

System G3C

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1 Height preserving admissibility of weakening on the left

- Case(s) rule \rightarrow_R

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4}}{\rightarrow_R} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\text{ax}}}{\frac{h_1 : \Delta_2, F_3, F_W \vdash \Delta_5, F_4}{\text{IH}}} \rightarrow_R}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \rightarrow F_4}$$

- Case(s) rule \wedge_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4}}{\wedge_R} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\text{ax}}}{\frac{h_1 : \Delta_2, F_W \vdash \Delta_5, F_3}{\text{IH}}} \quad \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{\text{ax}}}{\frac{h_1 : \Delta_2, F_W \vdash \Delta_5, F_4}{\text{IH}}} \wedge_R}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \wedge F_4}$$

- Case(s) rule \vee_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4}}{\vee_R} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\text{ax}}}{\frac{h_1 : \Delta_2, F_W \vdash \Delta_5, F_3, F_4}{\text{IH}}} \vee_R}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \vee F_4}$$

- Case(s) rule \perp_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3}}{\perp_R} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\text{ax}}}{\frac{h_1 : \Delta_2, F_W \vdash \Delta_3}{\text{IH}}} \perp_R}{\bullet h_1 : \Delta_2, F_W \vdash \perp, \Delta_3}$$

- Case(s) rule \top_R

$$\frac{\overline{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3}}{\top_R} \rightarrow \frac{\overline{\bullet h_1 : \Delta_2, F_W \vdash \top, \Delta_3}}{\top_R}$$

- Case(s) rule \rightarrow_L

$$\frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4}}{\rightarrow_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\text{ax}}}{\frac{h_1 : \Delta_5, F_W \vdash \Delta_4, F_2}{\text{IH}}} \quad \frac{\frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\text{ax}}}{\frac{h_1 : \Delta_5, F_3, F_W \vdash \Delta_4}{\text{IH}}} \rightarrow_L}{\bullet h_1 : \Delta_5, F_W, F_2 \rightarrow F_3 \vdash \Delta_4}$$

- Case(s) rule \wedge_L

$$\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4}}{\wedge_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\text{ax}}}{\frac{h_1 : \Delta_5, F_2, F_3, F_W \vdash \Delta_4}{\text{IH}}} \wedge_L}{\bullet h_1 : \Delta_5, F_W, F_2 \wedge F_3 \vdash \Delta_4}$$

- Case(s) rule \vee_L

$$\frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4}}{\vee_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\text{ax}}}{\frac{h_1 : \Delta_5, F_2, F_W \vdash \Delta_4}{\text{IH}}} \quad \frac{\frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\text{ax}}}{\frac{h_1 : \Delta_5, F_3, F_W \vdash \Delta_4}{\text{IH}}} \vee_L}{\bullet h_1 : \Delta_5, F_W, F_2 \vee F_3 \vdash \Delta_4}$$

- Case(s) rule \perp_L

$$\frac{\overline{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2}}{\perp_L} \rightarrow \frac{\overline{\bullet h_1 : \perp, \Delta_3, F_W \vdash \Delta_2}}{\perp_L}$$

- Case(s) rule I

$$\frac{\overline{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3}}{I} \rightarrow \frac{\overline{\bullet h_1 : \Delta_4, F_W, p_3 \vdash \Delta_2, p_3}}{I}$$

- Case(s) rule \top_L

$$\frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2}}{\top_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\text{ax}}}{\frac{h_1 : \Delta_3, F_W \vdash \Delta_2}{\text{IH}}} \top_L}{\bullet h_1 : \top, \Delta_3, F_W \vdash \Delta_2}$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule \rightarrow_R

$$\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \text{ ax}}{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \rightarrow F_4} \text{ IH}} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_W} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_4, F_W} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_W \quad h_1 : \Delta_2 \vdash \Delta_5, F_4, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \wedge F_4} \text{ IH}} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_W} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \vee F_4} \text{ IH}} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_W}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \text{ IH}} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, F_W} \top_R$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{h_1 : \Delta_5 \vdash \Delta_4, F_2, F_W} \text{ ax} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W} \text{ ax}}{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2, F_W \quad h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4, F_W} \text{ IH}} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4, F_W} \text{ ax}}{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4, F_W}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4, F_W} \text{ IH}} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{h_1 : \Delta_5, F_2 \vdash \Delta_4, F_W} \text{ ax} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W} \text{ ax}}{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4, F_W \quad h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4, F_W} \text{ IH}} \vee_L$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2, F_W} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, F_W, p_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \rightarrow \frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{h_1 : \Delta_3 \vdash \Delta_2, F_W} \text{ ax}}{\frac{h_1 : \Delta_3 \vdash \Delta_2, F_W}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2, F_W} \text{ IH}} \top_L$$

3 Measure of derivations

- Case(s) rule \rightarrow_R

$$\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R}{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R} \rightarrow$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \text{ax} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \quad \rightarrow \quad \frac{\frac{\frac{}{h_1 : \Delta_2 \vdash \Delta_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R$$

- Case(s) rule \top_R

$$\overline{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3}^{\top R} \quad \rightarrow \quad \overline{\bullet \bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3}^{\top R}$$

- Case(s) rule \rightarrow_L

$$\frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4}}{\bullet \bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \frac{\frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{IH} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{IH}}{\bullet \bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{IH}}{\bullet \bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} \text{IH}}{\bullet \bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \text{ax} \wedge_L$$

- Case(s) rule \forall_L

$$\frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4}}{\vee_L} \quad \rightarrow \quad \frac{\frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vdash \Delta_4} \text{IH} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{IH}}{\bullet \bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax}}{\vee_L} \text{ax}$$

- Case(s) rule \perp_L

$$\overline{\bullet \mathbf{h}_1 : \perp, \Delta_3 \vdash \Delta_2}^{\perp L} \rightarrow \overline{\bullet \bullet \mathbf{h}_1 : \perp, \Delta_3 \vdash \Delta_2}^{\perp L}$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \quad \rightarrow \quad \frac{\frac{\frac{}{h_1 : \Delta_3 \vdash \Delta_2} \text{ax}}{h_1 : \Delta_3 \vdash \Delta_2} \text{IH}}{\bullet h_1 : \Delta_3 \vdash \Delta_2} \top_L}{\bullet \bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L$$

4 Invertibility of Rules

4.1 Status of \rightarrow_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash \Delta_7, F_2, F_6}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \text{ax/ind} \rightarrow_R$$

$$\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{ax} \text{height}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \rightarrow F_2 \quad h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5} \text{ax/ind} \quad \overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \rightarrow F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash \Delta_5, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \rightarrow F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash \Delta_6, F_4, F_1 \rightarrow F_2 \quad h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \rightarrow F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_2, F_4} \text{ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6, F_2} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \rightarrow F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6, F_2} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1 \rightarrow F_2 \quad h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \rightarrow F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6, F_2} \text{ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6, F_2} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \rightarrow F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4, F_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \rightarrow F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, p_5 \vdash \Delta_4, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \rightarrow F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \rightarrow F_2} \top_L \rightarrow \frac{h_3 : \Delta_5, F_1 \vdash \Delta_4, F_2}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4, F_2} \top_L \text{ ax/ind}$$

4.2 Status of \wedge_R : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \rightarrow F_6} \rightarrow_R \text{ ax/ind}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ ax} \text{ height}$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6} \vee_R \text{ ax/ind}$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightarrow \frac{h_3 : \Delta_4 \vdash \Delta_5, F_1}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1} \perp_R \text{ ax/ind}$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash \Delta_6, F_4, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \wedge F_2} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \wedge F_2}}{\wedge_L} \rightarrow \frac{\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1}}{\wedge_L} \text{ ax/ind}$$

- Case rule \vee_L

$$\frac{\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \wedge F_2} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1 \wedge F_2}{\vee_L}}{\rightarrow} \frac{\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}{\vee_L}}{\vee_L} \text{ ax/ind}$$

- Case rule \perp_L

$$\frac{\frac{h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1}}{\perp_L} \rightarrow \frac{\frac{h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1}}{\perp_L}$$

- Case rule I

$$\frac{\frac{h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \wedge F_2}{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \wedge F_2}}{I} \rightarrow \frac{\frac{h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, p_5}{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, p_5}}{I}$$

- Case rule \top_L

$$\frac{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2}}{\top_L} \rightarrow \frac{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \text{ ax/ind}}{\top_L}$$

4.3 Status of \wedge_R (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6}}{\rightarrow_R} \rightarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_2, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \text{ ax/ind}}{\rightarrow_R}$$

- Case rule \wedge_R

$$\frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \quad \frac{h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \wedge F_2}{\wedge_R}}{\rightarrow} \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \wedge F_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_6}{\wedge_R}}{\wedge_R} \text{ ax/ind}$$

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{\wedge_R}}{\rightarrow} \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{ ax}}{\text{height}}$$

- Case rule \vee_R

$$\frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \quad \frac{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \wedge F_6}{\vee_R}}{\vee_R} \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \vee F_6} \text{ ax/ind}}{\vee_R}$$

- Case rule \perp_R

$$\frac{\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2}}{\perp_R} \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_5, F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_2} \text{ ax/ind}}{\perp_R}$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash \Delta_6, F_4, F_1 \wedge F_2 \quad h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \wedge F_2} \rightarrow_L \rightarrow \frac{\frac{}{h_3 : \Delta_7 \vdash \Delta_6, F_2, F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_2} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \wedge F_2} \wedge_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_2} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1 \wedge F_2 \quad h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \wedge F_2} \vee_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_2} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_2} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \wedge F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5 \vdash \Delta_4, F_2} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_2} \top_L$$

4.4 Status of \vee_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_6, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{}{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_2, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \vee F_2 \quad h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\frac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5} \text{ax/ind} \quad \frac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \vee F_6} \text{ax/ind}}{\vee_R}$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{ax}}{\text{height}}$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \vee F_2} \perp_R \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1, F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1, F_2} \text{ax/ind}}{\perp_R}$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \vee F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1, F_2} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash \Delta_6, F_4, F_1 \vee F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \vee F_2} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_2, F_4}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1, F_2} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1, F_2}{\text{ax/ind}}}{\rightarrow_L}$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1 \vee F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \vee F_2} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1, F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1, F_2} \text{ax/ind}}{\wedge_L}$$

- Case rule \vee_L

$$\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1 \vee F_2 \quad h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1 \vee F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \vee F_2} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1, F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1, F_2} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1, F_2}{\text{ax/ind}}}{\vee_L}$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \vee F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1, F_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \vee F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \vee F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \vee F_2} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1, F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1, F_2} \text{ax/ind}}{\top_L}$$

4.5 Status of \perp_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \Delta_2, F_3 \vdash \perp, \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \text{ax/ind} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \perp, \Delta_5, F_3 \quad h_1 : \Delta_2 \vdash \perp, \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3} \quad \overline{h_1 : \Delta_2 \vdash \Delta_5, F_4}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \text{ax/ind} \wedge_R$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \perp, \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \vee F_4} \vee_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \text{ax/ind} \vee_R$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{ax} \text{height}$$

- Case rule \top_R

$$\overline{\bullet h_1 : \Delta_2 \vdash \top, \perp, \Delta_3} \top_R \rightarrow \overline{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash \perp, \Delta_4, F_2 \quad h_1 : \Delta_5, F_3 \vdash \perp, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \perp, \Delta_4} \rightarrow_L \rightarrow \frac{\overline{h_1 : \Delta_5 \vdash \Delta_4, F_2} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ax/ind} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_1 : \Delta_5, F_2, F_3 \vdash \perp, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \perp, \Delta_4} \wedge_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \text{ax/ind} \wedge_L$$

- Case rule \vee_L

$$\frac{h_1 : \Delta_5, F_2 \vdash \perp, \Delta_4 \quad h_1 : \Delta_5, F_3 \vdash \perp, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \perp, \Delta_4} \vee_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2 \vdash \Delta_4} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax/ind} \vee_L$$

- Case rule \perp_L

$$\overline{\bullet h_1 : \perp, \Delta_3 \vdash \perp, \Delta_2} \perp_L \rightarrow \overline{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_4 \vdash p_3, \perp, \Delta_2} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I$$

- Case rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \perp, \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \perp, \Delta_2} \top_L \rightarrow \frac{\frac{}{h_1 : \Delta_3 \vdash \Delta_2} \text{ax/ind}}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L$$

4.6 Status of \top_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \Delta_2, F_3 \vdash \top, \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_5, F_3 \quad h_1 : \Delta_2 \vdash \top, \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \wedge F_4} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \vee F_4} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \top, \Delta_3} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash \top, \Delta_4, F_2 \quad h_1 : \Delta_5, F_3 \vdash \top, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \top, \Delta_4} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : \Delta_5, F_2, F_3 \vdash \top, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \top, \Delta_4} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : \Delta_5, F_2 \vdash \top, \Delta_4 \quad h_1 : \Delta_5, F_3 \vdash \top, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \top, \Delta_4} \vee_L \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \top, \Delta_2} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_4 \vdash p_3, \top, \Delta_2} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \top, \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \top, \Delta_2} \top_L \rightarrow \text{trivial}$$

4.7 Status of \rightarrow_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1, F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R}{\frac{\frac{}{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \wedge F_5} \wedge_R} \rightarrow$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{}{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4, F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{}{h_3 : \Delta_5 \vdash \Delta_4, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_5 \vdash \perp, \Delta_4, F_1} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5 \vdash \top, \Delta_4, F_1} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{}{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \rightarrow_L$$

$$\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{}{h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{ax}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{height}$$

- Case rule \wedge_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6}}{\wedge_L} \rightarrow \frac{\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\wedge_L}$$

- Case rule \vee_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \quad \frac{h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\vee_L}}{\rightarrow} \frac{\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}{\vee_L} \text{ ax/ind}}{\vee_L}$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \rightarrow F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, p_5} I$$

- Case rule \top_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}}{\top_L} \rightarrow \frac{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \text{ ax/ind}}{\top_L}$$

4.8 Status of \rightarrow_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \rightarrow F_5}}{\rightarrow_R} \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \text{ ax/ind}}{\rightarrow_R}$$

- Case rule \wedge_R

$$\frac{\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \wedge F_5} \quad \frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_5}{\wedge_R}}{\rightarrow} \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_5}{\wedge_R} \text{ ax/ind}}{\wedge_R}$$

- Case rule \vee_R

$$\frac{\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \vee F_5}}{\vee_R} \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \text{ ax/ind}}{\vee_R}$$

- Case rule \perp_R

$$\frac{\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \perp, \Delta_4}}{\perp_R} \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_2 \vdash \perp, \Delta_4} \text{ ax/ind}}{\perp_R}$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_2 \vdash \top, \Delta_4} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

$$\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{height}$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \rightarrow F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_2 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

4.9 Status of \wedge_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_7, F_4, F_1 \wedge F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6, F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R}{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \vee F_5} \text{ ax/ind}} \vee_R \rightarrow$$

- Case rule \perp_R

$$\frac{\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \wedge F_2 \vdash \perp, \Delta_4} \perp_R}{\frac{h_3 : \Delta_5, F_1, F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1, F_2 \vdash \perp, \Delta_4} \text{ ax/ind}} \perp_R \rightarrow$$

- Case rule \top_R

$$\frac{\overline{\bullet h_3 : \Delta_5, F_1 \wedge F_2 \vdash \top, \Delta_4}}{\overline{\bullet h_3 : \Delta_5, F_1, F_2 \vdash \top, \Delta_4}} \top_R \rightarrow$$

- Case rule \rightarrow_L

$$\frac{\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L}{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ ax/ind}} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ ax/ind}}{\frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_5, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L}{\frac{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash \Delta_6} \text{ ax/ind}} \wedge_L \rightarrow$$

$$\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L}{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} \text{ ax}} \text{ height} \rightarrow$$

- Case rule \vee_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L}{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ ax/ind}} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} \text{ ax/ind}}{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ ax/ind}} \vee_L$$

- Case rule \perp_L

$$\frac{\overline{\bullet h_3 : \perp, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}}{\overline{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash \Delta_4}} \perp_L \rightarrow$$

- Case rule I

$$\frac{\overline{\bullet h_3 : p_5, \Delta_6, F_1 \wedge F_2 \vdash p_5, \Delta_4}}{\overline{\bullet h_3 : \Delta_6, F_1, F_2, p_5 \vdash \Delta_4, p_5}} I \rightarrow$$

- Case rule \top_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \top_L}{\frac{h_3 : \Delta_5, F_1, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash \Delta_4} \text{ ax/ind}} \top_L \rightarrow$$

4.10 Status of \vee_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6, F_5}}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \rightarrow F_5} \xrightarrow{ax/ind} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4} \quad \overline{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_5}}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \wedge F_5} \xrightarrow{ax/ind} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4, F_5}}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \vee F_5} \xrightarrow{ax/ind} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_5, F_1 \vdash \Delta_4}}{\bullet h_3 : \Delta_5, F_1 \vdash \perp, \Delta_4} \xrightarrow{ax/ind} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_1 \vdash \top, \Delta_4} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \xrightarrow{ax/ind} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \xrightarrow{ax/ind} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \xrightarrow{ax/ind} \vee_L$$

$$\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2 \vdash \Delta_4}}{\bullet h_1 : \Delta_5, F_2 \vdash \Delta_4} \xrightarrow{ax \text{ height}} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \vee F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_1 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \top_L$$

4.11 Status of \vee_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4, F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_2 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \Delta_5, F_2 \vdash \perp, \Delta_4} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_2 \vdash \top, \Delta_4} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind}}{\vee_L}$$

$$\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4 \quad h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{height}$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \vee F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L$$

4.12 Status of \perp_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \perp, \Delta_5, F_2 \vdash \Delta_4, F_3}{\bullet h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \perp, \Delta_5 \vdash \Delta_4, F_3}{\bullet h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2, F_3}{\bullet h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \perp, \Delta_3 \vdash \perp, \Delta_2} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \top, \Delta_2} \top_R \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \perp, \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : \perp, \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : \perp, \Delta_5, F_2 \vdash \Delta_4 \quad h_1 : \perp, \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \perp, \Delta_4 \vdash p_3, \Delta_2} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \perp, \Delta_3 \vdash \Delta_2} \top_L \rightarrow \text{trivial}$$

4.13 Status of I : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \Delta_6, F_2, p_5 \vdash \Delta_4, F_3, p_5}{\bullet h_1 : \Delta_6, p_5 \vdash (\Delta_4, p_5), F_2 \rightarrow F_3} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_6, p_5 \vdash \Delta_4, F_2, p_5 \quad h_1 : \Delta_6, p_5 \vdash \Delta_4, F_3, p_5}{\bullet h_1 : \Delta_6, p_5 \vdash (\Delta_4, p_5), F_2 \wedge F_3} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_6, p_5 \vdash \Delta_4, F_2, F_3, p_5}{\bullet h_1 : \Delta_6, p_5 \vdash (\Delta_4, p_5), F_2 \vee F_3} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3}{\bullet h_1 : \Delta_4, p_3 \vdash \perp, \Delta_2, p_3} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \top, \Delta_2, p_3} \top_R \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \Delta_6, p_5 \vdash \Delta_4, F_2, p_5 \quad h_1 : \Delta_6, F_3, p_5 \vdash \Delta_4, p_5}{\bullet h_1 : (\Delta_6, p_5), F_2 \rightarrow F_3 \vdash \Delta_4, p_5} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : \Delta_6, F_2, F_3, p_5 \vdash \Delta_4, p_5}{\bullet h_1 : (\Delta_6, p_5), F_2 \wedge F_3 \vdash \Delta_4, p_5} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : \Delta_6, F_2, p_5 \vdash \Delta_4, p_5 \quad h_1 : \Delta_6, F_3, p_5 \vdash \Delta_4, p_5}{\bullet h_1 : (\Delta_6, p_5), F_2 \vee F_3 \vdash \Delta_4, p_5} \vee_L \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_4, p_3 \vdash \Delta_2, p_3} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_4, \Delta_5, p_3 \vdash p_4, \Delta_2, p_3} I \rightarrow \text{trivial}$$

$$\frac{}{\bullet h_1 : p_3, \Delta_4 \vdash p_3, \Delta_2} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3}{\bullet h_1 : \top, \Delta_4, p_3 \vdash \Delta_2, p_3} \top_L \rightarrow \text{trivial}$$

