Modal Logic K+T+4+5

May 11, 2021

Abstract

This system does not have cut-elimination.

Contents

1   Height preserving admissibility of weakening on the left

- Case(s) rule $\to_R$

$$\cfrac{\mathtt{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}}{\bullet\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4 \to F_5}} \to_R \quad\to\quad \cfrac{\cfrac{\overline{\mathtt{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}}\ \text{ax}}{\mathtt{h_1 : \Delta_2, F_4, F_W \vdash \Delta_3, F_5}}\ \text{IH}}{\bullet\mathtt{h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \to F_5}} \to_R$$

- Case(s) rule $\wedge_R$

$$\cfrac{\mathtt{h_1 : \Delta_2 \vdash F_4, \Delta_3} \quad \mathtt{h_1 : \Delta_2 \vdash F_5, \Delta_3}}{\bullet\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5}} \wedge_R \quad\to\quad \cfrac{\cfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4}}\ \text{ax}}{\mathtt{h_1 : \Delta_2, F_W \vdash \Delta_3, F_4}}\ \text{IH} \quad \cfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_5}}\ \text{ax}}{\mathtt{h_1 : \Delta_2, F_W \vdash \Delta_3, F_5}}\ \text{IH}}{\bullet\mathtt{h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \wedge F_5}} \wedge_R$$

- Case(s) rule $\vee_R$

$$\cfrac{\mathtt{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}}{\bullet\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5}} \vee_R \quad\to\quad \cfrac{\cfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}}\ \text{ax}}{\mathtt{h_1 : \Delta_2, F_W \vdash \Delta_3, F_4, F_5}}\ \text{IH}}{\bullet\mathtt{h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \vee F_5}} \vee_R$$

- Case(s) rule $\bot_R$

$$\cfrac{\mathtt{h_1 : \Delta_2 \vdash \Delta_3}}{\bullet\mathtt{h_1 : \Delta_2 \vdash \bot, \Delta_3}} \bot_R \quad\to\quad \cfrac{\cfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash \Delta_3}}\ \text{ax}}{\mathtt{h_1 : \Delta_2, F_W \vdash \Delta_3}}\ \text{IH}}{\bullet\mathtt{h_1 : \Delta_2, F_W \vdash \bot, \Delta_3}} \bot_R$$

- Case(s) rule $\top_R$

$$\cfrac{}{\bullet\mathtt{h_1 : \Delta_2 \vdash \top, \Delta_3}} \top_R \quad\to\quad \cfrac{}{\bullet\mathtt{h_1 : \Delta_2, F_W \vdash \top, \Delta_3}} \top_R$$

- Case(s) rule K

$$\cfrac{\mathtt{h_1 : unbox(\Box\Gamma_2) \vdash F_5}}{\bullet\mathtt{h_1 : \Box\Gamma_2, \Delta_3 \vdash \Delta_4, []F_5}} K \quad\to\quad \cfrac{\overline{\mathtt{h_1 : unbox(\Box\Gamma_2) \vdash F_5}}\ \text{ax}}{\bullet\mathtt{h_1 : \Delta_3, F_W, \Box\Gamma_2 \vdash \Delta_4, []F_5}} K$$

- Case(s) rule $A45$

$$\cfrac{\mathtt{h_1 : \Box\Gamma_2 \vdash \Box\Gamma_4, F_6}}{\bullet\mathtt{h_1 : \Box\Gamma_2, \Delta_3 \vdash \Box\Gamma_4, \Delta_5, []F_6}} A45 \quad\to\quad \cfrac{\overline{\mathtt{h_1 : \Box\Gamma_2 \vdash F_6, \Box\Gamma_4}}\ \text{ax}}{\bullet\mathtt{h_1 : \Delta_3, F_W, \Box\Gamma_2 \vdash \Delta_5, \Box\Gamma_4, []F_6}} A45$$

- Case(s) rule $\to_L$

$$\cfrac{\mathtt{h_1 : \Delta_2 \vdash F_3, \Delta_5} \quad \mathtt{h_1 : F_4, \Delta_2 \vdash \Delta_5}}{\bullet\mathtt{h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5}} \to_L \quad\to\quad \cfrac{\cfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash \Delta_5, F_3}}\ \text{ax}}{\mathtt{h_1 : \Delta_2, F_W \vdash \Delta_5, F_3}}\ \text{IH} \quad \cfrac{\overline{\mathtt{h_1 : \Delta_2, F_4 \vdash \Delta_5}}\ \text{ax}}{\mathtt{h_1 : \Delta_2, F_4, F_W \vdash \Delta_5}}\ \text{IH}}{\bullet\mathtt{h_1 : \Delta_2, F_W, F_3 \to F_4 \vdash \Delta_5}} \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}}{\bullet \mathtt{h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5}} \wedge_L \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}}\ \mathtt{ax}}{\mathtt{h_1 : \Delta_2, F_3, F_4, F_W \vdash \Delta_5}}\ \mathtt{IH}}{\bullet \mathtt{h_1 : \Delta_2, F_W, F_3 \wedge F_4 \vdash \Delta_5}} \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h_1 : F_3, \Delta_2 \vdash \Delta_5} \quad \mathtt{h_1 : F_4, \Delta_2 \vdash \Delta_5}}{\bullet \mathtt{h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5}} \vee_L \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h_1 : \Delta_2, F_3 \vdash \Delta_5}}\ \mathtt{ax}}{\mathtt{h_1 : \Delta_2, F_3, F_W \vdash \Delta_5}}\ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h_1 : \Delta_2, F_4 \vdash \Delta_5}}\ \mathtt{ax}}{\mathtt{h_1 : \Delta_2, F_4, F_W \vdash \Delta_5}}\ \mathtt{IH}}{\bullet \mathtt{h_1 : \Delta_2, F_W, F_3 \vee F_4 \vdash \Delta_5}} \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h_1 : F_3, \Delta_2, []F_3 \vdash \Delta_4}}{\bullet \mathtt{h_1 : \Delta_2, []F_3 \vdash \Delta_4}} AT \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4}}\ \mathtt{ax}}{\mathtt{h_1 : \Delta_2, F_3, F_W, []F_3 \vdash \Delta_4}}\ \mathtt{IH}}{\bullet \mathtt{h_1 : \Delta_2, F_W, []F_3 \vdash \Delta_4}} AT$$

- Case(s) rule $\perp_L$

$$\frac{}{\bullet \mathtt{h_1 : \perp, \Delta_2 \vdash \Delta_3}} \perp_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h_1 : \perp, \Delta_2, F_W \vdash \Delta_3}} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3}} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h_1 : \Delta_2, F_W, p_3 \vdash \Delta_4, p_3}} I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h_1 : \Delta_2 \vdash \Delta_3}}{\bullet \mathtt{h_1 : \top, \Delta_2 \vdash \Delta_3}} \top_L \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash \Delta_3}}\ \mathtt{ax}}{\mathtt{h_1 : \Delta_2, F_W \vdash \Delta_3}}\ \mathtt{IH}}{\bullet \mathtt{h_1 : \top, \Delta_2, F_W \vdash \Delta_3}} \top_L$$

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2 Height preserving admissibility of weakening on the right

- Case(s) rule $\to_R$

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \to F_5}\ \to_R \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}\ \text{ax}}{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5, F_W}\ \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \to F_5}\ \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5}\ \wedge_R \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_4}\ \text{ax}}{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_W}\ \text{IH} \quad \dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_5}\ \text{ax}}{h_1 : \Delta_2 \vdash \Delta_3, F_5, F_W}\ \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \wedge F_5}\ \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5}\ \vee_R \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}\ \text{ax}}{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5, F_W}\ \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \vee F_5}\ \vee_R$$

- Case(s) rule $\perp_R$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3}\ \perp_R \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3}\ \text{ax}}{h_1 : \Delta_2 \vdash \Delta_3, F_W}\ \text{IH}}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W}\ \perp_R$$

- Case(s) rule $\top_R$

$$\overline{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R \qquad \to \qquad \overline{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, F_W}\ \top_R$$

- Case(s) rule K

$$\frac{h_1 : unbox(\Box\Gamma_2) \vdash F_5}{\bullet h_1 : \Box\Gamma_2, \Delta_3 \vdash \Delta_4, []F_5}\ K \qquad \to \qquad \frac{\overline{h_1 : unbox(\Box\Gamma_2) \vdash F_5}\ \text{ax}}{\bullet h_1 : \Delta_3, \Box\Gamma_2 \vdash \Delta_4, F_W, []F_5}\ K$$

- Case(s) rule $A45$

$$\frac{h_1 : \Box\Gamma_2 \vdash \Box\Gamma_4, F_6}{\bullet h_1 : \Box\Gamma_2, \Delta_3 \vdash \Box\Gamma_4, \Delta_5, []F_6}\ A45 \qquad \to \qquad \frac{\overline{h_1 : \Box\Gamma_2 \vdash F_6, \Box\Gamma_4}\ \text{ax}}{\bullet h_1 : \Delta_3, \Box\Gamma_2 \vdash \Delta_5, F_W, \Box\Gamma_4, []F_6}\ A45$$

- Case(s) rule $\to_L$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5}\ \to_L \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3}\ \text{ax}}{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_W}\ \text{IH} \quad \dfrac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5}\ \text{ax}}{h_1 : \Delta_2, F_4 \vdash \Delta_5, F_W}\ \text{IH}}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5, F_W}\ \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5}\ \wedge_L \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}\ \text{ax}}{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5, F_W}\ \text{IH}}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5, F_W}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5}\ \vee_L \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2, F_3 \vdash \Delta_5}\ \text{ax}}{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_W}\ \text{IH} \quad \dfrac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5}\ \text{ax}}{h_1 : \Delta_2, F_4 \vdash \Delta_5, F_W}\ \text{IH}}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5, F_W}\ \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h_1 : F_3}, \Delta_2, \mathtt{[]F_3} \vdash \Delta_4}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{[]F_3} \vdash \Delta_4} \; AT \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{[]F_3} \vdash \Delta_4} \; \mathtt{ax}}{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{[]F_3} \vdash \Delta_4, \mathtt{F}_W} \; \mathtt{IH}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{[]F_3} \vdash \Delta_4, \mathtt{F}_W} \; AT$$

- Case(s) rule $\perp_L$

$$\frac{}{\bullet\mathtt{h_1} : \perp, \Delta_2 \vdash \Delta_3} \; \perp_L \qquad \rightarrow \qquad \frac{}{\bullet\mathtt{h_1} : \perp, \Delta_2 \vdash \Delta_3, \mathtt{F}_W} \; \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{p_3} \vdash \Delta_4, \mathtt{p_3}} \; I \qquad \rightarrow \qquad \frac{}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{p_3} \vdash \Delta_4, \mathtt{F}_W, \mathtt{p_3}} \; I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h_1} : \Delta_2 \vdash \Delta_3}{\bullet\mathtt{h_1} : \top, \Delta_2 \vdash \Delta_3} \; \top_L \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_1} : \Delta_2 \vdash \Delta_3} \; \mathtt{ax}}{\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F}_W} \; \mathtt{IH}}{\bullet\mathtt{h_1} : \top, \Delta_2 \vdash \Delta_3, \mathtt{F}_W} \; \top_L$$

3 Measure of derivations

- Case(s) rule $\to_R$

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \to F_5}\ \to_R \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}\ \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}\ \text{IH}}{\bullet\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \to F_5}\ \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5}\ \wedge_R \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_4}\ \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4}\ \text{IH} \quad \dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_5}\ \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5}\ \text{IH}}{\bullet\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5}\ \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5}\ \vee_R \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}\ \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}\ \text{IH}}{\bullet\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5}\ \vee_R$$

- Case(s) rule $\perp_R$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3}\ \perp_R \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3}\ \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3}\ \text{IH}}{\bullet\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3}\ \perp_R$$

- Case(s) rule $\top_R$

$$\overline{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R \qquad \to \qquad \overline{\bullet\bullet h_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R$$

- Case(s) rule K

$$\frac{h_1 : unbox(\Box\Gamma_2) \vdash F_5}{\bullet h_1 : \Box\Gamma_2, \Delta_3 \vdash \Delta_4, []F_5}\ K \qquad \to \qquad \frac{\dfrac{\overline{h_1 : unbox(\Box\Gamma_2) \vdash F_5}\ \text{ax}}{\bullet h_1 : unbox(\Box\Gamma_2) \vdash F_5}\ \text{IH}}{\bullet\bullet h_1 : \Delta_3, \Box\Gamma_2 \vdash \Delta_4, []F_5}\ K$$

- Case(s) rule $A45$

$$\frac{h_1 : \Box\Gamma_2 \vdash \Box\Gamma_4, F_6}{\bullet h_1 : \Box\Gamma_2, \Delta_3 \vdash \Box\Gamma_4, \Delta_5, []F_6}\ A45 \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Box\Gamma_2 \vdash F_6, \Box\Gamma_4}\ \text{ax}}{\bullet h_1 : \Box\Gamma_2 \vdash F_6, \Box\Gamma_4}\ \text{IH}}{\bullet\bullet h_1 : \Delta_3, \Box\Gamma_2 \vdash \Delta_5, \Box\Gamma_4, []F_6}\ A45$$

- Case(s) rule $\to_L$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5}\ \to_L \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3}\ \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3}\ \text{IH} \quad \dfrac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5}\ \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5}\ \text{IH}}{\bullet\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5}\ \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5}\ \wedge_L \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}\ \text{ax}}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}\ \text{IH}}{\bullet\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5}\ \vee_L \qquad \to \qquad \frac{\dfrac{\overline{h_1 : \Delta_2, F_3 \vdash \Delta_5}\ \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5}\ \text{IH} \quad \dfrac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5}\ \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5}\ \text{IH}}{\bullet\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5}\ \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h_1 : F_3, \Delta_2, []F_3 \vdash \Delta_4}}{\bullet\mathtt{h_1 : \Delta_2, []F_3 \vdash \Delta_4}} \; AT \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4}} \; {\scriptstyle ax}}{\bullet\mathtt{h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4}} \; {\scriptstyle IH}}{\bullet\bullet\mathtt{h_1 : \Delta_2, []F_3 \vdash \Delta_4}} \; AT$$

- Case(s) rule $\perp_L$

$$\overline{\bullet\mathtt{h_1 : \perp, \Delta_2 \vdash \Delta_3}} \; \perp L \qquad \rightarrow \qquad \overline{\bullet\bullet\mathtt{h_1 : \perp, \Delta_2 \vdash \Delta_3}} \; \perp L$$

- Case(s) rule I

$$\overline{\bullet\mathtt{h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3}} \; I \qquad \rightarrow \qquad \overline{\bullet\bullet\mathtt{h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3}} \; I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h_1 : \Delta_2 \vdash \Delta_3}}{\bullet\mathtt{h_1 : \top, \Delta_2 \vdash \Delta_3}} \; \top L \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash \Delta_3}} \; {\scriptstyle ax}}{\bullet\mathtt{h_1 : \Delta_2 \vdash \Delta_3}} \; {\scriptstyle IH}}{\bullet\bullet\mathtt{h_1 : \top, \Delta_2 \vdash \Delta_3}} \; \top L$$

4 Invertibility of Rules

4.1 Status of $\rightarrow_R$: : Invertible

- Case rule $\rightarrow_R$

$$\frac{\texttt{h}_3 : \texttt{F}_5, \Delta_4 \vdash \texttt{F}_6, \Delta_7, \texttt{F}_1 \rightarrow \texttt{F}_2}{\bullet \texttt{h}_3 : \Delta_4 \vdash (\Delta_7, \texttt{F}_1 \rightarrow \texttt{F}_2), \texttt{F}_5 \rightarrow \texttt{F}_6} \rightarrow_R \qquad \rightarrow \qquad \frac{\overline{\texttt{h}_3 : \Delta_4, \texttt{F}_1, \texttt{F}_5 \vdash \Delta_7, \texttt{F}_2, \texttt{F}_6}}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \Delta_7, \texttt{F}_2, \texttt{F}_5 \rightarrow \texttt{F}_6} \overset{\texttt{ax/ind}}{\rightarrow_R}$$

$$\frac{\texttt{h}_1 : \texttt{F}_4, \Delta_2 \vdash \texttt{F}_5, \Delta_3}{\bullet \texttt{h}_1 : \Delta_2 \vdash \Delta_3, \texttt{F}_4 \rightarrow \texttt{F}_5} \rightarrow_R \qquad \rightarrow \qquad \frac{\overline{\texttt{h}_1 : \Delta_2, \texttt{F}_4 \vdash \Delta_3, \texttt{F}_5}}{\bullet \texttt{h}_1 : \Delta_2, \texttt{F}_4 \vdash \Delta_3, \texttt{F}_5} \overset{\texttt{ax}}{\texttt{H}}$$

- Case rule $\wedge_R$

$$\frac{\texttt{h}_3 : \Delta_4 \vdash \texttt{F}_5, \Delta_7, \texttt{F}_1 \rightarrow \texttt{F}_2 \quad \texttt{h}_3 : \Delta_4 \vdash \texttt{F}_6, \Delta_7, \texttt{F}_1 \rightarrow \texttt{F}_2}{\bullet \texttt{h}_3 : \Delta_4 \vdash (\Delta_7, \texttt{F}_1 \rightarrow \texttt{F}_2), \texttt{F}_5 \wedge \texttt{F}_6} \wedge_R \qquad \rightarrow \qquad \frac{\overline{\texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \Delta_7, \texttt{F}_2, \texttt{F}_5}}{\texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \Delta_7, \texttt{F}_2, \texttt{F}_5} \overset{\texttt{ax/ind}}{} \quad \frac{\overline{\texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \Delta_7, \texttt{F}_2, \texttt{F}_6}}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \Delta_7, \texttt{F}_2, \texttt{F}_5 \wedge \texttt{F}_6} \overset{\texttt{ax/ind}}{\wedge_R}$$

- Case rule $\vee_R$

$$\frac{\texttt{h}_3 : \Delta_4 \vdash \texttt{F}_5, \texttt{F}_6, \Delta_7, \texttt{F}_1 \rightarrow \texttt{F}_2}{\bullet \texttt{h}_3 : \Delta_4 \vdash (\Delta_7, \texttt{F}_1 \rightarrow \texttt{F}_2), \texttt{F}_5 \vee \texttt{F}_6} \vee_R \qquad \rightarrow \qquad \frac{\overline{\texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \Delta_7, \texttt{F}_2, \texttt{F}_5, \texttt{F}_6}}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \Delta_7, \texttt{F}_2, \texttt{F}_5 \vee \texttt{F}_6} \overset{\texttt{ax/ind}}{\vee_R}$$

- Case rule $\perp_R$

$$\frac{\texttt{h}_3 : \Delta_4 \vdash \Delta_5, \texttt{F}_1 \rightarrow \texttt{F}_2}{\bullet \texttt{h}_3 : \Delta_4 \vdash \perp, \Delta_5, \texttt{F}_1 \rightarrow \texttt{F}_2} \perp_R \qquad \rightarrow \qquad \frac{\overline{\texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \Delta_5, \texttt{F}_2}}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \perp, \Delta_5, \texttt{F}_2} \overset{\texttt{ax/ind}}{\perp_R}$$

- Case rule $\top_R$

$$\frac{}{\bullet \texttt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \texttt{F}_1 \rightarrow \texttt{F}_2} \top_R \qquad \rightarrow \qquad \frac{}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_1 \vdash \top, \Delta_5, \texttt{F}_2} \top_R$$

- Case rule K

$$\frac{\texttt{h}_3 : unbox(\square\Gamma_4) \vdash \texttt{F}_6}{\bullet \texttt{h}_3 : \square\Gamma_4, \Delta_5 \vdash (\Delta_7, \texttt{F}_1 \rightarrow \texttt{F}_2), []\texttt{F}_6} K \qquad \rightarrow \qquad \frac{\overline{\texttt{h}_3 : unbox(\square\Gamma_4) \vdash \texttt{F}_6}}{\bullet \texttt{h}_3 : \Delta_5, \texttt{F}_1, \square\Gamma_4 \vdash \Delta_7, \texttt{F}_2, []\texttt{F}_6} \overset{\texttt{ax}}{K}$$

- Case rule $A45$

$$\frac{\texttt{h}_3 : \square\Gamma_4 \vdash \square\Gamma_6, \texttt{F}_7}{\bullet \texttt{h}_3 : \square\Gamma_4, \Delta_5 \vdash \square\Gamma_6, (\Delta_8, \texttt{F}_1 \rightarrow \texttt{F}_2), []\texttt{F}_7} A45 \qquad \rightarrow \qquad \frac{\overline{\texttt{h}_3 : \square\Gamma_4 \vdash \texttt{F}_7, \square\Gamma_6}}{\bullet \texttt{h}_3 : \Delta_5, \texttt{F}_1, \square\Gamma_4 \vdash \Delta_8, \texttt{F}_2, \square\Gamma_6, []\texttt{F}_7} \overset{\texttt{ax}}{A45}$$

- Case rule $\rightarrow_L$

$$\frac{\texttt{h}_4 : \Delta_5 \vdash \texttt{F}_6, \Delta_1, \texttt{F}_2 \rightarrow \texttt{F}_3 \quad \texttt{h}_4 : \texttt{F}_7, \Delta_5 \vdash \Delta_1, \texttt{F}_2 \rightarrow \texttt{F}_3}{\bullet \texttt{h}_4 : \Delta_5, \texttt{F}_6 \rightarrow \texttt{F}_7 \vdash \Delta_1, \texttt{F}_2 \rightarrow \texttt{F}_3} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\texttt{h}_4 : \Delta_5, \texttt{F}_2 \vdash \Delta_1, \texttt{F}_3, \texttt{F}_6}}{\texttt{h}_4 : \Delta_5, \texttt{F}_2 \vdash \Delta_1, \texttt{F}_3, \texttt{F}_6} \overset{\texttt{ax/ind}}{} \quad \frac{\overline{\texttt{h}_4 : \Delta_5, \texttt{F}_2, \texttt{F}_7 \vdash \Delta_1, \texttt{F}_3}}{\bullet \texttt{h}_4 : \Delta_5, \texttt{F}_2, \texttt{F}_6 \rightarrow \texttt{F}_7 \vdash \Delta_1, \texttt{F}_3} \overset{\texttt{ax/ind}}{\rightarrow_L}$$

- Case rule $\wedge_L$

$$\dfrac{\mathtt{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \to F_3}}{\bullet\mathtt{h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \to F_3}} \ \wedge_L \qquad \to \qquad \dfrac{\overline{\mathtt{h_4 : \Delta_5, F_2, F_6, F_7 \vdash \Delta_1, F_3}} \ \ \mathtt{ax/ind}}{\bullet\mathtt{h_4 : \Delta_5, F_2, F_6 \wedge F_7 \vdash \Delta_1, F_3}} \ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \to F_3} \quad \mathtt{h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \to F_3}}{\bullet\mathtt{h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \to F_3}} \ \vee_L \qquad \to \qquad \dfrac{\overline{\mathtt{h_4 : \Delta_5, F_2, F_6 \vdash \Delta_1, F_3}} \ \mathtt{ax/ind} \quad \overline{\mathtt{h_4 : \Delta_5, F_2, F_7 \vdash \Delta_1, F_3}} \ \mathtt{ax/ind}}{\bullet\mathtt{h_4 : \Delta_5, F_2, F_6 \vee F_7 \vdash \Delta_1, F_3}} \ \vee_L$$

- Case rule AT

$$\dfrac{\mathtt{h_4 : F_6, \Delta_5, []F_6 \vdash \Delta_1, F_2 \to F_3}}{\bullet\mathtt{h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_2 \to F_3}} \ AT \qquad \to \qquad \dfrac{\overline{\mathtt{h_4 : \Delta_5, F_2, F_6, []F_6 \vdash \Delta_1, F_3}} \ \mathtt{ax/ind}}{\bullet\mathtt{h_4 : \Delta_5, F_2, []F_6 \vdash \Delta_1, F_3}} \ AT$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet\mathtt{h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \to F_3}} \ \perp_L \qquad \to \qquad \dfrac{}{\bullet\mathtt{h_4 : \perp, \Delta_5, F_2 \vdash \Delta_1, F_3}} \ \perp_L$$

- Case rule I

$$\dfrac{}{\bullet\mathtt{h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \to F_2}} \ I \qquad \to \qquad \dfrac{}{\bullet\mathtt{h_3 : \Delta_4, F_1, p_5 \vdash \Delta_6, F_2, p_5}} \ I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h_4 : \Delta_5 \vdash \Delta_1, F_2 \to F_3}}{\bullet\mathtt{h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \to F_3}} \ \top_L \qquad \to \qquad \dfrac{\overline{\mathtt{h_4 : \Delta_5, F_2 \vdash \Delta_1, F_3}} \ \mathtt{ax/ind}}{\bullet\mathtt{h_4 : \top, \Delta_5, F_2 \vdash \Delta_1, F_3}} \ \top_L$$

4.2   Status of $\wedge_R$:   (Left Premise): `Invertible`

- Case rule $\to_R$

$$\dfrac{\mathtt{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}}{\bullet\mathtt{h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \to F_6}} \ \to_R \qquad \to \qquad \dfrac{\overline{\mathtt{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_6}} \ \mathtt{ax/ind}}{\bullet\mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \to F_6}} \ \to_R$$

- Case rule $\wedge_R$

$$\dfrac{\mathtt{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2} \quad \mathtt{h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}}{\bullet\mathtt{h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6}} \ \wedge_R \qquad \to \qquad \dfrac{\overline{\mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5}} \ \mathtt{ax/ind} \quad \overline{\mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_6}} \ \mathtt{ax/ind}}{\bullet\mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6}} \ \wedge_R$$

$$\dfrac{\mathtt{h_1 : \Delta_2 \vdash F_4, \Delta_3} \quad \mathtt{h_1 : \Delta_2 \vdash F_5, \Delta_3}}{\bullet\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5}} \ \wedge_R \qquad \to \qquad \dfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4}} \ \mathtt{ax}}{\bullet\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4}} \ \mathtt{H}$$

- Case rule $\vee_R$

$$\dfrac{\mathtt{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}}{\bullet\mathtt{h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6}} \ \vee_R \qquad \to \qquad \dfrac{\overline{\mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5, F_6}} \ \mathtt{ax/ind}}{\bullet\mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6}} \ \vee_R$$

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- Case rule $\perp_R$

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \bot, \Delta_5, F_1 \wedge F_2} \perp_R \qquad \to \qquad \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_1} \;\; \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \bot, \Delta_5, F_1} \perp_R$$

- Case rule $\top_R$

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \qquad \to \qquad \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1} \top_R$$

- Case rule K

$$\frac{h_3 : unbox(\Box\Gamma_4) \vdash F_6}{\bullet h_3 : \Box\Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \wedge F_2), []F_6} K \qquad \to \qquad \frac{\overline{h_3 : unbox(\Box\Gamma_4) \vdash F_6} \;\; \text{ax}}{\bullet h_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_7, F_1, []F_6} K$$

- Case rule $A45$

$$\frac{h_3 : \Box\Gamma_4 \vdash \Box\Gamma_6, F_7}{\bullet h_3 : \Box\Gamma_4, \Delta_5 \vdash \Box\Gamma_6, (\Delta_8, F_1 \wedge F_2), []F_7} A45 \qquad \to \qquad \frac{\overline{h_3 : \Box\Gamma_4 \vdash F_7, \Box\Gamma_6} \;\; \text{ax}}{\bullet h_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_8, F_1, \Box\Gamma_6, []F_7} A45$$

- Case rule $\to_L$

$$\frac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \to F_7 \vdash \Delta_1, F_2 \wedge F_3} \to_L \quad \to \quad \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_6} \;\text{ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2} \;\text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \to F_7 \vdash \Delta_1, F_2} \to_L$$

- Case rule $\wedge_L$

$$\frac{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3} \wedge_L \qquad \to \qquad \frac{\overline{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2} \;\; \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2} \wedge_L$$

- Case rule $\vee_L$

$$\frac{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3} \vee_L \quad \to \quad \frac{\overline{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2} \;\text{ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2} \;\text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2} \vee_L$$

- Case rule AT

$$\frac{h_4 : F_6, \Delta_5, []F_6 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_2 \wedge F_3} AT \qquad \to \qquad \frac{\overline{h_4 : \Delta_5, F_6, []F_6 \vdash \Delta_1, F_2} \;\; \text{ax/ind}}{\bullet h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_2} AT$$

- Case rule $\perp_L$

$$\frac{}{\bullet h_4 : \bot, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \perp_L \qquad \to \qquad \frac{}{\bullet h_4 : \bot, \Delta_5 \vdash \Delta_1, F_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \wedge F_2} I \qquad \to \qquad \frac{}{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_1, p_5} I$$

- Case rule $\top_L$

$$\dfrac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \ \top_L \qquad \rightarrow \qquad \dfrac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2} \ \ \text{ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2} \ \top_L$$

4.3 Status of $\wedge_R$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\dfrac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \ \rightarrow_R \qquad \rightarrow \qquad \dfrac{\overline{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_2, F_6} \ \ \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \ \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \ \wedge_R \quad \rightarrow \quad \dfrac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5} \ \text{ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \wedge F_6} \ \wedge_R$$

$$\dfrac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \ \wedge_R \quad \rightarrow \quad \dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_5} \ \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5} \ \text{H}$$

- Case rule $\vee_R$

$$\dfrac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \ \vee_R \qquad \rightarrow \qquad \dfrac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5, F_6} \ \ \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \vee F_6} \ \vee_R$$

- Case rule $\perp_R$

$$\dfrac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \ \perp_R \qquad \rightarrow \qquad \dfrac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_2} \ \ \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_2} \ \perp_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \ \top_R \qquad \rightarrow \qquad \dfrac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_2} \ \top_R$$

- Case rule K

$$\dfrac{h_3 : unbox(\Box\Gamma_4) \vdash F_6}{\bullet h_3 : \Box\Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \wedge F_2), []F_6} \ K \qquad \rightarrow \qquad \dfrac{\overline{h_3 : unbox(\Box\Gamma_4) \vdash F_6} \ \ \text{ax}}{\bullet h_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_7, F_2, []F_6} \ K$$

- Case rule $A45$

$$\dfrac{h_3 : \Box\Gamma_4 \vdash \Box\Gamma_6, F_7}{\bullet h_3 : \Box\Gamma_4, \Delta_5 \vdash \Box\Gamma_6, (\Delta_8, F_1 \wedge F_2), []F_7} \ A45 \quad \rightarrow \quad \dfrac{\overline{h_3 : \Box\Gamma_4 \vdash F_7, \Box\Gamma_6} \ \ \text{ax}}{\bullet h_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_8, F_2, \Box\Gamma_6, []F_7} \ A45$$

- Case rule $\rightarrow_L$

$$\dfrac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \wedge F_3} \ \rightarrow_L \quad \rightarrow \quad \dfrac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_3, F_6} \ \text{ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3} \ \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_3} \ \rightarrow_L$$

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- Case rule $\wedge_L$

$$\dfrac{\mathtt{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}}{\bullet \mathtt{h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3}} \wedge_L \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_3}} \;\; \text{ax/ind}}{\bullet \mathtt{h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_3}} \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \quad \mathtt{h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}}{\bullet \mathtt{h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3}} \vee_L \quad \rightarrow \quad \dfrac{\overline{\mathtt{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_3}} \; \text{ax/ind} \quad \overline{\mathtt{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3}} \; \text{ax/ind}}{\bullet \mathtt{h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_3}} \vee_L$$

- Case rule AT

$$\dfrac{\mathtt{h_4 : F_6, \Delta_5, []F_6 \vdash \Delta_1, F_2 \wedge F_3}}{\bullet \mathtt{h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_2 \wedge F_3}} \; AT \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_4 : \Delta_5, F_6, []F_6 \vdash \Delta_1, F_3}} \;\; \text{ax/ind}}{\bullet \mathtt{h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_3}} \; AT$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet \mathtt{h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}} \; \perp_L \qquad \rightarrow \qquad \dfrac{}{\bullet \mathtt{h_4 : \perp, \Delta_5 \vdash \Delta_1, F_3}} \; \perp_L$$

- Case rule I

$$\dfrac{}{\bullet \mathtt{h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \wedge F_2}} \; I \qquad \rightarrow \qquad \dfrac{}{\bullet \mathtt{h_3 : \Delta_4, p_5 \vdash \Delta_6, F_2, p_5}} \; I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}}{\bullet \mathtt{h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}} \; \top_L \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_4 : \Delta_5 \vdash \Delta_1, F_3}} \;\; \text{ax/ind}}{\bullet \mathtt{h_4 : \top, \Delta_5 \vdash \Delta_1, F_3}} \; \top_L$$

4.4   Status of $\vee_R$: :   Invertible

- Case rule $\rightarrow_R$

$$\dfrac{\mathtt{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}}{\bullet \mathtt{h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \rightarrow F_6}} \rightarrow_R \quad \rightarrow \quad \dfrac{\overline{\mathtt{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_2, F_6}} \;\; \text{ax/ind}}{\bullet \mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6}} \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\mathtt{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \vee F_2} \quad \mathtt{h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}}{\bullet \mathtt{h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \wedge F_6}} \wedge_R \quad \rightarrow \quad \dfrac{\overline{\mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5}} \; \text{ax/ind} \quad \overline{\mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_6}} \; \text{ax/ind}}{\bullet \mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6}} \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\mathtt{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \vee F_2}}{\bullet \mathtt{h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \vee F_6}} \vee_R \quad \rightarrow \quad \dfrac{\overline{\mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5, F_6}} \;\; \text{ax/ind}}{\bullet \mathtt{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \vee F_6}} \vee_R$$

$$\dfrac{\mathtt{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}}{\bullet \mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5}} \vee_R \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}} \;\; \text{ax}}{\bullet \mathtt{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}} \; \text{H}$$

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- Case rule $\perp_R$

$$\frac{\mathrm{h}_3 : \Delta_4 \vdash \Delta_5, \mathrm{F}_1 \vee \mathrm{F}_2}{\bullet\mathrm{h}_3 : \Delta_4 \vdash \perp, \Delta_5, \mathrm{F}_1 \vee \mathrm{F}_2} \; \perp_R \qquad \rightarrow \qquad \frac{\overline{\mathrm{h}_3 : \Delta_4 \vdash \Delta_5, \mathrm{F}_1, \mathrm{F}_2} \;\; \mathrm{ax/ind}}{\bullet\mathrm{h}_3 : \Delta_4 \vdash \perp, \Delta_5, \mathrm{F}_1, \mathrm{F}_2} \; \perp_R$$

- Case rule $\top_R$

$$\frac{}{\bullet\mathrm{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathrm{F}_1 \vee \mathrm{F}_2} \; \top_R \qquad \rightarrow \qquad \frac{}{\bullet\mathrm{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathrm{F}_1, \mathrm{F}_2} \; \top_R$$

- Case rule K

$$\frac{\mathrm{h}_3 : unbox(\Box\Gamma_4) \vdash \mathrm{F}_6}{\bullet\mathrm{h}_3 : \Box\Gamma_4, \Delta_5 \vdash (\Delta_7, \mathrm{F}_1 \vee \mathrm{F}_2), []\mathrm{F}_6} \; K \qquad \rightarrow \qquad \frac{\overline{\mathrm{h}_3 : unbox(\Box\Gamma_4) \vdash \mathrm{F}_6} \;\; \mathrm{ax}}{\bullet\mathrm{h}_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_7, \mathrm{F}_1, \mathrm{F}_2, []\mathrm{F}_6} \; K$$

- Case rule $A45$

$$\frac{\mathrm{h}_3 : \Box\Gamma_4 \vdash \Box\Gamma_6, \mathrm{F}_7}{\bullet\mathrm{h}_3 : \Box\Gamma_4, \Delta_5 \vdash \Box\Gamma_6, (\Delta_8, \mathrm{F}_1 \vee \mathrm{F}_2), []\mathrm{F}_7} \; A45 \qquad \rightarrow \qquad \frac{\overline{\mathrm{h}_3 : \Box\Gamma_4 \vdash \mathrm{F}_7, \Box\Gamma_6} \;\; \mathrm{ax}}{\bullet\mathrm{h}_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_8, \mathrm{F}_1, \mathrm{F}_2, \Box\Gamma_6, []\mathrm{F}_7} \; A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathrm{h}_4 : \Delta_5 \vdash \mathrm{F}_6, \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3 \quad \mathrm{h}_4 : \mathrm{F}_7, \Delta_5 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3}{\bullet\mathrm{h}_4 : \Delta_5, \mathrm{F}_6 \rightarrow \mathrm{F}_7 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3} \; \rightarrow_L \qquad \rightarrow \qquad \frac{\overline{\mathrm{h}_4 : \Delta_5 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3, \mathrm{F}_6} \;\mathrm{ax/ind}\quad \overline{\mathrm{h}_4 : \Delta_5, \mathrm{F}_7 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \;\mathrm{ax/ind}}{\bullet\mathrm{h}_4 : \Delta_5, \mathrm{F}_6 \rightarrow \mathrm{F}_7 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \; \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathrm{h}_4 : \mathrm{F}_6, \mathrm{F}_7, \Delta_5 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3}{\bullet\mathrm{h}_4 : \Delta_5, \mathrm{F}_6 \wedge \mathrm{F}_7 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3} \; \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathrm{h}_4 : \Delta_5, \mathrm{F}_6, \mathrm{F}_7 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \;\; \mathrm{ax/ind}}{\bullet\mathrm{h}_4 : \Delta_5, \mathrm{F}_6 \wedge \mathrm{F}_7 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \; \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathrm{h}_4 : \mathrm{F}_6, \Delta_5 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3 \quad \mathrm{h}_4 : \mathrm{F}_7, \Delta_5 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3}{\bullet\mathrm{h}_4 : \Delta_5, \mathrm{F}_6 \vee \mathrm{F}_7 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3} \; \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathrm{h}_4 : \Delta_5, \mathrm{F}_6 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \;\mathrm{ax/ind}\quad \overline{\mathrm{h}_4 : \Delta_5, \mathrm{F}_7 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \;\mathrm{ax/ind}}{\bullet\mathrm{h}_4 : \Delta_5, \mathrm{F}_6 \vee \mathrm{F}_7 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \; \vee_L$$

- Case rule AT

$$\frac{\mathrm{h}_4 : \mathrm{F}_6, \Delta_5, []\mathrm{F}_6 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3}{\bullet\mathrm{h}_4 : \Delta_5, []\mathrm{F}_6 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3} \; AT \qquad \rightarrow \qquad \frac{\overline{\mathrm{h}_4 : \Delta_5, \mathrm{F}_6, []\mathrm{F}_6 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \;\; \mathrm{ax/ind}}{\bullet\mathrm{h}_4 : \Delta_5, []\mathrm{F}_6 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \; AT$$

- Case rule $\perp_L$

$$\frac{}{\bullet\mathrm{h}_4 : \perp, \Delta_5 \vdash \Delta_1, \mathrm{F}_2 \vee \mathrm{F}_3} \; \perp_L \qquad \rightarrow \qquad \frac{}{\bullet\mathrm{h}_4 : \perp, \Delta_5 \vdash \Delta_1, \mathrm{F}_2, \mathrm{F}_3} \; \perp_L$$

- Case rule I

$$\frac{}{\bullet\mathrm{h}_3 : \mathrm{p}_5, \Delta_4 \vdash \mathrm{p}_5, \Delta_6, \mathrm{F}_1 \vee \mathrm{F}_2} \; I \qquad \rightarrow \qquad \frac{}{\bullet\mathrm{h}_3 : \Delta_4, \mathrm{p}_5 \vdash \Delta_6, \mathrm{F}_1, \mathrm{F}_2, \mathrm{p}_5} \; I$$

- Case rule $\top_L$

$$\dfrac{\texttt{h}_4 : \Delta_5 \vdash \Delta_1, \texttt{F}_2 \vee \texttt{F}_3}{\bullet\texttt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \texttt{F}_2 \vee \texttt{F}_3} \;\top_L \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_4 : \Delta_5 \vdash \Delta_1, \texttt{F}_2, \texttt{F}_3}\;\;\texttt{ax/ind}}{\bullet\texttt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \texttt{F}_2, \texttt{F}_3}\;\top_L$$

4.5 Status of $\perp_R$: : Invertible

- Case rule $\rightarrow_R$

$$\dfrac{\texttt{h}_1 : \texttt{F}_3, \Delta_2 \vdash \perp, \texttt{F}_4, \Delta_5}{\bullet\texttt{h}_1 : \Delta_2 \vdash (\perp, \Delta_5), \texttt{F}_3 \rightarrow \texttt{F}_4} \;\rightarrow_R \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_1 : \Delta_2, \texttt{F}_3 \vdash \Delta_5, \texttt{F}_4}\;\;\texttt{ax/ind}}{\bullet\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3 \rightarrow \texttt{F}_4}\;\rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\texttt{h}_1 : \Delta_2 \vdash \perp, \texttt{F}_3, \Delta_5 \quad \texttt{h}_1 : \Delta_2 \vdash \perp, \texttt{F}_4, \Delta_5}{\bullet\texttt{h}_1 : \Delta_2 \vdash (\perp, \Delta_5), \texttt{F}_3 \wedge \texttt{F}_4} \;\wedge_R \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3}\;\texttt{ax/ind} \quad \overline{\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_4}\;\texttt{ax/ind}}{\bullet\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3 \wedge \texttt{F}_4}\;\wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\texttt{h}_1 : \Delta_2 \vdash \perp, \texttt{F}_3, \texttt{F}_4, \Delta_5}{\bullet\texttt{h}_1 : \Delta_2 \vdash (\perp, \Delta_5), \texttt{F}_3 \vee \texttt{F}_4} \;\vee_R \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3, \texttt{F}_4}\;\;\texttt{ax/ind}}{\bullet\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3 \vee \texttt{F}_4}\;\vee_R$$

- Case rule $\perp_R$

$$\dfrac{\texttt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet\texttt{h}_1 : \Delta_2 \vdash \perp, \Delta_3} \;\perp_R \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_1 : \Delta_2 \vdash \Delta_3}\;\;\texttt{ax}}{\bullet\texttt{h}_1 : \Delta_2 \vdash \Delta_3}\;\texttt{H}$$

- Case rule $\top_R$

$$\dfrac{}{\bullet\texttt{h}_1 : \Delta_2 \vdash \top, \perp, \Delta_3} \;\top_R \qquad \rightarrow \qquad \dfrac{}{\bullet\texttt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\;\top_R$$

- Case rule K

$$\dfrac{\texttt{h}_1 : unbox(\Box\Gamma_2) \vdash \texttt{F}_4}{\bullet\texttt{h}_1 : \Box\Gamma_2, \Delta_3 \vdash (\perp, \Delta_5), []\texttt{F}_4} \;K \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_1 : unbox(\Box\Gamma_2) \vdash \texttt{F}_4}\;\;\texttt{ax}}{\bullet\texttt{h}_1 : \Delta_3, \Box\Gamma_2 \vdash \Delta_5, []\texttt{F}_4}\;K$$

- Case rule $A45$

$$\dfrac{\texttt{h}_1 : \Box\Gamma_2 \vdash \Box\Gamma_4, \texttt{F}_5}{\bullet\texttt{h}_1 : \Box\Gamma_2, \Delta_3 \vdash \Box\Gamma_4, (\perp, \Delta_6), []\texttt{F}_5} \;A45 \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_1 : \Box\Gamma_2 \vdash \texttt{F}_5, \Box\Gamma_4}\;\;\texttt{ax}}{\bullet\texttt{h}_1 : \Delta_3, \Box\Gamma_2 \vdash \Delta_6, \Box\Gamma_4, []\texttt{F}_5}\;A45$$

- Case rule $\rightarrow_L$

$$\dfrac{\texttt{h}_2 : \Delta_3 \vdash \perp, \texttt{F}_4, \Delta_1 \quad \texttt{h}_2 : \texttt{F}_5, \Delta_3 \vdash \perp, \Delta_1}{\bullet\texttt{h}_2 : \Delta_3, \texttt{F}_4 \rightarrow \texttt{F}_5 \vdash \perp, \Delta_1} \;\rightarrow_L \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_2 : \Delta_3 \vdash \Delta_1, \texttt{F}_4}\;\texttt{ax/ind} \quad \overline{\texttt{h}_2 : \Delta_3, \texttt{F}_5 \vdash \Delta_1}\;\texttt{ax/ind}}{\bullet\texttt{h}_2 : \Delta_3, \texttt{F}_4 \rightarrow \texttt{F}_5 \vdash \Delta_1}\;\rightarrow_L$$

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- Case rule $\wedge_L$

$$\frac{\mathtt{h_2} : \mathtt{F_4}, \mathtt{F_5}, \Delta_3 \vdash \bot, \Delta_1}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{F_4} \wedge \mathtt{F_5} \vdash \bot, \Delta_1} \ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} : \Delta_3, \mathtt{F_4}, \mathtt{F_5} \vdash \Delta_1} \ ^{\mathrm{ax/ind}}}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{F_4} \wedge \mathtt{F_5} \vdash \Delta_1} \ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h_2} : \mathtt{F_4}, \Delta_3 \vdash \bot, \Delta_1 \quad \mathtt{h_2} : \mathtt{F_5}, \Delta_3 \vdash \bot, \Delta_1}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{F_4} \vee \mathtt{F_5} \vdash \bot, \Delta_1} \ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} : \Delta_3, \mathtt{F_4} \vdash \Delta_1} \ ^{\mathrm{ax/ind}} \quad \overline{\mathtt{h_2} : \Delta_3, \mathtt{F_5} \vdash \Delta_1} \ ^{\mathrm{ax/ind}}}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{F_4} \vee \mathtt{F_5} \vdash \Delta_1} \ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h_2} : \mathtt{F_4}, \Delta_3, []\mathtt{F_4} \vdash \bot, \Delta_1}{\bullet\mathtt{h_2} : \Delta_3, []\mathtt{F_4} \vdash \bot, \Delta_1} \ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} : \Delta_3, \mathtt{F_4}, []\mathtt{F_4} \vdash \Delta_1} \ ^{\mathrm{ax/ind}}}{\bullet\mathtt{h_2} : \Delta_3, []\mathtt{F_4} \vdash \Delta_1} \ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h_2} : \bot, \Delta_3 \vdash \bot, \Delta_1} \ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h_2} : \bot, \Delta_3 \vdash \Delta_1} \ \bot_L$$

- Case rule I

$$\frac{}{\bullet\mathtt{h_1} : \mathtt{p_3}, \Delta_2 \vdash \mathtt{p_3}, \bot, \Delta_4} \ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{p_3} \vdash \Delta_4, \mathtt{p_3}} \ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h_2} : \Delta_3 \vdash \bot, \Delta_1}{\bullet\mathtt{h_2} : \top, \Delta_3 \vdash \bot, \Delta_1} \ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} : \Delta_3 \vdash \Delta_1} \ ^{\mathrm{ax/ind}}}{\bullet\mathtt{h_2} : \top, \Delta_3 \vdash \Delta_1} \ \top_L$$

4.6 Status of $\top_R$: : Invertible

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h_1} : \mathtt{F_3}, \Delta_2 \vdash \top, \mathtt{F_4}, \Delta_5}{\bullet\mathtt{h_1} : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F_3} \rightarrow \mathtt{F_4}} \ \rightarrow_R \quad \rightarrow \quad \text{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h_1} : \Delta_2 \vdash \top, \mathtt{F_3}, \Delta_5 \quad \mathtt{h_1} : \Delta_2 \vdash \top, \mathtt{F_4}, \Delta_5}{\bullet\mathtt{h_1} : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F_3} \wedge \mathtt{F_4}} \ \wedge_R \quad \rightarrow \quad \text{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h_1} : \Delta_2 \vdash \top, \mathtt{F_3}, \mathtt{F_4}, \Delta_5}{\bullet\mathtt{h_1} : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F_3} \vee \mathtt{F_4}} \ \vee_R \quad \rightarrow \quad \text{trivial}$$

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- Case rule $\perp_R$

$$\frac{\mathtt{h_1} : \Delta_2 \vdash \top, \Delta_3}{\bullet\mathtt{h_1} : \Delta_2 \vdash \perp, \top, \Delta_3} \;\perp_R \qquad \to \qquad \text{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet\mathtt{h_1} : \Delta_2 \vdash \top, \Delta_3} \;\top_R \qquad \to \qquad \text{trivial}$$

- Case rule K

$$\frac{\mathtt{h_1} : unbox(\Box\Gamma_2) \vdash \mathtt{F_4}}{\bullet\mathtt{h_1} : \Box\Gamma_2, \Delta_3 \vdash (\top, \Delta_5), []\mathtt{F_4}} \;K \qquad \to \qquad \text{trivial}$$

- Case rule $A45$

$$\frac{\mathtt{h_1} : \Box\Gamma_2 \vdash \Box\Gamma_4, \mathtt{F_5}}{\bullet\mathtt{h_1} : \Box\Gamma_2, \Delta_3 \vdash \Box\Gamma_4, (\top, \Delta_6), []\mathtt{F_5}} \;A45 \qquad \to \qquad \text{trivial}$$

- Case rule $\to_L$

$$\frac{\mathtt{h_2} : \Delta_3 \vdash \top, \mathtt{F_4}, \Delta_1 \quad \mathtt{h_2} : \mathtt{F_5}, \Delta_3 \vdash \top, \Delta_1}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{F_4} \to \mathtt{F_5} \vdash \top, \Delta_1} \;\to_L \qquad \to \qquad \text{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h_2} : \mathtt{F_4}, \mathtt{F_5}, \Delta_3 \vdash \top, \Delta_1}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{F_4} \wedge \mathtt{F_5} \vdash \top, \Delta_1} \;\wedge_L \qquad \to \qquad \text{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h_2} : \mathtt{F_4}, \Delta_3 \vdash \top, \Delta_1 \quad \mathtt{h_2} : \mathtt{F_5}, \Delta_3 \vdash \top, \Delta_1}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{F_4} \vee \mathtt{F_5} \vdash \top, \Delta_1} \;\vee_L \qquad \to \qquad \text{trivial}$$

- Case rule AT

$$\frac{\mathtt{h_2} : \mathtt{F_4}, \Delta_3, []\mathtt{F_4} \vdash \top, \Delta_1}{\bullet\mathtt{h_2} : \Delta_3, []\mathtt{F_4} \vdash \top, \Delta_1} \;AT \qquad \to \qquad \text{trivial}$$

- Case rule $\perp_L$

$$\frac{}{\bullet\mathtt{h_2} : \perp, \Delta_3 \vdash \top, \Delta_1} \;\perp_L \qquad \to \qquad \text{trivial}$$

- Case rule I

$$\frac{}{\bullet\mathtt{h_1} : \mathtt{p_3}, \Delta_2 \vdash \mathtt{p_3}, \top, \Delta_4} \;I \qquad \to \qquad \text{trivial}$$

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- Case rule $\top_L$

$$\frac{\mathtt{h_2} : \Delta_3 \vdash \top, \Delta_1}{\bullet \mathtt{h_2} : \top, \Delta_3 \vdash \top, \Delta_1} \ \top_L \qquad \rightarrow \qquad \mathtt{trivial}$$

4.7 Status of K: : Non invertible

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h_4} : \Box\Gamma_1, \mathtt{F_5}, \Delta_2 \vdash \mathtt{F_6}, \Delta_7, []\mathtt{F_3}}{\bullet \mathtt{h_4} : \Box\Gamma_1, \Delta_2 \vdash (\Delta_7, []\mathtt{F_3}), \mathtt{F_5} \rightarrow \mathtt{F_6}} \ \rightarrow_R \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_4} : unbox(\Box\Gamma_1) \vdash \mathtt{F_3}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_4} : unbox(\Box\Gamma_1) \vdash \mathtt{F_3}} \ \mathtt{H}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h_4} : \Box\Gamma_1, \Delta_2 \vdash \mathtt{F_5}, \Delta_7, []\mathtt{F_3} \quad \mathtt{h_4} : \Box\Gamma_1, \Delta_2 \vdash \mathtt{F_6}, \Delta_7, []\mathtt{F_3}}{\bullet \mathtt{h_4} : \Box\Gamma_1, \Delta_2 \vdash (\Delta_7, []\mathtt{F_3}), \mathtt{F_5} \wedge \mathtt{F_6}} \ \wedge_R \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_4} : unbox(\Box\Gamma_1) \vdash \mathtt{F_3}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_4} : unbox(\Box\Gamma_1) \vdash \mathtt{F_3}} \ \mathtt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h_4} : \Box\Gamma_1, \Delta_2 \vdash \mathtt{F_5}, \mathtt{F_6}, \Delta_7, []\mathtt{F_3}}{\bullet \mathtt{h_4} : \Box\Gamma_1, \Delta_2 \vdash (\Delta_7, []\mathtt{F_3}), \mathtt{F_5} \vee \mathtt{F_6}} \ \vee_R \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_4} : unbox(\Box\Gamma_1) \vdash \mathtt{F_3}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_4} : unbox(\Box\Gamma_1) \vdash \mathtt{F_3}} \ \mathtt{H}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h_4} : \Box\Gamma_1, \Delta_2 \vdash \Delta_5, []\mathtt{F_3}}{\bullet \mathtt{h_4} : \Box\Gamma_1, \Delta_2 \vdash \bot, \Delta_5, []\mathtt{F_3}} \ \bot_R \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_4} : unbox(\Box\Gamma_1) \vdash \mathtt{F_3}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_4} : unbox(\Box\Gamma_1) \vdash \mathtt{F_3}} \ \mathtt{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h_4} : \Box\Gamma_1, \Delta_2 \vdash \top, \Delta_5, []\mathtt{F_3}} \ \top_R \qquad \rightarrow \qquad \frac{\textcolor{red}{}}{\textcolor{red}{\bullet \mathtt{h_4} : unbox(\Box\Gamma_1) \vdash \mathtt{F_3}}} \ \textcolor{red}{\mathtt{fail}}$$

- Case rule K

$$\frac{\mathtt{h_2} : unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash \mathtt{F_3}}{\bullet \mathtt{h_2} : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []\mathtt{F_1}), []\mathtt{F_3}} \ K \qquad \rightarrow \qquad \frac{\textcolor{red}{}}{\textcolor{red}{\bullet \mathtt{h_2} : unbox(\Box\Gamma_5), unbox(\Box\Gamma_7) \vdash \mathtt{F_1}}} \ \textcolor{red}{\mathtt{fail}}$$

$$\frac{\mathtt{h_1} : unbox(\Box\Gamma_4), unbox(\Box\Gamma_5) \vdash \mathtt{F_3}}{\bullet \mathtt{h_1} : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_2, []\mathtt{F_3}} \ K \qquad \rightarrow \qquad \frac{\textcolor{red}{}}{\textcolor{red}{\bullet \mathtt{h_1} : unbox(\Box\Gamma_4), unbox(\Box\Gamma_6) \vdash \mathtt{F_3}}} \ \textcolor{red}{\mathtt{fail}}$$

- Case rule $A45$

$$\frac{\mathtt{h_2} : \Box\Gamma_6, \Box\Gamma_7 \vdash \Box\Gamma_5, \mathtt{F_4}, []\mathtt{F_1}}{\bullet \mathtt{h_2} : (\Box\Gamma_6, \Box\Gamma_7), \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_5, []\mathtt{F_1}), \Delta_3, []\mathtt{F_4}} \ A45 \qquad \rightarrow \qquad \frac{\textcolor{red}{}}{\textcolor{red}{\bullet \mathtt{h_2} : unbox(\Box\Gamma_6), unbox(\Box\Gamma_8) \vdash \mathtt{F_1}}} \ \textcolor{red}{\mathtt{fail}}$$

$$\frac{\mathtt{h_2} : \Box\Gamma_6, \Box\Gamma_7 \vdash \Box\Gamma_3, \mathtt{F_4}}{\bullet \mathtt{h_2} : (\Box\Gamma_6, \Box\Gamma_7), \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_3, (\Delta_5, []\mathtt{F_1}), []\mathtt{F_4}} \ A45 \qquad \rightarrow \qquad \frac{\textcolor{red}{}}{\textcolor{red}{\bullet \mathtt{h_2} : unbox(\Box\Gamma_6), unbox(\Box\Gamma_8) \vdash \mathtt{F_1}}} \ \textcolor{red}{\mathtt{fail}}$$

$$\frac{\mathtt{h_1} : \Box\Gamma_5, \Box\Gamma_6 \vdash \Box\Gamma_2, \mathtt{F_4}}{\bullet \mathtt{h_1} : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_2, \Delta_3, []\mathtt{F_4}} \ A45 \qquad \rightarrow \qquad \frac{\textcolor{red}{}}{\textcolor{red}{\bullet \mathtt{h_1} : unbox(\Box\Gamma_5), unbox(\Box\Gamma_7) \vdash \mathtt{F_4}}} \ \textcolor{red}{\mathtt{fail}}$$

- Case rule $\rightarrow_L$

$$\cfrac{\mathtt{h}_4 : \Box\Gamma_1, \Delta_7 \vdash \mathtt{F}_5, \Delta_2, []\mathtt{F}_3 \quad \mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_6, \Delta_7 \vdash \Delta_2, []\mathtt{F}_3}{\bullet\mathtt{h}_4 : (\Box\Gamma_1, \Delta_7), \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_2, []\mathtt{F}_3} \rightarrow_L \qquad \rightarrow \qquad \cfrac{\cfrac{}{\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{ax/ind}}{\bullet\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{H}$$

- Case rule $\wedge_L$

$$\cfrac{\mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_5, \mathtt{F}_6, \Delta_7 \vdash \Delta_2, []\mathtt{F}_3}{\bullet\mathtt{h}_4 : (\Box\Gamma_1, \Delta_7), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_2, []\mathtt{F}_3} \wedge_L \qquad \rightarrow \qquad \cfrac{\cfrac{}{\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{ax/ind}}{\bullet\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{H}$$

- Case rule $\vee_L$

$$\cfrac{\mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_5, \Delta_7 \vdash \Delta_2, []\mathtt{F}_3 \quad \mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_6, \Delta_7 \vdash \Delta_2, []\mathtt{F}_3}{\bullet\mathtt{h}_4 : (\Box\Gamma_1, \Delta_7), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_2, []\mathtt{F}_3} \vee_L \qquad \rightarrow \qquad \cfrac{\cfrac{}{\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{ax/ind}}{\bullet\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{H}$$

- Case rule AT

$$\cfrac{\mathtt{h}_4 : \Box\Gamma_6, \mathtt{F}_5, \Delta_1, []\mathtt{F}_5 \vdash \Delta_2, []\mathtt{F}_3}{\bullet\mathtt{h}_4 : (\Box\Gamma_6, \Delta_1), []\mathtt{F}_5 \vdash \Delta_2, []\mathtt{F}_3} \ AT \qquad \rightarrow \qquad \cfrac{\cfrac{}{\mathtt{h}_4 : \mathtt{F}_5, unbox(\Box\Gamma_6) \vdash \mathtt{F}_3}\ \text{ax/ind}}{\bullet\mathtt{h}_4 : \mathtt{F}_5, unbox(\Box\Gamma_6) \vdash \mathtt{F}_3}\ \text{H}$$

$$\cfrac{\mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_5, \Delta_6, []\mathtt{F}_5 \vdash \Delta_2, []\mathtt{F}_3}{\bullet\mathtt{h}_4 : (\Box\Gamma_1, \Delta_6), []\mathtt{F}_5 \vdash \Delta_2, []\mathtt{F}_3} \ AT \qquad \rightarrow \qquad \cfrac{\cfrac{}{\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{ax/ind}}{\bullet\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{H}$$

- Case rule $\perp_L$

$$\cfrac{}{\bullet\mathtt{h}_4 : \perp, \Box\Gamma_1, \Delta_5 \vdash \Delta_2, []\mathtt{F}_3} \perp_L \qquad \rightarrow \qquad \textcolor{red}{\cfrac{}{\bullet\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \texttt{fail}}$$

- Case rule I

$$\cfrac{}{\bullet\mathtt{h}_3 : \mathtt{p}_4, \Box\Gamma_1, \Delta_6 \vdash \mathtt{p}_4, \Delta_5, []\mathtt{F}_2} I \qquad \rightarrow \qquad \textcolor{red}{\cfrac{}{\bullet\mathtt{h}_3 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_2}\ \texttt{fail}}$$

- Case rule $\top_L$

$$\cfrac{\mathtt{h}_4 : \Box\Gamma_1, \Delta_5 \vdash \Delta_2, []\mathtt{F}_3}{\bullet\mathtt{h}_4 : \top, \Box\Gamma_1, \Delta_5 \vdash \Delta_2, []\mathtt{F}_3} \top_L \qquad \rightarrow \qquad \cfrac{\cfrac{}{\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{ax/ind}}{\bullet\mathtt{h}_4 : unbox(\Box\Gamma_1) \vdash \mathtt{F}_3}\ \text{H}$$

4.8 Status of $A45$: : Non invertible

- Case rule $\rightarrow_R$

$$\cfrac{\mathtt{h}_5 : \Box\Gamma_1, \mathtt{F}_6, \Delta_2 \vdash \Box\Gamma_3, \mathtt{F}_7, \Delta_8, []\mathtt{F}_4}{\bullet\mathtt{h}_5 : \Box\Gamma_1, \Delta_2 \vdash (\Box\Gamma_3, \Delta_8, []\mathtt{F}_4), \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R \qquad \rightarrow \qquad \cfrac{\cfrac{}{\mathtt{h}_5 : \Box\Gamma_1 \vdash \mathtt{F}_4, \Box\Gamma_3}\ \text{ax/ind}}{\bullet\mathtt{h}_5 : \Box\Gamma_1 \vdash \mathtt{F}_4, \Box\Gamma_3}\ \text{H}$$

- Case rule $\wedge_R$

$$\cfrac{\mathtt{h}_5 : \Box\Gamma_1, \Delta_2 \vdash \Box\Gamma_3, \mathtt{F}_6, \Delta_8, []\mathtt{F}_4 \quad \mathtt{h}_5 : \Box\Gamma_1, \Delta_2 \vdash \Box\Gamma_3, \mathtt{F}_7, \Delta_8, []\mathtt{F}_4}{\bullet\mathtt{h}_5 : \Box\Gamma_1, \Delta_2 \vdash (\Box\Gamma_3, \Delta_8, []\mathtt{F}_4), \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \qquad \rightarrow \qquad \cfrac{\cfrac{}{\mathtt{h}_5 : \Box\Gamma_1 \vdash \mathtt{F}_4, \Box\Gamma_3}\ \text{ax/ind}}{\bullet\mathtt{h}_5 : \Box\Gamma_1 \vdash \mathtt{F}_4, \Box\Gamma_3}\ \text{H}$$

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- Case rule $\vee_R$

$$\frac{h_5 : \Box\Gamma_1, \Delta_2 \vdash \Box\Gamma_3, F_6, F_7, \Delta_8, []F_4}{\bullet h_5 : \Box\Gamma_1, \Delta_2 \vdash (\Box\Gamma_3, \Delta_8, []F_4), F_6 \vee F_7}\ \vee_R \quad \to \quad \frac{\overline{h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_3}\ ^{ax/ind}}{\bullet h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_3}\ H$$

- Case rule $\perp_R$

$$\frac{h_5 : \Box\Gamma_1, \Delta_2 \vdash \Box\Gamma_3, \Delta_6, []F_4}{\bullet h_5 : \Box\Gamma_1, \Delta_2 \vdash \perp, \Box\Gamma_3, \Delta_6, []F_4}\ \perp_R \quad \to \quad \frac{\overline{h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_3}\ ^{ax/ind}}{\bullet h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_3}\ H$$

- Case rule $\top_R$

$$\frac{}{\bullet h_5 : \Box\Gamma_1, \Delta_2 \vdash \top, \Box\Gamma_3, \Delta_6, []F_4}\ \top_R \quad \to \quad \frac{}{\color{red}\bullet h_5 : \Box\Gamma_1 \vdash \Box\Gamma_3, F_4}\ ^{fail}$$

- Case rule K

$$\frac{h_3 : unbox(\Box\Gamma_6), unbox(\Box\Gamma_7) \vdash F_4}{\bullet h_3 : (\Box\Gamma_6, \Box\Gamma_7), \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_5, \Delta_1, []F_2), []F_4}\ K \quad \to \quad \frac{}{\color{red}\bullet h_3 : \Box\Gamma_6, \Box\Gamma_8 \vdash \Box\Gamma_5, F_2, []F_4}\ ^{fail}$$

$$\frac{h_3 : unbox(\Box\Gamma_6), unbox(\Box\Gamma_7) \vdash F_4}{\bullet h_3 : (\Box\Gamma_6, \Box\Gamma_7), \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_1, \Delta_5, []F_2), []F_4}\ K \quad \to \quad \frac{}{\color{red}\bullet h_3 : \Box\Gamma_6, \Box\Gamma_8 \vdash \Box\Gamma_1, F_2}\ ^{fail}$$

$$\frac{h_3 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash F_4}{\bullet h_3 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_1, \Delta_2), []F_4}\ K \quad \to \quad \frac{}{\color{red}\bullet h_3 : \Box\Gamma_5, \Box\Gamma_7 \vdash \Box\Gamma_1, F_4}\ ^{fail}$$

- Case rule $A45$

$$\frac{h_2 : \Box\Gamma_8, \Box\Gamma_9 \vdash \Box\Gamma_4, \Box\Gamma_5, F_3, []F_1}{\bullet h_2 : (\Box\Gamma_8, \Box\Gamma_9), \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_4, \Box\Gamma_5, []F_1), (\Box\Gamma_6, \Delta_7), []F_3}\ A45 \quad \to \quad \frac{}{\color{red}\bullet h_2 : \Box\Gamma_8, \Box\Gamma_{10} \vdash \Box\Gamma_4, \Box\Gamma_6, F_1, []F_3}\ ^{fail}$$

$$\frac{h_2 : \Box\Gamma_8, \Box\Gamma_9 \vdash \Box\Gamma_4, \Box\Gamma_5, F_3}{\bullet h_2 : (\Box\Gamma_8, \Box\Gamma_9), \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_4, \Box\Gamma_5), (\Box\Gamma_6, \Delta_7, []F_1), []F_3}\ A45 \quad \to \quad \frac{}{\color{red}\bullet h_2 : \Box\Gamma_8, \Box\Gamma_{10} \vdash \Box\Gamma_4, \Box\Gamma_6, F_1, []F_3}\ ^{fail}$$

$$\frac{h_2 : \Box\Gamma_8, \Box\Gamma_9 \vdash \Box\Gamma_4, \Box\Gamma_5, F_3, []F_1}{\bullet h_2 : (\Box\Gamma_8, \Box\Gamma_9), \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_4, \Box\Gamma_5, []F_1), (\Box\Gamma_6, \Delta_7), []F_3}\ A45 \quad \to \quad \frac{}{\color{red}\bullet h_2 : \Box\Gamma_8, \Box\Gamma_{10} \vdash \Box\Gamma_4, \Box\Gamma_6, F_1}\ ^{fail}$$

$$\frac{h_2 : \Box\Gamma_8, \Box\Gamma_9 \vdash \Box\Gamma_4, \Box\Gamma_5, F_3}{\bullet h_2 : (\Box\Gamma_8, \Box\Gamma_9), \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_4, \Box\Gamma_5), (\Box\Gamma_6, \Delta_7, []F_1), []F_3}\ A45 \quad \to \quad \frac{}{\color{red}\bullet h_2 : \Box\Gamma_8, \Box\Gamma_{10} \vdash \Box\Gamma_4, \Box\Gamma_6, F_1}\ ^{fail}$$

$$\frac{h_1 : \Box\Gamma_7, \Box\Gamma_8 \vdash \Box\Gamma_3, \Box\Gamma_4, F_2}{\bullet h_1 : (\Box\Gamma_7, \Box\Gamma_8), \Box\Gamma_9, \Delta_{10} \vdash (\Box\Gamma_3, \Box\Gamma_4), (\Box\Gamma_5, \Delta_6), []F_2}\ A45 \quad \to \quad \frac{}{\color{red}\bullet h_1 : \Box\Gamma_7, \Box\Gamma_9 \vdash \Box\Gamma_3, \Box\Gamma_5, F_2}\ ^{fail}$$

- Case rule $\to_L$

$$\frac{h_5 : \Box\Gamma_1, \Delta_8 \vdash \Box\Gamma_2, F_6, \Delta_3, []F_4 \quad h_5 : \Box\Gamma_1, F_7, \Delta_8 \vdash \Box\Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box\Gamma_1, \Delta_8), F_6 \to F_7 \vdash \Box\Gamma_2, \Delta_3, []F_4}\ \to_L \quad \to \quad \frac{\overline{h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2}\ ^{ax/ind}}{\bullet h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2}\ H$$

- Case rule $\wedge_L$

$$\frac{h_5 : \Box\Gamma_1, F_6, F_7, \Delta_8 \vdash \Box\Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box\Gamma_1, \Delta_8), F_6 \wedge F_7 \vdash \Box\Gamma_2, \Delta_3, []F_4} \wedge_L \quad \to \quad \frac{\overline{h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2}^{\ ax/ind}}{\bullet h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2} H$$

- Case rule $\vee_L$

$$\frac{h_5 : \Box\Gamma_1, F_6, \Delta_8 \vdash \Box\Gamma_2, \Delta_3, []F_4 \quad h_5 : \Box\Gamma_1, F_7, \Delta_8 \vdash \Box\Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box\Gamma_1, \Delta_8), F_6 \vee F_7 \vdash \Box\Gamma_2, \Delta_3, []F_4} \vee_L \quad \to \quad \frac{\overline{h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2}^{\ ax/ind}}{\bullet h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2} H$$

- Case rule AT

$$\frac{h_5 : \Box\Gamma_7, F_6, \Delta_1, []F_6 \vdash \Box\Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box\Gamma_7, \Delta_1), []F_6 \vdash \Box\Gamma_2, \Delta_3, []F_4} AT \quad \to \quad \frac{\overline{h_5 : \Box\Gamma_7, []F_6 \vdash F_4, \Box\Gamma_2}^{\ ax/ind}}{\bullet h_5 : \Box\Gamma_7, []F_6 \vdash F_4, \Box\Gamma_2} H$$

$$\frac{h_5 : \Box\Gamma_1, F_6, \Delta_7, []F_6 \vdash \Box\Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box\Gamma_1, \Delta_7), []F_6 \vdash \Box\Gamma_2, \Delta_3, []F_4} AT \quad \to \quad \frac{\overline{h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2}^{\ ax/ind}}{\bullet h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2} H$$

- Case rule $\perp_L$

$$\frac{}{\bullet h_5 : \perp, \Box\Gamma_1, \Delta_6 \vdash \Box\Gamma_2, \Delta_3, []F_4} \perp_L \quad \to \quad {\color{red}\frac{}{\bullet h_5 : \Box\Gamma_1 \vdash \Box\Gamma_2, F_4}^{\ fail}}$$

- Case rule I

$$\frac{}{\bullet h_4 : p_5, \Box\Gamma_1, \Delta_7 \vdash p_5, \Box\Gamma_2, \Delta_6, []F_3} I \quad \to \quad {\color{red}\frac{}{\bullet h_4 : \Box\Gamma_1 \vdash \Box\Gamma_2, F_3}^{\ fail}}$$

- Case rule $\top_L$

$$\frac{h_5 : \Box\Gamma_1, \Delta_6 \vdash \Box\Gamma_2, \Delta_3, []F_4}{\bullet h_5 : \top, \Box\Gamma_1, \Delta_6 \vdash \Box\Gamma_2, \Delta_3, []F_4} \top_L \quad \to \quad \frac{\overline{h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2}^{\ ax/ind}}{\bullet h_5 : \Box\Gamma_1 \vdash F_4, \Box\Gamma_2} H$$

