

Multiplicative-additive linear logic (MALL)

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1 Measure of derivations

- Case(s) rule 1

$$\frac{}{\bullet h_1 : \vdash 1} 1 \rightarrow \frac{}{\bullet \bullet h_1 : \vdash 1} 1$$

- Case(s) rule \$

$$\frac{h_1 : \vdash F_3, F_4, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \$ F_4} \$ \rightarrow \frac{\frac{}{h_1 : \vdash \Delta_2, F_3, F_4} \text{ax}}{\bullet h_1 : \vdash \Delta_2, F_3, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \$ F_4} \$$$

- Case(s) rule &

$$\frac{h_1 : \vdash F_3, \Delta_2 \quad h_1 : \vdash F_4, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \& F_4} \& \rightarrow \frac{\frac{\frac{}{h_1 : \vdash \Delta_2, F_3} \text{ax}}{\bullet h_1 : \vdash \Delta_2, F_3} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \& F_4} \& \quad \frac{\frac{}{h_1 : \vdash \Delta_2, F_4} \text{ax}}{\bullet h_1 : \vdash \Delta_2, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \& F_4} \&}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \& F_4} \&$$

- Case(s) rule \oplus_B

$$\frac{h_1 : \vdash F_4, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B \rightarrow \frac{\frac{\frac{}{h_1 : \vdash \Delta_2, F_4} \text{ax}}{\bullet h_1 : \vdash \Delta_2, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B$$

- Case(s) rule \oplus_A

$$\frac{h_1 : \vdash F_3, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A \rightarrow \frac{\frac{\frac{}{h_1 : \vdash \Delta_2, F_3} \text{ax}}{\bullet h_1 : \vdash \Delta_2, F_3} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A$$

- Case(s) rule \perp

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \rightarrow \frac{\frac{\frac{}{h_1 : \vdash \Delta_2} \text{ax}}{\bullet h_1 : \vdash \Delta_2} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, \perp} \perp$$

- Case(s) rule \top

$$\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \rightarrow \frac{}{\bullet \bullet h_1 : \vdash \Delta_2, \top} \top$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \vdash p(n_2), \wedge(n_2)} I \rightarrow \frac{}{\bullet \bullet h_1 : \vdash p(n_2), \wedge(n_2)} I$$

- Case(s) rule \otimes

$$\frac{h_1 : \vdash F_4, \Delta_2 \quad h_1 : \vdash F_5, \Delta_3}{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \otimes \rightarrow \frac{\frac{\frac{}{h_1 : \vdash \Delta_2, F_4} \text{ax}}{\bullet h_1 : \vdash \Delta_2, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \otimes \quad \frac{\frac{\frac{}{h_1 : \vdash \Delta_3, F_5} \text{ax}}{\bullet h_1 : \vdash \Delta_3, F_5} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \otimes}{\bullet \bullet h_1 : \vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \otimes$$

2 Invertibility of Rules

2.1 Status of 1: : Invertible

- Case rule 1

$$\frac{}{\bullet h_1 : \vdash 1} 1 \rightarrow \text{trivial}$$

- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

2.2 Status of \$: : Invertible

- Case rule 1
- Case rule \$

$$\frac{h_3 : \vdash F_4, F_5, \Delta_6, F_1 \$ F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \$ F_2), F_4 \$ F_5} \$ \rightarrow \frac{\frac{h_3 : \vdash \Delta_6, F_1, F_2, F_4, F_5}{\bullet h_3 : \vdash \Delta_6, F_1, F_2, F_4 \$ F_5} \text{ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_2, F_4 \$ F_5} \$$$

$$\frac{h_1 : \vdash F_3, F_4, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \$ F_4} \$ \rightarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3, F_4}{\bullet h_1 : \vdash \Delta_2, F_3, F_4} \text{ax}}{\bullet h_1 : \vdash \Delta_2, F_3, F_4} \text{height}$$

- Case rule &

$$\frac{\frac{h_3 : \vdash F_4, \Delta_6, F_1 \$ F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \$ F_2), F_4 \$ F_5} \quad h_3 : \vdash F_5, \Delta_6, F_1 \$ F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \$ F_2), F_4 \& F_5} \& \rightarrow \frac{\frac{h_3 : \vdash \Delta_6, F_1, F_2, F_4}{\bullet h_3 : \vdash \Delta_6, F_1, F_2, F_4 \& F_5} \text{ax/ind} \quad \frac{h_3 : \vdash \Delta_6, F_1, F_2, F_5}{\bullet h_3 : \vdash \Delta_6, F_1, F_2, F_4 \& F_5} \text{ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_2, F_4 \& F_5} \&$$

- Case rule \oplus_B

$$\frac{\frac{h_3 : \vdash F_5, \Delta_6, F_1 \$ F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \$ F_2), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\frac{h_3 : \vdash \Delta_6, F_1, F_2, F_5}{\bullet h_3 : \vdash \Delta_6, F_1, F_2, F_4 \oplus F_5} \text{ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_2, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash F_4, \Delta_6, F_1 \$F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \$F_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_2, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_2, F_4 \oplus F_5} \oplus_A$$

- Case rule \perp

$$\frac{h_3 : \vdash \Delta_4, F_1 \$F_2}{\bullet h_3 : \vdash \perp, \Delta_4, F_1 \$F_2} \perp \rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, F_1, F_2, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_3 : \vdash \top, \Delta_4, F_1 \$F_2} \top \rightarrow \overline{\bullet h_3 : \vdash \Delta_4, F_1, F_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\begin{aligned} \frac{h_3 : \vdash F_5, \Delta_7, F_1 \$F_2 \quad h_3 : \vdash F_6, \Delta_4}{\bullet h_3 : \vdash (\Delta_7, F_1 \$F_2), \Delta_4, F_5 \otimes F_6} \otimes &\rightarrow \frac{\overline{h_3 : \vdash \Delta_7, F_1, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_4, F_6} \text{ ax}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6} \otimes \\ \frac{h_3 : \vdash F_5, \Delta_4 \quad h_3 : \vdash F_6, \Delta_7, F_1 \$F_2}{\bullet h_3 : \vdash \Delta_4, (\Delta_7, F_1 \$F_2), F_5 \otimes F_6} \otimes &\rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_5} \text{ ax} \quad \overline{h_3 : \vdash \Delta_7, F_1, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6} \otimes \end{aligned}$$

2.3 Status of $\&$: (Left Premise): Invertible

- Case rule 1

- Case rule $\$$

$$\frac{h_3 : \vdash F_4, F_5, \Delta_6, F_1 \&F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \&F_2), F_4 \$F_5} \$ \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \$F_5} \$$$

- Case rule $\&$

$$\begin{aligned} \frac{h_3 : \vdash F_4, \Delta_6, F_1 \&F_2 \quad h_3 : \vdash F_5, \Delta_6, F_1 \&F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \&F_2), F_4 \&F_5} \& \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \&F_5} \& \\ \frac{h_1 : \vdash F_3, \Delta_2 \quad h_1 : \vdash F_4, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \&F_4} \& \rightarrow \frac{\overline{h_1 : \vdash \Delta_2, F_3} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, F_3} \text{ height} \end{aligned}$$

- Case rule \oplus_B

$$\frac{h_3 : \vdash F_5, \Delta_6, F_1 \&F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \&F_2), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash F_4, \Delta_6, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_A$$

- Case rule \perp

$$\frac{h_3 : \vdash \Delta_4, F_1 \& F_2}{\bullet h_3 : \vdash \perp, \Delta_4, F_1 \& F_2} \perp \rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_1} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, F_1, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_3 : \vdash \top, \Delta_4, F_1 \& F_2} \top \rightarrow \overline{\bullet h_3 : \vdash \Delta_4, F_1, \top} \top$$

- Case rule I

- Case rule \otimes

$$\begin{aligned} \frac{h_3 : \vdash F_5, \Delta_7, F_1 \& F_2 \quad h_3 : \vdash F_6, \Delta_4}{\bullet h_3 : \vdash (\Delta_7, F_1 \& F_2), \Delta_4, F_5 \otimes F_6} \otimes &\rightarrow \frac{\overline{h_3 : \vdash \Delta_7, F_1, F_5} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_4, F_6} \text{ ax}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes \\ \frac{h_3 : \vdash F_5, \Delta_4 \quad h_3 : \vdash F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 : \vdash \Delta_4, (\Delta_7, F_1 \& F_2), F_5 \otimes F_6} \otimes &\rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_5} \text{ ax} \quad \overline{h_3 : \vdash \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes \end{aligned}$$

2.4 Status of $\&$ (Right Premise): : Invertible

- Case rule 1

- Case rule $\$$

$$\frac{h_3 : \vdash F_4, F_5, \Delta_6, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \$ F_5} \$ \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_4, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \$ F_5} \$$$

- Case rule $\&$

$$\begin{aligned} \frac{h_3 : \vdash F_4, \Delta_6, F_1 \& F_2 \quad h_3 : \vdash F_5, \Delta_6, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \& F_5} \& \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_4} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \& F_5} \& \\ \frac{h_1 : \vdash F_3, \Delta_2 \quad h_1 : \vdash F_4, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \& F_4} \& \rightarrow \frac{\overline{h_1 : \vdash \Delta_2, F_4} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, F_4} \text{ height} \end{aligned}$$

- Case rule \oplus_B

$$\frac{h_3 : \vdash F_5, \Delta_6, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash F_4, \Delta_6, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \oplus F_5} \oplus_A$$

- Case rule \perp

$$\frac{h_3 : \vdash \Delta_4, F_1 \& F_2}{\bullet h_3 : \vdash \perp, \Delta_4, F_1 \& F_2} \perp \rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_2} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, F_2, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_3 : \vdash \top, \Delta_4, F_1 \& F_2} \top \rightarrow \overline{\bullet h_3 : \vdash \Delta_4, F_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\begin{aligned} \frac{h_3 : \vdash F_5, \Delta_7, F_1 \& F_2 \quad h_3 : \vdash F_6, \Delta_4}{\bullet h_3 : \vdash (\Delta_7, F_1 \& F_2), \Delta_4, F_5 \otimes F_6} \otimes &\rightarrow \frac{\overline{h_3 : \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_4, F_6} \text{ ax}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes \\ \frac{h_3 : \vdash F_5, \Delta_4 \quad h_3 : \vdash F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 : \vdash \Delta_4, (\Delta_7, F_1 \& F_2), F_5 \otimes F_6} \otimes &\rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_5} \text{ ax} \quad \overline{h_3 : \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes \end{aligned}$$

2.5 Status of \oplus_B : : Non invertible

- Case rule 1

- Case rule $\$$

$$\frac{h_3 : \vdash F_4, F_5, \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \$ F_5} \$ \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_4, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \$ F_5} \$$$

- Case rule $\&$

$$\frac{h_3 : \vdash F_4, \Delta_6, F_1 \oplus F_2 \quad h_3 : \vdash F_5, \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \& F_5} \& \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_4} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \& F_5} \&$$

