Modal Logic K+T+4+5

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Abstract

This system does not have cut-elimination.

Contents

1 Height preserving admissibility of weakening on the left

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \to \mathtt{F}_4} \to_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3 \to \mathtt{F}_4} \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3}^{\,\mathtt{IH}} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \bot, \Delta_3} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \top, \Delta_3} \top_R$$

- Case(s) rule K

$$\frac{\mathtt{h}_1 : unbox(\Box \Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [] \mathtt{F}_2} K \quad \to \quad \frac{\overline{\mathtt{h}_1 : unbox(\Box \Gamma_4) \vdash \mathtt{F}_2}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \Box \Gamma_4 \vdash \Delta_3, [] \mathtt{F}_2} K$$

- Case(s) rule $A45$

$$\frac{\mathtt{h}_1 : \Box \Gamma_5 \vdash \Box \Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box \Gamma_5, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, [] \mathtt{F}_2} A45 \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Box \Gamma_5 \vdash \mathtt{F}_2, \Box \Gamma_3}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_W, \Box \Gamma_5 \vdash \Delta_4, \Box \Gamma_3, [] \mathtt{F}_2} A45$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4} \to_L \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_W \vdash \Delta_4, \mathtt{F}_2}^{\,\mathtt{IH}} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_4}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4} \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_W \vdash \Delta_4}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, \mathtt{F}_W, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_W, []\mathtt{F}_2 \vdash \Delta_3}\ AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3, \mathtt{F}_W \vdash \Delta_2}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_W, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_W \vdash \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \top, \Delta_3, \mathtt{F}_W \vdash \Delta_2}\ \top_L$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_W, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R}$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\scriptstyle \wedge_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_W, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\scriptstyle \wedge_R}$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\scriptstyle \vee_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_W, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\scriptstyle \vee_R}$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\scriptstyle \bot_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \mathtt{F}_W}\ {\scriptstyle \bot_R}$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\scriptstyle \top_R} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3, \mathtt{F}_W}\ {\scriptstyle \top_R}$$

- Case(s) rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ {\scriptstyle K} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, \mathtt{F}_W, []\mathtt{F}_2}\ {\scriptstyle K}$$

- Case(s) rule $A45$

$$\frac{\mathtt{h}_1 : \Box\Gamma_5 \vdash \Box\Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_2}\ {\scriptstyle A45} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2, \Box\Gamma_3}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_6, \Box\Gamma_5 \vdash \Delta_4, \mathtt{F}_W, \Box\Gamma_3, []\mathtt{F}_2}\ {\scriptstyle A45}$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \rightarrow_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \rightarrow_L}$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \wedge_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \wedge_L}$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \vee_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \vee_L}$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet\mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3, \mathtt{F}_W}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3, \mathtt{F}_W}\ AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2, \mathtt{F}_W}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{F}_W, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2, \mathtt{F}_W}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2, \mathtt{F}_W}\ \top_L$$

3 Measure of derivations

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case(s) rule K

$$\frac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2} K \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_4) \vdash \mathtt{F}_2}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_4) \vdash \mathtt{F}_2}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2} K$$

- Case(s) rule $A45$

$$\frac{\mathtt{h}_1 : \Box\Gamma_5 \vdash \Box\Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_2} A45 \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2, \Box\Gamma_3}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2, \Box\Gamma_3}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_6, \Box\Gamma_5 \vdash \Delta_4, \Box\Gamma_3, []\mathtt{F}_2} A45$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet\mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\bullet\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet\bullet\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L$$

4 Invertibility of Rules

4.1 Status of $\rightarrow_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ {\scriptstyle \rightarrow_R}$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{H}}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ {\scriptstyle \wedge_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ {\scriptstyle \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ {\scriptstyle \wedge_R}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ {\scriptstyle \vee_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ {\scriptstyle \vee_R}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \bot_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \bot, \Delta_5, \mathtt{F}_2}\ {\scriptstyle \bot_R}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \top_R} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \top, \Delta_5, \mathtt{F}_2}\ {\scriptstyle \top_R}$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2), []\mathtt{F}_4}\ {\scriptstyle K} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_2, []\mathtt{F}_4}\ {\scriptstyle K}$$

- Case rule $A45$

$$\frac{\mathtt{h}_3 : \Box\Gamma_7 \vdash \Box\Gamma_5, \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_5, (\Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2), []\mathtt{F}_4}\ {\scriptstyle A45} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_7 \vdash \mathtt{F}_4, \Box\Gamma_5}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_1, \Box\Gamma_7 \vdash \Delta_6, \mathtt{F}_2, \Box\Gamma_5, []\mathtt{F}_4}\ {\scriptstyle A45}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{F}_4, \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \rightarrow_L} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\scriptstyle \rightarrow_L}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2}\ \wedge_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2}\ \vee_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2}\ AT \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_2}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}\ \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vdash \Delta_4, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}\ I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}\ \top_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vdash \Delta_4, \mathtt{F}_2}\ \top_L$$

4.2 Status of $\wedge_R$: (Left Premise): `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_7, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_4}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_3 : \Box\Gamma_7 \vdash \Box\Gamma_5, \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_5, (\Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4}\ A45 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_7 \vdash \mathtt{F}_4, \Box\Gamma_5}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_8, \Box\Gamma_7 \vdash \Delta_6, \mathtt{F}_1, \Box\Gamma_5, []\mathtt{F}_4}\ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{F}_4, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \top_L$$

4.3 Status of $\wedge_R$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_2}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, \mathtt{F}_2, []\mathtt{F}_4}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_3 : \Box \Gamma_7 \vdash \Box \Gamma_5, \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, (\Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4}\ A45 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box \Gamma_7 \vdash \mathtt{F}_4, \Box \Gamma_5}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_8, \Box \Gamma_7 \vdash \Delta_6, \mathtt{F}_2, \Box \Gamma_5, []\mathtt{F}_4}\ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{F}_4, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_2}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \top_L$$

4.4 Status of $\vee_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2, []\mathtt{F}_4}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_3 : \Box \Gamma_7 \vdash \Box \Gamma_5, \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, (\Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4}\ A45 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box \Gamma_7 \vdash \mathtt{F}_4, \Box \Gamma_5}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_8, \Box \Gamma_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \Box \Gamma_5, []\mathtt{F}_4}\ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{F}_4, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2}\ \top_L$$

4.5 Status of $\bot_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \bot, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \bot, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \bot, \Delta_3}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash (\bot, \Delta_3), []\mathtt{F}_2}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_1 : \Box\Gamma_5 \vdash \Box\Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \Delta_6 \vdash \Box\Gamma_3, (\bot, \Delta_4), []\mathtt{F}_2}\ A45 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2, \Box\Gamma_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_6, \Box\Gamma_5 \vdash \Delta_4, \Box\Gamma_3, []\mathtt{F}_2}\ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \bot, \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \bot, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \bot, \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \bot, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \bot, \Delta_4} \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \bot, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \bot, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \bot, \Delta_4} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \bot, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \bot, \Delta_3} AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3} AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \bot, \Delta_2} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_4 \vdash \mathtt{p}_3, \bot, \Delta_2} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \bot, \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \bot, \Delta_2} \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2} \top_L$$

4.6 Status of $\top_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \top, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \top, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \top, \Delta_3}\ \bot_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box \Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box \Gamma_4, \Delta_5 \vdash (\top, \Delta_3), [] \mathtt{F}_2}\ K \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $A45$

$$\frac{\mathtt{h}_1 : \Box \Gamma_5 \vdash \Box \Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box \Gamma_5, \Delta_6 \vdash \Box \Gamma_3, (\top, \Delta_4), [] \mathtt{F}_2}\ A45 \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \top, \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \top, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \top, \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \top, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \top, \Delta_4}\ \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \top, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \top, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \top, \Delta_4}\ \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, [] \mathtt{F}_2 \vdash \top, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_4, [] \mathtt{F}_2 \vdash \top, \Delta_3}\ AT \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \top, \Delta_2}\ \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_4 \vdash \mathtt{p}_3, \top, \Delta_2}\ I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \top, \Delta_2}{\bullet\mathtt{h}_1 : \top, \Delta_3 \vdash \top, \Delta_2}\ \top_L \quad \rightarrow \quad \text{trivial}$$

4.7 Status of K: : Non invertible

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \Delta_7 \vdash \mathtt{F}_4, \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathtt{F}_1), \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_3, \Delta_5, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_4, \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathtt{F}_1), \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathtt{F}_1), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \bot, \Delta_3, []\mathtt{F}_1}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \top, \Delta_3, []\mathtt{F}_1}\ \top_R \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \text{fail}$$

- Case rule K

$$\frac{\mathtt{h}_2 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash \mathtt{F}_3}{\bullet\mathtt{h}_2 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []\mathtt{F}_1), []\mathtt{F}_3}\ K \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_7) \vdash \mathtt{F}_1}\ \text{fail}$$

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_5) \vdash \mathtt{F}_2}{\bullet\mathtt{h}_1 : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_3, []\mathtt{F}_2}\ K \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_6) \vdash \mathtt{F}_2}\ \text{fail}$$

- Case rule $A45$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Box\Gamma_7 \vdash \Box\Gamma_4, \mathtt{F}_3, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Box\Gamma_7), \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_4, []\mathtt{F}_1), \Delta_5, []\mathtt{F}_3}\ A45 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6), unbox(\Box\Gamma_8) \vdash \mathtt{F}_1}\ \text{fail}$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Box\Gamma_7 \vdash \Box\Gamma_4, \mathtt{F}_3}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Box\Gamma_7), \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_4, (\Delta_5, []\mathtt{F}_1), []\mathtt{F}_3}\ A45 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6), unbox(\Box\Gamma_8) \vdash \mathtt{F}_1}\ \text{fail}$$

$$\frac{\mathtt{h}_1 : \Box\Gamma_5, \Box\Gamma_6 \vdash \Box\Gamma_3, \mathtt{F}_2}{\bullet\mathtt{h}_1 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_2}\ A45 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_7) \vdash \mathtt{F}_2}\ \text{fail}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_3, \Delta_5, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_4, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5, []\mathtt{F}_1}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \mathtt{F}_4, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5, []\mathtt{F}_1}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_4, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5, []\mathtt{F}_1}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule AT

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \mathtt{F}_3, \Delta_6, []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_5, \Delta_6), []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \mathtt{F}_3, unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : \mathtt{F}_3, unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{H}$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \mathtt{F}_3, \Delta_6, []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_5, \Delta_6), []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_2 : \bot, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \mathtt{fail}$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_2 : \mathtt{p}_4, \Box\Gamma_5, \Delta_6 \vdash \mathtt{p}_4, \Delta_3, []\mathtt{F}_1}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{fail}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \top, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \mathtt{H}$$

4.8 Status of $A45$: : `Non invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_7, \mathtt{F}_3, \Delta_8 \vdash \Box\Gamma_5, \mathtt{F}_4, \Delta_6, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_5, \Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \mathtt{H}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_5, \mathtt{F}_3, \Delta_6, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_5, \mathtt{F}_4, \Delta_6, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_5, \Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \mathtt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_5, \mathtt{F}_3, \mathtt{F}_4, \Delta_6, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_5, \Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \mathtt{H}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_5, \Delta_6 \vdash \bot, \Box\Gamma_3, \Delta_4, []\mathtt{F}_1}\ \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_5 \vdash \mathtt{F}_1, \Box\Gamma_3}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_5 \vdash \mathtt{F}_1, \Box\Gamma_3}\ \mathtt{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_5, \Delta_6 \vdash \top, \Box\Gamma_3, \Delta_4, []\mathtt{F}_1}\ \top_R \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_5 \vdash \Box\Gamma_3, \mathtt{F}_1}\ \text{fail}$$

- Case rule K

$$\frac{\mathtt{h}_2 : unbox(\Box\Gamma_6), unbox(\Box\Gamma_7) \vdash \mathtt{F}_3}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Box\Gamma_7), \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_4, \Delta_5, []\mathtt{F}_1), []\mathtt{F}_3}\ K \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_6, \Box\Gamma_8 \vdash \Box\Gamma_4, \mathtt{F}_1, []\mathtt{F}_3}\ \text{fail}$$

$$\frac{\mathtt{h}_2 : unbox(\Box\Gamma_6), unbox(\Box\Gamma_7) \vdash \mathtt{F}_3}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Box\Gamma_7), \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_4, \Delta_5, []\mathtt{F}_1), []\mathtt{F}_3}\ K \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_6, \Box\Gamma_8 \vdash \Box\Gamma_4, \mathtt{F}_1}\ \text{fail}$$

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash \mathtt{F}_2}{\bullet\mathtt{h}_1 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Box\Gamma_3, \Delta_4), []\mathtt{F}_2}\ K \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Box\Gamma_5, \Box\Gamma_7 \vdash \Box\Gamma_3, \mathtt{F}_2}\ \text{fail}$$

- Case rule $A45$

$$\frac{\mathtt{h}_2 : \Box\Gamma_8, \Box\Gamma_9 \vdash \Box\Gamma_4, \Box\Gamma_5, \mathtt{F}_3, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_8, \Box\Gamma_9), \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_4, \Box\Gamma_5, []\mathtt{F}_1), (\Box\Gamma_6, \Delta_7), []\mathtt{F}_3}\ A45 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_8, \Box\Gamma_{10} \vdash \Box\Gamma_4, \Box\Gamma_6, \mathtt{F}_1, []\mathtt{F}_3}\ \text{fail}$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_8, \Box\Gamma_9 \vdash \Box\Gamma_4, \Box\Gamma_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 : (\Box\Gamma_8, \Box\Gamma_9), \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_4, \Box\Gamma_5), (\Box\Gamma_6, \Delta_7, []\mathtt{F}_1), []\mathtt{F}_3}\ A45 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_8, \Box\Gamma_{10} \vdash \Box\Gamma_4, \Box\Gamma_6, \mathtt{F}_1, []\mathtt{F}_3}\ \text{fail}$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_8, \Box\Gamma_9 \vdash \Box\Gamma_4, \Box\Gamma_5, \mathtt{F}_3, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_8, \Box\Gamma_9), \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_4, \Box\Gamma_5, []\mathtt{F}_1), (\Box\Gamma_6, \Delta_7), []\mathtt{F}_3}\ A45 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_8, \Box\Gamma_{10} \vdash \Box\Gamma_4, \Box\Gamma_6, \mathtt{F}_1}\ \text{fail}$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_8, \Box\Gamma_9 \vdash \Box\Gamma_4, \Box\Gamma_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 : (\Box\Gamma_8, \Box\Gamma_9), \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_4, \Box\Gamma_5), (\Box\Gamma_6, \Delta_7, []\mathtt{F}_1), []\mathtt{F}_3}\ A45 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_8, \Box\Gamma_{10} \vdash \Box\Gamma_4, \Box\Gamma_6, \mathtt{F}_1}\ \text{fail}$$

$$\frac{\mathtt{h}_1 : \Box\Gamma_7, \Box\Gamma_8 \vdash \Box\Gamma_3, \Box\Gamma_4, \mathtt{F}_2}{\bullet\mathtt{h}_1 : (\Box\Gamma_7, \Box\Gamma_8), \Box\Gamma_9, \Delta_{10} \vdash (\Box\Gamma_3, \Box\Gamma_4), (\Box\Gamma_5, \Delta_6), []\mathtt{F}_2}\ A45 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Box\Gamma_7, \Box\Gamma_9 \vdash \Box\Gamma_3, \Box\Gamma_5, \mathtt{F}_2}\ \text{fail}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash \Box\Gamma_5, \mathtt{F}_3, \Delta_6, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_7, \mathtt{F}_4, \Delta_8 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_7, \Delta_8), \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_1}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_7, \mathtt{F}_3, \mathtt{F}_4, \Delta_8 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_7, \Delta_8), \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_1}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \text{H}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_7, \mathtt{F}_3, \Delta_8 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_7, \mathtt{F}_4, \Delta_8 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_7, \Delta_8), \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_1}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_1, \Box\Gamma_5}\ \text{H}$$

- Case rule AT

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \Delta_7, []\mathtt{F}_3 \vdash \Box\Gamma_4, \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), []\mathtt{F}_3 \vdash \Box\Gamma_4, \Delta_5, []\mathtt{F}_1}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_6, []\mathtt{F}_3 \vdash \mathtt{F}_1, \Box\Gamma_4}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_6, []\mathtt{F}_3 \vdash \mathtt{F}_1, \Box\Gamma_4}\ \text{H}$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \Delta_7, []\mathtt{F}_3 \vdash \Box\Gamma_4, \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), []\mathtt{F}_3 \vdash \Box\Gamma_4, \Delta_5, []\mathtt{F}_1}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1, \Box\Gamma_4}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1, \Box\Gamma_4}\ \text{H}$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_2 : \bot, \Box\Gamma_5, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_1}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_5 \vdash \Box\Gamma_3, \mathtt{F}_1}\ \text{fail}$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_2 : \mathtt{p}_5, \Box\Gamma_6, \Delta_7 \vdash \mathtt{p}_5, \Box\Gamma_3, \Delta_4, []\mathtt{F}_1}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_6 \vdash \Box\Gamma_3, \mathtt{F}_1}\ \text{fail}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \top, \Box\Gamma_5, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_1}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_5 \vdash \mathtt{F}_1, \Box\Gamma_3}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_5 \vdash \mathtt{F}_1, \Box\Gamma_3}\ \text{H}$$

4.9 Status of $\rightarrow_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_5 \vdash \bot, \Delta_4, \mathtt{F}_1}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5 \vdash \top, \Delta_4, \mathtt{F}_1}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ K \quad \to \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_7, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_4}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_3 : \Box\Gamma_7 \vdash \Box\Gamma_5, \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_7, \Delta_8, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_4}\ A45 \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_7 \vdash \mathtt{F}_4, \Box\Gamma_5}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_8, \Box\Gamma_7 \vdash \Delta_6, \mathtt{F}_1, \Box\Gamma_5, []\mathtt{F}_4}\ A45$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6}\ \to_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \to_L$$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4}\ \to_L \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \to \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \top_L$$

4.10 Status of $\to_L$ (Right Premise): : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \to \mathtt{F}_5}\ \to_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \to \mathtt{F}_5}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ K \quad \to \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_3 : \Box\Gamma_7 \vdash \Box\Gamma_5, \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_7, \Delta_8, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_4}\ A45 \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_7 \vdash \mathtt{F}_4, \Box\Gamma_5}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_2, \Box\Gamma_7 \vdash \Delta_6, \Box\Gamma_5, []\mathtt{F}_4}\ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L$$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_4} \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4} \top_L$$

4.11 Status of $\wedge_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_3 : \Box\Gamma_7 \vdash \Box\Gamma_5, \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_7, \Delta_8, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_4}\ A45 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_7 \vdash \mathtt{F}_4, \Box\Gamma_5}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_1, \mathtt{F}_2, \Box\Gamma_7 \vdash \Delta_6, \Box\Gamma_5, []\mathtt{F}_4}\ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{H}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.12 Status of $\vee_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \bot, \Delta_4}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \top, \Delta_4}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_3 : \Box\Gamma_7 \vdash \Box\Gamma_5, \mathtt{F}_4}{\bullet\mathtt{h}_3 : \Box\Gamma_7, \Delta_8, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_4}\ A45 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_7 \vdash \mathtt{F}_4, \Box\Gamma_5}\ \mathtt{ax}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_1, \Box\Gamma_7 \vdash \Delta_6, \Box\Gamma_5, []\mathtt{F}_4}\ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{H}$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet\mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \top_L$$

4.13 Status of $\vee_L$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \top, \Delta_4} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \top, \Delta_4} \top_R$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4} K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}^{\ ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4} K$$

- Case rule $A45$

$$\frac{\mathtt{h}_3 : \Box\Gamma_7 \vdash \Box\Gamma_5, \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_7, \Delta_8, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Box\Gamma_5, \Delta_6, []\mathtt{F}_4} A45 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_7 \vdash \mathtt{F}_4, \Box\Gamma_5}^{\ ax}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_2, \Box\Gamma_7 \vdash \Delta_6, \Box\Gamma_5, []\mathtt{F}_4} A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\,{\scriptstyle ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\,{\scriptstyle ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

$$\dfrac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,{\scriptstyle ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,{\scriptstyle H}$$

- Case rule AT

$$\dfrac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5} AT \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\,{\scriptstyle ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, []\mathtt{F}_4 \vdash \Delta_5} AT$$

- Case rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4} \bot_L \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4} \bot_L$$

- Case rule I

$$\dfrac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4} I \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5} I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4} \top_L \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\,{\scriptstyle ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4} \top_L$$