4.9 Status of $\to_L$: (Left Premise): `Invertible`

- Case rule $\to_R$

$$\frac{h_4 : F_6, \Delta_1, F_2 \to F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \to F_7} \to_R \quad \to \quad \frac{\overline{h_4 : \Delta_1, F_6 \vdash \Delta_5, F_2, F_7}^{\ ax/ind}}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \to F_7} \to_R$$

- Case rule $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \to F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \quad \to \quad \frac{\overline{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6}^{\ ax/ind} \quad \overline{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_7}^{\ ax/ind}}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \wedge F_7} \wedge_R$$

- Case rule $\vee_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \quad \to \quad \frac{\overline{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6, F_7}^{\ ax/ind}}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \vee F_7} \vee_R$$

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- Case rule $\perp_R$

$$\frac{\mathtt{h_4}:\Delta_1,\mathtt{F_2}\to\mathtt{F_3}\vdash\Delta_5}{\bullet\mathtt{h_4}:\Delta_1,\mathtt{F_2}\to\mathtt{F_3}\vdash\perp,\Delta_5}\ \perp_R \qquad\to\qquad \frac{\overline{\mathtt{h_4}:\Delta_1\vdash\Delta_5,\mathtt{F_2}}\ ^{\text{ax/ind}}}{\bullet\mathtt{h_4}:\Delta_1\vdash\perp,\Delta_5,\mathtt{F_2}}\ \perp_R$$

- Case rule $\top_R$

$$\frac{}{\bullet\mathtt{h_4}:\Delta_1,\mathtt{F_2}\to\mathtt{F_3}\vdash\top,\Delta_5}\ \top_R \qquad\to\qquad \frac{}{\bullet\mathtt{h_4}:\Delta_1\vdash\top,\Delta_5,\mathtt{F_2}}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h_3}:unbox(\Box\Gamma_4)\vdash\mathtt{F_6}}{\bullet\mathtt{h_3}:\Box\Gamma_4,\Delta_7,\mathtt{F_1}\to\mathtt{F_2}\vdash\Delta_5,[]\mathtt{F_6}}\ K \qquad\to\qquad \frac{\overline{\mathtt{h_3}:unbox(\Box\Gamma_4)\vdash\mathtt{F_6}}\ ^{\text{ax}}}{\bullet\mathtt{h_3}:\Delta_7,\Box\Gamma_4\vdash\Delta_5,\mathtt{F_1},[]\mathtt{F_6}}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h_3}:\Box\Gamma_4\vdash\Box\Gamma_5,\mathtt{F_7}}{\bullet\mathtt{h_3}:\Box\Gamma_4,\Delta_8,\mathtt{F_1}\to\mathtt{F_2}\vdash\Box\Gamma_5,\Delta_6,[]\mathtt{F_7}}\ A45 \qquad\to\qquad \frac{\overline{\mathtt{h_3}:\Box\Gamma_4\vdash\mathtt{F_7},\Box\Gamma_5}\ ^{\text{ax}}}{\bullet\mathtt{h_3}:\Delta_8,\Box\Gamma_4\vdash\Delta_6,\mathtt{F_1},\Box\Gamma_5,[]\mathtt{F_7}}\ A45$$

- Case rule $\to_L$

$$\frac{\mathtt{h_3}:\Delta_7,\mathtt{F_1}\to\mathtt{F_2}\vdash\mathtt{F_4},\Delta_6 \quad \mathtt{h_3}:\mathtt{F_5},\Delta_7,\mathtt{F_1}\to\mathtt{F_2}\vdash\Delta_6}{\bullet\mathtt{h_3}:(\Delta_7,\mathtt{F_1}\to\mathtt{F_2}),\mathtt{F_4}\to\mathtt{F_5}\vdash\Delta_6}\ \to_L \qquad\to\qquad \frac{\overline{\mathtt{h_3}:\Delta_7\vdash\Delta_6,\mathtt{F_1},\mathtt{F_4}}\ ^{\text{ax/ind}}\quad\overline{\mathtt{h_3}:\Delta_7,\mathtt{F_5}\vdash\Delta_6,\mathtt{F_1}}\ ^{\text{ax/ind}}}{\bullet\mathtt{h_3}:\Delta_7,\mathtt{F_4}\to\mathtt{F_5}\vdash\Delta_6,\mathtt{F_1}}\ \to_L$$

$$\frac{\mathtt{h_1}:\Delta_2\vdash\mathtt{F_3},\Delta_5 \quad \mathtt{h_1}:\mathtt{F_4},\Delta_2\vdash\Delta_5}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\to\mathtt{F_4}\vdash\Delta_5}\ \to_L \qquad\to\qquad \frac{\overline{\mathtt{h_1}:\Delta_2\vdash\Delta_5,\mathtt{F_3}}\ ^{\text{ax}}}{\bullet\mathtt{h_1}:\Delta_2\vdash\Delta_5,\mathtt{F_3}}\ \text{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h_3}:\mathtt{F_4},\mathtt{F_5},\Delta_7,\mathtt{F_1}\to\mathtt{F_2}\vdash\Delta_6}{\bullet\mathtt{h_3}:(\Delta_7,\mathtt{F_1}\to\mathtt{F_2}),\mathtt{F_4}\wedge\mathtt{F_5}\vdash\Delta_6}\ \wedge_L \qquad\to\qquad \frac{\overline{\mathtt{h_3}:\Delta_7,\mathtt{F_4},\mathtt{F_5}\vdash\Delta_6,\mathtt{F_1}}\ ^{\text{ax/ind}}}{\bullet\mathtt{h_3}:\Delta_7,\mathtt{F_4}\wedge\mathtt{F_5}\vdash\Delta_6,\mathtt{F_1}}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h_3}:\mathtt{F_4},\Delta_7,\mathtt{F_1}\to\mathtt{F_2}\vdash\Delta_6 \quad \mathtt{h_3}:\mathtt{F_5},\Delta_7,\mathtt{F_1}\to\mathtt{F_2}\vdash\Delta_6}{\bullet\mathtt{h_3}:(\Delta_7,\mathtt{F_1}\to\mathtt{F_2}),\mathtt{F_4}\vee\mathtt{F_5}\vdash\Delta_6}\ \vee_L \qquad\to\qquad \frac{\overline{\mathtt{h_3}:\Delta_7,\mathtt{F_4}\vdash\Delta_6,\mathtt{F_1}}\ ^{\text{ax/ind}}\quad\overline{\mathtt{h_3}:\Delta_7,\mathtt{F_5}\vdash\Delta_6,\mathtt{F_1}}\ ^{\text{ax/ind}}}{\bullet\mathtt{h_3}:\Delta_7,\mathtt{F_4}\vee\mathtt{F_5}\vdash\Delta_6,\mathtt{F_1}}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h_3}:\mathtt{F_4},\Delta_6,[]\mathtt{F_4},\mathtt{F_1}\to\mathtt{F_2}\vdash\Delta_5}{\bullet\mathtt{h_3}:(\Delta_6,\mathtt{F_1}\to\mathtt{F_2}),[]\mathtt{F_4}\vdash\Delta_5}\ AT \qquad\to\qquad \frac{\overline{\mathtt{h_3}:\Delta_6,\mathtt{F_4},[]\mathtt{F_4}\vdash\Delta_5,\mathtt{F_1}}\ ^{\text{ax/ind}}}{\bullet\mathtt{h_3}:\Delta_6,[]\mathtt{F_4}\vdash\Delta_5,\mathtt{F_1}}\ AT$$

- Case rule $\perp_L$

$$\frac{}{\bullet\mathtt{h_3}:\perp,\Delta_5,\mathtt{F_1}\to\mathtt{F_2}\vdash\Delta_4}\ \perp_L \qquad\to\qquad \frac{}{\bullet\mathtt{h_3}:\perp,\Delta_5\vdash\Delta_4,\mathtt{F_1}}\ \perp_L$$

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- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \to F_2 \vdash p_4, \Delta_5} \; I \quad \to \quad \frac{}{\bullet h_3 : \Delta_6, p_4 \vdash \Delta_5, F_1, p_4} \; I$$

- Case rule $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \to F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \to F_2 \vdash \Delta_4} \; \top_L \quad \to \quad \frac{\dfrac{}{h_3 : \Delta_5 \vdash \Delta_4, F_1} \; \text{ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \; \top_L$$

4.10 Status of $\to_L$ (Right Premise): : `Invertible`

- Case rule $\to_R$

$$\frac{h_4 : F_6, \Delta_1, F_2 \to F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \to F_7} \; \to_R \quad \to \quad \frac{\dfrac{}{h_4 : \Delta_1, F_3, F_6 \vdash \Delta_5, F_7} \; \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \to F_7} \; \to_R$$

- Case rule $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \to F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \wedge F_7} \; \wedge_R \quad \to \quad \frac{\dfrac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6} \; \text{ax/ind} \quad \dfrac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_7} \; \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \; \wedge_R$$

- Case rule $\vee_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \vee F_7} \; \vee_R \quad \to \quad \frac{\dfrac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6, F_7} \; \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \; \vee_R$$

- Case rule $\perp_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \perp, \Delta_5} \; \perp_R \quad \to \quad \frac{\dfrac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5} \; \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \perp, \Delta_5} \; \perp_R$$

- Case rule $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \top, \Delta_5} \; \top_R \quad \to \quad \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top, \Delta_5} \; \top_R$$

- Case rule K

$$\frac{h_3 : unbox(\Box\Gamma_4) \vdash F_6}{\bullet h_3 : \Box\Gamma_4, \Delta_7, F_1 \to F_2 \vdash \Delta_5, []F_6} \; K \quad \to \quad \frac{\dfrac{}{h_3 : unbox(\Box\Gamma_4) \vdash F_6} \; \text{ax}}{\bullet h_3 : \Delta_7, F_2, \Box\Gamma_4 \vdash \Delta_5, []F_6} \; K$$

- Case rule $A45$

$$\frac{h_3 : \Box\Gamma_4 \vdash \Box\Gamma_5, F_7}{\bullet h_3 : \Box\Gamma_4, \Delta_8, F_1 \to F_2 \vdash \Box\Gamma_5, \Delta_6, []F_7} \; A45 \quad \to \quad \frac{\dfrac{}{h_3 : \Box\Gamma_4 \vdash F_7, \Box\Gamma_5} \; \text{ax}}{\bullet h_3 : \Delta_8, F_2, \Box\Gamma_4 \vdash \Delta_6, \Box\Gamma_5, []F_7} \; A45$$

- Case rule $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \quad \rightarrow \quad \frac{\overline{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}^{\ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \quad \rightarrow \quad \frac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5}^{\ ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5}\ H$$

- Case rule $\wedge_L$

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \quad \rightarrow \quad \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule $\vee_L$

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \quad \rightarrow \quad \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6}^{\ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \rightarrow F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \rightarrow F_2), []F_4 \vdash \Delta_5} AT \quad \rightarrow \quad \frac{\overline{h_3 : \Delta_6, F_2, F_4, []F_4 \vdash \Delta_5}^{\ ax/ind}}{\bullet h_3 : \Delta_6, F_2, []F_4 \vdash \Delta_5} AT$$

- Case rule $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \quad \rightarrow \quad \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \rightarrow F_2 \vdash p_4, \Delta_5} I \quad \rightarrow \quad \frac{}{\bullet h_3 : \Delta_6, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \quad \rightarrow \quad \frac{\overline{h_3 : \Delta_5, F_2 \vdash \Delta_4}^{\ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

4.11 Status of $\wedge_L$: : Invertible

- Case rule $\rightarrow_R$

$$\frac{h_4 : F_6, \Delta_1, F_2 \wedge F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \quad \rightarrow \quad \frac{\overline{h_4 : \Delta_1, F_2, F_3, F_6 \vdash \Delta_5, F_7}^{\ ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \quad \rightarrow \quad \frac{\overline{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6}^{\ ax/ind} \quad \overline{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_7}^{\ ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule $\lor_R$

$$\frac{h_4 : \Delta_1, F_2 \land F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \land F_3 \vdash \Delta_5, F_6 \lor F_7} \ \lor_R \quad \to \quad \frac{\overline{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6, F_7} \ ^{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \lor F_7} \ \lor_R$$

- Case rule $\bot_R$

$$\frac{h_4 : \Delta_1, F_2 \land F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \land F_3 \vdash \bot, \Delta_5} \ \bot_R \quad \to \quad \frac{\overline{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5} \ ^{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \bot, \Delta_5} \ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \land F_3 \vdash \top, \Delta_5} \ \top_R \quad \to \quad \frac{}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \top, \Delta_5} \ \top_R$$

- Case rule K

$$\frac{h_3 : unbox(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \land F_2 \vdash \Delta_5, []F_6} \ K \quad \to \quad \frac{\overline{h_3 : unbox(\Box \Gamma_4) \vdash F_6} \ ^{ax}}{\bullet h_3 : \Delta_7, F_1, F_2, \Box \Gamma_4 \vdash \Delta_5, []F_6} \ K$$

- Case rule $A45$

$$\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_5, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_8, F_1 \land F_2 \vdash \Box \Gamma_5, \Delta_6, []F_7} \ A45 \quad \to \quad \frac{\overline{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_5} \ ^{ax}}{\bullet h_3 : \Delta_8, F_1, F_2, \Box \Gamma_4 \vdash \Delta_6, \Box \Gamma_5, []F_7} \ A45$$

- Case rule $\to_L$

$$\frac{h_3 : \Delta_7, F_1 \land F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \land F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \land F_2), F_4 \to F_5 \vdash \Delta_6} \ \to_L \quad \to \quad \frac{\overline{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4} \ ^{ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} \ ^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \to F_5 \vdash \Delta_6} \ \to_L$$

- Case rule $\land_L$

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \land F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \land F_2), F_4 \land F_5 \vdash \Delta_6} \ \land_L \quad \to \quad \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash \Delta_6} \ ^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \land F_5 \vdash \Delta_6} \ \land_L$$

$$\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \land F_4 \vdash \Delta_5} \ \land_L \quad \to \quad \frac{\overline{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5} \ ^{ax}}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5} \ \mathtt{H}$$

- Case rule $\lor_L$

$$\frac{h_3 : F_4, \Delta_7, F_1 \land F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \land F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \land F_2), F_4 \lor F_5 \vdash \Delta_6} \ \lor_L \quad \to \quad \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6} \ ^{ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} \ ^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \lor F_5 \vdash \Delta_6} \ \lor_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \land F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \land F_2), []F_4 \vdash \Delta_5} \ AT \quad \to \quad \frac{\overline{h_3 : \Delta_6, F_1, F_2, F_4, []F_4 \vdash \Delta_5} \ ^{ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_2, []F_4 \vdash \Delta_5} \ AT$$

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- Case rule $\perp_L$

$$\dfrac{}{\bullet\mathtt{h_3} : \perp, \Delta_5, \mathtt{F_1} \wedge \mathtt{F_2} \vdash \Delta_4} \; {\perp_L} \qquad \rightarrow \qquad \dfrac{}{\bullet\mathtt{h_3} : \perp, \Delta_5, \mathtt{F_1}, \mathtt{F_2} \vdash \Delta_4} \; {\perp_L}$$

- Case rule I

$$\dfrac{}{\bullet\mathtt{h_3} : \mathtt{p_4}, \Delta_6, \mathtt{F_1} \wedge \mathtt{F_2} \vdash \mathtt{p_4}, \Delta_5} \; {I} \qquad \rightarrow \qquad \dfrac{}{\bullet\mathtt{h_3} : \Delta_6, \mathtt{F_1}, \mathtt{F_2}, \mathtt{p_4} \vdash \Delta_5, \mathtt{p_4}} \; {I}$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h_3} : \Delta_5, \mathtt{F_1} \wedge \mathtt{F_2} \vdash \Delta_4}{\bullet\mathtt{h_3} : \top, \Delta_5, \mathtt{F_1} \wedge \mathtt{F_2} \vdash \Delta_4} \; {\top_L} \qquad \rightarrow \qquad \dfrac{\dfrac{}{\mathtt{h_3} : \Delta_5, \mathtt{F_1}, \mathtt{F_2} \vdash \Delta_4} \; {\mathtt{ax/ind}}}{\bullet\mathtt{h_3} : \top, \Delta_5, \mathtt{F_1}, \mathtt{F_2} \vdash \Delta_4} \; {\top_L}$$

4.12   Status of $\vee_L$:   (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\dfrac{\mathtt{h_4} : \mathtt{F_6}, \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_7}, \Delta_5}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \rightarrow \mathtt{F_7}} \; {\rightarrow_R} \qquad \rightarrow \qquad \dfrac{\dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_2}, \mathtt{F_6} \vdash \Delta_5, \mathtt{F_7}} \; {\mathtt{ax/ind}}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \Delta_5, \mathtt{F_6} \rightarrow \mathtt{F_7}} \; {\rightarrow_R}$$

- Case rule $\wedge_R$

$$\dfrac{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_6}, \Delta_5 \quad \mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_7}, \Delta_5}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \wedge \mathtt{F_7}} \; {\wedge_R} \qquad \rightarrow \qquad \dfrac{\dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \Delta_5, \mathtt{F_6}} \; {\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \Delta_5, \mathtt{F_7}} \; {\mathtt{ax/ind}}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \Delta_5, \mathtt{F_6} \wedge \mathtt{F_7}} \; {\wedge_R}$$

- Case rule $\vee_R$

$$\dfrac{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_6}, \mathtt{F_7}, \Delta_5}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \vee \mathtt{F_7}} \; {\vee_R} \qquad \rightarrow \qquad \dfrac{\dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \Delta_5, \mathtt{F_6}, \mathtt{F_7}} \; {\mathtt{ax/ind}}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \Delta_5, \mathtt{F_6} \vee \mathtt{F_7}} \; {\vee_R}$$

- Case rule $\perp_R$

$$\dfrac{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \Delta_5}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \perp, \Delta_5} \; {\perp_R} \qquad \rightarrow \qquad \dfrac{\dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \Delta_5} \; {\mathtt{ax/ind}}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \perp, \Delta_5} \; {\perp_R}$$

- Case rule $\top_R$

$$\dfrac{}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \top, \Delta_5} \; {\top_R} \qquad \rightarrow \qquad \dfrac{}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \top, \Delta_5} \; {\top_R}$$

- Case rule K

$$\dfrac{\mathtt{h_3} : unbox(\square\Gamma_4) \vdash \mathtt{F_6}}{\bullet\mathtt{h_3} : \square\Gamma_4, \Delta_7, \mathtt{F_1} \vee \mathtt{F_2} \vdash \Delta_5, []\mathtt{F_6}} \; {K} \qquad \rightarrow \qquad \dfrac{\dfrac{}{\mathtt{h_3} : unbox(\square\Gamma_4) \vdash \mathtt{F_6}} \; {\mathtt{ax}}}{\bullet\mathtt{h_3} : \Delta_7, \mathtt{F_1}, \square\Gamma_4 \vdash \Delta_5, []\mathtt{F_6}} \; {K}$$

- Case rule $A45$

$$\frac{h_3 : \Box\Gamma_4 \vdash \Box\Gamma_5, F_7}{\bullet h_3 : \Box\Gamma_4, \Delta_8, F_1 \lor F_2 \vdash \Box\Gamma_5, \Delta_6, []F_7} \ A45 \qquad \rightarrow \qquad \frac{\dfrac{}{h_3 : \Box\Gamma_4 \vdash F_7, \Box\Gamma_5} \ \text{ax}}{\bullet h_3 : \Delta_8, F_1, \Box\Gamma_4 \vdash \Delta_6, \Box\Gamma_5, []F_7} \ A45$$

- Case rule $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \lor F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \lor F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \lor F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4} \ \text{ax/ind} \quad \dfrac{}{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \lor F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \lor F_2), F_4 \wedge F_5 \vdash \Delta_6} \ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \ \wedge_L$$

- Case rule $\vee_L$

$$\frac{h_3 : F_4, \Delta_7, F_1 \lor F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \lor F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \lor F_2), F_4 \lor F_5 \vdash \Delta_6} \ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6} \ \text{ax/ind} \quad \dfrac{}{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \lor F_5 \vdash \Delta_6} \ \vee_L$$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \lor F_4 \vdash \Delta_5} \ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{h_1 : \Delta_2, F_3 \vdash \Delta_5} \ \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5} \ \text{H}$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \lor F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \lor F_2), []F_4 \vdash \Delta_5} \ AT \quad \rightarrow \quad \frac{\dfrac{}{h_3 : \Delta_6, F_1, F_4, []F_4 \vdash \Delta_5} \ \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1, []F_4 \vdash \Delta_5} \ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet h_3 : \bot, \Delta_5, F_1 \lor F_2 \vdash \Delta_4} \ \bot_L \quad \rightarrow \quad \frac{}{\bullet h_3 : \bot, \Delta_5, F_1 \vdash \Delta_4} \ \bot_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \lor F_2 \vdash p_4, \Delta_5} \ I \quad \rightarrow \quad \frac{}{\bullet h_3 : \Delta_6, F_1, p_4 \vdash \Delta_5, p_4} \ I$$

- Case rule $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \lor F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \lor F_2 \vdash \Delta_4} \ \top_L \quad \rightarrow \quad \frac{\dfrac{}{h_3 : \Delta_5, F_1 \vdash \Delta_4} \ \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \ \top_L$$

4.13   Status of $\vee_L$ (Right Premise): :   `Invertible`

- Case rule $\rightarrow_R$

$$\cfrac{\mathtt{h_4 : F_6, \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}}{\bullet\mathtt{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \rightarrow F_7}} \ \rightarrow_R \qquad \rightarrow \qquad \cfrac{\overline{\mathtt{h_4 : \Delta_1, F_3, F_6 \vdash \Delta_5, F_7}} \ ^{\text{ax/ind}}}{\bullet\mathtt{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7}} \ \rightarrow_R$$

- Case rule $\wedge_R$

$$\cfrac{\mathtt{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, \Delta_5} \quad \mathtt{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}}{\bullet\mathtt{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \wedge F_7}} \ \wedge_R \qquad \rightarrow \qquad \cfrac{\overline{\mathtt{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6}} \ ^{\text{ax/ind}} \quad \overline{\mathtt{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_7}} \ ^{\text{ax/ind}}}{\bullet\mathtt{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7}} \ \wedge_R$$

- Case rule $\vee_R$

$$\cfrac{\mathtt{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, F_7, \Delta_5}}{\bullet\mathtt{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \vee F_7}} \ \vee_R \qquad \rightarrow \qquad \cfrac{\overline{\mathtt{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6, F_7}} \ ^{\text{ax/ind}}}{\bullet\mathtt{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7}} \ \vee_R$$

- Case rule $\bot_R$

$$\cfrac{\mathtt{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5}}{\bullet\mathtt{h_4 : \Delta_1, F_2 \vee F_3 \vdash \bot, \Delta_5}} \ \bot_R \qquad \rightarrow \qquad \cfrac{\overline{\mathtt{h_4 : \Delta_1, F_3 \vdash \Delta_5}} \ ^{\text{ax/ind}}}{\bullet\mathtt{h_4 : \Delta_1, F_3 \vdash \bot, \Delta_5}} \ \bot_R$$

- Case rule $\top_R$

$$\cfrac{}{\bullet\mathtt{h_4 : \Delta_1, F_2 \vee F_3 \vdash \top, \Delta_5}} \ \top_R \qquad \rightarrow \qquad \cfrac{}{\bullet\mathtt{h_4 : \Delta_1, F_3 \vdash \top, \Delta_5}} \ \top_R$$

- Case rule K

$$\cfrac{\mathtt{h_3 : } unbox(\Box\Gamma_4) \vdash \mathtt{F_6}}{\bullet\mathtt{h_3 : \Box\Gamma_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, []F_6}} \ K \qquad \rightarrow \qquad \cfrac{\overline{\mathtt{h_3 : } unbox(\Box\Gamma_4) \vdash \mathtt{F_6}} \ ^{\text{ax}}}{\bullet\mathtt{h_3 : \Delta_7, F_2, \Box\Gamma_4 \vdash \Delta_5, []F_6}} \ K$$

- Case rule $A45$

$$\cfrac{\mathtt{h_3 : \Box\Gamma_4 \vdash \Box\Gamma_5, F_7}}{\bullet\mathtt{h_3 : \Box\Gamma_4, \Delta_8, F_1 \vee F_2 \vdash \Box\Gamma_5, \Delta_6, []F_7}} \ A45 \qquad \rightarrow \qquad \cfrac{\overline{\mathtt{h_3 : \Box\Gamma_4 \vdash F_7, \Box\Gamma_5}} \ ^{\text{ax}}}{\bullet\mathtt{h_3 : \Delta_8, F_2, \Box\Gamma_4 \vdash \Delta_6, \Box\Gamma_5, []F_7}} \ A45$$

- Case rule $\rightarrow_L$

$$\cfrac{\mathtt{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6} \quad \mathtt{h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}}{\bullet\mathtt{h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6}} \ \rightarrow_L \qquad \rightarrow \qquad \cfrac{\overline{\mathtt{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}} \ ^{\text{ax/ind}} \quad \overline{\mathtt{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}} \ ^{\text{ax/ind}}}{\bullet\mathtt{h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6}} \ \rightarrow_L$$

- Case rule $\wedge_L$

$$\cfrac{\mathtt{h_3 : F_4, F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}}{\bullet\mathtt{h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6}} \ \wedge_L \qquad \rightarrow \qquad \cfrac{\overline{\mathtt{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6}} \ ^{\text{ax/ind}}}{\bullet\mathtt{h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6}} \ \wedge_L$$

- Case rule $\vee_L$

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \quad \to \quad \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6}\ \text{ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}\ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \quad \to \quad \frac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5}\ \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5}\ \text{H}$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \vee F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \vee F_2), []F_4 \vdash \Delta_5} AT \quad \to \quad \frac{\overline{h_3 : \Delta_6, F_2, F_4, []F_4 \vdash \Delta_5}\ \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2, []F_4 \vdash \Delta_5} AT$$

- Case rule $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \quad \to \quad \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \vee F_2 \vdash p_4, \Delta_5} I \quad \to \quad \frac{}{\bullet h_3 : \Delta_6, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \quad \to \quad \frac{\overline{h_3 : \Delta_5, F_2 \vdash \Delta_4}\ \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

4.14 Status of AT: : Invertible

- Case rule $\to_R$

$$\frac{h_3 : F_5, \Delta_1, []F_2 \vdash F_6, \Delta_4}{\bullet h_3 : \Delta_1, []F_2 \vdash \Delta_4, F_5 \to F_6} \to_R \quad \to \quad \frac{\overline{h_3 : \Delta_1, F_2, F_5, []F_2 \vdash \Delta_4, F_6}\ \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \to F_6} \to_R$$

- Case rule $\wedge_R$

$$\frac{h_3 : \Delta_1, []F_2 \vdash F_5, \Delta_4 \quad h_3 : \Delta_1, []F_2 \vdash F_6, \Delta_4}{\bullet h_3 : \Delta_1, []F_2 \vdash \Delta_4, F_5 \wedge F_6} \wedge_R \quad \to \quad \frac{\overline{h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5}\ \text{ax/ind} \quad \overline{h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_6}\ \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \wedge F_6} \wedge_R$$

- Case rule $\vee_R$

$$\frac{h_3 : \Delta_1, []F_2 \vdash F_5, F_6, \Delta_4}{\bullet h_3 : \Delta_1, []F_2 \vdash \Delta_4, F_5 \vee F_6} \vee_R \quad \to \quad \frac{\overline{h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5, F_6}\ \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \vee F_6} \vee_R$$

- Case rule $\perp_R$

$$\frac{h_3 : \Delta_1, []F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_1, []F_2 \vdash \perp, \Delta_4} \perp_R \quad \to \quad \frac{\overline{h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4}\ \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \perp, \Delta_4} \perp_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet h_3 : \Delta_1, []F_2 \vdash \top, \Delta_4}\ \top_R \qquad \to \qquad \dfrac{}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \top, \Delta_4}\ \top_R$$

- Case rule K

$$\dfrac{h_2 : F_1, unbox(\Box\Gamma_6) \vdash F_5}{\bullet h_2 : (\Box\Gamma_6, []F_1), \Delta_3 \vdash \Delta_4, []F_5}\ K \qquad \to \qquad \dfrac{\overline{h_2 : F_1, unbox(\Box\Gamma_6) \vdash F_5}\ ^{ax}}{\bullet h_2 : \Delta_3, F_1, \Box\Gamma_6, []F_1 \vdash \Delta_4, []F_5}\ K$$

$$\dfrac{h_2 : unbox(\Box\Gamma_3) \vdash F_5}{\bullet h_2 : \Box\Gamma_3, \Delta_6, []F_1 \vdash \Delta_4, []F_5}\ K \qquad \to \qquad \dfrac{\overline{h_2 : unbox(\Box\Gamma_3) \vdash F_5}\ ^{ax}}{\bullet h_2 : \Delta_6, F_1, \Box\Gamma_3, []F_1 \vdash \Delta_4, []F_5}\ K$$

- Case rule $A45$

$$\dfrac{h_2 : \Box\Gamma_7, []F_1 \vdash \Box\Gamma_4, F_6}{\bullet h_2 : (\Box\Gamma_7, []F_1), \Delta_3 \vdash \Box\Gamma_4, \Delta_5, []F_6}\ A45 \qquad \to \qquad \dfrac{\overline{h_2 : \Box\Gamma_7, []F_1 \vdash F_6, \Box\Gamma_4}\ ^{ax}}{\bullet h_2 : \Delta_3, F_1, \Box\Gamma_7, []F_1 \vdash \Delta_5, \Box\Gamma_4, []F_6}\ A45$$

$$\dfrac{h_2 : \Box\Gamma_3 \vdash \Box\Gamma_4, F_6}{\bullet h_2 : \Box\Gamma_3, \Delta_7, []F_1 \vdash \Box\Gamma_4, \Delta_5, []F_6}\ A45 \qquad \to \qquad \dfrac{\overline{h_2 : \Box\Gamma_3 \vdash F_6, \Box\Gamma_4}\ ^{ax}}{\bullet h_2 : \Delta_7, F_1, \Box\Gamma_3, []F_1 \vdash \Delta_5, \Box\Gamma_4, []F_6}\ A45$$

- Case rule $\to_L$

$$\dfrac{h_2 : \Delta_6, []F_1 \vdash F_3, \Delta_5 \quad h_2 : F_4, \Delta_6, []F_1 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, []F_1), F_3 \to F_4 \vdash \Delta_5}\ \to_L \qquad \to \qquad \dfrac{\overline{h_2 : \Delta_6, F_1, []F_1 \vdash \Delta_5, F_3}\ ^{ax/ind} \quad \overline{h_2 : \Delta_6, F_1, F_4, []F_1 \vdash \Delta_5}\ ^{ax/ind}}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \to F_4 \vdash \Delta_5}\ \to_L$$

- Case rule $\wedge_L$

$$\dfrac{h_2 : F_3, F_4, \Delta_6, []F_1 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, []F_1), F_3 \wedge F_4 \vdash \Delta_5}\ \wedge_L \qquad \to \qquad \dfrac{\overline{h_2 : \Delta_6, F_1, F_3, F_4, []F_1 \vdash \Delta_5}\ ^{ax/ind}}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \wedge F_4 \vdash \Delta_5}\ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{h_2 : F_3, \Delta_6, []F_1 \vdash \Delta_5 \quad h_2 : F_4, \Delta_6, []F_1 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, []F_1), F_3 \vee F_4 \vdash \Delta_5}\ \vee_L \qquad \to \qquad \dfrac{\overline{h_2 : \Delta_6, F_1, F_3, []F_1 \vdash \Delta_5}\ ^{ax/ind} \quad \overline{h_2 : \Delta_6, F_1, F_4, []F_1 \vdash \Delta_5}\ ^{ax/ind}}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \vee F_4 \vdash \Delta_5}\ \vee_L$$

- Case rule AT

$$\dfrac{h_2 : F_3, \Delta_5, []F_1, []F_3 \vdash \Delta_4}{\bullet h_2 : (\Delta_5, []F_1), []F_3 \vdash \Delta_4}\ AT \qquad \to \qquad \dfrac{\overline{h_2 : \Delta_5, F_1, F_3, []F_1, []F_3 \vdash \Delta_4}\ ^{ax/ind}}{\bullet h_2 : \Delta_5, F_1, []F_1, []F_3 \vdash \Delta_4}\ AT$$

$$\dfrac{h_1 : F_3, \Delta_2, []F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_2, []F_3 \vdash \Delta_4}\ AT \qquad \to \qquad \dfrac{\overline{h_1 : \Delta_2, F_3, F_3, []F_3 \vdash \Delta_4}\ ^{ax/ind}}{\bullet h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4}\ AT$$

- Case rule $\bot_L$

$$\dfrac{}{\bullet h_2 : \bot, \Delta_4, []F_1 \vdash \Delta_3}\ \bot_L \qquad \to \qquad \dfrac{}{\bullet h_2 : \bot, \Delta_4, F_1, []F_1 \vdash \Delta_3}\ \bot_L$$

- Case rule I

$$\overline{\bullet\mathtt{h}_2 : \mathtt{p}_3, \Delta_5, []\mathtt{F}_1 \vdash \mathtt{p}_3, \Delta_4} \ I \quad \rightarrow \quad \overline{\bullet\mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{p}_3, []\mathtt{F}_1 \vdash \Delta_4, \mathtt{p}_3} \ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}{\bullet\mathtt{h}_2 : \top, \Delta_4, []\mathtt{F}_1 \vdash \Delta_3} \ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3} \ \mathrm{ax/ind}}{\bullet\mathtt{h}_2 : \top, \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3} \ \top_L$$

4.15 Status of $\bot_L$: : Invertible

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \bot, \mathtt{F}_4, \Delta_1 \vdash \mathtt{F}_5, \Delta_3}{\bullet\mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \ \rightarrow_R \quad \rightarrow \quad \mathrm{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_4, \Delta_3 \quad \mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_5, \Delta_3}{\bullet\mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \ \wedge_R \quad \rightarrow \quad \mathrm{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_3}{\bullet\mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \ \vee_R \quad \rightarrow \quad \mathrm{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3}{\bullet\mathtt{h}_2 : \bot, \Delta_1 \vdash \bot, \Delta_3} \ \bot_R \quad \rightarrow \quad \mathrm{trivial}$$

- Case rule $\top_R$

$$\overline{\bullet\mathtt{h}_2 : \bot, \Delta_1 \vdash \top, \Delta_3} \ \top_R \quad \rightarrow \quad \mathrm{trivial}$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_2) \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Box\Gamma_2, \bot, \Delta_5 \vdash \Delta_3, []\mathtt{F}_4} \ K \quad \rightarrow \quad \mathrm{trivial}$$

- Case rule $A45$

$$\frac{\mathtt{h}_1 : \Box\Gamma_2 \vdash \Box\Gamma_3, \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Box\Gamma_2, \bot, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_5} \ A45 \quad \rightarrow \quad \mathrm{trivial}$$

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- Case rule $\rightarrow_L$

$$\frac{\mathtt{h_1} : \bot, \Delta_5 \vdash \mathtt{F_2}, \Delta_4 \quad \mathtt{h_1} : \bot, \mathtt{F_3}, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h_1} : (\bot, \Delta_5), \mathtt{F_2} \rightarrow \mathtt{F_3} \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \text{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h_1} : \bot, \mathtt{F_2}, \mathtt{F_3}, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h_1} : (\bot, \Delta_5), \mathtt{F_2} \wedge \mathtt{F_3} \vdash \Delta_4} \wedge_L \quad \rightarrow \quad \text{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h_1} : \bot, \mathtt{F_2}, \Delta_5 \vdash \Delta_4 \quad \mathtt{h_1} : \bot, \mathtt{F_3}, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h_1} : (\bot, \Delta_5), \mathtt{F_2} \vee \mathtt{F_3} \vdash \Delta_4} \vee_L \quad \rightarrow \quad \text{trivial}$$

- Case rule AT

$$\frac{\mathtt{h_1} : \bot, \mathtt{F_2}, \Delta_4, []\mathtt{F_2} \vdash \Delta_3}{\bullet \mathtt{h_1} : (\bot, \Delta_4), []\mathtt{F_2} \vdash \Delta_3} \ AT \quad \rightarrow \quad \text{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h_1} : \bot, \Delta_2 \vdash \Delta_3} \ \bot_L \quad \rightarrow \quad \text{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h_1} : \mathtt{p_2}, \bot, \Delta_4 \vdash \mathtt{p_2}, \Delta_3} \ I \quad \rightarrow \quad \text{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h_1} : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h_1} : \top, \bot, \Delta_3 \vdash \Delta_2} \ \top_L \quad \rightarrow \quad \text{trivial}$$

4.16   Status of I: :   Invertible

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h_3} : \mathtt{F_4}, \Delta_1, \mathtt{p_2} \vdash \mathtt{F_5}, \Delta_6, \mathtt{p_2}}{\bullet \mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash (\Delta_6, \mathtt{p_2}), \mathtt{F_4} \rightarrow \mathtt{F_5}} \rightarrow_R \quad \rightarrow \quad \text{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash \mathtt{F_4}, \Delta_6, \mathtt{p_2} \quad \mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash \mathtt{F_5}, \Delta_6, \mathtt{p_2}}{\bullet \mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash (\Delta_6, \mathtt{p_2}), \mathtt{F_4} \wedge \mathtt{F_5}} \wedge_R \quad \rightarrow \quad \text{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash \mathtt{F_4}, \mathtt{F_5}, \Delta_6, \mathtt{p_2}}{\bullet \mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash (\Delta_6, \mathtt{p_2}), \mathtt{F_4} \vee \mathtt{F_5}} \vee_R \quad \rightarrow \quad \text{trivial}$$

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- Case rule $\perp_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \Delta_4, \mathtt{p}_2}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \perp, \Delta_4, \mathtt{p}_2} \; \perp_R \qquad \to \qquad \text{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \top, \Delta_4, \mathtt{p}_2} \; \top_R \qquad \to \qquad \text{trivial}$$

- Case rule K

$$\frac{\mathtt{h}_2 : unbox(\Box\Gamma_3) \vdash \mathtt{F}_4}{\bullet\mathtt{h}_2 : \Box\Gamma_3, \Delta_6, \mathtt{p}_1 \vdash (\Delta_5, \mathtt{p}_1), []\mathtt{F}_4} \; K \qquad \to \qquad \text{trivial}$$

- Case rule $A45$

$$\frac{\mathtt{h}_2 : \Box\Gamma_3 \vdash \Box\Gamma_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Box\Gamma_3, \Delta_7, \mathtt{p}_1 \vdash \Box\Gamma_4, (\Delta_6, \mathtt{p}_1), []\mathtt{F}_5} \; A45 \qquad \to \qquad \text{trivial}$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{p}_1 \vdash \mathtt{F}_4, \Delta_2, \mathtt{p}_1 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_6, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1} \; \to_L \qquad \to \qquad \text{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_6, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1} \; \wedge_L \qquad \to \qquad \text{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_6, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1} \; \vee_L \qquad \to \qquad \text{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_5, \mathtt{p}_1, []\mathtt{F}_4 \vdash \Delta_2, \mathtt{p}_1}{\bullet\mathtt{h}_3 : (\Delta_5, \mathtt{p}_1), []\mathtt{F}_4 \vdash \Delta_2, \mathtt{p}_1} \; AT \qquad \to \qquad \text{trivial}$$

- Case rule $\perp_L$

$$\frac{}{\bullet\mathtt{h}_3 : \perp, \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1} \; \perp_L \qquad \to \qquad \text{trivial}$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_2 : \mathtt{p}_3, \Delta_5, \mathtt{p}_1 \vdash \mathtt{p}_3, \Delta_4, \mathtt{p}_1} \; I \qquad \to \qquad \text{trivial}$$

$$\frac{}{\bullet\mathtt{h}_1 : \mathtt{p}_3, \Delta_2 \vdash \mathtt{p}_3, \Delta_4} \; I \qquad \to \qquad \text{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h_3} : \Delta_4, \mathtt{p_1} \vdash \Delta_2, \mathtt{p_1}}{\bullet \mathtt{h_3} : \top, \Delta_4, \mathtt{p_1} \vdash \Delta_2, \mathtt{p_1}} \ \top_L \qquad \rightarrow \qquad \mathtt{trivial}$$

4.17 Status of $\top_L$: : Invertible

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h_2} : \top, \mathtt{F_4}, \Delta_1 \vdash \mathtt{F_5}, \Delta_3}{\bullet \mathtt{h_2} : \top, \Delta_1 \vdash \Delta_3, \mathtt{F_4} \rightarrow \mathtt{F_5}} \ \rightarrow_R \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_2} : \Delta_1, \mathtt{F_4} \vdash \Delta_3, \mathtt{F_5}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_2} : \Delta_1 \vdash \Delta_3, \mathtt{F_4} \rightarrow \mathtt{F_5}} \ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h_2} : \top, \Delta_1 \vdash \mathtt{F_4}, \Delta_3 \quad \mathtt{h_2} : \top, \Delta_1 \vdash \mathtt{F_5}, \Delta_3}{\bullet \mathtt{h_2} : \top, \Delta_1 \vdash \Delta_3, \mathtt{F_4} \wedge \mathtt{F_5}} \ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} : \Delta_1 \vdash \Delta_3, \mathtt{F_4}} \ \mathtt{ax/ind} \quad \overline{\mathtt{h_2} : \Delta_1 \vdash \Delta_3, \mathtt{F_5}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_2} : \Delta_1 \vdash \Delta_3, \mathtt{F_4} \wedge \mathtt{F_5}} \ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h_2} : \top, \Delta_1 \vdash \mathtt{F_4}, \mathtt{F_5}, \Delta_3}{\bullet \mathtt{h_2} : \top, \Delta_1 \vdash \Delta_3, \mathtt{F_4} \vee \mathtt{F_5}} \ \vee_R \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_2} : \Delta_1 \vdash \Delta_3, \mathtt{F_4}, \mathtt{F_5}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_2} : \Delta_1 \vdash \Delta_3, \mathtt{F_4} \vee \mathtt{F_5}} \ \vee_R$$

- Case rule $\perp_R$

$$\frac{\mathtt{h_2} : \top, \Delta_1 \vdash \Delta_3}{\bullet \mathtt{h_2} : \top, \Delta_1 \vdash \perp, \Delta_3} \ \perp_R \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_2} : \Delta_1 \vdash \Delta_3} \ \mathtt{ax/ind}}{\bullet \mathtt{h_2} : \Delta_1 \vdash \perp, \Delta_3} \ \perp_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h_2} : \top, \Delta_1 \vdash \top, \Delta_3} \ \top_R \qquad \rightarrow \qquad \frac{}{\bullet \mathtt{h_2} : \Delta_1 \vdash \top, \Delta_3} \ \top_R$$

- Case rule K

$$\frac{\mathtt{h_1} : unbox(\Box\Gamma_2) \vdash \mathtt{F_4}}{\bullet \mathtt{h_1} : \Box\Gamma_2, \top, \Delta_5 \vdash \Delta_3, []\mathtt{F_4}} \ K \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_1} : unbox(\Box\Gamma_2) \vdash \mathtt{F_4}} \ \mathtt{ax}}{\bullet \mathtt{h_1} : \Delta_5, \Box\Gamma_2 \vdash \Delta_3, []\mathtt{F_4}} \ K$$

- Case rule $A45$

$$\frac{\mathtt{h_1} : \Box\Gamma_2 \vdash \Box\Gamma_3, \mathtt{F_5}}{\bullet \mathtt{h_1} : \Box\Gamma_2, \top, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F_5}} \ A45 \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_1} : \Box\Gamma_2 \vdash \mathtt{F_5}, \Box\Gamma_3} \ \mathtt{ax}}{\bullet \mathtt{h_1} : \Delta_6, \Box\Gamma_2 \vdash \Delta_4, \Box\Gamma_3, []\mathtt{F_5}} \ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h_1} : \top, \Delta_5 \vdash \mathtt{F_2}, \Delta_4 \quad \mathtt{h_1} : \top, \mathtt{F_3}, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h_1} : (\top, \Delta_5), \mathtt{F_2} \rightarrow \mathtt{F_3} \vdash \Delta_4} \ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h_1} : \Delta_5 \vdash \Delta_4, \mathtt{F_2}} \ \mathtt{ax/ind} \quad \overline{\mathtt{h_1} : \Delta_5, \mathtt{F_3} \vdash \Delta_4} \ \mathtt{ax/ind}}{\bullet \mathtt{h_1} : \Delta_5, \mathtt{F_2} \rightarrow \mathtt{F_3} \vdash \Delta_4} \ \rightarrow_L$$

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- Case rule $\wedge_L$

$$\frac{h_1 : \top, F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \ \wedge_L \qquad \rightarrow \qquad \frac{\overline{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} \ ^{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \ \wedge_L$$

- Case rule $\vee_L$

$$\frac{h_1 : \top, F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : \top, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \ \vee_L \qquad \rightarrow \qquad \frac{\overline{h_1 : \Delta_5, F_2 \vdash \Delta_4} \ ^{ax/ind} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4} \ ^{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \ \vee_L$$

- Case rule AT

$$\frac{h_1 : \top, F_2, \Delta_4, []F_2 \vdash \Delta_3}{\bullet h_1 : (\top, \Delta_4), []F_2 \vdash \Delta_3} \ AT \qquad \rightarrow \qquad \frac{\overline{h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3} \ ^{ax/ind}}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3} \ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet h_1 : \bot, \top, \Delta_3 \vdash \Delta_2} \ \bot_L \qquad \rightarrow \qquad \frac{}{\bullet h_1 : \bot, \Delta_3 \vdash \Delta_2} \ \bot_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_2, \top, \Delta_4 \vdash p_2, \Delta_3} \ I \qquad \rightarrow \qquad \frac{}{\bullet h_1 : \Delta_4, p_2 \vdash \Delta_3, p_2} \ I$$

- Case rule $\top_L$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \ \top_L \qquad \rightarrow \qquad \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3} \ ^{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \ H$$

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5 Identity-Expansion

$$\frac{\dfrac{}{-\,:\,\mathsf{F}_0 \vdash \mathsf{F}_0}\ \texttt{IH}}{-\,:\,[]\mathsf{F}_0 \vdash []\mathsf{F}_0}\ K$$

$$\frac{\dfrac{\dfrac{}{-\,:\,\mathsf{F}_0 \vdash \mathsf{F}_0}\ \texttt{IH}}{-\,:\,\mathsf{F}_0 \vdash \mathsf{F}_0, \mathsf{F}_1}\ W \qquad \dfrac{\dfrac{}{-\,:\,\mathsf{F}_1 \vdash \mathsf{F}_1}\ \texttt{IH}}{-\,:\,\mathsf{F}_1 \vdash \mathsf{F}_0, \mathsf{F}_1}\ W}{\dfrac{-\,:\,\mathsf{F}_0 \vee \mathsf{F}_1 \vdash \mathsf{F}_0, \mathsf{F}_1}{-\,:\,\mathsf{F}_0 \vee \mathsf{F}_1 \vdash \mathsf{F}_0 \vee \mathsf{F}_1}\ \vee_R}\ \vee_L$$

$$\frac{\dfrac{\dfrac{}{-\,:\,\mathsf{F}_0 \vdash \mathsf{F}_0}\ \texttt{IH}}{-\,:\,\mathsf{F}_0, \mathsf{F}_1 \vdash \mathsf{F}_0}\ W \qquad \dfrac{\dfrac{}{-\,:\,\mathsf{F}_1 \vdash \mathsf{F}_1}\ \texttt{IH}}{-\,:\,\mathsf{F}_0, \mathsf{F}_1 \vdash \mathsf{F}_1}\ W}{\dfrac{-\,:\,\mathsf{F}_0, \mathsf{F}_1 \vdash \mathsf{F}_0 \wedge \mathsf{F}_1}{-\,:\,\mathsf{F}_0 \wedge \mathsf{F}_1 \vdash \mathsf{F}_0 \wedge \mathsf{F}_1}\ \wedge_L}\ \wedge_R$$

$$\frac{\dfrac{\dfrac{}{-\,:\,\mathsf{F}_0 \vdash \mathsf{F}_0}\ \texttt{IH}}{-\,:\,\mathsf{F}_0 \vdash \mathsf{F}_0, \mathsf{F}_1}\ W \qquad \dfrac{\dfrac{}{-\,:\,\mathsf{F}_1 \vdash \mathsf{F}_1}\ \texttt{IH}}{-\,:\,\mathsf{F}_0, \mathsf{F}_1 \vdash \mathsf{F}_1}\ W}{\dfrac{-\,:\,\mathsf{F}_0, \mathsf{F}_0 \rightarrow \mathsf{F}_1 \vdash \mathsf{F}_1}{-\,:\,\mathsf{F}_0 \rightarrow \mathsf{F}_1 \vdash \mathsf{F}_0 \rightarrow \mathsf{F}_1}\ \rightarrow_R}\ \rightarrow_L$$

$$\frac{}{-\,:\,\top \vdash \top}\ \top_R$$

$$\frac{}{-\,:\,\bot \vdash \bot}\ \bot_L$$

6 Cut-Elimination

6.1 Status of $\to_R$: OK

- Case rule $\to_R$

$$
\cfrac{
\cfrac{h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \to F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7 \to F_8} \to_R
\quad
\cfrac{h_9 : F_{11}, \Delta_6, F_7 \to F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R
}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}} \text{Cut}
$$
$$\to$$
$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{h_1 : \Delta_6, F_{11}, F_7 \vdash \Delta_{10}, F_{12}, F_8} \text{inv-th/ax}}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \to F_8} \to_R
\quad
\cfrac{}{h_9 : \Delta_6, F_{11}, F_7 \to F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W}
}{
\cfrac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R
} \text{hCut}
$$

$$
\cfrac{
\cfrac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \to F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \to F_{13}), F_9 \to F_{10}), F_7} \to_R
\quad
\cfrac{h_{11} : F_7, F_{12}, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \to F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_9 \to F_{10}} \to_R
}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_9 \to F_{10}} \text{Cut}
$$
$$\to$$
$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{h_2 : \Delta_8, F_{12}, F_9 \vdash \Delta_{14}, F_{10}, F_{13}, F_7} \text{inv-th/ax}}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9 \to F_{10}} \to_R
\quad
\cfrac{}{h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \to F_{10}} \text{ax/W}
}{
\cfrac{- : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \to F_{10}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \to F_{13}, F_9 \to F_{10}} \to_R
} \text{hCut}
$$

$$
\cfrac{
\cfrac{h_2 : F_{11}, \Delta_8 \vdash F_7, F_{12}, \Delta_{10}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7} \to_R
\quad
\cfrac{h_9 : F_7, F_{11}, \Delta_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R
}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \to F_{12}} \text{Cut}
$$
$$\to$$
$$
\cfrac{
\cfrac{}{h_2 : \Delta_8, F_{11} \vdash \Delta_{10}, F_{12}, F_7} \text{ax/W}
\quad
\cfrac{
\cfrac{}{h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{10}, F_{12}} \text{ax/W}
}{\bullet h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{10}, F_{12}} \text{H}
}{
\cfrac{- : \Delta_8, F_{11} \vdash \Delta_{10}, F_{12}}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R
} \text{hCut}
$$

- Case rule $\wedge_R$

$$
\cfrac{
\cfrac{h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \to F_8} \to_R
\quad
\cfrac{h_9 : \Delta_6, F_7 \to F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Delta_6, F_7 \to F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R
}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut}
$$
$$\to$$
$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_8}\text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \to F_8}\to_R \quad \cfrac{}{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11}}\text{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, F_{11}}\text{hCut}
\quad
\cfrac{
\cfrac{\cfrac{}{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{12}, F_8}\text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7 \to F_8}\to_R \quad \cfrac{}{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{12}}\text{ax/}}{- : \Delta_6 \vdash \Delta_{10}, F_{12}}\text{hCu}
}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R
$$

$$
\cfrac{
\cfrac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \to F_{10}), F_7} \to_R
\quad
\cfrac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \to F_{10} \quad h_{11} : F_7, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \to F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \to F_{10}} \wedge_R
}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \to F_{10}} \text{Cut}
$$
$$\to$$
$$
\cfrac{
\cfrac{}{h_2 : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_7, F_{12} \wedge F_{13}}\text{ax/W}
\quad
\cfrac{
\cfrac{\cfrac{}{h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12}}\text{inv-th/ax} \quad \cfrac{}{h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{13}}\text{inv-th/ax}}{\bullet h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \wedge F_{13}}\wedge_R
}{
\cfrac{- : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \wedge F_{13}}{- : \Delta_8 \vdash \Delta_{14}, F_9 \to F_{10}, F_{12} \wedge F_{13}}\to_R
} \text{hCut}
$$

- Case rule $\vee_R$

$$\cfrac{\cfrac{\mathtt{h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \vee F_{12}}}{\mathtt{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \to F_8}} \to_R \quad \cfrac{\mathtt{h_9 : \Delta_6, F_7 \to F_8 \vdash F_{11}, F_{12}, \Delta_{10}}}{\mathtt{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}}} \vee_R}{\mathtt{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}}} \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}}{\mathtt{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \to F_8}} \to_R \quad \cfrac{}{\mathtt{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11}, F_{12}}} \text{ax/W}}{\cfrac{\mathtt{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}}}{\mathtt{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}}} \vee_R} \text{inv-th/ax}}{} \text{hCut}$$

$$\cfrac{\cfrac{\mathtt{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}}{\mathtt{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \to F_{10}), F_7}} \to_R \quad \cfrac{\mathtt{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \to F_{10}}}{\mathtt{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \to F_{10}}} \vee_R}{\mathtt{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \to F_{10}}} \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\mathtt{h_2 : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_7, F_{12} \vee F_{13}}} \text{ax/W} \quad \cfrac{\cfrac{\mathtt{h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}}}{\mathtt{\bullet h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \vee F_{13}}} \vee_R}{} \text{inv-th/ax}}{\cfrac{\cfrac{\mathtt{- : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \vee F_{13}}}{\mathtt{- : \Delta_8 \vdash \Delta_{14}, F_9 \to F_{10}, F_{12} \vee F_{13}}} \to_R} \text{hCut}}{}$$

- Case rule $\perp_R$

$$\cfrac{\cfrac{\mathtt{h_1 : F_7, \Delta_6 \vdash F_8, \perp, \Delta_{10}}}{\mathtt{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \to F_8}} \to_R \quad \cfrac{\mathtt{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}}}{\mathtt{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \perp, \Delta_{10}}} \perp_R}{\mathtt{- : \Delta_6 \vdash \perp, \Delta_{10}}} \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{\mathtt{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \to F_8}} \text{ax/W} \quad \cfrac{}{\mathtt{h_9 : \Delta_6, F_7 \to F_8 \vdash \perp, \Delta_{10}}} \text{ax/W}}{\mathtt{- : \Delta_6 \vdash \perp, \Delta_{10}}} \text{hCut}$$

$$\cfrac{\cfrac{\mathtt{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \perp, \Delta_{12}}}{\mathtt{\bullet h_2 : \Delta_8 \vdash ((\perp, \Delta_{12}), F_9 \to F_{10}), F_7}} \to_R \quad \cfrac{\mathtt{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \to F_{10}}}{\mathtt{\bullet h_{11} : \Delta_8, F_7 \vdash (\perp, \Delta_{12}), F_9 \to F_{10}}} \perp_R}{\mathtt{- : \Delta_8 \vdash (\perp, \Delta_{12}), F_9 \to F_{10}}} \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{\mathtt{\bullet h_2 : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \to F_{10}}} \text{ax/W} \quad \cfrac{}{\mathtt{h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \to F_{10}}} \text{ax/W}}{\mathtt{- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \to F_{10}}} \text{hCut}$$

- Case rule $\top_R$

$$\cfrac{\cfrac{\mathtt{h_1 : F_7, \Delta_6 \vdash F_8, \top, \Delta_{10}}}{\mathtt{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \to F_8}} \to_R \quad \cfrac{}{\mathtt{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \top, \Delta_{10}}} \top_R}{\mathtt{- : \Delta_6 \vdash \top, \Delta_{10}}} \text{Cut}$$

$$\to$$

$$\cfrac{}{\mathtt{- : \Delta_6 \vdash \top, \Delta_{10}}} \top_R$$

$$\cfrac{\cfrac{\mathtt{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \top, \Delta_{12}}}{\mathtt{\bullet h_2 : \Delta_8 \vdash ((\top, \Delta_{12}), F_9 \to F_{10}), F_7}} \to_R \quad \cfrac{}{\mathtt{\bullet h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \to F_{10}}} \top_R}{\mathtt{- : \Delta_8 \vdash (\top, \Delta_{12}), F_9 \to F_{10}}} \text{Cut}$$

$$\to$$

$$\cfrac{}{\mathtt{- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \to F_{10}}} \top_R$$

- Case rule K

$$\cfrac{\cfrac{\mathtt{h_1 : F_6, \Box\Gamma_9, \Delta_{12} \vdash F_7, \Delta_{10}, []F_{11}}}{\mathtt{\bullet h_1 : \Box\Gamma_9, \Delta_{12} \vdash (\Delta_{10}, []F_{11}), F_6 \to F_7}} \to_R \quad \cfrac{\mathtt{h_8 : \mathit{unbox}(\Box\Gamma_9) \vdash F_{11}}}{\mathtt{\bullet h_8 : (\Box\Gamma_9, \Delta_{12}), F_6 \to F_7 \vdash \Delta_{10}, []F_{11}}} K}{\mathtt{- : \Box\Gamma_9, \Delta_{12} \vdash \Delta_{10}, []F_{11}}} \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{\mathtt{- : \mathit{unbox}(\Box\Gamma_9) \vdash F_{11}}} \text{ax/W}}{\mathtt{- : \Delta_{12}, \Box\Gamma_9 \vdash \Delta_{10}, []F_{11}}} K$$

First derivation block:

$$\cfrac{\cfrac{h_2 : F_8, \Box\Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_9, \Delta_{13}, []F_{12}}{\bullet h_2 : \Box\Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, []F_{12}), F_8 \to F_9), \Box F_7} \to_R \quad \cfrac{h_{10} : unbox(\Box\Gamma_{14}), unbox(\Box F_7) \vdash F_{12}}{\bullet h_{10} : (\Box\Gamma_{14}, \Delta_{11}), \Box F_7 \vdash (\Delta_{13}, []F_{12}), F_8 \to F_9} K}{- : \Box\Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, []F_{12}), F_8 \to F_9} Cut$$

$$\to$$

$$\cfrac{\cfrac{h_2 : \Delta_{11}, F_8, \Box\Gamma_{14} \vdash \Box F_7, \Delta_{13}, F_9, []F_{12}}{} ax/W \quad \cfrac{\cfrac{h_{10} : unbox(\Box F_7), unbox(\Box\Gamma_{14}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, F_8, \Box\Gamma_{14} \vdash \Delta_{13}, F_9, []F_{12}} K}{} ax/W}{\cfrac{- : \Delta_{11}, F_8, \Box\Gamma_{14} \vdash \Delta_{13}, F_9, []F_{12}}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, []F_{12}, F_8 \to F_9} \to_R} hCut$$

$$\cfrac{\cfrac{h_2 : F_8, \Box\Gamma_{11}, \Delta_{14} \vdash F_7, F_9, \Delta_{13}, []F_{12}}{\bullet h_2 : \Box\Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, []F_{12}), F_8 \to F_9), F_7} \to_R \quad \cfrac{h_{10} : unbox(\Box\Gamma_{11}) \vdash F_{12}}{\bullet h_{10} : (\Box\Gamma_{11}, \Delta_{14}), F_7 \vdash (\Delta_{13}, []F_{12}), F_8 \to F_9} K}{- : \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_{13}, []F_{12}), F_8 \to F_9} Cut$$

$$\to$$

$$\cfrac{\cfrac{- : unbox(\Box\Gamma_{11}) \vdash F_{12}}{} ax/W}{- : \Delta_{14}, \Box\Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \to F_9} K$$

- Case rule $A45$

$$\cfrac{\cfrac{h_1 : F_6, \Box\Gamma_9, \Delta_{13} \vdash F_7, \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_1 : \Box\Gamma_9, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_6 \to F_7} \to_R \quad \cfrac{h_8 : \Box\Gamma_9 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_8 : (\Box\Gamma_9, \Delta_{13}), F_6 \to F_7 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} A45}{- : \Box\Gamma_9, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} Cut$$

$$\to$$

$$\cfrac{\cfrac{- : \Box\Gamma_9 \vdash F_{12}, \Box\Gamma_{10}}{} ax/W}{- : \Delta_{13}, \Box\Gamma_9 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} A45$$

$$\cfrac{\cfrac{h_2 : F_8, \Box\Gamma_{15}, \Delta_{11} \vdash \Box F_7, F_9, \Box\Gamma_{12}, \Delta_{14}, []F_{13}}{\bullet h_2 : \Box\Gamma_{15}, \Delta_{11} \vdash ((\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \to F_9), \Box F_7} \to_R \quad \cfrac{h_{10} : \Box\Gamma_{15}, \Box F_7 \vdash \Box\Gamma_{12}, F_{13}}{\bullet h_{10} : (\Box\Gamma_{15}, \Delta_{11}), \Box F_7 \vdash (\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \to F_9} A45}{- : \Box\Gamma_{15}, \Delta_{11} \vdash (\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \to F_9} Cut$$

$$\to$$

$$\cfrac{\cfrac{h_2 : \Delta_{11}, F_8, \Box\Gamma_{15} \vdash \Box F_7, \Delta_{14}, F_9, \Box\Gamma_{12}, []F_{13}}{} ax/W \quad \cfrac{\cfrac{h_{10} : \Box F_7, \Box\Gamma_{15} \vdash F_{13}, \Box\Gamma_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, F_8, \Box\Gamma_{15} \vdash \Delta_{14}, F_9, \Box\Gamma_{12}, []F_{13}} A45}{} ax/W}{\cfrac{- : \Delta_{11}, F_8, \Box\Gamma_{15} \vdash \Delta_{14}, F_9, \Box\Gamma_{12}, []F_{13}}{- : \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, \Box\Gamma_{12}, []F_{13}, F_8 \to F_9} \to_R} hCut$$

$$\cfrac{\cfrac{h_2 : F_8, \Box\Gamma_{11}, \Delta_{15} \vdash F_7, F_9, \Box\Gamma_{12}, \Delta_{14}, []F_{13}}{\bullet h_2 : \Box\Gamma_{11}, \Delta_{15} \vdash ((\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \to F_9), F_7} \to_R \quad \cfrac{h_{10} : \Box\Gamma_{11} \vdash \Box\Gamma_{12}, F_{13}}{\bullet h_{10} : (\Box\Gamma_{11}, \Delta_{15}), F_7 \vdash (\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \to F_9} A45}{- : \Box\Gamma_{11}, \Delta_{15} \vdash (\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \to F_9} Cut$$

$$\to$$

$$\cfrac{\cfrac{- : \Box\Gamma_{11} \vdash F_{13}, \Box\Gamma_{12}}{} ax/W}{- : \Delta_{15}, \Box\Gamma_{11} \vdash \Delta_{14}, \Box\Gamma_{12}, []F_{13}, F_8 \to F_9} A45$$

- Case rule $\to_L$

$$\cfrac{\cfrac{h_1 : F_6, \Delta_{12}, F_9 \to F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6 \to F_7} \to_R \quad \cfrac{h_8 : \Delta_{12}, F_6 \to F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \to F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash \Delta_{11}} \to_L}{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} Cut$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \to F_7} \to_R \quad \cfrac{h_8 : \Delta_{12}, F_6 \to F_7 \vdash \Delta_{11}, F_9}{} ax/W}{- : \Delta_{12} \vdash \Delta_{11}, F_9} hCut \quad \cfrac{\cfrac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \to F_7} \to_R \quad \cfrac{h_8 : \Delta_{12}, F_{10}, F_6 \to F_7 \vdash \Delta_{11}}{} a\,h}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}}}{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \to_L$$

with inv-th/ax annotations.

$$\cfrac{\cfrac{h_1 : F_8, \Delta_7 \vdash F_9, \Delta_{10}}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \to F_9} \to_R \quad \cfrac{h_6 : \Delta_7 \vdash F_8, \Delta_{10} \quad h_6 : F_9, \Delta_7 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash \Delta_{10}} \to_L}{- : \Delta_7 \vdash \Delta_{10}} Cut$$

$$\to$$

$$\cfrac{\cfrac{- : \Delta_7 \vdash \Delta_{10}, F_8}{} ax/W \quad \cfrac{\cfrac{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9}{} ax/W \quad \cfrac{- : \Delta_7, F_8, F_9 \vdash \Delta_{10}}{} ax/W}{- : \Delta_7, F_8 \vdash \Delta_{10}} sCut}{- : \Delta_7 \vdash \Delta_{10}} sCut$$

$$\frac{\dfrac{h_2 : F_8, \Delta_{11} \vdash F_{12} \to F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), F_{12} \to F_{13}} \to_R \qquad \dfrac{h_{10} : \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \to F_9 \qquad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \to F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \to F_{13} \vdash \Delta_7, F_8 \to F_9} \to_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{}{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \to F_{13}} \text{ax/W} \qquad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_8 \vdash \Delta_7, F_{12}, F_9}{} \text{inv-th/ax} \qquad \dfrac{h_{10} : \Delta_{11}, F_{13}, F_8 \vdash \Delta_7, F_9}{} \text{inv-th/ax}}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \to F_{13} \vdash \Delta_7, F_9} \to_L}{\dfrac{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \to_R} \text{hCut}$$

$$\frac{\dfrac{h_2 : F_9, \Delta_{14}, F_{12} \to F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \to F_{13} \vdash (\Delta_8, F_9 \to F_{10}), F_7} \to_R \qquad \dfrac{h_{11} : F_7, \Delta_{14} \vdash F_{12}, \Delta_8, F_9 \to F_{10} \qquad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \to F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \to F_{13}), F_7 \vdash \Delta_8, F_9 \to F_{10}} \to_L}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_9 \to F_{10}} \text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{}{h_2 : \Delta_{14}, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}, F_7} \text{ax/W} \qquad \dfrac{\dfrac{h_{11} : \Delta_{14}, F_7, F_9 \vdash \Delta_8, F_{10}, F_{12}}{} \text{inv-th/ax} \qquad \dfrac{h_{11} : \Delta_{14}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}{} \text{inv-th/ax}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}} \to_L}{\dfrac{- : \Delta_{14}, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_9 \to F_{10}} \to_R} \text{hCut}$$

- Case rule $\wedge_L$

$$\frac{\dfrac{h_1 : F_6, \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \to F_7} \to_R \qquad \dfrac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \to F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \to F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{h_1 : \Delta_{12}, F_{10}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \to F_7} \to_R \text{inv-th/ax} \qquad \dfrac{}{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \to F_7 \vdash \Delta_{11}} \text{ax/W}}{\dfrac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L} \text{hCut}$$

$$\frac{\dfrac{h_2 : F_8, \Delta_{11} \vdash F_{12} \wedge F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), F_{12} \wedge F_{13}} \to_R \qquad \dfrac{h_{10} : F_{12}, F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \to F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \to F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{}{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \wedge F_{13}} \text{ax/W} \qquad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12}, F_{13}, F_8 \vdash \Delta_7, F_9}{} \text{inv-th/ax}}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \wedge F_{13} \vdash \Delta_7, F_9} \wedge_L}{\dfrac{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \to_R} \text{hCut}$$

$$\frac{\dfrac{h_2 : F_9, \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \to F_{10}), F_7} \to_R \qquad \dfrac{h_{11} : F_7, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \to F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \to F_{10}} \wedge_L}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \to F_{10}} \text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{}{h_2 : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7} \text{ax/W} \qquad \dfrac{\dfrac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}{} \text{inv-th/ax}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}} \wedge_L}{\dfrac{- : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \to F_{10}} \to_R} \text{hCut}$$

- Case rule $\vee_L$

$$\frac{\dfrac{h_1 : F_6, \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \to F_7} \to_R \qquad \dfrac{h_8 : F_9, \Delta_{12}, F_6 \to F_7 \vdash \Delta_{11} \qquad h_8 : F_{10}, \Delta_{12}, F_6 \to F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \to F_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \to F_7} \to_R \text{inv-th/ax} \quad \dfrac{}{h_8 : \Delta_{12}, F_9, F_6 \to F_7 \vdash \Delta_{11}} \text{ax/W}}{\dfrac{- : \Delta_{12}, F_9 \vdash \Delta_{11}}{} \text{hCut}} \quad \dfrac{\dfrac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \to F_7} \to_R \text{inv-th/ax} \quad \dfrac{}{h_8 : \Delta_{12}, F_{10}, F_6 \to F_7 \vdash \Delta_{11}} \text{a}}{\dfrac{- : \Delta_{12}, F_{10} \vdash \Delta_{11}}{} \text{h}}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L$$

$$\dfrac{\dfrac{\texttt{h}_2 : \texttt{F}_8, \Delta_{11} \vdash \texttt{F}_{12} \vee \texttt{F}_{13}, \texttt{F}_9, \Delta_7}{\bullet\texttt{h}_2 : \Delta_{11} \vdash (\Delta_7, \texttt{F}_8 \to \texttt{F}_9), \texttt{F}_{12} \vee \texttt{F}_{13}} \to_R \qquad \dfrac{\texttt{h}_{10} : \texttt{F}_{12}, \Delta_{11} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9 \qquad \texttt{h}_{10} : \texttt{F}_{13}, \Delta_{11} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9}{\bullet\texttt{h}_{10} : \Delta_{11}, \texttt{F}_{12} \vee \texttt{F}_{13} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9} \vee_L}{- : \Delta_{11} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\texttt{h}_2 : \Delta_{11}, \texttt{F}_8 \vdash \Delta_7, \texttt{F}_9, \texttt{F}_{12} \vee \texttt{F}_{13}}{} \ \texttt{ax/W} \qquad \dfrac{\dfrac{\texttt{h}_{10} : \Delta_{11}, \texttt{F}_{12}, \texttt{F}_8 \vdash \Delta_7, \texttt{F}_9}{} \ \texttt{inv-th/ax} \qquad \dfrac{\texttt{h}_{10} : \Delta_{11}, \texttt{F}_{13}, \texttt{F}_8 \vdash \Delta_7, \texttt{F}_9}{} \ \texttt{inv-th/ax}}{\bullet\texttt{h}_{10} : \Delta_{11}, \texttt{F}_8, \texttt{F}_{12} \vee \texttt{F}_{13} \vdash \Delta_7, \texttt{F}_9} \vee_L}{- : \Delta_{11}, \texttt{F}_8 \vdash \Delta_7, \texttt{F}_9} \text{hCut}}{- : \Delta_{11} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9} \to_R$$

$$\dfrac{\dfrac{\texttt{h}_2 : \texttt{F}_9, \Delta_{14}, \texttt{F}_{12} \vee \texttt{F}_{13} \vdash \texttt{F}_7, \texttt{F}_{10}, \Delta_8}{\bullet\texttt{h}_2 : \Delta_{14}, \texttt{F}_{12} \vee \texttt{F}_{13} \vdash (\Delta_8, \texttt{F}_9 \to \texttt{F}_{10}), \texttt{F}_7} \to_R \qquad \dfrac{\texttt{h}_{11} : \texttt{F}_7, \texttt{F}_{12}, \Delta_{14} \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10} \qquad \texttt{h}_{11} : \texttt{F}_7, \texttt{F}_{13}, \Delta_{14} \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}}{\bullet\texttt{h}_{11} : (\Delta_{14}, \texttt{F}_{12} \vee \texttt{F}_{13}), \texttt{F}_7 \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}} \vee_L}{- : \Delta_{14}, \texttt{F}_{12} \vee \texttt{F}_{13} \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\texttt{h}_2 : \Delta_{14}, \texttt{F}_9, \texttt{F}_{12} \vee \texttt{F}_{13} \vdash \Delta_8, \texttt{F}_{10}, \texttt{F}_7}{} \ \texttt{ax/W} \qquad \dfrac{\dfrac{\texttt{h}_{11} : \Delta_{14}, \texttt{F}_{12}, \texttt{F}_7, \texttt{F}_9 \vdash \Delta_8, \texttt{F}_{10}}{} \ \texttt{inv-th/ax} \qquad \dfrac{\texttt{h}_{11} : \Delta_{14}, \texttt{F}_{13}, \texttt{F}_7, \texttt{F}_9 \vdash \Delta_8, \texttt{F}_{10}}{} \ \texttt{inv-th/ax}}{\bullet\texttt{h}_{11} : \Delta_{14}, \texttt{F}_7, \texttt{F}_9, \texttt{F}_{12} \vee \texttt{F}_{13} \vdash \Delta_8, \texttt{F}_{10}} \vee_L}{- : \Delta_{14}, \texttt{F}_9, \texttt{F}_{12} \vee \texttt{F}_{13} \vdash \Delta_8, \texttt{F}_{10}} \text{hCut}}{- : \Delta_{14}, \texttt{F}_{12} \vee \texttt{F}_{13} \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}} \to_R$$

