

System for Linear Logic

July 6, 2022

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1 Measure of derivations

- Case(s) rule 1

$$\frac{}{\bullet h_1 : \vdash 1} 1 \rightsquigarrow \frac{}{\bullet \bullet h_1 : \vdash 1} 1$$

- Case(s) rule !

$$\frac{h_1 : \vdash F_3, ?\Upsilon 2}{\bullet h_1 : \vdash ?\Upsilon 2, !F_3} ! \rightsquigarrow \frac{\frac{h_1 : \vdash ?\Upsilon 2, F_3}{\bullet h_1 : \vdash ?\Upsilon 2, F_3} \text{IH}}{\bullet \bullet h_1 : \vdash ?\Upsilon 2, !F_3} !$$

- Case(s) rule ?W

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?W \rightsquigarrow \frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, ?F_3} ?W$$

- Case(s) rule ?C

$$\frac{h_1 : \vdash \Delta_2, ?F_3, ?F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?C \rightsquigarrow \frac{\frac{h_1 : \vdash \Delta_2, ?F_3, ?F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3, ?F_3} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, ?F_3} ?C$$

- Case(s) rule ?

$$\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ? \rightsquigarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, ?F_3} ?$$

- Case(s) rule \$

$$\frac{h_1 : \vdash \Delta_2, F_3, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \$F_4} \$ \rightsquigarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3, F_4}{\bullet h_1 : \vdash \Delta_2, F_3, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \$F_4} \$$$

- Case(s) rule &

$$\frac{h_1 : \vdash \Delta_2, F_3 \quad h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \&F_4} \& \rightsquigarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3} \text{IH} \quad \frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \&F_4} \&$$

- Case(s) rule \oplus_B

$$\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B \rightsquigarrow \frac{\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B$$

- Case(s) rule \oplus_A

$$\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A \rightsquigarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A$$

- Case(s) rule \perp

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \rightsquigarrow \frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, \perp} \perp$$

- Case(s) rule \top

$$\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \rightsquigarrow \quad \frac{}{\bullet \bullet h_1 : \vdash \Delta_2, \top} \top$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \vdash p(n_2), \neg(n_2)} I \quad \rightsquigarrow \quad \frac{}{\bullet \bullet h_1 : \vdash p(n_2), \neg(n_2)} I$$

- Case(s) rule \otimes

$$\frac{h_1 : \vdash \Delta_2, F_4 \quad h_1 : \vdash \Delta_3, F_5}{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \otimes \quad \rightsquigarrow \quad \frac{\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_4} \text{IH} \quad \frac{h_1 : \vdash \Delta_3, F_5}{\bullet h_1 : \vdash \Delta_3, F_5} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \otimes$$

2 Weakening on bang: $\vdash \Gamma, !F$ implies $\vdash \Gamma, F$.

- Case(s) rule 1
- Case(s) rule !

$$\frac{h_1 : \vdash F_3, ?\Upsilon 2}{\bullet h_1 : \vdash !F_3, ?\Upsilon 2} ! \rightsquigarrow \frac{\frac{h_1 : \vdash ?\Upsilon 2, F_3}{\bullet h_1 : \vdash ?\Upsilon 2, F_3} \text{ax}}{\bullet h_1 : \vdash ?\Upsilon 2, F_3} \text{H}$$

- Case(s) rule ?W

$$\frac{h_2 : \vdash \Delta_4, !F_1}{\bullet h_2 : \vdash !F_1, \Delta_4, ?F_3} ?W \rightsquigarrow \frac{\frac{\frac{h_2 : \vdash \Delta_4, !F_1}{\bullet h_2 : \vdash \Delta_4, !F_1} \text{ax}}{h_2 : \vdash \Delta_4, F_1} \text{IH}}{\bullet h_2 : \vdash \Delta_4, F_1, ?F_3} ?W$$

- Case(s) rule ?C

$$\frac{h_2 : \vdash \Delta_4, !F_1, ?F_3, ?F_3}{\bullet h_2 : \vdash !F_1, \Delta_4, ?F_3} ?C \rightsquigarrow \frac{\frac{\frac{h_2 : \vdash \Delta_4, !F_1, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, !F_1, ?F_3, ?F_3} \text{ax}}{h_2 : \vdash \Delta_4, F_1, ?F_3, ?F_3} \text{IH}}{\bullet h_2 : \vdash \Delta_4, F_1, ?F_3} ?C$$

- Case(s) rule ?

$$\frac{h_2 : \vdash \Delta_4, F_3, !F_1}{\bullet h_2 : \vdash !F_1, \Delta_4, ?F_3} ? \rightsquigarrow \frac{\frac{\frac{h_2 : \vdash \Delta_4, F_3, !F_1}{\bullet h_2 : \vdash \Delta_4, F_3, !F_1} \text{ax}}{h_2 : \vdash \Delta_4, F_1, F_3} \text{IH}}{\bullet h_2 : \vdash \Delta_4, F_1, ?F_3} ?$$

- Case(s) rule \$

$$\frac{h_2 : \vdash \Delta_5, F_3, F_4, !F_1}{\bullet h_2 : \vdash !F_1, \Delta_5, F_3 \$F_4} \$ \rightsquigarrow \frac{\frac{\frac{h_2 : \vdash \Delta_5, F_3, F_4, !F_1}{\bullet h_2 : \vdash \Delta_5, F_3, F_4, !F_1} \text{ax}}{h_2 : \vdash \Delta_5, F_1, F_3, F_4} \text{IH}}{\bullet h_2 : \vdash \Delta_5, F_1, F_3 \$F_4} \$$$

- Case(s) rule &

$$\frac{h_2 : \vdash \Delta_5, F_3, !F_1 \quad h_2 : \vdash \Delta_5, F_4, !F_1}{\bullet h_2 : \vdash !F_1, \Delta_5, F_3 \&F_4} \& \rightsquigarrow \frac{\frac{\frac{h_2 : \vdash \Delta_5, F_3, !F_1}{\bullet h_2 : \vdash \Delta_5, F_3, !F_1} \text{ax}}{h_2 : \vdash \Delta_5, F_1, F_3} \text{IH} \quad \frac{\frac{h_2 : \vdash \Delta_5, F_4, !F_1}{\bullet h_2 : \vdash \Delta_5, F_4, !F_1} \text{ax}}{h_2 : \vdash \Delta_5, F_1, F_4} \text{IH}}{\bullet h_2 : \vdash \Delta_5, F_1, F_3 \&F_4} \&$$

- Case(s) rule \oplus_B

$$\frac{h_2 : \vdash \Delta_5, F_4, !F_1}{\bullet h_2 : \vdash !F_1, \Delta_5, F_3 \oplus F_4} \oplus_B \rightsquigarrow \frac{\frac{\frac{h_2 : \vdash \Delta_5, F_4, !F_1}{\bullet h_2 : \vdash \Delta_5, F_4, !F_1} \text{ax}}{h_2 : \vdash \Delta_5, F_1, F_4} \text{IH}}{\bullet h_2 : \vdash \Delta_5, F_1, F_3 \oplus F_4} \oplus_B$$

- Case(s) rule \oplus_A

$$\frac{h_2 : \vdash \Delta_5, F_3, !F_1}{\bullet h_2 : \vdash !F_1, \Delta_5, F_3 \oplus F_4} \oplus_A \rightsquigarrow \frac{\frac{\frac{h_2 : \vdash \Delta_5, F_3, !F_1}{\bullet h_2 : \vdash \Delta_5, F_3, !F_1} \text{ax}}{h_2 : \vdash \Delta_5, F_1, F_3} \text{IH}}{\bullet h_2 : \vdash \Delta_5, F_1, F_3 \oplus F_4} \oplus_A$$

- Case(s) rule \perp

$$\frac{h_2 : \vdash \Delta_3, !F_1}{\bullet h_2 : \vdash !F_1, \perp, \Delta_3} \perp \rightsquigarrow \frac{\frac{\frac{h_2 : \vdash \Delta_3, !F_1}{\bullet h_2 : \vdash \Delta_3, !F_1} \text{ax}}{h_2 : \vdash \Delta_3, F_1} \text{IH}}{\bullet h_2 : \vdash \Delta_3, F_1, \perp} \perp$$

- Case(s) rule \top

$$\frac{}{\bullet h_2 : \vdash !F_1, \top, \Delta_3} \top \rightsquigarrow \frac{}{\bullet h_2 : \vdash \Delta_3, F_1, \top} \top$$

- Case(s) rule I
- Case(s) rule \otimes

$$\begin{array}{c} \frac{h_2 : \vdash \Delta_6, F_4, !F_1 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash !F_1, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \rightsquigarrow \frac{\frac{h_2 : \vdash \Delta_6, F_4, !F_1}{h_2 : \vdash \Delta_6, F_1, F_4} \text{IH} \quad \frac{}{h_2 : \vdash \Delta_3, F_5} \text{ax}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes \\[10pt] \frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \Delta_6, F_5, !F_1}{\bullet h_2 : \vdash !F_1, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \rightsquigarrow \frac{\frac{}{h_2 : \vdash \Delta_3, F_4} \text{ax} \quad \frac{\frac{h_2 : \vdash \Delta_6, F_5, !F_1}{h_2 : \vdash \Delta_6, F_1, F_5} \text{IH}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes \end{array}$$

3 Invertibility of Rules

3.1 Status of 1: : Invertible

- Case rule 1

$$\frac{}{\bullet h_1 : \vdash 1} 1 \rightsquigarrow \text{trivial}$$

- Case rule !
- Case rule ?W
- Case rule ?C
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

3.2 Status of !: : Invertible

- Case rule 1
- Case rule !

$$\frac{h_1 : \vdash F_3, ?\Upsilon 2}{\bullet h_1 : \vdash ?\Upsilon 2, !F_3} ! \quad \rightsquigarrow \quad \frac{\overline{h_1 : \vdash ?\Upsilon 2, F_3}^{\text{ax}}}{\bullet h_1 : \vdash ?\Upsilon 2, F_3} \text{H}$$

- Case rule ?W

$$\frac{h_2 : \vdash ?\Upsilon 4, !F_1}{\bullet h_2 : \vdash (? \Upsilon 4, !F_1), ?F_3} ?W \quad \rightsquigarrow \quad \frac{\overline{h_2 : \vdash ?\Upsilon 4, F_1}^{\text{ax/ind}}}{\bullet h_2 : \vdash ?\Upsilon 4, F_1, ?F_3} ?W$$

- Case rule ?C

$$\frac{h_2 : \vdash ?\Upsilon 4, !F_1, ?F_3, ?F_3}{\bullet h_2 : \vdash (? \Upsilon 4, !F_1), ?F_3} ?C \quad \rightsquigarrow \quad \frac{\overline{h_2 : \vdash ?\Upsilon 4, F_1, ?F_3, ?F_3}^{\text{ax/ind}}}{\bullet h_2 : \vdash ?\Upsilon 4, F_1, ?F_3} ?C$$

- Case rule ?

$$\frac{h_2 : \vdash F_3, ?\Upsilon 4, !F_1}{\bullet h_2 : \vdash (? \Upsilon 4, !F_1), ?F_3} ? \quad \rightsquigarrow \quad \frac{\overline{h_2 : \vdash ?\Upsilon 4, F_3, !F_1}^{\text{ax}}}{\bullet h_2 : \vdash ?\Upsilon 4, !F_1, ?F_3} ? \quad \frac{\overline{\bullet h_2 : \vdash ?\Upsilon 4, !F_1, ?F_3}}{\bullet h_2 : \vdash ?\Upsilon 4, F_1, ?F_3} W!$$

- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

3.3 Status of $?W$: : Non invertible

- Case rule 1
- Case rule !

$$\frac{h_2 : \vdash F_3, ?\Upsilon 4, ?F_1}{\bullet h_2 : \vdash (? \Upsilon 4, ?F_1), !F_3} ! \rightsquigarrow \frac{\overline{h_2 : \vdash ?\Upsilon 4, F_3} \text{ ax/ind}}{\bullet h_2 : \vdash ?\Upsilon 4, !F_3} !$$

- Case rule $?W$

$$\frac{h_2 : \vdash \Delta_4, ?F_1}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ?W \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_4} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_4, ?F_3} ?W$$

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?W \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_2} \text{ ax}}{\bullet h_1 : \vdash \Delta_2} H$$

- Case rule $?C$

$$\frac{h_2 : \vdash \Delta_4, ?F_1, ?F_3, ?F_3}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ?C \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_4, ?F_3, ?F_3} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_4, ?F_3} ?C$$

$$\frac{h_1 : \vdash \Delta_2, ?F_3, ?F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?C \rightsquigarrow \frac{\overline{\bullet h_1 : \vdash \Delta_2} \text{ fail}}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?C$$

- Case rule $?$

$$\frac{h_2 : \vdash \Delta_4, F_3, ?F_1}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ? \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_4, F_3} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_4, ?F_3} ?$$

$$\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ? \rightsquigarrow \frac{\overline{\bullet h_1 : \vdash \Delta_2} \text{ fail}}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?$$

- Case rule $\$$

$$\frac{h_2 : \vdash \Delta_5, F_3, F_4, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \$F_4} \$ \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_5, F_3, F_4} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, F_3 \$F_4} \$$$

- Case rule $\&$

$$\frac{h_2 : \vdash \Delta_5, F_3, ?F_1 \quad h_2 : \vdash \Delta_5, F_4, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \&F_4} \& \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_5, F_3} \text{ ax/ind} \quad \overline{h_2 : \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, F_3 \&F_4} \&$$

- Case rule \oplus_B

$$\frac{h_2 : \vdash \Delta_5, F_4, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_B \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, F_3 \oplus F_4} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_2 : \vdash \Delta_5, F_3, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_A \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_5, F_3} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, F_3 \oplus F_4} \oplus_A$$

- Case rule \perp

$$\frac{h_2 : \vdash \Delta_3, ?F_1}{\bullet h_2 : \vdash \perp, \Delta_3, ?F_1} \perp \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_3} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_3, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_2 : \vdash \top, \Delta_3, ?F_1} \top \rightsquigarrow \overline{\bullet h_2 : \vdash \Delta_3, \top} \top$$

- Case rule I

- Case rule \otimes

$$\begin{aligned} \frac{h_2 : \vdash \Delta_6, F_4, ?F_1 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash (\Delta_6, ?F_1), \Delta_3, F_4 \otimes F_5} \otimes &\rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_6, F_4} \text{ ax/ind} \quad \overline{h_2 : \vdash \Delta_3, F_5} \text{ ax}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \\ \frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \Delta_6, F_5, ?F_1}{\bullet h_2 : \vdash \Delta_3, (\Delta_6, ?F_1), F_4 \otimes F_5} \otimes &\rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_3, F_4} \text{ ax} \quad \overline{h_2 : \vdash \Delta_6, F_5} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \end{aligned}$$

3.4 Status of $?C$: : Non invertible

- Case rule 1

- Case rule $!$

$$\frac{h_2 : \vdash F_3, ?\top 4, ?F_1}{\bullet h_2 : \vdash (? \top 4, ?F_1), !F_3} ! \rightsquigarrow \frac{\overline{h_2 : \vdash ? \top 4, F_3, ?F_1, ?F_1} \text{ ax/ind}}{\bullet h_2 : \vdash ? \top 4, !F_3, ?F_1, ?F_1} !$$

- Case rule $?W$

$$\begin{aligned} \frac{h_2 : \vdash \Delta_4, ?F_1}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ?W &\rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_4, ?F_1, ?F_1} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_4, ?F_1, ?F_1, ?F_3} ?W \\ \frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?W &\rightsquigarrow \frac{\overline{\text{fail}}}{\bullet h_1 : \vdash \Delta_2, ?F_3, ?F_3} \text{ fail} \end{aligned}$$

- Case rule $?C$

$$\begin{aligned} \frac{h_2 : \vdash \Delta_4, ?F_1, ?F_3, ?F_3}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ?C &\rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_4, ?F_1, ?F_1, ?F_3, ?F_3} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_4, ?F_1, ?F_1, ?F_3} ?C \\ \frac{h_1 : \vdash \Delta_2, ?F_3, ?F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?C &\rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_2, ?F_3, ?F_3, ?F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_2, ?F_3, ?F_3} ?C \end{aligned}$$

- Case rule ?

$$\frac{h_2 : \vdash \Delta_4, F_3, ?F_1}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ? \rightsquigarrow \frac{\frac{h_2 : \vdash \Delta_4, F_3, ?F_1}{\bullet h_2 : \vdash \Delta_4, ?F_1, ?F_1} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_4, ?F_1, ?F_3} ?$$

$$\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ? \rightsquigarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} \text{ fail}}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?$$

- Case rule \$

$$\frac{h_2 : \vdash \Delta_5, F_3, F_4, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \$F_4} \$ \rightsquigarrow \frac{\frac{h_2 : \vdash \Delta_5, F_3, F_4, ?F_1}{\bullet h_2 : \vdash \Delta_5, ?F_1, ?F_1, ?F_1} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, ?F_1, ?F_1, F_3 \$F_4} \$$$

- Case rule &

$$\frac{h_2 : \vdash \Delta_5, F_3, ?F_1 \quad h_2 : \vdash \Delta_5, F_4, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \&F_4} \& \rightsquigarrow \frac{\frac{h_2 : \vdash \Delta_5, F_3, ?F_1}{\bullet h_2 : \vdash \Delta_5, ?F_1, ?F_1, F_3 \&F_4} \text{ ax/ind} \quad \frac{h_2 : \vdash \Delta_5, F_4, ?F_1}{\bullet h_2 : \vdash \Delta_5, ?F_1, ?F_1, F_3 \&F_4} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, ?F_1, ?F_1, F_3 \&F_4} \&$$

- Case rule \oplus_B

$$\frac{h_2 : \vdash \Delta_5, F_4, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_B \rightsquigarrow \frac{\frac{h_2 : \vdash \Delta_5, F_4, ?F_1}{\bullet h_2 : \vdash \Delta_5, ?F_1, ?F_1, F_3 \oplus F_4} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, ?F_1, ?F_1, F_3 \oplus F_4} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_2 : \vdash \Delta_5, F_3, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_A \rightsquigarrow \frac{\frac{h_2 : \vdash \Delta_5, F_3, ?F_1}{\bullet h_2 : \vdash \Delta_5, ?F_1, ?F_1, F_3 \oplus F_4} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, ?F_1, ?F_1, F_3 \oplus F_4} \oplus_A$$

- Case rule \perp

$$\frac{h_2 : \vdash \Delta_3, ?F_1}{\bullet h_2 : \vdash \perp, \Delta_3, ?F_1} \perp \rightsquigarrow \frac{\frac{h_2 : \vdash \Delta_3, ?F_1}{\bullet h_2 : \vdash \Delta_3, \perp, ?F_1, ?F_1} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_3, \perp, ?F_1, ?F_1} \perp$$

- Case rule \top

$$\frac{}{\bullet h_2 : \vdash \top, \Delta_3, ?F_1} \top \rightsquigarrow \frac{}{\bullet h_2 : \vdash \Delta_3, \top, ?F_1, ?F_1} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_2 : \vdash \Delta_6, F_4, ?F_1 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash (\Delta_6, ?F_1), \Delta_3, F_4 \otimes F_5} \otimes \rightsquigarrow \frac{\frac{h_2 : \vdash \Delta_6, F_4, ?F_1}{\bullet h_2 : \vdash \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \text{ ax/ind} \quad \frac{h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \text{ ax}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \otimes$$

$$\frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \Delta_6, F_5, ?F_1}{\bullet h_2 : \vdash \Delta_3, (\Delta_6, ?F_1), F_4 \otimes F_5} \otimes \rightsquigarrow \frac{\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \text{ ax} \quad \frac{h_2 : \vdash \Delta_6, F_5, ?F_1}{\bullet h_2 : \vdash \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \otimes$$

3.5 Status of $? :$: Non invertible

- Case rule 1

- Case rule !

$$\frac{h_2 : \vdash F_3, ?\Upsilon 4, ?F_1}{\bullet h_2 : \vdash (? \Upsilon 4, ?F_1), !F_3} ! \rightsquigarrow \frac{}{\bullet h_2 : \vdash F_1, ?\Upsilon 4, !F_3} \text{fail}$$

- Case rule $?W$

$$\frac{h_2 : \vdash \Delta_4, ?F_1}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ?W \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_4, F_1} \text{ax/ind}}{\bullet h_2 : \vdash \Delta_4, F_1, ?F_3} ?W$$

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?W \rightsquigarrow \frac{}{\bullet h_1 : \vdash \Delta_2, F_3} \text{fail}$$

- Case rule $?C$

$$\frac{h_2 : \vdash \Delta_4, ?F_1, ?F_3, ?F_3}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ?C \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_4, F_1, ?F_3, ?F_3} \text{ax/ind}}{\bullet h_2 : \vdash \Delta_4, F_1, ?F_3} ?C$$

$$\frac{h_1 : \vdash \Delta_2, ?F_3, ?F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?C \rightsquigarrow \frac{}{\bullet h_1 : \vdash \Delta_2, F_3} \text{fail}$$

- Case rule $?$

$$\frac{h_2 : \vdash \Delta_4, F_3, ?F_1}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ? \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_4, F_1, F_3} \text{ax/ind}}{\bullet h_2 : \vdash \Delta_4, F_1, ?F_3} ?$$

$$\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ? \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_2, F_3} \text{ax}}{\bullet h_1 : \vdash \Delta_2, F_3} H$$

- Case rule $\$$

$$\frac{h_2 : \vdash \Delta_5, F_3, F_4, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \$F_4} \$ \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_5, F_1, F_3, F_4} \text{ax/ind}}{\bullet h_2 : \vdash \Delta_5, F_1, F_3 \$F_4} \$$$

- Case rule $\&$

$$\frac{h_2 : \vdash \Delta_5, F_3, ?F_1 \quad h_2 : \vdash \Delta_5, F_4, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \&F_4} \& \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_5, F_1, F_3} \text{ax/ind} \quad \overline{h_2 : \vdash \Delta_5, F_1, F_4} \text{ax/ind}}{\bullet h_2 : \vdash \Delta_5, F_1, F_3 \&F_4} \&$$

- Case rule \oplus_B

$$\frac{h_2 : \vdash \Delta_5, F_4, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_B \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_5, F_1, F_4} \text{ax/ind}}{\bullet h_2 : \vdash \Delta_5, F_1, F_3 \oplus F_4} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_2 : \vdash \Delta_5, F_3, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_A \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_5, F_1, F_3} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, F_1, F_3 \oplus F_4} \oplus_A$$

- Case rule \perp

$$\frac{h_2 : \vdash \Delta_3, ?F_1}{\bullet h_2 : \vdash \perp, \Delta_3, ?F_1} \perp \rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_3, F_1} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_3, F_1, \perp} \perp$$

- Case rule \top

$$\frac{\overline{\bullet h_2 : \vdash \top, \Delta_3, ?F_1}}{\bullet h_2 : \vdash \top, \Delta_3, ?F_1} \top \rightsquigarrow \frac{\overline{\bullet h_2 : \vdash \Delta_3, F_1, \top}}{\bullet h_2 : \vdash \Delta_3, F_1, \top} \top$$

- Case rule I

- Case rule \otimes

$$\begin{aligned} \frac{h_2 : \vdash \Delta_6, F_4, ?F_1 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash (\Delta_6, ?F_1), \Delta_3, F_4 \otimes F_5} \otimes &\rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_2 : \vdash \Delta_3, F_5} \text{ ax}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes \\ \frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \Delta_6, F_5, ?F_1}{\bullet h_2 : \vdash \Delta_3, (\Delta_6, ?F_1), F_4 \otimes F_5} \otimes &\rightsquigarrow \frac{\overline{h_2 : \vdash \Delta_3, F_4} \text{ ax} \quad \overline{h_2 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes \end{aligned}$$

3.6 Status of \$: : Invertible

- Case rule 1

- Case rule !