4.14 Status of AT: : `Invertible`

- Case rule $\rightarrow_R$

$$\dfrac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_4}\,{\scriptstyle ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3}\,{\scriptstyle ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_4}\,{\scriptstyle ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\,{\scriptstyle ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\mathtt{h}_2 : \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \Delta_4, []\mathtt{F}_1 \vdash \bot, \Delta_3} \bot_R \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}\,{\scriptstyle ax/ind}}{\bullet \mathtt{h}_2 : \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \bot, \Delta_3} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_4, \Box \mathtt{F}_1 \vdash \top, \Delta_3}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_4, \mathtt{F}_1, \Box \mathtt{F}_1 \vdash \top, \Delta_3}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_2 : \mathtt{F}_1, unbox(\Box \Gamma_5) \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : (\Box \Gamma_5, \Box \mathtt{F}_1), \Delta_6 \vdash \Delta_4, \Box \mathtt{F}_3}\ K \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \mathtt{F}_1, unbox(\Box \Gamma_5) \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box \Gamma_5, \Box \mathtt{F}_1 \vdash \Delta_4, \Box \mathtt{F}_3}\ K$$

$$\frac{\mathtt{h}_2 : unbox(\Box \Gamma_5) \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : \Box \Gamma_5, \Delta_6, \Box \mathtt{F}_1 \vdash \Delta_4, \Box \mathtt{F}_3}\ K \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : unbox(\Box \Gamma_5) \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box \Gamma_5, \Box \mathtt{F}_1 \vdash \Delta_4, \Box \mathtt{F}_3}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_2 : \Box \Gamma_6, \Box \mathtt{F}_1 \vdash \Box \Gamma_4, \mathtt{F}_3}{\bullet \mathtt{h}_2 : (\Box \Gamma_6, \Box \mathtt{F}_1), \Delta_7 \vdash \Box \Gamma_4, \Delta_5, \Box \mathtt{F}_3}\ A45 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box \Gamma_6, \Box \mathtt{F}_1 \vdash \mathtt{F}_3, \Box \Gamma_4}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_7, \mathtt{F}_1, \Box \Gamma_6, \Box \mathtt{F}_1 \vdash \Delta_5, \Box \Gamma_4, \Box \mathtt{F}_3}\ A45$$

$$\frac{\mathtt{h}_2 : \Box \Gamma_6 \vdash \Box \Gamma_4, \mathtt{F}_3}{\bullet \mathtt{h}_2 : \Box \Gamma_6, \Delta_7, \Box \mathtt{F}_1 \vdash \Box \Gamma_4, \Delta_5, \Box \mathtt{F}_3}\ A45 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box \Gamma_6 \vdash \mathtt{F}_3, \Box \Gamma_4}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_7, \mathtt{F}_1, \Box \Gamma_6, \Box \mathtt{F}_1 \vdash \Delta_5, \Box \Gamma_4, \Box \mathtt{F}_3}\ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Delta_6, \Box \mathtt{F}_1 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_6, \Box \mathtt{F}_1 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, \Box \mathtt{F}_1), \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box \mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \Box \mathtt{F}_1 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box \mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \mathtt{F}_4, \Delta_6, \Box \mathtt{F}_1 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, \Box \mathtt{F}_1), \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4, \Box \mathtt{F}_1 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box \mathtt{F}_1, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_6, \Box \mathtt{F}_1 \vdash \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_6, \Box \mathtt{F}_1 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, \Box \mathtt{F}_1), \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, \Box \mathtt{F}_1 \vdash \Delta_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \Box \mathtt{F}_1 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box \mathtt{F}_1, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_5, \Box \mathtt{F}_1, \Box \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_2 : (\Delta_5, \Box \mathtt{F}_1), \Box \mathtt{F}_3 \vdash \Delta_4}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{F}_3, \Box \mathtt{F}_1, \Box \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \Box \mathtt{F}_1, \Box \mathtt{F}_3 \vdash \Delta_4}\ AT$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, \Box \mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_4, \Box \mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, \mathtt{F}_2, \Box \mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_2, \Box \mathtt{F}_2 \vdash \Delta_3}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_4, \Box \mathtt{F}_1 \vdash \Delta_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_4, \mathtt{F}_1, \Box \mathtt{F}_1 \vdash \Delta_3}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_2 : \mathtt{p}_4, \Delta_5, []\mathtt{F}_1 \vdash \mathtt{p}_4, \Delta_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{p}_4, []\mathtt{F}_1 \vdash \Delta_3, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \top, \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \top, \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}\ \top_L$$

4.15 Status of $\bot_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \Delta_5 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \rightarrow \mathtt{F}_3}\ \rightarrow_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \bot, \Delta_5 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \mathtt{F}_2, \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \bot, \Delta_2}\ \bot_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \top, \Delta_2}\ \top_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \bot, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ K \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $A45$

$$\frac{\mathtt{h}_1 : \Box\Gamma_5 \vdash \Box\Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \bot, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_2}\ A45 \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \bot, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \bot, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : (\bot, \Delta_4), []\mathtt{F}_2 \vdash \Delta_3} AT \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2} \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \bot, \Delta_4 \vdash \mathtt{p}_3, \Delta_2} I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \bot, \Delta_3 \vdash \Delta_2} \top_L \quad \rightarrow \quad \texttt{trivial}$$

4.16 Status of I: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_3, \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash (\Delta_4, \mathtt{p}_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3} \rightarrow_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_2, \Delta_4, \mathtt{p}_5 \quad \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_3, \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash (\Delta_4, \mathtt{p}_5), \mathtt{F}_2 \wedge \mathtt{F}_3} \wedge_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_2, \mathtt{F}_3, \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash (\Delta_4, \mathtt{p}_5), \mathtt{F}_2 \vee \mathtt{F}_3} \vee_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \bot, \Delta_2, \mathtt{p}_3}\ \bot_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \top, \Delta_2, \mathtt{p}_3}\ \top_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \Delta_6, \mathtt{p}_4 \vdash (\Delta_3, \mathtt{p}_4), []\mathtt{F}_2}\ K \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $A45$

$$\frac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \Box\Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_6, \Delta_7, \mathtt{p}_5 \vdash \Box\Gamma_3, (\Delta_4, \mathtt{p}_5), []\mathtt{F}_2}\ A45 \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_2, \Delta_4, \mathtt{p}_5 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : (\Delta_6, \mathtt{p}_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4, \mathtt{p}_5}\ \rightarrow_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : (\Delta_6, \mathtt{p}_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \mathtt{p}_5}\ \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : (\Delta_6, \mathtt{p}_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \mathtt{p}_5}\ \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5, \mathtt{p}_4, []\mathtt{F}_2 \vdash \Delta_3, \mathtt{p}_4}{\bullet \mathtt{h}_1 : (\Delta_5, \mathtt{p}_4), []\mathtt{F}_2 \vdash \Delta_3, \mathtt{p}_4}\ AT \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_4, \Delta_5, \mathtt{p}_3 \vdash \mathtt{p}_4, \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \texttt{trivial}$$

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_4 \vdash \mathtt{p}_3, \Delta_2}\ I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}{\bullet \mathtt{h}_1 : \top, \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ \top_L \quad \rightarrow \quad \mathtt{trivial}$$

4.17 Status of $\top_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \Delta_5 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \rightarrow \mathtt{F}_3}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_3}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \rightarrow \mathtt{F}_3}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \top, \Delta_5 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_3}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \mathtt{F}_2, \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \bot, \Delta_2}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \bot, \Delta_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \top, \Delta_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top, \Delta_2}\ \top_R$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \top, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2}\ K$$

- Case rule $A45$

$$\frac{\mathtt{h}_1 : \Box\Gamma_5 \vdash \Box\Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \top, \Delta_6 \vdash \Box\Gamma_3, \Delta_4, []\mathtt{F}_2}\ A45 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2, \Box\Gamma_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_6, \Box\Gamma_5 \vdash \Delta_4, \Box\Gamma_3, []\mathtt{F}_2}\ A45$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \top, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \top, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : (\top, \Delta_4), []\mathtt{F}_2 \vdash \Delta_3} AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3} AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_3 \vdash \Delta_2} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \top, \Delta_4 \vdash \mathtt{p}_3, \Delta_2} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{H}$$

5 Identity-Expansion

$$\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \Box\mathtt{F}_0 \vdash \Box\mathtt{F}_0}\,K$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,\vee_L}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0 \vee \mathtt{F}_1}\,\vee_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_R}{- : \mathtt{F}_0 \wedge \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_1}\,\to_L}{- : \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_0 \to \mathtt{F}_1}\,\to_R$$

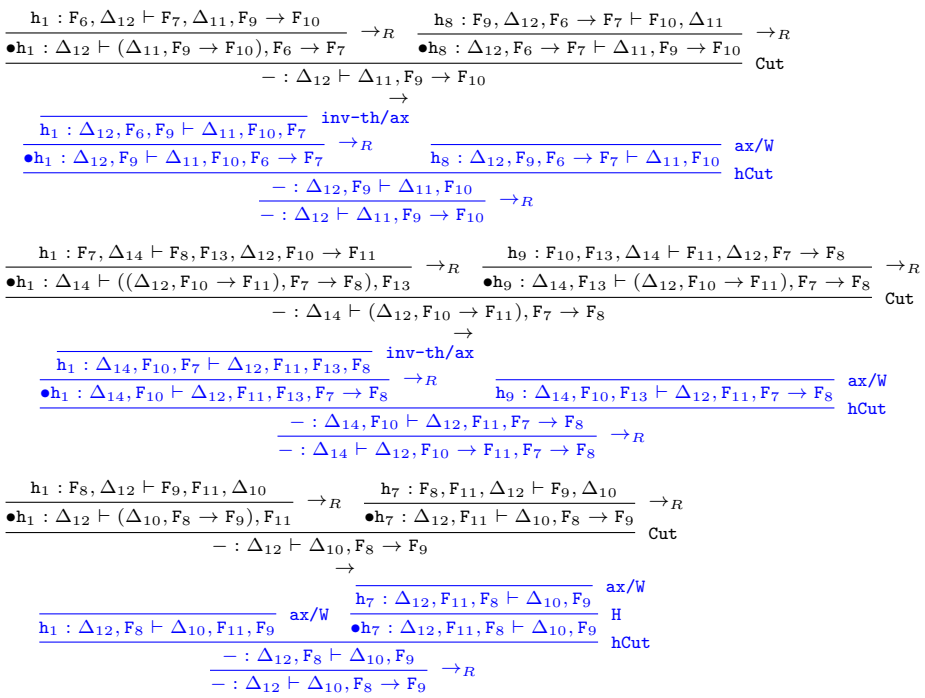
$$\dfrac{}{- : \top \vdash \top}\,\top_R$$

$$\dfrac{}{- : \bot \vdash \bot}\,\bot_L$$

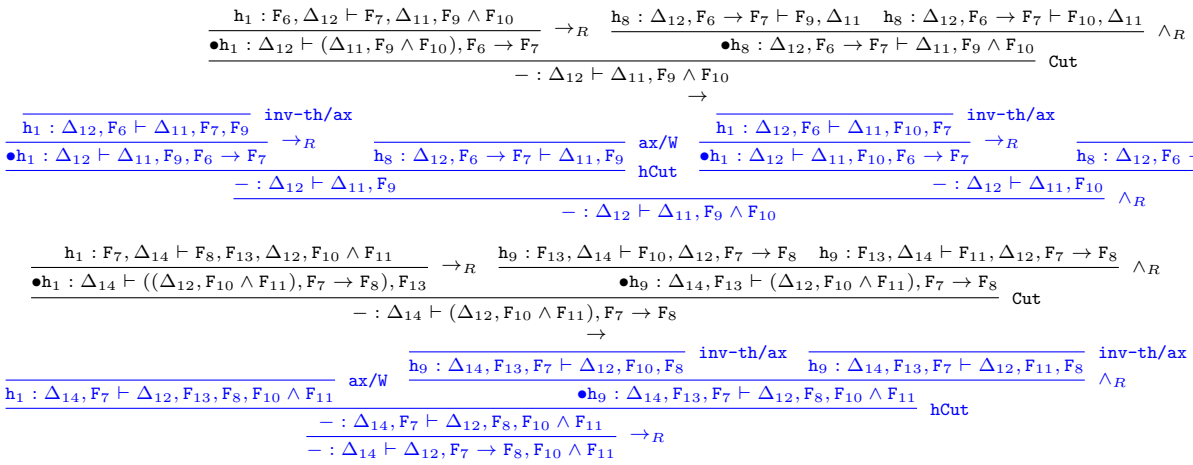
6 Cut-Elimination

6.1 Status of $\to_R$: OK

- Case rule $\to_R$



- Case rule $\wedge_R$



- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_R}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_R}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\vee_R}}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,{\vee_R}}{- : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,{\to_R}}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10} \vdash \mathtt{F}_7, \bot, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_9}\,{\bot_R}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11}}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\bot_R}}{- : \Delta_{12} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10} \vdash \mathtt{F}_7, \top, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \top, \Delta_9}\,{\top_R}}{- : \Delta_{10} \vdash \top, \Delta_9}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_9}\,{\top_R}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11}}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\top_R}}{- : \Delta_{12} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{12} \vdash \top, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,{\top_R}}$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}\,K}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_9}\,K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_8, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \Box\mathtt{F}_{12}} \to_R \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), \Box\mathtt{F}_{12} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} K}{- : \Box\Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : unbox(\Box\mathtt{F}_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}} K}{- : \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} K}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8} K} \to$$

- Case rule $A45$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Box\Gamma_{12}, \Delta_{13} \vdash \mathtt{F}_7, \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9} A45}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_9, \Box\Gamma_{10}}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_9} A45} \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{14}, \Delta_{15} \vdash \mathtt{F}_8, \Box\mathtt{F}_{13}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15} \vdash ((\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \Box\mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \Box\mathtt{F}_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}), \Box\mathtt{F}_{13} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} A45}{- : \Box\Gamma_{14}, \Delta_{15} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{15}, \mathtt{F}_7, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_{10}}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{13}, \Box\Gamma_{14} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{13}, \Delta_{15}, \mathtt{F}_7, \Box\Gamma_{14} \vdash \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_{10}} A45}{- : \Delta_{15}, \mathtt{F}_7, \Box\Gamma_{14} \vdash \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{13}, \Delta_{15} \vdash \mathtt{F}_8, \mathtt{F}_{14}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{15} \vdash ((\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{14}} \to_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_{14} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} A45}{- : \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{13} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}}\,\mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8} A45} \to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}} \to_L}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{a}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{h} \to_L}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}} \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{10} \vdash \mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7 \to \mathtt{F}_8} \to_R \quad \dfrac{\mathtt{h}_6 : \Delta_{10} \vdash \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9} \to_L}{- : \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{sCut}}{- : \Delta_{10} \vdash \Delta_9}\,\mathtt{sCut} \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13} \vdash \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \to_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}, \Delta_{12}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\to_L}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}\\ \hline - : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8\end{array}}\,\to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \to_L}{- : \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\to_L}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}\\ \hline - : \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8\end{array}}\,\to_R$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut}\\ \hline - : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}\end{array}}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \wedge_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \Delta_{12}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}\\ \hline - : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8\end{array}}\,\to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \wedge_L}{- : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \wedge_L}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}\\ \hline - : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8\end{array}}\,\to_R$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{a}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{h} \vee_L\\ \hline - : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}\end{array}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \vee_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}\ \vee_L}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8} \to_R}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \vee_L}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}\ \vee_L}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \to_R}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_7, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}}{\bullet \mathtt{h}_8 : (\Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}}\,AT}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\,AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_8, []\mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), []\mathtt{F}_{10}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_8}\,AT}{- : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_{12} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,AT}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \bot, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\bullet \mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}\,\bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\bot_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{11} \vdash \mathtt{F}_8, \bot, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \bot} \to_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{11}, \bot \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash \bot, \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8}\,\bot_L}{- : \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \bot, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \bot, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11}} \to_R \quad \dfrac{}{\bullet \mathtt{h}_9 : (\bot, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \bot_L}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \bot_L}\ \overset{\to}{}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9, \mathtt{p}_{10}} I}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}} I}\ \overset{\to}{}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{11}} \to_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8} I}{- : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}} \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}} I}{- : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}}\ \mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R}\ \overset{\to}{}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12}} \to_R \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8} I}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8} I}\ \overset{\to}{}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \top, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9} \top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7} \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9} \mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\ \mathtt{hCut}}\ \overset{\to}{}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{11} \vdash \mathtt{F}_8, \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \top} \to_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{11}, \top \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \top_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{ax/W}}\ \overset{\to}{}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \top, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\top, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \top_L}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{hCut}}\ \overset{\to}{}$$

6.2 Status of $\wedge_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}} \to_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6} \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_7} \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}} \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}} \to_R}\ \overset{\to}{}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11} \quad \mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\to_R}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : \Delta_{12}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{sCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{sCut}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_7}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}}}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11}}\wedge_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12} \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12} \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\wedge_R}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W}}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{H}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{H}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \quad \mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\vee_R}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_6, \bot, \Delta_9 \quad \mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_7, \bot, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_9}\bot_R}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \bot, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\bot, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\bot_R}{- : \Delta_{12} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_6, \top, \Delta_9 \quad \mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_7, \top, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_9}\top_R}{- : \Delta_{10} \vdash \top, \Delta_9}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_9}\top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \top, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11}}\wedge_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\top_R}{- : \Delta_{12} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{12} \vdash \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\top_R}$$

- Case rule K

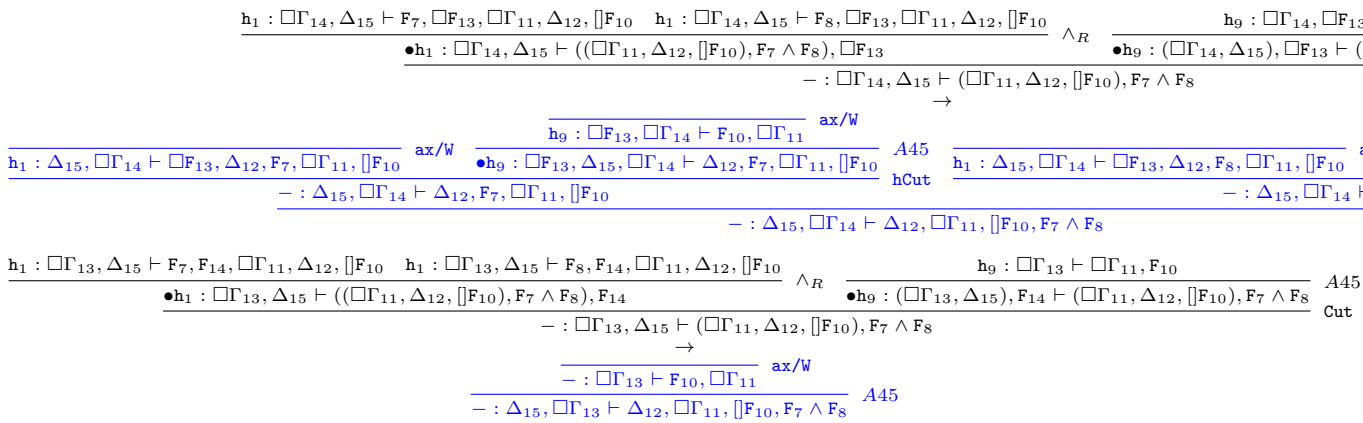
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_9 \quad \mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}K}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_9}K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_7, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10} \quad \mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_8, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \Box\mathtt{F}_{12}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \mathit{unbox}(\Box\Gamma_{13}), \mathit{unbox}(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), \Box\mathtt{F}_{12} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge}}{- : \Box\Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \mathit{unbox}(\Box\mathtt{F}_{12}), \mathit{unbox}(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}K}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \mathit{unbox}(\Box}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}$$

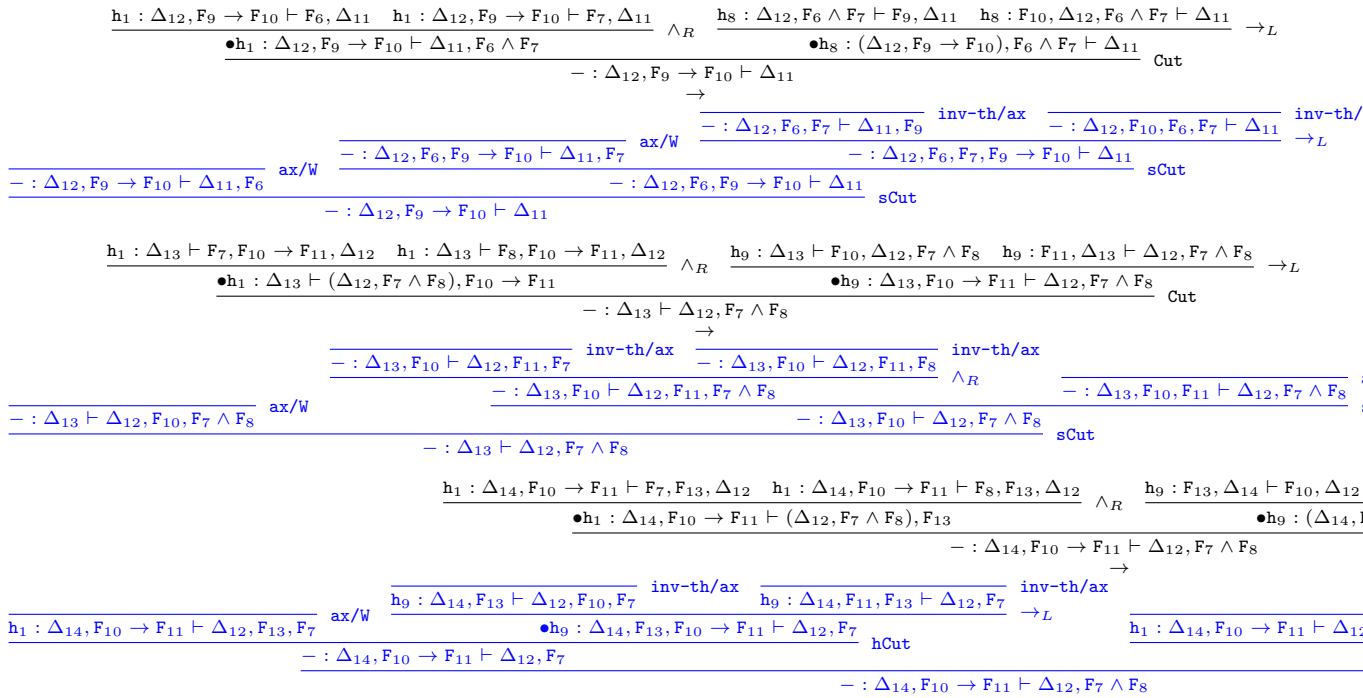
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10} \quad \mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}K}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}K}$$

- Case rule $A45$

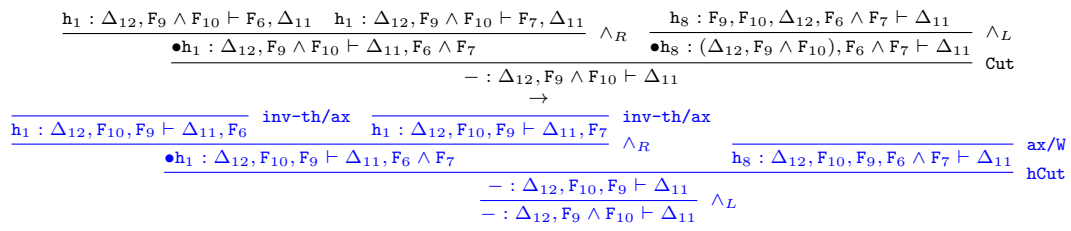
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash \mathtt{F}_6, \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9 \quad \mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash \mathtt{F}_7, \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}A45}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_9, \Box\Gamma_{10}}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_9}A45}$$