- Case rule \oplus_B

$$\begin{aligned} \frac{h_3 : \vdash F_5, \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_B &\rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \oplus F_5} \oplus_B \\ \frac{h_1 : \vdash F_4, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B &\rightarrow \frac{\overline{h_1 : \vdash \Delta_2, F_4} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, F_4} \text{ height} \end{aligned}$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash F_4, \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_2, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_2, F_4 \oplus F_5} \oplus_A$$

$$\frac{h_1 : \vdash F_3, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A \rightarrow \frac{\overline{\bullet h_1 : \vdash F_4, \Delta_2} \text{ fail}}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A$$

- Case rule \perp

$$\frac{h_3 : \vdash \Delta_4, F_1 \oplus F_2}{\bullet h_3 : \vdash \perp, \Delta_4, F_1 \oplus F_2} \perp \rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_2} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, F_2, \perp} \perp$$

- Case rule \top

$$\frac{\overline{\bullet h_3 : \vdash \top, \Delta_4, F_1 \oplus F_2}}{\bullet h_3 : \vdash \top, \Delta_4, F_1 \oplus F_2} \top \rightarrow \frac{\overline{\bullet h_3 : \vdash \Delta_4, F_2, \top}}{\bullet h_3 : \vdash \Delta_4, F_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 : \vdash F_5, \Delta_7, F_1 \oplus F_2 \quad h_3 : \vdash F_6, \Delta_4}{\bullet h_3 : \vdash (\Delta_7, F_1 \oplus F_2), \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 : \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_4, F_6} \text{ ax}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes$$

$$\frac{h_3 : \vdash F_5, \Delta_4 \quad h_3 : \vdash F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 : \vdash \Delta_4, (\Delta_7, F_1 \oplus F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_5} \text{ ax} \quad \overline{h_3 : \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes$$

2.6 Status of \oplus_A : : Non invertible

- Case rule 1

- Case rule $\$$

$$\frac{h_3 : \vdash F_4, F_5, \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \$ F_5} \$ \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \$ F_5} \$$$

- Case rule $\&$

$$\frac{h_3 : \vdash F_4, \Delta_6, F_1 \oplus F_2 \quad h_3 : \vdash F_5, \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \& F_5} \& \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \& F_5} \&$$

- Case rule \oplus_B

$$\frac{h_3 : \vdash F_5, \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_B$$

$$\frac{h_1 : \vdash F_4, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B \rightarrow \frac{\overline{\bullet h_1 : \vdash F_3, \Delta_2} \text{ fail}}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash F_4, \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_A$$

$$\frac{h_1 : \vdash F_3, \Delta_2}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A \rightarrow \frac{\overline{h_1 : \vdash \Delta_2, F_3} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, F_3} \text{ height}$$

- Case rule \perp

$$\frac{h_3 : \vdash \Delta_4, F_1 \oplus F_2}{\bullet h_3 : \vdash \perp, \Delta_4, F_1 \oplus F_2} \perp \rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_1} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, F_1, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_3 : \vdash \top, \Delta_4, F_1 \oplus F_2} \top \rightarrow \overline{\bullet h_3 : \vdash \Delta_4, F_1, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 : \vdash F_5, \Delta_7, F_1 \oplus F_2 \quad h_3 : \vdash F_6, \Delta_4}{\bullet h_3 : \vdash (\Delta_7, F_1 \oplus F_2), \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 : \vdash \Delta_7, F_1, F_5} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_4, F_6} \text{ ax}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes$$

$$\frac{h_3 : \vdash F_5, \Delta_4 \quad h_3 : \vdash F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 : \vdash \Delta_4, (\Delta_7, F_1 \oplus F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_5} \text{ ax} \quad \overline{h_3 : \vdash \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes$$

2.7 Status of \perp : : Invertible

- Case rule 1

- Case rule $\$$

$$\frac{h_1 : \vdash \perp, F_2, F_3, \Delta_4}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \$ F_3} \$ \rightarrow \frac{\overline{h_1 : \vdash \Delta_4, F_2, F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \$ F_3} \$$$

- Case rule $\&$

$$\frac{h_1 : \vdash \perp, F_2, \Delta_4 \quad h_1 : \vdash \perp, F_3, \Delta_4}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \& F_3} \& \rightarrow \frac{\overline{h_1 : \vdash \Delta_4, F_2} \text{ ax/ind} \quad \overline{h_1 : \vdash \Delta_4, F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \& F_3} \&$$

- Case rule \oplus_B

$$\frac{h_1 : \vdash \perp, F_3, \Delta_4}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \oplus F_3} \oplus_B \rightarrow \frac{\overline{h_1 : \vdash \Delta_4, F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \oplus F_3} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_1 : \vdash \perp, F_2, \Delta_4}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \oplus F_3} \oplus_A \rightarrow \frac{\overline{h_1 : \vdash \Delta_4, F_2} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \oplus F_3} \oplus_A$$

- Case rule \perp

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \rightarrow \frac{\overline{h_1 : \vdash \Delta_2} \text{ ax}}{\bullet h_1 : \vdash \Delta_2} \text{ height}$$

- Case rule \top

$$\overline{\bullet h_1 : \vdash \top, \perp, \Delta_2} \top \rightarrow \overline{\bullet h_1 : \vdash \Delta_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_1 : \vdash \perp, F_3, \Delta_5 \quad h_1 : \vdash F_4, \Delta_2}{\bullet h_1 : \vdash (\perp, \Delta_5), \Delta_2, F_3 \otimes F_4} \otimes \rightarrow \frac{\overline{h_1 : \vdash \Delta_5, F_3} \text{ ax/ind} \quad \overline{h_1 : \vdash \Delta_2, F_4} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \otimes$$

$$\frac{h_1 : \vdash F_3, \Delta_2 \quad h_1 : \vdash \perp, F_4, \Delta_5}{\bullet h_1 : \vdash \Delta_2, (\perp, \Delta_5), F_3 \otimes F_4} \otimes \rightarrow \frac{\overline{h_1 : \vdash \Delta_2, F_3} \text{ ax} \quad \overline{h_1 : \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \otimes$$

2.8 Status of \top : : Invertible

- Case rule 1

- Case rule $\$$

$$\frac{h_1 : \vdash \top, F_2, F_3, \Delta_4}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \$ F_3} \$ \rightarrow \text{trivial}$$

- Case rule $\&$

$$\frac{h_1 : \vdash \top, F_2, \Delta_4 \quad h_1 : \vdash \top, F_3, \Delta_4}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \& F_3} \& \rightarrow \text{trivial}$$

- Case rule \oplus_B

$$\frac{h_1 : \vdash \top, F_3, \Delta_4}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \oplus F_3} \oplus_B \rightarrow \text{trivial}$$

- Case rule \oplus_A

$$\frac{h_1 : \vdash \top, F_2, \Delta_4}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \oplus F_3} \oplus_A \rightarrow \text{trivial}$$

- Case rule \perp

$$\frac{h_1 : \vdash \top, \Delta_2}{\bullet h_1 : \vdash \perp, \top, \Delta_2} \perp \rightarrow \text{trivial}$$

- Case rule \top

$$\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \rightarrow \text{trivial}$$

- Case rule I

- Case rule \otimes

$$\frac{h_1 : \vdash \top, F_3, \Delta_5 \quad h_1 : \vdash F_4, \Delta_2}{\bullet h_1 : \vdash (\top, \Delta_5), \Delta_2, F_3 \otimes F_4} \otimes \rightarrow \text{trivial}$$

$$\frac{h_1 : \vdash F_3, \Delta_2 \quad h_1 : \vdash \top, F_4, \Delta_5}{\bullet h_1 : \vdash \Delta_2, (\top, \Delta_5), F_3 \otimes F_4} \otimes \rightarrow \text{trivial}$$

2.9 Status of I : : Invertible

- Case rule 1
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I

$$\frac{}{\bullet h_1 : \vdash p(n_2), \gamma(n_2)} I \rightarrow \text{trivial}$$

- Case rule \otimes

2.10 Status of \otimes : (Left Premise): Non invertible

- Case rule 1
- Case rule \$

$$\frac{h_4 : \vdash F_5, F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$ F_6} \$ \rightarrow \frac{\overline{h_4 : \vdash \Delta_7, F_2, F_5, F_6}}{\bullet h_4 : \vdash \Delta_7, F_2, F_5 \$ F_6} \text{ax/ind} \$$$

$$\frac{h_4 : \vdash F_5, F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$ F_6} \$ \rightarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2}}{\bullet h_4 : \vdash \Delta_1, F_2} \text{ax/ind height}$$

- Case rule &

$$\frac{h_4 : \vdash F_5, \Delta_1, \Delta_7, F_2 \otimes F_3 \quad h_4 : \vdash F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \& F_6} \& \rightarrow \frac{\overline{h_4 : \vdash \Delta_7, F_2, F_5} \text{ax/ind} \quad \overline{h_4 : \vdash \Delta_7, F_2, F_6} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_7, F_2, F_5 \& F_6} \&$$

$$\frac{h_4 : \vdash F_5, \Delta_1, \Delta_7, F_2 \otimes F_3 \quad h_4 : \vdash F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \& F_6} \& \rightarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} \text{height}$$

- Case rule \oplus_B

$$\frac{h_4 : \vdash F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_4 : \vdash \Delta_7, F_2, F_6} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_7, F_2, F_5 \oplus F_6} \oplus_B$$

$$\frac{h_4 : \vdash F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} \text{height}$$

- Case rule \oplus_A

$$\frac{h_4 : \vdash F_5, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_4 : \vdash \Delta_7, F_2, F_5} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_7, F_2, F_5 \oplus F_6} \oplus_A$$

$$\frac{h_4 : \vdash F_5, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} \text{height}$$

- Case rule \perp

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightarrow \frac{\overline{h_4 : \vdash \Delta_5, F_2} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_5, F_2, \perp} \perp$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightarrow \frac{\overline{h_4 : \vdash \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} \text{height}$$

- Case rule \top

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash \Delta_5, F_2, \top} \top$$

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash F_2, \Delta_1} \text{fail}$$

- Case rule I
- Case rule \otimes

$$\begin{array}{c}
\frac{h_3 : \vdash F_4, \Delta_6, \Delta_7, F_1 \otimes F_2 \quad h_3 : \vdash F_5, \Delta_8, \Delta_9}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash F_1, \Delta_6, \Delta_8, F_4 \otimes F_5} \text{ fail} \\
\\
\frac{h_3 : \vdash F_4, \Delta_6, \Delta_7 \quad h_3 : \vdash F_5, \Delta_8, \Delta_9, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash F_1, \Delta_6, \Delta_8, F_4 \otimes F_5} \text{ fail} \\
\\
\frac{h_3 : \vdash F_4, \Delta_6, \Delta_7, F_1 \otimes F_2 \quad h_3 : \vdash F_5, \Delta_8, \Delta_9}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash F_1, \Delta_6, \Delta_8} \text{ fail} \\
\\
\frac{h_3 : \vdash F_4, \Delta_6, \Delta_7 \quad h_3 : \vdash F_5, \Delta_8, \Delta_9, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash F_1, \Delta_6, \Delta_8} \text{ fail} \\
\\
\frac{h_1 : \vdash F_2, \Delta_4, \Delta_5 \quad h_1 : \vdash F_3, \Delta_6, \Delta_7}{\bullet h_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), F_2 \otimes F_3} \otimes \rightarrow \frac{}{\bullet h_1 : \vdash F_2, \Delta_4, \Delta_6} \text{ fail}
\end{array}$$

