}_4} \underbrace{?C}_{\mathbf{h}_{\mathbf{Cut}}} \end{aligned} \mathbf{h}_{\mathbf{Cut}}$$

$$\frac{ \begin{array}{l} \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5, \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \$ \mathbf{F}_6, \Delta_2 \end{array}}{ \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6) } \\ \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \$ \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8 \\ & - : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8 \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \$ \mathbf{F}_6 \end{array}} \xrightarrow{\mathbf{ax}} \begin{array}{l} - : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6) \\ \bullet \mathbf{h}_7 : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6) \\ \hline - : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8 \\ - : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8 \end{array}} \begin{array}{l} ?C \\ \bullet \mathbf{hCut} \\ \bullet \mathbf{hCut} \\ \end{array}$$

• Case rule?

• Case rule \$

\bullet Case rule &

$$\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4}}{} \$ \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6) \quad h_7 : \vdash \Delta_{10}, F_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \underbrace{Cut} \\ - : \vdash (\Delta_5, F_3 \$ F_4), \Delta_{10}, F_8 \& F_9} \\ \hline \frac{\rightarrow}{h_2 : \vdash \Delta_5, F_3, F_4, F_6} \underbrace{ax}_{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \& F_9} \\ \hline - : \vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \& F_9} \\ \hline \frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_{10}, F_8 \& F_9} \underbrace{cut} \\ \hline - : \vdash \Delta_2, \Delta_{10}, F_8, dual(F_6) \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_1, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, F_9, dual(F_6)}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, \Delta_2, F_8}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, \Delta_2, F_8}_{- : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{ax}_{h_7 : \vdash \Delta_{10}, \Delta_2$$

• Case rule \oplus_B

• Case rule \oplus_A

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \$ \mathbf{F}_4} \quad \$ & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \mathbf{Cut} \\ \hline & & \rightarrow \\ \hline \\ \underline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_6} & \mathbf{ax} & & \rightarrow \\ \hline \\ \underline{\mathbf{h}_2 : \vdash \Delta_1, \Delta_5, \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_6} & \mathbf{ax} & & \rightarrow \\ \hline \\ \underline{- : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_8 \oplus \mathbf{F}_9} & \\ \hline \\ \underline{- : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_8 \oplus \mathbf{F}_9} & \\ \hline \\ \underline{- : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_8 \oplus \mathbf{F}_9} & \\ \hline \\ \underline{- : \vdash \Delta_1, \Delta_5, \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_8 \oplus \mathbf{F}_9} & \\ \hline \\ \underline{- : \vdash \Delta_1, \Delta_1, \mathbf{F}_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)} & \oplus A \\ \hline \\ \underline{- : \vdash \Delta_2, \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \\ \hline \\ \underline{- : \vdash \Delta_1, \Delta_2, \mathbf{F}_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)} & \mathbf{ax} \\ \hline \\ \underline{- : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8} & \oplus A \\ \hline \\ \underline{- : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8} & \oplus A \\ \hline \end{array} \end{array} \right. \\ \begin{array}{c} \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \$ \mathbf{F}_6 & \mathbf{ax} \\ \hline \\ \underline{- : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8} & \oplus A \\ \hline \\ \underline{- : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8} & \oplus A \\ \end{array} \right. \\ \begin{array}{c} \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \$ \mathbf{F}_6 & \mathbf{ax} \\ \hline \\ \underline{- : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8} & \oplus A \\ \hline \\ \underline{- : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8} & \oplus A \\ \end{array} \right. \\ \begin{array}{c} \mathbf{h}_1 : \vdash \mathbf{h}_2 & \mathbf{h}_3 & \mathbf{h}_4 \\ \hline \\ \mathbf{h}_4 & \mathbf{h}_5 & \mathbf{h}_5 & \mathbf{h}_5 \\ \hline \\ \mathbf{h}_5 & \mathbf{h}_5 & \mathbf{h}_5 & \mathbf{h}_5 \\ \hline \\ \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{h}_8 & \mathbf{h}_9 & \mathbf{h}_8 \\ \hline \\ \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{h}_8 & \mathbf{h}_9 & \mathbf{h}_8 \\ \hline \end{array} \right.$$

• Case rule \perp

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \$ \mathbf{F}_4} \$ & \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \bot, \Delta_8} & \bot \\ \hline - : \vdash (\Delta_5, \mathbf{F}_3 \$ \mathbf{F}_4), \bot, \Delta_8 & \to \\ \hline \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_6}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \Delta_8, \mathbf{F}_3, \mathbf{F}_4, \bot} & \Rightarrow \\ \hline \frac{- : \vdash \Delta_5, \Delta_8, \mathbf{F}_3, \mathbf{F}_4, \bot}{- : \vdash \Delta_5, \Delta_8, \bot, \mathbf{F}_3 \$ \mathbf{F}_4} \$ & \text{hCut} \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \$ \mathbf{F}_6, \Delta_2} \$ & \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \$ \mathbf{F}_6), \bot, \Delta_8} & \bot \\ \hline - : \vdash \Delta_2, \bot, \Delta_8 & \to \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \$ \mathbf{F}_6} & \text{ax} & \to \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)} & \text{ax} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_2, \Delta_8 & \to \\ \hline - : \vdash \Delta_2, \Delta_8, \bot & \bot \\ \hline & \bullet \mathsf{Cut} \\ \hline \end{array}$$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathsf{F}_6, \mathsf{F}_3, \mathsf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathsf{F}_6, \Delta_5, \mathsf{F}_3 \$ \mathsf{F}_4} \quad \$ \quad & \frac{\bullet \mathbf{h}_7 : \vdash dual(\mathsf{F}_6), \top, \Delta_8}{\bullet \mathbf{h}_7 : \vdash dual(\mathsf{F}_6), \top, \Delta_8} \quad \mathsf{Cut} \\ & \frac{- : \vdash (\Delta_5, \mathsf{F}_3 \$ \mathsf{F}_4), \top, \Delta_8}{- : \vdash \Delta_5, \Delta_8, \top, \mathsf{F}_3 \$ \mathsf{F}_4} \quad \top \\ \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathsf{F}_5, \mathsf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathsf{F}_5 \$ \mathsf{F}_6, \Delta_2} \quad \$ \quad & \frac{\bullet \mathbf{h}_7 : \vdash dual(\mathsf{F}_5 \$ \mathsf{F}_6), \top, \Delta_8}{\bullet \mathbf{h}_7 : \vdash dual(\mathsf{F}_5 \$ \mathsf{F}_6), \top, \Delta_8} \quad \mathsf{Cut} \\ & \frac{- : \vdash \Delta_2, \top, \Delta_8}{- : \vdash \Delta_2, \Delta_8, \top} \quad \top \end{array}$$

- \bullet Case rule I
- Case rule \otimes

5.7 Status of &: OK

• Case rule 1

- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathsf{F}_6, \mathsf{F}_3 \quad \mathbf{h}_2 :\vdash \Delta_5, \mathsf{F}_6, \mathsf{F}_4}{\underbrace{\bullet \mathbf{h}_2 :\vdash \mathsf{F}_6, \Delta_5, \mathsf{F}_3 \& \mathsf{F}_4}_{} = \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathsf{F}_6), \Delta_9, ?\mathsf{F}_8}_{} } \underbrace{\phantom{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathsf{F}_6), \Delta_9, ?\mathsf{F}_8}_{} } \underbrace{\phantom{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathsf{F}_6), \Delta_9, ?\mathsf{F}_8}_{} }_{Cut} \\ \underbrace{\phantom{\bullet \mathbf{h}_2 :\vdash \Delta_5, \mathsf{F}_6, \mathsf{F}_3 \& \mathsf{F}_4}_{} \underbrace{\phantom{\bullet \mathbf{h}_3 :\vdash \Delta_5, \Delta_9, \mathsf{F}_3 \& \mathsf{F}_4}_{} } \underbrace{\phantom{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathsf{F}_6), \Delta_9, ?\mathsf{F}_8, \mathsf{F}_3 \& \mathsf{F}_4}_{} } \underbrace{\phantom{\bullet \mathbf{h}_1 :\vdash \Delta_2, \mathsf{F}_6, \Delta_2}}_{} \underbrace{\phantom{\bullet \mathbf{h}_1 :\vdash \Delta_2, \mathsf{F}_6, \Delta_2}}_{} \underbrace{\phantom{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathsf{F}_5) \oplus dual(\mathsf{F}_6), \Delta_9, ?\mathsf{F}_8, \mathsf{F}_6, \Delta_2, ?\mathsf{F}_8, \mathsf{F}_6, \Delta_9, ?\mathsf{F}_8, \mathsf{F}_6, \mathsf{$$

 \bullet Case rule ?C

• Case rule?

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4}{\underbrace{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4}_{} = \underbrace{\bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8}_{} ?}_{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} = \underbrace{\alpha}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \& \mathbf{F}_4}^{} = \alpha \mathbf{x} \xrightarrow{\bullet}_{h_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_6)}_{} \underbrace{\alpha}_{h_C u t}^{} = \frac{\alpha \mathbf{h}_1 : \vdash \Delta_5, \Delta_9, \mathbf{F}_8, \mathbf{F}_3 \& \mathbf{F}_4}_{} ?}_{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5, \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6} \underbrace{\delta}_{\bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{} \underbrace{\delta}_{\bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{} ?}_{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \& \mathbf{F}_6, \Delta_2} \underbrace{\delta}_{h_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{} \underbrace{\delta}_{h_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{} \end{aligned}_{\bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{} \underbrace{\delta}_{h_C u t}$$

• Case rule \$

$$\frac{ \begin{array}{c} \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4 \\ \hline \bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline - : \vdash (\Delta_5, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{10}, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline \end{array} \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_8 \& \mathbf{F}_9, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \& \mathbf{F}_6, \Delta_2 \\ \hline \end{array} \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10$$

• Case rule &

$$\frac{ \frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \quad \& \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6) \quad h_7 : \vdash \Delta_{10}, F_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \quad \text{Cut} } \\ \frac{- : \vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \& F_9}{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \& F_9} \quad \frac{\text{ax}}{\text{hCut}} \quad \frac{h_2 : \vdash \Delta_5, F_4, F_6}{\bullet h_7 : \vdash \Delta_{10}, \Delta_5, F_3, F_8 \& F_9} \quad \frac{\text{ax}}{\text{hCut}} \quad \frac{h_2 : \vdash \Delta_5, F_4, F_6}{\bullet h_7 : \vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \quad \frac{\text{ax}}{\text{hCut}} \quad \frac{h_2 : \vdash \Delta_5, F_4, F_6}{\bullet h_7 : \vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \quad \frac{\text{ax}}{\text{hCut}} \quad \frac{\text{hCut}}{\bullet h_7 : \vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \quad \frac{\text{ax}}{\text{hCut}} \quad \frac{\text{hCut}}{\bullet h_7 : \vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \quad \frac{\text{ax}}{\text{hCut}} \quad \frac{\text{hCut}}{\bullet h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \oplus dual(F_6)} \quad \& \quad \frac{\text{h}_7 : \vdash \Delta_1, F_9, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash \Delta_1, F_9, dual(F_5) \oplus dual(F_6)} \quad \& \quad \frac{\text{h}_7 : \vdash \Delta_1, F_9, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash \Delta_1, F_9, dual(F_5) \oplus dual(F_6)} \quad \frac{\text{ax}}{h_7 : \vdash \Delta_1, F_9, dual(F_5) \oplus dual(F_6)} \quad \frac{\text{ax}}{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \oplus dual(F_6)} \quad \frac{\text{ax}}{h_7 : \vdash \Delta_1, F_9, dual(F_5) \oplus dual(F_6)}} \quad \frac{\text{ax}}{h_{\text{Cut}}} \quad \frac{\text{hCut}}{- : \vdash \Delta_1, F_9, dual(F_5) \oplus dual(F_6)} \quad \frac{\text{ax}}{h_{\text{Cut}}} \quad \frac{\text{hCut}}{- : \vdash \Delta_1, F_9, dual(F_5) \oplus dual(F_6)} \quad \frac{\text{ax}}{h_{\text{Cut}}} \quad \frac{\text{hCut}}{- : \vdash \Delta_1, F_9, dual(F_5) \oplus dual(F_6)} \quad \frac{\text{ax}}{h_{\text{Cut}}} \quad \frac{\text{hCut}}{- : \vdash \Delta_1, F_9, dual(F_9)} \quad \frac{\text{ax}}{h_{\text{Cut}}} \quad \frac{\text{hCut}}{- : \vdash \Delta_1, F_9, dual(F_9)} \quad \frac{\text{ax}}{h_{\text{Cut}}} \quad \frac{\text{hCut}}{- : \vdash \Delta_1, F_9, dual(F_9)} \quad \frac{\text{ax}}{h_{\text{Cut}}} \quad \frac{\text{hCut}}{- : \vdash \Delta_1, F_9, dual(F_9)} \quad \frac{\text{ax}}{h_{\text{Cut}}} \quad \frac{\text{hCut}}{- : \vdash \Delta_1, F_9, dual(F_9)} \quad \frac{\text{Ax}}{h_{\text{Cut}}} \quad \frac{\text{hCut}}{- : \vdash \Delta_1, F_9, dual(F_9)} \quad \frac{\text{Ax}}{h_{\text{Cut}}} \quad \frac{\text{Ax}}{- : \vdash \Delta_1, F_9, dual(F_9)} \quad \frac{\text{Ax}}{h_{\text{Cut}}} \quad \frac{\text{Ax}}{h_{\text{Cut}}} \quad \frac{\text{Ax}}{- : \vdash \Delta_1, F_9, dual(F_9)} \quad \frac{\text{Ax}}{h_{\text{Cut}}} \quad \frac{\text{Ax}}{h_{\text{Cut}}} \quad \frac{\text{Ax}}{- : \vdash \Delta_1, F_9, dual(F_9)} \quad \frac{\text{Ax}}{h_{\text{Cut}}} \quad \frac{\text{Ax}}{h_{\text{Cut}}} \quad \frac{\text{Ax}}{h_{\text{Cut}}} \quad \frac{\text{Ax}}{h_{\text{Cu$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_2 :\vdash \Delta_5, F_6, F_3 \quad \mathbf{h}_2 :\vdash \Delta_5, F_6, F_4}{\bullet \mathbf{h}_2 :\vdash F_6, \Delta_5, F_3 \& F_4} \quad \& \quad \frac{\mathbf{h}_7 :\vdash \Delta_{10}, F_9, dual(F_6)}{\bullet \mathbf{h}_7 :\vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \\ \hline \bullet \frac{\bullet \mathbf{h}_2 :\vdash F_6, \Delta_5, F_3 \& F_4}{\bullet \mathbf{h}_2 :\vdash \Delta_5, F_6, F_3 \& F_4} \quad \text{ax} \quad \frac{\rightarrow}{\mathbf{h}_7 :\vdash \Delta_{10}, F_9, dual(F_6)} \quad \text{ax} \\ \hline \bullet \frac{\bullet \mathbf{h}_2 :\vdash \Delta_5, F_6, F_3 \& F_4}{\bullet \mathbf{h}_2 :\vdash \Delta_10, \Delta_5, F_9, F_3 \& F_4} \oplus \mathcal{B} \\ \hline \\ \frac{\bullet \mathbf{h}_1 :\vdash \Delta_2, F_5 \quad \mathbf{h}_1 :\vdash \Delta_2, F_6}{\bullet \mathbf{h}_1 :\vdash F_5 \& F_6, \Delta_2} \quad \& \quad \frac{\mathbf{h}_7 :\vdash \Delta_{10}, F_9, dual(F_5) \oplus dual(F_6)}{\bullet \mathbf{h}_7 :\vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \oplus F_9} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_5 \& \mathbf{h}_6 \quad \text{ax} \\ \hline \qquad \qquad - :\vdash \Delta_2, \Delta_10, F_9, dual(F_5) \oplus dual(F_6)} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_5 \& \mathbf{h}_6 \quad \text{ax} \\ \hline \qquad \qquad - :\vdash \Delta_{10}, \Delta_2, F_9 \\ \hline \qquad \qquad - :\vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9 \\ \hline \\ \bullet \mathbf{h}_1 :\vdash F_5 \& F_6, \Delta_2 \quad \& \quad \frac{\mathbf{h}_7 :\vdash \Delta_8, dual(F_6)}{\bullet \mathbf{h}_7 :\vdash dual(F_5 \& F_6), \Delta_8} \quad \oplus_{\mathbf{h}_5} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash F_5 \& F_6, \Delta_2 \quad \& \quad \frac{\mathbf{h}_7 :\vdash \Delta_8, dual(F_6)}{\bullet \mathbf{h}_7 :\vdash dual(F_5 \& F_6), \Delta_8} \quad \oplus_{\mathbf{h}_5} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash F_5 \& F_6, \Delta_2 \quad \& \quad \frac{\mathbf{h}_7 :\vdash \Delta_8, dual(F_6)}{\bullet \mathbf{h}_7 :\vdash dual(F_5 \& F_6), \Delta_8} \quad \oplus_{\mathbf{h}_5} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash F_5 \& F_6, \Delta_2 \quad \& \quad \frac{\mathbf{h}_7 :\vdash \Delta_8, dual(F_6)}{\bullet \mathbf{h}_7 :\vdash dual(F_5 \& F_6), \Delta_8} \quad \oplus_{\mathbf{h}_5} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash F_5 \& F_6, \Delta_2 \quad \& \quad \frac{\mathbf{h}_7 :\vdash \Delta_8, dual(F_6)}{\bullet \mathbf{h}_7 :\vdash dual(F_5 \& F_6), \Delta_8} \quad \oplus_{\mathbf{h}_5} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash F_5 \& F_6, \Delta_2 \quad \& \quad \mathbf{h}_7 :\vdash \Delta_8, dual(F_6) \quad \oplus_{\mathbf{h}_7 :\vdash \Delta_8, dual(F_6)} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_6 \quad \text{ax} \quad \rightarrow \quad \mathbf{h}_7 :\vdash \Delta_8, dual(F_6) \quad \oplus_{\mathbf{h}_7 :\vdash \Delta_8, dual(F_6)} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_6 \quad \text{ax} \quad \rightarrow \quad \mathbf{h}_7 :\vdash \Delta_8, dual(F_6) \quad \oplus_{\mathbf{h}_7 :\vdash \Delta_8, dual(F_6)} \\ \hline \\ \bullet \mathbf{h}_7 :\vdash \Delta_8, dual(F_6) \quad \oplus_{\mathbf{h}_8 \to \mathbf{h}_8} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_6 \quad \text{ax} \quad \rightarrow \quad \mathbf{h}_7 :\vdash \Delta_8, dual(F_6) \quad \oplus_{\mathbf{h}_8 \to \mathbf{h}_8} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_6 \quad \text{ax} \quad \rightarrow \quad \mathbf{h}_7 :\vdash \Delta_8, dual(F_6) \quad \oplus_{\mathbf{h}_8 \to \mathbf{h}_8} \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_6 \quad \text{ax} \quad \rightarrow \quad \mathbf{h}_8 \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_6 \quad \text{ax} \quad \rightarrow \quad \mathbf{h}_8 \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_6 \quad \text{ax} \quad \rightarrow \quad \mathbf{h}_8 \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_2, F_6 \quad \mathbf$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathsf{F}_6, \mathsf{F}_3 \quad \mathbf{h}_2 :\vdash \Delta_5, \mathsf{F}_6, \mathsf{F}_4}{\bullet \mathsf{h}_2 :\vdash \mathsf{F}_6, \Delta_5, \mathsf{F}_3 \& \mathsf{F}_4} \quad \& \quad \frac{\mathsf{h}_7 :\vdash \Delta_{10}, \mathsf{F}_8, dual(\mathsf{F}_6)}{\bullet \mathsf{h}_7 :\vdash dual(\mathsf{F}_6), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9} \\ - :\vdash (\Delta_5, \mathsf{F}_3 \& \mathsf{F}_4), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline \bullet \mathsf{h}_2 :\vdash \Delta_5, \mathsf{F}_6, \mathsf{F}_3 \& \mathsf{F}_4} \quad \text{ax} \quad \xrightarrow{\mathsf{h}_7 :\vdash \Delta_{10}, \mathsf{F}_8, dual(\mathsf{F}_6)} \quad \mathsf{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_5, \mathsf{F}_8, \mathsf{F}_3 \& \mathsf{F}_4} \\ \hline \bullet \mathsf{h}_1 :\vdash \Delta_2, \mathsf{F}_5 \quad \mathsf{h}_1 :\vdash \Delta_2, \mathsf{F}_6 \\ \hline \bullet \mathsf{h}_1 :\vdash \mathsf{F}_5 \& \mathsf{F}_6, \Delta_2} \quad \& \quad \frac{\mathsf{h}_7 :\vdash \Delta_{10}, \mathsf{F}_8, dual(\mathsf{F}_5) \oplus dual(\mathsf{F}_6)}{\bullet \mathsf{h}_7 :\vdash dual(\mathsf{F}_5 \& \mathsf{F}_6), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9} \\ \hline - :\vdash \Delta_2, \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline \bullet \mathsf{h}_1 :\vdash \Delta_2, \mathsf{F}_5 \& \mathsf{F}_6} \quad \mathsf{ax} \\ \hline - :\vdash \Delta_{10}, \Delta_2, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline - :\vdash \Delta_{10}, \Delta_2, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline - :\vdash \Delta_{10}, \Delta_2, \mathsf{F}_8 \oplus \mathsf{F}_9} \oplus A \\ \hline \bullet \mathsf{h}_1 :\vdash \mathsf{F}_5 \& \mathsf{F}_6, \Delta_2} \quad \& \quad \frac{\mathsf{h}_7 :\vdash \Delta_8, dual(\mathsf{F}_5) \oplus dual(\mathsf{F}_6)}{\bullet \mathsf{hCut}} \quad \mathsf{ax} \\ \hline \bullet \mathsf{h}_1 :\vdash \Delta_2, \mathsf{F}_5 & \mathsf{h}_1 :\vdash \Delta_2, \mathsf{F}_6 \\ \hline \bullet \mathsf{h}_7 :\vdash \Delta_{10}, \Delta_2, \mathsf{F}_8 \oplus \mathsf{F}_9} \quad \oplus A \\ \hline \bullet \mathsf{h}_1 :\vdash \mathsf{F}_5 \& \mathsf{F}_6, \Delta_2} \quad \& \quad \frac{\mathsf{h}_7 :\vdash \Delta_8, dual(\mathsf{F}_5)}{\bullet \mathsf{h}_7 :\vdash dual(\mathsf{F}_5 \& \mathsf{F}_6), \Delta_8} \quad \mathsf{Cut} \\ \hline \bullet \mathsf{h}_1 :\vdash \mathsf{F}_5 \& \mathsf{F}_6, \Delta_2} \quad \& \quad \frac{\mathsf{h}_7 :\vdash \Delta_8, dual(\mathsf{F}_5)}{\bullet \mathsf{h}_7 :\vdash dual(\mathsf{F}_5 \& \mathsf{F}_6), \Delta_8} \quad \mathsf{Cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline - :\vdash \Delta_2, \Delta_8 \quad \to \quad \mathsf{cut} \\ \hline \rightarrow \quad \mathsf{cut} \quad \to \quad \mathsf{cut} \\ \hline \to \mathsf{cut} \quad \to \quad \mathsf{cut} \quad \to \quad \mathsf{cut} \\ \hline \to \mathsf{cut} \quad \to \quad \mathsf{cut} \quad \to \quad \mathsf{cut} \\ \hline \to \mathsf$$