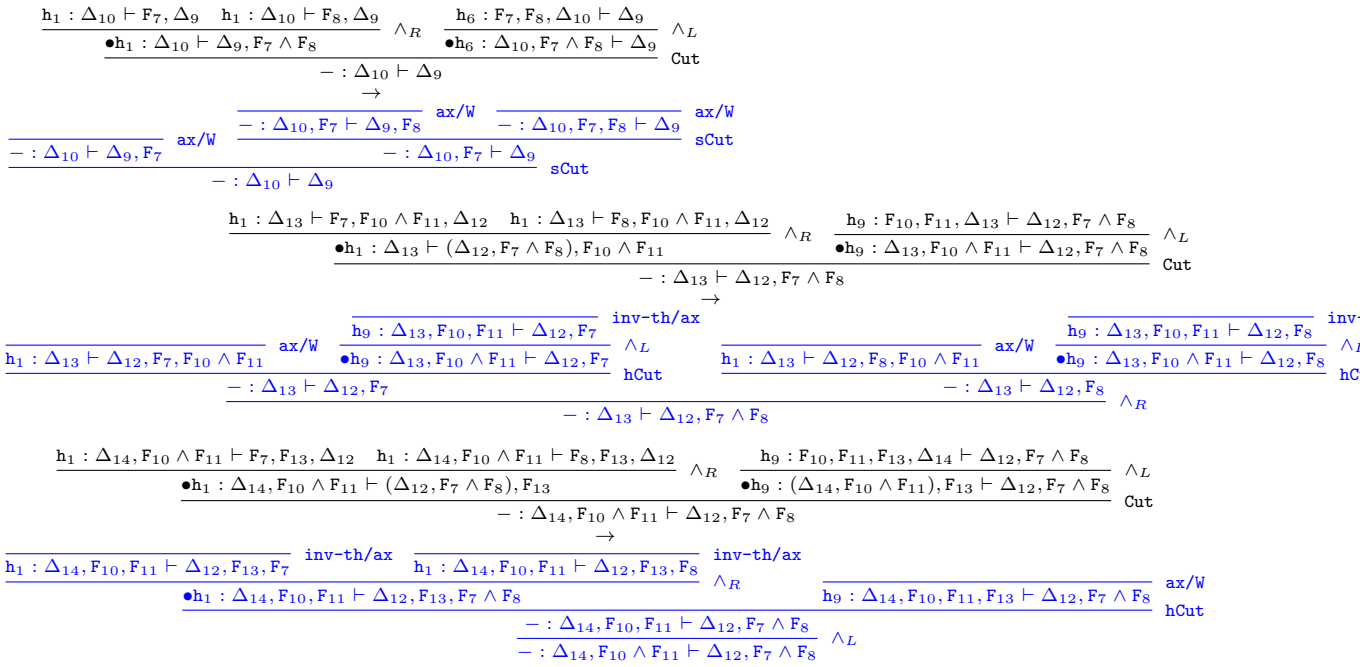


- Case rule $\rightarrow_L$

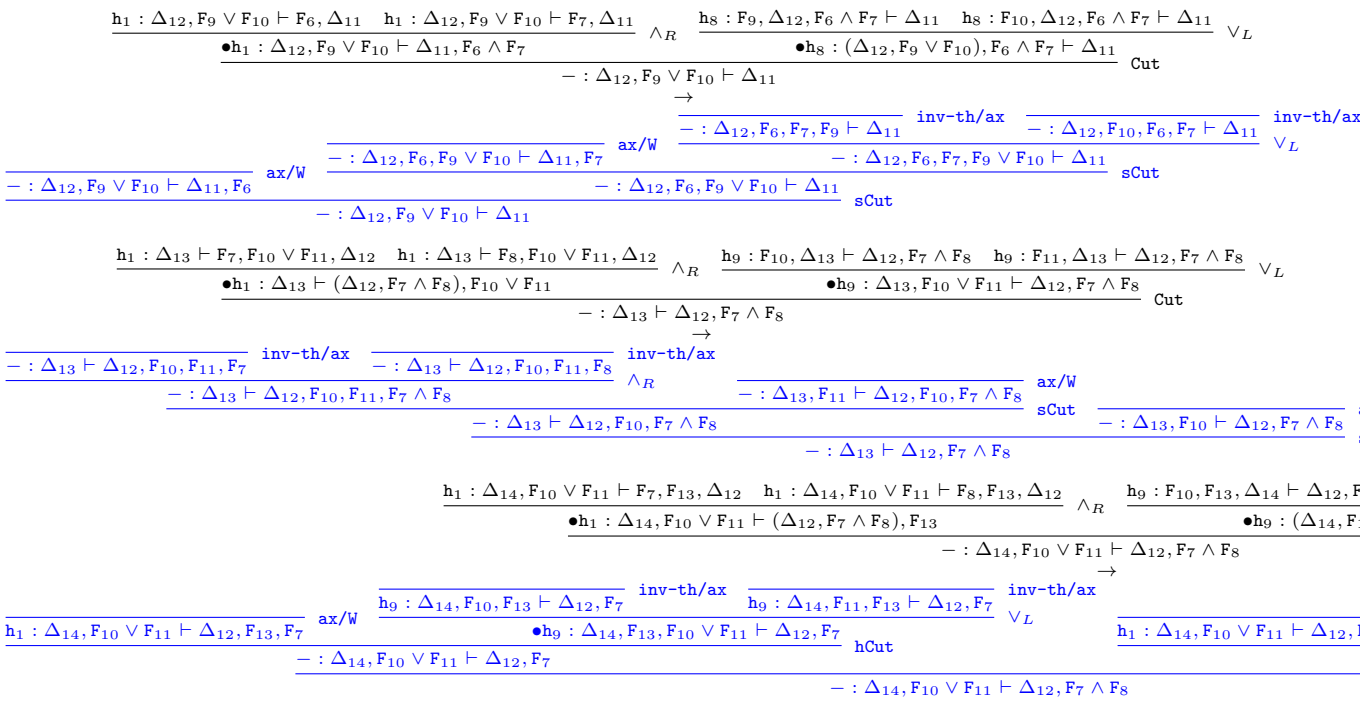


- Case rule $\wedge_L$





- Case rule $\vee_L$



- Case rule AT

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_6, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_7, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}}\,AT}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\,ATG$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, []\mathtt{F}_{10}, \Delta_{11} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, []\mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), []\mathtt{F}_{10}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}{}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}\,AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_8}{}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_8}\,AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{11} \quad \mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_{12} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,AT}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}{}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,ATG$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{10} \vdash \mathtt{F}_6, \Delta_9 \quad \mathtt{h}_1 : \bot, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\,\bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\to}}{\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\bot_L}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \bot, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_8, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \bot}\wedge_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{11}, \bot \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\,\bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8}\,\bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \bot, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : \bot, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11}}\wedge_R \quad \dfrac{}{\bullet\mathtt{h}_9 : (\bot, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\bot_L}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\to}}{\dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9, \mathtt{p}_{10}}\,I}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\,\mathtt{Cut}}{\to}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\,I}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{11}}\wedge_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,I}{- : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}, \mathtt{p}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\,I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}}\,I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12}}\ \wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\ I}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}$$

$$\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \mathtt{F}_6, \Delta_9 \quad \mathtt{h}_1 : \top, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\ \top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\ \mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \top, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_8, \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \top}\ \wedge_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{11}, \top \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \top_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}$$

$$\dfrac{}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \top, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11}}\ \wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\top, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \top_L}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{hCut}$$

6.3 Status of $\vee_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \to_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \to_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}}\ \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\ \to_R}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\ \to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \wedge_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9}\ \mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \wedge_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}} \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \wedge_R}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \wedge_R}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}} \vee_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}} \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{11}} \vee_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9} \vee_R}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{ax/W}}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{H}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9} \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \bot, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_9} \bot_R}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11}} \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8} \bot_R}{- : \Delta_{12} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \top, \Delta_9}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_9}\top_R}{- : \Delta_{10} \vdash \top, \Delta_9}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_9}\top_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11}}\vee_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\top_R}{- : \Delta_{12} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{12} \vdash \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\top_R$$

- Case rule K

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_9}{\bullet \mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}K}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_9}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_9}K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \Box\mathtt{F}_{12}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathit{unbox}(\Box\Gamma_{13}), \mathit{unbox}(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), \Box\mathtt{F}_{12} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}K}{- : \Box\Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \mathit{unbox}(\Box\mathtt{F}_{12}), \mathit{unbox}(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}K}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}K}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}K$$

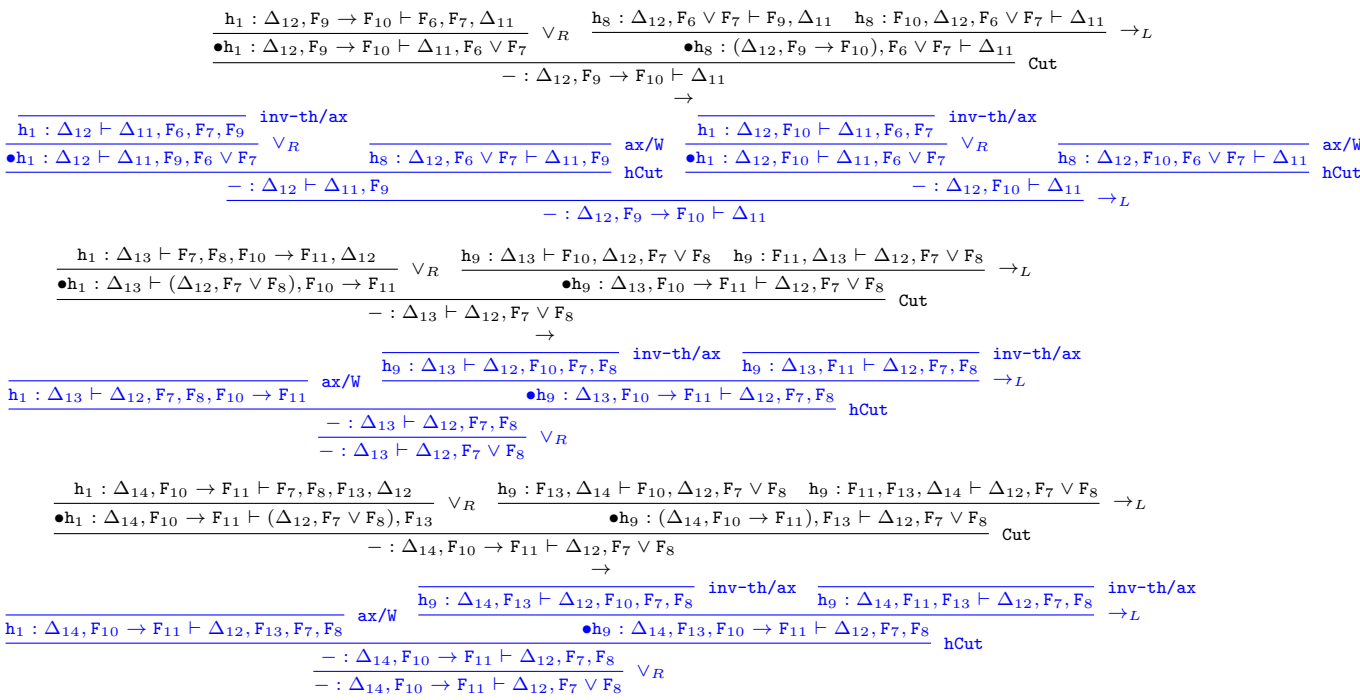
- Case rule $A45$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash \mathtt{F}_6, \mathtt{F}_7, \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}{\bullet \mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_9}{\bullet \mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}A45}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_9, \Box\Gamma_{10}}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_9}A45$$

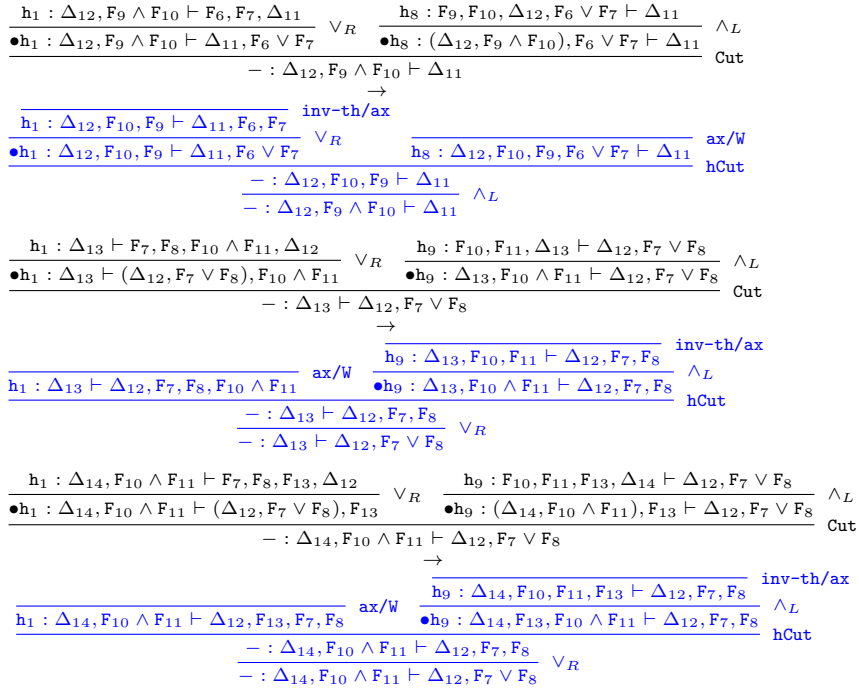
$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15} \vdash \mathtt{F}_7, \mathtt{F}_8, \Box\mathtt{F}_{13}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15} \vdash ((\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \Box\mathtt{F}_{13}}\vee_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \Box\mathtt{F}_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}), \Box\mathtt{F}_{13} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}A45}{- : \Box\Gamma_{14}, \Delta_{15} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{15}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_{10}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{13}, \Box\Gamma_{14} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}}\,\mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Box\mathtt{F}_{13}, \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_{10}}A45}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{15} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{14}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{15} \vdash ((\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14}}\vee_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_{14} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}A45}{- : \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{13} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}}\,\mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}A45$$

- Case rule $\rightarrow_L$



- Case rule $\wedge_L$



- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\vee_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \Delta_{10} \vdash \Delta_9 \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\vee_L}{- : \Delta_{10} \vdash \Delta_9}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_{10}, \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7}\mathtt{sCut} \quad \dfrac{}{- : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \Delta_9}\mathtt{sCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\vee_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\mathtt{hCut}$$

$$\dfrac{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_L}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\vee_L}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\mathtt{hCut}$$

$$\dfrac{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R$$

- Case rule AT

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}}\,AT}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}}\mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\,ATG$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), []\mathtt{F}_{10}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8}\,AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8}\mathtt{hCut}$$

$$\dfrac{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_{12} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,AT}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,ATG$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\bot_L}{\mathtt{Cut}}}{- : \bot, \Delta_{10} \vdash \Delta_9} \to \dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\bot_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \bot}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{11}, \bot \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L}{\mathtt{Cut}}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8}\bot_L}{\mathtt{hCut}}}{\dfrac{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : \bot, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11}}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : (\bot, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L}{\mathtt{Cut}}}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9, \mathtt{p}_{10}}I}{\mathtt{Cut}}}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}} \to \dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}I$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{11}}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}I}{\mathtt{Cut}}}{- : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}I}{\mathtt{hCut}}}{\dfrac{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12}}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}I}{\mathtt{Cut}}}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\top_L}{\mathtt{Cut}}}{- : \top, \Delta_{10} \vdash \Delta_9} \to \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\mathtt{ax/W}}{\mathtt{hCut}}}{- : \top, \Delta_{10} \vdash \Delta_9}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \top}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{11}, \top \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\top_L}{\mathtt{Cut}}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11}} \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\top, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \top_L}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}$$

6.4 Status of $\bot_R$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \rightarrow \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \rightarrow \mathtt{F}_6), \bot} \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \Delta_8 \vdash \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \bot \vdash \Delta_7, \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_9} \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6), \bot} \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \bot, \Delta_8 \vdash \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \bot \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9} \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6), \bot} \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \bot \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9} \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \bot, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_5), \bot} \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_6, \bot \vdash \bot, \Delta_5} \bot_R}{- : \Delta_6 \vdash \bot, \Delta_5}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6 \vdash \bot, \Delta_5}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 : \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet\mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_6}\ \bot_R}{- : \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_8 \vdash \bot, \Delta_6, \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \top, \Delta_5}{\bullet\mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_5), \bot}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_4 : \Delta_6, \bot \vdash \top, \Delta_5}\ \top_R}{- : \Delta_6 \vdash \top, \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_6 \vdash \top, \Delta_5}\ \top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \top, \Delta_6}{\bullet\mathtt{h}_1 : \Delta_8 \vdash (\bot, \top, \Delta_6), \mathtt{F}_7}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \bot, \top, \Delta_6}\ \top_R}{- : \Delta_8 \vdash \bot, \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8 \vdash \bot, \top, \Delta_6}\ \top_R}$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_7, \Delta_8 \vdash \Delta_6, []\mathtt{F}_5}{\bullet\mathtt{h}_1 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_6, []\mathtt{F}_5), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : unbox(\Box\Gamma_7) \vdash \mathtt{F}_5}{\bullet\mathtt{h}_4 : (\Box\Gamma_7, \Delta_8), \bot \vdash \Delta_6, []\mathtt{F}_5}\ K}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_6, []\mathtt{F}_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_6, []\mathtt{F}_5}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\bot, \Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \bot_R \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_9), unbox(\Box\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ K}{- : \Box\Gamma_9, \Delta_{10} \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Box\mathtt{F}_8, \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash \mathtt{F}_9, \Delta_7, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash (\bot, \Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ K}{- : \Box\Gamma_8, \Delta_{10} \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_8 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ K}$$

- Case rule $A45$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_5}{\bullet\mathtt{h}_1 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_6, \Delta_7, []\mathtt{F}_5), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \Box\Gamma_8 \vdash \Box\Gamma_6, \mathtt{F}_5}{\bullet\mathtt{h}_4 : (\Box\Gamma_8, \Delta_9), \bot \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_5}\ A45}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_7, \Box\Gamma_6, []\mathtt{F}_5}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash \Box\mathtt{F}_9, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\bot, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6), \Box\mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_{10}, \Box\mathtt{F}_9 \vdash \Box\Gamma_7, \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\Box\Gamma_{10}, \Delta_{11}), \Box\mathtt{F}_9 \vdash \bot, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ A45}{- : \Box\Gamma_{10}, \Delta_{11} \vdash \bot, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \Box\Gamma_{10} \vdash \bot, \Box\mathtt{F}_9, \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Box\mathtt{F}_9, \Delta_{11}, \Box\Gamma_{10} \vdash \bot, \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \bot, \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{11} \vdash \mathtt{F}_{10}, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{11} \vdash (\bot, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6), \mathtt{F}_{10}}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_9 \vdash \Box\Gamma_7, \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\Box\Gamma_9, \Delta_{11}), \mathtt{F}_{10} \vdash \bot, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ A45}{- : \Box\Gamma_9, \Delta_{11} \vdash \bot, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_9 \vdash \mathtt{F}_6, \Box\Gamma_7}\ \mathtt{ax/W}}{- : \Delta_{11}, \Box\Gamma_9 \vdash \bot, \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ A45$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \bot, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6), \bot \vdash \Delta_7}\ \rightarrow_L}{- : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \rightarrow \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8), \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9 \vdash \bot, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9 \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \bot, \Delta_8}\ \rightarrow_L}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash (\bot, \Delta_8), \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_9 \vdash \bot, \Delta_8}\ \rightarrow_L}{- : \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \bot, \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{hCut}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \bot \vdash \Delta_7}\ \wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8), \mathtt{F}_6 \wedge \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \Delta_9 \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \wedge_L}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash (\bot, \Delta_8), \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9 \vdash \bot, \Delta_8}\ \wedge_L}{- : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \Delta_8 \vdash \Delta_7 \quad \mathtt{h}_4 : \bot, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \bot \vdash \Delta_7}\ \vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \vee \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8), \mathtt{F}_6 \vee \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_9 \vdash \bot, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9 \vdash \bot, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \vee_L}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash (\bot, \Delta_8), \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9 \vdash \bot, \Delta_8}\ \vee_L}{- : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{hCut}}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}{\bullet \mathtt{h}_1 : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}{\bullet \mathtt{h}_4 : (\Delta_7, []\mathtt{F}_5), \bot \vdash \Delta_6}\ AT}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash []\mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\bot, \Delta_7), []\mathtt{F}_6}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_8, []\mathtt{F}_6 \vdash \bot, \Delta_7}{\bullet \mathtt{h}_5 : \Delta_8, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ AT}{- : \Delta_8 \vdash \bot, \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_8 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \bot, \Delta_7}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_7}{\bullet \mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash (\bot, \Delta_7), \mathtt{F}_8}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_8, \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_7}{\bullet \mathtt{h}_5 : (\Delta_9, []\mathtt{F}_6), \mathtt{F}_8 \vdash \bot, \Delta_7}\ AT}{- : \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_7, \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_8, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ \mathtt{ax/W}}{- : \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ \mathtt{hCut}}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_5, \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_6, \bot \vdash \Delta_5}\ \bot_L}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\bot, \Delta_6), \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_7, \bot \vdash \bot, \Delta_6}\ \bot_L}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \bot, \Delta_6}\ \bot_L}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash (\Delta_5, \mathtt{p}_6), \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\Delta_7, \mathtt{p}_6), \bot \vdash \Delta_5, \mathtt{p}_6}\ I}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ I}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{p}_7, \Delta_6, \mathtt{p}_7}{\bullet\mathtt{h}_1 : \Delta_8 \vdash (\bot, \Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_8, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8 \vdash \bot, \Delta_6, \mathtt{p}_7}\ \texttt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_8 \vdash \bot, \Delta_6, \mathtt{p}_7, \mathtt{p}_7}\ \texttt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_8, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8 \vdash \bot, \Delta_6, \mathtt{p}_7}\ \texttt{hCut}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{p}_7 \vdash \mathtt{F}_8, \Delta_6, \mathtt{p}_7}{\bullet\mathtt{h}_1 : \Delta_9, \mathtt{p}_7 \vdash (\bot, \Delta_6, \mathtt{p}_7), \mathtt{F}_8}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_5 : (\Delta_9, \mathtt{p}_7), \mathtt{F}_8 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_9, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ \texttt{Cut}}{\dfrac{}{- : \Delta_9, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}\ \to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_6 \vdash \Delta_5}{\bullet\mathtt{h}_4 : (\top, \Delta_6), \bot \vdash \Delta_5}\ \top_L}{- : \top, \Delta_6 \vdash \Delta_5}\ \texttt{Cut}}{\dfrac{}{- : \top, \Delta_6 \vdash \Delta_5}\ \texttt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \top, \Delta_6}{\bullet\mathtt{h}_1 : \Delta_7 \vdash (\bot, \Delta_6), \top}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_7 \vdash \bot, \Delta_6}{\bullet\mathtt{h}_5 : \Delta_7, \top \vdash \bot, \Delta_6}\ \top_L}{- : \Delta_7 \vdash \bot, \Delta_6}\ \texttt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \bot, \Delta_6}\ \texttt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 : \top, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \bot, \Delta_6}{\bullet\mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \bot, \Delta_6}\ \top_L}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \texttt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_8 \vdash \bot, \Delta_6, \mathtt{F}_7}\ \texttt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_6}\ \texttt{ax/W}}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \texttt{hCut}}\ \to$$

