System for Intuitionistic Linear Logic

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1 Measure of derivations

• Case(s) rule !R

• Case(s) rule $\mathbf{1}_R$

• Case(s) rule ⊤

• Case(s) rule $\&_R$

• Case(s) rule \multimap_R

• Case(s) rule \bigoplus_{R_2}

• Case(s) rule \bigoplus_{R_1}

• Case(s) rule $\mathbf{1}_L$

• Case(s) rule \otimes_R

$$\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2,\Delta_3 \vdash \mathbf{F}_4 \otimes \mathbf{F}_5} \ \otimes_R \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4} \quad \underset{\bullet}{\text{II}} \quad \frac{\frac{\mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_1:\Delta_2,\Delta_3 \vdash \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \underset{\otimes}{\text{III}} \quad \underset{\otimes}{\text{III}}$$

 \bullet Case(s) rule W

• Case(s) rule C

• Case(s) rule !L

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_2,!\mathbf{F}_3\vdash\mathbf{F}_4} & !L & \sim & \frac{\overline{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_4}}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_4} & \mathbf{IH} \\ \hline \bullet \bullet \mathbf{h}_1:\Delta_2,!\mathbf{F}_3\vdash\mathbf{F}_4 & !L \end{array}$$

• Case(s) rule $\&_{L2}$

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{f}_4\vdash\mathbf{f}_5}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{f}_3\&\mathbf{f}_4\vdash\mathbf{f}_5} \ \&_{L2} \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1:\Delta_2,\mathbf{f}_4\vdash\mathbf{f}_5}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{f}_4\vdash\mathbf{f}_5} \ ^{\mathrm{ax}}}{\bullet \ \mathbf{h}_1:\Delta_2,\mathbf{f}_3\&\mathbf{f}_4\vdash\mathbf{f}_5} \ \&_{L2}$$

• Case(s) rule $\&_{L1}$

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{f}_3\vdash\mathbf{f}_5}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{f}_3\&\mathbf{f}_4\vdash\mathbf{f}_5} \ \&_{L1} \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1:\Delta_2,\mathbf{f}_3\vdash\mathbf{f}_5}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{f}_3\vdash\mathbf{f}_5}}{\bullet\bullet\mathbf{h}_1:\Delta_2,\mathbf{f}_3\&\mathbf{f}_4\vdash\mathbf{f}_5} \ \&_{L1}$$

• Case(s) rule \otimes_L

$$\begin{array}{c|c} \mathbf{h}_1:\Delta_2,\mathbf{f}_3,\mathbf{f}_4\vdash\mathbf{f}_5\\ \bullet\mathbf{h}_1:\Delta_2,\mathbf{f}_3\otimes\mathbf{f}_4\vdash\mathbf{f}_5 \end{array} \otimes_L \qquad \leadsto \qquad \begin{array}{c|c} \overline{\mathbf{h}_1:\Delta_2,\mathbf{f}_3,\mathbf{f}_4\vdash\mathbf{f}_5}\\ \bullet\mathbf{h}_1:\Delta_2,\mathbf{f}_3,\mathbf{f}_4\vdash\mathbf{f}_5 \end{array} \xrightarrow[\mathbf{H}]{} \mathbf{h}_1:\Delta_2,\mathbf{f}_3,\mathbf{f}_4\vdash\mathbf{f}_5\\ \bullet\mathbf{h}_1:\Delta_2,\mathbf{f}_3\otimes\mathbf{f}_4\vdash\mathbf{f}_5 \end{array} \otimes_L$$

• Case(s) rule \oplus_L

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5\quad\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_5}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_5}\ \oplus_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5}\ \overset{\mathrm{ax}}{\mathbf{h}} \qquad \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_5}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_5} \qquad \overset{\mathrm{ax}}{\mathbf{h}} \qquad \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_5}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_5} \qquad \overset{\mathrm{ax}}{\mathbf{h}} \qquad \overset{\mathrm{a$$

• Case(s) rule \multimap_L

$$\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1:\Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_6} \quad \multimap_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4} \quad \frac{\mathbf{ax}}{\mathbf{IH}} \quad \frac{\mathbf{h}_1:\Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_6} \quad \frac{\mathbf{ax}}{\mathbf{IH}} \quad \square_1 \vdash \square_2 \vdash \square_2 \vdash \square_3 \vdash \square_3 \vdash \square_4 \vdash \square_4$$

• Case(s) rule I

2 Invertibility of Rules

2.1 Status of !R: : Non invertible

 $\bullet \;$ Case rule !R

- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- ullet Case rule W

$$\frac{\mathbf{h}_2: ! \Upsilon 3 \vdash ! \mathbf{F}_1}{\bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_1} \quad W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2: ! \Upsilon 3 \vdash \mathbf{F}_1}}{\bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4 \vdash \mathbf{F}_1} \quad W$$

 \bullet Case rule C

• Case rule !L

$$\frac{\mathtt{h}_2: \mathtt{F}_4, ! \Upsilon 3 \vdash ! \mathtt{F}_1}{\bullet \mathtt{h}_2: ! \Upsilon 3, ! \mathtt{F}_4 \vdash ! \mathtt{F}_1} \quad !L \qquad \rightsquigarrow \qquad \overline{\bullet \mathtt{h}_2: ! \Upsilon 3, ! \mathtt{F}_4 \vdash \mathtt{F}_1} \quad \mathtt{fail}$$

- Case rule $\&_{L2}$
- Case rule $\&_{L1}$

- Case rule \otimes_L Case rule \oplus_L Case rule $-\circ_L$
- $\bullet\,$ Case rule I

2.2 Status of 1_R : : Invertible

- Case rule !R
- Case rule $\mathbf{1}_R$

- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet\,$ Case rule W
- $\bullet\,$ Case rule C
- \bullet Case rule !L
- \bullet Case rule $\&_{L2}$

- Case rule \otimes_L
- Case rule \oplus_L
- $\bullet\,$ Case rule I

2.3 Status of \top : : Invertible

- \bullet Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\overline{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top} \ \top \qquad \leadsto \qquad \mathtt{trivial}$$

- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \top}{\bullet \mathbf{h}_1: \mathbf{1}, \Delta_2 \vdash \top} \ \mathbf{1}_L \qquad \leadsto \qquad \mathsf{trivial}$$

- Case rule \otimes_R
- $\bullet\,$ Case rule W

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \top}{\bullet \mathbf{h}_1: \Delta_2, !\mathbf{F}_3 \vdash \top} \ W \qquad \leadsto \qquad \mathsf{trivial}$$

 \bullet Case rule C

$$\frac{\mathbf{h}_1:\Delta_2, \mathbf{!f}_3, \mathbf{!f}_3 \vdash \top}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{!f}_3 \vdash \top} \ C \qquad \leadsto \qquad \mathsf{trivial}$$

 \bullet Case rule !L

$$\frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \top}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{!F}_3 \vdash \top} \ !L \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\top}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\&\mathbf{F}_4\vdash\top}~\&_{L2}~~\leadsto~~\mathsf{trivial}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\top}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\&\mathbf{F}_4\vdash\top}~\&_{L1}~~\leadsto~~\mathsf{trivial}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3,\mathbf{F}_4\vdash\top}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\top}\ \otimes_L \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\top\quad\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\top}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\top}\ \oplus_L \qquad \leadsto \qquad \text{trivial}$$

• Case rule \multimap_L

$$\begin{array}{ll} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 & \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \top \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \top \end{array} \ \multimap_L \qquad \leadsto \qquad \mathsf{trivial}$$

ullet Case rule I

2.4 Status of $\&_R$: (Left Premise): Invertible

• Case rule !R

• Case rule $\mathbf{1}_R$

 \bullet Case rule \top

• Case rule $\&_R$

• Case rule \multimap_R

- Case rule \oplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F_1} \& \mathbf{F_2}}{\bullet \mathbf{h}_3: \mathbf{1}, \Delta_4 \vdash \mathbf{F_1} \& \mathbf{F_2}} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F_1}}}{\bullet \mathbf{h}_3: \mathbf{1}, \Delta_4 \vdash \mathbf{F_1}} \ \mathbf{1}_L$$

- Case rule \otimes_R
- \bullet Case rule W

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_1\&\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4, !\mathbf{F}_5\vdash \mathbf{F}_1\&\mathbf{F}_2} \quad W \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_1} \quad \mathbf{ax/ind}}{\bullet\mathbf{h}_3:\Delta_4, !\mathbf{F}_5\vdash \mathbf{F}_1} \quad W$$

 \bullet Case rule C

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{l}_{\mathsf{F}_5},\mathbf{l}_{\mathsf{F}_5}\vdash\mathbf{F}_1\&\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{l}_{\mathsf{F}_5}\vdash\mathbf{F}_1\&\mathbf{F}_2} \quad C \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{l}_{\mathsf{F}_5},\mathbf{l}_{\mathsf{F}_5}\vdash\mathbf{F}_1}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{l}_{\mathsf{F}_5}\vdash\mathbf{F}_1} \quad \frac{\mathbf{ax/ind}}{C}$$

• Case rule !L

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{f}_6\vdash\mathbf{f}_1\&\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_1\&\mathbf{f}_2} \quad\&_{L2}\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{f}_6\vdash\mathbf{f}_1}\quad \mathbf{ax/ind}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_1} \&_{L2}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\&\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_1\&\mathbf{F}_2}~\&_{L1}~~\sim~\frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_1}^{\mathrm{ax/ind}}\&_{L1}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_1\&\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_1\&\mathbf{f}_2}\otimes_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_1}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_1}\otimes_L$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\&\mathbf{F}_2\quad\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1\&\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_6\ominus\mathbf{F}_1\&\mathbf{F}_2}\oplus_L\qquad \leadsto\qquad \frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_1} \xrightarrow{\mathbf{ax/ind}} \frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1}{\oplus_L} \xrightarrow{\mathbf{bh}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1}$$

• Case rule \multimap_L

 \bullet Case rule I

2.5 Status of $\&_R$ (Right Premise): : Invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$

$$\frac{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_3\quad \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_3\&\mathbf{F}_4}\quad\&_R\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_4}}{\bullet \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_4}\stackrel{\mathrm{ax}}{\vdash}$$

- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$

- Case rule \otimes_R
- \bullet Case rule W

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F_1}\&\mathbf{F_2}}{\bullet\mathbf{h}_3:\Delta_4, \mathbf{!F_5}\vdash \mathbf{F_1}\&\mathbf{F_2}} \quad W \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F_2}}{\bullet\mathbf{h}_3:\Delta_4, \mathbf{!F_5}\vdash \mathbf{F_2}}}{\bullet\mathbf{h}_3:\Delta_4, \mathbf{!F_5}\vdash \mathbf{F_2}} W$$

 \bullet Case rule C

• Case rule !L

$$\frac{\mathtt{h}_3:\Delta_4,\mathtt{F}_5 \vdash \mathtt{F}_1\&\mathtt{F}_2}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \vdash \mathtt{F}_1\&\mathtt{F}_2} \ \mathtt{!L} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_3:\Delta_4,\mathtt{F}_5 \vdash \mathtt{F}_2}}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{!F}_5 \vdash \mathtt{F}_2} \ \overset{\mathtt{ax/ind}}{}{} \ \mathtt{!L}$$

• Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3:\Delta_4,\mathtt{F}_6 \vdash \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_1 \& \mathtt{F}_2} \quad \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_3:\Delta_4,\mathtt{F}_6 \vdash \mathtt{F}_2}}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2} \overset{\mathrm{ax/ind}}{} \&_{L2}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{f}_5\vdash\mathbf{f}_1\&\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_1\&\mathbf{f}_2} \ \&_{L1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{f}_5\vdash\mathbf{f}_2}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_2} \overset{\mathrm{ax/ind}}{\&_{L1}}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_1\&\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_1\&\mathbf{f}_2}\otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_2}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_2}\overset{\mathrm{ax/ind}}{\otimes}_L$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\&\mathbf{F}_2\quad\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1\&\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_6\ominus\mathbf{F}_1\&\mathbf{F}_2}\oplus_L\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_2}\quad \text{ax/ind}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\ominus\mathbf{F}_6\vdash\mathbf{F}_2} \xrightarrow{\mathbf{ax/ind}}\quad \frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_2}{\oplus_L}$$

• Case rule \multimap_L

$$\frac{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_6 \quad \mathbf{h}_3: \Delta_5, \mathbf{F}_7 \vdash \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4, \Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_1 \& \mathbf{F}_2} \quad \multimap \\ \frac{\overline{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_6} \quad \text{ax}}{\bullet \mathbf{h}_3: \Delta_4, \Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_2} \quad \frac{\mathbf{ax/ind}}{\multimap L}$$

 $\bullet\,$ Case rule I

2.6 Status of \multimap_R : : Invertible

- Case rule !R
- Case rule $\mathbf{1}_{R}$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R

- Case rule \oplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F_1} \multimap \mathbf{F_2}}{\bullet \mathbf{h}_3: \mathbf{1}, \Delta_4 \vdash \mathbf{F_1} \multimap \mathbf{F_2}} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: \Delta_4, \mathbf{F_1} \vdash \mathbf{F_2}}}{\bullet \mathbf{h}_3: \mathbf{1}, \Delta_4, \mathbf{F_1} \vdash \mathbf{F_2}} \ \mathbf{1}_L$$

- Case rule \otimes_R
- \bullet Case rule W

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F_1}\multimap \mathbf{F_2}}{\bullet \mathbf{h}_3:\Delta_4,!\mathbf{F_5}\vdash \mathbf{F_1}\multimap \mathbf{F_2}} \ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F_1}\vdash \mathbf{F_2}}}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{F_1},!\mathbf{F_5}\vdash \mathbf{F_2}} \ W$$

 \bullet Case rule C

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{l}_{F_5},\mathbf{l}_{F_5}\vdash \mathbf{F}_1\multimap \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{l}_{F_5}\vdash \mathbf{F}_1\multimap \mathbf{F}_2} \quad C \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{l}_{F_5}\vdash \mathbf{F}_2}}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{l}_{F_5}\vdash \mathbf{F}_2} \quad \frac{\mathbf{ax/ind}}{C}$$

• Case rule !L

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1\multimap\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_1\multimap\mathbf{F}_2} \ \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{F}_6\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_2} \ \&_{L2}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\multimap\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_1\multimap\mathbf{F}_2} \ \&_{L1} \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{F}_5\vdash\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_2}} \overset{\mathrm{ax/ind}}{\bullet} \&_{L1}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_1\multimap\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_1\multimap\mathbf{f}_2}\otimes_L\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{f}_1,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_2}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_1,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_2}\overset{\mathrm{ax/ind}}{\otimes L}$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\multimap\mathbf{F}_2\quad\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1\multimap\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_6\uplus\mathbf{F}_6\vdash\mathbf{F}_1\multimap\mathbf{F}_2}\oplus_L\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{F}_5\vdash\mathbf{F}_2}\quad \text{ax/ind}\quad \overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{F}_6\vdash\mathbf{F}_2}\quad \frac{\text{ax/ind}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_1,\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_2}\quad \oplus_L$$

• Case rule \multimap_L

$$\begin{array}{l} \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6\quad \mathbf{h}_3:\Delta_5,\mathbf{F}_7\vdash \mathbf{F}_1\multimap \mathbf{F}_2} \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_1\multimap \mathbf{F}_2} \end{array} \ \, \multimap_L \qquad \leadsto \qquad \begin{array}{l} \overline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6} \quad \text{ax} \quad \overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1,\mathbf{F}_7\vdash \mathbf{F}_2} \\ \bullet \mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_1,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_2 \end{array} \ \, \xrightarrow{\mathrm{ax/ind}} \quad \neg_L \\ \end{array}$$

ullet Case rule I

2.7 Status of \oplus_{R_2} : : Non invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}

$$\begin{array}{cccc} & & & & & & \\ & \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_4 & \oplus_{R_2} & & \leadsto & & & \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_3 \oplus \mathbf{F}_4 & & & & \\ \end{array} \xrightarrow{\bullet} & & \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_4 & & \\ \end{array} \xrightarrow{\bullet} \mathbf{H}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4} \ \oplus_{R_1} \qquad \leadsto \qquad \overline{\bullet \mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_4} \ \mathsf{fail}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3: \mathbf{1}, \Delta_4 \vdash \mathbf{F}_1 \oplus \mathbf{F}_2} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_3: \mathbf{1}, \Delta_4 \vdash \mathbf{F}_2} \ \mathbf{1}_L$$

- Case rule \otimes_R
- ullet Case rule W

 \bullet Case rule C

$$\begin{array}{lll} \frac{\mathbf{h}_3:\Delta_4, \mathbb{I}_{\mathbf{F}_5}, \mathbb{I}_{\mathbf{F}_5} \vdash \mathbf{F}_1 \oplus \mathbb{F}_2}{\bullet \mathbf{h}_3:\Delta_4, \mathbb{I}_{\mathbf{F}_5} \vdash \mathbf{F}_1 \oplus \mathbb{F}_2} & C & & \Rightarrow & & \frac{\mathbf{h}_3:\Delta_4, \mathbb{I}_{\mathbf{F}_5}, \mathbb{I}_{\mathbf{F}_5} \vdash \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4, \mathbb{I}_{\mathbf{F}_5} \vdash \mathbb{F}_2} & & \\ \end{array} \begin{array}{ll} \mathbf{ax/ind} \\ C & & \end{array}$$

• Case rule !L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\oplus\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{IF}_5\vdash\mathbf{F}_1\oplus\mathbf{F}_2}\ !L\qquad \rightsquigarrow\qquad \frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{IF}_5\vdash\mathbf{F}_2}\ \frac{\mathbf{ax/ind}}{!L}$$

• Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3:\Delta_4,\mathtt{F}_6 \vdash \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_1 \oplus \mathtt{F}_2} \quad \&_{L2} \qquad \leadsto \qquad \frac{\frac{\mathtt{h}_3:\Delta_4,\mathtt{F}_6 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2} \overset{\mathrm{ax/ind}}{\bullet}$$

• Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3:\Delta_4,\mathtt{F}_5 \vdash \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_1 \oplus \mathtt{F}_2} \quad \&_{L1} \qquad \leadsto \qquad \frac{\frac{\mathtt{h}_3:\Delta_4,\mathtt{F}_5 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2} \stackrel{\mathrm{ax/ind}}{\bullet}_{L1}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_1\oplus\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_1\oplus\mathbf{f}_2}\otimes_L\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_2}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_2}\overset{\mathrm{ax/ind}}{\otimes}_L$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\oplus\mathbf{F}_2\quad\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1\oplus\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_6\oplus\mathbf{F}_6\vdash\mathbf{F}_1\oplus\mathbf{F}_2}\oplus_L\qquad \leadsto\qquad \frac{\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_6\oplus\mathbf{F}_6\vdash\mathbf{F}_2}\xrightarrow{\mathrm{ax/ind}}\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_2}$$

• Case rule \multimap_L

$$\frac{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_6 \quad \mathbf{h}_3: \Delta_5, \mathbf{F}_7 \vdash \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4, \Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_1 \oplus \mathbf{F}_2} \quad \circ_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Delta_4, \Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_4, \Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\multimap_L}$$

ullet Case rule I

2.8 Status of \bigoplus_{R_1} : Non invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R

• Case rule \oplus_{R_2}

$$\frac{\mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4} \ \oplus_{R_2} \quad \rightsquigarrow \quad \boxed{\bullet \mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3} \ \mathsf{fail}$$

• Case rule \oplus_{R_1}

$$\frac{\mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4} \ \oplus_{R_1} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3}}{\bullet \mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_3} \ ^{\mathsf{ax}}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3:\mathbf{1},\Delta_4 \vdash \mathbf{F}_1 \oplus \mathbf{F}_2} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_1}}{\bullet \mathbf{h}_3:\mathbf{1},\Delta_4 \vdash \mathbf{F}_1} \ \mathbf{1}_L$$

- Case rule \otimes_R
- \bullet Case rule W

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_1\oplus \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4,!\mathbf{F}_5\vdash \mathbf{F}_1\oplus \mathbf{F}_2} \ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_1} \ \ ^{\mathrm{ax/ind}}}{\bullet \mathbf{h}_3:\Delta_4,!\mathbf{F}_5\vdash \mathbf{F}_1} \ W$$

ullet Case rule C

 \bullet Case rule !L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\oplus\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\oplus\mathbf{F}_2}\text{ !L } \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1}\text{ ax/ind}$$

• Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3:\Delta_4,\mathtt{F}_6 \vdash \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_1 \oplus \mathtt{F}_2} \quad \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_3:\Delta_4,\mathtt{F}_6 \vdash \mathtt{F}_1}}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_1} \stackrel{\mathrm{ax/ind}}{} \&_{L2}$$

• Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3:\Delta_4,\mathtt{F}_5 \vdash \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_1 \oplus \mathtt{F}_2} \quad \&_{L1} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_3:\Delta_4,\mathtt{F}_5 \vdash \mathtt{F}_1}}{\bullet \mathtt{h}_3:\Delta_4,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_1} \stackrel{\mathrm{ax/ind}}{\&_{L1}}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_1\oplus\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_1\oplus\mathbf{f}_2} \ \otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_1}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_1} \overset{\mathrm{ax/ind}}{\otimes_L}$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1\oplus\mathbf{F}_2\quad\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1\oplus\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_1\oplus\mathbf{F}_2}\oplus_L\quad\quad\Leftrightarrow\quad\frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_5\vdash\mathbf{F}_1}\quad\text{ax/ind}\quad\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1}\quad\frac{\mathbf{ax/ind}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_6\vdash\mathbf{F}_1}\quad\oplus_L\quad\oplus_L$$

• Case rule \multimap_L

$$\begin{array}{l} \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6\quad \mathbf{h}_3:\Delta_5,\mathbf{F}_7\vdash \mathbf{F}_1\oplus \mathbf{F}_2} \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_1\oplus \mathbf{F}_2} \end{array} \ \, \\ \bullet \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6} \ \, \begin{array}{l} \mathrm{ax} \\ \hline \bullet \underline{\mathbf{h}_3:\Delta_5,\mathbf{F}_7\vdash \mathbf{F}_1} \end{array} \ \, \begin{array}{l} \mathrm{ax/ind} \\ \neg \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6} \end{array} \ \, \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_1} \end{array} \ \, \begin{array}{l} \mathrm{ax/ind} \\ \neg \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6} \end{array} \ \, \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_1} \end{array} \ \, \begin{array}{l} \mathrm{ax/ind} \\ \neg \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6} \end{array} \ \, \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_1} \end{array} \ \, \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_1} \end{array} \ \, \begin{array}{l} \mathrm{ax/ind} \\ \neg \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6} \end{array} \ \, \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_1} \end{array} \ \, \begin{array}{l} \mathrm{ax/ind} \\ \neg \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6} \end{array} \ \, \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_1} \end{array} \ \, \begin{array}{l} \mathrm{ax/ind} \\ \neg \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6} \end{array} \ \, \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6\multimap \mathbf{F}_7\vdash \mathbf{F}_1} \end{array} \ \, \begin{array}{l} \mathrm{ax/ind} \\ \neg \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6} \end{array} \ \, \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6} \longrightarrow \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6} \longrightarrow \underline{\mathbf{h}_3:\Delta_4} \longrightarrow \underline{\mathbf{h}_3:\Delta_4,\Delta_5,\mathbf{F}_6} \longrightarrow \underline{\mathbf{h}_3:\Delta_5,\mathbf{F}_6} \longrightarrow \underline{\mathbf{h}_3:\Delta_4} \longrightarrow \underline{\mathbf{h}_3:\Delta_5,\mathbf{F}_6} \longrightarrow \underline{\mathbf{h}_3:\Delta_5} \longrightarrow \underline{\mathbf{h}_3:\Delta_5}$$

ullet Case rule I

2.9 Status of 1_L : Invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

• Case rule $\&_R$

$$\frac{\mathbf{h}_2:\mathbf{1},\Delta_\mathbf{1}\vdash\mathbf{F}_3\quad\mathbf{h}_2:\mathbf{1},\Delta_\mathbf{1}\vdash\mathbf{F}_4}{\bullet\mathbf{h}_2:\mathbf{1},\Delta_\mathbf{1}\vdash\mathbf{F}_3\&\mathbf{F}_4}\quad\&_R\qquad\leadsto\qquad\frac{\overline{\mathbf{h}_2:\Delta_\mathbf{1}\vdash\mathbf{F}_3}\quad\overset{\mathrm{ax/ind}}{\bullet}\quad\overline{\mathbf{h}_2:\Delta_\mathbf{1}\vdash\mathbf{F}_4}\quad\overset{\mathrm{ax/ind}}{\&_R}\quad\&_R$$

• Case rule \multimap_R

$$\frac{\mathbf{h}_2:\mathbf{1},\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_2:\mathbf{1},\Delta_1\vdash\mathbf{F}_3\multimap\mathbf{F}_4}\ \neg\circ_R\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_2:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_4}\ \text{ax/ind}}{\bullet\mathbf{h}_2:\Delta_1\vdash\mathbf{F}_3\multimap\mathbf{F}_4}\stackrel{\text{ax/ind}}{\multimap_R}$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_2: \mathbf{1}, \Delta_1 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_2: \mathbf{1}, \Delta_1 \vdash \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_{R_2} \qquad \leadsto \qquad \frac{\mathbf{h}_2: \Delta_1 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_2: \Delta_1 \vdash \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_{R_2}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathbf{h}_2:\mathbf{1},\Delta_1 \vdash \mathbf{F}_3}{\bullet \mathbf{h}_2:\mathbf{1},\Delta_1 \vdash \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_{R_1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_1 \vdash \mathbf{F}_3} \ \text{ax/ind}}{\bullet \mathbf{h}_2:\Delta_1 \vdash \mathbf{F}_3 \oplus \mathbf{F}_4} \oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

• Case rule \otimes_R

 \bullet Case rule W

$$\frac{\mathbf{h}_1: \mathbf{1}, \Delta_4 \vdash \mathbf{F}_3}{\bullet \mathbf{h}_1: (\mathbf{1}, \Delta_4), !\mathbf{F}_2 \vdash \mathbf{F}_3} \quad W \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_4 \vdash \mathbf{F}_3} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_4, !\mathbf{F}_2 \vdash \mathbf{F}_3} \quad W$$

ullet Case rule C

$$\begin{array}{lll} \frac{\mathbf{h}_1:\mathbf{1},\Delta_4, \mathsf{!F}_2, \mathsf{!F}_2 \vdash \mathsf{F}_3}{\bullet \mathbf{h}_1:(\mathbf{1},\Delta_4), \mathsf{!F}_2 \vdash \mathsf{F}_3} & C & & \leadsto & & \frac{\mathbf{h}_1:\Delta_4, \mathsf{!F}_2, \mathsf{!F}_2 \vdash \mathsf{F}_3}{\bullet \mathbf{h}_1:\Delta_4, \mathsf{!F}_2 \vdash \mathsf{F}_3} & \frac{\mathsf{ax/ind}}{C} \end{array}$$

• Case rule !L

$$\frac{\mathtt{h}_1:\mathtt{1},\Delta_4,\mathtt{F}_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1:(\mathtt{1},\Delta_4),\mathtt{!F}_2 \vdash \mathtt{F}_3} \ !L \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_1:\Delta_4,\mathtt{F}_2 \vdash \mathtt{F}_3}}{\bullet \mathtt{h}_1:\Delta_4,\mathtt{!F}_2 \vdash \mathtt{F}_3} \ \frac{\mathtt{ax/ind}}{\mathtt{l}}$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_1:\mathbf{1},\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:(\mathbf{1},\Delta_5),\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_4} \quad\&_{L2}\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_4} \quad\&_{L2}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_1:\mathbf{1},\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:(\mathbf{1},\Delta_5),\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_4} \quad\&_{L1}\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4}}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_4} \quad\&_{L1}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_1: \mathbf{1}, \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1: (\mathbf{1}, \Delta_5), \mathbf{F}_2 \otimes \mathbf{F}_3 \vdash \mathbf{F}_4} \quad \otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \mathbf{F}_4}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3 \vdash \mathbf{F}_4} \overset{\mathsf{ax/ind}}{\otimes}_L$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_1:\mathbf{1},\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4\quad\mathbf{h}_1:\mathbf{1},\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:(\mathbf{1},\Delta_5),\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_4}\ \oplus_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4}\ \ \text{ax/ind}}{\bullet\mathbf{h}_1:\Delta_5,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_4}\ \ \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}{\oplus_L} \\ \oplus_L$$

• Case rule \multimap_L

$$\frac{\mathbf{h}_1: \mathbf{1}, \Delta_6 \vdash \mathbf{F}_3 \quad \mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: (\mathbf{1}, \Delta_6), \Delta_2, \mathbf{F}_3 \multimap \mathbf{F}_4 \vdash \mathbf{F}_5} \quad \multimap_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1: \Delta_6 \vdash \mathbf{F}_3}{\bullet \mathbf{h}_1: \Delta_2, \Delta_6, \mathbf{F}_3} \stackrel{\mathrm{ax/ind}}{\longrightarrow} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \Delta_2, \Delta_6, \mathbf{F}_3 \multimap \mathbf{F}_4 \vdash \mathbf{F}_5} \quad \multimap_L$$

$$\begin{array}{lll} \frac{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_3 & \mathbf{h}_1:\mathbf{1},\Delta_6,\mathbf{F}_4\vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2,(\mathbf{1},\Delta_6),\mathbf{F}_3\multimap \mathbf{F}_4\vdash \mathbf{F}_5} & \multimap_L & & \leadsto & & & & \frac{\overline{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_3} & \text{ax} & \overline{\mathbf{h}_1:\Delta_6,\mathbf{F}_4\vdash \mathbf{F}_5} & \text{ax/ind} \\ \bullet \mathbf{h}_1:\Delta_2,\Delta_6,\mathbf{F}_3\multimap \mathbf{F}_4\vdash \mathbf{F}_5} & & \multimap_L & & & & & & \\ \end{array}$$

 \bullet Case rule I

2.10 Status of \otimes_R : (Left Premise): Non invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_5 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_1,\Delta_5 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_5 \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_5 \vdash \mathbf{F}_2} \ \mathbf{1}_L$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_5\vdash \mathbf{F}_2\otimes \mathbf{F}_3}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_1,\Delta_5\vdash \mathbf{F}_2\otimes \mathbf{F}_3} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash \mathbf{F}_2}}{\bullet \mathbf{h}_4:\Delta_1\vdash \mathbf{F}_2} \ ^{\mathrm{ax/ind}}_{\mathrm{H}}$$

• Case rule \otimes_R

$$\begin{array}{cccc} \mathbf{h}_1:\Delta_4,\Delta_5 \vdash \mathbf{F}_2 & \mathbf{h}_1:\Delta_6,\Delta_7 \vdash \mathbf{F}_3 \\ \bullet \mathbf{h}_1:(\Delta_4,\Delta_5),\Delta_6,\Delta_7 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3 \end{array} \ \otimes_R \qquad \leadsto \qquad \overline{\bullet \mathbf{h}_1:\Delta_4,\Delta_6 \vdash \mathbf{F}_2} \quad \text{fail}$$

 \bullet Case rule W

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_6), !\mathbf{F}_5 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3} \quad W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_6 \vdash \mathbf{F}_2} \quad ^{\mathrm{ax/ind}}}{\bullet \mathbf{h}_4:\Delta_6, !\mathbf{F}_5 \vdash \mathbf{F}_2} \quad W$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6\vdash \mathbf{F}_2\otimes \mathbf{F}_3}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_6),!\mathbf{F}_5\vdash \mathbf{F}_2\otimes \mathbf{F}_3} \quad W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash \mathbf{F}_2}}{\bullet \mathbf{h}_4:\Delta_1\vdash \mathbf{F}_2} \quad \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

 $\bullet\,$ Case rule C

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6, \mathbb{IF}_5, \mathbb{IF}_5 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_6), \mathbb{IF}_5 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3} \quad C \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_6 \vdash \mathbf{F}_2} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_4:\Delta_6, \mathbb{IF}_5 \vdash \mathbf{F}_2} \frac{W}{W}$$

$$\begin{array}{lll} \frac{\mathbf{h}_4:\Delta_1,\Delta_6, \mathbb{IF}_5, \mathbb{IF}_5 \vdash \mathbb{F}_2 \otimes \mathbb{F}_3}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_6), \mathbb{IF}_5 \vdash \mathbb{F}_2 \otimes \mathbb{F}_3} & C & & \leadsto & & \frac{\overline{\mathbf{h}_4:\Delta_1 \vdash \mathbb{F}_2}}{\bullet \mathbf{h}_4:\Delta_1 \vdash \mathbb{F}_2} & \frac{\mathbf{ax/ind}}{\bullet} \end{array}$$

• Case rule !L

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_6),\mathbf{!F}_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3} \ !L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_6\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_4:\Delta_6,\mathbf{!F}_5\vdash\mathbf{F}_2} \frac{\mathbf{ax/ind}}{W}$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3} \quad\&_{L2}\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_4:\Delta_7,\mathbf{F}_6\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_4:\Delta_7,\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_2} \stackrel{\mathrm{ax/ind}}{\&_{L2}}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3} \ \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2} \ ^{\mathrm{ax/ind}}_{\mathrm{H}}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{f}_5\vdash\mathbf{f}_2\otimes\mathbf{f}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_2\otimes\mathbf{f}_3} \ \&_{L1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7,\mathbf{f}_5\vdash\mathbf{f}_2}}{\bullet\mathbf{h}_4:\Delta_7,\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_2} \ ^{\mathrm{ax/ind}}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_2\otimes\mathbf{f}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_2\otimes\mathbf{f}_3} \ \otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7,\mathbf{f}_5,\mathbf{f}_6\vdash\mathbf{f}_2}}{\bullet\mathbf{h}_4:\Delta_7,\mathbf{f}_5\otimes\mathbf{f}_6\vdash\mathbf{f}_2} \overset{\mathrm{ax/ind}}{\otimes_L}$$

• Case rule \oplus_L

$$\begin{array}{c} \mathbf{h}_4: \Delta_1, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3 \quad \mathbf{h}_4: \Delta_1, \Delta_7, \mathbf{F}_6 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3 \\ \bullet \mathbf{h}_4: (\Delta_1, \Delta_7), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3 \end{array} \\ \oplus \mathbf{h}_4: \Delta_7, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_2 \end{array} \\ \begin{array}{c} \bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_2 & \text{ax/ind} \\ \bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_2 & \text{ax/ind} \\ \bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_2 & \text{ax/ind} \\ \end{array} \\ \begin{array}{c} \bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_2 & \text{ax/ind} \\ \bullet \mathbf{h}_4: \Delta_7, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_2 & \text{ax/ind} \\ \end{array} \\ \begin{array}{c} \bullet \mathbf{h}_4: \Delta_7, \mathbf{h}_5 \oplus \mathbf{h}_6 \vdash \mathbf{h}_2 & \text{ax/ind} \\ \bullet \mathbf{h}_4: \Delta_7, \mathbf{h}_5 \oplus \mathbf{h}_6 \vdash \mathbf{h}_2 & \text{ax/ind} \\ \end{array} \\ \begin{array}{c} \bullet \mathbf{h}_4: \Delta_7, \mathbf{h}_5 \oplus \mathbf{h}_6 \vdash \mathbf{h}_2 & \text{ax/ind} \\ \bullet \mathbf{h}_4: \Delta_7, \mathbf{h}_5 \oplus \mathbf{h}_6 \vdash \mathbf{h}_2 & \text{ax/ind} \\ \bullet \mathbf{h}_4: \Delta_7, \mathbf{h}_5 \oplus \mathbf{h}_6 \vdash \mathbf{h}_2 & \text{ax/ind} \\ \end{array} \\ \begin{array}{c} \bullet \mathbf{h}_4: \Delta_7, \mathbf{h}_5 \oplus \mathbf{h}_6 \vdash \mathbf{h}_2 & \text{ax/ind} \\ \bullet \mathbf{h}_4: \Delta_7, \mathbf{h}_5 \oplus \mathbf{h}_6 \vdash \mathbf{h}_2 & \text{ax/ind} \\ \bullet \mathbf{h}_4: \Delta_7, \mathbf{h}_5 \oplus \mathbf{h}_6 \vdash \mathbf{h}_2 & \text{ax/ind} \\ \end{array}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3\quad\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3}\ \oplus_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2}\ _{\mathbf{H}}^{\mathbf{ax/ind}}$$

• Case rule \multimap_L

$$\begin{array}{lll} \underline{\mathbf{h}}_3: \Delta_6, \Delta_7 \vdash \mathbf{F}_4 & \underline{\mathbf{h}}_3: \Delta_8, \Delta_9, \mathbf{F}_5 \vdash \mathbf{F}_1 \otimes \mathbf{F}_2 \\ \bullet \mathbf{h}_3: (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_1 \otimes \mathbf{F}_2 \end{array} \quad \multimap_L \qquad \leadsto \qquad \begin{array}{ll} \bullet \mathbf{h}_3: \Delta_6, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_1 \\ \bullet \mathbf{h}_3: \Delta_6, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_1 \end{array} \quad \text{fail}$$

$$\begin{array}{lll} & \mathbf{h}_3: \Delta_6, \Delta_7 \vdash \mathbf{F}_4 & \mathbf{h}_3: \Delta_8, \Delta_9, \mathbf{F}_5 \vdash \mathbf{F}_1 \otimes \mathbf{F}_2 \\ & \bullet \mathbf{h}_3: (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_1 \otimes \mathbf{F}_2 \\ \end{array} \\ & \bullet \mathbf{h}_3: \Delta_6, \Delta_8 \vdash \mathbf{F}_1 \end{array} \quad \text{fail}$$

 \bullet Case rule I

2.11 Status of \otimes_R (Right Premise): : Non invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_5\vdash \mathbf{F}_2\otimes \mathbf{F}_3}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_1,\Delta_5\vdash \mathbf{F}_2\otimes \mathbf{F}_3} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4:\Delta_1\vdash \mathbf{F}_3} \ ^{\mathrm{ax/ind}}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:\mathbf{1},\Delta_1,\Delta_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_5\vdash\mathbf{F}_3}}{\bullet\mathbf{h}_4:\mathbf{1},\Delta_5\vdash\mathbf{F}_3} \ \mathbf{1}_L$$

• Case rule \otimes_R

$$\begin{array}{ll} \frac{\mathbf{h}_1:\Delta_4,\Delta_5 \vdash \mathbf{F}_2 \quad \mathbf{h}_1:\Delta_6,\Delta_7 \vdash \mathbf{F}_3}{\bullet \mathbf{h}_1:(\Delta_4,\Delta_5),\Delta_6,\Delta_7 \vdash \mathbf{F}_2 \otimes \mathbf{F}_3} \quad \otimes_R \qquad \leadsto \qquad \boxed{\bullet \mathbf{h}_1:\Delta_5,\Delta_7 \vdash \mathbf{F}_3} \quad \text{fail} \end{array}$$

ullet Case rule W

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6\vdash \mathbf{F}_2\otimes \mathbf{F}_3}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_6), !\mathbf{F}_5\vdash \mathbf{F}_2\otimes \mathbf{F}_3} \ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4:\Delta_1\vdash \mathbf{F}_3} \ \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6\vdash \mathbf{F}_2\otimes \mathbf{F}_3}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_6),!\mathbf{F}_5\vdash \mathbf{F}_2\otimes \mathbf{F}_3} \quad W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_6\vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4:\Delta_6,!\mathbf{F}_5\vdash \mathbf{F}_3} \overset{\mathrm{ax/ind}}{W}$$

 \bullet Case rule C

$$\begin{array}{lll} \frac{\mathbf{h}_4:\Delta_1,\Delta_6, \mathbb{IF}_5, \mathbb{IF}_5 \vdash \mathbb{F}_2 \otimes \mathbb{F}_3}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_6), \mathbb{IF}_5 \vdash \mathbb{F}_2 \otimes \mathbb{F}_3} & C & & \leadsto & & \frac{\overline{\mathbf{h}_4:\Delta_1 \vdash \mathbb{F}_3}}{\bullet \mathbf{h}_4:\Delta_1 \vdash \mathbb{F}_3} & \frac{\mathsf{ax/ind}}{\mathsf{I}} \end{array}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6,!\mathbf{F}_5,!\mathbf{F}_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_6),!\mathbf{F}_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3}\ C\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_4:\Delta_6\vdash\mathbf{F}_3}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_4:\Delta_6,!\mathbf{F}_5\vdash\mathbf{F}_3}\ W$$

• Case rule !L

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6,\mathbf{F}_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_6),\mathbf{!F}_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3} \ \mathbf{!}L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_6\vdash\mathbf{F}_3} \ \mathbf{ax/ind}}{\bullet\mathbf{h}_4:\Delta_6,\mathbf{!F}_5\vdash\mathbf{F}_3} \ W$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3} \ \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_3}}{\bullet\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_3} \ ^{\mathrm{ax/ind}}_{\mathrm{H}}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{f}_6\vdash\mathbf{f}_2\otimes\mathbf{f}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_2\otimes\mathbf{f}_3} \ \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7,\mathbf{f}_6\vdash\mathbf{f}_3}}{\bullet\mathbf{h}_4:\Delta_7,\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_3} \overset{\mathrm{ax/ind}}{\bullet}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3} \ \&_{L1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_3}}{\bullet\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_3} \ \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{f}_5\vdash\mathbf{f}_2\otimes\mathbf{f}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_2\otimes\mathbf{f}_3} \ \&_{L1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7,\mathbf{f}_5\vdash\mathbf{f}_3}}{\bullet\mathbf{h}_4:\Delta_7,\mathbf{f}_5\&\mathbf{f}_6\vdash\mathbf{f}_3} \ \&_{L1}$$

• Case rule \otimes_L

$$\frac{\mathtt{h}_4:\Delta_1,\Delta_7,\mathtt{F}_5,\mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4:(\Delta_1,\Delta_7),\mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3} \ \otimes_L \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_4:\Delta_1 \vdash \mathtt{F}_3}}{\bullet \mathtt{h}_4:\Delta_1 \vdash \mathtt{F}_3} \ \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathtt{h}_4: \Delta_1, \Delta_7, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4: (\Delta_1, \Delta_7), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3} \quad \otimes_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_4: \Delta_7, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_3}}{\bullet \mathtt{h}_4: \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_3} \overset{\mathtt{ax/ind}}{\otimes}_L$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_5\vdash\mathbf{F}_2\otimes\mathbf{F}_3\quad\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7),\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_2\otimes\mathbf{F}_3}\oplus_L\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_4:\Delta_7,\mathbf{F}_5\vdash\mathbf{F}_3}\quad \text{ax/ind}}{\bullet\mathbf{h}_4:\Delta_7,\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_3}\quad \frac{\mathbf{ax/ind}}{\oplus_L}$$

• Case rule \multimap_L

$$\begin{array}{l} \underline{\mathbf{h}}_3: \Delta_6, \Delta_7 \vdash \mathbf{F}_4 \quad \underline{\mathbf{h}}_3: \Delta_8, \Delta_9, \mathbf{F}_5 \vdash \mathbf{F}_1 \otimes \mathbf{F}_2 \\ \bullet \mathbf{h}_3: (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_1 \otimes \mathbf{F}_2 \end{array} \multimap_L \qquad \leadsto \qquad \overline{\bullet \mathbf{h}}_3: \Delta_7, \Delta_9 \vdash \mathbf{F}_2 \end{array} \quad \mathbf{fail}$$

$$\begin{array}{l} \mathbf{h}_3: \Delta_6, \Delta_7 \vdash \mathbf{F}_4 \quad \mathbf{h}_3: \Delta_8, \Delta_9, \mathbf{F}_5 \vdash \mathbf{F}_1 \otimes \mathbf{F}_2 \\ \bullet \mathbf{h}_3: (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_1 \otimes \mathbf{F}_2 \end{array} \quad \multimap_L \qquad \leadsto \qquad \frac{\bullet \mathbf{h}_3: \Delta_7, \Delta_9, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \Delta_9, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_2} \quad \mathbf{fail}$$

 \bullet Case rule I

2.12 Status of W: Non invertible

• Case rule !R

$$\frac{h_3: !\Upsilon\mathbf{1}, !F_2 \vdash F_4}{\bullet h_3: !\Upsilon\mathbf{1}, !F_2 \vdash !F_4} \ \, !R \qquad \rightsquigarrow \qquad \frac{\overline{h_3: !\Upsilon\mathbf{1} \vdash F_4}}{\bullet h_3: !\Upsilon\mathbf{1} \vdash !F_4} \ \, \frac{ax/ind}{!R}$$

- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{}{\bullet \mathbf{h}_3:\Delta_{\mathbf{1}},!\mathbf{F}_2\vdash \top} \quad \top \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_{\mathbf{1}}\vdash \top} \quad \top$$

• Case rule $\&_R$

$$\frac{\mathbf{h}_3:\Delta_1, \mathbf{l}_{F_2} \vdash \mathbf{F}_4 \quad \mathbf{h}_3:\Delta_1, \mathbf{l}_{F_2} \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3:\Delta_1, \mathbf{l}_{F_2} \vdash \mathbf{F}_4 \& \mathbf{F}_5} \quad \&_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_1 \vdash \mathbf{F}_4} \quad \text{ax/ind} \quad \overline{\mathbf{h}_3:\Delta_1 \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_3:\Delta_1 \vdash \mathbf{F}_4 \& \mathbf{F}_5} \quad \frac{\text{ax/ind}}{\&_R}$$

• Case rule \multimap_R

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{F}_4,\mathbf{i}\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{i}\mathbf{F}_2\vdash\mathbf{F}_4\multimap\mathbf{F}_5} \multimap_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_1,\mathbf{F}_4\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_3:\Delta_1\vdash\mathbf{F}_4\multimap\mathbf{F}_5} \stackrel{\mathrm{ax/ind}}{\multimap_R}$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{l}_{F_2}\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{l}_{F_2}\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_1\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_3:\Delta_1\vdash\mathbf{F}_4\oplus\mathbf{F}_5} \stackrel{\mathrm{ax/ind}}{\oplus_{R_2}}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{!F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{!F}_2\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_1\vdash\mathbf{F}_4}\ \text{ax/ind}}{\bullet\mathbf{h}_3:\Delta_1\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

• Case rule \otimes_R

$$\frac{\mathbf{h}_2:\Delta_3\vdash \mathbf{F}_4\quad \mathbf{h}_2:\Delta_6, \mathbf{!F_1}\vdash \mathbf{F}_5}{\bullet \mathbf{h}_2:\Delta_3,\Delta_6, \mathbf{!F_1}\vdash \mathbf{F}_4\otimes \mathbf{F}_5} \quad \otimes_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_2:\Delta_3\vdash \mathbf{F}_4}\quad \text{ax}\quad \overline{\mathbf{h}_2:\Delta_6\vdash \mathbf{F}_5}}{\bullet \mathbf{h}_2:\Delta_3,\Delta_6\vdash \mathbf{F}_4\otimes \mathbf{F}_5}\quad \frac{\text{av/ind}}{\otimes_R}$$

 \bullet Case rule W

$$\frac{\mathbf{h}_2:\Delta_5,\mathbf{!F_1}\vdash\mathbf{F}_4}{\bullet\mathbf{h}_2:(\Delta_5,\mathbf{!F_1}),\mathbf{!F}_3\vdash\mathbf{F}_4}\ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_5\vdash\mathbf{F}_4}\ \mathbf{ax/ind}}{\bullet\mathbf{h}_2:\Delta_5,\mathbf{!F}_3\vdash\mathbf{F}_4}\ W$$

$$\frac{\mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1:\Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4} \ W \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_4}}{\bullet \mathtt{h}_1:\Delta_2 \vdash \mathtt{F}_4} \ _{\mathtt{H}}^{\mathtt{ax}}$$

 \bullet Case rule C

$$\frac{\mathbf{h}_2:\Delta_5,\mathbf{l}_{F_1},\mathbf{l}_{F_3},\mathbf{l}_{F_3}\vdash \mathbf{F}_4}{\bullet \mathbf{h}_2:(\Delta_5,\mathbf{l}_{F_1}),\mathbf{l}_{F_3}\vdash \mathbf{F}_4} \ C \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_2:\Delta_5,\mathbf{l}_{F_3},\mathbf{l}_{F_3}\vdash \mathbf{F}_4}}{\bullet \mathbf{h}_2:\Delta_5,\mathbf{l}_{F_3}\vdash \mathbf{F}_4} \ \frac{\mathbf{ax/ind}}{C}$$

$$\begin{array}{ccc} \mathbf{h}_1:\Delta_2, \mathbf{l}\mathbf{F}_3, \mathbf{l}\mathbf{F}_3 \vdash \mathbf{F}_4 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{l}\mathbf{F}_3 \vdash \mathbf{F}_4 \end{array} \quad C \qquad \leadsto \qquad \overline{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4} \quad \mathbf{fail} \\ \end{array}$$

• Case rule !L

$$\frac{\mathbf{h}_2:\Delta_5,\mathbf{F}_3,\mathbf{i}\mathbf{F}_1\vdash\mathbf{F}_4}{\bullet\mathbf{h}_2:(\Delta_5,\mathbf{i}\mathbf{F}_1),\mathbf{i}\mathbf{F}_3\vdash\mathbf{F}_4}\ \mathbf{!}L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_5,\mathbf{F}_3\vdash\mathbf{F}_4}}{\bullet\mathbf{h}_2:\Delta_5,\mathbf{i}\mathbf{F}_3\vdash\mathbf{F}_4} \overset{\mathrm{ax/ind}}{\vdash}L$$

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_4} \ !L \qquad \rightsquigarrow \qquad \overline{\bullet\mathbf{h}_1:\Delta_2\vdash\mathbf{F}_4} \ \mathbf{fail}$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_2 : \Delta_6, \mathbf{F}_4, \mathbf{!F}_1 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : (\Delta_6, \mathbf{!F}_1), \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} \quad \&_{L2} \qquad \rightsquigarrow \qquad \frac{\frac{\mathbf{h}_2 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5}} \overset{\mathrm{ax/ind}}{\bullet} \\ \frac{\mathbf{h}_2 : \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{h}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{h}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5}{\bullet \mathbf{h}_4 \vdash \mathbf{h}_5} & & \\ \frac{\mathbf{h}_3 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5}{\bullet \mathbf{h}_4} & & \\ \frac{\mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5}{\bullet \mathbf{h}_4} & & \\ \frac{\mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5}{\bullet \mathbf{h}_4} & & \\ \frac{\mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5}{\bullet \mathbf{h}_4} & & \\ \frac{\mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5}{\bullet \mathbf{h}_4} &$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_2 : \Delta_6, \mathbf{F}_3, \mathbf{!F}_1 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : (\Delta_6, \mathbf{!F}_1), \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} \quad \&_{L1} \qquad \rightsquigarrow \qquad \frac{\mathbf{h}_2 : \Delta_6, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} \quad \&_{L1}$$

• Case rule \otimes_L

$$\frac{\mathtt{h}_2:\Delta_6,\mathtt{F}_3,\mathtt{F}_4,\mathtt{lF}_1 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2:(\Delta_6,\mathtt{lF}_1),\mathtt{F}_3\otimes \mathtt{F}_4 \vdash \mathtt{F}_5} \otimes_L \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_2:\Delta_6,\mathtt{F}_3,\mathtt{F}_4 \vdash \mathtt{F}_5}}{\bullet \mathtt{h}_2:\Delta_6,\mathtt{F}_3\otimes \mathtt{F}_4 \vdash \mathtt{F}_5} \overset{\mathrm{av}/\operatorname{ind}}{\otimes}_L$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_2:\Delta_6,\mathbf{F}_3,\mathbf{!F_1}\vdash\mathbf{F}_5\quad\mathbf{h}_2:\Delta_6,\mathbf{F}_4,\mathbf{!F_1}\vdash\mathbf{F}_5}{\bullet\mathbf{h}_2:(\Delta_6,\mathbf{!F_1}),\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_5}\oplus L\qquad \leadsto\qquad \frac{\mathbf{h}_2:\Delta_6,\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_5} \xrightarrow{\mathrm{ax/ind}} \frac{\mathbf{h}_2:\Delta_6,\mathbf{F}_4\vdash\mathbf{F}_5}{\oplus L} \xrightarrow{\bullet} L$$

• Case rule \multimap_L

$$\frac{\mathbf{h}_2:\Delta_7,\mathbf{lF}_1\vdash \mathbf{F}_4\quad \mathbf{h}_2:\Delta_3,\mathbf{F}_5\vdash \mathbf{F}_6}{\bullet \mathbf{h}_2:(\Delta_7,\mathbf{lF}_1),\Delta_3,\mathbf{F}_4\multimap \mathbf{F}_5\vdash \mathbf{F}_6}\quad \circ_L\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_2:\Delta_7\vdash \mathbf{F}_4}\quad \mathrm{ax/ind}\quad \overline{\mathbf{h}_2:\Delta_3,\mathbf{F}_5\vdash \mathbf{F}_6}}{\bullet \mathbf{h}_2:\Delta_3,\Delta_7,\mathbf{F}_4\multimap \mathbf{F}_5\vdash \mathbf{F}_6}\quad \circ_L$$

$$\begin{array}{l} \mathbf{h}_2: \Delta_3 \vdash \mathbf{F}_4 \quad \mathbf{h}_2: \Delta_7, \mathbf{F}_5, \mathbf{IF}_1 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_2: \Delta_3, (\Delta_7, \mathbf{IF}_1), \mathbf{F}_4 \vdash \mathbf{o} \mathbf{F}_5 \vdash \mathbf{F}_6 \end{array} \quad \circ_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2: \Delta_3 \vdash \mathbf{F}_4} \quad \mathbf{ax} \quad \overline{\mathbf{h}_2: \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6} \\ \bullet \mathbf{h}_2: \Delta_3, (\Delta_7, \mathbf{F}_4 \vdash \mathbf{o} \mathbf{F}_5 \vdash \mathbf{F}_6 \end{array} \quad \circ_L \quad \Longrightarrow \quad \frac{\overline{\mathbf{h}_2: \Delta_3 \vdash \mathbf{F}_4} \quad \mathbf{ax} \quad \overline{\mathbf{h}_2: \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6} \\ \bullet \mathbf{h}_2: \Delta_3, (\Delta_7, \mathbf{F}_4 \vdash \mathbf{o} \mathbf{F}_5 \vdash \mathbf{F}_6) \quad \circ_L \quad \Longrightarrow \quad \frac{\overline{\mathbf{h}_2: \Delta_3 \vdash \mathbf{F}_4} \quad \mathbf{ax} \quad \overline{\mathbf{h}_2: \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6} \\ \bullet \mathbf{h}_2: \Delta_3, (\Delta_7, \mathbf{F}_4 \vdash \mathbf{o} \mathbf{F}_5 \vdash \mathbf{F}_6) \quad \bullet_L \quad \Longrightarrow \quad \frac{\overline{\mathbf{h}_2: \Delta_3 \vdash \mathbf{F}_4} \quad \overline{\mathbf{h}_2: \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6} \\ \bullet \mathbf{h}_2: \Delta_3, (\Delta_7, \mathbf{h}_4 \vdash \mathbf{h}_2) \quad \bullet_L \quad \Longrightarrow \quad \frac{\overline{\mathbf{h}_2: \Delta_3 \vdash \mathbf{F}_4} \quad \overline{\mathbf{h}_2: \Delta_7, \mathbf{h}_4} \quad \overline{\mathbf{h}_2: \Delta_7, \mathbf{h}_5 \vdash \mathbf{F}_6} \\ \bullet \mathbf{h}_2: \Delta_3 \mid \Delta_7, \mathbf{h}_4 \mid \Delta_7, \mathbf{$$

ullet Case rule I

2.13 Status of C: Non invertible

• Case rule !R

$$\frac{\mathbf{h}_3: !\Upsilon\mathbf{1}, !\mathbf{F}_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_3: !\Upsilon\mathbf{1}, !\mathbf{F}_2 \vdash !\mathbf{F}_4} \ !R \qquad \leadsto \qquad \frac{\mathbf{h}_3: !\Upsilon\mathbf{1}, !\mathbf{F}_2, !\mathbf{F}_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_3: !\Upsilon\mathbf{1}, !\mathbf{F}_2, !\mathbf{F}_2 \vdash !\mathbf{F}_4} \ !R$$

- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\frac{}{\bullet \mathbf{h}_3:\Delta_1, \mathbf{IF}_2 \vdash \top} \ \top \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_1, \mathbf{IF}_2, \mathbf{IF}_2 \vdash \top} \ \top$$

• Case rule $\&_R$

$$\frac{\mathbf{a}_3:\Delta_1, \mathbf{i}_{\mathbf{F}_2} \vdash \mathbf{F}_4 \quad \mathbf{a}_3:\Delta_1, \mathbf{i}_{\mathbf{F}_2} \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3:\Delta_1, \mathbf{i}_{\mathbf{F}_2} \vdash \mathbf{F}_4 \& \mathbf{F}_5} \quad \&_R \qquad \leadsto \qquad \frac{\overline{\mathbf{a}_3:\Delta_1, \mathbf{i}_{\mathbf{F}_2}, \mathbf{i}_{\mathbf{F}_2} \vdash \mathbf{F}_4} \quad \text{ax/ind}}{\bullet \mathbf{a}_3:\Delta_1, \mathbf{i}_{\mathbf{F}_2}, \mathbf{i}_{\mathbf{F}_2} \vdash \mathbf{F}_5} \frac{\mathbf{ax/ind}}{\&_R} \quad \&_R$$

• Case rule \multimap_R

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{F}_4,\mathbf{!F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{!F}_2\vdash\mathbf{F}_4-\circ\mathbf{F}_5} \ \, \circ_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_1,\mathbf{F}_4,\mathbf{!F}_2,\mathbf{!F}_2\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{!F}_2,\mathbf{!F}_2\vdash\mathbf{F}_4-\circ\mathbf{F}_5} \ \, \overset{\mathrm{ax/ind}}{-\circ_R}$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{l}\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{l}\mathbf{F}_2\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_2} \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_1,\mathbf{l}\mathbf{F}_2,\mathbf{l}\mathbf{F}_2\vdash\mathbf{F}_5}\ ^\mathrm{ax/ind}}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{l}\mathbf{F}_2,\mathbf{l}\mathbf{F}_2\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_2}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{l}_{\mathsf{F}_2}\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{l}_{\mathsf{F}_2}\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_1}\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_3:\Delta_1,\mathbf{l}_{\mathsf{F}_2},\mathbf{l}_{\mathsf{F}_2}\vdash\mathbf{F}_4}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{l}_{\mathsf{F}_2},\mathbf{l}_{\mathsf{F}_2}\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_2:\Delta_4, \mathbf{!F_1} \vdash \mathbf{F_3}}{\bullet \mathbf{h}_2:\mathbf{1},\Delta_4, \mathbf{!F_1} \vdash \mathbf{F_3}} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_4, \mathbf{!F_1}, \mathbf{!F_1} \vdash \mathbf{F_3}}}{\bullet \mathbf{h}_2:\mathbf{1},\Delta_4, \mathbf{!F_1}, \mathbf{!F_1} \vdash \mathbf{F_3}} \ \mathbf{1}_L$$

• Case rule \otimes_R

 \bullet Case rule W

$$\frac{\mathbf{h}_2:\Delta_5, \mathbf{lF_1} \vdash \mathbf{F}_4}{\bullet \mathbf{h}_2:(\Delta_5, \mathbf{lF_1}), \mathbf{lF_3} \vdash \mathbf{F}_4} \quad W \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_2:\Delta_5, \mathbf{lF_1}, \mathbf{lF_1} \vdash \mathbf{F}_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_2:\Delta_5, \mathbf{lF_1}, \mathbf{lF_1}, \mathbf{lF_3} \vdash \mathbf{F}_4} \quad W$$

$$\begin{array}{ccc} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \\ \bullet \mathbf{h}_1: \Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_4 \end{array} \ W \qquad \leadsto \qquad \begin{array}{c} \bullet \mathbf{h}_1: \Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_4 \end{array} \ \mathbf{fail} \end{array}$$

 \bullet Case rule C

$$\begin{array}{lll} \frac{\mathbf{h}_2:\Delta_5, \mathbb{IF}_1, \mathbb{IF}_3, \mathbb{IF}_3 \vdash \mathbb{F}_4}{\bullet \mathbf{h}_2:(\Delta_5, \mathbb{IF}_1), \mathbb{IF}_3 \vdash \mathbb{F}_4} & C & & & & \frac{\mathbf{h}_2:\Delta_5, \mathbb{IF}_1, \mathbb{IF}_1, \mathbb{IF}_3, \mathbb{IF}_3 \vdash \mathbb{F}_4}{\bullet \mathbf{h}_2:\Delta_5, \mathbb{IF}_1, \mathbb{IF}_1, \mathbb{IF}_3 \vdash \mathbb{F}_4} & & & & C \\ \end{array} \\ \end{array}$$

$$\begin{array}{cccc} \frac{\mathbf{h}_1:\Delta_2, \mathbb{IF}_3, \mathbb{IF}_3 \vdash \mathbb{F}_4}{\bullet \mathbf{h}_1:\Delta_2, \mathbb{IF}_3 \vdash \mathbb{F}_4} & C & & & & & & & \frac{\mathbf{h}_1:\Delta_2, \mathbb{IF}_3, \mathbb{IF}_3 \vdash \mathbb{F}_4}{\bullet \mathbf{h}_1:\Delta_2, \mathbb{IF}_3, \mathbb{IF}_3 \vdash \mathbb{F}_4} & & & & & & & \\ \end{array} \begin{array}{c} \mathbf{ax/ind} \\ \bullet \mathbf{h}_1:\Delta_2, \mathbb{IF}_3, \mathbb{IF}_3 \vdash \mathbb{F}_4 & & & & \\ \end{array} \end{array}$$

• Case rule !L

$$\frac{\mathbf{h}_2: \Delta_5, \mathbf{F}_3, \mathbf{!F_1} \vdash \mathbf{F}_4}{\bullet \mathbf{h}_2: (\Delta_5, \mathbf{!F_1}), \mathbf{!F_3} \vdash \mathbf{F}_4} \ \mathbf{I}L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_2: \Delta_5, \mathbf{F}_3, \mathbf{!F_1}, \mathbf{!F_1} \vdash \mathbf{F}_4}}{\bullet \mathbf{h}_2: \Delta_5, \mathbf{!F_1}, \mathbf{!F_1}, \mathbf{!F_3} \vdash \mathbf{F}_4} \\ \mathbf{h}_2: \Delta_5, \mathbf{H}_1, \mathbf{H}_1, \mathbf{H}_2 \vdash \mathbf{H}_3 \\ \mathbf{h}_2: \Delta_5, \mathbf{H}_3, \mathbf{H}_4 \vdash \mathbf{H}_4 \\ \mathbf{h}_2: \Delta_5, \mathbf{H}_3, \mathbf{H}_4 \vdash \mathbf{H}_4 \\ \mathbf{h}_2: \Delta_5, \mathbf{H}_3, \mathbf{H}_4 \vdash \mathbf{H}_4 \\ \mathbf{h}_3: \Delta_5, \mathbf{H}_4 \vdash \mathbf{H}_4 \\ \mathbf{h}_3: \Delta_5, \mathbf{H}_4 \vdash \mathbf{H}_4 \\ \mathbf{h}_4: \Delta_5, \mathbf{H}_5 \vdash \mathbf{H}_4 \\ \mathbf{h}_5: \Delta_5, \mathbf{H}_5 \vdash \mathbf{H}_5 \\ \mathbf$$

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_4}{\bullet\mathbf{h}_1:\Delta_2,!\mathbf{F}_3\vdash\mathbf{F}_4} \ !L \qquad \leadsto \qquad \overline{\bullet\mathbf{h}_1:\Delta_2,!\mathbf{F}_3,!\mathbf{F}_3\vdash\mathbf{F}_4} \ \text{fail}$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_2 : \Delta_6, \mathbf{F}_4, \mathbf{l}_{\mathbf{F}_1} \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : (\Delta_6, \mathbf{l}_{\mathbf{F}_1}), \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} \quad \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2 : \Delta_6, \mathbf{F}_4, \mathbf{l}_{\mathbf{F}_1}, \mathbf{l}_{\mathbf{F}_1} \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_2 : \Delta_6, \mathbf{l}_{\mathbf{F}_1}, \mathbf{l}_{\mathbf{F}_1}, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5}} \overset{\mathrm{ax/ind}}{\bullet} \\$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{a}_2:\Delta_6,\mathbf{F}_3,\mathbf{!F}_1\vdash\mathbf{F}_5}{\bullet\mathbf{h}_2:(\Delta_6,\mathbf{!F}_1),\mathbf{F}_3\&\mathbf{F}_4\vdash\mathbf{F}_5} \quad\&_{L1}\qquad \leadsto\qquad \frac{\overline{\mathbf{a}_2:\Delta_6,\mathbf{F}_3,\mathbf{!F}_1,\mathbf{!F}_1\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_2:\Delta_6,\mathbf{!F}_1,\mathbf{!F}_1,\mathbf{F}_3\&\mathbf{F}_4\vdash\mathbf{F}_5} \quad\&_{L1}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_2:\Delta_6,\mathbf{F}_3,\mathbf{F}_4,\mathbf{!F_1}\vdash\mathbf{F}_5}{\bullet\mathbf{h}_2:(\Delta_6,\mathbf{!F_1}),\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_5}\otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_6,\mathbf{F}_3,\mathbf{F}_4,\mathbf{!F_1}\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_2:\Delta_6,\mathbf{!F_1},\mathbf{!F_1},\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_5} \overset{\mathsf{av}/\mathsf{ind}}{\otimes}_L$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_2:\Delta_6,\mathbf{F}_3,!\mathbf{F}_1\vdash\mathbf{F}_5\quad\mathbf{h}_2:\Delta_6,\mathbf{F}_4,!\mathbf{F}_1\vdash\mathbf{F}_5}{\bullet\mathbf{h}_2:(\Delta_6,!\mathbf{F}_1),\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_5}\ \oplus_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_6,\mathbf{F}_3,!\mathbf{F}_1,!\mathbf{F}_1\vdash\mathbf{F}_5}\quad \text{ax/ind}}{\bullet\mathbf{h}_2:\Delta_6,!\mathbf{F}_1,!\mathbf{F}_1\vdash\mathbf{F}_5} \xrightarrow[\bullet 1]{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_4,!\mathbf{F}_1,!\mathbf{F}_1\vdash\mathbf{F}_5} \xrightarrow[\bullet 1]{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_4,!\mathbf{F}_1,\mathbf{F}_1\vdash\mathbf{F}_5} \xrightarrow[\bullet 1]{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_4,!\mathbf{F}_1,\mathbf{F}_1\vdash\mathbf{F}_5} \xrightarrow[\bullet 1]{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_4,!\mathbf{F}_1,\mathbf{F}_1\vdash\mathbf{F}_5} \xrightarrow[\bullet 1]{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_4,!\mathbf{F}_1\vdash\mathbf{F}_5} \xrightarrow[\bullet 1]{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_4,!\mathbf{F}_1\vdash\mathbf{F}_1\vdash\mathbf{F}_1\vdash\mathbf{F}_2} \xrightarrow[\bullet 1]{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_4,!\mathbf{F}_1\vdash\mathbf{F}_1\vdash\mathbf{F}_1\vdash\mathbf{F}_1\vdash\mathbf{F}_1$$

• Case rule \multimap_L

$$\frac{\mathbf{h}_2: \Delta_7, !\mathsf{F}_1 \vdash \mathsf{F}_4 \quad \mathbf{h}_2: \Delta_3, \mathsf{F}_5 \vdash \mathsf{F}_6}{\bullet \mathbf{h}_2: (\Delta_7, !\mathsf{F}_1), \Delta_3, \mathsf{F}_4 \multimap \mathsf{F}_5 \vdash \mathsf{F}_6} \quad \multimap_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2: \Delta_7, !\mathsf{F}_1, !\mathsf{F}_1 \vdash \mathsf{F}_4} \quad \operatorname{ax/ind}}{\bullet \mathbf{h}_2: \Delta_3, \Delta_7, !\mathsf{F}_1, !\mathsf{F}_1, \mathsf{F}_4 \multimap \mathsf{F}_5 \vdash \mathsf{F}_6} \quad \multimap_L$$

$$\begin{array}{l} \mathbf{h}_2: \Delta_3 \vdash \mathbf{F}_4 \quad \mathbf{h}_2: \Delta_7, \mathbf{F}_5, ! \mathbf{F}_1 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_2: \Delta_3, (\Delta_7, ! \mathbf{F}_1), \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_6 \end{array} \quad \leadsto \quad \\ \begin{array}{l} \overline{\mathbf{h}_2: \Delta_3 \vdash \mathbf{F}_4} \quad \text{ax} \quad \overline{\mathbf{h}_2: \Delta_7, \mathbf{F}_5, ! \mathbf{F}_1, ! \mathbf{F}_1 \vdash \mathbf{F}_6} \\ \bullet \mathbf{h}_2: \Delta_3, \Delta_7, ! \mathbf{F}_1, ! \mathbf{F}_1 \vdash \mathbf{F}_6 \vdash \mathbf{F}_6 \end{array} \quad \overset{\text{av/ind}}{\bullet} \\ - \circ_L \end{array}$$

ullet Case rule I

2.14 Status of !L: : Non invertible

• Case rule !R

$$\begin{array}{c} \frac{\mathbf{h}_3: ! \Upsilon \mathbf{1}, ! \mathbf{F}_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_3: ! \Upsilon \mathbf{1}, ! \mathbf{F}_2 \vdash ! \mathbf{F}_4} & !R & \quad \leadsto & \quad \hline{\bullet \mathbf{h}_3: \mathbf{F}_2, ! \Upsilon \mathbf{1} \vdash ! \mathbf{F}_4} & \mathtt{fail} \end{array}$$

- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

• Case rule $\&_R$

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{!F}_2\vdash \mathbf{F}_4\quad \mathbf{h}_3:\Delta_1,\mathbf{!F}_2\vdash \mathbf{F}_5}{\bullet \mathbf{h}_3:\Delta_1,\mathbf{!F}_2\vdash \mathbf{F}_4\& \mathbf{F}_5} \quad \&_R \qquad \leadsto \qquad \frac{\mathbf{h}_3:\Delta_1,\mathbf{F}_2\vdash \mathbf{F}_4}{\bullet \mathbf{h}_3:\Delta_1,\mathbf{F}_2\vdash \mathbf{F}_4\& \mathbf{F}_5} \quad \frac{\mathbf{ax/ind}}{\&_R} \quad \frac{\mathbf{ax/ind}}{\lozenge}$$

