

Modal Logic K+T+4+5

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Abstract

This system does not have cut-elimination.

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1 Height preserving admissibility of weakening on the left

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \rightarrow F_5} \text{IH}}{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_5} \text{ax}} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_5} \text{IH}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \wedge F_5} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4, F_5} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \vee F_5} \text{ax}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \vee F_5} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3} \text{IH}}{\bullet h_1 : \Delta_2, F_W \vdash \perp, \Delta_3} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_W \vdash \top, \Delta_3} \top_R$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Delta_4, [\Box F_5]} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \Delta_3, F_W, \Box \Gamma_2 \vdash \Delta_4, [\Box F_5]} \text{ax}}{\bullet h_1 : \Delta_3, F_W, \Box \Gamma_2 \vdash \Delta_4, [\Box F_5]} K$$

- Case(s) rule $A45$

$$\frac{h_1 : \Box \Gamma_2 \vdash \Box \Gamma_4, F_6}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Box \Gamma_4, \Delta_5, [\Box F_6]} A45 \rightarrow \frac{\frac{h_1 : \Box \Gamma_2 \vdash F_6, \Box \Gamma_4}{\bullet h_1 : \Delta_3, F_W, \Box \Gamma_2 \vdash \Delta_5, \Box \Gamma_4, [\Box F_6]} \text{ax}}{\bullet h_1 : \Delta_3, F_W, \Box \Gamma_2 \vdash \Delta_5, \Box \Gamma_4, [\Box F_6]} A45$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3} \text{IH} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4, F_W \vdash \Delta_5} \text{ax}}{\bullet h_1 : \Delta_2, F_W, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5}}{\wedge_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\text{ax}}}{\frac{h_1 : \Delta_2, F_3, F_4, F_W \vdash \Delta_5}{\text{IH}}} \wedge_L}{\bullet h_1 : \Delta_2, F_W, F_3 \wedge F_4 \vdash \Delta_5}$$

- Case(s) rule \vee_L

$$\frac{\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\text{ax}}}{\frac{h_1 : \Delta_2, F_3, F_W \vdash \Delta_5}{\text{IH}}} \vee_L}{\frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\text{ax}}}{\frac{h_1 : \Delta_2, F_4, F_W \vdash \Delta_5}{\text{IH}}} \vee_L}$$

- Case(s) rule AT

$$\frac{\frac{h_1 : F_3, \Delta_2, []F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_2, []F_3 \vdash \Delta_4}}{AT} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4}{\text{ax}}}{\frac{h_1 : \Delta_2, F_3, F_W, []F_3 \vdash \Delta_4}{\text{IH}}} AT}{\bullet h_1 : \Delta_2, F_W, []F_3 \vdash \Delta_4}$$

- Case(s) rule \perp_L

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3}}{\perp_L} \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_2, F_W \vdash \Delta_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_W, p_3 \vdash \Delta_4, p_3} I$$

- Case(s) rule \top_L

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3}}{\top_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\text{ax}}}{\frac{h_1 : \Delta_2, F_W \vdash \Delta_3}{\text{IH}}} \top_L}{\bullet h_1 : \top, \Delta_2, F_W \vdash \Delta_3}$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \rightarrow F_5} \text{ IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \rightarrow F_5} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5, F_W} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_W \quad h_1 : \Delta_2 \vdash \Delta_3, F_5, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \wedge F_5} \text{ IH}} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \vee F_5} \text{ IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \vee F_5} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_W}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \text{ IH}}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, F_W} \top_R$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Delta_4, [] F_5} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_4, F_W, [] F_5} \text{ ax}}{\bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_4, F_W, [] F_5} K$$

- Case(s) rule $A45$

$$\frac{h_1 : \Box \Gamma_2 \vdash \Box \Gamma_4, F_6}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Box \Gamma_4, \Delta_5, [] F_6} A45 \rightarrow \frac{\frac{h_1 : \Box \Gamma_2 \vdash F_6, \Box \Gamma_4}{\bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_5, F_W, \Box \Gamma_4, [] F_6} \text{ ax}}{\bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_5, F_W, \Box \Gamma_4, [] F_6} A45$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5, F_W} \text{ ax}}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5, F_W} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5, F_W}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5, F_W} \text{ IH}}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5, F_W} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5, F_W} \text{ ax}}{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_W \quad h_1 : \Delta_2, F_4 \vdash \Delta_5, F_W}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5, F_W} \text{ IH}} \vee_L$$

- Case(s) rule AT

$$\frac{h_1 : F_3, \Delta_2, []F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_2, []F_3 \vdash \Delta_4} AT \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4}{h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4, F_W} IH}{\bullet h_1 : \Delta_2, []F_3 \vdash \Delta_4, F_W} AT$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3, F_W} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, F_W, p_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{h_1 : \Delta_2 \vdash \Delta_3, F_W} IH}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3, F_W} \top_L$$

3 Measure of derivations

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \text{ax} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \text{ax} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \text{ax} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \text{ax} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Delta_4, [\top] F_5} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5} \text{IH}}{\bullet \bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_4, [\top] F_5} \text{ax} K$$

- Case(s) rule $A45$

$$\frac{h_1 : \Box \Gamma_2 \vdash \Box \Gamma_4, F_6}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Box \Gamma_4, \Delta_5, [\top] F_6} A45 \rightarrow \frac{\frac{h_1 : \Box \Gamma_2 \vdash F_6, \Box \Gamma_4}{\bullet h_1 : \Box \Gamma_2 \vdash F_6, \Box \Gamma_4} \text{IH}}{\bullet \bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_5, \Box \Gamma_4, [\top] F_6} \text{ax} A45$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{IH} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{IH}}{\bullet \bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \text{ax} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5} \text{IH}}{\bullet \bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \text{ax} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5} \text{IH} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{IH}}{\bullet \bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \text{ax} \vee_L$$

- Case(s) rule AT

$$\frac{h_1 : F_3, \Delta_2, []F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_2, []F_3 \vdash \Delta_4} AT \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4} IH}{\bullet \bullet h_1 : \Delta_2, []F_3 \vdash \Delta_4} AT$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} IH}{\bullet \bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L$$

4 Invertibility of Rules

4.1 Status of \rightarrow_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash \Delta_7, F_2, F_6}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \text{ax/ind} \rightarrow_R$$

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5} \text{ax} \text{H}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \rightarrow F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5} \text{ax/ind} \quad \overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \rightarrow F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_5, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \rightarrow F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \rightarrow F_2), \Box F_6} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ax}}{\bullet h_3 : \Delta_5, F_1, \Box \Gamma_4 \vdash \Delta_7, F_2, \Box F_6} K$$

- Case rule $A45$

$$\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_6, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash \Box \Gamma_6, (\Delta_8, F_1 \rightarrow F_2), \Box F_7} A45 \rightarrow \frac{\overline{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_6} \text{ax}}{\bullet h_3 : \Delta_5, F_1, \Box \Gamma_4 \vdash \Delta_8, F_2, \Box \Gamma_6, \Box F_7} A45$$

- Case rule \rightarrow_L

$$\frac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \rightarrow F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \rightarrow F_3} \rightarrow_L \rightarrow \frac{\overline{h_4 : \Delta_5, F_2 \vdash \Delta_1, F_3, F_6} \text{ax/ind} \quad \overline{h_4 : \Delta_5, F_2, F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_2, F_6 \rightarrow F_7 \vdash \Delta_1, F_3} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \rightarrow F_3} \wedge_L \rightarrow \frac{\frac{}{h_4 : \Delta_5, F_2, F_6, F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_2, F_6 \wedge F_7 \vdash \Delta_1, F_3} \wedge_L$$

- Case rule \vee_L

$$\frac{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \rightarrow F_3} \vee_L \rightarrow \frac{\frac{}{h_4 : \Delta_5, F_2, F_6 \vdash \Delta_1, F_3} \text{ax/ind} \quad \frac{}{h_4 : \Delta_5, F_2, F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_2, F_6 \vee F_7 \vdash \Delta_1, F_3} \vee_L$$

- Case rule AT

$$\frac{h_4 : F_6, \Delta_5, []F_6 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_2 \rightarrow F_3} AT \rightarrow \frac{\frac{}{h_4 : \Delta_5, F_2, F_6, []F_6 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_2, []F_6 \vdash \Delta_1, F_3} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3} \perp_L \rightarrow \frac{}{\bullet h_4 : \perp, \Delta_5, F_2 \vdash \Delta_1, F_3} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \rightarrow F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1, p_5 \vdash \Delta_6, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3} \top_L \rightarrow \frac{\frac{}{h_4 : \Delta_5, F_2 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \top, \Delta_5, F_2 \vdash \Delta_1, F_3} \top_L$$

4.2 Status of \wedge_R : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{}{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\frac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5} \text{ax/ind} \quad \frac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash \Delta_3, F_4} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\frac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1} \perp_R$$

- Case rule \top_R

$$\overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightarrow \overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \wedge F_2), []F_6} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ ax}}{\bullet h_3 : \Delta_5, \Box \Gamma_4 \vdash \Delta_7, F_1, []F_6} K$$

- Case rule $A45$

$$\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_6, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash \Box \Gamma_6, (\Delta_8, F_1 \wedge F_2), []F_7} A45 \rightarrow \frac{\overline{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_6} \text{ ax}}{\bullet h_3 : \Delta_5, \Box \Gamma_4 \vdash \Delta_8, F_1, \Box \Gamma_6, []F_7} A45$$

- Case rule \rightarrow_L

$$\frac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \wedge F_3} \rightarrow_L \rightarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_6} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3} \wedge_L \rightarrow \frac{\overline{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2} \wedge_L$$

- Case rule \vee_L

$$\frac{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3} \vee_L \rightarrow \frac{\overline{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2} \vee_L$$

- Case rule AT

$$\frac{h_4 : F_6, \Delta_5, []F_6 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_2 \wedge F_3} AT \rightarrow \frac{\overline{h_4 : \Delta_5, F_6, []F_6 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_2} AT$$

- Case rule \perp_L

$$\overline{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \perp_L \rightarrow \overline{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2} \perp_L$$

- Case rule I

$$\overline{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \wedge F_2} I \rightarrow \overline{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_1, p_5} I$$

- Case rule \top_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \top_L \rightarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2} \top_L$$

4.3 Status of \wedge_R (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_5} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5} \text{ H}$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule \top_R

$$\overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightarrow \overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \wedge F_2), \Box F_6} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ ax}}{\bullet h_3 : \Delta_5, \Box \Gamma_4 \vdash \Delta_7, F_2, \Box F_6} K$$

- Case rule $A45$

$$\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_6, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash \Box \Gamma_6, (\Delta_8, F_1 \wedge F_2), \Box F_7} A45 \rightarrow \frac{\overline{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_6} \text{ ax}}{\bullet h_3 : \Delta_5, \Box \Gamma_4 \vdash \Delta_8, F_2, \Box \Gamma_6, \Box F_7} A45$$

- Case rule \rightarrow_L

$$\frac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \wedge F_3} \rightarrow_L \rightarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_3, F_6} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_3} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3} \wedge_L \rightarrow \frac{\frac{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_3} \wedge_L$$

- Case rule \vee_L

$$\frac{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3} \vee_L \rightarrow \frac{\frac{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_3} \text{ax/ind} \quad \frac{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_3} \vee_L$$

- Case rule AT

$$\frac{h_4 : F_6, \Delta_5, []F_6 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_2 \wedge F_3} AT \rightarrow \frac{\frac{h_4 : \Delta_5, F_6, []F_6 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, []F_6 \vdash \Delta_1, F_3} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \perp_L \rightarrow \frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_3} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \wedge F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \top_L \rightarrow \frac{\frac{h_4 : \Delta_5 \vdash \Delta_1, F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_3} \top_L$$

4.4 Status of \vee_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_2, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \vee F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \vee F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \vee F_6} \vee_R$$

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5} H$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \vee F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1, F_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \vee F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1, F_2} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \vee F_2), [] F_6} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ ax}}{\bullet h_3 : \Delta_5, \Box \Gamma_4 \vdash \Delta_7, F_1, F_2, [] F_6} K$$

- Case rule $A45$

$$\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_6, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash \Box \Gamma_6, (\Delta_8, F_1 \vee F_2), [] F_7} A45 \rightarrow \frac{\overline{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_6} \text{ ax}}{\bullet h_3 : \Delta_5, \Box \Gamma_4 \vdash \Delta_8, F_1, F_2, \Box \Gamma_6, [] F_7} A45$$

- Case rule \rightarrow_L

$$\frac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \vee F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \vee F_3} \rightarrow_L \rightarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_3, F_6} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2, F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2, F_3} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \vee F_3} \wedge_L \rightarrow \frac{\overline{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2, F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2, F_3} \wedge_L$$

- Case rule \vee_L

$$\frac{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \vee F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \vee F_3} \vee_L \rightarrow \frac{\overline{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2, F_3} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2, F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2, F_3} \vee_L$$

- Case rule AT

$$\frac{h_4 : F_6, \Delta_5, [] F_6 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, [] F_6 \vdash \Delta_1, F_2 \vee F_3} AT \rightarrow \frac{\overline{h_4 : \Delta_5, F_6, [] F_6 \vdash \Delta_1, F_2, F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_5, [] F_6 \vdash \Delta_1, F_2, F_3} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \vee F_3} \perp_L \rightarrow \frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2, F_3} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \vee F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_1, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \vee F_3} \top_L \rightarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_3} \text{ ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2, F_3} \top_L$$

4.5 Status of \perp_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash \perp, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \perp, F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash \perp, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ ax/ind} \quad \overline{h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \perp, F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \vee F_4} \vee_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{H}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \perp, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash (\perp, \Delta_5), \Box F_4} K \rightarrow \frac{\overline{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_5, \Box F_4} K$$

- Case rule $A45$

$$\frac{h_1 : \Box \Gamma_2 \vdash \Box \Gamma_4, F_5}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Box \Gamma_4, (\perp, \Delta_6), \Box F_5} A45 \rightarrow \frac{\overline{h_1 : \Box \Gamma_2 \vdash F_5, \Box \Gamma_4} \text{ ax}}{\bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_6, \Box \Gamma_4, \Box F_5} A45$$

- Case rule \rightarrow_L

$$\frac{h_2 : \Delta_3 \vdash \perp, F_4, \Delta_1 \quad h_2 : F_5, \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \perp, \Delta_1} \rightarrow_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash \Delta_1, F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash \Delta_1} \text{ ax/ind}}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \Delta_1} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_2 : F_4, F_5, \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \perp, \Delta_1} \wedge_L \rightarrow \frac{\overline{h_2 : \Delta_3, F_4, F_5 \vdash \Delta_1} \text{ ax/ind}}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \Delta_1} \wedge_L$$

- Case rule \vee_L

$$\frac{h_2 : F_4, \Delta_3 \vdash \perp, \Delta_1 \quad h_2 : F_5, \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \perp, \Delta_1} \vee_L \rightarrow \frac{\overline{h_2 : \Delta_3, F_4 \vdash \Delta_1} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash \Delta_1} \text{ ax/ind}}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \Delta_1} \vee_L$$

- Case rule AT

$$\frac{h_2 : F_4, \Delta_3, []F_4 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, []F_4 \vdash \perp, \Delta_1} AT \rightarrow \frac{\overline{h_2 : \Delta_3, F_4, []F_4 \vdash \Delta_1} \text{ ax/ind}}{\bullet h_2 : \Delta_3, []F_4 \vdash \Delta_1} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_2 : \perp, \Delta_3 \vdash \perp, \Delta_1} \perp_L \rightarrow \frac{}{\bullet h_2 : \perp, \Delta_3 \vdash \Delta_1} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3, \perp, \Delta_4} I \rightarrow \frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I$$

- Case rule \top_L

$$\frac{h_2 : \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \top, \Delta_3 \vdash \perp, \Delta_1} \top_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash \Delta_1} \text{ ax/ind}}{\bullet h_2 : \top, \Delta_3 \vdash \Delta_1} \top_L$$

4.6 Status of \top_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash \top, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \top, F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash \top, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \wedge F_4} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \top, F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \vee F_4} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \top, \Delta_3} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \text{trivial}$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash (\top, \Delta_5), []F_4} K \rightarrow \text{trivial}$$

- Case rule $A45$

$$\frac{h_1 : \Box \Gamma_2 \vdash \Box \Gamma_4, F_5}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Box \Gamma_4, (\top, \Delta_6), []F_5} A45 \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_2 : \Delta_3 \vdash \top, F_4, \Delta_1 \quad h_2 : F_5, \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \top, \Delta_1} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_2 : F_4, F_5, \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \top, \Delta_1} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_2 : F_4, \Delta_3 \vdash \top, \Delta_1 \quad h_2 : F_5, \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \top, \Delta_1} \vee_L \rightarrow \text{trivial}$$

- Case rule AT

$$\frac{h_2 : F_4, \Delta_3, []F_4 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, []F_4 \vdash \top, \Delta_1} AT \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_2 : \perp, \Delta_3 \vdash \top, \Delta_1} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3, \top, \Delta_4} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_2 : \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \top, \Delta_3 \vdash \top, \Delta_1} \top_L \rightarrow \text{trivial}$$

4.7 Status of K : : Non invertible

- Case rule \rightarrow_R

$$\frac{h_4 : \Box \Gamma_1, F_5, \Delta_2 \vdash F_6, \Delta_7, [] F_3}{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash (\Delta_7, [] F_3), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3}{\bullet h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3} \text{H}}{\text{ax/ind}}$$

- Case rule \wedge_R

$$\frac{\frac{h_4 : \Box \Gamma_1, \Delta_2 \vdash F_5, \Delta_7, [] F_3}{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash (\Delta_7, [] F_3), F_5 \wedge F_6} \wedge_R}{\frac{h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3}{\bullet h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3} \text{H}} \text{ax/ind}$$

- Case rule \vee_R

$$\frac{\frac{h_4 : \Box \Gamma_1, \Delta_2 \vdash F_5, F_6, \Delta_7, [] F_3}{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash (\Delta_7, [] F_3), F_5 \vee F_6} \vee_R}{\frac{h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3}{\bullet h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3} \text{H}} \text{ax/ind}$$

- Case rule \perp_R

$$\frac{\frac{h_4 : \Box \Gamma_1, \Delta_2 \vdash \Delta_5, [] F_3}{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash \perp, \Delta_5, [] F_3} \perp_R}{\frac{h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3}{\bullet h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3} \text{H}} \text{ax/ind}$$

- Case rule \top_R

$$\frac{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash \top, \Delta_5, [] F_3}{\bullet h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3} \top_R \rightarrow \frac{\bullet h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3}{\bullet h_4 : \text{unbox}(\Box \Gamma_1) \vdash F_3} \text{fail}$$

- Case rule K

$$\frac{\frac{h_2 : \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_6) \vdash F_3}{\bullet h_2 : (\Box \Gamma_5, \Box \Gamma_6), \Box \Gamma_7, \Delta_8 \vdash (\Delta_4, [] F_1), [] F_3} K}{\bullet h_2 : \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_7) \vdash F_1} \text{fail}$$

$$\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4), \text{unbox}(\Box \Gamma_5) \vdash F_3}{\bullet h_1 : (\Box \Gamma_4, \Box \Gamma_5), \Box \Gamma_6, \Delta_7 \vdash \Delta_2, [] F_3} K}{\bullet h_1 : \text{unbox}(\Box \Gamma_4), \text{unbox}(\Box \Gamma_6) \vdash F_3} \text{fail}$$

- Case rule $A45$

$$\frac{\frac{h_2 : \Box \Gamma_6, \Box \Gamma_7 \vdash \Box \Gamma_5, F_4, [] F_1}{\bullet h_2 : (\Box \Gamma_6, \Box \Gamma_7), \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_5, [] F_1), \Delta_3, [] F_4} A45}{\bullet h_2 : \text{unbox}(\Box \Gamma_6), \text{unbox}(\Box \Gamma_8) \vdash F_1} \text{fail}$$

$$\frac{\frac{h_2 : \Box \Gamma_6, \Box \Gamma_7 \vdash \Box \Gamma_3, F_4}{\bullet h_2 : (\Box \Gamma_6, \Box \Gamma_7), \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_3, (\Delta_5, [] F_1), [] F_4} A45}{\bullet h_2 : \text{unbox}(\Box \Gamma_6), \text{unbox}(\Box \Gamma_8) \vdash F_1} \text{fail}$$

$$\frac{\frac{h_1 : \Box \Gamma_5, \Box \Gamma_6 \vdash \Box \Gamma_2, F_4}{\bullet h_1 : (\Box \Gamma_5, \Box \Gamma_6), \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_2, \Delta_3, [] F_4} A45}{\bullet h_1 : \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_7) \vdash F_4} \text{fail}$$

- Case rule \rightarrow_L

$$\frac{h_4 : \Box \Gamma_1, \Delta_7 \vdash F_5, \Delta_2, []F_3 \quad h_4 : \Box \Gamma_1, F_6, \Delta_7 \vdash \Delta_2, []F_3}{\bullet h_4 : (\Box \Gamma_1, \Delta_7), F_5 \rightarrow F_6 \vdash \Delta_2, []F_3} \rightarrow_L \rightarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule \wedge_L

$$\frac{h_4 : \Box \Gamma_1, F_5, F_6, \Delta_7 \vdash \Delta_2, []F_3}{\bullet h_4 : (\Box \Gamma_1, \Delta_7), F_5 \wedge F_6 \vdash \Delta_2, []F_3} \wedge_L \rightarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule \vee_L

$$\frac{h_4 : \Box \Gamma_1, F_5, \Delta_7 \vdash \Delta_2, []F_3 \quad h_4 : \Box \Gamma_1, F_6, \Delta_7 \vdash \Delta_2, []F_3}{\bullet h_4 : (\Box \Gamma_1, \Delta_7), F_5 \vee F_6 \vdash \Delta_2, []F_3} \vee_L \rightarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule AT

$$\frac{h_4 : \Box \Gamma_6, F_5, \Delta_1, []F_5 \vdash \Delta_2, []F_3}{\bullet h_4 : (\Box \Gamma_6, \Delta_1), []F_5 \vdash \Delta_2, []F_3} AT \rightarrow \frac{\overline{h_4 : F_5, unbox(\Box \Gamma_6) \vdash F_3}}{\bullet h_4 : F_5, unbox(\Box \Gamma_6) \vdash F_3} \text{ax/ind}_H$$

$$\frac{h_4 : \Box \Gamma_1, F_5, \Delta_6, []F_5 \vdash \Delta_2, []F_3}{\bullet h_4 : (\Box \Gamma_1, \Delta_6), []F_5 \vdash \Delta_2, []F_3} AT \rightarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule \perp_L

$$\frac{}{\bullet h_4 : \perp, \Box \Gamma_1, \Delta_5 \vdash \Delta_2, []F_3} \perp_L \rightarrow \frac{}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{fail}$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Box \Gamma_1, \Delta_6 \vdash p_4, \Delta_5, []F_2} I \rightarrow \frac{}{\bullet h_3 : unbox(\Box \Gamma_1) \vdash F_2} \text{fail}$$

- Case rule \top_L

$$\frac{h_4 : \Box \Gamma_1, \Delta_5 \vdash \Delta_2, []F_3}{\bullet h_4 : \top, \Box \Gamma_1, \Delta_5 \vdash \Delta_2, []F_3} \top_L \rightarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

4.8 Status of A45: : Non invertible

- Case rule \rightarrow_R

$$\frac{h_5 : \Box \Gamma_1, F_6, \Delta_2 \vdash \Box \Gamma_3, F_7, \Delta_8, []F_4}{\bullet h_5 : \Box \Gamma_1, \Delta_2 \vdash (\Box \Gamma_3, \Delta_8, []F_4), F_6 \rightarrow F_7} \rightarrow_R \rightarrow \frac{\overline{h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_3}}{\bullet h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_3} \text{ax/ind}_H$$

- Case rule \wedge_R

$$\frac{h_5 : \Box \Gamma_1, \Delta_2 \vdash \Box \Gamma_3, F_6, \Delta_8, []F_4 \quad h_5 : \Box \Gamma_1, \Delta_2 \vdash \Box \Gamma_3, F_7, \Delta_8, []F_4}{\bullet h_5 : \Box \Gamma_1, \Delta_2 \vdash (\Box \Gamma_3, \Delta_8, []F_4), F_6 \wedge F_7} \wedge_R \rightarrow \frac{\overline{h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_3}}{\bullet h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_3} \text{ax/ind}_H$$

- Case rule \vee_R

$$\frac{h_5 : \Box \Gamma_1, \Delta_2 \vdash \Box \Gamma_3, F_6, F_7, \Delta_8, []F_4}{\bullet h_5 : \Box \Gamma_1, \Delta_2 \vdash (\Box \Gamma_3, \Delta_8, []F_4), F_6 \vee F_7} \vee_R \rightarrow \frac{\frac{h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_3}{\bullet h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_3} \text{H}}{\text{ax/ind}}$$

- Case rule \perp_R

$$\frac{h_5 : \Box \Gamma_1, \Delta_2 \vdash \Box \Gamma_3, \Delta_6, []F_4}{\bullet h_5 : \Box \Gamma_1, \Delta_2 \vdash \perp, \Box \Gamma_3, \Delta_6, []F_4} \perp_R \rightarrow \frac{\frac{h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_3}{\bullet h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_3} \text{H}}{\text{ax/ind}}$$

- Case rule \top_R

$$\frac{}{\bullet h_5 : \Box \Gamma_1, \Delta_2 \vdash \top, \Box \Gamma_3, \Delta_6, []F_4} \top_R \rightarrow \frac{}{\bullet h_5 : \Box \Gamma_1 \vdash \Box \Gamma_3, F_4} \text{fail}$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6), \text{unbox}(\Box \Gamma_7) \vdash F_4}{\bullet h_3 : (\Box \Gamma_6, \Box \Gamma_7), \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_5, \Delta_1, []F_2), []F_4} K \rightarrow \frac{}{\bullet h_3 : \Box \Gamma_6, \Box \Gamma_8 \vdash \Box \Gamma_5, F_2, []F_4} \text{fail}$$

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6), \text{unbox}(\Box \Gamma_7) \vdash F_4}{\bullet h_3 : (\Box \Gamma_6, \Box \Gamma_7), \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_1, \Delta_5, []F_2), []F_4} K \rightarrow \frac{}{\bullet h_3 : \Box \Gamma_6, \Box \Gamma_8 \vdash \Box \Gamma_1, F_2} \text{fail}$$

$$\frac{h_3 : \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : (\Box \Gamma_5, \Box \Gamma_6), \Box \Gamma_7, \Delta_8 \vdash (\Box \Gamma_1, \Delta_2), []F_4} K \rightarrow \frac{}{\bullet h_3 : \Box \Gamma_5, \Box \Gamma_7 \vdash \Box \Gamma_1, F_4} \text{fail}$$

- Case rule $A45$

$$\frac{h_2 : \Box \Gamma_8, \Box \Gamma_9 \vdash \Box \Gamma_4, \Box \Gamma_5, F_3, []F_1}{\bullet h_2 : (\Box \Gamma_8, \Box \Gamma_9), \Box \Gamma_{10}, \Delta_{11} \vdash (\Box \Gamma_4, \Box \Gamma_5, []F_1), (\Box \Gamma_6, \Delta_7), []F_3} A45 \rightarrow \frac{}{\bullet h_2 : \Box \Gamma_8, \Box \Gamma_{10} \vdash \Box \Gamma_4, \Box \Gamma_6, F_1, []F_3} \text{fail}$$

$$\frac{h_2 : \Box \Gamma_8, \Box \Gamma_9 \vdash \Box \Gamma_4, \Box \Gamma_5, F_3}{\bullet h_2 : (\Box \Gamma_8, \Box \Gamma_9), \Box \Gamma_{10}, \Delta_{11} \vdash (\Box \Gamma_4, \Box \Gamma_5), (\Box \Gamma_6, \Delta_7, []F_1), []F_3} A45 \rightarrow \frac{}{\bullet h_2 : \Box \Gamma_8, \Box \Gamma_{10} \vdash \Box \Gamma_4, \Box \Gamma_6, F_1, []F_3} \text{fail}$$

$$\frac{h_2 : \Box \Gamma_8, \Box \Gamma_9 \vdash \Box \Gamma_4, \Box \Gamma_5, F_3, []F_1}{\bullet h_2 : (\Box \Gamma_8, \Box \Gamma_9), \Box \Gamma_{10}, \Delta_{11} \vdash (\Box \Gamma_4, \Box \Gamma_5, []F_1), (\Box \Gamma_6, \Delta_7), []F_3} A45 \rightarrow \frac{}{\bullet h_2 : \Box \Gamma_8, \Box \Gamma_{10} \vdash \Box \Gamma_4, \Box \Gamma_6, F_1} \text{fail}$$

$$\frac{h_2 : \Box \Gamma_8, \Box \Gamma_9 \vdash \Box \Gamma_4, \Box \Gamma_5, F_3}{\bullet h_2 : (\Box \Gamma_8, \Box \Gamma_9), \Box \Gamma_{10}, \Delta_{11} \vdash (\Box \Gamma_4, \Box \Gamma_5), (\Box \Gamma_6, \Delta_7, []F_1), []F_3} A45 \rightarrow \frac{}{\bullet h_2 : \Box \Gamma_8, \Box \Gamma_{10} \vdash \Box \Gamma_4, \Box \Gamma_6, F_1} \text{fail}$$

$$\frac{h_1 : \Box \Gamma_7, \Box \Gamma_8 \vdash \Box \Gamma_3, \Box \Gamma_4, F_2}{\bullet h_1 : (\Box \Gamma_7, \Box \Gamma_8), \Box \Gamma_9, \Delta_{10} \vdash (\Box \Gamma_3, \Box \Gamma_4), (\Box \Gamma_5, \Delta_6), []F_2} A45 \rightarrow \frac{}{\bullet h_1 : \Box \Gamma_7, \Box \Gamma_9 \vdash \Box \Gamma_3, \Box \Gamma_5, F_2} \text{fail}$$

- Case rule \rightarrow_L

$$\frac{h_5 : \Box \Gamma_1, \Delta_8 \vdash \Box \Gamma_2, F_6, \Delta_3, []F_4 \quad h_5 : \Box \Gamma_1, F_7, \Delta_8 \vdash \Box \Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box \Gamma_1, \Delta_8), F_6 \rightarrow F_7 \vdash \Box \Gamma_2, \Delta_3, []F_4} \rightarrow_L \rightarrow \frac{\frac{h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2}{\bullet h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2} \text{H}}{\text{ax/ind}}$$

- Case rule \wedge_L

$$\frac{h_5 : \Box \Gamma_1, F_6, F_7, \Delta_8 \vdash \Box \Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box \Gamma_1, \Delta_8), F_6 \wedge F_7 \vdash \Box \Gamma_2, \Delta_3, []F_4} \wedge_L \rightarrow \frac{\frac{h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2}{\bullet h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2} \text{ax/ind}}{H}$$

- Case rule \vee_L

$$\frac{h_5 : \Box \Gamma_1, F_6, \Delta_8 \vdash \Box \Gamma_2, \Delta_3, []F_4 \quad h_5 : \Box \Gamma_1, F_7, \Delta_8 \vdash \Box \Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box \Gamma_1, \Delta_8), F_6 \vee F_7 \vdash \Box \Gamma_2, \Delta_3, []F_4} \vee_L \rightarrow \frac{\frac{h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2}{\bullet h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2} \text{ax/ind}}{H}$$

- Case rule AT

$$\frac{h_5 : \Box \Gamma_7, F_6, \Delta_1, []F_6 \vdash \Box \Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box \Gamma_7, \Delta_1), []F_6 \vdash \Box \Gamma_2, \Delta_3, []F_4} AT \rightarrow \frac{\frac{h_5 : \Box \Gamma_7, []F_6 \vdash F_4, \Box \Gamma_2}{\bullet h_5 : \Box \Gamma_7, []F_6 \vdash F_4, \Box \Gamma_2} \text{ax/ind}}{H}$$

$$\frac{h_5 : \Box \Gamma_1, F_6, \Delta_7, []F_6 \vdash \Box \Gamma_2, \Delta_3, []F_4}{\bullet h_5 : (\Box \Gamma_1, \Delta_7), []F_6 \vdash \Box \Gamma_2, \Delta_3, []F_4} AT \rightarrow \frac{\frac{h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2}{\bullet h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2} \text{ax/ind}}{H}$$

- Case rule \perp_L

$$\frac{}{\bullet h_5 : \perp, \Box \Gamma_1, \Delta_6 \vdash \Box \Gamma_2, \Delta_3, []F_4} \perp_L \rightarrow \frac{}{\bullet h_5 : \Box \Gamma_1 \vdash \Box \Gamma_2, F_4} \text{fail}$$

- Case rule I

$$\frac{}{\bullet h_4 : p_5, \Box \Gamma_1, \Delta_7 \vdash p_5, \Box \Gamma_2, \Delta_6, []F_3} I \rightarrow \frac{}{\bullet h_4 : \Box \Gamma_1 \vdash \Box \Gamma_2, F_3} \text{fail}$$

- Case rule \top_L

$$\frac{h_5 : \Box \Gamma_1, \Delta_6 \vdash \Box \Gamma_2, \Delta_3, []F_4}{\bullet h_5 : \top, \Box \Gamma_1, \Delta_6 \vdash \Box \Gamma_2, \Delta_3, []F_4} \top_L \rightarrow \frac{\frac{h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2}{\bullet h_5 : \Box \Gamma_1 \vdash F_4, \Box \Gamma_2} \text{ax/ind}}{H}$$

4.9 Status of \rightarrow_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_4 : F_6, \Delta_1, F_2 \rightarrow F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_6 \vdash \Delta_5, F_2, F_7}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \rightarrow F_7} \text{ax/ind}}{\rightarrow_R}$$

- Case rule \wedge_R

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightarrow \frac{\frac{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \wedge F_7} \text{ax/ind} \quad \frac{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_7}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_7} \text{ax/ind}}{\wedge_R}$$

- Case rule \vee_R

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightarrow \frac{\frac{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6, F_7}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \vee F_7} \text{ax/ind}}{\vee_R}$$

- Case rule \perp_R

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \perp, \Delta_5} \perp_R \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash \Delta_5, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \top, \Delta_5} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, []F_6} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_4 \vdash \Delta_5, F_1, []F_6} K$$

- Case rule $A45$

$$\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_5, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_8, F_1 \rightarrow F_2 \vdash \Box \Gamma_5, \Delta_6, []F_7} A45 \rightarrow \frac{\overline{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_5} \text{ ax}}{\bullet h_3 : \Delta_8, \Box \Gamma_4 \vdash \Delta_6, F_1, \Box \Gamma_5, []F_7} A45$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \rightarrow_L$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} H$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \vee_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \rightarrow F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \rightarrow F_2), []F_4 \vdash \Delta_5} AT \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, []F_4 \vdash \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, []F_4 \vdash \Delta_5, F_1} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \rightarrow F_2 \vdash p_4, \Delta_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, p_4 \vdash \Delta_5, F_1, p_4} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5 \vdash \Delta_4, F_1} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \top_L$$

4.10 Status of \rightarrow_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_4 : F_6, \Delta_1, F_2 \rightarrow F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3, F_6 \vdash \Delta_5, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6} \text{ax/ind} \quad \frac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule \vee_R

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R$$

- Case rule \perp_R

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \perp, \Delta_5} \perp_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \perp, \Delta_5} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \top, \Delta_5} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top, \Delta_5} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, [\Box F_6]} K \rightarrow \frac{\frac{}{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ax}}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_4 \vdash \Delta_5, [\Box F_6]} K$$

- Case rule $A45$

$$\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_5, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_8, F_1 \rightarrow F_2 \vdash \Box \Gamma_5, \Delta_6, [\Box F_7]} A45 \rightarrow \frac{\frac{}{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_5} \text{ax}}{\bullet h_3 : \Delta_8, F_2, \Box \Gamma_4 \vdash \Delta_6, \Box \Gamma_5, [\Box F_7]} A45$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{H}$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \rightarrow F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \rightarrow F_2), []F_4 \vdash \Delta_5} AT \rightarrow \frac{\frac{h_3 : \Delta_6, F_2, F_4, []F_4 \vdash \Delta_5}{\bullet h_3 : \Delta_6, F_2, []F_4 \vdash \Delta_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2, []F_4 \vdash \Delta_5} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \rightarrow F_2 \vdash p_4, \Delta_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

4.11 Status of \wedge_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_4 : F_6, \Delta_1, F_2 \wedge F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3, F_6 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind} \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule \vee_R

$$\frac{\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \vee F_7}}{\vee_R} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \vee F_7} \text{ ax/ind}}{\vee_R}$$

- Case rule \perp_R

$$\frac{\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \perp, \Delta_5}}{\perp_R} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \perp, \Delta_5} \text{ ax/ind}}{\perp_R}$$

- Case rule \top_R

$$\frac{\overline{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \top, \Delta_5}}{\top_R} \rightarrow \frac{\overline{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \top, \Delta_5}}{\top_R}$$

- Case rule K

$$\frac{\frac{h_3 : unbox(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \wedge F_2 \vdash \Delta_5, [\Gamma_6]} K}{\rightarrow} \frac{\frac{h_3 : unbox(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, \Box \Gamma_4 \vdash \Delta_5, [\Gamma_6]} \text{ ax}}{K}$$

- Case rule $A45$

$$\frac{\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_5, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_8, F_1 \wedge F_2 \vdash \Box \Gamma_5, \Delta_6, [\Gamma_7]} A45}{\rightarrow} \frac{\frac{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_5}{\bullet h_3 : \Delta_8, F_1, F_2, \Box \Gamma_4 \vdash \Delta_6, \Box \Gamma_5, [\Gamma_7]} \text{ ax}}{A45}$$

- Case rule \rightarrow_L

$$\frac{\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \rightarrow F_5 \vdash \Delta_6}}{\rightarrow_L} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ ax/ind}}{\rightarrow_L} \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\rightarrow_L}$$

- Case rule \wedge_L

$$\frac{\frac{h_3 : F_4, F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \wedge F_5 \vdash \Delta_6}}{\wedge_L} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash \Delta_6} \text{ ax/ind}}{\wedge_L}$$

$$\frac{\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5}}{\wedge_L} \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5} \text{ ax}}{H}$$

- Case rule \vee_L

$$\frac{\frac{h_3 : F_4, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash \Delta_6}}{\vee_L} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ ax/ind}}{\vee_L} \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\vee_L}$$

- Case rule AT

$$\frac{\frac{h_3 : F_4, \Delta_6, [\Gamma_4, F_1 \wedge F_2 \vdash \Delta_5]}{\bullet h_3 : (\Delta_6, F_1 \wedge F_2), [\Gamma_4] \vdash \Delta_5}}{AT} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2, F_4, [\Gamma_4] \vdash \Delta_5}{\bullet h_3 : \Delta_6, F_1, F_2, [\Gamma_4] \vdash \Delta_5} \text{ ax/ind}}{AT}$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \wedge F_2 \vdash p_4, \Delta_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \top_L \rightarrow \frac{h_3 : \Delta_5, F_1, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash \Delta_4} \top_L \text{ ax/ind}$$

4.12 Status of \vee_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_4 : F_6, \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightarrow \frac{h_4 : \Delta_1, F_2, F_6 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \text{ ax/ind}$$

- Case rule \wedge_R

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightarrow \frac{h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \text{ ax/ind} \quad h_4 : \Delta_1, F_2 \vdash \Delta_5, F_7 \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule \vee_R

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightarrow \frac{h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6, F_7 \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \vee F_7} \vee_R$$

- Case rule \perp_R

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \perp, \Delta_5} \perp_R \rightarrow \frac{h_4 : \Delta_1, F_2 \vdash \Delta_5 \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \perp, \Delta_5} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \top, \Delta_5} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2 \vdash \top, \Delta_5} \top_R$$

- Case rule K

$$\frac{h_3 : unbox(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, [\Box F_6]} K \rightarrow \frac{h_3 : unbox(\Box \Gamma_4) \vdash F_6 \text{ ax}}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_4 \vdash \Delta_5, [\Box F_6]} K$$

- Case rule $A45$

$$\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_5, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_8, F_1 \vee F_2 \vdash \Box \Gamma_5, \Delta_6, [] F_7} A45 \rightarrow \frac{\frac{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_5}{\bullet h_3 : \Delta_8, F_1, \Box \Gamma_4 \vdash \Delta_6, \Box \Gamma_5, [] F_7} ax}{\bullet h_3 : \Delta_8, F_1, \Box \Gamma_4 \vdash \Delta_6, \Box \Gamma_5, [] F_7} A45$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} ax/ind \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} ax/ind}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} ax/ind}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} ax/ind \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} ax/ind}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} ax}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} H$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, [] F_4, F_1 \vee F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \vee F_2), [] F_4 \vdash \Delta_5} AT \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_4, [] F_4 \vdash \Delta_5}{\bullet h_3 : \Delta_6, F_1, [] F_4 \vdash \Delta_5} ax/ind}{\bullet h_3 : \Delta_6, F_1, [] F_4 \vdash \Delta_5} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \vee F_2 \vdash p_4, \Delta_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, p_4 \vdash \Delta_5, p_4} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_1 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} ax/ind}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \top_L$$

4.13 Status of \vee_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_4 : F_6, \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_3, F_6 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind} \quad \frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule \vee_R

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R$$

- Case rule \perp_R

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \perp, \Delta_5} \perp_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_3 \vdash \perp, \Delta_5} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \perp, \Delta_5} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \top, \Delta_5} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top, \Delta_5} \top_R$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, []F_6} K \rightarrow \frac{\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_4 \vdash \Delta_5, []F_6} \text{ax}}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_4 \vdash \Delta_5, []F_6} K$$

- Case rule $A45$

$$\frac{h_3 : \Box \Gamma_4 \vdash \Box \Gamma_5, F_7}{\bullet h_3 : \Box \Gamma_4, \Delta_8, F_1 \vee F_2 \vdash \Box \Gamma_5, \Delta_6, []F_7} A45 \rightarrow \frac{\frac{h_3 : \Box \Gamma_4 \vdash F_7, \Box \Gamma_5}{\bullet h_3 : \Delta_8, F_2, \Box \Gamma_4 \vdash \Delta_6, \Box \Gamma_5, []F_7} \text{ax}}{\bullet h_3 : \Delta_8, F_2, \Box \Gamma_4 \vdash \Delta_6, \Box \Gamma_5, []F_7} A45$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{H}$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \vee F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \vee F_2), []F_4 \vdash \Delta_5} AT \rightarrow \frac{\frac{h_3 : \Delta_6, F_2, F_4, []F_4 \vdash \Delta_5}{\bullet h_3 : \Delta_6, F_2, []F_4 \vdash \Delta_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2, []F_4 \vdash \Delta_5} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \vee F_2 \vdash p_4, \Delta_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