- Case rule AT

$$\dfrac{\dfrac{\texttt{h}_1 : \texttt{F}_6, \Delta_{11}, []\texttt{F}_9 \vdash \texttt{F}_7, \Delta_{10}}{\bullet\texttt{h}_1 : \Delta_{11}, []\texttt{F}_9 \vdash \Delta_{10}, \texttt{F}_6 \to \texttt{F}_7} \to_R \qquad \dfrac{\texttt{h}_8 : \texttt{F}_9, \Delta_{11}, []\texttt{F}_9, \texttt{F}_6 \to \texttt{F}_7 \vdash \Delta_{10}}{\bullet\texttt{h}_8 : (\Delta_{11}, []\texttt{F}_9), \texttt{F}_6 \to \texttt{F}_7 \vdash \Delta_{10}} AT}{- : \Delta_{11}, []\texttt{F}_9 \vdash \Delta_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\bullet\texttt{h}_1 : \Delta_{11}, \texttt{F}_9, []\texttt{F}_9 \vdash \Delta_{10}, \texttt{F}_6 \to \texttt{F}_7}{} \ \texttt{ax/W} \qquad \dfrac{\texttt{h}_8 : \Delta_{11}, \texttt{F}_9, []\texttt{F}_9, \texttt{F}_6 \to \texttt{F}_7 \vdash \Delta_{10}}{} \ \texttt{ax/W}}{- : \Delta_{11}, \texttt{F}_9, []\texttt{F}_9 \vdash \Delta_{10}} \text{hCut}}{- : \Delta_{11}, []\texttt{F}_9 \vdash \Delta_{10}} AT$$

$$\dfrac{\dfrac{\texttt{h}_2 : \texttt{F}_8, \Delta_{11} \vdash []\texttt{F}_{12}, \texttt{F}_9, \Delta_7}{\bullet\texttt{h}_2 : \Delta_{11} \vdash (\Delta_7, \texttt{F}_8 \to \texttt{F}_9), []\texttt{F}_{12}} \to_R \qquad \dfrac{\texttt{h}_{10} : \texttt{F}_{12}, \Delta_{11}, []\texttt{F}_{12} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9}{\bullet\texttt{h}_{10} : \Delta_{11}, []\texttt{F}_{12} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9} AT}{- : \Delta_{11} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\texttt{h}_2 : \Delta_{11}, \texttt{F}_8 \vdash \Delta_7, \texttt{F}_9, []\texttt{F}_{12}}{} \ \texttt{ax/W} \qquad \dfrac{\dfrac{\texttt{h}_{10} : \Delta_{11}, \texttt{F}_{12}, \texttt{F}_8, []\texttt{F}_{12} \vdash \Delta_7, \texttt{F}_9}{} \ \texttt{inv-th/ax}}{\bullet\texttt{h}_{10} : \Delta_{11}, \texttt{F}_8, []\texttt{F}_{12} \vdash \Delta_7, \texttt{F}_9} AT}{- : \Delta_{11}, \texttt{F}_8 \vdash \Delta_7, \texttt{F}_9} \text{hCut}}{- : \Delta_{11} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9} \to_R$$

$$\dfrac{\dfrac{\texttt{h}_2 : \texttt{F}_9, \Delta_{13}, []\texttt{F}_{12} \vdash \texttt{F}_7, \texttt{F}_{10}, \Delta_8}{\bullet\texttt{h}_2 : \Delta_{13}, []\texttt{F}_{12} \vdash (\Delta_8, \texttt{F}_9 \to \texttt{F}_{10}), \texttt{F}_7} \to_R \qquad \dfrac{\texttt{h}_{11} : \texttt{F}_7, \texttt{F}_{12}, \Delta_{13}, []\texttt{F}_{12} \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}}{\bullet\texttt{h}_{11} : (\Delta_{13}, []\texttt{F}_{12}), \texttt{F}_7 \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}} AT}{- : \Delta_{13}, []\texttt{F}_{12} \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\bullet\texttt{h}_2 : \Delta_{13}, \texttt{F}_{12}, []\texttt{F}_{12} \vdash \Delta_8, \texttt{F}_7, \texttt{F}_9 \to \texttt{F}_{10}}{} \ \texttt{ax/W} \qquad \dfrac{\texttt{h}_{11} : \Delta_{13}, \texttt{F}_{12}, \texttt{F}_7, []\texttt{F}_{12} \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}}{} \ \texttt{ax/W}}{- : \Delta_{13}, \texttt{F}_{12}, []\texttt{F}_{12} \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}} \text{hCut}}{- : \Delta_{13}, []\texttt{F}_{12} \vdash \Delta_8, \texttt{F}_9 \to \texttt{F}_{10}} AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\texttt{h}_1 : \texttt{F}_6, \bot, \Delta_{10} \vdash \texttt{F}_7, \Delta_9}{\bullet\texttt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \texttt{F}_6 \to \texttt{F}_7} \to_R \qquad \dfrac{}{\bullet\texttt{h}_8 : (\bot, \Delta_{10}), \texttt{F}_6 \to \texttt{F}_7 \vdash \Delta_9} \bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9} \text{Cut}$$

$$\to$$

$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9} \bot_L$$

$$\dfrac{\dfrac{\texttt{h}_2 : \texttt{F}_8, \Delta_{11} \vdash \bot, \texttt{F}_9, \Delta_7}{\bullet\texttt{h}_2 : \Delta_{11} \vdash (\Delta_7, \texttt{F}_8 \to \texttt{F}_9), \bot} \to_R \qquad \dfrac{}{\bullet\texttt{h}_{10} : \Delta_{11}, \bot \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9} \bot_L}{- : \Delta_{11} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\texttt{h}_2 : \Delta_{11}, \texttt{F}_8 \vdash \bot, \Delta_7, \texttt{F}_9}{} \ \texttt{ax/W} \qquad \dfrac{}{\bullet\texttt{h}_{10} : \bot, \Delta_{11}, \texttt{F}_8 \vdash \Delta_7, \texttt{F}_9} \bot_L}{- : \Delta_{11}, \texttt{F}_8 \vdash \Delta_7, \texttt{F}_9} \text{hCut}}{- : \Delta_{11} \vdash \Delta_7, \texttt{F}_8 \to \texttt{F}_9} \to_R$$

$$\frac{\dfrac{\mathtt{h_2 : F_9, \bot, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}}{\bullet\mathtt{h_2 : \bot, \Delta_{12} \vdash (\Delta_8, F_9 \to F_{10}), F_7}} \;\to_R \qquad \dfrac{}{\bullet\mathtt{h_{11} : (\bot, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \to F_{10}}}\;\bot_L}{\dfrac{\dfrac{- : \bot, \Delta_{12} \vdash \Delta_8, F_9 \to F_{10}}{\to}}{- : \bot, \Delta_{12} \vdash \Delta_8, F_9 \to F_{10}}\;\bot_L}\;\mathtt{Cut}$$

- Case rule I

$$\frac{\dfrac{\mathtt{h_1 : F_6, \Delta_{11}, p_9 \vdash F_7, \Delta_{10}, p_9}}{\bullet\mathtt{h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \to F_7}}\;\to_R \qquad \dfrac{}{\bullet\mathtt{h_8 : (\Delta_{11}, p_9), F_6 \to F_7 \vdash \Delta_{10}, p_9}}\;I}{\dfrac{\dfrac{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9}{\to}}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9}\;I}\;\mathtt{Cut}$$

$$\frac{\dfrac{\mathtt{h_2 : F_7, \Delta_{10} \vdash p_{11}, F_8, \Delta_{12}, p_{11}}}{\bullet\mathtt{h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \to F_8), p_{11}}}\;\to_R \qquad \dfrac{}{\bullet\mathtt{h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \to F_8}}\;I}{\dfrac{\dfrac{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \to F_8}{\to}}{\dfrac{\dfrac{\mathtt{h_2 : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, p_{11}, p_{11}}}{}\;\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_9 : \Delta_{10}, F_7, p_{11} \vdash \Delta_{12}, F_8, p_{11}}}\;I}{\dfrac{- : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, p_{11}}{- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \to F_8}\;\to_R}\;\mathtt{hCut}}}\;\mathtt{Cut}$$

$$\frac{\dfrac{\mathtt{h_2 : F_8, \Delta_{13}, p_{11} \vdash F_7, F_9, \Delta_{12}, p_{11}}}{\bullet\mathtt{h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \to F_9), F_7}}\;\to_R \qquad \dfrac{}{\bullet\mathtt{h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \to F_9}}\;I}{\dfrac{\dfrac{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \to F_9}{\to}}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \to F_9}\;I}\;\mathtt{Cut}$$

- Case rule $\top_L$

$$\frac{\dfrac{\mathtt{h_1 : F_6, \top, \Delta_{10} \vdash F_7, \Delta_9}}{\bullet\mathtt{h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \to F_7}}\;\to_R \qquad \dfrac{\mathtt{h_8 : \Delta_{10}, F_6 \to F_7 \vdash \Delta_9}}{\bullet\mathtt{h_8 : (\top, \Delta_{10}), F_6 \to F_7 \vdash \Delta_9}}\;\top_L}{\dfrac{\dfrac{- : \top, \Delta_{10} \vdash \Delta_9}{\to}}{\dfrac{\dfrac{}{\bullet\mathtt{h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \to F_7}}\;\mathtt{ax/W} \quad \dfrac{}{\mathtt{h_8 : \top, \Delta_{10}, F_6 \to F_7 \vdash \Delta_9}}\;\mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\;\mathtt{hCut}}}\;\mathtt{Cut}$$

$$\frac{\dfrac{\mathtt{h_2 : F_8, \Delta_{11} \vdash \top, F_9, \Delta_7}}{\bullet\mathtt{h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), \top}}\;\to_R \qquad \dfrac{\mathtt{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \to F_9}}{\bullet\mathtt{h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \to F_9}}\;\top_L}{\dfrac{\dfrac{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9}{\to}}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9}\;\mathtt{ax/W}}\;\mathtt{Cut}$$

$$\frac{\dfrac{\mathtt{h_2 : F_9, \top, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}}{\bullet\mathtt{h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \to F_{10}), F_7}}\;\to_R \qquad \dfrac{\mathtt{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \to F_{10}}}{\bullet\mathtt{h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \to F_{10}}}\;\top_L}{\dfrac{\dfrac{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \to F_{10}}{\to}}{\dfrac{\dfrac{}{\bullet\mathtt{h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \to F_{10}}}\;\mathtt{ax/W} \quad \dfrac{}{\mathtt{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \to F_{10}}}\;\mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \to F_{10}}\;\mathtt{hCut}}}\;\mathtt{Cut}$$

6.2 Status of $\wedge_R$: OK

- Case rule $\to_R$

$$\frac{\dfrac{\mathtt{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \to F_{12}} \quad \mathtt{h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \to F_{12}}}{\bullet\mathtt{h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7 \wedge F_8}}\;\wedge_R \qquad \dfrac{\mathtt{h_9 : F_{11}, \Delta_6, F_7 \wedge F_8 \vdash F_{12}, \Delta_{10}}}{\bullet\mathtt{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \to F_{12}}}\;\to_R}{\dfrac{\dfrac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}}{\to}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7}}\;\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_8}}\;\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \wedge F_8}}\;\wedge_R \quad \dfrac{}{\mathtt{h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}}}\;\mathtt{ax/W}}{\dfrac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}}\;\to_R}\;\mathtt{hCut}}}\;\mathtt{Cut}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \to F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \to F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \to F_{13}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{h_{11} : F_7, F_{12}, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_9 \wedge F_{10}} \to_R}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_9 \wedge F_{10}} \text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\overline{h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9}}{} \text{inv-th/ax} \quad \dfrac{\overline{h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_7}}{} \text{inv-th/ax}}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9 \wedge F_{10}} \wedge_R \quad \dfrac{\overline{h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \wedge F_{10}}}{} \text{ax/W}}{\dfrac{- : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \wedge F_{10}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \to F_{13}, F_9 \wedge F_{10}} \to_R} \text{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \wedge F_{12} \quad h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \wedge F_8} \wedge_R \quad \dfrac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\overline{- : \Delta_6 \vdash \Delta_{10}, F_7, F_{11} \wedge F_{12}}}{- : \Delta_6 \vdash \Delta_{10}, F_7, F_{11} \wedge F_{12}} \text{ax/W} \quad \dfrac{\overline{- : \Delta_6, F_7 \vdash \Delta_{10}, F_8, F_{11} \wedge F_{12}}}{} \text{ax/W} \quad \dfrac{\dfrac{\overline{- : \Delta_6, F_7, F_8 \vdash \Delta_{10}, F_{11}}}{} \text{inv-th/ax} \quad \dfrac{\overline{- : \Delta_6, F_7, F_8 \vdash \Delta_{10}, F_{12}}}{} \text{inv-th/ax}}{- : \Delta_6, F_7, F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{\dfrac{- : \Delta_6, F_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{sCut}} \text{sCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \wedge F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta}{}}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\overline{h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_7, F_9}}{} \text{inv-th/ax} \quad \dfrac{\overline{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_7}}{} \text{inv-th/ax}}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_7, F_9 \wedge F_{10}} \wedge_R \quad \dfrac{\overline{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}}}{} \text{ax/W}}{\dfrac{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}}{} } \text{hCut} \quad \dfrac{\overline{h_2 : \Delta_8 \vdash \Delta_{14}, F_{13}, F_7, F_9}}{\bullet h_2 : \Delta_8 \vdash \Delta_1} \text{inv-}$$
$$- : \Delta_8 \vdash \Delta_{14}, F_{12} \wedge F_{13}, F_9 \wedge F_{10}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_{11}, \Delta_{10} \quad h_2 : \Delta_8 \vdash F_7, F_{12}, \Delta_{10}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7} \wedge_R \quad \dfrac{h_9 : F_7, \Delta_8 \vdash F_{11}, \Delta_{10} \quad h_9 : F_7, \Delta_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\overline{h_2 : \Delta_8 \vdash \Delta_{10}, F_{11}, F_7}}{} \text{ax/W} \quad \dfrac{\dfrac{\overline{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}} \text{ax/W}}{- : \Delta_8 \vdash \Delta_{10}, F_{11}} \text{H}}{\dfrac{- : \Delta_8 \vdash \Delta_{10}, F_{11}}{} } \text{hCut} \quad \dfrac{\overline{h_2 : \Delta_8 \vdash \Delta_{10}, F_{12}, F_7}}{} \text{ax/W} \quad \dfrac{\dfrac{\overline{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{12}}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{12}} \text{ax/W}}{- : \Delta_8 \vdash \Delta_{10}, F_{12}} \text{H}}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \vee F_{12} \quad h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \wedge F_8} \wedge_R \quad \dfrac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\overline{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7}}{} \text{inv-th/ax} \quad \dfrac{\overline{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}}{} \text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \wedge F_8} \wedge_R \quad \dfrac{\overline{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}, F_{12}}}{} \text{ax/W}}{\dfrac{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R} \text{hCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \vee F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}} \vee_R}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}} \text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\overline{h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9}}{} \text{inv-th/ax} \quad \dfrac{\overline{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_7}}{} \text{inv-th/ax}}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \wedge F_{10}} \wedge_R \quad \dfrac{\overline{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \wedge F_{10}}}{} \text{ax/W}}{\dfrac{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \wedge F_{10}}{- : \Delta_8 \vdash \Delta_{14}, F_9 \wedge F_{10}, F_{12} \vee F_{13}} \vee_R} \text{hCut}$$

- Case rule $\perp_R$

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$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, \bot, \Delta_{10} \quad h_1 : \Delta_6 \vdash F_8, \bot, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\bot, \Delta_{10}), F_7 \wedge F_8} \wedge_R \quad \dfrac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \bot, \Delta_{10}} \bot_R}{- : \Delta_6 \vdash \bot, \Delta_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_6 \vdash \bot, \Delta_{10}, F_7 \wedge F_8} \text{ax/W} \quad \dfrac{}{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \bot, \Delta_{10}} \text{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}} \text{hCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, \bot, \Delta_{12} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \bot, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\bot, \Delta_{12}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\bot, \Delta_{12}), F_9 \wedge F_{10}} \bot_R}{- : \Delta_8 \vdash (\bot, \Delta_{12}), F_9 \wedge F_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_8 \vdash \bot, \Delta_{12}, F_7, F_9 \wedge F_{10}} \text{ax/W} \quad \dfrac{}{h_{11} : \Delta_8, F_7 \vdash \bot, \Delta_{12}, F_9 \wedge F_{10}} \text{ax/W}}{- : \Delta_8 \vdash \bot, \Delta_{12}, F_9 \wedge F_{10}} \text{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, \top, \Delta_{10} \quad h_1 : \Delta_6 \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \wedge F_8} \wedge_R \quad \dfrac{}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \top, \Delta_{10}} \top_R}{- : \Delta_6 \vdash \top, \Delta_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}} \top_R$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, \top, \Delta_{12} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \top, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\top, \Delta_{12}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{}{\bullet h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \wedge F_{10}} \top_R}{- : \Delta_8 \vdash (\top, \Delta_{12}), F_9 \wedge F_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \wedge F_{10}} \top_R$$

- Case rule K

$$\dfrac{\dfrac{h_1 : \Box\Gamma_9, \Delta_{12} \vdash F_6, \Delta_{10}, []F_{11} \quad h_1 : \Box\Gamma_9, \Delta_{12} \vdash F_7, \Delta_{10}, []F_{11}}{\bullet h_1 : \Box\Gamma_9, \Delta_{12} \vdash (\Delta_{10}, []F_{11}), F_6 \wedge F_7} \wedge_R \quad \dfrac{h_8 : unbox(\Box\Gamma_9) \vdash F_{11}}{\bullet h_8 : (\Box\Gamma_9, \Delta_{12}), F_6 \wedge F_7 \vdash \Delta_{10}, []F_{11}} K}{- : \Box\Gamma_9, \Delta_{12} \vdash \Delta_{10}, []F_{11}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_9) \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Box\Gamma_9 \vdash \Delta_{10}, []F_{11}} K$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_8, \Delta_{13}, []F_{12} \quad h_2 : \Box\Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_9, \Delta_{13}, []F_{12}}{\bullet h_2 : \Box\Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, []F_{12}), F_8 \wedge F_9), \Box F_7} \wedge_R \quad \dfrac{h_{10} : unbox(\Box\Gamma_{14}), unbox(\Box F_7) \vdash F_{12}}{\bullet h_{10} : (\Box\Gamma_{14}, \Delta_{11}), \Box F_7 \vdash (\Delta_{13}, []F_{12}), F_8 \wedge F} }{- : \Box\Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, []F_{12}), F_8 \wedge F_9}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_{11}, \Box\Gamma_{14} \vdash \Box F_7, \Delta_{13}, F_8, []F_{12}} \text{ax/W} \quad \dfrac{\dfrac{}{h_{10} : unbox(\Box F_7), unbox(\Box\Gamma_{14}) \vdash F_{12}} \text{ax/W}}{\bullet h_{10} : \Box F_7, \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, F_8, []F_{12}} K}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, F_8, []F_{12}} \text{hCut} \quad \dfrac{\dfrac{}{h_2 : \Delta_{11}, \Box\Gamma_{14} \vdash \Box F_7, \Delta_{13}, F_9, []F_{12}} \text{ax/W} \quad \dfrac{\dfrac{}{h_{10} : unbox(\Box F}{\bullet h_{10} : \Box F_7, \Delta_{11}}}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, F_9, []F_{12}}}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, []F_{12}, F_8 \wedge F_9}$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_{11}, \Delta_{14} \vdash F_7, F_8, \Delta_{13}, []F_{12} \quad h_2 : \Box\Gamma_{11}, \Delta_{14} \vdash F_7, F_9, \Delta_{13}, []F_{12}}{\bullet h_2 : \Box\Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, []F_{12}), F_8 \wedge F_9), F_7} \wedge_R \quad \dfrac{h_{10} : unbox(\Box\Gamma_{11}) \vdash F_{12}}{\bullet h_{10} : (\Box\Gamma_{11}, \Delta_{14}), F_7 \vdash (\Delta_{13}, []F_{12}), F_8 \wedge F_9} K}{- : \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_{13}, []F_{12}), F_8 \wedge F_9} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash F_{12}} \text{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \wedge F_9} K$$

- Case rule $A45$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_9, \Delta_{13} \vdash F_6, \Box\Gamma_{10}, \Delta_{11}, []F_{12} \quad h_1 : \Box\Gamma_9, \Delta_{13} \vdash F_7, \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_1 : \Box\Gamma_9, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_6 \wedge F_7} \wedge_R \quad \dfrac{h_8 : \Box\Gamma_9 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_8 : (\Box\Gamma_9, \Delta_{13}), F_6 \wedge F_7 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} A45}{- : \Box\Gamma_9, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_9 \vdash F_{12}, \Box\Gamma_{10}} \text{ax/W}}{- : \Delta_{13}, \Box\Gamma_9 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} A45$$

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$$\cfrac{\cfrac{\cfrac{h_2 : \Box\Gamma_{15}, \Delta_{11} \vdash \Box F_7, F_8, \Box\Gamma_{12}, \Delta_{14}, []F_{13} \quad h_2 : \Box\Gamma_{15}, \Delta_{11} \vdash \Box F_7, F_9, \Box\Gamma_{12}, \Delta_{14}, []F_{13}}{\bullet h_2 : \Box\Gamma_{15}, \Delta_{11} \vdash ((\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \wedge F_9), \Box F_7} \wedge_R \quad \cfrac{h_{10} : \Box\Gamma_{15}, \Box F_7 \vdash}{\bullet h_{10} : (\Box\Gamma_{15}, \Delta_{11}), \Box F_7 \vdash (\Box}}{- : \Box\Gamma_{15}, \Delta_{11} \vdash (\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \wedge F_9}}{\rightarrow} A45$$

$$\cfrac{\cfrac{\cfrac{h_2 : \Delta_{11}, \Box\Gamma_{15} \vdash \Box F_7, \Delta_{14}, F_8, \Box\Gamma_{12}, []F_{13}}{- : \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, F_8, \Box\Gamma_{12}, []F_{13}} \text{ax/W} \quad \cfrac{\cfrac{h_{10} : \Box F_7, \Box\Gamma_{15} \vdash F_{13}, \Box\Gamma_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, F_8, \Box\Gamma_{12}, []F_{13}} \text{ax/W}}{- : \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, F_8, \Box\Gamma_{12}, []F_{13}} \text{hCut} \quad \cfrac{h_2 : \Delta_{11}, \Box\Gamma_{15} \vdash \Box F_7, \Delta_{14}, F_9, \Box\Gamma_{12}, []F_{13}}{- : \Delta_{11}, \Box\Gamma_{15} \vdash} \text{ax}}{- : \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, \Box\Gamma_{12}, []F_{13}, F_8 \wedge F_9}}{}$$

$$\cfrac{\cfrac{\cfrac{h_2 : \Box\Gamma_{11}, \Delta_{15} \vdash F_7, F_8, \Box\Gamma_{12}, \Delta_{14}, []F_{13} \quad h_2 : \Box\Gamma_{11}, \Delta_{15} \vdash F_7, F_9, \Box\Gamma_{12}, \Delta_{14}, []F_{13}}{\bullet h_2 : \Box\Gamma_{11}, \Delta_{15} \vdash ((\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \wedge F_9), F_7} \wedge_R \quad \cfrac{h_{10} : \Box\Gamma_{11} \vdash \Box\Gamma_{12}, F_{13}}{\bullet h_{10} : (\Box\Gamma_{11}, \Delta_{15}), F_7 \vdash (\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \wedge F_9} A45}{- : \Box\Gamma_{11}, \Delta_{15} \vdash (\Box\Gamma_{12}, \Delta_{14}, []F_{13}), F_8 \wedge F_9}}{\cfrac{\rightarrow}{\cfrac{\cfrac{}{- : \Box\Gamma_{11} \vdash F_{13}, \Box\Gamma_{12}} \text{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11} \vdash \Delta_{14}, \Box\Gamma_{12}, []F_{13}, F_8 \wedge F_9} A45}} \text{Cut}$$

- Case rule $\rightarrow_L$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \cfrac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \rightarrow_L}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut}}{\rightarrow}$$

$$\cfrac{\cfrac{\cfrac{}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6} \text{ax/W} \quad \cfrac{\cfrac{}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_7} \text{ax/W} \quad \cfrac{\cfrac{}{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9} \text{inv-th/ax} \quad \cfrac{}{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}} \text{inv-th/}}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut}$$

$$\cfrac{\cfrac{\cfrac{h_2 : \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \rightarrow F_{13}} \wedge_R \quad \cfrac{h_{10} : \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \wedge F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \wedge F_9} \rightarrow_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut}}{\rightarrow}$$

$$\cfrac{\cfrac{}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \wedge F_9} \text{ax/W} \quad \cfrac{\cfrac{\cfrac{}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_8} \text{inv-th/ax} \quad \cfrac{}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_9} \text{inv-th/ax}}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_8 \wedge F_9} \wedge_R \quad \cfrac{}{- : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8 \wedge F_9} \text{ax/W}}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} \text{sCut}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{sCut}$$

$$\cfrac{\cfrac{\cfrac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \cfrac{h_{11} : F_7, \Delta_{14} \vdash F_{12}, \Delta_8, F_9 \wedge}{\bullet h_{11} : (\Delta_{14}, F_{12}} }{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}}{\rightarrow}$$

$$\cfrac{\cfrac{}{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_9} \text{ax/W} \quad \cfrac{\cfrac{}{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{12}, F_9} \text{inv-th/ax} \quad \cfrac{}{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9} \text{inv-th/ax}}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9} \rightarrow_L \quad \cfrac{}{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9} \text{hCut}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \cfrac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut}}{\rightarrow}$$

$$\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6} \text{inv-th/ax} \quad \cfrac{}{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7} \text{inv-th/ax}}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \cfrac{}{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}} \text{ax/W}}{\cfrac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L} \text{hCut}$$

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$$\dfrac{\dfrac{h_1 : \Delta_7 \vdash F_8, \Delta_{10} \quad h_1 : \Delta_7 \vdash F_9, \Delta_{10}}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \wedge F_9} \, {\wedge_R} \quad \dfrac{h_6 : F_8, F_9, \Delta_7 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{10}} \, {\wedge_L}}{- : \Delta_7 \vdash \Delta_{10}} \, {\text{Cut}}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Delta_7 \vdash \Delta_{10}, F_8} \, {\text{ax/W}} \quad \dfrac{\dfrac{}{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9} \, {\text{ax/W}} \quad \dfrac{}{- : \Delta_7, F_8, F_9 \vdash \Delta_{10}} \, {\text{ax/W}}}{- : \Delta_7, F_8 \vdash \Delta_{10}} \, {\text{sCut}}}{- : \Delta_7 \vdash \Delta_{10}} \, {\text{sCut}}$$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash F_{12} \wedge F_{13}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash F_{12} \wedge F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \wedge F_{13}} \, {\wedge_R} \quad \dfrac{h_{10} : F_{12}, F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \wedge F_9} \, {\wedge_L}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \, {\text{Cut}}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_{12} \wedge F_{13}} \, {\text{ax/W}} \quad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8} \, {\wedge_L}}{\rule{0pt}{0pt}} \, {\text{inv-th/ax}}}{- : \Delta_{11} \vdash \Delta_7, F_8} \, {\text{hCut}} \quad \dfrac{\dfrac{}{h_2 : \Delta_{11} \vdash \Delta_7, F_9, F_{12} \wedge F_{13}} \, {\text{ax/W}} \quad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_9} \, {\wedge_L}}{\rule{0pt}{0pt}} \, {\text{inv-t}}}{- : \Delta_{11} \vdash \Delta_7, F_9} \, {\text{hCut}}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \, {\wedge_R}$$

$$\dfrac{\dfrac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \, {\wedge_R} \quad \dfrac{h_{11} : F_7, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \, {\wedge_L}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \, {\text{Cut}}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9} \, {\text{inv-th/ax}} \quad \dfrac{}{h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_{10}, F_7} \, {\text{inv-th/ax}}}{\bullet h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \, {\wedge_R}}{- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \quad \dfrac{\dfrac{}{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \, {\text{ax/W}}}{\rule{0pt}{0pt}} \, {\text{hCut}}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \, {\wedge_L}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \, {\wedge_R} \quad \dfrac{h_8 : F_9, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \, {\vee_L}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \, {\text{Cut}}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6} \, {\text{ax/W}} \quad \dfrac{\dfrac{}{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7} \, {\text{ax/W}} \quad \dfrac{\dfrac{}{- : \Delta_{12}, F_6, F_7, F_9 \vdash \Delta_{11}} \, {\text{inv-th/ax}} \quad \dfrac{}{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}} \, {\text{inv-th/ax}}}{- : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash \Delta_{11}} \, {\vee_L}}{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}} \, {\text{sCut}}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \, {\text{sCut}}$$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash F_{12} \vee F_{13}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash F_{12} \vee F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \vee F_{13}} \, {\wedge_R} \quad \dfrac{h_{10} : F_{12}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \wedge F_9} \, {\vee_L}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \, {\text{Cut}}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8} \, {\text{inv-th/ax}} \quad \dfrac{}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_9} \, {\text{inv-th/ax}}}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8 \wedge F_9} \, {\wedge_R}}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \wedge F_9} \quad \dfrac{\dfrac{}{- : \Delta_{11}, F_{13} \vdash \Delta_7, F_{12}, F_8 \wedge F_9} \, {\text{ax/W}}}{\rule{0pt}{0pt}} \, {\text{sCut}} \quad \dfrac{}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} \, {\text{ax/W}}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \, {\text{sCut}}$$

$$\dfrac{\dfrac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \, {\wedge_R} \quad \dfrac{h_{11} : F_7, F_{12}, \Delta_{14} \vdash \Delta_8, F_9 \wedge F}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee}{\rule{0pt}{0pt}}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9} \, {\text{ax/W}} \quad \dfrac{\dfrac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_9}{\rule{0pt}{0pt}} \, {\text{inv-th/ax}} \quad \dfrac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9}{\rule{0pt}{0pt}} \, {\text{inv-th/ax}}}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \, {\vee_L}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \, {\text{hCut}} \quad \dfrac{}{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F}{\rule{0pt}{0pt}}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}$$

- Case rule AT

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_{11},[]\mathtt{F_9}\vdash \mathtt{F_6},\Delta_{10}\quad \mathtt{h_1}:\Delta_{11},[]\mathtt{F_9}\vdash \mathtt{F_7},\Delta_{10}}{\bullet\mathtt{h_1}:\Delta_{11},[]\mathtt{F_9}\vdash\Delta_{10},\mathtt{F_6}\wedge \mathtt{F_7}}\ \wedge_R \quad \dfrac{\mathtt{h_8}:\mathtt{F_9},\Delta_{11},[]\mathtt{F_9},\mathtt{F_6}\wedge \mathtt{F_7}\vdash\Delta_{10}}{\bullet\mathtt{h_8}:(\Delta_{11},[]\mathtt{F_9}),\mathtt{F_6}\wedge \mathtt{F_7}\vdash\Delta_{10}}\ AT}{-:\Delta_{11},[]\mathtt{F_9}\vdash\Delta_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h_1}:\Delta_{11},\mathtt{F_9},[]\mathtt{F_9}\vdash\Delta_{10},\mathtt{F_6}\wedge \mathtt{F_7}}{}\ \mathtt{ax/W}\quad \dfrac{\mathtt{h_8}:\Delta_{11},\mathtt{F_9},[]\mathtt{F_9},\mathtt{F_6}\wedge \mathtt{F_7}\vdash\Delta_{10}}{}\ \mathtt{ax/W}}{-:\Delta_{11},\mathtt{F_9},[]\mathtt{F_9}\vdash\Delta_{10}}\ \mathtt{hCut}}{-:\Delta_{11},[]\mathtt{F_9}\vdash\Delta_{10}}\ ATG$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_{11}\vdash []\mathtt{F_{12}},\mathtt{F_8},\Delta_7\quad \mathtt{h_2}:\Delta_{11}\vdash []\mathtt{F_{12}},\mathtt{F_9},\Delta_7}{\bullet\mathtt{h_2}:\Delta_{11}\vdash(\Delta_7,\mathtt{F_8}\wedge \mathtt{F_9}),[]\mathtt{F_{12}}}\ \wedge_R\quad \dfrac{\mathtt{h_{10}}:\mathtt{F_{12}},\Delta_{11},[]\mathtt{F_{12}}\vdash\Delta_7,\mathtt{F_8}\wedge \mathtt{F_9}}{\bullet\mathtt{h_{10}}:\Delta_{11},[]\mathtt{F_{12}}\vdash\Delta_7,\mathtt{F_8}\wedge \mathtt{F_9}}\ AT}{-:\Delta_{11}\vdash\Delta_7,\mathtt{F_8}\wedge \mathtt{F_9}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Delta_{11}\vdash\Delta_7,\mathtt{F_8},[]\mathtt{F_{12}}}{}\ \mathtt{ax/W}\quad \dfrac{\dfrac{\mathtt{h_{10}}:\Delta_{11},\mathtt{F_{12}},[]\mathtt{F_{12}}\vdash\Delta_7,\mathtt{F_8}}{}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h_{10}}:\Delta_{11},[]\mathtt{F_{12}}\vdash\Delta_7,\mathtt{F_8}}\ AT}{-:\Delta_{11}\vdash\Delta_7,\mathtt{F_8}}\ \mathtt{hCut}\quad \dfrac{\dfrac{\mathtt{h_2}:\Delta_{11}\vdash\Delta_7,\mathtt{F_9},[]\mathtt{F_{12}}}{}\ \mathtt{ax/W}\quad \dfrac{\dfrac{\mathtt{h_{10}}:\Delta_{11},\mathtt{F_{12}},[]\mathtt{F_{12}}\vdash\Delta_7,\mathtt{F_9}}{}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h_{10}}:\Delta_{11},[]\mathtt{F_{12}}\vdash\Delta_7,\mathtt{F_9}}\ AT}{-:\Delta_{11}\vdash\Delta_7,\mathtt{F_9}}\ \mathtt{hCut}}{-:\Delta_{11}\vdash\Delta_7,\mathtt{F_8}\wedge \mathtt{F_9}}\ \wedge_R$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_{13},[]\mathtt{F_{12}}\vdash \mathtt{F_7},\mathtt{F_9},\Delta_8\quad \mathtt{h_2}:\Delta_{13},[]\mathtt{F_{12}}\vdash \mathtt{F_7},\mathtt{F_{10}},\Delta_8}{\bullet\mathtt{h_2}:\Delta_{13},[]\mathtt{F_{12}}\vdash(\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}),\mathtt{F_7}}\ \wedge_R\quad \dfrac{\mathtt{h_{11}}:\mathtt{F_7},\mathtt{F_{12}},\Delta_{13},[]\mathtt{F_{12}}\vdash\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}}{\bullet\mathtt{h_{11}}:(\Delta_{13},[]\mathtt{F_{12}}),\mathtt{F_7}\vdash\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}}\ AT}{-:\Delta_{13},[]\mathtt{F_{12}}\vdash\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h_2}:\Delta_{13},\mathtt{F_{12}},[]\mathtt{F_{12}}\vdash\Delta_8,\mathtt{F_7},\mathtt{F_9}\wedge \mathtt{F_{10}}}{}\ \mathtt{ax/W}\quad \dfrac{\mathtt{h_{11}}:\Delta_{13},\mathtt{F_{12}},\mathtt{F_7},[]\mathtt{F_{12}}\vdash\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}}{}\ \mathtt{ax/W}}{-:\Delta_{13},\mathtt{F_{12}},[]\mathtt{F_{12}}\vdash\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}}\ \mathtt{hCut}}{-:\Delta_{13},[]\mathtt{F_{12}}\vdash\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}}\ ATG$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{\mathtt{h_1}:\perp,\Delta_{10}\vdash \mathtt{F_6},\Delta_9\quad \mathtt{h_1}:\perp,\Delta_{10}\vdash \mathtt{F_7},\Delta_9}{\bullet\mathtt{h_1}:\perp,\Delta_{10}\vdash\Delta_9,\mathtt{F_6}\wedge \mathtt{F_7}}\ \wedge_R\quad \dfrac{}{\bullet\mathtt{h_8}:(\perp,\Delta_{10}),\mathtt{F_6}\wedge \mathtt{F_7}\vdash\Delta_9}\ \perp_L}{-:\perp,\Delta_{10}\vdash\Delta_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{-:\perp,\Delta_{10}\vdash\Delta_9}\ \perp_L$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_{11}\vdash\perp,\mathtt{F_8},\Delta_7\quad \mathtt{h_2}:\Delta_{11}\vdash\perp,\mathtt{F_9},\Delta_7}{\bullet\mathtt{h_2}:\Delta_{11}\vdash(\Delta_7,\mathtt{F_8}\wedge \mathtt{F_9}),\perp}\ \wedge_R\quad \dfrac{}{\bullet\mathtt{h_{10}}:\Delta_{11},\perp\vdash\Delta_7,\mathtt{F_8}\wedge \mathtt{F_9}}\ \perp_L}{-:\Delta_{11}\vdash\Delta_7,\mathtt{F_8}\wedge \mathtt{F_9}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Delta_{11}\vdash\perp,\Delta_7,\mathtt{F_8}}{}\ \mathtt{ax/W}\quad \dfrac{\bullet\mathtt{h_{10}}:\perp,\Delta_{11}\vdash\Delta_7,\mathtt{F_8}}{}\ \perp_L}{-:\Delta_{11}\vdash\Delta_7,\mathtt{F_8}}\ \mathtt{hCut}\quad \dfrac{\dfrac{\mathtt{h_2}:\Delta_{11}\vdash\perp,\Delta_7,\mathtt{F_9}}{}\ \mathtt{ax/W}\quad \dfrac{\bullet\mathtt{h_{10}}:\perp,\Delta_{11}\vdash\Delta_7,\mathtt{F_9}}{}\ \perp_L}{-:\Delta_{11}\vdash\Delta_7,\mathtt{F_9}}\ \mathtt{hCut}}{-:\Delta_{11}\vdash\Delta_7,\mathtt{F_8}\wedge \mathtt{F_9}}\ \wedge_R$$

$$\dfrac{\dfrac{\mathtt{h_2}:\perp,\Delta_{12}\vdash \mathtt{F_7},\mathtt{F_9},\Delta_8\quad \mathtt{h_2}:\perp,\Delta_{12}\vdash \mathtt{F_7},\mathtt{F_{10}},\Delta_8}{\bullet\mathtt{h_2}:\perp,\Delta_{12}\vdash(\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}),\mathtt{F_7}}\ \wedge_R\quad \dfrac{}{\bullet\mathtt{h_{11}}:(\perp,\Delta_{12}),\mathtt{F_7}\vdash\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}}\ \perp_L}{-:\perp,\Delta_{12}\vdash\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{-:\perp,\Delta_{12}\vdash\Delta_8,\mathtt{F_9}\wedge \mathtt{F_{10}}}\ \perp_L$$

- Case rule I

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_{11},\mathtt{p_9}\vdash \mathtt{F_6},\Delta_{10},\mathtt{p_9}\quad \mathtt{h_1}:\Delta_{11},\mathtt{p_9}\vdash \mathtt{F_7},\Delta_{10},\mathtt{p_9}}{\bullet\mathtt{h_1}:\Delta_{11},\mathtt{p_9}\vdash(\Delta_{10},\mathtt{p_9}),\mathtt{F_6}\wedge \mathtt{F_7}}\ \wedge_R\quad \dfrac{}{\bullet\mathtt{h_8}:(\Delta_{11},\mathtt{p_9}),\mathtt{F_6}\wedge \mathtt{F_7}\vdash\Delta_{10},\mathtt{p_9}}\ I}{-:\Delta_{11},\mathtt{p_9}\vdash\Delta_{10},\mathtt{p_9}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{-:\Delta_{11},\mathtt{p_9}\vdash\Delta_{10},\mathtt{p_9}}\ I$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_{10}\vdash \mathtt{p_{11}},\mathtt{F_7},\Delta_{12},\mathtt{p_{11}}\quad \mathtt{h_2}:\Delta_{10}\vdash \mathtt{p_{11}},\mathtt{F_8},\Delta_{12},\mathtt{p_{11}}}{\bullet\mathtt{h_2}:\Delta_{10}\vdash((\Delta_{12},\mathtt{p_{11}}),\mathtt{F_7}\wedge \mathtt{F_8}),\mathtt{p_{11}}}\ \wedge_R\quad \dfrac{}{\bullet\mathtt{h_9}:\Delta_{10},\mathtt{p_{11}}\vdash(\Delta_{12},\mathtt{p_{11}}),\mathtt{F_7}\wedge \mathtt{F_8}}\ I}{-:\Delta_{10}\vdash(\Delta_{12},\mathtt{p_{11}}),\mathtt{F_7}\wedge \mathtt{F_8}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Delta_{10}\vdash\Delta_{12},\mathtt{F_7},\mathtt{p_{11}},\mathtt{p_{11}}}{}\ \mathtt{ax/W}\quad \dfrac{\bullet\mathtt{h_9}:\Delta_{10},\mathtt{p_{11}}\vdash\Delta_{12},\mathtt{F_7},\mathtt{p_{11}}}{}\ I}{-:\Delta_{10}\vdash\Delta_{12},\mathtt{F_7},\mathtt{p_{11}}}\ \mathtt{hCut}\quad \dfrac{\dfrac{\mathtt{h_2}:\Delta_{10}\vdash\Delta_{12},\mathtt{F_8},\mathtt{p_{11}},\mathtt{p_{11}}}{}\ \mathtt{ax/W}\quad \dfrac{\bullet\mathtt{h_9}:\Delta_{10},\mathtt{p_{11}}\vdash\Delta_{12},\mathtt{F_8},\mathtt{p_{11}}}{}\ I}{-:\Delta_{10}\vdash\Delta_{12},\mathtt{F_8},\mathtt{p_{11}}}\ \mathtt{hCut}}{-:\Delta_{10}\vdash\Delta_{12},\mathtt{p_{11}},\mathtt{F_7}\wedge \mathtt{F_8}}\ \wedge_R$$

$$\dfrac{\dfrac{h_2 : \Delta_{13}, p_{11} \vdash F_7, F_8, \Delta_{12}, p_{11} \quad h_2 : \Delta_{13}, p_{11} \vdash F_7, F_9, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \wedge F_9), F_7} \wedge_R \quad \dfrac{}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9} I}{\begin{array}{c} - : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9 \\ \rightarrow \\ \hline - : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \wedge F_9 \end{array} I} \text{Cut}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{h_1 : \top, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \top, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \dfrac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \top_L}{\begin{array}{c} - : \top, \Delta_{10} \vdash \Delta_9 \\ \rightarrow \\ \dfrac{\dfrac{}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \text{ax/W} \quad \dfrac{}{h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut} \end{array}} \text{Cut}$$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash \top, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash \top, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \top} \wedge_R \quad \dfrac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \wedge F_9} \top_L}{\begin{array}{c} - : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \\ \rightarrow \\ \dfrac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{ax/W} \end{array}} \text{Cut}$$

$$\dfrac{\dfrac{h_2 : \top, \Delta_{12} \vdash F_7, F_9, \Delta_8 \quad h_2 : \top, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \top_L}{\begin{array}{c} - : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10} \\ \rightarrow \\ \dfrac{\dfrac{}{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \text{ax/W} \quad \dfrac{}{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{hCut} \end{array}} \text{Cut}$$

6.3 Status of $\vee_R$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \rightarrow F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \vee F_8} \vee_R \quad \dfrac{h_9 : F_{11}, \Delta_6, F_7 \vee F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R}{\begin{array}{c} - : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12} \\ \rightarrow \\ \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7, F_8} \text{inv-th/ax}}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \vee F_8} \vee_R \quad \dfrac{}{h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W}}{\dfrac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R} \text{hCut} \end{array}} \text{Cut}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \rightarrow F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}), F_7} \vee_R \quad \dfrac{h_{11} : F_7, F_{12}, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}} \rightarrow_R}{\begin{array}{c} - : \Delta_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10} \\ \rightarrow \\ \dfrac{\dfrac{\dfrac{}{h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_7, F_9} \text{inv-th/ax}}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9 \vee F_{10}} \vee_R \quad \dfrac{}{h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}} \text{ax/W}}{\dfrac{- : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \vee F_{10}} \rightarrow_R} \text{hCut} \end{array}} \text{Cut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \vee F_8} \vee_R \quad \dfrac{h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{\begin{array}{c} - : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12} \\ \rightarrow \\ \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7, F_8} \text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \vee F_8} \vee_R \quad \dfrac{}{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}} \text{ax/W}}{\dfrac{- : \Delta_6 \vdash \Delta_{10}, F_{11}} {}} \text{hCut} \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7, F_8} \text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7 \vee F_8} \vee_R \quad \dfrac{}{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W}}{\dfrac{- : \Delta_6 \vdash \Delta_{10}, F_{12}}{}} \text{hCut} \\ \hline - : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12} \end{array}} \wedge_R$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}), F_7} \vee_R \quad \dfrac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \vee F_{10} \quad h_{11} : F_7, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}} \wedge_R}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_7, F_9, F_{12} \wedge F_{13}} \text{ax/W} \quad \dfrac{\dfrac{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_{12}, F_9}{} \text{inv-th/ax} \quad \dfrac{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_{13}, F_9}{} \text{inv-th/ax}}{\bullet h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}} \wedge_R}{\dfrac{- : \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \wedge F_{13}, F_9 \vee F_{10}} \vee_R} \text{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \vee F_8} \vee_R \quad \dfrac{h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \vee F_8} \vee_R \text{inv-th/ax} \quad \dfrac{}{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}} \text{hCut}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}), F_7} \vee_R \quad \dfrac{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}} \vee_R}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_7, F_9}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \vee F_{10}} \vee_R \text{inv-th/ax} \quad \dfrac{}{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}} \text{hCut}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \vee F_{13}, F_9 \vee F_{10}} \vee_R$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_{11}, F_{12}, \Delta_{10}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7} \vee_R \quad \dfrac{h_9 : F_7, \Delta_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_8 \vdash \Delta_{10}, F_{11}, F_{12}, F_7} \text{ax/W} \quad \dfrac{\dfrac{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}, F_{12}} \text{H}}{} \text{ax/W}}{- : \Delta_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{hCut}}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \vee F_8} \vee_R \quad \dfrac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \perp_R}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \vee F_8} \text{ax/W} \quad \dfrac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}}{} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{hCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \perp, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\perp, \Delta_{12}), F_9 \vee F_{10}), F_7} \vee_R \quad \dfrac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\perp, \Delta_{12}), F_9 \vee F_{10}} \perp_R}{- : \Delta_8 \vdash (\perp, \Delta_{12}), F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \vee F_{10}} \text{ax/W} \quad \dfrac{h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}}{} \text{ax/W}}{- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}} \text{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_6\vdash \mathtt{F_7},\mathtt{F_8},\top,\Delta_{10}}{\bullet\mathtt{h_1}:\Delta_6\vdash (\top,\Delta_{10}),\mathtt{F_7}\vee\mathtt{F_8}}\ \vee_R \quad \dfrac{}{\bullet\mathtt{h_9}:\Delta_6,\mathtt{F_7}\vee\mathtt{F_8}\vdash \top,\Delta_{10}}\ \top_R}{-:\Delta_6\vdash \top,\Delta_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{-:\Delta_6\vdash \top,\Delta_{10}}\ \top_R$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_8\vdash \mathtt{F_7},\mathtt{F_9},\mathtt{F_{10}},\top,\Delta_{12}}{\bullet\mathtt{h_2}:\Delta_8\vdash ((\top,\Delta_{12}),\mathtt{F_9}\vee\mathtt{F_{10}}),\mathtt{F_7}}\ \vee_R \quad \dfrac{}{\bullet\mathtt{h_{11}}:\Delta_8,\mathtt{F_7}\vdash (\top,\Delta_{12}),\mathtt{F_9}\vee\mathtt{F_{10}}}\ \top_R}{-:\Delta_8\vdash (\top,\Delta_{12}),\mathtt{F_9}\vee\mathtt{F_{10}}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{-:\Delta_8\vdash \top,\Delta_{12},\mathtt{F_9}\vee\mathtt{F_{10}}}\ \top_R$$

- Case rule K

$$\dfrac{\dfrac{\mathtt{h_1}:\Box\Gamma_9,\Delta_{12}\vdash \mathtt{F_6},\mathtt{F_7},\Delta_{10},[]\mathtt{F_{11}}}{\bullet\mathtt{h_1}:\Box\Gamma_9,\Delta_{12}\vdash (\Delta_{10},[]\mathtt{F_{11}}),\mathtt{F_6}\vee\mathtt{F_7}}\ \vee_R \quad \dfrac{\mathtt{h_8}:unbox(\Box\Gamma_9)\vdash \mathtt{F_{11}}}{\bullet\mathtt{h_8}:(\Box\Gamma_9,\Delta_{12}),\mathtt{F_6}\vee\mathtt{F_7}\vdash \Delta_{10},[]\mathtt{F_{11}}}\ K}{-:\Box\Gamma_9,\Delta_{12}\vdash \Delta_{10},[]\mathtt{F_{11}}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{-:unbox(\Box\Gamma_9)\vdash \mathtt{F_{11}}}\ \mathtt{ax/W}}{-:\Delta_{12},\Box\Gamma_9\vdash \Delta_{10},[]\mathtt{F_{11}}}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{14},\Delta_{11}\vdash \Box\mathtt{F_7},\mathtt{F_8},\mathtt{F_9},\Delta_{13},[]\mathtt{F_{12}}}{\bullet\mathtt{h_2}:\Box\Gamma_{14},\Delta_{11}\vdash ((\Delta_{13},[]\mathtt{F_{12}}),\mathtt{F_8}\vee\mathtt{F_9}),\Box\mathtt{F_7}}\ \vee_R \quad \dfrac{\mathtt{h_{10}}:unbox(\Box\Gamma_{14}),unbox(\Box\mathtt{F_7})\vdash \mathtt{F_{12}}}{\bullet\mathtt{h_{10}}:(\Box\Gamma_{14},\Delta_{11}),\Box\mathtt{F_7}\vdash (\Delta_{13},[]\mathtt{F_{12}}),\mathtt{F_8}\vee\mathtt{F_9}}\ K}{-:\Box\Gamma_{14},\Delta_{11}\vdash (\Delta_{13},[]\mathtt{F_{12}}),\mathtt{F_8}\vee\mathtt{F_9}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_2}:\Delta_{11},\Box\Gamma_{14}\vdash \Box\mathtt{F_7},\Delta_{13},\mathtt{F_8},\mathtt{F_9},[]\mathtt{F_{12}}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h_{10}}:unbox(\Box\mathtt{F_7}),unbox(\Box\Gamma_{14})\vdash \mathtt{F_{12}}}\ \mathtt{ax/W}}{\bullet\mathtt{h_{10}}:\Box\mathtt{F_7},\Delta_{11},\Box\Gamma_{14}\vdash \Delta_{13},\mathtt{F_8},\mathtt{F_9},[]\mathtt{F_{12}}}\ K}{-:\Delta_{11},\Box\Gamma_{14}\vdash \Delta_{13},\mathtt{F_8},\mathtt{F_9},[]\mathtt{F_{12}}}\ \mathtt{hCut}}{-:\Delta_{11},\Box\Gamma_{14}\vdash \Delta_{13},[]\mathtt{F_{12}},\mathtt{F_8}\vee\mathtt{F_9}}\ \vee_R$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{11},\Delta_{14}\vdash \mathtt{F_7},\mathtt{F_8},\mathtt{F_9},\Delta_{13},[]\mathtt{F_{12}}}{\bullet\mathtt{h_2}:\Box\Gamma_{11},\Delta_{14}\vdash ((\Delta_{13},[]\mathtt{F_{12}}),\mathtt{F_8}\vee\mathtt{F_9}),\mathtt{F_7}}\ \vee_R \quad \dfrac{\mathtt{h_{10}}:unbox(\Box\Gamma_{11})\vdash \mathtt{F_{12}}}{\bullet\mathtt{h_{10}}:(\Box\Gamma_{11},\Delta_{14}),\mathtt{F_7}\vdash (\Delta_{13},[]\mathtt{F_{12}}),\mathtt{F_8}\vee\mathtt{F_9}}\ K}{-:\Box\Gamma_{11},\Delta_{14}\vdash (\Delta_{13},[]\mathtt{F_{12}}),\mathtt{F_8}\vee\mathtt{F_9}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{-:unbox(\Box\Gamma_{11})\vdash \mathtt{F_{12}}}\ \mathtt{ax/W}}{-:\Delta_{14},\Box\Gamma_{11}\vdash \Delta_{13},[]\mathtt{F_{12}},\mathtt{F_8}\vee\mathtt{F_9}}\ K$$

- Case rule $A45$

$$\dfrac{\dfrac{\mathtt{h_1}:\Box\Gamma_9,\Delta_{13}\vdash \mathtt{F_6},\mathtt{F_7},\Box\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}{\bullet\mathtt{h_1}:\Box\Gamma_9,\Delta_{13}\vdash (\Box\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}),\mathtt{F_6}\vee\mathtt{F_7}}\ \vee_R \quad \dfrac{\mathtt{h_8}:\Box\Gamma_9\vdash \Box\Gamma_{10},\mathtt{F_{12}}}{\bullet\mathtt{h_8}:(\Box\Gamma_9,\Delta_{13}),\mathtt{F_6}\vee\mathtt{F_7}\vdash \Box\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}\ A45}{-:\Box\Gamma_9,\Delta_{13}\vdash \Box\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{-:\Box\Gamma_9\vdash \mathtt{F_{12}},\Box\Gamma_{10}}\ \mathtt{ax/W}}{-:\Delta_{13},\Box\Gamma_9\vdash \Delta_{11},\Box\Gamma_{10},[]\mathtt{F_{12}}}\ A45$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{15},\Delta_{11}\vdash \Box\mathtt{F_7},\mathtt{F_8},\mathtt{F_9},\Box\Gamma_{12},\Delta_{14},[]\mathtt{F_{13}}}{\bullet\mathtt{h_2}:\Box\Gamma_{15},\Delta_{11}\vdash ((\Box\Gamma_{12},\Delta_{14},[]\mathtt{F_{13}}),\mathtt{F_8}\vee\mathtt{F_9}),\Box\mathtt{F_7}}\ \vee_R \quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{15},\Box\mathtt{F_7}\vdash \Box\Gamma_{12},\mathtt{F_{13}}}{\bullet\mathtt{h_{10}}:(\Box\Gamma_{15},\Delta_{11}),\Box\mathtt{F_7}\vdash (\Box\Gamma_{12},\Delta_{14},[]\mathtt{F_{13}}),\mathtt{F_8}\vee\mathtt{F_9}}\ A45}{-:\Box\Gamma_{15},\Delta_{11}\vdash (\Box\Gamma_{12},\Delta_{14},[]\mathtt{F_{13}}),\mathtt{F_8}\vee\mathtt{F_9}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_2}:\Delta_{11},\Box\Gamma_{15}\vdash \Box\mathtt{F_7},\Delta_{14},\mathtt{F_8},\mathtt{F_9},\Box\Gamma_{12},[]\mathtt{F_{13}}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h_{10}}:\Box\mathtt{F_7},\Box\Gamma_{15}\vdash \mathtt{F_{13}},\Box\Gamma_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h_{10}}:\Box\mathtt{F_7},\Delta_{11},\Box\Gamma_{15}\vdash \Delta_{14},\mathtt{F_8},\mathtt{F_9},\Box\Gamma_{12},[]\mathtt{F_{13}}}\ A45}{-:\Delta_{11},\Box\Gamma_{15}\vdash \Delta_{14},\mathtt{F_8},\mathtt{F_9},\Box\Gamma_{12},[]\mathtt{F_{13}}}\ \mathtt{hCut}}{-:\Delta_{11},\Box\Gamma_{15}\vdash \Delta_{14},\Box\Gamma_{12},[]\mathtt{F_{13}},\mathtt{F_8}\vee\mathtt{F_9}}\ \vee_R$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{11},\Delta_{15}\vdash \mathtt{F_7},\mathtt{F_8},\mathtt{F_9},\Box\Gamma_{12},\Delta_{14},[]\mathtt{F_{13}}}{\bullet\mathtt{h_2}:\Box\Gamma_{11},\Delta_{15}\vdash ((\Box\Gamma_{12},\Delta_{14},[]\mathtt{F_{13}}),\mathtt{F_8}\vee\mathtt{F_9}),\mathtt{F_7}}\ \vee_R \quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{11}\vdash \Box\Gamma_{12},\mathtt{F_{13}}}{\bullet\mathtt{h_{10}}:(\Box\Gamma_{11},\Delta_{15}),\mathtt{F_7}\vdash (\Box\Gamma_{12},\Delta_{14},[]\mathtt{F_{13}}),\mathtt{F_8}\vee\mathtt{F_9}}\ A45}{-:\Box\Gamma_{11},\Delta_{15}\vdash (\Box\Gamma_{12},\Delta_{14},[]\mathtt{F_{13}}),\mathtt{F_8}\vee\mathtt{F_9}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{-:\Box\Gamma_{11}\vdash \mathtt{F_{13}},\Box\Gamma_{12}}\ \mathtt{ax/W}}{-:\Delta_{15},\Box\Gamma_{11}\vdash \Delta_{14},\Box\Gamma_{12},[]\mathtt{F_{13}},\mathtt{F_8}\vee\mathtt{F_9}}\ A45$$

- Case rule $\to_L$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \Delta_{12}, F_9 \to F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6 \lor F_7} \lor_R
\quad
\cfrac{h_8 : \Delta_{12}, F_6 \lor F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \lor F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \to F_{10}), F_6 \lor F_7 \vdash \Delta_{11}} \to_L
}{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \text{Cut}
}{}
$$

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\to
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\cfrac{
\cfrac{
\cfrac{\cfrac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \lor F_7} \text{inv-th/ax} \ \lor_R \quad \cfrac{}{h_8 : \Delta_{12}, F_6 \lor F_7 \vdash \Delta_{11}, F_9} \text{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \text{hCut}
\quad
\cfrac{\cfrac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \lor F_7} \text{inv-th/ax} \ \lor_R \quad \cfrac{}{h_8 : \Delta_{12}, F_{10}, F_6 \lor F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \text{hCut}
}{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \to_L
}{}
$$

$$
\cfrac{
\cfrac{
\cfrac{h_2 : \Delta_{11} \vdash F_{12} \to F_{13}, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \lor F_9), F_{12} \to F_{13}} \lor_R
\quad
\cfrac{h_{10} : \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \lor F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \lor F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \to F_{13} \vdash \Delta_7, F_8 \lor F_9} \to_L
}{- : \Delta_{11} \vdash \Delta_7, F_8 \lor F_9} \text{Cut}
}{}
$$

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\to
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$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \to F_{13}} \text{ax/W} \quad \cfrac{\cfrac{h_{10} : \Delta_{11} \vdash \Delta_7, F_{12}, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \to F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax} \quad \cfrac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9}{} \text{inv-th/ax}}{} \to_L}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{hCut}
}{- : \Delta_{11} \vdash \Delta_7, F_8 \lor F_9} \lor_R
}{}
$$

$$
\cfrac{
\cfrac{
\cfrac{h_2 : \Delta_{14}, F_{12} \to F_{13} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \to F_{13} \vdash (\Delta_8, F_9 \lor F_{10}), F_7} \lor_R
\quad
\cfrac{h_{11} : F_7, \Delta_{14} \vdash F_{12}, \Delta_8, F_9 \lor F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \lor F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \to F_{13}), F_7 \vdash \Delta_8, F_9 \lor F_{10}} \to_L
}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_9 \lor F_{10}} \text{Cut}
}{}
$$

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\to
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$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{h_2 : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \text{ax/W} \quad \cfrac{\cfrac{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{10}, F_{12}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \to F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax} \quad \cfrac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{} \text{inv-th/ax}}{} \to_L}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_{10}, F_9} \text{hCut}
}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_9 \lor F_{10}} \lor_R
}{}
$$

- Case rule $\land_L$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \Delta_{12}, F_9 \land F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \land F_{10} \vdash \Delta_{11}, F_6 \lor F_7} \lor_R
\quad
\cfrac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \lor F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \land F_{10}), F_6 \lor F_7 \vdash \Delta_{11}} \land_L
}{- : \Delta_{12}, F_9 \land F_{10} \vdash \Delta_{11}} \text{Cut}
}{}
$$

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\to
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$$
\cfrac{
\cfrac{
\cfrac{\cfrac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \lor F_7} \text{inv-th/ax} \ \lor_R \quad \cfrac{}{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \lor F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \text{hCut}
}{- : \Delta_{12}, F_9 \land F_{10} \vdash \Delta_{11}} \land_L
}{}
$$

$$
\cfrac{
\cfrac{
\cfrac{h_2 : \Delta_{11} \vdash F_{12} \land F_{13}, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \lor F_9), F_{12} \land F_{13}} \lor_R
\quad
\cfrac{h_{10} : F_{12}, F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \lor F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \land F_{13} \vdash \Delta_7, F_8 \lor F_9} \land_L
}{- : \Delta_{11} \vdash \Delta_7, F_8 \lor F_9} \text{Cut}
}{}
$$

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\to
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$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \land F_{13}} \text{ax/W} \quad \cfrac{\cfrac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \land F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax} \ \land_L}{} }{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{hCut}
}{- : \Delta_{11} \vdash \Delta_7, F_8 \lor F_9} \lor_R
}{}
$$

$$
\cfrac{
\cfrac{
\cfrac{h_2 : \Delta_{14}, F_{12} \land F_{13} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \land F_{13} \vdash (\Delta_8, F_9 \lor F_{10}), F_7} \lor_R
\quad
\cfrac{h_{11} : F_7, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \lor F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \land F_{13}), F_7 \vdash \Delta_8, F_9 \lor F_{10}} \land_L
}{- : \Delta_{14}, F_{12} \land F_{13} \vdash \Delta_8, F_9 \lor F_{10}} \text{Cut}
}{}
$$

$$
\to
$$

$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{h_2 : \Delta_{14}, F_{12} \land F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \text{ax/W} \quad \cfrac{\cfrac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \land F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax} \ \land_L}{} }{- : \Delta_{14}, F_{12} \land F_{13} \vdash \Delta_8, F_{10}, F_9} \text{hCut}
}{- : \Delta_{14}, F_{12} \land F_{13} \vdash \Delta_8, F_9 \lor F_{10}} \lor_R
}{}
$$

- Case rule $\lor_L$

$$\dfrac{\dfrac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \vee F_7}\ \vee_R \quad \dfrac{h_8 : F_9, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}}\ \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \vee F_7}\ \vee_R \quad \dfrac{}{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}\ \text{ax/W}}{- : \Delta_{12}, F_9 \vdash \Delta_{11}}\ \text{hCut} \quad \dfrac{\dfrac{\dfrac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7}\ \vee_R \quad \dfrac{}{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}\ \text{ax/W}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}}\ \text{hCut}}{}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}}\ \vee_L$$

(the two inner steps labelled inv-th/ax)

$$\dfrac{\dfrac{h_1 : \Delta_7 \vdash F_8, F_9, \Delta_{10}}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \vee F_9}\ \vee_R \quad \dfrac{h_6 : F_8, \Delta_7 \vdash \Delta_{10} \quad h_6 : F_9, \Delta_7 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{10}}\ \vee_L}{- : \Delta_7 \vdash \Delta_{10}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_7 \vdash \Delta_{10}, F_8, F_9}\ \text{ax/W} \quad \dfrac{}{- : \Delta_7, F_9 \vdash \Delta_{10}, F_8}\ \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}, F_8}\ \text{sCut} \quad \dfrac{}{- : \Delta_7, F_8 \vdash \Delta_{10}}\ \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}}\ \text{sCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash F_{12} \vee F_{13}, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \vee F_{13}}\ \vee_R \quad \dfrac{h_{10} : F_{12}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \vee F_9}\ \vee_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \vee F_{13}}\ \text{ax/W} \quad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8, F_9}{} \quad \dfrac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9}{}}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8, F_9}\ \vee_L}{\dfrac{- : \Delta_{11} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}\ \vee_R}\ \text{hCut}}{}$$

(inner steps labelled inv-th/ax)

$$\dfrac{\dfrac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7}\ \vee_R \quad \dfrac{h_{11} : F_7, F_{12}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}}\ \vee_L}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}\ \text{ax/W} \quad \dfrac{\dfrac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_{10}, F_9}{} \quad \dfrac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{}}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9}\ \vee_L}{\dfrac{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10}}\ \vee_R}\ \text{hCut}}{}$$

(inner steps labelled inv-th/ax)

- Case rule AT

$$\dfrac{\dfrac{h_1 : \Delta_{11}, []F_9 \vdash F_6, F_7, \Delta_{10}}{\bullet h_1 : \Delta_{11}, []F_9 \vdash \Delta_{10}, F_6 \vee F_7}\ \vee_R \quad \dfrac{h_8 : F_9, \Delta_{11}, []F_9, F_6 \vee F_7 \vdash \Delta_{10}}{\bullet h_8 : (\Delta_{11}, []F_9), F_6 \vee F_7 \vdash \Delta_{10}}\ AT}{- : \Delta_{11}, []F_9 \vdash \Delta_{10}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}, F_6 \vee F_7}\ \text{ax/W} \quad \dfrac{}{h_8 : \Delta_{11}, F_9, []F_9, F_6 \vee F_7 \vdash \Delta_{10}}\ \text{ax/W}}{\dfrac{- : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}}{- : \Delta_{11}, []F_9 \vdash \Delta_{10}}\ ATG}\ \text{hCut}}{}$$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash []F_{12}, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), []F_{12}}\ \vee_R \quad \dfrac{h_{10} : F_{12}, \Delta_{11}, []F_{12} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, []F_{12} \vdash \Delta_7, F_8 \vee F_9}\ AT}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, []F_{12}}\ \text{ax/W} \quad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12}, []F_{12} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, []F_{12} \vdash \Delta_7, F_8, F_9}\ AT}{}}{\dfrac{- : \Delta_{11} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}\ \vee_R}\ \text{hCut}}{}$$

(inner step labelled inv-th/ax)

$$\dfrac{\dfrac{h_2 : \Delta_{13}, []F_{12} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{13}, []F_{12} \vdash (\Delta_8, F_9 \vee F_{10}), F_7}\ \vee_R \quad \dfrac{h_{11} : F_7, F_{12}, \Delta_{13}, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{13}, []F_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}}\ AT}{- : \Delta_{13}, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_2 : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_7, F_9 \vee F_{10}}\ \text{ax/W} \quad \dfrac{}{h_{11} : \Delta_{13}, F_{12}, F_7, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}}\ \text{ax/W}}{\dfrac{- : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}}{- : \Delta_{13}, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}}\ AT}\ \text{hCut}}{}$$

- Case rule $\bot_L$

$$
\dfrac{
\dfrac{\mathtt{h_1}:\bot,\Delta_{10}\vdash \mathtt{F_6},\mathtt{F_7},\Delta_9}{\bullet\mathtt{h_1}:\bot,\Delta_{10}\vdash \Delta_9,\mathtt{F_6}\vee \mathtt{F_7}}\ \vee_R
\qquad
\dfrac{}{\bullet\mathtt{h_8}:(\bot,\Delta_{10}),\mathtt{F_6}\vee \mathtt{F_7}\vdash \Delta_9}\ \bot_L
}{-:\bot,\Delta_{10}\vdash \Delta_9}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{}{-:\bot,\Delta_{10}\vdash \Delta_9}\ \bot_L
$$

$$
\dfrac{
\dfrac{\mathtt{h_2}:\Delta_{11}\vdash \bot,\mathtt{F_8},\mathtt{F_9},\Delta_7}{\bullet\mathtt{h_2}:\Delta_{11}\vdash (\Delta_7,\mathtt{F_8}\vee \mathtt{F_9}),\bot}\ \vee_R
\qquad
\dfrac{}{\bullet\mathtt{h_{10}}:\Delta_{11},\bot\vdash \Delta_7,\mathtt{F_8}\vee \mathtt{F_9}}\ \bot_L
}{-:\Delta_{11}\vdash \Delta_7,\mathtt{F_8}\vee \mathtt{F_9}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{
\dfrac{
\dfrac{}{\mathtt{h_2}:\Delta_{11}\vdash \bot,\Delta_7,\mathtt{F_8},\mathtt{F_9}}\ \mathtt{ax/W}
\qquad
\dfrac{}{\bullet\mathtt{h_{10}}:\bot,\Delta_{11}\vdash \Delta_7,\mathtt{F_8},\mathtt{F_9}}\ \bot_L
}{-:\Delta_{11}\vdash \Delta_7,\mathtt{F_8},\mathtt{F_9}}\ \mathtt{hCut}
}{-:\Delta_{11}\vdash \Delta_7,\mathtt{F_8}\vee \mathtt{F_9}}\ \vee_R
$$

$$
\dfrac{
\dfrac{\mathtt{h_2}:\bot,\Delta_{12}\vdash \mathtt{F_7},\mathtt{F_9},\mathtt{F_{10}},\Delta_8}{\bullet\mathtt{h_2}:\bot,\Delta_{12}\vdash (\Delta_8,\mathtt{F_9}\vee \mathtt{F_{10}}),\mathtt{F_7}}\ \vee_R
\qquad
\dfrac{}{\bullet\mathtt{h_{11}}:(\bot,\Delta_{12}),\mathtt{F_7}\vdash \Delta_8,\mathtt{F_9}\vee \mathtt{F_{10}}}\ \bot_L
}{-:\bot,\Delta_{12}\vdash \Delta_8,\mathtt{F_9}\vee \mathtt{F_{10}}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{}{-:\bot,\Delta_{12}\vdash \Delta_8,\mathtt{F_9}\vee \mathtt{F_{10}}}\ \bot_L
$$