• Case rule \multimap_R

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{F}_4,\mathbf{IF}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{IF}_2\vdash\mathbf{F}_4\multimap\mathbf{F}_5} \multimap_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_1,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_4\multimap\mathbf{F}_5} \stackrel{\mathrm{ax/ind}}{\multimap_R}$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_2}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{IF}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{IF}_2\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_4}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_4\oplus\mathbf{F}_5}\ \oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_2:\Delta_4, \mathbf{l}_{\mathsf{F}_1} \vdash \mathbf{f}_3}{\bullet \mathbf{h}_2:\mathbf{1},\Delta_4, \mathbf{l}_{\mathsf{F}_1} \vdash \mathbf{f}_3} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_4, \mathbf{f}_1 \vdash \mathbf{f}_3}}{\bullet \mathbf{h}_2:\mathbf{1},\Delta_4, \mathbf{f}_1 \vdash \mathbf{f}_3} \ \mathbf{1}_L$$

• Case rule \otimes_R

$$\begin{array}{c} \frac{\mathbf{h}_2:\Delta_6,!\mathbf{F}_1\vdash \mathbf{F}_4\quad \mathbf{h}_2:\Delta_3\vdash \mathbf{F}_5}{\bullet \mathbf{h}_2:(\Delta_6,!\mathbf{F}_1),\Delta_3\vdash \mathbf{F}_4\otimes \mathbf{F}_5} \ \otimes_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_6,\mathbf{F}_1\vdash \mathbf{F}_4}\quad ^{\mathrm{ax}/\mathrm{ind}}\quad \overline{\mathbf{h}_2:\Delta_3\vdash \mathbf{F}_5}}{\bullet \mathbf{h}_2:\Delta_3,\Delta_6,\mathbf{F}_1\vdash \mathbf{F}_4\otimes \mathbf{F}_5} \quad \otimes_R \end{array}$$

 \bullet Case rule W

$$\frac{\mathbf{h}_2:\Delta_5, |\mathbf{F_1}| + \mathbf{F_4}}{\bullet \mathbf{h}_2:(\Delta_5, |\mathbf{F_1}), |\mathbf{F}_3| + \mathbf{F_4}} \quad W \qquad \rightsquigarrow \qquad \frac{\frac{\mathbf{h}_2:\Delta_5, \mathbf{F_1} + \mathbf{F_4}}{\bullet \mathbf{h}_2:\Delta_5, \mathbf{F_1}, |\mathbf{F_3}| + \mathbf{F_4}} \quad W}{\bullet \mathbf{h}_2:\Delta_5, \mathbf{F_1}, |\mathbf{F_3}| + \mathbf{F_4}} \quad W$$

$$\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_4} \quad W \qquad \leadsto \qquad \overline{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_4} \quad \mathbf{fail}$$

 \bullet Case rule C

 \bullet Case rule !L

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_2 : \Delta_6, \mathbf{F}_4, !\mathbf{F}_1 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : (\Delta_6, !\mathbf{F}_1), \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} \quad \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2 : \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_2 : \Delta_6, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} \stackrel{\mathrm{ax/ind}}{\bullet} \&_{L2}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_2 : \Delta_6, \mathbf{F}_3, \mathbf{!F}_1 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : (\Delta_6, \mathbf{!F}_1), \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} \quad \&_{L1} \qquad \rightsquigarrow \qquad \frac{\frac{\mathbf{h}_2 : \Delta_6, \mathbf{F}_1, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2 : \Delta_6, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5}} \overset{\mathrm{ax/ind}}{\bullet \mathbf{h}_2 : \Delta_6, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} \\ & & \bullet \mathbf{h}_2 : \Delta_6, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} & & & \bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{F}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h}_4 : \Delta_6, \mathbf{h}_4 \vdash \mathbf{h}_5 & & & \bullet \mathbf{h$$

• Case rule \otimes_L

$$\frac{\mathtt{h}_2:\Delta_6,\mathtt{F}_3,\mathtt{F}_4,\mathtt{lF}_1 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2:(\Delta_6,\mathtt{lF}_1),\mathtt{F}_3\otimes \mathtt{F}_4 \vdash \mathtt{F}_5} \otimes_L \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_2:\Delta_6,\mathtt{F}_1,\mathtt{F}_3,\mathtt{F}_4 \vdash \mathtt{F}_5}}{\bullet \mathtt{h}_2:\Delta_6,\mathtt{F}_1,\mathtt{F}_3\otimes \mathtt{F}_4 \vdash \mathtt{F}_5} \overset{\mathsf{av}/\mathsf{ind}}{\otimes}_L$$

• Case rule \oplus_L

$$\frac{\mathtt{h}_2:\Delta_6,\mathtt{F}_3,\mathtt{lF}_1 \vdash \mathtt{F}_5 \quad \mathtt{h}_2:\Delta_6,\mathtt{F}_4,\mathtt{lF}_1 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2:\Delta_6,\mathtt{F}_4 \vdash \mathtt{F}_5} \ \oplus_L \qquad \rightsquigarrow \qquad \frac{\frac{\mathtt{h}_2:\Delta_6,\mathtt{F}_1,\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2:\Delta_6,\mathtt{F}_1,\mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}}{\bullet \mathtt{h}_2:\Delta_6,\mathtt{F}_1,\mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5} \ \oplus_L$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{\mathbf{h}_2 : \Delta_7, |\mathbf{f}_1 \vdash \mathbf{f}_4 \quad \mathbf{h}_2 : \Delta_3, \mathbf{f}_5 \vdash \mathbf{f}_6}{\bullet \mathbf{h}_2 : (\Delta_7, |\mathbf{f}_1|), \Delta_3, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6} \\ & \bullet \mathbf{h}_2 : (\Delta_7, |\mathbf{f}_1|), \Delta_3, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, (\Delta_7, |\mathbf{f}_1|), \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, (\Delta_7, |\mathbf{f}_1|), \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \multimap \mathbf{f}_5 \vdash \mathbf{f}_6 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{h}_1, \mathbf{h}_2 \multimap \mathbf{h}_2 \vdash \mathbf{h}_2 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{h}_1, \mathbf{h}_2 \multimap \mathbf{h}_2 \vdash \mathbf{h}_2 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{h}_1, \mathbf{h}_2 \multimap \mathbf{h}_2 \vdash \mathbf{h}_2 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{h}_1, \mathbf{h}_2 \multimap \mathbf{h}_2 \vdash \mathbf{h}_2 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{h}_1, \mathbf{h}_2 \multimap \mathbf{h}_2 \vdash \mathbf{h}_2 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{h}_1, \mathbf{h}_2 \multimap \mathbf{h}_2 \vdash \mathbf{h}_2 \\ & \bullet \mathbf{h}_2 : \Delta_3, \Delta_7, \mathbf{h}_1, \mathbf{h}_2 \multimap \mathbf{h}_2 \vdash \mathbf{h}_2 \\ & \bullet \mathbf{h}_2 \vdash \mathbf{h}_2 \\ & \bullet \mathbf{h}_2 \vdash \mathbf{h}_2 \vdash \mathbf{h$$

ullet Case rule I

2.15 Status of $\&_{L2}$: Non invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{}{\bullet \mathbf{h}_4 : \Delta_1, \mathbf{F}_2 \& \mathbf{F}_3 \vdash \top} \quad \top \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_4 : \Delta_1, \mathbf{F}_3 \vdash \top} \quad \top$$

• Case rule $\&_R$

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_5\quad\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_5\&\mathbf{F}_6} \quad\&_R\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_5}\quad ^{\mathrm{ax/ind}}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_6}\quad ^{\mathrm{ax/ind}}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_6}\quad \&_R$$

• Case rule \multimap_R

$$\frac{\mathsf{h}_4:\Delta_1,\mathsf{F}_5,\mathsf{F}_2\&\mathsf{F}_3\vdash\mathsf{F}_6}{\bullet\mathsf{h}_4:\Delta_1,\mathsf{F}_2\&\mathsf{F}_3\vdash\mathsf{F}_5\multimap\mathsf{F}_6} \ \multimap_R \qquad \leadsto \qquad \frac{\overline{\mathsf{h}_4:\Delta_1,\mathsf{F}_3,\mathsf{F}_5\vdash\mathsf{F}_6}}{\bullet\mathsf{h}_4:\Delta_1,\mathsf{F}_3\vdash\mathsf{F}_5\multimap\mathsf{F}_6} \stackrel{\mathsf{ax/ind}}{\multimap_R}$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\ \oplus_{R_2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_6}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\ \oplus_{R_2}$$

• Case rule \oplus_{R_1}

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

• Case rule \otimes_R

$$\begin{array}{lll} \frac{\mathbf{h}_3:\Delta_7, \mathbf{F}_1\&\mathbf{F}_2\vdash \mathbf{F}_5 & \mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6}{\bullet \mathbf{h}_3:(\Delta_7, \mathbf{F}_1\&\mathbf{F}_2), \Delta_4\vdash \mathbf{F}_5\otimes \mathbf{F}_6} & \otimes_R & \\ & \bullet \mathbf{h}_3:(\Delta_7, \mathbf{F}_2\vdash \mathbf{F}_5) & \mathbf{ax}/\mathbf{ind} & \frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6}{\mathbf{h}_3:\Delta_4, \Delta_7, \mathbf{F}_2\vdash \mathbf{F}_5\otimes \mathbf{F}_6} & \otimes_R \end{array}$$

$$\begin{array}{l} \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5\quad \mathbf{h}_3:\Delta_7,\mathbf{F}_1\&\mathbf{F}_2\vdash \mathbf{F}_6} \\ \bullet \mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_1\&\mathbf{F}_2\vdash \mathbf{F}_5\otimes \mathbf{F}_6 \end{array} \otimes_R \qquad \leadsto \qquad \frac{\underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5}\quad \mathrm{ax}}{\bullet \mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_2\vdash \mathbf{F}_5\otimes \mathbf{F}_6} \quad \frac{\mathrm{ax/ind}}{\otimes_R} \end{array}$$

 \bullet Case rule W

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\&\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:(\Delta_6,\mathbf{F}_1\&\mathbf{F}_2),\mathbf{F}_4\vdash\mathbf{F}_5}\ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_2\vdash\mathbf{F}_5}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_5}\ W$$

 \bullet Case rule C

$$\begin{array}{ll} \frac{\mathbf{h}_3:\Delta_6, \mathbb{IF}_4, \mathbb{IF}_4, \mathbb{F}_1 \& \mathbb{F}_2 \vdash \mathbb{F}_5}{\bullet \mathbf{h}_3:(\Delta_6, \mathbb{F}_1, \mathbb{E}_F_2), \mathbb{IF}_4 \vdash \mathbb{F}_5} & C & \longrightarrow & \frac{\overline{\mathbf{h}_3:\Delta_6, \mathbb{F}_2, \mathbb{IF}_4 \vdash \mathbb{F}_5}}{\bullet \mathbf{h}_3:\Delta_6, \mathbb{F}_2, \mathbb{IF}_4 \vdash \mathbb{F}_5} & \frac{\mathrm{ax/ind}}{C} \end{array}$$

• Case rule !L

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_4,\mathbf{F}_1\&\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:(\Delta_6,\mathbf{F}_1\&\mathbf{F}_2),!\mathbf{F}_4\vdash\mathbf{F}_5} \ !L \qquad \leadsto \qquad \frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_2,!\mathbf{F}_4\vdash\mathbf{F}_5} \xrightarrow{\mathrm{ax/ind}} !L$$

• Case rule $\&_{L2}$

• Case rule $\&_{L1}$

• Case rule \otimes_L

$$\frac{\mathtt{h}_3:\Delta_7,\mathtt{F}_4,\mathtt{F}_5,\mathtt{F}_1\&\mathtt{F}_2 + \mathtt{F}_6}{\bullet \mathtt{h}_3:(\Delta_7,\mathtt{F}_1\&\mathtt{F}_2),\mathtt{F}_4\otimes\mathtt{F}_5 + \mathtt{F}_6} \otimes_L \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_3:\Delta_7,\mathtt{F}_2,\mathtt{F}_4,\mathtt{F}_5 + \mathtt{F}_6}}{\bullet \mathtt{h}_3:\Delta_7,\mathtt{F}_2,\mathtt{F}_4\otimes\mathtt{F}_5 + \mathtt{F}_6} \otimes_L$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\&\mathbf{F}_2\vdash\mathbf{F}_6\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\&\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\&\mathbf{F}_2),\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6}\oplus\boldsymbol{L} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_6}\quad \text{ax/ind} \quad \overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_6}\quad \frac{\text{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6}\quad \oplus \boldsymbol{L}} \oplus \boldsymbol{L}$$

• Case rule \multimap_L

 \bullet Case rule I

2.16 Status of $\&_{L1}$: Non invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\overline{\bullet_{h_4:\Delta_1,\,F_2\&F_3\,\vdash\,\top}} \quad \top \qquad \leadsto \qquad \overline{\bullet_{h_4:\Delta_1,\,F_2\,\vdash\,\top}} \quad \top$$

• Case rule $\&_R$

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_5\quad\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_5\&\mathbf{F}_6} \quad\&_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5}\quad\text{ax/ind}\quad\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_6}\quad\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5\&\mathbf{F}_6}\quad\&_R$$

• Case rule \multimap_R

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_5,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\&\mathbf{F}_3\vdash\mathbf{F}_5\multimap\mathbf{F}_6} \ \ ^{\odot}R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5\multimap\mathbf{F}_6} \ \ ^{\mathrm{ax/ind}}$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathtt{h}_4:\Delta_1,\mathtt{F}_2\&\mathtt{F}_3\vdash\mathtt{F}_6}{\bullet\mathtt{h}_4:\Delta_1,\mathtt{F}_2\&\mathtt{F}_3\vdash\mathtt{F}_5\oplus\mathtt{F}_6}\oplus_{R_2}\quad \leadsto\quad \frac{\overline{\mathtt{h}_4:\Delta_1,\mathtt{F}_2\vdash\mathtt{F}_6}\quad \mathtt{ax/ind}}{\bullet\mathtt{h}_4:\Delta_1,\mathtt{F}_2\vdash\mathtt{F}_5\oplus\mathtt{F}_6}\oplus_{R_2}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathtt{h}_4:\Delta_1,\mathtt{F}_2\&\mathtt{F}_3\vdash\mathtt{F}_5}{\bullet\mathtt{h}_4:\Delta_1,\mathtt{F}_2\&\mathtt{F}_3\vdash\mathtt{F}_5\oplus\mathtt{F}_6}\oplus_{R_1} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_4:\Delta_1,\mathtt{F}_2\vdash\mathtt{F}_5} \quad \mathtt{ax/ind}}{\bullet\mathtt{h}_4:\Delta_1,\mathtt{F}_2\vdash\mathtt{F}_5\oplus\mathtt{F}_6} \oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

• Case rule \otimes_R

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\&\mathbf{F}_2\vdash\mathbf{F}_5\quad\mathbf{h}_3:\Delta_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\&\mathbf{F}_2),\Delta_4\vdash\mathbf{F}_5\otimes\mathbf{F}_6}\ \otimes_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\vdash\mathbf{F}_5}\quad ^{\mathrm{ax/ind}}\quad \overline{\mathbf{h}_3:\Delta_4\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_1\vdash\mathbf{F}_5\otimes\mathbf{F}_6}\ \otimes_R$$

 \bullet Case rule W

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\&\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:(\Delta_6,\mathbf{F}_1\&\mathbf{F}_2),\mathbf{F}_4\vdash\mathbf{F}_5}\ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_5} \ W$$

 \bullet Case rule C

• Case rule !L

$$\frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \& \mathbf{F}_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_1 \& \mathbf{F}_2), |\mathbf{F}_4 \vdash \mathbf{F}_5} \ !L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, |\mathbf{F}_4 \vdash \mathbf{F}_5} \ !L$$

• Case rule $\&_{L2}$

• Case rule $\&_{L1}$

• Case rule \otimes_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_5,\mathbf{F}_1\&\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\&\mathbf{F}_2),\mathbf{F}_4\otimes\mathbf{F}_5\vdash\mathbf{F}_6}\otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\otimes\mathbf{F}_5\vdash\mathbf{F}_6}\otimes_L$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\&\mathbf{F}_2\vdash\mathbf{F}_6\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\&\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\&\mathbf{F}_2),\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6}\oplus L}\quad \Rightarrow\quad \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6} \xrightarrow{\mathbf{ax/ind}} \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_5\vdash\mathbf{F}_6}{\oplus L} \xrightarrow{\bullet} L$$

• Case rule \multimap_L

$$\begin{array}{c} \mathbf{h}_3: \Delta_8, \mathbf{F}_1 \& \mathbf{F}_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_3: \Delta_4, \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_3: (\Delta_8, \mathbf{F}_1 \& \mathbf{F}_2), \Delta_4, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array} \multimap_L \\ & \stackrel{\mathbf{h}_3: \Delta_8, \mathbf{F}_1 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3: \Delta_4, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7} \overset{\mathbf{ax}}{\bullet \mathbf{h}_3: \Delta_4, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7} \overset{\mathbf{ax}}{\bullet \mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_5} \\ & \stackrel{\mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3: \Delta_4, (\Delta_8, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7} & \neg \mathbf{L} \end{array} \longrightarrow \begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_5 & \mathbf{ax} & \mathbf{h}_3: \Delta_8, \mathbf{F}_1, \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_3: \Delta_4, (\Delta_8, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array}}_{\bullet \mathbf{h}_3: \Delta_4, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7} \overset{\mathbf{ax}/\mathbf{ind}}{\multimap_L} \\ \end{array}}$$

 \bullet Case rule I

2.17 Status of \otimes_L : Invertible

- Case rule !R
- Case rule $\mathbf{1}_{R}$
- \bullet Case rule \top

$$\frac{}{\bullet^{}_{h_4}:\Delta_1,{\scriptscriptstyle F_2}\otimes{\scriptscriptstyle F_3}\vdash\top}\ \top \qquad \leadsto \qquad \frac{}{\bullet^{}_{h_4}:\Delta_1,{\scriptscriptstyle F_2},{\scriptscriptstyle F_3}\vdash\top}\ \top$$

• Case rule $\&_R$

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\otimes\mathbf{F}_3\vdash\mathbf{F}_5\quad\mathbf{h}_4:\Delta_1,\mathbf{F}_2\otimes\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\otimes\mathbf{F}_3\vdash\mathbf{F}_6}\quad\&_R\qquad\Leftrightarrow\qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_5}\quad\text{ax/ind}\quad\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_6}\quad\frac{\mathbf{ax/ind}}{\&_R}\quad\&_R$$

• Case rule \multimap_R

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_5,\mathbf{F}_2\otimes\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\otimes\mathbf{F}_3\vdash\mathbf{F}_5\multimap\mathbf{F}_6} \ \multimap_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_5\multimap\mathbf{F}_6} \ \multimap_R$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\otimes\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\otimes\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\mathbf{F}_6}\oplus_{R_2}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathtt{h}_4:\Delta_1,\mathtt{F}_2\otimes\mathtt{F}_3\vdash\mathtt{F}_5}{\bullet\mathtt{h}_4:\Delta_1,\mathtt{F}_2\otimes\mathtt{F}_3\vdash\mathtt{F}_5\oplus\mathtt{F}_6}\oplus_{R_1} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_4:\Delta_1,\mathtt{F}_2,\mathtt{F}_3\vdash\mathtt{F}_5}}{\bullet\mathtt{h}_4:\Delta_1,\mathtt{F}_2,\mathtt{F}_3\vdash\mathtt{F}_5\oplus\mathtt{F}_6}\oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\otimes\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\mathbf{1},\Delta_5,\mathbf{F}_1\otimes\mathbf{F}_2\vdash\mathbf{F}_4}\ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_4}}{\bullet\mathbf{h}_3:\mathbf{1},\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_4}\ \mathbf{1}_L$$

• Case rule \otimes_R

$$\begin{array}{lll} \underbrace{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\otimes\mathbf{F}_2\vdash\mathbf{F}_5\quad\mathbf{h}_3:\Delta_4\vdash\mathbf{F}_6}_{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\otimes\mathbf{F}_2),\Delta_4\vdash\mathbf{F}_5\otimes\mathbf{F}_6} \end{array} \otimes_R \qquad \leadsto & \begin{array}{ll} \underbrace{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_5}_{\bullet\mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_5\otimes\mathbf{F}_6} \end{array} \otimes_R \\ & \bullet\mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_5\otimes\mathbf{F}_6 \end{array} \otimes_R$$

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5\quad \mathbf{h}_3:\Delta_7,\mathbf{F}_1\otimes \mathbf{F}_2\vdash \mathbf{F}_6}{\bullet \mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_1\otimes \mathbf{F}_2\vdash \mathbf{F}_5\otimes \mathbf{F}_6}\otimes_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5}\quad ^{\mathrm{ax}}\quad \overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash \mathbf{F}_6}}{\bullet \mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash \mathbf{F}_5\otimes \mathbf{F}_6}\quad \overset{\mathrm{ax/ind}}{\otimes_R}$$

 \bullet Case rule W

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\otimes\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:(\Delta_6,\mathbf{F}_1\otimes\mathbf{F}_2),!\mathbf{F}_4\vdash\mathbf{F}_5}\ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_2\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1,\mathbf{F}_2,!\mathbf{F}_4\vdash\mathbf{F}_5}\ W$$

 \bullet Case rule C

$$\begin{array}{lll} \frac{\mathbf{h}_3:\Delta_6, \mathbf{l}_{F_4}, \mathbf{l}_{F_4}, \mathbf{r}_1 \otimes \mathbf{r}_2 \vdash \mathbf{r}_5}{\bullet \mathbf{h}_3:(\Delta_6, \mathbf{r}_1 \otimes \mathbf{r}_2), \mathbf{l}_{F_4} \vdash \mathbf{r}_5} & C & & \\ & & & \\ \bullet \mathbf{h}_3:\Delta_6, \mathbf{r}_1, \mathbf{r}_2, \mathbf{l}_{F_4}, \mathbf{l}_{F_4} \vdash \mathbf{r}_5} & \\ & & \\ & & \\ \bullet \mathbf{h}_3:\Delta_6, \mathbf{r}_1, \mathbf{r}_2, \mathbf{l}_{F_4} \vdash \mathbf{r}_5} & \\ & & \\ & & \\ \end{array} \right) \xrightarrow{\mathbf{ax/ind}} C$$

• Case rule !L

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\otimes\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\otimes\mathbf{F}_2),\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6} \quad \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6} \quad \&_{L2}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{f}_4,\mathbf{f}_1\otimes\mathbf{f}_2\vdash\mathbf{f}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{f}_1\otimes\mathbf{f}_2),\mathbf{f}_4\&\mathbf{f}_5\vdash\mathbf{f}_6} \ \&_{L1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{f}_1,\mathbf{f}_2,\mathbf{f}_4\vdash\mathbf{f}_6}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_1,\mathbf{f}_2,\mathbf{f}_4\&\mathbf{f}_5\vdash\mathbf{f}_6} \ \&_{L1}$$

• Case rule \otimes_L

• Case rule \oplus_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\otimes\mathbf{F}_2\vdash\mathbf{F}_6\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\otimes\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\otimes\mathbf{F}_2),\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6}\oplus L} \quad \Leftrightarrow \quad \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6}} \stackrel{\mathrm{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6}} \stackrel{\mathrm{ax/ind}}{\oplus L}$$

• Case rule \multimap_L

$$\begin{array}{c} \mathbf{h}_3:\Delta_8, \mathbf{F}_1\otimes \mathbf{F}_2\vdash \mathbf{F}_5 \quad \mathbf{h}_3:\Delta_4, \mathbf{F}_6\vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_3:(\Delta_8, \mathbf{F}_1\otimes \mathbf{F}_2), \Delta_4, \mathbf{F}_5\multimap \mathbf{F}_6\vdash \mathbf{F}_7 \end{array} \multimap_L \qquad \leadsto \qquad \begin{array}{c} \overline{\mathbf{h}_3:\Delta_8, \mathbf{F}_1, \mathbf{F}_2\vdash \mathbf{F}_5} \quad \mathrm{ax/ind} \quad \overline{\mathbf{h}_3:\Delta_4, \mathbf{F}_6\vdash \mathbf{F}_7} \\ \bullet \mathbf{h}_3:\Delta_4 \cup \mathbf{F}_5 \quad \mathbf{h}_3:\Delta_4, \mathbf{F}_6 \cup \mathbf{F}_7 \\ \bullet \mathbf{h}_3:\Delta_4 \cup \mathbf{F}_5 \quad \mathbf{h}_3:\Delta_8, \mathbf{F}_6, \mathbf{F}_1\otimes \mathbf{F}_2\vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_3:\Delta_4 \cup \mathbf{F}_5 \quad \mathbf{h}_3:\Delta_4, (\Delta_8, \mathbf{F}_1\otimes \mathbf{F}_2), \mathbf{F}_5 \multimap \mathbf{F}_6\vdash \mathbf{F}_7 \end{array} \multimap_L \qquad \leadsto \qquad \begin{array}{c} \overline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5} \quad \mathrm{ax} \\ \overline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5} \quad \mathrm{ax} \\ \overline{\mathbf{h}_3:\Delta_4 \cup \mathbf{F}_5} \quad \overline{\mathbf{h}_3:\Delta_8, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_6\vdash \mathbf{F}_7} \end{array} \longrightarrow_L \\ \bullet \mathbf{h}_3:\Delta_4 \cup \mathbf{h}_3:\Delta_4, (\Delta_8, \mathbf{F}_1, \mathbf{F}_2), \mathbf{F}_5 \multimap \mathbf{F}_6\vdash \mathbf{F}_7} \end{array} \multimap_L$$

 \bullet Case rule I

2.18 Status of \oplus_L : (Left Premise): Invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\overbrace{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \top} \quad \top \qquad \leadsto \qquad \overline{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \top} \quad \top \\$$

• Case rule $\&_R$

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5\quad\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_6}\quad\&_R\qquad\leadsto\qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5}\quad\text{ax/ind}\quad\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_6}\quad\frac{\mathbf{ax/ind}}{\&_R}\quad\&_R$$

• Case rule \multimap_R

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_5,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5\multimap\mathbf{F}_6} \ \multimap_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5\multimap\mathbf{F}_6} \ ^{\mathrm{ax/ind}} \ \multimap_R$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_2}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_1} \qquad \leadsto \qquad \frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\mathbf{1},\Delta_5,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_4}\ \mathbf{1}_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\mathbf{1},\Delta_5,\mathbf{F}_1\vdash\mathbf{F}_4}\ \mathbf{1}_L}{\mathbf{1}_L}$$

• Case rule \otimes_R

$$\begin{array}{lll} \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_5\quad\mathbf{h}_3:\Delta_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2),\Delta_4\vdash\mathbf{F}_5\otimes\mathbf{F}_6} & \otimes_R & & \leadsto & & \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\vdash\mathbf{F}_5}\quad^{\mathrm{ax/ind}}\quad\overline{\mathbf{h}_3:\Delta_4\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_1\vdash\mathbf{F}_5\otimes\mathbf{F}_6} & \otimes_R & & & & & & \\ \end{array}$$

$$\begin{array}{c} \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5\quad \mathbf{h}_3:\Delta_7,\mathbf{F}_1\oplus \mathbf{F}_2\vdash \mathbf{F}_6} \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_1\oplus \mathbf{F}_2\vdash \mathbf{F}_5\otimes \mathbf{F}_6} \end{array} \otimes_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5}\quad \text{ax}\quad \overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\vdash \mathbf{F}_6} \\ \bullet \underline{\mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_1\vdash \mathbf{F}_5\otimes \mathbf{F}_6} \end{array} \otimes_R$$

 \bullet Case rule W

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:(\Delta_6,\mathbf{F}_1\oplus\mathbf{F}_2),!\mathbf{F}_4\vdash\mathbf{F}_5}\ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{F}_1\vdash\mathbf{F}_5}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{F}_1,!\mathbf{F}_4\vdash\mathbf{F}_5} \ W$$

 \bullet Case rule C

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \Delta_6, \mathsf{!F}_4, \mathsf{!F}_4, \mathsf{F}_1 \oplus \mathsf{F}_2 \vdash \mathsf{F}_5}{\bullet \mathbf{h}_3 : (\Delta_6, \mathsf{F}_1 \oplus \mathsf{F}_2), \mathsf{!F}_4 \vdash \mathsf{F}_5} & C & & \leadsto & & & & & & \frac{\mathbf{h}_3 : \Delta_6, \mathsf{F}_1, \mathsf{!F}_4 \vdash \mathsf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathsf{F}_1, \mathsf{!F}_4 \vdash \mathsf{F}_5} & & & & & & & \\ \end{array} \right. \\ \frac{\mathbf{h}_3 : \Delta_6, \mathsf{F}_1, \mathsf{!F}_4 \vdash \mathsf{F}_5}{\bullet \mathbf{h}_3 : \Delta_6, \mathsf{F}_1, \mathsf{!F}_4 \vdash \mathsf{F}_5} & & & & & & \\ \end{array}$$

• Case rule !L

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2),\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6}~\&_{L2}~~\overset{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_5\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6}~^{\mathrm{ax/ind}}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2),\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6}~\&_{L1}~~\leadsto~~\frac{\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6}~\&_{L1}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_5,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2),\mathbf{F}_4\otimes\mathbf{F}_5\vdash\mathbf{F}_6}\otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\otimes\mathbf{F}_5\vdash\mathbf{F}_6}\otimes_L$$

• Case rule \oplus_L

$$\begin{array}{c} \underline{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_6\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_6} \\ \bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2),\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6} \end{array} \oplus_L \qquad \leadsto \qquad \begin{array}{c} \underline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_6} & \mathbf{ax/ind} \\ \bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6} \end{array} \xrightarrow[\bullet L]{} \underline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_6} & \mathbf{ax/ind} \\ \bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6} \end{array} \xrightarrow[\bullet L]{} \underline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\vdash\mathbf{F}_6} & \mathbf{ax/ind} \\ \bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6} \end{array} \xrightarrow[\bullet L]{} \underline{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5} \xrightarrow[\bullet L]{} \underline{\mathbf{h}_1:\Delta_2,\mathbf{h}_1:\Delta_2,\mathbf{h}_2:\Delta_2,\mathbf{h}_1:\Delta_2,\mathbf{h}_2:\Delta_2,\mathbf{h}_2:\Delta_2,\mathbf{h}_2:\Delta_2,\mathbf{h}_2:\Delta_2,\mathbf{h}_3:\Delta_2,\mathbf{h}_3:\Delta_2,\mathbf{h}_3:\Delta_2,\mathbf{h}_3:\Delta_2,\mathbf{h}_3:\Delta_$$

• Case rule \multimap_L

$$\begin{array}{c} \underline{\mathbf{h}}_3: \Delta_8, \mathbf{F}_1 \oplus \mathbf{F}_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_3: \Delta_4, \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_3: (\Delta_8, \mathbf{F}_1 \oplus \mathbf{F}_2), \Delta_4, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array} \stackrel{\circ}{-} \mathbf{L} \\ \\ & \underbrace{\begin{array}{c} \underline{\mathbf{h}}_3: \Delta_8, \mathbf{F}_1 \vdash \mathbf{F}_5 \quad \mathbf{ax}/\mathbf{ind} \\ \bullet \mathbf{h}_3: \Delta_4, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array}}_{\bullet \mathbf{h}_3: \Delta_4, \mathbf{F}_5} \stackrel{\mathbf{ax}/\mathbf{ind}}{-} \mathbf{L} \\ \\ \\ \underbrace{\underline{\mathbf{h}}_3: \Delta_4 \vdash \mathbf{F}_5 \quad \mathbf{h}_3: \Delta_8, \mathbf{F}_6, \mathbf{F}_1 \oplus \mathbf{F}_2 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_5 \quad \mathbf{h}_3: \Delta_8, \mathbf{F}_6, \mathbf{F}_1 \oplus \mathbf{F}_2 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_3: \Delta_4, (\Delta_8, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array}}_{\bullet \mathbf{h}_3: \Delta_4, \mathbf{h}_3: \Delta_4, \mathbf{h}_4, \mathbf{h}_5: \mathbf{h}_$$

 \bullet Case rule I

2.19 Status of \oplus_L (Right Premise): : Invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\overbrace{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \top} \quad \top \qquad \leadsto \qquad \overline{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \top} \quad \top \\$$

• Case rule $\&_R$

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5\quad\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_6}\quad\&_R\qquad\leadsto\qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_5}\quad\text{ax/ind}\quad\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_6}\quad\frac{\mathbf{ax/ind}}{\&_R}\quad\&_R$$

• Case rule \multimap_R

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_5,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5\multimap\mathbf{F}_6} \ \multimap_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_5\multimap\mathbf{F}_6} \ ^{\mathrm{ax/ind}} \ \multimap_R$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_2}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\oplus\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_1} \qquad \leadsto \qquad \frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_5\oplus\mathbf{F}_6}\oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\mathbf{1},\Delta_5,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_4} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4}{\bullet\mathbf{h}_3:\mathbf{1},\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4} \ \mathbf{ax/ind}}{\bullet\mathbf{h}_3:\mathbf{1},\Delta_5,\mathbf{F}_2\vdash\mathbf{F}_4} \ \mathbf{1}_L$$

• Case rule \otimes_R

$$\begin{array}{lll} \underbrace{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_5\quad\mathbf{h}_3:\Delta_4\vdash\mathbf{F}_6}_{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2),\Delta_4\vdash\mathbf{F}_5\otimes\mathbf{F}_6} \end{array} \otimes_R \qquad \leadsto \qquad \frac{\underbrace{\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\mathbf{F}_5}_{\bullet\mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_2\vdash\mathbf{F}_5}_{\bullet\mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_2\vdash\mathbf{F}_5\otimes\mathbf{F}_6} }_{\bullet\mathbf{h}_3:\Delta_4,\Delta_7,\mathbf{F}_2\vdash\mathbf{F}_5\otimes\mathbf{F}_6} \otimes_R$$

 \bullet Case rule W

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{f}_1\oplus\mathbf{f}_2\vdash\mathbf{f}_5}{\bullet\mathbf{h}_3:(\Delta_6,\mathbf{f}_1\oplus\mathbf{f}_2),!\mathbf{f}_4\vdash\mathbf{f}_5}\ W \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_6,\mathbf{f}_2\vdash\mathbf{f}_5}}{\bullet\mathbf{h}_3:\Delta_6,\mathbf{f}_2,!\mathbf{f}_4\vdash\mathbf{f}_5} \stackrel{\mathrm{ax/ind}}{W}$$

 \bullet Case rule C

$$\begin{array}{lll} \frac{\mathbf{h}_3:\Delta_6, \mathsf{!F}_4, \mathsf{!F}_4, \mathsf{F}_1 \oplus \mathsf{F}_2 \vdash \mathsf{F}_5}{\bullet \mathbf{h}_3:(\Delta_6, \mathsf{F}_1 \oplus \mathsf{F}_2), \mathsf{!F}_4 \vdash \mathsf{F}_5} & C & & \leadsto & & & & & & \frac{\mathbf{h}_3:\Delta_6, \mathsf{F}_2, \mathsf{!F}_4 \vdash \mathsf{F}_5}{\bullet \mathbf{h}_3:\Delta_6, \mathsf{F}_2, \mathsf{!F}_4 \vdash \mathsf{F}_5} & & & & \text{ax/ind} \\ \end{array}$$

• Case rule !L

$$\frac{\mathsf{h}_3:\Delta_6,\mathsf{F}_4,\mathsf{F}_1\oplus\mathsf{F}_2\vdash\mathsf{F}_5}{\bullet\mathsf{h}_3:(\Delta_6,\mathsf{F}_1\oplus\mathsf{F}_2),!\mathsf{F}_4\vdash\mathsf{F}_5} \ !L \qquad \leadsto \qquad \frac{\overline{\mathsf{h}_3:\Delta_6,\mathsf{F}_2,\mathsf{F}_4\vdash\mathsf{F}_5}}{\bullet\mathsf{h}_3:\Delta_6,\mathsf{F}_2,!\mathsf{F}_4\vdash\mathsf{F}_5} \ !L$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2),\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6} \ \ \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_5\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6} \ \ \&_{L2}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2),\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6}~\&_{L1}~~\leadsto~~\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\&\mathbf{F}_5\vdash\mathbf{F}_6}~\&_{L1}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{f}_4,\mathbf{f}_5,\mathbf{f}_1\oplus\mathbf{f}_2\vdash\mathbf{f}_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{f}_1\oplus\mathbf{f}_2),\mathbf{f}_4\otimes\mathbf{f}_5\vdash\mathbf{f}_6}\otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{f}_2,\mathbf{f}_4,\mathbf{f}_5\vdash\mathbf{f}_6}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_2,\mathbf{f}_4\otimes\mathbf{f}_5\vdash\mathbf{f}_6} \otimes_L$$

• Case rule \oplus_L

$$\begin{array}{c} \underline{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_6\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_5,\mathbf{F}_1\oplus\mathbf{F}_2\vdash\mathbf{F}_6} \\ \bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\oplus\mathbf{F}_2),\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6 \end{array}} \oplus_L \qquad \leadsto \qquad \begin{array}{c} \underline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\vdash\mathbf{F}_6} & \mathbf{ax/ind} \\ \bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6 \end{array}} \oplus_L \\ \\ \underline{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5\quad\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_5} \\ \bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_5 \end{array}} \oplus_L \qquad \leadsto \qquad \begin{array}{c} \underline{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_6} & \mathbf{ax/ind} \\ \bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\oplus\mathbf{F}_5\vdash\mathbf{F}_6 \end{array}} \\ \mathbf{h} \\ \end{array}$$

• Case rule \multimap_L

$$\begin{array}{c} \mathbf{h}_3: \Delta_8, \mathbf{F}_1 \oplus \mathbf{F}_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_3: \Delta_4, \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_3: (\Delta_8, \mathbf{F}_1 \oplus \mathbf{F}_2), \Delta_4, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array} \stackrel{\circ}{-} \mathbf{L} \\ & \underbrace{\begin{array}{c} \mathbf{h}_3: \Delta_8, \mathbf{F}_2 \vdash \mathbf{F}_5 \quad \mathbf{ax/ind} \\ \bullet \mathbf{h}_3: \Delta_4, \Delta_8, \mathbf{F}_2, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array}}_{\bullet \mathbf{h}_3: \Delta_4, \Delta_8, \mathbf{F}_2, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7} \stackrel{\mathbf{ax/ind}}{-} \mathbf{L} \\ \\ \underbrace{\begin{array}{c} \mathbf{h}_3: \Delta_4 \vdash \mathbf{F}_5 \quad \mathbf{h}_3: \Delta_8, \mathbf{F}_6, \mathbf{F}_1 \oplus \mathbf{F}_2 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_3: \Delta_4, (\Delta_8, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array}}_{\bullet \mathbf{h}_3: \Delta_4, \Delta_8, \mathbf{F}_2, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_7} \stackrel{\mathbf{ax/ind}}{-} \mathbf{L} \\ \end{array}$$

 \bullet Case rule I

2.20 Status of $-\circ_L$: (Left Premise): Non invertible

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

• Case rule $\&_R$

$$\frac{\mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3\multimap\mathbf{F}_4\vdash\mathbf{F}_6\quad\mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3\multimap\mathbf{F}_4\vdash\mathbf{F}_7}{\bullet\mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3\multimap\mathbf{F}_4\vdash\mathbf{F}_6\&\mathbf{F}_7}\quad\&_R\qquad\leadsto\qquad\frac{\overline{\mathbf{h}_5:\Delta_1\vdash\mathbf{F}_3}}{\bullet\mathbf{h}_5:\Delta_1\vdash\mathbf{F}_3}\overset{\mathrm{ax/ind}}{\vdash}$$

• Case rule \multimap_R

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3 \multimap \mathbf{F}_4 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3 \multimap \mathbf{F}_4 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \ \oplus_{R_2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5:\Delta_1 \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_5:\Delta_1 \vdash \mathbf{F}_3} \ \frac{\mathbf{ax/ind}}{\mathbf{H}}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathtt{h}_5:\Delta_1,\Delta_2,\mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5:\Delta_1,\Delta_2,\mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7} \ \oplus_{R_1} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_5:\Delta_1 \vdash \mathtt{F}_3}}{\bullet \mathtt{h}_5:\Delta_1 \vdash \mathtt{F}_3} \ \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_4:\mathbf{1},\Delta_1,\Delta_6,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_5} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_6\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_4:\mathbf{1},\Delta_6\vdash\mathbf{F}_2} \ \mathbf{1}_L$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_1,\Delta_6,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_5} \quad \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\mathrm{H}}$$

• Case rule \otimes_R

$$\begin{array}{ll} \underline{\mathbf{h}}_3: \Delta_6, \Delta_7, \mathbf{F}_1 \multimap \mathbf{F}_2 \vdash \mathbf{F}_4 & \underline{\mathbf{h}}_3: \Delta_8, \Delta_9 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_3: (\Delta_6, \Delta_7, \mathbf{F}_1 \multimap \mathbf{F}_2), \Delta_8, \Delta_9 \vdash \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \otimes_R \qquad \leadsto \qquad \overline{\bullet \mathbf{h}}_3: \Delta_6, \Delta_8 \vdash \mathbf{F}_1 \end{array} \ \, \mathbf{fail}$$

$$\begin{array}{lll} \underline{\mathtt{h}_3:\Delta_6,\Delta_7 \vdash \mathtt{F}_4} & \underline{\mathtt{h}_3:\Delta_8,\Delta_9,\mathtt{F}_1 \multimap \mathtt{F}_2 \vdash \mathtt{F}_5} \\ \underline{\bullet \mathtt{h}_3:(\Delta_6,\Delta_7),\Delta_8,\Delta_9,\mathtt{F}_1 \multimap \mathtt{F}_2 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5} \end{array} \otimes_R \qquad \leadsto \qquad \overline{\bullet \mathtt{h}_3:\Delta_6,\Delta_8 \vdash \mathtt{F}_1} \quad \mathtt{fail}$$

 \bullet Case rule W

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7,\mathbf{F}_2\multimap\mathbf{F}_3),|\mathbf{F}_5\vdash\mathbf{F}_6}\ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7\vdash\mathbf{F}_2}\ ^{\mathbf{ax}/\mathbf{ind}}}{\bullet\mathbf{h}_4:\Delta_7,|\mathbf{F}_5\vdash\mathbf{F}_2} \frac{\mathbf{h}_4:\Delta_7\vdash\mathbf{F}_2}{W}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_7,\mathbf{F}_2 \multimap \mathbf{F}_3),!\mathbf{F}_5 \vdash \mathbf{F}_6} \quad W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2} \quad \underset{\mathbb{H}}{\operatorname{ax/ind}}$$

 \bullet Case rule C

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbb{I}_{\mathsf{F}_5},\mathbb{I}_{\mathsf{F}_5},\mathbb{F}_2\multimap \mathbb{F}_3\vdash \mathbb{F}_6}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_7,\mathbb{F}_2\multimap \mathbb{F}_3),\mathbb{I}_{\mathsf{F}_5}\vdash \mathbb{F}_6} \ C \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7\vdash \mathbb{F}_2}}{\bullet \mathbf{h}_4:\Delta_7,\mathbb{I}_{\mathsf{F}_5}\vdash \mathbb{F}_2} \ W$$

$$\begin{array}{ll} \frac{\mathbf{h}_4:\Delta_1,\Delta_7, \mathsf{!F}_5, \mathsf{!F}_5, \mathsf{F}_2 \multimap \mathsf{F}_3 \vdash \mathsf{F}_6}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_7, \mathsf{F}_2 \multimap \mathsf{F}_3), \mathsf{!F}_5 \vdash \mathsf{F}_6} & C & \longrightarrow & \frac{\overline{\mathbf{h}_4:\Delta_1 \vdash \mathsf{F}_2}}{\bullet \mathbf{h}_4:\Delta_1 \vdash \mathsf{F}_2} & \mathbf{H} \end{array}$$

• Case rule !L

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_5,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7,\mathbf{F}_2\multimap\mathbf{F}_3),|\mathbf{F}_5\vdash\mathbf{F}_6} \ !L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7\vdash\mathbf{F}_2} \ \ \text{ax/ind}}{\bullet\mathbf{h}_4:\Delta_7,|\mathbf{F}_5\vdash\mathbf{F}_2} \ W$$

$$\begin{array}{ccc} \frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_5,\mathbf{F}_2-\mathbf{o}\,\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7,\mathbf{F}_2-\mathbf{o}\,\mathbf{F}_3),!\mathbf{F}_5\vdash\mathbf{F}_6} & !L & & \leadsto & & \frac{\overline{\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2} & \mathbf{H} \end{array}$$

• Case rule $\&_{L2}$

$$\frac{\mathtt{h}_4:\Delta_1,\Delta_8,\mathtt{F}_6,\mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4:(\Delta_1,\Delta_8,\mathtt{F}_2 \multimap \mathtt{F}_3),\mathtt{F}_5\&\mathtt{F}_6 \vdash \mathtt{F}_7} \quad \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_4:\Delta_8,\mathtt{F}_6 \vdash \mathtt{F}_2}}{\bullet \mathtt{h}_4:\Delta_8,\mathtt{F}_5\&\mathtt{F}_6 \vdash \mathtt{F}_2} \stackrel{\mathrm{ax/ind}}{\bullet}_{L2}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_6,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2 \multimap \mathbf{F}_3),\mathbf{F}_5\&\mathbf{F}_6 \vdash \mathbf{F}_7} \quad \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

• Case rule $\&_{L1}$

$$\frac{\mathtt{h}_4 : \Delta_1, \Delta_8, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_1, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7} \quad \&_{L1} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_4 : \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_2}}{\bullet \mathtt{h}_4 : \Delta_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2} \stackrel{\mathrm{ax/ind}}{\bullet}_{L1}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_5,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_7}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2\multimap\mathbf{F}_3),\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_7} \quad\&_{L1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\overset{\mathrm{ax/ind}}{\bullet}}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_5,\mathbf{F}_6,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_7}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2\multimap\mathbf{F}_3),\mathbf{F}_5\otimes\mathbf{F}_6\vdash\mathbf{F}_7}\otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_8,\mathbf{F}_5,\mathbf{F}_6\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_4:\Delta_8,\mathbf{F}_5\otimes\mathbf{F}_6\vdash\mathbf{F}_2} \overset{\mathrm{ax/ind}}{\otimes}_L$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_5,\mathbf{F}_6,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_7}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2\multimap\mathbf{F}_3),\mathbf{F}_5\otimes\mathbf{F}_6\vdash\mathbf{F}_7} \ \otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2}}{\bullet\mathbf{h}_4:\Delta_1\vdash\mathbf{F}_2} \ \overset{\mathrm{ax/ind}}{\overset{\mathrm{H}}{\to}}$$

• Case rule \oplus_L

$$\begin{array}{c} \underline{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_5,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_7 \quad \mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_6,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_7} \\ \bullet \mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2 \multimap \mathbf{F}_3),\mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array} \oplus L \qquad \\ \begin{array}{c} \underline{\mathbf{h}_4:\Delta_8,\mathbf{F}_5 \vdash \mathbf{F}_2} \quad \mathrm{ax/ind} \quad \overline{\mathbf{h}_4:\Delta_8,\mathbf{F}_6 \vdash \mathbf{F}_2} \\ \bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_2 \end{array} \end{array} \xrightarrow{\bullet} \underline{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_6 \vdash \mathbf{F}_2} \\ \underline{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_5,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_7 \quad \mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_6,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_7} \\ \bullet \mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2 \multimap \mathbf{F}_3),\mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array} \oplus L \qquad \\ \begin{array}{c} \underline{\mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2} \\ \bullet \mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2 \end{array} \xrightarrow{\mathbf{ax/ind}} \underline{\mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2} \\ \bullet \mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2 \end{array} \xrightarrow{\mathbf{ax/ind}} \underline{\mathbf{h}_4:\Delta_1 \vdash \mathbf{F}_2} \overset{\mathbf{ax/ind}}{\vdash} \underline{\mathbf{h}_4:\Delta_1 \vdash \mathbf{h}_4:\Delta_1 \vdash \mathbf{h}_4:\Delta_1$$

• Case rule \multimap_L

$$\begin{array}{l} \frac{\mathbf{h}_{3}:\Delta_{7},\Delta_{8},\mathbf{F}_{1} \multimap \mathbf{F}_{2} \vdash \mathbf{F}_{4} \quad \mathbf{h}_{3}:\Delta_{9},\Delta_{10},\mathbf{F}_{5} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{3}:(\Delta_{7},\Delta_{8},\mathbf{F}_{1} \multimap \mathbf{F}_{2}),(\Delta_{9},\Delta_{10}),\mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{6}} \quad \lnot_{L} \qquad \qquad \bullet \mathbf{h}_{3}:\Delta_{7},\Delta_{9},\mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{1}} \quad \mathbf{fail} \\ \frac{\mathbf{h}_{3}:\Delta_{7},\Delta_{8} \vdash \mathbf{F}_{4} \quad \mathbf{h}_{3}:\Delta_{9},\Delta_{10},\mathbf{F}_{5},\mathbf{F}_{1} \multimap \mathbf{F}_{2} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{3}:(\Delta_{7},\Delta_{8}),(\Delta_{9},\Delta_{10},\mathbf{F}_{1} \multimap \mathbf{F}_{2}),\mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{6}}} \quad \lnot_{L} \qquad \qquad \bullet \mathbf{h}_{3}:\Delta_{7},\Delta_{9},\mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{1}} \quad \mathbf{fail} \\ \frac{\mathbf{h}_{3}:\Delta_{7},\Delta_{8},\mathbf{F}_{1} \multimap \mathbf{F}_{2} \vdash \mathbf{F}_{4} \quad \mathbf{h}_{3}:\Delta_{9},\Delta_{10},\mathbf{F}_{5} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{3}:(\Delta_{7},\Delta_{8},\mathbf{F}_{1} \multimap \mathbf{F}_{2}),(\Delta_{9},\Delta_{10}),\mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{6}}} \quad \lnot_{L} \qquad \qquad \bullet \mathbf{h}_{3}:\Delta_{7},\Delta_{9} \vdash \mathbf{F}_{1}} \quad \mathbf{fail} \\ \frac{\mathbf{h}_{3}:\Delta_{7},\Delta_{8} \vdash \mathbf{F}_{4} \quad \mathbf{h}_{3}:\Delta_{9},\Delta_{10},\mathbf{F}_{5},\mathbf{F}_{1} \multimap \mathbf{F}_{2} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{3}:(\Delta_{7},\Delta_{8}),(\Delta_{9},\Delta_{10},\mathbf{F}_{1} \multimap \mathbf{F}_{2}),\mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{6}}} \quad \lnot_{L} \qquad \qquad \bullet \mathbf{h}_{3}:\Delta_{7},\Delta_{9} \vdash \mathbf{F}_{1}} \quad \mathbf{fail} \\ \frac{\mathbf{h}_{1}:\Delta_{5},\Delta_{6} \vdash \mathbf{F}_{2} \quad \mathbf{h}_{1}:\Delta_{7},\Delta_{8},\mathbf{F}_{1} \multimap \mathbf{F}_{2}),\mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{3}:(\Delta_{7},\Delta_{8}),(\Delta_{9},\Delta_{10},\mathbf{F}_{1} \multimap \mathbf{F}_{2}),\mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{6}}} \quad \lnot_{L} \qquad \qquad \bullet \mathbf{h}_{3}:\Delta_{7},\Delta_{9} \vdash \mathbf{F}_{1}} \quad \mathbf{fail} \\ \frac{\mathbf{h}_{1}:\Delta_{5},\Delta_{6} \vdash \mathbf{F}_{2} \quad \mathbf{h}_{1}:\Delta_{7},\Delta_{8},\mathbf{F}_{1} \multimap \mathbf{F}_{2}),\mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:(\Delta_{5},\Delta_{6}),(\Delta_{7},\Delta_{8}),\mathbf{F}_{2} \multimap \mathbf{F}_{3} \vdash \mathbf{F}_{4}}} \quad \lnot_{L} \qquad \qquad \bullet \mathbf{h}_{1}:\Delta_{5},\Delta_{7} \vdash \mathbf{F}_{2}} \quad \mathbf{fail} \\ \frac{\mathbf{h}_{1}:\Delta_{5},\Delta_{6} \vdash \mathbf{F}_{2} \quad \mathbf{h}_{1}:\Delta_{7},\Delta_{8},\mathbf{F}_{1} \multimap \mathbf{F}_{2} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:(\Delta_{5},\Delta_{6}),(\Delta_{7},\Delta_{8}),\mathbf{F}_{2} \multimap \mathbf{F}_{3} \vdash \mathbf{F}_{4}}} \quad \lnot_{L} \qquad \qquad \bullet \mathbf{h}_{1}:\Delta_{5},\Delta_{7} \vdash \mathbf{F}_{2}} \quad \mathbf{fail} \\ \frac{\mathbf{h}_{1}:\Delta_{5},\Delta_{6} \vdash \mathbf{F}_{2} \quad \mathbf{h}_{1}:\Delta_{7},\Delta_{8},\mathbf{F}_{1} \multimap \mathbf{F}_{2} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{7},\Delta_{8},\mathbf{F}_{1} \multimap \mathbf{F}_{2} \vdash \mathbf{F}_{6}} \quad \lnot_{L} \qquad \qquad \bullet \mathbf{h}_{1}:\Delta_{5},\Delta_{7} \vdash \mathbf{F}_{2}} \quad \bullet \mathbf{h}_{1}:\Delta_{7},\Delta_{8} \vdash \mathbf{h}_{1} \rightarrow \mathbf{h}_{1}:\Delta_{7},\Delta_{8} \vdash \mathbf{h}_{1} \rightarrow \mathbf{h}_{1} \rightarrow$$

ullet Case rule I

2.21 Status of \multimap_L (Right Premise): : Non invertible

- Case rule !R
- Case rule $\mathbf{1}_{R}$
- \bullet Case rule \top

$$\frac{}{\bullet \mathsf{h}_5 : \Delta_1, \Delta_2, \mathsf{F}_3 \multimap \mathsf{F}_4 \vdash \top} \ \top \qquad \leadsto \qquad \frac{}{\bullet \mathsf{h}_5 : \Delta_2, \mathsf{F}_4 \vdash \top} \ \top$$

 \bullet Case rule $\&_R$

$$\frac{\mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3\multimap\mathbf{F}_4\vdash\mathbf{F}_6\quad\mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3\multimap\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3\multimap\mathbf{F}_4\vdash\mathbf{F}_6\&\mathbf{F}_7}\quad\&_R\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_5:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_6}\quad\text{ax/ind}\quad\overline{\mathbf{h}_5:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_7}}{\bullet\mathbf{h}_5:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_6\&\mathbf{F}_7}\quad\&_R$$

• Case rule \multimap_R

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3 \multimap \mathbf{F}_4 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3 \multimap \mathbf{F}_4 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \ \oplus_{R_2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5:\Delta_2,\mathbf{F}_4 \vdash \mathbf{F}_7}}{\bullet \mathbf{h}_5:\Delta_2,\mathbf{F}_4 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \ \oplus_{R_2}$$

• Case rule \bigoplus_{R_1}

$$\frac{\mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3 \multimap \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_5:\Delta_1,\Delta_2,\mathbf{F}_3 \multimap \mathbf{F}_4 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \ \oplus_{R_1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5:\Delta_2,\mathbf{F}_4 \vdash \mathbf{F}_6}}{\bullet \mathbf{h}_5:\Delta_2,\mathbf{F}_4 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \ \oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_6,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_1,\Delta_6,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_5} \ \mathbf{1}_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_6,\mathbf{F}_3 \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_6,\mathbf{F}_3 \vdash \mathbf{F}_5} \ \mathbf{1}_L$$

• Case rule \otimes_R

$$\begin{array}{l} \underline{\mathbf{h}}_3: \Delta_6, \Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_3: \Delta_8, \Delta_9 \vdash \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_3: (\Delta_6, \Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2), \Delta_8, \Delta_9 \vdash \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \otimes_R \\ \qquad \leadsto \qquad \begin{array}{l} \bullet \mathbf{h}_3: \Delta_7, \Delta_9, \mathbf{F}_2 \vdash \mathbf{F}_4 \otimes \mathbf{F}_5 \\ \hline \end{array} \quad \text{fail}$$

$$\begin{array}{ll} \frac{\mathbf{h}_3:\Delta_6,\Delta_7 \vdash \mathbf{F}_4 \quad \mathbf{h}_3:\Delta_8,\Delta_9,\mathbf{F}_1 \multimap \mathbf{F}_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3:(\Delta_6,\Delta_7),\Delta_8,\Delta_9,\mathbf{F}_1 \multimap \mathbf{F}_2 \vdash \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes_R \qquad \leadsto \qquad \overline{\bullet \mathbf{h}_3:\Delta_7,\Delta_9,\mathbf{F}_2 \vdash \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \mathbf{fail}$$

 \bullet Case rule W

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_7,\mathbf{F}_2 \multimap \mathbf{F}_3),!\mathbf{F}_5 \vdash \mathbf{F}_6} \ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3 \vdash \mathbf{F}_6}}{\bullet \mathbf{h}_4:\Delta_1,\mathbf{F}_3 \vdash \mathbf{F}_6} \ \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7,\mathbf{F}_2\multimap\mathbf{F}_3),!\mathbf{F}_5\vdash\mathbf{F}_6}\ W \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7,\mathbf{F}_3\vdash\mathbf{F}_6}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_4:\Delta_7,\mathbf{F}_3,!\mathbf{F}_5\vdash\mathbf{F}_6}\ W$$

 \bullet Case rule C

$$\begin{array}{ll} \frac{\mathbf{h}_4:\Delta_1,\Delta_7,!\mathbf{F}_5,!\mathbf{F}_5,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7,\mathbf{F}_2\multimap\mathbf{F}_3),!\mathbf{F}_5\vdash\mathbf{F}_6} & C & \longrightarrow & \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_6} & \mathbf{H} \end{array}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbb{I}_5,\mathbb{I}_5,\mathbb{I}_5,\mathbb{F}_2-\mathbf{o}\,\mathbb{F}_3\vdash\mathbb{F}_6}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_7,\mathbb{F}_2-\mathbf{o}\,\mathbb{F}_3),\mathbb{I}_5\vdash\mathbb{F}_6} \ C \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7,\mathbb{F}_3\vdash\mathbb{F}_6}}{\bullet \mathbf{h}_4:\Delta_7,\mathbb{F}_3,\mathbb{I}_5\vdash\mathbb{F}_6} \ W$$

 \bullet Case rule !L

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_5,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_7,\mathbf{F}_2 \multimap \mathbf{F}_3),|\mathbf{F}_5 \vdash \mathbf{F}_6} : L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3 \vdash \mathbf{F}_6}}{\bullet \mathbf{h}_4:\Delta_1,\mathbf{F}_3 \vdash \mathbf{F}_6} \overset{\mathrm{ax/ind}}{\vdash \mathbf{h}_4:\Delta_1,\mathbf{F}_3 \vdash \mathbf{F}_6}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_7,\mathbf{F}_5,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_6}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_7,\mathbf{F}_2\multimap\mathbf{F}_3),|\mathbf{F}_5\vdash\mathbf{F}_6}} \ !L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_7,\mathbf{F}_3\vdash\mathbf{F}_6}}{\bullet\mathbf{h}_4:\Delta_7,\mathbf{F}_3,|\mathbf{F}_5\vdash\mathbf{F}_6}} \ ^{\mathrm{ax/ind}}W$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_6,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_7}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2\multimap\mathbf{F}_3),\mathbf{F}_5\&\mathbf{F}_6\vdash\mathbf{F}_7} \quad\&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_7}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_7} \quad \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathtt{h}_4:\Delta_1,\Delta_8,\mathtt{F}_6,\mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4:(\Delta_1,\Delta_8,\mathtt{F}_2 \multimap \mathtt{F}_3),\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7} \ \&_{L2} \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_4:\Delta_8,\mathtt{F}_3,\mathtt{F}_6 \vdash \mathtt{F}_7}}{\bullet \mathtt{h}_4:\Delta_8,\mathtt{F}_3,\mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7} \ \&_{L2}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_4 : \Delta_1, \Delta_8, \mathbf{f}_5, \mathbf{f}_2 \multimap \mathbf{f}_3 \vdash \mathbf{f}_7}{\bullet \mathbf{h}_4 : (\Delta_1, \Delta_8, \mathbf{f}_2 \multimap \mathbf{f}_3), \mathbf{f}_5 \& \mathbf{f}_6 \vdash \mathbf{f}_7} \quad \&_{L1} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4 : \Delta_8, \mathbf{f}_3, \mathbf{f}_5 \vdash \mathbf{f}_7}}{\bullet \mathbf{h}_4 : \Delta_8, \mathbf{f}_3, \mathbf{f}_5 \& \mathbf{f}_6 \vdash \mathbf{f}_7} \quad \&_{L1}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_5,\mathbf{F}_6,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_7}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2\multimap\mathbf{F}_3),\mathbf{F}_5\otimes\mathbf{F}_6\vdash\mathbf{F}_7}\otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_7}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\mathbf{F}_7} \overset{\mathrm{ax/ind}}{\vdash}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_5,\mathbf{F}_6,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2 \multimap \mathbf{F}_3),\mathbf{F}_5 \otimes \mathbf{F}_6 \vdash \mathbf{F}_7} \otimes_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_8,\mathbf{F}_3,\mathbf{F}_5,\mathbf{F}_6 \vdash \mathbf{F}_7}}{\bullet \mathbf{h}_4:\Delta_8,\mathbf{F}_3,\mathbf{F}_5 \otimes \mathbf{F}_6 \vdash \mathbf{F}_7} \otimes_L$$

• Case rule \oplus_L

$$\begin{array}{c} \mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_5,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_7 \quad \mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_6,\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2 \multimap \mathbf{F}_3),\mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array} \oplus_{\mathbf{h}_4:\Delta_1,\mathbf{F}_3 \vdash \mathbf{F}_7} \begin{array}{c} \mathbf{a}_4 \text{ 'ad}_1,\mathbf{F}_3 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_4:\Delta_1,\mathbf{F}_3 \vdash \mathbf{F}_7 \end{array} \end{array}$$

$$\frac{\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_5,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_7\quad\mathbf{h}_4:\Delta_1,\Delta_8,\mathbf{F}_6,\mathbf{F}_2\multimap\mathbf{F}_3\vdash\mathbf{F}_7}{\bullet\mathbf{h}_4:(\Delta_1,\Delta_8,\mathbf{F}_2\multimap\mathbf{F}_3),\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_7}\ \oplus_L\qquad \leadsto\qquad \frac{\frac{\mathbf{h}_4:\Delta_8,\mathbf{F}_3,\mathbf{F}_5\vdash\mathbf{F}_7}{\bullet\mathbf{h}_4:\Delta_8,\mathbf{F}_3,\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_7}\ \frac{\mathbf{ax/ind}}{\mathbf{h}_4:\Delta_8,\mathbf{F}_3,\mathbf{F}_5\oplus\mathbf{F}_6\vdash\mathbf{F}_7}\ \oplus_L$$

• Case rule \multimap_L

$$\begin{array}{l} \underline{\mathbf{h}}_3:\Delta_7,\Delta_8,\mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \mathbf{F}_4 \quad \underline{\mathbf{h}}_3:\Delta_9,\Delta_{10},\mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_3:(\Delta_7,\Delta_8,\mathbf{F}_1 \rightarrow \mathbf{F}_2),(\Delta_9,\Delta_{10}),\mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \mathbf{F}_6 \end{array} \rightarrow \mathbf{0}_L \qquad \leadsto \qquad \begin{array}{l} \bullet \mathbf{h}_3:\Delta_8,\Delta_{10},\mathbf{F}_2 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_3:\Delta_8,\Delta_{10},\mathbf{F}_2 \vdash \mathbf{F}_6 \end{array}$$

$$\begin{array}{lll} & & & & \\ \mathbf{h}_3:\Delta_7,\Delta_8 \vdash \mathbf{F}_4 & \mathbf{h}_3:\Delta_9,\Delta_{10},\mathbf{F}_5,\mathbf{F}_1 \multimap \mathbf{F}_2 \vdash \mathbf{F}_6 \\ & & & \\ \bullet \mathbf{h}_3:(\Delta_7,\Delta_8),(\Delta_9,\Delta_{10},\mathbf{F}_1 \multimap \mathbf{F}_2),\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_6 \end{array} \multimap_L \qquad \leadsto \qquad \begin{array}{ll} & & & \\ \bullet \mathbf{h}_3:\Delta_8,\Delta_{10},\mathbf{F}_2 \vdash \mathbf{F}_6 \end{array} \quad \text{fail}$$

$$\begin{array}{ll} \frac{\mathbf{h}_3:\Delta_7,\Delta_8,\mathbf{F}_1 \multimap \mathbf{F}_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_3:\Delta_9,\Delta_{10},\mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3:(\Delta_7,\Delta_8,\mathbf{F}_1 \multimap \mathbf{F}_2),(\Delta_9,\Delta_{10}),\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_6} \end{array} \quad \multimap_L \qquad \leadsto \qquad \begin{array}{ll} \bullet \mathbf{h}_3:\Delta_8,\Delta_{10},\mathbf{F}_2,\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_6} \end{array} \quad \mathbf{fail}$$

$$\begin{array}{lll} \underline{\mathbf{h}}_3:\Delta_7,\Delta_8 \vdash \mathbf{F}_4 & \underline{\mathbf{h}}_3:\Delta_9,\Delta_{10},\mathbf{F}_5,\mathbf{F}_1 \multimap \mathbf{F}_2 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_3:(\Delta_7,\Delta_8),(\Delta_9,\Delta_{10},\mathbf{F}_1 \multimap \mathbf{F}_2),\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_6 \end{array} \ \multimap_L \qquad \leadsto \qquad \overline{\bullet \mathbf{h}_3:\Delta_8,\Delta_{10},\mathbf{F}_2,\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_6} \ \text{fail}$$

$$\begin{array}{lll} \frac{\mathbf{h}_1:\Delta_5,\Delta_6 \vdash \mathbf{F}_2 & \mathbf{h}_1:\Delta_7,\Delta_8,\mathbf{F}_3 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:(\Delta_5,\Delta_6),(\Delta_7,\Delta_8),\mathbf{F}_2 \multimap \mathbf{F}_3 \vdash \mathbf{F}_4} & \multimap_L & \longrightarrow & \hline {\bullet \mathbf{h}_1:\Delta_6,\Delta_8,\mathbf{F}_3 \vdash \mathbf{F}_4} & \mathtt{fail} \end{array}$$

 \bullet Case rule I

2.22 Status of I: : Invertible

- $\bullet \;$ Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \bigoplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet \;$ Case rule W
- $\bullet\,$ Case rule C
- \bullet Case rule !L
- Case rule $\&_{L2}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- ullet Case rule I

 $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$

3 Identity-Expansion

$$\begin{array}{c|c} \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline -: F_0 \vdash F_0 \oplus F_1 \\ \hline \hline -: F_0 \vdash F_0 \oplus F_1 \\ \hline \hline -: F_0 \vdash F_0 \oplus F_1 \\ \hline \hline -: F_0 \vdash F_0 \oplus F_1 \\ \hline \hline -: F_0 \vdash F_0 \\ \hline \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_0 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline \hline \\ -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline \hline \\ \hline -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ \hline \hline \\ \hline \hline \\ -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\ -: F_0 \lor F_1 \vdash F_1 \\ \hline \hline \\$$

4 Weakening on bang: $\Gamma \vdash !F$ implies $\Gamma \vdash F$.

• Case(s) rule !R

- Case(s) rule $\mathbf{1}_R$
- \bullet Case(s) rule \top
- Case(s) rule $\&_R$
- Case(s) rule \multimap_R
- Case(s) rule \bigoplus_{R_2}
- Case(s) rule \bigoplus_{R_1}
- Case(s) rule $\mathbf{1}_L$

- Case(s) rule \otimes_R
- \bullet Case(s) rule W

• Case(s) rule C

• Case(s) rule !L

• Case(s) rule $\&_{L2}$

$$\frac{\underset{\bullet}{\mathbf{h}_2:\Delta_3,\mathsf{F}_5\vdash !\mathsf{F}_1}{\mathsf{h}_2:\Delta_3,\mathsf{F}_4\&\mathsf{F}_5\vdash !\mathsf{F}_1}}{\underset{\bullet}{\mathbf{h}_2:\Delta_3,\mathsf{F}_4\&\mathsf{F}_5\vdash \mathsf{F}_1}{\mathsf{h}_2:\Delta_3,\mathsf{F}_4\&\mathsf{F}_5\vdash \mathsf{F}_1}} \overset{\mathsf{ax}}{\underset{\mathsf{IH}}{\overset{\mathsf{h}_2:\Delta_3,\mathsf{F}_5\vdash !\mathsf{F}_1}{\mathsf{h}_2:\Delta_3,\mathsf{F}_4\&\mathsf{F}_5\vdash \mathsf{F}_1}}} \&_{L2}$$

• Case(s) rule $\&_{L1}$

$$\frac{\underset{\bullet}{\mathbf{h}_2:\Delta_3, \mathsf{F}_4 \vdash !\mathsf{F}_1}{\mathsf{h}_2:\Delta_3, \mathsf{F}_4 \& \mathsf{F}_5 \vdash !\mathsf{F}_1}}{\underset{\bullet}{\mathbf{h}_2:\Delta_3, \mathsf{F}_4 \& \mathsf{F}_5 \vdash !\mathsf{F}_1}{\mathsf{h}_2:\Delta_3, \mathsf{F}_4 \& \mathsf{F}_5 \vdash \mathsf{F}_1}} \overset{\mathsf{ax}}{\underset{\bullet}{\mathsf{h}_2:\Delta_3, \mathsf{F}_4 \& \mathsf{F}_5 \vdash \mathsf{F}_1}{\mathsf{h}_2:\Delta_3, \mathsf{F}_4 \& \mathsf{F}_5 \vdash \mathsf{F}_1}} \&_{L1}$$