6.5 Status of $\top_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \to \mathtt{F}_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \Delta_8 \vdash \mathtt{F}_6, \Delta_7}{\bullet\mathtt{h}_4 : \Delta_8, \top \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\ \texttt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vdash \top, \Delta_7, \mathtt{F}_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6}\ \texttt{ax/W}}{- : \Delta_8, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6}\ \texttt{hCut}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \texttt{Cut}}{\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \top_R}\ \to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_6, \Delta_7}{\bullet\mathtt{h}_4 : \Delta_8, \top \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \texttt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \texttt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \texttt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_6}\ \texttt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_6}\ \texttt{hCut}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \top_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \top \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \top_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_6, \top \vdash \bot, \Delta_5}\ \bot_R}{- : \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \bot, \top, \Delta_5}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\top, \bot, \Delta_6), \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \top, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \top, \bot, \Delta_6}\ \bot_R}{- : \Delta_8 \vdash \top, \bot, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \bot, \top, \Delta_6}\ \top_R$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_5), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_6, \top \vdash \top, \Delta_5}\ \top_R}{- : \Delta_6 \vdash \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \top, \Delta_6}\ \top_R}{- : \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule K

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_6, []\mathtt{F}_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \mathit{unbox}(\Box\Gamma_7) \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : (\Box\Gamma_7, \Delta_8), \top \vdash \Delta_6, []\mathtt{F}_5}\ K}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_6, []\mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_7) \vdash \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_6, []\mathtt{F}_5}\ K$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\top, \Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_9), unbox(\Box\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \top, \Delta_7, []\mathtt{F}_6}\ K}{- : \Box\Gamma_9, \Delta_{10} \vdash \top, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_7, []\mathtt{F}_6}\ \top_R$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash (\top, \Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \top, \Delta_7, []\mathtt{F}_6}\ K}{- : \Box\Gamma_8, \Delta_{10} \vdash \top, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{- : \Delta_{10}, \Box\Gamma_8 \vdash \top, \Delta_7, []\mathtt{F}_6}\ \top_R$$

- Case rule $A45$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_8, \Delta_9 \vdash (\Box\Gamma_6, \Delta_7, []\mathtt{F}_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \Box\Gamma_8 \vdash \Box\Gamma_6, \mathtt{F}_5}{\bullet \mathtt{h}_4 : (\Box\Gamma_8, \Delta_9), \top \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_5}\ A45}{- : \Box\Gamma_8, \Delta_9 \vdash \Box\Gamma_6, \Delta_7, []\mathtt{F}_5}\ \mathtt{Cut}}{\dfrac{}{- : \Box\Gamma_8 \vdash \mathtt{F}_5, \Box\Gamma_6}\ \mathtt{ax/W}}{- : \Delta_9, \Box\Gamma_8 \vdash \Delta_7, \Box\Gamma_6, []\mathtt{F}_5}\ A45$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\top, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6), \Box\mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_{10}, \Box\mathtt{F}_9 \vdash \Box\Gamma_7, \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_{10}, \Delta_{11}), \Box\mathtt{F}_9 \vdash \top, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ A45}{- : \Box\Gamma_{10}, \Delta_{11} \vdash \top, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ \mathtt{Cut}}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \top, \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \top_R$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_9, \Delta_{11} \vdash (\top, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6), \mathtt{F}_{10}}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_9 \vdash \Box\Gamma_7, \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_9, \Delta_{11}), \mathtt{F}_{10} \vdash \top, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ A45}{- : \Box\Gamma_9, \Delta_{11} \vdash \top, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ \mathtt{Cut}}{- : \Delta_{11}, \Box\Gamma_9 \vdash \top, \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \top, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6), \top \vdash \Delta_7}\ \rightarrow_L}{- : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7}\ \rightarrow_L$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8), \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9 \vdash \top, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9 \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \top, \Delta_8}\ \rightarrow_L}{- : \Delta_9 \vdash \top, \Delta_8}\ \mathtt{Cut}}{- : \Delta_9 \vdash \top, \Delta_8}\ \top_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash (\top, \Delta_8), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_9 \vdash \top, \Delta_8}\ \rightarrow_L}{- : \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \top, \Delta_8}\ \mathtt{Cut}}{- : \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \top \vdash \Delta_7}\ \wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \wedge_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8), \mathtt{F}_6 \wedge \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \Delta_9 \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_8}\ \wedge_L}{- : \Delta_9 \vdash \top, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_9 \vdash \top, \Delta_8}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash (\top, \Delta_8), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9 \vdash \top, \Delta_8}\ \wedge_L}{- : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \Delta_8 \vdash \Delta_7 \quad \mathtt{h}_4 : \top, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \top \vdash \Delta_7}\ \vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \vee_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8), \mathtt{F}_6 \vee \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_9 \vdash \top, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9 \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_8}\ \vee_L}{- : \Delta_9 \vdash \top, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_9 \vdash \top, \Delta_8}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash (\top, \Delta_8), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9 \vdash \top, \Delta_8}\ \vee_L}{- : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_8}\ \top_R$$

- Case rule AT

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}{\bullet \mathtt{h}_4 : (\Delta_7, []\mathtt{F}_5), \top \vdash \Delta_6}\ AT}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_5, []\mathtt{F}_5 \vdash \top, \Delta_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_7, \mathtt{F}_5, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_5, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{hCut}}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ ATG$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\top, \Delta_7), []\mathtt{F}_6}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_8, []\mathtt{F}_6 \vdash \top, \Delta_7}{\bullet \mathtt{h}_5 : \Delta_8, []\mathtt{F}_6 \vdash \top, \Delta_7}\ AT}{- : \Delta_8 \vdash \top, \Delta_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_8 \vdash \top, \Delta_7}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash (\top, \Delta_7), \mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_8, \Delta_9, []\mathtt{F}_6 \vdash \top, \Delta_7}{\bullet \mathtt{h}_5 : (\Delta_9, []\mathtt{F}_6), \mathtt{F}_8 \vdash \top, \Delta_7}\ AT}{- : \Delta_9, []\mathtt{F}_6 \vdash \top, \Delta_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_9, []\mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_6 \vdash \Delta_5, \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\bot, \Delta_6), \top \vdash \Delta_5}\ \bot_L}{- : \bot, \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \Delta_5}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \bot}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_7, \bot \vdash \top, \Delta_6}\ \bot_L}{- : \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \top, \Delta_6}\ \bot_L}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash (\Delta_5, \mathtt{p}_6), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\Delta_7, \mathtt{p}_6), \top \vdash \Delta_5, \mathtt{p}_6}\ I}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\top, \Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{p}_7 \vdash \top, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8 \vdash \top, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \top, \Delta_6, \mathtt{p}_7}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_7 \vdash (\top, \Delta_6, \mathtt{p}_7), \mathtt{F}_8}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_7), \mathtt{F}_8 \vdash \top, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_6, \mathtt{p}_7}\ \top_R$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_5, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_6, \top \vdash \Delta_5}\ \top_L}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_7 \vdash \top, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_7, \top \vdash \top, \Delta_6}\ \top_L}{- : \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \top, \Delta_6}{\bullet \mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \top, \Delta_6}\ \top_L}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

6.6 Status of K: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_6}\, K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_8, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R}}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6}\, K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_{13}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7}\,{\to_R}}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10}}\, K$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6}\, K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,{\wedge_R}}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, []\mathtt{F}_6}\, K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6}\, K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,{\wedge_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_{13}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7}\,{\wedge_R}}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\, K$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6}\, K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,{\vee_R}}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, []\mathtt{F}_6}\, K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,{\vee_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_{13}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7}\,{\vee_R}}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10}}\, K$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_9) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\bot, \Delta_8), []\mathtt{F}_6}\, K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_6 \vdash \bot, \Delta_8}\,{\bot_R}}{- : \Box\Gamma_9, \Delta_{10} \vdash \bot, \Delta_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_8, []\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_6 \vdash \bot, \Delta_8}\,\mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\bot, \Delta_9), []\mathtt{F}_7), \mathtt{F}_{11}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{10}, \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{F}_{11} \vdash (\bot, \Delta_9), []\mathtt{F}_7}\, \bot_R}{- : \Box\Gamma_{10}, \Delta_{12} \vdash (\bot, \Delta_9), []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \bot, \Delta_9, []\mathtt{F}_7}\, K}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_9) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\top, \Delta_8), []\mathtt{F}_6}\, K \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_6 \vdash \top, \Delta_8}\, \top_R}{- : \Box\Gamma_9, \Delta_{10} \vdash \top, \Delta_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_8}\, \top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\top, \Delta_9), []\mathtt{F}_7), \mathtt{F}_{11}}\, K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{F}_{11} \vdash (\top, \Delta_9), []\mathtt{F}_7}\, \top_R}{- : \Box\Gamma_{10}, \Delta_{12} \vdash (\top, \Delta_9), []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \top, \Delta_9, []\mathtt{F}_7}\, \top_R}$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\, K \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_6, \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{11}), \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_6, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \mathtt{F}_6, \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{11}), \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{11}), \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_8}\,\mathtt{sCut}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\, K$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\, K \quad \dfrac{\mathtt{h}_7 : \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\, K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}, \Box\Gamma_{14}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \Box\mathtt{F}_{12}}\, K \quad \dfrac{\mathtt{h}_8 : \mathit{unbox}(\Box\Gamma_{11}), \mathit{unbox}(\Box\Gamma_{13}), \mathit{unbox}(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathtt{F}_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\, K}{- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{11}), \mathit{unbox}(\Box\Gamma_{14}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\, K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}, \Box\Gamma_{13}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{11}}\, K \quad \dfrac{\mathtt{h}_7 : \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{12}), \mathit{unbox}(\Box\mathtt{F}_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box\mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{13}) \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, []\mathtt{F}_8}\, K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \mathtt{F}_{14}}\, K \quad \dfrac{\mathtt{h}_8 : \mathit{unbox}(\Box\Gamma_{11}), \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), \mathtt{F}_{14} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\, K}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{11}), \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\, K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{13}}\, K \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\, K}$$

- Case rule $A45$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8), []\mathtt{F}_6}\, K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \Box\Gamma_9, \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_6 \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\, A45}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_6}\, \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \mathtt{F}_8, \Box\Gamma_9, []\mathtt{F}_6}\, K \quad \dfrac{}{\mathtt{h}_7 : \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Box\Gamma_9}\, \mathtt{ax/W}}{- : \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \mathtt{F}_8, \Box\Gamma_9}\, \mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\, A45$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8), []\mathtt{F}_6}\, K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Box\Gamma_9, \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_6 \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\, A45}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_8, \Box\Gamma_9}\, \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\, A45}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}, \Box\Gamma_{15}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash ((\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7), \Box\mathtt{F}_{13}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \Box\Gamma_{14}, \Box\mathtt{F}_{13} \vdash \Box\Gamma_{10}, \mathtt{F}_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \Box\mathtt{F}_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7}\, A45}{- : (\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\, K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}, \Box\Gamma_{15}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash ((\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7), \Box\mathtt{F}_{13}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \Box\Gamma_{14}, \Box\mathtt{F}_{13} \vdash \Box\Gamma_{10}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \Box\mathtt{F}_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7}\, A45}{- : (\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\, K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}, \Box\Gamma_{14}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_8), \Box\mathtt{F}_{12}}\, K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\mathtt{F}_{12} \vdash \Box\Gamma_9, \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathtt{F}_{12} \vdash (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_8}\, A45}{- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\, K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}, \Box\Gamma_{14}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash ((\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7), \mathtt{F}_{15}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Box\Gamma_{10}, \mathtt{F}_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16}), \mathtt{F}_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7}\, A45}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\, K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}, \Box\Gamma_{14}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash ((\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7), \mathtt{F}_{15}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Box\Gamma_{10}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16}), \mathtt{F}_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7}\, A45}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\, K}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_8), \mathtt{F}_{14}}\ K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Box\Gamma_9, \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), \mathtt{F}_{14} \vdash (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_8}\ A45}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_8}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{11}), \mathit{unbox}(\Box\Gamma_{13}) \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\ K$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_6}\ K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\ \to_L}{- : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, []\mathtt{F}_6}\ K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8}\ \mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_6}\ K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\ \to_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_9 \to \mathtt{F}_{10}}\ K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \Delta_{13} \vdash \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ \to_L}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7}\ K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_{13}}\ K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\ \to_L}{- : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ K$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_6}\ K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_8, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\ \wedge_L}{- : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_6}\ K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\ \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ \wedge_L}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7}\ K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_{13}}\ K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\ \wedge_L}{- : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ K$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_8, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10} \quad \mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\vee_L}{- : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\vee_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_9 \vee \mathtt{F}_{10}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\vee_L}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7}\,K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_{13}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\vee_L}{- : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,K$$

- Case rule AT

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}, []\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, []\mathtt{F}_8), \Delta_{11} \vdash \Delta_9, []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \mathtt{F}_8, \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, []\mathtt{F}_8), \Delta_{11}), []\mathtt{F}_6 \vdash \Delta_9}\,AT}{- : (\Box\Gamma_{10}, []\mathtt{F}_8), \Delta_{11} \vdash \Delta_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{hCut}}{- : \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\,AT$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \mathtt{F}_8, \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6 \vdash \Delta_9}\,AT}{- : \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{hCut}}{- : \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\,AT$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_9) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_7}\,K \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_9, \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_6 : (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_7 \vdash \Delta_8}\,AT}{- : \Box\Gamma_9, \Delta_{10} \vdash \Delta_8}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_{10}, \Box\Gamma_9, unbox(\Box\Gamma_9) \vdash \Delta_8, \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8, \mathtt{F}_7}\,ATG \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7, \Box\Gamma_9 \vdash \Delta_8, []\mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \Box\Gamma_9, []\mathtt{F}_7 \vdash \Delta_8}\,\mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7, \Box\Gamma_9 \vdash \Delta_8}\,\mathtt{hCut}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8}\,\mathtt{sCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}, []\mathtt{F}_9) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_{13} \vdash (\Delta_{10}, []\mathtt{F}_7), \mathtt{F}_{12}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_{13}), \mathtt{F}_{12} \vdash \Delta_{10}, []\mathtt{F}_7}\,AT}{- : (\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_{13} \vdash \Delta_{10}, []\mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \mathtt{F}_9, unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\,K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_7), []\mathtt{F}_9}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\,AT}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_7}\,K$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9 \vdash (\Delta_{10}, []\mathtt{F}_7), \mathtt{F}_{12}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9), \mathtt{F}_{12} \vdash \Delta_{10}, []\mathtt{F}_7}\, AT}{- : \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\, \mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\, K}\ \rightarrow$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_9) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \bot, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_6}\, K \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \bot, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\, \bot_L}{- : \Box\Gamma_9, \bot, \Delta_{10} \vdash \Delta_8}\, \mathtt{Cut}}{- : \bot, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8}\, \bot_L\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, []\mathtt{F}_7), \bot}\, K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{11}), \bot \vdash \Delta_9, []\mathtt{F}_7}\, \bot_L}{- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathtt{F}_7}\, \mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\, K}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \bot, \Delta_{12} \vdash (\Delta_9, []\mathtt{F}_7), \mathtt{F}_{11}}\, K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \bot, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_7}\, \bot_L}{- : \Box\Gamma_{10}, \bot, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}\, \mathtt{Cut}}{- : \bot, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\, \bot_L\ \rightarrow$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_8, \mathtt{p}_9), []\mathtt{F}_6}\, K \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Box\Gamma_{10}, \Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, I}{- : \Box\Gamma_{10}, \Delta_{11}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, \mathtt{Cut}}{- : \Delta_{11}, \Box\Gamma_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, I\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash ((\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7), \mathtt{p}_{10}}\, K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\, I}{- : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\, \mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_9, \mathtt{p}_{10}, []\mathtt{F}_7}\, K}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7), \mathtt{F}_{12}}\, K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10}), \mathtt{F}_{12} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\, I}{- : \Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\, \mathtt{Cut}}{- : \Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, []\mathtt{F}_7}\, I\ \rightarrow$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_9) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_6}\, K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \top, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\, \top_L}{- : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8, []\mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8}\, \mathtt{hCut}}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, []\mathtt{F}_7), \top}\ K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{11}), \top \vdash \Delta_9, []\mathtt{F}_7}\ \top_L}{- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathtt{F}_7}\ \mathtt{Cut}}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \top, \Delta_{12} \vdash (\Delta_9, []\mathtt{F}_7), \mathtt{F}_{11}}\ K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{10}, \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \top, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_7}\ \top_L}{- : \Box\Gamma_{10}, \top, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}\ \mathtt{Cut}}{- : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \top, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\ K$$

6.7 Status of $A45$: `fail`

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash (\Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{13}, \mathtt{F}_9, \Delta_{14}, []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \mathtt{F}_{10}, \Delta_{12}}{\bullet\mathtt{h}_8 : (\Box\Gamma_{13}, \Delta_{14}), []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \to_R}{- : \Box\Gamma_{13}, \Delta_{14} \vdash \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \mathtt{F}_7, \Box\Gamma_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_{11}, []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13}, []\mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_{11}}{}\ \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_{11}}\ \mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \to_R}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8, \Box\mathtt{F}_{16}}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15} \vdash (\Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), []\mathtt{F}_8), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{10}, \Delta_{15}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, \mathtt{F}_{11}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}), \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), []\mathtt{F}_8}\ \to_R}{- : \Box\Gamma_{14}, \Delta_{15} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_8, \Box\Gamma_{12}}{\bullet\mathtt{h}_1 : \Delta_{15}, \mathtt{F}_{10}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \Delta_{13}, \mathtt{F}_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\mathtt{F}_{16}, \Delta_{15}, \mathtt{F}_{10}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}{}\ \mathtt{ax/W}}{- : \Delta_{15}, \mathtt{F}_{10}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}\ \to_R}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), []\mathtt{F}_8), \mathtt{F}_{15}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{10}, \mathtt{F}_{15}, \Delta_{16} \vdash \Box\Gamma_{12}, \mathtt{F}_{11}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{16}), \mathtt{F}_{15} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), []\mathtt{F}_8}\ \to_R}{- : \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{14} \vdash \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}\ A45}\ \to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash (\Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{13}, \Delta_{14}, []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \mathtt{F}_9, \Delta_{12} \quad \mathtt{h}_8 : \Box\Gamma_{13}, \Delta_{14}, []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \mathtt{F}_{10}, \Delta}{\bullet\mathtt{h}_8 : (\Box\Gamma_{13}, \Delta_{14}), []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{Cut}}{- : \Box\Gamma_{13}, \Delta_{14} \vdash \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}}{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \mathtt{F}_7, \Box\Gamma_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Delta_{14}, \Box\Gamma_{13}, []\mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}}{}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}}\ \mathtt{hCut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \mathtt{F}_7, \Box\Gamma_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_{11}, []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Delta_{14}, \Box\Gamma_{1}}{}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_{11}}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8, \Box\mathtt{F}_{16}}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15} \vdash (\Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), []\mathtt{F}_8), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \Delta_{15}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13}, []\mathtt{F}_8 \quad \mathtt{h}_9 : \Box\Gamma_{14}, \Delta_{15}, \Box}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}), \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \wedge}}{- : \Box\Gamma_{14}, \Delta_{15} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), []\mathtt{F}_8}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{15}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \Delta_{13}, \mathtt{F}_{10}, \Box\Gamma_{12}, []\mathtt{F}_8}\ A45 \quad \dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{16}, \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_{10}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_{10}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{15}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \Delta_{13}, \mathtt{F}_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash}$$

$$- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), []\mathtt{F}_8), \mathtt{F}_{15}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{15}, \Delta_{16} \vdash \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13}, []\mathtt{F}_8 \quad \mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{15}, \Delta_{16} \vdash \Box\Gamma_{12}, \mathtt{F}_{11}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{16}), \mathtt{F}_{15} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), []\mathtt{F}_8}\ \wedge_R}{- : \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14} \vdash \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ A45$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash (\Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{13}, \Delta_{14}, []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{12}}{\bullet\mathtt{h}_8 : (\Box\Gamma_{13}, \Delta_{14}), []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R}{- : \Box\Gamma_{13}, \Delta_{14} \vdash \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \mathtt{F}_7, \Box\Gamma_{11}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_7}\ A45 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{14}, \Box\Gamma_{13}, []\mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \Box\Gamma_{11}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \Box\Gamma_{11}}\ \mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8, \Box\mathtt{F}_{16}}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15} \vdash (\Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), []\mathtt{F}_8), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \Delta_{15}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}), \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), []\mathtt{F}_8}\ \vee_R}{- : \Box\Gamma_{14}, \Delta_{15} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{15}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}\ A45 \quad \dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{16}, \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\ \vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), []\mathtt{F}_8), \mathtt{F}_{15}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{15}, \Delta_{16} \vdash \Box\Gamma_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{16}), \mathtt{F}_{15} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), []\mathtt{F}_8}\ \vee_R}{- : \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_{12}, (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14} \vdash \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\ A45$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \Box\Gamma_9, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Box\Gamma_9, \bot, \Delta_{10}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_7 \vdash \Box\Gamma_9, \Delta_{10}}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \Box\Gamma_9, \bot, \Delta_{10}}\ \bot_R}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Box\Gamma_9, \bot, \Delta_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \bot, \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_7 \vdash \bot, \Delta_{10}, \Box\Gamma_9}\ \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \bot, \Delta_{10}, \Box\Gamma_9}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8, \Box\mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Box\Gamma_{10}, (\bot, \Delta_{11}), []\mathtt{F}_8), \Box\mathtt{F}_{14}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12}, \Delta_{13}, \Box\mathtt{F}_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{13}), \Box\mathtt{F}_{14} \vdash \Box\Gamma_{10}, (\bot, \Delta_{11}), []\mathtt{F}_8}\ \bot_R}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Box\Gamma_{10}, (\bot, \Delta_{11}), []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{13}, \Box\Gamma_{12} \vdash \bot, \Box\mathtt{F}_{14}, \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{14}, \Delta_{13}, \Box\Gamma_{12} \vdash \bot, \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \bot, \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash (\Box\Gamma_{10}, (\bot, \Delta_{11}), []\mathtt{F}_8), \mathtt{F}_{13}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash \Box\Gamma_{10}, (\bot, \Delta_{11}), []\mathtt{F}_8}\ \bot_R}{- : \Box\Gamma_{12}, \Delta_{14} \vdash \Box\Gamma_{10}, (\bot, \Delta_{11}), []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_8, \Box\Gamma_{10}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \bot, \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ A45$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \Box\Gamma_9, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Box\Gamma_9, \top, \Delta_{10}), []\mathtt{F}_7}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \Box\Gamma_9, \top, \Delta_{10}}\ \top_R}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Box\Gamma_9, \top, \Delta_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \top, \Delta_{10}, \Box\Gamma_9}\ \top_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8, \Box\mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Box\Gamma_{10}, (\top, \Delta_{11}), []\mathtt{F}_8), \Box\mathtt{F}_{14}}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{13}), \Box\mathtt{F}_{14} \vdash \Box\Gamma_{10}, (\top, \Delta_{11}), []\mathtt{F}_8}\ \top_R}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Box\Gamma_{10}, (\top, \Delta_{11}), []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \top, \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \top_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash (\Box\Gamma_{10}, (\top, \Delta_{11}), []\mathtt{F}_8), \mathtt{F}_{13}}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash \Box\Gamma_{10}, (\top, \Delta_{11}), []\mathtt{F}_8}\ \top_R}{- : \Box\Gamma_{12}, \Delta_{14} \vdash \Box\Gamma_{10}, (\top, \Delta_{11}), []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \top, \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \top_R$$