- Case rule I

$$
\dfrac{
\dfrac{\mathtt{h_1}:\Delta_{11},\mathtt{p_9}\vdash \mathtt{F_6},\mathtt{F_7},\Delta_{10},\mathtt{p_9}}{\bullet\mathtt{h_1}:\Delta_{11},\mathtt{p_9}\vdash (\Delta_{10},\mathtt{p_9}),\mathtt{F_6}\vee \mathtt{F_7}}\ \vee_R
\qquad
\dfrac{}{\bullet\mathtt{h_8}:(\Delta_{11},\mathtt{p_9}),\mathtt{F_6}\vee \mathtt{F_7}\vdash \Delta_{10},\mathtt{p_9}}\ I
}{-:\Delta_{11},\mathtt{p_9}\vdash \Delta_{10},\mathtt{p_9}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{}{-:\Delta_{11},\mathtt{p_9}\vdash \Delta_{10},\mathtt{p_9}}\ I
$$

$$
\dfrac{
\dfrac{\mathtt{h_2}:\Delta_{10}\vdash \mathtt{p_{11}},\mathtt{F_7},\mathtt{F_8},\Delta_{12},\mathtt{p_{11}}}{\bullet\mathtt{h_2}:\Delta_{10}\vdash ((\Delta_{12},\mathtt{p_{11}}),\mathtt{F_7}\vee \mathtt{F_8}),\mathtt{p_{11}}}\ \vee_R
\qquad
\dfrac{}{\bullet\mathtt{h_9}:\Delta_{10},\mathtt{p_{11}}\vdash (\Delta_{12},\mathtt{p_{11}}),\mathtt{F_7}\vee \mathtt{F_8}}\ I
}{-:\Delta_{10}\vdash (\Delta_{12},\mathtt{p_{11}}),\mathtt{F_7}\vee \mathtt{F_8}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{
\dfrac{
\dfrac{}{\mathtt{h_2}:\Delta_{10}\vdash \Delta_{12},\mathtt{F_7},\mathtt{F_8},\mathtt{p_{11}},\mathtt{p_{11}}}\ \mathtt{ax/W}
\qquad
\dfrac{}{\bullet\mathtt{h_9}:\Delta_{10},\mathtt{p_{11}}\vdash \Delta_{12},\mathtt{F_7},\mathtt{F_8},\mathtt{p_{11}}}\ I
}{-:\Delta_{10}\vdash \Delta_{12},\mathtt{F_7},\mathtt{F_8},\mathtt{p_{11}}}\ \mathtt{hCut}
}{-:\Delta_{10}\vdash \Delta_{12},\mathtt{p_{11}},\mathtt{F_7}\vee \mathtt{F_8}}\ \vee_R
$$

$$
\dfrac{
\dfrac{\mathtt{h_2}:\Delta_{13},\mathtt{p_{11}}\vdash \mathtt{F_7},\mathtt{F_8},\mathtt{F_9},\Delta_{12},\mathtt{p_{11}}}{\bullet\mathtt{h_2}:\Delta_{13},\mathtt{p_{11}}\vdash ((\Delta_{12},\mathtt{p_{11}}),\mathtt{F_8}\vee \mathtt{F_9}),\mathtt{F_7}}\ \vee_R
\quad
\dfrac{}{\bullet\mathtt{h_{10}}:(\Delta_{13},\mathtt{p_{11}}),\mathtt{F_7}\vdash (\Delta_{12},\mathtt{p_{11}}),\mathtt{F_8}\vee \mathtt{F_9}}\ I
}{-:\Delta_{13},\mathtt{p_{11}}\vdash (\Delta_{12},\mathtt{p_{11}}),\mathtt{F_8}\vee \mathtt{F_9}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{}{-:\Delta_{13},\mathtt{p_{11}}\vdash \Delta_{12},\mathtt{p_{11}},\mathtt{F_8}\vee \mathtt{F_9}}\ I
$$

- Case rule $\top_L$

$$
\dfrac{
\dfrac{\mathtt{h_1}:\top,\Delta_{10}\vdash \mathtt{F_6},\mathtt{F_7},\Delta_9}{\bullet\mathtt{h_1}:\top,\Delta_{10}\vdash \Delta_9,\mathtt{F_6}\vee \mathtt{F_7}}\ \vee_R
\qquad
\dfrac{\mathtt{h_8}:\Delta_{10},\mathtt{F_6}\vee \mathtt{F_7}\vdash \Delta_9}{\bullet\mathtt{h_8}:(\top,\Delta_{10}),\mathtt{F_6}\vee \mathtt{F_7}\vdash \Delta_9}\ \top_L
}{-:\top,\Delta_{10}\vdash \Delta_9}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{
\dfrac{}{\bullet\mathtt{h_1}:\top,\Delta_{10}\vdash \Delta_9,\mathtt{F_6}\vee \mathtt{F_7}}\ \mathtt{ax/W}
\qquad
\dfrac{}{\mathtt{h_8}:\top,\Delta_{10},\mathtt{F_6}\vee \mathtt{F_7}\vdash \Delta_9}\ \mathtt{ax/W}
}{-:\top,\Delta_{10}\vdash \Delta_9}\ \mathtt{hCut}
$$

$$
\dfrac{
\dfrac{\mathtt{h_2}:\Delta_{11}\vdash \top,\mathtt{F_8},\mathtt{F_9},\Delta_7}{\bullet\mathtt{h_2}:\Delta_{11}\vdash (\Delta_7,\mathtt{F_8}\vee \mathtt{F_9}),\top}\ \vee_R
\qquad
\dfrac{\mathtt{h_{10}}:\Delta_{11}\vdash \Delta_7,\mathtt{F_8}\vee \mathtt{F_9}}{\bullet\mathtt{h_{10}}:\Delta_{11},\top\vdash \Delta_7,\mathtt{F_8}\vee \mathtt{F_9}}\ \top_L
}{-:\Delta_{11}\vdash \Delta_7,\mathtt{F_8}\vee \mathtt{F_9}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{}{-:\Delta_{11}\vdash \Delta_7,\mathtt{F_8}\vee \mathtt{F_9}}\ \mathtt{ax/W}
$$

$$\dfrac{\dfrac{h_2 : \top, \Delta_{12} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \ \vee_R \quad \dfrac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \ \top_L}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}} \ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \vee F_{10}} \ \text{ax/W} \quad \dfrac{}{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \vee F_{10}} \ \text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}} \ \text{hCut}$$

6.4 Status of $\perp_R$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \rightarrow F_8), \perp} \ \perp_R \quad \dfrac{h_5 : \perp, F_7, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \rightarrow F_8} \ \rightarrow_R}{- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8} \ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8} \ \text{ax/W}$$

$$\dfrac{\dfrac{h_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \rightarrow F_9}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_{10}, F_8 \rightarrow F_9), F_5} \ \perp_R \quad \dfrac{h_7 : F_5, F_8, \Delta_6 \vdash \perp, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \ \rightarrow_R}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_6 \vdash \perp, \Delta_{10}, F_5, F_8 \rightarrow F_9} \ \text{ax/W} \quad \dfrac{}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \ \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \ \text{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \wedge F_8), \perp} \ \perp_R \quad \dfrac{h_5 : \perp, \Delta_4 \vdash F_7, \Delta_6 \quad h_5 : \perp, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \wedge F_8} \ \wedge_R}{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \ \text{ax/W}$$

$$\dfrac{\dfrac{h_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \wedge F_9}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_{10}, F_8 \wedge F_9), F_5} \ \perp_R \quad \dfrac{h_7 : F_5, \Delta_6 \vdash \perp, F_8, \Delta_{10} \quad h_7 : F_5, \Delta_6 \vdash \perp, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \ \wedge_R}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_6 \vdash \perp, \Delta_{10}, F_5, F_8 \wedge F_9} \ \text{ax/W} \quad \dfrac{}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \ \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \ \text{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \vee F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \vee F_8), \perp} \ \perp_R \quad \dfrac{h_5 : \perp, \Delta_4 \vdash F_7, F_8, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \vee F_8} \ \vee_R}{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \ \text{ax/W}$$

$$\dfrac{\dfrac{h_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \vee F_9}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_{10}, F_8 \vee F_9), F_5} \ \perp_R \quad \dfrac{h_7 : F_5, \Delta_6 \vdash \perp, F_8, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \ \vee_R}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_6 \vdash \perp, \Delta_{10}, F_5, F_8 \vee F_9} \ \text{ax/W} \quad \dfrac{}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \ \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \ \text{hCut}$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{h_1 : \Delta_4 \vdash \perp, \Delta_6}{\bullet h_1 : \Delta_4 \vdash (\perp, \Delta_6), \perp} \ \perp_R \quad \dfrac{h_5 : \perp, \Delta_4 \vdash \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \perp, \Delta_6} \ \perp_R}{- : \Delta_4 \vdash \perp, \Delta_6} \ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \perp, \Delta_6} \ \text{ax/W}$$

53

$$\cfrac{\cfrac{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{F}_5, \Delta_8}{\bullet\mathtt{h}_2 : \Delta_6 \vdash (\bot, \Delta_8), \mathtt{F}_5}\ \bot_R \qquad \cfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_8}\ \bot_R}{- : \Delta_6 \vdash \bot, \Delta_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{\mathtt{h}_2 : \Delta_6 \vdash \bot, \Delta_8, \mathtt{F}_5}\ \texttt{ax/W} \qquad \cfrac{}{\bullet\mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_8}\ \texttt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_8}\ \texttt{hCut}$$

- Case rule $\top_R$

$$\cfrac{\cfrac{\mathtt{h}_1 : \Delta_4 \vdash \top, \Delta_6}{\bullet\mathtt{h}_1 : \Delta_4 \vdash (\top, \Delta_6), \bot}\ \bot_R \qquad \cfrac{}{\bullet\mathtt{h}_5 : \Delta_4, \bot \vdash \top, \Delta_6}\ \top_R}{- : \Delta_4 \vdash \top, \Delta_6}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_4 \vdash \top, \Delta_6}\ \top_R$$

$$\cfrac{\cfrac{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{F}_5, \top, \Delta_8}{\bullet\mathtt{h}_2 : \Delta_6 \vdash (\bot, \top, \Delta_8), \mathtt{F}_5}\ \bot_R \qquad \cfrac{}{\bullet\mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \top, \Delta_8}\ \top_R}{- : \Delta_6 \vdash \bot, \top, \Delta_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_6 \vdash \bot, \top, \Delta_8}\ \top_R$$

- Case rule K

$$\cfrac{\cfrac{\mathtt{h}_1 : \Box\Gamma_5, \Delta_8 \vdash \Delta_6, []\mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_5, \Delta_8 \vdash (\Delta_6, []\mathtt{F}_7), \bot}\ \bot_R \qquad \cfrac{\mathtt{h}_4 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_4 : (\Box\Gamma_5, \Delta_8), \bot \vdash \Delta_6, []\mathtt{F}_7}\ K}{- : \Box\Gamma_5, \Delta_8 \vdash \Delta_6, []\mathtt{F}_7}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_8, \Box\Gamma_5 \vdash \Delta_6, []\mathtt{F}_7}\ \texttt{ax/W}$$

$$\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_{10}, \Delta_7 \vdash \Box\mathtt{F}_5, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_2 : \Box\Gamma_{10}, \Delta_7 \vdash (\bot, \Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_5}\ \bot_R \qquad \cfrac{\mathtt{h}_6 : unbox(\Box\Gamma_{10}), unbox(\Box\mathtt{F}_5) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Box\Gamma_{10}, \Delta_7), \Box\mathtt{F}_5 \vdash \bot, \Delta_9, []\mathtt{F}_8}\ K}{- : \Box\Gamma_{10}, \Delta_7 \vdash \bot, \Delta_9, []\mathtt{F}_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{\mathtt{h}_2 : \Delta_7, \Box\Gamma_{10} \vdash \bot, \Box\mathtt{F}_5, \Delta_9, []\mathtt{F}_8}\ \texttt{ax/W} \qquad \cfrac{}{\bullet\mathtt{h}_6 : \Box\mathtt{F}_5, \Delta_7, \Box\Gamma_{10} \vdash \bot, \Delta_9, []\mathtt{F}_8}\ \texttt{ax/W}}{- : \Delta_7, \Box\Gamma_{10} \vdash \bot, \Delta_9, []\mathtt{F}_8}\ \texttt{hCut}$$

$$\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_7, \Delta_{10} \vdash \mathtt{F}_5, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_{10} \vdash (\bot, \Delta_9, []\mathtt{F}_8), \mathtt{F}_5}\ \bot_R \qquad \cfrac{\mathtt{h}_6 : unbox(\Box\Gamma_7) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Box\Gamma_7, \Delta_{10}), \mathtt{F}_5 \vdash \bot, \Delta_9, []\mathtt{F}_8}\ K}{- : \Box\Gamma_7, \Delta_{10} \vdash \bot, \Delta_9, []\mathtt{F}_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- : unbox(\Box\Gamma_7) \vdash \mathtt{F}_8}\ \texttt{ax/W}}{- : \Delta_{10}, \Box\Gamma_7 \vdash \bot, \Delta_9, []\mathtt{F}_8}\ K$$

- Case rule $A45$

$$\cfrac{\cfrac{\mathtt{h}_1 : \Box\Gamma_5, \Delta_9 \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_5, \Delta_9 \vdash (\Box\Gamma_6, \Delta_7, []\mathtt{F}_8), \bot}\ \bot_R \qquad \cfrac{\mathtt{h}_4 : \Box\Gamma_5 \vdash \Box\Gamma_6, \mathtt{F}_8}{\bullet\mathtt{h}_4 : (\Box\Gamma_5, \Delta_9), \bot \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_8}\ A45}{- : \Box\Gamma_5, \Delta_9 \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_9, \Box\Gamma_5 \vdash \Delta_7, \Box\Gamma_6, []\mathtt{F}_8}\ \texttt{ax/W}$$

$$\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_{11}, \Delta_7 \vdash \Box\mathtt{F}_5, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9}{\bullet\mathtt{h}_2 : \Box\Gamma_{11}, \Delta_7 \vdash (\bot, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9), \Box\mathtt{F}_5}\ \bot_R \qquad \cfrac{\mathtt{h}_6 : \Box\Gamma_{11}, \Box\mathtt{F}_5 \vdash \Box\Gamma_8, \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Box\Gamma_{11}, \Delta_7), \Box\mathtt{F}_5 \vdash \bot, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9}\ A45}{- : \Box\Gamma_{11}, \Delta_7 \vdash \bot, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{\mathtt{h}_2 : \Delta_7, \Box\Gamma_{11} \vdash \bot, \Box\mathtt{F}_5, \Delta_{10}, \Box\Gamma_8, []\mathtt{F}_9}\ \texttt{ax/W} \qquad \cfrac{}{\bullet\mathtt{h}_6 : \Box\mathtt{F}_5, \Delta_7, \Box\Gamma_{11} \vdash \bot, \Delta_{10}, \Box\Gamma_8, []\mathtt{F}_9}\ \texttt{ax/W}}{- : \Delta_7, \Box\Gamma_{11} \vdash \bot, \Delta_{10}, \Box\Gamma_8, []\mathtt{F}_9}\ \texttt{hCut}$$

$$\frac{\dfrac{h_2 : \Box\Gamma_7, \Delta_{11} \vdash F_5, \Box\Gamma_8, \Delta_{10}, []F_9}{\bullet h_2 : \Box\Gamma_7, \Delta_{11} \vdash (\bot, \Box\Gamma_8, \Delta_{10}, []F_9), F_5}\ \bot_R \qquad \dfrac{h_6 : \Box\Gamma_7 \vdash \Box\Gamma_8, F_9}{\bullet h_6 : (\Box\Gamma_7, \Delta_{11}), F_5 \vdash \bot, \Box\Gamma_8, \Delta_{10}, []F_9}\ A45}{- : \Box\Gamma_7, \Delta_{11} \vdash \bot, \Box\Gamma_8, \Delta_{10}, []F_9}\ \text{Cut}$$

$$\xrightarrow{}$$

$$\frac{\dfrac{}{- : \Box\Gamma_7 \vdash F_9, \Box\Gamma_8}\ \text{ax/W}}{- : \Delta_{11}, \Box\Gamma_7 \vdash \bot, \Delta_{10}, \Box\Gamma_8, []F_9}\ A45$$

- Case rule $\to_L$

$$\frac{\dfrac{h_1 : \Delta_8, F_5 \to F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \to F_6 \vdash \Delta_7, \bot}\ \bot_R \qquad \dfrac{h_4 : \bot, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \bot, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \to F_6), \bot \vdash \Delta_7}\ \to_L}{- : \Delta_8, F_5 \to F_6 \vdash \Delta_7}\ \text{Cut}$$

$$\xrightarrow{}$$

$$\frac{}{- : \Delta_8, F_5 \to F_6 \vdash \Delta_7}\ \text{ax/W}$$

$$\frac{\dfrac{h_2 : \Delta_7 \vdash F_8 \to F_9, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\bot, \Delta_5), F_8 \to F_9}\ \bot_R \qquad \dfrac{h_6 : \Delta_7 \vdash \bot, F_8, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \bot, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash \bot, \Delta_5}\ \to_L}{- : \Delta_7 \vdash \bot, \Delta_5}\ \text{Cut}$$

$$\xrightarrow{}$$

$$\frac{\dfrac{}{h_2 : \Delta_7 \vdash \bot, \Delta_5, F_8 \to F_9}\ \text{ax/W} \qquad \dfrac{}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash \bot, \Delta_5}\ \text{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5}\ \text{hCut}$$

$$\frac{\dfrac{h_2 : \Delta_{10}, F_8 \to F_9 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_{10}, F_8 \to F_9 \vdash (\bot, \Delta_6), F_5}\ \bot_R \qquad \dfrac{h_7 : F_5, \Delta_{10} \vdash \bot, F_8, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \bot, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \to F_9), F_5 \vdash \bot, \Delta_6}\ \to_L}{- : \Delta_{10}, F_8 \to F_9 \vdash \bot, \Delta_6}\ \text{Cut}$$

$$\xrightarrow{}$$

$$\frac{\dfrac{}{h_2 : \Delta_{10}, F_8 \to F_9 \vdash \bot, \Delta_6, F_5}\ \text{ax/W} \qquad \dfrac{}{\bullet h_7 : \Delta_{10}, F_5, F_8 \to F_9 \vdash \bot, \Delta_6}\ \text{ax/W}}{- : \Delta_{10}, F_8 \to F_9 \vdash \bot, \Delta_6}\ \text{hCut}$$

- Case rule $\wedge_L$

$$\frac{\dfrac{h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \bot}\ \bot_R \qquad \dfrac{h_4 : \bot, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \bot \vdash \Delta_7}\ \wedge_L}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}\ \text{Cut}$$

$$\xrightarrow{}$$

$$\frac{}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}\ \text{ax/W}$$

$$\frac{\dfrac{h_2 : \Delta_7 \vdash F_8 \wedge F_9, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\bot, \Delta_5), F_8 \wedge F_9}\ \bot_R \qquad \dfrac{h_6 : F_8, F_9, \Delta_7 \vdash \bot, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \bot, \Delta_5}\ \wedge_L}{- : \Delta_7 \vdash \bot, \Delta_5}\ \text{Cut}$$

$$\xrightarrow{}$$

$$\frac{\dfrac{}{h_2 : \Delta_7 \vdash \bot, \Delta_5, F_8 \wedge F_9}\ \text{ax/W} \qquad \dfrac{}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \bot, \Delta_5}\ \text{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5}\ \text{hCut}$$

$$\frac{\dfrac{h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash (\bot, \Delta_6), F_5}\ \bot_R \qquad \dfrac{h_7 : F_5, F_8, F_9, \Delta_{10} \vdash \bot, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \wedge F_9), F_5 \vdash \bot, \Delta_6}\ \wedge_L}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \bot, \Delta_6}\ \text{Cut}$$

$$\xrightarrow{}$$

$$\frac{\dfrac{}{h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash \bot, \Delta_6, F_5}\ \text{ax/W} \qquad \dfrac{}{\bullet h_7 : \Delta_{10}, F_5, F_8 \wedge F_9 \vdash \bot, \Delta_6}\ \text{ax/W}}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \bot, \Delta_6}\ \text{hCut}$$

- Case rule $\vee_L$

$$\frac{\dfrac{h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \bot}\ \bot_R \qquad \dfrac{h_4 : \bot, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \bot, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \bot \vdash \Delta_7}\ \vee_L}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}\ \text{Cut}$$

$$\xrightarrow{}$$

$$\frac{}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}\ \text{ax/W}$$

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$$\cfrac{\cfrac{h_2 : \Delta_7 \vdash F_8 \vee F_9, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\bot, \Delta_5), F_8 \vee F_9} \bot_R \quad \cfrac{h_6 : F_8, \Delta_7 \vdash \bot, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \bot, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \bot, \Delta_5} \vee_L}{- : \Delta_7 \vdash \bot, \Delta_5} \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_2 : \Delta_7 \vdash \bot, \Delta_5, F_8 \vee F_9} \text{ax/W} \quad \cfrac{}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \bot, \Delta_5} \text{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5} \text{hCut}$$

$$\cfrac{\cfrac{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\bot, \Delta_6), F_5} \bot_R \quad \cfrac{h_7 : F_5, F_8, \Delta_{10} \vdash \bot, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \bot, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \bot, \Delta_6} \vee_L}{- : \Delta_{10}, F_8 \vee F_9 \vdash \bot, \Delta_6} \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash \bot, \Delta_6, F_5} \text{ax/W} \quad \cfrac{}{\bullet h_7 : \Delta_{10}, F_5, F_8 \vee F_9 \vdash \bot, \Delta_6} \text{ax/W}}{- : \Delta_{10}, F_8 \vee F_9 \vdash \bot, \Delta_6} \text{hCut}$$

- Case rule AT

$$\cfrac{\cfrac{h_1 : \Delta_7, []F_5 \vdash \Delta_6}{\bullet h_1 : \Delta_7, []F_5 \vdash \Delta_6, \bot} \bot_R \quad \cfrac{h_4 : \bot, F_5, \Delta_7, []F_5 \vdash \Delta_6}{\bullet h_4 : (\Delta_7, []F_5), \bot \vdash \Delta_6} AT}{- : \Delta_7, []F_5 \vdash \Delta_6} \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_7, []F_5 \vdash \Delta_6} \text{ax/W}$$

$$\cfrac{\cfrac{h_2 : \Delta_7 \vdash []F_8, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\bot, \Delta_5), []F_8} \bot_R \quad \cfrac{h_6 : F_8, \Delta_7, []F_8 \vdash \bot, \Delta_5}{\bullet h_6 : \Delta_7, []F_8 \vdash \bot, \Delta_5} AT}{- : \Delta_7 \vdash \bot, \Delta_5} \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_2 : \Delta_7 \vdash \bot, \Delta_5, []F_8} \text{ax/W} \quad \cfrac{}{\bullet h_6 : \Delta_7, []F_8 \vdash \bot, \Delta_5} \text{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5} \text{hCut}$$

$$\cfrac{\cfrac{h_2 : \Delta_9, []F_8 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_9, []F_8 \vdash (\bot, \Delta_6), F_5} \bot_R \quad \cfrac{h_7 : F_5, F_8, \Delta_9, []F_8 \vdash \bot, \Delta_6}{\bullet h_7 : (\Delta_9, []F_8), F_5 \vdash \bot, \Delta_6} AT}{- : \Delta_9, []F_8 \vdash \bot, \Delta_6} \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_2 : \Delta_9, []F_8 \vdash \bot, \Delta_6, F_5} \text{ax/W} \quad \cfrac{}{\bullet h_7 : \Delta_9, F_5, []F_8 \vdash \bot, \Delta_6} \text{ax/W}}{- : \Delta_9, []F_8 \vdash \bot, \Delta_6} \text{hCut}$$

- Case rule $\bot_L$

$$\cfrac{\cfrac{h_1 : \Delta_5 \vdash \Delta_6}{\bullet h_1 : \Delta_5 \vdash \Delta_6, \bot} \bot_R \quad \cfrac{}{\bullet h_4 : \Delta_5, \bot \vdash \Delta_6} \bot_L}{- : \Delta_5 \vdash \Delta_6} \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_5 \vdash \Delta_6} \text{ax/W}$$

$$\cfrac{\cfrac{h_2 : \Delta_7 \vdash \bot, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\bot, \Delta_5), \bot} \bot_R \quad \cfrac{}{\bullet h_6 : \Delta_7, \bot \vdash \bot, \Delta_5} \bot_L}{- : \Delta_7 \vdash \bot, \Delta_5} \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_7 \vdash \bot, \Delta_5} \text{ax/W}$$

$$\cfrac{\cfrac{h_2 : \bot, \Delta_8 \vdash F_5, \Delta_6}{\bullet h_2 : \bot, \Delta_8 \vdash (\bot, \Delta_6), F_5} \bot_R \quad \cfrac{}{\bullet h_7 : (\bot, \Delta_8), F_5 \vdash \bot, \Delta_6} \bot_L}{- : \bot, \Delta_8 \vdash \bot, \Delta_6} \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \bot, \Delta_8 \vdash \bot, \Delta_6} \bot_L$$

- Case rule I

$$\cfrac{\cfrac{h_1 : \Delta_7, p_5 \vdash \Delta_6, p_5}{\bullet h_1 : \Delta_7, p_5 \vdash (\Delta_6, p_5), \bot} \bot_R \quad \cfrac{}{\bullet h_4 : (\Delta_7, p_5), \bot \vdash \Delta_6, p_5} I}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} I$$

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$$\dfrac{\dfrac{\texttt{h}_2 : \Delta_6 \vdash \texttt{p}_7, \Delta_8, \texttt{p}_7}{\bullet\texttt{h}_2 : \Delta_6 \vdash (\bot, \Delta_8, \texttt{p}_7), \texttt{p}_7} \perp_R \quad \overline{\bullet\texttt{h}_5 : \Delta_6, \texttt{p}_7 \vdash \bot, \Delta_8, \texttt{p}_7} \; I}{-: \Delta_6 \vdash \bot, \Delta_8, \texttt{p}_7} \; \text{Cut}$$

$$\xrightarrow{\;\;}$$

$$\dfrac{\dfrac{}{\texttt{h}_2 : \Delta_6 \vdash \bot, \Delta_8, \texttt{p}_7, \texttt{p}_7} \; \texttt{ax/W} \quad \overline{\bullet\texttt{h}_5 : \Delta_6, \texttt{p}_7 \vdash \bot, \Delta_8, \texttt{p}_7} \; I}{-: \Delta_6 \vdash \bot, \Delta_8, \texttt{p}_7} \; \text{hCut}$$

$$\dfrac{\dfrac{\texttt{h}_2 : \Delta_9, \texttt{p}_7 \vdash \texttt{F}_5, \Delta_8, \texttt{p}_7}{\bullet\texttt{h}_2 : \Delta_9, \texttt{p}_7 \vdash (\bot, \Delta_8, \texttt{p}_7), \texttt{F}_5} \perp_R \quad \overline{\bullet\texttt{h}_6 : (\Delta_9, \texttt{p}_7), \texttt{F}_5 \vdash \bot, \Delta_8, \texttt{p}_7} \; I}{-: \Delta_9, \texttt{p}_7 \vdash \bot, \Delta_8, \texttt{p}_7} \; \text{Cut}$$

$$\xrightarrow{\;\;}$$

$$\dfrac{}{-: \Delta_9, \texttt{p}_7 \vdash \bot, \Delta_8, \texttt{p}_7} \; I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\texttt{h}_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet\texttt{h}_1 : \top, \Delta_6 \vdash \Delta_5, \bot} \perp_R \quad \dfrac{\texttt{h}_4 : \bot, \Delta_6 \vdash \Delta_5}{\bullet\texttt{h}_4 : (\top, \Delta_6), \bot \vdash \Delta_5} \; \top_L}{-: \top, \Delta_6 \vdash \Delta_5} \; \text{Cut}$$

$$\xrightarrow{\;\;}$$

$$\dfrac{}{-: \top, \Delta_6 \vdash \Delta_5} \; \texttt{ax/W}$$

$$\dfrac{\dfrac{\texttt{h}_2 : \Delta_7 \vdash \top, \Delta_5}{\bullet\texttt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \top} \perp_R \quad \dfrac{\texttt{h}_6 : \Delta_7 \vdash \bot, \Delta_5}{\bullet\texttt{h}_6 : \Delta_7, \top \vdash \bot, \Delta_5} \; \top_L}{-: \Delta_7 \vdash \bot, \Delta_5} \; \text{Cut}$$

$$\xrightarrow{\;\;}$$

$$\dfrac{}{-: \Delta_7 \vdash \bot, \Delta_5} \; \texttt{ax/W}$$

$$\dfrac{\dfrac{\texttt{h}_2 : \top, \Delta_8 \vdash \texttt{F}_5, \Delta_6}{\bullet\texttt{h}_2 : \top, \Delta_8 \vdash (\bot, \Delta_6), \texttt{F}_5} \perp_R \quad \dfrac{\texttt{h}_7 : \texttt{F}_5, \Delta_8 \vdash \bot, \Delta_6}{\bullet\texttt{h}_7 : (\top, \Delta_8), \texttt{F}_5 \vdash \bot, \Delta_6} \; \top_L}{-: \top, \Delta_8 \vdash \bot, \Delta_6} \; \text{Cut}$$

$$\xrightarrow{\;\;}$$

$$\dfrac{\dfrac{}{\texttt{h}_2 : \top, \Delta_8 \vdash \bot, \Delta_6, \texttt{F}_5} \; \texttt{ax/W} \quad \dfrac{}{\bullet\texttt{h}_7 : \top, \Delta_8, \texttt{F}_5 \vdash \bot, \Delta_6} \; \texttt{ax/W}}{-: \top, \Delta_8 \vdash \bot, \Delta_6} \; \text{hCut}$$

6.5 Status of $\top_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{}{\bullet\texttt{h}_1 : \Delta_4 \vdash (\Delta_6, \texttt{F}_7 \to \texttt{F}_8), \top} \; \top_R \quad \dfrac{\texttt{h}_5 : \top, \texttt{F}_7, \Delta_4 \vdash \texttt{F}_8, \Delta_6}{\bullet\texttt{h}_5 : \Delta_4, \top \vdash \Delta_6, \texttt{F}_7 \to \texttt{F}_8} \to_R}{-: \Delta_4 \vdash \Delta_6, \texttt{F}_7 \to \texttt{F}_8} \; \text{Cut}$$

$$\xrightarrow{\;\;}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\texttt{h}_1 : \Delta_4, \texttt{F}_7 \vdash \top, \Delta_6, \texttt{F}_8} \; \top_R \quad \dfrac{}{\texttt{h}_5 : \top, \Delta_4, \texttt{F}_7 \vdash \Delta_6, \texttt{F}_8} \; \texttt{ax/W}}{-: \Delta_4, \texttt{F}_7 \vdash \Delta_6, \texttt{F}_8} \; \text{hCut}}{-: \Delta_4 \vdash \Delta_6, \texttt{F}_7 \to \texttt{F}_8} \to_R$$

$$\dfrac{\dfrac{}{\bullet\texttt{h}_2 : \Delta_6 \vdash (\top, \Delta_{10}, \texttt{F}_8 \to \texttt{F}_9), \texttt{F}_5} \; \top_R \quad \dfrac{\texttt{h}_7 : \texttt{F}_5, \texttt{F}_8, \Delta_6 \vdash \top, \texttt{F}_9, \Delta_{10}}{\bullet\texttt{h}_7 : \Delta_6, \texttt{F}_5 \vdash \top, \Delta_{10}, \texttt{F}_8 \to \texttt{F}_9} \to_R}{-: \Delta_6 \vdash \top, \Delta_{10}, \texttt{F}_8 \to \texttt{F}_9} \; \text{Cut}$$

$$\xrightarrow{\;\;}$$

$$\dfrac{}{-: \Delta_6 \vdash \top, \Delta_{10}, \texttt{F}_8 \to \texttt{F}_9} \; \top_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet\texttt{h}_1 : \Delta_4 \vdash (\Delta_6, \texttt{F}_7 \wedge \texttt{F}_8), \top} \; \top_R \quad \dfrac{\texttt{h}_5 : \top, \Delta_4 \vdash \texttt{F}_7, \Delta_6 \quad \texttt{h}_5 : \top, \Delta_4 \vdash \texttt{F}_8, \Delta_6}{\bullet\texttt{h}_5 : \Delta_4, \top \vdash \Delta_6, \texttt{F}_7 \wedge \texttt{F}_8} \wedge_R}{-: \Delta_4 \vdash \Delta_6, \texttt{F}_7 \wedge \texttt{F}_8} \; \text{Cut}$$

$$\xrightarrow{\;\;}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\texttt{h}_1 : \Delta_4 \vdash \top, \Delta_6, \texttt{F}_7} \; \top_R \quad \dfrac{}{\texttt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \texttt{F}_7} \; \texttt{ax/W}}{-: \Delta_4 \vdash \Delta_6, \texttt{F}_7} \; \text{hCut} \quad \dfrac{\dfrac{}{\bullet\texttt{h}_1 : \Delta_4 \vdash \top, \Delta_6, \texttt{F}_8} \; \top_R \quad \dfrac{}{\texttt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \texttt{F}_8} \; \texttt{ax/W}}{-: \Delta_4 \vdash \Delta_6, \texttt{F}_8} \; \text{hCut}}{-: \Delta_4 \vdash \Delta_6, \texttt{F}_7 \wedge \texttt{F}_8} \wedge_R$$

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$$\cfrac{\cfrac{}{\bullet\mathbf{h}_2 : \Delta_6 \vdash (\top, \Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9), \mathbf{F}_5}\ \top_R \quad \cfrac{\mathbf{h}_7 : \mathbf{F}_5, \Delta_6 \vdash \top, \mathbf{F}_8, \Delta_{10} \quad \mathbf{h}_7 : \mathbf{F}_5, \Delta_6 \vdash \top, \mathbf{F}_9, \Delta_{10}}{\bullet\mathbf{h}_7 : \Delta_6, \mathbf{F}_5 \vdash \top, \Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9}\ \wedge_R}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9}\ \top_R$$

- Case rule $\vee_R$

$$\cfrac{\cfrac{}{\bullet\mathbf{h}_1 : \Delta_4 \vdash (\Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8), \top}\ \top_R \quad \cfrac{\mathbf{h}_5 : \top, \Delta_4 \vdash \mathbf{F}_7, \mathbf{F}_8, \Delta_6}{\bullet\mathbf{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8}\ \vee_R}{- : \Delta_4 \vdash \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet\mathbf{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathbf{F}_7, \mathbf{F}_8}\ \top_R \quad \cfrac{}{\mathbf{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_8}\ \text{ax/W}}{- : \Delta_4 \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_8}\ \text{hCut}}{- : \Delta_4 \vdash \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8}\ \vee_R$$

$$\cfrac{\cfrac{}{\bullet\mathbf{h}_2 : \Delta_6 \vdash (\top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9), \mathbf{F}_5}\ \top_R \quad \cfrac{\mathbf{h}_7 : \mathbf{F}_5, \Delta_6 \vdash \top, \mathbf{F}_8, \mathbf{F}_9, \Delta_{10}}{\bullet\mathbf{h}_7 : \Delta_6, \mathbf{F}_5 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9}\ \vee_R}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9}\ \top_R$$

- Case rule $\perp_R$

$$\cfrac{\cfrac{}{\bullet\mathbf{h}_1 : \Delta_4 \vdash (\perp, \Delta_6), \top}\ \top_R \quad \cfrac{\mathbf{h}_5 : \top, \Delta_4 \vdash \Delta_6}{\bullet\mathbf{h}_5 : \Delta_4, \top \vdash \perp, \Delta_6}\ \perp_R}{- : \Delta_4 \vdash \perp, \Delta_6}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{\bullet\mathbf{h}_1 : \Delta_4 \vdash \perp, \top, \Delta_6}\ \text{ax/W} \quad \cfrac{}{\mathbf{h}_5 : \top, \Delta_4 \vdash \perp, \Delta_6}\ \text{ax/W}}{- : \Delta_4 \vdash \perp, \Delta_6}\ \text{hCut}$$

$$\cfrac{\cfrac{}{\bullet\mathbf{h}_2 : \Delta_6 \vdash (\top, \perp, \Delta_8), \mathbf{F}_5}\ \top_R \quad \cfrac{\mathbf{h}_7 : \mathbf{F}_5, \Delta_6 \vdash \top, \Delta_8}{\bullet\mathbf{h}_7 : \Delta_6, \mathbf{F}_5 \vdash \top, \perp, \Delta_8}\ \perp_R}{- : \Delta_6 \vdash \top, \perp, \Delta_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_6 \vdash \perp, \top, \Delta_8}\ \top_R$$

- Case rule $\top_R$

$$\cfrac{\cfrac{}{\bullet\mathbf{h}_1 : \Delta_4 \vdash (\top, \Delta_6), \top}\ \top_R \quad \cfrac{}{\bullet\mathbf{h}_5 : \Delta_4, \top \vdash \top, \Delta_6}\ \top_R}{- : \Delta_4 \vdash \top, \Delta_6}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_4 \vdash \top, \Delta_6}\ \top_R$$

$$\cfrac{\cfrac{}{\bullet\mathbf{h}_2 : \Delta_6 \vdash (\top, \Delta_8), \mathbf{F}_5}\ \top_R \quad \cfrac{}{\bullet\mathbf{h}_7 : \Delta_6, \mathbf{F}_5 \vdash \top, \Delta_8}\ \top_R}{- : \Delta_6 \vdash \top, \Delta_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_6 \vdash \top, \Delta_8}\ \top_R$$

- Case rule K

$$\cfrac{\cfrac{}{\bullet\mathbf{h}_1 : \Box\Gamma_5, \Delta_8 \vdash (\Delta_6, []\mathbf{F}_7), \top}\ \top_R \quad \cfrac{\mathbf{h}_4 : unbox(\Box\Gamma_5) \vdash \mathbf{F}_7}{\bullet\mathbf{h}_4 : (\Box\Gamma_5, \Delta_8), \top \vdash \Delta_6, []\mathbf{F}_7}\ K}{- : \Box\Gamma_5, \Delta_8 \vdash \Delta_6, []\mathbf{F}_7}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- : unbox(\Box\Gamma_5) \vdash \mathbf{F}_7}\ \text{ax/W}}{- : \Delta_8, \Box\Gamma_5 \vdash \Delta_6, []\mathbf{F}_7}\ K$$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_2 : \Box\Gamma_{10}, \Delta_7 \vdash (\top, \Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_5} \ \top_R
  \qquad
  \cfrac{\mathtt{h}_6 : unbox(\Box\Gamma_{10}), unbox(\Box\mathtt{F}_5) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Box\Gamma_{10}, \Delta_7), \Box\mathtt{F}_5 \vdash \top, \Delta_9, []\mathtt{F}_8} \ K
}{
  - : \Box\Gamma_{10}, \Delta_7 \vdash \top, \Delta_9, []\mathtt{F}_8
} \ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{}{- : \Delta_7, \Box\Gamma_{10} \vdash \top, \Delta_9, []\mathtt{F}_8} \ \top_R
$$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_{10} \vdash (\top, \Delta_9, []\mathtt{F}_8), \mathtt{F}_5} \ \top_R
  \qquad
  \cfrac{\mathtt{h}_6 : unbox(\Box\Gamma_7) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Box\Gamma_7, \Delta_{10}), \mathtt{F}_5 \vdash \top, \Delta_9, []\mathtt{F}_8} \ K
}{
  - : \Box\Gamma_7, \Delta_{10} \vdash \top, \Delta_9, []\mathtt{F}_8
} \ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{}{- : \Delta_{10}, \Box\Gamma_7 \vdash \top, \Delta_9, []\mathtt{F}_8} \ \top_R
$$

- Case rule $A45$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_1 : \Box\Gamma_5, \Delta_9 \vdash (\Box\Gamma_6, \Delta_7, []\mathtt{F}_8), \top} \ \top_R
  \qquad
  \cfrac{\mathtt{h}_4 : \Box\Gamma_5 \vdash \Box\Gamma_6, \mathtt{F}_8}{\bullet\mathtt{h}_4 : (\Box\Gamma_5, \Delta_9), \top \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_8} \ A45
}{
  - : \Box\Gamma_5, \Delta_9 \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_8
} \ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{
  \cfrac{}{- : \Box\Gamma_5 \vdash \mathtt{F}_8, \Box\Gamma_6} \ \mathtt{ax/W}
}{
  - : \Delta_9, \Box\Gamma_5 \vdash \Delta_7, \Box\Gamma_6, []\mathtt{F}_8
} \ A45
$$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_2 : \Box\Gamma_{11}, \Delta_7 \vdash (\top, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9), \Box\mathtt{F}_5} \ \top_R
  \qquad
  \cfrac{\mathtt{h}_6 : \Box\Gamma_{11}, \Box\mathtt{F}_5 \vdash \Box\Gamma_8, \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Box\Gamma_{11}, \Delta_7), \Box\mathtt{F}_5 \vdash \top, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9} \ A45
}{
  - : \Box\Gamma_{11}, \Delta_7 \vdash \top, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9
} \ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{}{- : \Delta_7, \Box\Gamma_{11} \vdash \top, \Delta_{10}, \Box\Gamma_8, []\mathtt{F}_9} \ \top_R
$$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_{11} \vdash (\top, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9), \mathtt{F}_5} \ \top_R
  \qquad
  \cfrac{\mathtt{h}_6 : \Box\Gamma_7 \vdash \Box\Gamma_8, \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Box\Gamma_7, \Delta_{11}), \mathtt{F}_5 \vdash \top, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9} \ A45
}{
  - : \Box\Gamma_7, \Delta_{11} \vdash \top, \Box\Gamma_8, \Delta_{10}, []\mathtt{F}_9
} \ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{}{- : \Delta_{11}, \Box\Gamma_7 \vdash \top, \Delta_{10}, \Box\Gamma_8, []\mathtt{F}_9} \ \top_R
$$

- Case rule $\rightarrow_L$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7, \top} \ \top_R
  \qquad
  \cfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \top, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6), \top \vdash \Delta_7} \ \rightarrow_L
}{
  - : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7
} \ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5} \ \top_R \quad \cfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5} \ \mathtt{ax/W}
  }{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5} \ \mathtt{hCut}
  \qquad
  \cfrac{
  \cfrac{}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7} \ \top_R \quad \cfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7} \ \mathtt{ax/W}
  }{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7} \ \mathtt{hCut}
$$

$$
\overline{- : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7} \ \rightarrow_L
$$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \mathtt{F}_8 \rightarrow \mathtt{F}_9} \ \top_R
  \qquad
  \cfrac{\mathtt{h}_6 : \Delta_7 \vdash \top, \mathtt{F}_8, \Delta_5 \quad \mathtt{h}_6 : \mathtt{F}_9, \Delta_7 \vdash \top, \Delta_5}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \top, \Delta_5} \ \rightarrow_L
}{
  - : \Delta_7 \vdash \top, \Delta_5
} \ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{}{- : \Delta_7 \vdash \top, \Delta_5} \ \top_R
$$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash (\top, \Delta_6), \mathtt{F}_5} \ \top_R
  \qquad
  \cfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_{10} \vdash \top, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet\mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9), \mathtt{F}_5 \vdash \top, \Delta_6} \ \rightarrow_L
}{
  - : \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \top, \Delta_6
} \ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{}{- : \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \top, \Delta_6} \ \top_R
$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{h_4 : \top, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \top \vdash \Delta_7}\ \wedge_L}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_5, F_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}\ \text{ax/W}}{- : \Delta_8, F_5, F_6 \vdash \Delta_7}\ \text{hCut}}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}\ \wedge_L$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \wedge F_9}\ \top_R \quad \dfrac{h_6 : F_8, F_9, \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \top, \Delta_5}\ \wedge_L}{- : \Delta_7 \vdash \top, \Delta_5}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash (\top, \Delta_6), F_5}\ \top_R \quad \dfrac{h_7 : F_5, F_8, F_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \wedge F_9), F_5 \vdash \top, \Delta_6}\ \wedge_L}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6}\ \top_R$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{h_4 : \top, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \top \vdash \Delta_7}\ \vee_L}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7}\ \top_R\ \dfrac{}{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7}\ \text{ax/W}}{- : \Delta_8, F_5 \vdash \Delta_7}\ \text{hCut} \quad \dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7}\ \top_R\ \dfrac{}{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}\ \text{ax/W}}{- : \Delta_8, F_6 \vdash \Delta_7}\ \text{hCut}}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}\ \vee_L$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \vee F_9}\ \top_R \quad \dfrac{h_6 : F_8, \Delta_7 \vdash \top, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \top, \Delta_5}\ \vee_L}{- : \Delta_7 \vdash \top, \Delta_5}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\top, \Delta_6), F_5}\ \top_R \quad \dfrac{h_7 : F_5, F_8, \Delta_{10} \vdash \top, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \top, \Delta_6}\ \vee_L}{- : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6}\ \top_R$$

- Case rule AT

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_7, []F_5 \vdash \Delta_6, \top}\ \top_R \quad \dfrac{h_4 : \top, F_5, \Delta_7, []F_5 \vdash \Delta_6}{\bullet h_4 : (\Delta_7, []F_5), \top \vdash \Delta_6}\ AT}{- : \Delta_7, []F_5 \vdash \Delta_6}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_7, F_5, []F_5 \vdash \top, \Delta_6}\ \top_R \quad \dfrac{}{h_4 : \top, \Delta_7, F_5, []F_5 \vdash \Delta_6}\ \text{ax/W}}{- : \Delta_7, F_5, []F_5 \vdash \Delta_6}\ \text{hCut}}{- : \Delta_7, []F_5 \vdash \Delta_6}\ ATG$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), []F_8}\ \top_R \quad \dfrac{h_6 : F_8, \Delta_7, []F_8 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, []F_8 \vdash \top, \Delta_5}\ AT}{- : \Delta_7 \vdash \top, \Delta_5}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_9, []F_8 \vdash (\top, \Delta_6), F_5}\ \top_R \quad \dfrac{h_7 : F_5, F_8, \Delta_9, []F_8 \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_9, []F_8), F_5 \vdash \top, \Delta_6}\ AT}{- : \Delta_9, []F_8 \vdash \top, \Delta_6}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_9, []F_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule $\bot_L$

$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_6 \vdash \Delta_5, \top}\ \top_R \quad \dfrac{}{\bullet\mathtt{h}_4 : (\bot, \Delta_6), \top \vdash \Delta_5}\ \bot_L}{\dfrac{- : \bot, \Delta_6 \vdash \Delta_5}{\dfrac{\rightarrow}{- : \bot, \Delta_6 \vdash \Delta_5}\ \bot_L}}\ \mathtt{Cut}
$$

$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \bot}\ \top_R \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \bot \vdash \top, \Delta_5}\ \bot_L}{\dfrac{- : \Delta_7 \vdash \top, \Delta_5}{\dfrac{\rightarrow}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R}}\ \mathtt{Cut}
$$

$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \bot, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{}{\bullet\mathtt{h}_7 : (\bot, \Delta_8), \mathtt{F}_5 \vdash \top, \Delta_6}\ \bot_L}{\dfrac{- : \bot, \Delta_8 \vdash \top, \Delta_6}{\dfrac{\rightarrow}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \top_R}}\ \mathtt{Cut}
$$

- Case rule I

$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_7, \mathtt{p}_5 \vdash (\Delta_6, \mathtt{p}_5), \top}\ \top_R \quad \dfrac{}{\bullet\mathtt{h}_4 : (\Delta_7, \mathtt{p}_5), \top \vdash \Delta_6, \mathtt{p}_5}\ I}{\dfrac{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}{\dfrac{\rightarrow}{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}\ I}}\ \mathtt{Cut}
$$

$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_6 \vdash (\top, \Delta_8, \mathtt{p}_7), \mathtt{p}_7}\ \top_R \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{p}_7 \vdash \top, \Delta_8, \mathtt{p}_7}\ I}{\dfrac{- : \Delta_6 \vdash \top, \Delta_8, \mathtt{p}_7}{\dfrac{\rightarrow}{- : \Delta_6 \vdash \top, \Delta_8, \mathtt{p}_7}\ \top_R}}\ \mathtt{Cut}
$$

$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_9, \mathtt{p}_7 \vdash (\top, \Delta_8, \mathtt{p}_7), \mathtt{F}_5}\ \top_R \quad \dfrac{}{\bullet\mathtt{h}_6 : (\Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \top, \Delta_8, \mathtt{p}_7}\ I}{\dfrac{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_8, \mathtt{p}_7}{\dfrac{\rightarrow}{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_8, \mathtt{p}_7}\ \top_R}}\ \mathtt{Cut}
$$

- Case rule $\top_L$

$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_5 \vdash \Delta_6, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_6}{\bullet\mathtt{h}_4 : \Delta_5, \top \vdash \Delta_6}\ \top_L}{\dfrac{- : \Delta_5 \vdash \Delta_6}{\dfrac{\rightarrow}{- : \Delta_5 \vdash \Delta_6}\ \mathtt{ax/W}}}\ \mathtt{Cut}
$$

$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \top, \Delta_5}{\bullet\mathtt{h}_6 : \Delta_7, \top \vdash \top, \Delta_5}\ \top_L}{\dfrac{- : \Delta_7 \vdash \top, \Delta_5}{\dfrac{\rightarrow}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R}}\ \mathtt{Cut}
$$

$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \top, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_8 \vdash \top, \Delta_6}{\bullet\mathtt{h}_7 : (\top, \Delta_8), \mathtt{F}_5 \vdash \top, \Delta_6}\ \top_L}{\dfrac{- : \top, \Delta_8 \vdash \top, \Delta_6}{\dfrac{\rightarrow}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \top_R}}\ \mathtt{Cut}
$$

6.6 Status of K: `OK`

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash \mathtt{F_8}}{\bullet\mathtt{h_1}:\Box\Gamma_6,\Delta_7\vdash(\Delta_{10},\mathtt{F_{11}}\to\mathtt{F_{12}}),[]\mathtt{F_8}}\,K \quad \dfrac{\mathtt{h_9}:\Box\Gamma_6,\mathtt{F_{11}},\Delta_7,[]\mathtt{F_8}\vdash \mathtt{F_{12}},\Delta_{10}}{\bullet\mathtt{h_9}:(\Box\Gamma_6,\Delta_7),[]\mathtt{F_8}\vdash\Delta_{10},\mathtt{F_{11}}\to\mathtt{F_{12}}}\,{\to_R}}{-:\Box\Gamma_6,\Delta_7\vdash\Delta_{10},\mathtt{F_{11}}\to\mathtt{F_{12}}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\overline{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash \mathtt{F_8}}\,\mathtt{ax/W}}{\bullet\mathtt{h_1}:\Delta_7,\mathtt{F_{11}},\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{12}},[]\mathtt{F_8}}\,K \quad \overline{\mathtt{h_9}:\Delta_7,\mathtt{F_{11}},\Box\Gamma_6,[]\mathtt{F_8}\vdash\Delta_{10},\mathtt{F_{12}}}\,\mathtt{ax/W}}{-:\Delta_7,\mathtt{F_{11}},\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{12}}}\,\mathtt{hCut}}{-:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{11}}\to\mathtt{F_{12}}}\,{\to_R}$$

$$\dfrac{\dfrac{\mathtt{h_2}:unbox(\Box\Gamma_7)\vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}:\Box\Gamma_7,\Delta_9\vdash((\Delta_{14},\mathtt{F_{12}}\to\mathtt{F_{13}}),[]\mathtt{F_{10}}),\mathtt{F_8}}\,K \quad \dfrac{\mathtt{h_{11}}:\Box\Gamma_7,\mathtt{F_8},\mathtt{F_{12}},\Delta_9\vdash \mathtt{F_{13}},\Delta_{14},[]\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}:(\Box\Gamma_7,\Delta_9),\mathtt{F_8}\vdash(\Delta_{14},\mathtt{F_{12}}\to\mathtt{F_{13}}),[]\mathtt{F_{10}}}\,{\to_R}}{-:\Box\Gamma_7,\Delta_9\vdash(\Delta_{14},\mathtt{F_{12}}\to\mathtt{F_{13}}),[]\mathtt{F_{10}}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\overline{-:unbox(\Box\Gamma_7)\vdash \mathtt{F_{10}}}\,\mathtt{ax/W}}{-:\Delta_9,\Box\Gamma_7\vdash\Delta_{14},[]\mathtt{F_{10}},\mathtt{F_{12}}\to\mathtt{F_{13}}}}{}\,K$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash \mathtt{F_8}}{\bullet\mathtt{h_1}:\Box\Gamma_6,\Delta_7\vdash(\Delta_{10},\mathtt{F_{11}}\wedge\mathtt{F_{12}}),[]\mathtt{F_8}}\,K \quad \dfrac{\mathtt{h_9}:\Box\Gamma_6,\Delta_7,[]\mathtt{F_8}\vdash \mathtt{F_{11}},\Delta_{10} \quad \mathtt{h_9}:\Box\Gamma_6,\Delta_7,[]\mathtt{F_8}\vdash \mathtt{F_{12}},\Delta_{10}}{\bullet\mathtt{h_9}:(\Box\Gamma_6,\Delta_7),[]\mathtt{F_8}\vdash\Delta_{10},\mathtt{F_{11}}\wedge\mathtt{F_{12}}}\,{\wedge_R}}{-:\Box\Gamma_6,\Delta_7\vdash\Delta_{10},\mathtt{F_{11}}\wedge\mathtt{F_{12}}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\overline{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash \mathtt{F_8}}\,\mathtt{ax/W}}{\bullet\mathtt{h_1}:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{11}},[]\mathtt{F_8}}\,K \quad \overline{\mathtt{h_9}:\Delta_7,\Box\Gamma_6,[]\mathtt{F_8}\vdash\Delta_{10},\mathtt{F_{11}}}\,\mathtt{ax/W}}{-:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{11}}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\overline{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash \mathtt{F_8}}\,\mathtt{ax/W}}{\bullet\mathtt{h_1}:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{12}},[]\mathtt{F_8}}\,K \quad \overline{\mathtt{h_9}:\Delta_7,\Box\Gamma_6,[]\mathtt{F_8}\vdash\Delta_{10},\mathtt{F_{12}}}\,\mathtt{ax/W}}{-:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{12}}}\,\mathtt{hCut}}{-:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{11}}\wedge\mathtt{F_{12}}}\,{\wedge_R}$$

$$\dfrac{\dfrac{\mathtt{h_2}:unbox(\Box\Gamma_7)\vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}:\Box\Gamma_7,\Delta_9\vdash((\Delta_{14},\mathtt{F_{12}}\wedge\mathtt{F_{13}}),[]\mathtt{F_{10}}),\mathtt{F_8}}\,K \quad \dfrac{\mathtt{h_{11}}:\Box\Gamma_7,\mathtt{F_8},\Delta_9\vdash \mathtt{F_{12}},\Delta_{14},[]\mathtt{F_{10}} \quad \mathtt{h_{11}}:\Box\Gamma_7,\mathtt{F_8},\Delta_9\vdash \mathtt{F_{13}},\Delta_{14},[]\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}:(\Box\Gamma_7,\Delta_9),\mathtt{F_8}\vdash(\Delta_{14},\mathtt{F_{12}}\wedge\mathtt{F_{13}}),[]\mathtt{F_{10}}}\,{\wedge_R}}{-:\Box\Gamma_7,\Delta_9\vdash(\Delta_{14},\mathtt{F_{12}}\wedge\mathtt{F_{13}}),[]\mathtt{F_{10}}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\overline{-:unbox(\Box\Gamma_7)\vdash \mathtt{F_{10}}}\,\mathtt{ax/W}}{-:\Delta_9,\Box\Gamma_7\vdash\Delta_{14},[]\mathtt{F_{10}},\mathtt{F_{12}}\wedge\mathtt{F_{13}}}}{}\,K$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash \mathtt{F_8}}{\bullet\mathtt{h_1}:\Box\Gamma_6,\Delta_7\vdash(\Delta_{10},\mathtt{F_{11}}\vee\mathtt{F_{12}}),[]\mathtt{F_8}}\,K \quad \dfrac{\mathtt{h_9}:\Box\Gamma_6,\Delta_7,[]\mathtt{F_8}\vdash \mathtt{F_{11}},\mathtt{F_{12}},\Delta_{10}}{\bullet\mathtt{h_9}:(\Box\Gamma_6,\Delta_7),[]\mathtt{F_8}\vdash\Delta_{10},\mathtt{F_{11}}\vee\mathtt{F_{12}}}\,{\vee_R}}{-:\Box\Gamma_6,\Delta_7\vdash\Delta_{10},\mathtt{F_{11}}\vee\mathtt{F_{12}}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\overline{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash \mathtt{F_8}}\,\mathtt{ax/W}}{\bullet\mathtt{h_1}:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{11}},\mathtt{F_{12}},[]\mathtt{F_8}}\,K \quad \overline{\mathtt{h_9}:\Delta_7,\Box\Gamma_6,[]\mathtt{F_8}\vdash\Delta_{10},\mathtt{F_{11}},\mathtt{F_{12}}}\,\mathtt{ax/W}}{-:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{11}},\mathtt{F_{12}}}\,\mathtt{hCut}}{-:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_{11}}\vee\mathtt{F_{12}}}\,{\vee_R}$$

$$\dfrac{\dfrac{\mathtt{h_2}:unbox(\Box\Gamma_7)\vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}:\Box\Gamma_7,\Delta_9\vdash((\Delta_{14},\mathtt{F_{12}}\vee\mathtt{F_{13}}),[]\mathtt{F_{10}}),\mathtt{F_8}}\,K \quad \dfrac{\mathtt{h_{11}}:\Box\Gamma_7,\mathtt{F_8},\Delta_9\vdash \mathtt{F_{12}},\mathtt{F_{13}},\Delta_{14},[]\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}:(\Box\Gamma_7,\Delta_9),\mathtt{F_8}\vdash(\Delta_{14},\mathtt{F_{12}}\vee\mathtt{F_{13}}),[]\mathtt{F_{10}}}\,{\vee_R}}{-:\Box\Gamma_7,\Delta_9\vdash(\Delta_{14},\mathtt{F_{12}}\vee\mathtt{F_{13}}),[]\mathtt{F_{10}}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\overline{-:unbox(\Box\Gamma_7)\vdash \mathtt{F_{10}}}\,\mathtt{ax/W}}{-:\Delta_9,\Box\Gamma_7\vdash\Delta_{14},[]\mathtt{F_{10}},\mathtt{F_{12}}\vee\mathtt{F_{13}}}}{}\,K$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash \mathtt{F_8}}{\bullet\mathtt{h_1}:\Box\Gamma_6,\Delta_7\vdash(\bot,\Delta_{10}),[]\mathtt{F_8}}\,K \quad \dfrac{\mathtt{h_9}:\Box\Gamma_6,\Delta_7,[]\mathtt{F_8}\vdash\Delta_{10}}{\bullet\mathtt{h_9}:(\Box\Gamma_6,\Delta_7),[]\mathtt{F_8}\vdash\bot,\Delta_{10}}\,{\bot_R}}{-:\Box\Gamma_6,\Delta_7\vdash\bot,\Delta_{10}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\overline{\bullet\mathtt{h_1}:\Delta_7,\Box\Gamma_6\vdash\bot,\Delta_{10},[]\mathtt{F_8}}\,\mathtt{ax/W} \quad \overline{\mathtt{h_9}:\Delta_7,\Box\Gamma_6,[]\mathtt{F_8}\vdash\bot,\Delta_{10}}\,\mathtt{ax/W}}{-:\Delta_7,\Box\Gamma_6\vdash\bot,\Delta_{10}}\,\mathtt{hCut}$$

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$$\dfrac{\dfrac{\mathtt{h_2} : unbox(\Box\Gamma_7) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h_2} : \Box\Gamma_7, \Delta_9 \vdash ((\bot, \Delta_{12}), []\mathtt{F}_{10}), \mathtt{F}_8}\ K \qquad \dfrac{\mathtt{h_{11}} : \Box\Gamma_7, \mathtt{F}_8, \Delta_9 \vdash \Delta_{12}, []\mathtt{F}_{10}}{\bullet\mathtt{h_{11}} : (\Box\Gamma_7, \Delta_9), \mathtt{F}_8 \vdash (\bot, \Delta_{12}), []\mathtt{F}_{10}}\ \bot_R}{- : \Box\Gamma_7, \Delta_9 \vdash (\bot, \Delta_{12}), []\mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\Box\Gamma_7) \vdash \mathtt{F}_{10}}{- : \Delta_9, \Box\Gamma_7 \vdash \bot, \Delta_{12}, []\mathtt{F}_{10}}\ \mathtt{ax/W}}{}\ K$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h_1} : unbox(\Box\Gamma_6) \vdash \mathtt{F}_8}{\bullet\mathtt{h_1} : \Box\Gamma_6, \Delta_7 \vdash (\top, \Delta_{10}), []\mathtt{F}_8}\ K \qquad \dfrac{}{\bullet\mathtt{h_9} : (\Box\Gamma_6, \Delta_7), []\mathtt{F}_8 \vdash \top, \Delta_{10}}\ \top_R}{- : \Box\Gamma_6, \Delta_7 \vdash \top, \Delta_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_7, \Box\Gamma_6 \vdash \top, \Delta_{10}}\ \top_R$$

$$\dfrac{\dfrac{\mathtt{h_2} : unbox(\Box\Gamma_7) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h_2} : \Box\Gamma_7, \Delta_9 \vdash ((\top, \Delta_{12}), []\mathtt{F}_{10}), \mathtt{F}_8}\ K \qquad \dfrac{}{\bullet\mathtt{h_{11}} : (\Box\Gamma_7, \Delta_9), \mathtt{F}_8 \vdash (\top, \Delta_{12}), []\mathtt{F}_{10}}\ \top_R}{- : \Box\Gamma_7, \Delta_9 \vdash (\top, \Delta_{12}), []\mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_9, \Box\Gamma_7 \vdash \top, \Delta_{12}, []\mathtt{F}_{10}}\ \top_R$$

- Case rule K

$$\dfrac{\dfrac{\mathtt{h_1} : unbox(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_6}{\bullet\mathtt{h_1} : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_8, []\mathtt{F}_9), []\mathtt{F}_6}\ K \quad \dfrac{\mathtt{h_7} : \mathtt{F}_6, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}{\bullet\mathtt{h_7} : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_8, []\mathtt{F}_9}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_8, []\mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_6, \mathtt{F}_9}{\ }\ \mathtt{ax/W} \quad \dfrac{- : \mathtt{F}_6, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_9}{\ }\ \mathtt{ax/W}}{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_9}\ \mathtt{sCut}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_8, []\mathtt{F}_9}\ K$$

$$\dfrac{\dfrac{\mathtt{h_1} : unbox(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_6}{\bullet\mathtt{h_1} : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_8, []\mathtt{F}_9), []\mathtt{F}_6}\ K \quad \dfrac{\mathtt{h_7} : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}{\bullet\mathtt{h_7} : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_8, []\mathtt{F}_9}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_8, []\mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}{\ }\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_8, []\mathtt{F}_9}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2} : unbox(\Box\Gamma_{12}, \Box\Gamma_{14}) \vdash \mathtt{F}_8}{\bullet\mathtt{h_2} : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8), \Box\mathtt{F}_7}\ K \quad \dfrac{\mathtt{h_9} : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F}_7) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h_9} : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathtt{F}_7 \vdash (\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_8}{\ }\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2} : unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h_2} : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_{10}), \Box\mathtt{F}_7}\ K \quad \dfrac{\mathtt{h_8} : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}), unbox(\Box\mathtt{F}_7) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h_8} : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ K}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}{\ }\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2} : unbox(\Box\Gamma_{12}, \Box\Gamma_{14}) \vdash \mathtt{F}_8}{\bullet\mathtt{h_2} : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8), \mathtt{F}_7}\ K \quad \dfrac{\mathtt{h_9} : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h_9} : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_7 \vdash (\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}{\ }\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_{11},\Box\Gamma_{13}) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}: (\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14} \vdash (\Delta_9,[]\mathtt{F_{10}}),\mathtt{F_7}}\ K \quad \dfrac{\mathtt{h_8}: unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_8}: ((\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14}),\mathtt{F_7} \vdash \Delta_9,[]\mathtt{F_{10}}}\ K}{-: (\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14} \vdash \Delta_9,[]\mathtt{F_{10}}}\ \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{-: unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F_{10}}}{}\ \mathtt{ax/W}}{-: \Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13} \vdash \Delta_9,[]\mathtt{F_{10}}}\ K$$