• Case(s) rule \otimes_L

$$\begin{array}{c} \mathbf{h}_2:\Delta_3,\mathbf{F}_4,\mathbf{F}_5\vdash \mathbf{l}\mathbf{F}_1\\ \hline \bullet \mathbf{h}_2:\Delta_3,\mathbf{F}_4\otimes \mathbf{F}_5\vdash \mathbf{l}\mathbf{F}_1 \end{array} \otimes_L \qquad \leadsto \qquad \begin{array}{c} \overline{\mathbf{h}_2:\Delta_3,\mathbf{F}_4,\mathbf{F}_5\vdash \mathbf{l}\mathbf{F}_1}\\ \overline{\mathbf{h}_2:\Delta_3,\mathbf{F}_4,\mathbf{F}_5\vdash \mathbf{F}_1} \end{array} \overset{\mathrm{ax}}{\underset{\mathrm{lift}}{\mathbf{l}}} \otimes_L \\ \hline \bullet \mathbf{h}_2:\Delta_3,\mathbf{F}_4\otimes \mathbf{F}_5\vdash \mathbf{F}_1 \end{array}$$

• Case(s) rule \oplus_L

$$\frac{\mathbf{h}_2:\Delta_3,\mathbf{f}_4\vdash \mathbf{l}\mathbf{f}_1\quad \mathbf{h}_2:\Delta_3,\mathbf{f}_5\vdash \mathbf{l}\mathbf{f}_1}{\bullet \mathbf{h}_2:\Delta_3,\mathbf{f}_4\oplus \mathbf{f}_5\vdash \mathbf{l}\mathbf{f}_1}\ \oplus_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_2:\Delta_3,\mathbf{f}_4\vdash \mathbf{l}\mathbf{f}_1}{\mathbf{h}_2:\Delta_3,\mathbf{f}_4\vdash \mathbf{l}\mathbf{f}_1}\ \frac{\mathbf{h}}{\mathbf{h}_2:\Delta_3,\mathbf{f}_5\vdash \mathbf{l}\mathbf{f}_1}\ \frac{\mathbf{h}}{\mathbf{h}_2:\Delta_3,\mathbf{f}_5\vdash \mathbf{l}\mathbf{f}_1}\ \frac{\mathbf{h}}{\mathbf{h}_2:\Delta_3,\mathbf{f}_5\vdash \mathbf{l}\mathbf{f}_1}\ \oplus_L$$

• Case(s) rule \multimap_L

$$\underbrace{ \begin{array}{c} \mathbf{h}_2 : \Delta_3 \vdash \mathbf{F}_5 \quad \mathbf{h}_2 : \Delta_4, \mathbf{F}_6 \vdash \mathsf{lF}_1 \\ \bullet \mathbf{h}_2 : \Delta_3, \Delta_4, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathsf{lF}_1 \end{array}}_{\bullet \mathbf{h}_2 : \Delta_3, \Delta_4, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathsf{lF}_1} \ \, \stackrel{\mathbf{ax}}{\multimap}_L \qquad \\ \stackrel{\bullet}{\longrightarrow} \underbrace{ \begin{array}{c} \mathbf{h}_2 : \Delta_3 \vdash \mathbf{F}_5 \quad \mathbf{ax} \quad \frac{\mathbf{h}_2 : \Delta_4, \mathbf{F}_6 \vdash \mathsf{lF}_1}{\mathbf{h}_2 : \Delta_4, \mathbf{F}_6 \vdash \mathbf{F}_1} \quad \\ \bullet \mathbf{h}_2 : \Delta_3, \Delta_4, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_1 \end{array}} \ \, \stackrel{\mathbf{ax}}{\longrightarrow} L$$

• Case(s) rule I

5 Cut-Elimination

5.1 Status of !R: OK

• Case rule !R

$$\begin{array}{c} \frac{\mathbf{h}_{1}: ! \Upsilon 2 \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{5}} : R & \frac{\mathbf{h}_{6}: ! \Upsilon 4, ! \mathbf{F}_{5} \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{6}: ! \Upsilon 4, ! \mathbf{F}_{5} \vdash ! \mathbf{F}_{7}} & ! R \\ \hline - : ! \Upsilon 2, ! \Upsilon 4 \vdash ! \mathbf{F}_{7} & \cdots \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{5} & \mathbf{ax/W} & \overset{\bullet}{h}_{6}: ! \Upsilon 4, ! \mathbf{F}_{5} \vdash \mathbf{F}_{7} \\ \hline - : ! \Upsilon 2, ! \Upsilon 4 \vdash \mathbf{F}_{7} \\ \hline - : ! \Upsilon 2, ! \Upsilon 4 \vdash ! \mathbf{F}_{7} & ! R \end{array}$$

- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1: ! \Upsilon 2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: ! \Upsilon 2 \vdash ! \mathbf{F}_5} : R & \frac{}{\bullet \mathbf{h}_6: \Delta_4, ! \mathbf{F}_5 \vdash \top} & \top \\ & -: ! \Upsilon 2, \Delta_4 \vdash \top & \\ \hline & -: ! \Upsilon 2, \Delta_4 \vdash \top & \top \\ \end{array}$$

• Case rule $\&_R$

$$\frac{\underbrace{\frac{\mathbf{h}_1 : ! \Upsilon 2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_5}}_{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_5} :_{R} \underbrace{\frac{\mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_7 \quad \mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_7 \& \mathbf{F}_8}}_{\bullet \mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_7 \& \mathbf{F}_8} \underbrace{\mathbf{Cut}}_{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_5} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_8}}_{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_5} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_8}}_{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_5} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_8}}_{\bullet \mathbf{h}_1 : \mathbf{h}_1 : \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_8}}_{\bullet \mathbf{h}_1 : \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_8}}_{\bullet \mathbf{h}_1 : \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_8}}_{\bullet \mathbf{h}_1 : \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_2 \vdash \mathbf{h}_3} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_3 \vdash \mathbf{h}_4 \vdash \mathbf{h}_5} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_3 \vdash \mathbf{h}_4 \vdash \mathbf{h}_5} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_3 \vdash \mathbf{h}_4 \vdash \mathbf{h}_5} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_6 : \Delta_4, ! \mathbf{h}_5 \vdash \mathbf{h}_8}}_{\bullet \mathbf{h}_3 \vdash \mathbf{h}_5}$$

• Case rule \multimap_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : ! \Upsilon 2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_5} : R & \frac{\mathbf{h}_6 : \Delta_4, \mathbf{F}_7, ! \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6 : \Delta_4, ! \mathbf{F}_5 \vdash \mathbf{F}_7 \multimap \mathbf{F}_8} \\ \hline - : ! \Upsilon 2, \Delta_4 \vdash \mathbf{F}_7 \multimap \mathbf{F}_8 \\ \hline \frac{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_5}{\bullet} & \text{ax/W} & \frac{\leadsto}{\mathbf{h}_6 : \Delta_4, \mathbf{F}_7, ! \mathbf{F}_5 \vdash \mathbf{F}_8} \\ \hline \frac{- : ! \Upsilon 2, \Delta_4, \mathbf{F}_7 \vdash \mathbf{F}_8}{- : ! \Upsilon 2, \Delta_4 \vdash \mathbf{F}_7 \multimap \mathbf{F}_8} & - \circ_R \end{array} \right. \\ \begin{array}{c} \bullet \mathbf{h}_1 : \mathbf{h}_1 : \mathbf{h}_1 : \mathbf{h}_2 \vdash \mathbf{h}_3 \\ \hline \bullet \mathbf{h}_1 : \mathbf{h}_2 \vdash \mathbf{h}_3 \vdash \mathbf{h}_4 \vdash \mathbf{h}_4 \vdash \mathbf{h}_5 \vdash \mathbf{h}_8 \\ \hline - : \mathbf{h}_2 : \mathbf{h}_3 \vdash \mathbf{h}_4 \vdash \mathbf{h}_5 \vdash \mathbf{h}_8 \\ \hline - : \mathbf{h}_3 : \mathbf{h}_4 \vdash \mathbf{h}_5 \vdash \mathbf{h}_8 \vdash \mathbf{h}_4 \vdash \mathbf{h}_5 \vdash \mathbf{h}_8 \\ \hline - : \mathbf{h}_3 : \mathbf{h}_4 \vdash \mathbf{h}_5 \vdash \mathbf{h}_8 \vdash \mathbf{h}_6 \vdash \mathbf{h}_8 \vdash$$

• Case rule \oplus_{R_2}

$$\frac{ \begin{array}{c} \frac{\mathbf{h}_1: ! \Upsilon 2 \vdash F_5}{\bullet \mathbf{h}_1: ! \Upsilon 2 \vdash ! F_5} : R & \frac{\mathbf{h}_6: \Delta_4, ! F_5 \vdash F_8}{\bullet \mathbf{h}_6: \Delta_4, ! F_5 \vdash F_7 \oplus F_8} \\ \hline -: ! \Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8 \\ \hline \frac{\bullet \mathbf{h}_1: ! \Upsilon 2 \vdash ! F_5}{\bullet \mathbf{h}_1: ! \Upsilon 2 \vdash ! F_5} & \mathbf{ax/W} & \overbrace{\mathbf{h}_6: \Delta_4, ! F_5 \vdash F_8}^{\bullet \mathbf{h}_1: ! \Upsilon 2 \vdash ! F_5} & \mathbf{ax/W} \\ \hline \frac{-: ! \Upsilon 2, \Delta_4 \vdash F_8}{-: ! \Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8} & \oplus_{R_2} \end{array} \begin{array}{c} \oplus_{R_2} \end{array}$$

• Case rule \bigoplus_{R_1}

$$\frac{ \begin{array}{c} \mathbf{h}_1 : ! \Upsilon 2 \vdash F_5 \\ \bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_5 \end{array} : R \quad \frac{\mathbf{h}_6 : \Delta_4, ! F_5 \vdash F_7}{\bullet \mathbf{h}_6 : \Delta_4, ! F_5 \vdash F_7 \oplus F_8} \quad \begin{array}{c} \oplus_{R_1} \\ \text{Cut} \end{array} } \\ \hline \\ - : ! \Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8 \\ \hline \\ \bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_5} \quad \text{ax/W} \quad \stackrel{\longleftarrow}{\stackrel{\longleftarrow}{\mathbf{h}_6 : \Delta_4, ! F_5 \vdash F_7}} \quad \underset{h\text{Cut}}{\overset{\text{ax/W}}{\bullet}} \\ \hline \\ - : ! \Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8 \quad \oplus_{R_1} \end{array} \right.$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c} \frac{\mathbf{h}_1: !\Upsilon2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1: !\Upsilon2 \vdash !\mathbf{F}_4} & !R & \frac{\mathbf{h}_5: \Delta_7, !\mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_5: (\mathbf{1}, \Delta_7), !\mathbf{F}_4 \vdash \mathbf{F}_6} & \mathbf{Cut} \\ \hline -: !\Upsilon2, \mathbf{1}, \Delta_7 \vdash \mathbf{F}_6 & \\ \hline \frac{\bullet \mathbf{h}_1: !\Upsilon2 \vdash !\mathbf{F}_4}{\bullet \mathbf{h}_5: \mathbf{1}, \Delta_7, !\mathbf{F}_4 \vdash \mathbf{F}_6} & \mathbf{ax/W} \\ \hline -: \mathbf{1}, !\Upsilon2, \Delta_7 \vdash \mathbf{F}_6 & \\ \hline \end{pmatrix}$$

• Case rule \otimes_R

 \bullet Case rule W

$$\begin{array}{c} \frac{\mathbf{h}_{1}: ! \Upsilon 2 \vdash \mathbf{F}_{4}}{\bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{4}} & ! R & \frac{\mathbf{h}_{5}: \Delta_{8}, ! \mathbf{F}_{4} \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{5}: (\Delta_{8}, ! \mathbf{F}_{6}), ! \mathbf{F}_{4} \vdash \mathbf{F}_{7}} & W \\ \hline & -: ! \Upsilon 2, \Delta_{8}, ! \mathbf{F}_{6} \vdash \mathbf{F}_{7} & \overset{\bullet}{\Box} \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{4} & \mathsf{ax/W} & \overset{\sim}{\Box} \\ \hline & -: ! \Upsilon 2, \Delta_{8}, ! \mathbf{F}_{6} \vdash \mathbf{F}_{7} & \mathsf{ax/W} \\ \hline & -: ! \Upsilon 2 \vdash \mathbf{F}_{6} & ! R & \frac{\mathbf{h}_{4}: \Delta_{5} \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{4}: \Delta_{5}, ! \mathbf{F}_{6} \vdash \mathbf{F}_{7}} & W \\ \hline & -: ! \Upsilon 2, \Delta_{5} \vdash \mathbf{F}_{7} & & \mathsf{Cut} \\ \hline & & & -: ! \Upsilon 2, \Delta_{5} \vdash \mathbf{F}_{7} & \mathsf{ax/W} \\ \hline \end{array}$$

ullet Case rule C

$$\frac{\frac{\mathbf{h}_{1}: !\Upsilon2 \vdash F_{4}}{\bullet \mathbf{h}_{1}: !\Upsilon2 \vdash !F_{4}} : R \quad \frac{\mathbf{h}_{5}: \Delta_{8}, !F_{4}, !F_{6}, !F_{6} \vdash F_{7}}{\bullet \mathbf{h}_{5}: (\Delta_{8}, !F_{6}), !F_{4} \vdash F_{7}} \quad C}{-: !\Upsilon2, \Delta_{8}, !F_{6} \vdash F_{7}} \quad \frac{C}{\cot}$$

$$\frac{\bullet \mathbf{h}_{1}: !\Upsilon2 \vdash !F_{4}}{\bullet \mathbf{h}_{1}: !\Upsilon2 \vdash !F_{4}} \quad \text{ax/W} \quad \frac{\sim}{\mathbf{h}_{5}: \Delta_{8}, !F_{4}, !F_{6}, !F_{6} \vdash F_{7}}}{-: !\Upsilon2, \Delta_{8}, !F_{6} \vdash F_{7}} \quad C$$

$$\frac{-: !\Upsilon2, \Delta_{8}, !F_{6} \vdash F_{7}}{-: !\Upsilon2, \Delta_{8}, !F_{6} \vdash F_{7}} \quad C$$

$$\frac{ \begin{array}{l} \frac{\mathbf{h}_{1} : ! \Upsilon 2 \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{6}} & !R & \frac{\mathbf{h}_{4} : \Delta_{5}, ! \mathbf{F}_{6}, ! \mathbf{F}_{6} \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{4} : \Delta_{5}, ! \mathbf{F}_{6} \vdash \mathbf{F}_{7}} & C \\ \hline & - : ! \Upsilon 2, \Delta_{5} \vdash \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{6} & \mathsf{ax/W} & \overset{\leadsto}{\mathbf{h}_{4} : \Delta_{5}, ! \mathbf{F}_{6}, ! \mathbf{F}_{6} \vdash \mathbf{F}_{7}} \\ \hline & - : ! \Upsilon 2, \Delta_{5} \vdash \mathbf{F}_{7} \end{array} \quad \text{ax/W} \\ \hline \quad Cut$$

• Case rule !L

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : ! \Upsilon 2 \vdash F_4}{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_4} & !R & \frac{\mathbf{h}_5 : \Delta_8, F_6, ! F_4 \vdash F_7}{\bullet \mathbf{h}_5 : (\Delta_8, ! F_6), ! F_4 \vdash F_7} \\ \hline - : ! \Upsilon 2, \Delta_8, ! F_6 \vdash F_7 & \\ \hline \frac{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_4}{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_4} & \mathbf{ax/W} & \frac{\sim}{\mathbf{h}_5 : \Delta_8, F_6, ! F_4 \vdash F_7} \\ \hline \frac{- : ! \Upsilon 2, \Delta_8, F_6 \vdash F_7}{- : ! \Upsilon 2, \Delta_8, ! F_6 \vdash F_7} & !L \\ \hline \frac{\mathbf{h}_1 : ! \Upsilon 2 \vdash F_6}{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_6} & !R & \frac{\mathbf{h}_4 : \Delta_5, F_6 \vdash F_7}{\bullet \mathbf{h}_4 : \Delta_5, ! F_6 \vdash F_7} & !L \\ \hline - : ! \Upsilon 2, \Delta_5 \vdash F_7 & \mathbf{cut} \\ \hline \hline - : ! \Upsilon 2 \vdash F_6 & \mathbf{ax/W} & \mathbf{cut} \\ \hline - : ! \Upsilon 2, \Delta_5 \vdash F_7 & \mathbf{ax/W} \\ \hline - : ! \Upsilon 2, \Delta_5 \vdash F_7 & \mathbf{ax/W} \\ \hline - : ! \Upsilon 2, \Delta_5 \vdash F_7 & \mathbf{cut} \\ \hline \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_1 : ! \Upsilon 2 \vdash F_4}{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_4} : R \quad \frac{\mathbf{h}_5 : \Delta_9, F_7, ! F_4 \vdash F_8}{\bullet \mathbf{h}_5 : (\Delta_9, F_6 \& F_7), ! F_4 \vdash F_8} \quad \&_{L2} \\ \hline \\ - : ! \Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \\ \hline \\ \bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_4 \quad \text{ax/W} \quad \overset{\hookrightarrow}{\underbrace{\begin{array}{c} \bullet \\ \mathbf{h}_5 : \Delta_9, F_7, ! F_4 \vdash F_8 \\ \hline - : ! \Upsilon 2, \Delta_9, F_7 \vdash F_8 \\ \hline - : ! \Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \end{array}} \quad &_{hCut} \end{array}$$

• Case rule $\&_{L1}$

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : ! \Upsilon 2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_4} & !R & \frac{\mathbf{h}_5 : \Delta_9, \mathbf{F}_6, ! \mathbf{F}_4 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5 : (\Delta_9, \mathbf{F}_6 \& \mathbf{F}_7), ! \mathbf{F}_4 \vdash \mathbf{F}_8} & \&_{L1} \\ \hline \\ - : ! \Upsilon 2, \Delta_9, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_8 & \\ \hline \bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_4 & \mathsf{ax/W} & \overset{\leadsto}{\mathbf{h}_5 : \Delta_9, \mathbf{F}_6, ! \mathbf{F}_4 \vdash \mathbf{F}_8} \\ \hline \\ - : ! \Upsilon 2, \Delta_9, \mathbf{F}_6 \vdash \mathbf{F}_8 & \&_{L1} \\ \hline \\ - : ! \Upsilon 2, \Delta_9, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_8 & \&_{L1} \\ \hline \end{array}$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1: !\Upsilon2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1: !\Upsilon2 \vdash !\mathbf{F}_4} & !R & \frac{\mathbf{h}_5: \Delta_9, \mathbf{F}_6, \mathbf{F}_7, !\mathbf{F}_4 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5: (\Delta_9, \mathbf{F}_6 \otimes \mathbf{F}_7), !\mathbf{F}_4 \vdash \mathbf{F}_8} & \otimes L \\ \hline -: !\Upsilon2, \Delta_9, \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_8 & & \text{Cut} \\ \hline \bullet \mathbf{h}_1: !\Upsilon2 \vdash !\mathbf{F}_4 & \text{ax/W} & & & \\ \hline \bullet \mathbf{h}_5: \Delta_9, \mathbf{F}_6, \mathbf{F}_7, !\mathbf{F}_4 \vdash \mathbf{F}_8} & & \text{ax/W} \\ \hline -: !\Upsilon2, \Delta_9, \mathbf{F}_6, \mathbf{F}_7 \vdash \mathbf{F}_8} & \otimes L & & \text{hCut} \\ \hline -: !\Upsilon2, \Delta_9, \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_8 & \otimes L & & \\ \hline \end{array}$$

• Case rule \oplus_L

$$\frac{\underbrace{\frac{\mathbf{h}_{1} : ! \Upsilon 2 \vdash \mathbf{F}_{4}}{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4}}}_{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4}} : R \quad \underbrace{\frac{\mathbf{h}_{5} : \Delta_{9}, \mathbf{F}_{6}, ! \mathbf{F}_{4} \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{5} : (\Delta_{9}, \mathbf{F}_{6} \oplus \mathbf{F}_{7}), ! \mathbf{F}_{4} \vdash \mathbf{F}_{8}}}_{\bullet \mathbf{h}_{5} : (\Delta_{9}, \mathbf{F}_{6} \oplus \mathbf{F}_{7}), ! \mathbf{F}_{4} \vdash \mathbf{F}_{8}}} \underbrace{\mathbf{Cut}} \\ - : ! \Upsilon 2, \Delta_{9}, \mathbf{F}_{6} \oplus \mathbf{F}_{7} \vdash \mathbf{F}_{8}}_{\bullet \mathbf{h} \mathbf{Cut}} \underbrace{\underbrace{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4}}_{\bullet \mathbf{h}_{5} : \Delta_{9}, \mathbf{F}_{7}, ! \mathbf{F}_{4} \vdash \mathbf{F}_{8}}}_{\bullet \mathbf{h} \mathbf{Cut}} \underbrace{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4}}_{\bullet \mathbf{h}_{7} : ! \Upsilon 2, \Delta_{9}, \mathbf{F}_{7} \vdash \mathbf{F}_{8}}}_{\bullet \mathbf{h}_{7} : ! \Upsilon 2, \Delta_{9}, \mathbf{F}_{7} \vdash \mathbf{F}_{8}} \underbrace{\bullet \mathbf{h}_{8} : \Delta_{9}, \mathbf{h}_{7}, ! \mathbf{F}_{8} \vdash \mathbf{h}_{8}}_{\bullet \mathbf{h}_{8}}}_{\bullet \mathbf{h}_{8}} \underbrace{\bullet \mathbf{h}_{8} : \Delta_{9}, \mathbf{h}_{7}, ! \mathbf{h}_{8} \vdash \mathbf{h}_{8}}_{\bullet \mathbf{h}_{8}} \underbrace{\bullet \mathbf{h}_{8} : \Delta_{9}, \mathbf{h}_{7}, ! \mathbf{h}_{8} \vdash \mathbf{h}_{8}}_{\bullet \mathbf{h}_{8}} \underbrace{\bullet \mathbf{h}_{8} : \Delta_{9}, \mathbf{h}_{7}, ! \mathbf{h}_{8} \vdash \mathbf{h}_{8}}_{\bullet \mathbf{h}_{8}}}_{\bullet \mathbf{h}_{8}} \underbrace{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{h}_{4}}_{\bullet \mathbf{h}_{7} : \Delta_{9}, \mathbf{h}_{7}, ! \mathbf{h}_{8} \vdash \mathbf{h}_{8}}}_{\bullet \mathbf{h}_{8}} \underbrace{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{h}_{4}}_{\bullet \mathbf{h}_{9} : \Delta_{9}, \mathbf{h}_{7}, ! \mathbf{h}_{8} \vdash \mathbf{h}_{8}}}_{\bullet \mathbf{h}_{8}} \underbrace{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{h}_{4}}_{\bullet \mathbf{h}_{9} : \Delta_{9}, \mathbf{h}_{7}, ! \mathbf{h}_{8} \vdash \mathbf{h}_{8}}}_{\bullet \mathbf{h}_{9} : \Delta_{9}, \mathbf{h}_{7}, ! \mathbf{h}_{8} \vdash \mathbf{h}_{8}}$$

• Case rule \multimap_L

 $\bullet\,$ Case rule I

5.2 Status of 1_R : OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- Case rule \top

• Case rule $\&_R$

$$\frac{\underbrace{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}}_{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}} \ \mathbf{1}_R \ \ \frac{\mathbf{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : \Delta_2, \mathbf{1} \vdash \mathbf{F}_4 \& \mathbf{F}_5} \ \mathbf{Cut}} \ \&_R$$

$$\frac{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}}{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}} \ \frac{\mathbf{ax}/\mathbb{W}}{\mathbf{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathbf{F}_4} \ \frac{\mathbf{ax}/\mathbb{W}}{\mathbf{h}_1 : * \vdash \mathbf{1}} \ \frac{\mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}} \ \frac{\mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathbf{F}_5} \ \frac{\mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}} \ \frac{\mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_2 : \mathbf{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathbf{F}_5} \ \frac{\mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathbf{F}_5} \ \frac{\mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_2 : \mathbf{h}_3 : \mathbf{h}_$$

• Case rule \multimap_R

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1:\,*\vdash\mathbf{1}} & \mathbf{1}_R & \frac{\mathbf{h}_3:\,\mathbf{1},\,\Delta_2,\,\mathbf{F}_4\vdash\mathbf{F}_5}{\bullet\mathbf{h}_3:\,\Delta_2,\,\mathbf{1}\vdash\mathbf{F}_4\multimap\mathbf{F}_5} & -\circ_R \\ \hline & -:\,*,\,\Delta_2\vdash\mathbf{F}_4\multimap\mathbf{F}_5 & \text{Cut} \\ \hline \hline \bullet_{\mathbf{h}_1:\,*\vdash\mathbf{1}} & \text{ax/W} & \overset{\leadsto}{h_3:\,\mathbf{1},\,\Delta_2,\,\mathbf{F}_4\vdash\mathbf{F}_5} \\ \hline & \frac{-:\,\Delta_2,\,\mathbf{F}_4\vdash\mathbf{F}_5}{-:\,\Delta_2\vdash\mathbf{F}_4\multimap\mathbf{F}_5} & -\circ_R \end{array} \begin{array}{c} \mathsf{ax/W} \\ \mathsf{hCut} \end{array}$$

• Case rule \bigoplus_{R_2}

$$\begin{array}{c|c} \frac{\mathbf{h}_3: \mathbf{1}, \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3: \Delta_2, \mathbf{1} \vdash \mathbf{F}_4 \oplus \mathbf{F}_5} & \oplus_{R_2} \\ \hline -: *, \Delta_2 \vdash \mathbf{F}_4 \oplus \mathbf{F}_5 & \hookrightarrow \\ \hline \frac{\bullet \mathbf{h}_1: * \vdash \mathbf{1}}{\bullet \mathbf{h}_1: * \vdash \mathbf{1}} & \mathbf{1}_R & \xrightarrow{\mathbf{h}_3: \mathbf{1}, \Delta_2 \vdash \mathbf{F}_5} \\ \hline \frac{-: \Delta_2 \vdash \mathbf{F}_5}{-: \Delta_2 \vdash \mathbf{F}_4 \oplus \mathbf{F}_5} & \oplus_{R_2} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \bigoplus_{R_1}

$$\begin{array}{c|c} & \mathbf{h}_{1}: * \vdash \mathbf{1} & \mathbf{1}_{R} & \frac{\mathbf{h}_{3}: \mathbf{1}, \Delta_{2} \vdash \mathbf{F}_{4}}{\bullet \mathbf{h}_{3}: \Delta_{2}, \mathbf{1} \vdash \mathbf{F}_{4} \oplus \mathbf{F}_{5}} & \oplus_{R_{1}} \\ \hline & -: *, \Delta_{2} \vdash \mathbf{F}_{4} \oplus \mathbf{F}_{5} & & \text{Cut} \\ \hline & \frac{\bullet \mathbf{h}_{1}: * \vdash \mathbf{1}}{\bullet \mathbf{h}_{1}: * \vdash \mathbf{1}} & \mathbf{1}_{R} & & & \\ \hline & \frac{-: \Delta_{2} \vdash \mathbf{F}_{4}}{-: \Delta_{2} \vdash \mathbf{F}_{4} \oplus \mathbf{F}_{5}} & \oplus_{R_{1}} & & \text{ax/W} \\ \hline & & -: \Delta_{2} \vdash \mathbf{F}_{4} \oplus \mathbf{F}_{5} & & & \text{hCut} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\frac{ \underbrace{ \begin{array}{ccc} \bullet_{\mathbf{h}_1} : * \vdash \mathbf{1} \end{array} \mathbf{1}_R & \begin{array}{ccc} \bullet_{\mathbf{h}_2} : \Delta_3 \vdash \mathbf{F}_4 \\ \bullet_{\mathbf{h}_2} : \Delta_3, \mathbf{1} \vdash \mathbf{F}_4 \end{array}}_{-: *, \Delta_3 \vdash \mathbf{F}_4} & \mathbf{1}_L \\ \underbrace{ \begin{array}{ccc} \bullet \\ -: \Delta_3 \vdash \mathbf{F}_4 \end{array}}_{-: \Delta_3 \vdash \mathbf{F}_4} & \mathsf{ax/W} \end{array}}_{\mathbf{h}_{\mathbf{h}_2}}$$

• Case rule \otimes_R

 $\bullet\,$ Case rule W

$$\begin{array}{c|c} \frac{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}}{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}} & \frac{\mathbf{h}_2 : \mathbf{1}, \Delta_5 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_2 : (\Delta_5, ! \mathbf{F}_3), \mathbf{1} \vdash \mathbf{F}_4} & W \\ \hline & - : *, \Delta_5, ! \mathbf{F}_3 \vdash \mathbf{F}_4 \\ \hline & \frac{\bullet}{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}} & \mathbf{ax/W} & \frac{\leadsto}{\mathbf{h}_2 : \mathbf{1}, \Delta_5, ! \mathbf{F}_3 \vdash \mathbf{F}_4} & \mathbf{ax/W} \\ \hline & - : \Delta_5, ! \mathbf{F}_3 \vdash \mathbf{F}_4 & \mathbf{hCut} \end{array}$$

 \bullet Case rule C

• Case rule !L

$$\begin{array}{c|c} & \mathbf{h}_1 : * \vdash \mathbf{1} & \mathbf{1}_R & \frac{\mathbf{h}_2 : \mathbf{1}, \Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_2 : (\Delta_5, !\mathbf{F}_3), \mathbf{1} \vdash \mathbf{F}_4} & !L \\ \hline & - : *, \Delta_5, !\mathbf{F}_3 \vdash \mathbf{F}_4 & \cdots \\ \hline & \frac{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}}{\bullet} & \mathbf{1}_R & \frac{\mathbf{h}_2 : \mathbf{1}, \Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_4}{\mathbf{h}_2 : \mathbf{1}, \Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_4} & \mathbf{ax/W} \\ \hline & \frac{- : \Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_4}{- : \Delta_5, !\mathbf{F}_3 \vdash \mathbf{F}_4} & !L & \mathbf{hCut} \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c|c} & \frac{\mathbf{h}_{2}: \mathbf{1}, \Delta_{6}, \mathbf{F}_{4} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{2}: (\Delta_{6}, \mathbf{F}_{3} \& \mathbf{F}_{4}), \mathbf{1} \vdash \mathbf{F}_{5}} & \&_{L2} \\ \hline & -: *, \Delta_{6}, \mathbf{F}_{3} \& \mathbf{F}_{4} \vdash \mathbf{F}_{5} & \\ \hline & \frac{\bullet \mathbf{h}_{1}: * \vdash \mathbf{1}}{\bullet \mathbf{h}_{1}: * \vdash \mathbf{1}} & \mathbf{1}_{R} & \overset{\sim}{\mathbf{h}_{2}: \mathbf{1}, \Delta_{6}, \mathbf{F}_{4} \vdash \mathbf{F}_{5}} \\ & \frac{-: \Delta_{6}, \mathbf{F}_{4} \vdash \mathbf{F}_{5}}{-: \Delta_{6}, \mathbf{F}_{3} \& \mathbf{F}_{4} \vdash \mathbf{F}_{5}} & \&_{L2} \end{array} \right. \\ & \frac{\bullet \mathbf{h}_{1}: * \vdash \mathbf{1}}{\bullet \mathbf{h}_{1}: * \vdash \mathbf{1}} & \mathbf{h}_{1} & \overset{\leftarrow}{\mathbf{h}_{1}: * \vdash \mathbf{1}} & \mathbf{h}_{2} & \overset{\leftarrow}{\mathbf{h}_{1}: * \vdash \mathbf{1}} & \mathbf{h}_{2} & \overset{\leftarrow}{\mathbf{h}_{1}: * \vdash \mathbf{1}} & \overset{\leftarrow}{\mathbf{h}_{2}: * \vdash \mathbf{1}} & \overset{\leftarrow}{\mathbf{h}_{2}: * \vdash \mathbf{1}} & \overset{\leftarrow}{\mathbf{h}_{1}: * \vdash \mathbf{1}} & \overset{\leftarrow}{\mathbf{h}_{2}: * \vdash \mathbf{1}} & \overset{\leftarrow}{\mathbf{h}_{2}: * \vdash \mathbf{1}} & \overset{\leftarrow}{\mathbf{h}_{1}: * \vdash \mathbf{1}} & \overset{\leftarrow}{\mathbf{h}_{1}: * \vdash \mathbf{1}} & \overset{\leftarrow}{\mathbf{h}_{2}: * \vdash \mathbf{1}} & \overset{\leftarrow}{\mathbf{h}_$$

• Case rule $\&_{L1}$

$$\begin{array}{c|c} & \frac{\mathbf{h}_2: \mathbf{1}, \Delta_6, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2: (\Delta_6, \mathbf{F}_3 \& \mathbf{F}_4), \mathbf{1} \vdash \mathbf{F}_5} & \&_{L1} \\ \hline & -: *, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5 & \\ \hline & \bullet_{\mathbf{1}}: * \vdash \mathbf{1} & \mathsf{ax/W} & \overset{\leadsto}{\frac{\mathbf{h}_2: \mathbf{1}, \Delta_6, \mathbf{F}_3 \vdash \mathbf{F}_5}{1}} & \mathsf{ax/W} \\ \hline & \frac{-: \Delta_6, \mathbf{F}_3 \vdash \mathbf{F}_5}{-: \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_5} & \&_{L1} & \\ \hline \end{array} \right. \\ & \frac{\mathsf{hold}}{\bullet \mathsf{hold}}$$

• Case rule \otimes_L

$$\begin{array}{c|c} \hline \bullet h_1 : * \vdash 1 & 1_R & \frac{h_2 : 1, \Delta_6, F_3, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \otimes F_4), 1 \vdash F_5} & \otimes_L \\ \hline - : *, \Delta_6, F_3 \otimes F_4 \vdash F_5 & \\ \hline \bullet h_1 : * \vdash 1 & \text{ax/W} & \frac{}{h_2 : 1, \Delta_6, F_3, F_4 \vdash F_5} \\ \hline - : \Delta_6, F_3, F_4 \vdash F_5 & \otimes_L & \text{hCut} \\ \hline - : \Delta_6, F_3 \otimes F_4 \vdash F_5 & \otimes_L & \\ \hline \end{array}$$

• Case rule \oplus_L

• Case rule \multimap_L

 \bullet Case rule I

5.3 Status of \top : OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- Case rule T

$$\begin{array}{c|c} \hline { \bullet \mathbf{h}_1 : \Delta_2 \vdash \top} & \top & \hline { \bullet \mathbf{h}_4 : \Delta_3, \top \vdash \top} \\ \hline { - : \Delta_2, \Delta_3 \vdash \top} & \mathbf{Cut} \\ \hline \hline { & \overset{\leadsto}{-} : \Delta_2, \Delta_3 \vdash \top} & \top \\ \hline \end{array}$$

• Case rule $\&_R$

$$\frac{\underbrace{\bullet\mathbf{h}_1:\Delta_2\vdash\top}_{}\vdash \underbrace{-:\Delta_2,\Delta_3\vdash F_5}_{} \underbrace{\mathbf{h}_4:\top,\Delta_3\vdash F_5}_{} \underbrace{\mathbf{h}_4:\top,\Delta_3\vdash F_6}_{} \underbrace{\mathbf{Cut}}_{} \&_R}_{} \\ \underbrace{-:\Delta_2,\Delta_3\vdash F_5\&F_6}_{} \underbrace{-:\Delta_2,\Delta_3\vdash F_5}_{} \underbrace{\mathbf{h}_4:\Delta_2,\Delta_3,\top\vdash F_5}_{} \underbrace{\mathbf{h}_1:*\vdash\top}_{} \underbrace{-:\Delta_2,\Delta_3\vdash F_5}_{} \underbrace{\mathbf{h}_1:*\vdash\top}_{} \underbrace{-:\Delta_2,\Delta_3\vdash F_6}_{} \underbrace{\mathbb{k}_R}_{} \\ \underbrace{-:\Delta_2,\Delta_3\vdash F_5}_{} \underbrace{\mathbb{k}_R}_{} \\ \underbrace{-:\Delta_2,\Delta_3\vdash F_5}_{} \underbrace{\mathbb{k}_R}_{} \underbrace{\mathbb{k}_R}_{} \\ \underbrace{-:\Delta_2,\Delta_3\vdash F_5}_{} \underbrace{$$

• Case rule \multimap_R

$$\begin{array}{c|c} & \underbrace{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top} & \top & \frac{\mathbf{h}_4 : \top, \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_4 : \Delta_3, \top \vdash \mathbf{F}_5 \multimap \mathbf{F}_6} & \neg \circ_R \\ & - : \Delta_2, \Delta_3 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 & \\ \hline \bullet \mathbf{h}_1 : * \vdash \top & \top & \frac{}{\mathbf{h}_4 : \Delta_2, \Delta_3, \mathbf{F}_5, \top \vdash \mathbf{F}_6} \\ \hline & \frac{- : \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \vdash \mathbf{F}_6}{- : \Delta_2, \Delta_3 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6} & \neg \circ_R & \\ \hline \end{array} \right. \\ \begin{array}{c} \bullet \mathbf{h}_1 : * \vdash \top & \bullet \mathbf{h}_4 : \Delta_2, \Delta_3, \mathbf{F}_5, \nabla \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_2, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 & \bullet \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_3, \Delta_3 \vdash \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_4$$

• Case rule \bigoplus_{R_2}

$$\frac{ \underbrace{ \begin{array}{c} \mathbf{h}_{1} : \Delta_{2} \vdash \top}_{} \vdash \mathbf{h}_{4} : \top, \Delta_{3} \vdash \mathbf{F}_{6} \\ \bullet \mathbf{h}_{4} : \Delta_{3}, \top \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} \\ \hline - : \Delta_{2}, \Delta_{3} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{1} : * \vdash \top \end{array} \stackrel{\longleftarrow}{} \frac{\mathbf{h}_{4} : \Delta_{2}, \Delta_{3}, \top \vdash \mathbf{F}_{6}}_{} \underbrace{\begin{array}{c} \mathbf{h}_{1} : * \vdash \top \\ \bullet \mathbf{h}_{2} : \Delta_{2}, \Delta_{3} \vdash \mathbf{F}_{6} \\ \hline - : \Delta_{2}, \Delta_{3} \vdash \mathbf{F}_{6} \\ \hline - : \Delta_{2}, \Delta_{3} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} \end{array} }_{\mathbf{h} \mathbf{R}_{2}} \underbrace{ \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h} \mathbf{Cut} \\ \mathbf{h} \mathbf{$$

• Case rule \oplus_{R_1}

$$\begin{array}{c|c} \underline{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top} & \top & \frac{\mathbf{h}_4 : \top, \Delta_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_4 : \Delta_3, \top \vdash \mathbf{F}_5 \oplus \mathbf{F}_6} \\ \hline -: \Delta_2, \Delta_3 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \\ \hline \\ \underline{\bullet \mathbf{h}_1 : * \vdash \top} & \top & \frac{\leadsto}{\mathbf{h}_4 : \Delta_2, \Delta_3, \top \vdash \mathbf{F}_5} \\ \hline \frac{-: \Delta_2, \Delta_3 \vdash \mathbf{F}_5}{-: \Delta_2, \Delta_3 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6} & \oplus_{R_1} \end{array} \right. \\ \underline{\bullet \mathbf{cut}}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c|c} \frac{\mathbf{h}_3: \top, \Delta_5 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_3: (\mathbf{1}, \Delta_5), \top \vdash \mathbf{F}_4} & \mathbf{1}_L \\ \hline -: \Delta_2, \mathbf{1}, \Delta_5 \vdash \mathbf{F}_4 & \mathsf{Cut} \\ \hline \frac{\bullet \mathbf{h}_1: * \vdash \top}{\bullet \mathbf{1}} & \top & \frac{}{\mathbf{h}_3: \mathbf{1}, \Delta_2, \Delta_5, \top \vdash \mathbf{F}_4} \\ \hline -: \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathbf{F}_4 & \mathsf{hCut} \end{array}$$

• Case rule \otimes_R

$$\begin{array}{c} \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \top}{\bullet \mathbf{h}_3: \Delta_4, \Delta_7} \vdash \frac{\mathbf{h}_3: T, \Delta_7 \vdash F_5 \quad \mathbf{h}_3: \Delta_4 \vdash F_6}{\bullet \mathbf{h}_3: (\Delta_4, \Delta_7), \top \vdash F_5 \otimes F_6} \otimes_R \\ \hline -: \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6 \\ \hline \\ \hline \bullet \mathbf{h}_1: * \vdash \top \quad T \quad \xrightarrow{\bullet} \\ \hline \frac{\bullet \mathbf{h}_1: * \vdash \top}{\bullet \mathbf{h}_3: \Delta_7, \top \vdash F_5} \quad \text{ax/W} \\ \hline -: \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6 \\ \hline \hline -: \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6 \\ \hline \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \top \quad T \quad \xrightarrow{\bullet \mathbf{h}_3: \Delta_4 \vdash F_5 \quad \mathbf{h}_3: \top, \Delta_7 \vdash F_6} \otimes_R \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \top \quad T \quad \xrightarrow{\bullet \mathbf{h}_3: (\Delta_4, \Delta_7), \top \vdash F_5 \otimes F_6} \text{Cut} \\ \hline -: \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6 \\ \hline \hline -: \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6 \\ \hline -: \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6 \\ \hline -: \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6 \\ \hline -: \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6 \\ \hline \end{array}$$

 $\bullet \;$ Case rule W

$$\begin{array}{c} \frac{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top}{\bullet} \ \top \ \frac{\mathbf{h}_3 : \top, \Delta_6 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : (\Delta_6, !\mathbf{F}_4), \top \vdash \mathbf{F}_5} \\ \hline -: \Delta_2, \Delta_6, !\mathbf{F}_4 \vdash \mathbf{F}_5 \\ \hline \frac{\bullet \mathbf{h}_1 : * \vdash \top}{\bullet} \ \top \ \frac{\sim}{\mathbf{h}_3 : \Delta_2, \Delta_6, \top, !\mathbf{F}_4 \vdash \mathbf{F}_5} \\ -: \Delta_2, \Delta_6, !\mathbf{F}_4 \vdash \mathbf{F}_5 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathsf{hCut} \end{array}$$

 $\bullet\,$ Case rule C

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \Delta_2 \vdash \top \\ \bullet \bullet_{\mathbf{h}_3} : (\Delta_6, !\mathsf{F}_4, !\mathsf{F}_4 \vdash \mathsf{F}_5 \\ - : \Delta_2, \Delta_6, !\mathsf{F}_4 \vdash \mathsf{F}_5 \\ \end{array} }_{\bullet} \underbrace{ \begin{array}{c} C \\ \mathsf{Cut} \\ \bullet \mathsf{h}_1 : * \vdash \top \end{array} }_{\bullet} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_3} : \top, \Delta_6, !\mathsf{F}_4, !\mathsf{F}_4 \vdash \mathsf{F}_5 \\ - : \Delta_2, \Delta_6, !\mathsf{F}_4 \vdash \mathsf{F}_5 \\ \hline - : \Delta_2, \Delta_6, !\mathsf{F}_4, !\mathsf{F}_4 \vdash \mathsf{F}_5 \\ - : \Delta_2, \Delta_6, !\mathsf{F}_4, !\mathsf{F}_4 \vdash \mathsf{F}_5 \end{array} }_{\bullet} \underbrace{ \begin{array}{c} \mathsf{Cut} \\ \mathsf{hCut} \\ \mathsf{hCut} \\ \bullet \mathsf{Cut} \\ \mathsf{Cut} \\ \bullet \mathsf{Cu$$

• Case rule !L

$$\begin{array}{c|c} \bullet \mathbf{h}_1 : \Delta_2 \vdash \top & \frac{\mathbf{h}_3 : \top, \Delta_6, \mathbf{F}_4 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_3 : (\Delta_6, !\mathbf{F}_4), \top \vdash \mathbf{F}_5} & !L \\ \hline -: \Delta_2, \Delta_6, !\mathbf{F}_4 \vdash \mathbf{F}_5 & \\ \hline \bullet \mathbf{h}_1 : * \vdash \top & \overset{\sim}{} \frac{}{\mathbf{h}_3 : \Delta_2, \Delta_6, \mathbf{F}_4, \top \vdash \mathbf{F}_5} \\ \hline -: \Delta_2, \Delta_6, !\mathbf{F}_4 \vdash \mathbf{F}_5 & !L & \\ \hline & \frac{-: \Delta_2, \Delta_6, !\mathbf{F}_4 \vdash \mathbf{F}_5}{-: \Delta_2, \Delta_6, !\mathbf{F}_4 \vdash \mathbf{F}_5} & !L \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c|c} \underline{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top} & \top & \frac{\mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3 : (\Delta_7, \mathbf{F}_4 \& \mathbf{F}_5), \top \vdash \mathbf{F}_6} & \&_{L2} \\ \hline \\ -: \Delta_2, \Delta_7, \mathbf{F}_4 \& \mathbf{F}_5 \vdash \mathbf{F}_6 & & \\ & & \xrightarrow{\bullet} \\ \underline{\bullet \mathbf{h}_1 : * \vdash \top} & \top & \frac{}{\mathbf{h}_3 : \Delta_2, \Delta_7, \mathbf{F}_5, \top \vdash \mathbf{F}_6} & \mathbf{ax/W} \\ \hline \\ -: \Delta_2, \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 & \&_{L2} \\ \hline \\ -: \Delta_2, \Delta_7, \mathbf{F}_4 \& \mathbf{F}_5 \vdash \mathbf{F}_6 & \&_{L2} \\ \end{array}$$

• Case rule $\&_{L1}$

$$\frac{ \underbrace{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash \top}_{} \vdash \frac{\mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_6}_{} \\ \mathbf{h}_3 : (\Delta_7, \mathbf{F}_4 \& \mathbf{F}_5), \top \vdash \mathbf{F}_6}_{} \vdash \mathbf{F}_6 \\ - : \Delta_2, \Delta_7, \mathbf{F}_4 \& \mathbf{F}_5 \vdash \mathbf{F}_6}_{} \\ \xrightarrow{\bullet \mathbf{h}_1 : * \vdash \top}_{} \vdash \frac{\mathbf{h}_3 : \Delta_2, \Delta_7, \mathbf{F}_4, \top \vdash \mathbf{F}_6}_{} \\ \frac{- : \Delta_2, \Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_6}_{- : \Delta_2, \Delta_7, \mathbf{F}_4 \& \mathbf{F}_5 \vdash \mathbf{F}_6} \\ & \underbrace{ \begin{array}{c} \mathbf{ax} / \mathbf{w} \\ \mathbf{hCut} \end{array} } \end{array} } \right)$$

• Case rule \otimes_L

$$\begin{array}{c} \underbrace{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top}_{} \; \top \; \; \frac{\mathbf{h}_3 : \top, \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3 : (\Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5), \top \vdash \mathbf{F}_6}}_{} \; \; \underset{\bullet}{\otimes_L} \\ \underbrace{-: \Delta_2, \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \vdash \mathbf{F}_6}_{} \; \; \underset{\bullet}{\sim} \\ \underbrace{\bullet \mathbf{h}_1 : * \vdash \top}_{} \; \top \; \; \frac{\mathbf{h}_3 : \Delta_2, \Delta_7, \mathbf{F}_4, \mathbf{F}_5, \top \vdash \mathbf{F}_6}{\mathbf{h}_3 : \Delta_2, \Delta_7, \mathbf{F}_4, \mathbf{F}_5, \top \vdash \mathbf{F}_6}}_{} \; \underbrace{\bullet \mathbf{x}}_{} \; \; \text{hCut} \\ \underbrace{-: \Delta_2, \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \vdash \mathbf{F}_6}_{} \; \otimes_L} \end{array}$$

• Case rule \oplus_L

$$\frac{\bullet \mathbf{h}_1:\Delta_2 \vdash \top}{-:\Delta_2,\Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_6 \quad \mathbf{h}_3:\top,\Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6} \bullet_L \\ -:\Delta_2,\Delta_7, \mathbf{F}_4 \oplus \mathbf{F}_5 \vdash \mathbf{F}_6} \bullet_L \\ \hline \bullet \mathbf{h}_1:*\vdash \top \quad \frac{\bullet}{\mathbf{h}_3:\Delta_2,\Delta_7, \mathbf{F}_4,\top \vdash \mathbf{F}_6} \bullet_{\mathrm{Cut}} \\ \hline \bullet \mathbf{h}_1:*\vdash \top \quad \frac{\bullet}{\mathbf{h}_3:\Delta_2,\Delta_7, \mathbf{F}_4,\top \vdash \mathbf{F}_6} \bullet_{\mathrm{Cut}} \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_4 \oplus \mathbf{F}_5 \vdash \mathbf{F}_6} \bullet_{\mathrm{Cut}} \\ \hline \bullet \mathbf{h}_1:*\vdash \top \quad \frac{\bullet}{\mathbf{h}_3:\Delta_2,\Delta_7, \mathbf{F}_5,\top \vdash \mathbf{F}_6} \bullet_{\mathrm{Cut}} \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_6} \bullet_{\mathrm{Cut}} \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_6} \bullet_{\mathrm{Cut}} \\ \hline \bullet \mathbf{h}_1:*\vdash \top \quad \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \\ \hline \bullet \mathbf{h}_1:*\vdash \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \\ \hline \bullet \mathbf{h}_1:*\vdash \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \\ \hline \bullet \mathbf{h}_1:*\vdash \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \\ \hline \bullet \mathbf{h}_1:*\vdash \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \\ \hline \bullet \mathbf{h}_1:*\vdash \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}} \\ \hline \bullet \mathbf{h}_1:*\vdash \bullet_{\mathrm{Cut}} \bullet_{\mathrm{Cut}}$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{\bullet h_1 : \Delta_2 \vdash \top}{\bullet} \quad \frac{h_3 : \top, \Delta_8 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_4, \Delta_8, F_5 \multimap F_6), \top \vdash F_7} \stackrel{\multimap_L}{\cot} \\ \\ \frac{\bullet h_1 : \Delta_2 \vdash \top}{\bullet} \quad \frac{\bullet h_3 : (\Delta_4, \Delta_8, F_5 \multimap F_6), \top \vdash F_7}{\bullet} \quad \frac{\bullet L}{\cot} \\ \\ \frac{\bullet h_1 : * \vdash \top}{\bullet} \quad \frac{\vdash}{h_3 : \Delta_8, \top \vdash F_5} \quad \frac{\bullet x/W}{\bullet L} \\ \frac{- : \Delta_8 \vdash F_5}{\bullet} \quad \frac{\bullet x/W}{\bullet L} \\ \frac{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7}{\bullet} \quad \frac{\bullet L}{\bullet} \\ \\ \frac{\bullet h_1 : \Delta_2 \vdash \top}{\bullet} \quad \frac{\vdash}{\bullet} \quad \frac{\bullet h_3 : \Delta_4 \vdash F_5 \quad h_3 : \top, \Delta_8, F_6 \vdash F_7}{\bullet h_3 : (\Delta_4, \Delta_8, F_5 \multimap F_6), \top \vdash F_7} \quad \frac{\bullet L}{\bullet} \\ \frac{\bullet h_1 : \Delta_2 \vdash \top}{\bullet} \quad \frac{\bullet h_1 : * \vdash \top}{\bullet} \quad \frac{\bullet h_1 : * \vdash \top}{h_3 : \Delta_2, \Delta_8, F_6, \top \vdash F_7} \quad \frac{\bullet x/W}{\bullet L} \\ \frac{- : \Delta_4 \vdash F_5}{\bullet} \quad \frac{\bullet x/W}{\bullet} \quad \frac{\bullet h_1 : * \vdash \top}{\bullet} \quad \frac{h_3 : \Delta_2, \Delta_8, F_6, \top \vdash F_7}{\bullet} \quad \frac{\bullet x/W}{\bullet} \\ \frac{- : \Delta_4 \vdash F_5}{\bullet} \quad \frac{\bullet x/W}{\bullet} \quad \frac{\bullet h_1 : * \vdash \top}{\bullet} \quad \frac{\bullet h_1 : \bullet}{\bullet} \quad \frac{\bullet h_1$$

 \bullet Case rule I

5.4 Status of $\&_R$: OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_6 \quad \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_7}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_6 \& \mathsf{F}_7} \quad \&_R \quad \overbrace{\bullet \mathbf{h}_8:\Delta_5,\mathsf{F}_6 \& \mathsf{F}_7 \vdash \top}^{\bullet \mathbf{h}_8:\Delta_5,\mathsf{F}_6 \& \mathsf{F}_7 \vdash \top} \quad \mathsf{Cut} \\ & \stackrel{\sim}{-:\Delta_2,\Delta_5 \vdash \top} \quad \top \end{array}$$

• Case rule $\&_R$

$$\frac{\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7} \quad \&_R}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7} \quad \&_R} \quad \frac{\mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_9 \quad \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{9} \& \mathbf{F}_{10}}} \quad \&_R}{-: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10}} \quad \mathbf{Cut}} \\ \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7}{\bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_9}} \quad \frac{\mathbf{ax}/\mathbb{W}}{\mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{10}}} \quad \frac{\mathbf{ax}/\mathbb{W}}{\mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{10}}} \quad \frac{\mathbf{ax}/\mathbb{W}}{\mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{10}}} \\ -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10}} \quad &_R \\ \end{pmatrix} \quad \mathbf{hCut}$$

• Case rule \multimap_R

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7}{\underbrace{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7}_{} \quad \&_R \quad \underbrace{\frac{\mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}}}_{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7} \quad \underbrace{\frac{\mathbf{a}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_2: \Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7} \quad \underbrace{\frac{\mathbf{a}_8 \times \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_2} \quad \underbrace{\frac{\mathbf{a}_8 \times \Delta_5}{\bullet \mathbf{h}_2 \vdash \mathbf{h}_2}}_{\bullet \mathbf{h}_2} \quad \underbrace{\frac{\mathbf{a}_8 \times \Delta_5}{\bullet \mathbf{h}_2}}_{\bullet \mathbf{h}_2} \quad \underbrace{\frac{\mathbf{a}$$

• Case rule \oplus_{R_1}

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \quad \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7} \quad \&_R \quad \frac{\mathbf{h}_8:\Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8:\Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7}}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7} \quad \overset{\text{ax/W}}{\underbrace{\mathsf{h}_8:\Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 \vdash \mathbf{F}_9}}}_{\bullet \mathbf{h}_2:\Delta_2,\Delta_5 \vdash \mathbf{F}_9} \quad \frac{\mathbf{ax/W}}{\mathsf{h}_2:\Delta_2,\Delta_5 \vdash \mathbf{F}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_5 \vdash \mathbf{F}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_5 \vdash \mathbf{F}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \& \mathbf{F}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_5 \vdash \mathbf{F}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_5 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_5 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_6 \& \mathbf{h}_7}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}_9}}{\bullet \mathbf{h}_2:\Delta_2,\Delta_3 \vdash \mathbf{h}_9} \quad \overset{\bigoplus_{\mathbf{h}_1:\Delta_2 \vdash \mathbf{h}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \quad \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_6 \\ & \underbrace{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \& \mathsf{F}_6}_{\bullet \mathbf{h}_7: (\mathbf{1}, \Delta_9), \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_8}_{\bullet \mathbf{h}_7: (\mathbf{1}, \Delta_9), \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_8} & \mathsf{L}_L \\ & \underbrace{-: \Delta_2, \mathbf{1}, \Delta_9 \vdash \mathsf{F}_8}_{\bullet \mathbf{h}_7: (\mathbf{1}, \Delta_9, \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_8}_{\bullet \mathbf{h}_7: \mathbf{1}, \Delta_9, \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_8} & \mathsf{ax/W} \\ & \underbrace{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \& \mathsf{F}_6}_{\bullet \mathbf{1}, \mathbf{1}, \Delta_9, \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_8}_{\bullet \mathbf{1}, \mathbf{1}, \Delta_9, \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_8} & \mathsf{hCut} \end{array}$$

• Case rule \otimes_R

ullet Case rule W

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \& \mathbf{F}_6} \quad \&_R \quad \frac{\mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7 : (\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_9} \quad W \\ \hline - : \Delta_2, \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \& \mathbf{F}_6 \quad \text{ax/W} \quad & \overset{\longleftrightarrow}{h_7 : \Delta_{10}, !\mathbf{F}_8, \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_9} \quad \text{ax/W} \\ \hline - : \Delta_{10}, \Delta_2, !\mathbf{F}_8 \vdash \mathbf{F}_9 \quad & \text{hCut} \end{array}$$

 \bullet Case rule C

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \quad \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \& \mathsf{F}_6} \quad \&_R \quad \frac{\mathbf{h}_7: \Delta_{10}, !\mathsf{F}_8, !\mathsf{F}_8, \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_9}{\bullet \mathbf{h}_7: (\Delta_{10}, !\mathsf{F}_8), \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_9} \quad C \\ \hline -: \Delta_2, \Delta_{10}, !\mathsf{F}_8 \vdash \mathsf{F}_9 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \& \mathsf{F}_6} \quad \text{ax/W} \quad \frac{}{\mathsf{h}_7: \Delta_{10}, !\mathsf{F}_8, !\mathsf{F}_8, \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_9}}{\mathsf{h}_7: \Delta_{10}, \Delta_2, !\mathsf{F}_8, !\mathsf{F}_8 \vdash \mathsf{F}_9} \quad C \\ \hline \frac{-: \Delta_{10}, \Delta_2, !\mathsf{F}_8, !\mathsf{F}_8 \vdash \mathsf{F}_9}{-: \Delta_{10}, \Delta_2, !\mathsf{F}_8 \vdash \mathsf{F}_9} \quad C \\ \hline$$

$\bullet \;$ Case rule !L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2} \vdash \mathsf{F}_{5} \quad \mathbf{h}_{1}:\Delta_{2} \vdash \mathsf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathsf{F}_{5} \& \mathsf{F}_{6}} \quad \&_{R} \quad \frac{\mathbf{h}_{7}:\Delta_{10}, \mathsf{F}_{8}, \mathsf{F}_{5} \& \mathsf{F}_{6} \vdash \mathsf{F}_{9}}{\bullet \mathbf{h}_{7}:(\Delta_{10}, !\mathsf{F}_{8}), \mathsf{F}_{5} \& \mathsf{F}_{6} \vdash \mathsf{F}_{9}} \quad \mathsf{Cut} \\ \\ -:\Delta_{2},\Delta_{10}, !\mathsf{F}_{8} \vdash \mathsf{F}_{9} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathsf{F}_{5} \& \mathsf{F}_{6}} \quad \overset{\boldsymbol{\mathsf{ax/W}}}{\bullet} \quad \frac{\overset{\bullet}{\mathsf{h}_{7}:\Delta_{10}, \mathsf{F}_{8}, \mathsf{F}_{5} \& \mathsf{F}_{6} \vdash \mathsf{F}_{9}}{\mathsf{h}_{7}:\Delta_{10},\Delta_{2}, \mathsf{F}_{8} \vdash \mathsf{F}_{9}} \quad \mathsf{l}L \\ \\ -:\Delta_{10},\Delta_{2}, !\mathsf{F}_{8} \vdash \mathsf{F}_{9} \quad !L \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \quad \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \& \mathsf{F}_6} \quad \&_R \quad \frac{\mathbf{h}_7: \Delta_{11}, \mathsf{F}_9, \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_{10}}{\bullet \mathsf{h}_7: (\Delta_{11}, \mathsf{F}_8 \& \mathsf{F}_9), \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_{10}} \\ \hline -: \Delta_2, \Delta_{11}, \mathsf{F}_8 \& \mathsf{F}_9 \vdash \mathsf{F}_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \& \mathsf{F}_6} \quad \mathbf{ax/W} \quad \overset{\bullet}{\sim} \\ \hline \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \& \mathsf{F}_6}{-: \Delta_{11}, \Delta_2, \mathsf{F}_9 \vdash \mathsf{F}_{10}} \quad \&_{L2} \\ \hline -: \Delta_{11}, \Delta_2, \mathsf{F}_8 \& \mathsf{F}_9 \vdash \mathsf{F}_{10}} \quad \&_{L2} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_7 \quad \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_8} \quad \&_R \quad \frac{\mathbf{h}_5: \Delta_6, \mathsf{F}_8 \vdash \mathsf{F}_9}{\bullet \mathbf{h}_5: \Delta_6, \mathsf{F}_7 \& \mathsf{F}_8 \vdash \mathsf{F}_9} \quad \&_{L2} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_7 \& \mathsf{F}_8 \qquad \&_R \quad \frac{\mathbf{h}_5: \Delta_6, \mathsf{F}_7 \& \mathsf{F}_8 \vdash \mathsf{F}_9}{\bullet \mathsf{h}_5: \Delta_6, \mathsf{F}_7 \& \mathsf{F}_8 \vdash \mathsf{F}_9} \quad \&_{L2} \\ \hline -: \Delta_2, \Delta_6 \vdash \mathsf{F}_9 \qquad & \mathsf{ax/W} \\ \hline -: \Delta_2, \Delta_6 \vdash \mathsf{F}_9 \qquad & \mathsf{ax/W} \\ \hline -: \Delta_2, \Delta_6 \vdash \mathsf{F}_9 \qquad & \mathsf{sCut} \\ \hline \end{array}$$

• Case rule $\&_{L1}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_5 \quad \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_6}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_5 \& \mathsf{F}_6} \quad \&_R \quad \frac{\mathbf{h}_7:\Delta_{11},\mathsf{F}_8,\mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_{10}}{\bullet \mathbf{h}_7:(\Delta_{11},\mathsf{F}_8 \& \mathsf{F}_9),\mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_{10}} \quad \&_{L1} \\ \hline \\ -:\Delta_2,\Delta_{11},\mathsf{F}_8 \& \mathsf{F}_9 \vdash \mathsf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_5 \& \mathsf{F}_6 \quad \mathbf{ax/W} \quad \frac{\sim}{\mathbf{h}_7:\Delta_{11},\mathsf{F}_8,\mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_{10}} \quad \mathbf{ax/W} \\ \hline \\ -:\Delta_{11},\Delta_2,\mathsf{F}_8 \& \mathsf{F}_{10} \quad \&_{L1} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_7 \quad \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_8 \quad \&_R \quad \frac{\mathbf{h}_5:\Delta_6,\mathsf{F}_7 \vdash \mathsf{F}_9}{\bullet \mathbf{h}_5:\Delta_6,\mathsf{F}_7 \& \mathsf{F}_8 \vdash \mathsf{F}_9} \quad \&_{L1} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2 \vdash \mathsf{F}_7 \& \mathsf{F}_8 \quad \&_R \quad \frac{\mathbf{h}_5:\Delta_6,\mathsf{F}_7 \vdash \mathsf{F}_9}{\bullet \mathbf{h}_5:\Delta_6,\mathsf{F}_7 \& \mathsf{F}_8 \vdash \mathsf{F}_9} \quad \mathsf{Cut} \\ \hline \\ -:\Delta_2,\Delta_6 \vdash \mathsf{F}_9 \quad \mathbf{ax/W} \quad \frac{\sim}{-:\Delta_6,\mathsf{F}_7 \vdash \mathsf{F}_9} \quad \mathsf{ax/W} \\ \hline \\ -:\Delta_2,\Delta_6 \vdash \mathsf{F}_9 \quad \mathsf{sCut} \\ \hline \end{array}$$

• Case rule \otimes_L

$$\frac{ \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \quad \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_6 }{ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \& \mathbf{F}_6 } \quad \&_R \quad \frac{ \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_{10} }{ \bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9), \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_{10} } \quad \underbrace{ \circ \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} }_{\leftarrow \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_{10} }_{\leftarrow \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_{10} } \quad \underbrace{ \underbrace{ \mathbf{ax/W} }_{\mathbf{hCut}} }_{\leftarrow \mathbf{hCut}}$$

• Case rule \oplus_L

$$\frac{\underbrace{\frac{h_1:\Delta_2 \vdash F_5}{\bullet h_1:\Delta_2 \vdash F_5 \& F_6}}_{\bullet h_1:\Delta_2 \vdash F_5 \& F_6} \underbrace{\&_R}_{\bullet R} \underbrace{\frac{h_7:\Delta_{11},F_8,F_5\& F_6 \vdash F_{10}}{\bullet h_7:(\Delta_{11},F_8 \oplus F_9),F_5\& F_6 \vdash F_{10}}_{\bullet h_7:(\Delta_{11},F_8 \oplus F_9),F_5\& F_6 \vdash F_{10}} \underbrace{Cut}_{\bullet L} \oplus \underbrace{\underbrace{(L_1,F_8 \oplus F_9),F_5\& F_6 \vdash F_{10}}_{\bullet L}}_{\bullet L} \oplus \underbrace{\underbrace{(L_1,F_8 \oplus F_9),F_5\& F_6 \vdash F_{10}}_{\bullet L}}_{\bullet L} \oplus \underbrace{\underbrace{(L_1,F_8),F_5\& F_6 \vdash F_{10}}_{\bullet L}}_{\bullet L}}_{\bullet L} \oplus \underbrace{\underbrace{(L_1,F_8),F_5\& F_6 \vdash F_{10}}_{\bullet L}}_{\bullet L}}_{\bullet L}$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{5}\quad \mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{5}\&\mathsf{F}_{6}} & \&_{R} & \frac{\mathbf{h}_{7}:\Delta_{12},\mathsf{F}_{5}\&\mathsf{F}_{6}\vdash \mathsf{F}_{9}\quad \mathbf{h}_{7}:\Delta_{8},\mathsf{F}_{10}\vdash \mathsf{F}_{11}}{\bullet \mathbf{h}_{7}:(\Delta_{8},\Delta_{12},\mathsf{F}_{9}\multimap \mathsf{F}_{10}),\mathsf{F}_{5}\&\mathsf{F}_{6}\vdash \mathsf{F}_{11}} & \mathsf{Cut} \\ & -:\Delta_{2},\Delta_{8},\Delta_{12},\mathsf{F}_{9}\multimap \mathsf{F}_{10}\vdash \mathsf{F}_{11} & & \mathsf{Cut} \\ & & & & & & \\ \hline & \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{5}\&\mathsf{F}_{6} & \mathsf{ax}/\mathbb{W} & & & \\ & \frac{\bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{5}\&\mathsf{F}_{6}}{\bullet \mathsf{m}_{1}:\Delta_{2}\vdash \mathsf{F}_{9}} & \mathsf{ax}/\mathbb{W} & & & \\ & -:\Delta_{12},\Delta_{2}\vdash \mathsf{F}_{9} & \mathsf{h}_{1} & & & \\ \hline & -:\Delta_{12},\Delta_{2}\vdash \mathsf{F}_{9} & \mathsf{h}_{7}:\Delta_{12},\mathsf{F}_{10}\vdash \mathsf{F}_{11} & & \mathsf{ax}/\mathbb{W} \\ \hline & -:\Delta_{12}\vdash \mathsf{F}_{5}\&\mathsf{F}_{6} & \&_{R} & & & & \\ \hline & \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{5}&\mathsf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{6} & \&_{R} & & & \\ \hline & \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{5}\&\mathsf{F}_{6} & \mathsf{f}_{11} & & & \\ \hline & \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{5}\&\mathsf{F}_{6} & \&_{R} & & & \\ \hline & \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{5}\&\mathsf{F}_{6} & \mathsf{ax}/\mathbb{W} & & \\ \hline & -:\Delta_{2},\Delta_{8},\Delta_{12},\mathsf{F}_{9}\multimap \mathsf{F}_{10}\vdash \mathsf{F}_{11} & & \\ \hline & \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{5}\&\mathsf{F}_{6} & \mathsf{ax}/\mathbb{W} & & \\ \hline & -:\Delta_{2},\Delta_{8},\Delta_{12},\mathsf{F}_{9}\multimap \mathsf{F}_{10}\vdash \mathsf{F}_{11} & & \\ \hline & -:\Delta_{12},\Delta_{2},\mathsf{F}_{10}\vdash \mathsf{F}_{11} & & \\ \hline &$$

• Case rule I

5.5 Status of \multimap_R : OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \multimap \mathbf{F}_7} \stackrel{-\circ_R}{-} \frac{}{\bullet \mathbf{h}_8:\Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \top} \\ \hline -:\Delta_2, \Delta_5 \vdash \top \\ \hline \frac{}{-:\Delta_2, \Delta_5 \vdash \top} \top \end{array} \subset \mathbf{Cut}$$

• Case rule $\&_R$

$$\frac{\underbrace{\begin{array}{l} \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{6}\vdash\mathbf{F}_{7} \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{F}_{6}\multimap\mathbf{F}_{7} \\ \hline \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{F}_{6}\multimap\mathbf{F}_{7} \\ \hline \\ -:\Delta_{2},\Delta_{5}\vdash\mathbf{F}_{9}\&\mathbf{F}_{10} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{F}_{6}\multimap\mathbf{F}_{7} \\ \hline \\ \bullet\mathbf{h}_{2}:\Delta_{2}\vdash\mathbf{F}_{6}\multimap\mathbf{F}_{7} \\ \hline \\ \bullet\mathbf{h}_{3}:\Delta_{5},\mathbf{F}_{6}\multimap\mathbf{F}_{7}\vdash\mathbf{F}_{9}\&\mathbf{F}_{10} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{F}_{6}\multimap\mathbf{F}_{7} \\ \hline \\ \bullet\mathbf{h}_{2}:\Delta_{2}\vdash\mathbf{F}_{6}\multimap\mathbf{F}_{7} \\ \hline \\ \bullet\mathbf{h}_{2}:\Delta_{2}\vdash\mathbf{h}_{2}\vdash\mathbf{h}_{2} \\ \hline \\ \bullet\mathbf{h}_{3}:\Delta_{5}\vdash\mathbf{h}_{6}\multimap\mathbf{F}_{7} \\ \hline \\ \bullet\mathbf{h}_{3}:\Delta_{5}\vdash\mathbf{h}_{6}\multimap\mathbf{F}_{7} \\ \hline \\ \bullet\mathbf{h}_{3}:\Delta_{5}\vdash\mathbf{h}_{6}\multimap\mathbf{F}_{7} \\ \hline \\ \bullet\mathbf{h}_{3}:\Delta_{5}\vdash\mathbf{h}_{6}\multimap\mathbf{F}_{7} \\ \hline \\ \bullet\mathbf{h}_{3}:\Delta_{5}\vdash\mathbf{h}_{6}\multimap\mathbf{h}_{7} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{h}_{6}\multimap\mathbf{h}_{7} \\ \hline \\ \bullet\mathbf{h}_{2}:\Delta_{2}\vdash\mathbf{h}_{5}\vdash\mathbf{h}_{9} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{h}_{6}\multimap\mathbf{h}_{7} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{h}_{6}\multimap\mathbf{h}_{7} \\ \hline \\ \bullet\mathbf{h}_{2}:\Delta_{2}\vdash\mathbf{h}_{5}\vdash\mathbf{h}_{9} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{h}_{2}\vdash\mathbf{h}_{9} \\ \hline \\ \bullet\mathbf{h}_{2}:\Delta_{2}\vdash\mathbf{h}_{3}\vdash\mathbf{h}_{3} \\ \hline \\ \bullet\mathbf{h}_{3}:\Delta_{5}\vdash\mathbf{h}_{1} \\ \hline \\ \bullet\mathbf{h}_{2}:\Delta_{2}\vdash\mathbf{h}_{3}\vdash\mathbf{h}_{3} \\ \hline \\ \bullet\mathbf{h}_{3}:\Delta_{5}\vdash\mathbf{h}_{1} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{h}_{2}\vdash\mathbf{h}_{3}\vdash\mathbf{h}_{3} \\ \hline \\ \bullet\mathbf{h}_{2}\vdash\mathbf{h}_{3}\vdash\mathbf{h}_{3}\vdash\mathbf{h}_{3} \\ \hline \\ \bullet\mathbf{h}_{3}:\Delta_{5}\vdash\mathbf{h}_{2}\vdash\mathbf{h}_{3}\vdash\mathbf{h}_{3} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{h}_{2}\vdash\mathbf{h}_{3}\vdash\mathbf{h}_{3}\vdash\mathbf{h}_{3} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{h}_{2}\vdash\mathbf{h}_{3}$$