2.11 Status of \otimes (Right Premise): : Non invertible

- Case rule 1
- Case rule $\$$

$$\begin{array}{c}
\frac{h_4 : \vdash F_5, F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$ F_6} \$ \rightarrow \frac{}{h_4 : \vdash \Delta_1, F_3} \frac{\text{ax/ind}}{\text{height}} \\
\\
\frac{h_4 : \vdash F_5, F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$ F_6} \$ \rightarrow \frac{}{h_4 : \vdash \Delta_7, F_3, F_5, F_6} \frac{\text{ax/ind}}{\$}
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{h_4 : \vdash F_5, \Delta_1, \Delta_7, F_2 \otimes F_3 \quad h_4 : \vdash F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \& F_6} \& \rightarrow \frac{}{h_4 : \vdash \Delta_1, F_3} \frac{\text{ax/ind}}{\text{height}} \\
\\
\frac{h_4 : \vdash F_5, \Delta_1, \Delta_7, F_2 \otimes F_3 \quad h_4 : \vdash F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \& F_6} \& \rightarrow \frac{}{h_4 : \vdash \Delta_7, F_3, F_5} \frac{\text{ax/ind}}{\&} \frac{}{h_4 : \vdash \Delta_7, F_3, F_6} \frac{\text{ax/ind}}{\&}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_4 : \vdash F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus_B F_6} \oplus_B \rightarrow \frac{}{h_4 : \vdash \Delta_1, F_3} \frac{\text{ax/ind}}{\text{height}} \\
\\
\frac{h_4 : \vdash F_6, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus_B F_6} \oplus_B \rightarrow \frac{}{h_4 : \vdash \Delta_7, F_3, F_6} \frac{\text{ax/ind}}{\oplus_B}
\end{array}$$

- Case rule \oplus_A

$$\frac{h_4 : \vdash F_5, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus A \rightarrow \frac{\overline{h_4 : \vdash \Delta_1, F_3} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_3} \text{ height}$$

$$\frac{h_4 : \vdash F_5, \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus A \rightarrow \frac{\overline{h_4 : \vdash \Delta_7, F_3, F_5} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_7, F_3, F_5 \oplus F_6} \oplus A$$

- Case rule \perp

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightarrow \frac{\overline{h_4 : \vdash \Delta_1, F_3} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_3} \text{ height}$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightarrow \frac{\overline{h_4 : \vdash \Delta_5, F_3} \text{ ax/ind}}{\bullet h_4 : \vdash \Delta_5, F_3, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash F_3, \Delta_1} \text{ fail}$$

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash \Delta_5, F_3, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 : \vdash F_4, \Delta_6, \Delta_7, F_1 \otimes F_2 \quad h_3 : \vdash F_5, \Delta_8, \Delta_9}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash F_2, \Delta_7, \Delta_9} \text{ fail}$$

$$\frac{h_3 : \vdash F_4, \Delta_6, \Delta_7 \quad h_3 : \vdash F_5, \Delta_8, \Delta_9, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash F_2, \Delta_7, \Delta_9} \text{ fail}$$

$$\frac{h_3 : \vdash F_4, \Delta_6, \Delta_7, F_1 \otimes F_2 \quad h_3 : \vdash F_5, \Delta_8, \Delta_9}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash F_2, \Delta_7, \Delta_9, F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_3 : \vdash F_4, \Delta_6, \Delta_7 \quad h_3 : \vdash F_5, \Delta_8, \Delta_9, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash F_2, \Delta_7, \Delta_9, F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_1 : \vdash F_2, \Delta_4, \Delta_5 \quad h_1 : \vdash F_3, \Delta_6, \Delta_7}{\bullet h_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), F_2 \otimes F_3} \otimes \rightarrow \frac{}{\bullet h_1 : \vdash F_3, \Delta_5, \Delta_7} \text{ fail}$$

3 Identity-Expansion

$$\begin{array}{c}
\frac{}{- : \vdash \mathbf{0}, \top} \top \\
\\
\frac{\frac{}{- : \vdash \mathbf{1}} \mathbf{1}}{- : \vdash \mathbf{1}, \perp} \perp \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash dual(\mathbf{F}_0), \mathbf{F}_0 \oplus \mathbf{F}_1} \oplus_A \quad \frac{\frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash dual(\mathbf{F}_1), \mathbf{F}_0 \oplus \mathbf{F}_1} \oplus_B}{- : \vdash dual(\mathbf{F}_0) \& dual(\mathbf{F}_1), \mathbf{F}_0 \oplus \mathbf{F}_1} \& \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash dual(\mathbf{F}_0), dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \otimes \quad \frac{\frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash dual(\mathbf{F}_0), dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \$}{- : \vdash dual(\mathbf{F}_0) \$ dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \$ \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \oplus_A \quad \frac{\frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \oplus_B}{- : \vdash \mathbf{F}_0 \& \mathbf{F}_1, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \& \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash \mathbf{F}_0, \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \otimes \quad \frac{\frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash \mathbf{F}_0 \$ \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \$}{- : \vdash \mathbf{F}_0 \$ \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \$ \\
\\
\frac{\frac{}{- : \vdash \mathbf{1}} \mathbf{1}}{- : \vdash \mathbf{1}, \perp} \perp \\
\\
\frac{}{- : \vdash \mathbf{0}, \top} \top
\end{array}$$

4 Cut-Elimination

4.1 Status of 1: OK

- Case rule 1
- Case rule \$

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, F_3, F_4, \Delta_5}{\bullet h_2 : \vdash dual(1), \Delta_5, F_3 \$ F_4} \$ \\
 \hline
 - : \vdash *, \Delta_5, F_3 \$ F_4 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{\bullet h_1 : \vdash 1}{\bullet h_1 : \vdash 1} \text{ ax} \quad \frac{h_2 : \vdash \Delta_5, F_3, F_4, \perp}{h_2 : \vdash \Delta_5, F_3, F_4, \perp} \text{ ax}}{- : \vdash \Delta_5, F_3, F_4} \text{ hCut} \\
 \hline
 - : \vdash \Delta_5, F_3 \$ F_4 \quad \$
 \end{array}$$

- Case rule &

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, F_3, \Delta_5 \quad h_2 : \vdash \perp, F_4, \Delta_5}{\bullet h_2 : \vdash dual(1), \Delta_5, F_3 \& F_4} \& \\
 \hline
 - : \vdash *, \Delta_5, F_3 \& F_4 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{\bullet h_1 : \vdash 1}{\bullet h_1 : \vdash 1} \quad 1 \quad \frac{h_2 : \vdash \Delta_5, F_3, \perp}{h_2 : \vdash \Delta_5, F_3, \perp} \text{ ax}}{- : \vdash \Delta_5, F_3} \text{ hCut} \quad \frac{\frac{\bullet h_1 : \vdash 1}{\bullet h_1 : \vdash 1} \quad 1 \quad \frac{h_2 : \vdash \Delta_5, F_4, \perp}{h_2 : \vdash \Delta_5, F_4, \perp} \text{ ax}}{- : \vdash \Delta_5, F_4} \text{ hCut} \\
 \hline
 - : \vdash \Delta_5, F_3 \& F_4 \quad \&
 \end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, F_4, \Delta_5}{\bullet h_2 : \vdash dual(1), \Delta_5, F_3 \oplus F_4} \oplus_B \\
 \hline
 - : \vdash *, \Delta_5, F_3 \oplus F_4 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{\bullet h_1 : \vdash 1}{\bullet h_1 : \vdash 1} \text{ ax} \quad \frac{h_2 : \vdash \Delta_5, F_4, \perp}{h_2 : \vdash \Delta_5, F_4, \perp} \text{ ax}}{- : \vdash \Delta_5, F_4} \text{ hCut} \\
 \hline
 - : \vdash \Delta_5, F_3 \oplus F_4 \quad \oplus_B
 \end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, F_3, \Delta_5}{\bullet h_2 : \vdash dual(1), \Delta_5, F_3 \oplus F_4} \oplus_A \\
 \hline
 - : \vdash *, \Delta_5, F_3 \oplus F_4 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{\bullet h_1 : \vdash 1}{\bullet h_1 : \vdash 1} \text{ ax} \quad \frac{h_2 : \vdash \Delta_5, F_3, \perp}{h_2 : \vdash \Delta_5, F_3, \perp} \text{ ax}}{- : \vdash \Delta_5, F_3} \text{ hCut} \\
 \hline
 - : \vdash \Delta_5, F_3 \oplus F_4 \quad \oplus_A
 \end{array}$$

- Case rule \perp

$$\begin{array}{c}
 \frac{}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \Delta_3}{\bullet h_2 : \vdash dual(1), \Delta_3} \perp \\
 \hline
 - : \vdash *, \Delta_3 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{}{- : \vdash \Delta_3} \text{ ax}
 \end{array}$$