- Case rule $A45$

$$\dfrac{\dfrac{\mathtt{h_1}: unbox(\Box\Gamma_{11},\Box\Gamma_{13}) \vdash \mathtt{F_6}}{\bullet\mathtt{h_1}: (\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14} \vdash (\Box\mathtt{F_8},\Delta_9,[]\mathtt{F_{10}}),[]\mathtt{F_6}}\ K \quad \dfrac{\mathtt{h_7}: \Box\Gamma_{11},\Box\Gamma_{12},[]\mathtt{F_6} \vdash \Box\Gamma_8,\mathtt{F_{10}}}{\bullet\mathtt{h_7}: ((\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14}),[]\mathtt{F_6} \vdash \Box\mathtt{F_8},\Delta_9,[]\mathtt{F_{10}}}\ A45}{-: (\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14} \vdash \Box\mathtt{F_8},\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_1}: unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F_6}}{\bullet\mathtt{h_1}: \Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13} \vdash \mathtt{F_{10}},\Box\mathtt{F_8},[]\mathtt{F_6}}\ \mathtt{ax/W}}{}\ K \quad \dfrac{\mathtt{h_7}: \Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathtt{F_6} \vdash \mathtt{F_{10}},\Box\Gamma_8}{}\ \mathtt{ax/W}}{\dfrac{-: \Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13} \vdash \mathtt{F_{10}},\Box\Gamma_8}{-: \Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13} \vdash \Delta_9,\Box\mathtt{F_8},[]\mathtt{F_{10}}}\ A45}\ \mathrm{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_1}: unbox(\Box\Gamma_{11},\Box\Gamma_{13}) \vdash \mathtt{F_6}}{\bullet\mathtt{h_1}: (\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14} \vdash (\Box\mathtt{F_8},\Delta_9,[]\mathtt{F_{10}}),[]\mathtt{F_6}}\ K \quad \dfrac{\mathtt{h_7}: \Box\Gamma_{11},\Box\Gamma_{12} \vdash \Box\Gamma_8,\mathtt{F_{10}}}{\bullet\mathtt{h_7}: ((\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14}),[]\mathtt{F_6} \vdash \Box\mathtt{F_8},\Delta_9,[]\mathtt{F_{10}}}\ A45}{-: (\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14} \vdash \Box\mathtt{F_8},\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{-: \Box\Gamma_{11},\Box\Gamma_{12} \vdash \mathtt{F_{10}},\Box\Gamma_8}{}\ \mathtt{ax/W}}{-: \Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13} \vdash \Delta_9,\Box\mathtt{F_8},[]\mathtt{F_{10}}}\ A45$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_{13},\Box\Gamma_{15}) \vdash \mathtt{F_8}}{\bullet\mathtt{h_2}: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash ((\Box\Gamma_{12},\Delta_{10},[]\mathtt{F_{11}}),[]\mathtt{F_8}),\Box\mathtt{F_7}}\ K \quad \dfrac{\mathtt{h_9}: \Box\Gamma_{13},\Box\Gamma_{14},\Box\mathtt{F_7} \vdash \Box\Gamma_{12},\mathtt{F_{11}},[]\mathtt{F_8}}{\bullet\mathtt{h_9}: ((\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16}),\Box\mathtt{F_7} \vdash (\Box\Gamma_{12},\Delta_{10},[]\mathtt{F_{11}}),[]\mathtt{F_8}}\ A45}{-: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash (\Box\Gamma_{12},\Delta_{10},[]\mathtt{F_{11}}),[]\mathtt{F_8}}\ \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F_8}}{}\ \mathtt{ax/W}}{-: \Delta_{16},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{15} \vdash \Delta_{10},\Box\Gamma_{12},[]\mathtt{F_{11}},[]\mathtt{F_8}}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_{13},\Box\Gamma_{15}) \vdash \mathtt{F_8}}{\bullet\mathtt{h_2}: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash ((\Box\Gamma_{10},\Delta_{12},[]\mathtt{F_{11}}),[]\mathtt{F_8}),\Box\mathtt{F_7}}\ K \quad \dfrac{\mathtt{h_9}: \Box\Gamma_{13},\Box\Gamma_{14},\Box\mathtt{F_7} \vdash \Box\Gamma_{10},\mathtt{F_{11}}}{\bullet\mathtt{h_9}: ((\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16}),\Box\mathtt{F_7} \vdash (\Box\Gamma_{10},\Delta_{12},[]\mathtt{F_{11}}),[]\mathtt{F_8}}\ A45}{-: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash (\Box\Gamma_{10},\Delta_{12},[]\mathtt{F_{11}}),[]\mathtt{F_8}}\ \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F_8}}{}\ \mathtt{ax/W}}{-: \Delta_{16},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{15} \vdash \Delta_{12},\Box\Gamma_{10},[]\mathtt{F_{11}},[]\mathtt{F_8}}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_{12},\Box\Gamma_{14}) \vdash \mathtt{F_{11}}}{\bullet\mathtt{h_2}: (\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15} \vdash ((\Box\Gamma_9,\Delta_{10}),[]\mathtt{F_{11}}),\Box\mathtt{F_7}}\ K \quad \dfrac{\mathtt{h_8}: \Box\Gamma_{12},\Box\Gamma_{13},\Box\mathtt{F_7} \vdash \Box\Gamma_9,\mathtt{F_{11}}}{\bullet\mathtt{h_8}: ((\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}),\Box\mathtt{F_7} \vdash (\Box\Gamma_9,\Delta_{10}),[]\mathtt{F_{11}}}\ A45}{-: (\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15} \vdash (\Box\Gamma_9,\Delta_{10}),[]\mathtt{F_{11}}}\ \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F_{11}}}{}\ \mathtt{ax/W}}{-: \Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14} \vdash \Delta_{10},\Box\Gamma_9,[]\mathtt{F_{11}}}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_{13},\Box\Gamma_{15}) \vdash \mathtt{F_8}}{\bullet\mathtt{h_2}: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash ((\Box\Gamma_{12},\Delta_{10},[]\mathtt{F_{11}}),[]\mathtt{F_8}),\mathtt{F_7}}\ K \quad \dfrac{\mathtt{h_9}: \Box\Gamma_{13},\Box\Gamma_{14} \vdash \Box\Gamma_{12},\mathtt{F_{11}},[]\mathtt{F_8}}{\bullet\mathtt{h_9}: ((\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16}),\mathtt{F_7} \vdash (\Box\Gamma_{12},\Delta_{10},[]\mathtt{F_{11}}),[]\mathtt{F_8}}\ A45}{-: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash (\Box\Gamma_{12},\Delta_{10},[]\mathtt{F_{11}}),[]\mathtt{F_8}}\ \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F_8}}{}\ \mathtt{ax/W}}{-: \Delta_{16},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{15} \vdash \Delta_{10},\Box\Gamma_{12},[]\mathtt{F_{11}},[]\mathtt{F_8}}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_{13},\Box\Gamma_{15}) \vdash \mathtt{F_8}}{\bullet\mathtt{h_2}: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash ((\Box\Gamma_{10},\Delta_{12},[]\mathtt{F_{11}}),[]\mathtt{F_8}),\mathtt{F_7}}\ K \quad \dfrac{\mathtt{h_9}: \Box\Gamma_{13},\Box\Gamma_{14} \vdash \Box\Gamma_{10},\mathtt{F_{11}}}{\bullet\mathtt{h_9}: ((\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16}),\mathtt{F_7} \vdash (\Box\Gamma_{10},\Delta_{12},[]\mathtt{F_{11}}),[]\mathtt{F_8}}\ A45}{-: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash (\Box\Gamma_{10},\Delta_{12},[]\mathtt{F_{11}}),[]\mathtt{F_8}}\ \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F_8}}{}\ \mathtt{ax/W}}{-: \Delta_{16},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{15} \vdash \Delta_{12},\Box\Gamma_{10},[]\mathtt{F_{11}},[]\mathtt{F_8}}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\square\Gamma_{12}, \square\Gamma_{14}) \vdash \mathtt{F_{11}}}{\bullet\mathtt{h_2}: (\square\Gamma_{12}, \square\Gamma_{14}), \square\Gamma_{13}, \Delta_{15} \vdash ((\square\Gamma_9, \Delta_{10}), []\mathtt{F_{11}}), \mathtt{F_7}}\;K \quad \dfrac{\mathtt{h_8}: \square\Gamma_{12}, \square\Gamma_{13} \vdash \square\Gamma_9, \mathtt{F_{11}}}{\bullet\mathtt{h_8}: ((\square\Gamma_{12}, \square\Gamma_{14}), \square\Gamma_{13}, \Delta_{15}), \mathtt{F_7} \vdash (\square\Gamma_9, \Delta_{10}), []\mathtt{F_{11}}}\;A45}{- : (\square\Gamma_{12}, \square\Gamma_{14}), \square\Gamma_{13}, \Delta_{15} \vdash (\square\Gamma_9, \Delta_{10}), []\mathtt{F_{11}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\square\Gamma_{12}), unbox(\square\Gamma_{14}) \vdash \mathtt{F_{11}}}{}\;\mathtt{ax/W}}{- : \Delta_{15}, \square\Gamma_{12}, \square\Gamma_{13}, \square\Gamma_{14} \vdash \Delta_{10}, \square\Gamma_9, []\mathtt{F_{11}}}\;K$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h_1}: unbox(\square\Gamma_6) \vdash \mathtt{F_7}}{\bullet\mathtt{h_1}: \square\Gamma_6, \Delta_{12}, \mathtt{F_9} \rightarrow \mathtt{F_{10}} \vdash \Delta_{11}, []\mathtt{F_7}}\;K \quad \dfrac{\mathtt{h_8}: \square\Gamma_6, \Delta_{12}, []\mathtt{F_7} \vdash \mathtt{F_9}, \Delta_{11} \quad \mathtt{h_8}: \square\Gamma_6, \mathtt{F_{10}}, \Delta_{12}, []\mathtt{F_7} \vdash \Delta_{11}}{\bullet\mathtt{h_8}: (\square\Gamma_6, \Delta_{12}, \mathtt{F_9} \rightarrow \mathtt{F_{10}}), []\mathtt{F_7} \vdash \Delta_{11}}\;\rightarrow_L}{- : \square\Gamma_6, \Delta_{12}, \mathtt{F_9} \rightarrow \mathtt{F_{10}} \vdash \Delta_{11}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_1}: unbox(\square\Gamma_6) \vdash \mathtt{F_7}}{}\;\mathtt{ax/W}}{\bullet\mathtt{h_1}: \Delta_{12}, \square\Gamma_6 \vdash \Delta_{11}, \mathtt{F_9}, []\mathtt{F_7}}\;K \quad \dfrac{\mathtt{h_8}: \Delta_{12}, \square\Gamma_6, []\mathtt{F_7} \vdash \Delta_{11}, \mathtt{F_9}}{}\;\mathtt{ax/W}}{- : \Delta_{12}, \square\Gamma_6 \vdash \Delta_{11}, \mathtt{F_9}}\;\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{\mathtt{h_1}: unbox(\square\Gamma_6) \vdash \mathtt{F_7}}{}\;\mathtt{ax/W}}{\bullet\mathtt{h_1}: \Delta_{12}, \mathtt{F_{10}}, \square\Gamma_6 \vdash \Delta_{11}, []\mathtt{F_7}}\;K \quad \dfrac{\mathtt{h_8}: \Delta_{12}, \mathtt{F_{10}}, \square\Gamma_6, []\mathtt{F_7} \vdash \Delta_{11}}{}\;\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F_{10}}, \square\Gamma_6 \vdash \Delta_{11}}\;\mathtt{hCut}$$

$$\dfrac{}{- : \Delta_{12}, \square\Gamma_6, \mathtt{F_9} \rightarrow \mathtt{F_{10}} \vdash \Delta_{11}}\;\rightarrow_L$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\square\Gamma_7) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}: \square\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F_{10}}), \mathtt{F_{12}} \rightarrow \mathtt{F_{13}}}\;K \quad \dfrac{\mathtt{h_{11}}: \square\Gamma_7, \Delta_8 \vdash \mathtt{F_{12}}, \Delta_9, []\mathtt{F_{10}} \quad \mathtt{h_{11}}: \square\Gamma_7, \mathtt{F_{13}}, \Delta_8 \vdash \Delta_9, []\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}: (\square\Gamma_7, \Delta_8), \mathtt{F_{12}} \rightarrow \mathtt{F_{13}} \vdash \Delta_9, []\mathtt{F_{10}}}\;\rightarrow_L}{- : \square\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F_{10}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\square\Gamma_7) \vdash \mathtt{F_{10}}}{}\;\mathtt{ax/W}}{- : \Delta_8, \square\Gamma_7 \vdash \Delta_9, []\mathtt{F_{10}}}\;K$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\square\Gamma_7) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}: \square\Gamma_7, \Delta_{14}, \mathtt{F_{12}} \rightarrow \mathtt{F_{13}} \vdash (\Delta_9, []\mathtt{F_{10}}), \mathtt{F_8}}\;K \quad \dfrac{\mathtt{h_{11}}: \square\Gamma_7, \mathtt{F_8}, \Delta_{14} \vdash \mathtt{F_{12}}, \Delta_9, []\mathtt{F_{10}} \quad \mathtt{h_{11}}: \square\Gamma_7, \mathtt{F_8}, \mathtt{F_{13}}, \Delta_{14} \vdash \Delta_9, []\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}: (\square\Gamma_7, \Delta_{14}, \mathtt{F_{12}} \rightarrow \mathtt{F_{13}}), \mathtt{F_8} \vdash \Delta_9, []\mathtt{F_{10}}}\;\rightarrow_L}{- : \square\Gamma_7, \Delta_{14}, \mathtt{F_{12}} \rightarrow \mathtt{F_{13}} \vdash \Delta_9, []\mathtt{F_{10}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\square\Gamma_7) \vdash \mathtt{F_{10}}}{}\;\mathtt{ax/W}}{- : \Delta_{14}, \square\Gamma_7, \mathtt{F_{12}} \rightarrow \mathtt{F_{13}} \vdash \Delta_9, []\mathtt{F_{10}}}\;K$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h_1}: unbox(\square\Gamma_6) \vdash \mathtt{F_7}}{\bullet\mathtt{h_1}: \square\Gamma_6, \Delta_{12}, \mathtt{F_9} \wedge \mathtt{F_{10}} \vdash \Delta_{11}, []\mathtt{F_7}}\;K \quad \dfrac{\mathtt{h_8}: \square\Gamma_6, \mathtt{F_9}, \mathtt{F_{10}}, \Delta_{12}, []\mathtt{F_7} \vdash \Delta_{11}}{\bullet\mathtt{h_8}: (\square\Gamma_6, \Delta_{12}, \mathtt{F_9} \wedge \mathtt{F_{10}}), []\mathtt{F_7} \vdash \Delta_{11}}\;\wedge_L}{- : \square\Gamma_6, \Delta_{12}, \mathtt{F_9} \wedge \mathtt{F_{10}} \vdash \Delta_{11}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_1}: unbox(\square\Gamma_6) \vdash \mathtt{F_7}}{}\;\mathtt{ax/W}}{\bullet\mathtt{h_1}: \Delta_{12}, \mathtt{F_{10}}, \mathtt{F_9}, \square\Gamma_6 \vdash \Delta_{11}, []\mathtt{F_7}}\;K \quad \dfrac{\mathtt{h_8}: \Delta_{12}, \mathtt{F_{10}}, \mathtt{F_9}, \square\Gamma_6, []\mathtt{F_7} \vdash \Delta_{11}}{}\;\mathtt{ax/W}}{\dfrac{- : \Delta_{12}, \mathtt{F_{10}}, \mathtt{F_9}, \square\Gamma_6 \vdash \Delta_{11}}{- : \Delta_{12}, \square\Gamma_6, \mathtt{F_9} \wedge \mathtt{F_{10}} \vdash \Delta_{11}}\;\wedge_L}\;\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\square\Gamma_7) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}: \square\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F_{10}}), \mathtt{F_{12}} \wedge \mathtt{F_{13}}}\;K \quad \dfrac{\mathtt{h_{11}}: \square\Gamma_7, \mathtt{F_{12}}, \mathtt{F_{13}}, \Delta_8 \vdash \Delta_9, []\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}: (\square\Gamma_7, \Delta_8), \mathtt{F_{12}} \wedge \mathtt{F_{13}} \vdash \Delta_9, []\mathtt{F_{10}}}\;\wedge_L}{- : \square\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F_{10}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\square\Gamma_7) \vdash \mathtt{F_{10}}}{}\;\mathtt{ax/W}}{- : \Delta_8, \square\Gamma_7 \vdash \Delta_9, []\mathtt{F_{10}}}\;K$$

$$\dfrac{\dfrac{\mathtt{h_2}: unbox(\square\Gamma_7) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}: \square\Gamma_7, \Delta_{14}, \mathtt{F_{12}} \wedge \mathtt{F_{13}} \vdash (\Delta_9, []\mathtt{F_{10}}), \mathtt{F_8}}\;K \quad \dfrac{\mathtt{h_{11}}: \square\Gamma_7, \mathtt{F_8}, \mathtt{F_{12}}, \mathtt{F_{13}}, \Delta_{14} \vdash \Delta_9, []\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}: (\square\Gamma_7, \Delta_{14}, \mathtt{F_{12}} \wedge \mathtt{F_{13}}), \mathtt{F_8} \vdash \Delta_9, []\mathtt{F_{10}}}\;\wedge_L}{- : \square\Gamma_7, \Delta_{14}, \mathtt{F_{12}} \wedge \mathtt{F_{13}} \vdash \Delta_9, []\mathtt{F_{10}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{- : unbox(\square\Gamma_7) \vdash \mathtt{F_{10}}}{}\;\mathtt{ax/W}}{- : \Delta_{14}, \square\Gamma_7, \mathtt{F_{12}} \wedge \mathtt{F_{13}} \vdash \Delta_9, []\mathtt{F_{10}}}\;K$$

- Case rule $\vee_L$

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$$\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash \mathtt{F_7}}{\bullet\mathtt{h_1}:\Box\Gamma_6,\Delta_{12},\mathtt{F_9}\vee\mathtt{F_{10}}\vdash\Delta_{11},[]\mathtt{F_7}}\ K \qquad \dfrac{\mathtt{h_8}:\Box\Gamma_6,\mathtt{F_9},\Delta_{12},[]\mathtt{F_7}\vdash\Delta_{11} \quad \mathtt{h_8}:\Box\Gamma_6,\mathtt{F_{10}},\Delta_{12},[]\mathtt{F_7}\vdash\Delta_{11}}{\bullet\mathtt{h_8}:(\Box\Gamma_6,\Delta_{12},\mathtt{F_9}\vee\mathtt{F_{10}}),[]\mathtt{F_7}\vdash\Delta_{11}}\ \vee_L}{-:\Box\Gamma_6,\Delta_{12},\mathtt{F_9}\vee\mathtt{F_{10}}\vdash\Delta_{11}}\ \mathrm{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash\mathtt{F_7}}{\bullet\mathtt{h_1}:\Delta_{12},\mathtt{F_9},\Box\Gamma_6\vdash\Delta_{11},[]\mathtt{F_7}}\ \mathrm{ax/W}}{-:\Delta_{12},\mathtt{F_9},\Box\Gamma_6\vdash\Delta_{11}}\ K \quad \dfrac{\mathtt{h_8}:\Delta_{12},\mathtt{F_9},\Box\Gamma_6,[]\mathtt{F_7}\vdash\Delta_{11}}{}\ \mathrm{ax/W}}{-:\Delta_{12},\mathtt{F_9},\Box\Gamma_6\vdash\Delta_{11}}\ \mathrm{hCut}\qquad \dfrac{\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash\mathtt{F_7}}{\bullet\mathtt{h_1}:\Delta_{12},\mathtt{F_{10}},\Box\Gamma_6\vdash\Delta_{11},[]\mathtt{F_7}}\ \mathrm{ax/W}}{-:\Delta_{12},\mathtt{F_{10}},\Box\Gamma_6\vdash\Delta_{11}}\ K \quad \dfrac{\mathtt{h_8}:\Delta_{12},\mathtt{F_{10}},\Box\Gamma_6,[]\mathtt{F_7}\vdash\Delta_{11}}{}\ \mathrm{ax/W}}{-:\Delta_{12},\mathtt{F_{10}},\Box\Gamma_6\vdash\Delta_{11}}\ \mathrm{hCut}}{-:\Delta_{12},\Box\Gamma_6,\mathtt{F_9}\vee\mathtt{F_{10}}\vdash\Delta_{11}}\ \vee_L$$

$$\dfrac{\dfrac{\mathtt{h_2}:unbox(\Box\Gamma_7)\vdash\mathtt{F_{10}}}{\bullet\mathtt{h_2}:\Box\Gamma_7,\Delta_8\vdash(\Delta_9,[]\mathtt{F_{10}}),\mathtt{F_{12}}\vee\mathtt{F_{13}}}\ K \qquad \dfrac{\mathtt{h_{11}}:\Box\Gamma_7,\mathtt{F_{12}},\Delta_8\vdash\Delta_9,[]\mathtt{F_{10}} \quad \mathtt{h_{11}}:\Box\Gamma_7,\mathtt{F_{13}},\Delta_8\vdash\Delta_9,[]\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}:(\Box\Gamma_7,\Delta_8),\mathtt{F_{12}}\vee\mathtt{F_{13}}\vdash\Delta_9,[]\mathtt{F_{10}}}\ \vee_L}{-:\Box\Gamma_7,\Delta_8\vdash\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{Cut}$$
$$\to$$
$$\dfrac{\dfrac{-:unbox(\Box\Gamma_7)\vdash\mathtt{F_{10}}}{-:\Delta_8,\Box\Gamma_7\vdash\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{ax/W}}{-:\Delta_8,\Box\Gamma_7\vdash\Delta_9,[]\mathtt{F_{10}}}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2}:unbox(\Box\Gamma_7)\vdash\mathtt{F_{10}}}{\bullet\mathtt{h_2}:\Box\Gamma_7,\Delta_{14},\mathtt{F_{12}}\vee\mathtt{F_{13}}\vdash(\Delta_9,[]\mathtt{F_{10}}),\mathtt{F_8}}\ K \qquad \dfrac{\mathtt{h_{11}}:\Box\Gamma_7,\mathtt{F_8},\mathtt{F_{12}},\Delta_{14}\vdash\Delta_9,[]\mathtt{F_{10}} \quad \mathtt{h_{11}}:\Box\Gamma_7,\mathtt{F_8},\mathtt{F_{13}},\Delta_{14}\vdash\Delta_9,[]\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}:(\Box\Gamma_7,\Delta_{14},\mathtt{F_{12}}\vee\mathtt{F_{13}}),\mathtt{F_8}\vdash\Delta_9,[]\mathtt{F_{10}}}\ \vee_L}{-:\Box\Gamma_7,\Delta_{14},\mathtt{F_{12}}\vee\mathtt{F_{13}}\vdash\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{Cut}$$
$$\to$$
$$\dfrac{\dfrac{-:unbox(\Box\Gamma_7)\vdash\mathtt{F_{10}}}{-:\Delta_{14},\Box\Gamma_7,\mathtt{F_{12}}\vee\mathtt{F_{13}}\vdash\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{ax/W}}{-:\Delta_{14},\Box\Gamma_7,\mathtt{F_{12}}\vee\mathtt{F_{13}}\vdash\Delta_9,[]\mathtt{F_{10}}}\ K$$

- Case rule AT

$$\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_{11},[]\mathtt{F_9})\vdash\mathtt{F_7}}{\bullet\mathtt{h_1}:(\Box\Gamma_{11},[]\mathtt{F_9}),\Delta_6\vdash\Delta_{10},[]\mathtt{F_7}}\ K \quad \dfrac{\mathtt{h_8}:\Box\Gamma_{11},\mathtt{F_9},\Delta_6,[]\mathtt{F_7},[]\mathtt{F_9}\vdash\Delta_{10}}{\bullet\mathtt{h_8}:((\Box\Gamma_{11},[]\mathtt{F_9}),\Delta_6),[]\mathtt{F_7}\vdash\Delta_{10}}\ AT}{-:(\Box\Gamma_{11},[]\mathtt{F_9}),\Delta_6\vdash\Delta_{10}}\ \mathrm{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h_1}:\Delta_6,\mathtt{F_9},\Box\Gamma_{11},[]\mathtt{F_9}\vdash\Delta_{10},[]\mathtt{F_7}}{}\ \mathrm{ax/W}\quad \dfrac{\mathtt{h_8}:\Delta_6,\mathtt{F_9},\Box\Gamma_{11},[]\mathtt{F_7},[]\mathtt{F_9}\vdash\Delta_{10}}{}\ \mathrm{ax/W}}{-:\Delta_6,\mathtt{F_9},\Box\Gamma_{11},[]\mathtt{F_9}\vdash\Delta_{10}}\ \mathrm{hCut}}{-:\Delta_6,\Box\Gamma_{11},[]\mathtt{F_9}\vdash\Delta_{10}}\ AT$$

$$\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash\mathtt{F_7}}{\bullet\mathtt{h_1}:\Box\Gamma_6,\Delta_{11},[]\mathtt{F_9}\vdash\Delta_{10},[]\mathtt{F_7}}\ K \quad \dfrac{\mathtt{h_8}:\Box\Gamma_6,\mathtt{F_9},\Delta_{11},[]\mathtt{F_7},[]\mathtt{F_9}\vdash\Delta_{10}}{\bullet\mathtt{h_8}:(\Box\Gamma_6,\Delta_{11},[]\mathtt{F_9}),[]\mathtt{F_7}\vdash\Delta_{10}}\ AT}{-:\Box\Gamma_6,\Delta_{11},[]\mathtt{F_9}\vdash\Delta_{10}}\ \mathrm{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h_1}:\Delta_{11},\mathtt{F_9},\Box\Gamma_6,[]\mathtt{F_9}\vdash\Delta_{10},[]\mathtt{F_7}}{}\ \mathrm{ax/W}\quad \dfrac{\mathtt{h_8}:\Delta_{11},\mathtt{F_9},\Box\Gamma_6,[]\mathtt{F_7},[]\mathtt{F_9}\vdash\Delta_{10}}{}\ \mathrm{ax/W}}{-:\Delta_{11},\mathtt{F_9},\Box\Gamma_6,[]\mathtt{F_9}\vdash\Delta_{10}}\ \mathrm{hCut}}{-:\Delta_{11},\Box\Gamma_6,[]\mathtt{F_9}\vdash\Delta_{10}}\ ATG$$

$$\dfrac{\dfrac{\mathtt{h_1}:unbox(\Box\Gamma_6)\vdash\mathtt{F_9}}{\bullet\mathtt{h_1}:\Box\Gamma_6,\Delta_7\vdash\Delta_{10},[]\mathtt{F_9}}\ K \quad \dfrac{\mathtt{h_8}:\Box\Gamma_6,\mathtt{F_9},\Delta_7,[]\mathtt{F_9}\vdash\Delta_{10}}{\bullet\mathtt{h_8}:(\Box\Gamma_6,\Delta_7),[]\mathtt{F_9}\vdash\Delta_{10}}\ AT}{-:\Box\Gamma_6,\Delta_7\vdash\Delta_{10}}\ \mathrm{Cut}$$
$$\to$$
$$\dfrac{\dfrac{-:\Delta_7,\Box\Gamma_6,unbox(\Box\Gamma_6)\vdash\Delta_{10},\mathtt{F_9}}{-:\Delta_7,\Box\Gamma_6\vdash\Delta_{10},\mathtt{F_9}}\ \mathrm{ax/W}\ ATG \quad \dfrac{\dfrac{\bullet\mathtt{h_1}:\Delta_7,\mathtt{F_9},\Box\Gamma_6\vdash\Delta_{10},[]\mathtt{F_9}}{}\ \mathrm{ax/W}\quad \dfrac{\mathtt{h_8}:\Delta_7,\mathtt{F_9},\Box\Gamma_6,[]\mathtt{F_9}\vdash\Delta_{10}}{}\ \mathrm{ax/W}}{-:\Delta_7,\mathtt{F_9},\Box\Gamma_6\vdash\Delta_{10}}\ \mathrm{hCut}}{-:\Delta_7,\Box\Gamma_6\vdash\Delta_{10}}\ \mathrm{sCut}$$

$$\dfrac{\dfrac{\mathtt{h_2}:unbox(\Box\Gamma_{13},[]\mathtt{F_{12}})\vdash\mathtt{F_{10}}}{\bullet\mathtt{h_2}:(\Box\Gamma_{13},[]\mathtt{F_{12}}),\Delta_8\vdash(\Delta_9,[]\mathtt{F_{10}}),\mathtt{F_7}}\ K \quad \dfrac{\mathtt{h_{11}}:\Box\Gamma_{13},\mathtt{F_7},\mathtt{F_{12}},\Delta_8,[]\mathtt{F_{12}}\vdash\Delta_9,[]\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}:((\Box\Gamma_{13},[]\mathtt{F_{12}}),\Delta_8),\mathtt{F_7}\vdash\Delta_9,[]\mathtt{F_{10}}}\ AT}{-:(\Box\Gamma_{13},[]\mathtt{F_{12}}),\Delta_8\vdash\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{Cut}$$
$$\to$$
$$\dfrac{\dfrac{-:\mathtt{F_{12}},unbox(\Box\Gamma_{13})\vdash\mathtt{F_{10}}}{-:\Delta_8,\Box\Gamma_{13},[]\mathtt{F_{12}}\vdash\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{ax/W}}{-:\Delta_8,\Box\Gamma_{13},[]\mathtt{F_{12}}\vdash\Delta_9,[]\mathtt{F_{10}}}\ K$$

$$\dfrac{\dfrac{\mathtt{h_2}:unbox(\Box\Gamma_7)\vdash\mathtt{F_{10}}}{\bullet\mathtt{h_2}:\Box\Gamma_7,\Delta_8\vdash(\Delta_9,[]\mathtt{F_{10}}),[]\mathtt{F_{12}}}\ K \quad \dfrac{\mathtt{h_{11}}:\Box\Gamma_7,\mathtt{F_{12}},\Delta_8,[]\mathtt{F_{12}}\vdash\Delta_9,[]\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}:(\Box\Gamma_7,\Delta_8),[]\mathtt{F_{12}}\vdash\Delta_9,[]\mathtt{F_{10}}}\ AT}{-:\Box\Gamma_7,\Delta_8\vdash\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{Cut}$$
$$\to$$
$$\dfrac{\dfrac{-:unbox(\Box\Gamma_7)\vdash\mathtt{F_{10}}}{-:\Delta_8,\Box\Gamma_7\vdash\Delta_9,[]\mathtt{F_{10}}}\ \mathrm{ax/W}}{-:\Delta_8,\Box\Gamma_7\vdash\Delta_9,[]\mathtt{F_{10}}}\ K$$

$$\frac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_7) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}: \Box\Gamma_7, \Delta_{13}, []\mathtt{F_{12}} \vdash (\Delta_9, []\mathtt{F_{10}}), \mathtt{F_8}}\ K \qquad \dfrac{\mathtt{h_{11}}: \Box\Gamma_7, \mathtt{F_8}, \mathtt{F_{12}}, \Delta_{13}, []\mathtt{F_{12}} \vdash \Delta_9, []\mathtt{F_{10}}}{\bullet\mathtt{h_{11}}: (\Box\Gamma_7, \Delta_{13}, []\mathtt{F_{12}}), \mathtt{F_8} \vdash \Delta_9, []\mathtt{F_{10}}}\ AT}{- : \Box\Gamma_7, \Delta_{13}, []\mathtt{F_{12}} \vdash \Delta_9, []\mathtt{F_{10}}}\ Cut$$

$$\rightarrow$$

$$\frac{\dfrac{}{- : unbox(\Box\Gamma_7) \vdash \mathtt{F_{10}}}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_7, []\mathtt{F_{12}} \vdash \Delta_9, []\mathtt{F_{10}}}\ K$$

- Case rule $\bot_L$

$$\frac{\dfrac{\mathtt{h_1}: unbox(\Box\Gamma_6) \vdash \mathtt{F_7}}{\bullet\mathtt{h_1}: \Box\Gamma_6, \bot, \Delta_{10} \vdash \Delta_9, []\mathtt{F_7}}\ K \qquad \dfrac{}{\bullet\mathtt{h_8}: (\Box\Gamma_6, \bot, \Delta_{10}), []\mathtt{F_7} \vdash \Delta_9}\ \bot_L}{- : \Box\Gamma_6, \bot, \Delta_{10} \vdash \Delta_9}\ Cut$$

$$\rightarrow$$

$$\frac{}{- : \bot, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9}\ \bot_L$$

$$\frac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_7) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}: \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F_{10}}), \bot}\ K \qquad \dfrac{}{\bullet\mathtt{h_{11}}: (\Box\Gamma_7, \Delta_8), \bot \vdash \Delta_9, []\mathtt{F_{10}}}\ \bot_L}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F_{10}}}\ Cut$$

$$\rightarrow$$

$$\frac{\dfrac{}{- : unbox(\Box\Gamma_7) \vdash \mathtt{F_{10}}}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F_{10}}}\ K$$

$$\frac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_7) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2}: \Box\Gamma_7, \bot, \Delta_{12} \vdash (\Delta_9, []\mathtt{F_{10}}), \mathtt{F_8}}\ K \qquad \dfrac{}{\bullet\mathtt{h_{11}}: (\Box\Gamma_7, \bot, \Delta_{12}), \mathtt{F_8} \vdash \Delta_9, []\mathtt{F_{10}}}\ \bot_L}{- : \Box\Gamma_7, \bot, \Delta_{12} \vdash \Delta_9, []\mathtt{F_{10}}}\ Cut$$

$$\rightarrow$$

$$\frac{}{- : \bot, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F_{10}}}\ \bot_L$$

- Case rule I

$$\frac{\dfrac{\mathtt{h_1}: unbox(\Box\Gamma_6) \vdash \mathtt{F_7}}{\bullet\mathtt{h_1}: \Box\Gamma_6, \Delta_{11}, \mathtt{p_9} \vdash (\Delta_{10}, \mathtt{p_9}), []\mathtt{F_7}}\ K \qquad \dfrac{}{\bullet\mathtt{h_8}: (\Box\Gamma_6, \Delta_{11}, \mathtt{p_9}), []\mathtt{F_7} \vdash \Delta_{10}, \mathtt{p_9}}\ I}{- : \Box\Gamma_6, \Delta_{11}, \mathtt{p_9} \vdash \Delta_{10}, \mathtt{p_9}}\ Cut$$

$$\rightarrow$$

$$\frac{}{- : \Delta_{11}, \Box\Gamma_6, \mathtt{p_9} \vdash \Delta_{10}, \mathtt{p_9}}\ I$$

$$\frac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_7) \vdash \mathtt{F_9}}{\bullet\mathtt{h_2}: \Box\Gamma_7, \Delta_8 \vdash ((\Delta_{12}, \mathtt{p_{11}}), []\mathtt{F_9}), \mathtt{p_{11}}}\ K \qquad \dfrac{}{\bullet\mathtt{h_{10}}: (\Box\Gamma_7, \Delta_8), \mathtt{p_{11}} \vdash (\Delta_{12}, \mathtt{p_{11}}), []\mathtt{F_9}}\ I}{- : \Box\Gamma_7, \Delta_8 \vdash (\Delta_{12}, \mathtt{p_{11}}), []\mathtt{F_9}}\ Cut$$

$$\rightarrow$$

$$\frac{\dfrac{}{- : unbox(\Box\Gamma_7) \vdash \mathtt{F_9}}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{12}, \mathtt{p_{11}}, []\mathtt{F_9}}\ K$$

$$\frac{\dfrac{\mathtt{h_2}: unbox(\Box\Gamma_7) \vdash \mathtt{F_9}}{\bullet\mathtt{h_2}: \Box\Gamma_7, \Delta_{13}, \mathtt{p_{11}} \vdash ((\Delta_{12}, \mathtt{p_{11}}), []\mathtt{F_9}), \mathtt{F_8}}\ K \qquad \dfrac{}{\bullet\mathtt{h_{10}}: (\Box\Gamma_7, \Delta_{13}, \mathtt{p_{11}}), \mathtt{F_8} \vdash (\Delta_{12}, \mathtt{p_{11}}), []\mathtt{F_9}}\ I}{- : \Box\Gamma_7, \Delta_{13}, \mathtt{p_{11}} \vdash (\Delta_{12}, \mathtt{p_{11}}), []\mathtt{F_9}}\ Cut$$

$$\rightarrow$$

$$\frac{}{- : \Delta_{13}, \Box\Gamma_7, \mathtt{p_{11}} \vdash \Delta_{12}, \mathtt{p_{11}}, []\mathtt{F_9}}\ I$$

- Case rule $\top_L$

$$\frac{\dfrac{\mathtt{h_1}: unbox(\Box\Gamma_6) \vdash \mathtt{F_7}}{\bullet\mathtt{h_1}: \Box\Gamma_6, \top, \Delta_{10} \vdash \Delta_9, []\mathtt{F_7}}\ K \qquad \dfrac{\mathtt{h_8}: \Box\Gamma_6, \Delta_{10}, []\mathtt{F_7} \vdash \Delta_9}{\bullet\mathtt{h_8}: (\Box\Gamma_6, \top, \Delta_{10}), []\mathtt{F_7} \vdash \Delta_9}\ \top_L}{- : \Box\Gamma_6, \top, \Delta_{10} \vdash \Delta_9}\ Cut$$

$$\rightarrow$$

$$\frac{\dfrac{}{\bullet\mathtt{h_1}: \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9, []\mathtt{F_7}}\ \mathtt{ax/W} \qquad \dfrac{\mathtt{h_8}: \top, \Delta_{10}, \Box\Gamma_6, []\mathtt{F_7} \vdash \Delta_9}{}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9}\ \mathtt{hCut}$$

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$$\dfrac{\dfrac{\mathtt{h_2} : unbox(\Box\Gamma_7) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2} : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F_{10}}), \top} \ K \qquad \dfrac{\mathtt{h_{11}} : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F_{10}}}{\bullet\mathtt{h_{11}} : (\Box\Gamma_7, \Delta_8), \top \vdash \Delta_9, []\mathtt{F_{10}}} \ \top_L}{\dfrac{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F_{10}}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F_{10}}} \ \mathtt{ax/W}} \ \mathtt{Cut} \ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2} : unbox(\Box\Gamma_7) \vdash \mathtt{F_{10}}}{\bullet\mathtt{h_2} : \Box\Gamma_7, \top, \Delta_{12} \vdash (\Delta_9, []\mathtt{F_{10}}), \mathtt{F_8}} \ K \quad \dfrac{\mathtt{h_{11}} : \Box\Gamma_7, \mathtt{F_8}, \Delta_{12} \vdash \Delta_9, []\mathtt{F_{10}}}{\bullet\mathtt{h_{11}} : (\Box\Gamma_7, \top, \Delta_{12}), \mathtt{F_8} \vdash \Delta_9, []\mathtt{F_{10}}} \ \top_L}{\dfrac{- : \Box\Gamma_7, \top, \Delta_{12} \vdash \Delta_9, []\mathtt{F_{10}}}{- : unbox(\Box\Gamma_7) \vdash \mathtt{F_{10}}} \ \mathtt{ax/W}} \ \mathtt{Cut} \ \to}{- : \top, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F_{10}}} \ K$$

6.7 Status of $A45$: `fail`

- Case rule $\to_R$

$$\dfrac{\dfrac{\mathtt{h_1} : \Box\Gamma_7 \vdash \Box\Gamma_{10}, \mathtt{F_9}}{\bullet\mathtt{h_1} : \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_{10}, \Delta_{14}, \mathtt{F_{12}} \to \mathtt{F_{13}}), []\mathtt{F_9}} \ A45 \quad \dfrac{\mathtt{h_{11}} : \Box\Gamma_7, \mathtt{F_{12}}, \Delta_8, []\mathtt{F_9} \vdash \Box\Gamma_{10}, \mathtt{F_{13}}, \Delta_{14}}{\bullet\mathtt{h_{11}} : (\Box\Gamma_7, \Delta_8), []\mathtt{F_9} \vdash \Box\Gamma_{10}, \Delta_{14}, \mathtt{F_{12}} \to \mathtt{F_{13}}} \ \to_R}{- : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_{10}, \Delta_{14}, \mathtt{F_{12}} \to \mathtt{F_{13}}} \ \mathtt{Cut} \ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_1} : \Box\Gamma_7 \vdash \mathtt{F_9}, \Box\Gamma_{10}}{\bullet\mathtt{h_1} : \Delta_8, \mathtt{F_{12}}, \Box\Gamma_7 \vdash \Delta_{14}, \mathtt{F_{13}}, \Box\Gamma_{10}, []\mathtt{F_9}} \ \mathtt{ax/W} \ A45 \quad \dfrac{\mathtt{h_{11}} : \Delta_8, \mathtt{F_{12}}, \Box\Gamma_7, []\mathtt{F_9} \vdash \Delta_{14}, \mathtt{F_{13}}, \Box\Gamma_{10}}{} \ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F_{12}}, \Box\Gamma_7 \vdash \Delta_{14}, \mathtt{F_{13}}, \Box\Gamma_{10}} \ \mathtt{hCut}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, \Box\Gamma_{10}, \mathtt{F_{12}} \to \mathtt{F_{13}}} \ \to_R$$

$$\dfrac{\dfrac{\mathtt{h_2} : \Box\Gamma_8 \vdash \Box\Gamma_{11}, \mathtt{F_{12}}, \Box\mathtt{F_{10}}}{\bullet\mathtt{h_2} : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{11}, (\Delta_{16}, \mathtt{F_{14}} \to \mathtt{F_{15}}), []\mathtt{F_{12}}), \Box\mathtt{F_{10}}} \ A45 \quad \dfrac{\mathtt{h_{13}} : \Box\Gamma_8, \mathtt{F_{14}}, \Delta_9, \Box\mathtt{F_{10}} \vdash \Box\Gamma_{11}, \mathtt{F_{15}}, \Delta_{16}, []\mathtt{F_{12}}}{\bullet\mathtt{h_{13}} : (\Box\Gamma_8, \Delta_9), \Box\mathtt{F_{10}} \vdash \Box\Gamma_{11}, (\Delta_{16}, \mathtt{F_{14}} \to \mathtt{F_{15}}), []\mathtt{F_{12}}} \ \to_R}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{11}, (\Delta_{16}, \mathtt{F_{14}} \to \mathtt{F_{15}}), []\mathtt{F_{12}}} \ \mathtt{Cut} \ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2} : \Box\Gamma_8 \vdash \Box\mathtt{F_{10}}, \mathtt{F_{12}}, \Box\Gamma_{11}}{\bullet\mathtt{h_2} : \Delta_9, \mathtt{F_{14}}, \Box\Gamma_8 \vdash \Box\mathtt{F_{10}}, \Delta_{16}, \mathtt{F_{15}}, \Box\Gamma_{11}, []\mathtt{F_{12}}} \ \mathtt{ax/W} \ A45 \quad \dfrac{\mathtt{h_{13}} : \Box\mathtt{F_{10}}, \Delta_9, \mathtt{F_{14}}, \Box\Gamma_8 \vdash \Delta_{16}, \mathtt{F_{15}}, \Box\Gamma_{11}, []\mathtt{F_{12}}}{} \ \mathtt{ax/W}}{- : \Delta_9, \mathtt{F_{14}}, \Box\Gamma_8 \vdash \Delta_{16}, \mathtt{F_{15}}, \Box\Gamma_{11}, []\mathtt{F_{12}}} \ \mathtt{hCut}}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{16}, \Box\Gamma_{11}, []\mathtt{F_{12}}, \mathtt{F_{14}} \to \mathtt{F_{15}}} \ \to_R$$

$$\dfrac{\dfrac{\mathtt{h_2} : \Box\Gamma_8 \vdash \Box\Gamma_{11}, \mathtt{F_{12}}}{\bullet\mathtt{h_2} : \Box\Gamma_8, \Delta_{10} \vdash (\Box\Gamma_{11}, (\Delta_{16}, \mathtt{F_{14}} \to \mathtt{F_{15}}), []\mathtt{F_{12}}), \mathtt{F_9}} \ A45 \quad \dfrac{\mathtt{h_{13}} : \Box\Gamma_8, \mathtt{F_9}, \mathtt{F_{14}}, \Delta_{10} \vdash \Box\Gamma_{11}, \mathtt{F_{15}}, \Delta_{16}, []\mathtt{F_{12}}}{\bullet\mathtt{h_{13}} : (\Box\Gamma_8, \Delta_{10}), \mathtt{F_9} \vdash \Box\Gamma_{11}, (\Delta_{16}, \mathtt{F_{14}} \to \mathtt{F_{15}}), []\mathtt{F_{12}}} \ \to_R}{\dfrac{- : \Box\Gamma_8, \Delta_{10} \vdash \Box\Gamma_{11}, (\Delta_{16}, \mathtt{F_{14}} \to \mathtt{F_{15}}), []\mathtt{F_{12}}}{\dfrac{- : \Box\Gamma_8 \vdash \mathtt{F_{12}}, \Box\Gamma_{11}}{- : \Delta_{10}, \Box\Gamma_8 \vdash \Delta_{16}, \Box\Gamma_{11}, []\mathtt{F_{12}}, \mathtt{F_{14}} \to \mathtt{F_{15}}} \ \mathtt{ax/W}} \ A45} \ \mathtt{Cut} \ \to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h_1} : \Box\Gamma_7 \vdash \Box\Gamma_{10}, \mathtt{F_9}}{\bullet\mathtt{h_1} : \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_{10}, \Delta_{14}, \mathtt{F_{12}} \wedge \mathtt{F_{13}}), []\mathtt{F_9}} \ A45 \quad \dfrac{\mathtt{h_{11}} : \Box\Gamma_7, \Delta_8, []\mathtt{F_9} \vdash \Box\Gamma_{10}, \mathtt{F_{12}}, \Delta_{14} \quad \mathtt{h_{11}} : \Box\Gamma_7, \Delta_8, []\mathtt{F_9} \vdash \Box\Gamma_{10}, \mathtt{F_{13}}, \Delta_{14}}{\bullet\mathtt{h_{11}} : (\Box\Gamma_7, \Delta_8), []\mathtt{F_9} \vdash \Box\Gamma_{10}, \Delta_{14}, \mathtt{F_{12}} \wedge \mathtt{F_{13}}} \ \wedge_R}{- : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_{10}, \Delta_{14}, \mathtt{F_{12}} \wedge \mathtt{F_{13}}} \ \mathtt{Cut} \ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_1} : \Box\Gamma_7 \vdash \mathtt{F_9}, \Box\Gamma_{10}}{\bullet\mathtt{h_1} : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, \mathtt{F_{12}}, \Box\Gamma_{10}, []\mathtt{F_9}} \ \mathtt{ax/W} \ A45 \quad \dfrac{\mathtt{h_{11}} : \Delta_8, \Box\Gamma_7, []\mathtt{F_9} \vdash \Delta_{14}, \mathtt{F_{12}}, \Box\Gamma_{10}}{} \ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, \mathtt{F_{12}}, \Box\Gamma_{10}} \ \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h_1} : \Box\Gamma_7 \vdash \mathtt{F_9}, \Box\Gamma_{10}}{\bullet\mathtt{h_1} : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, \mathtt{F_{13}}, \Box\Gamma_{10}, []\mathtt{F_9}} \ \mathtt{ax/W} \ A45 \quad \mathtt{h_{11}} : \Delta_8, \Box\Gamma_7, []}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, \mathtt{F_{13}}, \Box\Gamma_{10}}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, \Box\Gamma_{10}, \mathtt{F_{12}} \wedge \mathtt{F_{13}}} \ \wedge_R$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\Gamma_{11},\mathtt{F}_{12},\Box\mathtt{F}_{10}}{\bullet\mathtt{h_2}:\Box\Gamma_8,\Delta_9\vdash(\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\wedge\mathtt{F}_{15}),[]\mathtt{F}_{12}),\Box\mathtt{F}_{10}}\,A45\quad\dfrac{\mathtt{h_{13}}:\Box\Gamma_8,\Delta_9,\Box\mathtt{F}_{10}\vdash\Box\Gamma_{11},\mathtt{F}_{14},\Delta_{16},[]\mathtt{F}_{12}\quad\mathtt{h_{13}}:\Box\Gamma_8,\Delta_9,\Box\mathtt{F}_{1}}{\bullet\mathtt{h_{13}}:(\Box\Gamma_8,\Delta_9),\Box\mathtt{F}_{10}\vdash\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\wedge\mathtt{F}_{1})}}{-:\Box\Gamma_8,\Delta_9\vdash\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\wedge\mathtt{F}_{15}),[]\mathtt{F}_{12}}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\overline{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\mathtt{F}_{10},\mathtt{F}_{12},\Box\Gamma_{11}}\,\mathtt{ax/W}}{\bullet\mathtt{h_2}:\Delta_9,\Box\Gamma_8\vdash\Box\mathtt{F}_{10},\Delta_{16},\mathtt{F}_{14},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,A45\quad\overline{\mathtt{h_{13}}:\Box\mathtt{F}_{10},\Delta_9,\Box\Gamma_8\vdash\Delta_{16},\mathtt{F}_{14},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,\mathtt{ax/W}}{-:\Delta_9,\Box\Gamma_8\vdash\Delta_{16},\mathtt{F}_{14},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,\mathtt{hCut}\quad\dfrac{\overline{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\mathtt{F}_{10},\mathtt{F}_{12},\Box\Gamma_{11}}\,\mathtt{ax/W}}{\bullet\mathtt{h_2}:\Delta_9,\Box\Gamma_8\vdash\Box\mathtt{F}_{10},\Delta_{16},\mathtt{F}_{15},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,A}{-:\Delta_9,\Box\Gamma_8\vdash\Delta}}{-:\Delta_9,\Box\Gamma_8\vdash\Delta_{16},\Box\Gamma_{11},[]\mathtt{F}_{12},\mathtt{F}_{14}\wedge\mathtt{F}_{15}}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\Gamma_{11},\mathtt{F}_{12}}{\bullet\mathtt{h_2}:\Box\Gamma_8,\Delta_{10}\vdash(\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\wedge\mathtt{F}_{15}),[]\mathtt{F}_{12}),\mathtt{F}_9}\,A45\quad\dfrac{\mathtt{h_{13}}:\Box\Gamma_8,\mathtt{F}_9,\Delta_{10}\vdash\Box\Gamma_{11},\mathtt{F}_{14},\Delta_{16},[]\mathtt{F}_{12}\quad\mathtt{h_{13}}:\Box\Gamma_8,\mathtt{F}_9,\Delta_{10}\vdash\Box\Gamma_{11},\mathtt{F}_{15},\Delta_{16},[]\mathtt{F}_{12}}{\bullet\mathtt{h_{13}}:(\Box\Gamma_8,\Delta_{10}),\mathtt{F}_9\vdash\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\wedge\mathtt{F}_{15}),[]\mathtt{F}_{12}}\,\mathtt{Cut}}{-:\Box\Gamma_8,\Delta_{10}\vdash\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\wedge\mathtt{F}_{15}),[]\mathtt{F}_{12}}}\,\wedge_R$$
$$\to$$
$$\dfrac{\dfrac{\overline{-:\Box\Gamma_8\vdash\mathtt{F}_{12},\Box\Gamma_{11}}\,\mathtt{ax/W}}{-:\Delta_{10},\Box\Gamma_8\vdash\Delta_{16},\Box\Gamma_{11},[]\mathtt{F}_{12},\mathtt{F}_{14}\wedge\mathtt{F}_{15}}\,A45}{}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h_1}:\Box\Gamma_7\vdash\Box\Gamma_{10},\mathtt{F}_9}{\bullet\mathtt{h_1}:\Box\Gamma_7,\Delta_8\vdash(\Box\Gamma_{10},\Delta_{14},\mathtt{F}_{12}\vee\mathtt{F}_{13}),[]\mathtt{F}_9}\,A45\quad\dfrac{\mathtt{h_{11}}:\Box\Gamma_7,\Delta_8,[]\mathtt{F}_9\vdash\Box\Gamma_{10},\mathtt{F}_{12},\mathtt{F}_{13},\Delta_{14}}{\bullet\mathtt{h_{11}}:(\Box\Gamma_7,\Delta_8),[]\mathtt{F}_9\vdash\Box\Gamma_{10},\Delta_{14},\mathtt{F}_{12}\vee\mathtt{F}_{13}}\,\vee_R}{-:\Box\Gamma_7,\Delta_8\vdash\Box\Gamma_{10},\Delta_{14},\mathtt{F}_{12}\vee\mathtt{F}_{13}}\,\mathtt{Cut}}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\overline{\mathtt{h_1}:\Box\Gamma_7\vdash\mathtt{F}_9,\Box\Gamma_{10}}\,\mathtt{ax/W}}{\bullet\mathtt{h_1}:\Delta_8,\Box\Gamma_7\vdash\Delta_{14},\mathtt{F}_{12},\mathtt{F}_{13},\Box\Gamma_{10},[]\mathtt{F}_9}\,A45\quad\overline{\mathtt{h_{11}}:\Delta_8,\Box\Gamma_7,[]\mathtt{F}_9\vdash\Delta_{14},\mathtt{F}_{12},\mathtt{F}_{13},\Box\Gamma_{10}}\,\mathtt{ax/W}}{-:\Delta_8,\Box\Gamma_7\vdash\Delta_{14},\mathtt{F}_{12},\mathtt{F}_{13},\Box\Gamma_{10}}\,\mathtt{hCut}}{-:\Delta_8,\Box\Gamma_7\vdash\Delta_{14},\Box\Gamma_{10},\mathtt{F}_{12}\vee\mathtt{F}_{13}}\,\vee_R}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\Gamma_{11},\mathtt{F}_{12},\Box\mathtt{F}_{10}}{\bullet\mathtt{h_2}:\Box\Gamma_8,\Delta_9\vdash(\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\vee\mathtt{F}_{15}),[]\mathtt{F}_{12}),\Box\mathtt{F}_{10}}\,A45\quad\dfrac{\mathtt{h_{13}}:\Box\Gamma_8,\Delta_9,\Box\mathtt{F}_{10}\vdash\Box\Gamma_{11},\mathtt{F}_{14},\mathtt{F}_{15},\Delta_{16},[]\mathtt{F}_{12}}{\bullet\mathtt{h_{13}}:(\Box\Gamma_8,\Delta_9),\Box\mathtt{F}_{10}\vdash\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\vee\mathtt{F}_{15}),[]\mathtt{F}_{12}}\,\vee_R}{-:\Box\Gamma_8,\Delta_9\vdash\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\vee\mathtt{F}_{15}),[]\mathtt{F}_{12}}\,\mathtt{Cut}}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\overline{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\mathtt{F}_{10},\mathtt{F}_{12},\Box\Gamma_{11}}\,\mathtt{ax/W}}{\bullet\mathtt{h_2}:\Delta_9,\Box\Gamma_8\vdash\Box\mathtt{F}_{10},\Delta_{16},\mathtt{F}_{14},\mathtt{F}_{15},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,A45\quad\overline{\mathtt{h_{13}}:\Box\mathtt{F}_{10},\Delta_9,\Box\Gamma_8\vdash\Delta_{16},\mathtt{F}_{14},\mathtt{F}_{15},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,\mathtt{ax/W}}{-:\Delta_9,\Box\Gamma_8\vdash\Delta_{16},\mathtt{F}_{14},\mathtt{F}_{15},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,\mathtt{hCut}}{-:\Delta_9,\Box\Gamma_8\vdash\Delta_{16},\Box\Gamma_{11},[]\mathtt{F}_{12},\mathtt{F}_{14}\vee\mathtt{F}_{15}}\,\vee_R}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\Gamma_{11},\mathtt{F}_{12}}{\bullet\mathtt{h_2}:\Box\Gamma_8,\Delta_{10}\vdash(\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\vee\mathtt{F}_{15}),[]\mathtt{F}_{12}),\mathtt{F}_9}\,A45\quad\dfrac{\mathtt{h_{13}}:\Box\Gamma_8,\mathtt{F}_9,\Delta_{10}\vdash\Box\Gamma_{11},\mathtt{F}_{14},\mathtt{F}_{15},\Delta_{16},[]\mathtt{F}_{12}}{\bullet\mathtt{h_{13}}:(\Box\Gamma_8,\Delta_{10}),\mathtt{F}_9\vdash\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\vee\mathtt{F}_{15}),[]\mathtt{F}_{12}}\,\vee_R}{-:\Box\Gamma_8,\Delta_{10}\vdash\Box\Gamma_{11},(\Delta_{16},\mathtt{F}_{14}\vee\mathtt{F}_{15}),[]\mathtt{F}_{12}}\,\mathtt{Cut}}$$
$$\to$$
$$\dfrac{\dfrac{\overline{-:\Box\Gamma_8\vdash\mathtt{F}_{12},\Box\Gamma_{11}}\,\mathtt{ax/W}}{-:\Delta_{10},\Box\Gamma_8\vdash\Delta_{16},\Box\Gamma_{11},[]\mathtt{F}_{12},\mathtt{F}_{14}\vee\mathtt{F}_{15}}\,A45}{}$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{\mathtt{h_1}:\Box\Gamma_7\vdash\Box\Gamma_{10},\mathtt{F}_9}{\bullet\mathtt{h_1}:\Box\Gamma_7,\Delta_8\vdash(\Box\Gamma_{10},\perp,\Delta_{12}),[]\mathtt{F}_9}\,A45\quad\dfrac{\mathtt{h_{11}}:\Box\Gamma_7,\Delta_8,[]\mathtt{F}_9\vdash\Box\Gamma_{10},\Delta_{12}}{\bullet\mathtt{h_{11}}:(\Box\Gamma_7,\Delta_8),[]\mathtt{F}_9\vdash\Box\Gamma_{10},\perp,\Delta_{12}}\,\perp_R}{-:\Box\Gamma_7,\Delta_8\vdash\Box\Gamma_{10},\perp,\Delta_{12}}\,\mathtt{Cut}}$$
$$\to$$
$$\dfrac{\dfrac{\overline{\bullet\mathtt{h_1}:\Delta_8,\Box\Gamma_7\vdash\perp,\Delta_{12},\Box\Gamma_{10},[]\mathtt{F}_9}\,\mathtt{ax/W}\quad\overline{\mathtt{h_{11}}:\Delta_8,\Box\Gamma_7,[]\mathtt{F}_9\vdash\perp,\Delta_{12},\Box\Gamma_{10}}\,\mathtt{ax/W}}{-:\Delta_8,\Box\Gamma_7\vdash\perp,\Delta_{12},\Box\Gamma_{10}}\,\mathtt{hCut}}{}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\Gamma_{11},\mathtt{F}_{12},\Box\mathtt{F}_{10}}{\bullet\mathtt{h_2}:\Box\Gamma_8,\Delta_9\vdash(\Box\Gamma_{11},(\perp,\Delta_{14}),[]\mathtt{F}_{12}),\Box\mathtt{F}_{10}}\,A45\quad\dfrac{\mathtt{h_{13}}:\Box\Gamma_8,\Delta_9,\Box\mathtt{F}_{10}\vdash\Box\Gamma_{11},\Delta_{14},[]\mathtt{F}_{12}}{\bullet\mathtt{h_{13}}:(\Box\Gamma_8,\Delta_9),\Box\mathtt{F}_{10}\vdash\Box\Gamma_{11},(\perp,\Delta_{14}),[]\mathtt{F}_{12}}\,\perp_R}{-:\Box\Gamma_8,\Delta_9\vdash\Box\Gamma_{11},(\perp,\Delta_{14}),[]\mathtt{F}_{12}}\,\mathtt{Cut}}$$
$$\to$$
$$\dfrac{\dfrac{\overline{\bullet\mathtt{h_2}:\Delta_9,\Box\Gamma_8\vdash\perp,\Box\mathtt{F}_{10},\Delta_{14},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,\mathtt{ax/W}\quad\overline{\mathtt{h_{13}}:\Box\mathtt{F}_{10},\Delta_9,\Box\Gamma_8\vdash\perp,\Delta_{14},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,\mathtt{ax/W}}{-:\Delta_9,\Box\Gamma_8\vdash\perp,\Delta_{14},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,\mathtt{hCut}}{}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\Gamma_{11},\mathtt{F}_{12}}{\bullet\mathtt{h_2}:\Box\Gamma_8,\Delta_{10}\vdash(\Box\Gamma_{11},(\bot,\Delta_{14}),[]\mathtt{F}_{12}),\mathtt{F}_9}\,A45 \quad \dfrac{\mathtt{h_{13}}:\Box\Gamma_8,\mathtt{F}_9,\Delta_{10}\vdash\Box\Gamma_{11},\Delta_{14},[]\mathtt{F}_{12}}{\bullet\mathtt{h_{13}}:(\Box\Gamma_8,\Delta_{10}),\mathtt{F}_9\vdash\Box\Gamma_{11},(\bot,\Delta_{14}),[]\mathtt{F}_{12}}\,\bot_R}{-:\Box\Gamma_8,\Delta_{10}\vdash\Box\Gamma_{11},(\bot,\Delta_{14}),[]\mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\xrightarrow{} $$

$$\dfrac{\dfrac{}{-:\Box\Gamma_8\vdash\mathtt{F}_{12},\Box\Gamma_{11}}\,\mathtt{ax/W}}{-:\Delta_{10},\Box\Gamma_8\vdash\bot,\Delta_{14},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,A45$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h_1}:\Box\Gamma_7\vdash\Box\Gamma_{10},\mathtt{F}_9}{\bullet\mathtt{h_1}:\Box\Gamma_7,\Delta_8\vdash(\Box\Gamma_{10},\top,\Delta_{12}),[]\mathtt{F}_9}\,A45 \quad \dfrac{}{\bullet\mathtt{h_{11}}:(\Box\Gamma_7,\Delta_8),[]\mathtt{F}_9\vdash\Box\Gamma_{10},\top,\Delta_{12}}\,\top_R}{-:\Box\Gamma_7,\Delta_8\vdash\Box\Gamma_{10},\top,\Delta_{12}}\,\mathtt{Cut}$$

$$\xrightarrow{}$$

$$\dfrac{}{-:\Delta_8,\Box\Gamma_7\vdash\top,\Delta_{12},\Box\Gamma_{10}}\,\top_R$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\Gamma_{11},\mathtt{F}_{12},\Box\mathtt{F}_{10}}{\bullet\mathtt{h_2}:\Box\Gamma_8,\Delta_9\vdash(\Box\Gamma_{11},(\top,\Delta_{14}),[]\mathtt{F}_{12}),\Box\mathtt{F}_{10}}\,A45 \quad \dfrac{}{\bullet\mathtt{h_{13}}:(\Box\Gamma_8,\Delta_9),\Box\mathtt{F}_{10}\vdash\Box\Gamma_{11},(\top,\Delta_{14}),[]\mathtt{F}_{12}}\,\top_R}{-:\Box\Gamma_8,\Delta_9\vdash\Box\Gamma_{11},(\top,\Delta_{14}),[]\mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\xrightarrow{}$$

$$\dfrac{}{-:\Delta_9,\Box\Gamma_8\vdash\top,\Delta_{14},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,\top_R$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_8\vdash\Box\Gamma_{11},\mathtt{F}_{12}}{\bullet\mathtt{h_2}:\Box\Gamma_8,\Delta_{10}\vdash(\Box\Gamma_{11},(\top,\Delta_{14}),[]\mathtt{F}_{12}),\mathtt{F}_9}\,A45 \quad \dfrac{}{\bullet\mathtt{h_{13}}:(\Box\Gamma_8,\Delta_{10}),\mathtt{F}_9\vdash\Box\Gamma_{11},(\top,\Delta_{14}),[]\mathtt{F}_{12}}\,\top_R}{-:\Box\Gamma_8,\Delta_{10}\vdash\Box\Gamma_{11},(\top,\Delta_{14}),[]\mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\xrightarrow{}$$

$$\dfrac{}{-:\Delta_{10},\Box\Gamma_8\vdash\top,\Delta_{14},\Box\Gamma_{11},[]\mathtt{F}_{12}}\,\top_R$$

- Case rule K

$$\dfrac{\dfrac{\mathtt{h_1}:\Box\Gamma_{12},\Box\Gamma_{14}\vdash(\Box\Gamma_{11},[]\mathtt{F}_{10}),\mathtt{F}_7}{\bullet\mathtt{h_1}:(\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}\vdash((\Box\Gamma_{11},[]\mathtt{F}_{10}),\Delta_8),[]\mathtt{F}_7}\,A45 \quad \dfrac{\mathtt{h_9}:\mathtt{F}_7,unbox(\Box\Gamma_{12}),unbox(\Box\Gamma_{13})\vdash\mathtt{F}_{10}}{\bullet\mathtt{h_9}:((\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F}_7\vdash(\Box\Gamma_{11},[]\mathtt{F}_{10}),\Delta_8}\,K}{-:(\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}\vdash(\Box\Gamma_{11},[]\mathtt{F}_{10}),\Delta_8}\,\mathtt{Cut} \quad\rightarrow\quad \dfrac{}{{\color{red}-:(\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}}}$$

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Axioms assumed:

inf : C:MSFormula |-- True ; C’:MSFormula
inf : False ; C:MSFormula |-- C’:MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C’:MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C’:MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C’:MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C’:MSFormula
```

$$\dfrac{\dfrac{\mathtt{h_1}:\Box\Gamma_{12},\Box\Gamma_{14}\vdash\Box\Gamma_8,\mathtt{F}_7}{\bullet\mathtt{h_1}:(\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}\vdash(\Box\Gamma_8,\Delta_{11},[]\mathtt{F}_{10}),[]\mathtt{F}_7}\,A45 \quad \dfrac{\mathtt{h_9}:\mathtt{F}_7,unbox(\Box\Gamma_{12}),unbox(\Box\Gamma_{13})\vdash\mathtt{F}_{10}}{\bullet\mathtt{h_9}:((\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F}_7\vdash\Box\Gamma_8,\Delta_{11},[]\mathtt{F}_{10}}\,K}{-:(\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}\vdash\Box\Gamma_8,\Delta_{11},[]\mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\xrightarrow{}$$

$$\dfrac{\dfrac{\dfrac{}{-:\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\mathtt{F}_{10},\mathtt{F}_7,\Box\Gamma_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{-:\mathtt{F}_7,\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},unbox(\Box\Gamma_{12}),unbox(\Box\Gamma_{13})\vdash\mathtt{F}_{10},\Box\Gamma_8}\,\mathtt{ax/W}}{-:\mathtt{F}_7,\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\mathtt{F}_{10},\Box\Gamma_8}\,ATG}{-:\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\mathtt{F}_{10},\Box\Gamma_8}\,\mathtt{sCut}}{-:\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_{11},\Box\Gamma_8,[]\mathtt{F}_{10}}\,A45$$

$$\dfrac{\dfrac{\mathtt{h_1}:\Box\Gamma_{12},\Box\Gamma_{14}\vdash(\Box\Gamma_{11},[]\mathtt{F}_{10}),\mathtt{F}_7}{\bullet\mathtt{h_1}:(\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}\vdash((\Box\Gamma_{11},[]\mathtt{F}_{10}),\Delta_8),[]\mathtt{F}_7}\,A45 \quad \dfrac{\mathtt{h_9}:unbox(\Box\Gamma_{12}),unbox(\Box\Gamma_{13})\vdash\mathtt{F}_{10}}{\bullet\mathtt{h_9}:((\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F}_7\vdash(\Box\Gamma_{11},[]\mathtt{F}_{10}),\Delta_8}\,K}{-:(\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}\vdash(\Box\Gamma_{11},[]\mathtt{F}_{10}),\Delta_8}\,\mathtt{Cut}$$

$$\xrightarrow{}$$

$$\dfrac{\dfrac{}{-:unbox(\Box\Gamma_{12}),unbox(\Box\Gamma_{13})\vdash\mathtt{F}_{10}}\,\mathtt{ax/W}}{-:\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_8,\Box\Gamma_{11},[]\mathtt{F}_{10}}\,K$$

$$\dfrac{\dfrac{\mathtt{h_1}:\Box\Gamma_{12},\Box\Gamma_{14}\vdash\Box\Gamma_8,\mathtt{F_7}}{\bullet\mathtt{h_1}:(\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}\vdash(\Box\Gamma_8,\Delta_{11},[]\mathtt{F_{10}}),[]\mathtt{F_7}}\,A45 \quad \dfrac{\mathtt{h_9}:unbox(\Box\Gamma_{12}),unbox(\Box\Gamma_{13})\vdash\mathtt{F_{10}}}{\bullet\mathtt{h_9}:((\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F_7}\vdash\Box\Gamma_8,\Delta_{11},[]\mathtt{F_{10}}}\,K}{-:(\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}\vdash\Box\Gamma_8,\Delta_{11},[]\mathtt{F_{10}}}\,\text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\overline{-:unbox(\Box\Gamma_{12}),unbox(\Box\Gamma_{13})\vdash\mathtt{F_{10}}}}{-:\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_{11},\Box\Gamma_8,[]\mathtt{F_{10}}}\,\text{ax/W}}{}\,K$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{14},\Box\Gamma_{16}\vdash(\Box\Gamma_{13},[]\mathtt{F_{12}}),\mathtt{F_{10}},\Box\mathtt{F_8}}{\bullet\mathtt{h_2}:(\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}\vdash((\Box\Gamma_{13},[]\mathtt{F_{12}}),\Delta_9,[]\mathtt{F_{10}}),\Box\mathtt{F_8}}\,A45 \quad \dfrac{\mathtt{h_{11}}:unbox(\Box\Gamma_{14}),unbox(\Box\Gamma_{15}),unbox(\Box\mathtt{F_8})\vdash\mathtt{F_{12}}}{\bullet\mathtt{h_{11}}:((\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}),\Box\mathtt{F_8}\vdash(\Box\Gamma_{13},[]\mathtt{F_{12}}),\Delta_9,[]\mathtt{F_{10}}}\,K}{-:(\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}\vdash(\Box\Gamma_{13},[]\mathtt{F_{12}}),\Delta_9,[]\mathtt{F_{10}}}\,\text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Box\mathtt{F_8},\mathtt{F_{10}},\Box\Gamma_{13},[]\mathtt{F_{12}}}{}\,\text{ax/W}\quad \dfrac{\dfrac{\mathtt{h_{11}}:unbox(\Box\mathtt{F_8}),unbox(\Box\Gamma_{14}),unbox(\Box\Gamma_{15})\vdash\mathtt{F_{12}}}{\bullet\mathtt{h_{11}}:\Box\mathtt{F_8},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\mathtt{F_{10}},\Box\Gamma_{13},[]\mathtt{F_{12}}}\,K}{}\,\text{ax/W}}{-:\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\mathtt{F_{10}},\Box\Gamma_{13},[]\mathtt{F_{12}}}\,\text{hCut}}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_9,\Box\Gamma_{13},[]\mathtt{F_{10}},[]\mathtt{F_{12}}}\,A45$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{14},\Box\Gamma_{16}\vdash\Box\Gamma_9,\mathtt{F_{10}},\Box\mathtt{F_8}}{\bullet\mathtt{h_2}:(\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}\vdash(\Box\Gamma_9,(\Delta_{13},[]\mathtt{F_{12}}),[]\mathtt{F_{10}}),\Box\mathtt{F_8}}\,A45 \quad \dfrac{\mathtt{h_{11}}:unbox(\Box\Gamma_{14}),unbox(\Box\Gamma_{15}),unbox(\Box\mathtt{F_8})\vdash\mathtt{F_{12}}}{\bullet\mathtt{h_{11}}:((\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}),\Box\mathtt{F_8}\vdash\Box\Gamma_9,(\Delta_{13},[]\mathtt{F_{12}}),[]\mathtt{F_{10}}}\,K}{-:(\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}\vdash\Box\Gamma_9,(\Delta_{13},[]\mathtt{F_{12}}),[]\mathtt{F_{10}}}\,\text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Box\mathtt{F_8},\mathtt{F_{10}},\Box\Gamma_9,[]\mathtt{F_{12}}}{}\,\text{ax/W}\quad \dfrac{\dfrac{\mathtt{h_{11}}:unbox(\Box\mathtt{F_8}),unbox(\Box\Gamma_{14}),unbox(\Box\Gamma_{15})\vdash\mathtt{F_{12}}}{\bullet\mathtt{h_{11}}:\Box\mathtt{F_8},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\mathtt{F_{10}},\Box\Gamma_9,[]\mathtt{F_{12}}}\,K}{}\,\text{ax/W}}{-:\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\mathtt{F_{10}},\Box\Gamma_9,[]\mathtt{F_{12}}}\,\text{hCut}}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_{13},\Box\Gamma_9,[]\mathtt{F_{10}},[]\mathtt{F_{12}}}\,A45$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{13},\Box\Gamma_{15}\vdash\Box\Gamma_9,\mathtt{F_{12}},\Box\mathtt{F_8}}{\bullet\mathtt{h_2}:(\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16}\vdash(\Box\Gamma_9,\Delta_{10},[]\mathtt{F_{12}}),\Box\mathtt{F_8}}\,A45 \quad \dfrac{\mathtt{h_{11}}:unbox(\Box\Gamma_{13}),unbox(\Box\Gamma_{14}),unbox(\Box\mathtt{F_8})\vdash\mathtt{F_{12}}}{\bullet\mathtt{h_{11}}:((\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16}),\Box\mathtt{F_8}\vdash\Box\Gamma_9,\Delta_{10},[]\mathtt{F_{12}}}\,K}{-:(\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16}\vdash\Box\Gamma_9,\Delta_{10},[]\mathtt{F_{12}}}\,\text{Cut} \quad\to\quad \color{red}{\overline{-:(\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16}\vdash}}$$

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Axioms assumed:

inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
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$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{14},\Box\Gamma_{16}\vdash(\Box\Gamma_{13},[]\mathtt{F_{12}}),\mathtt{F_{10}},\Box\mathtt{F_8}}{\bullet\mathtt{h_2}:(\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}\vdash((\Box\Gamma_{13},[]\mathtt{F_{12}}),\Delta_9,[]\mathtt{F_{10}}),\Box\mathtt{F_8}}\,A45 \quad \dfrac{\mathtt{h_{11}}:unbox(\Box\Gamma_{14}),unbox(\Box\Gamma_{15})\vdash\mathtt{F_{12}}}{\bullet\mathtt{h_{11}}:((\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}),\Box\mathtt{F_8}\vdash(\Box\Gamma_{13},[]\mathtt{F_{12}}),\Delta_9,[]\mathtt{F_{10}}}\,K}{-:(\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}\vdash(\Box\Gamma_{13},[]\mathtt{F_{12}}),\Delta_9,[]\mathtt{F_{10}}}\,\text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\overline{-:unbox(\Box\Gamma_{14}),unbox(\Box\Gamma_{15})\vdash\mathtt{F_{12}}}}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_9,\Box\Gamma_{13},[]\mathtt{F_{10}},[]\mathtt{F_{12}}}\,\text{ax/W}}{}\,K$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{14},\Box\Gamma_{16}\vdash\Box\Gamma_9,\mathtt{F_{10}},\Box\mathtt{F_8}}{\bullet\mathtt{h_2}:(\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}\vdash(\Box\Gamma_9,(\Delta_{13},[]\mathtt{F_{12}}),[]\mathtt{F_{10}}),\Box\mathtt{F_8}}\,A45 \quad \dfrac{\mathtt{h_{11}}:unbox(\Box\Gamma_{14}),unbox(\Box\Gamma_{15})\vdash\mathtt{F_{12}}}{\bullet\mathtt{h_{11}}:((\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}),\Box\mathtt{F_8}\vdash\Box\Gamma_9,(\Delta_{13},[]\mathtt{F_{12}}),[]\mathtt{F_{10}}}\,K}{-:(\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17}\vdash\Box\Gamma_9,(\Delta_{13},[]\mathtt{F_{12}}),[]\mathtt{F_{10}}}\,\text{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\overline{-:unbox(\Box\Gamma_{14}),unbox(\Box\Gamma_{15})\vdash\mathtt{F_{12}}}}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_{13},\Box\Gamma_9,[]\mathtt{F_{10}},[]\mathtt{F_{12}}}\,\text{ax/W}}{}\,K$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \Box\Gamma_9, \mathtt{F}_{12}, \Box\mathtt{F}_8}{\bullet\mathtt{h}_2 : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_9, \Delta_{10}, []\mathtt{F}_{12}), \Box\mathtt{F}_8} \; A45 \quad \cfrac{\mathtt{h}_{11} : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{11} : ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \Box\mathtt{F}_8 \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_{12}} \; K}{- : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_{12}} \; Cut}{\rightarrow}$$

$$\cfrac{\cfrac{\overline{- : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{12}}}{- : \Delta_{16}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_{12}} \; K}{} \; ax/W$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{13}, []\mathtt{F}_{12}), \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_9, []\mathtt{F}_{10}), \Box\mathtt{F}_8} \; A45 \quad \cfrac{\mathtt{h}_{11} : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}), unbox(\Box\mathtt{F}_8) \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{11} : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box\mathtt{F}_8 \vdash (\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_9, []\mathtt{F}_{10}} \; K}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_9, []\mathtt{F}_{10}} \; Cut}{\rightarrow}$$

$$\cfrac{\cfrac{\overline{- : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_{13}, []\mathtt{F}_{12}}}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_9, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_{12}} \; A45}{} \; ax/W$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \Box\Gamma_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_9, (\Delta_{13}, []\mathtt{F}_{12}), []\mathtt{F}_{10}), \Box\mathtt{F}_8} \; A45 \quad \cfrac{\mathtt{h}_{11} : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}), unbox(\Box\mathtt{F}_8) \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{11} : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box\mathtt{F}_8 \vdash \Box\Gamma_9, (\Delta_{13}, []\mathtt{F}_{12}), []\mathtt{F}_{10}} \; K}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash \Box\Gamma_9, (\Delta_{13}, []\mathtt{F}_{12}), []\mathtt{F}_{10}} \; Cut}{\rightarrow}$$

$$\cfrac{\cfrac{\overline{- : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_9}}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_9, []\mathtt{F}_{10}, []\mathtt{F}_{12}} \; A45}{} \; ax/W$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \Box\Gamma_9, \mathtt{F}_{12}}{\bullet\mathtt{h}_2 : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_9, \Delta_{10}, []\mathtt{F}_{12}), \Box\mathtt{F}_8} \; A45 \quad \cfrac{\mathtt{h}_{11} : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}), unbox(\Box\mathtt{F}_8) \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{11} : ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \Box\mathtt{F}_8 \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_{12}} \; K}{- : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_{12}} \; Cut}{\rightarrow}$$

$$\cfrac{\cfrac{\overline{- : \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \mathtt{F}_{12}, \Box\Gamma_9}}{- : \Delta_{16}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_{12}} \; A45}{} \; ax/W$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{13}, []\mathtt{F}_{12}), \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_9, []\mathtt{F}_{10}), \mathtt{F}_8} \; A45 \quad \cfrac{\mathtt{h}_{11} : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{11} : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \mathtt{F}_8 \vdash (\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_9, []\mathtt{F}_{10}} \; K}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_9, []\mathtt{F}_{10}} \; Cut}{\rightarrow}$$

$$\cfrac{\cfrac{\overline{- : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_{12}}}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_9, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_{12}} \; K}{} \; ax/W$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \Box\Gamma_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_9, (\Delta_{13}, []\mathtt{F}_{12}), []\mathtt{F}_{10}), \mathtt{F}_8} \; A45 \quad \cfrac{\mathtt{h}_{11} : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{11} : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \mathtt{F}_8 \vdash \Box\Gamma_9, (\Delta_{13}, []\mathtt{F}_{12}), []\mathtt{F}_{10}} \; K}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash \Box\Gamma_9, (\Delta_{13}, []\mathtt{F}_{12}), []\mathtt{F}_{10}} \; Cut}{\rightarrow}$$

$$\cfrac{\cfrac{\overline{- : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_{12}}}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_9, []\mathtt{F}_{10}, []\mathtt{F}_{12}} \; K}{} \; ax/W$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h}_2 : \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \Box\Gamma_9, \mathtt{F}_{12}}{\bullet\mathtt{h}_2 : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_9, \Delta_{10}, []\mathtt{F}_{12}), \mathtt{F}_8} \; A45 \quad \cfrac{\mathtt{h}_{11} : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{11} : ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \mathtt{F}_8 \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_{12}} \; K}{- : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_{12}} \; Cut}{\rightarrow}$$

$$\cfrac{\cfrac{\overline{- : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{12}}}{- : \Delta_{16}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_{12}} \; K}{} \; ax/W$$

- Case rule $A45$

$$\cfrac{\cfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_7} \; A45 \quad \cfrac{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{15}, []\mathtt{F}_7 \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), []\mathtt{F}_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \Box\Gamma_{11}, \Delta_{13}} \; A45}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \Box\Gamma_{11}, \Delta_{13}} \; Cut} \quad \rightarrow \quad \overline{}$$

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Axioms assumed:
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inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
```

$$\cfrac{\cfrac{\cfrac{h_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), F_7}{\bullet h_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []F_9), []F_7} \quad \cfrac{h_8 : \Box\Gamma_{14}, \Box\Gamma_{15}, []F_7 \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, F_9}{\bullet h_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), []F_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []F_9} \; A45}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []F_9}}{\rightarrow} \\ \cfrac{\cfrac{\cfrac{h_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash F_7, \Box\Gamma_{10}, \Box\Gamma_{12}}{\bullet h_1 : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []F_7} \; ax/W \quad A45 \quad \cfrac{h_8 : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16}, []F_7 \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}{} \; ax/W}{- : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}} \; hCut}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []F_9} \; A45$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []F_9), F_7}{\bullet h_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}, []F_9), \Box\Gamma_{11}, \Delta_{13}), []F_7} \quad \cfrac{h_8 : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, F_9}{\bullet h_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), []F_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []F_9), \Box\Gamma_{11}, \Delta_{13}} \; A45}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []F_9), \Box\Gamma_{11}, \Delta_{13}}}{\rightarrow} \\ \cfrac{\cfrac{- : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}}{} \; ax/W}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []F_9} \; A45$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), F_7}{\bullet h_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []F_9), []F_7} \quad \cfrac{h_8 : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, F_9}{\bullet h_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), []F_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []F_9} \; A45}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []F_9}}{\rightarrow} \\ \cfrac{\cfrac{- : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}}{} \; ax/W}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []F_9} \; A45$$

$$\cfrac{\cfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), F_9, \Box F_8}{\bullet h_2 : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_9), \Box F_8} \quad \cfrac{h_{10} : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box F_8 \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, F_{11}, []F_9}{\bullet h_{10} : ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_8 \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_9} \; A45}{- : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_9}$$