• Case rule \multimap_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \multimap \mathbf{F}_7} \multimap_R & \frac{\mathbf{h}_8: \Delta_5, \mathbf{F}_9, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10}} & \neg_O_R \\ \hline \\ -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} & & & & \\ \hline \\ \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \multimap \mathbf{F}_7}{\bullet} & \text{ax/W} & & & \\ \hline \\ \frac{-: \Delta_2, \Delta_5, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{-: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10}} & \neg_O_R \\ \end{array}$$

• Case rule \bigoplus_{R_2}

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2, \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \multimap \mathbf{F}_7 \end{array} \multimap_R \begin{array}{c} \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \end{array}} \begin{array}{c} \oplus_{R_2} \\ -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \end{array} \\ \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \multimap \mathbf{F}_7 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \hline -: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} \\ \hline -: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} \\ \hline -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \bigoplus_{R_1}

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \multimap \mathbf{F}_7} & \multimap_R & \frac{\mathbf{h}_8:\Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8:\Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \overset{\bigoplus_{R_1}}{\mathsf{Cut}} \\ \hline & -:\Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \hline & \bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \multimap \mathbf{F}_7 & \mathsf{ax/W} & \overset{\longleftarrow}{h_8:\Delta_5, \mathbf{F}_6 \multimap \mathbf{F}_7 \vdash \mathbf{F}_9} \\ \hline & \frac{-:\Delta_2, \Delta_5 \vdash \mathbf{F}_9}{-:\Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \oplus_{R_1} & \mathsf{ax/W} \\ \hline \end{array}$$

• Case rule $\mathbf{1}_L$

$$\frac{ \begin{array}{l} \mathbf{h}_1: \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \end{array} \multimap_R \quad \frac{\mathbf{h}_7: \Delta_9, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: (\mathbf{1}, \Delta_9), \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_8} \quad \frac{\mathbf{1}_L}{\mathsf{Cut}} \\ \hline \\ -: \Delta_2, \mathbf{1}, \Delta_9 \vdash \mathbf{F}_8 \\ \hline \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \quad \mathsf{ax/W} \quad \overset{\leadsto}{\underbrace{\begin{array}{c} \bullet \\ \mathbf{h}_7: \mathbf{1}, \Delta_9, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_8 \\ -: \mathbf{1}, \Delta_2, \Delta_9 \vdash \mathbf{F}_8 \end{array}}} \quad \mathbf{ax/W} \quad \overset{\mathsf{hCut}}{\mathsf{hCut}} \end{array}$$

• Case rule \otimes_R

 \bullet Case rule W

$$\begin{array}{c|c} \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{5}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{F}_{5}\multimap\mathbf{F}_{6}} & \circ_{R} & \frac{\mathbf{h}_{7}:\Delta_{10},\mathbf{F}_{5}\multimap\mathbf{F}_{6}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{7}:(\Delta_{10},!\mathbf{F}_{8}),\mathbf{F}_{5}\multimap\mathbf{F}_{6}\vdash\mathbf{F}_{9}} & Cut \\ \hline & -:\Delta_{2},\Delta_{10},!\mathbf{F}_{8}\vdash\mathbf{F}_{9} & \\ \hline & \bullet_{\mathbf{h}_{1}}:\Delta_{2}\vdash\mathbf{F}_{5}\multimap\mathbf{F}_{6} & \mathbf{ax/W} & \overset{\leadsto}{h_{7}:\Delta_{10},!\mathbf{F}_{8},\mathbf{F}_{5}\multimap\mathbf{F}_{6}\vdash\mathbf{F}_{9}} \\ \hline & -:\Delta_{10},\Delta_{2},!\mathbf{F}_{8}\vdash\mathbf{F}_{9} & \mathbf{ax/W} \\ \hline \end{array}$$

ullet Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{5}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{F}_{5}\multimap\mathbf{F}_{6}} \ \multimap_{R} \ \frac{\mathbf{h}_{7}:\Delta_{10},\mathbb{IF}_{8},\mathbb{IF}_{8},\mathbf{F}_{5}\multimap\mathbf{F}_{6}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{7}:(\Delta_{10},\mathbb{IF}_{8}),\mathbf{F}_{5}\multimap\mathbf{F}_{6}\vdash\mathbf{F}_{9}} \ Cut \\ \hline \\ -:\Delta_{2},\Delta_{10},\mathbb{IF}_{8}\vdash\mathbf{F}_{9} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{F}_{5}\multimap\mathbf{F}_{6} \ \frac{\mathsf{ax/W}}{\bullet} \ \frac{\sim}{\mathbf{h}_{7}:\Delta_{10},\mathbb{IF}_{8},\mathbb{IF}_{8},\mathbf{F}_{5}\multimap\mathbf{F}_{6}\vdash\mathbf{F}_{9}} \ Cut \\ \hline \\ \frac{-:\Delta_{10},\Delta_{2},\mathbb{IF}_{8}\vdash\mathbf{F}_{9}}{-:\Delta_{10},\Delta_{2},\mathbb{IF}_{8}\vdash\mathbf{F}_{9}} \ C \end{array}$$

• Case rule !L

$$\begin{array}{c|c} \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{5} \vdash \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5} \multimap \mathbf{F}_{6} \end{array} \rightarrow_{\mathbf{F}_{6}} \begin{array}{c} \mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{8}, \mathbf{F}_{5} \multimap \mathbf{F}_{6} \vdash \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{7}: (\Delta_{10}, !\mathbf{F}_{8}), \mathbf{F}_{5} \multimap \mathbf{F}_{6} \vdash \mathbf{F}_{9} \end{array} \begin{array}{c} !L \\ \mathrm{Cut} \\ \hline \hline \bullet \mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5} \multimap \mathbf{F}_{6} \end{array} \begin{array}{c} \mathbf{a}_{X}/\mathbb{W} \\ \hline \hline \bullet \mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5} \multimap \mathbf{F}_{6} \end{array} \begin{array}{c} \mathbf{a}_{X}/\mathbb{W} \\ \hline -: \Delta_{10}, \Delta_{2}, \mathbf{F}_{8} \vdash \mathbf{F}_{9} \\ -: \Delta_{10}, \Delta_{2}, !\mathbf{F}_{8} \vdash \mathbf{F}_{9} \end{array} \begin{array}{c} !L \\ \mathrm{hCut} \end{array}$$

• Case rule $\&_{L2}$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \end{array} \multimap_{\mathbf{R}} \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_9, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9), \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_{10} \end{array}}{ - : \Delta_2, \Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} } \begin{array}{c} \&_{L2} \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline - : \Delta_{11}, \Delta_2, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline - : \Delta_{11}, \Delta_2, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathsf{hCut} \end{array}$$

• Case rule $\&_{L1}$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \end{array} \multimap_R \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9), \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_{10} \end{array} \begin{array}{c} \&_{L1} \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \end{array} \begin{array}{c} \bullet \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} \begin{array}{c} \&_{L1} \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \end{array} \begin{array}{c} \bullet \mathbf{x} / \mathbb{W} \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \end{array} \begin{array}{c} \bullet \mathbf{x} / \mathbb{W} \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \end{array} \begin{array}{c} \bullet \mathbf{x} / \mathbb{W} \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \bullet : \Delta_{11}, \Delta_2, \mathbf{F}_8 \vdash \mathbf{F}_{10} \end{array} \begin{array}{c} \bullet \mathbf{h}_1 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{h}_1 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{h}_1 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{h}_1 \end{array} \begin{array}{c} \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{h}_1 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{h}_2 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{h}_3 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6} \multimap_{\mathbf{R}} & \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9), \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_{10}} & \otimes_L \\ & - : \Delta_2, \Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} & & \text{cut} \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \multimap \mathbf{F}_6 & \text{ax/W} & \frac{}{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \multimap \mathbf{F}_6 \vdash \mathbf{F}_{10}} & & \text{ax/W} \\ \hline & \frac{- : \Delta_{11}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{- : \Delta_{11}, \Delta_2, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \otimes_L & & & \text{hCut} \end{array}$$

• Case rule \oplus_L

$$\frac{\underbrace{\frac{h_1: \Delta_2, F_5 \vdash F_6}{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6} \vdash \circ_R}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6} \vdash \circ_R}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6} \vdash \circ_R}_{\bullet h_2: \Delta_1, F_8, F_5 \multimap F_6 \vdash F_{10}}_{\bullet h_7: (\Delta_{11}, F_8 \oplus F_9), F_5 \multimap F_6 \vdash F_{10}}_{\bullet h_7: (\Delta_{11}, F_8 \oplus F_9), F_5 \multimap F_6 \vdash F_{10}}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6}} \underbrace{\text{Cut}}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6} \underbrace{\text{ax/W}}_{h_7: \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6} \underbrace{\text{ax/W}}_{h_7: \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6} \underbrace{\text{ax/W}}_{h_7: \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6}_{\bullet h_1: \Delta_2 \vdash F_5 \multimap F_6}$$

• Case rule \multimap_L

ullet Case rule I

5.6 Status of \bigoplus_{R_2} : OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \ \oplus_{R_2} \ \hline \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \top \\ -: \Delta_2, \Delta_5 \vdash \top \\ \hline \hline -: \Delta_2, \Delta_5 \vdash \top \end{array} \ \top$$
 Cut

• Case rule $\&_R$

$$\frac{\underbrace{\begin{array}{l} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \oplus_{R_2} \quad \underbrace{\begin{array}{l} \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \quad \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \end{array}}_{=: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10}} \quad \mathbf{Cut}} \\ \underbrace{\begin{array}{l} \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \underbrace{\begin{array}{l} \mathbf{ax/W} \\ \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \\ \mathbf{h}_{Cut} \end{array}}_{=: \Delta_2, \Delta_5 \vdash \mathbf{F}_9} \underbrace{\begin{array}{l} \mathbf{ax/W} \\ \mathbf{h}_{Cut} \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array}}_{=: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10}} \underbrace{\begin{array}{l} \mathbf{ax/W} \\ \mathbf{h}_{Cut} \\ \bullet \mathbf{h}_{Cut} \\ \bullet$$

• Case rule \multimap_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \oplus_{R_2} \quad \begin{array}{c} \mathbf{h}_8: \Delta_5, \mathbf{F}_9, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} \end{array} \quad \begin{array}{c} - \circ_R \\ \text{Cut} \end{array} \\ \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \quad \text{ax/W} \quad \begin{array}{c} \hookrightarrow \\ \mathbf{h}_8: \Delta_5, \mathbf{F}_9, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \quad \text{ax/W} \end{array} \quad \begin{array}{c} \rightarrow \\ \mathbf{h}_8: \Delta_5, \mathbf{F}_9, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \hline -: \Delta_2, \Delta_5, \mathbf{F}_9 \vdash \mathbf{F}_{10} \quad - \circ_R \end{array} \quad \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \bigoplus_{R_2}

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \oplus_{R_2} & \frac{\mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}} \\ \hline & -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \hline \\ \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7}{\bullet \mathbf{m}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} & \mathbf{ax/W} & \overset{\longleftarrow}{\mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10}} \\ \hline & \frac{-: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10}}{-: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \oplus_{R_2} \end{array} \right. \\ \\ \frac{\mathsf{hCut}}{\mathsf{hCut}}$$

• Case rule \bigoplus_{R_1}

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \oplus_{R_2} \begin{array}{c} \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \\ \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \end{array} \\ -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \\ \hline -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \end{array} \begin{array}{c} \oplus_{R_1} \\ \mathbf{hCut} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\frac{ \begin{array}{l} \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \oplus \mathsf{F}_6 \end{array} \oplus_{R_2} \quad \frac{\mathbf{h}_7: \Delta_9, \mathsf{F}_5 \oplus \mathsf{F}_6 \vdash \mathsf{F}_8}{\bullet \mathbf{h}_7: (\mathbf{1}, \Delta_9), \mathsf{F}_5 \oplus \mathsf{F}_6 \vdash \mathsf{F}_8} \quad \mathbf{L}_{\mathsf{Cut}} \\ \hline -: \Delta_2, \mathbf{1}, \Delta_9 \vdash \mathsf{F}_8 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_5 \oplus \mathsf{F}_6 \quad \mathsf{ax/W} \quad \frac{\leadsto}{\mathbf{h}_7: \mathbf{1}, \Delta_9, \mathsf{F}_5 \oplus \mathsf{F}_6 \vdash \mathsf{F}_8} \quad \mathsf{ax/W} \\ -: \mathbf{1}, \Delta_2, \Delta_9 \vdash \mathsf{F}_8 \end{array} \quad \mathbf{hCut} \\ \end{array}$$

• Case rule \otimes_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_6 \end{array} \oplus_{R_2} \begin{array}{c} \frac{\mathbf{h}_7: \Delta_{11}, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9 \quad \mathbf{h}_7: \Delta_8 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7: (\Delta_8, \Delta_{11}), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10}} \\ \hline \\ -: \Delta_2, \Delta_8, \Delta_{11} \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus \mathcal{R}_2 \begin{array}{c} \mathbf{h}_7: \Delta_8 \vdash \mathbf{F}_9 \quad \mathbf{h}_7: \Delta_{11}, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_7: (\Delta_8, \Delta_{11}), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \end{array} \\ \hline -: \Delta_2, \Delta_8, \Delta_{11} \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline \hline -: \Delta_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_{10} \oplus \mathbf{F}_{10} \end{array} \\ \hline -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

 \bullet Case rule W

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus_{R_2} \quad \frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: (\Delta_{10}, \mathbf{!F}_8), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \quad W \\ \hline -: \Delta_2, \Delta_{10}, \mathbf{!F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \quad \text{ax/W} \quad \overset{\leadsto}{\frac{}{\mathbf{h}_7: \Delta_{10}, \mathbf{!F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9}} \\ -: \Delta_{10}, \Delta_2, \mathbf{!F}_8 \vdash \mathbf{F}_9 \end{array} \quad \mathbf{hCut}$$

 $\bullet\,$ Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus \mathbf{R}_2 & \frac{\mathbf{h}_7 : \Delta_{10}, !\mathbf{F}_8, !\mathbf{F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7 : (\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \ Cut \\ \hline & -: \Delta_2, \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \text{ax/W} & \overset{\sim}{h_7 : \Delta_{10}, !\mathbf{F}_8, !\mathbf{F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \\ \hline & \frac{-: \Delta_{10}, \Delta_2, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9}{-: \Delta_{10}, \Delta_2, !\mathbf{F}_8 \vdash \mathbf{F}_9} \ C \end{array} \right. \\ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \overset{\sim}{\mathbf{h}_7} : \Delta_{10}, !\mathbf{F}_8, !\mathbf{F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \\ \hline & & \bullet \\ \hline & -: \Delta_{10}, \Delta_2, !\mathbf{F}_8 \vdash \mathbf{F}_9 \end{array} \right. \\ \end{array}$$

 \bullet Case rule !L

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash \mathsf{F}_6 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathsf{F}_5 \oplus \mathsf{F}_6 \end{array} \oplus_{R_2} \quad \frac{\mathbf{h}_7 : \Delta_{10}, \mathsf{F}_8, \mathsf{F}_5 \oplus \mathsf{F}_6 \vdash \mathsf{F}_9}{\bullet \mathbf{h}_7 : (\Delta_{10}, !\mathsf{F}_8), \mathsf{F}_5 \oplus \mathsf{F}_6 \vdash \mathsf{F}_9} \quad \overset{!L}{\underbrace{\mathsf{Cut}}} \\ \\ - : \Delta_2, \Delta_{10}, !\mathsf{F}_8 \vdash \mathsf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathsf{F}_5 \oplus \mathsf{F}_6 \end{array} \quad \mathbf{ax/W} \quad \frac{\overset{\longrightarrow}{\mathsf{h}_7 : \Delta_{10}, \mathsf{F}_8, \mathsf{F}_5 \oplus \mathsf{F}_6 \vdash \mathsf{F}_9}}{\mathsf{h}_7 : \Delta_{10}, \mathsf{F}_8, \mathsf{F}_5 \oplus \mathsf{F}_6 \vdash \mathsf{F}_9}} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \\ - : \Delta_{10}, \Delta_2, !\mathsf{F}_8 \vdash \mathsf{F}_9} \quad !L \end{array}$$

• Case rule $\&_{L2}$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus_{R_2} \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_9, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_{10} \end{array} \\ \hline \\ - : \Delta_2, \Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ - : \Delta_{11}, \Delta_2, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ - : \Delta_{11}, \Delta_2, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule $\&_{L1}$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash F_6 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash F_5 \oplus F_6 \end{array} \oplus_{R_2} \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \\ \bullet \mathbf{h}_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10} \end{array}}{ \begin{array}{c} -: \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash F_5 \oplus F_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ \frac{-: \Delta_{11}, \Delta_2, F_8 \vdash F_{10}}{-: \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6} \oplus R_2 & \frac{\mathbf{h}_7: \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7: (\Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_{10}} & \text{Cut} \\ \hline & -: \Delta_2, \Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} & \\ \hline & \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \text{ax/W} & \frac{}{\mathbf{h}_7: \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_{10}} \\ \hline & \frac{-: \Delta_{11}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{-: \Delta_{11}, \Delta_2, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \otimes_L \end{array} \\ \end{array}$$

• Case rule \oplus_L

$$\frac{\begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash F_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash F_5 \oplus F_6 \end{array} \oplus_{R_2} \begin{array}{c} \mathbf{h}_7: \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} & \mathbf{h}_7: \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10} \\ \bullet \mathbf{h}_7: (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10} \end{array}}{\begin{array}{c} \bullet \mathbf{h}_7: \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \end{array}} \begin{array}{c} \oplus \mathbf{h}_7: \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10} \\ \bullet \mathbf{h}_7: (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10} \end{array}} \begin{array}{c} \mathbf{Cut} \end{array}$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6}} \oplus \mathbb{H}_{2} & \frac{\mathbf{h}_{7}:\Delta_{12}, \mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{9} \quad \mathbf{h}_{7}:\Delta_{8}, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_{7}:(\Delta_{8},\Delta_{12}, \mathbf{F}_{9} \multimap \mathbf{F}_{10}), \mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{11}} & -\circ_{L} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} & \mathbf{ax/W} & \frac{}{\mathbf{h}_{7}:\Delta_{12}, \mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{9}} \\ \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} & \frac{\mathbf{ax/W}}{\mathbf{h}_{7}:\Delta_{12}, \mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{9}} & \mathbf{ax/W} \\ \hline & -:\Delta_{12},\Delta_{2} \vdash \mathbf{F}_{9} & \mathbf{h}_{7}:\Delta_{12},\mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{6} & \oplus \mathbf{R}_{2} & \frac{\mathbf{h}_{7}:\Delta_{8} \vdash \mathbf{F}_{9} \cdot \mathbf{h}_{7}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_{7}:(\Delta_{8},\Delta_{12},\mathbf{F}_{9} \multimap \mathbf{F}_{10}),\mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{11}} & -\circ_{L} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} & \oplus \mathbf{R}_{2} & \frac{\mathbf{h}_{7}:\Delta_{8} \vdash \mathbf{F}_{9} \cdot \mathbf{h}_{7}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_{7}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{11}} & -\circ_{L} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{7}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{11}} & -\circ_{L} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{7}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{11}} & -\circ_{L} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{7}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{11}} & -\circ_{L} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5} \oplus \mathbf{F}_{6} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{7}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{5} \oplus \mathbf{F}_{6} \vdash \mathbf{F}_{11}} & -\circ_{L} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{h}_{5} \oplus \mathbf{h}_{7} \oplus \mathbf{h}_{7$$

ullet Case rule I

5.7 Status of \bigoplus_{R_1} : OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_{R_1} & \overline{\bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \top} \\ \hline -: \Delta_2, \Delta_5 \vdash \top & \\ \hline -: \Delta_2, \Delta_5 \vdash \top & \top \\ \hline \end{array} \begin{array}{c} \top \\ \mathrm{Cut} \end{array}$$

• Case rule $\&_R$

$$\frac{\underbrace{\begin{array}{l} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \oplus_{R_1} \begin{array}{l} \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 & \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \end{array}}_{\bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10}} \mathbf{Cut}} \underbrace{\begin{array}{l} \mathbf{\Phi}_{\mathbf{h}_1}: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 & \mathbf{ax/W} \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 & \mathbf{ax/W} & \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \\ \bullet \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 & \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} & \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} & \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} & \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} & \mathbf{h}_3: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} & \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} & \mathbf{h}_3: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} & \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_2: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} & \mathbf{h}_2: \Delta_3, \Delta_5 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_3: \Delta_5, \mathbf{h}_3: \Delta_5, \mathbf{h}_4: \Delta_5, \mathbf{h}_5: \Delta_5, \mathbf$$

• Case rule \multimap_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash F_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash F_6 \oplus F_7 \end{array} \oplus_{R_1} \quad \begin{array}{c} \mathbf{h}_8: \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10} \\ \bullet \mathbf{h}_8: \Delta_5, F_6 \oplus F_7 \vdash F_9 \multimap F_{10} \end{array} \\ \hline -: \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash F_6 \oplus F_7 \quad \text{ax/W} \quad \begin{array}{c} \hookrightarrow \\ h_8: \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash F_6 \oplus F_7 \end{array} \\ \hline -: \Delta_2, \Delta_5, F_9 \vdash F_{10} \\ \hline -: \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10} \end{array} \quad \begin{array}{c} \circ_R \\ \text{hCut} \end{array}$$

• Case rule \bigoplus_{R_2}

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \oplus_{R_1} \begin{array}{c} \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{9} \oplus \mathbf{F}_{10} \end{array}}{ \begin{array}{c} -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_{10} \end{array} \begin{array}{c} \oplus_{R_2} \\ \mathbf{Cut} \end{array} } \\ \frac{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7}{ \begin{array}{c} -: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} \\ -: \Delta_2, \Delta_5 \vdash \mathbf{F}_{10} \end{array} \oplus_{R_2} \end{array}} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \oplus_{R_1}

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \oplus_{R_1} \begin{array}{c} \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_8: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \end{array} \xrightarrow[\mathbf{cut}]{} \begin{array}{c} \oplus_{R_1} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \\ \hline -: \Delta_2, \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus_{R_1} \quad \frac{\mathbf{h}_7: \Delta_9, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: (\mathbf{1}, \Delta_9), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_8} \quad \mathbf{1}_L \\ \hline -: \Delta_2, \mathbf{1}, \Delta_9 \vdash \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \quad \text{ax/W} \quad \frac{\leadsto}{\mathbf{h}_7: \mathbf{1}, \Delta_9, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_8} \quad \text{ax/W} \\ -: \mathbf{1}, \Delta_2, \Delta_9 \vdash \mathbf{F}_8 \end{array} \quad \mathbf{hCut}$$

• Case rule \otimes_R

$$\frac{\underbrace{\begin{array}{l} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus \mathbb{R}_1 \quad \underbrace{\begin{array}{l} \mathbf{h}_7: \Delta_{11}, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9 \quad \mathbf{h}_7: \Delta_8 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_7: (\Delta_8, \Delta_{11}), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline \\ -: \Delta_2, \Delta_8, \Delta_{11} \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ & \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \quad \underbrace{\begin{array}{l} \mathbf{ax/W} \\ \hline \mathbf{h}_7: \Delta_{11}, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline \\ -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_9 \\ \hline \\ -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline \end{array}} \quad \underbrace{\begin{array}{l} \mathbf{ax/W} \\ \mathbf{hCut} \\ \hline \\ -: \Delta_8 \vdash \mathbf{F}_{10} \\ \hline \\ \otimes_R \\ \end{array}}_{\otimes R}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \to \mathbf{F}_6 \end{array} \oplus \mathcal{R}_1 & \begin{array}{c} \mathbf{h}_7: \Delta_8 \vdash \mathbf{F}_9 & \mathbf{h}_7: \Delta_{11}, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \otimes \mathcal{R} \\ \hline -: \Delta_2, \Delta_8, \Delta_{11} \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline \\ \hline -: \Delta_2, \Delta_8, \Delta_{11} \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \\ \hline \hline \\ \hline -: \Delta_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_{10} \\ \hline -: \Delta_{11}, \Delta_2 \vdash \mathbf{F}_{10} \\ \hline -: \Delta_{11}, \Delta_2, \Delta_8 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \end{array} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

 \bullet Case rule W

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus_{R_1} \quad \frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: (\Delta_{10}, \mathbf{!F}_8), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \quad W} \quad \mathbf{Cut} \\ \\ -: \Delta_2, \Delta_{10}, \mathbf{!F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \quad \mathbf{ax/W} \quad \frac{\sim}{\mathbf{h}_7: \Delta_{10}, \mathbf{!F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9}} \quad \frac{\mathbf{ax/W}}{\mathbf{hCut}} \\ \\ -: \Delta_{10}, \Delta_2, \mathbf{!F}_8 \vdash \mathbf{F}_9} \end{array} \quad \mathbf{hCut}$$

 $\bullet\,$ Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus \mathbf{R}_1 & \frac{\mathbf{h}_7 : \Delta_{10}, !\mathbf{F}_8, !\mathbf{F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7 : (\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \ Cut \\ \hline & -: \Delta_2, \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \text{ax/W} & \overset{\sim}{h_7 : \Delta_{10}, !\mathbf{F}_8, !\mathbf{F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \\ \hline & \frac{-: \Delta_{10}, \Delta_2, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9}{-: \Delta_{10}, \Delta_2, !\mathbf{F}_8 \vdash \mathbf{F}_9} \ C \end{array} \right. \\ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \overset{\sim}{\mathbf{h}_7} : \Delta_{10}, !\mathbf{F}_8, !\mathbf{F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \\ \hline & & \bullet \\ \hline & -: \Delta_{10}, \Delta_2, !\mathbf{F}_8 \vdash \mathbf{F}_9 \end{array} \right. \\ \end{array}$$

 \bullet Case rule !L

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus_{R_1} \quad \frac{\mathbf{h}_7 : \Delta_{10}, \mathbf{F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7 : (\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \quad \overset{!L}{\underbrace{\mathsf{Cut}}} \\ \\ - : \Delta_2, \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 \quad \mathsf{ax/W} \quad \frac{}{\mathbf{h}_7 : \Delta_{10}, \mathbf{F}_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_9} \\ \hline - : \Delta_{10}, \Delta_2, \mathbf{F}_8 \vdash \mathbf{F}_9 \quad !L \end{array} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}}$$

• Case rule $\&_{L2}$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash F_5 \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash F_5 \oplus F_6 \end{array} \oplus_{R_1} \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10} \\ \hline \bullet \mathbf{h}_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10} \\ \hline \\ - : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash F_5 \oplus F_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ \hline - : \Delta_{11}, \Delta_2, F_9 \vdash F_{10} \\ \hline - : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule $\&_{L1}$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash F_5 \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash F_5 \oplus F_6 \end{array} \oplus_{R_1} \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \\ \hline \bullet \mathbf{h}_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10} \end{array}} \begin{array}{c} \&_{L1} \\ \hline \\ - : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\ \hline \hline \\ \bullet \mathbf{h}_1 : \Delta_2 \vdash F_5 \oplus F_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ \hline \\ - : \Delta_{11}, \Delta_2, F_8 \vdash F_{10} \\ \hline \\ - : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10} \end{array} \begin{array}{c} \&_{L1} \\ \end{array} \end{array}$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6} \ \oplus_{R_1} \ \frac{\mathbf{h}_7: \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7: (\Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_{10}} \ \\ \hline -: \Delta_2, \Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \\ \hline -: \Delta_{11}, \Delta_2, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline -: \Delta_{11}, \Delta_2, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} \otimes_L \\ \end{array} \begin{array}{c} \otimes_L \\ \text{Cut} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5 \oplus \mathbf{F}_6 & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_1 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_2 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_2 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_2 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_2 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_2 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_2 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_2 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_2 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_2 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3 \otimes_L & \\ \hline \bullet \mathbf{h}$$

• Case rule \oplus_L

$$\frac{ \begin{array}{c} h_1: \Delta_2 \vdash F_5 \\ \bullet h_1: \Delta_2 \vdash F_5 \oplus F_6 \end{array} \oplus_{R_1} \begin{array}{c} h_7: \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \\ \bullet h_7: (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10} \end{array} \oplus_{L} \\ \hline -: \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\ \hline \\ \bullet h_1: \Delta_2 \vdash F_5 \oplus F_6 \end{array} \xrightarrow{ax/W} \begin{array}{c} h_7: \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \\ \hline -: \Delta_{11}, \Delta_2, F_8 \vdash F_{10} \end{array} \xrightarrow{\bullet h_1: \Delta_2 \vdash F_5 \oplus F_6} \begin{array}{c} ax/W \\ h_{Cut} \end{array} \xrightarrow{\bullet h_1: \Delta_2 \vdash F_5 \oplus F_6} \begin{array}{c} ax/W \\ h_7: \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10} \end{array} \xrightarrow{\bullet h_2: \Delta_1, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \\ \hline -: \Delta_{11}, \Delta_2, F_8 \mapsto F_{10} \end{array} \xrightarrow{\bullet h_1: \Delta_2 \vdash F_7} \begin{array}{c} ax/W \\ \bullet h_1: \Delta_2 \vdash F_7 \end{array} \xrightarrow{\bullet h_5: \Delta_6, F_7 \vdash F_9} \begin{array}{c} h_5: \Delta_6, F_8 \vdash F_9 \\ \bullet h_5: \Delta_6, F_7 \oplus F_8 \vdash F_9 \end{array} \xrightarrow{\bullet h_5: \Delta_6, F_7 \oplus F_8 \vdash F_9} Cut \end{array} \xrightarrow{\bullet h_1: \Delta_2 \vdash F_7} \begin{array}{c} ax/W \\ \bullet h_1: \Delta_2 \vdash F_7 \end{array} \xrightarrow{\bullet h_2: \Delta_2 \vdash F_7} \begin{array}{c} ax/W \\ \bullet h_2: \Delta_2 \vdash F_7 \end{array} \xrightarrow{\bullet h_3: \Delta_6, F_7 \vdash F_9} \begin{array}{c} ax/W \\ \bullet h_3: \Delta_2 \vdash F_7 \end{array} \xrightarrow{\bullet h_3: \Delta_6, F_7 \vdash F_9} Cut \end{array}$$

• Case rule \multimap_L

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: \Delta_{2} \vdash F_{5} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{2} \vdash F_{5} \oplus F_{6} \end{array} \oplus \mathbb{R}_{1} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{2} \vdash F_{5} \oplus F_{6} \end{array} \oplus \mathbb{R}_{1} \\ \hline \begin{array}{c} \mathbf{h}_{7}: \Delta_{12}, F_{5} \oplus F_{6} \vdash F_{9} \quad \mathbf{h}_{7}: \Delta_{8}, F_{10} \vdash F_{11} \\ \hline -: \Delta_{2}, \Delta_{8}, \Delta_{12}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} \end{array} \xrightarrow{\bullet \mathbf{h}_{7}: \Delta_{2}, F_{5} \oplus F_{6} \vdash F_{9} \\ \hline \begin{array}{c} \bullet \mathbf{h}_{1}: \Delta_{2} \vdash F_{5} \oplus F_{6} \end{array} & \mathbf{ax/W} \\ \hline \hline \begin{array}{c} \bullet \mathbf{h}_{7}: \Delta_{12}, F_{5} \oplus F_{6} \vdash F_{9} \\ \hline -: \Delta_{12}, \Delta_{2} \vdash F_{9} \end{array} & \mathbf{ax/W} \\ \hline \begin{array}{c} -: \Delta_{12}, \Delta_{2} \vdash F_{9} \\ \hline -: \Delta_{12}, \Delta_{2} \vdash F_{9} \end{array} & \mathbf{hCut} \\ \hline \begin{array}{c} \bullet \mathbf{h}_{1}: \Delta_{2} \vdash F_{5} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{2} \vdash F_{5} \oplus F_{6} \end{array} & \oplus \mathbb{R}_{1} & \begin{array}{c} \mathbf{h}_{7}: \Delta_{8} \vdash F_{9} \quad \mathbf{h}_{7}: \Delta_{12}, F_{10}, F_{5} \oplus F_{6} \vdash F_{11} \\ \hline \bullet \mathbf{h}_{7}: (\Delta_{8}, \Delta_{12}, F_{9} \multimap F_{10}) \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline \begin{array}{c} \bullet \mathbf{h}_{1}: \Delta_{2} \vdash F_{5} \oplus F_{6} \end{array} & \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_{7}: (\Delta_{8}, \Delta_{12}, F_{9} \multimap F_{10}) \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{2}, \Delta_{8}, \Delta_{12}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array} & \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_{2}, \Delta_{8}, F_{9} \multimap F_{10} \vdash F_{11} \end{array}$$

 \bullet Case rule I

5.8 Status of 1_L : OK

• Case rule !R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash 5}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash 5} \ \mathbf{1}_L \quad \frac{\mathbf{h}_6: ! \Upsilon 4, 5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6: ! \Upsilon 4, 5 \vdash ! \mathbf{F}_7} \\ -: (\mathbf{1},\Delta_2), ! \Upsilon 4 \vdash ! \mathbf{F}_7 \end{array} \begin{array}{c} ! R \\ \mathrm{Cut} \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash 5}{\bullet \mathbf{h}_6: \mathbf{1}, 5, ! \Upsilon 4 \vdash ! \mathbf{F}_7} \end{array} \begin{array}{c} \mathrm{ax/W} \\ -: \mathbf{1}, ! \Upsilon 4, \Delta_2 \vdash ! \mathbf{F}_7 \end{array} \end{array} \begin{array}{c} \mathrm{ax/W} \\ \mathrm{hCut} \end{array}$$

• Case rule $\mathbf{1}_R$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \mathbf{1}, \Delta_2 \vdash \mathbf{F}_5} \ \mathbf{1}_L & \frac{}{\bullet \mathbf{h}_6: \Delta_4, \mathbf{F}_5 \vdash \top} \\ \hline -: (\mathbf{1}, \Delta_2), \Delta_4 \vdash \top & \overset{\leadsto}{} \\ \hline -: \mathbf{1}, \Delta_2, \Delta_4 \vdash \top & \top \end{array}$$
 Cut

• Case rule $\&_R$

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \mathbf{1}, \Delta_2 \vdash \mathbf{F}_5} & \mathbf{1}_L & \frac{\mathbf{h}_6: \Delta_4, \mathbf{F}_5 \vdash \mathbf{F}_7 \quad \mathbf{h}_6: \Delta_4, \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6: \Delta_4, \mathbf{F}_5 \vdash \mathbf{F}_7 \& \mathbf{F}_8} \quad \mathbf{Cut} \\ \hline & -: (\mathbf{1}, \Delta_2), \Delta_4 \vdash \mathbf{F}_7 \& \mathbf{F}_8 \\ \hline & \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_6: \mathbf{1}, \Delta_4, \mathbf{F}_5 \vdash \mathbf{F}_7 \& \mathbf{F}_8} & \mathbf{ax/W} \\ & -: \mathbf{1}, \Delta_2, \Delta_4 \vdash \mathbf{F}_7 \& \mathbf{F}_8 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \multimap_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_5} \ \mathbf{1}_L & \frac{\mathbf{h}_6:\Delta_4,\mathbf{F}_5,\mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6:\Delta_4,\mathbf{F}_5 \vdash \mathbf{F}_7 \multimap \mathbf{F}_8} & \mathbf{Cut} \\ \hline -:(\mathbf{1},\Delta_2),\Delta_4 \vdash \mathbf{F}_7 \multimap \mathbf{F}_8 & \\ & \stackrel{\longleftarrow}{\bullet} \\ \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet} & \mathbf{ax/W} & \stackrel{\bullet}{\bullet} \\ \hline -:\mathbf{1},\Delta_2,\Delta_4 \vdash \mathbf{F}_7 \multimap \mathbf{F}_8 & \mathbf{ax/W} \\ \hline -:\mathbf{1},\Delta_2,\Delta_4 \vdash \mathbf{F}_7 \multimap \mathbf{F}_8 & \mathbf{bCut} \end{array}$$

• Case rule \bigoplus_{R_2}

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \mathbf{1}, \Delta_2 \vdash \mathbf{F}_5} \ \mathbf{1}_L \ \ \frac{\mathbf{h}_6: \Delta_4, \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6: \Delta_4, \mathbf{F}_5 \vdash \mathbf{F}_7 \oplus \mathbf{F}_8} \\ \hline -: (\mathbf{1}, \Delta_2), \Delta_4 \vdash \mathbf{F}_7 \oplus \mathbf{F}_8 \\ \hline \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_6: \mathbf{1}, \Delta_4, \mathbf{F}_5 \vdash \mathbf{F}_7 \oplus \mathbf{F}_8} \end{array} \begin{array}{c} \oplus_{R_2} \\ \mathrm{Cut} \\ \hline \bullet \mathbf{h}_6: \mathbf{1}, \Delta_4, \mathbf{F}_5 \vdash \mathbf{F}_7 \oplus \mathbf{F}_8 \\ \hline -: \mathbf{1}, \Delta_2, \Delta_4 \vdash \mathbf{F}_7 \oplus \mathbf{F}_8 \end{array} \begin{array}{c} \mathrm{dax/W} \\ \mathrm{hCut} \end{array}$$

• Case rule \bigoplus_{R_1}

$$\begin{array}{l} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_5} \ \mathbf{1}_L \ \ \frac{\mathbf{h}_6:\Delta_4,\mathbf{F}_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6:\Delta_4,\mathbf{F}_5 \vdash \mathbf{F}_7 \oplus \mathbf{F}_8} \\ -:(\mathbf{1},\Delta_2),\Delta_4 \vdash \mathbf{F}_7 \oplus \mathbf{F}_8 \end{array} \begin{array}{l} \oplus_{R_1} \\ \mathrm{Cut} \\ \\ \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet} \ \ \mathrm{ax/W} \\ \hline -:\mathbf{1},\Delta_2,\Delta_4 \vdash \mathbf{F}_7 \oplus \mathbf{F}_8 \end{array} \begin{array}{l} \oplus_{R_1} \\ \mathrm{Cut} \\ \end{array}$$

• Case rule $\mathbf{1}_L$

• Case rule \otimes_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_4} & \mathbf{1}_L & \frac{\mathbf{h}_5:\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \quad \mathbf{h}_5:\Delta_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5:(\Delta_6,\Delta_9),\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8} & \otimes_R \\ \hline -:(\mathbf{1},\Delta_2),\Delta_6,\Delta_9 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \text{Cut} \\ \hline & & & & & \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet} & \text{ax/W} & & & & \\ \hline -:\mathbf{1},\Delta_2,\Delta_6,\Delta_9 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & & \text{hCut} \\ \hline \bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4 & \mathbf{1}_L & & & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & & \\ \hline \bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_4 & \mathbf{1}_L & & & & \\ \hline \bullet \mathbf{h}_5:(\Delta_6,\Delta_9),\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & & \\ \hline \bullet \mathbf{h}_5:(\Delta_6,\Delta_9),\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & & \\ \hline -:(\mathbf{1},\Delta_2),\Delta_6,\Delta_9 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_4 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_6,\Delta_9,\mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\Delta_9,\mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\Delta_9,\mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\Delta_9,\mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\Delta_9,\Delta_9,\mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\Delta_9,\mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\Delta_9,\mathbf{F}_7 \otimes \mathbf{F}_8 & & \\ \hline \bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\Delta_9,\Delta_9,\mathbf{F}$$

\bullet Case rule W

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_4} & \mathbf{1}_L & \frac{\mathbf{h}_5:\Delta_8,\mathbf{F}_4 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_5:(\Delta_8,!\mathbf{F}_6),\mathbf{F}_4 \vdash \mathbf{F}_7} & W \\ \hline -:(\mathbf{1},\Delta_2),\Delta_8,!\mathbf{F}_6 \vdash \mathbf{F}_7 & \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_5:\mathbf{1},\Delta_8,\mathbf{F}_4,!\mathbf{F}_6 \vdash \mathbf{F}_7} & \text{ax/W} \\ \hline -:\mathbf{1},\Delta_2,\Delta_8,!\mathbf{F}_6 \vdash \mathbf{F}_7 & \\ \hline \bullet \mathbf{h}_1:\Delta_2 \vdash !\mathbf{F}_6 & \mathbf{1}_L & \frac{\mathbf{h}_4:\Delta_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4:\Delta_5,!\mathbf{F}_6 \vdash \mathbf{F}_7} & W \\ \hline -:(\mathbf{1},\Delta_2),\Delta_5 \vdash \mathbf{F}_7 & \\ \hline -:(\mathbf{1},\Delta_2),\Delta_5 \vdash \mathbf{F}_7 & \\ \hline -:\mathbf{1},\Delta_2,\Delta_5 \vdash \mathbf{F}_7 & \\ \hline \end{array}$$

\bullet Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_4} \quad \mathbf{1}_L \quad \frac{\mathbf{h}_5:\Delta_8,\mathbf{F}_4, |\mathbf{F}_6, |\mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_5:(\Delta_8, |\mathbf{F}_6),\mathbf{F}_4 \vdash \mathbf{F}_7} \quad C \\ \hline -:(\mathbf{1},\Delta_2),\Delta_8, |\mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{} \quad \mathbf{ax/W} \quad \frac{\bullet}{\bullet \mathbf{h}_5:\mathbf{1},\Delta_8,\mathbf{F}_4, |\mathbf{F}_6 \vdash \mathbf{F}_7} \quad \mathbf{ax/W} \\ \hline -:\mathbf{1},\Delta_2,\Delta_8, |\mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2 \vdash |\mathbf{F}_6}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash |\mathbf{F}_6} \quad \mathbf{1}_L \quad \frac{\mathbf{h}_4:\Delta_5, |\mathbf{F}_6, |\mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4:\Delta_5, |\mathbf{F}_6 \vdash \mathbf{F}_7} \quad C \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2 \vdash |\mathbf{F}_6}{} \quad \mathbf{ax/W} \quad \frac{\bullet}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_5, |\mathbf{F}_6 \vdash \mathbf{F}_7} \quad \mathbf{ax/W} \\ \hline \\ -:\mathbf{1},\Delta_2,\Delta_5 \vdash \mathbf{F}_7 \quad \mathbf{ax/W} \\ \hline \\ -:\mathbf{1},\Delta_2,\Delta_5 \vdash \mathbf{F}_7 \end{array}$$

• Case rule !L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathbf{F}_4} \quad \mathbf{1}_L \quad \frac{\mathbf{h}_5 : \Delta_8, \mathbf{F}_4, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_5 : (\Delta_8, !\mathbf{F}_6), \mathbf{F}_4 \vdash \mathbf{F}_7} \quad \overset{!L}{\cot} \\ \\ - : (\mathbf{1}, \Delta_2), \Delta_8, !\mathbf{F}_6 \vdash \mathbf{F}_7 \\ & \stackrel{}{\sim} \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_4}{-: \mathbf{1}, \Delta_2, \Delta_8, !\mathbf{F}_6 \vdash \mathbf{F}_7} \quad \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_5 : \mathbf{1}, \Delta_8, \mathbf{F}_4, !\mathbf{F}_6 \vdash \mathbf{F}_7} \quad \overset{\mathbf{ax/W}}{\bullet \mathbf{h}_C \mathbf{ut}} \\ \\ \frac{\mathbf{h}_1 : \Delta_2 \vdash !\mathbf{F}_6}{\bullet \mathbf{h}_1 : \mathbf{1}, \Delta_2 \vdash !\mathbf{F}_6} \quad \mathbf{1}_L \quad \frac{\mathbf{h}_4 : \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4 : \Delta_5, !\mathbf{F}_6 \vdash \mathbf{F}_7} \quad \overset{!L}{\cot} \\ \\ - : (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathbf{F}_7 \quad & \mathbf{ax/W} \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash !\mathbf{F}_6}{-: \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathbf{F}_7} \quad & \mathbf{ax/W} \\ \hline - : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathbf{F}_7 \quad & \mathbf{ax/W} \\ \hline \end{pmatrix}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_4} \ \mathbf{1}_L & \frac{\mathbf{h}_5:\Delta_9,\mathbf{F}_4,\mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5:(\Delta_9,\mathbf{F}_6\&\mathbf{F}_7),\mathbf{F}_4 \vdash \mathbf{F}_8} & \&_{L2} \\ \hline -:(\mathbf{1},\Delta_2),\Delta_9,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8 & \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\mathbf{F}_4,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8} & \text{ax/W} \\ \hline -:\mathbf{1},\Delta_2,\Delta_9,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8 & \text{hCut} \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6\&\mathbf{F}_7}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_6\&\mathbf{F}_7} & \mathbf{1}_L & \frac{\mathbf{h}_4:\Delta_5,\mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8} & \&_{L2} \\ \hline -:(\mathbf{1},\Delta_2),\Delta_5 \vdash \mathbf{F}_8 & \\ \hline -:(\mathbf{1},\Delta_2),\Delta_5 \vdash \mathbf{F}_8 & \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6\&\mathbf{F}_7}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_5,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8} & \text{ax/W} \\ \hline -:\mathbf{1},\Delta_2,\Delta_5 \vdash \mathbf{F}_8 & \\ \hline \end{pmatrix} \\ \mathbf{hCut} \end{array}$$

• Case rule $\&_{L1}$

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_4} & \mathbf{1}_L & \frac{\mathbf{h}_5:\Delta_9,\mathbf{F}_4,\mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5:(\Delta_9,\mathbf{F}_6\&\mathbf{F}_7),\mathbf{F}_4 \vdash \mathbf{F}_8} & \&_{L1} \\ \hline & -:(\mathbf{1},\Delta_2),\Delta_9,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8 & & \\ \hline & \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\mathbf{F}_4,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8} & \text{ax/W} \\ \hline & -:\mathbf{1},\Delta_2,\Delta_9,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8 & & \text{hCut} \\ \hline & \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6\&\mathbf{F}_7}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_6\&\mathbf{F}_7} & \mathbf{1}_L & \frac{\mathbf{h}_4:\Delta_5,\mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8} & \&_{L1} \\ \hline & -:(\mathbf{1},\Delta_2),\Delta_5 \vdash \mathbf{F}_8 & & \\ \hline & \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6\&\mathbf{F}_7}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_5,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8} & \text{ax/W} \\ \hline & \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6\&\mathbf{F}_7}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_5,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8} & \text{ax/W} \\ \hline & \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6\&\mathbf{F}_7}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_5,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8} & \text{ax/W} \\ \hline & \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6\&\mathbf{F}_7}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_5,\mathbf{F}_6\&\mathbf{F}_7 \vdash \mathbf{F}_8} & \text{ax/W} \\ \hline & \mathbf{h}_0 \\ \hline \end{array}$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_4} \quad \mathbf{1}_L \quad \frac{\mathbf{h}_5:\Delta_9,\mathbf{F}_4,\mathbf{F}_6,\mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5:(\Delta_9,\mathbf{F}_6 \otimes \mathbf{F}_7),\mathbf{F}_4 \vdash \mathbf{F}_8} \quad \underset{\mathsf{Cut}}{\otimes L} \\ \hline -:(\mathbf{1},\Delta_2),\Delta_9,\mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_5:\mathbf{1},\Delta_9,\mathbf{F}_4,\mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_8} \quad \frac{\mathbf{ax/W}}{\bullet \mathbf{hCut}} \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7}{\bullet \mathbf{h}_1:\mathbf{1},\Delta_2 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7} \quad \mathbf{1}_L \quad \frac{\mathbf{h}_4:\Delta_5,\mathbf{F}_6,\mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_8} \quad \underset{\mathsf{Cut}}{\otimes L} \\ \hline -:(\mathbf{1},\Delta_2),\Delta_5 \vdash \mathbf{F}_8 \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7}{\bullet \mathbf{h}_4:\mathbf{1},\Delta_5,\mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_8} \quad \underset{\mathsf{Cut}}{\otimes L} \\ \hline -:(\mathbf{1},\Delta_2),\Delta_5 \vdash \mathbf{F}_8 \\ \hline -:\mathbf{1},\Delta_2,\Delta_5 \vdash \mathbf{F}_8 \\ \hline \end{array}$$

• Case rule \oplus_L

$$\frac{\begin{array}{l} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \\ \bullet \mathbf{h}_1: \mathbf{1}, \Delta_2 \vdash \mathbf{F}_4 \end{array} \quad \mathbf{1}_L \quad \begin{array}{l} \mathbf{h}_5: \Delta_9, \mathbf{F}_4, \mathbf{F}_6 \vdash \mathbf{F}_8 \quad \mathbf{h}_5: \Delta_9, \mathbf{F}_4, \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_5: (\Delta_9, \mathbf{F}_6 \oplus \mathbf{F}_7), \mathbf{F}_4 \vdash \mathbf{F}_8 \end{array} \quad \oplus L}{\begin{array}{l} -: (\mathbf{1}, \Delta_2), \Delta_9, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \hline \\ \overline{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4} \quad \mathbf{ax/W} \end{array} \quad \begin{array}{l} \longrightarrow \\ \bullet \mathbf{h}_5: \mathbf{1}, \Delta_9, \mathbf{F}_4, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \hline \\ -: \mathbf{1}, \Delta_2, \Delta_9, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_8 \end{array} \quad \mathbf{ax/W} \quad \mathbf{hCut} \end{array}}$$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7}{\bullet \mathbf{h}_1: \mathbf{1}, \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \ \mathbf{1}_L & \frac{\mathbf{h}_4: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \quad \mathbf{h}_4: \Delta_5, \mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_4: \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_8} \ \mathbf{Cut} \\ & -: (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathbf{F}_8 \\ & \stackrel{\longleftrightarrow}{\underbrace{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7}} \ \mathbf{ax/W} & \stackrel{\longleftrightarrow}{\underbrace{\bullet \mathbf{h}_4: \mathbf{1}, \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \vdash \mathbf{F}_8}} \ \mathbf{ax/W} \\ & -: \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathbf{F}_8 \end{array} \ \text{hCut} \end{array}$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}\vdash \mathbf{F}_{4}}{\bullet \mathbf{h}_{1}:\mathbf{1},\Delta_{2}\vdash \mathbf{F}_{4}} & \mathbf{1}_{L} & \frac{\mathbf{h}_{5}:\Delta_{10},\mathbf{F}_{4}\vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{5}:(\Delta_{6},\Delta_{10},\mathbf{F}_{7}\multimap \mathbf{F}_{8}),\mathbf{F}_{4}\vdash \mathbf{F}_{9}} \\ -:(\mathbf{1},\Delta_{2}),\Delta_{6},\Delta_{10},\mathbf{F}_{7}\multimap \mathbf{F}_{8}\vdash \mathbf{F}_{9} & \\ & \overset{\bullet}{\bullet \mathbf{h}_{5}:\mathbf{1},\Delta_{10},\Delta_{6},\mathbf{F}_{4},\mathbf{F}_{7}\multimap \mathbf{F}_{8}\vdash \mathbf{F}_{9}} \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2}\vdash \mathbf{F}_{4}}{-:\mathbf{1},\Delta_{10},\Delta_{2},\Delta_{6},\mathbf{F}_{7}\multimap \mathbf{F}_{8}\vdash \mathbf{F}_{9}} & \mathbf{ax}/\mathbb{W} \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2}\vdash \mathbf{F}_{4}}{\bullet \mathbf{h}_{1}:\mathbf{1},\Delta_{2}\vdash \mathbf{F}_{4}} & \mathbf{1}_{L} & \frac{\mathbf{h}_{5}:\Delta_{6}\vdash \mathbf{F}_{7} \quad \mathbf{h}_{5}:\Delta_{10},\mathbf{F}_{4},\mathbf{F}_{8}\vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{5}:(\Delta_{6},\Delta_{10},\mathbf{F}_{7}\multimap \mathbf{F}_{8}),\mathbf{F}_{4}\vdash \mathbf{F}_{9}} & \mathbf{cut} \\ \hline -:(\mathbf{1},\Delta_{2}),\Delta_{6},\Delta_{10},\mathbf{F}_{7}\multimap \mathbf{F}_{8}\vdash \mathbf{F}_{9} & \mathbf{cut} \\ \hline -:(\mathbf{1},\Delta_{2}),\Delta_{6},\Delta_{10},\mathbf{F}_{7}\multimap \mathbf{F}_{8}\vdash \mathbf{F}_{9} & \mathbf{ax}/\mathbb{W} \\ \hline -:\mathbf{1},\Delta_{10},\Delta_{2},\Delta_{6},\mathbf{F}_{7}\multimap \mathbf{F}_{8}\vdash \mathbf{F}_{9} & \mathbf{ax}/\mathbb{W} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathbf{F}_{7}\multimap \mathbf{F}_{8} & \mathbf{1}_{L} & \frac{\mathbf{h}_{4}:\Delta_{5}\vdash \mathbf{F}_{7} \quad \mathbf{h}_{4}:\Delta_{6},\mathbf{F}_{8}\vdash \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathbf{F}_{7}\multimap \mathbf{F}_{8} & \mathbf{1}_{L} & \frac{\mathbf{h}_{4}:\Delta_{5}\vdash \mathbf{F}_{7} \quad \mathbf{h}_{4}:\Delta_{6},\mathbf{F}_{8}\vdash \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{4}:(\Delta_{5},\Delta_{6}),\mathbf{F}_{7}\multimap \mathbf{F}_{8}\vdash \mathbf{F}_{9} & \mathbf{cut} \\ \hline \hline -:(\mathbf{1},\Delta_{2}),\Delta_{5},\Delta_{6}\vdash \mathbf{F}_{9} & \mathbf{ax}/\mathbb{W} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathbf{F}_{7}\multimap \mathbf{F}_{8} & \mathbf{ax}/\mathbb{W} & \mathbf{h}_{Cut} \\ \hline \hline -:(\mathbf{1},\Delta_{2},\Delta_{5},\Delta_{6}\vdash \mathbf{F}_{9} & \mathbf{ax}/\mathbb{W} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}\vdash \mathbf{F}_{7}\multimap \mathbf{F}_{8} & \mathbf{ax}/\mathbb{W} & \mathbf{h}_{Cut} \\ \hline \hline \end{array}$$

ullet Case rule I

$$\frac{ \mathbf{h}_1 : \Delta_2 \vdash p(\mathbf{n}_5) }{ \bullet \mathbf{h}_1 : \mathbf{1}, \Delta_2 \vdash p(\mathbf{n}_5) } \ \mathbf{1}_L \quad \frac{}{\bullet \mathbf{h}_4 : *, p(\mathbf{n}_5) \vdash p(\mathbf{n}_5) } \quad I_{\mathbf{Cut}} \\ - : (\mathbf{1}, \Delta_2), * \vdash p(\mathbf{n}_5) \\ & \stackrel{\sim}{\longrightarrow} \quad \frac{}{- : \mathbf{1}, \Delta_2 \vdash p(\mathbf{n}_5)} \quad \mathbf{ax/W}$$

5.9 Status of \otimes_R : OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7 \quad \mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8} \quad \otimes_R \quad & \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8} & \otimes_R \quad & \\ \hline -: (\Delta_2, \Delta_3), \Delta_6 \vdash \top \\ \hline & & \\ \hline -: \Delta_2, \Delta_3, \Delta_6 \vdash \top \end{array} \quad \top$$

• Case rule $\&_R$

$$\frac{\underbrace{\frac{\mathbf{h}_1:\Delta_2\vdash \mathsf{F}_7\quad \mathbf{h}_1:\Delta_3\vdash \mathsf{F}_8}{\bullet \mathbf{h}_1:\Delta_2,\Delta_3\vdash \mathsf{F}_7\otimes \mathsf{F}_8}}_{\bullet \mathbf{h}_1:\Delta_2,\Delta_3\vdash \mathsf{F}_7\otimes \mathsf{F}_8}\otimes_R \quad \underbrace{\frac{\mathbf{h}_9:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{10}\quad \mathbf{h}_9:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}}_{\bullet \mathbf{h}_9:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{10}\&\mathsf{F}_{11}}} \underbrace{\mathbf{Cut}}_{-:(\Delta_2,\Delta_3),\Delta_6\vdash \mathsf{F}_{10}\&\mathsf{F}_{11}} \underbrace{\mathbf{cut}}_{\bullet \mathbf{h}_1:\Delta_2,\Delta_3\vdash \mathsf{F}_7\otimes \mathsf{F}_8} \underbrace{\mathsf{ax/W}}_{\mathsf{h}_9:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}} \underbrace{\mathsf{ax/W}}_{\mathsf{h}_2:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}}_{-:\Delta_2,\Delta_3,\Delta_6\vdash \mathsf{F}_{10}} \underbrace{\mathbf{ax/W}}_{\mathsf{h}_2:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}}}_{-:\Delta_2,\Delta_3,\Delta_6\vdash \mathsf{F}_{10}\&\mathsf{F}_{11}} \underbrace{\mathbf{ax/W}}_{\mathsf{h}_2:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}} \underbrace{\mathbf{ax/W}}_{\mathsf{h}_2:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}}_{-:\Delta_2,\Delta_3,\Delta_6\vdash \mathsf{F}_{10}\&\mathsf{F}_{11}} \underbrace{\mathbf{ax/W}}_{\mathsf{h}_2:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}}}_{-:\Delta_2,\Delta_3,\Delta_6\vdash \mathsf{F}_{11}} \underbrace{\mathbf{ax/W}}_{\mathsf{h}_2:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}}_{-:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}}}_{-:\Delta_6,\mathsf{F}_7\otimes \mathsf{F}_8\vdash \mathsf{F}_{11}}$$

• Case rule \multimap_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7 \quad \mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8} \otimes_R & \frac{\mathbf{h}_9: \Delta_6, \mathbf{F}_{10}, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_9: \Delta_6, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_{10} \multimap \mathbf{F}_{11}} & \neg \circ_R \\ & -: (\Delta_2, \Delta_3), \Delta_6 \vdash \mathbf{F}_{10} \multimap \mathbf{F}_{11} \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8 & \text{ax/W} & \\ \hline & \frac{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8}{\bullet} & \text{ax/W} & \frac{-: \Delta_2, \Delta_3, \Delta_6, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{-: \Delta_2, \Delta_3, \Delta_6 \vdash \mathbf{F}_{10} \multimap \mathbf{F}_{11}} & \neg \circ_R \end{array} \\ & \frac{\mathsf{hCut}}{\bullet} \\ \end{array}$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_7 \quad \mathbf{h}_1: \Delta_3 \vdash \mathsf{F}_8}{\overset{\bullet}{\bullet} \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathsf{F}_7 \otimes \mathsf{F}_8} \otimes_R \quad \frac{\mathbf{h}_9: \Delta_6, \mathsf{F}_7 \otimes \mathsf{F}_8 \vdash \mathsf{F}_{11}}{\overset{\bullet}{\bullet} \mathbf{h}_9: \Delta_6, \mathsf{F}_7 \otimes \mathsf{F}_8 \vdash \mathsf{F}_{10} \oplus \mathsf{F}_{11}} \\ -: (\Delta_2, \Delta_3), \Delta_6 \vdash \mathsf{F}_{10} \oplus \mathsf{F}_{11} \\ \\ \overset{\bullet}{\bullet} \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathsf{F}_7 \otimes \mathsf{F}_8} \quad \overset{\mathsf{ax/W}}{\overset{\bullet}{\bullet}} \frac{\mathbf{h}_9: \Delta_6, \mathsf{F}_7 \otimes \mathsf{F}_8 \vdash \mathsf{F}_{10}}{\overset{\bullet}{\bullet} \mathsf{h}_{11}} \overset{\mathsf{ax/W}}{\overset{\bullet}{\bullet}} \\ \frac{-: \Delta_2, \Delta_3, \Delta_6 \vdash \mathsf{F}_{11}}{-: \Delta_2, \Delta_3, \Delta_6 \vdash \mathsf{F}_{10} \oplus \mathsf{F}_{11}} \overset{\oplus}{\overset{\bullet}{\bullet}} \mathsf{R}_2$$

• Case rule \oplus_{R_1}

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_7 \quad \mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_8}{\underbrace{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8}} \otimes_R \quad \frac{\mathbf{h}_9: \Delta_6, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_9: \Delta_6, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_{10} \oplus \mathbf{F}_{11}} \quad \overset{\oplus}{\mathsf{Cut}} \\ \\ -: (\Delta_2, \Delta_3), \Delta_6 \vdash \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \\ \hline{\underbrace{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8}} \quad \underbrace{\mathsf{ax/W}} \quad \frac{\mathbf{h}_9: \Delta_6, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_{10}}{\mathbf{h}_9: \Delta_6, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_{10}} \quad \underset{\mathsf{hCut}}{\mathsf{ax/W}} \\ \\ -: \Delta_2, \Delta_3, \Delta_6 \vdash \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_6 \vdash \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \end{bmatrix} \oplus_{R_1}$$

• Case rule $\mathbf{1}_L$

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \quad \mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_7}{\underbrace{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7}} \otimes_R \quad \frac{\mathbf{h}_8: \Delta_{10}, \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8: (\mathbf{1}, \Delta_{10}), \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_9} \quad \mathbf{1}_L}{-: (\Delta_2, \Delta_3), \mathbf{1}, \Delta_{10} \vdash \mathbf{F}_9} \quad \mathbf{Cut}}$$

$$\underbrace{\frac{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7}_{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7} \quad \mathbf{ax/W}}_{h_8: \mathbf{1}, \Delta_{10}, \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_9}}_{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7} \quad \mathbf{ax/W}}_{\bullet \mathbf{h}\mathbf{Cut}}$$

• Case rule \otimes_R

$$\frac{\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2\vdash F_6\quad \mathbf{h}_1:\Delta_3\vdash F_7}{\bullet \mathbf{h}_1:\Delta_2,\Delta_3\vdash F_6\otimes F_7} \otimes_R & \frac{\mathbf{h}_8:\Delta_{12},F_6\otimes F_7\vdash F_{10}\quad \mathbf{h}_8:\Delta_9\vdash F_{11}}{\bullet \mathbf{h}_8:(\Delta_9,\Delta_{12}),F_6\otimes F_7\vdash F_{10}\otimes F_{11}} & \otimes_R \\ \hline & -:(\Delta_2,\Delta_3),\Delta_9,\Delta_{12}\vdash F_{10}\otimes F_{11} & & \mathbf{cut} \\ \hline & & & & & \\ \hline \bullet \mathbf{h}_1:\Delta_2,\Delta_3\vdash F_6\otimes F_7 & \mathbf{ax/W} & & & & \\ \hline & \frac{-:\Delta_{12},\Delta_2,\Delta_3\vdash F_{10}}{\bullet \mathbf{h}^2} & \mathbf{h}^2 \mathbf{cut} & & & \\ \hline & & -:\Delta_{12},\Delta_2,\Delta_3,\Delta_9\vdash F_{10}\otimes F_{11} & & & \\ \hline & & & -:\Delta_{12},\Delta_2,\Delta_3,\Delta_9\vdash F_{10}\otimes F_{11} & & \\ \hline \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_6 \quad \mathbf{h}_1: \Delta_3 \vdash \mathsf{F}_7 \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathsf{F}_6 \otimes \mathsf{F}_7 \end{array} \otimes_R \quad \begin{array}{c} \mathbf{h}_8: \Delta_9 \vdash \mathsf{F}_{10} \quad \mathbf{h}_8: \Delta_{12}, \mathsf{F}_6 \otimes \mathsf{F}_7 \vdash \mathsf{F}_{11} \\ \hline \bullet \mathbf{h}_8: (\Delta_9, \Delta_{12}), \mathsf{F}_6 \otimes \mathsf{F}_7 \vdash \mathsf{F}_{10} \otimes \mathsf{F}_{11} \end{array} \otimes_R \\ \hline -: (\Delta_2, \Delta_3), \Delta_9, \Delta_{12} \vdash \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \xrightarrow{\bullet} \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathsf{F}_6 \otimes \mathsf{F}_7} \quad \begin{array}{c} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_8: \Delta_{12}, \mathsf{F}_6 \otimes \mathsf{F}_7 \vdash \mathsf{F}_{11} \\ \hline -: \Delta_{12}, \Delta_2, \Delta_3 \vdash \mathsf{F}_{10} \otimes \mathsf{F}_{11} \end{array} \otimes_R \end{array} \quad \begin{array}{c} \mathsf{ax/W} \\ \mathsf{hCut} \end{array}$$

 \bullet Case rule W

$$\frac{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_6\quad \mathbf{h}_1:\Delta_3\vdash \mathbf{F}_7}{\underbrace{\bullet\mathbf{h}_1:\Delta_2,\Delta_3\vdash \mathbf{F}_6\otimes \mathbf{F}_7}}\otimes_R \quad \frac{\mathbf{h}_8:\Delta_{11},\mathbf{F}_6\otimes \mathbf{F}_7\vdash \mathbf{F}_{10}}{\bullet\mathbf{h}_8:(\Delta_{11},!\mathbf{F}_9),\mathbf{F}_6\otimes \mathbf{F}_7\vdash \mathbf{F}_{10}} \quad W \quad \mathsf{Cut}}{-:(\Delta_2,\Delta_3),\Delta_{11},!\mathbf{F}_9\vdash \mathbf{F}_{10}} \quad \underbrace{\bullet}_{\mathbf{h}_8:\Delta_{11},l\mathbf{F}_9,\mathbf{F}_6\otimes \mathbf{F}_7\vdash \mathbf{F}_{10}}^{\bullet\bullet} \quad \mathsf{ax/W}}_{\mathbf{h}:\Delta_1,\Delta_2,\Delta_3\vdash \mathbf{F}_6\otimes \mathbf{F}_7} \quad \mathsf{ax/W}}_{-:\Delta_{11},\Delta_2,\Delta_3,!\mathbf{F}_9\vdash \mathbf{F}_{10}} \quad \mathsf{ax/W}}$$

ullet Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \quad \mathbf{h}_1: \Delta_3 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7} \otimes_R \quad \frac{\mathbf{h}_8: \Delta_{11}, !\mathbf{F}_9, !\mathbf{F}_9, \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: (\Delta_{11}, !\mathbf{F}_9), \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{10}} \quad C \\ \hline -: (\Delta_2, \Delta_3), \Delta_{11}, !\mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7 \quad \text{ax/W} \quad \overbrace{\mathbf{h}_8: \Delta_{11}, !\mathbf{F}_9, !\mathbf{F}_9, \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{10}}^{\bullet \mathbf{xz/W}} \quad \frac{\mathbf{Ax/W}}{\mathbf{h}_8: \Delta_{11}, !\mathbf{F}_9, !\mathbf{F}_9, \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{10}} \\ \hline -: \Delta_{11}, \Delta_2, \Delta_3, !\mathbf{F}_9 \vdash \mathbf{F}_{10} \quad C \end{array}$$

• Case rule !L

$$\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6 \quad \mathbf{h}_1:\Delta_3 \vdash \mathbf{F}_7}{\underbrace{\bullet \mathbf{h}_1:\Delta_2,\Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7}} \otimes_R \quad \frac{\mathbf{h}_8:\Delta_{11},\mathbf{F}_9,\mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8:(\Delta_{11},\mathbb{IF}_9),\mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{10}} \quad !L \quad \text{cut}} \\ \underbrace{\frac{\bullet \mathbf{h}_1:\Delta_2,\Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7}{\bullet \mathbf{h}_1:\Delta_2,\Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7}}_{\bullet \mathbf{h}_1:\Delta_2,\Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7} \quad \mathbf{ax/W}}_{\bullet \mathbf{h}_2:\Delta_{11},\mathbf{F}_9,\mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{10}}} \underbrace{\frac{\mathbf{ax/W}}{\mathbf{h}_2:\Delta_{11},\Delta_2,\Delta_3,\mathbf{F}_9 \vdash \mathbf{F}_{10}}}_{\bullet:\Delta_{11},\Delta_2,\Delta_3,\mathbb{IF}_9 \vdash \mathbf{F}_{10}}}_{\bullet:L} \quad !L}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathsf{F}_6 \quad \mathbf{h}_1: \Delta_3 \vdash \mathsf{F}_7}{\bullet \mathsf{h}_1: \Delta_2, \Delta_3 \vdash \mathsf{F}_6 \otimes \mathsf{F}_7} \otimes_R & \frac{\mathbf{h}_8: \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_6 \otimes \mathsf{F}_7 \vdash \mathsf{F}_{11}}{\bullet \mathsf{h}_8: (\Delta_{12}, \mathsf{F}_9 \& \mathsf{F}_{10}), \mathsf{F}_6 \otimes \mathsf{F}_7 \vdash \mathsf{F}_{11}} & \mathcal{E}_{L2} \\ & -: (\Delta_2, \Delta_3), \Delta_{12}, \mathsf{F}_9 \& \mathsf{F}_{10} \vdash \mathsf{F}_{11} \\ \hline \bullet \mathsf{h}_1: \Delta_2, \Delta_3 \vdash \mathsf{F}_6 \otimes \mathsf{F}_7 & \mathsf{ax/W} & \overset{\leadsto}{h_8: \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_6 \otimes \mathsf{F}_7 \vdash \mathsf{F}_{11}} \\ & \frac{\bullet \mathsf{h}_1: \Delta_2, \Delta_3 \vdash \mathsf{F}_6 \otimes \mathsf{F}_7}{-: \Delta_{12}, \Delta_2, \Delta_3, \mathsf{F}_{10} \vdash \mathsf{F}_{11}} & \mathcal{E}_{L2} \\ \hline & -: \Delta_{12}, \Delta_2, \Delta_3, \mathsf{F}_{9} \& \mathsf{F}_{10} \vdash \mathsf{F}_{11} & \mathcal{E}_{L2} \\ \hline \end{array}$$

