Multiplicative-additive linear logic (MALL)

July 6, 2022

Contents

1   Measure of derivations

- Case(s) rule **1**

$$\frac{}{\bullet h_1 :\vdash \mathbf{1}} \; \mathbf{1} \quad \rightsquigarrow \quad \frac{}{\bullet\bullet h_1 :\vdash \mathbf{1}} \; \mathbf{1}$$

- Case(s) rule $

$$\frac{h_1 :\vdash F_3, F_4, \Delta_2}{\bullet h_1 :\vdash \Delta_2, F_3 \$ F_4} \; \$ \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{h_1 :\vdash \Delta_2, F_3, F_4} \; \text{ax}}{\bullet h_1 :\vdash \Delta_2, F_3, F_4} \; \text{IH}}{\bullet\bullet h_1 :\vdash \Delta_2, F_3 \$ F_4} \; \$$$

- Case(s) rule &

$$\frac{h_1 :\vdash F_3, \Delta_2 \quad h_1 :\vdash F_4, \Delta_2}{\bullet h_1 :\vdash \Delta_2, F_3 \& F_4} \; \& \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{h_1 :\vdash \Delta_2, F_3} \; \text{ax}}{\bullet h_1 :\vdash \Delta_2, F_3} \; \text{IH} \quad \dfrac{\dfrac{}{h_1 :\vdash \Delta_2, F_4} \; \text{ax}}{\bullet h_1 :\vdash \Delta_2, F_4} \; \text{IH}}{\bullet\bullet h_1 :\vdash \Delta_2, F_3 \& F_4} \; \&$$

- Case(s) rule $\oplus_B$

$$\frac{h_1 :\vdash F_4, \Delta_2}{\bullet h_1 :\vdash \Delta_2, F_3 \oplus F_4} \; \oplus_B \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{h_1 :\vdash \Delta_2, F_4} \; \text{ax}}{\bullet h_1 :\vdash \Delta_2, F_4} \; \text{IH}}{\bullet\bullet h_1 :\vdash \Delta_2, F_3 \oplus F_4} \; \oplus_B$$

- Case(s) rule $\oplus_A$

$$\frac{h_1 :\vdash F_3, \Delta_2}{\bullet h_1 :\vdash \Delta_2, F_3 \oplus F_4} \; \oplus_A \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{h_1 :\vdash \Delta_2, F_3} \; \text{ax}}{\bullet h_1 :\vdash \Delta_2, F_3} \; \text{IH}}{\bullet\bullet h_1 :\vdash \Delta_2, F_3 \oplus F_4} \; \oplus_A$$

- Case(s) rule $\perp$

$$\frac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \perp, \Delta_2} \; \perp \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{h_1 :\vdash \Delta_2} \; \text{ax}}{\bullet h_1 :\vdash \Delta_2} \; \text{IH}}{\bullet\bullet h_1 :\vdash \Delta_2, \perp} \; \perp$$

- Case(s) rule $\top$

$$\frac{}{\bullet h_1 :\vdash \top, \Delta_2} \; \top \quad \rightsquigarrow \quad \frac{}{\bullet\bullet h_1 :\vdash \Delta_2, \top} \; \top$$

- Case(s) rule I

$$\frac{}{\bullet h_1 :\vdash p(n_2), \hat{\ }(n_2)} \; I \quad \rightsquigarrow \quad \frac{}{\bullet\bullet h_1 :\vdash p(n_2), \hat{\ }(n_2)} \; I$$

- Case(s) rule $\otimes$

$$\frac{h_1 :\vdash F_4, \Delta_2 \quad h_1 :\vdash F_5, \Delta_3}{\bullet h_1 :\vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \; \otimes \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{h_1 :\vdash \Delta_2, F_4} \; \text{ax}}{\bullet h_1 :\vdash \Delta_2, F_4} \; \text{IH} \quad \dfrac{\dfrac{}{h_1 :\vdash \Delta_3, F_5} \; \text{ax}}{\bullet h_1 :\vdash \Delta_3, F_5} \; \text{IH}}{\bullet\bullet h_1 :\vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \; \otimes$$

2 Invertibility of Rules

2.1 Status of 1: : Invertible

- Case rule **1**

$$\frac{}{\bullet\mathtt{h_1} \,:\vdash\, \mathtt{1}}\;\mathtt{1} \quad\rightsquigarrow\quad \mathtt{trivial}$$

- Case rule $

- Case rule &

- Case rule $\oplus_B$

- Case rule $\oplus_A$

- Case rule $\bot$

- Case rule $\top$

- Case rule I

- Case rule $\otimes$

2.2 Status of $: : Invertible

- Case rule **1**

- Case rule $

$$\frac{\mathtt{h_3} \,:\vdash\, \mathtt{F_4, F_5, \Delta_6, F_1 \$ F_2}}{\bullet\mathtt{h_3} \,:\vdash\, \mathtt{(\Delta_6, F_1 \$ F_2), F_4 \$ F_5}}\;\$ \quad\rightsquigarrow\quad \frac{\dfrac{}{\mathtt{h_3} \,:\vdash\, \Delta_6, F_1, F_2, F_4, F_5}\;\mathtt{ax/ind}}{\bullet\mathtt{h_3} \,:\vdash\, \Delta_6, F_1, F_2, F_4 \$ F_5}\;\$$$

$$\frac{\mathtt{h_1} \,:\vdash\, \mathtt{F_3, F_4, \Delta_2}}{\bullet\mathtt{h_1} \,:\vdash\, \mathtt{\Delta_2, F_3 \$ F_4}}\;\$ \quad\rightsquigarrow\quad \frac{\dfrac{}{\mathtt{h_1} \,:\vdash\, \Delta_2, F_3, F_4}\;\mathtt{ax}}{\bullet\mathtt{h_1} \,:\vdash\, \Delta_2, F_3, F_4}\;\mathit{height}$$

- Case rule &

$$\frac{\mathtt{h_3} \,:\vdash\, \mathtt{F_4, \Delta_6, F_1 \$ F_2} \quad \mathtt{h_3} \,:\vdash\, \mathtt{F_5, \Delta_6, F_1 \$ F_2}}{\bullet\mathtt{h_3} \,:\vdash\, \mathtt{(\Delta_6, F_1 \$ F_2), F_4 \& F_5}}\;\& \quad\rightsquigarrow\quad \frac{\dfrac{}{\mathtt{h_3} \,:\vdash\, \Delta_6, F_1, F_2, F_4}\;\mathtt{ax/ind} \quad \dfrac{}{\mathtt{h_3} \,:\vdash\, \Delta_6, F_1, F_2, F_5}\;\mathtt{ax/ind}}{\bullet\mathtt{h_3} \,:\vdash\, \Delta_6, F_1, F_2, F_4 \& F_5}\;\&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h_3} \,:\vdash\, \mathtt{F_5, \Delta_6, F_1 \$ F_2}}{\bullet\mathtt{h_3} \,:\vdash\, \mathtt{(\Delta_6, F_1 \$ F_2), F_4 \oplus F_5}}\;\oplus_B \quad\rightsquigarrow\quad \frac{\dfrac{}{\mathtt{h_3} \,:\vdash\, \Delta_6, F_1, F_2, F_5}\;\mathtt{ax/ind}}{\bullet\mathtt{h_3} \,:\vdash\, \Delta_6, F_1, F_2, F_4 \oplus F_5}\;\oplus_B$$

3

- Case rule $\oplus_A$

$$\frac{h_3 :\vdash F_4, \Delta_6, F_1\$F_2}{\bullet h_3 :\vdash (\Delta_6, F_1\$F_2), F_4 \oplus F_5}\ \oplus_A \quad\rightsquigarrow\quad \frac{\overline{h_3 :\vdash \Delta_6, F_1, F_2, F_4}\ ^{ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_1, F_2, F_4 \oplus F_5}\ \oplus_A$$

- Case rule $\perp$

$$\frac{h_3 :\vdash \Delta_4, F_1\$F_2}{\bullet h_3 :\vdash \perp, \Delta_4, F_1\$F_2}\ \perp \quad\rightsquigarrow\quad \frac{\overline{h_3 :\vdash \Delta_4, F_1, F_2}\ ^{ax/ind}}{\bullet h_3 :\vdash \Delta_4, F_1, F_2, \perp}\ \perp$$

- Case rule $\top$

$$\frac{}{\bullet h_3 :\vdash \top, \Delta_4, F_1\$F_2}\ \top \quad\rightsquigarrow\quad \frac{}{\bullet h_3 :\vdash \Delta_4, F_1, F_2, \top}\ \top$$

- Case rule I

- Case rule $\otimes$

$$\frac{h_3 :\vdash F_5, \Delta_7, F_1\$F_2 \quad h_3 :\vdash F_6, \Delta_4}{\bullet h_3 :\vdash (\Delta_7, F_1\$F_2), \Delta_4, F_5 \otimes F_6}\ \otimes \quad\rightsquigarrow\quad \frac{\overline{h_3 :\vdash \Delta_7, F_1, F_2, F_5}\ ^{ax/ind} \quad \overline{h_3 :\vdash \Delta_4, F_6}\ ^{ax}}{\bullet h_3 :\vdash \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6}\ \otimes$$

$$\frac{h_3 :\vdash F_5, \Delta_4 \quad h_3 :\vdash F_6, \Delta_7, F_1\$F_2}{\bullet h_3 :\vdash \Delta_4, (\Delta_7, F_1\$F_2), F_5 \otimes F_6}\ \otimes \quad\rightsquigarrow\quad \frac{\overline{h_3 :\vdash \Delta_4, F_5}\ ^{ax} \quad \overline{h_3 :\vdash \Delta_7, F_1, F_2, F_6}\ ^{ax/ind}}{\bullet h_3 :\vdash \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6}\ \otimes$$

2.3 Status of $\&$: (Left Premise): `Invertible`

- Case rule $\mathbf{1}$

- Case rule $\$$

$$\frac{h_3 :\vdash F_4, F_5, \Delta_6, F_1\&F_2}{\bullet h_3 :\vdash (\Delta_6, F_1\&F_2), F_4\$F_5}\ \$ \quad\rightsquigarrow\quad \frac{\overline{h_3 :\vdash \Delta_6, F_1, F_4, F_5}\ ^{ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_1, F_4\$F_5}\ \$$$

- Case rule $\&$

$$\frac{h_3 :\vdash F_4, \Delta_6, F_1\&F_2 \quad h_3 :\vdash F_5, \Delta_6, F_1\&F_2}{\bullet h_3 :\vdash (\Delta_6, F_1\&F_2), F_4\&F_5}\ \& \quad\rightsquigarrow\quad \frac{\overline{h_3 :\vdash \Delta_6, F_1, F_4}\ ^{ax/ind} \quad \overline{h_3 :\vdash \Delta_6, F_1, F_5}\ ^{ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_1, F_4\&F_5}\ \&$$

$$\frac{h_1 :\vdash F_3, \Delta_2 \quad h_1 :\vdash F_4, \Delta_2}{\bullet h_1 :\vdash \Delta_2, F_3\&F_4}\ \& \quad\rightsquigarrow\quad \frac{\overline{h_1 :\vdash \Delta_2, F_3}\ ^{ax}}{\bullet h_1 :\vdash \Delta_2, F_3}\ _{height}$$

- Case rule $\oplus_B$

$$\frac{h_3 :\vdash F_5, \Delta_6, F_1\&F_2}{\bullet h_3 :\vdash (\Delta_6, F_1\&F_2), F_4 \oplus F_5}\ \oplus_B \quad\rightsquigarrow\quad \frac{\overline{h_3 :\vdash \Delta_6, F_1, F_5}\ ^{ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_1, F_4 \oplus F_5}\ \oplus_B$$

4

- Case rule $\oplus_A$

$$\frac{\mathtt{h_3} :\vdash \mathtt{F_4}, \Delta_6, \mathtt{F_1}\&\mathtt{F_2}}{\bullet\mathtt{h_3} :\vdash (\Delta_6, \mathtt{F_1}\&\mathtt{F_2}), \mathtt{F_4}\oplus\mathtt{F_5}}\ \oplus_A \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_1}, \mathtt{F_4}}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_1}, \mathtt{F_4}\oplus\mathtt{F_5}}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_1}\&\mathtt{F_2}}{\bullet\mathtt{h_3} :\vdash \bot, \Delta_4, \mathtt{F_1}\&\mathtt{F_2}}\ \bot \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_1}}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_1}, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h_3} :\vdash \top, \Delta_4, \mathtt{F_1}\&\mathtt{F_2}}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_1}, \top}\ \top$$

- Case rule I

- Case rule $\otimes$

$$\frac{\mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_7, \mathtt{F_1}\&\mathtt{F_2} \quad \mathtt{h_3} :\vdash \mathtt{F_6}, \Delta_4}{\bullet\mathtt{h_3} :\vdash (\Delta_7, \mathtt{F_1}\&\mathtt{F_2}), \Delta_4, \mathtt{F_5}\otimes\mathtt{F_6}}\ \otimes \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} :\vdash \Delta_7, \mathtt{F_1}, \mathtt{F_5}}\ ^{\mathtt{ax/ind}} \quad \overline{\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_6}}\ ^{\mathtt{ax}}}{\bullet\mathtt{h_3} :\vdash \Delta_4, \Delta_7, \mathtt{F_1}, \mathtt{F_5}\otimes\mathtt{F_6}}\ \otimes$$

$$\frac{\mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_4 \quad \mathtt{h_3} :\vdash \mathtt{F_6}, \Delta_7, \mathtt{F_1}\&\mathtt{F_2}}{\bullet\mathtt{h_3} :\vdash \Delta_4, (\Delta_7, \mathtt{F_1}\&\mathtt{F_2}), \mathtt{F_5}\otimes\mathtt{F_6}}\ \otimes \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_5}}\ ^{\mathtt{ax}} \quad \overline{\mathtt{h_3} :\vdash \Delta_7, \mathtt{F_1}, \mathtt{F_6}}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3} :\vdash \Delta_4, \Delta_7, \mathtt{F_1}, \mathtt{F_5}\otimes\mathtt{F_6}}\ \otimes$$

2.4   Status of $\&$ (Right Premise): :   Invertible

- Case rule $\mathbf{1}$

- Case rule $\$$

$$\frac{\mathtt{h_3} :\vdash \mathtt{F_4}, \mathtt{F_5}, \Delta_6, \mathtt{F_1}\&\mathtt{F_2}}{\bullet\mathtt{h_3} :\vdash (\Delta_6, \mathtt{F_1}\&\mathtt{F_2}), \mathtt{F_4}\$\mathtt{F_5}}\ \$ \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4}, \mathtt{F_5}}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4}\$\mathtt{F_5}}\ \$$$

- Case rule $\&$

$$\frac{\mathtt{h_3} :\vdash \mathtt{F_4}, \Delta_6, \mathtt{F_1}\&\mathtt{F_2} \quad \mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_6, \mathtt{F_1}\&\mathtt{F_2}}{\bullet\mathtt{h_3} :\vdash (\Delta_6, \mathtt{F_1}\&\mathtt{F_2}), \mathtt{F_4}\&\mathtt{F_5}}\ \& \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4}}\ ^{\mathtt{ax/ind}} \quad \overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_5}}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4}\&\mathtt{F_5}}\ \&$$

$$\frac{\mathtt{h_1} :\vdash \mathtt{F_3}, \Delta_2 \quad \mathtt{h_1} :\vdash \mathtt{F_4}, \Delta_2}{\bullet\mathtt{h_1} :\vdash \Delta_2, \mathtt{F_3}\&\mathtt{F_4}}\ \& \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_1} :\vdash \Delta_2, \mathtt{F_4}}\ ^{\mathtt{ax}}}{\bullet\mathtt{h_1} :\vdash \Delta_2, \mathtt{F_4}}\ _{height}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_6, \mathtt{F_1}\&\mathtt{F_2}}{\bullet\mathtt{h_3} :\vdash (\Delta_6, \mathtt{F_1}\&\mathtt{F_2}), \mathtt{F_4}\oplus\mathtt{F_5}}\ \oplus_B \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_5}}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4}\oplus\mathtt{F_5}}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_4}, \Delta_6, \mathtt{F_1 \& F_2}}{\bullet \mathtt{h_3} :\vdash (\Delta_6, \mathtt{F_1 \& F_2}), \mathtt{F_4} \oplus \mathtt{F_5}} \ \oplus_A \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4} \oplus \mathtt{F_5}} \ \oplus_A$$

- Case rule $\perp$

$$\dfrac{\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_1 \& F_2}}{\bullet \mathtt{h_3} :\vdash \perp, \Delta_4, \mathtt{F_1 \& F_2}} \ \perp \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_2}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_3} :\vdash \Delta_4, \mathtt{F_2}, \perp} \ \perp$$

- Case rule $\top$

$$\dfrac{}{\bullet \mathtt{h_3} :\vdash \top, \Delta_4, \mathtt{F_1 \& F_2}} \ \top \qquad \rightsquigarrow \qquad \dfrac{}{\bullet \mathtt{h_3} :\vdash \Delta_4, \mathtt{F_2}, \top} \ \top$$

- Case rule I

- Case rule $\otimes$

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_7, \mathtt{F_1 \& F_2} \quad \mathtt{h_3} :\vdash \mathtt{F_6}, \Delta_4}{\bullet \mathtt{h_3} :\vdash (\Delta_7, \mathtt{F_1 \& F_2}), \Delta_4, \mathtt{F_5} \otimes \mathtt{F_6}} \ \otimes \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_3} :\vdash \Delta_7, \mathtt{F_2}, \mathtt{F_5}} \ \mathtt{ax/ind} \quad \overline{\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_6}} \ \mathtt{ax}}{\bullet \mathtt{h_3} :\vdash \Delta_4, \Delta_7, \mathtt{F_2}, \mathtt{F_5} \otimes \mathtt{F_6}} \ \otimes$$

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_4 \quad \mathtt{h_3} :\vdash \mathtt{F_6}, \Delta_7, \mathtt{F_1 \& F_2}}{\bullet \mathtt{h_3} :\vdash \Delta_4, (\Delta_7, \mathtt{F_1 \& F_2}), \mathtt{F_5} \otimes \mathtt{F_6}} \ \otimes \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_3} :\vdash \Delta_4, \mathtt{F_5}} \ \mathtt{ax} \quad \overline{\mathtt{h_3} :\vdash \Delta_7, \mathtt{F_2}, \mathtt{F_6}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_3} :\vdash \Delta_4, \Delta_7, \mathtt{F_2}, \mathtt{F_5} \otimes \mathtt{F_6}} \ \otimes$$

2.5 Status of $\oplus_B$: : Non invertible

- Case rule $\mathbf{1}$

- Case rule \$

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_4}, \mathtt{F_5}, \Delta_6, \mathtt{F_1} \oplus \mathtt{F_2}}{\bullet \mathtt{h_3} :\vdash (\Delta_6, \mathtt{F_1} \oplus \mathtt{F_2}), \mathtt{F_4} \$ \mathtt{F_5}} \ \$ \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4}, \mathtt{F_5}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4} \$ \mathtt{F_5}} \ \$$$

- Case rule &

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_4}, \Delta_6, \mathtt{F_1} \oplus \mathtt{F_2} \quad \mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_6, \mathtt{F_1} \oplus \mathtt{F_2}}{\bullet \mathtt{h_3} :\vdash (\Delta_6, \mathtt{F_1} \oplus \mathtt{F_2}), \mathtt{F_4} \& \mathtt{F_5}} \ \& \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4}} \ \mathtt{ax/ind} \quad \overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_5}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4} \& \mathtt{F_5}} \ \&$$

- Case rule $\oplus_B$

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_6, \mathtt{F_1} \oplus \mathtt{F_2}}{\bullet \mathtt{h_3} :\vdash (\Delta_6, \mathtt{F_1} \oplus \mathtt{F_2}), \mathtt{F_4} \oplus \mathtt{F_5}} \ \oplus_B \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_5}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_3} :\vdash \Delta_6, \mathtt{F_2}, \mathtt{F_4} \oplus \mathtt{F_5}} \ \oplus_B$$

$$\dfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, \Delta_2}{\bullet \mathtt{h_1} :\vdash \Delta_2, \mathtt{F_3} \oplus \mathtt{F_4}} \ \oplus_B \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_1} :\vdash \Delta_2, \mathtt{F_4}} \ \mathtt{ax}}{\bullet \mathtt{h_1} :\vdash \Delta_2, \mathtt{F_4}} \ height$$