Axioms assumed:

$$\cfrac{\cfrac{\cfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), F_9, \Box F_8}{\bullet h_2 : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_9), \Box F_8} \quad \cfrac{h_{10} : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box F_8 \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, F_{11}}{\bullet h_{10} : ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_8 \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}),} \; A45}{- : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_9}}{\rightarrow} \\ \cfrac{\cfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Box F_8, F_9, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}}{} \; ax/W \quad \cfrac{\cfrac{h_{10} : \Box F_8, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}}{\bullet h_{10} : \Box F_8, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash F_9, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}} \; ax/W}{} \; A45}{\cfrac{- : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash F_9, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}}{- : \Delta_{19}, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}, []F_9} \; A45} \; hC}$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), F_9, \Box F_8}{\bullet h_2 : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_9), \Box F_8} \quad \dfrac{h_{10} : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box F_8 \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, F_{11}, []F_9}{\bullet h_{10} : ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_8 \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13},}}{} A45$$

$$- : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_9$$
$$\to$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \Box F_8, F_9, \Box\Gamma_{12}, \Box\Gamma_{14}}{\bullet h_2 : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Box F_8, F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_9} \; ax/W \quad \dfrac{h_{10} : \Box F_8, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_9}{} \; ax}{A45 \quad hC}$$

$$- : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_9$$
$$- : \Delta_{19}, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}, []F_9 \quad A45$$

$$\dfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), F_9, \Box F_8}{\bullet h_2 : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_9), \Box F_8} \quad A45 \quad \dfrac{h_{10} : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box F_8 \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, F_{11}}{\bullet h_{10} : ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_8 \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13},}$$

$$- : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_9$$
$$\to$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \Box F_8, F_9, \Box\Gamma_{12}, \Box\Gamma_{14}}{\bullet h_2 : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Box F_8, F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_9} \; ax/W \quad \dfrac{h_{10} : \Box F_8, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_9}{} \; ax}{A45 \quad hC}$$

$$- : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_9$$
$$- : \Delta_{19}, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}, []F_9 \quad A45$$

$$\dfrac{h_2 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), F_{10}, \Box F_8}{\bullet h_2 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), []F_{10}), \Box F_8} \quad A45 \quad \dfrac{h_9 : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box F_8 \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, F_{10}}{\bullet h_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18}), \Box F_8 \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14})}$$

$$- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), []F_{10}$$
$$\to$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Box F_8, F_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}}{} \; ax/W \quad \dfrac{\dfrac{h_9 : \Box F_8, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash F_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}}{\bullet h_9 : \Box F_8, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash F_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}} \; ax/W}{H}}{hCut}$$

$$- : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash F_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}$$
$$- : \Delta_{18}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []F_{10} \quad A45$$

$$\dfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), F_9, \Box F_8}{\bullet h_2 : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_9), \Box F_8} \quad A45 \quad \dfrac{h_{10} : \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, F_{11}, []F_9}{\bullet h_{10} : ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_8 \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (}$$

$$- : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_9$$
$$\to$$

$$\dfrac{\dfrac{- : \Box\Gamma_{16}, \Box\Gamma_{17} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []F_9}{} \; ax/W}{- : \Delta_{19}, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}, []F_9} \quad A45$$

$$\dfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), F_9, \Box F_8}{\bullet h_2 : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_9), \Box F_8} \quad A45 \quad \dfrac{h_{10} : \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, F_{11}}{\bullet h_{10} : ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_8 \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (}$$

$$- : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_9$$
$$\to$$

$$\dfrac{\dfrac{- : \Box\Gamma_{16}, \Box\Gamma_{17} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}}{} \; ax/W}{- : \Delta_{19}, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}, []F_9} \quad A45$$

$$\dfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), F_9, \Box F_8}{\bullet h_2 : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_9), \Box F_8} \quad A45 \quad \dfrac{h_{10} : \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, F_{11}, []F_9}{\bullet h_{10} : ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_8 \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13},}$$

$$- : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_9$$
$$\to$$

$$\dfrac{\dfrac{- : \Box\Gamma_{16}, \Box\Gamma_{17} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []F_9}{} \; ax/W}{- : \Delta_{19}, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}, []F_9} \quad A45$$

$$\dfrac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), F_9, \Box F_8}{\bullet h_2 : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_9), \Box F_8} \quad A45 \quad \dfrac{h_{10} : \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, F_{11}}{\bullet h_{10} : ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_8 \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13},}$$

$$- : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_9$$
$$\to$$

$$\dfrac{\dfrac{- : \Box\Gamma_{16}, \Box\Gamma_{17} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}}{} \; ax/W}{- : \Delta_{19}, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}, []F_9} \quad A45$$

$$\dfrac{h_2 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), F_{10}, \Box F_8}{\bullet h_2 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), []F_{10}), \Box F_8} \quad A45 \quad \dfrac{h_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, F_{10}}{\bullet h_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18}), \Box F_8 \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14})}$$

$$- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), []F_{10}$$
$$\to$$

$$\dfrac{\dfrac{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}{} \; ax/W}{- : \Delta_{18}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []F_{10}} \quad A45$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),\mathtt{F_9}}{\bullet\mathtt{h_2}:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash((\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F_9}),\Box\mathtt{F_8}}\quad A45\quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{16},\Box\Gamma_{17},\Box\mathtt{F_8}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F_{11}},[]\mathtt{F_9}}{\bullet\mathtt{h_{10}}:((\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}),\Box\mathtt{F_8}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(}}{-:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F_9}}\ \rightarrow}{\dfrac{\dfrac{}{-:\Box\Gamma_{16},\Box\Gamma_{18}\vdash\mathtt{F_9},\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}}\ \mathtt{ax/W}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathtt{F_{11}},[]\mathtt{F_9}}\ A45}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),\mathtt{F_9}}{\bullet\mathtt{h_2}:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash((\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F_9}),\Box\mathtt{F_8}}\quad A45\quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{16},\Box\Gamma_{17},\Box\mathtt{F_8}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F_{11}}}{\bullet\mathtt{h_{10}}:((\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}),\Box\mathtt{F_8}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(}}{-:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F_9}}\ \rightarrow}{\dfrac{\dfrac{}{-:\Box\Gamma_{16},\Box\Gamma_{18}\vdash\mathtt{F_9},\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}}\ \mathtt{ax/W}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathtt{F_{11}},[]\mathtt{F_9}}\ A45}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),\mathtt{F_9}}{\bullet\mathtt{h_2}:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash((\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15},[]\mathtt{F_{11}}),[]\mathtt{F_9}),\Box\mathtt{F_8}}\quad A45\quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{16},\Box\Gamma_{17},\Box\mathtt{F_8}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F_{11}},[]\mathtt{F_9}}{\bullet\mathtt{h_{10}}:((\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}),\Box\mathtt{F_8}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},}}{-:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15},[]\mathtt{F_{11}}),[]\mathtt{F_9}}\ \rightarrow}{\dfrac{\dfrac{}{-:\Box\Gamma_{16},\Box\Gamma_{18}\vdash\mathtt{F_9},\Box\Gamma_{12},\Box\Gamma_{14}}\ \mathtt{ax/W}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathtt{F_{11}},[]\mathtt{F_9}}\ A45}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),\mathtt{F_9}}{\bullet\mathtt{h_2}:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash((\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15},[]\mathtt{F_{11}}),[]\mathtt{F_9}),\Box\mathtt{F_8}}\quad A45\quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{16},\Box\Gamma_{17},\Box\mathtt{F_8}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F_{11}}}{\bullet\mathtt{h_{10}}:((\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}),\Box\mathtt{F_8}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},}}{-:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15},[]\mathtt{F_{11}}),[]\mathtt{F_9}}\ \rightarrow}{\dfrac{\dfrac{}{-:\Box\Gamma_{16},\Box\Gamma_{18}\vdash\mathtt{F_9},\Box\Gamma_{12},\Box\Gamma_{14}}\ \mathtt{ax/W}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathtt{F_{11}},[]\mathtt{F_9}}\ A45}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{15},\Box\Gamma_{17}\vdash(\Box\Gamma_{11},\Box\Gamma_{13}),\mathtt{F_{10}}}{\bullet\mathtt{h_2}:(\Box\Gamma_{15},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{18}\vdash((\Box\Gamma_{11},\Box\Gamma_{13}),(\Box\Gamma_{12},\Delta_{14}),[]\mathtt{F_{10}}),\Box\mathtt{F_8}}\quad A45\quad \dfrac{\mathtt{h_9}:\Box\Gamma_{15},\Box\Gamma_{16},\Box\mathtt{F_8}\vdash\Box\Gamma_{11},\Box\Gamma_{12},\mathtt{F_{10}}}{\bullet\mathtt{h_9}:((\Box\Gamma_{15},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{18}),\Box\mathtt{F_8}\vdash(\Box\Gamma_{11},\Box\Gamma_{13}),(\Box\Gamma_{12},\Delta_{14})}}{-:(\Box\Gamma_{15},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{18}\vdash(\Box\Gamma_{11},\Box\Gamma_{13}),(\Box\Gamma_{12},\Delta_{14}),[]\mathtt{F_{10}}}\ \rightarrow}{\dfrac{\dfrac{}{-:\Box\Gamma_{15},\Box\Gamma_{17}\vdash\mathtt{F_{10}},\Box\Gamma_{11},\Box\Gamma_{13}}\ \mathtt{ax/W}}{-:\Delta_{18},\Box\Gamma_{15},\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathtt{F_{10}}}\ A45}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),\mathtt{F_9}}{\bullet\mathtt{h_2}:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash((\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F_9}),\Box\mathtt{F_8}}\quad A45\quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F_{11}},[]\mathtt{F_9}}{\bullet\mathtt{h_{10}}:((\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}),\mathtt{F_8}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{1}}}{-:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F_9}}\ \rightarrow}{\dfrac{\dfrac{}{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\mathtt{F_{11}},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathtt{F_9}}\ \mathtt{ax/W}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathtt{F_{11}},[]\mathtt{F_9}}\ A45}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),\mathtt{F_9}}{\bullet\mathtt{h_2}:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash((\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F_9}),\Box\mathtt{F_8}}\quad A45\quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F_{11}}}{\bullet\mathtt{h_{10}}:((\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}),\mathtt{F_8}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{1}}}{-:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F_{11}}),(\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F_9}}\ \rightarrow}{\dfrac{\dfrac{}{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\mathtt{F_{11}},\Box\Gamma_{12},\Box\Gamma_{13}}\ \mathtt{ax/W}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathtt{F_{11}},[]\mathtt{F_9}}\ A45}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),\mathtt{F_9}}{\bullet\mathtt{h_2}:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash((\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15},[]\mathtt{F_{11}}),[]\mathtt{F_9}),\Box\mathtt{F_8}}\quad A45\quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F_{11}},[]\mathtt{F_9}}{\bullet\mathtt{h_{10}}:((\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}),\mathtt{F_8}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{1}}}{-:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15},[]\mathtt{F_{11}}),[]\mathtt{F_9}}\ \rightarrow}{\dfrac{\dfrac{}{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\mathtt{F_{11}},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathtt{F_9}}\ \mathtt{ax/W}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathtt{F_{11}},[]\mathtt{F_9}}\ A45}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2}:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),\mathtt{F_9}}{\bullet\mathtt{h_2}:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash((\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15},[]\mathtt{F_{11}}),[]\mathtt{F_9}),\mathtt{F_8}}\quad A45\quad \dfrac{\mathtt{h_{10}}:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F_{11}}}{\bullet\mathtt{h_{10}}:((\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}),\mathtt{F_8}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{1}}}{-:(\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15},[]\mathtt{F_{11}}),[]\mathtt{F_9}}\ \rightarrow}{\dfrac{\dfrac{}{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\mathtt{F_{11}},\Box\Gamma_{12},\Box\Gamma_{13}}\ \mathtt{ax/W}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathtt{F_{11}},[]\mathtt{F_9}}\ A45}$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), F_{10}}{\bullet h_2 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), []F_{10}), F_8} \; A45 \qquad \dfrac{h_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, F_{10}}{\bullet h_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18}), F_8 \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), []F\ldots}}{\dfrac{\dfrac{- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), []F_{10}}{\;}\;\to}{\dfrac{\dfrac{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}{\;}\;\text{ax/W}}{- : \Delta_{18}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []F_{10}}\;A45}}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \Delta_{14}, F_{12} \to F_{13} \vdash (\Box\Gamma_9, \Delta_{10}), []F_8}\;A45 \quad \dfrac{h_{11} : \Box\Gamma_7, \Delta_{14}, []F_8 \vdash \Box\Gamma_9, F_{12}, \Delta_{10} \quad h_{11} : \Box\Gamma_7, F_{13}, \Delta_{14}, []F_8 \vdash \Box\Gamma_9, \Delta_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \to F_{13}), []F_8 \vdash \Box\Gamma_9, \Delta_{10}}\;\text{Cut}}{- : \Box\Gamma_7, \Delta_{14}, F_{12} \to F_{13} \vdash \Box\Gamma_9, \Delta_{10} \quad \to}$$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_7 \vdash F_8, \Box\Gamma_9}{\bullet h_1 : \Delta_{14}, \Box\Gamma_7 \vdash \Delta_{10}, F_{12}, \Box\Gamma_9, []F_8}\;\text{ax/W} \; A45 \quad \dfrac{}{h_{11} : \Delta_{14}, \Box\Gamma_7, []F_8 \vdash \Delta_{10}, F_{12}, \Box\Gamma_9}\;\text{ax/W}}{- : \Delta_{14}, \Box\Gamma_7 \vdash \Delta_{10}, F_{12}, \Box\Gamma_9}\;\text{hCut} \quad \dfrac{\dfrac{h_1 : \Box\Gamma_7 \vdash F_8, \Box\Gamma_9}{\bullet h_1 : \Delta_{14}, F_{13}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9, []F_8}\;\text{ax/W} \; A45 \quad h_{11} : \Delta_{14}, F_{13}, \Box\ldots}{- : \Delta_{14}, F_{13}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9}$$
$$- : \Delta_{14}, \Box\Gamma_7, F_{12} \to F_{13} \vdash \Delta_{10}, \Box\Gamma_9 \quad \to_L$$

$$\dfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : \Box\Gamma_8, \Delta_{16}, F_{14} \to F_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), \Box F_9}\;A45 \quad \dfrac{h_{13} : \Box\Gamma_8, \Delta_{16}, \Box F_9 \vdash \Box\Gamma_{10}, F_{14}, \Delta_{11}, []F_{12} \quad h_{13} : \Box\Gamma_8, F_{15}, \Delta_{16}\ldots}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{16}, F_{14} \to F_{15}), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{1\ldots}}$$
$$- : \Box\Gamma_8, \Delta_{16}, F_{14} \to F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12} \quad \to$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_8 \vdash \Box F_9, F_{12}, \Box\Gamma_{10}}{\bullet h_2 : \Delta_{16}, \Box\Gamma_8 \vdash \Box F_9, \Delta_{11}, F_{14}, \Box\Gamma_{10}, []F_{12}}\;\text{ax/W} \; A45 \quad \dfrac{}{h_{13} : \Box F_9, \Delta_{16}, \Box\Gamma_8 \vdash \Delta_{11}, F_{14}, \Box\Gamma_{10}, []F_{12}}\;\text{ax/W}}{- : \Delta_{16}, \Box\Gamma_8 \vdash \Delta_{11}, F_{14}, \Box\Gamma_{10}, []F_{12}}\;\text{hCut} \quad \dfrac{\dfrac{h_2 : \Box\Gamma_8 \vdash \Box F_9, F_{12}, \Box\Gamma_{10}}{\bullet h_2 : \Delta_{16}, F_{15}, \Box\Gamma_8 \vdash \Box F_9, \Delta_{11}, \Box\Gamma_{10}, []F_{12}}\;\text{ax/W}\;A}{- : \Delta_{16}, F_{15}, \Box\Gamma_8\ldots}$$
$$- : \Delta_{16}, \Box\Gamma_8, F_{14} \to F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_{14} \to F_{15}}\;A45 \quad \dfrac{h_{13} : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, F_{14}, \Delta_{11}, []F_{12} \quad h_{13} : \Box\Gamma_8, F_{15}, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), F_{14} \to F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\;\to_L \; \text{Cut}}{\dfrac{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12} \quad \to}{\dfrac{\dfrac{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}{\;}\;\text{ax/W}}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}\;A45}}$$

$$\dfrac{\dfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_{16}, F_{14} \to F_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_9}\;A45 \quad \dfrac{h_{13} : \Box\Gamma_8, F_9, \Delta_{16} \vdash \Box\Gamma_{10}, F_{14}, \Delta_{11}, []F_{12} \quad h_{13} : \Box\Gamma_8, F_9, F_{15}, \Delta_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{16}, F_{14} \to F_{15}), F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\;\to_L \; \text{Cut}}{\dfrac{- : \Box\Gamma_8, \Delta_{16}, F_{14} \to F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12} \quad \to}{\dfrac{\dfrac{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}{\;}\;\text{ax/W}}{- : \Delta_{16}, \Box\Gamma_8, F_{14} \to F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}\;A45}}$$

- Case rule $\land_L$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \Delta_{14}, F_{12} \land F_{13} \vdash (\Box\Gamma_9, \Delta_{10}), []F_8}\;A45 \quad \dfrac{h_{11} : \Box\Gamma_7, F_{12}, F_{13}, \Delta_{14}, []F_8 \vdash \Box\Gamma_9, \Delta_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \land F_{13}), []F_8 \vdash \Box\Gamma_9, \Delta_{10}}\;\land_L}{- : \Box\Gamma_7, \Delta_{14}, F_{12} \land F_{13} \vdash \Box\Gamma_9, \Delta_{10}}\;\text{Cut} \quad \to}$$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_7 \vdash F_8, \Box\Gamma_9}{\bullet h_1 : \Delta_{14}, F_{12}, F_{13}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9, []F_8}\;\text{ax/W} \; A45 \quad \dfrac{}{h_{11} : \Delta_{14}, F_{12}, F_{13}, \Box\Gamma_7, []F_8 \vdash \Delta_{10}, \Box\Gamma_9}\;\text{ax/W}}{\dfrac{- : \Delta_{14}, F_{12}, F_{13}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9 \quad \text{hCut}}{- : \Delta_{14}, \Box\Gamma_7, F_{12} \land F_{13} \vdash \Delta_{10}, \Box\Gamma_9}\;\land_L}}$$

$$
\cfrac{
\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : \Box\Gamma_8, \Delta_{16}, F_{14} \wedge F_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), \Box F_9} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_{14}, F_{15}, \Delta_{16}, \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{16}, F_{14} \wedge F_{15}), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \wedge_L
}{- : \Box\Gamma_8, \Delta_{16}, F_{14} \wedge F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{\cfrac{}{h_2 : \Box\Gamma_8 \vdash \Box F_9, F_{12}, \Box\Gamma_{10}} \; \text{ax/W}}{\bullet h_2 : \Delta_{16}, F_{14}, F_{15}, \Box\Gamma_8 \vdash \Box F_9, \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; A45 \qquad \cfrac{}{h_{13} : \Box F_9, \Delta_{16}, F_{14}, F_{15}, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; \text{ax/W}
}{\cfrac{- : \Delta_{16}, F_{14}, F_{15}, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}{- : \Delta_{16}, \Box\Gamma_8, F_{14} \wedge F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; \wedge_L} \; \text{hCut}
$$

$$
\cfrac{
\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_{14} \wedge F_{15}} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_{14}, F_{15}, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), F_{14} \wedge F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \wedge_L
}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{\cfrac{}{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}} \; \text{ax/W}}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; A45
$$

$$
\cfrac{
\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_{16}, F_{14} \wedge F_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_9} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_9, F_{14}, F_{15}, \Delta_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{16}, F_{14} \wedge F_{15}), F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \wedge_L
}{- : \Box\Gamma_8, \Delta_{16}, F_{14} \wedge F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{\cfrac{}{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}} \; \text{ax/W}}{- : \Delta_{16}, \Box\Gamma_8, F_{14} \wedge F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; A45
$$

- Case rule $\vee_L$

$$
\cfrac{
\cfrac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \Delta_{14}, F_{12} \vee F_{13} \vdash (\Box\Gamma_9, \Delta_{10}), []F_8} \; A45 \qquad \cfrac{h_{11} : \Box\Gamma_7, F_{12}, \Delta_{14}, []F_8 \vdash \Box\Gamma_9, \Delta_{10} \quad h_{11} : \Box\Gamma_7, F_{13}, \Delta_{14}, []F_8 \vdash \Box\Gamma_9, \Delta_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \vee F_{13}), []F_8 \vdash \Box\Gamma_9, \Delta_{10}} \; \text{Cut}
}{- : \Box\Gamma_7, \Delta_{14}, F_{12} \vee F_{13} \vdash \Box\Gamma_9, \Delta_{10}}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{\cfrac{}{h_1 : \Box\Gamma_7 \vdash F_8, \Box\Gamma_9} \; \text{ax/W}}{\bullet h_1 : \Delta_{14}, F_{12}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9, []F_8} \; A45 \quad \cfrac{}{h_{11} : \Delta_{14}, F_{12}, \Box\Gamma_7, []F_8 \vdash \Delta_{10}, \Box\Gamma_9} \; \text{ax/W}}{\cfrac{- : \Delta_{14}, F_{12}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9}{} \; \text{hCut}} \quad \cfrac{\cfrac{\cfrac{}{h_1 : \Box\Gamma_7 \vdash F_8, \Box\Gamma_9} \; \text{ax/W}}{\bullet h_1 : \Delta_{14}, F_{13}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9, []F_8} \; A45 \quad \cfrac{}{h_{11} : \Delta_{14}, F_{13}, \Box} }{- : \Delta_{14}, F_{13}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9} \; \vee_L
}{- : \Delta_{14}, \Box\Gamma_7, F_{12} \vee F_{13} \vdash \Delta_{10}, \Box\Gamma_9}
$$

$$
\cfrac{
\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : \Box\Gamma_8, \Delta_{16}, F_{14} \vee F_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), \Box F_9} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_{14}, \Delta_{16}, \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12} \quad h_{13} : \Box\Gamma_8, F_{15}, \Delta_{16}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{16}, F_{14} \vee F_{15}), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{11}} }{- : \Box\Gamma_8, \Delta_{16}, F_{14} \vee F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{\cfrac{}{h_2 : \Box\Gamma_8 \vdash \Box F_9, F_{12}, \Box\Gamma_{10}} \; \text{ax/W}}{\bullet h_2 : \Delta_{16}, F_{14}, \Box\Gamma_8 \vdash \Box F_9, \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; A45 \quad \cfrac{}{h_{13} : \Box F_9, \Delta_{16}, F_{14}, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; \text{ax/W}}{\cfrac{- : \Delta_{16}, F_{14}, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}{} \; \text{hCut}} \quad \cfrac{\cfrac{}{h_2 : \Box\Gamma_8 \vdash \Box F_9, F_{12}, \Box\Gamma_{10}} \; \text{ax/W} }{\bullet h_2 : \Delta_{16}, F_{15}, \Box\Gamma_8 \vdash \Box F_9, \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; A
}{- : \Delta_{16}, \Box\Gamma_8, F_{14} \vee F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}
$$

$$
\cfrac{
\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_{14} \vee F_{15}} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_{14}, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12} \quad h_{13} : \Box\Gamma_8, F_{15}, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), F_{14} \vee F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \vee_L
}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{\cfrac{}{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}} \; \text{ax/W}}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; A45
$$

$$
\cfrac{
\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_{16}, F_{14} \vee F_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_9} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_9, F_{14}, \Delta_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12} \quad h_{13} : \Box\Gamma_8, F_9, F_{15}, \Delta_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{16}, F_{14} \vee F_{15}), F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \vee_L
}{- : \Box\Gamma_8, \Delta_{16}, F_{14} \vee F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{\cfrac{}{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}} \; \text{ax/W}}{- : \Delta_{16}, \Box\Gamma_8, F_{14} \vee F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; A45
$$

- Case rule AT

$$\cfrac{\cfrac{\cfrac{h_1 : \Box\Gamma_{13}, []F_{12} \vdash \Box\Gamma_9, F_8}{\bullet h_1 : (\Box\Gamma_{13}, []F_{12}), \Delta_7 \vdash (\Box\Gamma_9, \Delta_{10}), []F_8} \; A45 \qquad \cfrac{h_{11} : \Box\Gamma_{13}, F_{12}, \Delta_7, []F_8, []F_{12} \vdash \Box\Gamma_9, \Delta_{10}}{\bullet h_{11} : ((\Box\Gamma_{13}, []F_{12}), \Delta_7), []F_8 \vdash \Box\Gamma_9, \Delta_{10}} \; AT}{- : (\Box\Gamma_{13}, []F_{12}), \Delta_7 \vdash \Box\Gamma_9, \Delta_{10}} \; \text{Cut}}{\cfrac{\cfrac{}{\bullet h_1 : \Delta_7, F_{12}, \Box\Gamma_{13}, []F_{12} \vdash \Delta_{10}, \Box\Gamma_9, []F_8} \; \text{ax/W} \qquad \cfrac{}{h_{11} : \Delta_7, F_{12}, \Box\Gamma_{13}, []F_{12}, []F_8 \vdash \Delta_{10}, \Box\Gamma_9} \; \text{ax/W}}{\cfrac{- : \Delta_7, F_{12}, \Box\Gamma_{13}, []F_{12} \vdash \Delta_{10}, \Box\Gamma_9}{- : \Delta_7, \Box\Gamma_{13}, []F_{12} \vdash \Delta_{10}, \Box\Gamma_9} \; AT} \; \text{hCut}} \; \rightarrow$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \Delta_{13}, []F_{12} \vdash (\Box\Gamma_9, \Delta_{10}), []F_8} \; A45 \qquad \cfrac{h_{11} : \Box\Gamma_7, F_{12}, \Delta_{13}, []F_8, []F_{12} \vdash \Box\Gamma_9, \Delta_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{13}, []F_{12}), []F_8 \vdash \Box\Gamma_9, \Delta_{10}} \; AT}{- : \Box\Gamma_7, \Delta_{13}, []F_{12} \vdash \Box\Gamma_9, \Delta_{10}} \; \text{Cut}}{\cfrac{\cfrac{}{\bullet h_1 : \Delta_{13}, F_{12}, \Box\Gamma_7, []F_{12} \vdash \Delta_{10}, \Box\Gamma_9, []F_8} \; \text{ax/W} \qquad \cfrac{}{h_{11} : \Delta_{13}, F_{12}, \Box\Gamma_7, []F_{12}, []F_8 \vdash \Delta_{10}, \Box\Gamma_9} \; \text{ax/W}}{\cfrac{- : \Delta_{13}, F_{12}, \Box\Gamma_7, []F_{12} \vdash \Delta_{10}, \Box\Gamma_9}{- : \Delta_{13}, \Box\Gamma_7, []F_{12} \vdash \Delta_{10}, \Box\Gamma_9} \; AT} \; \text{hCut}} \; \rightarrow$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_{12}}{\bullet h_1 : \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_9, \Delta_{10}), []F_{12}} \; A45 \qquad \cfrac{h_{11} : \Box\Gamma_7, F_{12}, \Delta_8, []F_{12} \vdash \Box\Gamma_9, \Delta_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), []F_{12} \vdash \Box\Gamma_9, \Delta_{10}} \; AT}{- : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_9, \Delta_{10}} \; \text{Cut}}{\cfrac{\cfrac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{10}, F_{12}, \Box\Gamma_9} \; \text{ax/W} \qquad \cfrac{\cfrac{}{\bullet h_1 : \Delta_8, F_{12}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9, []F_{12}} \; \text{ax/W} \quad \cfrac{}{h_{11} : \Delta_8, F_{12}, \Box\Gamma_7, []F_{12} \vdash \Delta_{10}, \Box\Gamma_9} \; \text{ax/W}}{- : \Delta_8, F_{12}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9} \; \text{hCut}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9} \; \text{sCut}} \; \rightarrow$$

$$\cfrac{\cfrac{\cfrac{h_2 : \Box\Gamma_{15}, []F_{14} \vdash \Box\Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : (\Box\Gamma_{15}, []F_{14}), \Delta_8 \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), \Box F_9} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_{15}, F_{14}, \Delta_8, \Box F_9, []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : ((\Box\Gamma_{15}, []F_{14}), \Delta_8), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; AT}{- : (\Box\Gamma_{15}, []F_{14}), \Delta_8 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{Cut}}{\cfrac{\cfrac{}{\bullet h_2 : \Delta_8, F_{14}, \Box\Gamma_{15}, []F_{14} \vdash \Box F_9, \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; \text{ax/W} \qquad \cfrac{}{h_{13} : \Box F_9, \Delta_8, F_{14}, \Box\Gamma_{15}, []F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; \text{ax/W}}{\cfrac{- : \Delta_8, F_{14}, \Box\Gamma_{15}, []F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}{- : \Delta_8, \Box\Gamma_{15}, []F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; AT} \; \text{hCut}} \; \rightarrow$$

$$\cfrac{\cfrac{\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : \Box\Gamma_8, \Delta_{15}, []F_{14} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), \Box F_9} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_{14}, \Delta_{15}, \Box F_9, []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{15}, []F_{14}), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; AT}{- : \Box\Gamma_8, \Delta_{15}, []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{Cut}}{\cfrac{\cfrac{}{\bullet h_2 : \Delta_{15}, F_{14}, \Box\Gamma_8, []F_{14} \vdash \Box F_9, \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; \text{ax/W} \qquad \cfrac{}{h_{13} : \Box F_9, \Delta_{15}, F_{14}, \Box\Gamma_8, []F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; \text{ax/W}}{\cfrac{- : \Delta_{15}, F_{14}, \Box\Gamma_8, []F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}{- : \Delta_{15}, \Box\Gamma_8, []F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; ATG} \; \text{hCut}}$$

$$\cfrac{\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}, []F_{14}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), []F_{14}} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_{14}, \Delta_9, []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; AT}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{Cut} \qquad \rightarrow \qquad \cfrac{}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{fail}$$

Axioms assumed:

```
inf  : C:MSFormula |-- True ; C':MSFormula
inf  : False ; C:MSFormula |-- C':MSFormula
inf  : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
```

$$\cfrac{\cfrac{\cfrac{h_2 : \Box\Gamma_{15}, []F_{14} \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : (\Box\Gamma_{15}, []F_{14}), \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_8} \; A45 \qquad \cfrac{h_{13} : \Box\Gamma_{15}, F_8, F_{14}, \Delta_9, []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : ((\Box\Gamma_{15}, []F_{14}), \Delta_9), F_8 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; AT}{- : (\Box\Gamma_{15}, []F_{14}), \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; \text{Cut}}{\cfrac{\cfrac{}{- : \Box\Gamma_{15}, []F_{14} \vdash F_{12}, \Box\Gamma_{10}} \; \text{ax/W}}{- : \Delta_9, \Box\Gamma_{15}, []F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; A45} \; \rightarrow$$

$$\cfrac{\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), []F_{14}}\ A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_{14}, \Delta_9, []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ AT}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}\ ax/W}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}\ A45$$

$$\cfrac{\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_{15}, []F_{14} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_9}\ A45 \qquad \cfrac{h_{13} : \Box\Gamma_8, F_9, F_{14}, \Delta_{15}, []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{15}, []F_{14}), F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ AT}{- : \Box\Gamma_8, \Delta_{15}, []F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}\ ax/W}{- : \Delta_{15}, \Box\Gamma_8, []F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}\ A45$$

- Case rule $\bot_L$

$$\cfrac{\cfrac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \bot, \Delta_{12} \vdash (\Box\Gamma_9, \Delta_{10}), []F_8}\ A45 \qquad \cfrac{}{\bullet h_{11} : (\Box\Gamma_7, \bot, \Delta_{12}), []F_8 \vdash \Box\Gamma_9, \Delta_{10}}\ \bot_L}{- : \Box\Gamma_7, \bot, \Delta_{12} \vdash \Box\Gamma_9, \Delta_{10}}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \bot, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9}\ \bot_L$$

$$\cfrac{\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : \Box\Gamma_8, \bot, \Delta_{14} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), \Box F_9}\ A45 \qquad \cfrac{}{\bullet h_{13} : (\Box\Gamma_8, \bot, \Delta_{14}), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ \bot_L}{- : \Box\Gamma_8, \bot, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \bot, \Delta_{14}, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}\ \bot_L$$

$$\cfrac{\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), \bot}\ A45 \qquad \cfrac{}{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), \bot \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ \bot_L}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}\ ax/W}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}\ A45$$

$$\cfrac{\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \bot, \Delta_{14} \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_9}\ A45 \qquad \cfrac{}{\bullet h_{13} : (\Box\Gamma_8, \bot, \Delta_{14}), F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ \bot_L}{- : \Box\Gamma_8, \bot, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \bot, \Delta_{14}, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}}\ \bot_L$$

- Case rule I

$$\cfrac{\cfrac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash (\Box\Gamma_9, \Delta_{12}, p_{11}), []F_8}\ A45 \qquad \cfrac{}{\bullet h_{10} : (\Box\Gamma_7, \Delta_{13}, p_{11}), []F_8 \vdash \Box\Gamma_9, \Delta_{12}, p_{11}}\ I}{- : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash \Box\Gamma_9, \Delta_{12}, p_{11}}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_{13}, \Box\Gamma_7, p_{11} \vdash \Delta_{12}, \Box\Gamma_9, p_{11}}\ I$$

$$\cfrac{\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{11}, \Box F_9}{\bullet h_2 : \Box\Gamma_8, \Delta_{15}, p_{13} \vdash (\Box\Gamma_{10}, (\Delta_{14}, p_{13}), []F_{11}), \Box F_9}\ A45 \qquad \cfrac{}{\bullet h_{12} : (\Box\Gamma_8, \Delta_{15}, p_{13}), \Box F_9 \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), []F_{11}}\ I}{- : \Box\Gamma_8, \Delta_{15}, p_{13} \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), []F_{11}}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_{15}, \Box\Gamma_8, p_{13} \vdash \Delta_{14}, \Box\Gamma_{10}, p_{13}, []F_{11}}\ I$$

$$\cfrac{\cfrac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{11}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, (\Delta_{14}, p_{13}), []F_{11}), p_{13}}\ A45 \qquad \cfrac{}{\bullet h_{12} : (\Box\Gamma_8, \Delta_9), p_{13} \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), []F_{11}}\ I}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), []F_{11}}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{- : \Box\Gamma_8 \vdash F_{11}, \Box\Gamma_{10}}\ ax/W}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{14}, \Box\Gamma_{10}, p_{13}, []F_{11}}\ A45$$

$$\dfrac{\dfrac{\mathtt{h_2}:\square\Gamma_8\vdash\square\Gamma_{10},\mathtt{F_{11}}}{\bullet\mathtt{h_2}:\square\Gamma_8,\Delta_{15},\mathtt{p_{13}}\vdash(\square\Gamma_{10},(\Delta_{14},\mathtt{p_{13}}),[]\mathtt{F_{11}}),\mathtt{F_9}}\;A45 \quad \dfrac{}{\bullet\mathtt{h_{12}}:(\square\Gamma_8,\Delta_{15},\mathtt{p_{13}}),\mathtt{F_9}\vdash\square\Gamma_{10},(\Delta_{14},\mathtt{p_{13}}),[]\mathtt{F_{11}}}\;I}{-:\square\Gamma_8,\Delta_{15},\mathtt{p_{13}}\vdash\square\Gamma_{10},(\Delta_{14},\mathtt{p_{13}}),[]\mathtt{F_{11}}}\;\mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{}{-:\Delta_{15},\square\Gamma_8,\mathtt{p_{13}}\vdash\Delta_{14},\square\Gamma_{10},\mathtt{p_{13}},[]\mathtt{F_{11}}}\;I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h_1}:\square\Gamma_7\vdash\square\Gamma_9,\mathtt{F_8}}{\bullet\mathtt{h_1}:\square\Gamma_7,\top,\Delta_{12}\vdash(\square\Gamma_9,\Delta_{10}),[]\mathtt{F_8}}\;A45 \quad \dfrac{\mathtt{h_{11}}:\square\Gamma_7,\Delta_{12},[]\mathtt{F_8}\vdash\square\Gamma_9,\Delta_{10}}{\bullet\mathtt{h_{11}}:(\square\Gamma_7,\top,\Delta_{12}),[]\mathtt{F_8}\vdash\square\Gamma_9,\Delta_{10}}\;\top_L}{-:\square\Gamma_7,\top,\Delta_{12}\vdash\square\Gamma_9,\Delta_{10}}\;\mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_1}:\top,\Delta_{12},\square\Gamma_7\vdash\Delta_{10},\square\Gamma_9,[]\mathtt{F_8}}\;\mathtt{ax/W} \quad \dfrac{\mathtt{h_{11}}:\top,\Delta_{12},\square\Gamma_7,[]\mathtt{F_8}\vdash\Delta_{10},\square\Gamma_9}{}\;\mathtt{ax/W}}{-:\top,\Delta_{12},\square\Gamma_7\vdash\Delta_{10},\square\Gamma_9}\;\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\square\Gamma_8\vdash\square\Gamma_{10},\mathtt{F_{12}},\square\mathtt{F_9}}{\bullet\mathtt{h_2}:\square\Gamma_8,\top,\Delta_{14}\vdash(\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}),\square\mathtt{F_9}}\;A45 \quad \dfrac{\mathtt{h_{13}}:\square\Gamma_8,\Delta_{14},\square\mathtt{F_9}\vdash\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}{\bullet\mathtt{h_{13}}:(\square\Gamma_8,\top,\Delta_{14}),\square\mathtt{F_9}\vdash\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}\;\top_L}{-:\square\Gamma_8,\top,\Delta_{14}\vdash\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}\;\mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_2}:\top,\Delta_{14},\square\Gamma_8\vdash\square\mathtt{F_9},\Delta_{11},\square\Gamma_{10},[]\mathtt{F_{12}}}\;\mathtt{ax/W} \quad \dfrac{\mathtt{h_{13}}:\top,\square\mathtt{F_9},\Delta_{14},\square\Gamma_8\vdash\Delta_{11},\square\Gamma_{10},[]\mathtt{F_{12}}}{}\;\mathtt{ax/W}}{-:\top,\Delta_{14},\square\Gamma_8\vdash\Delta_{11},\square\Gamma_{10},[]\mathtt{F_{12}}}\;\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\square\Gamma_8\vdash\square\Gamma_{10},\mathtt{F_{12}}}{\bullet\mathtt{h_2}:\square\Gamma_8,\Delta_9\vdash(\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}),\top}\;A45 \quad \dfrac{\mathtt{h_{13}}:\square\Gamma_8,\Delta_9\vdash\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}{\bullet\mathtt{h_{13}}:(\square\Gamma_8,\Delta_9),\top\vdash\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}\;\top_L}{-:\square\Gamma_8,\Delta_9\vdash\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}\;\mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{}{-:\Delta_9,\square\Gamma_8\vdash\Delta_{11},\square\Gamma_{10},[]\mathtt{F_{12}}}\;\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\square\Gamma_8\vdash\square\Gamma_{10},\mathtt{F_{12}}}{\bullet\mathtt{h_2}:\square\Gamma_8,\top,\Delta_{14}\vdash(\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}),\mathtt{F_9}}\;A45 \quad \dfrac{\mathtt{h_{13}}:\square\Gamma_8,\mathtt{F_9},\Delta_{14}\vdash\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}{\bullet\mathtt{h_{13}}:(\square\Gamma_8,\top,\Delta_{14}),\mathtt{F_9}\vdash\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}\;\top_L}{-:\square\Gamma_8,\top,\Delta_{14}\vdash\square\Gamma_{10},\Delta_{11},[]\mathtt{F_{12}}}\;\mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{-:\square\Gamma_8\vdash\mathtt{F_{12}},\square\Gamma_{10}}\;\mathtt{ax/W}}{-:\top,\Delta_{14},\square\Gamma_8\vdash\Delta_{11},\square\Gamma_{10},[]\mathtt{F_{12}}}\;A45$$

6.8   Status of $\rightarrow_L$:   OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h_3}:\Delta_8\vdash\mathtt{F_7},\mathtt{F_9},\Delta_{12},\mathtt{F_{13}}\rightarrow\mathtt{F_{14}} \quad \mathtt{h_3}:\mathtt{F_{10}},\Delta_8\vdash\mathtt{F_7},\Delta_{12},\mathtt{F_{13}}\rightarrow\mathtt{F_{14}}}{\bullet\mathtt{h_3}:\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash(\Delta_{12},\mathtt{F_{13}}\rightarrow\mathtt{F_{14}}),\mathtt{F_7}}\;\rightarrow_L \quad \dfrac{\mathtt{h_{11}}:\mathtt{F_7},\mathtt{F_{13}},\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\mathtt{F_{14}},\Delta_{12}}{\bullet\mathtt{h_{11}}:(\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}),\mathtt{F_7}\vdash\Delta_{12},\mathtt{F_{13}}\rightarrow\mathtt{F_{14}}}\;\rightarrow_R}{-:\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{13}}\rightarrow\mathtt{F_{14}}}\;\mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_3}:\Delta_8,\mathtt{F_{13}}\vdash\Delta_{12},\mathtt{F_{14}},\mathtt{F_7},\mathtt{F_9}}\;\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h_3}:\Delta_8,\mathtt{F_{10}},\mathtt{F_{13}}\vdash\Delta_{12},\mathtt{F_{14}},\mathtt{F_7}}\;\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h_3}:\Delta_8,\mathtt{F_{13}},\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{14}},\mathtt{F_7}}\;\rightarrow_L \quad \dfrac{\mathtt{h_{11}}:\Delta_8,\mathtt{F_{13}},\mathtt{F_7},\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{14}}}{}\;\mathtt{ax/W}}{\dfrac{-:\Delta_8,\mathtt{F_{13}},\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{14}}}{-:\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{13}}\rightarrow\mathtt{F_{14}}}\;\rightarrow_R}\;\mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h_3}:\Delta_8\vdash\mathtt{F_7},\mathtt{F_9},\Delta_{12},\mathtt{F_{13}}\wedge\mathtt{F_{14}} \quad \mathtt{h_3}:\mathtt{F_{10}},\Delta_8\vdash\mathtt{F_7},\Delta_{12},\mathtt{F_{13}}\wedge\mathtt{F_{14}}}{\bullet\mathtt{h_3}:\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash(\Delta_{12},\mathtt{F_{13}}\wedge\mathtt{F_{14}}),\mathtt{F_7}}\;\rightarrow_L \quad \dfrac{\mathtt{h_{11}}:\mathtt{F_7},\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\mathtt{F_{13}},\Delta}{\bullet\mathtt{h_{11}}:(\Delta_8,\mathtt{F_9}\rightarrow}}{-:\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{13}}\wedge\mathtt{F_{14}}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_3}:\Delta_8\vdash\Delta_{12},\mathtt{F_{13}},\mathtt{F_7},\mathtt{F_9}}\;\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h_3}:\Delta_8,\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{13}},\mathtt{F_7}}\;\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h_3}:\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{13}},\mathtt{F_7}}\;\rightarrow_L \quad \dfrac{\mathtt{h_{11}}:\Delta_8,\mathtt{F_7},\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{13}}}{}\;\mathtt{ax/W}}{-:\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{13}}}\;\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h_3}:\Delta_8\vdash\Delta_{12},\mathtt{F_{14}},\mathtt{F_7},\mathtt{F_9}}\;\mathtt{inv}}{\bullet\mathtt{h_3}:\Delta_8,\mathtt{F_9}\rightarrow}$$

$$-:\Delta_8,\mathtt{F_9}\rightarrow\mathtt{F_{10}}\vdash\Delta_{12},\mathtt{F_{13}}\wedge\mathtt{F_{14}}$$

- Case rule $\vee_R$

$$
\cfrac{
\cfrac{
\cfrac{
\mathtt{h_3} : \Delta_8 \vdash \mathtt{F_7}, \mathtt{F_9}, \Delta_{12}, \mathtt{F_{13}} \vee \mathtt{F_{14}} \quad \mathtt{h_3} : \mathtt{F_{10}}, \Delta_8 \vdash \mathtt{F_7}, \Delta_{12}, \mathtt{F_{13}} \vee \mathtt{F_{14}}
}{
\bullet\mathtt{h_3} : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash (\Delta_{12}, \mathtt{F_{13}} \vee \mathtt{F_{14}}), \mathtt{F_7}
} \to_L
\quad
\cfrac{
\mathtt{h_{11}} : \mathtt{F_7}, \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \mathtt{F_{13}}, \mathtt{F_{14}}, \Delta_{12}
}{
\bullet\mathtt{h_{11}} : (\Delta_8, \mathtt{F_9} \to \mathtt{F_{10}}), \mathtt{F_7} \vdash \Delta_{12}, \mathtt{F_{13}} \vee \mathtt{F_{14}}
} \vee_R
}{
- : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \Delta_{12}, \mathtt{F_{13}} \vee \mathtt{F_{14}}
} \text{Cut}
}{
\to
}
$$

$$
\cfrac{
\cfrac{
\cfrac{
\cfrac{}{\mathtt{h_3} : \Delta_8 \vdash \Delta_{12}, \mathtt{F_{13}}, \mathtt{F_{14}}, \mathtt{F_7}, \mathtt{F_9}} \text{inv-th/ax} \quad \cfrac{}{\mathtt{h_3} : \Delta_8, \mathtt{F_{10}} \vdash \Delta_{12}, \mathtt{F_{13}}, \mathtt{F_{14}}, \mathtt{F_7}} \text{inv-th/ax}
}{
\bullet\mathtt{h_3} : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \Delta_{12}, \mathtt{F_{13}}, \mathtt{F_{14}}, \mathtt{F_7}
} \to_L
\quad
\cfrac{}{\mathtt{h_{11}} : \Delta_8, \mathtt{F_7}, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \Delta_{12}, \mathtt{F_{13}}, \mathtt{F_{14}}} \text{ax/W}
}{
- : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \Delta_{12}, \mathtt{F_{13}}, \mathtt{F_{14}}
} \text{hCut}
}{
- : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \Delta_{12}, \mathtt{F_{13}} \vee \mathtt{F_{14}}
} \vee_R
$$

- Case rule $\perp_R$

$$
\cfrac{
\cfrac{
\cfrac{
\mathtt{h_3} : \Delta_8 \vdash \mathtt{F_7}, \mathtt{F_9}, \perp, \Delta_{12} \quad \mathtt{h_3} : \mathtt{F_{10}}, \Delta_8 \vdash \mathtt{F_7}, \perp, \Delta_{12}
}{
\bullet\mathtt{h_3} : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash (\perp, \Delta_{12}), \mathtt{F_7}
} \to_L
\quad
\cfrac{
\mathtt{h_{11}} : \mathtt{F_7}, \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \Delta_{12}
}{
\bullet\mathtt{h_{11}} : (\Delta_8, \mathtt{F_9} \to \mathtt{F_{10}}), \mathtt{F_7} \vdash \perp, \Delta_{12}
} \perp_R
}{
- : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \perp, \Delta_{12}
} \text{Cut}
}{
\to
}
$$

$$
\cfrac{
\cfrac{}{\bullet\mathtt{h_3} : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \perp, \Delta_{12}, \mathtt{F_7}} \text{ax/W} \quad \cfrac{}{\mathtt{h_{11}} : \Delta_8, \mathtt{F_7}, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \perp, \Delta_{12}} \text{ax/W}
}{
- : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \perp, \Delta_{12}
} \text{hCut}
$$

- Case rule $\top_R$

$$
\cfrac{
\cfrac{
\mathtt{h_3} : \Delta_8 \vdash \mathtt{F_7}, \mathtt{F_9}, \top, \Delta_{12} \quad \mathtt{h_3} : \mathtt{F_{10}}, \Delta_8 \vdash \mathtt{F_7}, \top, \Delta_{12}
}{
\bullet\mathtt{h_3} : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash (\top, \Delta_{12}), \mathtt{F_7}
} \to_L
\quad
\cfrac{}{\bullet\mathtt{h_{11}} : (\Delta_8, \mathtt{F_9} \to \mathtt{F_{10}}), \mathtt{F_7} \vdash \top, \Delta_{12}} \top_R
}{
- : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \top, \Delta_{12}
} \text{Cut}
$$

$$
\cfrac{
\to
}{
\cfrac{}{- : \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}} \vdash \top, \Delta_{12}} \top_R
}
$$

- Case rule K

$$
\cfrac{
\mathtt{h_3} : \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F_7}, \mathtt{F_8}, \Delta_{11}, []\mathtt{F_{12}} \quad \mathtt{h_3} : \mathtt{F_9}, \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F_7}, \Delta_{11}, []\mathtt{F_{12}}
}{
\bullet\mathtt{h_3} : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F_8} \to \mathtt{F_9} \vdash (\Delta_{11}, []\mathtt{F_{12}}), \Box\mathtt{F_7}
} \to_L
\quad
\cfrac{
\mathtt{h_{10}} : unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F_7}) \vdash \mathtt{F_{12}}
}{
\bullet\mathtt{h_{10}} : ((\Box\Gamma_{13}, \Delta_{14}), \mathtt{F_8} \to \mathtt{F_9}), \Box\mathtt{F_7} \vdash \Delta_{11}, []\mathtt{F}
}
$$

$$
- : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F_8} \to \mathtt{F_9} \vdash \Delta_{11}, []\mathtt{F_{12}}
$$
$$
\to
$$

$$
\cfrac{
\cfrac{}{\mathtt{h_3} : \Delta_{14}, \Box\Gamma_{13} \vdash \Box\mathtt{F_7}, \Delta_{11}, \mathtt{F_8}, []\mathtt{F_{12}}} \text{ax/W} \quad \cfrac{\cfrac{\mathtt{h_{10}} : unbox(\Box\mathtt{F_7}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F_{12}}}{\bullet\mathtt{h_{10}} : \Box\mathtt{F_7}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F_8}, []\mathtt{F_{12}}} K}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F_8}, []\mathtt{F_{12}}}
}{
\cdots
}
$$

$$
\cfrac{
\cfrac{}{\mathtt{h_3} : \Delta_{14}, \mathtt{F_9}, \Box\Gamma_{13} \vdash \Box\mathtt{F_7}, \Delta_{11}, []\mathtt{F_{12}}} \text{ax/W} \quad \cfrac{\mathtt{h_{10}} : unbox(\Box}{\bullet\mathtt{h_{10}} : \Box\mathtt{F_7}, \Delta_{14}}}{- : \Delta_{14}, \mathtt{F_9}, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F_{12}}}
}{
- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{F_8} \to \mathtt{F_9} \vdash \Delta_{11}, []\mathtt{F_{12}}
}
$$

$$
\cfrac{
\mathtt{h_3} : \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F_7}, \mathtt{F_8}, \Delta_{12}, []\mathtt{F_{13}} \quad \mathtt{h_3} : \mathtt{F_9}, \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F_7}, \Delta_{12}, []\mathtt{F_{13}}
}{
\bullet\mathtt{h_3} : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F_8} \to \mathtt{F_9} \vdash (\Delta_{12}, []\mathtt{F_{13}}), \mathtt{F_7}
} \to_L
\quad
\cfrac{
\mathtt{h_{10}} : unbox(\Box\Gamma_{11}) \vdash \mathtt{F_{13}}
}{
\bullet\mathtt{h_{10}} : ((\Box\Gamma_{11}, \Delta_{14}), \mathtt{F_8} \to \mathtt{F_9}), \mathtt{F_7} \vdash \Delta_{12}, []\mathtt{F_{13}}
} K
$$

$$
\cfrac{
- : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F_8} \to \mathtt{F_9} \vdash \Delta_{12}, []\mathtt{F_{13}}
}{
\to
} \text{Cut}
$$

$$
\cfrac{
\cfrac{}{- : unbox(\Box\Gamma_{11}) \vdash \mathtt{F_{13}}} \text{ax/W}
}{
- : \Delta_{14}, \Box\Gamma_{11}, \mathtt{F_8} \to \mathtt{F_9} \vdash \Delta_{12}, []\mathtt{F_{13}}
} K
$$

- Case rule $A45$

$$
\cfrac{
\mathtt{h_3} : \Box\Gamma_{14}, \Delta_{15} \vdash \Box\mathtt{F_7}, \mathtt{F_8}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F_{13}} \quad \mathtt{h_3} : \mathtt{F_9}, \Box\Gamma_{14}, \Delta_{15} \vdash \Box\mathtt{F_7}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F_{13}}
}{
\bullet\mathtt{h_3} : (\Box\Gamma_{14}, \Delta_{15}), \mathtt{F_8} \to \mathtt{F_9} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F_{13}}), \Box\mathtt{F_7}
} \to_L
\quad
\cfrac{
\mathtt{h_{10}} : \Box\Gamma_{14}, \Box\mathtt{F_7} \vdash
}{
\bullet\mathtt{h_{10}} : ((\Box\Gamma_{14}, \Delta_{15}), \mathtt{F_8} \to \mathtt{F_9})
}
$$

$$
- : (\Box\Gamma_{14}, \Delta_{15}), \mathtt{F_8} \to \mathtt{F_9} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F_{13}}
$$
$$
\to
$$

$$
\cfrac{
\cfrac{}{\mathtt{h_3} : \Delta_{15}, \Box\Gamma_{14} \vdash \Box\mathtt{F_7}, \Delta_{12}, \mathtt{F_8}, \Box\Gamma_{11}, []\mathtt{F_{13}}} \text{ax/W} \quad \cfrac{\cfrac{\mathtt{h_{10}} : \Box\mathtt{F_7}, \Box\Gamma_{14} \vdash \mathtt{F_{13}}, \Box\Gamma_{11}}{\bullet\mathtt{h_{10}} : \Box\mathtt{F_7}, \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{12}, \mathtt{F_8}, \Box\Gamma_{11}, []\mathtt{F_{13}}} A45}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{12}, \mathtt{F_8}, \Box\Gamma_{11}, []\mathtt{F_{13}}} \text{hCut} \quad \cfrac{\mathtt{h_3} : \Delta_{15}, \mathtt{F_9}, \Box\Gamma_{14} \vdash \Box\mathtt{F_7}, \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F_{13}}}{- : \Delta_{15}, \mathtt{F_9}, \Box\Gamma_{14}} \text{ax}
}{
- : \Delta_{15}, \Box\Gamma_{14}, \mathtt{F_8} \to \mathtt{F_9} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F_{13}}
}
$$

$$\dfrac{\dfrac{h_3 : \Box\Gamma_{11}, \Delta_{15} \vdash F_7, F_8, \Box\Gamma_{12}, \Delta_{13}, []F_{14} \quad h_3 : F_9, \Box\Gamma_{11}, \Delta_{15} \vdash F_7, \Box\Gamma_{12}, \Delta_{13}, []F_{14}}{\bullet h_3 : (\Box\Gamma_{11}, \Delta_{15}), F_8 \to F_9 \vdash (\Box\Gamma_{12}, \Delta_{13}, []F_{14}), F_7} \to_L \quad \dfrac{h_{10} : \Box\Gamma_{11} \vdash \Box\Gamma_{12}, F_{14}}{\bullet h_{10} : ((\Box\Gamma_{11}, \Delta_{15}), F_8 \to F_9), F_7 \vdash \Box\Gamma_{12}, \Delta_{13}, []F_{14}} A45}{\dfrac{- : (\Box\Gamma_{11}, \Delta_{15}), F_8 \to F_9 \vdash \Box\Gamma_{12}, \Delta_{13}, []F_{14}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash F_{14}, \Box\Gamma_{12}} \text{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, F_8 \to F_9 \vdash \Delta_{13}, \Box\Gamma_{12}, []F_{14}} A45} \to} \text{Cut}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_3 : \Delta_7 \vdash F_{11} \to F_{12}, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \to F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \to F_{12}} \to_L \quad \dfrac{h_{10} : \Delta_7, F_8 \to F_9 \vdash F_{11}, \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \to F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), F_{11} \to F_{12} \vdash \Delta_{13}} \to_L}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}} \text{Cut}$$

$$\dfrac{\dfrac{}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11}} \text{ax/W} \quad \dfrac{\dfrac{\dfrac{}{- : \Delta_7, F_{11} \vdash \Delta_{13}, F_{12}, F_8} \text{inv-th/ax} \quad \dfrac{}{- : \Delta_7, F_{11}, F_9 \vdash \Delta_{13}, F_{12}}}{- : \Delta_7, F_{11}, F_8 \to F_9 \vdash \Delta_{13}, F_{12}} \to_L \quad \dfrac{}{- : \Delta_7, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}} \text{ax}}{- : \Delta_7, F_{11}, F_8 \to F_9 \vdash \Delta_{13}} \text{sC}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}} \text{sCut}$$

$$\dfrac{h_3 : \Delta_{14}, F_{11} \to F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \to F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \to F_{12}), F_8 \to F_9 \vdash \Delta_{13}, F_7} \to_L \quad \dfrac{h_{10} : F_7, \Delta_{14}, F_8 \to F_9 \vdash F_{11},}{\bullet h_{10} : ((\Delta_{14}, F_{11}}$$

$$- : (\Delta_{14}, F_{11} \to F_{12}), F_8 \to F_9 \vdash \Delta_{13}$$

$$\dfrac{\dfrac{}{h_3 : \Delta_{14} \vdash \Delta_{13}, F_{11}, F_7, F_8} \text{inv-th/ax} \quad \dfrac{}{h_3 : \Delta_{14}, F_9 \vdash \Delta_{13}, F_{11}, F_7} \text{inv-th/ax}}{\bullet h_3 : \Delta_{14}, F_8 \to F_9 \vdash \Delta_{13}, F_{11}, F_7} \to_L \quad \dfrac{}{h_{10} : \Delta_{14}, F_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11}} \text{ax/W} \quad \dfrac{}{h_3 : \Delta_{14}, F_{12} \vdash \Delta_{13}, F_7, F_8} \dfrac{}{\bullet h_3 : \Delta_{14}, F_1}$$

$$\dfrac{- : \Delta_{14}, F_8 \to F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_{14}, F_{11} \to F_{12}, F_8 \to F_9 \vdash \Delta_{13}} \text{hCut}$$

$$\dfrac{\dfrac{h_3 : \Delta_8 \vdash F_7, F_{10}, \Delta_{12} \quad h_3 : F_{11}, \Delta_8 \vdash F_7, \Delta_{12}}{\bullet h_3 : \Delta_8, F_{10} \to F_{11} \vdash \Delta_{12}, F_7} \to_L \quad \dfrac{h_9 : F_7, \Delta_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_7, F_{11}, \Delta_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_8, F_{10} \to F_{11}), F_7 \vdash \Delta_{12}} \to_L}{- : \Delta_8, F_{10} \to F_{11} \vdash \Delta_{12}} \text{Cut}$$

$$\dfrac{\dfrac{}{h_3 : \Delta_8 \vdash \Delta_{12}, F_{10}, F_7} \text{ax/W} \quad \dfrac{\dfrac{}{h_9 : \Delta_8, F_7 \vdash \Delta_{12}, F_{10}} \text{ax/W}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{12}, F_{10}} \text{H}}{- : \Delta_8 \vdash \Delta_{12}, F_{10}} \text{hCut} \quad \dfrac{\dfrac{}{h_3 : \Delta_8, F_{11} \vdash \Delta_{12}, F_7} \text{ax/W} \quad \dfrac{\dfrac{}{h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{12}} \text{ax/W}}{\bullet h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{12}} \text{H}}{- : \Delta_8, F_{11} \vdash \Delta_{12}} \text{hCut}}{- : \Delta_8, F_{10} \to F_{11} \vdash \Delta_{12}} \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_3 : \Delta_7 \vdash F_{11} \wedge F_{12}, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \wedge F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \to_L \quad \dfrac{h_{10} : F_{11}, F_{12}, \Delta_7, F_8 \to F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}} \text{Cut}$$

$$\dfrac{\dfrac{}{h_3 : \Delta_7 \vdash \Delta_{13}, F_8, F_{11} \wedge F_{12}} \text{ax/W} \quad \dfrac{\dfrac{}{h_{10} : \Delta_7, F_{11}, F_{12} \vdash \Delta_{13}, F_8} \text{inv-th/ax}}{\bullet h_{10} : \Delta_7, F_{11} \wedge F_{12} \vdash \Delta_{13}, F_8} \wedge_L}{- : \Delta_7 \vdash \Delta_{13}, F_8} \text{hCut} \quad \dfrac{\dfrac{}{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \text{ax/W} \quad \dfrac{\dfrac{}{h_{10} : \Delta_7, F_{11}, F_{12}, F_9 \vdash \Delta_{13}} \text{inv-th}}{\bullet h_{10} : \Delta_7, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_9 \vdash \Delta_{13}} \text{hCut}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}} \to_L$$

$$\dfrac{\dfrac{h_3 : \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \to F_9 \vdash \Delta_{13}, F_7} \to_L \quad \dfrac{h_{10} : F_7, F_{11}, F_{12}, \Delta_{14}, F_8 \to F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \to F_9), F_7 \vdash \Delta_{13}} \wedge_L}{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \to F_9 \vdash \Delta_{13}} \text{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{14}, F_{11}, F_{12} \vdash \Delta_{13}, F_7, F_8} \text{inv-th/ax} \quad \dfrac{}{h_3 : \Delta_{14}, F_{11}, F_{12}, F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax}}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_7} \to_L \quad \dfrac{}{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \to F_9 \vdash \Delta_{13}} \text{ax/W}}{- : \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}} \text{hCut}}{- : \Delta_{14}, F_8 \to F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_7 \vdash F_{11} \vee F_{12}, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \vee F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \to_L \quad \cfrac{h_{10} : F_{11}, \Delta_7, F_8 \to F_9 \vdash \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \to F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \vee_L}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}} \text{Cut}}{\to}$$

$$\cfrac{\cfrac{\cfrac{\cfrac{- : \Delta_7 \vdash \Delta_{13}, F_{11}, F_{12}, F_8}{} \text{inv-th/ax} \quad \cfrac{- : \Delta_7, F_9 \vdash \Delta_{13}, F_{11}, F_{12}}{} \text{inv-th/ax}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \to_L \quad \cfrac{- : \Delta_7, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_{11}}{} \text{ax/W}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11}} \text{sCut} \quad \cfrac{- : \Delta_7, F_{11}, F_8 \to F_9 \vdash \Delta_{13}}{} \text{ax}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}} \text{sC}$$

$$\cfrac{\cfrac{h_3 : \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \to F_9 \vdash \Delta_{13}, F_7} \to_L \quad \cfrac{h_{10} : F_7, F_{11}, \Delta_{14}, F_8 \to F_9 \vdash}{\bullet h_{10} : ((\Delta_{14}, F_{11}}{}}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \to F_9 \vdash \Delta_{13}}$$
$$\to$$

$$\cfrac{\cfrac{h_3 : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7, F_8}{} \text{ax/W} \quad \cfrac{\cfrac{h_{10} : \Delta_{14}, F_{11}, F_7 \vdash \Delta_{13}, F_8}{} \text{inv-th/ax} \quad \cfrac{h_{10} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_{13}, F_8}{} \text{inv-th/ax}}{\bullet h_{10} : \Delta_{14}, F_7, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \vee_L}{- : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \text{hCut} \quad \cfrac{h_3 : \Delta_{14}, F_9, F_{11} \vee F_{12} \vdash \Delta_1}{}}{- : \Delta_{14}, F_8 \to F_9, F_{11} \vee F_{12} \vdash \Delta_{13}}$$

- Case rule AT

$$\cfrac{\cfrac{h_3 : \Delta_7 \vdash []F_{11}, F_8, \Delta_{12} \quad h_3 : F_9, \Delta_7 \vdash []F_{11}, \Delta_{12}}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{12}, []F_{11}} \to_L \quad \cfrac{h_{10} : F_{11}, \Delta_7, []F_{11}, F_8 \to F_9 \vdash \Delta_{12}}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), []F_{11} \vdash \Delta_{12}} AT}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{12}} \text{Cut}$$
$$\to$$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_7 \vdash \Delta_{12}, F_8, []F_{11}}{} \text{ax/W} \quad \cfrac{\cfrac{h_{10} : \Delta_7, F_{11}, []F_{11} \vdash \Delta_{12}, F_8}{} \text{inv-th/ax}}{\bullet h_{10} : \Delta_7, []F_{11} \vdash \Delta_{12}, F_8} AT}{- : \Delta_7 \vdash \Delta_{12}, F_8} \text{hCut} \quad \cfrac{\cfrac{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, []F_{11}}{} \text{ax/W} \quad \cfrac{\cfrac{h_{10} : \Delta_7, F_{11}, F_9, []F_{11} \vdash \Delta_{12}}{} \text{inv-th/ax}}{\bullet h_{10} : \Delta_7, F_9, []F_{11} \vdash \Delta_{12}} AT}{- : \Delta_7, F_9 \vdash \Delta_{12}} \text{hCut}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{12}} \to_L$$

$$\cfrac{\cfrac{h_3 : \Delta_{13}, []F_{11} \vdash F_7, F_8, \Delta_{12} \quad h_3 : F_9, \Delta_{13}, []F_{11} \vdash F_7, \Delta_{12}}{\bullet h_3 : (\Delta_{13}, []F_{11}), F_8 \to F_9 \vdash \Delta_{12}, F_7} \to_L \quad \cfrac{h_{10} : F_7, F_{11}, \Delta_{13}, []F_{11}, F_8 \to F_9 \vdash \Delta_{12}}{\bullet h_{10} : ((\Delta_{13}, []F_{11}), F_8 \to F_9), F_7 \vdash \Delta_{12}} AT}{- : (\Delta_{13}, []F_{11}), F_8 \to F_9 \vdash \Delta_{12}} \text{Cut}$$
$$\to$$

$$\cfrac{\cfrac{\cfrac{\bullet h_3 : \Delta_{13}, F_{11}, []F_{11}, F_8 \to F_9 \vdash \Delta_{12}, F_7}{} \text{ax/W} \quad \cfrac{h_{10} : \Delta_{13}, F_{11}, F_7, []F_{11}, F_8 \to F_9 \vdash \Delta_{12}}{} \text{ax/W}}{- : \Delta_{13}, F_{11}, []F_{11}, F_8 \to F_9 \vdash \Delta_{12}} \text{hCut}}{- : \Delta_{13}, []F_{11}, F_8 \to F_9 \vdash \Delta_{12}} ATG$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{h_3 : \Delta_7 \vdash \perp, F_8, \Delta_{11} \quad h_3 : F_9, \Delta_7 \vdash \perp, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}, \perp} \to_L \quad \cfrac{}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), \perp \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}} \text{Cut}$$
$$\to$$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_7 \vdash \perp, \Delta_{11}, F_8}{} \text{ax/W} \quad \cfrac{}{\bullet h_{10} : \perp, \Delta_7 \vdash \Delta_{11}, F_8} \perp_L}{- : \Delta_7 \vdash \Delta_{11}, F_8} \text{hCut} \quad \cfrac{\cfrac{h_3 : \Delta_7, F_9 \vdash \perp, \Delta_{11}}{} \text{ax/W} \quad \cfrac{}{\bullet h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_9 \vdash \Delta_{11}} \text{hCut}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}} \to_L$$

$$\cfrac{\cfrac{h_3 : \perp, \Delta_{12} \vdash F_7, F_8, \Delta_{11} \quad h_3 : F_9, \perp, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \to F_9 \vdash \Delta_{11}, F_7} \to_L \quad \cfrac{}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \to F_9), F_7 \vdash \Delta_{11}} \perp_L}{- : (\perp, \Delta_{12}), F_8 \to F_9 \vdash \Delta_{11}} \text{Cut}$$
$$\to$$

$$\cfrac{}{- : \perp, \Delta_{12}, F_8 \to F_9 \vdash \Delta_{11}} \perp_L$$

- Case rule I

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$$\dfrac{\dfrac{h_3:\Delta_7\vdash p_{11},F_8,\Delta_{12},p_{11}\quad h_3:F_9,\Delta_7\vdash p_{11},\Delta_{12},p_{11}}{\bullet h_3:\Delta_7,F_8\to F_9\vdash(\Delta_{12},p_{11}),p_{11}}\to_L\quad \dfrac{}{\bullet h_{10}:(\Delta_7,F_8\to F_9),p_{11}\vdash\Delta_{12},p_{11}}I}{-:\Delta_7,F_8\to F_9\vdash\Delta_{12},p_{11}}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_3:\Delta_7\vdash\Delta_{12},F_8,p_{11},p_{11}}\text{ax/W}\quad\dfrac{}{\bullet h_{10}:\Delta_7,p_{11}\vdash\Delta_{12},F_8,p_{11}}I}{-:\Delta_7\vdash\Delta_{12},F_8,p_{11}}\text{hCut}\quad\dfrac{\dfrac{}{h_3:\Delta_7,F_9\vdash\Delta_{12},p_{11},p_{11}}\text{ax/W}\quad\dfrac{}{\bullet h_{10}:\Delta_7,F_9,p_{11}\vdash\Delta_{12},p_{11}}I}{-:\Delta_7,F_9\vdash\Delta_{12},p_{11}}\text{hCut}}{-:\Delta_7,F_8\to F_9\vdash\Delta_{12},p_{11}}\to_L$$

$$\dfrac{\dfrac{h_3:\Delta_{13},p_{11}\vdash F_7,F_8,\Delta_{12},p_{11}\quad h_3:F_9,\Delta_{13},p_{11}\vdash F_7,\Delta_{12},p_{11}}{\bullet h_3:(\Delta_{13},p_{11}),F_8\to F_9\vdash(\Delta_{12},p_{11}),F_7}\to_L\quad\dfrac{}{\bullet h_{10}:((\Delta_{13},p_{11}),F_8\to F_9),F_7\vdash\Delta_{12},p_{11}}I}{-:(\Delta_{13},p_{11}),F_8\to F_9\vdash\Delta_{12},p_{11}}\text{Cut}$$

$$\to$$

$$\dfrac{}{-:\Delta_{13},p_{11},F_8\to F_9\vdash\Delta_{12},p_{11}}I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{h_3:\Delta_7\vdash\top,F_8,\Delta_{11}\quad h_3:F_9,\Delta_7\vdash\top,\Delta_{11}}{\bullet h_3:\Delta_7,F_8\to F_9\vdash\Delta_{11},\top}\to_L\quad\dfrac{h_{10}:\Delta_7,F_8\to F_9\vdash\Delta_{11}}{\bullet h_{10}:(\Delta_7,F_8\to F_9),\top\vdash\Delta_{11}}\top_L}{-:\Delta_7,F_8\to F_9\vdash\Delta_{11}}\text{Cut}$$

$$\to$$

$$\dfrac{}{-:\Delta_7,F_8\to F_9\vdash\Delta_{11}}\text{ax/W}$$

$$\dfrac{\dfrac{h_3:\top,\Delta_{12}\vdash F_7,F_8,\Delta_{11}\quad h_3:F_9,\top,\Delta_{12}\vdash F_7,\Delta_{11}}{\bullet h_3:(\top,\Delta_{12}),F_8\to F_9\vdash\Delta_{11},F_7}\to_L\quad\dfrac{h_{10}:F_7,\Delta_{12},F_8\to F_9\vdash\Delta_{11}}{\bullet h_{10}:((\top,\Delta_{12}),F_8\to F_9),F_7\vdash\Delta_{11}}\top_L}{-:(\top,\Delta_{12}),F_8\to F_9\vdash\Delta_{11}}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet h_3:\top,\Delta_{12},F_8\to F_9\vdash\Delta_{11},F_7}\text{ax/W}\quad\dfrac{}{h_{10}:\top,\Delta_{12},F_7,F_8\to F_9\vdash\Delta_{11}}\text{ax/W}}{-:\top,\Delta_{12},F_8\to F_9\vdash\Delta_{11}}\text{hCut}$$