• Case rule $\&_{L1}$

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}\vdash \mathsf{F}_{6}\quad \mathbf{h}_{1}:\Delta_{3}\vdash \mathsf{F}_{7}}{\bullet \mathsf{h}_{1}:\Delta_{2},\Delta_{3}\vdash \mathsf{F}_{6}\otimes \mathsf{F}_{7}}\otimes_{R} & \frac{\mathbf{h}_{8}:\Delta_{12},\mathsf{F}_{9},\mathsf{F}_{6}\otimes \mathsf{F}_{7}\vdash \mathsf{F}_{11}}{\bullet \mathsf{h}_{8}:(\Delta_{12},\mathsf{F}_{9}\&\mathsf{F}_{10}),\mathsf{F}_{6}\otimes \mathsf{F}_{7}\vdash \mathsf{F}_{11}} & \mathcal{E}_{L1} \\ & -:(\Delta_{2},\Delta_{3}),\Delta_{12},\mathsf{F}_{9}\&\mathsf{F}_{10}\vdash \mathsf{F}_{11} & \overset{\longleftarrow}{\mathsf{h}_{8}:\Delta_{12},\mathsf{F}_{9},\mathsf{F}_{6}\otimes \mathsf{F}_{7}\vdash \mathsf{F}_{11}} & \mathsf{ax/W} \\ & \overset{\bullet}{\underbrace{\bullet \mathsf{h}_{1}:\Delta_{2},\Delta_{3}\vdash \mathsf{F}_{6}\otimes \mathsf{F}_{7}}} & \mathsf{ax/W} & \overset{\longleftarrow}{\underbrace{\mathsf{h}_{8}:\Delta_{12},\mathsf{F}_{9},\mathsf{F}_{6}\otimes \mathsf{F}_{7}\vdash \mathsf{F}_{11}}} & \mathsf{bCut} \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & \\ & & & & & & & & & & & & \\ & & & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & \\ & & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & \\ & \\ & & \\ & \\ & \\ & & \\ & \\ & & \\ & \\ & \\ & & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ &$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_6 \quad \mathbf{h}_1 : \Delta_3 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1 : \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7} \otimes_R \quad \frac{\mathbf{h}_8 : \Delta_{12}, \mathbf{F}_9, \mathbf{F}_{10}, \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_8 : (\Delta_{12}, \mathbf{F}_9 \otimes \mathbf{F}_{10}), \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{11}} \quad \underset{\bullet}{\otimes_L} \\ \hline -: (\Delta_2, \Delta_3), \Delta_{12}, \mathbf{F}_9 \otimes \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \Delta_3 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7 \quad \text{ax/W} \quad \frac{}{\mathbf{h}_8 : \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_9, \mathbf{F}_6 \otimes \mathbf{F}_7 \vdash \mathbf{F}_{11}} \\ \hline -: \Delta_{12}, \Delta_2, \Delta_3, \mathbf{F}_{10}, \mathbf{F}_9 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_{12}, \Delta_2, \Delta_3, \mathbf{F}_{10}, \mathbf{F}_9 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_{12}, \Delta_2, \Delta_3, \mathbf{F}_9 \otimes \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_8 \quad \mathbf{h}_1 : \Delta_3 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \Delta_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & \otimes_R \quad \frac{\mathbf{h}_6 : \Delta_7, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6 : \Delta_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \hline \bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_8 \quad \mathbf{h}_1 : \Delta_3 \vdash \mathbf{F}_9 \quad \underset{\bullet}{\bullet \mathbf{h}}_1 \otimes_{\mathbf{h}}_1 \otimes$$

• Case rule \oplus_L

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{2}\vdash \mathbf{F}_{6}\quad \mathbf{h}_{1}:\Delta_{3}\vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3}\vdash \mathbf{F}_{6}\otimes \mathbf{F}_{7}}}{\bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3}\vdash \mathbf{F}_{6}\otimes \mathbf{F}_{7}}}\otimes_{R} \quad \frac{\mathbf{h}_{8}:\Delta_{12},\mathbf{F}_{9},\mathbf{F}_{6}\otimes \mathbf{F}_{7}\vdash \mathbf{F}_{11}\quad \mathbf{h}_{8}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{6}\otimes \mathbf{F}_{7}\vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_{8}:(\Delta_{12},\mathbf{F}_{9}\oplus \mathbf{F}_{10}),\mathbf{F}_{6}\otimes \mathbf{F}_{7}\vdash \mathbf{F}_{11}}}{\mathbf{Cut}} \quad \oplus_{L}$$

$$\frac{\bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3}\vdash \mathbf{F}_{6}\otimes \mathbf{F}_{7}\quad \mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_{8}:\Delta_{12},\mathbf{F}_{9},\mathbf{F}_{6}\otimes \mathbf{F}_{7}\vdash \mathbf{F}_{11}}}{\bullet \mathbf{h}_{2}} \quad \frac{\mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_{2}}\quad \frac{\bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3}\vdash \mathbf{F}_{6}\otimes \mathbf{F}_{7}\quad \mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_{2}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{6}\otimes \mathbf{F}_{7}\vdash \mathbf{F}_{11}}}{\bullet \mathbf{h}_{2}} \quad \frac{\mathbf{ax}/\mathbb{W}}{\bullet \mathbf{h}_{2}:\Delta_{12},\Delta_{2},\Delta_{3},\mathbf{F}_{10}\vdash \mathbf{F}_{11}} \quad \oplus_{L}$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{6} \quad \mathbf{h}_{1}:\Delta_{3} \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3} \vdash \mathbf{F}_{6} \otimes \mathbf{F}_{7}} \quad \otimes_{R} \quad \frac{\mathbf{h}_{8}:\Delta_{13},\mathbf{F}_{6} \otimes \mathbf{F}_{7} \vdash \mathbf{F}_{10} \quad \mathbf{h}_{8}:\Delta_{9},\mathbf{F}_{11} \vdash \mathbf{F}_{12}}{\bullet \mathbf{h}_{8}:(\Delta_{9},\Delta_{13},\mathbf{F}_{10} \multimap \mathbf{F}_{11}),\mathbf{F}_{6} \otimes \mathbf{F}_{7} \vdash \mathbf{F}_{12}} \quad \mathbf{cut} \\ \hline -:(\Delta_{2},\Delta_{3}),\Delta_{9},\Delta_{13},\mathbf{F}_{10} \multimap \mathbf{F}_{11} \vdash \mathbf{F}_{12} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3} \vdash \mathbf{F}_{6} \otimes \mathbf{F}_{7} \quad \mathbf{ax/W} \quad \mathbf{h}_{8}:\Delta_{13},\mathbf{F}_{6} \otimes \mathbf{F}_{7} \vdash \mathbf{F}_{10} \\ \hline -:\Delta_{13},\Delta_{2},\Delta_{3} \vdash \mathbf{F}_{10} \quad \mathbf{h}_{8}:\Delta_{13},\mathbf{F}_{10} \multimap \mathbf{F}_{11} \vdash \mathbf{F}_{12} \\ \hline -:\Delta_{13},\Delta_{2},\Delta_{3} \vdash \mathbf{F}_{6} \quad \mathbf{h}_{1}:\Delta_{2},\Delta_{3} \vdash \mathbf{F}_{7} \quad \otimes_{R} \quad \frac{\mathbf{h}_{8}:\Delta_{9} \vdash \mathbf{F}_{10} \quad \mathbf{h}_{8}:\Delta_{13},\mathbf{F}_{11},\mathbf{F}_{6} \otimes \mathbf{F}_{7} \vdash \mathbf{F}_{12} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3} \vdash \mathbf{F}_{6} \otimes \mathbf{F}_{7} \quad \otimes_{R} \quad \frac{\mathbf{h}_{8}:\Delta_{9} \vdash \mathbf{F}_{10} \quad \mathbf{h}_{8}:\Delta_{13},\mathbf{F}_{11},\mathbf{F}_{6} \otimes \mathbf{F}_{7} \vdash \mathbf{F}_{12} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3} \vdash \mathbf{F}_{6} \otimes \mathbf{F}_{7} \quad \otimes_{R} \quad \frac{\mathbf{h}_{8}:\Delta_{9} \vdash \mathbf{F}_{10} \quad \mathbf{h}_{8}:\Delta_{13},\mathbf{F}_{11},\mathbf{F}_{6} \otimes \mathbf{F}_{7} \vdash \mathbf{F}_{12} \\ \hline -:(\Delta_{2},\Delta_{3}),\Delta_{9},\Delta_{13},\mathbf{F}_{10} \multimap \mathbf{F}_{11} \vdash \mathbf{F}_{12} \\ \hline -:\Delta_{13},\Delta_{2},\Delta_{3} \vdash \mathbf{F}_{6} \otimes \mathbf{F}_{7} \quad \mathbf{ax/W} \quad \mathbf{h}_{8}:\Delta_{13},\mathbf{F}_{11},\mathbf{F}_{6} \otimes \mathbf{F}_{7} \vdash \mathbf{F}_{12} \\ \hline -:\Delta_{13},\Delta_{2},\Delta_{3},\Delta_{9},\mathbf{F}_{10} \multimap \mathbf{F}_{11} \vdash \mathbf{F}_{12} \quad -\circ_{L} \\ \hline \end{array}$$

 \bullet Case rule I

5.10 Status of W: OK

• Case rule !R

$$\begin{array}{l} \frac{\mathbf{h}_1:\Delta_2\vdash 6}{\bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3\vdash 6} \ W \quad \frac{\mathbf{h}_7: !\Upsilon 5, 6\vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: !\Upsilon 5, 6\vdash !\mathbf{F}_8} \quad !R \\ \hline -: (\Delta_2, !\mathbf{F}_3), !\Upsilon 5\vdash !\mathbf{F}_8 \\ \hline \frac{\mathbf{h}_1:\Delta_2\vdash 6}{\bullet \mathbf{h}_7: 6, !\Upsilon 5, !\mathbf{F}_3\vdash !\mathbf{F}_8} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \hline -: !\Upsilon 5, \Delta_2, !\mathbf{F}_3\vdash !\mathbf{F}_8 \end{array} \quad \frac{\mathsf{hCut}}{\bullet \mathsf{hCut}}$$

• Case rule $\mathbf{1}_R$

• Case rule \top

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \Delta_2 , ! \mathbf{F}_3 \vdash \mathbf{F}_6 \end{array} W \quad \begin{array}{c} \bullet \mathbf{h}_7 : \Delta_5 , \mathbf{F}_6 \vdash \top \\ - : (\Delta_2 , ! \mathbf{F}_3), \Delta_5 \vdash \top \\ \hline \\ \hline - : \Delta_2 , \Delta_5 , ! \mathbf{F}_3 \vdash \top \end{array} \quad \mathbf{Cut}$$

• Case rule $\&_R$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6 \\ \underline{\bullet \mathbf{h}_1: \Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_6 \end{array}} W \begin{array}{c} \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 & \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \underline{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \end{array} \begin{array}{c} \mathbf{Cut} \\ \\ \underline{-: (\Delta_2, |\mathbf{F}_3), \Delta_5 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \\ \\ \underline{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \underline{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6, |\mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \\ \\ \underline{-: \Delta_2, \Delta_5, |\mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \multimap_R

$$\begin{array}{l} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_6} \quad W \quad \frac{\mathbf{h}_7: \Delta_5, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9} \quad \overset{\multimap_R}{\overset{\longleftarrow}{\bullet}} \quad \text{Cut} \\ \\ \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{} \quad \frac{\mathsf{ax/W}}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6, !\mathbf{F}_3 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \end{array}$$

• Case rule \bigoplus_{R_2}

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_6|} \ W & \frac{\mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline -: (\Delta_2, |\mathbf{F}_3), \Delta_5 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6, |\mathbf{F}_3 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9} \end{array} & \frac{\oplus R_2}{\mathsf{Cut}} \\ \hline -: (\Delta_2, \Delta_5, |\mathbf{F}_3 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9) & \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \hline \end{array}$$

• Case rule \bigoplus_{R_1}

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_6|} \ W & \frac{\mathbf{h}_7:\Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7:\Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline -:(\Delta_2, |\mathbf{F}_3), \Delta_5 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9 & \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_7:\Delta_5, \mathbf{F}_6, |\mathbf{F}_3 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9} & \frac{\mathsf{ax/W}}{\mathsf{hCut}} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2\vdash\mathbf{1}}{\bullet\mathbf{h}_1:\Delta_2, |\mathbf{F}_3\vdash\mathbf{1}} & W & \frac{\mathbf{h}_5:\Delta_6\vdash\mathbf{F}_7}{\bullet\mathbf{h}_5:\Delta_6,\mathbf{1}\vdash\mathbf{F}_7} & \mathbf{1}_L \\ & -:(\Delta_2, |\mathbf{F}_3), \Delta_6\vdash\mathbf{F}_7 & \mathbf{ax/W} \\ \hline & -:\Delta_2,\Delta_6, |\mathbf{F}_3\vdash\mathbf{F}_7| & \mathbf{ax/W} \\ \hline & \frac{\mathbf{h}_1:\Delta_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_1:\Delta_2, |\mathbf{F}_3\vdash\mathbf{F}_5|} & W & \frac{\mathbf{h}_6:\Delta_8,\mathbf{F}_5\vdash\mathbf{F}_7}{\bullet\mathbf{h}_6:(\mathbf{1},\Delta_8),\mathbf{F}_5\vdash\mathbf{F}_7} & \mathbf{1}_L \\ \hline & -:(\Delta_2, |\mathbf{F}_3),\mathbf{1},\Delta_8\vdash\mathbf{F}_7 & \mathbf{cut} \\ \hline & \frac{\mathbf{h}_1:\Delta_2\vdash\mathbf{F}_5}{\bullet\mathbf{h}_1:\Delta_2\vdash\mathbf{F}_5} & \mathbf{ax/W} & \frac{\mathbf{h}_6:\mathbf{1},\Delta_8,\mathbf{F}_5,|\mathbf{F}_3\vdash\mathbf{F}_7}{\bullet\mathbf{h}_6:\mathbf{1},\Delta_8,\mathbf{F}_5,|\mathbf{F}_3\vdash\mathbf{F}_7} & \mathbf{ax/W} \\ \hline & -:\mathbf{1},\Delta_2,\Delta_8,|\mathbf{F}_3\vdash\mathbf{F}_7 & \mathbf{ax/W} \\ \hline \end{array}$$

• Case rule \otimes_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_5|} \ W & \frac{\mathbf{h}_6:\Delta_{10}, \mathbf{F}_5 \vdash \mathbf{F}_8 \quad \mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:(\Delta_7,\Delta_{10}), \mathbf{F}_5 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} \\ \hline -:(\Delta_2, |\mathbf{F}_3),\Delta_7,\Delta_{10} \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & \mathbf{Cut} \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_6:\Delta_{10},\Delta_2,\Delta_7, |\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & \mathbf{Ax/W} \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, |\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 & W & \frac{\mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_8 \quad \mathbf{h}_6:\Delta_{10},\mathbf{F}_5 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:(\Delta_7,\Delta_{10}),\mathbf{F}_5 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & \mathbf{Cut} \\ \hline \bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 & \mathbf{Ax/W} & \bullet \mathbf{h}_6:\Delta_{10},\Delta_7,\mathbf{F}_5, |\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:(\Delta_2, |\mathbf{F}_3),\Delta_7,\Delta_{10} \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & \mathbf{Cut} \\ \hline \bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5 & \mathbf{Ax/W} & \bullet \mathbf{h}_6:\Delta_{10},\Delta_7,\mathbf{F}_5, |\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, |\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & \mathbf{hCut} \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, |\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & \mathbf{hCut} \\ \hline \end{array}$$

\bullet Case rule W

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash \mathbf{F}_5} & W & \frac{\mathbf{h}_6 : \Delta_9, \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6 : (\Delta_9, ! \mathbf{F}_7), \mathbf{F}_5 \vdash \mathbf{F}_8} & W \\ \hline -: (\Delta_2, ! \mathbf{F}_3), \Delta_9, ! \mathbf{F}_7 \vdash \mathbf{F}_8 & & \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5}{\bullet} & \mathbf{ax/W} & \frac{\bullet \mathbf{h}_6 : \Delta_9, \mathbf{F}_5, ! \mathbf{F}_3, ! \mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet} & \mathbf{ax/W} \\ \hline -: \Delta_2, \Delta_9, ! \mathbf{F}_3, ! \mathbf{F}_7 \vdash \mathbf{F}_8 & & \mathbf{hCut} \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash ! \mathbf{F}_7}{\bullet \mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash ! \mathbf{F}_7} & W & \frac{\mathbf{h}_5 : \Delta_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7 \vdash \mathbf{F}_8} & W \\ \hline -: (\Delta_2, ! \mathbf{F}_3), \Delta_6 \vdash \mathbf{F}_8 & & \\ \hline & -: \Delta_2, \Delta_6, ! \mathbf{F}_3 \vdash \mathbf{F}_8 & \mathbf{ax/W} \\ \hline \end{array}$$

• Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash \mathbf{F}_5} \ W & \frac{\mathbf{h}_6 : \Delta_9, \mathbf{F}_5, ! \mathbf{F}_7, ! \mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6 : (\Delta_9, ! \mathbf{F}_7), \mathbf{F}_5 \vdash \mathbf{F}_8} \\ \hline - : (\Delta_2, ! \mathbf{F}_3), \Delta_9, ! \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5}{\bullet} & \frac{\mathsf{ax}/\mathsf{W}}{\bullet} \\ \hline - : \Delta_2, \Delta_9, ! \mathbf{F}_3, ! \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \hline - : \Delta_2, \Delta_9, ! \mathbf{F}_3, ! \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash ! \mathbf{F}_7}{\bullet \mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash ! \mathbf{F}_7} \ W & \frac{\mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7, ! \mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7 \vdash \mathbf{F}_8} \ C \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash ! \mathbf{F}_7}{\bullet} & \frac{\mathsf{ax}/\mathsf{W}}{\bullet} \\ \hline \frac{\mathsf{ax}/\mathsf{W}}{\bullet} \\ \hline - : (\Delta_2, ! \mathbf{F}_3), \Delta_6 \vdash \mathbf{F}_8 \\ \hline - : (\Delta_2, \Delta_6, ! \mathbf{F}_3, ! \mathbf{F}_7 \vdash \mathbf{F}_8) \ h \mathsf{Cut} \\ \hline - : \Delta_2, \Delta_6, ! \mathbf{F}_3 \vdash \mathbf{F}_8 \end{array} & \frac{\mathsf{ax}/\mathsf{W}}{\bullet} \\ \hline - : \Delta_2, \Delta_6, ! \mathbf{F}_3 \vdash \mathbf{F}_8} & \mathsf{hCut} \\ \hline \end{array}$$

• Case rule !L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1 : \Delta_2 \mid \mathbf{F}_3 \vdash \mathbf{F}_5} \ W & \frac{\mathbf{h}_6 : \Delta_9, \mathbf{F}_5, \mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6 : (\Delta_9, !\mathbf{F}_7), \mathbf{F}_5 \vdash \mathbf{F}_8} \\ \hline - : (\Delta_2, !\mathbf{F}_3), \Delta_9, !\mathbf{F}_7 \vdash \mathbf{F}_8 \\ & \sim \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5}{\bullet} & \mathbf{ax/W} & \frac{\bullet \mathbf{h}_6 : \Delta_9, \mathbf{F}_5, !\mathbf{F}_3, !\mathbf{F}_7 \vdash \mathbf{F}_8}{\circ} \\ \hline - : \Delta_2, \Delta_9, !\mathbf{F}_3, !\mathbf{F}_7 \vdash \mathbf{F}_8 \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash !\mathbf{F}_7}{\bullet \mathbf{h}_1 : \Delta_2, !\mathbf{F}_3 \vdash !\mathbf{F}_7} \ W & \frac{\mathbf{h}_5 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5 : \Delta_6, !\mathbf{F}_7 \vdash \mathbf{F}_8} \\ \hline - : (\Delta_2, !\mathbf{F}_3), \Delta_6 \vdash \mathbf{F}_8 \\ \hline \frac{\mathbf{h}_1 : \Delta_2 \vdash !\mathbf{F}_7}{\bullet} & \mathbf{ax/W} \\ \hline \frac{\bullet \mathbf{h}_5 : \Delta_6, !\mathbf{F}_3, !\mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet} & \mathbf{ax/W} \\ \hline - : \Delta_2, \Delta_6, !\mathbf{F}_3 \vdash \mathbf{F}_8 \\ \hline - : \Delta_2, \Delta_6, !\mathbf{F}_3 \vdash \mathbf{F}_8 \\ \hline \end{array} \right. & \mathbf{ax/W} \\ \mathbf{hCut} \\ \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, ! \mathbf{F}_3 \vdash \mathbf{F}_5} \ W & \frac{\mathbf{h}_6:\Delta_{10}, \mathbf{F}_5, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:(\Delta_{10}, \mathbf{F}_7 \& \mathbf{F}_8), \mathbf{F}_5 \vdash \mathbf{F}_9} \\ & -:(\Delta_2, ! \mathbf{F}_3), \Delta_{10}, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9 \\ & \sim \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5} & \mathbf{ax/W} & \frac{\mathbf{h}_6:\Delta_{10}, \mathbf{F}_5, ! \mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_0:\Delta_1, \Delta_2, ! \mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9} & \mathbf{ax/W} \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7 \& \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2, ! \mathbf{F}_3 \vdash \mathbf{F}_7 \& \mathbf{F}_8} & W & \frac{\mathbf{h}_5:\Delta_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_5:\Delta_6, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9} & \mathcal{L}_2 \\ \hline -:(\Delta_2, ! \mathbf{F}_3), \Delta_6 \vdash \mathbf{F}_9 & \sim \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7 \& \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7 \& \mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_5:\Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline -:\Delta_2,\Delta_6, ! \mathbf{F}_3 \vdash \mathbf{F}_9 & \mathbf{h}_C \mathsf{ut} \\ \hline \end{array}$$

• Case rule $\&_{L1}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, ! \mathbf{F}_3 \vdash \mathbf{F}_5} \ W & \frac{\mathbf{h}_6:\Delta_{10}, \mathbf{F}_5, \mathbf{F}_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:(\Delta_{10}, \mathbf{F}_7 \& \mathbf{F}_8), \mathbf{F}_5 \vdash \mathbf{F}_9} \\ -:(\Delta_2, ! \mathbf{F}_3), \Delta_{10}, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9 \\ & \sim \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_6:\Delta_{10}, \mathbf{F}_5, ! \mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9} \\ -:\Delta_{10}, \Delta_2, ! \mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7 \& \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2, ! \mathbf{F}_3 \vdash \mathbf{F}_7 \& \mathbf{F}_8} & W & \frac{\mathbf{h}_5:\Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_5:\Delta_6, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7 \& \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7 \& \mathbf{F}_8} & \mathbf{ax/W} \\ \hline \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7 \& \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_7 \& \mathbf{F}_8} & \mathbf{ax/W} \\ \hline -:(\Delta_2, \Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9) & \mathbf{ax/W} \\ \hline \mathbf{h}_5:\Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \& \mathbf{F}_8 \vdash \mathbf{F}_9 & \mathbf{ax/W} \\ \hline -:\Delta_2,\Delta_6, ! \mathbf{F}_3 \vdash \mathbf{F}_9 & \mathbf{hCut} \\ \hline \end{array}$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash \mathbf{F}_5} \quad W \quad \frac{\mathbf{h}_6 : \Delta_{10}, \mathbf{F}_5, \mathbf{F}_7, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6 : (\Delta_{10}, \mathbf{F}_7 \otimes \mathbf{F}_8), \mathbf{F}_5 \vdash \mathbf{F}_9} \\ & \stackrel{\sim}{\bullet} \frac{\mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash \mathbf{F}_5} \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5}{\bullet} \quad \frac{\mathbf{ax}/W} \quad & \stackrel{\sim}{\bullet} \frac{\mathbf{h}_6 : \Delta_{10}, \mathbf{F}_5, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6 : \Delta_{10}, \Delta_2, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8}{\bullet \mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8} \quad W \quad & \frac{\mathbf{h}_5 : \Delta_6, \mathbf{F}_7, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_5 : \Delta_6, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8}{\bullet} \quad & \mathbf{xx}/W \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8}{\bullet} \quad & \mathbf{xx}/W \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8}{\bullet} \quad & \mathbf{xx}/W \\ \hline \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7, \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_7 : \Delta_2, \Delta_6, ! \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_7 : \Delta_7, \Delta_6, ! \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_7 : \Delta_7, \Delta_6, ! \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_7 : \Delta_7, \Delta_6, ! \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_7 : \Delta_7, \Delta_6, ! \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_7 : \Delta_7, \Delta_7, \mathbf{h}_7 \vdash \mathbf{h}_7 \otimes \mathbf{h}_7$$

• Case rule \oplus_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}: \Delta_{2}, |\mathbf{F}_{3} \vdash \mathbf{F}_{5}|} \ W & \frac{\mathbf{h}_{6}: \Delta_{10}, \mathbf{F}_{5}, \mathbf{F}_{7} \vdash \mathbf{F}_{9} \quad \mathbf{h}_{6}: \Delta_{10}, \mathbf{F}_{5}, \mathbf{F}_{8} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{6}: (\Delta_{10}, \mathbf{F}_{7} \oplus \mathbf{F}_{8}), \mathbf{F}_{5} \vdash \mathbf{F}_{9}} \ Cut \\ & -: (\Delta_{2}, |\mathbf{F}_{3}), \Delta_{10}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9} \\ & \stackrel{\longleftarrow}{\longrightarrow} \\ \frac{\mathbf{h}_{1}: \Delta_{2} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{6}: \Delta_{10}, \mathbf{F}_{5}, |\mathbf{F}_{3}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9}} \ \mathbf{ax/W} \\ & -: \Delta_{10}, \Delta_{2}, |\mathbf{F}_{3}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9} \end{array} \right) \text{ hCut} \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2 \vdash \mathbb{F}_7 \oplus \mathbb{F}_8 \\ \bullet \mathbf{h}_1 : \Delta_2 : \mathbb{F}_3 \vdash \mathbb{F}_7 \oplus \mathbb{F}_8 \end{array} W \quad \begin{array}{c} \mathbf{h}_5 : \Delta_6, \mathbb{F}_7 \vdash \mathbb{F}_9 \quad \mathbf{h}_5 : \Delta_6, \mathbb{F}_8 \vdash \mathbb{F}_9 \\ \bullet \mathbf{h}_5 : \Delta_6, \mathbb{F}_7 \oplus \mathbb{F}_8 \vdash \mathbb{F}_9 \end{array} \quad \oplus L \\ \\ - : (\Delta_2, \mathbb{F}_3), \Delta_6 \vdash \mathbb{F}_9 \\ \hline \\ \underline{\mathbf{h}_1 : \Delta_2 \vdash \mathbb{F}_7 \oplus \mathbb{F}_8} \quad \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_5 : \Delta_6, \mathbb{F}_3, \mathbb{F}_7 \oplus \mathbb{F}_8 \vdash \mathbb{F}_9 \\ \hline - : \Delta_2, \Delta_6, \mathbb{F}_3 \vdash \mathbb{F}_9 \end{array} \quad \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}:\Delta_{2}, |\mathbf{F}_{3} \vdash \mathbf{F}_{5}|} \ W \\ & \frac{\mathbf{h}_{6}:\Delta_{11}, \mathbf{F}_{5} \vdash \mathbf{F}_{8} \quad \mathbf{h}_{6}:\Delta_{7}, \mathbf{F}_{9} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{6}:(\Delta_{7},\Delta_{11}, \mathbf{F}_{8} \multimap \mathbf{F}_{9}), \mathbf{F}_{5} \vdash \mathbf{F}_{10}} \\ & -:(\Delta_{2}, |\mathbf{F}_{3}), \Delta_{7}, \Delta_{11}, \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ & \sim \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5}} \ & \frac{\mathbf{ax}/\mathbf{W}}{\bullet \mathbf{h}_{6}:\Delta_{11},\Delta_{7}, \mathbf{F}_{5}, |\mathbf{F}_{3}, \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10}} \\ & -:\Delta_{11},\Delta_{2},\Delta_{7}, |\mathbf{F}_{3}, \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}, |\mathbf{F}_{3} \vdash \mathbf{F}_{5}| \ & \frac{\mathbf{h}_{6}:\Delta_{7} \vdash \mathbf{F}_{8} \quad \mathbf{h}_{6}:\Delta_{11}, \mathbf{F}_{5}, \mathbf{F}_{9} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{6}:(\Delta_{7},\Delta_{11}, \mathbf{F}_{8} \multimap \mathbf{F}_{9}), \mathbf{F}_{5} \vdash \mathbf{F}_{10}} \ & -: L \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}, |\mathbf{F}_{3} \vdash \mathbf{F}_{5}| \ & \frac{\mathbf{ax}/\mathbf{W}}{\bullet \mathbf{h}_{6}:(\Delta_{7},\Delta_{11}, \mathbf{F}_{8} \multimap \mathbf{F}_{9}), \mathbf{F}_{5} \vdash \mathbf{F}_{10}} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{5}| \ & \mathbf{ax}/\mathbf{W} \ & \frac{\mathbf{ax}/\mathbf{W}}{\bullet \mathbf{h}_{6}:\Delta_{11},\Delta_{7}, \mathbf{F}_{5}, |\mathbf{F}_{3}, \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10}} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{8} \multimap \mathbf{F}_{9}| \ & W \ & \frac{\mathbf{h}_{6}:\Delta_{11},\Delta_{7},\mathbf{F}_{5}, |\mathbf{F}_{3}, \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{5}:\Delta_{6} \vdash \mathbf{F}_{8} \quad \mathbf{h}_{5}:\Delta_{7}, \mathbf{F}_{9} \vdash \mathbf{F}_{10}} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{8} \multimap \mathbf{F}_{9}| \ & W \ & \frac{\mathbf{h}_{5}:\Delta_{6} \vdash \mathbf{F}_{8} \quad \mathbf{h}_{5}:\Delta_{7}, \mathbf{F}_{9} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{5}:\Delta_{6},\Delta_{7}, |\mathbf{F}_{3},\mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10}} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{8} \multimap \mathbf{F}_{9}| \ & \mathbf{ax}/\mathbf{W} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{8} \multimap \mathbf{F}_{9}| \ & \mathbf{ax}/\mathbf{W} \\ \hline \bullet \mathbf{h}_{5}:\Delta_{6},\Delta_{7}, |\mathbf{F}_{3},\mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10}| \ & \mathbf{ax}/\mathbf{W} \\ \hline \bullet \mathbf{h}_{5}:\Delta_{6},\Delta_{7}, |\mathbf{F}_{3},\mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10}| \ & \mathbf{ax}/\mathbf{W} \\ \hline \bullet \mathbf{h}_{5}:\Delta_{6},\Delta_{7}, |\mathbf{F}_{3},\mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10}| \ & \mathbf{ax}/\mathbf{W} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{8} \multimap \mathbf{F}_{9}| \ & \mathbf{ax}/\mathbf{W} \\ \hline \bullet \mathbf{h}_{2}:\Delta_{2},\Delta_{6},\Delta_{7}, |\mathbf{F}_{3},\mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10}| \ & \mathbf{ax}/\mathbf{W} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{h}_{1} \multimap \mathbf{h}_{2} \lor \mathbf{h}_{2} \lor \mathbf{h}_{2} \lor \mathbf{h}_{2} \lor \mathbf{h}_{2} \lor \mathbf{h}_{2} \lor \mathbf{h}_{$$

 \bullet Case rule I

5.11 Status of C: OK

• Case rule !R

$$\frac{\frac{\mathbf{h}_1:\Delta_2, !\mathsf{F}_3, !\mathsf{F}_3 \vdash 6}{\bullet \mathbf{h}_1:\Delta_2, !\mathsf{F}_3 \vdash 6}}{C} \frac{\mathbf{h}_7: !\Upsilon 5, 6 \vdash \mathsf{F}_8}{\bullet \mathbf{h}_7: !\Upsilon 5, 6 \vdash !\mathsf{F}_8}}{-: (\Delta_2, !\mathsf{F}_3), !\Upsilon 5 \vdash !\mathsf{F}_8} \\ \frac{\frac{\mathbf{h}_1:\Delta_2, !\mathsf{F}_3, !\mathsf{F}_3 \vdash 6}{\bullet \mathbf{h}_7: 6, !\Upsilon 5 \vdash !\mathsf{F}_8}}{\bullet \mathbf{h}_7: 6, !\Upsilon 5 \vdash !\mathsf{F}_8} \\ \frac{-: !\Upsilon 5, \Delta_2, !\mathsf{F}_3, !\mathsf{F}_3 \vdash !\mathsf{F}_8}{-: !\Upsilon 5, \Delta_2, !\mathsf{F}_3 \vdash !\mathsf{F}_8} C$$

- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_6}{\underbrace{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_6}_{-:(\Delta_2, !\mathbf{F}_3), \Delta_5 \vdash \top}} \overset{\top}{\underbrace{-:(\Delta_2, !\mathbf{F}_3), \Delta_5 \vdash \top}_{-:\Delta_2, \Delta_5, !\mathbf{F}_3 \vdash \top}} \overset{\top}{\top}$$

• Case rule $\&_R$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_6} \ C \ \ \frac{\mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \quad \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \ \ \mathbf{Cut} \\ \\ \frac{-: (\Delta_2, !\mathbf{F}_3), \Delta_5 \vdash \mathbf{F}_8 \& \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \ \ \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \\ \frac{-: \Delta_2, \Delta_5, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9}{-: \Delta_2, \Delta_5, !\mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \ \ C \end{array} \right. \\ \frac{\mathbf{hCut}}{\bullet \mathbf{hCut}} \\ \end{array}$$

• Case rule \multimap_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_6} \ C & \frac{\mathbf{h}_7:\Delta_5, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9} & \neg \circ_R \\ \hline \\ -:(\Delta_2, !\mathbf{F}_3), \Delta_5 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9 & \mathsf{Cut} \\ \hline \\ \frac{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet} & \mathsf{ax/W} & \overset{\hookrightarrow}{\underset{h_7:\Delta_5, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{}} \\ \\ \frac{-:\Delta_2, \Delta_5, \mathbf{F}_8, !\mathbf{F}_3 \vdash \mathbf{F}_9}{-:\Delta_2,\Delta_5, !\mathbf{F}_3 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9} & \neg \circ_R \\ \end{array}$$

• Case rule \bigoplus_{R_2}

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{6}} \ C \ \frac{\mathbf{h}_{7}:\Delta_{5}, \mathbf{F}_{6} \vdash \mathbf{F}_{8} \oplus \mathbf{F}_{9}}{\bullet \mathbf{h}_{7}:\Delta_{5}, \mathbf{F}_{6} \vdash \mathbf{F}_{8} \oplus \mathbf{F}_{9}} \\ \hline -:(\Delta_{2}, !\mathbf{F}_{3}), \Delta_{5} \vdash \mathbf{F}_{8} \oplus \mathbf{F}_{9} \\ \hline \frac{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{6}}{\bullet \mathbf{x}^{\vee} \vee} \frac{\mathbf{a}^{\vee} \vee \vee}{\mathbf{h}_{7}:\Delta_{5}, \mathbf{F}_{6} \vdash \mathbf{F}_{9}} \\ \hline \frac{-:\Delta_{2},\Delta_{5}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}}{-:\Delta_{2},\Delta_{5}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \oplus_{R_{2}} \end{array} \begin{array}{c} \oplus_{R_{2}} \\ \text{hCut} \end{array}$$

• Case rule \oplus_{R_1}

$$\frac{ \begin{aligned} & \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_6} \ C & \frac{\mathbf{h}_7:\Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7:\Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9} \ \frac{\oplus R_1}{\mathsf{Cut}} \\ & -: (\Delta_2, !\mathbf{F}_3), \Delta_5 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9 \\ & \overset{\bullet}{\longrightarrow} \\ & \frac{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{x} / \mathbf{W}} \frac{\mathbf{h}_7:\Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8}{\mathbf{h}_7:\Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8} \ \frac{\mathsf{ax} / \mathsf{W}}{\mathsf{hCut}} \\ & \frac{-: \Delta_2, \Delta_5, !\mathbf{F}_3 \vdash \mathbf{F}_8}{-: \Delta_2, \Delta_5, !\mathbf{F}_3 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9} \ \oplus R_1 \end{aligned} \\ \end{aligned} \\ \frac{\mathsf{hCut}}{\mathsf{hCut}}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3}\vdash \mathbf{1}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}\vdash \mathbf{1}} \quad C \quad \frac{\mathbf{h}_{5}:\Delta_{6}\vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{5}:\Delta_{6}, \mathbf{1}\vdash \mathbf{F}_{7}} \quad \mathbf{1}_{L} \\ -:(\Delta_{2}, !\mathbf{F}_{3})\vdash \mathbf{h}_{2} \quad & \rightarrow \\ \hline -:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3}\vdash \mathbf{F}_{7} \quad \mathbf{ax/W} \\ \\ \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3}\vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}\vdash \mathbf{F}_{5}} \quad C \quad \frac{\mathbf{h}_{6}:\Delta_{8}, \mathbf{F}_{5}\vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{6}:(\mathbf{1},\Delta_{8}), \mathbf{F}_{5}\vdash \mathbf{F}_{7}} \quad \mathbf{Cut} \\ \hline -:(\Delta_{2}, !\mathbf{F}_{3})\cdot \mathbf{1},\Delta_{8}\vdash \mathbf{F}_{7} \quad & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}\vdash \mathbf{F}_{5} \quad \mathbf{ax/W} \quad & \mathbf{h}_{6}:\mathbf{1},\Delta_{8}, \mathbf{F}_{5}\vdash \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}\vdash \mathbf{F}_{5} \quad \mathbf{ax/W} \quad & \mathbf{h}_{6}:\mathbf{1},\Delta_{8}, \mathbf{F}_{5}\vdash \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}\vdash \mathbf{F}_{5} \quad \mathbf{ax/W} \quad & \mathbf{h}_{6}:\mathbf{1},\Delta_{8}, \mathbf{F}_{5}\vdash \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}\vdash \mathbf{F}_{5} \quad \mathbf{ax/W} \quad & \mathbf{h}_{1}:\Delta_{2}, \mathbf{h}_{2}\vdash \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}, \mathbf{h}_{2}\vdash \mathbf{h}_{3}\vdash \mathbf{h}_{7} \quad & \mathbf{h}_{2}\vdash \mathbf{h}_{3}\vdash \mathbf{h}_{7} \\ \hline \end{array}$$

• Case rule \otimes_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_5} \ C \\ \hline \frac{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_5}{-:(\Delta_2, !\mathbf{F}_3), \Delta_7, \Delta_{10} \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} \\ \hline \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_5}{-:(\Delta_{10}, \Delta_2, \Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9)} \\ \hline \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_5}{-:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} \\ \hline \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_5}{-:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} \\ \hline -:(\Delta_2, !\mathbf{F}_3),\Delta_7,\Delta_{10} \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:(\Delta_2, !\mathbf{F}_3),\Delta_7,\Delta_{10} \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:(\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_5 \otimes \mathbf{F}_9 \\ \hline -:(\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:(\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:(\Delta_1, \Delta_2, \Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline -:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \\ \hline \end{array}$$

\bullet Case rule W

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{5} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}} C & \frac{\mathbf{h}_{6}:\Delta_{9}, !\mathbf{F}_{7}), \mathbf{F}_{5} \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{6}:(\Delta_{9}, !\mathbf{F}_{7}), \mathbf{F}_{5} \vdash \mathbf{F}_{8}} & Cut \\ \hline & -:(\Delta_{2}, !\mathbf{F}_{3}), \Delta_{9}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8} \\ \hline & \frac{\bullet}{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}} & \mathbf{ax/W} & \frac{\leadsto}{\mathbf{h}_{6}:\Delta_{9}, \mathbf{F}_{5}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8}} & \mathbf{ax/W} \\ & -:\Delta_{2}, \Delta_{9}, !\mathbf{F}_{3}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8} & \mathbf{hCut} \\ \hline & \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{7} \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash !\mathbf{F}_{7}} & C & \frac{\mathbf{h}_{5}:\Delta_{6} \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{5}:\Delta_{6}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8}} & W \\ & \frac{-:(\Delta_{2}, !\mathbf{F}_{3}), \Delta_{6} \vdash \mathbf{F}_{8}}{\hookrightarrow} & \mathbf{Cut} \\ \hline & -:(\Delta_{2}, !\mathbf{F}_{3}), \Delta_{6} \vdash \mathbf{F}_{8} & \mathbf{ax/W} \\ \hline & -:\Delta_{2}, \Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{8} & \mathbf{ax/W} \\ \hline \end{array}$$

ullet Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}} & C & \frac{\mathbf{h}_{6}:\Delta_{9}, \mathbf{F}_{5}, !\mathbf{F}_{7}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{6}:\Delta_{9}, !\mathbf{F}_{7}, \mathbf{F}_{5} \vdash \mathbf{F}_{8}} & C \\ \hline -:(\Delta_{2}, !\mathbf{F}_{3}), \Delta_{9}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8} & \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{6}:\Delta_{9}, \mathbf{F}_{5}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8}} & \mathbf{ax/W} \\ \hline -:\Delta_{2}, \Delta_{9}, !\mathbf{F}_{3}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8} & C \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7}}{-:\Delta_{2},\Delta_{9}, !\mathbf{F}_{3}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8}} & C \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7}} & C & \frac{\mathbf{h}_{5}:\Delta_{6}, !\mathbf{F}_{7}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{5}:\Delta_{6}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8}} & C \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7} & \mathbf{ax/W} & \mathbf{h}_{5}:\Delta_{6}, !\mathbf{F}_{7}, !\mathbf{F}_{7} \vdash \mathbf{F}_{8}} & \mathbf{ax/W} \\ \hline -:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{8} & \mathbf{ax/W} & \mathbf{h}_{Cut} \\ \hline \end{array}$$

• Case rule !L

$$\frac{\frac{\mathbf{h}_1:\Delta_2, \mathsf{lF}_3, \mathsf{lF}_3 \vdash \mathsf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, \mathsf{lF}_3 \vdash \mathsf{F}_5}}{-:(\Delta_2, \mathsf{lF}_3), \Delta_9, \mathsf{lF}_7 \vdash \mathsf{F}_8}} \underbrace{\begin{array}{c} \mathbf{h}_6:\Delta_9, \mathsf{F}_5, \mathsf{F}_7 \vdash \mathsf{F}_8 \\ \bullet \mathbf{h}_6:(\Delta_9, \mathsf{lF}_7), \mathsf{F}_5 \vdash \mathsf{F}_8 \end{array}}_{\bullet \mathbf{h}_6:(\Delta_9, \mathsf{lF}_7), \mathsf{F}_5 \vdash \mathsf{F}_8} \underbrace{\begin{array}{c} \mathsf{l}_1:\Delta_2, \mathsf{lF}_3, \mathsf{lF}_3 \vdash \mathsf{F}_5 \\ \bullet \mathsf{h}_6:\Delta_9, \mathsf{F}_5, \mathsf{lF}_7 \vdash \mathsf{F}_8 \end{array}}_{\bullet \mathsf{h}_6:\Delta_9, \mathsf{F}_5, \mathsf{lF}_7 \vdash \mathsf{F}_8} \underbrace{\begin{array}{c} \mathsf{ax/W} \\ \mathsf{hCut} \end{array}}_{\bullet \mathsf{C}}$$

$$\frac{ \begin{array}{l} \frac{\mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3, ! \mathbf{F}_3 \vdash ! \mathbf{F}_7}{\bullet \mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash ! \mathbf{F}_7} \ C & \frac{\mathbf{h}_5 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7 \vdash \mathbf{F}_8} \end{array} }{ \cdot \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7 \vdash \mathbf{F}_8} & !L \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3, ! \mathbf{F}_3 \vdash ! \mathbf{F}_7}{- : (\Delta_2, \Delta_6, ! \mathbf{F}_3, ! \mathbf{F}_3 \vdash \mathbf{F}_8)} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7 \vdash \mathbf{F}_8} \\ \hline \\ \frac{- : \Delta_2, \Delta_6, ! \mathbf{F}_3, ! \mathbf{F}_3 \vdash \mathbf{F}_8}{- : \Delta_2, \Delta_6, ! \mathbf{F}_3 \vdash \mathbf{F}_8} \ C \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_5} \ C & \frac{\mathbf{h}_6:\Delta_{10}, \mathbf{F}_5, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:(\Delta_{10}, \mathbf{F}_7\&\mathbf{F}_8), \mathbf{F}_5 \vdash \mathbf{F}_9} \\ \hline -:(\Delta_2, !\mathbf{F}_3), \Delta_{10}, \mathbf{F}_7\&\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_6:\Delta_{10}, \mathbf{F}_5, \mathbf{F}_7\&\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline -:\Delta_{10}, \Delta_2, !\mathbf{F}_3, !\mathbf{F}_3, \mathbf{F}_7\&\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline -:\Delta_{10}, \Delta_2, !\mathbf{F}_3, \mathbf{F}_7\&\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline -:\Delta_{10}, \Delta_2, !\mathbf{F}_3, \mathbf{F}_7\&\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline -:(\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_7\&\mathbf{F}_8 \\ \hline -:(\Delta_2, !\mathbf{F}_3), \Delta_6 \vdash \mathbf{F}_9 \\ \hline \hline \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_7\&\mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3, \mathbf{F}_7\&\mathbf{F}_8} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_5:\Delta_6, \mathbf{F}_7\&\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, |\mathbf{F}_3 \vdash \mathbf{F}_7\&\mathbf{F}_8}{\bullet \mathbf{ax/W}} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_5:\Delta_6, \mathbf{F}_7\&\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline -:\Delta_2, \Delta_6, !\mathbf{F}_3, !\mathbf{F}_3 \vdash \mathbf{F}_9 \\ \hline -:\Delta_2, \Delta_6, !\mathbf{F}_3 \vdash \mathbf{F}_9 \\ \hline -:\Delta_2, \Delta_6, !\mathbf{F}_3 \vdash \mathbf{F}_9 \\ \hline \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \\ \end{array}$$

• Case rule $\&_{L1}$

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}} \ C \ \frac{\mathbf{h}_{6}:(\Delta_{10}, \mathbf{F}_{5}, \mathbf{F}_{7} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{6}:(\Delta_{10}, \mathbf{F}_{7}\&\mathbf{F}_{8}), \mathbf{F}_{5} \vdash \mathbf{F}_{9}} \ Cut \\ \hline \\ \underline{-:(\Delta_{2}, !\mathbf{F}_{3}), \Delta_{10}, \mathbf{F}_{7}\&\mathbf{F}_{8} \vdash \mathbf{F}_{9}} \\ \hline \\ \underline{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}} \ \mathbf{ax/W} \ \frac{\overset{\leadsto}{\bullet} \mathbf{h}_{6}:\Delta_{10}, \mathbf{F}_{5}, \mathbf{F}_{7}\&\mathbf{F}_{8} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{6}:\Delta_{10}, \Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3}, \mathbf{F}_{7}\&\mathbf{F}_{8} \vdash \mathbf{F}_{9}} \ C \\ \hline \\ \underline{-:\Delta_{10}, \Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3}, \mathbf{F}_{7}\&\mathbf{F}_{8} \vdash \mathbf{F}_{9}} \ C \\ \\ \underline{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7}\&\mathbf{F}_{8}} \ C \ \frac{\mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{7} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{7}\&\mathbf{F}_{8} \vdash \mathbf{F}_{9}} \ C \\ \hline \\ \underline{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7}\&\mathbf{F}_{8}} \ C \ \frac{\mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{7}\&\mathbf{F}_{8} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{7}\&\mathbf{F}_{8} \vdash \mathbf{F}_{9}} \ C \\ \hline \\ \underline{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7}\&\mathbf{F}_{8}} \ \mathbf{ax/W} \ \mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{7}\&\mathbf{F}_{8} \vdash \mathbf{F}_{9}} \ \mathbf{ax/W} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \end{array} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \ \mathbf{h}_{Cut} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \end{array} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \ \mathbf{h}_{Cut} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \ \mathbf{h}_{Cut} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \ \mathbf{h}_{Cut} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \ \mathbf{h}_{Cut} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \ \mathbf{h}_{Cut} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \ \mathbf{h}_{Cut} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \ \mathbf{h}_{Cut} \ \mathbf{h}_{Cut} \\ \hline \\ \underline{-:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9}} \ C \ \mathbf{h}_{Cut} \ \mathbf$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2, |\mathbf{F}_3, |\mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1 : \Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_5} \ C \ \frac{\mathbf{h}_6 : \Delta_{10}, \mathbf{F}_5, \mathbf{F}_7, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6 : (\Delta_{10}, \mathbf{F}_7 \otimes \mathbf{F}_8), \mathbf{F}_5 \vdash \mathbf{F}_9} \ \frac{\otimes_L}{\mathsf{Cut}} \\ \hline \\ -: (\Delta_2, |\mathbf{F}_3), \Delta_{10}, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2, |\mathbf{F}_3, |\mathbf{F}_3 \vdash \mathbf{F}_5|}{\bullet \mathbf{h}_6 : \Delta_{10}, \mathbf{F}_5, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9} \ \frac{\mathsf{ax/W}}{\bullet \mathbf{h}_6 : \Delta_{10}, \mathbf{F}_5, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9} \ C \\ \hline \\ \frac{-: \Delta_{10}, \Delta_2, |\mathbf{F}_3, |\mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9}{-: \Delta_{10}, \Delta_2, |\mathbf{F}_3, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9} \ C \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2, |\mathbf{F}_3, |\mathbf{F}_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8}{\bullet \mathbf{h}_1 : \Delta_2, |\mathbf{F}_3, |\mathbf{F}_7 \otimes \mathbf{F}_8} \ C \ \frac{\mathbf{h}_5 : \Delta_6, \mathbf{F}_7, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_5 : \Delta_6, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9} \ C \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2, |\mathbf{F}_3, |\mathbf{F}_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8}{\bullet \mathbf{h}_1 : \Delta_2, |\mathbf{F}_3, |\mathbf{F}_3 \vdash \mathbf{F}_7 \otimes \mathbf{F}_8} \ \mathbf{ax/W} \ \frac{\bullet}{\bullet \mathbf{h}_5 : \Delta_6, \mathbf{F}_7 \otimes \mathbf{F}_8 \vdash \mathbf{F}_9} \ C \\ \hline \\ \hline \\ \frac{-: \Delta_2, \Delta_6, |\mathbf{F}_3, |\mathbf{F}_3 \vdash \mathbf{F}_9}{-: \Delta_2, \Delta_6, |\mathbf{F}_3, |\mathbf{F}_9 \vdash \mathbf{F}_9} \ C \\ \hline \end{array}$$

• Case rule \oplus_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{5}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}} & C & \frac{\mathbf{h}_{6}:\Delta_{10}, \mathbf{F}_{5}, \mathbf{F}_{7} \vdash \mathbf{F}_{9} \quad \mathbf{h}_{6}:\Delta_{10}, \mathbf{F}_{5}, \mathbf{F}_{8} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{6}:(\Delta_{10}, \mathbf{F}_{7} \oplus \mathbf{F}_{8}), \mathbf{F}_{5} \vdash \mathbf{F}_{9}} \quad \mathbf{Cut} \\ \\ & -:(\Delta_{2}, !\mathbf{F}_{3}), \Delta_{10}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9} \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{5}} \quad \mathbf{ax/W} \\ \hline \frac{-:\Delta_{10},\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{8} \vdash \mathbf{F}_{9}}{-:\Delta_{10},\Delta_{2}, !\mathbf{F}_{3}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9}} \quad C \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7} \oplus \mathbf{F}_{8}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7} \oplus \mathbf{F}_{8}} \quad C \quad \frac{\mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{7} \vdash \mathbf{F}_{9} \quad \mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{8} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9}} \quad \mathbf{Cut} \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{7} \oplus \mathbf{F}_{8}}{\bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{7} \oplus \mathbf{F}_{8}} \quad C \quad \frac{\mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{5}:\Delta_{6}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9}} \quad \mathbf{Cut} \\ \hline -:(\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{7} \oplus \mathbf{F}_{8} \quad \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2}, !\mathbf{F}_{3}, !\mathbf{F}_{7} \oplus \mathbf{F}_{8} \quad \mathbf{ax/W} \\ \hline -:\Delta_{2},\Delta_{6}, !\mathbf{F}_{3}, !\mathbf{F}_{3} \vdash \mathbf{F}_{9} \quad C \\ \hline \end{array} \quad \mathbf{hCut} \\ \hline \end{array}$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, |\mathbf{F}_3, \mathbf{F}_5|}{\bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3| + \mathbf{F}_5} \quad C \quad \frac{\mathbf{h}_6:\Delta_{11}, \mathbf{F}_5 + \mathbf{F}_8 \quad \mathbf{h}_6:\Delta_7, \mathbf{F}_9 + \mathbf{F}_{10}}{\bullet \mathbf{h}_6:(\Delta_7,\Delta_{11}, \mathbf{F}_8 \multimap \mathbf{F}_9), \mathbf{F}_5 \vdash \mathbf{F}_{10}} \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3),\Delta_7,\Delta_{11},\mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ & \stackrel{\bullet}{\longrightarrow} \\ \hline \frac{\mathbf{h}_1:\Delta_2, |\mathbf{F}_3, \mathbf{F}_5 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_6:\Delta_{11},\Delta_7,\mathbf{F}_5,\mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} \quad \mathcal{O}_L \\ \hline \frac{\mathbf{h}_1:\Delta_2, |\mathbf{F}_3, \mathbf{F}_5 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2,\Delta_7, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} \quad \mathcal{O}_L \\ \hline \frac{\mathbf{h}_1:\Delta_2, |\mathbf{F}_3, \mathbf{F}_5 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3, \mathbf{F}_5 \vdash \mathbf{F}_6} \quad \mathcal{O}_L \\ \hline \frac{\mathbf{h}_1:\Delta_2, |\mathbf{F}_3, \mathbf{F}_5 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3, \mathbf{F}_5 \vdash \mathbf{F}_6} \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3),\Delta_7,\Delta_{11},\mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} \quad \mathcal{O}_L \\ \hline \frac{\mathbf{h}_1:\Delta_2, |\mathbf{F}_3, \mathbf{F}_5 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3, \mathbf{F}_5 \vdash \mathbf{F}_{10}} \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3),\Delta_7,\Delta_{11},\mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3, \mathbf{F}_5 \vdash \mathbf{F}_6 \vdash \mathbf{F}_{10}) \quad \mathcal{O}_L \\ \hline -:(\Delta_1,\Delta_2,\Delta_7, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}) \quad \mathcal{O}_L \\ \hline -:(\Delta_1,\Delta_2,\Delta_7, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}) \quad \mathcal{O}_L \\ \hline \bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9 \quad \mathcal{O}_L \\ \hline \bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9 \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3),\Delta_6,\Delta_7 \vdash \mathbf{F}_{10} \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \quad \mathcal{O}_L \\ \hline -:(\Delta_2, |\mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline -:(\Delta_2, \Delta_6, \Delta_7, \mathbf{F}_8, \mathbf{F}_{10} \vdash \mathbf{F}_{10} \\ \hline -:(\Delta_2, \Delta_6, \Delta_7, \mathbf{F}_8, \mathbf{F}_{10} \vdash \mathbf{F}_{10} \\ \hline -:(\Delta_2, \Delta_6, \Delta_7, \mathbf{F}_8, \mathbf{F}_{10} \vdash \mathbf{F}_{10} \\ \hline -:(\Delta_2, \Delta_6, \Delta_7, \mathbf{F}_8, \mathbf{F}_{10} \vdash \mathbf{F}_{10} \\ \hline -:(\Delta_2, \Delta_6, \Delta_7, \mathbf{F}_8, \mathbf{F}_{10} \vdash \mathbf{F}_{10} \\ \hline -:(\Delta_2, \Delta_6, \Delta_7, \mathbf{F}_8, \mathbf{F}_{10} \vdash \mathbf{F}_{10} \\ \hline -:(\Delta_2, \Delta_6, \Delta_7, \mathbf{F}_8, \mathbf{F}_{10} \vdash \mathbf{F}_{10} \\ \hline -:(\Delta_2, \Delta_6, \Delta_7, \mathbf{F}_8, \mathbf{F}_{10} \vdash \mathbf{F}_{10} \\ \hline -:(\Delta_2, \Delta_6, \Delta_7, \mathbf{F}_8,$$

ullet Case rule I

$$\frac{\mathbf{h}_1:\Delta_2, !\mathbf{F}_3, !\mathbf{F}_3 \vdash p(\mathbf{n}_6)}{\underbrace{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash p(\mathbf{n}_6)}_{-:(\Delta_2, !\mathbf{F}_3), * \vdash p(\mathbf{n}_6)} C \underbrace{\frac{\bullet \mathbf{h}_5:*, p(\mathbf{n}_6) \vdash p(\mathbf{n}_6)}_{-:\Delta_2, !\mathbf{F}_3 \vdash p(\mathbf{n}_6)}}_{-:\Delta_2, !\mathbf{F}_3 \vdash p(\mathbf{n}_6)} \mathbf{ax/W}}_{I}$$

5.12 Status of !L: OK

• Case rule !R

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash 6}{\bullet\mathbf{h}_{1}:\Delta_{2},!\mathbf{F}_{3}\vdash 6}}{\cdot (\Delta_{2},!\mathbf{F}_{3})\vdash 6}} \overset{!}{!}L \xrightarrow{\begin{array}{c} \mathbf{h}_{7}:!\Upsilon 5, 6\vdash \mathbf{F}_{8} \\ \bullet \mathbf{h}_{7}:!\Upsilon 5, 6\vdash !\mathbf{F}_{8} \end{array}}} \overset{!}{Cut}$$

$$\frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash 6}{\bullet\mathbf{h}_{7}:\Delta_{2},\mathbf{F}_{3}\vdash !\mathbf{F}_{8}}} \xrightarrow{\bullet\mathbf{h}_{7}:6,!\Upsilon 5\vdash !\mathbf{F}_{8}} \overset{\mathbf{ax/W}}{\bullet}$$

$$\frac{-:!\Upsilon 5,\Delta_{2},\mathbf{F}_{3}\vdash !\mathbf{F}_{8}}{-:!\Upsilon 5,\Delta_{2},!\mathbf{F}_{3}\vdash !\mathbf{F}_{8}} \overset{!}{!}L$$

- Case rule $\mathbf{1}_R$
- Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_6} \ \, !L \quad \\ \frac{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \top}{-: (\Delta_2, !\mathbf{F}_3), \Delta_5 \vdash \top} \quad \mathsf{Cut} \\ \\ \frac{}{-: \Delta_2, \Delta_5, !\mathbf{F}_3 \vdash \top} \ \, \top \end{array}$$

• Case rule $\&_R$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_6} \cdot !L & \frac{\mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \cdot \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \cdot \mathbf{Cut} \\ & -: (\Delta_2, !\mathbf{F}_3), \Delta_5 \vdash \mathbf{F}_8 \& \mathbf{F}_9} & \frac{\mathbf{c}}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \cdot \mathbf{c} \\ & \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \cdot \mathbf{h} \mathbf{c} \\ & \frac{-: \Delta_2, \Delta_5, \mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9}{-: \Delta_2, \Delta_5, !\mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9} \cdot !L \end{array} \right. \\ \end{array} \right. \\ \begin{array}{c} \bullet \mathbf{h} \mathbf{h} \cdot \mathbf{h}$$

• Case rule \multimap_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_6|} : L & \frac{\mathbf{h}_7: \Delta_5, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9} & \neg \circ_R \\ \hline -: (\Delta_2, |\mathbf{F}_3), \Delta_5 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9 & \neg \circ_R \\ \hline \bullet \mathbf{h}_1: \Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_6| & \text{ax/W} & \frac{}{\mathbf{h}_7: \Delta_5, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline -: \Delta_2, \Delta_5, \mathbf{F}_8, |\mathbf{F}_3 \vdash \mathbf{F}_9 & \neg \circ_R \\ \hline -: \Delta_2, \Delta_5, |\mathbf{F}_3 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9 & \neg \circ_R \\ \end{array} \right.$$

• Case rule \oplus_{R_2}

$$\frac{ \begin{array}{c|c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_6 \end{array} : L & \begin{array}{c} \mathbf{h}_7 : \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \bullet \mathbf{h}_7 : \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9 \end{array} & \begin{array}{c} \bigoplus_{R_2} \\ \mathsf{Cut} \end{array} \\ \hline - : (\Delta_2, |\mathbf{F}_3|), \Delta_5 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_6 \end{array} & \begin{array}{c} \mathsf{ax}/\mathbb{W} \\ \mathsf{h}_7 : \Delta_5, \mathbf{F}_6 \vdash \mathbf{F}_9 \\ \hline \\ - : \Delta_2, \Delta_5, |\mathbf{F}_3 \vdash \mathbf{F}_9 \\ \hline - : \Delta_2, \Delta_5, |\mathbf{F}_3 \vdash \mathbf{F}_9 \\ \hline - : \Delta_2, \Delta_5, |\mathbf{F}_3 \vdash \mathbf{F}_8 \oplus \mathbf{F}_9 \end{array} & \begin{array}{c} \mathsf{ax}/\mathbb{W} \\ \mathsf{hCut} \end{array}$$

• Case rule \bigoplus_{R_1}

$$\begin{array}{c} \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \vdash \mathbf{F}_{6} \\ \bullet \mathbf{h}_{1}: \Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{6} \end{array} \stackrel{!L}{\bullet} \mathbf{h}_{7}: \Delta_{5}, \mathbf{F}_{6} \vdash \mathbf{F}_{8} \oplus \mathbf{F}_{9} \\ \hline -: (\Delta_{2}, !\mathbf{F}_{3}), \Delta_{5} \vdash \mathbf{F}_{8} \oplus \mathbf{F}_{9} \\ \bullet \mathbf{h}_{1}: \Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{6} \end{array} \stackrel{\text{ax/W}}{\bullet} \mathbf{h}_{7}: \Delta_{5}, \mathbf{F}_{6} \vdash \mathbf{F}_{8} \\ \hline \frac{\bullet \mathbf{h}_{1}: \Delta_{2}, !\mathbf{F}_{3} \vdash \mathbf{F}_{6}}{-: \Delta_{2}, \Delta_{5}, !\mathbf{F}_{3} \vdash \mathbf{F}_{8}} \stackrel{\text{th}_{Cut}}{\bullet} \mathbf{h}_{Cut} \\ \hline -: \Delta_{2}, \Delta_{5}, !\mathbf{F}_{3} \vdash \mathbf{F}_{8} \oplus \mathbf{F}_{9} \\ \hline -: \Delta_{2}, \Delta_{5}, !\mathbf{F}_{3} \vdash \mathbf{F}_{8} \oplus \mathbf{F}_{9} \end{array} \stackrel{\text{th}_{Cut}}{\bullet} \mathbf{h}_{Cut}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{1}}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{IF}_3 \vdash \mathbf{1}} \cdot \mathbf{L} & \frac{\mathbf{h}_5:\Delta_6 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_5:\Delta_6, \mathbf{1} \vdash \mathbf{F}_7} \cdot \mathbf{L} \\ \hline -:(\Delta_2, \mathbf{IF}_3), \Delta_6 \vdash \mathbf{F}_7 & \mathbf{cut} \\ \hline -:\Delta_2, \Delta_6, \mathbf{IF}_3 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{IF}_3 \vdash \mathbf{F}_5} \cdot \mathbf{IL} & \frac{\mathbf{h}_6:\Delta_8, \mathbf{F}_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6:(\mathbf{1},\Delta_8), \mathbf{F}_5 \vdash \mathbf{F}_7} \cdot \mathbf{L} \\ \hline -:(\Delta_2, \mathbf{IF}_3), \mathbf{1}, \Delta_8 \vdash \mathbf{F}_7 & \mathbf{cut} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{IF}_3 \vdash \mathbf{F}_5 & \mathbf{ax/W} & \mathbf{h}_6:\mathbf{1},\Delta_8, \mathbf{F}_5 \vdash \mathbf{F}_7 \\ \hline -:\mathbf{1},\Delta_2,\Delta_8, \mathbf{IF}_3 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline -:\mathbf{1},\Delta_2,\Delta_8, \mathbf{IF}_3 \vdash \mathbf{F}_7 & \mathbf{ax/W} \\ \hline \end{array}$$

• Case rule \otimes_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_5} & !L & \frac{\mathbf{h}_6:\Delta_{10}, \mathbf{F}_5 \vdash \mathbf{F}_8 & \mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:(\Delta_7,\Delta_{10}), \mathbf{F}_5 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & \mathbb{C}ut \\ \hline \\ -:(\Delta_2, !\mathbf{F}_3), \Delta_7, \Delta_{10} \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & \cdots \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_6:\Delta_{10},\Delta_2,\Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & \mathbf{ax/W} \\ \hline \\ \frac{-:\Delta_{10},\Delta_2,\Delta_7, \mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9}{-:\Delta_{10},\Delta_2,\Delta_7, !\mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & !L \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_5} & !L & \frac{\mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_8 & \mathbf{h}_6:\Delta_{10}, \mathbf{F}_5 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:(\Delta_7,\Delta_{10}), \mathbf{F}_5 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & \mathbb{C}ut \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{-:(\Delta_2, !\mathbf{F}_3),\Delta_7,\Delta_{10} \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & \mathbf{Cut} \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{-:\Delta_{10},\Delta_2,\Delta_7, \mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & !L \\ \hline \end{array}$$

ullet Case rule W

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_5} & !L & \frac{\mathbf{h}_6:\Delta_9, \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6:(\Delta_9, |\mathbf{F}_7), \mathbf{F}_5 \vdash \mathbf{F}_8} & W \\ \hline -:(\Delta_2, |\mathbf{F}_3), \Delta_9, |\mathbf{F}_7 \vdash \mathbf{F}_8 & \leadsto \\ \hline \bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_5 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_9, \mathbf{F}_5, |\mathbf{F}_7 \vdash \mathbf{F}_8 \\ \hline -:\Delta_2, \Delta_9, |\mathbf{F}_3, |\mathbf{F}_7 \vdash \mathbf{F}_8 & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_7 & !L & \frac{\mathbf{h}_5:\Delta_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5:\Delta_6, |\mathbf{F}_7 \vdash \mathbf{F}_8} & W \\ \hline \bullet \mathbf{h}_1:\Delta_2, |\mathbf{F}_3 \vdash \mathbf{F}_7 & !L & \frac{\mathbf{h}_5:\Delta_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_5:\Delta_6, |\mathbf{F}_7 \vdash \mathbf{F}_8} & \mathbf{Cut} \\ \hline -:(\Delta_2, |\mathbf{F}_3), \Delta_6 \vdash \mathbf{F}_8 & & & & \\ \hline -:\Delta_2, \Delta_6, |\mathbf{F}_3 \vdash \mathbf{F}_8 & \mathbf{ax/W} & & \\ \hline \end{array}$$

\bullet Case rule C

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{5}}{\bullet\mathbf{h}_{1}:\Delta_{2},!\mathbf{F}_{3}\vdash\mathbf{F}_{5}}}{\bullet\mathbf{h}_{1}:\Delta_{2},!\mathbf{F}_{3}\vdash\mathbf{F}_{5}}} \overset{!}{L} \quad \frac{\mathbf{h}_{6}:\Delta_{9},\mathbf{F}_{5},!\mathbf{F}_{7},!\mathbf{F}_{7}\vdash\mathbf{F}_{8}}{\bullet\mathbf{h}_{6}:(\Delta_{9},!\mathbf{F}_{7}),\mathbf{F}_{5}\vdash\mathbf{F}_{8}}} \quad C \\ \overline{-:(\Delta_{2},!\mathbf{F}_{3}),\Delta_{9},!\mathbf{F}_{7}\vdash\mathbf{F}_{8}}} \quad \overset{\bullet}{\sim} \\ \underline{\bullet\mathbf{h}_{1}:\Delta_{2},!\mathbf{F}_{3}\vdash\mathbf{F}_{5}}} \quad \text{ax/W} \quad \overset{\bullet}{\sim} \\ \overline{-:\Delta_{2},\Delta_{9},!\mathbf{F}_{3},!\mathbf{F}_{7},!\mathbf{F}_{7}\vdash\mathbf{F}_{8}}} \quad C \\ \overline{-:\Delta_{2},\Delta_{9},!\mathbf{F}_{3},!\mathbf{F}_{7}\vdash\mathbf{F}_{8}}} \quad C \\ \end{array}$$

$$\frac{ \begin{array}{l} \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash ! \mathbf{F}_7 \\ \bullet \mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash ! \mathbf{F}_7 \end{array} : L \quad \begin{array}{l} \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7, ! \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7 \vdash \mathbf{F}_8 \end{array} C}{-: (\Delta_2, ! \mathbf{F}_3), \Delta_6 \vdash \mathbf{F}_8} \quad \mathbf{Cut} \\ \\ \bullet \mathbf{h}_1 : \Delta_2, ! \mathbf{F}_3 \vdash ! \mathbf{F}_7 \end{array} \quad \begin{array}{l} \bullet \mathbf{x} / \mathbb{W} \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7, ! \mathbf{F}_7 \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_5 : \Delta_6, ! \mathbf{F}_7, ! \mathbf{F}_7 \vdash \mathbf{F}_8 \end{array} \quad \mathbf{ax} / \mathbb{W} \\ \bullet \mathbf{nCut} \end{array}$$