- Case rule $\oplus_A$

$$\frac{\texttt{h}_3 : \vdash \texttt{F}_4, \Delta_6, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2}{\bullet\texttt{h}_3 : \vdash (\Delta_6, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2), \texttt{F}_4 \oplus \texttt{F}_5} \oplus_A \quad \rightsquigarrow \quad \frac{\overline{\texttt{h}_3 : \vdash \Delta_6, \texttt{F}_2, \texttt{F}_4} \ \texttt{ax/ind}}{\bullet\texttt{h}_3 : \vdash \Delta_6, \texttt{F}_2, \texttt{F}_4 \oplus \texttt{F}_5} \oplus_A$$

$$\frac{\texttt{h}_1 : \vdash \texttt{F}_3, \Delta_2}{\bullet\texttt{h}_1 : \vdash \Delta_2, \texttt{F}_3 \oplus \texttt{F}_4} \oplus_A \quad \rightsquigarrow \quad \overline{\textcolor{red}{\bullet\texttt{h}_1 : \vdash \texttt{F}_4, \Delta_2}} \ \textcolor{red}{\texttt{fail}}$$

- Case rule $\perp$

$$\frac{\texttt{h}_3 : \vdash \Delta_4, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2}{\bullet\texttt{h}_3 : \vdash \perp, \Delta_4, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2} \perp \quad \rightsquigarrow \quad \frac{\overline{\texttt{h}_3 : \vdash \Delta_4, \texttt{F}_2} \ \texttt{ax/ind}}{\bullet\texttt{h}_3 : \vdash \Delta_4, \texttt{F}_2, \perp} \perp$$

- Case rule $\top$

$$\overline{\bullet\texttt{h}_3 : \vdash \top, \Delta_4, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2} \ \top \quad \rightsquigarrow \quad \overline{\bullet\texttt{h}_3 : \vdash \Delta_4, \texttt{F}_2, \top} \ \top$$

- Case rule I

- Case rule $\otimes$

$$\frac{\texttt{h}_3 : \vdash \texttt{F}_5, \Delta_7, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2 \quad \texttt{h}_3 : \vdash \texttt{F}_6, \Delta_4}{\bullet\texttt{h}_3 : \vdash (\Delta_7, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2), \Delta_4, \texttt{F}_5 \otimes \texttt{F}_6} \otimes \quad \rightsquigarrow \quad \frac{\overline{\texttt{h}_3 : \vdash \Delta_7, \texttt{F}_2, \texttt{F}_5} \ \texttt{ax/ind} \quad \overline{\texttt{h}_3 : \vdash \Delta_4, \texttt{F}_6} \ \texttt{ax}}{\bullet\texttt{h}_3 : \vdash \Delta_4, \Delta_7, \texttt{F}_2, \texttt{F}_5 \otimes \texttt{F}_6} \otimes$$

$$\frac{\texttt{h}_3 : \vdash \texttt{F}_5, \Delta_4 \quad \texttt{h}_3 : \vdash \texttt{F}_6, \Delta_7, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2}{\bullet\texttt{h}_3 : \vdash \Delta_4, (\Delta_7, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2), \texttt{F}_5 \otimes \texttt{F}_6} \otimes \quad \rightsquigarrow \quad \frac{\overline{\texttt{h}_3 : \vdash \Delta_4, \texttt{F}_5} \ \texttt{ax} \quad \overline{\texttt{h}_3 : \vdash \Delta_7, \texttt{F}_2, \texttt{F}_6} \ \texttt{ax/ind}}{\bullet\texttt{h}_3 : \vdash \Delta_4, \Delta_7, \texttt{F}_2, \texttt{F}_5 \otimes \texttt{F}_6} \otimes$$

2.6 Status of $\oplus_A$: : Non invertible

- Case rule $\mathbf{1}$

- Case rule $\$$

$$\frac{\texttt{h}_3 : \vdash \texttt{F}_4, \texttt{F}_5, \Delta_6, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2}{\bullet\texttt{h}_3 : \vdash (\Delta_6, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2), \texttt{F}_4 \$ \texttt{F}_5} \$ \quad \rightsquigarrow \quad \frac{\overline{\texttt{h}_3 : \vdash \Delta_6, \texttt{F}_\mathbf{1}, \texttt{F}_4, \texttt{F}_5} \ \texttt{ax/ind}}{\bullet\texttt{h}_3 : \vdash \Delta_6, \texttt{F}_\mathbf{1}, \texttt{F}_4 \$ \texttt{F}_5} \$$$

- Case rule $\&$

$$\frac{\texttt{h}_3 : \vdash \texttt{F}_4, \Delta_6, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2 \quad \texttt{h}_3 : \vdash \texttt{F}_5, \Delta_6, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2}{\bullet\texttt{h}_3 : \vdash (\Delta_6, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2), \texttt{F}_4 \& \texttt{F}_5} \& \quad \rightsquigarrow \quad \frac{\overline{\texttt{h}_3 : \vdash \Delta_6, \texttt{F}_\mathbf{1}, \texttt{F}_4} \ \texttt{ax/ind} \quad \overline{\texttt{h}_3 : \vdash \Delta_6, \texttt{F}_\mathbf{1}, \texttt{F}_5} \ \texttt{ax/ind}}{\bullet\texttt{h}_3 : \vdash \Delta_6, \texttt{F}_\mathbf{1}, \texttt{F}_4 \& \texttt{F}_5} \&$$

- Case rule $\oplus_B$

$$\frac{\texttt{h}_3 : \vdash \texttt{F}_5, \Delta_6, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2}{\bullet\texttt{h}_3 : \vdash (\Delta_6, \texttt{F}_\mathbf{1} \oplus \texttt{F}_2), \texttt{F}_4 \oplus \texttt{F}_5} \oplus_B \quad \rightsquigarrow \quad \frac{\overline{\texttt{h}_3 : \vdash \Delta_6, \texttt{F}_\mathbf{1}, \texttt{F}_5} \ \texttt{ax/ind}}{\bullet\texttt{h}_3 : \vdash \Delta_6, \texttt{F}_\mathbf{1}, \texttt{F}_4 \oplus \texttt{F}_5} \oplus_B$$

$$\frac{\texttt{h}_1 : \vdash \texttt{F}_4, \Delta_2}{\bullet\texttt{h}_1 : \vdash \Delta_2, \texttt{F}_3 \oplus \texttt{F}_4} \oplus_B \quad \rightsquigarrow \quad \overline{\textcolor{red}{\bullet\texttt{h}_1 : \vdash \texttt{F}_3, \Delta_2}} \ \textcolor{red}{\texttt{fail}}$$

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- Case rule $\oplus_A$

$$\dfrac{h_3 :\vdash F_4, \Delta_6, F_1 \oplus F_2}{\bullet h_3 :\vdash (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \ \oplus_A \quad \leadsto \quad \dfrac{\overline{h_3 :\vdash \Delta_6, F_1, F_4} \ \text{ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_1, F_4 \oplus F_5} \ \oplus_A$$

$$\dfrac{h_1 :\vdash F_3, \Delta_2}{\bullet h_1 :\vdash \Delta_2, F_3 \oplus F_4} \ \oplus_A \quad \leadsto \quad \dfrac{\overline{h_1 :\vdash \Delta_2, F_3} \ \text{ax}}{\bullet h_1 :\vdash \Delta_2, F_3} \ height$$

- Case rule $\perp$

$$\dfrac{h_3 :\vdash \Delta_4, F_1 \oplus F_2}{\bullet h_3 :\vdash \perp, \Delta_4, F_1 \oplus F_2} \ \perp \quad \leadsto \quad \dfrac{\overline{h_3 :\vdash \Delta_4, F_1} \ \text{ax/ind}}{\bullet h_3 :\vdash \Delta_4, F_1, \perp} \ \perp$$

- Case rule $\top$

$$\dfrac{}{\bullet h_3 :\vdash \top, \Delta_4, F_1 \oplus F_2} \ \top \quad \leadsto \quad \dfrac{}{\bullet h_3 :\vdash \Delta_4, F_1, \top} \ \top$$

- Case rule I

- Case rule $\otimes$

$$\dfrac{h_3 :\vdash F_5, \Delta_7, F_1 \oplus F_2 \quad h_3 :\vdash F_6, \Delta_4}{\bullet h_3 :\vdash (\Delta_7, F_1 \oplus F_2), \Delta_4, F_5 \otimes F_6} \ \otimes \quad \leadsto \quad \dfrac{\overline{h_3 :\vdash \Delta_7, F_1, F_5} \ \text{ax/ind} \quad \overline{h_3 :\vdash \Delta_4, F_6} \ \text{ax}}{\bullet h_3 :\vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \ \otimes$$

$$\dfrac{h_3 :\vdash F_5, \Delta_4 \quad h_3 :\vdash F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 :\vdash \Delta_4, (\Delta_7, F_1 \oplus F_2), F_5 \otimes F_6} \ \otimes \quad \leadsto \quad \dfrac{\overline{h_3 :\vdash \Delta_4, F_5} \ \text{ax} \quad \overline{h_3 :\vdash \Delta_7, F_1, F_6} \ \text{ax/ind}}{\bullet h_3 :\vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \ \otimes$$

2.7   Status of $\perp$: :   Invertible

- Case rule $\mathbf{1}$

- Case rule $\$$

$$\dfrac{h_1 :\vdash \perp, F_2, F_3, \Delta_4}{\bullet h_1 :\vdash (\perp, \Delta_4), F_2 \$ F_3} \ \$ \quad \leadsto \quad \dfrac{\overline{h_1 :\vdash \Delta_4, F_2, F_3} \ \text{ax/ind}}{\bullet h_1 :\vdash \Delta_4, F_2 \$ F_3} \ \$$$

- Case rule $\&$

$$\dfrac{h_1 :\vdash \perp, F_2, \Delta_4 \quad h_1 :\vdash \perp, F_3, \Delta_4}{\bullet h_1 :\vdash (\perp, \Delta_4), F_2 \& F_3} \ \& \quad \leadsto \quad \dfrac{\overline{h_1 :\vdash \Delta_4, F_2} \ \text{ax/ind} \quad \overline{h_1 :\vdash \Delta_4, F_3} \ \text{ax/ind}}{\bullet h_1 :\vdash \Delta_4, F_2 \& F_3} \ \&$$

- Case rule $\oplus_B$

$$\dfrac{h_1 :\vdash \perp, F_3, \Delta_4}{\bullet h_1 :\vdash (\perp, \Delta_4), F_2 \oplus F_3} \ \oplus_B \quad \leadsto \quad \dfrac{\overline{h_1 :\vdash \Delta_4, F_3} \ \text{ax/ind}}{\bullet h_1 :\vdash \Delta_4, F_2 \oplus F_3} \ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h_1} :\vdash \bot, \mathtt{F_2}, \Delta_4}{\bullet\mathtt{h_1} :\vdash (\bot, \Delta_4), \mathtt{F_2} \oplus \mathtt{F_3}} \oplus_A \qquad \leadsto \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}} \; \mathtt{ax/ind}}{\bullet\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2} \oplus \mathtt{F_3}} \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h_1} :\vdash \Delta_2}{\bullet\mathtt{h_1} :\vdash \bot, \Delta_2} \bot \qquad \leadsto \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_2} \; \mathtt{ax}}{\bullet\mathtt{h_1} :\vdash \Delta_2} \; height$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h_1} :\vdash \top, \bot, \Delta_2} \top \qquad \leadsto \qquad \frac{}{\bullet\mathtt{h_1} :\vdash \Delta_2, \top} \top$$

- Case rule I

- Case rule $\otimes$

$$\frac{\mathtt{h_1} :\vdash \bot, \mathtt{F_3}, \Delta_5 \quad \mathtt{h_1} :\vdash \mathtt{F_4}, \Delta_2}{\bullet\mathtt{h_1} :\vdash (\bot, \Delta_5), \Delta_2, \mathtt{F_3} \otimes \mathtt{F_4}} \otimes \qquad \leadsto \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_5, \mathtt{F_3}} \; \mathtt{ax/ind} \quad \overline{\mathtt{h_1} :\vdash \Delta_2, \mathtt{F_4}} \; \mathtt{ax}}{\bullet\mathtt{h_1} :\vdash \Delta_2, \Delta_5, \mathtt{F_3} \otimes \mathtt{F_4}} \otimes$$

$$\frac{\mathtt{h_1} :\vdash \mathtt{F_3}, \Delta_2 \quad \mathtt{h_1} :\vdash \bot, \mathtt{F_4}, \Delta_5}{\bullet\mathtt{h_1} :\vdash \Delta_2, (\bot, \Delta_5), \mathtt{F_3} \otimes \mathtt{F_4}} \otimes \qquad \leadsto \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_2, \mathtt{F_3}} \; \mathtt{ax} \quad \overline{\mathtt{h_1} :\vdash \Delta_5, \mathtt{F_4}} \; \mathtt{ax/ind}}{\bullet\mathtt{h_1} :\vdash \Delta_2, \Delta_5, \mathtt{F_3} \otimes \mathtt{F_4}} \otimes$$

2.8   Status of $\top$: :   Invertible

- Case rule $\mathbf{1}$

- Case rule $\$$

$$\frac{\mathtt{h_1} :\vdash \top, \mathtt{F_2}, \mathtt{F_3}, \Delta_4}{\bullet\mathtt{h_1} :\vdash (\top, \Delta_4), \mathtt{F_2}\$\mathtt{F_3}} \$ \qquad \leadsto \qquad trivial$$

- Case rule $\&$

$$\frac{\mathtt{h_1} :\vdash \top, \mathtt{F_2}, \Delta_4 \quad \mathtt{h_1} :\vdash \top, \mathtt{F_3}, \Delta_4}{\bullet\mathtt{h_1} :\vdash (\top, \Delta_4), \mathtt{F_2}\&\mathtt{F_3}} \& \qquad \leadsto \qquad trivial$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h_1} :\vdash \top, \mathtt{F_3}, \Delta_4}{\bullet\mathtt{h_1} :\vdash (\top, \Delta_4), \mathtt{F_2} \oplus \mathtt{F_3}} \oplus_B \qquad \leadsto \qquad trivial$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h_1} :\vdash \top, \mathtt{F_2}, \Delta_4}{\bullet\mathtt{h_1} :\vdash (\top, \Delta_4), \mathtt{F_2} \oplus \mathtt{F_3}} \oplus_A \qquad \leadsto \qquad trivial$$

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- Case rule $\perp$

$$\dfrac{\mathtt{h_1} :\vdash \top, \Delta_2}{\bullet\mathtt{h_1} :\vdash \perp, \top, \Delta_2} \perp \qquad \leadsto \qquad \texttt{trivial}$$

- Case rule $\top$

$$\dfrac{}{\bullet\mathtt{h_1} :\vdash \top, \Delta_2} \top \qquad \leadsto \qquad \texttt{trivial}$$

- Case rule I

- Case rule $\otimes$

$$\dfrac{\mathtt{h_1} :\vdash \top, \mathtt{F_3}, \Delta_5 \quad \mathtt{h_1} :\vdash \mathtt{F_4}, \Delta_2}{\bullet\mathtt{h_1} :\vdash (\top, \Delta_5), \Delta_2, \mathtt{F_3} \otimes \mathtt{F_4}} \otimes \qquad \leadsto \qquad \texttt{trivial}$$

$$\dfrac{\mathtt{h_1} :\vdash \mathtt{F_3}, \Delta_2 \quad \mathtt{h_1} :\vdash \top, \mathtt{F_4}, \Delta_5}{\bullet\mathtt{h_1} :\vdash \Delta_2, (\top, \Delta_5), \mathtt{F_3} \otimes \mathtt{F_4}} \otimes \qquad \leadsto \qquad \texttt{trivial}$$

2.9 Status of I: : Invertible

- Case rule $\mathbf{1}$

- Case rule $\$$

- Case rule $\&$

- Case rule $\oplus_B$

- Case rule $\oplus_A$

- Case rule $\perp$

- Case rule $\top$

- Case rule I

$$\dfrac{}{\bullet\mathtt{h_1} :\vdash p(\mathtt{n_2}), \hat{}(\mathtt{n_2})} I \qquad \leadsto \qquad \texttt{trivial}$$

- Case rule $\otimes$

2.10   Status of ⊗:   (Left Premise): `Non invertible`

- Case rule **1**

- Case rule $

$$\frac{\mathtt{h_4} : \vdash \mathtt{F_5}, \mathtt{F_6}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash (\Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5}\$\mathtt{F_6}} \ \$ \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_7, \mathtt{F_2}, \mathtt{F_5}, \mathtt{F_6}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_7, \mathtt{F_2}, \mathtt{F_5}\$\mathtt{F_6}} \ \$$$

$$\frac{\mathtt{h_4} : \vdash \mathtt{F_5}, \mathtt{F_6}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash (\Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5}\$\mathtt{F_6}} \ \$ \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ {}_{height}$$

- Case rule &

$$\frac{\mathtt{h_4} : \vdash \mathtt{F_5}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3} \quad \mathtt{h_4} : \vdash \mathtt{F_6}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash (\Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5}\&\mathtt{F_6}} \ \& \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_7, \mathtt{F_2}, \mathtt{F_5}} \ \text{ax/ind} \quad \overline{\mathtt{h_4} : \vdash \Delta_7, \mathtt{F_2}, \mathtt{F_6}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_7, \mathtt{F_2}, \mathtt{F_5}\&\mathtt{F_6}} \ \&$$

$$\frac{\mathtt{h_4} : \vdash \mathtt{F_5}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3} \quad \mathtt{h_4} : \vdash \mathtt{F_6}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash (\Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5}\&\mathtt{F_6}} \ \& \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ {}_{height}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h_4} : \vdash \mathtt{F_6}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash (\Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5} \oplus \mathtt{F_6}} \ \oplus_B \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_7, \mathtt{F_2}, \mathtt{F_6}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_7, \mathtt{F_2}, \mathtt{F_5} \oplus \mathtt{F_6}} \ \oplus_B$$

$$\frac{\mathtt{h_4} : \vdash \mathtt{F_6}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash (\Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5} \oplus \mathtt{F_6}} \ \oplus_B \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ {}_{height}$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h_4} : \vdash \mathtt{F_5}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash (\Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5} \oplus \mathtt{F_6}} \ \oplus_A \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_7, \mathtt{F_2}, \mathtt{F_5}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_7, \mathtt{F_2}, \mathtt{F_5} \oplus \mathtt{F_6}} \ \oplus_A$$

$$\frac{\mathtt{h_4} : \vdash \mathtt{F_5}, \Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash (\Delta_1, \Delta_7, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5} \oplus \mathtt{F_6}} \ \oplus_A \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ {}_{height}$$

- Case rule ⊥

$$\frac{\mathtt{h_4} : \vdash \Delta_1, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash \perp, \Delta_1, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \ \perp \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_5, \mathtt{F_2}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_5, \mathtt{F_2}, \perp} \ \perp$$

$$\frac{\mathtt{h_4} : \vdash \Delta_1, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} : \vdash \perp, \Delta_1, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \ \perp \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ \text{ax/ind}}{\bullet\mathtt{h_4} : \vdash \Delta_1, \mathtt{F_2}} \ {}_{height}$$

- Case rule ⊤

$$\frac{}{\bullet\mathtt{h_4} : \vdash \top, \Delta_1, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \ \top \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h_4} : \vdash \Delta_5, \mathtt{F_2}, \top} \ \top$$

$$\frac{}{\bullet\mathtt{h_4} : \vdash \top, \Delta_1, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \ \top \quad \rightsquigarrow \quad {\color{red}\frac{}{\bullet\mathtt{h_4} : \vdash \mathtt{F_2}, \Delta_1} \ \text{fail}}$$