- Case rule \top

$$\frac{\frac{\bullet h_1 : \vdash 1, *}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{\bullet h_2 : \vdash dual(1), \top, \Delta_3}{\bullet h_2 : \vdash dual(1), \top, \Delta_3} \top}{- : \vdash *, \top, \Delta_3} \text{Cut}$$

$$\xrightarrow{\quad} \frac{}{- : \vdash \Delta_3, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{\frac{\bullet h_1 : \vdash 1, *}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash \perp, F_4, \Delta_6 \quad h_2 : \vdash F_5, \Delta_3}{\bullet h_2 : \vdash dual(1), \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes}{- : \vdash *, \Delta_3, \Delta_6, F_4 \otimes F_5} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{\bullet h_1 : \vdash 1}{\bullet h_1 : \vdash 1} \quad \frac{1 \quad h_2 : \vdash \Delta_6, F_4, \perp}{h_2 : \vdash \Delta_6, F_4, \perp} \text{ax}}{- : \vdash \Delta_6, F_4} \text{hCut} \quad \frac{\text{ax}}{- : \vdash \Delta_3, F_5} \text{ax}$$

$$\frac{}{- : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes$$

$$\frac{\frac{\bullet h_1 : \vdash 1, *}{\bullet h_1 : \vdash 1, *} \quad 1 \quad \frac{h_2 : \vdash F_4, \Delta_3 \quad h_2 : \vdash \perp, F_5, \Delta_6}{\bullet h_2 : \vdash dual(1), \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes}{- : \vdash *, \Delta_3, \Delta_6, F_4 \otimes F_5} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\text{ax} \quad \frac{\bullet h_1 : \vdash 1}{\bullet h_1 : \vdash 1} \quad \frac{1 \quad h_2 : \vdash \Delta_6, F_5, \perp}{h_2 : \vdash \Delta_6, F_5, \perp} \text{ax}}{- : \vdash \Delta_3, F_4} \text{hCut}$$

$$\frac{}{- : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes$$

4.2 Status of \$: OK

- Case rule 1

- Case rule \$

$$\frac{\frac{h_1 : \vdash F_5, F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 : \vdash F_8, F_9, \Delta_{10}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_{10}, F_8 \$ F_9} \$}{- : \vdash \Delta_2, \Delta_{10}, F_8 \$ F_9} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, dual(F_5) \otimes dual(F_6)}{h_7 : \vdash \Delta_{10}, F_8, F_9, dual(F_5) \otimes dual(F_6)} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8, F_9} \text{hCut}$$

$$\frac{}{- : \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \$$$

$$\frac{\frac{h_2 : \vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 : \vdash F_8, F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \$ F_9} \$}{- : \vdash (\Delta_5, F_3 \$ F_4), \Delta_{10}, F_8 \$ F_9} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_2 : \vdash \Delta_5, F_3, F_4, F_6}{h_2 : \vdash \Delta_5, F_3, F_4, F_6} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \$ F_9}{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \$ F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \$ F_9} \text{hCut}$$

$$\frac{}{- : \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \$ F_9} \$$$

- Case rule $\&$

$$\frac{\frac{h_1 : \vdash F_5, F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 : \vdash F_8, \Delta_{10}, dual(F_5) \otimes dual(F_6) \quad h_7 : \vdash F_9, \Delta_{10}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \$ F_6), \Delta_{10}, F_8 \& F_9} \&}{- : \vdash \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8} \text{hCut}$$

$$\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6}{\bullet h_1 : \vdash \Delta_2, F_5 \$ F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}{h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_9} \text{hCut}$$

$$\frac{}{- : \vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \&$$

$$\begin{array}{c}
\frac{h_2 \vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_6) \quad h_7 \vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- \vdash (\Delta_5, F_3 \$ F_4), \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_4, F_6}{- \vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \& F_9}{- \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \& F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \& F_9} \text{hCut} \$
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash F_5, F_6, \Delta_2}{\bullet h_1 \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 \vdash F_9, \Delta_{10}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 \vdash dual(F_5 \$ F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \$ F_6}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{hCut} \oplus_B \\
\hline
\frac{h_2 \vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 \vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash (\Delta_5, F_3 \$ F_4), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_4, F_6}{- \vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{- \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \oplus F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \oplus F_9} \text{hCut} \$
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash F_5, F_6, \Delta_2}{\bullet h_1 \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 \vdash dual(F_5 \$ F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \$ F_6}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{hCut} \oplus_A \\
\hline
\frac{h_2 \vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash (\Delta_5, F_3 \$ F_4), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_4, F_6}{- \vdash \Delta_{10}, \Delta_5, F_3, F_4, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{- \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \oplus F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_3 \$ F_4, F_8 \oplus F_9} \text{hCut} \$
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash F_5, F_6, \Delta_2}{\bullet h_1 \vdash F_5 \$ F_6, \Delta_2} \$ \quad \frac{h_7 \vdash \Delta_8, dual(F_5) \otimes dual(F_6)}{\bullet h_7 \vdash dual(F_5 \$ F_6), \perp, \Delta_8} \perp \\
\hline
- \vdash \Delta_2, \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \$ F_6}{- \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{h_7 \vdash \Delta_8, dual(F_5) \otimes dual(F_6)}{- \vdash \Delta_2, \Delta_8, \perp} \text{ax}}{- \vdash \Delta_2, \Delta_8, \perp} \text{hCut} \perp \\
\hline
\frac{h_2 \vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \quad \frac{h_7 \vdash \Delta_8, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \perp, \Delta_8} \perp \\
\hline
- \vdash (\Delta_5, F_3 \$ F_4), \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_4, F_6}{- \vdash \Delta_5, \Delta_8, F_3, F_4, \perp} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_8, \perp, dual(F_6)}{- \vdash \Delta_5, \Delta_8, \perp, F_3 \$ F_4} \text{ax}}{- \vdash \Delta_5, \Delta_8, \perp, F_3 \$ F_4} \text{hCut} \$
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash F_5, F_6, \Delta_2}{\bullet h_1 \vdash F_5 \$ F_6, \Delta_2} \$ \frac{}{\bullet h_7 \vdash dual(F_5 \$ F_6), \top, \Delta_8} \top \\
\hline
- \vdash \Delta_2, \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
- \vdash \Delta_2, \Delta_8, \top \quad \top
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \frac{}{\bullet h_7 \vdash dual(F_6), \top, \Delta_8} \top \\
\hline
- \vdash (\Delta_5, F_3 \$ F_4), \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
- \vdash \Delta_5, \Delta_8, \top, F_3 \$ F_4 \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash F_5, F_6, \Delta_2}{\bullet h_1 \vdash F_5 \$ F_6, \Delta_2} \$ \frac{h_7 \vdash F_9, \Delta_{11}, dual(F_5) \otimes dual(F_6) \quad h_7 \vdash F_{10}, \Delta_8}{\bullet h_7 \vdash dual(F_5 \$ F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{}{\bullet h_1 \vdash \Delta_2, F_5 \$ F_6} \text{ax} \quad \frac{h_7 \vdash F_9, \Delta_{11}, dual(F_5) \otimes dual(F_6)}{h_7 \vdash \Delta_{11}, F_9, dual(F_5) \otimes dual(F_6)} \text{ax}}{- \vdash \Delta_{11}, \Delta_2, F_9} \text{hCut} \quad \frac{}{- \vdash \Delta_8, F_{10}} \text{ax} \\
\hline
- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10} \quad \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash F_5, F_6, \Delta_2}{\bullet h_1 \vdash F_5 \$ F_6, \Delta_2} \$ \frac{h_7 \vdash F_9, \Delta_8 \quad h_7 \vdash F_{10}, \Delta_{11}, dual(F_5) \otimes dual(F_6)}{\bullet h_7 \vdash dual(F_5 \$ F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{}{- \vdash \Delta_8, F_9} \text{ax} \quad \frac{\frac{}{\bullet h_1 \vdash \Delta_2, F_5 \$ F_6} \text{ax} \quad \frac{h_7 \vdash F_{10}, \Delta_{11}, dual(F_5) \otimes dual(F_6)}{h_7 \vdash \Delta_{11}, F_{10}, dual(F_5) \otimes dual(F_6)} \text{ax}}{- \vdash \Delta_{11}, \Delta_2, F_{10}} \text{hCut} \\
\hline
- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10} \quad \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash F_5, F_6, \Delta_2}{\bullet h_1 \vdash F_5 \$ F_6, \Delta_2} \$ \frac{h_7 \vdash \Delta_8, dual(F_5) \quad h_7 \vdash \Delta_9, dual(F_6)}{\bullet h_7 \vdash dual(F_5 \$ F_6), \Delta_8, \Delta_9} \otimes \\
\hline
- \vdash \Delta_2, \Delta_8, \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{}{- \vdash \Delta_2, F_5, F_6} \text{ax} \quad \frac{}{- \vdash \Delta_9, dual(F_6)} \text{ax}}{- \vdash \Delta_2, \Delta_9, F_5} \text{sCut} \quad \frac{}{- \vdash \Delta_8, dual(F_5)} \text{ax} \\
\hline
- \vdash \Delta_2, \Delta_8, \Delta_9 \quad \text{sCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \frac{h_7 \vdash F_9, \Delta_{11}, dual(F_6) \quad h_7 \vdash F_{10}, \Delta_8}{\bullet h_7 \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash (\Delta_5, F_3 \$ F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{}{h_2 \vdash \Delta_5, F_3, F_4, F_6} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{\bullet h_7 \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}} \text{ax}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_4, F_9 \otimes F_{10}} \text{hCut} \\
\hline
- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \$ F_4, F_9 \otimes F_{10} \quad \$
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash F_6, F_3, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \$ F_4} \$ \frac{h_7 \vdash F_9, \Delta_8 \quad h_7 \vdash F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash (\Delta_5, F_3 \$ F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{}{h_2 \vdash \Delta_5, F_3, F_4, F_6} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{\bullet h_7 \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}} \text{ax}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_4, F_9 \otimes F_{10}} \text{hCut} \\
\hline
- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \$ F_4, F_9 \otimes F_{10} \quad \$
\end{array}$$

