System G3i

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1 Height preserving admissibility of weakening

• Case(s) rule \top_R

• Case(s) rule \rightarrow_R

• Case(s) rule \wedge_R

$$\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3 \quad \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3 \land \mathbf{F}_4} \quad \wedge_R \qquad \rightarrow \qquad \frac{\frac{\overline{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3}}{\mathbf{h}_1:\Delta_2,\mathbf{F}_0 \vdash \mathbf{F}_3} \quad \text{ax}}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_0 \vdash \mathbf{F}_3 \land \mathbf{F}_4} \quad \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\mathbf{h}_1:\Delta_2,\mathbf{F}_0 \vdash \mathbf{F}_3 \land \mathbf{F}_4} \quad \text{if} \quad \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\mathbf{h}_1:\Delta_2,\mathbf{F}_0 \vdash \mathbf{F}_3 \land \mathbf{F}_4} \quad \wedge_R$$

• Case(s) rule \vee_1

• Case(s) rule \vee_2

• Case(s) rule \rightarrow_L

$$\frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2\quad\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_4}\rightarrow_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2}\quad \mathbf{ax}}{\underbrace{\mathbf{h}_1:\Delta_5,\mathbf{F}_0,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2}\quad \mathbf{IH}\quad \frac{\overline{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}\quad \mathbf{ax}}{\mathbf{h}_1:\Delta_5,\mathbf{F}_0,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_4}\quad \mathbf{IH}\quad \mathbf{h}_1:\Delta_5,\mathbf{F}_0,\mathbf{F}_3\vdash\mathbf{F}_4}\rightarrow_L$$

• Case(s) rule \wedge_L

$$\frac{\mathbf{h}_1:\Delta_5,\mathbf{f}_2,\mathbf{f}_3\vdash\mathbf{f}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{f}_2\land\mathbf{f}_3\vdash\mathbf{f}_4} \ \land_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1:\Delta_5,\mathbf{f}_2,\mathbf{f}_3\vdash\mathbf{f}_4}}{\underbrace{\mathbf{h}_1:\Delta_5,\mathbf{f}_0,\mathbf{f}_2,\mathbf{f}_3\vdash\mathbf{f}_4}} \overset{\mathrm{ax}}{\to} \underbrace{\mathbf{h}_1:\Delta_5,\mathbf{f}_0,\mathbf{f}_2\land\mathbf{f}_3\vdash\mathbf{f}_4}^{\mathrm{ax}} \land_L$$

• Case(s) rule \vee_L

$$\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash\mathbf{F}_{4}\quad\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{4}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{F}_{4}}\ \vee_{L} \qquad\rightarrow\qquad \frac{\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash\mathbf{F}_{4}}{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{0}\vdash\mathbf{F}_{4}} \overset{\mathbf{ax}}{\mathbf{IH}} \frac{\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{4}}{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{0}\vdash\mathbf{F}_{4}} \overset{\mathbf{ax}}{\mathbf{IH}}}{\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{0}\vdash\mathbf{F}_{4}}{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{0}\vdash\mathbf{F}_{4}}} \overset{\mathbf{ax}}{\vee_{L}}$$

• Case(s) rule \perp_L

 \bullet Case(s) rule I

• Case(s) rule \top_L

2 Measure of derivations

• Case(s) rule \top_R

• Case(s) rule \rightarrow_R

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_2\vdash\mathbf{F}_3\to\mathbf{F}_4}\to_R \qquad \to \qquad \frac{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_4} \quad ^{\mathrm{ax}}}{\bullet \quad \bullet \quad \mathbf{h}_1:\Delta_2\vdash\mathbf{F}_3\to\mathbf{F}_4} \to_R$$

• Case(s) rule \wedge_R

$$\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3 \quad \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3 \land \mathbf{F}_4} \quad \wedge_R \qquad \rightarrow \qquad \frac{\overbrace{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3}^{} \quad \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3} \quad \overset{\mathbf{ax}}{\mathbf{H}} \quad \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{H}} \quad \overset{\mathbf{ax}}{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{H}} \quad \overset{\mathbf{ax}}{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_4} \quad \overset{\mathbf{ax}}{\mathbf{h}_1:\Delta_2$$

• Case(s) rule \vee_1

• Case(s) rule \vee_2

$$\frac{\begin{smallmatrix} \mathbf{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \lor \mathtt{F}_4 \end{smallmatrix}}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \lor \mathtt{F}_4} \ \lor_2 \qquad \rightarrow \qquad \frac{\begin{smallmatrix} \overline{\mathbf{h}_1 : \Delta_2 \vdash \mathtt{F}_4} \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \end{smallmatrix}}{\bullet \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \lor \mathtt{F}_4} \ \lor_2$$

• Case(s) rule \rightarrow_L

$$\frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2\quad\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_4}\rightarrow_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_4} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_4} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_4} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_4} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_4} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_2} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_2} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{h}_2}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{h}_2} \stackrel{\mathrm{ax}}{=} \qquad \frac{\mathbf{h}_1:\Delta_5,\mathbf{h}_1:\Delta_5,\mathbf{h}_2:\Delta_5,\mathbf{h}_1:\Delta_5,\mathbf{h}_2:\Delta_5,\mathbf{h}_1:\Delta_5,\mathbf{h}_2:\Delta_5,\mathbf{h}_2:\Delta_5,\mathbf{h}_1:\Delta_5,\mathbf{h}_2:\Delta_5,\mathbf{h}_$$

• Case(s) rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\land\mathbf{F}_3\vdash\mathbf{F}_4} & \wedge_L & \rightarrow & \begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_4} & \mathbf{ax} \\ \bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_4 & \mathbf{IH} \\ \hline \bullet\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_4 & \wedge_L \end{array}$$

• Case(s) rule \vee_L

$$\frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_4} \quad\vee_L \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4}\quad \mathbf{m}}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4}\quad \mathbf{m} \quad \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}\quad \mathbf{m} \quad \mathbf{m} \quad$$

• Case(s) rule \perp_L

• Case(s) rule I

• Case(s) rule \top_L

3 Invertibility of Rules

3.1 Status of \top_R : Invertible

• Case rule \top_R

$$\frac{}{\bullet^{\mathbf{h}_1}:\Delta_2\vdash\top}\ ^\top R\qquad\rightarrow\qquad \mathsf{trivial}$$

- Case rule \rightarrow_R
- Case rule \wedge_R
- Case rule \vee_1
- Case rule \vee_2
- Case rule \rightarrow_L

$$\begin{array}{ccc} \mathbf{h}_1 : \Delta_4, \mathbf{F}_2 \to \mathbf{F}_3 \vdash \mathbf{F}_2 & \mathbf{h}_1 : \Delta_4, \mathbf{F}_3 \vdash \top \\ & \bullet \mathbf{h}_1 : \Delta_4, \mathbf{F}_2 \to \mathbf{F}_3 \vdash \top \end{array} \to_L \qquad \to \qquad \mathsf{trivial}$$

• Case rule \wedge_L

$$\begin{array}{ccc} \frac{\mathbf{h}_1 : \Delta_4, \mathbf{F}_2, \mathbf{F}_3 \vdash \top}{\bullet \mathbf{h}_1 : \Delta_4, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \top} & \land_L & & \rightarrow & & \text{trivial} \end{array}$$

$$\frac{\mathbf{h}_1:\Delta_4,\mathbf{F}_2\vdash\top\quad \mathbf{h}_1:\Delta_4,\mathbf{F}_3\vdash\top}{\bullet\mathbf{h}_1:\Delta_4,\mathbf{F}_2\vee\mathbf{F}_3\vdash\top}\ \vee_L \qquad \rightarrow \qquad \mathsf{trivial}$$

- $\bullet\,$ Case rule I
- Case rule \top_L

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \top}{\bullet \mathbf{h}_1: \top, \Delta_2 \vdash \top} \ \top_L \qquad \rightarrow \qquad \mathsf{trivial}$$

3.2 Status of \rightarrow_R : Invertible

- Case rule \top_R
- Case rule \rightarrow_R

$$\frac{\mathsf{h}_1:\Delta_2,\mathsf{F}_3\vdash\mathsf{F}_4}{\bullet\mathsf{h}_1:\Delta_2\vdash\mathsf{F}_3\to\mathsf{F}_4}\to_R \qquad\to\qquad \frac{\overline{\mathsf{h}_1:\Delta_2,\mathsf{F}_3\vdash\mathsf{F}_4}}{\bullet\mathsf{h}_1:\Delta_2,\mathsf{F}_3\vdash\mathsf{F}_4} \overset{\mathsf{ax}}{\to}$$

- Case rule \wedge_R
- Case rule \vee_1
- Case rule \vee_2
- Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_4\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_1\to\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_1\to\mathbf{F}_2}\to_L \qquad \to \qquad \frac{\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_4}}W \xrightarrow{\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_5\vdash\mathbf{F}_2}W \xrightarrow{\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_5\vdash\mathbf{F}_2}\to_L$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{f}_4,\mathbf{f}_5\vdash\mathbf{f}_1\to\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_4\land\mathbf{f}_5\vdash\mathbf{f}_1\to\mathbf{f}_2} \ \land_L \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{f}_1,\mathbf{f}_4,\mathbf{f}_5\vdash\mathbf{f}_2}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_1,\mathbf{f}_4\land\mathbf{f}_5\vdash\mathbf{f}_2} \overset{\mathrm{ax/ind}}{\land}_L$$

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_1\to\mathbf{F}_2\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_1\to\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_1\to\mathbf{F}_2}\quad\forall_L\qquad\rightarrow\qquad\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_2}\quad\overset{\mathrm{ax/ind}}{\bullet}\quad\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_5\vdash\mathbf{F}_2}{\vee_L}\quad\overset{\mathrm{ax/ind}}{\vee_L$$

- ullet Case rule I
- Case rule \top_L

3.3 Status of \wedge_R : (Left Premise): Invertible

- Case rule \top_R
- Case rule \rightarrow_R
- Case rule \wedge_R

- Case rule \vee_1
- Case rule \vee_2
- Case rule \rightarrow_L

$$\frac{\mathtt{h}_3:\Delta_6,\mathtt{F}_4\to\mathtt{F}_5\vdash\mathtt{F}_4\quad\mathtt{h}_3:\Delta_6,\mathtt{F}_5\vdash\mathtt{F}_1\land\mathtt{F}_2}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_4\to\mathtt{F}_5\vdash\mathtt{F}_1\land\mathtt{F}_2} \ \to L \qquad \to \qquad \frac{\overline{\mathtt{h}_3:\Delta_6,\mathtt{F}_4\to\mathtt{F}_5\vdash\mathtt{F}_4}\quad \text{ax}\quad \overline{\mathtt{h}_3:\Delta_6,\mathtt{F}_5\vdash\mathtt{F}_1}\quad \frac{\mathtt{ax/ind}}{\to L} \\ \bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_4\to\mathtt{F}_5\vdash\mathtt{F}_1$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{f}_4,\mathbf{f}_5\vdash\mathbf{f}_1\wedge\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_4\wedge\mathbf{f}_5\vdash\mathbf{f}_1\wedge\mathbf{f}_2} \ \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{f}_4,\mathbf{f}_5\vdash\mathbf{f}_1}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_4\wedge\mathbf{f}_5\vdash\mathbf{f}_1} \overset{\mathrm{ax/ind}}{\wedge_L}$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_1\land\mathbf{F}_2\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_1\land\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_4\lor\mathbf{F}_5\vdash\mathbf{F}_1\land\mathbf{F}_2}\quad\vee_L\qquad\rightarrow\qquad\frac{\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_1}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_4\lor\mathbf{F}_5\vdash\mathbf{F}_1}\quad\overset{\mathrm{ax/ind}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_1}\quad\overset{\mathrm{ax/ind}}{\vee_L}\quad\vee_L$$

- \bullet Case rule I
- Case rule \top_L

3.4 Status of \wedge_R (Right Premise): : Invertible

- Case rule \top_R
- Case rule \rightarrow_R
- Case rule \wedge_R

- Case rule \vee_1
- Case rule \vee_2
- Case rule \rightarrow_L

$$\frac{\mathtt{h}_3:\Delta_6,\mathtt{F}_4\to\mathtt{F}_5\vdash\mathtt{F}_4\quad\mathtt{h}_3:\Delta_6,\mathtt{F}_5\vdash\mathtt{F}_1\land\mathtt{F}_2}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_4\to\mathtt{F}_5\vdash\mathtt{F}_1\land\mathtt{F}_2} \ \to L \qquad \to \qquad \frac{\overline{\mathtt{h}_3:\Delta_6,\mathtt{F}_4\to\mathtt{F}_5\vdash\mathtt{F}_4}\quad \text{ax} \quad \overline{\mathtt{h}_3:\Delta_6,\mathtt{F}_5\vdash\mathtt{F}_2}\quad \frac{\mathtt{ax/ind}}{\to L} \\ \bullet \mathtt{h}_3:\Delta_6,\mathtt{F}_4\to\mathtt{F}_5\vdash\mathtt{F}_2}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{f}_4,\mathbf{f}_5\vdash\mathbf{f}_1\wedge\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_4\wedge\mathbf{f}_5\vdash\mathbf{f}_1\wedge\mathbf{f}_2} \ \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{f}_4,\mathbf{f}_5\vdash\mathbf{f}_2}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_4\wedge\mathbf{f}_5\vdash\mathbf{f}_2} \overset{\mathrm{ax/ind}}{\wedge}_L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_1\land\mathbf{F}_2\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_1\land\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_4\lor\mathbf{F}_5\vdash\mathbf{F}_1\land\mathbf{F}_2}\ \lor_L \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_2}\quad \text{ax/ind} \quad \overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_2}\quad \frac{\text{ax/ind}}{\lor_L}\quad }{\lor_L}$$

- \bullet Case rule I
- Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3:\top,\Delta_4 \vdash \mathbf{F}_1 \land \mathbf{F}_2} \ \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_3:\top,\Delta_4 \vdash \mathbf{F}_2} \ ^{\mathrm{ax/ind}}$$

3.5 Status of \vee_1 : Non invertible

- Case rule \top_R
- Case rule \rightarrow_R
- Case rule \wedge_R
- Case rule \vee_1

• Case rule \vee_2

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{f}_4\rightarrow\mathbf{f}_5\vdash\mathbf{f}_4\quad\mathbf{h}_3:\Delta_6,\mathbf{f}_5\vdash\mathbf{f}_1\vee\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_4\rightarrow\mathbf{f}_5\vdash\mathbf{f}_1\vee\mathbf{f}_2}\rightarrow_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{f}_4\rightarrow\mathbf{f}_5\vdash\mathbf{f}_4}\quad \overset{\mathrm{ax}}{}\quad \overline{\mathbf{h}_3:\Delta_6,\mathbf{f}_5\vdash\mathbf{f}_1}\quad \overset{\mathrm{ax}/\mathrm{ind}}{\rightarrow}_L \qquad \rightarrow C} \rightarrow C$$

• Case rule \wedge_L

• Case rule \vee_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_1\vee\mathbf{F}_2\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_1\vee\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_1\vee\mathbf{F}_2}\quad\vee_L\qquad\rightarrow\qquad\frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_1}\quad\text{ax/ind}\quad\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_1}\quad\frac{\mathbf{ax/ind}}{\vee_L}\quad\vee_L$$

- ullet Case rule I
- Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_1 \vee \mathbf{F}_2}{\bullet \mathbf{h}_3:\top,\Delta_4 \vdash \mathbf{F}_1 \vee \mathbf{F}_2} \ \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_1}}{\bullet \mathbf{h}_3:\top,\Delta_4 \vdash \mathbf{F}_1} \ ^{\mathrm{ax/ind}} \top_L$$

3.6 Status of \vee_2 : Non invertible

- Case rule \top_R
- Case rule \rightarrow_R
- Case rule \wedge_R
- Case rule \vee_1

$$\frac{\mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4} \ \lor_1 \qquad \rightarrow \qquad \overline{\bullet \mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_4} \ \mathtt{fail}$$

• Case rule \vee_2

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{f}_4\to\mathbf{f}_5\vdash\mathbf{f}_4\quad\mathbf{h}_3:\Delta_6,\mathbf{f}_5\vdash\mathbf{f}_1\vee\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_4\to\mathbf{f}_5\vdash\mathbf{f}_1\vee\mathbf{f}_2} \ \to L \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{f}_4\to\mathbf{f}_5\vdash\mathbf{f}_4}\quad \text{ax} \quad \overline{\mathbf{h}_3:\Delta_6,\mathbf{f}_5\vdash\mathbf{f}_2}\quad \frac{\mathbf{ax}/\mathbf{ind}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_4\to\mathbf{f}_5\vdash\mathbf{f}_2} \ \to L$$

• Case rule \wedge_L

• Case rule \vee_L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_1\vee\mathbf{F}_2\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_1\vee\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_1\vee\mathbf{F}_2}\quad\vee_L\qquad\rightarrow\qquad\frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_2}\quad\text{ax/ind}\quad\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_2}\quad\text{ax/ind}\quad\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_2}\quad\vee_L$$

$$\frac{}{\bullet \mathsf{h}_3:\bot,\Delta_4 \vdash \mathsf{F}_1 \vee \mathsf{F}_2} \ ^{\bot}L \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3:\bot,\Delta_4 \vdash \mathsf{F}_2} \ ^{\bot}L$$

- ullet Case rule I
- Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_1 \vee \mathbf{F}_2}{\bullet \mathbf{h}_3:\top,\Delta_4 \vdash \mathbf{F}_1 \vee \mathbf{F}_2} \ \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_3:\top,\Delta_4 \vdash \mathbf{F}_2} \ ^{\mathrm{ax/ind}}$$

3.7 Status of \rightarrow_L : (Left Premise): Non invertible

• Case rule \top_R

• Case rule \rightarrow_R

$$\begin{array}{c} \begin{array}{c} \mathbf{h}_3: \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \to \mathbf{F}_2 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1 \to \mathbf{F}_2 \vdash \mathbf{F}_4 \to \mathbf{F}_5 \end{array} \to_R \qquad \to \qquad \begin{array}{c} \bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1 \to \mathbf{F}_2 \vdash \mathbf{F}_1 \end{array} \ \, \mathbf{fail} \end{array}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_4\land\mathbf{F}_5}\quad\wedge_R\qquad\rightarrow\qquad\frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_1}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_1}\overset{\mathrm{ax/ind}}{\to}$$

• Case rule \vee_1

$$\frac{\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_4}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_1 \qquad \to \qquad \frac{\overline{\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_1}}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_1}\ _{\mathtt{H}}^{\mathtt{ax/ind}}$$

• Case rule \vee_2

$$\frac{\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_5}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_2 \qquad\to\qquad \frac{\overline{\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_1}}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_1}\ ^{\mathtt{ax/ind}}_{\mathtt{H}}$$

• Case rule \rightarrow_L

$$\begin{array}{c} \underline{\mathbf{h}_3:\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2, \mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \mathbf{F}_4 \quad \mathbf{h}_3:\Delta_7, \mathbf{F}_5, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \mathbf{F}_6} \\ \bullet \underline{\mathbf{h}_3:(\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2), \mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \mathbf{F}_6} \end{array} \rightarrow_L \\ \rightarrow \begin{array}{c} \underline{\mathbf{h}_3:\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2, \mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \mathbf{F}_1} \\ \bullet \underline{\mathbf{h}_3:\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2, \mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \mathbf{F}_1} \end{array} \xrightarrow{\mathbf{ax/ind}} \\ \underline{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2 \quad \mathbf{h}_1:\Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_4} \\ \bullet \underline{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2} \xrightarrow{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_4} \end{array} \rightarrow_L \\ \rightarrow \begin{array}{c} \underline{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2} \\ \bullet \underline{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2} \end{array} \xrightarrow{\mathbf{ax/ind}} \\ \underline{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2} \xrightarrow{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_4} \end{array} \rightarrow_L \\ \rightarrow \begin{array}{c} \underline{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2} \\ \bullet \underline{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2} \end{array} \xrightarrow{\mathbf{h}_1:\Delta_2, \mathbf{h}_2 \rightarrow \mathbf{h}_2} \xrightarrow{\mathbf{h}_1:\Delta_3, \mathbf{h}_2 \rightarrow \mathbf{h}_2} \end{array} \xrightarrow{\mathbf{h}_1:\Delta_3, \mathbf{h}_2 \rightarrow \mathbf{h}_2} \xrightarrow{\mathbf{h}_1:\Delta_3, \mathbf{h}_2 \rightarrow \mathbf{h}_2} \xrightarrow{\mathbf{h}_1:\Delta_5, \mathbf{h}_2} \xrightarrow{\mathbf{h}_1:\Delta_5, \mathbf{h}_2 \rightarrow \mathbf{h}_2} \xrightarrow{\mathbf{h}_$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\to\mathbf{F}_2),\mathbf{F}_4\land\mathbf{F}_5\vdash\mathbf{F}_6}~\land L~~\rightarrow~~\frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_1}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1\to\mathbf{F}_2,\mathbf{F}_4\land\mathbf{F}_5\vdash\mathbf{F}_1}~\overset{\mathsf{ax/ind}}{\land} L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_6\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\to\mathbf{F}_2),\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}\quad\vee_L\qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_1}\quad \text{ax/ind}\quad \overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_1}\quad \mathbf{x}/\mathbf{h}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1\to\mathbf{F}_2,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_1}\quad \nabla_L$$

 \bullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_5, \Delta_4, \mathsf{F}_1 \to \mathsf{F}_2 \vdash \mathsf{p}_5} \quad I \qquad \to \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_4, \mathsf{p}_5, \mathsf{F}_1 \to \mathsf{F}_2 \vdash \mathsf{F}_1} \quad \mathsf{fail}$$

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_4}\ \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_1}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_1}\ ^{\mathrm{T}}_L$$

3.8 Status of \rightarrow_L (Right Premise): : Invertible

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{F}_1\to\mathbf{F}_2\vdash\top} \ ^\top R \qquad \to \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{F}_2\vdash\top} \ ^\top R$$

• Case rule \rightarrow_R

$$\frac{\mathsf{h}_3:\Delta_6,\mathsf{F}_4,\mathsf{F}_1\to\mathsf{F}_2\vdash\mathsf{F}_5}{\bullet\mathsf{h}_3:\Delta_6,\mathsf{F}_1\to\mathsf{F}_2\vdash\mathsf{F}_4\to\mathsf{F}_5}\to_R \qquad\to\qquad \frac{\overline{\mathsf{h}_3:\Delta_6,\mathsf{F}_2,\mathsf{F}_4\vdash\mathsf{F}_5}}{\bullet\mathsf{h}_3:\Delta_6,\mathsf{F}_2\vdash\mathsf{F}_4\to\mathsf{F}_5}\to_R$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_4\land\mathbf{F}_5}\quad\wedge_R\qquad\rightarrow\qquad\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_2\vdash\mathbf{F}_4\land\mathbf{F}_5}\quad\frac{\mathbf{ax/ind}}{\land_R}\quad\wedge_R$$

• Case rule \vee_1

$$\frac{\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_4}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1\to\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_1 \qquad\to\qquad \frac{\mathtt{h}_3:\Delta_6,\mathtt{F}_2\vdash\mathtt{F}_4}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_1$$

• Case rule \vee_2

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\rightarrow\mathbf{F}_2,\mathbf{F}_4\rightarrow\mathbf{F}_5\vdash\mathbf{F}_4\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\rightarrow\mathbf{F}_2),\mathbf{F}_4\rightarrow\mathbf{F}_5\vdash\mathbf{F}_6}\rightarrow_L\qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\rightarrow\mathbf{F}_5\vdash\mathbf{F}_4}\quad\text{ax/ind}\quad\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_6}\quad\text{ax/ind}\quad\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_6}\rightarrow_L$$

$$\frac{\mathtt{h}_1:\Delta_5,\mathtt{F}_2\to\mathtt{F}_3\vdash\mathtt{F}_2\quad\mathtt{h}_1:\Delta_5,\mathtt{F}_3\vdash\mathtt{F}_4}{\bullet\mathtt{h}_1:\Delta_5,\mathtt{F}_2\to\mathtt{F}_3\vdash\mathtt{F}_4}\ \to_L \qquad \to \qquad \frac{\frac{\mathtt{h}_1:\Delta_5,\mathtt{F}_3\vdash\mathtt{F}_4}{\bullet\mathtt{h}_1:\Delta_5,\mathtt{F}_3\vdash\mathtt{F}_4}\ ^{\mathtt{ax}}}{\bullet\mathtt{h}_1:\Delta_5,\mathtt{F}_3\vdash\mathtt{F}_4}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_5,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\rightarrow\mathbf{F}_2),\mathbf{F}_4\wedge\mathbf{F}_5\vdash\mathbf{F}_6}} \ \land_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\wedge\mathbf{F}_5\vdash\mathbf{F}_6}} \ \stackrel{\mathrm{ax/ind}}{\wedge}L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_6\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\rightarrow\mathbf{F}_2),\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}}\quad\vee_L\qquad\rightarrow\qquad\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}\quad\frac{\mathbf{ax/ind}}{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}\quad\vee_L$$

• Case rule \perp_L

 \bullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_5, \Delta_4, \mathsf{F}_1 \to \mathsf{F}_2 \vdash \mathsf{p}_5} \quad I \qquad \to \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_4, \mathsf{F}_2, \mathsf{p}_5 \vdash \mathsf{p}_5} \quad I$$

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\mathbf{F}_4}\ \top_L \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4}\ \top_L$$

3.9 Status of \wedge_L : Invertible

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\top} \ \top_R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{F}_2\vdash\top} \ \top_R$$

• Case rule \rightarrow_R

$$\frac{\mathsf{h}_3:\Delta_6,\mathsf{F}_4,\mathsf{F}_1\wedge\mathsf{F}_2\vdash\mathsf{F}_5}{\bullet\mathsf{h}_3:\Delta_6,\mathsf{F}_1\wedge\mathsf{F}_2\vdash\mathsf{F}_4\to\mathsf{F}_5}\to_R \qquad\to\qquad \frac{\overline{\mathsf{h}_3:\Delta_6,\mathsf{F}_1,\mathsf{F}_2,\mathsf{F}_4\vdash\mathsf{F}_5}}{\bullet\mathsf{h}_3:\Delta_6,\mathsf{F}_1,\mathsf{F}_2\vdash\mathsf{F}_4\to\mathsf{F}_5}\overset{\mathsf{ax/ind}}{\to_R}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R\qquad\rightarrow\qquad\frac{\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{ax/ind}\quad}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_5}\quad\frac{\mathbf{ax/ind}\quad}{\wedge_R}\quad\wedge_R}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R}$$

• Case rule \vee_1

$$\frac{\mathtt{h}_3:\Delta_6,\mathtt{F}_1\wedge\mathtt{F}_2\vdash\mathtt{F}_4}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1\wedge\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_1 \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3:\Delta_6,\mathtt{F}_1,\mathtt{F}_2\vdash\mathtt{F}_4}\ ^{\mathrm{ax/ind}}}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1,\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_1$$

• Case rule \vee_2

$$\frac{\mathtt{h}_3:\Delta_6,\mathtt{F}_1\wedge\mathtt{F}_2\vdash\mathtt{F}_5}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1\wedge\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_2\qquad\rightarrow\qquad\frac{\overline{\mathtt{h}_3:\Delta_6,\mathtt{F}_1,\mathtt{F}_2\vdash\mathtt{F}_5}\ ^{\mathsf{ax/ind}}}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1,\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_2$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4\to\mathbf{F}_5,\mathbf{F}_1\land\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\land\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\land\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_6}}\to_L \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_4}\quad \mathbf{ax/ind} \quad \overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_6}\quad \mathbf{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_6}\quad \to L} \to L$$

• Case rule \wedge_L

• Case rule \vee_L

$$\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\mathbf{F}_6\quad\mathbf{a}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\wedge\mathbf{F}_2),\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}}\ \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_6}\quad \mathrm{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}\quad \frac{\mathrm{ax/ind}}{\vee_L} \qquad \wedge_L = \frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_6}\quad \mathrm{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}$$

• Case rule \perp_L

ullet Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\mathbf{F}_4}\ \top_L \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_4}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_4}\ ^{\mathrm{ax/ind}}\ \top_L$$

3.10 Status of \vee_L : (Left Premise): Invertible

• Case rule \top_R

$$\underbrace{\bullet_{\mathsf{h}_3}:\Delta_4,\mathsf{F}_1\vee\mathsf{F}_2\vdash\top}^{} \ \top_R \qquad \rightarrow \qquad \underbrace{\bullet_{\mathsf{h}_3}:\Delta_4,\mathsf{F}_1\vdash\top}^{} \ \top_R$$

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4\to\mathbf{F}_5}\ \to_R \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vdash\mathbf{F}_4\to\mathbf{F}_5} \overset{\mathsf{ax/ind}}{\to_R}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R\qquad\rightarrow\qquad\frac{\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vdash\mathbf{F}_4\wedge\mathbf{F}_5}\quad\frac{\mathbf{ax/ind}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vdash\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vdash\mathbf{F}_4\wedge\mathbf{F}_5}$$

• Case rule \vee_1

$$\frac{ \begin{smallmatrix} \mathbf{h}_3 : \Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \mathbf{F}_4 \\ \bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \mathbf{F}_4 \vee \mathbf{F}_5 \end{smallmatrix}}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_1 \vdash \mathbf{F}_4} \overset{\mathsf{ax/ind}}{} \vee_1 \qquad \rightarrow \qquad \frac{ \begin{smallmatrix} \mathbf{h}_3 : \Delta_6, \mathbf{F}_1 \vdash \mathbf{F}_4 \\ \bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_1 \vdash \mathbf{F}_4 \vee \mathbf{F}_5 \end{smallmatrix}}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_1 \vdash \mathbf{F}_4 \vee \mathbf{F}_5} \overset{\mathsf{ax/ind}}{} \vee_1$$

• Case rule \vee_2

$$\frac{\mathtt{h}_3:\Delta_6,\mathtt{f}_1\vee\mathtt{f}_2\vdash\mathtt{f}_5}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{f}_1\vee\mathtt{f}_2\vdash\mathtt{f}_4\vee\mathtt{f}_5}\ \vee_2 \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3:\Delta_6,\mathtt{f}_1\vdash\mathtt{f}_5}\ ^{\mathrm{ax/ind}}}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{f}_1\vdash\mathtt{f}_4\vee\mathtt{f}_5}\ \vee_2$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{f}_4\to\mathbf{f}_5,\mathbf{f}_1\vee\mathbf{f}_2\vdash\mathbf{f}_4\quad\mathbf{h}_3:\Delta_7,\mathbf{f}_5,\mathbf{f}_1\vee\mathbf{f}_2\vdash\mathbf{f}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{f}_1\vee\mathbf{f}_2),\mathbf{f}_4\to\mathbf{f}_5\vdash\mathbf{f}_6} \ \to_L \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{f}_1,\mathbf{f}_4\to\mathbf{f}_5\vdash\mathbf{f}_4}\quad \frac{\mathbf{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_1,\mathbf{f}_4\to\mathbf{f}_5\vdash\mathbf{f}_6} \ \frac{\mathbf{ax/ind}}{\to_L} \ \to_L$$

• Case rule \wedge_L

• Case rule \vee_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_6\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2)\cdot\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}} \quad \vee_L \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}} \quad \overset{\mathrm{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}} \quad \vee_L \qquad \rightarrow \qquad \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vee\mathbf{F}_5\vdash\mathbf{F}_6}} \quad \overset{\mathrm{ax/ind}}{\vee_L} \quad \overset{\mathrm{ax/ind}}{\vee_L} \quad \vee_L \quad \overset{\mathrm{ax/ind}}{\vee_L} \quad \overset{\mathrm{ax/$$

$$\frac{\mathtt{h}_1:\Delta_5,\mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1:\Delta_5,\mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1:\Delta_5,\mathtt{F}_2 \lor \mathtt{F}_3 \vdash \mathtt{F}_4} \ \lor_L \qquad \to \qquad \frac{\overline{\mathtt{h}_1:\Delta_5,\mathtt{F}_2 \vdash \mathtt{F}_4}}{\bullet \mathtt{h}_1:\Delta_5,\mathtt{F}_2 \vdash \mathtt{F}_4} \ ^{\mathsf{ax}}_{\mathsf{H}}$$

• Case rule \perp_L

$$\frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{f}_1\vee\mathbf{f}_2\vdash\mathbf{f}_4} \ \bot_L \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{f}_1\vdash\mathbf{f}_4} \ \bot_L$$

ullet Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4}\ \top_L \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vdash\mathbf{F}_4}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\vdash\mathbf{F}_4}\ ^{\mathrm{T}}_L$$

3.11 Status of \vee_L (Right Premise): : Invertible

• Case rule \top_R

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4\to\mathbf{F}_5}\ \to_R \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_2\vdash\mathbf{F}_4\to\mathbf{F}_5} \overset{\mathrm{ax/ind}}{\to_R}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R\qquad\rightarrow\qquad\frac{\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_2\vdash\mathbf{F}_4\wedge\mathbf{F}_5}\quad\overset{\mathrm{ax/ind}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_2\vdash\mathbf{F}_4\wedge\mathbf{F}_5}\quad\overset{\mathrm{ax/ind}}{\wedge_R}$$

• Case rule \vee_1

$$\frac{\mathtt{h}_3:\Delta_6,\mathtt{F}_1\vee\mathtt{F}_2\vdash\mathtt{F}_4}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_1\vee\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_1 \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3:\Delta_6,\mathtt{F}_2\vdash\mathtt{F}_4}\ ^{\mathsf{ax/ind}}}{\bullet\mathtt{h}_3:\Delta_6,\mathtt{F}_2\vdash\mathtt{F}_4\vee\mathtt{F}_5}\ \vee_1$$

• Case rule \vee_2

$$\frac{ \begin{smallmatrix} \mathbf{h}_3 : \Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \mathbf{F}_4 \vee \mathbf{F}_5 \end{smallmatrix}}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_2 \vdash \mathbf{F}_5} \overset{ax/ind}{\bullet} \frac{ }{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_2 \vdash \mathbf{F}_5} \overset{ax/ind}{\vee} 2}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_4\to\mathbf{F}_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{a}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_6} \to_L \longrightarrow \frac{\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_4}\quad \overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_6}\quad \overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_6}\quad \overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_6}\quad \overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\mathbf{F}_6}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2),\mathbf{F}_4\wedge\mathbf{F}_5\vdash\mathbf{F}_6}\wedge_L\qquad\rightarrow\qquad\frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\wedge\mathbf{F}_5\vdash\mathbf{F}_6}\wedge_L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \mathbf{F}_6 \quad \mathbf{h}_3: \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2), \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \mathbf{F}_6}} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \mathbf{F}_6} \quad \text{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \mathbf{F}_6}} \quad \overset{\text{ax/ind}}{\vee_L} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \mathbf{F}_6}} \quad \overset{\text{ax/ind}}{\vee_L} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \mathbf{F}_6}} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \mathbf{F}_6}} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_2, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_2, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_2, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_2, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_2, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_2, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_3 \vdash \mathbf{h}_3} \quad \vee_L \quad \rightarrow \quad \frac{\mathbf{$$

$$\frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4\quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_4}\ \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}\ ^{\mathrm{ax}}$$

ullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_5, \Delta_4, \mathsf{F}_1 \vee \mathsf{F}_2 \vdash \mathsf{p}_5} \quad I \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_4, \mathsf{F}_2, \mathsf{p}_5 \vdash \mathsf{p}_5} \quad I$$

• Case rule \top_L

3.12 Status of \perp_L : Invertible

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_1:\bot,\Delta_2\vdash\top} \ ^\top R \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_1:\bot,\Delta_4,\mathbf{F}_2\vdash\mathbf{F}_3}{\bullet\mathbf{h}_1:\bot,\Delta_4\vdash\mathbf{F}_2\to\mathbf{F}_3}\ \to_R \qquad \to \qquad \mathsf{trivial}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_1:\bot,\Delta_4 \vdash \mathbf{F}_2 \quad \mathbf{h}_1:\bot,\Delta_4 \vdash \mathbf{F}_3}{\bullet \mathbf{h}_1:\bot,\Delta_4 \vdash \mathbf{F}_2 \land \mathbf{F}_3} \ \land_R \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \vee_1

$$\frac{\mathtt{h}_1:\bot,\Delta_4\vdash\mathtt{F}_2}{\bullet\mathtt{h}_1:\bot,\Delta_4\vdash\mathtt{F}_2\vee\mathtt{F}_3}\ \vee_1 \qquad \to \qquad \mathtt{trivial}$$