- Case rule ?W

$$\frac{h_3 : \vdash \Delta_5, F_1 \$ F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \$ F_2), ?F_4} ?W \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, F_2, ?F_4} ?W$$

- Case rule ?C

$$\frac{h_3 : \vdash \Delta_5, ?F_4, ?F_4, F_1 \$ F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \$ F_2), ?F_4} ?C \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1, F_2, ?F_4, ?F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, F_2, ?F_4} ?C$$

- Case rule ?

$$\frac{h_3 : \vdash \Delta_5, F_4, F_1 \$ F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \$ F_2), ?F_4} ? \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1, F_2, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, F_2, ?F_4} ?$$

- Case rule \$

$$\frac{h_3 \vdash \Delta_6, F_4, F_5, F_1 \$F_2}{\bullet h_3 \vdash (\Delta_6, F_1 \$F_2), F_4 \$F_5} \$ \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_6, F_1, F_2, F_4, F_5}}{\bullet h_3 \vdash \Delta_6, F_1, F_2, F_4 \$F_5} \text{ax/ind} \$$$

$$\frac{h_1 \vdash \Delta_2, F_3, F_4}{\bullet h_1 \vdash \Delta_2, F_3 \$F_4} \$ \rightsquigarrow \frac{\overline{h_1 \vdash \Delta_2, F_3, F_4}}{\bullet h_1 \vdash \Delta_2, F_3, F_4} \text{ax} \text{H}$$

- Case rule &

$$\frac{h_3 \vdash \Delta_6, F_4, F_1 \$F_2 \quad h_3 \vdash \Delta_6, F_5, F_1 \$F_2}{\bullet h_3 \vdash (\Delta_6, F_1 \$F_2), F_4 \&F_5} \& \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_6, F_1, F_2, F_4} \text{ax/ind} \quad \overline{h_3 \vdash \Delta_6, F_1, F_2, F_5} \text{ax/ind}}{\bullet h_3 \vdash \Delta_6, F_1, F_2, F_4 \&F_5} \&$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \Delta_6, F_5, F_1 \$F_2}{\bullet h_3 \vdash (\Delta_6, F_1 \$F_2), F_4 \oplus F_5} \oplus_B \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_6, F_1, F_2, F_5} \text{ax/ind}}{\bullet h_3 \vdash \Delta_6, F_1, F_2, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \vdash \Delta_6, F_4, F_1 \$F_2}{\bullet h_3 \vdash (\Delta_6, F_1 \$F_2), F_4 \oplus F_5} \oplus_A \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_6, F_1, F_2, F_4} \text{ax/ind}}{\bullet h_3 \vdash \Delta_6, F_1, F_2, F_4 \oplus F_5} \oplus_A$$

- Case rule \perp

$$\frac{h_3 \vdash \Delta_4, F_1 \$F_2}{\bullet h_3 \vdash \perp, \Delta_4, F_1 \$F_2} \perp \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_4, F_1, F_2} \text{ax/ind}}{\bullet h_3 \vdash \Delta_4, F_1, F_2, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 \vdash \top, \Delta_4, F_1 \$F_2} \top \rightsquigarrow \frac{}{\bullet h_3 \vdash \Delta_4, F_1, F_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 \vdash \Delta_7, F_5, F_1 \$F_2 \quad h_3 \vdash \Delta_4, F_6}{\bullet h_3 \vdash (\Delta_7, F_1 \$F_2), \Delta_4, F_5 \otimes F_6} \otimes \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_7, F_1, F_2, F_5} \text{ax/ind} \quad \overline{h_3 \vdash \Delta_4, F_6} \text{ax}}{\bullet h_3 \vdash \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6} \otimes$$

$$\frac{h_3 \vdash \Delta_4, F_5 \quad h_3 \vdash \Delta_7, F_6, F_1 \$F_2}{\bullet h_3 \vdash \Delta_4, (\Delta_7, F_1 \$F_2), F_5 \otimes F_6} \otimes \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_4, F_5} \text{ax} \quad \overline{h_3 \vdash \Delta_7, F_1, F_2, F_6} \text{ax/ind}}{\bullet h_3 \vdash \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6} \otimes$$

3.7 Status of &: (Left Premise): Invertible

- Case rule 1
- Case rule !
- Case rule ?W

$$\frac{h_3 : \vdash \Delta_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \& F_2), ?F_4} ?W \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, ?F_4} ?W$$

- Case rule ?C

$$\frac{h_3 : \vdash \Delta_5, ?F_4, ?F_4, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \& F_2), ?F_4} ?C \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1, ?F_4, ?F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, ?F_4} ?C$$

- Case rule ?

$$\frac{h_3 : \vdash \Delta_5, F_4, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \& F_2), ?F_4} ? \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, ?F_4} ?$$

- Case rule \$

$$\frac{h_3 : \vdash \Delta_6, F_4, F_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \$ F_5} \$ \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \$ F_5} \$$$

- Case rule &

$$\frac{h_3 : \vdash \Delta_6, F_4, F_1 \& F_2 \quad h_3 : \vdash \Delta_6, F_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \& F_5} \& \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \& F_5} \&$$

$$\frac{h_1 : \vdash \Delta_2, F_3 \quad h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \& F_4} \& \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_2, F_3} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, F_3} \text{ H}$$

- Case rule \oplus_B

$$\frac{h_3 : \vdash \Delta_6, F_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_B \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash \Delta_6, F_4, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_A \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_A$$

- Case rule \perp

$$\frac{h_3 : \vdash \Delta_4, F_1 \& F_2}{\bullet h_3 : \vdash \perp, \Delta_4, F_1 \& F_2} \perp \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_4, F_1} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, F_1, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 : \vdash \top, \Delta_4, F_1 \& F_2} \top \rightsquigarrow \frac{}{\bullet h_3 : \vdash \Delta_4, F_1, \top} \top$$

- Case rule I

- Case rule \otimes

$$\begin{aligned} \frac{h_3 : \vdash \Delta_7, F_5, F_1 \& F_2 \quad h_3 : \vdash \Delta_4, F_6}{\bullet h_3 : \vdash (\Delta_7, F_1 \& F_2), \Delta_4, F_5 \otimes F_6} \otimes &\rightsquigarrow \frac{\frac{}{h_3 : \vdash \Delta_7, F_1, F_5} \text{ax/ind} \quad \frac{}{h_3 : \vdash \Delta_4, F_6} \text{ax}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes \\ \frac{h_3 : \vdash \Delta_4, F_5 \quad h_3 : \vdash \Delta_7, F_6, F_1 \& F_2}{\bullet h_3 : \vdash \Delta_4, (\Delta_7, F_1 \& F_2), F_5 \otimes F_6} \otimes &\rightsquigarrow \frac{\frac{}{h_3 : \vdash \Delta_4, F_5} \text{ax} \quad \frac{}{h_3 : \vdash \Delta_7, F_1, F_6} \text{ax/ind}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes \end{aligned}$$

3.8 Status of $\&$ (Right Premise): : Invertible

- Case rule 1
- Case rule $!$
- Case rule $?W$

$$\frac{h_3 : \vdash \Delta_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \& F_2), ?F_4} ?W \rightsquigarrow \frac{\frac{}{h_3 : \vdash \Delta_5, F_2} \text{ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_2, ?F_4} ?W$$

- Case rule $?C$

$$\frac{h_3 : \vdash \Delta_5, ?F_4, ?F_4, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \& F_2), ?F_4} ?C \rightsquigarrow \frac{\frac{}{h_3 : \vdash \Delta_5, F_2, ?F_4, ?F_4} \text{ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_2, ?F_4} ?C$$

- Case rule $?$

$$\frac{h_3 : \vdash \Delta_5, F_4, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \& F_2), ?F_4} ? \rightsquigarrow \frac{\frac{}{h_3 : \vdash \Delta_5, F_2, F_4} \text{ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_2, ?F_4} ?$$

- Case rule $\$$

$$\frac{h_3 : \vdash \Delta_6, F_4, F_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \$ F_5} \$ \rightsquigarrow \frac{\frac{}{h_3 : \vdash \Delta_6, F_2, F_4, F_5} \text{ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \$ F_5} \$$$

- Case rule $\&$

$$\frac{h_3 : \vdash \Delta_6, F_4, F_1 \& F_2 \quad h_3 : \vdash \Delta_6, F_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \& F_5} \& \rightsquigarrow \frac{\frac{}{h_3 : \vdash \Delta_6, F_2, F_4} \text{ax/ind} \quad \frac{}{h_3 : \vdash \Delta_6, F_2, F_5} \text{ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \& F_5} \&$$

$$\frac{h_1 : \vdash \Delta_2, F_3 \quad h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \& F_4} \& \rightsquigarrow \frac{\frac{}{h_1 : \vdash \Delta_2, F_4} \text{ax}}{\bullet h_1 : \vdash \Delta_2, F_4} H$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \Delta_6, F_5, F_1 \& F_2}{\bullet h_3 \vdash (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_B \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_6, F_2, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \vdash \Delta_6, F_4, F_1 \& F_2}{\bullet h_3 \vdash (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_A \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_6, F_2, F_4} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_6, F_2, F_4 \oplus F_5} \oplus_A$$

- Case rule \perp

$$\frac{h_3 \vdash \Delta_4, F_1 \& F_2}{\bullet h_3 \vdash \perp, \Delta_4, F_1 \& F_2} \perp \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_4, F_2} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_4, F_2, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_3 \vdash \top, \Delta_4, F_1 \& F_2} \top \rightsquigarrow \overline{\bullet h_3 \vdash \Delta_4, F_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\begin{aligned} \frac{h_3 \vdash \Delta_7, F_5, F_1 \& F_2 \quad h_3 \vdash \Delta_4, F_6}{\bullet h_3 \vdash (\Delta_7, F_1 \& F_2), \Delta_4, F_5 \otimes F_6} \otimes &\rightsquigarrow \frac{\overline{h_3 \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 \vdash \Delta_4, F_6} \text{ ax}}{\bullet h_3 \vdash \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes \\ \frac{h_3 \vdash \Delta_4, F_5 \quad h_3 \vdash \Delta_7, F_6, F_1 \& F_2}{\bullet h_3 \vdash \Delta_4, (\Delta_7, F_1 \& F_2), F_5 \otimes F_6} \otimes &\rightsquigarrow \frac{\overline{h_3 \vdash \Delta_4, F_5} \text{ ax} \quad \overline{h_3 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes \end{aligned}$$

3.9 Status of \oplus_B : Non invertible

- Case rule 1
- Case rule $!$
- Case rule $?W$

$$\frac{h_3 \vdash \Delta_5, F_1 \oplus F_2}{\bullet h_3 \vdash (\Delta_5, F_1 \oplus F_2), ?F_4} ?W \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_5, F_2} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_5, F_2, ?F_4} ?W$$

- Case rule $?C$

$$\frac{h_3 \vdash \Delta_5, ?F_4, ?F_4, F_1 \oplus F_2}{\bullet h_3 \vdash (\Delta_5, F_1 \oplus F_2), ?F_4} ?C \rightsquigarrow \frac{\overline{h_3 \vdash \Delta_5, F_2, ?F_4, ?F_4} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_5, F_2, ?F_4} ?C$$

- Case rule ?

$$\frac{h_3 : \vdash \Delta_5, F_4, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \oplus F_2), ?F_4} ? \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_2, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_2, ?F_4} ?$$

- Case rule \$

$$\frac{h_3 : \vdash \Delta_6, F_4, F_5, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \$F_5} \$ \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_4, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \$F_5} \$$$

- Case rule &

$$\frac{h_3 : \vdash \Delta_6, F_4, F_1 \oplus F_2 \quad h_3 : \vdash \Delta_6, F_5, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \&F_5} \& \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_4} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \&F_5} \&$$

- Case rule \oplus_B

$$\frac{h_3 : \vdash \Delta_6, F_5, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_B \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \oplus F_5} \oplus_B$$

$$\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B \rightsquigarrow \frac{h_1 : \vdash \Delta_2, F_4 \text{ ax}}{\bullet h_1 : \vdash \Delta_2, F_4} H$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash \Delta_6, F_4, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_A \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \oplus F_5} \oplus_A$$

$$\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A \rightsquigarrow \frac{\bullet h_1 : \vdash \Delta_2, F_4 \text{ fail}}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A$$

- Case rule \perp

$$\frac{h_3 : \vdash \Delta_4, F_1 \oplus F_2}{\bullet h_3 : \vdash \perp, \Delta_4, F_1 \oplus F_2} \perp \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_4, F_2} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, F_2, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_3 : \vdash \top, \Delta_4, F_1 \oplus F_2} \top \rightsquigarrow \overline{\bullet h_3 : \vdash \Delta_4, F_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 : \vdash \Delta_7, F_5, F_1 \oplus F_2 \quad h_3 : \vdash \Delta_4, F_6}{\bullet h_3 : \vdash (\Delta_7, F_1 \oplus F_2), \Delta_4, F_5 \otimes F_6} \otimes \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_4, F_6} \text{ ax}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes$$

$$\frac{h_3 : \vdash \Delta_4, F_5 \quad h_3 : \vdash \Delta_7, F_6, F_1 \oplus F_2}{\bullet h_3 : \vdash \Delta_4, (\Delta_7, F_1 \oplus F_2), F_5 \otimes F_6} \otimes \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_4, F_5} \text{ ax} \quad \overline{h_3 : \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes$$

3.10 Status of \oplus_A : : Non invertible

- Case rule 1
- Case rule !
- Case rule ?W

$$\frac{h_3 : \vdash \Delta_5, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \oplus F_2), ?F_4} ?W \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, ?F_4} ?W$$

- Case rule ?C

$$\frac{h_3 : \vdash \Delta_5, ?F_4, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \oplus F_2), ?F_4} ?C \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1, ?F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, ?F_4} ?C$$

- Case rule ?

$$\frac{h_3 : \vdash \Delta_5, F_4, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \oplus F_2), ?F_4} ? \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, ?F_4} ?$$

- Case rule \$

$$\frac{h_3 : \vdash \Delta_6, F_4, F_5, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \$F_5} \$ \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \$F_5} \$$$

- Case rule &

$$\frac{h_3 : \vdash \Delta_6, F_4, F_1 \oplus F_2 \quad h_3 : \vdash \Delta_6, F_5, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \&F_5} \& \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \&F_5} \&$$

- Case rule \oplus_B

$$\frac{h_3 : \vdash \Delta_6, F_5, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_B \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_B$$

$$\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B \rightsquigarrow \frac{\overline{\bullet h_1 : \vdash \Delta_2, F_3} \text{ fail}}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash \Delta_6, F_4, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_A \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_A$$

$$\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_2, F_3} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, F_3} \text{ H}$$

- Case rule \perp

$$\frac{h_3 : \vdash \Delta_4, F_1 \oplus F_2}{\bullet h_3 : \vdash \perp, \Delta_4, F_1 \oplus F_2} \perp \rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_4, F_1} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, F_1, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_3 : \vdash \top, \Delta_4, F_1 \oplus F_2} \top \rightsquigarrow \overline{\bullet h_3 : \vdash \Delta_4, F_1, \top} \top$$

- Case rule I

- Case rule \otimes

$$\begin{aligned} \frac{h_3 : \vdash \Delta_7, F_5, F_1 \oplus F_2 \quad h_3 : \vdash \Delta_4, F_6}{\bullet h_3 : \vdash (\Delta_7, F_1 \oplus F_2), \Delta_4, F_5 \otimes F_6} \otimes &\rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_7, F_1, F_5} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_4, F_6} \text{ ax}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes \\ \frac{h_3 : \vdash \Delta_4, F_5 \quad h_3 : \vdash \Delta_7, F_6, F_1 \oplus F_2}{\bullet h_3 : \vdash \Delta_4, (\Delta_7, F_1 \oplus F_2), F_5 \otimes F_6} \otimes &\rightsquigarrow \frac{\overline{h_3 : \vdash \Delta_4, F_5} \text{ ax} \quad \overline{h_3 : \vdash \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes \end{aligned}$$

3.11 Status of \perp : Invertible

- Case rule 1
- Case rule !
- Case rule $?W$

$$\frac{h_1 : \vdash \perp, \Delta_3}{\bullet h_1 : \vdash (\perp, \Delta_3), ?F_2} ?W \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_3, ?F_2} ?W$$

- Case rule $?C$

$$\frac{h_1 : \vdash \perp, \Delta_3, ?F_2, ?F_2}{\bullet h_1 : \vdash (\perp, \Delta_3), ?F_2} ?C \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_3, ?F_2, ?F_2} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_3, ?F_2} ?C$$

- Case rule $?$

$$\frac{h_1 : \vdash \perp, \Delta_3, F_2}{\bullet h_1 : \vdash (\perp, \Delta_3), ?F_2} ? \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_3, F_2} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_3, ?F_2} ?$$

- Case rule $\$$

$$\frac{h_1 : \vdash \perp, \Delta_4, F_2, F_3}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \$F_3} \$ \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_4, F_2, F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \$F_3} \$$$

- Case rule $\&$

$$\frac{h_1 : \vdash \perp, \Delta_4, F_2 \quad h_1 : \vdash \perp, \Delta_4, F_3}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \& F_3} \& \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_4, F_2} \text{ ax/ind} \quad \overline{h_1 : \vdash \Delta_4, F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \& F_3} \&$$

- Case rule \oplus_B

$$\frac{h_1 : \vdash \perp, \Delta_4, F_3}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \oplus F_3} \oplus_B \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_4, F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \oplus F_3} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_1 : \vdash \perp, \Delta_4, F_2}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \oplus F_3} \oplus_A \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_4, F_2} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \oplus F_3} \oplus_A$$

- Case rule \perp

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_2} \text{ ax}}{\bullet h_1 : \vdash \Delta_2} \text{H}$$

- Case rule \top

$$\overline{\bullet h_1 : \vdash \top, \perp, \Delta_2} \top \rightsquigarrow \overline{\bullet h_1 : \vdash \Delta_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_1 : \vdash \perp, \Delta_5, F_3 \quad h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash (\perp, \Delta_5), \Delta_2, F_3 \otimes F_4} \otimes \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_5, F_3} \text{ ax/ind} \quad \overline{h_1 : \vdash \Delta_2, F_4} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \otimes$$

$$\frac{h_1 : \vdash \Delta_2, F_3 \quad h_1 : \vdash \perp, \Delta_5, F_4}{\bullet h_1 : \vdash \Delta_2, (\perp, \Delta_5), F_3 \otimes F_4} \otimes \rightsquigarrow \frac{\overline{h_1 : \vdash \Delta_2, F_3} \text{ ax} \quad \overline{h_1 : \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \otimes$$

3.12 Status of \top : Invertible

- Case rule 1

- Case rule $!$

- Case rule $?W$

$$\frac{h_1 : \vdash \top, \Delta_3}{\bullet h_1 : \vdash (\top, \Delta_3), ?F_2} ?W \rightsquigarrow \text{trivial}$$

- Case rule $?C$

$$\frac{h_1 : \vdash \top, \Delta_3, ?F_2, ?F_2}{\bullet h_1 : \vdash (\top, \Delta_3), ?F_2} ?C \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule $?$

$$\frac{h_1 : \vdash \top, \Delta_3, F_2}{\bullet h_1 : \vdash (\top, \Delta_3), ?F_2} ? \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule $\$$

$$\frac{h_1 : \vdash \top, \Delta_4, F_2, F_3}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \$F_3} \$ \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule $\&$

$$\frac{h_1 : \vdash \top, \Delta_4, F_2 \quad h_1 : \vdash \top, \Delta_4, F_3}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \&F_3} \& \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule \oplus_B

$$\frac{h_1 : \vdash \top, \Delta_4, F_3}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \oplus F_3} \oplus_B \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule \oplus_A

$$\frac{h_1 : \vdash \top, \Delta_4, F_2}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \oplus F_3} \oplus_A \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule \perp

$$\frac{h_1 : \vdash \top, \Delta_2}{\bullet h_1 : \vdash \perp, \top, \Delta_2} \perp \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule \top

$$\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule I

- Case rule \otimes

$$\frac{h_1 : \vdash \top, \Delta_5, F_3 \quad h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash (\top, \Delta_5), \Delta_2, F_3 \otimes F_4} \otimes \quad \rightsquigarrow \quad \text{trivial}$$

$$\frac{h_1 : \vdash \Delta_2, F_3 \quad h_1 : \vdash \top, \Delta_5, F_4}{\bullet h_1 : \vdash \Delta_2, (\top, \Delta_5), F_3 \otimes F_4} \otimes \quad \rightsquigarrow \quad \text{trivial}$$

3.13 Status of I : : Invertible

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I

$$\frac{}{\bullet h_1 : \vdash p(n_2), \sim(n_2)} I \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule \otimes

3.14 Status of \otimes : (Left Premise): Non invertible

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?W \quad \rightsquigarrow \quad \frac{\frac{}{h_4 : \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_6, F_2, ?F_5} ?W$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?W \quad \rightsquigarrow \quad \frac{\frac{}{h_4 : \vdash \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} H$$

- Case rule $?C$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, ?F_5, ?F_5, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?C \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_6, F_2, ?F_5} ?W$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, ?F_5, ?F_5, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?C \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} H$$

- Case rule $?$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_5, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ? \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_6, F_2, ?F_5} ?W$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_5, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ? \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} H$$

- Case rule $\$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_7, F_5, F_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$F_6} \$ \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_7, F_2, F_5, F_6} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_7, F_2, F_5 \$F_6} \$$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_7, F_5, F_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$F_6} \$ \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} H$$

- Case rule $\&$

$$\frac{h_4 : \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3 \quad h_4 : \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \&F_6} \& \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_4 : \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_7, F_2, F_5 \&F_6} \&$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3 \quad h_4 : \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \&F_6} \& \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} H$$

- Case rule \oplus_B

$$\frac{h_4 : \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_7, F_2, F_5 \oplus F_6} \oplus_B$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} H$$

- Case rule \oplus_A

$$\frac{h_4 : \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_7, F_2, F_5} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_7, F_2, F_5 \oplus F_6} \oplus_A$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} H$$

- Case rule \perp

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_5, F_2}}{\bullet h_4 : \vdash \Delta_5, F_2, \perp} \perp \text{ ax/ind}$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2}}{\bullet h_4 : \vdash \Delta_1, F_2} \text{ H ax/ind}$$

- Case rule \top

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightsquigarrow \frac{}{\bullet h_4 : \vdash \Delta_5, F_2, \top} \top$$

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightsquigarrow \frac{}{\bullet h_4 : \vdash \Delta_1, F_2} \text{ fail}$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightsquigarrow \frac{}{\bullet h_3 : \vdash \Delta_6, \Delta_8, F_1, F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightsquigarrow \frac{}{\bullet h_3 : \vdash \Delta_6, \Delta_8, F_1, F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightsquigarrow \frac{}{\bullet h_3 : \vdash \Delta_6, \Delta_8, F_1} \text{ fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightsquigarrow \frac{}{\bullet h_3 : \vdash \Delta_6, \Delta_8, F_1} \text{ fail}$$

$$\frac{h_1 : \vdash \Delta_4, \Delta_5, F_2 \quad h_1 : \vdash \Delta_6, \Delta_7, F_3}{\bullet h_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), F_2 \otimes F_3} \otimes \rightsquigarrow \frac{}{\bullet h_1 : \vdash \Delta_4, \Delta_6, F_2} \text{ fail}$$

3.15 Status of \otimes (Right Premise): : Non invertible

- Case rule 1
- Case rule $!$
- Case rule $?W$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?W \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_1, F_3}}{\bullet h_4 : \vdash \Delta_1, F_3} \text{ H ax/ind}$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?W \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_6, F_3}}{\bullet h_4 : \vdash \Delta_6, F_3, ?F_5} ?W \text{ ax/ind}$$

- Case rule $?C$

$$\frac{h_4 \vdash \Delta_1, \Delta_6, ?F_5, ?F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?C \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_1, F_3}}{\bullet h_4 \vdash \Delta_1, F_3} \text{H} \text{ ax/ind}$$

$$\frac{h_4 \vdash \Delta_1, \Delta_6, ?F_5, ?F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?C \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_6, F_3}}{\bullet h_4 \vdash \Delta_6, F_3, ?F_5} ?W \text{ ax/ind}$$

- Case rule $?$

$$\frac{h_4 \vdash \Delta_1, \Delta_6, F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ? \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_1, F_3}}{\bullet h_4 \vdash \Delta_1, F_3} \text{H} \text{ ax/ind}$$

$$\frac{h_4 \vdash \Delta_1, \Delta_6, F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ? \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_6, F_3}}{\bullet h_4 \vdash \Delta_6, F_3, ?F_5} ?W \text{ ax/ind}$$

- Case rule $\$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$F_6} \$ \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_1, F_3}}{\bullet h_4 \vdash \Delta_1, F_3} \text{H} \text{ ax/ind}$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$F_6} \$ \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_7, F_3, F_5, F_6}}{\bullet h_4 \vdash \Delta_7, F_3, F_5 \$F_6} \$ \text{ ax/ind}$$

- Case rule $\&$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3 \quad h_4 \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \&F_6} \& \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_1, F_3}}{\bullet h_4 \vdash \Delta_1, F_3} \text{H} \text{ ax/ind}$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3 \quad h_4 \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \&F_6} \& \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_7, F_3, F_5} \quad \overline{h_4 \vdash \Delta_7, F_3, F_6}}{\bullet h_4 \vdash \Delta_7, F_3, F_5 \&F_6} \& \text{ ax/ind}$$

- Case rule \oplus_B

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_1, F_3}}{\bullet h_4 \vdash \Delta_1, F_3} \text{H} \text{ ax/ind}$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_7, F_3, F_6}}{\bullet h_4 \vdash \Delta_7, F_3, F_5 \oplus F_6} \oplus_B \text{ ax/ind}$$

- Case rule \oplus_A

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_1, F_3}}{\bullet h_4 \vdash \Delta_1, F_3} \text{H} \text{ ax/ind}$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightsquigarrow \frac{\overline{h_4 \vdash \Delta_7, F_3, F_5}}{\bullet h_4 \vdash \Delta_7, F_3, F_5 \oplus F_6} \oplus_A \text{ ax/ind}$$

- Case rule \perp

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_1, F_3} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_3} \text{ H}$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightsquigarrow \frac{\overline{h_4 : \vdash \Delta_5, F_3} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_5, F_3, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightsquigarrow \frac{}{\bullet h_4 : \vdash \Delta_1, F_3} \text{ fail}$$

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightsquigarrow \frac{}{\bullet h_4 : \vdash \Delta_5, F_3, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightsquigarrow \frac{}{\bullet h_3 : \vdash \Delta_7, \Delta_9, F_2} \text{ fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightsquigarrow \frac{}{\bullet h_3 : \vdash \Delta_7, \Delta_9, F_2} \text{ fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightsquigarrow \frac{}{\bullet h_3 : \vdash \Delta_7, \Delta_9, F_2, F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightsquigarrow \frac{}{\bullet h_3 : \vdash \Delta_7, \Delta_9, F_2, F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_1 : \vdash \Delta_4, \Delta_5, F_2 \quad h_1 : \vdash \Delta_6, \Delta_7, F_3}{\bullet h_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), F_2 \otimes F_3} \otimes \rightsquigarrow \frac{}{\bullet h_1 : \vdash \Delta_5, \Delta_7, F_3} \text{ fail}$$

4 Identity-Expansion

$$\begin{array}{c}
\frac{}{- : \vdash \mathbf{0}, \top} \top \\[10pt]
\frac{\frac{}{- : \vdash \mathbf{1}} \mathbf{1}}{- : \vdash \mathbf{1}, \perp} \perp \\[10pt]
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash dual(\mathbf{F}_0), \mathbf{F}_0 \oplus \mathbf{F}_1} \oplus_A \quad \frac{\frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash dual(\mathbf{F}_1), \mathbf{F}_0 \oplus \mathbf{F}_1} \oplus_B}{- : \vdash dual(\mathbf{F}_0) \& dual(\mathbf{F}_1), \mathbf{F}_0 \oplus \mathbf{F}_1} \& \\[10pt]
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash dual(\mathbf{F}_0), dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \otimes \quad \frac{\frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \$}{- : \vdash dual(\mathbf{F}_0) \$ dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \$ \\[10pt]
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \oplus_A \quad \frac{\frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \oplus_B}{- : \vdash \mathbf{F}_0 \& \mathbf{F}_1, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \& \\[10pt]
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash \mathbf{F}_0, \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \otimes \quad \frac{\frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash \mathbf{F}_0 \$ \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \$}{- : \vdash \mathbf{F}_0 \$ \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \$ \\[10pt]
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash ?\mathbf{F}_0, dual(\mathbf{F}_0)} ?}{- : \vdash !dual(\mathbf{F}_0), ?\mathbf{F}_0} ! \\[10pt]
\frac{\frac{}{- : \vdash \mathbf{1}} \mathbf{1}}{- : \vdash \mathbf{1}, \perp} \perp \\[10pt]
\frac{}{- : \vdash \mathbf{0}, \top} \top \\[10pt]
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash \mathbf{F}_0, ?dual(\mathbf{F}_0)} ?}{- : \vdash !\mathbf{F}_0, ?dual(\mathbf{F}_0)} !
\end{array}$$

5 Cut-Elimination

5.1 Status of 1: OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, \Delta_4}{\bullet h_2 : \vdash dual(1), \Delta_4, ?F_3} \quad ?W \\
 \hline
 - : \vdash *, \Delta_4, ?F_3 \quad \text{Cut} \\
 \hline
 \rightsquigarrow \\
 \frac{\frac{}{\bullet h_1 : \vdash 1} \quad ax \quad \frac{h_2 : \vdash \Delta_4, \perp}{ax} \quad hCut}{- : \vdash \Delta_4} \quad ?W \\
 \hline
 - : \vdash \Delta_4, ?F_3
 \end{array}$$

- Case rule ?C

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, \Delta_4, ?F_3, ?F_3}{\bullet h_2 : \vdash dual(1), \Delta_4, ?F_3} \quad ?C \\
 \hline
 - : \vdash *, \Delta_4, ?F_3 \quad \text{Cut} \\
 \hline
 \rightsquigarrow \\
 \frac{\frac{}{\bullet h_1 : \vdash 1} \quad ax \quad \frac{h_2 : \vdash \Delta_4, \perp, ?F_3, ?F_3}{ax} \quad hCut}{- : \vdash \Delta_4, ?F_3, ?F_3} \quad ?C \\
 \hline
 - : \vdash \Delta_4, ?F_3
 \end{array}$$

- Case rule ?