\bullet Case rule \bot

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \quad \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \bot, \Delta_8} \\ & \xrightarrow{- : \vdash (\Delta_5, \mathbf{F}_3 \& \mathbf{F}_4), \bot, \Delta_8} \\ & \xrightarrow{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \& \mathbf{F}_4} \quad \xrightarrow{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_6)} \quad \underset{\mathsf{hCut}}{\underbrace{- : \vdash \Delta_5, \Delta_8, \mathbf{F}_3 \& \mathbf{F}_4}} \quad \bot \\ & \xrightarrow{- : \vdash \Delta_5, \Delta_8, \bot, \mathbf{F}_3 \& \mathbf{F}_4} \quad \bot \\ & \xrightarrow{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \quad \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6} \quad \& \quad \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \bot, \Delta_8} \quad \bot \\ & \xrightarrow{- : \vdash \Delta_2, \bot, \Delta_8} \\ & \xrightarrow{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \& \mathbf{F}_6} \quad \underset{\mathsf{hCut}}{\underbrace{- : \vdash \Delta_2, \Delta_8}} \quad \bot \\ & \xrightarrow{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \& \mathbf{F}_6} \quad \underset{\mathsf{hCut}}{\underbrace{- : \vdash \Delta_2, \Delta_8}} \quad \bot \end{array}$$

\bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \quad \frac{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \top, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \Delta_8} \quad \overset{\top}{\text{Cut}} \\ & \stackrel{- : \vdash (\Delta_5, \mathbf{F}_3 \& \mathbf{F}_4), \top, \Delta_8}{- : \vdash \Delta_5, \Delta_8, \top, \mathbf{F}_3 \& \mathbf{F}_4} \quad \top \\ \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \quad \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \& \mathbf{F}_6, \Delta_2} \quad \& \quad \frac{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \top, \Delta_8}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \top, \Delta_8} \quad \overset{\top}{\text{Cut}} \\ & \stackrel{- : \vdash \Delta_2, \top, \Delta_8}{- : \vdash \Delta_2, \Delta_8, \top} \quad \top \end{array}$$

- \bullet Case rule I
- \bullet Case rule \otimes

5.8 Status of \oplus_B : OK

- Case rule 1
- Case rule!
- ullet Case rule ?W

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \\ \hline \\ - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_9, ?\mathbf{F}_8 & \rightarrow \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \oplus \mathbf{F}_4} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6)} \\ \hline \\ - : \vdash \Delta_5, \Delta_9, \mathbf{F}_3 \oplus \mathbf{F}_4 \\ \hline \\ - : \vdash \Delta_5, \Delta_9, ?\mathbf{F}_8, \mathbf{F}_3 \oplus \mathbf{F}_4 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2} & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2} & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \xrightarrow{\mathbf{h}_7} & \rightarrow \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6} & \xrightarrow{\mathbf{h}_7} & \xrightarrow{\mathbf{h}_7} & \xrightarrow{\mathbf{h}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} & \xrightarrow{\mathbf{h}_7} & \xrightarrow{\mathbf{h}_7} & \xrightarrow{\mathbf{h}_7} & \xrightarrow{\mathbf{h}_9} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{h}_9, ?\mathbf{F}_8 & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{h}_9, ?\mathbf{F}_8 & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{h}_9, ?\mathbf{h}_8 & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{h}_9, ?\mathbf{h}_8 & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_9} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{h}_9, ?\mathbf{h}_8 & \xrightarrow{\mathbf{h}_9} & \xrightarrow{\mathbf{h}_$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \oplus B & \frac{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} ?C \\ \hline - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_9, ?\mathbf{F}_8 & \mathbf{Cut} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \oplus \mathbf{F}_4 & \mathbf{ax} & \rightarrow \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_6)} \\ \hline \frac{- : \vdash \Delta_5, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, \mathbf{F}_3 \oplus \mathbf{F}_4}{- : \vdash \Delta_5, \Delta_9, ?\mathbf{F}_8, \mathbf{F}_3 \oplus \mathbf{F}_4} ?C \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 & \oplus \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} \\ \hline - : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_8, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_7, ?\mathbf{h}_8 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_9, ?$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathsf{F}_6, \mathsf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathsf{F}_6, \Delta_5, \mathsf{F}_3 \oplus \mathsf{F}_4} \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_9, \mathsf{F}_8, dual(\mathsf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathsf{F}_6), \Delta_9, ?\mathsf{F}_8} \\ \hline & - : \vdash (\Delta_5, \mathsf{F}_3 \oplus \mathsf{F}_4), \Delta_9, ?\mathsf{F}_8 \\ \hline & \frac{\bullet}{\mathsf{h}_2 : \vdash \Delta_5, \mathsf{F}_6, \mathsf{F}_3 \oplus \mathsf{F}_4} & \text{ax} & \frac{\rightarrow}{\mathsf{h}_7 : \vdash \Delta_9, \mathsf{F}_8, dual(\mathsf{F}_6)} \\ \hline & \frac{- : \vdash \Delta_5, \Delta_9, \mathsf{F}_8, \mathsf{F}_3 \oplus \mathsf{F}_4}{- : \vdash \Delta_5, \Delta_9, ?\mathsf{F}_8, \mathsf{F}_3 \oplus \mathsf{F}_4} ? \\ \hline & \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathsf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathsf{F}_5 \oplus \mathsf{F}_6, \Delta_2} & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_9, \mathsf{F}_8, dual(\mathsf{F}_5) \& dual(\mathsf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathsf{F}_5 \oplus \mathsf{F}_6), \Delta_9, ?\mathsf{F}_8} & ? \\ \hline & \frac{\bullet}{\mathsf{h}_1 : \vdash \Delta_2, \mathsf{F}_5 \oplus \mathsf{F}_6} & \text{ax} & \frac{\rightarrow}{\mathsf{h}_7 : \vdash \Delta_9, \mathsf{F}_8, dual(\mathsf{F}_5) \& dual(\mathsf{F}_6)}{\bullet \mathsf{h}_7 : \vdash \Delta_9, \mathsf{F}_8, dual(\mathsf{F}_5) \& dual(\mathsf{F}_6)} \\ \hline & \frac{\bullet}{\mathsf{h}_1 : \vdash \Delta_2, \mathsf{F}_5 \oplus \mathsf{F}_6} & \text{ax} & \frac{\rightarrow}{\mathsf{h}_7 : \vdash \Delta_9, \mathsf{F}_8, dual(\mathsf{F}_5) \& dual(\mathsf{F}_6)}{\bullet \mathsf{h}_7 : \vdash \Delta_9, \mathsf{F}_8, dual(\mathsf{F}_5) \& dual(\mathsf{F}_6)} \\ \hline & \frac{- : \vdash \Delta_2, \Delta_9, \mathsf{F}_8}{- : \vdash \Delta_2, \Delta_9, ?\mathsf{F}_8} ? \\ \hline \end{array} \right.$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \oplus B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9} \\ \hline - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9} & \xrightarrow{\bullet} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \oplus \mathbf{F}_4} & \text{ax} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_6)} \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_3 \oplus \mathbf{F}_4} & \$ \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2} & \oplus B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9} \\ \hline - : \vdash \Delta_2, \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6} & \mathbf{ax} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{h}_7, \mathbf$$

• Case rule &

$$\frac{\underbrace{\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2}}_{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \underbrace{\frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9}}_{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9} \underbrace{Cut} \\ \underbrace{- : \vdash \Delta_2, \Delta_{10}, F_8 \& F_9}_{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}}_{\bullet h_7 : \vdash \Delta_{10}, \Delta_2, F_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_{10}, \Delta_2, F_9} \&}_{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \underbrace{\frac{h_7 : \vdash \Delta_8, dual(F_5) \land h_7 : \vdash \Delta_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_8}}_{\bullet h_7 : \vdash \Delta_8, dual(F_5)} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_8, dual(F_5)}}_{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8}}_{\bullet h_7 : \vdash \Delta_2, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_8, dual(F_6)} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_8, dual(F_6)} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_9, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_9, dual(F_6)} \underbrace{\frac{Ax}{h_7 : \vdash \Delta_9, \Delta_9, dual(F_6)}}_{\bullet h_7 : \vdash \Delta_9, \Delta_9, dual(F_6)} \underbrace{\frac{Ax}{h_7 : \vdash \Delta$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \oplus_B \\ \hline - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash (\Delta_5, \mathbf{F}_4, \mathbf{F}_6) & \mathbf{ax} & \xrightarrow{\bullet} \mathbf{h}_7 : \vdash \Delta_{10}, dual(\mathbf{F}_6), \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_4, \mathbf{F}_8 \oplus \mathbf{F}_9 & \oplus_B \\ \hline \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \oplus_B \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 & \oplus_{\bullet} & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \oplus_B \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \xrightarrow{\bullet} & \xrightarrow{\bullet} & \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \xrightarrow{\bullet} & \xrightarrow{\bullet} & \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} & \mathbf{ax} \\ \hline & & \xrightarrow{\bullet} & \xrightarrow{\bullet} & \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_9 & \oplus_B} & \oplus_B \\ \hline \end{array}$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_4, \mathbf{F}_6 & \mathbf{v} & \bullet \mathbf{h}_7 : \vdash \Delta_{10}, dual(\mathbf{F}_6), \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_4, \mathbf{F}_8 \oplus \mathbf{F}_9 & \oplus B \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 & \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_2, \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 & \to \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \to \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \to \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 & \oplus \mathbf{F}_9 & \oplus \mathbf{h}_7 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 &$$

• Case rule \perp

$$\frac{\begin{array}{c} \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4 \\ \bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4 \end{array} \oplus_B \quad \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \bot, \Delta_8} \\ - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \bot, \Delta_8 \\ & \xrightarrow{\bullet} \\ \underline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, \mathbf{F}_6} \quad \mathbf{ax} \quad \xrightarrow{\bullet} \\ \underline{\mathbf{h}_7 : \vdash \Delta_8, \bot, dual(\mathbf{F}_6)} \\ & \xrightarrow{\bullet} \\ \underline{\mathbf{h}_7 : \vdash \Delta_5, \Delta_8, \mathbf{F}_4, \bot} \\ \underline{- : \vdash \Delta_5, \Delta_8, \bot, \mathbf{F}_3 \oplus \mathbf{F}_4} \quad \oplus_B \end{array} \quad \mathbf{hCut}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 \end{array} \oplus_B \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \bot, \Delta_8 \end{array} \\ - : \vdash \Delta_2, \bot, \Delta_8 \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \xrightarrow[\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ - : \vdash \Delta_2, \Delta_8 \\ \hline - : \vdash \Delta_2, \Delta_8, \bot \end{array} \xrightarrow[\mathbf{h}_7 : \vdash \Delta_8, \Delta_8]{} \begin{array}{c} \Delta \mathbf{x} \\ \mathbf{h} \mathbf{Cut} \end{array}$$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B \quad & \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \top, \Delta_8}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \top, \Delta_8} \ \\ & \xrightarrow{-} : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \top, \Delta_8 \\ & \xrightarrow{-} : \vdash \Delta_5, \Delta_8, \top, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \top \\ \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2} \ \oplus_B \quad & \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \top, \Delta_8 \\ & \xrightarrow{-} : \vdash \Delta_2, \top, \Delta_8 \\ & \xrightarrow{-} : \vdash \Delta_2, \Delta_8, \top} \ \top \\ \\ \mathbf{Cut} \end{array}$$

$\bullet\,$ Case rule I

• Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, F_6, F_4}{\bullet \mathbf{h}_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{11}, F_9, dual(F_6) \quad \mathbf{h}_7 : \vdash \Delta_8, F_{10}}{\bullet \mathbf{h}_7 : \vdash \Delta_{11}, F_9 \otimes F_{10}} \otimes \\ \hline - : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \\ \hline \frac{\mathbf{h}_2 : \vdash \Delta_5, F_4, F_6}{\bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10}} & \text{ax} \\ \hline - : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10} \\ \hline - : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4 & \oplus_B \\ \hline \frac{\mathbf{h}_2 : \vdash \Delta_5, F_6, F_4}{\bullet \mathbf{h}_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_8, F_9 \otimes F_{10}}{\bullet \mathbf{h}_7 : \vdash \Delta_{11}, F_9 \otimes F_{10}} \otimes \\ \hline - : \vdash (\Delta_5, F_3 \oplus F_4) & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_8, F_9 \otimes F_{10}}{\bullet \mathbf{h}_7 : \vdash \Delta_{11}, F_9 \otimes F_{10}} & \text{cut} \\ \hline - : \vdash (\Delta_5, F_8, H_4) & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_8, F_9 \otimes F_{10}}{\bullet \mathbf{h}_7 : \vdash \Delta_{11}, A_8, dual(F_6), F_9 \otimes F_{10}} & \text{ax} \\ \hline - : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10} & \oplus_B \\ \hline - : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4 & \oplus_B \\ \hline & \frac{\mathbf{h}_1 : \vdash \Delta_2, F_6}{\bullet \mathbf{h}_1 : \vdash F_5 \oplus F_6, \Delta_2} & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{11}, F_9, dual(F_5) \& dual(F_6)}{\bullet \mathbf{h}_7 : \vdash dual(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} & \text{cut} \\ \hline & - : \vdash \Delta_{2}, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} & \oplus_B \\ \hline \bullet \mathbf{h}_1 : \vdash F_5 \oplus F_6, \Delta_2 & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{11}, F_9, dual(F_5) \& dual(F_6)}{\bullet \mathbf{h}_7 : \vdash dual(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} & \text{cut} \\ \hline & - : \vdash \Delta_{11}, \Delta_2, F_9 & h_7 : \vdash \Delta_{11}, F_{10}, dual(F_6) & \text{ax} \\ \hline & - : \vdash \Delta_{11}, \Delta_2, F_9 & h_7 : \vdash \Delta_{11}, F_{10}, dual(F_6) & \otimes \\ \hline & - : \vdash \Delta_{11}, \Delta_2, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_{11}, \Delta_2, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_{21}, \Delta_8, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_{21}, \Delta_8, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_1, \Delta_2, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_1, \Delta_2, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_1, \Delta_2, F_9 \otimes F_{10} & \text{cut} \\ \hline & - : \vdash \Delta_1, \Delta_2, F_$$