6.9 Status of $\wedge_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{h_3:F_9,F_{10},\Delta_8\vdash F_7,\Delta_{12},F_{13}\to F_{14}}{\bullet h_3:\Delta_8,F_9\wedge F_{10}\vdash(\Delta_{12},F_{13}\to F_{14}),F_7}\wedge_L\quad\dfrac{h_{11}:F_7,F_{13},\Delta_8,F_9\wedge F_{10}\vdash F_{14},\Delta_{12}}{\bullet h_{11}:(\Delta_8,F_9\wedge F_{10}),F_7\vdash\Delta_{12},F_{13}\to F_{14}}\to_R}{-:\Delta_8,F_9\wedge F_{10}\vdash\Delta_{12},F_{13}\to F_{14}}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_3:\Delta_8,F_{10},F_{13},F_9\vdash\Delta_{12},F_{14},F_7}\text{inv-th/ax}}{\bullet h_3:\Delta_8,F_{13},F_9\wedge F_{10}\vdash\Delta_{12},F_{14},F_7}\wedge_L\quad\dfrac{}{h_{11}:\Delta_8,F_{13},F_7,F_9\wedge F_{10}\vdash\Delta_{12},F_{14}}\text{ax/W}}{-:\Delta_8,F_{13},F_9\wedge F_{10}\vdash\Delta_{12},F_{14}}\text{hCut}}{-:\Delta_8,F_9\wedge F_{10}\vdash\Delta_{12},F_{13}\to F_{14}}\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_3:F_9,F_{10},\Delta_8\vdash F_7,\Delta_{12},F_{13}\wedge F_{14}}{\bullet h_3:\Delta_8,F_9\wedge F_{10}\vdash(\Delta_{12},F_{13}\wedge F_{14}),F_7}\wedge_L\quad\dfrac{h_{11}:F_7,\Delta_8,F_9\wedge F_{10}\vdash F_{13},\Delta_{12}\quad h_{11}:F_7,\Delta_8,F_9\wedge F_{10}\vdash F_{14},\Delta_{12}}{\bullet h_{11}:(\Delta_8,F_9\wedge F_{10}),F_7\vdash\Delta_{12},F_{13}\wedge F_{14}}\wedge_R}{-:\Delta_8,F_9\wedge F_{10}\vdash\Delta_{12},F_{13}\wedge F_{14}}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_3:\Delta_8,F_{10},F_9\vdash\Delta_{12},F_7,F_{13}\wedge F_{14}}\text{ax/W}\quad\dfrac{\dfrac{}{h_{11}:\Delta_8,F_{10},F_7,F_9\vdash\Delta_{12},F_{13}}\text{inv-th/ax}\quad\dfrac{}{h_{11}:\Delta_8,F_{10},F_7,F_9\vdash\Delta_{12},F_{14}}\text{inv-th/ax}}{\bullet h_{11}:\Delta_8,F_{10},F_7,F_9\vdash\Delta_{12},F_{13}\wedge F_{14}}\wedge_R}{-:\Delta_8,F_{10},F_9\vdash\Delta_{12},F_{13}\wedge F_{14}}\text{hCut}}{-:\Delta_8,F_9\wedge F_{10}\vdash\Delta_{12},F_{13}\wedge F_{14}}\wedge_L$$

- Case rule $\vee_R$

$$\frac{\dfrac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_7}\ \wedge_L \qquad \dfrac{h_{11} : F_7, \Delta_8, F_9 \wedge F_{10} \vdash F_{13}, F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \Delta_{12}, F_{13} \vee F_{14}}\ \vee_R}{- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}}\ \text{Cut}$$

$$\rightarrow$$

$$\frac{\dfrac{\dfrac{h_3 : \Delta_8, F_{10}, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7}\ \wedge_L \ ^{\text{inv-th/ax}} \qquad \dfrac{}{h_{11} : \Delta_8, F_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}\ \text{ax/W}}{- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}\ \text{hCut}}{- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}}\ \vee_R$$

- Case rule $\perp_R$

$$\frac{\dfrac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \perp, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\perp, \Delta_{12}), F_7}\ \wedge_L \qquad \dfrac{h_{11} : F_7, \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \perp, \Delta_{12}}\ \perp_R}{- : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}\ \text{Cut}$$

$$\rightarrow$$

$$\frac{\dfrac{}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}, F_7}\ \text{ax/W} \qquad \dfrac{}{h_{11} : \Delta_8, F_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}\ \text{ax/W}}{- : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}\ \text{hCut}$$

- Case rule $\top_R$

$$\frac{\dfrac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \top, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\top, \Delta_{12}), F_7}\ \wedge_L \qquad \dfrac{}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \top, \Delta_{12}}\ \top_R}{- : \Delta_8, F_9 \wedge F_{10} \vdash \top, \Delta_{12}}\ \text{Cut}$$

$$\rightarrow$$

$$\frac{}{- : \Delta_8, F_9 \wedge F_{10} \vdash \top, \Delta_{12}}\ \top_R$$

- Case rule K

$$\frac{\dfrac{h_3 : F_8, F_9, \Box\Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, []F_{12}}{\bullet h_3 : (\Box\Gamma_{13}, \Delta_{14}), F_8 \wedge F_9 \vdash (\Delta_{11}, []F_{12}), \Box F_7}\ \wedge_L \qquad \dfrac{h_{10} : unbox(\Box\Gamma_{13}), unbox(\Box F_7) \vdash F_{12}}{\bullet h_{10} : ((\Box\Gamma_{13}, \Delta_{14}), F_8 \wedge F_9), \Box F_7 \vdash \Delta_{11}, []F_{12}}\ K}{- : (\Box\Gamma_{13}, \Delta_{14}), F_8 \wedge F_9 \vdash \Delta_{11}, []F_{12}}\ \text{Cut}$$

$$\rightarrow$$

$$\frac{\dfrac{\dfrac{}{h_3 : \Delta_{14}, F_8, F_9, \Box\Gamma_{13} \vdash \Box F_7, \Delta_{11}, []F_{12}}\ \text{ax/W} \qquad \dfrac{\dfrac{}{h_{10} : unbox(\Box F_7), unbox(\Box\Gamma_{13}) \vdash F_{12}}\ \text{ax/W}}{\bullet h_{10} : \Box F_7, \Delta_{14}, F_8, F_9, \Box\Gamma_{13} \vdash \Delta_{11}, []F_{12}}\ K}{- : \Delta_{14}, F_8, F_9, \Box\Gamma_{13} \vdash \Delta_{11}, []F_{12}}\ \text{hCut}}{- : \Delta_{14}, \Box\Gamma_{13}, F_8 \wedge F_9 \vdash \Delta_{11}, []F_{12}}\ \wedge_L$$

$$\frac{\dfrac{h_3 : F_8, F_9, \Box\Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, []F_{13}}{\bullet h_3 : (\Box\Gamma_{11}, \Delta_{14}), F_8 \wedge F_9 \vdash (\Delta_{12}, []F_{13}), F_7}\ \wedge_L \qquad \dfrac{h_{10} : unbox(\Box\Gamma_{11}) \vdash F_{13}}{\bullet h_{10} : ((\Box\Gamma_{11}, \Delta_{14}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, []F_{13}}\ K}{- : (\Box\Gamma_{11}, \Delta_{14}), F_8 \wedge F_9 \vdash \Delta_{12}, []F_{13}}\ \text{Cut}$$

$$\rightarrow$$

$$\frac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash F_{13}}\ \text{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, []F_{13}}\ K$$

- Case rule $A45$

$$\frac{\dfrac{h_3 : F_8, F_9, \Box\Gamma_{14}, \Delta_{15} \vdash \Box F_7, \Box\Gamma_{11}, \Delta_{12}, []F_{13}}{\bullet h_3 : (\Box\Gamma_{14}, \Delta_{15}), F_8 \wedge F_9 \vdash (\Box\Gamma_{11}, \Delta_{12}, []F_{13}), \Box F_7}\ \wedge_L \qquad \dfrac{h_{10} : \Box\Gamma_{14}, \Box F_7 \vdash \Box\Gamma_{11}, F_{13}}{\bullet h_{10} : ((\Box\Gamma_{14}, \Delta_{15}), F_8 \wedge F_9), \Box F_7 \vdash \Box\Gamma_{11}, \Delta_{12}, []F_{13}}\ A45}{- : (\Box\Gamma_{14}, \Delta_{15}), F_8 \wedge F_9 \vdash \Box\Gamma_{11}, \Delta_{12}, []F_{13}}\ \text{Cut}$$

$$\rightarrow$$

$$\frac{\dfrac{\dfrac{}{h_3 : \Delta_{15}, F_8, F_9, \Box\Gamma_{14} \vdash \Box F_7, \Delta_{12}, \Box\Gamma_{11}, []F_{13}}\ \text{ax/W} \qquad \dfrac{\dfrac{}{h_{10} : \Box F_7, \Box\Gamma_{14} \vdash F_{13}, \Box\Gamma_{11}}\ \text{ax/W}}{\bullet h_{10} : \Box F_7, \Delta_{15}, F_8, F_9, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []F_{13}}\ A45}{- : \Delta_{15}, F_8, F_9, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []F_{13}}\ \text{hCut}}{- : \Delta_{15}, \Box\Gamma_{14}, F_8 \wedge F_9 \vdash \Delta_{12}, \Box\Gamma_{11}, []F_{13}}\ \wedge_L$$

$$\cfrac{\cfrac{\cfrac{\text{h}_3 : \text{F}_8, \text{F}_9, \Box\Gamma_{11}, \Delta_{15} \vdash \text{F}_7, \Box\Gamma_{12}, \Delta_{13}, []\text{F}_{14}}{\bullet\text{h}_3 : (\Box\Gamma_{11}, \Delta_{15}), \text{F}_8 \wedge \text{F}_9 \vdash (\Box\Gamma_{12}, \Delta_{13}, []\text{F}_{14}), \text{F}_7} \ {}_{\wedge_L} \quad \cfrac{\text{h}_{10} : \Box\Gamma_{11} \vdash \Box\Gamma_{12}, \text{F}_{14}}{\bullet\text{h}_{10} : ((\Box\Gamma_{11}, \Delta_{15}), \text{F}_8 \wedge \text{F}_9), \text{F}_7 \vdash \Box\Gamma_{12}, \Delta_{13}, []\text{F}_{14}} \ {}_{A45}}{- : (\Box\Gamma_{11}, \Delta_{15}), \text{F}_8 \wedge \text{F}_9 \vdash \Box\Gamma_{12}, \Delta_{13}, []\text{F}_{14}} \ {}_{\text{Cut}}}{\cfrac{\overset{\to}{\cfrac{\overline{- : \Box\Gamma_{11} \vdash \text{F}_{14}, \Box\Gamma_{12}}}{} \ {}_{\text{ax/W}}}{- : \Delta_{15}, \Box\Gamma_{11}, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}, \Box\Gamma_{12}, []\text{F}_{14}}} \ {}_{A45}}$$

- Case rule $\to_L$

$$\cfrac{\cfrac{\cfrac{\text{h}_3 : \text{F}_8, \text{F}_9, \Delta_7 \vdash \text{F}_{11} \to \text{F}_{12}, \Delta_{13}}{\bullet\text{h}_3 : \Delta_7, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}, \text{F}_{11} \to \text{F}_{12}} \ {}_{\wedge_L} \quad \cfrac{\text{h}_{10} : \Delta_7, \text{F}_8 \wedge \text{F}_9 \vdash \text{F}_{11}, \Delta_{13} \quad \text{h}_{10} : \text{F}_{12}, \Delta_7, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}}{\bullet\text{h}_{10} : (\Delta_7, \text{F}_8 \wedge \text{F}_9), \text{F}_{11} \to \text{F}_{12} \vdash \Delta_{13}} \ {}_{\to_L}}{- : \Delta_7, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}} \ {}_{\text{Cut}}}{\cfrac{\overset{\to}{\cfrac{\cfrac{\overline{\text{h}_3 : \Delta_7, \text{F}_8, \text{F}_9 \vdash \Delta_{13}, \text{F}_{11} \to \text{F}_{12}}}{} \ {}_{\text{ax/W}} \quad \cfrac{\cfrac{\text{h}_{10} : \Delta_7, \text{F}_8, \text{F}_9 \vdash \Delta_{13}, \text{F}_{11}}{} \ {}_{\text{inv-th/ax}} \quad \cfrac{\text{h}_{10} : \Delta_7, \text{F}_{12}, \text{F}_8, \text{F}_9 \vdash \Delta_{13}}{} \ {}_{\text{inv-th/ax}}}{\bullet\text{h}_{10} : \Delta_7, \text{F}_8, \text{F}_9, \text{F}_{11} \to \text{F}_{12} \vdash \Delta_{13}} \ {}_{\to_L}}{- : \Delta_7, \text{F}_8, \text{F}_9 \vdash \Delta_{13}} \ {}_{\text{hCut}}}{- : \Delta_7, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}}} \ {}_{\wedge_L}$$

$$\cfrac{\cfrac{\cfrac{\text{h}_3 : \text{F}_8, \text{F}_9, \Delta_{14}, \text{F}_{11} \to \text{F}_{12} \vdash \text{F}_7, \Delta_{13}}{\bullet\text{h}_3 : (\Delta_{14}, \text{F}_{11} \to \text{F}_{12}), \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}, \text{F}_7} \ {}_{\wedge_L} \quad \cfrac{\text{h}_{10} : \text{F}_7, \Delta_{14}, \text{F}_8 \wedge \text{F}_9 \vdash \text{F}_{11}, \Delta_{13} \quad \text{h}_{10} : \text{F}_7, \text{F}_{12}, \Delta_{14}, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}}{\bullet\text{h}_{10} : ((\Delta_{14}, \text{F}_{11} \to \text{F}_{12}), \text{F}_8 \wedge \text{F}_9), \text{F}_7 \vdash \Delta_{13}} \ {}_{\to_L}}{- : (\Delta_{14}, \text{F}_{11} \to \text{F}_{12}), \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}} \ {}_{\text{Cut}}}{\cfrac{\overset{\to}{\cfrac{\cfrac{\overline{\text{h}_3 : \Delta_{14}, \text{F}_8, \text{F}_9, \text{F}_{11} \to \text{F}_{12} \vdash \Delta_{13}, \text{F}_7}}{} \ {}_{\text{ax/W}} \quad \cfrac{\cfrac{\text{h}_{10} : \Delta_{14}, \text{F}_7, \text{F}_8, \text{F}_9 \vdash \Delta_{13}, \text{F}_{11}}{} \ {}_{\text{inv-th/ax}} \quad \cfrac{\text{h}_{10} : \Delta_{14}, \text{F}_{12}, \text{F}_7, \text{F}_8, \text{F}_9 \vdash \Delta_{13}}{} \ {}_{\text{inv-th/ax}}}{\bullet\text{h}_{10} : \Delta_{14}, \text{F}_7, \text{F}_8, \text{F}_9, \text{F}_{11} \to \text{F}_{12} \vdash \Delta_{13}} \ {}_{\to_L}}{- : \Delta_{14}, \text{F}_8, \text{F}_9, \text{F}_{11} \to \text{F}_{12} \vdash \Delta_{13}} \ {}_{\text{hCut}}}{- : \Delta_{14}, \text{F}_{11} \to \text{F}_{12}, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}}} \ {}_{\wedge_L}$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{\cfrac{\text{h}_3 : \text{F}_8, \text{F}_9, \Delta_7 \vdash \text{F}_{11} \wedge \text{F}_{12}, \Delta_{13}}{\bullet\text{h}_3 : \Delta_7, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}, \text{F}_{11} \wedge \text{F}_{12}} \ {}_{\wedge_L} \quad \cfrac{\text{h}_{10} : \text{F}_{11}, \text{F}_{12}, \Delta_7, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}}{\bullet\text{h}_{10} : (\Delta_7, \text{F}_8 \wedge \text{F}_9), \text{F}_{11} \wedge \text{F}_{12} \vdash \Delta_{13}} \ {}_{\wedge_L}}{- : \Delta_7, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}} \ {}_{\text{Cut}}}{\cfrac{\overset{\to}{\cfrac{\cfrac{\overline{\text{h}_3 : \Delta_7, \text{F}_8, \text{F}_9 \vdash \Delta_{13}, \text{F}_{11} \wedge \text{F}_{12}}}{} \ {}_{\text{ax/W}} \quad \cfrac{\cfrac{\text{h}_{10} : \Delta_7, \text{F}_{11}, \text{F}_{12}, \text{F}_8, \text{F}_9 \vdash \Delta_{13}}{} \ {}_{\text{inv-th/ax}}}{\bullet\text{h}_{10} : \Delta_7, \text{F}_8, \text{F}_9, \text{F}_{11} \wedge \text{F}_{12} \vdash \Delta_{13}} \ {}_{\wedge_L}}{- : \Delta_7, \text{F}_8, \text{F}_9 \vdash \Delta_{13}} \ {}_{\text{hCut}}}{- : \Delta_7, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}}} \ {}_{\wedge_L}$$

$$\cfrac{\cfrac{\cfrac{\text{h}_3 : \text{F}_8, \text{F}_9, \Delta_{14}, \text{F}_{11} \wedge \text{F}_{12} \vdash \text{F}_7, \Delta_{13}}{\bullet\text{h}_3 : (\Delta_{14}, \text{F}_{11} \wedge \text{F}_{12}), \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}, \text{F}_7} \ {}_{\wedge_L} \quad \cfrac{\text{h}_{10} : \text{F}_7, \text{F}_{11}, \text{F}_{12}, \Delta_{14}, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}}{\bullet\text{h}_{10} : ((\Delta_{14}, \text{F}_{11} \wedge \text{F}_{12}), \text{F}_8 \wedge \text{F}_9), \text{F}_7 \vdash \Delta_{13}} \ {}_{\wedge_L}}{- : (\Delta_{14}, \text{F}_{11} \wedge \text{F}_{12}), \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}} \ {}_{\text{Cut}}}{\cfrac{\overset{\to}{\cfrac{\cfrac{\cfrac{\text{h}_3 : \Delta_{14}, \text{F}_{11}, \text{F}_{12}, \text{F}_8, \text{F}_9 \vdash \Delta_{13}, \text{F}_7}{} \ {}_{\text{inv-th/ax}}}{\bullet\text{h}_3 : \Delta_{14}, \text{F}_{11}, \text{F}_{12}, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}, \text{F}_7} \ {}_{\wedge_L} \quad \cfrac{\overline{\text{h}_{10} : \Delta_{14}, \text{F}_{11}, \text{F}_{12}, \text{F}_7, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}}}{} \ {}_{\text{ax/W}}}{- : \Delta_{14}, \text{F}_{11}, \text{F}_{12}, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}} \ {}_{\text{hCut}}}{- : \Delta_{14}, \text{F}_{11} \wedge \text{F}_{12}, \text{F}_8 \wedge \text{F}_9 \vdash \Delta_{13}}} \ {}_{\wedge_L}$$

$$\cfrac{\cfrac{\cfrac{\text{h}_3 : \text{F}_{10}, \text{F}_{11}, \Delta_8 \vdash \text{F}_7, \Delta_{12}}{\bullet\text{h}_3 : \Delta_8, \text{F}_{10} \wedge \text{F}_{11} \vdash \Delta_{12}, \text{F}_7} \ {}_{\wedge_L} \quad \cfrac{\text{h}_9 : \text{F}_7, \text{F}_{10}, \text{F}_{11}, \Delta_8 \vdash \Delta_{12}}{\bullet\text{h}_9 : (\Delta_8, \text{F}_{10} \wedge \text{F}_{11}), \text{F}_7 \vdash \Delta_{12}} \ {}_{\wedge_L}}{- : \Delta_8, \text{F}_{10} \wedge \text{F}_{11} \vdash \Delta_{12}} \ {}_{\text{Cut}}}{\cfrac{\overset{\to}{\cfrac{\cfrac{\overline{\text{h}_3 : \Delta_8, \text{F}_{10}, \text{F}_{11} \vdash \Delta_{12}, \text{F}_7}}{} \ {}_{\text{ax/W}} \quad \cfrac{\cfrac{\overline{\text{h}_9 : \Delta_8, \text{F}_{10}, \text{F}_{11}, \text{F}_7 \vdash \Delta_{12}}}{} \ {}_{\text{ax/W}}}{\bullet\text{h}_9 : \Delta_8, \text{F}_{10}, \text{F}_{11}, \text{F}_7 \vdash \Delta_{12}} \ {}_{\text{H}}}{- : \Delta_8, \text{F}_{10}, \text{F}_{11} \vdash \Delta_{12}} \ {}_{\text{hCut}}}{- : \Delta_8, \text{F}_{10} \wedge \text{F}_{11} \vdash \Delta_{12}}} \ {}_{\wedge_L}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\texttt{h}_3 : \texttt{F}_8, \texttt{F}_9, \Delta_7 \vdash \texttt{F}_{11} \vee \texttt{F}_{12}, \Delta_{13}}{\bullet\texttt{h}_3 : \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{13}, \texttt{F}_{11} \vee \texttt{F}_{12}} \wedge_L \quad \dfrac{\texttt{h}_{10} : \texttt{F}_{11}, \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{13} \quad \texttt{h}_{10} : \texttt{F}_{12}, \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{13}}{\bullet\texttt{h}_{10} : (\Delta_7, \texttt{F}_8 \wedge \texttt{F}_9), \texttt{F}_{11} \vee \texttt{F}_{12} \vdash \Delta_{13}} \vee_L}{- : \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{13}} \, \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\texttt{h}_3 : \Delta_7, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{13}, \texttt{F}_{11} \vee \texttt{F}_{12}} \texttt{ax/W} \quad \dfrac{\dfrac{\texttt{h}_{10} : \Delta_7, \texttt{F}_{11}, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{13}}{} \texttt{inv-th/ax} \quad \dfrac{\texttt{h}_{10} : \Delta_7, \texttt{F}_{12}, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{13}}{} \texttt{inv-th/ax}}{\bullet\texttt{h}_{10} : \Delta_7, \texttt{F}_8, \texttt{F}_9, \texttt{F}_{11} \vee \texttt{F}_{12} \vdash \Delta_{13}} \vee_L}{- : \Delta_7, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{13}} \texttt{hCut}}{- : \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{13}} \wedge_L$$

$$\dfrac{\dfrac{\texttt{h}_3 : \texttt{F}_8, \texttt{F}_9, \Delta_{14}, \texttt{F}_{11} \vee \texttt{F}_{12} \vdash \texttt{F}_7, \Delta_{13}}{\bullet\texttt{h}_3 : (\Delta_{14}, \texttt{F}_{11} \vee \texttt{F}_{12}), \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{13}, \texttt{F}_7} \wedge_L \quad \dfrac{\texttt{h}_{10} : \texttt{F}_7, \texttt{F}_{11}, \Delta_{14}, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{13} \quad \texttt{h}_{10} : \texttt{F}_7, \texttt{F}_{12}, \Delta_{14}, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{13}}{\bullet\texttt{h}_{10} : ((\Delta_{14}, \texttt{F}_{11} \vee \texttt{F}_{12}), \texttt{F}_8 \wedge \texttt{F}_9), \texttt{F}_7 \vdash \Delta_{13}} \vee_L}{- : (\Delta_{14}, \texttt{F}_{11} \vee \texttt{F}_{12}), \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{13}} \, \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\texttt{h}_3 : \Delta_{14}, \texttt{F}_8, \texttt{F}_9, \texttt{F}_{11} \vee \texttt{F}_{12} \vdash \Delta_{13}, \texttt{F}_7} \texttt{ax/W} \quad \dfrac{\dfrac{\texttt{h}_{10} : \Delta_{14}, \texttt{F}_{11}, \texttt{F}_7, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{13}}{} \texttt{inv-th/ax} \quad \dfrac{\texttt{h}_{10} : \Delta_{14}, \texttt{F}_{12}, \texttt{F}_7, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{13}}{} \texttt{inv-th/ax}}{\bullet\texttt{h}_{10} : \Delta_{14}, \texttt{F}_7, \texttt{F}_8, \texttt{F}_9, \texttt{F}_{11} \vee \texttt{F}_{12} \vdash \Delta_{13}} \vee_L}{- : \Delta_{14}, \texttt{F}_8, \texttt{F}_9, \texttt{F}_{11} \vee \texttt{F}_{12} \vdash \Delta_{13}} \texttt{hCut}}{- : \Delta_{14}, \texttt{F}_8 \wedge \texttt{F}_9, \texttt{F}_{11} \vee \texttt{F}_{12} \vdash \Delta_{13}} \wedge_L$$

- Case rule AT

$$\dfrac{\dfrac{\texttt{h}_3 : \texttt{F}_8, \texttt{F}_9, \Delta_7 \vdash []\texttt{F}_{11}, \Delta_{12}}{\bullet\texttt{h}_3 : \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}, []\texttt{F}_{11}} \wedge_L \quad \dfrac{\texttt{h}_{10} : \texttt{F}_{11}, \Delta_7, []\texttt{F}_{11}, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}}{\bullet\texttt{h}_{10} : (\Delta_7, \texttt{F}_8 \wedge \texttt{F}_9), []\texttt{F}_{11} \vdash \Delta_{12}} AT}{- : \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}} \, \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\texttt{h}_3 : \Delta_7, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{12}, []\texttt{F}_{11}} \texttt{ax/W} \quad \dfrac{\dfrac{\texttt{h}_{10} : \Delta_7, \texttt{F}_{11}, \texttt{F}_8, \texttt{F}_9, []\texttt{F}_{11} \vdash \Delta_{12}}{} \texttt{inv-th/ax}}{\bullet\texttt{h}_{10} : \Delta_7, \texttt{F}_8, \texttt{F}_9, []\texttt{F}_{11} \vdash \Delta_{12}} AT}{- : \Delta_7, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{12}} \texttt{hCut}}{- : \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}} \wedge_L$$

$$\dfrac{\dfrac{\texttt{h}_3 : \texttt{F}_8, \texttt{F}_9, \Delta_{13}, []\texttt{F}_{11} \vdash \texttt{F}_7, \Delta_{12}}{\bullet\texttt{h}_3 : (\Delta_{13}, []\texttt{F}_{11}), \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}, \texttt{F}_7} \wedge_L \quad \dfrac{\texttt{h}_{10} : \texttt{F}_7, \texttt{F}_{11}, \Delta_{13}, []\texttt{F}_{11}, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}}{\bullet\texttt{h}_{10} : ((\Delta_{13}, []\texttt{F}_{11}), \texttt{F}_8 \wedge \texttt{F}_9), \texttt{F}_7 \vdash \Delta_{12}} AT}{- : (\Delta_{13}, []\texttt{F}_{11}), \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}} \, \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\texttt{h}_3 : \Delta_{13}, \texttt{F}_{11}, []\texttt{F}_{11}, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}, \texttt{F}_7} \texttt{ax/W} \quad \dfrac{}{\texttt{h}_{10} : \Delta_{13}, \texttt{F}_{11}, \texttt{F}_7, []\texttt{F}_{11}, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}} \texttt{ax/W}}{- : \Delta_{13}, \texttt{F}_{11}, []\texttt{F}_{11}, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}} \texttt{hCut}}{- : \Delta_{13}, []\texttt{F}_{11}, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{12}} ATG$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{\texttt{h}_3 : \texttt{F}_8, \texttt{F}_9, \Delta_7 \vdash \perp, \Delta_{11}}{\bullet\texttt{h}_3 : \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{11}, \perp} \wedge_L \quad \dfrac{}{\bullet\texttt{h}_{10} : (\Delta_7, \texttt{F}_8 \wedge \texttt{F}_9), \perp \vdash \Delta_{11}} \perp_L}{- : \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{11}} \, \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\texttt{h}_3 : \Delta_7, \texttt{F}_8, \texttt{F}_9 \vdash \perp, \Delta_{11}} \texttt{ax/W} \quad \dfrac{}{\bullet\texttt{h}_{10} : \perp, \Delta_7, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{11}} \perp_L}{- : \Delta_7, \texttt{F}_8, \texttt{F}_9 \vdash \Delta_{11}} \texttt{hCut}}{- : \Delta_7, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{11}} \wedge_L$$

$$\dfrac{\dfrac{\texttt{h}_3 : \texttt{F}_8, \texttt{F}_9, \perp, \Delta_{12} \vdash \texttt{F}_7, \Delta_{11}}{\bullet\texttt{h}_3 : (\perp, \Delta_{12}), \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{11}, \texttt{F}_7} \wedge_L \quad \dfrac{}{\bullet\texttt{h}_{10} : ((\perp, \Delta_{12}), \texttt{F}_8 \wedge \texttt{F}_9), \texttt{F}_7 \vdash \Delta_{11}} \perp_L}{- : (\perp, \Delta_{12}), \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{11}} \, \mathrm{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \perp, \Delta_{12}, \texttt{F}_8 \wedge \texttt{F}_9 \vdash \Delta_{11}} \perp_L$$

- Case rule I

$$\dfrac{\dfrac{h_3 : F_8, F_9, \Delta_7 \vdash p_{11}, \Delta_{12}, p_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash (\Delta_{12}, p_{11}), p_{11}} \wedge_L \quad \overline{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), p_{11} \vdash \Delta_{12}, p_{11}} \; I}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \text{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\overline{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{12}, p_{11}, p_{11}} \; \text{ax/W} \quad \overline{\bullet h_{10} : \Delta_7, F_8, F_9, p_{11} \vdash \Delta_{12}, p_{11}} \; I}{- : \Delta_7, F_8, F_9 \vdash \Delta_{12}, p_{11}} \text{hCut}}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \wedge_L$$

$$\dfrac{\dfrac{h_3 : F_8, F_9, \Delta_{13}, p_{11} \vdash F_7, \Delta_{12}, p_{11}}{\bullet h_3 : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash (\Delta_{12}, p_{11}), F_7} \wedge_L \quad \overline{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, p_{11}} \; I}{- : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \text{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{13}, p_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \; I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{h_3 : F_8, F_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \top} \wedge_L \quad \dfrac{h_{10} : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \top \vdash \Delta_{11}} \top_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \; \text{ax/W}$$

$$\dfrac{\dfrac{h_3 : F_8, F_9, \top, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \wedge_L \quad \dfrac{h_{10} : F_7, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}} \top_L}{- : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut}$$
$$\rightarrow$$
$$\dfrac{\overline{\bullet h_3 : \top, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \; \text{ax/W} \quad \overline{h_{10} : \top, \Delta_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{11}} \; \text{ax/W}}{- : \top, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}} \text{hCut}$$

6.10 Status of $\vee_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{h_3 : F_9, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \rightarrow F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \rightarrow F_{14}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_7} \vee_L \quad \dfrac{h_{11} : F_7, F_{13}, \Delta_8, F_9 \vee F_{10} \vdash F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \text{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{\overline{h_3 : \Delta_8, F_{13}, F_9 \vdash \Delta_{12}, F_{14}, F_7} \; \text{inv-th/ax} \quad \overline{h_3 : \Delta_8, F_{10}, F_{13} \vdash \Delta_{12}, F_{14}, F_7} \; \text{inv-th/ax}}{\bullet h_3 : \Delta_8, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}, F_7} \vee_L \quad \overline{h_{11} : \Delta_8, F_{13}, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}} \; \text{ax/W}}{- : \Delta_8, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}} \text{hCut}}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_3 : F_9, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \wedge F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \wedge F_{14}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_7} \vee_L \quad \dfrac{h_{11} : F_7, \Delta_8, F_9 \vee F_{10} \vdash F_{13}, \Delta_{1\cdots}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_1\cdots}}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{\overline{h_3 : \Delta_8, F_9 \vdash \Delta_{12}, F_{13}, F_7} \; \text{inv-th/ax} \quad \overline{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_7} \; \text{inv-th/ax}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_7} \vee_L \quad \overline{h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}} \; \text{ax/W}}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}} \text{hCut} \quad \dfrac{\overline{h_3 : \Delta_8, F_9 \vdash \Delta_{12}, F_{14}, F_7} \; \text{inv-}}{\bullet h_3 : \Delta_8, F_9 \vee }}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{h_3 : F_9, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_7} \vee_L \quad \dfrac{h_{11} : F_7, \Delta_8, F_9 \vee F_{10} \vdash F_{13}, F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_8, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \text{inv-th/ax} \quad \dfrac{}{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \text{inv-th/ax}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \vee_L \quad \dfrac{}{h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W}}{\dfrac{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R} \text{hCut}$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{h_3 : F_9, \Delta_8 \vdash F_7, \perp, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \perp, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\perp, \Delta_{12}), F_7} \vee_L \quad \dfrac{h_{11} : F_7, \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \perp, \Delta_{12}} \perp_R}{- : \Delta_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12}, F_7} \text{ax/W} \quad \dfrac{}{h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}} \text{ax/W}}{- : \Delta_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12}} \text{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{h_3 : F_9, \Delta_8 \vdash F_7, \top, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \top, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\top, \Delta_{12}), F_7} \vee_L \quad \dfrac{}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \top, \Delta_{12}} \top_R}{- : \Delta_8, F_9 \vee F_{10} \vdash \top, \Delta_{12}} \text{Cut}$$

$$\to$$

$$\dfrac{}{\dfrac{- : \Delta_8, F_9 \vee F_{10} \vdash \top, \Delta_{12}}{}} \top_R$$

- Case rule K

$$\dfrac{\dfrac{h_3 : F_8, \Box\Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, []F_{12} \quad h_3 : F_9, \Box\Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, []F_{12}}{\bullet h_3 : (\Box\Gamma_{13}, \Delta_{14}), F_8 \vee F_9 \vdash (\Delta_{11}, []F_{12}), \Box F_7} \vee_L \quad \dfrac{h_{10} : unbox(\Box\Gamma_{13}), unbox(\Box F_7) \vdash F_{12}}{\bullet h_{10} : ((\Box\Gamma_{13}, \Delta_{14}), F_8 \vee F_9), \Box F_7 \vdash \Delta_{11}, []F_1} K}{- : (\Box\Gamma_{13}, \Delta_{14}), F_8 \vee F_9 \vdash \Delta_{11}, []F_{12}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{14}, F_8, \Box\Gamma_{13} \vdash \Box F_7, \Delta_{11}, []F_{12}} \text{ax/W} \quad \dfrac{\dfrac{}{h_{10} : unbox(\Box F_7), unbox(\Box\Gamma_{13}) \vdash F_{12}} \text{ax/W}}{\bullet h_{10} : \Box F_7, \Delta_{14}, F_8, \Box\Gamma_{13} \vdash \Delta_{11}, []F_{12}} K}{\dfrac{- : \Delta_{14}, F_8, \Box\Gamma_{13} \vdash \Delta_{11}, []F_{12}}{}} \text{hCut} \quad \dfrac{\dfrac{}{h_3 : \Delta_{14}, F_9, \Box\Gamma_{13} \vdash \Box F_7, \Delta_{11}, []F_{12}} \text{ax/W} \quad \dfrac{}{\bullet h_{10} : \Box F_7, \Delta_{14}}{}} \dfrac{\dfrac{}{h_{10} : unbox(\Box F_{-}}{}}}{- : \Delta_{14}, \Box\Gamma_{13}, F_8 \vee F_9 \vdash \Delta_{11}, []F_{12}} \vee$$

$$\dfrac{\dfrac{h_3 : F_8, \Box\Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, []F_{13} \quad h_3 : F_9, \Box\Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, []F_{13}}{\bullet h_3 : (\Box\Gamma_{11}, \Delta_{14}), F_8 \vee F_9 \vdash (\Delta_{12}, []F_{13}), F_7} \vee_L \quad \dfrac{h_{10} : unbox(\Box\Gamma_{11}) \vdash F_{13}}{\bullet h_{10} : ((\Box\Gamma_{11}, \Delta_{14}), F_8 \vee F_9), F_7 \vdash \Delta_{12}, []F_{13}} K}{- : (\Box\Gamma_{11}, \Delta_{14}), F_8 \vee F_9 \vdash \Delta_{12}, []F_{13}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash F_{13}} \text{ax/W}}{}}{- : \Delta_{14}, \Box\Gamma_{11}, F_8 \vee F_9 \vdash \Delta_{12}, []F_{13}} K$$

- Case rule $A45$

$$\dfrac{\dfrac{h_3 : F_8, \Box\Gamma_{14}, \Delta_{15} \vdash \Box F_7, \Box\Gamma_{11}, \Delta_{12}, []F_{13} \quad h_3 : F_9, \Box\Gamma_{14}, \Delta_{15} \vdash \Box F_7, \Box\Gamma_{11}, \Delta_{12}, []F_{13}}{\bullet h_3 : (\Box\Gamma_{14}, \Delta_{15}), F_8 \vee F_9 \vdash (\Box\Gamma_{11}, \Delta_{12}, []F_{13}), \Box F_7} \vee_L \quad \dfrac{h_{10} : \Box\Gamma_{14}, \Box F_7 \vdash}{\bullet h_{10} : ((\Box\Gamma_{14}, \Delta_{15}), F_8 \vee F_9),}{}}{- : (\Box\Gamma_{14}, \Delta_{15}), F_8 \vee F_9 \vdash \Box\Gamma_{11}, \Delta_{12}, []F_{13}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{15}, F_8, \Box\Gamma_{14} \vdash \Box F_7, \Delta_{12}, \Box\Gamma_{11}, []F_{13}} \text{ax/W} \quad \dfrac{\dfrac{}{h_{10} : \Box F_7, \Box\Gamma_{14} \vdash F_{13}, \Box\Gamma_{11}} \text{ax/W}}{\bullet h_{10} : \Box F_7, \Delta_{15}, F_8, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []F_{13}} A45}{\dfrac{- : \Delta_{15}, F_8, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []F_{13}}{}} \text{hCut} \quad \dfrac{\dfrac{}{h_3 : \Delta_{15}, F_9, \Box\Gamma_{14} \vdash \Box F_7, \Delta_{12}, \Box\Gamma_{11}, []F_{13}} \text{ax}}{- : \Delta_{15}, F_9, \Box\Gamma_{14}}}{- : \Delta_{15}, \Box\Gamma_{14}, F_8 \vee F_9 \vdash \Delta_{12}, \Box\Gamma_{11}, []F_{13}}$$

$$\dfrac{\dfrac{\mathtt{h_3} : \mathtt{F_8}, \Box\Gamma_{11}, \Delta_{15} \vdash \mathtt{F_7}, \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F_{14}} \quad \mathtt{h_3} : \mathtt{F_9}, \Box\Gamma_{11}, \Delta_{15} \vdash \mathtt{F_7}, \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F_{14}}}{\bullet\mathtt{h_3} : (\Box\Gamma_{11}, \Delta_{15}), \mathtt{F_8} \vee \mathtt{F_9} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F_{14}}), \mathtt{F_7}} \vee_L \quad \dfrac{\mathtt{h_{10}} : \Box\Gamma_{11} \vdash \Box\Gamma_{12}, \mathtt{F_{14}}}{\bullet\mathtt{h_{10}} : ((\Box\Gamma_{11}, \Delta_{15}), \mathtt{F_8} \vee \mathtt{F_9}), \mathtt{F_7} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F_{14}}} A45}{\dfrac{- : (\Box\Gamma_{11}, \Delta_{15}), \mathtt{F_8} \vee \mathtt{F_9} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F_{14}}}{\dfrac{\rightarrow}{\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F_{14}}, \Box\Gamma_{12}} \text{ ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F_{14}}} A45}} \text{Cut}}$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_3} : \mathtt{F_8}, \Delta_7 \vdash \mathtt{F_{11}} \rightarrow \mathtt{F_{12}}, \Delta_{13} \quad \mathtt{h_3} : \mathtt{F_9}, \Delta_7 \vdash \mathtt{F_{11}} \rightarrow \mathtt{F_{12}}, \Delta_{13}}{\bullet\mathtt{h_3} : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}} \rightarrow \mathtt{F_{12}}} \vee_L \quad \dfrac{\mathtt{h_{10}} : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \mathtt{F_{11}}, \Delta_{13} \quad \mathtt{h_{10}} : \mathtt{F_{12}}, \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}{\bullet\mathtt{h_{10}} : (\Delta_7, \mathtt{F_8} \vee \mathtt{F_9}), \mathtt{F_{11}} \rightarrow \mathtt{F_{12}} \vdash \Delta_{13}} \rightarrow_L}{- : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}} \text{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}}} \text{ ax/W} \quad \dfrac{\dfrac{\dfrac{}{- : \Delta_7, \mathtt{F_{11}}, \mathtt{F_8} \vdash \Delta_{13}, \mathtt{F_{12}}} \text{ inv-th/ax} \quad \dfrac{}{- : \Delta_7, \mathtt{F_{11}}, \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{12}}} \text{ inv-th/ax}}{- : \Delta_7, \mathtt{F_{11}}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{12}}} \vee_L \quad \dfrac{}{- : \Delta_7, \mathtt{F_{11}}, \mathtt{F_{12}}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}} \text{ ax/W}}{- : \Delta_7, \mathtt{F_{11}}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}} \text{sCut}}{- : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}} \text{sCut}}$$

$$\dfrac{\dfrac{\mathtt{h_3} : \mathtt{F_8}, \Delta_{14}, \mathtt{F_{11}} \rightarrow \mathtt{F_{12}} \vdash \mathtt{F_7}, \Delta_{13} \quad \mathtt{h_3} : \mathtt{F_9}, \Delta_{14}, \mathtt{F_{11}} \rightarrow \mathtt{F_{12}} \vdash \mathtt{F_7}, \Delta_{13}}{\bullet\mathtt{h_3} : (\Delta_{14}, \mathtt{F_{11}} \rightarrow \mathtt{F_{12}}), \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_7}} \vee_L \quad \dfrac{\mathtt{h_{10}} : \mathtt{F_7}, \Delta_{14}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \mathtt{F_{11}}, \Delta}{\bullet\mathtt{h_{10}} : ((\Delta_{14}, \mathtt{F_{11}} \rightarrow}}{\dfrac{- : (\Delta_{14}, \mathtt{F_{11}} \rightarrow \mathtt{F_{12}}), \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}{\rightarrow}}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_3} : \Delta_{14}, \mathtt{F_8} \vdash \Delta_{13}, \mathtt{F_{11}}, \mathtt{F_7}} \text{ inv-th/ax} \quad \dfrac{}{\mathtt{h_3} : \Delta_{14}, \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}}, \mathtt{F_7}} \text{ inv-th/ax}}{\bullet\mathtt{h_3} : \Delta_{14}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}}, \mathtt{F_7}} \vee_L \quad \dfrac{}{\mathtt{h_{10}} : \Delta_{14}, \mathtt{F_7}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}}} \text{ ax/W} \quad \dfrac{}{\mathtt{h_3} : \Delta_{14}, \mathtt{F_{12}}, \mathtt{F_8} \vdash \Delta_{13}, \mathtt{F_7}} \text{ in}}{\dfrac{- : \Delta_{14}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}}}{- : \Delta_{14}, \mathtt{F_{11}} \rightarrow \mathtt{F_{12}}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}} \text{hCut} \quad \bullet\mathtt{h_3} : \Delta_{14}, \mathtt{F_1}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_3} : \mathtt{F_8}, \Delta_7 \vdash \mathtt{F_{11}} \wedge \mathtt{F_{12}}, \Delta_{13} \quad \mathtt{h_3} : \mathtt{F_9}, \Delta_7 \vdash \mathtt{F_{11}} \wedge \mathtt{F_{12}}, \Delta_{13}}{\bullet\mathtt{h_3} : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}} \wedge \mathtt{F_{12}}} \vee_L \quad \dfrac{\mathtt{h_{10}} : \mathtt{F_{11}}, \mathtt{F_{12}}, \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}{\bullet\mathtt{h_{10}} : (\Delta_7, \mathtt{F_8} \vee \mathtt{F_9}), \mathtt{F_{11}} \wedge \mathtt{F_{12}} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}} \text{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h_3} : \Delta_7, \mathtt{F_8} \vdash \Delta_{13}, \mathtt{F_{11}} \wedge \mathtt{F_{12}}} \text{ ax/W} \quad \dfrac{\dfrac{}{\mathtt{h_{10}} : \Delta_7, \mathtt{F_{11}}, \mathtt{F_{12}}, \mathtt{F_8} \vdash \Delta_{13}} \text{ inv-th/ax}}{\bullet\mathtt{h_{10}} : \Delta_7, \mathtt{F_8}, \mathtt{F_{11}} \wedge \mathtt{F_{12}} \vdash \Delta_{13}} \wedge_L}{\dfrac{- : \Delta_7, \mathtt{F_8} \vdash \Delta_{13}}{}} \text{hCut} \quad \dfrac{\dfrac{}{\mathtt{h_3} : \Delta_7, \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}} \wedge \mathtt{F_{12}}} \text{ ax/W} \quad \dfrac{\dfrac{}{\mathtt{h_{10}} : \Delta_7, \mathtt{F_{11}}, \mathtt{F_{12}}, \mathtt{F_9} \vdash \Delta_{13}} \text{ inv-t}}{\bullet\mathtt{h_{10}} : \Delta_7, \mathtt{F_9}, \mathtt{F_{11}} \wedge \mathtt{F_{12}} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, \mathtt{F_9} \vdash \Delta_{13}} \text{hCut}}{- : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}} \vee_L$$

$$\dfrac{\dfrac{\mathtt{h_3} : \mathtt{F_8}, \Delta_{14}, \mathtt{F_{11}} \wedge \mathtt{F_{12}} \vdash \mathtt{F_7}, \Delta_{13} \quad \mathtt{h_3} : \mathtt{F_9}, \Delta_{14}, \mathtt{F_{11}} \wedge \mathtt{F_{12}} \vdash \mathtt{F_7}, \Delta_{13}}{\bullet\mathtt{h_3} : (\Delta_{14}, \mathtt{F_{11}} \wedge \mathtt{F_{12}}), \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_7}} \vee_L \quad \dfrac{\mathtt{h_{10}} : \mathtt{F_7}, \mathtt{F_{11}}, \mathtt{F_{12}}, \Delta_{14}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}{\bullet\mathtt{h_{10}} : ((\Delta_{14}, \mathtt{F_{11}} \wedge \mathtt{F_{12}}), \mathtt{F_8} \vee \mathtt{F_9}), \mathtt{F_7} \vdash \Delta_{13}} \wedge_L}{\dfrac{- : (\Delta_{14}, \mathtt{F_{11}} \wedge \mathtt{F_{12}}), \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}{\rightarrow}} \text{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_3} : \Delta_{14}, \mathtt{F_{11}}, \mathtt{F_{12}}, \mathtt{F_8} \vdash \Delta_{13}, \mathtt{F_7}} \text{ inv-th/ax} \quad \dfrac{}{\mathtt{h_3} : \Delta_{14}, \mathtt{F_{11}}, \mathtt{F_{12}}, \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_7}} \text{ inv-th/ax}}{\bullet\mathtt{h_3} : \Delta_{14}, \mathtt{F_{11}}, \mathtt{F_{12}}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_7}} \vee_L \quad \dfrac{}{\mathtt{h_{10}} : \Delta_{14}, \mathtt{F_{11}}, \mathtt{F_{12}}, \mathtt{F_7}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}} \text{ ax/W}}{\dfrac{- : \Delta_{14}, \mathtt{F_{11}}, \mathtt{F_{12}}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}{- : \Delta_{14}, \mathtt{F_{11}} \wedge \mathtt{F_{12}}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}} \text{hCut} \wedge_L}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_3} : \mathtt{F_8}, \Delta_7 \vdash \mathtt{F_{11}} \vee \mathtt{F_{12}}, \Delta_{13} \quad \mathtt{h_3} : \mathtt{F_9}, \Delta_7 \vdash \mathtt{F_{11}} \vee \mathtt{F_{12}}, \Delta_{13}}{\bullet\mathtt{h_3} : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}} \vee \mathtt{F_{12}}} \vee_L \quad \dfrac{\mathtt{h_{10}} : \mathtt{F_{11}}, \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13} \quad \mathtt{h_{10}} : \mathtt{F_{12}}, \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}{\bullet\mathtt{h_{10}} : (\Delta_7, \mathtt{F_8} \vee \mathtt{F_9}), \mathtt{F_{11}} \vee \mathtt{F_{12}} \vdash \Delta_{13}} \vee_L}{- : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}} \text{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_7, \mathtt{F_8} \vdash \Delta_{13}, \mathtt{F_{11}}, \mathtt{F_{12}}} \text{ inv-th/ax} \quad \dfrac{}{- : \Delta_7, \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}}, \mathtt{F_{12}}} \text{ inv-th/ax}}{- : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}}, \mathtt{F_{12}}} \vee_L \quad \dfrac{}{- : \Delta_7, \mathtt{F_{12}}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}}} \text{ ax/W}}{\dfrac{- : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}, \mathtt{F_{11}}}{- : \Delta_7, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}}} \text{sCut} \quad \dfrac{}{- : \Delta_7, \mathtt{F_{11}}, \mathtt{F_8} \vee \mathtt{F_9} \vdash \Delta_{13}} \text{ ax/W} \text{sCut}}$$

$$\frac{h_3 : F_8, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \quad \frac{h_{10} : F_7, F_{11}, \Delta_{14}, F_8 \vee F_9 \vdash \Delta}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee}$$

$$- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}$$
$$\rightarrow$$

$$\frac{}{h_3 : \Delta_{14}, F_{11}, F_8 \vdash \Delta_{13}, F_7} \text{inv-th/ax} \quad \frac{}{h_3 : \Delta_{14}, F_{11}, F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax}$$
$$\frac{\bullet h_3 : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}, F_7}{} \vee_L \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}}{} \text{ax/W} \quad \frac{}{h_3 : \Delta_{14}, F_{12}, F_8 \vdash \Delta_{13}, F_7} \text{in}$$
$$\frac{- : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}}{} \text{hCut} \quad \frac{\bullet h_3 : \Delta_{14}, F_1}{}$$
$$- : \Delta_{14}, F_{11} \vee F_{12}, F_8 \vee F_9 \vdash \Delta_{13}$$

$$\frac{h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12} \quad h_3 : F_{11}, \Delta_8 \vdash F_7, \Delta_{12}}{\bullet h_3 : \Delta_8, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \vee_L \quad \frac{h_9 : F_7, F_{10}, \Delta_8 \vdash \Delta_{12} \quad h_9 : F_7, F_{11}, \Delta_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_8, F_{10} \vee F_{11}), F_7 \vdash \Delta_{12}} \vee_L$$
$$- : \Delta_8, F_{10} \vee F_{11} \vdash \Delta_{12} \quad \text{Cut}$$
$$\rightarrow$$

$$\frac{}{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_7} \text{ax/W} \quad \frac{\frac{}{h_9 : \Delta_8, F_{10}, F_7 \vdash \Delta_{12}} \text{ax/W}}{\bullet h_9 : \Delta_8, F_{10}, F_7 \vdash \Delta_{12}} \text{H} \quad \frac{}{h_3 : \Delta_8, F_{11} \vdash \Delta_{12}, F_7} \text{ax/W} \quad \frac{\frac{}{h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{12}} \text{ax/W}}{\bullet h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{12}} \text{H}$$
$$\frac{- : \Delta_8, F_{10} \vdash \Delta_{12}}{} \text{hCut} \quad \frac{- : \Delta_8, F_{11} \vdash \Delta_{12}}{} \text{hCut}$$
$$- : \Delta_8, F_{10} \vee F_{11} \vdash \Delta_{12} \quad \vee_L$$

- Case rule AT

$$\frac{h_3 : F_8, \Delta_7 \vdash []F_{11}, \Delta_{12} \quad h_3 : F_9, \Delta_7 \vdash []F_{11}, \Delta_{12}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, []F_{11}} \vee_L \quad \frac{h_{10} : F_{11}, \Delta_7, []F_{11}, F_8 \vee F_9 \vdash \Delta_{12}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), []F_{11} \vdash \Delta_{12}} AT$$
$$- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12} \quad \text{Cut}$$
$$\rightarrow$$

$$\frac{}{h_3 : \Delta_7, F_8 \vdash \Delta_{12}, []F_{11}} \text{ax/W} \quad \frac{\frac{}{h_{10} : \Delta_7, F_{11}, F_8, []F_{11} \vdash \Delta_{12}} \text{inv-th/ax}}{\bullet h_{10} : \Delta_7, F_8, []F_{11} \vdash \Delta_{12}} AT \quad \frac{}{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, []F_{11}} \text{ax/W} \quad \frac{\frac{}{h_{10} : \Delta_7, F_{11}, F_9, []F_{11} \vdash \Delta_{12}} \text{inv-th/ax}}{\bullet h_{10} : \Delta_7, F_9, []F_{11} \vdash \Delta_{12}} AT$$
$$\frac{- : \Delta_7, F_8 \vdash \Delta_{12}}{} \text{hCut} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{12}}{} \text{hCut}$$
$$- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12} \quad \vee_L$$

$$\frac{h_3 : F_8, \Delta_{13}, []F_{11} \vdash F_7, \Delta_{12} \quad h_3 : F_9, \Delta_{13}, []F_{11} \vdash F_7, \Delta_{12}}{\bullet h_3 : (\Delta_{13}, []F_{11}), F_8 \vee F_9 \vdash \Delta_{12}, F_7} \vee_L \quad \frac{h_{10} : F_7, F_{11}, \Delta_{13}, []F_{11}, F_8 \vee F_9 \vdash \Delta_{12}}{\bullet h_{10} : ((\Delta_{13}, []F_{11}), F_8 \vee F_9), F_7 \vdash \Delta_{12}} AT$$
$$- : (\Delta_{13}, []F_{11}), F_8 \vee F_9 \vdash \Delta_{12} \quad \text{Cut}$$
$$\rightarrow$$

$$\frac{}{\bullet h_3 : \Delta_{13}, F_{11}, []F_{11}, F_8 \vee F_9 \vdash \Delta_{12}, F_7} \text{ax/W} \quad \frac{}{h_{10} : \Delta_{13}, F_{11}, F_7, []F_{11}, F_8 \vee F_9 \vdash \Delta_{12}} \text{ax/W}$$
$$\frac{- : \Delta_{13}, F_{11}, []F_{11}, F_8 \vee F_9 \vdash \Delta_{12}}{} \text{hCut}$$
$$- : \Delta_{13}, []F_{11}, F_8 \vee F_9 \vdash \Delta_{12} \quad AT$$

- Case rule $\perp_L$

$$\frac{h_3 : F_8, \Delta_7 \vdash \perp, \Delta_{11} \quad h_3 : F_9, \Delta_7 \vdash \perp, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}, \perp} \vee_L \quad \frac{}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), \perp \vdash \Delta_{11}} \perp_L$$
$$- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11} \quad \text{Cut}$$
$$\rightarrow$$

$$\frac{}{h_3 : \Delta_7, F_8 \vdash \perp, \Delta_{11}} \text{ax/W} \quad \frac{}{\bullet h_{10} : \perp, \Delta_7, F_8 \vdash \Delta_{11}} \perp_L \quad \frac{}{h_3 : \Delta_7, F_9 \vdash \perp, \Delta_{11}} \text{ax/W} \quad \frac{}{\bullet h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}} \perp_L$$
$$\frac{- : \Delta_7, F_8 \vdash \Delta_{11}}{} \text{hCut} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{11}}{} \text{hCut}$$
$$- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11} \quad \vee_L$$

$$\frac{h_3 : F_8, \perp, \Delta_{12} \vdash F_7, \Delta_{11} \quad h_3 : F_9, \perp, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}, F_7} \vee_L \quad \frac{}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}} \perp_L$$
$$- : (\perp, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11} \quad \text{Cut}$$
$$\rightarrow$$
$$\frac{}{- : \perp, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}} \perp_L$$

- Case rule I

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$$\dfrac{\dfrac{\mathtt{h_3}: F_8, \Delta_7 \vdash \mathtt{p_{11}}, \Delta_{12}, \mathtt{p_{11}} \quad \mathtt{h_3}: F_9, \Delta_7 \vdash \mathtt{p_{11}}, \Delta_{12}, \mathtt{p_{11}}}{\bullet\mathtt{h_3}: \Delta_7, F_8 \vee F_9 \vdash (\Delta_{12}, \mathtt{p_{11}}), \mathtt{p_{11}}}\ \vee_L \quad \dfrac{}{\bullet\mathtt{h_{10}}: (\Delta_7, F_8 \vee F_9), \mathtt{p_{11}} \vdash \Delta_{12}, \mathtt{p_{11}}}\ I}{-: \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, \mathtt{p_{11}}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_3}: \Delta_7, F_8 \vdash \Delta_{12}, \mathtt{p_{11}}, \mathtt{p_{11}}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_{10}}: \Delta_7, F_8, \mathtt{p_{11}} \vdash \Delta_{12}, \mathtt{p_{11}}}\ I}{-: \Delta_7, F_8 \vdash \Delta_{12}, \mathtt{p_{11}}}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h_3}: \Delta_7, F_9 \vdash \Delta_{12}, \mathtt{p_{11}}, \mathtt{p_{11}}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_{10}}: \Delta_7, F_9, \mathtt{p_{11}} \vdash \Delta_{12}, \mathtt{p_{11}}}\ I}{-: \Delta_7, F_9 \vdash \Delta_{12}, \mathtt{p_{11}}}\ \mathtt{hCut}}{-: \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, \mathtt{p_{11}}}\ \vee_L$$

$$\dfrac{\dfrac{\mathtt{h_3}: F_8, \Delta_{13}, \mathtt{p_{11}} \vdash F_7, \Delta_{12}, \mathtt{p_{11}} \quad \mathtt{h_3}: F_9, \Delta_{13}, \mathtt{p_{11}} \vdash F_7, \Delta_{12}, \mathtt{p_{11}}}{\bullet\mathtt{h_3}: (\Delta_{13}, \mathtt{p_{11}}), F_8 \vee F_9 \vdash (\Delta_{12}, \mathtt{p_{11}}), F_7}\ \vee_L \quad \dfrac{}{\bullet\mathtt{h_{10}}: ((\Delta_{13}, \mathtt{p_{11}}), F_8 \vee F_9), F_7 \vdash \Delta_{12}, \mathtt{p_{11}}}\ I}{-: (\Delta_{13}, \mathtt{p_{11}}), F_8 \vee F_9 \vdash \Delta_{12}, \mathtt{p_{11}}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{-: \Delta_{13}, \mathtt{p_{11}}, F_8 \vee F_9 \vdash \Delta_{12}, \mathtt{p_{11}}}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h_3}: F_8, \Delta_7 \vdash \top, \Delta_{11} \quad \mathtt{h_3}: F_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet\mathtt{h_3}: \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}, \top}\ \vee_L \quad \dfrac{\mathtt{h_{10}}: \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet\mathtt{h_{10}}: (\Delta_7, F_8 \vee F_9), \top \vdash \Delta_{11}}\ \top_L}{-: \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{-: \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h_3}: F_8, \top, \Delta_{12} \vdash F_7, \Delta_{11} \quad \mathtt{h_3}: F_9, \top, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet\mathtt{h_3}: (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}, F_7}\ \vee_L \quad \dfrac{\mathtt{h_{10}}: F_7, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet\mathtt{h_{10}}: ((\top, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}}\ \top_L}{-: (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_3}: \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}, F_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h_{10}}: \top, \Delta_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{11}}\ \mathtt{ax/W}}{-: \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}}\ \mathtt{hCut}$$

6.11 Status of AT: `OK`

- Case rule $\to_R$

$$\dfrac{\dfrac{\mathtt{h_3}: F_8, \Delta_7, []F_8 \vdash F_6, \Delta_{10}, F_{11} \to F_{12}}{\bullet\mathtt{h_3}: \Delta_7, []F_8 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_6}\ AT \quad \dfrac{\mathtt{h_9}: F_6, F_{11}, \Delta_7, []F_8 \vdash F_{12}, \Delta_{10}}{\bullet\mathtt{h_9}: (\Delta_7, []F_8), F_6 \vdash \Delta_{10}, F_{11} \to F_{12}}\ \to_R}{-: \Delta_7, []F_8 \vdash \Delta_{10}, F_{11} \to F_{12}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_3}: \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_6, F_{11} \to F_{12}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_9}: \Delta_7, F_6, F_8, []F_8 \vdash \Delta_{10}, F_{11} \to F_{12}}\ \mathtt{ax/W}}{-: \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_{11} \to F_{12}}\ \mathtt{hCut}}{-: \Delta_7, []F_8 \vdash \Delta_{10}, F_{11} \to F_{12}}\ AT$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h_3}: F_8, \Delta_7, []F_8 \vdash F_6, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet\mathtt{h_3}: \Delta_7, []F_8 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_6}\ AT \quad \dfrac{\mathtt{h_9}: F_6, \Delta_7, []F_8 \vdash F_{11}, \Delta_{10} \quad \mathtt{h_9}: F_6, \Delta_7, []F_8 \vdash F_{12}, \Delta_{10}}{\bullet\mathtt{h_9}: (\Delta_7, []F_8), F_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}}\ \wedge_R}{-: \Delta_7, []F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_3}: \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_6, F_{11} \wedge F_{12}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_9}: \Delta_7, F_6, F_8, []F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}}\ \mathtt{ax/W}}{-: \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}}\ \mathtt{hCut}}{-: \Delta_7, []F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}}\ AT$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h_3}: F_8, \Delta_7, []F_8 \vdash F_6, \Delta_{10}, F_{11} \vee F_{12}}{\bullet\mathtt{h_3}: \Delta_7, []F_8 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_6}\ AT \quad \dfrac{\mathtt{h_9}: F_6, \Delta_7, []F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet\mathtt{h_9}: (\Delta_7, []F_8), F_6 \vdash \Delta_{10}, F_{11} \vee F_{12}}\ \vee_R}{-: \Delta_7, []F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_3}: \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_6, F_{11} \vee F_{12}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_9}: \Delta_7, F_6, F_8, []F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}}\ \mathtt{ax/W}}{-: \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}}\ \mathtt{hCut}}{-: \Delta_7, []F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}}\ ATG$$

- Case rule $\perp_R$

$$
\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_6, \perp, \Delta_{10}}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_8 \vdash (\perp, \Delta_{10}), \mathtt{F}_6} \; AT \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_7, []\mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_7, []\mathtt{F}_8), \mathtt{F}_6 \vdash \perp, \Delta_{10}} \; \perp_R}{- : \Delta_7, []\mathtt{F}_8 \vdash \perp, \Delta_{10}} \; \text{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_8 \vdash \perp, \Delta_{10}, \mathtt{F}_6} \; \text{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_7, \mathtt{F}_6, []\mathtt{F}_8 \vdash \perp, \Delta_{10}} \; \text{ax/W}}{- : \Delta_7, []\mathtt{F}_8 \vdash \perp, \Delta_{10}} \; \text{hCut}
$$

- Case rule $\top_R$

$$
\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_6, \top, \Delta_{10}}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_6} \; AT \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_7, []\mathtt{F}_8), \mathtt{F}_6 \vdash \top, \Delta_{10}} \; \top_R}{- : \Delta_7, []\mathtt{F}_8 \vdash \top, \Delta_{10}} \; \text{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{}{- : \Delta_7, []\mathtt{F}_8 \vdash \top, \Delta_{10}} \; \top_R
$$

- Case rule K

$$
\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\square\Gamma_{12}, \Delta_9), []\mathtt{F}_7 \vdash \square\mathtt{F}_6, \Delta_{10}, []\mathtt{F}_{11}}{\bullet\mathtt{h}_3 : (\square\Gamma_{12}, \Delta_9), []\mathtt{F}_7 \vdash (\Delta_{10}, []\mathtt{F}_{11}), \square\mathtt{F}_6} \; AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_7, unbox(\square\Gamma_{12}), unbox(\square\mathtt{F}_6) \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : ((\square\Gamma_{12}, \Delta_9), []\mathtt{F}_7), \square\mathtt{F}_6 \vdash \Delta_{10}, []\mathtt{F}_{11}} \; K}{- : (\square\Gamma_{12}, \Delta_9), []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}} \; \text{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_9, \mathtt{F}_7, \square\Gamma_{12}, []\mathtt{F}_7 \vdash \square\mathtt{F}_6, \Delta_{10}, []\mathtt{F}_{11}} \; \text{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \square\mathtt{F}_6, \Delta_9, \mathtt{F}_7, \square\Gamma_{12}, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}} \; \text{ax/W}}{- : \Delta_9, \mathtt{F}_7, \square\Gamma_{12}, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}} \; \text{hCut}}{- : \Delta_9, \square\Gamma_{12}, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}} \; ATG
$$

$$
\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\square\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_3 : (\square\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_6} \; AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_7, unbox(\square\Gamma_{11}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : ((\square\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_{10}} \; K}{- : (\square\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}} \; \text{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_7, unbox(\square\Gamma_{11}) \vdash \mathtt{F}_{10}} \; \text{ax/W}}{- : \Delta_{12}, \square\Gamma_{11}, []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}}{} \; K
$$

$$
\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\square\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \square\mathtt{F}_6, \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_3 : (\square\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash (\Delta_9, []\mathtt{F}_{10}), \square\mathtt{F}_6} \; AT \quad \dfrac{\mathtt{h}_8 : unbox(\square\Gamma_{11}), unbox(\square\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : ((\square\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7), \square\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_{10}} \; K}{- : (\square\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}} \; \text{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{12}, \mathtt{F}_7, \square\Gamma_{11}, []\mathtt{F}_7 \vdash \square\mathtt{F}_6, \Delta_9, []\mathtt{F}_{10}} \; \text{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \square\mathtt{F}_6, \Delta_{12}, \mathtt{F}_7, \square\Gamma_{11}, []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}} \; \text{ax/W}}{- : \Delta_{12}, \mathtt{F}_7, \square\Gamma_{11}, []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}} \; \text{hCut}}{- : \Delta_{12}, \square\Gamma_{11}, []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}} \; AT
$$

$$
\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\square\Gamma_9, \Delta_{12}), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_{11}}{\bullet\mathtt{h}_3 : (\square\Gamma_9, \Delta_{12}), []\mathtt{F}_7 \vdash (\Delta_{10}, []\mathtt{F}_{11}), \mathtt{F}_6} \; AT \quad \dfrac{\mathtt{h}_8 : unbox(\square\Gamma_9) \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : ((\square\Gamma_9, \Delta_{12}), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_{10}, []\mathtt{F}_{11}} \; K}{- : (\square\Gamma_9, \Delta_{12}), []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}} \; \text{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{\dfrac{\dfrac{}{- : unbox(\square\Gamma_9) \vdash \mathtt{F}_{11}} \; \text{ax/W}}{- : \Delta_{12}, \square\Gamma_9, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}}{} \; K
$$

- Case rule $A45$

$$
\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\square\Gamma_{13}, \Delta_9), []\mathtt{F}_7 \vdash \square\mathtt{F}_6, \square\Gamma_{10}, \Delta_{11}, []\mathtt{F}_{12}}{\bullet\mathtt{h}_3 : (\square\Gamma_{13}, \Delta_9), []\mathtt{F}_7 \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathtt{F}_{12}), \square\mathtt{F}_6} \; AT \quad \dfrac{\mathtt{h}_8 : \square\Gamma_{13}, \square\mathtt{F}_6, []\mathtt{F}_7 \vdash \square\Gamma_{10}, \mathtt{F}_{12}}{\bullet\mathtt{h}_8 : ((\square\Gamma_{13}, \Delta_9), []\mathtt{F}_7), \square\mathtt{F}_6 \vdash \square\Gamma_{10}, \Delta_{11}, []\mathtt{F}_{12}} \; A45}{- : (\square\Gamma_{13}, \Delta_9), []\mathtt{F}_7 \vdash \square\Gamma_{10}, \Delta_{11}, []\mathtt{F}_{12}} \; \text{Cut}
$$
$$
\rightarrow
$$
$$
\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_9, \mathtt{F}_7, \square\Gamma_{13}, []\mathtt{F}_7 \vdash \square\mathtt{F}_6, \Delta_{11}, \square\Gamma_{10}, []\mathtt{F}_{12}} \; \text{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \square\mathtt{F}_6, \Delta_9, \mathtt{F}_7, \square\Gamma_{13}, []\mathtt{F}_7 \vdash \Delta_{11}, \square\Gamma_{10}, []\mathtt{F}_{12}} \; \text{ax/W}}{- : \Delta_9, \mathtt{F}_7, \square\Gamma_{13}, []\mathtt{F}_7 \vdash \Delta_{11}, \square\Gamma_{10}, []\mathtt{F}_{12}} \; \text{hCut}}{- : \Delta_9, \square\Gamma_{13}, []\mathtt{F}_7 \vdash \Delta_{11}, \square\Gamma_{10}, []\mathtt{F}_{12}} \; ATG
$$