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, \Delta_4, F_3}{\bullet h_2 : \vdash dual(1), \Delta_4, ?F_3} \quad ? \\
 \hline
 - : \vdash *, \Delta_4, ?F_3 \quad \text{Cut} \\
 \hline
 \rightsquigarrow \\
 \frac{\frac{}{\bullet h_1 : \vdash 1} \quad ax \quad \frac{h_2 : \vdash \Delta_4, F_3, \perp}{ax} \quad hCut}{- : \vdash \Delta_4, F_3} \quad ? \\
 \hline
 - : \vdash \Delta_4, ?F_3
 \end{array}$$

- Case rule \$

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, \Delta_5, F_3, F_4}{\bullet h_2 : \vdash dual(1), \Delta_5, F_3 \$ F_4} \quad \$ \\
 \hline
 - : \vdash *, \Delta_5, F_3 \$ F_4 \quad \text{Cut} \\
 \hline
 \rightsquigarrow \\
 \frac{\frac{}{\bullet h_1 : \vdash 1} \quad 1 \quad \frac{h_2 : \vdash \Delta_5, F_3, F_4, \perp}{ax} \quad hCut}{- : \vdash \Delta_5, F_3, F_4} \quad \$ \\
 \hline
 - : \vdash \Delta_5, F_3 \$ F_4
 \end{array}$$

- Case rule &

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, \Delta_5, F_3 \quad h_2 : \vdash \perp, \Delta_5, F_4}{\bullet h_2 : \vdash dual(1), \Delta_5, F_3 \& F_4} \quad \& \\
 \hline
 - : \vdash *, \Delta_5, F_3 \& F_4 \quad \text{Cut} \\
 \hline
 \rightsquigarrow \\
 \frac{\frac{}{\bullet h_1 : \vdash 1} \quad 1 \quad \frac{h_2 : \vdash \Delta_5, F_3, \perp}{ax} \quad hCut \quad \frac{\frac{}{\bullet h_1 : \vdash 1} \quad 1 \quad \frac{h_2 : \vdash \Delta_5, F_4, \perp}{ax} \quad hCut}{- : \vdash \Delta_5, F_4} \quad \&}{- : \vdash \Delta_5, F_3 \& F_4} \quad \&
 \end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash 1, *} \mathbf{1} \quad \frac{h_2 : \vdash \perp, \Delta_5, F_4}{\bullet h_2 : \vdash dual(1), \Delta_5, F_3 \oplus F_4} \oplus_B \\
\hline
- : \vdash *, \Delta_5, F_3 \oplus F_4 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \vdash 1} \mathbf{ax} \quad \frac{h_2 : \vdash \Delta_5, F_4, \perp}{- : \vdash \Delta_5, F_4} \mathbf{ax} \\
\hline
- : \vdash \Delta_5, F_3 \oplus F_4 \quad \oplus_B \\
\hline
- : \vdash \Delta_5, F_3 \oplus F_4 \quad \text{hCut}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash 1, *} \mathbf{1} \quad \frac{h_2 : \vdash \perp, \Delta_5, F_3}{\bullet h_2 : \vdash dual(1), \Delta_5, F_3 \oplus F_4} \oplus_A \\
\hline
- : \vdash *, \Delta_5, F_3 \oplus F_4 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \vdash 1} \mathbf{1} \quad \frac{h_2 : \vdash \Delta_5, F_3, \perp}{- : \vdash \Delta_5, F_3} \mathbf{ax} \\
\hline
- : \vdash \Delta_5, F_3 \oplus F_4 \quad \oplus_A \\
\hline
- : \vdash \Delta_5, F_3 \oplus F_4 \quad \text{hCut}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash 1, *} \mathbf{1} \quad \frac{h_2 : \vdash \Delta_3}{\bullet h_2 : \vdash dual(1), \Delta_3} \perp \\
\hline
- : \vdash *, \Delta_3 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3 \quad \mathbf{ax}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash 1, *} \mathbf{1} \quad \frac{}{\bullet h_2 : \vdash dual(1), \top, \Delta_3} \top \\
\hline
- : \vdash *, \top, \Delta_3 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \top \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash 1, *} \mathbf{1} \quad \frac{h_2 : \vdash \perp, \Delta_6, F_4 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash dual(1), \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \\
\hline
- : \vdash *, \Delta_3, \Delta_6, F_4 \otimes F_5 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \vdash 1} \mathbf{1} \quad \frac{h_2 : \vdash \Delta_6, F_4, \perp}{- : \vdash \Delta_6, F_4} \mathbf{ax} \quad \frac{}{- : \vdash \Delta_3, F_5} \mathbf{ax} \\
\hline
- : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5 \quad \otimes \\
\hline
\frac{}{\bullet h_1 : \vdash 1, *} \mathbf{1} \quad \frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \perp, \Delta_6, F_5}{\bullet h_2 : \vdash dual(1), \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \\
\hline
- : \vdash *, \Delta_3, \Delta_6, F_4 \otimes F_5 \quad \text{Cut} \\
\hline
\frac{}{- : \vdash \Delta_3, F_4} \mathbf{ax} \quad \frac{}{\bullet h_1 : \vdash 1} \mathbf{ax} \quad \frac{h_2 : \vdash \Delta_6, F_5, \perp}{- : \vdash \Delta_6, F_5} \mathbf{ax} \\
\hline
- : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5 \quad \otimes \\
\hline
- : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5 \quad \text{hCut}
\end{array}$$

5.2 Status of !: OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
 \frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash F_6, ?\Upsilon 7, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), ?\Upsilon 7, !F_6} ! \\
 \hline
 - \vdash ?\Upsilon 2, ?\Upsilon 7, !F_6 \quad \text{Cut} \\
 \sim \\
 \frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- \vdash ?\Upsilon 2, ?\Upsilon 7, F_6} \text{ax} \quad \frac{h_5 \vdash ?\Upsilon 7, F_6, ?dual(F_4)}{- \vdash ?\Upsilon 2, ?\Upsilon 7, !F_6} \text{ax} \\
 \hline
 - \vdash ?\Upsilon 2, ?\Upsilon 7, !F_6 \quad \text{hCut} \\
 \hline
 - \vdash ?\Upsilon 2, ?\Upsilon 7, !F_6 \quad !
 \end{array}$$

- Case rule ?W

$$\begin{array}{c}
 \frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_8, dual(5)}{\bullet h_6 \vdash dual(5), \Delta_8, ?F_7} ?W \\
 \hline
 - \vdash (? \Upsilon 4, !F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
 \sim \\
 \frac{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3}{- \vdash ?\Upsilon 4, \Delta_8, !F_3} \text{ax} \quad \frac{h_6 \vdash \Delta_8, dual(5)}{- \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7} \text{ax} \\
 \hline
 - \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7 \quad \text{hCut} \\
 \hline
 - \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7 \quad W \\
 \\
 \frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_7, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_7, ?F_6} ?W \\
 \hline
 - \vdash ?\Upsilon 2, \Delta_7, ?F_6 \quad \text{Cut} \\
 \sim \\
 \frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- \vdash ?\Upsilon 2, \Delta_7} \text{ax} \quad \frac{h_5 \vdash \Delta_7, ?dual(F_4)}{- \vdash ?\Upsilon 2, \Delta_7, ?F_6} \text{ax} \\
 \hline
 - \vdash ?\Upsilon 2, \Delta_7, ?F_6 \quad \text{hCut} \\
 \hline
 - \vdash ?\Upsilon 2, \Delta_7, ?F_6 \quad ?W \\
 \\
 \frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_6}{\bullet h_5 \vdash dual(!F_4), \Delta_6} ?W \\
 \hline
 - \vdash ?\Upsilon 2, \Delta_6 \quad \text{Cut} \\
 \sim \\
 \frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- \vdash ?\Upsilon 2, \Delta_6} \text{ax} \\
 \hline
 - \vdash ?\Upsilon 2, \Delta_6 \quad W
 \end{array}$$

- Case rule ?C

$$\begin{array}{c}
 \frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, dual(5)}{\bullet h_6 \vdash dual(5), \Delta_8, ?F_7} ?C \\
 \hline
 - \vdash (? \Upsilon 4, !F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
 \sim \\
 \frac{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3}{- \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7, ?F_7} \text{ax} \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, dual(5)}{- \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7} \text{ax} \\
 \hline
 - \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7, ?F_7 \quad \text{hCut} \\
 \hline
 - \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7 \quad ?C \\
 \\
 \frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_7, ?F_6, ?F_6, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_7, ?F_6} ?C \\
 \hline
 - \vdash ?\Upsilon 2, \Delta_7, ?F_6 \quad \text{Cut} \\
 \sim \\
 \frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- \vdash ?\Upsilon 2, \Delta_7, ?F_6, ?F_6} \text{ax} \quad \frac{h_5 \vdash \Delta_7, ?F_6, ?F_6, ?dual(F_4)}{- \vdash ?\Upsilon 2, \Delta_7, ?F_6, ?F_6} \text{ax} \\
 \hline
 - \vdash ?\Upsilon 2, \Delta_7, ?F_6, ?F_6 \quad \text{hCut} \\
 \hline
 - \vdash ?\Upsilon 2, \Delta_7, ?F_6 \quad ?C
 \end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_6, ?dual(F_4), ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_6} ?C}{- : \vdash ?\Upsilon 2, \Delta_6} Cut \\
\sim\!\!\!\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, !F_4} ax \quad \frac{h_5 : \vdash \Delta_6, ?dual(F_4), ?dual(F_4)}{- : \vdash ?\Upsilon 2, \Delta_6} ax}{- : \vdash ?\Upsilon 2, \Delta_6} mCut
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 : \vdash \Delta_8, F_7, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_8, ?F_7} ?}{- : \vdash (? \Upsilon 4, !F_3), \Delta_8, ?F_7} Cut \\
\sim\!\!\!\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3}{- : \vdash ?\Upsilon 4, \Delta_8, !F_3} ax \quad \frac{h_6 : \vdash \Delta_8, F_7, dual(5)}{- : \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7} ax}{- : \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7} hCut \\
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_7, F_6, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_7, ?F_6} ?}{- : \vdash ?\Upsilon 2, \Delta_7, ?F_6} Cut \\
\sim\!\!\!\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, \Delta_7, F_6} ax \quad \frac{h_5 : \vdash \Delta_7, F_6, ?dual(F_4)}{- : \vdash ?\Upsilon 2, \Delta_7, ?F_6} ax}{- : \vdash ?\Upsilon 2, \Delta_7, ?F_6} hCut \\
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_6, dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_6} ?}{- : \vdash ?\Upsilon 2, \Delta_6} Cut \\
\sim\!\!\!\rightarrow \\
\frac{\frac{- : \vdash ?\Upsilon 2, F_4}{- : \vdash ?\Upsilon 2, \Delta_6} ax \quad \frac{- : \vdash \Delta_6, dual(F_4)}{- : \vdash ?\Upsilon 2, \Delta_6} ax}{- : \vdash ?\Upsilon 2, \Delta_6} sCut
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 : \vdash \Delta_9, F_7, F_8, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_9, F_7 \$F_8} \$}{- : \vdash (? \Upsilon 4, !F_3), \Delta_9, F_7 \$F_8} Cut \\
\sim\!\!\!\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3}{- : \vdash ?\Upsilon 4, \Delta_9, F_7, F_8, !F_3} ax \quad \frac{h_6 : \vdash \Delta_9, F_7, F_8, dual(5)}{- : \vdash ?\Upsilon 4, \Delta_9, !F_3, F_7 \$F_8} ax}{- : \vdash ?\Upsilon 4, \Delta_9, !F_3, F_7 \$F_8} hCut \\
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_8, F_6, F_7, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_8, F_6 \$F_7} \$}{- : \vdash ?\Upsilon 2, \Delta_8, F_6 \$F_7} Cut \\
\sim\!\!\!\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, \Delta_8, F_6, F_7} ax \quad \frac{h_5 : \vdash \Delta_8, F_6, F_7, ?dual(F_4)}{- : \vdash ?\Upsilon 2, \Delta_8, F_6 \$F_7} ax}{- : \vdash ?\Upsilon 2, \Delta_8, F_6 \$F_7} hCut
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 : \vdash \Delta_9, F_7, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_9, F_7 \&F_8} \&}{- : \vdash (? \Upsilon 4, !F_3), \Delta_9, F_7 \&F_8} Cut \\
\sim\!\!\!\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3}{- : \vdash ?\Upsilon 4, \Delta_9, F_7, !F_3} ax \quad \frac{h_6 : \vdash \Delta_9, F_7, dual(5)}{- : \vdash ?\Upsilon 4, \Delta_9, F_8, !F_3} ax}{- : \vdash ?\Upsilon 4, \Delta_9, F_7 \&F_8} hCut
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_8, F_6, ?dual(F_4) \quad h_5 : \vdash \Delta_8, F_7, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_8, F_6 \& F_7} \&}{- : \vdash ?\Upsilon 2, \Delta_8, F_6 \& F_7} \text{Cut} \\
\\
\frac{\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, \Delta_8, F_6} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, ?dual(F_4)}{h_5 : \vdash \Delta_8, F_7, ?dual(F_4)} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_8, F_6} \text{hCut} \quad \frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, \Delta_8, F_7} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_7, ?dual(F_4)}{h_5 : \vdash \Delta_8, F_6 \& F_7} \text{hCut}}{- : \vdash ?\Upsilon 2, \Delta_8, F_6 \& F_7} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 : \vdash \Delta_9, F_8, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_9, F_7 \oplus F_8} \oplus_B}{- : \vdash (?\Upsilon 4, !F_3), \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\\
\frac{\frac{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3}{- : \vdash ?\Upsilon 4, \Delta_9, !F_3} \text{ax} \quad \frac{h_6 : \vdash \Delta_9, F_8, dual(5)}{h_6 : \vdash \Delta_9, F_7 \oplus F_8} \text{ax}}{- : \vdash ?\Upsilon 4, \Delta_9, !F_3} \text{hCut} \\
\\
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_8, F_7, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_8, F_6 \oplus F_7} \oplus_B}{- : \vdash ?\Upsilon 2, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\\
\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, \Delta_8, F_7} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_7, ?dual(F_4)}{h_5 : \vdash \Delta_8, F_6 \oplus F_7} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_8, F_7} \text{hCut} \\
\\
\frac{- : \vdash ?\Upsilon 2, \Delta_8, F_7}{- : \vdash ?\Upsilon 2, \Delta_8, F_6 \oplus F_7} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 : \vdash \Delta_9, F_7, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_9, F_7 \oplus F_8} \oplus_A}{- : \vdash (?\Upsilon 4, !F_3), \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\\
\frac{\frac{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3}{- : \vdash ?\Upsilon 4, \Delta_9, !F_3} \text{ax} \quad \frac{h_6 : \vdash \Delta_9, F_7, dual(5)}{h_6 : \vdash \Delta_9, F_7 \oplus F_8} \text{ax}}{- : \vdash ?\Upsilon 4, \Delta_9, !F_3} \text{hCut} \\
\\
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_8, F_6, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_8, F_6 \oplus F_7} \oplus_A}{- : \vdash ?\Upsilon 2, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\\
\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, \Delta_8, F_6} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, ?dual(F_4)}{h_5 : \vdash \Delta_8, F_6 \oplus F_7} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_8, F_6} \text{hCut} \\
\\
\frac{- : \vdash ?\Upsilon 2, \Delta_8, F_6}{- : \vdash ?\Upsilon 2, \Delta_8, F_6 \oplus F_7} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 : \vdash \Delta_7, dual(5)}{\bullet h_6 : \vdash dual(5), \perp, \Delta_7} \perp}{- : \vdash (?\Upsilon 4, !F_3), \perp, \Delta_7} \text{Cut} \\
\\
\frac{\frac{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3}{- : \vdash ?\Upsilon 4, \Delta_7, !F_3} \text{ax} \quad \frac{h_6 : \vdash \Delta_7, dual(5)}{h_6 : \vdash \Delta_7, \perp, \Delta_7} \text{ax}}{- : \vdash ?\Upsilon 4, \Delta_7, !F_3} \text{hCut} \\
\\
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_6, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \perp, \Delta_6} \perp}{- : \vdash ?\Upsilon 2, \perp, \Delta_6} \text{Cut} \\
\\
\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, \Delta_6} \text{ax} \quad \frac{h_5 : \vdash \Delta_6, ?dual(F_4)}{h_5 : \vdash \Delta_6, \perp, \Delta_6} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_6} \text{hCut} \\
\\
\frac{- : \vdash ?\Upsilon 2, \Delta_6}{- : \vdash ?\Upsilon 2, \Delta_6, \perp} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{}{\bullet h_6 : \vdash dual(5), \top, \Delta_7} \top}{- : \vdash (? \Upsilon 4, !F_3), \top, \Delta_7} \text{Cut} \\
\sim \\
\frac{}{- : \vdash ?\Upsilon 4, \Delta_7, \top, !F_3} \top
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{}{\bullet h_5 : \vdash dual(!F_4), \top, \Delta_6} \top}{- : \vdash ?\Upsilon 2, \top, \Delta_6} \text{Cut} \\
\sim \\
\frac{}{- : \vdash ?\Upsilon 2, \Delta_6, \top} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{\frac{h_6 : \vdash \Delta_{10}, F_8, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \quad \frac{h_6 : \vdash \Delta_7, F_9}{\bullet h_6 : \vdash dual(5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{- : \vdash (? \Upsilon 4, !F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} \text{ax} \quad \frac{h_6 : \vdash \Delta_{10}, F_8, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{ax}}{- : \vdash ?\Upsilon 4, \Delta_{10}, F_8, !F_3} \text{hCut} \quad \frac{}{- : \vdash \Delta_7, F_9} \text{ax}}{- : \vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !F_3, F_8 \otimes F_9} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{\frac{h_6 : \vdash \Delta_7, F_8}{\bullet h_6 : \vdash dual(5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \quad \frac{h_6 : \vdash \Delta_{10}, F_9, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{- : \vdash (? \Upsilon 4, !F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut} \\
\sim \\
\frac{\frac{}{- : \vdash \Delta_7, F_8} \text{ax} \quad \frac{\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} \text{ax} \quad \frac{h_6 : \vdash \Delta_{10}, F_9, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{ax}}{- : \vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !F_3, F_8 \otimes F_9} \text{hCut} \quad \otimes
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{\frac{h_5 : \vdash \Delta_9, F_7, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \quad \frac{h_5 : \vdash \Delta_6, F_8}{\bullet h_5 : \vdash dual(!F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} \text{ax} \quad \frac{h_5 : \vdash \Delta_9, F_7, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_9, F_7} \text{hCut} \quad \frac{}{- : \vdash \Delta_6, F_8} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{\frac{h_5 : \vdash \Delta_6, F_7}{\bullet h_5 : \vdash dual(!F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \quad \frac{h_5 : \vdash \Delta_9, F_8, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\sim \\
\frac{\frac{}{- : \vdash \Delta_6, F_7} \text{ax} \quad \frac{\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} \text{ax} \quad \frac{h_5 : \vdash \Delta_9, F_8, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_9, F_8} \text{hCut} \quad \otimes
\end{array}$$