• Case rule \vee_2

$$\frac{\mathtt{h}_1:\bot,\Delta_4\vdash\mathtt{F}_3}{\bullet\mathtt{h}_1:\bot,\Delta_4\vdash\mathtt{F}_2\vee\mathtt{F}_3}\ \vee_2\qquad\rightarrow\qquad\mathsf{trivial}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_1:\bot,\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_2\quad\mathbf{h}_1:\bot,\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:(\bot,\Delta_5),\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_4}\ \to_L \qquad \to \qquad \mathsf{trivial}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_1:\bot,\Delta_5,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:(\bot,\Delta_5),\mathbf{F}_2\wedge\mathbf{F}_3\vdash\mathbf{F}_4} \ \wedge_L \qquad \rightarrow \qquad \text{trivial}$$

• Case rule \vee_L

$$\frac{\mathbf{h}_1:\bot,\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{h}_1:\bot,\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:(\bot,\Delta_5),\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_4}\ \vee_L \qquad \rightarrow \qquad \text{trivial}$$

• Case rule \perp_L

 $\bullet\,$ Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_1:\bot,\Delta_3\vdash\mathbf{F}_2}{\bullet\mathbf{h}_1:\top,\bot,\Delta_3\vdash\mathbf{F}_2}\ \top_L \qquad \rightarrow \qquad \mathsf{trivial}$$

3.13 Status of I: Invertible

- Case rule \top_R
- Case rule \rightarrow_R
- Case rule \wedge_R
- Case rule \vee_1
- Case rule \vee_2
- Case rule \rightarrow_L

$$\frac{\mathbf{h}_1:\Delta_4,\mathbf{p}_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2\quad \mathbf{h}_1:\Delta_4,\mathbf{F}_3,\mathbf{p}_5\vdash\mathbf{p}_5}{\bullet\mathbf{h}_1:(\Delta_4,\mathbf{p}_5),\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{p}_5}\ \rightarrow_L \qquad \rightarrow \qquad \text{trivial}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_1:\Delta_4,\mathbf{F}_2,\mathbf{F}_3,\mathbf{p}_5 \vdash \mathbf{p}_5}{\bullet \mathbf{h}_1:(\Delta_4,\mathbf{p}_5),\mathbf{F}_2 \wedge \mathbf{F}_3 \vdash \mathbf{p}_5} \ \wedge_L \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \vee_L

$$\frac{\mathbf{h}_1:\Delta_4,\mathbf{F}_2,\mathbf{p}_5\vdash\mathbf{p}_5\quad\mathbf{h}_1:\Delta_4,\mathbf{F}_3,\mathbf{p}_5\vdash\mathbf{p}_5}{\bullet\mathbf{h}_1:(\Delta_4,\mathbf{p}_5),\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{p}_5}\ \vee_L \qquad \rightarrow \qquad \mathsf{trivial}$$

ullet Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{p}_3\vdash\mathbf{p}_3}{\bullet\mathbf{h}_1:\top,\Delta_2,\mathbf{p}_3\vdash\mathbf{p}_3}\ \top_L \qquad \rightarrow \qquad \mathsf{trivial}$$

3.14 Status of \top_L : Invertible

• Case rule \top_R

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_1: \top, \Delta_4, \mathbf{F}_2 \vdash \mathbf{F}_3}{\bullet \mathbf{h}_1: \top, \Delta_4 \vdash \mathbf{F}_2 \to \mathbf{F}_3} \ \to_R \qquad \to \qquad \frac{\overline{\mathbf{h}_1: \Delta_4, \mathbf{F}_2 \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_1: \Delta_4 \vdash \mathbf{F}_2 \to \mathbf{F}_3} \overset{\mathrm{ax/ind}}{\to}_R$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_1: \top, \Delta_4 \vdash \mathbf{F}_2 \quad \mathbf{h}_1: \top, \Delta_4 \vdash \mathbf{F}_3}{\bullet \mathbf{h}_1: \top, \Delta_4 \vdash \mathbf{F}_2 \land \mathbf{F}_3} \quad \wedge_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_4 \vdash \mathbf{F}_2} \quad \text{ax/ind} \quad \overline{\mathbf{h}_1: \Delta_4 \vdash \mathbf{F}_3} \quad \frac{\mathbf{ax/ind}}{\land_R} \quad \wedge_R \quad \wedge_R$$

• Case rule \vee_1

$$\frac{\mathtt{h}_1: \top, \Delta_4 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1: \top, \Delta_4 \vdash \mathtt{F}_2 \vee \mathtt{F}_3} \ \vee_1 \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_1: \Delta_4 \vdash \mathtt{F}_2} \ \text{ax/ind}}{\bullet \mathtt{h}_1: \Delta_4 \vdash \mathtt{F}_2 \vee \mathtt{F}_3} \vee_1$$

• Case rule \vee_2

$$\frac{\mathtt{h}_1: \top, \Delta_4 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1: \top, \Delta_4 \vdash \mathtt{F}_2 \vee \mathtt{F}_3} \ \lor_2 \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_1: \Delta_4 \vdash \mathtt{F}_3}}{\bullet \mathtt{h}_1: \Delta_4 \vdash \mathtt{F}_2 \vee \mathtt{F}_3} \ \lor_2$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_1: \top, \Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2 \quad \mathbf{h}_1: \top, \Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1: (\top, \Delta_5), \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_4} \quad \rightarrow L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \rightarrow \mathbf{h}_1: \Delta_5, \mathbf{h}_2: \Delta_5, \mathbf$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_1: \top, \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1: (\top, \Delta_5), \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_4} \ \land_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_4}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_4} \ \stackrel{\mathsf{ax/ind}}{\wedge}_L$$

$$\frac{\mathbf{h}_1: \top, \Delta_5, \mathbf{F}_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \top, \Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1: (\top, \Delta_5), \mathbf{F}_3 \lor \mathbf{F}_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \mathbf{F}_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \lor \mathbf{F}_3 \vdash \mathbf{F}_4} \quad \frac{\mathbf{ax/ind}}{\vee_L} \quad \vee_L \quad \vee_L$$

• Case rule \perp_L

 $\bullet\,$ Case rule I

• Case rule \top_L

4 Height preserving admissibility of contraction

• Case(s) rule \top_R

$$\frac{}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3, \mathbf{F}_3 \vdash \top} \ ^{\top}R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \top} \ ^{\top}R$$

• Case(s) rule \rightarrow_R

$$\frac{\mathbf{a}_1:\Delta_4,\mathbf{F}_2,\mathbf{F}_5,\mathbf{F}_5\vdash\mathbf{F}_3}{\bullet\mathbf{h}_1:\Delta_4,\mathbf{F}_5,\mathbf{F}_5\vdash\mathbf{F}_2\to\mathbf{F}_3}\to_R \qquad \to \qquad \frac{\mathbf{a}_1:\Delta_4,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_3}{\mathbf{a}_1:\Delta_4,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_3} & \overset{\mathbf{a}_K}{\to}\\ \frac{\mathbf{h}_1:\Delta_4,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_3}{\bullet\mathbf{h}_1:\Delta_4,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_3} & \to R \end{cases}$$

• Case(s) rule \wedge_R

$$\frac{\mathbf{h}_1:\Delta_4,\mathbf{f}_5,\mathbf{f}_5\vdash\mathbf{f}_2\quad \mathbf{h}_1:\Delta_4,\mathbf{f}_5,\mathbf{f}_5\vdash\mathbf{f}_3}{\bullet\mathbf{h}_1:\Delta_4,\mathbf{f}_5,\mathbf{f}_5\vdash\mathbf{f}_2\wedge\mathbf{f}_3} \quad \wedge_R \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1:\Delta_4,\mathbf{f}_5,\mathbf{f}_5\vdash\mathbf{f}_2}{\mathbf{h}_1:\Delta_4,\mathbf{f}_5\vdash\mathbf{f}_2}\quad \mathbf{m}}{\bullet\mathbf{h}_1:\Delta_4,\mathbf{f}_5\vdash\mathbf{f}_2\wedge\mathbf{f}_3} \quad \mathbf{m} \quad \frac{\mathbf{h}_1:\Delta_4,\mathbf{f}_5\vdash\mathbf{f}_2}{\mathbf{h}_1:\Delta_4,\mathbf{f}_5\vdash\mathbf{f}_2\wedge\mathbf{f}_3} \quad \mathbf{m} \quad \mathbf{m}$$

• Case(s) rule \vee_1

• Case(s) rule \vee_2

• Case(s) rule \rightarrow_L

$$\frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3,\mathbf{F}_2\rightarrow\mathbf{F}_2\rightarrow\mathbf{F}_3,\mathbf{F}_2\rightarrow\mathbf$$

• Case(s) rule \wedge_L

• Case(s) rule \vee_L

$$\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{F}_{4}\quad\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{F}_{4}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{F}_{4}}\vee_{L}}\rightarrow \frac{\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{2}\vdash\mathbf{F}_{4}}{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}}\frac{\mathbf{inv-th/ax}}{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{F}_{4}}} \vee_{L} \rightarrow \frac{\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{2}\vdash\mathbf{F}_{4}}{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{F}_{4}}}{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{F}_{4}}} \vee_{L} \rightarrow \frac{\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{6}\vdash\mathbf{F}_{4}}{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{F}_{4}}}{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3},\mathbf{F}_{6}\vdash\mathbf{F}_{4}}} \vee_{L} \vee_{L$$

• Case(s) rule \perp_L

$$\frac{}{\bullet \mathtt{h}_1: (\bot, \Delta_3), \mathtt{F}_4, \mathtt{F}_4 \vdash \mathtt{F}_2} \ ^\bot L \qquad \rightarrow \qquad \frac{}{\bullet \mathtt{h}_1: \bot, \Delta_3, \mathtt{F}_4 \vdash \mathtt{F}_2} \ ^\bot L$$

• Case(s) rule I

• Case(s) rule \top_L

$$\begin{array}{ccc} \frac{\mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{F}_2}{\bullet \mathbf{h}_1: \Delta_3, \top, \top \vdash \mathbf{F}_2} & \top_L & \rightarrow & & \frac{\overline{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{F}_2} & \top_L \end{array}$$

$$\frac{\mathbf{h}_1:\Delta_3,\mathbf{f}_4,\mathbf{f}_4\vdash\mathbf{f}_2}{\bullet\mathbf{h}_1:(\top,\Delta_3),\mathbf{f}_4,\mathbf{f}_4\vdash\mathbf{f}_2}\ \top_L \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1:\Delta_3,\mathbf{f}_4,\mathbf{f}_4\vdash\mathbf{f}_2}{\mathbf{h}_1:\Delta_3,\mathbf{f}_4\vdash\mathbf{f}_2}}{\bullet\mathbf{h}_1:\top,\Delta_3,\mathbf{f}_4\vdash\mathbf{f}_2} \quad \frac{\mathbf{ax}}{\top_L}$$

5 Identity-Expansion

$$\begin{array}{c|c} \hline -: F_0 \vdash F_0 & \text{IH} & \hline -: F_1 \vdash F_1 & \text{IH} \\ \hline -: F_0 \vdash F_0 \lor F_1 & \lor_1 & \hline -: F_1 \vdash F_0 \lor F_1 & \lor_2 \\ \hline -: F_0 \vdash F_0 \lor F_1 \vdash F_0 \lor F_1 & \lor_L \\ \hline \hline \hline -: F_0 \vdash F_0 & W & \hline -: F_0, F_1 \vdash F_1 & W \\ \hline \hline -: F_0, F_1 \vdash F_0 & W & \hline -: F_0, F_1 \vdash F_1 & W \\ \hline \hline -: F_0, F_1 \vdash F_0 \land F_1 & \land_L & \\ \hline \hline \hline -: F_0 \vdash F_0 & \text{IH} & \hline -: F_0, F_1 \vdash F_1 & W \\ \hline \hline -: F_0, F_0 \to F_1 \vdash F_0 & W & \hline -: F_0, F_1 \vdash F_1 & W \\ \hline \hline -: F_0, F_0 \to F_1 \vdash F_0 \to F_1 & \to_R \\ \hline \hline -: F_0 \to F_1 \vdash F_0 \to F_1 & \to_R \\ \hline \hline -: T \vdash T & \top_R \\ \hline \hline \hline -: \bot \vdash \bot & \bot_L \\ \hline \hline \end{array}$$

6 Cut-Elimination

6.1 Status of \top_R : OK

• Case rule \top_R

• Case rule \rightarrow_R

$$\begin{array}{c|c} \frac{ \bullet \mathbf{h}_1 : \Delta_6 \vdash \top}{\bullet \mathbf{h}_1 : \Delta_6 \vdash \top} \; \top_R & \frac{\mathbf{h}_3 : \top, \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \top \vdash \mathbf{F}_4 \to \mathbf{F}_5} & \neg_R \\ \hline - : \Delta_6 \vdash \mathbf{F}_4 \to \mathbf{F}_5 & \neg_R \\ \hline \frac{\bullet \mathbf{h}_1 : \Delta_6, \mathbf{F}_4 \vdash \top}{\bullet \mathbf{h}_3 : \top, \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \hline \frac{- : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{- : \Delta_6 \vdash \mathbf{F}_4 \to \mathbf{F}_5} \to_R \end{array}$$

• Case rule \wedge_R

• Case rule \vee_1

$$\begin{array}{c|c} \underline{\bullet_{\mathbf{h}_1}:\Delta_6 \vdash \top} & \top_R & \frac{\mathbf{h}_3:\top,\Delta_6 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_3:\Delta_6,\top \vdash \mathbf{F}_4 \vee \mathbf{F}_5} & \mathsf{Cut} \\ \hline -:\Delta_6 \vdash \mathbf{F}_4 \vee \mathbf{F}_5 & \to \\ \hline \bullet_{\mathbf{h}_1}:\Delta_6 \vdash \top & \top_R & \frac{\mathbf{h}_3:\top,\Delta_6 \vdash \mathbf{F}_4}{\mathbf{h}_3:\top,\Delta_6 \vdash \mathbf{F}_4} & \mathsf{ax/W} \\ \hline -:\Delta_6 \vdash \mathbf{F}_4 \vee \mathbf{F}_5 & \vee_1 \\ \hline -:\Delta_6 \vdash \mathbf{F}_4 \vee \mathbf{F}_5 & \vee_1 \end{array}$$

• Case rule \vee_2

$$\begin{array}{c|c} \underline{\bullet \mathbf{h}_1 : \Delta_6 \vdash \top} & \top_R & \frac{\mathbf{h}_3 : \top, \Delta_6 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \top \vdash \mathbf{F}_4 \vee \mathbf{F}_5} & \nabla_2 \\ \hline -: \Delta_6 \vdash \mathbf{F}_4 \vee \mathbf{F}_5 & \rightarrow \\ \underline{\bullet \mathbf{h}_1 : \Delta_6 \vdash \top} & \top_R & \frac{\lambda_3 : \top, \Delta_6 \vdash \mathbf{F}_5}{\mathbf{h}_3 : \top, \Delta_6 \vdash \mathbf{F}_5} & \mathrm{ax/W} \\ \hline -: \Delta_6 \vdash \mathbf{F}_5 & \nabla_2 & \mathrm{hCut} \end{array}$$

• Case rule \rightarrow_L

$$\frac{\underbrace{\bullet \mathbf{h}_1: \Delta_7, \mathbf{F}_4 \to \mathbf{F}_5 \vdash \top}_{\mathbf{h}_1: \Delta_7, \mathbf{F}_4 \to \mathbf{F}_5 \vdash \mathbf{F}_4} \quad \mathbf{h}_3: \top, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6}_{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_4 \to \mathbf{F}_5), \top \vdash \mathbf{F}_6} \quad \mathbf{Cut}} \\ -: \Delta_7, \mathbf{F}_4 \to \mathbf{F}_5 \vdash \mathbf{F}_6} \\ \underbrace{\bullet \mathbf{h}_1: \Delta_7, \mathbf{F}_4 \to \mathbf{F}_5 \vdash \top}_{\mathbf{h}_3: \top, \Delta_7, \mathbf{F}_4 \to \mathbf{F}_5 \vdash \mathbf{F}_4} \quad \mathbf{ax/W}}_{\mathbf{h}_3: \top, \Delta_7, \mathbf{F}_4 \to \mathbf{F}_5 \vdash \mathbf{F}_4} \\ \underbrace{-: \Delta_7, \mathbf{F}_4 \to \mathbf{F}_5 \vdash \mathbf{F}_4}_{-: \Delta_7, \mathbf{F}_4 \to \mathbf{F}_5 \vdash \mathbf{F}_6} \quad \mathbf{h}_{\mathbf{Cut}} \\ \underbrace{-: \Delta_7, \mathbf{F}_4 \to \mathbf{F}_5 \vdash \mathbf{F}_6}_{-: \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6} \to_L} \quad \mathbf{h}_{\mathbf{Cut}}$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\bullet \mathbf{h}_1 : \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \top}{\bullet \mathbf{h}_1 : \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \top} & \frac{\mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3 : (\Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5), \top \vdash \mathbf{F}_6} & \wedge L \\ \hline & -: \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \hline & \bullet \mathbf{h}_1 : \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \top & \top_R & \frac{\rightarrow}{\mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \mathbf{F}_6} \\ \hline & \frac{-: \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \mathbf{F}_6}{-: \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \mathbf{F}_6} & \wedge_L & \text{ax/W} \\ \hline \end{array}$$

• Case rule \vee_L

$$\frac{\underbrace{\begin{array}{l} \bullet \mathbf{h}_1 : \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \top}_{\bullet} \quad \mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_6 \quad \mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_3 : (\Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5), \top \vdash \mathbf{F}_6 \end{array}}_{\bullet} \quad \mathbf{Cut}}{-: \Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_6} \quad \underbrace{\begin{array}{l} \bullet \mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_3 : (\Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5), \top \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_4 : \Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_6 \end{array}}_{\bullet} \quad \underbrace{\begin{array}{l} \bullet \mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_7 \vdash \mathbf{h}_7 \\ \bullet \mathbf{h}_2 : \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \end{array}}_{\bullet} \quad \underbrace{\begin{array}{l} \bullet \mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_4 : \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_5 : \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \end{array}}_{\bullet} \quad \mathbf{h}_{\bullet} \mathbf$$

• Case rule \perp_L

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \bot, \Delta_5 \vdash \top & \top_R & \hline \bullet_{\mathbf{h}_3}: (\bot, \Delta_5), \top \vdash \mathbf{F}_4 \\ \hline -: \bot, \Delta_5 \vdash \mathbf{F}_4 \\ \hline \rightarrow \\ \hline -: \bot, \Delta_5 \vdash \mathbf{F}_4 & \bot_L \end{array} \quad \mathbf{Cut}$$

ullet Case rule I

• Case rule \top_L

$$\frac{\bullet_{\mathbf{h}_1}:\Delta_5\vdash\top}{-:\Delta_5\vdash \mathsf{F}_4} \xrightarrow{\begin{array}{c} \mathbf{h}_3:\Delta_5\vdash \mathsf{F}_4\\ \bullet_{\mathbf{h}_3}:\Delta_5,\top\vdash \mathsf{F}_4\\ -:\Delta_5\vdash \mathsf{F}_4\\ \hline -:\Delta_5\vdash \mathsf{F}_4 \end{array}}_{\bullet} \xrightarrow{\mathsf{ax/W}} \mathsf{Cut}$$

6.2 Status of \rightarrow_R : OK

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_8,\mathbf{F}_5\vdash\mathbf{F}_6}{\bullet\mathbf{h}_1:\Delta_8\vdash\mathbf{F}_5\to\mathbf{F}_6} \to_R & \\ \hline \bullet\mathbf{h}_7:\Delta_8\vdash\mathbf{F}_5\to\mathbf{F}_6\vdash\top \\ \hline -:\Delta_8\vdash\top \\ \hline -:\Delta_8\vdash\top \\ \hline -:\Delta_8\vdash\top \end{array} \begin{array}{c} \top_R \\ \mathtt{Cut} \end{array}$$

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_{10},\mathbf{F}_5\vdash\mathbf{F}_6}{\bullet\mathbf{h}_1:\Delta_{10}\vdash\mathbf{F}_5\to\mathbf{F}_6} \to_R & \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_8,\mathbf{F}_5\to\mathbf{F}_6\vdash\mathbf{F}_9}{\bullet\mathbf{h}_7:\Delta_{10},\mathbf{F}_5\to\mathbf{F}_6\vdash\mathbf{F}_8\to\mathbf{F}_9} & \to_R \\ \hline -:\Delta_{10}\vdash\mathbf{F}_8\to\mathbf{F}_9 & \to \\ \hline \bullet\mathbf{h}_1:\Delta_{10},\mathbf{F}_8\vdash\mathbf{F}_5\to\mathbf{F}_6 & \mathbf{ax/W} & \to \\ \hline \frac{-:\Delta_{10},\mathbf{F}_8\vdash\mathbf{F}_9}{-:\Delta_{10}\vdash\mathbf{F}_8\to\mathbf{F}_9} \to_R \\ \hline -:\Delta_{10}\vdash\mathbf{F}_8\to\mathbf{F}_9 & \to_R \end{array} \qquad \mathbf{ax/W}$$

• Case rule \wedge_R

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{10},\mathbf{F}_{5}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{10}\vdash\mathbf{F}_{5}\to\mathbf{F}_{6}}}{\bullet\mathbf{h}_{1}:\Delta_{10}\vdash\mathbf{F}_{5}\to\mathbf{F}_{6}}}{\bullet\mathbf{h}_{1}:\Delta_{10}\vdash\mathbf{F}_{5}\to\mathbf{F}_{6}}\to\mathbf{h}} \xrightarrow{\mathbf{h}_{7}:\Delta_{10},\mathbf{F}_{5}\to\mathbf{F}_{6}\vdash\mathbf{F}_{8}} \underbrace{\mathbf{h}_{7}:\Delta_{10},\mathbf{F}_{5}\to\mathbf{F}_{6}\vdash\mathbf{F}_{9}}_{\mathbf{Cut}} \land \mathbf{h}} \land \mathbf{h}$$

$$\frac{-:\Delta_{10}\vdash\mathbf{F}_{8}\wedge\mathbf{F}_{9}}{\bullet\mathbf{h}_{1}:\Delta_{10}\vdash\mathbf{F}_{5}\to\mathbf{F}_{6}} \xrightarrow{\mathbf{ax/W}} \underbrace{\frac{-:\Delta_{10}\vdash\mathbf{F}_{8}}{\bullet\mathbf{h}_{1}:\Delta_{10}\vdash\mathbf{F}_{5}\to\mathbf{F}_{6}}}_{-:\Delta_{10}\vdash\mathbf{F}_{8}\wedge\mathbf{F}_{9}} \land \mathbf{h}$$

$$\frac{-:\Delta_{10}\vdash\mathbf{F}_{8}\wedge\mathbf{F}_{9}}{\bullet\mathbf{h}_{1}:\Delta_{10}\vdash\mathbf{F}_{8}\wedge\mathbf{F}_{9}} \land \mathbf{h}$$

• Case rule \vee_1

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_{10}, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \Delta_{10} \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \rightarrow_R \quad \frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8 \vee \mathbf{F}_9} \quad \begin{array}{c} \mathsf{Cut} \\ \\ -: \Delta_{10} \vdash \mathbf{F}_8 \vee \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1: \Delta_{10} \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \quad \frac{\mathsf{ax/W}}{\mathsf{h}_7: \Delta_{10}, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \hline \\ -: \Delta_{10} \vdash \mathbf{F}_8 \\ \hline -: \Delta_{10} \vdash \mathbf{F}_8 \vee \mathbf{F}_9 \end{array} \quad \forall_1$$

• Case rule \vee_2

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_{10},\mathbf{F}_5\vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_{10}\vdash \mathbf{F}_5\to \mathbf{F}_6}\to_R & \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_5\to \mathbf{F}_6\vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5\to \mathbf{F}_6\vdash \mathbf{F}_8\vee \mathbf{F}_9} & \mathbf{Cut} \\ \hline & -:\Delta_{10}\vdash \mathbf{F}_8\vee \mathbf{F}_9 \\ \hline & \bullet \mathbf{h}_1:\Delta_{10}\vdash \mathbf{F}_5\to \mathbf{F}_6 & \mathbf{ax/W} & \frac{-}{\mathbf{h}_7:\Delta_{10},\mathbf{F}_5\to \mathbf{F}_6\vdash \mathbf{F}_9} \\ \hline & -:\Delta_{10}\vdash \mathbf{F}_9 \\ \hline & -:\Delta_{10}\vdash \mathbf{F}_8\vee \mathbf{F}_9 & \vee_2 \\ \hline \end{array}$$

• Case rule \rightarrow_L

$$\frac{\underbrace{\frac{h_1:(\Delta_{11},F_8\to F_9),F_5\vdash F_6}{\bullet h_1:\Delta_{11},F_8\to F_9\vdash F_5\to F_6}}_{\bullet h_1:\Delta_{11},F_8\to F_9\vdash F_5\to F_6}\to \underbrace{\frac{h_7:\Delta_{11},F_5\to F_6,F_8\to F_9\vdash F_8}{\bullet h_7:(\Delta_{11},F_8\to F_9),F_5\to F_6\vdash F_{10}}_{\bullet h_7:(\Delta_{11},F_8\to F_9),F_5\to F_6\vdash F_{10}}}_{Cut}\to L} \\ \frac{\bullet h_1:\Delta_{11},F_8\to F_9\vdash F_5\to F_6}}{-:\Delta_{11},F_8\to F_9\vdash F_8} \underbrace{\frac{ax/W}{hCut}} \to \underbrace{\frac{h_1:\Delta_{11},F_5,F_9\vdash F_6}{\bullet h_1:\Delta_{11},F_9\vdash F_5\to F_6}}_{\bullet h_1:\Delta_{11},F_9\vdash F_5\to F_6} \to \underbrace{\frac{h_1:\Delta_{11},F_9\to F_6\to F_6}{\bullet h_1:\Delta_{11},F_9\vdash F_5\to F_6}}_{\bullet h_1:\Delta_{11},F_9\vdash F_5\to F_6} \to \underbrace{\frac{ax/W}{hCut}}_{-:\Delta_{11},F_9\vdash F_10}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7}\to \underbrace{\frac{h_1:\Delta_9,F_6\vdash F_7}{\bullet h_1:\Delta_9\vdash F_6\to F_7}\to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7\vdash F_6}{\bullet h_5:\Delta_9,F_6\to F_7\vdash F_8}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7}{\bullet h_1:\Delta_9\vdash F_6\to F_7}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7}{\bullet h_1:\Delta_9\vdash F_6\to F_7}}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7}{\bullet h_1:\Delta_9\vdash F_6\to F_7}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7}{\bullet h_1:\Delta_9\vdash F_6\to F_7}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5:\Delta_9,F_6\to F_7}{\bullet h_1:\Delta_9\vdash F_6\to F_7}}_{\bullet h_1:\Delta_9\vdash F_6\to F_7} \to \underbrace{\frac{h_5$$

• Case rule \wedge_L

$$\frac{ \begin{array}{c} \mathbf{h}_1 : (\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9), \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \rightarrow_R \quad \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9), \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_{10}} \quad \begin{array}{c} \wedge_L \\ \bullet \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_5, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_6 \end{array} \rightarrow_R \quad \frac{\bullet}{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_{10}} \\ \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \rightarrow_R \quad \frac{\bullet}{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_{10}} \\ \bullet \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline - : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline - : \Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_{10}} \quad \wedge_L \end{array}$$

$$\frac{\underbrace{\frac{h_1:(\Delta_{11},F_8\vee F_9),F_5\vdash F_6}{\bullet h_1:\Delta_{11},F_8\vee F_9\vdash F_5\to F_6}}_{\bullet h_1:\Delta_{11},F_8\vee F_9\vdash F_5\to F_6}\to_R \underbrace{\frac{h_7:\Delta_{11},F_8,F_5\to F_6\vdash F_{10}}{\bullet h_7:(\Delta_{11},F_8\vee F_9),F_5\to F_6\vdash F_{10}}}_{-:\Delta_{11},F_8\vdash F_5\to F_6}\underbrace{Cut}$$

$$\frac{-:\Delta_{11},F_8\vee F_9\vdash F_{10}}{h_1:\Delta_{11},F_5,F_8\vdash F_6}\underbrace{\frac{inv-th/ax}{h_7:\Delta_{11},F_8,F_5\to F_6\vdash F_{10}}}_{h_7:\Delta_{11},F_8,F_5\to F_6\vdash F_{10}}\underbrace{\frac{ax/W}{\bullet h_1:\Delta_{11},F_9\vdash F_5\to F_6}}_{\bullet h_1:\Delta_{11},F_9\vdash F_5\to F_6}\underbrace{\frac{inv-th/ax}{h_7:\Delta_{11},F_9,F_5\to F_6\vdash F_{10}}}_{-:\Delta_{11},F_8\vee F_9\vdash F_{10}}\underbrace{\frac{ax/W}{\bullet h_7:\Delta_{11},F_9\vdash F_5\to F_6\vdash F_{10}}}_{-:\Delta_{11},F_9\vdash F_{10}}\vee_L$$

• Case rule \perp_L

$$\begin{array}{c} \frac{h_1: (\bot, \Delta_9), F_5 \vdash F_6}{\bullet h_1: \bot, \Delta_9 \vdash F_5 \to F_6} \to_{\mathit{R}} & \\ \frac{\bullet h_7: (\bot, \Delta_9), F_5 \to F_6 \vdash F_8}{-: \bot, \Delta_9 \vdash F_8} & \bot_{\mathit{L}} & \\ & \xrightarrow{-: \bot, \Delta_9 \vdash F_8} & \bot_{\mathit{L}} & \\ \end{array}$$

ullet Case rule I

$$\frac{ \frac{\mathbf{h}_1: (\Delta_8, \mathbf{p}_9), \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_8, \mathbf{p}_9 \vdash \mathbf{F}_5 \rightarrow \mathbf{F}_6} \rightarrow_R \frac{}{\bullet \mathbf{h}_7: (\Delta_8, \mathbf{p}_9), \mathbf{F}_5 \rightarrow \mathbf{F}_6 \vdash \mathbf{p}_9} }{-: \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9} \underbrace{}_{-: \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9} I$$

• Case rule \top_L

$$\begin{array}{c} \frac{\mathbf{h}_1: (\top, \Delta_9), \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \to_R & \frac{\mathbf{h}_7: \Delta_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8} & \mathsf{Cut} \\ \hline & -: \top, \Delta_9 \vdash \mathbf{F}_8 & \to \\ \hline & \bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \to \mathbf{F}_6 & \mathsf{ax/W} & \\ \hline & -: \top, \Delta_9 \vdash \mathbf{F}_8 & \mathsf{hCut} \end{array}$$

6.3 Status of \wedge_R : OK

• Case rule \top_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_8 \vdash \mathbf{F}_5 \quad \mathbf{h}_1:\Delta_8 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \wedge_R & \frac{\bullet}{\bullet \mathbf{h}_7:\Delta_8, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \top} & \top_R \\ \hline & -:\Delta_8 \vdash \top \\ \hline & \frac{-:\Delta_8 \vdash \top}{-:\Delta_8 \vdash \top} & \top_R \end{array}$$

• Case rule \rightarrow_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_{10}\vdash \mathbf{F}_5 \quad \mathbf{h}_1:\Delta_{10}\vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_{10}\vdash \mathbf{F}_5 \wedge \mathbf{F}_6} & \wedge_R & \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_8,\mathbf{F}_5 \wedge \mathbf{F}_6\vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5 \wedge \mathbf{F}_6\vdash \mathbf{F}_8 \to \mathbf{F}_9} & \rightarrow_R \\ \hline & -:\Delta_{10}\vdash \mathbf{F}_8 \to \mathbf{F}_9 \\ \hline & \bullet \mathbf{h}_1:\Delta_{10},\mathbf{F}_8\vdash \mathbf{F}_5 \wedge \mathbf{F}_6} & \text{ax/W} & \frac{}{\mathbf{h}_7:\Delta_{10},\mathbf{F}_8,\mathbf{F}_5 \wedge \mathbf{F}_6\vdash \mathbf{F}_9}}{\mathbf{h}_7:\Delta_{10},\mathbf{F}_8,\mathbf{F}_5 \wedge \mathbf{F}_6\vdash \mathbf{F}_9}} & \text{ax/W} \\ \hline & \frac{-:\Delta_{10}\vdash \mathbf{F}_8 \to \mathbf{F}_9}{-:\Delta_{10}\vdash \mathbf{F}_8 \to \mathbf{F}_9} & \rightarrow_R \end{array}$$

• Case rule \wedge_R

$$\frac{\underbrace{\begin{array}{l} \frac{h_1:\Delta_{10}\vdash F_5 \quad h_1:\Delta_{10}\vdash F_6}{\bullet h_1:\Delta_{10}\vdash F_6} \quad \wedge_R \quad \frac{h_7:\Delta_{10},F_5\wedge F_6\vdash F_8 \quad h_7:\Delta_{10},F_5\wedge F_6\vdash F_9}{\bullet h_7:\Delta_{10},F_5\wedge F_6\vdash F_8\wedge F_9} \quad \wedge_R \\ \hline \\ \frac{\bullet h_1:\Delta_{10}\vdash F_5\wedge F_6}{\bullet h_1:\Delta_{10}\vdash F_5\wedge F_6} \quad \underbrace{\begin{array}{l} \frac{\bullet h_7:\Delta_{10},F_5\wedge F_6\vdash F_8}{\bullet h_7:\Delta_{10},F_5\wedge F_6\vdash F_9} \\ \hline \\ -:\Delta_{10}\vdash F_8 \\ \hline \\ -:\Delta_{10}\vdash F_8 \\ \hline \end{array} \quad \underbrace{\begin{array}{l} \frac{\bullet h_7:\Delta_{10},F_5\wedge F_6\vdash F_9}{\bullet h_1:\Delta_{10}\vdash F_5\wedge F_6} \quad \underbrace{\begin{array}{l} \frac{\bullet x}{h_7:\Delta_{10},F_5\wedge F_6\vdash F_9} \\ h_1:\Delta_{10}\vdash F_5\wedge F_6 \\ \hline \\ -:\Delta_{10}\vdash F_8 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_5\wedge F_6} \quad h_7:\Delta_{10},F_5\wedge F_6\vdash F_9}{\bullet h_1:\Delta_{10}\vdash F_5\wedge F_6} \quad \underbrace{\begin{array}{l} \frac{\bullet x}{h_7:\Delta_{10},F_5\wedge F_6\vdash F_9} \\ h_1:\Delta_{10}\vdash F_5\wedge F_6 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_5\wedge F_6} \quad h_7:\Delta_{10},F_5\wedge F_6\vdash F_9}{h_1:\Delta_{10}\vdash F_5\wedge F_6} \quad \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_5\wedge F_6} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_5\wedge F_6} \quad h_7:\Delta_{10},F_5\wedge F_6\vdash F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_6} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_6} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_{10}\vdash F_8\wedge F_9 \\ \hline \end{array}}_{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ \underbrace{\begin{array}{l} \frac{\bullet x}{h_1:\Delta_{10}\vdash F_8\wedge F_9} \\ h_7:\Delta_$$

• Case rule \vee_1

$$\frac{\mathbf{h}_1 : \Delta_{10} \vdash \mathbf{F}_5 \quad \mathbf{h}_1 : \Delta_{10} \vdash \mathbf{F}_6}{\underbrace{\bullet \mathbf{h}_1 : \Delta_{10} \vdash \mathbf{F}_5 \land \mathbf{F}_6}_{\bullet \mathbf{h}_1 : \Delta_{10} \vdash \mathbf{F}_5 \land \mathbf{F}_6}} \quad \wedge_R \quad \frac{\mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8 \lor \mathbf{F}_9} \quad \forall_1 \\ \frac{- : \Delta_{10} \vdash \mathbf{F}_8 \lor \mathbf{F}_9}{\bullet \mathbf{h}_1 : \Delta_{10} \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8} \quad \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{Cut}} \\ \frac{- : \Delta_{10} \vdash \mathbf{F}_8}{- : \Delta_{10} \vdash \mathbf{F}_8 \lor \mathbf{F}_9} \quad \forall_1 \\ \end{array}$$

• Case rule \vee_2

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \Delta_{10} \vdash \mathbf{F}_5 \quad \mathbf{h}_1 : \Delta_{10} \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_{10} \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \wedge_R \quad \frac{\mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8 \lor \mathbf{F}_9} \quad \mathcal{V}_2 \\ \hline \\ -: \Delta_{10} \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \Delta_{10} \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \underset{\mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet} \quad \underset{\mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet} \quad \underset{\mathbf{h}_7 : \Delta_{10} \vdash \mathbf{F}_8 \lor \mathbf{F}_9}{\bullet} \\ \hline \\ -: \Delta_{10} \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \quad \vee_2 \end{array} \quad \underset{\mathbf{h}_7 : \Delta_{10} \vdash \mathbf{F}_8 \lor \mathbf{F}_9}{\bullet} \quad \mathcal{V}_2 \end{array}$$

