

# Modal Logic K

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## Contents

<b>1</b>	<b>Height preserving admissibility of weakening on the left</b>	<b>2</b>
<b>2</b>	<b>Height preserving admissibility of weakening on the right</b>	<b>4</b>
<b>3</b>	<b>Measure of derivations</b>	<b>6</b>
<b>4</b>	<b>Invertibility of Rules</b>	<b>8</b>
4.1	Status of $\rightarrow_R$ : : Invertible . . . . .	8
4.2	Status of $\wedge_R$ : (Left Premise): Invertible . . . . .	9
4.3	Status of $\wedge_R$ (Right Premise): : Invertible . . . . .	10
4.4	Status of $\vee_R$ : : Invertible . . . . .	12
4.5	Status of $\perp_R$ : : Invertible . . . . .	13
4.6	Status of $\top_R$ : : Invertible . . . . .	14
4.7	Status of $K$ : : Non invertible . . . . .	16
4.8	Status of $\rightarrow_L$ : (Left Premise): Invertible . . . . .	17
4.9	Status of $\rightarrow_L$ (Right Premise): : Invertible . . . . .	18
4.10	Status of $\wedge_L$ : : Invertible . . . . .	20
4.11	Status of $\vee_L$ : (Left Premise): Invertible . . . . .	21
4.12	Status of $\vee_L$ (Right Premise): : Invertible . . . . .	22
4.13	Status of $\perp_L$ : : Invertible . . . . .	24
4.14	Status of $I$ : : Invertible . . . . .	25
4.15	Status of $\top_L$ : : Invertible . . . . .	26
<b>5</b>	<b>Height preserving admissibility of contraction on the left</b>	<b>28</b>
<b>6</b>	<b>Height preserving admissibility of contraction on the Right</b>	<b>30</b>
<b>7</b>	<b>Identity-Expansion</b>	<b>32</b>
<b>8</b>	<b>Cut-Elimination</b>	<b>33</b>
8.1	Status of $\rightarrow_R$ : OK . . . . .	33
8.2	Status of $\wedge_R$ : OK . . . . .	38
8.3	Status of $\vee_R$ : OK . . . . .	42
8.4	Status of $\perp_R$ : OK . . . . .	47
8.5	Status of $\top_R$ : OK . . . . .	51
8.6	Status of $K$ : OK . . . . .	55
8.7	Status of $\rightarrow_L$ : OK . . . . .	59
8.8	Status of $\wedge_L$ : OK . . . . .	62
8.9	Status of $\vee_L$ : OK . . . . .	66

8.10 Status of $\perp_L$ : OK . . . . .	69
8.11 Status of $I$ : OK . . . . .	71
8.12 Status of $\top_L$ : OK . . . . .	75

## 1 Height preserving admissibility of weakening on the left

- Case(s) rule  $\rightarrow_R$

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightsquigarrow \frac{\frac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_5} \text{ax} \quad \frac{\overline{h_1 : \Delta_2, F_4, F_W \vdash \Delta_3, F_5}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \rightarrow F_5} \text{IH}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightsquigarrow \frac{\frac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_4}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4} \text{ax} \quad \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_5}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_5} \text{ax}}{\frac{\overline{h_1 : \Delta_2, F_W \vdash \Delta_3, F_4} \text{IH} \quad \frac{\overline{h_1 : \Delta_2, F_W \vdash \Delta_3, F_5} \text{IH}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \wedge F_5} \wedge_R} \wedge_R$$

- Case(s) rule  $\vee_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightsquigarrow \frac{\frac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4, F_5} \text{ax} \quad \frac{\overline{h_1 : \Delta_2, F_W \vdash \Delta_3, F_4, F_5}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \vee F_5} \text{IH}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \vee F_5} \vee_R$$

- Case(s) rule  $\perp_R$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightsquigarrow \frac{\frac{\overline{h_1 : \Delta_2 \vdash \Delta_3}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3} \text{ax} \quad \frac{\overline{h_1 : \Delta_2, F_W \vdash \Delta_3}}{\bullet h_1 : \Delta_2, F_W \vdash \perp, \Delta_3} \text{IH}}{\bullet h_1 : \Delta_2, F_W \vdash \perp, \Delta_3} \perp_R$$

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightsquigarrow \frac{}{\bullet h_1 : \Delta_2, F_W \vdash \top, \Delta_3} \top_R$$

- Case(s) rule  $K$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Delta_4, [] F_5} K \rightsquigarrow \frac{\overline{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5} \text{ax}}{\bullet h_1 : \Delta_3, F_W, \Box \Gamma_2 \vdash \Delta_4, [] F_5} K$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightsquigarrow \frac{\frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3} \text{ax} \quad \frac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5}}{\bullet h_1 : \Delta_2, F_4, F_W \vdash \Delta_5} \text{ax}}{\frac{\overline{h_1 : \Delta_2, F_W \vdash \Delta_5, F_3} \text{IH} \quad \frac{\overline{h_1 : \Delta_2, F_4, F_W \vdash \Delta_5} \text{IH}}{\bullet h_1 : \Delta_2, F_W, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightsquigarrow \frac{\frac{\overline{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}}{\bullet h_1 : \Delta_2, F_3, F_4, F_W \vdash \Delta_5} \text{ax} \quad \frac{\overline{h_1 : \Delta_2, F_3, F_4, F_W \vdash \Delta_5}}{\bullet h_1 : \Delta_2, F_W, F_3 \wedge F_4 \vdash \Delta_5} \text{IH}}{\bullet h_1 : \Delta_2, F_W, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightsquigarrow \frac{\frac{\overline{h_1 : \Delta_2, F_3 \vdash \Delta_5}}{\bullet h_1 : \Delta_2, F_3, F_W \vdash \Delta_5} \text{ax} \quad \frac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5}}{\bullet h_1 : \Delta_2, F_4, F_W \vdash \Delta_5} \text{ax}}{\frac{\overline{h_1 : \Delta_2, F_3, F_W \vdash \Delta_5} \text{IH} \quad \frac{\overline{h_1 : \Delta_2, F_4, F_W \vdash \Delta_5} \text{IH}}{\bullet h_1 : \Delta_2, F_W, F_3 \vee F_4 \vdash \Delta_5} \vee_L} \vee_L$$

- Case(s) rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \quad \rightsquigarrow \quad \frac{}{\bullet h_1 : \perp, \Delta_2, F_W \vdash \Delta_3} \perp_L$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I \quad \rightsquigarrow \quad \frac{}{\bullet h_1 : \Delta_2, F_W, p_3 \vdash \Delta_4, p_3} I$$

- Case(s) rule  $\top_L$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \quad \rightsquigarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{h_1 : \Delta_2, F_W \vdash \Delta_3} \text{IH}}{\bullet h_1 : \top, \Delta_2, F_W \vdash \Delta_3} \top_L$$

## 2 Height preserving admissibility of weakening on the right

- Case(s) rule  $\rightarrow_R$

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightsquigarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \rightarrow F_5} \text{ IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \rightarrow F_5} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightsquigarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5}{h_1 : \Delta_2 \vdash \Delta_3, F_5, F_W} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_W \quad h_1 : \Delta_2 \vdash \Delta_3, F_5, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \wedge F_5} \text{ IH}} \wedge_R$$

- Case(s) rule  $\vee_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightsquigarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \vee F_5} \text{ IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \vee F_5} \vee_R$$

- Case(s) rule  $\perp_R$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightsquigarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{h_1 : \Delta_2 \vdash \Delta_3, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_W}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \text{ IH}}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \perp_R$$

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightsquigarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, F_W} \top_R$$

- Case(s) rule  $K$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Delta_4, [] F_5} K \rightsquigarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_4, F_W, [] F_5} \text{ ax}}{\bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_4, F_W, [] F_5} K$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightsquigarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{h_1 : \Delta_2, F_4 \vdash \Delta_5, F_W} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_W \quad h_1 : \Delta_2, F_4 \vdash \Delta_5, F_W}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5, F_W} \text{ IH}} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightsquigarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5, F_W}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5, F_W} \text{ IH}}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5, F_W} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightsquigarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{h_1 : \Delta_2, F_4 \vdash \Delta_5, F_W} \text{ ax}}{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_W \quad h_1 : \Delta_2, F_4 \vdash \Delta_5, F_W}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5, F_W} \text{ IH}} \vee_L$$

- Case(s) rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \rightsquigarrow \frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3, F_W} \perp_L$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, P_3} I \quad \rightsquigarrow \quad \frac{}{\bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, F_W, P_3} I$$

- Case(s) rule  $\top_L$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \quad \rightsquigarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{h_1 : \Delta_2 \vdash \Delta_3, F_W} \text{IH}}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3, F_W} \top_L$$

### 3 Measure of derivations

- Case(s) rule  $\rightarrow_R$

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightsquigarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \text{ IH} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightsquigarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \text{ IH} \wedge_R$$

- Case(s) rule  $\vee_R$

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightsquigarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \text{ IH} \vee_R$$

- Case(s) rule  $\perp_R$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightsquigarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{ ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \text{ IH} \perp_R$$

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightsquigarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case(s) rule  $K$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash \Delta_4, [\Gamma_5]} K \rightsquigarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5}{\bullet h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_4, [\Gamma_5]} \text{ IH} K$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightsquigarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \text{ IH} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightsquigarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \text{ IH} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightsquigarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \text{ IH} \vee_L$$

- Case(s) rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \rightsquigarrow \frac{}{\bullet \bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I \quad \rightsquigarrow \quad \frac{}{\bullet \bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I$$

- Case(s) rule  $\top_L$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \quad \rightsquigarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{IH}}{\bullet \bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L$$

## 4 Invertibility of Rules

### 4.1 Status of $\rightarrow_R$ : : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \rightarrow F_6} \rightarrow_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash \Delta_7, F_2, F_6}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \text{ax/ind} \rightarrow_R$$

$$\frac{h_1 : F_4, \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightsquigarrow \frac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5} \text{ax} \text{H}$$

- Case rule  $\wedge_R$

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \rightarrow F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \wedge F_6} \wedge_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5} \text{ax/ind} \quad \overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule  $\vee_R$

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \vee F_6} \vee_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \rightarrow F_2} \perp_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_5, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \rightarrow F_2} \top_R \rightsquigarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule  $K$

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \rightarrow F_2), \Box F_6} K \rightsquigarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ax}}{\bullet h_3 : \Delta_5, F_1, \Box \Gamma_4 \vdash \Delta_7, F_2, \Box F_6} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \rightarrow F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \rightarrow F_3} \rightarrow_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5, F_2 \vdash \Delta_1, F_3, F_6} \text{ax/ind} \quad \overline{h_4 : \Delta_5, F_2, F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_2, F_6 \rightarrow F_7 \vdash \Delta_1, F_3} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \rightarrow F_3} \wedge_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5, F_2, F_6, F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_2, F_6 \wedge F_7 \vdash \Delta_1, F_3} \wedge_L$$



- Case rule  $\vee_L$

$$\frac{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \rightarrow F_3} \vee_L \rightsquigarrow \frac{\frac{h_4 : \Delta_5, F_2, F_6 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, F_2, F_6 \vee F_7 \vdash \Delta_1, F_3} \text{ax/ind} \quad \frac{h_4 : \Delta_5, F_2, F_7 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, F_2, F_6 \vee F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_2, F_6 \vee F_7 \vdash \Delta_1, F_3} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3} \perp_L \rightsquigarrow \frac{}{\bullet h_4 : \perp, \Delta_5, F_2 \vdash \Delta_1, F_3} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \rightarrow F_2} I \rightsquigarrow \frac{}{\bullet h_3 : \Delta_4, F_1, p_5 \vdash \Delta_6, F_2, p_5} I$$

- Case rule  $\top_L$

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3} \top_L \rightsquigarrow \frac{\frac{h_4 : \Delta_5, F_2 \vdash \Delta_1, F_3}{\bullet h_4 : \top, \Delta_5, F_2 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \top, \Delta_5, F_2 \vdash \Delta_1, F_3} \top_L$$

## 4.2 Status of $\wedge_R$ : (Left Premise): Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightsquigarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \rightarrow F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightsquigarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightsquigarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4} H$$

- Case rule  $\vee_R$

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightsquigarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightsquigarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1} \perp_R$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightsquigarrow \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1} \top_R$$

- Case rule  $K$

$$\frac{h_3 : unbox(\Box\Gamma_4) \vdash F_6}{\bullet h_3 : \Box\Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \wedge F_2), []F_6} K \rightsquigarrow \frac{\overline{h_3 : unbox(\Box\Gamma_4) \vdash F_6} \text{ ax}}{\bullet h_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_7, F_1, []F_6} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \wedge F_3} \rightarrow_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_6} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3} \wedge_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3} \vee_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2} \vee_L$$

- Case rule  $\perp_L$

$$\overline{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \perp_L \rightsquigarrow \overline{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2} \perp_L$$

- Case rule  $I$

$$\overline{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \wedge F_2} I \rightsquigarrow \overline{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_1, p_5} I$$

- Case rule  $\top_L$

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \top_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2} \top_L$$

### 4.3 Status of $\wedge_R$ (Right Premise): : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash F_4, \Delta_3 \quad h_1 : \Delta_2 \vdash F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightsquigarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_5} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5} H$$

- Case rule  $\vee_R$

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule  $\top_R$

$$\overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightsquigarrow \overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule  $K$

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \wedge F_2), \Box F_6} K \rightsquigarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ ax}}{\bullet h_3 : \Delta_5, \Box \Gamma_4 \vdash \Delta_7, F_2, \Box F_6} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \wedge F_3} \rightarrow_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_3, F_6} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_3} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3} \wedge_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_3} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3} \vee_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_3} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_3} \vee_L$$

- Case rule  $\perp_L$

$$\overline{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \perp_L \rightsquigarrow \overline{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_3} \perp_L$$

- Case rule  $I$

$$\overline{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \wedge F_2} I \rightsquigarrow \overline{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_2, p_5} I$$

- Case rule  $\top_L$

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \top_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_3} \text{ ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_3} \top_L$$

#### 4.4 Status of $\vee_R$ : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \rightarrow F_6} \rightarrow_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_2, F_6}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \text{ax/ind} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \vee F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \wedge F_6} \wedge_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5} \text{ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule  $\vee_R$

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \vee F_6} \vee_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \vee F_6} \vee_R$$

$$\frac{h_1 : \Delta_2 \vdash F_4, F_5, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightsquigarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5} \text{H}$$

- Case rule  $\perp_R$

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \vee F_2} \perp_R \rightsquigarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_1, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1, F_2} \perp_R$$

- Case rule  $\top_R$

$$\overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \vee F_2} \top_R \rightsquigarrow \overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1, F_2} \top_R$$

- Case rule  $K$

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, F_1 \vee F_2), [\Box F_6]} K \rightsquigarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ax}}{\bullet h_3 : \Delta_5, \Box \Gamma_4 \vdash \Delta_7, F_1, F_2, [\Box F_6]} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_4 : \Delta_5 \vdash F_6, \Delta_1, F_2 \vee F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \vee F_3} \rightarrow_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_3, F_6} \text{ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2, F_3} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_4 : F_6, F_7, \Delta_5 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \vee F_3} \wedge_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2, F_3} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_4 : F_6, \Delta_5 \vdash \Delta_1, F_2 \vee F_3 \quad h_4 : F_7, \Delta_5 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \vee F_3} \vee_L \rightsquigarrow \frac{\overline{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2, F_3} \text{ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2, F_3} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \vee F_3} \perp_L \rightsquigarrow \frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2, F_3} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \vee F_2} I \rightsquigarrow \frac{}{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_1, F_2, p_5} I$$

- Case rule  $\top_L$

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \vee F_3} \top_L \rightsquigarrow \frac{\frac{}{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_3} \text{ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2, F_3} \top_L$$

#### 4.5 Status of $\perp_R$ : : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_1 : F_3, \Delta_2 \vdash \perp, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \rightsquigarrow \frac{\frac{}{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash \perp, F_3, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3} \wedge_R \quad \frac{h_1 : \Delta_2 \vdash \perp, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \wedge F_4} \wedge_R}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \wedge F_4} \wedge_R \rightsquigarrow \frac{\frac{\frac{}{h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \wedge_R \quad \frac{\frac{}{h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_4} \wedge_R}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R$$

- Case rule  $\vee_R$

$$\frac{h_1 : \Delta_2 \vdash \perp, F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \vee F_4} \vee_R \rightsquigarrow \frac{\frac{\frac{}{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \vee_R}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightsquigarrow \frac{\frac{}{h_1 : \Delta_2 \vdash \Delta_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{H}$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \perp, \Delta_3} \top_R \rightsquigarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case rule  $K$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash (\perp, \Delta_5), [\Box F_4]} K \rightsquigarrow \frac{\frac{}{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_5, [\Box F_4]} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_2 : \Delta_3 \vdash \perp, F_4, \Delta_1 \quad h_2 : F_5, \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \perp, \Delta_1} \rightarrow_L \quad \rightsquigarrow \quad \frac{\overline{h_2 : \Delta_3 \vdash \Delta_1, F_4} \quad \text{ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash \Delta_1} \quad \text{ax/ind}}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \Delta_1} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_2 : F_4, F_5, \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \perp, \Delta_1} \wedge_L \quad \rightsquigarrow \quad \frac{\overline{h_2 : \Delta_3, F_4, F_5 \vdash \Delta_1} \quad \text{ax/ind}}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \Delta_1} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_2 : F_4, \Delta_3 \vdash \perp, \Delta_1 \quad h_2 : F_5, \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \perp, \Delta_1} \vee_L \quad \rightsquigarrow \quad \frac{\overline{h_2 : \Delta_3, F_4 \vdash \Delta_1} \quad \text{ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash \Delta_1} \quad \text{ax/ind}}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \Delta_1} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_2 : \perp, \Delta_3 \vdash \perp, \Delta_1} \perp_L \quad \rightsquigarrow \quad \frac{}{\bullet h_2 : \perp, \Delta_3 \vdash \Delta_1} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3, \perp, \Delta_4} I \quad \rightsquigarrow \quad \frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I$$

- Case rule  $\top_L$

$$\frac{h_2 : \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \top, \Delta_3 \vdash \perp, \Delta_1} \top_L \quad \rightsquigarrow \quad \frac{\overline{h_2 : \Delta_3 \vdash \Delta_1} \quad \text{ax/ind}}{\bullet h_2 : \top, \Delta_3 \vdash \Delta_1} \top_L$$

## 4.6 Status of $\top_R$ : : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_1 : F_3, \Delta_2 \vdash \top, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash \top, F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash \top, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \wedge F_4} \wedge_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\vee_R$

$$\frac{h_1 : \Delta_2 \vdash \top, F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \vee F_4} \vee_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\perp_R$

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \top, \Delta_3} \perp_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $K$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4}{\bullet h_1 : \Box \Gamma_2, \Delta_3 \vdash (\top, \Delta_5), [\Box F_4]} K \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\rightarrow_L$

$$\frac{h_2 : \Delta_3 \vdash \top, F_4, \Delta_1 \quad h_2 : F_5, \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \top, \Delta_1} \rightarrow_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\wedge_L$

$$\frac{h_2 : F_4, F_5, \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \top, \Delta_1} \wedge_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\vee_L$

$$\frac{h_2 : F_4, \Delta_3 \vdash \top, \Delta_1 \quad h_2 : F_5, \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \top, \Delta_1} \vee_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_2 : \perp, \Delta_3 \vdash \top, \Delta_1} \perp_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $I$

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3, \top, \Delta_4} I \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\top_L$

$$\frac{h_2 : \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \top, \Delta_3 \vdash \top, \Delta_1} \top_L \quad \rightsquigarrow \quad \text{trivial}$$

## 4.7 Status of $K$ : : Non invertible

- Case rule  $\rightarrow_R$

$$\frac{h_4 : \Box \Gamma_1, F_5, \Delta_2 \vdash F_6, \Delta_7, []F_3}{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash (\Delta_7, []F_3), F_5 \rightarrow F_6} \rightarrow_R \rightsquigarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule  $\wedge_R$

$$\frac{h_4 : \Box \Gamma_1, \Delta_2 \vdash F_5, \Delta_7, []F_3 \quad h_4 : \Box \Gamma_1, \Delta_2 \vdash F_6, \Delta_7, []F_3}{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash (\Delta_7, []F_3), F_5 \wedge F_6} \wedge_R \rightsquigarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule  $\vee_R$

$$\frac{h_4 : \Box \Gamma_1, \Delta_2 \vdash F_5, F_6, \Delta_7, []F_3}{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash (\Delta_7, []F_3), F_5 \vee F_6} \vee_R \rightsquigarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule  $\perp_R$

$$\frac{h_4 : \Box \Gamma_1, \Delta_2 \vdash \Delta_5, []F_3}{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash \perp, \Delta_5, []F_3} \perp_R \rightsquigarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_4 : \Box \Gamma_1, \Delta_2 \vdash \top, \Delta_5, []F_3} \top_R \rightsquigarrow \frac{}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{fail}$$

- Case rule  $K$

$$\frac{h_2 : unbox(\Box \Gamma_5), unbox(\Box \Gamma_6) \vdash F_3}{\bullet h_2 : (\Box \Gamma_5, \Box \Gamma_6), \Box \Gamma_7, \Delta_8 \vdash (\Delta_4, []F_1), []F_3} K \rightsquigarrow \frac{}{\bullet h_2 : unbox(\Box \Gamma_5), unbox(\Box \Gamma_7) \vdash F_1} \text{fail}$$

$$\frac{h_1 : unbox(\Box \Gamma_4), unbox(\Box \Gamma_5) \vdash F_3}{\bullet h_1 : (\Box \Gamma_4, \Box \Gamma_5), \Box \Gamma_6, \Delta_7 \vdash \Delta_2, []F_3} K \rightsquigarrow \frac{}{\bullet h_1 : unbox(\Box \Gamma_4), unbox(\Box \Gamma_6) \vdash F_3} \text{fail}$$

- Case rule  $\rightarrow_L$

$$\frac{h_4 : \Box \Gamma_1, \Delta_7 \vdash F_5, \Delta_2, []F_3 \quad h_4 : \Box \Gamma_1, F_6, \Delta_7 \vdash \Delta_2, []F_3}{\bullet h_4 : (\Box \Gamma_1, \Delta_7), F_5 \rightarrow F_6 \vdash \Delta_2, []F_3} \rightarrow_L \rightsquigarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule  $\wedge_L$

$$\frac{h_4 : \Box \Gamma_1, F_5, F_6, \Delta_7 \vdash \Delta_2, []F_3}{\bullet h_4 : (\Box \Gamma_1, \Delta_7), F_5 \wedge F_6 \vdash \Delta_2, []F_3} \wedge_L \rightsquigarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$

- Case rule  $\vee_L$

$$\frac{h_4 : \Box \Gamma_1, F_5, \Delta_7 \vdash \Delta_2, []F_3 \quad h_4 : \Box \Gamma_1, F_6, \Delta_7 \vdash \Delta_2, []F_3}{\bullet h_4 : (\Box \Gamma_1, \Delta_7), F_5 \vee F_6 \vdash \Delta_2, []F_3} \vee_L \rightsquigarrow \frac{\overline{h_4 : unbox(\Box \Gamma_1) \vdash F_3}}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}_H$$



- Case rule  $\perp_L$

$$\frac{}{\bullet h_4 : \perp, \Box \Gamma_1, \Delta_5 \vdash \Delta_2, [] F_3} \perp_L \rightsquigarrow \frac{}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{fail}$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_4, \Box \Gamma_1, \Delta_6 \vdash p_4, \Delta_5, [] F_2} I \rightsquigarrow \frac{}{\bullet h_3 : unbox(\Box \Gamma_1) \vdash F_2} \text{fail}$$

- Case rule  $\top_L$

$$\frac{h_4 : \Box \Gamma_1, \Delta_5 \vdash \Delta_2, [] F_3}{\bullet h_4 : \top, \Box \Gamma_1, \Delta_5 \vdash \Delta_2, [] F_3} \top_L \rightsquigarrow \frac{h_4 : unbox(\Box \Gamma_1) \vdash F_3}{\bullet h_4 : unbox(\Box \Gamma_1) \vdash F_3} \text{ax/ind}^H$$

#### 4.8 Status of $\rightarrow_L$ : (Left Premise): Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_4 : F_6, \Delta_1, F_2 \rightarrow F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightsquigarrow \frac{h_4 : \Delta_1, F_6 \vdash \Delta_5, F_2, F_7}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \rightarrow F_7} \text{ax/ind} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightsquigarrow \frac{\frac{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6} \text{ax/ind} \quad \frac{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_7}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \wedge F_7} \wedge_R$$

- Case rule  $\vee_R$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightsquigarrow \frac{\frac{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6, F_7}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \vee F_7} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \perp, \Delta_5} \perp_R \rightsquigarrow \frac{h_4 : \Delta_1 \vdash \Delta_5, F_2}{\bullet h_4 : \Delta_1 \vdash \perp, \Delta_5, F_2} \text{ax/ind} \perp_R$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \top, \Delta_5} \top_R \rightsquigarrow \frac{}{\bullet h_4 : \Delta_1 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule  $K$

$$\frac{h_3 : unbox(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, [] F_6} K \rightsquigarrow \frac{h_3 : unbox(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Delta_7, \Box \Gamma_4 \vdash \Delta_5, F_1, [] F_6} \text{ax} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \rightarrow_L$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \quad \rightsquigarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{H}$$

- Case rule  $\wedge_L$

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \rightarrow F_2 \vdash p_4, \Delta_5} I \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \Delta_6, p_4 \vdash \Delta_5, F_1, p_4} I$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \top_L$$

## 4.9 Status of $\rightarrow_L$ (Right Premise): : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_4 : F_6, \Delta_1, F_2 \rightarrow F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \quad \rightsquigarrow \quad \frac{\frac{h_4 : \Delta_1, F_3, F_6 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \quad \rightsquigarrow \quad \frac{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind} \quad \frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule  $\vee_R$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \quad \rightsquigarrow \quad \frac{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \perp, \Delta_5} \perp_R \quad \rightsquigarrow \quad \frac{\overline{h_4 : \Delta_1, F_3 \vdash \Delta_5} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \perp, \Delta_5} \perp_R$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \top, \Delta_5} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top, \Delta_5} \top_R$$

- Case rule  $K$

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, []F_6} K \quad \rightsquigarrow \quad \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ ax}}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_4 \vdash \Delta_5, []F_6} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \quad \rightsquigarrow \quad \frac{\overline{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \quad \rightsquigarrow \quad \frac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} H$$

- Case rule  $\wedge_L$

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \quad \rightsquigarrow \quad \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \quad \rightsquigarrow \quad \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \rightarrow F_2 \vdash p_4, \Delta_5} I \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \Delta_6, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \quad \rightsquigarrow \quad \frac{\overline{h_3 : \Delta_5, F_2 \vdash \Delta_4} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

#### 4.10 Status of $\wedge_L$ : : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_4 : F_6, \Delta_1, F_2 \wedge F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightsquigarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3, F_6 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightsquigarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind} \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule  $\vee_R$

$$\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightsquigarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \vee F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \perp, \Delta_5} \perp_R \rightsquigarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \perp, \Delta_5} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \perp, \Delta_5} \perp_R$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \top, \Delta_5} \top_R \rightsquigarrow \frac{}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \top, \Delta_5} \top_R$$

- Case rule  $K$

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \wedge F_2 \vdash \Delta_5, [] F_6} K \rightsquigarrow \frac{\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, \Box \Gamma_4 \vdash \Delta_5, [] F_6} \text{ax}}{\bullet h_3 : \Delta_7, F_1, F_2, \Box \Gamma_4 \vdash \Delta_5, [] F_6} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightsquigarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightsquigarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

$$\frac{h_1 : F_3, F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightsquigarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5} \text{ax}}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5} \text{H}$$