5.3 Status of $?W$: OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 : \vdash ?\Upsilon 6, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(?F_4), ?\Upsilon 6} ! \\
\hline
- : \vdash \Delta_2, ?\Upsilon 6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{- : \vdash \Delta_2}{- : \vdash ?\Upsilon 6, \Delta_2} \text{ ax} \\
\hline
- : \vdash ?\Upsilon 6, \Delta_2 \quad W
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 : \vdash \Delta_8, \text{dual}(F_5)}{\bullet h_6 : \vdash \text{dual}(F_5), \Delta_8, ?F_7} \text{ ?W} \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{- : \vdash \Delta_4, \Delta_8, ?F_7} \text{ ax} \quad \frac{\bullet h_6 : \vdash \Delta_8, ?F_7, \text{dual}(F_5)}{- : \vdash \Delta_4, \Delta_8, ?F_7, \text{dual}(F_5)} \text{ ax}}{- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ hCut} \\
\hline
- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7 \quad W
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 : \vdash \Delta_7, !\text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(?F_4), \Delta_7, ?F_6} \text{ ?W} \\
\hline
- : \vdash \Delta_2, \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{- : \vdash \Delta_2, \Delta_7} \text{ ax} \quad \frac{h_5 : \vdash \Delta_7, !\text{dual}(F_4)}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ ax}}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ hCut} \\
\hline
- : \vdash \Delta_2, \Delta_7, ?F_6 \quad W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 : \vdash \Delta_8, ?F_7, ?F_7, \text{dual}(F_5)}{\bullet h_6 : \vdash \text{dual}(F_5), \Delta_8, ?F_7} \text{ ?C} \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{- : \vdash \Delta_4, \Delta_8, ?F_7} \text{ ax} \quad \frac{\bullet h_6 : \vdash \Delta_8, ?F_7, \text{dual}(F_5)}{- : \vdash \Delta_4, \Delta_8, ?F_7, \text{dual}(F_5)} \text{ ax}}{- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ hCut} \\
\hline
- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7 \quad ?W
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 : \vdash \Delta_7, !\text{dual}(F_4), ?F_6, ?F_6}{\bullet h_5 : \vdash \text{dual}(?F_4), \Delta_7, ?F_6} \text{ ?C} \\
\hline
- : \vdash \Delta_2, \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ ax} \quad \frac{h_5 : \vdash \Delta_7, !\text{dual}(F_4), ?F_6, ?F_6}{- : \vdash \Delta_2, \Delta_7, ?F_6, ?F_6} \text{ ax}}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ hCut} \\
\hline
- : \vdash \Delta_2, \Delta_7, ?F_6 \quad ?C
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 : \vdash \Delta_8, F_7, \text{dual}(F_5)}{\bullet h_6 : \vdash \text{dual}(F_5), \Delta_8, ?F_7} ? \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{- : \vdash \Delta_4, \Delta_8, ?F_7} \text{ ax} \quad \frac{\bullet h_6 : \vdash \Delta_8, ?F_7, \text{dual}(F_5)}{- : \vdash \Delta_4, \Delta_8, ?F_7, \text{dual}(F_5)} \text{ ax}}{- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ hCut} \\
\hline
- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7 \quad W
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 : \vdash \Delta_7, F_6, !\text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(?F_4), \Delta_7, ?F_6} ? \\
\hline
- : \vdash \Delta_2, \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{- : \vdash \Delta_2, \Delta_7, F_6} \text{ ax} \quad \frac{h_5 : \vdash \Delta_7, F_6, !\text{dual}(F_4)}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ ax}}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ hCut} \\
\hline
- : \vdash \Delta_2, \Delta_7, ?F_6 \quad ?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_4, F_5}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?W \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \$F_8} \$}{- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \$F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_5}{- \vdash \Delta_4, \Delta_9, F_7 \$F_8} \text{ax} \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \$F_8}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \$F_8} \text{ax}}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \$F_8} \text{hCut} \\
\frac{\frac{h_1 \vdash \Delta_2}{\bullet h_1 \vdash ?F_4, \Delta_2} ?W \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \$F_7} \$}{- \vdash \Delta_2, \Delta_8, F_6 \$F_7} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_8, F_6, F_7} \text{ax} \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)}{- \vdash \Delta_2, \Delta_8, F_6 \$F_7} \text{ax}}{- \vdash \Delta_2, \Delta_8, F_6 \$F_7} \text{hCut} \\
\frac{}{- \vdash \Delta_2, \Delta_8, F_6 \$F_7} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_4, F_5}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?W \quad \frac{h_6 \vdash \Delta_9, F_7, dual(F_5) \quad h_6 \vdash \Delta_9, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \&F_8} \&}{- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \&F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_5}{- \vdash \Delta_4, \Delta_9, F_7 \&F_8} \text{ax} \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \&F_8}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \&F_8} \text{ax}}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \&F_8} \text{hCut} \\
\frac{\frac{h_1 \vdash \Delta_2}{\bullet h_1 \vdash ?F_4, \Delta_2} ?W \quad \frac{h_5 \vdash \Delta_8, F_6, !dual(F_4) \quad h_5 \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \&F_7} \&}{- \vdash \Delta_2, \Delta_8, F_6 \&F_7} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_8, F_6} \text{ax} \quad \frac{h_5 \vdash \Delta_8, F_6, !dual(F_4)}{- \vdash \Delta_2, \Delta_8, F_7} \text{ax}}{- \vdash \Delta_2, \Delta_8, F_6 \&F_7} \text{hCut} \\
\frac{}{- \vdash \Delta_2, \Delta_8, F_6 \&F_7} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_4, F_5}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?W \quad \frac{h_6 \vdash \Delta_9, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \oplus F_8} \oplus_B}{- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_5}{- \vdash \Delta_4, \Delta_9, F_7 \oplus F_8} \text{ax} \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \oplus F_8}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} \text{ax}}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} \text{hCut} \\
\frac{\frac{h_1 \vdash \Delta_2}{\bullet h_1 \vdash ?F_4, \Delta_2} ?W \quad \frac{h_5 \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \oplus F_7} \oplus_B}{- \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_8, F_7} \text{ax} \quad \frac{h_5 \vdash \Delta_8, F_7, !dual(F_4)}{- \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \text{ax}}{- \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \text{hCut} \\
\frac{}{- \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \quad ?W \quad \frac{h_6 : \vdash \Delta_9, F_7, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_9, F_7 \oplus F_8} \quad \oplus A}{- : \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash \Delta_4, F_5} \quad \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_9, dual(F_5), F_7 \oplus F_8}{- : \vdash \Delta_4, \Delta_9, F_7 \oplus F_8} \quad \text{ax}}{- : \vdash \Delta_4, \Delta_9, F_7 \oplus F_8} \quad \text{hCut} \\
\frac{- : \vdash \Delta_4, \Delta_9, F_7 \oplus F_8}{- : \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} \quad ?W \\
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \quad ?W \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_8, F_6 \oplus F_7} \quad \oplus A}{- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4} \quad \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{h_5 : \vdash \Delta_8, F_6, !dual(F_4)} \quad \text{ax}}{- : \vdash \Delta_2, \Delta_8, F_6} \quad \text{hCut} \\
\frac{- : \vdash \Delta_2, \Delta_8, F_6}{- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \quad \oplus A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \quad ?W \quad \frac{h_6 : \vdash \Delta_7, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \perp, \Delta_7} \quad \perp}{- : \vdash (\Delta_4, ?F_3), \perp, \Delta_7} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash \Delta_4, F_5} \quad \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_7, \perp, dual(F_5)}{\bullet h_6 : \vdash \Delta_7, \perp, dual(F_5)} \quad \text{ax}}{- : \vdash \Delta_4, \Delta_7, \perp} \quad \text{hCut} \\
\frac{- : \vdash \Delta_4, \Delta_7, \perp}{- : \vdash \Delta_4, \Delta_7, \perp, ?F_3} \quad W \\
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \quad ?W \quad \frac{h_5 : \vdash \Delta_6, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \perp, \Delta_6} \quad \perp}{- : \vdash \Delta_2, \perp, \Delta_6} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4} \quad \text{ax} \quad \frac{h_5 : \vdash \Delta_6, !dual(F_4)}{h_5 : \vdash \Delta_6, !dual(F_4)} \quad \text{ax}}{- : \vdash \Delta_2, \Delta_6} \quad \text{hCut} \\
\frac{- : \vdash \Delta_2, \Delta_6}{- : \vdash \Delta_2, \Delta_6, \perp} \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \quad ?W \quad \frac{\bullet h_6 : \vdash dual(F_5), \top, \Delta_7}{\bullet h_6 : \vdash dual(F_5), \top, \Delta_7} \quad \top}{- : \vdash (\Delta_4, ?F_3), \top, \Delta_7} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{- : \vdash \Delta_4, \Delta_7, \top, ?F_3}{- : \vdash \Delta_4, \Delta_7, \top, ?F_3} \quad \top \\
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \quad ?W \quad \frac{\bullet h_5 : \vdash dual(?F_4), \top, \Delta_6}{\bullet h_5 : \vdash dual(?F_4), \top, \Delta_6} \quad \top}{- : \vdash \Delta_2, \top, \Delta_6} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{- : \vdash \Delta_2, \Delta_6, \top}{- : \vdash \Delta_2, \Delta_6, \top} \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \quad ?W \quad \frac{h_6 : \vdash \Delta_{10}, F_8, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \quad h_6 : \vdash \Delta_7, F_9}{- : \vdash (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \quad \otimes \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash \Delta_4, F_5} \quad \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_{10}, \Delta_7, dual(F_5), F_8 \otimes F_9}{\bullet h_6 : \vdash \Delta_{10}, \Delta_7, dual(F_5), F_8 \otimes F_9} \quad \text{ax}}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, F_8 \otimes F_9} \quad \text{hCut} \\
\frac{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, F_8 \otimes F_9}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \quad ?W
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?W \quad \frac{h_6 : \vdash \Delta_7, F_8 \quad h_6 : \vdash \Delta_{10}, F_9, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{- : \vdash (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{- : \vdash \Delta_4, F_5} \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_{10}, \Delta_7, dual(F_5), F_8 \otimes F_9}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, F_8 \otimes F_9} \text{hCut}}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} ?W \\
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?W \quad \frac{h_5 : \vdash \Delta_9, F_7, !dual(F_4) \quad h_5 : \vdash \Delta_6, F_8}{\bullet h_5 : \vdash dual(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\sim \\
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_9, F_7, !dual(F_4)}{h_5 : \vdash \Delta_9, F_7, !dual(F_4)} \text{ax}}{- : \vdash \Delta_2, \Delta_9, F_7} \text{hCut} \quad \frac{- : \vdash \Delta_6, F_8}{- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?W \quad \frac{h_5 : \vdash \Delta_6, F_7 \quad h_5 : \vdash \Delta_9, F_8, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\sim \\
\frac{- : \vdash \Delta_6, F_7}{- : \vdash \Delta_6, F_7} \text{ax} \quad \frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{- : \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_9, F_8, !dual(F_4)}{- : \vdash \Delta_2, \Delta_9, F_8} \text{ax}}{- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{hCut} \otimes
\end{array}$$

5.4 Status of ?C: OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash ?\Upsilon 6, dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), ?\Upsilon 6} !}{- : \vdash \Delta_2, ?\Upsilon 6} \text{Cut} \\
\sim \\
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4, ?F_4} \text{ax} \quad \frac{h_5 : \vdash ?\Upsilon 6, dual(F_4)}{\bullet h_5 : \vdash ?\Upsilon 6, !dual(F_4)} !}{- : \vdash ?\Upsilon 6, \Delta_2} \text{mCut}
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 : \vdash \Delta_8, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_8, ?F_7} ?W}{- : \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 : \vdash \Delta_4, F_5, ?F_3}{- : \vdash \Delta_4, F_5, ?F_3} \text{ax} \quad \frac{h_6 : \vdash \Delta_8, dual(F_5)}{- : \vdash \Delta_8, dual(F_5)} \text{ax}}{- : \vdash \Delta_4, \Delta_8, ?F_3} \text{hCut} \\
- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7 \quad W \\
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_7, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_7, ?F_6} ?W}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{- : \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_7, !dual(F_4)}{- : \vdash \Delta_7, !dual(F_4)} \text{ax}}{- : \vdash \Delta_2, \Delta_7} \text{hCut} \\
- : \vdash \Delta_2, \Delta_7, ?F_6 \quad ?W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_8, ?F_7} ?C \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_3, ?F_7} ax \quad \frac{\bullet h_6 \vdash \Delta_8, ?F_7, dual(F_5)}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} ax \\
\hline
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7 \quad ?C \quad hCut
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 \vdash \Delta_7, !dual(F_4), ?F_6, ?F_6}{\bullet h_5 \vdash dual(?F_4), \Delta_7, ?F_6} ?C \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_7, ?F_6, ?F_6} ax \quad \frac{h_5 \vdash \Delta_7, !dual(F_4), ?F_6, ?F_6}{- \vdash \Delta_2, \Delta_7, ?F_6} ax \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6 \quad hCut
\end{array}
\end{array}$$

• Case rule ?

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \vdash \Delta_8, F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_8, ?F_7} ? \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_3, ?F_7} ax \quad \frac{\bullet h_6 \vdash \Delta_8, ?F_7, dual(F_5)}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} ax \\
\hline
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7 \quad ?C \quad hCut
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 \vdash \Delta_7, F_6, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_7, ?F_6} ? \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_7, F_6} ax \quad \frac{h_5 \vdash \Delta_7, F_6, !dual(F_4)}{- \vdash \Delta_2, \Delta_7, F_6} ax \\
\hline
- \vdash \Delta_2, \Delta_7, F_6 \quad hCut
\end{array}
\end{array}$$

• Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \$F_8} \$ \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \$F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{- \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7 \$F_8} ax \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \$F_8}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \$F_8} ax \\
\hline
- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \$F_8 \quad ?C \quad hCut
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \$F_7} \$ \\
\hline
- \vdash \Delta_2, \Delta_8, F_6 \$F_7 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_8, F_6, F_7} ax \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)}{- \vdash \Delta_2, \Delta_8, F_6 \$F_7} ax \\
\hline
- \vdash \Delta_2, \Delta_8, F_6, F_7 \quad hCut \\
- \vdash \Delta_2, \Delta_8, F_6 \$F_7 \quad \$
\end{array}
\end{array}$$

• Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \vdash \Delta_9, F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \&F_8} \& \quad h_6 \vdash \Delta_9, F_8, dual(F_5) \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \&F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{- \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7 \&F_8} ax \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \&F_8}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \&F_8} ax \\
\hline
- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \&F_8 \quad ?C \quad hCut
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4) \quad h_5 : \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_8, F_6 \& F_7} \& \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \& F_7 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{\bullet h_5 : \vdash \Delta_8, !dual(F_4), F_6 \& F_7} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \& F_7 \quad \text{mCut} \quad \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 : \vdash \Delta_9, F_8, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_9, F_7 \oplus F_8} \oplus_B \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3} \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_9, dual(F_5), F_7 \oplus F_8}{\bullet h_6 : \vdash \Delta_9, dual(F_5), F_7 \oplus F_8} \text{ax} \\
\hline
- : \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7 \oplus F_8 \quad \text{hCut} \\
\hline
- : \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8 \quad ?C \\
\hline
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_8, F_6 \oplus F_7} \oplus_B \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_7, !dual(F_4)}{h_5 : \vdash \Delta_8, F_7, !dual(F_4)} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8, F_7 \quad \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 : \vdash \Delta_9, F_7, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_9, F_7 \oplus F_8} \oplus_A \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3} \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_9, dual(F_5), F_7 \oplus F_8}{\bullet h_6 : \vdash \Delta_9, dual(F_5), F_7 \oplus F_8} \text{ax} \\
\hline
- : \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7 \oplus F_8 \quad \text{hCut} \\
\hline
- : \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8 \quad ?C \\
\hline
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_8, F_6 \oplus F_7} \oplus_A \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{h_5 : \vdash \Delta_8, F_6, !dual(F_4)} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \quad \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 : \vdash \Delta_7, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \perp, \Delta_7} \perp \\
\hline
- : \vdash (\Delta_4, ?F_3), \perp, \Delta_7 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3} \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_7, \perp, dual(F_5)}{\bullet h_6 : \vdash \Delta_7, \perp, dual(F_5)} \text{ax} \\
\hline
- : \vdash \Delta_4, \Delta_7, \perp, ?F_3, ?F_3 \quad \text{hCut} \\
\hline
- : \vdash \Delta_4, \Delta_7, \perp, ?F_3 \quad ?C \\
\hline
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_6, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \perp, \Delta_6} \perp \\
\hline
- : \vdash \Delta_2, \perp, \Delta_6 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_6, !dual(F_4)}{\bullet h_5 : \vdash \Delta_6, \perp, !dual(F_4)} \perp \\
\hline
- : \vdash \Delta_2, \Delta_6, \perp \quad \text{mCut}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{}{\bullet h_6 : \vdash dual(F_5), \top, \Delta_7} \top \\
\hline
- : \vdash (\Delta_4, ?F_3), \top, \Delta_7 \quad \text{Cut} \\
\hline
\sim \\
- : \vdash \Delta_4, \Delta_7, \top, ?F_3 \quad \top
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{}{\bullet h_5 : \vdash dual(?F_4), \top, \Delta_6} \top \\
\hline
- : \vdash \Delta_2, \top, \Delta_6 \quad \text{Cut} \\
\hline
\sim \\
- : \vdash \Delta_2, \Delta_6, \top \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 : \vdash \Delta_{10}, F_8, dual(F_5) \quad h_6 : \vdash \Delta_7, F_9}{\bullet h_6 : \vdash dual(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, ?F_3, F_8 \otimes F_9} \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_{10}, \Delta_7, dual(F_5), F_8 \otimes F_9}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \text{ax} \\
\hline
- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9 \quad \text{hCut} \\
\hline
- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9 \quad ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 : \vdash \Delta_7, F_8 \quad h_6 : \vdash \Delta_{10}, F_9, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, ?F_3, F_8 \otimes F_9} \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_{10}, \Delta_7, dual(F_5), F_8 \otimes F_9}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \text{ax} \\
\hline
- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9 \quad \text{hCut} \\
\hline
- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9 \quad ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_9, F_7, !dual(F_4) \quad h_5 : \vdash \Delta_6, F_8}{\bullet h_5 : \vdash dual(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax} \quad \frac{h_5 : \vdash \Delta_9, F_7, !dual(F_4) \quad h_5 : \vdash \Delta_6, F_8}{\bullet h_5 : \vdash \Delta_6, \Delta_9, !dual(F_4), F_7 \otimes F_8} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{mCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_6, F_7 \quad h_5 : \vdash \Delta_9, F_8, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax} \quad \frac{h_5 : \vdash \Delta_6, F_7 \quad h_5 : \vdash \Delta_9, F_8, !dual(F_4)}{\bullet h_5 : \vdash \Delta_6, \Delta_9, !dual(F_4), F_7 \otimes F_8} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{mCut}
\end{array}$$

5.5 Status of $?$: OK

- Case rule 1
- Case rule $!$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash ?\Upsilon 6, dual(F_4)}{\bullet h_5 \vdash dual(?F_4), ?\Upsilon 6} ! \\
\hline
- \vdash \Delta_2, ?\Upsilon 6 \quad \text{Cut} \\
\hline
\sim\sim \\
\frac{- \vdash \Delta_2, F_4 \quad \text{ax}}{- \vdash ?\Upsilon 6, \Delta_2} \quad \frac{- \vdash ?\Upsilon 6, dual(F_4) \quad \text{ax}}{- \vdash ?\Upsilon 6, \Delta_2} \text{sCut}
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_8, ?F_7} ?W \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
\hline
\sim\sim \\
\frac{\bullet h_2 \vdash \Delta_4, F_5, ?F_3 \quad \text{ax} \quad h_6 \vdash \Delta_8, dual(F_5) \quad \text{ax}}{- \vdash \Delta_4, \Delta_8, ?F_3} \text{hCut} \\
\hline
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7 \quad W \\
\hline
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_7, ?F_6} ?W \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\sim\sim \\
\frac{\bullet h_1 \vdash \Delta_2, ?F_4 \quad \text{ax} \quad h_5 \vdash \Delta_7, !dual(F_4) \quad \text{ax}}{- \vdash \Delta_2, \Delta_7} \text{hCut} \\
\hline
- \vdash \Delta_2, \Delta_7 \quad W \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_8, ?F_7} ?C \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
\hline
\sim\sim \\
\frac{\bullet h_2 \vdash \Delta_4, F_5, ?F_3 \quad \text{ax} \quad h_6 \vdash \Delta_8, ?F_7, ?F_7, dual(F_5) \quad \text{ax}}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7, ?F_7} \text{hCut} \\
\hline
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7 \quad ?C \\
\hline
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_7, !dual(F_4), ?F_6, ?F_6}{\bullet h_5 \vdash dual(?F_4), \Delta_7, ?F_6} ?C \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\sim\sim \\
\frac{\bullet h_1 \vdash \Delta_2, ?F_4 \quad \text{ax} \quad h_5 \vdash \Delta_7, !dual(F_4), ?F_6, ?F_6 \quad \text{ax}}{- \vdash \Delta_2, \Delta_7, ?F_6, ?F_6} \text{hCut} \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6 \quad ?C \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_8, F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_8, ?F_7} ? \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
\hline
\sim\sim \\
\frac{h_2 \vdash \Delta_4, F_3, F_5 \quad \text{ax} \quad \bullet h_6 \vdash \Delta_8, ?F_7, dual(F_5) \quad \text{ax}}{- \vdash \Delta_4, \Delta_8, F_3, ?F_7} \text{hCut} \\
\hline
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7 \quad ? \\
\hline
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_7, F_6, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_7, ?F_6} ? \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\sim\sim \\
\frac{\bullet h_1 \vdash \Delta_2, ?F_4 \quad \text{ax} \quad h_5 \vdash \Delta_7, F_6, !dual(F_4) \quad \text{ax}}{- \vdash \Delta_2, \Delta_7, F_6} \text{hCut} \\
\hline
- \vdash \Delta_2, \Delta_7, F_6 \quad ? \\
\hline
- \vdash \Delta_2, \Delta_7, ?F_6
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \$ F_8} \$ \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \$ F_8 \quad \text{Cut} \\
\hline
\frac{}{h_2 \vdash \Delta_4, F_3, F_5} \text{ax} \quad \frac{}{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \$ F_8} \text{ax} \\
\hline
- \vdash \Delta_4, \Delta_9, F_3, F_7 \$ F_8 \quad ? \\
- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \$ F_8 \\
\hline
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \$ F_7} \$ \\
\hline
- \vdash \Delta_2, \Delta_8, F_6 \$ F_7 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{}{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)} \text{ax} \\
\hline
- \vdash \Delta_2, \Delta_8, F_6, F_7 \quad ? \\
- \vdash \Delta_2, \Delta_8, F_6 \$ F_7 \quad \$ \\
\hline
- \vdash \Delta_2, \Delta_8, F_6 \$ F_7
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_9, F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \& F_8} \& \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \& F_8 \quad \text{Cut} \\
\hline
\frac{}{h_2 \vdash \Delta_4, F_3, F_5} \text{ax} \quad \frac{}{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \& F_8} \text{ax} \\
\hline
- \vdash \Delta_4, \Delta_9, F_3, F_7 \& F_8 \quad ? \\
- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \& F_8 \\
\hline
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_8, F_6, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \& F_7} \& \\
\hline
- \vdash \Delta_2, \Delta_8, F_6 \& F_7 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{}{h_5 \vdash \Delta_8, F_6, !dual(F_4)} \text{ax} \\
\hline
- \vdash \Delta_2, \Delta_8, F_6 \quad ? \\
- \vdash \Delta_2, \Delta_8, F_6 \& F_7 \quad \& \\
\hline
- \vdash \Delta_2, \Delta_8, F_6 \& F_7
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_9, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \oplus F_8} \oplus_B \\
\hline
- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\frac{}{h_2 \vdash \Delta_4, F_3, F_5} \text{ax} \quad \frac{}{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \oplus F_8} \text{ax} \\
\hline
- \vdash \Delta_4, \Delta_9, F_3, F_7 \oplus F_8 \quad ? \\
- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8 \\
\hline
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \oplus F_7} \oplus_B \\
\hline
- \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{}{h_5 \vdash \Delta_8, F_7, !dual(F_4)} \text{ax} \\
\hline
- \vdash \Delta_2, \Delta_8, F_7 \quad ? \\
- \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \oplus_B \\
\hline
- \vdash \Delta_2, \Delta_8, F_6 \oplus F_7
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_4, F_5, F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 : \vdash \Delta_9, F_7, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_9, F_7 \oplus F_8} \oplus A}{- : \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_4, F_3, F_5}{\bullet h_2 : \vdash \Delta_4, F_3, F_5} \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_9, dual(F_5), F_7 \oplus F_8}{\bullet h_6 : \vdash \Delta_9, F_3, F_7 \oplus F_8} \text{ax}}{- : \vdash \Delta_4, \Delta_9, F_3, F_7 \oplus F_8} \text{hCut} \\
\frac{- : \vdash \Delta_4, \Delta_9, F_3, F_7 \oplus F_8}{- : \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} ? \\
\frac{\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_8, F_6 \oplus F_7} \oplus A}{- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{h_5 : \vdash \Delta_8, F_6, !dual(F_4)} \text{ax}}{- : \vdash \Delta_2, \Delta_8, F_6} \text{hCut} \\
\frac{- : \vdash \Delta_2, \Delta_8, F_6}{- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \oplus A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_4, F_5, F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 : \vdash \Delta_7, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \perp, \Delta_7} \perp}{- : \vdash (\Delta_4, ?F_3), \perp, \Delta_7} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_4, F_3, F_5}{\bullet h_2 : \vdash \Delta_4, F_3, F_5} \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_7, \perp, dual(F_5)}{\bullet h_6 : \vdash \Delta_7, \perp, dual(F_5)} \text{ax}}{- : \vdash \Delta_4, \Delta_7, F_3, \perp} \text{hCut} \\
\frac{- : \vdash \Delta_4, \Delta_7, F_3, \perp}{- : \vdash \Delta_4, \Delta_7, \perp, ?F_3} ? \\
\frac{\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 : \vdash \Delta_6, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \perp, \Delta_6} \perp}{- : \vdash \Delta_2, \perp, \Delta_6} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_6, !dual(F_4)}{h_5 : \vdash \Delta_6, !dual(F_4)} \text{ax}}{- : \vdash \Delta_2, \Delta_6} \text{hCut} \\
\frac{- : \vdash \Delta_2, \Delta_6}{- : \vdash \Delta_2, \Delta_6, \perp} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_4, F_5, F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 : \vdash \Delta_7, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \top, \Delta_7} \top}{- : \vdash (\Delta_4, ?F_3), \top, \Delta_7} \text{Cut} \\
\sim \\
\frac{- : \vdash \Delta_4, \Delta_7, \top, ?F_3}{- : \vdash \Delta_4, \Delta_7, \top, ?F_3} \top \\
\frac{\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 : \vdash \Delta_6, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \top, \Delta_6} \top}{- : \vdash \Delta_2, \top, \Delta_6} \text{Cut} \\
\sim \\
\frac{- : \vdash \Delta_2, \Delta_6, \top}{- : \vdash \Delta_2, \Delta_6, \top} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_4, F_5, F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 : \vdash \Delta_{10}, F_8, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \quad h_6 : \vdash \Delta_7, F_9}{- : \vdash (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_4, F_3, F_5}{\bullet h_2 : \vdash \Delta_4, F_3, F_5} \text{ax} \quad \frac{\bullet h_6 : \vdash \Delta_{10}, \Delta_7, dual(F_5), F_8 \otimes F_9}{\bullet h_6 : \vdash \Delta_{10}, \Delta_7, F_3, F_8 \otimes F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, F_3, F_8 \otimes F_9} \text{hCut} \\
\frac{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, F_3, F_8 \otimes F_9}{- : \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} ?
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{\frac{h_6 \vdash \Delta_7, F_8 \quad h_6 \vdash \Delta_{10}, F_9, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{- \vdash (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_2 \vdash \Delta_4, F_3, F_5}{- \vdash \Delta_{10}, \Delta_4, \Delta_7, F_3, F_8 \otimes F_9} \text{ax} \quad \frac{\bullet h_6 \vdash \Delta_{10}, \Delta_7, dual(F_5), F_8 \otimes F_9}{- \vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_4, \Delta_7, F_3, F_8 \otimes F_9} \text{hCut} \\
\sim \\
\frac{\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{\frac{h_5 \vdash \Delta_9, F_7, !dual(F_4) \quad h_5 \vdash \Delta_6, F_8}{\bullet h_5 \vdash dual(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_9, F_7} \text{ax} \quad \frac{\frac{h_5 \vdash \Delta_9, F_7, !dual(F_4)}{h_5 \vdash \Delta_6, F_8, !dual(F_4)} \text{ax}}{- \vdash \Delta_2, \Delta_9, F_7} \text{hCut} \quad \frac{- \vdash \Delta_6, F_8}{- \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax} \otimes \\
\sim \\
\frac{\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{\frac{h_5 \vdash \Delta_6, F_7 \quad h_5 \vdash \Delta_9, F_8, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\sim \\
\frac{- \vdash \Delta_6, F_7}{- \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax} \quad \frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_9, F_7} \text{ax} \quad \frac{h_5 \vdash \Delta_9, F_8, !dual(F_4)}{- \vdash \Delta_2, \Delta_9, F_8} \text{ax}}{- \vdash \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{hCut} \otimes
\end{array}$$