4.14 Status of \top_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \top, \Delta_5, F_2 \vdash \Delta_4, F_3}{\bullet h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_2 \vdash \Delta_4, F_3} \text{ax/ind}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \top, \Delta_5 \vdash \Delta_4, F_3}{\bullet h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R \rightarrow \frac{\frac{}{h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{ax/ind} \quad \frac{}{h_1 : \Delta_5 \vdash \Delta_4, F_3} \text{ax/ind}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R$$

- Case rule \vee_R

$$\frac{h_1 : \top, \Delta_5 \vdash \Delta_4, F_2, F_3}{\bullet h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2, F_3}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \text{ax/ind}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \vee_R$$

- Case rule \perp_R

$$\frac{h_1 : \top, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \perp, \Delta_2} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \Delta_3 \vdash \perp, \Delta_2} \text{ax/ind}}{\bullet h_1 : \Delta_3 \vdash \perp, \Delta_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \top, \Delta_3 \vdash \top, \Delta_2} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_3 \vdash \top, \Delta_2} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \top, \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_1 : \top, \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L$$

- Case rule \vee_L

$$\frac{h_1 : \top, \Delta_5, F_2 \vdash \Delta_4 \quad h_1 : \top, \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \top, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \top, \Delta_4 \vdash p_3, \Delta_2} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I$$

- Case rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \rightarrow \frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \Delta_3 \vdash \Delta_2} \text{ax}}{\bullet h_1 : \Delta_3 \vdash \Delta_2} \text{height}$$

5 Height preserving admissibility of contraction on the left

- Case(s) rule \rightarrow_R

$$\frac{\frac{h_1 : \Delta_5, F_2, F_6, F_6 \vdash \Delta_4, F_3}{\bullet h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_2 \rightarrow F_3}}{\rightarrow_R} \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_6, F_6 \vdash \Delta_4, F_3}{h_1 : \Delta_5, F_2, F_6 \vdash \Delta_4, F_3} \text{IH}}{\bullet h_1 : \Delta_5, F_6 \vdash \Delta_4, F_2 \rightarrow F_3} \text{ax} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{\frac{h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_2 \quad h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_3}{\bullet h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_2 \wedge F_3}}{\wedge_R} \quad \rightarrow \quad \frac{\frac{\frac{h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_2}{h_1 : \Delta_5, F_6 \vdash \Delta_4, F_2} \text{IH} \quad \frac{h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_3}{h_1 : \Delta_5, F_6 \vdash \Delta_4, F_3} \text{IH}}{\bullet h_1 : \Delta_5, F_6 \vdash \Delta_4, F_2 \wedge F_3}}{\wedge_R} \text{ax}$$

- Case(s) rule \vee_R

$$\frac{\frac{h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_2, F_3}{\bullet h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_2 \vee F_3} \vee R}{\frac{h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_2, F_3}{h_1 : \Delta_5, F_6 \vdash \Delta_4, F_2, F_3} \text{IH} \quad \frac{h_1 : \Delta_5, F_6 \vdash \Delta_4, F_2 \vee F_3}{\bullet h_1 : \Delta_5, F_6 \vdash \Delta_4, F_2 \vee F_3} \vee R} \rightarrow$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_3, F_4, F_4 \vdash \Delta_2}{\bullet h_1 : \Delta_3, F_4, F_4 \vdash \perp, \Delta_2} \perp_R \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3, F_4, F_4 \vdash \Delta_2}{h_1 : \Delta_3, F_4 \vdash \Delta_2} \text{IH}}{\bullet h_1 : \Delta_3, F_4 \vdash \perp, \Delta_2} \perp_R \text{ax}$$

- Case(s) rule \top_R

$$\frac{\bullet h_1 : \Delta_3, F_4, F_4 \vdash \top, \Delta_2}{\bullet h_1 : \Delta_3, F_4 \vdash \top, \Delta_2} \top_R \quad \rightarrow \quad \frac{\bullet h_1 : \Delta_3, F_4 \vdash \top, \Delta_2}{\bullet h_1 : \Delta_3, F_4 \vdash \top, \Delta_2} \top_R$$

- Case(s) rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4, F_2 \quad h_1 : \Delta_5, F_3, F_2 \rightarrow F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2, F_2}{h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{inv-th/ax} \quad \frac{h_1 : \Delta_5, F_3, F_3 \vdash \Delta_4}{h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{inv-th/ax}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{IH-Mutual} \rightarrow_L \\
\\
\frac{h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_2 \quad h_1 : \Delta_5, F_3, F_6, F_6 \vdash \Delta_4}{\bullet h_1 : (\Delta_5, F_2 \rightarrow F_3), F_6, F_6 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5, F_6, F_6 \vdash \Delta_4, F_2}{h_1 : \Delta_5, F_6 \vdash \Delta_4, F_2} \text{ax} \quad \frac{h_1 : \Delta_5, F_3, F_6, F_6 \vdash \Delta_4}{h_1 : \Delta_5, F_3, F_6 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, F_6, F_2 \rightarrow F_3 \vdash \Delta_4} \text{IH} \rightarrow_L
\end{array}$$

- Case(s) rule \wedge_L

$$\begin{array}{ccc}
\frac{\frac{h_1 : \Delta_5, F_2, F_3, F_2 \wedge F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3, F_2 \wedge F_3 \vdash \Delta_4}}{\wedge_L} & \rightarrow & \frac{\frac{\frac{h_1 : \Delta_5, F_2, F_2, F_3 \vdash \Delta_4}{\wedge_L} \text{ IH}}{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} \text{ IH}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \text{ inv-th/ax} \\
\\
\frac{h_1 : \Delta_5, F_2, F_3, F_6, F_6 \vdash \Delta_4}{\bullet h_1 : (\Delta_5, F_2 \wedge F_3), F_6, F_6 \vdash \Delta_4} \wedge_L & \rightarrow & \frac{\frac{h_1 : \Delta_5, F_2, F_3, F_6, F_6 \vdash \Delta_4}{h_1 : \Delta_5, F_2, F_3, F_6 \vdash \Delta_4} \text{ IH}}{\bullet h_1 : \Delta_5, F_6, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \text{ ax}
\end{array}$$

- Case(s) rule \forall_L

$$\frac{\frac{h_1 : \Delta_5, F_2, F_2 \vee F_3 \vdash \Delta_4 \quad h_1 : \Delta_5, F_3, F_2 \vee F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3, F_2 \vee F_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5, F_2, F_2 \vdash \Delta_4}{h_1 : \Delta_5, F_2 \vdash \Delta_4} \text{IH} \quad \frac{\frac{h_1 : \Delta_5, F_3, F_3 \vdash \Delta_4}{h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{IH} \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \text{IH}$$

$$\frac{\frac{h_1 : \Delta_5, F_2, F_6, F_6 \vdash \Delta_4 \quad h_1 : \Delta_5, F_3, F_6, F_6 \vdash \Delta_4}{\bullet h_1 : (\Delta_5, F_2 \vee F_3), F_6, F_6 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_5, F_2, F_6, F_6 \vdash \Delta_4}{h_1 : \Delta_5, F_2, F_6 \vdash \Delta_4} \text{IH} \quad \frac{\frac{h_1 : \Delta_5, F_3, F_6, F_6 \vdash \Delta_4}{h_1 : \Delta_5, F_3, F_6 \vdash \Delta_4} \text{IH} \quad \text{ax}}{\bullet h_1 : \Delta_5, F_6, F_2 \vee F_3 \vdash \Delta_4} \vee_L \text{IH}$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \Delta_3, \perp, \perp \vdash \Delta_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

$$\frac{}{\bullet h_1 : (\perp, \Delta_3), F_4, F_4 \vdash \Delta_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3, F_4 \vdash \Delta_2} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_4, P_3, P_3 \vdash \Delta_2, P_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, P_3 \vdash \Delta_2, P_3} I$$

$$\frac{}{\bullet h_1 : (\Delta_4, P_3), F_5, F_5 \vdash \Delta_2, P_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, F_5, P_3 \vdash \Delta_2, P_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \top, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \Delta_3, \top, \top \vdash \Delta_2} \top_L \rightarrow \frac{\frac{}{h_1 : \Delta_3 \vdash \Delta_2} \text{inv-th/ax}}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L$$

$$\frac{h_1 : \Delta_3, F_4, F_4 \vdash \Delta_2}{\bullet h_1 : (\top, \Delta_3), F_4, F_4 \vdash \Delta_2} \top_L \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_3, F_4, F_4 \vdash \Delta_2} \text{ax}}{h_1 : \Delta_3, F_4 \vdash \Delta_2} \text{IH}}{\bullet h_1 : \top, \Delta_3, F_4 \vdash \Delta_2} \top_L$$

6 Height preserving admissibility of contraction on the Right

- Case(s) rule \rightarrow_R

$$\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4, F_3 \rightarrow F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_3 \vdash \Delta_5, F_4, F_4}{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4, F_4} \text{IH-Mutual} \quad \frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R$$

$$\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4, F_6, F_6}{\bullet h_1 : \Delta_2 \vdash (\Delta_5, F_3 \rightarrow F_4), F_6, F_6} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4, F_6, F_6}{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4, F_6} \text{ax} \quad \frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4, F_6}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_6, F_3 \rightarrow F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_6, F_3 \rightarrow F_4} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_3 \wedge F_4 \quad h_1 : \Delta_2 \vdash \Delta_5, F_4, F_3 \wedge F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_3}{h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4, F_4}{h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_6, F_6 \quad h_1 : \Delta_2 \vdash \Delta_5, F_4, F_6, F_6}{\bullet h_1 : \Delta_2 \vdash (\Delta_5, F_3 \wedge F_4), F_6, F_6} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_6, F_6}{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_6} \text{ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4, F_6, F_6}{h_1 : \Delta_2 \vdash \Delta_5, F_4, F_6} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_6, F_3 \wedge F_4} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_3 \vee F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_3, F_4, F_4}{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_3, F_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_6, F_6}{\bullet h_1 : \Delta_2 \vdash (\Delta_5, F_3 \vee F_4), F_6, F_6} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_6, F_6}{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_6} \text{ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_6}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_6, F_3 \vee F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_6, F_3 \vee F_4} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \perp, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, \perp, \perp} \perp_R \rightarrow \frac{h_1 : \Delta_2 \vdash \perp, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_4}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_3), F_4, F_4} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_4}{h_1 : \Delta_2 \vdash \Delta_3, F_4} \text{ax} \quad \frac{h_1 : \Delta_2 \vdash \perp, \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_4} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \Delta_3, \top, \top} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_3), F_4, F_4} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, F_4} \top_R$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_6 \vdash \Delta_4, F_2, F_5, F_5 \quad h_1 : \Delta_6, F_3 \vdash \Delta_4, F_5, F_5}{\bullet h_1 : \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4, F_5, F_5} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_6 \vdash \Delta_4, F_2, F_5, F_5}{h_1 : \Delta_6 \vdash \Delta_4, F_2, F_5} \text{ax} \quad \frac{h_1 : \Delta_6, F_3 \vdash \Delta_4, F_5, F_5}{h_1 : \Delta_6, F_3 \vdash \Delta_4, F_5} \text{ax}}{\bullet h_1 : \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4, F_5} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{\frac{h_1 : \Delta_6, F_2, F_3 \vdash \Delta_4, F_5, F_5}{\bullet h_1 : \Delta_6, F_2 \wedge F_3 \vdash \Delta_4, F_5, F_5}}{\wedge_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_6, F_2, F_3 \vdash \Delta_4, F_5, F_5}{\bullet h_1 : \Delta_6, F_2 \wedge F_3 \vdash \Delta_4, F_5}}{\wedge_L} \quad \frac{\frac{h_1 : \Delta_6, F_2, F_3 \vdash \Delta_4, F_5, F_5}{\bullet h_1 : \Delta_6, F_2 \wedge F_3 \vdash \Delta_4, F_5, F_5}}{\wedge_L} \quad \text{ax} \quad \text{IH}}{\wedge_L}$$

- Case(s) rule \vee_L

$$\frac{\frac{h_1 : \Delta_6, F_2 \vdash \Delta_4, F_5, F_5}{\bullet h_1 : \Delta_6, F_2 \vee F_3 \vdash \Delta_4, F_5, F_5} \quad \frac{h_1 : \Delta_6, F_3 \vdash \Delta_4, F_5, F_5}{\bullet h_1 : \Delta_6, F_2 \vee F_3 \vdash \Delta_4, F_5, F_5}}{\vee_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_6, F_2 \vdash \Delta_4, F_5, F_5}{\bullet h_1 : \Delta_6, F_2 \vee F_3 \vdash \Delta_4, F_5, F_5}}{\vee_L} \quad \frac{\frac{h_1 : \Delta_6, F_3 \vdash \Delta_4, F_5, F_5}{\bullet h_1 : \Delta_6, F_2 \vee F_3 \vdash \Delta_4, F_5, F_5}}{\vee_L} \quad \text{ax} \quad \text{IH} \quad \text{IH}}{\vee_L}$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash \Delta_2, F_3, F_3} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_4 \vdash \Delta_2, F_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_4, P_3 \vdash \Delta_2, P_3, P_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, P_3 \vdash \Delta_2, P_3} I$$

$$\frac{}{\bullet h_1 : \Delta_5, P_4 \vdash (\Delta_2, P_4), F_3, F_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_5, P_4 \vdash \Delta_2, F_3, P_4} I$$

- Case(s) rule \top_L

$$\frac{\frac{h_1 : \Delta_4 \vdash \Delta_2, F_3, F_3}{\bullet h_1 : \top, \Delta_4 \vdash \Delta_2, F_3, F_3}}{\top_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_4 \vdash \Delta_2, F_3, F_3}{\bullet h_1 : \top, \Delta_4 \vdash \Delta_2, F_3}}{\top_L} \quad \text{ax} \quad \text{IH}}{\top_L}$$

7 Identity-Expansion

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0, F_1} W}{- : F_0 \vee F_1 \vdash F_0, F_1} \vee_L}{- : F_0 \vee F_1 \vdash F_0 \vee F_1} \vee_R$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_1 \vdash F_0} W \quad \frac{\frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{- : F_0, F_1 \vdash F_0 \wedge F_1} \wedge_R}{- : F_0 \wedge F_1 \vdash F_0 \wedge F_1} \wedge_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{- : F_0, F_0 \rightarrow F_1 \vdash F_1} \rightarrow_L}{- : F_0 \rightarrow F_1 \vdash F_0 \rightarrow F_1} \rightarrow_R$$