- Case rule  $\vee_L$

$$\frac{h_3 : F_4, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightsquigarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \perp_L \rightsquigarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash \Delta_4} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \wedge F_2 \vdash p_4, \Delta_5} I \rightsquigarrow \frac{}{\bullet h_3 : \Delta_6, F_1, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \top_L \rightsquigarrow \frac{h_3 : \Delta_5, F_1, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash \Delta_4} \top_L \text{ ax/ind}$$

#### 4.11 Status of $\vee_L$ : (Left Premise): Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_4 : F_6, \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightsquigarrow \frac{h_4 : \Delta_1, F_2, F_6 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \text{ ax/ind}$$

- Case rule  $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightsquigarrow \frac{h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \text{ ax/ind} \quad h_4 : \Delta_1, F_2 \vdash \Delta_5, F_7 \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule  $\vee_R$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightsquigarrow \frac{h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6, F_7 \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \vee F_7} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \perp, \Delta_5} \perp_R \rightsquigarrow \frac{h_4 : \Delta_1, F_2 \vdash \Delta_5 \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \perp, \Delta_5} \perp_R$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \top, \Delta_5} \top_R \rightsquigarrow \frac{}{\bullet h_4 : \Delta_1, F_2 \vdash \top, \Delta_5} \top_R$$

- Case rule  $K$

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, [\Gamma_6]} K \rightsquigarrow \frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6 \text{ ax}}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_4 \vdash \Delta_5, [\Gamma_6]} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightsquigarrow \frac{\frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightsquigarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightsquigarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightsquigarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \text{ ax}}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightsquigarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \vee F_2 \vdash p_4, \Delta_5} I \rightsquigarrow \frac{}{\bullet h_3 : \Delta_6, F_1, p_4 \vdash \Delta_5, p_4} I$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightsquigarrow \frac{\frac{h_3 : \Delta_5, F_1 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \top_L$$

## 4.12 Status of $\vee_L$ (Right Premise): : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_4 : F_6, \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightsquigarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, \Delta_5 \quad h_4 : \Delta_1, F_2 \vee F_3 \vdash F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightsquigarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ ax/ind} \quad \frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule  $\vee_R$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6, F_7, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightsquigarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \perp, \Delta_5} \perp_R \rightsquigarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash \Delta_5} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \perp, \Delta_5} \perp_R$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \top, \Delta_5} \top_R \rightsquigarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top, \Delta_5} \top_R$$

- Case rule  $K$

$$\frac{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6}{\bullet h_3 : \Box \Gamma_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, []F_6} K \rightsquigarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_4) \vdash F_6} \text{ ax}}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_4 \vdash \Delta_5, []F_6} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightsquigarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightsquigarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightsquigarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{h_1 : F_3, \Delta_2 \vdash \Delta_5 \quad h_1 : F_4, \Delta_2 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightsquigarrow \frac{\overline{h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} H$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightsquigarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \vee F_2 \vdash p_4, \Delta_5} I \rightsquigarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightsquigarrow \frac{\overline{h_3 : \Delta_5, F_2 \vdash \Delta_4} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

### 4.13 Status of $\perp_L$ : : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_2 : \perp, F_4, \Delta_1 \vdash F_5, \Delta_3}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\wedge_R$

$$\frac{h_2 : \perp, \Delta_1 \vdash F_4, \Delta_3 \quad h_2 : \perp, \Delta_1 \vdash F_5, \Delta_3}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\vee_R$

$$\frac{h_2 : \perp, \Delta_1 \vdash F_4, F_5, \Delta_3}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \vee F_5} \vee_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\perp_R$

$$\frac{h_2 : \perp, \Delta_1 \vdash \Delta_3}{\bullet h_2 : \perp, \Delta_1 \vdash \perp, \Delta_3} \perp_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_2 : \perp, \Delta_1 \vdash \top, \Delta_3} \top_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $K$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4}{\bullet h_1 : \Box \Gamma_2, \perp, \Delta_5 \vdash \Delta_3, [\Gamma_4]} K \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\rightarrow_L$

$$\frac{h_1 : \perp, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \perp, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\wedge_L$

$$\frac{h_1 : \perp, F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\vee_L$

$$\frac{h_1 : \perp, F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : \perp, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \quad \rightsquigarrow \quad \text{trivial}$$



- Case rule  $I$

$$\frac{}{\bullet h_1 : p_2, \perp, \Delta_4 \vdash p_2, \Delta_3} I \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\top_L$

$$\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \perp, \Delta_3 \vdash \Delta_2} \top_L \quad \rightsquigarrow \quad \text{trivial}$$

#### 4.14 Status of $I$ : : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_3 : F_4, \Delta_1, p_2 \vdash F_5, \Delta_6, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash (\Delta_6, p_2), F_4 \rightarrow F_5} \rightarrow_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\wedge_R$

$$\frac{h_3 : \Delta_1, p_2 \vdash F_4, \Delta_6, p_2 \quad h_3 : \Delta_1, p_2 \vdash F_5, \Delta_6, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash (\Delta_6, p_2), F_4 \wedge F_5} \wedge_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\vee_R$

$$\frac{h_3 : \Delta_1, p_2 \vdash F_4, F_5, \Delta_6, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash (\Delta_6, p_2), F_4 \vee F_5} \vee_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\perp_R$

$$\frac{h_3 : \Delta_1, p_2 \vdash \Delta_4, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash \perp, \Delta_4, p_2} \perp_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_1, p_2 \vdash \top, \Delta_4, p_2} \top_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $K$

$$\frac{h_2 : \text{unbox}(\Box \Gamma_3) \vdash F_4}{\bullet h_2 : \Box \Gamma_3, \Delta_6, p_1 \vdash (\Delta_5, p_1), \Box F_4} K \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_6, p_1 \vdash F_4, \Delta_2, p_1 \quad h_3 : F_5, \Delta_6, p_1 \vdash \Delta_2, p_1}{\bullet h_3 : (\Delta_6, p_1), F_4 \rightarrow F_5 \vdash \Delta_2, p_1} \rightarrow_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\wedge_L$

$$\frac{h_3 : F_4, F_5, \Delta_6, p_1 \vdash \Delta_2, p_1}{\bullet h_3 : (\Delta_6, p_1), F_4 \wedge F_5 \vdash \Delta_2, p_1} \wedge_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule  $\vee_L$

$$\frac{h_3 : F_4, \Delta_6, P_1 \vdash \Delta_2, P_1 \quad h_3 : F_5, \Delta_6, P_1 \vdash \Delta_2, P_1}{\bullet h_3 : (\Delta_6, P_1), F_4 \vee F_5 \vdash \Delta_2, P_1} \vee_L \rightsquigarrow \text{trivial}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4, P_1 \vdash \Delta_2, P_1} \perp_L \rightsquigarrow \text{trivial}$$

- Case rule  $I$

$$\frac{}{\bullet h_2 : P_3, \Delta_5, P_1 \vdash P_3, \Delta_4, P_1} I \rightsquigarrow \text{trivial}$$

$$\frac{}{\bullet h_1 : P_3, \Delta_2 \vdash P_3, \Delta_4} I \rightsquigarrow \text{trivial}$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_4, P_1 \vdash \Delta_2, P_1}{\bullet h_3 : \top, \Delta_4, P_1 \vdash \Delta_2, P_1} \top_L \rightsquigarrow \text{trivial}$$

#### 4.15 Status of $\top_L$ : : Invertible

- Case rule  $\rightarrow_R$

$$\frac{h_2 : \top, F_4, \Delta_1 \vdash F_5, \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightsquigarrow \frac{\frac{}{h_2 : \Delta_1, F_4 \vdash \Delta_3, F_5} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_2 : \top, \Delta_1 \vdash F_4, \Delta_3 \quad h_2 : \top, \Delta_1 \vdash F_5, \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightsquigarrow \frac{\frac{}{h_2 : \Delta_1 \vdash \Delta_3, F_4} \text{ax/ind} \quad \frac{}{h_2 : \Delta_1 \vdash \Delta_3, F_5} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R$$

- Case rule  $\vee_R$

$$\frac{h_2 : \top, \Delta_1 \vdash F_4, F_5, \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightsquigarrow \frac{\frac{}{h_2 : \Delta_1 \vdash \Delta_3, F_4, F_5} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \vee F_5} \vee_R$$

- Case rule  $\perp_R$

$$\frac{h_2 : \top, \Delta_1 \vdash \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \perp, \Delta_3} \perp_R \rightsquigarrow \frac{\frac{}{h_2 : \Delta_1 \vdash \Delta_3} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash \perp, \Delta_3} \perp_R$$

- Case rule  $\top_R$

$$\frac{}{\bullet h_2 : \top, \Delta_1 \vdash \top, \Delta_3} \top_R \rightsquigarrow \frac{}{\bullet h_2 : \Delta_1 \vdash \top, \Delta_3} \top_R$$

- Case rule  $K$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4}{\bullet h_1 : \Box \Gamma_2, \top, \Delta_5 \vdash \Delta_3, []F_4} K \rightsquigarrow \frac{\overline{h_1 : \text{unbox}(\Box \Gamma_2) \vdash F_4}^{\text{ax}}}{\bullet h_1 : \Delta_5, \Box \Gamma_2 \vdash \Delta_3, []F_4} K$$

- Case rule  $\rightarrow_L$

$$\frac{h_1 : \top, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \top, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightsquigarrow \frac{\overline{h_1 : \Delta_5 \vdash \Delta_4, F_2}^{\text{ax/ind}} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4}^{\text{ax/ind}}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_1 : \top, F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightsquigarrow \frac{\overline{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}^{\text{ax/ind}}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_1 : \top, F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : \top, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightsquigarrow \frac{\overline{h_1 : \Delta_5, F_2 \vdash \Delta_4}^{\text{ax/ind}} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4}^{\text{ax/ind}}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L$$

- Case rule  $\perp_L$

$$\overline{\bullet h_1 : \perp, \top, \Delta_3 \vdash \Delta_2}^{\perp_L} \rightsquigarrow \overline{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2}^{\perp_L}$$

- Case rule  $I$

$$\overline{\bullet h_1 : p_2, \top, \Delta_4 \vdash p_2, \Delta_3}^I \rightsquigarrow \overline{\bullet h_1 : \Delta_4, p_2 \vdash \Delta_3, p_2}^I$$

- Case rule  $\top_L$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \rightsquigarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3}^{\text{ax}}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{H}$$

## 5 Height preserving admissibility of contraction on the left

- Case(s) rule  $\rightarrow_R$

$$\frac{h_3 : F_5, \Delta_1, \Delta_2, \Delta_2 \vdash F_6, \Delta_4}{\bullet h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, F_5 \rightarrow F_6} \rightarrow_R \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_1, \Delta_2, \Delta_2, F_5 \vdash \Delta_4, F_6}{\bullet h_3 : \Delta_1, \Delta_2, F_5 \vdash \Delta_4, F_6} \text{IH} \quad \frac{}{h_3 : \Delta_1, \Delta_2, \Delta_2, F_5 \vdash \Delta_4, F_6} \text{ax}}{\bullet h_3 : \Delta_1, \Delta_2 \vdash \Delta_4, F_5 \rightarrow F_6} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash F_5, \Delta_4 \quad h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash F_6, \Delta_4}{\bullet h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, F_5 \wedge F_6} \wedge_R \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, F_5}{h_3 : \Delta_1, \Delta_2 \vdash \Delta_4, F_5} \text{IH} \quad \frac{\frac{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, F_6}{h_3 : \Delta_1, \Delta_2 \vdash \Delta_4, F_6} \text{IH} \quad \frac{}{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, F_5 \wedge F_6} \text{ax}}{\bullet h_3 : \Delta_1, \Delta_2 \vdash \Delta_4, F_5 \wedge F_6} \wedge_R$$

- Case(s) rule  $\vee_R$

$$\frac{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash F_5, F_6, \Delta_4}{\bullet h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, F_5 \vee F_6} \vee_R \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, F_5, F_6}{h_3 : \Delta_1, \Delta_2 \vdash \Delta_4, F_5 \vee F_6} \vee_R \quad \frac{}{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, F_5 \vee F_6} \text{ax}}{\bullet h_3 : \Delta_1, \Delta_2 \vdash \Delta_4, F_5 \vee F_6} \vee_R$$

- Case(s) rule  $\perp_R$

$$\frac{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4}{\bullet h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \perp, \Delta_4} \perp_R \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4}{h_3 : \Delta_1, \Delta_2 \vdash \Delta_4} \text{IH} \quad \frac{}{h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4} \text{ax}}{\bullet h_3 : \Delta_1, \Delta_2 \vdash \perp, \Delta_4} \perp_R$$

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \top, \Delta_4} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \Delta_1, \Delta_2 \vdash \top, \Delta_4} \top_R$$

- Case(s) rule  $K$

$$\frac{h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_6) \vdash F_3}{\bullet h_1 : (\Box\Gamma_4, \Delta_7), (\Box\Gamma_5, \Box\Gamma_6, \Delta_8), \Box\Gamma_5, \Box\Gamma_6, \Delta_8 \vdash \Delta_2, [\Box F_3]} K \quad \rightsquigarrow \quad \frac{\frac{h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_6) \vdash F_3}{h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_6) \vdash F_3} K \quad \frac{}{h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_6) \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_7, \Delta_8, \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_6 \vdash \Delta_2, [\Box F_3]} K$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_2 : \Delta_1, \Delta_6, \Delta_6, F_3 \rightarrow F_4 \vdash F_3, \Delta_5 \quad h_2 : F_4, \Delta_1, \Delta_6, \Delta_6, F_3 \rightarrow F_4 \vdash \Delta_5}{\bullet h_2 : \Delta_1, (\Delta_6, F_3 \rightarrow F_4), \Delta_6, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \quad \rightsquigarrow \quad \frac{\frac{h_2 : \Delta_1, \Delta_6, \Delta_6 \vdash \Delta_5, F_3, F_3}{h_2 : \Delta_1, \Delta_6 \vdash \Delta_5, F_3, F_3} \text{IH} \quad \frac{h_2 : \Delta_1, \Delta_6, \Delta_6, F_4, F_4 \vdash \Delta_5}{h_2 : \Delta_1, \Delta_6, F_4 \vdash \Delta_5} \text{inv-th/ax}}{\frac{h_2 : \Delta_1, \Delta_6 \vdash \Delta_5, F_3}{\bullet h_2 : \Delta_1, \Delta_6, F_3 \rightarrow F_4 \vdash \Delta_5} \text{IH-Mutual} \quad \frac{h_2 : \Delta_1, \Delta_6, F_4 \vdash \Delta_5}{h_2 : \Delta_1, \Delta_6, F_3 \rightarrow F_4 \vdash \Delta_5} \text{IH}}{\bullet h_2 : \Delta_1, \Delta_6, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L$$

$$\frac{h_2 : \Delta_1, \Delta_1, \Delta_6 \vdash F_3, \Delta_5 \quad h_2 : F_4, \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, F_3 \rightarrow F_4), \Delta_1, \Delta_1 \vdash \Delta_5} \rightarrow_L \quad \rightsquigarrow \quad \frac{\frac{h_2 : \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5, F_3}{h_2 : \Delta_1, \Delta_6 \vdash \Delta_5, F_3} \text{IH} \quad \frac{h_2 : \Delta_1, \Delta_1, \Delta_6, F_4 \vdash \Delta_5}{h_2 : \Delta_1, \Delta_6, F_4 \vdash \Delta_5} \text{ax}}{\bullet h_2 : \Delta_1, \Delta_6, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_2 : F_3, F_4, \Delta_1, \Delta_6, \Delta_6, F_3 \wedge F_4 \vdash \Delta_5}{\bullet h_2 : \Delta_1, (\Delta_6, F_3 \wedge F_4), \Delta_6, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \quad \rightsquigarrow \quad \frac{\frac{h_2 : \Delta_1, \Delta_6, \Delta_6, F_3, F_3, F_4, F_4 \vdash \Delta_5}{h_2 : \Delta_1, \Delta_6, F_3, F_4 \vdash \Delta_5} \text{inv-th/ax} \quad \frac{}{h_2 : \Delta_1, \Delta_6, F_3, F_4 \vdash \Delta_5} \text{IH}}{\bullet h_2 : \Delta_1, \Delta_6, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L$$

$$\frac{h_2 : F_3, F_4, \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, F_3 \wedge F_4), \Delta_1, \Delta_1 \vdash \Delta_5} \wedge_L \quad \rightsquigarrow \quad \frac{\frac{h_2 : \Delta_1, \Delta_1, \Delta_6, F_3, F_4 \vdash \Delta_5}{h_2 : \Delta_1, \Delta_6, F_3, F_4 \vdash \Delta_5} \text{ax} \quad \frac{}{h_2 : \Delta_1, \Delta_6, F_3, F_4 \vdash \Delta_5} \text{IH}}{\bullet h_2 : \Delta_1, \Delta_6, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\begin{array}{c}
\frac{h_2 : F_3, \Delta_1, \Delta_6, \Delta_6, F_3 \vee F_4 \vdash \Delta_5 \quad h_2 : F_4, \Delta_1, \Delta_6, \Delta_6, F_3 \vee F_4 \vdash \Delta_5}{\bullet h_2 : \Delta_1, (\Delta_6, F_3 \vee F_4), \Delta_6, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightsquigarrow \frac{\frac{h_2 : \Delta_1, \Delta_6, \Delta_6, F_3, F_3 \vdash \Delta_5}{h_2 : \Delta_1, \Delta_6, F_3 \vdash \Delta_5} \text{inv-th/ax} \quad \frac{h_2 : \Delta_1, \Delta_6, \Delta_6, F_4, F_4 \vdash \Delta_5}{h_2 : \Delta_1, \Delta_6, F_4 \vdash \Delta_5} \text{inv-th/ax}}{\bullet h_2 : \Delta_1, \Delta_6, F_3 \vee F_4 \vdash \Delta_5} \text{IH} \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_3, \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5 \quad h_2 : F_4, \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, F_3 \vee F_4), \Delta_1, \Delta_1 \vdash \Delta_5} \vee_L \rightsquigarrow \frac{\frac{h_2 : \Delta_1, \Delta_1, \Delta_6, F_3 \vdash \Delta_5}{h_2 : \Delta_1, \Delta_6, F_3 \vdash \Delta_5} \text{ax} \quad \frac{h_2 : \Delta_1, \Delta_1, \Delta_6, F_4 \vdash \Delta_5}{h_2 : \Delta_1, \Delta_6, F_4 \vdash \Delta_5} \text{ax}}{\bullet h_2 : \Delta_1, \Delta_6, F_3 \vee F_4 \vdash \Delta_5} \text{IH} \vee_L
\end{array}$$

- Case(s) rule  $\perp_L$

$$\begin{array}{c}
\frac{}{\bullet h_2 : (\perp, \Delta_4), \Delta_1, \Delta_1 \vdash \Delta_3} \perp_L \rightsquigarrow \frac{}{\bullet h_2 : \perp, \Delta_1, \Delta_4 \vdash \Delta_3} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_2 : \Delta_1, (\perp, \Delta_4), \perp, \Delta_4 \vdash \Delta_3} \perp_L \rightsquigarrow \frac{}{\bullet h_2 : \perp, \Delta_1, \Delta_4 \vdash \Delta_3} \perp_L
\end{array}$$

- Case(s) rule  $I$

$$\begin{array}{c}
\frac{}{\bullet h_2 : (\Delta_5, p_3), \Delta_1, \Delta_1 \vdash \Delta_4, p_3} I \rightsquigarrow \frac{}{\bullet h_2 : \Delta_1, \Delta_5, p_3 \vdash \Delta_4, p_3} I
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_2 : \Delta_1, (\Delta_5, p_3), \Delta_5, p_3 \vdash \Delta_4, p_3} I \rightsquigarrow \frac{}{\bullet h_2 : \Delta_1, \Delta_5, p_3 \vdash \Delta_4, p_3} I
\end{array}$$

- Case(s) rule  $\top_L$

$$\begin{array}{c}
\frac{h_2 : \Delta_1, \Delta_1, \Delta_4 \vdash \Delta_3}{\bullet h_2 : (\top, \Delta_4), \Delta_1, \Delta_1 \vdash \Delta_3} \top_L \rightsquigarrow \frac{\frac{h_2 : \Delta_1, \Delta_1, \Delta_4 \vdash \Delta_3}{h_2 : \Delta_1, \Delta_4 \vdash \Delta_3} \text{ax}}{\bullet h_2 : \top, \Delta_1, \Delta_4 \vdash \Delta_3} \text{IH} \top_L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \top, \Delta_1, \Delta_4, \Delta_4 \vdash \Delta_3}{\bullet h_2 : \Delta_1, (\top, \Delta_4), \top, \Delta_4 \vdash \Delta_3} \top_L \rightsquigarrow \frac{\frac{h_2 : \Delta_1, \Delta_4, \Delta_4 \vdash \Delta_3}{h_2 : \Delta_1, \Delta_4 \vdash \Delta_3} \text{inv-th/ax}}{\bullet h_2 : \top, \Delta_1, \Delta_4 \vdash \Delta_3} \text{IH} \top_L
\end{array}$$

## 6 Height preserving admissibility of contraction on the Right

- Case(s) rule  $\rightarrow_R$

$$\begin{array}{c}
 \frac{h_2 : F_4, \Delta_3 \vdash F_5, \Delta_1, \Delta_6, \Delta_6, F_4 \rightarrow F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, (\Delta_6, F_4 \rightarrow F_5), \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \quad \rightsquigarrow \quad \frac{\frac{h_2 : \Delta_3, F_4, F_4 \vdash \Delta_1, \Delta_6, \Delta_6, F_5, F_5}{h_2 : \Delta_3, F_4 \vdash \Delta_1, \Delta_6, \Delta_6, F_5, F_5} \text{IH-Mutual} \quad \frac{h_2 : \Delta_3, F_4 \vdash \Delta_1, \Delta_6, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, F_4 \rightarrow F_5} \rightarrow_R}{\frac{h_2 : \Delta_3, F_4 \vdash \Delta_1, \Delta_1, \Delta_6}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \rightarrow F_5), \Delta_1, \Delta_1} \rightarrow_R} \text{IH}
 \end{array}$$

- Case(s) rule  $\wedge_R$

$$\begin{array}{c}
 \frac{h_2 : \Delta_3 \vdash F_4, \Delta_1, \Delta_6, \Delta_6, F_4 \wedge F_5 \quad h_2 : \Delta_3 \vdash F_5, \Delta_1, \Delta_6, \Delta_6, F_4 \wedge F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, (\Delta_6, F_4 \wedge F_5), \Delta_6, F_4 \wedge F_5} \wedge_R \quad \rightsquigarrow \quad \frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \Delta_6, F_4, F_4}{h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, F_4} \text{IH} \quad \frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \Delta_6, F_5, F_5}{h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, F_5} \text{IH}}{\bullet h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, F_4 \wedge F_5} \wedge_R}{\frac{h_2 : \Delta_3 \vdash F_4, \Delta_1, \Delta_1, \Delta_6 \quad h_2 : \Delta_3 \vdash F_5, \Delta_1, \Delta_1, \Delta_6}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \wedge F_5), \Delta_1, \Delta_1} \wedge_R} \text{ax}
 \end{array}$$

- Case(s) rule  $\vee_R$

$$\begin{array}{c}
 \frac{h_2 : \Delta_3 \vdash F_4, F_5, \Delta_1, \Delta_6, \Delta_6, F_4 \vee F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, (\Delta_6, F_4 \vee F_5), \Delta_6, F_4 \vee F_5} \vee_R \quad \rightsquigarrow \quad \frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \Delta_6, F_4, F_4, F_5, F_5}{h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, F_4, F_5} \text{IH} \quad \frac{h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, F_4, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, \Delta_6, F_4 \vee F_5} \vee_R}{\frac{h_2 : \Delta_3 \vdash F_4, F_5, \Delta_1, \Delta_1, \Delta_6}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \vee F_5), \Delta_1, \Delta_1} \vee_R} \text{ax}
 \end{array}$$

- Case(s) rule  $\perp_R$

$$\begin{array}{c}
 \frac{h_2 : \Delta_3 \vdash \Delta_1, \Delta_1, \Delta_4}{\bullet h_2 : \Delta_3 \vdash (\perp, \Delta_4), \Delta_1, \Delta_1} \perp_R \quad \rightsquigarrow \quad \frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, \Delta_1, \Delta_4}{h_2 : \Delta_3 \vdash \Delta_1, \Delta_4} \text{IH} \quad \frac{h_2 : \Delta_3 \vdash \perp, \Delta_1, \Delta_4}{\bullet h_2 : \Delta_3 \vdash \perp, \Delta_1, \Delta_4} \perp_R}{\frac{h_2 : \Delta_3 \vdash \perp, \Delta_1, \Delta_4, \Delta_4}{\bullet h_2 : \Delta_3 \vdash \Delta_1, (\perp, \Delta_4), \perp, \Delta_4} \perp_R} \text{inv-th/ax}
 \end{array}$$

- Case(s) rule  $\top_R$

$$\begin{array}{c}
 \frac{}{\bullet h_2 : \Delta_3 \vdash (\top, \Delta_4), \Delta_1, \Delta_1} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet h_2 : \Delta_3 \vdash \top, \Delta_1, \Delta_4} \top_R \\
 \frac{}{\bullet h_2 : \Delta_3 \vdash \Delta_1, (\top, \Delta_4), \top, \Delta_4} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet h_2 : \Delta_3 \vdash \top, \Delta_1, \Delta_4} \top_R
 \end{array}$$

- Case(s) rule  $K$

$$\begin{array}{c}
 \frac{h_2 : \text{unbox}(\Box \Gamma_3) \vdash F_5}{\bullet h_2 : \Box \Gamma_3, \Delta_4 \vdash \Delta_1, (\Delta_6, [\Box F_5]), \Delta_6, [\Box F_5]} K \quad \rightsquigarrow \quad \frac{\frac{h_2 : \text{unbox}(\Box \Gamma_3) \vdash F_5}{\bullet h_2 : \Delta_4, \Box \Gamma_3 \vdash \Delta_1, \Delta_6, [\Box F_5]} \text{ax}}{\bullet h_2 : \Delta_4, \Box \Gamma_3 \vdash \Delta_1, \Delta_6, [\Box F_5]} K \\
 \frac{h_2 : \text{unbox}(\Box \Gamma_3) \vdash F_5}{\bullet h_2 : \Box \Gamma_3, \Delta_4 \vdash (\Delta_6, [\Box F_5]), \Delta_1, \Delta_1} K \quad \rightsquigarrow \quad \frac{\frac{h_2 : \text{unbox}(\Box \Gamma_3) \vdash F_5}{\bullet h_2 : \Delta_4, \Box \Gamma_3 \vdash \Delta_1, \Delta_6, [\Box F_5]} \text{ax}}{\bullet h_2 : \Delta_4, \Box \Gamma_3 \vdash \Delta_1, \Delta_6, [\Box F_5]} K
 \end{array}$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_1, \Delta_2, \Delta_2 \quad h_3 : F_6, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, \Delta_2, \Delta_2} \rightarrow_L \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2, F_5}{h_3 : \Delta_4 \vdash \Delta_1, \Delta_2, F_5} \text{ax} \quad \frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, \Delta_2, \Delta_2}{h_3 : \Delta_4, F_6 \vdash \Delta_1, \Delta_2} \text{IH}}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, \Delta_2} \rightarrow_L \text{IH}$$

- Case(s) rule  $\wedge_L$

$$\frac{h_3 : F_5, F_6, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, \Delta_2, \Delta_2} \wedge_L \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, \Delta_2, \Delta_2}{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, \Delta_2} \text{ax}}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, \Delta_2} \wedge_L \text{IH}$$

- Case(s) rule  $\vee_L$

$$\frac{h_3 : F_5, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2 \quad h_3 : F_6, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, \Delta_2, \Delta_2} \vee_L \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_1, \Delta_2, \Delta_2}{h_3 : \Delta_4, F_5 \vdash \Delta_1, \Delta_2} \text{ax} \quad \frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, \Delta_2, \Delta_2}{h_3 : \Delta_4, F_6 \vdash \Delta_1, \Delta_2} \text{ax}}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, \Delta_2} \vee_L \text{IH}$$