5.9 Status of \bigoplus_A : OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} & \oplus_A & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \\ \hline \\ - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_9, ?\mathbf{F}_8 \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \oplus \mathbf{F}_4} & \xrightarrow{\mathbf{a}_7} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6)} \\ \hline \\ \frac{- : \vdash \Delta_5, \Delta_9, \mathbf{F}_3 \oplus \mathbf{F}_4}{- : \vdash \Delta_5, \Delta_9, ?\mathbf{F}_8, \mathbf{F}_3 \oplus \mathbf{F}_4} & W \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 & \oplus_A & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} & ?W \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \xrightarrow{\mathbf{a}_8} & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} & \text{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \xrightarrow{\mathbf{a}_8} & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} & \text{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \xrightarrow{\mathbf{a}_8} & \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) & \text{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{h}_9, ?\mathbf{F}_8 & \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) & \text{ax} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \Delta_9, ?\mathbf{F}_8 & \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) & \text{ax} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \Delta_9, ?\mathbf{F}_8 & \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) & \text{ax} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \Delta_9, ?\mathbf{F}_8 & \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) & \text{ax} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \Delta_9, ?\mathbf{F}_8 & \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) & \text{ax} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \Delta_9, ?\mathbf{F}_8 & \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6) & \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \oplus_A & \frac{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} ?C \\ \hline & - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_9, ?\mathbf{F}_8 \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \oplus \mathbf{F}_4 & \text{ax} & \xrightarrow{\bullet} \\ \hline & \frac{- : \vdash \Delta_5, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, \mathbf{F}_3 \oplus \mathbf{F}_4}{- : \vdash \Delta_5, \Delta_9, ?\mathbf{F}_8, \mathbf{F}_3 \oplus \mathbf{F}_4} ?C \\ \hline & \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 & \oplus_{\bullet} & \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \xrightarrow{\bullet} \\ \hline & \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8 \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_6) \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{h}_9, ?\mathbf{h}_9 \\ \hline & \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} & \oplus_A & \frac{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} & ?\\ \hline \\ \frac{- : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_9, ?\mathbf{F}_8}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \oplus \mathbf{F}_4} & \text{ax} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_6)} & \mathbf{hCut} \\ \hline \\ \frac{- : \vdash \Delta_5, \Delta_9, \mathbf{F}_8, \mathbf{F}_3 \oplus \mathbf{F}_4}{- : \vdash \Delta_5, \Delta_9, ?\mathbf{F}_8, \mathbf{F}_3 \oplus \mathbf{F}_4} ? & \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2} & \oplus_{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_2, \Delta_9, ?\mathbf{F}_8} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6} & \text{ax} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6} & \text{ax} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{h}_7) \& dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{h}_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_8 : \mathbf{h}_8$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A \ \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9} \ \\ \hline - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3 \oplus \mathbf{F}_4 \ \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_3 \oplus \mathbf{F}_4 \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_8 \$ \mathbf{F}_9, \mathbf{F}_3 \oplus \mathbf{F}_4 \ \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \ \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 \ \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 \ \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \ \\ \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9 \ \\ \hline - : \vdash \Delta_2, \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9 \ \\ \hline - : \vdash \Delta_2, \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \ \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \ \\ \hline - : \vdash \Delta_2, \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \ \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \ \\ \hline - : \vdash \Delta_1, \mathbf{h}_9, \mathbf{h}_9, \mathbf{h}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \ \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \ \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \ \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \ \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \ \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \ \\ \hline \end{array} \right] \ \mathbf{h}_{\mathbf{Cut}} \$$

• Case rule &

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, F_6, F_3}{\bullet \mathbf{h}_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_{A} & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, F_8, dual(F_6) \quad \mathbf{h}_7 : \vdash \Delta_{10}, F_8, \&F_9}{\bullet \mathbf{h}_7 : \vdash dual(F_6), \Delta_{10}, F_8 \&F_9} \\ \hline\\ & - : \vdash (\Delta_5, F_3, F_6) & \mathbf{ax} & \xrightarrow{\bullet} \\ \hline\\ & \frac{-}{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_3, F_6} & \mathbf{ax} & \xrightarrow{\bullet} \\ \hline\\ & - : \vdash \Delta_{10}, \Delta_5, F_8, \&F_9 \\ \hline\\ & - : \vdash \Delta_{10}, \Delta_5, F_8, \&F_9, F_3 \oplus F_4 & \oplus_{A} \\ \hline\\ & \frac{\mathbf{h}_1 : \vdash \Delta_2, F_5}{\bullet \mathbf{h}_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_{A} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_8 \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_2 : \vdash \Delta_2, \Delta_8 \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_2 : \vdash \Delta_2, A_8 \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_5 \oplus_{A} \\ \hline\\ & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_8 \\ \hline\\ & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_8 \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2, A_8 \\ \hline\\ & \bullet \mathbf{h}_1 : \vdash \Delta_2,$$

• Case rule \oplus_B

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} & \oplus_{\mathbf{A}} & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \oplus_{\mathbf{A}} \\ \hline - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 & \to \\ \hline \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, dual(\mathbf{F}_6), \mathbf{F}_8 \oplus \mathbf{F}_9} & \text{ax} \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9 & \oplus_{\mathbf{A}} \\ \hline - : \vdash \Delta_{10}, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4, \mathbf{F}_8 \oplus \mathbf{F}_9 & \oplus_{\mathbf{A}} \\ \hline \bullet_{\mathbf{h}_1} : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 & \oplus_{\mathbf{h}_7} : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet_{\mathbf{h}_1} : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 & \oplus_{\mathbf{h}_7} : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \bullet_{\mathbf{h}_1} : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \text{ax} & \to \\ \hline \bullet_{\mathbf{h}_1} : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \text{ax} & \to \\ \hline \bullet_{\mathbf{h}_1} : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \text{ax} & \to \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \oplus \mathbf{F}_9 & \oplus_{\mathbf{A}} \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \oplus \mathbf{F}_9 & \oplus_{\mathbf{A}} \\ \hline - : \vdash \Delta_{10}, \Delta_2, \mathbf{F}_8 \oplus \mathbf{F}_9 & \oplus_{\mathbf{A}} \\ \hline \end{array}$$

• Case rule \perp

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_6, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_6, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} & \oplus_A & \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \perp, \Delta_8} & \bot \\ \hline \\ - : \vdash (\Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4), \bot, \Delta_8 & \to \\ \hline \underline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_6} & \mathbf{ax} & \xrightarrow{\bullet} \\ \hline \underline{\mathbf{h}_7 : \vdash \Delta_8, \bot, dual(\mathbf{F}_6)} & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_5, \Delta_8, \mathbf{F}_3, \bot \\ \hline - : \vdash \Delta_5, \Delta_8, \bot, \mathbf{F}_3 \oplus \mathbf{F}_4 & \oplus_A \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_2 & \oplus_{\bullet} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \bot, \Delta_8 \\ \hline \\ - : \vdash \Delta_2, \bot, \Delta_8 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \xrightarrow{\bullet} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual$$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, F_6, F_3}{\bullet \mathbf{h}_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_A & \overline{\bullet \mathbf{h}_7 : \vdash dual(F_6), \top, \Delta_8} & \top \\ \hline - : \vdash (\Delta_5, F_3 \oplus F_4), \top, \Delta_8 & \rightarrow \\ \hline - : \vdash \Delta_5, \Delta_8, \top, F_3 \oplus F_4 & \top \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, F_5}{\bullet \mathbf{h}_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_A & \overline{\bullet \mathbf{h}_7 : \vdash dual(F_5 \oplus F_6), \top, \Delta_8} & \top \\ \hline - : \vdash \Delta_2, \top, \Delta_8 & \rightarrow \\ \hline - : \vdash \Delta_2, \Delta_8, \top & \top \\ \hline \end{array}$$

- \bullet Case rule I
- \bullet Case rule \otimes

5.10 Status of \perp : OK

 \bullet Case rule 1

$$\begin{array}{cccc} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \ \bot & \frac{\bullet}{\bullet \mathbf{h}_3 : \vdash dual(\bot), *} & \mathbf{1} \\ & - : \vdash \Delta_2, * \\ & \frac{\rightarrow}{- : \vdash \Delta_2} & \mathtt{ax} \end{array} \quad \mathtt{Cut}$$

- Case rule!
- Case rule ?W

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \perp & \frac{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_5}{\bullet \mathbf{h}_3 : \vdash dual(\bot), \Delta_5, ?F_4} \\ \hline - : \vdash \Delta_2, \Delta_5, ?F_4 \\ \hline \bullet \underline{\mathbf{h}_1 : \vdash \Delta_2, \bot} & \text{ax} & \frac{\rightarrow}{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_5} & \text{ax} \\ \hline - : \vdash \Delta_2, \Delta_5, ?F_4 & \text{hCut} \\ \hline \hline - : \vdash \Delta_2, \Delta_5, ?F_4 & W \\ \hline \bullet \underline{\mathbf{h}_2 : \vdash \Delta_3, F_4} & \bot & \frac{\mathbf{h}_5 : \vdash \Delta_7, dual(F_4)}{\bullet \mathbf{h}_5 : \vdash dual(F_4), \Delta_7, ?F_6} & ?W \\ \hline - : \vdash (\bot, \Delta_3), \Delta_7, ?F_6 & \xrightarrow{\bullet} \underline{\mathbf{h}_2 : \vdash \Delta_3, F_4, \bot} & \text{ax} & \xrightarrow{\bullet} \underline{\mathbf{h}_5 : \vdash \Delta_7, dual(F_4)} \\ \hline \bullet \underline{\mathbf{h}_2 : \vdash \Delta_3, F_4, \bot} & \text{ax} & \xrightarrow{\bullet} \underline{\mathbf{h}_5 : \vdash \Delta_7, dual(F_4)} & \text{ax} \\ \hline - : \vdash \Delta_3, \Delta_7, \bot \\ - : \vdash \Delta_3, \Delta_7, \bot, ?F_6 & W \\ \hline \end{array}$$

• Case rule ?C

• Case rule?

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \perp & \frac{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_5, \mathbf{F}_4}{\bullet \mathbf{h}_3 : \vdash dual(\bot), \Delta_5, ?\mathbf{F}_4} \\ \hline \\ - : \vdash \Delta_2, \Delta_5, ?\mathbf{F}_4 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \bot & \text{ax} & \frac{\rightarrow}{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_5, \mathbf{F}_4} & \text{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \bot & \text{ax} & \frac{\rightarrow}{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_5, \mathbf{F}_4} & \text{ax} \\ \hline \\ - : \vdash \Delta_2, \Delta_5, ?\mathbf{F}_4 & ? \\ \hline \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \mathbf{h}_3, \mathbf{h}_4 & \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} & ? \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{F}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4, \bot & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_7, \bot, ?\mathbf{h}_6 : \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_3 : \vdash \Delta_3, \Delta_7, \bot, ?\mathbf{h}_6 : \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_3 : \vdash \Delta_3, \Delta_7, \bot, ?\mathbf{h}_6 : \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_3 : \vdash \Delta_3, \Delta_7, \bot, ?\mathbf{h}_6 : \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_3 : \vdash \Delta_3, \Delta_7, \bot, ?\mathbf{h}_6 : \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_3 : \vdash \Delta_7, \mathbf{h}_7 : \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{h}_6, dual(\mathbf{h}_4)} \\ \hline \\ \bullet \mathbf{h}_3 : \vdash \Delta_7, \mathbf{h}_7 : \frac{\rightarrow}{\mathbf{h}_7 : \vdash \Delta_7, \mathbf{h}_7 : \mathbf{h}_7 : \frac{\rightarrow}{\mathbf{h}_7 : \vdash \Delta_7, \mathbf{h}_7 : \frac{\rightarrow}{\mathbf{h}_7 : \vdash \Delta_7, \mathbf{h}_7}$$

• Case rule \$

• Case rule &

$$\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \bot, \Delta_2} \ \bot \ \frac{h_3 : \vdash 1, \Delta_6, F_4 \ h_3 : \vdash 1, \Delta_6, F_5}{\bullet h_3 : \vdash dual(\bot), \Delta_6, F_4\&F_5} \ \&}{- : \vdash \Delta_2, \Delta_6, F_4\&F_5} \ \text{Cut}} \xrightarrow{\bullet h_1 : \vdash \Delta_2, \bot} \ \frac{\text{ax}}{h_3 : \vdash 1, \Delta_6, F_4} \ \frac{\rightarrow}{h_1 : \vdash \Delta_2, \bot} \ \frac{\text{ax}}{h_3 : \vdash 1, \Delta_6, F_5} \ \frac{\rightarrow}{h_1 : \vdash \Delta_2, \bot} \ \frac{\text{ax}}{h_3 : \vdash 1, \Delta_6, F_5} \ \frac{\text{ax}}{h_2 : \vdash \Delta_2, \Delta_6, F_4} \ \frac{\rightarrow}{h_2 : \vdash \Delta_2, \Delta_6, F_4} \ \frac{\rightarrow}{h_2 : \vdash \Delta_3, F_4} \ \bot \ \frac{h_5 : \vdash \Delta_8, F_6, dual(F_4) \ h_5 : \vdash \Delta_8, F_7, dual(F_4)}{\bullet h_5 : \vdash \Delta_8, F_7, dual(F_4)} \ \&}{- : \vdash (\bot, \Delta_3), \Delta_8, F_6\&F_7} \ \cot} \xrightarrow{\bullet h_5 : \vdash \Delta_8, dual(F_4), F_6\&F_7} \ \cot} \ \frac{\rightarrow}{h_2 : \vdash \Delta_3, F_4} \ \frac{\rightarrow}{h_5 : \vdash \Delta_8, dual(F_4), F_6\&F_7} \ \Delta_8, \int_{h_3 : \vdash \Delta_8, h_3} \ \Delta_8, \int_{h_3 : \vdash$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \ \bot \ \frac{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_6, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash dual(\bot), \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5} \\ \hline - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \\ \hline \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \bot}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_5} \ \frac{\to}{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_6, \mathbf{F}_5} \ \frac{\mathsf{ax}}{\mathsf{hCut}} \\ \hline - : \vdash \Delta_2, \Delta_6, \mathbf{F}_5 \\ \hline - : \vdash \Delta_2, \Delta_6, \mathbf{F}_5 \oplus B \end{array} \begin{array}{c} \mathsf{ax} \\ \mathsf{hCut} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \ \bot \ \frac{\mathsf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} \\ \hline - : \vdash (\bot, \Delta_3), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4, \bot \ \mathbf{ax} \ \frac{\to}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4)} \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4) \ \end{array} \begin{array}{c} \oplus_B \\ \mathsf{Cut} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4, \bot \ \mathbf{ax} \ \frac{\to}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4)} \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4) \ \end{array} \begin{array}{c} \mathsf{hCut} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_8, \mathbf{h}_7, \bot \ \\ \hline - : \vdash \Delta_3, \Delta_8, \mathbf{h}_7, \bot \ \end{array} \begin{array}{c} \mathsf{hCut} \\ \hline - : \vdash \Delta_3, \Delta_8, \mathbf{h}_7, \bot \ \end{array} \begin{array}{c} \mathsf{hCut} \\ \mathsf{hCut} \end{array}$$

• Case rule \bigoplus_A

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \perp & \frac{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_6, \mathbf{F}_4}{\bullet \mathbf{h}_3 : \vdash dual(\bot), \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5} \\ - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \\ \hline \bullet \underline{\bullet}_1 : \vdash \Delta_2, \bot & \text{ax} & \frac{\rightarrow}{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_6, \mathbf{F}_4} & \text{ax} \\ \hline - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 & \oplus A \\ \hline - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 & \oplus A \\ \hline \bullet \underline{\bullet}_1 : \vdash \mathbf{h}_2 : \vdash \Delta_3, \mathbf{h}_4 & \bot & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{h}_6, dual(\mathbf{f}_4)}{\bullet \mathbf{h}_5 : \vdash dual(\mathbf{f}_4), \Delta_8, \mathbf{h}_6 \oplus \mathbf{f}_7} & \mathbf{h}_5 \\ \hline \bullet \underline{\bullet}_1 : \vdash \mathbf{h}_3, \mathbf{h}_4, \bot & \mathbf{h}_5 : \vdash \mathbf{h}_3, \mathbf{h}_6, \mathbf{h}_4 \\ \hline \bullet \underline{\bullet}_1 : \vdash \Delta_3, \mathbf{h}_4, \bot & \mathbf{h}_5 : \vdash \mathbf{h}_8, \mathbf{h}_6, dual(\mathbf{f}_4) \\ \hline \bullet \underline{\bullet}_1 : \vdash \Delta_3, \mathbf{h}_4, \bot & \mathbf{h}_5 : \vdash \mathbf{h}_8, \mathbf{h}_6, dual(\mathbf{f}_4) \\ \hline \bullet \underline{\bullet}_1 : \vdash \Delta_3, \mathbf{h}_8, \mathbf{h}_6, \bot \\ \hline - : \vdash \Delta_3, \Delta_8, \mathbf{h}_8, \mathbf{h}_6, \bot \\ \hline - : \vdash \Delta_3, \Delta_8, \bot, \mathbf{h}_6 \oplus \mathbf{h}_7 & \oplus A \\ \hline \end{array} \quad \begin{array}{c} \mathbf{h}_2 : \vdash \mathbf{h}_3, \mathbf{h}_8, \mathbf{h}_6, \bot \\ \hline - : \vdash \Delta_3, \Delta_8, \bot, \mathbf{h}_6 \oplus \mathbf{h}_7 & \oplus A \\ \hline \end{array} \quad \mathbf{h}_6 : \mathbf{h}_4 : \mathbf{h}_5 : \mathbf{h}_4 : \mathbf{h}_6 : \mathbf{$$

• Case rule \perp

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \ \bot \ \frac{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_4}{\bullet \mathbf{h}_3 : \vdash dual(\bot), \bot, \Delta_4} \ \\ \hline - : \vdash \Delta_2, \bot, \Delta_4 \\ \hline \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \bot}{\bullet \mathbf{h}_3 : \vdash \mathbf{1}, \Delta_4} \ \frac{\mathbf{ax}}{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_4} \ \mathbf{hCut} \\ \hline \frac{- : \vdash \Delta_2, \Delta_4}{- : \vdash \Delta_2, \Delta_4, \bot} \ \bot \end{array}$$

$$\frac{\begin{array}{c} \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \\ \bullet \mathbf{h}_2 : \vdash \mathbf{F}_4, \bot, \Delta_3 \end{array} \bot \begin{array}{c} \mathbf{h}_5 : \vdash \Delta_6, dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 : \vdash dual(\mathbf{F}_4), \bot, \Delta_6 \end{array} }{\begin{array}{c} \vdash (\bot, \Delta_3), \bot, \Delta_6 \\ \bullet \mathbf{h}_5 : \vdash \Delta_6, \bot, dual(\mathbf{F}_4) \end{array}} \begin{array}{c} \bot \\ \mathsf{Cut} \\ \bullet \mathsf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \end{array} \begin{array}{c} \mathbf{ax} \\ \bullet \mathsf{h}_5 : \vdash \Delta_6, \bot, dual(\mathbf{F}_4) \\ \bullet \mathsf{h}_5 : \vdash \Delta_6, \bot, dual(\mathbf{F}_4) \end{array} \begin{array}{c} \mathsf{ax} \\ \mathsf{hCut} \\ \bullet \mathsf{cut} \end{array}$$

\bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \ \bot \ \hline \bullet \mathbf{h}_3 : \vdash dual(\bot), \top, \Delta_4 \\ \hline - : \vdash \Delta_2, \top, \Delta_4 \\ \hline - : \vdash \Delta_2, \Delta_4, \top \end{array} \\ \top \\ \frac{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{f}_4}{\bullet \mathbf{h}_2 : \vdash \mathbf{f}_4, \bot, \Delta_3} \ \bot \ \hline \bullet \mathbf{h}_5 : \vdash dual(\mathbf{f}_4), \top, \Delta_6 \\ \hline - : \vdash (\bot, \Delta_3), \top, \Delta_6 \\ \hline - : \vdash \Delta_3, \Delta_6, \bot, \top \end{array} \\ \top \\ \mathbf{Cut}$$