4.3 Status of $\&$: OK

- Case rule 1
- Case rule $\$$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_5, \Delta_2 \quad h_1 \vdash F_6, \Delta_2}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash F_8, F_9, \Delta_{10}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \$ F_9} \$}{\frac{}{- \vdash \Delta_2, \Delta_{10}, F_8 \$ F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \& F_6}{- \vdash \Delta_{10}, \Delta_2, F_8, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_5) \oplus dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \text{ax}}{\frac{}{- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \$} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 \vdash F_6, F_3, \Delta_5 \quad h_2 \vdash F_6, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 \vdash F_8, F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \$ F_9} \$}{\frac{}{- \vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \$ F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \& F_4}{- \vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_6)}{- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \& F_4} \text{ax}}{\frac{}{- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \& F_4} \$} \text{hCut}
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_5, \Delta_2 \quad h_1 \vdash F_6, \Delta_2}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_5) \oplus dual(F_6) \quad h_7 \vdash F_9, \Delta_{10}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \& F_9} \&}{\frac{}{- \vdash \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \& F_6}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_5) \oplus dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax}}{\frac{}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{hCut}} \& \quad \frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \& F_6}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \oplus dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax}}{\frac{}{- \vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \&} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 \vdash F_6, F_3, \Delta_5 \quad h_2 \vdash F_6, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_6) \quad h_7 \vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \&}{\frac{}{- \vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \& F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_6}{- \vdash \Delta_{10}, \Delta_5, F_3, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \& F_9}{- \vdash \Delta_{10}, \Delta_5, F_3, F_8 \& F_9} \text{ax}}{\frac{}{- \vdash \Delta_{10}, \Delta_5, F_3, F_8 \& F_9} \text{hCut}} \& \quad \frac{\frac{h_2 \vdash \Delta_5, F_4, F_6}{- \vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \& F_9}{- \vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \text{ax}}{\frac{}{- \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \& F_9} \&} \text{hCut}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_5, \Delta_2 \quad h_1 \vdash F_6, \Delta_2}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash F_9, \Delta_{10}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{\frac{}{- \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \& F_6}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \oplus dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ax}}{\frac{}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus_B} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_5, \Delta_2 \quad h_1 \vdash F_6, \Delta_2}{\bullet h_1 \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 \vdash \Delta_8, dual(F_6)}{\bullet h_7 \vdash dual(F_5 \& F_6), \Delta_8} \oplus_B}{\frac{}{- \vdash \Delta_2, \Delta_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{}{- \vdash \Delta_2, F_6} \text{ax} \quad \frac{}{- \vdash \Delta_8, dual(F_6)} \text{ax}}{\frac{}{- \vdash \Delta_2, \Delta_8} \text{sCut}}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_6, F_3, \Delta_5 \quad h_2 : \vdash F_6, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- : \vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \oplus F_9 \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_{10}, \Delta_5, F_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_9, dual(F_6)}{- : \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_5, F_9, F_3 \& F_4} \text{hCut} \\
\hline
- : \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9 \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 : \vdash F_5, \Delta_2 \quad h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash F_8, \Delta_{10}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- : \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9 \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{hCut} \\
\hline
- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9 \oplus_A
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash F_5, \Delta_2 \quad h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_8, dual(F_5)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_8} \oplus_A \\
\hline
- : \vdash \Delta_2, \Delta_8 \\
\hline
\rightarrow \\
\frac{\frac{- : \vdash \Delta_2, F_5}{- : \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{- : \vdash \Delta_8, dual(F_5)}{- : \vdash \Delta_2, \Delta_8} \text{ax}}{- : \vdash \Delta_2, \Delta_8} \text{sCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_6, F_3, \Delta_5 \quad h_2 : \vdash F_6, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash F_8, \Delta_{10}, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- : \vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \oplus F_9 \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_{10}, \Delta_5, F_8, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6)}{- : \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9} \text{ax}}{- : \vdash \Delta_{10}, \Delta_5, F_8, F_3 \& F_4} \text{hCut} \\
\hline
- : \vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9 \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 : \vdash F_5, \Delta_2 \quad h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \perp, \Delta_8} \perp \\
\hline
- : \vdash \Delta_2, \perp, \Delta_8 \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_2, \Delta_8} \text{ax}}{- : \vdash \Delta_2, \Delta_8} \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_8, \perp \perp
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_6, F_3, \Delta_5 \quad h_2 : \vdash F_6, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash \Delta_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \perp, \Delta_8} \perp \\
\hline
- : \vdash (\Delta_5, F_3 \& F_4), \perp, \Delta_8 \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_5, \Delta_8, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_8, dual(F_6)}{- : \vdash \Delta_5, \Delta_8, \perp, F_3 \& F_4} \text{ax}}{- : \vdash \Delta_5, \Delta_8, F_3 \& F_4} \text{hCut} \\
\hline
- : \vdash \Delta_5, \Delta_8, \perp, F_3 \& F_4 \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash F_5, \Delta_2 \quad h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{}{\bullet h_7 : \vdash dual(F_5 \& F_6), \top, \Delta_8} \top \\
\hline
- : \vdash \Delta_2, \top, \Delta_8 \\
\hline
\rightarrow \\
\frac{}{- : \vdash \Delta_2, \Delta_8, \top} \top
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_6, F_3, \Delta_5 \quad h_2 : \vdash F_6, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{}{\bullet h_7 : \vdash dual(F_6), \top, \Delta_8} \top \\
\hline
- : \vdash (\Delta_5, F_3 \& F_4), \top, \Delta_8 \\
\rightarrow \\
- : \vdash \Delta_5, \Delta_8, \top, F_3 \& F_4 \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash F_5, \Delta_2 \quad h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash F_9, \Delta_{11}, dual(F_5) \oplus dual(F_6) \quad h_7 : \vdash F_{10}, \Delta_8}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, F_5 \& F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_{11}, F_9, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_{11}, \Delta_2, F_9} \text{hCut} \quad \frac{}{- : \vdash \Delta_8, F_{10}} \text{ax}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\\
\frac{h_1 : \vdash F_5, \Delta_2 \quad h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash F_9, \Delta_8 \quad h_7 : \vdash F_{10}, \Delta_{11}, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{}{- : \vdash \Delta_8, F_9} \text{ax} \quad \frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, F_5 \& F_6} \text{ax} \quad \frac{h_7 : \vdash \Delta_{11}, F_{10}, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_{11}, \Delta_2, F_{10}} \text{hCut}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\\
\frac{h_2 : \vdash F_6, F_3, \Delta_5 \quad h_2 : \vdash F_6, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash F_9, \Delta_{11}, dual(F_6) \quad h_7 : \vdash F_{10}, \Delta_8}{\bullet h_7 : \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash (\Delta_5, F_3 \& F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_{11}, \Delta_5, F_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_{11}, F_9, dual(F_6)}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \text{hCut} \quad \frac{}{- : \vdash \Delta_8, F_{10}} \text{ax}}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \otimes \\
\\
\frac{h_2 : \vdash F_6, F_3, \Delta_5 \quad h_2 : \vdash F_6, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash F_9, \Delta_8 \quad h_7 : \vdash F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- : \vdash (\Delta_5, F_3 \& F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{}{- : \vdash \Delta_8, F_9} \text{ax} \quad \frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_{11}, \Delta_5, F_{10}, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_{11}, F_{10}, dual(F_6)}{- : \vdash \Delta_{11}, \Delta_5, F_{10}, F_3 \& F_4} \text{hCut}}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \otimes
\end{array}$$

4.4 Status of \oplus_B : OK

- Case rule 1

- Case rule $\$$

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash F_8, F_9, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \$ F_9} \$ \\
\hline
- : \vdash \Delta_2, \Delta_{10}, F_8 \$ F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_{10}, \Delta_2, F_8, F_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, dual(F_5) \& dual(F_6)}{- : \vdash \Delta_{10}, \Delta_2, F_8, F_9} \text{hCut}}{- : \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} \$
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_6, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 : \vdash F_8, F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \$ \\
\hline
- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \oplus F_4 \quad ax \quad h_7 : \vdash \Delta_{10}, F_8, F_9, dual(F_6) \quad ax}{- : \vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3 \oplus F_4} hCut \\
\hline
- : \vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4 \quad \$
\end{array}$$

• Case rule &

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash F_8, \Delta_{10}, dual(F_5) \& dual(F_6) \quad h_7 : \vdash F_9, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- : \vdash \Delta_2, \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6 \quad ax \quad h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6) \quad ax}{- : \vdash \Delta_{10}, \Delta_2, F_8} hCut \quad \frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6 \quad ax \quad h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6) \quad ax}{- : \vdash \Delta_{10}, \Delta_2, F_9} hCut \\
\hline
- : \vdash \Delta_{10}, \Delta_2, F_8 \& F_9 \quad \&
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash \Delta_8, dual(F_5) \quad h_7 : \vdash \Delta_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_8} \& \\
\hline
- : \vdash \Delta_2, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \vdash \Delta_2, F_6 \quad ax \quad - : \vdash \Delta_8, dual(F_6) \quad ax}{- : \vdash \Delta_2, \Delta_8} sCut
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_6, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 : \vdash F_8, \Delta_{10}, dual(F_6) \quad h_7 : \vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \vdash \Delta_5, F_4, F_6 \quad ax \quad \bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \& F_9 \quad ax}{- : \vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} hCut \\
\hline
- : \vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4 \quad \oplus_B
\end{array}$$

• Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash F_9, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- : \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6 \quad ax \quad h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6) \quad ax}{- : \vdash \Delta_{10}, \Delta_2, F_9} hCut \\
\hline
- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9 \quad \oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_6, F_4, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 : \vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \vdash \Delta_5, F_4, F_6 \quad ax \quad \bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9 \quad ax}{- : \vdash \Delta_{10}, \Delta_5, F_4, F_8 \oplus F_9} hCut \\
\hline
- : \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9 \quad \oplus_B
\end{array}$$

• Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 : \vdash F_8, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- : \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6 \quad ax \quad h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6) \quad ax}{- : \vdash \Delta_{10}, \Delta_2, F_8} hCut \\
\hline
- : \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9 \quad \oplus_A
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash F_6, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_4, F_6}{- \vdash \Delta_{10}, \Delta_5, F_4, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \text{hCut} \oplus_B
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash F_6, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash \Delta_8, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \perp, \Delta_8} \perp \\
\hline
- \vdash \Delta_2, \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{h_7 \vdash \Delta_8, dual(F_5) \& dual(F_6)}{- \vdash \Delta_2, \Delta_8, \perp} \text{ax}}{- \vdash \Delta_2, \Delta_8, \perp} \text{hCut} \perp \\
\hline
\frac{h_2 \vdash F_6, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash \Delta_8, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \perp, \Delta_8} \perp \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_4, F_6}{- \vdash \Delta_5, \Delta_8, F_4, \perp} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_8, \perp, dual(F_6)}{- \vdash \Delta_5, \Delta_8, \perp, F_3 \oplus F_4} \text{ax}}{- \vdash \Delta_5, \Delta_8, \perp, F_3 \oplus F_4} \text{hCut} \oplus_B
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash F_6, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \top, \Delta_8} \top \\
\hline
- \vdash \Delta_2, \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \Delta_2, \Delta_8, \top} \top \\
\hline
\frac{h_2 \vdash F_6, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{}{\bullet h_7 \vdash dual(F_6), \top, \Delta_8} \top \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \Delta_5, \Delta_8, \top, F_3 \oplus F_4} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash F_6, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash F_9, \Delta_{11}, dual(F_5) \& dual(F_6) \quad h_7 \vdash F_{10}, \Delta_8}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{11}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_9, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{ax}}{- \vdash \Delta_{11}, \Delta_2, F_9} \text{hCut} \quad \frac{}{- \vdash \Delta_8, F_{10}} \text{ax} \otimes \\
\hline
\frac{h_1 \vdash F_6, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash F_9, \Delta_8 \quad h_7 \vdash F_{10}, \Delta_{11}, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \Delta_8, F_9} \text{ax} \quad \frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{11}, \Delta_2, F_{10}} \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_{10}, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{11}, \Delta_2, F_{10}} \text{ax}}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{hCut} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash F_6, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash F_9, \Delta_{11}, dual(F_6) \quad h_7 \vdash F_{10}, \Delta_8}{\bullet h_7 \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_4, F_6}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10}} ax \quad \frac{\bullet h_7 \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} ax}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} hCut \oplus_B \\
\hline
\frac{h_2 \vdash F_6, F_4, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash F_9, \Delta_8 \quad h_7 \vdash F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_4, F_6}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10}} ax \quad \frac{\bullet h_7 \vdash \Delta_{11}, \Delta_8, dual(F_6), F_9 \otimes F_{10}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} ax}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} hCut \oplus_B
\end{array}$$