• Case rule !L

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{IF}_3\vdash\mathbf{F}_5} & !L & \frac{\mathbf{h}_6:\Delta_9,\mathbf{F}_5,\mathbf{F}_7\vdash\mathbf{F}_8}{\bullet\mathbf{h}_6:(\Delta_9,\mathbf{IF}_7),\mathbf{F}_5\vdash\mathbf{F}_8} & !L \\ \hline -:(\Delta_2,\mathbf{IF}_3),\Delta_9,\mathbf{IF}_7\vdash\mathbf{F}_8 & \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_6:\Delta_9,\mathbf{F}_5,\mathbf{IF}_7\vdash\mathbf{F}_8} & \frac{\mathbf{ax/W}}{\bullet\mathbf{h}_6:\Delta_9,\mathbf{F}_5,\mathbf{IF}_7\vdash\mathbf{F}_8} \\ \hline \frac{-:\Delta_2,\Delta_9,\mathbf{F}_3,\mathbf{IF}_7\vdash\mathbf{F}_8}{-:\Delta_2,\Delta_9,\mathbf{IF}_3,\mathbf{IF}_7\vdash\mathbf{F}_8} & !L & \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{IF}_7}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{IF}_3\vdash\mathbf{IF}_7} & !L & \frac{\mathbf{h}_5:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_8}{\bullet\mathbf{h}_5:\Delta_6,\mathbf{IF}_7\vdash\mathbf{F}_8} & !L \\ \hline -:(\Delta_2,\mathbf{IF}_3),\Delta_6\vdash\mathbf{F}_8 & \frac{\mathbf{ax/W}}{\bullet\mathbf{h}_5:\Delta_6,\mathbf{IF}_7\vdash\mathbf{F}_8} & \frac{\mathbf{ax/W}}{\bullet\mathbf{h}_5:\Delta_6,\mathbf{IF}_7\vdash\mathbf{F}_8} \\ \hline \frac{-:\Delta_2,\Delta_6,\mathbf{F}_3\vdash\mathbf{F}_8}{-:\Delta_2,\Delta_6,\mathbf{IF}_3\vdash\mathbf{F}_8} & !L & \\ \hline \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_1:\Delta_2,!\mathbf{F}_3\vdash\mathbf{F}_5} : L & \frac{\mathbf{h}_6:\Delta_{10},\mathbf{F}_5,\mathbf{F}_8\vdash\mathbf{F}_9}{\bullet\mathbf{h}_6:(\Delta_{10},\mathbf{F}_7\&\mathbf{F}_8),\mathbf{F}_5\vdash\mathbf{F}_9} & \&_{L2} \\ \hline -:(\Delta_2,!\mathbf{F}_3),\Delta_{10},\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9 & & \mathbf{cut} \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_2} & \frac{\bullet\mathbf{h}_6:\Delta_{10},\mathbf{F}_5,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9}{\bullet\mathbf{h}_6:\Delta_{10},\mathbf{F}_5,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9} & \mathbf{ax/W} \\ \hline \\ \frac{-:\Delta_{10},\Delta_2,\mathbf{F}_3,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9}{-:\Delta_{10},\Delta_2,!\mathbf{F}_3,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9} : L & \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_7\&\mathbf{F}_8}{\bullet\mathbf{h}_1:\Delta_2,!\mathbf{F}_3\vdash\mathbf{F}_7\&\mathbf{F}_8} : L & \frac{\mathbf{h}_5:\Delta_6,\mathbf{F}_8\vdash\mathbf{F}_9}{\bullet\mathbf{h}_5:\Delta_6,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9} & \&_{L2} \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_7\&\mathbf{F}_8}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_7\&\mathbf{F}_8} & \mathbf{ax/W} & \\ \hline \\ \frac{-:\Delta_2,\Delta_6,\mathbf{F}_3\vdash\mathbf{F}_9}{-:\Delta_2,\Delta_6,\mathbf{F}_3\vdash\mathbf{F}_9} : L & \\ \hline \end{array}$$

• Case rule $\&_{L1}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_1:\Delta_2,|\mathbf{F}_3\vdash\mathbf{F}_5|} : L & \frac{\mathbf{h}_6:\Delta_{10},\mathbf{F}_5,\mathbf{F}_7\vdash\mathbf{F}_9}{\bullet\mathbf{h}_6:(\Delta_{10},\mathbf{F}_7\&\mathbf{F}_8),\mathbf{F}_5\vdash\mathbf{F}_9} & \&_{L1} \\ \hline -:(\Delta_2,|\mathbf{F}_3),\Delta_{10},\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9 & \\ \hline \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_5}{\bullet\mathbf{h}_6:\Delta_{10},\Delta_2,\mathbf{F}_3,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9} & \mathbf{ax/W} \\ \hline \frac{-:\Delta_{10},\Delta_2,\mathbf{F}_3,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9}{-:\Delta_{10},\Delta_2,|\mathbf{F}_3,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9} : L & \mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_7\&\mathbf{F}_8 & |L| \\ \hline \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_7\&\mathbf{F}_8}{\bullet\mathbf{h}_1:\Delta_2,|\mathbf{F}_3\mid\mathbf{F}_7\&\mathbf{F}_8} & |L| & \frac{\mathbf{h}_5:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9}{\bullet\mathbf{h}_5:\Delta_6,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9} & \&_{L1} \\ \hline -:(\Delta_2,|\mathbf{F}_3),\Delta_6\vdash\mathbf{F}_9 & \\ \hline -:(\Delta_2,|\mathbf{F}_3),\Delta_6\vdash\mathbf{F}_9 & \\ \hline \hline \mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_7\&\mathbf{F}_8 & \mathbf{ax/W} & \\ \hline -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash\mathbf{F}_9 & |L| & \\ \hline \end{pmatrix} & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_5:\Delta_6,\mathbf{F}_7\&\mathbf{F}_8\vdash\mathbf{F}_9 & \\ \hline -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash\mathbf{F}_9 & |L| & \\ \hline \end{array}$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2,!\mathbf{F}_3\vdash \mathbf{F}_5} : L & \frac{\mathbf{h}_6:\Delta_{10},\mathbf{F}_5,\mathbf{F}_7,\mathbf{F}_8\vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:(\Delta_{10},\mathbf{F}_7\otimes \mathbf{F}_8),\mathbf{F}_5\vdash \mathbf{F}_9} \\ \hline -:(\Delta_2,!\mathbf{F}_3),\Delta_{10},\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 & \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_5}{\bullet \mathbf{h}_2} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_6:\Delta_{10},\mathbf{F}_5,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9} \\ \hline \\ \frac{-:\Delta_{10},\Delta_2,\mathbf{F}_3,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9}{-:\Delta_{10},\Delta_2,!\mathbf{F}_3,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9} : L \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_7\otimes \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2,!\mathbf{F}_3\vdash \mathbf{F}_7\otimes \mathbf{F}_8} & !L & \frac{\mathbf{h}_5:\Delta_6,\mathbf{F}_7,\mathbf{F}_8\vdash \mathbf{F}_9}{\bullet \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,!\mathbf{F}_3\vdash \mathbf{F}_7\otimes \mathbf{F}_8 & !L & \frac{\mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9}{\bullet \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9} & \mathbf{Cut} \\ \hline \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_7\otimes \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_7\otimes \mathbf{F}_8} & \mathbf{ax/W} & \bullet \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8\vdash \mathbf{F}_9 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{F}_7\otimes \mathbf{F}_8 \\ \hline \\ -:\Delta_2,\Delta_6,\mathbf{F}_3\vdash \mathbf{F}_9 & !L & \mathbf{h}_5:\Delta_6,\mathbf{$$

• Case rule \oplus_L

• Case rule \multimap_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{IF}_3 \vdash \mathbf{F}_5} & \mathbf{I}_L & \frac{\mathbf{h}_6:\Delta_{11}, \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6:(\Delta_7,\Delta_{11}, \mathbf{F}_8 \multimap \mathbf{F}_9), \mathbf{F}_5 \vdash \mathbf{F}_{10}} & -\circ_L \\ \hline -:(\Delta_2, !\mathbf{F}_3), \Delta_7, \Delta_{11}, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10} & -\circ_L \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5} & \mathbf{ax/W} & \frac{\bullet \mathbf{h}_6:\Delta_{11}, \Delta_7, \mathbf{F}_5, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6:\Delta_{11},\Delta_2,\Delta_7, \mathbf{IF}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathbf{ax/W} \\ \hline \frac{-:\Delta_{11},\Delta_2,\Delta_7, \mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}}{-:\Delta_{11},\Delta_2,\Delta_7, \mathbf{IF}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathbf{IL} \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_5} & !L & \frac{\mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_8 \quad \mathbf{h}_6:\Delta_{11}, \mathbf{F}_5, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6:(\Delta_7,\Delta_{11}, \mathbf{F}_8 \multimap \mathbf{F}_9), \mathbf{F}_5 \vdash \mathbf{F}_{10}} & -\circ_L \\ \hline -:(\Delta_2, !\mathbf{F}_3), \Delta_7, \Delta_{11}, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} & -\circ_L \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2, \Delta_7, \mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathbf{ax/W} \\ \hline -:\Delta_{11},\Delta_2,\Delta_7, \mathbf{F}_3, \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} & !L \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_2, !\mathbf{F}_3 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9} & !L & \frac{\mathbf{h}_5:\Delta_6 \vdash \mathbf{F}_8 \quad \mathbf{h}_5:\Delta_7, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_5:(\Delta_6,\Delta_7), \mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} & -\circ_L \\ \hline -:(\Delta_2, !\mathbf{F}_3),\Delta_6,\Delta_7 \vdash \mathbf{F}_{10} & -\circ_L \\ \hline \hline -:(\Delta_2, !\mathbf{F}_3),\Delta_6,\Delta_7,\mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} & -\circ_L \\ \hline \hline \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_2,\Delta_7,\mathbf{F}_3 \vdash \mathbf{F}_{10}} & -\circ_L \\ \hline -:(\Delta_2, !\mathbf{F}_3),\Delta_6,\Delta_7 \vdash \mathbf{F}_{10}} & -\circ_L \\ \hline \hline -:(\Delta_2, !\mathbf{F}_3),\Delta_6,\Delta_7,\mathbf{F}_8 \multimap \mathbf{F}_9 \vdash \mathbf{F}_{10}} & -\circ_L \\ \hline \hline -:(\Delta_2, \Delta_6,\Delta_7,\mathbf{F}_3 \vdash \mathbf{F}_{10}) & -\circ_L \\ \hline -:\Delta_2,\Delta_6,\Delta_7,\mathbf{F}_3 \vdash \mathbf{F}_{10} & !L \\ \hline \end{array}$$

 \bullet Case rule I

$$\frac{ \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash p(\mathbf{n}_6) }{ \bullet \mathbf{h}_1 : \Delta_2, !\mathbf{F}_3 \vdash p(\mathbf{n}_6) } : L \quad \underbrace{ \bullet \mathbf{h}_5 : *, p(\mathbf{n}_6) \vdash p(\mathbf{n}_6) }_{\leftarrow : (\Delta_2, !\mathbf{F}_3), * \vdash p(\mathbf{n}_6)} \quad \mathbf{Cut}$$

$$\frac{ }{- : \Delta_2, !\mathbf{F}_3 \vdash p(\mathbf{n}_6)} \quad \mathbf{ax/W}$$

5.13 Status of $\&_{L2}$: OK

• Case rule !R

$$\frac{ \begin{array}{l} \mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash 7 \\ \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash 7 \end{array} \&_{L2} \quad \frac{\mathbf{h}_8 : ! \Upsilon 6, 7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8 : ! \Upsilon 6, 7 \vdash ! \mathbf{F}_9} \\ - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), ! \Upsilon 6 \vdash ! \mathbf{F}_9 \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash 7 \quad \text{ax/W} \quad \bullet_{\mathbf{h}_8 : 7, ! \Upsilon 6 \vdash ! \mathbf{F}_9}}{\bullet \mathbf{h}_8 : 7, ! \Upsilon 6 \vdash ! \mathbf{F}_9} \quad \underset{\mathbf{h} \mathsf{Cut}}{\mathsf{ax/W}} \\ \hline \\ \frac{- : ! \Upsilon 6, \Delta_2, \mathbf{F}_4 \vdash ! \mathbf{F}_9}{- : ! \Upsilon 6, \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash ! \mathbf{F}_9} \quad \&_{L2} \end{array} \right. \\ \\ \end{array}$$

- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{ \mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_7 }{ \underbrace{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7 }_{ - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_6 \vdash \top} \underbrace{ \begin{matrix} \top \\ \bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \top \end{matrix} }_{ \sim } \underbrace{ \begin{matrix} \top \\ \mathsf{Cut} \end{matrix} }_{ - : \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \top}$$

• Case rule $\&_R$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7 \end{array}}{ \bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 } \begin{array}{c} \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \end{array}} \begin{array}{c} \mathcal{E}_R \\ \bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \end{array}} \begin{array}{c} \mathcal{E}_R \\ \bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ \bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \end{array}} \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{hCut} \\ \bullet \mathbf{hCut} \end{array}$$

• Case rule \multimap_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7} & \&_{L2} & \frac{\mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10}} & \neg \circ_R \\ \hline & -: (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_6 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} \\ \hline & \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7 & \text{ax/W} & \hline \\ & \frac{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \text{ax/W} \\ \hline & \frac{-: \Delta_2, \Delta_6, \mathbf{F}_9, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10}}{-: \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{9} \multimap \mathbf{F}_{10}} & \neg \circ_R \end{array} \right. \\ \end{array}$$

• Case rule \bigoplus_{R_2}

$$\begin{array}{c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \underline{\bullet} \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \end{array} & \underbrace{\frac{\mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_{9} \oplus \mathbf{F}_{10}}}_{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}} \\ \underline{- : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7)}_{\bullet \mathbf{x} \times \mathsf{W}} \underbrace{\frac{\bullet}{\mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_{10}} \\ \underline{- : \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10}}_{- : \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}} \oplus_{R_2} \\ \end{array}} \underbrace{\mathbf{h}_{\mathbf{Cut}}}_{\bullet \mathbf{Cut}}$$

• Case rule \oplus_{R_1}

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7} \&_{L2} & \frac{\mathbf{h}_8: \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}} \\ & -: (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_6 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ & \stackrel{\bullet}{\bullet} \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7} & \frac{\Rightarrow}{\mathbf{h}_8: \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9} \\ & \frac{-: \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9}{-: \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9} & \oplus_{R_1} \end{array} \right. \mathbf{ax/W} \\ & \frac{\mathbf{h}_8: \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9}{-: \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9} & \oplus_{R_1} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{1}}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{1}} & \&_{L2} & \frac{\mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_6:\Delta_7, \mathbf{1} \vdash \mathbf{F}_8} & \mathbf{1}_L \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_7 \vdash \mathbf{F}_8 & \\ \hline -:\Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_8 & \mathbf{ax/W} \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L2} & \frac{\mathbf{h}_7:\Delta_9, \mathbf{F}_6 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7:(\mathbf{1},\Delta_9), \mathbf{F}_6 \vdash \mathbf{F}_8} & \mathbf{1}_L \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \mathbf{1}, \Delta_9 \vdash \mathbf{F}_8 & \mathbf{cut} \\ \hline \\ \frac{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{1}_7:\Delta_9, \mathbf{F}_6 \vdash \mathbf{F}_8} & \mathbf{ax/W} \\ \hline -:1,\Delta_2,\Delta_9,\mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_8 & \mathbf{hCut} \\ \hline \end{array}$$

• Case rule \otimes_R

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L2} & \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_6 \vdash \mathbf{F}_9 & \mathbf{h}_7 : \Delta_8 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : (\Delta_8, \Delta_{11}), \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10}} & \mathbf{Cut} \\ \hline \\ - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_8, \Delta_{11} \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_7 : \Delta_{11}, \Delta_2, \Delta_8, \mathbf{F}_4 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10}} & & \mathbf{ax/W} \\ \hline \\ - : \Delta_{11}, \Delta_2, \Delta_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline - : \Delta_{11}, \Delta_2, \Delta_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_8 \vdash \mathbf{F}_9 & \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_6 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6 & & & & \\ \hline \bullet \mathbf{h}_7 : (\Delta_8, \Delta_{11}), \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_8, \Delta_{11} \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \Delta_{8}, \mathbf{F}_4 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_6 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_8 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_8 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_8 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_8 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_7 : \Delta_{11}, \Delta_8, \mathbf{F}_8 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_$$

 \bullet Case rule W

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L2} & \frac{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:(\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_6 \vdash \mathbf{F}_9} & W \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 & \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6 & \mathbf{ax/W} & \\ \hline -:\Delta_{10},\Delta_2, !\mathbf{F}_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9 & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash !\mathbf{F}_8 & \&_{L2} & \frac{\mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9} & W \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_7 \vdash \mathbf{F}_9 & \\ \hline & \ddots & \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_7 \vdash \mathbf{F}_9 & \mathbf{ax/W} \\ \hline \end{array}$$

• Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L2} & \frac{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_6, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:(\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_6 \vdash \mathbf{F}_9} & C \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 & \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6 & \mathbf{ax/W} & \\ \hline \frac{-:\Delta_{10}, \Delta_2, !\mathbf{F}_8, !\mathbf{F}_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9}{-:\Delta_{10}, \Delta_2, !\mathbf{F}_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9} & C \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash !\mathbf{F}_8 & \&_{L2} & \frac{\mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9} & C \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \&_{L2} & \frac{\mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9} & C \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{h}_7 & \mathbf{h}_7 & \mathbf{h}_8 &$$

• Case rule !L

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L2} & \frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: (\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_6 \vdash \mathbf{F}_9} & !L \\ \hline -: (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 & \frac{\mathbf{ax/W}}{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9} & \frac{\mathbf{ax/W}}{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \frac{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6}{-: \Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9} & !L \\ \hline \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash !\mathbf{F}_8}{-: \Delta_10, \Delta_2, !\mathbf{F}_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9} & !L \\ \hline \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8} & \&_{L2} & \frac{\mathbf{h}_6: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6: \Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9} & !L \\ \hline -: (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_7 \vdash \mathbf{F}_9 & \mathbf{cut} \\ \hline \hline \bullet_1: \Delta_2, \mathbf{F}_4 \vdash !\mathbf{F}_8} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_6: \Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9} & \mathbf{ax/W} \\ \hline \bullet_1: \Delta_2, \mathbf{A}_7, \mathbf{F}_4 \vdash \mathbf{F}_9} & \&_{L2} \\ \hline -: \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9} & \&_{L2} \\ \hline \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L2} & \frac{\mathbf{h}_7:\Delta_{11}, \mathbf{F}_6, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7:(\Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9), \mathbf{F}_6 \vdash \mathbf{F}_{10}} & \mathcal{E}_{L2} \\ \hline & -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \cdots & \cdots \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_7:\Delta_{11}, \Delta_2, \mathbf{F}_4, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathbf{ax/W} \\ \hline & -:\Delta_{11}, \Delta_2, \mathbf{F}_4, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathcal{E}_{L2} \\ \hline & -:\Delta_{11}, \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathcal{E}_{L2} \\ \hline & \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_8 \& \mathbf{F}_9} & \mathcal{E}_{L2} & \frac{\mathbf{h}_6:\Delta_7, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathcal{E}_{L2} \\ \hline & -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_7 \vdash \mathbf{F}_{10} \\ \hline & \bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathcal{E}_{L2} \\ \hline & \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_8 \& \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathbf{ax/W} \\ \hline & -:\Delta_2,\Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_{10} & \mathcal{E}_{L2} \\ \hline \end{array}$$

• Case rule $\&_{L1}$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6 \end{array}}{ \bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_{10} \\ \hline \\ - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array}} \begin{array}{c} \&_{L1} \\ \hline \\ \mathbf{Cut} \\ \hline \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{ } & \mathbf{ax/W} & \xrightarrow{\bullet} \\ \hline \\ \frac{- : \Delta_{11}, \Delta_2, \mathbf{F}_4, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ - : \Delta_{11}, \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ - : \Delta_{11}, \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array}} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \\ \hline \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{4} \vdash \mathbf{F}_{8} \& \mathbf{F}_{9} \\ \bullet \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \& \mathbf{F}_{4} \vdash \mathbf{F}_{8} \& \mathbf{F}_{9} \end{array}}{ \bullet \mathbf{h}_{6}: \Delta_{7}, \mathbf{F}_{8} \& \mathbf{F}_{9} \vdash \mathbf{F}_{10} } \begin{array}{c} \mathcal{L}_{L1} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \mathbf{F}_{8} \& \mathbf{F}_{9} \vdash \mathbf{F}_{10} \end{array}} \begin{array}{c} \mathcal{L}_{L1} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \mathbf{F}_{8} \& \mathbf{F}_{9} \vdash \mathbf{F}_{10} \end{array} \\ \overline{\mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{4} \vdash \mathbf{F}_{8} \& \mathbf{F}_{9}} \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \mathbf{F}_{8} \& \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ \hline -: \Delta_{2}, \Delta_{7}, \mathbf{F}_{4} \vdash \mathbf{F}_{10} \\ \overline{-: \Delta_{2}, \Delta_{7}, \mathbf{F}_{3} \& \mathbf{F}_{4} \vdash \mathbf{F}_{10}} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L2} & \frac{\mathbf{h}_7:\Delta_{11}, \mathbf{F}_6, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7:(\Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9), \mathbf{F}_6 \vdash \mathbf{F}_{10}} & \otimes_L \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} & & \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{m}_7} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_7:\Delta_{11}, \mathbf{F}_6, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \mathbf{ax/W} \\ \hline -:\Delta_{11}, \Delta_2, \mathbf{F}_4, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} & \&_{L2} & & \mathbf{hCut} \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9}{-:\Delta_{11},\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \&_{L2} & & \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & \&_{L2} & & & \\ \hline \frac{\mathbf{h}_6:\Delta_7, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \otimes_L \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_7 \vdash \mathbf{F}_{10} & & \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} & & \\ \hline -:\Delta_2, \Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_{10} & & & \\ \hline -:\Delta_2, \Delta_7, \mathbf{F}_4 \vdash \mathbf{F}_{10} & & & \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \end{array}$$

• Case rule \oplus_L

$$\begin{array}{c} \frac{h_1:\Delta_2,F_4\vdash F_6}{\bullet h_1:\Delta_2,F_3\&F_4\vdash F_6} &\&_{L2} &\frac{h_7:\Delta_{11},F_6,F_8\vdash F_{10}}{\bullet h_7:(\Delta_{11},F_8\oplus F_9),F_6\vdash F_{10}}\\ \hline\\ -:(\Delta_2,F_3\&F_4),\Delta_{11},F_8\oplus F_9\vdash F_{10} \\\hline\\ \frac{h_1:\Delta_2,F_4\vdash F_6}{\bullet h_1:\Delta_2,F_4\vdash F_6} &ax/W &\frac{\bullet}{\bullet h_7:\Delta_{11},F_6,F_8\oplus F_9\vdash F_{10}}\\ \hline\\ -:\Delta_{11},\Delta_2,F_4\vdash F_6 &F_9\vdash F_{10}\\ \hline\\ -:\Delta_{11},\Delta_2,F_4\vdash F_8\oplus F_9\vdash F_{10}\\\hline\\ \bullet h_1:\Delta_2,F_4\vdash F_8\oplus F_9\\\hline\\ \bullet h_1:\Delta_2,F_3\&F_4\vdash F_8\oplus F_9\\\hline\\ \bullet h_1:\Delta_2,F_3\&F_4\vdash F_8\oplus F_9\\\hline\\ \bullet h_1:\Delta_2,F_3\&F_4\vdash F_8\oplus F_9\\\hline\\ \bullet h_1:\Delta_2,F_3\&F_4\vdash F_{10}\\\hline\\ -:(\Delta_2,F_3\&F_4),\Delta_7\vdash F_{10}\\\hline\\ \hline\\ -:(\Delta_2,F_3\&F_4),\Delta_7\vdash F_{10}\\\hline\\ \hline\\ \bullet h_6:\Delta_7,F_8\oplus F_9\vdash F_{10}\\\hline\\ \bullet h_6:\Delta_7,F_8\oplus F_9\vdash F_{10}\\\hline\\ \bullet h_6:\Delta_7,F_8\oplus F_9\vdash F_{10}\\\hline\\ -:(\Delta_2,A_7,F_3\&F_4\vdash F_{10}\\\hline\\ -:\Delta_2,\Delta_7,F_3\&F_4\vdash F_{10}\\\hline\\ \end{array}$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{h_1:\Delta_2,F_4\vdash F_6}{\bullet h_1:\Delta_2,F_3\&F_4\vdash F_6} &\&_{L2} &\frac{h_7:\Delta_{12},F_6\vdash F_9}{\bullet h_7:(\Delta_8,\Delta_{12},F_9\multimap F_{10}),F_6\vdash F_{11}} \\ -:(\Delta_2,F_3\&F_4),\Delta_8,\Delta_{12},F_9\multimap F_{10}\vdash F_{11} \\ \hline \\ \frac{h_1:\Delta_2,F_4\vdash F_6}{\bullet h_2,\Delta_2,\Delta_8,F_4,F_9\multimap F_{10}\vdash F_{11}} &&_{L2} \\ \hline \\ \frac{h_1:\Delta_2,F_4\vdash F_6}{\bullet h_2,\Delta_2,\Delta_8,F_4,F_9\multimap F_{10}\vdash F_{11}} &&_{L2} \\ \hline \\ \frac{h_1:\Delta_2,F_4\vdash F_6}{\bullet h_1:\Delta_2,F_3\&F_4\vdash F_6} &&_{L2} &&_{h_7:\Delta_8\vdash F_9} &h_7:\Delta_{12},F_6,F_{10}\vdash F_{11} \\ \hline \\ \bullet h_1:\Delta_2,F_3\&F_4\vdash F_6 &&_{L2} &&_{h_7:\Delta_8\vdash F_9} &h_7:\Delta_{12},F_6,F_{10}\vdash F_{11} \\ \hline \\ \bullet h_7:(\Delta_8,\Delta_{12},F_9\multimap F_{10}\vdash F_{11}) &&_{Cut} \\ \hline \\ -:(\Delta_2,F_3\&F_4),\Delta_8,\Delta_{12},F_9\multimap F_{10}\vdash F_{11} &&_{Cut} \\ \hline \\ \hline \\ h_1:\Delta_2,F_4\vdash F_6 &&_{M_1} &&_{M_2} &&_{M_2} \\ \hline \\ -:(\Delta_2,F_3\&F_4),\Delta_8,\Delta_{12},F_9\multimap F_{10}\vdash F_{11} &&_{M_2} \\ \hline \\ -:\Delta_{12},\Delta_2,\Delta_8,F_4,F_9\multimap F_{10}\vdash F_{11} &&_{L2} \\ \hline \\ \hline \\ -:\Delta_{12},\Delta_2,\Delta_8,F_3\&F_4,F_9\multimap F_{10}\vdash F_{11} &&_{L2} \\ \hline \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{4} \vdash \mathbf{F}_{9} \multimap \mathbf{F}_{10} \\ \bullet \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \& \mathbf{F}_{4} \vdash \mathbf{F}_{9} \multimap \mathbf{F}_{10} \end{array}}{\bullet \mathbf{h}_{6}: \Delta_{7} \vdash \mathbf{F}_{9} \quad \mathbf{h}_{6}: \Delta_{8}, \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ -: (\Delta_{2}, \mathbf{F}_{3} \& \mathbf{F}_{4}), \Delta_{7}, \Delta_{8} \vdash \mathbf{F}_{11} \end{array}} \overset{\bullet}{\bullet} \mathbf{h}_{6}: (\Delta_{7}, \Delta_{8}), \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: (\Delta_{7}, \Delta_{8}), \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{6}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_{1}: \Delta_{7}, \Delta_{8}, \mathbf{F}_{1} \vdash \mathbf{h}_{1}$$

 \bullet Case rule I

$$\frac{ \mathbf{h}_1 : \Delta_2, \mathbf{F}_4 \vdash p(\mathbf{n}_7)}{\underbrace{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash p(\mathbf{n}_7)}_{\quad \leftarrow 1} \ \, \underbrace{ \begin{array}{c} \&_{L2} \\ \bullet \mathbf{h}_6 : *, p(\mathbf{n}_7) \vdash p(\mathbf{n}_7) \end{array}}_{\quad \leftarrow \mathbf{Cut}} \ \, \underbrace{ \begin{array}{c} I \\ - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), * \vdash p(\mathbf{n}_7) \\ \hline \\ - : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash p(\mathbf{n}_7) \end{array}}_{\quad \text{ax/W}} \ \, \underbrace{ \begin{array}{c} I \\ \text{Cut} \\ \text{Cut} \\ \end{array} }_{\quad \leftarrow 1} \ \, \underbrace{ \begin{array}{c} I \\ \text{Cut} \\ \text{Cut} \\ \text{Cut} \\ \end{array}$$

5.14 Status of $\&_{L1}$: OK

• Case rule !R

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash 7 \\ \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash 7 \end{array} \&_{L1} \quad \frac{\mathbf{h}_8 : ! \Upsilon 6, 7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8 : ! \Upsilon 6, 7 \vdash ! \mathbf{F}_9} \\ - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), ! \Upsilon 6 \vdash ! \mathbf{F}_9 \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash 7}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash 7} \quad \underset{\bullet}{\mathsf{ax/W}} \quad \overset{\bullet}{\bullet} \mathbf{h}_8 : 7, ! \Upsilon 6 \vdash ! \mathbf{F}_9} \\ \frac{- : ! \Upsilon 6, \Delta_2, \mathbf{F}_3 \vdash ! \mathbf{F}_9}{- : ! \Upsilon 6, \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash ! \mathbf{F}_9} \quad \&_{L1} \end{array} \quad \overset{\mathsf{ax/W}}{\bullet}$$

- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_7 \\ \underline{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7 } \end{array} \&_{L1} \quad \frac{}{\bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7 \vdash \top} \quad \mathbf{Cut} \\ -: (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_6 \vdash \top \\ & \stackrel{\leadsto}{-: \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \top} \quad \top \end{array}$$

• Case rule $\&_R$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_7 \\ \underline{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7} \end{array} \&_{L1} & \begin{array}{c} \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 & \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \underline{\bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10}} \end{array} & \mathbf{Cut} \\ \\ \hline - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_6 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ \underline{\bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \& \mathbf{F}_{10}} \\ \hline - : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_7 & \mathbf{ax}/\mathbb{W} & \mathbf{hCut} \\ \hline - : \Delta_2, \Delta_6, \mathbf{F}_3 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ - : \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ \hline - : \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ \end{array} & \&_{L1} \end{array}$$

• Case rule \multimap_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7} & \&_{L1} & \frac{\mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10}} & \neg \circ_R \\ \hline & -: (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_6 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} & \text{Cut} \\ \hline \\ \frac{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7} & \mathbf{ax/W} & \frac{\bullet}{\mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \hline & \frac{-: \Delta_2, \Delta_6, \mathbf{F}_9, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10}}{-: \Delta_2, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10}} & \neg \circ_R \\ \end{array} \quad \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

• Case rule \bigoplus_{R_2}

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7 \end{array} \&_{L1} \quad \frac{\mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10}} \quad \begin{array}{c} \oplus_{R_2} \\ \bullet \mathbf{h}_1 : \Delta_2, \bullet \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7 \end{array} & \overset{\bullet}{\mathbf{h}_8} : \Delta_6, \bullet \mathbf{F}_7 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_1 : \Delta_2, \bullet \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7 \end{array} & \overset{\bullet}{\mathbf{ax/W}} \quad \frac{\bullet}{\mathbf{h}_8 : \Delta_6, \bullet} \quad \overset{\bullet}{\mathbf{F}_7 \vdash \mathbf{F}_{10}} \\ \bullet \mathbf{h}_1 : \Delta_2, \bullet \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_7 \end{array} & \overset{\bullet}{\mathbf{ax/W}} \quad \overset{\bullet}{\mathbf{h}_{10}} \quad \overset{\bullet}{\mathbf{h}_{10}} \quad \overset{\bullet}{\mathbf{h}_{10}} \quad \overset{\bullet}{\mathbf{h}_{10}} \\ \bullet \mathbf{h}_1 : \Delta_2, \bullet \mathbf{h}_1 \otimes \mathbf{h}_2 \otimes \mathbf{h}_2 \otimes \mathbf{h}_3 \otimes \mathbf{h}_4 \vdash \mathbf{h}_{10} \\ \bullet \mathbf{h}_2 \otimes \mathbf{h}_3 \otimes \mathbf{h}_4 \vdash \mathbf{h}_{10} \otimes \mathbf{h}_2 \otimes \mathbf{h}_3 \otimes \mathbf{h}_4 \vdash \mathbf{h}_{10} \otimes \mathbf{h}_2 \otimes \mathbf{h}_3 \otimes \mathbf{h}_4 \\ \bullet \mathbf{h}_1 \otimes \mathbf{h}_2 \otimes \mathbf{h}_3 \otimes \mathbf{h}_4 \otimes$$

• Case rule \oplus_{R_1}

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \vdash \mathbf{F}_{7} \\ \bullet \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \& \mathbf{F}_{4} \vdash \mathbf{F}_{7} \end{array} \&_{L1} \quad \frac{\mathbf{h}_{8}: \Delta_{6}, \mathbf{F}_{7} \vdash \mathbf{F}_{9} \\ \bullet \mathbf{h}_{8}: \Delta_{6}, \mathbf{F}_{7} \vdash \mathbf{F}_{9} \oplus \mathbf{F}_{10} \\ -: (\Delta_{2}, \mathbf{F}_{3} \& \mathbf{F}_{4}), \Delta_{6} \vdash \mathbf{F}_{9} \oplus \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \& \mathbf{F}_{4} \vdash \mathbf{F}_{7} \quad \text{ax/W} \\ \hline \\ \frac{-: \Delta_{2}, \Delta_{6}, \mathbf{F}_{3} \& \mathbf{F}_{4} \vdash \mathbf{F}_{9} \\ -: \Delta_{2}, \Delta_{6}, \mathbf{F}_{3} \& \mathbf{F}_{4} \vdash \mathbf{F}_{9} \oplus \mathbf{F}_{10} \end{array} \oplus_{R_{1}} \quad \begin{array}{c} \oplus_{R_{1}} \\ \text{ax/W} \\ \text{hCut} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c} \frac{h_1:\Delta_2,F_3\vdash 1}{\bullet h_1:\Delta_2,F_3\&F_4\vdash 1} &\&_{L1} & \frac{h_6:\Delta_7\vdash F_8}{\bullet h_6:\Delta_7,1\vdash F_8} & \mathbf{1}_L \\ \hline -:(\Delta_2,F_3\&F_4),\Delta_7\vdash F_8 & \\ \hline -:\Delta_2,\Delta_7,F_3\&F_4\vdash F_8 & ax/W \\ \hline \\ \frac{h_1:\Delta_2,F_3\vdash F_6}{\bullet h_1:\Delta_2,F_3\&F_4\vdash F_6} &\&_{L1} & \frac{h_7:\Delta_9,F_6\vdash F_8}{\bullet h_7:(\mathbf{1},\Delta_9),F_6\vdash F_8} & \mathbf{1}_L \\ \hline -:(\Delta_2,F_3\&F_4),\mathbf{1},\Delta_9\vdash F_8 & \\ \hline \bullet h_1:\Delta_2,F_3\&F_4\vdash F_6 & ax/W & \\ \hline \\ \hline \bullet h_1:\Delta_2,F_3\&F_4\vdash F_6 & ax/W & \\ \hline \\ \hline \bullet h_1:\Delta_2,F_3\&F_4\vdash F_6 & ax/W & \\ \hline \end{pmatrix} \begin{array}{c} \mathbf{1}_L \\ \mathbf{0}_L \\$$

• Case rule \otimes_R

 \bullet Case rule W

\bullet Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L1} & \frac{\mathbf{h}_7 : \Delta_{10}, \mathbf{F}_6, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7 : (\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_6 \vdash \mathbf{F}_9} & C \\ \hline - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 & \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6 & \mathbf{ax/W} & \\ \hline \frac{- : \Delta_{10}, \Delta_2, !\mathbf{F}_8, !\mathbf{F}_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9}{- : \Delta_{10}, \Delta_2, !\mathbf{F}_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9} & C \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_8 & \&_{L1} & \frac{\mathbf{h}_6 : \Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6 : \Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9} & C \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \&_{L1} & \frac{\mathbf{h}_6 : \Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6 : \Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9} & C \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6 : \Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6 : \Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{ax/W} & \mathbf{h}_6 : \Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline - : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9 & \mathbf{ax/W} & \mathbf{h}_6 : \Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline - : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9 & \mathbf{ax/W} & \mathbf{h}_6 : \Delta_7, !\mathbf{F}_8, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \end{pmatrix}$$

\bullet Case rule !L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L1} & \frac{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:(\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_6 \vdash \mathbf{F}_9} & !L \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 & \frac{\bullet}{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9} & \text{ax/W} \\ \hline \frac{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{x}/\mathbf{W}} & \frac{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9} & !L \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_8} & \&_{L1} & \frac{\mathbf{h}_6:\Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9} & !L \\ \hline -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_7 \vdash \mathbf{F}_9 & \text{cut} \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7:\Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_9} & \mathbf{ax/W} & \mathbf{h}_7:\Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7:\Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_9} & \&_{L1} \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9} & \&_{L1} \\ \hline \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\&\mathbf{F}_{4}\vdash\mathbf{F}_{6}} & \&_{L1} & \frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{9}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\&\mathbf{F}_{9}),\mathbf{F}_{6}\vdash\mathbf{F}_{10}} & \&_{L2} \\ \hline \\ -:(\Delta_{2},\mathbf{F}_{3}\&\mathbf{F}_{4}),\Delta_{11},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash\mathbf{F}_{10} & \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\&\mathbf{F}_{4}\vdash\mathbf{F}_{6} & \mathbf{ax/W} & \mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{9}\vdash\mathbf{F}_{10} \\ \hline \\ -:\Delta_{11},\Delta_{2},\mathbf{F}_{9},\mathbf{F}_{3}\&\mathbf{F}_{4}\vdash\mathbf{F}_{10} & \&_{L2} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{8}\&\mathbf{F}_{9} & \&_{L1} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{9}\vdash\mathbf{F}_{10} \\ \hline \bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\&\mathbf{F}_{4}\vdash\mathbf{F}_{8}\&\mathbf{F}_{9} & \&_{L1} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash\mathbf{F}_{10} \\ \hline -:(\Delta_{2},\mathbf{F}_{3}\&\mathbf{F}_{4}),\Delta_{7}\vdash\mathbf{F}_{10} & & \&_{L2} \\ \hline \\ \bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{8}\&\mathbf{F}_{9} & \mathbf{ax/W} & \bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash\mathbf{F}_{10} \\ \hline \bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash\mathbf{F}_{10} & & \mathbf{ax/W} \\ \hline \bullet\mathbf{h}_{7}:\Delta_{7},\mathbf{h}_{7}:\Delta_{7},\mathbf{h}_{7}:\Delta_{7}:\Delta_{7},\mathbf{h}_{7}:\Delta_{7}:\Delta_{7},\mathbf{h}_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7$$

• Case rule $\&_{L1}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L1} & \frac{\mathbf{h}_7:\Delta_{11}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7:(\Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9), \mathbf{F}_6 \vdash \mathbf{F}_{10}} & \mathcal{E}_{L1} \\ \hline \\ -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet} & \mathbf{ax/W} & \bullet \mathbf{h}_7:\Delta_{11}, \mathbf{F}_6, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ -:\Delta_{11},\Delta_2, \mathbf{F}_3, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} & \&_{L1} \\ \hline \\ -:\Delta_{11},\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} & \&_{L1} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9 & \&_{L1} & \bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_8 \& \mathbf{F}_9 & \&_{L1} & \bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ -:(\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4),\Delta_7 \vdash \mathbf{F}_{10} & \&_{L1} \\ \hline \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9 & \mathbf{ax/W} & \bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9 & \mathbf{ax/W} & \bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9 & \mathbf{ax/W} & \bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \& \mathbf{F}_9 & \mathbf{ax/W} & \bullet \mathbf{h}_{Cut} \\ \hline \\ -:\Delta_2,\Delta_7, \mathbf{F}_3 \vdash \mathbf{F}_{10} & \&_{L1} \\ \hline \end{array}$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L1} & \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_6, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9), \mathbf{F}_6 \vdash \mathbf{F}_{10}} & \otimes_{L} \\ \hline \\ - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} & & \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet} & \mathbf{ax/W} & \bullet \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_6, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ - : \Delta_{11}, \Delta_2, \mathbf{F}_3, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} & & & \\ \hline - : \Delta_{11}, \Delta_2, \mathbf{F}_3, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} & & & \\ \hline - : \Delta_{11}, \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & & & & & \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & & & & \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & & & & \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_8 \otimes \mathbf{F}_9 & & & & \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & & & & \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & & & & \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & & & \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 & & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \vdash \mathbf{F}_{10} & & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{h}_3 \& \mathbf{h}_4 \vdash \mathbf{h}_{10} & & \\ \hline \bullet \mathbf{h}_2 : \Delta_2, \Delta_7, \mathbf{h}_3 \& \mathbf{h}_4 \vdash \mathbf{h$$

• Case rule \oplus_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\& \mathbf{F}_{4}\vdash \mathbf{F}_{6}} & \&_{L1} & \frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{8}\vdash \mathbf{F}_{10} \quad \mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{9}\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\oplus \mathbf{F}_{9}),\mathbf{F}_{6}\vdash \mathbf{F}_{10}} & \mathbf{Cut} \\ & -:(\Delta_{2},\mathbf{F}_{3}\& \mathbf{F}_{4}),\Delta_{11},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \\ & \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\& \mathbf{F}_{4}\vdash \mathbf{F}_{6}} & \mathbf{ax/W} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10}} & \mathbf{ax/W} \\ & -:\Delta_{11},\Delta_{2},\mathbf{F}_{3},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \&_{L1} \\ & \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash \mathbf{F}_{8}\oplus \mathbf{F}_{9}}{-:\Delta_{11},\Delta_{2},\mathbf{F}_{3}\& \mathbf{F}_{4}\vdash \mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10}} & \mathbb{h}_{6}:\Delta_{7},\mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{cut} \\ & -:(\Delta_{2},\mathbf{F}_{3}\& \mathbf{F}_{4}),\Delta_{7}\vdash \mathbf{F}_{10} & \\ & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{cut} \\ & \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash \mathbf{F}_{8}\oplus \mathbf{F}_{9} & \mathbf{ax/W} & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus \mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7},\mathbf{F}_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta_{7}:\Delta$$

• Case rule \multimap_L

$$\frac{\begin{array}{l} \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash \mathbf{F}_{6} \\ \underline{\bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\& \mathbf{F}_{4}\vdash \mathbf{F}_{6} \\ } & \&_{L1} & \frac{\mathbf{h}_{7}:\Delta_{12},\mathbf{F}_{6}\vdash \mathbf{F}_{9} & \mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{10}\vdash \mathbf{F}_{11} \\ \underline{\bullet \mathbf{h}_{7}:(\Delta_{8},\Delta_{12},\mathbf{F}_{9}\multimap \mathbf{F}_{10}),\mathbf{F}_{6}\vdash \mathbf{F}_{11} \\ } & -:(\Delta_{2},\mathbf{F}_{3}\& \mathbf{F}_{4}),\Delta_{8},\Delta_{12},\mathbf{F}_{9}\multimap \mathbf{F}_{10}\vdash \mathbf{F}_{11} \\ \underline{\bullet} \\ \underline{\bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash \mathbf{F}_{6}} & \mathbf{ax/W} & \underline{\bullet} \\ \underline{\bullet \mathbf{h}_{7}:\Delta_{12},\Delta_{8},\mathbf{F}_{6},\mathbf{F}_{9}\multimap \mathbf{F}_{10}\vdash \mathbf{F}_{11} \\ \underline{-:\Delta_{12},\Delta_{2},\Delta_{8},\mathbf{F}_{3},\mathbf{F}_{9}\multimap \mathbf{F}_{10}\vdash \mathbf{F}_{11} \\ \underline{-:\Delta_{12},\Delta_{2},\Delta_{8},\mathbf{F}_{3}\& \mathbf{F}_{4},\mathbf{F}_{9}\multimap \mathbf{F}_{10}\vdash \mathbf{F}_{11} \\ \underline{-:\Delta_{12},\Delta_{2},\Delta_{8},\mathbf{F}_{3}\& \mathbf{F}_{4},\mathbf{F}_{9}\multimap \mathbf{F}_{10}\vdash \mathbf{F}_{11} \\ \underline{\bullet} \\ \end{array}} & \underbrace{\mathbf{ax/W}}_{\mathbf{hCut}}$$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_6} & \&_{L1} & \frac{\mathbf{h}_7: \Delta_8 \vdash \mathbf{F}_9 \quad \mathbf{h}_7: \Delta_{12}, \mathbf{F}_6, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_7: (\Delta_8, \Delta_{12}, \mathbf{F}_9 \multimap \mathbf{F}_{10}), \mathbf{F}_6 \vdash \mathbf{F}_{11}} & \mathbf{cut} \\ \hline \\ -: (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), \Delta_8, \Delta_{12}, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_7: \Delta_{12}, \Delta_2, \Delta_8, \mathbf{F}_6, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11}} & \frac{\mathbf{ax/W}}{\bullet \mathbf{hCut}} \\ \hline \\ -: \Delta_{12}, \Delta_2, \Delta_8, \mathbf{F}_3 \& \mathbf{F}_4, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & \&_{L1} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} & & \frac{\mathbf{h}_6: \Delta_7 \vdash \mathbf{F}_9 \quad \mathbf{h}_6: \Delta_8, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_6: (\Delta_7, \Delta_8), \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11}} & \mathbf{cut} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} & & & \frac{\mathbf{h}_6: \Delta_7 \vdash \mathbf{F}_9 \quad \mathbf{h}_6: \Delta_8, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_6: (\Delta_7, \Delta_8), \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11}} & \mathbf{cut} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} & & & & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & \\ \hline \bullet \mathbf{h}_6: \Delta_7, \Delta_8, \mathbf{F}_9 \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} & & \\ \hline \bullet \mathbf{h}_9 \vdash \mathbf{h}_9 \vdash$$

 \bullet Case rule I

$$\frac{ \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \vdash p(\mathbf{n}_7) }{ \underbrace{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash p(\mathbf{n}_7) }_{\quad - : (\Delta_2, \mathbf{F}_3 \& \mathbf{F}_4), * \vdash p(\mathbf{n}_7) } \underbrace{ \begin{matrix} I \\ \bullet \mathbf{h}_6 : *, p(\mathbf{n}_7) \vdash p(\mathbf{n}_7) \end{matrix} }_{\quad - : \Delta_2, \mathbf{F}_3 \& \mathbf{F}_4 \vdash p(\mathbf{n}_7) } \mathbf{ax/W}$$

5.15 Status of \otimes_L : OK

• Case rule !R

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3,\mathbf{F}_4\vdash 7}{\underbrace{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash 7}}\otimes L \quad \frac{\mathbf{h}_8:!\Upsilon\mathbf{6},7\vdash \mathbf{F}_9}{\underbrace{\bullet\mathbf{h}_8:!\Upsilon\mathbf{6},7\vdash !\mathbf{F}_9}}}{-:(\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4),!\Upsilon\mathbf{6}\vdash !\mathbf{F}_9} \quad \overset{!R}{\underbrace{\mathsf{Cut}}}$$

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3,\mathbf{F}_4\vdash 7}{\underbrace{\bullet\mathbf{h}_8:7,!\Upsilon\mathbf{6}\vdash !\mathbf{F}_9}} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}}$$

$$\frac{-:!\Upsilon\mathbf{6},\Delta_2,\mathbf{F}_3,\mathbf{F}_4\vdash !\mathbf{F}_9}{-:!\Upsilon\mathbf{6},\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash !\mathbf{F}_9} \quad \otimes_L$$

- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_7 \end{array} \otimes_L \quad \frac{}{\bullet \mathbf{h}_8 : \Delta_6, \mathbf{F}_7 \vdash \top} \quad \mathbf{Cut} \\ - : (\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4), \Delta_6 \vdash \top \\ & \stackrel{\leadsto}{} \\ \hline - : \Delta_2, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \top \quad \top \end{array} }$$

• Case rule $\&_R$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3,\mathbf{F}_4\vdash\mathbf{F}_7}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_7}\otimes_L & \frac{\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9 & \mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9\&\mathbf{F}_{10}} \\ & -:(\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4),\Delta_6\vdash\mathbf{F}_9\&\mathbf{F}_{10}} \\ \hline \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3,\mathbf{F}_4\vdash\mathbf{F}_7}{\bullet\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9\&\mathbf{F}_{10}} & \frac{\mathbf{ax/W}}{\bullet\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9\&\mathbf{F}_{10}} \\ \hline \frac{-:\Delta_2,\Delta_6,\mathbf{F}_3,\mathbf{F}_4\vdash\mathbf{F}_9\&\mathbf{F}_{10}}{-:\Delta_2,\Delta_6,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_9\&\mathbf{F}_{10}} \otimes_L \end{array} \\ \bullet \mathbf{h}_1:\mathbf{h}_1:\mathbf{h}_2:\mathbf{h}_2:\mathbf{h}_2:\mathbf{h}_3:\mathbf{h}_3:\mathbf{h}_3:\mathbf{h}_3:\mathbf{h}_4:\mathbf{h}_3:$$

• Case rule \multimap_R

$$\begin{array}{c} \mathbf{h}_1: \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_7 \end{array} \otimes_L \quad \begin{array}{c} \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} \end{array} \quad \begin{array}{c} -\circ_R \\ \text{Cut} \end{array} \\ \\ \hline \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_7 \end{array} \quad \begin{array}{c} \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} \end{array} \quad \begin{array}{c} -\circ_R \\ \text{Cut} \end{array} \\ \\ \hline \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_7 \end{array} \quad \begin{array}{c} \mathbf{a}_R / \mathbb{W} \\ \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} \quad \begin{array}{c} \mathbf{a}_R / \mathbb{W} \\ \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} \quad \begin{array}{c} \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{h}_{10} \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{h}_9 \vdash \mathbf{h}_9 \vdash \mathbf{h}_9 \vdash \mathbf{h}_9 \\ \hline \bullet \mathbf{h}_8: \Delta_6, \mathbf{h}_9 \vdash \mathbf{h}_9 \vdash \mathbf{h}_9 \vdash \mathbf{h}_9 \vdash \mathbf{h}_9 \vdash \mathbf{h}_9 \\ \hline \bullet \mathbf{h}_9 \vdash \mathbf$$

• Case rule \bigoplus_{R_2}

$$\begin{array}{c} \mathbf{h}_1: \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \hline & -: (\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4), \Delta_6 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \hline & \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_7 \\ \hline & -: \Delta_2, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_{10} \\ \hline & -: \Delta_2, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_{9} \oplus \mathbf{F}_{10} \\ \hline \end{array} \begin{array}{c} \oplus \mathbf{h}_1: \Delta_2, \mathbf{h}_3 \otimes \mathbf{h}_4 \vdash \mathbf{h}_7 \\ \hline & \bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_3 \otimes \mathbf{h}_4 \vdash \mathbf{h}_{10} \\ \hline & \bullet \mathbf{h}_2 \end{array} \begin{array}{c} \bullet \mathbf{h}_3: \Delta_6, \mathbf{h}_7 \vdash \mathbf{h}_{10} \\ \bullet \mathbf{h}_3: \Delta_6, \mathbf{h}_7 \vdash \mathbf{h}_{10} \\ \hline & \bullet \mathbf{h}_3: \Delta_6, \mathbf{h}_7 \otimes \mathbf{h}_{10} \end{array} \begin{array}{c} \bullet \mathbf{h}_7 \otimes \mathbf{h}$$

• Case rule \bigoplus_{R_1}

• Case rule $\mathbf{1}_L$

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3,\mathbf{F}_4\vdash\mathbf{1}}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{1}}\otimes_L & \frac{\mathbf{h}_6:\Delta_7\vdash\mathbf{F}_8}{\bullet\mathbf{h}_6:\Delta_7,\mathbf{1}\vdash\mathbf{F}_8} & \mathbf{1}_L\\ \hline -:(\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4),\Delta_7\vdash\mathbf{F}_8 & \\ \hline -:\Delta_2,\Delta_7,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_8 & \mathbf{ax/W} \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3,\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_6}\otimes_L & \frac{\mathbf{h}_7:\Delta_9,\mathbf{F}_6\vdash\mathbf{F}_8}{\bullet\mathbf{h}_7:(\mathbf{1},\Delta_9),\mathbf{F}_6\vdash\mathbf{F}_8} & \mathbf{1}_L\\ \hline -:(\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4),\mathbf{1},\Delta_9\vdash\mathbf{F}_8 & \mathbf{1}_L\\ \hline -:(\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_6 & \mathbf{ax/W}\\ \hline & \frac{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_6}{\bullet\mathbf{h}_2:\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_6} & \mathbf{1}_L\\ \hline -:(\Delta_2,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_6 & \mathbf{1}_L) & \mathbf{1}_L\\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{h}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_6 & \mathbf{h}_7:\mathbf{h}_2,\mathbf{h}_3\in\mathbf{F}_6\vdash\mathbf{F}_8\\ \hline -:\mathbf{1},\Delta_2,\Delta_9,\mathbf{F}_3\otimes\mathbf{F}_4\vdash\mathbf{F}_8 & \mathbf{h}_7 \\ \hline \end{array}$$

• Case rule \otimes_R

\bullet Case rule W

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_6} \otimes_L & \frac{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_6 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:(\Delta_{10}, \mathbf{!F}_8), \mathbf{F}_6 \vdash \mathbf{F}_9} & W \\ \hline -:(\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4), \Delta_{10}, \mathbf{!F}_8 \vdash \mathbf{F}_9 & \text{Cut} \\ \hline \bullet \underline{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_6} & \mathbf{ax/W} & \overline{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_6, \mathbf{!F}_8 \vdash \mathbf{F}_9} \\ \hline -:\Delta_{10}, \Delta_2, \mathbf{!F}_8, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_9 & \mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{!F}_8 & \otimes_L & \underline{\mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_9} \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{!F}_8 & \otimes_L & \underline{\mathbf{h}_6:\Delta_7 \vdash \mathbf{F}_9} \\ \hline -:(\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4), \Delta_7 \vdash \mathbf{F}_9 & \mathbf{cut} \\ \hline & -:\Delta_2, \Delta_7, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_9 & \mathbf{ax/W} \\ \hline \end{array}$$

\bullet Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3,\mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_6} \otimes L & \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_6,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:(\Delta_{10},!\mathbf{F}_8),\mathbf{F}_6 \vdash \mathbf{F}_9} & C \\ \hline -:(\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4),\Delta_{10},!\mathbf{F}_8 \vdash \mathbf{F}_9 & \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_6 & \mathbf{ax/W} & \\ \hline -:\Delta_{10},\Delta_2,!\mathbf{F}_8,!\mathbf{F}_8,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_9 & C \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3,\mathbf{F}_4 \vdash !\mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} \otimes L & \frac{\mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7,!\mathbf{F}_8 \vdash \mathbf{F}_9} & C \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} \otimes L & \frac{\mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7,!\mathbf{F}_8 \vdash \mathbf{F}_9} & C \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8} & \mathbf{ax/W} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8 & \mathbf{h}_7 \otimes \mathbf{h}_8 \vdash \mathbf{h}_9 \\ \hline \end{pmatrix} & \mathbf{h}_6:\Delta_7,!\mathbf{F}_8,!\mathbf{F}_8 \vdash \mathbf{F}_9 & \mathbf{h}_9 \\ \hline \end{pmatrix} = \mathbf{h}_1:\Delta_2,\mathbf{h}_3 \otimes \mathbf{h}_4 \vdash \mathbf{h}_8 & \mathbf{h}_9 \otimes \mathbf{$$

• Case rule !L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_6} \otimes_L & \frac{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:(\Delta_{10}, !\mathbf{F}_8), \mathbf{F}_6 \vdash \mathbf{F}_9} \\ \hline -:(\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4), \Delta_{10}, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \underline{\mathbf{h}_1:\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_6} & \mathbf{ax/W} & \frac{\mathbf{h}_7:\Delta_{10}, \mathbf{F}_6, \mathbf{F}_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:\Delta_{10}, \Delta_2, \mathbf{F}_8, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_9} & \mathbf{tL} \\ \hline -:\Delta_{10}, \Delta_2, !\mathbf{F}_8, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_9 \\ -:\Delta_{10}, \Delta_2, !\mathbf{F}_8, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_9 \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash !\mathbf{F}_8 \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8 \\ \bullet \mathbf{h}_1:\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash !\mathbf{F}_8 \\ \hline -:(\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4), \Delta_7 \vdash \mathbf{F}_9 \\ \hline \hline \mathbf{h}_1:\Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash !\mathbf{F}_8 \\ \hline \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_6:\Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_6:\Delta_7, !\mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_9 \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_9 \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_9 \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_9 \\ \hline -:\Delta_2,\Delta_7, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_9 \\ \hline \end{array}$$

• Case rule $\&_{L2}$

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3}, \mathbf{F}_{4} \vdash \mathbf{F}_{6} \\ \bullet \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \otimes \mathbf{F}_{4} \vdash \mathbf{F}_{6} \end{array} \otimes_{L} \quad \frac{\mathbf{h}_{7}: \Delta_{11}, \mathbf{F}_{6}, \mathbf{F}_{9} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{7}: (\Delta_{11}, \mathbf{F}_{8} \& \mathbf{F}_{9}), \mathbf{F}_{6} \vdash \mathbf{F}_{10}} \quad \frac{\&_{L2}}{\mathsf{cut}} \\ \\ -: (\Delta_{2}, \mathbf{F}_{3} \otimes \mathbf{F}_{4}), \Delta_{11}, \mathbf{F}_{8} \& \mathbf{F}_{9} \vdash \mathbf{F}_{10}} \\ \hline \frac{\bullet \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \otimes \mathbf{F}_{4} \vdash \mathbf{F}_{6}}{\mathsf{ax} / \mathsf{w}} \quad \frac{\mathsf{h}_{7}: \Delta_{11}, \mathbf{F}_{6}, \mathbf{F}_{9} \vdash \mathbf{F}_{10}}{\mathsf{h}_{7}: \Delta_{11}, \Delta_{2}, \mathbf{F}_{9}, \mathbf{F}_{3} \otimes \mathbf{F}_{4} \vdash \mathbf{F}_{10}} \quad \frac{\mathsf{ax} / \mathsf{w}}{\mathsf{hCut}} \\ \hline \frac{-: \Delta_{11}, \Delta_{2}, \mathbf{F}_{9}, \mathbf{F}_{3} \otimes \mathbf{F}_{4} \vdash \mathbf{F}_{10}}{-: \Delta_{11}, \Delta_{2}, \mathbf{F}_{8} \& \mathbf{F}_{9}, \mathbf{F}_{3} \otimes \mathbf{F}_{4} \vdash \mathbf{F}_{10}} \quad \&_{L2} \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3}, \mathbf{F}_{4} \vdash \mathbf{F}_{8} \& \mathbf{F}_{9} \\ \bullet \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \otimes \mathbf{F}_{4} \vdash \mathbf{F}_{8} \& \mathbf{F}_{9} \end{array} \otimes_{L} \quad \frac{\mathbf{h}_{6}: \Delta_{7}, \mathbf{F}_{9} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{6}: \Delta_{7}, \mathbf{F}_{8} \& \mathbf{F}_{9} \vdash \mathbf{F}_{10}} \quad \underset{\bullet}{\otimes}_{\mathbf{Cut}} \\ \\ \frac{-: (\Delta_{2}, \mathbf{F}_{3} \otimes \mathbf{F}_{4}), \Delta_{7} \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{6}: \Delta_{7}, \mathbf{F}_{8} \& \mathbf{F}_{9} \vdash \mathbf{F}_{10}} \quad \underset{\bullet}{\text{ax/W}} \\ \frac{-: \Delta_{2}, \Delta_{7}, \mathbf{F}_{3}, \mathbf{F}_{4} \vdash \mathbf{F}_{10}}{-: \Delta_{2}, \Delta_{7}, \mathbf{F}_{3} \otimes \mathbf{F}_{4} \vdash \mathbf{F}_{10}} \quad \otimes_{L}} \quad \underset{\bullet}{\text{ax/W}} \\ \\ \frac{-: \Delta_{2}, \Delta_{7}, \mathbf{F}_{3} \otimes \mathbf{F}_{4} \vdash \mathbf{F}_{10}}{-: \Delta_{2}, \Delta_{7}, \mathbf{F}_{3} \otimes \mathbf{F}_{4} \vdash \mathbf{F}_{10}} \otimes_{L}} \end{array}$$

• Case rule $\&_{L1}$

$$\begin{array}{c} \frac{h_1:\Delta_2,F_3,F_4\vdash F_6}{\bullet h_1:\Delta_2,F_3\otimes F_4\vdash F_6}\otimes L & \frac{h_7:\Delta_{11},F_6,F_8\vdash F_{10}}{\bullet h_7:(\Delta_{11},F_8\&F_9),F_6\vdash F_{10}} & \&_{L1} \\ \hline \\ -:(\Delta_2,F_3\otimes F_4),\Delta_{11},F_8\&F_9\vdash F_{10} & \\ \hline \\ \frac{\bullet h_1:\Delta_2,F_3\otimes F_4\vdash F_6}{-:\Delta_{11},\Delta_2,F_8,F_3\otimes F_4\vdash F_{10}} & \frac{\mathsf{ax/W}}{\mathsf{h}_7:\Delta_{11},F_6,F_8\vdash F_{10}} & \mathsf{hCut} \\ \hline \\ \frac{-:\Delta_{11},\Delta_2,F_8\&F_9,F_3\otimes F_4\vdash F_{10}}{-:\Delta_{11},\Delta_2,F_8\&F_9} \otimes L & \frac{h_6:\Delta_7,F_8\vdash F_{10}}{\bullet h_6:\Delta_7,F_8\&F_9\vdash F_{10}} & \&_{L1} \\ \hline \\ \frac{\bullet h_1:\Delta_2,F_3\otimes F_4\vdash F_8\&F_9}{-:(\Delta_2,F_3\otimes F_4\vdash F_8\&F_9)} \otimes L & \frac{h_6:\Delta_7,F_8\&F_9\vdash F_{10}}{\bullet h_6:\Delta_7,F_8\&F_9\vdash F_{10}} & \&_{L1} \\ \hline \\ \frac{h_1:\Delta_2,F_3,F_4\vdash F_8\&F_9}{-:\Delta_2,\Delta_7,F_3\otimes F_4\vdash F_{10}} & & \mathsf{ax/W} \\ \hline \\ \frac{-:\Delta_2,\Delta_7,F_3\otimes F_4\vdash F_{10}}{-:\Delta_2,\Delta_7,F_3\otimes F_4\vdash F_{10}} \otimes L & & \mathsf{hCut} \\ \hline \end{array}$$

• Case rule \otimes_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_6} \otimes_L & \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_6, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9), \mathbf{F}_6 \vdash \mathbf{F}_{10}} & \otimes_L \\ \hline \\ \underline{\mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_6} & \mathbf{ax/W} & \frac{}{\bullet \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_6, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \hline \\ \underline{\mathbf{h}_1 : \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_6} & \mathbf{ax/W} & \frac{}{\bullet \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_6, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \hline \\ \underline{- : \Delta_{11}, \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ - : \Delta_{11}, \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} & \otimes_L \\ \hline \\ \underline{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & \otimes_L & \frac{\mathbf{h}_6 : \Delta_7, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9} & \otimes_L & \frac{\mathbf{h}_6 : \Delta_7, \mathbf{F}_8, \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \hline \\ \underline{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9}} & \mathbf{ax/W} & \frac{}{\bullet \mathbf{h}_6 : \Delta_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \hline \\ \underline{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9}} & \mathbf{ax/W} & \frac{}{\bullet \mathbf{h}_6 : \Delta_7, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \hline \\ \underline{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_8 \otimes \mathbf{F}_9}} & \mathbf{ax/W} & \mathbf{hCut} \\ \hline \\ \underline{- : \Delta_2, \Delta_7, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_{10}} & \otimes_L \\ \hline \\ \underline{- : \Delta_2, \Delta_7, \mathbf{F}_3, \mathbf{F}_4 \vdash \mathbf{F}_{10}} & \otimes_L \\ \hline \end{array}$$

• Case rule \oplus_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3},\mathbf{F}_{4}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\otimes\mathbf{F}_{4}\vdash\mathbf{F}_{6}} \otimes_{L} & \frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{8}\vdash\mathbf{F}_{10} \quad \mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{9}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\oplus\mathbf{F}_{9}),\mathbf{F}_{6}\vdash\mathbf{F}_{10}} & \mathbf{Cut} \\ \hline \\ -:(\Delta_{2},\mathbf{F}_{3}\otimes\mathbf{F}_{4}),\Delta_{11},\mathbf{F}_{8}\oplus\mathbf{F}_{9}\vdash\mathbf{F}_{10} & & \\ \hline \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3},\mathbf{F}_{4}\vdash\mathbf{F}_{6}}{\bullet\mathbf{x}'} & \frac{\bullet\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{8}\oplus\mathbf{F}_{9}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{7}:\Delta_{11},\Delta_{2},\mathbf{F}_{3},\mathbf{F}_{4},\mathbf{F}_{8}\oplus\mathbf{F}_{9}\vdash\mathbf{F}_{10}} & \mathbf{h}^{\mathbf{x}}\mathbf{x}'^{\mathbf{y}} \\ \hline -:\Delta_{11},\Delta_{2},\mathbf{F}_{3},\mathbf{F}_{4}\vdash\mathbf{F}_{8}\oplus\mathbf{F}_{9} & \otimes_{L} & \frac{\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{9}\vdash\mathbf{F}_{10}} & \otimes_{L} \\ \hline \bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3},\mathbf{F}_{4}\vdash\mathbf{F}_{8}\oplus\mathbf{F}_{9} & \otimes_{L} & \frac{\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus\mathbf{F}_{9}\vdash\mathbf{F}_{10}} & \mathbf{cut} \\ \hline -:(\Delta_{2},\mathbf{F}_{3}\otimes\mathbf{F}_{4}),\Delta_{7}\vdash\mathbf{F}_{10} & & \mathbf{x}'^{\mathbf{y}} \\ \hline \bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus\mathbf{F}_{9}\vdash\mathbf{F}_{10} & & \mathbf{x}^{\mathbf{y}} \\ \hline \bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus\mathbf{F}_{9}\vdash\mathbf{F}_{10} & & \mathbf{h}^{\mathbf{x}}\mathbf{x}'^{\mathbf{y}} \\ \hline -:(\Delta_{2},\mathbf{F}_{3},\mathbf{F}_{4}\vdash\mathbf{F}_{8}\oplus\mathbf{F}_{9} & & \mathbf{x}'^{\mathbf{y}} \\ \hline \bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\oplus\mathbf{F}_{9}\vdash\mathbf{F}_{10} & & \mathbf{h}^{\mathbf{x}}\mathbf{x}'^{\mathbf{y}} \\ \hline -:(\Delta_{2},\mathbf{F}_{3},\mathbf{F}_{4}\vdash\mathbf{F}_{8}\oplus\mathbf{F}_{9} & & \mathbf{x}'^{\mathbf{y}} \\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{3},\mathbf{F}_{4}\vdash\mathbf{F}_{10} & \otimes_{L} \\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{3}\otimes\mathbf{F}_{4}\vdash\mathbf{F}_{10} & \otimes_{L} \\ \hline \end{array}$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{h_1:\Delta_2,F_3,F_4\vdash F_6}{\bullet h_1:\Delta_2,F_3\otimes F_4\vdash F_6} & \otimes_L & \frac{h_7:\Delta_{12},F_6\vdash F_9 & h_7:\Delta_8,F_{10}\vdash F_{11}}{\bullet h_7:(\Delta_8,\Delta_{12},F_9\multimap F_{10}),F_6\vdash F_{11}} & \circlearrowleft_L \\ \hline -:(\Delta_2,F_3\otimes F_4),\Delta_8,\Delta_{12},F_9\multimap F_{10}\vdash F_{11} & & \\ \hline \frac{h_1:\Delta_2,F_3,F_4\vdash F_6}{\bullet h_1:\Delta_2,F_3,F_4\vdash F_6} & \text{ax/W} & \frac{\neg}{\bullet h_7:\Delta_{12},\Delta_8,F_6,F_9\multimap F_{10}\vdash F_{11}} & \text{ax/W} \\ \hline -:\Delta_{12},\Delta_2,\Delta_8,F_3,F_4\vdash F_9 & & \underbrace{h_7:\Delta_{12},\Delta_8,F_6,F_9\multimap F_{10}\vdash F_{11}}_{\bullet h_7:\Delta_2,F_3,F_4\vdash F_6} & \otimes_L & \frac{h_7:\Delta_8\vdash F_9 & h_7:\Delta_{12},F_6,F_{10}\vdash F_{11}}{\bullet h_7:(\Delta_8,\Delta_{12},F_9\multimap F_{10})\vdash F_{11}} & \xrightarrow{\bullet_L} \\ \hline -:(\Delta_2,F_3\otimes F_4),\Delta_8,\Delta_{12},F_9\multimap F_{10}\vdash F_{11}} & & \xrightarrow{\bullet_L} \\ \hline -:(\Delta_2,F_3\otimes F_4),\Delta_8,\Delta_{12},F_9\multimap F_{10}\vdash F_{11}} & & \xrightarrow{\bullet_L} \\ \hline -:(\Delta_2,F_3\otimes F_4),\Delta_8,\Delta_{12},F_9\multimap F_{10}\vdash F_{11}} & & \xrightarrow{\bullet_L} \\ \hline -:\Delta_{12},\Delta_2,\Delta_8,F_3,F_4\vdash F_9\multimap F_{10}\vdash F_{11}} & & \xrightarrow{\bullet_L} \\ \hline -:\Delta_{12},\Delta_2,\Delta_8,F_3,F_4\vdash F_9\multimap F_{10}\vdash F_{11}} & \otimes_L \\ \hline -:(\Delta_2,F_3\otimes F_4),\Delta_7,\Delta_8,F_3,F_4\vdash F_9\multimap F_{10}\vdash F_{11}} & & \xrightarrow{\bullet_L} \\ \hline -:(\Delta_2,F_3\otimes F_4),\Delta_7,\Delta_8\vdash F_{11}} & & \xrightarrow{\bullet_L} \\ \hline -:(\Delta_2,F_3\otimes F_4),\Delta_7,\Delta_8\vdash F_{11}} & & \xrightarrow{\bullet_L} \\ \hline -:(\Delta_2,F_3\otimes F_4),\Delta_7,\Delta_8\vdash F_{11}} & & \xrightarrow{\bullet_L} \\ \hline -:(\Delta_2,F_3,F_4\vdash F_9\multimap F_{10}} & & \xrightarrow{\bullet_L} \\ \hline -:(\Delta_2,A_7,\Delta_8,F_3,F_4\vdash F_{11})} & \otimes_L \\ \hline -:(\Delta_2,A_7,\Delta_8,F_3,F_4\vdash F_{11})} & & \xrightarrow{\bullet_L} \\ \hline -:\Delta_2,\Delta_7,\Delta_8,F_3,F_4\vdash F_{11}} & \otimes_L \\ \hline \end{array}$$