- Case rule K

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Box\Gamma_{14} \vdash (\Box\Gamma_{10}, []\mathtt{F}_9), \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Box\Gamma_{10}, []\mathtt{F}_9), \Delta_{11}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_7, unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), []\mathtt{F}_7 \vdash (\Box\Gamma_{10}, []\mathtt{F}_9), \Delta_{11}}\ K}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{10}, []\mathtt{F}_9), \Delta_{11}}\ \mathtt{Cut} \quad \rightarrow \quad \dfrac{}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}}$$

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Axioms assumed:

inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
```

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \Box\Gamma_{10}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_7, unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), []\mathtt{F}_7 \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\ K}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \mathtt{F}_7, \mathtt{F}_9, \Box\Gamma_{10}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \mathtt{F}_7, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9, \Box\Gamma_{10}}\ \mathtt{ax/W}}{- : \mathtt{F}_7, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \mathtt{F}_9, \Box\Gamma_{10}}\ ATG}{- : \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \mathtt{F}_9, \Box\Gamma_{10}}\ \mathtt{sCut}}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_9}\ A45$$

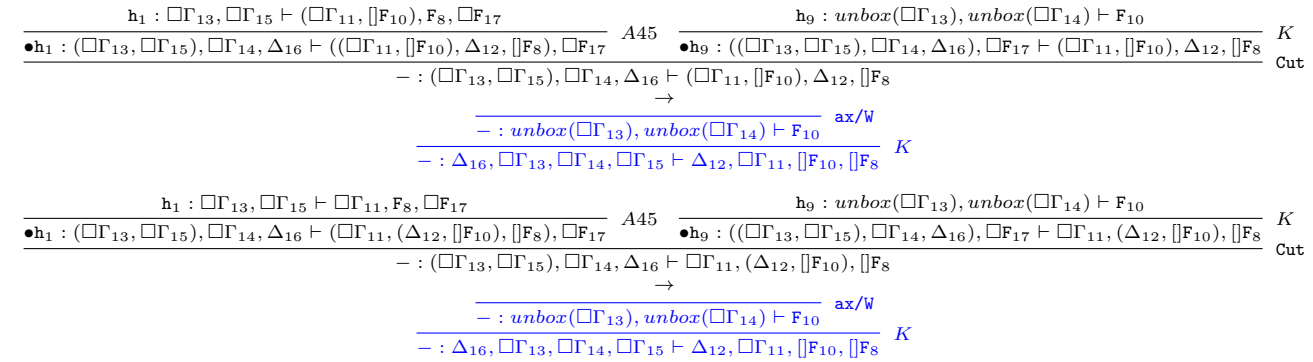
$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Box\Gamma_{14} \vdash (\Box\Gamma_{10}, []\mathtt{F}_9), \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Box\Gamma_{10}, []\mathtt{F}_9), \Delta_{11}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), []\mathtt{F}_7 \vdash (\Box\Gamma_{10}, []\mathtt{F}_9), \Delta_{11}}\ K}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{10}, []\mathtt{F}_9), \Delta_{11}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_9}\ K$$

```
Axioms assumed:

inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
```



$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \Box\Gamma_{10}, \mathtt{F}_9, \Box\mathtt{F}_{16}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathtt{F}_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\ K}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\ \mathtt{Cut}}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_9}\ K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Box\Gamma_{16} \vdash (\Box\Gamma_{11}, []\mathtt{F}_{10}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{11}, []\mathtt{F}_{10}), \Delta_{12}, []\mathtt{F}_8), \Box\mathtt{F}_{14}}\ A45 \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}), unbox(\Box\mathtt{F}_{14}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box\mathtt{F}_{14} \vdash (\Box\Gamma_{11}, []\mathtt{F}_{10}), \Delta_{12}, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{11}, []\mathtt{F}_{10}), \Delta_{12}, []\mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{13}, \Box\Gamma_{16} \vdash \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{17}, \Box\Gamma_{13}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{11}, (\Delta_{12}, []\mathtt{F}_{10}), []\mathtt{F}_8), \Box\mathtt{F}_{14}}\ A45 \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}), unbox(\Box\mathtt{F}_{14}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box\mathtt{F}_{14} \vdash \Box\Gamma_{11}, (\Delta_{12}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash \Box\Gamma_{11}, (\Delta_{12}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{13}, \Box\Gamma_{16} \vdash \mathtt{F}_8, \Box\Gamma_{11}}\ \mathtt{ax/W}}{- : \Delta_{17}, \Box\Gamma_{13}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \mathtt{F}_9}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), \Box\mathtt{F}_{13}}\ A45 \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{14}), unbox(\Box\mathtt{F}_{13}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \Box\mathtt{F}_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\ K}{- : (\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{12}, \Box\Gamma_{15} \vdash \mathtt{F}_9, \Box\Gamma_{10}}\ \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_9}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Box\Gamma_{15} \vdash (\Box\Gamma_{11}, []\mathtt{F}_{10}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{17} \vdash ((\Box\Gamma_{11}, []\mathtt{F}_{10}), \Delta_{12}, []\mathtt{F}_8), \mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{17}), \mathtt{F}_{16} \vdash (\Box\Gamma_{11}, []\mathtt{F}_{10}), \Delta_{12}, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{17} \vdash (\Box\Gamma_{11}, []\mathtt{F}_{10}), \Delta_{12}, []\mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{17}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \Box\Gamma_{11}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{17} \vdash (\Box\Gamma_{11}, (\Delta_{12}, []\mathtt{F}_{10}), []\mathtt{F}_8), \mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{17}), \mathtt{F}_{16} \vdash \Box\Gamma_{11}, (\Delta_{12}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{17} \vdash \Box\Gamma_{11}, (\Delta_{12}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{17}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \Box\Gamma_{10}, \mathtt{F}_9}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9), \mathtt{F}_{15}}\ A45 \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16}), \mathtt{F}_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\ K}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_9}\ K$$

- Case rule $A45$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{15}, []\mathtt{F}_7 \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), []\mathtt{F}_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \Box\Gamma_{11}, \Delta_{13}}\ A45}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \Box\Gamma_{11}, \Delta_{13}}\ \mathtt{Cut}$$
$$\rightarrow$$

Axioms assumed:

```
inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
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$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{15}, []\mathtt{F}_7 \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), []\mathtt{F}_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9}}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9}\ \to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \mathtt{F}_7, \Box\Gamma_{10}, \Box\Gamma_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_7}\ A45 \quad \dfrac{}{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16}, []\mathtt{F}_7 \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\ \mathtt{hCut}}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_9}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), []\mathtt{F}_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \Box\Gamma_{11}, \Delta_{13}}}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}, []\mathtt{F}_9), \Box\Gamma_{11}, \Delta_{13}}\ \to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}}\ \mathtt{ax/W}}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_9}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), []\mathtt{F}_7 \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9}}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9}\ \to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}}\ \mathtt{ax/W}}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_9}\ A45$$

$$\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8, \Box\mathtt{F}_{17}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8), \Box\mathtt{F}_{17}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\mathtt{F}_{17} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19}), \Box\mathtt{F}_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8}$$

$$- : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8\ \to$$

Axioms assumed:

```
inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
```

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8, \Box\mathtt{F}_{17}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8), \Box\mathtt{F}_{17}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\mathtt{F}_{17} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19}), \Box\mathtt{F}_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}),}}{- : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8}\ \to$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \Box\mathtt{F}_{17}, \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{17}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{17}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}}\ A}{- : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}}\ \mathtt{h}$$

$$\dfrac{- : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{F}_8, \Box\mathtt{F}_{17}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8), \Box\mathtt{F}_{17}}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\mathtt{F}_{17} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19}), \Box\mathtt{F}_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}}}{- : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8}\,\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash \Box\mathtt{F}_{17}, \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{13}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \Box\mathtt{F}_{17}, \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_8}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\mathtt{F}_{17}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_8}{\mathtt{h}}\,\mathtt{a}}{- : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_8}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\,A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{F}_8, \Box\mathtt{F}_{17}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8), \Box\mathtt{F}_{17}}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\mathtt{F}_{17} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19}), \Box\mathtt{F}_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}}}{- : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8}\,\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash \Box\mathtt{F}_{17}, \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{13}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \Box\mathtt{F}_{17}, \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_8}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\mathtt{F}_{17}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_8}{\mathtt{h}}\,\mathtt{a}}{- : \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_8}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\,A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \mathtt{F}_9, \Box\mathtt{F}_{16}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Box\Gamma_{17}), \Box\Gamma_{15}, \Delta_{18} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_9), \Box\mathtt{F}_{16}}\,A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{14}, \Box\Gamma_{17}), \Box\Gamma_{15}, \Delta_{18}), \Box\mathtt{F}_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}}}{- : (\Box\Gamma_{14}, \Box\Gamma_{17}), \Box\Gamma_{15}, \Delta_{18} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_9}\,\to$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{17} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_8 : \Box\mathtt{F}_{16}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{17} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_8 : \Box\mathtt{F}_{16}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{17} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\,\mathtt{H}}{- : \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{17} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\,\mathtt{hCut}}{- : \Delta_{18}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{17} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_9}\,A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8, \Box\mathtt{F}_{19}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8), \Box\mathtt{F}_{19}}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18}), \Box\mathtt{F}_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}),}}{- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8}\,\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{18}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\,A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8, \Box\mathtt{F}_{19}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8), \Box\mathtt{F}_{19}}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18}), \Box\mathtt{F}_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}),}}{- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8}\,\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\,\mathtt{ax/W}}{- : \Delta_{18}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\,A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{F}_8, \Box\mathtt{F}_{19}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8), \Box\mathtt{F}_{19}}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18}), \Box\mathtt{F}_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}}}{- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8}\,\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{18}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\,A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{F}_8, \Box\mathtt{F}_{19}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8), \Box\mathtt{F}_{19}}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18}), \Box\mathtt{F}_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}}}{- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8}\,\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\,\mathtt{ax/W}}{- : \Delta_{18}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\,A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \mathtt{F}_9, \Box\mathtt{F}_{18}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_9), \Box\mathtt{F}_{18}}\,A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box\mathtt{F}_{18} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}}}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_9}\,\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}}\,\mathtt{ax/W}}{- : \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_9}\,A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{17}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box\mathtt{F}_{16} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}),}}{- : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{17}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box\mathtt{F}_{16} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}),}}{- : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{17}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box\mathtt{F}_{16} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}}}{- : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{13}}\ \mathtt{ax/W}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{17}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box\mathtt{F}_{16} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}}}{- : (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{18} \vdash \mathtt{F}_8, \Box\Gamma_{11}, \Box\Gamma_{13}}\ \mathtt{ax/W}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \mathtt{F}_9}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_9), \Box\mathtt{F}_{15}}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{16}, \Box\mathtt{F}_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{14}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18}), \Box\mathtt{F}_{15} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}}}{- : (\Box\Gamma_{14}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_9}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{14}, \Box\Gamma_{17} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{18}, \Box\Gamma_{14}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_9}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8), \mathtt{F}_{18}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19}), \mathtt{F}_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma}}{- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8), \mathtt{F}_{18}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19}), \mathtt{F}_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma}}{- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), (\Box\Gamma_{12}, \Delta_{14}), []\mathtt{F}_8}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8), \mathtt{F}_{18}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19}), \mathtt{F}_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta}}{- : (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{10}), []\mathtt{F}_8}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{19}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \mathtt{F}_9}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{18} \vdash ((\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_9), \mathtt{F}_{17}}\,A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{18}), \mathtt{F}_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []}}{- : (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{18} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_9}\,\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \mathtt{F}_9, \Box\Gamma_{10}, \Box\Gamma_{11}}\,\mathtt{ax/W}}{- : \Delta_{18}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_9}\,A45$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7}\,A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{13}, \Delta_{14}, []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \mathtt{F}_9, \Delta_{12} \quad \mathtt{h}_8 : \Box\Gamma_{13}, \mathtt{F}_{10}, \Delta_{14}, []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta}{\bullet\mathtt{h}_8 : (\Box\Gamma_{13}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta_{12}}\,\mathtt{Cut}}{- : \Box\Gamma_{13}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Box\Gamma_{11}, \Delta_{12}}\,\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \mathtt{F}_7, \Box\Gamma_{11}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_7}\,A45 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{14}, \Box\Gamma_{13}, []\mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \mathtt{F}_7, \Box\Gamma_{11}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_7}\,A45 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{14}, \mathtt{F}_{10},}}{- : \Delta_{14}, \mathtt{F}_{10}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}}\,\mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \Box\Gamma_{11}}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8, \Box\mathtt{F}_{16}}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8), \Box\mathtt{F}_{16}}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \Delta_{15}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13}, []\mathtt{F}_8 \quad \mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{11}, \Delta_1}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, \Delta}}{- : \Box\Gamma_{14}, \Delta_{15}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\,\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_8, \Box\Gamma_{12}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{15}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \Delta_{13}, \mathtt{F}_{10}, \Box\Gamma_{12}, []\mathtt{F}_8}\,A45 \quad \dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{16}, \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_{10}, \Box\Gamma_{12}, []\mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_{10}, \Box\Gamma_{12}, []\mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_8, \Box\Gamma_{12}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{15}, \mathtt{F}_{11}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\,A45}{- : \Delta_{15}, \Box\Gamma_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11}}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \Delta_{15} \vdash \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13}, []\mathtt{F}_8 \quad \mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{11}, \Delta_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}), \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\,\mathtt{Cut}}{- : \Box\Gamma_{14}, \Delta_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\,\to_L$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14} \vdash \mathtt{F}_8, \Box\Gamma_{12}}\,\mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\,A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{16}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8), \mathtt{F}_{15}}\,A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{15}, \Delta_{16} \vdash \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13}, []\mathtt{F}_8 \quad \mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{11}, \mathtt{F}_{15}, \Delta_{16} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{16}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\,\mathtt{Cut}}{- : \Box\Gamma_{14}, \Delta_{16}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\,\to_L$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14} \vdash \mathtt{F}_8, \Box\Gamma_{12}}\,\mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\,A45$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7}\,A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{13}, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{14}, []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta_{12}}{\bullet\mathtt{h}_8 : (\Box\Gamma_{13}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta_{12}}\,\wedge_L}{- : \Box\Gamma_{13}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Box\Gamma_{11}, \Delta_{12}}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \mathtt{F}_7, \Box\Gamma_{11}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_7}\,A45 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_9, \Box\Gamma_{13}, []\mathtt{F}_7 \vdash \Delta_{12}, \Box\Gamma_{11}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}}\,\mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \Box\Gamma_{11}}\,\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8, \Box\mathtt{F}_{16}}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{15}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \wedge_L}{- : \Box\Gamma_{14}, \Delta_{15}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{15}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ A45 \quad \dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{16}, \Delta_{15}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{15}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_{15}, \Box\Gamma_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \wedge_L}{- : \Box\Gamma_{14}, \Delta_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14} \vdash \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{16}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8), \mathtt{F}_{15}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{15}, \Delta_{16} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{16}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \wedge_L}{- : \Box\Gamma_{14}, \Delta_{16}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{14} \vdash \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ A45$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{13}, \mathtt{F}_9, \Delta_{14}, []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta_{12} \quad \mathtt{h}_8 : \Box\Gamma_{13}, \mathtt{F}_{10}, \Delta_{14}, []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta}{\bullet\mathtt{h}_8 : (\Box\Gamma_{13}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Box\Gamma_{11}, \Delta_{12}}\ \mathtt{Cut}}{- : \Box\Gamma_{13}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Box\Gamma_{11}, \Delta_{12}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \mathtt{F}_7, \Box\Gamma_{11}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_7}\ A45 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13}, []\mathtt{F}_7 \vdash \Delta_{12}, \Box\Gamma_{11}}\ \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \mathtt{F}_7, \Box\Gamma_{11}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_7}\ A45 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{14}, \mathtt{F}_{10},}$$

$$- : \Delta_{14}, \mathtt{F}_{10}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}$$

$$- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \Box\Gamma_{11}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8, \Box\mathtt{F}_{16}}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8), \Box\mathtt{F}_{16}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{10}, \Delta_{15}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8 \quad \mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{11}, \Delta_1}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \Box\mathtt{F}_{16} \vdash \Box\Gamma_{12}, \Delta}$$

$$- : \Box\Gamma_{14}, \Delta_{15}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{15}, \mathtt{F}_{10}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ A45 \quad \dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{16}, \Delta_{15}, \mathtt{F}_{10}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{15}, \mathtt{F}_{10}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{15}, \mathtt{F}_{11}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{16}, \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}$$

$$- : \Delta_{15}, \mathtt{F}_{11}, \Box$$

$$- : \Delta_{15}, \Box\Gamma_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{15} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{10}, \Delta_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8 \quad \mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{11}, \Delta_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{15}), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \vee_L}{- : \Box\Gamma_{14}, \Delta_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

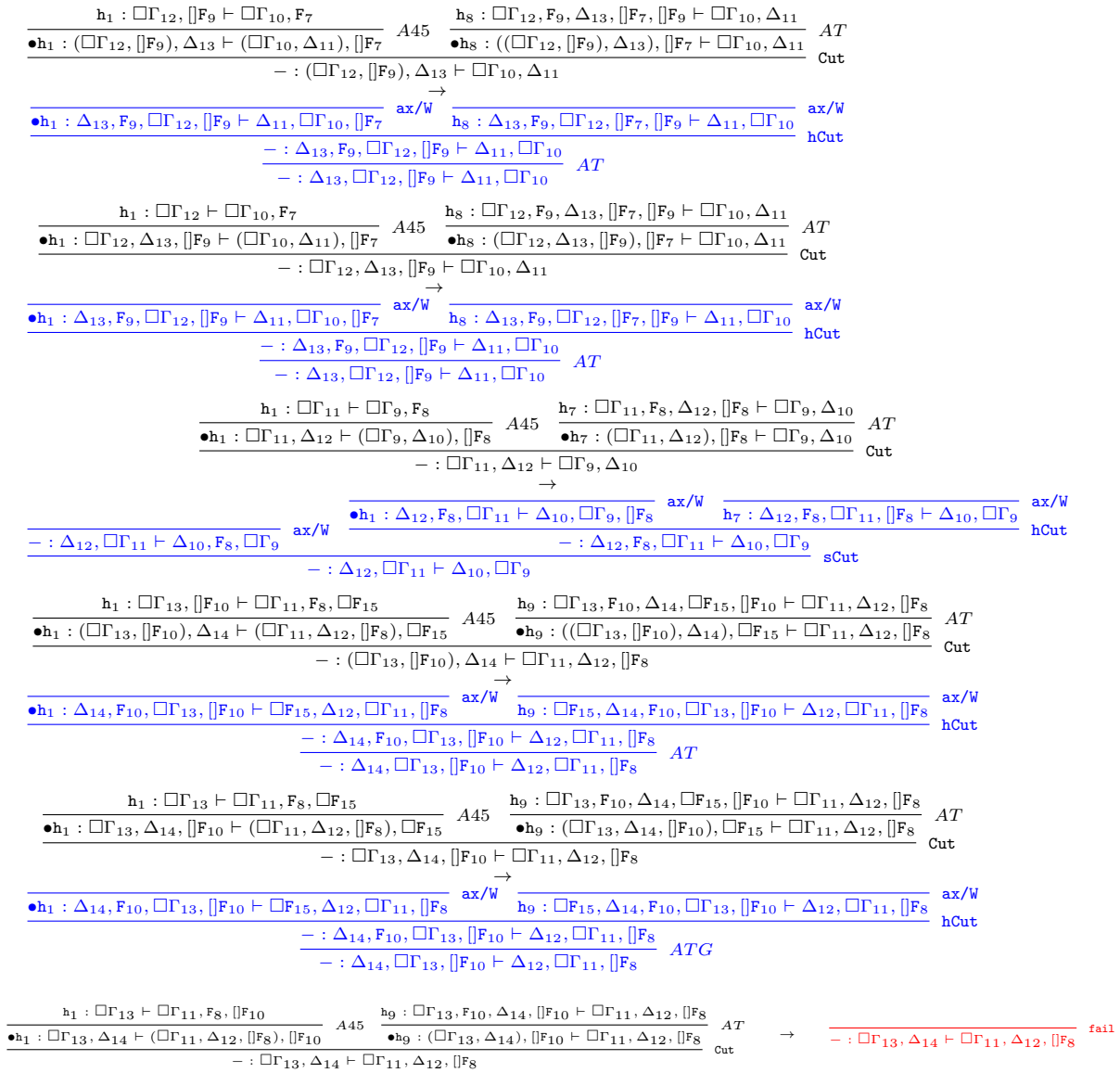
$$\dfrac{\dfrac{}{- : \Box\Gamma_{14} \vdash \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{14} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{14}, \Delta_{16}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8), \mathtt{F}_{15}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{10}, \mathtt{F}_{15}, \Delta_{16} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8 \quad \mathtt{h}_9 : \Box\Gamma_{14}, \mathtt{F}_{11}, \mathtt{F}_{15}, \Delta_{16} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{14}, \Delta_{16}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{15} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \vee_L}{- : \Box\Gamma_{14}, \Delta_{16}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