4.14 Status of AT : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_1, []F_2 \vdash F_6, \Delta_4}{\bullet h_3 : \Delta_1, []F_2 \vdash \Delta_4, F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_1, F_2, F_5, []F_2 \vdash \Delta_4, F_6}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \rightarrow F_6} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_1, []F_2 \vdash F_5, \Delta_4 \quad h_3 : \Delta_1, []F_2 \vdash F_6, \Delta_4}{\bullet h_3 : \Delta_1, []F_2 \vdash \Delta_4, F_5 \wedge F_6} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \wedge F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_6}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \wedge F_6} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_1, []F_2 \vdash F_5, F_6, \Delta_4}{\bullet h_3 : \Delta_1, []F_2 \vdash \Delta_4, F_5 \vee F_6} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5, F_6}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \vee F_6} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_1, []F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_1, []F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{h_3 : \Delta_1, F_2, []F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \perp, \Delta_4} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \perp, \Delta_4} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_1, []F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_1, F_2, []F_2 \vdash \top, \Delta_4} \top_R$$

- Case rule K

$$\frac{h_2 : F_1, unbox(\Box \Gamma_6) \vdash F_5}{\bullet h_2 : (\Box \Gamma_6, []F_1), \Delta_3 \vdash \Delta_4, []F_5} K \rightarrow \frac{\frac{}{h_2 : F_1, unbox(\Box \Gamma_6) \vdash F_5} ax}{\bullet h_2 : \Delta_3, F_1, \Box \Gamma_6, []F_1 \vdash \Delta_4, []F_5} K$$

$$\frac{h_2 : unbox(\Box \Gamma_3) \vdash F_5}{\bullet h_2 : \Box \Gamma_3, \Delta_6, []F_1 \vdash \Delta_4, []F_5} K \rightarrow \frac{\frac{}{h_2 : unbox(\Box \Gamma_3) \vdash F_5} ax}{\bullet h_2 : \Delta_6, F_1, \Box \Gamma_3, []F_1 \vdash \Delta_4, []F_5} K$$

- Case rule $A45$

$$\frac{h_2 : \Box \Gamma_7, []F_1 \vdash \Box \Gamma_4, F_6}{\bullet h_2 : (\Box \Gamma_7, []F_1), \Delta_3 \vdash \Box \Gamma_4, \Delta_5, []F_6} A45 \rightarrow \frac{\frac{}{h_2 : \Box \Gamma_7, []F_1 \vdash F_6, \Box \Gamma_4} ax}{\bullet h_2 : \Delta_3, F_1, \Box \Gamma_7, []F_1 \vdash \Delta_5, \Box \Gamma_4, []F_6} A45$$

$$\frac{h_2 : \Box \Gamma_3 \vdash \Box \Gamma_4, F_6}{\bullet h_2 : \Box \Gamma_3, \Delta_7, []F_1 \vdash \Box \Gamma_4, \Delta_5, []F_6} A45 \rightarrow \frac{\frac{}{h_2 : \Box \Gamma_3 \vdash F_6, \Box \Gamma_4} ax}{\bullet h_2 : \Delta_7, F_1, \Box \Gamma_3, []F_1 \vdash \Delta_5, \Box \Gamma_4, []F_6} A45$$

- Case rule \rightarrow_L

$$\frac{h_2 : \Delta_6, []F_1 \vdash F_3, \Delta_5 \quad h_2 : F_4, \Delta_6, []F_1 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, []F_1), F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\frac{}{h_2 : \Delta_6, F_1, []F_1 \vdash \Delta_5, F_3} ax/ind \quad \frac{}{h_2 : \Delta_6, F_1, F_4, []F_1 \vdash \Delta_5} ax/ind}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_2 : F_3, F_4, \Delta_6, []F_1 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, []F_1), F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightarrow \frac{\frac{}{h_2 : \Delta_6, F_1, F_3, F_4, []F_1 \vdash \Delta_5} ax/ind}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L$$

- Case rule \vee_L

$$\frac{h_2 : F_3, \Delta_6, []F_1 \vdash \Delta_5 \quad h_2 : F_4, \Delta_6, []F_1 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, []F_1), F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightarrow \frac{\frac{}{h_2 : \Delta_6, F_1, F_3, []F_1 \vdash \Delta_5} ax/ind \quad \frac{}{h_2 : \Delta_6, F_1, F_4, []F_1 \vdash \Delta_5} ax/ind}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \vee F_4 \vdash \Delta_5} \vee_L$$

- Case rule AT

$$\frac{h_2 : F_3, \Delta_5, []F_1, []F_3 \vdash \Delta_4}{\bullet h_2 : (\Delta_5, []F_1), []F_3 \vdash \Delta_4} AT \rightarrow \frac{\frac{}{h_2 : \Delta_5, F_1, F_3, []F_1, []F_3 \vdash \Delta_4} ax/ind}{\bullet h_2 : \Delta_5, F_1, []F_1, []F_3 \vdash \Delta_4} AT$$

$$\frac{h_1 : F_3, \Delta_2, []F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_2, []F_3 \vdash \Delta_4} AT \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3, F_3, []F_3 \vdash \Delta_4} ax/ind}{\bullet h_1 : \Delta_2, F_3, []F_3 \vdash \Delta_4} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_2 : \perp, \Delta_4, []F_1 \vdash \Delta_3} \perp_L \rightarrow \frac{}{\bullet h_2 : \perp, \Delta_4, F_1, []F_1 \vdash \Delta_3} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_2 : p_3, \Delta_5, []F_1 \vdash p_3, \Delta_4} I \quad \rightarrow \quad \frac{}{\bullet h_2 : \Delta_5, F_1, p_3, []F_1 \vdash \Delta_4, p_3} I$$

- Case rule \top_L

$$\frac{h_2 : \Delta_4, []F_1 \vdash \Delta_3}{\bullet h_2 : \top, \Delta_4, []F_1 \vdash \Delta_3} \top_L \quad \rightarrow \quad \frac{\frac{}{h_2 : \Delta_4, F_1, []F_1 \vdash \Delta_3} \text{ax/ind}}{\bullet h_2 : \top, \Delta_4, F_1, []F_1 \vdash \Delta_3} \top_L$$

4.15 Status of \perp_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_2 : \perp, F_4, \Delta_1 \vdash F_5, \Delta_3}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \quad \rightarrow \quad \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_2 : \perp, \Delta_1 \vdash F_4, \Delta_3 \quad h_2 : \perp, \Delta_1 \vdash F_5, \Delta_3}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \quad \rightarrow \quad \text{trivial}$$

- Case rule \vee_R

$$\frac{h_2 : \perp, \Delta_1 \vdash F_4, F_5, \Delta_3}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \vee F_5} \vee_R \quad \rightarrow \quad \text{trivial}$$

- Case rule \perp_R

$$\frac{h_2 : \perp, \Delta_1 \vdash \Delta_3}{\bullet h_2 : \perp, \Delta_1 \vdash \perp, \Delta_3} \perp_R \quad \rightarrow \quad \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_2 : \perp, \Delta_1 \vdash \top, \Delta_3} \top_R \quad \rightarrow \quad \text{trivial}$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box\Gamma_2) \vdash F_4}{\bullet h_1 : \Box\Gamma_2, \perp, \Delta_5 \vdash \Delta_3, []F_4} K \quad \rightarrow \quad \text{trivial}$$

- Case rule $A45$

$$\frac{h_1 : \Box\Gamma_2 \vdash \Box\Gamma_3, F_5}{\bullet h_1 : \Box\Gamma_2, \perp, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []F_5} A45 \quad \rightarrow \quad \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \perp, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \perp, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : \perp, F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : \perp, F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : \perp, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \text{trivial}$$

- Case rule AT

$$\frac{h_1 : \perp, F_2, \Delta_4, []F_2 \vdash \Delta_3}{\bullet h_1 : (\perp, \Delta_4), []F_2 \vdash \Delta_3} AT \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_2, \perp, \Delta_4 \vdash p_2, \Delta_3} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \perp, \Delta_3 \vdash \Delta_2} \top_L \rightarrow \text{trivial}$$

4.16 Status of I : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_4, \Delta_1, p_2 \vdash F_5, \Delta_6, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash (\Delta_6, p_2), F_4 \rightarrow F_5} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_1, p_2 \vdash F_4, \Delta_6, p_2 \quad h_3 : \Delta_1, p_2 \vdash F_5, \Delta_6, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash (\Delta_6, p_2), F_4 \wedge F_5} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_1, p_2 \vdash F_4, F_5, \Delta_6, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash (\Delta_6, p_2), F_4 \vee F_5} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_1, p_2 \vdash \Delta_4, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash \perp, \Delta_4, p_2} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_1, p_2 \vdash \top, \Delta_4, p_2} \top_R \rightarrow \text{trivial}$$

- Case rule K

$$\frac{h_2 : \text{unbox}(\Box \Gamma_3) \vdash F_4}{\bullet h_2 : \Box \Gamma_3, \Delta_6, p_1 \vdash (\Delta_5, p_1), [\Box F_4]} K \rightarrow \text{trivial}$$

- Case rule $A45$

$$\frac{h_2 : \Box \Gamma_3 \vdash \Box \Gamma_4, F_5}{\bullet h_2 : \Box \Gamma_3, \Delta_7, p_1 \vdash \Box \Gamma_4, (\Delta_6, p_1), [\Box F_5]} A45 \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_6, p_1 \vdash F_4, \Delta_2, p_1 \quad h_3 : F_5, \Delta_6, p_1 \vdash \Delta_2, p_1}{\bullet h_3 : (\Delta_6, p_1), F_4 \rightarrow F_5 \vdash \Delta_2, p_1} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_6, p_1 \vdash \Delta_2, p_1}{\bullet h_3 : (\Delta_6, p_1), F_4 \wedge F_5 \vdash \Delta_2, p_1} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_6, p_1 \vdash \Delta_2, p_1 \quad h_3 : F_5, \Delta_6, p_1 \vdash \Delta_2, p_1}{\bullet h_3 : (\Delta_6, p_1), F_4 \vee F_5 \vdash \Delta_2, p_1} \vee_L \rightarrow \text{trivial}$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_5, p_1, [\Box F_4] \vdash \Delta_2, p_1}{\bullet h_3 : (\Delta_5, p_1), [\Box F_4] \vdash \Delta_2, p_1} AT \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_4, p_1 \vdash \Delta_2, p_1} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_2 : p_3, \Delta_5, p_1 \vdash p_3, \Delta_4, p_1} I \rightarrow \text{trivial}$$

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3, \Delta_4} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_3 : \Delta_4, p_1 \vdash \Delta_2, p_1}{\bullet h_3 : \top, \Delta_4, p_1 \vdash \Delta_2, p_1} \top_L \rightarrow \text{trivial}$$

4.17 Status of \top_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_2 : \top, F_4, \Delta_1 \vdash F_5, \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\overline{h_2 : \Delta_1, F_4 \vdash \Delta_3, F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_2 : \top, \Delta_1 \vdash F_4, \Delta_3 \quad h_2 : \top, \Delta_1 \vdash F_5, \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\overline{h_2 : \Delta_1 \vdash \Delta_3, F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_1 \vdash \Delta_3, F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_2 : \top, \Delta_1 \vdash F_4, F_5, \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\overline{h_2 : \Delta_1 \vdash \Delta_3, F_4, F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_2 : \top, \Delta_1 \vdash \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\overline{h_2 : \Delta_1 \vdash \Delta_3} \text{ ax/ind}}{\bullet h_2 : \Delta_1 \vdash \perp, \Delta_3} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_2 : \top, \Delta_1 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_2 : \Delta_1 \vdash \top, \Delta_3} \top_R$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4}{\bullet h_1 : \Box \Gamma_2, \top, \Delta_5 \vdash \Delta_3, [\Gamma_4]} K \rightarrow \frac{\overline{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_5, \Box \Gamma_2 \vdash \Delta_3, [\Gamma_4]} K$$

- Case rule $A45$

$$\frac{h_1 : \Box \Gamma_2 \vdash \Box \Gamma_3, F_5}{\bullet h_1 : \Box \Gamma_2, \top, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, [\Gamma_5]} A45 \rightarrow \frac{\overline{h_1 : \Box \Gamma_2 \vdash F_5, \Box \Gamma_3} \text{ ax}}{\bullet h_1 : \Delta_6, \Box \Gamma_2 \vdash \Delta_4, \Box \Gamma_3, [\Gamma_5]} A45$$

- Case rule \rightarrow_L

$$\frac{h_1 : \top, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \top, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\overline{h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{ ax/ind} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_1 : \top, F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L^{\text{ax/ind}}$$

- Case rule \vee_L

$$\frac{h_1 : \top, F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : \top, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2 \vdash \Delta_4} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L^{\text{ax/ind}}$$

- Case rule AT

$$\frac{h_1 : \top, F_2, \Delta_4, []F_2 \vdash \Delta_3}{\bullet h_1 : (\top, \Delta_4), []F_2 \vdash \Delta_3} AT \rightarrow \frac{\overline{h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3}}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3} AT^{\text{ax/ind}}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \top, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_2, \top, \Delta_4 \vdash p_2, \Delta_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, p_2 \vdash \Delta_3, p_2} I$$

- Case rule \top_L

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \top_L^{\text{ax}}$$

5 Identity-Expansion

$$\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : \Box F_0 \vdash \Box F_0} K$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0, F_1} W}{\frac{- : F_0 \vee F_1 \vdash F_0, F_1}{- : F_0 \vee F_1 \vdash F_0 \vee F_1} \vee_R} \vee_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_1 \vdash F_0} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{\frac{- : F_0, F_1 \vdash F_0 \wedge F_1}{- : F_0 \wedge F_1 \vdash F_0 \wedge F_1} \wedge_L} \wedge_R$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{\frac{- : F_0, F_0 \rightarrow F_1 \vdash F_1}{- : F_0 \rightarrow F_1 \vdash F_0 \rightarrow F_1} \rightarrow_R} \rightarrow_L$$

$$\overline{- : \top \vdash \top} \top_R$$

$$\overline{- : \perp \vdash \perp} \perp_L$$

6 Cut-Elimination

6.1 Status of \rightarrow_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \rightarrow F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : F_{11}, \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_6, F_{11}, F_7 \vdash \Delta_{10}, F_{12}, F_8}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \rightarrow F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}}{\text{ax/W}}}{\frac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{\rightarrow_R} \text{hCut}} \\
\frac{\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \rightarrow F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \rightarrow F_{10}} \rightarrow_R}{\frac{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \rightarrow F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_2 : \Delta_8, F_{12}, F_9 \vdash \Delta_{14}, F_{10}, F_{13}, F_7}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9 \rightarrow F_{10}} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \rightarrow F_{10}}{\text{ax/W}}}{\frac{- : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \rightarrow F_{10}}{\rightarrow_R} \text{hCut}} \\
\frac{\frac{h_2 : F_{11}, \Delta_8 \vdash F_7, F_{12}, \Delta_{10}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7} \rightarrow_R \quad \frac{h_9 : F_7, F_{11}, \Delta_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R}{\frac{- : \Delta_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_2 : \Delta_8, F_{11} \vdash \Delta_{10}, F_{12}, F_7}{\text{ax/W}} \quad \frac{h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{10}, F_{12}} \text{ax/W}}{\frac{- : \Delta_8, F_{11} \vdash \Delta_{10}, F_{12}}{\rightarrow_R} \text{H}} \\
\frac{- : \Delta_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\rightarrow_R} \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \rightarrow F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}}{\text{ax/W}}}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11}}{\rightarrow_R} \text{hCut}} \quad \frac{\frac{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{12}, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7 \rightarrow F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}}{\text{ax/W}}}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{12}}{\rightarrow_R} \text{hCut}} \\
\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}}{\wedge_R} \\
\frac{\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \quad h_{11} : F_7, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \rightarrow F_{10}} \wedge_R}{\frac{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \rightarrow F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_2 : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_7, F_{12} \wedge F_{13}}{\text{ax/W}} \quad \frac{\frac{h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12}}{\bullet h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \wedge F_{13}} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{13}}{\text{inv-th/ax}}}{\frac{- : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \wedge F_{13}}{\rightarrow_R} \text{hCut}} \\
\frac{- : \Delta_8 \vdash \Delta_{14}, F_9 \rightarrow F_{10}, F_{12} \wedge F_{13}}{\rightarrow_R}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \rightarrow F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W} \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12} \quad \vee_R \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12} \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \rightarrow F_{10}} \vee_R \\
\hline
- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \rightarrow F_{10} \\
\rightarrow \\
\frac{h_2 : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_7, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_7, F_{12} \vee F_{13}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}}{\bullet h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \vee F_{13}} \text{inv-th/ax} \\
\hline
- : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \vee F_{13} \quad \vee_R \\
- : \Delta_8 \vdash \Delta_{14}, F_9 \rightarrow F_{10}, F_{12} \vee F_{13} \quad \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_6 \vdash F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}} \text{ax/W} \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10} \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \perp, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\perp, \Delta_{12}), F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\perp, \Delta_{12}), F_9 \rightarrow F_{10}} \perp_R \\
\hline
- : \Delta_8 \vdash (\perp, \Delta_{12}), F_9 \rightarrow F_{10} \\
\rightarrow \\
\frac{\bullet h_2 : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \rightarrow F_{10}}{- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10}} \text{ax/W} \\
\hline
- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10} \quad \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_6 \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10} \\
\rightarrow \\
- : \Delta_6 \vdash \top, \Delta_{10} \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \top, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\top, \Delta_{12}), F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \rightarrow F_{10}} \top_R \\
\hline
- : \Delta_8 \vdash (\top, \Delta_{12}), F_9 \rightarrow F_{10} \\
\rightarrow \\
- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \rightarrow F_{10} \quad \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : F_6, \Box \Gamma_9, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_{11}}{\bullet h_1 : \Box \Gamma_9, \Delta_{12} \vdash (\Delta_{10}, \Box F_{11}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_9) \vdash F_{11}}{\bullet h_8 : (\Box \Gamma_9, \Delta_{12}), F_6 \rightarrow F_7 \vdash \Delta_{10}, \Box F_{11}} K \\
\hline
- : \Box \Gamma_9, \Delta_{12} \vdash \Delta_{10}, \Box F_{11} \\
\rightarrow \\
- : \text{unbox}(\Box \Gamma_9) \vdash F_{11} \quad \text{ax/W} \\
- : \Delta_{12}, \Box \Gamma_9 \vdash \Delta_{10}, \Box F_{11} \quad K
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_8, \Box\Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_9, \Delta_{13}, \Box F_{12}}{\bullet h_2 : \Box\Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, \Box F_{12}), F_8 \rightarrow F_9), \Box F_7} \rightarrow_R \quad \frac{h_{10} : unbox(\Box\Gamma_{14}), unbox(\Box F_7) \vdash F_{12}}{\bullet h_{10} : (\Box\Gamma_{14}, \Delta_{11}), \Box F_7 \vdash (\Delta_{13}, \Box F_{12}), F_8 \rightarrow F_9} \quad \begin{array}{l} K \\ \text{Cut} \end{array} \\
\hline
- : \Box\Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, \Box F_{12}), F_8 \rightarrow F_9 \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11}, F_8, \Box\Gamma_{14} \vdash \Box F_7, \Delta_{13}, F_9, \Box F_{12}}{\quad} \text{ax/W} \quad \frac{h_{10} : unbox(\Box F_7), unbox(\Box\Gamma_{14}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, F_8, \Box\Gamma_{14} \vdash \Delta_{13}, F_9, \Box F_{12}} \quad \begin{array}{l} K \\ \text{hCut} \end{array}}{\frac{- : \Delta_{11}, F_8, \Box\Gamma_{14} \vdash \Delta_{13}, F_9, \Box F_{12}}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box F_{12}, F_8 \rightarrow F_9}} \rightarrow_R \\
\\
\frac{h_2 : F_8, \Box\Gamma_{11}, \Delta_{14} \vdash F_7, F_9, \Delta_{13}, \Box F_{12}}{\bullet h_2 : \Box\Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, \Box F_{12}), F_8 \rightarrow F_9), F_7} \rightarrow_R \quad \frac{h_{10} : unbox(\Box\Gamma_{11}) \vdash F_{12}}{\bullet h_{10} : (\Box\Gamma_{11}, \Delta_{14}), F_7 \vdash (\Delta_{13}, \Box F_{12}), F_8 \rightarrow F_9} \quad \begin{array}{l} K \\ \text{Cut} \end{array} \\
\hline
- : \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_{13}, \Box F_{12}), F_8 \rightarrow F_9 \\
\rightarrow \\
\frac{- : unbox(\Box\Gamma_{11}) \vdash F_{12}}{- : \Delta_{14}, \Box\Gamma_{11} \vdash \Delta_{13}, \Box F_{12}, F_8 \rightarrow F_9} \quad \begin{array}{l} \text{ax/W} \\ K \end{array}
\end{array}$$

• Case rule A45

$$\begin{array}{c}
\frac{h_1 : F_6, \Box\Gamma_9, \Delta_{13} \vdash F_7, \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}}{\bullet h_1 : \Box\Gamma_9, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Box\Gamma_9 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_8 : (\Box\Gamma_9, \Delta_{13}), F_6 \rightarrow F_7 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}} \quad \begin{array}{l} A45 \\ \text{Cut} \end{array} \\
\hline
- : \Box\Gamma_9, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12} \\
\rightarrow \\
\frac{- : \Box\Gamma_9 \vdash F_{12}, \Box\Gamma_{10}}{- : \Delta_{13}, \Box\Gamma_9 \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad \begin{array}{l} \text{ax/W} \\ A45 \end{array} \\
\\
\frac{h_2 : F_8, \Box\Gamma_{15}, \Delta_{11} \vdash \Box F_7, F_9, \Box\Gamma_{12}, \Delta_{14}, \Box F_{13}}{\bullet h_2 : \Box\Gamma_{15}, \Delta_{11} \vdash ((\Box\Gamma_{12}, \Delta_{14}), \Box F_{13}), F_8 \rightarrow F_9} \rightarrow_R \quad \frac{h_{10} : \Box\Gamma_{15}, \Box F_7 \vdash \Box\Gamma_{12}, F_{13}}{\bullet h_{10} : (\Box\Gamma_{15}, \Delta_{11}), \Box F_7 \vdash (\Box\Gamma_{12}, \Delta_{14}), \Box F_{13}), F_8 \rightarrow F_9} \quad \begin{array}{l} A45 \\ \text{Cut} \end{array} \\
\hline
- : \Box\Gamma_{15}, \Delta_{11} \vdash (\Box\Gamma_{12}, \Delta_{14}), \Box F_{13}), F_8 \rightarrow F_9 \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11}, F_8, \Box\Gamma_{15} \vdash \Box F_7, \Delta_{14}, F_9, \Box\Gamma_{12}, \Box F_{13}}{\quad} \text{ax/W} \quad \frac{h_{10} : \Box F_7, \Box\Gamma_{15} \vdash F_{13}, \Box\Gamma_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, F_8, \Box\Gamma_{15} \vdash \Delta_{14}, F_9, \Box\Gamma_{12}, \Box F_{13}} \quad \begin{array}{l} A45 \\ \text{hCut} \end{array}}{\frac{- : \Delta_{11}, F_8, \Box\Gamma_{15} \vdash \Delta_{14}, F_9, \Box\Gamma_{12}, \Box F_{13}}{- : \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, \Box\Gamma_{12}, \Box F_{13}, F_8 \rightarrow F_9}} \rightarrow_R \\
\\
\frac{h_2 : F_8, \Box\Gamma_{11}, \Delta_{15} \vdash F_7, F_9, \Box\Gamma_{12}, \Delta_{14}, \Box F_{13}}{\bullet h_2 : \Box\Gamma_{11}, \Delta_{15} \vdash ((\Box\Gamma_{12}, \Delta_{14}), \Box F_{13}), F_8 \rightarrow F_9} \rightarrow_R \quad \frac{h_{10} : \Box\Gamma_{11} \vdash \Box\Gamma_{12}, F_{13}}{\bullet h_{10} : (\Box\Gamma_{11}, \Delta_{15}), F_7 \vdash (\Box\Gamma_{12}, \Delta_{14}), \Box F_{13}), F_8 \rightarrow F_9} \quad \begin{array}{l} A45 \\ \text{Cut} \end{array} \\
\hline
- : \Box\Gamma_{11}, \Delta_{15} \vdash (\Box\Gamma_{12}, \Delta_{14}), \Box F_{13}), F_8 \rightarrow F_9 \\
\rightarrow \\
\frac{- : \Box\Gamma_{11} \vdash F_{13}, \Box\Gamma_{12}}{- : \Delta_{15}, \Box\Gamma_{11} \vdash \Delta_{14}, \Box\Gamma_{12}, \Box F_{13}, F_8 \rightarrow F_9} \quad \begin{array}{l} \text{ax/W} \\ A45 \end{array}
\end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \quad \begin{array}{l} \text{Cut} \\ \rightarrow_L \end{array} \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \quad \frac{\text{inv-th/ax}}{\quad} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9} \quad \begin{array}{l} \text{ax/W} \\ \text{hCut} \end{array} \quad \frac{\frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \quad \frac{\text{inv-th/ax}}{\quad} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}} \quad \begin{array}{l} \text{ax/W} \\ \text{sCut} \end{array}}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \quad - : \Delta_{12}, F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}}} \rightarrow_L \\
\\
\frac{h_1 : F_8, \Delta_7 \vdash F_9, \Delta_{10}}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \quad \frac{h_6 : \Delta_7 \vdash F_8, \Delta_{10} \quad h_6 : F_9, \Delta_7 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{10}} \quad \begin{array}{l} \text{Cut} \\ \rightarrow_L \end{array} \\
\hline
- : \Delta_7 \vdash \Delta_{10} \\
\rightarrow \\
\frac{- : \Delta_7 \vdash \Delta_{10}, F_8 \quad \frac{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9} \quad \frac{- : \Delta_7, F_8, F_9 \vdash \Delta_{10}}{- : \Delta_7, F_8 \vdash \Delta_{10}} \quad \begin{array}{l} \text{ax/W} \\ \text{sCut} \end{array}}{- : \Delta_7 \vdash \Delta_{10}}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_8, \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), F_{12} \rightarrow F_{13}} \rightarrow_R \quad \frac{h_{10} : \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \rightarrow F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \rightarrow F_9} \rightarrow_L \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \rightarrow F_{13}}{\bullet h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \rightarrow F_{13}} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_8 \vdash \Delta_7, F_{12}, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{11}, F_{13}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_{13}, F_8 \vdash \Delta_7, F_9} \text{inv-th/ax} \\
\hline
- : \Delta_{11}, F_8 \vdash \Delta_7, F_9 \quad \text{hCut} \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9 \quad \rightarrow_R \\
\hline
\frac{h_2 : F_9, \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, \Delta_{14} \vdash F_{12}, \Delta_8, F_9 \rightarrow F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_L \\
\hline
- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_{14}, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7}{\bullet h_2 : \Delta_{14}, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_7, F_9 \vdash \Delta_8, F_{10}, F_{12}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}{\bullet h_{11} : \Delta_{14}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}} \text{inv-th/ax} \\
\hline
- : \Delta_{14}, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10} \quad \rightarrow_R
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \wedge_L \\
\hline
- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_{10}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \\
\hline
- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11} \quad \text{hCut} \\
\hline
- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \quad \wedge_L \\
\hline
\frac{h_2 : F_8, \Delta_{11} \vdash F_{12} \wedge F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), F_{12} \wedge F_{13}} \rightarrow_R \quad \frac{h_{10} : F_{12}, F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \rightarrow F_9} \wedge_L \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \wedge F_{13}} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_{13}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \wedge F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax} \quad \wedge_L \\
\hline
- : \Delta_{11}, F_8 \vdash \Delta_7, F_9 \quad \text{hCut} \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9 \quad \rightarrow_R \\
\hline
\frac{h_2 : F_9, \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \wedge_L \\
\hline
- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7}{\bullet h_2 : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}} \text{inv-th/ax} \quad \wedge_L \\
\hline
- : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10} \quad \rightarrow_R
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \vee_L \\
\hline
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \quad \wedge_L \\
\hline
- : \Delta_{12}, F_9 \vdash \Delta_{11} \quad \text{hCut} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{hCut} \\
\hline
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \quad \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_8, \Delta_{11} \vdash F_{12} \vee F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), F_{12} \vee F_{13}} \rightarrow_R \quad \frac{h_{10} : F_{12}, \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \rightarrow F_9} \vee_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \vee F_{13}} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \vee F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{11}, F_{13}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \vee F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax} \quad \vee_L \\
\frac{}{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \text{hCut} \\
\rightarrow_R \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_9, \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_{14} \vdash \Delta_8, F_9 \rightarrow F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \vee_L \\
\frac{}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7}{\bullet h_2 : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_7, F_9 \vdash \Delta_8, F_{10}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}} \text{inv-th/ax} \quad \vee_L \\
\frac{}{- : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}} \text{hCut} \\
\rightarrow_R \\
\frac{}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}}
\end{array}$$

• Case rule AT

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{11}, [F_9 \vdash F_7, \Delta_{10}]}{\bullet h_1 : \Delta_{11}, [F_9 \vdash \Delta_{10}, F_6 \rightarrow F_7]} \rightarrow_R \quad \frac{h_8 : F_9, \Delta_{11}, [F_9, F_6 \rightarrow F_7 \vdash \Delta_{10}]}{\bullet h_8 : (\Delta_{11}, [F_9]), F_6 \rightarrow F_7 \vdash \Delta_{10}} AT \\
\frac{}{- : \Delta_{11}, [F_9 \vdash \Delta_{10}]} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}, F_6 \rightarrow F_7]}{\bullet h_1 : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}, F_6 \rightarrow F_7]} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_9, [F_9, F_6 \rightarrow F_7 \vdash \Delta_{10}]}{\bullet h_8 : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}, F_6 \rightarrow F_7]} \text{ax/W} \\
\frac{}{- : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}]} \text{hCut} \\
\frac{}{- : \Delta_{11}, [F_9 \vdash \Delta_{10}]} ATG
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_8, \Delta_{11} \vdash [F_{12}, F_9, \Delta_7]}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), [F_{12}]} \rightarrow_R \quad \frac{h_{10} : F_{12}, \Delta_{11}, [F_{12} \vdash \Delta_7, F_8 \rightarrow F_9]}{\bullet h_{10} : \Delta_{11}, [F_{12} \vdash \Delta_7, F_8 \rightarrow F_9]} AT \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, [F_{12}]}{\bullet h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, [F_{12}]} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_8, [F_{12} \vdash \Delta_7, F_8 \rightarrow F_9]}{\bullet h_{10} : \Delta_{11}, F_8, [F_{12} \vdash \Delta_7, F_8 \rightarrow F_9]} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_8, [F_{12} \vdash \Delta_7, F_8 \rightarrow F_9]}{\bullet h_{10} : \Delta_{11}, F_8, [F_{12} \vdash \Delta_7, F_8 \rightarrow F_9]} AT \\
\frac{}{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \text{hCut} \\
\rightarrow_R \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_9, \Delta_{13}, [F_{12} \vdash F_7, F_{10}, \Delta_8]}{\bullet h_2 : \Delta_{13}, [F_{12} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7]} \rightarrow_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_{13}, [F_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}]}{\bullet h_{11} : (\Delta_{13}, [F_{12}]), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} AT \\
\frac{}{- : \Delta_{13}, [F_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}]} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : \Delta_{13}, F_{12}, [F_{12} \vdash \Delta_8, F_7, F_9 \rightarrow F_{10}]}{\bullet h_2 : \Delta_{13}, F_{12}, [F_{12} \vdash \Delta_8, F_7, F_9 \rightarrow F_{10}]} \text{ax/W} \quad \frac{h_{11} : \Delta_{13}, F_{12}, F_7, [F_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}]}{\bullet h_{11} : \Delta_{13}, F_{12}, F_7, [F_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}]} \text{ax/W} \\
\frac{}{- : \Delta_{13}, F_{12}, [F_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}]} \text{hCut} \\
\frac{}{- : \Delta_{13}, [F_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}]} ATG
\end{array}$$

• Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : F_6, \perp, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \perp_L \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_8, \Delta_{11} \vdash \perp, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), \perp} \rightarrow_R \quad \frac{h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \rightarrow F_9} \perp_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_{11}, F_8 \vdash \perp, \Delta_7, F_9}{\bullet h_2 : \Delta_{11}, F_8 \vdash \perp, \Delta_7, F_9} \text{ax/W} \quad \frac{h_{10} : \perp, \Delta_{11}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \perp, \Delta_{11}, F_8 \vdash \Delta_7, F_9} \perp_L \\
\frac{}{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \text{hCut} \\
\rightarrow_R \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_9, \perp, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}}{\perp_L} \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{11}, p_9 \vdash F_7, \Delta_{10}, p_9}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{\bullet h_8 : (\Delta_{11}, p_9), F_6 \rightarrow F_7 \vdash \Delta_{10}, p_9}{I} \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I \\
\\
\frac{h_2 : F_7, \Delta_{10} \vdash p_{11}, F_8, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \rightarrow F_8), p_{11}} \rightarrow_R \quad \frac{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \rightarrow F_8}{I} \\
\frac{}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \rightarrow F_8} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, p_{11}, p_{11}}{\bullet h_2 : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{10}, F_7, p_{11} \vdash \Delta_{12}, F_8, p_{11}}{I} \\
\frac{}{- : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, p_{11}} \text{hCut} \\
\rightarrow_R \\
\frac{}{- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \rightarrow F_8} \\
\\
\frac{h_2 : F_8, \Delta_{13}, p_{11} \vdash F_7, F_9, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \rightarrow F_9), F_7} \rightarrow_R \quad \frac{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \rightarrow F_9}{I} \\
\frac{}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \rightarrow F_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \rightarrow F_9} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : F_6, \top, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \top_L \\
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7}{\top_L} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9} \text{ax/W} \\
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut} \\
\\
\frac{h_2 : F_8, \Delta_{11} \vdash \top, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), \top} \rightarrow_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \rightarrow F_9} \top_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{ax/W} \\
\\
\frac{h_2 : F_9, \top, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \top_L \\
\frac{}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \rightarrow F_{10}}{\top_L} \text{ax/W} \quad \frac{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W} \\
\frac{}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{hCut}
\end{array}$$

6.2 Status of \wedge_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \rightarrow F_{12} \quad h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \rightarrow F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : F_{11}, \Delta_6, F_7 \wedge F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_8}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \wedge F_8} \text{inv-th/ax} \\
\frac{}{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}} \wedge_R \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W} \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \rightarrow F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \rightarrow F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}} \rightarrow_R \\
\frac{}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \frac{h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_7}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_7} \text{inv-th/ax}}{\frac{}{- : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \wedge F_{10}} \wedge_R} \text{ax/W} \\
\frac{}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \wedge F_{10}} \rightarrow_R \quad \frac{}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \wedge F_{10}} \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \wedge F_{12} \quad h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \Delta_6, F_7 \vdash \Delta_{10}, F_8, F_{11} \wedge F_{12}} \text{ax/W} \quad \frac{}{- : \Delta_6, F_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{ax/W}}{\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_7, F_{11} \wedge F_{12}} \text{ax/W}} \quad \frac{\frac{}{- : \Delta_6, F_7, F_8 \vdash \Delta_{10}, F_{11}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}} \text{inv-th/ax} \quad \frac{}{- : \Delta_6, F_7, F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}} \text{inv-th/ax}}{\frac{}{- : \Delta_6, F_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R} \text{sCut} \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{sCut} \\
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \wedge F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}} \wedge_R \\
\frac{}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_7, F_9}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_7, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_7}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_7} \text{inv-th/ax}}{\frac{}{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}} \wedge_R} \text{ax/W} \\
\frac{}{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}} \text{hCut} \\
\frac{}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \wedge F_{13}, F_9 \wedge F_{10}} \text{hCut} \\
\frac{h_2 : \Delta_8 \vdash F_7, F_{11}, \Delta_{10} \quad h_2 : \Delta_8 \vdash F_7, F_{12}, \Delta_{10}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7} \wedge_R \quad \frac{h_9 : F_7, \Delta_8 \vdash F_{11}, \Delta_{10} \quad h_9 : F_7, \Delta_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R \\
\frac{}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{10}, F_{11}, F_7}{\bullet h_2 : \Delta_8 \vdash \Delta_{10}, F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}} \text{H}}{\frac{}{- : \Delta_8 \vdash \Delta_{10}, F_{11}} \text{hCut}} \quad \frac{\frac{h_2 : \Delta_8 \vdash \Delta_{10}, F_{12}, F_7}{\bullet h_2 : \Delta_8 \vdash \Delta_{10}, F_{12}} \text{ax/W} \quad \frac{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{12}} \text{H}}{\frac{}{- : \Delta_8 \vdash \Delta_{10}, F_{12}} \text{hCut}} \wedge_R \\
\frac{}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \vee F_{12} \quad h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_8} \text{inv-th/ax}}{\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}} \wedge_R} \text{ax/W} \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R \\
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \vee F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}} \wedge_R \\
\frac{}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_7}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_7} \text{inv-th/ax}}{\frac{}{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \wedge F_{10}} \wedge_R} \text{ax/W} \\
\frac{}{- : \Delta_8 \vdash \Delta_{14}, F_9 \wedge F_{10}, F_{12} \vee F_{13}} \vee_R \quad \frac{}{- : \Delta_8 \vdash \Delta_{14}, F_9 \wedge F_{10}, F_{12} \vee F_{13}} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, \perp, \Delta_{10} \quad h_1 : \Delta_6 \vdash F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \perp, \Delta_{10}} \perp_R \\
\frac{}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \wedge F_8 \quad \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}} \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \perp, \Delta_{10} \quad \text{ax/W}}{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \perp, \Delta_{10}} \text{hCut} \\
\frac{}{- : \Delta_6 \vdash \perp, \Delta_{10}}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \perp, \Delta_{12} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \perp, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\perp, \Delta_{12}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\perp, \Delta_{12}), F_9 \wedge F_{10}} \perp_R \\
\frac{}{- : \Delta_8 \vdash (\perp, \Delta_{12}), F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \wedge F_{10} \quad \text{ax/W}}{- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \wedge F_{10}} \quad \frac{h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \wedge F_{10} \quad \text{ax/W}}{h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \wedge F_{10}} \text{hCut} \\
\frac{}{- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \wedge F_{10}}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, \top, \Delta_{10} \quad h_1 : \Delta_6 \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \wedge F_8} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \top, \Delta_{10}} \top_R \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_{10}} \top_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \top, \Delta_{12} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \top, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\top, \Delta_{12}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{}{\bullet h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \wedge F_{10}} \top_R \\
\frac{}{- : \Delta_8 \vdash (\top, \Delta_{12}), F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \wedge F_{10}} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_9, \Delta_{12} \vdash F_6, \Delta_{10}, [\Gamma_{11}] \quad h_1 : \Box \Gamma_9, \Delta_{12} \vdash F_7, \Delta_{10}, [\Gamma_{11}]}{\bullet h_1 : \Box \Gamma_9, \Delta_{12} \vdash (\Delta_{10}, [\Gamma_{11}]), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_9) \vdash F_{11}}{\bullet h_8 : (\Box \Gamma_9, \Delta_{12}), F_6 \wedge F_7 \vdash \Delta_{10}, [\Gamma_{11}]} K \\
\frac{}{- : \Box \Gamma_9, \Delta_{12} \vdash \Delta_{10}, [\Gamma_{11}]} \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_9) \vdash F_{11}} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box \Gamma_9 \vdash \Delta_{10}, [\Gamma_{11}]} K
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_8, \Delta_{13}, [\Gamma_{12}] \quad h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_9, \Delta_{13}, [\Gamma_{12}]}{\bullet h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, [\Gamma_{12}]), F_8 \wedge F_9), \Box F_7} \wedge_R \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{14}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : (\Box \Gamma_{14}, \Delta_{11}), \Box F_7 \vdash (\Delta_{13}, [\Gamma_{12}]), F_8 \wedge F_9} \\
\frac{}{- : \Box \Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, [\Gamma_{12}]), F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_{14}), \text{unbox}(\Box F_7) \vdash F_{12}} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box \Gamma_{14} \vdash \Delta_{13}, [\Gamma_{12}]} K \\
\frac{}{- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8, [\Gamma_{12}]} \text{ax/W} \\
\frac{}{- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_9, [\Gamma_{12}]} \text{ax/W} \\
\frac{}{- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8 \wedge F_9} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box \Gamma_{11}, \Delta_{14} \vdash F_7, F_8, \Delta_{13}, [\Gamma_{12}] \quad h_2 : \Box \Gamma_{11}, \Delta_{14} \vdash F_7, F_9, \Delta_{13}, [\Gamma_{12}]}{\bullet h_2 : \Box \Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, [\Gamma_{12}]), F_8 \wedge F_9), F_7} \wedge_R \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{11}) \vdash F_{12}}{\bullet h_{10} : (\Box \Gamma_{11}, \Delta_{14}), F_7 \vdash (\Delta_{13}, [\Gamma_{12}]), F_8 \wedge F_9} K \\
\frac{}{- : \Box \Gamma_{11}, \Delta_{14} \vdash (\Delta_{13}, [\Gamma_{12}]), F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_{12}} \text{ax/W} \\
\frac{}{- : \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, [\Gamma_{12}], F_8 \wedge F_9} K
\end{array}$$