• Case rule \rightarrow_L

$$\frac{\underbrace{\frac{h_1:\Delta_{11},F_8\to F_9\vdash F_5}{\bullet h_1:\Delta_{11},F_8\to F_9\vdash F_6}}_{\bullet h_1:\Delta_{11},F_8\to F_9\vdash F_5\wedge F_6} \land \underbrace{\frac{h_7:\Delta_{11},F_8\to F_9,F_5\wedge F_6\vdash F_8}{\bullet h_7:(\Delta_{11},F_8\to F_9),F_5\wedge F_6\vdash F_{10}}}_{\bullet h_7:(\Delta_{11},F_8\to F_9),F_5\wedge F_6\vdash F_{10}} \underbrace{\frac{-:\Delta_{11},F_8\to F_9\vdash F_8}{\bullet h_7:\Delta_{11},F_8\to F_9\vdash F_8}}_{\bullet h_2:\Delta_{11},F_8\to F_9\vdash F_8} \underbrace{\frac{h_7:\Delta_{11},F_8\to F_9\vdash F_6}{\bullet h_1:\Delta_{11},F_9\vdash F_5}}_{\bullet h_2:\Delta_{11},F_9\vdash F_5\wedge F_6} \underbrace{\frac{inv-th/ax}{h_1:\Delta_{11},F_9\vdash F_6}}_{h_7:\Delta_{11},F_9\vdash F_{10}} \underbrace{\frac{-:\Delta_{11},F_9\vdash F_8}{\bullet h_2:\Delta_{11},F_9\vdash F_{10}}}_{-:\Delta_{11},F_9\vdash F_{10}} \underbrace{-:\Delta_{11},F_9\vdash F_{10}}_{\bullet h_2:\Delta_{11},F_9\vdash F_{10}} \underbrace{-:\Delta_{11},F_9\vdash F_{10}}_{\bullet h_2:\Delta_{11},F_9\vdash F_{10}}$$

• Case rule \wedge_L

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\wedge\mathbf{F}_{9}\vdash\mathbf{F}_{5}\quad\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\wedge\mathbf{F}_{9}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\wedge\mathbf{F}_{9}\vdash\mathbf{F}_{5}\wedge\mathbf{F}_{6}}} \wedge_{R} \quad \frac{\frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{9},\mathbf{F}_{5}\wedge\mathbf{F}_{6}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\wedge\mathbf{F}_{9}),\mathbf{F}_{5}\wedge\mathbf{F}_{6}\vdash\mathbf{F}_{10}}} \quad \wedge_{L} \quad \text{Cut}}{\frac{-:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{9}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{9}\vdash\mathbf{F}_{6}}} \quad \frac{\mathbf{inv}-\mathbf{th}/\mathbf{ax}}{\wedge_{R}} \quad \frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{9},\mathbf{F}_{5}\wedge\mathbf{F}_{6}\vdash\mathbf{F}_{10}}}{\frac{-:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{9}\vdash\mathbf{F}_{10}}{-:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{9}\vdash\mathbf{F}_{10}}} \wedge_{L} \quad \mathbf{ax}/\mathbf{W}} \quad \mathbf{hCut}$$

$$\frac{\begin{array}{l} \mathbf{h}_1: \Delta_9 \vdash \mathbf{F}_6 \quad \mathbf{h}_1: \Delta_9 \vdash \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1: \Delta_9 \vdash \mathbf{F}_6 \land \mathbf{F}_7 \end{array} \wedge_R \quad \frac{\begin{array}{l} \mathbf{h}_5: \Delta_9, \mathbf{F}_6, \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_5: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_8 \end{array}}{-: \Delta_9 \vdash \mathbf{F}_6} \wedge_L \\ \hline -: \Delta_9 \vdash \mathbf{F}_8 \\ \hline -: \Delta_9 \vdash \mathbf{F}_6 \end{array} \xrightarrow[-: \Delta_9, \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array}} \begin{array}{l} \mathbf{ax/W} \\ \hline -: \Delta_9, \mathbf{F}_6, \mathbf{F}_7 \vdash \mathbf{F}_8 \end{array} \xrightarrow[\mathbf{SCut}]{} \mathbf{ax/W} \\ \hline -: \Delta_9 \vdash \mathbf{F}_8 \end{array}$$

• Case rule \vee_L

$$\frac{\underbrace{\frac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_6}}_{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5} \quad h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \wedge F_6} \quad \wedge_R \quad \underbrace{\frac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \wedge F_6 \vdash F_{10}}}_{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \wedge F_6 \vdash F_{10}} \quad \wedge_L \quad \\ \underbrace{\frac{-: \Delta_{11}, F_8 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \wedge F_6}}_{\bullet h_7 : \Delta_{11}, F_8 \vdash F_5 \wedge F_6 \vdash F_{10}} \quad \underbrace{\frac{-: \Delta_{11}, F_8 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5}}_{\bullet h_{10}} \underbrace{\frac{-: \Delta_{11}, F_8 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5}}_{\bullet h_{10}} \underbrace{\frac{-: \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5}}_{\bullet h_{10}} \underbrace{\frac{-: \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6}}_{\bullet h_{10}} \underbrace{\frac{-: \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6}}_{\bullet h_{10}} \underbrace{\frac{-: \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6}}_{\bullet h_{10}} \underbrace{\frac{-: \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6}}_{\bullet h_{10}} \underbrace{\frac{-: \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6}}_{\bullet h_{10}} \underbrace{\frac{-: \Delta_{11}, F_9 \vdash F_5}{\bullet h_1}}_{\bullet h_{10}} \underbrace{\frac{-: \Delta_{11}, F$$

• Case rule \perp_L

$$\begin{array}{c} \mathbf{h}_1: \bot, \Delta_9 \vdash \mathbf{F}_5 \quad \mathbf{h}_1: \bot, \Delta_9 \vdash \mathbf{F}_6 \\ \\ \underline{\bullet \mathbf{h}_1: \bot, \Delta_9 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \\ \hline \\ -: \bot, \Delta_9 \vdash \mathbf{F}_8 \\ \hline \\ -: \bot, \Delta_9 \vdash \mathbf{F}_8 \\ \hline \\ -: \bot, \Delta_9 \vdash \mathbf{F}_8 \\ \hline \\ \bot_L \end{array} \quad \begin{array}{c} \bot_L \\ \text{Cut} \\ \hline \end{array}$$

 \bullet Case rule I

$$\frac{\mathbf{h}_1:\Delta_8,\mathbf{p}_9\vdash \mathbf{F}_5\quad \mathbf{h}_1:\Delta_8,\mathbf{p}_9\vdash \mathbf{F}_6}{\underbrace{\begin{array}{c}\bullet \mathbf{h}_1:\Delta_8,\mathbf{p}_9\vdash \mathbf{F}_6\\\\ -:\Delta_8,\mathbf{p}_9\vdash \mathbf{p}_9\\\\ -:\Delta_8,\mathbf{p}_9\vdash \mathbf{p}_9\end{array}}_{}I} \quad \underbrace{\begin{array}{c}I\\ \text{Cut}\end{array}}$$

$$\frac{\mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \quad \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_6}{\underbrace{\bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \land \mathbf{F}_6}_{\bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8}_{\bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8}} \underbrace{\begin{array}{c} \top_L \\ \bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \end{array}}_{\bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \underbrace{\begin{array}{c} -: \top, \Delta_9 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_7: \top, \Delta_9, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8 \end{array}}_{\bullet \mathbf{h}_7: \top, \Delta_9, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8} \underbrace{\begin{array}{c} \mathbf{ax/W} \\ \mathsf{hCut} \end{array}}_{\bullet \mathsf{Lut}}$$

6.4 Status of \vee_1 : OK

• Case rule \top_R

• Case rule \rightarrow_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_5 \vee \mathbf{F}_6} & \vee_1 & \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_8,\mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \to \mathbf{F}_9} & \rightarrow_R \\ \hline & -:\Delta_{10} \vdash \mathbf{F}_8 \to \mathbf{F}_9 \\ & & \rightarrow \\ \hline \bullet \mathbf{h}_1:\Delta_{10},\mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 & \mathsf{ax/W} & \\ \hline & \frac{-:\Delta_{10},\mathbf{F}_8 \vdash \mathbf{F}_9}{-:\Delta_{10} \vdash \mathbf{F}_8 \to \mathbf{F}_9} & \rightarrow_R \\ \hline & & -:\Delta_{10} \vdash \mathbf{F}_8 \to \mathbf{F}_9 \\ \hline & -:\Delta_{10} \vdash \mathbf{F}_8 \to \mathbf{F}_9 & \rightarrow_R \\ \hline \end{array}$$

• Case rule \wedge_R

$$\frac{\underbrace{\begin{array}{l} \mathbf{h}_1: \Delta_{10} \vdash F_5 \\ \bullet \mathbf{h}_1: \Delta_{10} \vdash F_5 \\ \end{array}}_{\bullet \mathbf{h}_1: \Delta_{10} \vdash F_5 \vee F_6} \vee_1 \underbrace{\begin{array}{l} \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_8 & \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_8 \wedge F_9 \\ \end{array}}_{\bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \wedge F_9} \wedge_R \\ \underbrace{\begin{array}{l} \bullet \mathbf{h}_1: \Delta_{10} \vdash F_5 \vee F_6 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_8 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_8 \\ \bullet \mathbf{h}_{10} \\ \bullet \mathbf{h}_{10} \\ \hline \\ -: \Delta_{10} \vdash F_8 \wedge F_9 \\ \end{array}}_{\bullet \mathbf{h}_{10}} \underbrace{\begin{array}{l} \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_{10} \\ \hline \\ -: \Delta_{10} \vdash F_8 \wedge F_9 \\ \end{array}}_{\bullet \mathbf{h}_{10}} \wedge_R \\ \underbrace{\begin{array}{l} \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_{10} \\ \hline \\ -: \Delta_{10} \vdash F_8 \wedge F_9 \\ \end{array}}_{\bullet \mathbf{h}_{10}} \wedge_R \\ \underbrace{\begin{array}{l} \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_{10} \\ \hline \\ -: \Delta_{10} \vdash F_8 \wedge F_9 \\ \hline \end{array}}_{\bullet \mathbf{h}_{10}} \wedge_R \\ \underbrace{\begin{array}{l} \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_{10} \\ \hline \end{array}}_{\bullet \mathbf{h}_{10}} \wedge_R \\ \underbrace{\begin{array}{l} \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \hline \end{array}}_{\bullet \mathbf{h}_{10}} \wedge_R \\ \underbrace{\begin{array}{l} \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \hline \end{array}}_{\bullet \mathbf{h}_{10}} \wedge_R \\ \underbrace{\begin{array}{l} \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \hline \end{array}}_{\bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \vee F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_7 \vee F_9 \\ \bullet \mathbf{h}_7 \vee \mathbf{h}_7 \vee \mathbf{h}_7 \vee \mathbf{h}_7 \vee \mathbf{h}_7 \\ \bullet \mathbf{h}_7 \vee \mathbf{h}_7 \vee \mathbf{h}_7 \vee \mathbf{h}_7 \\ \bullet \mathbf{h}_7 \vee \mathbf$$

• Case rule \vee_1

$$\frac{ \begin{array}{c|c} \mathbf{h}_{1}: \Delta_{10} \vdash \mathbf{F}_{5} \\ \bullet \mathbf{h}_{1}: \Delta_{10} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6} \end{array} \lor_{1} & \frac{\mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{8} \lor \mathbf{F}_{9}} \\ \hline -: \Delta_{10} \vdash \mathbf{F}_{8} \lor \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{10} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6} & \text{ax/W} \\ \hline -: \Delta_{10} \vdash \mathbf{F}_{8} \\ \hline \end{array} \lor_{1} \\ \hline \end{array} \quad \begin{array}{c} \mathsf{Ax/W} \\ \mathsf{hCut} \\ \hline \end{array}$$

• Case rule \vee_2

$$\frac{ \begin{array}{c|c} \mathbf{h}_1: \Delta_{10} \vdash \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_1: \Delta_{10} \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \vee_1 & \begin{array}{c} \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \\ \hline -: \Delta_{10} \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \\ \hline \hline \bullet \mathbf{h}_1: \Delta_{10} \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} & \begin{array}{c} \mathbf{ax/W} \\ \hline \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline -: \Delta_{10} \vdash \mathbf{F}_9 \\ \hline -: \Delta_{10} \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \end{array} & \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \rightarrow_L

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{5}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{5}\vee\mathbf{F}_{6}}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{5}\vee\mathbf{F}_{6}}} \vee_{1} \frac{\frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9},\mathbf{F}_{5}\vee\mathbf{F}_{6}\vdash\mathbf{F}_{8}}{\bullet\mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}),\mathbf{F}_{5}\vee\mathbf{F}_{6}\vdash\mathbf{F}_{10}}} \operatorname{Cut} \\ -:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{10} \\ \to \\ \frac{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{5}\vee\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9},\mathbf{F}_{5}\vee\mathbf{F}_{6}\vdash\mathbf{F}_{8}} \operatorname{ax/W} \\ -:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{10}} \\ \to \\ -:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{10}} \to_{L} \end{aligned} \qquad \mathbf{ax/W}$$

• Case rule \wedge_L

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \vee_1 \quad \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ & - : \Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ & \rightarrow \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_5 \end{array} \quad \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ & \rightarrow \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \quad \vee_1 \qquad \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ & \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ & - : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ & - : \Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} \quad \wedge_L \end{array} \quad \mathbf{h}_7 : \mathbf$$

• Case rule \vee_L

$$\frac{\begin{array}{c} h_1: \Delta_{11}, F_8 \vee F_9 \vdash F_5 \\ \hline \bullet h_1: \Delta_{11}, F_8 \vee F_9 \vdash F_5 \vee F_6 \end{array}}{\bullet h_1: \Delta_{11}, F_8 \vee F_9 \vdash F_5 \vee F_6 \end{array}} \vee_1 \quad \frac{\begin{array}{c} h_7: \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \\ \hline \bullet h_7: (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10} \end{array}}{\bullet h_7: (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \quad \vee_L \\ \hline \\ \hline \begin{array}{c} -: \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \\ \hline \\ \hline \bullet h_1: \Delta_{11}, F_8 \vdash F_5 \end{array} \\ \hline \bullet h_1: \Delta_{11}, F_8 \vdash F_5 \end{array} \\ \hline \bullet h_1: \Delta_{11}, F_8 \vdash F_5 \vee F_6 \end{array} \vee_1 \quad \frac{\bullet h_7: \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{h_7: \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}} \quad \frac{\bullet h_1: \Delta_{11}, F_9 \vdash F_5 \vee F_6}{h_1: \Delta_{11}, F_9 \vdash F_5 \vee F_6} \vee_1 \qquad \frac{h_7: \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{h_7: \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \\ \hline \\ \hline -: \Delta_{11}, F_8 \vdash F_{10} \qquad \qquad -: \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \end{array} \\ \hline -: \Delta_{11}, F_8 \vdash F_9 \vdash F_{10} \end{array} \vee_L \\ \hline \begin{array}{c} h_1: \Delta_9 \vdash F_6 \\ \hline \bullet h_1: \Delta_9 \vdash F_6 \\ \hline \bullet h_1: \Delta_9 \vdash F_6 \\ \hline \bullet h_1: \Delta_9 \vdash F_6 \end{array} \quad V_1 \quad \frac{h_5: \Delta_9, F_6 \vdash F_8}{\bullet h_5: \Delta_9, F_6 \vee F_7 \vdash F_8} \quad Cut \\ \hline -: \Delta_9 \vdash F_8 \\ \hline -: \Delta_9 \vdash F_6 \end{array} \quad X_W \quad \frac{\bullet}{-: \Delta_9 \vdash F_8} \quad \text{ax/W} \\ \hline -: \Delta_9 \vdash F_8 \end{array} \quad X_W \quad SCut$$

• Case rule \perp_L

$$\begin{array}{c|c} \frac{h_1:\bot,\Delta_9\vdash F_5}{\bullet h_1:\bot,\Delta_9\vdash F_5\vee F_6} & \vee_1 & \hline\\ -:\bot,\Delta_9\vdash F_8 & \to \\ \hline\\ -:\bot,\Delta_9\vdash F_8 & \bot_L \\ \hline\\ -:\bot,\Delta_9\vdash F_8 & \bot_L \end{array}$$
 Cut

 $\bullet\,$ Case rule I

$$\frac{ \mathbf{h}_1 : \Delta_8, \mathbf{p}_9 \vdash \mathbf{F}_5 }{ \underbrace{\bullet \mathbf{h}_1 : \Delta_8, \mathbf{p}_9 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 }_{} \quad \vee_1 \quad \underbrace{\bullet \mathbf{h}_7 : (\Delta_8, \mathbf{p}_9), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{p}_9 }_{} \quad \mathcal{C}ut}_{} \underbrace{ \begin{array}{c} -: \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9 \\ \longrightarrow \\ -: \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9 \end{array} I}_{}$$

• Case rule \top_L

$$\frac{ \begin{array}{c} \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \vee_1 \quad \frac{\mathbf{h}_7: \Delta_9, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8} \quad \frac{\top_L}{\mathsf{Cut}} \\ \hline -: \top, \Delta_9 \vdash \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \quad \frac{\mathsf{ax/W}}{\mathsf{h}_7: \top, \Delta_9, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8} \quad \frac{\mathsf{ax/W}}{\mathsf{h}_\mathsf{Cut}} \\ \hline -: \top, \Delta_9 \vdash \mathbf{F}_8 \end{array}$$

6.5 Status of \vee_2 : OK

• Case rule \top_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_8 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_8 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} & \vee_2 & \\ \hline -: \Delta_8 \vdash \top \\ & \xrightarrow{\bullet} \\ \hline -: \Delta_8 \vdash \top \\ \hline -: \Delta_8 \vdash \top \end{array} \uparrow_R \\ \mathbf{Cut} \\$$

• Case rule \rightarrow_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_5 \vee \mathbf{F}_6} & \vee_2 & \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_8,\mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \to \mathbf{F}_9} & \rightarrow_R \\ \hline -:\Delta_{10} \vdash \mathbf{F}_8 \to \mathbf{F}_9 & \rightarrow \\ \hline \bullet \mathbf{h}_1:\Delta_{10},\mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 & \operatorname{ax/W} & \rightarrow \\ \hline -:\Delta_{10},\mathbf{F}_8 \vdash \mathbf{F}_9 & \rightarrow_R \\ \hline -:\Delta_{10} \vdash \mathbf{F}_8 \to \mathbf{F}_9 & \rightarrow_R \\ \hline -:\Delta_{10} \vdash \mathbf{F}_8 \to \mathbf{F}_9 & \rightarrow_R \end{array}$$

• Case rule \wedge_R

$$\frac{\underbrace{\begin{array}{l} \mathbf{h}_1: \Delta_{10} \vdash F_6 \\ \bullet \mathbf{h}_1: \Delta_{10} \vdash F_5 \lor F_6 \end{array}}_{\bullet \mathbf{h}_1: \Delta_{10} \vdash F_5 \lor F_6} \lor_2 \underbrace{\begin{array}{l} \mathbf{h}_7: \Delta_{10}, F_5 \lor F_6 \vdash F_8 & \mathbf{h}_7: \Delta_{10}, F_5 \lor F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10}, F_5 \lor F_6 \vdash F_8 \land F_9 \end{array}}_{\bullet \mathbf{h}_7: \Delta_{10}, F_5 \lor F_6 \vdash F_8 \land F_9} \mathsf{Cut} \\ \\ \underbrace{\begin{array}{l} \bullet \mathbf{h}_1: \Delta_{10} \vdash F_5 \lor F_6 \\ \bullet \mathbf{h}_1: \Delta_{10} \vdash F_5 \lor F_6 \end{array}}_{\bullet \mathbf{h}_7: \Delta_{10}, F_5 \lor F_6 \vdash F_8} \underbrace{\begin{array}{l} \mathbf{ax/W} \\ \bullet \mathbf{h}_7: \Delta_{10} \vdash F_5 \lor F_6 \end{array}}_{\bullet \mathbf{h}_7: \Delta_{10}, F_5 \lor F_6 \vdash F_9} \underbrace{\begin{array}{l} \mathbf{ax/W} \\ \bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \lor F_6 \\ \bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \lor F_9 \end{array}}_{\bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \lor F_6 \vdash F_9} \underbrace{\begin{array}{l} \mathbf{ax/W} \\ \bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \lor F_6 \\ \bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \lor F_6 \\ \bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \lor F_9 \end{aligned}}_{\bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \lor F_9} \underbrace{\begin{array}{l} \mathbf{ax/W} \\ \bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \lor F_6 \vdash F_9 \\ \bullet \mathbf{h}_7: \Delta_{10} \vdash F_8 \lor F_9 \\ \bullet \mathbf{h}_7: \Delta_{10} \vdash \mathbf{h}_8 \\ \bullet \mathbf{$$

• Case rule \vee_1

$$\frac{ \begin{array}{c|c} \mathbf{h}_{1}: \Delta_{10} \vdash \mathbf{F}_{6} \\ \bullet \mathbf{h}_{1}: \Delta_{10} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6} \end{array} \lor_{2} & \frac{\mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{8} \lor \mathbf{F}_{9}} \\ \hline -: \Delta_{10} \vdash \mathbf{F}_{8} \lor \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{10} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6} & \text{ax/W} \\ \hline -: \Delta_{10} \vdash \mathbf{F}_{8} \\ \hline \end{array} \lor_{1} \\ \hline \end{array} \quad \begin{array}{c} \mathsf{Ax/W} \\ \mathsf{hCut} \\ \hline \end{array}$$

• Case rule \vee_2

$$\frac{\begin{array}{c} \mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \vee_2 \quad \frac{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10}, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \vee \mathbf{F}_9} \quad \begin{array}{c} \vee_2 \\ \bullet \mathbf{h}_7:\Delta_{10}, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \vee \mathbf{F}_9 \end{array} & \mathcal{C}ut} \\ \\ \bullet \underbrace{\begin{array}{c} \bullet \mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \bullet \mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} & \mathbf{ax/W}}_{\quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10}, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline -:\Delta_{10} \vdash \mathbf{F}_9 \\ \hline -:\Delta_{10} \vdash \mathbf{F}_8 \vee \mathbf{F}_9 \end{array}}_{\quad \begin{array}{c} \vee_2 \\ \mathbf{h}_7:\Delta_{10}, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline -:\Delta_{10} \vdash \mathbf{F}_8 \vee \mathbf{F}_9 \end{array}} \end{array}} \quad \mathbf{ax/W}}_{\quad \mathbf{h}Cut}$$

• Case rule \rightarrow_L

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{5}\vee\mathbf{F}_{6}}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{5}\vee\mathbf{F}_{6}}} \vee_{2} \frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9},\mathbf{F}_{5}\vee\mathbf{F}_{6}\vdash\mathbf{F}_{8}}{\bullet\mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}),\mathbf{F}_{5}\vee\mathbf{F}_{6}\vdash\mathbf{F}_{10}}} \operatorname{Cut} \\ -:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{10}} \to \frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{5}\vee\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{8}\to\mathbf{F}_{9}\vdash\mathbf{F}_{8}} \frac{\mathbf{inv}-\mathbf{th}/\mathbf{ax}}{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{9}\vdash\mathbf{F}_{6}} \frac{\mathbf{inv}-\mathbf{th}/\mathbf{ax}}{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{9}\vdash\mathbf{F}_{5}\vee\mathbf{F}_{6}} \vee_{2} \frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{9},\mathbf{F}_{5}\vee\mathbf{F}_{6}\vdash\mathbf{F}_{10}}{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{9}\to\mathbf{F}_{5}\vee\mathbf{F}_{6}\vdash\mathbf{F}_{10}} \to L$$

• Case rule \wedge_L

$$\frac{ \begin{array}{c} \mathbf{h}_1:\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1:\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \end{array}}{ \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_7:(\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \end{array}} \begin{array}{c} \wedge_L \\ \text{Cut} \\ \hline \\ -:\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_6 \\ \hline \\ \bullet \mathbf{h}_1:\Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline \\ -:\Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ -:\Delta_{11}, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ -:\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ -:\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \end{array} \begin{array}{c} \wedge_L \\ \text{hCut} \\ \hline \end{array}$$

• Case rule \vee_L

• Case rule \perp_L

$$\begin{array}{c} \frac{h_1:\bot,\Delta_9\vdash F_6}{\bullet h_1:\bot,\Delta_9\vdash F_5\vee F_6} \;\; \vee_2 \quad \\ \frac{\bullet h_7:(\bot,\Delta_9),F_5\vee F_6\vdash F_8}{-:\bot,\Delta_9\vdash F_8} \quad \\ \frac{-:\bot,\Delta_9\vdash F_8}{-:\bot,\Delta_9\vdash F_8} \;\; \bot_L \end{array} \quad \text{Cut}$$

 $\bullet\,$ Case rule I

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_8, \mathbf{p}_9 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \Delta_8, \mathbf{p}_9 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \vee_2 \quad \frac{}{\bullet \mathbf{h}_7: (\Delta_8, \mathbf{p}_9), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{p}_9} \\ -: \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9 \\ \hline -: \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9 \quad I \end{array} \quad \mathbf{Cut}$$

• Case rule \top_L

$$\frac{ \begin{array}{c|c} \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \vee_2 \quad \begin{array}{c} \mathbf{h}_7: \Delta_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \end{array}}{-: \top, \Delta_9 \vdash \mathbf{F}_8} \quad \begin{array}{c} \top_L \\ \mathsf{Cut} \\ \bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \\ \bullet \mathbf{h}_1: \top, \Delta_9 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \quad \begin{array}{c} \bullet \mathbf{A}_7 \vee \mathbf{A}_9 \vee \mathbf{F}_7 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \\ \bullet \mathbf{A}_7: \top, \Delta_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \end{array} \quad \begin{array}{c} \bullet \mathbf{A}_7 \vee \mathbf{A}_9 \vee \vee$$

6.6 Status of \rightarrow_L : OK

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_9, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \mathbf{F}_6 \quad \mathbf{h}_1:\Delta_9, \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1:\Delta_9, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \mathbf{F}_{10}} \rightarrow_L \\ \frac{\bullet \mathbf{h}_1:\Delta_9, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \mathbf{F}_{10}}{-:\Delta_9, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \top} \xrightarrow{\top_R} \\ & \xrightarrow{-:\Delta_9, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \top} \top_R \end{array} \quad \begin{array}{c} \top_R \\ \text{Cut} \end{array}$$

• Case rule \rightarrow_R

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_6 \quad \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_7 \vdash \mathbf{F}_{12} \\ \hline \\ & \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{12} \\ \hline \\ & - : \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_9 \to \mathbf{F}_{10} \\ \hline \\ & - : \Delta_{11}, \mathbf{F}_9, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{12} \\ \hline \\ & \bullet \mathbf{h}_8 : (\Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7), \mathbf{F}_{12} \vdash \mathbf{F}_9 \to \mathbf{F}_{10} \\ \hline \\ & \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_9, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{12} \\ \hline \\ & \bullet \mathbf{h}_8 : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_9, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \hline \\ & - : \Delta_{11}, \mathbf{F}_9, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \hline \\ & - : \Delta_{11}, \mathbf{F}_9, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \hline \\ & - : \Delta_{11}, \mathbf{F}_9, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \hline \end{array} \right) \rightarrow_R \\ \end{array} } \right.$$

• Case rule \wedge_R

$$\frac{ \begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{6}\quad\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{7}\vdash\mathbf{F}_{12}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{12}} \to_{L} & \frac{\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{9}\quad\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{8}:(\Delta_{11},\mathbf{F}_{6}\to\mathbf{F}_{7}),\mathbf{F}_{12}\vdash\mathbf{F}_{9}\wedge\mathbf{F}_{10}} & \mathbf{Cut} \\ \hline \\ \frac{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{12}\quad\mathbf{ax/W}\quad & \bullet\\ \frac{-:\Delta_{11},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{9}}{\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{10}} & \mathbf{ax/W}\\ \hline \\ \frac{-:\Delta_{11},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{10}} & -:\Delta_{11},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{10}\\ \hline \\ -:\Delta_{11},\mathbf{F}_{6}\to\mathbf{F}_{7}\vdash\mathbf{F}_{9}\wedge\mathbf{F}_{10} & \wedge_{R} \end{array}} \\ \end{array}}$$

• Case rule \vee_1

$$\frac{\mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_6 \quad \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_7 \vdash \mathbf{F}_{12}}{\bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{12}} \to_L \quad \frac{\mathbf{h}_8 : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8 : (\Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7), \mathbf{F}_{12} \vdash \mathbf{F}_9 \vee \mathbf{F}_{10}} \quad \text{Cut} \\ \frac{-: \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_9 \vee \mathbf{F}_{10}}{\bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_1} \quad \frac{\mathbf{ax/W}}{\mathbf{h}_8 : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_9}} \\ \frac{-: \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_9}{-: \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_9} \vee \mathbf{f}_{10}} \quad \vee_1 \\ \end{array} \right.$$

• Case rule \vee_2

$$\frac{\mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_6 \quad \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_7 \vdash \mathbf{F}_{12}}{\bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{12}} \to_L \quad \frac{\mathbf{h}_8 : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8 : (\Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7), \mathbf{F}_{12} \vdash \mathbf{F}_9 \vee \mathbf{F}_{10}} \quad \underbrace{-: \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_9 \vee \mathbf{F}_{10}}_{\bullet \mathbf{h}_8 : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{10}} \quad \underbrace{-: \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{10}}_{\bullet \mathbf{h}_8 : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{10}} \quad \underbrace{-: \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{10}}_{\bullet : \Delta_{11}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{9} \vee \mathbf{F}_{10}} \quad \vee_2$$

• Case rule \rightarrow_L

$$\frac{h_1: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_6}{\bullet h_1: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{13}} \to L \xrightarrow{h_8: \Delta_{12}, F_{13}, F_6 \to F_7, F_9 \to F_{10} \vdash F_9} \xrightarrow{h_8: \Delta_{12}, F_{10}, F_{13}, F_6 \to F_7 \vdash F_{11}} Cut$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_9$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_9$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_9$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_9$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_9$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_9$$

$$-: (\Delta_{12}, F_9 \to F_{10}), F_9 \to F_{10} \vdash F_{11}$$

$$-: (\Delta_{12}, F_9 \to$$

$$\frac{\mathbf{h}_{1}:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{7}\quad\mathbf{h}_{1}:\Delta_{10},\mathbf{F}_{8}\vdash\mathbf{F}_{11}}{\bullet}\to_{L}\quad\frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{7}\quad\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{8},\mathbf{F}_{11}\vdash\mathbf{F}_{9}}{\bullet}_{L}}\to_{L}\\ \frac{\bullet\mathbf{h}_{1}:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{11}}{-:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}\quad\mathbf{Cut}\\ \frac{-:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}{\bullet}\to_{L}\\ \frac{-:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{7}}{\bullet}_{L}$$

• Case rule \wedge_L

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \rightarrow F_{7} \vdash F_{6} \quad \mathbf{h}_{1}: (\Delta_{12}, F_{9} \wedge F_{10}), F_{7} \vdash F_{13} \\ \bullet \mathbf{h}_{1}: (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \rightarrow F_{7} \vdash F_{13} \\ \end{array}}{ \begin{array}{c} \bullet \mathbf{h}_{1}: (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \rightarrow F_{7} \vdash F_{13} \\ \end{array}} \\ -: (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11} \\ \rightarrow \\ \\ -: (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11} \\ \end{array}} \\ -: (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11} \\ \rightarrow \\ \\ -: \Delta_{12}, F_{9} \wedge F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11} \\ \hline \rightarrow \\ \\ -: \Delta_{12}, F_{9} \wedge F_{10} \vdash F_{13} \\ \end{array}} \\ \xrightarrow{\begin{array}{c} h_{1}: \Delta_{12}, F_{10}, F_{13}, F_{13}, F_{13}, F_{13} \vdash F_{11} \\ \hline \bullet \mathbf{h}_{8}: \Delta_{12}, F_{13}, F_{7}, F_{9} \wedge F_{10} \vdash F_{11} \\ \hline -: \Delta_{12}, F_{6} \rightarrow F_{7}, F_{9} \wedge F_{10} \vdash F_{11} \\ \hline -: \Delta_{12}, F_{6} \rightarrow F_{7}, F_{9} \wedge F_{10} \vdash F_{11} \\ \hline -: \Delta_{12}, F_{6} \rightarrow F_{7} \vdash F_{9} \wedge F_{10} \\ \hline -: \Delta_{12}, F_{6} \rightarrow F_{7} \vdash F_{9} \wedge F_{10} \\ \hline -: \Delta_{12}, F_{6} \rightarrow F_{7} \vdash F_{9} \wedge F_{10} \\ \hline -: \Delta_{12}, F_{6} \rightarrow F_{7} \vdash F_{9} \wedge F_{10} \\ \hline -: \Delta_{12}, F_{7} \vdash F_{9} \wedge F_{10} \\ \hline -: \Delta_{12$$

• Case rule \vee_L

$$\frac{\mathbf{h}_{1}: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{6} \quad \mathbf{h}_{1}: (\Delta_{12}, F_{9} \vee F_{10}), F_{7} \vdash F_{13}}{\bullet \mathbf{h}_{1}: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{13}} \rightarrow_{L} \frac{\mathbf{h}_{8}: \Delta_{12}, F_{9}, F_{13}, F_{6} \rightarrow F_{7} \vdash F_{11}}{\bullet \mathbf{h}_{8}: ((\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7}), F_{13} \vdash F_{11}}} \underbrace{\mathbf{cut}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \rightarrow F_{7} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{11}} \\ -: (\Delta_{12}, F_{9} \vee F_{10}), F_{9} \vee F_{10} \vdash F_{$$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\mathbf{F}_6\quad\mathbf{h}_1:\Delta_{10},\mathbf{F}_7\vdash\bot}{\bullet\mathbf{h}_1:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\bot}\to_L \quad \frac{\bullet\mathbf{h}_8:(\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7),\bot\vdash\mathbf{F}_9}{\bullet\mathbf{h}_8:(\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7),\bot\vdash\mathbf{F}_9} \stackrel{\bot_L}{\subset} \\ & \xrightarrow{-:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\mathbf{F}_6} \quad \frac{\to}{\bullet\mathbf{h}_1:\Delta_{10},\mathbf{F}_7\vdash\bot} \quad \mathbf{ax/W} \quad \frac{\bullet\mathbf{h}_8:\bot,\Delta_{10},\mathbf{F}_7\vdash\mathbf{F}_9}{\bullet\mathbf{h}_8:\bot,\Delta_{10},\mathbf{F}_7\vdash\mathbf{F}_9} \quad \overset{\bot_L}{\to} \\ & \xrightarrow{-:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\mathbf{F}_9} \quad \to_L \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_1: (\bot, \Delta_{10}), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_6 \quad \mathbf{h}_1: (\bot, \Delta_{10}), \mathbf{F}_7 \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_1: (\bot, \Delta_{10}), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{11}} \\ \to L \\ \frac{\bullet \mathbf{h}_1: (\bot, \Delta_{10}), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_{11}}{-: (\bot, \Delta_{10}), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_9} \\ \to \\ \frac{\bot_L}{-: \bot, \Delta_{10}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \mathbf{F}_9} \\ \bot_L \end{array}$$

 \bullet Case rule I

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: (\Delta_{9}, \mathbf{p}_{11}), \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{F}_{6} \quad \mathbf{h}_{1}: (\Delta_{9}, \mathbf{p}_{11}), \mathbf{F}_{7} \vdash \mathbf{F}_{10} \\ & \underbrace{ \begin{array}{c} \bullet \mathbf{h}_{1}: (\Delta_{9}, \mathbf{p}_{11}), \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{F}_{10} \\ & -: (\Delta_{9}, \mathbf{p}_{11}), \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{p}_{11} \\ & -: \Delta_{9}, \mathbf{p}_{11}, \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{p}_{11} \end{array} \begin{array}{c} I \\ \\ \bullet \mathbf{h}_{8}: ((\Delta_{9}, \mathbf{p}_{11}), \mathbf{F}_{6} \to \mathbf{F}_{7}), \mathbf{F}_{10} \vdash \mathbf{p}_{10} \\ & -: \Delta_{9}, \mathbf{p}_{11}, \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{p}_{11} \end{array} \begin{array}{c} I \\ \\ \bullet \mathbf{h}_{1}: \Delta_{9}, \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{F}_{6} \quad \mathbf{h}_{1}: \Delta_{9}, \mathbf{F}_{7} \vdash \mathbf{p}_{10} \\ & -: \Delta_{9}, \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{p}_{10} \\ & -: \Delta_{9}, \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{p}_{10} \\ & -: \Delta_{9}, \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{p}_{10} \\ & -: \Delta_{9}, \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{p}_{10} \end{array} \begin{array}{c} I \\ \text{Cut} \\ \\ \bullet \mathbf{h}_{8}: (\Delta_{9}, \mathbf{F}_{6} \to \mathbf{F}_{7}), \mathbf{p}_{10} \vdash \mathbf{p}_{10} \\ & -: \Delta_{9}, \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{p}_{10} \\ & -: \Delta_{9}, \mathbf{F}_{6} \to \mathbf{F}_{7} \vdash \mathbf{p}_{10} \end{array} \begin{array}{c} I \\ \text{Cut} \\ \end{array}$$