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- Case rule I

- Case rule $\otimes$

$$\dfrac{\texttt{h}_3 :\vdash \texttt{F}_4, \Delta_6, \Delta_7, \texttt{F}_\mathbf{1} \otimes \texttt{F}_2 \quad \texttt{h}_3 :\vdash \texttt{F}_5, \Delta_8, \Delta_9}{\bullet\texttt{h}_3 :\vdash (\Delta_6, \Delta_7, \texttt{F}_\mathbf{1} \otimes \texttt{F}_2), (\Delta_8, \Delta_9), \texttt{F}_4 \otimes \texttt{F}_5} \otimes \quad \leadsto \quad \dfrac{}{\color{red}\bullet\texttt{h}_3 :\vdash \texttt{F}_\mathbf{1}, \Delta_6, \Delta_8, \texttt{F}_4 \otimes \texttt{F}_5} \texttt{ fail}$$

$$\dfrac{\texttt{h}_3 :\vdash \texttt{F}_4, \Delta_6, \Delta_7 \quad \texttt{h}_3 :\vdash \texttt{F}_5, \Delta_8, \Delta_9, \texttt{F}_\mathbf{1} \otimes \texttt{F}_2}{\bullet\texttt{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \texttt{F}_\mathbf{1} \otimes \texttt{F}_2), \texttt{F}_4 \otimes \texttt{F}_5} \otimes \quad \leadsto \quad \dfrac{}{\color{red}\bullet\texttt{h}_3 :\vdash \texttt{F}_\mathbf{1}, \Delta_6, \Delta_8, \texttt{F}_4 \otimes \texttt{F}_5} \texttt{ fail}$$

$$\dfrac{\texttt{h}_3 :\vdash \texttt{F}_4, \Delta_6, \Delta_7, \texttt{F}_\mathbf{1} \otimes \texttt{F}_2 \quad \texttt{h}_3 :\vdash \texttt{F}_5, \Delta_8, \Delta_9}{\bullet\texttt{h}_3 :\vdash (\Delta_6, \Delta_7, \texttt{F}_\mathbf{1} \otimes \texttt{F}_2), (\Delta_8, \Delta_9), \texttt{F}_4 \otimes \texttt{F}_5} \otimes \quad \leadsto \quad \dfrac{}{\color{red}\bullet\texttt{h}_3 :\vdash \texttt{F}_\mathbf{1}, \Delta_6, \Delta_8} \texttt{ fail}$$

$$\dfrac{\texttt{h}_3 :\vdash \texttt{F}_4, \Delta_6, \Delta_7 \quad \texttt{h}_3 :\vdash \texttt{F}_5, \Delta_8, \Delta_9, \texttt{F}_\mathbf{1} \otimes \texttt{F}_2}{\bullet\texttt{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \texttt{F}_\mathbf{1} \otimes \texttt{F}_2), \texttt{F}_4 \otimes \texttt{F}_5} \otimes \quad \leadsto \quad \dfrac{}{\color{red}\bullet\texttt{h}_3 :\vdash \texttt{F}_\mathbf{1}, \Delta_6, \Delta_8} \texttt{ fail}$$

$$\dfrac{\texttt{h}_1 :\vdash \texttt{F}_2, \Delta_4, \Delta_5 \quad \texttt{h}_1 :\vdash \texttt{F}_3, \Delta_6, \Delta_7}{\bullet\texttt{h}_1 :\vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), \texttt{F}_2 \otimes \texttt{F}_3} \otimes \quad \leadsto \quad \dfrac{}{\color{red}\bullet\texttt{h}_1 :\vdash \texttt{F}_2, \Delta_4, \Delta_6} \texttt{ fail}$$

2.11 Status of $\otimes$ (Right Premise): : Non invertible

- Case rule $\mathbf{1}$

- Case rule $\$$

$$\dfrac{\texttt{h}_4 :\vdash \texttt{F}_5, \texttt{F}_6, \Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3}{\bullet\texttt{h}_4 :\vdash (\Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3), \texttt{F}_5\$\texttt{F}_6} \$ \quad \leadsto \quad \dfrac{\dfrac{}{\texttt{h}_4 :\vdash \Delta_\mathbf{1}, \texttt{F}_3} \texttt{ ax/ind}}{\bullet\texttt{h}_4 :\vdash \Delta_\mathbf{1}, \texttt{F}_3} \, height$$

$$\dfrac{\texttt{h}_4 :\vdash \texttt{F}_5, \texttt{F}_6, \Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3}{\bullet\texttt{h}_4 :\vdash (\Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3), \texttt{F}_5\$\texttt{F}_6} \$ \quad \leadsto \quad \dfrac{\dfrac{}{\texttt{h}_4 :\vdash \Delta_7, \texttt{F}_3, \texttt{F}_5, \texttt{F}_6} \texttt{ ax/ind}}{\bullet\texttt{h}_4 :\vdash \Delta_7, \texttt{F}_3, \texttt{F}_5\$\texttt{F}_6} \$$$

- Case rule $\&$

$$\dfrac{\texttt{h}_4 :\vdash \texttt{F}_5, \Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3 \quad \texttt{h}_4 :\vdash \texttt{F}_6, \Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3}{\bullet\texttt{h}_4 :\vdash (\Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3), \texttt{F}_5\&\texttt{F}_6} \& \quad \leadsto \quad \dfrac{\dfrac{}{\texttt{h}_4 :\vdash \Delta_\mathbf{1}, \texttt{F}_3} \texttt{ ax/ind}}{\bullet\texttt{h}_4 :\vdash \Delta_\mathbf{1}, \texttt{F}_3} \, height$$

$$\dfrac{\texttt{h}_4 :\vdash \texttt{F}_5, \Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3 \quad \texttt{h}_4 :\vdash \texttt{F}_6, \Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3}{\bullet\texttt{h}_4 :\vdash (\Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3), \texttt{F}_5\&\texttt{F}_6} \& \quad \leadsto \quad \dfrac{\dfrac{}{\texttt{h}_4 :\vdash \Delta_7, \texttt{F}_3, \texttt{F}_5} \texttt{ ax/ind} \quad \dfrac{}{\texttt{h}_4 :\vdash \Delta_7, \texttt{F}_3, \texttt{F}_6} \texttt{ ax/ind}}{\bullet\texttt{h}_4 :\vdash \Delta_7, \texttt{F}_3, \texttt{F}_5\&\texttt{F}_6} \&$$

- Case rule $\oplus_B$

$$\dfrac{\texttt{h}_4 :\vdash \texttt{F}_6, \Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3}{\bullet\texttt{h}_4 :\vdash (\Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3), \texttt{F}_5 \oplus \texttt{F}_6} \oplus_B \quad \leadsto \quad \dfrac{\dfrac{}{\texttt{h}_4 :\vdash \Delta_\mathbf{1}, \texttt{F}_3} \texttt{ ax/ind}}{\bullet\texttt{h}_4 :\vdash \Delta_\mathbf{1}, \texttt{F}_3} \, height$$

$$\dfrac{\texttt{h}_4 :\vdash \texttt{F}_6, \Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3}{\bullet\texttt{h}_4 :\vdash (\Delta_\mathbf{1}, \Delta_7, \texttt{F}_2 \otimes \texttt{F}_3), \texttt{F}_5 \oplus \texttt{F}_6} \oplus_B \quad \leadsto \quad \dfrac{\dfrac{}{\texttt{h}_4 :\vdash \Delta_7, \texttt{F}_3, \texttt{F}_6} \texttt{ ax/ind}}{\bullet\texttt{h}_4 :\vdash \Delta_7, \texttt{F}_3, \texttt{F}_5 \oplus \texttt{F}_6} \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\mathtt{h_4} :\vdash \mathtt{F_5}, \Delta_{\mathbf{1}}, \Delta_{\mathbf{7}}, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} :\vdash (\Delta_{\mathbf{1}}, \Delta_{\mathbf{7}}, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5} \oplus \mathtt{F_6}} \; \oplus_A \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_4} :\vdash \Delta_{\mathbf{1}}, \mathtt{F_3}}}{\bullet\mathtt{h_4} :\vdash \Delta_{\mathbf{1}}, \mathtt{F_3}} \begin{smallmatrix} \mathrm{ax/ind} \\ \\ height \end{smallmatrix}$$

$$\dfrac{\mathtt{h_4} :\vdash \mathtt{F_5}, \Delta_{\mathbf{1}}, \Delta_{\mathbf{7}}, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} :\vdash (\Delta_{\mathbf{1}}, \Delta_{\mathbf{7}}, \mathtt{F_2} \otimes \mathtt{F_3}), \mathtt{F_5} \oplus \mathtt{F_6}} \; \oplus_A \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_4} :\vdash \Delta_{\mathbf{7}}, \mathtt{F_3}, \mathtt{F_5}}}{\bullet\mathtt{h_4} :\vdash \Delta_{\mathbf{7}}, \mathtt{F_3}, \mathtt{F_5} \oplus \mathtt{F_6}} \begin{smallmatrix} \mathrm{ax/ind} \\ \\ \oplus_A \end{smallmatrix}$$

- Case rule $\bot$

$$\dfrac{\mathtt{h_4} :\vdash \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} :\vdash \bot, \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \; \bot \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_4} :\vdash \Delta_{\mathbf{1}}, \mathtt{F_3}}}{\bullet\mathtt{h_4} :\vdash \Delta_{\mathbf{1}}, \mathtt{F_3}} \begin{smallmatrix} \mathrm{ax/ind} \\ \\ height \end{smallmatrix}$$

$$\dfrac{\mathtt{h_4} :\vdash \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}}{\bullet\mathtt{h_4} :\vdash \bot, \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \; \bot \qquad \rightsquigarrow \qquad \dfrac{\overline{\mathtt{h_4} :\vdash \Delta_5, \mathtt{F_3}}}{\bullet\mathtt{h_4} :\vdash \Delta_5, \mathtt{F_3}, \bot} \begin{smallmatrix} \mathrm{ax/ind} \\ \\ \bot \end{smallmatrix}$$

- Case rule $\top$

$$\dfrac{}{\bullet\mathtt{h_4} :\vdash \top, \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \; \top \qquad \rightsquigarrow \qquad {\color{red}\dfrac{}{\bullet\mathtt{h_4} :\vdash \mathtt{F_3}, \Delta_{\mathbf{1}}} \; \mathtt{fail}}$$

$$\dfrac{}{\bullet\mathtt{h_4} :\vdash \top, \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \; \top \qquad \rightsquigarrow \qquad \dfrac{}{\bullet\mathtt{h_4} :\vdash \Delta_5, \mathtt{F_3}, \top} \; \top$$

- Case rule I

- Case rule $\otimes$

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_4}, \Delta_6, \Delta_{\mathbf{7}}, \mathtt{F_{\mathbf{1}}} \otimes \mathtt{F_2} \quad \mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_8, \Delta_9}{\bullet\mathtt{h_3} :\vdash (\Delta_6, \Delta_{\mathbf{7}}, \mathtt{F_{\mathbf{1}}} \otimes \mathtt{F_2}), (\Delta_8, \Delta_9), \mathtt{F_4} \otimes \mathtt{F_5}} \; \otimes \qquad \rightsquigarrow \qquad {\color{red}\dfrac{}{\bullet\mathtt{h_3} :\vdash \mathtt{F_2}, \Delta_{\mathbf{7}}, \Delta_9} \; \mathtt{fail}}$$

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_4}, \Delta_6, \Delta_{\mathbf{7}} \quad \mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_8, \Delta_9, \mathtt{F_{\mathbf{1}}} \otimes \mathtt{F_2}}{\bullet\mathtt{h_3} :\vdash (\Delta_6, \Delta_{\mathbf{7}}), (\Delta_8, \Delta_9, \mathtt{F_{\mathbf{1}}} \otimes \mathtt{F_2}), \mathtt{F_4} \otimes \mathtt{F_5}} \; \otimes \qquad \rightsquigarrow \qquad {\color{red}\dfrac{}{\bullet\mathtt{h_3} :\vdash \mathtt{F_2}, \Delta_{\mathbf{7}}, \Delta_9} \; \mathtt{fail}}$$

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_4}, \Delta_6, \Delta_{\mathbf{7}}, \mathtt{F_{\mathbf{1}}} \otimes \mathtt{F_2} \quad \mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_8, \Delta_9}{\bullet\mathtt{h_3} :\vdash (\Delta_6, \Delta_{\mathbf{7}}, \mathtt{F_{\mathbf{1}}} \otimes \mathtt{F_2}), (\Delta_8, \Delta_9), \mathtt{F_4} \otimes \mathtt{F_5}} \; \otimes \qquad \rightsquigarrow \qquad {\color{red}\dfrac{}{\bullet\mathtt{h_3} :\vdash \mathtt{F_2}, \Delta_{\mathbf{7}}, \Delta_9, \mathtt{F_4} \otimes \mathtt{F_5}} \; \mathtt{fail}}$$

$$\dfrac{\mathtt{h_3} :\vdash \mathtt{F_4}, \Delta_6, \Delta_{\mathbf{7}} \quad \mathtt{h_3} :\vdash \mathtt{F_5}, \Delta_8, \Delta_9, \mathtt{F_{\mathbf{1}}} \otimes \mathtt{F_2}}{\bullet\mathtt{h_3} :\vdash (\Delta_6, \Delta_{\mathbf{7}}), (\Delta_8, \Delta_9, \mathtt{F_{\mathbf{1}}} \otimes \mathtt{F_2}), \mathtt{F_4} \otimes \mathtt{F_5}} \; \otimes \qquad \rightsquigarrow \qquad {\color{red}\dfrac{}{\bullet\mathtt{h_3} :\vdash \mathtt{F_2}, \Delta_{\mathbf{7}}, \Delta_9, \mathtt{F_4} \otimes \mathtt{F_5}} \; \mathtt{fail}}$$

$$\dfrac{\mathtt{h_1} :\vdash \mathtt{F_2}, \Delta_4, \Delta_5 \quad \mathtt{h_1} :\vdash \mathtt{F_3}, \Delta_6, \Delta_{\mathbf{7}}}{\bullet\mathtt{h_1} :\vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_{\mathbf{7}}), \mathtt{F_2} \otimes \mathtt{F_3}} \; \otimes \qquad \rightsquigarrow \qquad {\color{red}\dfrac{}{\bullet\mathtt{h_1} :\vdash \mathtt{F_3}, \Delta_5, \Delta_{\mathbf{7}}} \; \mathtt{fail}}$$

3 Identity-Expansion

$$\frac{}{-\,:\vdash \mathbf{0}, \top} \; \top$$

$$\frac{\dfrac{}{-\,:\vdash \mathbf{1}} \; \mathbf{1}}{-\,:\vdash \mathbf{1}, \bot} \; \bot$$

$$\frac{\dfrac{\dfrac{}{-\,:\vdash \mathbf{F_0}, dual(\mathbf{F_0})} \; \text{IH}}{-\,:\vdash dual(\mathbf{F_0}), \mathbf{F_0} \oplus \mathbf{F_1}} \; \oplus_A \qquad \dfrac{\dfrac{}{-\,:\vdash \mathbf{F_1}, dual(\mathbf{F_1})} \; \text{IH}}{-\,:\vdash dual(\mathbf{F_1}), \mathbf{F_0} \oplus \mathbf{F_1}} \; \oplus_B}{-\,:\vdash dual(\mathbf{F_0}) \& dual(\mathbf{F_1}), \mathbf{F_0} \oplus \mathbf{F_1}} \; \&$$

$$\frac{\dfrac{\dfrac{}{-\,:\vdash \mathbf{F_0}, dual(\mathbf{F_0})} \; \text{IH} \qquad \dfrac{}{-\,:\vdash \mathbf{F_1}, dual(\mathbf{F_1})} \; \text{IH}}{-\,:\vdash dual(\mathbf{F_0}), dual(\mathbf{F_1}), \mathbf{F_0} \otimes \mathbf{F_1}} \; \otimes}{-\,:\vdash dual(\mathbf{F_0}) \$ dual(\mathbf{F_1}), \mathbf{F_0} \otimes \mathbf{F_1}} \; \$$$

$$\frac{\dfrac{\dfrac{}{-\,:\vdash \mathbf{F_0}, dual(\mathbf{F_0})} \; \text{IH}}{-\,:\vdash \mathbf{F_0}, dual(\mathbf{F_0}) \oplus dual(\mathbf{F_1})} \; \oplus_A \qquad \dfrac{\dfrac{}{-\,:\vdash \mathbf{F_1}, dual(\mathbf{F_1})} \; \text{IH}}{-\,:\vdash \mathbf{F_1}, dual(\mathbf{F_0}) \oplus dual(\mathbf{F_1})} \; \oplus_B}{-\,:\vdash \mathbf{F_0} \& \mathbf{F_1}, dual(\mathbf{F_0}) \oplus dual(\mathbf{F_1})} \; \&$$

$$\frac{\dfrac{\dfrac{}{-\,:\vdash \mathbf{F_0}, dual(\mathbf{F_0})} \; \text{IH} \qquad \dfrac{}{-\,:\vdash \mathbf{F_1}, dual(\mathbf{F_1})} \; \text{IH}}{-\,:\vdash \mathbf{F_0}, \mathbf{F_1}, dual(\mathbf{F_0}) \otimes dual(\mathbf{F_1})} \; \otimes}{-\,:\vdash \mathbf{F_0} \$ \mathbf{F_1}, dual(\mathbf{F_0}) \otimes dual(\mathbf{F_1})} \; \$$$

$$\frac{\dfrac{}{-\,:\vdash \mathbf{1}} \; \mathbf{1}}{-\,:\vdash \mathbf{1}, \bot} \; \bot$$

$$\frac{}{-\,:\vdash \mathbf{0}, \top} \; \top$$

4 Cut-Elimination

4.1 Status of 1: `OK`

- Case rule **1**

- Case rule $

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1}
  \qquad
  \cfrac{\mathtt{h}_2 :\vdash \bot, \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$
}{
  - :\vdash *, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4
}\ \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \mathtt{ax}
    \qquad
    \cfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \bot}\ \mathtt{ax}
  }{
    - :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4
  }\ \mathtt{hCut}
}{
  - :\vdash \Delta_5, \mathtt{F}_3\$\mathtt{F}_4
}\ \$
$$

- Case rule &

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1}
  \qquad
  \cfrac{\mathtt{h}_2 :\vdash \bot, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_2 :\vdash \bot, \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_5, \mathtt{F}_3\&\mathtt{F}_4}\ \&
}{
  - :\vdash *, \Delta_5, \mathtt{F}_3\&\mathtt{F}_4
}\ \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1}
    \quad
    \cfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \bot}\ \mathtt{ax}
  }{- :\vdash \Delta_5, \mathtt{F}_3}\ \mathtt{hCut}
  \qquad
  \cfrac{
    \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1}
    \quad
    \cfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \bot}\ \mathtt{ax}
  }{- :\vdash \Delta_5, \mathtt{F}_4}\ \mathtt{hCut}
}{
  - :\vdash \Delta_5, \mathtt{F}_3\&\mathtt{F}_4
}\ \&
$$

- Case rule $\oplus_B$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1}
  \qquad
  \cfrac{\mathtt{h}_2 :\vdash \bot, \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_5, \mathtt{F}_3\oplus\mathtt{F}_4}\ \oplus_B
}{
  - :\vdash *, \Delta_5, \mathtt{F}_3\oplus\mathtt{F}_4
}\ \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \mathtt{ax}
    \qquad
    \cfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \bot}\ \mathtt{ax}
  }{- :\vdash \Delta_5, \mathtt{F}_4}\ \mathtt{hCut}
}{
  - :\vdash \Delta_5, \mathtt{F}_3\oplus\mathtt{F}_4
}\ \oplus_B
$$

- Case rule $\oplus_A$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1}
  \qquad
  \cfrac{\mathtt{h}_2 :\vdash \bot, \mathtt{F}_3, \Delta_5}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_5, \mathtt{F}_3\oplus\mathtt{F}_4}\ \oplus_A
}{
  - :\vdash *, \Delta_5, \mathtt{F}_3\oplus\mathtt{F}_4
}\ \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \mathtt{ax}
    \qquad
    \cfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \bot}\ \mathtt{ax}
  }{- :\vdash \Delta_5, \mathtt{F}_3}\ \mathtt{hCut}
}{
  - :\vdash \Delta_5, \mathtt{F}_3\oplus\mathtt{F}_4
}\ \oplus_A
$$

- Case rule $\bot$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1}
  \qquad
  \cfrac{\mathtt{h}_2 :\vdash \Delta_3}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_3}\ \bot
}{
  - :\vdash *, \Delta_3
}\ \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{}{- :\vdash \Delta_3}\ \mathtt{ax}
$$