- Case(s) rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2} \perp_L \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash \Delta_1, \Delta_2} \perp_L$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_2 : \Delta_3, p_4 \vdash \Delta_1, (\Delta_5, p_4), \Delta_5, p_4} I \quad \rightsquigarrow \quad \frac{}{\bullet h_2 : \Delta_3, p_4 \vdash \Delta_1, \Delta_5, p_4} I$$

$$\frac{}{\bullet h_2 : \Delta_3, p_4 \vdash (\Delta_5, p_4), \Delta_1, \Delta_1} I \quad \rightsquigarrow \quad \frac{}{\bullet h_2 : \Delta_3, p_4 \vdash \Delta_1, \Delta_5, p_4} I$$

- Case(s) rule  $\top_L$

$$\frac{h_3 : \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}{\bullet h_3 : \top, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2} \top_L \quad \rightsquigarrow \quad \frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}{h_3 : \Delta_4 \vdash \Delta_1, \Delta_2} \text{ax}}{\bullet h_3 : \top, \Delta_4 \vdash \Delta_1, \Delta_2} \top_L \text{IH}$$

## 7 Identity-Expansion

$$\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : \Box F_0 \vdash \Box F_0} K$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0, F_1} W}{\frac{- : F_0 \vee F_1 \vdash F_0, F_1}{- : F_0 \vee F_1 \vdash F_0 \vee F_1} \vee_R} \vee_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_1 \vdash F_0} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{\frac{- : F_0, F_1 \vdash F_0 \wedge F_1}{- : F_0 \wedge F_1 \vdash F_0 \wedge F_1} \wedge_L} \wedge_R$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{\frac{- : F_0, F_0 \rightarrow F_1 \vdash F_1}{- : F_0 \rightarrow F_1 \vdash F_0 \rightarrow F_1} \rightarrow_R} \rightarrow_L$$

$$\overline{- : \top \vdash \top} \top_R$$

$$\overline{- : \perp \vdash \perp} \perp_L$$



## 8 Cut-Elimination

### 8.1 Status of $\rightarrow_R$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \rightarrow F_{12} \quad \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \rightarrow F_8}{\rightarrow_R} \quad \frac{h_9 : F_{11}, \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}, \Delta_{10} \quad \bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\rightarrow_R} \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{h_1 : \Delta_6, F_{11}, F_7 \vdash \Delta_{10}, F_{12}, F_8}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \rightarrow F_8} \xrightarrow{\text{inv-th/ax}} \xrightarrow{\rightarrow_R} \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}}{\text{ax/W}}}{\xrightarrow{\rightarrow_R} \quad \frac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}} \quad \text{hCut} \\
\hline
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \rightarrow F_{13} \quad \bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \rightarrow F_{10}), F_7}{\rightarrow_R} \quad \frac{h_{11} : F_7, F_{12}, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \rightarrow F_{10} \quad \bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \rightarrow F_{10}}{\rightarrow_R} \\
\hline
- : \Delta_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{h_2 : \Delta_8, F_{12}, F_9 \vdash \Delta_{14}, F_{10}, F_{13}, F_7}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9 \rightarrow F_{10}} \xrightarrow{\text{inv-th/ax}} \xrightarrow{\rightarrow_R} \quad \frac{h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \rightarrow F_{10}}{\text{ax/W}}}{\xrightarrow{\rightarrow_R} \quad \frac{- : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \rightarrow F_{10}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \rightarrow F_{10}}} \quad \text{hCut} \\
\hline
\frac{h_2 : F_{11}, \Delta_8 \vdash F_7, F_{12}, \Delta_{10} \quad \bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7}{\rightarrow_R} \quad \frac{h_9 : F_7, F_{11}, \Delta_8 \vdash F_{12}, \Delta_{10} \quad \bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\rightarrow_R} \\
\hline
- : \Delta_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{h_2 : \Delta_8, F_{11} \vdash \Delta_{10}, F_{12}, F_7}{\text{ax/W}} \quad \frac{h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{10}, F_{12}} \xrightarrow{\text{ax/W}} \xrightarrow{\text{H}} \xrightarrow{\text{hCut}}}{\xrightarrow{\rightarrow_R} \quad \frac{- : \Delta_8, F_{11} \vdash \Delta_{10}, F_{12}}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \wedge F_{12} \quad \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \rightarrow F_8}{\rightarrow_R} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \quad \wedge_R \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \rightarrow F_8} \xrightarrow{\text{inv-th/ax}} \xrightarrow{\rightarrow_R} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}}{\text{ax/W}}}{\xrightarrow{\rightarrow_R} \quad \frac{- : \Delta_6 \vdash \Delta_{10}, F_{11}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}}} \quad \text{hCut} \\
\hline
\frac{\frac{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{12}, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7 \rightarrow F_8} \xrightarrow{\text{inv-th/ax}} \xrightarrow{\rightarrow_R} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}}{\text{ax/W}}}{\xrightarrow{\rightarrow_R} \quad \frac{- : \Delta_6 \vdash \Delta_{10}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}}} \quad \text{hCut} \\
\hline
\wedge_R \\
\hline
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \wedge F_{13} \quad \bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \rightarrow F_{10}), F_7}{\rightarrow_R} \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \quad h_{11} : F_7, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \rightarrow F_{10}} \quad \wedge_R \\
\hline
- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{h_2 : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_7, F_{12} \wedge F_{13}}{\text{ax/W}} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12}}{\bullet h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \wedge F_{13}} \xrightarrow{\text{inv-th/ax}} \xrightarrow{\rightarrow_R} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{13}}{\text{inv-th/ax}}}{\xrightarrow{\rightarrow_R} \quad \frac{- : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \wedge F_{13}}{- : \Delta_8 \vdash \Delta_{14}, F_9 \rightarrow F_{10}, F_{12} \wedge F_{13}}} \quad \text{hCut}
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12} \quad \text{Cut} \\
\hline
\frac{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \rightarrow F_8} \xrightarrow{\text{inv-th/ax}} \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W} \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12} \quad \text{hCut} \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12} \quad \vee_R \\
\hline
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \rightarrow F_{10}} \vee_R \\
\hline
- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_7 \rightarrow F_{10}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}}{\bullet h_{11} : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \vee F_{13}} \text{inv-th/ax} \\
\hline
- : \Delta_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \vee F_{13} \quad \text{hCut} \\
\hline
- : \Delta_8 \vdash \Delta_{14}, F_9 \rightarrow F_{10}, F_{12} \vee F_{13} \quad \rightarrow_R \\
\hline
- : \Delta_8 \vdash \Delta_{14}, F_9 \rightarrow F_{10}, F_{12} \vee F_{13}
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_6 \vdash F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10} \quad \text{Cut} \\
\hline
\frac{h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}} \text{ax/W} \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10} \quad \text{hCut} \\
\hline
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \perp, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\perp, \Delta_{12}), F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\perp, \Delta_{12}), F_9 \rightarrow F_{10}} \perp_R \\
\hline
- : \Delta_8 \vdash (\perp, \Delta_{12}), F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \perp, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\perp, \Delta_{12}), F_9 \rightarrow F_{10}), F_7} \text{ax/W} \quad \frac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\perp, \Delta_{12}), F_9 \rightarrow F_{10}} \text{ax/W} \\
\hline
- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10} \quad \text{hCut} \\
\hline
- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10}
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_6 \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10} \quad \text{Cut} \\
\hline
\frac{h_1 : F_7, \Delta_6 \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8} \xrightarrow{\text{inv-th/ax}} \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}} \text{ax/W} \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10} \quad \text{hCut} \\
\hline
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \top, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\top, \Delta_{12}), F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \rightarrow F_{10}} \top_R \\
\hline
- : \Delta_8 \vdash (\top, \Delta_{12}), F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\frac{h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \top, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\top, \Delta_{12}), F_9 \rightarrow F_{10}), F_7} \text{ax/W} \quad \frac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \rightarrow F_{10}} \text{ax/W} \\
\hline
- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \rightarrow F_{10} \quad \text{hCut} \\
\hline
- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \rightarrow F_{10}
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{h_1 : F_6, \Box \Gamma_9, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_{11}}{\bullet h_1 : \Box \Gamma_9, \Delta_{12} \vdash (\Delta_{10}, \Box F_{11}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_9) \vdash F_{11}}{\bullet h_8 : (\Box \Gamma_9, \Delta_{12}), F_6 \rightarrow F_7 \vdash \Delta_{10}, \Box F_{11}} K \\
\hline
- : \Box \Gamma_9, \Delta_{12} \vdash \Delta_{10}, \Box F_{11} \quad \text{Cut} \\
\hline
\frac{h_1 : F_6, \Box \Gamma_9, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_{11}}{\bullet h_1 : \Box \Gamma_9, \Delta_{12} \vdash (\Delta_{10}, \Box F_{11}), F_6 \rightarrow F_7} \xrightarrow{\text{inv-th/ax}} \frac{h_8 : \text{unbox}(\Box \Gamma_9) \vdash F_{11}}{\bullet h_8 : (\Box \Gamma_9, \Delta_{12}), F_6 \rightarrow F_7 \vdash \Delta_{10}, \Box F_{11}} \text{ax/W} \\
\hline
- : \text{unbox}(\Box \Gamma_9) \vdash F_{11} \quad \text{hCut} \\
\hline
- : \Delta_{12}, \Box \Gamma_9 \vdash \Delta_{10}, \Box F_{11} \quad K \\
\hline
- : \Delta_{12}, \Box \Gamma_9 \vdash \Delta_{10}, \Box F_{11}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_8, \Box \Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_9, \Delta_{13}, \llbracket F_{12} \rrbracket \rightarrow_R \frac{h_{10} : \text{unbox}(\Box \Gamma_{14}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : (\Box \Gamma_{14}, \Delta_{11}), \Box F_7 \vdash (\Delta_{13}, \llbracket F_{12} \rrbracket), F_8 \rightarrow F_9} K}{\bullet h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, \llbracket F_{12} \rrbracket), F_8 \rightarrow F_9), \Box F_7} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : F_8, \Box \Gamma_{14}, \Delta_{14} \vdash F_7, F_9, \Delta_{13}, \llbracket F_{12} \rrbracket}{\bullet h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, \llbracket F_{12} \rrbracket), F_8 \rightarrow F_9), \Box F_7} \text{ax/W} \quad \frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{14}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, F_8, \Box \Gamma_{14} \vdash \Delta_{13}, F_9, \llbracket F_{12} \rrbracket} \text{ax/W}}{\frac{- : \Delta_{11}, F_8, \Box \Gamma_{14} \vdash \Delta_{13}, F_9, \llbracket F_{12} \rrbracket}{- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, \llbracket F_{12} \rrbracket, F_8 \rightarrow F_9} \text{hCut}} \rightarrow_R \\
\frac{h_2 : F_8, \Box \Gamma_{11}, \Delta_{14} \vdash F_7, F_9, \Delta_{13}, \llbracket F_{12} \rrbracket \rightarrow_R \frac{h_{10} : \text{unbox}(\Box \Gamma_{11}) \vdash F_{12}}{\bullet h_{10} : (\Box \Gamma_{11}, \Delta_{14}), F_7 \vdash (\Delta_{13}, \llbracket F_{12} \rrbracket), F_8 \rightarrow F_9} K}{\bullet h_2 : \Box \Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, \llbracket F_{12} \rrbracket), F_8 \rightarrow F_9), F_7} \text{Cut} \\
\sim \\
\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_{12} \text{ ax/W}}{- : \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, \llbracket F_{12} \rrbracket, F_8 \rightarrow F_9} K
\end{array}$$

• Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_7, \Delta_{11} \rightarrow_R \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \rightarrow_L}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{Cut} \\
\sim \\
\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}{\bullet h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W}}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{hCut}} \rightarrow_R \\
\frac{h_1 : F_8, \Delta_7 \vdash F_9, \Delta_{10} \rightarrow_R \frac{h_6 : \Delta_7 \vdash F_8, \Delta_{10} \quad h_6 : F_9, \Delta_7 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{10}} \rightarrow_L}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \rightarrow F_9} \text{Cut} \\
\sim \\
\frac{- : \Delta_7 \vdash \Delta_{10}, F_8 \text{ ax/W} \quad \frac{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9}{- : \Delta_7, F_8 \vdash \Delta_{10}} \text{ax/W} \quad \frac{- : \Delta_7, F_8, F_9 \vdash \Delta_{10}}{- : \Delta_7, F_8 \vdash \Delta_{10}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}} \text{sCut} \\
\frac{h_2 : F_8, \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_9, \Delta_7 \rightarrow_R \frac{h_{10} : \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \rightarrow F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \rightarrow F_9} \rightarrow_L}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), F_{12} \rightarrow F_{13}} \text{Cut} \\
\sim \\
\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \rightarrow F_{13} \text{ ax/W} \quad \frac{h_{10} : \Delta_{11}, F_8 \vdash \Delta_7, F_{12}, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax}}{\frac{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{hCut}} \rightarrow_R \\
\frac{h_2 : F_9, \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_{10}, \Delta_8 \rightarrow_R \frac{h_{11} : F_7, \Delta_{14} \vdash F_{12}, \Delta_8, F_9 \rightarrow F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_L}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \text{Cut} \\
\sim \\
\frac{h_2 : \Delta_{14}, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7 \text{ ax/W} \quad \frac{h_{11} : \Delta_{14}, F_7, F_9 \vdash \Delta_8, F_{10}, F_{12}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{hCut}} \rightarrow_R
\end{array}$$

• Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11} \quad \bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7 \quad \rightarrow_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11} \quad \bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11} \quad \wedge_L}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{Cut}}{\vdash : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \\
\sim \\
\frac{\frac{h_1 : \Delta_{12}, F_{10}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \quad \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\vdash : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \text{ax/W}}{\vdash : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{hCut} \\
\sim \\
\frac{h_2 : F_8, \Delta_{11} \vdash F_{12} \wedge F_{13}, F_9, \Delta_7 \quad \bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), F_{12} \wedge F_{13} \quad \rightarrow_R \quad \frac{h_{10} : F_{12}, F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9 \quad \bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \rightarrow F_9 \quad \wedge_L}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \rightarrow F_9}}{\vdash : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \wedge F_{13}}{\vdash : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_{13}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \wedge F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax}}{\vdash : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \wedge_L \text{hCut} \\
\sim \\
\frac{h_2 : F_9, \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_{10}, \Delta_8 \quad \bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7 \quad \rightarrow_R \quad \frac{h_{11} : F_7, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \rightarrow F_{10} \quad \bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \wedge_L}{\vdash : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7}{\vdash : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}} \text{inv-th/ax}}{\vdash : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \wedge_L \text{hCut} \\
\sim \\
\frac{\vdash : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}}{\vdash : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11} \quad \bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7 \quad \rightarrow_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11} \quad \vee_L}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{Cut}}{\vdash : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \\
\sim \\
\frac{\frac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \quad \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\vdash : \Delta_{12}, F_9 \vdash \Delta_{11}} \text{ax/W}}{\vdash : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{hCut} \\
\sim \\
\frac{\frac{h_2 : F_8, \Delta_{11} \vdash F_{12} \vee F_{13}, F_9, \Delta_7 \quad \bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), F_{12} \vee F_{13} \quad \rightarrow_R \quad \frac{h_{10} : F_{12}, \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9 \quad \vee_L}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \rightarrow F_9}}{\vdash : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \vee F_{13}}{\vdash : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \vee F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax}}{\vdash : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \wedge_L \text{hCut} \\
\sim \\
\frac{h_2 : F_9, \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_{10}, \Delta_8 \quad \bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7 \quad \rightarrow_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_{14} \vdash \Delta_8, F_9 \rightarrow F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \rightarrow F_{10} \quad \vee_L}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut}}{\vdash : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \\
\sim \\
\frac{\frac{h_2 : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7}{\vdash : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_7, F_9 \vdash \Delta_8, F_{10}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}} \text{inv-th/ax}}{\vdash : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \wedge_L \text{hCut} \\
\sim \\
\frac{\vdash : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}}{\vdash : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : F_6, \perp, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{\bullet h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9}{\perp_L} \\
\hline
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\hline
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{ax/W} \\
\hline
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_8, \Delta_{11} \vdash \perp, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), \perp} \rightarrow_R \quad \frac{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \rightarrow F_9}{\perp_L} \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_{11}, F_8 \vdash \perp, \Delta_7, F_9}{\bullet h_{10} : \perp, \Delta_{11}, F_8 \vdash \Delta_7, F_9} \perp_L \\
\hline
\frac{}{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \text{hCut} \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \rightarrow_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_9, \perp, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}}{\perp_L} \\
\hline
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut} \\
\hline
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W} \\
\hline
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \perp_L
\end{array}$$

• Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{11}, p_9 \vdash F_7, \Delta_{10}, p_9}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{\bullet h_8 : (\Delta_{11}, p_9), F_6 \rightarrow F_7 \vdash \Delta_{10}, p_9}{I} \\
\hline
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_7, \Delta_{10} \vdash p_{11}, F_8, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \rightarrow F_8), p_{11}} \rightarrow_R \quad \frac{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \rightarrow F_8}{I} \\
\hline
\frac{}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \rightarrow F_8} \text{Cut} \\
\hline
\frac{}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \rightarrow F_8} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, p_{11}, p_{11}}{\bullet h_9 : \Delta_{10}, F_7, p_{11} \vdash \Delta_{12}, F_8, p_{11}} I \\
\hline
\frac{}{- : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, p_{11}} \text{hCut} \\
\hline
\frac{}{- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \rightarrow F_8} \rightarrow_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_8, \Delta_{13}, p_{11} \vdash F_7, F_9, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \rightarrow F_9), F_7} \rightarrow_R \quad \frac{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \rightarrow F_9}{I} \\
\hline
\frac{}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \rightarrow F_9} \text{Cut} \\
\hline
\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \rightarrow F_9} I
\end{array}$$

• Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : F_6, \top, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \top_L \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W} \\
\hline
\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7}{\bullet h_8 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9} \text{ax/W} \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_8, \Delta_{11} \vdash \top, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), \top} \rightarrow_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \rightarrow F_9} \top_L \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_9, \top, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \top_L \\
\hline
\frac{}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W} \\
\hline
\frac{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \rightarrow F_{10}}{\bullet h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W} \\
\hline
\frac{}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{hCut}
\end{array}$$

## 8.2 Status of $\wedge_R$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \rightarrow F_{12} \quad h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \rightarrow F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : F_{11}, \Delta_6, F_7 \wedge F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\sim} \text{Cut}} \\
\\
\frac{\frac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_8}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \wedge F_8} \text{inv-th/ax}}{\frac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{\sim} \wedge_R} \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W}}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\sim} \rightarrow_R} \text{hCut} \\
\\
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \rightarrow F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \rightarrow F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}} \rightarrow_R}{\frac{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}}{\sim} \text{Cut}} \\
\\
\frac{\frac{h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \frac{h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_7}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_7 \wedge F_{10}} \text{inv-th/ax}}{\frac{- : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \wedge F_{10}}{\sim} \wedge_R} \quad \frac{h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \wedge F_{10}} \text{ax/W}}{\frac{- : \Delta_8 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \wedge F_{10}}{\sim} \rightarrow_R} \text{hCut}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \wedge F_{12} \quad h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}}{\sim} \text{Cut}} \\
\\
\frac{\frac{- : \Delta_6 \vdash \Delta_{10}, F_7, F_{11} \wedge F_{12}}{\sim} \text{ax/W} \quad \frac{- : \Delta_6, F_7 \vdash \Delta_{10}, F_8, F_{11} \wedge F_{12}}{\sim} \text{ax/W} \quad \frac{- : \Delta_6, F_7, F_8 \vdash \Delta_{10}, F_{11}}{\sim} \text{inv-th/ax} \quad \frac{- : \Delta_6, F_7, F_8 \vdash \Delta_{10}, F_{12}}{\sim} \text{inv-th/ax}}{\frac{- : \Delta_6, F_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}}{\sim} \wedge_R} \text{sCut} \\
\\
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \wedge F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}} \rightarrow_R}{\frac{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}}{\sim} \text{Cut}} \\
\\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_7, F_9}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_7, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_7}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_7 \wedge F_{10}} \text{inv-th/ax}}{\frac{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}}{\sim} \wedge_R} \quad \frac{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}} \text{ax/W}}{\frac{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}}{\sim} \wedge_R} \text{hCut} \\
\\
\frac{h_2 : \Delta_8 \vdash F_7, F_{11}, \Delta_{10} \quad h_2 : \Delta_8 \vdash F_7, F_{12}, \Delta_{10}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7} \wedge_R \quad \frac{h_9 : F_7, \Delta_8 \vdash F_{11}, \Delta_{10} \quad h_9 : F_7, \Delta_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{\frac{- : \Delta_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}}{\sim} \text{Cut}} \\
\\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{10}, F_{11}, F_7}{\sim} \text{ax/W} \quad \frac{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}} \text{H}}{\frac{- : \Delta_8 \vdash \Delta_{10}, F_{11}}{\sim} \wedge_R} \text{hCut} \quad \frac{\frac{h_2 : \Delta_8 \vdash \Delta_{10}, F_{12}, F_7}{\sim} \text{ax/W} \quad \frac{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{12}} \text{H}}{\frac{- : \Delta_8 \vdash \Delta_{10}, F_{12}}{\sim} \wedge_R} \text{hCut} \\
\\
\frac{- : \Delta_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}}{\sim} \wedge_R
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \vee F_{12} \quad h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}}{\sim} \text{Cut}} \\
\\
\frac{\frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \wedge F_8} \text{inv-th/ax}}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}}{\sim} \wedge_R} \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}}{\sim} \vee_R} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \vee F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}} \vee_R \\
\hline
- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_7}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_7} \text{inv-th/ax}}{\frac{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \wedge F_{10}}{- : \Delta_8 \vdash \Delta_{14}, F_9 \wedge F_{10}, F_{12} \vee F_{13}} \vee_R} \wedge_R \quad \frac{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \wedge F_{10}} \text{ax/W} \\
\hline
- : \Delta_8 \vdash \Delta_{14}, F_9 \wedge F_{10}, F_{12} \vee F_{13} \quad \text{hCut}
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, \perp, \Delta_{10} \quad h_1 : \Delta_6 \vdash F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \wedge F_8}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \perp, \Delta_{10}} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{hCut} \\
\hline
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \perp, \Delta_{12} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \perp, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\perp, \Delta_{12}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\perp, \Delta_{12}), F_9 \wedge F_{10}} \perp_R \\
\hline
- : \Delta_8 \vdash (\perp, \Delta_{12}), F_9 \wedge F_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{\bullet h_2 : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \wedge F_{10}}{- : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \wedge F_{10}} \text{ax/W}}{- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \wedge F_{10}} \text{hCut}
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, \top, \Delta_{10} \quad h_1 : \Delta_6 \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \top, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{- : \Delta_6 \vdash \top, \Delta_{10}}{- : \Delta_6 \vdash \top, \Delta_{10}} \top_R \\
\hline
\frac{h_2 : \Delta_8 \vdash F_7, F_9, \top, \Delta_{12} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \top, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\top, \Delta_{12}), F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \wedge F_{10}} \top_R \\
\hline
- : \Delta_8 \vdash (\top, \Delta_{12}), F_9 \wedge F_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \wedge F_{10}}{- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \wedge F_{10}} \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_9, \Delta_{12} \vdash F_6, \Delta_{10}, \Box F_{11} \quad h_1 : \Box \Gamma_9, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_{11}}{\bullet h_1 : \Box \Gamma_9, \Delta_{12} \vdash (\Delta_{10}, \Box F_{11}), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_9) \vdash F_{11}}{\bullet h_8 : (\Box \Gamma_9, \Delta_{12}), F_6 \wedge F_7 \vdash \Delta_{10}, \Box F_{11}} K \\
\hline
- : \Box \Gamma_9, \Delta_{12} \vdash \Delta_{10}, \Box F_{11} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{- : \text{unbox}(\Box \Gamma_9) \vdash F_{11}}{- : \Delta_{12}, \Box \Gamma_9 \vdash \Delta_{10}, \Box F_{11}} \text{ax/W} \\
\hline
\frac{h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_8, \Delta_{13}, \Box F_{12} \quad h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_9, \Delta_{13}, \Box F_{12}}{\bullet h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, \Box F_{12}), F_8 \wedge F_9), \Box F_7} \wedge_R \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{14}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : (\Box \Gamma_{14}, \Delta_{11}), \Box F_7 \vdash (\Delta_{13}, \Box F_{12}), F_8 \wedge F_9} K \\
\hline
- : \Box \Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, \Box F_{12}), F_8 \wedge F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_8, \Delta_{13}, \Box F_{12}}{\bullet h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_8, \Delta_{13}, \Box F_{12}} \text{ax/W} \quad \frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{14}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8, \Box F_{12}} K \\
\hline
\frac{h_2 : \Delta_{11}, \Box \Gamma_{14} \vdash \Box F_7, \Delta_{13}, F_8, \Box F_{12}}{- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8, \Box F_{12}} \text{ax/W} \quad \frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{14}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8, \Box F_{12}} K \\
\hline
- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8, \Box F_{12} \quad \text{hCut} \quad \frac{h_2 : \Delta_{11}, \Box \Gamma_{14} \vdash \Box F_7, \Delta_{13}, F_9, \Box F_{12}}{- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_9, \Box F_{12}} \text{ax/W} \quad \frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{14}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8, \Box F_{12}} K \\
\hline
- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, \Box F_{12}, F_8 \wedge F_9
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box\Gamma_{11}, \Delta_{14} \vdash F_7, F_8, \Delta_{13}, [\Box F_{12} \quad h_2 : \Box\Gamma_{11}, \Delta_{14} \vdash F_7, F_9, \Delta_{13}, [\Box F_{12}}{\bullet h_2 : \Box\Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, [\Box F_{12}]), F_8 \wedge F_9), F_7} \wedge_R \quad \frac{h_{10} : unbox(\Box\Gamma_{11}) \vdash F_{12}}{\bullet h_{10} : (\Box\Gamma_{11}, \Delta_{14}), F_7 \vdash (\Delta_{13}, [\Box F_{12}]), F_8 \wedge F_9} K \\
\hline
- : \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_{13}, [\Box F_{12}]), F_8 \wedge F_9 \\
\hline
\sim \\
\frac{- : unbox(\Box\Gamma_{11}) \vdash F_{12} \quad ax/W}{- : \Delta_{14}, \Box\Gamma_{11} \vdash \Delta_{13}, [\Box F_{12}, F_8 \wedge F_9} K
\end{array}$$

• Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \rightarrow_L \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \\
\hline
\sim \\
\frac{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9 \quad inv\text{-th}/ax \quad - : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L \\
\hline
\frac{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \quad ax/W \quad - : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_7 \quad ax/W \quad - : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} sCut \\
\hline
\frac{h_2 : \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \rightarrow F_{13}} \wedge_R \quad \frac{h_{10} : \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \wedge F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \wedge F_9} \rightarrow_L \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \\
\hline
\sim \\
\frac{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_8 \quad inv\text{-th}/ax \quad - : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_9 \quad inv\text{-th}/ax}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_8 \wedge F_9} \wedge_R \quad \frac{- : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8 \wedge F_9 \quad ax/W}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} sCut \\
\hline
\frac{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \wedge F_9 \quad ax/W \quad - : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} sCut \\
\hline
\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_{14} \vdash F_{12}, \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}) \vdash \Delta_8, F_9 \wedge F_{10}} \rightarrow_L \\
\hline
- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10} \\
\hline
\sim \\
\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_9 \quad ax/W \quad h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{12}, F_9 \quad inv\text{-th}/ax \quad h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9 \quad inv\text{-th}/ax}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9} \rightarrow_L \\
\hline
\frac{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \quad hCut}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}
\end{array}$$

• Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L \\
\hline
- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \\
\hline
\sim \\
\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \quad inv\text{-th}/ax \quad h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7 \quad inv\text{-th}/ax}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}}{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}} ax/W \\
\hline
- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11} \quad \wedge_L \\
\hline
\frac{h_1 : \Delta_7 \vdash F_8, \Delta_{10} \quad h_1 : \Delta_7 \vdash F_9, \Delta_{10}}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R \quad \frac{h_6 : F_8, F_9, \Delta_7 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{10}} \wedge_L \\
\hline
- : \Delta_7 \vdash \Delta_{10} \\
\hline
\sim \\
\frac{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9 \quad ax/W \quad - : \Delta_7, F_8, F_9 \vdash \Delta_{10} \quad ax/W}{- : \Delta_7 \vdash \Delta_{10}, F_8} sCut \\
\hline
- : \Delta_7 \vdash \Delta_{10}
\end{array}$$



$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_{11} \vdash F_{12} \wedge F_{13}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash F_{12} \wedge F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \wedge F_{13}} \wedge_R \quad \frac{h_{10} : F_{12}, F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \wedge F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_{12} \wedge F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8} \text{inv-th/ax} \quad \frac{}{- : \Delta_{11} \vdash \Delta_7, F_8} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8} \text{hCut} \quad \frac{h_2 : \Delta_{11} \vdash \Delta_7, F_9, F_{12} \wedge F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_9} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax} \quad \frac{}{- : \Delta_{11} \vdash \Delta_7, F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_9} \text{hCut}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \wedge_R \\
\\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \frac{h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_{10}, F_7}{\bullet h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \text{inv-th/ax}}{- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \wedge_R \\
\frac{}{- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{hCut} \\
\frac{}{- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_7, F_9 \vdash \Delta_{11}} \text{inv-th/ax} \quad \frac{}{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}} \text{inv-th/ax}}{- : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L \\
\frac{}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6} \text{sCut} \quad \frac{}{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}} \text{sCut} \\
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \\
\\
\frac{\frac{h_2 : \Delta_{11} \vdash F_{12} \vee F_{13}, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash F_{12} \vee F_{13}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \vee F_{13}} \wedge_R \quad \frac{h_{10} : F_{12}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \wedge F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8} \text{inv-th/ax} \quad \frac{}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_9} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8 \wedge F_9} \wedge_R \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8 \wedge F_9} \text{ax/W} \quad \frac{}{- : \Delta_{11}, F_{13} \vdash \Delta_7, F_{12}, F_8 \wedge F_9} \text{sCut} \quad \frac{}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} \text{sCut} \\
- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \\
\\
\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_9, \Delta_8 \quad h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_{14} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \vee_L \\
\frac{}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \text{hCut} \quad \frac{}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9} \text{ax/W} \quad \frac{}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{sCut} \\
- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \perp, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \perp_L}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightsquigarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_{11} \vdash \perp, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash \perp, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \perp} \wedge_R \quad \frac{}{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \wedge F_9} \perp_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_8}{- : \Delta_{11} \vdash \Delta_7, F_8} \text{ax/W} \quad \frac{\bullet h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_8}{- : \Delta_{11} \vdash \Delta_7, F_8} \perp_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{hCut} \quad \frac{\frac{h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_9}{- : \Delta_{11} \vdash \Delta_7, F_9} \text{ax/W} \quad \frac{\bullet h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_9}{- : \Delta_{11} \vdash \Delta_7, F_9} \perp_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{hCut} \\
\wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \perp, \Delta_{12} \vdash F_7, F_9, \Delta_8 \quad h_2 : \perp, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{}{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \perp_L \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\sim \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, p_9 \vdash F_6, \Delta_{10}, p_9 \quad h_1 : \Delta_{11}, p_9 \vdash F_7, \Delta_{10}, p_9}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : (\Delta_{11}, p_9), F_6 \wedge F_7 \vdash \Delta_{10}, p_9} I \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{Cut} \\
\sim \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I \\
\frac{h_2 : \Delta_{10} \vdash p_{11}, F_7, \Delta_{12}, p_{11} \quad h_2 : \Delta_{10} \vdash p_{11}, F_8, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \wedge F_8), p_{11}} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8} I \\
\frac{}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8} \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}, p_{11}}{- : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_7, p_{11}}{- : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}} I}{- : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}} \text{hCut} \quad \frac{\frac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}, p_{11}}{- : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_8, p_{11}}{- : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}} I}{- : \Delta_{10} \vdash \Delta_{12}, F_7 \wedge F_8} \text{hCut} \\
\wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_{13}, p_{11} \vdash F_7, F_8, \Delta_{12}, p_{11} \quad h_2 : \Delta_{13}, p_{11} \vdash F_7, F_9, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \wedge F_9), F_7} \wedge_R \quad \frac{}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9} I \\
\frac{}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9} \text{Cut} \\
\sim \\
\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \wedge F_9} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \top, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \top_L \\
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut} \\
\frac{h_2 : \Delta_{11} \vdash \top, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash \top, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \top} \wedge_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \wedge F_9} \top_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\sim \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{ax/W} \\
\frac{h_2 : \top, \Delta_{12} \vdash F_7, F_9, \Delta_8 \quad h_2 : \top, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \top_L \\
\frac{}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \wedge F_{10}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{hCut}
\end{array}$$

### 8.3 Status of $\vee_R$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \rightarrow F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : F_{11}, \Delta_6, F_7 \vee F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12} \quad \text{Cut} \\
\hline
\frac{\frac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7, F_8}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W}}{\frac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R} \text{hCut} \\
\hline
\frac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \rightarrow F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}} \rightarrow_R \\
\hline
- : \Delta_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\frac{\frac{h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_7, F_9}{\bullet h_2 : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7, F_9 \vee F_{10}} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}} \text{ax/W}}{\frac{- : \Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \vee F_{10}} \rightarrow_R} \text{hCut}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \wedge F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12} \quad \text{Cut} \\
\hline
\frac{\frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}}{\bullet h_9 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7 \vee F_8} \text{ax/W}}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R} \text{hCut} \\
\hline
\frac{\frac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \vee F_{10} \quad h_{11} : F_7, \Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}} \wedge_R \\
\hline
- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_7, F_9, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_7, F_9, F_{12} \wedge F_{13}} \text{ax/W} \quad \frac{\frac{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_{12}, F_9}{\bullet h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_{13}, F_9}{\bullet h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_{13}, F_9} \text{inv-th/ax}}{\frac{- : \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \wedge F_{13}, F_9 \vee F_{10}} \vee_R} \text{hCut}
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R \\
\hline
- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12} \quad \text{Cut} \\
\hline
\frac{\frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R} \text{hCut} \\
\hline
\frac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}} \vee_R \\
\hline
- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_7, F_9}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \vee F_{10}} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}} \text{ax/W}}{\frac{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \vee F_{13}, F_9 \vee F_{10}} \vee_R} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_8 \vdash F_7, F_{11}, F_{12}, \Delta_{10}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7} \vee_R \quad \frac{h_9 : F_7, \Delta_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R \\
\hline
- : \Delta_8 \vdash \Delta_{10}, F_{11} \vee F_{12} \\
\sim \\
\frac{\frac{h_2 : \Delta_8 \vdash \Delta_{10}, F_{11}, F_{12}, F_7}{- : \Delta_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ ax/W} \quad \frac{\frac{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ ax/W} \quad \text{H}}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{ hCut} \\
\vee_R
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10} \\
\sim \\
\frac{\frac{h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \vee F_8}{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \vee F_8} \text{ ax/W} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{ ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{ hCut} \\
\sim \\
\frac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \perp, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\perp, \Delta_{12}), F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\perp, \Delta_{12}), F_9 \vee F_{10}} \perp_R \\
\hline
- : \Delta_8 \vdash (\perp, \Delta_{12}), F_9 \vee F_{10} \\
\sim \\
\frac{\frac{h_2 : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \vee F_{10}}{\bullet h_2 : \Delta_8 \vdash \perp, \Delta_{12}, F_7, F_9 \vee F_{10}} \text{ ax/W} \quad \frac{h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}} \text{ ax/W}}{- : \Delta_8 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}} \text{ hCut}
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7, F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \top, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10} \\
\sim \\
\frac{- : \Delta_6 \vdash \top, \Delta_{10}}{- : \Delta_6 \vdash \top, \Delta_{10}} \top_R \\
\sim \\
\frac{h_2 : \Delta_8 \vdash F_7, F_9, F_{10}, \top, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\top, \Delta_{12}), F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \vee F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\top, \Delta_{12}), F_9 \vee F_{10}} \top_R \\
\hline
- : \Delta_8 \vdash (\top, \Delta_{12}), F_9 \vee F_{10} \\
\sim \\
\frac{- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \vee F_{10}}{- : \Delta_8 \vdash \top, \Delta_{12}, F_9 \vee F_{10}} \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_9, \Delta_{12} \vdash F_6, F_7, \Delta_{10}, [\Box F_{11}]}{\bullet h_1 : \Box \Gamma_9, \Delta_{12} \vdash (\Delta_{10}, [\Box F_{11}]), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_9) \vdash F_{11}}{\bullet h_8 : (\Box \Gamma_9, \Delta_{12}), F_6 \vee F_7 \vdash \Delta_{10}, [\Box F_{11}]} K \\
\hline
- : \Box \Gamma_9, \Delta_{12} \vdash \Delta_{10}, [\Box F_{11}] \\
\sim \\
\frac{- : \text{unbox}(\Box \Gamma_9) \vdash F_{11}}{- : \Delta_{12}, \Box \Gamma_9 \vdash \Delta_{10}, [\Box F_{11}]} \text{ ax/W} \\
K \\
\frac{h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_8, F_9, \Delta_{13}, [\Box F_{12}]}{\bullet h_2 : \Box \Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, [\Box F_{12}]), F_8 \vee F_9), \Box F_7} \vee_R \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{14}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : (\Box \Gamma_{14}, \Delta_{11}), \Box F_7 \vdash (\Delta_{13}, [\Box F_{12}]), F_8 \vee F_9} K \\
\hline
- : \Box \Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, [\Box F_{12}]), F_8 \vee F_9 \\
\sim \\
\frac{\frac{h_2 : \Delta_{11}, \Box \Gamma_{14} \vdash \Box F_7, \Delta_{13}, F_8, F_9, [\Box F_{12}]}{\bullet h_2 : \Delta_{11}, \Box \Gamma_{14} \vdash \Box F_7, \Delta_{13}, F_8, F_9, [\Box F_{12}]} \text{ ax/W} \quad \frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{14}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8, F_9, [\Box F_{12}]} K}{- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8, F_9, [\Box F_{12}]} \text{ hCut} \\
\vee_R \\
- : \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, [\Box F_{12}], F_8 \vee F_9
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Box \Gamma_{11}, \Delta_{14} \vdash F_7, F_8, F_9, \Delta_{13}, [\Box F_{12}] \quad \vee_R \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{11}) \vdash F_{12}}{\bullet h_{10} : (\Box \Gamma_{11}, \Delta_{14}), F_7 \vdash (\Delta_{13}, [\Box F_{12}]), F_8 \vee F_9}}{K} \\
\frac{\bullet h_2 : \Box \Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, [\Box F_{12}]), F_8 \vee F_9), F_7}{- : \Box \Gamma_{11}, \Delta_{14} \vdash (\Delta_{13}, [\Box F_{12}]), F_8 \vee F_9} \quad \text{Cut} \\
\sim \\
\frac{\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_{12}}{- : \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, [\Box F_{12}], F_8 \vee F_9} \text{ ax/W}}{K}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, F_7, \Delta_{11} \quad \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7 \vdash \Delta_{11}}}{\rightarrow_L} \\
\frac{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \vee F_7}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \quad \text{Cut} \\
\sim \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{ inv-th/ax} \quad \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \text{ ax/W} \quad \text{hCut}}{\sim} \\
\frac{\frac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \text{ inv-th/ax} \quad \vee_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \text{ ax/W}}{\rightarrow_L} \\
\frac{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L \\
\frac{h_2 : \Delta_{11} \vdash F_{12} \rightarrow F_{13}, F_8, F_9, \Delta_7 \quad \vee_R \quad \frac{h_{10} : \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \vee F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \vee F_9}}{\rightarrow_L} \\
\frac{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \rightarrow F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \quad \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \rightarrow F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{ ax/W} \quad \frac{\frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_{12}, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8, F_9} \text{ inv-th/ax} \quad \frac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9} \text{ inv-th/ax}}{\rightarrow_L} \text{ hCut}}{\rightarrow_L} \\
\frac{- : \Delta_{11} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R \\
\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash F_7, F_9, F_{10}, \Delta_8 \quad \vee_R \quad \frac{h_{11} : F_7, \Delta_{14} \vdash F_{12}, \Delta_8, F_9 \vee F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}}}{\rightarrow_L} \\
\frac{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \quad \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \text{ ax/W} \quad \frac{\frac{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{10}, F_{12}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_9} \text{ inv-th/ax} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9} \text{ inv-th/ax}}{\rightarrow_L} \text{ hCut}}{\rightarrow_L} \\
\frac{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \vee_R
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_6, F_7, \Delta_{11} \quad \vee_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash \Delta_{11}}}{\wedge_L} \\
\frac{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \vee F_7}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \quad \text{Cut} \\
\sim \\
\frac{\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{ inv-th/ax} \quad \vee_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \text{ ax/W}}{\wedge_L} \text{ hCut} \\
\frac{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L \\
\frac{h_2 : \Delta_{11} \vdash F_{12} \wedge F_{13}, F_8, F_9, \Delta_7 \quad \vee_R \quad \frac{h_{10} : F_{12}, F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \vee F_9}}{\wedge_L} \\
\frac{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \wedge F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \quad \text{Cut} \\
\sim \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \wedge F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{ ax/W} \quad \frac{\frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8, F_9} \text{ inv-th/ax} \quad \wedge_L}{\wedge_L} \text{ hCut}}{\wedge_L} \\
\frac{- : \Delta_{11} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \wedge_L \\
\hline
- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9} \text{ax/W} \quad \frac{\frac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_7, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9} \wedge_L}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \vee_R \quad \text{hCut}
\end{array}$$

• Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \vee_L \\
\hline
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \vdash \Delta_{11}} \vee_R \quad \frac{\frac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \vee_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{ax/W} \quad \text{hCut}
\end{array}$$
  

$$\begin{array}{c}
\frac{h_1 : \Delta_7 \vdash F_8, F_9, \Delta_{10}}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \vee F_9} \vee_R \quad \frac{h_6 : F_8, \Delta_7 \vdash \Delta_{10} \quad h_6 : F_9, \Delta_7 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{10}} \vee_L \\
\hline
- : \Delta_7 \vdash \Delta_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{- : \Delta_7 \vdash \Delta_{10}, F_8, F_9}{- : \Delta_7 \vdash \Delta_{10}, F_8} \text{ax/W} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{10}, F_8}{- : \Delta_7 \vdash \Delta_{10}, F_8} \text{ax/W} \quad \frac{- : \Delta_7, F_8 \vdash \Delta_{10}}{- : \Delta_7 \vdash \Delta_{10}} \text{sCut}
\end{array}$$
  

$$\begin{array}{c}
\frac{h_2 : \Delta_{11} \vdash F_{12} \vee F_{13}, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \vee F_{13}} \vee_R \quad \frac{h_{10} : F_{12}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9 \quad h_{10} : F_{13}, \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \vee F_9} \vee_L \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \vee F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax}}{- : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_L \quad \text{hCut}
\end{array}$$
  

$$\begin{array}{c}
\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, F_{12}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10} \quad h_{11} : F_7, F_{13}, \Delta_{14} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \vee_L \\
\hline
- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9} \text{ax/W} \quad \frac{\frac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_{10}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \vee_R \quad \text{hCut}
\end{array}$$

• Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_{10} \vdash F_6, F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{h_8 : (\perp, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \perp_L \\
\hline
- : \perp, \Delta_{10} \vdash \Delta_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
- : \perp, \Delta_{10} \vdash \Delta_9 \quad \perp_L
\end{array}$$
  

$$\begin{array}{c}
\frac{h_2 : \Delta_{11} \vdash \perp, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), \perp} \vee_R \quad \frac{h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \vee F_9} \perp_L \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_8, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{ax/W} \quad \frac{h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_8, F_9} \perp_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \perp, \Delta_{12} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{}{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \perp_L \\
\hline
- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10} \\
\hline
- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10} \quad \perp_L \\
\hline
- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, p_9 \vdash F_6, F_7, \Delta_{10}, p_9}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \vee F_7} \vee_R \quad \frac{}{\bullet h_8 : (\Delta_{11}, p_9), F_6 \vee F_7 \vdash \Delta_{10}, p_9} I \\
\hline
- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9 \\
\hline
- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9 \quad I \\
\hline
- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_{10} \vdash p_{11}, F_7, F_8, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \vee F_8), p_{11}} \vee_R \quad \frac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \vee F_8} I \\
\hline
- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \vee F_8 \\
\hline
- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \vee F_8 \\
\hline
\frac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, F_8, p_{11}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, F_8, p_{11}} \text{ax/W} \quad \frac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_7, F_8, p_{11}} I \\
\hline
- : \Delta_{10} \vdash \Delta_{12}, F_7, F_8, p_{11} \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \vee F_8 \quad \vee_R \\
\hline
- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \vee F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_{13}, p_{11} \vdash F_7, F_8, F_9, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \vee F_9), F_7} \vee_R \quad \frac{}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \vee F_9} I \\
\hline
- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \vee F_9 \\
\hline
- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \vee F_9 \\
\hline
- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \vee F_9 \quad I \\
\hline
- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \vee F_9
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_{10} \vdash F_6, F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \top_L \\
\hline
- : \top, \Delta_{10} \vdash \Delta_9 \\
\hline
- : \top, \Delta_{10} \vdash \Delta_9 \\
\hline
\frac{}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \text{ax/W} \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \text{hCut} \\
\hline
- : \top, \Delta_{10} \vdash \Delta_9
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_{11} \vdash \top, F_8, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), \top} \vee_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \vee F_9} \top_L \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9 \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9 \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9 \quad \text{ax/W} \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \top, \Delta_{12} \vdash F_7, F_9, F_{10}, \Delta_8}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \top_L \\
\hline
- : \top, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10} \\
\hline
- : \top, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10} \\
\hline
\frac{}{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \vee F_{10}} \text{ax/W} \quad \frac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \text{hCut} \\
\hline
- : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \vee F_{10}
\end{array}$$

## 8.4 Status of $\perp_R$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \rightarrow F_8), \perp} \perp_R \quad \frac{h_5 : \perp, F_7, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \rightarrow F_8} \rightarrow_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \quad \text{ax/W} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \rightarrow F_9}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_{10}, F_8 \rightarrow F_9), F_5} \perp_R \quad \frac{h_7 : F_5, F_8, \Delta_6 \vdash \perp, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{h_2 : \Delta_6 \vdash \perp, \Delta_{10}, F_5, F_8 \rightarrow F_9}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{hCut}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \wedge F_8), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash F_7, \Delta_6 \quad h_5 : \perp, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \wedge F_8} \wedge_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8}{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \wedge F_9}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_{10}, F_8 \wedge F_9), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \perp, F_8, \Delta_{10} \quad h_7 : F_5, \Delta_6 \vdash \perp, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{h_2 : \Delta_6 \vdash \perp, \Delta_{10}, F_5, F_8 \wedge F_9}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{hCut}
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \vee F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \vee F_8), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash F_7, F_8, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \vee F_8} \vee_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8}{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \vee F_9}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_{10}, F_8 \vee F_9), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \perp, F_8, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \vee_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{h_2 : \Delta_6 \vdash \perp, \Delta_{10}, F_5, F_8 \vee F_9}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{hCut}
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \perp, \Delta_6}{\bullet h_1 : \Delta_4 \vdash (\perp, \Delta_6), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \perp, \Delta_6} \perp_R \\
\hline
- : \Delta_4 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{- : \Delta_4 \vdash \perp, \Delta_6}{- : \Delta_4 \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_6 \vdash F_5, \Delta_8}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_8), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \Delta_8}{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_8} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{h_2 : \Delta_6 \vdash \perp, \Delta_8, F_5}{- : \Delta_6 \vdash \perp, \Delta_8, F_5} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_6, F_5 \vdash \perp, \Delta_8}{- : \Delta_6 \vdash \perp, \Delta_8, F_5} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_8, F_5} \text{hCut}
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \top, \Delta_6}{\bullet h_1 : \Delta_4 \vdash (\top, \Delta_6), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash \top, \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \top, \Delta_6} \top_R \\
\hline
- : \Delta_4 \vdash \top, \Delta_6 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{- : \Delta_4 \vdash \top, \Delta_6}{- : \Delta_4 \vdash \top, \Delta_6} \top_R
\end{array}$$



$$\begin{array}{c}
\frac{h_2 : \Delta_6 \vdash F_5, \top, \Delta_8}{\bullet h_2 : \Delta_6 \vdash (\perp, \top, \Delta_8), F_5} \perp_R \quad \frac{h_7 : \Delta_6, F_5 \vdash \perp, \top, \Delta_8}{\bullet h_7 : \Delta_6 \vdash \perp, \top, \Delta_8} \top_R \\
\hline
- : \Delta_6 \vdash \perp, \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_6 \vdash \perp, \top, \Delta_8} \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_5, \Delta_8 \vdash \Delta_6, [\Box F_7]}{\bullet h_1 : \Box \Gamma_5, \Delta_8 \vdash (\Delta_6, [\Box F_7]), \perp} \perp_R \quad \frac{h_4 : \text{unbox}(\Box \Gamma_5) \vdash F_7}{\bullet h_4 : (\Box \Gamma_5, \Delta_8), \perp \vdash \Delta_6, [\Box F_7]} K \\
\hline
- : \Box \Gamma_5, \Delta_8 \vdash \Delta_6, [\Box F_7] \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_8, \Box \Gamma_5 \vdash \Delta_6, [\Box F_7]} \text{ax/W} \\
\\
\frac{h_2 : \Box \Gamma_{10}, \Delta_7 \vdash \Box F_5, \Delta_9, [\Box F_8]}{\bullet h_2 : \Box \Gamma_{10}, \Delta_7 \vdash (\perp, \Delta_9, [\Box F_8]), \Box F_5} \perp_R \quad \frac{h_6 : \text{unbox}(\Box \Gamma_{10}), \text{unbox}(\Box F_5) \vdash F_8}{\bullet h_6 : (\Box \Gamma_{10}, \Delta_7), \Box F_5 \vdash \perp, \Delta_9, [\Box F_8]} K \\
\hline
- : \Box \Gamma_{10}, \Delta_7 \vdash \perp, \Delta_9, [\Box F_8] \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{h_2 : \Delta_7, \Box \Gamma_{10} \vdash \perp, \Box F_5, \Delta_9, [\Box F_8]}{- : \Delta_7, \Box \Gamma_{10} \vdash \perp, \Delta_9, [\Box F_8]} \text{ax/W} \quad \frac{h_6 : \Box F_5, \Delta_7, \Box \Gamma_{10} \vdash \perp, \Delta_9, [\Box F_8]}{\bullet h_6 : \Box F_5, \Delta_7, \Box \Gamma_{10} \vdash \perp, \Delta_9, [\Box F_8]} \text{ax/W} \\
\hline
\text{hCut} \\
\\
\frac{h_2 : \Box \Gamma_7, \Delta_{10} \vdash F_5, \Delta_9, [\Box F_8]}{\bullet h_2 : \Box \Gamma_7, \Delta_{10} \vdash (\perp, \Delta_9, [\Box F_8]), F_5} \perp_R \quad \frac{h_6 : \text{unbox}(\Box \Gamma_7) \vdash F_8}{\bullet h_6 : (\Box \Gamma_7, \Delta_{10}), F_5 \vdash \perp, \Delta_9, [\Box F_8]} K \\
\hline
- : \Box \Gamma_7, \Delta_{10} \vdash \perp, \Delta_9, [\Box F_8] \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_7) \vdash F_8} \text{ax/W} \\
\hline
\frac{}{- : \Delta_{10}, \Box \Gamma_7 \vdash \perp, \Delta_9, [\Box F_8]} K
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \perp, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \perp \vdash \Delta_7} \rightarrow_L \\
\hline
- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{ax/W} \\
\\
\frac{h_2 : \Delta_7 \vdash F_8 \rightarrow F_9, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), F_8 \rightarrow F_9} \perp_R \quad \frac{h_6 : \Delta_7 \vdash \perp, F_8, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \perp, \Delta_5} \rightarrow_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \rightarrow F_9}{\bullet h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \rightarrow F_9} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \perp, \Delta_5} \text{ax/W} \\
\hline
\text{hCut} \\
\hline
\rightsquigarrow \\
\frac{h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_{10} \vdash \perp, F_8, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \rightarrow F_9), F_5 \vdash \perp, \Delta_6} \rightarrow_L \\
\hline
- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6, F_5}{\bullet h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6, F_5} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5, F_8 \rightarrow F_9 \vdash \perp, \Delta_6}{\bullet h_7 : \Delta_{10}, F_5, F_8 \rightarrow F_9 \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
\text{hCut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \perp \vdash \Delta_7} \wedge_L \\
\hline
- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_7 \vdash F_8 \wedge F_9, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), F_8 \wedge F_9} \perp_R \quad \frac{h_6 : F_8, F_9, \Delta_7 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \perp, \Delta_5} \wedge_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \wedge F_9}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \perp, \Delta_5}{\text{hCut}} \text{ax/W}}{} \\
\\
\frac{h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : F_5, F_8, F_9, \Delta_{10} \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \wedge F_9), F_5 \vdash \perp, \Delta_6} \wedge_L \\
\hline
- : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6, F_5}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{10}, F_5, F_8 \wedge F_9 \vdash \perp, \Delta_6}{\text{hCut}} \text{ax/W}}{}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \perp, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \perp \vdash \Delta_7} \vee_L \\
\hline
- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{ax/W} \\
\\
\frac{h_2 : \Delta_7 \vdash F_8 \vee F_9, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), F_8 \vee F_9} \perp_R \quad \frac{h_6 : F_8, \Delta_7 \vdash \perp, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \perp, \Delta_5} \vee_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \vee F_9}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \perp, \Delta_5}{\text{hCut}} \text{ax/W}}{} \\
\\
\frac{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash F_5, \Delta_6}{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : F_5, F_8, \Delta_{10} \vdash \perp, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \perp, \Delta_6} \vee_L \\
\hline
- : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6, F_5}{- : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{10}, F_5, F_8 \vee F_9 \vdash \perp, \Delta_6}{\text{hCut}} \text{ax/W}}{}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash \Delta_6}{\bullet h_1 : \Delta_5 \vdash \Delta_6, \perp} \perp_R \quad \frac{}{\bullet h_4 : \Delta_5, \perp \vdash \Delta_6} \perp_L \\
\hline
- : \Delta_5 \vdash \Delta_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_5 \vdash \Delta_6} \text{ax/W} \\
\\
\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), \perp} \perp_R \quad \frac{}{\bullet h_6 : \Delta_7, \perp \vdash \perp, \Delta_5} \perp_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \\
\\
\frac{h_2 : \perp, \Delta_8 \vdash F_5, \Delta_6}{\bullet h_2 : \perp, \Delta_8 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{}{\bullet h_7 : (\perp, \Delta_8), F_5 \vdash \perp, \Delta_6} \perp_L \\
\hline
- : \perp, \Delta_8 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_7, p_5 \vdash \Delta_6, p_5}{\bullet h_1 : \Delta_7, p_5 \vdash (\Delta_6, p_5), \perp} \perp_R \quad \frac{}{\bullet h_4 : (\Delta_7, p_5), \perp \vdash \Delta_6, p_5} I \\
\hline
- : \Delta_7, p_5 \vdash \Delta_6, p_5 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} I
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_6 \vdash p_7, \Delta_8, p_7}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_8, p_7), p_7} \perp_R \quad \frac{}{\bullet h_5 : \Delta_6, p_7 \vdash \perp, \Delta_8, p_7} I \\
\hline
- : \Delta_6 \vdash \perp, \Delta_8, p_7 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_5 : \Delta_6, p_7 \vdash \perp, \Delta_8, p_7} I \\
\frac{h_2 : \Delta_6 \vdash \perp, \Delta_8, p_7, p_7}{- : \Delta_6 \vdash \perp, \Delta_8, p_7} \text{ax/W} \quad \frac{}{\bullet h_5 : \Delta_6, p_7 \vdash \perp, \Delta_8, p_7} I \\
\hline
- : \Delta_6 \vdash \perp, \Delta_8, p_7 \quad \text{hCut} \\
\hline
\frac{h_2 : \Delta_9, p_7 \vdash F_5, \Delta_8, p_7}{\bullet h_2 : \Delta_9, p_7 \vdash (\perp, \Delta_8, p_7), F_5} \perp_R \quad \frac{}{\bullet h_6 : (\Delta_9, p_7), F_5 \vdash \perp, \Delta_8, p_7} I \\
\hline
- : \Delta_9, p_7 \vdash \perp, \Delta_8, p_7 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_6 : (\Delta_9, p_7), F_5 \vdash \perp, \Delta_8, p_7} I \\
\frac{}{- : \Delta_9, p_7 \vdash \perp, \Delta_8, p_7} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet h_1 : \top, \Delta_6 \vdash \Delta_5, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_6 \vdash \Delta_5}{\bullet h_4 : (\top, \Delta_6), \perp \vdash \Delta_5} \top_L \\
\hline
- : \top, \Delta_6 \vdash \Delta_5 \quad \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_6 \vdash \Delta_5} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_7 \vdash \top, \Delta_5}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), \top} \perp_R \quad \frac{h_6 : \Delta_7 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, \top \vdash \perp, \Delta_5} \top_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\hline
\frac{}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \\
\hline
\frac{h_2 : \top, \Delta_8 \vdash F_5, \Delta_6}{\bullet h_2 : \top, \Delta_8 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : F_5, \Delta_8 \vdash \perp, \Delta_6}{\bullet h_7 : (\top, \Delta_8), F_5 \vdash \perp, \Delta_6} \top_L \\
\hline
- : \top, \Delta_8 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \\
\frac{h_2 : \top, \Delta_8 \vdash \perp, \Delta_6, F_5}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{h_7 : \top, \Delta_8, F_5 \vdash \perp, \Delta_6}{\bullet h_7 : \top, \Delta_8, F_5 \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
- : \top, \Delta_8 \vdash \perp, \Delta_6 \quad \text{hCut}
\end{array}$$