• Case rule \top_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{10},F_{6}\to F_{7}\vdash F_{6}\quad h_{1}:\Delta_{10},F_{7}\vdash \top}{\bullet h_{1}:\Delta_{10},F_{6}\to F_{7}\vdash F_{9}} \to L \quad \frac{\mathbf{h}_{8}:\Delta_{10},F_{6}\to F_{7}\vdash F_{9}}{\bullet h_{8}:(\Delta_{10},F_{6}\to F_{7}),\top\vdash F_{9}} \\ \frac{\bullet h_{1}:\Delta_{10},F_{6}\to F_{7}\vdash F_{9}}{-:\Delta_{10},F_{6}\to F_{7}\vdash F_{9}} \quad \text{ax/W} \\ \\ \frac{h_{1}:(\top,\Delta_{10}),F_{6}\to F_{7}\vdash F_{6}\quad h_{1}:(\top,\Delta_{10}),F_{7}\vdash F_{11}}{\bullet h_{8}:((\top,\Delta_{10}),F_{6}\to F_{7}\vdash F_{9})} \\ \frac{\bullet h_{1}:(\top,\Delta_{10}),F_{6}\to F_{7}\vdash F_{11}}{-:(\top,\Delta_{10}),F_{6}\to F_{7}\vdash F_{9}} \\ \frac{\bullet h_{1}:(\top,\Delta_{10},F_{6}\to F_{7}\vdash F_{11}) \quad \text{ax/W}}{\bullet h_{8}:\top,\Delta_{10},F_{11},F_{6}\to F_{7}\vdash F_{9}} \\ \frac{\bullet h_{1}:(\top,\Delta_{10},F_{6}\to F_{7}\vdash F_{11}) \quad \text{ax/W}}{-:(\top,\Delta_{10},F_{6}\to F_{7}\vdash F_{9})} \quad \text{ax/W}}{h_{Cut}} \end{array}$$

6.7 Status of \wedge_L : OK

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_9, \mathbf{F}_6, \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_{10}} \ \land_L & \frac{}{\bullet \mathbf{h}_8: (\Delta_9, \mathbf{F}_6 \land \mathbf{F}_7), \mathbf{F}_{10} \vdash \top} \\ -: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \top \\ & \xrightarrow{} \\ -: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \top \end{array} \top_{\mathit{R}} \end{array} \subset \mathsf{Cut}$$

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_{11}, \mathbf{F}_6, \mathbf{F}_7 \vdash \mathbf{F}_{12}}{\bullet \mathbf{h}_1:\Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_{12}} \land_L & \frac{\mathbf{h}_8:\Delta_{11}, \mathbf{F}_9, \mathbf{F}_{12}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8:(\Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7), \mathbf{F}_{12} \vdash \mathbf{F}_9 \to \mathbf{F}_{10}} & \mathsf{Cut} \\ \hline -:\Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_9 \to \mathbf{F}_{10} & \\ \hline \bullet \mathbf{h}_1:\Delta_{11}, \mathbf{F}_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_{12} & \mathsf{ax/W} & \\ \hline -:\Delta_{11}, \mathbf{F}_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_{10} & \\ \hline -:\Delta_{11}, \mathbf{F}_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_{10} & \\ -:\Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_9 \to \mathbf{F}_{10} & \\ \hline \end{array} & \uparrow_R \\ \end{array}$$

• Case rule \wedge_R

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{7}\vdash\mathbf{F}_{12}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\wedge\mathbf{F}_{7}\vdash\mathbf{F}_{12}}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\wedge\mathbf{F}_{7}\vdash\mathbf{F}_{12}}} \wedge_{L} \frac{\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6}\wedge\mathbf{F}_{7}\vdash\mathbf{F}_{9}\quad\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6}\wedge\mathbf{F}_{7}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{8}:(\Delta_{11},\mathbf{F}_{6}\wedge\mathbf{F}_{7}),\mathbf{F}_{12}\vdash\mathbf{F}_{9}\wedge\mathbf{F}_{10}} \text{ Cut}}{\frac{-:\Delta_{11},\mathbf{F}_{6}\wedge\mathbf{F}_{7}\vdash\mathbf{F}_{9}\wedge\mathbf{F}_{10}}{\bullet\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6},\mathbf{F}_{7}\vdash\mathbf{F}_{9}} \text{ inv-th/ax } \frac{-:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{7}\vdash\mathbf{F}_{9}\wedge\mathbf{F}_{10}}{\bullet\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6},\mathbf{F}_{7}\vdash\mathbf{F}_{9}\wedge\mathbf{F}_{10}} \text{ hCut}}{\frac{-:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{7}\vdash\mathbf{F}_{9}\wedge\mathbf{F}_{10}}{-:\Delta_{11},\mathbf{F}_{6}\wedge\mathbf{F}_{7}\vdash\mathbf{F}_{9}\wedge\mathbf{F}_{10}}} \wedge_{L}}$$

• Case rule \vee_1

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6, \mathbf{F}_7 \vdash \mathbf{F}_{12} \\ \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_{12} \end{array} \wedge_L \quad \frac{\mathbf{h}_8 : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8 : (\Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7), \mathbf{F}_{12} \vdash \mathbf{F}_9 \vee \mathbf{F}_{10}} \quad \\ - : \Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_9 \vee \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_{12} \quad \text{ax/W} \quad \frac{\rightarrow}{\mathbf{h}_8 : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_9} \\ \hline - : \Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_9} \quad \vee_1 \\ \hline - : \Delta_{11}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_9 \vee \mathbf{F}_{10} \end{array} \vee_1$$

• Case rule \vee_2

• Case rule \rightarrow_L

$$\frac{\frac{h_{1}:(\Delta_{12},F_{9}\to F_{10}),F_{6},F_{7}\vdash F_{13}}{\bullet h_{1}:(\Delta_{12},F_{9}\to F_{10}),F_{6}\wedge F_{7}\vdash F_{13}}}{\bullet h_{1}:(\Delta_{12},F_{9}\to F_{10}),F_{6}\wedge F_{7}\vdash F_{13}}} \wedge_{L} \frac{h_{8}:\Delta_{12},F_{13},F_{9}\to F_{10},F_{6}\wedge F_{7}\vdash F_{9}}{\bullet h_{8}:((\Delta_{12},F_{9}\to F_{10}),F_{6}\wedge F_{7}),F_{13}\vdash F_{11}}}{\bullet h_{8}:(\Delta_{12},F_{13},F_{6}\to F_{7}\vdash F_{11}}} \wedge_{L} \\ -:(\Delta_{12},F_{9}\to F_{10}),F_{6}\wedge F_{7}\vdash F_{11}} \wedge_{L} \\ \frac{h_{1}:\Delta_{12},F_{6},F_{7},F_{9}\to F_{10}\vdash F_{13}}{\bullet h_{8}:\Delta_{12},F_{13},F_{6},F_{7},F_{9}\to F_{10}\vdash F_{9}}} \text{ inv-th/ax } \frac{h_{8}:\Delta_{12},F_{13},F_{6},F_{7}\vdash F_{11}}}{\bullet h_{8}:\Delta_{12},F_{13},F_{6}\to F_{7}\vdash F_{11}}} \wedge_{L} \\ \frac{-:\Delta_{12},F_{6},F_{7},F_{9}\to F_{10}\vdash F_{11}}{-:\Delta_{12},F_{9}\to F_{10}\to F_{9}} \wedge_{L} \frac{h_{8}:\Delta_{12},F_{13},F_{6},F_{7}\vdash F_{9}\to h_{8}:\Delta_{12},F_{10},F_{6}\wedge F_{7}\vdash F_{11}}}{\bullet h_{8}:\Delta_{12},F_{6}\wedge F_{7}\vdash F_{9}\to h_{8}:\Delta_{12},F_{10},F_{6}\wedge F_{7}\vdash F_{11}}} \wedge_{L} \\ \frac{h_{1}:\Delta_{12},F_{6}\wedge F_{7}\vdash F_{9}\to F_{10}}{\bullet h_{8}:\Delta_{12},F_{6}\wedge F_{7}\vdash F_{9}\to h_{8}:\Delta_{12},F_{10},F_{6}\wedge F_{7}\vdash F_{11}}} \wedge_{L} \\ \frac{-:\Delta_{12},F_{6}\wedge F_{7}\vdash F_{11}}{\bullet h_{8}:\Delta_{12},F_{6}\wedge F_{7}\vdash F_{9}\to F_{10}\vdash F_{9}}} \text{ inv-th/ax } \frac{h_{8}:\Delta_{12},F_{10},F_{6},F_{7}\vdash F_{11}}}{h_{8}:\Delta_{12},F_{10},F_{6},F_{7}\vdash F_{11}}} \wedge_{L} \\ \frac{-:\Delta_{12},F_{6}\wedge F_{7}\vdash F_{11}}{\bullet h_{8}:\Delta_{12},F_{6},F_{7},F_{9}\to F_{10}\vdash F_{9}}} \text{ inv-th/ax } \frac{h_{8}:\Delta_{12},F_{10},F_{6},F_{7}\vdash F_{11}}}{h_{8}:\Delta_{12},F_{10},F_{6},F_{7}\vdash F_{11}}} \wedge_{L} \\ \frac{-:\Delta_{12},F_{6}\wedge F_{7}\vdash F_{11}}{\bullet h_{8}:\Delta_{12},F_{6},F_{7},F_{9}\to F_{10}\vdash F_{9}}} \text{ hCut}}{h_{1}:\Delta_{12},F_{6}\wedge F_{7}\vdash F_{11}}} \wedge_{L}}$$

• Case rule \wedge_L

$$\frac{ \begin{array}{c} \mathbf{h}_1: (\Delta_{12}, F_9 \wedge F_{10}), F_6, F_7 \vdash F_{13} \\ \bullet \mathbf{h}_1: (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash F_{13} \end{array} \wedge_L \begin{array}{c} \mathbf{h}_8: \Delta_{12}, F_9, F_{10}, F_{13}, F_6 \wedge F_7 \vdash F_{11} \\ \bullet \mathbf{h}_8: ((\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7), F_{13} \vdash F_{11} \end{array} \wedge_L \\ \\ -: (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash F_{11} \\ & \xrightarrow{\bullet \mathbf{h}_8: (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7, F_9 \vdash F_{11}} \\ \underline{\mathbf{h}_1: \Delta_{12}, F_6, F_7, F_9 \wedge F_{10} \vdash F_{13}} \begin{array}{c} \bullet \mathbf{h}_8: \Delta_{12}, F_{10}, F_{13}, F_6, F_7, F_9 \vdash F_{11} \\ \bullet \mathbf{h}_8: \Delta_{12}, F_{10}, F_{13}, F_6, F_7, F_9 \wedge F_{10} \vdash F_{11} \\ \hline -: \Delta_{12}, F_6, F_7, F_9 \wedge F_{10} \vdash F_{11} \\ \hline -: \Delta_{12}, F_6 \wedge F_7, F_9 \wedge F_{10} \vdash F_{11} \end{array} \wedge_L \\ \end{array} \wedge_L \\ \bullet \mathbf{h} \mathbf{Cut}$$

$$\begin{array}{c} \frac{h_1:\Delta_{12},F_6,F_7\vdash F_9\wedge F_{10}}{\bullet h_1:\Delta_{12},F_6\wedge F_7\vdash F_9\wedge F_{10}} \wedge_L & \frac{h_8:\Delta_{12},F_9,F_{10},F_6\wedge F_7\vdash F_{11}}{\bullet h_8:(\Delta_{12},F_6\wedge F_7),F_9\wedge F_{10}\vdash F_{11}} & \wedge_L \\ \hline & -:\Delta_{12},F_6\wedge F_7\vdash F_{11} \\ & \xrightarrow{} \\ \frac{\to}{h_8:\Delta_{12},F_{10},F_6,F_7,F_9\vdash F_{11}} & \text{inv-th/ax} \\ \hline & \frac{h_1:\Delta_{12},F_6,F_7\vdash F_9\wedge F_{10}}{-:\Delta_{12},F_6,F_7\vdash F_{11}} \wedge_L & \frac{-:\Delta_{12},F_6,F_7\vdash F_{11}}{-:\Delta_{12},F_6\wedge F_7\vdash F_{11}} \wedge_L \\ \hline & \frac{-:\Delta_{12},F_6\wedge F_7\vdash F_{11}}{-:\Delta_{12},F_6\wedge F_7\vdash F_{11}} \wedge_L \\ \hline & \frac{h_1:\Delta_{10},F_7,F_8\vdash F_{11}}{\bullet h_1:\Delta_{10},F_7\wedge F_8\vdash F_{11}} & \wedge_L & \frac{h_6:\Delta_{10},F_7,F_8,F_{11}\vdash F_9}{\bullet h_6:(\Delta_{10},F_7\wedge F_8),F_{11}\vdash F_9} \wedge_L \\ \hline & \frac{-:\Delta_{10},F_7\wedge F_8\vdash F_9}{\bullet h_6:\Delta_{10},F_{11},F_7,F_8\vdash F_9} & H \\ \hline & \frac{-:\Delta_{10},F_7,F_8\vdash F_9}{-:\Delta_{10},F_7\wedge F_8\vdash F_9} & \wedge_L \\ \hline \end{array}$$

• Case rule \vee_L

$$\frac{\frac{h_1: (\Delta_{12}, F_9 \vee F_{10}), F_6, F_7 \vdash F_{13}}{\bullet h_1: (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash F_{13}}}{\circ h_1: (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash F_{13}}} \wedge_L \frac{h_8: \Delta_{12}, F_9, F_{13}, F_6 \wedge F_7 \vdash F_{11}}{\bullet h_8: ((\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7), F_{13} \vdash F_{11}}} \circ_{h_8: ((\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7), F_{13} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7)} \circ_{h_8: (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7), F_{13} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_{13}, F_6, F_7), F_9 \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_{13}, F_6, F_7), F_9 \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_{13}, F_6, F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_{13}, F_6, F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_{13}, F_6, F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}}} \circ_{h_8: (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}$$

• Case rule \perp_L

$$\begin{array}{c} \frac{h_1:\Delta_{10},F_6,F_7\vdash\bot}{\bullet h_1:\Delta_{10},F_6\land F_7\vdash\bot} \; \land_L \; & \frac{\bullet h_8:(\Delta_{10},F_6\land F_7),\bot\vdash F_9}{\bullet h_8:(\Delta_{10},F_6\land F_7),\bot\vdash F_9} \; \underset{L_L}{ \subset \Delta_{10},F_6\land F_7\vdash F_9} \\ & \frac{\to}{h_1:\Delta_{10},F_6,F_7\vdash\bot} \; \underset{\bullet h_8:\bot,\Delta_{10},F_6,F_7\vdash F_9}{\bullet h_8:\bot,\Delta_{10},F_6,F_7\vdash F_9} \; \underset{hCut}{ \to L } \\ & \frac{-:\Delta_{10},F_6,F_7\vdash F_9}{-:\Delta_{10},F_6\land F_7\vdash F_9} \; \land_L \\ \\ \frac{h_1:(\bot,\Delta_{10}),F_6,F_7\vdash F_{11}}{\bullet h_1:(\bot,\Delta_{10}),F_6\land F_7\vdash F_{11}} \; \land_L \; & \frac{\bullet h_8:((\bot,\Delta_{10}),F_6\land F_7),F_{11}\vdash F_9}{\bullet h_8:((\bot,\Delta_{10}),F_6\land F_7),F_{11}\vdash F_9} \; \underset{\vdash}{ \to L } \\ & \frac{\to}{-:\bot,\Delta_{10},F_6\land F_7\vdash F_9} \; \bot_L \end{array}$$

ullet Case rule I

$$\frac{ \frac{ \mathsf{h}_1 : (\Delta_9, \mathsf{p}_{11}), \mathsf{F}_6, \mathsf{F}_7 \vdash \mathsf{F}_{10} }{ \bullet \mathsf{h}_1 : (\Delta_9, \mathsf{p}_{11}), \mathsf{F}_6 \land \mathsf{F}_7 \vdash \mathsf{F}_{10} } \ \, \wedge_L \ \, \frac{ }{ \bullet \mathsf{h}_8 : ((\Delta_9, \mathsf{p}_{11}), \mathsf{F}_6 \land \mathsf{F}_7), \mathsf{F}_{10} \vdash \mathsf{p}_{11} } }{ - : (\Delta_9, \mathsf{p}_{11}), \mathsf{F}_6 \land \mathsf{F}_7 \vdash \mathsf{p}_{11} } \ \, - : \Delta_9, \mathsf{p}_{11}, \mathsf{F}_6 \land \mathsf{F}_7 \vdash \mathsf{p}_{11} } \ \, I } \ \, \mathcal{L}$$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_9, \mathbf{F}_6, \mathbf{F}_7 \vdash \mathbf{p}_{10}}{\bullet \mathbf{h}_1: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} \land_L & \frac{}{\bullet \mathbf{h}_8: (\Delta_9, \mathbf{F}_6 \land \mathbf{F}_7), \mathbf{p}_{10} \vdash \mathbf{p}_{10}} & I \\ & \frac{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}}{\to} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf{p}_{10} \\ & \frac{}{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf{p}_{10} \\ & \frac{}{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf{f}_{10} \\ & \frac{}{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf{f}_{10} \\ & \frac{}{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf{f}_{10} \\ & \frac{}{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf{f}_{10} \\ & \frac{}{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf{f}_{10} \\ & \frac{}{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf{f}_{10} \\ & \frac{}{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf{f}_{10} \\ & \frac{}{-: \Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{p}_{10}} & \Delta \wedge \mathbf{f}_7 \vdash \mathbf$$

• Case rule \top_L

6.8 Status of \vee_L : OK

• Case rule \top_R

$$\frac{ \mathbf{h}_1 : \Delta_9, \mathbf{F}_6 \vdash \mathbf{F}_{10} \quad \mathbf{h}_1 : \Delta_9, \mathbf{F}_7 \vdash \mathbf{F}_{10} }{ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : \Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \\ - : \Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \top \\ \\ \hline \\ - : \Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \top \end{array} } \begin{array}{c} \top_R \\ \mathsf{Cut} \\ \\ \hline \end{array} }$$

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\vdash\mathbf{F}_{12}\quad\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{7}\vdash\mathbf{F}_{12}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\mathbf{F}_{12}} \vee_{L} \quad \frac{\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{9},\mathbf{F}_{12},\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{8}:(\Delta_{11},\mathbf{F}_{6}\vee\mathbf{F}_{7}),\mathbf{F}_{12}\vdash\mathbf{F}_{9}\to\mathbf{F}_{10}} \\ \frac{-:\Delta_{11},\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\mathbf{F}_{9}\to\mathbf{F}_{10}}{\bullet\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{9},\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\mathbf{F}_{10}} \\ \frac{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{9},\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\mathbf{F}_{12}}{\bullet\mathbf{h}_{2}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{9},\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\mathbf{F}_{10}} \to_{R} \\ \frac{-:\Delta_{11},\mathbf{F}_{9},\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\mathbf{F}_{10}}{-:\Delta_{11},\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\mathbf{F}_{9}\to\mathbf{F}_{10}} \to_{R}$$

• Case rule \wedge_R

$$\frac{ \frac{\mathbf{h}_{1} : \Delta_{11}, \mathbf{F}_{6} \vdash \mathbf{F}_{12} \quad \mathbf{h}_{1} : \Delta_{11}, \mathbf{F}_{7} \vdash \mathbf{F}_{12}}{\bullet \mathbf{h}_{1} : \Delta_{11}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \mathbf{F}_{12}} \quad \vee_{L} \quad \frac{\mathbf{h}_{8} : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \mathbf{F}_{9} \quad \mathbf{h}_{8} : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{8} : (\Delta_{11}, \mathbf{F}_{6} \lor \mathbf{F}_{7}), \mathbf{F}_{12} \vdash \mathbf{F}_{9} \land \mathbf{F}_{10}} \quad \mathbf{Cut} \\ \\ \underline{- : \Delta_{11}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \mathbf{F}_{9}} \quad \mathbf{ax} / \mathbf{W} \quad \frac{\Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{10}} \quad \mathbf{Ax} / \mathbf{W} \quad \mathbf{h}_{8} : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \mathbf{F}_{10}} \quad \mathbf{Ax} / \mathbf{W} \quad \mathbf{h}_{11} : \Delta_{11}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \mathbf{F}_{12} \quad \mathbf{ax} / \mathbf{W} \quad \mathbf{h}_{12} : \Delta_{11}, \mathbf{F}_{12}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \mathbf{F}_{10}} \\ \underline{- : \Delta_{11}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \mathbf{F}_{9} \land \mathbf{F}_{10}} \quad \wedge_{R} \quad \mathbf{h}_{11} : \Delta_{11}, \mathbf{F}_{12} \lor \mathbf{F}_{11} : \Delta_{11}, \mathbf{F}_{12} \lor \mathbf{F}_{12} : \Delta_{11}, \mathbf{F}_{12} : \Delta_{11}, \mathbf{F}_{12} \lor \mathbf{F}_{12} : \Delta_{11}, \mathbf{F}$$

• Case rule \vee_1

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\vdash\mathbf{F}_{12}\quad\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{7}\vdash\mathbf{F}_{12}}{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{12}} \\ & \underbrace{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{12}}_{\bullet\mathbf{h}_{3}:(\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}),\mathbf{F}_{12}\vdash\mathbf{F}_{9}\lor\mathbf{F}_{10}}_{\bullet\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}\lor\mathbf{F}_{10}} \\ & \underbrace{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{12}}_{\bullet\mathbf{h}_{1}:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{12}} \underbrace{\bullet\mathbf{x}/\mathbb{W}}_{\mathbf{h}_{8}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}}_{\bullet\mathbf{h}_{2}:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}} \underbrace{\bullet\mathbf{x}/\mathbb{W}}_{\mathbf{h}_{Cut}} \\ & \underbrace{-:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}}_{-:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}\lor\mathbf{F}_{10}}^{\bullet\mathbf{h}_{2}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}} \\ & \underbrace{-:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}\lor\mathbf{F}_{10}}_{\bullet\mathbf{h}_{2}:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}\lor\mathbf{F}_{10}}^{\bullet\mathbf{h}_{2}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}} \\ & \underbrace{-:\Delta_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}\lor\mathbf{F}_{10}}_{\bullet\mathbf{h}_{2}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}}_{\bullet\mathbf{h}_{21}}^{\bullet\mathbf{h}_{21}} \underbrace{\bullet\mathbf{h}_{21},\mathbf{h}_{21}}_{\bullet\mathbf{h}_{21}}^{\bullet\mathbf{h$$

• Case rule \vee_2

$$\frac{\mathbf{h}_1 : \Delta_{11}, F_6 \vdash F_{12} \quad \mathbf{h}_1 : \Delta_{11}, F_7 \vdash F_{12}}{\bullet \mathbf{h}_1 : \Delta_{11}, F_6 \lor F_7 \vdash F_{12}} \quad \bigvee_L \quad \frac{\mathbf{h}_8 : \Delta_{11}, F_{12}, F_6 \lor F_7 \vdash F_{10}}{\bullet \mathbf{h}_8 : (\Delta_{11}, F_6 \lor F_7), F_{12} \vdash F_9 \lor F_{10}} \quad \bigvee_{-: \Delta_{11}, F_6 \lor F_7 \vdash F_9 \lor F_{10}} \quad Cut \\ \frac{-: \Delta_{11}, F_6 \lor F_7 \vdash F_{10}}{\bullet \mathbf{h}_1 : \Delta_{11}, F_6 \lor F_7 \vdash F_{12}} \quad \text{ax/W} \quad \frac{-: \Delta_{11}, F_6 \lor F_7 \vdash F_{10}}{-: \Delta_{11}, F_6 \lor F_7 \vdash F_{10}} \quad \bigvee_{2} \quad \text{ax/W} \quad hCut}$$

• Case rule \rightarrow_L

$$\frac{\mathsf{h}_1: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}), \mathsf{F}_6 \vdash \mathsf{F}_{13} \quad \mathsf{h}_1: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}), \mathsf{F}_7 \vdash \mathsf{F}_{13}}{\bullet \mathsf{h}_1: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}), \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{13}} \quad \mathsf{v}_L \quad \frac{\mathsf{h}_8: \Delta_{12}, \mathsf{F}_{13}, \mathsf{F}_9 \to \mathsf{F}_{10}, \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_9}{\bullet \mathsf{h}_8: ((\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}), \mathsf{F}_6 \lor \mathsf{F}_7), \mathsf{F}_{13} \vdash \mathsf{F}_{13}} \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}), \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11} \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}), \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11} \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}), \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11} \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}), \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11} \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}, \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11}) \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}, \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11}) \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}, \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11}) \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}, \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11}) \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}, \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11}) \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}) \\ -: (\Delta_{12}, \mathsf{F}_9 \to \mathsf{F}_{10}) \\ -: (\Delta_{12}, \mathsf{F}_6 \lor \mathsf{F}_7 \vdash \mathsf{F}_{11}) \\ -: (\Delta_{12}, \mathsf{F}_6$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_{1}: (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \vdash F_{13} \quad \mathbf{h}_{1}: (\Delta_{12}, F_{9} \wedge F_{10}), F_{7} \vdash F_{13}}{\bullet \mathbf{h}_{1}: (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \vee F_{7} \vdash F_{13}} \quad \vee_{L} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{9}, F_{10}, F_{13}, F_{6} \vee F_{7} \vdash F_{11}}{\bullet \mathbf{h}_{8}: (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \vee F_{7}), F_{13} \vdash F_{11}} \quad \wedge_{L} \quad \text{Cut}} \\ - : (\Delta_{12}, F_{9} \wedge F_{10}), F_{6} \vee F_{7} \vdash F_{11}} \\ \hline \frac{\mathbf{h}_{1}: \Delta_{12}, F_{10}, F_{6}, F_{9} \vdash F_{13}}{\bullet \mathbf{h}_{1}: \Delta_{12}, F_{10}, F_{7}, F_{9} \vdash F_{13}} \quad \text{inv-th/ax}}{\bullet \mathbf{h}_{1}: \Delta_{12}, F_{10}, F_{9}, F_{6} \vee F_{7} \vdash F_{11}} \quad \wedge_{L} \\ \hline \frac{\mathbf{h}_{1}: \Delta_{12}, F_{10}, F_{9}, F_{6} \vee F_{7} \vdash F_{13}}{\bullet \mathbf{h}_{1}: \Delta_{12}, F_{10}, F_{9}, F_{9} \wedge F_{10}} \quad \nabla_{L} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{10}, F_{13}, F_{9}, F_{6} \vee F_{7} \vdash F_{11}}{\bullet \mathbf{h}_{1}: \Delta_{12}, F_{6} \vee F_{7} \vdash F_{9} \wedge F_{10}} \quad \nabla_{L} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{9}, F_{10}, F_{6} \vee F_{7} \vdash F_{11}}{\bullet \mathbf{h}_{8}: (\Delta_{12}, F_{6} \vee F_{7}), F_{9} \wedge F_{10} \vdash F_{11}} \quad \wedge_{L} \\ \hline \mathbf{h}_{1}: \Delta_{12}, F_{6} \vdash F_{9} \wedge F_{10} \quad \mathbf{h}_{1}: \Delta_{12}, F_{7} \vdash F_{9} \wedge F_{10}} \quad \nabla_{L} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{9}, F_{10}, F_{6} \vee F_{7} \vdash F_{11}}{\bullet \mathbf{h}_{8}: (\Delta_{12}, F_{6} \vee F_{7}), F_{9} \wedge F_{10} \vdash F_{11}} \quad \wedge_{L} \\ \hline \mathbf{h}_{1}: \Delta_{12}, F_{6} \vdash F_{9} \wedge F_{10} \quad \mathbf{h}_{1}: \Delta_{12}, F_{7} \vdash F_{9} \wedge F_{10}} \quad \mathbf{h}_{Cut} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{10}, F_{10}, F_{10} \vdash F_{11}}{\bullet \mathbf{h}_{8}: \Delta_{12}, F_{10}, F_{10}, F_{10} \vdash F_{11}} \quad \wedge_{L} \\ \hline \mathbf{h}_{1}: \Delta_{12}, F_{6} \vdash F_{9} \wedge F_{10} \quad \mathbf{h}_{1}: \Delta_{12}, F_{6} \vee F_{7} \vdash F_{11}} \quad \mathbf{h}_{Cut} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{10}, F_{7}, F_{9} \vdash F_{11}}{\bullet \mathbf{h}_{1}: \Delta_{12}, F_{7} \vdash F_{9} \wedge F_{10}} \quad \mathbf{h}_{Cut} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{10}, F_{7}, F_{9} \vdash F_{11}}{\bullet \mathbf{h}_{Cut}} \quad \mathbf{h}_{Cut} \quad \mathbf{h}$$

• Case rule \vee_L

$$\frac{\mathbf{h}_{1}: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \vdash F_{13} \quad \mathbf{h}_{1}: (\Delta_{12}, F_{9} \vee F_{10}), F_{7} \vdash F_{13}}{\mathbf{e}\mathbf{h}_{1}: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \vee F_{7} \vdash F_{13}} \quad \mathbf{v}_{L} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{9}, F_{13}, F_{6} \vee F_{7} \vdash F_{11} \quad \mathbf{h}_{8}: \Delta_{12}, F_{10}, F_{10}, F_{10}}{\mathbf{h}_{1}: \Delta_{12}, F_{9} \vee F_{10}), F_{6} \vee F_{7} \vdash F_{11}} \quad \mathbf{v}_{L} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{13}, F_{6}, F_{9} \vdash F_{11}}{\mathbf{h}_{1}: \Delta_{12}, F_{6}, F_{9} \vee F_{10} \vdash F_{11}} \quad \mathbf{hout} \quad \mathbf{v}_{L} \quad \mathbf{h}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash F_{13}} \quad \mathbf{ax/W} \quad \frac{\mathbf{h}_{8}: \Delta_{12}, F_{13}, F_{6}, F_{9} \vee F_{10} \vdash F_{11}}{\mathbf{h}_{1}: \Delta_{12}, F_{6}, F_{9} \vee F_{10} \vdash F_{11}} \quad \mathbf{hout} \quad \mathbf{v}_{L} \quad \mathbf{h}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash F_{13}} \quad \mathbf{ax/W} \quad \mathbf{h}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash F_{11}} \quad \mathbf{hout} \quad \mathbf{v}_{L} \quad \mathbf{h}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash F_{11}} \quad \mathbf{hout} \quad \mathbf{v}_{L} \quad \mathbf{h}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash F_{11}} \quad \mathbf{hout} \quad \mathbf{v}_{L} \quad \mathbf{h}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash F_{11}} \quad \mathbf{hout} \quad \mathbf{v}_{L} \quad \mathbf{h}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash F_{11}} \quad \mathbf{hout} \quad \mathbf{v}_{L} \quad \mathbf{hout} \quad$$

• Case rule \perp_L

$$\frac{\frac{h_1:\Delta_{10},F_6\vdash\bot\ h_1:\Delta_{10},F_7\vdash\bot}{\bullet h_1:\Delta_{10},F_6\lor F_7\vdash\bot}\ \lor_L}{\frac{\bullet h_1:\Delta_{10},F_6\lor F_7\vdash\bot}{\bullet h_8:(\Delta_{10},F_6\lor F_7),\bot\vdash F_9}} \xrightarrow{\bot_L}_{Cut}}{\frac{-:\Delta_{10},F_6\lor F_7\vdash F_9}{\bullet h_8:\bot,\Delta_{10},F_6\vdash F_9}} \xrightarrow{\bot_L}_{hCut} \xrightarrow{\bullet h_1:\Delta_{10},F_7\vdash\bot} \frac{ax/W}{\bullet h_8:\bot,\Delta_{10},F_7\vdash F_9} \xrightarrow{\bot_L}_{hCut}}_{-:\Delta_{10},F_7\vdash F_9} \lor_L$$

• Case rule I

$$\frac{ \frac{\mathbf{h}_1 : (\Delta_9, \mathbf{p}_{11}), \mathbf{F}_6 \vdash \mathbf{F}_{10} \quad \mathbf{h}_1 : (\Delta_9, \mathbf{p}_{11}), \mathbf{F}_7 \vdash \mathbf{F}_{10} }{ \bullet \mathbf{h}_1 : (\Delta_9, \mathbf{p}_{11}), \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \mathbf{F}_{10} } } \vee_L \quad \frac{\bullet \mathbf{h}_8 : ((\Delta_9, \mathbf{p}_{11}), \mathbf{F}_6 \lor \mathbf{F}_7), \mathbf{F}_{10} \vdash \mathbf{p}_{11} }{ - : (\Delta_9, \mathbf{p}_{11}), \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \mathbf{p}_{11} } \quad \mathbf{Cut} } \\ \frac{- : (\Delta_9, \mathbf{p}_{11}), \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \mathbf{p}_{11} }{ - : \Delta_9, \mathbf{p}_{11}, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \mathbf{p}_{11} } \quad I \\ \\ \frac{\mathbf{h}_1 : \Delta_9, \mathbf{F}_6 \vdash \mathbf{p}_{10} \quad \mathbf{h}_1 : \Delta_9, \mathbf{F}_7 \vdash \mathbf{p}_{10} }{ \bullet \mathbf{h}_8 : (\Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7), \mathbf{p}_{10} \vdash \mathbf{p}_{10} } \quad \mathbf{Cut} \\ \frac{- : \Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \mathbf{p}_{10} }{ - : \Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \mathbf{p}_{10} } \quad \mathbf{ax/W} \\ \hline \\ \frac{- : \Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \mathbf{p}_{10} }{ - : \Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \mathbf{p}_{10} } \quad \mathbf{ax/W} \\ \hline \end{array}$$

• Case rule \top_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{10},\mathbf{F}_{6}\vdash\top\ \mathbf{h}_{1}:\Delta_{10},\mathbf{F}_{7}\vdash\top\ }{\bullet\mathbf{h}_{1}:\Delta_{10},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\top\ } \vee_{L} \quad \frac{\mathbf{h}_{8}:\Delta_{10},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{8}:(\Delta_{10},\mathbf{F}_{6}\lor\mathbf{F}_{7}),\top\vdash\mathbf{F}_{9}} \quad \top_{L} \\ \hline -:\Delta_{10},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9} \\ \hline -:\Delta_{10},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9} \quad \mathbf{ax}/\mathsf{W} \\ \\ \frac{\mathbf{h}_{1}:(\top,\Delta_{10}),\mathbf{F}_{6}\vdash\mathbf{F}_{11} \quad \mathbf{h}_{1}:(\top,\Delta_{10}),\mathbf{F}_{7}\vdash\mathbf{F}_{11}}{\bullet\mathbf{h}_{1}:(\top,\Delta_{10}),\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{11}} \vee_{L} \quad \frac{\mathbf{h}_{8}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{8}:((\top,\Delta_{10}),\mathbf{F}_{6}\lor\mathbf{F}_{7}),\mathbf{F}_{11}\vdash\mathbf{F}_{9}} \quad \top_{L} \\ \hline -:(\top,\Delta_{10}),\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9} \\ \hline \bullet_{1}:(\top,\Delta_{10},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{11} \quad \mathbf{ax}/\mathsf{W} \quad \frac{\rightarrow}{\mathbf{h}_{8}:\top,\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9}} \quad \mathbf{ax}/\mathsf{W} \\ \hline -:\top,\Delta_{10},\mathbf{F}_{6}\lor\mathbf{F}_{7}\vdash\mathbf{F}_{9} \quad \mathbf{ax}/\mathsf{W} \quad \mathbf{hCut} \\ \end{array}$$

6.9 Status of \perp_L : OK

• Case rule \top_R

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \bot, \Delta_5 \vdash \mathbf{F}_6 & \bot_L & \hline \bullet_{\mathbf{h}_4}: (\bot, \Delta_5), \mathbf{F}_6 \vdash \top & \top_R \\ \hline -: \bot, \Delta_5 \vdash \top & \\ \hline -: \bot, \Delta_5 \vdash \top & \top_R \\ \hline \end{array}$$
 Cut