4.5 Status of \oplus_A : OK

- Case rule 1
- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash F_5, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \vdash F_8, F_9, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \$ F_9} \$ \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \$ F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_8, F_9} ax \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} ax}{- \vdash \Delta_{10}, \Delta_2, F_8 \$ F_9} hCut \$ \\
\hline
\frac{h_2 \vdash F_6, F_3, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \vdash F_8, F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \$ F_9} \$ \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \$ F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \oplus F_4}{- \vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3 \oplus F_4} ax \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, dual(F_6)}{- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \oplus F_4} ax}{- \vdash \Delta_{10}, \Delta_5, F_8 \$ F_9, F_3 \oplus F_4} hCut \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_1 \vdash F_5, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_5) \& dual(F_6) \quad h_7 \vdash F_9, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_8} ax \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8} hCut \quad \frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_9} ax \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_9} hCut}{- \vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \& \\
\hline
\frac{h_1 \vdash F_5, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \vdash \Delta_8, dual(F_5) \quad h_7 \vdash \Delta_8, dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_8} \& \\
\hline
- \vdash \Delta_2, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- \vdash \Delta_2, F_5}{- \vdash \Delta_2, \Delta_8} ax \quad \frac{- \vdash \Delta_8, dual(F_5)}{- \vdash \Delta_2, \Delta_8} ax}{- \vdash \Delta_2, \Delta_8} sCut
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash F_6, F_3, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_6) \quad h_7 \vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_5, F_3, F_6}{- \vdash \Delta_{10}, \Delta_5, F_3, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \& F_9}{- \vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \text{hCut} \oplus_A
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash F_5, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \vdash F_9, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ax} \\
\hline
- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9 \quad \text{hCut} \oplus_B \\
\rightarrow \\
\frac{h_2 \vdash F_6, F_3, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \vdash F_9, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \Delta_5, F_6, F_3 \oplus F_4}{- \vdash \Delta_{10}, \Delta_5, F_9, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, dual(F_6)}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \text{ax} \\
\hline
- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9 \quad \text{hCut} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash F_5, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ax} \\
\hline
- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9 \quad \text{hCut} \oplus_A \\
\rightarrow \\
\frac{h_2 \vdash F_6, F_3, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \vdash F_8, \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \Delta_5, F_3, F_6}{- \vdash \Delta_{10}, \Delta_5, F_3, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \text{ax} \\
\hline
- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9 \quad \text{hCut} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash F_5, \Delta_2}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \vdash \Delta_8, dual(F_5) \& dual(F_6)}{\bullet h_7 \vdash dual(F_5 \oplus F_6), \perp, \Delta_8} \perp \\
\hline
- \vdash \Delta_2, \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{h_7 \vdash \Delta_8, dual(F_5) \& dual(F_6)}{- \vdash \Delta_2, \Delta_8, \perp} \text{ax} \\
\hline
- \vdash \Delta_2, \Delta_8, \perp \quad \text{hCut} \perp \\
\rightarrow \\
\frac{h_2 \vdash F_6, F_3, \Delta_5}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \vdash \Delta_8, dual(F_6)}{\bullet h_7 \vdash dual(F_6), \perp, \Delta_8} \perp \\
\hline
- \vdash (\Delta_5, F_3 \oplus F_4), \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \Delta_5, F_3, F_6}{- \vdash \Delta_5, \Delta_8, F_3, \perp} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_8, \perp, dual(F_6)}{- \vdash \Delta_5, \Delta_8, \perp, F_3 \oplus F_4} \text{ax} \\
\hline
- \vdash \Delta_5, \Delta_8, \perp, F_3 \oplus F_4 \quad \text{hCut} \oplus_A
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash F_5, \Delta_2}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{}{\bullet h_7 : \vdash \text{dual}(F_5 \oplus F_6), \top, \Delta_8} \top}{- : \vdash \Delta_2, \top, \Delta_8} \text{Cut} \\
\rightarrow \\
- : \vdash \Delta_2, \Delta_8, \top \quad \top
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_6, F_3, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{}{\bullet h_7 : \vdash \text{dual}(F_6), \top, \Delta_8} \top}{- : \vdash (\Delta_5, F_3 \oplus F_4), \top, \Delta_8} \text{Cut} \\
\rightarrow \\
- : \vdash \Delta_5, \Delta_8, \top, F_3 \oplus F_4 \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash F_5, \Delta_2}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 : \vdash F_9, \Delta_{11}, \text{dual}(F_5) \& \text{dual}(F_6) \quad h_7 : \vdash F_{10}, \Delta_8}{\bullet h_7 : \vdash \text{dual}(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6}{- : \vdash \Delta_{11}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_{11}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{hCut} \quad \frac{}{- : \vdash \Delta_8, F_{10}} \text{ax}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash F_5, \Delta_2}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 : \vdash F_9, \Delta_8 \quad h_7 : \vdash F_{10}, \Delta_{11}, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 : \vdash \text{dual}(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \vdash \Delta_8, F_9} \text{ax} \quad \frac{\bullet h_1 : \vdash \Delta_2, F_5 \oplus F_6 \quad h_7 : \vdash \Delta_{11}, F_{10}, \text{dual}(F_5) \& \text{dual}(F_6)}{- : \vdash \Delta_{11}, \Delta_2, F_{10}} \text{hCut}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_6, F_3, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 : \vdash F_9, \Delta_{11}, \text{dual}(F_6) \quad h_7 : \vdash F_{10}, \Delta_8}{\bullet h_7 : \vdash \text{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_5, F_3, F_6}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{11}, \Delta_8, \text{dual}(F_6), F_9 \otimes F_{10}}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{hCut}}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \oplus A
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash F_6, F_3, \Delta_5}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 : \vdash F_9, \Delta_8 \quad h_7 : \vdash F_{10}, \Delta_{11}, \text{dual}(F_6)}{\bullet h_7 : \vdash \text{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_5, F_3, F_6}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{11}, \Delta_8, \text{dual}(F_6), F_9 \otimes F_{10}}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{hCut}}{- : \vdash \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \oplus A
\end{array}$$

4.6 Status of \perp : OK

- Case rule 1

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{}{\bullet h_3 : \vdash \text{dual}(\perp), *} 1}{- : \vdash \Delta_2, *} \text{Cut} \\
\rightarrow \\
- : \vdash \Delta_2 \quad \text{ax}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, F_4, F_5, \Delta_6}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \$ F_5} \$}{- : \vdash \Delta_2, \Delta_6, F_4 \$ F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_4, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4, F_5}{h_3 : \vdash \mathbf{1}, \Delta_6, F_4, F_5} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \$ F_5} \text{hCut} \$ \\
\\
\frac{\frac{h_2 : \vdash F_4, \Delta_3}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash F_6, F_7, \Delta_8, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \$ F_7} \$}{- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \$ F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_8, F_6, F_7, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, F_7, dual(F_4)}{h_5 : \vdash \Delta_8, F_6, F_7, dual(F_4)} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \$ F_7} \text{hCut} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, F_4, \Delta_6 \quad h_3 : \vdash \mathbf{1}, F_5, \Delta_6}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \& F_5} \&}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4}{h_3 : \vdash \mathbf{1}, \Delta_6, F_4} \text{ax} \quad \frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{h_3 : \vdash \mathbf{1}, \Delta_6, F_5} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \text{hCut} \& \\
\\
\frac{\frac{h_2 : \vdash F_4, \Delta_3}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash F_6, \Delta_8, dual(F_4) \quad h_5 : \vdash F_7, \Delta_8, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \& F_7} \&}{- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \& F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, \Delta_8, F_6 \& F_7} \text{ax} \quad \frac{\bullet h_5 : \vdash \Delta_8, dual(F_4), F_6 \& F_7}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \& F_7} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \& F_7} \text{hCut} \perp
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, F_5, \Delta_6}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \oplus F_5} \oplus_B}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{h_3 : \vdash \mathbf{1}, \Delta_6, F_5} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{hCut} \oplus_B \\
\\
\frac{\frac{h_2 : \vdash F_4, \Delta_3}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash F_7, \Delta_8, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \oplus F_7} \oplus_B}{- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_8, F_7, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_7, dual(F_4)}{h_5 : \vdash \Delta_8, F_7, dual(F_4)} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7} \text{hCut} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, F_4, \Delta_6}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \oplus F_5} \oplus_A \\
\hline
- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{hCut} \oplus_A \\
\\
\frac{h_2 : \vdash F_4, \Delta_3}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash F_6, \Delta_8, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \oplus F_7} \oplus_A \\
\hline
- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_8, F_6, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, dual(F_4)}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7} \text{hCut} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_4}{\bullet h_3 : \vdash dual(\perp), \perp, \Delta_4} \perp \\
\hline
- : \vdash \Delta_2, \perp, \Delta_4 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_4}{- : \vdash \Delta_2, \Delta_4, \perp} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \perp} \text{hCut} \perp \\
\\
\frac{h_2 : \vdash F_4, \Delta_3}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_6, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \perp, \Delta_6} \perp \\
\hline
- : \vdash (\perp, \Delta_3), \perp, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, \Delta_6, \perp} \text{ax} \quad \frac{\bullet h_5 : \vdash \Delta_6, \perp, dual(F_4)}{- : \vdash \Delta_3, \Delta_6, \perp, \perp} \text{ax}}{- : \vdash \Delta_3, \Delta_6, \perp, \perp} \text{hCut} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{}{\bullet h_3 : \vdash dual(\perp), \top, \Delta_4} \top \\
\hline
- : \vdash \Delta_2, \top, \Delta_4 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \Delta_2, \Delta_4, \top} \top \\
\\
\frac{h_2 : \vdash F_4, \Delta_3}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{}{\bullet h_5 : \vdash dual(F_4), \top, \Delta_6} \top \\
\hline
- : \vdash (\perp, \Delta_3), \top, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \Delta_3, \Delta_6, \perp, \top} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, F_5, \Delta_7 \quad h_3 : \vdash F_6, \Delta_4}{\bullet h_3 : \vdash dual(\perp), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \\
\hline
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_7, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_7, F_5}{- : \vdash \Delta_4, F_6} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{hCut} \otimes
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{0}, F_5, \Delta_6}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_6, F_4 \oplus F_5} \oplus_B \\
\hline
- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \Delta_2, \top} \top \quad \frac{}{h_3 : \vdash \mathbf{0}, \Delta_6, F_5} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_6, F_5 \quad \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5 \quad \oplus_B
\end{array} \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash F_7, \Delta_8, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_8, F_6 \oplus F_7} \oplus_B \\
\hline
- : \vdash (\top, \Delta_3), \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_3, \Delta_8, \top, F_6 \oplus F_7} \top
\end{array}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{0}, F_4, \Delta_6}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_6, F_4 \oplus F_5} \oplus_A \\
\hline
- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \Delta_2, \top} \top \quad \frac{}{h_3 : \vdash \mathbf{0}, \Delta_6, F_4} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_6, F_4 \quad \text{hCut} \\
\hline
- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5 \quad \oplus_A
\end{array} \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash F_6, \Delta_8, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_8, F_6 \oplus F_7} \oplus_A \\
\hline
- : \vdash (\top, \Delta_3), \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_3, \Delta_8, \top, F_6 \oplus F_7} \top
\end{array}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_4}{\bullet h_3 : \vdash \text{dual}(\top), \perp, \Delta_4} \perp \\
\hline
- : \vdash \Delta_2, \perp, \Delta_4 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \Delta_2, \perp, \top} \top \quad \frac{}{h_3 : \vdash \mathbf{0}, \Delta_4} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_4, \perp \quad \text{hCut}
\end{array} \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_6, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \perp, \Delta_6} \perp \\
\hline
- : \vdash (\top, \Delta_3), \perp, \Delta_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_3, \Delta_6, \perp, \top} \top
\end{array}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{}{\bullet h_3 : \vdash \text{dual}(\top), \top, \Delta_4} \top \\
\hline
- : \vdash \Delta_2, \top, \Delta_4 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_2, \Delta_4, \top} \top
\end{array} \\
\hline
\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{}{\bullet h_5 : \vdash \text{dual}(F_4), \top, \Delta_6} \top \\
\hline
- : \vdash (\top, \Delta_3), \top, \Delta_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_3, \Delta_6, \top, \top} \top
\end{array}
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{O}, F_5, \Delta_7 \quad h_3 : \vdash F_6, \Delta_4}{\bullet h_3 : \vdash dual(\top), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_7, F_5} \top \quad \frac{h_3 : \vdash \mathbf{O}, \Delta_7, F_5}{hCut} \text{ax} \quad \frac{}{- : \vdash \Delta_4, F_6} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \\
\\
\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash F_5, \Delta_4 \quad h_3 : \vdash \mathbf{O}, F_6, \Delta_7}{\bullet h_3 : \vdash dual(\top), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \vdash \Delta_4, F_5} \text{ax} \quad \frac{\bullet h_1 : \vdash \Delta_2, \top \quad h_3 : \vdash \mathbf{O}, \Delta_7, F_6}{- : \vdash \Delta_2, \Delta_7, F_6} \text{ax} \quad hCut}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \\
\\
\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash F_7, \Delta_9, dual(F_4) \quad h_5 : \vdash F_8, \Delta_6}{\bullet h_5 : \vdash dual(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash (\top, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_7 \otimes F_8} \top \\
\\
\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash F_7, \Delta_6 \quad h_5 : \vdash F_8, \Delta_9, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash (\top, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_7 \otimes F_8} \top
\end{array}$$