## 8.5 Status of $\top_R$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \rightarrow F_8), \top} \top_R \quad \frac{h_5 : \top, F_7, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \rightarrow F_8} \rightarrow_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \Delta_4, F_7 \vdash \top, \Delta_6, F_8} \top_R \quad \frac{h_5 : \top, \Delta_4, F_7 \vdash \Delta_6, F_8}{\bullet h_5 : \Delta_4, F_7 \vdash \Delta_6, F_8} \text{ax/W} \\
\hline
- : \Delta_4, F_7 \vdash \Delta_6, F_8 \quad \text{hCut} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \quad \rightarrow_R \\
\hline
\frac{h_2 : \Delta_6 \vdash (\top, \Delta_{10}, F_8 \rightarrow F_9), F_5}{\bullet h_2 : \Delta_6 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9} \top_R \quad \frac{h_7 : F_5, F_8, \Delta_6 \vdash \top, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\frac{}{- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9} \text{ax/W} \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9 \quad \top_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \wedge F_8), \top} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash F_7, \Delta_6 \quad h_5 : \top, \Delta_4 \vdash F_8, \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \wedge F_8} \wedge_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \quad \text{Cut} \\
\hline
\frac{}{\bullet h_1 : \Delta_4 \vdash \top, \Delta_6, F_7} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_7}{\bullet h_5 : \Delta_4 \vdash \top, \Delta_6, F_7} \text{ax/W} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \quad \text{hCut} \\
\hline
\frac{}{\bullet h_1 : \Delta_4 \vdash \top, \Delta_6, F_8} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_8}{\bullet h_5 : \Delta_4 \vdash \top, \Delta_6, F_8} \text{ax/W} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_8 \quad \text{hCut} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \quad \wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_{10}, F_8 \wedge F_9), F_5} \top_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \top, F_8, \Delta_{10} \quad h_7 : F_5, \Delta_6 \vdash \top, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \Delta_{10}, F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \wedge F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \wedge F_9} \top_R
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \vee F_8), \top} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash F_7, F_8, \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \vee F_8} \vee_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_4 \vdash \top, \Delta_6, F_7, F_8} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_7, F_8}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \vee F_8} \text{ax/W}}{- : \Delta_4 \vdash \Delta_6, F_7, F_8} \text{hCut} \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8 \quad \vee_R \\
\hline
\frac{}{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_{10}, F_8 \vee F_9), F_5} \top_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \top, F_8, F_9, \Delta_{10}}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \Delta_{10}, F_8 \vee F_9} \vee_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_{10}, F_8 \vee F_9} \top_R
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\perp, \Delta_6), \top} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \perp, \Delta_6} \perp_R \\
\hline
- : \Delta_4 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_4 \vdash \perp, \top, \Delta_6} \text{ax/W} \quad \frac{h_5 : \top, \Delta_4 \vdash \perp, \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \perp, \Delta_6} \text{ax/W}}{- : \Delta_4 \vdash \perp, \Delta_6} \text{hCut} \\
\hline
\frac{}{\bullet h_2 : \Delta_6 \vdash (\top, \perp, \Delta_8), F_5} \top_R \quad \frac{h_7 : F_5, \Delta_6 \vdash \top, \Delta_8}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \perp, \Delta_8} \perp_R \\
\hline
- : \Delta_6 \vdash \top, \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_6 \vdash \perp, \top, \Delta_8} \top_R
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\top, \Delta_6), \top} \top_R \quad \frac{}{\bullet h_5 : \Delta_4, \top \vdash \top, \Delta_6} \top_R \\
\hline
- : \Delta_4 \vdash \top, \Delta_6 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_4 \vdash \top, \Delta_6} \top_R \\
\hline
\frac{}{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_8), F_5} \top_R \quad \frac{}{\bullet h_7 : \Delta_6, F_5 \vdash \top, \Delta_8} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Box \Gamma_5, \Delta_8 \vdash (\Delta_6, \llbracket F_7 \rrbracket), \top} \top_R \quad \frac{h_4 : \text{unbox}(\Box \Gamma_5) \vdash F_7}{\bullet h_4 : (\Box \Gamma_5, \Delta_8), \top \vdash \Delta_6, \llbracket F_7 \rrbracket} K \\
\hline
- : \Box \Gamma_5, \Delta_8 \vdash \Delta_6, \llbracket F_7 \rrbracket \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_5) \vdash F_7} \text{ax/W} \\
\hline
- : \Delta_8, \Box \Gamma_5 \vdash \Delta_6, \llbracket F_7 \rrbracket \quad K
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_2 : \Box \Gamma_{10}, \Delta_7 \vdash (\top, \Delta_9, [\Box F_8], \Box F_5)} \top_R \quad \frac{h_6 : \text{unbox}(\Box \Gamma_{10}), \text{unbox}(\Box F_5) \vdash F_8}{\bullet h_6 : (\Box \Gamma_{10}, \Delta_7), \Box F_5 \vdash \top, \Delta_9, [\Box F_8]} K}{- : \Box \Gamma_{10}, \Delta_7 \vdash \top, \Delta_9, [\Box F_8]} \text{Cut} \\
\sim \\
\frac{}{- : \Delta_7, \Box \Gamma_{10} \vdash \top, \Delta_9, [\Box F_8]} \top_R \\
\\
\frac{\frac{}{\bullet h_2 : \Box \Gamma_7, \Delta_{10} \vdash (\top, \Delta_9, [\Box F_8], F_5)} \top_R \quad \frac{h_6 : \text{unbox}(\Box \Gamma_7) \vdash F_8}{\bullet h_6 : (\Box \Gamma_7, \Delta_{10}), F_5 \vdash \top, \Delta_9, [\Box F_8]} K}{- : \Box \Gamma_7, \Delta_{10} \vdash \top, \Delta_9, [\Box F_8]} \text{Cut} \\
\sim \\
\frac{}{- : \Delta_{10}, \Box \Gamma_7 \vdash \top, \Delta_9, [\Box F_8]} \top_R
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \top \vdash \Delta_7} \rightarrow_L}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{Cut} \\
\sim \\
\frac{\frac{}{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5}{\bullet h_4 : \Delta_8, F_6 \vdash \top, \Delta_7} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \top_R \quad h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{- : \Delta_8, F_6 \vdash \Delta_7} \text{hCut}}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \rightarrow_L \\
\\
\frac{\frac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \rightarrow F_9} \top_R \quad \frac{h_6 : \Delta_7 \vdash \top, F_8, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \top, \Delta_5} \rightarrow_L}{- : \Delta_7 \vdash \top, \Delta_5} \text{Cut} \\
\sim \\
\frac{}{- : \Delta_7 \vdash \top, \Delta_5} \top_R \\
\\
\frac{\frac{}{\bullet h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash (\top, \Delta_6), F_5} \top_R \quad \frac{h_7 : F_5, \Delta_{10} \vdash \top, F_8, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \rightarrow F_9), F_5 \vdash \top, \Delta_6} \rightarrow_L}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6} \text{Cut} \\
\sim \\
\frac{}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6} \top_R
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \top} \top_R \quad \frac{h_4 : \top, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \top \vdash \Delta_7} \wedge_L}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{Cut} \\
\sim \\
\frac{\frac{}{\bullet h_1 : \Delta_8, F_5, F_6 \vdash \top, \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}{\bullet h_4 : \Delta_8, F_5, F_6 \vdash \Delta_7} \text{ax/W}}{- : \Delta_8, F_5, F_6 \vdash \Delta_7} \text{hCut} \\
\wedge_L \\
\frac{}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \\
\\
\frac{\frac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \wedge F_9} \top_R \quad \frac{h_6 : F_8, F_9, \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \top, \Delta_5} \wedge_L}{- : \Delta_7 \vdash \top, \Delta_5} \text{Cut} \\
\sim \\
\frac{}{- : \Delta_7 \vdash \top, \Delta_5} \top_R \\
\\
\frac{\frac{}{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash (\top, \Delta_6), F_5} \top_R \quad \frac{h_7 : F_5, F_8, F_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \wedge F_9), F_5 \vdash \top, \Delta_6} \wedge_L}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6} \text{Cut} \\
\sim \\
\frac{}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6} \top_R
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \top}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \top \vdash \Delta_7} \vee_L}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7}{\vdash : \Delta_8, F_5 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7}{\vdash : \Delta_8, F_5 \vdash \Delta_7} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7}{\vdash : \Delta_8, F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{\vdash : \Delta_8, F_6 \vdash \Delta_7} \text{ax/W}}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{hCut} \\
\sim \\
\frac{\frac{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \vee F_9}{\vdash : \Delta_7 \vdash \top, \Delta_5} \top_R \quad \frac{h_6 : F_8, \Delta_7 \vdash \top, \Delta_5 \quad h_6 : F_9, \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \top, \Delta_5} \vee_L}{\vdash : \Delta_7 \vdash \top, \Delta_5} \text{Cut} \\
\sim \\
\vdash : \Delta_7 \vdash \top, \Delta_5 \quad \top_R \\
\sim \\
\frac{\frac{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\top, \Delta_6), F_5}{\vdash : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6} \top_R \quad \frac{h_7 : F_5, F_8, \Delta_{10} \vdash \top, \Delta_6 \quad h_7 : F_5, F_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \top, \Delta_6} \vee_L}{\vdash : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6} \text{Cut} \\
\sim \\
\vdash : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_6 \vdash \Delta_5, \top}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \top_R \quad \frac{\bullet h_4 : (\perp, \Delta_6), \top \vdash \Delta_5}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \perp_L}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \text{Cut} \\
\sim \\
\vdash : \perp, \Delta_6 \vdash \Delta_5 \quad \perp_L \\
\sim \\
\frac{\frac{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), \perp}{\vdash : \Delta_7 \vdash \top, \Delta_5} \top_R \quad \frac{\bullet h_6 : \Delta_7, \perp \vdash \top, \Delta_5}{\vdash : \Delta_7 \vdash \top, \Delta_5} \perp_L}{\vdash : \Delta_7 \vdash \top, \Delta_5} \text{Cut} \\
\sim \\
\vdash : \Delta_7 \vdash \top, \Delta_5 \quad \top_R \\
\sim \\
\frac{\frac{\bullet h_2 : \perp, \Delta_8 \vdash (\top, \Delta_6), F_5}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \top_R \quad \frac{\bullet h_7 : (\perp, \Delta_8), F_5 \vdash \top, \Delta_6}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \perp_L}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \text{Cut} \\
\sim \\
\vdash : \perp, \Delta_8 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_5 \vdash (\Delta_6, p_5), \top}{\vdash : \Delta_7, p_5 \vdash \Delta_6, p_5} \top_R \quad \frac{\bullet h_4 : (\Delta_7, p_5), \top \vdash \Delta_6, p_5}{\vdash : \Delta_7, p_5 \vdash \Delta_6, p_5} I}{\vdash : \Delta_7, p_5 \vdash \Delta_6, p_5} \text{Cut} \\
\sim \\
\vdash : \Delta_7, p_5 \vdash \Delta_6, p_5 \quad I \\
\sim \\
\frac{\frac{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_8, p_7), p_7}{\vdash : \Delta_6 \vdash \top, \Delta_8, p_7} \top_R \quad \frac{\bullet h_5 : \Delta_6, p_7 \vdash \top, \Delta_8, p_7}{\vdash : \Delta_6 \vdash \top, \Delta_8, p_7} I}{\vdash : \Delta_6 \vdash \top, \Delta_8, p_7} \text{Cut} \\
\sim \\
\vdash : \Delta_6 \vdash \top, \Delta_8, p_7 \quad \top_R \\
\sim \\
\frac{\frac{\bullet h_2 : \Delta_9, p_7 \vdash (\top, \Delta_8, p_7), F_5}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_8, p_7} \top_R \quad \frac{\bullet h_6 : (\Delta_9, p_7), F_5 \vdash \top, \Delta_8, p_7}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_8, p_7} I}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_8, p_7} \text{Cut} \\
\sim \\
\vdash : \Delta_9, p_7 \vdash \top, \Delta_8, p_7 \quad \top_R
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5 \vdash \Delta_6, \top}{\vdash : \Delta_5 \vdash \Delta_6} \top_R \quad \frac{h_4 : \Delta_5 \vdash \Delta_6}{\bullet h_4 : \Delta_5, \top \vdash \Delta_6} \top_L}{\vdash : \Delta_5 \vdash \Delta_6} \text{Cut} \\
\sim \\
\vdash : \Delta_5 \vdash \Delta_6 \quad \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), \top} \top_R \quad \frac{h_6 : \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, \top \vdash \top, \Delta_5} \top_L}{- : \Delta_7 \vdash \top, \Delta_5} \text{Cut}}{\sim} \\
\frac{}{- : \Delta_7 \vdash \top, \Delta_5} \top_R \\
\\
\frac{\frac{\frac{}{\bullet h_2 : \top, \Delta_8 \vdash (\top, \Delta_6), F_5} \top_R \quad \frac{h_7 : F_5, \Delta_8 \vdash \top, \Delta_6}{\bullet h_7 : (\top, \Delta_8), F_5 \vdash \top, \Delta_6} \top_L}{- : \top, \Delta_8 \vdash \top, \Delta_6} \text{Cut}}{\sim} \\
\frac{}{- : \top, \Delta_8 \vdash \top, \Delta_6} \top_R
\end{array}$$

## 8.6 Status of $K$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_8}{\bullet h_1 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), \Box F_8} K \quad \frac{h_9 : \Box \Gamma_6, F_{11}, \Delta_7, \Box F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : (\Box \Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R}{- : \Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_8}{\bullet h_1 : \Delta_7, F_{11}, \Box \Gamma_6 \vdash \Delta_{10}, F_{12}, \Box F_8} \text{ax/W} \quad \frac{h_9 : \Delta_7, F_{11}, \Box \Gamma_6, \Box F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : (\Box \Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \text{ax/W}}{\frac{- : \Delta_7, F_{11}, \Box \Gamma_6 \vdash \Delta_{10}, F_{12}}{- : \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R} \text{hCut} \\
\\
\frac{\frac{h_2 : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box \Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), \Box F_{10}), F_8} K \quad \frac{h_{11} : \Box \Gamma_7, F_8, F_{12}, \Delta_9 \vdash F_{13}, \Delta_{14}, \Box F_{10}}{\bullet h_{11} : (\Box \Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), \Box F_{10}} \rightarrow_R}{- : \Box \Gamma_7, \Delta_9 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), \Box F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{}{- : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box \Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), \Box F_{10}), F_8} \text{ax/W}}{\frac{}{- : \Delta_9, \Box \Gamma_7 \vdash \Delta_{14}, \Box F_{10}, F_{12} \rightarrow F_{13}} K} K
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_8}{\bullet h_1 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), \Box F_8} K \quad \frac{h_9 : \Box \Gamma_6, \Delta_7, \Box F_8 \vdash F_{11}, \Delta_{10} \quad h_9 : \Box \Gamma_6, \Delta_7, \Box F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : (\Box \Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{- : \Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_8}{\bullet h_1 : \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, \Box F_8} \text{ax/W} \quad \frac{h_9 : \Delta_7, \Box \Gamma_6, \Box F_8 \vdash \Delta_{10}, F_{11}}{\bullet h_9 : (\Box \Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{ax/W}}{\frac{- : \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}}{- : \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R} \text{hCut} \\
\\
\frac{\frac{h_2 : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box \Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), \Box F_{10}), F_8} K \quad \frac{h_{11} : \Box \Gamma_7, F_8, \Delta_9 \vdash F_{12}, \Delta_{14}, \Box F_{10} \quad h_{11} : \Box \Gamma_7, F_8, \Delta_9 \vdash F_{13}, \Delta_{14}, \Box F_{10}}{\bullet h_{11} : (\Box \Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), \Box F_{10}} \wedge_R}{- : \Box \Gamma_7, \Delta_9 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), \Box F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{}{- : \text{unbox}(\Box \Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box \Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), \Box F_{10}), F_8} \text{ax/W}}{\frac{}{- : \Delta_9, \Box \Gamma_7 \vdash \Delta_{14}, \Box F_{10}, F_{12} \wedge F_{13}} K} K
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_8}{\bullet h_1 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_{10}, F_{11} \vee F_{12}), \Box F_8} K \quad \frac{h_9 : \Box \Gamma_6, \Delta_7, \Box F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : (\Box \Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box \Gamma_6) \vdash F_8}{\bullet h_1 : \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, F_{12}, \Box F_8} \text{ax/W} \quad \frac{h_9 : \Delta_7, \Box \Gamma_6, \Box F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : (\Box \Gamma_6, \Delta_7), \Box F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{ax/W}}{\frac{- : \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, F_{12}}{- : \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), \Box F_{10}), F_8} \quad K \quad \frac{h_{11} : \Box\Gamma_7, F_8, \Delta_9 \vdash F_{12}, F_{13}, \Delta_{14}, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), \Box F_{10}} \quad \vee_R \\
\hline
- : \Box\Gamma_7, \Delta_9 \vdash (\Delta_{14}, F_{12} \vee F_{13}), \Box F_{10} \\
\hline
\rightsquigarrow \\
\frac{\text{ax/W}}{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}} \quad K \\
\hline
- : \Delta_9, \Box\Gamma_7 \vdash \Delta_{14}, \Box F_{10}, F_{12} \vee F_{13}
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Box\Gamma_6, \Delta_7 \vdash (\perp, \Delta_{10}), \Box F_8} \quad K \quad \frac{h_9 : \Box\Gamma_6, \Delta_7, \Box F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \perp, \Delta_{10}} \quad \perp_R \\
\hline
- : \Box\Gamma_6, \Delta_7 \vdash \perp, \Delta_{10} \\
\hline
\rightsquigarrow \\
\frac{\text{ax/W}}{\bullet h_1 : \Delta_7, \Box\Gamma_6 \vdash \perp, \Delta_{10}, \Box F_8} \quad \frac{\text{ax/W}}{h_9 : \Delta_7, \Box\Gamma_6, \Box F_8 \vdash \perp, \Delta_{10}} \quad \text{hCut} \\
\hline
- : \Delta_7, \Box\Gamma_6 \vdash \perp, \Delta_{10}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_9 \vdash ((\perp, \Delta_{12}), \Box F_{10}), F_8} \quad K \quad \frac{h_{11} : \Box\Gamma_7, F_8, \Delta_9 \vdash \Delta_{12}, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_9), F_8 \vdash (\perp, \Delta_{12}), \Box F_{10}} \quad \perp_R \\
\hline
- : \Box\Gamma_7, \Delta_9 \vdash (\perp, \Delta_{12}), \Box F_{10} \\
\hline
\rightsquigarrow \\
\frac{\text{ax/W}}{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}} \quad K \\
\hline
- : \Delta_9, \Box\Gamma_7 \vdash \perp, \Delta_{12}, \Box F_{10}
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_8}{\bullet h_1 : \Box\Gamma_6, \Delta_7 \vdash (\top, \Delta_{10}), \Box F_8} \quad K \quad \frac{}{\bullet h_9 : (\Box\Gamma_6, \Delta_7), \Box F_8 \vdash \top, \Delta_{10}} \quad \top_R \\
\hline
- : \Box\Gamma_6, \Delta_7 \vdash \top, \Delta_{10} \\
\hline
\rightsquigarrow \\
- : \Delta_7, \Box\Gamma_6 \vdash \top, \Delta_{10} \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_9 \vdash ((\top, \Delta_{12}), \Box F_{10}), F_8} \quad K \quad \frac{}{\bullet h_{11} : (\Box\Gamma_7, \Delta_9), F_8 \vdash (\top, \Delta_{12}), \Box F_{10}} \quad \top_R \\
\hline
- : \Box\Gamma_7, \Delta_9 \vdash (\top, \Delta_{12}), \Box F_{10} \\
\hline
\rightsquigarrow \\
- : \Delta_9, \Box\Gamma_7 \vdash \top, \Delta_{12}, \Box F_{10} \quad \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_8, \Box F_9), \Box F_6} \quad K \quad \frac{h_7 : F_6, \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}) \vdash F_9}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), \Box F_6 \vdash \Delta_8, \Box F_9} \quad K \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_8, \Box F_9 \\
\hline
\rightsquigarrow \\
\frac{\text{ax/W}}{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_6, F_9} \quad \frac{\text{ax/W}}{- : F_6, \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_9} \quad \text{sCut} \\
\hline
\frac{}{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_9} \quad K \\
\hline
- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_8, \Box F_9
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_8, \Box F_9), \Box F_6} \quad K \quad \frac{h_7 : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}) \vdash F_9}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), \Box F_6 \vdash \Delta_8, \Box F_9} \quad K \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_8, \Box F_9 \\
\hline
\rightsquigarrow \\
\frac{\text{ax/W}}{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{11}) \vdash F_9} \quad K \\
\hline
- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_8, \Box F_9
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \text{unbox}(\Box\Gamma_{12}, \Box\Gamma_{14}) \vdash F_8}{\bullet h_2 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{11}, \Box F_{10}), \Box F_8), \Box F_7} \quad K \quad \frac{h_9 : \text{unbox}(\Box\Gamma_{12}), \text{unbox}(\Box\Gamma_{13}), \text{unbox}(\Box F_7) \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box F_7 \vdash (\Delta_{11}, \Box F_{10}), \Box F_8} \quad K \\
\hline
- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{11}, \Box F_{10}), \Box F_8 \\
\hline
\rightsquigarrow \\
\frac{\text{ax/W}}{- : \text{unbox}(\Box\Gamma_{12}), \text{unbox}(\Box\Gamma_{14}) \vdash F_8} \quad K \\
\hline
- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box F_{10}, \Box F_8
\end{array}$$



$$\begin{array}{c}
\frac{h_2 : \text{unbox}(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash F_{10}}{\bullet h_2 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, \Box F_{10}), \Box F_7} K \quad \frac{h_8 : \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}), \text{unbox}(\Box F_7) \vdash F_{10}}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box F_7 \vdash \Delta_9, \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, \Box F_{10} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{13}) \vdash F_{10}}{- : \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, \Box F_{10}} K \quad \text{ax/W}}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, \Box F_{10}} K \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_{12}, \Box\Gamma_{14}) \vdash F_8}{\bullet h_2 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{11}, \Box F_{10}), \Box F_8), F_7} K \quad \frac{h_9 : \text{unbox}(\Box\Gamma_{12}), \text{unbox}(\Box\Gamma_{13}) \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), F_7 \vdash (\Delta_{11}, \Box F_{10}), \Box F_8} K \\
\hline
- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{11}, \Box F_{10}), \Box F_8 \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{12}), \text{unbox}(\Box\Gamma_{13}) \vdash F_{10}}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box F_{10}, \Box F_8} K \quad \text{ax/W}}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{11}, \Box F_{10}), \Box F_8} K \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash F_{10}}{\bullet h_2 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, \Box F_{10}), F_7} K \quad \frac{h_8 : \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_{10}}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), F_7 \vdash \Delta_9, \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, \Box F_{10} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{12}) \vdash F_{10}}{- : \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, \Box F_{10}} K \quad \text{ax/W}}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, \Box F_{10}} K
\end{array}$$

• Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, \Box F_7} K \quad \frac{h_8 : \Box\Gamma_6, \Delta_{12}, \Box F_7 \vdash F_9, \Delta_{11} \quad h_8 : \Box\Gamma_6, F_{10}, \Delta_{12}, \Box F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Box\Gamma_6, \Delta_{12}, F_9 \rightarrow F_{10}), \Box F_7 \vdash \Delta_{11}} K \\
\hline
- : \Box\Gamma_6, \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Delta_{12}, \Box\Gamma_6 \vdash \Delta_{11}, F_9, \Box F_7} K \quad \frac{h_8 : \Delta_{12}, \Box\Gamma_6, \Box F_7 \vdash \Delta_{11}, F_9}{\bullet h_8 : \Delta_{12}, F_{10}, \Box\Gamma_6 \vdash \Delta_{11}, \Box F_7} K \quad \frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Delta_{12}, F_{10}, \Box\Gamma_6 \vdash \Delta_{11}, \Box F_7} K \quad \frac{h_8 : \Delta_{12}, F_{10}, \Box\Gamma_6, \Box F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, \Box\Gamma_6 \vdash \Delta_{11}} K}{- : \Delta_{12}, \Box\Gamma_6 \vdash \Delta_{11}, F_9} \text{hCut} \quad \frac{- : \Delta_{12}, F_{10}, \Box\Gamma_6 \vdash \Delta_{11}}{- : \Delta_{12}, \Box\Gamma_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ax/W} \rightarrow_L \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, \Box F_{10}), F_{12} \rightarrow F_{13}} K \quad \frac{h_{11} : \Box\Gamma_7, \Delta_8 \vdash F_{12}, \Delta_9, \Box F_{10} \quad h_{11} : \Box\Gamma_7, F_{13}, \Delta_8 \vdash \Delta_9, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), F_{12} \rightarrow F_{13} \vdash \Delta_9, \Box F_{10}} K \\
\hline
- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, \Box F_{10} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, \Box F_{10}} K \quad \text{ax/W}}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, \Box F_{10}} K \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_9, \Box F_{10}), F_8} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, \Delta_{14} \vdash F_{12}, \Delta_9, \Box F_{10} \quad h_{11} : \Box\Gamma_7, F_8, F_{13}, \Delta_{14} \vdash \Delta_9, \Box F_{10}}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13}), F_8 \vdash \Delta_9, \Box F_{10}} K \\
\hline
- : \Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_9, \Box F_{10} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_{14}, \Box\Gamma_7, F_{12} \rightarrow F_{13} \vdash \Delta_9, \Box F_{10}} K \quad \text{ax/W}}{- : \Box\Gamma_7, \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_9, \Box F_{10}} K
\end{array}$$

• Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, \Box F_7} K \quad \frac{h_8 : \Box\Gamma_6, F_9, F_{10}, \Delta_{12}, \Box F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Box\Gamma_6, \Delta_{12}, F_9 \wedge F_{10}), \Box F_7 \vdash \Delta_{11}} K \\
\hline
- : \Box\Gamma_6, \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\sim \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9, \Box\Gamma_6 \vdash \Delta_{11}, \Box F_7} K \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, \Box\Gamma_6, \Box F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_9, \Box\Gamma_6 \vdash \Delta_{11}} K}{- : \Delta_{12}, F_{10}, F_9, \Box\Gamma_6 \vdash \Delta_{11}} \text{hCut} \quad \frac{- : \Delta_{12}, F_{10}, F_9, \Box\Gamma_6 \vdash \Delta_{11}}{- : \Delta_{12}, \Box\Gamma_6, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{ax/W} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, [\Box F_{10}], F_{12} \wedge F_{13})} K \quad \frac{h_{11} : \Box\Gamma_7, F_{12}, F_{13}, \Delta_8 \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), F_{12} \wedge F_{13} \vdash \Delta_9, [\Box F_{10}]} \wedge_L \\
\hline
- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, [\Box F_{10}] \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, [\Box F_{10}]} \text{ax/w}}{K} \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_9, [\Box F_{10}], F_8)} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, F_{12}, F_{13}, \Delta_{14} \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \wedge F_{13}), F_8 \vdash \Delta_9, [\Box F_{10}]} \wedge_L \\
\hline
- : \Box\Gamma_7, \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_9, [\Box F_{10}] \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_{14}, \Box\Gamma_7, F_{12} \wedge F_{13} \vdash \Delta_9, [\Box F_{10}]} \text{ax/w}}{K}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, [\Box F_7]} K \quad \frac{h_8 : \Box\Gamma_6, F_9, \Delta_{12}, [\Box F_7] \vdash \Delta_{11} \quad h_8 : \Box\Gamma_6, F_{10}, \Delta_{12}, [\Box F_7] \vdash \Delta_{11}}{\bullet h_8 : (\Box\Gamma_6, \Delta_{12}, F_9 \vee F_{10}), [\Box F_7] \vdash \Delta_{11}} \vee_L \\
\hline
- : \Box\Gamma_6, \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Delta_{12}, F_9, \Box\Gamma_6 \vdash \Delta_{11}, [\Box F_7]} \text{ax/w}}{K} \quad \frac{h_8 : \Delta_{12}, F_9, \Box\Gamma_6, [\Box F_7] \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, \Box\Gamma_6 \vdash \Delta_{11}, [\Box F_7]} \text{ax/w} \quad \frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Delta_{12}, F_{10}, \Box\Gamma_6 \vdash \Delta_{11}, [\Box F_7]} \text{ax/w} \\
\hline
- : \Delta_{12}, F_9, \Box\Gamma_6 \vdash \Delta_{11} \quad \text{hCut} \quad - : \Delta_{12}, F_{10}, \Box\Gamma_6 \vdash \Delta_{11} \quad \text{hCut} \\
\hline
- : \Delta_{12}, \Box\Gamma_6, F_9 \vee F_{10} \vdash \Delta_{11} \quad \vee_L \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, [\Box F_{10}], F_{12} \vee F_{13})} K \quad \frac{h_{11} : \Box\Gamma_7, F_{12}, \Delta_8 \vdash \Delta_9, [\Box F_{10}] \quad h_{11} : \Box\Gamma_7, F_{13}, \Delta_8 \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), F_{12} \vee F_{13} \vdash \Delta_9, [\Box F_{10}]} \vee_L \\
\hline
- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, [\Box F_{10}] \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, [\Box F_{10}]} \text{ax/w}}{K} \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_9, [\Box F_{10}], F_8)} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, F_{12}, \Delta_{14} \vdash \Delta_9, [\Box F_{10}] \quad h_{11} : \Box\Gamma_7, F_8, F_{13}, \Delta_{14} \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : (\Box\Gamma_7, \Delta_{14}, F_{12} \vee F_{13}), F_8 \vdash \Delta_9, [\Box F_{10}]} \vee_L \\
\hline
- : \Box\Gamma_7, \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_9, [\Box F_{10}] \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_{14}, \Box\Gamma_7, F_{12} \vee F_{13} \vdash \Delta_9, [\Box F_{10}]} \text{ax/w}}{K}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \perp, \Delta_{10} \vdash \Delta_9, [\Box F_7]} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_6, \perp, \Delta_{10}), [\Box F_7] \vdash \Delta_9} \perp_L \\
\hline
- : \Box\Gamma_6, \perp, \Delta_{10} \vdash \Delta_9 \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{}{- : \perp, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9} \perp_L \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, [\Box F_{10}], \perp)} K \quad \frac{}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), \perp \vdash \Delta_9, [\Box F_{10}]} \perp_L \\
\hline
- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, [\Box F_{10}] \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, [\Box F_{10}]} \text{ax/w}}{K} \\
\hline
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \perp, \Delta_{12} \vdash (\Delta_9, [\Box F_{10}], F_8)} K \quad \frac{}{\bullet h_{11} : (\Box\Gamma_7, \perp, \Delta_{12}), F_8 \vdash \Delta_9, [\Box F_{10}]} \perp_L \\
\hline
- : \Box\Gamma_7, \perp, \Delta_{12} \vdash \Delta_9, [\Box F_{10}] \quad \text{Cut} \\
\hline
\sim\!\!\sim \\
\frac{}{- : \perp, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_9, [\Box F_{10}]} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), [\Box F_7]} K \quad \frac{\bullet h_8 : (\Box\Gamma_6, \Delta_{11}, p_9), [\Box F_7] \vdash \Delta_{10}, p_9}{- : \Box\Gamma_6, \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I}{- : \Box\Gamma_6, \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{Cut} \\
\frac{}{- : \Delta_{11}, \Box\Gamma_6, p_9 \vdash \Delta_{10}, p_9} I \\
\frac{}{- : \Delta_{11}, \Box\Gamma_6, p_9 \vdash \Delta_{10}, p_9} I \\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_9}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash ((\Delta_{12}, p_{11}), [\Box F_9]), p_{11}} K \quad \frac{\bullet h_{10} : (\Box\Gamma_7, \Delta_8), p_{11} \vdash (\Delta_{12}, p_{11}), [\Box F_9]}{- : \Box\Gamma_7, \Delta_8 \vdash (\Delta_{12}, p_{11}), [\Box F_9]} I}{- : \Box\Gamma_7, \Delta_8 \vdash (\Delta_{12}, p_{11}), [\Box F_9]} \text{Cut} \\
\frac{}{- : \text{unbox}(\Box\Gamma_7) \vdash F_9} \text{ax/W} \\
\frac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{12}, p_{11}, [\Box F_9]} K \\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_9}{\bullet h_2 : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), [\Box F_9]), F_8} K \quad \frac{\bullet h_{10} : (\Box\Gamma_7, \Delta_{13}, p_{11}), F_8 \vdash (\Delta_{12}, p_{11}), [\Box F_9]}{- : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), [\Box F_9]} I}{- : \Box\Gamma_7, \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), [\Box F_9]} \text{Cut} \\
\frac{}{- : \Delta_{13}, \Box\Gamma_7, p_{11} \vdash \Delta_{12}, p_{11}, [\Box F_9]} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_6) \vdash F_7}{\bullet h_1 : \Box\Gamma_6, \top, \Delta_{10} \vdash \Delta_9, [\Box F_7]} K \quad \frac{h_8 : \Box\Gamma_6, \Delta_{10}, [\Box F_7] \vdash \Delta_9}{\bullet h_8 : (\Box\Gamma_6, \top, \Delta_{10}), [\Box F_7] \vdash \Delta_9} \top_L}{- : \Box\Gamma_6, \top, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\frac{}{- : \Box\Gamma_6, \top, \Delta_{10} \vdash \Delta_9} \text{ax/W} \\
\frac{\bullet h_1 : \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9, [\Box F_7]}{- : \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, \Box\Gamma_6, [\Box F_7] \vdash \Delta_9}{\bullet h_8 : (\Box\Gamma_6, \top, \Delta_{10}), [\Box F_7] \vdash \Delta_9} \text{hCut}}{- : \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9} \\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, [\Box F_{10}]), \top} K \quad \frac{h_{11} : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : (\Box\Gamma_7, \Delta_8), \top \vdash \Delta_9, [\Box F_{10}]} \top_L}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, [\Box F_{10}]} \text{Cut} \\
\frac{}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, [\Box F_{10}]} \text{ax/W} \\
\frac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, [\Box F_{10}]} \text{ax/W} \\
\frac{h_2 : \text{unbox}(\Box\Gamma_7) \vdash F_{10}}{\bullet h_2 : \Box\Gamma_7, \top, \Delta_{12} \vdash (\Delta_9, [\Box F_{10}]), F_8} K \quad \frac{h_{11} : \Box\Gamma_7, F_8, \Delta_{12} \vdash \Delta_9, [\Box F_{10}]}{\bullet h_{11} : (\Box\Gamma_7, \top, \Delta_{12}), F_8 \vdash \Delta_9, [\Box F_{10}]} \top_L}{- : \Box\Gamma_7, \top, \Delta_{12} \vdash \Delta_9, [\Box F_{10}]} \text{Cut} \\
\frac{}{- : \Box\Gamma_7, \top, \Delta_{12} \vdash \Delta_9, [\Box F_{10}]} \text{ax/W} \\
\frac{}{- : \text{unbox}(\Box\Gamma_7) \vdash F_{10}} K \\
\frac{}{- : \top, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_9, [\Box F_{10}]} K
\end{array}$$

## 8.7 Status of $\rightarrow_L$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_3 : \Delta_8 \vdash F_7, F_9, \Delta_{12}, F_{13} \rightarrow F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \rightarrow F_{14}}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_7} \rightarrow_L \quad \frac{h_{11} : F_7, F_{13}, \Delta_8, F_9 \rightarrow F_{10} \vdash F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \text{Cut} \\
\frac{}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \text{inv-th/ax} \\
\frac{h_3 : \Delta_8, F_{13} \vdash \Delta_{12}, F_{14}, F_7, F_9}{\bullet h_3 : \Delta_8, F_{13}, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{14}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_8, F_{10}, F_{13} \vdash \Delta_{12}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_{13}, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{14}, F_7} \text{inv-th/ax}}{- : \Delta_8, F_{13}, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{14}, F_7} \rightarrow_L \\
\frac{}{- : \Delta_8, F_{13}, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{14}} \text{ax/W} \quad \frac{}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \text{hCut}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_8 \vdash F_7, F_9, \Delta_{12}, F_{13} \wedge F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \wedge F_{14}}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_7} \rightarrow_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \rightarrow F_{10} \vdash F_{13}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}) \vdash F_{13}, \Delta_{12}} \rightarrow_R}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_3 : \Delta_8 \vdash \Delta_{12}, F_{13}, F_7, F_9}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_7}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_7} \text{inv-th/ax}}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_7} \rightarrow_L \quad \frac{h_{11} : \Delta_8, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}) \vdash \Delta_{12}, F_{13}} \text{ax/W}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}} \text{hCut} \\
\sim \\
\frac{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{Cut}
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_8 \vdash F_7, F_9, \Delta_{12}, F_{13} \vee F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14}}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_7} \rightarrow_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \rightarrow F_{10} \vdash F_{13}, F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \Delta_{12}, F_{13} \vee F_{14}} \rightarrow_R}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_3 : \Delta_8 \vdash \Delta_{12}, F_{13}, F_{14}, F_7, F_9}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \text{inv-th/ax}}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \rightarrow_L \quad \frac{h_{11} : \Delta_8, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}) \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \text{hCut} \\
\sim \\
\frac{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_8 \vdash F_7, F_9, \perp, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \perp, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\perp, \Delta_{12}), F_7} \rightarrow_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \rightarrow F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \perp, \Delta_{12}} \rightarrow_R}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}, F_7}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}, F_7} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}) \vdash \perp, \Delta_{12}} \text{ax/W}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \perp, \Delta_{12}} \text{hCut}
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_8 \vdash F_7, F_9, \top, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \top, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \rightarrow F_{10} \vdash (\top, \Delta_{12}), F_7} \rightarrow_L \quad \frac{h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \top, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \rightarrow F_{10}), F_7 \vdash \top, \Delta_{12}} \rightarrow_R}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \top, \Delta_{12}} \text{Cut} \\
\sim \\
\frac{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \top, \Delta_{12}}{- : \Delta_8, F_9 \rightarrow F_{10} \vdash \top, \Delta_{12}} \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{\frac{h_3 : \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, F_8, \Delta_{11}, \Box F_{12} \quad h_3 : F_9, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, \Box F_{12}}{\bullet h_3 : (\Box \Gamma_{13}, \Delta_{14}), F_8 \rightarrow F_9 \vdash (\Delta_{11}, \Box F_{12}), \Box F_7} \rightarrow_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : ((\Box \Gamma_{13}, \Delta_{14}), F_8 \rightarrow F_9), \Box F_7 \vdash \Delta_{11}, \Box F_{12}} \rightarrow_R}{- : (\Box \Gamma_{13}, \Delta_{14}), F_8 \rightarrow F_9 \vdash \Delta_{11}, \Box F_{12}} \text{Cut} \\
\sim \\
\frac{\frac{\frac{h_3 : \Delta_{14}, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, F_8, \Box F_{12}}{\bullet h_3 : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{12}} \text{ax/W} \quad \frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{13}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{12}} \text{ax/W}}{\bullet h_{10} : \Box F_7, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{12}} \text{K} \\
\sim \\
\frac{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{12}}{- : \Delta_{14}, \Box \Gamma_{13}, F_8 \rightarrow F_9 \vdash \Delta_{11}, \Box F_{12}} \text{hCut} \\
\sim \\
\frac{\frac{h_3 : \Box \Gamma_{11}, \Delta_{14} \vdash F_7, F_8, \Delta_{12}, \Box F_{13} \quad h_3 : F_9, \Box \Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, \Box F_{13}}{\bullet h_3 : (\Box \Gamma_{11}, \Delta_{14}), F_8 \rightarrow F_9 \vdash (\Delta_{12}, \Box F_{13}), F_7} \rightarrow_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}}{\bullet h_{10} : ((\Box \Gamma_{11}, \Delta_{14}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{12}, \Box F_{13}} \rightarrow_R}{- : (\Box \Gamma_{11}, \Delta_{14}), F_8 \rightarrow F_9 \vdash \Delta_{12}, \Box F_{13}} \text{Cut} \\
\sim \\
\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}}{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, \Box F_{13}} \text{ax/W} \\
\sim \\
\frac{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, \Box F_{13}}{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, \Box F_{13}} K
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_3 : \Delta_7 \vdash F_{11} \rightarrow F_{12}, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \rightarrow F_{12}, \Delta_{13} \rightarrow_L \quad \frac{h_{10} : \Delta_7, F_8 \rightarrow F_9 \vdash F_{11}, \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13} \rightarrow_L}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \rightarrow_L}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{Cut} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_{11} \vdash \Delta_{13}, F_{12}, F_8} \text{inv-th/ax} \quad \frac{- : \Delta_7, F_{11}, F_9 \vdash \Delta_{13}, F_{12}}{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{12}} \text{inv-th/ax} \rightarrow_L \quad \frac{- : \Delta_7, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{ax/W} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{sCut} \\
\frac{h_3 : \Delta_{14}, F_{11} \rightarrow F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \rightarrow F_{12} \vdash F_7, \Delta_{13} \rightarrow_L \quad \frac{h_{10} : F_7, \Delta_{14}, F_8 \rightarrow F_9 \vdash F_{11}, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7} \rightarrow_L}{- : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{Cut} \\
\frac{h_3 : \Delta_{14} \vdash \Delta_{13}, F_{11}, F_7, F_8 \text{ inv-th/ax} \quad h_3 : \Delta_{14}, F_9 \vdash \Delta_{13}, F_{11}, F_7 \text{ inv-th/ax} \rightarrow_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}}{\bullet h_3 : \Delta_{14}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}, F_7} \text{ax/W}}{- : \Delta_{14}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \text{hCut} \\
\frac{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{hCut} \\
\frac{h_3 : \Delta_8 \vdash F_7, F_{10}, \Delta_{12} \quad h_3 : F_{11}, \Delta_8 \vdash F_7, \Delta_{12} \rightarrow_L \quad \frac{h_9 : F_7, \Delta_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_7, F_{11}, \Delta_8 \vdash \Delta_{12} \rightarrow_L}{\bullet h_3 : \Delta_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \rightarrow_L}{- : \Delta_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \text{Cut} \\
\frac{h_3 : \Delta_8 \vdash \Delta_{12}, F_{10}, F_7 \text{ ax/W} \quad \frac{h_9 : \Delta_8, F_7 \vdash \Delta_{12}, F_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{12}, F_{10}} \text{ax/W} \quad \frac{h_3 : \Delta_8, F_{11} \vdash \Delta_{12}, F_7 \text{ ax/W} \quad \frac{h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{12}}{\bullet h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{12}} \text{ax/W}}{- : \Delta_8 \vdash \Delta_{12}, F_{10}} \text{hCut} \\
\frac{- : \Delta_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}}{- : \Delta_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_3 : \Delta_7 \vdash F_{11} \wedge F_{12}, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \wedge F_{12}, \Delta_{13} \rightarrow_L \quad \frac{h_{10} : F_{11}, F_{12}, \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \rightarrow_L}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \wedge_L \text{Cut} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_{11}, F_{12} \vdash \Delta_{13}, F_8} \text{inv-th/ax} \quad \frac{- : \Delta_7, F_{11}, F_{12} \vdash \Delta_{13}, F_8}{\bullet h_{10} : \Delta_7, F_{11} \wedge F_{12} \vdash \Delta_{13}, F_8} \wedge_L \text{hCut} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{- : \Delta_7, F_9 \vdash \Delta_{13}} \text{ax/W} \quad \frac{- : \Delta_7, F_{11}, F_{12}, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L \text{hCut} \\
\frac{- : \Delta_7 \vdash \Delta_{13}, F_8}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \rightarrow_L \\
\frac{h_3 : \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, \Delta_{13} \rightarrow_L \quad \frac{h_{10} : F_7, F_{11}, F_{12}, \Delta_{14}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7} \rightarrow_L}{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}} \wedge_L \text{Cut} \\
\frac{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_{11}, F_{12} \vdash \Delta_{13}, F_7, F_8} \text{inv-th/ax} \quad \frac{- : \Delta_{14}, F_{11}, F_{12}, F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax} \rightarrow_L \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{ax/W} \\
\frac{- : \Delta_{14}, F_8 \rightarrow F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}}{- : \Delta_{14}, F_8 \rightarrow F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L \text{hCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_3 : \Delta_7 \vdash F_{11} \vee F_{12}, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \vee F_{12}, \Delta_{13} \rightarrow_L \quad \frac{h_{10} : F_{11}, \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \rightarrow_L}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \vee_L \text{Cut} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \text{inv-th/ax} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{13}, F_{11}, F_{12}}{- : \Delta_7, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \text{inv-th/ax} \rightarrow_L \quad \frac{- : \Delta_7, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \text{ax/W} \\
\frac{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{sCut} \quad \frac{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \text{ax/sCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7} \rightarrow_L \quad \frac{h_{10} : F_7, F_{11}, \Delta_{14}, F_8 \rightarrow F_9 \vdash}{\bullet h_{10} : ((\Delta_{14}, F_{11}), F_8 \rightarrow F_9 \vdash \Delta_{13})} \\
\frac{}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}} \rightsquigarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7, F_8}{- : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \text{ ax/W} \quad \frac{\frac{h_{10} : \Delta_{14}, F_{11}, F_7 \vdash \Delta_{13}, F_8}{\bullet h_{10} : \Delta_{14}, F_7, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \text{ inv-th/ax} \quad \frac{h_{10} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_{13}, F_8}{\bullet h_{10} : \Delta_{14}, F_7, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \text{ inv-th/ax}}{\frac{}{- : \Delta_{14}, F_7, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \vee_L} \text{ hCut} \\
\frac{}{- : \Delta_{14}, F_8 \rightarrow F_9, F_{11} \vee F_{12} \vdash \Delta_{13}}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7 \vdash \perp, F_8, \Delta_{11} \quad h_3 : F_9, \Delta_7 \vdash \perp, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}, \perp} \rightarrow_L \quad \frac{}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), \perp \vdash \Delta_{11}} \perp_L}{\frac{}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ Cut}} \rightsquigarrow \\
\frac{\frac{h_3 : \Delta_7 \vdash \perp, \Delta_{11}, F_8}{- : \Delta_7 \vdash \Delta_{11}, F_8} \text{ ax/W} \quad \frac{\frac{\bullet h_{10} : \perp, \Delta_7 \vdash \Delta_{11}, F_8}{- : \Delta_7 \vdash \Delta_{11}, F_8} \perp_L}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ hCut} \quad \frac{\frac{h_3 : \Delta_7, F_9 \vdash \perp, \Delta_{11}}{- : \Delta_7, F_9 \vdash \Delta_{11}} \text{ ax/W} \quad \frac{\frac{\bullet h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}}{- : \Delta_7, F_9 \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ hCut}} \\
\frac{}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \\
\frac{\frac{h_3 : \perp, \Delta_{12} \vdash F_7, F_8, \Delta_{11} \quad h_3 : F_9, \perp, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}, F_7} \rightarrow_L \quad \frac{}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{11}} \perp_L}{\frac{}{- : (\perp, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ Cut}} \rightsquigarrow \\
\frac{}{- : \perp, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7 \vdash p_{11}, F_8, \Delta_{12}, p_{11} \quad h_3 : F_9, \Delta_7 \vdash p_{11}, \Delta_{12}, p_{11}}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash (\Delta_{12}, p_{11}), p_{11}} \rightarrow_L \quad \frac{}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), p_{11} \vdash \Delta_{12}, p_{11}} I}{\frac{}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}, p_{11}} \text{ Cut}} \rightsquigarrow \\
\frac{\frac{h_3 : \Delta_7 \vdash \Delta_{12}, F_8, p_{11}, p_{11}}{- : \Delta_7 \vdash \Delta_{12}, F_8, p_{11}} \text{ ax/W} \quad \frac{\frac{\bullet h_{10} : \Delta_7, p_{11} \vdash \Delta_{12}, F_8, p_{11}}{- : \Delta_7, p_{11} \vdash \Delta_{12}, F_8, p_{11}} I}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}, p_{11}} \text{ hCut} \quad \frac{\frac{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}, p_{11}}{- : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}} \text{ ax/W} \quad \frac{\frac{\bullet h_{10} : \Delta_7, F_9, p_{11} \vdash \Delta_{12}, p_{11}}{- : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}, p_{11}} \text{ hCut}} \\
\frac{}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}, p_{11}} \\
\frac{\frac{h_3 : \Delta_{13}, p_{11} \vdash F_7, F_8, \Delta_{12}, p_{11} \quad h_3 : F_9, \Delta_{13}, p_{11} \vdash F_7, \Delta_{12}, p_{11}}{\bullet h_3 : (\Delta_{13}, p_{11}), F_8 \rightarrow F_9 \vdash (\Delta_{12}, p_{11}), F_7} \rightarrow_L \quad \frac{}{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{12}, p_{11}} I}{\frac{}{- : (\Delta_{13}, p_{11}), F_8 \rightarrow F_9 \vdash \Delta_{12}, p_{11}} \text{ Cut}} \rightsquigarrow \\
\frac{}{- : \Delta_{13}, p_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, p_{11}} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7 \vdash \top, F_8, \Delta_{11} \quad h_3 : F_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}, \top} \rightarrow_L \quad \frac{h_{10} : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), \top \vdash \Delta_{11}} \top_L}{\frac{}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ Cut}} \rightsquigarrow \\
\frac{}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ ax/W} \\
\frac{\frac{h_3 : \top, \Delta_{12} \vdash F_7, F_8, \Delta_{11} \quad h_3 : F_9, \top, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\top, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}, F_7} \rightarrow_L \quad \frac{h_{10} : F_7, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{11}} \top_L}{\frac{}{- : (\top, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ Cut}} \rightsquigarrow \\
\frac{\frac{\bullet h_3 : \top, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}, F_7}{- : \top, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ ax/W} \quad \frac{h_{10} : \top, \Delta_{12}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{11}}{- : \top, \Delta_{12}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ hCut}}
\end{array}$$

## 8.8 Status of $\wedge_L$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \rightarrow F_{14}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_7} \wedge_L \quad \frac{h_{11} : F_7, F_{13}, \Delta_8, F_9 \wedge F_{10} \vdash F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_8, F_{10}, F_{13}, F_9 \vdash \Delta_{12}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_{13}, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{14}, F_7} \wedge_L \quad \frac{h_{11} : \Delta_8, F_{13}, F_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{14}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \text{ax/W} \quad \text{hCut}}{- : \Delta_8, F_{13}, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{14}} \rightarrow_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \wedge F_{14}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_7} \wedge_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \wedge F_{10} \vdash F_{13}, \Delta_{12} \quad h_{11} : F_7, \Delta_8, F_9 \wedge F_{10} \vdash F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_8, F_{10}, F_9 \vdash \Delta_{12}, F_7, F_{13} \wedge F_{14}}{\bullet h_3 : \Delta_8, F_{10}, F_9 \vdash \Delta_{12}, F_7, F_{13} \wedge F_{14}} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_{10}, F_7, F_9 \vdash \Delta_{12}, F_{13}}{\bullet h_{11} : \Delta_8, F_{10}, F_7, F_9 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{inv-th/ax} \quad \text{hCut}}{- : \Delta_8, F_{10}, F_9 \vdash \Delta_{12}, F_7, F_{13} \wedge F_{14}} \wedge_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_7} \wedge_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \wedge F_{10} \vdash F_{13}, F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_8, F_{10}, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : \Delta_8, F_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W} \quad \text{hCut}}{- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \vee_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \perp, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\perp, \Delta_{12}), F_7} \wedge_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \wedge F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \perp, \Delta_{12}} \perp_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12} \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}, F_7}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}, F_7} \text{ax/W} \quad \frac{h_{11} : \Delta_8, F_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}{\bullet h_{11} : \Delta_8, F_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}} \text{hCut}}{- : \Delta_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_3 : F_9, F_{10}, \Delta_8 \vdash F_7, \top, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \wedge F_{10} \vdash (\top, \Delta_{12}), F_7} \wedge_L \quad \frac{h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \top, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \wedge F_{10}), F_7 \vdash \top, \Delta_{12}} \top_R \\
\hline
- : \Delta_8, F_9 \wedge F_{10} \vdash \top, \Delta_{12} \quad \text{Cut} \\
\rightsquigarrow \\
- : \Delta_8, F_9 \wedge F_{10} \vdash \top, \Delta_{12} \quad \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}]}{\bullet h_3 : (\Box \Gamma_{13}, \Delta_{14}), F_8 \wedge F_9 \vdash (\Delta_{11}, [\Box F_{12}]), \Box F_7} \wedge_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : ((\Box \Gamma_{13}, \Delta_{14}), F_8 \wedge F_9), \Box F_7 \vdash \Delta_{11}, [\Box F_{12}]} K \\
\hline
- : (\Box \Gamma_{13}, \Delta_{14}), F_8 \wedge F_9 \vdash \Delta_{11}, [\Box F_{12}] \quad \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_8, F_9, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}]}{\bullet h_3 : \Delta_{14}, F_8, F_9, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, [\Box F_{12}]} \text{ax/W} \quad \frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{13}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{14}, F_8, F_9, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{12}]} K \quad \text{hCut}}{- : \Delta_{14}, F_8, F_9, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{12}]} \wedge_L \\
\hline
- : \Delta_{14}, \Box \Gamma_{13}, F_8 \wedge F_9 \vdash \Delta_{11}, [\Box F_{12}]
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Box \Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, [\Box F_{13}}{\bullet h_3 : (\Box \Gamma_{11}, \Delta_{14}), F_8 \wedge F_9 \vdash (\Delta_{12}, [\Box F_{13}), F_7} \wedge_L \quad \frac{h_{10} : unbox(\Box \Gamma_{11}) \vdash F_{13}}{\bullet h_{10} : ((\Box \Gamma_{11}, \Delta_{14}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, [\Box F_{13}} K \\
\hline
- : (\Box \Gamma_{11}, \Delta_{14}), F_8 \wedge F_9 \vdash \Delta_{12}, [\Box F_{13} \\
\hline
\sim \\
\frac{- : unbox(\Box \Gamma_{11}) \vdash F_{13}}{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, [\Box F_{13}} ax/W \quad K
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Delta_7 \vdash F_{11} \rightarrow F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \wedge_L \quad \frac{h_{10} : \Delta_7, F_8 \wedge F_9 \vdash F_{11}, \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \rightarrow_L \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \\
\hline
\sim \\
\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} ax/W \quad \frac{h_{10} : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : F_{12}, \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}} inv-th/ax \rightarrow_L \\
\hline
- : \Delta_7, F_8, F_9 \vdash \Delta_{13} \quad hCut \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \quad \wedge_L \\
\hline
\frac{h_3 : F_8, F_9, \Delta_{14}, F_{11} \rightarrow F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \wedge_L \quad \frac{h_{10} : F_7, \Delta_{14}, F_8 \wedge F_9 \vdash F_{11}, \Delta_{13} \quad h_{10} : F_7, F_{12}, \Delta_{14}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \rightarrow_L \\
\hline
- : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9 \vdash \Delta_{13} \\
\hline
\sim \\
\frac{h_3 : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_7} ax/W \quad \frac{h_{10} : \Delta_{14}, F_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : F_7, F_{12}, \Delta_{14}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}} inv-th/ax \rightarrow_L \\
\hline
- : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13} \quad hCut \\
\hline
- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \wedge F_9 \vdash \Delta_{13} \quad \wedge_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \Delta_7 \vdash F_{11} \wedge F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \wedge_L \quad \frac{h_{10} : F_{11}, F_{12}, \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \\
\hline
\sim \\
\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} ax/W \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} inv-th/ax \wedge_L \\
\hline
- : \Delta_7, F_8, F_9 \vdash \Delta_{13} \quad hCut \\
\hline
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \quad \wedge_L \\
\hline
\frac{h_3 : F_8, F_9, \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \wedge_L \quad \frac{h_{10} : F_7, F_{11}, F_{12}, \Delta_{14}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \wedge_L \\
\hline
- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9 \vdash \Delta_{13} \\
\hline
\sim \\
\frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}, F_7} inv-th/ax \wedge_L \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{13}} ax/W \\
\hline
- : \Delta_{14}, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13} \quad hCut \\
\hline
- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \wedge F_9 \vdash \Delta_{13} \quad \wedge_L \\
\hline
\frac{h_3 : F_{10}, F_{11}, \Delta_8 \vdash F_7, \Delta_{12}}{\bullet h_3 : \Delta_8, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7} \wedge_L \quad \frac{h_9 : F_7, F_{10}, F_{11}, \Delta_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_8, F_{10} \wedge F_{11}), F_7 \vdash \Delta_{12}} \wedge_L \\
\hline
- : \Delta_8, F_{10} \wedge F_{11} \vdash \Delta_{12} \\
\hline
\sim \\
\frac{h_3 : \Delta_8, F_{10}, F_{11} \vdash \Delta_{12}, F_7}{\bullet h_3 : \Delta_8, F_{10}, F_{11} \vdash \Delta_{12}, F_7} ax/W \quad \frac{h_9 : \Delta_8, F_{10}, F_{11}, F_7 \vdash \Delta_{12}}{\bullet h_9 : \Delta_8, F_{10}, F_{11}, F_7 \vdash \Delta_{12}} H \\
\hline
- : \Delta_8, F_{10}, F_{11} \vdash \Delta_{12} \quad hCut \\
\hline
- : \Delta_8, F_{10} \wedge F_{11} \vdash \Delta_{12} \quad \wedge_L
\end{array}$$