- Case rule $A45$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_9, \Delta_{13} \vdash F_6, \Box \Gamma_{10}, \Delta_{11}, [\Gamma_{12}] \quad h_1 : \Box \Gamma_9, \Delta_{13} \vdash F_7, \Box \Gamma_{10}, \Delta_{11}, [\Gamma_{12}]}{\bullet h_1 : \Box \Gamma_9, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [\Gamma_{12}]), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Box \Gamma_9 \vdash \Box \Gamma_{10}, F_{12}}{\bullet h_8 : (\Box \Gamma_9, \Delta_{13}), F_6 \wedge F_7 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Gamma_{12}]} A45 \\
\frac{}{- : \Box \Gamma_9, \Delta_{13} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Gamma_{12}]} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Box \Gamma_9 \vdash F_{12}, \Box \Gamma_{10}} \text{ax/W} \\
\frac{}{- : \Delta_{13}, \Box \Gamma_9 \vdash \Delta_{11}, \Box \Gamma_{10}, [\Gamma_{12}]} A45
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_{15}, \Delta_{11} \vdash \Box F_7, F_8, \Box\Gamma_{12}, \Delta_{14}, \Box F_{13} \quad h_2 : \Box\Gamma_{15}, \Delta_{11} \vdash \Box F_7, F_9, \Box\Gamma_{12}, \Delta_{14}, \Box F_{13}}{\bullet h_2 : \Box\Gamma_{15}, \Delta_{11} \vdash ((\Box\Gamma_{12}, \Delta_{14}, \Box F_{13}), F_8 \wedge F_9), \Box F_7} \wedge_R \quad \frac{h_{10} : \Box\Gamma_{15}, \Box F_7 \vdash \Box F_9}{\bullet h_{10} : (\Box\Gamma_{15}, \Delta_{11}), \Box F_7 \vdash \Box F_9} \wedge_R \\
\frac{\quad}{- : \Box\Gamma_{15}, \Delta_{11} \vdash (\Box\Gamma_{12}, \Delta_{14}, \Box F_{13}), F_8 \wedge F_9} \rightarrow \\
\frac{\frac{h_2 : \Delta_{11}, \Box\Gamma_{15} \vdash \Box F_7, \Delta_{14}, F_8, \Box\Gamma_{12}, \Box F_{13}}{- : \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, F_8, \Box\Gamma_{12}, \Box F_{13}} \text{ ax/W} \quad \frac{h_{10} : \Box F_7, \Box\Gamma_{15} \vdash F_{13}, \Box\Gamma_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, F_8, \Box\Gamma_{12}, \Box F_{13}} \text{ ax/W} \quad \frac{A45}{\text{hCut}} \quad \frac{h_2 : \Delta_{11}, \Box\Gamma_{15} \vdash \Box F_7, \Delta_{14}, F_9, \Box\Gamma_{12}, \Box F_{13}}{- : \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, \Box\Gamma_{12}, \Box F_{13}, F_8 \wedge F_9} \text{ ax/W}}{- : \Delta_{11}, \Box\Gamma_{15} \vdash \Delta_{14}, \Box\Gamma_{12}, \Box F_{13}, F_8 \wedge F_9} \\
\frac{h_2 : \Box\Gamma_{11}, \Delta_{15} \vdash F_7, F_8, \Box\Gamma_{12}, \Delta_{14}, \Box F_{13} \quad h_2 : \Box\Gamma_{11}, \Delta_{15} \vdash F_7, F_9, \Box\Gamma_{12}, \Delta_{14}, \Box F_{13}}{\bullet h_2 : \Box\Gamma_{11}, \Delta_{15} \vdash ((\Box\Gamma_{12}, \Delta_{14}, \Box F_{13}), F_8 \wedge F_9), F_7} \wedge_R \quad \frac{h_{10} : \Box\Gamma_{11} \vdash \Box\Gamma_{12}, F_{13}}{\bullet h_{10} : (\Box\Gamma_{11}, \Delta_{15}), F_7 \vdash (\Box\Gamma_{12}, \Delta_{14}, \Box F_{13}), F_8 \wedge F_9} \wedge_R \\
\frac{\quad}{- : \Box\Gamma_{11}, \Delta_{15} \vdash (\Box\Gamma_{12}, \Delta_{14}, \Box F_{13}), F_8 \wedge F_9} \rightarrow \\
\frac{\quad}{- : \Box\Gamma_{11} \vdash F_{13}, \Box\Gamma_{12}} \text{ ax/W} \quad \frac{A45}{- : \Delta_{15}, \Box\Gamma_{11} \vdash \Delta_{14}, \Box\Gamma_{12}, \Box F_{13}, F_8 \wedge F_9} \text{ Cut}
\end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_R \\
\frac{\quad}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow \\
\frac{\quad}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ Cut} \\
\frac{\quad}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, \Delta_{11}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6} \text{ ax/W} \quad \frac{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_7}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ ax/W} \quad \frac{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ inv-th/ax} \quad \frac{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ inv-th/ax}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ sCut} \\
\frac{\quad}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ sCut} \\
\frac{h_2 : \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \rightarrow F_{13}} \wedge_R \quad \frac{h_{10} : \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \wedge F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \wedge F_9} \wedge_R \\
\frac{\quad}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \rightarrow \\
\frac{\quad}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_8} \text{ inv-th/ax} \quad \frac{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_9}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_8 \wedge F_9} \text{ inv-th/ax} \quad \frac{- : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8 \wedge F_9}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} \text{ ax/W} \\
\frac{\quad}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \wedge F_9} \text{ ax/W} \quad \frac{\quad}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{ sCut} \\
\frac{\quad}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \rightarrow \\
\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_{14} \vdash F_{12}, \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R \\
\frac{\quad}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \rightarrow \\
\frac{\quad}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_9}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_9} \text{ ax/W} \quad \frac{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{12}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9} \text{ inv-th/ax} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9}{- : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9} \text{ inv-th/ax}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9} \text{ hCut} \\
\frac{\quad}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{ ax/W}
\end{array}$$

• Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_R \\
\frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \rightarrow \\
\frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{ Cut} \\
\frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \wedge F_7} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7} \text{ inv-th/ax} \quad \frac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{ ax/W}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{ hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_7 \vdash F_8, \Delta_{10} \quad h_1 : \Delta_7 \vdash F_9, \Delta_{10}}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R \quad \frac{h_6 : F_8, F_9, \Delta_7 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{10}} \wedge_L}{\frac{- : \Delta_7 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{- : \Delta_7 \vdash \Delta_{10}, F_8 \quad \frac{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9 \quad \text{ax/W}}{- : \Delta_7, F_8 \vdash \Delta_{10}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}} \text{sCut} \\
\frac{\frac{h_2 : \Delta_{11} \vdash F_{12} \wedge F_{13}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash F_{12} \wedge F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \wedge F_{13}} \wedge_R \quad \frac{h_{10} : F_{12}, F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \wedge F_9} \wedge_L}{\frac{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_{12} \wedge F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8} \text{inv-th/ax}}{\frac{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\wedge_L} \text{hCut}} \quad \frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_9, F_{12} \wedge F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_9} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax}}{\frac{- : \Delta_{11} \vdash \Delta_7, F_9 \wedge F_8}{\wedge_L} \text{hCut}}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \wedge_R \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L}{\frac{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \frac{h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_{10}, F_7}{\bullet h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}{\wedge_L} \text{hCut}} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L}{\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}}{\rightarrow} \text{Cut}} \\
\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \quad \frac{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7 \quad \text{ax/W}}{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{sCut} \\
\frac{h_2 : \Delta_{11} \vdash F_{12} \vee F_{13}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash F_{12} \vee F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \vee F_{13}} \wedge_R \quad \frac{h_{10} : F_{12}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \wedge F_9} \wedge_L}{\frac{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\rightarrow} \text{Cut}} \\
\frac{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8}{\text{inv-th/ax}} \quad \frac{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_9}{\text{inv-th/ax}}}{\frac{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8 \wedge F_9}{\wedge_R} \text{ax/W}} \quad \frac{- : \Delta_{11}, F_{13} \vdash \Delta_7, F_{12}, F_8 \wedge F_9}{\text{sCut}} \quad \frac{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9}{\text{ax/W}}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \wedge_R \\
\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_{14} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L}{\frac{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}{\rightarrow} \text{Cut}} \\
\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9}{\text{ax/W}} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9}{\wedge_L} \text{hCut}} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9}{\text{inv-th/ax}}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, []F_9 \vdash F_6, \Delta_{10} \quad h_1 : \Delta_{11}, []F_9 \vdash F_7, \Delta_{10}}{\bullet h_1 : \Delta_{11}, []F_9 \vdash \Delta_{10}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, \Delta_{11}, []F_9, F_6 \wedge F_7 \vdash \Delta_{10}}{\bullet h_8 : (\Delta_{11}, []F_9), F_6 \wedge F_7 \vdash \Delta_{10}} AT \\
\frac{}{- : \Delta_{11}, []F_9 \vdash \Delta_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}, F_6 \wedge F_7}{- : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}} ax/W \quad \frac{h_8 : \Delta_{11}, F_9, []F_9, F_6 \wedge F_7 \vdash \Delta_{10}}{h_8 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}} ax/W}{- : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}} hCut \\
\frac{}{- : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}} ATG \\
\frac{}{- : \Delta_{11}, []F_9 \vdash \Delta_{10}} \\
\frac{h_2 : \Delta_{11} \vdash []F_{12}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash []F_{12}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), []F_{12}} \wedge_R \quad \frac{h_{10} : F_{12}, \Delta_{11}, []F_{12} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, []F_{12} \vdash \Delta_7, F_8 \wedge F_9} AT \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_2 : \Delta_{11} \vdash []F_{12}, F_8, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash \Delta_7, F_8, []F_{12}} ax/W \quad \frac{h_{10} : \Delta_{11}, F_{12}, []F_{12} \vdash \Delta_7, F_8}{\bullet h_{10} : \Delta_{11}, []F_{12} \vdash \Delta_7, F_8} inv-th/ax \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8} hCut \quad \frac{h_2 : \Delta_{11} \vdash \Delta_7, F_9, []F_{12}}{\bullet h_2 : \Delta_{11}, []F_{12} \vdash \Delta_7, F_9} ax/W \quad \frac{h_{10} : \Delta_{11}, F_{12}, []F_{12} \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, []F_{12} \vdash \Delta_7, F_9} inv-th/ax \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} hCut \quad \frac{}{- : \Delta_{11} \vdash \Delta_7, F_9} \wedge_R \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \\
\frac{h_2 : \Delta_{13}, []F_{12} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{13}, []F_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{13}, []F_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_{13}, []F_{12} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{13}, []F_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} AT \\
\frac{}{- : \Delta_{13}, []F_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\bullet h_2 : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_7, F_9 \wedge F_{10}}{- : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_9 \wedge F_{10}} ax/W \quad \frac{h_{11} : \Delta_{13}, F_{12}, F_7, []F_{12} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_{13}, F_{12}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}} ax/W \\
\frac{}{- : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_9 \wedge F_{10}} hCut \quad \frac{}{- : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_9 \wedge F_{10}} ATG \\
\frac{}{- : \Delta_{13}, []F_{12} \vdash \Delta_8, F_9 \wedge F_{10}}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \perp, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \perp_L \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \\
\frac{h_2 : \Delta_{11} \vdash \perp, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash \perp, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \perp} \wedge_R \quad \frac{h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \wedge F_9} \perp_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_8}{\bullet h_2 : \perp, \Delta_{11} \vdash \Delta_7, F_8} ax/W \quad \frac{h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_8}{\bullet h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_8} \perp_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8} hCut \quad \frac{h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_9}{\bullet h_2 : \perp, \Delta_{11} \vdash \Delta_7, F_9} ax/W \quad \frac{h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_9}{\bullet h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_9} \perp_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} hCut \quad \frac{}{- : \Delta_{11} \vdash \Delta_7, F_9} \wedge_R \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \\
\frac{h_2 : \perp, \Delta_{12} \vdash F_7, F_9, \Delta_8 \quad h_2 : \perp, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \perp_L \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \perp_L \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, p_9 \vdash F_6, \Delta_{10}, p_9 \quad h_1 : \Delta_{11}, p_9 \vdash F_7, \Delta_{10}, p_9}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : (\Delta_{11}, p_9), F_6 \wedge F_7 \vdash \Delta_{10}, p_9}{\bullet h_8 : (\Delta_{11}, p_9), F_6 \wedge F_7 \vdash \Delta_{10}, p_9} I \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \\
\frac{h_2 : \Delta_{10} \vdash p_{11}, F_7, \Delta_{12}, p_{11} \quad h_2 : \Delta_{10} \vdash p_{11}, F_8, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \wedge F_8), p_{11}} \wedge_R \quad \frac{h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8}{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8} I \\
\frac{}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}} ax/W \quad \frac{h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_7, p_{11}}{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_7, p_{11}} I \\
\frac{}{- : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}} hCut \quad \frac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}} ax/W \quad \frac{h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_8, p_{11}}{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_8, p_{11}} I \\
\frac{}{- : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}} hCut \quad \frac{}{- : \Delta_{10} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \wedge F_8}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_{13}, p_{11} \vdash F_7, F_8, \Delta_{12}, p_{11} \quad h_2 : \Delta_{13}, p_{11} \vdash F_7, F_9, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \wedge F_9), F_7} \wedge_R \quad \frac{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9} I \\
\text{Cut} \\
\rightarrow \\
- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \wedge F_9 \quad I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \top, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \top_L \\
\text{Cut} \\
- : \top, \Delta_{10} \vdash \Delta_9 \\
\rightarrow \\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{h_{\text{Cut}}} \text{ax/W} \\
\rightarrow \\
\frac{h_2 : \Delta_{11} \vdash \top, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash \top, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \top} \wedge_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \wedge F_9} \top_L \\
\text{Cut} \\
- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \\
\rightarrow \\
- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \quad \text{ax/W} \\
\rightarrow \\
\frac{h_2 : \top, \Delta_{12} \vdash F_7, F_9, \Delta_8 \quad h_2 : \top, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \top_L \\
\text{Cut} \\
- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10} \\
\rightarrow \\
\frac{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \wedge F_{10}}{- : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{h_{\text{Cut}}} \text{ax/W} \\
\rightarrow \\
- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}
\end{array}$$

6.3 Status of \vee_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \rightarrow F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : F_{11}, \Delta_6, F_7 \vee F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R \\
\text{Cut} \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7, F_8}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}}{h_{\text{Cut}}} \text{ax/W} \\
\vee_R \\
- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12} \quad \rightarrow_R \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12} \\
\rightarrow \\
\frac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \rightarrow F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}} \rightarrow_R \\
\text{Cut} \\
- : \Delta_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10} \\
\rightarrow \\
\frac{h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_7, F_9}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9 \vee F_{10}} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}}{h_{\text{Cut}}} \text{ax/W} \\
\vee_R \\
- : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10} \quad \rightarrow_R \\
- : \Delta_8 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \vee F_{10}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R \\
\text{Cut} \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12} \\
\rightarrow \\
\frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}}{\bullet h_9 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \vee F_8} \text{ax/W} \\
\vee_R \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \quad \rightarrow_R \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12} \\
\rightarrow \\
\frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}}{\bullet h_9 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \vee F_8} \text{ax/W} \\
\vee_R \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \quad \rightarrow_R \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}), F_7} \vee_R \quad \frac{\frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \vee F_{10} \quad h_{11} : F_7, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}} \wedge_R}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_7, F_9, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_7, F_9, F_{12} \wedge F_{13}} \text{ax/W} \quad \frac{\frac{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_{12}, F_9 \quad h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_{13}, F_9}{\bullet h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}} \text{inv-th/ax}}{- : \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}} \wedge_R}{- : \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}} \text{hCut} \\
\rightarrow \\
\frac{- : \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \wedge F_{13}, F_9 \vee F_{10}} \vee_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}} \vee_R \\
\rightarrow \\
\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R \\
\frac{\frac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}} \vee_R}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_7, F_9}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \vee F_{10}} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}} \vee_R \\
\rightarrow \\
\frac{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \vee F_{13}, F_9 \vee F_{10}} \vee_R \\
\frac{\frac{h_2 : \Delta_8 \vdash F_7, F_{11}, F_{12}, \Delta_{10}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7} \vee_R \quad \frac{h_9 : F_7, \Delta_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{10}, F_{11}, F_{12}, F_7}{\bullet h_2 : \Delta_8 \vdash \Delta_{10}, F_{11}, F_{12}, F_7} \text{ax/W} \quad \frac{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{- : \Delta_8 \vdash \Delta_{10}, F_{11}, F_{12}} \vee_R \\
\rightarrow \\
\frac{- : \Delta_8 \vdash \Delta_{10}, F_{11}, F_{12}}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash F_7, F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \perp_R}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \vee F_8}{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \vee F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{hCut} \\
\frac{\frac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \perp, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\perp, \Delta_{12}), F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\perp, \Delta_{12}), F_9 \vee F_{10}} \perp_R}{- : \Delta_8 \vdash (\perp, \Delta_{12}), F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \vee F_{10}}{\bullet h_2 : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \vee F_{10}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}} \text{hCut}
\end{array}$$

- Case rule \top_R

- Case rule K

- Case rule *A45*

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- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \rightarrow_L}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{- : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \vee_R}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{hCut} \\
\rightarrow \\
\frac{h_2 : \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \rightarrow F_{13}} \vee_R \quad \frac{h_{10} : \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \vee F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \vee F_9} \rightarrow_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \rightarrow F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{ax/W} \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_{12}, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{hCut} \\
\rightarrow \\
\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, \Delta_{14} \vdash F_{12}, \Delta_8, F_9 \vee F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \rightarrow_L}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_9} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{10}, F_{12}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \text{hCut} \\
\rightarrow \\
\frac{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \vee_R
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{hCut} \\
\rightarrow \\
\frac{h_2 : \Delta_{11} \vdash F_{12} \wedge F_{13}, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \wedge F_{13}} \vee_R \quad \frac{h_{10} : F_{12}, F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \vee F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \wedge F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{hCut} \\
\rightarrow \\
\frac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \wedge_L}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9} \text{hCut} \\
\rightarrow \\
\frac{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \vee_R
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \vee_L}{\neg : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{\text{inv-th/ax}}{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}} \quad \frac{\text{ax/W}}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{\text{inv-th/ax}}{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}} \vee_L}{\neg : \Delta_{12}, F_9 \vdash \Delta_{11}} \text{hCut} \\
\rightarrow \\
\frac{\neg : \Delta_{12}, F_9 \vdash \Delta_{11}}{\neg : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L \\
\\
\frac{\frac{h_1 : \Delta_7 \vdash F_8, F_9, \Delta_{10}}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \vee F_9} \vee_R \quad \frac{h_6 : F_8, \Delta_7 \vdash \Delta_{10} \quad h_6 : F_9, \Delta_7 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{10}} \vee_L}{\neg : \Delta_7 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\neg : \Delta_7 \vdash \Delta_{10}, F_8, F_9}{\neg : \Delta_7 \vdash \Delta_{10}, F_8} \text{ax/W} \quad \frac{\neg : \Delta_7, F_9 \vdash \Delta_{10}, F_8}{\neg : \Delta_7 \vdash \Delta_{10}, F_8} \text{sCut} \quad \frac{\neg : \Delta_7, F_8 \vdash \Delta_{10}}{\neg : \Delta_7 \vdash \Delta_{10}} \text{sCut} \\
\rightarrow \\
\frac{\neg : \Delta_7 \vdash \Delta_{10}}{\neg : \Delta_7 \vdash \Delta_{10}} \\
\\
\frac{\frac{h_2 : \Delta_{11} \vdash F_{12} \vee F_{13}, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \vee F_{13}} \vee_R \quad \frac{h_{10} : F_{12}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \vee F_9} \vee_L}{\neg : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \vee F_{13}} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax}}{\neg : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{hCut} \\
\rightarrow \\
\frac{\neg : \Delta_{11} \vdash \Delta_7, F_8, F_9}{\neg : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R \\
\\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \vee_L}{\neg : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_{10}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{\neg : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9} \text{hCut} \\
\rightarrow \\
\frac{\neg : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9}{\neg : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \vee_R
\end{array}$$

• Case rule AT

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, []F_9 \vdash F_6, F_7, \Delta_{10}}{\bullet h_1 : \Delta_{11}, []F_9 \vdash \Delta_{10}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, \Delta_{11}, []F_9, F_6 \vee F_7 \vdash \Delta_{10}}{\bullet h_8 : (\Delta_{11}, []F_9), F_6 \vee F_7 \vdash \Delta_{10}} AT}{\neg : \Delta_{11}, []F_9 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}, F_6 \vee F_7}{\bullet h_1 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}, F_6 \vee F_7} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_9, []F_9, F_6 \vee F_7 \vdash \Delta_{10}}{\bullet h_8 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}, F_6 \vee F_7} \text{hCut}}{\neg : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}} \text{ATG} \\
\rightarrow \\
\frac{\neg : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}}{\neg : \Delta_{11}, []F_9 \vdash \Delta_{10}} \\
\\
\frac{\frac{h_2 : \Delta_{11} \vdash []F_{12}, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), []F_{12}} \vee_R \quad \frac{h_{10} : F_{12}, \Delta_{11}, []F_{12} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, []F_{12} \vdash \Delta_7, F_8 \vee F_9} AT}{\neg : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, []F_{12}}{\bullet h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, []F_{12}} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12}, []F_{12} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, []F_{12} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax}}{\neg : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{hCut} \\
\rightarrow \\
\frac{\neg : \Delta_{11} \vdash \Delta_7, F_8, F_9}{\neg : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R \\
\\
\frac{\frac{h_2 : \Delta_{13}, []F_{12} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{13}, []F_{12} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_{13}, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{13}, []F_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} AT}{\neg : \Delta_{13}, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_7, F_9 \vee F_{10}}{\bullet h_2 : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_7, F_9 \vee F_{10}} \text{ax/W} \quad \frac{h_{11} : \Delta_{13}, F_{12}, F_7, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_{13}, F_{12}, F_7, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}} \text{hCut}}{\neg : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}} \text{AT} \\
\rightarrow \\
\frac{\neg : \Delta_{13}, F_{12}, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}}{\neg : \Delta_{13}, []F_{12} \vdash \Delta_8, F_9 \vee F_{10}}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \perp, \Delta_{10} \vdash F_6, F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \perp_L}{\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut}} \\
\frac{}{\rightarrow} \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L \\
\\
\frac{\frac{h_2 : \Delta_{11} \vdash \perp, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), \perp} \vee_R \quad \frac{}{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \vee F_9} \perp_L}{\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut}} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_8, F_9} \text{ax/W} \quad \frac{}{\bullet h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_8, F_9} \perp_L}{\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{hCut}} \\
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R \\
\\
\frac{\frac{h_2 : \perp, \Delta_{12} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{}{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \perp_L}{\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut}} \\
\frac{}{\rightarrow} \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, p_9 \vdash F_6, F_7, \Delta_{10}, p_9}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \vee F_7} \vee_R \quad \frac{}{\bullet h_8 : (\Delta_{11}, p_9), F_6 \vee F_7 \vdash \Delta_{10}, p_9} I}{\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{Cut}} \\
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I \\
\\
\frac{\frac{h_2 : \Delta_{10} \vdash p_{11}, F_7, F_8, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \vee F_8), p_{11}} \vee_R \quad \frac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \vee F_8} I}{\frac{}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \vee F_8} \text{Cut}} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, F_8, p_{11}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, F_8, p_{11}, p_{11}} \text{ax/W} \quad \frac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_7, F_8, p_{11}} I}{\frac{}{- : \Delta_{10} \vdash \Delta_{12}, F_7, F_8, p_{11}} \text{hCut}} \\
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \vee F_8} \vee_R \\
\\
\frac{\frac{h_2 : \Delta_{13}, p_{11} \vdash F_7, F_8, F_9, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \vee F_9), F_7} \vee_R \quad \frac{}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \vee F_9} I}{\frac{}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \vee F_9} \text{Cut}} \\
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \vee F_9} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : \top, \Delta_{10} \vdash F_6, F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \top_L}{\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut}} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9} \text{ax/W}}{\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}} \\
\\
\frac{\frac{h_2 : \Delta_{11} \vdash \top, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), \top} \vee_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \vee F_9} \top_L}{\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut}} \\
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \top, \Delta_{12} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \top_L \\
\hline
- : \top, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \vee F_{10} \quad \text{ax/W} \quad \frac{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : \top, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}} \text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}} \text{hCut}
\end{array}$$

6.4 Status of \perp_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \rightarrow F_8), \perp} \perp_R \quad \frac{h_5 : \perp, F_7, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \rightarrow F_8} \rightarrow_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \quad \text{ax/W}}{- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \rightarrow F_9}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_{10}, F_8 \rightarrow F_9), F_5} \perp_R \quad \frac{h_7 : F_5, F_8, \Delta_6 \vdash \perp, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_6 \vdash \perp, \Delta_{10}, F_5, F_8 \rightarrow F_9 \quad \text{ax/W} \quad \frac{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \wedge F_8), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash F_7, \Delta_6 \quad h_5 : \perp, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \wedge F_8} \wedge_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \quad \text{ax/W}}{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \wedge F_9}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_{10}, F_8 \wedge F_9), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \perp, F_8, \Delta_{10} \quad h_7 : F_5, \Delta_6 \vdash \perp, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_6 \vdash \perp, \Delta_{10}, F_5, F_8 \wedge F_9 \quad \text{ax/W} \quad \frac{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{hCut}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \vee F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \vee F_8), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash F_7, F_8, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \vee F_8} \vee_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8 \quad \text{ax/W}}{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \vee F_9}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_{10}, F_8 \vee F_9), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \perp, F_8, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \vee_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_6 \vdash \perp, \Delta_{10}, F_5, F_8 \vee F_9 \quad \text{ax/W} \quad \frac{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \perp, \Delta_6}{\bullet h_1 : \Delta_4 \vdash (\perp, \Delta_6), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \perp, \Delta_6} \perp_R \\
\hline
- : \Delta_4 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_4 \vdash \perp, \Delta_6 \quad \text{ax/W}}{- : \Delta_4 \vdash \perp, \Delta_6} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_6 \vdash F_5, \Delta_8}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_8), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \Delta_8}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_8} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_6 \vdash \perp, \Delta_8, F_5}{- : \Delta_6 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_8}{- : \Delta_6 \vdash \perp, \Delta_8} \text{ax/W} \\
\hline
\text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \top, \Delta_6}{\bullet h_1 : \Delta_4 \vdash (\top, \Delta_6), \perp} \perp_R \quad \frac{}{\bullet h_5 : \Delta_4, \perp \vdash \top, \Delta_6} \top_R \\
\hline
- : \Delta_4 \vdash \top, \Delta_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_4 \vdash \top, \Delta_6} \top_R \\
\hline
\frac{h_2 : \Delta_6 \vdash F_5, \top, \Delta_8}{\bullet h_2 : \Delta_6 \vdash (\perp, \top, \Delta_8), F_5} \perp_R \quad \frac{}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \top, \Delta_8} \top_R \\
\hline
- : \Delta_6 \vdash \perp, \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_6 \vdash \perp, \top, \Delta_8} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_5, \Delta_8 \vdash \Delta_6, [\Box F_7]}{\bullet h_1 : \Box \Gamma_5, \Delta_8 \vdash (\Delta_6, [\Box F_7]), \perp} \perp_R \quad \frac{h_4 : \text{unbox}(\Box \Gamma_5) \vdash F_7}{\bullet h_4 : (\Box \Gamma_5, \Delta_8), \perp \vdash \Delta_6, [\Box F_7]} K \\
\hline
- : \Box \Gamma_5, \Delta_8 \vdash \Delta_6, [\Box F_7] \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_8, \Box \Gamma_5 \vdash \Delta_6, [\Box F_7]} \text{ax/W} \\
\hline
\frac{h_2 : \Box \Gamma_{10}, \Delta_7 \vdash \Box F_5, \Delta_9, [\Box F_8]}{\bullet h_2 : \Box \Gamma_{10}, \Delta_7 \vdash (\perp, \Delta_9, [\Box F_8]), \Box F_5} \perp_R \quad \frac{h_6 : \text{unbox}(\Box \Gamma_{10}), \text{unbox}(\Box F_5) \vdash F_8}{\bullet h_6 : (\Box \Gamma_{10}, \Delta_7), \Box F_5 \vdash \perp, \Delta_9, [\Box F_8]} K \\
\hline
- : \Box \Gamma_{10}, \Delta_7 \vdash \perp, \Delta_9, [\Box F_8] \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_7, \Box \Gamma_{10} \vdash \perp, \Box F_5, \Delta_9, [\Box F_8]}{- : \Delta_7, \Box \Gamma_{10} \vdash \perp, \Delta_9, [\Box F_8]} \text{ax/W} \quad \frac{\bullet h_6 : \Box F_5, \Delta_7, \Box \Gamma_{10} \vdash \perp, \Delta_9, [\Box F_8]}{- : \Delta_7, \Box \Gamma_{10} \vdash \perp, \Delta_9, [\Box F_8]} \text{ax/W} \\
\hline
\text{hCut} \\
\hline
\frac{h_2 : \Box \Gamma_7, \Delta_{10} \vdash F_5, \Delta_9, [\Box F_8]}{\bullet h_2 : \Box \Gamma_7, \Delta_{10} \vdash (\perp, \Delta_9, [\Box F_8]), F_5} \perp_R \quad \frac{h_6 : \text{unbox}(\Box \Gamma_7) \vdash F_8}{\bullet h_6 : (\Box \Gamma_7, \Delta_{10}), F_5 \vdash \perp, \Delta_9, [\Box F_8]} K \\
\hline
- : \Box \Gamma_7, \Delta_{10} \vdash \perp, \Delta_9, [\Box F_8] \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_7) \vdash F_8} \text{ax/W} \\
\hline
\frac{}{- : \Delta_{10}, \Box \Gamma_7 \vdash \perp, \Delta_9, [\Box F_8]} K
\end{array}$$

- Case rule $A45$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_5, \Delta_9 \vdash \Box \Gamma_6, \Delta_7, [\Box F_8]}{\bullet h_1 : \Box \Gamma_5, \Delta_9 \vdash (\Box \Gamma_6, \Delta_7, [\Box F_8]), \perp} \perp_R \quad \frac{h_4 : \Box \Gamma_5 \vdash \Box \Gamma_6, F_8}{\bullet h_4 : (\Box \Gamma_5, \Delta_9), \perp \vdash \Box \Gamma_6, \Delta_7, [\Box F_8]} A45 \\
\hline
- : \Box \Gamma_5, \Delta_9 \vdash \Box \Gamma_6, \Delta_7, [\Box F_8] \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_9, \Box \Gamma_5 \vdash \Delta_7, \Box \Gamma_6, [\Box F_8]} \text{ax/W} \\
\hline
\frac{h_2 : \Box \Gamma_{11}, \Delta_7 \vdash \Box F_5, \Box \Gamma_8, \Delta_{10}, [\Box F_9]}{\bullet h_2 : \Box \Gamma_{11}, \Delta_7 \vdash (\perp, \Box \Gamma_8, \Delta_{10}, [\Box F_9]), \Box F_5} \perp_R \quad \frac{h_6 : \Box \Gamma_{11}, \Box F_5 \vdash \Box \Gamma_8, F_9}{\bullet h_6 : (\Box \Gamma_{11}, \Delta_7), \Box F_5 \vdash \perp, \Box \Gamma_8, \Delta_{10}, [\Box F_9]} A45 \\
\hline
- : \Box \Gamma_{11}, \Delta_7 \vdash \perp, \Box \Gamma_8, \Delta_{10}, [\Box F_9] \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 : \Delta_7, \Box \Gamma_{11} \vdash \perp, \Box F_5, \Delta_{10}, \Box \Gamma_8, [\Box F_9]}{- : \Delta_7, \Box \Gamma_{11} \vdash \perp, \Delta_{10}, \Box \Gamma_8, [\Box F_9]} \text{ax/W} \quad \frac{\bullet h_6 : \Box F_5, \Delta_7, \Box \Gamma_{11} \vdash \perp, \Delta_{10}, \Box \Gamma_8, [\Box F_9]}{- : \Delta_7, \Box \Gamma_{11} \vdash \perp, \Delta_{10}, \Box \Gamma_8, [\Box F_9]} \text{ax/W} \\
\hline
\text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_7, \Delta_{11} \vdash F_5, \Box\Gamma_8, \Delta_{10}, \Box F_9}{\bullet h_2 : \Box\Gamma_7, \Delta_{11} \vdash (\perp, \Box\Gamma_8, \Delta_{10}, \Box F_9), F_5} \perp_R \quad \frac{h_6 : \Box\Gamma_7 \vdash \Box\Gamma_8, F_9}{\bullet h_6 : (\Box\Gamma_7, \Delta_{11}), F_5 \vdash \perp, \Box\Gamma_8, \Delta_{10}, \Box F_9} A45 \\
\hline
- : \Box\Gamma_7, \Delta_{11} \vdash \perp, \Box\Gamma_8, \Delta_{10}, \Box F_9 \\
\rightarrow \\
\frac{\perp : \Box\Gamma_7 \vdash F_9, \Box\Gamma_8}{- : \Delta_{11}, \Box\Gamma_7 \vdash \perp, \Delta_{10}, \Box\Gamma_8, \Box F_9} \text{ax/W} \quad A45
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \perp, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \perp \vdash \Delta_7} \rightarrow_L \\
\hline
- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7 \\
\rightarrow \\
- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7 \quad \text{ax/W} \\
\hline
\frac{h_2 : \Delta_7 \vdash F_8 \rightarrow F_9, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), F_8 \rightarrow F_9} \perp_R \quad \frac{h_6 : \Delta_7 \vdash \perp, F_8, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \perp, \Delta_5} \rightarrow_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_5 \\
\rightarrow \\
\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \rightarrow F_9}{\bullet h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \rightarrow F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \perp, \Delta_5}{- : \Delta_7 \vdash \perp, \Delta_5} \text{hCut} \\
\hline
\frac{h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_{10} \vdash \perp, F_8, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \rightarrow F_9), F_5 \vdash \perp, \Delta_6} \rightarrow_L \\
\hline
- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6 \\
\rightarrow \\
\frac{h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6, F_5}{\bullet h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6, F_5} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{10}, F_5, F_8 \rightarrow F_9 \vdash \perp, \Delta_6}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6} \text{hCut}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \perp \vdash \Delta_7} \wedge_L \\
\hline
- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7 \\
\rightarrow \\
- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7 \quad \text{ax/W} \\
\hline
\frac{h_2 : \Delta_7 \vdash F_8 \wedge F_9, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), F_8 \wedge F_9} \perp_R \quad \frac{h_6 : F_8, F_9, \Delta_7 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \perp, \Delta_5} \wedge_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_5 \\
\rightarrow \\
\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \wedge F_9}{\bullet h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \wedge F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \perp, \Delta_5}{- : \Delta_7 \vdash \perp, \Delta_5} \text{hCut} \\
\hline
\frac{h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : F_5, F_8, F_9, \Delta_{10} \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \wedge F_9), F_5 \vdash \perp, \Delta_6} \wedge_L \\
\hline
- : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6 \\
\rightarrow \\
\frac{h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6, F_5}{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6, F_5} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{10}, F_5, F_8 \wedge F_9 \vdash \perp, \Delta_6}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6} \text{hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \perp, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \perp \vdash \Delta_7} \vee_L \\
\hline
- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7 \\
\rightarrow \\
- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7 \quad \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_7 \vdash F_8 \vee F_9, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), F_8 \vee F_9} \perp_R \quad \frac{h_6 : F_8, \Delta_7 \vdash \perp, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \perp, \Delta_5} \vee_L}{- : \Delta_7 \vdash \perp, \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \vee F_9}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \perp, \Delta_5}{h\text{Cut}} \text{ax/W}}{- : \Delta_7 \vdash \perp, \Delta_5} \\
\\
\frac{\frac{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : F_5, F_8, \Delta_{10} \vdash \perp, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \perp, \Delta_6} \vee_L}{- : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6, F_5}{- : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{10}, F_5, F_8 \vee F_9 \vdash \perp, \Delta_6}{h\text{Cut}} \text{ax/W}}{- : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6}
\end{array}$$

• Case rule AT

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_7, []F_5 \vdash \Delta_6}{\bullet h_1 : \Delta_7, []F_5 \vdash \Delta_6, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, \Delta_7, []F_5 \vdash \Delta_6}{\bullet h_4 : (\Delta_7, []F_5), \perp \vdash \Delta_6} AT}{- : \Delta_7, []F_5 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7, []F_5 \vdash \Delta_6}{- : \Delta_7, []F_5 \vdash \Delta_6} \text{ax/W} \\
\\
\frac{\frac{h_2 : \Delta_7 \vdash []F_8, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), []F_8} \perp_R \quad \frac{h_6 : F_8, \Delta_7, []F_8 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, []F_8 \vdash \perp, \Delta_5} AT}{- : \Delta_7 \vdash \perp, \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5, []F_8}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, []F_8 \vdash \perp, \Delta_5}{h\text{Cut}} \text{ax/W}}{- : \Delta_7 \vdash \perp, \Delta_5} \\
\\
\frac{\frac{h_2 : \Delta_9, []F_8 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_9, []F_8 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : F_5, F_8, \Delta_9, []F_8 \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_9, []F_8), F_5 \vdash \perp, \Delta_6} AT}{- : \Delta_9, []F_8 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_9, []F_8 \vdash \perp, \Delta_6, F_5}{- : \Delta_9, []F_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_9, F_5, []F_8 \vdash \perp, \Delta_6}{h\text{Cut}} \text{ax/W}}{- : \Delta_9, []F_8 \vdash \perp, \Delta_6}
\end{array}$$

• Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5 \vdash \Delta_6}{\bullet h_1 : \Delta_5 \vdash \Delta_6, \perp} \perp_R \quad \frac{\bullet h_4 : \Delta_5, \perp \vdash \Delta_6}{\bullet h_4 : \Delta_5, \perp \vdash \Delta_6} \perp_L}{- : \Delta_5 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_5 \vdash \Delta_6}{- : \Delta_5 \vdash \Delta_6} \text{ax/W} \\
\\
\frac{\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), \perp} \perp_R \quad \frac{\bullet h_6 : \Delta_7, \perp \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, \perp \vdash \perp, \Delta_5} \perp_L}{- : \Delta_7 \vdash \perp, \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7 \vdash \perp, \Delta_5}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \\
\\
\frac{\frac{h_2 : \perp, \Delta_8 \vdash F_5, \Delta_6}{\bullet h_2 : \perp, \Delta_8 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{\bullet h_7 : (\perp, \Delta_8), F_5 \vdash \perp, \Delta_6}{\bullet h_7 : (\perp, \Delta_8), F_5 \vdash \perp, \Delta_6} \perp_L}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{- : \perp, \Delta_8 \vdash \perp, \Delta_6}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \perp_L
\end{array}$$

• Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_7, p_5 \vdash \Delta_6, p_5}{\bullet h_1 : \Delta_7, p_5 \vdash (\Delta_6, p_5), \perp} \perp_R \quad \frac{\bullet h_4 : (\Delta_7, p_5), \perp \vdash \Delta_6, p_5}{\bullet h_4 : (\Delta_7, p_5), \perp \vdash \Delta_6, p_5} I}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7, p_5 \vdash \Delta_6, p_5}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} I
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_6 \vdash p_7, \Delta_8, p_7}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_8, p_7), p_7} \perp_R \quad \frac{}{\bullet h_5 : \Delta_6, p_7 \vdash \perp, \Delta_8, p_7} I \\
\hline
- : \Delta_6 \vdash \perp, \Delta_8, p_7 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_2 : \Delta_6 \vdash \perp, \Delta_8, p_7, p_7}{- : \Delta_6 \vdash \perp, \Delta_8, p_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, p_7 \vdash \perp, \Delta_8, p_7}{- : \Delta_6 \vdash \perp, \Delta_8, p_7} I \\
\hline
- : \Delta_6 \vdash \perp, \Delta_8, p_7 \quad \text{hCut} \\
\hline
\frac{h_2 : \Delta_9, p_7 \vdash F_5, \Delta_8, p_7}{\bullet h_2 : \Delta_9, p_7 \vdash (\perp, \Delta_8, p_7), F_5} \perp_R \quad \frac{}{\bullet h_6 : (\Delta_9, p_7), F_5 \vdash \perp, \Delta_8, p_7} I \\
\hline
- : \Delta_9, p_7 \vdash \perp, \Delta_8, p_7 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
- : \Delta_9, p_7 \vdash \perp, \Delta_8, p_7 \quad I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet h_1 : \top, \Delta_6 \vdash \Delta_5, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_6 \vdash \Delta_5}{\bullet h_4 : (\top, \Delta_6), \perp \vdash \Delta_5} \top_L \\
\hline
- : \top, \Delta_6 \vdash \Delta_5 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
- : \top, \Delta_6 \vdash \Delta_5 \quad \text{ax/W} \\
\hline
\frac{h_2 : \Delta_7 \vdash \top, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), \top} \perp_R \quad \frac{h_6 : \Delta_7 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, \top \vdash \perp, \Delta_5} \top_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
- : \Delta_7 \vdash \perp, \Delta_5 \quad \text{ax/W} \\
\hline
\frac{h_2 : \top, \Delta_8 \vdash F_5, \Delta_6}{\bullet h_2 : \top, \Delta_8 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_8 \vdash \perp, \Delta_6}{\bullet h_7 : (\top, \Delta_8), F_5 \vdash \perp, \Delta_6} \top_L \\
\hline
- : \top, \Delta_8 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_2 : \top, \Delta_8 \vdash \perp, \Delta_6, F_5}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_8, F_5 \vdash \perp, \Delta_6}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
- : \top, \Delta_8 \vdash \perp, \Delta_6 \quad \text{hCut}
\end{array}$$