 \bullet Case rule I

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_2, \mathbf{F}_3, \mathbf{F}_4 \vdash p(\mathbf{n}_7) \\ \bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash p(\mathbf{n}_7) \end{array} \otimes_L \quad \frac{}{\bullet \mathbf{h}_6: *, p(\mathbf{n}_7) \vdash p(\mathbf{n}_7)} \quad \begin{array}{c} I \\ \bullet \mathbf{h}_6: *, p(\mathbf{n}_7) \vdash p(\mathbf{n}_7) \\ \hline \\ -: (\Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4), * \vdash p(\mathbf{n}_7) \\ \hline \\ -: \Delta_2, \mathbf{F}_3 \otimes \mathbf{F}_4 \vdash p(\mathbf{n}_7) \end{array} \quad \text{ax/W} \end{array}$$

5.16 Status of \oplus_L : OK

• Case rule !R

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash 7\quad\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash 7}{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash 7}\oplus_{L}\quad\frac{\mathbf{h}_{8}:!\Upsilon6,7\vdash \mathbf{F}_{9}}{\bullet\mathbf{h}_{8}:!\Upsilon6,7\vdash \mathbf{F}_{9}}}{(\mathbf{h}_{8}:!\Upsilon6,7\vdash \mathbf{F}_{9})}\underbrace{\begin{array}{c}!R\\\mathsf{Cut}\end{array}}\\ \frac{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash 7\quad\mathsf{ax/W}}{\bullet\mathbf{h}_{8}:7,!\Upsilon6\vdash !\mathbf{F}_{9}}\xrightarrow{\mathsf{ax/W}}\underbrace{\begin{array}{c}\mathsf{ax/W}\\\mathsf{hCut}\end{array}}\frac{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash 7\quad\mathsf{ax/W}}{\bullet\mathbf{h}_{8}:7,!\Upsilon6\vdash !\mathbf{F}_{9}}\xrightarrow{\mathsf{ax/W}}\underbrace{\begin{array}{c}\mathsf{ax/W}\\\mathsf{hCut}\end{array}}\frac{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash 7\quad\mathsf{ax/W}}{\bullet\cdot\mathbf{h}_{8}:7,!\Upsilon6\vdash !\mathbf{F}_{9}}\xrightarrow{\mathsf{ax/W}}\underbrace{\begin{array}{c}\mathsf{ax/W}\\\mathsf{hCut}\end{array}}$$

- Case rule $\mathbf{1}_R$
- Case rule \top

$$\begin{array}{c|c} \underline{h_1:\Delta_2,F_3\vdash F_7 \quad h_1:\Delta_2,F_4\vdash F_7} \\ \underline{\bullet h_1:\Delta_2,F_3\oplus F_4\vdash F_7} & \oplus_L & \\ \underline{\bullet h_8:\Delta_6,F_7\vdash \top} \\ -:(\Delta_2,F_3\oplus F_4),\Delta_6\vdash \top \\ & \xrightarrow{} \\ \underline{-:\Delta_2,\Delta_6,F_3\oplus F_4\vdash \top} & \top \end{array}$$

• Case rule $\&_R$

• Case rule \multimap_R

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{7}\quad\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash\mathbf{F}_{7}}{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{7}}\oplus\mathbf{L} \quad \frac{\mathbf{h}_{8}:\Delta_{6},\mathbf{F}_{7},\mathbf{F}_{9}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{8}:\Delta_{6},\mathbf{F}_{7}\vdash\mathbf{F}_{9}\multimap\mathbf{F}_{10}} \quad \neg \circ_{R} \\ \hline \\ -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{7}) \quad \xrightarrow{\bullet} \mathbf{h}_{8}:\Delta_{6},\mathbf{F}_{7}\vdash\mathbf{F}_{9}\multimap\mathbf{F}_{10}} \\ \hline \\ \frac{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{7}}{\bullet\mathbf{x}^{\vee}\vee} \quad \xrightarrow{\mathbf{h}_{8}:\Delta_{6},\mathbf{F}_{7},\mathbf{F}_{9}\vdash\mathbf{F}_{10}} \\ \hline \\ -:\Delta_{2},\Delta_{6},\mathbf{F}_{9},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{10} \\ \hline \\ -:\Delta_{2},\Delta_{6},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{9}\multimap\mathbf{F}_{10} \end{array} \quad \xrightarrow{\bullet}_{R} \begin{array}{c} \mathbf{a}\mathbf{x}/\mathbf{w} \\ \mathbf{h}\mathbf{C}\mathbf{u}\mathbf{t} \end{array}$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_7\quad\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_7}{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_7}\oplus_L \quad \frac{\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9\oplus\mathbf{F}_{10}} \quad \oplus_{\mathbf{R}_2} \\ \frac{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_7}{-:(\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_7)} \quad \frac{\bullet\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_{10}} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \frac{-:\Delta_2,\Delta_6,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_{10}}{-:\Delta_2,\Delta_6,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_{10}} \quad \oplus_{\mathbf{R}_2} \\ \frac{-:\Delta_2,\Delta_6,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_{10}}{-:\Delta_2,\Delta_6,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_{10}} \quad \oplus_{\mathbf{R}_2} \\ \end{array}$$

• Case rule \bigoplus_{R_1}

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash\mathbf{F}_7\quad\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash\mathbf{F}_7}{\bullet}\oplus_L & \frac{\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9}{\bullet\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9\oplus\mathbf{F}_{10}} \\ & \frac{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_7}{-:(\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4),\Delta_6\vdash\mathbf{F}_9\oplus\mathbf{F}_{10}} & \overset{\oplus\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9}{\bullet}\oplus_{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_7} & \overset{\bullet}{\mathbf{ax/W}} & \frac{\bullet}{\mathbf{h}_8:\Delta_6,\mathbf{F}_7\vdash\mathbf{F}_9} \\ & \frac{-:\Delta_2,\Delta_6,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_9}{-:\Delta_2,\Delta_6,\mathbf{F}_3\oplus\mathbf{F}_4\vdash\mathbf{F}_9\oplus\mathbf{F}_{10}} & \oplus_{R_1} & \overset{\bullet}{\mathbf{h}}_{\mathbf{h}} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{1}\quad\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash\mathbf{1}}{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{1}}\oplus_{L} & \frac{\mathbf{h}_{6}:\Delta_{7}\vdash\mathbf{F}_{8}}{\bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{1}\vdash\mathbf{F}_{8}} & \mathbf{1}_{L} \\ \hline & -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}),\Delta_{7}\vdash\mathbf{F}_{8} & \mathbf{cut} \\ \hline & -:\Delta_{2},\Delta_{7},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{8} & \mathbf{ax/W} \\ \hline \\ \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{6}\quad\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{6}} \oplus_{\mathbf{h}_{7}:(\mathbf{1},\Delta_{9}),\mathbf{F}_{6}\vdash\mathbf{F}_{8}} & \mathbf{1}_{L} \\ \hline & -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{6} & \mathbf{ax/W} \\ \hline & -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{6} & \mathbf{ax/W} \\ \hline & -:\mathbf{1},\Delta_{2},\Delta_{9},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{8} & \mathbf{ax/W} \\ \hline \end{array} \begin{array}{c} \mathbf{h}_{Cut} \\ \hline \end{array}$$

• Case rule \otimes_R

\bullet Case rule W

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{6}\quad\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{6}} \oplus L \quad \frac{\mathbf{h}_{7}:\Delta_{10},\mathbf{F}_{6}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{7}:(\Delta_{10},!\mathbf{F}_{8}),\mathbf{F}_{6}\vdash\mathbf{F}_{9}} \quad W \\ \hline -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}),\Delta_{10},!\mathbf{F}_{8}\vdash\mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{6} \quad \mathbf{ax/W} \quad \mathbf{h}_{7}:\Delta_{10},\mathbf{F}_{6},!\mathbf{F}_{8}\vdash\mathbf{F}_{9} \\ \hline -:\Delta_{10},\Delta_{2},!\mathbf{F}_{8},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{9} \quad \mathbf{h}_{6}:\Delta_{7}\vdash\mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash!\mathbf{F}_{8} \quad \oplus L \quad \frac{\mathbf{h}_{6}:\Delta_{7}\vdash\mathbf{F}_{9}}{\bullet\mathbf{h}_{6}:\Delta_{7},!\mathbf{F}_{8}\vdash\mathbf{F}_{9}} \quad W \\ \hline -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}),\Delta_{7}\vdash\mathbf{F}_{9} \quad \mathbf{cut} \\ \hline -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}),\Delta_{7}\vdash\mathbf{F}_{9} \quad \mathbf{ax/W} \\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{9} \quad \mathbf{ax/W} \end{array}$$

ullet Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_6 \quad \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_6} \oplus L \quad \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_6,\mathbf{!F}_8,\mathbf{!F}_8\vdash \mathbf{F}_9}{\bullet \mathbf{h}_7:(\Delta_{10},\mathbf{!F}_8),\mathbf{F}_6\vdash \mathbf{F}_9} \quad C \\ \hline -:(\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4),\Delta_{10},\mathbf{!F}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_6 \quad \mathbf{ax/W} \quad \frac{\sim}{\mathbf{h}_7:\Delta_{10},\mathbf{F}_6,\mathbf{!F}_8,\mathbf{!F}_8\vdash \mathbf{F}_9} \quad \mathbf{ax/W} \\ \hline -:\Delta_{10},\Delta_2,\mathbf{!F}_8,\mathbf{!F}_8,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_9 \quad C \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{!F}_8 \quad \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{!F}_8 \quad \oplus L \quad \frac{\mathbf{h}_6:\Delta_7,\mathbf{!F}_8,\mathbf{!F}_8\vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{!F}_8\vdash \mathbf{F}_9} \quad C \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{!F}_8 \quad \oplus L \quad \frac{\mathbf{h}_6:\Delta_7,\mathbf{!F}_8,\mathbf{!F}_8\vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{!F}_8\vdash \mathbf{F}_9} \quad C \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{!F}_8 \quad \mathbf{ax/W} \quad \mathbf{h}_6:\Delta_7,\mathbf{!F}_8,\mathbf{IF}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{!F}_8 \quad \mathbf{ax/W} \quad \mathbf{h}_6:\Delta_7,\mathbf{!F}_8,\mathbf{!F}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{!F}_8 \quad \mathbf{ax/W} \quad \mathbf{h}_6:\Delta_7,\mathbf{!F}_8,\mathbf{!F}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{!F}_8 \quad \mathbf{ax/W} \quad \mathbf{h}_6:\Delta_7,\mathbf{!F}_8,\mathbf{IF}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{!F}_8 \quad \mathbf{ax/W} \quad \mathbf{h}_6:\Delta_7,\mathbf{IF}_8,\mathbf{IF}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{IF}_8 \quad \mathbf{ax/W} \quad \mathbf{h}_6:\Delta_7,\mathbf{IF}_8,\mathbf{IF}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{IF}_8 \quad \mathbf{ax/W} \quad \mathbf{h}_6:\Delta_7,\mathbf{IF}_8,\mathbf{IF}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{IF}_8 \quad \mathbf{ax/W} \quad \mathbf{h}_6:\Delta_7,\mathbf{IF}_8,\mathbf{IF}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{IF}_8 \quad \mathbf{ax/W} \quad \mathbf{h}_6:\Delta_7,\mathbf{IF}_8,\mathbf{IF}_8\vdash \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{IF}_8 \quad \mathbf{h}_1 \vdash \mathbf{h}_1 \vdash \mathbf{h}_1 \vdash \mathbf{h}_1 \vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{IF}_8 \quad \mathbf{h}_1 \vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{h}_1 \vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{h}_1 \vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{h}_1 \vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{h}_1 \vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{h}_2 \vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{h}_2 \vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{h}_2 \vdash \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1:\Delta_2,\mathbf{h$$

• Case rule !L

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_6\quad \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_6}{\underbrace{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_6}}_{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_6}\oplus L\quad \frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_6,\mathbf{F}_8\vdash \mathbf{F}_9}{\bullet\mathbf{h}_7:(\Delta_{10},!\mathbf{F}_8),\mathbf{F}_6\vdash \mathbf{F}_9}}_{\bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_6}\xrightarrow{ax/W}_{\bullet}\frac{\mathbf{h}_7:\Delta_{10},\mathbf{F}_6,\mathbf{F}_8\vdash \mathbf{F}_9}_{\bullet\mathbf{h}_7:\Delta_{10},\mathbf{F}_6,\mathbf{F}_8\vdash \mathbf{F}_9}}_{\bullet\mathbf{h}_7:\Delta_{10},\mathbf{F}_6,\mathbf{F}_8\vdash \mathbf{F}_9}\underbrace{-:\Delta_{10},\Delta_2,\mathbf{F}_8,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_9}_{-:\Delta_{10},\Delta_2,!\mathbf{F}_8,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_9}_{!L}$$

$$\frac{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{l}\mathbf{F}_8\quad \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{l}\mathbf{F}_8}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{l}\mathbf{F}_8}\oplus L\quad \frac{\mathbf{h}_6:\Delta_7,\mathbf{F}_8\vdash \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{l}\mathbf{F}_8\vdash \mathbf{F}_9}}_{\bullet \mathbf{h}_6:\Delta_7,\mathbf{l}\mathbf{F}_8\vdash \mathbf{F}_9}}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{l}\mathbf{F}_8}{\bullet \mathbf{h}_2}\oplus \mathbf{h}_2}}_{\bullet \mathbf{h}_6:\Delta_7,\mathbf{l}\mathbf{F}_8\vdash \mathbf{F}_9}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{l}\mathbf{F}_8}{\bullet \mathbf{h}_2}\oplus \mathbf{h}_2}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{l}\mathbf{h}_8}{\bullet \mathbf{h}_2}\oplus \mathbf{h}_2}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{l}\mathbf{h}_8}{\bullet \mathbf{h}_2}\oplus \mathbf{h}_2}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{l}\mathbf{h}_8}{\bullet \mathbf{h}_2}\oplus \mathbf{h}_2}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{h}_4\vdash \mathbf{h}_8}{\bullet \mathbf{h}_2}\oplus \mathbf{h}_2}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{h}_4\vdash \mathbf{h}_8}{\bullet \mathbf{h}_2}\oplus \mathbf{h}_2}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{h}_4\vdash \mathbf{h}_4\vdash \mathbf{h}_4\vdash \mathbf{h}_4}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{h}_4\vdash \mathbf{h}_4\vdash \mathbf{h}_4}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{h}_4\vdash \mathbf{h}_4}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{h}_4\vdash \mathbf{h}_4}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{h}_4\vdash \mathbf{h}_4}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{h}_4\vdash \mathbf{h}_4}}_{\bullet \mathbf{h}_2}\underbrace{\phantom{\frac{\mathbf{h}_1:\Delta_2,\mathbf{h}_4\vdash \mathbf{h}$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{6}\quad\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash\mathbf{F}_{6}}{\bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{6}}\oplus\mathbf{L}\quad\frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{9}\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\&\mathbf{F}_{9}),\mathbf{F}_{6}\vdash\mathbf{F}_{10}}\\ -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4})\vdash\Delta_{11},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{11},\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{6}\quad\mathbf{ax/W}\quad h_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{9}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{11},\Delta_{2},\mathbf{F}_{9},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{11},\Delta_{2},\mathbf{F}_{8}\&\mathbf{F}_{9},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{11},\Delta_{2},\mathbf{F}_{8}\&\mathbf{F}_{9},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{8}\&\mathbf{F}_{9}\quad\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash\mathbf{F}_{8}\&\mathbf{F}_{9}\\ \hline -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}),\Delta_{7}\vdash\mathbf{F}_{10}\\ \hline \bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{8}\&\mathbf{F}_{9}\quad\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash\mathbf{F}_{8}\&\mathbf{F}_{9}\\ \hline -:(\Delta_{2},\mathbf{F}_{3}\oplus\mathbf{F}_{4}),\Delta_{7}\vdash\mathbf{F}_{10}\\ \hline \bullet\mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash\mathbf{F}_{10}\\ \hline \bullet\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash\mathbf{F}_{8}\&\mathbf{F}_{9}\quad\mathbf{h}_{2}\\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{3}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{3}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{3}\oplus\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\ \hline -:\Delta_{2},\Delta_{7},\mathbf{F}_{4}\vdash\mathbf{F}_{10}\\$$

• Case rule $\&_{L1}$

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash \mathbf{F}_{6}\quad \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus \mathbf{F}_{4}\vdash \mathbf{F}_{6}} \oplus_{\mathbf{h}} \frac{\mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{8}\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{7}:(\Delta_{11},\mathbf{F}_{8}\&\mathbf{F}_{9}),\mathbf{F}_{6}\vdash \mathbf{F}_{10}} \\ \hline -:(\Delta_{2},\mathbf{F}_{3}\oplus \mathbf{F}_{4}),\Delta_{11},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ \hline -:(\Delta_{2},\mathbf{F}_{3}\oplus \mathbf{F}_{4}\vdash \mathbf{F}_{6}) & \mathbf{ax/W} & \mathbf{h}_{7}:\Delta_{11},\mathbf{F}_{6},\mathbf{F}_{8}\vdash \mathbf{F}_{10} \\ \hline -:\Delta_{11},\Delta_{2},\mathbf{F}_{8},\mathbf{F}_{3}\oplus \mathbf{F}_{4}\vdash \mathbf{F}_{10} & \mathbf{h}_{Cut} \\ \hline -:\Delta_{11},\Delta_{2},\mathbf{F}_{8}\&\mathbf{F}_{9},\mathbf{F}_{3}\oplus \mathbf{F}_{4}\vdash \mathbf{F}_{10} & \mathbf{h}_{L1} \\ \hline -:\Delta_{11},\Delta_{2},\mathbf{F}_{8}\&\mathbf{F}_{9} & \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash \mathbf{F}_{8}\&\mathbf{F}_{9} & \mathbf{h}_{L} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus \mathbf{F}_{4}\vdash \mathbf{F}_{8}\&\mathbf{F}_{9} & \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash \mathbf{F}_{8}\&\mathbf{F}_{9} & \mathbf{h}_{L} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\oplus \mathbf{F}_{4}\vdash \mathbf{F}_{8}\&\mathbf{F}_{9} & \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash \mathbf{F}_{8}\&\mathbf{F}_{9} & \mathbf{ax/W} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash \mathbf{F}_{8}\&\mathbf{F}_{9} & \mathbf{ax/W} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{3}\vdash \mathbf{F}_{10} & \mathbf{h}_{1}:\Delta_{2},\mathbf{F}_{4}\vdash \mathbf{F}_{8}\&\mathbf{F}_{9} & \mathbf{ax/W} & \mathbf{h}_{6}:\Delta_{7},\mathbf{F}_{8}\&\mathbf{F}_{9}\vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{7},\mathbf{h}_{9}\vdash \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{7},\mathbf{h}_{9}\vdash \mathbf{h}_{1} & \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{7}\vdash \mathbf{h}_{1} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{7},\mathbf{h}_{9}\vdash \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{7},\mathbf{h}_{9}\vdash \mathbf{h}_{1} & \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{1}\vdash \mathbf{h}_{1} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{7},\mathbf{h}_{9}\vdash \mathbf{h}_{1} & \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{1}\vdash \mathbf{h}_{1} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{1}\vdash \mathbf{h}_{1} & \mathbf{h}_{1}\vdash \mathbf{h}_{1} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{2},\mathbf{h}_{1}\vdash \mathbf{h}_{1} & \mathbf{h}_{2}\vdash \mathbf{h}_{1} \\ \hline \bullet \mathbf{h}_{2}:\Delta_{1},\mathbf{h}_{2}\vdash \mathbf{h}_{1} & \mathbf{h}_{2}\vdash \mathbf{h}_{2} \\ \hline \bullet \mathbf{h}_{2}:\Delta_{1}\vdash \mathbf{h}_{2}\vdash \mathbf{h}_{2} & \mathbf{h}_{2}\vdash \mathbf{h}_{2} \\ \hline \bullet \mathbf{h}_{2}:\Delta_{1}\vdash \mathbf{h}_{2}\vdash \mathbf{h}_{2} & \mathbf{h}_{2}\vdash \mathbf{h}_{2} \\ \hline \bullet \mathbf{h}_{2}:\Delta_{1}\vdash \mathbf{h}_{2}\vdash \mathbf{h}_{2} & \mathbf{h}_{2}\vdash \mathbf{h}_{2}\vdash \mathbf{h}_{2}\vdash \mathbf{h}_{2} \\ \hline \bullet \mathbf{h}_{2}:\Delta_{2}\vdash \mathbf{h}_{2}\vdash \mathbf{h}_{2}\vdash \mathbf{h}_{2}\vdash \mathbf{h}_{2} \\ \hline \bullet$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_6\quad \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_6}{\underbrace{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_6}}\oplus L\quad \frac{\mathbf{h}_7:\Delta_{11},\mathbf{F}_6,\mathbf{F}_8,\mathbf{F}_9\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7:\left(\Delta_{11},\mathbf{F}_8\otimes \mathbf{F}_9\right),\mathbf{F}_6\vdash \mathbf{F}_{10}}\\ -:\left(\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\right),\Delta_{11},\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}\\ \hline\\ \underbrace{\frac{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_8,\mathbf{F}_9,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_{10}}}^{\bullet \mathbf{h}_7:\Delta_{11},\mathbf{F}_6,\mathbf{F}_8,\mathbf{F}_9\vdash \mathbf{F}_{10}}} \underbrace{\mathbf{ax}/\mathbf{W}}_{\mathbf{h}\mathbf{Cut}} \\ -:\Delta_{11},\Delta_2,\mathbf{F}_8\otimes \mathbf{F}_9,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_{10}}^{\bullet \mathbf{h}_7:\Delta_{11},\mathbf{F}_6,\mathbf{F}_8,\mathbf{F}_9\vdash \mathbf{F}_{10}}} \otimes L\\ \underbrace{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_8\otimes \mathbf{F}_9,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_8\otimes \mathbf{F}_9} \oplus L\quad \underbrace{\frac{\mathbf{h}_6:\Delta_7,\mathbf{F}_8,\mathbf{F}_9\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}} \otimes L} \\ \underbrace{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_8\otimes \mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_9\otimes \mathbf{F}_9} \oplus L\quad \underbrace{\frac{\mathbf{h}_6:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}} \otimes L} \\ \underbrace{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_8\otimes \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}}} \otimes L} \underbrace{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_8\otimes \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_0:\mathbf{h}_0:\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_8\otimes \mathbf{F}_9} \otimes \mathbf{h}_0:\mathbf{h}_0:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_0:\mathbf{h}_0:\Delta_7,\mathbf{F}_8\otimes \mathbf{F}_9\vdash \mathbf{F}_{10}}} \oplus L$$

• Case rule \oplus_L

$$\frac{\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_6\quad \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_6}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_6}}{\bullet \mathbf{h}_2}}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_4\vdash \mathbf{F}_6}} \oplus_L \frac{\mathbf{h}_7:\Delta_{11},\mathbf{F}_6,\mathbf{F}_8\vdash \mathbf{F}_{10}\quad \mathbf{h}_7:\Delta_{11},\mathbf{F}_6,\mathbf{F}_9\vdash \mathbf{F}_{10}}}{\bullet \mathbf{h}_7:(\Delta_{11},\mathbf{F}_8\oplus \mathbf{F}_9),\mathbf{F}_6\vdash \mathbf{F}_{10}}} Cut} \oplus_L \\ \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_6}}{\bullet \mathbf{h}_7:\Delta_{11},\mathbf{F}_6,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}}} \frac{\mathbf{ax}/\mathbf{W}}{\mathbf{h}_{Cut}} \frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_6}{\bullet \mathbf{h}_7:\Delta_{11},\mathbf{F}_6,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}}} \oplus_L \\ \frac{-:\Delta_{11},\Delta_2,\mathbf{F}_3,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}} \oplus_L \\ \frac{-:\Delta_{11},\Delta_2,\mathbf{F}_3,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_8\oplus \mathbf{F}_9} \oplus_L \frac{\mathbf{h}_6:\Delta_7,\mathbf{F}_8\vdash \mathbf{F}_{10}\quad \mathbf{h}_6:\Delta_7,\mathbf{F}_9\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}} \oplus_L \\ \frac{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_9\vdash \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_8\oplus \mathbf{F}_9}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}} \oplus_L \frac{\mathbf{h}_6:\Delta_7,\mathbf{F}_8\vdash \mathbf{F}_{10}\quad \mathbf{h}_6:\Delta_7,\mathbf{F}_9\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}} \oplus_L \\ \frac{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus \mathbf{F}_9\vdash \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \mathbf{F}_8\oplus \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}} \oplus_L \frac{\mathbf{h}_6:\Delta_7,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}} \oplus_L \\ \frac{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash \mathbf{F}_8\oplus \mathbf{F}_9}{\bullet \mathbf{h}_6:\Delta_7,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}} \oplus_L \frac{\mathbf{h}_6:\Delta_7,\mathbf{F}_8\oplus \mathbf{F}_9\vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{Cut}} \oplus_L \\ \frac{\bullet \mathbf{h}_1:\Delta_2,\mathbf{h}_1\vdash \mathbf{h}_1:\Delta_2,\mathbf{h}_1$$

• Case rule \multimap_L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2, F_3 \vdash F_6 \quad \mathbf{h}_1 : \Delta_2, F_4 \vdash F_6}{\bullet \mathbf{h}_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \quad \oplus L \quad \frac{\mathbf{h}_7 : \Delta_{12}, F_6 \vdash F_9 \quad \mathbf{h}_7 : \Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}}{\bullet \mathbf{h}_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}} \\ \hline - : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\ \hline \bullet \mathbf{h}_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6 \quad \mathbf{ax}/W \quad \frac{\mathbf{h}_7 : \Delta_{12}, F_6 \vdash F_9}{\bullet \mathbf{h}_7 : \Delta_12, F_6 \vdash F_9} \quad \mathbf{ax}/W \\ \hline - : \Delta_{12}, \Delta_2, F_3 \oplus F_4 \vdash F_9 \quad \mathbf{ax}/W \\ \hline - : \Delta_{12}, \Delta_2, F_3 \oplus F_4 \vdash F_9 \quad \oplus L \quad \frac{\mathbf{h}_7 : \Delta_8 \vdash F_9 \multimap F_1 \cap F_{11}}{\bullet \mathbf{h}_7 : \Delta_8, F_9 \multimap F_{10}, F_3 \oplus F_4 \vdash F_{11}} \quad -\circ_L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, F_3 \vdash F_6 \quad \mathbf{h}_1 : \Delta_2, F_4 \vdash F_6 \quad \oplus L \quad \frac{\mathbf{h}_7 : \Delta_8 \vdash F_9 \multimap F_1 \cap F_1 \cap F_{11}}{\bullet \mathbf{h}_7 : \Delta_8, \Delta_{12}, F_9 \multimap F_{10}, F_6 \vdash F_{11}} \quad -\circ_L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6 \quad \oplus L \quad \frac{\mathbf{h}_7 : \Delta_8 \vdash F_9 \multimap F_1 \cap F_{10} \vdash F_{11}}{\bullet \mathbf{h}_7 : \Delta_8, \Delta_{12}, F_9 \multimap F_{10}, F_6 \vdash F_{11}} \quad -\circ_L \\ \hline - : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \quad -\circ_L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6 \quad \mathbf{ax}/W \quad \mathbf{h}_7 : \Delta_{12}, F_{10}, F_6 \vdash F_{11}} \quad -\circ_L \\ \hline - : \Delta_{12}, \Delta_2, F_3 \oplus F_4 \vdash F_9 \quad -\circ F_{10} \vdash F_{11}} \quad -\circ_L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, F_3 \vdash F_9 \multimap F_{10} \quad \mathbf{h}_1 : \Delta_2, F_3 \oplus F_4 \vdash F_9 \multimap F_{10} \quad \oplus L \quad \frac{\mathbf{h}_6 : \Delta_7 \vdash F_9 \quad \mathbf{h}_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet \mathbf{h}_6 : (\Delta_7, \Delta_8), F_9 \multimap F_{10} \vdash F_{11}} \quad -\circ_L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, F_3 \oplus F_4 \vdash F_9 \multimap F_{10} \quad \oplus L \quad \frac{\mathbf{h}_6 : \Delta_7 \vdash F_9 \quad \mathbf{h}_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet \mathbf{h}_6 : (\Delta_7, \Delta_8), F_9 \multimap F_{10} \vdash F_{11}} \quad -\circ_L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, F_3 \oplus F_9 \multimap F_{10} \quad \mathbf{ax}/W \quad \bullet \mathbf{h}_6 : \Delta_7, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \quad -\circ L \\ \hline - : \Delta_2, \Delta_7, \Delta_8, F_3 \vdash F_{11} \quad - : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{11}} \quad \oplus L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, F_3 \vdash F_9 \multimap F_{10} \quad \bullet \mathbf{h}_6 : \Delta_7, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \quad - : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{11}} \quad \oplus L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \Delta_7, \Delta_8, F_3 \vdash F_{11} \quad - : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{11}} \quad \oplus L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, A_7, \Delta_8, F_3 \vdash F_{11} \quad - : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{11}} \quad \oplus L \\ \hline \bullet \mathbf{h}_1 : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{11} \quad - : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{1$$

 \bullet Case rule I

$$\frac{\mathbf{h}_1:\Delta_2,\mathbf{F}_3\vdash p(\mathbf{n}_7)\quad \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash p(\mathbf{n}_7)}{\underbrace{\begin{array}{c} \bullet\mathbf{h}_1:\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4\vdash p(\mathbf{n}_7)\\ \\ -:(\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4),*\vdash p(\mathbf{n}_7) \end{array}}_{\bullet}} \underbrace{\begin{array}{c} \bullet\mathbf{h}_6:*,p(\mathbf{n}_7)\vdash p(\mathbf{n}_7)\\ \\ -:\Delta_2,\mathbf{F}_3\oplus\mathbf{F}_4),*\vdash p(\mathbf{n}_7) \end{array}}_{\bullet} \mathbf{Cut}$$

5.17 Status of \multimap_L : OK

• Case rule !R

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}\vdash\mathbf{F}_{4}\quad\mathbf{h}_{1}:\Delta_{3},\mathbf{F}_{5}\vdash8}{\bullet\mathbf{h}_{1}:\Delta_{2},\Delta_{3},\mathbf{F}_{4}\multimap\mathbf{F}_{5}\vdash8} \multimap_{L} \quad \frac{\mathbf{h}_{9}:!\Upsilon7,8\vdash\mathbf{F}_{10}}{\bullet\mathbf{h}_{9}:!\Upsilon7,8\vdash\mathbf{F}_{10}} \quad !R\\ \hline -:(\Delta_{2},\Delta_{3},\mathbf{F}_{4}\multimap\mathbf{F}_{5}),!\Upsilon7\vdash!\mathbf{F}_{10} \\ & \stackrel{\longrightarrow}{\longrightarrow} \quad \mathbf{Cut} \\ \hline \frac{-:\Delta_{2}\vdash\mathbf{F}_{4}}{\bullet\mathbf{h}_{2}:\mathbf{h}_{2}} \quad \frac{\mathbf{h}_{1}:\Delta_{3},\mathbf{F}_{5}\vdash8} \quad \mathbf{ax/W} \quad \frac{\bullet\mathbf{h}_{9}:8,!\Upsilon7\vdash!\mathbf{F}_{10}}{\bullet\mathbf{h}_{9}:8,!\Upsilon7\vdash!\mathbf{F}_{10}} \quad \mathbf{ax/W} \\ \hline -:!\Upsilon7,\Delta_{2},\Delta_{3},\mathbf{F}_{4}\multimap\mathbf{F}_{5}\vdash!\mathbf{F}_{10} \quad -\circ_{L} \end{array}$$

- Case rule $\mathbf{1}_R$
- Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_8} \quad \multimap_L \quad \frac{\bullet \mathbf{h}_9: \Delta_7, \mathbf{F}_8 \vdash \top}{\bullet \mathbf{h}_9: \Delta_7, \mathbf{F}_8 \vdash \top} \quad \top \\ & \qquad \qquad \cdot : (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_7 \vdash \top \\ & \qquad \qquad \cdots \\ & \qquad \qquad - : \Delta_2, \Delta_3, \Delta_7, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \top \end{array} \quad \top$$

• Case rule $\&_R$

$$\frac{\mathbf{h}_{1}:\Delta_{2} \vdash \mathbf{F}_{4} \quad \mathbf{h}_{1}:\Delta_{3}, \mathbf{F}_{5} \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3}, \mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{8}} \multimap_{L} \quad \frac{\mathbf{h}_{9}:\Delta_{7}, \mathbf{F}_{8} \vdash \mathbf{F}_{10} \quad \mathbf{h}_{9}:\Delta_{7}, \mathbf{F}_{8} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_{9}:\Delta_{7}, \mathbf{F}_{8} \vdash \mathbf{F}_{10} \& \mathbf{F}_{11}} \quad \mathbf{Cut}} \quad & \&_{R} \\ \hline -:(\Delta_{2},\Delta_{3}, \mathbf{F}_{4} \multimap \mathbf{F}_{5}), \Delta_{7} \vdash \mathbf{F}_{10} \& \mathbf{F}_{11}} \quad \mathbf{Cut} \\ \hline -:\Delta_{2} \vdash \mathbf{F}_{4} \quad \mathbf{ax/W} \quad & \bullet \mathbf{h}_{9}:\Delta_{7}, \mathbf{F}_{8} \vdash \mathbf{F}_{10} \& \mathbf{F}_{11}} \\ \hline -:\Delta_{3},\Delta_{7}, \mathbf{F}_{5} \vdash \mathbf{F}_{10} \& \mathbf{F}_{11}} \quad & -\diamond_{L} \\ \hline -:\Delta_{2},\Delta_{3},\Delta_{7}, \mathbf{F}_{4} \multimap \mathbf{F}_{5} \vdash \mathbf{F}_{10} \& \mathbf{F}_{11}} \quad & -\diamond_{L} \\ \hline \end{array}$$

• Case rule \multimap_R

$$\begin{array}{c} \frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1 : \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1 : \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_8} \quad \multimap_L \quad \frac{\mathbf{h}_9 : \Delta_7, \mathbf{F}_8, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_9 : \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_{10} \multimap \mathbf{F}_{11}} \quad \multimap_R \\ \hline \\ - : (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_7 \vdash \mathbf{F}_{10} \multimap \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_8} \quad \text{ax/W} \quad \frac{\bullet}{\mathbf{h}_9 : \Delta_7, \mathbf{F}_{10}, \mathbf{F}_8 \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_9 : \Delta_7, \mathbf{F}_{10}, \mathbf{F}_8 \vdash \mathbf{F}_{11}} \quad \bullet_R \\ \hline \\ - : \Delta_2, \Delta_3, \Delta_7, \mathbf{F}_{10}, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ - : \Delta_2, \Delta_3, \Delta_7, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \multimap \mathbf{F}_{11} \quad \multimap_R \end{array} \quad \text{ax/W}$$

• Case rule \oplus_{R_2}

• Case rule \bigoplus_{R_1}

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_8} \quad \multimap_L \quad \frac{\mathbf{h}_9: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_9: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_{10} \oplus \mathbf{F}_{11}} \quad \overset{\bigoplus_{R_1}}{\circ} \\ -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_7 \vdash \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ & \stackrel{\bullet}{\longrightarrow} \\ \frac{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_8}{\bullet \mathbf{x}} \quad \frac{\mathsf{ax/W}}{\mathsf{h}_9: \Delta_7, \mathbf{F}_8 \vdash \mathbf{F}_{10}} \quad \underset{\mathsf{hCut}}{\bullet} \\ & \frac{-: \Delta_2, \Delta_3, \Delta_7, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10}}{-: \Delta_2, \Delta_3, \Delta_7, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \oplus \mathbf{F}_{11}} \quad \oplus_{R_1} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{1}}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{1}} \quad \multimap_L \quad \frac{\mathbf{h}_7: \Delta_8 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_8, \mathbf{1} \vdash \mathbf{F}_9} \quad \mathbf{1}_L \\ \quad -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_8 \vdash \mathbf{F}_9 \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_9} \quad \mathbf{ax/W} \\ \\ \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7} \quad \multimap_L \quad \frac{\mathbf{h}_8: \Delta_{10}, \mathbf{F}_7 \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8: (\mathbf{1}, \Delta_{10}), \mathbf{F}_7 \vdash \mathbf{F}_9} \quad \mathbf{1}_L \\ \hline \quad -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7 \quad \bullet \mathbf{xx/W} \quad \overset{\longleftrightarrow}{\mathbf{h}_8: (\mathbf{1}, \Delta_{10}), \mathbf{F}_7 \vdash \mathbf{F}_9} \quad \mathbf{ax/W} \\ \hline \quad \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7 \quad \bullet \mathbf{xx/W} \quad \overset{\longleftrightarrow}{\mathbf{h}_8: (\mathbf{1}, \Delta_{10}, \mathbf{F}_7 \vdash \mathbf{F}_9)} \quad \mathbf{ax/W} \\ \hline \quad -: \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_9 \quad \mathbf{hCut} \end{array}$$

• Case rule \otimes_R

$\bullet\,$ Case rule W

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7} \quad \multimap_L \quad \frac{\mathbf{h}_8: \Delta_{11}, \mathbf{F}_7 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: (\Delta_{11}, !\mathbf{F}_9), \mathbf{F}_7 \vdash \mathbf{F}_{10}} \quad W \\ \hline -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7) \quad \frac{\bullet}{\mathbf{h}_8: \Delta_{11}, \mathbf{F}_7, !\mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7 \quad \mathbf{ax/W} \quad \overline{\mathbf{h}_8: \Delta_{11}, \mathbf{F}_7, !\mathbf{F}_9 \vdash \mathbf{F}_{10}} \quad \mathbf{ax/W} \\ \hline -: \Delta_{11}, \Delta_2, \Delta_3, !\mathbf{F}_9, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \quad \mathbf{hCut} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash !\mathbf{F}_9 \quad \multimap_L \quad \overline{\mathbf{h}_7: \Delta_8 \vdash \mathbf{F}_{10}} \quad W \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash !\mathbf{F}_9 \quad \multimap_L \quad \overline{\mathbf{h}_7: \Delta_8, !\mathbf{F}_9 \vdash \mathbf{F}_{10}} \quad W \\ \hline -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_8 \vdash \mathbf{F}_{10} \quad \cdots \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \quad \mathbf{ax/W} \\ \hline \end{array}$$

\bullet Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7} \quad \multimap_L \quad \frac{\mathbf{h}_8: \Delta_{11}, \mathbf{F}_7, !\mathbf{F}_9, !\mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8: (\Delta_{11}, !\mathbf{F}_9), \mathbf{F}_7 \vdash \mathbf{F}_{10}} \quad C \\ \hline \\ -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_{11}, !\mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7} \quad \overset{\mathsf{ax/W}}{\longrightarrow} \\ \hline \\ \frac{-: \Delta_{11}, \Delta_2, \Delta_3, !\mathbf{F}_9, !\mathbf{F}_9, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10}}{-: \Delta_{11}, \Delta_2, \Delta_3, !\mathbf{F}_9, !\mathbf{F}_9, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10}} \quad C \\ \hline \end{array} \quad \overset{\mathsf{ax/W}}{\longleftarrow} \quad \mathsf{hCut}$$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash !\mathbf{F}_9}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash !\mathbf{F}_9} \\ -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_8 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_8 \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash !\mathbf{F}_9} \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash !\mathbf{F}_9} \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash !\mathbf{F}_9} \\ -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \\ \hline \end{array} \quad \begin{array}{c} \mathbf{ax/W} \\ \mathbf{mCut} \end{array}$$

• Case rule !L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1:\Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1:\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7} \quad \multimap_L \quad \frac{\mathbf{h}_8:\Delta_{11}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8:(\Delta_{11}, !\mathbf{F}_9), \mathbf{F}_7 \vdash \mathbf{F}_{10}} \quad !L \\ \hline \quad -:(\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5),\Delta_{11}, !\mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \quad \bullet \mathbf{h}_8:\Delta_{11}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \quad -:\Delta_{11},\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7 \quad \mathbf{ax/W} \quad \mathbf{h}_8:\Delta_{11}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{10} \\ \hline \quad -:\Delta_{11},\Delta_2,\Delta_3, \mathbf{F}_9, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \\ \hline \quad -:\Delta_{11},\Delta_2,\Delta_3, \mathbf{F}_9, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \\ \hline \quad \bullet \mathbf{h}_1:\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \\ \hline \quad \bullet \mathbf{h}_1:\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \\ \hline \quad \bullet \mathbf{h}_1:\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_9 \\ \hline \quad \bullet \mathbf{h}_1:\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_9 \\ \hline \quad \bullet \mathbf{h}_1:\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \\ \hline \quad -:(\Delta_2,\Delta_3,\mathbf{F}_4 \multimap \mathbf{F}_5),\Delta_8 \vdash \mathbf{F}_{10} \\ \hline \quad -:\Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{ax/W} \\ \hline \quad -:\Delta_3,\Delta_8,\mathbf{F}_5 \vdash \mathbf{F}_{10} \\ \hline \quad -:\Delta_3,\Delta_8,\mathbf{F}_5 \vdash \mathbf{F}_{10} \\ \hline \quad -:\Delta_2,\Delta_3,\Delta_8,\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{10} \\ \hline \end{array} \quad \mathbf{ax/W} \quad \mathbf{hCut} \\ \hline \end{array}$$

• Case rule $\&_{L2}$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7} \quad \multimap_L \quad \frac{\mathbf{h}_8: \Delta_{12}, \mathbf{F}_7, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_8: (\Delta_{12}, \mathbf{F}_9 \& \mathbf{F}_{10}), \mathbf{F}_7 \vdash \mathbf{F}_{11}} \quad \&_{L2} \\ \hline \quad -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_{12}, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7 \quad \text{ax/W} \quad \frac{\sim}{\mathbf{h}_8: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_7 \vdash \mathbf{F}_{11}} \quad \text{ax/W} \\ \hline \quad -: \Delta_{12}, \Delta_2, \Delta_3, \mathbf{F}_{10}, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad \&_{L2} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \quad \multimap_L \quad \frac{\mathbf{h}_7: \Delta_8, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \quad \&_{L2} \\ \hline \quad -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_8 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \\ \hline \quad -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_8 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \\ \hline \quad -: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{ax/W} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad &_{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \quad -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline \quad -: \Delta$$

• Case rule $\&_{L1}$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1:\Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1:\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7} \quad \multimap_L \quad \frac{\mathbf{h}_8:\Delta_{12}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_8:(\Delta_{12}, \mathbf{F}_9 \& \mathbf{F}_{10}), \mathbf{F}_7 \vdash \mathbf{F}_{11}} \quad \&_{L1} \\ \hline \\ -:(\Delta_2,\Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5),\Delta_{12}, \mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8:\Delta_{12}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8:\Delta_{12}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8:\Delta_{12}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8:\Delta_{12}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8:\Delta_{12}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_9:\Delta_1,\Delta_2,\Delta_3, \mathbf{F}_9 \lor \mathbf{F}_{10} \\ \hline \\ -:\Delta_{12},\Delta_2,\Delta_3, \mathbf{F}_9 \lor \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\Delta_3,\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2,\Delta_3,\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_9 \& \mathbf{F}_{10} \\ \hline \\ -:(\Delta_2,\Delta_3,\mathbf{F}_4 \multimap \mathbf{F}_5),\Delta_8 \vdash \mathbf{F}_{11} \\ \hline \\ -:(\Delta_2,\Delta_3,\mathbf{F}_4 \multimap \mathbf{F}_5),\Delta_8 \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ -:\Delta_2 \vdash \mathbf{F}_4 \\ \hline \\ \bullet \mathbf{h}_7:\Delta_3,\Delta_8,\mathbf{F}_9 \lor \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ -:\Delta_2,\Delta_3,\Delta_8,\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline \\ -:\Delta_2,\Delta_3,\Delta_8,\mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{h}_7 \vdash \mathbf{h}_7 \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{F}_9 \& \mathbf{h}_7 \vdash \mathbf{h}_7 \\ \hline \\ \bullet \mathbf{h}_7:\Delta_8,\mathbf{h}_7 \vdash \mathbf{h}_7 \vdash \mathbf{h}_7 \\ \hline \\ \bullet \mathbf{h}_7 \vdash \mathbf{h}_7 \vdash \mathbf{h}_7 \vdash$$

• Case rule \otimes_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7} \quad \multimap_L \quad \frac{\mathbf{h}_8: \Delta_{12}, \mathbf{F}_7, \mathbf{F}_9, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_8: (\Delta_{12}, \mathbf{F}_9 \otimes \mathbf{F}_{10}), \mathbf{F}_7 \vdash \mathbf{F}_{11}} \quad \bigotimes_L \\ \hline -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_{12}, \mathbf{F}_9 \otimes \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \hline \bullet \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_7 \quad \mathbf{ax/W} \quad \frac{}{\mathbf{h}_8: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_7, \mathbf{F}_9 \vdash \mathbf{F}_{11}} \quad \mathbf{ax/W} \\ \hline -: \Delta_{12}, \Delta_2, \Delta_3, \mathbf{F}_{10}, \mathbf{F}_9, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad \bigotimes_L \\ \hline -: \Delta_{12}, \Delta_2, \Delta_3, \mathbf{F}_{10}, \mathbf{F}_9, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \quad \bigotimes_L \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash \mathbf{F}_9 \otimes \mathbf{F}_{10} \quad -\circ_L \quad \frac{\mathbf{h}_7: \Delta_8, \mathbf{F}_9, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_7: \Delta_8, \mathbf{F}_9 \otimes \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \quad \bigotimes_L \\ \hline -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_8 \vdash \mathbf{F}_{11} \\ \hline -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_8 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{ax/W} \quad \frac{\bullet}{\mathbf{h}_7: \Delta_8, \mathbf{F}_9 \otimes \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_7: \Delta_8, \mathbf{F}_9 \otimes \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \quad \underbrace{\otimes_L \\ -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), \Delta_8 \vdash \mathbf{F}_{11}}_{-: \Delta_2 \vdash \mathbf{F}_4} \quad \mathbf{ax/W} \quad \underbrace{\bullet \mathbf{h}_7: \Delta_8, \mathbf{F}_9 \otimes \mathbf{F}_{10} \vdash \mathbf{F}_{11}}_{-: \Delta_2} \quad \mathbf{ax/W} \\ -: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{ax/W} \quad -: \Delta_3, \Delta_8, \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3, \Delta_8, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash \mathbf{F}_{11} \\ \hline -: \Delta_2, \Delta_3,$$

• Case rule \oplus_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{2}\vdash \mathbf{F}_{4}\quad \mathbf{h}_{1}:\Delta_{3},\mathbf{F}_{5}\vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3},\mathbf{F}_{4}\multimap \mathbf{F}_{5}\vdash \mathbf{F}_{7}} \multimap_{L} & \frac{\mathbf{h}_{8}:\Delta_{12},\mathbf{F}_{7},\mathbf{F}_{9}\vdash \mathbf{F}_{11}\quad \mathbf{h}_{8}:\Delta_{12},\mathbf{F}_{7},\mathbf{F}_{10}\vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_{8}:(\Delta_{12},\mathbf{F}_{9}\oplus \mathbf{F}_{10}),\mathbf{F}_{7}\vdash \mathbf{F}_{11}} & \mathbf{Cut} \\ & -:(\Delta_{2},\Delta_{3},\mathbf{F}_{4}\multimap \mathbf{F}_{5}),\Delta_{12},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11} & \mathbf{ax}/\mathbf{W} \\ & \frac{\mathbf{ax}/\mathbf{W}}{\bullet \mathbf{h}_{1}:\Delta_{3},\mathbf{F}_{5}\vdash \mathbf{F}_{7}} & \frac{\mathbf{ax}/\mathbf{W}}{\bullet \mathbf{h}_{8}:\Delta_{12},\mathbf{F}_{7},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11}} & \mathbf{ax}/\mathbf{W} \\ & -:\Delta_{12},\Delta_{2},\Delta_{3},\mathbf{F}_{4}\multimap \mathbf{F}_{5},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11} & \mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{10}\vdash \mathbf{F}_{11} \\ & -:\Delta_{12},\Delta_{2},\Delta_{3},\mathbf{F}_{4}\multimap \mathbf{F}_{5},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11} & \mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11} \\ & \bullet \mathbf{h}_{1}:\Delta_{2},\Delta_{3},\mathbf{F}_{4}\multimap \mathbf{F}_{5}\vdash \mathbf{F}_{9}\oplus \mathbf{F}_{10} & -\circ_{L} & \frac{\mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11}} & \mathbf{Cut} \\ & -:(\Delta_{2},\Delta_{3},\mathbf{F}_{4}\multimap \mathbf{F}_{5}),\Delta_{8}\vdash \mathbf{F}_{9}\oplus \mathbf{F}_{10} & \mathbf{ax}/\mathbf{W} & \bullet_{\mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11} & \mathbf{ax}/\mathbf{W} \\ & \bullet_{\mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11} & -\circ_{L} & \bullet_{\mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11} & \mathbf{ax}/\mathbf{W} \\ & -:\Delta_{2},\Delta_{3},\Delta_{8},\mathbf{F}_{4}\multimap \mathbf{F}_{5}\vdash \mathbf{F}_{9}\oplus \mathbf{F}_{10} & -\circ_{L} & \bullet_{\mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash \mathbf{F}_{11} & \bullet_{\mathbf{h}_{7}:\Delta_{8},\mathbf{F}_{9}\oplus \mathbf{F}_{10}\vdash$$

• Case rule \multimap_L

ullet Case rule I

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4 \quad \mathbf{h}_1: \Delta_3, \mathbf{F}_5 \vdash p(\mathbf{n}_8)}{\overset{\bullet}{\bullet} \mathbf{h}_1: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash p(\mathbf{n}_8)} \\ -: (\Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5), * \vdash p(\mathbf{n}_8) \\ &\overset{\bullet}{\longrightarrow} \\ \hline -: \Delta_2, \Delta_3, \mathbf{F}_4 \multimap \mathbf{F}_5 \vdash p(\mathbf{n}_8)} \quad \text{ax/W}$$

5.18 Status of I: OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\begin{aligned} & \frac{\bullet \mathbf{h}_1 : p(\mathbf{n}_4) \vdash p(\mathbf{n}_4)}{-: p(\mathbf{n}_4), \Delta_3 \vdash \top} & \overset{\top}{\bullet} \mathbf{h}_5 : \Delta_3, p(\mathbf{n}_4) \vdash \top} & \overset{\top}{\mathsf{Cut}} \\ & & \overset{\leadsto}{-: \Delta_3, p(\mathbf{n}_4) \vdash \top} & \top \end{aligned}$$

• Case rule $\&_R$

$$\frac{\bullet \mathbf{h}_1:p(\mathbf{n}_4) \vdash p(\mathbf{n}_4)}{\bullet \mathbf{h}_1:p(\mathbf{n}_4) \vdash p(\mathbf{n}_4)} \ I \ \frac{\mathbf{h}_5:\Delta_3,p(\mathbf{n}_4) \vdash \mathbf{F}_6 \quad \mathbf{h}_5:\Delta_3,p(\mathbf{n}_4) \vdash \mathbf{F}_7}{\bullet \mathbf{h}_5:\Delta_3,p(\mathbf{n}_4) \vdash \mathbf{F}_6\&\mathbf{F}_7} \ \mathbf{Cut} \\ -:p(\mathbf{n}_4),\Delta_3 \vdash \mathbf{F}_6\&\mathbf{F}_7 \\ & \stackrel{\smile}{-}:\Delta_3,p(\mathbf{n}_4) \vdash \mathbf{F}_6\&\mathbf{F}_7 \ \mathbf{ax/W} \\ \end{array}$$

• Case rule \multimap_R

$$\frac{\underbrace{\frac{\mathbf{h}_5:\Delta_3,\mathbf{F}_6,p(\mathbf{n}_4)\vdash\mathbf{F}_7}{\bullet\mathbf{h}_5:\Delta_3,p(\mathbf{n}_4)\vdash\mathbf{F}_6\multimap\mathbf{F}_7}}_{-:p(\mathbf{n}_4),\Delta_3\vdash\mathbf{F}_6\multimap\mathbf{F}_7} \overset{-\circ_R}{\leftarrow} \text{Cut}}_{-:\Delta_3,p(\mathbf{n}_4)\vdash\mathbf{F}_6\multimap\mathbf{F}_7} \text{ax/W}}$$

• Case rule \bigoplus_{R_2}

$$\frac{ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : p(\mathbf{n}_4) \vdash p(\mathbf{n}_4) \\ - : p(\mathbf{n}_4) \vdash p(\mathbf{n}_4) \end{array} I \quad \begin{array}{c} \mathbf{h}_5 : \Delta_3, p(\mathbf{n}_4) \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_5 : \Delta_3, p(\mathbf{n}_4) \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline - : p(\mathbf{n}_4), \Delta_3 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline - : \Delta_3, p(\mathbf{n}_4) \vdash \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \quad \begin{array}{c} \oplus_{R_2} \\ \mathsf{Cut} \end{array} }$$

• Case rule \bigoplus_{R_1}

$$\frac{\bullet \mathbf{h}_1:p(\mathbf{n}_4) \vdash p(\mathbf{n}_4)}{\bullet \mathbf{h}_1:p(\mathbf{n}_4) \vdash p(\mathbf{n}_4)} \ I \quad \frac{\mathbf{h}_5:\Delta_3,p(\mathbf{n}_4) \vdash \mathbf{F}_6 \oplus \mathbf{F}_7}{\bullet \mathbf{h}_5:\Delta_3,p(\mathbf{n}_4) \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \quad \frac{\oplus_{R_1}}{\cot} \quad \mathbf{Cut}$$

$$\frac{-:p(\mathbf{n}_4),\Delta_3 \vdash \mathbf{F}_6 \oplus \mathbf{F}_7}{-:\Delta_3,p(\mathbf{n}_4) \vdash \mathbf{F}_6 \oplus \mathbf{F}_7} \quad \mathbf{ax/W}$$

• Case rule $\mathbf{1}_L$

$$\frac{\underbrace{\bullet \mathbf{h}_1:p(\mathbf{n}_3)\vdash p(\mathbf{n}_3)}_{} \ I \ \frac{\mathbf{h}_4:\Delta_6,p(\mathbf{n}_3)\vdash \mathbf{F}_5}{\bullet \mathbf{h}_4:(\mathbf{1},\Delta_6),p(\mathbf{n}_3)\vdash \mathbf{F}_5}_{} \ \frac{\mathbf{1}_L}{\mathsf{Cut}} \\ \frac{-:p(\mathbf{n}_3),\mathbf{1},\Delta_6\vdash \mathbf{F}_5}{-:\mathbf{1},\Delta_6,p(\mathbf{n}_3)\vdash \mathbf{F}_5} \ \mathsf{ax/W}$$

• Case rule \otimes_R

$$\begin{array}{c} \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \\ } I & \underbrace{ \begin{array}{c} \mathbf{h}_4 : \Delta_8, p(\mathbf{n}_3) \vdash \mathbf{F}_6 \quad \mathbf{h}_4 : \Delta_5 \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_4 : (\Delta_5, \Delta_8), p(\mathbf{n}_3) \vdash \mathbf{F}_6 \otimes \mathbf{F}_7 \\ \hline \\ - : p(\mathbf{n}_3), \Delta_5, \Delta_8 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7 \\ \hline \\ \hline \\ - : \Delta_5, \Delta_8, p(\mathbf{n}_3) \vdash \mathbf{F}_6 \otimes \mathbf{F}_7 \\ \hline \end{array} } \begin{array}{c} \otimes_R \\ \mathrm{Cut} \\ \\ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \\ \hline \\ \bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \\ \hline \end{array} } I & \underbrace{ \begin{array}{c} \mathbf{h}_4 : \Delta_5 \vdash \mathbf{F}_6 \quad \mathbf{h}_4 : \Delta_8, p(\mathbf{n}_3) \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_4 : (\Delta_5, \Delta_8), p(\mathbf{n}_3) \vdash \mathbf{F}_6 \otimes \mathbf{F}_7 \\ \hline \\ - : p(\mathbf{n}_3), \Delta_5, \Delta_8 \vdash \mathbf{F}_6 \otimes \mathbf{F}_7 \\ \hline \\ \hline \\ - : \Delta_5, \Delta_8, p(\mathbf{n}_3) \vdash \mathbf{F}_6 \otimes \mathbf{F}_7 \\ \hline \\ \hline \end{array} } \begin{array}{c} \otimes_R \\ \mathrm{Cut} \\ \\ \mathrm{Cut} \\ \end{array}$$

 $\bullet\,$ Case rule W

$$\frac{\underbrace{\bullet_{\text{h}_1}:p(\text{n}_3) \vdash p(\text{n}_3)}_{\bullet \text{h}_4:p(\text{n}_3) \vdash \text{F}_6} I \quad \frac{\text{h}_4:\Delta_7,p(\text{n}_3) \vdash \text{F}_6}{\bullet_{\text{h}_4}:(\Delta_7,\text{l}^{\text{F}_5}),p(\text{n}_3) \vdash \text{F}_6}} \quad \frac{W}{\text{Cut}}}{\underbrace{-:\Delta_7,\text{l}^{\text{F}_5},p(\text{n}_3) \vdash \text{F}_6}_{\bullet}} \quad \text{ax/W}}$$

ullet Case rule C

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \\ \hline \\ \bullet \bullet_{\mathbf{h}_1} : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \\ \hline \\ - : p(\mathbf{n}_3), \Delta_{7}, ! \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \hline \\ \hline \\ - : \Delta_{7}, ! \mathbf{F}_5, p(\mathbf{n}_3) \vdash \mathbf{F}_6 \\ \hline \\ \hline \\ - : \Delta_{7}, ! \mathbf{F}_5, p(\mathbf{n}_3) \vdash \mathbf{F}_6 \\ \hline \end{array} } \begin{array}{c} C \\ \text{Cut} \\ \\ \hline \end{array}$$

• Case rule !L

$$\frac{ \underbrace{ \begin{array}{c} \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \\ - : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \end{array} I \quad \begin{array}{c} \mathbf{h}_4 : (\Delta_7, \mathbf{F}_5, p(\mathbf{n}_3) \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_4 : (\Delta_7, \mathbf{F}_5), p(\mathbf{n}_3) \vdash \mathbf{F}_6 \\ \hline \\ - : p(\mathbf{n}_3), \Delta_7, \mathbf{F}_5 \vdash \mathbf{F}_6 \\ \hline \\ - : \Delta_7, \mathbf{F}_5, p(\mathbf{n}_3) \vdash \mathbf{F}_6 \end{array}} \quad \underbrace{ \begin{array}{c} !L \\ \text{Cut} \\ \bullet \\ \hline \\ - : \Delta_7, \mathbf{F}_5, p(\mathbf{n}_3) \vdash \mathbf{F}_6 \end{array}}$$

• Case rule $\&_{L2}$

$$\frac{ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \\ - : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \end{array} I \quad \begin{array}{c} \mathbf{h}_4 : \Delta_8, \mathsf{F}_6, p(\mathbf{n}_3) \vdash \mathsf{F}_7 \\ \bullet \mathbf{h}_4 : (\Delta_8, \mathsf{F}_5 \& \mathsf{F}_6), p(\mathbf{n}_3) \vdash \mathsf{F}_7 \\ \end{array}}_{- : p(\mathbf{n}_3), \Delta_8, \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_7 \\ \hline \\ \hline - : \Delta_8, p(\mathbf{n}_3), \mathsf{F}_5 \& \mathsf{F}_6 \vdash \mathsf{F}_7 \end{array}}_{\mathbf{ax/W}} \quad \begin{array}{c} \&_{L2} \\ \mathsf{Cut} \\ \\ \bullet \\ \hline \end{array}$$

• Case rule $\&_{L1}$

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3) \\ \end{array}}_{} I \quad \frac{ \begin{array}{c} \mathbf{h}_4 : \Delta_8, \mathbf{F}_5, p(\mathbf{n}_3) \vdash \mathbf{F}_7 \\ \bullet \mathbf{h}_4 : (\Delta_8, \mathbf{F}_5 \& \mathbf{F}_6), p(\mathbf{n}_3) \vdash \mathbf{F}_7 \\ \end{array}}_{} \quad \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : p(\mathbf{n}_3), \Delta_8, \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline \\ \hline \\ - : p(\mathbf{n}_3), \Delta_8, \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_7 \\ \hline \\ \hline \\ - : \Delta_8, p(\mathbf{n}_3), \mathbf{F}_5 \& \mathbf{F}_6 \vdash \mathbf{F}_7 \end{array}}_{} \quad \mathbf{ax/W}$$

• Case rule \otimes_L

$$\frac{\bullet \mathbf{h}_1:p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)}{I} \frac{\mathbf{h}_4:\Delta_8, \mathsf{F}_5, \mathsf{F}_6, p(\mathbf{n}_3) \vdash \mathsf{F}_7}{\bullet \mathbf{h}_4:(\Delta_8, \mathsf{F}_5 \otimes \mathsf{F}_6), p(\mathbf{n}_3) \vdash \mathsf{F}_7} \underset{\frown}{\otimes_L} \mathsf{Cut}$$

$$\frac{-:p(\mathbf{n}_3),\Delta_8, \mathsf{F}_5 \otimes \mathsf{F}_6 \vdash \mathsf{F}_7}{-:\Delta_8, p(\mathbf{n}_3), \mathsf{F}_5 \otimes \mathsf{F}_6 \vdash \mathsf{F}_7} \overset{\mathsf{ax/W}}{}$$

• Case rule \oplus_L

$$\frac{\bullet \mathbf{h}_1: p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)}{I} \quad \frac{\mathbf{h}_4: \Delta_8, \mathbf{F}_5, p(\mathbf{n}_3) \vdash \mathbf{F}_7 \quad \mathbf{h}_4: \Delta_8, \mathbf{F}_6, p(\mathbf{n}_3) \vdash \mathbf{F}_7}{\bullet \mathbf{h}_4: (\Delta_8, \mathbf{F}_5 \oplus \mathbf{F}_6), p(\mathbf{n}_3) \vdash \mathbf{F}_7} \quad \oplus_L \\ -: p(\mathbf{n}_3), \Delta_8, \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_7 \\ & \stackrel{\longleftarrow}{\longrightarrow} \\ \hline -: \Delta_8, p(\mathbf{n}_3), \mathbf{F}_5 \oplus \mathbf{F}_6 \vdash \mathbf{F}_7} \quad \mathbf{ax/W}$$

• Case rule \multimap_L

ullet Case rule I

6 Cut-Elimination

6.1 Status of !R: OK

• Case rule !R

$$\frac{\underset{\bullet h_1 : ! \Upsilon 2 \vdash F_5}{\underline{\bullet h_1 : ! \Upsilon 2 \vdash ! F_5}} :_{R} \quad \frac{\underset{\bullet h_7 : ! \Upsilon 4, ! F_5, contract(n_6, ! F_5) \vdash F_8}{\underline{\bullet h_7 : contract(sn_6, ! F_5), ! \Upsilon 4 \vdash ! F_8}} \underset{\leftarrow}{\underbrace{\hspace{0.5cm} ! R}} \quad \underbrace{\underset{\bullet h_7 : ! \Upsilon 2, ! \Upsilon 4 \vdash ! F_8}{\underline{\bullet h_7 : ! \Upsilon 2, ! \Upsilon 4 \vdash ! F_8}} \underset{\leftarrow}{\underline{\bullet h_7 : ! \Upsilon 2, ! \Upsilon 4 \vdash F_8}} \underset{\vdash}{\underline{h_7 : ! \Upsilon 2, ! \Upsilon 4 \vdash F_8}} \quad \underset{\leftarrow}{\underline{ax}} \quad \underset{hCut}{\underline{hCut}}$$

- Case rule $\mathbf{1}_R$
- Case rule \top

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : ! \Upsilon 2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! \mathbf{F}_5} & ! R & \frac{}{\bullet \mathbf{h}_7 : contract(sn_6, ! \mathbf{F}_5), \Delta_4 \vdash \top} & \top \\ \hline & - : ! \Upsilon 2, \Delta_4 \vdash \top \\ \hline & \frac{}{- : ! \Upsilon 2, \Delta_4 \vdash \top} & \top \\ \end{array}$$

• Case rule $\&_R$

$$\frac{\underbrace{\frac{h_{1} : ! \Upsilon 2 \vdash F_{5}}{\bullet h_{1} : ! \Upsilon 2 \vdash ! F_{5}}}_{\bullet h_{1} : ! \Upsilon 2 \vdash ! F_{5}} ! R \frac{\frac{h_{7} : \Delta_{4}, ! F_{5}, contract(n_{6}, ! F_{5}) \vdash F_{8}}{\bullet h_{7} : contract(sn_{6}, ! F_{5}), \Delta_{4} \vdash F_{8} \& F_{9}}}{- : ! \Upsilon 2, \Delta_{4} \vdash F_{8} \& F_{9}} \underbrace{\frac{- : ! \Upsilon 2, \Delta_{4} \vdash F_{8}}{h_{7} : \Delta_{4}, ! F_{5}, contract(n_{6}, ! F_{5}) \vdash F_{8}}}_{h_{Cut}} \underbrace{\frac{a_{1}}{h_{7} : \Delta_{4}, ! F_{5}, contract(n_{6}, ! F_{5}) \vdash F_{9}}}{- : ! \Upsilon 2, \Delta_{4} \vdash F_{8}}}_{h_{Cut}} \underbrace{\frac{a_{1}}{h_{7} : \Delta_{4}, ! F_{5}, contract(n_{6}, ! F_{5}) \vdash F_{9}}}{- : ! \Upsilon 2, \Delta_{4} \vdash F_{8}}}_{h_{Cut}} \underbrace{\frac{a_{1}}{h_{7} : \Delta_{4}, ! F_{5}, contract(n_{6}, ! F_{5}) \vdash F_{9}}}{- : ! \Upsilon 2, \Delta_{4} \vdash F_{8} \& F_{9}}}_{h_{Cut}}$$

• Case rule \multimap_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1: !\Upsilon2 \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: !\Upsilon2 \vdash !\mathbf{F}_5} & !R & \frac{\mathbf{h}_7: \Delta_4, \mathbf{F}_8, !\mathbf{F}_5, contract(\mathbf{n}_6, !\mathbf{F}_5) \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: contract(\mathbf{s}\mathbf{n}_6, !\mathbf{F}_5), \Delta_4 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9} & \mathbf{Cut} \\ \hline \\ \bullet \mathbf{h}_1: !\Upsilon2 \vdash !\mathbf{F}_5 & \mathbf{ax} & & \\ \hline \bullet \mathbf{h}_1: !\Upsilon2 \vdash !\mathbf{F}_5 & \mathbf{ax} & & \\ \hline \\ \bullet \mathbf{h}_7: \Delta_4, \mathbf{F}_8, !\mathbf{F}_5, contract(\mathbf{n}_6, !\mathbf{F}_5) \vdash \mathbf{F}_9} \\ \hline \\ & & -: !\Upsilon2, \Delta_4, \mathbf{F}_8 \vdash \mathbf{F}_9 \\ \hline \\ & -: !\Upsilon2, \Delta_4 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9 & \\ \hline \\ & -: !\Upsilon2, \Delta_4 \vdash \mathbf{F}_8 \multimap \mathbf{F}_9 & \\ \hline \end{array}$$

• Case rule \oplus_{R_2}

$$\frac{\mathbf{h}_1: !\Upsilon2 \vdash F_5}{\bullet \mathbf{h}_1: !\Upsilon2 \vdash !F_5}: R \quad \frac{\mathbf{h}_7: \Delta_4, !F_5, contract(\mathbf{n}_6, !F_5) \vdash F_9}{\bullet \mathbf{h}_7: contract(\mathbf{s}_{\mathbf{n}_6}, !F_5), \Delta_4 \vdash F_8 \oplus F_9} \quad \overset{\bigoplus}{\mathsf{Cut}} \\ -: !\Upsilon2, \Delta_4 \vdash F_8 \oplus F_9 \quad \overset{\longleftarrow}{\mathsf{m}_1: !\Upsilon2 \vdash !F_5} \quad \mathsf{ax} \quad \frac{\overset{\longleftarrow}{\mathsf{h}_7: \Delta_4, !F_5, contract(\mathbf{n}_6, !F_5) \vdash F_9}}{-: !\Upsilon2, \Delta_4 \vdash F_8 \oplus F_9} \quad \overset{\bigoplus}{\oplus}_{R_2} \\ -: !\Upsilon2, \Delta_4 \vdash F_8 \oplus F_9 \quad \overset{\bigoplus}{\oplus}_{R_2}$$

• Case rule \oplus_{R_1}

$$\frac{ \begin{array}{c} \mathbf{h}_1 : ! \Upsilon 2 \vdash F_5 \\ \bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_5 \end{array} : R \quad \begin{array}{c} \mathbf{h}_7 : \Delta_4, ! F_5, contract(\mathbf{n}_6, ! F_5) \vdash F_8 \\ \bullet \mathbf{h}_7 : contract(s\mathbf{n}_6, ! F_5), \Delta_4 \vdash F_8 \oplus F_9 \end{array} \quad \begin{array}{c} \oplus_{R_1} \\ \bullet \mathbf{h}_7 : \Delta_4, ! F_8 \oplus F_9 \end{array} \quad \begin{array}{c} - : ! \Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9 \\ \bullet \mathbf{h}_1 : ! \Upsilon 2 \vdash ! F_5 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \hline - : ! \Upsilon 2, \Delta_4 \vdash F_8 \\ \hline - : ! \Upsilon 2, \Delta_4 \vdash F_8 \\ \hline - : ! \Upsilon 2, \Delta_4 \vdash F_8 \\ \hline - : ! \Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \mathsf{hCut} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c|c} \frac{\mathbf{h}_1: !\Upsilon 2 \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1: !\Upsilon 2 \vdash !\mathbf{F}_4} & !R & \frac{\mathbf{h}_6: \Delta_8, !\mathbf{F}_4, contract(\mathbf{n}_5, !\mathbf{F}_4) \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6: contract(\mathbf{s}_{\mathbf{n}_5}, !\mathbf{F}_4), \mathbf{1}, \Delta_8 \vdash \mathbf{F}_7} & \mathsf{Cut} \\ \hline \\ \bullet \mathbf{h}_1: !\Upsilon 2 \vdash !\mathbf{F}_4 & \mathsf{ax} & \overset{\bullet}{\longrightarrow} & \frac{-: !\Upsilon 2, \mathbf{1}, \Delta_8 \vdash \mathbf{F}_7}{\bullet \mathbf{h}_6: \Delta_8, !\mathbf{F}_4, contract(\mathbf{n}_5, !\mathbf{F}_4) \vdash \mathbf{F}_7} & \mathsf{ax} \\ \hline \\ & & \frac{-: !\Upsilon 2, \Delta_8 \vdash \mathbf{F}_7}{-: \mathbf{1}, !\Upsilon 2, \Delta_8 \vdash \mathbf{F}_7} & \mathbf{1}_L \end{array}$$