- Case rule $\top$

$$\frac{\overline{\bullet h_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \qquad \overline{\bullet h_2 :\vdash dual(\mathbf{1}), \top, \Delta_3}\ \top}{- :\vdash *, \top, \Delta_3}\ \text{Cut}$$

$$\leadsto$$

$$\frac{}{- :\vdash \Delta_3, \top}\ \top$$

- Case rule I

- Case rule $\otimes$

$$\frac{\overline{\bullet h_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \qquad \dfrac{h_2 :\vdash \bot, F_4, \Delta_6 \quad h_2 :\vdash F_5, \Delta_3}{\bullet h_2 :\vdash dual(\mathbf{1}), \Delta_3, \Delta_6, F_4 \otimes F_5}\ \otimes}{- :\vdash *, \Delta_3, \Delta_6, F_4 \otimes F_5}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{\overline{\bullet h_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \overline{h_2 :\vdash \Delta_6, F_4, \bot}\ \text{ax}}{- :\vdash \Delta_6, F_4}\ \text{hCut} \qquad \overline{- :\vdash \Delta_3, F_5}\ \text{ax}}{- :\vdash \Delta_3, \Delta_6, F_4 \otimes F_5}\ \otimes$$

$$\frac{\overline{\bullet h_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \qquad \dfrac{h_2 :\vdash F_4, \Delta_3 \quad h_2 :\vdash \bot, F_5, \Delta_6}{\bullet h_2 :\vdash dual(\mathbf{1}), \Delta_3, \Delta_6, F_4 \otimes F_5}\ \otimes}{- :\vdash *, \Delta_3, \Delta_6, F_4 \otimes F_5}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\overline{- :\vdash \Delta_3, F_4}\ \text{ax} \qquad \dfrac{\overline{\bullet h_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \overline{h_2 :\vdash \Delta_6, F_5, \bot}\ \text{ax}}{- :\vdash \Delta_6, F_5}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_6, F_4 \otimes F_5}\ \otimes$$

4.2 Status of $: OK

- Case rule $\mathbf{1}$

- Case rule $

$$\frac{\dfrac{h_1 :\vdash F_5, F_6, \Delta_2}{\bullet h_1 :\vdash F_5\$F_6, \Delta_2}\ \$ \qquad \dfrac{h_7 :\vdash F_8, F_9, \Delta_{10}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_{10}, F_8\$F_9}\ \$}{- :\vdash \Delta_2, \Delta_{10}, F_8\$F_9}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{\overline{\bullet h_1 :\vdash \Delta_2, F_5\$F_6}\ \text{ax} \qquad \overline{h_7 :\vdash \Delta_{10}, F_8, F_9, dual(F_5) \otimes dual(F_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, F_8, F_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_2, F_8\$F_9}\ \$$$

$$\frac{\dfrac{h_2 :\vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\$F_4}\ \$ \qquad \dfrac{h_7 :\vdash F_8, F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8\$F_9}\ \$}{- :\vdash (\Delta_5, F_3\$F_4), \Delta_{10}, F_8\$F_9}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{\overline{h_2 :\vdash \Delta_5, F_3, F_4, F_6}\ \text{ax} \qquad \overline{\bullet h_7 :\vdash \Delta_{10}, dual(F_6), F_8\$F_9}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8\$F_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_5, F_3\$F_4, F_8\$F_9}\ \$$$

- Case rule $\&$

$$\frac{\dfrac{h_1 :\vdash F_5, F_6, \Delta_2}{\bullet h_1 :\vdash F_5\$F_6, \Delta_2}\ \$ \qquad \dfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_5) \otimes dual(F_6) \quad h_7 :\vdash F_9, \Delta_{10}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_{10}, F_8\&F_9}\ \&}{- :\vdash \Delta_2, \Delta_{10}, F_8\&F_9}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{\overline{\bullet h_1 :\vdash \Delta_2, F_5\$F_6}\ \text{ax} \ \ \overline{h_7 :\vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, F_8}\ \text{hCut} \quad \dfrac{\overline{\bullet h_1 :\vdash \Delta_2, F_5\$F_6}\ \text{ax} \ \ \overline{h_7 :\vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, F_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_2, F_8\&F_9}\ \&$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\$F_4}\,\$ \quad \dfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_6) \quad h_7 :\vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8\&F_9}\,\&}{- :\vdash (\Delta_5, F_3\$F_4), \Delta_{10}, F_8\&F_9}\,\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_5, F_3, F_4, F_6}\,\text{ax} \quad \dfrac{}{\bullet h_7 :\vdash \Delta_{10}, dual(F_6), F_8\&F_9}\,\text{ax}}{- :\vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8\&F_9}\,\text{hCut}}{- :\vdash \Delta_{10}, \Delta_5, F_3\$F_4, F_8\&F_9}\,\$$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{h_1 :\vdash F_5, F_6, \Delta_2}{\bullet h_1 :\vdash F_5\$F_6, \Delta_2}\,\$ \quad \dfrac{h_7 :\vdash F_9, \Delta_{10}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_{10}, F_8 \oplus F_9}\,\oplus_B}{- :\vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9}\,\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5\$F_6}\,\text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}\,\text{ax}}{- :\vdash \Delta_{10}, \Delta_2, F_9}\,\text{hCut}}{- :\vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9}\,\oplus_B$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\$F_4}\,\$ \quad \dfrac{h_7 :\vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9}\,\oplus_B}{- :\vdash (\Delta_5, F_3\$F_4), \Delta_{10}, F_8 \oplus F_9}\,\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_5, F_3, F_4, F_6}\,\text{ax} \quad \dfrac{}{\bullet h_7 :\vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}\,\text{ax}}{- :\vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \oplus F_9}\,\text{hCut}}{- :\vdash \Delta_{10}, \Delta_5, F_3\$F_4, F_8 \oplus F_9}\,\$$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{h_1 :\vdash F_5, F_6, \Delta_2}{\bullet h_1 :\vdash F_5\$F_6, \Delta_2}\,\$ \quad \dfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_{10}, F_8 \oplus F_9}\,\oplus_A}{- :\vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9}\,\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5\$F_6}\,\text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}\,\text{ax}}{- :\vdash \Delta_{10}, \Delta_2, F_8}\,\text{hCut}}{- :\vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9}\,\oplus_A$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\$F_4}\,\$ \quad \dfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9}\,\oplus_A}{- :\vdash (\Delta_5, F_3\$F_4), \Delta_{10}, F_8 \oplus F_9}\,\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_5, F_3, F_4, F_6}\,\text{ax} \quad \dfrac{}{\bullet h_7 :\vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}\,\text{ax}}{- :\vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \oplus F_9}\,\text{hCut}}{- :\vdash \Delta_{10}, \Delta_5, F_3\$F_4, F_8 \oplus F_9}\,\$$$

- Case rule $\perp$

$$\dfrac{\dfrac{h_1 :\vdash F_5, F_6, \Delta_2}{\bullet h_1 :\vdash F_5\$F_6, \Delta_2}\,\$ \quad \dfrac{h_7 :\vdash \Delta_8, dual(F_5) \otimes dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \perp, \Delta_8}\,\perp}{- :\vdash \Delta_2, \perp, \Delta_8}\,\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5\$F_6}\,\text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_8, dual(F_5) \otimes dual(F_6)}\,\text{ax}}{- :\vdash \Delta_2, \Delta_8}\,\text{hCut}}{- :\vdash \Delta_2, \Delta_8, \perp}\,\perp$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\$F_4}\,\$ \quad \dfrac{h_7 :\vdash \Delta_8, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \perp, \Delta_8}\,\perp}{- :\vdash (\Delta_5, F_3\$F_4), \perp, \Delta_8}\,\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_5, F_3, F_4, F_6}\,\text{ax} \quad \dfrac{}{\bullet h_7 :\vdash \Delta_8, \perp, dual(F_6)}\,\text{ax}}{- :\vdash \Delta_5, \Delta_8, F_3, F_4, \perp}\,\text{hCut}}{- :\vdash \Delta_5, \Delta_8, \perp, F_3\$F_4}\,\$$$

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• Case rule $\top$

$$\dfrac{\dfrac{h_1 :\vdash F_5, F_6, \Delta_2}{\bullet h_1 :\vdash F_5\$F_6, \Delta_2} \ \$ \quad \dfrac{}{\bullet h_7 :\vdash dual(F_5\$F_6), \top, \Delta_8} \ \top}{- :\vdash \Delta_2, \top, \Delta_8} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- :\vdash \Delta_2, \Delta_8, \top} \ \top$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\$F_4} \ \$ \quad \dfrac{}{\bullet h_7 :\vdash dual(F_6), \top, \Delta_8} \ \top}{- :\vdash (\Delta_5, F_3\$F_4), \top, \Delta_8} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- :\vdash \Delta_5, \Delta_8, \top, F_3\$F_4} \ \top$$

• Case rule I

• Case rule $\otimes$

$$\dfrac{\dfrac{h_1 :\vdash F_5, F_6, \Delta_2}{\bullet h_1 :\vdash F_5\$F_6, \Delta_2} \ \$ \quad \dfrac{h_7 :\vdash F_9, \Delta_{11}, dual(F_5) \otimes dual(F_6) \quad h_7 :\vdash F_{10}, \Delta_8}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \ \otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5\$F_6} \ \text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_{11}, F_9, dual(F_5) \otimes dual(F_6)} \ \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, F_9} \ \text{hCut} \quad \dfrac{}{- :\vdash \Delta_8, F_{10}} \ \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \ \otimes$$

$$\dfrac{\dfrac{h_1 :\vdash F_5, F_6, \Delta_2}{\bullet h_1 :\vdash F_5\$F_6, \Delta_2} \ \$ \quad \dfrac{h_7 :\vdash F_9, \Delta_8 \quad h_7 :\vdash F_{10}, \Delta_{11}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \ \otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_8, F_9} \ \text{ax} \quad \dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5\$F_6} \ \text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_{11}, F_{10}, dual(F_5) \otimes dual(F_6)} \ \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, F_{10}} \ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \ \otimes$$

$$\dfrac{\dfrac{h_1 :\vdash F_5, F_6, \Delta_2}{\bullet h_1 :\vdash F_5\$F_6, \Delta_2} \ \$ \quad \dfrac{h_7 :\vdash \Delta_8, dual(F_5) \quad h_7 :\vdash \Delta_9, dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_8, \Delta_9} \ \otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_9} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \Delta_2, F_5, F_6} \ \text{ax} \quad \dfrac{}{- :\vdash \Delta_9, dual(F_6)} \ \text{ax}}{- :\vdash \Delta_2, \Delta_9, F_5} \ \text{sCut} \quad \dfrac{}{- :\vdash \Delta_8, dual(F_5)} \ \text{ax}}{- :\vdash \Delta_2, \Delta_8, \Delta_9} \ \text{sCut}$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\$F_4} \ \$ \quad \dfrac{h_7 :\vdash F_9, \Delta_{11}, dual(F_6) \quad h_7 :\vdash F_{10}, \Delta_8}{\bullet h_7 :\vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \ \otimes}{- :\vdash (\Delta_5, F_3\$F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_5, F_3, F_4, F_6} \ \text{ax} \quad \dfrac{}{\bullet h_7 :\vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}} \ \text{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_4, F_9 \otimes F_{10}} \ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_3\$F_4, F_9 \otimes F_{10}} \ \$$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\$F_4} \ \$ \quad \dfrac{h_7 :\vdash F_9, \Delta_8 \quad h_7 :\vdash F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \ \otimes}{- :\vdash (\Delta_5, F_3\$F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_5, F_3, F_4, F_6} \ \text{ax} \quad \dfrac{}{\bullet h_7 :\vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}} \ \text{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_4, F_9 \otimes F_{10}} \ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_3\$F_4, F_9 \otimes F_{10}} \ \$$$

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4.3 Status of &: `OK`

- Case rule **1**

- Case rule $

$$
\cfrac{
\cfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5\&F_6, \Delta_2}\ \&
\quad
\cfrac{h_7 :\vdash F_8, F_9, \Delta_{10}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 :\vdash dual(F_5\&F_6), \Delta_{10}, F_8\$F_9}\ \$
}{- :\vdash \Delta_2, \Delta_{10}, F_8\$F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{\ }{\bullet h_1 :\vdash \Delta_2, F_5\&F_6}\ \text{ax}
\quad
\cfrac{\ }{h_7 :\vdash \Delta_{10}, F_8, F_9, dual(F_5) \oplus dual(F_6)}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_2, F_8, F_9}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_2, F_8\$F_9}\ \$
$$

$$
\cfrac{
\cfrac{h_2 :\vdash F_6, F_3, \Delta_5 \quad h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\&F_4}\ \&
\quad
\cfrac{h_7 :\vdash F_8, F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8\$F_9}\ \$
}{- :\vdash (\Delta_5, F_3\&F_4), \Delta_{10}, F_8\$F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{\ }{\bullet h_2 :\vdash \Delta_5, F_6, F_3\&F_4}\ \text{ax}
\quad
\cfrac{\ }{h_7 :\vdash \Delta_{10}, F_8, F_9, dual(F_6)}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3\&F_4}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_5, F_8\$F_9, F_3\&F_4}\ \$
$$

- Case rule &

$$
\cfrac{
\cfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5\&F_6, \Delta_2}\ \&
\quad
\cfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_5) \oplus dual(F_6) \quad h_7 :\vdash F_9, \Delta_{10}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 :\vdash dual(F_5\&F_6), \Delta_{10}, F_8\&F_9}\ \&
}{- :\vdash \Delta_2, \Delta_{10}, F_8\&F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{\ }{\bullet h_1 :\vdash \Delta_2, F_5\&F_6}\ \text{ax}\ \ \cfrac{\ }{h_7 :\vdash \Delta_{10}, F_8, dual(F_5)\oplus dual(F_6)}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_2, F_8}\ \text{hCut}
\quad
\cfrac{
\cfrac{\ }{\bullet h_1 :\vdash \Delta_2, F_5\&F_6}\ \text{ax}\ \ \cfrac{\ }{h_7 :\vdash \Delta_{10}, F_9, dual(F_5)\oplus dual(F_6)}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_2, F_9}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_2, F_8\&F_9}\ \&
$$

$$
\cfrac{
\cfrac{h_2 :\vdash F_6, F_3, \Delta_5 \quad h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3\&F_4}\ \&
\quad
\cfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_6) \quad h_7 :\vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8\&F_9}\ \&
}{- :\vdash (\Delta_5, F_3\&F_4), \Delta_{10}, F_8\&F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{\ }{h_2 :\vdash \Delta_5, F_3, F_6}\ \text{ax}\ \ \cfrac{\ }{\bullet h_7 :\vdash \Delta_{10}, dual(F_6), F_8\&F_9}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_5, F_3, F_8\&F_9}\ \text{hCut}
\quad
\cfrac{
\cfrac{\ }{h_2 :\vdash \Delta_5, F_4, F_6}\ \text{ax}\ \ \cfrac{\ }{\bullet h_7 :\vdash \Delta_{10}, dual(F_6), F_8\&F_9}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_5, F_4, F_8\&F_9}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_5, F_3\&F_4, F_8\&F_9}\ \&
$$

- Case rule $\oplus_B$

$$
\cfrac{
\cfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5\&F_6, \Delta_2}\ \&
\quad
\cfrac{h_7 :\vdash F_9, \Delta_{10}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 :\vdash dual(F_5\&F_6), \Delta_{10}, F_8 \oplus F_9}\ \oplus_B
}{- :\vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{\ }{\bullet h_1 :\vdash \Delta_2, F_5\&F_6}\ \text{ax}
\quad
\cfrac{\ }{h_7 :\vdash \Delta_{10}, F_9, dual(F_5) \oplus dual(F_6)}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_2, F_9}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9}\ \oplus_B
$$

$$
\cfrac{
\cfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5\&F_6, \Delta_2}\ \&
\quad
\cfrac{h_7 :\vdash \Delta_8, dual(F_6)}{\bullet h_7 :\vdash dual(F_5\&F_6), \Delta_8}\ \oplus_B
}{- :\vdash \Delta_2, \Delta_8}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{\ }{- :\vdash \Delta_2, F_6}\ \text{ax}
\quad
\cfrac{\ }{- :\vdash \Delta_8, dual(F_6)}\ \text{ax}
}{- :\vdash \Delta_2, \Delta_8}\ \text{sCut}
$$

$$\frac{\dfrac{h_2 :\vdash F_6, F_3, \Delta_5 \quad h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \& F_4}\ \& \quad \dfrac{h_7 :\vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9}\ \oplus_B}{- :\vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \oplus F_9}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{}{\bullet h_2 :\vdash \Delta_5, F_6, F_3 \& F_4}\ \text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_{10}, F_9, dual(F_6)}\ \text{ax}}{\dfrac{- :\vdash \Delta_{10}, \Delta_5, F_9, F_3 \& F_4}{- :\vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9}\ \oplus_B}\ \text{hCut}$$

- Case rule $\oplus_A$

$$\frac{\dfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5 \& F_6, \Delta_2}\ \& \quad \dfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \oplus F_9}\ \oplus_A}{- :\vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5 \& F_6}\ \text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_{10}, F_8, dual(F_5) \oplus dual(F_6)}\ \text{ax}}{\dfrac{- :\vdash \Delta_{10}, \Delta_2, F_8}{- :\vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9}\ \oplus_A}\ \text{hCut}$$

$$\frac{\dfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5 \& F_6, \Delta_2}\ \& \quad \dfrac{h_7 :\vdash \Delta_8, dual(F_5)}{\bullet h_7 :\vdash dual(F_5 \& F_6), \Delta_8}\ \oplus_A}{- :\vdash \Delta_2, \Delta_8}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{}{- :\vdash \Delta_2, F_5}\ \text{ax} \quad \dfrac{}{- :\vdash \Delta_8, dual(F_5)}\ \text{ax}}{- :\vdash \Delta_2, \Delta_8}\ \text{sCut}$$

$$\frac{\dfrac{h_2 :\vdash F_6, F_3, \Delta_5 \quad h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \& F_4}\ \& \quad \dfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9}\ \oplus_A}{- :\vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \oplus F_9}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{}{\bullet h_2 :\vdash \Delta_5, F_6, F_3 \& F_4}\ \text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_{10}, F_8, dual(F_6)}\ \text{ax}}{\dfrac{- :\vdash \Delta_{10}, \Delta_5, F_8, F_3 \& F_4}{- :\vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9}\ \oplus_A}\ \text{hCut}$$

- Case rule $\perp$

$$\frac{\dfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5 \& F_6, \Delta_2}\ \& \quad \dfrac{h_7 :\vdash \Delta_8, dual(F_5) \oplus dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \& F_6), \perp, \Delta_8}\ \perp}{- :\vdash \Delta_2, \perp, \Delta_8}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5 \& F_6}\ \text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_8, dual(F_5) \oplus dual(F_6)}\ \text{ax}}{\dfrac{- :\vdash \Delta_2, \Delta_8}{- :\vdash \Delta_2, \Delta_8, \perp}\ \perp}\ \text{hCut}$$

$$\frac{\dfrac{h_2 :\vdash F_6, F_3, \Delta_5 \quad h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \& F_4}\ \& \quad \dfrac{h_7 :\vdash \Delta_8, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \perp, \Delta_8}\ \perp}{- :\vdash (\Delta_5, F_3 \& F_4), \perp, \Delta_8}\ \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{}{\bullet h_2 :\vdash \Delta_5, F_6, F_3 \& F_4}\ \text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_8, dual(F_6)}\ \text{ax}}{\dfrac{- :\vdash \Delta_5, \Delta_8, F_3 \& F_4}{- :\vdash \Delta_5, \Delta_8, \perp, F_3 \& F_4}\ \perp}\ \text{hCut}$$

- Case rule $\top$

$$\frac{\dfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5 \& F_6, \Delta_2}\ \& \quad \dfrac{}{\bullet h_7 :\vdash dual(F_5 \& F_6), \top, \Delta_8}\ \top}{- :\vdash \Delta_2, \top, \Delta_8}\ \text{Cut}$$

$$\leadsto$$

$$\frac{}{- :\vdash \Delta_2, \Delta_8, \top}\ \top$$

$$\cfrac{\cfrac{h_2 :\vdash F_6, F_3, \Delta_5 \quad h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \cfrac{}{\bullet h_7 :\vdash dual(F_6), \top, \Delta_8} \top}{\cfrac{- :\vdash (\Delta_5, F_3 \& F_4), \top, \Delta_8}{\cfrac{\rightsquigarrow}{- :\vdash \Delta_5, \Delta_8, \top, F_3 \& F_4} \top}} \text{Cut}$$