- ullet Case rule I
- \bullet Case rule \otimes

5.11 Status of \top : OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\begin{array}{c|c} & \mathbf{h_3} : \vdash \mathbf{0}, \Delta_5 \\ \hline \bullet \mathbf{h_1} : \vdash \top, \Delta_2 & \top & \bullet \mathbf{h_3} : \vdash dual(\top), \Delta_5, ?F_4 \\ \hline & -: \vdash \Delta_2, \Delta_5, ?F_4 \\ \hline \bullet \mathbf{h_1} : \vdash \Delta_2, \top, ?F_4 & \top & \mathbf{h_3} : \vdash \mathbf{0}, \Delta_5 \\ \hline & \bullet \mathbf{h_1} : \vdash \Delta_2, \top, ?F_4 & \top \\ \hline & \bullet \mathbf{h_1} : \vdash \Delta_2, \top, ?F_4 & \top \\ \hline & \bullet \mathbf{h_2} : \vdash F_4, \top, \Delta_3 & \top & \bullet \mathbf{h_5} : \vdash \Delta_7, dual(F_4) \\ \hline & \bullet \mathbf{h_5} : \vdash dual(F_4), \Delta_7, ?F_6 \\ \hline & -: \vdash (\top, \Delta_3), \Delta_7, ?F_6 & \top \\ \hline & -: \vdash \Delta_3, \Delta_7, \top, ?F_6 & \top \\ \hline \end{array}$$

 \bullet Case rule ?C

$$\begin{array}{c|c} \frac{\bullet_{\mathbf{h}_1} : \vdash \top, \Delta_2}{\bullet_{\mathbf{h}_3} : \vdash \mathbf{0}, \Delta_5, ?\mathsf{F}_4, ?\mathsf{F}_4} & ?C \\ \hline - : \vdash \Delta_2, \Delta_5, ?\mathsf{F}_4 \\ \hline \bullet_{\mathbf{h}_1} : \vdash \Delta_2, \top & \top & \frac{\rightarrow}{\mathbf{h}_3} : \vdash \mathbf{0}, \Delta_5, ?\mathsf{F}_4 \\ \hline \bullet_{\mathbf{h}_1} : \vdash \Delta_2, \top & \top & \frac{\rightarrow}{\mathbf{h}_3} : \vdash \mathbf{0}, \Delta_5, ?\mathsf{F}_4, ?\mathsf{F}_4 \\ \hline - : \vdash \Delta_2, \Delta_5, ?\mathsf{F}_4, ?\mathsf{F}_4 \\ \hline - : \vdash \Delta_2, \Delta_5, ?\mathsf{F}_4, ?\mathsf{F}_4 \\ \hline - : \vdash \Delta_2, \Delta_5, ?\mathsf{F}_4 \\ \hline \hline \bullet_{\mathbf{h}_5} : \vdash \Delta_7, ?\mathsf{F}_6, ?\mathsf{F}_6, dual(\mathsf{F}_4) \\ \hline \bullet_{\mathbf{h}_5} : \vdash dual(\mathsf{F}_4), \Delta_7, ?\mathsf{F}_6 \\ \hline - : \vdash (\top, \Delta_3), \Delta_7, ?\mathsf{F}_6 \\ \hline - : \vdash \Delta_3, \Delta_7, \top, ?\mathsf{F}_6 \\ \hline \end{array} \begin{array}{c} ?C \\ \mathsf{Cut} \\ \hline \end{array}$$

• Case rule ?

• Case rule \$

\bullet Case rule &

$$\frac{\underbrace{\bullet_{h_1} : \vdash \top, \Delta_2} \ \top \ \frac{h_3 : \vdash 0, \Delta_6, F_4 \quad h_3 : \vdash 0, \Delta_6, F_5}{\bullet h_3 : \vdash dual(\top), \Delta_6, F_4\&F_5} \ \&}_{- : \vdash \Delta_2, \Delta_6, F_4\&F_5} \ \text{Cut}} \xrightarrow{\bullet_{h_1} : \vdash \Delta_2, \top} \ \top \ \frac{\rightarrow}{h_3 : \vdash 0, \Delta_6, F_4} \ \text{ax}}_{- : \vdash \Delta_2, \Delta_6, F_4} \ \frac{\rightarrow}{h_3 : \vdash 0, \Delta_6, F_5} \ \&}_{- : \vdash \Delta_2, \Delta_6, F_4} \ \frac{\rightarrow}{h_3 : \vdash 0, \Delta_6, F_5} \ \&}_{- : \vdash \Delta_2, \Delta_6, F_5} \ \&}$$

$$\underbrace{\bullet_{h_1} : \vdash \Delta_2, \top}_{- : \vdash \Delta_2, \Delta_6, F_4} \ \top \ \frac{\rightarrow}{h_3 : \vdash \Delta_8, F_6, dual(F_4) \quad h_5 : \vdash \Delta_8, F_7, dual(F_4)}_{- : \vdash (\top, \Delta_3), \Delta_8, F_6\&F_7} \ Cut}_{- : \vdash (\top, \Delta_3), \Delta_8, F_6\&F_7} \ \top$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{ \begin{array}{c} \mathbf{h}_3 : \vdash \mathbf{0}, \Delta_6, \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_3 : \vdash \mathbf{d}ual(\top), \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \\ \hline \\ - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \\ \hline \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \top \end{array} \begin{array}{c} \mathbf{a}x \\ \hline \\ - : \vdash \Delta_2, \Delta_6, \mathbf{F}_5 \\ \hline \\ \hline \\ - : \vdash \Delta_2, \Delta_6, \mathbf{F}_5 \\ \hline \\ - : \vdash \Delta_2, \Delta_6, \mathbf{F}_5 \end{array} \begin{array}{c} \mathbf{a}x \\ \mathbf{h}\mathbf{C}ut \\ \hline \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \mathbf{F}_4, \top, \Delta_3 \end{array} \begin{array}{c} \mathbf{T} \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4) \\ \hline \\ \bullet \mathbf{h}_5 : \vdash dual(\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \\ - : \vdash (\top, \Delta_3), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \top, \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \begin{array}{c} \oplus_B \\ \mathbf{C}ut \\ \hline \end{array}$$

• Case rule \oplus_A

$$\begin{array}{c} \underbrace{\begin{array}{c} \bullet_{h_1} : \vdash \top, \Delta_2 \end{array} \top \begin{array}{c} \bullet_{h_3} : \vdash \mathbf{0}, \Delta_6, \mathbf{F}_4 \\ \bullet_{h_3} : \vdash dual(\top), \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \end{array}}_{\bullet h_3 : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5} \begin{array}{c} \oplus_A \\ \mathsf{Cut} \end{array}} \\ \\ \underbrace{\begin{array}{c} - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \\ \bullet_{h_1} : \vdash \Delta_2, \top \end{array} \begin{array}{c} \to \\ h_3 : \vdash \mathbf{0}, \Delta_6, \mathbf{F}_4 \\ \hline - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \end{array}}_{\bullet h_3} \begin{array}{c} \mathsf{ax} \\ \mathsf{hCut} \end{array}} \\ \underbrace{\begin{array}{c} - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \\ \hline - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \end{array}}_{\bullet h_5 : \vdash \Delta_8, \mathbf{F}_6, dual(\mathbf{F}_4)} \\ \bullet_{h_5} : \vdash dual(\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array}}_{\bullet L} \begin{array}{c} \oplus_A \\ \mathsf{Cut} \\ \underbrace{\begin{array}{c} \bullet_{h_2} : \vdash \mathbf{F}_4, \top, \Delta_3 \end{array}}_{- : \vdash (\top, \Delta_3), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} \end{array}}_{\bullet L} \begin{array}{c} \oplus_A \\ \mathsf{Cut} \end{array}$$

• Case rule \perp

$$\begin{array}{c} \underbrace{\begin{array}{c} \bullet_{h_1} : \vdash \top, \Delta_2 \end{array} \top \begin{array}{c} h_3 : \vdash \mathbf{0}, \Delta_4 \\ \bullet_{h_3} : \vdash dual(\top), \bot, \Delta_4 \end{array}}_{\bullet h_3 : \vdash \Delta_2, \bot, \Delta_4} \begin{array}{c} \bot \\ \bullet_{h_1} : \vdash \Delta_2, \bot, \Delta_4 \end{array} \begin{array}{c} \mathsf{Cut} \\ \\ \bullet_{h_1} : \vdash \Delta_2, \bot, \top \end{array} \begin{array}{c} \to \\ h_3 : \vdash \mathbf{0}, \Delta_4 \end{array} \begin{array}{c} \mathsf{ax} \\ \mathsf{hCut} \\ \\ - : \vdash \Delta_2, \Delta_4, \bot \end{array} \\ \\ \underbrace{\begin{array}{c} \bullet_{h_1} : \vdash F_4, \top, \Delta_3 \end{array}}_{\bullet h_2 : \vdash F_4, \top, \Delta_3} \begin{array}{c} \top \\ \bullet_{h_5} : \vdash \Delta_6, dual(F_4) \\ \bullet_{h_5} : \vdash dual(F_4), \bot, \Delta_6 \end{array} \begin{array}{c} \bot \\ \mathsf{Cut} \\ \\ - : \vdash (\top, \Delta_3), \bot, \Delta_6 \\ \\ \hline - : \vdash \Delta_3, \Delta_6, \bot, \top \end{array} \begin{array}{c} \top \end{array} \begin{array}{c} \mathsf{Cut} \end{array}$$

\bullet Case rule \top

ullet Case rule I

$\bullet \;$ Case rule \otimes

5.12 Status of I: OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W

 \bullet Case rule ?C

• Case rule?

• Case rule \$

• Case rule &

• Case rule \oplus_B

$$\frac{ \underbrace{\bullet_{h_1} :\vdash p(n_3), \, \hat{\ }_{(n_3)} } }_{\bullet h_1 :\vdash p(n_3), \, \hat{\ }_{(n_3)} } I \underbrace{ \begin{array}{c} h_4 :\vdash \Delta_7, F_6, \, \hat{\ }_{(n_3)} \\ \bullet_{h_4} :\vdash dual(p(n_3)), \Delta_7, F_5 \oplus F_6 \\ \hline \\ - :\vdash \hat{\ }_{(n_3)}, \Delta_7, F_5 \oplus F_6 \\ \hline \\ - :\vdash \Delta_7, F_6, \, \hat{\ }_{(n_3)} \\ \hline \\ - :\vdash \Delta_7, \, \hat{\ }_{(n_3)}, F_5 \oplus F_6 \\ \hline \\ \bullet_{h_1} :\vdash \hat{\ }_{(n_3)}, p(n_3) \end{array} }_{\bullet h_4 :\vdash \Delta_7, F_6, \, p(n_3)} \underbrace{ \begin{array}{c} \oplus_B \\ \\ \oplus_{h_1} :\vdash \hat{\ }_{(n_3)}, p(n_3) \\ \hline \\ - :\vdash p(n_3), \Delta_7, F_5 \oplus F_6 \\ \hline \\ - :\vdash \Delta_7, F_6, p(n_3) \\ \hline \\ - :\vdash \Delta_7, F_6, p(n_3) \\ \hline \\ - :\vdash \Delta_7, F_6, p(n_3) \\ \hline \\ - :\vdash \Delta_7, p(n_3), F_5 \oplus F_6 \\ \hline \end{array} }_{\bullet B}$$

• Case rule \oplus_A

• Case rule \perp

$$\begin{array}{c|c} \bullet_{\mathbf{h}_1} : \vdash p(\mathbf{n}_3), \, \hat{\ } (\mathbf{n}_3) & I & \frac{\mathbf{h}_4 : \vdash \Delta_5, \, \hat{\ } (\mathbf{n}_3)}{\bullet \mathbf{h}_4 : \vdash dual(p(\mathbf{n}_3)), \, \bot, \, \Delta_5} & \bot \\ & & & \\ & & & \\ & & & \\ \hline & & & - : \vdash \hat{\ } (\mathbf{n}_3), \, \bot, \, \Delta_5} & \\ & & & & \\ \hline & & & - : \vdash \Delta_5, \, \hat{\ } (\mathbf{n}_3)} & \bot & \\ \hline & & & & - : \vdash \Delta_5, \, \bot, \, \hat{\ } (\mathbf{n}_3)} & \bot & \\ \hline & & & & \\ \hline & & \bullet_{\mathbf{h}_1} : \vdash \hat{\ } (\mathbf{n}_3), \, p(\mathbf{n}_3) & I & \frac{\mathbf{h}_4 : \vdash \Delta_5, \, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 : \vdash dual(\hat{\ } (\mathbf{n}_3)), \, \bot, \, \Delta_5} & \bot \\ & & & & \\ \hline & & & & - : \vdash p(\mathbf{n}_3), \, \bot, \, \Delta_5} & \\ & & & & & \\ \hline & & & & - : \vdash \Delta_5, \, p(\mathbf{n}_3) & \bot \\ \hline & & & & - : \vdash \Delta_5, \, L, \, p(\mathbf{n}_3) & \bot \\ \hline & & & & - : \vdash \Delta_5, \, \bot, \, p(\mathbf{n}_3) & \bot \\ \hline \end{array}$$

\bullet Case rule \top

$$\begin{array}{c|c} \hline { \bullet \mathbf{h}_1 : \vdash p(\mathbf{n}_3), \hat{\ } (\mathbf{n}_3) } & I & \bullet \mathbf{h}_4 : \vdash dual(p(\mathbf{n}_3)), \top, \Delta_5 \\ \hline & - : \vdash \hat{\ } (\mathbf{n}_3), \top, \Delta_5 \\ \hline & - : \vdash \Delta_5, \top, \hat{\ } (\mathbf{n}_3) \\ \hline \hline { \bullet \mathbf{h}_1 : \vdash \hat{\ } (\mathbf{n}_3), p(\mathbf{n}_3) } & I & \bullet \mathbf{h}_4 : \vdash dual(\hat{\ } (\mathbf{n}_3)), \top, \Delta_5 \\ \hline & - : \vdash p(\mathbf{n}_3), \top, \Delta_5 \\ \hline & - : \vdash p(\mathbf{n}_3), \top, \Delta_5 \\ \hline & - : \vdash \Delta_5, \top, p(\mathbf{n}_3) \\ \hline \end{array} \right. \end{array}$$

\bullet Case rule I

$$\begin{array}{c|c} \hline { \bullet \mathbf{h}_1 : \vdash p(\mathbf{n}_4), \widehat{}(\mathbf{n}_4) } & I & \hline {\bullet \mathbf{h}_3 : \vdash dual(p(\mathbf{n}_4)), p(\mathbf{n}_4) } \\ & & - : \vdash \widehat{}(\mathbf{n}_4), p(\mathbf{n}_4) \\ \hline & & - : \vdash p(\mathbf{n}_4), \widehat{}(\mathbf{n}_4) \\ \hline {\bullet \mathbf{h}_1 : \vdash \widehat{}(\mathbf{n}_4), p(\mathbf{n}_4) } & I & \hline {\bullet \mathbf{h}_3 : \vdash dual(\widehat{}(\mathbf{n}_4)), \widehat{}(\mathbf{n}_4) } \\ \hline {\bullet \mathbf{h}_1 : \vdash \widehat{}(\mathbf{n}_4), p(\mathbf{n}_4) } & I & \hline {\bullet \mathbf{h}_3 : \vdash dual(\widehat{}(\mathbf{n}_4)), \widehat{}(\mathbf{n}_4) } & Cut \\ \hline & - : \vdash p(\mathbf{n}_4), \widehat{}(\mathbf{n}_4) \\ \hline & - : \vdash p(\mathbf{n}_4), \widehat{}(\mathbf{n}_4) \end{array}$$

\bullet Case rule \otimes

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \vdash p(n_3), \, \hat{\ } (n_3) \\ \bullet}_{h_4} : \vdash \Delta_8, F_6, \, \hat{\ } (n_3) \\ \bullet_{h_4} : \vdash dual(p(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7 \\ \hline \\ - : \vdash \hat{\ } (n_3), \Delta_5, \Delta_8, F_6 \otimes F_7 \\ \hline \\ - : \vdash \Delta_8, F_6, \, \hat{\ } (n_3) \\ \hline \\ - : \vdash \Delta_5, \Delta_8, \, \hat{\ } (n_3), F_6 \otimes F_7 \\ \hline \\ \bullet_{h_1} : \vdash p(n_3), \, \hat{\ } (n_3) \\ \hline \\ \bullet_{h_4} : \vdash dual(p(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7 \\ \hline \\ \bullet_{h_4} : \vdash dual(p(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7 \\ \hline \\ - : \vdash \hat{\ } (n_3), \Delta_5, \Delta_8, F_6 \otimes F_7 \\ \hline \\ - : \vdash \Delta_5, F_6 \\ \hline \\ \bullet \\ - : \vdash \Delta_5, F_6 \\ \hline \\ - : \vdash \Delta_5, F_6 \\ \hline \\ - : \vdash \Delta_5, A_8, \, \hat{\ } (n_3), F_6 \otimes F_7 \\ \hline \\ \hline \\ - : \vdash \Delta_5, F_6 \\ \hline \\ - : \vdash \Delta_5, A_8, \, \hat{\ } (n_3), F_6 \otimes F_7 \\ \hline \\ \hline \end{array} \right]$$

5.13 Status of \otimes : OK

- \bullet Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} \\ \hline & - : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5), \Delta_{10}, ?\mathbf{F}_9 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_7)} \mathbf{hCut} \\ \hline & - : \vdash \Delta_{10}, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & - : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \mathbf{h}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_2, \Delta_3 \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline & \bullet \mathbf{h}_3 : \vdash \Delta_{10}, du$$