- Case rule  $\vee_L$



$$\begin{array}{c}
\frac{\frac{h_3 : F_8, F_9, \Delta_7 \vdash F_{11} \vee F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \wedge_L \quad \frac{h_{10} : F_{11}, \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \vee_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_7, F_{11}, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_7, F_{12}, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_7, F_8, F_9 \vdash \Delta_{13} \quad - : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{hCut} \\
\\
\frac{\frac{h_3 : F_8, F_9, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \wedge_L \quad \frac{h_{10} : F_7, F_{11}, \Delta_{14}, F_8 \wedge F_9 \vdash \Delta_{13} \quad h_{10} : F_7, F_{12}, \Delta_{14}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \vee_L}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_8 \wedge F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_{14}, F_{11}, F_7, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{14}, F_{12}, F_7, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13} \quad - : \Delta_{14}, F_8 \wedge F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_{14}, F_8 \wedge F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{hCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, F_9, \Delta_7 \vdash \perp, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \perp} \wedge_L \quad \frac{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \perp \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \perp \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \perp, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \perp} \text{ax/W} \quad \frac{\bullet h_{10} : \perp, \Delta_7, F_8, F_9 \vdash \Delta_{11}}{\bullet h_{10} : \perp, \Delta_7, F_8, F_9 \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8, F_9 \vdash \Delta_{11} \quad - : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{hCut} \\
\\
\frac{\frac{h_3 : F_8, F_9, \perp, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \wedge_L \quad \frac{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}} \perp_L}{- : (\perp, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightsquigarrow \\
\frac{\bullet h_{10} : (\perp, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\perp, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, F_9, \Delta_7 \vdash p_{11}, \Delta_{12}, p_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash (\Delta_{12}, p_{11}), p_{11}} \wedge_L \quad \frac{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), p_{11} \vdash \Delta_{12}, p_{11}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), p_{11} \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \text{Cut} \\
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{12}, p_{11}, p_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}, p_{11}} \text{ax/W} \quad \frac{\bullet h_{10} : \Delta_7, F_8, F_9, p_{11} \vdash \Delta_{12}, p_{11}}{\bullet h_{10} : \Delta_7, F_8, F_9, p_{11} \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_8, F_9 \vdash \Delta_{12}, p_{11} \quad - : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \text{hCut} \\
\\
\frac{\frac{h_3 : F_8, F_9, \Delta_{13}, p_{11} \vdash F_7, \Delta_{12}, p_{11}}{\bullet h_3 : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash (\Delta_{12}, p_{11}), F_7} \wedge_L \quad \frac{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, p_{11}}{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, p_{11}} I}{- : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \text{Cut} \\
\rightsquigarrow \\
\frac{\bullet h_{10} : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, F_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \top} \wedge_L \quad \frac{h_{10} : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \top \vdash \Delta_{11}} \top_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightsquigarrow \\
\frac{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \top \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \top \vdash \Delta_{11}} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : F_8, F_9, \top, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \wedge_L \quad \frac{h_{10} : F_7, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}} \top_L \\
\hline
- : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{\bullet h_3 : \top, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}, F_7}{- : \top, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}} \text{ax/W}}{\frac{h_{10} : \top, \Delta_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{11}}{- : \top, \Delta_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{hCut}} \text{hCut}
\end{array}$$

## 8.9 Status of $\vee_L$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \rightarrow F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \rightarrow F_{14}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_7} \vee_L \quad \frac{h_{11} : F_7, F_{13}, \Delta_8, F_9 \vee F_{10} \vdash F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_8, F_{13}, F_9 \vdash \Delta_{12}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}, F_7} \text{inv-th/ax}}{\frac{h_3 : \Delta_8, F_{10}, F_{13} \vdash \Delta_{12}, F_{14}, F_7}{- : \Delta_8, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}, F_7} \vee_L} \text{inv-th/ax} \\
\frac{h_{11} : \Delta_8, F_{13}, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}}{- : \Delta_8, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}} \text{ax/W} \\
\hline
\frac{- : \Delta_8, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14} \quad - : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R \quad \text{hCut}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \wedge F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \wedge F_{14}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_7} \vee_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \vee F_{10} \vdash F_{13}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_8, F_9 \vdash \Delta_{12}, F_{13}, F_7}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_7} \text{inv-th/ax}}{\frac{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_7}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_7} \vee_L} \text{inv-th/ax} \\
\frac{h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}}{- : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}} \text{ax/W} \\
\hline
\frac{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \quad - : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R \quad \text{hCut}
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \vee F_{14}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_7} \vee_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \vee F_{10} \vdash F_{13}, F_{14}, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_8, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_7}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \text{inv-th/ax}}{\frac{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_7} \vee_L} \text{inv-th/ax} \\
\frac{h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{- : \Delta_8, F_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W} \\
\hline
\frac{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14} \quad - : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}}{- : \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R \quad \text{hCut}
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \perp, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \perp, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\perp, \Delta_{12}), F_7} \vee_L \quad \frac{h_{11} : F_7, \Delta_8, F_9 \vee F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \perp, \Delta_{12}} \perp_R \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{\frac{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12}, F_7}{- : \Delta_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12}} \text{ax/W}}{\frac{h_{11} : \Delta_8, F_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}}{- : \Delta_8, F_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}} \text{ax/W}} \text{hCut}
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_3 : F_9, \Delta_8 \vdash F_7, \top, \Delta_{12} \quad h_3 : F_{10}, \Delta_8 \vdash F_7, \top, \Delta_{12}}{\bullet h_3 : \Delta_8, F_9 \vee F_{10} \vdash (\top, \Delta_{12}), F_7} \vee_L \quad \frac{h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \top, \Delta_{12}}{\bullet h_{11} : (\Delta_8, F_9 \vee F_{10}), F_7 \vdash \top, \Delta_{12}} \top_R \\
\hline
- : \Delta_8, F_9 \vee F_{10} \vdash \top, \Delta_{12} \quad \text{Cut} \\
\hline
\rightsquigarrow \\
\frac{- : \Delta_8, F_9 \vee F_{10} \vdash \top, \Delta_{12}}{- : \Delta_8, F_9 \vee F_{10} \vdash \top, \Delta_{12}} \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{h_3 : F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, [] F_{12} \quad h_3 : F_9, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, [] F_{12}}{\bullet h_3 : (\Box \Gamma_{13}, \Delta_{14}), F_8 \vee F_9 \vdash (\Delta_{11}, [] F_{12}), \Box F_7} \vee_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_7) \vdash F_{12}}{\bullet h_{10} : ((\Box \Gamma_{13}, \Delta_{14}), F_8 \vee F_9), \Box F_7 \vdash \Delta_{11}, [] F_{12}} \\
\hline
- : (\Box \Gamma_{13}, \Delta_{14}), F_8 \vee F_9 \vdash \Delta_{11}, [] F_{12} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, [] F_{12}}{- : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [] F_{12}} \text{ ax/W} \quad \frac{\frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{13}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [] F_{12}} K}{- : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [] F_{12}} \text{ hCut} \quad \frac{\frac{h_3 : \Delta_{14}, F_9, \Box \Gamma_{13} \vdash \Box F_7, \Delta_{11}, [] F_{12}}{- : \Delta_{14}, F_9, \Box \Gamma_{13} \vdash \Delta_{11}, [] F_{12}} \text{ ax/W} \quad \frac{h_{10} : \text{unbox}(\Box F_7), \text{unbox}(\Box \Gamma_{13}) \vdash F_{12}}{\bullet h_{10} : \Box F_7, \Delta_{14}, F_9, \Box \Gamma_{13} \vdash \Delta_{11}, [] F_{12}} K}{- : \Delta_{14}, F_9, \Box \Gamma_{13} \vdash \Delta_{11}, [] F_{12}} \text{ hCut} \\
\hline
- : \Delta_{14}, \Box \Gamma_{13}, F_8 \vee F_9 \vdash \Delta_{11}, [] F_{12} \\
\hline
\frac{h_3 : F_8, \Box \Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, [] F_{13} \quad h_3 : F_9, \Box \Gamma_{11}, \Delta_{14} \vdash F_7, \Delta_{12}, [] F_{13}}{\bullet h_3 : (\Box \Gamma_{11}, \Delta_{14}), F_8 \vee F_9 \vdash (\Delta_{12}, [] F_{13}), F_7} \vee_L \quad \frac{h_{10} : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}}{\bullet h_{10} : ((\Box \Gamma_{11}, \Delta_{14}), F_8 \vee F_9), F_7 \vdash \Delta_{12}, [] F_{13}} K \\
\hline
- : (\Box \Gamma_{11}, \Delta_{14}), F_8 \vee F_9 \vdash \Delta_{12}, [] F_{13} \\
\hline
\rightsquigarrow \\
\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_{13}}{- : \Delta_{14}, \Box \Gamma_{11}, F_8 \vee F_9 \vdash \Delta_{12}, [] F_{13}} \text{ ax/W} \quad K
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_3 : F_8, \Delta_7 \vdash F_{11} \rightarrow F_{12}, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \rightarrow F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \vee_L \quad \frac{h_{10} : \Delta_7, F_8 \vee F_9 \vdash F_{11}, \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \rightarrow_L \\
\hline
- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13} \\
\hline
\rightsquigarrow \\
\frac{\frac{- : \Delta_7, F_{11}, F_8 \vdash \Delta_{13}, F_{12}}{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}, F_{12}} \text{ inv-th/ax} \quad \frac{- : \Delta_7, F_{11}, F_9 \vdash \Delta_{13}, F_{12}}{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}, F_{12}} \text{ inv-th/ax}}{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}, F_{12}} \vee_L \quad \frac{- : \Delta_7, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}} \text{ sCut} \\
\hline
- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13} \\
\hline
\frac{h_3 : F_8, \Delta_{14}, F_{11} \rightarrow F_{12} \vdash F_7, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \rightarrow F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \quad \frac{h_{10} : F_7, \Delta_{14}, F_8 \vee F_9 \vdash F_{11}, \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vee F_9 \vdash \Delta_{13}), F_7 \vdash \Delta_{13}} \text{ Cut} \\
\hline
- : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vee F_9 \vdash \Delta_{13} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_8 \vdash \Delta_{13}, F_{11}, F_7}}{\bullet h_3 : \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_7} \text{ inv-th/ax} \quad \frac{h_3 : \Delta_{14}, F_9 \vdash \Delta_{13}, F_{11}, F_7}}{\bullet h_3 : \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_7} \text{ inv-th/ax}}{- : \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}} \vee_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \text{ ax/W} \quad \frac{h_3 : \Delta_{14}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7}}{\bullet h_3 : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \text{ hCut} \\
\hline
- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_3 : F_8, \Delta_7 \vdash F_{11} \wedge F_{12}, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \wedge F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \vee_L \quad \frac{h_{10} : F_{11}, F_{12}, \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L \\
\hline
- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{- : \Delta_7, F_8 \vdash \Delta_{13}} \text{ ax/W} \quad \frac{\frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_{11} \wedge F_{12} \vdash \Delta_{13}} \text{ inv-th/ax}}{- : \Delta_7, F_8, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{ hCut} \quad \frac{\frac{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{- : \Delta_7, F_9 \vdash \Delta_{13}} \text{ ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \text{ inv-t}}{- : \Delta_7, F_9 \vdash \Delta_{13}} \wedge_L \\
\hline
- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13} \\
\hline
\frac{h_3 : F_8, \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \wedge F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \quad \frac{h_{10} : F_7, F_{11}, F_{12}, \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{13}} \wedge_L \\
\hline
- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9 \vdash \Delta_{13} \\
\hline
\rightsquigarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vdash \Delta_{13}, F_7}}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \text{ inv-th/ax} \quad \frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_9 \vdash \Delta_{13}, F_7}}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \text{ inv-th/ax}}{- : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \text{ ax/W} \quad \text{hCut} \\
\hline
- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \vee F_9 \vdash \Delta_{13}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, \Delta_7 \vdash F_{11} \vee F_{12}, \Delta_{13} \quad h_3 : F_9, \Delta_7 \vdash F_{11} \vee F_{12}, \Delta_{13}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \vee_L \quad \frac{\frac{h_{10} : F_{11}, \Delta_7, F_8 \vee F_9 \vdash \Delta_{13} \quad h_{10} : F_{12}, \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \vee_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{Cut} \\
\\
\frac{\frac{\frac{- : \Delta_7, F_8 \vdash \Delta_{13}, F_{11}, F_{12}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \text{inv-th/ax} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{13}, F_{11}, F_{12}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}} \text{inv-th/ax}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}} \vee_L \quad \frac{\frac{- : \Delta_7, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{ax/W} \quad \frac{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{sCut}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \\
\\
\frac{\frac{h_3 : F_8, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13} \quad h_3 : F_9, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \quad \frac{h_{10} : F_7, F_{11}, \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9) \vdash \Delta_{13}} \vee_L}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}} \text{Cut} \\
\\
\frac{\frac{h_3 : \Delta_{14}, F_{11}, F_8 \vdash \Delta_{13}, F_7 \quad h_3 : \Delta_{14}, F_{11}, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax} \quad \frac{\frac{h_3 : \Delta_{14}, F_{11}, F_9 \vdash \Delta_{13}, F_7}{h_{10} : \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}} \vee_L \quad \frac{\frac{h_3 : \Delta_{14}, F_{12}, F_8 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \text{ax/W} \quad \frac{- : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_{11} \vee F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \text{hCut}}{- : \Delta_{14}, F_{11} \vee F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \\
\\
\frac{\frac{h_3 : F_{10}, \Delta_8 \vdash F_7, \Delta_{12} \quad h_3 : F_{11}, \Delta_8 \vdash F_7, \Delta_{12}}{\bullet h_3 : \Delta_8, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \vee_L \quad \frac{h_9 : F_7, F_{10}, \Delta_8 \vdash \Delta_{12} \quad h_9 : F_7, F_{11}, \Delta_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_8, F_{10} \vee F_{11}), F_7 \vdash \Delta_{12}} \vee_L}{- : \Delta_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{Cut} \\
\\
\frac{\frac{h_3 : \Delta_8, F_{10} \vdash \Delta_{12}, F_7 \quad h_9 : \Delta_8, F_{10}, F_7 \vdash \Delta_{12}}{- : \Delta_8, F_{10} \vdash \Delta_{12}} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_8, F_{10}, F_7 \vdash \Delta_{12}}{\bullet h_9 : \Delta_8, F_{10}, F_7 \vdash \Delta_{12}} \text{H} \quad \frac{- : \Delta_8, F_{11} \vdash \Delta_{12}}{- : \Delta_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{hCut}}{- : \Delta_8, F_{10} \vdash \Delta_{12}} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{12}}{\bullet h_9 : \Delta_8, F_{11}, F_7 \vdash \Delta_{12}} \text{H} \quad \frac{- : \Delta_8, F_{11} \vdash \Delta_{12}}{- : \Delta_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{hCut}}{- : \Delta_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, \Delta_7 \vdash \perp, \Delta_{11} \quad h_3 : F_9, \Delta_7 \vdash \perp, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}, \perp} \vee_L \quad \frac{\frac{h_{10} : (\Delta_7, F_8 \vee F_9), \perp \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), \perp \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \text{Cut} \\
\\
\frac{\frac{h_3 : \Delta_7, F_8 \vdash \perp, \Delta_{11}}{- : \Delta_7, F_8 \vdash \perp, \Delta_{11}} \text{ax/W} \quad \frac{\frac{\bullet h_{10} : \perp, \Delta_7, F_8 \vdash \Delta_{11}}{- : \Delta_7, F_8 \vdash \Delta_{11}} \perp_L \quad \frac{h_3 : \Delta_7, F_9 \vdash \perp, \Delta_{11}}{- : \Delta_7, F_9 \vdash \perp, \Delta_{11}} \text{ax/W} \quad \frac{\bullet h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}}{- : \Delta_7, F_9 \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \text{hCut} \vee_L \\
\\
\frac{\frac{h_3 : F_8, \perp, \Delta_{12} \vdash F_7, \Delta_{11} \quad h_3 : F_9, \perp, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}, F_7} \vee_L \quad \frac{\frac{h_{10} : ((\perp, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}} \perp_L}{- : (\perp, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}} \text{Cut} \\
\\
\frac{- : \perp, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}}{- : \perp, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, \Delta_7 \vdash p_{11}, \Delta_{12}, p_{11} \quad h_3 : F_9, \Delta_7 \vdash p_{11}, \Delta_{12}, p_{11}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash (\Delta_{12}, p_{11}), p_{11}} \vee_L \quad \frac{\frac{h_{10} : (\Delta_7, F_8 \vee F_9), p_{11} \vdash \Delta_{12}, p_{11}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), p_{11} \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, p_{11}} \text{Cut} \\
\\
\frac{\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{12}, p_{11}, p_{11}}{- : \Delta_7, F_8 \vdash \Delta_{12}, p_{11}} \text{ax/W} \quad \frac{\frac{\bullet h_{10} : \Delta_7, F_8, p_{11} \vdash \Delta_{12}, p_{11}}{- : \Delta_7, F_8 \vdash \Delta_{12}, p_{11}} I \quad \frac{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}, p_{11}}{- : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}} \text{ax/W} \quad \frac{\bullet h_{10} : \Delta_7, F_9, p_{11} \vdash \Delta_{12}, p_{11}}{- : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, p_{11}} \text{hCut} \vee_L \\
\\
\frac{\frac{h_3 : F_8, \Delta_{13}, p_{11} \vdash F_7, \Delta_{12}, p_{11} \quad h_3 : F_9, \Delta_{13}, p_{11} \vdash F_7, \Delta_{12}, p_{11}}{\bullet h_3 : (\Delta_{13}, p_{11}), F_8 \vee F_9 \vdash (\Delta_{12}, p_{11}), F_7} \vee_L \quad \frac{\frac{h_{10} : ((\Delta_{13}, p_{11}), F_8 \vee F_9), F_7 \vdash \Delta_{12}, p_{11}}{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \vee F_9), F_7 \vdash \Delta_{12}, p_{11}} I}{- : (\Delta_{13}, p_{11}), F_8 \vee F_9 \vdash \Delta_{12}, p_{11}} \text{Cut} \\
\\
\frac{- : \Delta_{13}, p_{11}, F_8 \vee F_9 \vdash \Delta_{12}, p_{11}}{- : \Delta_{13}, p_{11}, F_8 \vee F_9 \vdash \Delta_{12}, p_{11}} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_3 : F_8, \Delta_7 \vdash \top, \Delta_{11} \quad h_3 : F_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}, \top} \vee_L \quad \frac{h_{10} : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), \top \vdash \Delta_{11}} \top_L}{\frac{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}{\sim\!\!\sim} \text{Cut}} \\
\frac{}{\frac{}{\frac{}{\frac{}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \text{ax/W}}}} \\
\frac{\frac{h_3 : F_8, \top, \Delta_{12} \vdash F_7, \Delta_{11} \quad h_3 : F_9, \top, \Delta_{12} \vdash F_7, \Delta_{11}}{\bullet h_3 : (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}, F_7} \vee_L \quad \frac{h_{10} : F_7, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}} \top_L}{\frac{- : (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}}{\sim\!\!\sim} \text{Cut}} \\
\frac{\frac{\bullet h_3 : \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}, F_7}{\text{ax/W}} \quad \frac{h_{10} : \top, \Delta_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{11}}{\text{hCut}}}{\frac{}{\frac{}{- : \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}}}}
\end{array}$$

## 8.10 Status of $\perp_L$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_5} \perp_L \quad \frac{h_7 : \perp, F_5, F_9, \Delta_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R}{\frac{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \rightarrow F_{10}}{\sim\!\!\sim} \text{Cut}} \\
\frac{}{\frac{}{\frac{}{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \perp_L}}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\Delta_8, F_9 \wedge F_{10}), F_5} \perp_L \quad \frac{h_7 : \perp, F_5, \Delta_6 \vdash F_9, \Delta_8 \quad h_7 : \perp, F_5, \Delta_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R}{\frac{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \wedge F_{10}}{\sim\!\!\sim} \text{Cut}} \\
\frac{}{\frac{}{\frac{}{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \wedge F_{10}} \perp_L}}
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\Delta_8, F_9 \vee F_{10}), F_5} \perp_L \quad \frac{h_7 : \perp, F_5, \Delta_6 \vdash F_9, F_{10}, \Delta_8}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R}{\frac{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10}}{\sim\!\!\sim} \text{Cut}} \\
\frac{}{\frac{}{\frac{}{- : \perp, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10}} \perp_L}}
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\perp, \Delta_8), F_5} \perp_L \quad \frac{h_7 : \perp, F_5, \Delta_6 \vdash \Delta_8}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \perp, \Delta_8} \perp_R}{\frac{- : \perp, \Delta_6 \vdash \perp, \Delta_8}{\sim\!\!\sim} \text{Cut}} \\
\frac{}{\frac{}{\frac{}{- : \perp, \Delta_6 \vdash \perp, \Delta_8} \perp_L}}
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash (\top, \Delta_8), F_5} \perp_L \quad \frac{}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \top, \Delta_8} \top_R}{\frac{- : \perp, \Delta_6 \vdash \top, \Delta_8}{\sim\!\!\sim} \text{Cut}} \\
\frac{}{\frac{}{\frac{}{- : \perp, \Delta_6 \vdash \top, \Delta_8} \top_R}}
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, [\Box F_8], \Box F_5)} \perp_L \quad \frac{h_6 : unbox(\Box \Gamma_9), unbox(\Box F_5) \vdash F_8}{\bullet h_6 : (\perp, \Box \Gamma_9, \Delta_{10}), \Box F_5 \vdash \Delta_7, [\Box F_8]} K}{\frac{}{- : \perp, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, [\Box F_8]} \text{Cut}} \perp_L \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [\Box F_8]} \perp_L \\
\\
\frac{\frac{}{\bullet h_3 : \perp, \Box \Gamma_7, \Delta_{10} \vdash (\Delta_8, [\Box F_9], F_5)} \perp_L \quad \frac{h_6 : unbox(\Box \Gamma_7) \vdash F_9}{\bullet h_6 : (\perp, \Box \Gamma_7, \Delta_{10}), F_5 \vdash \Delta_8, [\Box F_9]} K}{\frac{}{- : \perp, \Box \Gamma_7, \Delta_{10} \vdash \Delta_8, [\Box F_9]} \text{Cut}} \perp_L \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [\Box F_9]} \perp_L
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8} \perp_L \quad \frac{h_6 : \perp, \Delta_5 \vdash F_7, \Delta_9 \quad h_6 : \perp, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_5), F_7 \rightarrow F_8 \vdash \Delta_9} \rightarrow_L}{\frac{}{- : \perp, \Delta_5 \vdash \Delta_9} \text{Cut}} \perp_L \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_5 \vdash \Delta_9} \perp_L \\
\\
\frac{\frac{}{\bullet h_3 : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5} \perp_L \quad \frac{h_6 : \perp, F_5, \Delta_{10} \vdash F_7, \Delta_9 \quad h_6 : \perp, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_{10}, F_7 \rightarrow F_8), F_5 \vdash \Delta_9} \rightarrow_L}{\frac{}{- : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{Cut}} \perp_L \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \perp_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8} \perp_L \quad \frac{h_6 : \perp, F_7, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_5), F_7 \wedge F_8 \vdash \Delta_9} \wedge_L}{\frac{}{- : \perp, \Delta_5 \vdash \Delta_9} \text{Cut}} \perp_L \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_5 \vdash \Delta_9} \perp_L \\
\\
\frac{\frac{}{\bullet h_3 : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \perp_L \quad \frac{h_6 : \perp, F_5, F_7, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \wedge_L}{\frac{}{- : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{Cut}} \perp_L \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \perp_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \perp_L \quad \frac{h_6 : \perp, F_7, \Delta_5 \vdash \Delta_9 \quad h_6 : \perp, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_5), F_7 \vee F_8 \vdash \Delta_9} \vee_L}{\frac{}{- : \perp, \Delta_5 \vdash \Delta_9} \text{Cut}} \perp_L \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_5 \vdash \Delta_9} \perp_L \\
\\
\frac{\frac{}{\bullet h_3 : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \perp_L \quad \frac{h_6 : \perp, F_5, F_7, \Delta_{10} \vdash \Delta_9 \quad h_6 : \perp, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_{10}, F_7 \vee F_8), F_5 \vdash \Delta_9} \vee_L}{\frac{}{- : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{Cut}} \perp_L \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \perp_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_3 : \perp, \Delta_6 \vdash \Delta_8, F_5} \perp_L \quad \frac{}{\bullet h_7 : (\perp, \Delta_6), F_5 \vdash \Delta_8} \perp_L}{\frac{}{- : \perp, \Delta_6 \vdash \Delta_8} \text{Cut}} \perp_L \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_6 \vdash \Delta_8} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{\bullet h_3 : \perp, \Delta_5 \vdash (\Delta_8, p_7), p_7}{\vdash : \perp, \Delta_5 \vdash \Delta_8, p_7} \perp_L \quad \frac{\bullet h_6 : (\perp, \Delta_5), p_7 \vdash \Delta_8, p_7}{\vdash : \perp, \Delta_5 \vdash \Delta_8, p_7} I}{\vdash : \perp, \Delta_5 \vdash \Delta_8, p_7} \text{Cut} \\
\sim \\
\vdash : \perp, \Delta_5 \vdash \Delta_8, p_7 \quad \perp_L \\
\vdash : \perp, \Delta_5 \vdash \Delta_8, p_7
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_3 : \perp, \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7} \perp_L \quad \frac{\bullet h_6 : (\perp, \Delta_9, p_7), F_5 \vdash \Delta_8, p_7}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7} I}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\sim \\
\vdash : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7 \quad \perp_L \\
\vdash : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_7, \top}{\vdash : \perp, \Delta_5 \vdash \Delta_7} \perp_L \quad \frac{h_6 : \perp, \Delta_5 \vdash \Delta_7}{\bullet h_6 : (\perp, \Delta_5), \top \vdash \Delta_7} \top_L}{\vdash : \perp, \Delta_5 \vdash \Delta_7} \text{Cut} \\
\sim \\
\vdash : \perp, \Delta_5 \vdash \Delta_7 \quad \perp_L \\
\vdash : \perp, \Delta_5 \vdash \Delta_7
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_3 : \perp, \top, \Delta_8 \vdash \Delta_7, F_5}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_7} \perp_L \quad \frac{h_6 : \perp, F_5, \Delta_8 \vdash \Delta_7}{\bullet h_6 : (\perp, \top, \Delta_8), F_5 \vdash \Delta_7} \top_L}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_7} \text{Cut} \\
\sim \\
\vdash : \perp, \top, \Delta_8 \vdash \Delta_7 \quad \perp_L \\
\vdash : \perp, \top, \Delta_8 \vdash \Delta_7
\end{array}$$