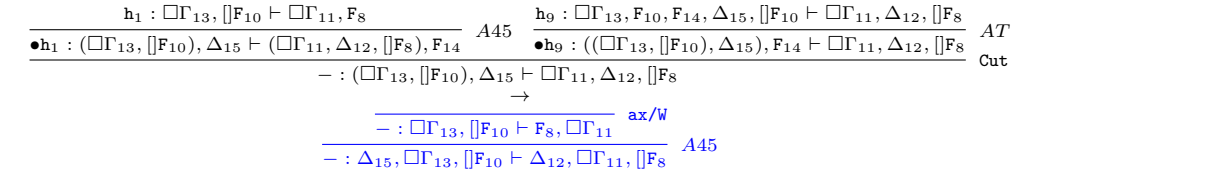
$$\dfrac{\dfrac{}{- : \Box\Gamma_{14} \vdash \mathtt{F}_8, \Box\Gamma_{12}}\ \mathtt{ax/W}}{- : \Delta_{16}, \Box\Gamma_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_8}\ A45$$

- Case rule AT



```
Axioms assumed:

inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula |-- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
```



$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_8), []\mathtt{F}_{10}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13}, \mathtt{F}_{10}, \Delta_{14}, []\mathtt{F}_{10} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), []\mathtt{F}_{10} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_8}\ AT}{- : \Box\Gamma_{13}, \Delta_{14} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{- : \Box\Gamma_{13} \vdash \mathtt{F}_8, \Box\Gamma_{11}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_8}\ A45}\ \mathtt{ax/W}}{}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{15}, []\mathtt{F}_{10} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_8), \mathtt{F}_{14}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \Delta_{15}, []\mathtt{F}_{10} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{15}, []\mathtt{F}_{10}), \mathtt{F}_{14} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_8}\ AT}{- : \Box\Gamma_{13}, \Delta_{15}, []\mathtt{F}_{10} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{- : \Box\Gamma_{13} \vdash \mathtt{F}_8, \Box\Gamma_{11}}{- : \Delta_{15}, \Box\Gamma_{13}, []\mathtt{F}_{10} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_8}\ A45}\ \mathtt{ax/W}}{}\ \rightarrow$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \Box\Gamma_9, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \bot, \Delta_{12} \vdash (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_7}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \bot, \Delta_{12}), []\mathtt{F}_7 \vdash \Box\Gamma_9, \Delta_{10}}\ \bot_L}{- : \Box\Gamma_{11}, \bot, \Delta_{12} \vdash \Box\Gamma_9, \Delta_{10}}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \Box\Gamma_9}\ \bot_L}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8, \Box\mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \bot, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8), \Box\mathtt{F}_{14}}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \bot, \Delta_{13}), \Box\mathtt{F}_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \bot_L}{- : \Box\Gamma_{12}, \bot, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \bot_L}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8), \bot}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{13}), \bot \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \bot_L}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{- : \Box\Gamma_{12} \vdash \mathtt{F}_8, \Box\Gamma_{10}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ A45}\ \mathtt{ax/W}}{}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \bot, \Delta_{14} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8), \mathtt{F}_{13}}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \bot, \Delta_{14}), \mathtt{F}_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \bot_L}{- : \Box\Gamma_{12}, \bot, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \bot_L}\ \rightarrow$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_9, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13}, \mathtt{p}_{11} \vdash (\Box\Gamma_9, \Delta_{10}, \mathtt{p}_{11}), []\mathtt{F}_7}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}, \mathtt{p}_{11}), []\mathtt{F}_7 \vdash \Box\Gamma_9, \Delta_{10}, \mathtt{p}_{11}}\ I}{- : \Box\Gamma_{12}, \Delta_{13}, \mathtt{p}_{11} \vdash \Box\Gamma_9, \Delta_{10}, \mathtt{p}_{11}}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{13}, \Box\Gamma_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \Box\Gamma_9, \mathtt{p}_{11}}\ I}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{10}, \mathtt{F}_8, \Box\mathtt{F}_{15}}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14}, \mathtt{p}_{12} \vdash (\Box\Gamma_{10}, (\Delta_{11}, \mathtt{p}_{12}), []\mathtt{F}_8), \Box\mathtt{F}_{15}}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}, \mathtt{p}_{12}), \Box\mathtt{F}_{15} \vdash \Box\Gamma_{10}, (\Delta_{11}, \mathtt{p}_{12}), []\mathtt{F}_8}\ I}{- : \Box\Gamma_{13}, \Delta_{14}, \mathtt{p}_{12} \vdash \Box\Gamma_{10}, (\Delta_{11}, \mathtt{p}_{12}), []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{p}_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, \mathtt{p}_{12}, []\mathtt{F}_8}\ I}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{10}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash (\Box\Gamma_{10}, (\Delta_{11}, \mathtt{p}_{12}), []\mathtt{F}_8), \mathtt{p}_{12}}\ A45 \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{p}_{12} \vdash \Box\Gamma_{10}, (\Delta_{11}, \mathtt{p}_{12}), []\mathtt{F}_8}\ I}{- : \Box\Gamma_{13}, \Delta_{14} \vdash \Box\Gamma_{10}, (\Delta_{11}, \mathtt{p}_{12}), []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{- : \Box\Gamma_{13} \vdash \mathtt{F}_8, \Box\Gamma_{10}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \Box\Gamma_{10}, \mathtt{p}_{12}, []\mathtt{F}_8}\ A45}\ \mathtt{ax/W}}{}\ \rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13} \vdash \Box\Gamma_{10}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{15}, \mathtt{p}_{12} \vdash (\Box\Gamma_{10}, (\Delta_{11}, \mathtt{p}_{12}), []\mathtt{F}_8), \mathtt{F}_{14}}\ A45 \quad \dfrac{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{15}, \mathtt{p}_{12}), \mathtt{F}_{14} \vdash \Box\Gamma_{10}, (\Delta_{11}, \mathtt{p}_{12}), []\mathtt{F}_8}{}\ I}{- : \Box\Gamma_{13}, \Delta_{15}, \mathtt{p}_{12} \vdash \Box\Gamma_{10}, (\Delta_{11}, \mathtt{p}_{12}), []\mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_{15}, \Box\Gamma_{13}, \mathtt{p}_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, \mathtt{p}_{12}, []\mathtt{F}_8}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \Box\Gamma_9, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \top, \Delta_{12} \vdash (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_7}\ A45 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_7 \vdash \Box\Gamma_9, \Delta_{10}}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \top, \Delta_{12}), []\mathtt{F}_7 \vdash \Box\Gamma_9, \Delta_{10}}\ \top_L}{- : \Box\Gamma_{11}, \top, \Delta_{12} \vdash \Box\Gamma_9, \Delta_{10}}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_7 \vdash \Delta_{10}, \Box\Gamma_9}\ \mathtt{ax/W}}{- : \top, \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \Box\Gamma_9}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8, \Box\mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \top, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8), \Box\mathtt{F}_{14}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12}, \Delta_{13}, \Box\mathtt{F}_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \top, \Delta_{13}), \Box\mathtt{F}_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \top_L}{- : \Box\Gamma_{12}, \top, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{13}, \Box\Gamma_{12} \vdash \Box\mathtt{F}_{14}, \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Box\mathtt{F}_{14}, \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \top, \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8), \top}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12}, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{13}), \top \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \top_L}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \top, \Delta_{14} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8), \mathtt{F}_{13}}\ A45 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \top, \Delta_{14}), \mathtt{F}_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \top_L}{- : \Box\Gamma_{12}, \top, \Delta_{14} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_8, \Box\Gamma_{10}}\ \mathtt{ax/W}}{- : \top, \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8}\ A45$$

6.8 Status of $\rightarrow_L$: OK

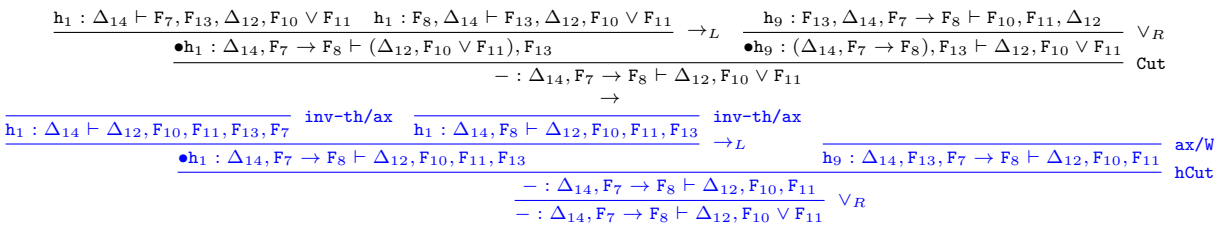
- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_{13}}\ \rightarrow_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\ \rightarrow_R}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\ \rightarrow_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\ \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\ \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\ \rightarrow_R$$

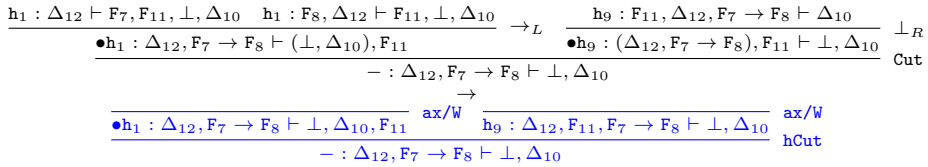
- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{13}}\ \rightarrow_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \mathtt{F}_1}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\ \rightarrow_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\ \mathtt{hCut} \quad \dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{14},}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}$$