$$\frac{}{- : \top \vdash \top} \top_R$$

$$\frac{}{- : \perp \vdash \perp} \perp_L$$

8 Cut-Elimination

8.1 Status of \rightarrow_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_8, F_{14}}{\bullet h_1 : \Delta_{13} \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8), F_{14}} \rightarrow_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8} \rightarrow_R \\
\frac{}{- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_{11}, F_{14}, F_8}{\bullet h_1 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_{14}, F_7 \rightarrow F_8} \text{inv-th/ax}}{\rightarrow_R} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8} \text{ax/W} \\
\frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8} \text{hCut} \\
\frac{}{\rightarrow_R} \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \rightarrow F_8} \\
\frac{h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, F_9, F_{12}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_8 \rightarrow F_9), F_{12}} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_{12} \vdash \Delta_{10}, F_9}{\bullet h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_8 \rightarrow F_9} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, F_{12}, F_9}{\bullet h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, F_{12}, F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_{12}, F_8 \vdash \Delta_{10}, F_9}{\bullet h_7 : \Delta_{11}, F_{12}, F_8 \vdash \Delta_{10}, F_9} \text{ax/W} \\
\frac{}{- : \Delta_{11}, F_8 \vdash \Delta_{10}, F_9} \text{height} \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_8 \rightarrow F_9} \text{hCut} \\
\frac{}{\rightarrow_R} \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_8 \rightarrow F_9} \\
\frac{h_1 : \Delta_{12}, F_6 \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_7}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_{10}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6 \rightarrow F_7} \text{inv-th/ax} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}} \text{ax/W} \\
\frac{}{- : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}} \text{hCut} \\
\frac{}{\rightarrow_R} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_8, F_{14}}{\bullet h_1 : \Delta_{13} \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8), F_{14}} \rightarrow_R \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_7 \rightarrow F_8 \quad h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8} \wedge_R \\
\frac{}{- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{14}, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{14}, F_8, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \vdash \Delta_{12}, F_{10}, F_8}{\bullet h_9 : \Delta_{13}, F_{14}, F_7 \vdash \Delta_{12}, F_{10}, F_8} \text{inv-th/ax} \\
\frac{}{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{14}, F_8, F_{10} \wedge F_{11}} \text{hCut} \\
\frac{}{\rightarrow_R} \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8, F_{10} \wedge F_{11}} \\
\frac{h_1 : \Delta_{12}, F_6 \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_7}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \wedge_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \text{inv-th/ax} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9} \text{ax/W} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \text{hCut} \\
\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_{10}, F_7}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6 \rightarrow F_7} \text{inv-th/ax} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}} \text{ax/W} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_{10}} \text{hCut} \\
\frac{}{\wedge_R} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_8, F_{14}}{\bullet h_1 : \Delta_{13} \vdash ((\Delta_{12}, F_{10} \vee F_{11}), F_7 \rightarrow F_8), F_{14}} \rightarrow_R \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \rightarrow F_8} \vee_R \\
\hline
- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{14}, F_8, F_{10} \vee F_{11}}{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11}} \text{ ax/W} \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \vdash \Delta_{12}, F_{10}, F_{11}, F_8}{\bullet h_9 : \Delta_{13}, F_{14}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11}} \text{ inv-th/ax} \vee_R}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8, F_{10} \vee F_{11}} \text{ hCut} \\
\rightarrow_R \\
\frac{h_1 : \Delta_{12}, F_6 \vdash (\Delta_{11}, F_9 \vee F_{10}), F_7}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_{10}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9, F_6 \rightarrow F_7} \text{ inv-th/ax} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}, F_9}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}, F_9} \text{ ax/W} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9 \quad \text{hCut} \\
\rightarrow_R \\
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \quad \vee_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7 \vdash (\perp, \Delta_{10}), F_8, F_{12}}{\bullet h_1 : \Delta_{11} \vdash ((\perp, \Delta_{10}), F_7 \rightarrow F_8), F_{12}} \rightarrow_R \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{11}, F_{12} \vdash (\perp, \Delta_{10}), F_7 \rightarrow F_8} \perp_R \\
\hline
- : \Delta_{11} \vdash (\perp, \Delta_{10}), F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_{12}, F_7 \rightarrow F_8}{- : \Delta_{11} \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8} \text{ ax/W} \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{11}, F_{12} \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8} \text{ hCut} \\
\rightarrow_R \\
\frac{h_1 : \Delta_{10}, F_6 \vdash (\perp, \Delta_9), F_7}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_9} \perp_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \rightarrow F_7}{- : \Delta_{10} \vdash \perp, \Delta_9} \text{ ax/W} \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_9} \text{ hCut} \\
\rightarrow_R \\
- : \Delta_{10} \vdash \perp, \Delta_9
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7 \vdash (\top, \Delta_{10}), F_8, F_{12}}{\bullet h_1 : \Delta_{11} \vdash ((\top, \Delta_{10}), F_7 \rightarrow F_8), F_{12}} \rightarrow_R \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{11}, F_{12} \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8} \top_R \\
\hline
- : \Delta_{11} \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{11} \vdash \top, \Delta_{10}, F_7 \rightarrow F_8}{- : \Delta_{11} \vdash \top, \Delta_{10}, F_7 \rightarrow F_8} \top_R \\
\rightarrow_R \\
\frac{h_1 : \Delta_{10}, F_6 \vdash (\top, \Delta_9), F_7}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_9} \top_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_9 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{10} \vdash \top, \Delta_9}{- : \Delta_{10} \vdash \top, \Delta_9} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vdash \Delta_{12}, F_8, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{14}} \rightarrow_R \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_7 \rightarrow F_8 \quad h_9 : \Delta_{13}, F_{11}, F_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow_L \\
\hline
- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{14}, F_8}{- : \Delta_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \text{ ax/W} \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \vdash \Delta_{12}, F_{10}, F_8}{\bullet h_9 : \Delta_{13}, F_{14}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \text{ inv-th/ax} \rightarrow_L \\
\hline
- : \Delta_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8 \quad \text{hCut} \\
\rightarrow_R \\
- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{10} \rightarrow F_{11}} \rightarrow_R \quad \frac{h_9 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \rightarrow F_8 \quad h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_9 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8} \text{inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8} \text{inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \text{hCut} \\
\frac{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow_R \\
\frac{\frac{h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \rightarrow_L}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}{\bullet h_1 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9} \text{ax/W}}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \text{hCut} \\
\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9}{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9} \rightarrow_R \\
\frac{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, F_7 \vdash \Delta_9, F_8}{\bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_6 : \Delta_{10} \vdash \Delta_9, F_7 \quad h_6 : \Delta_{10}, F_8 \vdash \Delta_9}{\bullet h_6 : \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \rightarrow_L}{- : \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{10}, F_7 \vdash \Delta_9, F_8}{- : \Delta_{10} \vdash \Delta_9, F_7} \text{ax/W} \quad \frac{- : \Delta_{10}, F_7, F_8 \vdash \Delta_9}{- : \Delta_{10}, F_7 \vdash \Delta_9} \text{ax/W}}{- : \Delta_{10} \vdash \Delta_9, F_7} \text{sCut} \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7}{- : \Delta_{10} \vdash \Delta_9} \text{sCut}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vdash \Delta_{12}, F_8, F_{14} \quad h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_1 : \Delta_{13}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{14} \quad \bullet h_9 : (\Delta_{13}, F_{10} \wedge F_{11}), F_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8} \wedge_L \text{Cut} \\
\frac{}{- : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow \\
\frac{h_1 : \Delta_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_{14}, F_8 \quad \bullet h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14}, F_7 \vdash \Delta_{12}, F_8}{h_1 : \Delta_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_{14}, F_8 \quad \bullet h_9 : \Delta_{13}, F_{14}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \text{ax/W} \quad \text{inv-th/ax} \\
\frac{}{- : \Delta_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \wedge_L \\
\frac{}{- : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \text{hCut} \quad \rightarrow_R \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11} \quad h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{10} \wedge F_{11} \quad \bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow_R \quad \wedge_L \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \text{Cut} \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11} \quad \bullet h_9 : \Delta_{13}, F_{10}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11} \quad \bullet h_9 : \Delta_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \text{ax/W} \quad \text{inv-th/ax} \\
\frac{}{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} \wedge_L \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \text{hCut} \quad \rightarrow_R \\
\frac{h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vdash \Delta_{11}, F_7 \quad h_8 : \Delta_{12}, F_9, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7 \quad \bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \rightarrow_R \quad \wedge_L \\
\frac{}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut} \\
\frac{}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \rightarrow \\
\frac{h_1 : \Delta_{12}, F_{10}, F_6, F_9 \vdash \Delta_{11}, F_7 \quad h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7 \quad h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{inv-th/ax} \quad \text{ax/W} \\
\frac{}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \wedge_L \\
\frac{}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{hCut} \quad \wedge_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vdash \Delta_{12}, F_8, F_{14} \rightarrow_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : \Delta_{13}, F_{11}, F_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : (\Delta_{13}, F_{10} \vee F_{11}), F_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8} \vee_L}{\bullet h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{14} \rightarrow_R \quad \bullet h_9 : (\Delta_{13}, F_{10} \vee F_{11}), F_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8} \text{Cut} \\
\frac{}{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{14}, F_8}{\bullet h_1 : \Delta_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{14}, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_{14}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_{14}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_{11}, F_{14}, F_7 \vdash \Delta_{12}, F_8} \text{inv-th/ax}}{\frac{}{- : \Delta_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \rightarrow_R \quad \frac{}{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \text{hCut}}{\frac{}{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} \rightarrow_R} \vee_L \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11} \rightarrow_R \quad \frac{h_9 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \vee_L}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{10} \vee F_{11} \rightarrow_R \quad \bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \text{Cut} \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8} \text{inv-th/ax}}{\frac{}{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} \rightarrow_R \quad \frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \text{hCut}}{\frac{}{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} \rightarrow_R} \vee_L \\
\frac{h_1 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vdash \Delta_{11}, F_7 \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \vee_L}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7 \rightarrow_R \quad \bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{Cut} \\
\frac{}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7} \text{inv-th/ax}}{\frac{}{- : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7} \rightarrow_R \quad \frac{}{- : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{hCut} \quad \frac{}{- : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7} \rightarrow_R} \vee_L \\
\frac{}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7 \vdash \Delta_{10}, F_8, \perp \rightarrow_R \quad \frac{}{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \rightarrow F_8} \perp_L}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \rightarrow F_8), \perp \rightarrow_R \quad \bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{Cut} \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}, F_8}{\bullet h_1 : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}, F_8} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}, F_8}{\bullet h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}, F_8} \perp_L}{\frac{}{- : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}, F_8} \rightarrow_R \quad \frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{hCut}}{\frac{}{- : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}, F_8} \rightarrow_R} \vee_L \\
\frac{h_1 : (\perp, \Delta_{11}), F_7 \vdash \Delta_{10}, F_8, F_{12} \rightarrow_R \quad \frac{}{\bullet h_9 : (\perp, \Delta_{11}), F_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8} \perp_L}{\bullet h_1 : \perp, \Delta_{11} \vdash (\Delta_{10}, F_7 \rightarrow F_8), F_{12} \rightarrow_R \quad \bullet h_9 : (\perp, \Delta_{11}), F_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{Cut} \\
\frac{}{- : \perp, \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \rightarrow \\
\frac{}{- : \perp, \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \perp_L \\
\frac{h_1 : (\perp, \Delta_{10}), F_6 \vdash \Delta_9, F_7 \rightarrow_R \quad \frac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \perp_L}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7 \rightarrow_R \quad \bullet h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \text{Cut} \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \rightarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_7 \vdash (\Delta_{10}, p_{11}), F_8, p_{11} \rightarrow_R \quad \frac{}{\bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8} I}{\bullet h_1 : \Delta_{12} \vdash ((\Delta_{10}, p_{11}), F_7 \rightarrow F_8), p_{11} \rightarrow_R \quad \bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8} \text{Cut} \\
\frac{}{- : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8} \rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_7 \vdash \Delta_{10}, F_8, p_{11}, p_{11}}{\bullet h_1 : \Delta_{12}, F_7 \vdash \Delta_{10}, F_8, p_{11}, p_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_7, p_{11} \vdash \Delta_{10}, F_8, p_{11}}{\bullet h_9 : \Delta_{12}, F_7, p_{11} \vdash \Delta_{10}, F_8, p_{11}} I}{\frac{}{- : \Delta_{12}, F_7 \vdash \Delta_{10}, F_8, p_{11}, p_{11}} \rightarrow_R \quad \frac{}{- : \Delta_{12} \vdash \Delta_{10}, p_{11}, F_7 \rightarrow F_8} \text{hCut}}{\frac{}{- : \Delta_{12}, F_7 \vdash \Delta_{10}, F_8, p_{11}, p_{11}} \rightarrow_R} \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{12}, p_{11}), F_7 \vdash (\Delta_{10}, p_{11}), F_8, F_{13}}{\bullet h_1 : \Delta_{12}, p_{11} \vdash ((\Delta_{10}, p_{11}), F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : (\Delta_{12}, p_{11}), F_{13} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8}{\bullet h_9 : (\Delta_{12}, p_{11}), F_{13} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8} I \\
\hline
\frac{}{- : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \rightarrow F_8} \text{Cut} \\
\hline
\frac{}{- : \Delta_{12}, p_{11} \vdash \Delta_{10}, p_{11}, F_7 \rightarrow F_8} I
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, p_{10}), F_6 \vdash (\Delta_9, p_{10}), F_7}{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : (\Delta_{11}, p_{10}), F_6 \rightarrow F_7 \vdash \Delta_9, p_{10}}{\bullet h_8 : (\Delta_{11}, p_{10}), F_6 \rightarrow F_7 \vdash \Delta_9, p_{10}} I \\
\hline
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7 \vdash \Delta_{10}, F_8, \top}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \rightarrow F_8), \top} \rightarrow_R \quad \frac{h_9 : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{11}, \top \vdash \Delta_{10}, F_7 \rightarrow F_8} \top_L \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{ax/w}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\top, \Delta_{11}), F_7 \vdash \Delta_{10}, F_8, F_{12}}{\bullet h_1 : \top, \Delta_{11} \vdash (\Delta_{10}, F_7 \rightarrow F_8), F_{12}} \rightarrow_R \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : (\top, \Delta_{11}), F_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8} \top_L \\
\hline
\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{ax/w}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\top, \Delta_{11}), F_7 \vdash \Delta_{10}, F_8, F_{12}}{\bullet h_1 : \top, \Delta_{11} \vdash (\Delta_{10}, F_7 \rightarrow F_8), F_{12}} \rightarrow_R \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : (\top, \Delta_{11}), F_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8} \top_L \\
\hline
\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8} \text{ax/w}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\top, \Delta_{10}), F_6 \vdash \Delta_9, F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \top_L \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/w}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\top, \Delta_{10}), F_6 \vdash \Delta_9, F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \top_L \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/w}
\end{array}$$

8.2 Status of \wedge_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7, F_{14} \quad h_1 : \Delta_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_8, F_{14}}{\bullet h_1 : \Delta_{13} \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8), F_{14}} \wedge_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8} \rightarrow_R \\
\hline
\frac{}{- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8} \text{Cut} \\
\hline
\frac{}{- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8} \text{inv-th/ax}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_{14}, F_7}{\bullet h_1 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_{14}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_{14}, F_8}{\bullet h_1 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_{14}, F_8} \text{inv-th/ax} \\
\hline
\frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8} \wedge_R \quad \frac{}{- : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8} \text{ax/w} \\
\hline
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6 \quad h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_7}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R \\
\hline
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \text{Cut} \\
\hline
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \text{inv-th/ax}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6 \wedge F_7} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_7} \text{inv-th/ax} \\
\hline
\frac{}{- : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6 \wedge F_7} \wedge_R \quad \frac{}{- : \Delta_{12}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}, F_{10}} \text{ax/w} \\
\hline
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7, F_{14} \quad h_1 : \Delta_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_8, F_{14}}{\bullet h_1 : \Delta_{13} \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), F_7 \wedge F_8), F_{14}} \wedge_R \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, \quad \bullet h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}}{\rightarrow} \\
\frac{- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \wedge F_8}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{14}, F_7}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{14}, F_7 \wedge F_8} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{14}, F_8}{\wedge_R} \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8}{\text{ax/W}} \quad \frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_{11}, F_{14}, F_7}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_{11}, F_{14}, F_7} \text{ inv-th/ax}}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8}{\text{hCut}} \quad \frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8}{\text{hCut}}} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \wedge F_8}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_8, F_{12} \quad h_1 : \Delta_{11} \vdash \Delta_{10}, F_9, F_{12}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_8 \wedge F_9), F_{12}} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8 \quad h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_9}{\bullet h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R}{\frac{- : \Delta_{11} \vdash \Delta_{10}, F_8 \wedge F_9}{\text{Cut}}} \\
\frac{- : \Delta_{11} \vdash \Delta_{10}, F_8 \wedge F_9}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_{12}, F_8}{\text{ax/W}} \quad \frac{h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8}{\bullet h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_9}{\bullet h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_9} \text{ ax/W}}{\frac{- : \Delta_{11} \vdash \Delta_{10}, F_8}{\text{height}} \quad \frac{- : \Delta_{11} \vdash \Delta_{10}, F_9}{\text{height}}} \\
\frac{- : \Delta_{11} \vdash \Delta_{10}, F_8 \wedge F_9}{\wedge_R} \\
\frac{\frac{h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \quad h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_7}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \wedge_R}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}}{\text{Cut}}} \\
\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}}{\rightarrow} \\
\frac{\frac{- : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9 \wedge F_{10}}{\text{ax/W}} \quad \frac{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9}{\text{inv-th/ax}} \quad \frac{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_{10}}{\text{inv-th/ax}}}{\frac{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}}{\text{sCut}}} \\
\frac{- : \Delta_{12} \vdash \Delta_{11}, F_6, F_9 \wedge F_{10}}{\text{ax/W}} \quad \frac{- : \Delta_{12}, F_6 \vdash \Delta_{11}, F_9 \wedge F_{10}}{\text{sCut}} \\
\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}}{\text{sCut}}
\end{array}$$

• Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7, F_{14} \quad h_1 : \Delta_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_8, F_{14}}{\bullet h_1 : \Delta_{13} \vdash ((\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8), F_{14}} \wedge_R \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8}}{\frac{- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8}{\text{Cut}}} \\
\frac{- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}, F_7}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}, F_7 \wedge F_8} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}, F_8}{\wedge_R} \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\text{ax/W}} \quad \frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}, F_8}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}, F_8} \text{ inv-th/ax}}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\text{hCut}} \quad \frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\text{hCut}}} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\vee_R} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8, F_{10} \vee F_{11}}{\vee_R} \\
\frac{\frac{h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \quad h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_7}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \wedge_R}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}}{\text{Cut}}} \\
\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9, F_6 \wedge F_7} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_7, F_9}{\wedge_R} \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_{10}, F_9}{\text{ax/W}} \quad \frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_7, F_9} \text{ inv-th/ax}}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9}{\text{hCut}} \quad \frac{- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9}{\text{hCut}}} \\
\frac{- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9}{\vee_R} \\
\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}}{\vee_R}
\end{array}$$

• Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11} \vdash (\perp, \Delta_{10}), F_7, F_{12} \quad h_1 : \Delta_{11} \vdash (\perp, \Delta_{10}), F_8, F_{12}}{\bullet h_1 : \Delta_{11} \vdash ((\perp, \Delta_{10}), F_7 \wedge F_8), F_{12}} \wedge_R \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{11}, F_{12} \vdash (\perp, \Delta_{10}), F_7 \wedge F_8}}{\frac{- : \Delta_{11} \vdash (\perp, \Delta_{10}), F_7 \wedge F_8}{\text{Cut}}} \\
\frac{- : \Delta_{11} \vdash (\perp, \Delta_{10}), F_7 \wedge F_8}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_{12}, F_7 \wedge F_8}{\text{ax/W}} \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \perp, \Delta_{10}, F_7 \wedge F_8}{\text{ax/W}}}{\frac{- : \Delta_{11} \vdash \perp, \Delta_{10}, F_7 \wedge F_8}{\text{hCut}}}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \quad h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_7}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_9} \perp_R \\
\hline
\frac{}{- : \Delta_{10} \vdash \perp, \Delta_9} \text{Cut} \\
\hline
\frac{}{\frac{}{- : \Delta_{10} \vdash \perp, \Delta_9, F_6 \wedge F_7} \text{ax/W} \quad \frac{}{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_9} \text{ax/W}}{- : \Delta_{10} \vdash \perp, \Delta_9} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash (\top, \Delta_{10}), F_7, F_{12} \quad h_1 : \Delta_{11} \vdash (\top, \Delta_{10}), F_8, F_{12}}{\bullet h_1 : \Delta_{11} \vdash ((\top, \Delta_{10}), F_7 \wedge F_8), F_{12}} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_{11}, F_{12} \vdash (\top, \Delta_{10}), F_7 \wedge F_8} \top_R \\
\hline
\frac{}{- : \Delta_{11} \vdash (\top, \Delta_{10}), F_7 \wedge F_8} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11} \vdash \top, \Delta_{10}, F_7 \wedge F_8} \top_R \\
\hline
\frac{h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6 \quad h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_7}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \top, \Delta_9} \top_R \\
\hline
\frac{}{- : \Delta_{10} \vdash \top, \Delta_9} \text{Cut} \\
\hline
\frac{}{- : \Delta_{10} \vdash \top, \Delta_9} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_{14} \quad h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{14}} \wedge_R \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}}{\bullet h_9 : (\Delta_{13}, F_{10} \rightarrow F_{11}) \vdash \Delta_{12}, F_7 \wedge F_8} \rightarrow_L \\
\hline
\frac{}{- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \text{Cut} \\
\hline
\frac{h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{14}, F_7}{\bullet h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_7}{\bullet h_9 : \Delta_{13}, F_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_{14} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{13}, F_{11}, F_{14} \vdash \Delta_{12}, F_7} \text{inv-th/ax} \\
\hline
\frac{}{- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \text{hCut} \quad \frac{}{h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \rightarrow_L \\
\hline
\frac{}{- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \text{Cut} \\
\hline
\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \rightarrow F_{11} \quad h_1 : \Delta_{13} \vdash \Delta_{12}, F_8, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \rightarrow F_{11}} \wedge_R \quad \frac{h_9 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8 \quad h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \rightarrow_L \\
\hline
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \text{Cut} \\
\hline
\frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7} \text{inv-th/ax} \quad \frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_8} \text{inv-th/ax} \\
\hline
\frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8} \wedge_R \quad \frac{}{- : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \text{sCut} \\
\hline
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8} \text{ax/W} \quad \frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \text{sCut} \\
\hline
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \text{Cut} \\
\hline
\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \quad h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \rightarrow_L \\
\hline
\frac{}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut} \\
\hline
\frac{}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_7} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{inv-th/ax} \quad \frac{}{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}} \text{inv-th/ax} \\
\hline
\frac{}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut} \quad \frac{}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut} \\
\hline
\frac{}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_{14} \quad h_1 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{14}} \wedge_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : (\Delta_{13}, F_{10} \wedge F_{11}), F_{14} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L \\
\hline
\frac{}{- : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \text{Cut} \\
\hline
\frac{}{- : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_{14}, F_7} \text{inv-th/ax} \quad \frac{}{h_1 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_{14}, F_8} \text{inv-th/ax} \\
\hline
\frac{}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_{14}, F_7 \wedge F_8} \wedge_R \quad \frac{}{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14} \vdash \Delta_{12}, F_7 \wedge F_8} \text{ax/W} \\
\hline
\frac{}{- : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \text{hCut} \quad \frac{}{- : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \wedge F_{11} \quad h_1 : \Delta_{13} \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \wedge F_{11}} \wedge_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \wedge F_{11}}{- : \Delta_{13} \vdash \Delta_{12}, F_7} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax} \quad \frac{- : \Delta_{13} \vdash \Delta_{12}, F_7}{\wedge_L} \quad \frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}}{- : \Delta_{13} \vdash \Delta_{12}, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax}}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\wedge_R} \text{hCut}} \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \quad h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L}{\frac{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \wedge F_7} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7} \text{inv-th/ax}}{\frac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}}{\wedge_R} \wedge_L} \\
\frac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{ax/W} \\
\frac{h_1 : \Delta_{10} \vdash \Delta_9, F_7 \quad h_1 : \Delta_{10} \vdash \Delta_9, F_8}{\bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \wedge F_8} \wedge_R \quad \frac{h_6 : \Delta_{10}, F_7, F_8 \vdash \Delta_9}{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \wedge_L}{\frac{- : \Delta_{10} \vdash \Delta_9}{\rightarrow} \text{Cut}} \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7 \quad \frac{- : \Delta_{10}, F_7 \vdash \Delta_9, F_8}{\text{ax/W}} \quad \frac{- : \Delta_{10}, F_7, F_8 \vdash \Delta_9}{\text{sCut}}}{- : \Delta_{10} \vdash \Delta_9} \text{sCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_{14} \quad h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{14}} \wedge_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : (\Delta_{13}, F_{10} \vee F_{11}), F_{14} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{14}, F_7}{\bullet h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{13}, F_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_{14} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{13}, F_{11}, F_{14} \vdash \Delta_{12}, F_7} \text{inv-th/ax}}{\frac{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7}{\wedge_L} \text{hCut}} \\
\frac{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \vee F_{11} \quad h_1 : \Delta_{13} \vdash \Delta_{12}, F_8, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \vee F_{11}} \wedge_R \quad \frac{h_9 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \wedge F_8 \quad h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \quad \frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_8}{\text{inv-th/ax}} \quad \frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_8}{\text{inv-th/ax}}}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\wedge_R} \wedge_L} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8 \quad \frac{- : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8}{\text{ax/W}}}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \text{sCut} \\
\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \quad h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \wedge F_7 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L}{\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}}{\rightarrow} \text{Cut}} \\
\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \quad \frac{- : \Delta_{12}, F_6, F_7, F_9 \vdash \Delta_{11}}{\text{ax/W}} \quad \frac{- : \Delta_{12}, F_6, F_7, F_9 \vdash \Delta_{11}}{\text{inv-th/ax}} \quad \frac{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}}{\text{inv-th/ax}}}{\frac{- : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash \Delta_{11}}{\wedge_L} \wedge_L} \\
\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \quad \frac{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7}{\text{ax/W}} \quad \frac{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}}{\text{sCut}}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{sCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_7, \perp \quad h_1 : \Delta_{11} \vdash \Delta_{10}, F_8, \perp}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \wedge F_8), \perp} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \wedge F_8} \perp_L}{\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7}{- : \Delta_{11} \vdash \Delta_{10}, F_7} \text{ax/W} \quad \frac{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7}{- : \Delta_{11} \vdash \Delta_{10}, F_7} \perp_L}{\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{hCut}} \quad \frac{\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_8}{- : \Delta_{11} \vdash \Delta_{10}, F_8} \text{ax/W} \quad \frac{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_8}{- : \Delta_{11} \vdash \Delta_{10}, F_8} \perp_L}{\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{hCut}} \wedge_R \\
\rightarrow \\
\frac{\frac{h_1 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7, F_{12} \quad h_1 : \perp, \Delta_{11} \vdash \Delta_{10}, F_8, F_{12}}{\bullet h_1 : \perp, \Delta_{11} \vdash (\Delta_{10}, F_7 \wedge F_8), F_{12}} \wedge_R \quad \frac{}{\bullet h_9 : (\perp, \Delta_{11}), F_{12} \vdash \Delta_{10}, F_7 \wedge F_8} \perp_L}{\frac{}{- : \perp, \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \perp_L \\
\frac{\frac{h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \quad h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_7}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \perp_L}{\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L
\end{array}$$

• Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7, p_{11} \quad h_1 : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_8, p_{11}}{\bullet h_1 : \Delta_{12} \vdash ((\Delta_{10}, p_{11}), F_7 \wedge F_8), p_{11}} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8} I}{\frac{}{- : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}, p_{11}}{- : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, p_{11}}{- : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}} I}{\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}} \text{hCut}} \quad \frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_8, p_{11}, p_{11}}{- : \Delta_{12} \vdash \Delta_{10}, F_8, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_8, p_{11}}{- : \Delta_{12} \vdash \Delta_{10}, F_8, p_{11}} I}{\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_7 \wedge F_8} \text{hCut}} \wedge_R \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7, F_{13} \quad h_1 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_8, F_{13}}{\bullet h_1 : \Delta_{12}, p_{11} \vdash ((\Delta_{10}, p_{11}), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{}{\bullet h_9 : (\Delta_{12}, p_{11}), F_{13} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8} I}{\frac{}{- : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \Delta_{12}, p_{11} \vdash \Delta_{10}, p_{11}, F_7 \wedge F_8} I \\
\frac{\frac{h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6 \quad h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7}{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : (\Delta_{11}, p_{10}), F_6 \wedge F_7 \vdash \Delta_9, p_{10}} I}{\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I
\end{array}$$

• Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_7, \top \quad h_1 : \Delta_{11} \vdash \Delta_{10}, F_8, \top}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \wedge F_8), \top} \wedge_R \quad \frac{h_9 : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{11}, \top \vdash \Delta_{10}, F_7 \wedge F_8} \top_L}{\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{ax/W} \\
\frac{\frac{h_1 : \top, \Delta_{11} \vdash \Delta_{10}, F_7, F_{12} \quad h_1 : \top, \Delta_{11} \vdash \Delta_{10}, F_8, F_{12}}{\bullet h_1 : \top, \Delta_{11} \vdash (\Delta_{10}, F_7 \wedge F_8), F_{12}} \wedge_R \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : (\top, \Delta_{11}), F_{12} \vdash \Delta_{10}, F_7 \wedge F_8} \top_L}{\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_{11} \vdash \Delta_{10}, F_{12}, F_7 \wedge F_8}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{ax/W} \quad \frac{h_9 : \top, \Delta_{11}, F_{12} \vdash \Delta_{10}, F_7 \wedge F_8}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{ax/W}}{\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{hCut}} \\
\frac{\frac{h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \quad h_1 : \top, \Delta_{10} \vdash \Delta_9, F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \top_L}{\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W}}{\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}}
\end{array}$$

8.3 Status of \vee_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7, F_8, F_{14}}{\bullet h_1 : \Delta_{13} \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \vee F_8), F_{14}} \vee_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \vee F_8} \rightarrow_R \\
\hline
- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_{14}, F_7, F_8}{\bullet h_1 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_{14}, F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8} \text{ax/w}}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8} \text{hCut} \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \quad \rightarrow_R \\
\hline
\frac{h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6, F_7}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}} \text{ax/w}}{- : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}} \text{hCut} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10} \quad \rightarrow_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7, F_8, F_{14}}{\bullet h_1 : \Delta_{13} \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), F_7 \vee F_8), F_{14}} \vee_R \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_7 \vee F_8 \quad h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \vee F_8} \wedge_R \\
\hline
- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_{14}, F_7, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_{14}, F_7, F_8, F_{10} \wedge F_{11}} \text{ax/w} \quad \frac{\frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{11}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{11} \wedge F_{11}} \text{inv-th/ax}}{- : \Delta_{13} \vdash \Delta_{12}, F_{14}, F_7, F_8, F_{10} \wedge F_{11}} \text{hCut} \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \vee F_8 \quad \vee_R \\
\hline
\frac{h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6, F_7}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \wedge_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9} \text{ax/w}}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \text{hCut} \\
\hline
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6, F_7}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}} \text{ax/w}}{- : \Delta_{12} \vdash \Delta_{11}, F_{10}} \text{hCut} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10} \quad \wedge_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7, F_8, F_{14}}{\bullet h_1 : \Delta_{13} \vdash ((\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8), F_{14}} \vee_R \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8} \vee_R \\
\hline
- : \Delta_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}, F_7, F_8}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8} \text{ax/w}}{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8} \text{hCut} \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \quad \vee_R \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_8, F_9, F_{12}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_8 \vee F_9), F_{12}} \vee_R \quad \frac{h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8, F_9}{\bullet h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8 \vee F_9} \vee_R \\
\hline
- : \Delta_{11} \vdash \Delta_{10}, F_8 \vee F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_{12}, F_8, F_9}{\bullet h_1 : \Delta_{11} \vdash \Delta_{10}, F_{12}, F_8, F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8, F_9}{\bullet h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8, F_9} \text{ax/W}}{- : \Delta_{11} \vdash \Delta_{10}, F_8, F_9} \text{height} \\
\hline
- : \Delta_{11} \vdash \Delta_{10}, F_8 \vee F_9 \quad \vee_R \\
\hline
\frac{h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6, F_7}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}, F_9}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}, F_9} \text{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9} \text{hCut} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \quad \vee_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash (\perp, \Delta_{10}), F_7, F_8, F_{12}}{\bullet h_1 : \Delta_{11} \vdash ((\perp, \Delta_{10}), F_7 \vee F_8), F_{12}} \vee_R \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{11}, F_{12} \vdash (\perp, \Delta_{10}), F_7 \vee F_8} \perp_R \\
\hline
- : \Delta_{11} \vdash (\perp, \Delta_{10}), F_7 \vee F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_{12}, F_7 \vee F_8}{\bullet h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7 \vee F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \perp, \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{11}, F_{12} \vdash \perp, \Delta_{10}, F_7 \vee F_8} \text{ax/W}}{- : \Delta_{11} \vdash \perp, \Delta_{10}, F_7 \vee F_8} \text{hCut} \\
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6, F_7}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_9} \perp_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \vee F_7}{\bullet h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \vee F_7} \text{ax/W} \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_9} \text{ax/W}}{- : \Delta_{10} \vdash \perp, \Delta_9} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash (\top, \Delta_{10}), F_7, F_8, F_{12}}{\bullet h_1 : \Delta_{11} \vdash ((\top, \Delta_{10}), F_7 \vee F_8), F_{12}} \vee_R \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash (\top, \Delta_{10}), F_7 \vee F_8}{\bullet h_9 : \Delta_{11}, F_{12} \vdash (\top, \Delta_{10}), F_7 \vee F_8} \top_R \\
\hline
- : \Delta_{11} \vdash (\top, \Delta_{10}), F_7 \vee F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{11} \vdash \top, \Delta_{10}, F_7 \vee F_8}{\bullet h_1 : \Delta_{11} \vdash \top, \Delta_{10}, F_7 \vee F_8} \top_R \\
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6, F_7}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_9} \top_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_9 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{10} \vdash \top, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash \top, \Delta_9} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{14}} \vee_R \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_7 \vee F_8 \quad h_9 : \Delta_{13}, F_{11}, F_{14} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_{14} \vdash \Delta_{12}, F_7 \vee F_8} \rightarrow_L \\
\hline
- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \vee F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{14}, F_7, F_8}{\bullet h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{14}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \\
\hline
- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8 \quad \text{hCut} \\
\rightarrow_L \\
\frac{- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8}{\bullet h_1 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{14} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \\
\hline
- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8 \quad \vee_R \\
\hline
- : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \vee F_8
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \rightarrow F_{11}} \vee_R \quad \frac{h_9 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \vee F_8 \quad h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \rightarrow_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \rightarrow F_{11}} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax}}{- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \vee_R}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \rightarrow_L}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{\bullet h_8 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \vee_R \quad \frac{\frac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10} \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \vee_R \quad \frac{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{14}} \vee_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : (\Delta_{13}, F_{10} \wedge F_{11}), F_{14} \vdash \Delta_{12}, F_7 \vee F_8} \wedge_L}{- : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_{14}, F_7, F_8}{\bullet h_1 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_{14}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax}}{- : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \wedge_L \quad \frac{- : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8}{- : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \vee_R}{- : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \wedge F_{11}} \vee_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \wedge_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax}}{- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \wedge_L \quad \frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \vee_R}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \vee_R \quad \frac{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{14}} \vee_R \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_7 \vee F_8 \quad h_9 : \Delta_{13}, F_{11}, F_{14} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : (\Delta_{13}, F_{10} \vee F_{11}), F_{14} \vdash \Delta_{12}, F_7 \vee F_8} \vee_L}{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{14}, F_7, F_8}{\bullet h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{14}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax}}{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \vee_L \quad \frac{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8}{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \vee_R}{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \vee F_{11}} \vee_R \quad \frac{h_9 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \vee F_8 \quad h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \vee_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \vee F_{11}} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax}}{- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \text{hCut} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8 \quad - : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \vee_R \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_9 \vdash \Delta_{11}} \text{hCut} \\
\frac{- : \Delta_{12}, F_9 \vdash \Delta_{11} \quad - : \Delta_{12}, F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10} \vdash \Delta_9, F_7, F_8}{\bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \vee F_8} \vee_R \quad \frac{h_6 : \Delta_{10}, F_7 \vdash \Delta_9 \quad h_6 : \Delta_{10}, F_8 \vdash \Delta_9}{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \vee_L}{- : \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7, F_8 \quad - : \Delta_{10}, F_8 \vdash \Delta_9, F_7}{- : \Delta_{10} \vdash \Delta_9, F_7} \text{ax/W} \quad \frac{- : \Delta_{10}, F_8 \vdash \Delta_9, F_7}{- : \Delta_{10}, F_7 \vdash \Delta_9} \text{ax/W} \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7 \quad - : \Delta_{10}, F_7 \vdash \Delta_9}{- : \Delta_{10} \vdash \Delta_9} \text{sCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_7, F_8, \perp}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \vee F_8), \perp} \vee_R \quad \frac{h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \vee F_8} \perp_L}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7, F_8}{\bullet h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7, F_8} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7, F_8}{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7, F_8} \perp_L}{- : \Delta_{11} \vdash \Delta_{10}, F_7, F_8} \text{hCut} \\
\frac{- : \Delta_{11} \vdash \Delta_{10}, F_7, F_8 \quad - : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \vee_R \\
\rightarrow \\
\frac{\frac{h_1 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7, F_8, F_{12}}{\bullet h_1 : \perp, \Delta_{11} \vdash (\Delta_{10}, F_7 \vee F_8), F_{12}} \vee_R \quad \frac{h_9 : (\perp, \Delta_{11}), F_{12} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : (\perp, \Delta_{11}), F_{12} \vdash \Delta_{10}, F_7 \vee F_8} \perp_L}{- : \perp, \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{- : \perp, \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8}{- : \perp, \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \perp_L \\
\rightarrow \\
\frac{\frac{h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6, F_7}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{h_8 : (\perp, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \perp_L}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{- : \perp, \Delta_{10} \vdash \Delta_9}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7, F_8, p_{11}}{\bullet h_1 : \Delta_{12} \vdash ((\Delta_{10}, p_{11}), F_7 \vee F_8), p_{11}} \vee_R \quad \frac{h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8}{\bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} I}{- : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11}, p_{11}}{\bullet h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11}, p_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, F_8, p_{11}}{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, F_8, p_{11}} I}{- : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11}} \text{hCut} \\
\frac{- : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11} \quad - : \Delta_{12} \vdash \Delta_{10}, p_{11}, F_7 \vee F_8}{- : \Delta_{12} \vdash \Delta_{10}, p_{11}, F_7 \vee F_8} \vee_R \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7, F_8, F_{13}}{\bullet h_1 : \Delta_{12}, p_{11} \vdash ((\Delta_{10}, p_{11}), F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : (\Delta_{12}, p_{11}), F_{13} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8}{\bullet h_9 : (\Delta_{12}, p_{11}), F_{13} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} I}{- : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{12}, p_{11} \vdash \Delta_{10}, p_{11}, F_7 \vee F_8}{- : \Delta_{12}, p_{11} \vdash \Delta_{10}, p_{11}, F_7 \vee F_8} I
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6, F_7}{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6 \vee F_7} \vee_R \quad \frac{}{\bullet h_8 : (\Delta_{11}, p_{10}), F_6 \vee F_7 \vdash \Delta_9, p_{10}} I \\
\hline
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \rightarrow \\
\hline
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_7, F_8, \top}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \vee F_8), \top} \vee_R \quad \frac{h_9 : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{11}, \top \vdash \Delta_{10}, F_7 \vee F_8} \top_L \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \rightarrow \\
\hline
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{ax/W} \\
\hline
\frac{h_1 : \top, \Delta_{11} \vdash \Delta_{10}, F_7, F_8, F_{12}}{\bullet h_1 : \top, \Delta_{11} \vdash (\Delta_{10}, F_7 \vee F_8), F_{12}} \vee_R \quad \frac{h_9 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : (\top, \Delta_{11}), F_{12} \vdash \Delta_{10}, F_7 \vee F_8} \top_L \\
\hline
\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \rightarrow \\
\hline
\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{ax/W} \quad \frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{ax/W} \\
\hline
\frac{}{- : \top, \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{hCut} \\
\hline
\frac{h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6, F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \top_L \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \rightarrow \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W} \quad \frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W} \\
\hline
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}
\end{array}$$

8.4 Status of \perp_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \rightarrow F_6), \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8, F_5 \vdash \Delta_7, F_6}{\bullet h_4 : \Delta_8, \perp \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow_R \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{Cut} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_9 \vdash (\Delta_8, F_6 \rightarrow F_7), F_{10}}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8, F_6 \rightarrow F_7), F_{10}} \perp_R \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash \perp, \Delta_8, F_7}{\bullet h_5 : \Delta_9, F_{10} \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \rightarrow_R \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \text{Cut} \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \rightarrow \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \quad \frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \wedge F_6), \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash \Delta_7, F_5 \quad h_4 : \perp, \Delta_8 \vdash \Delta_7, F_6}{\bullet h_4 : \Delta_8, \perp \vdash \Delta_7, F_5 \wedge F_6} \wedge_R \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \text{Cut} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \rightarrow \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_9 \vdash (\Delta_8, F_6 \wedge F_7), F_{10}}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8, F_6 \wedge F_7), F_{10}} \perp_R \quad \frac{h_5 : \Delta_9, F_{10} \vdash \perp, \Delta_8, F_6 \quad h_5 : \Delta_9, F_{10} \vdash \perp, \Delta_8, F_7}{\bullet h_5 : \Delta_9, F_{10} \vdash \perp, \Delta_8, F_6 \wedge F_7} \wedge_R \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \wedge F_7} \text{Cut} \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \wedge F_7} \rightarrow \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \wedge F_7} \text{ax/W} \quad \frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \wedge F_7} \text{ax/W} \\
\hline
\frac{}{- : \Delta_9 \vdash \perp, \Delta_8, F_6 \wedge F_7} \text{hCut}
\end{array}$$