• Case rule ?C

$$\frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} \underbrace{ ?C}_{\mathsf{Cut}} \\ - : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5), \Delta_{10}, ?\mathbf{F}_9} \xrightarrow{\bullet}_{\mathsf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_7)} \underbrace{ ?C}_{\mathsf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_7)} \\ - : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathbf{F}_9, ?\mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5} \underbrace{ ?C}_{\mathsf{h}_8 : \vdash \Delta_{10}, 2, 2, 2, 3, 2, 4, 2, 5, 5} ?C}$$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \otimes \quad \frac{\mathbf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} \\ \hline - : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5), \Delta_{10}, ?\mathbf{F}_9 \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \mathbf{ax} \quad \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_7)} \quad \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathbf{F}_9, ?\mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5} \\ \hline - : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathbf{F}_9, ?\mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad ?C \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_2, \Delta_3} \otimes \quad \begin{array}{c} \mathbf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline \bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9 \\ \hline - : \vdash (\Delta_2, \Delta_3), \Delta_{10}, ?\mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathbf{F}_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} \\ \hline & - : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5), \Delta_{10}, ?\mathbf{F}_9 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \xrightarrow{\mathbf{ax}} & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)} \\ \hline & \frac{- : \vdash \Delta_{10}, \Delta_3, \Delta_6, \mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5}{- : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5} ? \\ \hline \\ \frac{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_5}{- : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} ? \\ \hline \\ \frac{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5}{- : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5)} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} ? \\ \hline \\ \frac{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5}{- : \vdash \Delta_{10}, \Delta_3, \Delta_6, \mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5} ? & \frac{\mathbf{ax}}{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{10}, \Delta_2, \Delta_3, \mathbf{F}_9 ? ? \\ \hline \\ \bullet \mathbf{h}_3 : \vdash \Delta_{10}, \mathbf{h}_2, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_2, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_2, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_2, \mathbf{h}_2, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_2, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_2, \mathbf{h}_2, \mathbf{h}_2$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, \mathbf{F}_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \$ \mathbf{F}_{10}} \\ & - : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5), \Delta_{11}, \mathbf{F}_9 \$ \mathbf{F}_{10} \\ & \rightarrow \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \frac{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_9 \$ \mathbf{F}_{10}, \mathbf{F}_4 \otimes \mathbf{F}_5} & \$ \\ \hline & \frac{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_5}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_9 \$ \mathbf{F}_{10}, \mathbf{F}_4 \otimes \mathbf{F}_5} & \$ \\ \hline & \frac{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_5}{- : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5)} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, \mathbf{F}_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \$ \mathbf{F}_{10}} & \$ \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_3,$$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7}{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_2, \Delta_3} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, \mathbf{F}_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \$ \mathbf{F}_{10}} \overset{\$}{\text{Cut}} \\ \hline - : \vdash (\Delta_2, \Delta_3), \Delta_{11}, \mathbf{F}_9 \$ \mathbf{F}_{10} & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7 & \text{ax} & \rightarrow \\ \hline - : \vdash \Delta_{11}, \Delta_2, \Delta_3, \mathbf{F}_{10}, \mathbf{F}_9 & \text{aval}(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline - : \vdash \Delta_{11}, \Delta_2, \Delta_3, \mathbf{F}_{10}, \mathbf{F}_9 & \$ \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 & \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_2, \Delta_3 & \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6), dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \Delta_9} & \$ \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_2, \Delta_3 & \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6), dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \Delta_9} & \$ \\ \hline - : \vdash (\Delta_2, \Delta_3), \Delta_9 & & \rightarrow \\ \hline - : \vdash \Delta_2, \mathbf{F}_6 & & \rightarrow \\ \hline - : \vdash \Delta_3, \mathbf{F}_7 & \text{ax} & & \rightarrow \\ \hline - : \vdash \Delta_9, dual(\mathbf{F}_6), dual(\mathbf{F}_7) & \text{ax} \\ \hline - : \vdash \Delta_2, \Delta_3, \Delta_9, dual(\mathbf{F}_6) & \text{sCut} \\ \hline \end{array}$$

• Case rule &

$$\frac{\mathbf{h}_{2} : \vdash \Delta_{6}, \mathbf{F}_{7}, \mathbf{F}_{4} \quad \mathbf{h}_{2} : \vdash \Delta_{3}, \mathbf{F}_{5}}{\bullet \mathbf{h}_{2} : \vdash \mathbf{F}_{7}, \Delta_{3}, \Delta_{6}, \mathbf{F}_{4} \otimes \mathbf{F}_{5}} \otimes \frac{\mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{9}, dual(\mathbf{F}_{7}) \quad \mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_{7})}{\bullet \mathbf{h}_{8} : \vdash dual(\mathbf{F}_{7}), \Delta_{11}, \mathbf{F}_{9} \& \mathbf{F}_{10}} \\ - : \vdash (\Delta_{3}, \Delta_{6}, \mathbf{F}_{4} \otimes \mathbf{F}_{5}), \Delta_{11}, \mathbf{F}_{9} \& \mathbf{F}_{10} \\ - : \vdash \Delta_{11}, \Delta_{6}, \mathbf{F}_{4}, \mathbf{F}_{7} \quad \mathbf{ax} \quad \bullet \mathbf{h}_{8} : \vdash \Delta_{11}, dual(\mathbf{F}_{7}), \mathbf{F}_{9} \& \mathbf{F}_{10} \\ - : \vdash \Delta_{11}, \Delta_{6}, \mathbf{F}_{4}, \mathbf{F}_{9} \& \mathbf{F}_{10} \\ - : \vdash \Delta_{11}, \Delta_{3}, \Delta_{6}, \mathbf{F}_{9} \& \mathbf{F}_{10}, \mathbf{F}_{4} \otimes \mathbf{F}_{5} \\ - : \vdash \Delta_{11}, \Delta_{3}, \Delta_{6}, \mathbf{F}_{9} \& \mathbf{F}_{10}, \mathbf{F}_{4} \otimes \mathbf{F}_{5} \\ \bullet \mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{9}, dual(\mathbf{F}_{7}) \quad \mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_{7}) \\ \bullet \mathbf{h}_{8} : \vdash \mathbf{h}_{3}, \mathbf{F}_{4} \quad \mathbf{h}_{2} : \vdash \mathbf{h}_{3}, \Delta_{6}, \mathbf{F}_{4} \otimes \mathbf{F}_{5} \\ \bullet \mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{9}, dual(\mathbf{F}_{7}), \Delta_{11}, \mathbf{F}_{9} \& \mathbf{F}_{10} \\ - : \vdash (\Delta_{3}, \Delta_{6}, \mathbf{F}_{4} \otimes \mathbf{F}_{5}), \Delta_{11}, \mathbf{F}_{9} \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_{8} : \vdash \Delta_{11}, \Delta_{3}, \Delta_{6}, \mathbf{F}_{4} \otimes \mathbf{F}_{5}, \mathbf{F}_{7} \quad \bullet \mathbf{h}_{8} : \vdash \Delta_{11}, dual(\mathbf{F}_{7}), \mathbf{F}_{9} \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_{8} : \vdash \Delta_{11}, \Delta_{3}, \Delta_{6}, \mathbf{F}_{9} \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_{8} : \vdash \Delta_{11}, \Delta_{3}, \Delta_{6}, \mathbf{F}_{9} \& \mathbf{F}_{10} \otimes \mathbf{h}_{8} : \vdash \Delta_{11}, dual(\mathbf{F}_{7}), \mathbf{F}_{9} \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_{1} : \vdash \Delta_{2}, \mathbf{F}_{6} \quad \mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{7} \quad \bullet \mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{9}, dual(\mathbf{F}_{7}) \& \mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_{6}) \& \mathbf{h}_{20} \\ \bullet \mathbf{h}_{1} : \vdash \mathbf{h}_{2}, \Delta_{3}, \mathbf{F}_{6} \otimes \mathbf{F}_{7} \quad \mathbf{ax} \quad \bullet \mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{9}, dual(\mathbf{F}_{7}) \& \mathbf{h}_{20} \\ \bullet \mathbf{h}_{1} : \vdash \Delta_{2}, \Delta_{3}, \mathbf{F}_{6} \otimes \mathbf{F}_{7} \quad \mathbf{ax} \quad \bullet \mathbf{h}_{1} : \vdash \Delta_{2}, \Delta_{3}, \mathbf{F}_{9} \& \mathbf{h}_{20} \\ \bullet \mathbf{h}_{1} : \vdash \Delta_{2}, \Delta_{3}, \mathbf{F}_{6} \otimes \mathbf{F}_{7} \quad \mathbf{ax} \quad \bullet \mathbf{h}_{1} : \vdash \Delta_{2}, \Delta_{3}, \mathbf{F}_{6} \otimes \mathbf{F}_{7} \quad \mathbf{h}_{8} : \vdash \Delta_{11}, \Delta_{2}, \Delta_{3}, \mathbf{F}_{10} \\ \bullet \mathbf{h}_{1} : \vdash \Delta_{2}, \Delta_{3}, \mathbf{F}_{9} \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_{1} : \vdash \Delta_{2}, \Delta_{3}, \mathbf{F}_{9} \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_{1} : \vdash \Delta_{2}, \Delta_{3}, \mathbf{F}_{9} \& \mathbf$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \mathbf{Cut} \\ \hline \\ - : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10} & \rightarrow \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7)} & \mathbf{hCut} \\ \hline \\ - : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_4 \otimes \mathbf{F}_5 & \oplus \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7) \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 & \bullet \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7) \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \mathbf{h}_3, \Delta_6, \mathbf{h}_4 \otimes \mathbf{h}_5 & \bullet \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{h}_{10}, dual(\mathbf{F}_7) \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \bullet \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7) \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \bullet \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7) \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \bullet \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7) \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \bullet \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7) \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline \\ - : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline \\ - : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5, \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline \\ - : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5, \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5, \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_8 \otimes \mathbf{F}_5 \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_8 \otimes \mathbf{F}_5 \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_8 \otimes \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_2 : \vdash \Delta_1, \Delta_1, \Delta_2, \Delta_6, \mathbf{F}_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline$$

$$\frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7}{\underbrace{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_2, \Delta_3}} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_6)\$dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10}} \\ - : \vdash (\Delta_2, \Delta_3), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \xrightarrow{\bullet} \underbrace{\mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7} \quad \underbrace{\mathbf{ax}} \quad \frac{\rightarrow}{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_6)\$dual(\mathbf{F}_7)}_{\bullet \mathbf{h}_2 : \vdash \Delta_{11}, \Delta_2, \Delta_3, \mathbf{F}_{10}} \quad \underbrace{\mathbf{h}_{Cut}} \quad \underbrace{\mathbf{h}_{Cut}}$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \oplus_A \\ \hline - : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10} & \text{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5} & \text{ax} & \frac{\rightarrow}{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, dual(\mathbf{F}_7)} & \text{hCut} \\ \hline - : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5 & \text{odd}(\mathbf{F}_7) & \oplus_A \\ \hline - : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5, \mathbf{F}_9 \oplus \mathbf{F}_{10} & \oplus_A \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_5 & \text{odd}(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9, dual(\mathbf{F}_7) & \oplus_{\mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10}} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 & \text{odd}(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \text{odd}(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \text{odd}(\mathbf{F}_7), \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, dual(\mathbf{F}_7) & \text{hCut} \\ \hline - : \vdash \Delta_{11}, \Delta_3, \Delta_6, \mathbf{F}_9, \mathbf{F}_4 \otimes \mathbf{F}_5 & \oplus_{\mathbf{h}_3} : \vdash \Delta_{11}, \mathbf{F}_9, dual(\mathbf{F}_6) \otimes dual(\mathbf{F}_7) \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 & \text{odd}(\mathbf{F}_7, \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, dual(\mathbf{F}_6) \otimes dual(\mathbf{F}_7) \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_2, \Delta_3, \mathbf{h}_6 \otimes \mathbf{F}_7 & \text{odd}(\mathbf{h}_7, \mathbf{h}_7, \mathbf{h$$

• Case rule \perp

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \bot, \Delta_9} \xrightarrow{\mathbf{Cut}} \\ \hline - : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5), \bot, \Delta_9 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \mathbf{ax} \\ \hline - : \vdash \Delta_3, \Delta_6, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline - : \vdash \Delta_3, \Delta_6, \Delta_9, \bot, \mathbf{F}_4 \otimes \mathbf{F}_5 & \bot \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \otimes \mathbf{F}_5 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_3 : \vdash \Delta_9, \mathbf{dual}(\mathbf{F}_7), \bot, \Delta_9 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_4 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 \quad \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7 \quad \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_2, \Delta_3 & \mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline \bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \bot, \Delta_9 & \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \bot, \Delta_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \Delta_9, \bot & \mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_2, \Delta_3, \Delta_9, \bot & \mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \Delta_9, \bot & \mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) & \mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7) \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, \Delta_9, \bot & \mathbf{h}_1 : \vdash \Delta_9, \Delta_9, \Delta_9, \bot \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_9, \Delta_9, \Delta_9, \bot & \mathbf{h}_1 : \vdash \Delta_9, \Delta_9, \Delta_9, \bot$$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes & \frac{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \top, \Delta_9}{\bullet \mathbf{h}_8 : \vdash \mathbf{f}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} & \top \\ \hline \\ - : \vdash (\Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5), \top, \Delta_9 \\ \hline \\ - : \vdash \Delta_3, \Delta_6, \Delta_9, \top, \mathbf{F}_4 \otimes \mathbf{F}_5} & \top \\ \hline \\ \frac{\mathbf{h}_2 : \vdash \Delta_3, \mathbf{F}_4 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \mathbf{F}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes & \frac{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \top, \Delta_9}{\bullet \mathbf{h}_8 : \vdash \mathbf{f}_7, \Delta_3, \Delta_6, \mathbf{F}_4 \otimes \mathbf{F}_5} & \top \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \top, \mathbf{F}_4 \otimes \mathbf{F}_5} & \top \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_7}{- : \vdash (\Delta_2, \Delta_3), \top, \Delta_9} & \frac{\top}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \top, \Delta_9} & \top \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_2, \mathbf{h}_3, \Delta_9, \top, \Delta_9}{- : \vdash \Delta_2, \Delta_3, \Delta_9, \top} & \top \\ \hline \end{array}$$

ullet Case rule I

• Case rule ⊗

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_6, F_7, F_4}{\bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \\ \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline - : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5) \\ \hline & \bullet \mathbf{h}_3 : \vdash \mathbf{h}_{al}(F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \\ \hline & \bullet \mathbf{h}_2 : \vdash A_3, \Delta_6, F_7, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_{12}, \Delta_3, \Delta_6, F_{10}, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_{12}, \Delta_3, \Delta_6, F_{10}, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_6, F_7, F_4 & \mathbf{h}_2 : \vdash \Delta_3, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_5 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_7 \\ \hline & \bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_7, F_7 \\ \hline & \bullet \mathbf{h}_2 : \vdash$$

6 Cut-Elimination

6.1 Status of 1: OK

- ullet Case rule $oldsymbol{1}$
- Case rule!
- \bullet Case rule ?W
- \bullet Case rule ?C
- Case rule?
- Case rule \$
- \bullet Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- \bullet Case rule I
- \bullet Case rule \otimes

6.2 Status of !: OK

- $\bullet \;$ Case rule ${\bf 1}$
- Case rule!

$$\frac{ \begin{array}{c} \frac{h_1 : \vdash F_4,?\Upsilon2}{\bullet h_1 : \vdash !F_4,?\Upsilon2} \ ! & \frac{h_6 : \vdash F_7,?\Upsilon8,?dual(F_4),contract(n_5,?dual(F_4))}{\bullet h_6 : \vdash contract(sn_5,?dual(F_4)),?\Upsilon8,!F_7} \\ \hline \\ \frac{\bullet h_1 : \vdash ?\Upsilon2,!F_4}{\bullet h_1 : \vdash ?\Upsilon2,!F_4} \ ax & \frac{- : \vdash ?\Upsilon2,?\Upsilon8,F_7}{h_6 : \vdash ?\Upsilon2,?\Upsilon8,F_7} \ ! \end{array} \begin{array}{c} ax \\ h\text{Cut} \end{array}$$

\bullet Case rule ?W

\bullet Case rule ?C

$$\frac{ \underbrace{ \begin{array}{c} \mathbf{h}_1 : \vdash \mathbf{F}_4, ?\Upsilon2}_{\bullet \mathbf{h}_1 : \vdash ! \vdash \mathbf{F}_4, ?\Upsilon2} : \begin{array}{c} \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, ?\mathbf{f}_7, ?dual(\mathbf{F}_4), contract(\mathbf{n}_5, ?dual(\mathbf{F}_4))} \\ & \underbrace{ \begin{array}{c} \bullet \mathbf{h}_6 : \vdash contract(\mathbf{s}\mathbf{n}_5, ?dual(\mathbf{F}_4)), \Delta_8, ?\mathbf{F}_7 \\ & \rightarrow \\ \\ \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, !\mathbf{F}_4 \end{array} \begin{array}{c} \mathbf{ax} \\ & \underbrace{ \begin{array}{c} \bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, ?\mathbf{f}_7, ?dual(\mathbf{F}_4), contract(\mathbf{n}_5, ?dual(\mathbf{F}_4)) \\ & \rightarrow \\ \\ \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, !\mathbf{F}_4 \end{array} \begin{array}{c} \mathbf{ax} \\ & \underbrace{ \begin{array}{c} - : \vdash ?\Upsilon2, \Delta_8, ?\mathbf{F}_7, ?\mathbf{f}_7 \\ - : \vdash ?\Upsilon2, \Delta_8, ?\mathbf{F}_7 \end{array} \begin{array}{c} ?C \\ \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, ?\Upsilon2 \\ \hline \bullet \mathbf{h}_1 : \vdash !\mathbf{F}_5, ?\Upsilon2 \end{array} \begin{array}{c} \underbrace{ \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \hline \bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ?dual(\mathbf{F}_5)), \Delta_4 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ?\Upsilon2, !\mathbf{F}_5 \end{array} \begin{array}{c} \mathbf{ax} \\ \\ \bullet \mathbf{h}_7 : \vdash \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash 2 \Delta_4, ?dual(\mathbf{F}_5), ?dual(\mathbf$$

• Case rule?

$$\frac{\underbrace{\frac{h_1 :\vdash F_4,?\Upsilon2}{\bullet h_1 :\vdash !F_4,?\Upsilon2}}_{\bullet h_1 :\vdash !F_4,?\Upsilon2} : \underbrace{\frac{h_6 :\vdash \Delta_8, F_7,?dual(F_4), contract(n_5,?dual(F_4))}{\bullet h_6 :\vdash contract(sn_5,?dual(F_4)), \Delta_8,?F_7}}_{\bullet h_1 :\vdash !\top \Upsilon2,!F_4} \underbrace{\frac{-:\vdash ?\Upsilon2, \Delta_8, F_7}{\bullet h_6 :\vdash \Delta_8, F_7,?dual(F_4), contract(n_5,?dual(F_4))}}_{h_6 :\vdash \Delta_8, F_7,?dual(F_4), contract(n_5,?dual(F_4))} \underbrace{\frac{-:\vdash ?\Upsilon2, \Delta_8, F_7}{-:\vdash ?\Upsilon2, \Delta_8, F_7}?}_{h_1 :\vdash !F_5,?\Upsilon2} : \underbrace{\frac{h_1 :\vdash F_5,?\Upsilon2}{\bullet h_1 :\vdash !F_5,?\Upsilon2} !}_{\bullet h_7 :\vdash contract(sn_6,?dual(F_5)), \Delta_4} \underbrace{\frac{-:\vdash ?\Upsilon2, \Delta_4}{\bullet h_7 :\vdash contract(sn_6,?dual(F_5)), \Delta_4}}_{h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))} \underbrace{\frac{-:\vdash ?\Upsilon2, \Delta_4}{\bullet h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))}}_{h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))} \underbrace{\frac{-:\vdash ?\Upsilon2, \Delta_4}{\bullet h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))}}_{h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))} \underbrace{\frac{-:\vdash ?\Upsilon2, \Delta_4}{\bullet h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))}}_{h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))} \underbrace{\frac{-:\vdash ?\Upsilon2, \Delta_4}{\bullet h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))}}_{h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))} \underbrace{\frac{-:\vdash ?\Upsilon2, \Delta_4}{\bullet h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))}}_{h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))}} \underbrace{\frac{-:\vdash ?\Upsilon2, \Delta_4}{\bullet h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))}}_{h_7 :\vdash \Delta_4, dual(F_5), contract(n_6,?dual(F_5))}}$$