6.5 Status of \top_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \rightarrow F_8), \top} \top_R \quad \frac{h_5 : \top, F_7, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \rightarrow F_8} \rightarrow_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_4, F_7 \vdash \top, \Delta_6, F_8}{- : \Delta_4, F_7 \vdash \Delta_6, F_8} \top_R \quad \frac{h_5 : \top, \Delta_4, F_7 \vdash \Delta_6, F_8}{- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8} \text{ax/W} \\
\hline
- : \Delta_4, F_7 \vdash \Delta_6, F_8 \quad \text{hCut} \\
\hline
\frac{}{\rightarrow} \\
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \quad \rightarrow_R \\
\hline
\frac{h_2 : \Delta_6 \vdash (\top, \Delta_{10}, F_8 \rightarrow F_9), F_5}{\bullet h_2 : \Delta_6 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9} \top_R \quad \frac{h_7 : F_5, F_8, \Delta_6 \vdash \top, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9 \quad \top_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \wedge F_8), \top} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash F_7, \Delta_6 \quad h_5 : \top, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \wedge F_8} \wedge_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_4 \vdash \top, \Delta_6, F_7}{- : \Delta_4 \vdash \Delta_6, F_7} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_7}{- : \Delta_4 \vdash \Delta_6, F_7} \text{ax/W} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \quad \text{hCut} \\
\hline
\frac{\bullet h_1 : \Delta_4 \vdash \top, \Delta_6, F_8}{- : \Delta_4 \vdash \Delta_6, F_8} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_8}{- : \Delta_4 \vdash \Delta_6, F_8} \text{ax/W} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_8 \quad \text{hCut} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \quad \wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_{10}, F_8 \wedge F_9), F_5} \top_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \top, F_8, \Delta_{10} \quad h_7 : F_5, \Delta_6 \vdash \top, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \Delta_{10}, F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \wedge F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \wedge F_9} \top_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \vee F_8), \top} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash F_7, F_8, \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \vee F_8} \vee_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : \Delta_4 \vdash \top, \Delta_6, F_7, F_8} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_7, F_8}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \vee F_8} \text{ax/W} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7, F_8 \quad \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \vee_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_{10}, F_8 \vee F_9), F_5} \top_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \top, F_8, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \Delta_{10}, F_8 \vee F_9} \vee_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \vee F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \vee F_9} \top_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\perp, \Delta_6), \top} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \perp, \Delta_6} \perp_R \\
\hline
- : \Delta_4 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : \Delta_4 \vdash \perp, \top, \Delta_6} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \perp, \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
- : \Delta_4 \vdash \perp, \Delta_6 \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_2 : \Delta_6 \vdash (\top, \perp, \Delta_8), F_5} \top_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \top, \Delta_8}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \perp, \Delta_8} \perp_R \\
\hline
- : \Delta_6 \vdash \top, \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6 \vdash \perp, \top, \Delta_8} \top_R
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\top, \Delta_6), \top} \top_R \quad \frac{}{\bullet h_5 : \Delta_4, \top \vdash \top, \Delta_6} \top_R \\
\hline
- : \Delta_4 \vdash \top, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_4 \vdash \top, \Delta_6} \top_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_8), F_5} \top_R \quad \frac{}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \Delta_8} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Box \Gamma_5, \Delta_8 \vdash (\Delta_6, [\top] F_7), \top} \top_R \quad \frac{h_4 : \text{unbox}(\Box \Gamma_5) \vdash F_7}{\bullet h_4 : (\Box \Gamma_5, \Delta_8), \top \vdash \Delta_6, [\top] F_7} K \\
\hline
- : \Box \Gamma_5, \Delta_8 \vdash \Delta_6, [\top] F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_5) \vdash F_7} \text{ax/W} \\
\rightarrow \\
\frac{}{- : \Delta_8, \Box \Gamma_5 \vdash \Delta_6, [\top] F_7} K
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_2 : \Box \Gamma_{10}, \Delta_7 \vdash (\top, \Delta_9, [\Box F_8]), \Box F_5} \top_R \quad \frac{h_6 : \text{unbox}(\Box \Gamma_{10}), \text{unbox}(\Box F_5) \vdash F_8}{\bullet h_6 : (\Box \Gamma_{10}, \Delta_7), \Box F_5 \vdash \top, \Delta_9, [\Box F_8]} K}{\frac{}{- : \Box \Gamma_{10}, \Delta_7 \vdash \top, \Delta_9, [\Box F_8]} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_7, \Box \Gamma_{10} \vdash \top, \Delta_9, [\Box F_8]} \top_R \\
\frac{}{\bullet h_2 : \Box \Gamma_7, \Delta_{10} \vdash (\top, \Delta_9, [\Box F_8]), F_5} \top_R \quad \frac{h_6 : \text{unbox}(\Box \Gamma_7) \vdash F_8}{\bullet h_6 : (\Box \Gamma_7, \Delta_{10}), F_5 \vdash \top, \Delta_9, [\Box F_8]} K}{\frac{}{- : \Box \Gamma_7, \Delta_{10} \vdash \top, \Delta_9, [\Box F_8]} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_{10}, \Box \Gamma_7 \vdash \top, \Delta_9, [\Box F_8]} \top_R
\end{array}$$

- Case rule $A45$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Box \Gamma_5, \Delta_9 \vdash (\Box \Gamma_6, \Delta_7, [\Box F_8]), \top} \top_R \quad \frac{h_4 : \Box \Gamma_5 \vdash \Box \Gamma_6, F_8}{\bullet h_4 : (\Box \Gamma_5, \Delta_9), \top \vdash \Box \Gamma_6, \Delta_7, [\Box F_8]} A45}{\frac{}{- : \Box \Gamma_5, \Delta_9 \vdash \Box \Gamma_6, \Delta_7, [\Box F_8]} \text{Cut}} \rightarrow \\
\frac{}{- : \Box \Gamma_5 \vdash F_8, \Box \Gamma_6} \text{ax/w} \\
\frac{}{- : \Delta_9, \Box \Gamma_5 \vdash \Delta_7, \Box \Gamma_6, [\Box F_8]} A45 \\
\frac{}{\bullet h_2 : \Box \Gamma_{11}, \Delta_7 \vdash (\top, \Box \Gamma_8, \Delta_{10}, [\Box F_9]), \Box F_5} \top_R \quad \frac{h_6 : \Box \Gamma_{11}, \Box F_5 \vdash \Box \Gamma_8, F_9}{\bullet h_6 : (\Box \Gamma_{11}, \Delta_7), \Box F_5 \vdash \top, \Box \Gamma_8, \Delta_{10}, [\Box F_9]} A45}{\frac{}{- : \Box \Gamma_{11}, \Delta_7 \vdash \top, \Box \Gamma_8, \Delta_{10}, [\Box F_9]} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_7, \Box \Gamma_{11} \vdash \top, \Delta_{10}, \Box \Gamma_8, [\Box F_9]} \top_R \\
\frac{}{\bullet h_2 : \Box \Gamma_7, \Delta_{11} \vdash (\top, \Box \Gamma_8, \Delta_{10}, [\Box F_9]), F_5} \top_R \quad \frac{h_6 : \Box \Gamma_7 \vdash \Box \Gamma_8, F_9}{\bullet h_6 : (\Box \Gamma_7, \Delta_{11}), F_5 \vdash \top, \Box \Gamma_8, \Delta_{10}, [\Box F_9]} A45}{\frac{}{- : \Box \Gamma_7, \Delta_{11} \vdash \top, \Box \Gamma_8, \Delta_{10}, [\Box F_9]} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_{11}, \Box \Gamma_7 \vdash \top, \Delta_{10}, \Box \Gamma_8, [\Box F_9]} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \top \vdash \Delta_7} \rightarrow_L}{\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5}{\bullet h_1 : \Delta_8 \vdash \Delta_7, F_5} \text{ax/w} \quad \frac{}{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : \Delta_8, F_6 \vdash \Delta_7} \text{ax/w}}{\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{hCut}} \rightarrow_L \\
\frac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \rightarrow F_9} \top_R \quad \frac{h_6 : \Delta_7 \vdash \top, F_8, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \top, \Delta_5} \rightarrow_L}{\frac{}{- : \Delta_7 \vdash \top, \Delta_5} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_7 \vdash \top, \Delta_5} \top_R \\
\frac{}{\bullet h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash (\top, \Delta_6), F_5} \top_R \quad \frac{h_7 : F_5, \Delta_{10} \vdash \top, F_8, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \rightarrow F_9), F_5 \vdash \top, \Delta_6} \rightarrow_L}{\frac{}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6} \top_R
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \top}{\vdash : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \top \vdash \Delta_7} \wedge_L}{\vdash : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5, F_6 \vdash \top, \Delta_7}{\vdash : \Delta_8, F_5, F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}{\vdash : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{ax/W}}{\vdash : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{hCut} \wedge_L \\
\\
\frac{\frac{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \wedge F_9}{\vdash : \Delta_7 \vdash \top, \Delta_5} \top_R \quad \frac{h_6 : F_8, F_9, \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \top, \Delta_5} \wedge_L}{\vdash : \Delta_7 \vdash \top, \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_5 \top_R \\
\\
\frac{\frac{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash (\top, \Delta_6), F_5}{\vdash : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6} \top_R \quad \frac{h_7 : F_5, F_8, F_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \wedge F_9), F_5 \vdash \top, \Delta_6} \wedge_L}{\vdash : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6 \top_R
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \top}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \top \vdash \Delta_7} \vee_L}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7}{\vdash : \Delta_8, F_5 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{ax/W}}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{hCut} \vee_L \\
\\
\frac{\frac{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \vee F_9}{\vdash : \Delta_7 \vdash \top, \Delta_5} \top_R \quad \frac{h_6 : F_8, \Delta_7 \vdash \top, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \top, \Delta_5} \vee_L}{\vdash : \Delta_7 \vdash \top, \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_5 \top_R \\
\\
\frac{\frac{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\top, \Delta_6), F_5}{\vdash : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6} \top_R \quad \frac{h_7 : F_5, F_8, \Delta_{10} \vdash \top, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \top, \Delta_6} \vee_L}{\vdash : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6 \top_R
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, \boxed{F_5} \vdash \Delta_6, \top}{\vdash : \Delta_7, \boxed{F_5} \vdash \Delta_6} \top_R \quad \frac{h_4 : \top, F_5, \Delta_7, \boxed{F_5} \vdash \Delta_6}{\bullet h_4 : (\Delta_7, \boxed{F_5}), \top \vdash \Delta_6} AT}{\vdash : \Delta_7, \boxed{F_5} \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_7, F_5, \boxed{F_5} \vdash \top, \Delta_6}{\vdash : \Delta_7, F_5, \boxed{F_5} \vdash \Delta_6} \text{ax/W} \quad \frac{h_4 : \top, \Delta_7, F_5, \boxed{F_5} \vdash \Delta_6}{\vdash : \Delta_7, \boxed{F_5} \vdash \Delta_6} \text{hCut}}{\vdash : \Delta_7, \boxed{F_5} \vdash \Delta_6} AT \\
\\
\frac{\frac{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), \boxed{F_8}}{\vdash : \Delta_7 \vdash \top, \Delta_5} \top_R \quad \frac{h_6 : F_8, \Delta_7, \boxed{F_8} \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, \boxed{F_8} \vdash \top, \Delta_5} AT}{\vdash : \Delta_7 \vdash \top, \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_5 \top_R \\
\\
\frac{\frac{\bullet h_2 : \Delta_9, \boxed{F_8} \vdash (\top, \Delta_6), F_5}{\vdash : \Delta_9, \boxed{F_8} \vdash \top, \Delta_6} \top_R \quad \frac{h_7 : F_5, F_8, \Delta_9, \boxed{F_8} \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_9, \boxed{F_8}), F_5 \vdash \top, \Delta_6} AT}{\vdash : \Delta_9, \boxed{F_8} \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_9, \boxed{F_8} \vdash \top, \Delta_6 \top_R
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_6 \vdash \Delta_5, \top}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \top_R \quad \frac{\bullet h_4 : (\perp, \Delta_6), \top \vdash \Delta_5}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \perp_L}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_6 \vdash \Delta_5 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), \perp}{\vdash : \Delta_7 \vdash \top, \Delta_5} \top_R \quad \frac{\bullet h_6 : \Delta_7, \perp \vdash \top, \Delta_5}{\vdash : \Delta_7 \vdash \top, \Delta_5} \perp_L}{\vdash : \Delta_7 \vdash \top, \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_5 \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \perp, \Delta_8 \vdash (\top, \Delta_6), F_5}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \top_R \quad \frac{\bullet h_7 : (\perp, \Delta_8), F_5 \vdash \top, \Delta_6}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \perp_L}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_8 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_5 \vdash (\Delta_6, p_5), \top}{\vdash : \Delta_7, p_5 \vdash \Delta_6, p_5} \top_R \quad \frac{\bullet h_4 : (\Delta_7, p_5), \top \vdash \Delta_6, p_5}{\vdash : \Delta_7, p_5 \vdash \Delta_6, p_5} I}{\vdash : \Delta_7, p_5 \vdash \Delta_6, p_5} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7, p_5 \vdash \Delta_6, p_5 \quad I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_8, p_7), p_7}{\vdash : \Delta_6 \vdash \top, \Delta_8, p_7} \top_R \quad \frac{\bullet h_5 : \Delta_6, p_7 \vdash \top, \Delta_8, p_7}{\vdash : \Delta_6 \vdash \top, \Delta_8, p_7} I}{\vdash : \Delta_6 \vdash \top, \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_6 \vdash \top, \Delta_8, p_7 \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_9, p_7 \vdash (\top, \Delta_8, p_7), F_5}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_8, p_7} \top_R \quad \frac{\bullet h_6 : (\Delta_9, p_7), F_5 \vdash \top, \Delta_8, p_7}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_8, p_7} I}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_9, p_7 \vdash \top, \Delta_8, p_7 \quad \top_R
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5 \vdash \Delta_6, \top}{\vdash : \Delta_5 \vdash \Delta_6} \top_R \quad \frac{\frac{h_4 : \Delta_5 \vdash \Delta_6}{\bullet h_4 : \Delta_5, \top \vdash \Delta_6} \top_L}{\vdash : \Delta_5 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_5 \vdash \Delta_6 \quad \text{ax/w}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), \top}{\vdash : \Delta_7 \vdash \top, \Delta_5} \top_R \quad \frac{\frac{h_6 : \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, \top \vdash \top, \Delta_5} \top_L}{\vdash : \Delta_7 \vdash \top, \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_5 \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \top, \Delta_8 \vdash (\top, \Delta_6), F_5}{\vdash : \top, \Delta_8 \vdash \top, \Delta_6} \top_R \quad \frac{\frac{h_7 : F_5, \Delta_8 \vdash \top, \Delta_6}{\bullet h_7 : (\top, \Delta_8), F_5 \vdash \top, \Delta_6} \top_L}{\vdash : \top, \Delta_8 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \top, \Delta_8 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

6.6 Status of K : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), \Box F_8} K \quad \frac{h_9 : \Box\Gamma_6, F_{11}, \Delta_7, \Box F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R \\
\hline
- : \Box\Gamma_6, \Delta_7 \vdash \Delta_{10}, F_{11} \rightarrow F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Delta_7, F_{11}, \Box\Gamma_6 \vdash \Delta_{10}, F_{12}, \Box F_8} \text{ax/W} \quad \frac{h_9 : \Delta_7, F_{11}, \Box\Gamma_6, \Box F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \text{ax/W}}{\frac{- : \Delta_7, F_{11}, \Box\Gamma_6 \vdash \Delta_{10}, F_{12}}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R} \text{hCut} \\
\rightarrow \\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), \Box F_{10}), F_8} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, F_{12}, \Delta_9 \vdash F_{13}, \Delta_{14}, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), \Box F_{10}} \rightarrow_R \\
\hline
- : \Box\Gamma_7, \Delta_9 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), \Box F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_9, \Box\Gamma_7 \vdash \Delta_{14}, \Box F_{10}, F_{12} \rightarrow F_{13}} \text{ax/W} \quad K
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), \Box F_8} K \quad \frac{h_9 : \Box\Gamma_6, \Delta_7, \Box F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Box\Gamma_6, \Delta_7, \Box F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R \\
\hline
- : \Box\Gamma_6, \Delta_7 \vdash \Delta_{10}, F_{11} \wedge F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, F_{11}, \Box F_8} \text{ax/W} \quad \frac{h_9 : \Delta_7, \Box\Gamma_6, \Box F_8 \vdash \Delta_{10}, F_{11}}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{ax/W}}{\frac{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, F_{11}}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R} \text{hCut} \\
\rightarrow \\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), \Box F_{10}), F_8} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, \Delta_9 \vdash F_{12}, \Delta_{14}, \Box F_{10} \quad h_{11} : \Box\Gamma_7, F_8, \Delta_9 \vdash F_{13}, \Delta_{14}, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), \Box F_{10}} \wedge_R \\
\hline
- : \Box\Gamma_7, \Delta_9 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), \Box F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_9, \Box\Gamma_7 \vdash \Delta_{14}, \Box F_{10}, F_{12} \wedge F_{13}} \text{ax/W} \quad K
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_{10}, F_{11} \vee F_{12}), \Box F_8} K \quad \frac{h_9 : \Box\Gamma_6, \Delta_7, \Box F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R \\
\hline
- : \Box\Gamma_6, \Delta_7 \vdash \Delta_{10}, F_{11} \vee F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, F_{11}, F_{12}, \Box F_8} \text{ax/W} \quad \frac{h_9 : \Delta_7, \Box\Gamma_6, \Box F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{ax/W}}{\frac{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, F_{11}, F_{12}}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R} \text{hCut} \\
\rightarrow \\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), \Box F_{10}), F_8} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, \Delta_9 \vdash F_{12}, F_{13}, \Delta_{14}, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), \Box F_{10}} \vee_R \\
\hline
- : \Box\Gamma_7, \Delta_9 \vdash (\Delta_{14}, F_{12} \vee F_{13}), \Box F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_9, \Box\Gamma_7 \vdash \Delta_{14}, \Box F_{10}, F_{12} \vee F_{13}} \text{ax/W} \quad K
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Box\Gamma_6, \Delta_7 \vdash (\perp, \Delta_{10}), \Box F_8} K \quad \frac{h_9 : \Box\Gamma_6, \Delta_7, \Box F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Box\Gamma_6, \Delta_7 \vdash \perp, \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Delta_7, \Box\Gamma_6 \vdash \perp, \Delta_{10}, \Box F_8} \text{ax/W} \quad \frac{h_9 : \Delta_7, \Box\Gamma_6, \Box F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \perp, \Delta_{10}} \text{ax/W}}{\frac{- : \Delta_7, \Box\Gamma_6 \vdash \perp, \Delta_{10}}{- : \Delta_7, \Box\Gamma_6 \vdash \perp, \Delta_{10}} \perp_R} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \text{unbox}(\Box\Gamma_{12}, \Box\Gamma_{14}) \vdash F_{11}}{\bullet h_2 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Box\Gamma_9, \Delta_{10}), \Box F_{11}), F_7} K \quad \frac{h_8 : \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Box\Gamma_9, F_{11}}{\bullet h_8 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), F_7 \vdash (\Box\Gamma_9, \Delta_{10}), \Box F_{11}} A45 \\
\hline
\frac{}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_9, \Delta_{10}), \Box F_{11}} \text{Cut} \\
\hline
\frac{}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, \Box\Gamma_9, \Box F_{11}} \text{ax/W} \\
\hline
\frac{}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, \Box\Gamma_9, \Box F_{11}} K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, \Box F_7} K \quad \frac{h_8 : \Box\Gamma_6, \Delta_{12}, \Box F_7 \vdash F_9, \Delta_{11} \quad h_8 : \Box\Gamma_6, F_{10}, \Delta_{12}, \Box F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Box\Gamma_6, \Delta_{12}, F_9 \rightarrow F_{10}), \Box F_7 \vdash \Delta_{11}} \text{Cut} \\
\hline
\frac{}{- : \Box\Gamma_6, \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L \\
\hline
\frac{}{- : \Delta_{12}, \Box\Gamma_6 \vdash \Delta_{11}, F_9} \text{ax/W} \quad \frac{}{- : \Delta_{12}, \Box\Gamma_6, \Box F_7 \vdash \Delta_{11}, F_9} K \quad \frac{}{- : \Delta_{12}, \Box\Gamma_6, \Box F_7 \vdash \Delta_{11}, F_9} \text{hCut} \quad \frac{}{- : \Delta_{12}, F_{10}, \Box\Gamma_6 \vdash \Delta_{11}, \Box F_7} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_{10}, \Box\Gamma_6, \Box F_7 \vdash \Delta_{11}} K \quad \frac{}{- : \Delta_{12}, F_{10}, \Box\Gamma_6, \Box F_7 \vdash \Delta_{11}} \text{hCut} \\
\hline
\frac{}{- : \Delta_{12}, \Box\Gamma_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, \Box F_{10}), F_{12} \rightarrow F_{13}} K \quad \frac{h_{11} : \Box\Gamma_7, \Delta_8 \vdash F_{12}, \Delta_9, \Box F_{10} \quad h_{11} : \Box\Gamma_7, F_{13}, \Delta_8 \vdash \Delta_9, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), F_{12} \rightarrow F_{13} \vdash \Delta_9, \Box F_{10}} \text{Cut} \\
\hline
\frac{}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, \Box F_{10}} \rightarrow_L \\
\hline
\frac{}{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, \Box F_{10}} K \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_9, \Box F_{10}), F_8} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, \Delta_{14} \vdash F_{12}, \Delta_9, \Box F_{10} \quad h_{11} : \Box\Gamma_7, F_8, F_{13}, \Delta_{14} \vdash \Delta_9, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13}), F_8 \vdash \Delta_9, \Box F_{10}} \text{Cut} \\
\hline
\frac{}{- : \Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_9, \Box F_{10}} \rightarrow_L \\
\hline
\frac{}{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{14}, \Box\Gamma_7, F_{12} \rightarrow F_{13} \vdash \Delta_9, \Box F_{10}} K
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, \Box F_7} K \quad \frac{h_8 : \Box\Gamma_6, F_9, F_{10}, \Delta_{12}, \Box F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Box\Gamma_6, \Delta_{12}, F_9 \wedge F_{10}), \Box F_7 \vdash \Delta_{11}} \wedge_L \\
\hline
\frac{}{- : \Box\Gamma_6, \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut} \\
\hline
\frac{}{- : \Delta_{12}, F_{10}, F_9, \Box\Gamma_6 \vdash \Delta_{11}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_{10}, F_9, \Box\Gamma_6, \Box F_7 \vdash \Delta_{11}} K \quad \frac{}{- : \Delta_{12}, F_{10}, F_9, \Box\Gamma_6 \vdash \Delta_{11}} \text{hCut} \\
\hline
\frac{}{- : \Delta_{12}, \Box\Gamma_6, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, \Box F_{10}), F_{12} \wedge F_{13}} K \quad \frac{h_{11} : \Box\Gamma_7, F_{12}, F_{13}, \Delta_8 \vdash \Delta_9, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), F_{12} \wedge F_{13} \vdash \Delta_9, \Box F_{10}} \wedge_L \\
\hline
\frac{}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, \Box F_{10}} \text{Cut} \\
\hline
\frac{}{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, \Box F_{10}} K \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_9, \Box F_{10}), F_8} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_9, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \wedge F_{13}), F_8 \vdash \Delta_9, \Box F_{10}} \wedge_L \\
\hline
\frac{}{- : \Box\Gamma_7, \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_9, \Box F_{10}} \text{Cut} \\
\hline
\frac{}{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{14}, \Box\Gamma_7, F_{12} \wedge F_{13} \vdash \Delta_9, \Box F_{10}} K
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_7}{\bullet h_1 : \Box \Gamma_6, \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, [\Box F_7]} K \quad \frac{h_8 : \Box \Gamma_6, F_9, \Delta_{12}, [\Box F_7] \vdash \Delta_{11} \quad h_8 : \Box \Gamma_6, F_{10}, \Delta_{12}, [\Box F_7] \vdash \Delta_{11}}{\bullet h_8 : (\Box \Gamma_6, \Delta_{12}, F_9 \vee F_{10}), [\Box F_7] \vdash \Delta_{11}} \vee_L}{- : \Box \Gamma_6, \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_7}{\bullet h_1 : \Delta_{12}, F_9, \Box \Gamma_6 \vdash \Delta_{11}, [\Box F_7]} K \quad \frac{h_8 : \Delta_{12}, F_9, \Box \Gamma_6, [\Box F_7] \vdash \Delta_{11}}{- : \Delta_{12}, F_9, \Box \Gamma_6 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_9, \Box \Gamma_6 \vdash \Delta_{11}} K \quad \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_7}{\bullet h_1 : \Delta_{12}, F_{10}, \Box \Gamma_6 \vdash \Delta_{11}, [\Box F_7]} K \quad \frac{h_8 : \Delta_{12}, F_{10}, \Box \Gamma_6, [\Box F_7] \vdash \Delta_{11}}{- : \Delta_{12}, F_{10}, \Box \Gamma_6 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10}, \Box \Gamma_6 \vdash \Delta_{11}} K \quad \frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, \Box \Gamma_6, F_9 \vee F_{10} \vdash \Delta_{11}} \text{hCut} \\
\\
\frac{\frac{h_2 : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box \Gamma_7, \Delta_8 \vdash (\Delta_9, [\Box F_{10}]), F_{12} \vee F_{13}} K \quad \frac{h_{11} : \Box \Gamma_7, F_{12}, \Delta_8 \vdash \Delta_9, [\Box F_{10}] \quad h_{11} : \Box \Gamma_7, F_{13}, \Delta_8 \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : (\Box \Gamma_7, \Delta_8), F_{12} \vee F_{13} \vdash \Delta_9, [\Box F_{10}]} \vee_L}{- : \Box \Gamma_7, \Delta_8 \vdash \Delta_9, [\Box F_{10}]} \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_9, [\Box F_{10}]} \text{ax/W} \quad \frac{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_9, [\Box F_{10}]}{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_9, [\Box F_{10}]} K \\
\\
\frac{\frac{h_2 : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box \Gamma_7, \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_9, [\Box F_{10}]), F_8} K \quad \frac{h_{11} : \Box \Gamma_7, F_8, F_{12}, \Delta_{14} \vdash \Delta_9, [\Box F_{10}] \quad h_{11} : \Box \Gamma_7, F_8, F_{13}, \Delta_{14} \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : (\Box \Gamma_7, \Delta_{14}, F_{12} \vee F_{13}), F_8 \vdash \Delta_9, [\Box F_{10}]} \vee_L}{- : \Box \Gamma_7, \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_9, [\Box F_{10}]} \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{- : \Delta_{14}, \Box \Gamma_7, F_{12} \vee F_{13} \vdash \Delta_9, [\Box F_{10}]} \text{ax/W} \quad \frac{- : \Delta_{14}, \Box \Gamma_7, F_{12} \vee F_{13} \vdash \Delta_9, [\Box F_{10}]}{- : \Delta_{14}, \Box \Gamma_7, F_{12} \vee F_{13} \vdash \Delta_9, [\Box F_{10}]} K
\end{array}$$

• Case rule AT

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_{11}, [\Box F_9]) \vdash F_7}{\bullet h_1 : (\Box \Gamma_{11}, [\Box F_9]), \Delta_6 \vdash \Delta_{10}, [\Box F_7]} K \quad \frac{h_8 : \Box \Gamma_{11}, F_9, \Delta_6, [\Box F_7], [\Box F_9] \vdash \Delta_{10}}{\bullet h_8 : ((\Box \Gamma_{11}, [\Box F_9]), \Delta_6), [\Box F_7] \vdash \Delta_{10}} AT}{- : (\Box \Gamma_{11}, [\Box F_9]), \Delta_6 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_6, F_9, \Box \Gamma_{11}, [\Box F_9] \vdash \Delta_{10}, [\Box F_7]}{\bullet h_1 : \Delta_6, F_9, \Box \Gamma_{11}, [\Box F_9] \vdash \Delta_{10}, [\Box F_7]} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_9, \Box \Gamma_{11}, [\Box F_7], [\Box F_9] \vdash \Delta_{10}}{\bullet h_8 : \Delta_6, F_9, \Box \Gamma_{11}, [\Box F_7], [\Box F_9] \vdash \Delta_{10}} \text{ax/W}}{- : \Delta_6, F_9, \Box \Gamma_{11}, [\Box F_9] \vdash \Delta_{10}, [\Box F_7]} \text{hCut} \\
\frac{- : \Delta_6, F_9, \Box \Gamma_{11}, [\Box F_9] \vdash \Delta_{10}, [\Box F_7]}{- : \Delta_6, \Box \Gamma_{11}, [\Box F_9] \vdash \Delta_{10}} AT \\
\\
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_7}{\bullet h_1 : \Box \Gamma_6, \Delta_{11}, [\Box F_9] \vdash \Delta_{10}, [\Box F_7]} K \quad \frac{h_8 : \Box \Gamma_6, F_9, \Delta_{11}, [\Box F_7], [\Box F_9] \vdash \Delta_{10}}{\bullet h_8 : (\Box \Gamma_6, \Delta_{11}, [\Box F_9]), [\Box F_7] \vdash \Delta_{10}} AT}{- : \Box \Gamma_6, \Delta_{11}, [\Box F_9] \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_{11}, F_9, \Box \Gamma_6, [\Box F_9] \vdash \Delta_{10}, [\Box F_7]}{\bullet h_1 : \Delta_{11}, F_9, \Box \Gamma_6, [\Box F_9] \vdash \Delta_{10}, [\Box F_7]} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_9, \Box \Gamma_6, [\Box F_7], [\Box F_9] \vdash \Delta_{10}}{\bullet h_8 : \Delta_{11}, F_9, \Box \Gamma_6, [\Box F_7], [\Box F_9] \vdash \Delta_{10}} \text{ax/W}}{- : \Delta_{11}, F_9, \Box \Gamma_6, [\Box F_9] \vdash \Delta_{10}, [\Box F_7]} \text{hCut} \\
\frac{- : \Delta_{11}, F_9, \Box \Gamma_6, [\Box F_9] \vdash \Delta_{10}, [\Box F_7]}{- : \Delta_{11}, \Box \Gamma_6, [\Box F_9] \vdash \Delta_{10}} AT \\
\\
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_9}{\bullet h_1 : \Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, [\Box F_9]} K \quad \frac{h_8 : \Box \Gamma_6, F_9, \Delta_7, [\Box F_9] \vdash \Delta_{10}}{\bullet h_8 : (\Box \Gamma_6, \Delta_7), [\Box F_9] \vdash \Delta_{10}} AT}{- : \Box \Gamma_6, \Delta_7 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7, \Box \Gamma_6, \text{unbox}(\Box \Gamma_6) \vdash \Delta_{10}, F_9}{- : \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_9} \text{ax/W} \quad \frac{\frac{\frac{h_1 : \Delta_7, F_9, \Box \Gamma_6 \vdash \Delta_{10}, [\Box F_9]}{\bullet h_1 : \Delta_7, F_9, \Box \Gamma_6 \vdash \Delta_{10}, [\Box F_9]} \text{ax/W} \quad \frac{h_8 : \Delta_7, F_9, \Box \Gamma_6, [\Box F_9] \vdash \Delta_{10}}{\bullet h_8 : \Delta_7, F_9, \Box \Gamma_6, [\Box F_9] \vdash \Delta_{10}} \text{ax/W}}{- : \Delta_7, F_9, \Box \Gamma_6 \vdash \Delta_{10}, [\Box F_9]} \text{hCut} \\
\frac{- : \Delta_7, F_9, \Box \Gamma_6 \vdash \Delta_{10}, [\Box F_9]}{- : \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}} \text{sCut} \\
\\
\frac{\frac{h_2 : \text{unbox}(\Box \Gamma_{13}, [\Box F_{12}]) \vdash F_{10}}{\bullet h_2 : (\Box \Gamma_{13}, [\Box F_{12}]), \Delta_8 \vdash (\Delta_9, [\Box F_{10}]), F_7} K \quad \frac{h_{11} : \Box \Gamma_{13}, F_7, F_{12}, \Delta_8, [\Box F_{12}] \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : ((\Box \Gamma_{13}, [\Box F_{12}]), \Delta_8), F_7 \vdash \Delta_9, [\Box F_{10}]} AT}{- : (\Box \Gamma_{13}, [\Box F_{12}]), \Delta_8 \vdash \Delta_9, [\Box F_{10}]} \text{Cut} \\
\rightarrow \\
\frac{- : F_{12}, \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{- : \Delta_8, \Box \Gamma_{13}, [\Box F_{12}] \vdash \Delta_9, [\Box F_{10}]} \text{ax/W} \quad \frac{- : \Delta_8, \Box \Gamma_{13}, [\Box F_{12}] \vdash \Delta_9, [\Box F_{10}]}{- : \Delta_8, \Box \Gamma_{13}, [\Box F_{12}] \vdash \Delta_9, [\Box F_{10}]} K \\
\\
\frac{\frac{h_2 : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box \Gamma_7, \Delta_8 \vdash (\Delta_9, [\Box F_{10}]), [\Box F_{12}]} K \quad \frac{h_{11} : \Box \Gamma_7, F_{12}, \Delta_8, [\Box F_{12}] \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : (\Box \Gamma_7, \Delta_8), [\Box F_{12}] \vdash \Delta_9, [\Box F_{10}]} AT}{- : \Box \Gamma_7, \Delta_8 \vdash \Delta_9, [\Box F_{10}]} \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_9, [\Box F_{10}]} \text{ax/W} \quad \frac{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_9, [\Box F_{10}]}{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_9, [\Box F_{10}]} K
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_{13}, [\Box F_{12} \vdash (\Delta_9, [\Box F_{10}]), F_8} \quad K \quad \frac{h_{11} : \Box\Gamma_7, F_8, F_{12}, \Delta_{13}, [\Box F_{12} \vdash \Delta_9, [\Box F_{10}]}{ \bullet h_{11} : (\Box\Gamma_7, \Delta_{13}, [\Box F_{12}]), F_8 \vdash \Delta_9, [\Box F_{10}]} \quad AT}{\frac{- : \Box\Gamma_7, \Delta_{13}, [\Box F_{12} \vdash \Delta_9, [\Box F_{10}]}{\rightarrow} \quad \text{Cut}}{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_{13}, \Box\Gamma_7, [\Box F_{12} \vdash \Delta_9, [\Box F_{10}]} \quad K} \quad \text{ax/W}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \perp, \Delta_{10} \vdash \Delta_9, [\Box F_7]} \quad K \quad \frac{\bullet h_8 : (\Box\Gamma_6, \perp, \Delta_{10}), [\Box F_7 \vdash \Delta_9]}{- : \Box\Gamma_6, \perp, \Delta_{10} \vdash \Delta_9} \quad \perp_L \quad \text{Cut}}{\frac{- : \perp, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9}{\rightarrow} \quad \perp_L} \\
\\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, [\Box F_{10}]), \perp} \quad K \quad \frac{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), \perp \vdash \Delta_9, [\Box F_{10}]}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, [\Box F_{10}]} \quad \perp_L \quad \text{Cut}}{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, [\Box F_{10}]} \quad K} \quad \text{ax/W} \\
\\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \perp, \Delta_{12} \vdash (\Delta_9, [\Box F_{10}]), F_8} \quad K \quad \frac{\bullet h_{11} : (\Box\Gamma_7, \perp, \Delta_{12}), F_8 \vdash \Delta_9, [\Box F_{10}]}{- : \Box\Gamma_7, \perp, \Delta_{12} \vdash \Delta_9, [\Box F_{10}]} \quad \perp_L \quad \text{Cut}}{\frac{- : \perp, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_9, [\Box F_{10}]}{\rightarrow} \quad \perp_L}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), [\Box F_7]} \quad K \quad \frac{\bullet h_8 : (\Box\Gamma_6, \Delta_{11}, p_9), [\Box F_7 \vdash \Delta_{10}, p_9]}{- : \Box\Gamma_6, \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \quad I \quad \text{Cut}}{\frac{- : \Delta_{11}, \Box\Gamma_6, p_9 \vdash \Delta_{10}, p_9}{\rightarrow} \quad I} \\
\\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_9}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash ((\Delta_{12}, p_{11}), [\Box F_9]), p_{11}} \quad K \quad \frac{\bullet h_{10} : (\Box\Gamma_7, \Delta_8), p_{11} \vdash (\Delta_{12}, p_{11}), [\Box F_9]}{- : \Box\Gamma_7, \Delta_8 \vdash (\Delta_{12}, p_{11}), [\Box F_9]} \quad I \quad \text{Cut}}{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_9}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{12}, p_{11}, [\Box F_9]} \quad K} \quad \text{ax/W} \\
\\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_9}{\bullet h_2 : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), [\Box F_9]), F_8} \quad K \quad \frac{\bullet h_{10} : (\Box\Gamma_7, \Delta_{13}, p_{11}), F_8 \vdash (\Delta_{12}, p_{11}), [\Box F_9]}{- : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), [\Box F_9]} \quad I \quad \text{Cut}}{\frac{- : \Delta_{13}, \Box\Gamma_7, p_{11} \vdash \Delta_{12}, p_{11}, [\Box F_9]}{\rightarrow} \quad I}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \top, \Delta_{10} \vdash \Delta_9, [\Box F_7]} \quad K \quad \frac{h_8 : \Box\Gamma_6, \Delta_{10}, [\Box F_7 \vdash \Delta_9]}{\bullet h_8 : (\Box\Gamma_6, \top, \Delta_{10}), [\Box F_7 \vdash \Delta_9]} \quad \top_L \quad \text{Cut}}{\frac{- : \Box\Gamma_6, \top, \Delta_{10} \vdash \Delta_9}{\rightarrow} \quad \text{ax/W}} \\
\\
\frac{\bullet h_1 : \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9, [\Box F_7]}{- : \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9} \quad \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, \Box\Gamma_6, [\Box F_7 \vdash \Delta_9]}{\bullet h_8 : \top, \Delta_{10}, \Box\Gamma_6, [\Box F_7 \vdash \Delta_9]} \quad \text{hCut}}{- : \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, \Box F_{10}), \top} K \quad \frac{h_{11} : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), \top \vdash \Delta_9, \Box F_{10}} \top_L \\
\hline
- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, \Box F_{10} \\
\rightarrow \\
\frac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, \Box F_{10}} \text{ax/W} \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \top, \Delta_{12} \vdash (\Delta_9, \Box F_{10}), F_8} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, \Delta_{12} \vdash \Delta_9, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \top, \Delta_{12}), F_8 \vdash \Delta_9, \Box F_{10}} \top_L \\
\hline
- : \Box\Gamma_7, \top, \Delta_{12} \vdash \Delta_9, \Box F_{10} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}} \text{ax/W} \\
\frac{}{- : \top, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_9, \Box F_{10}} K
\end{array}$$

6.7 Status of A45: fail

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_{10}, F_9}{\bullet h_1 : \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_{10}, \Delta_{14}, F_{12} \rightarrow F_{13}), \Box F_9} A45 \quad \frac{h_{11} : \Box\Gamma_7, F_{12}, \Delta_8, \Box F_9 \vdash \Box\Gamma_{10}, F_{13}, \Delta_{14}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{14}, F_{12} \rightarrow F_{13}} \rightarrow_R \\
\hline
- : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_{10}, \Delta_{14}, F_{12} \rightarrow F_{13} \\
\rightarrow \\
\frac{}{h_1 : \Box\Gamma_7 \vdash F_9, \Box\Gamma_{10}} \text{ax/W} \\
\frac{}{\bullet h_1 : \Delta_8, F_{12}, \Box\Gamma_7 \vdash \Delta_{14}, F_{13}, \Box\Gamma_{10}, \Box F_9} A45 \quad \frac{h_{11} : \Delta_8, F_{12}, \Box\Gamma_7, \Box F_9 \vdash \Delta_{14}, F_{13}, \Box\Gamma_{10}}{\bullet h_{11} : (\Delta_8, F_{12}, \Box\Gamma_7, \Box F_9) \vdash \Delta_{14}, F_{13}, \Box\Gamma_{10}} \text{ax/W} \\
\hline
- : \Delta_8, F_{12}, \Box\Gamma_7 \vdash \Delta_{14}, F_{13}, \Box\Gamma_{10} \\
\rightarrow_R \\
- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, \Box\Gamma_{10}, F_{12} \rightarrow F_{13} \\
\rightarrow \\
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{11}, F_{12}, \Box F_{10}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{11}, (\Delta_{16}, F_{14} \rightarrow F_{15}), \Box F_{12}), \Box F_{10}} A45 \quad \frac{h_{13} : \Box\Gamma_8, F_{14}, \Delta_9, \Box F_{10} \vdash \Box\Gamma_{11}, F_{15}, \Delta_{16}, \Box F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), \Box F_{10} \vdash \Box\Gamma_{11}, (\Delta_{16}, F_{14} \rightarrow F_{15}), \Box F_{12}} \rightarrow_R \\
\hline
- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{11}, (\Delta_{16}, F_{14} \rightarrow F_{15}), \Box F_{12} \\
\rightarrow \\
\frac{}{h_2 : \Box\Gamma_8 \vdash \Box F_{10}, F_{12}, \Box\Gamma_{11}} \text{ax/W} \\
\frac{}{\bullet h_2 : \Delta_9, F_{14}, \Box\Gamma_8 \vdash \Box F_{10}, \Delta_{16}, F_{15}, \Box\Gamma_{11}, \Box F_{12}} A45 \quad \frac{h_{13} : \Box F_{10}, \Delta_9, F_{14}, \Box\Gamma_8 \vdash \Delta_{16}, F_{15}, \Box\Gamma_{11}, \Box F_{12}}{\bullet h_{13} : (\Box F_{10}, \Delta_9, F_{14}, \Box\Gamma_8) \vdash \Delta_{16}, F_{15}, \Box\Gamma_{11}, \Box F_{12}} \text{ax/W} \\
\hline
- : \Delta_9, F_{14}, \Box\Gamma_8 \vdash \Delta_{16}, F_{15}, \Box\Gamma_{11}, \Box F_{12} \\
\rightarrow_R \\
- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{16}, \Box\Gamma_{11}, \Box F_{12}, F_{14} \rightarrow F_{15} \\
\rightarrow \\
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{11}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_{10} \vdash (\Box\Gamma_{11}, (\Delta_{16}, F_{14} \rightarrow F_{15}), \Box F_{12}), F_9} A45 \quad \frac{h_{13} : \Box\Gamma_8, F_9, F_{14}, \Delta_{10} \vdash \Box\Gamma_{11}, F_{15}, \Delta_{16}, \Box F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{10}), F_9 \vdash \Box\Gamma_{11}, (\Delta_{16}, F_{14} \rightarrow F_{15}), \Box F_{12}} \rightarrow_R \\
\hline
- : \Box\Gamma_8, \Delta_{10} \vdash \Box\Gamma_{11}, (\Delta_{16}, F_{14} \rightarrow F_{15}), \Box F_{12} \\
\rightarrow \\
\frac{}{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{11}} \text{ax/W} \\
\frac{}{- : \Delta_{10}, \Box\Gamma_8 \vdash \Delta_{16}, \Box\Gamma_{11}, \Box F_{12}, F_{14} \rightarrow F_{15}} A45
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_{10}, F_9}{\bullet h_1 : \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_{10}, \Delta_{14}, F_{12} \wedge F_{13}), \Box F_9} A45 \quad \frac{h_{11} : \Box\Gamma_7, \Delta_8, \Box F_9 \vdash \Box\Gamma_{10}, F_{12}, \Delta_{14} \quad h_{11} : \Box\Gamma_7, \Delta_8, \Box F_9 \vdash \Box\Gamma_{10}, F_{13}, \Delta_{14}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{14}, F_{12} \wedge F_{13}} \text{Cut} \\
\hline
- : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_{10}, \Delta_{14}, F_{12} \wedge F_{13} \\
\rightarrow \\
\frac{}{h_1 : \Box\Gamma_7 \vdash F_9, \Box\Gamma_{10}} \text{ax/W} \\
\frac{}{\bullet h_1 : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, F_{12}, \Box\Gamma_{10}, \Box F_9} A45 \quad \frac{h_{11} : \Delta_8, \Box\Gamma_7, \Box F_9 \vdash \Delta_{14}, F_{12}, \Box\Gamma_{10}}{\bullet h_{11} : (\Delta_8, \Box\Gamma_7, \Box F_9) \vdash \Delta_{14}, F_{12}, \Box\Gamma_{10}} \text{ax/W} \\
\hline
- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, F_{12}, \Box\Gamma_{10} \\
\rightarrow \\
\frac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{14}, \Box\Gamma_{10}, F_{12} \wedge F_{13}} \wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{11}, F_{12}, \Box F_{10}}{\bullet h_2 : \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{11}, (\Delta_{16}, F_{14} \wedge F_{15}), \Box F_{12}), \Box F_{10}} A45 \quad \frac{h_{13} : \Box \Gamma_8, \Delta_9, \Box F_{10} \vdash \Box \Gamma_{11}, F_{14}, \Delta_{16}, \Box F_{12} \quad h_{13} : \Box \Gamma_8, \Delta_9, \Box F_{10} \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \wedge F_{15}), \Box F_{12}}{\bullet h_{13} : (\Box \Gamma_8, \Delta_9), \Box F_{10} \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \wedge F_{15}), \Box F_{12}} \wedge_R \\
\hline
- : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \wedge F_{15}), \Box F_{12} \quad \rightarrow \\
\frac{\frac{h_2 : \Box \Gamma_8 \vdash \Box F_{10}, F_{12}, \Box \Gamma_{11}}{\bullet h_2 : \Delta_9, \Box \Gamma_8 \vdash \Box F_{10}, \Delta_{16}, F_{14}, \Box \Gamma_{11}, \Box F_{12}} ax/W \quad \frac{h_{13} : \Box F_{10}, \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, \Box \Gamma_{11}, \Box F_{12}}{\bullet h_{13} : (\Box \Gamma_8, \Delta_9), \Box F_{10} \vdash \Delta_{16}, F_{14}, \Box \Gamma_{11}, \Box F_{12}} ax/W}{- : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, \Box \Gamma_{11}, \Box F_{12}} hCut \\
\hline
- : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, \Box F_{12}, F_{14} \wedge F_{15} \\
\hline
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{11}, F_{12}}{\bullet h_2 : \Box \Gamma_8, \Delta_{10} \vdash (\Box \Gamma_{11}, (\Delta_{16}, F_{14} \wedge F_{15}), \Box F_{12}), F_9} A45 \quad \frac{h_{13} : \Box \Gamma_8, F_9, \Delta_{10} \vdash \Box \Gamma_{11}, F_{14}, \Delta_{16}, \Box F_{12} \quad h_{13} : \Box \Gamma_8, F_9, \Delta_{10} \vdash \Box \Gamma_{11}, F_{15}, \Delta_{16}, \Box F_{12}}{\bullet h_{13} : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \wedge F_{15}), \Box F_{12}} \wedge_R \\
\hline
- : \Box \Gamma_8, \Delta_{10} \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \wedge F_{15}), \Box F_{12} \quad \rightarrow \\
\frac{- : \Box \Gamma_8 \vdash F_{12}, \Box \Gamma_{11}}{- : \Delta_{10}, \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, \Box F_{12}, F_{14} \wedge F_{15}} ax/W \quad A45
\end{array}$$

• Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_7 \vdash \Box \Gamma_{10}, F_9}{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Box \Gamma_{10}, \Delta_{14}, F_{12} \vee F_{13}), \Box F_9} A45 \quad \frac{h_{11} : \Box \Gamma_7, \Delta_8, \Box F_9 \vdash \Box \Gamma_{10}, F_{12}, F_{13}, \Delta_{14}}{\bullet h_{11} : (\Box \Gamma_7, \Delta_8), \Box F_9 \vdash \Box \Gamma_{10}, \Delta_{14}, F_{12} \vee F_{13}} \vee_R \\
\hline
- : \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_{10}, \Delta_{14}, F_{12} \vee F_{13} \quad \rightarrow \\
\frac{h_1 : \Box \Gamma_7 \vdash F_9, \Box \Gamma_{10}}{\bullet h_1 : \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{12}, F_{13}, \Box \Gamma_{10}, \Box F_9} ax/W \quad \frac{h_{11} : \Delta_8, \Box \Gamma_7, \Box F_9 \vdash \Delta_{14}, F_{12}, F_{13}, \Box \Gamma_{10}}{\bullet h_{11} : (\Delta_8, \Box \Gamma_7), \Box F_9 \vdash \Delta_{14}, F_{12}, F_{13}, \Box \Gamma_{10}} ax/W}{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{12}, F_{13}, \Box \Gamma_{10}} hCut \\
\hline
- : \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, \Box \Gamma_{10}, F_{12} \vee F_{13} \quad \vee_R \\
\hline
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{11}, F_{12}, \Box F_{10}}{\bullet h_2 : \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{11}, (\Delta_{16}, F_{14} \vee F_{15}), \Box F_{12}), \Box F_{10}} A45 \quad \frac{h_{13} : \Box \Gamma_8, \Delta_9, \Box F_{10} \vdash \Box \Gamma_{11}, F_{14}, F_{15}, \Delta_{16}, \Box F_{12}}{\bullet h_{13} : (\Box \Gamma_8, \Delta_9), \Box F_{10} \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \vee F_{15}), \Box F_{12}} \vee_R \\
\hline
- : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \vee F_{15}), \Box F_{12} \quad \rightarrow \\
\frac{h_2 : \Box \Gamma_8 \vdash \Box F_{10}, F_{12}, \Box \Gamma_{11}}{\bullet h_2 : \Delta_9, \Box \Gamma_8 \vdash \Box F_{10}, \Delta_{16}, F_{14}, F_{15}, \Box \Gamma_{11}, \Box F_{12}} ax/W \quad \frac{h_{13} : \Box F_{10}, \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, F_{15}, \Box \Gamma_{11}, \Box F_{12}}{\bullet h_{13} : (\Delta_9, \Box \Gamma_8), \Box F_{10} \vdash \Delta_{16}, F_{14}, F_{15}, \Box \Gamma_{11}, \Box F_{12}} ax/W}{- : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, F_{15}, \Box \Gamma_{11}, \Box F_{12}} hCut \\
\hline
- : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, \Box F_{12}, F_{14} \vee F_{15} \quad \vee_R \\
\hline
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{11}, F_{12}}{\bullet h_2 : \Box \Gamma_8, \Delta_{10} \vdash (\Box \Gamma_{11}, (\Delta_{16}, F_{14} \vee F_{15}), \Box F_{12}), F_9} A45 \quad \frac{h_{13} : \Box \Gamma_8, F_9, \Delta_{10} \vdash \Box \Gamma_{11}, F_{14}, F_{15}, \Delta_{16}, \Box F_{12}}{\bullet h_{13} : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \vee F_{15}), \Box F_{12}} \vee_R \\
\hline
- : \Box \Gamma_8, \Delta_{10} \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \vee F_{15}), \Box F_{12} \quad \rightarrow \\
\frac{- : \Box \Gamma_8 \vdash F_{12}, \Box \Gamma_{11}}{- : \Delta_{10}, \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, \Box F_{12}, F_{14} \vee F_{15}} ax/W \quad A45
\end{array}$$

• Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_7 \vdash \Box \Gamma_{10}, F_9}{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Box \Gamma_{10}, \perp, \Delta_{12}), \Box F_9} A45 \quad \frac{h_{11} : \Box \Gamma_7, \Delta_8, \Box F_9 \vdash \Box \Gamma_{10}, \Delta_{12}}{\bullet h_{11} : (\Box \Gamma_7, \Delta_8), \Box F_9 \vdash \Box \Gamma_{10}, \perp, \Delta_{12}} \perp_R \\
\hline
- : \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_{10}, \perp, \Delta_{12} \quad \rightarrow \\
\frac{h_1 : \Box \Gamma_7 \vdash \perp, \Delta_{12}, \Box \Gamma_{10}, \Box F_9}{\bullet h_1 : \Delta_8, \Box \Gamma_7 \vdash \perp, \Delta_{12}, \Box \Gamma_{10}} ax/W \quad \frac{h_{11} : \Delta_8, \Box \Gamma_7, \Box F_9 \vdash \perp, \Delta_{12}, \Box \Gamma_{10}}{\bullet h_{11} : (\Delta_8, \Box \Gamma_7), \Box F_9 \vdash \perp, \Delta_{12}, \Box \Gamma_{10}} ax/W}{- : \Delta_8, \Box \Gamma_7 \vdash \perp, \Delta_{12}, \Box \Gamma_{10}} hCut \\
\hline
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{11}, F_{12}, \Box F_{10}}{\bullet h_2 : \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{11}, (\perp, \Delta_{14}), \Box F_{12}), \Box F_{10}} A45 \quad \frac{h_{13} : \Box \Gamma_8, \Delta_9, \Box F_{10} \vdash \Box \Gamma_{11}, \Delta_{14}, \Box F_{12}}{\bullet h_{13} : (\Box \Gamma_8, \Delta_9), \Box F_{10} \vdash \Box \Gamma_{11}, (\perp, \Delta_{14}), \Box F_{12}} \perp_R \\
\hline
- : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{11}, (\perp, \Delta_{14}), \Box F_{12} \quad \rightarrow \\
\frac{h_2 : \Delta_9, \Box \Gamma_8 \vdash \perp, \Box F_{10}, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}}{\bullet h_2 : \Delta_9, \Box \Gamma_8 \vdash \perp, \Box F_{10}, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}} ax/W \quad \frac{h_{13} : \Box F_{10}, \Delta_9, \Box \Gamma_8 \vdash \perp, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}}{\bullet h_{13} : (\Delta_9, \Box \Gamma_8), \Box F_{10} \vdash \perp, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}} ax/W}{- : \Delta_9, \Box \Gamma_8 \vdash \perp, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}} hCut
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{11}, F_{12}}{\bullet h_2 : \Box \Gamma_8, \Delta_{10} \vdash (\Box \Gamma_{11}, (\perp, \Delta_{14}), \Box F_{12}), F_9} \text{A45} \quad \frac{h_{13} : \Box \Gamma_8, F_9, \Delta_{10} \vdash \Box \Gamma_{11}, \Delta_{14}, \Box F_{12}}{\bullet h_{13} : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \Box \Gamma_{11}, (\perp, \Delta_{14}), \Box F_{12}} \text{A45} \\
\hline
- : \Box \Gamma_8, \Delta_{10} \vdash \Box \Gamma_{11}, (\perp, \Delta_{14}), \Box F_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Box \Gamma_8 \vdash F_{12}, \Box \Gamma_{11}}{- : \Delta_{10}, \Box \Gamma_8 \vdash \perp, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}} \text{ax/W} \quad \text{A45}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_7 \vdash \Box \Gamma_{10}, F_9}{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Box \Gamma_{10}, \top, \Delta_{12}), \Box F_9} \text{A45} \quad \frac{\bullet h_{11} : (\Box \Gamma_7, \Delta_8), \Box F_9 \vdash \Box \Gamma_{10}, \top, \Delta_{12}}{\bullet h_{11} : (\Box \Gamma_7, \Delta_8), \Box F_9 \vdash \Box \Gamma_{10}, \top, \Delta_{12}} \text{A45} \\
\hline
- : \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_{10}, \top, \Delta_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_8, \Box \Gamma_7 \vdash \top, \Delta_{12}, \Box \Gamma_{10}}{- : \Delta_8, \Box \Gamma_7 \vdash \top, \Delta_{12}, \Box \Gamma_{10}} \top_R \\
\hline
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{11}, F_{12}, \Box F_{10}}{\bullet h_2 : \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{11}, (\top, \Delta_{14}), \Box F_{12}), \Box F_{10}} \text{A45} \quad \frac{\bullet h_{13} : (\Box \Gamma_8, \Delta_9), \Box F_{10} \vdash \Box \Gamma_{11}, (\top, \Delta_{14}), \Box F_{12}}{\bullet h_{13} : (\Box \Gamma_8, \Delta_9), \Box F_{10} \vdash \Box \Gamma_{11}, (\top, \Delta_{14}), \Box F_{12}} \text{A45} \\
\hline
- : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{11}, (\top, \Delta_{14}), \Box F_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_9, \Box \Gamma_8 \vdash \top, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}}{- : \Delta_9, \Box \Gamma_8 \vdash \top, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}} \top_R \\
\hline
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{11}, F_{12}}{\bullet h_2 : \Box \Gamma_8, \Delta_{10} \vdash (\Box \Gamma_{11}, (\top, \Delta_{14}), \Box F_{12}), F_9} \text{A45} \quad \frac{\bullet h_{13} : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \Box \Gamma_{11}, (\top, \Delta_{14}), \Box F_{12}}{\bullet h_{13} : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \Box \Gamma_{11}, (\top, \Delta_{14}), \Box F_{12}} \text{A45} \\
\hline
- : \Box \Gamma_8, \Delta_{10} \vdash \Box \Gamma_{11}, (\top, \Delta_{14}), \Box F_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_{10}, \Box \Gamma_8 \vdash \top, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}}{- : \Delta_{10}, \Box \Gamma_8 \vdash \top, \Delta_{14}, \Box \Gamma_{11}, \Box F_{12}} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{12}, \Box \Gamma_{14} \vdash (\Box \Gamma_{11}, \Box F_{10}), F_7}{\bullet h_1 : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash ((\Box \Gamma_{11}, \Box F_{10}), \Delta_8), \Box F_7} \text{A45} \quad \frac{h_9 : F_7, \text{unbox}(\Box \Gamma_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), \Box F_7 \vdash (\Box \Gamma_{11}, \Box F_{10}), \Delta_8} \text{A45} \\
\hline
- : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Box F_{10}), \Delta_8 \quad \text{Cut} \quad \rightarrow \quad - : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}
\end{array}$$

Axioms assumed:

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inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula

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$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{12}, \Box \Gamma_{14} \vdash \Box \Gamma_8, F_7}{\bullet h_1 : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_8, \Delta_{11}, \Box F_{10}), \Box F_7} \text{A45} \quad \frac{h_9 : F_7, \text{unbox}(\Box \Gamma_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), \Box F_7 \vdash \Box \Gamma_8, \Delta_{11}, \Box F_{10}} \text{A45} \\
\hline
- : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash \Box \Gamma_8, \Delta_{11}, \Box F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14} \vdash F_{10}, F_7, \Box \Gamma_8}{- : \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14} \vdash F_{10}, F_7, \Box \Gamma_8} \text{ax/W} \quad \frac{- : F_7, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, \text{unbox}(\Box \Gamma_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}, \Box \Gamma_8}{- : F_7, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14} \vdash F_{10}, \Box \Gamma_8} \text{ax/W} \\
\hline
\frac{- : \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14} \vdash F_{10}, \Box \Gamma_8}{- : \Delta_{15}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14} \vdash \Delta_{11}, \Box \Gamma_8, \Box F_{10}} \text{sCut} \quad \text{ATG} \\
\hline
\frac{- : \Delta_{15}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14} \vdash \Delta_{11}, \Box \Gamma_8, \Box F_{10}}{- : \Delta_{15}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14} \vdash \Delta_{11}, \Box \Gamma_8, \Box F_{10}} \text{A45} \\
\hline
\frac{h_1 : \Box \Gamma_{12}, \Box \Gamma_{14} \vdash (\Box \Gamma_{11}, \Box F_{10}), F_7}{\bullet h_1 : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash ((\Box \Gamma_{11}, \Box F_{10}), \Delta_8), \Box F_7} \text{A45} \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), \Box F_7 \vdash (\Box \Gamma_{11}, \Box F_{10}), \Delta_8} \text{A45} \\
\hline
- : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Box F_{10}), \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{- : \Delta_{15}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14} \vdash \Delta_8, \Box \Gamma_{11}, \Box F_{10}} \text{ax/W} \quad \text{K}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \Box\Gamma_8, F_7}{\bullet h_1 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_8, \Delta_{11}, \Box F_{10}), \Box F_7} A45 \quad \frac{h_9 : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box F_7 \vdash \Box\Gamma_8, \Delta_{11}, \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash \Box\Gamma_8, \Delta_{11}, \Box F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash F_{10}} ax/W \\
\frac{}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box\Gamma_8, \Box F_{10}} K
\end{array} \\
\hline
\frac{h_2 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{13}, \Box F_{12}), F_{10}, \Box F_8}{\bullet h_2 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{13}, \Box F_{12}), \Delta_9, \Box F_{10}), \Box F_8} A45 \quad \frac{h_{11} : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}), unbox(\Box F_8) \vdash F_{12}}{\bullet h_{11} : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box F_8 \vdash (\Box\Gamma_{13}, \Box F_{12}), \Delta_9, \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{13}, \Box F_{12}), \Delta_9, \Box F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : unbox(\Box F_8), unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash F_{12}} ax/W \\
\frac{}{\bullet h_{11} : \Box F_8, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_{10}, \Box\Gamma_{13}, \Box F_{12}} K \\
\frac{}{- : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_{10}, \Box\Gamma_{13}, \Box F_{12}} hCut \\
\frac{}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_9, \Box\Gamma_{13}, \Box F_{10}, \Box F_{12}} A45
\end{array} \\
\hline
\frac{h_2 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \Box\Gamma_9, F_{10}, \Box F_8}{\bullet h_2 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_9, (\Delta_{13}, \Box F_{12}), \Box F_{10}), \Box F_8} A45 \quad \frac{h_{11} : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}), unbox(\Box F_8) \vdash F_{12}}{\bullet h_{11} : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box F_8 \vdash \Box\Gamma_9, (\Delta_{13}, \Box F_{12}), \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash \Box\Gamma_9, (\Delta_{13}, \Box F_{12}), \Box F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : unbox(\Box F_8), unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash F_{12}} ax/W \\
\frac{}{\bullet h_{11} : \Box F_8, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_{10}, \Box\Gamma_9, \Box F_{12}} K \\
\frac{}{- : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_{10}, \Box\Gamma_9, \Box F_{12}} hCut \\
\frac{}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_9, \Box F_{10}, \Box F_{12}} A45
\end{array} \\
\hline
\frac{h_2 : \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \Box\Gamma_9, F_{12}, \Box F_8}{\bullet h_2 : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_9, \Delta_{10}, \Box F_{12}), \Box F_8} A45 \quad \frac{h_{11} : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}), unbox(\Box F_8) \vdash F_{12}}{\bullet h_{11} : ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \Box F_8 \vdash \Box\Gamma_9, \Delta_{10}, \Box F_{12}} K \\
\hline
- : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_9, \Delta_{10}, \Box F_{12} \quad \text{Cut} \quad \rightarrow \quad - : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash
\end{array}$$

Axioms assumed:

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inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula

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$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{13}, \Box F_{12}), F_{10}, \Box F_8}{\bullet h_2 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{13}, \Box F_{12}), \Delta_9, \Box F_{10}), \Box F_8} A45 \quad \frac{h_{11} : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash F_{12}}{\bullet h_{11} : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box F_8 \vdash (\Box\Gamma_{13}, \Box F_{12}), \Delta_9, \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{13}, \Box F_{12}), \Delta_9, \Box F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash F_{12}} ax/W \\
\frac{}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_9, \Box\Gamma_{13}, \Box F_{10}, \Box F_{12}} K
\end{array} \\
\hline
\frac{h_2 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \Box\Gamma_9, F_{10}, \Box F_8}{\bullet h_2 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_9, (\Delta_{13}, \Box F_{12}), \Box F_{10}), \Box F_8} A45 \quad \frac{h_{11} : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash F_{12}}{\bullet h_{11} : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box F_8 \vdash \Box\Gamma_9, (\Delta_{13}, \Box F_{12}), \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash \Box\Gamma_9, (\Delta_{13}, \Box F_{12}), \Box F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash F_{12}} ax/W \\
\frac{}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_9, \Box F_{10}, \Box F_{12}} K
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), F_7}{\bullet h_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, [\Box F_9], [\Box F_7])} \text{A45} \quad \frac{h_8 : \Box\Gamma_{14}, \Box\Gamma_{15}, [\Box F_7 \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, F_9]}{\bullet h_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), [\Box F_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, [\Box F_9]]} \\
\frac{}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, [\Box F_9]} \rightarrow \\
\frac{\frac{h_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash F_7, \Box\Gamma_{10}, \Box\Gamma_{12}}{\bullet h_1 : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, [\Box F_7]} \text{A45} \quad \frac{h_8 : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16}, [\Box F_7 \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}]}{\text{hCut}}}{\frac{}{- : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}} \text{A45}} \text{A45} \\
\frac{}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, [\Box F_9]} \\
\frac{h_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, [\Box F_9]), F_7}{\bullet h_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}, [\Box F_9]), \Box\Gamma_{11}, \Delta_{13}), [\Box F_7]} \text{A45} \quad \frac{h_8 : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, F_9}{\bullet h_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), [\Box F_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, [\Box F_9]), \Box\Gamma_{11}, \Delta_{13}]} \\
\frac{}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, [\Box F_9]), \Box\Gamma_{11}, \Delta_{13}} \rightarrow \\
\frac{\frac{}{- : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}}{\text{ax/W}}}{\frac{}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, [\Box F_9]} \text{A45}} \\
\frac{h_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), F_7}{\bullet h_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, [\Box F_9]), [\Box F_7]} \text{A45} \quad \frac{h_8 : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, F_9}{\bullet h_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), [\Box F_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, [\Box F_9]]} \\
\frac{}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, [\Box F_9]} \rightarrow \\
\frac{\frac{}{- : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}}{\text{ax/W}}}{\frac{}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, [\Box F_9]} \text{A45}} \\
\frac{h_2 : \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, [\Box F_{11}]), F_9, \Box F_8}{\bullet h_2 : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}, [\Box F_{11}]), (\Box\Gamma_{13}, \Delta_{15}), [\Box F_9]), \Box F_8} \text{A45} \quad \frac{h_{10} : \Box\Gamma_{16}, \Box\Gamma_{17}, \Box F_8 \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, F_{11}, [\Box F_9]}{\bullet h_{10} : ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_8 \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, [\Box F_{11}]), (\Box\Gamma_{13}, \Delta_{15}), [\Box F_9]} \\
\frac{}{- : (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, [\Box F_{11}]), (\Box\Gamma_{13}, \Delta_{15}), [\Box F_9]}
\end{array}$$

```

inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula

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73

75

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), F_{10}}{\bullet h_2 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), \Box F_{10}), F_8} \quad A45 \quad \frac{h_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, F_{10}}{\bullet h_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18}), F_8 \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), \Box F_{10}} \\
\hline
- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}), \Box F_{10} \\
\hline
\rightarrow \\
\frac{\frac{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash F_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}{- : \Delta_{18}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box F_{10}} \quad ax/W}{- : \Delta_{18}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box F_{10}} \quad A45
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Box\Gamma_9, \Delta_{10}), \Box F_8} \quad A45 \quad \frac{h_{11} : \Box\Gamma_7, \Delta_{14}, \Box F_8 \vdash \Box\Gamma_9, F_{12}, \Delta_{10} \quad h_{11} : \Box\Gamma_7, F_{13}, \Delta_{14}, \Box F_8 \vdash \Box\Gamma_9, \Delta_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13}), \Box F_8 \vdash \Box\Gamma_9, \Delta_{10}} \quad Cut \\
\hline
- : \Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Box\Gamma_9, \Delta_{10} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Box\Gamma_7 \vdash F_8, \Box\Gamma_9}{\bullet h_1 : \Delta_{14}, \Box\Gamma_7 \vdash \Delta_{10}, F_{12}, \Box\Gamma_9, \Box F_8} \quad ax/W \quad A45 \quad \frac{h_{11} : \Delta_{14}, \Box\Gamma_7, \Box F_8 \vdash \Delta_{10}, F_{12}, \Box\Gamma_9}{- : \Delta_{14}, \Box\Gamma_7 \vdash \Delta_{10}, F_{12}, \Box\Gamma_9} \quad ax/W}{- : \Delta_{14}, \Box\Gamma_7, F_{12} \rightarrow F_{13} \vdash \Delta_{10}, \Box\Gamma_9} \quad hCut \\
\hline
- : \Delta_{14}, \Box\Gamma_7, F_{12} \rightarrow F_{13} \vdash \Delta_{10}, \Box\Gamma_9 \\
\hline
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : \Box\Gamma_8, \Delta_{16}, F_{14} \rightarrow F_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, \Box F_{12}), \Box F_9} \quad A45 \quad \frac{h_{13} : \Box\Gamma_8, \Delta_{16}, \Box F_9 \vdash \Box\Gamma_{10}, F_{14}, \Delta_{11}, \Box F_{12} \quad h_{13} : \Box\Gamma_8, F_{15}, \Delta_{16}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{16}, F_{14} \rightarrow F_{15}), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}} \quad Cut \\
\hline
- : \Box\Gamma_8, \Delta_{16}, F_{14} \rightarrow F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : \Box\Gamma_8 \vdash \Box F_9, F_{12}, \Box\Gamma_{10}}{\bullet h_2 : \Delta_{16}, \Box\Gamma_8 \vdash \Box F_9, \Delta_{11}, F_{14}, \Box\Gamma_{10}, \Box F_{12}} \quad ax/W \quad A45 \quad \frac{h_{13} : \Box F_9, \Delta_{16}, \Box\Gamma_8 \vdash \Delta_{11}, F_{14}, \Box\Gamma_{10}, \Box F_{12}}{- : \Delta_{16}, \Box\Gamma_8 \vdash \Delta_{11}, F_{14}, \Box\Gamma_{10}, \Box F_{12}} \quad ax/W}{- : \Delta_{16}, \Box\Gamma_8, F_{14} \rightarrow F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad hCut \\
\hline
- : \Delta_{16}, \Box\Gamma_8, F_{14} \rightarrow F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12} \\
\hline
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, \Box F_{12}), F_{14} \rightarrow F_{15}} \quad A45 \quad \frac{h_{13} : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, F_{14}, \Delta_{11}, \Box F_{12} \quad h_{13} : \Box\Gamma_8, F_{15}, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), F_{14} \rightarrow F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}} \quad Cut \\
\hline
- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12} \\
\hline
\rightarrow \\
\frac{\frac{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad ax/W}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad A45 \\
\hline
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_{16}, F_{14} \rightarrow F_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, \Box F_{12}), F_9} \quad A45 \quad \frac{h_{13} : \Box\Gamma_8, F_9, \Delta_{16} \vdash \Box\Gamma_{10}, F_{14}, \Delta_{11}, \Box F_{12} \quad h_{13} : \Box\Gamma_8, F_9, F_{15}, \Delta_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{16}, F_{14} \rightarrow F_{15}), F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}} \quad Cut \\
\hline
- : \Box\Gamma_8, \Delta_{16}, F_{14} \rightarrow F_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12} \\
\hline
\rightarrow \\
\frac{\frac{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}{- : \Delta_{16}, \Box\Gamma_8, F_{14} \rightarrow F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad ax/W}{- : \Delta_{16}, \Box\Gamma_8, F_{14} \rightarrow F_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad A45
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Box\Gamma_9, \Delta_{10}), \Box F_8} \quad A45 \quad \frac{h_{11} : \Box\Gamma_7, F_{12}, F_{13}, \Delta_{14}, \Box F_8 \vdash \Box\Gamma_9, \Delta_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \wedge F_{13}), \Box F_8 \vdash \Box\Gamma_9, \Delta_{10}} \quad \wedge_L \\
\hline
- : \Box\Gamma_7, \Delta_{14}, F_{12} \wedge F_{13} \vdash \Box\Gamma_9, \Delta_{10} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Box\Gamma_7 \vdash F_8, \Box\Gamma_9}{\bullet h_1 : \Delta_{14}, F_{12}, F_{13}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9, \Box F_8} \quad ax/W \quad A45 \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_{13}, \Box\Gamma_7, \Box F_8 \vdash \Delta_{10}, \Box\Gamma_9}{- : \Delta_{14}, F_{12}, F_{13}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9} \quad ax/W}{- : \Delta_{14}, \Box\Gamma_7, F_{12} \wedge F_{13} \vdash \Delta_{10}, \Box\Gamma_9} \quad hCut \\
\hline
- : \Delta_{14}, \Box\Gamma_7, F_{12} \wedge F_{13} \vdash \Delta_{10}, \Box\Gamma_9 \\
\hline
\wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{13}, [\Box \Gamma_9, F_8]}{\bullet h_1 : (\Box \Gamma_{13}, [\Box \Gamma_9, \Delta_{10}], [\Box F_8])} A45 \quad \frac{h_{11} : \Box \Gamma_{13}, F_{12}, \Delta_7, [\Box F_8, [\Box F_{12} \vdash \Box \Gamma_9, \Delta_{10}]]}{\bullet h_{11} : ((\Box \Gamma_{13}, [\Box F_{12}], \Delta_7), [\Box F_8 \vdash \Box \Gamma_9, \Delta_{10}])} AT \\
\hline
- : (\Box \Gamma_{13}, [\Box F_{12}], \Delta_7 \vdash \Box \Gamma_9, \Delta_{10}) \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_7, F_{12}, \Box \Gamma_{13}, [\Box F_{12} \vdash \Delta_{10}, \Box \Gamma_9, [\Box F_8]]}{- : \Delta_7, F_{12}, \Box \Gamma_{13}, [\Box F_{12} \vdash \Delta_{10}, \Box \Gamma_9]} ax/W \quad \frac{h_{11} : \Delta_7, F_{12}, \Box \Gamma_{13}, [\Box F_{12}, [\Box F_8 \vdash \Delta_{10}, \Box \Gamma_9]]}{- : \Delta_7, \Box \Gamma_{13}, [\Box F_{12} \vdash \Delta_{10}, \Box \Gamma_9]} hCut \\
\hline
- : \Delta_7, \Box \Gamma_{13}, [\Box F_{12} \vdash \Delta_{10}, \Box \Gamma_9] \quad AT \\
\hline
\frac{h_1 : \Box \Gamma_7 \vdash \Box \Gamma_9, F_8}{\bullet h_1 : \Box \Gamma_7, \Delta_{13}, [\Box F_{12} \vdash (\Box \Gamma_9, \Delta_{10}), [\Box F_8]]} A45 \quad \frac{h_{11} : \Box \Gamma_7, F_{12}, \Delta_{13}, [\Box F_8, [\Box F_{12} \vdash \Box \Gamma_9, \Delta_{10}]]}{\bullet h_{11} : (\Box \Gamma_7, \Delta_{13}, [\Box F_{12}], [\Box F_8 \vdash \Box \Gamma_9, \Delta_{10}])} AT \\
\hline
- : \Box \Gamma_7, \Delta_{13}, [\Box F_{12} \vdash \Box \Gamma_9, \Delta_{10}] \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{13}, F_{12}, \Box \Gamma_7, [\Box F_{12} \vdash \Delta_{10}, \Box \Gamma_9, [\Box F_8]]}{- : \Delta_{13}, F_{12}, \Box \Gamma_7, [\Box F_{12} \vdash \Delta_{10}, \Box \Gamma_9]} ax/W \quad \frac{h_{11} : \Delta_{13}, F_{12}, \Box \Gamma_7, [\Box F_{12}, [\Box F_8 \vdash \Delta_{10}, \Box \Gamma_9]]}{- : \Delta_{13}, \Box \Gamma_7, [\Box F_{12} \vdash \Delta_{10}, \Box \Gamma_9]} hCut \\
\hline
- : \Delta_{13}, \Box \Gamma_7, [\Box F_{12} \vdash \Delta_{10}, \Box \Gamma_9] \quad ATG \\
\hline
\frac{h_1 : \Box \Gamma_7 \vdash \Box \Gamma_9, F_{12}}{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Box \Gamma_9, \Delta_{10}), [\Box F_{12}]} A45 \quad \frac{h_{11} : \Box \Gamma_7, F_{12}, \Delta_8, [\Box F_{12} \vdash \Box \Gamma_9, \Delta_{10}]]}{\bullet h_{11} : (\Box \Gamma_7, \Delta_8), [\Box F_{12} \vdash \Box \Gamma_9, \Delta_{10}]} AT \\
\hline
- : \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_9, \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_{10}, F_{12}, \Box \Gamma_9}{\bullet h_1 : \Delta_8, F_{12}, \Box \Gamma_7 \vdash \Delta_{10}, \Box \Gamma_9, [\Box F_{12}]} ax/W \quad \frac{h_{11} : \Delta_8, F_{12}, \Box \Gamma_7, [\Box F_{12} \vdash \Delta_{10}, \Box \Gamma_9]}{- : \Delta_8, F_{12}, \Box \Gamma_7 \vdash \Delta_{10}, \Box \Gamma_9} ax/W \\
\hline
- : \Delta_8, \Box \Gamma_7 \vdash \Delta_{10}, \Box \Gamma_9 \quad sCut \\
\hline
\frac{h_2 : \Box \Gamma_{15}, [\Box F_{14} \vdash \Box \Gamma_{10}, F_{12}, \Box F_9]}{\bullet h_2 : (\Box \Gamma_{15}, [\Box F_{14}], \Delta_8 \vdash (\Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]), \Box F_9)} A45 \quad \frac{h_{13} : \Box \Gamma_{15}, F_{14}, \Delta_8, \Box F_9, [\Box F_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]]}{\bullet h_{13} : ((\Box \Gamma_{15}, [\Box F_{14}]), \Delta_8), \Box F_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} AT \\
\hline
- : (\Box \Gamma_{15}, [\Box F_{14}], \Delta_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]) \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 : \Delta_8, F_{14}, \Box \Gamma_{15}, [\Box F_{14} \vdash \Box F_9, \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]]}{- : \Delta_8, F_{14}, \Box \Gamma_{15}, [\Box F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]]} ax/W \quad \frac{h_{13} : \Box F_9, \Delta_8, F_{14}, \Box \Gamma_{15}, [\Box F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]]}{- : \Delta_8, \Box \Gamma_{15}, [\Box F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]]} hCut \\
\hline
- : \Delta_8, \Box \Gamma_{15}, [\Box F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]] \quad ATG \\
\hline
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : \Box \Gamma_8, \Delta_{15}, [\Box F_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]), \Box F_9]} A45 \quad \frac{h_{13} : \Box \Gamma_8, F_{14}, \Delta_{15}, \Box F_9, [\Box F_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]]}{\bullet h_{13} : (\Box \Gamma_8, \Delta_{15}, [\Box F_{14}], \Box F_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}])} AT \\
\hline
- : \Box \Gamma_8, \Delta_{15}, [\Box F_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]] \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 : \Delta_{15}, F_{14}, \Box \Gamma_8, [\Box F_{14} \vdash \Box F_9, \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]]}{- : \Delta_{15}, F_{14}, \Box \Gamma_8, [\Box F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]]} ax/W \quad \frac{h_{13} : \Box F_9, \Delta_{15}, F_{14}, \Box \Gamma_8, [\Box F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]]}{- : \Delta_{15}, \Box \Gamma_8, [\Box F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]]} hCut \\
\hline
- : \Delta_{15}, \Box \Gamma_8, [\Box F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]] \quad ATG \\
\hline
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12}, [\Box F_{14}]}{\bullet h_2 : \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]), [\Box F_{14}]} A45 \quad \frac{h_{13} : \Box \Gamma_8, F_{14}, \Delta_9, [\Box F_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]]}{\bullet h_{13} : (\Box \Gamma_8, \Delta_9), [\Box F_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]]} AT \\
\hline
- : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}] \quad \text{Cut} \quad \rightarrow \quad - : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}] \quad fail
\end{array}$$

Axioms assumed:

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inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula

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$$\begin{array}{c}
\frac{h_2 : \Box \Gamma_{15}, [\Box F_{14} \vdash \Box \Gamma_{10}, F_{12}]}{\bullet h_2 : (\Box \Gamma_{15}, [\Box F_{14}]), \Delta_9 \vdash (\Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]), F_8} A45 \quad \frac{h_{13} : \Box \Gamma_{15}, F_8, F_{14}, \Delta_9, [\Box F_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]]}{\bullet h_{13} : ((\Box \Gamma_{15}, [\Box F_{14}]), \Delta_9), F_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} AT \\
\hline
- : (\Box \Gamma_{15}, [\Box F_{14}]), \Delta_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}] \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Box \Gamma_{15}, [\Box F_{14} \vdash F_{12}, \Box \Gamma_{10}]}{- : \Delta_9, \Box \Gamma_{15}, [\Box F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]]} ax/W \quad A45
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, \Box F_{12}), \Box F_{14}} \quad A45 \quad \frac{h_{13} : \Box\Gamma_8, F_{14}, \Delta_9, \Box F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), \Box F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}} \quad AT \\
\hline
- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad ax/W}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad A45
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_{15}, \Box F_{14} \vdash (\Box\Gamma_{10}, \Delta_{11}, \Box F_{12}), F_9} \quad A45 \quad \frac{h_{13} : \Box\Gamma_8, F_9, F_{14}, \Delta_{15}, \Box F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}}{\bullet h_{13} : (\Box\Gamma_8, \Delta_{15}, \Box F_{14}), F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}} \quad AT \\
\hline
- : \Box\Gamma_8, \Delta_{15}, \Box F_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}{- : \Delta_{15}, \Box\Gamma_8, \Box F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad ax/W}{- : \Delta_{15}, \Box\Gamma_8, \Box F_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad A45
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \perp, \Delta_{12} \vdash (\Box\Gamma_9, \Delta_{10}), \Box F_8} \quad A45 \quad \frac{\bullet h_{11} : (\Box\Gamma_7, \perp, \Delta_{12}), \Box F_8 \vdash \Box\Gamma_9, \Delta_{10}}{- : \Box\Gamma_7, \perp, \Delta_{12} \vdash \Box\Gamma_9, \Delta_{10}} \quad \perp_L \\
\hline
- : \Box\Gamma_7, \perp, \Delta_{12} \vdash \Box\Gamma_9, \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_{10}, \Box\Gamma_9 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : \Box\Gamma_8, \perp, \Delta_{14} \vdash (\Box\Gamma_{10}, \Delta_{11}, \Box F_{12}), \Box F_9} \quad A45 \quad \frac{\bullet h_{13} : (\Box\Gamma_8, \perp, \Delta_{14}), \Box F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}}{- : \Box\Gamma_8, \perp, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}} \quad \perp_L \\
\hline
- : \Box\Gamma_8, \perp, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12} \quad \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_{14}, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12} \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, \Delta_{11}, \Box F_{12}), \perp} \quad A45 \quad \frac{\bullet h_{13} : (\Box\Gamma_8, \Delta_9), \perp \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}} \quad \perp_L \\
\hline
- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Box\Gamma_8 \vdash F_{12}, \Box\Gamma_{10}}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad ax/W}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12}} \quad A45
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_2 : \Box\Gamma_8, \perp, \Delta_{14} \vdash (\Box\Gamma_{10}, \Delta_{11}, \Box F_{12}), F_9} \quad A45 \quad \frac{\bullet h_{13} : (\Box\Gamma_8, \perp, \Delta_{14}), F_9 \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}}{- : \Box\Gamma_8, \perp, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12}} \quad \perp_L \\
\hline
- : \Box\Gamma_8, \perp, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, \Box F_{12} \quad \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_{14}, \Box\Gamma_8 \vdash \Delta_{11}, \Box\Gamma_{10}, \Box F_{12} \quad \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_7 \vdash \Box\Gamma_9, F_8}{\bullet h_1 : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash (\Box\Gamma_9, \Delta_{12}, p_{11}), \Box F_8} \quad A45 \quad \frac{\bullet h_{10} : (\Box\Gamma_7, \Delta_{13}, p_{11}), \Box F_8 \vdash \Box\Gamma_9, \Delta_{12}, p_{11}}{- : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash \Box\Gamma_9, \Delta_{12}, p_{11}} \quad I \\
\hline
- : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash \Box\Gamma_9, \Delta_{12}, p_{11} \quad \text{Cut} \\
\rightarrow \\
- : \Delta_{13}, \Box\Gamma_7, p_{11} \vdash \Delta_{12}, \Box\Gamma_9, p_{11} \quad I
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{11}, \Box F_9}{\bullet h_2 : \Box\Gamma_8, \Delta_{15}, p_{13} \vdash (\Box\Gamma_{10}, (\Delta_{14}, p_{13}), \Box F_{11}), \Box F_9} \quad A45 \quad \frac{\bullet h_{12} : (\Box\Gamma_8, \Delta_{15}, p_{13}), \Box F_9 \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), \Box F_{11}}{- : \Box\Gamma_8, \Delta_{15}, p_{13} \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), \Box F_{11}} \quad I \\
\hline
- : \Box\Gamma_8, \Delta_{15}, p_{13} \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), \Box F_{11} \quad \text{Cut} \\
\rightarrow \\
- : \Delta_{15}, \Box\Gamma_8, p_{13} \vdash \Delta_{14}, \Box\Gamma_{10}, p_{13}, \Box F_{11} \quad I
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_8 \vdash \Box\Gamma_{10}, F_{11}}{\bullet h_2 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_{10}, (\Delta_{14}, p_{13}), \Box F_{11}), p_{13}} \quad A45 \quad \frac{\bullet h_{12} : (\Box\Gamma_8, \Delta_9), p_{13} \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), \Box F_{11}}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), \Box F_{11}} \quad I \\
\hline
- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_{10}, (\Delta_{14}, p_{13}), \Box F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Box\Gamma_8 \vdash F_{11}, \Box\Gamma_{10}}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{14}, \Box\Gamma_{10}, p_{13}, \Box F_{11}} \quad ax/W}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_{14}, \Box\Gamma_{10}, p_{13}, \Box F_{11}} \quad A45
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{11}}{\bullet h_2 : \Box \Gamma_8, \Delta_{15}, p_{13} \vdash (\Box \Gamma_{10}, (\Delta_{14}, p_{13}), [\Box F_{11}], F_9)} \text{A45} \quad \frac{\bullet h_{12} : (\Box \Gamma_8, \Delta_{15}, p_{13}), F_9 \vdash \Box \Gamma_{10}, (\Delta_{14}, p_{13}), [\Box F_{11}]}{\bullet h_{12} : (\Box \Gamma_8, \Delta_{15}, p_{13}) \vdash \Box \Gamma_{10}, (\Delta_{14}, p_{13}), [\Box F_{11}]} \text{Cut} \\
\frac{}{- : \Box \Gamma_8, \Delta_{15}, p_{13} \vdash \Box \Gamma_{10}, (\Delta_{14}, p_{13}), [\Box F_{11}]} \rightarrow \\
\frac{}{- : \Delta_{15}, \Box \Gamma_8, p_{13} \vdash \Delta_{14}, \Box \Gamma_{10}, p_{13}, [\Box F_{11}]} \text{I}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_7 \vdash \Box \Gamma_9, F_8}{\bullet h_1 : \Box \Gamma_7, \top, \Delta_{12} \vdash (\Box \Gamma_9, \Delta_{10}), [\Box F_8]} \text{A45} \quad \frac{h_{11} : \Box \Gamma_7, \Delta_{12}, [\Box F_8 \vdash \Box \Gamma_9, \Delta_{10}]}{\bullet h_{11} : (\Box \Gamma_7, \top, \Delta_{12}), [\Box F_8 \vdash \Box \Gamma_9, \Delta_{10}]} \top_L \\
\frac{}{- : \Box \Gamma_7, \top, \Delta_{12} \vdash \Box \Gamma_9, \Delta_{10}} \text{Cut} \\
\frac{}{- : \Box \Gamma_7, \top, \Delta_{12} \vdash \Box \Gamma_9, \Delta_{10}} \rightarrow \\
\frac{\bullet h_1 : \top, \Delta_{12}, \Box \Gamma_7 \vdash \Delta_{10}, \Box \Gamma_9, [\Box F_8]}{- : \top, \Delta_{12}, \Box \Gamma_7 \vdash \Delta_{10}, \Box \Gamma_9} \text{ax/W} \quad \frac{h_{11} : \top, \Delta_{12}, \Box \Gamma_7, [\Box F_8 \vdash \Delta_{10}, \Box \Gamma_9]}{- : \top, \Delta_{12}, \Box \Gamma_7 \vdash \Delta_{10}, \Box \Gamma_9} \text{hCut} \\
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12}, \Box F_9}{\bullet h_2 : \Box \Gamma_8, \top, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}], \Box F_9)} \text{A45} \quad \frac{h_{13} : \Box \Gamma_8, \Delta_{14}, \Box F_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]}{\bullet h_{13} : (\Box \Gamma_8, \top, \Delta_{14}), \Box F_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} \top_L \\
\frac{}{- : \Box \Gamma_8, \top, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} \text{Cut} \\
\frac{}{- : \Box \Gamma_8, \top, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} \rightarrow \\
\frac{\bullet h_2 : \top, \Delta_{14}, \Box \Gamma_8 \vdash \Box F_9, \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]}{- : \top, \Delta_{14}, \Box \Gamma_8 \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]} \text{ax/W} \quad \frac{h_{13} : \top, \Box F_9, \Delta_{14}, \Box \Gamma_8 \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]}{- : \top, \Delta_{14}, \Box \Gamma_8 \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]} \text{hCut} \\
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12}}{\bullet h_2 : \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}], \top)} \text{A45} \quad \frac{h_{13} : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]}{\bullet h_{13} : (\Box \Gamma_8, \Delta_9), \top \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} \top_L \\
\frac{}{- : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} \text{Cut} \\
\frac{}{- : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} \rightarrow \\
\frac{}{- : \Delta_9, \Box \Gamma_8 \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]} \text{ax/W} \\
\frac{h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12}}{\bullet h_2 : \Box \Gamma_8, \top, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}], F_9)} \text{A45} \quad \frac{h_{13} : \Box \Gamma_8, F_9, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]}{\bullet h_{13} : (\Box \Gamma_8, \top, \Delta_{14}), F_9 \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} \top_L \\
\frac{}{- : \Box \Gamma_8, \top, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} \text{Cut} \\
\frac{}{- : \Box \Gamma_8, \top, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [\Box F_{12}]} \rightarrow \\
\frac{}{- : \Box \Gamma_8 \vdash F_{12}, \Box \Gamma_{10}} \text{ax/W} \\
\frac{}{- : \top, \Delta_{14}, \Box \Gamma_8 \vdash \Delta_{11}, \Box \Gamma_{10}, [\Box F_{12}]} \text{A45}
\end{array}$$

6.8 Status of \rightarrow_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_3 : \Delta_8 \vdash F_7, F_9, \Delta_{12}, F_{13} \rightarrow F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \rightarrow F_{14}}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_7} \rightarrow_L \quad \frac{h_{11} : F_7, F_{13}, \Delta_8, F_9 \rightarrow F_{10} \vdash F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R \\
\frac{}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \text{Cut} \\
\frac{}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow \\
\frac{h_3 : \Delta_8, F_{13} \vdash \Delta_{12}, F_{14}, F_7, F_9}{\bullet h_3 : \Delta_8, F_{13}, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{14}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_8, F_{10}, F_{13} \vdash \Delta_{12}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_{13}, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{14}, F_7} \text{inv-th/ax} \\
\frac{}{- : \Delta_8, F_{13}, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{14}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_{13}, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{14}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \text{hCut} \\
\frac{}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_3 : \Delta_8 \vdash F_7, F_9, \Delta_{12}, F_{13} \wedge F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \wedge F_{14}}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_7} \rightarrow_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \rightarrow F_{10} \vdash F_{13}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \rightarrow_R \\
\frac{}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{Cut} \\
\frac{}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \rightarrow \\
\frac{h_3 : \Delta_8 \vdash \Delta_{12}, F_{13}, F_7, F_9}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_7}{\bullet h_3 : \Delta_8, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}} \text{inv-th/ax} \\
\frac{}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{hCut} \\
\frac{}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \rightarrow_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_3 : \Delta_8 \vdash F_7, F_9, \Delta_{12}, F_{13} \vee F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14} \rightarrow_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \rightarrow F_{10} \vdash F_{13}, F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R}{\frac{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_7}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \rightarrow \quad \frac{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \Delta_{12}, F_{13} \vee F_{14}}{\text{Cut}} \vee_R} \\
\frac{\frac{h_3 : \Delta_8 \vdash \Delta_{12}, F_{13}, F_{14}, F_7, F_9}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \text{inv-th/ax} \rightarrow_L \quad \frac{h_{11} : \Delta_8, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_8, F_7, F_9 \rightarrow F_{10}), F_7 \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W}}{\frac{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_3 : \Delta_8 \vdash F_7, F_9, \perp, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \perp, \Delta_{12} \rightarrow_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \perp, \Delta_{12}} \perp_R}{\frac{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\perp, \Delta_{12}), F_7}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}} \rightarrow \quad \frac{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \perp, \Delta_{12}}{\text{Cut}} \perp_R} \\
\frac{\frac{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}, F_7}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_7, F_9 \rightarrow F_{10}), F_7 \vdash \perp, \Delta_{12}} \text{ax/W}}{\frac{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}} \text{hCut}}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_3 : \Delta_8 \vdash F_7, F_9, \top, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \top, \Delta_{12} \rightarrow_L \quad \frac{h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \top, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \top, \Delta_{12}} \top_R}{\frac{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\top, \Delta_{12}), F_7}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \top, \Delta_{12}} \rightarrow \quad \frac{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \top, \Delta_{12}}{\text{Cut}} \top_R} \\
\frac{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \top, \Delta_{12}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \top, \Delta_{12}} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_3 : \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, F_8, \Delta_{11}, \llbracket F_{12} \quad h_3 : F_9, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, \llbracket F_{12} \rightarrow_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : ((\Box \Gamma_{13}, \Delta_{14}), F_8 \rightarrow F_9), \Box F_7 \vdash \Delta_{11}, \llbracket F_{12}} \rightarrow_R}{\frac{\bullet h_3 : (\Box \Gamma_{13}, \Delta_{14}), F_8 \rightarrow F_9 \vdash (\Delta_{11}, \llbracket F_{12}), \Box F_7}{- : (\Box \Gamma_{13}, \Delta_{14}), F_8 \rightarrow F_9 \vdash \Delta_{11}, \llbracket F_{12}} \rightarrow \quad \frac{\bullet h_{10} : ((\Box \Gamma_{13}, \Delta_{14}), F_8 \rightarrow F_9), \Box F_7 \vdash \Delta_{11}, \llbracket F_{12}}{\text{Cut}} \rightarrow_R} \\
\frac{\frac{h_3 : \Delta_{14}, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, F_8, \llbracket F_{12}}{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \llbracket F_{12}} \text{ax/W} \quad \frac{\frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{13}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \llbracket F_{12}} \text{ax/W} \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \llbracket F_{12}} K}{\frac{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \llbracket F_{12}}{- : \Delta_{14}, \Box \Gamma_{13}, F_8 \rightarrow F_9 \vdash \Delta_{11}, \llbracket F_{12}} \text{hCut} \quad \frac{- : \Delta_{14}, F_9, \Box \Gamma_{13} \vdash \Delta_{11}, \llbracket F_{12}}{- : \Delta_{14}, F_9, \Box \Gamma_{13} \vdash \Delta_{11}, \llbracket F_{12}} \text{ax/W}} \\
\frac{h_3 : \Box \Gamma_{11}, \Delta_{14} \vdash F_7, F_8, \Delta_{12}, \llbracket F_{13} \quad h_3 : F_9, \Box \Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, \llbracket F_{13} \rightarrow_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}}{\bullet h_{10} : ((\Box \Gamma_{11}, \Delta_{14}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{12}, \llbracket F_{13}} K}{\frac{\bullet h_3 : (\Box \Gamma_{11}, \Delta_{14}), F_8 \rightarrow F_9 \vdash (\Delta_{12}, \llbracket F_{13}), F_7}{- : (\Box \Gamma_{11}, \Delta_{14}), F_8 \rightarrow F_9 \vdash \Delta_{12}, \llbracket F_{13}} \rightarrow \quad \frac{\bullet h_{10} : ((\Box \Gamma_{11}, \Delta_{14}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{12}, \llbracket F_{13}}{\text{Cut}} \rightarrow_R} \\
\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}}{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, \llbracket F_{13}} \text{ax/W} \quad \frac{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, \llbracket F_{13}}{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, \llbracket F_{13}} K
\end{array}$$

- Case rule $A45$

$$\begin{array}{c}
\frac{h_3 : \Box \Gamma_{14}, \Delta_{15} \vdash \Box F_7, F_8, \Box \Gamma_{11}, \Delta_{12}, \llbracket F_{13} \quad h_3 : F_9, \Box \Gamma_{14}, \Delta_{15} \vdash \Box F_7, \Box \Gamma_{11}, \Delta_{12}, \llbracket F_{13} \rightarrow_L \quad \frac{h_{10} : \Box \Gamma_{14}, \Box F_7 \vdash F_{13}}{\bullet h_{10} : ((\Box \Gamma_{14}, \Delta_{15}), F_8 \rightarrow F_9), \Box F_7 \vdash \Box \Gamma_{11}, \Delta_{12}, \llbracket F_{13}} \rightarrow_R}{\frac{\bullet h_3 : (\Box \Gamma_{14}, \Delta_{15}), F_8 \rightarrow F_9 \vdash (\Box \Gamma_{11}, \Delta_{12}, \llbracket F_{13}), \Box F_7}{- : (\Box \Gamma_{14}, \Delta_{15}), F_8 \rightarrow F_9 \vdash \Box \Gamma_{11}, \Delta_{12}, \llbracket F_{13}} \rightarrow \quad \frac{\bullet h_{10} : ((\Box \Gamma_{14}, \Delta_{15}), F_8 \rightarrow F_9), \Box F_7 \vdash \Box \Gamma_{11}, \Delta_{12}, \llbracket F_{13}}{\text{Cut}} \rightarrow_R} \\
\frac{\frac{h_3 : \Delta_{15}, \Box \Gamma_{14} \vdash \Box F_7, \Delta_{12}, F_8, \Box \Gamma_{11}, \llbracket F_{13}}{- : \Delta_{15}, \Box \Gamma_{14} \vdash \Delta_{12}, F_8, \Box \Gamma_{11}, \llbracket F_{13}} \text{ax/W} \quad \frac{\frac{h_{10} : \Box F_7, \Box \Gamma_{14} \vdash F_{13}, \Box \Gamma_{11}}{\bullet h_{10} : \Box F_7, \Delta_{15}, \Box \Gamma_{14} \vdash \Delta_{12}, F_8, \Box \Gamma_{11}, \llbracket F_{13}} \text{ax/W} \quad \frac{h_{10} : \Box \Gamma_{14}, \Box F_7 \vdash F_{13}}{\bullet h_{10} : \Box F_7, \Delta_{15}, \Box \Gamma_{14} \vdash \Delta_{12}, F_8, \Box \Gamma_{11}, \llbracket F_{13}} A45}{\frac{- : \Delta_{15}, \Box \Gamma_{14} \vdash \Delta_{12}, F_8, \Box \Gamma_{11}, \llbracket F_{13}}{- : \Delta_{15}, \Box \Gamma_{14}, F_8 \rightarrow F_9 \vdash \Delta_{12}, \Box \Gamma_{11}, \llbracket F_{13}} \text{hCut} \quad \frac{- : \Delta_{15}, F_9, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, \llbracket F_{13}}{- : \Delta_{15}, F_9, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, \llbracket F_{13}} \text{ax/W}}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : \Box\Gamma_{11}, \Delta_{15} \vdash F_7, F_8, \Box\Gamma_{12}, \Delta_{13}, \Box F_{14} \quad h_3 : F_9, \Box\Gamma_{11}, \Delta_{15} \vdash F_7, \Box\Gamma_{12}, \Delta_{13}, \Box F_{14}}{\bullet h_3 : (\Box\Gamma_{11}, \Delta_{15}), F_8 \rightarrow F_9 \vdash (\Box\Gamma_{12}, \Delta_{13}, \Box F_{14}), F_7} \rightarrow_L \quad \frac{h_{10} : \Box\Gamma_{11} \vdash \Box\Gamma_{12}, F_{14}}{\bullet h_{10} : ((\Box\Gamma_{11}, \Delta_{15}), F_8 \rightarrow F_9), F_7 \vdash \Box\Gamma_{12}, \Delta_{13}, \Box F_{14}} \text{A45} \\
\frac{\bullet h_3 : (\Box\Gamma_{11}, \Delta_{15}), F_8 \rightarrow F_9 \vdash (\Box\Gamma_{12}, \Delta_{13}, \Box F_{14}), F_7}{- : (\Box\Gamma_{11}, \Delta_{15}), F_8 \rightarrow F_9 \vdash \Box\Gamma_{12}, \Delta_{13}, \Box F_{14}} \text{Cut} \\
\frac{\frac{- : \Box\Gamma_{11} \vdash F_{14}, \Box\Gamma_{12}}{- : \Delta_{15}, \Box\Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}, \Box\Gamma_{12}, \Box F_{14}} \text{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}, \Box\Gamma_{12}, \Box F_{14}} \text{A45}
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_3 : \Delta_7 \vdash F_{11} \rightarrow F_{12}, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \rightarrow F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \rightarrow_L \quad \frac{h_{10} : \Delta_7, F_8 \rightarrow F_9 \vdash F_{11}, \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \text{Cut} \\
\frac{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \rightarrow \\
\frac{\frac{- : \Delta_7, F_{11} \vdash \Delta_{13}, F_{12}, F_8}{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{12}} \text{inv-th/ax} \quad \frac{- : \Delta_7, F_{11}, F_9 \vdash \Delta_{13}, F_{12}}{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{12}} \text{inv-th/ax}}{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{12}} \rightarrow_L \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{ax/W} \quad \frac{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{sCut} \\
\frac{h_3 : \Delta_{14}, F_{11} \rightarrow F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \rightarrow F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7} \rightarrow_L \quad \frac{h_{10} : F_7, \Delta_{14}, F_8 \rightarrow F_9 \vdash F_{11}, \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \rightarrow F_9) \vdash \Delta_{13}} \text{Cut} \\
\frac{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7}{- : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}} \rightarrow \\
\frac{h_3 : \Delta_{14} \vdash \Delta_{13}, F_{11}, F_7, F_8}{\bullet h_3 : \Delta_{14}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_{14}, F_9 \vdash \Delta_{13}, F_{11}, F_7}{h_{10} : \Delta_{14}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \text{inv-th/ax} \rightarrow_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}}{h_{10} : \Delta_{14}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \text{ax/W} \\
\frac{\bullet h_3 : \Delta_{14}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}, F_7}{- : \Delta_{14}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \text{hCut} \quad \frac{h_3 : \Delta_{14}, F_{12} \vdash \Delta_{13}, F_7, F_8}{\bullet h_3 : \Delta_{14}, F_{12} \vdash \Delta_{13}, F_7, F_8} \text{inv-th/ax} \\
\frac{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}} \rightarrow_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_3 : \Delta_7 \vdash F_{11} \wedge F_{12}, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \wedge F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \rightarrow_L \quad \frac{h_{10} : F_{11}, F_{12}, \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L \\
\frac{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{Cut} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \rightarrow \\
\frac{h_3 : \Delta_7 \vdash \Delta_{13}, F_8, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_{11}, F_{12} \vdash \Delta_{13}, F_8} \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12} \vdash \Delta_{13}, F_8}{\bullet h_{10} : \Delta_7, F_{11} \wedge F_{12} \vdash \Delta_{13}, F_8} \text{inv-th/ax} \wedge_L \\
\frac{\bullet h_3 : \Delta_7, F_{11}, F_{12} \vdash \Delta_{13}, F_8}{- : \Delta_7 \vdash \Delta_{13}, F_8} \text{hCut} \quad \frac{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \text{ax/W} \wedge_L \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \rightarrow_L \\
\frac{h_3 : \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7} \rightarrow_L \quad \frac{h_{10} : F_7, F_{11}, F_{12}, \Delta_{14}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{13}} \wedge_L \\
\frac{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7}{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{Cut} \\
\frac{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_{11}, F_{12} \vdash \Delta_{13}, F_7} \rightarrow \\
\frac{h_3 : \Delta_{14}, F_{11}, F_{12} \vdash \Delta_{13}, F_7, F_8}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_9 \vdash \Delta_{13}, F_7}{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{inv-th/ax} \rightarrow_L \\
\frac{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7}{- : \Delta_{14}, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{hCut} \quad \frac{- : \Delta_{14}, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_8 \rightarrow F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_3 : \Delta_7 \vdash F_{11} \vee F_{12}, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \vee F_{12}, \Delta_{13} \rightarrow_L \quad \frac{h_{10} : F_{11}, \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \vee_L}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \rightarrow_L \quad \frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \text{Cut} \\
\frac{- : \Delta_7 \vdash \Delta_{13}, F_{11}, F_{12}, F_8}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \text{inv-th/ax} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{13}, F_{11}, F_{12}}{- : \Delta_7, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \text{inv-th/ax} \rightarrow_L \quad \frac{- : \Delta_7, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \text{ax/W} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{sCut} \quad \frac{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{ax/sCut} \\
\frac{h_3 : \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13} \rightarrow_L \quad \frac{h_{10} : F_7, F_{11}, \Delta_{14}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7}} \rightarrow_L \quad \frac{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee F_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{13}} \text{Cut} \\
\frac{h_3 : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7, F_8}{- : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \text{ax/W} \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_7 \vdash \Delta_{13}, F_8}{\bullet h_{10} : \Delta_{14}, F_7, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_{13}, F_8}{- : \Delta_{14}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_8} \text{inv-th/ax} \rightarrow_L \\
\frac{- : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8}{- : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \text{hCut} \quad \frac{h_3 : \Delta_{14}, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}}{- : \Delta_{14}, F_8 \rightarrow F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{ax/W}
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{h_3 : \Delta_7 \vdash [F_{11}, F_8, \Delta_{12} \quad h_3 : F_9, \Delta_7 \vdash [F_{11}, \Delta_{12} \rightarrow_L \quad \frac{h_{10} : F_{11}, \Delta_7, [F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12} \quad \bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), [F_{11} \vdash \Delta_{12}]}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), [F_{11} \vdash \Delta_{12}]} \text{AT}}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}, [F_{11}]} \rightarrow_L \quad \frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), [F_{11} \vdash \Delta_{12}]} \text{Cut} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), [F_{11} \vdash \Delta_{12}]} \text{inv-th/ax} \rightarrow_L \\
\frac{h_3 : \Delta_7 \vdash \Delta_{12}, F_8, [F_{11}]}{- : \Delta_7 \vdash \Delta_{12}, F_8} \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, [F_{11} \vdash \Delta_{12}, F_8]}{\bullet h_{10} : \Delta_7, [F_{11} \vdash \Delta_{12}, F_8]} \text{inv-th/ax} \quad \frac{- : \Delta_7, F_{11}, F_9, [F_{11} \vdash \Delta_{12}]}{\bullet h_{10} : \Delta_7, F_9, [F_{11} \vdash \Delta_{12}]} \text{inv-th/ax} \rightarrow_L \\
\frac{- : \Delta_7, F_9 \vdash \Delta_{12}, [F_{11}]}{- : \Delta_7, F_9 \vdash \Delta_{12}} \text{ax/W} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{12}}{\bullet h_{10} : \Delta_7, F_9, [F_{11} \vdash \Delta_{12}]} \text{AT} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{12}}{\bullet h_{10} : \Delta_7, F_9, [F_{11} \vdash \Delta_{12}]} \text{hCut} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), [F_{11} \vdash \Delta_{12}]} \text{AT} \\
\frac{h_3 : \Delta_{13}, [F_{11} \vdash F_7, F_8, \Delta_{12} \quad h_3 : F_9, \Delta_{13}, [F_{11} \vdash F_7, \Delta_{12} \rightarrow_L \quad \frac{h_{10} : F_7, F_{11}, \Delta_{13}, [F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12} \quad \bullet h_{10} : ((\Delta_{13}, [F_{11}]), F_8 \rightarrow F_9), F_7 \vdash \Delta_{12}}{\bullet h_{10} : ((\Delta_{13}, [F_{11}]), F_8 \rightarrow F_9), F_7 \vdash \Delta_{12}} \text{AT}}{\bullet h_3 : (\Delta_{13}, [F_{11}]), F_8 \rightarrow F_9 \vdash \Delta_{12}, F_7}} \rightarrow_L \quad \frac{- : (\Delta_{13}, [F_{11}]), F_8 \rightarrow F_9 \vdash \Delta_{12}}{\bullet h_{10} : ((\Delta_{13}, [F_{11}]), F_8 \rightarrow F_9), F_7 \vdash \Delta_{12}} \text{Cut} \\
\frac{- : (\Delta_{13}, [F_{11}]), F_8 \rightarrow F_9 \vdash \Delta_{12}}{\bullet h_3 : \Delta_{13}, F_{11}, [F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, F_7]} \text{ax/W} \quad \frac{h_{10} : \Delta_{13}, F_{11}, F_7, [F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}]}{\bullet h_{10} : \Delta_{13}, F_{11}, F_7, [F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}]} \text{ax/W} \rightarrow_L \\
\frac{- : \Delta_{13}, F_{11}, [F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, F_7]}{- : \Delta_{13}, [F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}]} \text{hCut} \quad \frac{- : \Delta_{13}, [F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}]}{- : \Delta_{13}, [F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}]} \text{AT}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_3 : \Delta_7 \vdash \perp, F_8, \Delta_{11} \quad h_3 : F_9, \Delta_7 \vdash \perp, \Delta_{11} \rightarrow_L \quad \frac{h_{10} : (\Delta_7, F_8 \rightarrow F_9), \perp \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), \perp \vdash \Delta_{11}} \perp_L}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}, \perp} \rightarrow_L \quad \frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), \perp \vdash \Delta_{11}} \text{Cut} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), \perp \vdash \Delta_{11}} \text{inv-th/ax} \rightarrow_L \\