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$$\dfrac{\dfrac{h_3 : F_7, (\Box\Gamma_{12}, \Delta_{13}), []F_7 \vdash F_6, \Box\Gamma_9, \Delta_{10}, []F_{11}}{\bullet h_3 : (\Box\Gamma_{12}, \Delta_{13}), []F_7 \vdash (\Box\Gamma_9, \Delta_{10}, []F_{11}), F_6} \; AT \qquad \dfrac{h_8 : \Box\Gamma_{12}, []F_7 \vdash \Box\Gamma_9, F_{11}}{\bullet h_8 : ((\Box\Gamma_{12}, \Delta_{13}), []F_7), F_6 \vdash \Box\Gamma_9, \Delta_{10}, []F_{11}} \; A45}{\dfrac{- : (\Box\Gamma_{12}, \Delta_{13}), []F_7 \vdash \Box\Gamma_9, \Delta_{10}, []F_{11}}{\dfrac{\to}{\dfrac{\dfrac{}{- : \Box\Gamma_{12}, []F_7 \vdash F_{11}, \Box\Gamma_9} \; ax/W}{- : \Delta_{13}, \Box\Gamma_{12}, []F_7 \vdash \Delta_{10}, \Box\Gamma_9, []F_{11}} \; A45}} \; Cut}$$

$$\dfrac{\dfrac{h_3 : F_7, (\Box\Gamma_{12}, \Delta_{13}), []F_7 \vdash \Box F_6, \Box\Gamma_9, \Delta_{10}, []F_{11}}{\bullet h_3 : (\Box\Gamma_{12}, \Delta_{13}), []F_7 \vdash (\Box\Gamma_9, \Delta_{10}, []F_{11}), \Box F_6} \; AT \qquad \dfrac{h_8 : \Box\Gamma_{12}, \Box F_6 \vdash \Box\Gamma_9, F_{11}}{\bullet h_8 : ((\Box\Gamma_{12}, \Delta_{13}), []F_7), \Box F_6 \vdash \Box\Gamma_9, \Delta_{10}, []F_{11}} \; A45}{\dfrac{- : (\Box\Gamma_{12}, \Delta_{13}), []F_7 \vdash \Box\Gamma_9, \Delta_{10}, []F_{11}}{\dfrac{\to}{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{13}, F_7, \Box\Gamma_{12}, []F_7 \vdash \Box F_6, \Delta_{10}, \Box\Gamma_9, []F_{11}} \; ax/W \quad \dfrac{}{\bullet h_8 : \Box F_6, \Delta_{13}, F_7, \Box\Gamma_{12}, []F_7 \vdash \Delta_{10}, \Box\Gamma_9, []F_{11}} \; ax/W}{- : \Delta_{13}, F_7, \Box\Gamma_{12}, []F_7 \vdash \Delta_{10}, \Box\Gamma_9, []F_{11}} \; hCut}{- : \Delta_{13}, \Box\Gamma_{12}, []F_7 \vdash \Delta_{10}, \Box\Gamma_9, []F_{11}} \; AT}} \; Cut}$$

$$\dfrac{\dfrac{h_3 : F_7, (\Box\Gamma_9, \Delta_{13}), []F_7 \vdash F_6, \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\bullet h_3 : (\Box\Gamma_9, \Delta_{13}), []F_7 \vdash (\Box\Gamma_{10}, \Delta_{11}, []F_{12}), F_6} \; AT \qquad \dfrac{h_8 : \Box\Gamma_9 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_8 : ((\Box\Gamma_9, \Delta_{13}), []F_7), F_6 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}} \; A45}{\dfrac{- : (\Box\Gamma_9, \Delta_{13}), []F_7 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}}{\dfrac{\to}{\dfrac{\dfrac{}{- : \Box\Gamma_9 \vdash F_{12}, \Box\Gamma_{10}} \; ax/W}{- : \Delta_{13}, \Box\Gamma_9, []F_7 \vdash \Delta_{11}, \Box\Gamma_{10}, []F_{12}} \; A45}} \; Cut}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_3 : F_7, \Delta_6, []F_7 \vdash F_9 \to F_{10}, \Delta_{11}}{\bullet h_3 : \Delta_6, []F_7 \vdash \Delta_{11}, F_9 \to F_{10}} \; AT \qquad \dfrac{h_8 : \Delta_6, []F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_6, []F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_6, []F_7), F_9 \to F_{10} \vdash \Delta_{11}} \; \to_L}{\dfrac{- : \Delta_6, []F_7 \vdash \Delta_{11}}{\dfrac{\to}{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_6, F_7, []F_7 \vdash \Delta_{11}, F_9 \to F_{10}} \; ax/W \quad \dfrac{}{\bullet h_8 : \Delta_6, F_7, []F_7, F_9 \to F_{10} \vdash \Delta_{11}} \; ax/W}{- : \Delta_6, F_7, []F_7 \vdash \Delta_{11}} \; hCut}{- : \Delta_6, []F_7 \vdash \Delta_{11}} \; ATG}} \; Cut}$$

$$\dfrac{\dfrac{h_3 : F_7, (\Delta_{12}, F_9 \to F_{10}), []F_7 \vdash F_6, \Delta_{11}}{\bullet h_3 : (\Delta_{12}, F_9 \to F_{10}), []F_7 \vdash \Delta_{11}, F_6} \; AT \qquad \dfrac{h_8 : F_6, \Delta_{12}, []F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_6, F_{10}, \Delta_{12}, []F_7 \vdash \Delta_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \to F_{10}), []F_7), F_6 \vdash \Delta_{11}} \; \to_L}{\dfrac{- : (\Delta_{12}, F_9 \to F_{10}), []F_7 \vdash \Delta_{11}}{\dfrac{\to}{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{12}, F_7, []F_7, F_9 \to F_{10} \vdash \Delta_{11}, F_6} \; ax/W \quad \dfrac{}{\bullet h_8 : \Delta_{12}, F_6, F_7, []F_7, F_9 \to F_{10} \vdash \Delta_{11}} \; ax/W}{- : \Delta_{12}, F_7, []F_7, F_9 \to F_{10} \vdash \Delta_{11}} \; hCut}{- : \Delta_{12}, []F_7, F_9 \to F_{10} \vdash \Delta_{11}} \; ATG}} \; Cut}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_3 : F_7, \Delta_6, []F_7 \vdash F_9 \wedge F_{10}, \Delta_{11}}{\bullet h_3 : \Delta_6, []F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \; AT \qquad \dfrac{h_8 : F_9, F_{10}, \Delta_6, []F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_6, []F_7), F_9 \wedge F_{10} \vdash \Delta_{11}} \; \wedge_L}{\dfrac{- : \Delta_6, []F_7 \vdash \Delta_{11}}{\dfrac{\to}{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_6, F_7, []F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \; ax/W \quad \dfrac{}{\bullet h_8 : \Delta_6, F_7, []F_7, F_9 \wedge F_{10} \vdash \Delta_{11}} \; ax/W}{- : \Delta_6, F_7, []F_7 \vdash \Delta_{11}} \; hCut}{- : \Delta_6, []F_7 \vdash \Delta_{11}} \; ATG}} \; Cut}$$

$$\dfrac{\dfrac{h_3 : F_7, (\Delta_{12}, F_9 \wedge F_{10}), []F_7 \vdash F_6, \Delta_{11}}{\bullet h_3 : (\Delta_{12}, F_9 \wedge F_{10}), []F_7 \vdash \Delta_{11}, F_6} \; AT \qquad \dfrac{h_8 : F_6, F_9, F_{10}, \Delta_{12}, []F_7 \vdash \Delta_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \wedge F_{10}), []F_7), F_6 \vdash \Delta_{11}} \; \wedge_L}{\dfrac{- : (\Delta_{12}, F_9 \wedge F_{10}), []F_7 \vdash \Delta_{11}}{\dfrac{\to}{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{12}, F_7, []F_7, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6} \; ax/W \quad \dfrac{}{\bullet h_8 : \Delta_{12}, F_6, F_7, []F_7, F_9 \wedge F_{10} \vdash \Delta_{11}} \; ax/W}{- : \Delta_{12}, F_7, []F_7, F_9 \wedge F_{10} \vdash \Delta_{11}} \; hCut}{- : \Delta_{12}, []F_7, F_9 \wedge F_{10} \vdash \Delta_{11}} \; ATG}} \; Cut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_3 : F_7, \Delta_6, []F_7 \vdash F_9 \vee F_{10}, \Delta_{11}}{\bullet h_3 : \Delta_6, []F_7 \vdash \Delta_{11}, F_9 \vee F_{10}}\; AT \quad \dfrac{h_8 : F_9, \Delta_6, []F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_6, []F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_6, []F_7), F_9 \vee F_{10} \vdash \Delta_{11}}\; \vee_L}{- : \Delta_6, []F_7 \vdash \Delta_{11}}\; \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_6, F_7, []F_7 \vdash \Delta_{11}, F_9 \vee F_{10}}\; \text{ax/W} \quad \dfrac{}{\bullet h_8 : \Delta_6, F_7, []F_7, F_9 \vee F_{10} \vdash \Delta_{11}}\; \text{ax/W}}{- : \Delta_6, F_7, []F_7 \vdash \Delta_{11}}\; \text{hCut}}{- : \Delta_6, []F_7 \vdash \Delta_{11}}\; AT$$

$$\dfrac{\dfrac{h_3 : F_7, (\Delta_{12}, F_9 \vee F_{10}), []F_7 \vdash F_6, \Delta_{11}}{\bullet h_3 : (\Delta_{12}, F_9 \vee F_{10}), []F_7 \vdash \Delta_{11}, F_6}\; AT \quad \dfrac{h_8 : F_6, F_9, \Delta_{12}, []F_7 \vdash \Delta_{11} \quad h_8 : F_6, F_{10}, \Delta_{12}, []F_7 \vdash \Delta_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \vee F_{10}), []F_7), F_6 \vdash \Delta_{11}}\; \vee_L}{- : (\Delta_{12}, F_9 \vee F_{10}), []F_7 \vdash \Delta_{11}}\; \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{12}, F_7, []F_7, F_9 \vee F_{10} \vdash \Delta_{11}, F_6}\; \text{ax/W} \quad \dfrac{}{\bullet h_8 : \Delta_{12}, F_6, F_7, []F_7, F_9 \vee F_{10} \vdash \Delta_{11}}\; \text{ax/W}}{- : \Delta_{12}, F_7, []F_7, F_9 \vee F_{10} \vdash \Delta_{11}}\; \text{hCut}}{- : \Delta_{12}, []F_7, F_9 \vee F_{10} \vdash \Delta_{11}}\; ATG}$$

- Case rule AT

$$\dfrac{\dfrac{h_3 : F_7, \Delta_6, []F_7 \vdash []F_9, \Delta_{10}}{\bullet h_3 : \Delta_6, []F_7 \vdash \Delta_{10}, []F_9}\; AT \quad \dfrac{h_8 : F_9, \Delta_6, []F_7, []F_9 \vdash \Delta_{10}}{\bullet h_8 : (\Delta_6, []F_7), []F_9 \vdash \Delta_{10}}\; AT}{- : \Delta_6, []F_7 \vdash \Delta_{10}}\; \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_6, F_7, []F_7 \vdash \Delta_{10}, []F_9}\; \text{ax/W} \quad \dfrac{}{\bullet h_8 : \Delta_6, F_7, []F_7, []F_9 \vdash \Delta_{10}}\; \text{ax/W}}{- : \Delta_6, F_7, []F_7 \vdash \Delta_{10}}\; \text{hCut}}{- : \Delta_6, []F_7 \vdash \Delta_{10}}\; ATG$$

$$\dfrac{\dfrac{h_3 : F_7, (\Delta_{11}, []F_9), []F_7 \vdash F_6, \Delta_{10}}{\bullet h_3 : (\Delta_{11}, []F_9), []F_7 \vdash \Delta_{10}, F_6}\; AT \quad \dfrac{h_8 : F_6, F_9, \Delta_{11}, []F_7, []F_9 \vdash \Delta_{10}}{\bullet h_8 : ((\Delta_{11}, []F_9), []F_7), F_6 \vdash \Delta_{10}}\; AT}{- : (\Delta_{11}, []F_9), []F_7 \vdash \Delta_{10}}\; \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{11}, F_7, []F_7, []F_9 \vdash \Delta_{10}, F_6}\; \text{ax/W} \quad \dfrac{}{\bullet h_8 : \Delta_{11}, F_6, F_7, []F_7, []F_9 \vdash \Delta_{10}}\; \text{ax/W}}{- : \Delta_{11}, F_7, []F_7, []F_9 \vdash \Delta_{10}}\; \text{hCut}}{- : \Delta_{11}, []F_7, []F_9 \vdash \Delta_{10}}\; AT$$

$$\dfrac{\dfrac{h_3 : F_9, \Delta_7, []F_9 \vdash F_6, \Delta_{10}}{\bullet h_3 : \Delta_7, []F_9 \vdash \Delta_{10}, F_6}\; AT \quad \dfrac{h_8 : F_6, F_9, \Delta_7, []F_9 \vdash \Delta_{10}}{\bullet h_8 : (\Delta_7, []F_9), F_6 \vdash \Delta_{10}}\; AT}{- : \Delta_7, []F_9 \vdash \Delta_{10}}\; \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_7, F_9, []F_9 \vdash \Delta_{10}, F_6}\; \text{ax/W} \quad \dfrac{}{\bullet h_8 : \Delta_7, F_6, F_9, []F_9 \vdash \Delta_{10}}\; \text{ax/W}}{- : \Delta_7, F_9, []F_9 \vdash \Delta_{10}}\; \text{hCut}}{- : \Delta_7, []F_9 \vdash \Delta_{10}}\; AT$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{h_3 : F_7, \Delta_6, []F_7 \vdash \perp, \Delta_9}{\bullet h_3 : \Delta_6, []F_7 \vdash \Delta_9, \perp}\; AT \quad \dfrac{}{\bullet h_8 : (\Delta_6, []F_7), \perp \vdash \Delta_9}\; \perp_L}{- : \Delta_6, []F_7 \vdash \Delta_9}\; \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_6, F_7, []F_7 \vdash \perp, \Delta_9}\; \text{ax/W} \quad \dfrac{}{\bullet h_8 : \perp, \Delta_6, F_7, []F_7 \vdash \Delta_9}\; \perp_L}{- : \Delta_6, F_7, []F_7 \vdash \Delta_9}\; \text{hCut}}{- : \Delta_6, []F_7 \vdash \Delta_9}\; ATG$$

$$\dfrac{\dfrac{h_3 : F_7, (\perp, \Delta_{10}), []F_7 \vdash F_6, \Delta_9}{\bullet h_3 : (\perp, \Delta_{10}), []F_7 \vdash \Delta_9, F_6}\; AT \quad \dfrac{}{\bullet h_8 : ((\perp, \Delta_{10}), []F_7), F_6 \vdash \Delta_9}\; \perp_L}{- : (\perp, \Delta_{10}), []F_7 \vdash \Delta_9}\; \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \perp, \Delta_{10}, []F_7 \vdash \Delta_9}\; \perp_L$$

- Case rule I

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$$
\cfrac{
  \cfrac{
    \cfrac{\mathtt{h_3}: \mathtt{F_7}, \Delta_6, []\mathtt{F_7} \vdash \mathtt{p_9}, \Delta_{10}, \mathtt{p_9}}
         {\bullet\mathtt{h_3}: \Delta_6, []\mathtt{F_7} \vdash (\Delta_{10}, \mathtt{p_9}), \mathtt{p_9}}\ AT
    \qquad
    \cfrac{}{\bullet\mathtt{h_8}: (\Delta_6, []\mathtt{F_7}), \mathtt{p_9} \vdash \Delta_{10}, \mathtt{p_9}}\ I
  }{- : \Delta_6, []\mathtt{F_7} \vdash \Delta_{10}, \mathtt{p_9}}\ Cut
}{
  \cfrac{
    \cfrac{}{\mathtt{h_3}: \Delta_6, \mathtt{F_7}, []\mathtt{F_7} \vdash \Delta_{10}, \mathtt{p_9}, \mathtt{p_9}}\ ax/W
    \qquad
    \cfrac{}{\bullet\mathtt{h_8}: \Delta_6, \mathtt{F_7}, \mathtt{p_9}, []\mathtt{F_7} \vdash \Delta_{10}, \mathtt{p_9}}\ I
  }{
    \cfrac{- : \Delta_6, \mathtt{F_7}, []\mathtt{F_7} \vdash \Delta_{10}, \mathtt{p_9}}
         {- : \Delta_6, []\mathtt{F_7} \vdash \Delta_{10}, \mathtt{p_9}}\ AT
  }\ hCut
}\ \to
$$

$$
\cfrac{
  \cfrac{\mathtt{h_3}: \mathtt{F_7}, (\Delta_{11}, \mathtt{p_9}), []\mathtt{F_7} \vdash \mathtt{F_6}, \Delta_{10}, \mathtt{p_9}}
       {\bullet\mathtt{h_3}: (\Delta_{11}, \mathtt{p_9}), []\mathtt{F_7} \vdash (\Delta_{10}, \mathtt{p_9}), \mathtt{F_6}}\ AT
  \qquad
  \cfrac{}{\bullet\mathtt{h_8}: ((\Delta_{11}, \mathtt{p_9}), []\mathtt{F_7}), \mathtt{F_6} \vdash \Delta_{10}, \mathtt{p_9}}\ I
}{
  \cfrac{- : (\Delta_{11}, \mathtt{p_9}), []\mathtt{F_7} \vdash \Delta_{10}, \mathtt{p_9}}
       {- : \Delta_{11}, \mathtt{p_9}, []\mathtt{F_7} \vdash \Delta_{10}, \mathtt{p_9}}\ I
}\ Cut \quad \to
$$

- Case rule $\top_L$

$$
\cfrac{
  \cfrac{\mathtt{h_3}: \mathtt{F_7}, \Delta_6, []\mathtt{F_7} \vdash \top, \Delta_9}
       {\bullet\mathtt{h_3}: \Delta_6, []\mathtt{F_7} \vdash \Delta_9, \top}\ AT
  \qquad
  \cfrac{\mathtt{h_8}: \Delta_6, []\mathtt{F_7} \vdash \Delta_9}
       {\bullet\mathtt{h_8}: (\Delta_6, []\mathtt{F_7}), \top \vdash \Delta_9}\ \top_L
}{
  \cfrac{- : \Delta_6, []\mathtt{F_7} \vdash \Delta_9}
       {- : \Delta_6, []\mathtt{F_7} \vdash \Delta_9}\ ax/W
}\ Cut \quad \to
$$

$$
\cfrac{
  \cfrac{\mathtt{h_3}: \mathtt{F_7}, (\top, \Delta_{10}), []\mathtt{F_7} \vdash \mathtt{F_6}, \Delta_9}
       {\bullet\mathtt{h_3}: (\top, \Delta_{10}), []\mathtt{F_7} \vdash \Delta_9, \mathtt{F_6}}\ AT
  \qquad
  \cfrac{\mathtt{h_8}: \mathtt{F_6}, \Delta_{10}, []\mathtt{F_7} \vdash \Delta_9}
       {\bullet\mathtt{h_8}: ((\top, \Delta_{10}), []\mathtt{F_7}), \mathtt{F_6} \vdash \Delta_9}\ \top_L
}{
  \cfrac{- : (\top, \Delta_{10}), []\mathtt{F_7} \vdash \Delta_9}{}\ \to
}\ Cut
$$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h_3}: \top, \Delta_{10}, []\mathtt{F_7} \vdash \Delta_9, \mathtt{F_6}}\ ax/W
  \qquad
  \cfrac{}{\mathtt{h_8}: \top, \Delta_{10}, \mathtt{F_6}, []\mathtt{F_7} \vdash \Delta_9}\ ax/W
}{- : \top, \Delta_{10}, []\mathtt{F_7} \vdash \Delta_9}\ hCut
$$

6.12   Status of $\perp_L$:   OK

- Case rule $\to_R$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h_3}: \perp, \Delta_6 \vdash (\Delta_8, \mathtt{F_9} \to \mathtt{F_{10}}), \mathtt{F_5}}\ \perp_L
  \qquad
  \cfrac{\mathtt{h_7}: \perp, \mathtt{F_5}, \mathtt{F_9}, \Delta_6 \vdash \mathtt{F_{10}}, \Delta_8}
       {\bullet\mathtt{h_7}: (\perp, \Delta_6), \mathtt{F_5} \vdash \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}}}\ \to_R
}{
  \cfrac{- : \perp, \Delta_6 \vdash \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}}}
       {- : \perp, \Delta_6 \vdash \Delta_8, \mathtt{F_9} \to \mathtt{F_{10}}}\ \perp_L
}\ Cut \quad \to
$$

- Case rule $\wedge_R$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h_3}: \perp, \Delta_6 \vdash (\Delta_8, \mathtt{F_9} \wedge \mathtt{F_{10}}), \mathtt{F_5}}\ \perp_L
  \qquad
  \cfrac{\mathtt{h_7}: \perp, \mathtt{F_5}, \Delta_6 \vdash \mathtt{F_9}, \Delta_8 \quad \mathtt{h_7}: \perp, \mathtt{F_5}, \Delta_6 \vdash \mathtt{F_{10}}, \Delta_8}
       {\bullet\mathtt{h_7}: (\perp, \Delta_6), \mathtt{F_5} \vdash \Delta_8, \mathtt{F_9} \wedge \mathtt{F_{10}}}\ \wedge_R
}{
  \cfrac{- : \perp, \Delta_6 \vdash \Delta_8, \mathtt{F_9} \wedge \mathtt{F_{10}}}
       {- : \perp, \Delta_6 \vdash \Delta_8, \mathtt{F_9} \wedge \mathtt{F_{10}}}\ \perp_L
}\ Cut \quad \to
$$

- Case rule $\vee_R$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h_3}: \perp, \Delta_6 \vdash (\Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}), \mathtt{F_5}}\ \perp_L
  \qquad
  \cfrac{\mathtt{h_7}: \perp, \mathtt{F_5}, \Delta_6 \vdash \mathtt{F_9}, \mathtt{F_{10}}, \Delta_8}
       {\bullet\mathtt{h_7}: (\perp, \Delta_6), \mathtt{F_5} \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}}\ \vee_R
}{
  \cfrac{- : \perp, \Delta_6 \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}}
       {- : \perp, \Delta_6 \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}}\ \perp_L
}\ Cut \quad \to
$$

- Case rule $\perp_R$

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$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_6 \vdash (\bot, \Delta_8), \mathtt{F}_5} \ {}^{\bot_L} \quad \cfrac{\mathtt{h}_7 : \bot, \mathtt{F}_5, \Delta_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : (\bot, \Delta_6), \mathtt{F}_5 \vdash \bot, \Delta_8} \ {}^{\bot_R}}{\cfrac{- : \bot, \Delta_6 \vdash \bot, \Delta_8}{\cfrac{\rightarrow}{- : \bot, \Delta_6 \vdash \bot, \Delta_8} \ {}^{\bot_L}}} \ {}^{\text{Cut}}$$

- Case rule $\top_R$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_6 \vdash (\top, \Delta_8), \mathtt{F}_5} \ {}^{\bot_L} \quad \cfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_6), \mathtt{F}_5 \vdash \top, \Delta_8} \ {}^{\top_R}}{\cfrac{- : \bot, \Delta_6 \vdash \top, \Delta_8}{\cfrac{\rightarrow}{- : \bot, \Delta_6 \vdash \top, \Delta_8} \ {}^{\top_R}}} \ {}^{\text{Cut}}$$

- Case rule K

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_8), \Box\mathtt{F}_5} \ {}^{\bot_L} \quad \cfrac{\mathtt{h}_6 : unbox(\Box\Gamma_9), unbox(\Box\mathtt{F}_5) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\bot, \Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_5 \vdash \Delta_7, []\mathtt{F}_8} \ {}^{K}}{\cfrac{- : \bot, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_8}{\cfrac{\rightarrow}{- : \bot, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_8} \ {}^{\bot_L}}} \ {}^{\text{Cut}}$$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Box\Gamma_7, \Delta_{10} \vdash (\Delta_8, []\mathtt{F}_9), \mathtt{F}_5} \ {}^{\bot_L} \quad \cfrac{\mathtt{h}_6 : unbox(\Box\Gamma_7) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : (\bot, \Box\Gamma_7, \Delta_{10}), \mathtt{F}_5 \vdash \Delta_8, []\mathtt{F}_9} \ {}^{K}}{\cfrac{- : \bot, \Box\Gamma_7, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_9}{\cfrac{\rightarrow}{- : \bot, \Delta_{10}, \Box\Gamma_7 \vdash \Delta_8, []\mathtt{F}_9} \ {}^{\bot_L}}} \ {}^{\text{Cut}}$$

- Case rule $A45$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_7, \Delta_8, []\mathtt{F}_9), \Box\mathtt{F}_5} \ {}^{\bot_L} \quad \cfrac{\mathtt{h}_6 : \Box\Gamma_{10}, \Box\mathtt{F}_5 \vdash \Box\Gamma_7, \mathtt{F}_9}{\bullet \mathtt{h}_6 : (\bot, \Box\Gamma_{10}, \Delta_{11}), \Box\mathtt{F}_5 \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_9} \ {}^{A45}}{\cfrac{- : \bot, \Box\Gamma_{10}, \Delta_{11} \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_9}{\cfrac{\rightarrow}{- : \bot, \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_8, \Box\Gamma_7, []\mathtt{F}_9} \ {}^{\bot_L}}} \ {}^{\text{Cut}}$$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Box\Gamma_7, \Delta_{11} \vdash (\Box\Gamma_8, \Delta_9, []\mathtt{F}_{10}), \mathtt{F}_5} \ {}^{\bot_L} \quad \cfrac{\mathtt{h}_6 : \Box\Gamma_7 \vdash \Box\Gamma_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : (\bot, \Box\Gamma_7, \Delta_{11}), \mathtt{F}_5 \vdash \Box\Gamma_8, \Delta_9, []\mathtt{F}_{10}} \ {}^{A45}}{\cfrac{- : \bot, \Box\Gamma_7, \Delta_{11} \vdash \Box\Gamma_8, \Delta_9, []\mathtt{F}_{10}}{\cfrac{\rightarrow}{- : \bot, \Delta_{11}, \Box\Gamma_7 \vdash \Delta_9, \Box\Gamma_8, []\mathtt{F}_{10}} \ {}^{\bot_L}}} \ {}^{\text{Cut}}$$

- Case rule $\rightarrow_L$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8} \ {}^{\bot_L} \quad \cfrac{\mathtt{h}_6 : \bot, \Delta_5 \vdash \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 : \bot, \mathtt{F}_8, \Delta_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9} \ {}^{\rightarrow_L}}{\cfrac{- : \bot, \Delta_5 \vdash \Delta_9}{\cfrac{\rightarrow}{- : \bot, \Delta_5 \vdash \Delta_9} \ {}^{\bot_L}}} \ {}^{\text{Cut}}$$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5} \ {}^{\bot_L} \quad \cfrac{\mathtt{h}_6 : \bot, \mathtt{F}_5, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 : \bot, \mathtt{F}_5, \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9} \ {}^{\rightarrow_L}}{\cfrac{- : \bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9}{\cfrac{\rightarrow}{- : \bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9} \ {}^{\bot_L}}} \ {}^{\text{Cut}}$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8} \ {}^{\bot_L} \quad \cfrac{\mathtt{h}_6 : \bot, \mathtt{F}_7, \mathtt{F}_8, \Delta_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9} \ {}^{\wedge_L}}{\cfrac{- : \bot, \Delta_5 \vdash \Delta_9}{\cfrac{\rightarrow}{- : \bot, \Delta_5 \vdash \Delta_9} \ {}^{\bot_L}}} \ {}^{\text{Cut}}$$

$$\cfrac{\cfrac{\overline{\bullet h_3 : \bot, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5}}{} \perp_L \quad \cfrac{h_6 : \bot, F_5, F_7, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\bot, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \wedge_L}{\cfrac{- : \bot, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9}{\cfrac{\rightarrow}{- : \bot, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \perp_L}} \text{Cut}$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{\overline{\bullet h_3 : \bot, \Delta_5 \vdash \Delta_9, F_7 \vee F_8}}{} \perp_L \quad \cfrac{h_6 : \bot, F_7, \Delta_5 \vdash \Delta_9 \quad h_6 : \bot, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\bot, \Delta_5), F_7 \vee F_8 \vdash \Delta_9} \vee_L}{\cfrac{- : \bot, \Delta_5 \vdash \Delta_9}{\cfrac{\rightarrow}{- : \bot, \Delta_5 \vdash \Delta_9} \perp_L}} \text{Cut}$$

$$\cfrac{\cfrac{\overline{\bullet h_3 : \bot, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5}}{} \perp_L \quad \cfrac{h_6 : \bot, F_5, F_7, \Delta_{10} \vdash \Delta_9 \quad h_6 : \bot, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\bot, \Delta_{10}, F_7 \vee F_8), F_5 \vdash \Delta_9} \vee_L}{\cfrac{- : \bot, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9}{\cfrac{\rightarrow}{- : \bot, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \perp_L}} \text{Cut}$$

- Case rule AT

$$\cfrac{\cfrac{\overline{\bullet h_3 : \bot, \Delta_5 \vdash \Delta_8, []F_7}}{} \perp_L \quad \cfrac{h_6 : \bot, F_7, \Delta_5, []F_7 \vdash \Delta_8}{\bullet h_6 : (\bot, \Delta_5), []F_7 \vdash \Delta_8} AT}{\cfrac{- : \bot, \Delta_5 \vdash \Delta_8}{\cfrac{\rightarrow}{- : \bot, \Delta_5 \vdash \Delta_8} \perp_L}} \text{Cut}$$

$$\cfrac{\cfrac{\overline{\bullet h_3 : \bot, \Delta_9, []F_7 \vdash \Delta_8, F_5}}{} \perp_L \quad \cfrac{h_6 : \bot, F_5, F_7, \Delta_9, []F_7 \vdash \Delta_8}{\bullet h_6 : (\bot, \Delta_9, []F_7), F_5 \vdash \Delta_8} AT}{\cfrac{- : \bot, \Delta_9, []F_7 \vdash \Delta_8}{\cfrac{\rightarrow}{- : \bot, \Delta_9, []F_7 \vdash \Delta_8} \perp_L}} \text{Cut}$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{\overline{\bullet h_3 : \bot, \Delta_6 \vdash \Delta_8, F_5}}{} \perp_L \quad \cfrac{\overline{\bullet h_7 : (\bot, \Delta_6), F_5 \vdash \Delta_8}}{} \perp_L}{\cfrac{- : \bot, \Delta_6 \vdash \Delta_8}{\cfrac{\rightarrow}{- : \bot, \Delta_6 \vdash \Delta_8} \perp_L}} \text{Cut}$$

- Case rule I

$$\cfrac{\cfrac{\overline{\bullet h_3 : \bot, \Delta_5 \vdash (\Delta_8, p_7), p_7}}{} \perp_L \quad \cfrac{\overline{\bullet h_6 : (\bot, \Delta_5), p_7 \vdash \Delta_8, p_7}}{} I}{\cfrac{- : \bot, \Delta_5 \vdash \Delta_8, p_7}{\cfrac{\rightarrow}{- : \bot, \Delta_5 \vdash \Delta_8, p_7} \perp_L}} \text{Cut}$$

$$\cfrac{\cfrac{\overline{\bullet h_3 : \bot, \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5}}{} \perp_L \quad \cfrac{\overline{\bullet h_6 : (\bot, \Delta_9, p_7), F_5 \vdash \Delta_8, p_7}}{} I}{\cfrac{- : \bot, \Delta_9, p_7 \vdash \Delta_8, p_7}{\cfrac{\rightarrow}{- : \bot, \Delta_9, p_7 \vdash \Delta_8, p_7} \perp_L}} \text{Cut}$$

- Case rule $\top_L$

$$\cfrac{\cfrac{\overline{\bullet h_3 : \bot, \Delta_5 \vdash \Delta_7, \top}}{} \perp_L \quad \cfrac{h_6 : \bot, \Delta_5 \vdash \Delta_7}{\bullet h_6 : (\bot, \Delta_5), \top \vdash \Delta_7} \top_L}{\cfrac{- : \bot, \Delta_5 \vdash \Delta_7}{\cfrac{\rightarrow}{- : \bot, \Delta_5 \vdash \Delta_7} \perp_L}} \text{Cut}$$

$$\cfrac{\cfrac{}{\bullet h_3 : \bot, \top, \Delta_8 \vdash \Delta_7, F_5} \; \bot_L \qquad \cfrac{h_6 : \bot, F_5, \Delta_8 \vdash \Delta_7}{\bullet h_6 : (\bot, \top, \Delta_8), F_5 \vdash \Delta_7} \; \top_L}{\cfrac{- : \bot, \top, \Delta_8 \vdash \Delta_7}{\cfrac{\to}{- : \bot, \top, \Delta_8 \vdash \Delta_7} \; \bot_L} } \; \text{Cut}$$

6.13 Status of I: `OK`

- Case rule $\to_R$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \to F_{10}), p_6} \; I \qquad \cfrac{h_7 : F_9, \Delta_5, p_6, p_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \to F_{10}} \; \to_R}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \to F_{10}} \; \text{Cut}$$
$$\to$$
$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10}, p_6} \; I \qquad \cfrac{}{h_7 : \Delta_5, F_9, p_6, p_6 \vdash \Delta_8, F_{10}} \; \text{ax/W}}{- : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10}} \; \text{hCut}}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \to F_{10}} \; \to_R$$

$$\cfrac{\cfrac{}{\bullet h_2 : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \to F_{11}), p_8), F_6} \; I \qquad \cfrac{h_9 : F_6, F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \to F_{11}), p_8} \; \to_R}{\cfrac{- : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \to F_{11}), p_8}{\cfrac{\to}{- : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \to F_{11}} \; I}} \; \text{Cut}$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \wedge F_{10}), p_6} \; I \qquad \cfrac{h_7 : \Delta_5, p_6, p_6 \vdash F_9, \Delta_8 \quad h_7 : \Delta_5, p_6, p_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \; \wedge_R}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \; \text{Cut}$$
$$\to$$
$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_9, p_6} \; I \quad \cfrac{}{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_9} \; \text{ax/W}}{- : \Delta_5, p_6 \vdash \Delta_8, F_9} \; \text{hCut} \qquad \cfrac{\cfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_{10}, p_6} \; I \quad \cfrac{}{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_{10}} \; \text{ax/W}}{- : \Delta_5, p_6 \vdash \Delta_8, F_{10}} \; \text{hCut}}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \; \wedge_R$$

$$\cfrac{\cfrac{}{\bullet h_2 : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), p_8), F_6} \; I \qquad \cfrac{h_9 : F_6, \Delta_7, p_8 \vdash F_{10}, \Delta_{12}, p_8 \quad h_9 : F_6, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8} \; \wedge_R}{\cfrac{- : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8}{\cfrac{\to}{- : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \wedge F_{11}} \; I}} \; \text{Cut}$$

- Case rule $\vee_R$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \vee F_{10}), p_6} \; I \qquad \cfrac{h_7 : \Delta_5, p_6, p_6 \vdash F_9, F_{10}, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \vee F_{10}} \; \vee_R}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}} \; \text{Cut}$$
$$\to$$
$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9, p_6} \; I \qquad \cfrac{}{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_{10}, F_9} \; \text{ax/W}}{- : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9} \; \text{hCut}}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}} \; \vee_R$$

$$\cfrac{\cfrac{}{\bullet h_2 : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \vee F_{11}), p_8), F_6} \; I \qquad \cfrac{h_9 : F_6, \Delta_7, p_8 \vdash F_{10}, F_{11}, \Delta_{12}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8} \; \vee_R}{\cfrac{- : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8}{\cfrac{\to}{- : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \vee F_{11}} \; I}} \; \text{Cut}$$

- Case rule $\bot_R$

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$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\bot, \Delta_8), p_6}\ I \qquad \dfrac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \bot, \Delta_8}\ \bot_R}{- : \Delta_5, p_6 \vdash \bot, \Delta_8}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash \bot, \Delta_8, p_6}\ I \qquad \dfrac{}{h_7 : \Delta_5, p_6, p_6 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_5, p_6 \vdash \bot, \Delta_8}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7, p_8 \vdash ((\bot, \Delta_{10}), p_8), F_6}\ I \qquad \dfrac{h_9 : F_6, \Delta_7, p_8 \vdash \Delta_{10}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\bot, \Delta_{10}), p_8}\ \bot_R}{- : \Delta_7, p_8 \vdash (\bot, \Delta_{10}), p_8}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_7, p_8 \vdash \bot, \Delta_{10}, p_8}\ I$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\top, \Delta_8), p_6}\ I \qquad \dfrac{}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \top, \Delta_8}\ \top_R}{- : \Delta_5, p_6 \vdash \top, \Delta_8}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_5, p_6 \vdash \top, \Delta_8}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7, p_8 \vdash ((\top, \Delta_{10}), p_8), F_6}\ I \qquad \dfrac{}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\top, \Delta_{10}), p_8}\ \top_R}{- : \Delta_7, p_8 \vdash (\top, \Delta_{10}), p_8}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_7, p_8 \vdash \top, \Delta_{10}, p_8}\ \top_R$$

- Case rule K

$$\dfrac{\dfrac{}{\bullet h_1 : (\Box\Gamma_7, \Delta_{10}), p_5 \vdash (\Delta_8, []F_9), p_5}\ I \qquad \dfrac{h_6 : unbox(\Box\Gamma_7) \vdash F_9}{\bullet h_6 : ((\Box\Gamma_7, \Delta_{10}), p_5), p_5 \vdash \Delta_8, []F_9}\ K}{- : (\Box\Gamma_7, \Delta_{10}), p_5 \vdash \Delta_8, []F_9}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_7) \vdash F_9}\ \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_7, p_5 \vdash \Delta_8, []F_9}\ K$$

$$\dfrac{\dfrac{}{\bullet h_2 : (\Box\Gamma_{11}, \Delta_{12}), p_7 \vdash ((\Delta_{10}, []F_9), p_7), \Box F_6}\ I \qquad \dfrac{h_8 : unbox(\Box\Gamma_{11}), unbox(\Box F_6) \vdash F_9}{\bullet h_8 : ((\Box\Gamma_{11}, \Delta_{12}), p_7), \Box F_6 \vdash (\Delta_{10}, []F_9), p_7}\ K}{- : (\Box\Gamma_{11}, \Delta_{12}), p_7 \vdash (\Delta_{10}, []F_9), p_7}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_{12}, \Box\Gamma_{11}, p_7 \vdash \Delta_{10}, p_7, []F_9}\ I$$

$$\dfrac{\dfrac{}{\bullet h_2 : (\Box\Gamma_9, \Delta_{12}), p_7 \vdash ((\Delta_{11}, []F_{10}), p_7), F_6}\ I \qquad \dfrac{h_8 : unbox(\Box\Gamma_9) \vdash F_{10}}{\bullet h_8 : ((\Box\Gamma_9, \Delta_{12}), p_7), F_6 \vdash (\Delta_{11}, []F_{10}), p_7}\ K}{- : (\Box\Gamma_9, \Delta_{12}), p_7 \vdash (\Delta_{11}, []F_{10}), p_7}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_{12}, \Box\Gamma_9, p_7 \vdash \Delta_{11}, p_7, []F_{10}}\ I$$

- Case rule $A45$

$$\dfrac{\dfrac{}{\bullet h_1 : (\Box\Gamma_7, \Delta_{11}), p_5 \vdash (\Box\Gamma_8, \Delta_9, []F_{10}), p_5}\ I \qquad \dfrac{h_6 : \Box\Gamma_7 \vdash \Box\Gamma_8, F_{10}}{\bullet h_6 : ((\Box\Gamma_7, \Delta_{11}), p_5), p_5 \vdash \Box\Gamma_8, \Delta_9, []F_{10}}\ A45}{- : (\Box\Gamma_7, \Delta_{11}), p_5 \vdash \Box\Gamma_8, \Delta_9, []F_{10}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash F_{10}, \Box\Gamma_8}\ \mathtt{ax/W}}{- : \Delta_{11}, \Box\Gamma_7, p_5 \vdash \Delta_9, \Box\Gamma_8, []F_{10}}\ A45$$

$$\dfrac{\dfrac{}{\bullet h_2 : (\Box\Gamma_{12}, \Delta_{13}), p_7 \vdash ((\Box\Gamma_9, \Delta_{11}, []F_{10}), p_7), \Box F_6}\ I \qquad \dfrac{h_8 : \Box\Gamma_{12}, \Box F_6 \vdash \Box\Gamma_9, F_{10}}{\bullet h_8 : ((\Box\Gamma_{12}, \Delta_{13}), p_7), \Box F_6 \vdash (\Box\Gamma_9, \Delta_{11}, []F_{10}), p_7}\ A45}{- : (\Box\Gamma_{12}, \Delta_{13}), p_7 \vdash (\Box\Gamma_9, \Delta_{11}, []F_{10}), p_7}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_{13}, \Box\Gamma_{12}, p_7 \vdash \Delta_{11}, \Box\Gamma_9, p_7, []F_{10}}\ I$$

$$\cfrac{\cfrac{}{\bullet h_2 : (\Box\Gamma_9, \Delta_{13}), p_7 \vdash ((\Box\Gamma_{10}, \Delta_{12}, []F_{11}), p_7), F_6}\ I \quad \cfrac{h_8 : \Box\Gamma_9 \vdash \Box\Gamma_{10}, F_{11}}{\bullet h_8 : ((\Box\Gamma_9, \Delta_{13}), p_7), F_6 \vdash (\Box\Gamma_{10}, \Delta_{12}, []F_{11}), p_7}\ A45}{\cfrac{\cfrac{- : (\Box\Gamma_9, \Delta_{13}), p_7 \vdash (\Box\Gamma_{10}, \Delta_{12}, []F_{11}), p_7}{\to}}{- : \Delta_{13}, \Box\Gamma_9, p_7 \vdash \Delta_{12}, \Box\Gamma_{10}, p_7, []F_{11}}\ I}\ Cut$$

- Case rule $\to_L$

$$\cfrac{\cfrac{}{\bullet h_1 : (\Delta_{10}, F_7 \to F_8), p_5 \vdash \Delta_9, p_5}\ I \quad \cfrac{h_6 : \Delta_{10}, p_5, p_5 \vdash F_7, \Delta_9 \quad h_6 : F_8, \Delta_{10}, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \to F_8), p_5), p_5 \vdash \Delta_9}\ \to_L}{\cfrac{- : (\Delta_{10}, F_7 \to F_8), p_5 \vdash \Delta_9}{\to}}\ Cut$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_{10}, p_5 \vdash \Delta_9, F_7, p_5}\ I \quad \cfrac{h_6 : \Delta_{10}, p_5, p_5 \vdash \Delta_9, F_7}{}\ ax/W}{- : \Delta_{10}, p_5 \vdash \Delta_9, F_7}\ hCut \quad \cfrac{\cfrac{}{\bullet h_1 : \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5}\ I \quad \cfrac{h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9}{}\ ax/W}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9}\ hCut}{- : \Delta_{10}, p_5, F_7 \to F_8 \vdash \Delta_9}\ \to_L$$

$$\cfrac{\cfrac{}{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \to F_{11}}\ I \quad \cfrac{h_9 : \Delta_6, p_7 \vdash F_{10}, \Delta_8, p_7 \quad h_9 : F_{11}, \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \to F_{11} \vdash \Delta_8, p_7}\ \to_L}{\cfrac{\cfrac{- : \Delta_6, p_7 \vdash \Delta_8, p_7}{\to}}{- : \Delta_6, p_7 \vdash \Delta_8, p_7}\ I}\ Cut$$

$$\cfrac{\cfrac{}{\bullet h_2 : (\Delta_{12}, F_{10} \to F_{11}), p_7 \vdash (\Delta_8, p_7), F_6}\ I \quad \cfrac{h_9 : F_6, \Delta_{12}, p_7 \vdash F_{10}, \Delta_8, p_7 \quad h_9 : F_6, F_{11}, \Delta_{12}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \to F_{11}), p_7), F_6 \vdash \Delta_8, p_7}\ \to_L}{\cfrac{\cfrac{- : (\Delta_{12}, F_{10} \to F_{11}), p_7 \vdash \Delta_8, p_7}{\to}}{- : \Delta_{12}, p_7, F_{10} \to F_{11} \vdash \Delta_8, p_7}\ I}\ Cut$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{}{\bullet h_1 : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9, p_5}\ I \quad \cfrac{h_6 : F_7, F_8, \Delta_{10}, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \wedge F_8), p_5), p_5 \vdash \Delta_9}\ \wedge_L}{\cfrac{- : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9}{\to}}\ Cut$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_{10}, F_7, F_8, p_5 \vdash \Delta_9, p_5}\ I \quad \cfrac{h_6 : \Delta_{10}, F_7, F_8, p_5, p_5 \vdash \Delta_9}{}\ ax/W}{- : \Delta_{10}, F_7, F_8, p_5 \vdash \Delta_9}\ hCut}{- : \Delta_{10}, p_5, F_7 \wedge F_8 \vdash \Delta_9}\ \wedge_L$$

$$\cfrac{\cfrac{}{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \wedge F_{11}}\ I \quad \cfrac{h_9 : F_{10}, F_{11}, \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \wedge F_{11} \vdash \Delta_8, p_7}\ \wedge_L}{\cfrac{\cfrac{- : \Delta_6, p_7 \vdash \Delta_8, p_7}{\to}}{- : \Delta_6, p_7 \vdash \Delta_8, p_7}\ I}\ Cut$$

$$\cfrac{\cfrac{}{\bullet h_2 : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash (\Delta_8, p_7), F_6}\ I \quad \cfrac{h_9 : F_6, F_{10}, F_{11}, \Delta_{12}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \wedge F_{11}), p_7), F_6 \vdash \Delta_8, p_7}\ \wedge_L}{\cfrac{\cfrac{- : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash \Delta_8, p_7}{\to}}{- : \Delta_{12}, p_7, F_{10} \wedge F_{11} \vdash \Delta_8, p_7}\ I}\ Cut$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{}{\bullet h_1 : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9, p_5}\ I \quad \cfrac{h_6 : F_7, \Delta_{10}, p_5, p_5 \vdash \Delta_9 \quad h_6 : F_8, \Delta_{10}, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \vee F_8), p_5), p_5 \vdash \Delta_9}\ \vee_L}{\cfrac{- : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9}{\to}}\ Cut$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_{10}, F_7, p_5 \vdash \Delta_9, p_5}\ I \quad \cfrac{h_6 : \Delta_{10}, F_7, p_5, p_5 \vdash \Delta_9}{}\ ax/W}{- : \Delta_{10}, F_7, p_5 \vdash \Delta_9}\ hCut \quad \cfrac{\cfrac{}{\bullet h_1 : \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5}\ I \quad \cfrac{h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9}{}\ ax/W}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9}\ hCut}{- : \Delta_{10}, p_5, F_7 \vee F_8 \vdash \Delta_9}\ \vee_L$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \vee F_{11}} \; I \quad \cfrac{h_9 : F_{10}, \Delta_6, p_7 \vdash \Delta_8, p_7 \quad h_9 : F_{11}, \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \vee F_{11} \vdash \Delta_8, p_7} \; \vee_L}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; \mathtt{Cut}}{\cfrac{\to}{- : \Delta_6, p_7 \vdash \Delta_8, p_7}} \; I$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 : (\Delta_{12}, F_{10} \vee F_{11}), p_7 \vdash (\Delta_8, p_7), F_6} \; I \quad \cfrac{h_9 : F_6, F_{10}, \Delta_{12}, p_7 \vdash \Delta_8, p_7 \quad h_9 : F_6, F_{11}, \Delta_{12}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \vee F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \; \vee_L}{- : (\Delta_{12}, F_{10} \vee F_{11}), p_7 \vdash \Delta_8, p_7} \; \mathtt{Cut}}{\cfrac{\to}{- : \Delta_{12}, p_7, F_{10} \vee F_{11} \vdash \Delta_8, p_7}} \; I$$

- Case rule AT

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : (\Delta_9, []F_7), p_5 \vdash \Delta_8, p_5} \; I \quad \cfrac{h_6 : F_7, \Delta_9, p_5, p_5, []F_7 \vdash \Delta_8}{\bullet h_6 : ((\Delta_9, []F_7), p_5), p_5 \vdash \Delta_8} \; AT}{- : (\Delta_9, []F_7), p_5 \vdash \Delta_8} \; \mathtt{Cut}}{\cfrac{\cfrac{\to}{\cfrac{}{\bullet h_1 : \Delta_9, F_7, p_5, []F_7 \vdash \Delta_8, p_5} \; I \quad \cfrac{h_6 : \Delta_9, F_7, p_5, p_5, []F_7 \vdash \Delta_8}{} \; \mathtt{ax/W}}{\cfrac{- : \Delta_9, F_7, p_5, []F_7 \vdash \Delta_8}{- : \Delta_9, p_5, []F_7 \vdash \Delta_8} \; AT} \; \mathtt{hCut}}{}}$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), []F_{10}} \; I \quad \cfrac{h_9 : F_{10}, \Delta_6, p_7, []F_{10} \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), []F_{10} \vdash \Delta_8, p_7} \; AT}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; \mathtt{Cut}}{\cfrac{\to}{- : \Delta_6, p_7 \vdash \Delta_8, p_7}} \; I$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 : (\Delta_{11}, []F_{10}), p_7 \vdash (\Delta_8, p_7), F_6} \; I \quad \cfrac{h_9 : F_6, F_{10}, \Delta_{11}, p_7, []F_{10} \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{11}, []F_{10}), p_7), F_6 \vdash \Delta_8, p_7} \; AT}{- : (\Delta_{11}, []F_{10}), p_7 \vdash \Delta_8, p_7} \; \mathtt{Cut}}{\cfrac{\to}{- : \Delta_{11}, p_7, []F_{10} \vdash \Delta_8, p_7}} \; I$$

- Case rule $\bot_L$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : (\bot, \Delta_8), p_5 \vdash \Delta_7, p_5} \; I \quad \cfrac{}{\bullet h_6 : ((\bot, \Delta_8), p_5), p_5 \vdash \Delta_7} \; \bot_L}{- : (\bot, \Delta_8), p_5 \vdash \Delta_7} \; \mathtt{Cut}}{\cfrac{\to}{- : \bot, \Delta_8, p_5 \vdash \Delta_7} \; \bot_L}$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \bot} \; I \quad \cfrac{}{\bullet h_9 : (\Delta_6, p_7), \bot \vdash \Delta_8, p_7} \; \bot_L}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; \mathtt{Cut}}{\cfrac{\to}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; I}$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 : (\bot, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6} \; I \quad \cfrac{}{\bullet h_9 : ((\bot, \Delta_{10}), p_7), F_6 \vdash \Delta_8, p_7} \; \bot_L}{- : (\bot, \Delta_{10}), p_7 \vdash \Delta_8, p_7} \; \mathtt{Cut}}{\cfrac{\to}{- : \bot, \Delta_{10}, p_7 \vdash \Delta_8, p_7} \; \bot_L}$$

- Case rule I

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_5, p_7 \vdash (\Delta_8, p_7), p_7} \; I \quad \cfrac{}{\bullet h_6 : (\Delta_5, p_7), p_7 \vdash \Delta_8, p_7} \; I}{- : \Delta_5, p_7 \vdash \Delta_8, p_7} \; \mathtt{Cut}}{\cfrac{\to}{- : \Delta_5, p_7 \vdash \Delta_8, p_7} \; I}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : (\Delta_9, \mathtt{p}_7), \mathtt{p}_5 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_5} \; I \quad \dfrac{}{\bullet\mathtt{h}_6 : ((\Delta_9, \mathtt{p}_7), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_8, \mathtt{p}_7} \; I}{\dfrac{- : (\Delta_9, \mathtt{p}_7), \mathtt{p}_5 \vdash \Delta_8, \mathtt{p}_7}{\dfrac{\rightarrow}{- : \Delta_9, \mathtt{p}_5, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7} \; I}} \; \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash ((\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7), \mathtt{p}_9} \; I \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_6, \mathtt{p}_7), \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7} \; I}{\dfrac{- : \Delta_6, \mathtt{p}_7 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7}{\dfrac{\rightarrow}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_{10}, \mathtt{p}_7, \mathtt{p}_9} \; I}} \; \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : (\Delta_{11}, \mathtt{p}_9), \mathtt{p}_7 \vdash ((\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7), \mathtt{F}_6} \; I \quad \dfrac{}{\bullet\mathtt{h}_8 : ((\Delta_{11}, \mathtt{p}_9), \mathtt{p}_7), \mathtt{F}_6 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7} \; I}{\dfrac{- : (\Delta_{11}, \mathtt{p}_9), \mathtt{p}_7 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7}{\dfrac{\rightarrow}{- : \Delta_{11}, \mathtt{p}_7, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_7, \mathtt{p}_9} \; I}} \; \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_7, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6} \; I \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_7, \mathtt{p}_9), \mathtt{F}_6 \vdash \Delta_{10}, \mathtt{p}_9} \; I}{\dfrac{- : \Delta_7, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}{\dfrac{\rightarrow}{- : \Delta_7, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9} \; I}} \; \mathtt{Cut}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : (\top, \Delta_8), \mathtt{p}_5 \vdash \Delta_7, \mathtt{p}_5} \; I \quad \dfrac{\mathtt{h}_6 : \Delta_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_7}{\bullet\mathtt{h}_6 : ((\top, \Delta_8), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_7} \; \top_L}{\dfrac{- : (\top, \Delta_8), \mathtt{p}_5 \vdash \Delta_7}{\dfrac{\rightarrow}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_8, \mathtt{p}_5 \vdash \Delta_7, \mathtt{p}_5} \; I \quad \dfrac{\mathtt{h}_6 : \top, \Delta_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_7}{\mathtt{ax/W}}}{- : \top, \Delta_8, \mathtt{p}_5 \vdash \Delta_7} \; \mathtt{hCut}}}} \; \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \top} \; I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet\mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \top \vdash \Delta_8, \mathtt{p}_7} \; \top_L}{\dfrac{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\dfrac{\rightarrow}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7} \; I}} \; \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : (\top, \Delta_{10}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6} \; I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_{10}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet\mathtt{h}_9 : ((\top, \Delta_{10}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7} \; \top_L}{\dfrac{- : (\top, \Delta_{10}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\dfrac{\rightarrow}{- : \top, \Delta_{10}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7} \; I}} \; \mathtt{Cut}$$

6.14   Status of $\top_L$: `OK`

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_6 \vdash \mathtt{F}_5, \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}{\bullet\mathtt{h}_3 : \top, \Delta_6 \vdash (\Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}), \mathtt{F}_5} \; \top_L \quad \dfrac{\mathtt{h}_7 : \top, \mathtt{F}_5, \mathtt{F}_9, \Delta_6 \vdash \mathtt{F}_{10}, \Delta_8}{\bullet\mathtt{h}_7 : (\top, \Delta_6), \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}} \; \rightarrow_R}{\dfrac{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}{\dfrac{\rightarrow}{\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_5, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}{\mathtt{ax/W}} \quad \dfrac{\bullet\mathtt{h}_7 : \top, \Delta_6, \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}{\mathtt{ax/W}}}{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}} \; \mathtt{hCut}}}} \; \mathtt{Cut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_6 \vdash \mathtt{F}_5, \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\bullet\mathtt{h}_3 : \top, \Delta_6 \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_5} \; \top_L \quad \dfrac{\mathtt{h}_7 : \top, \mathtt{F}_5, \Delta_6 \vdash \mathtt{F}_9, \Delta_8 \quad \mathtt{h}_7 : \top, \mathtt{F}_5, \Delta_6 \vdash \mathtt{F}_{10}, \Delta_8}{\bullet\mathtt{h}_7 : (\top, \Delta_6), \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}} \; \wedge_R}{\dfrac{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\dfrac{\rightarrow}{\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_5, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\mathtt{ax/W}} \quad \dfrac{\bullet\mathtt{h}_7 : \top, \Delta_6, \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\mathtt{ax/W}}}{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}} \; \mathtt{hCut}}}} \; \mathtt{Cut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{h_3 : \Delta_6 \vdash F_5, \Delta_8, F_9 \vee F_{10}}{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \vee F_{10}), F_5}\ \top_L \qquad \dfrac{h_7 : \top, F_5, \Delta_6 \vdash F_9, F_{10}, \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \vee F_{10}}\ \vee_R}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10}}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_6 \vdash \Delta_8, F_5, F_9 \vee F_{10}}\ \text{ax/W} \qquad \dfrac{}{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8, F_9 \vee F_{10}}\ \text{ax/W}}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10}}\ \text{hCut}$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{h_3 : \Delta_6 \vdash F_5, \perp, \Delta_8}{\bullet h_3 : \top, \Delta_6 \vdash (\perp, \Delta_8), F_5}\ \top_L \qquad \dfrac{h_7 : \top, F_5, \Delta_6 \vdash \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \perp, \Delta_8}\ \perp_R}{- : \top, \Delta_6 \vdash \perp, \Delta_8}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_6 \vdash \perp, \Delta_8, F_5}\ \text{ax/W} \qquad \dfrac{}{\bullet h_7 : \top, \Delta_6, F_5 \vdash \perp, \Delta_8}\ \text{ax/W}}{- : \top, \Delta_6 \vdash \perp, \Delta_8}\ \text{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{h_3 : \Delta_6 \vdash F_5, \top, \Delta_8}{\bullet h_3 : \top, \Delta_6 \vdash (\top, \Delta_8), F_5}\ \top_L \qquad \dfrac{}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \top, \Delta_8}\ \top_R}{- : \top, \Delta_6 \vdash \top, \Delta_8}\ \text{Cut}$$

$$\to$$

$$\dfrac{}{- : \top, \Delta_6 \vdash \top, \Delta_8}\ \top_R$$

- Case rule K

$$\dfrac{\dfrac{h_3 : \Box\Gamma_9, \Delta_{10} \vdash \Box F_5, \Delta_7, []F_8}{\bullet h_3 : \top, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []F_8), \Box F_5}\ \top_L \quad \dfrac{h_6 : unbox(\Box\Gamma_9), unbox(\Box F_5) \vdash F_8}{\bullet h_6 : (\top, \Box\Gamma_9, \Delta_{10}), \Box F_5 \vdash \Delta_7, []F_8}\ K}{- : \top, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []F_8}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Box F_5, \Delta_7, []F_8}\ \text{ax/W} \quad \dfrac{}{\bullet h_6 : \top, \Box F_5, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []F_8}\ \text{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []F_8}\ \text{hCut}$$

$$\dfrac{\dfrac{h_3 : \Box\Gamma_7, \Delta_{10} \vdash F_5, \Delta_8, []F_9}{\bullet h_3 : \top, \Box\Gamma_7, \Delta_{10} \vdash (\Delta_8, []F_9), F_5}\ \top_L \quad \dfrac{h_6 : unbox(\Box\Gamma_7) \vdash F_9}{\bullet h_6 : (\top, \Box\Gamma_7, \Delta_{10}), F_5 \vdash \Delta_8, []F_9}\ K}{- : \top, \Box\Gamma_7, \Delta_{10} \vdash \Delta_8, []F_9}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_7) \vdash F_9}\ \text{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_7 \vdash \Delta_8, []F_9}\ K$$

- Case rule $A45$

$$\dfrac{\dfrac{h_3 : \Box\Gamma_{10}, \Delta_{11} \vdash \Box F_5, \Box\Gamma_7, \Delta_8, []F_9}{\bullet h_3 : \top, \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_7, \Delta_8, []F_9), \Box F_5}\ \top_L \quad \dfrac{h_6 : \Box\Gamma_{10}, \Box F_5 \vdash \Box\Gamma_7, F_9}{\bullet h_6 : (\top, \Box\Gamma_{10}, \Delta_{11}), \Box F_5 \vdash \Box\Gamma_7, \Delta_8, []F_9}\ A45}{- : \top, \Box\Gamma_{10}, \Delta_{11} \vdash \Box\Gamma_7, \Delta_8, []F_9}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_{11}, \Box\Gamma_{10} \vdash \Box F_5, \Delta_8, \Box\Gamma_7, []F_9}\ \text{ax/W} \quad \dfrac{}{\bullet h_6 : \top, \Box F_5, \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_8, \Box\Gamma_7, []F_9}\ \text{ax/W}}{- : \top, \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_8, \Box\Gamma_7, []F_9}\ \text{hCut}$$

$$\dfrac{\dfrac{h_3 : \Box\Gamma_7, \Delta_{11} \vdash F_5, \Box\Gamma_8, \Delta_9, []F_{10}}{\bullet h_3 : \top, \Box\Gamma_7, \Delta_{11} \vdash (\Box\Gamma_8, \Delta_9, []F_{10}), F_5}\ \top_L \quad \dfrac{h_6 : \Box\Gamma_7 \vdash \Box\Gamma_8, F_{10}}{\bullet h_6 : (\top, \Box\Gamma_7, \Delta_{11}), F_5 \vdash \Box\Gamma_8, \Delta_9, []F_{10}}\ A45}{- : \top, \Box\Gamma_7, \Delta_{11} \vdash \Box\Gamma_8, \Delta_9, []F_{10}}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash F_{10}, \Box\Gamma_8}\ \text{ax/W}}{- : \top, \Delta_{11}, \Box\Gamma_7 \vdash \Delta_9, \Box\Gamma_8, []F_{10}}\ A45$$

- Case rule $\to_L$

$$
\cfrac{
  \cfrac{h_3 : \Delta_5 \vdash F_7 \to F_8, \Delta_9}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \to F_8} \; \top_L
  \qquad
  \cfrac{h_6 : \top, \Delta_5 \vdash F_7, \Delta_9 \quad h_6 : \top, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \to F_8 \vdash \Delta_9} \; \to_L
}{- : \top, \Delta_5 \vdash \Delta_9} \; \text{Cut}
$$

$$
\to
$$

$$
\cfrac{
  \cfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \to F_8} \; \text{ax/W}
  \qquad
  \cfrac{}{\bullet h_6 : \top, \Delta_5, F_7 \to F_8 \vdash \Delta_9} \; \text{ax/W}
}{- : \top, \Delta_5 \vdash \Delta_9} \; \text{hCut}
$$

$$
\cfrac{
  \cfrac{h_3 : \Delta_{10}, F_7 \to F_8 \vdash F_5, \Delta_9}{\bullet h_3 : \top, \Delta_{10}, F_7 \to F_8 \vdash \Delta_9, F_5} \; \top_L
  \qquad
  \cfrac{h_6 : \top, F_5, \Delta_{10} \vdash F_7, \Delta_9 \quad h_6 : \top, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \to F_8), F_5 \vdash \Delta_9} \; \to_L
}{- : \top, \Delta_{10}, F_7 \to F_8 \vdash \Delta_9} \; \text{Cut}
$$

$$
\to
$$

$$
\cfrac{
  \cfrac{}{h_3 : \top, \Delta_{10}, F_7 \to F_8 \vdash \Delta_9, F_5} \; \text{ax/W}
  \qquad
  \cfrac{}{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \to F_8 \vdash \Delta_9} \; \text{ax/W}
}{- : \top, \Delta_{10}, F_7 \to F_8 \vdash \Delta_9} \; \text{hCut}
$$

- Case rule $\wedge_L$

$$
\cfrac{
  \cfrac{h_3 : \Delta_5 \vdash F_7 \wedge F_8, \Delta_9}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8} \; \top_L
  \qquad
  \cfrac{h_6 : \top, F_7, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \wedge F_8 \vdash \Delta_9} \; \wedge_L
}{- : \top, \Delta_5 \vdash \Delta_9} \; \text{Cut}
$$

$$
\to
$$

$$
\cfrac{
  \cfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8} \; \text{ax/W}
  \qquad
  \cfrac{}{\bullet h_6 : \top, \Delta_5, F_7 \wedge F_8 \vdash \Delta_9} \; \text{ax/W}
}{- : \top, \Delta_5 \vdash \Delta_9} \; \text{hCut}
$$

$$
\cfrac{
  \cfrac{h_3 : \Delta_{10}, F_7 \wedge F_8 \vdash F_5, \Delta_9}{\bullet h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \; \top_L
  \qquad
  \cfrac{h_6 : \top, F_5, F_7, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \; \wedge_L
}{- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \; \text{Cut}
$$

$$
\to
$$

$$
\cfrac{
  \cfrac{}{h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \; \text{ax/W}
  \qquad
  \cfrac{}{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \wedge F_8 \vdash \Delta_9} \; \text{ax/W}
}{- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \; \text{hCut}
$$

- Case rule $\vee_L$

$$
\cfrac{
  \cfrac{h_3 : \Delta_5 \vdash F_7 \vee F_8, \Delta_9}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \; \top_L
  \qquad
  \cfrac{h_6 : \top, F_7, \Delta_5 \vdash \Delta_9 \quad h_6 : \top, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \vee F_8 \vdash \Delta_9} \; \vee_L
}{- : \top, \Delta_5 \vdash \Delta_9} \; \text{Cut}
$$

$$
\to
$$

$$
\cfrac{
  \cfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \; \text{ax/W}
  \qquad
  \cfrac{}{\bullet h_6 : \top, \Delta_5, F_7 \vee F_8 \vdash \Delta_9} \; \text{ax/W}
}{- : \top, \Delta_5 \vdash \Delta_9} \; \text{hCut}
$$

$$
\cfrac{
  \cfrac{h_3 : \Delta_{10}, F_7 \vee F_8 \vdash F_5, \Delta_9}{\bullet h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \; \top_L
  \qquad
  \cfrac{h_6 : \top, F_5, F_7, \Delta_{10} \vdash \Delta_9 \quad h_6 : \top, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \vee F_8), F_5 \vdash \Delta_9} \; \vee_L
}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \; \text{Cut}
$$

$$
\to
$$

$$
\cfrac{
  \cfrac{}{h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \; \text{ax/W}
  \qquad
  \cfrac{}{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \vee F_8 \vdash \Delta_9} \; \text{ax/W}
}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \; \text{hCut}
$$

- Case rule AT

$$
\cfrac{
  \cfrac{h_3 : \Delta_5 \vdash []F_7, \Delta_8}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_8, []F_7} \; \top_L
  \qquad
  \cfrac{h_6 : \top, F_7, \Delta_5, []F_7 \vdash \Delta_8}{\bullet h_6 : (\top, \Delta_5), []F_7 \vdash \Delta_8} \; AT
}{- : \top, \Delta_5 \vdash \Delta_8} \; \text{Cut}
$$

$$
\to
$$

$$
\cfrac{
  \cfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_8, []F_7} \; \text{ax/W}
  \qquad
  \cfrac{}{\bullet h_6 : \top, \Delta_5, []F_7 \vdash \Delta_8} \; \text{ax/W}
}{- : \top, \Delta_5 \vdash \Delta_8} \; \text{hCut}
$$

$$\cfrac{\cfrac{\mathtt{h_3}:\Delta_9,[]\mathtt{F_7}\vdash \mathtt{F_5},\Delta_8}{\bullet\mathtt{h_3}:\top,\Delta_9,[]\mathtt{F_7}\vdash \Delta_8,\mathtt{F_5}}\ \top_L \quad \cfrac{\mathtt{h_6}:\top,\mathtt{F_5},\mathtt{F_7},\Delta_9,[]\mathtt{F_7}\vdash \Delta_8}{\bullet\mathtt{h_6}:(\top,\Delta_9,[]\mathtt{F_7}),\mathtt{F_5}\vdash \Delta_8}\ AT}{-:\top,\Delta_9,[]\mathtt{F_7}\vdash \Delta_8}\ \text{Cut}$$
$$\to$$
$$\cfrac{\cfrac{}{\mathtt{h_3}:\top,\Delta_9,[]\mathtt{F_7}\vdash \Delta_8,\mathtt{F_5}}\ \mathtt{ax/W} \quad \cfrac{}{\bullet\mathtt{h_6}:\top,\Delta_9,\mathtt{F_5},[]\mathtt{F_7}\vdash \Delta_8}\ \mathtt{ax/W}}{-:\top,\Delta_9,[]\mathtt{F_7}\vdash \Delta_8}\ \mathtt{hCut}$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{\mathtt{h_3}:\Delta_5\vdash \perp,\Delta_7}{\bullet\mathtt{h_3}:\top,\Delta_5\vdash \Delta_7,\perp}\ \top_L \quad \cfrac{}{\bullet\mathtt{h_6}:(\top,\Delta_5),\perp\vdash \Delta_7}\ \perp_L}{-:\top,\Delta_5\vdash \Delta_7}\ \text{Cut}$$
$$\to$$
$$\cfrac{\cfrac{}{\mathtt{h_3}:\top,\Delta_5\vdash \perp,\Delta_7}\ \mathtt{ax/W} \quad \cfrac{}{\bullet\mathtt{h_6}:\perp,\top,\Delta_5\vdash \Delta_7}\ \perp_L}{-:\top,\Delta_5\vdash \Delta_7}\ \mathtt{hCut}$$

$$\cfrac{\cfrac{\mathtt{h_3}:\perp,\Delta_8\vdash \mathtt{F_5},\Delta_7}{\bullet\mathtt{h_3}:\top,\perp,\Delta_8\vdash \Delta_7,\mathtt{F_5}}\ \top_L \quad \cfrac{}{\bullet\mathtt{h_6}:(\top,\perp,\Delta_8),\mathtt{F_5}\vdash \Delta_7}\ \perp_L}{-:\top,\perp,\Delta_8\vdash \Delta_7}\ \text{Cut}$$
$$\to$$
$$\cfrac{}{-:\perp,\top,\Delta_8\vdash \Delta_7}\ \perp_L$$

- Case rule I

$$\cfrac{\cfrac{\mathtt{h_3}:\Delta_5\vdash \mathtt{p_7},\Delta_8,\mathtt{p_7}}{\bullet\mathtt{h_3}:\top,\Delta_5\vdash (\Delta_8,\mathtt{p_7}),\mathtt{p_7}}\ \top_L \quad \cfrac{}{\bullet\mathtt{h_6}:(\top,\Delta_5),\mathtt{p_7}\vdash \Delta_8,\mathtt{p_7}}\ I}{-:\top,\Delta_5\vdash \Delta_8,\mathtt{p_7}}\ \text{Cut}$$
$$\to$$
$$\cfrac{\cfrac{}{\mathtt{h_3}:\top,\Delta_5\vdash \Delta_8,\mathtt{p_7},\mathtt{p_7}}\ \mathtt{ax/W} \quad \cfrac{}{\bullet\mathtt{h_6}:\top,\Delta_5,\mathtt{p_7}\vdash \Delta_8,\mathtt{p_7}}\ I}{-:\top,\Delta_5\vdash \Delta_8,\mathtt{p_7}}\ \mathtt{hCut}$$

$$\cfrac{\cfrac{\mathtt{h_3}:\Delta_9,\mathtt{p_7}\vdash \mathtt{F_5},\Delta_8,\mathtt{p_7}}{\bullet\mathtt{h_3}:\top,\Delta_9,\mathtt{p_7}\vdash (\Delta_8,\mathtt{p_7}),\mathtt{F_5}}\ \top_L \quad \cfrac{}{\bullet\mathtt{h_6}:(\top,\Delta_9,\mathtt{p_7}),\mathtt{F_5}\vdash \Delta_8,\mathtt{p_7}}\ I}{-:\top,\Delta_9,\mathtt{p_7}\vdash \Delta_8,\mathtt{p_7}}\ \text{Cut}$$
$$\to$$
$$\cfrac{}{-:\top,\Delta_9,\mathtt{p_7}\vdash \Delta_8,\mathtt{p_7}}\ I$$

- Case rule $\top_L$

$$\cfrac{\cfrac{\mathtt{h_3}:\Delta_6\vdash \mathtt{F_5},\Delta_8}{\bullet\mathtt{h_3}:\top,\Delta_6\vdash \Delta_8,\mathtt{F_5}}\ \top_L \quad \cfrac{\mathtt{h_7}:\mathtt{F_5},\Delta_6\vdash \Delta_8}{\bullet\mathtt{h_7}:(\top,\Delta_6),\mathtt{F_5}\vdash \Delta_8}\ \top_L}{-:\top,\Delta_6\vdash \Delta_8}\ \text{Cut}$$
$$\to$$
$$\cfrac{\cfrac{}{\mathtt{h_3}:\top,\Delta_6\vdash \Delta_8,\mathtt{F_5}}\ \mathtt{ax/W} \quad \cfrac{}{\bullet\mathtt{h_7}:\top,\Delta_6,\mathtt{F_5}\vdash \Delta_8}\ \mathtt{ax/W}}{-:\top,\Delta_6\vdash \Delta_8}\ \mathtt{hCut}$$