5.6 Status of \$: OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \$F_4} \$ \quad \frac{\frac{h_7 \vdash \Delta_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_9, ?F_8} ?W}{- \vdash (\Delta_5, F_3 \$F_4), \Delta_9, ?F_8} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \$F_4}{- \vdash \Delta_5, \Delta_9, F_3 \$F_4} \text{ax} \quad \frac{\frac{h_7 \vdash \Delta_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_9, ?F_8} \text{ax}}{- \vdash \Delta_5, \Delta_9, F_3 \$F_4} \text{hCut} \\
\sim \\
\frac{- \vdash \Delta_5, \Delta_9, F_3 \$F_4}{- \vdash \Delta_5, \Delta_9, ?F_8, F_3 \$F_4} W \\
\sim \\
\frac{\frac{h_1 \vdash \Delta_2, F_5, F_6}{\bullet h_1 \vdash F_5 \$F_6, \Delta_2} \$ \quad \frac{\frac{h_7 \vdash \Delta_9, dual(F_5) \otimes dual(F_6)}{\bullet h_7 \vdash dual(F_5 \$F_6), \Delta_9, ?F_8} ?W}{- \vdash \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \$F_6}{- \vdash \Delta_2, \Delta_9} \text{ax} \quad \frac{\frac{h_7 \vdash \Delta_9, dual(F_5) \otimes dual(F_6)}{\bullet h_7 \vdash dual(F_5 \$F_6), \Delta_9, ?F_8} \text{ax}}{- \vdash \Delta_2, \Delta_9} \text{hCut} \\
\sim \\
\frac{- \vdash \Delta_2, \Delta_9}{- \vdash \Delta_2, \Delta_9, ?F_8} ?W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \$F_4} \$ \quad \frac{\frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_9, ?F_8} ?C}{- \vdash (\Delta_5, F_3 \$F_4), \Delta_9, ?F_8} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \$F_4}{- \vdash \Delta_5, \Delta_9, ?F_8, ?F_8, dual(F_6)} \text{ax} \quad \frac{\frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_9, ?F_8} \text{ax}}{- \vdash \Delta_5, \Delta_9, ?F_8, ?F_8, dual(F_6)} \text{hCut} \\
\sim \\
\frac{- \vdash \Delta_5, \Delta_9, ?F_8, ?F_8, dual(F_6)}{- \vdash \Delta_5, \Delta_9, ?F_8, F_3 \$F_4} ?C
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_9, ?F_8} ?C}{- : \vdash \Delta_2, \Delta_9, ?F_8} Cut \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{- : \vdash \Delta_2, \Delta_9, ?F_8} ax \quad \frac{\frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \otimes dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8} ax}{- : \vdash \Delta_2, \Delta_9, ?F_8} hCut
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} ?}{- : \vdash (\Delta_5, F_3 \$ F_4), \Delta_9, ?F_8} Cut \\
\sim \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \$ F_4}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \$ F_4} ax \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_6)}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \$ F_4} ax}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \$ F_4} hCut \\
? \\
\frac{\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_9, ?F_8} ?}{- : \vdash \Delta_2, \Delta_9, ?F_8} Cut \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{- : \vdash \Delta_2, \Delta_9, F_8} ax \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_5) \otimes dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8} ax}{- : \vdash \Delta_2, \Delta_9, ?F_8} hCut
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \$ F_9} \$}{- : \vdash (\Delta_5, F_3 \$ F_4), \Delta_{10}, F_8 \$ F_9} Cut \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_5, F_3, F_4, F_6}{- : \vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \$ F_9} ax \quad \frac{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \$ F_9}{- : \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \$ F_9} ax}{- : \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \$ F_9} hCut \\
\$ \\
\frac{\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_{10}, F_8 \$ F_9} \$}{- : \vdash \Delta_2, \Delta_{10}, F_8 \$ F_9} Cut \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{- : \vdash \Delta_{10}, \Delta_2, F_8, F_9} ax \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, dual(F_5) \otimes dual(F_6)}{- : \vdash \Delta_{10}, \Delta_2, F_8, F_9} ax}{- : \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} hCut
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6) \quad h_7 : \vdash \Delta_{10}, F_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \&}{- : \vdash (\Delta_5, F_3 \$ F_4), \Delta_{10}, F_8 \& F_9} Cut \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_5, F_3, F_4, F_6}{- : \vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \& F_9} ax \quad \frac{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \& F_9}{- : \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \& F_9} ax}{- : \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \& F_9} hCut \\
\$ \\
\frac{\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6) \quad h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_{10}, F_8 \& F_9} \&}{- : \vdash \Delta_2, \Delta_{10}, F_8 \& F_9} Cut \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{- : \vdash \Delta_{10}, \Delta_2, F_8} ax \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}{- : \vdash \Delta_{10}, \Delta_2, F_8} hCut \quad \frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{- : \vdash \Delta_{10}, \Delta_2, F_9} ax \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}{- : \vdash \Delta_{10}, \Delta_2, F_9} hCut}{- : \vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9}}{- : \vdash (\Delta_5, F_3 \$ F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \quad \oplus_B \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_5, F_3, F_4, F_6}{- : \vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{- : \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \oplus F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \oplus F_9} \text{hCut} \$ \\
\frac{\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_{10}, F_8 \oplus F_9}}{- : \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \quad \oplus_B \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{- : \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}{- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{hCut} \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9}}{- : \vdash (\Delta_5, F_3 \$ F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \quad \oplus_A \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_5, F_3, F_4, F_6}{- : \vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{- : \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \oplus F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \oplus F_9} \text{hCut} \$ \\
\frac{\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_{10}, F_8 \oplus F_9}}{- : \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \quad \oplus_A \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{- : \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}{- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{hCut} \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 : \vdash \Delta_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \perp, \Delta_8}}{- : \vdash (\Delta_5, F_3 \$ F_4), \perp, \Delta_8} \text{Cut} \quad \perp \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_5, F_3, F_4, F_6}{- : \vdash \Delta_5, \Delta_8, F_3, F_4, \perp} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_8, \perp, dual(F_6)}{- : \vdash \Delta_5, \Delta_8, \perp, F_3 \$ F_4} \text{ax}}{- : \vdash \Delta_5, \Delta_8, \perp, F_3 \$ F_4} \text{hCut} \$ \\
\frac{\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \perp, \Delta_8}}{- : \vdash \Delta_2, \perp, \Delta_8} \text{Cut} \quad \perp \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{- : \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \otimes dual(F_6)}{- : \vdash \Delta_2, \Delta_8, \perp} \text{ax}}{- : \vdash \Delta_2, \Delta_8, \perp} \text{hCut} \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \frac{}{\bullet h_7 : \vdash dual(F_6), \top, \Delta_8} \top \\
\hline
- : \vdash (\Delta_5, F_3 \$ F_4), \top, \Delta_8 \quad \text{Cut} \\
\hline
- : \vdash \Delta_5, \Delta_8, \top, F_3 \$ F_4 \quad \top \\
\hline
\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \frac{}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \top, \Delta_8} \top \\
\hline
- : \vdash \Delta_2, \top, \Delta_8 \quad \text{Cut} \\
\hline
- : \vdash \Delta_2, \Delta_8, \top \quad \top
\end{array}$$

• Case rule I

• Case rule \otimes

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \frac{h_7 : \vdash \Delta_{11}, F_9, dual(F_6) \quad h_7 : \vdash \Delta_8, F_{10}}{\bullet h_7 : \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash (\Delta_5, F_3 \$ F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\frac{h_2 : \vdash \Delta_5, F_3, F_4, F_6}{\bullet h_2 : \vdash \Delta_5, F_3, F_4, F_6} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{\bullet h_7 : \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}} \text{ax} \\
\hline
- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_4, F_9 \otimes F_{10} \quad \text{hCut} \\
\hline
- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \$ F_4, F_9 \otimes F_{10} \quad \$
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \frac{h_7 : \vdash \Delta_8, F_9 \quad h_7 : \vdash \Delta_{11}, F_{10}, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash (\Delta_5, F_3 \$ F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\frac{h_2 : \vdash \Delta_5, F_3, F_4, F_6}{\bullet h_2 : \vdash \Delta_5, F_3, F_4, F_6} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{\bullet h_7 : \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}} \text{ax} \\
\hline
- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_4, F_9 \otimes F_{10} \quad \text{hCut} \\
\hline
- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \$ F_4, F_9 \otimes F_{10} \quad \$
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \frac{h_7 : \vdash \Delta_{11}, F_9, dual(F_5) \otimes dual(F_6) \quad h_7 : \vdash \Delta_8, F_{10}}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_{11}, F_9, dual(F_5) \otimes dual(F_6)}{h_7 : \vdash \Delta_{11}, F_9, dual(F_5) \otimes dual(F_6)} \text{ax} \\
\hline
- : \vdash \Delta_{11}, \Delta_2, F_9 \quad \text{hCut} \quad - : \vdash \Delta_8, F_{10} \quad \text{ax} \\
\hline
- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10} \quad \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \frac{h_7 : \vdash \Delta_8, F_9 \quad h_7 : \vdash \Delta_{11}, F_{10}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_{11}, F_{10}, dual(F_5) \otimes dual(F_6)}{h_7 : \vdash \Delta_{11}, F_{10}, dual(F_5) \otimes dual(F_6)} \text{ax} \\
\hline
- : \vdash \Delta_8, F_9 \quad \text{hCut} \quad - : \vdash \Delta_{11}, \Delta_2, F_{10} \quad \text{ax} \\
\hline
- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10} \quad \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, F_5, F_6}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \frac{h_7 : \vdash \Delta_8, dual(F_5) \quad h_7 : \vdash \Delta_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_8, \Delta_9} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_8, \Delta_9 \quad \text{Cut} \\
\hline
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, dual(F_6)}{h_7 : \vdash \Delta_9, dual(F_6)} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_9, F_5 \quad \text{sCut} \quad - : \vdash \Delta_8, dual(F_5) \quad \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8, \Delta_9 \quad \text{sCut}
\end{array}$$

5.7 Status of $\&$: OK

• Case rule 1

- Case rule !

- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash \Delta_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} ?W}{- : \vdash (\Delta_5, F_3 \& F_4), \Delta_9, ?F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_5, \Delta_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, dual(F_6)}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{ax}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{hCut}}{W} \\
\\
\frac{\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_9, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_9, ?F_8} ?W}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, \Delta_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ax}}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{hCut} ?W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} ?C}{- : \vdash (\Delta_5, F_3 \& F_4), \Delta_9, ?F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{ax}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{hCut} ?C \\
\\
\frac{\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_9, ?F_8} ?C}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ax}}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{hCut} ?C
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} ?}{- : \vdash (\Delta_5, F_3 \& F_4), \Delta_9, ?F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_5, \Delta_9, F_8, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_6)}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{ax}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{hCut} ? \\
\\
\frac{\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_9, ?F_8} ?}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, \Delta_9, F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ax}}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{hCut} ?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_5, F_6, F_3 \quad h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \$ F_9} \$ \\
\hline
- \vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \$ F_9 \quad \text{Cut} \\
\hline
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \& F_4}{- \vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_6)}{- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \& F_4} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \& F_4} \text{hCut} \\
\hline
\frac{h_1 \vdash \Delta_2, F_5 \quad h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_5) \oplus dual(F_6)}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \$ F_9} \$ \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \$ F_9 \quad \text{Cut} \\
\hline
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \& F_6}{- \vdash \Delta_{10}, \Delta_2, F_8, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_5) \oplus dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \text{hCut} \\
\hline
- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9 \quad \$
\end{array}$$

• Case rule $\&$

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_5, F_6, F_3 \quad h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_6) \quad h_7 \vdash \Delta_{10}, F_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- \vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_6}{- \vdash \Delta_{10}, \Delta_5, F_3, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \& F_9}{- \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \& F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \& F_9} \text{hCut} \\
\hline
\frac{h_1 \vdash \Delta_2, F_5 \quad h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_5) \oplus dual(F_6) \quad h_7 \vdash \Delta_{10}, F_9, dual(F_5) \oplus dual(F_6)}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \& F_6}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_5) \oplus dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \text{hCut} \\
\hline
- \vdash \Delta_{10}, \Delta_2, F_8 \& F_9 \quad \&
\end{array}$$

• Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_5, F_6, F_3 \quad h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \& F_4}{- \vdash \Delta_{10}, \Delta_5, F_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_6)}{- \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9} \text{hCut} \\
\hline
\frac{h_1 \vdash \Delta_2, F_5 \quad h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \oplus dual(F_6)}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \& F_6}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \oplus dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{hCut} \\
\hline
\frac{h_1 \vdash \Delta_2, F_5 \quad h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash \Delta_8, dual(F_6)}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_8} \oplus_B \\
\hline
- \vdash \Delta_2, \Delta_8 \quad \text{Cut} \\
\hline
\frac{\frac{- \vdash \Delta_2, F_6}{- \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{- \vdash \Delta_8, dual(F_6)}{- \vdash \Delta_2, \Delta_8} \text{ax}}{- \vdash \Delta_2, \Delta_8} \text{sCut}
\end{array}$$

• Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- : \vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_{10}, \Delta_5, F_8, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6)}{- : \vdash \Delta_{10}, F_8, dual(F_6)} \text{ax} \\
\hline
- : \vdash \Delta_{10}, \Delta_5, F_8, F_3 \& F_4 \quad \text{hCut} \\
\hline
- : \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9 \quad \oplus_A
\end{array} \\
\\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- : \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_{10}, F_8, dual(F_5) \oplus dual(F_6)} \text{ax} \\
\hline
- : \vdash \Delta_{10}, \Delta_2, F_8 \quad \text{hCut} \\
\hline
- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9 \quad \oplus_A
\end{array} \\
\\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_8, dual(F_5)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_8} \oplus_A \\
\hline
- : \vdash \Delta_2, \Delta_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_2, F_5} \text{ax} \quad \frac{}{- : \vdash \Delta_8, dual(F_5)} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8 \quad \text{sCut}
\end{array}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash \Delta_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \perp, \Delta_8} \perp \\
\hline
- : \vdash (\Delta_5, F_3 \& F_4), \perp, \Delta_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_5, \Delta_8, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_8, dual(F_6)}{- : \vdash \Delta_8, dual(F_6)} \text{ax} \\
\hline
- : \vdash \Delta_5, \Delta_8, F_3 \& F_4 \quad \text{hCut} \\
\hline
- : \vdash \Delta_5, \Delta_8, \perp, F_3 \& F_4 \quad \perp
\end{array} \\
\\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \perp, \Delta_8} \perp \\
\hline
- : \vdash \Delta_2, \perp, \Delta_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_8, dual(F_5) \oplus dual(F_6)} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8 \quad \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_8, \perp \quad \perp
\end{array}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{}{\bullet h_7 : \vdash dual(F_6), \top, \Delta_8} \top \\
\hline
- : \vdash (\Delta_5, F_3 \& F_4), \top, \Delta_8 \quad \text{Cut} \\
\hline
\frac{}{- : \vdash \Delta_5, \Delta_8, \top, F_3 \& F_4} \top \\
\\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{}{\bullet h_7 : \vdash dual(F_5 \& F_6), \top, \Delta_8} \top \\
\hline
- : \vdash \Delta_2, \top, \Delta_8 \quad \text{Cut} \\
\hline
\frac{}{- : \vdash \Delta_2, \Delta_8, \top} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3 \quad h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 \vdash \Delta_{11}, F_9, dual(F_6) \quad h_7 \vdash \Delta_8, F_{10}}{\bullet h_7 \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash (\Delta_5, F_3 \& F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \& F_4}{- \vdash \Delta_{11}, \Delta_5, F_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, dual(F_6)}{hCut} \text{ax} \quad \frac{- \vdash \Delta_8, F_{10}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \otimes}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \otimes \\
\\
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3 \quad h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash (\Delta_5, F_3 \& F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{- \vdash \Delta_8, F_9}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \& F_4}{- \vdash \Delta_{11}, \Delta_5, F_{10}, F_3 \& F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_{10}, dual(F_6)}{hCut} \text{ax}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \otimes \\
\\
\frac{\frac{h_1 \vdash \Delta_2, F_5 \quad h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash \Delta_{11}, F_9, dual(F_5) \oplus dual(F_6) \quad h_7 \vdash \Delta_8, F_{10}}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \& F_6}{- \vdash \Delta_{11}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, dual(F_5) \oplus dual(F_6)}{hCut} \text{ax} \quad \frac{- \vdash \Delta_8, F_{10}}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\\
\frac{\frac{h_1 \vdash \Delta_2, F_5 \quad h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{- \vdash \Delta_8, F_9}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_1 \vdash \Delta_2, F_5 \& F_6}{- \vdash \Delta_{11}, \Delta_2, F_{10}} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_{10}, dual(F_5) \oplus dual(F_6)}{hCut} \text{ax}}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes
\end{array}$$

5.8 Status of \oplus_B : OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash \Delta_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_9, ?F_8} ?W}{- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \oplus F_4}{- \vdash \Delta_5, \Delta_9, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_9, dual(F_6)}{hCut} \text{ax}}{- \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} ?W \\
\\
\frac{\frac{h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash \Delta_9, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_9, ?F_8} ?W}{- \vdash \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_2, \Delta_9} \text{ax} \quad \frac{h_7 \vdash \Delta_9, dual(F_5) \& dual(F_6)}{hCut} \text{ax}}{- \vdash \Delta_2, \Delta_9, ?F_8} W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_9, ?F_8} ?C \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \oplus F_4}{- \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{- \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{ax} \\
\hline
- \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4 \quad \text{hCut} \\
\hline
- \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4 \quad ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_9, ?F_8} ?C \\
\hline
- \vdash \Delta_2, \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \& dual(F_6)}{- \vdash \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ax} \\
\hline
- \vdash \Delta_2, \Delta_9, ?F_8, ?F_8 \quad \text{hCut} \\
\hline
- \vdash \Delta_2, \Delta_9, ?F_8 \quad ?C
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash \Delta_9, F_8, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_9, ?F_8} ? \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \oplus F_4}{- \vdash \Delta_5, \Delta_9, F_8, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_9, F_8, dual(F_6)}{- \vdash \Delta_5, \Delta_9, F_8, F_3 \oplus F_4} \text{ax} \\
\hline
- \vdash \Delta_5, \Delta_9, F_8, F_3 \oplus F_4 \quad \text{hCut} \\
\hline
- \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4 \quad ?
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash \Delta_9, F_8, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_9, ?F_8} ? \\
\hline
- \vdash \Delta_2, \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_2, \Delta_9, F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_9, F_8, dual(F_5) \& dual(F_6)}{- \vdash \Delta_2, \Delta_9, F_8} \text{ax} \\
\hline
- \vdash \Delta_2, \Delta_9, F_8 \quad \text{hCut} \\
\hline
- \vdash \Delta_2, \Delta_9, ?F_8 \quad ?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \$ F_9} \$ \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \$ F_9 \quad \text{Cut} \\
\hline
\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \oplus F_4}{- \vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_6)}{- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \oplus F_4} \text{ax} \\
\hline
- \vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3 \oplus F_4 \quad \text{hCut} \\
\hline
- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \oplus F_4 \quad \$
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \$ F_9} \$ \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \$ F_9 \quad \text{Cut} \\
\hline
\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_8, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \text{ax} \\
\hline
- \vdash \Delta_{10}, \Delta_2, F_8, F_9 \quad \text{hCut} \\
\hline
- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9 \quad \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \quad h_7 \vdash \Delta_{10}, F_9, dual(F_6) \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\frac{h_2 \vdash \Delta_5, F_4, F_6}{- \vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \& F_9}{- \vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \text{ax} \\
\hline
- \vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9 \quad \text{hCut} \\
\hline
- \vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4 \quad \oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6) \quad h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9} \&}{- : \vdash \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{hCut} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8} \& \quad \frac{\frac{h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{hCut} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \&}{- : \vdash \Delta_2, \Delta_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \quad h_7 : \vdash \Delta_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_8} \&}{- : \vdash \Delta_2, \Delta_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_1 : \vdash \Delta_2, F_6}{- : \vdash \Delta_2, F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_8, dual(F_6)}{- : \vdash \Delta_8, dual(F_6)} \text{ax}}{- : \vdash \Delta_2, \Delta_8} \text{sCut}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 : \vdash \Delta_5, F_4, F_6}{- : \vdash \Delta_{10}, \Delta_5, F_4, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{hCut} \text{ax}}{- : \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \oplus_B \\
\rightsquigarrow \\
\frac{\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- : \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{hCut} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A}{- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 : \vdash \Delta_5, F_4, F_6}{- : \vdash \Delta_{10}, \Delta_5, F_4, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{hCut} \text{ax}}{- : \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \oplus_B \\
\rightsquigarrow \\
\frac{\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A}{- : \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{hCut} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 : \vdash \Delta_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \perp, \Delta_8} \perp}{- : \vdash (\Delta_5, F_3 \oplus F_4), \perp, \Delta_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 : \vdash \Delta_5, F_4, F_6}{- : \vdash \Delta_5, \Delta_8, F_4, \perp} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_8, \perp, dual(F_6)}{hCut} \text{ax}}{- : \vdash \Delta_5, \Delta_8, \perp, F_3 \oplus F_4} \oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \perp, \Delta_8} \perp \\
\hline
- : \vdash \Delta_2, \perp, \Delta_8 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \& dual(F_6)}{- : \vdash \Delta_2, \Delta_8} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8, \perp \quad \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_8, \perp \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{}{\bullet h_7 : \vdash dual(F_6), \top, \Delta_8} \top \\
\hline
- : \vdash (\Delta_5, F_3 \oplus F_4), \top, \Delta_8 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
- : \vdash \Delta_5, \Delta_8, \top, F_3 \oplus F_4 \quad \top \\
\hline
\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \top, \Delta_8} \top \\
\hline
- : \vdash \Delta_2, \top, \Delta_8 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
- : \vdash \Delta_2, \Delta_8, \top \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 : \vdash \Delta_{11}, F_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{h_2 : \vdash \Delta_5, F_4, F_6}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{ax} \\
\hline
- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4 \quad \oplus_B \\
\hline
\frac{h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 : \vdash \Delta_8, F_9 \quad h_7 : \vdash \Delta_{11}, F_{10}, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{h_2 : \vdash \Delta_5, F_4, F_6}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{ax} \\
\hline
- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4 \quad \oplus_B \\
\hline
\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash \Delta_{11}, F_9, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_{11}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_{11}, F_9, dual(F_5) \& dual(F_6)}{- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{ax} \\
\hline
- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10} \quad \otimes \\
\hline
\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash \Delta_8, F_9 \quad h_7 : \vdash \Delta_{11}, F_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{}{- : \vdash \Delta_8, F_9} \text{ax} \quad \frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_{11}, \Delta_2, F_{10}} \text{ax} \quad \frac{h_7 : \vdash \Delta_{11}, F_{10}, dual(F_5) \& dual(F_6)}{- : \vdash \Delta_{11}, \Delta_2, F_{10}} \text{ax} \\
\hline
- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10} \quad \otimes
\end{array}$$