- Case rule I

- Case rule $\otimes$

$$\cfrac{\cfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5 \& F_6, \Delta_2} \& \quad \cfrac{h_7 :\vdash F_9, \Delta_{11}, dual(F_5) \oplus dual(F_6) \quad h_7 :\vdash F_{10}, \Delta_8}{\bullet h_7 :\vdash dual(F_5 \& F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, F_5 \& F_6} \text{ax} \quad \cfrac{}{h_7 :\vdash \Delta_{11}, F_9, dual(F_5) \oplus dual(F_6)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, F_9} \text{hCut} \quad \cfrac{}{- :\vdash \Delta_8, F_{10}} \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes$$

$$\cfrac{\cfrac{h_1 :\vdash F_5, \Delta_2 \quad h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5 \& F_6, \Delta_2} \& \quad \cfrac{h_7 :\vdash F_9, \Delta_8 \quad h_7 :\vdash F_{10}, \Delta_{11}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \& F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}$$

$$\cfrac{\cfrac{}{- :\vdash \Delta_8, F_9} \text{ax} \quad \cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, F_5 \& F_6} \text{ax} \quad \cfrac{}{h_7 :\vdash \Delta_{11}, F_{10}, dual(F_5) \oplus dual(F_6)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, F_{10}} \text{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes$$

$$\cfrac{\cfrac{h_2 :\vdash F_6, F_3, \Delta_5 \quad h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \cfrac{h_7 :\vdash F_9, \Delta_{11}, dual(F_6) \quad h_7 :\vdash F_{10}, \Delta_8}{\bullet h_7 :\vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- :\vdash (\Delta_5, F_3 \& F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 :\vdash \Delta_5, F_6, F_3 \& F_4} \text{ax} \quad \cfrac{}{h_7 :\vdash \Delta_{11}, F_9, dual(F_6)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_5, F_9, F_3 \& F_4} \text{hCut} \quad \cfrac{}{- :\vdash \Delta_8, F_{10}} \text{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \otimes$$

$$\cfrac{\cfrac{h_2 :\vdash F_6, F_3, \Delta_5 \quad h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \cfrac{h_7 :\vdash F_9, \Delta_8 \quad h_7 :\vdash F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- :\vdash (\Delta_5, F_3 \& F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}$$

$$\cfrac{\cfrac{}{- :\vdash \Delta_8, F_9} \text{ax} \quad \cfrac{\cfrac{}{\bullet h_2 :\vdash \Delta_5, F_6, F_3 \& F_4} \text{ax} \quad \cfrac{}{h_7 :\vdash \Delta_{11}, F_{10}, dual(F_6)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_5, F_{10}, F_3 \& F_4} \text{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \otimes$$

4.4 Status of $\oplus_B$: OK

- Case rule $\mathbf{1}$

- Case rule $

$$\cfrac{\cfrac{h_1 :\vdash F_6, \Delta_2}{\bullet h_1 :\vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \cfrac{h_7 :\vdash F_8, F_9, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \$ F_9} \$}{- :\vdash \Delta_2, \Delta_{10}, F_8 \$ F_9} \text{Cut}$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, F_5 \oplus F_6} \text{ax} \quad \cfrac{}{h_7 :\vdash \Delta_{10}, F_8, F_9, dual(F_5) \& dual(F_6)} \text{ax}}{\cfrac{- :\vdash \Delta_{10}, \Delta_2, F_8, F_9}{- :\vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \$} \text{hCut}}{}$$

$$\dfrac{\dfrac{\texttt{h}_2 :\vdash \texttt{F}_6, \texttt{F}_4, \Delta_5}{\bullet\texttt{h}_2 :\vdash \texttt{F}_6, \Delta_5, \texttt{F}_3 \oplus \texttt{F}_4}\ \oplus_B \quad \dfrac{\texttt{h}_7 :\vdash \texttt{F}_8, \texttt{F}_9, \Delta_{10}, dual(\texttt{F}_6)}{\bullet\texttt{h}_7 :\vdash dual(\texttt{F}_6), \Delta_{10}, \texttt{F}_8\$\texttt{F}_9}\ \$}{- :\vdash (\Delta_5, \texttt{F}_3 \oplus \texttt{F}_4), \Delta_{10}, \texttt{F}_8\$\texttt{F}_9}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_6, \texttt{F}_3 \oplus \texttt{F}_4}\ \text{ax} \quad \dfrac{}{\texttt{h}_7 :\vdash \Delta_{10}, \texttt{F}_8, \texttt{F}_9, dual(\texttt{F}_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_5, \texttt{F}_8, \texttt{F}_9, \texttt{F}_3 \oplus \texttt{F}_4}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \texttt{F}_8\$\texttt{F}_9, \texttt{F}_3 \oplus \texttt{F}_4}\ \$$$

- Case rule &

$$\dfrac{\dfrac{\texttt{h}_1 :\vdash \texttt{F}_6, \Delta_2}{\bullet\texttt{h}_1 :\vdash \texttt{F}_5 \oplus \texttt{F}_6, \Delta_2}\ \oplus_B \quad \dfrac{\texttt{h}_7 :\vdash \texttt{F}_8, \Delta_{10}, dual(\texttt{F}_5)\&dual(\texttt{F}_6) \quad \texttt{h}_7 :\vdash \texttt{F}_9, \Delta_{10}, dual(\texttt{F}_5)\&dual(\texttt{F}_6)}{\bullet\texttt{h}_7 :\vdash dual(\texttt{F}_5 \oplus \texttt{F}_6), \Delta_{10}, \texttt{F}_8\&\texttt{F}_9}\ \&}{- :\vdash \Delta_2, \Delta_{10}, \texttt{F}_8\&\texttt{F}_9}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\texttt{h}_1 :\vdash \Delta_2, \texttt{F}_5 \oplus \texttt{F}_6}\ \text{ax} \quad \dfrac{}{\texttt{h}_7 :\vdash \Delta_{10}, \texttt{F}_8, dual(\texttt{F}_5)\&dual(\texttt{F}_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, \texttt{F}_8}\ \text{hCut} \quad \dfrac{\dfrac{}{\bullet\texttt{h}_1 :\vdash \Delta_2, \texttt{F}_5 \oplus \texttt{F}_6}\ \text{ax} \quad \dfrac{}{\texttt{h}_7 :\vdash \Delta_{10}, \texttt{F}_9, dual(\texttt{F}_5)\&dual(\texttt{F}_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, \texttt{F}_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \texttt{F}_8\&\texttt{F}_9}\ \&$$

$$\dfrac{\dfrac{\texttt{h}_1 :\vdash \texttt{F}_6, \Delta_2}{\bullet\texttt{h}_1 :\vdash \texttt{F}_5 \oplus \texttt{F}_6, \Delta_2}\ \oplus_B \quad \dfrac{\texttt{h}_7 :\vdash \Delta_8, dual(\texttt{F}_5) \quad \texttt{h}_7 :\vdash \Delta_8, dual(\texttt{F}_6)}{\bullet\texttt{h}_7 :\vdash dual(\texttt{F}_5 \oplus \texttt{F}_6), \Delta_8}\ \&}{- :\vdash \Delta_2, \Delta_8}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_2, \texttt{F}_6}\ \text{ax} \quad \dfrac{}{- :\vdash \Delta_8, dual(\texttt{F}_6)}\ \text{ax}}{- :\vdash \Delta_2, \Delta_8}\ \text{sCut}$$

$$\dfrac{\dfrac{\texttt{h}_2 :\vdash \texttt{F}_6, \texttt{F}_4, \Delta_5}{\bullet\texttt{h}_2 :\vdash \texttt{F}_6, \Delta_5, \texttt{F}_3 \oplus \texttt{F}_4}\ \oplus_B \quad \dfrac{\texttt{h}_7 :\vdash \texttt{F}_8, \Delta_{10}, dual(\texttt{F}_6) \quad \texttt{h}_7 :\vdash \texttt{F}_9, \Delta_{10}, dual(\texttt{F}_6)}{\bullet\texttt{h}_7 :\vdash dual(\texttt{F}_6), \Delta_{10}, \texttt{F}_8\&\texttt{F}_9}\ \&}{- :\vdash (\Delta_5, \texttt{F}_3 \oplus \texttt{F}_4), \Delta_{10}, \texttt{F}_8\&\texttt{F}_9}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_4, \texttt{F}_6}\ \text{ax} \quad \dfrac{}{\bullet\texttt{h}_7 :\vdash \Delta_{10}, dual(\texttt{F}_6), \texttt{F}_8\&\texttt{F}_9}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_5, \texttt{F}_4, \texttt{F}_8\&\texttt{F}_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \texttt{F}_8\&\texttt{F}_9, \texttt{F}_3 \oplus \texttt{F}_4}\ \oplus_B$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\texttt{h}_1 :\vdash \texttt{F}_6, \Delta_2}{\bullet\texttt{h}_1 :\vdash \texttt{F}_5 \oplus \texttt{F}_6, \Delta_2}\ \oplus_B \quad \dfrac{\texttt{h}_7 :\vdash \texttt{F}_9, \Delta_{10}, dual(\texttt{F}_5)\&dual(\texttt{F}_6)}{\bullet\texttt{h}_7 :\vdash dual(\texttt{F}_5 \oplus \texttt{F}_6), \Delta_{10}, \texttt{F}_8 \oplus \texttt{F}_9}\ \oplus_B}{- :\vdash \Delta_2, \Delta_{10}, \texttt{F}_8 \oplus \texttt{F}_9}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\texttt{h}_1 :\vdash \Delta_2, \texttt{F}_5 \oplus \texttt{F}_6}\ \text{ax} \quad \dfrac{}{\texttt{h}_7 :\vdash \Delta_{10}, \texttt{F}_9, dual(\texttt{F}_5)\&dual(\texttt{F}_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, \texttt{F}_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \texttt{F}_8 \oplus \texttt{F}_9}\ \oplus_B$$

$$\dfrac{\dfrac{\texttt{h}_2 :\vdash \texttt{F}_6, \texttt{F}_4, \Delta_5}{\bullet\texttt{h}_2 :\vdash \texttt{F}_6, \Delta_5, \texttt{F}_3 \oplus \texttt{F}_4}\ \oplus_B \quad \dfrac{\texttt{h}_7 :\vdash \texttt{F}_9, \Delta_{10}, dual(\texttt{F}_6)}{\bullet\texttt{h}_7 :\vdash dual(\texttt{F}_6), \Delta_{10}, \texttt{F}_8 \oplus \texttt{F}_9}\ \oplus_B}{- :\vdash (\Delta_5, \texttt{F}_3 \oplus \texttt{F}_4), \Delta_{10}, \texttt{F}_8 \oplus \texttt{F}_9}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_4, \texttt{F}_6}\ \text{ax} \quad \dfrac{}{\bullet\texttt{h}_7 :\vdash \Delta_{10}, dual(\texttt{F}_6), \texttt{F}_8 \oplus \texttt{F}_9}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_5, \texttt{F}_4, \texttt{F}_8 \oplus \texttt{F}_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \texttt{F}_3 \oplus \texttt{F}_4, \texttt{F}_8 \oplus \texttt{F}_9}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\texttt{h}_1 :\vdash \texttt{F}_6, \Delta_2}{\bullet\texttt{h}_1 :\vdash \texttt{F}_5 \oplus \texttt{F}_6, \Delta_2}\ \oplus_B \quad \dfrac{\texttt{h}_7 :\vdash \texttt{F}_8, \Delta_{10}, dual(\texttt{F}_5)\&dual(\texttt{F}_6)}{\bullet\texttt{h}_7 :\vdash dual(\texttt{F}_5 \oplus \texttt{F}_6), \Delta_{10}, \texttt{F}_8 \oplus \texttt{F}_9}\ \oplus_A}{- :\vdash \Delta_2, \Delta_{10}, \texttt{F}_8 \oplus \texttt{F}_9}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\texttt{h}_1 :\vdash \Delta_2, \texttt{F}_5 \oplus \texttt{F}_6}\ \text{ax} \quad \dfrac{}{\texttt{h}_7 :\vdash \Delta_{10}, \texttt{F}_8, dual(\texttt{F}_5)\&dual(\texttt{F}_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, \texttt{F}_8}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \texttt{F}_8 \oplus \texttt{F}_9}\ \oplus_A$$

$$
\cfrac{
\cfrac{\mathtt{h_2} :\vdash \mathtt{F_6}, \mathtt{F_4}, \Delta_5}{\bullet \mathtt{h_2} :\vdash \mathtt{F_6}, \Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}} \oplus_B
\quad
\cfrac{\mathtt{h_7} :\vdash \mathtt{F_8}, \Delta_{10}, dual(\mathtt{F_6})}{\bullet \mathtt{h_7} :\vdash dual(\mathtt{F_6}), \Delta_{10}, \mathtt{F_8} \oplus \mathtt{F_9}} \oplus_A
}{- :\vdash (\Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}), \Delta_{10}, \mathtt{F_8} \oplus \mathtt{F_9}} \mathtt{Cut}
$$

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\rightsquigarrow
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_4}, \mathtt{F_6}} \mathtt{ax}
\quad
\cfrac{}{\bullet \mathtt{h_7} :\vdash \Delta_{10}, dual(\mathtt{F_6}), \mathtt{F_8} \oplus \mathtt{F_9}} \mathtt{ax}
}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F_4}, \mathtt{F_8} \oplus \mathtt{F_9}} \mathtt{hCut}
}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}, \mathtt{F_8} \oplus \mathtt{F_9}} \oplus_B
$$

- Case rule $\bot$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_2}{\bullet \mathtt{h_1} :\vdash \mathtt{F_5} \oplus \mathtt{F_6}, \Delta_2} \oplus_B
\quad
\cfrac{\mathtt{h_7} :\vdash \Delta_8, dual(\mathtt{F_5}) \& dual(\mathtt{F_6})}{\bullet \mathtt{h_7} :\vdash dual(\mathtt{F_5} \oplus \mathtt{F_6}), \bot, \Delta_8} \bot
}{- :\vdash \Delta_2, \bot, \Delta_8} \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet \mathtt{h_1} :\vdash \Delta_2, \mathtt{F_5} \oplus \mathtt{F_6}} \mathtt{ax}
\quad
\cfrac{}{\mathtt{h_7} :\vdash \Delta_8, dual(\mathtt{F_5}) \& dual(\mathtt{F_6})} \mathtt{ax}
}{- :\vdash \Delta_2, \Delta_8} \mathtt{hCut}
}{- :\vdash \Delta_2, \Delta_8, \bot} \bot
$$

$$
\cfrac{
\cfrac{\mathtt{h_2} :\vdash \mathtt{F_6}, \mathtt{F_4}, \Delta_5}{\bullet \mathtt{h_2} :\vdash \mathtt{F_6}, \Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}} \oplus_B
\quad
\cfrac{\mathtt{h_7} :\vdash \Delta_8, dual(\mathtt{F_6})}{\bullet \mathtt{h_7} :\vdash dual(\mathtt{F_6}), \bot, \Delta_8} \bot
}{- :\vdash (\Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}), \bot, \Delta_8} \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_4}, \mathtt{F_6}} \mathtt{ax}
\quad
\cfrac{}{\bullet \mathtt{h_7} :\vdash \Delta_8, \bot, dual(\mathtt{F_6})} \mathtt{ax}
}{- :\vdash \Delta_5, \Delta_8, \mathtt{F_4}, \bot} \mathtt{hCut}
}{- :\vdash \Delta_5, \Delta_8, \bot, \mathtt{F_3} \oplus \mathtt{F_4}} \oplus_B
$$

- Case rule $\top$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_2}{\bullet \mathtt{h_1} :\vdash \mathtt{F_5} \oplus \mathtt{F_6}, \Delta_2} \oplus_B
\quad
\cfrac{}{\bullet \mathtt{h_7} :\vdash dual(\mathtt{F_5} \oplus \mathtt{F_6}), \top, \Delta_8} \top
}{- :\vdash \Delta_2, \top, \Delta_8} \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{}{- :\vdash \Delta_2, \Delta_8, \top} \top
$$

$$
\cfrac{
\cfrac{\mathtt{h_2} :\vdash \mathtt{F_6}, \mathtt{F_4}, \Delta_5}{\bullet \mathtt{h_2} :\vdash \mathtt{F_6}, \Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}} \oplus_B
\quad
\cfrac{}{\bullet \mathtt{h_7} :\vdash dual(\mathtt{F_6}), \top, \Delta_8} \top
}{- :\vdash (\Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}), \top, \Delta_8} \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{}{- :\vdash \Delta_5, \Delta_8, \top, \mathtt{F_3} \oplus \mathtt{F_4}} \top
$$

- Case rule I

- Case rule $\otimes$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_2}{\bullet \mathtt{h_1} :\vdash \mathtt{F_5} \oplus \mathtt{F_6}, \Delta_2} \oplus_B
\quad
\cfrac{\mathtt{h_7} :\vdash \mathtt{F_9}, \Delta_{11}, dual(\mathtt{F_5}) \& dual(\mathtt{F_6}) \quad \mathtt{h_7} :\vdash \mathtt{F_{10}}, \Delta_8}{\bullet \mathtt{h_7} :\vdash dual(\mathtt{F_5} \oplus \mathtt{F_6}), \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet \mathtt{h_1} :\vdash \Delta_2, \mathtt{F_5} \oplus \mathtt{F_6}} \mathtt{ax}
\quad
\cfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, dual(\mathtt{F_5}) \& dual(\mathtt{F_6})} \mathtt{ax}
}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F_9}} \mathtt{hCut}
\quad
\cfrac{}{- :\vdash \Delta_8, \mathtt{F_{10}}} \mathtt{ax}
}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
$$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_2}{\bullet \mathtt{h_1} :\vdash \mathtt{F_5} \oplus \mathtt{F_6}, \Delta_2} \oplus_B
\quad
\cfrac{\mathtt{h_7} :\vdash \mathtt{F_9}, \Delta_8 \quad \mathtt{h_7} :\vdash \mathtt{F_{10}}, \Delta_{11}, dual(\mathtt{F_5}) \& dual(\mathtt{F_6})}{\bullet \mathtt{h_7} :\vdash dual(\mathtt{F_5} \oplus \mathtt{F_6}), \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
\cfrac{}{- :\vdash \Delta_8, \mathtt{F_9}} \mathtt{ax}
\quad
\cfrac{
\cfrac{}{\bullet \mathtt{h_1} :\vdash \Delta_2, \mathtt{F_5} \oplus \mathtt{F_6}} \mathtt{ax}
\quad
\cfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_{10}}, dual(\mathtt{F_5}) \& dual(\mathtt{F_6})} \mathtt{ax}
}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F_{10}}} \mathtt{hCut}
}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \oplus F_4}\ \oplus_B \qquad \dfrac{h_7 :\vdash F_9, \Delta_{11}, dual(F_6) \quad h_7 :\vdash F_{10}, \Delta_8}{\bullet h_7 :\vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}}\ \otimes}{- :\vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_5, F_4, F_6}\ \text{ax} \qquad \dfrac{}{\bullet h_7 :\vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4}\ \oplus_B$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_4, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \oplus F_4}\ \oplus_B \qquad \dfrac{h_7 :\vdash F_9, \Delta_8 \quad h_7 :\vdash F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}}\ \otimes}{- :\vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_5, F_4, F_6}\ \text{ax} \qquad \dfrac{}{\bullet h_7 :\vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4}\ \oplus_B$$

4.5 Status of $\oplus_A$: OK

- Case rule **1**

- Case rule $

$$\dfrac{\dfrac{h_1 :\vdash F_5, \Delta_2}{\bullet h_1 :\vdash F_5 \oplus F_6, \Delta_2}\ \oplus_A \qquad \dfrac{h_7 :\vdash F_8, F_9, \Delta_{10}, dual(F_5)\&dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8\$F_9}\ \$}{- :\vdash \Delta_2, \Delta_{10}, F_8\$F_9}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5 \oplus F_6}\ \text{ax} \qquad \dfrac{}{h_7 :\vdash \Delta_{10}, F_8, F_9, dual(F_5)\&dual(F_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, F_8, F_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_2, F_8\$F_9}\ \$$$

$$\dfrac{\dfrac{h_2 :\vdash F_6, F_3, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \oplus F_4}\ \oplus_A \qquad \dfrac{h_7 :\vdash F_8, F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8\$F_9}\ \$}{- :\vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8\$F_9}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_2 :\vdash \Delta_5, F_6, F_3 \oplus F_4}\ \text{ax} \qquad \dfrac{}{h_7 :\vdash \Delta_{10}, F_8, F_9, dual(F_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3 \oplus F_4}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_5, F_8\$F_9, F_3 \oplus F_4}\ \$$$

- Case rule &

$$\dfrac{\dfrac{h_1 :\vdash F_5, \Delta_2}{\bullet h_1 :\vdash F_5 \oplus F_6, \Delta_2}\ \oplus_A \qquad \dfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_5)\&dual(F_6) \quad h_7 :\vdash F_9, \Delta_{10}, dual(F_5)\&dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8\&F_9}\ \&}{- :\vdash \Delta_2, \Delta_{10}, F_8\&F_9}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5 \oplus F_6}\ \text{ax}\ \dfrac{}{h_7 :\vdash \Delta_{10}, F_8, dual(F_5)\&dual(F_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, F_8}\ \text{hCut} \quad \dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, F_5 \oplus F_6}\ \text{ax}\ \dfrac{}{h_7 :\vdash \Delta_{10}, F_9, dual(F_5)\&dual(F_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_2, F_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_2, F_8\&F_9}\ \&$$

$$\dfrac{\dfrac{h_1 :\vdash F_5, \Delta_2}{\bullet h_1 :\vdash F_5 \oplus F_6, \Delta_2}\ \oplus_A \qquad \dfrac{h_7 :\vdash \Delta_8, dual(F_5) \quad h_7 :\vdash \Delta_8, dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \oplus F_6), \Delta_8}\ \&}{- :\vdash \Delta_2, \Delta_8}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_2, F_5}\ \text{ax} \qquad \dfrac{}{- :\vdash \Delta_8, dual(F_5)}\ \text{ax}}{- :\vdash \Delta_2, \Delta_8}\ \text{sCut}$$