• Case rule \$

$$\frac{ \begin{array}{c|c} \mathbf{h}_1 : \vdash \mathbf{F}_4,? \Upsilon 2}{\bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4,? \Upsilon 2} : \begin{array}{c|c} \mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8, ?dual(\mathbf{F}_4), contract(\mathbf{n}_5,?dual(\mathbf{F}_4))} \\ \hline \bullet \mathbf{h}_6 : \vdash contract(\mathbf{s}\mathbf{n}_5,?dual(\mathbf{F}_4)), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8} \\ \hline & - : \vdash ?\Upsilon 2, \Delta_9, \mathbf{F}_7\$\mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash ?\Upsilon 2, ! \mathbf{F}_4 \end{array} \\ \mathbf{o}_{\mathbf{h}_1} : \vdash ?\Upsilon 2, ! \mathbf{F}_4 \end{array} \text{ax} \begin{array}{c} \mathbf{n}_{\mathbf{h}_2} : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8, ?dual(\mathbf{F}_4), contract(\mathbf{n}_5,?dual(\mathbf{F}_4))} \\ \hline & \frac{- : \vdash ?\Upsilon 2, \Delta_9, \mathbf{F}_7, \mathbf{F}_8}{- : \vdash ?\Upsilon 2, \Delta_9, \mathbf{F}_7, \mathbf{F}_8} \$ \end{array} \\ \mathbf{n}_{\mathbf{h}_2} : \vdash \mathbf{n}_{\mathbf{h}_3} : \vdash \mathbf{n}_{\mathbf{h}_4} : \vdash \mathbf{n}$$

• Case rule &

$$\frac{\underbrace{\frac{h_1 :\vdash F_4,?\Upsilon2}{\bullet h_1 :\vdash !F_4,?\Upsilon2} : \frac{h_6 :\vdash \Delta_9, F_7,?dual(F_4), contract(n_5,?dual(F_4))}{\bullet h_6 :\vdash contract(sn_5,?dual(F_4)), \Delta_9, F_7\&F_8}}{-:\vdash?\Upsilon2, \Delta_9, F_7\&F_8}}_{\underbrace{\frac{-:\vdash?\Upsilon2,\Delta_9, F_7,?dual(F_4), contract(n_5,?dual(F_4))}{hCut}}_{hCut} \underbrace{\frac{h_6 :\vdash \Delta_9, F_7,?dual(F_4), contract(n_5,?dual(F_4))}{h_6 :\vdash \Delta_9, F_8,?dual(F_4), contract(n_5,?dual(F_4))}}_{h_6 :\vdash \Delta_9, F_8,?dual(F_4), contract(n_5,?dual(F_4))}_{h_6 :\vdash \Delta_9, F_8,?dual(F_4), contract(n_5,?dual(F_4))}_{h_6 :\vdash \Delta_9, F_8,?dual(F_4), contract(n_5,?dual(F_4))}}$$

• Case rule \oplus_B

• Case rule \oplus_A

$$\frac{ \frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon2}{\bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4,?\Upsilon2} : \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7,?dual(\mathbf{F}_4), contract(\mathbf{n}_5,?dual(\mathbf{F}_4))}{\bullet \mathbf{h}_6 : \vdash contract(\mathbf{s}_{\mathbf{n}_5},?dual(\mathbf{F}_4)), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \underbrace{- : \vdash ?\Upsilon2, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8}_{\bullet \mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7,?dual(\mathbf{F}_4), contract(\mathbf{n}_5,?dual(\mathbf{F}_4))}_{\bullet \mathbf{h}_6 : \vdash \Delta_9, \mathbf{h}_7,?dual(\mathbf{h}_4), contract(\mathbf{h}_5,?dual(\mathbf{h}_4))}_{\bullet \mathbf{h}_6 : \vdash \Delta_9, \mathbf{h}_7, \mathbf{h}_7, \mathbf{h}_8}_{\bullet \mathbf{$$

 \bullet Case rule \bot

 \bullet Case rule \top

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon2}{\bullet \mathbf{h}_1 : \vdash !\mathbf{F}_4,?\Upsilon2} & ! & \hline\\ \bullet \mathbf{h}_6 : \vdash contract(s\mathbf{n}_5,?dual(\mathbf{F}_4)),\top,\Delta_7} \\ & & - : \vdash ?\Upsilon2,\top,\Delta_7 \\ & & \hline\\ & & - : \vdash ?\Upsilon2,\Delta_7,\top \end{array} \top$$

- \bullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 2}{\bullet \mathbf{h}_{1} : \vdash \vdash F_{4}, ? \Upsilon 2} : \frac{\mathbf{h}_{6} : \vdash \Delta_{10}, F_{8}, ?dual(F_{4}), contract(\mathbf{n}_{5}, ?dual(F_{4})), \Delta_{7}, \Delta_{10}, F_{8} \otimes F_{9}}{\bullet \mathbf{h}_{6} : \vdash contract(\mathbf{n}_{5}, ?dual(F_{4})), \Delta_{7}, \Delta_{10}, F_{8} \otimes F_{9}} \\ & - : \vdash ? \Upsilon 2, \Delta_{7}, \Delta_{10}, F_{8} \otimes F_{9} \\ \hline \bullet \mathbf{h}_{1} : \vdash ? \Upsilon 2, \vdash F_{4} & \mathbf{ax} & \mathbf{h}_{6} : \vdash \Delta_{10}, F_{8}, ?dual(F_{4}), contract(\mathbf{n}_{5}, ?dual(F_{4})) & \mathbf{h}_{6} : \vdash \Delta_{7}, F_{9} \\ \bullet \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 2 & \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ?dual(F_{4})) & \mathbf{h}_{6} : \vdash \Delta_{10}, F_{8}, ?dual(F_{4}) \\ \bullet \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 2 & \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ?dual(F_{4})), \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8} \\ \hline \bullet \mathbf{h}_{1} : \vdash ? \Upsilon 2, \Delta_{9}, F_{7} & \mathbf{e} & \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 2 & \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ?dual(F_{4})), \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8} \\ \hline \bullet \mathbf{h}_{1} : \vdash ? \Upsilon 2, \Delta_{9}, F_{7} & \mathbf{e} & \mathbf{h}_{1} : \vdash ? \Upsilon 2, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \\ \hline \bullet \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 2 & \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ?dual(F_{4})) & \mathbf{h}_{6} : \vdash \Delta_{10}, F_{8}, ?dual(F_{4}) \\ \hline \bullet \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 2 & \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ?dual(F_{4})) & \mathbf{h}_{6} : \vdash ? \Upsilon 2, \mathbf{h}_{10}, F_{9}, ?dual(F_{4}) \\ \hline \bullet \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 2 & \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, ?dual(F_{4}) & \mathbf{h}_{6} : \vdash \Delta_{10}, F_{8}, contract(\mathbf{n}_{5}, ?dual(F_{4})) \\ \hline \bullet \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 2 & \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, ?dual(F_{4}) & \mathbf{h}_{6} : \vdash \Delta_{10}, F_{8}, contract(\mathbf{n}_{5}, ?dual(F_{4})) \\ \hline \bullet \mathbf{h}_{1} : \vdash \vdash \mathsf{T} 2, \Delta_{9}, F_{7} & \mathbf{h}_{10} : \vdash \mathsf{T} 2, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \\ \hline \hline \bullet \mathbf{h}_{1} : \vdash \vdash \mathsf{T} 2, \Delta_{9}, F_{7} & \mathbf{h}_{10} : \vdash \mathsf{T} 2, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \\ \hline \bullet \mathbf{h}_{1} : \vdash \vdash \mathsf{T} 2, \Delta_{9}, F_{7} & \mathbf{h}_{10} : \vdash \mathsf{T} 2, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \\ \hline \bullet \mathbf{h}_{1} : \vdash \vdash \mathsf{T} 2, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \\ \hline \bullet \mathbf{h}_{1} : \vdash \vdash \mathsf{T} 2, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \\ \hline \bullet \mathbf{h}_{1} : \vdash \vdash \mathsf{T} 2, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \\ \hline \bullet \mathbf{h}_{1} : \vdash \vdash \mathsf{T} 2, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8}$$

6.3 Status of ?W: OK

- Case rule 1
- Case rule!

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} ? W \qquad \frac{\mathbf{h}_7 : \vdash F_8, ? \Upsilon 9, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_{\mathbf{n}_6}, ? dual(F_5)), ? \Upsilon 9, ! F_8} Cut} ? \\ \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3 \qquad \mathbf{ax} \qquad \frac{- : \vdash ? \Upsilon 4, ? F_3, ? \Upsilon 9, ! F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\bullet \mathbf{h}_7 : \vdash ? \Upsilon 9, F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \qquad \mathbf{ax} \\ \frac{- : \vdash ? \Upsilon 4, ? \Upsilon 9, F_8, ? F_3}{- : \vdash ? \Upsilon 4, ? \Upsilon 9, ! F_8, ? F_3} !$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\underbrace{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}}_{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ! \mathsf{F}_5} ? W \quad \frac{\mathbf{h}_7 : \vdash \Delta_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5)), \Delta_9, ? \mathsf{F}_8}{\bullet \mathsf{h}_7 : \vdash contract(\mathsf{s}_\mathsf{n}_6, ? dual(\mathsf{F}_5)), \Delta_9, ? \mathsf{F}_8} \\ \underbrace{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_9, ? \mathsf{F}_8}_{\mathsf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3} \quad \underbrace{- : \vdash ? \Upsilon 4, \Delta_9, ? \mathsf{F}_3}_{\mathsf{h}_2 : \vdash ? \Upsilon 4, \Delta_9, ? \mathsf{F}_3, ? \mathsf{F}_8} ? W \quad \underbrace{- : \vdash ? \Upsilon 4, \Delta_9, ? \mathsf{F}_3, ? \mathsf{F}_8}_{\mathsf{h}_2 : \vdash ? \Upsilon 4, \Delta_9, ? \mathsf{F}_3, ? \mathsf{F}_8} ? W$$

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_6}{\bullet \mathbf{h}_2 : \vdash ! \mathsf{F}_6, ? \Upsilon 4, ? \mathsf{F}_3} ? W \quad \frac{\mathbf{h}_8 : \vdash \Delta_5, contract(\mathbf{n}_7, ?dual(\mathsf{F}_6))}{\bullet \mathbf{h}_8 : \vdash contract(\mathsf{sn}_7, ?dual(\mathsf{F}_6)), \Delta_5} ? W \\ - : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_5 \\ \xrightarrow{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_6, ? \mathsf{F}_3} \quad \underset{\bullet}{\mathsf{ax}} \quad \frac{\rightarrow}{\mathbf{h}_8 : \vdash \Delta_5, contract(\mathbf{n}_7, ?dual(\mathsf{F}_6))}} \quad \underset{\mathsf{hCut}}{\mathsf{ax}} \quad \underset{\mathsf{hCut}}{\bullet}$$

\bullet Case rule ?C

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} ? W \quad \frac{\mathbf{h}_7 : \vdash \Delta_9, ? F_8, ? F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_{\mathbf{n}_6}, ? dual(F_5)), \Delta_9, ? F_8} Cut} ? C \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3 \quad \mathbf{ax} \quad \frac{- : \vdash (? \Upsilon 4, ? F_3), \Delta_9, ? F_8}{\mathbf{h}_7 : \vdash \Delta_9, ? F_8, ? F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\mathbf{h}_7 : \vdash \Delta_9, ? F_8, ? F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \mathbf{h}_{\mathbf{cut}} \\ \hline \frac{- : \vdash ? \Upsilon 4, \Delta_9, ? F_3, ? F_8, ? F_8}{- : \vdash ? \Upsilon 4, \Delta_9, ? F_3, ? F_8} ? C \\ \hline \bullet \mathbf{h}_2 : \vdash ! F_6, ? \Upsilon 4, ? F_3 \end{cases} ? W \quad \frac{\mathbf{h}_8 : \vdash \Delta_5, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))}{\bullet \mathbf{h}_8 : \vdash contract(\mathbf{s}_{\mathbf{n}_7}, ? dual(F_6)), \Delta_5} Cut} \quad ? C \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_6, ? F_3 \end{cases} \quad \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_6, ? F_3 \end{cases} \quad \mathbf{ax} \\ \hline \bullet \mathbf{h}_3 : \vdash \Delta_5, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_5, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_5, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? dual(F_6), contract(\mathbf{n}_7, ? dual(F_6))} \\ \hline \bullet \mathbf{h}_8 : \vdash \Delta_7, ? dual(F_6), ? d$$

• Case rule?

$$\frac{\begin{array}{c} \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} \end{array}{?W} \quad \frac{\begin{array}{c} \mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_{\mathbf{n}_6}, ? dual(\mathbf{F}_5)), \Delta_9, ? F_8} \end{array}{?} \\ \frac{- : \vdash (? \Upsilon 4, ? F_3), \Delta_9, ? F_8}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3} \end{array} \xrightarrow{\mathbf{ax}} \frac{- : \vdash (? \Upsilon 4, \Delta_9, \mathbf{F}_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))} \end{array}{} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))} \\ \hline \\ \frac{- : \vdash : \vdash : \Upsilon 4, \Delta_9, \mathbf{F}_8, ? F_3}{- : \vdash : \vdash : \Upsilon 4, \Delta_9, ? F_3, ? F_8} ? \end{array}{} \\ \frac{\mathbf{h}_2 : \vdash : \vdash : \Upsilon 4, ! F_6}{\bullet \mathbf{h}_2 : \vdash : \vdash : F_6, ? \Upsilon 4, ? F_3} \end{array}{} \begin{array}{c} \mathbf{w} \\ \mathbf{h}_8 : \vdash \Delta_5, dual(\mathbf{F}_6), contract(\mathbf{n}_7, ? dual(\mathbf{F}_6)), \Delta_5} \\ \hline \\ - : \vdash : \vdash : \vdash : \vdash : \Upsilon 4, \Delta_5, ? dual(\mathbf{F}_6), contract(\mathbf{n}_7, ? dual(\mathbf{F}_6))} \\ \hline \\ \mathbf{h}_8 : \vdash \Delta_5, dual(\mathbf{F}_6), contract(\mathbf{n}_7, ? dual(\mathbf{F}_6))} \\ \hline \\ - : \vdash : \vdash : \Upsilon 4, \Delta_5, ? H_3} \end{aligned}{} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_{\text{Cut}} \\ \hline \\ - : \vdash : \Upsilon 4, \Delta_5, ? F_3 \end{array}{} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_{\text{Cut}} \\ \hline \\ - : \vdash : \Upsilon 4, \Delta_5, ? F_3} \end{aligned}{} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_{\text{Cut}} \\ \hline \end{array}{} \end{array}$$

• Case rule \$

$$\frac{\begin{array}{c} \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5 \\ \bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3 \end{array}}{\bullet \mathbf{h}_2 : \vdash \mathsf{P}_5, ? \Upsilon 4, ? F_3} \ ?W \ \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_{10}, F_8, F_9, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \\ \bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ? dual(F_5)), \Delta_{10}, F_8 \$ F_9 \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3 \end{array}} \ \mathbf{ax} \ \begin{array}{c} - : \vdash (? \Upsilon 4, ? F_3), \Delta_{10}, F_8 \$ F_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, F_8, F_9, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \\ \hline \\ & \frac{- : \vdash ? \Upsilon 4, \Delta_{10}, F_8, F_9, ? f_3}{- : \vdash ? \Upsilon 4, \Delta_{10}, ? F_3, F_8 \$ F_9} \ \$ \end{array}$$

• Case rule &

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} \quad ?W \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ?dual(\mathbf{F}_5)), \Delta_{10}, \mathbf{F}_8 \& F_9} \quad \mathbf{Cut} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))} \quad \mathbf{ax} \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))} \quad \mathbf{ax} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))} \quad \mathbf{ax} \\ \\ \frac{- : \vdash ? \Upsilon 4, \Delta_{10}, \mathbf{F}_8 \& F_9}{- : \vdash ? \Upsilon 4, \Delta_{10}, ? F_3, F_8 \& F_9} \quad ?W \\ \end{aligned} \quad \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{h}_7 : \vdash \Delta$$

• Case rule \oplus_B

$$\frac{ \begin{array}{c} \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5 \\ \bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3 \end{array}}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, F_9, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \\ \hline \\ \frac{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3 \end{array}}{\bullet \mathbf{h}_7 : \vdash Contract(\mathbf{s}\mathbf{n}_6, ? dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \\ \hline \\ \frac{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3} \end{array} \begin{array}{c} \mathbf{ax} \\ - : \vdash ? \Upsilon 4, \Delta_{10}, F_9, ? H_3 \\ \hline - : \vdash ? \Upsilon 4, \Delta_{10}, F_9, ? F_3 \\ \hline - : \vdash ? \Upsilon 4, \Delta_{10}, ? F_3, F_8 \oplus F_9 \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{hCut} \end{array}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} \quad ?W \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, F_8, ?dual(F_5), contract(\mathbf{n}_6, ?dual(F_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s} \mathbf{n}_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \quad \frac{- : \vdash (? \Upsilon 4, ? F_3), \Delta_{10}, F_8 \oplus F_9}{\bullet} \quad \frac{-}{\mathbf{h}_7 : \vdash \Delta_{10}, F_8, ?dual(F_5), contract(\mathbf{n}_6, ?dual(F_5))}}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, F_8, ?dual(F_5), contract(\mathbf{n}_6, ?dual(F_5))} \quad \frac{\mathsf{ax}}{\mathsf{hCut}} \quad \frac{- : \vdash ? \Upsilon 4, \Delta_{10}, F_8, ? F_3}{- : \vdash ? \Upsilon 4, \Delta_{10}, ? F_8, F_8} \quad \oplus_A$$

• Case rule \perp

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5}{\underbrace{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3}} \ ?W \ \frac{\mathbf{h}_7 : \vdash \Delta_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_\mathbf{n}_6, ?dual(\mathbf{F}_5)), \bot, \Delta_8} \ \Box \\ - : \vdash (? \Upsilon 4, ? \mathbf{F}_3), \bot, \Delta_8 \\ \rightarrow \\ \frac{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3}{\bullet \mathbf{h}_7 : \vdash \Delta_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))} \ \mathbf{ax} \\ \frac{- : \vdash ? \Upsilon 4, \Delta_8, ? \mathbf{F}_3}{- : \vdash ? \Upsilon 4, \Delta_8, \bot, ? \mathbf{F}_3} \ \bot$$

• Case rule \top

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash : \Upsilon 4, \mathsf{IF}_5}{\bullet \mathbf{h}_2 : \vdash : \mathsf{IF}_5, : \Upsilon 4, : \mathsf{F}_3} & ?W & \hline \bullet \mathbf{h}_7 : \vdash contract(s\mathbf{n}_6, ?dual(\mathbf{F}_5)), \top, \Delta_8} \\ & - : \vdash (: \Upsilon 4, : \mathsf{F}_3), \top, \Delta_8 \\ \hline & - : \vdash : \Upsilon 4, \Delta_8, \top, : \mathsf{F}_3} & \top \end{array}$$
 Cut

- ullet Case rule I
- Case rule \otimes

$$\frac{\underbrace{\frac{h_2 : \vdash ?\Upsilon 4, !F_5}{\bullet h_2 : \vdash !F_5, ?\Upsilon 4, ?F_3}}_{\bullet h_2 : \vdash !F_5, ?\Upsilon 4, !F_5} ?W \quad \frac{h_7 : \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5)) \quad h_7 : \vdash \Delta_8, F_{10}}{\bullet h_7 : \vdash contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \quad \text{Cut}} \\ - : \vdash (?\Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \\ - : \vdash (?\Upsilon 4, !F_5, ?F_3) \quad \text{ax} \quad \frac{}{h_7 : \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}_{\bullet h_7 : \vdash \Delta_8, F_{10}} \quad \text{ax} \\ - : \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3 \quad hCut} \\ - : \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \\ & \otimes \\ \\$$