- Case rule \vee_R

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8, F_5, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \perp \vdash \Delta_7} \wedge_L}{\neg : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{}{\neg : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{ax/W} \\
\\
\frac{\frac{h_1 : \Delta_9, F_6 \wedge F_7 \vdash \Delta_8, F_{10}}{\bullet h_1 : \Delta_9, F_6 \wedge F_7 \vdash (\perp, \Delta_8), F_{10}} \perp_R \quad \frac{h_5 : \Delta_9, F_6, F_7, F_{10} \vdash \perp, \Delta_8}{\bullet h_5 : (\Delta_9, F_6 \wedge F_7), F_{10} \vdash \perp, \Delta_8} \wedge_L}{\neg : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8, F_{10}}{\neg : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8} \text{ax/W}}{\neg : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_9 \vdash \Delta_8, F_6 \wedge F_7}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8), F_6 \wedge F_7} \perp_R \quad \frac{h_5 : \Delta_9, F_6, F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8} \wedge_L}{\neg : \Delta_9 \vdash \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \wedge F_7}{\neg : \Delta_9 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8} \text{ax/W}}{\neg : \Delta_9 \vdash \perp, \Delta_8} \text{hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8, F_5 \vdash \Delta_7 \quad h_4 : \perp, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \perp \vdash \Delta_7} \vee_L}{\neg : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{}{\neg : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{ax/W} \\
\\
\frac{\frac{h_1 : \Delta_9, F_6 \vee F_7 \vdash \Delta_8, F_{10}}{\bullet h_1 : \Delta_9, F_6 \vee F_7 \vdash (\perp, \Delta_8), F_{10}} \perp_R \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash \perp, \Delta_8 \quad h_5 : \Delta_9, F_7, F_{10} \vdash \perp, \Delta_8}{\bullet h_5 : (\Delta_9, F_6 \vee F_7), F_{10} \vdash \perp, \Delta_8} \vee_L}{\neg : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8, F_{10}}{\neg : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8} \text{ax/W}}{\neg : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_9 \vdash \Delta_8, F_6 \vee F_7}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8), F_6 \vee F_7} \perp_R \quad \frac{h_5 : \Delta_9, F_6 \vdash \perp, \Delta_8 \quad h_5 : \Delta_9, F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8} \vee_L}{\neg : \Delta_9 \vdash \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \vee F_7}{\neg : \Delta_9 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8} \text{ax/W}}{\neg : \Delta_9 \vdash \perp, \Delta_8} \text{hCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash \Delta_5}{\bullet h_1 : \Delta_6 \vdash \Delta_5, \perp} \perp_R \quad \frac{}{\bullet h_4 : \Delta_6, \perp \vdash \Delta_5} \perp_L}{\neg : \Delta_6 \vdash \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{}{\neg : \Delta_6 \vdash \Delta_5} \text{ax/W} \\
\\
\frac{\frac{h_1 : \Delta_7 \vdash \Delta_6, \perp}{\bullet h_1 : \Delta_7 \vdash (\perp, \Delta_6), \perp} \perp_R \quad \frac{}{\bullet h_5 : \Delta_7, \perp \vdash \perp, \Delta_6} \perp_L}{\neg : \Delta_7 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{\neg : \Delta_7 \vdash \perp, \Delta_6} \text{ax/W} \\
\\
\frac{\frac{h_1 : \perp, \Delta_7 \vdash \Delta_6, F_8}{\bullet h_1 : \perp, \Delta_7 \vdash (\perp, \Delta_6), F_8} \perp_R \quad \frac{}{\bullet h_5 : (\perp, \Delta_7), F_8 \vdash \perp, \Delta_6} \perp_L}{\neg : \perp, \Delta_7 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{\neg : \perp, \Delta_7 \vdash \perp, \Delta_6} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_7, p_6 \vdash \Delta_5, p_6}{\bullet h_1 : \Delta_7, p_6 \vdash (\Delta_5, p_6), \perp} \perp_R \quad \frac{}{\bullet h_4 : (\Delta_7, p_6), \perp \vdash \Delta_5, p_6} I \\
\hline
\frac{}{- : \Delta_7, p_6 \vdash \Delta_5, p_6} \text{Cut} \\
\hline
\frac{}{- : \Delta_7, p_6 \vdash \Delta_5, p_6} \rightarrow \\
\hline
\frac{}{- : \Delta_7, p_6 \vdash \Delta_5, p_6} I \\
\\
\frac{h_1 : \Delta_8 \vdash (\Delta_6, p_7), p_7}{\bullet h_1 : \Delta_8 \vdash (\perp, \Delta_6, p_7), p_7} \perp_R \quad \frac{}{\bullet h_5 : \Delta_8, p_7 \vdash \perp, \Delta_6, p_7} I \\
\hline
\frac{}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} \text{Cut} \\
\hline
\frac{}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} \rightarrow \\
\hline
\frac{h_1 : \Delta_8 \vdash \perp, \Delta_6, p_7, p_7}{\bullet h_1 : \Delta_8 \vdash \perp, \Delta_6, p_7, p_7} \text{ax/W} \quad \frac{}{\bullet h_5 : \Delta_8, p_7 \vdash \perp, \Delta_6, p_7} I \\
\hline
\frac{}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} \text{hCut} \\
\\
\frac{h_1 : \Delta_8, p_7 \vdash (\Delta_6, p_7), F_9}{\bullet h_1 : \Delta_8, p_7 \vdash (\perp, \Delta_6, p_7), F_9} \perp_R \quad \frac{}{\bullet h_5 : (\Delta_8, p_7), F_9 \vdash \perp, \Delta_6, p_7} I \\
\hline
\frac{}{- : \Delta_8, p_7 \vdash \perp, \Delta_6, p_7} \text{Cut} \\
\hline
\frac{}{- : \Delta_8, p_7 \vdash \perp, \Delta_6, p_7} \rightarrow \\
\hline
\frac{}{- : \Delta_8, p_7 \vdash \perp, \Delta_6, p_7} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet h_1 : \top, \Delta_6 \vdash \Delta_5, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_6 \vdash \Delta_5}{\bullet h_4 : (\top, \Delta_6), \perp \vdash \Delta_5} \top_L \\
\hline
\frac{}{- : \top, \Delta_6 \vdash \Delta_5} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_6 \vdash \Delta_5} \rightarrow \\
\hline
\frac{}{- : \top, \Delta_6 \vdash \Delta_5} \text{ax/W} \\
\\
\frac{h_1 : \Delta_7 \vdash \Delta_6, \top}{\bullet h_1 : \Delta_7 \vdash (\perp, \Delta_6), \top} \perp_R \quad \frac{h_5 : \Delta_7 \vdash \perp, \Delta_6}{\bullet h_5 : \Delta_7, \top \vdash \perp, \Delta_6} \top_L \\
\hline
\frac{}{- : \Delta_7 \vdash \perp, \Delta_6} \text{Cut} \\
\hline
\frac{}{- : \Delta_7 \vdash \perp, \Delta_6} \rightarrow \\
\hline
\frac{}{- : \Delta_7 \vdash \perp, \Delta_6} \text{ax/W} \\
\\
\frac{h_1 : \top, \Delta_7 \vdash \Delta_6, F_8}{\bullet h_1 : \top, \Delta_7 \vdash (\perp, \Delta_6), F_8} \perp_R \quad \frac{h_5 : \Delta_7, F_8 \vdash \perp, \Delta_6}{\bullet h_5 : (\top, \Delta_7), F_8 \vdash \perp, \Delta_6} \top_L \\
\hline
\frac{}{- : \top, \Delta_7 \vdash \perp, \Delta_6} \text{Cut} \\
\hline
\frac{}{- : \top, \Delta_7 \vdash \perp, \Delta_6} \rightarrow \\
\hline
\frac{h_1 : \top, \Delta_7 \vdash \perp, \Delta_6, F_8}{\bullet h_1 : \top, \Delta_7 \vdash \perp, \Delta_6, F_8} \text{ax/W} \quad \frac{h_5 : \top, \Delta_7, F_8 \vdash \perp, \Delta_6}{\bullet h_5 : \top, \Delta_7, F_8 \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
\frac{}{- : \top, \Delta_7 \vdash \perp, \Delta_6} \text{hCut}
\end{array}$$

8.5 Status of \top_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \rightarrow F_6), \top} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7, F_6}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow_R \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{Cut} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow_R \\
\\
\frac{}{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7, F_6} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7, F_6}{\bullet h_4 : \top, \Delta_8, F_5 \vdash \Delta_7, F_6} \text{ax/W} \\
\hline
\frac{}{- : \Delta_8, F_5 \vdash \Delta_7, F_6} \text{hCut} \\
\hline
\frac{}{- : \Delta_8, F_5 \vdash \Delta_7, F_6} \rightarrow_R \\
\\
\frac{}{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8, F_6 \rightarrow F_7), F_{10}} \top_R \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash \top, \Delta_8, F_7}{\bullet h_5 : \Delta_9, F_{10} \vdash \top, \Delta_8, F_6 \rightarrow F_7} \rightarrow_R \\
\hline
\frac{}{- : \Delta_9 \vdash \top, \Delta_8, F_6 \rightarrow F_7} \text{Cut} \\
\hline
\frac{}{- : \Delta_9 \vdash \top, \Delta_8, F_6 \rightarrow F_7} \rightarrow \\
\hline
\frac{}{- : \Delta_9 \vdash \top, \Delta_8, F_6 \rightarrow F_7} \top_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \wedge F_6), \top}{\vdash : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \top_R \quad \frac{\frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5 \quad h_4 : \top, \Delta_8 \vdash \Delta_7, F_6}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \wedge F_6} \wedge_R}{\vdash : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5}{\vdash : \Delta_8 \vdash \Delta_7, F_5} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5}{\vdash : \Delta_8 \vdash \Delta_7, F_5} \text{ax/W}}{\vdash : \Delta_8 \vdash \Delta_7, F_5} \text{hCut} \quad \frac{\frac{\frac{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_6}{\vdash : \Delta_8 \vdash \Delta_7, F_6} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_6}{\vdash : \Delta_8 \vdash \Delta_7, F_6} \text{ax/W}}{\vdash : \Delta_8 \vdash \Delta_7, F_6} \text{hCut}}{\vdash : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \wedge_R \\
\\
\frac{\frac{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8, F_6 \wedge F_7), F_{10}}{\vdash : \Delta_9 \vdash \top, \Delta_8, F_6 \wedge F_7} \top_R \quad \frac{\frac{h_5 : \Delta_9, F_{10} \vdash \top, \Delta_8, F_6 \quad h_5 : \Delta_9, F_{10} \vdash \top, \Delta_8, F_7}{\bullet h_5 : \Delta_9, F_{10} \vdash \top, \Delta_8, F_6 \wedge F_7} \wedge_R}{\vdash : \Delta_9 \vdash \top, \Delta_8, F_6 \wedge F_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_9 \vdash \top, \Delta_8, F_6 \wedge F_7} \top_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \vee F_6), \top}{\vdash : \Delta_8 \vdash \Delta_7, F_5 \vee F_6} \top_R \quad \frac{\frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5, F_6}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \vee F_6} \vee_R}{\vdash : \Delta_8 \vdash \Delta_7, F_5 \vee F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5, F_6}{\vdash : \Delta_8 \vdash \Delta_7, F_5, F_6} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5, F_6}{\vdash : \Delta_8 \vdash \Delta_7, F_5, F_6} \text{ax/W}}{\vdash : \Delta_8 \vdash \Delta_7, F_5, F_6} \text{hCut}}{\vdash : \Delta_8 \vdash \Delta_7, F_5 \vee F_6} \vee_R \\
\\
\frac{\frac{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8, F_6 \vee F_7), F_{10}}{\vdash : \Delta_9 \vdash \top, \Delta_8, F_6 \vee F_7} \top_R \quad \frac{\frac{h_5 : \Delta_9, F_{10} \vdash \top, \Delta_8, F_6, F_7}{\bullet h_5 : \Delta_9, F_{10} \vdash \top, \Delta_8, F_6 \vee F_7} \vee_R}{\vdash : \Delta_9 \vdash \top, \Delta_8, F_6 \vee F_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_9 \vdash \top, \Delta_8, F_6 \vee F_7} \top_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_5), \top}{\vdash : \Delta_6 \vdash \perp, \Delta_5} \top_R \quad \frac{\frac{h_4 : \top, \Delta_6 \vdash \Delta_5}{\bullet h_4 : \Delta_6, \top \vdash \perp, \Delta_5} \perp_R}{\vdash : \Delta_6 \vdash \perp, \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_6 \vdash \perp, \top, \Delta_5}{\vdash : \Delta_6 \vdash \perp, \Delta_5} \top_R \quad \frac{h_4 : \top, \Delta_6 \vdash \perp, \Delta_5}{\vdash : \Delta_6 \vdash \perp, \Delta_5} \text{ax/W}}{\vdash : \Delta_6 \vdash \perp, \Delta_5} \text{hCut} \\
\\
\frac{\frac{\bullet h_1 : \Delta_7 \vdash (\top, \perp, \Delta_6), F_8}{\vdash : \Delta_7 \vdash \top, \perp, \Delta_6} \top_R \quad \frac{\frac{h_5 : \Delta_7, F_8 \vdash \top, \Delta_6}{\bullet h_5 : \Delta_7, F_8 \vdash \top, \perp, \Delta_6} \perp_R}{\vdash : \Delta_7 \vdash \top, \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_7 \vdash \perp, \top, \Delta_6} \top_R
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_5), \top}{\vdash : \Delta_6 \vdash \top, \Delta_5} \top_R \quad \frac{\frac{h_4 : \Delta_6, \top \vdash \top, \Delta_5}{\bullet h_4 : \Delta_6, \top \vdash \top, \Delta_5} \top_R}{\vdash : \Delta_6 \vdash \top, \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_6 \vdash \top, \Delta_5} \top_R \\
\\
\frac{\frac{\bullet h_1 : \Delta_7 \vdash (\top, \Delta_6), F_8}{\vdash : \Delta_7 \vdash \top, \Delta_6} \top_R \quad \frac{\frac{h_5 : \Delta_7, F_8 \vdash \top, \Delta_6}{\bullet h_5 : \Delta_7, F_8 \vdash \top, \Delta_6} \top_R}{\vdash : \Delta_7 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \Delta_7 \vdash \top, \Delta_6} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5 \quad h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \top \vdash \Delta_7} \rightarrow_L}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5}{- : \Delta_8 \vdash \Delta_7, F_5} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5}{- : \Delta_8 \vdash \Delta_7, F_5} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7}{- : \Delta_8, F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{- : \Delta_8, F_6 \vdash \Delta_7} \text{ax/W}}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{hCut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, F_6 \rightarrow F_7 \vdash (\top, \Delta_8), F_{10}}{- : \Delta_9, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : \Delta_9, F_{10} \vdash \top, \Delta_8, F_6 \quad h_5 : \Delta_9, F_7, F_{10} \vdash \top, \Delta_8}{\bullet h_5 : (\Delta_9, F_6 \rightarrow F_7), F_{10} \vdash \top, \Delta_8} \rightarrow_L}{- : \Delta_9, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_9, F_6 \rightarrow F_7 \vdash \top, \Delta_8}{- : \Delta_9, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \top_R \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \rightarrow F_7}{- : \Delta_9 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : \Delta_9 \vdash \top, \Delta_8, F_6 \quad h_5 : \Delta_9, F_7 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \rightarrow_L}{- : \Delta_9 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_9 \vdash \top, \Delta_8}{- : \Delta_9 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \top}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \top \vdash \Delta_7} \wedge_L}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5, F_6 \vdash \top, \Delta_7}{- : \Delta_8, F_5, F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}{- : \Delta_8, F_5, F_6 \vdash \Delta_7} \text{ax/W}}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{hCut} \\
\wedge_L \\
\frac{\frac{\bullet h_1 : \Delta_9, F_6 \wedge F_7 \vdash (\top, \Delta_8), F_{10}}{- : \Delta_9, F_6 \wedge F_7 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : \Delta_9, F_6, F_7, F_{10} \vdash \top, \Delta_8}{\bullet h_5 : (\Delta_9, F_6 \wedge F_7), F_{10} \vdash \top, \Delta_8} \wedge_L}{- : \Delta_9, F_6 \wedge F_7 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_9, F_6 \wedge F_7 \vdash \top, \Delta_8}{- : \Delta_9, F_6 \wedge F_7 \vdash \top, \Delta_8} \top_R \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \wedge F_7}{- : \Delta_9 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : \Delta_9, F_6, F_7 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash \top, \Delta_8} \wedge_L}{- : \Delta_9 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_9 \vdash \top, \Delta_8}{- : \Delta_9 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \top}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7 \quad h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \top \vdash \Delta_7} \vee_L}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7}{- : \Delta_8, F_5 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7}{- : \Delta_8, F_5 \vdash \Delta_7} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7}{- : \Delta_8, F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{- : \Delta_8, F_6 \vdash \Delta_7} \text{ax/W}}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{hCut} \\
\vee_L \\
\frac{\frac{\bullet h_1 : \Delta_9, F_6 \vee F_7 \vdash (\top, \Delta_8), F_{10}}{- : \Delta_9, F_6 \vee F_7 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash \top, \Delta_8 \quad h_5 : \Delta_9, F_7, F_{10} \vdash \top, \Delta_8}{\bullet h_5 : (\Delta_9, F_6 \vee F_7), F_{10} \vdash \top, \Delta_8} \vee_L}{- : \Delta_9, F_6 \vee F_7 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_9, F_6 \vee F_7 \vdash \top, \Delta_8}{- : \Delta_9, F_6 \vee F_7 \vdash \top, \Delta_8} \top_R \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \vee F_7}{- : \Delta_9 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : \Delta_9, F_6 \vdash \top, \Delta_8 \quad h_5 : \Delta_9, F_7 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \top, \Delta_8} \vee_L}{- : \Delta_9 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_9 \vdash \top, \Delta_8}{- : \Delta_9 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_6 \vdash \Delta_5, \top}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \top_R \quad \frac{\bullet h_4 : (\perp, \Delta_6), \top \vdash \Delta_5}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \perp_L}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_6 \vdash \Delta_5 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7 \vdash (\top, \Delta_6), \perp}{\vdash : \Delta_7 \vdash \top, \Delta_6} \top_R \quad \frac{\bullet h_5 : \Delta_7, \perp \vdash \top, \Delta_6}{\vdash : \Delta_7 \vdash \top, \Delta_6} \perp_L}{\vdash : \Delta_7 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_7 \vdash (\top, \Delta_6), F_8}{\vdash : \perp, \Delta_7 \vdash \top, \Delta_6} \top_R \quad \frac{\bullet h_5 : (\perp, \Delta_7), F_8 \vdash \top, \Delta_6}{\vdash : \perp, \Delta_7 \vdash \top, \Delta_6} \perp_L}{\vdash : \perp, \Delta_7 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_7 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_6 \vdash (\Delta_5, p_6), \top}{\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6} \top_R \quad \frac{\bullet h_4 : (\Delta_7, p_6), \top \vdash \Delta_5, p_6}{\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6} I}{\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6 \quad I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8 \vdash (\top, \Delta_6, p_7), p_7}{\vdash : \Delta_8 \vdash \top, \Delta_6, p_7} \top_R \quad \frac{\bullet h_5 : \Delta_8, p_7 \vdash \top, \Delta_6, p_7}{\vdash : \Delta_8 \vdash \top, \Delta_6, p_7} I}{\vdash : \Delta_8 \vdash \top, \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_8 \vdash \top, \Delta_6, p_7 \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, p_7 \vdash (\top, \Delta_6, p_7), F_9}{\vdash : \Delta_8, p_7 \vdash \top, \Delta_6, p_7} \top_R \quad \frac{\bullet h_5 : (\Delta_8, p_7), F_9 \vdash \top, \Delta_6, p_7}{\vdash : \Delta_8, p_7 \vdash \top, \Delta_6, p_7} I}{\vdash : \Delta_8, p_7 \vdash \top, \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_8, p_7 \vdash \top, \Delta_6, p_7 \quad \top_R
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_6 \vdash \Delta_5, \top}{\vdash : \Delta_6 \vdash \Delta_5} \top_R \quad \frac{h_4 : \Delta_6 \vdash \Delta_5}{\bullet h_4 : \Delta_6, \top \vdash \Delta_5} \top_L}{\vdash : \Delta_6 \vdash \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_6 \vdash \Delta_5 \quad \text{ax/w}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7 \vdash (\top, \Delta_6), \top}{\vdash : \Delta_7 \vdash \top, \Delta_6} \top_R \quad \frac{h_5 : \Delta_7 \vdash \top, \Delta_6}{\bullet h_5 : \Delta_7, \top \vdash \top, \Delta_6} \top_L}{\vdash : \Delta_7 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \top, \Delta_7 \vdash (\top, \Delta_6), F_8}{\vdash : \top, \Delta_7 \vdash \top, \Delta_6} \top_R \quad \frac{h_5 : \Delta_7, F_8 \vdash \top, \Delta_6}{\bullet h_5 : (\top, \Delta_7), F_8 \vdash \top, \Delta_6} \top_L}{\vdash : \top, \Delta_7 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \top, \Delta_7 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