5.9 Status of \oplus_A : OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 : \vdash \Delta_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} ?W}{- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} ?W \text{ Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \oplus F_4}{- : \vdash \Delta_5, \Delta_9, F_3 \oplus F_4} \text{ ax} \quad \frac{h_7 : \vdash \Delta_9, dual(F_6)}{- : \vdash \Delta_9, ?F_8, F_3 \oplus F_4} \text{ ax}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{ hCut}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} W
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2, F_5}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 : \vdash \Delta_9, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_9, ?F_8} ?W}{- : \vdash \Delta_2, \Delta_9, ?F_8} ?W \text{ Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_2, \Delta_9} \text{ ax} \quad \frac{h_7 : \vdash \Delta_9, dual(F_5) \& dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ ax}}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ hCut}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} ?C}{- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} ?C \text{ Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \oplus F_4}{- : \vdash \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \oplus F_4} \text{ ax} \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{ ax}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{ hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2, F_5}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_9, ?F_8} ?C}{- : \vdash \Delta_2, \Delta_9, ?F_8} ?C \text{ Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ ax} \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \& dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ ax}}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ hCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} ?}{- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} ? \text{ Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \oplus F_4}{- : \vdash \Delta_5, \Delta_9, F_8, F_3 \oplus F_4} \text{ ax} \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_6)}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{ ax}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{ hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2, F_5}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_9, ?F_8} ?}{- : \vdash \Delta_2, \Delta_9, ?F_8} ? \text{ Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_2, \Delta_9, F_8} \text{ ax} \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_5) \& dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ ax}}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ hCut}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \$ F_9} \$}{- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \$ F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \oplus F_4}{- \vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_6)}{- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \oplus F_4} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \oplus F_4} \$ \text{hCut} \\
\\
\frac{\frac{h_1 \vdash \Delta_2, F_5}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \$ F_9} \$}{- \vdash \Delta_2, \Delta_{10}, F_8 \$ F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_8, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \$ \text{hCut}
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_6) \quad h_7 \vdash \Delta_{10}, F_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \&}{- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_6}{- \vdash \Delta_{10}, \Delta_5, F_3, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \& F_9}{- \vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \oplus A \\
\\
\frac{\frac{h_1 \vdash \Delta_2, F_5}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6) \quad h_7 \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9} \&}{- \vdash \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{hCut} \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \& \text{hCut} \\
\\
\frac{\frac{h_1 \vdash \Delta_2, F_5}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \vdash \Delta_8, dual(F_5) \quad h_7 \vdash \Delta_8, dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_8} \&}{- \vdash \Delta_2, \Delta_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{- \vdash \Delta_2, F_5}{- \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{- \vdash \Delta_8, dual(F_5)}{- \vdash \Delta_2, \Delta_8} \text{ax}}{- \vdash \Delta_2, \Delta_8} \text{sCut}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \oplus F_4}{- \vdash \Delta_{10}, \Delta_5, F_9, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_6)}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \text{hCut} \\
\\
\frac{\frac{h_1 \vdash \Delta_2, F_5}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus_B \text{hCut}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus A}{- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3}{\bullet h_2 : \vdash \Delta_5, F_6, F_3} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_5, F_3, F_8 \oplus F_9} \text{hCut} \\
\oplus A \\
- : \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2, F_5}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus A}{- : \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_1 : \vdash \Delta_2, F_5}{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8} \text{hCut} \\
\oplus A \\
- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 : \vdash \Delta_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \perp, \Delta_8} \perp}{- : \vdash (\Delta_5, F_3 \oplus F_4), \perp, \Delta_8} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3}{\bullet h_2 : \vdash \Delta_5, F_6, F_3} \text{ax} \quad \frac{h_7 : \vdash \Delta_8, \perp, dual(F_6)}{\bullet h_7 : \vdash \Delta_8, \perp, dual(F_6)} \text{ax}}{- : \vdash \Delta_5, \Delta_8, F_3, \perp} \text{hCut} \\
\oplus A \\
- : \vdash \Delta_5, \Delta_8, \perp, F_3 \oplus F_4
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2, F_5}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \perp, \Delta_8} \perp}{- : \vdash \Delta_2, \perp, \Delta_8} \text{Cut} \\
\sim \\
\frac{\frac{h_1 : \vdash \Delta_2, F_5}{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \& dual(F_6)}{h_7 : \vdash \Delta_8, dual(F_5) \& dual(F_6)} \text{ax}}{- : \vdash \Delta_2, \Delta_8} \text{hCut} \\
\perp \\
- : \vdash \Delta_2, \Delta_8, \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 : \vdash dual(F_6), \top, \Delta_8}{\bullet h_7 : \vdash dual(F_6), \top, \Delta_8} \top}{- : \vdash (\Delta_5, F_3 \oplus F_4), \top, \Delta_8} \text{Cut} \\
\sim \\
- : \vdash \Delta_5, \Delta_8, \top, F_3 \oplus F_4 \quad \top
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2, F_5}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 : \vdash dual(F_5 \oplus F_6), \top, \Delta_8}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \top, \Delta_8} \top}{- : \vdash \Delta_2, \top, \Delta_8} \text{Cut} \\
\sim \\
- : \vdash \Delta_2, \Delta_8, \top \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \vdash \Delta_{11}, F_9, dual(F_6) \quad h_7 \vdash \Delta_8, F_{10}}{\bullet h_7 \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_6}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{ax}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \oplus A \quad \text{hCut} \\
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_6}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{ax}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \oplus A \quad \text{hCut} \\
\frac{h_1 \vdash \Delta_2, F_5}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \vdash \Delta_{11}, F_9, dual(F_5) \& dual(F_6) \quad h_7 \vdash \Delta_8, F_{10}}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim \\
\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{11}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{ax} \quad \frac{- \vdash \Delta_8, F_{10}}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\frac{h_1 \vdash \Delta_2, F_5}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim \\
\frac{- \vdash \Delta_8, F_9}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{11}, \Delta_2, F_{10}} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_{10}, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{11}, \Delta_2, F_{10}} \text{ax}}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes
\end{array}$$

5.10 Status of \perp : OK

- Case rule 1

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2}{\bullet h_1 \vdash \perp, \Delta_2} \perp \quad \frac{}{\bullet h_3 \vdash dual(\perp), *} 1}{- \vdash \Delta_2, *} \text{Cut} \\
\sim \\
- \vdash \Delta_2 \quad \text{ax}
\end{array}$$

- Case rule !

- Case rule ?W

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2}{\bullet h_1 \vdash \perp, \Delta_2} \perp \quad \frac{h_3 \vdash \mathbf{1}, \Delta_5}{\bullet h_3 \vdash dual(\perp), \Delta_5, ?F_4} ?W}{- \vdash \Delta_2, \Delta_5, ?F_4} \text{Cut} \\
\sim \\
\frac{\bullet h_1 \vdash \Delta_2, \perp}{- \vdash \Delta_2, \Delta_5} \text{ax} \quad \frac{h_3 \vdash \mathbf{1}, \Delta_5}{- \vdash \Delta_2, \Delta_5, ?F_4} \text{ax}}{- \vdash \Delta_2, \Delta_5, ?F_4} \text{hCut} \quad W \\
\frac{h_2 \vdash \Delta_3, F_4}{\bullet h_2 \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 \vdash \Delta_7, dual(F_4)}{\bullet h_5 \vdash dual(F_4), \Delta_7, ?F_6} ?W}{- \vdash (\perp, \Delta_3), \Delta_7, ?F_6} \text{Cut} \\
\sim \\
\frac{\bullet h_2 \vdash \Delta_3, F_4, \perp}{- \vdash \Delta_3, \Delta_7, \perp} \text{ax} \quad \frac{h_5 \vdash \Delta_7, dual(F_4)}{- \vdash \Delta_3, \Delta_7, \perp, ?F_6} \text{ax}}{- \vdash \Delta_3, \Delta_7, \perp, ?F_6} \text{hCut} \quad W
\end{array}$$

- Case rule $?C$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_5, ?F_4, ?F_4}{\bullet h_3 : \vdash dual(\perp), \Delta_5, ?F_4} ?C \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_5, ?F_4, ?F_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_5, ?F_4, ?F_4}{- : \vdash \Delta_2, \Delta_5, ?F_4, ?F_4} \text{ax}}{- : \vdash \Delta_2, \Delta_5, ?F_4} \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_7, ?F_6, ?F_6, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_7, ?F_6} ?C \\
\hline
- : \vdash (\perp, \Delta_3), \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_7, \perp, ?F_6, ?F_6} \text{ax} \quad \frac{h_5 : \vdash \Delta_7, ?F_6, ?F_6, dual(F_4)}{- : \vdash \Delta_3, \Delta_7, \perp, ?F_6, ?F_6} \text{ax}}{- : \vdash \Delta_3, \Delta_7, \perp, ?F_6} \text{hCut} \\
\hline
- : \vdash \Delta_3, \Delta_7, \perp, ?F_6 \quad ?C
\end{array}$$

- Case rule $?$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_5, F_4}{\bullet h_3 : \vdash dual(\perp), \Delta_5, ?F_4} ? \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_5, F_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_5, F_4}{- : \vdash \Delta_2, \Delta_5, ?F_4} \text{ax}}{- : \vdash \Delta_2, \Delta_5, ?F_4} \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad ?
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_7, F_6, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_7, ?F_6} ? \\
\hline
- : \vdash (\perp, \Delta_3), \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_7, F_6, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_7, F_6, dual(F_4)}{- : \vdash \Delta_3, \Delta_7, \perp, ?F_6} \text{ax}}{- : \vdash \Delta_3, \Delta_7, \perp, ?F_6} \text{hCut} \\
\hline
- : \vdash \Delta_3, \Delta_7, \perp, ?F_6 \quad ?
\end{array}$$

- Case rule $\$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4, F_5}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \$F_5} \$ \\
\hline
- : \vdash \Delta_2, \Delta_6, F_4 \$F_5 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_4, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4, F_5}{- : \vdash \Delta_2, \Delta_6, F_4 \$F_5} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \$F_5} \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_6, F_4 \$F_5 \quad \$
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_8, F_6, F_7, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \$F_7} \$ \\
\hline
- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \$F_7 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_8, F_6, F_7, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, F_7, dual(F_4)}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \$F_7} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \$F_7} \text{hCut} \\
\hline
- : \vdash \Delta_3, \Delta_8, \perp, F_6 \$F_7 \quad \$
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4 \quad h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \& F_5} \&}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4}{hCut} \text{ax} \quad \frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{hCut} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \& \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_8, F_6, dual(F_4) \quad h_5 : \vdash \Delta_8, F_7, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \& F_7} \&}{- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \& F_7} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, \Delta_8, F_6 \& F_7} \text{ax} \quad \frac{\bullet h_5 : \vdash \Delta_8, dual(F_4), F_6 \& F_7}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \& F_7} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \& F_7} \text{hCut} \perp
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \oplus F_5} \oplus_B}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{hCut} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \oplus_B \\
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_8, F_7, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \oplus F_7} \oplus_B}{- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_8, F_7, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_7, dual(F_4)}{hCut} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \oplus F_5} \oplus_A}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4}{hCut} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \oplus_A \\
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_8, F_6, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \oplus F_7} \oplus_A}{- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_8, F_6, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, dual(F_4)}{hCut} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_4}{\bullet h_3 : \vdash dual(\perp), \perp, \Delta_4} \perp}{- : \vdash \Delta_2, \perp, \Delta_4} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_4}{hCut} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \perp} \perp
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_6, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \perp, \Delta_6} \perp \\
\hline
- : \vdash (\perp, \Delta_3), \perp, \Delta_6 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, F_4} \text{ ax} \quad \frac{\bullet h_5 : \vdash \Delta_6, \perp, dual(F_4)}{- : \vdash \Delta_6, \perp, dual(F_4)} \text{ ax}}{- : \vdash \Delta_3, \Delta_6, \perp} \text{ hCut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \perp, \perp \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{}{\bullet h_3 : \vdash dual(\perp), \top, \Delta_4} \top \\
\hline
- : \vdash \Delta_2, \top, \Delta_4 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
- : \vdash \Delta_2, \Delta_4, \top \quad \top \\
\hline
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{}{\bullet h_5 : \vdash dual(F_4), \top, \Delta_6} \top \\
\hline
- : \vdash (\perp, \Delta_3), \top, \Delta_6 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
- : \vdash \Delta_3, \Delta_6, \perp, \top \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_7, F_5 \quad h_3 : \vdash \Delta_4, F_6}{\bullet h_3 : \vdash dual(\perp), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \perp} \text{ ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_7, F_5}{- : \vdash \Delta_7, F_5} \text{ ax}}{- : \vdash \Delta_2, \Delta_7, F_5} \text{ hCut} \quad \frac{}{- : \vdash \Delta_4, F_6} \text{ ax} \\
\hline
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \quad \otimes \\
\hline
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \Delta_4, F_5 \quad h_3 : \vdash \mathbf{1}, \Delta_7, F_6}{\bullet h_3 : \vdash dual(\perp), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{}{- : \vdash \Delta_4, F_5} \text{ ax} \quad \frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \perp} \text{ ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_7, F_6}{- : \vdash \Delta_7, F_6} \text{ ax} \\
\hline
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \quad \otimes \quad \text{hCut} \\
\hline
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_9, F_7, dual(F_4) \quad h_5 : \vdash \Delta_6, F_8}{\bullet h_5 : \vdash dual(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\hline
- : \vdash (\perp, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, F_4} \text{ ax} \quad \frac{\bullet h_5 : \vdash \Delta_6, \Delta_9, dual(F_4), F_7 \otimes F_8}{- : \vdash \Delta_6, \Delta_9, dual(F_4), F_7 \otimes F_8} \text{ ax} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{hCut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_7 \otimes F_8 \quad \perp \\
\hline
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_6, F_7 \quad h_5 : \vdash \Delta_9, F_8, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\hline
- : \vdash (\perp, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, F_4} \text{ ax} \quad \frac{\bullet h_5 : \vdash \Delta_6, \Delta_9, dual(F_4), F_7 \otimes F_8}{- : \vdash \Delta_6, \Delta_9, dual(F_4), F_7 \otimes F_8} \text{ ax} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{hCut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_7 \otimes F_8 \quad \perp
\end{array}$$

5.11 Status of \top : OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_5}{\bullet h_3 : \vdash dual(\top), \Delta_5, ?F_4} ?W \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \vdash \Delta_2, \top, ?F_4} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_5}{\bullet h_3 : \vdash dual(\top), \Delta_5, ?F_4} ax \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad \text{hCut} \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_7, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_7, ?F_6} ?W \\
\hline
- : \vdash (\top, \Delta_3), \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \Delta_7, \top, ?F_6 \quad \top
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_5, ?F_4, ?F_4}{\bullet h_3 : \vdash dual(\top), \Delta_5, ?F_4} ?C \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \vdash \Delta_2, \top} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_5, ?F_4, ?F_4}{\bullet h_3 : \vdash dual(\top), \Delta_5, ?F_4} ax \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4, ?F_4 \quad \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad ?C \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_7, ?F_6, ?F_6, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_7, ?F_6} ?C \\
\hline
- : \vdash (\top, \Delta_3), \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \Delta_7, \top, ?F_6 \quad \top
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_5, F_4}{\bullet h_3 : \vdash dual(\top), \Delta_5, ?F_4} ? \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \vdash \Delta_2, \top} ax \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_5, F_4}{\bullet h_3 : \vdash dual(\top), \Delta_5, ?F_4} ax \\
\hline
- : \vdash \Delta_2, \Delta_5, F_4 \quad \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_5, ?F_4 \quad ? \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_7, F_6, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_7, ?F_6} ? \\
\hline
- : \vdash (\top, \Delta_3), \Delta_7, ?F_6 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \Delta_7, \top, ?F_6 \quad \top
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_6, F_4, F_5}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_6, F_4 \& F_5} \text{\$}}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_6, F_4, F_5} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_6, F_4, F_5}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \text{hCut} \text{\$} \\
\\
\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_8, F_6, F_7, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_8, F_6 \& F_7} \text{\$}}{- : \vdash (\top, \Delta_3), \Delta_8, F_6 \& F_7} \text{Cut} \\
\sim \\
- : \vdash \Delta_3, \Delta_8, \top, F_6 \& F_7 \top
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_6, F_4 \quad h_3 : \vdash \mathbf{O}, \Delta_6, F_5}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_6, F_4 \& F_5} \&}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_6, F_4} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_6, F_4}{- : \vdash \Delta_2, \Delta_6, F_4} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4} \text{hCut} \quad \frac{\frac{\bullet h_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_6, F_5} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_6, F_5}{- : \vdash \Delta_2, \Delta_6, F_5} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_5} \text{hCut} \\
- : \vdash \Delta_2, \Delta_6, F_4 \& F_5 \& \\
\\
\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_8, F_6, \text{dual}(F_4) \quad h_5 : \vdash \Delta_8, F_7, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_8, F_6 \& F_7} \&}{- : \vdash (\top, \Delta_3), \Delta_8, F_6 \& F_7} \text{Cut} \\
\sim \\
- : \vdash \Delta_3, \Delta_8, \top, F_6 \& F_7 \top
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_6, F_5}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_6, F_4 \oplus F_5} \oplus_B}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_6, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_6, F_5}{- : \vdash \Delta_2, \Delta_6, F_5} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{hCut} \oplus_B \\
\\
\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_8, F_7, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_8, F_6 \oplus F_7} \oplus_B}{- : \vdash (\top, \Delta_3), \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\sim \\
- : \vdash \Delta_3, \Delta_8, \top, F_6 \oplus F_7 \top
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_6, F_4}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_6, F_4 \oplus F_5} \oplus_A}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_6, F_4} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_6, F_4}{- : \vdash \Delta_2, \Delta_6, F_4} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{hCut} \oplus_A \\
\\
\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_8, F_6, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_8, F_6 \oplus F_7} \oplus_A}{- : \vdash (\top, \Delta_3), \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\sim \\
- : \vdash \Delta_3, \Delta_8, \top, F_6 \oplus F_7 \top
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_4}{\bullet h_3 : \vdash \text{dual}(\top), \perp, \Delta_4} \perp \\
\hline
- : \vdash \Delta_2, \perp, \Delta_4 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \vdash \Delta_2, \perp, \top} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_4}{\bullet h_3 : \vdash \mathbf{0}, \Delta_4} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_4, \perp \quad \text{hCut} \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_6, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \perp, \Delta_6} \perp \\
\hline
- : \vdash (\top, \Delta_3), \perp, \Delta_6 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \perp, \top \quad \top
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{}{\bullet h_3 : \vdash \text{dual}(\top), \top, \Delta_4} \top \\
\hline
- : \vdash \Delta_2, \top, \Delta_4 \quad \text{Cut} \\
\hline
- : \vdash \Delta_2, \Delta_4, \top \quad \top \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{}{\bullet h_5 : \vdash \text{dual}(F_4), \top, \Delta_6} \top \\
\hline
- : \vdash (\top, \Delta_3), \top, \Delta_6 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \top, \top \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_7, F_5 \quad h_3 : \vdash \Delta_4, F_6}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \vdash \Delta_2, \top} \text{ax} \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_7, F_5}{\bullet h_3 : \vdash \mathbf{0}, \Delta_7, F_5} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_7, F_5 \quad \text{hCut} \quad - : \vdash \Delta_4, F_6 \quad \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \quad \otimes \\
\hline
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \Delta_4, F_5 \quad h_3 : \vdash \mathbf{0}, \Delta_7, F_6}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \vdash \Delta_4, F_5} \text{ax} \quad \frac{}{\bullet h_1 : \vdash \Delta_2, \top} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_7, F_6}{\bullet h_3 : \vdash \mathbf{0}, \Delta_7, F_6} \text{ax} \\
\hline
- : \vdash \Delta_4, F_5 \quad - : \vdash \Delta_2, \Delta_7, F_6 \quad \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \quad \otimes \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_9, F_7, \text{dual}(F_4) \quad h_5 : \vdash \Delta_6, F_8}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\hline
- : \vdash (\top, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_7 \otimes F_8 \quad \top \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_6, F_7 \quad h_5 : \vdash \Delta_9, F_8, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\hline
- : \vdash (\top, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_7 \otimes F_8 \quad \top
\end{array}$$

5.12 Status of I : OK

- Case rule 1
- Case rule !
- Case rule ? W

$$\begin{array}{c}
\frac{\frac{}{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \wedge(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, \wedge(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(p(\mathbf{n}_3)), \Delta_6, ?\mathbf{F}_5} ?W}{- \vdash \wedge(\mathbf{n}_3), \Delta_6, ?\mathbf{F}_5} \text{Cut} \\
\frac{}{- \vdash \Delta_6, \wedge(\mathbf{n}_3)} \text{ax} \\
\frac{}{- \vdash \Delta_6, ?\mathbf{F}_5, \wedge(\mathbf{n}_3)} W
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \wedge(\mathbf{n}_3), p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(\wedge(\mathbf{n}_3)), \Delta_6, ?\mathbf{F}_5} ?W}{- \vdash p(\mathbf{n}_3), \Delta_6, ?\mathbf{F}_5} \text{Cut} \\
\frac{}{- \vdash \Delta_6, p(\mathbf{n}_3)} \text{ax} \\
\frac{}{- \vdash \Delta_6, ?\mathbf{F}_5, p(\mathbf{n}_3)} ?W
\end{array}$$

- Case rule ? C

$$\begin{array}{c}
\frac{\frac{}{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \wedge(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, ?\mathbf{F}_5, ?\mathbf{F}_5, \wedge(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(p(\mathbf{n}_3)), \Delta_6, ?\mathbf{F}_5} ?C}{- \vdash \wedge(\mathbf{n}_3), \Delta_6, ?\mathbf{F}_5} \text{Cut} \\
\frac{}{- \vdash \Delta_6, ?\mathbf{F}_5, ?\mathbf{F}_5, \wedge(\mathbf{n}_3)} \text{ax} \\
\frac{}{- \vdash \Delta_6, ?\mathbf{F}_5, \wedge(\mathbf{n}_3)} ?C
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \wedge(\mathbf{n}_3), p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, ?\mathbf{F}_5, ?\mathbf{F}_5, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(\wedge(\mathbf{n}_3)), \Delta_6, ?\mathbf{F}_5} ?C}{- \vdash p(\mathbf{n}_3), \Delta_6, ?\mathbf{F}_5} \text{Cut} \\
\frac{}{- \vdash \Delta_6, ?\mathbf{F}_5, ?\mathbf{F}_5, p(\mathbf{n}_3)} \text{ax} \\
\frac{}{- \vdash \Delta_6, ?\mathbf{F}_5, p(\mathbf{n}_3)} ?C
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{}{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \wedge(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, \mathbf{F}_5, \wedge(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(p(\mathbf{n}_3)), \Delta_6, ?\mathbf{F}_5} ?}{- \vdash \wedge(\mathbf{n}_3), \Delta_6, ?\mathbf{F}_5} \text{Cut} \\
\frac{}{- \vdash \Delta_6, \mathbf{F}_5, \wedge(\mathbf{n}_3)} \text{ax} \\
\frac{}{- \vdash \Delta_6, ?\mathbf{F}_5, \wedge(\mathbf{n}_3)} ?
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \wedge(\mathbf{n}_3), p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, \mathbf{F}_5, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(\wedge(\mathbf{n}_3)), \Delta_6, ?\mathbf{F}_5} ?}{- \vdash p(\mathbf{n}_3), \Delta_6, ?\mathbf{F}_5} \text{Cut} \\
\frac{}{- \vdash \Delta_6, \mathbf{F}_5, p(\mathbf{n}_3)} \text{ax} \\
\frac{}{- \vdash \Delta_6, ?\mathbf{F}_5, p(\mathbf{n}_3)} ?
\end{array}$$

- Case rule \$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash p(n_3), \wedge(n_3)} I \quad \frac{h_4 : \vdash \Delta_5, \wedge(n_3)}{\bullet h_4 : \vdash dual(p(n_3)), \perp, \Delta_5} \perp}{- : \vdash \wedge(n_3), \perp, \Delta_5} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{}{- : \vdash \Delta_5, \wedge(n_3)} \text{ax}}{- : \vdash \Delta_5, \perp, \wedge(n_3)} \perp \\
\\
\frac{\frac{}{\bullet h_1 : \vdash \wedge(n_3), p(n_3)} I \quad \frac{h_4 : \vdash \Delta_5, p(n_3)}{\bullet h_4 : \vdash dual(\wedge(n_3)), \perp, \Delta_5} \perp}{- : \vdash p(n_3), \perp, \Delta_5} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{}{- : \vdash \Delta_5, p(n_3)} \text{ax}}{- : \vdash \Delta_5, \perp, p(n_3)} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash p(n_3), \wedge(n_3)} I \quad \frac{}{\bullet h_4 : \vdash dual(p(n_3)), \top, \Delta_5} \top}{- : \vdash \wedge(n_3), \top, \Delta_5} \text{Cut} \\
\rightsquigarrow \\
\frac{}{- : \vdash \Delta_5, \top, \wedge(n_3)} \top \\
\\
\frac{\frac{}{\bullet h_1 : \vdash \wedge(n_3), p(n_3)} I \quad \frac{}{\bullet h_4 : \vdash dual(\wedge(n_3)), \top, \Delta_5} \top}{- : \vdash p(n_3), \top, \Delta_5} \text{Cut} \\
\rightsquigarrow \\
\frac{}{- : \vdash \Delta_5, \top, p(n_3)} \top
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash p(n_4), \wedge(n_4)} I \quad \frac{}{\bullet h_3 : \vdash dual(p(n_4)), p(n_4)} I}{- : \vdash \wedge(n_4), p(n_4)} \text{Cut} \\
\rightsquigarrow \\
\frac{}{- : \vdash p(n_4), \wedge(n_4)} \text{ax} \\
\\
\frac{\frac{}{\bullet h_1 : \vdash \wedge(n_4), p(n_4)} I \quad \frac{}{\bullet h_3 : \vdash dual(\wedge(n_4)), \wedge(n_4)} I}{- : \vdash p(n_4), \wedge(n_4)} \text{Cut} \\
\rightsquigarrow \\
\frac{}{- : \vdash p(n_4), \wedge(n_4)} I
\end{array}$$