## 8.11 Status of $I$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \rightarrow F_{10}), p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} I \quad \frac{h_7 : F_9, \Delta_5, p_6, p_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10}, p_6}{\vdash : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10}} I \quad \frac{h_7 : \Delta_5, F_9, p_6, p_6 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/w}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{hCut} \\
\rightarrow_R \\
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), p_8), F_6}{\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8} I \quad \frac{h_9 : F_6, F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8} \rightarrow_R}{\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8} \text{Cut} \\
\sim \\
\vdash : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \rightarrow F_{11} \quad I \\
\vdash : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \rightarrow F_{11}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \wedge F_{10}), p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash F_9, \Delta_8 \quad h_7 : \Delta_5, p_6, p_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\sim \\
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_9, p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_9}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/w}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9} \text{hCut} \\
\wedge_R \\
\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), p_8), F_6}{\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8} I \quad \frac{h_9 : F_6, \Delta_7, p_8 \vdash F_{10}, \Delta_{12}, p_8 \quad h_9 : F_6, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8} \wedge_R}{\vdash : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8} \text{Cut} \\
\sim \\
\vdash : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \wedge F_{11} \quad I \\
\vdash : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \wedge F_{11}
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \vee F_{10}), p_6}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash F_9, F_{10}, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\\
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9, p_6}{- : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_{10}, F_9}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_{10}, F_9} \text{ax/W}}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R \quad \text{hCut} \\
\\
\frac{\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \vee F_{11}), p_8), F_6}{- : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8} I \quad \frac{h_9 : F_6, \Delta_7, p_8 \vdash F_{10}, F_{11}, \Delta_{12}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8} \vee_R}{- : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8} \text{Cut} \\
\\
\frac{}{- : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \vee F_{11}} I
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\perp, \Delta_8), p_6}{- : \Delta_5, p_6 \vdash \perp, \Delta_8} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \perp, \Delta_8} \perp_R}{- : \Delta_5, p_6 \vdash \perp, \Delta_8} \text{Cut} \\
\\
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash \perp, \Delta_8, p_6}{- : \Delta_5, p_6 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \perp, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \perp, \Delta_8} \text{ax/W}}{- : \Delta_5, p_6 \vdash \perp, \Delta_8} \text{hCut} \\
\\
\frac{\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\perp, \Delta_{10}), p_8), F_6}{- : \Delta_7, p_8 \vdash (\perp, \Delta_{10}), p_8} I \quad \frac{h_9 : F_6, \Delta_7, p_8 \vdash \Delta_{10}, p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\perp, \Delta_{10}), p_8} \perp_R}{- : \Delta_7, p_8 \vdash (\perp, \Delta_{10}), p_8} \text{Cut} \\
\\
\frac{}{- : \Delta_7, p_8 \vdash \perp, \Delta_{10}, p_8} I
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\top, \Delta_8), p_6}{- : \Delta_5, p_6 \vdash \top, \Delta_8} I \quad \frac{h_7 : (\Delta_5, p_6), p_6 \vdash \top, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \top, \Delta_8} \top_R}{- : \Delta_5, p_6 \vdash \top, \Delta_8} \text{Cut} \\
\\
\frac{}{- : \Delta_5, p_6 \vdash \top, \Delta_8} \top_R \\
\\
\frac{\frac{\bullet h_2 : \Delta_7, p_8 \vdash ((\top, \Delta_{10}), p_8), F_6}{- : \Delta_7, p_8 \vdash (\top, \Delta_{10}), p_8} I \quad \frac{h_9 : (\Delta_7, p_8), F_6 \vdash (\top, \Delta_{10}), p_8}{\bullet h_9 : (\Delta_7, p_8), F_6 \vdash (\top, \Delta_{10}), p_8} \top_R}{- : \Delta_7, p_8 \vdash (\top, \Delta_{10}), p_8} \text{Cut} \\
\\
\frac{}{- : \Delta_7, p_8 \vdash \top, \Delta_{10}, p_8} \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Box \Gamma_7, \Delta_{10}), p_5 \vdash (\Delta_8, \Box F_9), p_5}{- : (\Box \Gamma_7, \Delta_{10}), p_5 \vdash \Delta_8, \Box F_9} I \quad \frac{h_6 : \text{unbox}(\Box \Gamma_7) \vdash F_9}{\bullet h_6 : ((\Box \Gamma_7, \Delta_{10}), p_5), p_5 \vdash \Delta_8, \Box F_9} K}{- : (\Box \Gamma_7, \Delta_{10}), p_5 \vdash \Delta_8, \Box F_9} \text{Cut} \\
\\
\frac{}{- : \text{unbox}(\Box \Gamma_7) \vdash F_9} \text{ax/W} \\
\frac{}{- : \Delta_{10}, \Box \Gamma_7, p_5 \vdash \Delta_8, \Box F_9} K \\
\\
\frac{\frac{\bullet h_2 : (\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash ((\Delta_{10}, \Box F_9), p_7), \Box F_6}{- : (\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash (\Delta_{10}, \Box F_9), p_7} I \quad \frac{h_8 : \text{unbox}(\Box \Gamma_{11}), \text{unbox}(\Box F_6) \vdash F_9}{\bullet h_8 : ((\Box \Gamma_{11}, \Delta_{12}), p_7), \Box F_6 \vdash (\Delta_{10}, \Box F_9), p_7} K}{- : (\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash (\Delta_{10}, \Box F_9), p_7} \text{Cut} \\
\\
\frac{}{- : \Delta_{12}, \Box \Gamma_{11}, p_7 \vdash \Delta_{10}, p_7, \Box F_9} I
\end{array}$$



$$\begin{array}{c}
\frac{\bullet h_2 : (\Box \Gamma_9, \Delta_{12}), p_7 \vdash ((\Delta_{11}, \Box F_{10}), p_7), F_6}{- : (\Box \Gamma_9, \Delta_{12}), p_7 \vdash (\Delta_{11}, \Box F_{10}), p_7} I \quad \frac{h_8 : unbox(\Box \Gamma_9) \vdash F_{10}}{\bullet h_8 : ((\Box \Gamma_9, \Delta_{12}), p_7), F_6 \vdash (\Delta_{11}, \Box F_{10}), p_7} K \\
\text{Cut} \\
\frac{}{- : (\Box \Gamma_9, \Delta_{12}), p_7 \vdash (\Delta_{11}, \Box F_{10}), p_7} \rightsquigarrow \\
\frac{}{- : \Delta_{12}, \Box \Gamma_9, p_7 \vdash \Delta_{11}, p_7, \Box F_{10}} I
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{10}, F_7 \rightarrow F_8), p_5 \vdash \Delta_9, p_5}{- : (\Delta_{10}, F_7 \rightarrow F_8), p_5 \vdash \Delta_9} I \quad \frac{h_6 : \Delta_{10}, p_5, p_5 \vdash F_7, \Delta_9 \quad h_6 : F_8, \Delta_{10}, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \rightarrow F_8), p_5), p_5 \vdash \Delta_9} \rightarrow_L \\
\text{Cut} \\
\frac{}{- : (\Delta_{10}, F_7 \rightarrow F_8), p_5 \vdash \Delta_9} \rightsquigarrow \\
\frac{\bullet h_1 : \Delta_{10}, p_5 \vdash \Delta_9, F_7, p_5 \quad h_6 : \Delta_{10}, p_5, p_5 \vdash \Delta_9, F_7}{- : \Delta_{10}, p_5 \vdash \Delta_9, F_7} I \quad \frac{ax/W}{hCut} \quad \frac{\bullet h_1 : \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5 \quad h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9} I \quad \frac{ax/W}{hCut} \\
\frac{}{- : \Delta_{10}, p_5, F_7 \rightarrow F_8 \vdash \Delta_9} \rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \rightarrow F_{11}}{- : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \rightarrow F_{11}} I \quad \frac{h_9 : \Delta_6, p_7 \vdash F_{10}, \Delta_8, p_7 \quad h_9 : F_{11}, \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \rightarrow F_{11} \vdash \Delta_8, p_7} \rightarrow_L \\
\text{Cut} \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \rightsquigarrow \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash (\Delta_8, p_7), F_6}{- : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash (\Delta_8, p_7), F_6} I \quad \frac{h_9 : F_6, \Delta_{12}, p_7 \vdash F_{10}, \Delta_8, p_7 \quad h_9 : F_6, F_{11}, \Delta_{12}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \rightarrow F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \rightarrow_L \\
\text{Cut} \\
\frac{}{- : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash \Delta_8, p_7} \rightsquigarrow \\
\frac{}{- : \Delta_{12}, p_7, F_{10} \rightarrow F_{11} \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9, p_5}{- : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9} I \quad \frac{h_6 : F_7, F_8, \Delta_{10}, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \wedge F_8), p_5), p_5 \vdash \Delta_9} \wedge_L \\
\text{Cut} \\
\frac{}{- : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9} \rightsquigarrow \\
\frac{\bullet h_1 : \Delta_{10}, F_7, F_8, p_5 \vdash \Delta_9, p_5 \quad h_6 : \Delta_{10}, F_7, F_8, p_5, p_5 \vdash \Delta_9}{- : \Delta_{10}, F_7, F_8, p_5 \vdash \Delta_9} I \quad \frac{ax/W}{hCut} \\
\frac{}{- : \Delta_{10}, p_5, F_7 \wedge F_8 \vdash \Delta_9} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \wedge F_{11}}{- : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \wedge F_{11}} I \quad \frac{h_9 : F_{10}, F_{11}, \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \wedge F_{11} \vdash \Delta_8, p_7} \wedge_L \\
\text{Cut} \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \rightsquigarrow \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash (\Delta_8, p_7), F_6}{- : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash (\Delta_8, p_7), F_6} I \quad \frac{h_9 : F_6, F_{10}, F_{11}, \Delta_{12}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \wedge F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \wedge_L \\
\text{Cut} \\
\frac{}{- : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash \Delta_8, p_7} \rightsquigarrow \\
\frac{}{- : \Delta_{12}, p_7, F_{10} \wedge F_{11} \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9, p_5}{- : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9} I \quad \frac{h_6 : F_7, \Delta_{10}, p_5, p_5 \vdash \Delta_9 \quad h_6 : F_8, \Delta_{10}, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \vee F_8), p_5), p_5 \vdash \Delta_9} \vee_L \\
\text{Cut} \\
\frac{}{- : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9} \rightsquigarrow \\
\frac{\bullet h_1 : \Delta_{10}, F_7, p_5 \vdash \Delta_9, p_5 \quad h_6 : \Delta_{10}, F_7, p_5, p_5 \vdash \Delta_9}{- : \Delta_{10}, F_7, p_5 \vdash \Delta_9} I \quad \frac{ax/W}{hCut} \quad \frac{\bullet h_1 : \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5 \quad h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9} I \quad \frac{ax/W}{hCut} \\
\frac{}{- : \Delta_{10}, p_5, F_7 \vee F_8 \vdash \Delta_9} \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \vee F_{11}}{I} \quad \frac{h_9 : F_{10}, \Delta_6, p_7 \vdash \Delta_8, p_7 \quad h_9 : F_{11}, \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \vee F_{11} \vdash \Delta_8, p_7} \vee_L}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : (\Delta_{12}, F_{10} \vee F_{11}), p_7 \vdash (\Delta_8, p_7), F_6}{I} \quad \frac{h_9 : F_6, F_{10}, \Delta_{12}, p_7 \vdash \Delta_8, p_7 \quad h_9 : F_6, F_{11}, \Delta_{12}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \vee F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \vee_L}{- : (\Delta_{12}, F_{10} \vee F_{11}), p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \Delta_{12}, p_7, F_{10} \vee F_{11} \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\perp, \Delta_8), p_5 \vdash \Delta_7, p_5}{I} \quad \frac{\bullet h_6 : ((\perp, \Delta_8), p_5), p_5 \vdash \Delta_7}{I}}{- : (\perp, \Delta_8), p_5 \vdash \Delta_7} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_8, p_5 \vdash \Delta_7} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \perp}{I} \quad \frac{\bullet h_9 : (\Delta_6, p_7), \perp \vdash \Delta_8, p_7}{I}}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : (\perp, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6}{I} \quad \frac{\bullet h_9 : ((\perp, \Delta_{10}), p_7), F_6 \vdash \Delta_8, p_7}{I}}{- : (\perp, \Delta_{10}), p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \perp, \Delta_{10}, p_7 \vdash \Delta_8, p_7} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5, p_7 \vdash (\Delta_8, p_7), p_7}{I} \quad \frac{\bullet h_6 : (\Delta_5, p_7), p_7 \vdash \Delta_8, p_7}{I}}{- : \Delta_5, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \Delta_5, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_9, p_7), p_5 \vdash (\Delta_8, p_7), p_5}{I} \quad \frac{\bullet h_6 : ((\Delta_9, p_7), p_5), p_5 \vdash \Delta_8, p_7}{I}}{- : (\Delta_9, p_7), p_5 \vdash \Delta_8, p_7} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \Delta_9, p_5, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_6, p_7 \vdash ((\Delta_{10}, p_9), p_7), p_9}{I} \quad \frac{\bullet h_8 : (\Delta_6, p_7), p_9 \vdash (\Delta_{10}, p_9), p_7}{I}}{- : \Delta_6, p_7 \vdash (\Delta_{10}, p_9), p_7} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_{10}, p_7, p_9} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : (\Delta_{11}, p_9), p_7 \vdash ((\Delta_{10}, p_9), p_7), F_6}{I} \quad \frac{\bullet h_8 : ((\Delta_{11}, p_9), p_7), F_6 \vdash (\Delta_{10}, p_9), p_7}{I}}{- : (\Delta_{11}, p_9), p_7 \vdash (\Delta_{10}, p_9), p_7} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \Delta_{11}, p_7, p_9 \vdash \Delta_{10}, p_7, p_9} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_2 : \Delta_7, p_9 \vdash (\Delta_{10}, p_9), F_6}{I} \quad \frac{\bullet h_8 : (\Delta_7, p_9), F_6 \vdash \Delta_{10}, p_9}{I}}{- : \Delta_7, p_9 \vdash \Delta_{10}, p_9} \text{Cut} \\
\frac{}{\sim} \\
\frac{}{- : \Delta_7, p_9 \vdash \Delta_{10}, p_9} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\top, \Delta_8), p_5 \vdash \Delta_7, p_5}{- : (\top, \Delta_8), p_5 \vdash \Delta_7} I \quad \frac{h_6 : \Delta_8, p_5, p_5 \vdash \Delta_7}{\bullet h_6 : ((\top, \Delta_8), p_5), p_5 \vdash \Delta_7} \top_L}{\text{Cut}} \\
\frac{\bullet h_1 : \top, \Delta_8, p_5 \vdash \Delta_7, p_5}{- : \top, \Delta_8, p_5 \vdash \Delta_7} I \quad \frac{h_6 : \top, \Delta_8, p_5, p_5 \vdash \Delta_7}{\bullet h_6 : ((\top, \Delta_8), p_5), p_5 \vdash \Delta_7} \text{ax/W}}{\text{hCut}} \\
\\
\frac{\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \top}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I \quad \frac{h_9 : \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), \top \vdash \Delta_8, p_7} \top_L}{\text{Cut}} \\
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \top}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I \\
\\
\frac{\frac{\bullet h_2 : (\top, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6}{- : (\top, \Delta_{10}), p_7 \vdash \Delta_8, p_7} I \quad \frac{h_9 : F_6, \Delta_{10}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\top, \Delta_{10}), p_7), F_6 \vdash \Delta_8, p_7} \top_L}{\text{Cut}} \\
\frac{\bullet h_2 : (\top, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6}{- : (\top, \Delta_{10}), p_7 \vdash \Delta_8, p_7} I
\end{array}$$

## 8.12 Status of $\top_L$ : OK

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_6 \vdash F_5, \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_5} \top_L \quad \frac{h_7 : \top, F_5, F_9, \Delta_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R}{\text{Cut}} \\
\frac{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_5}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W}}{\text{hCut}} \\
- : \top, \Delta_6 \vdash \Delta_8, F_9 \rightarrow F_{10}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_6 \vdash F_5, \Delta_8, F_9 \wedge F_{10}}{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \wedge F_{10}), F_5} \top_L \quad \frac{h_7 : \top, F_5, \Delta_6 \vdash F_9, \Delta_8 \quad h_7 : \top, F_5, \Delta_6 \vdash F_{10}, \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R}{\text{Cut}} \\
\frac{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \wedge F_{10}), F_5}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W}}{\text{hCut}} \\
- : \top, \Delta_6 \vdash \Delta_8, F_9 \wedge F_{10}
\end{array}$$

- Case rule  $\vee_R$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_6 \vdash F_5, \Delta_8, F_9 \vee F_{10}}{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \vee F_{10}), F_5} \top_L \quad \frac{h_7 : \top, F_5, \Delta_6 \vdash F_9, F_{10}, \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R}{\text{Cut}} \\
\frac{\bullet h_3 : \top, \Delta_6 \vdash (\Delta_8, F_9 \vee F_{10}), F_5}{- : \top, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8, F_9 \vee F_{10}} \text{ax/W}}{\text{hCut}} \\
- : \top, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10}
\end{array}$$

- Case rule  $\perp_R$

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_6 \vdash F_5, \perp, \Delta_8}{\bullet h_3 : \top, \Delta_6 \vdash (\perp, \Delta_8), F_5} \top_L \quad \frac{h_7 : \top, F_5, \Delta_6 \vdash \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \perp, \Delta_8} \perp_R}{\text{Cut}} \\
\frac{\bullet h_3 : \top, \Delta_6 \vdash (\perp, \Delta_8), F_5}{- : \top, \Delta_6 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \perp, \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \perp, \Delta_8} \text{ax/W}}{\text{hCut}} \\
- : \top, \Delta_6 \vdash \perp, \Delta_8
\end{array}$$

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_3 : \Delta_6 \vdash F_5, \top, \Delta_8}{\bullet h_3 : \top, \Delta_6 \vdash (\top, \Delta_8), F_5} \top_L \quad \frac{}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \top, \Delta_8} \top_R \\
\hline
- : \top, \Delta_6 \vdash \top, \Delta_8 \\
\sim \\
\frac{}{- : \top, \Delta_6 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule  $K$

$$\begin{array}{c}
\frac{h_3 : \Box \Gamma_9, \Delta_{10} \vdash \Box F_5, \Delta_7, [] F_8}{\bullet h_3 : \top, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, [] F_8), \Box F_5} \top_L \quad \frac{h_6 : \text{unbox}(\Box \Gamma_9), \text{unbox}(\Box F_5) \vdash F_8}{\bullet h_6 : (\top, \Box \Gamma_9, \Delta_{10}), \Box F_5 \vdash \Delta_7, [] F_8} K \\
\hline
- : \top, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, [] F_8 \\
\sim \\
\frac{h_3 : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Box F_5, \Delta_7, [] F_8}{- : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [] F_8} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Box F_5, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [] F_8}{\text{hCut}} \text{ax/W} \\
\hline
\frac{h_3 : \Box \Gamma_7, \Delta_{10} \vdash F_5, \Delta_8, [] F_9}{\bullet h_3 : \top, \Box \Gamma_7, \Delta_{10} \vdash (\Delta_8, [] F_9), F_5} \top_L \quad \frac{h_6 : \text{unbox}(\Box \Gamma_7) \vdash F_9}{\bullet h_6 : (\top, \Box \Gamma_7, \Delta_{10}), F_5 \vdash \Delta_8, [] F_9} K \\
\hline
- : \top, \Box \Gamma_7, \Delta_{10} \vdash \Delta_8, [] F_9 \\
\sim \\
\frac{- : \text{unbox}(\Box \Gamma_7) \vdash F_9}{- : \top, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] F_9} \text{ax/W} \quad K
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_3 : \Delta_5 \vdash F_7 \rightarrow F_8, \Delta_9}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8} \top_L \quad \frac{h_6 : \top, \Delta_5 \vdash F_7, \Delta_9 \quad h_6 : \top, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \rightarrow F_8 \vdash \Delta_9} \rightarrow_L \\
\hline
- : \top, \Delta_5 \vdash \Delta_9 \\
\sim \\
\frac{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8}{- : \top, \Delta_5 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, F_7 \rightarrow F_8 \vdash \Delta_9}{\text{hCut}} \text{ax/W} \\
\hline
\frac{h_3 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_5, \Delta_9}{\bullet h_3 : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5} \top_L \quad \frac{h_6 : \top, F_5, \Delta_{10} \vdash F_7, \Delta_9 \quad h_6 : \top, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \rightarrow F_8), F_5 \vdash \Delta_9} \rightarrow_L \\
\hline
- : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9 \\
\sim \\
\frac{h_3 : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5}{- : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \rightarrow F_8 \vdash \Delta_9}{\text{hCut}} \text{ax/W}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_3 : \Delta_5 \vdash F_7 \wedge F_8, \Delta_9}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8} \top_L \quad \frac{h_6 : \top, F_7, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \wedge F_8 \vdash \Delta_9} \wedge_L \\
\hline
- : \top, \Delta_5 \vdash \Delta_9 \\
\sim \\
\frac{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8}{- : \top, \Delta_5 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, F_7 \wedge F_8 \vdash \Delta_9}{\text{hCut}} \text{ax/W} \\
\hline
\frac{h_3 : \Delta_{10}, F_7 \wedge F_8 \vdash F_5, \Delta_9}{\bullet h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \top_L \quad \frac{h_6 : \top, F_5, F_7, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \wedge_L \\
\hline
- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9 \\
\sim \\
\frac{h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5}{- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \wedge F_8 \vdash \Delta_9}{\text{hCut}} \text{ax/W}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_3 : \Delta_5 \vdash F_7 \vee F_8, \Delta_9}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \top_L \quad \frac{h_6 : \top, F_7, \Delta_5 \vdash \Delta_9 \quad h_6 : \top, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \vee F_8 \vdash \Delta_9} \vee_L \\
\hline
- : \top, \Delta_5 \vdash \Delta_9 \\
\sim \\
\frac{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8}{- : \top, \Delta_5 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, F_7 \vee F_8 \vdash \Delta_9}{\text{hCut}} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : \Delta_{10}, F_7 \vee F_8 \vdash F_5, \Delta_9}{\bullet h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \top_L \quad \frac{h_6 : \top, F_5, F_7, \Delta_{10} \vdash \Delta_9 \quad h_6 : \top, F_5, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \vee F_8), F_5 \vdash \Delta_9} \vee_L \\
\hline
- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9 \quad \text{Cut} \\
\hline
\frac{\frac{h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \vee F_8 \vdash \Delta_9}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{hCut}}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{hCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_3 : \Delta_5 \vdash \perp, \Delta_7}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_7, \perp} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_5), \perp \vdash \Delta_7} \perp_L \\
\hline
- : \top, \Delta_5 \vdash \Delta_7 \quad \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_5 \vdash \Delta_7} \rightsquigarrow \\
\frac{h_3 : \top, \Delta_5 \vdash \perp, \Delta_7}{- : \top, \Delta_5 \vdash \Delta_7} \text{ax/W} \quad \frac{\bullet h_6 : \perp, \top, \Delta_5 \vdash \Delta_7}{- : \top, \Delta_5 \vdash \Delta_7} \perp_L \\
\hline
\frac{h_3 : \perp, \Delta_8 \vdash F_5, \Delta_7}{\bullet h_3 : \top, \perp, \Delta_8 \vdash \Delta_7, F_5} \top_L \quad \frac{}{\bullet h_6 : (\top, \perp, \Delta_8), F_5 \vdash \Delta_7} \perp_L \\
\hline
- : \top, \perp, \Delta_8 \vdash \Delta_7 \quad \text{Cut} \\
\hline
\frac{}{- : \top, \perp, \Delta_8 \vdash \Delta_7} \rightsquigarrow \\
\frac{}{- : \perp, \top, \Delta_8 \vdash \Delta_7} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_3 : \Delta_5 \vdash p_7, \Delta_8, p_7}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, p_7), p_7} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_5), p_7 \vdash \Delta_8, p_7} I \\
\hline
- : \top, \Delta_5 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_5 \vdash \Delta_8, p_7} \rightsquigarrow \\
\frac{h_3 : \top, \Delta_5 \vdash \Delta_8, p_7, p_7}{- : \top, \Delta_5 \vdash \Delta_8, p_7} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, p_7 \vdash \Delta_8, p_7}{- : \top, \Delta_5 \vdash \Delta_8, p_7} I \\
\hline
\frac{h_3 : \Delta_9, p_7 \vdash F_5, \Delta_8, p_7}{\bullet h_3 : \top, \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_9, p_7), F_5 \vdash \Delta_8, p_7} I \\
\hline
- : \top, \Delta_9, p_7 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_9, p_7 \vdash \Delta_8, p_7} \rightsquigarrow \\
\frac{}{- : \top, \Delta_9, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_3 : \Delta_6 \vdash F_5, \Delta_8}{\bullet h_3 : \top, \Delta_6 \vdash \Delta_8, F_5} \top_L \quad \frac{h_7 : F_5, \Delta_6 \vdash \Delta_8}{\bullet h_7 : (\top, \Delta_6), F_5 \vdash \Delta_8} \top_L \\
\hline
- : \top, \Delta_6 \vdash \Delta_8 \quad \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_6 \vdash \Delta_8} \rightsquigarrow \\
\frac{h_3 : \top, \Delta_6 \vdash \Delta_8, F_5}{- : \top, \Delta_6 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_6, F_5 \vdash \Delta_8}{- : \top, \Delta_6 \vdash \Delta_8} \text{hCut} \\
\hline
\frac{}{- : \top, \Delta_6 \vdash \Delta_8} \text{hCut}
\end{array}$$