\frac{h_3 : \Delta_7 \vdash \perp, \Delta_{11}, F_8}{- : \Delta_7 \vdash \perp, \Delta_{11}, F_8} \text{ax/W} \quad \frac{h_{10} : \perp, \Delta_7 \vdash \Delta_{11}, F_8}{\bullet h_{10} : \perp, \Delta_7 \vdash \Delta_{11}, F_8} \perp_L \quad \frac{h_3 : \Delta_7, F_9 \vdash \perp, \Delta_{11}}{- : \Delta_7, F_9 \vdash \perp, \Delta_{11}} \text{ax/W} \quad \frac{h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}}{\bullet h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}} \perp_L \rightarrow_L \\
\frac{- : \Delta_7 \vdash \perp, \Delta_{11}, F_8}{- : \Delta_7 \vdash \perp, \Delta_{11}, F_8} \text{hCut} \quad \frac{- : \Delta_7, F_9 \vdash \perp, \Delta_{11}}{- : \Delta_7, F_9 \vdash \perp, \Delta_{11}} \text{hCut} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), \perp \vdash \Delta_{11}} \text{AT} \\
\frac{h_3 : \perp, \Delta_{12} \vdash F_7, F_8, \Delta_{11} \quad h_3 : F_9, \perp, \Delta_{12} \vdash F_7, \Delta_{11} \rightarrow_L \quad \frac{h_{10} : ((\perp, \Delta_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{11}}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{11}} \perp_L}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}, F_7}} \rightarrow_L \quad \frac{- : (\perp, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{11}} \text{Cut} \\
\frac{- : (\perp, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}, F_7}} \text{inv-th/ax} \rightarrow_L \\
\frac{- : \perp, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}}{- : \perp, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_7} \wedge_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \wedge F_{10} \vdash F_{13}, F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_8, F_{10}, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W}}{\frac{- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \perp, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\perp, \Delta_{12}), F_7} \wedge_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \perp, \Delta_{12}} \perp_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12} \\
\rightarrow \\
\frac{\frac{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}, F_7}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \perp, \Delta_{12}} \text{ax/W}}{\frac{- : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}{- : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}} \text{hCut}}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \top, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\top, \Delta_{12}), F_7} \wedge_L \quad \frac{h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \top, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \top, \Delta_{12}} \top_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \top, \Delta_{12} \\
\rightarrow \\
\frac{- : \Delta_8, F_9 \wedge F_{10} \vdash \top, \Delta_{12}}{- : \Delta_8, F_9 \wedge F_{10} \vdash \top, \Delta_{12}} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}]}{\bullet h_3 : (\Box \Gamma_{13}, \Delta_{14}), F_8 \wedge F_9 \vdash (\Delta_{11}, [\Box F_{12}]), \Box F_7} \wedge_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : ((\Box \Gamma_{13}, \Delta_{14}), F_8 \wedge F_9), \Box F_7 \vdash \Delta_{11}, [\Box F_{12}]} K \\
\hline
- : (\Box \Gamma_{13}, \Delta_{14}), F_8 \wedge F_9 \vdash \Delta_{11}, [\Box F_{12}] \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_8, F_9, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}]}{\bullet h_3 : \Delta_{14}, F_8, F_9, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}]} \text{ax/W} \quad \frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{13}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{14}, F_8, F_9, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{12}]} K}{\frac{- : \Delta_{14}, F_8, F_9, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}]}{- : \Delta_{14}, \Box \Gamma_{13}, F_8 \wedge F_9 \vdash \Delta_{11}, [\Box F_{12}]} \wedge_L} \text{hCut} \\
\rightarrow \\
\frac{h_3 : F_8, F_9, \Box \Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, [\Box F_{13}]}{\bullet h_3 : (\Box \Gamma_{11}, \Delta_{14}), F_8 \wedge F_9 \vdash (\Delta_{12}, [\Box F_{13}]), F_7} \wedge_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}}{\bullet h_{10} : ((\Box \Gamma_{11}, \Delta_{14}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, [\Box F_{13}]} K \\
\hline
- : (\Box \Gamma_{11}, \Delta_{14}), F_8 \wedge F_9 \vdash \Delta_{12}, [\Box F_{13}] \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}}{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, [\Box F_{13}]} \text{ax/W} \\
\frac{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, [\Box F_{13}]}{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, [\Box F_{13}]} K
\end{array}$$

- Case rule $A45$

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Box \Gamma_{14}, \Delta_{15} \vdash \Box F_7, \Box \Gamma_{11}, \Delta_{12}, [\Box F_{13}]}{\bullet h_3 : (\Box \Gamma_{14}, \Delta_{15}), F_8 \wedge F_9 \vdash (\Box \Gamma_{11}, \Delta_{12}, [\Box F_{13}]), \Box F_7} \wedge_L \quad \frac{h_{10} : \Box \Gamma_{14}, \Box F_7 \vdash \Box \Gamma_{11}, F_{13}}{\bullet h_{10} : ((\Box \Gamma_{14}, \Delta_{15}), F_8 \wedge F_9), \Box F_7 \vdash \Box \Gamma_{11}, \Delta_{12}, [\Box F_{13}]} A45 \\
\hline
- : (\Box \Gamma_{14}, \Delta_{15}), F_8 \wedge F_9 \vdash \Box \Gamma_{11}, \Delta_{12}, [\Box F_{13}] \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{15}, F_8, F_9, \Box \Gamma_{14} \vdash \Box F_7, \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]}{\bullet h_3 : \Delta_{15}, F_8, F_9, \Box \Gamma_{14} \vdash \Box F_7, \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]} \text{ax/W} \quad \frac{h_{10} : \Box F_7, \Box \Gamma_{14} \vdash F_{13}, \Box \Gamma_{11}}{\bullet h_{10} : \Box F_7, \Delta_{15}, F_8, F_9, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]} A45}{\frac{- : \Delta_{15}, F_8, F_9, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]}{- : \Delta_{15}, \Box \Gamma_{14}, F_8 \wedge F_9 \vdash \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]} \wedge_L} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Box\Gamma_{11}, \Delta_{15} \vdash F_7, \Box\Gamma_{12}, \Delta_{13}, \Box F_{14}}{\bullet h_3 : (\Box\Gamma_{11}, \Delta_{15}), F_8 \wedge F_9 \vdash (\Box\Gamma_{12}, \Delta_{13}, \Box F_{14}), F_7} \wedge_L \quad \frac{h_{10} : \Box\Gamma_{11} \vdash \Box\Gamma_{12}, F_{14}}{\bullet h_{10} : ((\Box\Gamma_{11}, \Delta_{15}), F_8 \wedge F_9), F_7 \vdash \Box\Gamma_{12}, \Delta_{13}, \Box F_{14}} A45 \\
\hline
- : (\Box\Gamma_{11}, \Delta_{15}), F_8 \wedge F_9 \vdash \Box\Gamma_{12}, \Delta_{13}, \Box F_{14} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Box\Gamma_{11} \vdash F_{14}, \Box\Gamma_{12}}{- : \Delta_{15}, \Box\Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{13}, \Box\Gamma_{12}, \Box F_{14}} \text{ax/W} \quad A45
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Delta_7 \vdash F_{11} \rightarrow F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \wedge_L \quad \frac{h_{10} : \Delta_7, F_8 \wedge F_9 \vdash F_{11}, \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \rightarrow_L \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : \Delta_7, F_{12}, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \text{inv-th/ax} \rightarrow_L \\
\hline
- : \Delta_7, F_8, F_9 \vdash \Delta_{13} \quad \text{hCut} \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \quad \wedge_L \\
\hline
\frac{h_3 : F_8, F_9, \Delta_{14}, F_{11} \rightarrow F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \wedge_L \quad \frac{h_{10} : F_7, \Delta_{14}, F_8 \wedge F_9 \vdash F_{11}, \Delta_{13} \quad h_{10} : F_7, F_{12}, \Delta_{14}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \rightarrow_L \\
\hline
- : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9 \vdash \Delta_{13} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_3 : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_7} \text{ax/W} \quad \frac{h_{10} : \Delta_{14}, F_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : \Delta_{14}, F_{12}, F_7, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \text{inv-th/ax} \rightarrow_L \\
\hline
- : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13} \quad \text{hCut} \\
\hline
- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \wedge F_9 \vdash \Delta_{13} \quad \wedge_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Delta_7 \vdash F_{11} \wedge F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \wedge_L \quad \frac{h_{10} : F_{11}, F_{12}, \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8, F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_7, F_8, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \text{inv-th/ax} \wedge_L \\
\hline
- : \Delta_7, F_8, F_9 \vdash \Delta_{13} \quad \text{hCut} \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \quad \wedge_L \\
\hline
\frac{h_3 : F_8, F_9, \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \wedge_L \quad \frac{h_{10} : F_7, F_{11}, F_{12}, \Delta_{14}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \wedge_L \\
\hline
- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9 \vdash \Delta_{13} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_8, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax} \wedge_L \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{ax/W} \\
\hline
- : \Delta_{14}, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13} \quad \text{hCut} \\
\hline
- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \wedge F_9 \vdash \Delta_{13} \quad \wedge_L \\
\hline
\frac{h_3 : F_{10}, F_{11}, \Delta_8 \vdash F_7, \Delta_{12}}{\bullet h_3 : \Delta_8, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7} \wedge_L \quad \frac{h_9 : F_7, F_{10}, F_{11}, \Delta_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_8, F_{10} \wedge F_{11}), F_7 \vdash \Delta_{12}} \wedge_L \\
\hline
- : \Delta_8, F_{10} \wedge F_{11} \vdash \Delta_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_3 : \Delta_8, F_{10}, F_{11} \vdash \Delta_{12}, F_7}{\bullet h_3 : \Delta_8, F_{10}, F_{11} \vdash \Delta_{12}, F_7} \text{ax/W} \quad \frac{h_9 : \Delta_8, F_{10}, F_{11}, F_7 \vdash \Delta_{12}}{\bullet h_9 : \Delta_8, F_{10}, F_{11}, F_7 \vdash \Delta_{12}} \text{H} \\
\hline
- : \Delta_8, F_{10}, F_{11} \vdash \Delta_{12} \quad \text{hCut} \\
\hline
- : \Delta_8, F_{10} \wedge F_{11} \vdash \Delta_{12} \quad \wedge_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, F_9, \Delta_7 \vdash F_{11} \vee F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \wedge_L \quad \frac{h_{10} : F_{11}, \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \vee_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_7, F_{11}, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_7, F_{12}, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_7, F_8, F_9 \vdash \Delta_{13} \quad - : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8, F_9 \vdash \Delta_{13}} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_3 : F_8, F_9, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \wedge_L \quad \frac{h_{10} : F_7, F_{11}, \Delta_{14}, F_8 \wedge F_9 \vdash \Delta_{13} \quad h_{10} : F_7, F_{12}, \Delta_{14}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \vee_L}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_{14}, F_{11}, F_7, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{14}, F_{12}, F_7, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13} \quad - : \Delta_{14}, F_8 \wedge F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{hCut}
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, F_9, \Delta_7 \vdash [F_{11}, \Delta_{12}]}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, [F_{11}]} \wedge_L \quad \frac{h_{10} : F_{11}, \Delta_7, [F_{11}, F_8 \wedge F_9] \vdash \Delta_{12}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), [F_{11} \vdash \Delta_{12}]} AT}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{12}, [F_{11}]}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{12}, [F_{11}]} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_7, F_{11}, F_8, F_9, [F_{11} \vdash \Delta_{12}]}{\bullet h_{10} : \Delta_7, F_8, F_9, [F_{11} \vdash \Delta_{12}]} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_7, F_{11}, F_8, F_9, [F_{11} \vdash \Delta_{12}]}{\bullet h_{10} : \Delta_7, F_8, F_9, [F_{11} \vdash \Delta_{12}]} AT}{- : \Delta_7, F_8, F_9 \vdash \Delta_{12} \quad - : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}} \wedge_L}{- : \Delta_7, F_8, F_9 \vdash \Delta_{12}} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_3 : F_8, F_9, \Delta_{13}, [F_{11} \vdash F_7, \Delta_{12}]}{\bullet h_3 : (\Delta_{13}, [F_{11}]), F_8 \wedge F_9 \vdash \Delta_{12}, F_7} \wedge_L \quad \frac{h_{10} : F_7, F_{11}, \Delta_{13}, [F_{11}, F_8 \wedge F_9] \vdash \Delta_{12}}{\bullet h_{10} : ((\Delta_{13}, [F_{11}]), F_8 \wedge F_9), F_7 \vdash \Delta_{12}} AT}{- : (\Delta_{13}, [F_{11}]), F_8 \wedge F_9 \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_3 : \Delta_{13}, F_{11}, [F_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, F_7]}{\bullet h_3 : \Delta_{13}, F_{11}, [F_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, F_7]} \text{ax/W} \quad \frac{h_{10} : \Delta_{13}, F_{11}, F_7, [F_{11}, F_8 \wedge F_9 \vdash \Delta_{12}]}{\bullet h_{10} : \Delta_{13}, F_{11}, F_7, [F_{11}, F_8 \wedge F_9 \vdash \Delta_{12}]} \text{ax/W}}{- : \Delta_{13}, F_{11}, [F_{11}, F_8 \wedge F_9 \vdash \Delta_{12}] \quad - : \Delta_{13}, [F_{11}, F_8 \wedge F_9 \vdash \Delta_{12}]} \text{hCut} \\
\rightarrow \\
- : \Delta_{13}, [F_{11}, F_8 \wedge F_9 \vdash \Delta_{12}] \quad - : \Delta_{13}, [F_{11}, F_8 \wedge F_9 \vdash \Delta_{12}] \quad \text{ATG}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, F_9, \Delta_7 \vdash \perp, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \perp} \wedge_L \quad \frac{}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \perp \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \perp, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \perp, \Delta_{11}} \text{ax/W} \quad \frac{\frac{}{\bullet h_{10} : \perp, \Delta_7, F_8, F_9 \vdash \Delta_{11}} \perp_L}{\bullet h_{10} : \perp, \Delta_7, F_8, F_9 \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8, F_9 \vdash \perp, \Delta_{11} \quad - : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \wedge_L}{- : \Delta_7, F_8, F_9 \vdash \perp, \Delta_{11}} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_3 : F_8, F_9, \perp, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \wedge_L \quad \frac{}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}} \perp_L}{- : (\perp, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11} \quad \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Delta_7 \vdash p_{11}, \Delta_{12}, p_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash (\Delta_{12}, p_{11}), p_{11}} \wedge_L \quad \frac{}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), p_{11} \vdash \Delta_{12}, p_{11}} I \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), p_{11} \vdash \Delta_{12}, p_{11}} I \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11} \quad \wedge_L \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11} \quad \text{hCut}
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Delta_{13}, p_{11} \vdash F_7, \Delta_{12}, p_{11}}{\bullet h_3 : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash (\Delta_{12}, p_{11}), F_7} \wedge_L \quad \frac{}{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, p_{11}} I \\
\hline
- : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash \Delta_{12}, p_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, p_{11}} I \\
\hline
- : \Delta_{13}, p_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}
\end{array}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \top} \wedge_L \quad \frac{h_{10} : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \top \vdash \Delta_{11}} \top_L \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \top \vdash \Delta_{11}} \top_L \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11} \quad \text{ax/W}
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \top, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \wedge_L \quad \frac{h_{10} : F_7, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}} \top_L \\
\hline
- : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}} \top_L \\
\hline
- : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11} \quad \text{ax/W}
\end{array}
\end{array}$$

6.10 Status of \vee_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \rightarrow F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \rightarrow F_{14}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_7} \vee_L \quad \frac{h_{11} : F_7, F_{13}, \Delta_8, F_9 \vee F_{10} \vdash F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : \Delta_8, F_{13}, F_9 \vdash \Delta_{12}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_8, F_{10}, F_{13} \vdash \Delta_{12}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_{10}, F_{13} \vee F_{10} \vdash \Delta_{12}, F_{14}, F_7} \text{inv-th/ax} \\
\hline
\frac{}{\bullet h_{11} : \Delta_8, F_{13}, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}} \text{ax/W} \\
\hline
- : \Delta_8, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14} \quad \text{hCut} \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14} \quad \rightarrow_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \wedge F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \wedge F_{14}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_7} \vee_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \vee F_{10} \vdash F_{13}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : \Delta_8, F_9 \vdash \Delta_{12}, F_{13}, F_7}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_7}{\bullet h_3 : \Delta_8, F_{10} \vee F_{10} \vdash \Delta_{12}, F_{13}, F_7} \text{inv-th/ax} \\
\hline
\frac{}{\bullet h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}} \text{ax/W} \\
\hline
- : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \quad \text{hCut} \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_7} \vee_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \vee F_{10} \vdash F_{13}, F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R \\
\frac{\quad}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \Delta_8, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_7 \quad \text{inv-th/ax} \quad h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \vee_L \quad \frac{h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W} \\
\frac{\quad}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \text{hCut} \\
\frac{\quad}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \perp, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \perp, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\perp, \Delta_{12}), F_7} \vee_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \perp, \Delta_{12}} \perp_R \\
\frac{\quad}{- : \Delta_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12} \quad \text{ax/W} \quad h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}}{- : \Delta_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12}} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \top, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \top, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\top, \Delta_{12}), F_7} \vee_L \quad \frac{h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \top, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \top, \Delta_{12}} \top_R \\
\frac{\quad}{- : \Delta_8, F_9 \vee F_{10} \vdash \top, \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\quad}{- : \Delta_8, F_9 \vee F_{10} \vdash \top, \Delta_{12}} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_3 : F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}] \quad h_3 : F_9, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}]}{\bullet h_3 : (\Box \Gamma_{13}, \Delta_{14}), F_8 \vee F_9 \vdash (\Delta_{11}, [\Box F_{12}]), \Box F_7} \vee_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : ((\Box \Gamma_{13}, \Delta_{14}), F_8 \vee F_9), \Box F_7 \vdash \Delta_{11}, [\Box F_{12}]} \text{Cut} \\
\frac{\quad}{- : (\Box \Gamma_{13}, \Delta_{14}), F_8 \vee F_9 \vdash \Delta_{11}, [\Box F_{12}]} \rightarrow \\
\frac{h_3 : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}] \quad \text{ax/W} \quad h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{13}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{12}]} \text{K} \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{12}]} \text{hCut} \\
\frac{\quad}{- : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{12}]} \text{ax/W} \quad \frac{\quad}{- : \Delta_{14}, F_9, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}]} \text{ax/W} \quad \frac{\quad}{- : \Delta_{14}, F_9, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{12}]} \vee \\
\frac{\quad}{- : \Delta_{14}, \Box \Gamma_{13}, F_8 \vee F_9 \vdash \Delta_{11}, [\Box F_{12}]} \rightarrow \\
\frac{h_3 : F_8, \Box \Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, [\Box F_{13}] \quad h_3 : F_9, \Box \Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, [\Box F_{13}]}{\bullet h_3 : (\Box \Gamma_{11}, \Delta_{14}), F_8 \vee F_9 \vdash (\Delta_{12}, [\Box F_{13}]), F_7} \vee_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}}{\bullet h_{10} : ((\Box \Gamma_{11}, \Delta_{14}), F_8 \vee F_9), F_7 \vdash \Delta_{12}, [\Box F_{13}]} \text{K} \\
\frac{\quad}{- : (\Box \Gamma_{11}, \Delta_{14}), F_8 \vee F_9 \vdash \Delta_{12}, [\Box F_{13}]} \text{Cut} \\
\rightarrow \\
\frac{\quad}{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}} \text{ax/W} \\
\frac{\quad}{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \vee F_9 \vdash \Delta_{12}, [\Box F_{13}]} \text{K}
\end{array}$$

- Case rule $A45$

$$\begin{array}{c}
\frac{h_3 : F_8, \Box \Gamma_{14}, \Delta_{15} \vdash \Box F_7, \Box \Gamma_{11}, \Delta_{12}, [\Box F_{13}] \quad h_3 : F_9, \Box \Gamma_{14}, \Delta_{15} \vdash \Box F_7, \Box \Gamma_{11}, \Delta_{12}, [\Box F_{13}]}{\bullet h_3 : (\Box \Gamma_{14}, \Delta_{15}), F_8 \vee F_9 \vdash (\Box \Gamma_{11}, \Delta_{12}, [\Box F_{13}]), \Box F_7} \vee_L \quad \frac{h_{10} : \Box \Gamma_{14}, \Box F_7 \vdash F_{13}}{\bullet h_{10} : ((\Box \Gamma_{14}, \Delta_{15}), F_8 \vee F_9), \Box \Gamma_{11}, \Delta_{12}, [\Box F_{13}]} \text{Cut} \\
\frac{\quad}{- : (\Box \Gamma_{14}, \Delta_{15}), F_8 \vee F_9 \vdash \Box \Gamma_{11}, \Delta_{12}, [\Box F_{13}]} \rightarrow \\
\frac{h_3 : \Delta_{15}, F_8, \Box \Gamma_{14} \vdash \Box F_7, \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}] \quad \text{ax/W} \quad h_{10} : \Box F_7, \Box \Gamma_{14} \vdash F_{13}, \Box \Gamma_{11}}{\bullet h_{10} : \Box F_7, \Delta_{15}, F_8, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]} \text{A45} \quad \frac{h_{10} : \Box \Gamma_{14}, \Box F_7 \vdash F_{13}}{\bullet h_{10} : \Box F_7, \Delta_{15}, F_8, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]} \text{hCut} \\
\frac{\quad}{- : \Delta_{15}, F_8, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]} \text{ax/W} \quad \frac{\quad}{- : \Delta_{15}, F_9, \Box \Gamma_{14} \vdash \Box F_7, \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]} \text{ax/W} \\
\frac{\quad}{- : \Delta_{15}, \Box \Gamma_{14}, F_8 \vee F_9 \vdash \Delta_{12}, \Box \Gamma_{11}, [\Box F_{13}]} \text{hCut}
\end{array}$$

$$\frac{\frac{h_3 : F_8, \Box \Gamma_{11}, \Delta_{15} \vdash F_7, \Box \Gamma_{12}, \Delta_{13}, [\Box F_{14}] \quad h_3 : F_9, \Box \Gamma_{11}, \Delta_{15} \vdash F_7, \Box \Gamma_{12}, \Delta_{13}, [\Box F_{14}]}{\bullet h_3 : (\Box \Gamma_{11}, \Delta_{15}), F_8 \vee F_9 \vdash (\Box \Gamma_{12}, \Delta_{13}, [\Box F_{14}]), F_7} \vee_L \quad \frac{h_{10} : \Box \Gamma_{11} \vdash \Box \Gamma_{12}, F_{14}}{\bullet h_{10} : ((\Box \Gamma_{11}, \Delta_{15}), F_8 \vee F_9), F_7 \vdash \Box \Gamma_{12}, \Delta_{13}, [\Box F_{14}]} A45}{\frac{- : (\Box \Gamma_{11}, \Delta_{15}), F_8 \vee F_9 \vdash \Box \Gamma_{12}, \Delta_{13}, [\Box F_{14}]}{\rightarrow} \quad \frac{- : \Box \Gamma_{11} \vdash F_{14}, \Box \Gamma_{12} \quad ax/W}{- : \Delta_{15}, \Box \Gamma_{11}, F_8 \vee F_9 \vdash \Delta_{13}, \Box \Gamma_{12}, [\Box F_{14}]} A45} Cut$$

- Case rule \rightarrow_L

[illegible]

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_3 : F_8, \Delta_7 \vdash F_{11} \wedge F_{12}, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \wedge F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \vee_L \quad \frac{h_{10} : F_{11}, F_{12}, \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L \\
\frac{\quad}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{Cut} \\
\frac{\quad}{\quad} \rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\quad} \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \vdash \Delta_{13}} \text{hCut} \quad \frac{\frac{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\quad} \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_9 \vdash \Delta_{13}} \text{hCut} \\
\frac{\quad}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \vee_L \\
\frac{\quad}{\quad} \rightarrow \\
\frac{h_3 : F_8, \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \quad \frac{h_{10} : F_7, F_{11}, F_{12}, \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{13}} \wedge_L \\
\frac{\quad}{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9 \vdash \Delta_{13}} \text{Cut} \\
\frac{\quad}{\quad} \rightarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vdash \Delta_{13}, F_7}}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_9 \vdash \Delta_{13}, F_7}}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{inv-th/ax} \\
\frac{\quad}{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \vee_L \quad \frac{\quad}{\quad} \text{ax/W} \\
\frac{\quad}{- : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \text{hCut} \\
\frac{\quad}{- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \wedge_L
\end{array}$$

- Case rule \forall_L

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, \Delta_7 \vdash F_{11} \vee F_{12}, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \vee F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \vee_L \quad \frac{h_{10} : F_{11}, \Delta_7, F_8 \vee F_9 \vdash \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \vee_L \\
\frac{\quad}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{Cut} \\
\frac{\quad}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \rightarrow \\
\frac{\frac{- : \Delta_7, F_8 \vdash \Delta_{13}, F_{11}, F_{12}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \text{inv-th/ax} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{13}, F_{11}, F_{12}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \text{inv-th/ax}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \vee_L \quad \frac{- : \Delta_7, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}} \text{ax/W} \\
\frac{\quad}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}} \text{sCut} \quad \frac{\quad}{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}} \text{ax/W} \\
\frac{\quad}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{sCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, \Delta_7 \vdash P_{11}, \Delta_{12}, P_{11} \quad h_3 : F_9, \Delta_7 \vdash P_{11}, \Delta_{12}, P_{11}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash (\Delta_{12}, P_{11}), P_{11}} \vee_L \quad \frac{}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), P_{11} \vdash \Delta_{12}, P_{11}} I}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, P_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{12}, P_{11}, P_{11}}{- : \Delta_7, F_8 \vdash \Delta_{12}, P_{11}} \text{ax/W} \quad \frac{\bullet h_{10} : \Delta_7, F_8, P_{11} \vdash \Delta_{12}, P_{11}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, P_{11}} I}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, P_{11}} \text{hCut} \\
\frac{}{- : \Delta_7, F_9 \vdash \Delta_{12}, P_{11}} \text{ax/W} \quad \frac{\bullet h_{10} : \Delta_7, F_9, P_{11} \vdash \Delta_{12}, P_{11}}{- : \Delta_7, F_9 \vdash \Delta_{12}, P_{11}} I}{- : \Delta_7, F_9 \vdash \Delta_{12}, P_{11}} \text{hCut} \\
\vee_L \\
\frac{}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, P_{11}} \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : F_8, \Delta_{13}, P_{11} \vdash F_7, \Delta_{12}, P_{11} \quad h_3 : F_9, \Delta_{13}, P_{11} \vdash F_7, \Delta_{12}, P_{11}}{\bullet h_3 : (\Delta_{13}, P_{11}), F_8 \vee F_9 \vdash (\Delta_{12}, P_{11}), F_7} \vee_L \quad \frac{}{\bullet h_{10} : ((\Delta_{13}, P_{11}), F_8 \vee F_9), F_7 \vdash \Delta_{12}, P_{11}} I}{- : (\Delta_{13}, P_{11}), F_8 \vee F_9 \vdash \Delta_{12}, P_{11}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{13}, P_{11}, F_8 \vee F_9 \vdash \Delta_{12}, P_{11}} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_3 : F_8, \Delta_7 \vdash \top, \Delta_{11} \quad h_3 : F_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}, \top} \vee_L \quad \frac{h_{10} : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), \top \vdash \Delta_{11}} \top_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \text{ax/W} \\
\frac{h_3 : F_8, \top, \Delta_{12} \vdash F_7, \Delta_{11} \quad h_3 : F_9, \top, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}, F_7} \vee_L \quad \frac{h_{10} : F_7, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}} \top_L}{- : (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_3 : \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}, F_7 \quad h_{10} : \top, \Delta_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{11}}{- : \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}} \text{ax/W} \quad \text{hCut}
\end{array}$$

6.11 Status of AT : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_3 : F_8, \Delta_7, []F_8 \vdash F_6, \Delta_{10}, F_{11} \rightarrow F_{12}}{\bullet h_3 : \Delta_7, []F_8 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_6} AT \quad \frac{h_9 : F_6, F_{11}, \Delta_7, []F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : (\Delta_7, []F_8), F_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R}{- : \Delta_7, []F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_6, F_{11} \rightarrow F_{12}}{- : \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_7, F_6, F_8, []F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{- : \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \text{hCut} \\
AT
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_3 : F_8, \Delta_7, []F_8 \vdash F_6, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, []F_8 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_6} AT \quad \frac{h_9 : F_6, \Delta_7, []F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : F_6, \Delta_7, []F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : (\Delta_7, []F_8), F_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{- : \Delta_7, []F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_6, F_{11} \wedge F_{12}}{- : \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_7, F_6, F_8, []F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}}{- : \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{hCut} \\
ATG
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_3 : F_8, \Delta_7, []F_8 \vdash F_6, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_3 : \Delta_7, []F_8 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_6} AT \quad \frac{h_9 : F_6, \Delta_7, []F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : (\Delta_7, []F_8), F_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_7, []F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_6, F_{11} \vee F_{12}}{- : \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_7, F_6, F_8, []F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}}{- : \Delta_7, F_8, []F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{hCut} \\
AT
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, \Delta_7, [F_8 \vdash F_6, \perp, \Delta_{10}] \quad \bullet h_3 : (\Delta_7, [F_8 \vdash (\perp, \Delta_{10}), F_6]}{ - : \Delta_7, [F_8 \vdash \perp, \Delta_{10}] } AT \quad \frac{h_9 : F_6, \Delta_7, [F_8 \vdash \Delta_{10}] \quad \bullet h_9 : (\Delta_7, [F_8], F_6 \vdash \perp, \Delta_{10})}{ - : \Delta_7, [F_8 \vdash \perp, \Delta_{10}] } \perp_R}{ - : \Delta_7, [F_8 \vdash \perp, \Delta_{10}] } Cut \\
\rightarrow \\
\frac{\frac{\bullet h_3 : \Delta_7, [F_8 \vdash \perp, \Delta_{10}, F_6]}{ - : \Delta_7, [F_8 \vdash \perp, \Delta_{10}] } ax/W \quad \frac{h_9 : \Delta_7, F_6, [F_8 \vdash \perp, \Delta_{10}] \quad \bullet h_9 : (\Delta_7, F_6, [F_8 \vdash \perp, \Delta_{10}])}{ - : \Delta_7, [F_8 \vdash \perp, \Delta_{10}] } ax/W}{ - : \Delta_7, [F_8 \vdash \perp, \Delta_{10}] } hCut
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_3 : F_8, \Delta_7, [F_8 \vdash F_6, \top, \Delta_{10}] \quad \bullet h_3 : (\Delta_7, [F_8 \vdash (\top, \Delta_{10}), F_6]}{ - : \Delta_7, [F_8 \vdash \top, \Delta_{10}] } AT \quad \frac{h_9 : (\Delta_7, [F_8], F_6 \vdash \top, \Delta_{10})}{ - : \Delta_7, [F_8 \vdash \top, \Delta_{10}] } \top_R}{ - : \Delta_7, [F_8 \vdash \top, \Delta_{10}] } Cut \\
\rightarrow \\
\frac{}{ - : \Delta_7, [F_8 \vdash \top, \Delta_{10}] } \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_3 : F_7, (\Box \Gamma_{12}, \Delta_9), [F_7 \vdash \Box F_6, \Delta_{10}, [F_{11}]] \quad \bullet h_3 : ((\Box \Gamma_{12}, \Delta_9), [F_7 \vdash (\Delta_{10}, [F_{11}]), \Box F_6])}{ - : (\Box \Gamma_{12}, \Delta_9), [F_7 \vdash \Delta_{10}, [F_{11}]] } AT \quad \frac{h_8 : F_7, unbox(\Box \Gamma_{12}), unbox(\Box F_6) \vdash F_{11} \quad \bullet h_8 : ((\Box \Gamma_{12}, \Delta_9), [F_7], \Box F_6 \vdash \Delta_{10}, [F_{11}])}{ - : (\Box \Gamma_{12}, \Delta_9), [F_7 \vdash \Delta_{10}, [F_{11}]] } K}{ - : (\Box \Gamma_{12}, \Delta_9), [F_7 \vdash \Delta_{10}, [F_{11}]] } Cut \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_9, F_7, \Box \Gamma_{12}, [F_7 \vdash \Box F_6, \Delta_{10}, [F_{11}]] \quad \bullet h_3 : (\Delta_9, F_7, \Box \Gamma_{12}, [F_7 \vdash \Delta_{10}, [F_{11}]])}{ - : \Delta_9, F_7, \Box \Gamma_{12}, [F_7 \vdash \Delta_{10}, [F_{11}]] } ax/W \quad \frac{h_8 : \Box F_6, \Delta_9, F_7, \Box \Gamma_{12}, [F_7 \vdash \Delta_{10}, [F_{11}]] \quad \bullet h_8 : (\Box F_6, \Delta_9, F_7, \Box \Gamma_{12}, [F_7 \vdash \Delta_{10}, [F_{11}]])}{ - : \Delta_9, F_7, \Box \Gamma_{12}, [F_7 \vdash \Delta_{10}, [F_{11}]] } ax/W}{ - : \Delta_9, F_7, \Box \Gamma_{12}, [F_7 \vdash \Delta_{10}, [F_{11}]] } hCut \\
\rightarrow \\
\frac{}{ - : \Delta_9, \Box \Gamma_{12}, [F_7 \vdash \Delta_{10}, [F_{11}]] } ATG \\
\frac{h_3 : F_7, (\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash F_6, \Delta_9, [F_{10}]] \quad \bullet h_3 : ((\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash (\Delta_9, [F_{10}]), F_6])}{ - : (\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash \Delta_9, [F_{10}]] } AT \quad \frac{h_8 : F_7, unbox(\Box \Gamma_{11}) \vdash F_{10} \quad \bullet h_8 : ((\Box \Gamma_{11}, \Delta_{12}), [F_7], F_6 \vdash \Delta_9, [F_{10}])}{ - : (\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash \Delta_9, [F_{10}]] } K}{ - : (\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash \Delta_9, [F_{10}]] } Cut \\
\rightarrow \\
\frac{}{ - : F_7, unbox(\Box \Gamma_{11}) \vdash F_{10} } ax/W \\
\frac{}{ - : \Delta_{12}, \Box \Gamma_{11}, [F_7 \vdash \Delta_9, [F_{10}]] } K \\
\frac{h_3 : F_7, (\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash \Box F_6, \Delta_9, [F_{10}]] \quad \bullet h_3 : ((\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash (\Delta_9, [F_{10}]), \Box F_6])}{ - : (\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash \Delta_9, [F_{10}]] } AT \quad \frac{h_8 : unbox(\Box \Gamma_{11}), unbox(\Box F_6) \vdash F_{10} \quad \bullet h_8 : ((\Box \Gamma_{11}, \Delta_{12}), [F_7], \Box F_6 \vdash \Delta_9, [F_{10}])}{ - : (\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash \Delta_9, [F_{10}]] } K}{ - : (\Box \Gamma_{11}, \Delta_{12}), [F_7 \vdash \Delta_9, [F_{10}]] } Cut \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{12}, F_7, \Box \Gamma_{11}, [F_7 \vdash \Box F_6, \Delta_9, [F_{10}]] \quad \bullet h_3 : (\Delta_{12}, F_7, \Box \Gamma_{11}, [F_7 \vdash \Delta_9, [F_{10}]])}{ - : \Delta_{12}, F_7, \Box \Gamma_{11}, [F_7 \vdash \Delta_9, [F_{10}]] } ax/W \quad \frac{h_8 : \Box F_6, \Delta_{12}, F_7, \Box \Gamma_{11}, [F_7 \vdash \Delta_9, [F_{10}]] \quad \bullet h_8 : (\Box F_6, \Delta_{12}, F_7, \Box \Gamma_{11}, [F_7 \vdash \Delta_9, [F_{10}]])}{ - : \Delta_{12}, F_7, \Box \Gamma_{11}, [F_7 \vdash \Delta_9, [F_{10}]] } ax/W}{ - : \Delta_{12}, F_7, \Box \Gamma_{11}, [F_7 \vdash \Delta_9, [F_{10}]] } hCut \\
\rightarrow \\
\frac{}{ - : \Delta_{12}, \Box \Gamma_{11}, [F_7 \vdash \Delta_9, [F_{10}]] } ATG \\
\frac{h_3 : F_7, (\Box \Gamma_9, \Delta_{12}), [F_7 \vdash F_6, \Delta_{10}, [F_{11}]] \quad \bullet h_3 : ((\Box \Gamma_9, \Delta_{12}), [F_7 \vdash (\Delta_{10}, [F_{11}]), F_6])}{ - : (\Box \Gamma_9, \Delta_{12}), [F_7 \vdash \Delta_{10}, [F_{11}]] } AT \quad \frac{h_8 : unbox(\Box \Gamma_9) \vdash F_{11} \quad \bullet h_8 : ((\Box \Gamma_9, \Delta_{12}), [F_7], F_6 \vdash \Delta_{10}, [F_{11}])}{ - : (\Box \Gamma_9, \Delta_{12}), [F_7 \vdash \Delta_{10}, [F_{11}]] } K}{ - : (\Box \Gamma_9, \Delta_{12}), [F_7 \vdash \Delta_{10}, [F_{11}]] } Cut \\
\rightarrow \\
\frac{}{ - : unbox(\Box \Gamma_9) \vdash F_{11} } ax/W \\
\frac{}{ - : \Delta_{12}, \Box \Gamma_9, [F_7 \vdash \Delta_{10}, [F_{11}]] } K
\end{array}$$

- Case rule $A45$

$$\begin{array}{c}
\frac{h_3 : F_7, (\Box \Gamma_{13}, \Delta_9), [F_7 \vdash \Box F_6, \Box \Gamma_{10}, \Delta_{11}, [F_{12}]] \quad \bullet h_3 : ((\Box \Gamma_{13}, \Delta_9), [F_7 \vdash (\Box \Gamma_{10}, \Delta_{11}, [F_{12}]), \Box F_6])}{ - : (\Box \Gamma_{13}, \Delta_9), [F_7 \vdash \Box \Gamma_{10}, \Delta_{11}, [F_{12}]] } AT \quad \frac{h_8 : \Box \Gamma_{13}, \Box F_6, [F_7 \vdash \Box \Gamma_{10}, F_{12}] \quad \bullet h_8 : ((\Box \Gamma_{13}, \Delta_9), [F_7], \Box F_6 \vdash \Box \Gamma_{10}, \Delta_{11}, [F_{12}])}{ - : (\Box \Gamma_{13}, \Delta_9), [F_7 \vdash \Box \Gamma_{10}, \Delta_{11}, [F_{12}]] } A45}{ - : (\Box \Gamma_{13}, \Delta_9), [F_7 \vdash \Box \Gamma_{10}, \Delta_{11}, [F_{12}]] } Cut \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_9, F_7, \Box \Gamma_{13}, [F_7 \vdash \Box F_6, \Delta_{11}, \Box \Gamma_{10}, [F_{12}]] \quad \bullet h_3 : (\Delta_9, F_7, \Box \Gamma_{13}, [F_7 \vdash \Delta_{11}, \Box \Gamma_{10}, [F_{12}]])}{ - : \Delta_9, F_7, \Box \Gamma_{13}, [F_7 \vdash \Delta_{11}, \Box \Gamma_{10}, [F_{12}]] } ax/W \quad \frac{h_8 : \Box F_6, \Delta_9, F_7, \Box \Gamma_{13}, [F_7 \vdash \Delta_{11}, \Box \Gamma_{10}, [F_{12}]] \quad \bullet h_8 : (\Box F_6, \Delta_9, F_7, \Box \Gamma_{13}, [F_7 \vdash \Delta_{11}, \Box \Gamma_{10}, [F_{12}]])}{ - : \Delta_9, F_7, \Box \Gamma_{13}, [F_7 \vdash \Delta_{11}, \Box \Gamma_{10}, [F_{12}]] } ax/W}{ - : \Delta_9, F_7, \Box \Gamma_{13}, [F_7 \vdash \Delta_{11}, \Box \Gamma_{10}, [F_{12}]] } hCut \\
\rightarrow \\
\frac{}{ - : \Delta_9, \Box \Gamma_{13}, [F_7 \vdash \Delta_{11}, \Box \Gamma_{10}, [F_{12}]] } ATG
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : F_7, (\Box\Gamma_{12}, \Delta_{13}), [F_7 \vdash F_6, \Box\Gamma_9, \Delta_{10}, [F_{11}]}{\bullet h_3 : (\Box\Gamma_{12}, \Delta_{13}), [F_7 \vdash (\Box\Gamma_9, \Delta_{10}, [F_{11}]), F_6} \text{ AT} \quad \frac{h_8 : \Box\Gamma_{12}, [F_7 \vdash \Box\Gamma_9, F_{11}]}{\bullet h_8 : ((\Box\Gamma_{12}, \Delta_{13}), [F_7]), F_6 \vdash \Box\Gamma_9, \Delta_{10}, [F_{11}]} \text{ A45} \\
\hline
- : (\Box\Gamma_{12}, \Delta_{13}), [F_7 \vdash \Box\Gamma_9, \Delta_{10}, [F_{11}]] \quad \text{Cut} \\
\hline
\begin{array}{c}
\rightarrow \\
\frac{- : \Box\Gamma_{12}, [F_7 \vdash F_{11}, \Box\Gamma_9]}{- : \Delta_{13}, \Box\Gamma_{12}, [F_7 \vdash \Delta_{10}, \Box\Gamma_9, [F_{11}]]} \text{ ax/W} \\
\text{A45}
\end{array} \\
\\
\frac{h_3 : F_7, (\Box\Gamma_{12}, \Delta_{13}), [F_7 \vdash \Box F_6, \Box\Gamma_9, \Delta_{10}, [F_{11}]}{\bullet h_3 : (\Box\Gamma_{12}, \Delta_{13}), [F_7 \vdash (\Box\Gamma_9, \Delta_{10}, [F_{11}]), \Box F_6} \text{ AT} \quad \frac{h_8 : \Box\Gamma_{12}, \Box F_6 \vdash \Box\Gamma_9, F_{11}}{\bullet h_8 : ((\Box\Gamma_{12}, \Delta_{13}), [F_7]), \Box F_6 \vdash \Box\Gamma_9, \Delta_{10}, [F_{11}]} \text{ A45} \\
\hline
- : (\Box\Gamma_{12}, \Delta_{13}), [F_7 \vdash \Box\Gamma_9, \Delta_{10}, [F_{11}]] \quad \text{Cut} \\
\hline
\begin{array}{c}
\rightarrow \\
\frac{h_3 : \Delta_{13}, F_7, \Box\Gamma_{12}, [F_7 \vdash \Box F_6, \Delta_{10}, \Box\Gamma_9, [F_{11}]]}{- : \Delta_{13}, F_7, \Box\Gamma_{12}, [F_7 \vdash \Delta_{10}, \Box\Gamma_9, [F_{11}]]} \text{ ax/W} \\
\frac{\bullet h_8 : \Box F_6, \Delta_{13}, F_7, \Box\Gamma_{12}, [F_7 \vdash \Delta_{10}, \Box\Gamma_9, [F_{11}]]}{- : \Delta_{13}, \Box\Gamma_{12}, [F_7 \vdash \Delta_{10}, \Box\Gamma_9, [F_{11}]]} \text{ ax/W} \\
\text{hCut} \\
\text{AT}
\end{array} \\
\\
\frac{h_3 : F_7, (\Box\Gamma_9, \Delta_{13}), [F_7 \vdash F_6, \Box\Gamma_{10}, \Delta_{11}, [F_{12}]}{\bullet h_3 : (\Box\Gamma_9, \Delta_{13}), [F_7 \vdash (\Box\Gamma_{10}, \Delta_{11}, [F_{12}]), F_6} \text{ AT} \quad \frac{h_8 : \Box\Gamma_9 \vdash \Box\Gamma_{10}, F_{12}}{\bullet h_8 : ((\Box\Gamma_9, \Delta_{13}), [F_7]), F_6 \vdash \Box\Gamma_{10}, \Delta_{11}, [F_{12}]} \text{ A45} \\
\hline
- : (\Box\Gamma_9, \Delta_{13}), [F_7 \vdash \Box\Gamma_{10}, \Delta_{11}, [F_{12}]] \quad \text{Cut} \\
\hline
\begin{array}{c}
\rightarrow \\
\frac{- : \Box\Gamma_9 \vdash F_{12}, \Box\Gamma_{10}}{- : \Delta_{13}, \Box\Gamma_9, [F_7 \vdash \Delta_{11}, \Box\Gamma_{10}, [F_{12}]]} \text{ ax/W} \\
\text{A45}
\end{array}
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_3 : F_7, \Delta_6, [F_7 \vdash F_9 \rightarrow F_{10}, \Delta_{11}]}{\bullet h_3 : \Delta_6, [F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}]} \text{ AT} \quad \frac{h_8 : \Delta_6, [F_7 \vdash F_9, \Delta_{11}] \quad h_8 : F_{10}, \Delta_6, [F_7 \vdash \Delta_{11}]}{\bullet h_8 : (\Delta_6, [F_7]), F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L \\
\hline
- : \Delta_6, [F_7 \vdash \Delta_{11}] \quad \text{Cut} \\
\hline
\begin{array}{c}
\rightarrow \\
\frac{h_3 : \Delta_6, F_7, [F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}]}{- : \Delta_6, F_7, [F_7 \vdash \Delta_{11}]} \text{ ax/W} \\
\frac{\bullet h_8 : \Delta_6, F_7, [F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}]}{- : \Delta_6, [F_7 \vdash \Delta_{11}]} \text{ ax/W} \\
\text{hCut} \\
\text{AT}
\end{array} \\
\\
\frac{h_3 : F_7, (\Delta_{12}, F_9 \rightarrow F_{10}), [F_7 \vdash F_6, \Delta_{11}]}{\bullet h_3 : (\Delta_{12}, F_9 \rightarrow F_{10}), [F_7 \vdash \Delta_{11}, F_6]} \text{ AT} \quad \frac{h_8 : F_6, \Delta_{12}, [F_7 \vdash F_9, \Delta_{11}] \quad h_8 : F_6, F_{10}, \Delta_{12}, [F_7 \vdash \Delta_{11}]}{\bullet h_8 : ((\Delta_{12}, F_9 \rightarrow F_{10}), [F_7]), F_6 \vdash \Delta_{11}} \rightarrow_L \\
\hline
- : (\Delta_{12}, F_9 \rightarrow F_{10}), [F_7 \vdash \Delta_{11}] \quad \text{Cut} \\
\hline
\begin{array}{c}
\rightarrow \\
\frac{h_3 : \Delta_{12}, F_7, [F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6]}{- : \Delta_{12}, F_7, [F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}]} \text{ ax/W} \\
\frac{\bullet h_8 : \Delta_{12}, F_6, F_7, [F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}]}{- : \Delta_{12}, [F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}]} \text{ ax/W} \\
\text{hCut} \\
\text{ATG}
\end{array}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_3 : F_7, \Delta_6, [F_7 \vdash F_9 \wedge F_{10}, \Delta_{11}]}{\bullet h_3 : \Delta_6, [F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}]} \text{ AT} \quad \frac{h_8 : F_9, F_{10}, \Delta_6, [F_7 \vdash \Delta_{11}]}{\bullet h_8 : (\Delta_6, [F_7]), F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L \\
\hline
- : \Delta_6, [F_7 \vdash \Delta_{11}] \quad \text{Cut} \\
\hline
\begin{array}{c}
\rightarrow \\
\frac{h_3 : \Delta_6, F_7, [F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}]}{- : \Delta_6, F_7, [F_7 \vdash \Delta_{11}]} \text{ ax/W} \\
\frac{\bullet h_8 : \Delta_6, F_7, [F_7, F_9 \wedge F_{10} \vdash \Delta_{11}]}{- : \Delta_6, [F_7 \vdash \Delta_{11}]} \text{ ax/W} \\
\text{hCut} \\
\text{ATG}
\end{array} \\
\\
\frac{h_3 : F_7, (\Delta_{12}, F_9 \wedge F_{10}), [F_7 \vdash F_6, \Delta_{11}]}{\bullet h_3 : (\Delta_{12}, F_9 \wedge F_{10}), [F_7 \vdash \Delta_{11}, F_6]} \text{ AT} \quad \frac{h_8 : F_6, F_9, F_{10}, \Delta_{12}, [F_7 \vdash \Delta_{11}]}{\bullet h_8 : ((\Delta_{12}, F_9 \wedge F_{10}), [F_7]), F_6 \vdash \Delta_{11}} \wedge_L \\
\hline
- : (\Delta_{12}, F_9 \wedge F_{10}), [F_7 \vdash \Delta_{11}] \quad \text{Cut} \\
\hline
\begin{array}{c}
\rightarrow \\
\frac{h_3 : \Delta_{12}, F_7, [F_7, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6]}{- : \Delta_{12}, F_7, [F_7, F_9 \wedge F_{10} \vdash \Delta_{11}]} \text{ ax/W} \\
\frac{\bullet h_8 : \Delta_{12}, F_6, F_7, [F_7, F_9 \wedge F_{10} \vdash \Delta_{11}]}{- : \Delta_{12}, [F_7, F_9 \wedge F_{10} \vdash \Delta_{11}]} \text{ ax/W} \\
\text{hCut} \\
\text{ATG}
\end{array}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_3 : F_7, \Delta_6, [\Box F_7 \vdash F_9 \vee F_{10}, \Delta_{11}] \quad AT \quad \frac{h_8 : F_9, \Delta_6, [\Box F_7 \vdash \Delta_{11}] \quad h_8 : F_{10}, \Delta_6, [\Box F_7 \vdash \Delta_{11}] \quad \vee_L}{\bullet h_8 : (\Delta_6, [\Box F_7], F_9 \vee F_{10}) \vdash \Delta_{11}} \text{Cut}}{\neg : \Delta_6, [\Box F_7 \vdash \Delta_{11}]} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_6, F_7, [\Box F_7 \vdash \Delta_{11}, F_9 \vee F_{10}] \quad ax/W \quad \bullet h_8 : \Delta_6, F_7, [\Box F_7, F_9 \vee F_{10}] \vdash \Delta_{11} \quad ax/W}{\neg : \Delta_6, F_7, [\Box F_7 \vdash \Delta_{11}] \quad hCut} \quad AT}{\neg : \Delta_6, [\Box F_7 \vdash \Delta_{11}]} \\
\\
\frac{h_3 : F_7, (\Delta_{12}, F_9 \vee F_{10}), [\Box F_7 \vdash F_6, \Delta_{11}] \quad AT \quad \frac{h_8 : F_6, F_9, \Delta_{12}, [\Box F_7 \vdash \Delta_{11}] \quad h_8 : F_6, F_{10}, \Delta_{12}, [\Box F_7 \vdash \Delta_{11}] \quad \vee_L}{\bullet h_8 : ((\Delta_{12}, F_9 \vee F_{10}), [\Box F_7], F_6) \vdash \Delta_{11}} \text{Cut}}{\neg : (\Delta_{12}, F_9 \vee F_{10}), [\Box F_7 \vdash \Delta_{11}]} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{12}, F_7, [\Box F_7, F_9 \vee F_{10}] \vdash \Delta_{11}, F_6 \quad ax/W \quad \bullet h_8 : \Delta_{12}, F_6, F_7, [\Box F_7, F_9 \vee F_{10}] \vdash \Delta_{11} \quad ax/W}{\neg : \Delta_{12}, F_7, [\Box F_7, F_9 \vee F_{10}] \vdash \Delta_{11} \quad hCut} \quad ATG}{\neg : \Delta_{12}, [\Box F_7, F_9 \vee F_{10}] \vdash \Delta_{11}}