• Case rule \rightarrow_R

$$\frac{ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : \bot, \Delta_7 \vdash \mathbf{F}_8 \end{array} \bot_L \quad \begin{array}{c} \mathbf{h}_4 : \bot, \Delta_7, \mathbf{F}_5, \mathbf{F}_8 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_4 : (\bot, \Delta_7), \mathbf{F}_8 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \\ \hline - : \bot, \Delta_7 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \\ \hline - : \bot, \Delta_7 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \ \Delta_L}{\mathbf{Cut}}$$

• Case rule \wedge_R

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \bot, \Delta_7 \vdash F_8 \end{array}}_{} \ \bot_L \ \begin{array}{c} \underline{ \begin{array}{c} \mathbf{h}_4 : \bot, \Delta_7, F_8 \vdash F_5 \\ \bullet \mathbf{h}_4 : (\bot, \Delta_7), F_8 \vdash F_5 \land F_6 \end{array}}_{} \ Cut \\ - : \bot, \Delta_7 \vdash F_5 \land F_6 \\ \hline - : \bot, \Delta_7 \vdash F_5 \land F_6 \end{array} \right. \bot_L \\ \end{array}$$

• Case rule \vee_1

$$\frac{\underbrace{\begin{array}{l} \bullet\mathbf{h}_1:\bot,\Delta_7\vdash \mathbf{F}_8 \\ -:\bot,\Delta_7\vdash \mathbf{F}_8 \end{array} \bot_L \quad \frac{\mathbf{h}_4:\bot,\Delta_7,\mathbf{F}_8\vdash \mathbf{F}_5}{\bullet\mathbf{h}_4:(\bot,\Delta_7),\mathbf{F}_8\vdash \mathbf{F}_5\vee \mathbf{F}_6} \quad \mathbf{Cut}}_{-:\bot,\Delta_7\vdash \mathbf{F}_5\vee \mathbf{F}_6} \quad \bot_L \\ \end{array}}$$

• Case rule \vee_2

$$\frac{\underbrace{\begin{array}{l} \bullet\mathbf{h}_1:\bot,\Delta_7\vdash\mathbf{F}_8 \\ \bullet \mathbf{h}_4:\bot,\Delta_7,\mathbf{F}_8\vdash\mathbf{F}_6 \\ -:\bot,\Delta_7\vdash\mathbf{F}_5\lor\mathbf{F}_6 \\ \hline -:\bot,\Delta_7\vdash\mathbf{F}_5\lor\mathbf{F}_6 \\ \hline -:\bot,\Delta_7\vdash\mathbf{F}_5\lor\mathbf{F}_6 \end{array}}_{-:\bot,\Delta_7\vdash\mathbf{F}_5\lor\mathbf{F}_6} \begin{array}{l} \lor_2 \\ \mathsf{Cut} \end{array}}$$

• Case rule \rightarrow_L

$$\begin{array}{c} \underbrace{\bullet \mathbf{h}_1: \bot, \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_5 \quad \mathbf{h}_4: \bot, \Delta_8, \mathbf{F}_6, \mathbf{F}_9 \vdash \mathbf{F}_7}_{ \bullet \mathbf{h}_4: \bot, \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_5} \quad \mathbf{h}_4: \bot, \Delta_8, \mathbf{F}_6, \mathbf{F}_9 \vdash \mathbf{F}_7}_{\bullet \mathbf{h}_4: \bot, \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7} \\ & -: \bot, \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7}_{\bullet -: \bot, \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7} \quad \bot_L \\ \\ \underbrace{\bullet \mathbf{h}_1: \bot, \Delta_8 \vdash \mathbf{F}_5 \to \mathbf{F}_6}_{\bullet \mathbf{h}_4: \bot, \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_5} \quad \mathbf{h}_4: \bot, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7}_{\bullet \mathbf{h}_4: \bot, \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7} \quad Cut \\ \hline -: \bot, \Delta_8 \vdash \mathbf{F}_7}_{\bullet -: \bot, \Delta_8 \vdash \mathbf{F}_7} \quad \bot_L \end{array}$$

• Case rule \wedge_L

$$\begin{array}{c|c} \underline{\bullet \mathbf{h}_1: \bot, \Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_9} & \bot_L & \frac{\mathbf{h}_4: \bot, \Delta_8, \mathbf{F}_5, \mathbf{F}_6, \mathbf{F}_9 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4: (\bot, \Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6), \mathbf{F}_9 \vdash \mathbf{F}_7} & \land_L \\ \hline & -: \bot, \Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 & \\ \hline & -: \bot, \Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 & \bot_L \\ \hline \\ \underline{\bullet \mathbf{h}_1: \bot, \Delta_8 \vdash \mathbf{F}_5 \wedge \mathbf{F}_6} & \bot_L & \frac{\mathbf{h}_4: \bot, \Delta_8, \mathbf{F}_5, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4: (\bot, \Delta_8), \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7} & \land_L \\ \hline & -: \bot, \Delta_8 \vdash \mathbf{F}_7 & \bot_L \\ \hline & -: \bot, \Delta_8 \vdash \mathbf{F}_7 & \bot_L \end{array}$$

• Case rule \vee_L

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \bot, \Delta_8, F_5 \vee F_6 \vdash F_9 \\ \bullet h_4 : \bot, \Delta_8, F_5 \vee F_6), F_9 \vdash F_7 \\ \bullet h_4 : (\bot, \Delta_8, F_5 \vee F_6), F_9 \vdash F_7 \\ \bullet \\ - : \bot, \Delta_8, F_5 \vee F_6 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8, F_5 \vee F_6 \vdash F_7 \\ \hline \\ \bullet h_4 : \bot, \Delta_8, F_5 \vee F_6 \vdash F_7 \\ \hline \\ \bullet h_4 : \bot, \Delta_8, F_6 \vdash F_7 \\ \hline \\ \bullet h_4 : \bot, \Delta_8, F_5 \vee F_6 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ \end{array} \begin{array}{c} \bullet_{h_4} : \bot, \Delta_8, F_5 \vee F_6 \vdash F_7 \\ \bullet h_4 : \bot, \Delta_8, F_6 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \end{array} \begin{array}{c} \bullet_{h_4} : \bot, \Delta_8, F_6 \vdash F_7 \\ \bullet \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \end{array} \begin{array}{c} \bullet_{h_4} : \bot, \Delta_8, F_6 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \end{array} \begin{array}{c} \bullet_{h_4} : \bot, \Delta_8, F_6 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \end{array} \begin{array}{c} \bullet_{h_4} : \bot, \Delta_8, F_6 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \end{array} \begin{array}{c} \bullet_{h_4} : \bot, \Delta_8, F_6 \vdash F_7 \\ \hline \\ - : \bot, \Delta_8 \vdash F_7 \\ \hline \end{array}$$

• Case rule \perp_L

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \bot, \Delta_6 \vdash \mathsf{F}_7 & \bot_L & \hline \bullet_{\mathbf{h}_4}: (\bot, \Delta_6), \mathsf{F}_7 \vdash \mathsf{F}_5 \\ \hline -: \bot, \Delta_6 \vdash \mathsf{F}_5 \\ \hline -: \bot, \Delta_6 \vdash \mathsf{F}_5 & \bot_L \\ \end{array}$$
 Cut

ullet Case rule I

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \bot, \Delta_5, \mathbf{p}_7 \vdash \mathbf{F}_6 & \bot_L & \hline \bullet_{\mathbf{h}_4}: (\bot, \Delta_5, \mathbf{p}_7), \mathbf{F}_6 \vdash \mathbf{p}_7 \\ \hline & -: \bot, \Delta_5, \mathbf{p}_7 \vdash \mathbf{p}_7 \\ \hline & -: \bot, \Delta_5, \mathbf{p}_7 \vdash \mathbf{p}_7 \\ \hline & -: \bot, \Delta_5, \mathbf{p}_7 \vdash \mathbf{p}_7 & \bot_L \\ \hline \hline \bullet_{\mathbf{h}_1}: \bot, \Delta_5 \vdash \mathbf{p}_6 & \bot_L & \hline \bullet_{\mathbf{h}_4}: (\bot, \Delta_5), \mathbf{p}_6 \vdash \mathbf{p}_6 & I \\ \hline & -: \bot, \Delta_5 \vdash \mathbf{p}_6 & \\ \hline & -: \bot, \Delta_5 \vdash \mathbf{p}_6 & \bot_L \\ \hline \hline & -: \bot, \Delta_5 \vdash \mathbf{p}_6 & \bot_L \\ \hline \end{array}$$

• Case rule \top_L

$$\begin{array}{c|c} \hline \bullet \mathbf{h}_1 : \bot, \Delta_6 \vdash \top & \bot_L & \frac{\mathbf{h}_4 : \bot, \Delta_6 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4 : (\bot, \Delta_6), \top \vdash \mathbf{F}_5} & \top_L \\ \hline & -: \bot, \Delta_6 \vdash \mathbf{F}_5 \\ \hline & -: \bot, \Delta_6 \vdash \mathbf{F}_5 & \bot_L \\ \hline \\ \hline \bullet \mathbf{h}_1 : \bot, \top, \Delta_6 \vdash \mathbf{F}_7 & \bot_L & \frac{\mathbf{h}_4 : \bot, \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4 : (\bot, \top, \Delta_6), \mathbf{F}_7 \vdash \mathbf{F}_5} & \top_L \\ \hline & -: \bot, \top, \Delta_6 \vdash \mathbf{F}_5 & \bot_L \\ \hline & -: \bot, \top, \Delta_6 \vdash \mathbf{F}_5 & \bot_L \\ \hline \end{array}$$

6.10 Status of I: OK

• Case rule \top_R

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{\bullet \mathbf{h}_1:\Delta_7,\mathbf{p}_8\vdash\mathbf{p}_8}{\bullet} \ I \ \ \frac{\mathbf{h}_4:\Delta_7,\mathbf{F}_5,\mathbf{p}_8,\mathbf{p}_8\vdash\mathbf{F}_6}{\bullet \mathbf{h}_4:(\Delta_7,\mathbf{p}_8),\mathbf{p}_8\vdash\mathbf{F}_5\to\mathbf{F}_6} \\ -:\Delta_7,\mathbf{p}_8\vdash\mathbf{F}_5\to\mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1:\Delta_7,\mathbf{F}_5,\mathbf{p}_8\vdash\mathbf{p}_8 \ \ \text{ax/W} \ \ \frac{\rightarrow}{\mathbf{h}_4:\Delta_7,\mathbf{F}_5,\mathbf{p}_8,\mathbf{p}_8\vdash\mathbf{F}_6} \\ \hline -:\Delta_7,\mathbf{F}_5,\mathbf{p}_8\vdash\mathbf{F}_6 \ \ -:\Delta_7,\mathbf{p}_8\vdash\mathbf{F}_5\to\mathbf{F}_6 \end{array} \ \begin{array}{c} \rightarrow_R \\ \text{Cut} \\ \hline \bullet \text{hCut} \\ \hline \end{array}$$

• Case rule \wedge_R

$$\frac{\frac{\mathbf{a} + \mathbf{b}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8}{\mathbf{a}} \ I \ \frac{\mathbf{b}_4 : \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \mathbf{F}_5 \quad \mathbf{b}_4 : \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \mathbf{F}_6}{\bullet \mathbf{b}_4 : (\Delta_7, \mathbf{p}_8), \mathbf{p}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \mathbf{Cut}} \\ - : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \\ \frac{\bullet \mathbf{b}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8}{\bullet \mathbf{b}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8} \ \frac{\mathbf{a} \times / \mathbf{w}}{\mathbf{b}_4 : \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \mathbf{F}_5} \quad \frac{\mathbf{a} \times / \mathbf{w}}{\mathbf{b}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8} \ \frac{\mathbf{a} \times / \mathbf{w}}{\mathbf{b}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_6} \quad \frac{\mathbf{a} \times / \mathbf{w}}{\mathbf{b}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_6} \quad \wedge_R \\ - : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \\ - : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \\ \end{pmatrix} \\ \wedge_R$$

• Case rule \vee_1

$$\frac{ \bullet \mathbf{h}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8 }{ - : \Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8 } \begin{array}{c} I & \frac{\mathbf{h}_4 : \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4 : (\Delta_7, \mathbf{p}_8), \mathbf{p}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \\ \hline - : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8 \end{array} \begin{array}{c} I & \xrightarrow{\mathbf{h}_4 : \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \mathbf{F}_5} \\ \hline \mathbf{h}_4 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5} \\ \hline - : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline - : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \vee_2

$$\frac{ \begin{array}{c} \bullet \mathbf{h}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8 \end{array} I \begin{array}{c} \mathbf{h}_4 : \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_4 : (\Delta_7, \mathbf{p}_8), \mathbf{p}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array}}{ \begin{array}{c} -: \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_6 \vee \mathbf{F}_6 \end{array}} \begin{array}{c} \nabla_2 \\ \mathrm{Cut} \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8 \end{array} \begin{array}{c} I \begin{array}{c} \rightarrow \\ \bullet \mathbf{h}_4 : \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \mathbf{F}_6 \end{array}} \begin{array}{c} \Delta_7 / \mathbf{p}_8 + \mathbf{p}_8 \wedge \mathbf{p$$

• Case rule \rightarrow_L

$$\frac{\underbrace{\frac{\bullet h_1 : (\Delta_8, F_5 \to F_6), p_9 \vdash p_9}{\bullet h_4 : ((\Delta_8, F_5 \to F_6), p_9) \vdash F_7}}_{\bullet h_4 : (\Delta_8, F_6, p_9), p_9 \vdash F_7} \cap \underbrace{\frac{\bullet h_1 : \Delta_8, p_9, F_5 \to F_6 \vdash p_9}{\bullet h_4 : (\Delta_8, F_6, F_9), p_9 \vdash F_7}}_{h_4 : \Delta_8, p_9, F_5 \to F_6 \vdash F_5} \cap \underbrace{\frac{\bullet h_1 : \Delta_8, p_9, F_5 \to F_6 \vdash F_5}{\bullet h_4 : \Delta_8, p_9, F_5 \to F_6 \vdash F_5}}_{\bullet h_1 : \Delta_8, p_9, F_5 \to F_6 \vdash F_7} \cap \underbrace{\frac{\bullet h_1 : \Delta_8, F_6, p_9 \vdash p_9}{\bullet h_1 : \Delta_8, F_6, p_9 \vdash F_7}}_{\bullet L} \cap \underbrace{\frac{\bullet h_1 : \Delta_8, F_6, p_9 \vdash p_9}{\bullet h_1 : \Delta_8, F_6, p_9 \vdash F_7}}_{\bullet L} \cap \underbrace{\frac{\bullet h_1 : \Delta_8, F_6, p_9 \vdash p_9}{\bullet h_1 : \Delta_8, F_6, p_9 \vdash F_7}}_{\bullet L} \cap \underbrace{\frac{\bullet h_1 : \Delta_8, F_6, p_9 \vdash p_9}{\bullet h_1 : \Delta_8, F_6, p_9 \vdash F_7}}_{\bullet L} \cap \underbrace{\frac{\bullet h_1 : \Delta_8, F_6, P_9 \vdash F_7}{\bullet h_1 : \Delta_8, F_6, P_9 \vdash F_7}}_{\bullet L}$$

• Case rule \wedge_L

$$\frac{ \begin{array}{c} \bullet_{h_1} : (\Delta_8, F_5, F_6, p_9, p_9 \vdash F_7 \\ \hline \bullet_{h_1} : (\Delta_8, F_5, F_6, p_9, p_9 \vdash F_7 \\ \hline \\ - : (\Delta_8, F_5, \land F_6), p_9 \vdash F_7 \\ \hline \\ \bullet_{h_1} : \Delta_8, F_5, F_6, p_9 \vdash p_9 \end{array} \begin{array}{c} I & \begin{array}{c} h_4 : \Delta_8, F_5, F_6, p_9, p_9 \vdash F_7 \\ \bullet_{h_4} : ((\Delta_8, F_5, F_6), p_9), p_9 \vdash F_7 \\ \hline \\ \bullet_{h_1} : \Delta_8, F_5, F_6, p_9 \vdash p_9 \end{array} \begin{array}{c} I & \\ \hline \\ \bullet_{h_2} : \Delta_8, F_5, F_6, p_9 \vdash F_7 \\ \hline \\ - : \Delta_8, P_9, F_5, \land F_6 \vdash F_7 \end{array} \begin{array}{c} \land_L \\ \\ \land L \end{array} \end{array}$$

• Case rule \vee_L

$$\frac{\underbrace{\bullet \mathbf{h}_{1}: (\Delta_{8}, \mathbf{F}_{5} \vee \mathbf{F}_{6}), \mathbf{p}_{9} \vdash \mathbf{p}_{9}}_{} I \xrightarrow{\mathbf{h}_{4}: \Delta_{8}, \mathbf{F}_{5}, \mathbf{p}_{9}, \mathbf{p}_{9} \vdash \mathbf{F}_{7}}_{\bullet \mathbf{h}_{4}: ((\Delta_{8}, \mathbf{F}_{5} \vee \mathbf{F}_{6}), \mathbf{p}_{9}), \mathbf{p}_{9} \vdash \mathbf{F}_{7}}_{\bullet \mathbf{h}_{4}: ((\Delta_{8}, \mathbf{F}_{5} \vee \mathbf{F}_{6}), \mathbf{p}_{9}), \mathbf{p}_{9} \vdash \mathbf{F}_{7}}_{\bullet \mathbf{h}_{4}: ((\Delta_{8}, \mathbf{F}_{5} \vee \mathbf{F}_{6}), \mathbf{p}_{9}), \mathbf{p}_{9} \vdash \mathbf{F}_{7}}_{\bullet \mathbf{h}_{4}: \Delta_{8}, \mathbf{F}_{5}, \mathbf{p}_{9} \vdash \mathbf{F}_{7}}_{\bullet \mathbf{h}_{4}: \Delta_{8}, \mathbf{F}_{5}, \mathbf{p}_{9}, \mathbf{p}_{9} \vdash \mathbf{F}_{7}}_{\bullet \mathbf{h}_{4}: \Delta_{8}, \mathbf{F}_{6}, \mathbf{p}_{9} \vdash \mathbf{p}_{9}}_{\bullet \mathbf{h}_{1}: \Delta_{8}, \mathbf{F}_{6}, \mathbf{p}_{9} \vdash \mathbf{p}_{9}}_{\bullet \mathbf{h}_{1}: \Delta_{8}, \mathbf{F}_{6}, \mathbf{p}_{9} \vdash \mathbf{F}_{7}}_{\bullet \mathbf{h}_{4}: \Delta_{8}, \mathbf{F}_{6}, \mathbf{p}_{9} \vdash \mathbf{F}_{7}}_{\bullet \mathbf{h}_{7}}_{\bullet \mathbf{h}_{7$$

• Case rule \perp_L

$$\frac{ \underbrace{ \bullet_{\mathbf{h}_1} : (\bot, \Delta_6), \mathbf{p}_7 \vdash \mathbf{p}_7}_{\bullet \mathbf{h}_4 : ((\bot, \Delta_6), \mathbf{p}_7), \mathbf{p}_7 \vdash \mathbf{F}_5} \overset{\bot_L}{-: (\bot, \Delta_6), \mathbf{p}_7 \vdash \mathbf{F}_5} \overset{\bot_L}{-: \bot, \Delta_6, \mathbf{p}_7 \vdash \mathbf{F}_5} \ \bot_L }$$
 Cut

 $\bullet\,$ Case rule I

• Case rule \top_L

$$\frac{\underbrace{\bullet \mathbf{h}_1: (\top, \Delta_6), \mathbf{p}_7 \vdash \mathbf{p}_7}_{} I \quad \frac{\mathbf{h}_4: \Delta_6, \mathbf{p}_7, \mathbf{p}_7 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4: ((\top, \Delta_6), \mathbf{p}_7), \mathbf{p}_7 \vdash \mathbf{F}_5}_{} \quad \frac{\top_L}{\mathsf{Cut}}}_{} \\ -: (\top, \Delta_6), \mathbf{p}_7 \vdash \mathbf{F}_5} \quad \frac{\rightarrow}{\mathbf{h}_4: \top, \Delta_6, \mathbf{p}_7, \mathbf{p}_7 \vdash \mathbf{F}_5}}_{} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}}}_{}$$

6.11 Status of \top_L : OK

• Case rule \top_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \top, \Delta_5 \vdash \mathbf{F}_6} & \top_L & \frac{}{\bullet \mathbf{h}_4: (\top, \Delta_5), \mathbf{F}_6 \vdash \top} & \top_R \\ & -: \top, \Delta_5 \vdash \top & \rightarrow \\ & \frac{}{-: \top, \Delta_5 \vdash \top} & \top_R \end{array}$$
 Cut

• Case rule \rightarrow_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \top, \Delta_7 \vdash \mathbf{F}_8} & \top_L & \frac{\mathbf{h}_4: \top, \Delta_7, \mathbf{F}_5, \mathbf{F}_8 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_4: (\top, \Delta_7), \mathbf{F}_8 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \\ \hline & -: \top, \Delta_7 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \\ \hline & \frac{}{\bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \\ \hline & \frac{}{\bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \\ \hline & -: \top, \Delta_7 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \wedge_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_7 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_1: \top, \Delta_7 \vdash \mathbf{F}_8 \end{array}}{ \begin{array}{c} \bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 & \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_4: (\top, \Delta_7), \mathbf{F}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ \hline -: \top, \Delta_7 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ \hline -: \top, \Delta_7 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \end{array}} \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{hCut} \end{array}$$

• Case rule \vee_1

$$\frac{ \begin{array}{l} \mathbf{h}_1: \Delta_7 \vdash \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1: \top, \Delta_7 \vdash \mathbf{F}_8 \end{array} \top_L \quad \frac{\mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4: (\top, \Delta_7), \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \\ \hline -: \top, \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline \\ \underline{\mathbf{h}_1: \top, \Delta_7 \vdash \mathbf{F}_8} \quad \mathbf{ax/W} \quad \frac{\rightarrow}{\bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \\ -: \top, \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \quad \mathbf{ax/W} \\ \underline{\mathbf{h}_0: \top, \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \quad \mathbf{bCut}$$

• Case rule \vee_2

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_7 \vdash \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1 : \top, \Delta_7 \vdash \mathbf{F}_8 \end{array} \ \top_L \ \begin{array}{c} \mathbf{h}_4 : \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_4 : (\top, \Delta_7), \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline -: \top, \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline \hline \bullet \mathbf{h}_4 : \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline -: \top, \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{v}_2 \\ \mathrm{Cut} \\ \hline \bullet \mathbf{h}_4 : \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline -: \top, \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathrm{hCut} \end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_9}{\bullet\mathbf{h}_1:\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_9} \ \, \top_L \ \, \frac{\mathbf{h}_4:\top,\Delta_8,\mathbf{F}_9,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_5 \quad \mathbf{h}_4:\top,\Delta_8,\mathbf{F}_6,\mathbf{F}_9\vdash\mathbf{F}_7}{\bullet\mathbf{h}_4:(\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6),\mathbf{F}_9\vdash\mathbf{F}_7} \ \, \mathbf{Cut} \\ \\ \frac{-:\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_7}{\bullet\mathbf{h}_1:\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_9} \ \, \mathbf{ax/W} \ \, \frac{\rightarrow}{\bullet\mathbf{h}_4:\top,\Delta_8,\mathbf{F}_9,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_7} \ \, \mathbf{ax/W} \\ -:\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_7} \ \, \mathbf{hCut} \\ \\ \frac{\mathbf{h}_1:\Delta_8\vdash\mathbf{F}_5\rightarrow\mathbf{F}_6}{\bullet\mathbf{h}_1:\top,\Delta_8\vdash\mathbf{F}_5\rightarrow\mathbf{F}_6} \ \, \top_L \ \, \frac{\mathbf{h}_4:\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_5}{\bullet\mathbf{h}_4:(\top,\Delta_8),\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_7} \ \, \mathbf{Cut} \\ \\ \frac{-:\top,\Delta_8\vdash\mathbf{F}_7}{\bullet\mathbf{h}_4:\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_7} \ \, \mathbf{ax/W} \\ \hline \quad \, \frac{\rightarrow}{\bullet\mathbf{h}_4:\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_7} \ \, \mathbf{ax/W} \\ \hline \quad \, \frac{\rightarrow}{\bullet\mathbf{h}_4:\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_7} \ \, \mathbf{ax/W} \\ \hline \quad \, \frac{\rightarrow}{\bullet\mathbf{h}_4:\top,\Delta_8,\mathbf{F}_5\rightarrow\mathbf{F}_6\vdash\mathbf{F}_7} \ \, \mathbf{ax/W} \\ \hline \quad \, -:\top,\Delta_8\vdash\mathbf{F}_7 \ \, \mathbf{ax/W} \\ \hline \quad \, -:\top,\Delta_8\vdash\mathbf{F}_7 \ \, \mathbf{ax/W} \\ \hline \end{array}$$

• Case rule \wedge_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_1:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_9} & \top_L & \frac{\mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5, \mathbf{F}_6, \mathbf{F}_9 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4:(\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6), \mathbf{F}_9 \vdash \mathbf{F}_7} & \wedge_L \\ \hline & -:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \frac{\mathbf{h}_1:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_9, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7} & \mathbf{ax/W} \\ \hline & -:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1:\Delta_8 \vdash \mathbf{F}_5 \wedge \mathbf{F}_6 \\ \hline & \bullet \mathbf{h}_1:\top,\Delta_8 \vdash \mathbf{F}_5 \wedge \mathbf{F}_6 \\ \hline & \bullet \mathbf{h}_1:\top,\Delta_8 \vdash \mathbf{F}_5 \wedge \mathbf{F}_6 \\ \hline & -:\top,\Delta_8 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8 \vdash \mathbf{F}_5 \wedge \mathbf{F}_6 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{h}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \mathbf{h}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{h}_5 \wedge \mathbf{h}_6 \vdash \mathbf{h}_7 \\ \hline & \bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{h}_5 \wedge \mathbf{h}_6 \vdash \mathbf{h}_7 \\ \hline & \bullet \mathbf{h}_4:\top,$$

• Case rule \vee_L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_1 : \top, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_9} & \top_L & \frac{\mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_5, \mathbf{F}_9 \vdash \mathbf{F}_7 & \mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_6, \mathbf{F}_9 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4 : (\top, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6), \mathbf{F}_9 \vdash \mathbf{F}_7} & \mathbf{Cut} \\ & & \xrightarrow{\bullet} \frac{\rightarrow}{\mathbf{h}_1 : \top, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_9} & \mathbf{ax/W} & \xrightarrow{\bullet} \frac{\rightarrow}{\bullet \mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} & \mathbf{ax/W} \\ & & \xrightarrow{\bullet} \frac{\mathbf{h}_1 : \top, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_9} & \mathbf{T}_L & \frac{\mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_5 \vdash \mathbf{F}_7 & \mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1 : \top, \Delta_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} & \top_L & \frac{\mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_5 \vdash \mathbf{F}_7 & \mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4 : (\top, \Delta_8), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} & \mathbf{Cut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} & \mathbf{ax/W} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{ax/W} & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} & \mathbf{ax/W} \\ & \xrightarrow{\bullet} \mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} & \mathbf{ax/W} & \mathbf{hCut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{hCut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{hCut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{hCut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{hCut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{hCut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{hCut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{hCut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{hCut} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{F}_6} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{Ax/W}} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{Ax/W}} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{Ax/W}} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} & \mathbf{Ax/W} \\ & \xrightarrow{\bullet} \frac{-: \top, \Delta_8 \vdash \mathbf{F}_7 \vee \mathbf{Ax/W}} & \mathbf{Ax/W} & \mathbf{Ax/W}$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_6 \vdash \bot}{\bullet \mathbf{h}_1:\top,\Delta_6 \vdash \bot} \; \top_L \quad & \bullet \mathbf{h}_4:(\top,\Delta_6),\bot \vdash \mathbf{F}_5 \\ \hline -:\top,\Delta_6 \vdash \mathbf{F}_5 \\ \hline \frac{\mathbf{h}_1:\top,\Delta_6 \vdash \bot}{\bullet \mathbf{h}_4:\bot,\top,\Delta_6 \vdash \mathbf{F}_5} \; \frac{\bot_L}{\bullet \mathbf{h}_4:\bot,\top,\Delta_6 \vdash \mathbf{F}_5} \; \frac{\bot_L}{\bullet \mathbf{h}_4:\bot,\top,\Delta_6 \vdash \mathbf{F}_5} \\ \hline \bullet \mathbf{h}_1:\bot,\Delta_6 \vdash \mathbf{F}_7 \quad & \bullet \mathbf{h}_4:(\top,\bot,\Delta_6),\mathbf{F}_7 \vdash \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_1:\top,\bot,\Delta_6 \vdash \mathbf{F}_7 \quad & \bullet \mathbf{h}_4:(\top,\bot,\Delta_6),\mathbf{F}_7 \vdash \mathbf{F}_5 \\ \hline -:\top,\bot,\Delta_6 \vdash \mathbf{F}_5 \quad & \bot_L \\ \hline -:\top,\bot,\Delta_6 \vdash \mathbf{F}_5 \quad & \bot_L \\ \hline -:\bot,\top,\Delta_6 \vdash \mathbf{F}_5 \quad & \bot_L \\ \hline \end{array}$$

ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{p}_7 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\top,\Delta_5,\mathbf{p}_7 \vdash \mathbf{F}_6} \quad \top_L \quad \overbrace{\bullet \mathbf{h}_4:(\top,\Delta_5,\mathbf{p}_7),\mathbf{F}_6 \vdash \mathbf{p}_7}^{} \quad I \\ \hline -:\top,\Delta_5,\mathbf{p}_7 \vdash \mathbf{p}_7 \\ \hline -:\top,\Delta_5,\mathbf{p}_7 \vdash \mathbf{p}_7 \quad I \\ \hline \\ \frac{\mathbf{h}_1:\Delta_5 \vdash \mathbf{p}_6}{\bullet \mathbf{h}_1:\top,\Delta_5 \vdash \mathbf{p}_6} \quad \top_L \quad \overbrace{\bullet \mathbf{h}_4:(\top,\Delta_5),\mathbf{p}_6 \vdash \mathbf{p}_6}^{} \quad I \\ \hline -:\top,\Delta_5 \vdash \mathbf{p}_6 \\ \hline -:\top,\Delta_5 \vdash \mathbf{p}_6 \\ \hline -:\top,\Delta_5 \vdash \mathbf{p}_6 \end{array}$$

• Case rule \top_L

$$\frac{ \begin{array}{l} \mathbf{h}_1 : \Delta_6 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1 : \top, \Delta_6 \vdash \mathbf{F}_7 \end{array}}{ \begin{array}{l} \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_4 : (\top, \Delta_6), \mathbf{F}_7 \vdash \mathbf{F}_5 \end{array}} \begin{array}{l} \top_L \\ \hline - : \top, \Delta_6 \vdash \mathbf{F}_5 \\ \hline \\ \mathbf{h}_1 : \top, \Delta_6 \vdash \mathbf{F}_7 \end{array}} \begin{array}{l} \mathbf{ax/W} \\ \hline - : \top, \Delta_6 \vdash \mathbf{F}_5 \end{array}} \begin{array}{l} \mathbf{ax/W} \\ \bullet \mathbf{h}_4 : \top, \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_5 \end{array}} \begin{array}{l} \mathbf{ax/W} \\ \mathsf{hCut} \end{array}$$

7 Cut-Elimination

7.1 Status of \top_R : OK

• Case rule \top_R

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1 : \Delta_2 \vdash \top} & \top_R & \hline \bullet_{\mathbf{h}_3 : \Delta_4, \top \vdash \top} & \top_R \\ \hline - : \Delta_2, \Delta_4 \vdash \top & \\ \hline - : \Delta_2, \Delta_4 \vdash \top & \top_R \\ \hline \hline - : \Delta_2, \Delta_4 \vdash \top & \top_R \end{array}$$

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{\mathbf{h}_3: \top, \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \top} \ \, \frac{\mathbf{h}_3: \Delta_6, \top \vdash \mathbf{F}_4 \to \mathbf{F}_5}{\bullet \mathbf{h}_3: \Delta_6, \top \vdash \mathbf{F}_4 \to \mathbf{F}_5} \\ -: \Delta_2, \Delta_6 \vdash \mathbf{F}_4 \to \mathbf{F}_5 \\ \hline \frac{\bullet \mathbf{h}_1: * \vdash \top}{\bullet} \ \, \frac{\top_R}{\mathbf{h}_3: \top, \Delta_2, \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} \\ \hline \frac{-: \Delta_2, \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{-: \Delta_2, \Delta_6, \mathbf{F}_4 \to \mathbf{F}_5} \ \, \to_R \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \wedge_R

$$\frac{\underbrace{\bullet_{\mathbf{h}_1}:\Delta_2\vdash\top}_{\bullet\mathbf{h}_1:\Delta_2\vdash\top} \top_R \quad \frac{\mathbf{h}_3:\top,\Delta_6\vdash \mathbf{F}_4\quad \mathbf{h}_3:\top,\Delta_6\vdash \mathbf{F}_5}_{\bullet\mathbf{h}_3:\Delta_6,\top\vdash \mathbf{F}_4\land \mathbf{F}_5} \quad \mathbf{Cut}}_{-:\Delta_2,\Delta_6\vdash \mathbf{F}_4\land \mathbf{F}_5} \quad \mathbf{Cut}} \\ \xrightarrow{\bullet_{\mathbf{h}_1}:*\vdash\top} \overset{\top_R}{\xrightarrow{\mathbf{h}_3}:\top,\Delta_2,\Delta_6\vdash \mathbf{F}_4}} \underbrace{\mathbf{ax/W}}_{\mathbf{hCut}} \xrightarrow{\bullet_{\mathbf{h}_1}:*\vdash\top} \overset{\top_R}{\xrightarrow{\mathbf{h}_3}:\top,\Delta_2,\Delta_6\vdash \mathbf{F}_5}}_{-:\Delta_2,\Delta_6\vdash \mathbf{F}_4} \underbrace{\mathbf{ax/W}}_{\mathbf{hCut}} \\ \xrightarrow{-:\Delta_2,\Delta_6\vdash \mathbf{F}_4\land \mathbf{F}_5}}^{\bullet\mathbf{hA}:*\vdash\top} \overset{\wedge}{\xrightarrow{\mathbf{h}_3}:\top,\Delta_2,\Delta_6\vdash \mathbf{F}_5}}_{\land_R} \\ \xrightarrow{\bullet_{\mathbf{h}_1}:*\vdash\top}_{-:\Delta_2,\Delta_6\vdash \mathbf{F}_4\land \mathbf{F}_5}}^{\bullet\mathbf{hCut}}$$

• Case rule \vee_1

$$\begin{array}{c} \frac{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top}{\bullet} \; \top_R \; \frac{\mathbf{h}_3 : \top, \Delta_6 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_3 : \Delta_6, \top \vdash \mathbf{F}_4 \vee \mathbf{F}_5} \; \underset{\mathsf{Cut}}{\leftarrow} \\ \frac{-: \Delta_2, \Delta_6 \vdash \mathbf{F}_4 \vee \mathbf{F}_5}{\to} \; \frac{\rightarrow}{\mathbf{h}_3 : \top, \Delta_2, \Delta_6 \vdash \mathbf{F}_4} \\ \frac{-: \Delta_2, \Delta_6 \vdash \mathbf{F}_4}{-: \Delta_2, \Delta_6 \vdash \mathbf{F}_4 \vee \mathbf{F}_5} \; \vee_1 \end{array} \begin{array}{c} \mathsf{ax/W} \\ \mathsf{hCut} \end{array}$$

• Case rule \vee_2

$$\begin{array}{c|c} & \frac{\mathbf{h}_3: \top, \Delta_6 \vdash F_5}{\bullet \mathbf{h}_3: \Delta_6, \top \vdash F_4 \lor F_5} & \bigvee_2 \\ \hline -: \Delta_2, \Delta_6 \vdash F_4 \lor F_5 \\ \hline -: \Delta_2, \Delta_6 \vdash F_4 \lor F_5 \\ \hline \rightarrow & \\ \hline \frac{\bullet \mathbf{h}_1: * \vdash \top}{\bullet \mathbf{h}_3: \top, \Delta_2, \Delta_6 \vdash F_5} & \underset{\mathsf{hCut}}{\bullet \mathbf{cut}} \\ \hline -: \Delta_2, \Delta_6 \vdash F_5 \\ \hline -: \Delta_2, \Delta_6 \vdash F_4 \lor F_5 & \bigvee_2 \end{array}$$