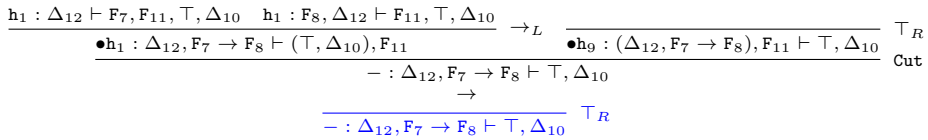
- Case rule $\vee_R$



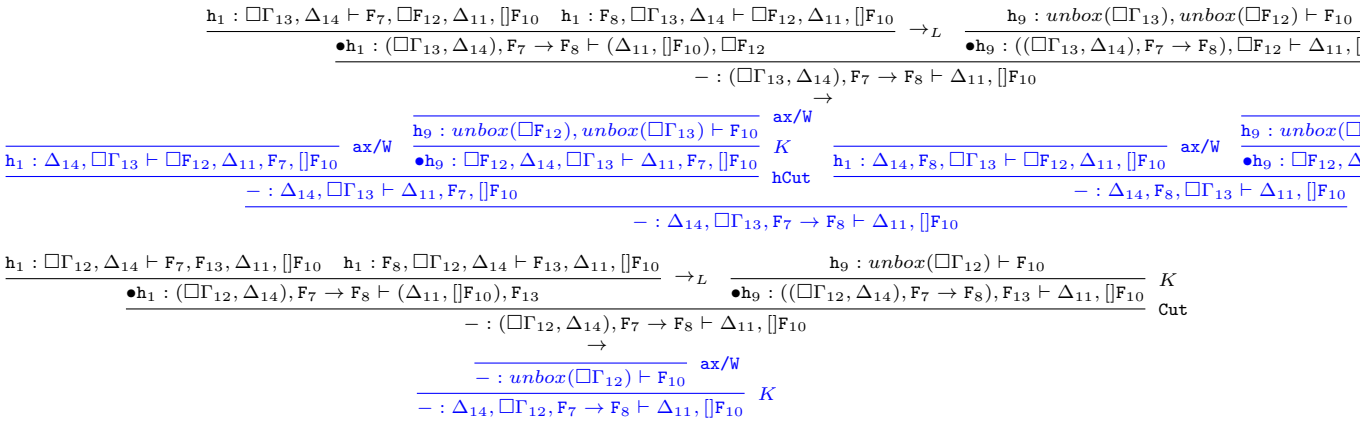
- Case rule $\bot_R$



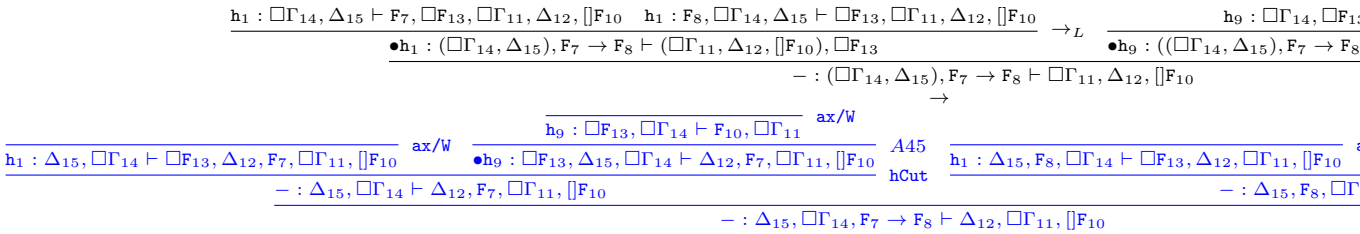
- Case rule $\top_R$

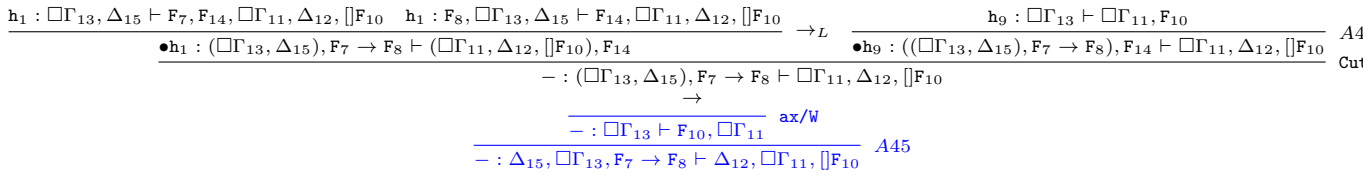


- Case rule K

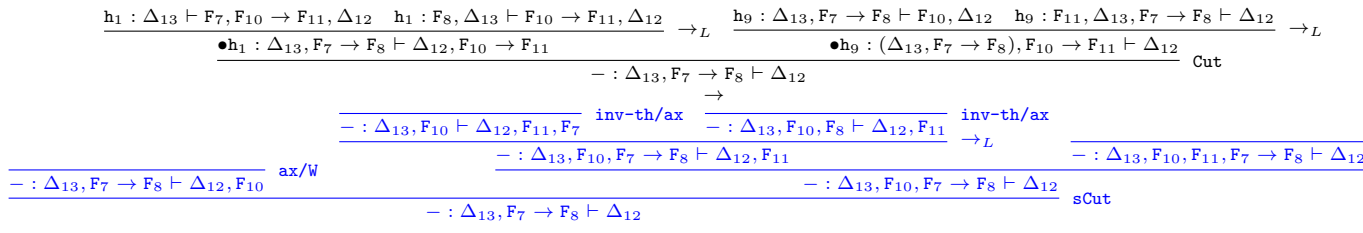


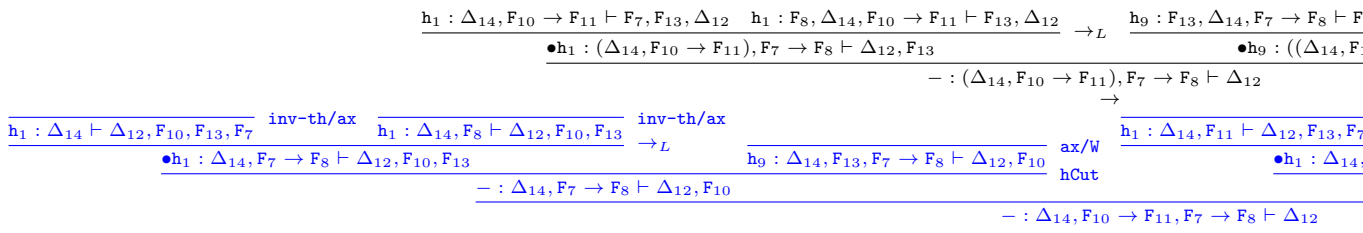
- Case rule $A45$

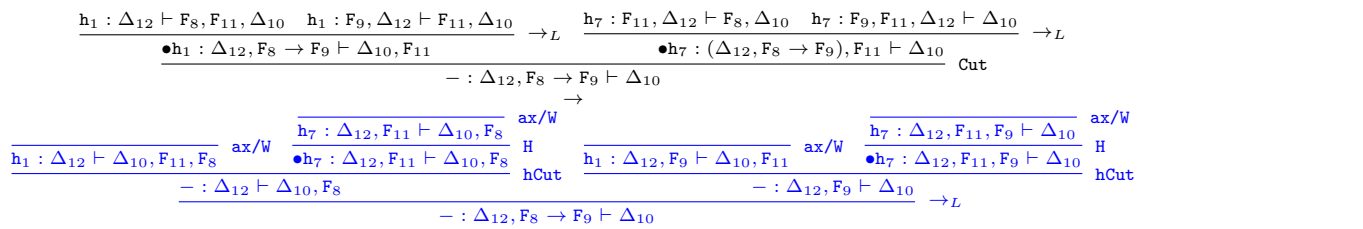




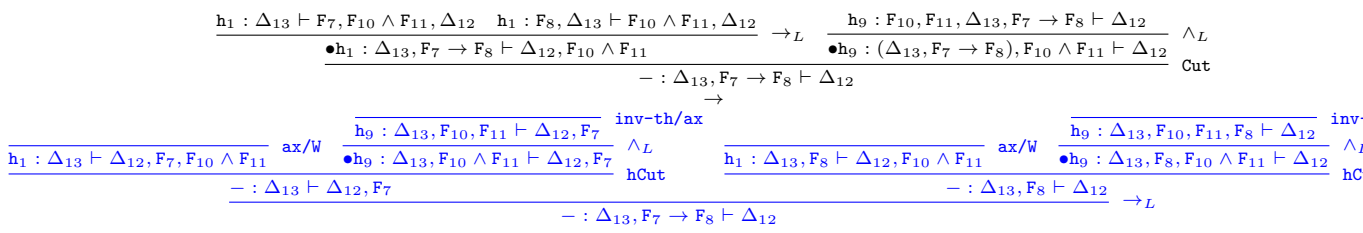
- Case rule $\rightarrow_L$

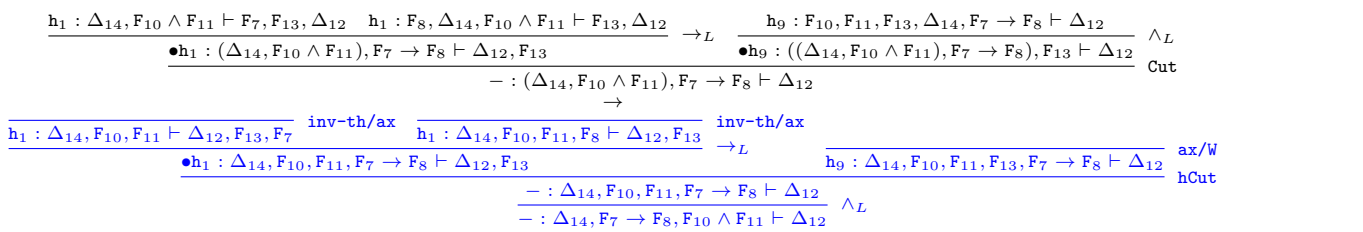




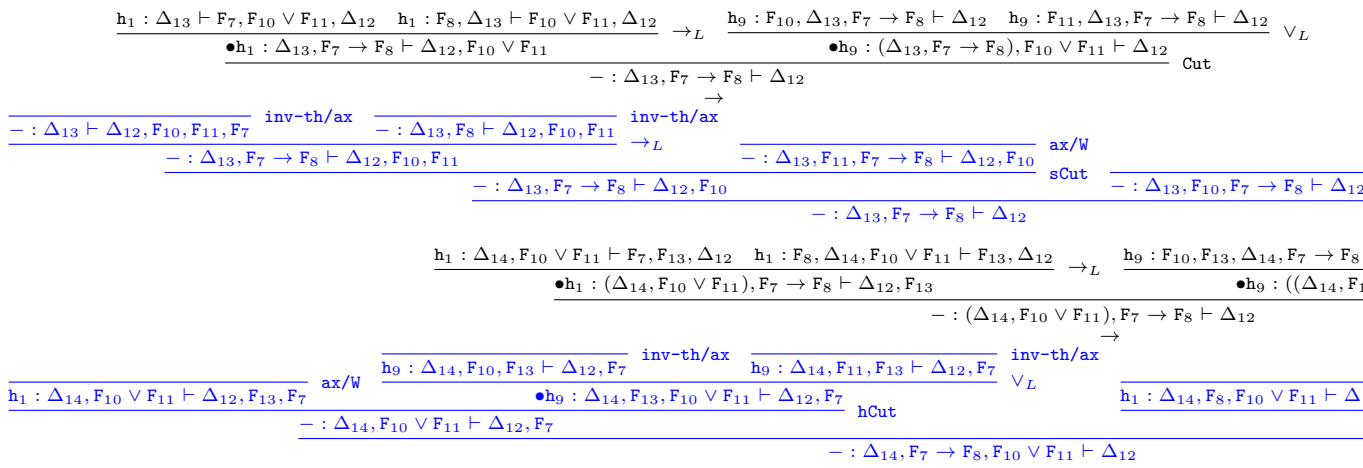


- Case rule $\wedge_L$

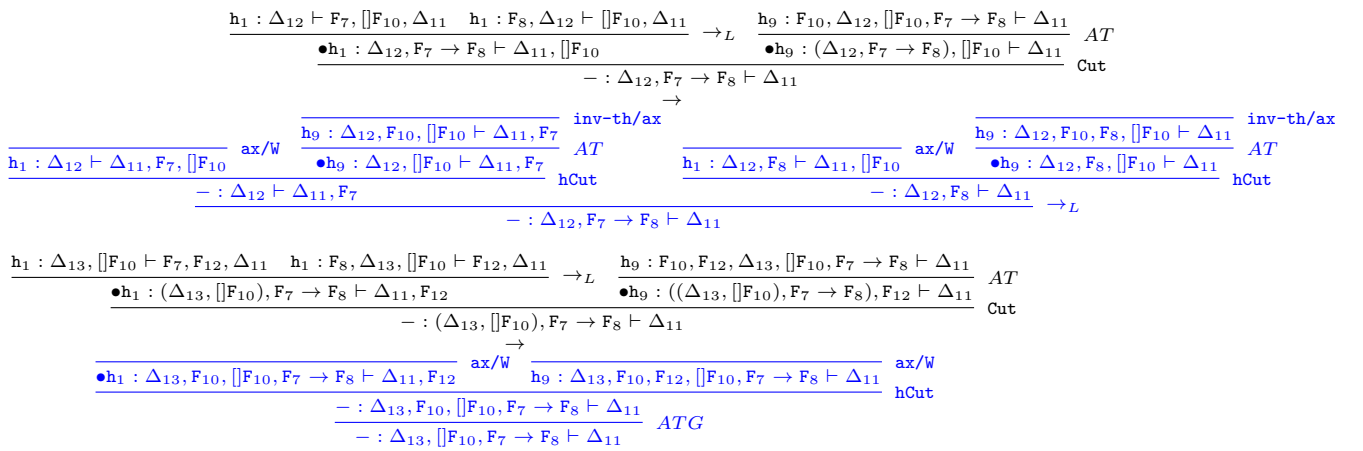




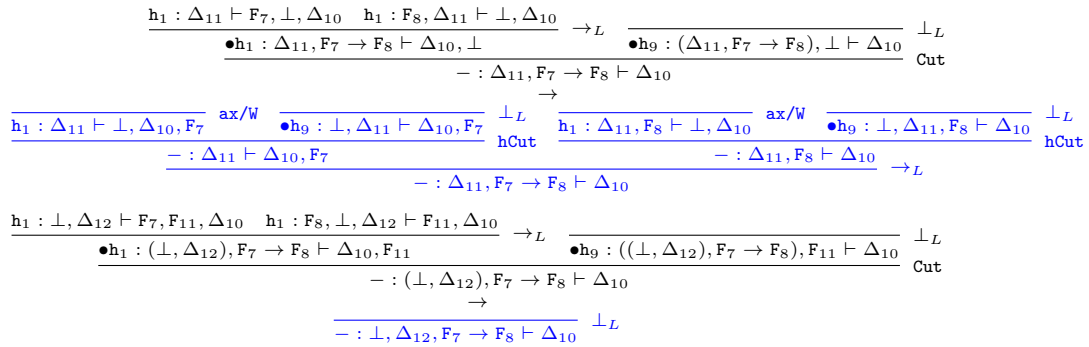
- Case rule $\vee_L$



- Case rule AT



- Case rule $\bot_L$



- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{12} \vdash \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}} \to_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}, \mathtt{p}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\, I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\to_L}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{12}} \to_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\to\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \top, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \top} \to_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \top \vdash \Delta_{10}}\, \top_L}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{}\,\mathtt{ax/W}}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \top, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\top, \Delta_{12}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}} \to_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : ((\top, \Delta_{12}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11} \vdash \Delta_{10}}\, \top_L}{- : (\top, \Delta_{12}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \top, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{}\,\mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{hCut}}\,\to$$

6.9 Status of $\wedge_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{13}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}} \to_R}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}{}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}} \wedge_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\to_R}\,\to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{13}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \wedge_R}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \wedge_L}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}{}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \quad \dfrac{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}{}\,\mathtt{inv\text{-}th/ax}}{\wedge_R}\,\to$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{13}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_R}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}} \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}} \wedge_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}} \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}} \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{12} \vdash \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{11}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11} \vdash \bot, \Delta_{10}} \bot_R}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}, \mathtt{F}_{11}} \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{12} \vdash \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{11}} \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11} \vdash \top, \Delta_{10}} \top_R}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \top, \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \top, \Delta_{10}} \top_R$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \Box\mathtt{F}_{12}} \wedge_L \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : ((\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8), \Box\mathtt{F}_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}} K}{- : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}} \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : unbox(\Box\mathtt{F}_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}} \mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}} K}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}} \mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}} \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_{13}} \wedge_L \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : ((\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}} K}{- : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}} \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}} K$$

- Case rule $A45$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{14}, \Delta_{15} \vdash \Box\mathtt{F}_{13}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : (\Box\Gamma_{14}, \Delta_{15}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \Box\mathtt{F}_{13}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \Box\mathtt{F}_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : ((\Box\Gamma_{14}, \Delta_{15}), \mathtt{F}_7 \wedge \mathtt{F}_8), \Box\mathtt{F}_{13} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}} A45}{- : (\Box\Gamma_{14}, \Delta_{15}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{15}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{13}, \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}} \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{13}, \Box\Gamma_{14} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}} \mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Box\mathtt{F}_{13}, \Delta_{15}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}} A45}{- : \Delta_{15}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}} \mathtt{hCut}}{- : \Delta_{15}, \Box\Gamma_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}} \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13}, \Delta_{15} \vdash \mathtt{F}_{14}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_{14}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}\ A45}{- : (\Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{13} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}}\ A45}\ \to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}}\ \to_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}}\ \to_L}\ \mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \wedge_L}\ \to$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet\mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}}\ \to_L}{- : (\Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}}\ \to_L}\ \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}}\ \wedge_L}{- : \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \wedge_L}\ \to$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\ \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\ \wedge_L}\ \mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \wedge_L}\ \to$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet\mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}}\ \wedge_L}{- : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\ \wedge_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \wedge_L}\ \to$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_8, \mathtt{F}_9, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \wedge_L \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11} \vdash \Delta_{10}}\ \wedge_L}{- : \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{H}}\ \mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\ \wedge_L}{- : \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\ \wedge_L}\ \to$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}} \vee_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{inv\text{-}th/ax} \vee_L}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}} \mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}} \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}} \vee_L}{- : (\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}} \mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{inv\text{-}th/ax} \vee_L}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}} \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}} \wedge_L}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{12} \vdash []\mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), []\mathtt{F}_{10} \vdash \Delta_{11}} AT}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}} \mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10} \vdash \Delta_{11}}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10} \vdash \Delta_{11}} AT}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10} \vdash \Delta_{11}} \mathtt{inv\text{-}th/ax} \mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_{12}, \Delta_{11}}{\bullet \mathtt{h}_1 : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, \mathtt{F}_{12}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}{\bullet \mathtt{h}_9 : ((\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{11}} AT}{- : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, \mathtt{F}_{12}} \mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}} \mathtt{ax/W} \mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}} \mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}} ATG$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{11} \vdash \bot, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \bot} \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \bot \vdash \Delta_{10}} \bot_L}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}} \bot_L}{\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}} \wedge_L}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \bot, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : (\bot, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}} \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : ((\bot, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11} \vdash \Delta_{10}} \bot_L}{- : (\bot, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}} \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{12} \vdash \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}} I}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}} I}{- : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\wedge_L}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{12}}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{p}_{11}} I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\rightarrow I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \top}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \top \vdash \Delta_{10}}\top_L}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\rightarrow \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \top, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : (\top, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : ((\top, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11} \vdash \Delta_{10}}\top_L}{- : (\top, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{hCut}}\,\rightarrow$$

6.10 Status of $\vee_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_{13}}\vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\rightarrow_R}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\rightarrow_R}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\,\rightarrow$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{13}}\vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7}}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}$$

$$\bullet \mathtt{h}_1 : \Delta_{14},$$

$$- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{13}} \vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_R}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}} \vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\ \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\ \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_{11}, \bot, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{12} \vdash \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{11}} \vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11} \vdash \bot, \Delta_{10}} \bot_R}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}, \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}}\ \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_{11}, \top, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{12} \vdash \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{11}} \vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11} \vdash \top, \Delta_{10}} \top_R}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \top, \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \top, \Delta_{10}} \top_R$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \Box\mathtt{F}_{12}} \vee_L \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \vee \mathtt{F}_8), \Box\mathtt{F}_{12} \vdash \Delta_{11}, []}}{- : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : unbox(\Box\mathtt{F}_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}} K}{- : \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : unbox(\Box \quad \bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta}}{- : \Delta_{14}, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}}}{- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_{13}} \vee_L \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}} K}{- : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\ K$$

- Case rule $A45$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{14}, \Delta_{15} \vdash \Box\mathtt{F}_{13}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Box\Gamma_{14}, \Delta_{15} \vdash \Box\mathtt{F}_{13}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{14}, \Delta_{15}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \Box\mathtt{F}_{13}} \vee_L \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{14}, \Box\mathtt{F}_{13}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{14}, \Delta_{15}), \mathtt{F}_7 \vee \mathtt{F}_8)}}{- : (\Box\Gamma_{14}, \Delta_{15}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{15}, \mathtt{F}_7, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{13}, \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{13}, \Box\Gamma_{14} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{13}, \Delta_{15}, \mathtt{F}_7, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}} A45}{- : \Delta_{15}, \mathtt{F}_7, \Box\Gamma_{14} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}}\ \mathtt{hCut} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{15}, \mathtt{F}_8, \Box\Gamma_{14} \vdash \Box\mathtt{F}_{13}, \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}}}{- : \Delta_{15}, \mathtt{F}_8, \Box\Gamma}}{- : \Delta_{15}, \Box\Gamma_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{13}, \Delta_{15} \vdash \mathtt{F}_{14}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Box\Gamma_{13}, \Delta_{15} \vdash \mathtt{F}_{14}, \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}), \mathtt{F}_{14}}\vee_L \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}A45}{- : (\Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{10}}\mathtt{Cut}}{\to}}{\dfrac{\dfrac{}{- : \Box\Gamma_{13} \vdash \mathtt{F}_{10}, \Box\Gamma_{11}}\mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}}A45}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}, \Delta_{12} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}}\to_L}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\dfrac{}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{11}}\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\mathtt{inv\text{-}th/ax}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\vee_L \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\mathtt{sCut}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet\mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_{10}}}{- : (\Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}}{\to}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}}{- : \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}$$

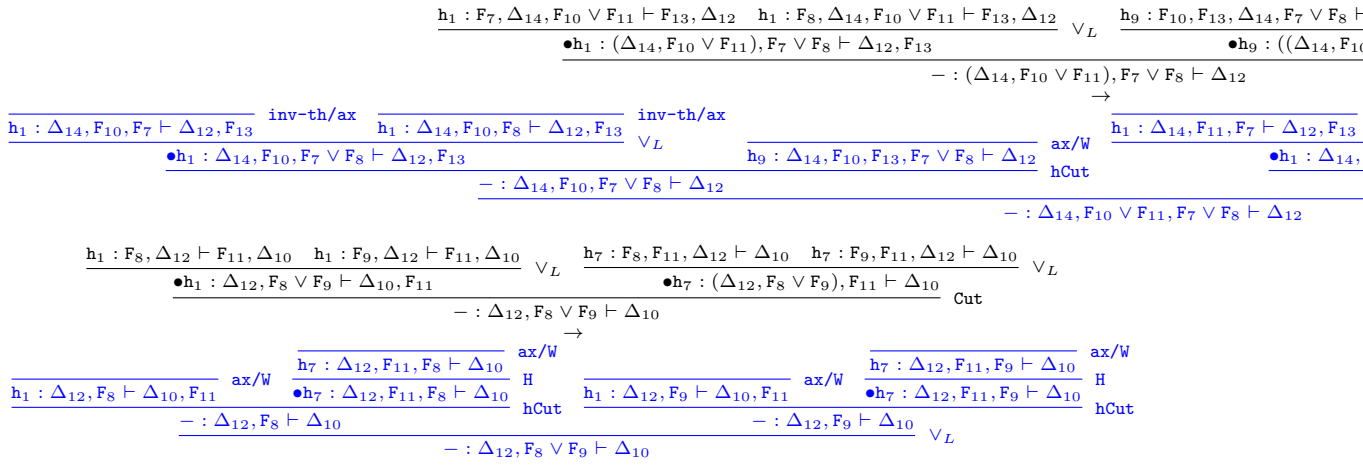
- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \Delta_{12} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}}\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}}\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{12}}\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\wedge_L}{- : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}}\mathtt{hC}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\vee_L$$

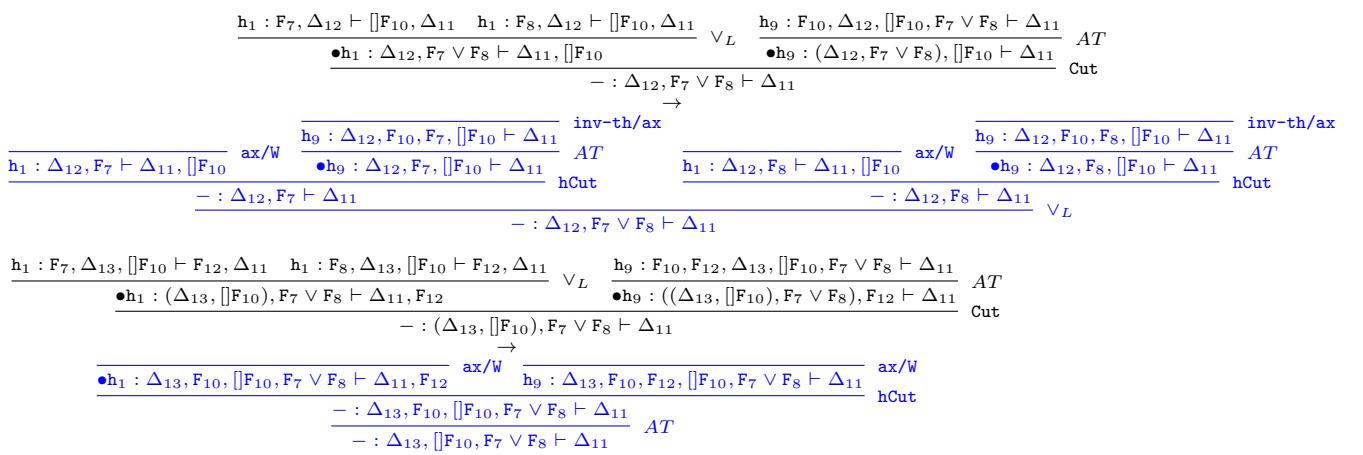
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet\mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}}\wedge_L}{- : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13}}\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\wedge_L$$

- Case rule $\vee_L$

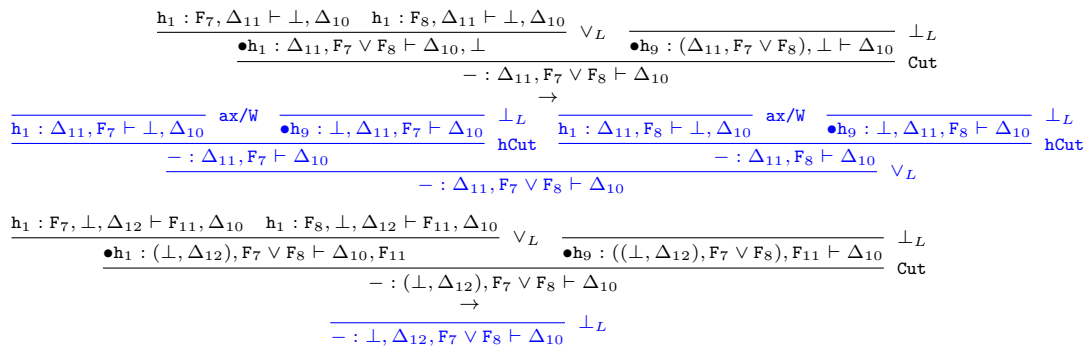
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\vee_L}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\dfrac{}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\mathtt{inv\text{-}th/ax}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\vee_L \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\mathtt{sCut} \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}$$



- Case rule AT



- Case rule $\bot_L$



- Case rule I

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{12} \vdash \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}\vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}{- : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}{- : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\vee_L}}{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{12}}\vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{11} \vdash \top, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \top}\vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \top \vdash \Delta_{10}}\top_L}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \top, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \top, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\top, \Delta_{12}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : ((\top, \Delta_{12}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11} \vdash \Delta_{10}}\top_L}{- : (\top, \Delta_{12}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\bullet\mathtt{h}_1 : \top, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}{- : \top, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{- : \top, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{hCut}}$$

6.11 Status of AT: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12}, []\mathtt{F}_6 \vdash (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{12}, []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\to_R}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\, AT}\,\mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12}, []\mathtt{F}_6 \vdash (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{12}, []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\wedge_R}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\, ATG$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12}, []\mathtt{F}_6 \vdash (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{12}, []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\vee_R}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\, ATG$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \bot, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash (\bot, \Delta_8), \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\Delta_{10}, []\mathtt{F}_6), \mathtt{F}_9 \vdash \bot, \Delta_8}\, \bot_R}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \bot, \Delta_8}\, \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash \bot, \Delta_8, \mathtt{F}_9}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6 \vdash \bot, \Delta_8}\, \mathtt{ax/W}}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \bot, \Delta_8}\, \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \top, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash (\top, \Delta_8), \mathtt{F}_9}\, AT \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Delta_{10}, []\mathtt{F}_6), \mathtt{F}_9 \vdash \top, \Delta_8}\, \top_R}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \top, \Delta_8}\, \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \top, \Delta_8}\, \top_R$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{10}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_6, unbox(\Box\Gamma_{11}), unbox(\Box\mathtt{F}_{10}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6), \Box\mathtt{F}_{10} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Box\mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_6, unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : \mathtt{F}_6, unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, K$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{10}}\, AT \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{11}), unbox(\Box\mathtt{F}_{10}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6), \Box\mathtt{F}_{10} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Box\mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, K$$

- Case rule $A45$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{12}, \Delta_{13}), []\mathtt{F}_6 \vdash \Box\mathtt{F}_{11}, \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Delta_{13}), []\mathtt{F}_6 \vdash (\Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8), \Box\mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{12}, \Box\mathtt{F}_{11}, []\mathtt{F}_6 \vdash \Box\Gamma_9, \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{12}, \Delta_{13}), []\mathtt{F}_6), \Box\mathtt{F}_{11} \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\, A45}{- : (\Box\Gamma_{12}, \Delta_{13}), []\mathtt{F}_6 \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_6, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \Box\mathtt{F}_{11}, \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Box\mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_6, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_6, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\, \mathtt{hCut}}{- : \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\, ATG$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \mathtt{F}_{12}, \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash (\Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8), \mathtt{F}_{12}}\ AT \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Box\Gamma_9, \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6), \mathtt{F}_{12} \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\ A45}{- : (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\ \mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Box\Gamma_9}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\ A45$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{12}, \Delta_{13}), []\mathtt{F}_6 \vdash \Box\mathtt{F}_{11}, \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Delta_{13}), []\mathtt{F}_6 \vdash (\Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8), \Box\mathtt{F}_{11}}\ AT \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{12}, \Box\mathtt{F}_{11} \vdash \Box\Gamma_9, \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{12}, \Delta_{13}), []\mathtt{F}_6), \Box\mathtt{F}_{11} \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\ A45}{- : (\Box\Gamma_{12}, \Delta_{13}), []\mathtt{F}_6 \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\ \mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_6, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \Box\mathtt{F}_{11}, \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Box\mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_6, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_6, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_{13}, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\ AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \mathtt{F}_{12}, \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash (\Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8), \mathtt{F}_{12}}\ AT \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11} \vdash \Box\Gamma_9, \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6), \mathtt{F}_{12} \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\ A45}{- : (\Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_8}\ \mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_8, \Box\Gamma_9}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_8}\ A45$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8 \to \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\ AT \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, []\mathtt{F}_6), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\ \to_L}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\ \mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\ AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\ AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : ((\Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}}\ \to_L}{- : (\Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\ \mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\ ATG$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, []\mathtt{F}_6), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\ \wedge_L}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\ \mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\ ATG$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\ AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : ((\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}}\ \wedge_L}{- : (\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\ \mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\ AT$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8 \vee \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, []\mathtt{F}_6), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\, \vee_L}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}{}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}{}\, \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}}\, \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\, ATG}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : ((\Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}}\, \vee_L}{- : (\Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}{}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}{}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\, \mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\, AT}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_6 \vdash []\mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \Delta_{10}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, []\mathtt{F}_6), []\mathtt{F}_8 \vdash \Delta_9}\, AT}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_9}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}{}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_9}\, ATG}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6 \vdash \mathtt{F}_{10}, \Delta_9}{\bullet\mathtt{h}_1 : (\Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6 \vdash \Delta_9, \mathtt{F}_{10}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : ((\Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6), \mathtt{F}_{10} \vdash \Delta_9}\, AT}{- : (\Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6 \vdash \Delta_9}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_{10}}{}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{}\, \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\, AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_6 : (\Delta_{10}, []\mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\, AT}{- : \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}{}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_9, []\mathtt{F}_7 \vdash \Delta_8}{}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8}\, AT}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8, \bot}\, AT \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Delta_9, []\mathtt{F}_6), \bot \vdash \Delta_8}\, \bot_L}{- : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6, []\mathtt{F}_6 \vdash \bot, \Delta_8}{}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : \bot, \Delta_9, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_8}{}\, \bot_L}{- : \Delta_9, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}\, AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\bot, \Delta_{10}), []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : (\bot, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9}\, AT \quad \dfrac{}{\bullet\mathtt{h}_7 : ((\bot, \Delta_{10}), []\mathtt{F}_6), \mathtt{F}_9 \vdash \Delta_8}\, \bot_L}{- : (\bot, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}\, \bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_6 \vdash \mathtt{p}_9, \Delta_8, \mathtt{p}_9}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash (\Delta_8, \mathtt{p}_9), \mathtt{p}_9}\, AT \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Delta_{10}, []\mathtt{F}_6), \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, I}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9, \mathtt{p}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, I}{- : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\,\mathtt{hCut}}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6 \vdash \mathtt{F}_{10}, \Delta_8, \mathtt{p}_9}{\bullet\mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6 \vdash (\Delta_8, \mathtt{p}_9), \mathtt{F}_{10}}\, AT \quad \dfrac{}{\bullet\mathtt{h}_7 : ((\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{p}_9}\, I}{- : (\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\,\mathtt{Cut}}{- : \Delta_{11}, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_9, []\mathtt{F}_6 \vdash \top, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8, \top}\, AT \quad \dfrac{\mathtt{h}_7 : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\Delta_9, []\mathtt{F}_6), \top \vdash \Delta_8}\,\top_L}{- : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}\,\mathtt{Cut}}{- : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\top, \Delta_{10}), []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : (\top, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : ((\top, \Delta_{10}), []\mathtt{F}_6), \mathtt{F}_9 \vdash \Delta_8}\,\top_L}{- : (\top, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6 \vdash \Delta_8}\,\mathtt{ax/W}}{- : \top, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}\,\mathtt{hCut}}$$