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{h_3 : F_7, \Delta_6, [\Box F_7 \vdash \Box F_9, \Delta_{10}] \quad AT \quad \frac{h_8 : F_9, \Delta_6, [\Box F_7, \Box F_9 \vdash \Delta_{10}] \quad AT}{\bullet h_8 : (\Delta_6, [\Box F_7], \Box F_9) \vdash \Delta_{10}} \text{Cut}}{\neg : \Delta_6, [\Box F_7 \vdash \Delta_{10}]} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_6, F_7, [\Box F_7 \vdash \Delta_{10}, \Box F_9] \quad ax/W \quad \bullet h_8 : \Delta_6, F_7, [\Box F_7, \Box F_9] \vdash \Delta_{10} \quad ax/W}{\neg : \Delta_6, F_7, [\Box F_7 \vdash \Delta_{10}] \quad hCut} \quad AT}{\neg : \Delta_6, [\Box F_7 \vdash \Delta_{10}]} \\
\\
\frac{h_3 : F_7, (\Delta_{11}, \Box F_9), [\Box F_7 \vdash F_6, \Delta_{10}] \quad AT \quad \frac{h_8 : F_6, F_9, \Delta_{11}, [\Box F_7, \Box F_9 \vdash \Delta_{10}] \quad AT}{\bullet h_8 : ((\Delta_{11}, \Box F_9), [\Box F_7], F_6) \vdash \Delta_{10}} \text{Cut}}{\neg : (\Delta_{11}, \Box F_9), [\Box F_7 \vdash \Delta_{10}]} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{11}, F_7, [\Box F_7, \Box F_9] \vdash \Delta_{10}, F_6 \quad ax/W \quad \bullet h_8 : \Delta_{11}, F_6, F_7, [\Box F_7, \Box F_9] \vdash \Delta_{10} \quad ax/W}{\neg : \Delta_{11}, F_7, [\Box F_7, \Box F_9] \vdash \Delta_{10} \quad hCut} \quad ATG}{\neg : \Delta_{11}, [\Box F_7, \Box F_9] \vdash \Delta_{10}} \\
\\
\frac{h_3 : F_9, \Delta_7, [\Box F_9 \vdash F_6, \Delta_{10}] \quad AT \quad \frac{h_8 : F_6, F_9, \Delta_7, [\Box F_9 \vdash \Delta_{10}] \quad AT}{\bullet h_8 : (\Delta_7, [\Box F_9], F_6) \vdash \Delta_{10}} \text{Cut}}{\neg : \Delta_7, [\Box F_9 \vdash \Delta_{10}]} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_9, [\Box F_9 \vdash \Delta_{10}, F_6] \quad ax/W \quad \bullet h_8 : \Delta_7, F_6, F_9, [\Box F_9 \vdash \Delta_{10}] \quad ax/W}{\neg : \Delta_7, F_9, [\Box F_9 \vdash \Delta_{10}] \quad hCut} \quad AT}{\neg : \Delta_7, [\Box F_9 \vdash \Delta_{10}]}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_3 : F_7, \Delta_6, [\Box F_7 \vdash \perp, \Delta_9] \quad AT \quad \frac{\bullet h_8 : (\Delta_6, [\Box F_7], \perp) \vdash \Delta_9}{\neg : \Delta_6, [\Box F_7 \vdash \Delta_9]} \perp_L}{\neg : \Delta_6, [\Box F_7 \vdash \Delta_9]} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_6, F_7, [\Box F_7 \vdash \perp, \Delta_9] \quad ax/W \quad \bullet h_8 : \perp, \Delta_6, F_7, [\Box F_7 \vdash \Delta_9] \quad \perp_L}{\neg : \Delta_6, F_7, [\Box F_7 \vdash \Delta_9] \quad hCut} \quad AT}{\neg : \Delta_6, [\Box F_7 \vdash \Delta_9]} \\
\\
\frac{h_3 : F_7, (\perp, \Delta_{10}), [\Box F_7 \vdash F_6, \Delta_9] \quad AT \quad \frac{\bullet h_8 : ((\perp, \Delta_{10}), [\Box F_7], F_6) \vdash \Delta_9}{\neg : (\perp, \Delta_{10}), [\Box F_7 \vdash \Delta_9]} \perp_L}{\neg : (\perp, \Delta_{10}), [\Box F_7 \vdash \Delta_9]} \text{Cut} \\
\rightarrow \\
\frac{\neg : \perp, \Delta_{10}, [\Box F_7 \vdash \Delta_9]}{\neg : \perp, \Delta_{10}, [\Box F_7 \vdash \Delta_9]} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_3 : F_7, \Delta_6, []F_7 \vdash p_9, \Delta_{10}, p_9}{\bullet h_3 : \Delta_6, []F_7 \vdash (\Delta_{10}, p_9), p_9} AT \quad \frac{}{\bullet h_8 : (\Delta_6, []F_7), p_9 \vdash \Delta_{10}, p_9} I}{- : \Delta_6, []F_7 \vdash \Delta_{10}, p_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_6, F_7, []F_7 \vdash \Delta_{10}, p_9, p_9}{- : \Delta_6, F_7, []F_7 \vdash \Delta_{10}, p_9} \text{ax/W} \quad \frac{\bullet h_8 : \Delta_6, F_7, p_9, []F_7 \vdash \Delta_{10}, p_9}{- : \Delta_6, []F_7 \vdash \Delta_{10}, p_9} I}{- : \Delta_6, F_7, []F_7 \vdash \Delta_{10}, p_9} \text{hCut} \\
\frac{}{- : \Delta_6, []F_7 \vdash \Delta_{10}, p_9} ATG
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : F_7, (\Delta_{11}, p_9), []F_7 \vdash F_6, \Delta_{10}, p_9}{\bullet h_3 : (\Delta_{11}, p_9), []F_7 \vdash (\Delta_{10}, p_9), F_6} AT \quad \frac{}{\bullet h_8 : ((\Delta_{11}, p_9), []F_7), F_6 \vdash \Delta_{10}, p_9} I}{- : (\Delta_{11}, p_9), []F_7 \vdash \Delta_{10}, p_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_9, []F_7 \vdash \Delta_{10}, p_9} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_3 : F_7, \Delta_6, []F_7 \vdash \top, \Delta_9}{\bullet h_3 : \Delta_6, []F_7 \vdash \Delta_9, \top} AT \quad \frac{h_8 : \Delta_6, []F_7 \vdash \Delta_9}{\bullet h_8 : (\Delta_6, []F_7), \top \vdash \Delta_9} \top_L}{- : \Delta_6, []F_7 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6, []F_7 \vdash \Delta_9} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : F_7, (\top, \Delta_{10}), []F_7 \vdash F_6, \Delta_9}{\bullet h_3 : (\top, \Delta_{10}), []F_7 \vdash \Delta_9, F_6} AT \quad \frac{h_8 : F_6, \Delta_{10}, []F_7 \vdash \Delta_9}{\bullet h_8 : ((\top, \Delta_{10}), []F_7), F_6 \vdash \Delta_9} \top_L}{- : (\top, \Delta_{10}), []F_7 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_3 : \top, \Delta_{10}, []F_7 \vdash \Delta_9, F_6}{- : \top, \Delta_{10}, []F_7 \vdash \Delta_9} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_6, []F_7 \vdash \Delta_9}{- : \top, \Delta_{10}, []F_7 \vdash \Delta_9} \text{hCut}
\end{array}$$

6.12 Status of \perp_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_5} \perp_L \quad \frac{h_7 : \perp, F_5, F_9, \Delta_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R}{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \perp_L
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\Delta_8, F_9 \wedge F_{10}), F_5} \perp_L \quad \frac{h_7 : \perp, F_5, \Delta_6 \vdash F_9, \Delta_8 \quad h_7 : \perp, F_5, \Delta_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R}{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \wedge F_{10}} \perp_L
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\Delta_8, F_9 \vee F_{10}), F_5} \perp_L \quad \frac{h_7 : \perp, F_5, \Delta_6 \vdash F_9, F_{10}, \Delta_8}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R}{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10}} \perp_L
\end{array}$$

- Case rule \perp_R

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\perp, \Delta_8), F_5}}{\vdash : \perp, \Delta_6 \vdash \perp, \Delta_8} \perp_L \quad \frac{\frac{h_7 : \perp, F_5, \Delta_6 \vdash \Delta_8}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \perp, \Delta_8} \perp_R}{\vdash : \perp, \Delta_6 \vdash \perp, \Delta_8} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_6 \vdash \perp, \Delta_8} \perp_L$$

- Case rule \top_R

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\top, \Delta_8), F_5}}{\vdash : \perp, \Delta_6 \vdash \top, \Delta_8} \perp_L \quad \frac{\frac{}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \top, \Delta_8} \top_R}{\vdash : \perp, \Delta_6 \vdash \top, \Delta_8} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_6 \vdash \top, \Delta_8} \top_R$$

- Case rule K

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, \Box F_8), \Box F_5}}{\vdash : \perp, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, \Box F_8} \perp_L \quad \frac{\frac{h_6 : \text{unbox}(\Box \Gamma_9), \text{unbox}(\Box F_5) \vdash F_8}{\bullet h_6 : (\perp, \Box \Gamma_9, \Delta_{10}), \Box F_5 \vdash \Delta_7, \Box F_8} K}{\vdash : \perp, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, \Box F_8} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_8} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Box \Gamma_7, \Delta_{10} \vdash (\Delta_8, \Box F_9), F_5}}{\vdash : \perp, \Box \Gamma_7, \Delta_{10} \vdash \Delta_8, \Box F_9} \perp_L \quad \frac{\frac{h_6 : \text{unbox}(\Box \Gamma_7) \vdash F_9}{\bullet h_6 : (\perp, \Box \Gamma_7, \Delta_{10}), F_5 \vdash \Delta_8, \Box F_9} K}{\vdash : \perp, \Box \Gamma_7, \Delta_{10} \vdash \Delta_8, \Box F_9} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, \Box F_9} \perp_L$$

- Case rule $A45$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Box \Gamma_{10}, \Delta_{11} \vdash (\Box \Gamma_7, \Delta_8, \Box F_9), \Box F_5}}{\vdash : \perp, \Box \Gamma_{10}, \Delta_{11} \vdash \Box \Gamma_7, \Delta_8, \Box F_9} \perp_L \quad \frac{\frac{h_6 : \Box \Gamma_{10}, \Box F_5 \vdash \Box \Gamma_7, F_9}{\bullet h_6 : (\perp, \Box \Gamma_{10}, \Delta_{11}), \Box F_5 \vdash \Box \Gamma_7, \Delta_8, \Box F_9} A45}{\vdash : \perp, \Box \Gamma_{10}, \Delta_{11} \vdash \Box \Gamma_7, \Delta_8, \Box F_9} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_8, \Box \Gamma_7, \Box F_9} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Box \Gamma_7, \Delta_{11} \vdash (\Box \Gamma_8, \Delta_9, \Box F_{10}), F_5}}{\vdash : \perp, \Box \Gamma_7, \Delta_{11} \vdash \Box \Gamma_8, \Delta_9, \Box F_{10}} \perp_L \quad \frac{\frac{h_6 : \Box \Gamma_7 \vdash \Box \Gamma_8, F_{10}}{\bullet h_6 : (\perp, \Box \Gamma_7, \Delta_{11}), F_5 \vdash \Box \Gamma_8, \Delta_9, \Box F_{10}} A45}{\vdash : \perp, \Box \Gamma_7, \Delta_{11} \vdash \Box \Gamma_8, \Delta_9, \Box F_{10}} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_{11}, \Box \Gamma_7 \vdash \Delta_9, \Box \Gamma_8, \Box F_{10}} \perp_L$$

- Case rule \rightarrow_L

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8}}{\vdash : \perp, \Delta_5 \vdash \Delta_9} \perp_L \quad \frac{\frac{h_6 : \perp, \Delta_5 \vdash F_7, \Delta_9 \quad h_6 : \perp, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_5), F_7 \rightarrow F_8 \vdash \Delta_9} \rightarrow_L}{\vdash : \perp, \Delta_5 \vdash \Delta_9} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_5 \vdash \Delta_9} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5}}{\vdash : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \perp_L \quad \frac{\frac{h_6 : \perp, F_5, \Delta_{10} \vdash F_7, \Delta_9 \quad h_6 : \perp, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_{10}, F_7 \rightarrow F_8), F_5 \vdash \Delta_9} \rightarrow_L}{\vdash : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \perp_L$$

- Case rule \wedge_L

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8}}{\vdash : \perp, \Delta_5 \vdash \Delta_9} \perp_L \quad \frac{\frac{h_6 : \perp, F_7, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_5), F_7 \wedge F_8 \vdash \Delta_9} \wedge_L}{\vdash : \perp, \Delta_5 \vdash \Delta_9} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_5 \vdash \Delta_9} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \perp_L \quad \frac{h_6 : \perp, F_5, F_7, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \wedge_L}{\frac{}{- : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \rightarrow} \text{Cut} \quad \frac{}{- : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \perp_L$$

- Case rule \vee_L

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \perp_L \quad \frac{h_6 : \perp, F_7, \Delta_5 \vdash \Delta_9 \quad h_6 : \perp, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_5), F_7 \vee F_8 \vdash \Delta_9} \vee_L}{\frac{}{- : \perp, \Delta_5 \vdash \Delta_9} \rightarrow} \text{Cut} \quad \frac{}{- : \perp, \Delta_5 \vdash \Delta_9} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \perp_L \quad \frac{h_6 : \perp, F_5, F_7, \Delta_{10} \vdash \Delta_9 \quad h_6 : \perp, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_{10}, F_7 \vee F_8), F_5 \vdash \Delta_9} \vee_L}{\frac{}{- : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \rightarrow} \text{Cut} \quad \frac{}{- : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \perp_L$$

- Case rule AT

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_8, []F_7} \perp_L \quad \frac{h_6 : \perp, F_7, \Delta_5, []F_7 \vdash \Delta_8}{\bullet h_6 : (\perp, \Delta_5), []F_7 \vdash \Delta_8} AT}{\frac{}{- : \perp, \Delta_5 \vdash \Delta_8} \rightarrow} \text{Cut} \quad \frac{}{- : \perp, \Delta_5 \vdash \Delta_8} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_9, []F_7 \vdash \Delta_8, F_5} \perp_L \quad \frac{h_6 : \perp, F_5, F_7, \Delta_9, []F_7 \vdash \Delta_8}{\bullet h_6 : (\perp, \Delta_9, []F_7), F_5 \vdash \Delta_8} AT}{\frac{}{- : \perp, \Delta_9, []F_7 \vdash \Delta_8} \rightarrow} \text{Cut} \quad \frac{}{- : \perp, \Delta_9, []F_7 \vdash \Delta_8} \perp_L$$

- Case rule \perp_L

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash \Delta_8, F_5} \perp_L \quad \frac{}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \Delta_8} \perp_L}{\frac{}{- : \perp, \Delta_6 \vdash \Delta_8} \rightarrow} \text{Cut} \quad \frac{}{- : \perp, \Delta_6 \vdash \Delta_8} \perp_L$$

- Case rule I

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash (\Delta_8, p_7), p_7} \perp_L \quad \frac{}{\bullet h_6 : (\perp, \Delta_5), p_7 \vdash \Delta_8, p_7} I}{\frac{}{- : \perp, \Delta_5 \vdash \Delta_8, p_7} \rightarrow} \text{Cut} \quad \frac{}{- : \perp, \Delta_5 \vdash \Delta_8, p_7} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5} \perp_L \quad \frac{}{\bullet h_6 : (\perp, \Delta_9, p_7), F_5 \vdash \Delta_8, p_7} I}{\frac{}{- : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7} \rightarrow} \text{Cut} \quad \frac{}{- : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7} \perp_L$$

- Case rule \top_L

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_7, \top} \perp_L \quad \frac{h_6 : \perp, \Delta_5 \vdash \Delta_7}{\bullet h_6 : (\perp, \Delta_5), \top \vdash \Delta_7} \top_L}{\frac{}{- : \perp, \Delta_5 \vdash \Delta_7} \rightarrow} \text{Cut} \quad \frac{}{- : \perp, \Delta_5 \vdash \Delta_7} \perp_L$$

$$\begin{array}{c}
\frac{\bullet h_3 : \perp, \top, \Delta_8 \vdash \Delta_7, F_5}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_7} \perp_L \quad \frac{h_6 : \perp, F_5, \Delta_8 \vdash \Delta_7}{\bullet h_6 : (\perp, \top, \Delta_8), F_5 \vdash \Delta_7} \top_L \\
\hline
\vdash : \perp, \top, \Delta_8 \vdash \Delta_7 \quad \text{Cut} \\
\hline
\vdash : \perp, \top, \Delta_8 \vdash \Delta_7 \quad \perp_L
\end{array}$$

6.13 Status of I : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \rightarrow F_{10}), p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} I \quad \frac{h_7 : F_9, \Delta_5, p_6, p_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10} \\
\hline
\frac{\bullet h_1 : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10}, p_6}{\vdash : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10}} I \quad \frac{h_7 : \Delta_5, F_9, p_6, p_6 \vdash \Delta_8, F_{10}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10} \quad \text{hCut} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10} \quad \rightarrow_R \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), p_8), F_6}{\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8} I \quad \frac{h_9 : F_6, F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8} \rightarrow_R \\
\hline
\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8 \quad \text{Cut} \\
\hline
\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8 \\
\hline
\vdash : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \rightarrow F_{11} \quad I
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \wedge F_{10}), p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash F_9, \Delta_8 \quad h_7 : \Delta_5, p_6, p_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10} \quad \text{Cut} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10} \\
\hline
\frac{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_9, p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_9}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10} \quad \text{hCut} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10} \\
\hline
\frac{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_{10}, p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10}} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_{10}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10} \wedge F_{11}} \text{ax/W} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10} \wedge F_{11} \quad \text{hCut} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10} \wedge F_{11} \quad \wedge_R \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10} \wedge F_{11}
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), p_8), F_6}{\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8} I \quad \frac{h_9 : F_6, \Delta_7, p_8 \vdash F_{10}, \Delta_{12}, p_8 \quad h_9 : F_6, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8} \wedge_R \\
\hline
\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8 \quad \text{Cut} \\
\hline
\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8 \\
\hline
\vdash : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \wedge F_{11} \quad I
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \vee F_{10}), p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash F_9, F_{10}, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10} \\
\hline
\frac{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9, p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_{10}, F_9}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10} \vee F_{11}} \text{ax/W} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9 \quad \text{hCut} \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10} \vee F_{11} \quad \vee_R \\
\hline
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10} \vee F_{11}
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \vee F_{11}), p_8), F_6}{\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8} I \quad \frac{h_9 : F_6, \Delta_7, p_8 \vdash F_{10}, F_{11}, \Delta_{12}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8} \vee_R \\
\hline
\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8 \quad \text{Cut} \\
\hline
\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8 \\
\hline
\vdash : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \vee F_{11} \quad I
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\perp, \Delta_8), p_6}{- : \Delta_5, p_6 \vdash \perp, \Delta_8} I \quad \frac{\frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \perp, \Delta_8} \perp_R}{\text{Cut}} \\
\frac{\bullet h_1 : \Delta_5, p_6 \vdash \perp, \Delta_8, p_6}{- : \Delta_5, p_6 \vdash \perp, \Delta_8} \xrightarrow{\text{ax/W}} \frac{h_7 : \Delta_5, p_6, p_6 \vdash \perp, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \perp, \Delta_8} \text{hCut} \\
\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\perp, \Delta_{10}), p_8), F_6}{- : \Delta_7, p_8 \vdash (\perp, \Delta_{10}), p_8} I \quad \frac{\frac{h_9 : F_6, \Delta_7, p_8 \vdash \Delta_{10}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\perp, \Delta_{10}), p_8} \perp_R}{\text{Cut}} \\
\frac{- : \Delta_7, p_8 \vdash (\perp, \Delta_{10}), p_8}{- : \Delta_7, p_8 \vdash \perp, \Delta_{10}, p_8} \xrightarrow{\text{I}}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\top, \Delta_8), p_6}{- : \Delta_5, p_6 \vdash \top, \Delta_8} I \quad \frac{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \top, \Delta_8}{\text{Cut}} \top_R \\
\frac{- : \Delta_5, p_6 \vdash \top, \Delta_8}{- : \Delta_5, p_6 \vdash \top, \Delta_8} \xrightarrow{\text{I}} \\
\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\top, \Delta_{10}), p_8), F_6}{- : \Delta_7, p_8 \vdash (\top, \Delta_{10}), p_8} I \quad \frac{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\top, \Delta_{10}), p_8}{\text{Cut}} \top_R \\
\frac{- : \Delta_7, p_8 \vdash (\top, \Delta_{10}), p_8}{- : \Delta_7, p_8 \vdash \top, \Delta_{10}, p_8} \xrightarrow{\text{I}}
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Box \Gamma_7, \Delta_{10}), p_5 \vdash (\Delta_8, \llbracket F_9 \rrbracket), p_5}{- : (\Box \Gamma_7, \Delta_{10}), p_5 \vdash \Delta_8, \llbracket F_9 \rrbracket} I \quad \frac{\frac{h_6 : \text{unbox}(\Box \Gamma_7) \vdash F_9}{\bullet h_6 : ((\Box \Gamma_7, \Delta_{10}), p_5), p_5 \vdash \Delta_8, \llbracket F_9 \rrbracket} K}{\text{Cut}} \\
\frac{- : (\Box \Gamma_7, \Delta_{10}), p_5 \vdash \Delta_8, \llbracket F_9 \rrbracket}{- : \Delta_{10}, \Box \Gamma_7, p_5 \vdash \Delta_8, \llbracket F_9 \rrbracket} \xrightarrow{\text{ax/W}} \\
\frac{- : \Delta_{10}, \Box \Gamma_7, p_5 \vdash \Delta_8, \llbracket F_9 \rrbracket}{- : \Delta_{10}, \Box \Gamma_7, p_5 \vdash \Delta_8, \llbracket F_9 \rrbracket} K \\
\frac{\frac{\bullet h_2 : (\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash ((\Delta_{10}, \llbracket F_9 \rrbracket), p_7), \Box F_6}{- : (\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash (\Delta_{10}, \llbracket F_9 \rrbracket), p_7} I \quad \frac{\frac{h_8 : \text{unbox}(\Box \Gamma_{11}), \text{unbox}(\Box F_6) \vdash F_9}{\bullet h_8 : ((\Box \Gamma_{11}, \Delta_{12}), p_7), \Box F_6 \vdash (\Delta_{10}, \llbracket F_9 \rrbracket), p_7} K}{\text{Cut}} \\
\frac{- : (\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash (\Delta_{10}, \llbracket F_9 \rrbracket), p_7}{- : \Delta_{12}, \Box \Gamma_{11}, p_7 \vdash \Delta_{10}, p_7, \llbracket F_9 \rrbracket} \xrightarrow{\text{I}} \\
\frac{\bullet h_2 : (\Box \Gamma_9, \Delta_{12}), p_7 \vdash ((\Delta_{11}, \llbracket F_{10} \rrbracket), p_7), F_6}{- : (\Box \Gamma_9, \Delta_{12}), p_7 \vdash (\Delta_{11}, \llbracket F_{10} \rrbracket), p_7} I \quad \frac{\frac{h_8 : \text{unbox}(\Box \Gamma_9) \vdash F_{10}}{\bullet h_8 : ((\Box \Gamma_9, \Delta_{12}), p_7), F_6 \vdash (\Delta_{11}, \llbracket F_{10} \rrbracket), p_7} K}{\text{Cut}} \\
\frac{- : (\Box \Gamma_9, \Delta_{12}), p_7 \vdash (\Delta_{11}, \llbracket F_{10} \rrbracket), p_7}{- : \Delta_{12}, \Box \Gamma_9, p_7 \vdash \Delta_{11}, p_7, \llbracket F_{10} \rrbracket} \xrightarrow{\text{I}}
\end{array}$$

- Case rule $A45$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Box \Gamma_7, \Delta_{11}), p_5 \vdash (\Box \Gamma_8, \Delta_9, \llbracket F_{10} \rrbracket), p_5}{- : (\Box \Gamma_7, \Delta_{11}), p_5 \vdash \Box \Gamma_8, \Delta_9, \llbracket F_{10} \rrbracket} I \quad \frac{\frac{h_6 : \Box \Gamma_7 \vdash \Box \Gamma_8, F_{10}}{\bullet h_6 : ((\Box \Gamma_7, \Delta_{11}), p_5), p_5 \vdash \Box \Gamma_8, \Delta_9, \llbracket F_{10} \rrbracket} A45}{\text{Cut}} \\
\frac{- : (\Box \Gamma_7, \Delta_{11}), p_5 \vdash \Box \Gamma_8, \Delta_9, \llbracket F_{10} \rrbracket}{- : \Delta_{11}, \Box \Gamma_7, p_5 \vdash \Delta_9, \Box \Gamma_8, \llbracket F_{10} \rrbracket} \xrightarrow{\text{ax/W}} \\
\frac{- : \Delta_{11}, \Box \Gamma_7, p_5 \vdash \Delta_9, \Box \Gamma_8, \llbracket F_{10} \rrbracket}{- : \Delta_{11}, \Box \Gamma_7, p_5 \vdash \Delta_9, \Box \Gamma_8, \llbracket F_{10} \rrbracket} A45 \\
\frac{\frac{\bullet h_2 : (\Box \Gamma_{12}, \Delta_{13}), p_7 \vdash ((\Box \Gamma_9, \Delta_{11}, \llbracket F_{10} \rrbracket), p_7), \Box F_6}{- : (\Box \Gamma_{12}, \Delta_{13}), p_7 \vdash (\Box \Gamma_9, \Delta_{11}, \llbracket F_{10} \rrbracket), p_7} I \quad \frac{\frac{h_8 : \Box \Gamma_{12}, \Box F_6 \vdash \Box \Gamma_9, F_{10}}{\bullet h_8 : ((\Box \Gamma_{12}, \Delta_{13}), p_7), \Box F_6 \vdash (\Box \Gamma_9, \Delta_{11}, \llbracket F_{10} \rrbracket), p_7} A45}{\text{Cut}} \\
\frac{- : (\Box \Gamma_{12}, \Delta_{13}), p_7 \vdash (\Box \Gamma_9, \Delta_{11}, \llbracket F_{10} \rrbracket), p_7}{- : \Delta_{13}, \Box \Gamma_{12}, p_7 \vdash \Delta_{11}, \Box \Gamma_9, p_7, \llbracket F_{10} \rrbracket} \xrightarrow{\text{I}}
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : (\Box \Gamma_9, \Delta_{13}), p_7 \vdash ((\Box \Gamma_{10}, \Delta_{12}, \Box F_{11}), p_7), F_6}{- : (\Box \Gamma_9, \Delta_{13}), p_7 \vdash (\Box \Gamma_{10}, \Delta_{12}, \Box F_{11}), p_7} I \quad \frac{h_8 : \Box \Gamma_9 \vdash \Box \Gamma_{10}, F_{11}}{\bullet h_8 : ((\Box \Gamma_9, \Delta_{13}), p_7), F_6 \vdash (\Box \Gamma_{10}, \Delta_{12}, \Box F_{11}), p_7} A45 \\
\hline
- : (\Box \Gamma_9, \Delta_{13}), p_7 \vdash (\Box \Gamma_{10}, \Delta_{12}, \Box F_{11}), p_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{13}, \Box \Gamma_9, p_7 \vdash \Delta_{12}, \Box \Gamma_{10}, p_7, \Box F_{11}} I
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{10}, F_7 \rightarrow F_8), p_5 \vdash \Delta_9, p_5}{- : (\Delta_{10}, F_7 \rightarrow F_8), p_5 \vdash \Delta_9} I \quad \frac{h_6 : \Delta_{10}, p_5, p_5 \vdash F_7, \Delta_9 \quad h_6 : F_8, \Delta_{10}, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \rightarrow F_8), p_5), p_5 \vdash \Delta_9} \rightarrow_L \\
\hline
- : (\Delta_{10}, F_7 \rightarrow F_8), p_5 \vdash \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10}, p_5 \vdash \Delta_9, F_7, p_5}{- : \Delta_{10}, p_5 \vdash \Delta_9, F_7} I \quad \frac{h_6 : \Delta_{10}, p_5, p_5 \vdash \Delta_9, F_7}{- : \Delta_{10}, p_5, F_7 \rightarrow F_8 \vdash \Delta_9} \text{ax/w} \quad \frac{\bullet h_1 : \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9} I \quad \frac{h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9} \text{hCut} \\
\hline
- : \Delta_{10}, p_5, F_7 \rightarrow F_8 \vdash \Delta_9 \quad \rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \rightarrow F_{11}}{- : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \rightarrow F_{11}} I \quad \frac{h_9 : \Delta_6, p_7 \vdash F_{10}, \Delta_8, p_7 \quad h_9 : F_{11}, \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \rightarrow F_{11} \vdash \Delta_8, p_7} \rightarrow_L \\
\hline
- : \Delta_6, p_7 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash (\Delta_8, p_7), F_6}{- : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash (\Delta_8, p_7), F_6} I \quad \frac{h_9 : F_6, \Delta_{12}, p_7 \vdash F_{10}, \Delta_8, p_7 \quad h_9 : F_6, F_{11}, \Delta_{12}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \rightarrow F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \rightarrow_L \\
\hline
- : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{12}, p_7, F_{10} \rightarrow F_{11} \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9, p_5}{- : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9} I \quad \frac{h_6 : F_7, F_8, \Delta_{10}, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \wedge F_8), p_5), p_5 \vdash \Delta_9} \wedge_L \\
\hline
- : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10}, F_7, F_8, p_5 \vdash \Delta_9, p_5}{- : \Delta_{10}, F_7, F_8, p_5 \vdash \Delta_9} I \quad \frac{h_6 : \Delta_{10}, F_7, F_8, p_5, p_5 \vdash \Delta_9}{- : \Delta_{10}, p_5, F_7 \wedge F_8 \vdash \Delta_9} \text{ax/w} \\
\hline
- : \Delta_{10}, F_7, F_8, p_5 \vdash \Delta_9 \quad \wedge_L \\
\hline
- : \Delta_{10}, p_5, F_7 \wedge F_8 \vdash \Delta_9
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \wedge F_{11}}{- : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \wedge F_{11}} I \quad \frac{h_9 : F_{10}, F_{11}, \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \wedge F_{11} \vdash \Delta_8, p_7} \wedge_L \\
\hline
- : \Delta_6, p_7 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash (\Delta_8, p_7), F_6}{- : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash (\Delta_8, p_7), F_6} I \quad \frac{h_9 : F_6, F_{10}, F_{11}, \Delta_{12}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \wedge F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \wedge_L \\
\hline
- : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{12}, p_7, F_{10} \wedge F_{11} \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9, p_5}{- : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9} I \quad \frac{h_6 : F_7, \Delta_{10}, p_5, p_5 \vdash \Delta_9 \quad h_6 : F_8, \Delta_{10}, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \vee F_8), p_5), p_5 \vdash \Delta_9} \vee_L \\
\hline
- : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10}, F_7, p_5 \vdash \Delta_9, p_5}{- : \Delta_{10}, F_7, p_5 \vdash \Delta_9} I \quad \frac{h_6 : \Delta_{10}, F_7, p_5, p_5 \vdash \Delta_9}{- : \Delta_{10}, F_7, p_5 \vdash \Delta_9} \text{ax/w} \quad \frac{\bullet h_1 : \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9} I \quad \frac{h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9} \text{hCut} \\
\hline
- : \Delta_{10}, p_5, F_7 \vee F_8 \vdash \Delta_9 \quad \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \vee F_{11}}{\vdash : \Delta_6, p_7 \vdash \Delta_8, p_7} I \quad \frac{\frac{h_9 : F_{10}, \Delta_6, p_7 \vdash \Delta_8, p_7 \quad h_9 : F_{11}, \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \vee F_{11} \vdash \Delta_8, p_7} \vee_L}{\vdash : \Delta_6, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : (\Delta_{12}, F_{10} \vee F_{11}), p_7 \vdash (\Delta_8, p_7), F_6}{\vdash : (\Delta_{12}, F_{10} \vee F_{11}), p_7 \vdash \Delta_8, p_7} I \quad \frac{\frac{h_9 : F_6, F_{10}, \Delta_{12}, p_7 \vdash \Delta_8, p_7 \quad h_9 : F_6, F_{11}, \Delta_{12}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \vee F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \vee_L}{\vdash : (\Delta_{12}, F_{10} \vee F_{11}), p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_{12}, p_7, F_{10} \vee F_{11} \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, \llbracket F_7 \rrbracket, p_5 \vdash \Delta_8, p_5)}{\vdash : (\Delta_9, \llbracket F_7 \rrbracket, p_5 \vdash \Delta_8} I \quad \frac{\frac{h_6 : F_7, \Delta_9, p_5, p_5, \llbracket F_7 \rrbracket \vdash \Delta_8}{\bullet h_6 : ((\Delta_9, \llbracket F_7 \rrbracket, p_5), p_5 \vdash \Delta_8} AT}{\vdash : (\Delta_9, \llbracket F_7 \rrbracket, p_5 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_9, F_7, p_5, \llbracket F_7 \rrbracket \vdash \Delta_8, p_5}{\vdash : \Delta_9, F_7, p_5, \llbracket F_7 \rrbracket \vdash \Delta_8} \text{ax/W} \quad \frac{\frac{h_6 : \Delta_9, F_7, p_5, p_5, \llbracket F_7 \rrbracket \vdash \Delta_8}{\bullet h_6 : ((\Delta_9, \llbracket F_7 \rrbracket, p_5), p_5 \vdash \Delta_8} \text{ax/W}}{\vdash : \Delta_9, p_5, \llbracket F_7 \rrbracket \vdash \Delta_8} \text{hCut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_9, p_5, \llbracket F_7 \rrbracket \vdash \Delta_8} ATG
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \llbracket F_{10} \rrbracket}{\vdash : \Delta_6, p_7 \vdash \Delta_8, p_7} I \quad \frac{\frac{h_9 : F_{10}, \Delta_6, p_7, \llbracket F_{10} \rrbracket \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), \llbracket F_{10} \rrbracket \vdash \Delta_8, p_7} AT}{\vdash : \Delta_6, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : (\Delta_{11}, \llbracket F_{10} \rrbracket, p_7 \vdash (\Delta_8, p_7), F_6)}{\vdash : (\Delta_{11}, \llbracket F_{10} \rrbracket, p_7 \vdash \Delta_8, p_7} I \quad \frac{\frac{h_9 : F_6, F_{10}, \Delta_{11}, p_7, \llbracket F_{10} \rrbracket \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{11}, \llbracket F_{10} \rrbracket, p_7), F_6 \vdash \Delta_8, p_7} AT}{\vdash : (\Delta_{11}, \llbracket F_{10} \rrbracket, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_{11}, p_7, \llbracket F_{10} \rrbracket \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\perp, \Delta_8), p_5 \vdash \Delta_7, p_5}{\vdash : (\perp, \Delta_8), p_5 \vdash \Delta_7} I \quad \frac{\frac{h_6 : ((\perp, \Delta_8), p_5), p_5 \vdash \Delta_7}{\bullet h_6 : ((\perp, \Delta_8), p_5), p_5 \vdash \Delta_7} \perp_L}{\vdash : (\perp, \Delta_8), p_5 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_8, p_5 \vdash \Delta_7} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \perp}{\vdash : \Delta_6, p_7 \vdash \Delta_8, p_7} I \quad \frac{\frac{h_9 : (\Delta_6, p_7), \perp \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), \perp \vdash \Delta_8, p_7} \perp_L}{\vdash : \Delta_6, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : (\perp, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6}{\vdash : (\perp, \Delta_{10}), p_7 \vdash \Delta_8, p_7} I \quad \frac{\frac{h_9 : ((\perp, \Delta_{10}), p_7), F_6 \vdash \Delta_8, p_7}{\bullet h_9 : ((\perp, \Delta_{10}), p_7), F_6 \vdash \Delta_8, p_7} \perp_L}{\vdash : (\perp, \Delta_{10}), p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_{10}, p_7 \vdash \Delta_8, p_7} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\bullet h_1 : \Delta_5, p_7 \vdash (\Delta_8, p_7), p_7}{\vdash : \Delta_5, p_7 \vdash \Delta_8, p_7} I \quad \frac{\frac{h_6 : (\Delta_5, p_7), p_7 \vdash \Delta_8, p_7}{\bullet h_6 : (\Delta_5, p_7), p_7 \vdash \Delta_8, p_7} I}{\vdash : \Delta_5, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_5, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_9, p_7), p_5 \vdash (\Delta_8, p_7), p_5}{- : (\Delta_9, p_7), p_5 \vdash \Delta_8, p_7} I}{\frac{\bullet h_6 : ((\Delta_9, p_7), p_5), p_5 \vdash \Delta_8, p_7}{- : (\Delta_9, p_7), p_5 \vdash \Delta_8, p_7} I} \text{Cut} \\
\frac{\bullet h_2 : \Delta_6, p_7 \vdash ((\Delta_{10}, p_9), p_7), p_9}{- : \Delta_6, p_7 \vdash (\Delta_{10}, p_9), p_7} I}{\frac{\bullet h_8 : (\Delta_6, p_7), p_9 \vdash (\Delta_{10}, p_9), p_7}{- : \Delta_6, p_7 \vdash (\Delta_{10}, p_9), p_7} I} \text{Cut} \\
\frac{\bullet h_2 : (\Delta_{11}, p_9), p_7 \vdash ((\Delta_{10}, p_9), p_7), F_6}{- : (\Delta_{11}, p_9), p_7 \vdash (\Delta_{10}, p_9), p_7} I}{\frac{\bullet h_8 : ((\Delta_{11}, p_9), p_7), F_6 \vdash (\Delta_{10}, p_9), p_7}{- : (\Delta_{11}, p_9), p_7 \vdash (\Delta_{10}, p_9), p_7} I} \text{Cut} \\
\frac{\bullet h_2 : \Delta_7, p_9 \vdash (\Delta_{10}, p_9), F_6}{- : \Delta_7, p_9 \vdash \Delta_{10}, p_9} I}{\frac{\bullet h_8 : (\Delta_7, p_9), F_6 \vdash \Delta_{10}, p_9}{- : \Delta_7, p_9 \vdash \Delta_{10}, p_9} I} \text{Cut}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\top, \Delta_8), p_5 \vdash \Delta_7, p_5}{- : (\top, \Delta_8), p_5 \vdash \Delta_7} I}{\frac{\bullet h_6 : ((\top, \Delta_8), p_5), p_5 \vdash \Delta_7}{- : (\top, \Delta_8), p_5 \vdash \Delta_7} I} \text{Cut} \\
\frac{\bullet h_1 : \top, \Delta_8, p_5 \vdash \Delta_7, p_5}{- : \top, \Delta_8, p_5 \vdash \Delta_7} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_8, p_5, p_5 \vdash \Delta_7}{- : \top, \Delta_8, p_5 \vdash \Delta_7} \text{hCut} \\
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \top}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I}{\frac{\bullet h_9 : (\Delta_6, p_7), \top \vdash \Delta_8, p_7}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I} \text{Cut} \\
\frac{\bullet h_2 : (\top, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6}{- : (\top, \Delta_{10}), p_7 \vdash \Delta_8, p_7} I}{\frac{\bullet h_9 : (\top, \Delta_{10}), p_7, F_6 \vdash \Delta_8, p_7}{- : \top, \Delta_{10}, p_7 \vdash \Delta_8, p_7} I} \text{Cut}
\end{array}$$

6.14 Status of \top_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_6 \vdash F_5, \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_5} \top_L}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \frac{h_7 : \top, F_5, F_9, \Delta_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R \\
\frac{\bullet h_3 : \top, \Delta_6 \vdash \Delta_8, F_5, F_9 \rightarrow F_{10}}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8, F_9 \rightarrow F_{10}}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_6 \vdash F_5, \Delta_8, F_9 \wedge F_{10}}{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \wedge F_{10}), F_5} \top_L}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \wedge F_{10}} \frac{h_7 : \top, F_5, \Delta_6 \vdash F_9, \Delta_8 \quad h_7 : \top, F_5, \Delta_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R \\
\frac{\bullet h_3 : \top, \Delta_6 \vdash \Delta_8, F_5, F_9 \wedge F_{10}}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8, F_9 \wedge F_{10}}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \wedge F_{10}} \text{hCut}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_3 : \Delta_6 \vdash F_5, \Delta_8, F_9 \vee F_{10}}{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \vee F_{10}), F_5} \top_L \quad \frac{h_7 : \top, F_5, \Delta_6 \vdash F_9, F_{10}, \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R \\
\hline
- : \top, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_3 : \top, \Delta_6 \vdash \Delta_8, F_5, F_9 \vee F_{10}}{\bullet h_3 : \top, \Delta_6 \vdash \Delta_8, F_5, F_9 \vee F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8, F_9 \vee F_{10}} \text{ax/W} \\
\hline
- : \top, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \quad \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_3 : \Delta_6 \vdash F_5, \perp, \Delta_8}{\bullet h_3 : \top, \Delta_6 \vdash (\perp, \Delta_8), F_5} \top_L \quad \frac{h_7 : \top, F_5, \Delta_6 \vdash \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \perp, \Delta_8} \perp_R \\
\hline
- : \top, \Delta_6 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_3 : \top, \Delta_6 \vdash \perp, \Delta_8, F_5}{\bullet h_3 : \top, \Delta_6 \vdash \perp, \Delta_8, F_5} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \perp, \Delta_8}{\bullet h_7 : \top, \Delta_6, F_5 \vdash \perp, \Delta_8} \text{ax/W} \\
\hline
- : \top, \Delta_6 \vdash \perp, \Delta_8 \quad \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_3 : \Delta_6 \vdash F_5, \top, \Delta_8}{\bullet h_3 : \top, \Delta_6 \vdash (\top, \Delta_8), F_5} \top_L \quad \frac{}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \top, \Delta_8} \top_R \\
\hline
- : \top, \Delta_6 \vdash \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \top, \Delta_8} \top_R \\
\hline
- : \top, \Delta_6 \vdash \top, \Delta_8
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_3 : \Box \Gamma_9, \Delta_{10} \vdash \Box F_5, \Delta_7, \Box F_8}{\bullet h_3 : \top, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, \Box F_8), \Box F_5} \top_L \quad \frac{h_6 : \text{unbox}(\Box \Gamma_9), \text{unbox}(\Box F_5) \vdash F_8}{\bullet h_6 : (\top, \Box \Gamma_9, \Delta_{10}), \Box F_5 \vdash \Delta_7, \Box F_8} K \\
\hline
- : \top, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, \Box F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_3 : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Box F_5, \Delta_7, \Box F_8}{\bullet h_3 : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Box F_5, \Delta_7, \Box F_8} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Box F_5, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_8}{\bullet h_6 : \top, \Box F_5, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_8} \text{ax/W} \\
\hline
- : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_8 \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : \Box \Gamma_7, \Delta_{10} \vdash F_5, \Delta_8, \Box F_9}{\bullet h_3 : \top, \Box \Gamma_7, \Delta_{10} \vdash (\Delta_8, \Box F_9), F_5} \top_L \quad \frac{h_6 : \text{unbox}(\Box \Gamma_7) \vdash F_9}{\bullet h_6 : (\top, \Box \Gamma_7, \Delta_{10}), F_5 \vdash \Delta_8, \Box F_9} K \\
\hline
- : \top, \Box \Gamma_7, \Delta_{10} \vdash \Delta_8, \Box F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{\bullet h_6 : (\top, \Box \Gamma_7, \Delta_{10}), F_5 \vdash \Delta_8, \Box F_9} K \\
\hline
- : \text{unbox}(\Box \Gamma_7) \vdash F_9 \quad \text{ax/W} \\
\hline
- : \top, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, \Box F_9
\end{array}$$

- Case rule $A45$

$$\begin{array}{c}
\frac{h_3 : \Box \Gamma_{10}, \Delta_{11} \vdash \Box F_5, \Box \Gamma_7, \Delta_8, \Box F_9}{\bullet h_3 : \top, \Box \Gamma_{10}, \Delta_{11} \vdash (\Box \Gamma_7, \Delta_8, \Box F_9), \Box F_5} \top_L \quad \frac{h_6 : \Box \Gamma_{10}, \Box F_5 \vdash \Box \Gamma_7, F_9}{\bullet h_6 : (\top, \Box \Gamma_{10}, \Delta_{11}), \Box F_5 \vdash \Box \Gamma_7, \Delta_8, \Box F_9} A45 \\
\hline
- : \top, \Box \Gamma_{10}, \Delta_{11} \vdash \Box \Gamma_7, \Delta_8, \Box F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_3 : \top, \Delta_{11}, \Box \Gamma_{10} \vdash \Box F_5, \Delta_8, \Box \Gamma_7, \Box F_9}{\bullet h_3 : \top, \Delta_{11}, \Box \Gamma_{10} \vdash \Box F_5, \Delta_8, \Box \Gamma_7, \Box F_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Box F_5, \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_8, \Box \Gamma_7, \Box F_9}{\bullet h_6 : \top, \Box F_5, \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_8, \Box \Gamma_7, \Box F_9} \text{ax/W} \\
\hline
- : \top, \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_8, \Box \Gamma_7, \Box F_9 \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : \Box \Gamma_7, \Delta_{11} \vdash F_5, \Box \Gamma_8, \Delta_9, \Box F_{10}}{\bullet h_3 : \top, \Box \Gamma_7, \Delta_{11} \vdash (\Box \Gamma_8, \Delta_9, \Box F_{10}), F_5} \top_L \quad \frac{h_6 : \Box \Gamma_7 \vdash \Box \Gamma_8, F_{10}}{\bullet h_6 : (\top, \Box \Gamma_7, \Delta_{11}), F_5 \vdash \Box \Gamma_8, \Delta_9, \Box F_{10}} A45 \\
\hline
- : \top, \Box \Gamma_7, \Delta_{11} \vdash \Box \Gamma_8, \Delta_9, \Box F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{\bullet h_6 : (\top, \Box \Gamma_7, \Delta_{11}), F_5 \vdash \Box \Gamma_8, \Delta_9, \Box F_{10}} A45 \\
\hline
- : \Box \Gamma_7 \vdash F_{10}, \Box \Gamma_8 \quad \text{ax/W} \\
\hline
- : \top, \Delta_{11}, \Box \Gamma_7 \vdash \Delta_9, \Box \Gamma_8, \Box F_{10}
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_5 \vdash F_7 \rightarrow F_8, \Delta_9}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8} \top_L \quad \frac{h_6 : \top, \Delta_5 \vdash F_7, \Delta_9 \quad h_6 : \top, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \rightarrow F_8 \vdash \Delta_9} \rightarrow_L}{- : \top, \Delta_5 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8}{- : \top, \Delta_5 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, F_7 \rightarrow F_8 \vdash \Delta_9}{- : \top, \Delta_5 \vdash \Delta_9} \text{hCut}}{- : \top, \Delta_5 \vdash \Delta_9}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_5, \Delta_9}{\bullet h_3 : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5} \top_L \quad \frac{h_6 : \top, F_5, \Delta_{10} \vdash F_7, \Delta_9 \quad h_6 : \top, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \rightarrow F_8), F_5 \vdash \Delta_9} \rightarrow_L}{- : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5}{- : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \rightarrow F_8 \vdash \Delta_9}{- : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{hCut}}{- : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_5 \vdash F_7 \wedge F_8, \Delta_9}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8} \top_L \quad \frac{h_6 : \top, F_7, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \wedge F_8 \vdash \Delta_9} \wedge_L}{- : \top, \Delta_5 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8}{- : \top, \Delta_5 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, F_7 \wedge F_8 \vdash \Delta_9}{- : \top, \Delta_5 \vdash \Delta_9} \text{hCut}}{- : \top, \Delta_5 \vdash \Delta_9}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_{10}, F_7 \wedge F_8 \vdash F_5, \Delta_9}{\bullet h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \top_L \quad \frac{h_6 : \top, F_5, F_7, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \wedge_L}{- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5}{- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \wedge F_8 \vdash \Delta_9}{- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{hCut}}{- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_5 \vdash F_7 \vee F_8, \Delta_9}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \top_L \quad \frac{h_6 : \top, F_7, \Delta_5 \vdash \Delta_9 \quad h_6 : \top, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \vee F_8 \vdash \Delta_9} \vee_L}{- : \top, \Delta_5 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8}{- : \top, \Delta_5 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, F_7 \vee F_8 \vdash \Delta_9}{- : \top, \Delta_5 \vdash \Delta_9} \text{hCut}}{- : \top, \Delta_5 \vdash \Delta_9}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_{10}, F_7 \vee F_8 \vdash F_5, \Delta_9}{\bullet h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \top_L \quad \frac{h_6 : \top, F_5, F_7, \Delta_{10} \vdash \Delta_9 \quad h_6 : \top, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \vee F_8), F_5 \vdash \Delta_9} \vee_L}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \vee F_8 \vdash \Delta_9}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{hCut}}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9}
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_5 \vdash \Box F_7, \Delta_8}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_8, \Box F_7} \top_L \quad \frac{h_6 : \top, F_7, \Delta_5, \Box F_7 \vdash \Delta_8}{\bullet h_6 : (\top, \Delta_5), \Box F_7 \vdash \Delta_8} AT}{- : \top, \Delta_5 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_8, \Box F_7}{- : \top, \Delta_5 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, \Box F_7 \vdash \Delta_8}{- : \top, \Delta_5 \vdash \Delta_8} \text{hCut}}{- : \top, \Delta_5 \vdash \Delta_8}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : \Delta_9, [\Box F_7 \vdash F_5, \Delta_8]}{\bullet h_3 : \top, \Delta_9, [\Box F_7 \vdash \Delta_8, F_5]} \top_L \quad \frac{h_6 : \top, F_5, F_7, \Delta_9, [\Box F_7 \vdash \Delta_8]}{\bullet h_6 : (\top, \Delta_9, [\Box F_7]), F_5 \vdash \Delta_8} AT \\
\hline
- : \top, \Delta_9, [\Box F_7 \vdash \Delta_8] \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_9, [\Box F_7 \vdash \Delta_8, F_5]}{\quad} ax/W \quad \frac{\bullet h_6 : \top, \Delta_9, F_5, [\Box F_7 \vdash \Delta_8]}{\quad} hCut}{- : \top, \Delta_9, [\Box F_7 \vdash \Delta_8]} ax/W
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_3 : \Delta_5 \vdash \perp, \Delta_7}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_7, \perp} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_5), \perp \vdash \Delta_7} \perp_L \\
\hline
- : \top, \Delta_5 \vdash \Delta_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_5 \vdash \perp, \Delta_7}{\quad} ax/W \quad \frac{\bullet h_6 : \perp, \top, \Delta_5 \vdash \Delta_7}{\quad} \perp_L}{- : \top, \Delta_5 \vdash \Delta_7} hCut \\
\\
\frac{h_3 : \perp, \Delta_8 \vdash F_5, \Delta_7}{\bullet h_3 : \top, \perp, \Delta_8 \vdash \Delta_7, F_5} \top_L \quad \frac{}{\bullet h_6 : (\top, \perp, \Delta_8), F_5 \vdash \Delta_7} \perp_L \\
\hline
- : \top, \perp, \Delta_8 \vdash \Delta_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \perp, \top, \Delta_8 \vdash \Delta_7} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_3 : \Delta_5 \vdash p_7, \Delta_8, p_7}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, p_7), p_7} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_5), p_7 \vdash \Delta_8, p_7} I \\
\hline
- : \top, \Delta_5 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_8, p_7, p_7}{\quad} ax/W \quad \frac{\bullet h_6 : \top, \Delta_5, p_7 \vdash \Delta_8, p_7}{\quad} I}{- : \top, \Delta_5 \vdash \Delta_8, p_7} hCut \\
\\
\frac{h_3 : \Delta_9, p_7 \vdash F_5, \Delta_8, p_7}{\bullet h_3 : \top, \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_9, p_7), F_5 \vdash \Delta_8, p_7} I \\
\hline
- : \top, \Delta_9, p_7 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \top, \Delta_9, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_3 : \Delta_6 \vdash F_5, \Delta_8}{\bullet h_3 : \top, \Delta_6 \vdash \Delta_8, F_5} \top_L \quad \frac{h_7 : F_5, \Delta_6 \vdash \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8} \top_L \\
\hline
- : \top, \Delta_6 \vdash \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_3 : \top, \Delta_6 \vdash \Delta_8, F_5}{\quad} ax/W \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8}{\quad} hCut}{- : \top, \Delta_6 \vdash \Delta_8} ax/W
\end{array}$$