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\bullet \mathbf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3} \quad ?W \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, contract(\mathsf{n}_6, ?dual(\mathsf{F}_5)), \ \, \mathbf{h}_7 : \vdash \Delta_{11}, \mathsf{F}_9, ?dual(\mathsf{F}_5)}{\bullet \mathsf{h}_7 : \vdash contract(\mathsf{sn}_6, ?dual(\mathsf{F}_5)), \ \, \Delta_{10}, \Delta_{11}, \mathsf{F}_8 \otimes \mathsf{F}_9} \quad \mathsf{Cut} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5} \quad \mathsf{ax} \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, contract(\mathsf{n}_6, ?dual(\mathsf{F}_5)), \ \, \mathsf{ax}}{\bullet \mathsf{h}_7 : \vdash \Delta_{10}, \Delta_{11}, ?dual(\mathsf{F}_5), \mathsf{F}_8 \otimes \mathsf{F}_9, contract(\mathsf{n}_6, ?dual(\mathsf{F}_5))} \quad \mathsf{ax}} \quad \mathsf{ax} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3} \quad ?W \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ?dual(\mathsf{F}_5), \mathsf{F}_8 \otimes \mathsf{F}_9, contract(\mathsf{n}_6, ?dual(\mathsf{F}_5))}{\bullet \mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ?dual(\mathsf{F}_5), \mathsf{h}_7 : \vdash \Delta_{11}, \mathsf{F}_9, contract(\mathsf{n}_6, ?dual(\mathsf{F}_5))} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3} \quad \mathsf{ax} \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ?dual(\mathsf{F}_5)}{\bullet \mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ?dual(\mathsf{F}_5)} \quad \mathsf{ax} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3} \quad \mathsf{ax} \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ?dual(\mathsf{F}_5)}{\bullet \mathsf{h}_7 : \vdash \Delta_{10}, \Delta_{11}, ?dual(\mathsf{F}_5), \mathsf{f}_8 \otimes \mathsf{F}_9} \quad \mathsf{h}_7 : \vdash \Delta_{11}, \mathsf{F}_9, contract(\mathsf{n}_6, ?dual(\mathsf{F}_5))}{\bullet \mathsf{h}_7 : \vdash \Delta_{10}, \Delta_{11}, ?dual(\mathsf{F}_5), \mathsf{f}_8 \otimes \mathsf{F}_9, contract(\mathsf{n}_6, ?dual(\mathsf{F}_5))} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\bullet \mathsf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5} \quad \mathsf{ax} \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \Delta_{11}, ?dual(\mathsf{F}_5), \mathsf{F}_8 \otimes \mathsf{F}_9}{\bullet \mathsf{h}_7 : \vdash \Delta_{11}, \mathsf{F}_{10}, ?dual(\mathsf{F}_5), \mathsf{contract}(\mathsf{n}_6, ?dual(\mathsf{F}_5))} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\bullet \mathsf{h}_2 : \vdash ! \vdash \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3} \quad ?W \quad \frac{\mathbf{h}_7 : \vdash \Delta_8, \mathsf{F}_9 \quad \mathsf{h}_7 : \vdash \Delta_{11}, \mathsf{F}_{10}, ?dual(\mathsf{F}_5), \mathsf{contract}(\mathsf{n}_6, ?dual(\mathsf{F}_5))}{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{s}_6, ?dual(\mathsf{F}_5)), \Delta_8, \Delta_{11}, \mathsf{F}_9 \otimes \mathsf{F}_{10}} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, \Delta_{10}, \Delta_{11}, \mathsf{F}_8, ? \mathsf{F}_9 \otimes \mathsf{F}_{10}}{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{s}_6, ?dual(\mathsf{F}_5), \mathsf{contract}(\mathsf{n}_6, ?dual(\mathsf{F}_5))} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, \Delta_{11}, \Delta_8, ? \mathsf{F}_9, \mathsf{F}_9 \otimes$$

6.4 Status of ?*C*: OK

- Case rule 1
- Case rule!

$$\frac{\begin{array}{c} \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3, ? \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3} \end{array} ? C \quad \frac{\begin{array}{c} \mathbf{h}_7 : \vdash \mathbf{F}_8, ? \Upsilon 9, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ? dual(\mathbf{F}_5)), ? \Upsilon 9, ! \mathbf{F}_8} \\ \\ \underline{\begin{array}{c} - : \vdash (? \Upsilon 4, ? \mathbf{F}_3), ? \Upsilon 9, ! \mathbf{F}_8 \\ \rightarrow \\ \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3 \end{array}} \quad \underset{\mathbf{ax}}{\mathbf{ax}} \quad \frac{- : \vdash ? \Upsilon 9, \mathbf{F}_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash ? \Upsilon 9, \mathbf{F}_8, ? \mathbf{f}_3} \\ \underline{- : \vdash ? \Upsilon 4, ? \Upsilon 9, ! \mathbf{F}_8, ? \mathbf{F}_3} \end{array} \quad ! \\ \\ \begin{array}{c} \mathbf{h}_7 : \vdash \mathbf{h}_7 \mathbf{h}_7 : \vdash \mathbf{h}_7 \mathbf{h}_7 \cdot \mathbf{h}$$

 \bullet Case rule ?W

 \bullet Case rule ?C

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3}{\underbrace{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}} ? C \quad \frac{\mathbf{h}_7 : \vdash \Delta_9, ? \mathsf{F}_8, ? \mathsf{f}_8, ? \mathsf{f}_0 : \vdash \mathsf{contract}(\mathsf{n}_6, ? \mathsf{dual}(\mathsf{F}_5)), \Delta_9, ? \mathsf{F}_8}{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5)), \Delta_9, ? \mathsf{F}_8} \underbrace{\mathsf{Cut}} ? C \\ \\ \underbrace{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_9, ? \mathsf{F}_8}_{\bullet \mathsf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3} \overset{\mathsf{ax}}{=} \underbrace{\frac{- : \vdash (? \Upsilon 4, ? \mathsf{F}_8, ? \mathsf{F}_8, ? \mathsf{f}_0 \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{n}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \Delta_9, ? \mathsf{F}_8, ? \mathsf{F}_8, ? \mathsf{f}_8, ? \mathsf{f}_8} ? C \\ \\ \underbrace{\frac{- : \vdash ? \Upsilon 4, \Delta_9, ? \mathsf{F}_3, ? \mathsf{F}_8}{- : \vdash ? \Upsilon 4, \Delta_9, ? \mathsf{F}_3, ? \mathsf{F}_8} ? C}_{\bullet \mathsf{h}_8 : \vdash \Delta_5, ? \mathsf{dual}(\mathsf{F}_6), ? \mathsf{dual}(\mathsf{F}_6), \mathsf{contract}(\mathsf{n}_7, ? \mathsf{dual}(\mathsf{F}_6))} ? C \\ \underbrace{\frac{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_5}{\bullet \mathsf{h}_8 : \vdash \mathsf{contract}(\mathsf{sn}_7, ? \mathsf{dual}(\mathsf{F}_6)), \Delta_5}_{\bullet \mathsf{h}_8 : \vdash \mathsf{contract}(\mathsf{sn}_7, ? \mathsf{dual}(\mathsf{F}_6))} \overset{\mathsf{ax}}{\to} \\ \underbrace{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_5}_{\bullet \mathsf{h}_8 : \vdash \Delta_5, ? \mathsf{dual}(\mathsf{F}_6), ? \mathsf{dual}(\mathsf{F}_6), \mathsf{contract}(\mathsf{n}_7, ? \mathsf{dual}(\mathsf{F}_6))}_{\bullet \mathsf{h}_8 : \vdash \Delta_5, ? \mathsf{dual}(\mathsf{F}_6), ? \mathsf{dual}(\mathsf{F}_6), \mathsf{contract}(\mathsf{n}_7, ? \mathsf{dual}(\mathsf{F}_6))} \overset{\mathsf{ax}}{\to} \\ \underbrace{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_5}_{\bullet \mathsf{h}_8 : \vdash \Delta_5, ? \mathsf{dual}(\mathsf{F}_6), ? \mathsf{dual}(\mathsf{F}_6), \mathsf{contract}(\mathsf{n}_7, ? \mathsf{dual}(\mathsf{F}_6))}_{\bullet \mathsf{h}_8 : \vdash \Delta_5, ? \mathsf{dual}(\mathsf{F}_6), ? \mathsf{dual}(\mathsf{F}_6), \mathsf{contract}(\mathsf{n}_7, ? \mathsf{dual}(\mathsf{F}_6))} \overset{\mathsf{ax}}{\to} \\ \underbrace{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_5}_{\bullet \mathsf{h}_8 : \vdash \Delta_5, ? \mathsf{dual}(\mathsf{F}_6), ? \mathsf{dual}(\mathsf{F}_6), \mathsf{contract}(\mathsf{n}_7, ? \mathsf{dual}(\mathsf{F}_6))}_{\bullet \mathsf{h}_8} \overset{\mathsf{ax}}{\to} \\ \underbrace{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_5}_{\bullet \mathsf{h}_8 : \vdash \Delta_5, ? \mathsf{dual}(\mathsf{F}_6), ? \mathsf{dual}(\mathsf{F}_6), \mathsf{contract}(\mathsf{n}_7, ? \mathsf{dual}(\mathsf{F}_6))}_{\bullet \mathsf{h}_8} \overset{\mathsf{ax}}{\to} \\ \underbrace{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3, ?$$

• Case rule?

• Case rule \$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}, ? F_{3}, ? F_{3}}{\underbrace{\bullet \mathbf{h}_{2} : \vdash ! F_{5}, ? \Upsilon 4, ? F_{3}}_{\bullet \mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}, ? F_{3}}} ? C \xrightarrow{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, F_{9}, ? dual(F_{5}), contract(\mathbf{n}_{6}, ? dual(F_{5}))}_{\bullet \mathbf{h}_{7} : \vdash contract(\mathbf{s}\mathbf{n}_{6}, ? dual(F_{5})), \Delta_{10}, F_{8} \$ F_{9}} Cut} \$ \\ \underbrace{- : \vdash (? \Upsilon 4, ? F_{3}), \Delta_{10}, F_{8} \$ F_{9}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, F_{9}, ? dual(F_{5}), contract(\mathbf{n}_{6}, ? dual(F_{5}))}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, F_{9}, ? dual(F_{5}), contract(\mathbf{n}_{6}, ? dual(F_{5}))}} _{\bullet \mathbf{h}_{Cut}} \underbrace{- : \vdash ? \Upsilon 4, \Delta_{10}, F_{8}, F_{9}, ? F_{3}}_{\bullet \mathbf{h}_{Cut}} \$ }$$

• Case rule &

$$\frac{\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3}{\bullet \mathbf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}}{\circ \mathbf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}} ?C \xrightarrow{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \bullet \mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \& \\ \frac{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_{10}, \mathsf{F}_8 \& \mathsf{F}_9}{\bullet \mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \bullet \mathsf{ax} \xrightarrow{\mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \bullet \mathsf{h}_7 : \vdash \Delta_{10}, ? \mathsf{h}_7 : \vdash \Delta_$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3}{\underbrace{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}}_{\bullet \mathsf{h}_2 : \vdash : \vdash \mathsf{F}_5, ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3} ? C \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash contract(\mathsf{s}_\mathsf{n}_6, ? dual(\mathsf{F}_5)), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9} \\ - : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \to \\ - : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ - : \vdash ? \Upsilon 4, \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}_{\mathsf{h}_2} \xrightarrow{\mathsf{h}_2}_{\mathsf{h}_2}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3}{\underbrace{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}_{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3}} \ ?C \ \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash contract(\mathsf{s}_{\mathsf{n}_6}, ? dual(\mathsf{F}_5)), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9} \ Cut} \\ = \underbrace{\frac{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9}_{\mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}_{\mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}} \ \frac{\mathsf{ax}}{\mathsf{hCut}} \\ \frac{- : \vdash ? \Upsilon 4, \Delta_{10}, \mathsf{F}_8, ? \mathsf{F}_3}{- : \vdash ? \Upsilon 4, \Delta_{10}, ? \mathsf{F}_3, \mathsf{F}_8 \oplus \mathsf{F}_9} \ \oplus_{A}$$

• Case rule \perp

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3, ? \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3} ? C \xrightarrow{\begin{array}{c} \mathbf{h}_7 : \vdash \Delta_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ? dual(\mathbf{F}_5)), \bot, \Delta_8} & \mathsf{Cut} \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3 & \mathsf{ax} & \xrightarrow{} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \mathsf{cut} \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3 & \mathsf{ax} & \xrightarrow{} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \mathsf{cut} \\ \hline & & & & & & & & & & \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \Delta_8, ? \mathbf{F}_3 & \bot & & & & & & \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \Delta_8, ? \mathbf{F}_3 & \bot & & & & & \\ \hline \end{array}$$

• Case rule T

$$\frac{ \begin{array}{c} \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3, ? F_3 \\ \hline \bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3 \end{array} ? C \quad \overline{ \begin{array}{c} \bullet \mathbf{h}_7 : \vdash contract(sn_6, ?dual(F_5)), \top, \Delta_8 \\ \hline \\ - : \vdash (? \Upsilon 4, ? F_3), \top, \Delta_8 \\ \hline \\ - : \vdash ? \Upsilon 4, \Delta_8, \top, ? F_3 \end{array} \top }$$

- \bullet Case rule I
- Case rule \otimes

$$\frac{\frac{h_{2}:\vdash ?\Upsilon 4, !_{F_{5}}, ?_{F_{3}}, ?_{F_{3}}}{\bullet h_{2}:\vdash !_{F_{5}}, ?\Upsilon 4, ?_{F_{3}}}}{\circ h_{2}:\vdash !_{F_{5}}, ?\Upsilon 4, ?_{F_{3}}}} ?C \xrightarrow{\begin{array}{c} h_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5}), contract(n_{6}, ?dual(F_{5})), \Delta_{8}, \Delta_{11}, F_{9} \otimes F_{10} \\ & - :\vdash (?\Upsilon 4, ?_{F_{3}}), \Delta_{8}, \Delta_{11}, F_{9} \otimes F_{10} \\ & \rightarrow \\ \hline \\ \hline \bullet h_{2}:\vdash ?\Upsilon 4, !_{F_{5}}, ?_{F_{3}} \end{array}} \underbrace{\begin{array}{c} ax \\ h_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5}), contract(n_{6}, ?dual(F_{5})) \\ & \rightarrow \\ \hline \\ \hline - :\vdash ?\Upsilon 4, \Delta_{11}, F_{9}, ?_{F_{3}} \\ \hline - :\vdash ?\Upsilon 4, \Delta_{11}, \Delta_{8}, ?_{F_{3}}, F_{9} \otimes F_{10} \\ \hline \end{array}} \underbrace{\begin{array}{c} ax \\ h_{Cut} \\ \hline - :\vdash \Delta_{8}, F_{10} \\ \hline - :\vdash ?\Upsilon 4, \Delta_{11}, A_{8}, ?_{F_{3}}, F_{9} \otimes F_{10} \\ \hline \\ \bullet h_{2}:\vdash ?\Upsilon 4, !_{F_{5}}, ?_{F_{3}}, ?_{F_{3}} \\ \hline \bullet h_{2}:\vdash !_{F_{5}}, ?\Upsilon 4, ?_{F_{3}} \end{array}} ?C \underbrace{\begin{array}{c} h_{7}:\vdash \Delta_{10}, F_{8}, contract(n_{6}, ?dual(F_{5})), h_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5})}{\bullet h_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5})} \otimes \\ \hline \\ - :\vdash (?\Upsilon 4, ?_{F_{3}}), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9} \\ \hline \rightarrow \\ h_{7}:\vdash \Delta_{10}, F_{8}, contract(n_{6}, ?dual(F_{5})), h_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5}) \\ \hline \bullet h_{7}:\vdash \Delta_{10}, A_{11}, ?_{10}, A_{11}, F_{8} \otimes F_{9} \\ \hline \hline \\ \bullet h_{7}:\vdash \Delta_{10}, A_{11}, ?dual(F_{5}), F_{8} \otimes F_{9}, contract(n_{6}, ?dual(F_{5})) \\ \hline \bullet h_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5}) \\ \hline \\ \bullet h_{7}:\vdash \Delta_{10}, A_{11}, ?dual(F_{5}), F_{8} \otimes F_{9}, contract(n_{6}, ?dual(F_{5})) \\ \hline \\ \bullet h_{7}:\vdash \Delta_{10}, A_{11}, ?dual(F_{5}), F_{8} \otimes F_{9}, contract(n_{6}, ?dual(F_{5})) \\ \hline \\ \bullet h_{7}:\vdash (?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_{3}, ?F_{3}, F_{8} \otimes F_{9}, contract(n_{6}, ?dual(F_{5})) \\ \hline \\ \bullet h_{7}:\vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_{3}, F_{3}, F_{8} \otimes F_{9} \\ \hline \\ \bullet h_{7}:\vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_{3}, F_{3}, F_{8} \otimes F_{9} \\ \hline \\ \bullet h_{7}:\vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_{3}, F_{3}, F_{8} \otimes F_{9} \\ \hline \\ \bullet h_{7}:\vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_{3}, F_{3}, F_{3} \otimes F_{9} \\ \hline \\ \bullet h_{7}:\vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_{3}, F_{3}, F_{3} \otimes F_{9} \\ \hline \end{array}$$

6.5 Status of ?: OK

- Case rule 1
- Case rule!

$$\frac{\frac{\mathbf{h}_2 :\vdash F_3,?\Upsilon 4,!F_5}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3}}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3}? \frac{\frac{\mathbf{h}_7 :\vdash F_8,?\Upsilon 9,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}{\bullet \mathbf{h}_7 :\vdash contract(\mathbf{s}\mathbf{n}_6,?dual(F_5)),?\Upsilon 9,!F_8}}{\bullet} \underbrace{\overset{-}{\cdot \vdash (\Upsilon 4,?F_3),?\Upsilon 9,!F_8}}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,!F_5,?F_3}} \underbrace{\overset{-}{\cdot \vdash (\Upsilon 4,?\Upsilon 9,F_8,?F_3}}_{\bullet \mathbf{h}_7 :\vdash ?\Upsilon 4,?\Upsilon 9,!F_8,?F_3} !}$$