24

$$
\cfrac{
\cfrac{h_2 :\vdash F_6, F_3, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \oplus F_4}\ \oplus_A
\quad
\cfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_6) \quad h_7 :\vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8 \& F_9}\ \&
}{- :\vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{h_2 :\vdash \Delta_5, F_3, F_6}{}\ \text{ax}
\quad
\cfrac{\bullet h_7 :\vdash \Delta_{10}, dual(F_6), F_8 \& F_9}{}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_5, F_3, F_8 \& F_9}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4}\ \oplus_A
$$

- Case rule $\oplus_B$

$$
\cfrac{
\cfrac{h_1 :\vdash F_5, \Delta_2}{\bullet h_1 :\vdash F_5 \oplus F_6, \Delta_2}\ \oplus_A
\quad
\cfrac{h_7 :\vdash F_9, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9}\ \oplus_B
}{- :\vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{\bullet h_1 :\vdash \Delta_2, F_5 \oplus F_6}{}\ \text{ax}
\quad
\cfrac{h_7 :\vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_2, F_9}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9}\ \oplus_B
$$

$$
\cfrac{
\cfrac{h_2 :\vdash F_6, F_3, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \oplus F_4}\ \oplus_A
\quad
\cfrac{h_7 :\vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9}\ \oplus_B
}{- :\vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{\bullet h_2 :\vdash \Delta_5, F_6, F_3 \oplus F_4}{}\ \text{ax}
\quad
\cfrac{h_7 :\vdash \Delta_{10}, F_9, dual(F_6)}{}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_5, F_9, F_3 \oplus F_4}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9}\ \oplus_B
$$

- Case rule $\oplus_A$

$$
\cfrac{
\cfrac{h_1 :\vdash F_5, \Delta_2}{\bullet h_1 :\vdash F_5 \oplus F_6, \Delta_2}\ \oplus_A
\quad
\cfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9}\ \oplus_A
}{- :\vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{\bullet h_1 :\vdash \Delta_2, F_5 \oplus F_6}{}\ \text{ax}
\quad
\cfrac{h_7 :\vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_2, F_8}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9}\ \oplus_A
$$

$$
\cfrac{
\cfrac{h_2 :\vdash F_6, F_3, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \oplus F_4}\ \oplus_A
\quad
\cfrac{h_7 :\vdash F_8, \Delta_{10}, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9}\ \oplus_A
}{- :\vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{h_2 :\vdash \Delta_5, F_3, F_6}{}\ \text{ax}
\quad
\cfrac{\bullet h_7 :\vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{}\ \text{ax}
}{- :\vdash \Delta_{10}, \Delta_5, F_3, F_8 \oplus F_9}\ \text{hCut}
}{- :\vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9}\ \oplus_A
$$

- Case rule $\perp$

$$
\cfrac{
\cfrac{h_1 :\vdash F_5, \Delta_2}{\bullet h_1 :\vdash F_5 \oplus F_6, \Delta_2}\ \oplus_A
\quad
\cfrac{h_7 :\vdash \Delta_8, dual(F_5) \& dual(F_6)}{\bullet h_7 :\vdash dual(F_5 \oplus F_6), \perp, \Delta_8}\ \perp
}{- :\vdash \Delta_2, \perp, \Delta_8}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{\bullet h_1 :\vdash \Delta_2, F_5 \oplus F_6}{}\ \text{ax}
\quad
\cfrac{h_7 :\vdash \Delta_8, dual(F_5) \& dual(F_6)}{}\ \text{ax}
}{- :\vdash \Delta_2, \Delta_8}\ \text{hCut}
}{- :\vdash \Delta_2, \Delta_8, \perp}\ \perp
$$

$$
\cfrac{
\cfrac{h_2 :\vdash F_6, F_3, \Delta_5}{\bullet h_2 :\vdash F_6, \Delta_5, F_3 \oplus F_4}\ \oplus_A
\quad
\cfrac{h_7 :\vdash \Delta_8, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \perp, \Delta_8}\ \perp
}{- :\vdash (\Delta_5, F_3 \oplus F_4), \perp, \Delta_8}\ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\cfrac{
\cfrac{
\cfrac{h_2 :\vdash \Delta_5, F_3, F_6}{}\ \text{ax}
\quad
\cfrac{\bullet h_7 :\vdash \Delta_8, \perp, dual(F_6)}{}\ \text{ax}
}{- :\vdash \Delta_5, \Delta_8, F_3, \perp}\ \text{hCut}
}{- :\vdash \Delta_5, \Delta_8, \perp, F_3 \oplus F_4}\ \oplus_A
$$

- Case rule $\top$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_2}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5} \oplus \mathtt{F_6}, \Delta_2} \oplus_A \quad \cfrac{}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_5} \oplus \mathtt{F_6}), \top, \Delta_8} \top
}{- :\vdash \Delta_2, \top, \Delta_8} \mathtt{Cut}
$$
$$
\leadsto
$$
$$
\cfrac{}{- :\vdash \Delta_2, \Delta_8, \top} \top
$$

$$
\cfrac{
\cfrac{\mathtt{h_2} :\vdash \mathtt{F_6}, \mathtt{F_3}, \Delta_5}{\bullet\mathtt{h_2} :\vdash \mathtt{F_6}, \Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}} \oplus_A \quad \cfrac{}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_6}), \top, \Delta_8} \top
}{- :\vdash (\Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}), \top, \Delta_8} \mathtt{Cut}
$$
$$
\leadsto
$$
$$
\cfrac{}{- :\vdash \Delta_5, \Delta_8, \top, \mathtt{F_3} \oplus \mathtt{F_4}} \top
$$

- Case rule I

- Case rule $\otimes$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_2}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5} \oplus \mathtt{F_6}, \Delta_2} \oplus_A \quad \cfrac{\mathtt{h_7} :\vdash \mathtt{F_9}, \Delta_{11}, dual(\mathtt{F_5}) \& dual(\mathtt{F_6}) \quad \mathtt{h_7} :\vdash \mathtt{F_{10}}, \Delta_8}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_5} \oplus \mathtt{F_6}), \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{Cut}
$$
$$
\leadsto
$$
$$
\cfrac{
\cfrac{
\cfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_2, \mathtt{F_5} \oplus \mathtt{F_6}} \mathtt{ax} \quad \cfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, dual(\mathtt{F_5}) \& dual(\mathtt{F_6})} \mathtt{ax}
}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F_9}} \mathtt{hCut} \quad \cfrac{}{- :\vdash \Delta_8, \mathtt{F_{10}}} \mathtt{ax}
}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
$$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_2}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5} \oplus \mathtt{F_6}, \Delta_2} \oplus_A \quad \cfrac{\mathtt{h_7} :\vdash \mathtt{F_9}, \Delta_8 \quad \mathtt{h_7} :\vdash \mathtt{F_{10}}, \Delta_{11}, dual(\mathtt{F_5}) \& dual(\mathtt{F_6})}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_5} \oplus \mathtt{F_6}), \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{Cut}
$$
$$
\leadsto
$$
$$
\cfrac{
\cfrac{}{- :\vdash \Delta_8, \mathtt{F_9}} \mathtt{ax} \quad \cfrac{
\cfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_2, \mathtt{F_5} \oplus \mathtt{F_6}} \mathtt{ax} \quad \cfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_{10}}, dual(\mathtt{F_5}) \& dual(\mathtt{F_6})} \mathtt{ax}
}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F_{10}}} \mathtt{hCut}
}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
$$

$$
\cfrac{
\cfrac{\mathtt{h_2} :\vdash \mathtt{F_6}, \mathtt{F_3}, \Delta_5}{\bullet\mathtt{h_2} :\vdash \mathtt{F_6}, \Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}} \oplus_A \quad \cfrac{\mathtt{h_7} :\vdash \mathtt{F_9}, \Delta_{11}, dual(\mathtt{F_6}) \quad \mathtt{h_7} :\vdash \mathtt{F_{10}}, \Delta_8}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_6}), \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
}{- :\vdash (\Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}), \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{Cut}
$$
$$
\leadsto
$$
$$
\cfrac{
\cfrac{
\cfrac{}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}, \mathtt{F_6}} \mathtt{ax} \quad \cfrac{}{\bullet\mathtt{h_7} :\vdash \Delta_{11}, \Delta_8, dual(\mathtt{F_6}), \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{ax}
}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F_3}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{hCut}
}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F_9} \otimes \mathtt{F_{10}}, \mathtt{F_3} \oplus \mathtt{F_4}} \oplus_A
$$

$$
\cfrac{
\cfrac{\mathtt{h_2} :\vdash \mathtt{F_6}, \mathtt{F_3}, \Delta_5}{\bullet\mathtt{h_2} :\vdash \mathtt{F_6}, \Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}} \oplus_A \quad \cfrac{\mathtt{h_7} :\vdash \mathtt{F_9}, \Delta_8 \quad \mathtt{h_7} :\vdash \mathtt{F_{10}}, \Delta_{11}, dual(\mathtt{F_6})}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_6}), \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \otimes
}{- :\vdash (\Delta_5, \mathtt{F_3} \oplus \mathtt{F_4}), \Delta_8, \Delta_{11}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{Cut}
$$
$$
\leadsto
$$
$$
\cfrac{
\cfrac{
\cfrac{}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}, \mathtt{F_6}} \mathtt{ax} \quad \cfrac{}{\bullet\mathtt{h_7} :\vdash \Delta_{11}, \Delta_8, dual(\mathtt{F_6}), \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{ax}
}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F_3}, \mathtt{F_9} \otimes \mathtt{F_{10}}} \mathtt{hCut}
}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F_9} \otimes \mathtt{F_{10}}, \mathtt{F_3} \oplus \mathtt{F_4}} \oplus_A
$$

4.6 Status of $\bot$: OK

- Case rule $\mathbf{1}$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \Delta_2}{\bullet\mathtt{h_1} :\vdash \bot, \Delta_2} \bot \quad \cfrac{}{\bullet\mathtt{h_3} :\vdash dual(\bot), *} \mathbf{1}
}{- :\vdash \Delta_2, *} \mathtt{Cut}
$$
$$
\leadsto
$$
$$
\cfrac{}{- :\vdash \Delta_2} \mathtt{ax}
$$

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- Case rule $

$$\dfrac{\dfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \quad \dfrac{h_3 :\vdash \mathbf{1}, F_4, F_5, \Delta_6}{\bullet h_3 :\vdash dual(\bot), \Delta_6, F_4\$F_5}\ \$}{- :\vdash \Delta_2, \Delta_6, F_4\$F_5}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \bot}\ \text{ax} \quad \dfrac{}{h_3 :\vdash \mathbf{1}, \Delta_6, F_4, F_5}\ \text{ax}}{- :\vdash \Delta_2, \Delta_6, F_4, F_5}\ \text{hCut}}{- :\vdash \Delta_2, \Delta_6, F_4\$F_5}\ \$$$

$$\dfrac{\dfrac{h_2 :\vdash F_4, \Delta_3}{\bullet h_2 :\vdash F_4, \bot, \Delta_3}\ \bot \quad \dfrac{h_5 :\vdash F_6, F_7, \Delta_8, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, F_6\$F_7}\ \$}{- :\vdash (\bot, \Delta_3), \Delta_8, F_6\$F_7}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_2 :\vdash \Delta_3, F_4, \bot}\ \text{ax} \quad \dfrac{}{h_5 :\vdash \Delta_8, F_6, F_7, dual(F_4)}\ \text{ax}}{\dfrac{- :\vdash \Delta_3, \Delta_8, F_6, F_7, \bot}{- :\vdash \Delta_3, \Delta_8, \bot, F_6\$F_7}\ \$}\ \text{hCut}}{}$$

- Case rule &

$$\dfrac{\dfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \quad \dfrac{h_3 :\vdash \mathbf{1}, F_4, \Delta_6 \quad h_3 :\vdash \mathbf{1}, F_5, \Delta_6}{\bullet h_3 :\vdash dual(\bot), \Delta_6, F_4\&F_5}\ \&}{- :\vdash \Delta_2, \Delta_6, F_4\&F_5}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \bot}\ \text{ax} \quad \dfrac{}{h_3 :\vdash \mathbf{1}, \Delta_6, F_4}\ \text{ax}}{- :\vdash \Delta_2, \Delta_6, F_4}\ \text{hCut} \quad \dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \bot}\ \text{ax} \quad \dfrac{}{h_3 :\vdash \mathbf{1}, \Delta_6, F_5}\ \text{ax}}{- :\vdash \Delta_2, \Delta_6, F_5}\ \text{hCut}}{- :\vdash \Delta_2, \Delta_6, F_4\&F_5}\ \&$$

$$\dfrac{\dfrac{h_2 :\vdash F_4, \Delta_3}{\bullet h_2 :\vdash F_4, \bot, \Delta_3}\ \bot \quad \dfrac{h_5 :\vdash F_6, \Delta_8, dual(F_4) \quad h_5 :\vdash F_7, \Delta_8, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, F_6\&F_7}\ \&}{- :\vdash (\bot, \Delta_3), \Delta_8, F_6\&F_7}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_3, F_4}\ \text{ax} \quad \dfrac{}{\bullet h_5 :\vdash \Delta_8, dual(F_4), F_6\&F_7}\ \text{ax}}{\dfrac{- :\vdash \Delta_3, \Delta_8, F_6\&F_7}{- :\vdash \Delta_3, \Delta_8, \bot, F_6\&F_7}\ \bot}\ \text{hCut}}{}$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \quad \dfrac{h_3 :\vdash \mathbf{1}, F_5, \Delta_6}{\bullet h_3 :\vdash dual(\bot), \Delta_6, F_4 \oplus F_5}\ \oplus_B}{- :\vdash \Delta_2, \Delta_6, F_4 \oplus F_5}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \bot}\ \text{ax} \quad \dfrac{}{h_3 :\vdash \mathbf{1}, \Delta_6, F_5}\ \text{ax}}{\dfrac{- :\vdash \Delta_2, \Delta_6, F_5}{- :\vdash \Delta_2, \Delta_6, F_4 \oplus F_5}\ \oplus_B}\ \text{hCut}}{}$$

$$\dfrac{\dfrac{h_2 :\vdash F_4, \Delta_3}{\bullet h_2 :\vdash F_4, \bot, \Delta_3}\ \bot \quad \dfrac{h_5 :\vdash F_7, \Delta_8, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, F_6 \oplus F_7}\ \oplus_B}{- :\vdash (\bot, \Delta_3), \Delta_8, F_6 \oplus F_7}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_2 :\vdash \Delta_3, F_4, \bot}\ \text{ax} \quad \dfrac{}{h_5 :\vdash \Delta_8, F_7, dual(F_4)}\ \text{ax}}{\dfrac{- :\vdash \Delta_3, \Delta_8, F_7, \bot}{- :\vdash \Delta_3, \Delta_8, \bot, F_6 \oplus F_7}\ \oplus_B}\ \text{hCut}}{}$$

- Case rule $\oplus_A$

$$\frac{\dfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \perp, \Delta_2}\perp \qquad \dfrac{h_3 :\vdash \mathbf{1}, F_4, \Delta_6}{\bullet h_3 :\vdash dual(\perp), \Delta_6, F_4 \oplus F_5}\oplus_A}{- :\vdash \Delta_2, \Delta_6, F_4 \oplus F_5}\text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \perp}\text{ax} \qquad \dfrac{}{h_3 :\vdash \mathbf{1}, \Delta_6, F_4}\text{ax}}{- :\vdash \Delta_2, \Delta_6, F_4}\text{hCut}}{- :\vdash \Delta_2, \Delta_6, F_4 \oplus F_5}\oplus_A$$

$$\frac{\dfrac{h_2 :\vdash F_4, \Delta_3}{\bullet h_2 :\vdash F_4, \perp, \Delta_3}\perp \qquad \dfrac{h_5 :\vdash F_6, \Delta_8, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, F_6 \oplus F_7}\oplus_A}{- :\vdash (\perp, \Delta_3), \Delta_8, F_6 \oplus F_7}\text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{\dfrac{}{\bullet h_2 :\vdash \Delta_3, F_4, \perp}\text{ax} \qquad \dfrac{}{h_5 :\vdash \Delta_8, F_6, dual(F_4)}\text{ax}}{- :\vdash \Delta_3, \Delta_8, F_6, \perp}\text{hCut}}{- :\vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7}\oplus_A$$

- Case rule $\perp$

$$\frac{\dfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \perp, \Delta_2}\perp \qquad \dfrac{h_3 :\vdash \mathbf{1}, \Delta_4}{\bullet h_3 :\vdash dual(\perp), \perp, \Delta_4}\perp}{- :\vdash \Delta_2, \perp, \Delta_4}\text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \perp}\text{ax} \qquad \dfrac{}{h_3 :\vdash \mathbf{1}, \Delta_4}\text{ax}}{- :\vdash \Delta_2, \Delta_4}\text{hCut}}{- :\vdash \Delta_2, \Delta_4, \perp}\perp$$

$$\frac{\dfrac{h_2 :\vdash F_4, \Delta_3}{\bullet h_2 :\vdash F_4, \perp, \Delta_3}\perp \qquad \dfrac{h_5 :\vdash \Delta_6, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \perp, \Delta_6}\perp}{- :\vdash (\perp, \Delta_3), \perp, \Delta_6}\text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_3, F_4}\text{ax} \qquad \dfrac{}{\bullet h_5 :\vdash \Delta_6, \perp, dual(F_4)}\text{ax}}{- :\vdash \Delta_3, \Delta_6, \perp}\text{hCut}}{- :\vdash \Delta_3, \Delta_6, \perp, \perp}\perp$$

- Case rule $\top$

$$\frac{\dfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \perp, \Delta_2}\perp \qquad \dfrac{}{\bullet h_3 :\vdash dual(\perp), \top, \Delta_4}\top}{- :\vdash \Delta_2, \top, \Delta_4}\text{Cut}$$

$$\rightsquigarrow$$

$$\frac{}{- :\vdash \Delta_2, \Delta_4, \top}\top$$

$$\frac{\dfrac{h_2 :\vdash F_4, \Delta_3}{\bullet h_2 :\vdash F_4, \perp, \Delta_3}\perp \qquad \dfrac{}{\bullet h_5 :\vdash dual(F_4), \top, \Delta_6}\top}{- :\vdash (\perp, \Delta_3), \top, \Delta_6}\text{Cut}$$

$$\rightsquigarrow$$

$$\frac{}{- :\vdash \Delta_3, \Delta_6, \perp, \top}\top$$

- Case rule I

- Case rule $\otimes$

$$\frac{\dfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \perp, \Delta_2}\perp \qquad \dfrac{h_3 :\vdash \mathbf{1}, F_5, \Delta_7 \quad h_3 :\vdash F_6, \Delta_4}{\bullet h_3 :\vdash dual(\perp), \Delta_4, \Delta_7, F_5 \otimes F_6}\otimes}{- :\vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6}\text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \perp}\text{ax} \quad \dfrac{}{h_3 :\vdash \mathbf{1}, \Delta_7, F_5}\text{ax}}{- :\vdash \Delta_2, \Delta_7, F_5}\text{hCut} \qquad \dfrac{}{- :\vdash \Delta_4, F_6}\text{ax}}{- :\vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6}\otimes$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \qquad \dfrac{h_3 :\vdash F_5, \Delta_4 \quad h_3 :\vdash \mathbf{1}, F_6, \Delta_7}{\bullet h_3 :\vdash dual(\bot), \Delta_4, \Delta_7, F_5 \otimes F_6}\ \otimes}{- :\vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_4, F_5}\ \text{ax} \qquad \dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \bot}\ \text{ax} \quad \dfrac{}{h_3 :\vdash \mathbf{1}, \Delta_7, F_6}\ \text{ax}}{- :\vdash \Delta_2, \Delta_7, F_6}\ \text{hCut}}{- :\vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6}\ \otimes$$

$$\dfrac{\dfrac{h_2 :\vdash F_4, \Delta_3}{\bullet h_2 :\vdash F_4, \bot, \Delta_3}\ \bot \qquad \dfrac{h_5 :\vdash F_7, \Delta_9, dual(F_4) \quad h_5 :\vdash F_8, \Delta_6}{\bullet h_5 :\vdash dual(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8}\ \otimes}{- :\vdash (\bot, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_3, F_4}\ \text{ax} \qquad \dfrac{}{\bullet h_5 :\vdash \Delta_6, \Delta_9, dual(F_4), F_7 \otimes F_8}\ \text{ax}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \bot, F_7 \otimes F_8}\ \bot$$

$$\dfrac{\dfrac{h_2 :\vdash F_4, \Delta_3}{\bullet h_2 :\vdash F_4, \bot, \Delta_3}\ \bot \qquad \dfrac{h_5 :\vdash F_7, \Delta_6 \quad h_5 :\vdash F_8, \Delta_9, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8}\ \otimes}{- :\vdash (\bot, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 :\vdash \Delta_3, F_4}\ \text{ax} \qquad \dfrac{}{\bullet h_5 :\vdash \Delta_6, \Delta_9, dual(F_4), F_7 \otimes F_8}\ \text{ax}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \bot, F_7 \otimes F_8}\ \bot$$