• Case rule \rightarrow_L

$$\frac{\underbrace{\begin{array}{l} \bullet_{h_1}: \Delta_2 \vdash \top} \quad \top_R \quad \frac{h_3: \top, \Delta_7, F_4 \to F_5 \vdash F_4 \quad h_3: \top, \Delta_7, F_5 \vdash F_6 \\ \bullet h_3: (\Delta_7, F_4 \to F_5), \top \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_4} \quad \underbrace{\begin{array}{l} \bullet_{h_3}: \top, \Delta_7, F_5 \vdash F_6 \\ \bullet h_3: (\Delta_7, F_4 \to F_5), \top \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_4} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \top \quad \top_R \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet h_1: * \vdash \top \quad T_R \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \top \quad \top_R \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet h_1: * \vdash \top \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \top \quad T_R \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet h_1: * \vdash \top \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \top \quad T_R \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet h_1: * \vdash \top \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \top \quad T_R \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet \quad h_1: * \vdash \vdash \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \top \quad T_R \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet \quad h_1: * \vdash \vdash \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \vdash \quad T_R \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet \quad h_1: * \vdash \vdash \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \vdash \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet \quad h_1: * \vdash \vdash \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \vdash \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet \quad h_1: * \vdash \vdash \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \vdash \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \\ \bullet \quad h_1: * \vdash \vdash \quad h_3: \top, \Delta_2, \Delta_7, F_5 \vdash F_6 \end{array}}_{-: \Delta_2, \Delta_7, F_5 \vdash F_6} \quad \underbrace{\begin{array}{l} \bullet_{h_1}: * \vdash \vdash \quad h_3: \Delta_2, \Delta_7, F_5 \vdash \vdash \quad h_3: \Delta_2, \Delta_7, F_5 \vdash \vdash \quad h_3: \Delta_2, \Delta_7, F_5 \vdash \vdash \quad h_3: \Delta_7, \Delta_7, A_7 \vdash$$

• Case rule \wedge_L

$$\begin{array}{c|c} & \frac{\mathbf{h}_3: \top, \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_4 \land \mathbf{F}_5), \top \vdash \mathbf{F}_6} & \wedge_L \\ \hline & -: \Delta_2, \Delta_7, \mathbf{F}_4 \land \mathbf{F}_5 \vdash \mathbf{F}_6} & -: \Delta_2, \Delta_7, \mathbf{F}_4 \land \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: * \vdash \top & \top_R & \xrightarrow{h_3: \top, \Delta_2, \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \mathbf{F}_6} \\ \hline & \frac{-: \Delta_2, \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \mathbf{F}_6}{-: \Delta_2, \Delta_7, \mathbf{F}_4 \land \mathbf{F}_5 \vdash \mathbf{F}_6} & \wedge_L \end{array} \quad \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \vee_L

$$\frac{\underbrace{\begin{array}{c} \bullet_{\mathbf{h}_1:\,\Delta_2\,\vdash\,\top} \\ \bullet_{\mathbf{h}_1:\,\Delta_2\,\vdash\,\top} \end{array}}_{\mathbf{h}_1:\,\Delta_2\,\vdash\,\top} \frac{\mathbf{h}_3:\,\top,\,\Delta_7,\,\mathbf{F}_4\,\vdash\,\mathbf{F}_6 \quad \mathbf{h}_3:\,\top,\,\Delta_7,\,\mathbf{F}_5\,\vdash\,\mathbf{F}_6}{\bullet\,\mathbf{h}_3:\,(\Delta_7,\,\mathbf{F}_4\,\vee\,\mathbf{F}_5),\,\top\,\vdash\,\mathbf{F}_6} \underbrace{\mathbf{Cut}}_{\mathbf{c}} \\ \underbrace{\begin{array}{c} \bullet_{\mathbf{h}_1:\,\ast\,\vdash\,\top} \\ \bullet_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_4\,\vdash\,\mathbf{F}_6} \\ \bullet_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_4\,\vdash\,\mathbf{F}_6} \end{array}}_{\mathbf{h}_{\mathbf{C}}\mathbf{u}} \underbrace{\begin{array}{c} \bullet_{\mathbf{h}_1:\,\ast\,\vdash\,\top} \\ \bullet_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_5\,\vdash\,\mathbf{F}_6} \\ \bullet_{\mathbf{h}_1:\,\ast\,\vdash\,\top} \end{array}}_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_5\,\vdash\,\mathbf{F}_6} \underbrace{\begin{array}{c} \bullet_{\mathbf{x}}/\mathcal{W} \\ \bullet_{\mathbf{h}}/\mathcal{W} \\ \bullet_{\mathbf{h}}/\mathcal{W}} \end{array}}_{\mathbf{h}_{\mathbf{C}}\mathbf{u}} \underbrace{\begin{array}{c} \bullet_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_5\,\vdash\,\mathbf{F}_6} \\ \bullet_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_5\,\vdash\,\mathbf{F}_6} \end{array}}_{\mathbf{h}_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_5\,\vdash\,\mathbf{F}_6}} \underbrace{\begin{array}{c} \bullet_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_5\,\vdash\,\mathbf{F}_6} \\ \bullet_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_5\,\vdash\,\mathbf{F}_6} \end{aligned}}_{\mathbf{h}_{\mathbf{h}_3:\,\top,\,\Delta_2,\,\Delta_7,\,\mathbf{F}_5\,\vdash\,\mathbf{F}_6}} \underbrace{\begin{array}{c} \bullet_{\mathbf{h}_3:$$

• Case rule \perp_L

$$\begin{array}{c|c} \hline { \bullet \mathbf{h}_1 : \Delta_2 \vdash \top} & \top_R & \hline { \bullet \mathbf{h}_3 : (\bot, \Delta_5), \top \vdash \mathbf{F}_4 } \\ \hline {- : \Delta_2, \bot, \Delta_5 \vdash \mathbf{F}_4} \\ \hline { \rightarrow \\ \hline {- : \bot, \Delta_2, \Delta_5 \vdash \mathbf{F}_4 } } & \bot_L \end{array} \quad \mathbf{Cut}$$

ullet Case rule I

$$\begin{array}{c|c} \hline { \bullet \mathbf{h}_1 : \Delta_2 \vdash \top} & \top_R & \hline { \bullet \mathbf{h}_3 : (\Delta_4, \mathbf{p}_5), \top \vdash \mathbf{p}_5} & I \\ \hline { - : \Delta_2, \Delta_4, \mathbf{p}_5 \vdash \mathbf{p}_5} & \\ \hline { - : \Delta_2, \Delta_4, \mathbf{p}_5 \vdash \mathbf{p}_5} & I \\ \hline \end{array}$$
 Cut

• Case rule \top_L

$$\begin{array}{c|c} \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \top}{\bullet} & \top_R & \frac{\mathbf{h}_3: \Delta_5 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_3: \Delta_5, \top \vdash \mathbf{F}_4} & \top_L \\ \hline & -: \Delta_2, \Delta_5 \vdash \mathbf{F}_4 \\ \hline & \rightarrow \\ \hline & -: \Delta_2, \Delta_5 \vdash \mathbf{F}_4 & \mathsf{ax/W} \end{array}$$

7.2 Status of \rightarrow_R : OK

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_5\vdash\mathbf{F}_6}{\bullet\mathbf{h}_1:\Delta_2\vdash\mathbf{F}_5\to\mathbf{F}_6}\to_R & \frac{}{\bullet\mathbf{h}_7:\Delta_8,\mathbf{F}_5\to\mathbf{F}_6\vdash\top} & \top_R \\ \hline & -:\Delta_2,\Delta_8\vdash\top & \to \\ \hline & \frac{}{-:\Delta_2,\Delta_8\vdash\top} & \top_R \end{array}$$
 Cut

• Case rule \rightarrow_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \rightarrow_R \quad \frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8 \to \mathbf{F}_9 \\ \hline -: \Delta_2, \Delta_{10} \vdash \mathbf{F}_8 \to \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \xrightarrow[\mathbf{h}_7: \Delta_{10}, \mathbf{F}_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline -: \Delta_{10}, \Delta_2, \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline -: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \to \mathbf{F}_9 \end{array} \xrightarrow[\mathbf{h}_7: \Delta_{10}, \mathbf{F}_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline -: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \to \mathbf{F}_9 \end{array} \xrightarrow[\mathbf{h}_7: \Delta_{10}, \mathbf{h}_7] \xrightarrow[\mathbf{h}_7: \mathbf{h}_7]} \xrightarrow[\mathbf{h}_7: \mathbf{h}_7: \mathbf{h}_7$$

• Case rule \wedge_R

• Case rule \vee_1

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \rightarrow_R \begin{array}{c} \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8 \vee \mathbf{F}_9 \end{array} \begin{array}{c} \vee_1 \\ \mathsf{Cut} \end{array} \\ \\ \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \begin{array}{c} \mathsf{ax/W} \\ \hline -: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \\ \hline -: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \vee \mathbf{F}_9 \end{array} \begin{array}{c} \vee_1 \\ \mathsf{hCut} \end{array}$$

• Case rule \vee_2

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \xrightarrow{} \mathbf{A}_R & \frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8 \vee \mathbf{F}_9} & \nabla_2 \\ \hline & -: \Delta_2, \Delta_{10} \vdash \mathbf{F}_8 \vee \mathbf{F}_9 \\ \xrightarrow{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6} & \xrightarrow{\mathbf{ax/W}} & \xrightarrow{\bullet} \\ \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6}{\bullet \mathbf{x}_1} & \xrightarrow{\mathbf{ax/W}} & \xrightarrow{\bullet} \\ \hline & -: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_9 \\ \hline & -: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \vee \mathbf{F}_9 & \nabla_2 \\ \end{array} & \xrightarrow{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6} & \mathbf{hCut} \end{array}$$

• Case rule \rightarrow_L

$$\frac{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_5\vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_5\to \mathbf{F}_6}}{\bullet \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_5\to \mathbf{F}_6}} \to_R \frac{\mathbf{h}_7:\Delta_{11},\mathbf{F}_5\to \mathbf{F}_6,\mathbf{F}_8\to \mathbf{F}_9\vdash \mathbf{F}_8}{\bullet \mathbf{h}_7:(\Delta_{11},\mathbf{F}_8\to \mathbf{F}_9),\mathbf{F}_5\to \mathbf{F}_6\vdash \mathbf{F}_{10}}} \subset_{\mathbf{Cut}} \\ -:\Delta_2,\Delta_{11},\mathbf{F}_8\to \mathbf{F}_9\vdash \mathbf{F}_{10}} \subset_{\mathbf{Cut}} \\ -:\Delta_2,\Delta_{11},\mathbf{F}_8\to \mathbf{F}_9\vdash \mathbf{F}_{10}} \\ -:\Delta_1,\Delta_1,\mathbf{F}_8\to \mathbf{F}_9\vdash \mathbf{F}_{10}} \\ -:\Delta_{11},\Delta_2,\mathbf{F}_8\to \mathbf{F}_9\vdash \mathbf{F}_8} \xrightarrow{\mathbf{ax/W}} \frac{\mathbf{h}_7:\Delta_{11},\mathbf{F}_9,\mathbf{F}_5\to \mathbf{F}_6\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_5\to \mathbf{F}_6}} \xrightarrow{\mathbf{ax/W}} \frac{\mathbf{h}_7:\Delta_{11},\mathbf{F}_9,\mathbf{F}_5\to \mathbf{F}_6\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_9\to \mathbf{F}_9\vdash \mathbf{F}_{10}} \to_L \\ -:\Delta_{11},\Delta_2,\mathbf{F}_8\to \mathbf{F}_9\vdash \mathbf{F}_{10} \\ \underbrace{\frac{\mathbf{h}_1:\Delta_2}{\bullet \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_6\to \mathbf{F}_7}}_{\bullet \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_6\to \mathbf{F}_7} \to_R \xrightarrow{\mathbf{h}_5:\Delta_9,\mathbf{F}_6\to \mathbf{F}_7\vdash \mathbf{F}_6}_{\bullet \mathbf{h}_5:\Delta_9,\mathbf{F}_6\to \mathbf{F}_7\vdash \mathbf{F}_8} \subset_{\mathbf{Cut}} \\ -:\Delta_2,\Delta_9\vdash \mathbf{F}_8} \xrightarrow{\mathbf{ax/W}}_{\bullet \mathbf{h}\mathbf{Cut}} \xrightarrow{-:\Delta_2,\Delta_9\vdash \mathbf{F}_8}_{\bullet \mathbf{Cut}} \xrightarrow{\mathbf{ax/W}}_{\bullet \mathbf{Cut}} \xrightarrow{-:\Delta_2,\Delta_9\vdash \mathbf{F}_8}_{\bullet \mathbf{Cut}} \xrightarrow{\mathbf{ax/W}}_{\bullet \mathbf{Cut}} \\ \underbrace{-:\Delta_2,\Delta_9\vdash \mathbf{F}_6}_{\bullet \mathbf{Cut}} \xrightarrow{-:\Delta_2,\Delta_9\vdash \mathbf{F}_8}_{\bullet \mathbf{Cut}} \xrightarrow{\mathbf{ax/W}}_{\bullet \mathbf{Cut}} \xrightarrow{-:\Delta_2,\Delta_9\vdash \mathbf{F}_8}_{\bullet \mathbf{Cut}} \xrightarrow{\mathbf{ax/W}}_{\bullet \mathbf{Cut}}}_{\bullet \mathbf{Cut}} \xrightarrow{\mathbf{ax/W}}_{\bullet \mathbf{Cut}} \xrightarrow{\mathbf{ax/W}}_{\bullet \mathbf{Cut}}$$

• Case rule \wedge_L

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \rightarrow_R \begin{array}{c} \mathbf{h}_7: \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_7: (\Delta_{11}, \mathbf{F}_8 \land \mathbf{F}_9), \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_{10} \end{array} }{ \begin{array}{c} \wedge_L \\ \bullet \mathbf{h}_7: (\Delta_{11}, \mathbf{F}_8 \land \mathbf{F}_9), \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_{10} \end{array} } \begin{array}{c} \wedge_L \\ \mathsf{Cut} \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \begin{array}{c} \mathsf{ax/W} \\ \hline \bullet \mathbf{h}_7: \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_7: \Delta_{11}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline -: \Delta_{11}, \Delta_2, \mathbf{F}_8 \land \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} } \begin{array}{c} \mathsf{AL} \\ \mathsf{hCut} \end{array}$$

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{5}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{F}_{5}\to\mathbf{F}_{6}}\to_{R}}{\bullet\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{5}\to\mathbf{F}_{6}\vdash\mathbf{F}_{10}}}{\bullet\mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\vee\mathbf{F}_{9}),\mathbf{F}_{5}\to\mathbf{F}_{6}\vdash\mathbf{F}_{10}}}_{\bullet\mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\vee\mathbf{F}_{9}),\mathbf{F}_{5}\to\mathbf{F}_{6}\vdash\mathbf{F}_{10}}}}_{\bullet\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8}\vee\mathbf{F}_{9}\vdash\mathbf{F}_{10}}}_{\bullet\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{5}\to\mathbf{F}_{6}\vdash\mathbf{F}_{10}}}_{\bullet\mathbf{h}_{Cut}}\to_{\mathbf{h}_{7}:\Delta_{11},\Delta_{2},\mathbf{F}_{8}\vdash\mathbf{F}_{10}}}_{\bullet\mathbf{h}_{11}:\Delta_{2}\vdash\mathbf{F}_{5}\to\mathbf{F}_{6}}}_{\bullet\mathbf{h}_{11}:\Delta_{2}\vdash\mathbf{F}_{5}\to\mathbf{F}_{6}}}_{\bullet\mathbf{h}_{11}:\Delta_{2}\vdash\mathbf{F}_{5}\to\mathbf{F}_{6}}^{\bullet\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{9},\mathbf{F}_{5}\to\mathbf{F}_{6}\vdash\mathbf{F}_{10}}}_{\bullet\mathbf{h}_{Cut}}$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \to_R & \frac{}{\bullet \mathbf{h}_7:(\bot,\Delta_9), \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8} \\ -:\Delta_2, \bot, \Delta_9 \vdash \mathbf{F}_8 & \xrightarrow{} \\ -:\bot, \Delta_2, \Delta_9 \vdash \mathbf{F}_8 & \bot_L \end{array} \quad \text{Cut}$$

 \bullet Case rule I

$$\frac{ \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \rightarrow \mathbf{F}_6} \rightarrow_R \frac{}{\bullet \mathbf{h}_7 : (\Delta_8, \mathbf{p}_9), \mathbf{F}_5 \rightarrow \mathbf{F}_6 \vdash \mathbf{p}_9}}{-: \Delta_2, \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9} \stackrel{I}{} \text{Cut}}$$

• Case rule \top_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \xrightarrow{} \rightarrow_R & \frac{\mathbf{h}_7: \Delta_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_8} & \overset{\top_L}{\cot} \\ \hline & -: \Delta_2, \top, \Delta_9 \vdash \mathbf{F}_8 \\ \hline & \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6 & \mathsf{ax/W} & \overset{\to}{\rightarrow} \\ & -: \top, \Delta_2, \Delta_9 \vdash \mathbf{F}_8 & \mathsf{ax/W} \\ \hline & -: \top, \Delta_2, \Delta_9 \vdash \mathbf{F}_8 & \mathsf{hCut} \end{array}$$

7.3 Status of \wedge_R : OK

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \wedge_R \quad \frac{\bullet \mathbf{h}_7: \Delta_8, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \top}{-: \Delta_2, \Delta_8 \vdash \top} \quad \overset{\top_R}{\mathsf{Cut}} \\ & \stackrel{\longleftarrow}{-: \Delta_2, \Delta_8 \vdash \top} \quad \top_R \end{array}$$

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \wedge_R \quad \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_8,\mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8 \rightarrow \mathbf{F}_9} \quad \xrightarrow{\bullet} \quad \frac{-:\Delta_2,\Delta_{10} \vdash \mathbf{F}_8 \rightarrow \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_8,\mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9} \quad \text{ax/W} \\ \hline \frac{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \text{ax/W} \quad \xrightarrow{\bullet} \quad \frac{-:\Delta_{10},\Delta_2,\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet -:\Delta_{10},\Delta_2 \vdash \mathbf{F}_8 \rightarrow \mathbf{F}_9} \quad \xrightarrow{\bullet}_R \\ \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_5 \land \mathbf{h}_6 \vdash \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \hline -:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\mathbf{h}_8,\mathbf{h}_9 \vdash \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \hline -:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_8 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \\ \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h}_9 \rightarrow \mathbf{h}_9 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{h$$

• Case rule \wedge_R

$$\frac{\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \wedge_R \quad \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8 \quad \mathbf{h}_7:\Delta_{10},\mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8 \land \mathbf{F}_9} \quad \mathbf{Cut}}{-:\Delta_2,\Delta_{10} \vdash \mathbf{F}_8 \land \mathbf{F}_9} \quad \mathbf{Cut}} \\ \frac{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6}{\bullet \mathbf{m}_1:\Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \mathbf{m}_7:\Delta_{10},\mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9}} \quad \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \mathbf{ax/W}}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9}} \quad \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_9}}{-:\Delta_{10},\Delta_2 \vdash \mathbf{F}_9} \quad \mathbf{h}_R} \\ \frac{-:\Delta_{10},\Delta_2 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7:\Delta_{10},\Delta_2 \vdash \mathbf{F}_8 \land \mathbf{F}_9} \quad \mathbf{h}_{10} \quad \mathbf{h}_{1$$

• Case rule \vee_1

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\underbrace{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6}_{\bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8}}_{\bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8 \lor \mathbf{F}_9}} \underbrace{\begin{array}{c} \vee_1 \\ \vee_1 \\ \vee_2 \\ \vee_3 \\ \vee_4 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \end{array}}_{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \underbrace{\begin{array}{c} \Delta_2 \vee_1 \\ \vee_1 \\ \vee_2 \\ \vee_3 \\ \vee_4 \\ \vee_5 \\ \vee_5 \\ \vee_5 \\ \vee_5 \end{array}}_{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \underbrace{\begin{array}{c} \Delta_2 \vee_1 \\ \vee_2 \\ \vee_3 \\ \vee_4 \\ \vee_5 \\$$

• Case rule \vee_2

$$\frac{\mathbf{h}_1:\Delta_2 \vdash F_5 \quad \mathbf{h}_1:\Delta_2 \vdash F_6}{\underbrace{\bullet \mathbf{h}_1:\Delta_2 \vdash F_5 \land F_6}} \quad \wedge_R \quad \frac{\mathbf{h}_7:\Delta_{10},F_5 \land F_6 \vdash F_9}{\bullet \mathbf{h}_7:\Delta_{10},F_5 \land F_6 \vdash F_8 \lor F_9}}{\underbrace{-:\Delta_2,\Delta_{10} \vdash F_8 \lor F_9}}_{\bullet \mathbf{h}_7:\Delta_{10},F_5 \land F_6 \vdash F_9} \quad \text{out}}_{\bullet \mathbf{h}_7:\Delta_1 \cup F_5 \land F_6 \vdash F_9} \quad \text{ax/W}}_{\bullet \mathbf{h}_7:\Delta_1 \cup F_5 \land F_6 \vdash F_9}$$

• Case rule \rightarrow_L

$$\frac{\underbrace{\frac{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \quad \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{6}}_{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{6}}}_{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \land \mathbf{F}_{6}} \land_{R} \quad \underbrace{\frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8} \to \mathbf{F}_{9},\mathbf{F}_{5} \land \mathbf{F}_{6} \vdash \mathbf{F}_{8} \quad \mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{9},\mathbf{F}_{5} \land \mathbf{F}_{6} \vdash \mathbf{F}_{10}}_{\bullet \mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8} \to \mathbf{F}_{9}),\mathbf{F}_{5} \land \mathbf{F}_{6} \vdash \mathbf{F}_{10}} \underbrace{\mathbf{Cut}}_{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \land \mathbf{F}_{6}} \quad \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8} \to \mathbf{F}_{9},\mathbf{F}_{5} \land \mathbf{F}_{6} \vdash \mathbf{F}_{8}}_{\bullet \mathbf{h}_{2}}}_{\bullet \mathbf{h}_{2}} \quad \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{9},\mathbf{F}_{5} \land \mathbf{F}_{6} \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{2}} \quad \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_{7}:\Delta_{11},\Delta_{2},\mathbf{F}_{9} \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{2}} \quad \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_{7}:\Delta_{11},\Delta_{2},\mathbf{F}_{9} \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{2}} \quad \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_{2}}}_{\bullet \mathbf{h}_{3}:\Delta_{2} \vdash \mathbf{F}_{5} \land \mathbf{F}_{6}} \quad \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_{7}:\Delta_{11},\Delta_{2},\mathbf{F}_{9} \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{2}:\Delta_{2},\mathbf{F}_{9} \vdash \mathbf{F}_{10}} \quad \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_{2}:\Delta_{2}}}_{\bullet \mathbf{h}_{3}:\Delta_{2},\mathbf{F}_{9} \vdash \mathbf{F}_{10}}$$

• Case rule \wedge_L

$$\frac{\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \quad \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \land \mathbf{F}_7} \quad \wedge_R \quad \frac{\mathbf{h}_5:\Delta_9, \mathbf{F}_6, \mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5:\Delta_9, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \mathbf{F}_8} \quad \wedge_L \\ \\ -:\Delta_2,\Delta_9 \vdash \mathbf{F}_8 \\ \hline -:\Delta_2 \vdash \mathbf{F}_6 \end{array} \\ \frac{-:\Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{m}_1:\Delta_2 \vdash \mathbf{F}_7} \quad \frac{\rightarrow}{\mathbf{ax/W}} \quad \frac{-:\Delta_9,\Delta_9, \mathbf{F}_6, \mathbf{F}_7 \vdash \mathbf{F}_8}{-:\Delta_2,\Delta_9,\Delta_9,\mathbf{F}_6 \vdash \mathbf{F}_8} \quad \text{sCut} \\ \frac{-:\Delta_2,\Delta_9,\Delta_9 \vdash \mathbf{F}_8}{-:\Delta_2,\Delta_9 \vdash \mathbf{F}_8} \quad C \end{array}$$

• Case rule \vee_L

$$\frac{\underbrace{\frac{\mathbf{h}_{1}:\Delta_{2}\vdash F_{5}\quad \mathbf{h}_{1}:\Delta_{2}\vdash F_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2}\vdash F_{5}\wedge F_{6}}}_{\bullet \mathbf{h}_{1}:\Delta_{2}\vdash F_{5}\wedge F_{6}} \wedge_{R} \underbrace{\frac{\mathbf{h}_{7}:\Delta_{11},F_{8},F_{5}\wedge F_{6}\vdash F_{10}\quad \mathbf{h}_{7}:\Delta_{11},F_{9},F_{5}\wedge F_{6}\vdash F_{10}}{\bullet \mathbf{h}_{7}:(\Delta_{11},F_{8}\vee F_{9}),F_{5}\wedge F_{6}\vdash F_{10}}}_{\bullet \mathbf{h}_{1}:\Delta_{2}\vdash F_{5}\wedge F_{6}}} \underbrace{\mathbf{Cut}}_{-:\Delta_{2},\Delta_{11},F_{8},F_{5}\wedge F_{6}\vdash F_{10}} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_{1}:\Delta_{2}\vdash F_{5}\wedge F_{6}}}_{\bullet \mathbf{h}_{1}:\Delta_{2}\vdash F_{5}\wedge F_{6}} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_{7}:\Delta_{11},F_{9},F_{5}\wedge F_{6}\vdash F_{10}}}_{-:\Delta_{11},\Delta_{2},F_{9}\vdash F_{10}} \underbrace{\mathbf{Ax/W}}_{\bullet \mathbf{h}\mathbf{Cut}}$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \quad \wedge_R \quad \frac{\bullet \mathbf{h}_7:(\bot,\Delta_9), \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet} \quad \frac{\bot_L}{\mathsf{Cut}} \\ & \xrightarrow{-:\Delta_2,\bot,\Delta_9 \vdash \mathbf{F}_8} \quad \bot_L \end{array}$$

 \bullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_5 \quad \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_6}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_5 \land \mathsf{F}_6} \quad \bigwedge_R \quad \frac{\bullet \mathbf{h}_7:(\Delta_8,\mathbf{p}_9),\mathsf{F}_5 \land \mathsf{F}_6 \vdash \mathbf{p}_9}{\bullet \mathbf{h}_7:(\Delta_8,\mathbf{p}_9),\mathsf{F}_5 \land \mathsf{F}_6 \vdash \mathbf{p}_9} \quad \text{Cut} \\ & \stackrel{-:\Delta_2,\Delta_8,\mathbf{p}_9 \vdash \mathbf{p}_9}{-:\Delta_2,\Delta_8,\mathbf{p}_9 \vdash \mathbf{p}_9} \quad I \end{array}$$

• Case rule \top_L

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\underbrace{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6}_{\bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8}}_{\bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8}} \frac{\top_L}{\mathsf{Cut}}$$

$$\frac{-: \Delta_2, \top, \Delta_9 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \land \mathbf{F}_6} \underbrace{\mathsf{ax/W}}_{\bullet \mathbf{f}_7: \top, \Delta_9, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_8}}^{\bullet \mathbf{ax/W}} \underbrace{\mathsf{ax/W}}_{\mathsf{hCut}}$$

$$-: \top, \Delta_2, \Delta_9 \vdash \mathbf{F}_8$$

7.4 Status of \vee_1 : OK

• Case rule \top_R

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \\ \underline{\bullet} \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \lor_1 \quad \overline{ \begin{array}{c} \bullet \mathbf{h}_7 : \Delta_8, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \top \\ - : \Delta_2, \Delta_8 \vdash \top \\ \hline \\ - : \Delta_2, \Delta_8 \vdash \top \end{array} } \begin{array}{c} \top_R \\ \mathbf{Cut} \end{array}$$

• Case rule \rightarrow_R

$$\frac{ \begin{array}{c|c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \lor_1 & \begin{array}{c} \mathbf{h}_7: \Delta_{10}, \mathbf{F}_8, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \to \mathbf{F}_9 \\ \hline \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} & \begin{array}{c} \mathbf{A} \mathbf{x} / \mathbb{W} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} & \begin{array}{c} \mathbf{A} \mathbf{x} / \mathbb{W} \\ \hline \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} & \mathbf{A} \mathbf{x} / \mathbb{W} \\ \hline \\ \hline \begin{array}{c} -: \Delta_{10}, \Delta_2, \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline -: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \to \mathbf{F}_9 \end{array} & \rightarrow_R \end{array} & \begin{array}{c} \mathbf{A} \mathbf{x} / \mathbb{W} \\ \mathbf{h} \mathbf{C} \mathbf{u} \mathbf{t} \\ \hline \end{array} \\ \end{array}$$

• Case rule \wedge_R

$$\frac{\frac{\mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6}}}{\bullet \mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6}}} \bigvee_{1} \frac{\mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{8} & \mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{8} \land \mathbf{F}_{9}}} Cut} \\ \frac{-: \Delta_{2}, \Delta_{10} \vdash \mathbf{F}_{8} \land \mathbf{F}_{9}}{\mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{8}} \xrightarrow{\mathbf{ax/W}} \frac{\mathbf{ax/W}}{\mathbf{h}_{Cut}} \xrightarrow{\bullet \mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6}} \mathbf{ax/W}} \xrightarrow{\mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{9}} \wedge_{R}} \mathbf{ax/W}} \xrightarrow{\mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6}} \wedge_{R}} \mathbf{h}_{1}$$

• Case rule \vee_1

• Case rule \vee_2

$$\begin{array}{c|c} \mathbf{h}_1: \Delta_2 \vdash F_5 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash F_5 \lor F_6 \\ \hline \\ -: \Delta_2, \Delta_{10} \vdash F_8 \lor F_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash F_5 \lor F_6 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash F_5 \lor F_6 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash F_5 \lor F_6 \\ \hline \\ -: \Delta_{10}, \Delta_2 \vdash F_9 \\ \hline \\ -: \Delta_{10}, \Delta_2 \vdash F_8 \lor F_9 \\ \hline \\ -: \Delta_{10}, \Delta_2 \vdash F_8 \lor F_9 \\ \hline \end{array} \begin{array}{c} \lor_2 \\ \mathsf{Cut} \\ \hline \\ \bullet \mathsf{h}_1: \Delta_2 \vdash F_5 \lor F_6 \\ \hline \\ \bullet \mathsf{h}_1: \Delta_2 \vdash F_5 \lor F_6 \\ \hline \\ -: \Delta_{10}, \Delta_2 \vdash F_9 \\ \hline \\ -: \Delta_{10}, \Delta_2 \vdash F_8 \lor F_9 \\ \hline \end{array} \begin{array}{c} \mathsf{dax/W} \\ \mathsf{hCut} \\ \hline \end{array}$$

• Case rule \rightarrow_L

$$\frac{\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} \lor_1 \quad \frac{\mathbf{h}_7: \Delta_{11}, \mathbf{F}_8 \to \mathbf{F}_9, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \quad \mathbf{h}_7: \Delta_{11}, \mathbf{F}_9, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7: (\Delta_{11}, \mathbf{F}_8 \to \mathbf{F}_9), \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_{10}} \quad \mathbf{Cut}} \to_L \\ \frac{-: \Delta_2, \Delta_{11}, \mathbf{F}_8 \to \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} \quad \frac{\mathsf{ax/W}}{\mathsf{h}_7: \Delta_{11}, \mathbf{F}_8 \to \mathbf{F}_9, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \frac{-: \Delta_{11}, \Delta_2, \mathbf{F}_8 \to \mathbf{F}_9 \vdash \mathbf{F}_8}{-: \Delta_{11}, \Delta_2, \mathbf{F}_8 \to \mathbf{F}_9 \vdash \mathbf{F}_{10}} \to_L \\ \hline$$

• Case rule \wedge_L

$$\frac{ \begin{array}{c|c} \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 & \vee_1 & \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 & \bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \hline & - : \Delta_2, \Delta_{11}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 & \mathsf{ax/W} & \frac{\rightarrow}{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10}} \\ \hline & \frac{- : \Delta_{11}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{- : \Delta_{11}, \Delta_2, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \wedge_L \end{array} \\ & \xrightarrow{\mathbf{h}_{Cut}} \begin{array}{c} \mathbf{Ax/W} \\ \mathbf{h}_{Cut} \\ \hline \end{array}$$

• Case rule \vee_L

$$\frac{\frac{h_1: \Delta_2 \vdash F_5}{\bullet h_1: \Delta_2 \vdash F_5 \lor F_6}}{\bullet h_1: \Delta_2 \vdash F_5 \lor F_6} \lor_1 \quad \frac{\frac{h_7: \Delta_{11}, F_8, F_5 \lor F_6 \vdash F_{10}}{\bullet h_7: (\Delta_{11}, F_8 \lor F_9), F_5 \lor F_6 \vdash F_{10}}}{\bullet h_7: (\Delta_{11}, F_8 \lor F_9), F_5 \lor F_6 \vdash F_{10}} \quad \text{Cut}} \lor_L \\ \frac{\bullet h_1: \Delta_2 \vdash F_5 \lor F_6}{\bullet h_7: \Delta_{11}, F_8, F_5 \lor F_6 \vdash F_{10}}}{h_7: \Delta_{11}, F_8, F_5 \lor F_6 \vdash F_{10}} \quad \frac{\text{ax/W}}{\text{hCut}} \quad \frac{\bullet h_1: \Delta_2 \vdash F_5 \lor F_6}{\bullet h_1: \Delta_2 \vdash F_5 \lor F_6} \quad \text{ax/W}} \quad \frac{h_7: \Delta_{11}, F_9, F_5 \lor F_6 \vdash F_{10}}{h_7: \Delta_{11}, F_9, F_5 \lor F_6 \vdash F_{10}} \quad \text{Ax/W}}{h_{\text{Cut}}} \quad \frac{\bullet h_1: \Delta_2 \vdash F_5 \lor F_6}{\bullet h_1: \Delta_2 \vdash F_5 \lor F_6} \quad \text{ax/W}}{h_7: \Delta_{11}, F_9, F_5 \lor F_6 \vdash F_{10}} \quad \frac{\bullet \text{Ax/W}}{h_{\text{Cut}}} \quad \frac{\bullet h_1: \Delta_2 \vdash F_5 \lor F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \lor_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \lor_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \bigvee_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \nabla_L \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad \frac{\bullet h_1: \Delta_2 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_6 \lor F_7} \quad$$

• Case rule \perp_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} & \vee_1 & \\ \hline \bullet \mathbf{h}_7: (\bot, \Delta_9), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \\ \hline -: \Delta_2, \bot, \Delta_9 \vdash \mathbf{F}_8 \\ \hline -: \bot, \Delta_2, \Delta_9 \vdash \mathbf{F}_8 & \bot_L \end{array} \quad \begin{array}{c} \bot_L \\ \text{Cut} \\ \hline \end{array}$$

 \bullet Case rule I

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \lor \mathsf{F}_6 \end{array} \lor_1 \quad \frac{}{\bullet \mathbf{h}_7: (\Delta_8, \mathbf{p}_9), \mathsf{F}_5 \lor \mathsf{F}_6 \vdash \mathbf{p}_9} \\ -: \Delta_2, \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9 \\ \hline -: \Delta_2, \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9 \end{array} \quad \mathbf{Cut}$$

• Case rule \top_L

$$\frac{ \begin{array}{c|c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \lor_1 & \begin{array}{c} \mathbf{h}_7: \Delta_9, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \end{array} }{ \begin{array}{c} -: \Delta_2, \top, \Delta_9 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} } & \begin{array}{c} \to \\ \mathbf{ax/W} \\ \bullet \mathbf{h}_7: \top, \Delta_9, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \end{array} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{hCut} \end{array}$$

7.5 Status of \vee_2 : OK

• Case rule \top_R

$$\frac{ \begin{array}{c|c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \lor_2 \begin{array}{c} \hline \bullet \mathbf{h}_7: \Delta_8, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \top \\ \hline -: \Delta_2, \Delta_8 \vdash \top \\ \hline \hline -: \Delta_2, \Delta_8 \vdash \top \end{array} } \begin{array}{c} \top_R \\ \mathbf{Cut} \end{array}$$

• Case rule \rightarrow_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} & \vee_2 & \frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_8, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \to \mathbf{F}_9} & \mathbf{Cut} \\ \hline & -: \Delta_2, \Delta_{10} \vdash \mathbf{F}_8 \to \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} & \mathbf{ax/W} & \frac{\rightarrow}{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_8, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_9}} \\ \hline & \frac{-: \Delta_{10}, \Delta_2, \mathbf{F}_8 \vdash \mathbf{F}_9}{-: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \to \mathbf{F}_9} & \rightarrow_R \end{array} & \mathbf{ax/W} \\ \hline & \frac{-: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \to \mathbf{F}_9}{-: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \to \mathbf{F}_9} & \rightarrow_R \end{array}$$

• Case rule \wedge_R

$$\frac{\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6}}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6}} \ \lor_2 \ \frac{\frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8} \land \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \land \mathbf{F}_9}} \ \mathbf{Cut} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} \ \frac{\mathbf{ax} / \mathbb{W}}{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8} \ \frac{\mathbf{ax} / \mathbb{W}}{\mathbf{h}_C \mathbf{ut}} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} \ \frac{\mathbf{ax} / \mathbb{W}}{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_9} \ \frac{\mathbf{ax} / \mathbb{W}}{\mathbf{h}_C \mathbf{ut}} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} \ \frac{\mathbf{ax} / \mathbb{W}}{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_9} \ \wedge_R \\ \hline \\ \bullet \mathbf{h}_C \mathbf{ut} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_8 \lor \mathbf{h}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf$$

• Case rule \vee_1

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \vee_2 \quad \frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \lor \mathbf{F}_9} \quad \begin{array}{c} \vee_1 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \quad \frac{\vee_1}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} \quad \frac{\mathsf{ax/W}}{\bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8} \\ \frac{-: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8}{-: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \lor \mathbf{F}_9} \quad \vee_1 \end{array} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}}$$

• Case rule \vee_2

$$\frac{ \begin{array}{c|c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} \vee_2 & \begin{array}{c} \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \end{array} & \begin{array}{c} \vee_2 \\ \hline \bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \end{array} & \begin{array}{c} \vee_2 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 \end{array} & \mathbf{ax/W} & \begin{array}{c} \rightarrow \\ \hline \mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline -: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_9 \\ \hline -: \Delta_{10}, \Delta_2 \vdash \mathbf{F}_8 \lor \mathbf{F}_9 \end{array} & \vee_2 \end{array} & \mathbf{ax/W} \\ & \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \rightarrow_L

$$\frac{\underbrace{\frac{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6}}}_{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6}} \lor_{2} \quad \underbrace{\frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8} \to \mathbf{F}_{9},\mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8} \to \mathbf{F}_{9}),\mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8} \to \mathbf{F}_{9} \vdash \mathbf{F}_{10})} \underbrace{\mathbf{Cut}}_{-:\Delta_{2},\Delta_{11},\mathbf{F}_{8} \to \mathbf{F}_{9},\mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{8}}_{\bullet \mathbf{ax}/\mathbf{W}} \underbrace{\frac{\bullet}{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6}}_{\bullet} \underbrace{\frac{\mathbf{ax}/\mathbf{W}}{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{9},\mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{10}}_{-:\Delta_{11},\Delta_{2},\mathbf{F}_{9} \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{Lut}} \underbrace{\frac{\bullet}{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6}}_{\bullet} \underbrace{\frac{\mathbf{ax}/\mathbf{W}}{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{9},\mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{Lut}}}_{\bullet \mathbf{Lut}}$$

• Case rule \wedge_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} & \vee_2 & \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : \left(\Delta_{11}, \mathbf{F}_8 \land \mathbf{F}_9\right), \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_{10}} & \wedge_L \\ \hline & -: \Delta_2, \Delta_{11}, \mathbf{F}_8 \land \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline & \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6 & \mathbf{ax/W} & \frac{\rightarrow}{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{F}_{10}} \\ \hline & \frac{-: \Delta_{11}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{-: \Delta_{11}, \Delta_2, \mathbf{F}_8 \land \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \wedge_L \end{array} \quad \begin{array}{c} \wedge_L \\ \text{hCut} \end{array}$$

• Case rule \vee_L

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{6}}}{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \lor \mathbf{F}_{6}}} \lor_{2} \quad \frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8} \lor \mathbf{F}_{9}),\mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{10}}} \quad \mathbf{Cut} \\ \frac{-:\Delta_{2},\Delta_{11},\mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{8},\mathbf{F}_{5} \lor \mathbf{F}_{6} \vdash \mathbf{F}_{10}} \quad \mathbf{ax/W} \quad \mathbf{hCut}}{\bullet \mathbf{hCut}} \\ \frac{-:\Delta_{11},\Delta_{2},\mathbf{F}_{8} \vdash \mathbf{F}_{10}}{-:\Delta_{11},\Delta_{2},\mathbf{F}_{8} \vdash \mathbf{F}_{10}} \quad \vee_{L}} \quad \mathbf{hCut}$$

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_7}{\bullet \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_6\vee \mathbf{F}_7} & \bigvee_2 & \frac{\mathbf{h}_5:\Delta_9,\mathbf{F}_6\vdash \mathbf{F}_8}{\bullet \mathbf{h}_5:\Delta_9,\mathbf{F}_6\vee \mathbf{F}_7\vdash \mathbf{F}_8} \\ & -:\Delta_2,\Delta_9\vdash \mathbf{F}_8 \\ \hline & \frac{-:\Delta_2,\Delta_9\vdash \mathbf{F}_8}{-:\Delta_2\vdash \mathbf{F}_7} & \mathbf{ax/W} & \frac{-:\Delta_9,\mathbf{F}_7\vdash \mathbf{F}_8}{\bullet \mathbf{cut}} & \mathbf{sCut} \end{array} \quad \bigvee_L$$

• Case rule \perp_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} & \vee_2 & \\ \hline -: \Delta_2, \bot, \Delta_9 \vdash \mathbf{F}_8 & \to \\ \hline -: \bot, \Delta_2, \Delta_9 \vdash \mathbf{F}_8 & \bot_L \end{array} \quad \text{Cut}$$

ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \lor \mathbf{F}_6} & \vee_2 & \\ \hline \bullet \mathbf{h}_7: (\Delta_8, \mathbf{p}_9), \mathbf{F}_5 \lor \mathbf{F}_6 \vdash \mathbf{p}_9 \\ \hline & -: \Delta_2, \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9 \\ \hline & -: \Delta_2, \Delta_8, \mathbf{p}_9 \vdash \mathbf{p}_9 \end{array} \quad \mathbf{Cut} \\ \hline \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \; \vee_2 \; \; \frac{\mathbf{h}_7: \Delta_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: (\top, \Delta_9), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8} \; \frac{\top_L}{\mathsf{Cut}} \\ \\ \frac{-: \Delta_2, \top, \Delta_9 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \; \frac{\mathsf{ax/W}}{\bullet \mathbf{h}_7: \top, \Delta_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8} \\ \\ \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \vee \mathbf{F}_6}{-: \top, \Delta_2, \Delta_9 \vdash \mathbf{F}_8} \; \frac{\mathsf{ax/W}}{\mathsf{hCut}} \end{array}$$

7.6 Status of \rightarrow_L : OK

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \to \mathbf{F}_3 \vdash \mathbf{F}_2 \quad \mathbf{h}_1:\Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \to \mathbf{F}_3 \vdash \mathbf{F}_8} \xrightarrow{} \bot_L \begin{array}{c} \bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \vdash \top \\ -:(\Delta_5, \mathbf{F}_2 \to \mathbf{F}_3), \Delta_7 \vdash \top \\ & \to \\ \hline -:\Delta_5, \Delta_7, \mathbf{F}_2 \to \mathbf{F}_3 \vdash \top \end{array} \begin{array}{c} \top_R \end{array}$$

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2 \quad \mathbf{h}_1:\Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_{10}} \rightarrow_L & \frac{\mathbf{h}_6:\Delta_9, \mathbf{F}_7, \mathbf{F}_{10} \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6:\Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7 \rightarrow \mathbf{F}_8} \\ \hline -:(\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3), \Delta_9 \vdash \mathbf{F}_7 \rightarrow \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_{10}} & \frac{\mathbf{ax}/\mathbf{W}}{\mathbf{ax}/\mathbf{W}} & \frac{\mathbf{ax}/\mathbf{W}}{\mathbf{h}_6:\Delta_9, \mathbf{F}_{10}, \mathbf{F}_7 \vdash \mathbf{F}_8} \\ \hline -:\Delta_5,\Delta_9, \mathbf{F}_7, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_8 \\ \hline -:\Delta_5,\Delta_9, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_8} \rightarrow_R \end{array} \begin{array}{c} \mathbf{ax}/\mathbf{W} \\ \mathbf{h}\mathbf{Cut} \end{array}$$

• Case rule \wedge_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2\quad\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_{10}} \rightarrow_L & \frac{\mathbf{h}_6:\Delta_9,\mathbf{F}_{10}\vdash\mathbf{F}_7\quad\mathbf{h}_6:\Delta_9,\mathbf{F}_{10}\vdash\mathbf{F}_8}{\bullet\mathbf{h}_6:\Delta_9,\mathbf{F}_{10}\vdash\mathbf{F}_7\wedge\mathbf{F}_8} \text{ Cut} \\ & -:(\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3),\Delta_9\vdash\mathbf{F}_7\wedge\mathbf{F}_8 \\ & \rightarrow \\ & -:\Delta_5,\Delta_9,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2 & \mathbf{ax/W} & \frac{\bullet\mathbf{h}_6:\Delta_9,\mathbf{F}_{10}\vdash\mathbf{F}_7\wedge\mathbf{F}_8}{\bullet\mathbf{h}_6:\Delta_9,\mathbf{F}_{10}\vdash\mathbf{F}_7\wedge\mathbf{F}_8} \rightarrow_L \\ & -:\Delta_5,\Delta_9,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2\rightarrow\mathbf{F}_2\vdash\mathbf{F}_7\wedge\mathbf{F}_8 \\ & -:\Delta_5,\Delta_9,\mathbf{F}_2\rightarrow\mathbf{F}_2\vdash\mathbf{F}_7\wedge\mathbf{F}_8 \end{array} \rightarrow_L \\ \end{array}$$

• Case rule \vee_1

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_2 \quad \mathbf{h}_1:\Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_{10}} \rightarrow_L & \frac{\mathbf{h}_6:\Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6:\Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7 \vee \mathbf{F}_8} & \vee_1 \\ \hline & -:(\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3), \Delta_9 \vdash \mathbf{F}_7 \vee \mathbf{F}_8} \\ \hline \bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_{10} & \frac{\rightarrow}{\mathbf{ax/W}} & \frac{\mathbf{ax/W}}{\mathbf{h}_6:\Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7} & \mathbf{ax/W} \\ \hline & -:\Delta_5, \Delta_9, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_7 \\ \hline & -:\Delta_5, \Delta_9, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_7 \vee \mathbf{F}_8 & \vee_1 \end{array}$$

• Case rule \vee_2

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_2\quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_{10}} \rightarrow_L & \frac{\mathbf{h}_6:\Delta_9,\mathbf{F}_{10}\vdash\mathbf{F}_8}{\bullet\mathbf{h}_6:\Delta_9,\mathbf{F}_{10}\vdash\mathbf{F}_7\vee\mathbf{F}_8} & \nabla_2\\ \hline & -:(\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3),\Delta_9\vdash\mathbf{F}_7\vee\mathbf{F}_8\\ \hline & \bullet\\ \bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_{10} & \mathbf{ax/W} & \frac{\bullet}{\mathbf{h}_6:\Delta_9,\mathbf{F}_{10}\vdash\mathbf{F}_8} & \mathbf{ax/W}\\ \hline & -:\Delta_5,\Delta_9,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_8\\ \hline & -:\Delta_5,\Delta_9,\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\mathbf{F}_8 & \nabla_2 \end{array}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{2}\quad\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{11}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{11}}\to_{L}\frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{7}\quad\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{8},\mathbf{F}_{11}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:(\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}),\mathbf{F}_{11}\vdash\mathbf{F}_{9}}\mathbf{Cut}}\to_{L}$$

$$\frac{-:(\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}),\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{11}}\frac{\mathbf{ax}/\mathbf{W}}{\bullet\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}\mathbf{Cut}}$$

$$\frac{-:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\to\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}\to_{L}$$

$$\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{2}\to\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{7}\to\mathbf{F}_{8}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{7}\to\mathbf{F}_{8}}\to_{L}$$

$$\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{2}\to\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{7}\to\mathbf{F}_{8}}{\bullet\mathbf{h}_{1}:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}$$

$$\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{7}\to\mathbf{F}_{8}}{\bullet\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}$$

$$\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{7}\to\mathbf{F}_{8}\to\mathbf{F}_{9}}$$

$$\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{7}\to\mathbf{F}_{8}\to\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}$$

$$\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9}\to\mathbf{F}_{8}\to\mathbf{F}_{9}}$$

$$\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9}\to\mathbf{F}_{8}\to\mathbf{F}_{9}$$

$$\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9}\to\mathbf{F}_{8}\to\mathbf{F}_{9}\to\mathbf{F}_{9}$$

$$\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9}\to\mathbf{F}_{8}\to\mathbf{F}_{9}\to\mathbf{F$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{2}\quad\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{11}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{11}} \to_{L} & \frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7},\mathbf{F}_{8},\mathbf{F}_{11}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:(\Delta_{10},\mathbf{F}_{7}\land\mathbf{F}_{8}),\mathbf{F}_{11}\vdash\mathbf{F}_{9}} & \wedge_{L} \\ & -:(\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}),\Delta_{10},\mathbf{F}_{7}\land\mathbf{F}_{8}\vdash\mathbf{F}_{9} & \to \\ & \xrightarrow{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{11}} & \mathbf{ax/W} & \frac{\bullet}{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7},\mathbf{F}_{8}\vdash\mathbf{F}_{9}} & \mathbf{ax/W} \\ & & & \underline{-:\Delta_{10},\Delta_{5},\mathbf{F}_{7},\mathbf{F}_{8},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9}} & \wedge_{L} \\ & & & & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3},\mathbf{F}_{7}\land\mathbf{F}_{8}\vdash\mathbf{F}_{9}} & \wedge_{L} \end{array}$$

$$\frac{\frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_2\quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_7\land\mathbf{F}_8}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_7\land\mathbf{F}_8}\to_L \quad \frac{\mathbf{h}_6:\Delta_{10},\mathbf{F}_7,\mathbf{F}_8\vdash\mathbf{F}_9}{\bullet\mathbf{h}_6:\Delta_{10},\mathbf{F}_7\land\mathbf{F}_8\vdash\mathbf{F}_9}}{\bullet\mathbf{h}_6:\Delta_{10},\mathbf{F}_7\land\mathbf{F}_8\vdash\mathbf{F}_9}} \quad \underset{\leftarrow}{\land L} \quad$$

• Case rule \vee_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{2}\quad\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{11}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{11}} \to_{L} & \frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7},\mathbf{F}_{11}\vdash\mathbf{F}_{9}\quad\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{8},\mathbf{F}_{11}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:(\Delta_{10},\mathbf{F}_{7}\vee\mathbf{F}_{8}),\mathbf{F}_{11}\vdash\mathbf{F}_{9}} & \mathbf{Cut} \\ & -:(\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}),\Delta_{10},\mathbf{F}_{7}\vee\mathbf{F}_{8}\vdash\mathbf{F}_{9} \\ & \to \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3},\mathbf{F}_{7}\vee\mathbf{F}_{8}\vdash\mathbf{F}_{2} & \mathbf{ax/W} & \frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{11}}{\bullet\mathbf{h}_{6}:\Delta_{10},\Delta_{5},\mathbf{F}_{11},\mathbf{F}_{7}\vee\mathbf{F}_{8}\vdash\mathbf{F}_{9}} & \mathbf{ax/W} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3},\mathbf{F}_{7}\vee\mathbf{F}_{8}\vdash\mathbf{F}_{9} & \mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7}\vee\mathbf{F}_{8}\vdash\mathbf{F}_{9} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{2} & \mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{7}\vee\mathbf{F}_{8} & \to_{L} & \frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\vdash\mathbf{F}_{9}\quad\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{8}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{7}\vee\mathbf{F}_{8}} & \to_{L} & \frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\vee\mathbf{F}_{8}\vdash\mathbf{F}_{9}\quad\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{8}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{7}\vee\mathbf{F}_{8}} & \to_{L} & \frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\vee\mathbf{F}_{8}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\vee\mathbf{F}_{8}\vdash\mathbf{F}_{9}} & \mathbf{Cut} \\ & -:(\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}),\Delta_{10}\vdash\mathbf{F}_{9} & \bullet_{h}_{6}:\Delta_{10},\mathbf{F}_{7}\vee\mathbf{F}_{8}\vdash\mathbf{F}_{9}} & \mathbf{ax/W} & \bullet_{L} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{7}\vee\mathbf{F}_{8} & \mathbf{ax/W} & \bullet_{L} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9} & \bullet_{L} & \bullet_{L} & \bullet_{L} & \bullet_{L} & \bullet_{L} & \bullet_{L} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9} & \bullet_{L} & \bullet_{L} & \bullet_{L} & \bullet_{L} & \bullet_{L} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9} & \bullet_{L} & \bullet_{L} & \bullet_{L} & \bullet_{L} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9} & \bullet_{L} & \bullet_{L} & \bullet_{L} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9} & \bullet_{L} & \bullet_{L} & \bullet_{L} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9} & \bullet_{L} & \bullet_{L} & \bullet_{L} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9} & \bullet_{L} & \bullet_{L} & \bullet_{L} \\ & -:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\to\mathbf{F}_{3}\vdash\mathbf{F}_{9} & \bullet_{L} & \bullet_{L} \\ & -:\Delta_$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_2}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\bot} \to L & \frac{\bullet\mathbf{h}_6:\Delta_8,\bot\vdash\mathbf{F}_7}{\bullet\mathbf{h}_6:\Delta_8,\bot\vdash\mathbf{F}_7} & \overset{\bot_L}{\mathsf{Cut}} \\ & -:(\Delta_5,\mathbf{F}_2\to\mathbf{F}_3),\Delta_8\vdash\mathbf{F}_7 & \\ & \to \\ & \frac{\bullet}{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\bot} & \mathbf{ax/W} & \frac{\bullet}{\bullet\mathbf{h}_6:\bot,\Delta_8\vdash\mathbf{F}_7} & \overset{\bot_L}{\mathsf{hCut}} \\ \hline -:\Delta_5,\Delta_8,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_2 & \mathbf{ax/W} & \frac{\bullet}{\bullet\mathbf{h}_6:\bot,\Delta_8\vdash\mathbf{F}_7} & \to L \\ \hline & -:\Delta_5,\Delta_8,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_7 & \to L \\ \hline & \frac{\bullet}{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_2} & \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_9 & \to L & \frac{\bullet}{\bullet\mathbf{h}_6:(\bot,\Delta_8),\mathbf{F}_9\vdash\mathbf{F}_7} & \overset{\bot_L}{\mathsf{Cut}} \\ \hline & -:(\Delta_5,\mathbf{F}_2\to\mathbf{F}_3)\vdash\mathbf{F}_9 & \to L & \frac{\bullet}{\bullet\mathbf{h}_6:(\bot,\Delta_8),\mathbf{F}_9\vdash\mathbf{F}_7} & \overset{\bot_L}{\mathsf{Cut}} \\ \hline & -:(\Delta_5,\mathbf{F}_2\to\mathbf{F}_3),\bot,\Delta_8\vdash\mathbf{F}_7 & \to L \\ \hline & -:(\Delta_5,\mathbf{F}_2\to\mathbf{F}_3),\bot,\Delta_8\vdash\mathbf{F}_7 & \bot_L \\ \hline \end{array}$$

ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_2}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_8}\to_L & \frac{\bullet\mathbf{h}_6:(\Delta_7,\mathbf{p}_9),\mathbf{F}_8\vdash\mathbf{p}_9}{\bullet\mathbf{h}_6:(\Delta_7,\mathbf{p}_9),\mathbf{F}_8\vdash\mathbf{p}_9} & I\\ \hline & -:(\Delta_5,\mathbf{F}_2\to\mathbf{F}_3),\Delta_7,\mathbf{p}_9\vdash\mathbf{p}_9\\ \hline & -:\Delta_5,\Delta_7,\mathbf{p}_9,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{p}_9 & I\\ \hline \\ \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_2}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{p}_8}\to_L & \frac{\bullet\mathbf{h}_6:\Delta_7,\mathbf{p}_8\vdash\mathbf{p}_8}{\bullet\mathbf{h}_6:\Delta_7,\mathbf{p}_8\vdash\mathbf{p}_8} & I\\ \hline & -:(\Delta_5,\mathbf{F}_2\to\mathbf{F}_3),\Delta_7\vdash\mathbf{p}_8\\ \hline & -:(\Delta_5,\mathbf{F}_2\to\mathbf{F}_3)\to\mathbf{p}_8 & \mathbf{ax/W} \\ \hline \end{array}$$

• Case rule \top_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_2\quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\top}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\top} \to_L & \frac{\mathbf{h}_6:\Delta_8\vdash\mathbf{F}_7}{\bullet\mathbf{h}_6:\Delta_8,\top\vdash\mathbf{F}_7} & \top_L \\ \hline -:(\Delta_5,\mathbf{F}_2\to\mathbf{F}_3),\Delta_8\vdash\mathbf{F}_7 & \mathbf{ax/W} \\ \hline -:\Delta_5,\Delta_8,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_7 & \mathbf{ax/W} \\ \hline \\ \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_2\quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_9}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_9} \to_L & \frac{\mathbf{h}_6:\Delta_8,\mathbf{F}_9\vdash\mathbf{F}_7}{\bullet\mathbf{h}_6:(\top,\Delta_8),\mathbf{F}_9\vdash\mathbf{F}_7} & \top_L \\ \hline \\ \frac{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_9}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_9} & \mathbf{ax/W} & \mathbf{h}_6:\top,\Delta_8,\mathbf{F}_9\vdash\mathbf{F}_7 \\ \hline \\ \bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_9 & \mathbf{ax/W} & \mathbf{h}_6:\top,\Delta_8,\mathbf{F}_9\vdash\mathbf{F}_7 \\ \hline \\ \bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_9 & \mathbf{ax/W} & \mathbf{h}_6:\top,\Delta_8,\mathbf{F}_9\vdash\mathbf{F}_7 \\ \hline \\ \bullet\mathbf{h}_1:\Delta_5,\mathbf{h}_2\to\mathbf{h}_3\vdash\mathbf{F}_9 & \mathbf{h}_1\to\mathbf{h}_1\to\mathbf{h}_1\to\mathbf{h}_2\to\mathbf{h}_2\to\mathbf{h}_1\to\mathbf{h}_2\to$$

7.7 Status of \wedge_L : OK

• Case rule \top_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_8 \end{array} \land_L \quad \frac{}{\bullet \mathbf{h}_6: \Delta_7, \mathbf{F}_8 \vdash \top} \quad \begin{array}{c} \top_R \\ \text{Cut} \\ \hline \\ -: (\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \Delta_7 \vdash \top \\ \hline \\ -: \Delta_5, \Delta_7, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \top \end{array} }$$

• Case rule \rightarrow_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10} \end{array} \wedge_L \quad \begin{array}{c} \mathbf{h}_6: \Delta_9, \mathbf{F}_7, \mathbf{F}_{10} \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7 \to \mathbf{F}_8 \end{array} \\ \hline -: (\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \Delta_9 \vdash \mathbf{F}_7 \to \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10} \quad \text{ax/W} \quad \begin{array}{c} \rightarrow \\ \mathbf{h}_6: \Delta_9, \mathbf{F}_{10}, \mathbf{F}_7 \vdash \mathbf{F}_8 \end{array} \\ \hline -: \Delta_5, \Delta_9, \mathbf{F}_7, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_8 \\ \hline -: \Delta_5, \Delta_9, \mathbf{F}_7, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_8 \end{array} \rightarrow_R \quad \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \wedge_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10}} \ \land_L \ \frac{\mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7 \quad \mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7 \land \mathbf{F}_8} \ \\ \frac{-: (\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \Delta_9 \vdash \mathbf{F}_7 \land \mathbf{F}_8}{\bullet \mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7 \land \mathbf{F}_8} \ \\ \frac{\mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7 \land \mathbf{F}_8} \ \\ \frac{-: \Delta_5, \Delta_9, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_7 \land \mathbf{F}_8}{-: \Delta_5, \Delta_9, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_7 \land \mathbf{F}_8} \ \land_L \end{array} \right. \\ \frac{\mathsf{ax/W}}{\mathsf{hCut}}$$

• Case rule \vee_1

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10} \end{array} \land_L \quad \frac{\mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7 \lor \mathbf{F}_8} \quad \begin{array}{c} \vee_1 \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10} \end{array} \\ \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10} \quad \xrightarrow{\mathsf{ax/W}} \quad \frac{\rightarrow}{\mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7} \\ \hline \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10} \quad & \bullet \mathbf{h}_1 \vdash \mathbf{h}_2 \vdash \mathbf{h}_3 \vdash \mathbf{h}_3$$

• Case rule \vee_2

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10} \end{array} \land L \quad \begin{array}{c} \mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_7 \lor \mathbf{F}_8 \end{array} \quad \begin{array}{c} \vee_2 \\ \mathsf{Cut} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10} \end{array} \quad \begin{array}{c} \to \mathsf{ax/W} \\ \bullet \mathbf{h}_6: \Delta_9, \mathbf{F}_{10} \vdash \mathbf{F}_8 \end{array} \quad \begin{array}{c} \mathsf{ax/W} \\ \bullet \mathsf{Cut} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{10} \end{array} \quad \begin{array}{c} \to \mathsf{ax/W} \\ \bullet \mathsf{Cut} \\ \hline \\ \bullet \mathsf{Cut} \end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{11}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\land\mathbf{F}_{3}\vdash\mathbf{F}_{11}} \wedge_{L} & \frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{7} & \mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{8},\mathbf{F}_{11}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:(\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}),\mathbf{F}_{11}\vdash\mathbf{F}_{9}} & \mathbf{Cut} \\ & -:(\Delta_{5},\mathbf{F}_{2}\land\mathbf{F}_{3}),\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9} & \\ & \frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{11}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{11}} & \mathbf{ax/W} & \frac{\rightarrow}{\bullet\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}} & \mathbf{ax/W} \\ & \frac{-:\Delta_{10},\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{3},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}{-:\Delta_{10},\Delta_{5},\mathbf{F}_{7}\to\mathbf{F}_{8},\mathbf{F}_{2}\land\mathbf{F}_{3}\vdash\mathbf{F}_{9}} & \wedge_{L} \\ & \frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{7}\to\mathbf{F}_{8}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\land\mathbf{F}_{3}\vdash\mathbf{F}_{7}\to\mathbf{F}_{8}} & \wedge_{L} & \frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}} & \mathbf{Cut} \\ & \frac{-:(\Delta_{5},\mathbf{F}_{2}\land\mathbf{F}_{3}),\Delta_{10}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}} & \mathbf{ax/W} \\ & \frac{-:\Delta_{10},\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\to\mathbf{F}_{8}\vdash\mathbf{F}_{9}} & \mathbf{hCut} \\ & \frac{-:\Delta_{10},\Delta_{5},\mathbf{F}_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{9}}{-:\Delta_{10},\Delta_{5},\mathbf{F}_{2}\land\mathbf{F}_{3}\vdash\mathbf{F}_{9}} & \wedge_{L} \\ \end{array}$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_{11}} \ \land_L & \frac{\mathbf{h}_6:\Delta_{10}, \mathbf{F}_7, \mathbf{F}_8, \mathbf{F}_{11} \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:(\Delta_{10}, \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{11} \vdash \mathbf{F}_9} \ \land_L \\ \hline \\ \frac{-:(\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \Delta_{10}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_{10}, \mathbf{F}_{11}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \frac{-:\Delta_{10}, \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_{11}}{-:\Delta_{10}, \Delta_5, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \mathbf{F}_9} \ \land_L \\ \hline \\ \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_7 \land \mathbf{F}_8}{-:\Delta_{10}, \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \mathbf{F}_9} \ \land_L \\ \hline \\ \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_7 \land \mathbf{F}_8}{-:\Delta_{10}, \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \Delta_{10} \vdash \mathbf{F}_9} \ \land_L \\ \hline \\ \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_7 \land \mathbf{F}_8}{-:\Delta_{10}, \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_9} \ \land_L \\ \hline \\ \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_7 \land \mathbf{F}_8}{-:\Delta_{10}, \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_9} \ \land_L \\ \hline \\ \frac{-:\Delta_{10}, \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_9}{-:\Delta_{10}, \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_9} \ \land_L \\ \hline \\ \frac{-:\Delta_{10}, \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_9}{-:\Delta_{10}, \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_9} \ \land_L \\ \hline \end{array} \ \ \begin{array}{c} \mathbf{Ax/W} \\ \mathbf{hCut} \\ \hline \end{array} \ \ \\ \mathbf{hCut} \\ \hline \end{array}$$

• Case rule \perp_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \bot}{\bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \bot} & \Delta_L & \frac{\bullet \mathbf{h}_6:\Delta_8, \bot \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6:\Delta_8, \bot \vdash \mathbf{F}_7} & \overset{\bot_L}{\mathsf{Cut}} \\ \hline -:(\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \Delta_8 \vdash \mathbf{F}_7 & \\ \hline \frac{\rightarrow}{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \bot} & \mathsf{ax/W} & \frac{\rightarrow}{\bullet \mathbf{h}_6:\bot,\Delta_8 \vdash \mathbf{F}_7} & \overset{\bot_L}{\mathsf{hCut}} \\ \hline -:\Delta_5, \Delta_8, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_7 & \wedge_L & \\ \hline \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_9}{-:\Delta_5, \Delta_8, \mathbf{F}_2 \land \mathbf{F}_3), \bot,\Delta_8 \vdash \mathbf{F}_7} & \overset{\bot_L}{\bullet \mathbf{h}_6:(\bot,\Delta_8), \mathbf{F}_9 \vdash \mathbf{F}_7} & \mathsf{Cut} \\ \hline -:(\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \bot,\Delta_8 \vdash \mathbf{F}_7 & & & \\ \hline -:(\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \bot,\Delta_8 \vdash \mathbf{F}_7 & & & \\ \hline -:\bot,\Delta_5,\Delta_8, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_7 & & & \\ \hline -:\bot,\Delta_5,\Delta_8, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_7 & & & \\ \hline \end{array}$$

ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_8} \ \land L & \frac{\bullet \mathbf{h}_6:(\Delta_7, \mathbf{p}_9), \mathbf{F}_8 \vdash \mathbf{p}_9}{\bullet \mathbf{h}_6:(\Delta_7, \mathbf{p}_9), \mathbf{F}_8 \vdash \mathbf{p}_9} & \mathbf{Cut} \\ & \xrightarrow{-:(\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \Delta_7, \mathbf{p}_9 \vdash \mathbf{p}_9} & I \\ & \xrightarrow{\bullet} \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{p}_8} \land L & \frac{\bullet}{\bullet \mathbf{h}_6:\Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8} & I \\ & \xrightarrow{\bullet} \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{p}_8} \land L & \frac{\bullet}{\bullet \mathbf{h}_6:\Delta_7, \mathbf{p}_8 \vdash \mathbf{p}_8} & \mathbf{Cut} \\ & \xrightarrow{-:(\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \Delta_7 \vdash \mathbf{p}_8} & \xrightarrow{\bullet} \frac{\bullet}{-:\Delta_5, \Delta_7, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{p}_8} & \mathbf{ax/W} \end{array}$$

• Case rule \top_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \top}{\bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \top} & \wedge_L & \frac{\mathbf{h}_6:\Delta_8 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6:\Delta_8, \top \vdash \mathbf{F}_7} & \top_L \\ \hline -:(\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \Delta_8 \vdash \mathbf{F}_7 & \\ \hline -:\Delta_5, \Delta_8, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline \\ \frac{\mathbf{h}_1:\Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_9} & \wedge_L & \frac{\mathbf{h}_6:\Delta_8, \mathbf{F}_9 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6:(\top,\Delta_8), \mathbf{F}_9 \vdash \mathbf{F}_7} & \nabla_L \\ \hline -:(\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3), \top, \Delta_8 \vdash \mathbf{F}_7 & \\ \hline \bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_9 & \mathbf{ax/W} & \\ \hline \bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_9 & \mathbf{ax/W} & \\ \hline -:\top, \Delta_5, \Delta_8, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline -:\top, \Delta_5, \Delta_8, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline \end{array}$$

7.8 Status of \vee_L : OK

• Case rule \top_R

$$\frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_8\quad\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_8}{\underbrace{\begin{array}{c}\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_8\\ -:(\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3),\Delta_7\vdash\top\\ &\rightarrow\\ \hline -:\Delta_5,\Delta_7,\mathbf{F}_2\vee\mathbf{F}_3\vdash\top\\ \end{array}}}_{} \overset{\bullet}{\mathbf{h}_6:\Delta_7,\mathbf{F}_8\vdash\top} \overset{\top}{\leftarrow}_R$$

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash \mathbf{F}_{10}\quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash \mathbf{F}_{10}}{\underbrace{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_{10}}_{-:(\Delta_5,\mathbf{F}_2\vee \mathbf{F}_3),\Delta_9\vdash \mathbf{F}_7\to \mathbf{F}_8}} \overset{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_{10}}{-:(\Delta_5,\mathbf{F}_2\vee \mathbf{F}_3),\Delta_9\vdash \mathbf{F}_7\to \mathbf{F}_8}} \overset{\bullet}{\underset{h_0:\Delta_9,\mathbf{F}_{10},\mathbf{F}_7\vdash \mathbf{F}_8}{\to}}} \overset{\bullet}{\underset{h_0:\Delta_9,\mathbf{F}_{10},\mathbf{F}_7\vdash \mathbf{F}_8}{\to}}} \overset{\bullet}{\underset{h_0:\Delta_9,\mathbf{F}_{10},\mathbf{F}_7\vdash \mathbf{F}_8}{\to}}} \overset{\bullet}{\underset{h_0:\Delta_9,\mathbf{F}_{10},\mathbf{F}_7\vdash \mathbf{F}_8}{\to}}} \overset{\bullet}{\underset{h_0:\Delta_9,\mathbf{F}_{10},\mathbf{F}_7\vdash \mathbf{F}_8}{\to}}} \overset{\bullet}{\underset{h_0:\Delta_9,\mathbf{F}_1,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_8}}{\to}} \overset{\bullet}{\underset{h_0:\Delta_9,\mathbf{F}_1,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_8}}}{\to} \overset{\bullet}{\underset{h_0:\Delta_9,\mathbf{F}_1,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_1,\mathbf{F}_1,\mathbf{F}_1,\mathbf{F}_1,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_1,\mathbf{F}_1,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_1,\mathbf{F}_1,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_1,\mathbf{F}_1,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_1,\mathbf{F}_1,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_1,\mathbf{F}_1,\mathbf{F}_2\vee \mathbf{F}_3\vdash \mathbf{F}_3\vdash \mathbf{F}_3}}$$

• Case rule \wedge_R

$$\frac{\underbrace{\begin{array}{l} \mathbf{h}_{1}: \Delta_{5}, \mathbf{F}_{2} \vdash \mathbf{F}_{10} \quad \mathbf{h}_{1}: \Delta_{5}, \mathbf{F}_{3} \vdash \mathbf{F}_{10} \\ \underline{\bullet}\mathbf{h}_{1}: \Delta_{5}, \mathbf{F}_{2} \vee \mathbf{F}_{3} \vdash \mathbf{F}_{10} \\ -: (\Delta_{5}, \mathbf{F}_{2} \vee \mathbf{F}_{3}), \Delta_{9} \vdash \mathbf{F}_{7} \wedge \mathbf{F}_{8} \\ \hline \\ \underline{\bullet}\mathbf{h}_{1}: \Delta_{5}, \mathbf{F}_{2} \vee \mathbf{F}_{3} \vdash \mathbf{F}_{10} \\ \underline{\bullet}\mathbf{h}_{1}: \Delta_{5}, \mathbf{F}_{2} \vee \mathbf{F}_{3} \vdash \mathbf{F}_{10} \\ -: (\Delta_{5}, \mathbf{F}_{2} \vee \mathbf{F}_{3}), \Delta_{9} \vdash \mathbf{F}_{7} \wedge \mathbf{F}_{8} \\ \hline \\ \underline{\bullet}\mathbf{h}_{1}: \Delta_{5}, \mathbf{F}_{2} \vee \mathbf{F}_{3} \vdash \mathbf{F}_{10} \\ \underline{\bullet}\mathbf{h}_{2}: \Delta_{9}, \mathbf{F}_{10} \vdash \mathbf{F}_{7} \\ \mathbf{hCut} \\ \hline \\ \underline{-: \Delta_{5}, \Delta_{9}, \mathbf{F}_{2} \vee \mathbf{F}_{3} \vdash \mathbf{F}_{10} \\ -: \Delta_{5}, \Delta_{9}, \mathbf{F}_{2} \vee \mathbf{F}_{3} \vdash \mathbf{F}_{8} \\ -: \Delta_{5}, \Delta_{9}, \mathbf{F}_{2} \vee \mathbf{F}_{3} \vdash \mathbf{F}_{7} \wedge \mathbf{F}_{8} \\ \hline \\ \underline{-: \Delta_{5}, \Delta_{9}, \mathbf{F}_{2} \vee \mathbf{F}_{3} \vdash \mathbf{F}_{8} \\ \wedge R} \\ \end{array}} \xrightarrow{\bullet \mathbf{h}_{1}: \Delta_{5}, \mathbf{F}_{2} \vee \mathbf{F}_{3} \vdash \mathbf{F}_{10}} \underbrace{\bullet \mathbf{h}_{6}: \Delta_{9}, \mathbf{F}_{10} \vdash \mathbf{F}_{8}}_{h_{C}} \wedge \mathbf{h}_{C} \\ \bullet \mathbf{h}_{C} \vee \mathbf{h}_$$

• Case rule \vee_1

• Case rule \vee_2

• Case rule \rightarrow_L

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash \mathbf{F}_{11} \quad \mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash \mathbf{F}_{11}}{\bullet} \quad \forall_{L} \quad \frac{\mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \mathbf{F}_{7} \quad \mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{8},\mathbf{F}_{11}\vdash \mathbf{F}_{9}}{\bullet} \quad \rightarrow_{L} \quad \\ \frac{\bullet \mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\lor \mathbf{F}_{3}\vdash \mathbf{F}_{11}}{\bullet} \quad \frac{\bullet \mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \mathbf{F}_{9}} \quad \mathbf{Cut} \quad \\ \frac{\bullet \mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\lor \mathbf{F}_{3}\vdash \mathbf{F}_{11}}{\bullet} \quad \frac{\bullet \mathbf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \mathbf{F}_{7}} \quad \frac{\bullet}{\mathsf{ax}/\mathsf{W}} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\lor \mathbf{F}_{3}\vdash \mathbf{F}_{11}} \quad \mathbf{ax}/\mathsf{W} \quad \frac{\bullet}{\mathsf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{8}\vdash \mathbf{F}_{9}}{\bullet} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\lor \mathbf{F}_{3}\vdash \mathbf{F}_{11}} \quad \mathbf{ax}/\mathsf{W} \quad \frac{\bullet}{\mathsf{h}_{6}:\Delta_{10},\mathbf{F}_{11},\mathbf{F}_{8}\vdash \mathbf{F}_{9}}{\bullet} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\lor \mathbf{F}_{3}\vdash \mathbf{F}_{9}}{\bullet} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\lor \mathbf{F}_{3}\vdash \mathbf{F}_{9}}{\bullet} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\lor \mathbf{F}_{3}\vdash \mathbf{F}_{9}}{\bullet} \quad \mathbf{Ax}/\mathsf{W}} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\lor \mathbf{F}_{3}\vdash \mathbf{F}_{9}}{\bullet} \quad \mathbf{Ax}/\mathsf{W} \quad \frac{\bullet}{\mathsf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \mathbf{F}_{9}}{\bullet} \quad \mathbf{Ax}/\mathsf{W}} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\lor \mathbf{F}_{3}\vdash \mathbf{F}_{9}}{\bullet} \quad \mathbf{Ax}/\mathsf{W}} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash \mathbf{F}_{7}\to \mathbf{F}_{8}} \quad \mathbf{ax}/\mathsf{W} \quad \frac{\bullet}{\mathsf{h}_{6}:\Delta_{10},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \mathbf{F}_{9}}{\bullet} \quad \mathbf{Ax}/\mathsf{W}} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash \mathbf{F}_{7}\to \mathbf{F}_{8}} \quad \mathbf{ax}/\mathsf{W}} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash \mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \mathbf{F}_{9}} \quad \mathbf{ax}/\mathsf{W}} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash \mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \mathbf{F}_{9}} \quad \mathbf{ax}/\mathsf{W}} \quad \frac{\bullet}{\mathsf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash \mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \mathbf{F}_{9}}$$

• Case rule \wedge_L

• Case rule \vee_L

$$\frac{\frac{h_{1}:\Delta_{5},F_{2}\vdash F_{11}}{e^{h_{1}}:\Delta_{5},F_{2}\lor F_{3}\vdash F_{11}}}{e^{h_{1}:\Delta_{5},F_{2}\lor F_{3}\vdash F_{11}}}\vee_{L}\frac{h_{6}:\Delta_{10},F_{7},F_{11}\vdash F_{9}}{e^{h_{6}:(\Delta_{10},F_{7}\lor F_{8}),F_{11}\vdash F_{9}}}}{e^{h_{6}:(\Delta_{10},F_{7}\lor F_{8}),F_{11}\vdash F_{9}}}\det_{L}\vee_{L}\frac{e^{h_{1}:\Delta_{5},F_{2}\lor F_{3}\vdash F_{11}}}{e^{h_{1}:\Delta_{5},F_{2}\lor F_{3}}}\vee_{L}}$$

$$\frac{-:(\Delta_{5},F_{2}\lor F_{3}),\Delta_{10},F_{7}\lor F_{8}\vdash F_{9}}{e^{h_{1}:\Delta_{5},F_{2}\lor F_{11}}}\underbrace{ax/W}\underbrace{e^{h_{6}:\Delta_{10},F_{11},F_{7}\lor F_{8}\vdash F_{9}}}_{e^{h_{6}:\Delta_{10},F_{11},F_{7}\lor F_{8}\vdash F_{9}}}\underbrace{ax/W}_{e^{h_{6}:\Delta_{10},\Delta_{5},F_{3},F_{7}\lor F_{8}\vdash F_{9}}}_{e^{h_{1}:\Delta_{5},F_{2}\lor F_{3}\vdash F_{7}\lor F_{8}}}\vee_{L}\underbrace{e^{h_{1}:\Delta_{5},F_{2}\lor F_{3}\vdash F_{7}\lor F_{8}}}_{e^{h_{1}:\Delta_{5},F_{2}\lor F_{3}\vdash F_{7}\lor F_{8}}}\underbrace{e^{h_{1}:\Delta_{5},F_{2}\lor F_{3}\lor F_{3}}}_{e^{h_{1}:\Delta_{5},F_{2}\lor F_{3}}}\underbrace{e^{$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\bot \quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\bot}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\bot} \quad \vee_L \quad \frac{\bullet \mathbf{h}_6:\Delta_8,\bot\vdash\mathbf{F}_7}{\bullet \mathbf{h}_6:\Delta_8,\bot\vdash\mathbf{F}_7} \quad \overset{\bot_L}{\mathsf{Cut}} \\ \\ \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\bot}{\bullet \mathbf{h}_6:\bot,\Delta_8\vdash\mathbf{F}_7} \quad \frac{\mathsf{ax/W}}{\bullet \mathbf{h}_6:\bot,\Delta_8\vdash\mathbf{F}_7} \quad \frac{\mathsf{ax/W}}{\mathsf{h}_1:\Delta_5,\mathbf{F}_3\vdash\bot} \quad \frac{\mathsf{ax/W}}{\bullet \mathbf{h}_6:\bot,\Delta_8\vdash\mathbf{F}_7} \quad \overset{\bot_L}{\mathsf{h}_{\mathsf{Cut}}} \\ \\ \frac{-:\Delta_5,\Delta_8,\mathbf{F}_2\vdash\mathbf{F}_7}{-:\Delta_5,\Delta_8,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_7} \quad \vee_L \\ \\ \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_9 \quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_9} \quad \vee_L \quad \overset{\bullet}{\bullet \mathbf{h}_6:(\bot,\Delta_8),\mathbf{F}_9\vdash\mathbf{F}_7} \quad \overset{\bot_L}{\mathsf{Cut}} \\ \\ \frac{\bullet}{-:(\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3),\bot,\Delta_8\vdash\mathbf{F}_7} \quad \overset{\bot_L}{\to} \\ \\ \frac{-:(\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3),\bot,\Delta_8\vdash\mathbf{F}_7}{-:\bot,\Delta_5,\Delta_8,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_7} \quad \overset{\bot_L}{\to} \\ \\ \frac{-:(\Delta_5,\Delta_8,\mathbf{F}_2\vee\mathbf{F}_3),\bot,\Delta_8\vdash\mathbf{F}_7}{-:\bot,\Delta_5,\Delta_8,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_7} \quad \overset{\bot_L}{\to} \\ \end{array}$$

$\bullet\,$ Case rule I

$$\begin{array}{c|c} \frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash\mathbf{F}_{8} & \mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{F}_{8}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{F}_{8}} & \vee_{L} & \frac{\bullet\mathbf{h}_{6}:(\Delta_{7},\mathbf{p}_{9}),\mathbf{F}_{8}\vdash\mathbf{p}_{9}}{\bullet\mathbf{h}_{6}:(\Delta_{7},\mathbf{p}_{9}),\mathbf{F}_{8}\vdash\mathbf{p}_{9}} & I\\ & & -:(\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}),\Delta_{7},\mathbf{p}_{9}\vdash\mathbf{p}_{9}\\ & & -:\Delta_{5},\Delta_{7},\mathbf{p}_{9},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{p}_{9} & I\\ \hline \frac{\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vdash\mathbf{p}_{8} & \mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{3}\vdash\mathbf{p}_{8}}{\bullet\mathbf{h}_{1}:\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{p}_{8}} & \vee_{L} & \frac{\bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{p}_{8}\vdash\mathbf{p}_{8}}{\bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{p}_{8}\vdash\mathbf{p}_{8}} & I\\ & & -:(\Delta_{5},\mathbf{F}_{2}\vee\mathbf{F}_{3}),\Delta_{7}\vdash\mathbf{p}_{8}\\ & & \rightarrow \\ & & -:\Delta_{5},\Delta_{7},\mathbf{F}_{2}\vee\mathbf{F}_{3}\vdash\mathbf{p}_{8} & \mathbf{ax/W} \end{array}$$

• Case rule \top_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\top \quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\top}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\top} \vee_L \quad \frac{\mathbf{h}_6:\Delta_8\vdash\mathbf{F}_7}{\bullet \mathbf{h}_6:\Delta_8,\top\vdash\mathbf{F}_7} \quad \top_L \\ \qquad \qquad -:(\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3),\Delta_8\vdash\mathbf{F}_7 \\ \qquad \qquad -:\Delta_5,\Delta_8,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_7 \end{array} \quad \mathbf{ax/W} \\ \\ \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_9 \quad \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_9} \quad \vee_L \quad \frac{\mathbf{h}_6:\Delta_8,\mathbf{F}_9\vdash\mathbf{F}_7}{\bullet \mathbf{h}_6:(\top,\Delta_8),\mathbf{F}_9\vdash\mathbf{F}_7} \quad \frac{\top_L}{\mathbf{Cut}} \\ \qquad \qquad -:(\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3),\top,\Delta_8\vdash\mathbf{F}_7 \\ \qquad \qquad \rightarrow \\ \frac{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_9} \quad \mathbf{ax/W} \\ \qquad \qquad \qquad \rightarrow \\ \frac{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_9} \quad \mathbf{ax/W} \\ \qquad \qquad -:\top,\Delta_5,\Delta_8,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_7 \end{array} \quad \mathbf{ax/W} \\ \qquad \qquad \mathbf{h}_6:\top,\Delta_8,\mathbf{F}_9\vdash\mathbf{F}_7 \\ \qquad \qquad -:\top,\Delta_5,\Delta_8,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_7 \end{array} \quad \mathbf{ax/W}$$

7.9 Status of \perp_L : OK

• Case rule \top_R

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \bot, \Delta_3 \vdash \mathsf{F}_6 & \bot_L & \hline \bullet_{\mathbf{h}_4}: \Delta_5, \mathsf{F}_6 \vdash \top \\ \hline -: (\bot, \Delta_3), \Delta_5 \vdash \top & \mathsf{Cut} \\ \hline & \hline -: \bot, \Delta_3, \Delta_5 \vdash \top & \top_R \end{array}$$

• Case rule \rightarrow_R

$$\frac{\underbrace{\bullet \mathbf{h}_1: \bot, \Delta_3 \vdash \mathbf{F}_8}_{} \ \bot_L \ \ \frac{\mathbf{h}_4: \Delta_7, \mathbf{F}_5, \mathbf{F}_8 \vdash \mathbf{F}_6}_{\bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \to \mathbf{F}_6}}_{-: (\bot, \Delta_3), \Delta_7 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \ \frac{\to_R}{\mathsf{cut}}$$

• Case rule \wedge_R

$$\frac{ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : \bot, \Delta_3 \vdash \mathbf{F}_8 \\ \end{array} \bot_L \quad \begin{array}{c} \underline{\mathbf{h}_4 : \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \quad \mathbf{h}_4 : \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_4 : \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ \hline - : (\bot, \Delta_3), \Delta_7 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ \hline - : \bot, \Delta_3, \Delta_7 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \end{array} \ \Delta_L} \quad \text{Cut}$$

• Case rule \vee_1

$$\begin{array}{c|c} \frac{\bullet \mathbf{h}_1: \bot, \Delta_3 \vdash \mathbf{F}_8}{\bullet} & \bot_L & \frac{\mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \\ \hline -: (\bot, \Delta_3), \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline & \rightarrow \\ \hline -: \bot, \Delta_3, \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 & \bot_L \end{array}$$

• Case rule \vee_2

$$\begin{array}{c|c} \underline{\bullet \mathbf{h}_1 : \bot, \Delta_3 \vdash \mathbf{F}_8} & \bot_L & \underline{\bullet \mathbf{h}_4 : \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_6} \\ \underline{\bullet \mathbf{h}_1 : \bot, \Delta_3 \vdash \mathbf{F}_8} & \bot_L & \underline{\bullet \mathbf{h}_4 : \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \\ - : (\bot, \Delta_3), \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \underline{\hspace{1cm}} \\ - : \bot, \Delta_3, \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \end{array} \ \underline{\hspace{1cm}} \quad \mathbf{Cut} \\ \end{array}$$

• Case rule \rightarrow_L

$$\frac{ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : \bot, \Delta_3 \vdash \mathbf{F}_9 \end{array} \bot_L \begin{array}{c} \mathbf{h}_4 : \Delta_8, \mathbf{F}_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_5 & \mathbf{h}_4 : \Delta_8, \mathbf{F}_6, \mathbf{F}_9 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_4 : (\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6), \mathbf{F}_9 \vdash \mathbf{F}_7 \end{array} }{-: (\bot, \Delta_3), \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7} \begin{array}{c} -: (\bot, \Delta_3), \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline -: \bot, \Delta_3, \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array} } \begin{array}{c} \bot_L \\ \hline \bullet \mathbf{h}_4 : \bot, \Delta_3 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \begin{array}{c} \bot_L \\ \bullet \mathbf{h}_4 : \Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_5 & \mathbf{h}_4 : \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline -: (\bot, \Delta_3), \Delta_8 \vdash \mathbf{F}_7 \end{array} \begin{array}{c} \bot_L \\ \hline -: (\bot, \Delta_3), \Delta_8 \vdash \mathbf{F}_7 \end{array} \begin{array}{c} \bot_L \\ \hline -: (\bot, \Delta_3), \Delta_8 \vdash \mathbf{F}_7 \end{array} \begin{array}{c} \bot_L \end{array}$$

• Case rule \wedge_L

$$\begin{array}{c|c} \bullet_{\mathbf{h}_1}: \bot, \Delta_3 \vdash F_9 & \bot_L & \frac{\mathbf{h}_4: \Delta_8, F_5, F_6, F_9 \vdash F_7}{\bullet \mathbf{h}_4: (\Delta_8, F_5 \land F_6), F_9 \vdash F_7} & \land_L \\ \hline -: (\bot, \Delta_3), \Delta_8, F_5 \land F_6 \vdash F_7 & \bot_L \\ \hline -: \bot, \Delta_3, \Delta_8, F_5 \land F_6 \vdash F_7 & \bot_L \\ \hline \bullet_{\mathbf{h}_1}: \bot, \Delta_3 \vdash F_5 \land F_6 & \bot_L & \frac{\mathbf{h}_4: \Delta_8, F_5, F_6 \vdash F_7}{\bullet \mathbf{h}_4: \Delta_8, F_5 \land F_6 \vdash F_7} & \land_L \\ \hline -: (\bot, \Delta_3), \Delta_8 \vdash F_7 & \bot_L \\ \hline -: \bot, \Delta_3, \Delta_8 \vdash F_7 & \bot_L \end{array}$$

• Case rule \vee_L

• Case rule \perp_L

$$\begin{array}{c|c} \hline \bullet \mathbf{h}_1 : \bot, \Delta_3 \vdash \bot & \bullet \mathbf{h}_4 : \Delta_6, \bot \vdash \mathbf{F}_5 \\ \hline - : (\bot, \Delta_3), \Delta_6 \vdash \mathbf{F}_5 \\ \hline - : \bot, \Delta_3, \Delta_6 \vdash \mathbf{F}_5 & \bot_L \\ \hline \hline \bullet \mathbf{h}_1 : \bot, \Delta_3 \vdash \mathbf{F}_7 & \bot_L & \bullet \mathbf{h}_4 : (\bot, \Delta_6), \mathbf{F}_7 \vdash \mathbf{F}_5 \\ \hline - : (\bot, \Delta_3), \bot, \Delta_6 \vdash \mathbf{F}_5 & \to \\ \hline - : \bot, \bot, \Delta_3, \Delta_6 \vdash \mathbf{F}_5 & \bot_L \\ \hline \end{array}$$

ullet Case rule I

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \bot, \Delta_3 \vdash \mathbf{F}_6 & \bot_L & \hline \bullet_{\mathbf{h}_4}: (\Delta_5, \mathbf{p}_7), \mathbf{F}_6 \vdash \mathbf{p}_7 \\ \hline -: (\bot, \Delta_3), \Delta_5, \mathbf{p}_7 \vdash \mathbf{p}_7 \\ \hline -: \bot, \Delta_3, \Delta_5, \mathbf{p}_7 \vdash \mathbf{p}_7 & \bot_L \end{array} \quad \mathbf{Cut}$$

$$\begin{array}{c|c} \hline { \bullet \mathbf{h}_1 : \bot, \Delta_3 \vdash \mathbf{p}_6} & \bot_L & \hline { \bullet \mathbf{h}_4 : \Delta_5, \mathbf{p}_6 \vdash \mathbf{p}_6 } \\ \hline { - : (\bot, \Delta_3), \Delta_5 \vdash \mathbf{p}_6} & \to \\ \hline { \hline { - : \bot, \Delta_3, \Delta_5 \vdash \mathbf{p}_6 } } & \bot_L \\ \hline \end{array}$$

• Case rule \top_L

$$\begin{array}{c|c} \frac{\mathbf{h}_{1}:\bot,\Delta_{3}\vdash\top}{-:(\bot,\Delta_{3}),\Delta_{6}\vdash F_{5}} & \top_{L} \\ \hline -:(\bot,\Delta_{3}),\Delta_{6}\vdash F_{5} & \mathbf{Cut} \\ \hline -:(\bot,\Delta_{3}),\Delta_{6}\vdash F_{5} & \bot_{L} \\ \hline \\ \hline -:\bot,\Delta_{3},\Delta_{6}\vdash F_{5} & \bot_{L} \\ \hline \\ \bullet \mathbf{h}_{1}:\bot,\Delta_{3}\vdash F_{7} & \bot_{L} & \frac{\mathbf{h}_{4}:\Delta_{6},F_{7}\vdash F_{5}}{\bullet \mathbf{h}_{4}:(\top,\Delta_{6}),F_{7}\vdash F_{5}} & \top_{L} \\ \hline -:(\bot,\Delta_{3}),\top,\Delta_{6}\vdash F_{5} & \\ \hline -:\bot,\top,\Delta_{3},\Delta_{6}\vdash F_{5} & \bot_{L} \\ \hline \end{array}$$

7.10 Status of I: OK

• Case rule \top_R

• Case rule \rightarrow_R

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\text{h}_1} : \Delta_2, p_8 \vdash p_8 \\ \hline \\ - : (\Delta_2, p_8), \Delta_7 \vdash F_5 \rightarrow F_6 \\ \hline \\ - : \Delta_2, \Delta_7, p_8 \vdash F_5 \rightarrow F_6 \\ \hline \\ - : \Delta_2, \Delta_7, p_8 \vdash F_5 \rightarrow F_6 \\ \hline \end{array}}_{\text{ax/W}} \xrightarrow{\text{ax/W}}$$

• Case rule \wedge_R

$$\frac{ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : \Delta_2, \mathbf{p}_8 \vdash \mathbf{p}_8 \\ \end{array} I \quad \begin{array}{c} \mathbf{h}_4 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \quad \mathbf{h}_4 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_4 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ \hline - : (\Delta_2, \mathbf{p}_8), \Delta_7 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ \hline - : \Delta_2, \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \end{array}} \quad \mathbf{cut} \\ \\ \begin{array}{c} \Delta_7 \\ \bullet \mathbf{h}_4 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ \hline - : \Delta_2, \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \end{array} \quad \mathbf{ax/W} \\ \end{array}$$

• Case rule \vee_1

$$\frac{\underbrace{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{p}_8 \vdash \mathbf{p}_8}_{} \ I \quad \frac{\mathbf{h}_4 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4 : \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6}}_{-: (\Delta_2, \mathbf{p}_8), \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \quad \mathbf{Cut}} \\ \frac{-: (\Delta_2, \mathbf{p}_8), \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6}{-: \Delta_2, \Delta_7, \mathbf{p}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6}} \quad \mathbf{ax/W}$$

• Case rule \vee_2

$$\frac{ \frac{\mathbf{h}_4:\Delta_7,\mathbf{p}_8 \vdash \mathbf{F}_6}{\mathbf{e}\mathbf{h}_1:\Delta_2,\mathbf{p}_8 \vdash \mathbf{p}_8} \ I \ \frac{\mathbf{h}_4:\Delta_7,\mathbf{p}_8 \vdash \mathbf{F}_6}{\mathbf{e}\mathbf{h}_4:\Delta_7,\mathbf{p}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \ \frac{\mathbf{v}_2}{\mathbf{cut}} }{ \frac{-:(\Delta_2,\mathbf{p}_8),\Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6}{\rightarrow} \ \mathbf{ax/W} } \ \mathbf{cut}$$

• Case rule \rightarrow_L

• Case rule \wedge_L

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \Delta_2, p_9 \vdash p_9 \\ \hline \bullet (\Delta_2, p_9 \vdash p_9 \end{array} I \begin{array}{c} \frac{h_4 : \Delta_8, F_5, F_6, p_9 \vdash F_7}{\bullet (A_8, F_5 \land F_6), p_9 \vdash F_7} \\ \hline - : (\Delta_2, p_9), \Delta_8, F_5 \land F_6 \vdash F_7 \\ \hline - : \Delta_2, \Delta_8, p_9, F_5 \land F_6 \vdash F_7 \end{array}} \begin{array}{c} \land_L \\ \text{Cut} \\ \hline \end{array}}$$

• Case rule \vee_L

$$\frac{ \underbrace{ \bullet \mathbf{h}_1 : \Delta_2, \mathbf{p}_9 \vdash \mathbf{p}_9}_{} \ I \ \frac{ \mathbf{h}_4 : \Delta_8, \mathbf{F}_5, \mathbf{p}_9 \vdash \mathbf{F}_7 \quad \mathbf{h}_4 : \Delta_8, \mathbf{F}_6, \mathbf{p}_9 \vdash \mathbf{F}_7}_{}{\bullet \mathbf{h}_4 : (\Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6), \mathbf{p}_9 \vdash \mathbf{F}_7}_{} \ \mathbf{Cut} \\ \underbrace{ - : (\Delta_2, \mathbf{p}_9), \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7}_{}_{} \underbrace{}_{} \ \mathbf{cv} \\ \underbrace{ - : \Delta_2, \Delta_8, \mathbf{p}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7}_{} \ \mathbf{ax/W}$$

• Case rule \perp_L

$$\begin{array}{c|c} \hline \\ \bullet \mathbf{h}_1 : \Delta_2, \mathbf{p}_7 \vdash \mathbf{p}_7 & \hline \\ \bullet \mathbf{h}_4 : (\bot, \Delta_6), \mathbf{p}_7 \vdash \mathbf{F}_5 \\ \hline \\ - : (\Delta_2, \mathbf{p}_7), \bot, \Delta_6 \vdash \mathbf{F}_5 \\ \hline \\ \hline \\ - : \bot, \Delta_2, \Delta_6, \mathbf{p}_7 \vdash \mathbf{F}_5 \\ \hline \end{array} \begin{array}{c} \bot_L \\ \mathsf{Cut} \\ \hline \\ \hline \end{array}$$

 $\bullet\,$ Case rule I

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1} : \Delta_2, \mathbf{p}_6 \vdash \mathbf{p}_6 & I & \hline \bullet_{\mathbf{h}_4} : (\Delta_5, \mathbf{p}_7), \mathbf{p}_6 \vdash \mathbf{p}_7 \\ \hline -: (\Delta_2, \mathbf{p}_6), \Delta_5, \mathbf{p}_7 \vdash \mathbf{p}_7 & \\ \hline -: \Delta_2, \Delta_5, \mathbf{p}_6, \mathbf{p}_7 \vdash \mathbf{p}_7 & I \\ \hline \hline \bullet_{\mathbf{h}_1} : \Delta_2, \mathbf{p}_6 \vdash \mathbf{p}_6 & I & \hline \bullet_{\mathbf{h}_4} : \Delta_5, \mathbf{p}_6 \vdash \mathbf{p}_6 & I \\ \hline -: (\Delta_2, \mathbf{p}_6), \Delta_5 \vdash \mathbf{p}_6 & \\ \hline -: (\Delta_2, \mathbf{p}_6), \Delta_5 \vdash \mathbf{p}_6 & \\ \hline -: \Delta_2, \Delta_5, \mathbf{p}_6 \vdash \mathbf{p}_6 & I \\ \hline \end{array}$$

$$\frac{ \frac{\mathbf{h}_4 : \Delta_6, \mathbf{p}_7 \vdash \mathbf{F}_5}{\mathbf{e} \mathbf{h}_4 : (\top, \Delta_6), \mathbf{p}_7 \vdash \mathbf{F}_5}}{ - : (\Delta_2, \mathbf{p}_7), \top, \Delta_6 \vdash \mathbf{F}_5} \\ \frac{- : (\Delta_2, \mathbf{p}_7), \top, \Delta_6 \vdash \mathbf{F}_5}{- : \top, \Delta_2, \Delta_6, \mathbf{p}_7 \vdash \mathbf{F}_5} \\ \mathbf{ax/W} \\$$

7.11 Status of \top_L : OK

• Case rule \top_R

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_3 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \top, \Delta_3 \vdash \mathbf{F}_6 \end{array} \top_L \begin{array}{c} \bullet \mathbf{h}_4 : \Delta_5, \mathbf{F}_6 \vdash \top \\ - : (\top, \Delta_3), \Delta_5 \vdash \top \\ \hline - : \top, \Delta_3, \Delta_5 \vdash \top \end{array} \begin{array}{c} \top_R \\ \mathsf{Cut} \end{array} }$$

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{F}_8} \quad \top_L \quad \frac{\mathbf{h}_4: \Delta_7, \mathbf{F}_5, \mathbf{F}_8 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \quad \overset{\rightarrow}{\mathrm{Cut}} \\ \hline -: (\top, \Delta_3), \Delta_7 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \\ \hline \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \to \mathbf{F}_6} \quad \frac{\mathrm{ax/W}}{\mathrm{hCut}} \\ \hline -: \top, \Delta_3, \Delta_7 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array}$$

• Case rule \wedge_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{F}_8} & \top_L & \frac{\mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \quad \mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6} & \mathbf{Cut} \\ \hline & -: (\top, \Delta_3), \Delta_7 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \\ & \xrightarrow{\bullet} \frac{}{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8} & \mathbf{ax/W} & \xrightarrow{\bullet} \frac{}{\bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \land \mathbf{F}_6} & \mathbf{ax/W} \\ \hline & -: \top, \Delta_3, \Delta_7 \vdash \mathbf{F}_5 \land \mathbf{F}_6 & \mathbf{hCut} \end{array}$$

• Case rule \vee_1

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{F}_8} \quad \top_L \quad \frac{\mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \quad \overset{\vee}{\text{Cut}} \\ \hline -: (\top, \Delta_3), \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \hline \bullet \\ \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8}{\bullet} \quad \frac{\mathsf{ax/W}}{\bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \end{array}$$

• Case rule \vee_2

$$\frac{ \begin{array}{l} \mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8 \\ \underline{\bullet \mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{F}_8} \end{array} \top_L \begin{array}{l} \mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_6 \\ \underline{\bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \\ -: (\top, \Delta_3), \Delta_7 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \\ \underline{\to} \\ \underline{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8} \begin{array}{l} \mathbf{ax/W} \\ \underline{\bullet \mathbf{h}_4: \top, \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \end{array} \begin{array}{l} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_1:\top,\Delta_3 \vdash \mathbf{F}_9} & \top_L & \frac{\mathbf{h}_4:\Delta_8, \mathbf{F}_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4:(\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6), \mathbf{F}_9 \vdash \mathbf{F}_7} & \Delta_L \\ \hline & -:(\top,\Delta_3),\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \frac{\mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_9, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7} & \mathbf{ax/W} \\ \hline & -:\top,\Delta_3,\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & -:\top,\Delta_3,\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{h}_4:\Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1:\top,\Delta_3 \vdash \mathbf{F}_5 \to \mathbf{F}_6 & \top_L & \frac{\mathbf{h}_4:\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4:\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7} & \mathbf{Cut} \\ \hline & -:(\top,\Delta_3),\Delta_8 \vdash \mathbf{F}_7 & \rightarrow \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \to \mathbf{F}_6 \vdash \mathbf{F}_7 & \mathbf{h}_8 \vdash \mathbf{h$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_1:\top,\Delta_3 \vdash \mathbf{F}_9} \quad \top_L \quad \frac{\mathbf{h}_4:\Delta_8, \mathbf{F}_5, \mathbf{F}_6, \mathbf{F}_9 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4:(\Delta_8, \mathbf{F}_5 \land \mathbf{F}_6), \mathbf{F}_9 \vdash \mathbf{F}_7} \quad \wedge_L \\ \hline -:(\top,\Delta_3),\Delta_8, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline \rightarrow \\ \frac{\mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_9}{\bullet} \quad \mathbf{ax}/\mathbb{W} \quad \frac{\rightarrow}{\bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_9, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_7} \quad \mathbf{ax}/\mathbb{W} \\ -:\top,\Delta_3,\Delta_8, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_5 \land \mathbf{F}_6 \quad \top_L \quad \frac{\mathbf{h}_4:\Delta_8, \mathbf{F}_5, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4:\Delta_8, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_7} \quad \wedge_L \\ \hline -:(\top,\Delta_3),\Delta_8 \vdash \mathbf{F}_7 \\ \hline \rightarrow \\ \frac{\mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_5 \land \mathbf{F}_6}{\bullet} \quad \mathbf{ax}/\mathbb{W} \quad \frac{\rightarrow}{\bullet \mathbf{h}_4:\top,\Delta_8, \mathbf{F}_5 \land \mathbf{F}_6 \vdash \mathbf{F}_7} \quad \mathbf{ax}/\mathbb{W} \\ \hline -:(\top,\Delta_3,\Delta_8 \vdash \mathbf{F}_7) \quad \mathbf{ax}/\mathbb{W} \\ \hline -:(\top,\Delta_3,\Delta_8 \vdash \mathbf{F}_7) \quad \mathbf{ax}/\mathbb{W} \\ \hline \end{pmatrix}$$

• Case rule \vee_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{F}_9} \; \top_L \; & \frac{\mathbf{h}_4: \Delta_8, \mathbf{F}_5, \mathbf{F}_9 \vdash \mathbf{F}_7 \quad \mathbf{h}_4: \Delta_8, \mathbf{F}_6, \mathbf{F}_9 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4: (\Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6), \mathbf{F}_9 \vdash \mathbf{F}_7} \; \mathbf{Cut} \\ & -: (\top, \Delta_3), \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \\ & \xrightarrow{} \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_9} \; \mathbf{ax/W} \; & \xrightarrow{} \frac{\partial}{\bullet \mathbf{h}_4: \top, \Delta_8, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} \; \mathbf{ax/W} \\ & -: \top, \Delta_3, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \; & \top_L \; & \frac{\mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vdash \mathbf{F}_7 \quad \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} \; \mathbf{Cut} \\ \hline & -: (\top, \Delta_3), \Delta_8 \vdash \mathbf{F}_7 \; & \xrightarrow{} \frac{\mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} \\ \hline & \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_5 \vee \mathbf{F}_6}{\bullet} \; & \mathbf{ax/W} \; & \xrightarrow{} \frac{\mathbf{h}_4: \top, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} \\ \hline & \mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \; & \mathbf{ax/W} \; & \xrightarrow{} \frac{\mathbf{h}_4: \top, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} \\ \hline & -: \top, \Delta_3, \Delta_8 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \; & \mathbf{ax/W} \; & \xrightarrow{} \frac{\mathbf{h}_4: \top, \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7} \\ \hline & \mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_5 \vee \mathbf{F}_6 \; & \mathbf{ax/W} \; & \mathbf{h}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{h}_7 \; & \mathbf{h}_4: \Delta_8, \mathbf{F}_6 \vdash \mathbf{h}_7 \; \\ \hline & \mathbf{h}_4: \Delta_8,$$

• Case rule \perp_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_3\vdash\bot}{\bullet\mathbf{h}_1:\top,\Delta_3\vdash\bot} & \top_L & \frac{}{\bullet\mathbf{h}_4:\Delta_6,\bot\vdash F_5} & \bot_L \\ & -:(\top,\Delta_3),\Delta_6\vdash F_5 & \\ \hline \frac{}{\mathbf{h}_1:\Delta_3\vdash\bot} & \mathbf{ax/W} & \frac{}{\bullet\mathbf{h}_4:\bot,\top,\Delta_6\vdash F_5} & \bot_L \\ & -:\top,\Delta_3,\Delta_6\vdash F_5 & \mathbf{hCut} \\ \hline \\ \frac{}{\bullet\mathbf{h}_1:\top,\Delta_3\vdash F_7} & \top_L & \frac{}{\bullet\mathbf{h}_4:(\bot,\Delta_6),F_7\vdash F_5} & \bot_L \\ \hline \\ \frac{}{-:(\top,\Delta_3),\bot,\Delta_6\vdash F_5} & \bot_L & \mathbf{Cut} \\ \hline \\ \frac{}{-:\bot,\top,\Delta_3,\Delta_6\vdash F_5} & \bot_L \\ \hline \end{array}$$

ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{F}_6} \quad \top_L \quad & \frac{\bullet}{\bullet \mathbf{h}_4: (\Delta_5, \mathbf{p}_7), \mathbf{F}_6 \vdash \mathbf{p}_7} \\ -: (\top, \Delta_3), \Delta_5, \mathbf{p}_7 \vdash \mathbf{p}_7 \\ & \xrightarrow{-: \top, \Delta_3, \Delta_5, \mathbf{p}_7 \vdash \mathbf{p}_7} I \end{array} \\ \\ \frac{\mathbf{h}_1: \Delta_3 \vdash \mathbf{p}_6}{\bullet \mathbf{h}_1: \top, \Delta_3 \vdash \mathbf{p}_6} \quad \top_L \quad & \frac{\bullet}{\bullet \mathbf{h}_4: \Delta_5, \mathbf{p}_6 \vdash \mathbf{p}_6} \quad I \\ -: (\top, \Delta_3), \Delta_5 \vdash \mathbf{p}_6 \quad & \mathbf{Cut} \\ & \xrightarrow{-: \top, \Delta_3, \Delta_5 \vdash \mathbf{p}_6} \quad \mathbf{ax/W} \end{array}$$

• Case rule \top_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_3 \vdash \top}{\bullet \mathbf{h}_1:\top,\Delta_3 \vdash \top} \;\; \top_L \quad \frac{\mathbf{h}_4:\Delta_6 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4:\Delta_6,\top \vdash \mathbf{F}_5} \quad \top_L \\ \hline -:(\top,\Delta_3),\Delta_6 \vdash \mathbf{F}_5 \\ \hline -:\top,\Delta_3,\Delta_6 \vdash \mathbf{F}_5 \end{array} \quad \mathbf{ax/W} \\ \\ \frac{\mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1:\top,\Delta_3 \vdash \mathbf{F}_7} \;\; \top_L \quad \frac{\mathbf{h}_4:\Delta_6,\mathbf{F}_7 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4:(\top,\Delta_6),\mathbf{F}_7 \vdash \mathbf{F}_5} \quad \top_L \\ \hline -:(\top,\Delta_3),\top,\Delta_6 \vdash \mathbf{F}_5 \\ \hline -:(\top,\Delta_3 \vdash \mathbf{F}_7) \quad \mathbf{ax/W} \\ \hline -:(\top,\Delta_3 \vdash \mathbf{F}_7) \quad \mathbf{ax/W} \\ \hline -:(\top,\Delta_3,\Delta_6 \vdash \mathbf{F}_5) \quad \mathbf{ax/W} \\ \hline \end{array}$$