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash p(n_3), \wedge(n_3)} I \quad \frac{h_4 : \vdash \Delta_8, F_6, \wedge(n_3) \quad h_4 : \vdash \Delta_5, F_7}{\bullet h_4 : \vdash dual(p(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes}{- : \vdash \wedge(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{}{- : \vdash \Delta_8, F_6, \wedge(n_3)} \text{ax} \quad \frac{}{- : \vdash \Delta_5, F_7} \text{ax}}{- : \vdash \Delta_5, \Delta_8, \wedge(n_3), F_6 \otimes F_7} \otimes \\
\\
\frac{\frac{}{\bullet h_1 : \vdash p(n_3), \wedge(n_3)} I \quad \frac{h_4 : \vdash \Delta_5, F_6 \quad h_4 : \vdash \Delta_8, F_7, \wedge(n_3)}{\bullet h_4 : \vdash dual(p(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes}{- : \vdash \wedge(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{}{- : \vdash \Delta_5, F_6} \text{ax} \quad \frac{}{- : \vdash \Delta_8, F_7, \wedge(n_3)} \text{ax}}{- : \vdash \Delta_5, \Delta_8, \wedge(n_3), F_6 \otimes F_7} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet \mathbf{h}_1 : \vdash \wedge(\mathbf{n}_3), p(\mathbf{n}_3)}{\vdash : \vdash p(\mathbf{n}_3), \Delta_5, \Delta_8, F_6 \otimes F_7} I \quad \frac{\frac{\mathbf{h}_4 : \vdash \Delta_8, F_6, p(\mathbf{n}_3) \quad \mathbf{h}_4 : \vdash \Delta_5, F_7}{\bullet \mathbf{h}_4 : \vdash dual(\wedge(\mathbf{n}_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes}{\vdash : \vdash p(\mathbf{n}_3), \Delta_5, \Delta_8, F_6 \otimes F_7} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\vdash : \vdash \Delta_8, F_6, p(\mathbf{n}_3)}{\vdash : \vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \text{ax} \quad \frac{\vdash : \vdash \Delta_5, F_7}{\vdash : \vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \text{ax}}{\vdash : \vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \otimes \\
\\
\frac{\frac{\bullet \mathbf{h}_1 : \vdash \wedge(\mathbf{n}_3), p(\mathbf{n}_3)}{\vdash : \vdash p(\mathbf{n}_3), \Delta_5, \Delta_8, F_6 \otimes F_7} I \quad \frac{\frac{\mathbf{h}_4 : \vdash \Delta_5, F_6 \quad \mathbf{h}_4 : \vdash \Delta_8, F_7, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 : \vdash dual(\wedge(\mathbf{n}_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes}{\vdash : \vdash p(\mathbf{n}_3), \Delta_5, \Delta_8, F_6 \otimes F_7} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\vdash : \vdash \Delta_5, F_6}{\vdash : \vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \text{ax} \quad \frac{\vdash : \vdash \Delta_8, F_7, p(\mathbf{n}_3)}{\vdash : \vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \text{ax}}{\vdash : \vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \otimes
\end{array}$$

5.13 Status of \otimes : OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
\frac{\frac{\mathbf{h}_2 : \vdash \Delta_6, F_7, F_4 \quad \mathbf{h}_2 : \vdash \Delta_3, F_5}{\bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(F_7)}{\bullet \mathbf{h}_8 : \vdash dual(F_7), \Delta_{10}, ?F_9} ?W}{\vdash : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{10}, ?F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, F_4 \otimes F_5} \text{ax} \quad \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(F_7)}{\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \text{ax}}{\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \text{hCut} \\
\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, F_4 \otimes F_5 \quad ?W \\
\\
\frac{\frac{\mathbf{h}_2 : \vdash \Delta_3, F_4 \quad \mathbf{h}_2 : \vdash \Delta_6, F_7, F_5}{\bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(F_7)}{\bullet \mathbf{h}_8 : \vdash dual(F_7), \Delta_{10}, ?F_9} ?W}{\vdash : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{10}, ?F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, F_4 \otimes F_5} \text{ax} \quad \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(F_7)}{\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \text{ax}}{\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \text{hCut} \\
\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, F_4 \otimes F_5 \quad ?W \\
\\
\frac{\frac{\mathbf{h}_1 : \vdash \Delta_2, F_6 \quad \mathbf{h}_1 : \vdash \Delta_3, F_7}{\bullet \mathbf{h}_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(F_6) \$ dual(F_7)}{\bullet \mathbf{h}_8 : \vdash dual(F_6 \otimes F_7), \Delta_{10}, ?F_9} ?W}{\vdash : \vdash (\Delta_2, \Delta_3), \Delta_{10}, ?F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{\vdash : \vdash \Delta_{10}, \Delta_2, \Delta_3} \text{ax} \quad \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(F_6) \$ dual(F_7)}{\vdash : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?F_9} \text{ax}}{\vdash : \vdash \Delta_{10}, \Delta_2, \Delta_3, ?F_9} \text{hCut} \\
\vdash : \vdash \Delta_{10}, \Delta_2, \Delta_3 \quad ?W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\frac{\mathbf{h}_2 : \vdash \Delta_6, F_7, F_4 \quad \mathbf{h}_2 : \vdash \Delta_3, F_5}{\bullet \mathbf{h}_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{\mathbf{h}_8 : \vdash \Delta_{10}, ?F_9, ?F_9, dual(F_7)}{\bullet \mathbf{h}_8 : \vdash dual(F_7), \Delta_{10}, ?F_9} ?C}{\vdash : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{10}, ?F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet \mathbf{h}_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, ?F_9, F_4 \otimes F_5} \text{ax} \quad \frac{\mathbf{h}_8 : \vdash \Delta_{10}, ?F_9, ?F_9, dual(F_7)}{\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \text{ax}}{\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \text{hCut} \\
\vdash : \vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, ?F_9, F_4 \otimes F_5 \quad ?C
\end{array}$$

[illegible]

- Case rule ?

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_6, F_7, F_4 \quad h_2 \vdash \Delta_3, F_5 \quad \otimes \quad \frac{h_8 \vdash \Delta_{10}, F_9, dual(F_7) \quad ?}{\bullet h_8 \vdash dual(F_7), \Delta_{10}, ?F_9}}{\vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{10}, ?F_9} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5 \quad \text{ax} \quad \frac{h_8 \vdash \Delta_{10}, F_9, dual(F_7)}{h_8 \vdash dual(F_7), \Delta_{10}, ?F_9} \quad \text{hCut}}{\vdash \Delta_{10}, \Delta_3, \Delta_6, F_9, F_4 \otimes F_5} \quad ?}{\vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \\
\\
\frac{h_2 \vdash \Delta_3, F_4 \quad h_2 \vdash \Delta_6, F_7, F_5 \quad \otimes \quad \frac{h_8 \vdash \Delta_{10}, F_9, dual(F_7) \quad ?}{\bullet h_8 \vdash dual(F_7), \Delta_{10}, ?F_9}}{\vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{10}, ?F_9} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5 \quad \text{ax} \quad \frac{h_8 \vdash \Delta_{10}, F_9, dual(F_7)}{h_8 \vdash dual(F_7), \Delta_{10}, ?F_9} \quad \text{hCut}}{\vdash \Delta_{10}, \Delta_3, \Delta_6, F_9, F_4 \otimes F_5} \quad ?}{\vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \\
\\
\frac{h_1 \vdash \Delta_2, F_6 \quad h_1 \vdash \Delta_3, F_7 \quad \otimes \quad \frac{h_8 \vdash \Delta_{10}, F_9, dual(F_6) \$ dual(F_7) \quad ?}{\bullet h_8 \vdash dual(F_6 \otimes F_7), \Delta_{10}, ?F_9}}{\vdash (\Delta_2, \Delta_3), \Delta_{10}, ?F_9} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, \Delta_3, F_6 \otimes F_7 \quad \text{ax} \quad \frac{h_8 \vdash \Delta_{10}, F_9, dual(F_6) \$ dual(F_7)}{h_8 \vdash dual(F_6 \otimes F_7), \Delta_{10}, ?F_9} \quad \text{hCut}}{\vdash \Delta_{10}, \Delta_2, \Delta_3, F_9} \quad ?}{\vdash \Delta_{10}, \Delta_2, \Delta_3, ?F_9}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_6, F_7, F_4 \quad h_2 \vdash \Delta_3, F_5}{\bullet h_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 \vdash \Delta_{11}, F_9, F_{10}, dual(F_7)}{\bullet h_8 \vdash dual(F_7), \Delta_{11}, F_9 \$ F_{10}} \$ \\
\frac{}{- \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \$ F_{10}} \\
\sim \\
\frac{\bullet h_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5 \quad ax \quad h_8 \vdash \Delta_{11}, F_{10}, F_9, dual(F_7) \quad ax}{- \vdash \Delta_{11}, \Delta_3, \Delta_6, F_{10}, F_9, F_4 \otimes F_5} hCut \\
\frac{}{- \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \$ F_{10}, F_4 \otimes F_5} \$ \\
\sim \\
\frac{h_2 \vdash \Delta_3, F_4 \quad h_2 \vdash \Delta_6, F_7, F_5}{\bullet h_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 \vdash \Delta_{11}, F_9, F_{10}, dual(F_7)}{\bullet h_8 \vdash dual(F_7), \Delta_{11}, F_9 \$ F_{10}} \$ \\
\frac{}{- \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \$ F_{10}} \\
\sim \\
\frac{\bullet h_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5 \quad ax \quad h_8 \vdash \Delta_{11}, F_{10}, F_9, dual(F_7) \quad ax}{- \vdash \Delta_{11}, \Delta_3, \Delta_6, F_{10}, F_9, F_4 \otimes F_5} hCut \\
\frac{}{- \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \$ F_{10}, F_4 \otimes F_5} \$
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2, F_6 \quad h_1 : \vdash \Delta_3, F_7}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_9, F_{10}, dual(F_6) \$ dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_{11}, F_9 \$ F_{10}} \$}{- : \vdash (\Delta_2, \Delta_3), \Delta_{11}, F_9 \$ F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_{10}, F_9} \text{ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, F_9, dual(F_6) \$ dual(F_7)}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9 \$ F_{10}} \text{ax}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9 \$ F_{10}} \text{hCut} \\
\$ \\
\frac{h_1 : \vdash \Delta_2, F_6 \quad h_1 : \vdash \Delta_3, F_7}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash \Delta_9, dual(F_6), dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_9} \$}{- : \vdash (\Delta_2, \Delta_3), \Delta_9} \text{Cut} \\
\sim \\
\frac{\frac{- : \vdash \Delta_2, F_6}{- : \vdash \Delta_2, F_6} \text{ax} \quad \frac{- : \vdash \Delta_3, F_7}{- : \vdash \Delta_3, F_7} \text{ax} \quad \frac{- : \vdash \Delta_9, dual(F_6), dual(F_7)}{- : \vdash \Delta_3, \Delta_9, dual(F_6)} \text{ax}}{- : \vdash \Delta_2, \Delta_3, \Delta_9} \text{sCut}
\end{array}$$

• Case rule &

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_6, F_7, F_4 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_7) \quad h_8 : \vdash \Delta_{11}, F_{10}, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \& F_{10}} \&}{- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \& F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \vdash \Delta_6, F_4, F_7}{- : \vdash \Delta_{11}, \Delta_6, F_4, F_9 \& F_{10}} \text{ax} \quad \frac{\bullet h_8 : \vdash \Delta_{11}, dual(F_7), F_9 \& F_{10}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \& F_{10}, F_4 \otimes F_5} \text{ax}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \& F_{10}, F_4 \otimes F_5} \text{hCut} \\
\otimes \\
\frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \Delta_6, F_7, F_5}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_7) \quad h_8 : \vdash \Delta_{11}, F_{10}, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \& F_{10}} \&}{- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \& F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{- : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, F_4} \text{ax} \quad \frac{h_2 : \vdash \Delta_6, F_5, F_7}{- : \vdash \Delta_{11}, \Delta_6, F_5, F_9 \& F_{10}} \text{ax} \quad \frac{\bullet h_8 : \vdash \Delta_{11}, dual(F_7), F_9 \& F_{10}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \& F_{10}, F_4 \otimes F_5} \text{ax}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \& F_{10}, F_4 \otimes F_5} \text{hCut} \\
\otimes \\
\frac{h_1 : \vdash \Delta_2, F_6 \quad h_1 : \vdash \Delta_3, F_7}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_6) \$ dual(F_7) \quad h_8 : \vdash \Delta_{11}, F_{10}, dual(F_6) \$ dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_{11}, F_9 \& F_{10}} \&}{- : \vdash (\Delta_2, \Delta_3), \Delta_{11}, F_9 \& F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9} \text{ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_6) \$ dual(F_7)}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9} \text{ax} \quad \frac{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_{10}} \text{ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, dual(F_6) \$ dual(F_7)}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_{10}} \text{ax}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9 \& F_{10}} \text{hCut} \\
\&
\end{array}$$

• Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_6, F_7, F_4 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_B}{- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \oplus F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_{10}, F_4 \otimes F_5} \text{ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, dual(F_7)}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_5, F_9 \oplus F_{10}} \text{ax}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_4 \otimes F_5, F_9 \oplus F_{10}} \text{hCut} \\
\oplus_B \\
\frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \Delta_6, F_7, F_5}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_B}{- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \oplus F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_{10}, F_4 \otimes F_5} \text{ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, dual(F_7)}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_5, F_9 \oplus F_{10}} \text{ax}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_4 \otimes F_5, F_9 \oplus F_{10}} \text{hCut} \\
\oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, F_6 \quad h_1 : \vdash \Delta_3, F_7}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, dual(F_6) \$ dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_B \\
\hline
- : \vdash (\Delta_2, \Delta_3), \Delta_{11}, F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\sim \rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_{10}} \text{ ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, dual(F_6) \$ dual(F_7)}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_{10}} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9 \oplus F_{10}} \text{ hCut} \oplus_B
\end{array}$$

• Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_6, F_7, F_4 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_A \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\sim \rightsquigarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9, F_4 \otimes F_5} \text{ ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_7)}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9, F_4 \otimes F_5} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_4 \otimes F_5, F_9 \oplus F_{10}} \text{ hCut} \oplus_A \\
\hline
\frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \Delta_6, F_7, F_5}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_A \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\sim \rightsquigarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9, F_4 \otimes F_5} \text{ ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_7)}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9, F_4 \otimes F_5} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_4 \otimes F_5, F_9 \oplus F_{10}} \text{ hCut} \oplus_A \\
\hline
\frac{h_1 : \vdash \Delta_2, F_6 \quad h_1 : \vdash \Delta_3, F_7}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_6) \$ dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_A \\
\hline
- : \vdash (\Delta_2, \Delta_3), \Delta_{11}, F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\sim \rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9} \text{ ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_6) \$ dual(F_7)}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9 \oplus F_{10}} \text{ hCut} \oplus_A
\end{array}$$

• Case rule \perp

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_6, F_7, F_4 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_9, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \perp, \Delta_9} \perp \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \perp, \Delta_9 \quad \text{Cut} \\
\hline
\sim \rightsquigarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{- : \vdash \Delta_3, \Delta_6, \Delta_9, F_4 \otimes F_5} \text{ ax} \quad \frac{h_8 : \vdash \Delta_9, dual(F_7)}{- : \vdash \Delta_3, \Delta_6, \Delta_9, F_4 \otimes F_5} \text{ ax}}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_4 \otimes F_5} \text{ hCut} \perp \\
\hline
\frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \Delta_6, F_7, F_5}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_9, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \perp, \Delta_9} \perp \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \perp, \Delta_9 \quad \text{Cut} \\
\hline
\sim \rightsquigarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{- : \vdash \Delta_3, \Delta_6, \Delta_9, F_4 \otimes F_5} \text{ ax} \quad \frac{h_8 : \vdash \Delta_9, dual(F_7)}{- : \vdash \Delta_3, \Delta_6, \Delta_9, F_4 \otimes F_5} \text{ ax}}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_4 \otimes F_5} \text{ hCut} \perp \\
\hline
\frac{h_1 : \vdash \Delta_2, F_6 \quad h_1 : \vdash \Delta_3, F_7}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash \Delta_9, dual(F_6) \$ dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \perp, \Delta_9} \perp \\
\hline
- : \vdash (\Delta_2, \Delta_3), \perp, \Delta_9 \quad \text{Cut} \\
\hline
\sim \rightsquigarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- : \vdash \Delta_2, \Delta_3, \Delta_9} \text{ ax} \quad \frac{h_8 : \vdash \Delta_9, dual(F_6) \$ dual(F_7)}{- : \vdash \Delta_2, \Delta_3, \Delta_9} \text{ ax}}{- : \vdash \Delta_2, \Delta_3, \Delta_9, \perp} \text{ hCut} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_6, F_7, F_4 \quad h_2 \vdash \Delta_3, F_5}{\bullet h_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \frac{\bullet h_8 \vdash dual(F_7), \top, \Delta_9}{\text{Cut}}}{- \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \top, \Delta_9} \\
\sim \\
- \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_4 \otimes F_5 \quad \top \\
\\
\frac{\frac{h_2 \vdash \Delta_3, F_4 \quad h_2 \vdash \Delta_6, F_7, F_5}{\bullet h_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \frac{\bullet h_8 \vdash dual(F_7), \top, \Delta_9}{\text{Cut}}}{- \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \top, \Delta_9} \\
\sim \\
- \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_4 \otimes F_5 \quad \top \\
\\
\frac{\frac{h_1 \vdash \Delta_2, F_6 \quad h_1 \vdash \Delta_3, F_7}{\bullet h_1 \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \frac{\bullet h_8 \vdash dual(F_6 \otimes F_7), \top, \Delta_9}{\text{Cut}}}{- \vdash (\Delta_2, \Delta_3), \top, \Delta_9} \\
\sim \\
- \vdash \Delta_2, \Delta_3, \Delta_9, \top \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_6, F_7, F_4 \quad h_2 \vdash \Delta_3, F_5}{\bullet h_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 \vdash \Delta_{12}, F_{10}, dual(F_7) \quad h_8 \vdash \Delta_9, F_{11}}{\bullet h_8 \vdash dual(F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5 \quad \text{ax} \quad h_8 \vdash \Delta_{12}, F_{10}, dual(F_7)}{- \vdash \Delta_{12}, \Delta_3, \Delta_6, F_{10}, F_4 \otimes F_5} \text{ax} \quad \frac{\text{ax}}{- \vdash \Delta_9, F_{11}} \text{hCut} \\
\hline
- \vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, F_{10} \otimes F_{11}, F_4 \otimes F_5 \quad \otimes \\
\hline
\frac{h_2 \vdash \Delta_6, F_7, F_4 \quad h_2 \vdash \Delta_3, F_5}{\bullet h_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 \vdash \Delta_9, F_{10} \quad h_8 \vdash \Delta_{12}, F_{11}, dual(F_7)}{\bullet h_8 \vdash dual(F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5 \quad \text{ax} \quad h_8 \vdash \Delta_{12}, F_{11}, dual(F_7)}{- \vdash \Delta_{12}, \Delta_3, \Delta_6, F_{11}, F_4 \otimes F_5} \text{ax} \quad \frac{\text{ax}}{- \vdash \Delta_9, F_{10}} \text{hCut} \\
\hline
- \vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, F_{10} \otimes F_{11}, F_4 \otimes F_5 \quad \otimes \\
\hline
\frac{h_2 \vdash \Delta_3, F_4 \quad h_2 \vdash \Delta_6, F_7, F_5}{\bullet h_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 \vdash \Delta_{12}, F_{10}, dual(F_7) \quad h_8 \vdash \Delta_9, F_{11}}{\bullet h_8 \vdash dual(F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5 \quad \text{ax} \quad h_8 \vdash \Delta_{12}, F_{10}, dual(F_7)}{- \vdash \Delta_{12}, \Delta_3, \Delta_6, F_{10}, F_4 \otimes F_5} \text{ax} \quad \frac{\text{ax}}{- \vdash \Delta_9, F_{11}} \text{hCut} \\
\hline
- \vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, F_{10} \otimes F_{11}, F_4 \otimes F_5 \quad \otimes \\
\hline
\frac{h_2 \vdash \Delta_3, F_4 \quad h_2 \vdash \Delta_6, F_7, F_5}{\bullet h_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 \vdash \Delta_9, F_{10} \quad h_8 \vdash \Delta_{12}, F_{11}, dual(F_7)}{\bullet h_8 \vdash dual(F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5 \quad \text{ax} \quad h_8 \vdash \Delta_{12}, F_{11}, dual(F_7)}{- \vdash \Delta_{12}, \Delta_3, \Delta_6, F_{11}, F_4 \otimes F_5} \text{ax} \quad \frac{\text{ax}}{- \vdash \Delta_9, F_{10}} \text{hCut} \\
\hline
- \vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, F_{10} \otimes F_{11}, F_4 \otimes F_5 \quad \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, F_6 \quad h_1 \vdash \Delta_3, F_7}{\bullet h_1 \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 \vdash \Delta_{12}, F_{10}, dual(F_6) \$ dual(F_7) \quad h_8 \vdash \Delta_9, F_{11}}{\bullet h_8 \vdash dual(F_6 \otimes F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\hline
\sim\!\!\!\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- \vdash \Delta_{12}, \Delta_2, \Delta_3, F_{10}} \text{ ax} \quad \frac{h_8 \vdash \Delta_{12}, F_{10}, dual(F_6) \$ dual(F_7)}{hCut} \text{ ax}}{- \vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11}} \otimes \\
\hline
\frac{h_1 \vdash \Delta_2, F_6 \quad h_1 \vdash \Delta_3, F_7}{\bullet h_1 \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 \vdash \Delta_9, F_{10} \quad h_8 \vdash \Delta_{12}, F_{11}, dual(F_6) \$ dual(F_7)}{\bullet h_8 \vdash dual(F_6 \otimes F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\hline
\sim\!\!\!\rightarrow \\
\frac{\frac{- \vdash \Delta_9, F_{10}}{- \vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11}} \text{ ax} \quad \frac{\bullet h_1 \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- \vdash \Delta_{12}, \Delta_2, \Delta_3, F_{11}} \text{ ax} \quad \frac{h_8 \vdash \Delta_{12}, F_{11}, dual(F_6) \$ dual(F_7)}{hCut} \text{ ax}}{- \vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11}} \otimes
\end{array}$$

6 Cut-Elimination

6.1 Status of 1: OK

- Case rule 1
- Case rule !
- Case rule ?W
- Case rule ?C
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.2 Status of !: OK

- Case rule 1
- Case rule !