6.12 Status of $\bot_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_9}\,\bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 : (\bot, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\to_R}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{Cut}}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\bot_L$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9}\,\bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 : (\bot, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\wedge_R}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{Cut}}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\bot_L$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9}\,\bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 : (\bot, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\vee_R}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{Cut}}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\bot_L$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \bot, \Delta_6}\ \bot_R}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \bot_L$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \top, \Delta_6}\ \top_R}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule K

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_9), unbox(\Box\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \Delta_7, []\mathtt{F}_6}\ K}{- : \bot, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Box\Gamma_8, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_7, []\mathtt{F}_6}\ K}{- : \bot, \Box\Gamma_8, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_8 \vdash \Delta_7, []\mathtt{F}_6}\ \bot_L$$

- Case rule $A45$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_7, \Delta_8, []\mathtt{F}_6), \Box\mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_{10}, \Box\mathtt{F}_9 \vdash \Box\Gamma_7, \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \Box\Gamma_{10}, \Delta_{11}), \Box\mathtt{F}_9 \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ A45}{- : \bot, \Box\Gamma_{10}, \Delta_{11} \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Box\Gamma_9, \Delta_{11} \vdash (\Box\Gamma_7, \Delta_8, []\mathtt{F}_6), \mathtt{F}_{10}}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_9 \vdash \Box\Gamma_7, \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \Box\Gamma_9, \Delta_{11}), \mathtt{F}_{10} \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ A45}{- : \bot, \Box\Gamma_9, \Delta_{11} \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{11}, \Box\Gamma_9 \vdash \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \bot_L$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9 \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \rightarrow_L}{- : \bot, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \rightarrow_L}{- : \bot, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \bot_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \wedge_L}{- : \bot, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \wedge_L}{- : \bot, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \bot_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \Delta_9 \vdash \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \vee_L}{- : \bot, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \vee_L}{- : \bot, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \bot_L$$

- Case rule AT

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash \Delta_7, []\mathtt{F}_6}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \Delta_8, []\mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), []\mathtt{F}_6 \vdash \Delta_7}\ AT}{- : \bot, \Delta_8 \vdash \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \Delta_7}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7, \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_8, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, []\mathtt{F}_6), \mathtt{F}_8 \vdash \Delta_7}\ AT}{- : \bot, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}\ \bot_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \Delta_6}\ \bot_L}{- : \bot, \Delta_8 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \Delta_6}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ I}{- : \bot, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{F}_8}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, \mathtt{p}_7), \mathtt{F}_8 \vdash \Delta_6, \mathtt{p}_7}\ I}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ \bot_L$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_7 \vdash \Delta_6, \top}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_7 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \top \vdash \Delta_6}\ \top_L}{- : \bot, \Delta_7 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_7 \vdash \Delta_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\bot, \top, \Delta_8), \mathtt{F}_7 \vdash \Delta_6}\ \top_L}{- : \bot, \top, \Delta_8 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \bot, \top, \Delta_8 \vdash \Delta_6}\ \bot_L$$

6.13 Status of I: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash (\Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_{10}}\ I \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11}}\ I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_8, \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{12}, \mathtt{p}_{10}), \mathtt{F}_{11} \vdash (\Delta_9, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10}}\ \to_R}{- : \Delta_{12}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10}}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ I$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10}}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11}}\ I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_8, \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{12}, \mathtt{p}_{10}), \mathtt{F}_{11} \vdash (\Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}}\ \wedge_R}{- : \Delta_{12}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ I$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10}}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\ \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11}}\ I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{12}, \mathtt{p}_{10}), \mathtt{F}_{11} \vdash (\Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}}\ \vee_R}{- : \Delta_{12}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\ I$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_8 \vdash (\bot, \Delta_6), \mathtt{p}_8}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_8), \mathtt{p}_8 \vdash \bot, \Delta_6}\, \bot_R}{- : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_6, \mathtt{p}_8}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \bot, \Delta_6}\, \mathtt{ax/W}}{- : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_6}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_8 \vdash ((\bot, \Delta_7), \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_9, \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_8), \mathtt{F}_9 \vdash (\bot, \Delta_7), \mathtt{p}_8}\, \bot_R}{- : \Delta_{10}, \mathtt{p}_8 \vdash (\bot, \Delta_7), \mathtt{p}_8}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8 \vdash \bot, \Delta_7, \mathtt{p}_8}\, I$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_8 \vdash (\top, \Delta_6), \mathtt{p}_8}\, I \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_8), \mathtt{p}_8 \vdash \top, \Delta_6}\, \top_R}{- : \Delta_7, \mathtt{p}_8 \vdash \top, \Delta_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_7, \mathtt{p}_8 \vdash \top, \Delta_6}\, \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_8 \vdash ((\top, \Delta_7), \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_8), \mathtt{F}_9 \vdash (\top, \Delta_7), \mathtt{p}_8}\, \top_R}{- : \Delta_{10}, \mathtt{p}_8 \vdash (\top, \Delta_7), \mathtt{p}_8}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8 \vdash \top, \Delta_7, \mathtt{p}_8}\, \top_R$$

- Case rule K

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_8, \Delta_9), \mathtt{p}_{10} \vdash (\Delta_7, []\mathtt{F}_6), \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : ((\Box\Gamma_8, \Delta_9), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_7, []\mathtt{F}_6}\, K}{- : (\Box\Gamma_8, \Delta_9), \mathtt{p}_{10} \vdash \Delta_7, []\mathtt{F}_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}\, \mathtt{ax/W}}{- : \Delta_9, \Box\Gamma_8, \mathtt{p}_{10} \vdash \Delta_7, []\mathtt{F}_6}\, K$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{p}_9 \vdash ((\Delta_8, []\mathtt{F}_7), \mathtt{p}_9), \Box\mathtt{F}_{10}}\, I \quad \dfrac{\mathtt{h}_6 : unbox(\Box\Gamma_{11}), unbox(\Box\mathtt{F}_{10}) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : ((\Box\Gamma_{11}, \Delta_{12}), \mathtt{p}_9), \Box\mathtt{F}_{10} \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, K}{- : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{p}_9 \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{12}, \Box\Gamma_{11}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9, []\mathtt{F}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{p}_9 \vdash ((\Delta_8, []\mathtt{F}_7), \mathtt{p}_9), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : ((\Box\Gamma_{10}, \Delta_{12}), \mathtt{p}_9), \mathtt{F}_{11} \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, K}{- : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{p}_9 \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{12}, \Box\Gamma_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9, []\mathtt{F}_7}\, I$$

- Case rule $A45$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_9, \Delta_{10}), \mathtt{p}_{11} \vdash (\Box\Gamma_7, \Delta_8, []\mathtt{F}_6), \mathtt{p}_{11}}\, I \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_9 \vdash \Box\Gamma_7, \mathtt{F}_6}{\bullet \mathtt{h}_5 : ((\Box\Gamma_9, \Delta_{10}), \mathtt{p}_{11}), \mathtt{p}_{11} \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\, A45}{- : (\Box\Gamma_9, \Delta_{10}), \mathtt{p}_{11} \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_9 \vdash \mathtt{F}_6, \Box\Gamma_7}\, \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_9, \mathtt{p}_{11} \vdash \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\, A45$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{p}_{10} \vdash ((\Box\Gamma_8, \Delta_9, []\mathtt{F}_7), \mathtt{p}_{10}), \Box\mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_{12}, \Box\mathtt{F}_{11} \vdash \Box\Gamma_8, \mathtt{F}_7}{\bullet \mathtt{h}_6 : ((\Box\Gamma_{12}, \Delta_{13}), \mathtt{p}_{10}), \Box\mathtt{F}_{11} \vdash (\Box\Gamma_8, \Delta_9, []\mathtt{F}_7), \mathtt{p}_{10}}\, A45}{- : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{p}_{10} \vdash (\Box\Gamma_8, \Delta_9, []\mathtt{F}_7), \mathtt{p}_{10}}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{13}, \Box\Gamma_{12}, \mathtt{p}_{10} \vdash \Delta_9, \Box\Gamma_8, \mathtt{p}_{10}, []\mathtt{F}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{13}), \mathtt{p}_{10} \vdash ((\Box\Gamma_8, \Delta_9, []\mathtt{F}_7), \mathtt{p}_{10}), \mathtt{F}_{12}}\, I \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_{11} \vdash \Box\Gamma_8, \mathtt{F}_7}{\bullet \mathtt{h}_6 : ((\Box\Gamma_{11}, \Delta_{13}), \mathtt{p}_{10}), \mathtt{F}_{12} \vdash (\Box\Gamma_8, \Delta_9, []\mathtt{F}_7), \mathtt{p}_{10}}\, A45}{- : (\Box\Gamma_{11}, \Delta_{13}), \mathtt{p}_{10} \vdash (\Box\Gamma_8, \Delta_9, []\mathtt{F}_7), \mathtt{p}_{10}}\, \mathtt{Cut}}{- : \Delta_{13}, \Box\Gamma_{11}, \mathtt{p}_{10} \vdash \Delta_9, \Box\Gamma_8, \mathtt{p}_{10}, []\mathtt{F}_7}\, I}\, \to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8}\, \to_L}{- : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\, \to_L}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, \to_L}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I}\, \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : ((\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11} \vdash \Delta_9, \mathtt{p}_{10}}\, \to_L}{- : (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}}{- : \Delta_{12}, \mathtt{p}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, I}\, \to$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8}\, \wedge_L}{- : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\, \wedge_L}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, \wedge_L}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I}\, \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : ((\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11} \vdash \Delta_9, \mathtt{p}_{10}}\, \wedge_L}{- : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}}{- : \Delta_{12}, \mathtt{p}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, I}\, \to$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8}\, \vee_L}{- : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\, \vee_L}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, \vee_L}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : ((\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11} \vdash \Delta_9, \mathtt{p}_{10}}\, \vee_L}{- : (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, I}\, \to$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_8, []\mathtt{F}_6), \mathtt{p}_9 \vdash \Delta_7, \mathtt{p}_9}\, I \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_8, \mathtt{p}_9, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_5 : ((\Delta_8, []\mathtt{F}_6), \mathtt{p}_9), \mathtt{p}_9 \vdash \Delta_7}\, AT}{- : (\Delta_8, []\mathtt{F}_6), \mathtt{p}_9 \vdash \Delta_7}\, \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7, \mathtt{p}_9}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_8, \mathtt{F}_6, \mathtt{p}_9, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7}\, \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7}\, \mathtt{hCut}}{- : \Delta_8, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7}\, ATG}{\dfrac{}{- : \Delta_8, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7}\, I}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_9 \vdash (\Delta_8, \mathtt{p}_9), []\mathtt{F}_7}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \Delta_{10}, \mathtt{p}_9, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{p}_9}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_9), []\mathtt{F}_7 \vdash \Delta_8, \mathtt{p}_9}\, AT}{- : \Delta_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, I}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{11}, []\mathtt{F}_7), \mathtt{p}_9 \vdash (\Delta_8, \mathtt{p}_9), \mathtt{F}_{10}}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_{10}, \Delta_{11}, \mathtt{p}_9, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{p}_9}{\bullet \mathtt{h}_6 : ((\Delta_{11}, []\mathtt{F}_7), \mathtt{p}_9), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{p}_9}\, AT}{- : (\Delta_{11}, []\mathtt{F}_7), \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_9, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{p}_9}\, I}\, \to$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_7), \mathtt{p}_8 \vdash \Delta_6, \mathtt{p}_8}\, I \quad \dfrac{}{\bullet \mathtt{h}_5 : ((\bot, \Delta_7), \mathtt{p}_8), \mathtt{p}_8 \vdash \Delta_6}\, \bot_L}{- : (\bot, \Delta_7), \mathtt{p}_8 \vdash \Delta_6}\, \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_7, \mathtt{p}_8 \vdash \Delta_6}\, \bot_L}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \bot}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_8), \bot \vdash \Delta_7, \mathtt{p}_8}\, \bot_L}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, I}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_{10}), \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : ((\bot, \Delta_{10}), \mathtt{p}_8), \mathtt{F}_9 \vdash \Delta_7, \mathtt{p}_8}\, \bot_L}{- : (\bot, \Delta_{10}), \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \bot_L}\, \to$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}\, I \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_8, \mathtt{p}_7), \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\, I}{- : \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\, I}\, \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_8, \mathtt{p}_7), \mathtt{p}_9 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_5 : ((\Delta_8, \mathtt{p}_7), \mathtt{p}_9), \mathtt{p}_9 \vdash \Delta_6, \mathtt{p}_7}\, I}{- : (\Delta_8, \mathtt{p}_7), \mathtt{p}_9 \vdash \Delta_6, \mathtt{p}_7}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_8, \mathtt{p}_7, \mathtt{p}_9 \vdash \Delta_6, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_8 \vdash ((\Delta_7, \mathtt{p}_9), \mathtt{p}_8), \mathtt{p}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_8), \mathtt{p}_9 \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\, I}{- : \Delta_{10}, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8, \mathtt{p}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_9), \mathtt{p}_8 \vdash ((\Delta_7, \mathtt{p}_9), \mathtt{p}_8), \mathtt{F}_{10}}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : ((\Delta_{11}, \mathtt{p}_9), \mathtt{p}_8), \mathtt{F}_{10} \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\, I}{- : (\Delta_{11}, \mathtt{p}_9), \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_8, \mathtt{p}_9 \vdash \Delta_7, \mathtt{p}_8, \mathtt{p}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_8), \mathtt{F}_9 \vdash \Delta_7, \mathtt{p}_8}\, I}{- : \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\top, \Delta_7), \mathtt{p}_8 \vdash \Delta_6, \mathtt{p}_8}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : ((\top, \Delta_7), \mathtt{p}_8), \mathtt{p}_8 \vdash \Delta_6}\, \top_L}{- : (\top, \Delta_7), \mathtt{p}_8 \vdash \Delta_6}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_7, \mathtt{p}_8 \vdash \Delta_6, \mathtt{p}_8}\, I \quad \dfrac{}{\mathtt{h}_5 : \top, \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \Delta_6}\, \mathtt{ax/W}}{- : \top, \Delta_7, \mathtt{p}_8 \vdash \Delta_6}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \top}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_8), \top \vdash \Delta_7, \mathtt{p}_8}\, \top_L}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\top, \Delta_{10}), \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_9, \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}{\bullet \mathtt{h}_6 : ((\top, \Delta_{10}), \mathtt{p}_8), \mathtt{F}_9 \vdash \Delta_7, \mathtt{p}_8}\, \top_L}{- : (\top, \Delta_{10}), \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \top, \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, I$$

6.14 Status of $\top_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_9}\, \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\top, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\, \rightarrow_R}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\, \mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\, \mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9}\, \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\top, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\, \wedge_R}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\, \mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\, \mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \bot, \Delta_6}{\bullet\mathtt{h}_1 : \top, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet\mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \bot, \Delta_6}\ \bot_R}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_8 \vdash \bot, \Delta_6, \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \top, \Delta_6}{\bullet\mathtt{h}_1 : \top, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \top, \Delta_6}\ \top_R}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \top, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \top_L \quad \dfrac{\mathtt{h}_5 : \mathit{unbox}(\Box\Gamma_9), \mathit{unbox}(\Box\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\top, \Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \Delta_7, []\mathtt{F}_6}\ K}{- : \top, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Box\mathtt{F}_8, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash \mathtt{F}_9, \Delta_7, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \top, \Box\Gamma_8, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \mathit{unbox}(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\top, \Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_7, []\mathtt{F}_6}\ K}{- : \top, \Box\Gamma_8, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_8) \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_8 \vdash \Delta_7, []\mathtt{F}_6}\ K$$

- Case rule $A45$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash \Box\mathtt{F}_9, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \top, \Box\Gamma_{10}, \Delta_{11} \vdash (\Box\Gamma_7, \Delta_8, []\mathtt{F}_6), \Box\mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_{10}, \Box\mathtt{F}_9 \vdash \Box\Gamma_7, \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\top, \Box\Gamma_{10}, \Delta_{11}), \Box\mathtt{F}_9 \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ A45}{- : \top, \Box\Gamma_{10}, \Delta_{11} \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_{11}, \Box\Gamma_{10} \vdash \Box\mathtt{F}_9, \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Box\mathtt{F}_9, \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \mathtt{ax/W}}{- : \top, \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{11} \vdash \mathtt{F}_{10}, \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \top, \Box\Gamma_9, \Delta_{11} \vdash (\Box\Gamma_7, \Delta_8, []\mathtt{F}_6), \mathtt{F}_{10}}\ \top_L \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_9 \vdash \Box\Gamma_7, \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\top, \Box\Gamma_9, \Delta_{11}), \mathtt{F}_{10} \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ A45}{- : \top, \Box\Gamma_9, \Delta_{11} \vdash \Box\Gamma_7, \Delta_8, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_9 \vdash \mathtt{F}_6, \Box\Gamma_7}\ \mathtt{ax/W}}{- : \top, \Delta_{11}, \Box\Gamma_9 \vdash \Delta_8, \Box\Gamma_7, []\mathtt{F}_6}\ A45$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \rightarrow \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9 \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \rightarrow_L}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \rightarrow_L}{- : \top, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}{\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{hCut}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \wedge_L}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \wedge_L}{- : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}{\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{hCut}}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \vee \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \Delta_9 \vdash \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \vee_L}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \vee_L}{- : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}{\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{hCut}}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash []\mathtt{F}_6, \Delta_7}{\bullet\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_7, []\mathtt{F}_6}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \Delta_8, []\mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_5 : (\top, \Delta_8), []\mathtt{F}_6 \vdash \Delta_7}\ AT}{- : \top, \Delta_8 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_7, []\mathtt{F}_6}{\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_8, []\mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_5 : \top, \Delta_8, []\mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \top, \Delta_8 \vdash \Delta_7}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_7}{\bullet\mathtt{h}_1 : \top, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7, \mathtt{F}_8}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_8, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_5 : (\top, \Delta_9, []\mathtt{F}_6), \mathtt{F}_8 \vdash \Delta_7}\,AT}{- : \top, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}\,\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_8, []\mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}}{- : \top, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}\,\mathtt{hCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \bot, \Delta_6}{\bullet\mathtt{h}_1 : \top, \Delta_7 \vdash \Delta_6, \bot}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_7), \bot \vdash \Delta_6}\,\bot_L}{- : \top, \Delta_7 \vdash \Delta_6}\,\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_7 \vdash \bot, \Delta_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \bot, \top, \Delta_7 \vdash \Delta_6}\,\bot_L}{- : \top, \Delta_7 \vdash \Delta_6}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 : \top, \bot, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \bot, \Delta_8), \mathtt{F}_7 \vdash \Delta_6}\,\bot_L}{- : \top, \bot, \Delta_8 \vdash \Delta_6}\,\mathtt{Cut}}{\to}$$
$$\dfrac{}{- : \bot, \top, \Delta_8 \vdash \Delta_6}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{p}_7, \Delta_6, \mathtt{p}_7}{\bullet\mathtt{h}_1 : \top, \Delta_8 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_8), \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,I}{- : \top, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\,\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_6, \mathtt{p}_7, \mathtt{p}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,I}{- : \top, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{p}_7 \vdash \mathtt{F}_8, \Delta_6, \mathtt{p}_7}{\bullet\mathtt{h}_1 : \top, \Delta_9, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{F}_8}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_9, \mathtt{p}_7), \mathtt{F}_8 \vdash \Delta_6, \mathtt{p}_7}\,I}{- : \top, \Delta_9, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,\mathtt{Cut}}{\to}$$
$$\dfrac{}{- : \top, \Delta_9, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet\mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \Delta_6}\,\top_L}{- : \top, \Delta_8 \vdash \Delta_6}\,\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_8, \mathtt{F}_7 \vdash \Delta_6}\,\mathtt{ax/W}}{- : \top, \Delta_8 \vdash \Delta_6}\,\mathtt{hCut}$$