8.6 Status of \rightarrow_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \rightarrow F_{11} \quad h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \quad h_9 : \Delta_{13}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12} \rightarrow_L}{\bullet h_9 : (\Delta_{13}, F_7 \rightarrow F_8), F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \text{Cut}}{\bullet h_1 : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_L \\
\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{\frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7} \text{inv-th/ax} \quad \frac{}{- : \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}} \text{inv-th/ax}}{\frac{}{- : \Delta_{13}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}} \rightarrow_L} \rightarrow_L \\
\frac{\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{13}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \text{sCut}}{\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}}} \\
\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_8, F_{12} \quad h_1 : \Delta_{11}, F_9 \vdash \Delta_{10}, F_{12} \rightarrow_L \quad \frac{h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8 \quad h_7 : \Delta_{11}, F_9, F_{12} \vdash \Delta_{10} \rightarrow_L}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_{12} \vdash \Delta_{10}} \text{Cut}}{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10}, F_{12}} \rightarrow_L \\
\frac{}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10}} \rightarrow \\
\frac{\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_{12}, F_8 \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8 \text{ ax/W}}{\bullet h_7 : \Delta_{11}, F_{12} \vdash \Delta_{10}, F_8} \text{height}}{\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_8} \text{hCut}} \text{height} \quad \frac{\frac{h_1 : \Delta_{11}, F_9 \vdash \Delta_{10}, F_{12} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_{12}, F_9 \vdash \Delta_{10} \text{ ax/W}}{\bullet h_7 : \Delta_{11}, F_{12}, F_9 \vdash \Delta_{10}} \text{height}}{\frac{}{- : \Delta_{11}, F_9 \vdash \Delta_{10}} \text{hCut}} \text{height}}{\frac{}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10}} \rightarrow_L}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_{14} \quad h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_8 \vdash \Delta_{12}, F_{14} \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8), F_{14} \vdash \Delta_{12}} \wedge_L}{\bullet h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{14}} \text{Cut} \\
\frac{}{- : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_{14}, F_7 \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{13}, F_{10}, F_{11}, F_8 \vdash \Delta_{12}, F_{14} \text{ inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{14}} \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8), F_{14} \vdash \Delta_{12}} \text{ax/W}}{\frac{}{- : \Delta_{13}, F_{10}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \text{hCut}} \wedge_L \\
\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8, F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L \\
\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \wedge F_{11} \quad h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11} \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \rightarrow F_8), F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L}{\bullet h_1 : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{Cut} \\
\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \wedge F_{11} \text{ ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \text{ inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7} \wedge_L}{\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7} \text{hCut}} \wedge_L \quad \frac{\frac{h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11} \text{ ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_8 \vdash \Delta_{12} \text{ inv}}{\bullet h_9 : \Delta_{13}, F_8, F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L}{\frac{}{- : \Delta_{13}, F_8 \vdash \Delta_{12}} \text{hCut}} \wedge_L}{\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \rightarrow_L}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_{14} \quad h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_8 \vdash \Delta_{12}, F_{14} \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \rightarrow F_8}{\bullet h_9 : ((\Delta_{13}, F_{10} \vee F_{11}), F_7 \rightarrow F_8), F_{14} \vdash \Delta_{12}} \vee_L}{\bullet h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{14}} \text{Cut} \\
\frac{}{- : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{14}, F_7 \text{ ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14} \vdash \Delta_{12}, F_7 \text{ inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{11}, F_{14} \vdash \Delta_{12}, F_7 \text{ inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \vee_L}{\frac{}{- : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{hCut}} \vee_L \\
\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L \\
\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \vee F_{11} \quad h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \vee F_{11} \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12} \quad h_9 : \Delta_{13}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \rightarrow F_8), F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L}{\bullet h_1 : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{Cut} \\
\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \rightarrow \\
\frac{\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7} \text{inv-th/ax} \quad \frac{}{- : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{inv-th/ax}}{\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \rightarrow_L} \rightarrow_L \quad \frac{\frac{}{- : \Delta_{13}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{13}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}} \text{sCut}}{\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}} \rightarrow_L} \rightarrow_L \\
\frac{}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_7, \perp \quad h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, \perp}{\bullet h_1 : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, \perp} \rightarrow_L \quad \frac{}{\bullet h_9 : (\Delta_{11}, F_7 \rightarrow F_8), \perp \vdash \Delta_{10}} \perp_L}{\frac{}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7}{- : \Delta_{11} \vdash \Delta_{10}, F_7} \text{ax/W} \quad \frac{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7}{- : \Delta_{11}, F_8 \vdash \Delta_{10}} \perp_L}{\frac{}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{hCut}} \quad \frac{\frac{h_1 : \Delta_{11}, F_8 \vdash \perp, \Delta_{10}}{- : \Delta_{11}, F_8 \vdash \Delta_{10}} \text{ax/W} \quad \frac{\bullet h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}}{- : \Delta_{11}, F_8 \vdash \Delta_{10}} \perp_L}{\frac{}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{hCut}} \\
\rightarrow \\
\frac{h_1 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7, F_{12} \quad h_1 : (\perp, \Delta_{11}), F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : (\perp, \Delta_{11}), F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}} \rightarrow_L \quad \frac{}{\bullet h_9 : ((\perp, \Delta_{11}), F_7 \rightarrow F_8), F_{12} \vdash \Delta_{10}} \perp_L}{\frac{}{- : (\perp, \Delta_{11}), F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7, p_{11} \quad h_1 : \Delta_{12}, F_8 \vdash (\Delta_{10}, p_{11}), p_{11}}{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash (\Delta_{10}, p_{11}), p_{11}} \rightarrow_L \quad \frac{}{\bullet h_9 : (\Delta_{12}, F_7 \rightarrow F_8), p_{11} \vdash \Delta_{10}, p_{11}} I}{\frac{}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}, p_{11}}{- : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, p_{11}}{- : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}} I}{\frac{}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \text{hCut}} \quad \frac{\frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}, p_{11}}{- : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{12}, F_8, p_{11} \vdash \Delta_{10}, p_{11}}{- : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}} I}{\frac{}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \text{hCut}} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7, F_{13} \quad h_1 : (\Delta_{12}, p_{11}), F_8 \vdash (\Delta_{10}, p_{11}), F_{13}}{\bullet h_1 : (\Delta_{12}, p_{11}), F_7 \rightarrow F_8 \vdash (\Delta_{10}, p_{11}), F_{13}} \rightarrow_L \quad \frac{}{\bullet h_9 : ((\Delta_{12}, p_{11}), F_7 \rightarrow F_8), F_{13} \vdash \Delta_{10}, p_{11}} I}{\frac{}{- : (\Delta_{12}, p_{11}), F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \Delta_{12}, p_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash \Delta_{10}, F_7, \top \quad h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, \top}{\bullet h_1 : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, \top} \rightarrow_L \quad \frac{h_9 : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \rightarrow F_8), \top \vdash \Delta_{10}} \top_L}{\frac{}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ax/W} \\
\rightarrow \\
\frac{h_1 : \top, \Delta_{11} \vdash \Delta_{10}, F_7, F_{12} \quad h_1 : (\top, \Delta_{11}), F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : (\top, \Delta_{11}), F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}} \rightarrow_L \quad \frac{h_9 : \Delta_{11}, F_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : ((\top, \Delta_{11}), F_7 \rightarrow F_8), F_{12} \vdash \Delta_{10}} \top_L}{\frac{}{- : (\top, \Delta_{11}), F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}}{- : \top, \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \top, \Delta_{11}, F_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{- : \top, \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ax/W}}{\frac{}{- : \top, \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{hCut}}
\end{array}$$

8.7 Status of \wedge_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{14} \quad h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{14}} \wedge_L \quad \frac{}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R}{\frac{}{- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10}, F_7, F_8 \vdash \Delta_{12}, F_{11}, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}, F_{14}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}}{- : \Delta_{13}, F_{10}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}} \text{ax/W}}{\frac{}{- : \Delta_{13}, F_{10}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}} \wedge_L} \quad \frac{}{- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{14}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \quad h_9 : \Delta_{13}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_R \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{14}, F_{10} \wedge F_{11}}{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{14}, F_{10} \wedge F_{11}} \text{ ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{14}, F_7, F_8 \vdash \Delta_{12}, F_{10}}{h_9 : \Delta_{13}, F_{14}, F_7, F_8 \vdash \Delta_{12}, F_{10}} \text{ inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7, F_8 \vdash \Delta_{12}, F_{11}}{h_9 : \Delta_{13}, F_{14}, F_7, F_8 \vdash \Delta_{12}, F_{11}} \text{ inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_{14}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_R \\
\hline
- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11} \quad \text{hCut} \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11} \quad \wedge_L
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{14}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}} \text{ inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{h_9 : \Delta_{13}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{ ax/W}}{\bullet h_9 : \Delta_{13}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \wedge_L \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11} \quad \text{hCut} \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11} \quad \vee_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7, F_8 \vdash (\perp, \Delta_{10}), F_{12}}{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash (\perp, \Delta_{10}), F_{12}} \wedge_L \quad \frac{h_9 : \Delta_{11}, F_{12}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \wedge F_8), F_{12} \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_{11}, F_7 \wedge F_8 \vdash \perp, \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}, F_{12}}{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}, F_{12}} \text{ ax/W} \quad \frac{h_9 : \Delta_{11}, F_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}}{h_9 : \Delta_{11}, F_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}} \text{ ax/W} \\
\hline
- : \Delta_{11}, F_7 \wedge F_8 \vdash \perp, \Delta_{10} \quad \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7, F_8 \vdash (\top, \Delta_{10}), F_{12}}{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash (\top, \Delta_{10}), F_{12}} \wedge_L \quad \frac{}{\bullet h_9 : (\Delta_{11}, F_7 \wedge F_8), F_{12} \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_{11}, F_7 \wedge F_8 \vdash \top, \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_{11}, F_7 \wedge F_8 \vdash \top, \Delta_{10} \quad \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7, F_8 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{14}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \quad h_9 : \Delta_{13}, F_{11}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8), F_{14} \vdash \Delta_{12}} \rightarrow_L \\
\hline
- : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash \Delta_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{14}}{h_1 : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{14}} \text{ ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{14}, F_7, F_8 \vdash \Delta_{12}, F_{10}}{h_9 : \Delta_{13}, F_{14}, F_7, F_8 \vdash \Delta_{12}, F_{10}} \text{ inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_{14}, F_7, F_8 \vdash \Delta_{12}}{h_9 : \Delta_{13}, F_{11}, F_{14}, F_7, F_8 \vdash \Delta_{12}} \text{ inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_{14}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \wedge_L \\
\hline
- : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12} \quad \text{hCut} \\
\hline
- : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8 \vdash \Delta_{12} \quad \wedge_L \\
\hline
\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \quad h_9 : \Delta_{13}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \rightarrow_L \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}}{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \text{ ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10}}{h_9 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10}} \text{ inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_7, F_8 \vdash \Delta_{12}}{h_9 : \Delta_{13}, F_{11}, F_7, F_8 \vdash \Delta_{12}} \text{ inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \wedge_L \\
\hline
- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \quad \text{hCut} \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \quad \wedge_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7, F_8 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{14}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8), F_{14} \vdash \Delta_{12}} \wedge_L \\
\hline
- : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash \Delta_{12} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10}, F_{11}, F_7, F_8 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{14}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8), F_{14} \vdash \Delta_{12}} \text{ax/W}}{\frac{- : \Delta_{13}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}}{- : \Delta_{13}, F_{10} \wedge F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}} \wedge_L} \text{hCut} \\
\hline
\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \wedge F_{11} \vdash \Delta_{12}} \text{inv-th/ax}}{\frac{- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}}{- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}} \wedge_L} \text{hCut} \\
\hline
\frac{h_1 : \Delta_{11}, F_8, F_9 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash \Delta_{10}, F_{12}} \wedge_L \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_{12} \vdash \Delta_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_{12} \vdash \Delta_{10}} \wedge_L \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash \Delta_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_8, F_9 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash \Delta_{10}, F_{12}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_{12}, F_8, F_9 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{11}, F_{12}, F_8, F_9 \vdash \Delta_{10}} \text{height}}{\frac{- : \Delta_{11}, F_8, F_9 \vdash \Delta_{10}}{- : \Delta_{11}, F_8 \wedge F_9 \vdash \Delta_{10}} \wedge_L} \text{hCut}
\end{array}$$

• Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7, F_8 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{14}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12} \quad h_9 : \Delta_{13}, F_{11}, F_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \vee F_{11}), F_7 \wedge F_8), F_{14} \vdash \Delta_{12}} \vee_L \\
\hline
- : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \wedge F_8 \vdash \Delta_{12} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{14}}{\bullet h_1 : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{14}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_{14}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{inv-th/ax}}{\frac{- : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}}{- : \Delta_{13}, F_7 \wedge F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \wedge_L} \text{hCut} \\
\hline
\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_7 \wedge F_8 \vdash \Delta_{12} \quad h_9 : \Delta_{13}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{inv-th/ax}}{\frac{- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}}{- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}} \wedge_L} \text{hCut}
\end{array}$$

• Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7, F_8 \vdash \Delta_{10}, \perp}{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, \perp} \wedge_L \quad \frac{}{\bullet h_9 : (\Delta_{11}, F_7 \wedge F_8), \perp \vdash \Delta_{10}} \perp_L \\
\hline
- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_7, F_8 \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7, F_8 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{}{\bullet h_9 : \perp, \Delta_{11}, F_7, F_8 \vdash \Delta_{10}} \perp_L}{\frac{- : \Delta_{11}, F_7, F_8 \vdash \Delta_{10}}{- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}} \wedge_L} \text{hCut} \\
\hline
\frac{h_1 : (\perp, \Delta_{11}), F_7, F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : (\perp, \Delta_{11}), F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}} \wedge_L \quad \frac{}{\bullet h_9 : ((\perp, \Delta_{11}), F_7 \wedge F_8), F_{12} \vdash \Delta_{10}} \perp_L \\
\hline
- : (\perp, \Delta_{11}), F_7 \wedge F_8 \vdash \Delta_{10} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_7, F_8 \vdash (\Delta_{10}, P_{11}), P_{11}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash (\Delta_{10}, P_{11}), P_{11}} \wedge_L \quad \frac{}{\bullet h_9 : (\Delta_{12}, F_7 \wedge F_8), P_{11} \vdash \Delta_{10}, P_{11}} I}{- : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}, P_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_7, F_8 \vdash \Delta_{10}, P_{11}, P_{11}}{- : \Delta_{12}, F_7, F_8 \vdash \Delta_{10}, P_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{12}, F_7, F_8, P_{11} \vdash \Delta_{10}, P_{11}}{\bullet h_9 : (\Delta_{12}, F_7 \wedge F_8), P_{11} \vdash \Delta_{10}, P_{11}} I}{- : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}, P_{11}} \wedge_L \text{hCut} \\
\frac{h_1 : (\Delta_{12}, P_{11}), F_7, F_8 \vdash (\Delta_{10}, P_{11}), F_{13}}{\bullet h_1 : (\Delta_{12}, P_{11}), F_7 \wedge F_8 \vdash (\Delta_{10}, P_{11}), F_{13}} \wedge_L \quad \frac{}{\bullet h_9 : ((\Delta_{12}, P_{11}), F_7 \wedge F_8), F_{13} \vdash \Delta_{10}, P_{11}} I}{- : (\Delta_{12}, P_{11}), F_7 \wedge F_8 \vdash \Delta_{10}, P_{11}} \text{Cut} \\
\rightarrow \\
- : \Delta_{12}, P_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, P_{11} \quad I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7, F_8 \vdash \Delta_{10}, \top}{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, \top} \wedge_L \quad \frac{h_9 : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \wedge F_8), \top \vdash \Delta_{10}} \top_L}{- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10} \quad \text{ax/W} \\
\frac{h_1 : (\top, \Delta_{11}), F_7, F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : (\top, \Delta_{11}), F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}} \wedge_L \quad \frac{h_9 : \Delta_{11}, F_{12}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : ((\top, \Delta_{11}), F_7 \wedge F_8), F_{12} \vdash \Delta_{10}} \top_L}{- : (\top, \Delta_{11}), F_7 \wedge F_8 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : \top, \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W} \quad \frac{h_9 : \top, \Delta_{11}, F_{12}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : \top, \Delta_{11}, F_{12}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{ax/W}}{- : \top, \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{hCut}
\end{array}$$