4.8 Status of I : OK

- Case rule 1
- Case rule $\$$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \wedge(n_3), p(n_3)} I \quad \frac{h_4 : \vdash F_5, F_6, \Delta_7, p(n_3)}{\bullet h_4 : \vdash dual(\wedge(n_3)), \Delta_7, F_5 \$ F_6} \$}{- : \vdash p(n_3), \Delta_7, F_5 \$ F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \vdash \Delta_7, F_5, F_6, p(n_3)} \text{ax} \quad \$}{- : \vdash \Delta_7, p(n_3), F_5 \$ F_6} \\
\\
\frac{\frac{}{\bullet h_1 : \vdash p(n_3), \wedge(n_3)} I \quad \frac{h_4 : \vdash F_5, F_6, \Delta_7, \wedge(n_3)}{\bullet h_4 : \vdash dual(p(n_3)), \Delta_7, F_5 \$ F_6} \$}{- : \vdash \wedge(n_3), \Delta_7, F_5 \$ F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \vdash \Delta_7, F_5, F_6, \wedge(n_3)} \text{ax} \quad \$}{- : \vdash \Delta_7, \wedge(n_3), F_5 \$ F_6}
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \wedge(n_3), p(n_3)} I \quad \frac{h_4 : \vdash F_5, \Delta_7, p(n_3) \quad h_4 : \vdash F_6, \Delta_7, p(n_3)}{\bullet h_4 : \vdash dual(\wedge(n_3)), \Delta_7, F_5 \& F_6} \&}{- : \vdash p(n_3), \Delta_7, F_5 \& F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \vdash \Delta_7, F_5, p(n_3)} \text{ax} \quad \frac{}{- : \vdash \Delta_7, F_6, p(n_3)} \text{ax}}{- : \vdash \Delta_7, p(n_3), F_5 \& F_6} \&
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash p(n_3), \wedge(n_3)} I \quad \frac{h_4 : \vdash F_5, \Delta_7, \wedge(n_3) \quad h_4 : \vdash F_6, \Delta_7, \wedge(n_3)}{\bullet h_4 : \vdash dual(p(n_3)), \Delta_7, F_5 \& F_6} \& \\
\hline
- : \vdash \wedge(n_3), \Delta_7, F_5 \& F_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_7, F_5, \wedge(n_3)} \text{ax} \quad \frac{}{- : \vdash \Delta_7, F_6, \wedge(n_3)} \text{ax} \\
\hline
- : \vdash \Delta_7, \wedge(n_3), F_5 \& F_6 \quad \&
\end{array}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \wedge(n_3), p(n_3)} I \quad \frac{h_4 : \vdash F_6, \Delta_7, p(n_3)}{\bullet h_4 : \vdash dual(\wedge(n_3)), \Delta_7, F_5 \oplus F_6} \oplus_B \\
\hline
- : \vdash p(n_3), \Delta_7, F_5 \oplus F_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_7, F_6, p(n_3)} \text{ax} \\
\hline
- : \vdash \Delta_7, p(n_3), F_5 \oplus F_6 \quad \oplus_B
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash p(n_3), \wedge(n_3)} I \quad \frac{h_4 : \vdash F_6, \Delta_7, \wedge(n_3)}{\bullet h_4 : \vdash dual(p(n_3)), \Delta_7, F_5 \oplus F_6} \oplus_B \\
\hline
- : \vdash \wedge(n_3), \Delta_7, F_5 \oplus F_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_7, F_6, \wedge(n_3)} \text{ax} \\
\hline
- : \vdash \Delta_7, \wedge(n_3), F_5 \oplus F_6 \quad \oplus_B
\end{array}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \wedge(n_3), p(n_3)} I \quad \frac{h_4 : \vdash F_5, \Delta_7, p(n_3)}{\bullet h_4 : \vdash dual(\wedge(n_3)), \Delta_7, F_5 \oplus F_6} \oplus_A \\
\hline
- : \vdash p(n_3), \Delta_7, F_5 \oplus F_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_7, F_5, p(n_3)} \text{ax} \\
\hline
- : \vdash \Delta_7, p(n_3), F_5 \oplus F_6 \quad \oplus_A
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash p(n_3), \wedge(n_3)} I \quad \frac{h_4 : \vdash F_5, \Delta_7, \wedge(n_3)}{\bullet h_4 : \vdash dual(p(n_3)), \Delta_7, F_5 \oplus F_6} \oplus_A \\
\hline
- : \vdash \wedge(n_3), \Delta_7, F_5 \oplus F_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_7, F_5, \wedge(n_3)} \text{ax} \\
\hline
- : \vdash \Delta_7, \wedge(n_3), F_5 \oplus F_6 \quad \oplus_A
\end{array}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \wedge(n_3), p(n_3)} I \quad \frac{h_4 : \vdash \Delta_5, p(n_3)}{\bullet h_4 : \vdash dual(\wedge(n_3)), \perp, \Delta_5} \perp \\
\hline
- : \vdash p(n_3), \perp, \Delta_5 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_5, p(n_3)} \text{ax} \\
\hline
- : \vdash \Delta_5, \perp, p(n_3) \quad \perp
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash p(n_3), \wedge(n_3)} I \quad \frac{h_4 : \vdash \Delta_5, \wedge(n_3)}{\bullet h_4 : \vdash dual(p(n_3)), \perp, \Delta_5} \perp \\
\hline
- : \vdash \wedge(n_3), \perp, \Delta_5 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_5, \wedge(n_3)} \text{ax} \\
\hline
- : \vdash \Delta_5, \perp, \wedge(n_3) \quad \perp
\end{array}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \wedge(n_3), p(n_3)} I \quad \frac{}{\bullet h_4 : \vdash dual(\wedge(n_3)), \top, \Delta_5} \top \\
\hline
- : \vdash p(n_3), \top, \Delta_5 \quad \text{Cut} \\
\hline
\frac{}{- : \vdash \Delta_5, \top, p(n_3)} \top
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 \vdash p(n_3), \wedge(n_3)}{\vdash \vdash \wedge(n_3), \top, \Delta_5} I \quad \frac{\bullet h_4 \vdash dual(p(n_3)), \top, \Delta_5}{\vdash \vdash \wedge(n_3), \top, \Delta_5} \top \\
\text{Cut} \\
\rightarrow \\
\vdash \vdash \Delta_5, \top, \wedge(n_3) \quad \top
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \wedge(n_4), p(n_4)}{\vdash \vdash p(n_4), \wedge(n_4)} I \quad \frac{\bullet h_3 \vdash dual(\wedge(n_4)), \wedge(n_4)}{\vdash \vdash p(n_4), \wedge(n_4)} I \\
\text{Cut} \\
\rightarrow \\
\vdash \vdash p(n_4), \wedge(n_4) \quad I \\
\\
\frac{\bullet h_1 \vdash p(n_4), \wedge(n_4)}{\vdash \vdash \wedge(n_4), p(n_4)} I \quad \frac{\bullet h_3 \vdash dual(p(n_4)), p(n_4)}{\vdash \vdash \wedge(n_4), p(n_4)} I \\
\text{Cut} \\
\rightarrow \\
\vdash \vdash p(n_4), \wedge(n_4) \quad \text{ax}
\end{array}$$