 \bullet Case rule ?W

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ? \Upsilon 4, ! \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3} ? \\ \frac{\mathbf{h}_7 : \vdash \Delta_9, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_{\mathbf{n}_6}, ? dual(\mathbf{F}_5)), \Delta_9, ? \mathbf{F}_8} \\ \\ - : \vdash (? \Upsilon 4, ? \mathbf{F}_3), \Delta_9, ? \mathbf{F}_8 \\ \\ \rightarrow \\ \mathbf{h}_7 : \vdash \Delta_9, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5)) \\ \\ - : \vdash ? \Upsilon 4, \Delta_9, ? \mathbf{F}_3, ? \mathbf{F}_8 \\ \\ \hline \bullet \mathbf{h}_2 : \vdash \mathbf{F}_3, ? \Upsilon 4, ! \mathbf{F}_6 \\ \bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_6, ? \Upsilon 4, ? \mathbf{F}_3 \\ \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \Delta_9, ? \mathbf{F}_3, ? \mathbf{F}_8 \\ \\ \hline \bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_6, ? \Upsilon 4, ? \mathbf{F}_3 \\ \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ? \mathbf{F}_3 \\ \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ? \mathbf{F}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash \Delta_5, contract(\mathbf{n}_7, ? dual(\mathbf{F}_6)), \Delta_5 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash (? \Upsilon 4, ? \mathbf{F}_3), \Delta_5 \\ \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_6, ? \mathbf{F}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash \Delta_5, contract(\mathbf{n}_7, ? dual(\mathbf{F}_6)), \Delta_5 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash (? \Upsilon 4, ? \mathbf{F}_3), \Delta_5 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash \Delta_5, contract(\mathbf{n}_7, ? dual(\mathbf{F}_6)) \\ \\ \hline \bullet \mathbf{h}_3 : \vdash ? \Upsilon 4, ! \mathbf{F}_6, ? \mathbf{F}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash \Delta_5, contract(\mathbf{n}_7, ? dual(\mathbf{F}_6)) \\ \\ \hline \bullet \mathbf{h}_3 : \vdash \Delta_5, contract(\mathbf{n}_7, ? dual(\mathbf{F}_6)) \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{h}_3 \\ \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{h}_3, 2 \mathbf{$$

• Case rule ?C

$$\frac{\underbrace{\frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_5}{\mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, ?\mathbf{F}_3}}_{\mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, !\mathbf{F}_5}? \underbrace{\frac{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{f}_8, ?\mathbf{dual}(\mathbf{F}_5), contract(\mathbf{n}_6, ?\mathbf{dual}(\mathbf{F}_5))}{\mathbf{h}_7 : \vdash contract(\mathbf{s}_6, ?\mathbf{dual}(\mathbf{F}_5)), \Delta_9, ?\mathbf{F}_8}}\underbrace{\frac{- : \vdash (?\Upsilon 4, ?\mathbf{F}_3), \Delta_9, ?\mathbf{F}_8}{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{f}_8, ?\mathbf{dual}(\mathbf{F}_5), contract(\mathbf{n}_6, ?\mathbf{dual}(\mathbf{F}_5))}}_{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, ?\mathbf{f}_8, ?\mathbf{F}_8}}\underbrace{\frac{- : \vdash ?\Upsilon 4, \Delta_9, ?\mathbf{F}_3, ?\mathbf{F}_8}{\mathbf{h}_7 : \vdash ?\Upsilon 4, \Delta_9, ?\mathbf{F}_3, ?\mathbf{F}_8}}_{\mathbf{h}_7 : \vdash ?\Upsilon 4, \Delta_9, ?\mathbf{F}_3, ?\mathbf{F}_8}}?C}$$

$$\frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_6}{\bullet \mathbf{h}_2 : \vdash !\mathbf{F}_6, ?\Upsilon 4, ?\mathbf{F}_3} ? \frac{\mathbf{h}_8 : \vdash \Delta_5, ?dual(\mathbf{F}_6), ?dual(\mathbf{F}_6), contract(\mathbf{n}_7, ?dual(\mathbf{F}_6))}{\bullet \mathbf{h}_8 : \vdash contract(s\mathbf{n}_7, ?dual(\mathbf{F}_6)), \Delta_5} Cut \\ \frac{- : \vdash (?\Upsilon 4, ?\mathbf{F}_3), \Delta_5}{\to} \frac{-}{\mathbf{h}_8 : \vdash \Delta_5, ?dual(\mathbf{F}_6), ?dual(\mathbf{F}_6), contract(\mathbf{n}_7, ?dual(\mathbf{F}_6))}{\to} ax}{- : \vdash : \vdash \Upsilon 4, \Delta_5, ?\mathbf{F}_3}$$

• Case rule?

$$\frac{\mathbf{h}_2 :\vdash \mathbf{F}_3, ? \Upsilon 4, ! \mathbf{F}_5}{\bullet \mathbf{h}_2 :\vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3} ? \frac{\mathbf{h}_7 :\vdash \Delta_9, \mathbf{F}_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 :\vdash contract(\mathbf{s}_{\mathbf{n}_6}, ? dual(\mathbf{F}_5)), \Delta_9, ? \mathbf{F}_8} \underbrace{ \mathbf{Cut} \\ - :\vdash (? \Upsilon 4, ? \mathbf{F}_3), \Delta_9, ? \mathbf{F}_8} \\ \bullet \mathbf{h}_2 :\vdash ! ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3 \underbrace{ \mathbf{h}_7 :\vdash \Delta_9, \mathbf{F}_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, \mathbf{F}_8, ? \mathbf{f}_3} \underbrace{ \mathbf{h}_7 :\vdash \Delta_9, \mathbf{F}_8, ? \mathbf{f}_3}_{\bullet \mathbf{h}_7 :\vdash ? \Upsilon 4, \Delta_9, ? \mathbf{F}_8, ? \mathbf{f}_3} ? \underbrace{ \mathbf{h}_8 :\vdash \Delta_5, dual(\mathbf{F}_6), contract(\mathbf{n}_7, ? dual(\mathbf{F}_6))}_{\bullet \mathbf{h}_8 :\vdash contract(\mathbf{s}_{\mathbf{n}_7}, ? dual(\mathbf{F}_6)), \Delta_5} \underbrace{ \mathbf{Cut} \\ \mathbf{h}_8 :\vdash ? \mathbf{f}_8, ? \mathbf{f}_8, ? \mathbf{f}_8, ? \mathbf{f}_8 :\vdash contract(\mathbf{s}_{\mathbf{n}_7}, ? dual(\mathbf{f}_6)), \Delta_5}_{\bullet \mathbf{h}_8 :\vdash contract(\mathbf{s}_{\mathbf{n}_7}, ? dual(\mathbf{f}_6)), \Delta_5} \underbrace{ \mathbf{Cut} \\ \mathbf{h}_8 :\vdash ? \mathbf{f}_8, ? \mathbf{f$$

• Case rule \$

$$\frac{\underbrace{\frac{\mathbf{h}_2 :\vdash F_3,?\Upsilon 4,!F_5}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3}}_{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,!F_5}? \cdot \frac{\frac{\mathbf{h}_7 :\vdash \Delta_{10},F_8,F_9,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}{\bullet \mathbf{h}_7 :\vdash contract(\mathbf{s}_\mathbf{n}_6,?dual(F_5)),\Delta_{10},F_8\$F_9} \cdot \underbrace{\frac{-:\vdash (\Upsilon 4,?F_3),\Delta_{10},F_8\$F_9}{\to}}_{\mathsf{h}_7 :\vdash \Delta_{10},F_8,F_9,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}}_{\mathsf{h}_{\mathsf{Cut}}} \underbrace{\frac{-:\vdash ?\Upsilon 4,\Delta_{10},F_8,F_9,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}{-:\vdash ?\Upsilon 4,\Delta_{10},?F_3,F_8\$F_9}}_{\mathsf{h}_{\mathsf{Cut}}}$$

 \bullet Case rule &

$$\frac{\frac{\mathbf{h}_2 :\vdash F_3,?\Upsilon 4,!F_5}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3}}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3} ? \frac{\mathbf{h}_7 :\vdash \Delta_{10},F_8,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}{\bullet \mathbf{h}_7 :\vdash contract(\mathbf{s}\mathbf{n}_6,?dual(F_5)),\Delta_{10},F_8\&F_9} \underbrace{-:\vdash (?\Upsilon 4,?F_3),\Delta_{10},F_8\&F_9}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,!F_5,?F_3} \underbrace{-:\vdash ?\Upsilon 4,\Delta_{10},F_8,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}_{\bullet \mathbf{h}\mathbf{Cut}} \underbrace{-:\vdash ?\Upsilon 4,\Delta_{10},F_8,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}_{\bullet \mathbf{h}\mathbf{Cut}} \underbrace{-:\vdash ?\Upsilon 4,\Delta_{10},F_9,?f_3}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,\Delta_{10},F_9,?F_3} \underbrace{\&}$$

• Case rule \oplus_B

$$\frac{\frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, ?\mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash ?\Upsilon 4, !\mathbf{F}_5, ?\mathbf{F}_3} ? \frac{\frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ?dual(\mathbf{F}_5)), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \underbrace{\frac{-}{\mathsf{cut}} \cdot (?\Upsilon 4, ?\mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9}_{\bullet \mathbf{h}_2 : \vdash ?\Upsilon 4, !\mathbf{F}_5, ?\mathbf{F}_3} \underbrace{\frac{-}{\mathsf{c}} \vdash ?\Upsilon 4, \Delta_{10}, \mathbf{F}_9, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}_{\bullet \mathbf{h}_2 : \vdash ?\Upsilon 4, \Delta_{10}, ?\mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9} \underbrace{\frac{-}{\mathsf{c}} \vdash ?\Upsilon 4, \Delta_{10}, ?\mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9}_{\bullet B}}_{\bullet B}$$

• Case rule \oplus_A

$$\frac{\begin{array}{c} \mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_5 \\ \bullet \mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, ?\mathbf{F}_3 \end{array}}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))} \\ \bullet \mathbf{h}_7 : \vdash contract(s\mathbf{n}_6, ?dual(\mathbf{F}_5)), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ & - : \vdash (?\Upsilon 4, ?\mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \bullet \mathbf{h}_2 : \vdash ?\Upsilon 4, !\mathbf{F}_5, ?\mathbf{F}_3 \end{array}} \begin{array}{c} \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))} \\ & - : \vdash ?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3 \\ \hline & - : \vdash ?\Upsilon 4, \Delta_{10}, ?\mathbf{F}_8, ?\mathbf{F}_3 \\ \hline & - : \vdash ?\Upsilon 4, \Delta_{10}, ?\mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9 \end{array} \begin{array}{c} \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{h}_8 : \mathbf{h}_8 : \mathbf{h}_8 : \vdash \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{h}_8 : \mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{h}_8 : \vdash \Delta_{10$$

 \bullet Case rule \bot

$$\frac{\underbrace{\frac{\mathbf{h}_2 :\vdash \mathbf{F}_3,?\Upsilon 4,!\mathbf{F}_5}{\bullet \mathbf{h}_2 :\vdash !\mathbf{F}_5,?\Upsilon 4,?\mathbf{F}_3}}_{\bullet \mathbf{h}_2 :\vdash !\mathbf{F}_5,?\Upsilon 4,?\mathbf{F}_3}? \underbrace{\frac{\mathbf{h}_7 :\vdash \Delta_8,?dual(\mathbf{F}_5),contract(\mathbf{n}_6,?dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 :\vdash contract(\mathbf{s}\mathbf{n}_6,?dual(\mathbf{F}_5)),\bot,\Delta_8}_{\bullet \mathbf{t}} \underbrace{- :\vdash (?\Upsilon 4,?\mathbf{F}_3),\bot,\Delta_8}_{\bullet \mathbf{t}_7 :\vdash \Delta_8,?dual(\mathbf{F}_5),contract(\mathbf{n}_6,?dual(\mathbf{F}_5))}_{\bullet \mathbf{h}\mathsf{Cut}} \underbrace{\frac{- :\vdash ?\Upsilon 4,\Delta_8,?\mathbf{F}_3}{- :\vdash ?\Upsilon 4,\Delta_8,\bot,?\mathbf{F}_3}}_{\bullet \mathsf{h}\mathsf{Cut}}$$

• Case rule \top

$$\frac{ \begin{array}{c} \mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_5 \\ \bullet \mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, ?\mathbf{F}_3 \end{array} ? \quad \overline{\bullet \mathbf{h}_7 : \vdash contract(s\mathbf{n}_6, ?dual(\mathbf{F}_5)), \top, \Delta_8} \quad \overline{\top} \\ - : \vdash (?\Upsilon 4, ?\mathbf{F}_3), \top, \Delta_8 \\ \hline - : \vdash ?\Upsilon 4, \Delta_8, \top, ?\mathbf{F}_3 \end{array} \top \\ \end{array}} \quad \overline{\top}$$

- \bullet Case rule I
- Case rule \otimes

$$\frac{\mathbf{h}_{2} : \vdash \mathbf{F}_{3}, ? \Upsilon 4, ! \mathbf{F}_{5}}{\bullet \mathbf{h}_{2} : \vdash ! \mathbf{F}_{5}, ? \Upsilon 4, ? \mathbf{F}_{3}} ? \frac{\mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, ? dual(\mathbf{F}_{5}), contract(\mathbf{n}_{6}, ? dual(\mathbf{F}_{5})), \Delta_{8}, \Delta_{11}, \mathbf{F}_{9} \otimes \mathbf{F}_{10}}{\bullet \mathbf{h}_{7} : \vdash contract(sn_{6}, ? dual(\mathbf{F}_{5})), \Delta_{8}, \Delta_{11}, \mathbf{F}_{9} \otimes \mathbf{F}_{10}} \underbrace{- : \vdash (? \Upsilon 4, ? \mathbf{F}_{3}), \Delta_{8}, \Delta_{11}, \mathbf{F}_{9} \otimes \mathbf{F}_{10}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, ? dual(\mathbf{F}_{5}), contract(\mathbf{n}_{6}, ? dual(\mathbf{F}_{5}))} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{2} : \vdash ? \Upsilon 4, \mathbf{h}_{11}, \mathbf{F}_{9}, ? \mathbf{f}_{3}} \underbrace{- : \vdash ? \Upsilon 4, \Delta_{11}, \mathbf{F}_{9}, ? \mathbf{f}_{3}}_{\bullet \vdash \mathbf{h}_{2} : \vdash ! \mathbf{f}_{5}, ? \Upsilon 4, ? \mathbf{f}_{5}} ? \underbrace{\mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, ? dual(\mathbf{F}_{5}))}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, ? \mathbf{f}_{3}} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, ? dual(\mathbf{F}_{5}))}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, ? \mathbf{f}_{3}} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, ? dual(\mathbf{F}_{5}))}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, ? \mathbf{f}_{3}} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, ? dual(\mathbf{F}_{5}))}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, ? \mathbf{f}_{3}} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, ? \mathbf{f}_{3}} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, ? \mathbf{f}_{3}} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, ? dual(\mathbf{F}_{5}))}_{\bullet \mathbf{h}_{11}, ? \mathbf{f}_{3}, ? \mathbf{f}_{8} \otimes \mathbf{F}_{9}} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, ? \mathbf{f}_{3}} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, contract(\mathbf{n}_{6}, ? dual(\mathbf{f}_{5})), \Delta_{10}, \Delta_{11}, ? \mathbf{f}_{8} \otimes \mathbf{F}_{9}} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, contract(\mathbf{n}_{6}, ? dual(\mathbf{f}_{5}))} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, contract(\mathbf{n}_{6}, ? dual(\mathbf{f}_{5}))}_{\bullet \mathbf{h}_{7} : \vdash (? \Upsilon 4, \Delta_{10}, \Delta_{11}, ? \mathbf{f}_{8}, ? \mathbf{f}_{3})} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{11}, \mathbf{F}_{9}, contract(\mathbf{n}_{6}, ? dual(\mathbf{f}_{5}))}_{\bullet \mathbf{h}_{7} : \vdash (? \Upsilon 4, \Delta_{10}, \Delta_{$$

$$\frac{\frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ? \Upsilon 4, ! \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3}} ? \frac{\mathbf{h}_7 : \vdash \Delta_8, \mathbf{F}_9 \quad \mathbf{h}_7 : \vdash \Delta_{11}, \mathbf{F}_{10}, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_{16}, ?dual(\mathbf{F}_5)), \Delta_8, \Delta_{11}, \mathbf{F}_9 \otimes \mathbf{F}_{10}}}{\circ \mathbf{h}_7 : \vdash (? \Upsilon 4, ? \mathbf{F}_3), \Delta_8, \Delta_{11}, \mathbf{F}_9 \otimes \mathbf{F}_{10}} \underbrace{\mathsf{Cut}} \\ - : \vdash (? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3) \xrightarrow{\mathsf{ax}} \frac{\rightarrow}{\mathbf{h}_7 : \vdash \Delta_{11}, \mathbf{F}_{10}, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}}{\circ \mathbf{h}_7 : \vdash \Delta_{11}, \mathbf{F}_{10}, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}} \xrightarrow{\mathsf{hCut}} \\ - : \vdash ? \Upsilon 4, \Delta_{11}, \Delta_8, ? \mathbf{F}_3, \mathbf{F}_9 \otimes \mathbf{F}_{10}} \otimes$$

6.6 Status of \$: OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W
- \bullet Case rule ?C
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- \bullet Case rule I
- \bullet Case rule \otimes

6.7 Status of &: OK

- Case rule 1
- Case rule!

•	Case rule ?W	
•	Case rule $?C$	
•	Case rule?	
•	Case rule \$	
•	Case rule &	
•	Case rule \oplus_B	
•	Case rule \oplus_A	
•	Case rule \perp	
•	Case rule \top	
•	Case rule I	
•	Case rule \otimes	
		OK
6.8	Case rule \otimes Status of \oplus_B : Case rule 1	OK
6.8	Status of \oplus_B :	OK
6.8	Status of \bigoplus_B : Case rule 1	OK
6.8	Status of \bigoplus_B : Case rule 1 Case rule !	OK
6.8	Status of \bigoplus_B : Case rule 1 Case rule ! Case rule ? W	OK
•	Status of \bigoplus_B : Case rule 1 Case rule ! Case rule ? W Case rule ? C	OK
6.8	Status of \bigoplus_B : Case rule 1 Case rule ! Case rule ? W Case rule ? C	OK

•	Case rule \top
•	Case rule I
•	Case rule \otimes
6.9	Status of \oplus_A :
•	Case rule 1
•	Case rule!
•	Case rule $?W$
•	Case rule $?C$
•	Case rule ?
•	Case rule \$
•	Case rule &
•	Case rule \oplus_B
•	Case rule \oplus_A

 \bullet Case rule \bot

 $\bullet \;$ Case rule \top

 $\bullet \;$ Case rule I

 $\bullet \;$ Case rule \otimes

OK

• Case rule \oplus_A

 \bullet Case rule \bot

6.10 Status of \perp : OK

- \bullet Case rule $\mathbf{1}$
- Case rule!
- \bullet Case rule ?W
- $\bullet \;$ Case rule ?C
- Case rule?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I
- \bullet Case rule \otimes

6.11 Status of \top : OK

- ullet Case rule $oldsymbol{1}$
- Case rule!
- ullet Case rule ?W
- \bullet Case rule ?C
- Case rule ?

ullet Case rule I		
\bullet Case rule \otimes		
6.12 Status of I: OK • Case rule 1		
• Case rule !		
• Case rule $?W$		
ullet Case rule $?C$		
• Case rule ?		
• Case rule \$		
• Case rule &		
• Case rule \oplus_B		
• Case rule \oplus_A		
\bullet Case rule \bot		
\bullet Case rule \top		
	90	

• Case rule \$

• Case rule &

• Case rule \oplus_B

• Case rule \oplus_A

 \bullet Case rule \bot

 \bullet Case rule \top

- ullet Case rule I
- \bullet Case rule \otimes

6.13 Status of \otimes : OK

- Case rule 1
- Case rule!
- $\bullet \;$ Case rule ?W
- \bullet Case rule ?C
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I
- \bullet Case rule \otimes