4.7   Status of $\top$: `OK`

- Case rule $\mathbf{1}$

- Case rule $

$$\dfrac{\dfrac{}{\bullet h_1 :\vdash \top, \Delta_2}\ \top \qquad \dfrac{h_3 :\vdash \mathbf{0}, F_4, F_5, \Delta_6}{\bullet h_3 :\vdash dual(\top), \Delta_6, F_4 \$ F_5}\ \$}{- :\vdash \Delta_2, \Delta_6, F_4 \$ F_5}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \top}\ \top \qquad \dfrac{}{h_3 :\vdash \mathbf{0}, \Delta_6, F_4, F_5}\ \text{ax}}{- :\vdash \Delta_2, \Delta_6, F_4, F_5}\ \text{hCut}}{- :\vdash \Delta_2, \Delta_6, F_4 \$ F_5}\ \$$$

$$\dfrac{\dfrac{}{\bullet h_2 :\vdash F_4, \top, \Delta_3}\ \top \qquad \dfrac{h_5 :\vdash F_6, F_7, \Delta_8, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, F_6 \$ F_7}\ \$}{- :\vdash (\top, \Delta_3), \Delta_8, F_6 \$ F_7}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- :\vdash \Delta_3, \Delta_8, \top, F_6 \$ F_7}\ \top$$

- Case rule &

$$\dfrac{\dfrac{}{\bullet h_1 :\vdash \top, \Delta_2}\ \top \qquad \dfrac{h_3 :\vdash \mathbf{0}, F_4, \Delta_6 \quad h_3 :\vdash \mathbf{0}, F_5, \Delta_6}{\bullet h_3 :\vdash dual(\top), \Delta_6, F_4 \& F_5}\ \&}{- :\vdash \Delta_2, \Delta_6, F_4 \& F_5}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \top}\ \text{ax} \quad \dfrac{}{h_3 :\vdash \mathbf{0}, \Delta_6, F_4}\ \text{ax}}{- :\vdash \Delta_2, \Delta_6, F_4}\ \text{hCut} \qquad \dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_2, \top}\ \text{ax} \quad \dfrac{}{h_3 :\vdash \mathbf{0}, \Delta_6, F_5}\ \text{ax}}{- :\vdash \Delta_2, \Delta_6, F_5}\ \text{hCut}}{- :\vdash \Delta_2, \Delta_6, F_4 \& F_5}\ \&$$

$$\dfrac{\dfrac{}{\bullet h_2 :\vdash F_4, \top, \Delta_3}\ \top \qquad \dfrac{h_5 :\vdash F_6, \Delta_8, dual(F_4) \quad h_5 :\vdash F_7, \Delta_8, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, F_6 \& F_7}\ \&}{- :\vdash (\top, \Delta_3), \Delta_8, F_6 \& F_7}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- :\vdash \Delta_3, \Delta_8, \top, F_6 \& F_7}\ \top$$

- Case rule $\oplus_B$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \top, \Delta_2}\ \top \quad \cfrac{h_3 :\vdash \mathbf{0}, F_5, \Delta_6}{\bullet h_3 :\vdash dual(\top), \Delta_6, F_4 \oplus F_5}\ \oplus_B}{- :\vdash \Delta_2, \Delta_6, F_4 \oplus F_5}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, \top}\ \top \quad \cfrac{}{h_3 :\vdash \mathbf{0}, \Delta_6, F_5}\ \text{ax}}{- :\vdash \Delta_2, \Delta_6, F_5}\ \text{hCut}}{- :\vdash \Delta_2, \Delta_6, F_4 \oplus F_5}\ \oplus_B$$

$$\cfrac{\cfrac{}{\bullet h_2 :\vdash F_4, \top, \Delta_3}\ \top \quad \cfrac{h_5 :\vdash F_7, \Delta_8, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, F_6 \oplus F_7}\ \oplus_B}{- :\vdash (\top, \Delta_3), \Delta_8, F_6 \oplus F_7}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- :\vdash \Delta_3, \Delta_8, \top, F_6 \oplus F_7}\ \top$$

- Case rule $\oplus_A$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \top, \Delta_2}\ \top \quad \cfrac{h_3 :\vdash \mathbf{0}, F_4, \Delta_6}{\bullet h_3 :\vdash dual(\top), \Delta_6, F_4 \oplus F_5}\ \oplus_A}{- :\vdash \Delta_2, \Delta_6, F_4 \oplus F_5}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, \top}\ \top \quad \cfrac{}{h_3 :\vdash \mathbf{0}, \Delta_6, F_4}\ \text{ax}}{- :\vdash \Delta_2, \Delta_6, F_4}\ \text{hCut}}{- :\vdash \Delta_2, \Delta_6, F_4 \oplus F_5}\ \oplus_A$$

$$\cfrac{\cfrac{}{\bullet h_2 :\vdash F_4, \top, \Delta_3}\ \top \quad \cfrac{h_5 :\vdash F_6, \Delta_8, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, F_6 \oplus F_7}\ \oplus_A}{- :\vdash (\top, \Delta_3), \Delta_8, F_6 \oplus F_7}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- :\vdash \Delta_3, \Delta_8, \top, F_6 \oplus F_7}\ \top$$

- Case rule $\bot$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \top, \Delta_2}\ \top \quad \cfrac{h_3 :\vdash \mathbf{0}, \Delta_4}{\bullet h_3 :\vdash dual(\top), \bot, \Delta_4}\ \bot}{- :\vdash \Delta_2, \bot, \Delta_4}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, \bot, \top}\ \top \quad \cfrac{}{h_3 :\vdash \mathbf{0}, \Delta_4}\ \text{ax}}{- :\vdash \Delta_2, \Delta_4, \bot}\ \text{hCut}$$

$$\cfrac{\cfrac{}{\bullet h_2 :\vdash F_4, \top, \Delta_3}\ \top \quad \cfrac{h_5 :\vdash \Delta_6, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \bot, \Delta_6}\ \bot}{- :\vdash (\top, \Delta_3), \bot, \Delta_6}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- :\vdash \Delta_3, \Delta_6, \bot, \top}\ \top$$

- Case rule $\top$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \top, \Delta_2}\ \top \quad \cfrac{}{\bullet h_3 :\vdash dual(\top), \top, \Delta_4}\ \top}{- :\vdash \Delta_2, \top, \Delta_4}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- :\vdash \Delta_2, \Delta_4, \top}\ \top$$

$$\cfrac{\cfrac{}{\bullet h_2 :\vdash F_4, \top, \Delta_3}\ \top \quad \cfrac{}{\bullet h_5 :\vdash dual(F_4), \top, \Delta_6}\ \top}{- :\vdash (\top, \Delta_3), \top, \Delta_6}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- :\vdash \Delta_3, \Delta_6, \top, \top}\ \top$$

- Case rule I

- Case rule $\otimes$

$$
\cfrac{
\cfrac{}{\bullet\mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \qquad
\cfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \mathtt{F}_5, \Delta_7 \qquad \mathtt{h}_3 :\vdash \mathtt{F}_6, \Delta_4}{\bullet\mathtt{h}_3 :\vdash dual(\top), \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes
}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \top}\ \top \qquad
\cfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_7, \mathtt{F}_5}\ \text{ax}
}{- :\vdash \Delta_2, \Delta_7, \mathtt{F}_5}\ \text{hCut} \qquad
\cfrac{}{- :\vdash \Delta_4, \mathtt{F}_6}\ \text{ax}
}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes
$$

$$
\cfrac{
\cfrac{}{\bullet\mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \qquad
\cfrac{\mathtt{h}_3 :\vdash \mathtt{F}_5, \Delta_4 \qquad \mathtt{h}_3 :\vdash \mathbf{0}, \mathtt{F}_6, \Delta_7}{\bullet\mathtt{h}_3 :\vdash dual(\top), \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes
}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
\cfrac{}{- :\vdash \Delta_4, \mathtt{F}_5}\ \text{ax} \qquad
\cfrac{
\cfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \top}\ \top \qquad
\cfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_7, \mathtt{F}_6}\ \text{ax}
}{- :\vdash \Delta_2, \Delta_7, \mathtt{F}_6}\ \text{hCut}
}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes
$$

$$
\cfrac{
\cfrac{}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \qquad
\cfrac{\mathtt{h}_5 :\vdash \mathtt{F}_7, \Delta_9, dual(\mathtt{F}_4) \qquad \mathtt{h}_5 :\vdash \mathtt{F}_8, \Delta_6}{\bullet\mathtt{h}_5 :\vdash dual(\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes
}{- :\vdash (\top, \Delta_3), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \top, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \top
$$

$$
\cfrac{
\cfrac{}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \qquad
\cfrac{\mathtt{h}_5 :\vdash \mathtt{F}_7, \Delta_6 \qquad \mathtt{h}_5 :\vdash \mathtt{F}_8, \Delta_9, dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes
}{- :\vdash (\top, \Delta_3), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \top, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \top
$$

4.8 Status of I: `OK`

- Case rule $\mathbf{1}$

- Case rule $

$$
\cfrac{
\cfrac{}{\bullet\mathtt{h}_1 :\vdash \hat{\ }(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \qquad
\cfrac{\mathtt{h}_4 :\vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, p(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(\hat{\ }(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$
}{- :\vdash p(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
\cfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, p(\mathtt{n}_3)}\ \text{ax}
}{- :\vdash \Delta_7, p(\mathtt{n}_3), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$
$$

$$
\cfrac{
\cfrac{}{\bullet\mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{\ }(\mathtt{n}_3)}\ I \qquad
\cfrac{\mathtt{h}_4 :\vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \hat{\ }(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$
}{- :\vdash \hat{\ }(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
\cfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \hat{\ }(\mathtt{n}_3)}\ \text{ax}
}{- :\vdash \Delta_7, \hat{\ }(\mathtt{n}_3), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$
$$

- Case rule $\&$

$$
\cfrac{
\cfrac{}{\bullet\mathtt{h}_1 :\vdash \hat{\ }(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \qquad
\cfrac{\mathtt{h}_4 :\vdash \mathtt{F}_5, \Delta_7, p(\mathtt{n}_3) \qquad \mathtt{h}_4 :\vdash \mathtt{F}_6, \Delta_7, p(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(\hat{\ }(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\ \&
}{- :\vdash p(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\cfrac{
\cfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, p(\mathtt{n}_3)}\ \text{ax} \qquad
\cfrac{}{- :\vdash \Delta_7, \mathtt{F}_6, p(\mathtt{n}_3)}\ \text{ax}
}{- :\vdash \Delta_7, p(\mathtt{n}_3), \mathtt{F}_5 \& \mathtt{F}_6}\ \&
$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \mathtt{F}_5, \Delta_7, \hat{}(\mathtt{n}_3) \quad \mathtt{h}_4 :\vdash \mathtt{F}_6, \Delta_7, \hat{}(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\ \&}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, \hat{}(\mathtt{n}_3)}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_7, \mathtt{F}_6, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, \hat{}(\mathtt{n}_3), \mathtt{F}_5 \& \mathtt{F}_6}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \mathtt{F}_6, \Delta_7, p(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B}{- :\vdash p(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_6, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, p(\mathtt{n}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \mathtt{F}_6, \Delta_7, \hat{}(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_6, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, \hat{}(\mathtt{n}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \mathtt{F}_5, \Delta_7, p(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A}{- :\vdash p(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, p(\mathtt{n}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \mathtt{F}_5, \Delta_7, \hat{}(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, \hat{}(\mathtt{n}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_5, p(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \bot, \Delta_5}\ \bot}{- :\vdash p(\mathtt{n}_3), \bot, \Delta_5}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_5, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_5, \bot, p(\mathtt{n}_3)}\ \bot$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_5, \hat{}(\mathtt{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \bot, \Delta_5}\ \bot}{- :\vdash \hat{}(\mathtt{n}_3), \bot, \Delta_5}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_5, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_5, \bot, \hat{}(\mathtt{n}_3)}\ \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{}{\bullet\mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \top, \Delta_5}\ \top}{- :\vdash p(\mathtt{n}_3), \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- :\vdash \Delta_5, \top, p(\mathtt{n}_3)}\ \top$$

$$\dfrac{\dfrac{}{\bullet h_1 :\vdash p(n_3), \hat{}(n_3)}\ I \quad \dfrac{}{\bullet h_4 :\vdash dual(p(n_3)), \top, \Delta_5}\ \top}{- :\vdash \hat{}(n_3), \top, \Delta_5}\ \text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- :\vdash \Delta_5, \top, \hat{}(n_3)}\ \top$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet h_1 :\vdash \hat{}(n_4), p(n_4)}\ I \quad \dfrac{}{\bullet h_3 :\vdash dual(\hat{}(n_4)), \hat{}(n_4)}\ I}{- :\vdash p(n_4), \hat{}(n_4)}\ \text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- :\vdash p(n_4), \hat{}(n_4)}\ I$$

$$\dfrac{\dfrac{}{\bullet h_1 :\vdash p(n_4), \hat{}(n_4)}\ I \quad \dfrac{}{\bullet h_3 :\vdash dual(p(n_4)), p(n_4)}\ I}{- :\vdash \hat{}(n_4), p(n_4)}\ \text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- :\vdash p(n_4), \hat{}(n_4)}\ \text{ax}$$

- Case rule $\otimes$

$$\dfrac{\dfrac{}{\bullet h_1 :\vdash \hat{}(n_3), p(n_3)}\ I \quad \dfrac{h_4 :\vdash F_6, \Delta_8, p(n_3) \quad h_4 :\vdash F_7, \Delta_5}{\bullet h_4 :\vdash dual(\hat{}(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7}\ \otimes}{- :\vdash p(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7}\ \text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_8, F_6, p(n_3)}\ \text{ax} \quad \dfrac{}{- :\vdash \Delta_5, F_7}\ \text{ax}}{- :\vdash \Delta_5, \Delta_8, p(n_3), F_6 \otimes F_7}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet h_1 :\vdash \hat{}(n_3), p(n_3)}\ I \quad \dfrac{h_4 :\vdash F_6, \Delta_5 \quad h_4 :\vdash F_7, \Delta_8, p(n_3)}{\bullet h_4 :\vdash dual(\hat{}(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7}\ \otimes}{- :\vdash p(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7}\ \text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_5, F_6}\ \text{ax} \quad \dfrac{}{- :\vdash \Delta_8, F_7, p(n_3)}\ \text{ax}}{- :\vdash \Delta_5, \Delta_8, p(n_3), F_6 \otimes F_7}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet h_1 :\vdash p(n_3), \hat{}(n_3)}\ I \quad \dfrac{h_4 :\vdash F_6, \Delta_8, \hat{}(n_3) \quad h_4 :\vdash F_7, \Delta_5}{\bullet h_4 :\vdash dual(p(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7}\ \otimes}{- :\vdash \hat{}(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7}\ \text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_8, F_6, \hat{}(n_3)}\ \text{ax} \quad \dfrac{}{- :\vdash \Delta_5, F_7}\ \text{ax}}{- :\vdash \Delta_5, \Delta_8, \hat{}(n_3), F_6 \otimes F_7}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet h_1 :\vdash p(n_3), \hat{}(n_3)}\ I \quad \dfrac{h_4 :\vdash F_6, \Delta_5 \quad h_4 :\vdash F_7, \Delta_8, \hat{}(n_3)}{\bullet h_4 :\vdash dual(p(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7}\ \otimes}{- :\vdash \hat{}(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7}\ \text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_5, F_6}\ \text{ax} \quad \dfrac{}{- :\vdash \Delta_8, F_7, \hat{}(n_3)}\ \text{ax}}{- :\vdash \Delta_5, \Delta_8, \hat{}(n_3), F_6 \otimes F_7}\ \otimes$$

4.9   Status of $\otimes$: OK

- Case rule $\mathbf{1}$

- Case rule $

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_2 \quad \mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_3}{\bullet\mathtt{h_1} :\vdash \mathtt{F_6} \otimes \mathtt{F_7}, \Delta_2, \Delta_3}\ \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_9}, \mathtt{F_{10}}, \Delta_{11}, dual(\mathtt{F_6})\$dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_6} \otimes \mathtt{F_7}), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}}\ \$}{- :\vdash (\Delta_2, \Delta_3), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}}\ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_2, \Delta_3, \mathtt{F_6} \otimes \mathtt{F_7}}\ \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{11}, \mathtt{F_{10}}, \mathtt{F_9}, dual(\mathtt{F_6})\$dual(\mathtt{F_7})}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F_{10}}, \mathtt{F_9}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F_9}\$\mathtt{F_{10}}}\ \$$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_2 \quad \mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_3}{\bullet\mathtt{h_1} :\vdash \mathtt{F_6} \otimes \mathtt{F_7}, \Delta_2, \Delta_3}\ \otimes \quad \dfrac{\mathtt{h_8} :\vdash \Delta_9, dual(\mathtt{F_6}), dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_6} \otimes \mathtt{F_7}), \Delta_9}\ \$}{- :\vdash (\Delta_2, \Delta_3), \Delta_9}\ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_2, \mathtt{F_6}}\ \text{ax} \quad \dfrac{\dfrac{}{- :\vdash \Delta_3, \mathtt{F_7}}\ \text{ax} \quad \dfrac{}{- :\vdash \Delta_9, dual(\mathtt{F_6}), dual(\mathtt{F_7})}\ \text{ax}}{- :\vdash \Delta_3, \Delta_9, dual(\mathtt{F_6})}\ \text{sCut}}{- :\vdash \Delta_2, \Delta_3, \Delta_9}\ \text{sCut}$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash \mathtt{F_7}, \mathtt{F_4}, \Delta_6 \quad \mathtt{h_2} :\vdash \mathtt{F_5}, \Delta_3}{\bullet\mathtt{h_2} :\vdash \mathtt{F_7}, \Delta_3, \Delta_6, \mathtt{F_4} \otimes \mathtt{F_5}}\ \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_9}, \mathtt{F_{10}}, \Delta_{11}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}}\ \$}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F_4} \otimes \mathtt{F_5}), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}}\ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash \Delta_3, \Delta_6, \mathtt{F_7}, \mathtt{F_4} \otimes \mathtt{F_5}}\ \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{11}, \mathtt{F_{10}}, \mathtt{F_9}, dual(\mathtt{F_7})}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F_{10}}, \mathtt{F_9}, \mathtt{F_4} \otimes \mathtt{F_5}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F_9}\$\mathtt{F_{10}}, \mathtt{F_4} \otimes \mathtt{F_5}}\ \$$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash \mathtt{F_4}, \Delta_3 \quad \mathtt{h_2} :\vdash \mathtt{F_7}, \mathtt{F_5}, \Delta_6}{\bullet\mathtt{h_2} :\vdash \mathtt{F_7}, \Delta_3, \Delta_6, \mathtt{F_4} \otimes \mathtt{F_5}}\ \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_9}, \mathtt{F_{10}}, \Delta_{11}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}}\ \$}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F_4} \otimes \mathtt{F_5}), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}}\ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash \Delta_3, \Delta_6, \mathtt{F_7}, \mathtt{F_4} \otimes \mathtt{F_5}}\ \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{11}, \mathtt{F_{10}}, \mathtt{F_9}, dual(\mathtt{F_7})}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F_{10}}, \mathtt{F_9}, \mathtt{F_4} \otimes \mathtt{F_5}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F_9}\$\mathtt{F_{10}}, \mathtt{F_4} \otimes \mathtt{F_5}}\ \$$$

- Case rule &

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_2 \quad \mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_3}{\bullet\mathtt{h_1} :\vdash \mathtt{F_6} \otimes \mathtt{F_7}, \Delta_2, \Delta_3}\ \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_9}, \Delta_{11}, dual(\mathtt{F_6})\$dual(\mathtt{F_7}) \quad \mathtt{h_8} :\vdash \mathtt{F_{10}}, \Delta_{11}, dual(\mathtt{F_6})\$dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_6} \otimes \mathtt{F_7}), \Delta_{11}, \mathtt{F_9}\&\mathtt{F_{10}}}\ \&}{- :\vdash (\Delta_2, \Delta_3), \Delta_{11}, \mathtt{F_9}\&\mathtt{F_{10}}}\ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_2, \Delta_3, \mathtt{F_6} \otimes \mathtt{F_7}}\ \text{ax}\ \dfrac{}{\mathtt{h_8} :\vdash \Delta_{11}, \mathtt{F_9}, dual(\mathtt{F_6})\$dual(\mathtt{F_7})}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F_9}}\ \text{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_2, \Delta_3, \mathtt{F_6} \otimes \mathtt{F_7}}\ \text{ax}\ \dfrac{}{\mathtt{h_8} :\vdash \Delta_{11}, \mathtt{F_{10}}, dual(\mathtt{F_6})\$dual(\mathtt{F_7})}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F_{10}}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F_9}\&\mathtt{F_{10}}}\ \&$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash \mathtt{F_7}, \mathtt{F_4}, \Delta_6 \quad \mathtt{h_2} :\vdash \mathtt{F_5}, \Delta_3}{\bullet\mathtt{h_2} :\vdash \mathtt{F_7}, \Delta_3, \Delta_6, \mathtt{F_4} \otimes \mathtt{F_5}}\ \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_9}, \Delta_{11}, dual(\mathtt{F_7}) \quad \mathtt{h_8} :\vdash \mathtt{F_{10}}, \Delta_{11}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{11}, \mathtt{F_9}\&\mathtt{F_{10}}}\ \&}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F_4} \otimes \mathtt{F_5}), \Delta_{11}, \mathtt{F_9}\&\mathtt{F_{10}}}\ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F_4}, \mathtt{F_7}}\ \text{ax}\ \dfrac{}{\bullet\mathtt{h_8} :\vdash \Delta_{11}, dual(\mathtt{F_7}), \mathtt{F_9}\&\mathtt{F_{10}}}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_6, \mathtt{F_4}, \mathtt{F_9}\&\mathtt{F_{10}}}\ \text{hCut} \quad \dfrac{}{- :\vdash \Delta_3, \mathtt{F_5}}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F_9}\&\mathtt{F_{10}}, \mathtt{F_4} \otimes \mathtt{F_5}}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash \mathtt{F_4}, \Delta_3 \quad \mathtt{h_2} :\vdash \mathtt{F_7}, \mathtt{F_5}, \Delta_6}{\bullet\mathtt{h_2} :\vdash \mathtt{F_7}, \Delta_3, \Delta_6, \mathtt{F_4} \otimes \mathtt{F_5}}\ \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_9}, \Delta_{11}, dual(\mathtt{F_7}) \quad \mathtt{h_8} :\vdash \mathtt{F_{10}}, \Delta_{11}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{11}, \mathtt{F_9}\&\mathtt{F_{10}}}\ \&}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F_4} \otimes \mathtt{F_5}), \Delta_{11}, \mathtt{F_9}\&\mathtt{F_{10}}}\ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_3, \mathtt{F_4}}\ \text{ax} \quad \dfrac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F_5}, \mathtt{F_7}}\ \text{ax}\ \dfrac{}{\bullet\mathtt{h_8} :\vdash \Delta_{11}, dual(\mathtt{F_7}), \mathtt{F_9}\&\mathtt{F_{10}}}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_6, \mathtt{F_5}, \mathtt{F_9}\&\mathtt{F_{10}}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F_9}\&\mathtt{F_{10}}, \mathtt{F_4} \otimes \mathtt{F_5}}\ \otimes$$

- Case rule $\oplus_B$

$$\cfrac{\cfrac{h_1 :\vdash F_6, \Delta_2 \quad h_1 :\vdash F_7, \Delta_3}{\bullet h_1 :\vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \cfrac{h_8 :\vdash F_{10}, \Delta_{11}, dual(F_6)\$dual(F_7)}{\bullet h_8 :\vdash dual(F_6 \otimes F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_B}{- :\vdash (\Delta_2, \Delta_3), \Delta_{11}, F_9 \oplus F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, \Delta_3, F_6 \otimes F_7} \text{ax} \quad \cfrac{}{h_8 :\vdash \Delta_{11}, F_{10}, dual(F_6)\$dual(F_7)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, F_{10}} \text{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, F_9 \oplus F_{10}} \oplus_B$$

$$\cfrac{\cfrac{h_2 :\vdash F_7, F_4, \Delta_6 \quad h_2 :\vdash F_5, \Delta_3}{\bullet h_2 :\vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \cfrac{h_8 :\vdash F_{10}, \Delta_{11}, dual(F_7)}{\bullet h_8 :\vdash dual(F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_B}{- :\vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \oplus F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 :\vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5} \text{ax} \quad \cfrac{}{h_8 :\vdash \Delta_{11}, F_{10}, dual(F_7)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, F_{10}, F_4 \otimes F_5} \text{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, F_4 \otimes F_5, F_9 \oplus F_{10}} \oplus_B$$

$$\cfrac{\cfrac{h_2 :\vdash F_4, \Delta_3 \quad h_2 :\vdash F_7, F_5, \Delta_6}{\bullet h_2 :\vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \cfrac{h_8 :\vdash F_{10}, \Delta_{11}, dual(F_7)}{\bullet h_8 :\vdash dual(F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_B}{- :\vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \oplus F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 :\vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5} \text{ax} \quad \cfrac{}{h_8 :\vdash \Delta_{11}, F_{10}, dual(F_7)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, F_{10}, F_4 \otimes F_5} \text{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, F_4 \otimes F_5, F_9 \oplus F_{10}} \oplus_B$$

- Case rule $\oplus_A$

$$\cfrac{\cfrac{h_1 :\vdash F_6, \Delta_2 \quad h_1 :\vdash F_7, \Delta_3}{\bullet h_1 :\vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \cfrac{h_8 :\vdash F_9, \Delta_{11}, dual(F_6)\$dual(F_7)}{\bullet h_8 :\vdash dual(F_6 \otimes F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_A}{- :\vdash (\Delta_2, \Delta_3), \Delta_{11}, F_9 \oplus F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, \Delta_3, F_6 \otimes F_7} \text{ax} \quad \cfrac{}{h_8 :\vdash \Delta_{11}, F_9, dual(F_6)\$dual(F_7)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, F_9} \text{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, F_9 \oplus F_{10}} \oplus_A$$

$$\cfrac{\cfrac{h_2 :\vdash F_7, F_4, \Delta_6 \quad h_2 :\vdash F_5, \Delta_3}{\bullet h_2 :\vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \cfrac{h_8 :\vdash F_9, \Delta_{11}, dual(F_7)}{\bullet h_8 :\vdash dual(F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_A}{- :\vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \oplus F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 :\vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5} \text{ax} \quad \cfrac{}{h_8 :\vdash \Delta_{11}, F_9, dual(F_7)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, F_9, F_4 \otimes F_5} \text{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, F_4 \otimes F_5, F_9 \oplus F_{10}} \oplus_A$$

$$\cfrac{\cfrac{h_2 :\vdash F_4, \Delta_3 \quad h_2 :\vdash F_7, F_5, \Delta_6}{\bullet h_2 :\vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \cfrac{h_8 :\vdash F_9, \Delta_{11}, dual(F_7)}{\bullet h_8 :\vdash dual(F_7), \Delta_{11}, F_9 \oplus F_{10}} \oplus_A}{- :\vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \oplus F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_2 :\vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5} \text{ax} \quad \cfrac{}{h_8 :\vdash \Delta_{11}, F_9, dual(F_7)} \text{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, F_9, F_4 \otimes F_5} \text{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, F_4 \otimes F_5, F_9 \oplus F_{10}} \oplus_A$$

- Case rule $\perp$

$$\cfrac{\cfrac{h_1 :\vdash F_6, \Delta_2 \quad h_1 :\vdash F_7, \Delta_3}{\bullet h_1 :\vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \cfrac{h_8 :\vdash \Delta_9, dual(F_6)\$dual(F_7)}{\bullet h_8 :\vdash dual(F_6 \otimes F_7), \perp, \Delta_9} \perp}{- :\vdash (\Delta_2, \Delta_3), \perp, \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, \Delta_3, F_6 \otimes F_7} \text{ax} \quad \cfrac{}{h_8 :\vdash \Delta_9, dual(F_6)\$dual(F_7)} \text{ax}}{- :\vdash \Delta_2, \Delta_3, \Delta_9} \text{hCut}}{- :\vdash \Delta_2, \Delta_3, \Delta_9, \perp} \perp$$

$$\dfrac{\dfrac{\text{h}_2 :\vdash \text{F}_7, \text{F}_4, \Delta_6 \quad \text{h}_2 :\vdash \text{F}_5, \Delta_3}{\bullet\text{h}_2 :\vdash \text{F}_7, \Delta_3, \Delta_6, \text{F}_4 \otimes \text{F}_5} \otimes \quad \dfrac{\text{h}_8 :\vdash \Delta_9, dual(\text{F}_7)}{\bullet\text{h}_8 :\vdash dual(\text{F}_7), \bot, \Delta_9} \bot}{- :\vdash (\Delta_3, \Delta_6, \text{F}_4 \otimes \text{F}_5), \bot, \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\text{h}_2 :\vdash \Delta_3, \Delta_6, \text{F}_7, \text{F}_4 \otimes \text{F}_5} \text{ax} \quad \dfrac{}{\text{h}_8 :\vdash \Delta_9, dual(\text{F}_7)} \text{ax}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \text{F}_4 \otimes \text{F}_5} \text{hCut}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \bot, \text{F}_4 \otimes \text{F}_5} \bot$$

$$\dfrac{\dfrac{\text{h}_2 :\vdash \text{F}_4, \Delta_3 \quad \text{h}_2 :\vdash \text{F}_7, \text{F}_5, \Delta_6}{\bullet\text{h}_2 :\vdash \text{F}_7, \Delta_3, \Delta_6, \text{F}_4 \otimes \text{F}_5} \otimes \quad \dfrac{\text{h}_8 :\vdash \Delta_9, dual(\text{F}_7)}{\bullet\text{h}_8 :\vdash dual(\text{F}_7), \bot, \Delta_9} \bot}{- :\vdash (\Delta_3, \Delta_6, \text{F}_4 \otimes \text{F}_5), \bot, \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\text{h}_2 :\vdash \Delta_3, \Delta_6, \text{F}_7, \text{F}_4 \otimes \text{F}_5} \text{ax} \quad \dfrac{}{\text{h}_8 :\vdash \Delta_9, dual(\text{F}_7)} \text{ax}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \text{F}_4 \otimes \text{F}_5} \text{hCut}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \bot, \text{F}_4 \otimes \text{F}_5} \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\text{h}_1 :\vdash \text{F}_6, \Delta_2 \quad \text{h}_1 :\vdash \text{F}_7, \Delta_3}{\bullet\text{h}_1 :\vdash \text{F}_6 \otimes \text{F}_7, \Delta_2, \Delta_3} \otimes \quad \dfrac{}{\bullet\text{h}_8 :\vdash dual(\text{F}_6 \otimes \text{F}_7), \top, \Delta_9} \top}{- :\vdash (\Delta_2, \Delta_3), \top, \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- :\vdash \Delta_2, \Delta_3, \Delta_9, \top} \top$$

$$\dfrac{\dfrac{\text{h}_2 :\vdash \text{F}_7, \text{F}_4, \Delta_6 \quad \text{h}_2 :\vdash \text{F}_5, \Delta_3}{\bullet\text{h}_2 :\vdash \text{F}_7, \Delta_3, \Delta_6, \text{F}_4 \otimes \text{F}_5} \otimes \quad \dfrac{}{\bullet\text{h}_8 :\vdash dual(\text{F}_7), \top, \Delta_9} \top}{- :\vdash (\Delta_3, \Delta_6, \text{F}_4 \otimes \text{F}_5), \top, \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \top, \text{F}_4 \otimes \text{F}_5} \top$$

$$\dfrac{\dfrac{\text{h}_2 :\vdash \text{F}_4, \Delta_3 \quad \text{h}_2 :\vdash \text{F}_7, \text{F}_5, \Delta_6}{\bullet\text{h}_2 :\vdash \text{F}_7, \Delta_3, \Delta_6, \text{F}_4 \otimes \text{F}_5} \otimes \quad \dfrac{}{\bullet\text{h}_8 :\vdash dual(\text{F}_7), \top, \Delta_9} \top}{- :\vdash (\Delta_3, \Delta_6, \text{F}_4 \otimes \text{F}_5), \top, \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \top, \text{F}_4 \otimes \text{F}_5} \top$$

- Case rule I

- Case rule $\otimes$

$$\dfrac{\dfrac{\text{h}_1 :\vdash \text{F}_6, \Delta_2 \quad \text{h}_1 :\vdash \text{F}_7, \Delta_3}{\bullet\text{h}_1 :\vdash \text{F}_6 \otimes \text{F}_7, \Delta_2, \Delta_3} \otimes \quad \dfrac{\text{h}_8 :\vdash \text{F}_{10}, \Delta_{12}, dual(\text{F}_6)\$dual(\text{F}_7) \quad \text{h}_8 :\vdash \text{F}_{11}, \Delta_9}{\bullet\text{h}_8 :\vdash dual(\text{F}_6 \otimes \text{F}_7), \Delta_9, \Delta_{12}, \text{F}_{10} \otimes \text{F}_{11}} \otimes}{- :\vdash (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, \text{F}_{10} \otimes \text{F}_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\text{h}_1 :\vdash \Delta_2, \Delta_3, \text{F}_6 \otimes \text{F}_7} \text{ax} \quad \dfrac{}{\text{h}_8 :\vdash \Delta_{12}, \text{F}_{10}, dual(\text{F}_6)\$dual(\text{F}_7)} \text{ax}}{- :\vdash \Delta_{12}, \Delta_2, \Delta_3, \text{F}_{10}} \text{hCut} \quad \dfrac{}{- :\vdash \Delta_9, \text{F}_{11}} \text{ax}}{- :\vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, \text{F}_{10} \otimes \text{F}_{11}} \otimes$$

$$\dfrac{\dfrac{\text{h}_1 :\vdash \text{F}_6, \Delta_2 \quad \text{h}_1 :\vdash \text{F}_7, \Delta_3}{\bullet\text{h}_1 :\vdash \text{F}_6 \otimes \text{F}_7, \Delta_2, \Delta_3} \otimes \quad \dfrac{\text{h}_8 :\vdash \text{F}_{10}, \Delta_9 \quad \text{h}_8 :\vdash \text{F}_{11}, \Delta_{12}, dual(\text{F}_6)\$dual(\text{F}_7)}{\bullet\text{h}_8 :\vdash dual(\text{F}_6 \otimes \text{F}_7), \Delta_9, \Delta_{12}, \text{F}_{10} \otimes \text{F}_{11}} \otimes}{- :\vdash (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, \text{F}_{10} \otimes \text{F}_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_9, \text{F}_{10}} \text{ax} \quad \dfrac{\dfrac{}{\bullet\text{h}_1 :\vdash \Delta_2, \Delta_3, \text{F}_6 \otimes \text{F}_7} \text{ax} \quad \dfrac{}{\text{h}_8 :\vdash \Delta_{12}, \text{F}_{11}, dual(\text{F}_6)\$dual(\text{F}_7)} \text{ax}}{- :\vdash \Delta_{12}, \Delta_2, \Delta_3, \text{F}_{11}} \text{hCut}}{- :\vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, \text{F}_{10} \otimes \text{F}_{11}} \otimes$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash \mathtt{F_7, F_4}, \Delta_6 \quad \mathtt{h_2} :\vdash \mathtt{F_5}, \Delta_3}{\bullet\mathtt{h_2} :\vdash \mathtt{F_7}, \Delta_3, \Delta_6, \mathtt{F_4 \otimes F_5}} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_{10}}, \Delta_{12}, dual(\mathtt{F_7}) \quad \mathtt{h_8} :\vdash \mathtt{F_{11}}, \Delta_9}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_9, \Delta_{12}, \mathtt{F_{10} \otimes F_{11}}} \otimes}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F_4 \otimes F_5}), \Delta_9, \Delta_{12}, \mathtt{F_{10} \otimes F_{11}}} \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash \Delta_3, \Delta_6, \mathtt{F_7, F_4 \otimes F_5}} \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{12}, \mathtt{F_{10}}, dual(\mathtt{F_7})} \text{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \mathtt{F_{10}, F_4 \otimes F_5}} \text{hCut} \quad \dfrac{}{- :\vdash \Delta_9, \mathtt{F_{11}}} \text{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, \mathtt{F_{10} \otimes F_{11}, F_4 \otimes F_5}} \otimes$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash \mathtt{F_7, F_4}, \Delta_6 \quad \mathtt{h_2} :\vdash \mathtt{F_5}, \Delta_3}{\bullet\mathtt{h_2} :\vdash \mathtt{F_7}, \Delta_3, \Delta_6, \mathtt{F_4 \otimes F_5}} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_{10}}, \Delta_9 \quad \mathtt{h_8} :\vdash \mathtt{F_{11}}, \Delta_{12}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_9, \Delta_{12}, \mathtt{F_{10} \otimes F_{11}}} \otimes}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F_4 \otimes F_5}), \Delta_9, \Delta_{12}, \mathtt{F_{10} \otimes F_{11}}} \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_9, \mathtt{F_{10}}} \text{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash \Delta_3, \Delta_6, \mathtt{F_7, F_4 \otimes F_5}} \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{12}, \mathtt{F_{11}}, dual(\mathtt{F_7})} \text{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \mathtt{F_{11}, F_4 \otimes F_5}} \text{hCut}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, \mathtt{F_{10} \otimes F_{11}, F_4 \otimes F_5}} \otimes$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash \mathtt{F_4}, \Delta_3 \quad \mathtt{h_2} :\vdash \mathtt{F_7, F_5}, \Delta_6}{\bullet\mathtt{h_2} :\vdash \mathtt{F_7}, \Delta_3, \Delta_6, \mathtt{F_4 \otimes F_5}} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_{10}}, \Delta_{12}, dual(\mathtt{F_7}) \quad \mathtt{h_8} :\vdash \mathtt{F_{11}}, \Delta_9}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_9, \Delta_{12}, \mathtt{F_{10} \otimes F_{11}}} \otimes}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F_4 \otimes F_5}), \Delta_9, \Delta_{12}, \mathtt{F_{10} \otimes F_{11}}} \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash \Delta_3, \Delta_6, \mathtt{F_7, F_4 \otimes F_5}} \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{12}, \mathtt{F_{10}}, dual(\mathtt{F_7})} \text{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \mathtt{F_{10}, F_4 \otimes F_5}} \text{hCut} \quad \dfrac{}{- :\vdash \Delta_9, \mathtt{F_{11}}} \text{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, \mathtt{F_{10} \otimes F_{11}, F_4 \otimes F_5}} \otimes$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash \mathtt{F_4}, \Delta_3 \quad \mathtt{h_2} :\vdash \mathtt{F_7, F_5}, \Delta_6}{\bullet\mathtt{h_2} :\vdash \mathtt{F_7}, \Delta_3, \Delta_6, \mathtt{F_4 \otimes F_5}} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \mathtt{F_{10}}, \Delta_9 \quad \mathtt{h_8} :\vdash \mathtt{F_{11}}, \Delta_{12}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_9, \Delta_{12}, \mathtt{F_{10} \otimes F_{11}}} \otimes}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F_4 \otimes F_5}), \Delta_9, \Delta_{12}, \mathtt{F_{10} \otimes F_{11}}} \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_9, \mathtt{F_{10}}} \text{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash \Delta_3, \Delta_6, \mathtt{F_7, F_4 \otimes F_5}} \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{12}, \mathtt{F_{11}}, dual(\mathtt{F_7})} \text{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \mathtt{F_{11}, F_4 \otimes F_5}} \text{hCut}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, \mathtt{F_{10} \otimes F_{11}, F_4 \otimes F_5}} \otimes$$