8.8 Status of \vee_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{14} \quad h_1 : \Delta_{13}, F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_{11}, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}, F_{14}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}, F_{14}} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}, F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \text{ax/W}}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \text{hCut} \\
\rightarrow_R \\
- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{14} \quad h_1 : \Delta_{13}, F_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_R}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{14}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10}, F_{14}}{\bullet h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10}, F_{14}} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{ax/W}}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}} \text{hCut} \\
- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{14} \quad h_1 : \Delta_{13}, F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{ax/W} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{hCut} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7 \vdash (\perp, \Delta_{10}), F_{12} \quad h_1 : \Delta_{11}, F_8 \vdash (\perp, \Delta_{10}), F_{12}}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash (\perp, \Delta_{10}), F_{12}} \vee_L \quad \frac{h_9 : \Delta_{11}, F_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \vee F_8), F_{12} \vdash \perp, \Delta_{10}} \perp_R \\
\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \perp, \Delta_{10}, F_{12}}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \Delta_{11}, F_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \vee F_8), F_{12} \vdash \perp, \Delta_{10}} \text{ax/W}}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_7 \vdash (\top, \Delta_{10}), F_{12} \quad h_1 : \Delta_{11}, F_8 \vdash (\top, \Delta_{10}), F_{12}}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash (\top, \Delta_{10}), F_{12}} \vee_L \quad \frac{h_9 : (\Delta_{11}, F_7 \vee F_8), F_{12} \vdash \top, \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \vee F_8), F_{12} \vdash \top, \Delta_{10}} \top_R \\
\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \top, \Delta_{10}} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vdash \Delta_{12}, F_{14} \quad h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_8 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \vdash \Delta_{12}, F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vee F_8) \vdash \Delta_{12}} \rightarrow_R \\
\frac{}{- : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{14}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10}, F_{14}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{14}} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{\bullet h_9 : ((\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vee F_8) \vdash \Delta_{12}} \text{ax/W} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}} \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash \Delta_{12}}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \quad h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \quad h_9 : \Delta_{13}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \rightarrow_R \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_{11}} \text{inv-th/ax} \quad \frac{}{- : \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}} \text{inv-th/ax} \\
\frac{}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}} \vee_L \quad \frac{}{- : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ax/W} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}} \text{sCut} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vdash \Delta_{12}, F_{14} \quad h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_8 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash \Delta_{12}, F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8), F_{14} \vdash \Delta_{12}} \wedge_L \\
\frac{}{- : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{14}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{13}, F_{10}, F_{11}, F_8 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{14}} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8), F_{14} \vdash \Delta_{12}} \text{ax/W} \\
\frac{}{- : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{hCut} \\
\frac{}{- : \Delta_{13}, F_{10} \wedge F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10} \wedge F_{11} \quad h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{- : \Delta_{13}, F_7 \vdash \Delta_{12}} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L \quad \frac{h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_8, F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L}{\rightarrow} \text{hCut} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \vee_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vdash \Delta_{12}, F_{14} \quad h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_8 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vee F_8 \vdash \Delta_{12}, F_{14}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \vee F_8}{\bullet h_9 : ((\Delta_{13}, F_{10} \vee F_{11}), F_7 \vee F_8) \vdash \Delta_{12}} \wedge_L}{\rightarrow} \text{Cut} \\
\frac{}{- : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vee F_8 \vdash \Delta_{12}} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{14}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{14}} \text{inv-th/ax}}{\rightarrow} \vee_L \\
\frac{}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_{10}, F_{14}, F_7 \vee F_8 \vdash \Delta_{12}} \text{hCut} \quad \frac{h_1 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_{14}}{\bullet h_1 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_{14}} \text{hCut} \\
\frac{}{- : \Delta_{13}, F_{10} \vee F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \\
\frac{\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10} \vee F_{11} \quad h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12} \quad h_9 : \Delta_{13}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \\
\frac{}{\rightarrow} \\
\frac{\frac{}{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_{11}} \text{inv-th/ax} \quad \frac{}{- : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{inv-th/ax}}{\rightarrow} \vee_L \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{ax/W} \quad \frac{}{- : \Delta_{13}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}} \text{sCut} \quad \frac{}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}} \text{sCut} \\
\frac{}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \\
\frac{\frac{h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, F_{12} \quad h_1 : \Delta_{11}, F_9 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash \Delta_{10}, F_{12}} \vee_L \quad \frac{h_7 : \Delta_{11}, F_8, F_{12} \vdash \Delta_{10} \quad h_7 : \Delta_{11}, F_9, F_{12} \vdash \Delta_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_{12} \vdash \Delta_{10}} \vee_L}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{11}, F_8 \vee F_9 \vdash \Delta_{10}} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_{12}, F_8 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{11}, F_{12}, F_8 \vdash \Delta_{10}} \text{height}}{\rightarrow} \text{hCut} \quad \frac{h_1 : \Delta_{11}, F_9 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : \Delta_{11}, F_9 \vdash \Delta_{10}, F_{12}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_{12}, F_9 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{11}, F_{12}, F_9 \vdash \Delta_{10}} \text{height}}{\rightarrow} \text{hCut} \\
\frac{}{- : \Delta_{11}, F_8 \vdash \Delta_{10}} \vee_L \\
\frac{}{- : \Delta_{11}, F_8 \vee F_9 \vdash \Delta_{10}}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, F_7 \vdash \Delta_{10}, \perp \quad h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, \perp}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}, \perp} \vee_L \quad \frac{}{\bullet h_9 : (\Delta_{11}, F_7 \vee F_8), \perp \vdash \Delta_{10}} \perp_L}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \\
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}}{\bullet h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}} \perp_L}{\rightarrow} \text{hCut} \quad \frac{h_1 : \Delta_{11}, F_8 \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_8 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}}{\bullet h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}} \perp_L}{\rightarrow} \text{hCut} \\
\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \vee_L \\
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : (\perp, \Delta_{11}), F_7 \vdash \Delta_{10}, F_{12} \quad h_1 : (\perp, \Delta_{11}), F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : (\perp, \Delta_{11}), F_7 \vee F_8 \vdash \Delta_{10}, F_{12}} \vee_L \quad \frac{}{\bullet h_9 : ((\perp, \Delta_{11}), F_7 \vee F_8), F_{12} \vdash \Delta_{10}} \perp_L}{\rightarrow} \text{Cut} \\
\frac{}{- : (\perp, \Delta_{11}), F_7 \vee F_8 \vdash \Delta_{10}} \\
\frac{}{\rightarrow} \\
\frac{}{- : \perp, \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_7 \vdash (\Delta_{10}, p_{11}), p_{11} \quad h_1 : \Delta_{12}, F_8 \vdash (\Delta_{10}, p_{11}), p_{11}}{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash (\Delta_{10}, p_{11}), p_{11}} \vee_L \quad \frac{}{\bullet h_9 : (\Delta_{12}, F_7 \vee F_8), p_{11} \vdash \Delta_{10}, p_{11}} I}{\frac{}{- : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}, p_{11}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_7 \vdash \Delta_{10}, p_{11}, p_{11}}{- : \Delta_{12}, F_7 \vdash \Delta_{10}, p_{11}} \text{ax/w} \quad \frac{\bullet h_9 : \Delta_{12}, F_7, p_{11} \vdash \Delta_{10}, p_{11}}{- : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}, p_{11}} I \quad \frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}, p_{11}}{- : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}} \text{ax/w} \quad \frac{\bullet h_9 : \Delta_{12}, F_8, p_{11} \vdash \Delta_{10}, p_{11}}{- : \Delta_{12}, F_8 \vee F_7 \vdash \Delta_{10}, p_{11}} I}{\frac{}{- : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}, p_{11}} \text{hCut} \vee_L} \\
\\
\frac{\frac{h_1 : (\Delta_{12}, p_{11}), F_7 \vdash (\Delta_{10}, p_{11}), F_{13} \quad h_1 : (\Delta_{12}, p_{11}), F_8 \vdash (\Delta_{10}, p_{11}), F_{13}}{\bullet h_1 : (\Delta_{12}, p_{11}), F_7 \vee F_8 \vdash (\Delta_{10}, p_{11}), F_{13}} \vee_L \quad \frac{}{\bullet h_9 : ((\Delta_{12}, p_{11}), F_7 \vee F_8), F_{13} \vdash \Delta_{10}, p_{11}} I}{\frac{}{- : (\Delta_{12}, p_{11}), F_7 \vee F_8 \vdash \Delta_{10}, p_{11}} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \Delta_{12}, p_{11}, F_7 \vee F_8 \vdash \Delta_{10}, p_{11}} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, F_7 \vdash \Delta_{10}, \top \quad h_1 : \Delta_{11}, F_8 \vdash \Delta_{10}, \top}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}, \top} \vee_L \quad \frac{h_9 : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \vee F_8), \top \vdash \Delta_{10}} \top_L}{\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \text{ax/w} \\
\\
\frac{\frac{h_1 : (\top, \Delta_{11}), F_7 \vdash \Delta_{10}, F_{12} \quad h_1 : (\top, \Delta_{11}), F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : (\top, \Delta_{11}), F_7 \vee F_8 \vdash \Delta_{10}, F_{12}} \vee_L \quad \frac{h_9 : \Delta_{11}, F_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : ((\top, \Delta_{11}), F_7 \vee F_8), F_{12} \vdash \Delta_{10}} \top_L}{\frac{}{- : (\top, \Delta_{11}), F_7 \vee F_8 \vdash \Delta_{10}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}}{- : \top, \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \text{ax/w} \quad \frac{h_9 : \top, \Delta_{11}, F_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{- : \top, \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \text{ax/w}}{\frac{}{- : \top, \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \text{hCut}}
\end{array}$$

8.9 Status of \perp_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_9 \vdash (\Delta_8, F_6 \rightarrow F_7), F_{10}} \perp_L \quad \frac{h_5 : \perp, \Delta_9, F_6, F_{10} \vdash \Delta_8, F_7}{\bullet h_5 : (\perp, \Delta_9), F_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \rightarrow_R}{\frac{}{- : \perp, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7} \perp_L
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_9 \vdash (\Delta_8, F_6 \wedge F_7), F_{10}} \perp_L \quad \frac{h_5 : \perp, \Delta_9, F_{10} \vdash \Delta_8, F_6 \quad h_5 : \perp, \Delta_9, F_{10} \vdash \Delta_8, F_7}{\bullet h_5 : (\perp, \Delta_9), F_{10} \vdash \Delta_8, F_6 \wedge F_7} \wedge_R}{\frac{}{- : \perp, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7} \perp_L
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_9 \vdash (\Delta_8, F_6 \vee F_7), F_{10}} \perp_L \quad \frac{h_5 : \perp, \Delta_9, F_{10} \vdash \Delta_8, F_6, F_7}{\bullet h_5 : (\perp, \Delta_9), F_{10} \vdash \Delta_8, F_6 \vee F_7} \vee_R}{\frac{}{- : \perp, \Delta_9 \vdash \Delta_8, F_6 \vee F_7} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_9 \vdash \Delta_8, F_6 \vee F_7} \perp_L
\end{array}$$

- Case rule \perp_R

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_7 \vdash (\perp, \Delta_6), F_8}}{\vdash : \perp, \Delta_7 \vdash \perp, \Delta_6} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_7, F_8 \vdash \Delta_6}{\bullet h_5 : (\perp, \Delta_7), F_8 \vdash \perp, \Delta_6} \perp_R}{\vdash : \perp, \Delta_7 \vdash \perp, \Delta_6} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_7 \vdash \perp, \Delta_6} \perp_L$$

- Case rule \top_R

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_7 \vdash (\top, \Delta_6), F_8}}{\vdash : \perp, \Delta_7 \vdash \top, \Delta_6} \perp_L \quad \frac{\frac{h_5 : (\perp, \Delta_7), F_8 \vdash \top, \Delta_6}{\bullet h_5 : (\perp, \Delta_7), F_8 \vdash \top, \Delta_6} \top_R}{\vdash : \perp, \Delta_7 \vdash \top, \Delta_6} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_7 \vdash \top, \Delta_6} \top_R$$

- Case rule \rightarrow_L

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8, F_{10}}} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_9, F_{10} \vdash \Delta_8, F_6 \quad h_5 : \perp, \Delta_9, F_7, F_{10} \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_9, F_6 \rightarrow F_7), F_{10} \vdash \Delta_8} \rightarrow_L}{\vdash : \perp, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7}} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_9 \vdash \Delta_8, F_6 \quad h_5 : \perp, \Delta_9, F_7 \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_9), F_6 \rightarrow F_7 \vdash \Delta_8} \rightarrow_L}{\vdash : \perp, \Delta_9 \vdash \Delta_8} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_9 \vdash \Delta_8} \perp_L$$

- Case rule \wedge_L

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8, F_{10}}} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_9, F_6, F_7, F_{10} \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_9, F_6 \wedge F_7), F_{10} \vdash \Delta_8} \wedge_L}{\vdash : \perp, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7}} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_9, F_6, F_7 \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_9), F_6 \wedge F_7 \vdash \Delta_8} \wedge_L}{\vdash : \perp, \Delta_9 \vdash \Delta_8} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_9 \vdash \Delta_8} \perp_L$$

- Case rule \vee_L

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_9, F_6 \vee F_7 \vdash \Delta_8, F_{10}}} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_9, F_6, F_{10} \vdash \Delta_8 \quad h_5 : \perp, \Delta_9, F_7, F_{10} \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_9, F_6 \vee F_7), F_{10} \vdash \Delta_8} \vee_L}{\vdash : \perp, \Delta_9, F_6 \vee F_7 \vdash \Delta_8} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_9, F_6 \vee F_7 \vdash \Delta_8} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_9 \vdash \Delta_8, F_6 \vee F_7}} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_9, F_6 \vdash \Delta_8 \quad h_5 : \perp, \Delta_9, F_7 \vdash \Delta_8}{\bullet h_5 : (\perp, \Delta_9), F_6 \vee F_7 \vdash \Delta_8} \vee_L}{\vdash : \perp, \Delta_9 \vdash \Delta_8} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_9 \vdash \Delta_8} \perp_L$$

- Case rule \perp_L

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_7 \vdash \Delta_6, F_8}}{\vdash : \perp, \Delta_7 \vdash \Delta_6} \perp_L \quad \frac{\frac{h_5 : (\perp, \Delta_7), F_8 \vdash \Delta_6}{\bullet h_5 : (\perp, \Delta_7), F_8 \vdash \Delta_6} \perp_L}{\vdash : \perp, \Delta_7 \vdash \Delta_6} \text{Cut} \rightarrow \frac{}{\vdash : \perp, \Delta_7 \vdash \Delta_6} \perp_L$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_8 \vdash (\Delta_6, p_7), p_7}{\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7} \perp_L \quad \frac{\bullet h_5 : (\perp, \Delta_8), p_7 \vdash \Delta_6, p_7}{\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7} I}{\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_8, p_7 \vdash (\Delta_6, p_7), F_9}{\vdash : \perp, \Delta_8, p_7 \vdash \Delta_6, p_7} \perp_L \quad \frac{\bullet h_5 : (\perp, \Delta_8, p_7), F_9 \vdash \Delta_6, p_7}{\vdash : \perp, \Delta_8, p_7 \vdash \Delta_6, p_7} I}{\vdash : \perp, \Delta_8, p_7 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_8, p_7 \vdash \Delta_6, p_7 \quad \perp_L
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_7 \vdash \Delta_6, \top}{\vdash : \perp, \Delta_7 \vdash \Delta_6} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_7 \vdash \Delta_6}{\bullet h_5 : (\perp, \Delta_7), \top \vdash \Delta_6} \top_L}{\vdash : \perp, \Delta_7 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_7 \vdash \Delta_6 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \top, \Delta_7 \vdash \Delta_6, F_8}{\vdash : \perp, \top, \Delta_7 \vdash \Delta_6} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_7, F_8 \vdash \Delta_6}{\bullet h_5 : (\perp, \top, \Delta_7), F_8 \vdash \Delta_6} \top_L}{\vdash : \perp, \top, \Delta_7 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \top, \Delta_7 \vdash \Delta_6 \quad \perp_L
\end{array}$$

8.10 Status of I : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash ((\Delta_9, F_7 \rightarrow F_8), p_{10}), F_{12}}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, F_7 \rightarrow F_8), p_{10}} I \quad \frac{h_6 : \Delta_{11}, F_7, F_{12}, p_{10} \vdash \Delta_9, F_8, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_{12} \vdash (\Delta_9, F_7 \rightarrow F_8), p_{10}} \rightarrow_R}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, F_7 \rightarrow F_8), p_{10}} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}, F_7 \rightarrow F_8 \quad I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \rightarrow F_7), p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} I \quad \frac{h_5 : \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8, F_7}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \rightarrow_R}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, F_6, p_{10} \vdash \Delta_8, F_7, p_{10}}{\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8, F_7} I \quad \frac{h_5 : \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8, F_7}{\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8, F_7} \text{ax/w}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{hCut} \rightarrow_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash ((\Delta_9, F_7 \wedge F_8), p_{10}), F_{12}}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, F_7 \wedge F_8), p_{10}} I \quad \frac{h_6 : \Delta_{11}, F_{12}, p_{10} \vdash \Delta_9, F_7, p_{10} \quad h_6 : \Delta_{11}, F_{12}, p_{10} \vdash \Delta_9, F_8, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_{12} \vdash (\Delta_9, F_7 \wedge F_8), p_{10}} \wedge_R}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, F_7 \wedge F_8), p_{10}} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}, F_7 \wedge F_8 \quad I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \wedge F_7), p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \wedge F_7} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6 \quad h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_7}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \wedge F_7} \wedge_R}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} \text{ax/w}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} \text{hCut} \quad \frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_7, p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_7} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_7}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_7} \text{ax/w}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_7} \text{hCut} \wedge_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash ((\Delta_9, F_7 \vee F_8), p_{10}), F_{12}}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10}} I \quad \frac{h_6 : \Delta_{11}, F_{12}, p_{10} \vdash \Delta_9, F_7, F_8, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_{12} \vdash (\Delta_9, F_7 \vee F_8), p_{10}} \vee_R}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10}} \text{Cut} \\
\rightarrow \\
\frac{\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}, F_7 \vee F_8}{\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}, F_7 \vee F_8} I \\
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \vee F_7), p_{10}}{\vdash : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \vee F_7), p_{10}} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6, F_7}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \vee F_7} \vee_R}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \vee F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7, p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7, p_{10}} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6, F_7}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6, F_7} \text{ax/W}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7} \text{hCut} \\
\vee_R \\
\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \vee F_7
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_8 \vdash ((\perp, \Delta_7), p_8), F_{10}}{\vdash : \Delta_9, p_8 \vdash (\perp, \Delta_7), p_8} I \quad \frac{h_6 : \Delta_9, F_{10}, p_8 \vdash \Delta_7, p_8}{\bullet h_6 : (\Delta_9, p_8), F_{10} \vdash (\perp, \Delta_7), p_8} \perp_R}{\vdash : \Delta_9, p_8 \vdash (\perp, \Delta_7), p_8} \text{Cut} \\
\rightarrow \\
\frac{\vdash : \Delta_9, p_8 \vdash \perp, \Delta_7, p_8}{\vdash : \Delta_9, p_8 \vdash \perp, \Delta_7, p_8} I \\
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash (\perp, \Delta_6), p_8}{\vdash : \Delta_7, p_8 \vdash (\perp, \Delta_6), p_8} I \quad \frac{h_5 : \Delta_7, p_8, p_8 \vdash \Delta_6}{\bullet h_5 : (\Delta_7, p_8), p_8 \vdash \perp, \Delta_6} \perp_R}{\vdash : \Delta_7, p_8 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash \perp, \Delta_6, p_8}{\vdash : \Delta_7, p_8 \vdash \perp, \Delta_6, p_8} I \quad \frac{h_5 : \Delta_7, p_8, p_8 \vdash \perp, \Delta_6}{\bullet h_5 : (\Delta_7, p_8), p_8 \vdash \perp, \Delta_6} \text{ax/W}}{\vdash : \Delta_7, p_8 \vdash \perp, \Delta_6} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_8 \vdash ((\top, \Delta_7), p_8), F_{10}}{\vdash : \Delta_9, p_8 \vdash (\top, \Delta_7), p_8} I \quad \frac{h_6 : (\Delta_9, p_8), F_{10} \vdash (\top, \Delta_7), p_8}{\bullet h_6 : (\Delta_9, p_8), F_{10} \vdash (\top, \Delta_7), p_8} \top_R}{\vdash : \Delta_9, p_8 \vdash (\top, \Delta_7), p_8} \text{Cut} \\
\rightarrow \\
\frac{\vdash : \Delta_9, p_8 \vdash \top, \Delta_7, p_8}{\vdash : \Delta_9, p_8 \vdash \top, \Delta_7, p_8} \top_R \\
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash (\top, \Delta_6), p_8}{\vdash : \Delta_7, p_8 \vdash (\top, \Delta_6), p_8} I \quad \frac{h_5 : (\Delta_7, p_8), p_8 \vdash \top, \Delta_6}{\bullet h_5 : (\Delta_7, p_8), p_8 \vdash \top, \Delta_6} \top_R}{\vdash : \Delta_7, p_8 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\vdash : \Delta_7, p_8 \vdash \top, \Delta_6}{\vdash : \Delta_7, p_8 \vdash \top, \Delta_6} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_{11}, F_7 \rightarrow F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{12}}{\vdash : (\Delta_{11}, F_7 \rightarrow F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{12}} I \quad \frac{h_6 : \Delta_{11}, F_{12}, p_{10} \vdash \Delta_9, F_7, p_{10} \quad h_6 : \Delta_{11}, F_8, F_{12}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : ((\Delta_{11}, F_7 \rightarrow F_8), p_{10}), F_{12} \vdash \Delta_9, p_{10}} \rightarrow_L}{\vdash : (\Delta_{11}, F_7 \rightarrow F_8), p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\rightarrow \\
\frac{\vdash : \Delta_{11}, p_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, p_{10}}{\vdash : \Delta_{11}, p_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, p_{10}} I \\
\frac{\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \rightarrow F_8}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \rightarrow F_8} I \quad \frac{h_6 : \Delta_{11}, p_{10} \vdash \Delta_9, F_7, p_{10} \quad h_6 : \Delta_{11}, F_8, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \rightarrow F_8 \vdash \Delta_9, p_{10}} \rightarrow_L}{\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\rightarrow \\
\frac{\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}}{\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8, p_{10}}{\vdash : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6 \quad h_5 : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_5 : ((\Delta_9, F_6 \rightarrow F_7), p_{10}), p_{10} \vdash \Delta_8} \rightarrow_L \\
\vdash : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8 \quad \text{Cut} \\
\vdash : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8 \\
\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} \text{ax/w} \quad \frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6 \quad h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} \text{hCut} \\
\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10} \quad \frac{h_5 : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8} I \quad \frac{\bullet h_1 : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10} \quad h_5 : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8} \text{ax/w} \\
\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8 \quad \text{hCut} \\
\vdash : \Delta_9, p_{10}, F_6 \rightarrow F_7 \vdash \Delta_8 \quad \rightarrow_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{11}, F_7 \wedge F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{12}}{\vdash : (\Delta_{11}, F_7 \wedge F_8), p_{10} \vdash \Delta_9, p_{10}} I \quad \frac{h_6 : \Delta_{11}, F_7, F_8, F_{12}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : ((\Delta_{11}, F_7 \wedge F_8), p_{10}), F_{12} \vdash \Delta_9, p_{10}} \wedge_L \\
\vdash : (\Delta_{11}, F_7 \wedge F_8), p_{10} \vdash \Delta_9, p_{10} \quad \text{Cut} \\
\vdash : (\Delta_{11}, F_7 \wedge F_8), p_{10} \vdash \Delta_9, p_{10} \\
\vdash : \Delta_{11}, p_{10}, F_7 \wedge F_8 \vdash \Delta_9, p_{10} \quad I \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \\
\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \wedge F_8 \quad I \quad \frac{h_6 : \Delta_{11}, F_7, F_8, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \wedge F_8 \vdash \Delta_9, p_{10}} \wedge_L \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \quad \text{Cut} \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \\
\vdash : \Delta_9, F_6 \wedge F_7, p_{10} \vdash \Delta_8, p_{10} \quad I \quad \frac{h_5 : \Delta_9, F_6, F_7, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_5 : ((\Delta_9, F_6 \wedge F_7), p_{10}), p_{10} \vdash \Delta_8} \wedge_L \\
\vdash : (\Delta_9, F_6 \wedge F_7), p_{10} \vdash \Delta_8 \quad \text{Cut} \\
\vdash : (\Delta_9, F_6 \wedge F_7), p_{10} \vdash \Delta_8 \\
\vdash : \Delta_9, F_6, F_7, p_{10} \vdash \Delta_8, p_{10} \quad I \quad \frac{h_5 : \Delta_9, F_6, F_7, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_6, F_7, p_{10} \vdash \Delta_8} \text{ax/w} \\
\vdash : \Delta_9, F_6, F_7, p_{10} \vdash \Delta_8 \quad \text{hCut} \\
\vdash : \Delta_9, p_{10}, F_6 \wedge F_7 \vdash \Delta_8 \quad \wedge_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{11}, F_7 \vee F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{12}}{\vdash : (\Delta_{11}, F_7 \vee F_8), p_{10} \vdash \Delta_9, p_{10}} I \quad \frac{h_6 : \Delta_{11}, F_7, F_{12}, p_{10} \vdash \Delta_9, p_{10} \quad h_6 : \Delta_{11}, F_8, F_{12}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : ((\Delta_{11}, F_7 \vee F_8), p_{10}), F_{12} \vdash \Delta_9, p_{10}} \vee_L \\
\vdash : (\Delta_{11}, F_7 \vee F_8), p_{10} \vdash \Delta_9, p_{10} \quad \text{Cut} \\
\vdash : (\Delta_{11}, F_7 \vee F_8), p_{10} \vdash \Delta_9, p_{10} \\
\vdash : \Delta_{11}, p_{10}, F_7 \vee F_8 \vdash \Delta_9, p_{10} \quad I \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \\
\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \vee F_8 \quad I \quad \frac{h_6 : \Delta_{11}, F_7, p_{10} \vdash \Delta_9, p_{10} \quad h_6 : \Delta_{11}, F_8, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \vee F_8 \vdash \Delta_9, p_{10}} \vee_L \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \quad \text{Cut} \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \\
\vdash : \Delta_9, F_6 \vee F_7, p_{10} \vdash \Delta_8, p_{10} \quad I \quad \frac{h_5 : \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8 \quad h_5 : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_5 : ((\Delta_9, F_6 \vee F_7), p_{10}), p_{10} \vdash \Delta_8} \vee_L \\
\vdash : (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8 \quad \text{Cut} \\
\vdash : (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8 \\
\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8, p_{10} \quad I \quad \frac{h_5 : \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8} \text{ax/w} \\
\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8 \quad \text{hCut} \\
\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10} \quad I \quad \frac{h_5 : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8} \text{ax/w} \\
\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8 \quad \text{hCut} \\
\vdash : \Delta_9, p_{10}, F_6 \vee F_7 \vdash \Delta_8 \quad \vee_L
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\bullet h_1 : \Delta_9, p_8 \vdash (\Delta_7, p_8), \perp}{\vdash : \Delta_9, p_8 \vdash \Delta_7, p_8} I \quad \frac{\bullet h_6 : (\Delta_9, p_8), \perp \vdash \Delta_7, p_8}{\vdash : \Delta_9, p_8 \vdash \Delta_7, p_8} \perp_L \\
\vdash : \Delta_9, p_8 \vdash \Delta_7, p_8 \quad \text{Cut} \\
\vdash : \Delta_9, p_8 \vdash \Delta_7, p_8 \\
\vdash : \Delta_9, p_8 \vdash \Delta_7, p_8 \quad I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\perp, \Delta_9), p_8 \vdash (\Delta_7, p_8), F_{10}}{- : (\perp, \Delta_9), p_8 \vdash \Delta_7, p_8} I}{\frac{\bullet h_6 : ((\perp, \Delta_9), p_8), F_{10} \vdash \Delta_7, p_8}{\rightarrow} \text{Cut}} \perp_L \\
\frac{\frac{\bullet h_1 : (\perp, \Delta_7), p_8 \vdash \Delta_6, p_8}{- : (\perp, \Delta_7), p_8 \vdash \Delta_6} I}{\frac{\bullet h_5 : ((\perp, \Delta_7), p_8), p_8 \vdash \Delta_6}{\rightarrow} \text{Cut}} \perp_L \\
\frac{\frac{\frac{\frac{\bullet h_1 : (\perp, \Delta_9), p_8 \vdash (\Delta_7, p_8), F_{10}}{- : (\perp, \Delta_9), p_8 \vdash \Delta_7, p_8} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_6 : ((\perp, \Delta_9), p_8), F_{10} \vdash \Delta_7, p_8}{\rightarrow} \text{Cut}} \perp_L \\
\frac{\frac{\frac{\bullet h_1 : (\perp, \Delta_7), p_8 \vdash \Delta_6, p_8}{- : (\perp, \Delta_7), p_8 \vdash \Delta_6} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_5 : ((\perp, \Delta_7), p_8), p_8 \vdash \Delta_6}{\rightarrow} \text{Cut}} \perp_L
\end{array}$$

• Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash ((\Delta_7, p_9), p_8), p_9}{- : \Delta_{10}, p_8 \vdash (\Delta_7, p_9), p_8} I}{\frac{\bullet h_6 : (\Delta_{10}, p_8), p_9 \vdash (\Delta_7, p_9), p_8}{\rightarrow} \text{Cut}} I \\
\frac{\frac{\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash ((\Delta_7, p_9), p_8), p_9}{- : \Delta_{10}, p_8 \vdash (\Delta_7, p_9), p_8} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_6 : ((\Delta_{10}, p_8), p_9) \vdash (\Delta_7, p_9), p_8}{\rightarrow} \text{Cut}} I \\
\frac{\frac{\frac{\bullet h_1 : (\Delta_{10}, p_9), p_8 \vdash ((\Delta_7, p_9), p_8), F_{11}}{- : (\Delta_{10}, p_9), p_8 \vdash (\Delta_7, p_9), p_8} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_6 : ((\Delta_{10}, p_9), p_8), F_{11} \vdash (\Delta_7, p_9), p_8}{\rightarrow} \text{Cut}} I \\
\frac{\frac{\frac{\bullet h_1 : \Delta_9, p_8 \vdash (\Delta_7, p_8), F_{10}}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_6 : (\Delta_9, p_8), F_{10} \vdash \Delta_7, p_8}{\rightarrow} \text{Cut}} I \\
\frac{\frac{\frac{\bullet h_1 : \Delta_8, p_7 \vdash (\Delta_6, p_7), p_7}{- : \Delta_8, p_7 \vdash \Delta_6, p_7} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_5 : (\Delta_8, p_7), p_7 \vdash \Delta_6, p_7}{\rightarrow} \text{Cut}} I \\
\frac{\frac{\frac{\bullet h_1 : (\Delta_8, p_7), p_9 \vdash (\Delta_6, p_7), p_9}{- : (\Delta_8, p_7), p_9 \vdash \Delta_6, p_7} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_5 : ((\Delta_8, p_7), p_9), p_9 \vdash \Delta_6, p_7}{\rightarrow} \text{Cut}} I
\end{array}$$

• Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\frac{\bullet h_1 : \Delta_9, p_8 \vdash (\Delta_7, p_8), \top}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_6 : (\Delta_9, p_8), \top \vdash \Delta_7, p_8}{\rightarrow} \text{Cut}} \top_L \\
\frac{\frac{\frac{\frac{\bullet h_1 : (\top, \Delta_9), p_8 \vdash (\Delta_7, p_8), F_{10}}{- : (\top, \Delta_9), p_8 \vdash \Delta_7, p_8} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_6 : ((\top, \Delta_9), p_8), F_{10} \vdash \Delta_7, p_8}{\rightarrow} \text{Cut}} \top_L \\
\frac{\frac{\frac{\frac{\bullet h_1 : (\top, \Delta_7), p_8 \vdash \Delta_6, p_8}{- : (\top, \Delta_7), p_8 \vdash \Delta_6} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_5 : ((\top, \Delta_7), p_8), p_8 \vdash \Delta_6}{\rightarrow} \text{Cut}} \top_L \\
\frac{\frac{\frac{\frac{\bullet h_1 : \top, \Delta_7, p_8 \vdash \Delta_6, p_8}{- : \top, \Delta_7, p_8 \vdash \Delta_6} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_5 : \top, \Delta_7, p_8, p_8 \vdash \Delta_6}{\rightarrow} \text{Cut}} \text{ax/W} \\
\frac{\frac{\frac{\frac{\bullet h_1 : \top, \Delta_7, p_8 \vdash \Delta_6, p_8}{- : \top, \Delta_7, p_8 \vdash \Delta_6} I}{\rightarrow} \text{Cut}}{\frac{\bullet h_5 : \top, \Delta_7, p_8, p_8 \vdash \Delta_6}{\rightarrow} \text{Cut}} \text{hCut}
\end{array}$$

8.11 Status of \top_L : OK

- Case rule \rightarrow_R

$$\frac{\frac{h_1 : \Delta_9 \vdash (\Delta_8, F_6 \rightarrow F_7), F_{10}}{\bullet h_1 : \top, \Delta_9 \vdash (\Delta_8, F_6 \rightarrow F_7), F_{10}} \top_L \quad \frac{h_5 : \top, \Delta_9, F_6, F_{10} \vdash \Delta_8, F_7}{\bullet h_5 : (\top, \Delta_9), F_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \rightarrow_R}{- : \top, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_{10}, F_6 \rightarrow F_7}{- : \top, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_6 \rightarrow F_7}{\bullet h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7} \text{hCut}$$

- Case rule \wedge_R

$$\frac{\frac{h_1 : \Delta_9 \vdash (\Delta_8, F_6 \wedge F_7), F_{10}}{\bullet h_1 : \top, \Delta_9 \vdash (\Delta_8, F_6 \wedge F_7), F_{10}} \top_L \quad \frac{h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_6 \quad h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_7}{\bullet h_5 : (\top, \Delta_9), F_{10} \vdash \Delta_8, F_6 \wedge F_7} \wedge_R}{- : \top, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_{10}, F_6 \wedge F_7}{- : \top, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_6 \wedge F_7}{\bullet h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7} \text{hCut}$$

- Case rule \vee_R

$$\frac{\frac{h_1 : \Delta_9 \vdash (\Delta_8, F_6 \vee F_7), F_{10}}{\bullet h_1 : \top, \Delta_9 \vdash (\Delta_8, F_6 \vee F_7), F_{10}} \top_L \quad \frac{h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_6, F_7}{\bullet h_5 : (\top, \Delta_9), F_{10} \vdash \Delta_8, F_6 \vee F_7} \vee_R}{- : \top, \Delta_9 \vdash \Delta_8, F_6 \vee F_7} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_{10}, F_6 \vee F_7}{- : \top, \Delta_9 \vdash \Delta_8, F_6 \vee F_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_6 \vee F_7}{\bullet h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_6 \vee F_7} \text{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8, F_6 \vee F_7} \text{hCut}$$

- Case rule \perp_R

$$\frac{\frac{h_1 : \Delta_7 \vdash (\perp, \Delta_6), F_8}{\bullet h_1 : \top, \Delta_7 \vdash (\perp, \Delta_6), F_8} \top_L \quad \frac{h_5 : \top, \Delta_7, F_8 \vdash \Delta_6}{\bullet h_5 : (\top, \Delta_7), F_8 \vdash \perp, \Delta_6} \perp_R}{- : \top, \Delta_7 \vdash \perp, \Delta_6} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_7 \vdash \perp, \Delta_6, F_8}{- : \top, \Delta_7 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_7, F_8 \vdash \perp, \Delta_6}{\bullet h_5 : \top, \Delta_7, F_8 \vdash \perp, \Delta_6} \text{ax/W}}{- : \top, \Delta_7 \vdash \perp, \Delta_6} \text{hCut}$$

- Case rule \top_R

$$\frac{\frac{h_1 : \Delta_7 \vdash (\top, \Delta_6), F_8}{\bullet h_1 : \top, \Delta_7 \vdash (\top, \Delta_6), F_8} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_7), F_8 \vdash \top, \Delta_6} \top_R}{- : \top, \Delta_7 \vdash \top, \Delta_6} \text{Cut}$$

$$\xrightarrow{\quad} \frac{}{- : \top, \Delta_7 \vdash \top, \Delta_6} \top_R$$

- Case rule \rightarrow_L

$$\frac{\frac{h_1 : \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8, F_{10}}{\bullet h_1 : \top, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8, F_{10}} \top_L \quad \frac{h_5 : \top, \Delta_9, F_{10} \vdash \Delta_8, F_6 \quad h_5 : \top, \Delta_9, F_7, F_{10} \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9, F_6 \rightarrow F_7), F_{10} \vdash \Delta_8} \rightarrow_L}{- : \top, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8, F_{10}}{- : \top, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_8}{\bullet h_5 : \top, \Delta_9, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_8} \text{ax/W}}{- : \top, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8} \text{hCut}$$

$$\frac{\frac{h_1 : \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7}{\bullet h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7} \top_L \quad \frac{h_5 : \top, \Delta_9 \vdash \Delta_8, F_6 \quad h_5 : \top, \Delta_9, F_7 \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9), F_6 \rightarrow F_7 \vdash \Delta_8} \rightarrow_L}{- : \top, \Delta_9 \vdash \Delta_8} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7}{- : \top, \Delta_9 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8}{\bullet h_5 : \top, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8} \text{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8} \text{hCut}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9, F_6 \wedge F_7 \vdash \Delta_8, F_{10}}{\bullet h_1 : \top, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8, F_{10}} \top_L \quad \frac{h_5 : \top, \Delta_9, F_6, F_7, F_{10} \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9, F_6 \wedge F_7), F_{10} \vdash \Delta_8} \wedge_L}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8, F_{10}}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_{10}, F_6 \wedge F_7 \vdash \Delta_8}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8} \text{hCut}}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9 \vdash \Delta_8, F_6 \wedge F_7}{\bullet h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7} \top_L \quad \frac{h_5 : \top, \Delta_9, F_6, F_7 \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9), F_6 \wedge F_7 \vdash \Delta_8} \wedge_L}{- : \top, \Delta_9 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7}{- : \top, \Delta_9 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8}{- : \top, \Delta_9 \vdash \Delta_8} \text{hCut}}{- : \top, \Delta_9 \vdash \Delta_8}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9, F_6 \vee F_7 \vdash \Delta_8, F_{10}}{\bullet h_1 : \top, \Delta_9, F_6 \vee F_7 \vdash \Delta_8, F_{10}} \top_L \quad \frac{\frac{h_5 : \top, \Delta_9, F_6, F_{10} \vdash \Delta_8} \quad \frac{h_5 : \top, \Delta_9, F_7, F_{10} \vdash \Delta_8}}{\bullet h_5 : (\top, \Delta_9, F_6 \vee F_7), F_{10} \vdash \Delta_8} \vee_L}{- : \top, \Delta_9, F_6 \vee F_7 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9, F_6 \vee F_7 \vdash \Delta_8, F_{10}}{- : \top, \Delta_9, F_6 \vee F_7 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_{10}, F_6 \vee F_7 \vdash \Delta_8}{- : \top, \Delta_9, F_6 \vee F_7 \vdash \Delta_8} \text{hCut}}{- : \top, \Delta_9, F_6 \vee F_7 \vdash \Delta_8}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9 \vdash \Delta_8, F_6 \vee F_7}{\bullet h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \vee F_7} \top_L \quad \frac{\frac{h_5 : \top, \Delta_9, F_6 \vdash \Delta_8} \quad \frac{h_5 : \top, \Delta_9, F_7 \vdash \Delta_8}}{\bullet h_5 : (\top, \Delta_9), F_6 \vee F_7 \vdash \Delta_8} \vee_L}{- : \top, \Delta_9 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \vee F_7}{- : \top, \Delta_9 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_6 \vee F_7 \vdash \Delta_8}{- : \top, \Delta_9 \vdash \Delta_8} \text{hCut}}{- : \top, \Delta_9 \vdash \Delta_8}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_7 \vdash \Delta_6, \perp}{\bullet h_1 : \top, \Delta_7 \vdash \Delta_6, \perp} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_7), \perp \vdash \Delta_6} \perp_L}{- : \top, \Delta_7 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_7 \vdash \perp, \Delta_6}{- : \top, \Delta_7 \vdash \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \perp, \top, \Delta_7 \vdash \Delta_6}{- : \top, \Delta_7 \vdash \Delta_6} \perp_L}{- : \top, \Delta_7 \vdash \Delta_6} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \perp, \Delta_7 \vdash \Delta_6, F_8}{\bullet h_1 : \top, \perp, \Delta_7 \vdash \Delta_6, F_8} \top_L \quad \frac{}{\bullet h_5 : (\top, \perp, \Delta_7), F_8 \vdash \Delta_6} \perp_L}{- : \top, \perp, \Delta_7 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \top, \Delta_7 \vdash \Delta_6} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8 \vdash (\Delta_6, p_7), p_7}{\bullet h_1 : \top, \Delta_8 \vdash (\Delta_6, p_7), p_7} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_8), p_7 \vdash \Delta_6, p_7} I}{- : \top, \Delta_8 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_8 \vdash \Delta_6, p_7, p_7}{- : \top, \Delta_8 \vdash \Delta_6, p_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, p_7 \vdash \Delta_6, p_7}{- : \top, \Delta_8 \vdash \Delta_6, p_7} I}{- : \top, \Delta_8 \vdash \Delta_6, p_7} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8, p_7 \vdash (\Delta_6, p_7), F_9}{\bullet h_1 : \top, \Delta_8, p_7 \vdash (\Delta_6, p_7), F_9} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_8, p_7), F_9 \vdash \Delta_6, p_7} I}{- : \top, \Delta_8, p_7 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_8, p_7 \vdash \Delta_6, p_7} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_7 \vdash \Delta_6, F_8}{\bullet h_1 : \top, \Delta_7 \vdash \Delta_6, F_8} \top_L \quad \frac{h_5 : \Delta_7, F_8 \vdash \Delta_6}{\bullet h_5 : (\top, \Delta_7), F_8 \vdash \Delta_6} \top_L}{- : \top, \Delta_7 \vdash \Delta_6} \text{Cut} \\
\begin{array}{c} \rightarrow \\ \hline \frac{h_1 : \top, \Delta_7 \vdash \Delta_6, F_8}{- : \top, \Delta_7 \vdash \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_7, F_8 \vdash \Delta_6}{- : \top, \Delta_7 \vdash \Delta_6} \text{ax/W} \end{array} \text{hCut}
\end{array}$$