- Case rule \otimes

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \wedge(n_3), p(n_3)}{\vdash \vdash p(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7} I \quad \frac{h_4 \vdash F_6, \Delta_8, p(n_3) \quad h_4 \vdash F_7, \Delta_5}{\bullet h_4 \vdash dual(\wedge(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes \\
\text{Cut} \\
\rightarrow \\
\vdash \vdash \Delta_8, F_6, p(n_3) \quad \text{ax} \quad \vdash \vdash \Delta_5, F_7 \quad \text{ax} \\
\vdash \vdash \Delta_5, \Delta_8, p(n_3), F_6 \otimes F_7 \quad \otimes \\
\\
\frac{\bullet h_1 \vdash \wedge(n_3), p(n_3)}{\vdash \vdash p(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7} I \quad \frac{h_4 \vdash F_6, \Delta_5 \quad h_4 \vdash F_7, \Delta_8, p(n_3)}{\bullet h_4 \vdash dual(\wedge(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes \\
\text{Cut} \\
\rightarrow \\
\vdash \vdash \Delta_5, F_6 \quad \text{ax} \quad \vdash \vdash \Delta_8, F_7, p(n_3) \quad \text{ax} \\
\vdash \vdash \Delta_5, \Delta_8, p(n_3), F_6 \otimes F_7 \quad \otimes \\
\\
\frac{\bullet h_1 \vdash p(n_3), \wedge(n_3)}{\vdash \vdash \wedge(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7} I \quad \frac{h_4 \vdash F_6, \Delta_8, \wedge(n_3) \quad h_4 \vdash F_7, \Delta_5}{\bullet h_4 \vdash dual(p(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes \\
\text{Cut} \\
\rightarrow \\
\vdash \vdash \Delta_8, F_6, \wedge(n_3) \quad \text{ax} \quad \vdash \vdash \Delta_5, F_7 \quad \text{ax} \\
\vdash \vdash \Delta_5, \Delta_8, \wedge(n_3), F_6 \otimes F_7 \quad \otimes \\
\\
\frac{\bullet h_1 \vdash p(n_3), \wedge(n_3)}{\vdash \vdash \wedge(n_3), \Delta_5, \Delta_8, F_6 \otimes F_7} I \quad \frac{h_4 \vdash F_6, \Delta_5 \quad h_4 \vdash F_7, \Delta_8, \wedge(n_3)}{\bullet h_4 \vdash dual(p(n_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes \\
\text{Cut} \\
\rightarrow \\
\vdash \vdash \Delta_5, F_6 \quad \text{ax} \quad \vdash \vdash \Delta_8, F_7, \wedge(n_3) \quad \text{ax} \\
\vdash \vdash \Delta_5, \Delta_8, \wedge(n_3), F_6 \otimes F_7 \quad \otimes
\end{array}$$

4.9 Status of \otimes : OK

- Case rule 1
- Case rule $\$$

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2 \quad h_1 : \vdash F_7, \Delta_3}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash F_9, F_{10}, \Delta_{11}, dual(F_6) \$ dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_{11}, F_9 \$ F_{10}} \$ \\
\hline
- : \vdash (\Delta_2, \Delta_3), \Delta_{11}, F_9 \$ F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7} \text{ ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, F_9, dual(F_6) \$ dual(F_7)}{h_8 : \vdash \Delta_{11}, F_{10}, F_9, dual(F_6) \$ dual(F_7)} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_{10}, F_9} \text{ hCut} \\
\hline
- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9 \$ F_{10} \quad \$ \\
\hline
\frac{h_1 : \vdash F_6, \Delta_2 \quad h_1 : \vdash F_7, \Delta_3}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash \Delta_9, dual(F_6), dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_9} \$ \\
\hline
- : \vdash (\Delta_2, \Delta_3), \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash \Delta_2, F_6} \text{ ax} \quad \frac{- : \vdash \Delta_3, F_7}{- : \vdash \Delta_3, F_7} \text{ ax} \quad \frac{- : \vdash \Delta_9, dual(F_6), dual(F_7)}{- : \vdash \Delta_9, dual(F_6), dual(F_7)} \text{ ax}}{- : \vdash \Delta_2, \Delta_3, \Delta_9} \text{ sCut} \\
\hline
- : \vdash \Delta_2, \Delta_3, \Delta_9 \quad \text{sCut} \\
\hline
\frac{h_2 : \vdash F_7, F_4, \Delta_6 \quad h_2 : \vdash F_5, \Delta_3}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash F_9, F_{10}, \Delta_{11}, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \$ F_{10}} \$ \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \$ F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5} \text{ ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, F_9, dual(F_7)}{h_8 : \vdash \Delta_{11}, F_{10}, F_9, dual(F_7)} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_{10}, F_9, F_4 \otimes F_5} \text{ hCut} \\
\hline
- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \$ F_{10}, F_4 \otimes F_5 \quad \$ \\
\hline
\frac{h_2 : \vdash F_4, \Delta_3 \quad h_2 : \vdash F_7, F_5, \Delta_6}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash F_9, F_{10}, \Delta_{11}, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \$ F_{10}} \$ \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \$ F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5} \text{ ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, F_9, dual(F_7)}{h_8 : \vdash \Delta_{11}, F_{10}, F_9, dual(F_7)} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_{10}, F_9, F_4 \otimes F_5} \text{ hCut} \\
\hline
- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \$ F_{10}, F_4 \otimes F_5 \quad \$ \\
\hline
- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \$ F_{10}, F_4 \otimes F_5
\end{array}$$

• Case rule &

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2 \quad h_1 : \vdash F_7, \Delta_3}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash F_9, \Delta_{11}, dual(F_6) \$ dual(F_7) \quad h_8 : \vdash F_{10}, \Delta_{11}, dual(F_6) \$ dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_{11}, F_9 \& F_{10}} \$ \\
\hline
- : \vdash (\Delta_2, \Delta_3), \Delta_{11}, F_9 \& F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7} \text{ ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_9, dual(F_6) \$ dual(F_7)}{h_8 : \vdash \Delta_{11}, F_9, dual(F_6) \$ dual(F_7)} \text{ ax} \quad \frac{h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7} \text{ ax} \quad \frac{h_8 : \vdash \Delta_{11}, F_{10}, dual(F_6) \$ dual(F_7)}{h_8 : \vdash \Delta_{11}, F_{10}, dual(F_6) \$ dual(F_7)} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9} \text{ hCut} \\
\hline
- : \vdash \Delta_{11}, \Delta_2, \Delta_3, F_9 \& F_{10} \quad \& \\
\hline
\frac{h_2 : \vdash F_7, F_4, \Delta_6 \quad h_2 : \vdash F_5, \Delta_3}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash F_9, \Delta_{11}, dual(F_7) \quad h_8 : \vdash F_{10}, \Delta_{11}, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \& F_{10}} \$ \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \& F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_6, F_4, F_7}{\bullet h_2 : \vdash \Delta_6, F_4, F_7} \text{ ax} \quad \frac{\bullet h_8 : \vdash \Delta_{11}, dual(F_7), F_9 \& F_{10}}{\bullet h_8 : \vdash \Delta_{11}, dual(F_7), F_9 \& F_{10}} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_6, F_4, F_9 \& F_{10}} \text{ hCut} \\
\hline
- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \& F_{10}, F_4 \otimes F_5 \quad \otimes \\
\hline
\frac{h_2 : \vdash F_4, \Delta_3 \quad h_2 : \vdash F_7, F_5, \Delta_6}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash F_9, \Delta_{11}, dual(F_7) \quad h_8 : \vdash F_{10}, \Delta_{11}, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \Delta_{11}, F_9 \& F_{10}} \$ \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \Delta_{11}, F_9 \& F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash \Delta_3, F_4} \text{ ax} \quad \frac{h_2 : \vdash \Delta_6, F_5, F_7}{h_2 : \vdash \Delta_6, F_5, F_7} \text{ ax} \quad \frac{\bullet h_8 : \vdash \Delta_{11}, dual(F_7), F_9 \& F_{10}}{\bullet h_8 : \vdash \Delta_{11}, dual(F_7), F_9 \& F_{10}} \text{ ax}}{- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_5, F_9 \& F_{10}} \text{ hCut} \\
\hline
- : \vdash \Delta_{11}, \Delta_3, \Delta_6, F_9 \& F_{10}, F_4 \otimes F_5 \quad \otimes
\end{array}$$

• Case rule \oplus_B

- Case rule \oplus_A

- Case rule \perp

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$$\begin{array}{c}
\frac{h_2 : \vdash F_7, F_4, \Delta_6 \quad h_2 : \vdash F_5, \Delta_3}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_9, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \perp, \Delta_9} \perp \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \perp, \Delta_9 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{- : \vdash \Delta_3, \Delta_6, \Delta_9, F_4 \otimes F_5} \text{ax} \quad \frac{h_8 : \vdash \Delta_9, dual(F_7)}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_4 \otimes F_5} \text{ax} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_4 \otimes F_5 \quad \text{hCut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_4 \otimes F_5 \quad \perp
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_4, \Delta_3 \quad h_2 : \vdash F_7, F_5, \Delta_6}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{h_8 : \vdash \Delta_9, dual(F_7)}{\bullet h_8 : \vdash dual(F_7), \perp, \Delta_9} \perp \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \perp, \Delta_9 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{- : \vdash \Delta_3, \Delta_6, \Delta_9, F_4 \otimes F_5} \text{ax} \quad \frac{h_8 : \vdash \Delta_9, dual(F_7)}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_4 \otimes F_5} \text{ax} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_4 \otimes F_5 \quad \text{hCut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_4 \otimes F_5 \quad \perp
\end{array}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2 \quad h_1 : \vdash F_7, \Delta_3}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \top, \Delta_9} \top \\
\hline
- : \vdash (\Delta_2, \Delta_3), \top, \Delta_9 \quad \text{Cut} \\
\hline
- : \vdash \Delta_2, \Delta_3, \Delta_9, \top \quad \top
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_7, F_4, \Delta_6 \quad h_2 : \vdash F_5, \Delta_3}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{}{\bullet h_8 : \vdash dual(F_7), \top, \Delta_9} \top \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \top, \Delta_9 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_4 \otimes F_5 \quad \top
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash F_4, \Delta_3 \quad h_2 : \vdash F_7, F_5, \Delta_6}{\bullet h_2 : \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{}{\bullet h_8 : \vdash dual(F_7), \top, \Delta_9} \top \\
\hline
- : \vdash (\Delta_3, \Delta_6, F_4 \otimes F_5), \top, \Delta_9 \quad \text{Cut} \\
\hline
- : \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_4 \otimes F_5 \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2 \quad h_1 : \vdash F_7, \Delta_3}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash F_{10}, \Delta_{12}, dual(F_6) \$ dual(F_7) \quad h_8 : \vdash F_{11}, \Delta_9}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- : \vdash (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- : \vdash \Delta_{12}, \Delta_2, \Delta_3, F_{10}} \text{ax} \quad \frac{h_8 : \vdash \Delta_{12}, F_{10}, dual(F_6) \$ dual(F_7)}{- : \vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11}} \text{ax} \\
\hline
- : \vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11} \quad \text{hCut} \quad - : \vdash \Delta_9, F_{11} \quad \text{ax} \\
\hline
- : \vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11} \quad \otimes
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash F_6, \Delta_2 \quad h_1 : \vdash F_7, \Delta_3}{\bullet h_1 : \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 : \vdash F_{10}, \Delta_9 \quad h_8 : \vdash F_{11}, \Delta_{12}, dual(F_6) \$ dual(F_7)}{\bullet h_8 : \vdash dual(F_6 \otimes F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- : \vdash (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \vdash \Delta_9, F_{10}} \text{ax} \quad \frac{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{- : \vdash \Delta_{12}, \Delta_2, \Delta_3, F_{11}} \text{ax} \quad \frac{h_8 : \vdash \Delta_{12}, F_{11}, dual(F_6) \$ dual(F_7)}{- : \vdash \Delta_{12}, \Delta_2, \Delta_3, F_{11}} \text{ax} \\
\hline
- : \vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11} \quad \text{hCut} \\
\hline
- : \vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11} \quad \otimes
\end{array}
\end{array}$$

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