$$\begin{array}{c}
 \frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} ! \quad \frac{h_6 \vdash F_7, ?Y8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), ?Y8, !F_7} !}{\frac{}{- \vdash ?Y2, ?Y8, !F_7} \text{Cut}} \\
 \frac{}{\sim} \\
 \frac{\frac{\bullet h_1 \vdash ?Y2, !F_4}{\bullet h_1 \vdash ?Y2, !F_4} \text{ax} \quad \frac{h_6 \vdash ?Y8, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?Y2, ?Y8, F_7} \text{ax}}{\frac{}{- \vdash ?Y2, ?Y8, !F_7} !} \text{hCut}
 \end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} ! \quad \frac{h_6 \vdash \Delta_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_8, ?F_7} ?W \\
\hline
- : \vdash ?Y2, \Delta_8, ?F_7 \quad \text{Cut} \\
\sim \\
\frac{\bullet h_1 \vdash ?Y2, !F_4}{\bullet h_1 \vdash ?Y2, !F_4} \text{ax} \quad \frac{h_6 \vdash \Delta_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- : \vdash ?Y2, \Delta_8} \text{ax} \\
\hline
- : \vdash ?Y2, \Delta_8, ?F_7 \quad ?W \\
\hline
\frac{h_1 \vdash F_5, ?Y2}{\bullet h_1 \vdash !F_5, ?Y2} ! \quad \frac{h_7 \vdash \Delta_4, contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_4} ?W \\
\hline
- : \vdash ?Y2, \Delta_4 \quad \text{Cut} \\
\sim \\
\frac{\bullet h_1 \vdash ?Y2, !F_5}{\bullet h_1 \vdash ?Y2, !F_5} \text{ax} \quad \frac{h_7 \vdash \Delta_4, contract(n_6, ?dual(F_5))}{- : \vdash ?Y2, \Delta_4} \text{ax} \\
\hline
- : \vdash ?Y2, \Delta_4 \quad \text{hCut}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} ! \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_8, ?F_7} ?C \\
\hline
- : \vdash ?Y2, \Delta_8, ?F_7 \quad \text{Cut} \\
\sim \\
\frac{\bullet h_1 \vdash ?Y2, !F_4}{\bullet h_1 \vdash ?Y2, !F_4} \text{ax} \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- : \vdash ?Y2, \Delta_8, ?F_7, ?F_7} \text{ax} \\
\hline
- : \vdash ?Y2, \Delta_8, ?F_7 \quad ?C \\
\hline
\frac{h_1 \vdash F_5, ?Y2}{\bullet h_1 \vdash !F_5, ?Y2} ! \quad \frac{h_7 \vdash \Delta_4, ?dual(F_5), ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_4} ?C \\
\hline
- : \vdash ?Y2, \Delta_4 \quad \text{Cut} \\
\sim \\
\frac{\bullet h_1 \vdash ?Y2, !F_5}{\bullet h_1 \vdash ?Y2, !F_5} \text{ax} \quad \frac{h_7 \vdash \Delta_4, ?dual(F_5), ?dual(F_5), contract(n_6, ?dual(F_5))}{- : \vdash ?Y2, \Delta_4} \text{ax} \\
\hline
- : \vdash ?Y2, \Delta_4 \quad \text{hCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} ! \quad \frac{h_6 \vdash \Delta_8, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_8, ?F_7} ? \\
\hline
- : \vdash ?Y2, \Delta_8, ?F_7 \quad \text{Cut} \\
\sim \\
\frac{\bullet h_1 \vdash ?Y2, !F_4}{\bullet h_1 \vdash ?Y2, !F_4} \text{ax} \quad \frac{h_6 \vdash \Delta_8, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- : \vdash ?Y2, \Delta_8, F_7} \text{ax} \\
\hline
- : \vdash ?Y2, \Delta_8, ?F_7 \quad ? \\
\hline
\frac{h_1 \vdash F_5, ?Y2}{\bullet h_1 \vdash !F_5, ?Y2} ! \quad \frac{h_7 \vdash \Delta_4, dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_4} ? \\
\hline
- : \vdash ?Y2, \Delta_4 \quad \text{Cut} \\
\sim \\
\frac{\bullet h_1 \vdash ?Y2, !F_5}{\bullet h_1 \vdash ?Y2, !F_5} \text{ax} \quad \frac{h_7 \vdash \Delta_4, dual(F_5), contract(n_6, ?dual(F_5))}{- : \vdash ?Y2, \Delta_4, dual(F_5)} \text{ax} \\
\hline
- : \vdash ?Y2, F_5 \quad \text{ax} \quad - : \vdash ?Y2, \Delta_4, dual(F_5) \quad \text{mCut} \\
\hline
- : \vdash ?Y2, \Delta_4
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} ! \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, F_7 \$F_8} \$ \\
\hline
- : \vdash ?Y2, \Delta_9, F_7 \$F_8 \quad \text{Cut} \\
\sim \\
\frac{\bullet h_1 \vdash ?Y2, !F_4}{\bullet h_1 \vdash ?Y2, !F_4} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- : \vdash ?Y2, \Delta_9, F_7, F_8} \text{ax} \\
\hline
- : \vdash ?Y2, \Delta_9, F_7 \$F_8 \quad \$ \\
\hline
- : \vdash ?Y2, \Delta_9, F_7 \$F_8
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4)) \quad h_6 \vdash \Delta_9, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, F_7 \& F_8} \&}{- \vdash ?\Upsilon 2, \Delta_9, F_7 \& F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_9, F_7} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_9, F_7 \& F_8} \text{hCut} \quad \frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_9, F_8} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_9, F_7 \& F_8} \text{hCut} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_6 \vdash \Delta_9, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, F_7 \oplus F_8} \oplus_B}{- \vdash ?\Upsilon 2, \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_9, F_8} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_9, F_7 \oplus F_8} \text{hCut} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, F_7 \oplus F_8} \oplus_A}{- \vdash ?\Upsilon 2, \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_9, F_7} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_9, F_7 \oplus F_8} \text{hCut} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_6 \vdash \Delta_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \perp, \Delta_7} \perp}{- \vdash ?\Upsilon 2, \perp, \Delta_7} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ax} \quad \frac{h_6 \vdash \Delta_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_7} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_7, \perp} \text{hCut} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_6 \vdash \Delta_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \top, \Delta_7} \top}{- \vdash ?\Upsilon 2, \top, \Delta_7} \text{Cut} \\
\rightsquigarrow \\
\frac{- \vdash ?\Upsilon 2, \Delta_7, \top}{- \vdash ?\Upsilon 2, \Delta_7, \top} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} ! \quad \frac{h_6 \vdash \Delta_{10}, F_8, ?dual(F_4), contract(n_5, ?dual(F_4)) \quad h_6 \vdash \Delta_7, F_9}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{- \vdash ?Y2, \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} \text{ax} \quad \frac{h_6 \vdash \Delta_{10}, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?Y2, \Delta_{10}, F_8} \text{ax}}{- \vdash ?Y2, \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{hCut} \quad \frac{- \vdash \Delta_7, F_9}{- \vdash ?Y2, \Delta_{10}, F_8 \otimes F_9} \text{ax}}{\otimes} \\
\\
\frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} ! \quad \frac{h_6 \vdash \Delta_9, F_7, contract(n_5, ?dual(F_4)) \quad h_6 \vdash \Delta_{10}, F_8, ?dual(F_4)}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, \Delta_{10}, F_7 \otimes F_8} \otimes}{- \vdash ?Y2, \Delta_9, \Delta_{10}, F_7 \otimes F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, contract(n_5, ?dual(F_4))}{- \vdash ?Y2, \Delta_9, F_7} \text{ax}}{- \vdash ?Y2, ?Y2, \Delta_{10}, \Delta_9, F_7 \otimes F_8} \text{hCut} \quad \frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} \text{ax} \quad \frac{h_6 \vdash \Delta_{10}, F_8, ?dual(F_4)}{- \vdash ?Y2, \Delta_{10}, F_8} \text{ax}}{- \vdash ?Y2, \Delta_{10}, \Delta_9, F_7 \otimes F_8} \text{hCut}}{\otimes} \\
\rightsquigarrow \\
\frac{- \vdash ?Y2, ?Y2, \Delta_{10}, \Delta_9, F_7 \otimes F_8}{- \vdash ?Y2, \Delta_{10}, \Delta_9, F_7 \otimes F_8} C \\
\\
\frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} ! \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4) \quad h_6 \vdash \Delta_{10}, F_8, contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, \Delta_{10}, F_7 \otimes F_8} \otimes}{- \vdash ?Y2, \Delta_9, \Delta_{10}, F_7 \otimes F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4)}{- \vdash ?Y2, \Delta_9, F_7} \text{ax}}{- \vdash ?Y2, ?Y2, \Delta_{10}, \Delta_9, F_7 \otimes F_8} \text{hCut} \quad \frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} \text{ax} \quad \frac{h_6 \vdash \Delta_{10}, F_8, contract(n_5, ?dual(F_4))}{- \vdash ?Y2, \Delta_{10}, F_8} \text{ax}}{- \vdash ?Y2, \Delta_{10}, \Delta_9, F_7 \otimes F_8} \text{hCut}}{\otimes} \\
\rightsquigarrow \\
\frac{- \vdash ?Y2, ?Y2, \Delta_{10}, \Delta_9, F_7 \otimes F_8}{- \vdash ?Y2, \Delta_{10}, \Delta_9, F_7 \otimes F_8} C \\
\\
\frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} ! \quad \frac{h_6 \vdash \Delta_7, F_8 \quad h_6 \vdash \Delta_{10}, F_9, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{- \vdash ?Y2, \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{- \vdash \Delta_7, F_8}{- \vdash ?Y2, \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{ax} \quad \frac{\frac{h_1 \vdash F_4, ?Y2}{\bullet h_1 \vdash !F_4, ?Y2} \text{ax} \quad \frac{h_6 \vdash \Delta_{10}, F_9, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?Y2, \Delta_{10}, F_9} \text{ax}}{- \vdash ?Y2, \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{hCut}}{\otimes}
\end{array}$$

6.3 Status of ?W: OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{\frac{h_2 \vdash ?Y4, !F_5}{\bullet h_2 \vdash !F_5, ?Y4, ?F_3} ?W \quad \frac{h_7 \vdash F_8, ?Y9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), ?Y9, !F_8} !}{- \vdash (?Y4, ?F_3), ?Y9, !F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 \vdash ?Y4, !F_5, ?F_3}{\bullet h_2 \vdash !F_5, ?Y4, ?F_3} \text{ax} \quad \frac{h_7 \vdash ?Y9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?Y4, ?Y9, F_8, ?F_3} \text{ax}}{- \vdash ?Y4, ?Y9, !F_8, ?F_3} \text{hCut} \\
!
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_2 \vdash ?Y4, !F_5}{\bullet h_2 \vdash !F_5, ?Y4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ?W}{- \vdash (?Y4, ?F_3), \Delta_9, ?F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 \vdash ?Y4, !F_5, ?F_3}{\bullet h_2 \vdash !F_5, ?Y4, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?Y4, \Delta_9, ?F_3} \text{ax}}{- \vdash ?Y4, \Delta_9, ?F_3, ?F_8} \text{hCut} \\
?W
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash ?\Upsilon 4, !F_6}{\bullet h_2 : \vdash !F_6, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_8 : \vdash \Delta_5, \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 : \vdash \text{contract}(sn_7, ?dual(F_6)), \Delta_5} \text{ ?W} \\
\hline
- : \vdash (? \Upsilon 4, ?F_3), \Delta_5 \quad \text{Cut} \\
\hline
\frac{\bullet h_2 : \vdash ?\Upsilon 4, !F_6, ?F_3}{- : \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ ax} \quad \frac{h_8 : \vdash \Delta_5, \text{contract}(n_7, ?dual(F_6))}{- : \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ ax} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_5, ?F_3 \quad \text{hCut}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 : \vdash ?\Upsilon 4, !F_5}{\bullet h_2 : \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 : \vdash \text{contract}(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \text{ ?C} \\
\hline
- : \vdash (? \Upsilon 4, ?F_3), \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\frac{\bullet h_2 : \vdash ?\Upsilon 4, !F_5, ?F_3}{- : \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{ ax} \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{- : \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8, ?F_8} \text{ ax} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8 \quad \text{hCut} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8 \quad \text{?C} \\
\hline
\frac{h_2 : \vdash ?\Upsilon 4, !F_6}{\bullet h_2 : \vdash !F_6, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_8 : \vdash \Delta_5, ?dual(F_6), ?dual(F_6), \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 : \vdash \text{contract}(sn_7, ?dual(F_6)), \Delta_5} \text{ ?C} \\
\hline
- : \vdash (? \Upsilon 4, ?F_3), \Delta_5 \quad \text{Cut} \\
\hline
\frac{\bullet h_2 : \vdash ?\Upsilon 4, !F_6, ?F_3}{- : \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ ax} \quad \frac{h_8 : \vdash \Delta_5, ?dual(F_6), ?dual(F_6), \text{contract}(n_7, ?dual(F_6))}{- : \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ ax} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_5, ?F_3 \quad \text{hCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 : \vdash ?\Upsilon 4, !F_5}{\bullet h_2 : \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_7 : \vdash \Delta_9, F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 : \vdash \text{contract}(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \text{ ?} \\
\hline
- : \vdash (? \Upsilon 4, ?F_3), \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\frac{\bullet h_2 : \vdash ?\Upsilon 4, !F_5, ?F_3}{- : \vdash ?\Upsilon 4, \Delta_9, F_8, ?F_3} \text{ ax} \quad \frac{h_7 : \vdash \Delta_9, F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{- : \vdash ?\Upsilon 4, \Delta_9, F_8, ?F_3} \text{ ax} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_9, F_8, ?F_3 \quad \text{hCut} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8 \quad \text{?} \\
\hline
\frac{h_2 : \vdash ?\Upsilon 4, !F_6}{\bullet h_2 : \vdash !F_6, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_8 : \vdash \Delta_5, dual(F_6), \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 : \vdash \text{contract}(sn_7, ?dual(F_6)), \Delta_5} \text{ ?} \\
\hline
- : \vdash (? \Upsilon 4, ?F_3), \Delta_5 \quad \text{Cut} \\
\hline
\frac{\bullet h_2 : \vdash ?\Upsilon 4, !F_6}{- : \vdash ?\Upsilon 4, \Delta_5} \text{ ax} \quad \frac{h_8 : \vdash \Delta_5, dual(F_6), \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 : \vdash \Delta_5, ?dual(F_6), \text{contract}(n_7, ?dual(F_6))} \text{ ax} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_5 \quad \text{?W} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_5, ?F_3 \quad \text{hCut}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 : \vdash ?\Upsilon 4, !F_5}{\bullet h_2 : \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 : \vdash \text{contract}(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \$ F_9} \text{ \$} \\
\hline
- : \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \$ F_9 \quad \text{Cut} \\
\hline
\frac{\bullet h_2 : \vdash ?\Upsilon 4, !F_5, ?F_3}{- : \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \text{ ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{- : \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \text{ ax} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3 \quad \text{hCut} \\
\hline
- : \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$ F_9 \quad \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5)) \quad h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{\frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{10}, ?dual(F_5), F_8 \& F_9, contract(n_6, ?dual(F_5))} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{10}, ?dual(F_5), F_8 \& F_9, contract(n_6, ?dual(F_5))} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8 \& F_9} \& \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \& F_9 \quad ?W \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \& F_9 \quad \text{hCut}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{10}, ?dual(F_5), F_8 \oplus F_9, contract(n_6, ?dual(F_5))} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9 \quad \text{hCut}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{10}, ?dual(F_5), F_8 \oplus F_9, contract(n_6, ?dual(F_5))} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9 \quad \text{hCut}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \perp, \Delta_8} \perp \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_8, ?dual(F_5), \perp, \Delta_8, contract(n_6, ?dual(F_5))} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_8, ?F_3} \perp \\
\hline
- \vdash ?\Upsilon 4, \Delta_8, \perp, ?F_3 \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \top, \Delta_8} \top \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
- \vdash ?\Upsilon 4, \Delta_8, \top, ?F_3 \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5)) \quad h_7 \vdash \Delta_8, F_{10}}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{11}, ?dual(F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}, contract(n_6, ?dual(F_5))} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \otimes \\
\hline
- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10} \quad \otimes
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_{10}, F_8, \text{contract}(n_6, ?dual(F_5)) \quad h_7 \vdash \Delta_{11}, F_9, ?dual(F_5)}{\bullet h_7 \vdash \text{contract}(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes}{- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ax} \quad \frac{\frac{h_7 \vdash \Delta_{10}, F_8, \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, \text{contract}(n_6, ?dual(F_5))} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5)}{h_7 \vdash \Delta_{11}, F_9, \text{contract}(n_6, ?dual(F_5))} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9} \text{hCut} \\
\sim \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5) \quad h_7 \vdash \Delta_{11}, F_9, \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \vdash \text{contract}(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes}{- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ax} \quad \frac{\frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5)}{\bullet h_7 \vdash \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, \text{contract}(n_6, ?dual(F_5))} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, \text{contract}(n_6, ?dual(F_5))}{h_7 \vdash \Delta_{11}, F_9, \text{contract}(n_6, ?dual(F_5))} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9} \text{hCut} \\
\sim \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?W \quad \frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \vdash \text{contract}(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ax} \quad \frac{\frac{h_7 \vdash \Delta_{11}, F_{10}, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{11}, F_{10}, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \text{hCut} \\
\sim \\
- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}
\end{array}$$

6.4 Status of ?C: OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash F_8, ?\Upsilon 9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \vdash \text{contract}(sn_6, ?dual(F_5)), ?\Upsilon 9, !F_8} !}{- \vdash (? \Upsilon 4, ?F_3), ?\Upsilon 9, !F_8} \text{Cut} \\
\sim \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ax} \quad \frac{h_7 \vdash ?\Upsilon 9, F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, ?\Upsilon 9, F_8, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, ?\Upsilon 9, !F_8, ?F_3} \text{hCut} \\
\sim \\
- \vdash ?\Upsilon 4, ?\Upsilon 9, !F_8, ?F_3
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \vdash \text{contract}(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ?W}{- \vdash (? \Upsilon 4, ?F_3), \Delta_9, ?F_8} \text{Cut} \\
\sim \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{hCut} \\
\sim \\
- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8 \\
\sim \\
\frac{h_2 \vdash ?\Upsilon 4, !F_6, ?F_3, ?F_3}{\bullet h_2 \vdash !F_6, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_8 \vdash \Delta_5, \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 \vdash \text{contract}(sn_7, ?dual(F_6)), \Delta_5} ?W}{- \vdash (? \Upsilon 4, ?F_3), \Delta_5} \text{Cut} \\
\sim \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_6, ?F_3}{\bullet h_2 \vdash !F_6, ?\Upsilon 4, ?F_3} \text{ax} \quad \frac{h_8 \vdash \Delta_5, \text{contract}(n_7, ?dual(F_6))}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{hCut} \\
\sim \\
- \vdash ?\Upsilon 4, \Delta_5, ?F_3
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ?C}{- \vdash (? \Upsilon 4, ?F_3), \Delta_9, ?F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{\frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{ax} \quad \frac{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} ?C}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{hCut} \\
\\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_6, ?F_3, ?F_3}{\bullet h_2 \vdash !F_6, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_8 \vdash \Delta_5, ?dual(F_6), ?dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \vdash contract(sn_7, ?dual(F_6)), \Delta_5} ?C}{- \vdash (? \Upsilon 4, ?F_3), \Delta_5} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_6, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_6, ?F_3} \text{ax} \quad \frac{h_8 \vdash \Delta_5, ?dual(F_6), ?dual(F_6), contract(n_7, ?dual(F_6))}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{hCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ?}{- \vdash (? \Upsilon 4, ?F_3), \Delta_9, ?F_8} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_9, F_8, ?F_3} \text{ax} \quad \frac{- \vdash ?\Upsilon 4, \Delta_9, F_8, ?F_3}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} ?}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{hCut} \\
\\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_6, ?F_3, ?F_3}{\bullet h_2 \vdash !F_6, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_8 \vdash \Delta_5, dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \vdash contract(sn_7, ?dual(F_6)), \Delta_5} ?}{- \vdash (? \Upsilon 4, ?F_3), \Delta_5} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_6, ?F_3, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_6, ?F_3, ?F_3} \text{ax} \quad \frac{\frac{h_8 \vdash \Delta_5, dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \vdash \Delta_5, ?dual(F_6), contract(n_7, ?dual(F_6))} \text{ax} \quad \frac{- \vdash ?\Upsilon 4, \Delta_5, ?F_3, ?F_3}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3, ?F_3} ?}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3, ?F_3} \text{hCut} \\
\rightsquigarrow \\
- \vdash ?\Upsilon 4, \Delta_5, ?F_3
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \$ F_9} \$}{- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \$ F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \text{ax} \quad \frac{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$ F_9} \$}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$ F_9} \text{hCut}
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \& F_9} \& \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \& F_9} \&}{- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3} \text{ax} \quad \frac{\frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{10}, ?dual(F_5), F_8 \& F_9, contract(n_6, ?dual(F_5))} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{10}, ?dual(F_5), F_8 \& F_9, contract(n_6, ?dual(F_5))} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, ?F_3, F_8 \& F_9} \text{hCut} \\
\rightsquigarrow \\
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \& F_9
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3 \quad \text{hCut} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9 \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3 \quad \text{hCut} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9 \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \perp, \Delta_8} \perp \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_8, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_8, ?F_3 \quad \text{hCut} \\
\hline
- \vdash ?\Upsilon 4, \Delta_8, \perp, ?F_3 \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \top, \Delta_8} \top \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
- \vdash ?\Upsilon 4, \Delta_8, \top, ?F_3 \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5)) \quad h_7 \vdash \Delta_8, F_{10}}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3 \quad \text{hCut} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10} \quad \otimes \\
\hline
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_{10}, F_8, contract(n_6, ?dual(F_5)) \quad h_7 \vdash \Delta_{11}, F_9, ?dual(F_5)}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, contract(n_6, ?dual(F_5))} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9 \quad \text{hCut} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9 \quad ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5) \quad h_7 \vdash \Delta_{11}, F_9, contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\sim \\
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ax \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5)}{\bullet h_7 \vdash \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, contract(n_6, ?dual(F_5))} ax \quad \frac{h_7 \vdash \Delta_{11}, F_9, contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, contract(n_6, ?dual(F_5))} \otimes \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3 \otimes F_9 \quad hCut \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9 \quad ?C \\
\hline
\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\sim \\
\frac{- \vdash \Delta_8, F_9}{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3} ax \quad \frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{h_7 \vdash \Delta_{11}, F_{10}, ?dual(F_5), contract(n_6, ?dual(F_5))} ax \\
\hline
- \vdash ?\Upsilon 4, \Delta_{11}, F_{10}, ?F_3 \quad hCut \\
\hline
- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10} \quad \otimes
\end{array}$$

6.5 Status of ? : OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash F_8, ?\Upsilon 9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), ?\Upsilon 9, !F_8} ! \\
\hline
- \vdash (? \Upsilon 4, ?F_3), ?\Upsilon 9, !F_8 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{h_7 \vdash ?\Upsilon 9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))} ax \quad \frac{h_7 \vdash ?\Upsilon 9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, ?\Upsilon 9, F_8, ?F_3} ax \\
\hline
- \vdash ?\Upsilon 4, ?\Upsilon 9, F_8, ?F_3 \quad hCut \\
\hline
- \vdash ?\Upsilon 4, ?\Upsilon 9, !F_8, ?F_3 \quad !
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ?W \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{h_7 \vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))} ax \quad \frac{h_7 \vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3} ax \\
\hline
- \vdash ?\Upsilon 4, \Delta_9, ?F_3 \quad ?W \\
\hline
- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8 \\
\hline
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_6}{\bullet h_2 \vdash !F_6, ?\Upsilon 4, ?F_3} ? \quad \frac{h_8 \vdash \Delta_5, contract(n_7, ?dual(F_6))}{\bullet h_8 \vdash contract(sn_7, ?dual(F_6)), \Delta_5} ?W \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_5 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_6, ?F_3}{h_8 \vdash \Delta_5, contract(n_7, ?dual(F_6))} ax \quad \frac{h_8 \vdash \Delta_5, contract(n_7, ?dual(F_6))}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} ax \\
\hline
- \vdash ?\Upsilon 4, \Delta_5, ?F_3 \quad hCut
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ?C \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{h_7 \vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))} ax \quad \frac{h_7 \vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} ax \\
\hline
- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8 \quad hCut \\
\hline
- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8 \quad ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_6}{\bullet h_2 \vdash !F_6, ?\Upsilon 4, ?F_3} ? \quad \frac{h_8 \vdash \Delta_5, ?dual(F_6), ?dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \vdash contract(sn_7, ?dual(F_6)), \Delta_5} ?C \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_5 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_6, ?F_3}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ax} \quad \frac{h_8 \vdash \Delta_5, ?dual(F_6), ?dual(F_6), contract(n_7, ?dual(F_6))}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_5, ?F_3 \quad \text{hCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ? \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_9, F_8, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_9, F_8, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_9, F_8, ?F_3 \quad \text{hCut} \\
\hline
- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8 ? \\
\hline
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_6}{\bullet h_2 \vdash !F_6, ?\Upsilon 4, ?F_3} ? \quad \frac{h_8 \vdash \Delta_5, dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \vdash contract(sn_7, ?dual(F_6)), \Delta_5} ? \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_5 \quad \text{Cut} \\
\hline
\sim \\
\frac{- \vdash ?\Upsilon 4, !F_6, ?F_3}{- \vdash ?\Upsilon 4, F_6, ?F_3} \text{ax} \quad \frac{\bullet h_2 \vdash ?\Upsilon 4, !F_6, ?F_3}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3, dual(F_6)} \text{ax} \quad \frac{h_8 \vdash \Delta_5, dual(F_6), contract(n_7, ?dual(F_6))}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3, dual(F_6)} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_5, ?F_3 \quad \text{bInv} \quad \text{mCut} \quad \text{hCut}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \$F_9} \$ \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \$F_9 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3 \quad \text{hCut} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$F_9 \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \&F_9} \& \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \&F_9 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3 \quad \text{hCut} \\
\hline
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \&F_9 \quad \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\sim \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3 \quad \text{hCut} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9 \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3 \quad \oplus_A \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \perp, \Delta_8} \perp \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_8, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_8, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_8, \perp, ?F_3 \quad \perp \\
\hline
- \vdash ?\Upsilon 4, \Delta_8, \perp, ?F_3
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash contract(sn_6, ?dual(F_5)), \top, \Delta_8}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \top, \Delta_8} \top \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
- \vdash ?\Upsilon 4, \Delta_8, \top, ?F_3 \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5)) \quad h_7 \vdash \Delta_8, F_{10}}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3 \quad \text{hCut} \quad \frac{- \vdash \Delta_8, F_{10}}{- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10} \\
\\
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, contract(n_6, ?dual(F_5)) \quad h_7 \vdash \Delta_{11}, F_9, ?dual(F_5)}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3 \quad \text{hCut} \quad \frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5)}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9 \quad \text{C} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9 \\
\\
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5) \quad h_7 \vdash \Delta_{11}, F_9, contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes \\
\hline
- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5)}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3 \quad \text{hCut} \quad \frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \text{ax} \\
\hline
- \vdash ?\Upsilon 4, ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9 \quad \text{C} \\
\hline
- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{\frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\sim\!\!\!\rightarrow \\
\frac{\frac{}{- \vdash \Delta_8, F_9} \text{ax} \quad \frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_{10}, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{11}, F_{10}, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \otimes \text{hCut}
\end{array}$$

6.6 Status of \$: OK

- Case rule 1
- Case rule !
- Case rule ?W
- Case rule ?C
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.7 Status of &: OK

- Case rule 1
- Case rule !

- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.8 Status of \oplus_B : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B

- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.9 Status of \oplus_A : OK

- Case rule 1
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.10 Status of \perp : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.11 Status of \top : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$

- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.12 Status of I : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top

- Case rule I
- Case rule \otimes

6.13 Status of \otimes : OK

- Case rule **1**
- Case rule **!**
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes