

Modal Logic K

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Contents

| | | |
|----------|--|-----------|
| 1 | Height preserving admissibility of weakening on the left | 2 |
| 2 | Height preserving admissibility of weakening on the right | 4 |
| 3 | Measure of derivations | 6 |
| 4 | Invertibility of Rules | 8 |
| 4.1 | Status of \rightarrow_R : : Invertible | 8 |
| 4.2 | Status of \wedge_R : (Left Premise): Invertible | 9 |
| 4.3 | Status of \wedge_R (Right Premise): : Invertible | 10 |
| 4.4 | Status of \vee_R : : Invertible | 12 |
| 4.5 | Status of \perp_R : : Invertible | 13 |
| 4.6 | Status of \top_R : : Invertible | 14 |
| 4.7 | Status of K : : Non invertible | 16 |
| 4.8 | Status of \rightarrow_L : (Left Premise): Invertible | 17 |
| 4.9 | Status of \rightarrow_L (Right Premise): : Invertible | 18 |
| 4.10 | Status of \wedge_L : : Invertible | 20 |
| 4.11 | Status of \vee_L : (Left Premise): Invertible | 21 |
| 4.12 | Status of \vee_L (Right Premise): : Invertible | 22 |
| 4.13 | Status of \perp_L : : Invertible | 24 |
| 4.14 | Status of I : : Invertible | 25 |
| 4.15 | Status of \top_L : : Invertible | 26 |
| 5 | Height preserving admissibility of contraction on the left | 28 |
| 6 | Height preserving admissibility of contraction on the Right | 30 |
| 7 | Identity-Expansion | 32 |
| 8 | Cut-Elimination | 33 |
| 8.1 | Status of \rightarrow_R : OK | 33 |
| 8.2 | Status of \wedge_R : OK | 38 |
| 8.3 | Status of \vee_R : OK | 42 |
| 8.4 | Status of \perp_R : OK | 47 |
| 8.5 | Status of \top_R : OK | 51 |
| 8.6 | Status of K : OK | 55 |
| 8.7 | Status of \rightarrow_L : OK | 59 |
| 8.8 | Status of \wedge_L : OK | 62 |
| 8.9 | Status of \vee_L : OK | 66 |

| | |
|---|----|
| 8.10 Status of \perp_L : OK | 69 |
| 8.11 Status of I : OK | 71 |
| 8.12 Status of \top_L : OK | 75 |

1 Height preserving admissibility of weakening on the left

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \rightarrow F_4} \text{IH}}{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \rightarrow F_4} \text{ax}} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_4} \text{IH}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \wedge F_4} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \vee F_4} \text{IH}}{\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \vee F_4} \text{ax}} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2, F_W \vdash \perp, \Delta_3} \text{IH}}{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2, F_W \vdash \perp, \Delta_3} \text{ax}} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_W \vdash \top, \Delta_3} \top_R$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [] F_2} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Delta_5, F_W, \Box \Gamma_4 \vdash \Delta_3, [] F_2} \text{ax}}{\bullet h_1 : \Delta_5, F_W, \Box \Gamma_4 \vdash \Delta_3, [] F_2} K$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4}{\bullet h_1 : \Delta_5, F_W, F_2 \rightarrow F_3 \vdash \Delta_4} \text{IH} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_W, F_2 \rightarrow F_3 \vdash \Delta_4} \text{IH}}{\bullet h_1 : \Delta_5, F_W, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2, F_3, F_W \vdash \Delta_4} \text{IH}}{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_W, F_2 \wedge F_3 \vdash \Delta_4} \text{ax}} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2, F_W \vdash \Delta_4} \text{IH} \quad \frac{h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3, F_W \vdash \Delta_4} \text{IH}}{\bullet h_1 : \Delta_5, F_W, F_2 \vee F_3 \vdash \Delta_4} \vee_L$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L \quad \rightarrow \quad \frac{}{\bullet h_1 : \perp, \Delta_3, F_W \vdash \Delta_2} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_4, F_W, p_3 \vdash \Delta_2, p_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{h_1 : \Delta_3, F_W \vdash \Delta_2} \text{IH}}{\bullet h_1 : \top, \Delta_3, F_W \vdash \Delta_2} \top_L$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule \rightarrow_R

$$\frac{\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R}{\frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \text{ ax} \quad \frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \rightarrow F_4} \text{ IH}}{\rightarrow_R}}$$

- Case(s) rule \wedge_R

$$\frac{\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \quad \frac{h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R}{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_4, F_W} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \wedge F_4} \text{ IH}} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R}{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \vee F_4} \text{ IH}}{\vee_R}}$$

- Case(s) rule \perp_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R}{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_W}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \text{ IH}}{\perp_R}}$$

- Case(s) rule \top_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R}{\frac{h_1 : \Delta_2 \vdash \top, \Delta_3, F_W}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, F_W} \top_R}$$

- Case(s) rule K

$$\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [] F_2} K}{\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, F_W, [] F_2} \text{ ax} \quad \frac{h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, F_W, [] F_2}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, F_W, [] F_2} K} K$$

- Case(s) rule \rightarrow_L

$$\frac{\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L}{\frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2, F_W} \text{ ax} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W} \text{ ax}}{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2, F_W}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4, F_W} \text{ IH}} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{\frac{h_1 : F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L}{\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4, F_W} \text{ ax} \quad \frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4, F_W}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4, F_W} \text{ IH}}{\wedge_L}}$$

- Case(s) rule \vee_L

$$\frac{\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L}{\frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vdash \Delta_4, F_W} \text{ ax} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W} \text{ ax}}{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4, F_W}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4, F_W} \text{ IH}} \vee_L$$

- Case(s) rule \perp_L

$$\frac{\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L}{\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2, F_W}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2, F_W} \perp_L}$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_4, P_3 \vdash \Delta_2, P_3} I \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_4, P_3 \vdash \Delta_2, F_W, P_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{h_1 : \Delta_3 \vdash \Delta_2, F_W} \text{IH}}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2, F_W} \top_L$$

3 Measure of derivations

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \text{IH} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \text{IH} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \text{IH} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \text{IH} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [\Gamma_2]} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2} \text{ax}}{\bullet \bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, [\Gamma_2]} \text{IH} K$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{ax} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{ax}}{\bullet \bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{IH} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} \text{ax}}{\bullet \bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \text{IH} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vdash \Delta_4} \text{ax} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{ax}}{\bullet \bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{IH} \vee_L$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \frac{}{\bullet \bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \Delta_3 \vdash \Delta_2} \text{IH}}{\bullet \bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L$$

4 Invertibility of Rules

4.1 Status of \rightarrow_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash \Delta_7, F_2, F_6}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \text{ax/ind} \rightarrow_R$$

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{ax} \text{H}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \rightarrow F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5} \text{ax/ind} \quad \overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \rightarrow F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_5, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \rightarrow F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \rightarrow F_2), \Box F_4} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4} \text{ax}}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, F_2, \Box F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash F_4, \Delta_6, F_1 \rightarrow F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \rightarrow F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_2, F_4} \text{ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6, F_2} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7 \vdash \Delta_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \rightarrow F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6, F_2} \wedge_L$$

- Case rule \vee_L

$$\frac{\frac{h_3 : F_4, \Delta_7 \vdash \Delta_6, F_1 \rightarrow F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \rightarrow F_2} \vee_L \quad \rightarrow \quad \frac{\frac{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6, F_2}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6, F_2} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6, F_2}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6, F_2} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \rightarrow F_2} \perp_L \quad \rightarrow \quad \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4, F_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \rightarrow F_2} I \quad \rightarrow \quad \frac{}{\bullet h_3 : \Delta_6, F_1, p_5 \vdash \Delta_4, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \rightarrow F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \rightarrow F_2} \top_L \quad \rightarrow \quad \frac{\frac{h_3 : \Delta_5, F_1 \vdash \Delta_4, F_2}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4, F_2} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4, F_2} \top_L$$

4.2 Status of \wedge_R : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \quad \rightarrow \quad \frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \rightarrow F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \quad \rightarrow \quad \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{H}$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \quad \rightarrow \quad \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \quad \rightarrow \quad \frac{\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \quad \rightarrow \quad \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \wedge F_2), [] F_4} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4} \text{ ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, [] F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash F_4, \Delta_6, F_1 \wedge F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \wedge F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \wedge F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \wedge F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \wedge F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash \Delta_4, F_1} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \top_L$$

4.3 Status of \wedge_R (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_4} H$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5, F_6}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \vee F_6} \text{ax/ind} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_2}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_2} \text{ax/ind} \perp_R$$

- Case rule \top_R

$$\overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightarrow \overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \wedge F_2), [\Gamma_4]} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_2, [\Gamma_4]} \text{ax} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash F_4, \Delta_6, F_1 \wedge F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \wedge F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash \Delta_6, F_2, F_4}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_2} \text{ax/ind} \frac{\overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_2}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_2} \text{ax/ind} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \wedge F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_2}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_2} \text{ax/ind} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \wedge F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_2}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_2} \text{ax/ind} \frac{\overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_2}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_2} \text{ax/ind} \vee_L$$

- Case rule \perp_L

$$\overline{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \perp_L \rightarrow \overline{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_2} \perp_L$$

- Case rule I

$$\overline{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \wedge F_2} I \rightarrow \overline{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash \Delta_4, F_2}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_2} \text{ax/ind} \top_L$$

4.4 Status of \vee_R : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_2, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \vee F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \vee F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \vee F_6} \vee_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \text{H}$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \vee F_2} \perp_R \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1, F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1, F_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \vee F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1, F_2} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \vee F_2), \Box F_4} K \rightarrow \frac{\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, F_2, \Box F_4} \text{ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, F_2, \Box F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash F_4, \Delta_6, F_1 \vee F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \vee F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \vee F_2} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_2, F_4}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1, F_2} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1, F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1, F_2} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7 \vdash \Delta_6, F_1 \vee F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \vee F_2} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1, F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1, F_2} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7 \vdash \Delta_6, F_1 \vee F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \vee F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \vee F_2} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1, F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1, F_2} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1, F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1, F_2} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \vee F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1, F_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \vee F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, F_2, p_5} I$$

- Case rule \top_L

$$\frac{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \vee F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \vee F_2} \top_L}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1, F_2} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1, F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1, F_2} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1, F_2} \top_L$$

4.5 Status of \perp_R : : Invertible

- Case rule \rightarrow_R

$$\frac{\frac{h_1 : F_3, \Delta_2 \vdash \perp, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \text{ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \perp, F_3, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \wedge F_4} \wedge_R}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \text{ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R$$

- Case rule \vee_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \perp, F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \vee F_4} \vee_R}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \text{ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R$$

- Case rule \perp_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{H}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \perp, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case rule K

$$\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash (\perp, \Delta_3), [\Box F_2]} K}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash (\perp, \Delta_3), [\Box F_2]} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, [\Box F_2]} \text{ax}}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, [\Box F_2]} K$$

- Case rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash \perp, F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \perp, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \perp, \Delta_4} \rightarrow_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \perp, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \perp, \Delta_4} \wedge_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L$$

- Case rule \vee_L

$$\frac{h_1 : F_2, \Delta_5 \vdash \perp, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \perp, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \perp, \Delta_4} \vee_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \perp, \Delta_2} \perp_L \quad \rightarrow \quad \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_4 \vdash p_3, \perp, \Delta_2} I \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I$$

- Case rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \perp, \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \perp, \Delta_2} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \text{ax/ind}}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L$$

4.6 Status of \top_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash \top, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \quad \rightarrow \quad \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \top, F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash \top, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \wedge F_4} \wedge_R \quad \rightarrow \quad \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \top, F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \vee F_4} \vee_R \quad \rightarrow \quad \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \top, \Delta_3} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \text{trivial}$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash (\top, \Delta_3), [\Box F_2]} K \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash \top, F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \top, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \top, \Delta_4} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \top, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \top, \Delta_4} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : F_2, \Delta_5 \vdash \top, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \top, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \top, \Delta_4} \vee_L \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \top, \Delta_2} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_4 \vdash p_3, \top, \Delta_2} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \top, \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \top, \Delta_2} \top_L \rightarrow \text{trivial}$$

4.7 Status of K : : Non invertible

- Case rule \rightarrow_R

$$\frac{h_2 : \Box\Gamma_6, F_3, \Delta_7 \vdash F_4, \Delta_5, []F_1}{\bullet h_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []F_1), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_2 : unbox(\Box\Gamma_6) \vdash F_1}}{\bullet h_2 : unbox(\Box\Gamma_6) \vdash F_1} \text{ax/ind}_H$$

- Case rule \wedge_R

$$\frac{h_2 : \Box\Gamma_6, \Delta_7 \vdash F_3, \Delta_5, []F_1 \quad h_2 : \Box\Gamma_6, \Delta_7 \vdash F_4, \Delta_5, []F_1}{\bullet h_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []F_1), F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_2 : unbox(\Box\Gamma_6) \vdash F_1}}{\bullet h_2 : unbox(\Box\Gamma_6) \vdash F_1} \text{ax/ind}_H$$

- Case rule \vee_R

$$\frac{h_2 : \Box\Gamma_6, \Delta_7 \vdash F_3, F_4, \Delta_5, []F_1}{\bullet h_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []F_1), F_3 \vee F_4} \vee_R \rightarrow \frac{\overline{h_2 : unbox(\Box\Gamma_6) \vdash F_1}}{\bullet h_2 : unbox(\Box\Gamma_6) \vdash F_1} \text{ax/ind}_H$$

- Case rule \perp_R

$$\frac{h_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []F_1}{\bullet h_2 : \Box\Gamma_4, \Delta_5 \vdash \perp, \Delta_3, []F_1} \perp_R \rightarrow \frac{\overline{h_2 : unbox(\Box\Gamma_4) \vdash F_1}}{\bullet h_2 : unbox(\Box\Gamma_4) \vdash F_1} \text{ax/ind}_H$$

- Case rule \top_R

$$\frac{}{\bullet h_2 : \Box\Gamma_4, \Delta_5 \vdash \top, \Delta_3, []F_1} \top_R \rightarrow \frac{}{\bullet h_2 : unbox(\Box\Gamma_4) \vdash F_1} \text{fail}$$

- Case rule K

$$\frac{h_2 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash F_3}{\bullet h_2 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []F_1), []F_3} K \rightarrow \frac{}{\bullet h_2 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_7) \vdash F_1} \text{fail}$$

$$\frac{h_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_5) \vdash F_2}{\bullet h_1 : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_3, []F_2} K \rightarrow \frac{}{\bullet h_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_6) \vdash F_2} \text{fail}$$

- Case rule \rightarrow_L

$$\frac{h_2 : \Box\Gamma_6, \Delta_7 \vdash F_3, \Delta_5, []F_1 \quad h_2 : \Box\Gamma_6, F_4, \Delta_7 \vdash \Delta_5, []F_1}{\bullet h_2 : (\Box\Gamma_6, \Delta_7), F_3 \rightarrow F_4 \vdash \Delta_5, []F_1} \rightarrow_L \rightarrow \frac{\overline{h_2 : unbox(\Box\Gamma_6) \vdash F_1}}{\bullet h_2 : unbox(\Box\Gamma_6) \vdash F_1} \text{ax/ind}_H$$

- Case rule \wedge_L

$$\frac{h_2 : \Box\Gamma_6, F_3, F_4, \Delta_7 \vdash \Delta_5, []F_1}{\bullet h_2 : (\Box\Gamma_6, \Delta_7), F_3 \wedge F_4 \vdash \Delta_5, []F_1} \wedge_L \rightarrow \frac{\overline{h_2 : unbox(\Box\Gamma_6) \vdash F_1}}{\bullet h_2 : unbox(\Box\Gamma_6) \vdash F_1} \text{ax/ind}_H$$

- Case rule \vee_L

$$\frac{h_2 : \Box\Gamma_6, F_3, \Delta_7 \vdash \Delta_5, []F_1 \quad h_2 : \Box\Gamma_6, F_4, \Delta_7 \vdash \Delta_5, []F_1}{\bullet h_2 : (\Box\Gamma_6, \Delta_7), F_3 \vee F_4 \vdash \Delta_5, []F_1} \vee_L \rightarrow \frac{\overline{h_2 : unbox(\Box\Gamma_6) \vdash F_1}}{\bullet h_2 : unbox(\Box\Gamma_6) \vdash F_1} \text{ax/ind}_H$$

- Case rule \perp_L

$$\frac{}{\bullet h_2 : \perp, \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [] F_1} \perp_L \rightarrow \frac{}{\bullet h_2 : unbox(\Box \Gamma_4) \vdash F_1} \text{fail}$$

- Case rule I

$$\frac{}{\bullet h_2 : p_4, \Box \Gamma_5, \Delta_6 \vdash p_4, \Delta_3, [] F_1} I \rightarrow \frac{}{\bullet h_2 : unbox(\Box \Gamma_5) \vdash F_1} \text{fail}$$

- Case rule \top_L

$$\frac{h_2 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [] F_1}{\bullet h_2 : \top, \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [] F_1} \top_L \rightarrow \frac{h_2 : unbox(\Box \Gamma_4) \vdash F_1}{\bullet h_2 : unbox(\Box \Gamma_4) \vdash F_1} \text{ax/ind}$$

4.8 Status of \rightarrow_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_7, \Delta_7, F_1 \rightarrow F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1, F_5}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \rightarrow F_5} \text{ax/ind}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4} \text{ax/ind} \quad \frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_5}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4, F_5}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \vee F_5} \text{ax/ind}$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{h_3 : \Delta_5 \vdash \Delta_4, F_1}{\bullet h_3 : \Delta_5 \vdash \perp, \Delta_4, F_1} \text{ax/ind}$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5 \vdash \top, \Delta_4, F_1} \top_R$$

- Case rule K

$$\frac{h_3 : unbox(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, [] F_4} K \rightarrow \frac{h_3 : unbox(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, [] F_4} \text{ax}$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \rightarrow_L$$

$$\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{ax}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{H}$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \rightarrow F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \top_L$$

4.9 Status of \rightarrow_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_5, F_2 \vdash \Delta_4} \text{ ax/ind}}{\bullet h_3 : \Delta_5, F_2 \vdash \perp, \Delta_4} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_2 \vdash \top, \Delta_4} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, [] F_4} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4} \text{ ax}}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_6 \vdash \Delta_5, [] F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

$$\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{ ax}}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} H$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \rightarrow F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5, F_2 \vdash \Delta_4} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

4.10 Status of \wedge_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_4, \Delta_7, F_1 \wedge F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash F_4, \Delta_6 \quad h_3 : \Delta_7, F_1 \wedge F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash F_4, F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \vee F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \wedge F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{h_3 : \Delta_5, F_1, F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1, F_2 \vdash \perp, \Delta_4} \text{ ax/ind}}{\bullet h_3 : \Delta_5, F_1, F_2 \vdash \perp, \Delta_4} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \wedge F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_1, F_2 \vdash \top, \Delta_4} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \wedge F_2 \vdash \Delta_5, [] F_4} K \rightarrow \frac{\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Delta_7, F_1, F_2, \Box \Gamma_6 \vdash \Delta_5, [] F_4} \text{ ax}}{\bullet h_3 : \Delta_7, F_1, F_2, \Box \Gamma_6 \vdash \Delta_5, [] F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} \text{ ax}}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} \text{ H}$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \wedge F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, F_2, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \top_L \rightarrow \frac{h_3 : \Delta_5, F_1, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash \Delta_4} \top_L \text{ ax/ind}$$

4.11 Status of \vee_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \text{ ax/ind}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : \Delta_7, F_1 \vee F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \text{ ax/ind} \quad h_3 : \Delta_7, F_1 \vdash \Delta_6, F_5 \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4, F_5 \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{h_3 : \Delta_5, F_1 \vdash \Delta_4 \text{ ax/ind}}{\bullet h_3 : \Delta_5, F_1 \vdash \perp, \Delta_4} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_1 \vdash \top, \Delta_4} \top_R$$

- Case rule K

$$\frac{h_3 : unbox(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, [\Gamma_4]} K \rightarrow \frac{h_3 : unbox(\Box \Gamma_6) \vdash F_4 \text{ ax}}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, [\Gamma_4]} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \vee F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_1 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \top_L$$

4.12 Status of \vee_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : \Delta_7, F_1 \vee F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_5, F_2 \vdash \Delta_4} \text{ ax/ind}}{\bullet h_3 : \Delta_5, F_2 \vdash \perp, \Delta_4} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_2 \vdash \top, \Delta_4} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, [] F_4} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4} \text{ ax}}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_6 \vdash \Delta_5, [] F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{ ax}}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} H$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \vee F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5, F_2 \vdash \Delta_4} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

4.13 Status of \perp_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \perp, F_2, \Delta_5 \vdash F_3, \Delta_4}{\bullet h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \perp, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \perp, \Delta_5 \vdash F_3, \Delta_4}{\bullet h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \perp, \Delta_5 \vdash F_2, F_3, \Delta_4}{\bullet h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \perp, \Delta_3 \vdash \perp, \Delta_2} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \top, \Delta_2} \top_R \rightarrow \text{trivial}$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \perp, \Delta_5 \vdash \Delta_3, [\Gamma_4] F_2} K \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \perp, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \perp, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : \perp, F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : \perp, F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : \perp, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \perp, \Delta_4 \vdash p_3, \Delta_2} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \perp, \Delta_3 \vdash \Delta_2} \top_L \rightarrow \text{trivial}$$

4.14 Status of I : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : F_2, \Delta_6, p_5 \vdash F_3, \Delta_4, p_5}{\bullet h_1 : \Delta_6, p_5 \vdash (\Delta_4, p_5), F_2 \rightarrow F_3} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_6, p_5 \vdash F_2, \Delta_4, p_5 \quad h_1 : \Delta_6, p_5 \vdash F_3, \Delta_4, p_5}{\bullet h_1 : \Delta_6, p_5 \vdash (\Delta_4, p_5), F_2 \wedge F_3} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_6, p_5 \vdash F_2, F_3, \Delta_4, p_5}{\bullet h_1 : \Delta_6, p_5 \vdash (\Delta_4, p_5), F_2 \vee F_3} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3}{\bullet h_1 : \Delta_4, p_3 \vdash \perp, \Delta_2, p_3} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \top, \Delta_2, p_3} \top_R \rightarrow \text{trivial}$$

- Case rule K

$$\frac{h_1 : unbox(\Box \Gamma_5) \vdash F_2}{\bullet h_1 : \Box \Gamma_5, \Delta_6, p_4 \vdash (\Delta_3, p_4), \Box F_2} K \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \Delta_6, p_5 \vdash F_2, \Delta_4, p_5 \quad h_1 : F_3, \Delta_6, p_5 \vdash \Delta_4, p_5}{\bullet h_1 : (\Delta_6, p_5), F_2 \rightarrow F_3 \vdash \Delta_4, p_5} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_6, p_5 \vdash \Delta_4, p_5}{\bullet h_1 : (\Delta_6, p_5), F_2 \wedge F_3 \vdash \Delta_4, p_5} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : F_2, \Delta_6, P_5 \vdash \Delta_4, P_5 \quad h_1 : F_3, \Delta_6, P_5 \vdash \Delta_4, P_5}{\bullet h_1 : (\Delta_6, P_5), F_2 \vee F_3 \vdash \Delta_4, P_5} \vee_L \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_4, P_3 \vdash \Delta_2, P_3} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : P_4, \Delta_5, P_3 \vdash P_4, \Delta_2, P_3} I \rightarrow \text{trivial}$$

$$\frac{}{\bullet h_1 : P_3, \Delta_4 \vdash P_3, \Delta_2} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \Delta_4, P_3 \vdash \Delta_2, P_3}{\bullet h_1 : \top, \Delta_4, P_3 \vdash \Delta_2, P_3} \top_L \rightarrow \text{trivial}$$

4.15 Status of \top_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \top, F_2, \Delta_5 \vdash F_3, \Delta_4}{\bullet h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_2 \vdash \Delta_4, F_3} \text{ax/ind}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_1 : \top, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \top, \Delta_5 \vdash F_3, \Delta_4}{\bullet h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R \rightarrow \frac{\frac{}{h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{ax/ind} \quad \frac{}{h_1 : \Delta_5 \vdash \Delta_4, F_3} \text{ax/ind}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R$$

- Case rule \vee_R

$$\frac{h_1 : \top, \Delta_5 \vdash F_2, F_3, \Delta_4}{\bullet h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \vee_R \rightarrow \frac{\frac{}{h_1 : \Delta_5 \vdash \Delta_4, F_2, F_3} \text{ax/ind}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \vee_R$$

- Case rule \perp_R

$$\frac{h_1 : \top, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \perp, \Delta_2} \perp_R \rightarrow \frac{\frac{}{h_1 : \Delta_3 \vdash \Delta_2} \text{ax/ind}}{\bullet h_1 : \Delta_3 \vdash \perp, \Delta_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \top, \Delta_3 \vdash \top, \Delta_2} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_3 \vdash \top, \Delta_2} \top_R$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \top, \Delta_5 \vdash \Delta_3, []F_2} K \rightarrow \frac{\overline{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}^{\text{ax}}}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, []F_2} K$$

- Case rule \rightarrow_L

$$\frac{h_1 : \top, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \top, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\overline{h_1 : \Delta_5 \vdash \Delta_4, F_2}^{\text{ax/ind}} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4}^{\text{ax/ind}}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_1 : \top, F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}^{\text{ax/ind}}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L$$

- Case rule \vee_L

$$\frac{h_1 : \top, F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : \top, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2 \vdash \Delta_4}^{\text{ax/ind}} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4}^{\text{ax/ind}}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L$$

- Case rule \perp_L

$$\overline{\bullet h_1 : \perp, \top, \Delta_3 \vdash \Delta_2}^{\perp_L} \rightarrow \overline{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2}^{\perp_L}$$

- Case rule I

$$\overline{\bullet h_1 : p_3, \top, \Delta_4 \vdash p_3, \Delta_2}^I \rightarrow \overline{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3}^I$$

- Case rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \rightarrow \frac{\overline{h_1 : \Delta_3 \vdash \Delta_2}^{\text{ax}}}{\bullet h_1 : \Delta_3 \vdash \Delta_2} \text{H}$$

5 Height preserving admissibility of contraction on the left

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_2, \Delta_5, \Delta_6, \Delta_6 \vdash F_3, \Delta_4}{\bullet h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2 \vdash \Delta_4, F_3}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \vdash \Delta_4, F_3} \text{IH}}{\bullet h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \rightarrow F_3} \text{ax} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash F_2, \Delta_4 \quad h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash F_3, \Delta_4}{\bullet h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2} \text{IH} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_3}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_3} \text{IH}}{\bullet h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash F_2, F_3, \Delta_4}{\bullet h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2 \vee F_3} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2, F_3}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \vee F_3} \text{IH}}{\bullet h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \vee F_3} \text{ax} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \perp, \Delta_2} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{h_1 : \Delta_3, \Delta_4 \vdash \Delta_2} \text{IH}}{\bullet h_1 : \Delta_3, \Delta_4 \vdash \perp, \Delta_2} \text{ax} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \top, \Delta_2} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_3, \Delta_4 \vdash \top, \Delta_2} \top_R$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_6) \vdash F_2}{\bullet h_1 : (\Box\Gamma_4, \Delta_7), (\Box\Gamma_5, \Box\Gamma_6, \Delta_8) \vdash F_3, \Box\Gamma_5, \Box\Gamma_6, \Delta_8 \vdash \Delta_3, \Box\Gamma_5} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_6) \vdash F_2}{h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_6) \vdash F_2} \text{IH}}{\bullet h_1 : \Delta_7, \Delta_8, \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_6 \vdash \Delta_3, \Box\Gamma_5} K \text{ax}$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2 \rightarrow F_3 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5, \Delta_6, \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, (\Delta_6, F_2 \rightarrow F_3), \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2, F_2}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2, F_2} \text{IH} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_3 \vdash F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_3 \vdash \Delta_4} \text{IH}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4} \text{inv-th/ax} \rightarrow_L$$

$$\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4}{\bullet h_1 : (\Delta_5, F_2 \rightarrow F_3), \Delta_6, \Delta_6 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2} \text{IH} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_3 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4} \text{IH} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_5, \Delta_6, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, (\Delta_6, F_2 \wedge F_3), \Delta_6, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2, F_2, F_3 \vdash F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2, F_3 \vdash \Delta_4} \text{IH} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4} \text{inv-th/ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L$$

$$\frac{h_1 : F_2, F_3, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4}{\bullet h_1 : (\Delta_5, F_2 \wedge F_3), \Delta_6, \Delta_6 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2, F_3 \vdash F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2, F_3 \vdash \Delta_4} \text{IH} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L$$

- Case(s) rule \vee_L

$$\begin{array}{c}
\frac{h_1 : F_2, \Delta_5, \Delta_6, \Delta_6, F_2 \vee F_3 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5, \Delta_6, \Delta_6, F_2 \vee F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, (\Delta_6, F_2 \vee F_3), \Delta_6, F_2 \vee F_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2, F_2 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2 \vdash \Delta_4} \text{inv-th/ax} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_3, F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_3 \vdash \Delta_4} \text{inv-th/ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \vee F_3 \vdash \Delta_4} \text{IH} \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_2, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4}{\bullet h_1 : (\Delta_5, F_2 \vee F_3), \Delta_6, \Delta_6 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2 \vdash \Delta_4} \text{ax} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_3 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \vee F_3 \vdash \Delta_4} \text{IH} \vee_L
\end{array}$$

- Case(s) rule \perp_L

$$\begin{array}{c}
\frac{}{\bullet h_1 : (\perp, \Delta_3), \Delta_4, \Delta_4 \vdash \Delta_2} \perp_L \quad \rightarrow \quad \frac{}{\bullet h_1 : \perp, \Delta_3, \Delta_4 \vdash \Delta_2} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_3, (\perp, \Delta_4), \perp, \Delta_4 \vdash \Delta_2} \perp_L \quad \rightarrow \quad \frac{}{\bullet h_1 : \perp, \Delta_3, \Delta_4 \vdash \Delta_2} \perp_L
\end{array}$$

- Case(s) rule I

$$\begin{array}{c}
\frac{}{\bullet h_1 : (\Delta_4, p_3), \Delta_5, \Delta_5 \vdash \Delta_2, p_3} I \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_4, \Delta_5, p_3 \vdash \Delta_2, p_3} I
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4, (\Delta_5, p_3), \Delta_5, p_3 \vdash \Delta_2, p_3} I \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_4, \Delta_5, p_3 \vdash \Delta_2, p_3} I
\end{array}$$

- Case(s) rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet h_1 : (\top, \Delta_3), \Delta_4, \Delta_4 \vdash \Delta_2} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{h_1 : \Delta_3, \Delta_4 \vdash \Delta_2} \text{ax}}{\bullet h_1 : \top, \Delta_3, \Delta_4 \vdash \Delta_2} \text{IH} \top_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet h_1 : \Delta_3, (\top, \Delta_4), \top, \Delta_4 \vdash \Delta_2} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{h_1 : \Delta_3, \Delta_4 \vdash \Delta_2} \text{inv-th/ax}}{\bullet h_1 : \top, \Delta_3, \Delta_4 \vdash \Delta_2} \text{IH} \top_L
\end{array}$$

6 Height preserving admissibility of contraction on the Right

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5, \Delta_6, \Delta_6, F_3 \rightarrow F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, (\Delta_6, F_3 \rightarrow F_4), \Delta_6, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_3 \vdash \Delta_5, \Delta_6, \Delta_6, F_4, F_4}{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, \Delta_6, F_4, F_4} \text{IH-Mutual} \quad \frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \rightarrow F_4} \text{IH}}{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, \Delta_6, F_4} \text{inv-th/ax} \rightarrow_R$$

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5, \Delta_6, \Delta_6}{\bullet h_1 : \Delta_2 \vdash (\Delta_5, F_3 \rightarrow F_4), \Delta_6, \Delta_6} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, \Delta_6, F_4}{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, F_4} \text{IH} \quad \frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, \Delta_6, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \rightarrow F_4} \text{ax}}{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, \Delta_6, F_4} \text{IH} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5, \Delta_6, \Delta_6, F_3 \wedge F_4 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5, \Delta_6, \Delta_6, F_3 \wedge F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, (\Delta_6, F_3 \wedge F_4), \Delta_6, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_3}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_4, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \wedge F_4} \text{inv-th/ax} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5, \Delta_6, \Delta_6 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5, \Delta_6, \Delta_6}{\bullet h_1 : \Delta_2 \vdash (\Delta_5, F_3 \wedge F_4), \Delta_6, \Delta_6} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \wedge F_4} \text{ax} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5, \Delta_6, \Delta_6, F_3 \vee F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, (\Delta_6, F_3 \vee F_4), \Delta_6, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_3, F_4, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3, F_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_4, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \vee F_4} \text{inv-th/ax}}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \vee F_4} \vee_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5, \Delta_6, \Delta_6}{\bullet h_1 : \Delta_2 \vdash (\Delta_5, F_3 \vee F_4), \Delta_6, \Delta_6} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3, F_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \vee F_4} \text{ax}}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \vee F_4} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4, \Delta_4}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_3), \Delta_4, \Delta_4} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4, \Delta_4}{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, \Delta_4} \text{ax}}{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4} \perp_R$$

$$\frac{h_1 : \Delta_2 \vdash \perp, \Delta_3, \Delta_4, \Delta_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, (\perp, \Delta_4), \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4, \Delta_4}{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, \Delta_4} \text{inv-th/ax}}{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_3), \Delta_4, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, \Delta_4} \top_R$$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \Delta_3, (\top, \Delta_4), \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, \Delta_4} \top_R$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_5) \vdash F_2}{\bullet h_1 : \Box \Gamma_5, \Delta_6 \vdash \Delta_3, (\Delta_4, [\Box F_2]), \Delta_4, [\Box F_2]} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_5) \vdash F_2}{\bullet h_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_3, \Delta_4, [\Box F_2]} \text{ax}}{\bullet h_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_3, \Delta_4, [\Box F_2]} K$$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_5) \vdash F_2}{\bullet h_1 : \Box \Gamma_5, \Delta_6 \vdash (\Delta_3, [\Box F_2]), \Delta_4, \Delta_4} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_5) \vdash F_2}{\bullet h_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_3, \Delta_4, [\Box F_2]} \text{ax}}{\bullet h_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_3, \Delta_4, [\Box F_2]} K$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_6 \vdash F_2, \Delta_4, \Delta_5, \Delta_5 \quad h_1 : F_3, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5}{\bullet h_1 : \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4, \Delta_5, \Delta_5} \rightarrow_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5, F_2}{h_1 : \Delta_6 \vdash \Delta_4, \Delta_5, F_2} \text{ax} \quad \frac{h_1 : \Delta_6, F_3 \vdash \Delta_4, \Delta_5, \Delta_5}{h_1 : \Delta_6, F_3 \vdash \Delta_4, \Delta_5} \text{IH}}{\bullet h_1 : \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4, \Delta_5} \text{IH} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5}{\bullet h_1 : \Delta_6, F_2 \wedge F_3 \vdash \Delta_4, \Delta_5, \Delta_5} \wedge_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_6, F_2, F_3 \vdash \Delta_4, \Delta_5, \Delta_5}{h_1 : \Delta_6, F_2, F_3 \vdash \Delta_4, \Delta_5} \text{ax}}{\bullet h_1 : \Delta_6, F_2 \wedge F_3 \vdash \Delta_4, \Delta_5} \text{IH} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : F_2, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5 \quad h_1 : F_3, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5}{\bullet h_1 : \Delta_6, F_2 \vee F_3 \vdash \Delta_4, \Delta_5, \Delta_5} \vee_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_6, F_2 \vdash \Delta_4, \Delta_5, \Delta_5}{h_1 : \Delta_6, F_2 \vdash \Delta_4, \Delta_5} \text{ax} \quad \frac{h_1 : \Delta_6, F_3 \vdash \Delta_4, \Delta_5, \Delta_5}{h_1 : \Delta_6, F_3 \vdash \Delta_4, \Delta_5} \text{ax}}{\bullet h_1 : \Delta_6, F_2 \vee F_3 \vdash \Delta_4, \Delta_5} \text{IH} \vee_L$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3} \perp_L \quad \rightarrow \quad \frac{}{\bullet h_1 : \perp, \Delta_4 \vdash \Delta_2, \Delta_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_5, p_4 \vdash \Delta_2, (\Delta_3, p_4), \Delta_3, p_4} I \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_5, p_4 \vdash \Delta_2, \Delta_3, p_4} I$$

$$\frac{}{\bullet h_1 : \Delta_5, p_4 \vdash (\Delta_2, p_4), \Delta_3, \Delta_3} I \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_5, p_4 \vdash \Delta_2, \Delta_3, p_4} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3}{\bullet h_1 : \top, \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3}{h_1 : \Delta_4 \vdash \Delta_2, \Delta_3} \text{ax}}{\bullet h_1 : \top, \Delta_4 \vdash \Delta_2, \Delta_3} \text{IH} \top_L$$

7 Identity-Expansion

$$\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : \Box F_0 \vdash \Box F_0} K$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0, F_1} W}{\frac{- : F_0 \vee F_1 \vdash F_0, F_1}{- : F_0 \vee F_1 \vdash F_0 \vee F_1} \vee_R} \vee_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_1 \vdash F_0} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{\frac{- : F_0, F_1 \vdash F_0 \wedge F_1}{- : F_0 \wedge F_1 \vdash F_0 \wedge F_1} \wedge_L} \wedge_R$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{\frac{- : F_0, F_0 \rightarrow F_1 \vdash F_1}{- : F_0 \rightarrow F_1 \vdash F_0 \rightarrow F_1} \rightarrow_R} \rightarrow_L$$

$$\overline{- : \top \vdash \top} \top_R$$

$$\overline{- : \perp \vdash \perp} \perp_L$$

8 Cut-Elimination

8.1 Status of \rightarrow_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{h_1 : F_6, \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \rightarrow F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_{10}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}}{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}} \text{ax/W}}{\frac{- : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}}{\rightarrow_R} \text{hCut}} \\
\frac{\frac{h_1 : F_7, \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8} \rightarrow_R}{\frac{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_7 \vdash \Delta_{12}, F_{11}, F_{13}, F_8}{\bullet h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8}{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8} \text{ax/W}}{\frac{- : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8}{\rightarrow_R} \text{hCut}} \\
\frac{\frac{h_1 : F_8, \Delta_{12} \vdash F_9, F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{10}, F_8 \rightarrow F_9), F_{11}} \rightarrow_R \quad \frac{h_7 : F_8, F_{11}, \Delta_{12} \vdash F_9, \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R}{\frac{- : \Delta_{12} \vdash \Delta_{10}, F_8 \rightarrow F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, F_{11}, F_9}{\bullet h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, F_{11}, F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9}{\bullet h_7 : \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} \text{ax/W}}{\frac{- : \Delta_{12}, F_8 \vdash \Delta_{10}, F_9}{\rightarrow_R} \text{hCut}}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{h_1 : F_6, \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \wedge F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9, \Delta_{11} \quad h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \rightarrow_R}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9} \text{ax/W}}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9}{\rightarrow_R} \text{hCut}} \\
\frac{\frac{h_1 : F_7, \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8} \rightarrow_R}{\frac{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{13}, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{13}, F_8, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}}{\rightarrow_R} \text{hCut}}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \vee F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9, F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_{10}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9, F_6 \rightarrow F_7} \text{inv-th/ax}}{\rightarrow_R} \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}, F_9}{\text{ax/W}} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9 \quad \text{hCut} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \quad \vee_R \\
\rightarrow \\
\frac{h_1 : F_7, \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \vee F_{11}), F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, F_{11}, \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \rightarrow F_8} \vee_R \\
\hline
- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{13}, F_8, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{13}, F_8, F_{10} \vee F_{11}} \text{ax/W}}{\rightarrow_R} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_{11}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11}} \text{inv-th/ax} \\
\hline
- : \Delta_{14}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11} \quad \text{hCut} \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8, F_{10} \vee F_{11} \quad \vee_R \\
\rightarrow_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{10} \vdash F_7, \perp, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_9} \perp_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_9 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \rightarrow F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_9 \quad \text{hCut} \\
\rightarrow \\
\frac{h_1 : F_7, \Delta_{12} \vdash F_8, F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\perp, \Delta_{10}), F_7 \rightarrow F_8), F_{11}} \rightarrow_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\perp, \Delta_{10}), F_7 \rightarrow F_8} \perp_R \\
\hline
- : \Delta_{12} \vdash (\perp, \Delta_{10}), F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{12} \vdash \perp, \Delta_{10}, F_{11}, F_7 \rightarrow F_8}{\text{ax/W}} \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8} \text{ax/W} \\
\hline
- : \Delta_{12} \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8 \quad \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{10} \vdash F_7, \top, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_9} \top_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_9 \\
\rightarrow \\
- : \Delta_{10} \vdash \top, \Delta_9 \quad \top_R \\
\rightarrow \\
\frac{h_1 : F_7, \Delta_{12} \vdash F_8, F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), F_7 \rightarrow F_8), F_{11}} \rightarrow_R \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8} \top_R \\
\hline
- : \Delta_{12} \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8 \\
\rightarrow \\
- : \Delta_{12} \vdash \top, \Delta_{10}, F_7 \rightarrow F_8 \quad \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : F_6, \Box \Gamma_{11}, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_9}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \Box F_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_{11}) \vdash F_9}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{12}), F_6 \rightarrow F_7 \vdash \Delta_{10}, \Box F_9} K \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \Box F_9 \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_9}{\text{ax/W}} \\
\hline
- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_9 \quad K
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Box \Gamma_{13}, \Delta_{14} \vdash F_8, \Box F_{12}, \Delta_{11}, \Box F_{10} \quad \bullet h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8), \Box F_{12}}{\rightarrow_R} \quad \frac{h_9 : unbox(\Box \Gamma_{13}), unbox(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{13}, \Delta_{14}), \Box F_{12} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8} \quad K \\
\hline
- : \Box \Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_8, \Box F_{10}}{ax/W} \quad \frac{h_9 : unbox(\Box F_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}} \quad K}{\frac{- : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}}{hCut}} \quad \frac{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, \Box F_{10}, F_7 \rightarrow F_8}{\rightarrow_R} \\
\rightarrow \\
\frac{h_1 : F_7, \Box \Gamma_{12}, \Delta_{14} \vdash F_8, F_{13}, \Delta_{11}, \Box F_{10} \quad \bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8), F_{13}}{\rightarrow_R} \quad \frac{h_9 : unbox(\Box \Gamma_{12}) \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8} \quad K \\
\hline
- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- : unbox(\Box \Gamma_{12}) \vdash F_{10}}{ax/W} \quad K \\
\hline
- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_{10}, F_7 \rightarrow F_8
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_7, \Delta_{11} \quad \bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7}{\rightarrow_R} \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \quad K \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}}{\rightarrow_R} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\rightarrow_R} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{hCut} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \quad \rightarrow_L \\
\rightarrow \\
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \\
\rightarrow \\
\frac{h_1 : F_7, \Delta_{10} \vdash F_8, \Delta_9 \quad \bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \rightarrow F_8}{\rightarrow_R} \quad \frac{h_6 : \Delta_{10} \vdash F_7, \Delta_9 \quad h_6 : F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \quad K \\
\hline
- : \Delta_{10} \vdash \Delta_9 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7}{ax/W} \quad \frac{- : \Delta_{10}, F_7 \vdash \Delta_9, F_8}{ax/W} \quad \frac{- : \Delta_{10}, F_7, F_8 \vdash \Delta_9}{sCut} \quad \frac{- : \Delta_{10}, F_7 \vdash \Delta_9}{sCut} \\
\hline
- : \Delta_{10} \vdash \Delta_9 \\
\rightarrow \\
\frac{h_1 : F_7, \Delta_{13} \vdash F_8, F_{10} \rightarrow F_{11}, \Delta_{12} \quad \bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{10} \rightarrow F_{11}}{\rightarrow_R} \quad \frac{h_9 : \Delta_{13} \vdash F_{10}, \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \quad K \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \rightarrow F_{11}}{ax/W} \quad \frac{h_9 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{\rightarrow_L} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{hCut} \\
\hline
- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8 \quad \rightarrow_R \\
\rightarrow \\
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{h_1 : F_7, \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_8, F_{13}, \Delta_{12} \quad \bullet h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{13}}{\rightarrow_R} \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \quad K \\
\hline
- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}, F_8}{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8}{\rightarrow_L} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8}{hCut} \\
\hline
- : \Delta_{14}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8 \quad \rightarrow_R \\
\rightarrow \\
- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11} \quad \bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \wedge_L \\
\hline
- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_{10}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_{10}, F_9), F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \\
\hline
- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11} \quad \wedge_L \\
- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \quad \text{hCut} \\
\hline
\frac{h_1 : F_7, \Delta_{13} \vdash F_8, F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{10} \wedge F_{11}} \rightarrow_R \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \wedge_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \\
\hline
h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11} \quad \text{ax/W} \quad \wedge_L \\
- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8 \quad \text{hCut} \\
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \rightarrow_R \\
\hline
\frac{h_1 : F_7, \Delta_{14}, F_{10} \wedge F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \wedge F_{11}), F_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \wedge_L \\
\hline
- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \\
\hline
h_1 : \Delta_{14}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_{13}, F_8 \quad \text{ax/W} \quad \wedge_L \\
- : \Delta_{14}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8 \quad \text{hCut} \\
- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \rightarrow_R \\
\hline
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \vee_L \\
\hline
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9), F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \\
\hline
- : \Delta_{12}, F_9 \vdash \Delta_{11} \quad \text{hCut} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \\
\hline
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \quad \vee_L \\
\hline
\frac{h_1 : F_7, \Delta_{13} \vdash F_8, F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{10} \vee F_{11}} \rightarrow_R \quad \frac{h_9 : F_{10}, \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \vee_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{h_9 : \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8} \text{inv-th/ax} \\
\hline
h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11} \quad \text{ax/W} \quad \vee_L \\
- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8 \quad \text{hCut} \\
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \rightarrow_R \\
\hline
\frac{h_1 : F_7, \Delta_{14}, F_{10} \vee F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \vee F_{11}), F_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \vee_L \\
\hline
- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \text{inv-th/ax} \\
\hline
h_1 : \Delta_{14}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_8 \quad \text{ax/W} \quad \vee_L \\
- : \Delta_{14}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8 \quad \text{hCut} \\
- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \rightarrow_R \\
\hline
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : F_6, \perp, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{\bullet h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9}{\perp_L} \\
\hline
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \rightarrow \\
\hline
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L \\
\\
\frac{h_1 : F_7, \Delta_{11} \vdash F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \rightarrow F_8), \perp} \rightarrow_R \quad \frac{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \rightarrow F_8}{\perp_L} \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \rightarrow \\
\hline
\frac{\frac{h_1 : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}, F_8}{\text{ax/W}} \quad \frac{\bullet h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}, F_8}{\perp_L}}{\frac{}{- : \Delta_{11}, F_7 \vdash \Delta_{10}, F_8} \rightarrow_R} \text{hCut} \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \rightarrow_R \\
\\
\frac{h_1 : F_7, \perp, \Delta_{12} \vdash F_8, F_{11}, \Delta_{10}}{\bullet h_1 : \perp, \Delta_{12} \vdash (\Delta_{10}, F_7 \rightarrow F_8), F_{11}} \rightarrow_R \quad \frac{\bullet h_9 : (\perp, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\perp_L} \\
\hline
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8} \rightarrow \\
\hline
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{11}, p_{10} \vdash F_7, \Delta_9, p_{10}}{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{\bullet h_8 : (\Delta_{11}, p_{10}), F_6 \rightarrow F_7 \vdash \Delta_9, p_{10}}{I} \\
\hline
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \rightarrow \\
\hline
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I \\
\\
\frac{h_1 : F_7, \Delta_{12} \vdash F_8, p_{11}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{12} \vdash ((\Delta_{10}, p_{11}), F_7 \rightarrow F_8), p_{11}} \rightarrow_R \quad \frac{\bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8}{\perp_L} \\
\hline
\frac{}{- : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8} \rightarrow \\
\hline
\frac{\frac{h_1 : \Delta_{12}, F_7 \vdash \Delta_{10}, F_8, p_{11}, p_{11}}{\text{ax/W}} \quad \frac{\bullet h_9 : \Delta_{12}, F_7, p_{11} \vdash \Delta_{10}, F_8, p_{11}}{I}}{\frac{}{- : \Delta_{12}, F_7 \vdash \Delta_{10}, F_8, p_{11}} \rightarrow_R} \text{hCut} \\
\hline
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, p_{11}, F_7 \rightarrow F_8} \rightarrow_R \\
\\
\frac{h_1 : F_7, \Delta_{13}, p_{11} \vdash F_8, F_{12}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{13}, p_{11} \vdash ((\Delta_{10}, p_{11}), F_7 \rightarrow F_8), F_{12}} \rightarrow_R \quad \frac{\bullet h_9 : (\Delta_{13}, p_{11}), F_{12} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8}{\perp_L} \\
\hline
\frac{}{- : \Delta_{13}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8} \rightarrow \\
\hline
\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{10}, p_{11}, F_7 \rightarrow F_8} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : F_6, \top, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \top_L \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \rightarrow \\
\hline
\frac{\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7}{\text{ax/W}} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\text{ax/W}}}{\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \rightarrow_R} \text{hCut} \\
\\
\frac{h_1 : F_7, \Delta_{11} \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \rightarrow F_8), \top} \rightarrow_R \quad \frac{h_9 : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{11}, \top \vdash \Delta_{10}, F_7 \rightarrow F_8} \top_L \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \rightarrow \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{ax/W} \\
\\
\frac{h_1 : F_7, \top, \Delta_{12} \vdash F_8, F_{11}, \Delta_{10}}{\bullet h_1 : \top, \Delta_{12} \vdash (\Delta_{10}, F_7 \rightarrow F_8), F_{11}} \rightarrow_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : (\top, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \top_L \\
\hline
\frac{}{- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8} \rightarrow \\
\hline
\frac{\frac{\bullet h_1 : \top, \Delta_{12} \vdash \Delta_{10}, F_{11}, F_7 \rightarrow F_8}{\text{ax/W}} \quad \frac{h_9 : \top, \Delta_{12}, F_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\text{ax/W}}}{\frac{}{- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8} \rightarrow_R} \text{hCut}
\end{array}$$

8.2 Status of \wedge_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, \Delta_{11}, F_9 \rightarrow F_{10} \quad h_1 : \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \rightarrow F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \wedge F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6 \wedge F_7} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_7} \text{inv-th/ax}}{\frac{}{- : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}}} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}} \text{ax/W} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R \quad \frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R \\
\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11} \quad h_1 : \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8} \rightarrow_R \\
\frac{}{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_8}{\bullet h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_8} \text{inv-th/ax}}{\frac{}{- : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7 \wedge F_8}} \wedge_R \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8} \text{ax/W} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8} \rightarrow_R \quad \frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8} \rightarrow_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, \Delta_{11}, F_9 \wedge F_{10} \quad h_1 : \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \wedge F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9, \Delta_{11} \quad h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \wedge_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_6, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9} \text{inv-th/ax} \quad \frac{}{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_{10}} \text{inv-th/ax}}{\frac{}{- : \Delta_{12}, F_6 \vdash \Delta_{11}, F_9 \wedge F_{10}}} \wedge_R \quad \frac{}{- : \Delta_{12}, F_6 \vdash \Delta_{11}, F_9 \wedge F_{10}} \text{sCut} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}} \text{sCut} \\
\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \wedge F_{11} \quad h_1 : \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \wedge F_8} \rightarrow_R \\
\frac{}{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_7}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_8}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_8} \text{inv-th/ax}}{\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_7 \wedge F_8}} \wedge_R \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8} \text{ax/W} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \wedge F_8} \text{hCut} \quad \frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \wedge F_8} \text{hCut} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \wedge F_8} \rightarrow_R \\
\frac{h_1 : \Delta_{12} \vdash F_8, F_{11}, \Delta_{10} \quad h_1 : \Delta_{12} \vdash F_9, F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{10}, F_8 \wedge F_9), F_{11}} \wedge_R \quad \frac{h_7 : F_{11}, \Delta_{12} \vdash F_8, \Delta_{10} \quad h_7 : F_{11}, \Delta_{12} \vdash F_9, \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_8}{\bullet h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8} \text{ax/W}}{\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_8}} \text{hCut} \quad \frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_9}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_9} \text{ax/W}}{\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_9}} \text{hCut} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, \Delta_{11}, F_9 \vee F_{10} \quad h_1 : \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \vee F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9, F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9, F_6 \wedge F_7} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_7, F_9} \text{inv-th/ax}}{\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9}} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_{10}, F_9}{\bullet h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_{10}, F_9} \text{ax/W} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9} \text{hCut} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \vee F_{11} \quad h_1 : \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, F_{11}, \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8} \vee_R}{\frac{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_8}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_8} \text{inv-th/ax}}{\frac{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\rightarrow} \wedge_R} \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8} \text{ax/w}}{\frac{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\rightarrow} \vee_R} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{10} \vdash F_6, \perp, \Delta_9 \quad h_1 : \Delta_{10} \vdash F_7, \perp, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_9} \perp_R}{\frac{- : \Delta_{10} \vdash \perp, \Delta_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \wedge F_7}{\rightarrow} \text{ax/w} \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_9} \text{ax/w}}{\frac{- : \Delta_{10} \vdash \perp, \Delta_9}{\rightarrow} \text{hCut}} \\
\frac{\frac{h_1 : \Delta_{12} \vdash F_7, F_{11}, \perp, \Delta_{10} \quad h_1 : \Delta_{12} \vdash F_8, F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\perp, \Delta_{10}), F_7 \wedge F_8), F_{11}} \wedge_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\perp, \Delta_{10}), F_7 \wedge F_8} \perp_R}{\frac{- : \Delta_{12} \vdash (\perp, \Delta_{10}), F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_{12} \vdash \perp, \Delta_{10}, F_{11}, F_7 \wedge F_8}{\rightarrow} \text{ax/w} \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \wedge F_8} \text{ax/w}}{\frac{- : \Delta_{12} \vdash \perp, \Delta_{10}, F_7 \wedge F_8}{\rightarrow} \text{hCut}}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{10} \vdash F_6, \top, \Delta_9 \quad h_1 : \Delta_{10} \vdash F_7, \top, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \top, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \top, \Delta_9} \top_R}{\frac{- : \Delta_{10} \vdash \top, \Delta_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{10} \vdash \top, \Delta_9}{\rightarrow} \top_R}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{12} \vdash F_7, F_{11}, \top, \Delta_{10} \quad h_1 : \Delta_{12} \vdash F_8, F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), F_7 \wedge F_8), F_{11}} \wedge_R \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash (\top, \Delta_{10}), F_7 \wedge F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\top, \Delta_{10}), F_7 \wedge F_8} \top_R}{\frac{- : \Delta_{12} \vdash (\top, \Delta_{10}), F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{12} \vdash \top, \Delta_{10}, F_7 \wedge F_8}{\rightarrow} \top_R}{\rightarrow}
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{\frac{h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash F_6, \Delta_{10}, \Box F_9 \quad h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_9}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \Box F_9), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_{11}) \vdash F_9}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{12}), F_6 \wedge F_7 \vdash \Delta_{10}, \Box F_9} K}{\frac{- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \Box F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_9}{\rightarrow} \text{ax/w} \quad \frac{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_9}{\rightarrow} K}{\rightarrow} \\
\frac{\frac{h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash F_7, \Box F_{12}, \Delta_{11}, \Box F_{10} \quad h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash F_8, \Box F_{12}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \wedge F_8), \Box F_{12}} \wedge_R \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{13}, \Delta_{14}), \Box F_{12} \vdash (\Delta_{11}, \Box F_{10}), F_7 \wedge F_8} K}{\frac{- : \Box \Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14}, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_7, \Box F_{10}}{\rightarrow} \text{ax/w} \quad \frac{h_9 : \text{unbox}(\Box F_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} K}{\frac{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}}{\rightarrow} \text{ax/w}} \quad \frac{\frac{h_1 : \Delta_{14}, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_8, \Box F_{10}}{\rightarrow} \text{ax/w} \quad \frac{h_9 : \text{unbox}(\Box F_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}} K}{\frac{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}}{\rightarrow} \text{ax/w}} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash F_7, F_{13}, \Delta_{11}, [\Box F_{10} \quad h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash F_8, F_{13}, \Delta_{11}, [\Box F_{10}}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, [\Box F_{10}]), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : \text{unbox}(\Box\Gamma_{12}) \vdash F_{10}}{\bullet h_9 : (\Box\Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, [\Box F_{10}]), F_7 \wedge F_8} K \\
\frac{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, [\Box F_{10}]), F_7 \wedge F_8), F_{13} \quad \bullet h_9 : (\Box\Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, [\Box F_{10}]), F_7 \wedge F_8}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, [\Box F_{10}]), F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, [\Box F_{10}, F_7 \wedge F_8} K
\end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \rightarrow_L \\
\frac{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \wedge F_7 \quad \bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9 \quad \text{inv-th/ax} \quad - : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{inv-th/} \\
\text{ax/W} \quad \frac{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_7 \quad \text{ax/W} \quad - : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut} \\
\frac{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \quad \text{ax/W} \quad - : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut} \\
\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \rightarrow F_{11}, \Delta_{12} \quad h_1 : \Delta_{13} \vdash F_8, F_{10} \rightarrow F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \rightarrow F_{11}} \wedge_R \quad \frac{h_9 : \Delta_{13} \vdash F_{10}, \Delta_{12}, F_7 \wedge F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \rightarrow_L \\
\frac{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \rightarrow F_{11} \quad \bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \quad \text{inv-th/ax} \quad - : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_8 \quad \text{inv-th/ax}}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8} \wedge_R \quad \frac{- : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \text{sCut} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8 \quad \text{ax/W} \quad - : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \text{sCut} \\
\frac{h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_{10} \rightarrow F_{11}) \vdash \Delta_{12}, F_7 \wedge F_8} \rightarrow_L \\
\frac{\bullet h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{13} \quad \bullet h_9 : (\Delta_{14}, F_{10} \rightarrow F_{11}) \vdash \Delta_{12}, F_7 \wedge F_8}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}, F_7 \quad \text{ax/W} \quad h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7 \quad \text{inv-th/ax} \quad h_9 : \Delta_{14}, F_{11}, F_{13} \vdash \Delta_{12}, F_7 \quad \text{inv-th/ax}}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \rightarrow_L \\
\frac{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \quad \bullet h_9 : \Delta_{14}, F_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \quad \text{hCut}}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \text{sCut}
\end{array}$$

• Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L \\
\frac{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \wedge F_7 \quad \bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \quad \text{inv-th/ax} \quad h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11} \quad \text{ax/W}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{hCut} \\
\frac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \wedge F_7 \quad - : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L \\
\frac{h_1 : \Delta_{10} \vdash F_7, \Delta_9 \quad h_1 : \Delta_{10} \vdash F_8, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \wedge F_8} \wedge_R \quad \frac{h_6 : F_7, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \wedge_L \\
\frac{\bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \wedge F_8 \quad \bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9}{- : \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7 \quad \text{ax/W} \quad - : \Delta_{10}, F_7 \vdash \Delta_9, F_8 \quad \text{ax/W} \quad - : \Delta_{10}, F_7, F_8 \vdash \Delta_9 \quad \text{ax/W}}{- : \Delta_{10} \vdash \Delta_9, F_7} \text{sCut} \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7 \quad - : \Delta_{10}, F_7 \vdash \Delta_9}{- : \Delta_{10} \vdash \Delta_9} \text{sCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \wedge F_{11}, \Delta_{12} \quad h_1 : \Delta_{13} \vdash F_8, F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \wedge F_{11}} \wedge_R \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \wedge F_{11}}{\rightarrow} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7} \wedge_L \quad \frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}}{\rightarrow} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \wedge_L}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{hCut}} \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \wedge F_{11}), F_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_{13}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_{13}, F_8}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_{13}, F_8} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \wedge_R} \\
\frac{\frac{- : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \text{hCut}}{\frac{- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \wedge_L}
\end{array}$$

• Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L}{\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6}{\rightarrow} \text{ax/W} \quad \frac{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7}{\rightarrow} \text{ax/W} \quad \frac{- : \Delta_{12}, F_6, F_7, F_9 \vdash \Delta_{11}}{\rightarrow} \text{inv-th/ax} \quad \frac{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}}{\rightarrow} \text{inv-th/ax}}{\frac{- : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash \Delta_{11}}{\rightarrow} \text{sCut}} \\
\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6}{\rightarrow} \text{sCut} \\
\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \vee F_{11}, \Delta_{12} \quad h_1 : \Delta_{13} \vdash F_8, F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \vee F_{11}} \wedge_R \quad \frac{h_9 : F_{10}, \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7}{\rightarrow} \text{inv-th/ax} \quad \frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_8}{\rightarrow} \text{inv-th/ax}}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\rightarrow} \wedge_R} \\
\frac{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\rightarrow} \text{ax/W} \quad \frac{- : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8}{\rightarrow} \text{sCut}}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8}{\rightarrow} \wedge_L} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \vee F_{11}), F_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_7}{\rightarrow} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7}{\rightarrow} \wedge_L} \\
\frac{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7}{\rightarrow} \text{hCut} \\
\frac{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7}{\rightarrow} \\
\frac{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow}
\end{array}$$

• Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \perp, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \perp_L}{\frac{- : \perp, \Delta_{10} \vdash \Delta_9}{\rightarrow} \text{Cut}} \\
\frac{- : \perp, \Delta_{10} \vdash \Delta_9}{\rightarrow} \\
\frac{- : \perp, \Delta_{10} \vdash \Delta_9}{\rightarrow} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11} \vdash F_7, \perp, \Delta_{10} \quad h_1 : \Delta_{11} \vdash F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \wedge F_8), \perp} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \wedge F_8}}{\perp_L} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7}{\perp_L} \quad \frac{ax/W \quad \bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7}{hCut} \quad \frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_8}{\perp_L} \quad \frac{ax/W \quad \bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_8}{hCut}}{\perp_L} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{\perp_L} \text{Cut}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, p_{10} \vdash F_6, \Delta_9, p_{10} \quad h_1 : \Delta_{11}, p_{10} \vdash F_7, \Delta_9, p_{10}}{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : (\Delta_{11}, p_{10}), F_6 \wedge F_7 \vdash \Delta_9, p_{10}} I}{\perp_L} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{\perp_L} I \\
\frac{\frac{h_1 : \Delta_{12} \vdash F_7, p_{11}, \Delta_{10}, p_{11} \quad h_1 : \Delta_{12} \vdash F_8, p_{11}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{12} \vdash ((\Delta_{10}, p_{11}), F_7 \wedge F_8), p_{11}} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8} I}{\perp_L} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}, p_{11}}{\perp_L} \quad \frac{ax/W \quad \bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, p_{11}}{hCut} \quad \frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_8, p_{11}, p_{11}}{\perp_L} \quad \frac{ax/W \quad \bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_8, p_{11}}{hCut}}{\perp_L} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{\perp_L} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : \top, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \top, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9}}{\top_L} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7}{\perp_L} \quad \frac{ax/W \quad h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{hCut} \\
\frac{}{\perp_L} I \\
\frac{\frac{h_1 : \Delta_{11} \vdash F_7, \top, \Delta_{10} \quad h_1 : \Delta_{11} \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \wedge F_8), \top} \wedge_R \quad \frac{h_9 : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{11}, \top \vdash \Delta_{10}, F_7 \wedge F_8}}{\top_L} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{\perp_L} I \\
\frac{\frac{h_1 : \top, \Delta_{12} \vdash F_7, F_{11}, \Delta_{10} \quad h_1 : \top, \Delta_{12} \vdash F_8, F_{11}, \Delta_{10}}{\bullet h_1 : \top, \Delta_{12} \vdash (\Delta_{10}, F_7 \wedge F_8), F_{11}} \wedge_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : (\top, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \wedge F_8}}{\top_L} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \top, \Delta_{12} \vdash \Delta_{10}, F_{11}, F_7 \wedge F_8}{\perp_L} \quad \frac{ax/W \quad h_9 : \top, \Delta_{12}, F_{11} \vdash \Delta_{10}, F_7 \wedge F_8}{hCut} \\
\frac{}{\perp_L} I
\end{array}$$

8.3 Status of \vee_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, F_7, \Delta_{11}, F_9 \rightarrow F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \vee F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6, F_7}}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}}{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}} \text{ax/W} \\
\hline
- : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10} \quad \rightarrow_R \\
\hline
\frac{h_1 : \Delta_{14} \vdash F_7, F_8, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \vee F_8} \rightarrow_R \\
\hline
- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7, F_8}}{\bullet h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8}{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8} \text{ax/W} \\
\hline
- : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8 \quad \text{hCut} \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \quad \rightarrow_R \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \vee F_8
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, F_7, \Delta_{11}, F_9 \wedge F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_9, \Delta_{11} \quad h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \wedge_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9} \text{ax/W} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \quad \text{hCut} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10} \quad \wedge_R \\
\hline
\frac{h_1 : \Delta_{14} \vdash F_7, F_8, F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \vee F_8} \wedge_R \\
\hline
- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{13}, F_7, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{13}, F_7, F_8, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \wedge F_{11}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{11}, F_7, F_8}{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{11}, F_7, F_8} \text{inv-th/ax} \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_{13}, F_7, F_8, F_{10} \wedge F_{11} \quad \text{hCut} \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \vee F_8 \quad \vee_R \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \vee F_8
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, F_7, \Delta_{11}, F_9 \vee F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_9, F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6, F_7, F_9}}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}, F_9}{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}, F_9} \text{ax/W} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9 \quad \text{hCut} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \quad \vee_R \\
\hline
\frac{h_1 : \Delta_{14} \vdash F_7, F_8, F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, F_{11}, \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8} \vee_R \\
\hline
- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7, F_8}}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8}{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8} \text{ax/W} \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8 \quad \text{hCut} \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \quad \vee_R \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_8, F_9, F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{10}, F_8 \vee F_9), F_{11}} \vee_R \quad \frac{h_7 : F_{11}, \Delta_{12} \vdash F_8, F_9, \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8 \vee F_9} \vee_R \\
\hline
- : \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_8, F_9}{- : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_8, F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8, F_9}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8, F_9} \text{H}}{- : \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9} \text{hCut} \\
\vee_R \\
- : \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_6, F_7, \perp, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_9} \perp_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \vee F_7}{- : \Delta_{10} \vdash \perp, \Delta_9, F_6 \vee F_7} \text{ax/W} \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_9} \text{hCut}}{- : \Delta_{10} \vdash \perp, \Delta_9} \\
\\
\frac{h_1 : \Delta_{12} \vdash F_7, F_8, F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\perp, \Delta_{10}), F_7 \vee F_8), F_{11}} \vee_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\perp, \Delta_{10}), F_7 \vee F_8} \perp_R \\
\hline
- : \Delta_{12} \vdash (\perp, \Delta_{10}), F_7 \vee F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \perp, \Delta_{10}, F_{11}, F_7 \vee F_8}{- : \Delta_{12} \vdash \perp, \Delta_{10}, F_{11}, F_7 \vee F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \vee F_8} \text{hCut}}{- : \Delta_{12} \vdash \perp, \Delta_{10}, F_7 \vee F_8}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_6, F_7, \top, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_9} \top_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_9 \\
\rightarrow \\
- : \Delta_{10} \vdash \top, \Delta_9 \quad \top_R \\
\\
\frac{h_1 : \Delta_{12} \vdash F_7, F_8, F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), F_7 \vee F_8), F_{11}} \vee_R \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash (\top, \Delta_{10}), F_7 \vee F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\top, \Delta_{10}), F_7 \vee F_8} \top_R \\
\hline
- : \Delta_{12} \vdash (\top, \Delta_{10}), F_7 \vee F_8 \\
\rightarrow \\
- : \Delta_{12} \vdash \top, \Delta_{10}, F_7 \vee F_8 \quad \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash F_6, F_7, \Delta_{10}, \Box F_9}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \Box F_9), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_{11}) \vdash F_9}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{12}), F_6 \vee F_7 \vdash \Delta_{10}, \Box F_9} K \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \Box F_9 \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_9}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_9} \text{ax/W} \\
K \\
\\
\frac{h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash F_7, F_8, \Box F_{12}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \vee F_8), \Box F_{12}} \vee_R \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{13}, \Delta_{14}), \Box F_{12} \vdash (\Delta_{11}, \Box F_{10}), F_7 \vee F_8} K \\
\hline
- : \Box \Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \vee F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, F_7, F_8, \Box F_{10}}{- : \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, F_7, F_8, \Box F_{10}} \text{ax/W} \quad \frac{h_9 : \text{unbox}(\Box F_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, F_8, \Box F_{10}} K}{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, F_8, \Box F_{10}} \text{hCut} \\
\vee_R \\
- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, \Box F_{10}, F_7 \vee F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash F_7, F_8, F_{13}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : unbox(\Box \Gamma_{12}) \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, \Box F_{10}), F_7 \vee F_8} K \\
\hline
- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \vee F_8 \\
\rightarrow \\
\frac{- : unbox(\Box \Gamma_{12}) \vdash F_{10}}{- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_{10}, F_7 \vee F_8} \text{ax/W} \\
\hline
- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_{10}, F_7 \vee F_8 \quad K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \rightarrow_L \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \\
\rightarrow \\
\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{inv-th/ax} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{\bullet h_8 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \vee_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10} \vdash \Delta_{11}} \text{ax/W} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \quad \text{hCut} \quad - : \Delta_{12}, F_{10} \vdash \Delta_{11} \\
\rightarrow \\
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{13} \vdash F_7, F_8, F_{10} \rightarrow F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \rightarrow F_{11}} \vee_R \quad \frac{h_9 : \Delta_{13} \vdash F_{10}, \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \rightarrow_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8 \\
\rightarrow \\
\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8 \quad \text{hCut} \\
\rightarrow \\
- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_7, F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_{13} \vdash \Delta_{12}, F_7 \vee F_8} \rightarrow_L \\
\hline
- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \vee F_8 \\
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}, F_7, F_8}{\bullet h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \\
\hline
- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8 \quad \text{hCut} \\
\rightarrow \\
- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \vee F_8
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \wedge_L \\
\hline
- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \vee_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \text{ax/W} \\
\hline
- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11} \quad \text{hCut} \\
\rightarrow \\
- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{13} \vdash F_7, F_8, F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \wedge F_{11}} \vee_R \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \wedge_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8 \\
\rightarrow \\
\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \wedge_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8 \quad \text{hCut} \\
\rightarrow \\
- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash F_7, F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \wedge F_{11}), F_{13} \vdash \Delta_{12}, F_7 \vee F_8} \wedge_L \\
\hline
- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_{13}, F_7, F_8}{- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \wedge_L}{- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \text{hCut} \\
\vee_R \\
- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \vee_L \\
\hline
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \vee_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}} \text{ax/W} \vee_R}{- : \Delta_{12}, F_9 \vdash \Delta_{11}} \text{hCut} \\
\vee_L \\
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \\
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash F_7, F_8, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \vee F_8} \vee_R \quad \frac{h_6 : F_7, \Delta_{10} \vdash \Delta_9 \quad h_6 : F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \vee_L \\
\hline
- : \Delta_{10} \vdash \Delta_9 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7, F_8}{- : \Delta_{10} \vdash \Delta_9, F_7} \text{ax/W} \quad \frac{- : \Delta_{10}, F_8 \vdash \Delta_9, F_7}{- : \Delta_{10}, F_7 \vdash \Delta_9} \text{ax/W} \\
\text{sCut} \quad \text{sCut} \\
- : \Delta_{10} \vdash \Delta_9 \\
\rightarrow \\
\frac{h_1 : \Delta_{13} \vdash F_7, F_8, F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \vee F_{11}} \vee_R \quad \frac{h_9 : F_{10}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \vee_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \vee_L \\
\text{hCut} \\
- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8 \\
\vee_R \\
- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8 \\
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash F_7, F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \vee F_{11}), F_{13} \vdash \Delta_{12}, F_7 \vee F_8} \vee_L \\
\hline
- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_7, F_8}{\bullet h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \vee_L \\
\text{hCut} \\
- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8 \\
\vee_R \\
- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_{10} \vdash F_6, F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \perp_L \\
\hline
- : \perp, \Delta_{10} \vdash \Delta_9 \quad \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_{10} \vdash \Delta_9 \quad \perp_L \\
\rightarrow \\
\frac{h_1 : \Delta_{11} \vdash F_7, F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \vee F_8), \perp} \vee_R \quad \frac{}{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \vee F_8} \perp_L \\
\hline
- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7, F_8}{\bullet h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7, F_8} \text{ax/W} \quad \frac{}{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7, F_8} \perp_L \\
\text{hCut} \\
- : \Delta_{11} \vdash \Delta_{10}, F_7, F_8 \\
\vee_R \\
- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_{12} \vdash F_7, F_8, F_{11}, \Delta_{10}}{\bullet h_1 : \perp, \Delta_{12} \vdash (\Delta_{10}, F_7 \vee F_8), F_{11}} \vee_R \quad \frac{}{\bullet h_9 : (\perp, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \vee F_8} \perp_L \\
\hline
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8} \rightarrow \\
\hline
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, p_{10} \vdash F_6, F_7, \Delta_9, p_{10}}{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6 \vee F_7} \vee_R \quad \frac{}{\bullet h_8 : (\Delta_{11}, p_{10}), F_6 \vee F_7 \vdash \Delta_9, p_{10}} I \\
\hline
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \rightarrow \\
\hline
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I \\
\\
\frac{h_1 : \Delta_{12} \vdash F_7, F_8, p_{11}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{12} \vdash ((\Delta_{10}, p_{11}), F_7 \vee F_8), p_{11}} \vee_R \quad \frac{}{\bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} I \\
\hline
\frac{}{- : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} \rightarrow \\
\hline
\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11}, p_{11}}{\bullet h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11}} \text{ax/W} \quad \frac{}{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, F_8, p_{11}} I \\
\hline
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11}} \text{hCut} \\
\hline
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, p_{11}, F_7 \vee F_8} \vee_R \\
\\
\frac{h_1 : \Delta_{13}, p_{11} \vdash F_7, F_8, F_{12}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{13}, p_{11} \vdash ((\Delta_{10}, p_{11}), F_7 \vee F_8), F_{12}} \vee_R \quad \frac{}{\bullet h_9 : (\Delta_{13}, p_{11}), F_{12} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} I \\
\hline
\frac{}{- : \Delta_{13}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} \rightarrow \\
\hline
\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{10}, p_{11}, F_7 \vee F_8} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_{10} \vdash F_6, F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \top_L \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \rightarrow \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W} \quad \frac{}{h_8 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9} \text{hCut} \\
\hline
\\
\frac{h_1 : \Delta_{11} \vdash F_7, F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \vee F_8), \top} \vee_R \quad \frac{h_9 : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{11}, \top \vdash \Delta_{10}, F_7 \vee F_8} \top_L \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \rightarrow \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{ax/W} \\
\\
\frac{h_1 : \top, \Delta_{12} \vdash F_7, F_8, F_{11}, \Delta_{10}}{\bullet h_1 : \top, \Delta_{12} \vdash (\Delta_{10}, F_7 \vee F_8), F_{11}} \vee_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : (\top, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \vee F_8} \top_L \\
\hline
\frac{}{- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8} \rightarrow \\
\hline
\frac{}{- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8} \text{ax/W} \quad \frac{}{h_9 : \top, \Delta_{12}, F_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{hCut} \\
\hline
\frac{}{- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8}
\end{array}$$

8.4 Status of \perp_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \rightarrow F_6), \perp} \perp_R \quad \frac{h_4 : \perp, F_5, \Delta_8 \vdash F_6, \Delta_7}{\bullet h_4 : \Delta_8, \perp \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow_R \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \rightarrow F_7}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_8, F_6 \rightarrow F_7), F_9} \perp_R \quad \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \perp, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \rightarrow_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \rightarrow F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash \perp, \Delta_8, F_9, F_6 \rightarrow F_7}{- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7}{- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \rightarrow F_7 \quad \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \wedge F_6), \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \perp, \Delta_8 \vdash F_6, \Delta_7}{\bullet h_4 : \Delta_8, \perp \vdash \Delta_7, F_5 \wedge F_6} \wedge_R \\
\hline
- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_5 \wedge F_6}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_8, F_5 \wedge F_6), F_9} \perp_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \perp, F_6, \Delta_8 \quad h_5 : F_9, \Delta_{10} \vdash \perp, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_5 \wedge F_6} \wedge_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_8, F_5 \wedge F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash \perp, \Delta_8, F_9, F_5 \wedge F_6}{- : \Delta_{10} \vdash \perp, \Delta_8, F_5 \wedge F_6} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_5 \wedge F_6}{- : \Delta_{10} \vdash \perp, \Delta_8, F_5 \wedge F_6} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_8, F_5 \wedge F_6 \quad \text{hCut}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash \Delta_7, F_5 \vee F_6}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \vee F_6), \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash F_5, F_6, \Delta_7}{\bullet h_4 : \Delta_8, \perp \vdash \Delta_7, F_5 \vee F_6} \vee_R \\
\hline
- : \Delta_8 \vdash \Delta_7, F_5 \vee F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_5 \vee F_6}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_8, F_5 \vee F_6), F_9} \perp_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \perp, F_6, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_5 \vee F_6} \vee_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_8, F_5 \vee F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash \perp, \Delta_8, F_9, F_5 \vee F_6}{- : \Delta_{10} \vdash \perp, \Delta_8, F_5 \vee F_6} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_5 \vee F_6}{- : \Delta_{10} \vdash \perp, \Delta_8, F_5 \vee F_6} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_8, F_5 \vee F_6 \quad \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash \perp, \Delta_5}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_5), \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_6 \vdash \Delta_5}{\bullet h_4 : \Delta_6, \perp \vdash \perp, \Delta_5} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \Delta_8 \vdash (\perp, \Delta_6), F_7} \perp_R \quad \frac{h_5 : F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : \Delta_8, F_7 \vdash \perp, \Delta_6} \perp_R \\
\hline
- : \Delta_8 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_8 \vdash \perp, \Delta_6, F_7}{- : \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, F_7 \vdash \perp, \Delta_6}{- : \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
- : \Delta_8 \vdash \perp, \Delta_6 \quad \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash \top, \Delta_5}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_5), \perp} \perp_R \quad \frac{h_4 : \Delta_6, \perp \vdash \top, \Delta_5}{\bullet h_4 : \Delta_6, \perp \vdash \top, \Delta_5} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_5 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_6 \vdash \top, \Delta_5}{- : \Delta_6 \vdash \top, \Delta_5} \top_R
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash F_7, \top, \Delta_6}{\bullet h_1 : \Delta_8 \vdash (\perp, \top, \Delta_6), F_7} \perp_R \quad \frac{}{\bullet h_5 : \Delta_8, F_7 \vdash \perp, \top, \Delta_6} \top_R \\
\hline
\frac{}{- : \Delta_8 \vdash \perp, \top, \Delta_6} \rightarrow \\
\hline
\frac{}{- : \Delta_8 \vdash \perp, \top, \Delta_6} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_7, \Delta_8 \vdash \Delta_6, [\Box F_5]}{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Delta_6, [\Box F_5]), \perp} \perp_R \quad \frac{h_4 : unbox(\Box \Gamma_7) \vdash F_5}{\bullet h_4 : (\Box \Gamma_7, \Delta_8), \perp \vdash \Delta_6, [\Box F_5]} K \\
\hline
\frac{}{- : \Box \Gamma_7, \Delta_8 \vdash \Delta_6, [\Box F_5]} \rightarrow \\
\hline
\frac{}{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_6, [\Box F_5]} ax/W \\
\\
\frac{h_1 : \Box \Gamma_9, \Delta_{10} \vdash \Box F_8, \Delta_7, [\Box F_6]}{\bullet h_1 : \Box \Gamma_9, \Delta_{10} \vdash (\perp, \Delta_7, [\Box F_6]), \Box F_8} \perp_R \quad \frac{h_5 : unbox(\Box \Gamma_9), unbox(\Box F_8) \vdash F_6}{\bullet h_5 : (\Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \perp, \Delta_7, [\Box F_6]} K \\
\hline
\frac{}{- : \Box \Gamma_9, \Delta_{10} \vdash \perp, \Delta_7, [\Box F_6]} \rightarrow \\
\hline
\frac{h_1 : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Box F_8, \Delta_7, [\Box F_6]}{\bullet h_1 : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, [\Box F_6]} ax/W \quad \frac{h_5 : \Box F_8, \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, [\Box F_6]}{\bullet h_5 : \Box F_8, \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, [\Box F_6]} ax/W \\
\hline
\frac{}{- : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, [\Box F_6]} hCut \\
\\
\frac{h_1 : \Box \Gamma_8, \Delta_{10} \vdash F_9, \Delta_7, [\Box F_6]}{\bullet h_1 : \Box \Gamma_8, \Delta_{10} \vdash (\perp, \Delta_7, [\Box F_6]), F_9} \perp_R \quad \frac{h_5 : unbox(\Box \Gamma_8) \vdash F_6}{\bullet h_5 : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \perp, \Delta_7, [\Box F_6]} K \\
\hline
\frac{}{- : \Box \Gamma_8, \Delta_{10} \vdash \perp, \Delta_7, [\Box F_6]} \rightarrow \\
\hline
\frac{}{- : unbox(\Box \Gamma_8) \vdash F_6} ax/W \\
\hline
\frac{}{- : \Delta_{10}, \Box \Gamma_8 \vdash \perp, \Delta_7, [\Box F_6]} K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \perp, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \perp \vdash \Delta_7} \rightarrow_L \\
\hline
\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \rightarrow \\
\hline
\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} ax/W \\
\\
\frac{h_1 : \Delta_9 \vdash F_6 \rightarrow F_7, \Delta_8}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8), F_6 \rightarrow F_7} \perp_R \quad \frac{h_5 : \Delta_9 \vdash \perp, F_6, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8} \rightarrow_L \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8} \rightarrow \\
\hline
\frac{h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7}{\bullet h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7} ax/W \quad \frac{h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8} ax/W \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8} hCut \\
\\
\frac{h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash (\perp, \Delta_8), F_9} \perp_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \perp, F_6, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \perp, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \perp, \Delta_8} \rightarrow_L \\
\hline
\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_8} \rightarrow \\
\hline
\frac{h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_8, F_9}{\bullet h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_8, F_9} ax/W \quad \frac{h_5 : \Delta_{10}, F_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8} ax/W \\
\hline
\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_8} hCut
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \perp \vdash \Delta_7} \wedge_L \\
\hline
\frac{}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \rightarrow \\
\hline
\frac{}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} ax/W
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_9 \vdash F_6 \wedge F_7, \Delta_8}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8), F_6 \wedge F_7} \perp_R \quad \frac{h_5 : F_6, F_7, \Delta_9 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8} \wedge_L \\
\hline
- : \Delta_9 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \wedge F_7}{- : \Delta_9 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8}{\text{hCut}} \text{ax/W}}{- : \Delta_9 \vdash \perp, \Delta_8} \\
\\
\frac{h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash (\perp, \Delta_8), F_9} \perp_R \quad \frac{h_5 : F_6, F_7, F_9, \Delta_{10} \vdash \perp, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \wedge F_7), F_9 \vdash \perp, \Delta_8} \wedge_L \\
\hline
- : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8, F_9}{- : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, F_9, F_6 \wedge F_7 \vdash \perp, \Delta_8}{\text{hCut}} \text{ax/W}}{- : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \perp, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \perp \vdash \Delta_7} \vee_L \\
\hline
- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{ax/W} \\
\\
\frac{h_1 : \Delta_9 \vdash F_6 \vee F_7, \Delta_8}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8), F_6 \vee F_7} \perp_R \quad \frac{h_5 : F_6, \Delta_9 \vdash \perp, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8} \vee_L \\
\hline
- : \Delta_9 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \vee F_7}{- : \Delta_9 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8}{\text{hCut}} \text{ax/W}}{- : \Delta_9 \vdash \perp, \Delta_8} \\
\\
\frac{h_1 : \Delta_{10}, F_6 \vee F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \Delta_{10}, F_6 \vee F_7 \vdash (\perp, \Delta_8), F_9} \perp_R \quad \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \perp, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \perp, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \vee F_7), F_9 \vdash \perp, \Delta_8} \vee_L \\
\hline
- : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8, F_9}{- : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, F_9, F_6 \vee F_7 \vdash \perp, \Delta_8}{\text{hCut}} \text{ax/W}}{- : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash \Delta_5}{\bullet h_1 : \Delta_6 \vdash \Delta_5, \perp} \perp_R \quad \frac{}{\bullet h_4 : \Delta_6, \perp \vdash \Delta_5} \perp_L \\
\hline
- : \Delta_6 \vdash \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_6 \vdash \Delta_5}{- : \Delta_6 \vdash \Delta_5} \text{ax/W} \\
\\
\frac{h_1 : \Delta_7 \vdash \perp, \Delta_6}{\bullet h_1 : \Delta_7 \vdash (\perp, \Delta_6), \perp} \perp_R \quad \frac{}{\bullet h_5 : \Delta_7, \perp \vdash \perp, \Delta_6} \perp_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7 \vdash \perp, \Delta_6}{- : \Delta_7 \vdash \perp, \Delta_6} \text{ax/W} \\
\\
\frac{h_1 : \perp, \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \perp, \Delta_8 \vdash (\perp, \Delta_6), F_7} \perp_R \quad \frac{}{\bullet h_5 : (\perp, \Delta_8), F_7 \vdash \perp, \Delta_6} \perp_L \\
\hline
- : \perp, \Delta_8 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \perp, \Delta_8 \vdash \perp, \Delta_6}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_7, p_6 \vdash \Delta_5, p_6}{\bullet h_1 : \Delta_7, p_6 \vdash (\Delta_5, p_6), \perp} \perp_R \quad \frac{}{\bullet h_4 : (\Delta_7, p_6), \perp \vdash \Delta_5, p_6} I \\
\hline
- : \Delta_7, p_6 \vdash \Delta_5, p_6 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7, p_6 \vdash \Delta_5, p_6}{- : \Delta_7, p_6 \vdash \Delta_5, p_6} I
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash p_7, \Delta_6, p_7}{\bullet h_1 : \Delta_8 \vdash (\perp, \Delta_6, p_7), p_7} \perp_R \quad \frac{}{\bullet h_5 : \Delta_8, p_7 \vdash \perp, \Delta_6, p_7} I \\
\hline
\frac{}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \Delta_8 \vdash \perp, \Delta_6, p_7, p_7}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, p_7 \vdash \perp, \Delta_6, p_7}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} I \\
\hline
\frac{}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} \text{hCut} \\
\hline
\frac{h_1 : \Delta_9, p_7 \vdash F_8, \Delta_6, p_7}{\bullet h_1 : \Delta_9, p_7 \vdash (\perp, \Delta_6, p_7), F_8} \perp_R \quad \frac{}{\bullet h_5 : (\Delta_9, p_7), F_8 \vdash \perp, \Delta_6, p_7} I \\
\hline
\frac{}{- : \Delta_9, p_7 \vdash \perp, \Delta_6, p_7} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_9, p_7 \vdash \perp, \Delta_6, p_7} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet h_1 : \top, \Delta_6 \vdash \Delta_5, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_6 \vdash \Delta_5}{\bullet h_4 : (\top, \Delta_6), \perp \vdash \Delta_5} \top_L \\
\hline
\frac{}{- : \top, \Delta_6 \vdash \Delta_5} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \top, \Delta_6 \vdash \Delta_5} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_7 \vdash \top, \Delta_6}{\bullet h_1 : \Delta_7 \vdash (\perp, \Delta_6), \top} \perp_R \quad \frac{h_5 : \Delta_7 \vdash \perp, \Delta_6}{\bullet h_5 : \Delta_7, \top \vdash \perp, \Delta_6} \top_L \\
\hline
\frac{}{- : \Delta_7 \vdash \perp, \Delta_6} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_7 \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
\frac{h_1 : \top, \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \top, \Delta_8 \vdash (\perp, \Delta_6), F_7} \perp_R \quad \frac{h_5 : F_7, \Delta_8 \vdash \perp, \Delta_6}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \perp, \Delta_6} \top_L \\
\hline
\frac{}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \top, \Delta_8 \vdash \perp, \Delta_6, F_7}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, F_7 \vdash \perp, \Delta_6}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{hCut}
\end{array}$$

8.5 Status of \top_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \rightarrow F_6), \top} \top_R \quad \frac{h_4 : \top, F_5, \Delta_8 \vdash F_6, \Delta_7}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow_R \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7, F_6}{- : \Delta_8, F_5 \vdash \Delta_7, F_6} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7, F_6}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{ax/W} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{hCut} \\
\hline
\frac{}{\rightarrow_R} \\
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow_R \\
\hline
\frac{h_1 : \Delta_{10} \vdash (\top, \Delta_8, F_6 \rightarrow F_7), F_9}{\bullet h_1 : \Delta_{10} \vdash \top, \Delta_8, F_6 \rightarrow F_7} \top_R \quad \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \top, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \top, \Delta_8, F_6 \rightarrow F_7} \rightarrow_R \\
\hline
\frac{}{- : \Delta_{10} \vdash \top, \Delta_8, F_6 \rightarrow F_7} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{10} \vdash \top, \Delta_8, F_6 \rightarrow F_7} \top_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \wedge F_6), \top} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \top, \Delta_8 \vdash F_6, \Delta_7}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \wedge F_6} \wedge_R \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5}{- : \Delta_8 \vdash \Delta_7, F_5} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5}{- : \Delta_8 \vdash \Delta_7, F_5} \text{ax/W} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5} \text{hCut} \\
\hline
\frac{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_6}{- : \Delta_8 \vdash \Delta_7, F_6} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_6}{- : \Delta_8 \vdash \Delta_7, F_6} \text{ax/W} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_6} \text{hCut} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_8, F_6 \wedge F_7), F_9} \top_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \top, F_6, \Delta_8 \quad h_5 : F_9, \Delta_{10} \vdash \top, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \top, \Delta_8, F_6 \wedge F_7} \wedge_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_8, F_6 \wedge F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{10} \vdash \top, \Delta_8, F_6 \wedge F_7} \top_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \vee F_6), \top} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash F_5, F_6, \Delta_7}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \vee F_6} \vee_R \\
\hline
- : \Delta_8 \vdash \Delta_7, F_5 \vee F_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5, F_6} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5, F_6}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \vee F_6} \text{ax/W} \\
\hline
- : \Delta_8 \vdash \Delta_7, F_5, F_6 \quad \text{hCut} \\
\rightarrow \\
- : \Delta_8 \vdash \Delta_7, F_5 \vee F_6 \quad \vee_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_8, F_6 \vee F_7), F_9} \top_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \top, F_6, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \top, \Delta_8, F_6 \vee F_7} \vee_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_8, F_6 \vee F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{10} \vdash \top, \Delta_8, F_6 \vee F_7} \top_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_5), \top} \top_R \quad \frac{h_4 : \top, \Delta_6 \vdash \Delta_5}{\bullet h_4 : \Delta_6, \top \vdash \perp, \Delta_5} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : \Delta_6 \vdash \perp, \top, \Delta_5} \top_R \quad \frac{h_4 : \top, \Delta_6 \vdash \perp, \Delta_5}{\bullet h_4 : \Delta_6, \top \vdash \perp, \Delta_5} \text{ax/W} \\
\hline
- : \Delta_6 \vdash \perp, \Delta_5 \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\top, \perp, \Delta_6), F_7} \top_R \quad \frac{h_5 : F_7, \Delta_8 \vdash \top, \Delta_6}{\bullet h_5 : \Delta_8, F_7 \vdash \top, \perp, \Delta_6} \perp_R \\
\hline
- : \Delta_8 \vdash \top, \perp, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8 \vdash \perp, \top, \Delta_6} \top_R
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_5), \top} \top_R \quad \frac{}{\bullet h_4 : \Delta_6, \top \vdash \top, \Delta_5} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_5} \top_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\top, \Delta_6), F_7} \top_R \quad \frac{}{\bullet h_5 : \Delta_8, F_7 \vdash \top, \Delta_6} \top_R \\
\hline
- : \Delta_8 \vdash \top, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8 \vdash \top, \Delta_6} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Delta_6, [\top] F_5), \top} \top_R \quad \frac{h_4 : \text{unbox}(\Box \Gamma_7) \vdash F_5}{\bullet h_4 : (\Box \Gamma_7, \Delta_8), \top \vdash \Delta_6, [\top] F_5} K \\
\hline
- : \Box \Gamma_7, \Delta_8 \vdash \Delta_6, [\top] F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_7) \vdash F_5} \text{ax/W} \\
\hline
- : \Delta_8, \Box \Gamma_7 \vdash \Delta_6, [\top] F_5 \quad K
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Box \Gamma_9, \Delta_{10} \vdash (\top, \Delta_7, [\Box F_6], \Box F_8)} \top_R \quad \frac{h_5 : unbox(\Box \Gamma_9), unbox(\Box F_8) \vdash F_6}{\bullet h_5 : (\Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \top, \Delta_7, [\Box F_6]} K}{\frac{}{- : \Box \Gamma_9, \Delta_{10} \vdash \top, \Delta_7, [\Box F_6]} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, \Box \Gamma_9 \vdash \top, \Delta_7, [\Box F_6]} \top_R \\
\frac{}{\bullet h_1 : \Box \Gamma_8, \Delta_{10} \vdash (\top, \Delta_7, [\Box F_6], F_9)} \top_R \quad \frac{h_5 : unbox(\Box \Gamma_8) \vdash F_6}{\bullet h_5 : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \top, \Delta_7, [\Box F_6]} K}{\frac{}{- : \Box \Gamma_8, \Delta_{10} \vdash \top, \Delta_7, [\Box F_6]} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, \Box \Gamma_8 \vdash \top, \Delta_7, [\Box F_6]} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \top \vdash \Delta_7} \rightarrow_L}{\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5}{\bullet h_4 : \Delta_8, F_6 \vdash \top, \Delta_7} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \top_R \quad h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : \Delta_8, F_6 \vdash \Delta_7} \text{hCut}}{\frac{}{- : \Delta_8 \vdash \Delta_7, F_5} \rightarrow} \text{hCut} \\
\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \rightarrow_L \\
\frac{\frac{}{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \rightarrow F_7} \top_R \quad \frac{h_5 : \Delta_9 \vdash \top, F_6, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \rightarrow_L}{\frac{}{- : \Delta_9 \vdash \top, \Delta_8} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_9 \vdash \top, \Delta_8} \top_R \\
\frac{\frac{}{\bullet h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash (\top, \Delta_8), F_9} \top_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \top, F_6, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \top, \Delta_8} \rightarrow_L}{\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \top} \top_R \quad \frac{h_4 : \top, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \top \vdash \Delta_7} \wedge_L}{\frac{}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : \Delta_8, F_5, F_6 \vdash \top, \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}{\bullet h_4 : \Delta_8, F_5, F_6 \vdash \Delta_7} \text{ax/W}}{\frac{}{- : \Delta_8, F_5, F_6 \vdash \Delta_7} \rightarrow} \text{hCut} \\
\frac{}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \wedge_L \\
\frac{\frac{}{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \wedge F_7} \top_R \quad \frac{h_5 : F_6, F_7, \Delta_9 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash \top, \Delta_8} \wedge_L}{\frac{}{- : \Delta_9 \vdash \top, \Delta_8} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_9 \vdash \top, \Delta_8} \top_R \\
\frac{\frac{}{\bullet h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash (\top, \Delta_8), F_9} \top_R \quad \frac{h_5 : F_6, F_7, F_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \wedge F_7), F_9 \vdash \top, \Delta_8} \wedge_L}{\frac{}{- : \Delta_{10}, F_6 \wedge F_7 \vdash \top, \Delta_8} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, F_6 \wedge F_7 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \top}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \top \vdash \Delta_7} \vee_L}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7}{\vdash : \Delta_8, F_5 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7}{\vdash : \Delta_8, F_5 \vdash \Delta_7} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7}{\vdash : \Delta_8, F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{\vdash : \Delta_8, F_6 \vdash \Delta_7} \text{ax/W}}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{hCut} \\
\\
\frac{\frac{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \vee F_7}{\vdash : \Delta_9 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : F_6, \Delta_9 \vdash \top, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \top, \Delta_8} \vee_L}{\vdash : \Delta_9 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_9 \vdash \top, \Delta_8 \quad \top_R \\
\\
\frac{\frac{\bullet h_1 : \Delta_{10}, F_6 \vee F_7 \vdash (\top, \Delta_8), F_9}{\vdash : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \top, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \vee F_7), F_9 \vdash \top, \Delta_8} \vee_L}{\vdash : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_8 \quad \top_R
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_6 \vdash \Delta_5, \top}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \top_R \quad \frac{\bullet h_4 : (\perp, \Delta_6), \top \vdash \Delta_5}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \perp_L}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_6 \vdash \Delta_5 \quad \perp_L \\
\\
\frac{\frac{\bullet h_1 : \Delta_7 \vdash (\top, \Delta_6), \perp}{\vdash : \Delta_7 \vdash \top, \Delta_6} \top_R \quad \frac{\bullet h_5 : \Delta_7, \perp \vdash \top, \Delta_6}{\vdash : \Delta_7 \vdash \top, \Delta_6} \perp_L}{\vdash : \Delta_7 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_6 \quad \top_R \\
\\
\frac{\frac{\bullet h_1 : \perp, \Delta_8 \vdash (\top, \Delta_6), F_7}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \top_R \quad \frac{\bullet h_5 : (\perp, \Delta_8), F_7 \vdash \top, \Delta_6}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \perp_L}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_8 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_6 \vdash (\Delta_5, p_6), \top}{\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6} \top_R \quad \frac{\bullet h_4 : (\Delta_7, p_6), \top \vdash \Delta_5, p_6}{\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6} I}{\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6 \quad I \\
\\
\frac{\frac{\bullet h_1 : \Delta_8 \vdash (\top, \Delta_6, p_7), p_7}{\vdash : \Delta_8 \vdash \top, \Delta_6, p_7} \top_R \quad \frac{\bullet h_5 : \Delta_8, p_7 \vdash \top, \Delta_6, p_7}{\vdash : \Delta_8 \vdash \top, \Delta_6, p_7} I}{\vdash : \Delta_8 \vdash \top, \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_8 \vdash \top, \Delta_6, p_7 \quad \top_R \\
\\
\frac{\frac{\bullet h_1 : \Delta_9, p_7 \vdash (\top, \Delta_6, p_7), F_8}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_6, p_7} \top_R \quad \frac{\bullet h_5 : (\Delta_9, p_7), F_8 \vdash \top, \Delta_6, p_7}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_6, p_7} I}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_9, p_7 \vdash \top, \Delta_6, p_7 \quad \top_R
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_6 \vdash \Delta_5, \top}{\vdash : \Delta_6 \vdash \Delta_5} \top_R \quad \frac{h_4 : \Delta_6 \vdash \Delta_5}{\bullet h_4 : \Delta_6, \top \vdash \Delta_5} \top_L}{\vdash : \Delta_6 \vdash \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_6 \vdash \Delta_5 \quad \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : \Delta_7 \vdash (\top, \Delta_6), \top} \top_R \quad \frac{h_5 : \Delta_7 \vdash \top, \Delta_6}{\bullet h_5 : \Delta_7, \top \vdash \top, \Delta_6} \top_L}{- : \Delta_7 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_7 \vdash \top, \Delta_6} \top_R \\
\\
\frac{\frac{\frac{}{\bullet h_1 : \top, \Delta_8 \vdash (\top, \Delta_6), F_7} \top_R \quad \frac{h_5 : F_7, \Delta_8 \vdash \top, \Delta_6}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \top, \Delta_6} \top_L}{- : \top, \Delta_8 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_8 \vdash \top, \Delta_6} \top_R
\end{array}$$

8.6 Status of K : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \rightarrow F_9), \Box F_6} K \quad \frac{h_7 : \Box \Gamma_{11}, F_8, \Delta_{12}, \Box F_6 \vdash F_9, \Delta_{10}}{\bullet h_7 : (\Box \Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R}{- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \rightarrow F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, F_8, \Box \Gamma_{11} \vdash \Delta_{10}, F_9, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_8, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}, F_9}{\bullet h_7 : (\Delta_{12}, F_8, \Box \Gamma_{11}), \Box F_6 \vdash \Delta_{10}, F_9} K}{- : \Delta_{12}, F_8, \Box \Gamma_{11} \vdash \Delta_{10}, F_9} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \\
\\
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \rightarrow F_{10}), \Box F_7), F_{13}} K \quad \frac{h_8 : \Box \Gamma_{12}, F_9, F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), \Box F_7} \rightarrow_R}{- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), \Box F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \text{unbox}(\Box \Gamma_{12}) \vdash F_7} \text{ax/W}}{- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_7, F_9 \rightarrow F_{10}} K
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \wedge F_9), \Box F_6} K \quad \frac{h_7 : \Box \Gamma_{11}, \Delta_{12}, \Box F_6 \vdash F_8, \Delta_{10} \quad h_7 : \Box \Gamma_{11}, \Delta_{12}, \Box F_6 \vdash F_9, \Delta_{10}}{\bullet h_7 : (\Box \Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R}{- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}, F_8}{\bullet h_7 : (\Delta_{12}, \Box \Gamma_{11}), \Box F_6 \vdash \Delta_{10}, F_8} K}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8} \text{ax/W} \\
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_9, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}, F_9}{\bullet h_7 : (\Delta_{12}, \Box \Gamma_{11}), \Box F_6 \vdash \Delta_{10}, F_9} K}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_9} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R \\
\\
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \wedge F_{10}), \Box F_7), F_{13}} K \quad \frac{h_8 : \Box \Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, \Delta_{11}, \Box F_7 \quad h_8 : \Box \Gamma_{12}, F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, F_9 \wedge F_{10}), \Box F_7} \wedge_R}{- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \wedge F_{10}), \Box F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \text{unbox}(\Box \Gamma_{12}) \vdash F_7} \text{ax/W}}{- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_7, F_9 \wedge F_{10}} K
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \vee F_9), \Box F_6} K \quad \frac{h_7 : \Box \Gamma_{11}, \Delta_{12}, \Box F_6 \vdash F_8, F_9, \Delta_{10}}{\bullet h_7 : (\Box \Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \vee F_9} \vee_R}{- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, F_9, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}, F_8, F_9}{\bullet h_7 : (\Delta_{12}, \Box \Gamma_{11}), \Box F_6 \vdash \Delta_{10}, F_8, F_9} K}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, F_9} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8 \vee F_9} \vee_R
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \vee F_{10}), \Box F_7), F_{13}} \quad K \quad \frac{h_8 : \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, F_{10}, \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, F_9 \vee F_{10}), \Box F_7} \quad \vee_R \\
\hline
- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \vee F_{10}), \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7 \quad \text{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7, F_9 \vee F_{10}} \quad K
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_9) \vdash F_6}{\bullet h_1 : \Box\Gamma_9, \Delta_{10} \vdash (\perp, \Delta_8), \Box F_6} \quad K \quad \frac{h_7 : \Box\Gamma_9, \Delta_{10}, \Box F_6 \vdash \Delta_8}{\bullet h_7 : (\Box\Gamma_9, \Delta_{10}), \Box F_6 \vdash \perp, \Delta_8} \quad \perp_R \\
\hline
- : \Box\Gamma_9, \Delta_{10} \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_9) \vdash F_6 \quad \text{ax/W}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \perp, \Delta_8, \Box F_6} \quad \text{ax/W} \\
\frac{- : \Delta_{10}, \Box\Gamma_9 \vdash \perp, \Delta_8 \quad h_7 : \Delta_{10}, \Box\Gamma_9, \Box F_6 \vdash \perp, \Delta_8}{- : \Delta_{10}, \Box\Gamma_9 \vdash \perp, \Delta_8} \quad \text{hCut} \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\perp, \Delta_9), \Box F_7), F_{11}} \quad K \quad \frac{h_8 : \Box\Gamma_{10}, F_{11}, \Delta_{12} \vdash \Delta_9, \Box F_7}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{12}), F_{11} \vdash (\perp, \Delta_9), \Box F_7} \quad \perp_R \\
\hline
- : \Box\Gamma_{10}, \Delta_{12} \vdash (\perp, \Delta_9), \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{10}) \vdash F_7 \quad \text{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \perp, \Delta_9, \Box F_7} \quad K
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_9) \vdash F_6}{\bullet h_1 : \Box\Gamma_9, \Delta_{10} \vdash (\top, \Delta_8), \Box F_6} \quad K \quad \frac{h_7 : (\Box\Gamma_9, \Delta_{10}), \Box F_6 \vdash \top, \Delta_8}{\bullet h_7 : (\Box\Gamma_9, \Delta_{10}), \Box F_6 \vdash \top, \Delta_8} \quad \top_R \\
\hline
- : \Box\Gamma_9, \Delta_{10} \vdash \top, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_8}{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_8} \quad \top_R \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\top, \Delta_9), \Box F_7), F_{11}} \quad K \quad \frac{h_8 : (\Box\Gamma_{10}, \Delta_{12}), F_{11} \vdash (\top, \Delta_9), \Box F_7}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{12}), F_{11} \vdash (\top, \Delta_9), \Box F_7} \quad \top_R \\
\hline
- : \Box\Gamma_{10}, \Delta_{12} \vdash (\top, \Delta_9), \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{12}, \Box\Gamma_{10} \vdash \top, \Delta_9, \Box F_7}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \top, \Delta_9, \Box F_7} \quad \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, \Box F_8), \Box F_6} \quad K \quad \frac{h_7 : F_6, \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}) \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), \Box F_6 \vdash \Delta_9, \Box F_8} \quad K \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, \Box F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_6, F_8 \quad \text{ax/W}}{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_8} \quad \text{ax/W} \\
\frac{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_8 \quad h_7 : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, \Box F_8}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, \Box F_8} \quad K \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, \Box F_8), \Box F_6} \quad K \quad \frac{h_7 : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}) \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), \Box F_6 \vdash \Delta_9, \Box F_8} \quad K \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, \Box F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}) \vdash F_8 \quad \text{ax/W}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, \Box F_8} \quad K \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}, \Box\Gamma_{14}) \vdash F_7}{\bullet h_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{10}, \Box F_9), \Box F_7), \Box F_{12}} \quad K \quad \frac{h_8 : \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{13}), \text{unbox}(\Box F_{12}) \vdash F_9}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box F_{12} \vdash (\Delta_{10}, \Box F_9), \Box F_7} \quad K \\
\hline
- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{10}, \Box F_9), \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{14}) \vdash F_7 \quad \text{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, \Box F_7, \Box F_9} \quad K
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{13}) \vdash F_8}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, \llbracket F_8 \rrbracket, \Box F_{11})} K \quad \frac{h_7 : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{12}), \text{unbox}(\Box F_{11}) \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box F_{11} \vdash \Delta_9, \llbracket F_8 \rrbracket} K}{\frac{- : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, \llbracket F_8 \rrbracket}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{13}) \vdash F_8}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, \llbracket F_8 \rrbracket} \text{ax/W}}{K} \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash F_7}{\bullet h_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Delta_{10}, \llbracket F_9 \rrbracket, \llbracket F_7 \rrbracket), F_{14})} K \quad \frac{h_8 : \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_9}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), F_{14} \vdash (\Delta_{10}, \llbracket F_9 \rrbracket, \llbracket F_7 \rrbracket)} K}{\frac{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Delta_{10}, \llbracket F_9 \rrbracket, \llbracket F_7 \rrbracket)}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_9}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, \llbracket F_7 \rrbracket, \llbracket F_9 \rrbracket} \text{ax/W}}{K} \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash F_8}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_9, \llbracket F_8 \rrbracket, F_{13})} K \quad \frac{h_7 : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}) \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), F_{13} \vdash \Delta_9, \llbracket F_8 \rrbracket} K}{\frac{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash \Delta_9, \llbracket F_8 \rrbracket}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}) \vdash F_8}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, \llbracket F_8 \rrbracket} \text{ax/W}}{K}
\end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10}, \llbracket F_6 \rrbracket} K \quad \frac{h_7 : \Box\Gamma_{11}, \Delta_{12}, \llbracket F_6 \rrbracket \vdash F_8, \Delta_{10} \quad h_7 : \Box\Gamma_{11}, F_9, \Delta_{12}, \llbracket F_6 \rrbracket \vdash \Delta_{10}}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9), \llbracket F_6 \rrbracket \vdash \Delta_{10}} \rightarrow_L}{\frac{- : \Box\Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{- : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, \llbracket F_6 \rrbracket} \text{ax/W}}{K} \quad \frac{h_7 : \Delta_{12}, \Box\Gamma_{11}, \llbracket F_6 \rrbracket \vdash \Delta_{10}, F_8}{h_7 : \Delta_{12}, \Box\Gamma_{11}, \llbracket F_6 \rrbracket \vdash \Delta_{10}, F_8} \text{ax/W}}{\frac{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8}{\rightarrow} \text{hCut}} \\
\frac{\frac{- : \Delta_{12}, \Box\Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10}}{\rightarrow} \text{hCut}}{\rightarrow_L} \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, \llbracket F_7 \rrbracket, F_9) \rightarrow F_{10}} K \quad \frac{h_8 : \Box\Gamma_{12}, \Delta_{13} \vdash F_9, \Delta_{11}, \llbracket F_7 \rrbracket \quad h_8 : \Box\Gamma_{12}, F_{10}, \Delta_{13} \vdash \Delta_{11}, \llbracket F_7 \rrbracket}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{13}), F_9 \rightarrow F_{10} \vdash \Delta_{11}, \llbracket F_7 \rrbracket} \rightarrow_L}{\frac{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, \llbracket F_7 \rrbracket}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \llbracket F_7 \rrbracket} \text{ax/W}}{K} \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \vdash (\Delta_{11}, \llbracket F_7 \rrbracket, F_{13})} K \quad \frac{h_8 : \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, \Delta_{11}, \llbracket F_7 \rrbracket \quad h_8 : \Box\Gamma_{12}, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, \llbracket F_7 \rrbracket}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10}), F_{13} \vdash \Delta_{11}, \llbracket F_7 \rrbracket} \rightarrow_L}{\frac{- : \Box\Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, \llbracket F_7 \rrbracket}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{14}, \Box\Gamma_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, \llbracket F_7 \rrbracket} \text{ax/W}}{K}
\end{array}$$

• Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}, \llbracket F_6 \rrbracket} K \quad \frac{h_7 : \Box\Gamma_{11}, F_8, F_9, \Delta_{12}, \llbracket F_6 \rrbracket \vdash \Delta_{10}}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9), \llbracket F_6 \rrbracket \vdash \Delta_{10}} \wedge_L}{\frac{- : \Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, \llbracket F_6 \rrbracket} \text{ax/W}}{K} \quad \frac{h_7 : \Delta_{12}, F_8, F_9, \Box\Gamma_{11}, \llbracket F_6 \rrbracket \vdash \Delta_{10}}{h_7 : \Delta_{12}, F_8, F_9, \Box\Gamma_{11}, \llbracket F_6 \rrbracket \vdash \Delta_{10}} \text{ax/W}}{\frac{- : \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}}{\rightarrow} \wedge_L} \\
\frac{- : \Delta_{12}, \Box\Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{10}}{\rightarrow}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, \Box F_7), F_9 \wedge F_{10}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_9, F_{10}, \Delta_{13} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{13}), F_9 \wedge F_{10} \vdash \Delta_{11}, \Box F_7} \wedge_L \\
\hline
- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7} K
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash (\Delta_{11}, \Box F_7), F_{13}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_9, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10}), F_{13} \vdash \Delta_{11}, \Box F_7} \wedge_L \\
\hline
- : \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{14}, \Box\Gamma_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, \Box F_7} K
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}, \Box F_6} K \quad \frac{h_7 : \Box\Gamma_{11}, F_8, \Delta_{12}, \Box F_6 \vdash \Delta_{10} \quad h_7 : \Box\Gamma_{11}, F_9, \Delta_{12}, \Box F_6 \vdash \Delta_{10}}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \vee F_9), \Box F_6 \vdash \Delta_{10}} \vee_L \\
\hline
- : \Box\Gamma_{11}, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{11}) \vdash F_6} \text{ax/W} \\
\frac{}{- : \Delta_{12}, F_8, \Box\Gamma_{11} \vdash \Delta_{10}, \Box F_6} K \quad \frac{h_7 : \Delta_{12}, F_8, \Box\Gamma_{11}, \Box F_6 \vdash \Delta_{10}}{h_7 : \Delta_{12}, F_8, \Box\Gamma_{11}, \Box F_6 \vdash \Delta_{10}} \text{ax/W} \\
\frac{}{- : \Delta_{12}, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, \Box F_6} K \quad \frac{h_7 : \Delta_{12}, F_9, \Box\Gamma_{11}, \Box F_6 \vdash \Delta_{10}}{h_7 : \Delta_{12}, F_9, \Box\Gamma_{11}, \Box F_6 \vdash \Delta_{10}} \text{ax/W} \\
\hline
- : \Delta_{12}, F_8, \Box\Gamma_{11} \vdash \Delta_{10} \quad - : \Delta_{12}, F_9, \Box\Gamma_{11} \vdash \Delta_{10} \quad \vee_L \\
\rightarrow \\
- : \Delta_{12}, \Box\Gamma_{11}, F_8 \vee F_9 \vdash \Delta_{10}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, \Box F_7), F_9 \vee F_{10}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_9, \Delta_{13} \vdash \Delta_{11}, \Box F_7 \quad h_8 : \Box\Gamma_{12}, F_{10}, \Delta_{13} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{13}), F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7} \vee_L \\
\hline
- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7} K
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14}, F_9 \vee F_{10} \vdash (\Delta_{11}, \Box F_7), F_{13}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_9, F_{13}, \Delta_{14} \vdash \Delta_{11}, \Box F_7 \quad h_8 : \Box\Gamma_{12}, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}, F_9 \vee F_{10}), F_{13} \vdash \Delta_{11}, \Box F_7} \vee_L \\
\hline
- : \Box\Gamma_{12}, \Delta_{14}, F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{14}, \Box\Gamma_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7} K
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_9) \vdash F_6}{\bullet h_1 : \Box\Gamma_9, \perp, \Delta_{10} \vdash \Delta_8, \Box F_6} K \quad \frac{}{\bullet h_7 : (\Box\Gamma_9, \perp, \Delta_{10}), \Box F_6 \vdash \Delta_8} \perp_L \\
\hline
- : \Box\Gamma_9, \perp, \Delta_{10} \vdash \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, \Box F_7), \perp} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{11}), \perp \vdash \Delta_9, \Box F_7} \perp_L \\
\hline
- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{10}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, \Box F_7} K
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \perp, \Delta_{12} \vdash (\Delta_9, \Box F_7), F_{11}} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_{10}, \perp, \Delta_{12}), F_{11} \vdash \Delta_9, \Box F_7} \perp_L \\
\hline
- : \Box\Gamma_{10}, \perp, \Delta_{12} \vdash \Delta_9, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, \Box F_7} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{11}, p_9 \vdash (\Delta_8, p_9), \llbracket F_6 \rrbracket} K \quad \frac{}{\bullet h_7 : (\Box\Gamma_{10}, \Delta_{11}, p_9), \llbracket F_6 \rrbracket \vdash \Delta_8, p_9} I \\
\hline
- : \Box\Gamma_{10}, \Delta_{11}, p_9 \vdash \Delta_8, p_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{11}, \Box\Gamma_{10}, p_9 \vdash \Delta_8, p_9} I
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12} \vdash ((\Delta_9, p_{10}), \llbracket F_7 \rrbracket), p_{10}} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_{11}, \Delta_{12}), p_{10} \vdash (\Delta_9, p_{10}), \llbracket F_7 \rrbracket} I \\
\hline
- : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_9, p_{10}), \llbracket F_7 \rrbracket \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{11}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_9, p_{10}, \llbracket F_7 \rrbracket} K
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{13}, p_{10} \vdash ((\Delta_9, p_{10}), \llbracket F_7 \rrbracket), F_{12}} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_{11}, \Delta_{13}, p_{10}), F_{12} \vdash (\Delta_9, p_{10}), \llbracket F_7 \rrbracket} I \\
\hline
- : \Box\Gamma_{11}, \Delta_{13}, p_{10} \vdash (\Delta_9, p_{10}), \llbracket F_7 \rrbracket \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{13}, \Box\Gamma_{11}, p_{10} \vdash \Delta_9, p_{10}, \llbracket F_7 \rrbracket} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_9) \vdash F_6}{\bullet h_1 : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8, \llbracket F_6 \rrbracket} K \quad \frac{h_7 : \Box\Gamma_9, \Delta_{10}, \llbracket F_6 \rrbracket \vdash \Delta_8}{\bullet h_7 : (\Box\Gamma_9, \top, \Delta_{10}), \llbracket F_6 \rrbracket \vdash \Delta_8} \top_L \\
\hline
- : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{\bullet h_1 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8, \llbracket F_6 \rrbracket} \text{ax/W} \quad \frac{h_7 : \top, \Delta_{10}, \Box\Gamma_9, \llbracket F_6 \rrbracket \vdash \Delta_8}{\bullet h_7 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, \llbracket F_7 \rrbracket), \top} K \quad \frac{h_8 : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, \llbracket F_7 \rrbracket}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{11}), \top \vdash \Delta_9, \llbracket F_7 \rrbracket} \top_L \\
\hline
- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, \llbracket F_7 \rrbracket \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, \llbracket F_7 \rrbracket} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \top, \Delta_{12} \vdash (\Delta_9, \llbracket F_7 \rrbracket), F_{11}} K \quad \frac{h_8 : \Box\Gamma_{10}, F_{11}, \Delta_{12} \vdash \Delta_9, \llbracket F_7 \rrbracket}{\bullet h_8 : (\Box\Gamma_{10}, \top, \Delta_{12}), F_{11} \vdash \Delta_9, \llbracket F_7 \rrbracket} \top_L \\
\hline
- : \Box\Gamma_{10}, \top, \Delta_{12} \vdash \Delta_9, \llbracket F_7 \rrbracket \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{10}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \top, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, \llbracket F_7 \rrbracket} K
\end{array}$$

8.7 Status of \rightarrow_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{13}} \rightarrow_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \rightarrow F_8 \vdash F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R \\
\hline
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \text{inv-th/ax} \\
\hline
\frac{}{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}} \text{ax/W} \\
\hline
\frac{}{- : \Delta_{14}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}} \rightarrow_R \\
\hline
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \wedge F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{13}} \rightarrow_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \rightarrow F_8 \vdash F_{13}}{\bullet h_9 : (\Delta_{14}, F_7 \rightarrow F_8) \vdash F_{13}}}{\rightarrow} \quad \frac{}{- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \\
\frac{\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{13}}{\rightarrow_L} \quad \frac{\frac{h_9 : \Delta_{14}, F_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}}{\text{ax/W}} \quad \frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{11}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \text{ inv-th/ax}}{\rightarrow} \quad \frac{}{- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}} \\
\frac{}{- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{ hCut}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \vee F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{13}} \rightarrow_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \rightarrow F_8 \vdash F_{10}, F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}}}{\rightarrow} \quad \frac{}{- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R \\
\frac{\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}}{\rightarrow_L} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\text{ax/W}}}{\rightarrow} \quad \frac{}{- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \vee_R \\
\frac{}{- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{ hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12} \vdash F_7, F_{11}, \perp, \Delta_{10} \quad h_1 : F_8, \Delta_{12} \vdash F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash (\perp, \Delta_{10}), F_{11}} \rightarrow_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \rightarrow F_8), F_{11} \vdash \perp, \Delta_{10}}}{\rightarrow} \quad \frac{}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}} \perp_R \\
\frac{\frac{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}, F_{11}}{\text{ax/W}} \quad \frac{h_9 : \Delta_{12}, F_{11}, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}}{\text{hCut}}}{\rightarrow} \quad \frac{}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}} \text{ Cut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12} \vdash F_7, F_{11}, \top, \Delta_{10} \quad h_1 : F_8, \Delta_{12} \vdash F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash (\top, \Delta_{10}), F_{11}} \rightarrow_L \quad \frac{h_9 : (\Delta_{12}, F_7 \rightarrow F_8), F_{11} \vdash \top, \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \rightarrow F_8), F_{11} \vdash \top, \Delta_{10}}}{\rightarrow} \quad \frac{}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}} \top_R \\
\frac{}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}} \text{ Cut}
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{\frac{h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash F_7, \Box F_{12}, \Delta_{11}, \Box F_{10} \quad h_1 : F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : (\Box \Gamma_{13}, \Delta_{14}), F_7 \rightarrow F_8 \vdash (\Delta_{11}, \Box F_{10}), \Box F_{12}} \rightarrow_L \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{13}, \Delta_{14}), F_7 \rightarrow F_8), \Box F_{12} \vdash \Delta_{11}, \Box F_{10}}}{\rightarrow} \quad \frac{}{- : (\Box \Gamma_{13}, \Delta_{14}), F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \\
\frac{\frac{h_1 : \Delta_{14}, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_7, \Box F_{10}}{\text{ax/W}} \quad \frac{\frac{h_9 : \text{unbox}(\Box F_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} \text{ K}}{\rightarrow} \quad \frac{}{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} \text{ hCut} \\
\frac{}{- : \Delta_{14}, \Box \Gamma_{13}, F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \\
\frac{\frac{h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash F_7, F_{13}, \Delta_{11}, \Box F_{10} \quad h_1 : F_8, \Box \Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : (\Box \Gamma_{12}, \Delta_{14}), F_7 \rightarrow F_8 \vdash (\Delta_{11}, \Box F_{10}), F_{13}} \rightarrow_L \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{12}, \Delta_{14}), F_7 \rightarrow F_8), F_{13} \vdash \Delta_{11}, \Box F_{10}}}{\rightarrow} \quad \frac{}{- : (\Box \Gamma_{12}, \Delta_{14}), F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} K \\
\frac{}{- : \text{unbox}(\Box \Gamma_{12}) \vdash F_{10}} \text{ ax/W} \\
\frac{}{- : \Delta_{14}, \Box \Gamma_{12}, F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \vee F_{11} \vdash F_{13}, \Delta_{12} \rightarrow_L \quad h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \rightarrow F_8}{\bullet h_1 : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{13}} \rightarrow_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \rightarrow F_8}{\bullet h_9 : ((\Delta_{14}, F_{10} \vee F_{11}), F_7 \rightarrow F_8) \vdash \Delta_{12}} \rightarrow_L \\
\frac{\bullet h_1 : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{13} \quad \bullet h_9 : ((\Delta_{14}, F_{10} \vee F_{11}), F_7 \rightarrow F_8) \vdash \Delta_{12}}{- : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_7}{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{ ax/W} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{14}, F_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{ inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{14}, F_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}}{h_1 : \Delta_{14}, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{ } \vee_L}{- : \Delta_{14}, F_7 \rightarrow F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{ hCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash F_7, \perp, \Delta_{10} \quad h_1 : F_8, \Delta_{11} \vdash \perp, \Delta_{10} \rightarrow_L \quad \frac{\bullet h_1 : (\Delta_{11}, F_7 \rightarrow F_8), \perp \vdash \Delta_{10}}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \rightarrow_L \quad \frac{\bullet h_9 : (\Delta_{11}, F_7 \rightarrow F_8), \perp \vdash \Delta_{10}}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \perp_L}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ Cut} \\
\frac{\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7}{- : \Delta_{11} \vdash \Delta_{10}, F_7} \text{ ax/W} \quad \frac{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7}{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7} \text{ } \perp_L \quad \frac{h_1 : \Delta_{11}, F_8 \vdash \perp, \Delta_{10}}{h_1 : \Delta_{11}, F_8 \vdash \perp, \Delta_{10}} \text{ ax/W} \quad \frac{\bullet h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}}{\bullet h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}} \text{ } \perp_L}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ hCut} \\
\frac{h_1 : \perp, \Delta_{12} \vdash F_7, F_{11}, \Delta_{10} \quad h_1 : F_8, \perp, \Delta_{12} \vdash F_{11}, \Delta_{10} \rightarrow_L \quad \frac{\bullet h_1 : (\perp, \Delta_{12}), F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}}{- : (\perp, \Delta_{12}), F_7 \rightarrow F_8 \vdash \Delta_{10}} \rightarrow_L \quad \frac{\bullet h_9 : ((\perp, \Delta_{12}), F_7 \rightarrow F_8), F_{11} \vdash \Delta_{10}}{- : (\perp, \Delta_{12}), F_7 \rightarrow F_8 \vdash \Delta_{10}} \perp_L}{- : \perp, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ Cut}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_7, p_{11}, \Delta_{10}, p_{11} \quad h_1 : F_8, \Delta_{12} \vdash p_{11}, \Delta_{10}, p_{11} \rightarrow_L \quad \frac{\bullet h_1 : (\Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}), p_{11}}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \rightarrow_L \quad \frac{\bullet h_9 : (\Delta_{12}, F_7 \rightarrow F_8), p_{11} \vdash \Delta_{10}, p_{11}}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} I}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \text{ Cut} \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}, p_{11}}{- : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}} \text{ ax/W} \quad \frac{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, p_{11}}{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, p_{11}} I \quad \frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}, p_{11}}{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}, p_{11}} \text{ ax/W} \quad \frac{\bullet h_9 : \Delta_{12}, F_8, p_{11} \vdash \Delta_{10}, p_{11}}{\bullet h_9 : \Delta_{12}, F_8, p_{11} \vdash \Delta_{10}, p_{11}} I}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \text{ hCut} \\
\frac{h_1 : \Delta_{13}, p_{11} \vdash F_7, F_{12}, \Delta_{10}, p_{11} \quad h_1 : F_8, \Delta_{13}, p_{11} \vdash F_{12}, \Delta_{10}, p_{11} \rightarrow_L \quad \frac{\bullet h_1 : (\Delta_{13}, p_{11}), F_7 \rightarrow F_8 \vdash (\Delta_{10}, p_{11}), F_{12}}{- : (\Delta_{13}, p_{11}), F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \rightarrow_L \quad \frac{\bullet h_9 : ((\Delta_{13}, p_{11}), F_7 \rightarrow F_8), F_{12} \vdash \Delta_{10}, p_{11}}{- : (\Delta_{13}, p_{11}), F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} I}{- : \Delta_{13}, p_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \text{ Cut}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash F_7, \top, \Delta_{10} \quad h_1 : F_8, \Delta_{11} \vdash \top, \Delta_{10} \rightarrow_L \quad \frac{h_9 : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \rightarrow F_8), \top \vdash \Delta_{10}} \rightarrow_L \quad \frac{\bullet h_1 : (\Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}), \top}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ } \top_L}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ Cut} \\
\frac{h_1 : \top, \Delta_{12} \vdash F_7, F_{11}, \Delta_{10} \quad h_1 : F_8, \top, \Delta_{12} \vdash F_{11}, \Delta_{10} \rightarrow_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : ((\top, \Delta_{12}), F_7 \rightarrow F_8), F_{11} \vdash \Delta_{10}} \rightarrow_L \quad \frac{\bullet h_1 : (\top, \Delta_{12}), F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}}{- : (\top, \Delta_{12}), F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ } \top_L}{- : \top, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ hCut} \\
\frac{\frac{\bullet h_1 : \top, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}}{- : \top, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ ax/W} \quad \frac{h_9 : \top, \Delta_{12}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{h_9 : \top, \Delta_{12}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ ax/W}}{- : \top, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ hCut}
\end{array}$$

8.8 Status of \wedge_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \wedge F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{13}} \wedge_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \wedge F_8), F_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R \\
\hline
- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_7, F_8 \vdash \Delta_{12}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \wedge_L \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}}{\bullet h_9 : (\Delta_{14}, F_{10}, F_{13}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}} \text{ax/W}}{\frac{- : \Delta_{14}, F_{10}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}}{- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R} \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \wedge F_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{13}} \wedge_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \wedge F_8), F_{13} \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_R \\
\hline
- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, F_8 \vdash \Delta_{12}, F_{13}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14}, F_7, F_8 \vdash \Delta_{12}, F_{13}, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10}}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_L} \text{hCut}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \wedge F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{13}} \wedge_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{10}, F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \wedge F_8), F_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R \\
\hline
- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}} \wedge_L \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\bullet h_9 : \Delta_{14}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{ax/W}}{\frac{- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{12} \vdash F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash (\perp, \Delta_{10}), F_{11}} \wedge_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \wedge F_8), F_{11} \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}, F_{11}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_{11}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : \Delta_{12}, F_{11}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}} \text{hCut} \\
\hline
- : \Delta_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{12} \vdash F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash (\top, \Delta_{10}), F_{11}} \wedge_L \quad \frac{h_9 : (\Delta_{12}, F_7 \wedge F_8), F_{11} \vdash \top, \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \wedge F_8), F_{11} \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_{12}, F_7 \wedge F_8 \vdash \top, \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
- : \Delta_{12}, F_7 \wedge F_8 \vdash \top, \Delta_{10} \quad \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}]}{\bullet h_1 : (\Box \Gamma_{13}, \Delta_{14}), F_7 \wedge F_8 \vdash (\Delta_{11}, [\Box F_{10}]), \Box F_{12}} \wedge_L \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{13}, \Delta_{14}), F_7 \wedge F_8), \Box F_{12} \vdash \Delta_{11}, [\Box F_{10}]} K \\
\hline
- : (\Box \Gamma_{13}, \Delta_{14}), F_7 \wedge F_8 \vdash \Delta_{11}, [\Box F_{10}] \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, F_8, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}]}{\bullet h_1 : \Delta_{14}, F_7, F_8, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}]} \text{ax/W} \quad \frac{h_9 : \text{unbox}(\Box F_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_7, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}]} K}{\frac{- : \Delta_{14}, F_7, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}]}{- : \Delta_{14}, \Box \Gamma_{13}, F_7 \wedge F_8 \vdash \Delta_{11}, [\Box F_{10}]} \wedge_L} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Box\Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : (\Box\Gamma_{12}, \Delta_{14}), F_7 \wedge F_8 \vdash (\Delta_{11}, \Box F_{10}), F_{13}} \wedge_L \quad \frac{h_9 : unbox(\Box\Gamma_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{12}, \Delta_{14}), F_7 \wedge F_8), F_{13} \vdash \Delta_{11}, \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{12}, \Delta_{14}), F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10} \\
\hline
\rightarrow \\
\frac{}{- : unbox(\Box\Gamma_{12}) \vdash F_{10}} ax/W \\
\hline
- : \Delta_{14}, \Box\Gamma_{12}, F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10} K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{13} \vdash F_{10} \rightarrow F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_7 \wedge F_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \rightarrow_L \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} ax/W \quad \frac{h_9 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \quad h_9 : F_{11}, \Delta_{13}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} inv-th/ax \\
\hline
- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \quad hCut \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \wedge_L \\
\hline
\frac{h_1 : F_7, F_8, \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{13}} \wedge_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_{11}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8), F_{13} \vdash \Delta_{12}} \rightarrow_L \\
\hline
- : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash \Delta_{12} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}} ax/W \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \quad h_9 : F_{11}, \Delta_{14}, F_{13}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} inv-th/ax \\
\hline
- : \Delta_{14}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12} \quad hCut \\
\hline
- : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8 \vdash \Delta_{12} \wedge_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{13} \vdash F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_L \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} ax/W \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \wedge F_{11} \vdash \Delta_{12}} inv-th/ax \\
\hline
- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \quad hCut \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \wedge_L \\
\hline
\frac{h_1 : F_7, F_8, \Delta_{14}, F_{10} \wedge F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{13}} \wedge_L \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \wedge F_{11}), F_7 \wedge F_8), F_{13} \vdash \Delta_{12}} \wedge_L \\
\hline
- : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash \Delta_{12} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10}, F_{11}, F_7, F_8 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{13}} inv-th/ax \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}} ax/W \\
\hline
- : \Delta_{14}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{12} \quad hCut \\
\hline
- : \Delta_{14}, F_{10} \wedge F_{11}, F_7 \wedge F_8 \vdash \Delta_{12} \wedge_L \\
\hline
\frac{h_1 : F_8, F_9, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}, F_{11}} \wedge_L \quad \frac{h_7 : F_8, F_9, F_{11}, \Delta_{12} \vdash \Delta_{10}}{\bullet h_7 : (\Delta_{12}, F_8 \wedge F_9), F_{11} \vdash \Delta_{10}} \wedge_L \\
\hline
- : \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_8, F_9 \vdash \Delta_{10}, F_{11}}{\bullet h_1 : \Delta_{12}, F_8, F_9 \vdash \Delta_{10}, F_{11}} ax/W \quad \frac{h_7 : \Delta_{12}, F_{11}, F_8, F_9 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11}, F_8, F_9 \vdash \Delta_{10}} H \\
\hline
- : \Delta_{12}, F_8, F_9 \vdash \Delta_{10} \quad hCut \\
\hline
- : \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10} \wedge_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, F_8, \Delta_{13} \vdash F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \wedge_L \quad \frac{\frac{h_9 : F_{10}, \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L}{\frac{- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}}{\text{ax/W}} \quad \frac{\frac{h_9 : \Delta_{13}, F_{10}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_7, F_8 \vdash \Delta_{12}}{\text{inv-th/ax}} \vee_L}{\frac{- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}}{- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}} \wedge_L \text{hCut}} \\
\frac{\frac{h_1 : F_7, F_8, \Delta_{14}, F_{10} \vee F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{13}} \wedge_L \quad \frac{\frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12} \quad h_9 : F_{11}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \vee F_{11}), F_7 \wedge F_8), F_{13} \vdash \Delta_{12}} \vee_L}{\frac{- : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}}{\text{ax/W}} \quad \frac{\frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13}, F_7, F_8 \vdash \Delta_{12}}{\text{inv-th/ax}} \vee_L}{\frac{- : \Delta_{14}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}}{- : \Delta_{14}, F_7 \wedge F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \wedge_L \text{hCut}}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, F_8, \Delta_{11} \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, \perp} \wedge_L \quad \frac{\bullet h_9 : (\Delta_{11}, F_7 \wedge F_8), \perp \vdash \Delta_{10}}{\text{Cut}} \perp_L}{\frac{- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{11}, F_7, F_8 \vdash \perp, \Delta_{10}}{\text{ax/W}} \quad \frac{\bullet h_9 : \perp, \Delta_{11}, F_7, F_8 \vdash \Delta_{10}}{\text{hCut}} \perp_L}{\frac{- : \Delta_{11}, F_7, F_8 \vdash \Delta_{10}}{- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}} \wedge_L} \\
\frac{\frac{h_1 : F_7, F_8, \perp, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\perp, \Delta_{12}), F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}} \wedge_L \quad \frac{\bullet h_9 : ((\perp, \Delta_{12}), F_7 \wedge F_8), F_{11} \vdash \Delta_{10}}{\text{Cut}} \perp_L}{\frac{- : (\perp, \Delta_{12}), F_7 \wedge F_8 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{- : \perp, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}}{\perp_L}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, F_8, \Delta_{12} \vdash P_{11}, \Delta_{10}, P_{11}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash (\Delta_{10}, P_{11}), P_{11}} \wedge_L \quad \frac{\bullet h_9 : (\Delta_{12}, F_7 \wedge F_8), P_{11} \vdash \Delta_{10}, P_{11}}{\text{Cut}} I}{\frac{- : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}, P_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{12}, F_7, F_8 \vdash \Delta_{10}, P_{11}, P_{11}}{\text{ax/W}} \quad \frac{\bullet h_9 : \Delta_{12}, F_7, F_8, P_{11} \vdash \Delta_{10}, P_{11}}{\text{hCut}} I}{\frac{- : \Delta_{12}, F_7, F_8 \vdash \Delta_{10}, P_{11}}{- : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}, P_{11}} \wedge_L} \\
\frac{\frac{h_1 : F_7, F_8, \Delta_{13}, P_{11} \vdash F_{12}, \Delta_{10}, P_{11}}{\bullet h_1 : (\Delta_{13}, P_{11}), F_7 \wedge F_8 \vdash (\Delta_{10}, P_{11}), F_{12}} \wedge_L \quad \frac{\bullet h_9 : ((\Delta_{13}, P_{11}), F_7 \wedge F_8), F_{12} \vdash \Delta_{10}, P_{11}}{\text{Cut}} I}{\frac{- : (\Delta_{13}, P_{11}), F_7 \wedge F_8 \vdash \Delta_{10}, P_{11}}{\rightarrow} \text{Cut}} \\
\frac{- : \Delta_{13}, P_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, P_{11}}{I}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, F_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, \top} \wedge_L \quad \frac{\frac{h_9 : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \wedge F_8), \top \vdash \Delta_{10}} \top_L}{\frac{- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}}{\text{ax/W}}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \top, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\top, \Delta_{12}), F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}} \wedge_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : ((\top, \Delta_{12}), F_7 \wedge F_8), F_{11} \vdash \Delta_{10}} \top_L \\
\hline
- : (\top, \Delta_{12}), F_7 \wedge F_8 \vdash \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}}{- : \top, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \top, \Delta_{12}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{10}}{- : \top, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{ax/W}}{- : \top, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{hCut}
\end{array}$$

8.9 Status of \vee_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{13}} \vee_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \vee F_8), F_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R \\
\hline
- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_7 \vdash \Delta_{12}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \text{inv-th/ax}}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \vee_L \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}}{- : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}} \text{ax/W}}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \text{hCut} \\
\hline
- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \quad \rightarrow_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \wedge F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{13}} \vee_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash F_{10}, F_{11}}{\bullet h_9 : (\Delta_{14}, F_7 \vee F_8), F_{13} \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_R \\
\hline
- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{10}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \text{inv-th/ax}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \vee_L \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}} \text{ax/W}}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{hCut} \\
\hline
- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \vee F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{13}} \vee_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash F_{10}, F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \vee F_8), F_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R \\
\hline
- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}} \text{inv-th/ax}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}} \vee_L \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{ax/W}}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{hCut} \\
\hline
- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11} \quad \vee_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{12} \vdash F_{11}, \perp, \Delta_{10} \quad h_1 : F_8, \Delta_{12} \vdash F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash (\perp, \Delta_{10}), F_{11}} \vee_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \vee F_8), F_{11} \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_{12}, F_7 \vee F_8 \vdash \perp, \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash \perp, \Delta_{10}, F_{11}}{- : \Delta_{12}, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_{11}, F_7 \vee F_8 \vdash \perp, \Delta_{10}}{- : \Delta_{12}, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{ax/W}}{- : \Delta_{12}, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{12} \vdash F_{11}, \top, \Delta_{10} \quad h_1 : F_8, \Delta_{12} \vdash F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash (\top, \Delta_{10}), F_{11}} \vee_L \quad \frac{h_9 : (\Delta_{12}, F_7 \vee F_8), F_{11} \vdash \top, \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \vee F_8), F_{11} \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_{12}, F_7 \vee F_8 \vdash \top, \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_{12}, F_7 \vee F_8 \vdash \top, \Delta_{10}}{- : \Delta_{12}, F_7 \vee F_8 \vdash \top, \Delta_{10}} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : F_7, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}] \quad h_1 : F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}] \quad \vee_L \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8), \Box F_{12} \vdash \Delta_{11}, [\Box F_{10}]} \rightarrow \\
\frac{\bullet h_1 : ((\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8) \vdash (\Delta_{11}, [\Box F_{10}]), \Box F_{12}}{- : ((\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8) \vdash \Delta_{11}, [\Box F_{10}]} \rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}]}{- : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}]} \text{ ax/W} \quad \frac{h_9 : \text{unbox}(\Box F_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}]} \text{ K} \quad \frac{h_1 : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}]}{- : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}]} \text{ ax/W} \quad \frac{h_9 : \text{unbox}(\Box F_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}]} \text{ K} \\
\frac{}{- : \Delta_{14}, \Box \Gamma_{13}, F_7 \vee F_8 \vdash \Delta_{11}, [\Box F_{10}]} \text{ hCut} \\
\frac{h_1 : F_7, \Box \Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, [\Box F_{10}] \quad h_1 : F_8, \Box \Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, [\Box F_{10}] \quad \vee_L \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8), F_{13} \vdash \Delta_{11}, [\Box F_{10}]} \text{ K} \\
\frac{\bullet h_1 : ((\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8) \vdash (\Delta_{11}, [\Box F_{10}]), F_{13}}{- : ((\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8) \vdash \Delta_{11}, [\Box F_{10}]} \text{ Cut} \\
\frac{}{- : \Delta_{14}, \Box \Gamma_{12}, F_7 \vee F_8 \vdash \Delta_{11}, [\Box F_{10}]} \rightarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_{12}) \vdash F_{10}} \text{ ax/W} \\
\frac{}{- : \Delta_{14}, \Box \Gamma_{12}, F_7 \vee F_8 \vdash \Delta_{11}, [\Box F_{10}]} \text{ K}
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{13} \vdash F_{10} \rightarrow F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \rightarrow F_{11}, \Delta_{12} \quad \vee_L \quad \frac{h_9 : \Delta_{13}, F_7 \vee F_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_7 \vee F_8), F_{10} \rightarrow F_{11}) \vdash \Delta_{12}} \rightarrow_L \\
\frac{\bullet h_1 : (\Delta_{13}, F_7 \vee F_8) \vdash \Delta_{12}, F_{10} \rightarrow F_{11}}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ Cut} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{}{- : \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_{11}} \text{ inv-th/ax} \quad \frac{}{- : \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}} \text{ inv-th/ax} \\
\frac{}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}} \vee_L \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}} \text{ ax/W} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ sCut} \\
\frac{h_1 : F_7, \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_{13}, \Delta_{12} \quad \vee_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash F_{10}}{\bullet h_9 : ((\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \vee F_8) \vdash \Delta_{12}} \rightarrow_L \\
\frac{\bullet h_1 : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \vdash \Delta_{12}, F_{13}}{- : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \vdash \Delta_{12}} \text{ Cut} \\
\frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{10}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{13}}{\bullet h_1 : \Delta_{14}, F_8 \vee F_7 \vdash \Delta_{12}, F_{10}, F_{13}} \text{ inv-th/ax} \\
\frac{}{- : \Delta_{14}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}} \vee_L \\
\frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \text{ ax/W} \\
\frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ hCut} \\
\frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ Cut}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{13} \vdash F_{10} \wedge F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \wedge F_{11}, \Delta_{12} \quad \vee_L \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_7 \vee F_8), F_{10} \wedge F_{11}) \vdash \Delta_{12}} \wedge_L \\
\frac{\bullet h_1 : (\Delta_{13}, F_7 \vee F_8) \vdash \Delta_{12}, F_{10} \wedge F_{11}}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ Cut} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{ ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}} \text{ inv-th/ax} \\
\frac{}{- : \Delta_{13}, F_7 \vdash \Delta_{12}} \text{ ax/W} \quad \frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \wedge_L \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ hCut} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \vee_L \\
\frac{h_1 : F_7, \Delta_{14}, F_{10} \wedge F_{11} \vdash F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \wedge F_{11} \vdash F_{13}, \Delta_{12} \quad \vee_L \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \wedge F_{11}), F_7 \vee F_8), F_{13} \vdash \Delta_{12}} \wedge_L \\
\frac{\bullet h_1 : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash \Delta_{12}, F_{13}}{- : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash \Delta_{12}} \text{ Cut} \\
\frac{}{- : \Delta_{14}, F_{10} \wedge F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_{11}, F_8 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11}, F_8 \vee F_7 \vdash \Delta_{12}, F_{13}} \text{ inv-th/ax} \\
\frac{}{- : \Delta_{14}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \vee_L \\
\frac{}{- : \Delta_{14}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ ax/W} \\
\frac{}{- : \Delta_{14}, F_{10} \wedge F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \wedge_L \\
\frac{}{- : \Delta_{14}, F_{10} \wedge F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, \Delta_{13} \vdash F_{10} \vee F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_L \quad \frac{h_9 : F_{10}, \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L}{\frac{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_{11}}{\rightarrow} \text{inv-th/ax} \quad \frac{- : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\rightarrow} \text{inv-th/ax}}{\frac{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\vee_L}} \quad \frac{- : \Delta_{13}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{\text{ax/W}} \quad \frac{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{\text{sCut}} \quad \frac{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}}{\rightarrow} \\
\frac{h_1 : F_7, \Delta_{14}, F_{10} \vee F_{11} \vdash F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \vee F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \vee_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \vee F_{11}), F_7 \vee F_8) \vdash \Delta_{12}} \vee_L}{\rightarrow} \\
\frac{h_1 : \Delta_{14}, F_{10}, F_7 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_8 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_8 \vee F_7 \vdash \Delta_{12}, F_{13}} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}}{\vee_L}} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\text{ax/W}} \quad \frac{h_1 : \Delta_{14}, F_{11}, F_7 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \text{hCut}}{\rightarrow} \\
\frac{- : \Delta_{14}, F_{10} \vee F_{11}, F_7 \vee F_8 \vdash \Delta_{12}}{\rightarrow} \\
\frac{h_1 : F_8, \Delta_{12} \vdash F_{11}, \Delta_{10} \quad h_1 : F_9, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}, F_{11}} \vee_L \quad \frac{h_7 : F_8, F_{11}, \Delta_{12} \vdash \Delta_{10} \quad h_7 : F_9, F_{11}, \Delta_{12} \vdash \Delta_{10}}{\bullet h_7 : (\Delta_{12}, F_8 \vee F_9), F_{11} \vdash \Delta_{10}} \vee_L}{\rightarrow} \\
\frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, F_{11}}{\bullet h_1 : \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}, F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11}, F_8 \vee F_9 \vdash \Delta_{10}} \text{H}}{\frac{- : \Delta_{12}, F_8 \vdash \Delta_{10}}{\text{hCut}}} \quad \frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{10}, F_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_8 \vdash \Delta_{10}, F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11}, F_9 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11}, F_9 \vee F_8 \vdash \Delta_{10}} \text{H}}{\frac{- : \Delta_{12}, F_9 \vdash \Delta_{10}}{\text{hCut}}} \quad \frac{- : \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}}{\vee_L}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{11} \vdash \perp, \Delta_{10} \quad h_1 : F_8, \Delta_{11} \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}, \perp} \vee_L \quad \frac{h_9 : (\Delta_{11}, F_7 \vee F_8), \perp \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \vee F_8), \perp \vdash \Delta_{10}} \vee_L}{\rightarrow} \\
\frac{h_1 : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}}{\bullet h_9 : \perp, \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \perp_L}{\frac{- : \Delta_{11}, F_7 \vdash \Delta_{10}}{\text{hCut}}} \quad \frac{h_1 : \Delta_{11}, F_8 \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_8 \vee F_7 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}}{\bullet h_9 : \perp, \Delta_{11}, F_8 \vee F_7 \vdash \Delta_{10}} \perp_L}{\frac{- : \Delta_{11}, F_8 \vdash \Delta_{10}}{\text{hCut}}} \quad \frac{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}}{\vee_L} \\
\frac{h_1 : F_7, \perp, \Delta_{12} \vdash F_{11}, \Delta_{10} \quad h_1 : F_8, \perp, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\perp, \Delta_{12}), F_7 \vee F_8 \vdash \Delta_{10}, F_{11}} \vee_L \quad \frac{h_9 : ((\perp, \Delta_{12}), F_7 \vee F_8), F_{11} \vdash \Delta_{10}}{\bullet h_9 : ((\perp, \Delta_{12}), F_7 \vee F_8), F_{11} \vdash \Delta_{10}} \vee_L}{\rightarrow} \\
\frac{- : (\perp, \Delta_{12}), F_7 \vee F_8 \vdash \Delta_{10}}{\rightarrow} \quad \frac{- : \perp, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{\perp_L}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{12} \vdash p_{11}, \Delta_{10}, p_{11} \quad h_1 : F_8, \Delta_{12} \vdash p_{11}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash (\Delta_{10}, p_{11}), p_{11}} \vee_L \quad \frac{h_9 : (\Delta_{12}, F_7 \vee F_8), p_{11} \vdash \Delta_{10}, p_{11}}{\bullet h_9 : (\Delta_{12}, F_7 \vee F_8), p_{11} \vdash \Delta_{10}, p_{11}} I}{\rightarrow} \\
\frac{h_1 : \Delta_{12}, F_7 \vdash \Delta_{10}, p_{11}, p_{11}}{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}, p_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_7, p_{11} \vdash \Delta_{10}, p_{11}}{\bullet h_9 : \Delta_{12}, F_7 \vee F_8, p_{11} \vdash \Delta_{10}, p_{11}} I}{\frac{- : \Delta_{12}, F_7 \vdash \Delta_{10}, p_{11}}{\text{hCut}}} \quad \frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}, p_{11}}{\bullet h_1 : \Delta_{12}, F_8 \vee F_7 \vdash \Delta_{10}, p_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_8, p_{11} \vdash \Delta_{10}, p_{11}}{\bullet h_9 : \Delta_{12}, F_8 \vee F_7, p_{11} \vdash \Delta_{10}, p_{11}} I}{\frac{- : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}}{\text{hCut}}} \quad \frac{- : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}, p_{11}}{\vee_L} \\
\frac{h_1 : F_7, \Delta_{13}, p_{11} \vdash F_{12}, \Delta_{10}, p_{11} \quad h_1 : F_8, \Delta_{13}, p_{11} \vdash F_{12}, \Delta_{10}, p_{11}}{\bullet h_1 : (\Delta_{13}, p_{11}), F_7 \vee F_8 \vdash (\Delta_{10}, p_{11}), F_{12}} \vee_L \quad \frac{h_9 : ((\Delta_{13}, p_{11}), F_7 \vee F_8), F_{12} \vdash \Delta_{10}, p_{11}}{\bullet h_9 : ((\Delta_{13}, p_{11}), F_7 \vee F_8), F_{12} \vdash \Delta_{10}, p_{11}} I}{\rightarrow} \\
\frac{- : (\Delta_{13}, p_{11}), F_7 \vee F_8 \vdash \Delta_{10}, p_{11}}{\rightarrow} \quad \frac{- : \Delta_{13}, p_{11}, F_7 \vee F_8 \vdash \Delta_{10}, p_{11}}{I}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, \Delta_{11} \vdash \top, \Delta_{10} \quad h_1 : F_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}, \top} \vee_L \quad \frac{h_9 : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \vee F_8), \top \vdash \Delta_{10}} \top_L}{\frac{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{}{\frac{}{\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \text{ax/W}}} \\
\frac{\frac{h_1 : F_7, \top, \Delta_{12} \vdash F_{11}, \Delta_{10} \quad h_1 : F_8, \top, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\top, \Delta_{12}), F_7 \vee F_8 \vdash \Delta_{10}, F_{11}} \vee_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : ((\top, \Delta_{12}), F_7 \vee F_8), F_{11} \vdash \Delta_{10}} \top_L}{\frac{- : (\top, \Delta_{12}), F_7 \vee F_8 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \top, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}}{\text{ax/W}} \quad \frac{h_9 : \top, \Delta_{12}, F_{11}, F_7 \vee F_8 \vdash \Delta_{10}}{\text{ax/W}}}{\frac{}{- : \top, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}} \text{hCut}}
\end{array}$$

8.10 Status of \perp_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_{10} \vdash (\Delta_8, F_6 \rightarrow F_7), F_9} \perp_L \quad \frac{h_5 : \perp, F_6, F_9, \Delta_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\perp, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \rightarrow F_7} \rightarrow_R}{\frac{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7}{\rightarrow} \text{Cut}} \\
\frac{}{\frac{}{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \perp_L}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_{10} \vdash (\Delta_8, F_6 \wedge F_7), F_9} \perp_L \quad \frac{h_5 : \perp, F_9, \Delta_{10} \vdash F_6, \Delta_8 \quad h_5 : \perp, F_9, \Delta_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\perp, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \wedge F_7} \wedge_R}{\frac{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7}{\rightarrow} \text{Cut}} \\
\frac{}{\frac{}{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7} \perp_L}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_{10} \vdash (\Delta_8, F_6 \vee F_7), F_9} \perp_L \quad \frac{h_5 : \perp, F_9, \Delta_{10} \vdash F_6, F_7, \Delta_8}{\bullet h_5 : (\perp, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \vee F_7} \vee_R}{\frac{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7}{\rightarrow} \text{Cut}} \\
\frac{}{\frac{}{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7} \perp_L}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_8 \vdash (\perp, \Delta_6), F_7} \perp_L \quad \frac{h_5 : \perp, F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : (\perp, \Delta_8), F_7 \vdash \perp, \Delta_6} \perp_R}{\frac{- : \perp, \Delta_8 \vdash \perp, \Delta_6}{\rightarrow} \text{Cut}} \\
\frac{}{\frac{}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \perp_L}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_8 \vdash (\top, \Delta_6), F_7} \perp_L \quad \frac{h_5 : (\perp, \Delta_8), F_7 \vdash \top, \Delta_6}{\bullet h_5 : (\perp, \Delta_8), F_7 \vdash \top, \Delta_6} \top_R}{\frac{- : \perp, \Delta_8 \vdash \top, \Delta_6}{\rightarrow} \text{Cut}} \\
\frac{}{\frac{}{- : \perp, \Delta_8 \vdash \top, \Delta_6} \top_R}
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, [\Box F_6], \Box F_8)} \perp_L \quad \frac{h_5 : \text{unbox}(\Box \Gamma_9), \text{unbox}(\Box F_8) \vdash F_6}{\bullet h_5 : (\perp, \Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \Delta_7, [\Box F_6]} K}{\frac{}{- : \perp, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, [\Box F_6]} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [\Box F_6]} \perp_L \\
\\
\frac{\frac{}{\bullet h_1 : \perp, \Box \Gamma_8, \Delta_{10} \vdash (\Delta_7, [\Box F_6], F_9)} \perp_L \quad \frac{h_5 : \text{unbox}(\Box \Gamma_8) \vdash F_6}{\bullet h_5 : (\perp, \Box \Gamma_8, \Delta_{10}), F_9 \vdash \Delta_7, [\Box F_6]} K}{\frac{}{- : \perp, \Box \Gamma_8, \Delta_{10} \vdash \Delta_7, [\Box F_6]} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_{10}, \Box \Gamma_8 \vdash \Delta_7, [\Box F_6]} \perp_L
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7} \perp_L \quad \frac{h_5 : \perp, \Delta_9 \vdash F_6, \Delta_8 \quad h_5 : \perp, F_7, \Delta_9 \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_9), F_6 \rightarrow F_7 \vdash \Delta_8} \rightarrow_L}{\frac{}{- : \perp, \Delta_9 \vdash \Delta_8} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_9 \vdash \Delta_8} \perp_L \\
\\
\frac{\frac{}{\bullet h_1 : \perp, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8, F_9} \perp_L \quad \frac{h_5 : \perp, F_9, \Delta_{10} \vdash F_6, \Delta_8 \quad h_5 : \perp, F_7, F_9, \Delta_{10} \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \Delta_8} \rightarrow_L}{\frac{}{- : \perp, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8} \perp_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7} \perp_L \quad \frac{h_5 : \perp, F_6, F_7, \Delta_9 \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_9), F_6 \wedge F_7 \vdash \Delta_8} \wedge_L}{\frac{}{- : \perp, \Delta_9 \vdash \Delta_8} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_9 \vdash \Delta_8} \perp_L \\
\\
\frac{\frac{}{\bullet h_1 : \perp, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8, F_9} \perp_L \quad \frac{h_5 : \perp, F_6, F_7, F_9, \Delta_{10} \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_{10}, F_6 \wedge F_7), F_9 \vdash \Delta_8} \wedge_L}{\frac{}{- : \perp, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8} \perp_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_9 \vdash \Delta_8, F_6 \vee F_7} \perp_L \quad \frac{h_5 : \perp, F_6, \Delta_9 \vdash \Delta_8 \quad h_5 : \perp, F_7, \Delta_9 \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_9), F_6 \vee F_7 \vdash \Delta_8} \vee_L}{\frac{}{- : \perp, \Delta_9 \vdash \Delta_8} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_9 \vdash \Delta_8} \perp_L \\
\\
\frac{\frac{}{\bullet h_1 : \perp, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8, F_9} \perp_L \quad \frac{h_5 : \perp, F_6, F_9, \Delta_{10} \vdash \Delta_8 \quad h_5 : \perp, F_7, F_9, \Delta_{10} \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_{10}, F_6 \vee F_7), F_9 \vdash \Delta_8} \vee_L}{\frac{}{- : \perp, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8} \perp_L
\end{array}$$

- Case rule \perp_L

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_8 \vdash \Delta_6, F_7} \perp_L \quad \frac{}{\bullet h_5 : (\perp, \Delta_8), F_7 \vdash \Delta_6} \perp_L}{\frac{}{- : \perp, \Delta_8 \vdash \Delta_6} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_8 \vdash \Delta_6} \perp_L$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_8 \vdash (\Delta_6, p_7), p_7}{\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7} \perp_L \quad \frac{\bullet h_5 : (\perp, \Delta_8), p_7 \vdash \Delta_6, p_7}{\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7} I}{\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_9, p_7 \vdash (\Delta_6, p_7), F_8}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_6, p_7} \perp_L \quad \frac{\bullet h_5 : (\perp, \Delta_9, p_7), F_8 \vdash \Delta_6, p_7}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_6, p_7} I}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_9, p_7 \vdash \Delta_6, p_7 \quad \perp_L
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_7 \vdash \Delta_6, \top}{\vdash : \perp, \Delta_7 \vdash \Delta_6} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_7 \vdash \Delta_6}{\bullet h_5 : (\perp, \Delta_7), \top \vdash \Delta_6} \top_L}{\vdash : \perp, \Delta_7 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_7 \vdash \Delta_6 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \top, \Delta_8 \vdash \Delta_6, F_7}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_6} \perp_L \quad \frac{\frac{h_5 : \perp, F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : (\perp, \top, \Delta_8), F_7 \vdash \Delta_6} \top_L}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \top, \Delta_8 \vdash \Delta_6 \quad \perp_L
\end{array}$$

8.11 Status of I : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \rightarrow F_7), p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} I \quad \frac{\frac{h_5 : F_6, \Delta_9, p_{10}, p_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \rightarrow_R}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, F_6, p_{10} \vdash \Delta_8, F_7, p_{10}}{\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8, F_7} I \quad \frac{h_5 : \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8, F_7}{\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8, F_7} \text{ax/w}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{hCut} \\
\rightarrow_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{12}, p_{10} \vdash ((\Delta_9, F_7 \rightarrow F_8), p_{10}), F_{11}}{\vdash : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \rightarrow F_8), p_{10}} I \quad \frac{\frac{h_6 : F_7, F_{11}, \Delta_{12}, p_{10} \vdash F_8, \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{12}, p_{10}), F_{11} \vdash (\Delta_9, F_7 \rightarrow F_8), p_{10}} \rightarrow_R}{\vdash : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \rightarrow F_8), p_{10}} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}, F_7 \rightarrow F_8 \quad I
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \wedge F_7), p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \wedge F_7} I \quad \frac{\frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash F_6, \Delta_8}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \wedge F_7} \wedge_R}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} \text{ax/w}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} \text{hCut} \\
\wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_7, p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_7} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_7}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_7} \text{ax/w}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_7} \text{hCut} \\
\wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{12}, p_{10} \vdash ((\Delta_9, F_7 \wedge F_8), p_{10}), F_{11}}{\vdash : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \wedge F_8), p_{10}} I \quad \frac{\frac{h_6 : F_{11}, \Delta_{12}, p_{10} \vdash F_7, \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{12}, p_{10}), F_{11} \vdash (\Delta_9, F_7 \wedge F_8), p_{10}} \wedge_R}{\vdash : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \wedge F_8), p_{10}} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}, F_7 \wedge F_8 \quad I
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \vee F_7), p_{10}}{- : \Delta_9, p_{10} \vdash \Delta_8, F_6 \vee F_7} I \quad \frac{\frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash F_6, F_7, \Delta_8}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \vee F_7} \vee_R}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7, p_{10}}{- : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6, F_7}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \vee F_7} \text{ax/W}}{- : \Delta_9, p_{10} \vdash \Delta_8, F_6 \vee F_7} \text{hCut} \\
\vee_R \\
\frac{\frac{\bullet h_1 : \Delta_{12}, p_{10} \vdash ((\Delta_9, F_7 \vee F_8), p_{10}), F_{11}}{- : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10}} I \quad \frac{\frac{h_6 : F_{11}, \Delta_{12}, p_{10} \vdash F_7, F_8, \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{12}, p_{10}), F_{11} \vdash (\Delta_9, F_7 \vee F_8), p_{10}} \vee_R}{\text{Cut}} \\
\rightarrow \\
\frac{- : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10}}{- : \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}, F_7 \vee F_8} I
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash (\perp, \Delta_6), p_8}{- : \Delta_7, p_8 \vdash \perp, \Delta_6} I \quad \frac{\frac{h_5 : \Delta_7, p_8, p_8 \vdash \Delta_6}{\bullet h_5 : (\Delta_7, p_8), p_8 \vdash \perp, \Delta_6} \perp_R}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash \perp, \Delta_6, p_8}{- : \Delta_7, p_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{h_5 : \Delta_7, p_8, p_8 \vdash \perp, \Delta_6}{\bullet h_5 : (\Delta_7, p_8), p_8 \vdash \perp, \Delta_6} \text{ax/W}}{- : \Delta_7, p_8 \vdash \perp, \Delta_6} \text{hCut} \\
\perp_R \\
\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash ((\perp, \Delta_7), p_8), F_9}{- : \Delta_{10}, p_8 \vdash (\perp, \Delta_7), p_8} I \quad \frac{\frac{h_6 : F_9, \Delta_{10}, p_8 \vdash \Delta_7, p_8}{\bullet h_6 : (\Delta_{10}, p_8), F_9 \vdash (\perp, \Delta_7), p_8} \perp_R}{\text{Cut}} \\
\rightarrow \\
\frac{- : \Delta_{10}, p_8 \vdash (\perp, \Delta_7), p_8}{- : \Delta_{10}, p_8 \vdash \perp, \Delta_7, p_8} I
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash (\top, \Delta_6), p_8}{- : \Delta_7, p_8 \vdash \top, \Delta_6} I \quad \frac{\frac{h_5 : (\Delta_7, p_8), p_8 \vdash \top, \Delta_6}{\bullet h_5 : (\Delta_7, p_8), p_8 \vdash \top, \Delta_6} \top_R}{\text{Cut}} \\
\rightarrow \\
\frac{- : \Delta_7, p_8 \vdash \top, \Delta_6}{- : \Delta_7, p_8 \vdash \top, \Delta_6} \top_R \\
\top_R \\
\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash ((\top, \Delta_7), p_8), F_9}{- : \Delta_{10}, p_8 \vdash (\top, \Delta_7), p_8} I \quad \frac{\frac{h_6 : (\Delta_{10}, p_8), F_9 \vdash (\top, \Delta_7), p_8}{\bullet h_6 : (\Delta_{10}, p_8), F_9 \vdash (\top, \Delta_7), p_8} \top_R}{\text{Cut}} \\
\rightarrow \\
\frac{- : \Delta_{10}, p_8 \vdash (\top, \Delta_7), p_8}{- : \Delta_{10}, p_8 \vdash \top, \Delta_7, p_8} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Box \Gamma_8, \Delta_9), p_{10} \vdash (\Delta_7, \Box F_6), p_{10}}{- : (\Box \Gamma_8, \Delta_9), p_{10} \vdash \Delta_7, \Box F_6} I \quad \frac{\frac{h_5 : \text{unbox}(\Box \Gamma_8) \vdash F_6}{\bullet h_5 : ((\Box \Gamma_8, \Delta_9), p_{10}), p_{10} \vdash \Delta_7, \Box F_6} K}{\text{Cut}} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_8) \vdash F_6}{- : \Delta_9, \Box \Gamma_8, p_{10} \vdash \Delta_7, \Box F_6} \text{ax/W} \\
K \\
\frac{\frac{\bullet h_1 : (\Box \Gamma_{11}, \Delta_{12}), p_9 \vdash ((\Delta_8, \Box F_7), p_9), \Box F_{10}}{- : (\Box \Gamma_{11}, \Delta_{12}), p_9 \vdash (\Delta_8, \Box F_7), p_9} I \quad \frac{\frac{h_6 : \text{unbox}(\Box \Gamma_{11}), \text{unbox}(\Box F_{10}) \vdash F_7}{\bullet h_6 : ((\Box \Gamma_{11}, \Delta_{12}), p_9), \Box F_{10} \vdash (\Delta_8, \Box F_7), p_9} K}{\text{Cut}} \\
\rightarrow \\
\frac{- : (\Box \Gamma_{11}, \Delta_{12}), p_9 \vdash (\Delta_8, \Box F_7), p_9}{- : \Delta_{12}, \Box \Gamma_{11}, p_9 \vdash \Delta_8, p_9, \Box F_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \Box F_7), p_9), F_{11}}{\vdash : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \Box F_7), p_9)} I \quad \frac{h_6 : \text{unbox}(\Box \Gamma_{10}) \vdash F_7}{\bullet h_6 : ((\Box \Gamma_{10}, \Delta_{12}), p_9), F_{11} \vdash (\Delta_8, \Box F_7), p_9} K \\
\vdash : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \Box F_7), p_9) \quad \text{Cut} \\
\vdash : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \Box F_7), p_9) \xrightarrow{\rightarrow} \\
\vdash : \Delta_{12}, \Box \Gamma_{10}, p_9 \vdash \Delta_8, p_9, \Box F_7 I
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8, p_{10}}{\vdash : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash F_6, \Delta_8 \quad h_5 : F_7, \Delta_9, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_5 : ((\Delta_9, F_6 \rightarrow F_7), p_{10}), p_{10} \vdash \Delta_8} \rightarrow_L \\
\vdash : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8 \quad \text{Cut} \\
\vdash : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8 \xrightarrow{\rightarrow} \\
\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, p_{10} \quad h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} I \quad \frac{\text{ax/W}}{\text{hCut}} \quad \frac{\bullet h_1 : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10} \quad h_5 : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8} I \quad \frac{\text{ax/W}}{\text{hCut}} \\
\vdash : \Delta_9, p_{10}, F_6 \rightarrow F_7 \vdash \Delta_8 \rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \rightarrow F_8}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \rightarrow F_8} I \quad \frac{h_6 : \Delta_{11}, p_{10} \vdash F_7, \Delta_9, p_{10} \quad h_6 : F_8, \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \rightarrow F_8 \vdash \Delta_9, p_{10}} \rightarrow_L \\
\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \rightarrow F_8 \quad \text{Cut} \\
\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \rightarrow F_8 \xrightarrow{\rightarrow} \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{12}, F_7 \rightarrow F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}}{\vdash : (\Delta_{12}, F_7 \rightarrow F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}} I \quad \frac{h_6 : F_{11}, \Delta_{12}, p_{10} \vdash F_7, \Delta_9, p_{10} \quad h_6 : F_8, F_{11}, \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : ((\Delta_{12}, F_7 \rightarrow F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}} \rightarrow_L \\
\vdash : (\Delta_{12}, F_7 \rightarrow F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11} \quad \text{Cut} \\
\vdash : (\Delta_{12}, F_7 \rightarrow F_8), p_{10} \vdash \Delta_9, p_{10} \xrightarrow{\rightarrow} \\
\vdash : \Delta_{12}, p_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, p_{10} I
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, F_6 \wedge F_7), p_{10} \vdash \Delta_8, p_{10}}{\vdash : (\Delta_9, F_6 \wedge F_7), p_{10} \vdash \Delta_8} I \quad \frac{h_5 : F_6, F_7, \Delta_9, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_5 : ((\Delta_9, F_6 \wedge F_7), p_{10}), p_{10} \vdash \Delta_8} \wedge_L \\
\vdash : (\Delta_9, F_6 \wedge F_7), p_{10} \vdash \Delta_8 \quad \text{Cut} \\
\vdash : (\Delta_9, F_6 \wedge F_7), p_{10} \vdash \Delta_8 \xrightarrow{\rightarrow} \\
\frac{\bullet h_1 : \Delta_9, F_6, F_7, p_{10} \vdash \Delta_8, p_{10} \quad h_5 : \Delta_9, F_6, F_7, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_6, F_7, p_{10} \vdash \Delta_8} I \quad \frac{\text{ax/W}}{\text{hCut}} \\
\vdash : \Delta_9, p_{10}, F_6 \wedge F_7 \vdash \Delta_8 \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \wedge F_8}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \wedge F_8} I \quad \frac{h_6 : F_7, F_8, \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \wedge F_8 \vdash \Delta_9, p_{10}} \wedge_L \\
\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \wedge F_8 \quad \text{Cut} \\
\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \wedge F_8 \xrightarrow{\rightarrow} \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{12}, F_7 \wedge F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}}{\vdash : (\Delta_{12}, F_7 \wedge F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}} I \quad \frac{h_6 : F_7, F_8, F_{11}, \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : ((\Delta_{12}, F_7 \wedge F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}} \wedge_L \\
\vdash : (\Delta_{12}, F_7 \wedge F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11} \quad \text{Cut} \\
\vdash : (\Delta_{12}, F_7 \wedge F_8), p_{10} \vdash \Delta_9, p_{10} \xrightarrow{\rightarrow} \\
\vdash : \Delta_{12}, p_{10}, F_7 \wedge F_8 \vdash \Delta_9, p_{10} I
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8, p_{10}}{\vdash : (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8} I \quad \frac{h_5 : F_6, \Delta_9, p_{10}, p_{10} \vdash \Delta_8 \quad h_5 : F_7, \Delta_9, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_5 : ((\Delta_9, F_6 \vee F_7), p_{10}), p_{10} \vdash \Delta_8} \vee_L \\
\vdash : (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8 \quad \text{Cut} \\
\vdash : (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8 \xrightarrow{\rightarrow} \\
\frac{\bullet h_1 : \Delta_9, F_6, p_{10} \vdash \Delta_8, p_{10} \quad h_5 : \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8} I \quad \frac{\text{ax/W}}{\text{hCut}} \quad \frac{\bullet h_1 : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10} \quad h_5 : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8} I \quad \frac{\text{ax/W}}{\text{hCut}} \\
\vdash : \Delta_9, p_{10}, F_6 \vee F_7 \vdash \Delta_8 \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \vee F_8}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I \quad \frac{\frac{h_6 : F_7, \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \quad h_6 : F_8, \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \vee F_8 \vdash \Delta_9, p_{10}} \vee_L}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_{12}, F_7 \vee F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}}{- : (\Delta_{12}, F_7 \vee F_8), p_{10} \vdash \Delta_9, p_{10}} I \quad \frac{\frac{h_6 : F_7, F_{11}, \Delta_{12}, p_{10} \vdash \Delta_9, p_{10} \quad h_6 : F_8, F_{11}, \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : ((\Delta_{12}, F_7 \vee F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}} \vee_L}{- : (\Delta_{12}, F_7 \vee F_8), p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{12}, p_{10}, F_7 \vee F_8 \vdash \Delta_9, p_{10}} I
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\perp, \Delta_7), p_8 \vdash \Delta_6, p_8}{- : (\perp, \Delta_7), p_8 \vdash \Delta_6} I \quad \frac{\bullet h_5 : ((\perp, \Delta_7), p_8), p_8 \vdash \Delta_6}{- : (\perp, \Delta_7), p_8 \vdash \Delta_6} \perp_L}{- : (\perp, \Delta_7), p_8 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_7, p_8 \vdash \Delta_6} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_8 \vdash (\Delta_7, p_8), \perp}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I \quad \frac{\bullet h_6 : (\Delta_9, p_8), \perp \vdash \Delta_7, p_8}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} \perp_L}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\perp, \Delta_{10}), p_8 \vdash (\Delta_7, p_8), F_9}{- : (\perp, \Delta_{10}), p_8 \vdash \Delta_7, p_8} I \quad \frac{\bullet h_6 : ((\perp, \Delta_{10}), p_8), F_9 \vdash \Delta_7, p_8}{- : (\perp, \Delta_{10}), p_8 \vdash \Delta_7, p_8} \perp_L}{- : (\perp, \Delta_{10}), p_8 \vdash \Delta_7, p_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10}, p_8 \vdash \Delta_7, p_8} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, p_7 \vdash (\Delta_6, p_7), p_7}{- : \Delta_8, p_7 \vdash \Delta_6, p_7} I \quad \frac{\bullet h_5 : (\Delta_8, p_7), p_7 \vdash \Delta_6, p_7}{- : \Delta_8, p_7 \vdash \Delta_6, p_7} I}{- : \Delta_8, p_7 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8, p_7 \vdash \Delta_6, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_8, p_7), p_9 \vdash (\Delta_6, p_7), p_9}{- : (\Delta_8, p_7), p_9 \vdash \Delta_6, p_7} I \quad \frac{\bullet h_5 : ((\Delta_8, p_7), p_9), p_9 \vdash \Delta_6, p_7}{- : (\Delta_8, p_7), p_9 \vdash \Delta_6, p_7} I}{- : (\Delta_8, p_7), p_9 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8, p_7, p_9 \vdash \Delta_6, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash ((\Delta_7, p_9), p_8), p_9}{- : \Delta_{10}, p_8 \vdash (\Delta_7, p_9), p_8} I \quad \frac{\bullet h_6 : (\Delta_{10}, p_8), p_9 \vdash (\Delta_7, p_9), p_8}{- : \Delta_{10}, p_8 \vdash (\Delta_7, p_9), p_8} I}{- : \Delta_{10}, p_8 \vdash (\Delta_7, p_9), p_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{10}, p_8 \vdash \Delta_7, p_8, p_9} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_{11}, p_9), p_8 \vdash ((\Delta_7, p_9), p_8), F_{10}}{- : (\Delta_{11}, p_9), p_8 \vdash (\Delta_7, p_9), p_8} I \quad \frac{\bullet h_6 : ((\Delta_{11}, p_9), p_8), F_{10} \vdash (\Delta_7, p_9), p_8}{- : (\Delta_{11}, p_9), p_8 \vdash (\Delta_7, p_9), p_8} I}{- : (\Delta_{11}, p_9), p_8 \vdash (\Delta_7, p_9), p_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_8, p_9 \vdash \Delta_7, p_8, p_9} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash (\Delta_7, p_8), F_9}{- : \Delta_{10}, p_8 \vdash \Delta_7, p_8} I \quad \frac{\bullet h_6 : (\Delta_{10}, p_8), F_9 \vdash \Delta_7, p_8}{- : \Delta_{10}, p_8 \vdash \Delta_7, p_8} I}{- : \Delta_{10}, p_8 \vdash \Delta_7, p_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{10}, p_8 \vdash \Delta_7, p_8} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\top, \Delta_7), p_8 \vdash \Delta_6, p_8}{- : (\top, \Delta_7), p_8 \vdash \Delta_6} I \quad \frac{h_5 : \Delta_7, p_8, p_8 \vdash \Delta_6}{\bullet h_5 : ((\top, \Delta_7), p_8), p_8 \vdash \Delta_6} \top_L}{\text{Cut}} \\
\frac{\bullet h_1 : \top, \Delta_7, p_8 \vdash \Delta_6, p_8}{- : \top, \Delta_7, p_8 \vdash \Delta_6} I \quad \frac{h_5 : \top, \Delta_7, p_8, p_8 \vdash \Delta_6}{\bullet h_5 : ((\top, \Delta_7), p_8), p_8 \vdash \Delta_6} \text{ax/W}}{\text{hCut}} \\
\\
\frac{\frac{\bullet h_1 : \Delta_9, p_8 \vdash (\Delta_7, p_8), \top}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I \quad \frac{h_6 : \Delta_9, p_8 \vdash \Delta_7, p_8}{\bullet h_6 : (\Delta_9, p_8), \top \vdash \Delta_7, p_8} \top_L}{\text{Cut}} \\
\frac{\bullet h_1 : \Delta_9, p_8 \vdash (\Delta_7, p_8), \top}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I \\
\\
\frac{\frac{\bullet h_1 : (\top, \Delta_{10}), p_8 \vdash (\Delta_7, p_8), F_9}{- : (\top, \Delta_{10}), p_8 \vdash \Delta_7, p_8} I \quad \frac{h_6 : F_9, \Delta_{10}, p_8 \vdash \Delta_7, p_8}{\bullet h_6 : ((\top, \Delta_{10}), p_8), F_9 \vdash \Delta_7, p_8} \top_L}{\text{Cut}} \\
\frac{\bullet h_1 : (\top, \Delta_{10}), p_8 \vdash (\Delta_7, p_8), F_9}{- : (\top, \Delta_{10}), p_8 \vdash \Delta_7, p_8} I
\end{array}$$

8.12 Status of \top_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \rightarrow F_7}{\bullet h_1 : \top, \Delta_{10} \vdash (\Delta_8, F_6 \rightarrow F_7), F_9} \top_L \quad \frac{h_5 : \top, F_6, F_9, \Delta_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\top, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \rightarrow F_7} \rightarrow_R}{\text{Cut}} \\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash (\Delta_8, F_6 \rightarrow F_7), F_9}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \rightarrow \\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_8, F_9, F_6 \rightarrow F_7}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9 \vdash \Delta_8, F_6 \rightarrow F_7}{\bullet h_5 : (\top, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \rightarrow F_7} \text{ax/W}}{\text{hCut}}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \wedge F_7}{\bullet h_1 : \top, \Delta_{10} \vdash (\Delta_8, F_6 \wedge F_7), F_9} \top_L \quad \frac{h_5 : \top, F_9, \Delta_{10} \vdash F_6, \Delta_8 \quad h_5 : \top, F_9, \Delta_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\top, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \wedge F_7} \wedge_R}{\text{Cut}} \\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash (\Delta_8, F_6 \wedge F_7), F_9}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7} \rightarrow \\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_8, F_9, F_6 \wedge F_7}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9 \vdash \Delta_8, F_6 \wedge F_7}{\bullet h_5 : (\top, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \wedge F_7} \text{ax/W}}{\text{hCut}}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \vee F_7}{\bullet h_1 : \top, \Delta_{10} \vdash (\Delta_8, F_6 \vee F_7), F_9} \top_L \quad \frac{h_5 : \top, F_9, \Delta_{10} \vdash F_6, F_7, \Delta_8}{\bullet h_5 : (\top, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \vee F_7} \vee_R}{\text{Cut}} \\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash (\Delta_8, F_6 \vee F_7), F_9}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7} \rightarrow \\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_8, F_9, F_6 \vee F_7}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9 \vdash \Delta_8, F_6 \vee F_7}{\bullet h_5 : (\top, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \vee F_7} \text{ax/W}}{\text{hCut}}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8 \vdash F_7, \perp, \Delta_6}{\bullet h_1 : \top, \Delta_8 \vdash (\perp, \Delta_6), F_7} \top_L \quad \frac{h_5 : \top, F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \perp, \Delta_6} \perp_R}{\text{Cut}} \\
\frac{\bullet h_1 : \top, \Delta_8 \vdash (\perp, \Delta_6), F_7}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \rightarrow \\
\frac{\bullet h_1 : \top, \Delta_8 \vdash \perp, \Delta_6, F_7}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, F_7 \vdash \perp, \Delta_6}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \perp, \Delta_6} \text{ax/W}}{\text{hCut}}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash F_7, \top, \Delta_6}{\bullet h_1 : \top, \Delta_8 \vdash (\top, \Delta_6), F_7} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \top, \Delta_6} \top_R \\
\hline
- : \top, \Delta_8 \vdash \top, \Delta_6 \\
\rightarrow \\
\frac{}{- : \top, \Delta_8 \vdash \top, \Delta_6} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_9, \Delta_{10} \vdash \Box F_8, \Delta_7, \Box F_6}{\bullet h_1 : \top, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, \Box F_6), \Box F_8} \top_L \quad \frac{h_5 : \text{unbox}(\Box \Gamma_9), \text{unbox}(\Box F_8) \vdash F_6}{\bullet h_5 : (\top, \Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \Delta_7, \Box F_6} K \\
\hline
- : \top, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, \Box F_6 \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Box F_8, \Delta_7, \Box F_6}{- : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_6} \text{ax/W}}{\frac{}{- : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_6} \text{hCut}} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_8, \Delta_{10} \vdash F_9, \Delta_7, \Box F_6}{\bullet h_1 : \top, \Box \Gamma_8, \Delta_{10} \vdash (\Delta_7, \Box F_6), F_9} \top_L \quad \frac{h_5 : \text{unbox}(\Box \Gamma_8) \vdash F_6}{\bullet h_5 : (\top, \Box \Gamma_8, \Delta_{10}), F_9 \vdash \Delta_7, \Box F_6} K \\
\hline
- : \top, \Box \Gamma_8, \Delta_{10} \vdash \Delta_7, \Box F_6 \\
\rightarrow \\
\frac{\frac{}{- : \text{unbox}(\Box \Gamma_8) \vdash F_6} \text{ax/W}}{\frac{}{- : \top, \Delta_{10}, \Box \Gamma_8 \vdash \Delta_7, \Box F_6} K} K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_9 \vdash F_6 \rightarrow F_7, \Delta_8}{\bullet h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7} \top_L \quad \frac{h_5 : \top, \Delta_9 \vdash F_6, \Delta_8 \quad h_5 : \top, F_7, \Delta_9 \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9), F_6 \rightarrow F_7 \vdash \Delta_8} \rightarrow_L \\
\hline
- : \top, \Delta_9 \vdash \Delta_8 \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7}{- : \top, \Delta_9 \vdash \Delta_8} \text{ax/W}}{\frac{}{- : \top, \Delta_9 \vdash \Delta_8} \text{hCut}} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8, F_9} \top_L \quad \frac{h_5 : \top, F_9, \Delta_{10} \vdash F_6, \Delta_8 \quad h_5 : \top, F_7, F_9, \Delta_{10} \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \Delta_8} \rightarrow_L \\
\hline
- : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8 \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8, F_9}{- : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8} \text{ax/W}}{\frac{}{- : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8} \text{hCut}} \text{ax/W}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_9 \vdash F_6 \wedge F_7, \Delta_8}{\bullet h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7} \top_L \quad \frac{h_5 : \top, F_6, F_7, \Delta_9 \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9), F_6 \wedge F_7 \vdash \Delta_8} \wedge_L \\
\hline
- : \top, \Delta_9 \vdash \Delta_8 \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7}{- : \top, \Delta_9 \vdash \Delta_8} \text{ax/W}}{\frac{}{- : \top, \Delta_9 \vdash \Delta_8} \text{hCut}} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8, F_9} \top_L \quad \frac{h_5 : \top, F_6, F_7, F_9, \Delta_{10} \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_{10}, F_6 \wedge F_7), F_9 \vdash \Delta_8} \wedge_L \\
\hline
- : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8 \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8, F_9}{- : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8} \text{ax/W}}{\frac{}{- : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8} \text{hCut}} \text{ax/W}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : \Delta_9 \vdash F_6 \vee F_7, \Delta_8}{\bullet h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \vee F_7} \top_L \quad \frac{h_5 : \top, F_6, \Delta_9 \vdash \Delta_8 \quad h_5 : \top, F_7, \Delta_9 \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9), F_6 \vee F_7 \vdash \Delta_8} \vee_L \\
\hline
- : \top, \Delta_9 \vdash \Delta_8 \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \vee F_7}{- : \top, \Delta_9 \vdash \Delta_8} \text{ax/W}}{\frac{}{- : \top, \Delta_9 \vdash \Delta_8} \text{hCut}} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_6 \vee F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8, F_9} \top_L \quad \frac{h_5 : \top, F_6, F_9, \Delta_{10} \vdash \Delta_8 \quad h_5 : \top, F_7, F_9, \Delta_{10} \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_{10}, F_6 \vee F_7), F_9 \vdash \Delta_8} \vee_L \\
\hline
- : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8, F_9}{- : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9, F_6 \vee F_7 \vdash \Delta_8}{- : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8} \text{hCut} \\
\hline
- : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8
\end{array}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \Delta_7 \vdash \perp, \Delta_6}{\bullet h_1 : \top, \Delta_7 \vdash \Delta_6, \perp} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_7), \perp \vdash \Delta_6} \perp_L \\
\hline
- : \top, \Delta_7 \vdash \Delta_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \top, \Delta_7 \vdash \perp, \Delta_6}{- : \top, \Delta_7 \vdash \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \perp, \top, \Delta_7 \vdash \Delta_6}{- : \top, \Delta_7 \vdash \Delta_6} \perp_L \\
\hline
- : \top, \Delta_7 \vdash \Delta_6
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \top, \perp, \Delta_8 \vdash \Delta_6, F_7} \top_L \quad \frac{}{\bullet h_5 : (\top, \perp, \Delta_8), F_7 \vdash \Delta_6} \perp_L \\
\hline
- : \top, \perp, \Delta_8 \vdash \Delta_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \perp, \top, \Delta_8 \vdash \Delta_6} \perp_L \\
\hline
- : \perp, \top, \Delta_8 \vdash \Delta_6
\end{array}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash p_7, \Delta_6, p_7}{\bullet h_1 : \top, \Delta_8 \vdash (\Delta_6, p_7), p_7} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_8), p_7 \vdash \Delta_6, p_7} I \\
\hline
- : \top, \Delta_8 \vdash \Delta_6, p_7 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \top, \Delta_8 \vdash \Delta_6, p_7, p_7}{- : \top, \Delta_8 \vdash \Delta_6, p_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, p_7 \vdash \Delta_6, p_7}{- : \top, \Delta_8 \vdash \Delta_6, p_7} I \\
\hline
- : \top, \Delta_8 \vdash \Delta_6, p_7
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_9, p_7 \vdash F_8, \Delta_6, p_7}{\bullet h_1 : \top, \Delta_9, p_7 \vdash (\Delta_6, p_7), F_8} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_9, p_7), F_8 \vdash \Delta_6, p_7} I \\
\hline
- : \top, \Delta_9, p_7 \vdash \Delta_6, p_7 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \top, \Delta_9, p_7 \vdash \Delta_6, p_7} I \\
\hline
- : \top, \Delta_9, p_7 \vdash \Delta_6, p_7
\end{array}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \top, \Delta_8 \vdash \Delta_6, F_7} \top_L \quad \frac{h_5 : F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \Delta_6} \top_L \\
\hline
- : \top, \Delta_8 \vdash \Delta_6 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \top, \Delta_8 \vdash \Delta_6, F_7}{- : \top, \Delta_8 \vdash \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, F_7 \vdash \Delta_6}{- : \top, \Delta_8 \vdash \Delta_6} \text{hCut} \\
\hline
- : \top, \Delta_8 \vdash \Delta_6
\end{array}
\end{array}$$