• Case rule \otimes_R

 $\bullet\,$ Case rule W

$$\begin{array}{c|c} \frac{\mathbf{h}_{1}: ! \Upsilon 2 \vdash \mathbf{F}_{4}}{\bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{4}} & !R & \frac{\mathbf{h}_{6}: \Delta_{9}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{6}: contract(s\mathbf{n}_{5}, ! \mathbf{F}_{4}), \Delta_{9}, ! \mathbf{F}_{7} \vdash \mathbf{F}_{8}} & Cut \\ \hline & -: ! \Upsilon 2, \Delta_{9}, ! \mathbf{F}_{7} \vdash \mathbf{F}_{8} & \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{4} & ax & \frac{\sim}{\mathbf{h}_{6}: \Delta_{9}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{8}} & ax \\ \hline & -: ! \Upsilon 2, \Delta_{9} \vdash \mathbf{F}_{8} & W \\ \hline & -: ! \Upsilon 2, \Delta_{9}, ! \mathbf{F}_{7} \vdash \mathbf{F}_{8} & W \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{7} & ! R & \frac{\mathbf{h}_{6}: \Delta_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{7}) \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{6}: contract(s\mathbf{n}_{5}, ! \mathbf{F}_{7}), \Delta_{4} \vdash \mathbf{F}_{8}} & W \\ \hline & -: ! \Upsilon 2, \Delta_{4} \vdash \mathbf{F}_{8} & W \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{7} & ax & \frac{\sim}{\mathbf{h}_{6}: \Delta_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{7}) \vdash \mathbf{F}_{8}} & ax \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{7} & ax & \frac{\sim}{\mathbf{h}_{6}: \Delta_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{7}) \vdash \mathbf{F}_{8}} & ax \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{7} & ax & \frac{\sim}{\mathbf{h}_{6}: \Delta_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{7}) \vdash \mathbf{F}_{8}} & ax \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{7} & ax & \frac{\sim}{\mathbf{h}_{6}: \Delta_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{7}) \vdash \mathbf{F}_{8}} & ax \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{7} & ax & \frac{\sim}{\mathbf{h}_{6}: \Delta_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{7}) \vdash \mathbf{F}_{8}} & ax \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{7} & ax & \frac{\sim}{\mathbf{h}_{6}: \Delta_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{7}) \vdash \mathbf{F}_{8}} & ax \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{7} & ax & \frac{\sim}{\mathbf{h}_{1}: \Delta_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{7}) \vdash \mathbf{F}_{8}} & ax \\ \hline \bullet \mathbf{h}_{1}: ! \Upsilon 2 \vdash ! \mathbf{F}_{7} & ax & \frac{\sim}{\mathbf{h}_{1}: \Delta_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{7}) \vdash \mathbf{F}_{8}} & ax \\ \hline \bullet \mathbf{h}_{1}: \mathbf{h}_{1}: \Delta_{4} \vdash \mathbf{h}_{2}: \Delta_{4} \vdash \mathbf{h}_{1}: \Delta_{4} \vdash \mathbf{h}_{2}: \Delta_{4} \vdash \mathbf{h}_{3}: \Delta_{4} \vdash \mathbf{h}_{4}: \Delta_{4} \vdash \mathbf{h}_{4}: \Delta_{4} \vdash \mathbf{h$$

$\bullet\,$ Case rule C

\bullet Case rule !L

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_{1} : ! \Upsilon 2 \vdash \mathbf{F}_{4}}{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4}} : R \quad \frac{\mathbf{h}_{6} : \Delta_{10}, \mathbf{F}_{8}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{6} : contract(s\mathbf{n}_{5}, ! \mathbf{F}_{4}), \Delta_{10}, \mathbf{F}_{7} \& \mathbf{F}_{8} \vdash \mathbf{F}_{9}} \quad \mathcal{E}_{L2}} \\ - : ! \Upsilon 2, \Delta_{10}, \mathbf{F}_{7} \& \mathbf{F}_{8} \vdash \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4} \quad \text{ax} \quad \frac{}{\mathbf{h}_{6} : \Delta_{10}, \mathbf{F}_{8}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{9}} \\ - : ! \Upsilon 2, \Delta_{10}, \mathbf{F}_{8} \vdash \mathbf{F}_{9} \quad \&_{L2}} \quad \mathbf{hCut}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_{1} : ! \Upsilon 2 \vdash \mathbf{F}_{4}}{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4}} : R \quad \frac{\mathbf{h}_{6} : \Delta_{10}, \mathbf{F}_{7}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{6} : contract(s\mathbf{n}_{5}, ! \mathbf{F}_{4}), \Delta_{10}, \mathbf{F}_{7} \& \mathbf{F}_{8} \vdash \mathbf{F}_{9}} \quad \underbrace{\mathcal{L}_{11}}_{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4}} \quad \text{ax} \quad \frac{- : ! \Upsilon 2, \Delta_{10}, \mathbf{F}_{7} \& \mathbf{F}_{8} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{6} : \Delta_{10}, \mathbf{F}_{7}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{9}} \quad \underbrace{\mathcal{L}_{11}}_{\bullet \mathbf{Cut}}$$

$$\frac{- : ! \Upsilon 2, \Delta_{10}, \mathbf{F}_{7} \vdash \mathbf{F}_{9}}{- : ! \Upsilon 2, \Delta_{10}, \mathbf{F}_{7} \& \mathbf{F}_{8} \vdash \mathbf{F}_{9}} \quad \&_{L1}$$

• Case rule \otimes_L

$$\begin{array}{c|c} \frac{h_1: !\Upsilon2 \vdash F_4}{\bullet h_1: !\Upsilon2 \vdash !F_4} & !R & \frac{h_6: \Delta_{10}, F_7, F_8, !F_4, contract(n_5, !F_4) \vdash F_9}{\bullet h_6: contract(sn_5, !F_4), \Delta_{10}, F_7 \otimes F_8 \vdash F_9} & \otimes_L \\ \hline \bullet h_1: !\Upsilon2 \vdash !F_4 & ax & \frac{-: !\Upsilon2, \Delta_{10}, F_7, F_8, !F_4, contract(n_5, !F_4) \vdash F_9}{h_6: \Delta_{10}, F_7, F_8, !F_4, contract(n_5, !F_4) \vdash F_9} & ax \\ \hline & \frac{-: !\Upsilon2, \Delta_{10}, F_7, F_8 \vdash F_9}{-: !\Upsilon2, \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L \end{array}$$

• Case rule \oplus_L

$$\frac{\frac{\mathbf{h}_{1} : ! \Upsilon 2 \vdash \mathbf{F}_{4}}{\bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash \mathbf{F}_{4}} : R}{\bullet \mathbf{h}_{6} : \Delta_{10}, \mathbf{F}_{7}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{9} \quad \mathbf{h}_{6} : \Delta_{10}, \mathbf{F}_{8}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{6} : contract(s\mathbf{n}_{5}, ! \mathbf{F}_{4}), \Delta_{10}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9}} \quad \mathbf{Cut}} \\ - : ! \Upsilon 2, \Delta_{10}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \vdash \mathbf{F}_{9} \\ \bullet \mathbf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4} \quad \mathbf{ax} \quad \frac{\mathbf{h}_{6} : \Delta_{10}, \mathbf{F}_{7}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{9}}{\mathsf{hCut}} \quad \frac{\mathbf{ax}}{\mathsf{h}_{1} : ! \Upsilon 2 \vdash ! \mathbf{F}_{4}} \quad \mathbf{ax} \quad \frac{\mathbf{h}_{6} : \Delta_{10}, \mathbf{F}_{8}, ! \mathbf{F}_{4}, contract(\mathbf{n}_{5}, ! \mathbf{F}_{4}) \vdash \mathbf{F}_{9}}{\mathsf{hCut}} \quad \mathbf{hCut} \\ - : ! \Upsilon 2, \Delta_{10}, \mathbf{F}_{7} \vdash \mathbf{F}_{9} \quad \oplus_{L} \quad \mathbf{hCut} \quad$$

• Case rule \multimap_L

 \bullet Case rule I

6.2 Status of 1_R : OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- \bullet Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet\,$ Case rule W
- Case rule C
- Case rule !L
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L

- Case rule \oplus_L
- Case rule \multimap_L
- ullet Case rule I

6.3 Status of \top : OK

- \bullet Case rule !R
- Case rule $\mathbf{1}_R$
- $\bullet \;$ Case rule \top
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- ullet Case rule W
- $\bullet\,$ Case rule C
- \bullet Case rule !L
- \bullet Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L

- Case rule \multimap_L
- $\bullet\,$ Case rule I

6.4 Status of $\&_R$: OK

- $\bullet \;$ Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- ullet Case rule W
- $\bullet\,$ Case rule C
- $\bullet \;$ Case rule !L
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- $\bullet\,$ Case rule I

6.5 Status of \multimap_R : OK

- $\bullet \;$ Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet \;$ Case rule W
- $\bullet\,$ Case rule C
- $\bullet \;$ Case rule !L
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- $\bullet\,$ Case rule I

6.6 Status of \bigoplus_{R_2} : OK

- $\bullet \;$ Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet \;$ Case rule W
- $\bullet\,$ Case rule C
- $\bullet \;$ Case rule !L
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- $\bullet\,$ Case rule I

6.7 Status of \bigoplus_{R_1} : OK

- $\bullet \;$ Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet \;$ Case rule W
- $\bullet\,$ Case rule C
- \bullet Case rule !L
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- $\bullet\,$ Case rule I

6.8 Status of 1_L : OK

- $\bullet \;$ Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet \;$ Case rule W
- $\bullet\,$ Case rule C
- $\bullet \;$ Case rule !L
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- $\bullet\,$ Case rule I

6.9 Status of \otimes_R : OK

- $\bullet \;$ Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet \;$ Case rule W
- $\bullet\,$ Case rule C
- $\bullet \;$ Case rule !L
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- ullet Case rule I

6.10 Status of W: OK

• Case rule !R

- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{ \begin{array}{c} \mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathbf{F}_6 \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_6 \end{array} W \quad \overbrace{ \begin{array}{c} \bullet \mathbf{h}_8 : contract(s\mathbf{n}_7, ! \mathbf{F}_6), \Delta_5 \vdash \top \\ - : (! \Upsilon 3, ! \mathbf{F}_4), \Delta_5 \vdash \top \\ \hline \\ \hline - : ! \Upsilon 3, \Delta_5, ! \mathbf{F}_4 \vdash \top \end{array} } \quad \top \quad \mathsf{Cut}$$

• Case rule $\&_R$

$$\frac{\mathbf{h}_{2} : ! \Upsilon 3 \vdash ! \mathsf{F}_{6}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3 \vdash ! \mathsf{F}_{6}} \quad W \quad \frac{\mathbf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{9} \quad \mathbf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{10}}{\bullet \mathbf{h}_{8} : contract(\mathbf{s}\mathbf{n}_{7}, ! \mathsf{F}_{6}), \Delta_{5} \vdash \mathsf{F}_{9} \& \mathsf{F}_{10}} \quad \mathsf{Cut} \\ \\ \frac{\mathbf{h}_{2} : ! \Upsilon 3 \vdash ! \mathsf{F}_{6}}{\mathbf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{9}} \quad \mathsf{ax} \quad \frac{\mathbf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{10}}{\bullet \mathbf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{10}} \quad \mathsf{k}_{R} \\ \\ \frac{- : ! \Upsilon 3, \Delta_{5} \vdash \mathsf{F}_{9} \& \mathsf{F}_{10}}{- : ! \Upsilon 3, \Delta_{5}, ! \mathsf{F}_{4} \vdash \mathsf{F}_{9} \& \mathsf{F}_{10}} \quad W$$

• Case rule \multimap_R

$$\begin{array}{c} \frac{\mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathbf{F}_6}{\bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_6} \ W & \frac{\mathbf{h}_8 : \Delta_5, \mathbf{F}_9, ! \mathbf{F}_6, contract(\mathbf{n}_7, ! \mathbf{F}_6) \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_8 : contract(\mathbf{s}\mathbf{n}_7, ! \mathbf{F}_6), \Delta_5 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10}} & \mathbf{Cut} \\ \hline & - : (! \Upsilon 3, ! \mathbf{F}_4), \Delta_5 \vdash \mathbf{F}_9 \multimap \mathbf{F}_{10} & \\ \hline & \bullet \mathbf{h}_8 : \Delta_5, \mathbf{F}_9, ! \mathbf{F}_6, contract(\mathbf{n}_7, ! \mathbf{F}_6) \vdash \mathbf{F}_{10}} \\ \hline & \frac{- : ! \Upsilon 3, \Delta_5, \mathbf{F}_9, ! \mathbf{F}_6, contract(\mathbf{n}_7, ! \mathbf{F}_6) \vdash \mathbf{F}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathbf{F}_4 \vdash \mathbf{F}_{10}} & \circ_{\mathbf{R}} \end{array} \right. \\ & \frac{- : ! \Upsilon 3, \Delta_5, \mathbf{F}_9, ! \mathbf{F}_4 \vdash \mathbf{F}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathbf{F}_4 \vdash \mathbf{F}_{10}} & \circ_{\mathbf{R}} \end{array}$$

• Case rule \oplus_{R_2}

$$\frac{ \begin{array}{c} \mathbf{h}_2 : ! \Upsilon 3 \vdash ! F_6 \\ \bullet \mathbf{h}_2 : ! \Upsilon 3 , ! F_4 \vdash ! F_6 \end{array} W \quad \begin{array}{c} \mathbf{h}_8 : \Delta_5, ! F_6, contract(\mathbf{n}_7, ! F_6) \vdash F_{10} \\ \bullet \mathbf{h}_8 : contract(s\mathbf{n}_7, ! F_6), \Delta_5 \vdash F_9 \oplus F_{10} \end{array} \quad \begin{array}{c} \oplus_{R_2} \\ \mathsf{Cut} \end{array} \\ \\ \overline{ \begin{array}{c} \bullet \mathbf{h}_2 : ! \Upsilon 3, ! F_4 \vdash ! F_6 \end{array}} \quad \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_8 : \Delta_5, ! F_6, contract(\mathbf{n}_7, ! F_6) \vdash F_{10} \\ \hline \bullet \mathbf{h}_2 : ! \Upsilon 3, ! F_4 \vdash ! F_6 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, \Delta_5, ! F_4 \vdash F_{10} \\ \hline - : ! \Upsilon 3, \Delta_5, ! F_4 \vdash F_{10} \end{array} \quad \oplus_{R_2} \end{array} \quad \begin{array}{c} \mathsf{ax} \\ \mathsf{hCut} \end{array}$$

• Case rule \oplus_{R_1}

$$\frac{ \begin{array}{c} \mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathbf{F}_6 \\ \bullet \mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathbf{F}_6 \end{array} W \quad \begin{array}{c} \mathbf{h}_8 : \Delta_5, ! \mathbf{F}_6, contract(\mathbf{n}_7, ! \mathbf{F}_6) \vdash \mathbf{F}_9 \\ \bullet \mathbf{h}_8 : contract(s\mathbf{n}_7, ! \mathbf{F}_6), \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \end{array} \quad \begin{array}{c} \oplus_{R_1} \\ \mathsf{Cut} \end{array} \\ \hline \\ - : (! \Upsilon 3, ! \mathbf{F}_4), \Delta_5 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ & \longrightarrow \\ \hline \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_6 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_8 : \Delta_5, ! \mathbf{F}_6, contract(\mathbf{n}_7, ! \mathbf{F}_6) \vdash \mathbf{F}_9 \\ \hline - : ! \Upsilon 3, \Delta_5, ! \mathbf{F}_4 \vdash \mathbf{F}_9 \\ \hline - : ! \Upsilon 3, \Delta_5, ! \mathbf{F}_4 \vdash \mathbf{F}_9 \oplus \mathbf{F}_{10} \end{array} \quad \begin{array}{c} \oplus_{R_1} \end{array} \quad \text{act} \\ \mathbf{hCut} \end{array}$$

• Case rule $\mathbf{1}_L$

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathbf{F}_5}{\bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5} & W & \frac{\mathbf{h}_7 : \Delta_9, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7 : contract(s\mathbf{n}_6, ! \mathbf{F}_5), \mathbf{1}, \Delta_9 \vdash \mathbf{F}_8} & \mathbf{1}_L \\ \hline & - : (! \Upsilon 3, ! \mathbf{F}_4), \mathbf{1}, \Delta_9 \vdash \mathbf{F}_8 \\ \hline & \bullet_{\mathbf{h}_2} : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 & \mathbf{ax} & \overset{\sim}{\mathbf{h}_7 : \Delta_9, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_8}} & \mathbf{ax} \\ \hline & \frac{- : ! \Upsilon 3, \Delta_9, ! \mathbf{F}_4 \vdash \mathbf{F}_8}{- : \mathbf{1}, ! \Upsilon 3, \Delta_9, ! \mathbf{F}_4 \vdash \mathbf{F}_8} & \mathbf{1}_L \end{array}$$

• Case rule \otimes_R

 \bullet Case rule W

$$\begin{array}{c} \frac{\mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathbf{F}_5}{\bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5} & W & \frac{\mathbf{h}_7 : \Delta_{10}, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7 : contract(\mathbf{s}\mathbf{n}_6, ! \mathbf{F}_5), \Delta_{10}, ! \mathbf{F}_8 \vdash \mathbf{F}_9} & Cut \\ \hline & - : (! \Upsilon 3, ! \mathbf{F}_4), \Delta_{10}, ! \mathbf{F}_8 \vdash \mathbf{F}_9 & \\ \hline \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 & \mathbf{ax} & \\ \hline & - : ! \Upsilon 3, \Delta_{10}, ! \mathbf{F}_4 \vdash \mathbf{F}_9 & \\ \hline & - : ! \Upsilon 3, \Delta_{10}, ! \mathbf{F}_4 \vdash \mathbf{F}_9 & W \\ \hline & - : ! \Upsilon 3, \Delta_{10}, ! \mathbf{F}_4 \vdash \mathbf{F}_9 & W \\ \hline & \mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathbf{F}_8 & W & \frac{\mathbf{h}_7 : \Delta_5, contract(\mathbf{n}_6, ! \mathbf{F}_8) \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7 : contract(\mathbf{s}\mathbf{n}_6, ! \mathbf{F}_8), \Delta_5 \vdash \mathbf{F}_9} & W \\ \hline & - : (! \Upsilon 3, ! \mathbf{F}_4), \Delta_5 \vdash \mathbf{F}_9 & Cut \\ \hline & \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_8 & \mathbf{ax} & \\ \hline & - : (! \Upsilon 3, \Delta_5, ! \mathbf{F}_4 \vdash \mathbf{F}_9 & \mathbf{ax} \\ \hline & - : ! \Upsilon 3, \Delta_5, ! \mathbf{F}_4 \vdash \mathbf{F}_9 & \mathbf{h}_{Cut} \\ \hline \end{array}$$

\bullet Case rule C

• Case rule !L

$$\frac{ \begin{array}{c} \mathbf{h}_{2} : | \Upsilon 3 \vdash | \mathbf{F}_{5} \\ \bullet \mathbf{h}_{2} : | \Upsilon 3, | \mathbf{F}_{4} \vdash | \mathbf{F}_{5} \end{array} W & \begin{array}{c} \mathbf{h}_{7} : \Delta_{10}, \mathbf{F}_{8}, | \mathbf{F}_{5}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{9} \\ \bullet \mathbf{h}_{7} : contract(s\mathbf{n}_{6}, | \mathbf{F}_{5}), \Delta_{10}, | \mathbf{F}_{8} \vdash \mathbf{F}_{9} \end{array} & \begin{array}{c} !L \\ \text{Cut} \\ \hline \\ \bullet \mathbf{h}_{2} : | \Upsilon 3, | \mathbf{F}_{4} \vdash | \mathbf{F}_{5} \end{array} & \begin{array}{c} \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{10}, \mathbf{F}_{8}, | \mathbf{F}_{5}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{10}, \mathbf{F}_{8}, | \mathbf{F}_{5}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : (\mathbf{Y}, \mathbf{A}, \mathbf{h}_{10}, \mathbf{F}_{8}, | \mathbf{F}_{8} \vdash \mathbf{F}_{9} \end{array} & \mathbf{h}_{1} \\ \hline \\ \bullet \mathbf{h}_{2} : | \Upsilon 3 \vdash | \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_{2} : | \Upsilon 3, | \mathbf{F}_{4} \vdash | \mathbf{F}_{8} \end{array} & W & \begin{array}{c} \mathbf{h}_{7} : \Delta_{5}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \bullet \mathbf{h}_{7} : contract(s\mathbf{n}_{6}, | \mathbf{F}_{8}), \Delta_{5} \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, | \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, | \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, | \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, | \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, | \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, | \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, | \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, | \mathbf{F}_{8}, contract(\mathbf{n}_{6}, | \mathbf{F}_{8}) \vdash \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{7} : \Delta_{5}, | \mathbf{F}_{8}, | \mathbf{h}_{7} \vdash \mathbf{h}_{9} \\ \hline \\ \hline \end{array}$$

• Case rule $\&_{L2}$

$$\frac{ \begin{array}{c} \mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathbf{F}_5 \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 \end{array} W & \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_9, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_7 : contract(\mathbf{s}\mathbf{n}_6, ! \mathbf{F}_5), \Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} & \begin{array}{c} \&_{L2} \\ \text{Cut} \\ \hline \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 \end{array} & \mathbf{ax} & \begin{array}{c} \longrightarrow \\ \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_9, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 \end{array} & \mathbf{ax} \\ \hline \\ & \begin{array}{c} - : ! \Upsilon 3, \Delta_{11}, \mathbf{F}_9, ! \mathbf{F}_4 \vdash \mathbf{F}_{10} \\ \hline \\ - : ! \Upsilon 3, \Delta_{11}, ! \mathbf{F}_4, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \end{array} & \&_{L2} \end{array} & \mathbf{hCut} \\ \end{array}$$

• Case rule $\&_{L1}$

$$\frac{ \begin{array}{c} \mathbf{h}_2 : ! \Upsilon 3 : ! F_5 \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, ! F_4 \vdash ! F_5 \end{array} W \quad \begin{array}{c} \mathbf{h}_7 : \Delta_{11}, F_8, ! F_5, contract(\mathbf{n}_6, ! F_5) \vdash F_{10} \\ \bullet \mathbf{h}_7 : contract(s\mathbf{n}_6, ! F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \end{array} \\ \begin{array}{c} - : (! \Upsilon 3, ! F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, ! F_4 \vdash ! F_5 \end{array} & \mathbf{ax} \quad \begin{array}{c} \bullet \\ \mathbf{h}_7 : \Delta_{11}, F_8, ! F_5, contract(\mathbf{n}_6, ! F_5) \vdash F_{10} \\ \bullet \mathbf{h}_7 : \Delta_{11}, F_8, ! F_4 \vdash F_{10} \\ \hline - : ! \Upsilon 3, \Delta_{11}, ! F_4, F_8 \& F_9 \vdash F_{10} \end{array} & \&_{L1} \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \mathsf{hCut} \end{array}$$

• Case rule \otimes_L

$$\frac{ \begin{array}{c} \mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathbf{F}_5 \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 \end{array} W \quad \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : contract(s\mathbf{n}_6, ! \mathbf{F}_5), \Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9 \vdash \mathbf{F}_{10}} \quad \begin{array}{c} \otimes_L \\ \text{Cut} \\ \hline \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 \end{array} \quad \text{ax} \quad \frac{}{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_{10}} \\ \hline \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 \end{array} \quad \frac{\mathbf{ax}}{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_{10}} \\ \hline \\ \bullet \mathbf{h}_2 : ! \Upsilon 3, \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, ! \mathbf{F}_4 \vdash \mathbf{F}_{10}} \\ \hline \\ \bullet \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, ! \mathbf{F}_9 \vdash \mathbf{F}_{10}} \otimes_L \end{array} \quad \frac{\mathbf{ax}}{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, \mathbf{F}_9, ! \mathbf{F}_9 \vdash \mathbf{F}_{10}} \\ \hline \end{array}$$

• Case rule \oplus_L

$$\frac{\mathbf{h}_{2}: ! \Upsilon 3 \vdash ! \mathsf{F}_{5}}{\bullet \mathsf{h}_{2}: ! \Upsilon 3 \vdash ! \mathsf{F}_{5}} \quad W \quad \frac{\mathbf{h}_{7}: \Delta_{11}, \mathsf{F}_{8}, ! \mathsf{F}_{5}, contract(\mathsf{n}_{6}, ! \mathsf{F}_{5}) \vdash \mathsf{F}_{10} \quad \mathsf{h}_{7}: \Delta_{11}, \mathsf{F}_{9}, ! \mathsf{F}_{5}, contract(\mathsf{n}_{6}, ! \mathsf{F}_{5}) \vdash \mathsf{F}_{10}}{\bullet \mathsf{h}_{7}: contract(s \mathsf{n}_{6}, ! \mathsf{F}_{5}), \Delta_{11}, \mathsf{F}_{8} \oplus \mathsf{F}_{9} \vdash \mathsf{F}_{10}} \quad \mathsf{Cut} \\ \\ \frac{-: (! \Upsilon 3, ! \mathsf{F}_{4}), \Delta_{11}, \mathsf{F}_{8} \oplus \mathsf{F}_{9} \vdash \mathsf{F}_{10}}{\circ} \quad \overset{\mathsf{ax}}{\longrightarrow} \quad \frac{\mathsf{h}_{7}: \Delta_{11}, \mathsf{F}_{8}, ! \mathsf{F}_{5}, contract(\mathsf{n}_{6}, ! \mathsf{F}_{5}) \vdash \mathsf{F}_{10}}{\circ} \quad \mathsf{ax}} \quad \frac{\mathsf{ax}}{\mathsf{h}_{7}: \Delta_{11}, \mathsf{F}_{9}, ! \mathsf{F}_{5}, contract(\mathsf{n}_{6}, ! \mathsf{F}_{5}) \vdash \mathsf{F}_{10}}} \quad \mathsf{dx} \\ & \frac{-: ! \Upsilon 3, \Delta_{11}, \mathsf{F}_{8}, ! \mathsf{F}_{5}, contract(\mathsf{n}_{6}, ! \mathsf{F}_{5}) \vdash \mathsf{F}_{10}}{\mathsf{h}_{11}} \quad \mathsf{h}_{12}}{\mathsf{h}_{13}} \quad \mathsf{dx} \\ & \frac{-: ! \Upsilon 3, \Delta_{11}, \mathsf{F}_{8} \oplus \mathsf{F}_{9} \vdash \mathsf{F}_{10}}{-: ! \Upsilon 3, \Delta_{11}, ! \mathsf{F}_{4}, \mathsf{F}_{8} \oplus \mathsf{F}_{9} \vdash \mathsf{F}_{10}} \quad W$$

• Case rule \multimap_L

$$\frac{ \begin{array}{c} h_2 : ! \Upsilon 3 \vdash ! F_5 \\ \bullet h_2 : ! \Upsilon 3, | F_4 \vdash ! F_5 \end{array} }{ \bullet h_2 : ! \Upsilon 3, | F_4 \vdash ! F_5 } \end{array} W \begin{array}{c} h_7 : \Delta_{12}, | F_5, contract(n_6, | F_5) \vdash F_9 & h_7 : \Delta_8, F_{10} \vdash F_{11} \\ \bullet h_7 : contract(sn_6, | F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \end{array} }{ \bullet h_7 : \Delta_{12}, | F_9 \multimap F_{10} \vdash F_{11} \end{aligned} } \overset{\circ}{\text{Cut}} \overset{\circ}{\text{Cut}} \\ \hline \bullet h_2 : ! \Upsilon 3, | F_4 \vdash ! F_5 \end{array} \begin{array}{c} \text{ax} & \overset{\circ}{\text{h}_7} : \Delta_{12}, | F_5, contract(n_6, | F_5) \vdash F_9 \\ h_7 : \Delta_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ & - : ! \Upsilon 3, \Delta_{12}, | F_4 \vdash F_9 \\ & - : ! \Upsilon 3, \Delta_{12}, \Delta_8, | F_4, F_9 \multimap F_{10} \vdash F_{11} \end{array} \end{array} \begin{array}{c} \text{ax} \\ - : \Delta_8, F_{10} \vdash F_{11} \\ \hline \bullet h_2 : ! \Upsilon 3 \vdash ! F_5 \\ \hline \bullet h_2 : ! \Upsilon 3, | F_4 \vdash ! F_5 \end{array} W \begin{array}{c} h_7 : \Delta_{11}, contract(n_6, | F_5) \vdash F_8 & h_7 : \Delta_{12}, F_9, | F_5 \vdash F_{10} \\ \hline \bullet h_7 : contract(sn_6, | F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, contract(n_6, | F_5) \vdash F_8 & h_7 : \Delta_{12}, F_9, | F_5 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, contract(n_6, | F_5) \vdash F_8 & h_7 : \Delta_{12}, F_9, | F_5 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, \Delta_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, \Delta_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, \Delta_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : Contract(sn_6, | F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12}, | F_8 \vdash F_9 \vdash F_{10} \\ \hline \bullet h_7 : \Delta_{11}, | A_{12},$$

$$\frac{\frac{\mathbf{h}_2 : ! \Upsilon 3 \vdash ! \mathsf{F}_5}{\bullet \mathbf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5}}{\bullet \mathbf{h}_7 : \Delta_8 \vdash \mathsf{F}_9 \quad \mathbf{h}_7 : \Delta_{12}, \mathsf{F}_{10}, ! \mathsf{F}_5, contract(\mathbf{n}_6, ! \mathsf{F}_5) \vdash \mathsf{F}_{11}}{\bullet \mathbf{h}_7 : contract(\mathbf{s}_{16}, ! \mathsf{F}_5), \Delta_8, \Delta_{12}, \mathsf{F}_9 \multimap \mathsf{F}_{10} \vdash \mathsf{F}_{11}} \quad \mathsf{Cut}} \\ - : (! \Upsilon 3, ! \mathsf{F}_4), \Delta_8, \Delta_{12}, \mathsf{F}_9 \multimap \mathsf{F}_{10} \vdash \mathsf{F}_{11} \\ \hline - : (! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5) \quad \mathsf{ax} \quad \xrightarrow{\mathbf{h}_7 : \Delta_{12}, \mathsf{F}_{10}, ! \mathsf{F}_5, contract(\mathbf{n}_6, ! \mathsf{F}_5) \vdash \mathsf{F}_{11}} \quad \mathsf{ax} \\ - : ! \Upsilon 3, \Delta_{12}, \Delta_{12}, \mathsf{F}_{10}, ! \mathsf{F}_4 \vdash \mathsf{F}_{11} \quad \multimap_L \\ \hline - : ! \Upsilon 3, \Delta_{12}, \Delta_8, ! \mathsf{F}_4, \mathsf{F}_9 \multimap \mathsf{F}_{10} \vdash \mathsf{F}_{11} \quad \multimap_L$$

 \bullet Case rule I

6.11 Status of *C*: OK

• Case rule !R

$$\frac{\begin{array}{l} \mathbf{h}_{2}: ! \Upsilon 3, ! \mathbf{F}_{4}, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{6} \\ \bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathbf{F}_{4}, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{6} \end{array} C \quad \begin{array}{l} \mathbf{h}_{8}: ! \Upsilon 5, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{9} \\ \bullet \mathbf{h}_{8}: contract(s\mathbf{n}_{7}, ! \mathbf{F}_{6}), ! \Upsilon 5 \vdash ! \mathbf{F}_{9} \end{array} \quad \begin{array}{l} ! R \\ \text{Cut} \\ \\ \bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{6} \end{array} \quad \begin{array}{l} \bullet \mathbf{h}_{8}: ! \Upsilon 5, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{9} \\ \bullet \mathbf{h}_{8}: ! \Upsilon 5, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{9} \end{array} \quad \begin{array}{l} \mathbf{ax} \\ \mathbf{h} \mathbf{Cut} \\ \hline - : ! \Upsilon 3, ! \Upsilon 5, ! \mathbf{F}_{4} \vdash \mathbf{F}_{9} \end{array} \quad \mathbf{l} R \end{array}$$

- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_2: ! \Upsilon 3, ! F_4, ! F_4 \vdash ! F_6}{\bullet \mathbf{h}_2: ! \Upsilon 3, ! F_4 \vdash ! F_6} \quad \mathcal{C} \quad \\ \frac{\bullet \mathbf{h}_2: ! \Upsilon 3, ! F_4 \vdash ! F_6}{-: (! \Upsilon 3, ! F_4), \Delta_5 \vdash \top} \quad \mathcal{C} \\ \\ \frac{-: (! \Upsilon 3, ! F_4), \Delta_5 \vdash \top}{-: ! \Upsilon 3, \Delta_5, ! F_4 \vdash \top} \quad \top \end{array}$$

• Case rule $\&_R$

$$\frac{\mathbf{h}_{2} : ! \Upsilon 3, ! \mathsf{F}_{4}, ! \mathsf{F}_{4} \vdash ! \mathsf{F}_{6}}{\bullet \mathsf{h}_{2} : ! \Upsilon 3, ! \mathsf{F}_{4} \vdash ! \mathsf{F}_{6}} C \xrightarrow{\mathbf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathsf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{9} \quad \mathsf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathsf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{10}}{\bullet \mathsf{h}_{8} : contract(\mathsf{sn}_{7}, ! \mathsf{F}_{6}), \Delta_{5} \vdash \mathsf{F}_{9} \& \mathsf{F}_{10}} \underbrace{\mathsf{Cut}} \\ - : (! \Upsilon 3, ! \mathsf{F}_{4}), \Delta_{5} \vdash \mathsf{F}_{9} \& \mathsf{F}_{10} \\ \xrightarrow{\bullet} \\ \frac{\mathsf{h}_{2} : ! \Upsilon 3, ! \mathsf{F}_{4}, ! \mathsf{F}_{4} \vdash ! \mathsf{F}_{6}}{\bullet} \mathsf{ax} \xrightarrow{\mathbf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathsf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{9}} \underbrace{\mathsf{ax}}_{\mathsf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathsf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{10}} \underbrace{\mathsf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathsf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{9} \& \mathsf{F}_{10}}_{\mathsf{h}_{8} : \Delta_{5}, ! \mathsf{F}_{4}, ! \mathsf{F}_{4} \vdash \mathsf{F}_{9} \& \mathsf{F}_{10}} \underbrace{\mathsf{h}_{1}}_{\mathsf{Cut}} \underbrace{\mathsf{h}_{1} \mathsf{Cut}}_{\mathsf{h}_{1} \mathsf{Cut}}$$

• Case rule \multimap_R

$$\frac{\mathbf{h}_2: !\Upsilon 3, !\mathsf{F}_4, !\mathsf{F}_4 \vdash !\mathsf{F}_6}{\underbrace{\bullet \mathbf{h}_2: !\Upsilon 3, !\mathsf{F}_4 \vdash !\mathsf{F}_6}} \ C \quad \frac{\mathbf{h}_8: \Delta_5, \mathsf{F}_9, !\mathsf{F}_6, contract(\mathsf{n}_7, !\mathsf{F}_6) \vdash \mathsf{F}_{10}}{\bullet \mathbf{h}_8: contract(\mathsf{s}\mathsf{n}_7, !\mathsf{F}_6), \Delta_5 \vdash \mathsf{F}_9 \multimap \mathsf{F}_{10}} \quad \underbrace{-\circ_R}_{\mathsf{Cut}} \\ -: (!\Upsilon 3, !\mathsf{F}_4), \Delta_5 \vdash \mathsf{F}_9 \multimap \mathsf{F}_{10} \\ & \stackrel{\bullet}{\longrightarrow} \\ \frac{\bullet \mathbf{h}_2: !\Upsilon 3, !\mathsf{F}_4 \vdash !\mathsf{F}_6}{} \quad \underset{\mathsf{n}_8: \Delta_5, \mathsf{F}_9, !\mathsf{F}_6, contract(\mathsf{n}_7, !\mathsf{F}_6) \vdash \mathsf{F}_{10}}{} \quad \underset{\mathsf{n}_8: \Delta_5, \mathsf{F}_9, !\mathsf{F}_4 \vdash \mathsf{F}_{10}}{} \quad -\circ_R}_{\mathsf{n}_{\mathsf{Cut}}} \\ & \frac{-: !\Upsilon 3, \Delta_5, \mathsf{F}_9, !\mathsf{F}_4 \vdash \mathsf{F}_{10}}{-: !\Upsilon 3, \Delta_5, !\mathsf{F}_4 \vdash \mathsf{F}_9 \multimap \mathsf{F}_{10}} \quad -\circ_R}_{\mathsf{n}_{\mathsf{Cut}}}$$

• Case rule \bigoplus_{R_2}

$$\frac{\mathbf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4, ! \mathsf{F}_4 \vdash ! \mathsf{F}_6}{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_6} \quad C \quad \frac{\mathbf{h}_8 : \Delta_5, ! \mathsf{F}_6, contract(\mathsf{n}_7, ! \mathsf{F}_6) \vdash \mathsf{F}_{10}}{\bullet \mathsf{h}_8 : contract(\mathsf{sn}_7, ! \mathsf{F}_6), \Delta_5 \vdash \mathsf{F}_9 \oplus \mathsf{F}_{10}} \quad \frac{\oplus R_2}{\mathsf{Cut}} \\ \\ \frac{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_6}{\bullet \mathsf{h}_8 : \Delta_5, ! \mathsf{F}_6, contract(\mathsf{n}_7, ! \mathsf{F}_6) \vdash \mathsf{F}_{10}} \quad \overset{\mathsf{ax}}{\mathsf{h}_8 : \Delta_5, ! \mathsf{F}_6, contract(\mathsf{n}_7, ! \mathsf{F}_6) \vdash \mathsf{F}_{10}}} \\ \frac{- : ! \Upsilon 3, \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_9 \oplus \mathsf{F}_{10}} \quad \oplus R_2} \\ \\ \\ \frac{\mathsf{h}_2 : ! \Upsilon 3, \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_{10}} \quad \oplus R_2} \\ \\ \\ \frac{\mathsf{h}_3 : \mathsf{h}_3 : \mathsf{h}_4 \vdash \mathsf{h}_4 \vdash \mathsf{h}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_{10}} \quad \oplus R_2} \\ \\ \\ \frac{\mathsf{h}_3 : \mathsf{h}_4 \vdash \mathsf{h}_5 \vdash \mathsf{h}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{h}_4 \vdash \mathsf{h}_{10}} \quad \oplus R_2} \\ \\ \\ \frac{\mathsf{h}_3 : \mathsf{h}_4 \vdash \mathsf{h}_5 \vdash \mathsf{h}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{h}_4 \vdash \mathsf{h}_{10}} \quad \oplus R_2} \\ \\ \frac{\mathsf{h}_3 : \mathsf{h}_4 \vdash \mathsf{h}_5 \vdash \mathsf{h}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{h}_4 \vdash \mathsf{h}_{10}} \quad \oplus R_2} \\ \\ \frac{\mathsf{h}_4 : \mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{h}_4 \vdash \mathsf{h}_{10}} \quad \oplus R_2} \\ \\ \frac{\mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{h}_4 \vdash \mathsf{h}_{10}} \quad \oplus R_2} \\ \\ \frac{\mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_{10}}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{h}_4 \vdash \mathsf{h}_{10}} \quad \oplus R_2} \\ \\ \frac{\mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5}{- \mathsf{h}_5 \vdash \mathsf{h}_5} \quad \oplus \mathsf{h}_5 \vdash \mathsf{h}_5} \\ \frac{\mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5}{- \mathsf{h}_5 \vdash \mathsf{h}_5} \quad \oplus \mathsf{h}_5} \\ \frac{\mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5}{- \mathsf{h}_5 \vdash \mathsf{h}_5} \quad \oplus \mathsf{h}_5} \\ \frac{\mathsf{h}_5 \vdash \mathsf{h}_5 \vdash \mathsf{h}_5}{- \mathsf{h}_5 \vdash \mathsf{h}_5} \quad \oplus \mathsf{h}_5} \\ \frac{\mathsf{h}_5 \vdash \mathsf{h}_5}{- \mathsf{h}_5 \vdash \mathsf{h}_5} \quad \oplus \mathsf{h}_5} \\ \frac{\mathsf{h}_5 \vdash \mathsf{h}_5}{- \mathsf{h}_5 \vdash \mathsf{h}_5} \quad \oplus \mathsf{h}_5} \\ \frac{\mathsf{h}_5 \vdash \mathsf{h}_5}{- \mathsf{$$

• Case rule \oplus_{R_1}

$$\frac{\mathbf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4, ! \mathsf{F}_4 \vdash ! \mathsf{F}_6}{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_6} \quad C \quad \frac{\mathbf{h}_8 : \Delta_5, ! \mathsf{F}_6, contract(\mathsf{n}_7, ! \mathsf{F}_6) \vdash \mathsf{F}_9}{\bullet \mathsf{h}_8 : contract(\mathsf{s}_{\mathsf{n}_7}, ! \mathsf{F}_6), \Delta_5 \vdash \mathsf{F}_9 \oplus \mathsf{F}_{10}} \quad \frac{\oplus_{R_1}}{\mathsf{Cut}} \\ - : (! \Upsilon 3, ! \mathsf{F}_4), \Delta_5 \vdash \mathsf{F}_9 \oplus \mathsf{F}_{10} \\ & \xrightarrow{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_6} \quad \overset{\mathsf{ax}}{=} \quad \frac{\mathsf{n}_8 : \Delta_5, ! \mathsf{F}_6, contract(\mathsf{n}_7, ! \mathsf{F}_6) \vdash \mathsf{F}_9}{\mathsf{h}_8 : \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_9} \quad \overset{\mathsf{d}_{R_1}}{=} \quad \overset{\mathsf{h}_{\mathsf{Cut}}}{\mathsf{h}_{\mathsf{Cut}}} \\ & \frac{- : ! \Upsilon 3, \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_9}{- : ! \Upsilon 3, \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_9 \oplus \mathsf{F}_{10}} \quad \oplus_{R_1} \\ \end{array}$$

• Case rule $\mathbf{1}_L$

$$\frac{\begin{array}{c} \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 \\ \underline{\bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5} \end{array} C \quad \begin{array}{c} \mathbf{h}_7: \Delta_9, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_7: contract(\mathbf{s}\mathbf{n}_6, ! \mathbf{F}_5), \mathbf{1}, \Delta_9 \vdash \mathbf{F}_8 \\ \\ -: (! \Upsilon 3, ! \mathbf{F}_4), \mathbf{1}, \Delta_9 \vdash \mathbf{F}_8 \\ \\ \underline{\bullet} \mathbf{h}_7: \Delta_9, ! \mathbf{F}_4 \vdash \mathbf{F}_8 \\ \hline -: 1, ! \Upsilon 3, \Delta_9, ! \mathbf{F}_4 \vdash \mathbf{F}_8 \\ \hline -: \mathbf{1}, ! \Upsilon 3, \Delta_9, ! \mathbf{F}_4 \vdash \mathbf{F}_8 \end{array} \mathbf{1}_L \end{array} \quad \mathbf{hCut}$$

• Case rule \otimes_R

$$\frac{\frac{\mathbf{h}_{2} : ! \Upsilon 3, ! F_{4}, ! F_{4} + ! F_{5}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! F_{4} + ! F_{5}}}{\circ \mathbf{h}_{2} : ! \Upsilon 3, ! F_{4} + ! F_{5}}} C \frac{\mathbf{h}_{7} : \Delta_{11}, ! F_{5}, contract(\mathbf{n}_{6}, ! F_{5}), \Delta_{8}, \Delta_{11} \vdash F_{9} \otimes F_{10}}{\bullet \mathbf{h}_{7} : contract(\mathbf{n}_{6}, ! F_{5}), \Delta_{8}, \Delta_{11} \vdash F_{9} \otimes F_{10}}} \underbrace{\mathbf{cut}}^{\bullet \mathbf{n}_{7} : \Delta_{11}, ! F_{5}} \frac{\mathbf{n}_{7} : \Delta_{11}, ! F_{5}, contract(\mathbf{n}_{6}, ! F_{5}) \vdash F_{9}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! F_{4} \vdash ! F_{5}} \underbrace{\mathbf{n}_{7} : \Delta_{11}, ! F_{5}, contract(\mathbf{n}_{6}, ! F_{5}) \vdash F_{9}}_{- : ! \Upsilon 3, \Delta_{11}, \Delta_{8}, ! F_{4} \vdash F_{9} \otimes F_{10}} \underbrace{\mathbf{n}_{7} : \Delta_{11}, ! F_{5} \vdash F_{9}}_{- : ! \Upsilon 3, \Delta_{11}, \Delta_{8}, ! F_{4} \vdash F_{9} \otimes F_{10}} \underbrace{\mathbf{n}_{8} : \mathcal{N}_{11}, \mathcal{N}_{11},$$

$$\frac{\mathbf{h}_{2} : ! \Upsilon 3, ! F_{4}, ! F_{4} \vdash ! F_{5}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! F_{4} \vdash ! F_{5}} C \xrightarrow{\mathbf{h}_{7} : \Delta_{8} \vdash F_{9} \quad \mathbf{h}_{7} : \Delta_{11}, ! F_{5}, contract(\mathbf{n}_{6}, ! F_{5}) \vdash F_{10}}{\bullet \mathbf{h}_{7} : contract(\mathbf{s}\mathbf{n}_{6}, ! F_{5}), \Delta_{8}, \Delta_{11} \vdash F_{9} \otimes F_{10}} \xrightarrow{\mathbf{Cut}} C_{\mathbf{u}\mathbf{t}} = \frac{-: (! \Upsilon 3, ! F_{4}), \Delta_{8}, \Delta_{11} \vdash F_{9} \otimes F_{10}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! F_{4} \vdash ! F_{5}} \xrightarrow{\mathbf{ax}} \underbrace{\overset{\leadsto}{\mathbf{h}_{7} : \Delta_{11}, ! F_{5}, contract(\mathbf{n}_{6}, ! F_{5}) \vdash F_{10}}}_{-: ! \Upsilon 3, \Delta_{11}, \Delta_{8}, ! F_{4} \vdash F_{9} \otimes F_{10}} \otimes_{R}$$

\bullet Case rule W

$$\begin{array}{c} \frac{\mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4}, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{5}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{5}} & C & \frac{\mathbf{h}_{7} : \Delta_{10}, ! \mathbf{F}_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{5}) \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{7} : contract(\mathbf{s}\mathbf{n}_{6}, ! \mathbf{F}_{5}), \Delta_{10}, ! \mathbf{F}_{8} \vdash \mathbf{F}_{9}} & Cut \\ \hline & - : (! \Upsilon 3, ! \mathbf{F}_{4}), \Delta_{10}, ! \mathbf{F}_{8} \vdash \mathbf{F}_{9} & \\ \hline & \bullet \mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{5} & \mathbf{ax} & \\ \hline & - : ! \Upsilon 3, \Delta_{10}, ! \mathbf{F}_{4} \vdash \mathbf{F}_{9} & W \\ \hline & - : ! \Upsilon 3, \Delta_{10}, ! \mathbf{F}_{4} \vdash \mathbf{F}_{9} & W \\ \hline & \bullet \mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{8} & C & \frac{\mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{7} : contract(\mathbf{s}\mathbf{n}_{6}, ! \mathbf{F}_{8}), \Delta_{5} \vdash \mathbf{F}_{9}} & W \\ \hline & \bullet \mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{8} & C & \frac{\mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{s}\mathbf{n}_{6}, ! \mathbf{F}_{8}), \Delta_{5} \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{7} : contract(\mathbf{s}\mathbf{n}_{6}, ! \mathbf{F}_{8}), \Delta_{5} \vdash \mathbf{F}_{9}} & Cut \\ \hline & \bullet \mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{8} & \mathbf{ax} & \mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9} & \mathbf{ax} \\ & \bullet \mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{8} & \mathbf{ax} & \mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{8} & \mathbf{ax} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{h}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{5}, \mathbf{h}_{5} \vdash \mathbf{h}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{5}, \mathbf{h}_{7} \vdash \mathbf{h}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{7}, \mathbf{h}_{7} \vdash \mathbf{h}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{7}, \mathbf{h}_{7} \vdash \mathbf{h}_{7} \vdash \mathbf{h}_{9} & \mathbf{h}_{C} \\ \hline \bullet \mathbf{h}_{7} : \Delta_{7}, \mathbf{h}_{7} \vdash \mathbf{h}_{9} & \mathbf{h}_{C} \\$$

\bullet Case rule C

$$\begin{array}{c} \frac{\mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5}{\bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5} & C & \frac{\mathbf{h}_7: \Delta_{10}, ! \mathbf{F}_5, ! \mathbf{F}_8, ! \mathbf{F}_8, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: contract(s\mathbf{n}_6, ! \mathbf{F}_5), \Delta_{10}, ! \mathbf{F}_8 \vdash \mathbf{F}_9} & Cut \\ & -: (! \Upsilon 3, ! \mathbf{F}_4), \Delta_{10}, ! \mathbf{F}_8 \vdash \mathbf{F}_9 & \\ & & & & \\ \hline \bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 & \mathbf{ax} & \\ \hline & \frac{-: ! \Upsilon 3, \Delta_{10}, ! \mathbf{F}_5, ! \mathbf{F}_8, ! \mathbf{F}_8, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: \Delta_{10}, ! \mathbf{F}_4, ! \mathbf{F}_8 \vdash \mathbf{F}_9} & C \\ \hline & \frac{-: ! \Upsilon 3, \Delta_{10}, ! \mathbf{F}_4, ! \mathbf{F}_8 \vdash \mathbf{F}_9}{-: ! \Upsilon 3, \Delta_{10}, ! \mathbf{F}_4, ! \mathbf{F}_8 \vdash \mathbf{F}_9} & C \\ \hline & \frac{\mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4, ! \mathbf{F}_8}{\bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_8} & C & \frac{\mathbf{h}_7: \Delta_5, ! \mathbf{F}_8, ! \mathbf{F}_8, contract(\mathbf{n}_6, ! \mathbf{F}_8) \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: contract(s\mathbf{n}_6, ! \mathbf{F}_8), \Delta_5 \vdash \mathbf{F}_9} & Cut \\ \hline & -: (! \Upsilon 3, ! \mathbf{F}_4), \Delta_5 \vdash \mathbf{F}_9 & \\ \hline \bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_8 & \mathbf{ax} & \\ \hline \bullet \mathbf{h}_7: \Delta_5, ! \mathbf{F}_8, ! \mathbf{F}_8, contract(\mathbf{n}_6, ! \mathbf{F}_8) \vdash \mathbf{F}_9} & \mathbf{ax} \\ \hline & -: ! \Upsilon 3, \Delta_5, ! \mathbf{F}_4 \vdash \mathbf{F}_9 & \mathbf{h} \mathsf{Cut} \\ \hline \end{array}$$

ullet Case rule !L

$$\frac{\mathbf{h}_{2} : ! \Upsilon 3, ! F_{4}, ! F_{4} \vdash ! F_{5}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! F_{4} \vdash ! F_{5}} \quad C \quad \frac{\mathbf{h}_{7} : \Delta_{10}, F_{8}, ! F_{5}, contract(\mathbf{n}_{6}, ! F_{5}) \vdash F_{9}}{\bullet \mathbf{h}_{7} : contract(\mathbf{s}\mathbf{n}_{6}, ! F_{5}), \Delta_{10}, ! F_{8} \vdash F_{9}} \quad cut} \quad \frac{\mathbf{L}}{\mathbf{L}} \\ \frac{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! F_{4} \vdash ! F_{5}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! F_{4} \vdash ! F_{8}} \quad ax \quad \frac{\overset{\bullet}{\mathbf{h}_{7}} : \Delta_{10}, F_{8}, ! F_{5}, contract(\mathbf{n}_{6}, ! F_{5}) \vdash F_{9}}{\bullet \mathbf{h}_{7} : \Delta_{10}, F_{8}, ! F_{4} \vdash F_{9}} \quad !L} \\ \frac{\mathbf{h}_{2} : ! \Upsilon 3, ! F_{4}, ! F_{4} \vdash ! F_{8}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! F_{4} \vdash ! F_{8}} \quad C \quad \overset{\mathbf{h}_{7} : \Delta_{5}, F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}}{\bullet \mathbf{h}_{7} : contract(\mathbf{s}\mathbf{n}_{6}, ! F_{8}), \Delta_{5} \vdash F_{9}} \quad !L} \\ - : (! \Upsilon 3, ! F_{4}), \Delta_{5} \vdash F_{9} \\ \overset{\bullet}{\bullet} \mathbf{h}_{7} : \Delta_{5}, F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}}{\bullet \mathbf{h}_{7} : \Delta_{5}, F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad the cut} \\ \frac{\mathbf{h}_{2} : ! \Upsilon 3, ! F_{4}, ! F_{4} \vdash ! F_{8}}{\bullet \mathbf{h}_{7} : \Delta_{5}, F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad dx}{\bullet \mathbf{h}_{7} : \Delta_{5}, F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad the cut} \\ \frac{\mathbf{h}_{2} : ! \Upsilon 3, ! F_{4}, ! F_{4} \vdash ! F_{8}}{\bullet \mathbf{h}_{7} : \Delta_{5}, F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad dx}{\bullet \mathbf{h}_{7} : \Delta_{5}, F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad the cut} \\ \frac{\mathbf{h}_{2} : ! \Upsilon 3, ! F_{4}, ! F_{4} \vdash ! F_{8}}{\bullet \mathbf{h}_{7} : \Delta_{5}, ! F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad dx}{\bullet \mathbf{h}_{7} : \Delta_{5}, F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad the cut} \\ \frac{\mathbf{h}_{2} : ! \Upsilon 3, ! F_{4}, ! F_{4} \vdash ! F_{8}}{\bullet \mathbf{h}_{7} : \Delta_{5}, ! F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad the cut} \\ \frac{\mathbf{h}_{2} : ! \Upsilon 3, \Delta_{5}, ! F_{4} \vdash F_{9}}{\bullet \mathbf{h}_{7} : \Delta_{5}, F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad the cut} \\ \frac{\mathbf{h}_{2} : ! \Upsilon 3, \Delta_{5}, ! F_{4} \vdash F_{9}}{\bullet \mathbf{h}_{7} : \Delta_{5}, ! F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad the cut} \\ \frac{\mathbf{h}_{2} : ! \Upsilon 3, \Delta_{5}, ! F_{4} \vdash F_{9}}{\bullet \mathbf{h}_{7} : \Delta_{5}, ! F_{8}, contract(\mathbf{n}_{6}, ! F_{8}) \vdash F_{9}} \quad the cut} \\ \frac{\mathbf{h}_{2} : \mathbf{h}_{3} : \mathbf{h}_{4} : \mathbf{h}_{4} \vdash \mathbf{h}_{5}}{\bullet \mathbf{h}_{5}} \quad$$

• Case rule $\&_{L2}$

$$\frac{\mathbf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5}{\underbrace{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5}}_{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5} C \xrightarrow{\bullet \mathsf{h}_7 : \Delta_{11}, \mathsf{F}_9, ! \mathsf{F}_5, contract(\mathsf{n}_6, ! \mathsf{F}_5) \vdash \mathsf{F}_{10}}_{\bullet \mathsf{h}_7 : contract(\mathsf{s}_{\mathsf{n}_6}, ! \mathsf{F}_5), \Delta_{11}, \mathsf{F}_8 \& \mathsf{F}_9 \vdash \mathsf{F}_{10}} Cut} \underbrace{\bullet}_{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5}^{\bullet \mathsf{L}_1} \underbrace{\bullet}_{\mathsf{h}_7 : \Delta_{11}, \mathsf{F}_9, ! \mathsf{F}_5, contract(\mathsf{n}_6, ! \mathsf{F}_5) \vdash \mathsf{F}_{10}}_{\mathsf{h}_7 : \Delta_{11}, \mathsf{F}_9, ! \mathsf{F}_4 \vdash \mathsf{F}_{10}}^{\bullet \mathsf{L}_2} \underbrace{\bullet}_{\mathsf{h}\mathsf{Cut}}$$

• Case rule $\&_{L1}$

$$\frac{\mathbf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5}{\underbrace{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5}}_{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5} C \quad \underbrace{\frac{\mathsf{h}_7 : \Delta_{11}, \mathsf{F}_8, ! \mathsf{F}_5, contract(\mathsf{n}_6, ! \mathsf{F}_5) \vdash \mathsf{F}_{10}}_{\bullet \mathsf{h}_7 : contract(\mathsf{s}_{\mathsf{n}_6}, ! \mathsf{F}_5), \Delta_{11}, \mathsf{F}_8 \& \mathsf{F}_9 \vdash \mathsf{F}_{10}}}_{\bullet \mathsf{cut}} \underbrace{\frac{}{\mathsf{e}_{\mathsf{h}_2} : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5}}_{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5} \underbrace{\frac{\mathsf{ax}}{\mathsf{h}_7 : \Delta_{11}, \mathsf{F}_8, ! \mathsf{F}_5, contract(\mathsf{n}_6, ! \mathsf{F}_5) \vdash \mathsf{F}_{10}}}_{\bullet \mathsf{h}_2 : ! \Upsilon 3, \Delta_{11}, ! \mathsf{F}_8, ! \mathsf{F}_4 \vdash \mathsf{F}_{10}}}_{\bullet \mathsf{h}_2 : ! \Upsilon 3, \Delta_{11}, ! \mathsf{F}_4, \mathsf{F}_8 \& \mathsf{F}_9 \vdash \mathsf{F}_{10}}} \underbrace{\mathcal{k}_{L1}}_{\bullet \mathsf{cut}}$$

• Case rule \otimes_L

$$\frac{\mathbf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5}{\underbrace{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5}}_{\bullet \mathsf{h}_2 : ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_5} C \xrightarrow{\bullet \mathsf{h}_7 : \Delta_{11}, \mathsf{F}_8, \mathsf{F}_9, ! \mathsf{F}_5, contract(\mathsf{n}_6, ! \mathsf{F}_5) \vdash \mathsf{F}_{10}}_{\bullet \mathsf{h}_7 : contract(\mathsf{s}_{\mathsf{n}_6}, ! \mathsf{F}_5), \Delta_{11}, \mathsf{F}_8 \otimes \mathsf{F}_9 \vdash \mathsf{F}_{10}}_{\bullet \mathsf{cut}} \\ \underbrace{- : (! \Upsilon 3, ! \mathsf{F}_4) \vdash ! \mathsf{F}_5}_{\bullet \mathsf{h}_7 : \Delta_{11}, \mathsf{F}_8, \mathsf{F}_9, ! \mathsf{F}_5, contract(\mathsf{n}_6, ! \mathsf{F}_5) \vdash \mathsf{F}_{10}}_{\bullet \mathsf{h}_7 : \Delta_{11}, \mathsf{F}_8, \mathsf{F}_9, ! \mathsf{F}_5, contract(\mathsf{n}_6, ! \mathsf{F}_5) \vdash \mathsf{F}_{10}}_{\bullet \mathsf{cut}} \xrightarrow{\mathsf{hCut}}_{\bullet \mathsf{cut}}$$

• Case rule \oplus_L

$$\frac{\frac{\mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4}, ! \mathbf{F}_{4} + ! \mathbf{F}_{5}}{\bullet \mathbf{h}_{2} : ! \Upsilon 3, ! \mathbf{F}_{4} + ! \mathbf{F}_{5}}}{C} \underbrace{\frac{\mathbf{h}_{7} : \Delta_{11}, \mathbf{F}_{8}, ! \mathbf{F}_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{5}) \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_{7} : contract(\mathbf{s}_{\mathbf{n}_{6}}, ! \mathbf{F}_{5}), \Delta_{11}, \mathbf{F}_{8} \oplus \mathbf{F}_{9} \vdash \mathbf{F}_{10}}}_{-: (! \Upsilon 3, ! \mathbf{F}_{4}), \Delta_{11}, \mathbf{F}_{8} \oplus \mathbf{F}_{9} \vdash \mathbf{F}_{10}}} \underbrace{\mathbf{Cut}} \underbrace{-: (! \Upsilon 3, ! \mathbf{F}_{4}), \Delta_{11}, \mathbf{F}_{8} \oplus \mathbf{F}_{9} \vdash \mathbf{F}_{10}}_{\bullet \mathbf{m}_{7} : \Delta_{11}, \mathbf{F}_{8}, ! \mathbf{F}_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{5}) \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{7} : \Delta_{11}, \mathbf{F}_{8}, ! \mathbf{F}_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{5}) \vdash \mathbf{F}_{10}}} \underbrace{\mathbf{n}_{\mathbf{h}_{7} : \Delta_{11}, \mathbf{F}_{9}, ! \mathbf{F}_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{5}) \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{7} : \Delta_{11}, \mathbf{F}_{9}, ! \mathbf{F}_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{5}) \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{2}} \underbrace{\mathbf{n}_{\mathbf{h}_{7} : \Delta_{11}, \mathbf{F}_{9}, ! \mathbf{F}_{5}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{5}) \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{2}}}_{\bullet \mathbf{h}_{2}}$$

• Case rule \multimap_L

$$\frac{\mathbf{h}_{2} : | \Upsilon 3, | \mathbf{F}_{4}, | \mathbf{F}_{4} \vdash | \mathbf{F}_{5}}{\bullet \mathbf{h}_{2} : | \Upsilon 3, | \mathbf{F}_{4} \vdash | \mathbf{F}_{5}} C \xrightarrow{\mathbf{h}_{7} : \Delta_{12}, | \mathbf{F}_{5}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{9} \quad \mathbf{h}_{7} : \Delta_{8}, \mathbf{F}_{10} \vdash \mathbf{F}_{11}}{\bullet \mathbf{h}_{7} : contract(\mathbf{s}_{16}, | \mathbf{F}_{5}), \Delta_{8}, \Delta_{12}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11}} \xrightarrow{\mathbf{cut}} \neg \circ_{L} \\ - : (| \Upsilon 3, | \mathbf{F}_{4}), \Delta_{8}, \Delta_{12}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ \sim \\ - : (| \Upsilon 3, | \mathbf{F}_{4}), \Delta_{12}, | \mathbf{F}_{5}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{9} \\ - : | \Upsilon 3, \Delta_{12}, | \mathbf{F}_{5}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{9} \\ - : | \Upsilon 3, \Delta_{12}, | \mathbf{F}_{4} \vdash \mathbf{F}_{9} \\ - : | \Upsilon 3, \Delta_{12}, | \mathbf{F}_{4} \vdash \mathbf{F}_{9} \\ - : | \Upsilon 3, \Delta_{12}, \Delta_{8}, | \mathbf{F}_{4}, \mathbf{F}_{9} \multimap \mathbf{F}_{10} \vdash \mathbf{F}_{11} \\ - : | \Upsilon 3, | \mathbf{F}_{4}, | \mathbf{F}_{4} \vdash | \mathbf{F}_{5} \\ \bullet \mathbf{h}_{7} : \Delta_{11}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{8} \quad \mathbf{h}_{7} : \Delta_{12}, \mathbf{F}_{9}, | \mathbf{F}_{5} \vdash \mathbf{F}_{10} \\ - : (| \Upsilon 3, | \mathbf{F}_{4}), \Delta_{11}, \Delta_{12}, \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : (| \Upsilon 3, | \mathbf{F}_{4}), \Delta_{11}, \Delta_{12}, \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_{7} : \Delta_{11}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{8} \quad \mathbf{ax} \\ \bullet \mathbf{h}_{7} : \Delta_{11}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{8} \quad \mathbf{ax} \\ \bullet \mathbf{h}_{7} : \Delta_{11}, contract(\mathbf{n}_{6}, | \mathbf{F}_{5}) \vdash \mathbf{F}_{8} \quad \mathbf{ax} \\ \bullet \mathbf{h}_{7} : \Delta_{11}, \Delta_{12}, | \mathbf{F}_{8}, | \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : | \Upsilon 3, \Delta_{11}, \Delta_{12}, | \mathbf{F}_{4}, | \mathbf{F}_{4}, | \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : | \Upsilon 3, \Delta_{11}, \Delta_{12}, | \mathbf{F}_{4}, | \mathbf{F}_{4}, | \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : | \Upsilon 3, \Delta_{11}, \Delta_{12}, | \mathbf{F}_{4}, | \mathbf{F}_{4}, | \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : | \Upsilon 3, \Delta_{11}, \Delta_{12}, | \mathbf{F}_{4}, | \mathbf{F}_{4}, | \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : | \Upsilon 3, \Delta_{11}, \Delta_{12}, | \mathbf{F}_{4}, | \mathbf{F}_{4}, | \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : | \Upsilon 3, \Delta_{11}, \Delta_{12}, | \mathbf{F}_{4}, | \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : | \Upsilon 3, \Delta_{11}, \Delta_{12}, | \mathbf{F}_{4}, | \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : | \Upsilon 3, \Delta_{11}, \Delta_{12}, | \mathbf{F}_{4}, | \mathbf{F}_{8} \multimap \mathbf{F}_{9} \vdash \mathbf{F}_{10} \\ - : | \Upsilon 3, \Delta_{11}, \Delta_{12}, | \mathbf{F}_{4}, | \mathbf$$

$$\frac{\frac{h_2 : ! \Upsilon 3, ! F_4, ! F_4 \vdash ! F_5}{\bullet h_2 : ! \Upsilon 3, ! F_4 \vdash ! F_5}}{\bullet h_2 : ! \Upsilon 3, ! F_4 \vdash ! F_5} C \xrightarrow{\begin{array}{c} h_7 : \Delta_{11}, ! F_5 \vdash F_8 & h_7 : \Delta_{12}, F_9, contract(n_6, ! F_5) \vdash F_{10} \\ \bullet h_7 : contract(sn_6, ! F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\ \\ \hline \\ - : (! \Upsilon 3, ! F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\ \\ \hline \\ \begin{array}{c} h_7 : \Delta_{11}, ! F_5 \vdash F_8 & \text{ax} \\ \hline \\ \bullet h_7 : \Delta_{11}, \Delta_{12}, ! F_5, F_8 \multimap F_9, contract(n_6, ! F_5) \vdash F_{10} \\ \hline \\ \bullet h_7 : \Delta_{11}, \Delta_{12}, ! F_5, F_8 \multimap F_9, contract(n_6, ! F_5) \vdash F_{10} \\ \hline \\ \hline \\ - : ! \Upsilon 3, \Delta_{11}, \Delta_{12}, ! F_4, ! F_4, F_8 \multimap F_9 \vdash F_{10} \\ \hline \\ - : ! \Upsilon 3, \Delta_{11}, \Delta_{12}, ! F_4, F_8 \multimap F_9 \vdash F_{10} \\ \hline \\ \bullet h_2 : ! \Upsilon 3, ! F_4, ! F_4 \vdash ! F_5 \\ \hline \\ \bullet h_2 : ! \Upsilon 3, ! F_4 \vdash ! F_5 \\ \hline \end{array} C \xrightarrow{\begin{array}{c} h_7 : \Delta_8 \vdash F_9 & h_7 : \Delta_{12}, F_{10}, ! F_5, contract(n_6, ! F_5) \vdash F_{11} \\ \hline \\ \bullet h_7 : contract(sn_6, ! F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\ \hline \\ - : (! \Upsilon 3, ! F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\ \hline \\ \hline \end{array} Cut$$

 \bullet Case rule I

6.12 Status of !L: OK

• Case rule !R

- Case rule $\mathbf{1}_R$
- \bullet Case rule \top

$$\frac{\frac{\mathbf{h}_2: F_4, !\Upsilon 3 \vdash !F_6}{\bullet \mathbf{h}_2: !\Upsilon 3, !F_4 \vdash !F_6}}{-: (!\Upsilon 3, !F_4), \Delta_5 \vdash \top} \begin{array}{c} \bot \\ \bullet \mathbf{h}_8: contract(s\mathbf{n}_7, !F_6), \Delta_5 \vdash \top \\ \hline -: (!\Upsilon 3, !F_4), \Delta_5 \vdash \top \\ \hline -: !\Upsilon 3, \Delta_5, !F_4 \vdash \top \end{array} \top$$

• Case rule $\&_R$

$$\frac{\frac{\mathbf{h}_{2}: \mathbf{F}_{4}, ! \Upsilon 3 \vdash ! \mathbf{F}_{6}}{\bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{6}}}{\bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{6}}} : L \xrightarrow{\mathbf{h}_{8}: \Delta_{5}, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{9}} \underbrace{\mathbf{h}_{8}: \Delta_{5}, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{10}}}_{\bullet \mathbf{h}_{8}: contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}), \Delta_{5} \vdash \mathbf{F}_{9} \& \mathbf{F}_{10}} \underbrace{\mathbf{Cut}}_{-: (! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{6}} \underbrace{\mathbf{n}_{8}: \Delta_{5}, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{9}}_{\bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{6}} \underbrace{\mathbf{n}_{8}: \Delta_{5}, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{10}}_{\bullet \mathbf{h}_{2}: ! \Upsilon 3, \Delta_{5}, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{6}} \underbrace{\mathbf{n}_{8}: \Delta_{5}, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{10}}_{\bullet \mathbf{h}_{2}: ! \Upsilon 3, \Delta_{5}, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{6}} \underbrace{\mathbf{n}_{8}: \Delta_{5}, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{10}}_{\bullet \mathbf{h}_{2}: ! \Upsilon 3, \Delta_{5}, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{10}} \underbrace{\mathbf{n}_{8}: \Delta_{5}, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{10}}_{\bullet \mathbf{h}_{2}: ! \Upsilon 3, \Delta_{5}, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{10}} \underbrace{\mathbf{n}_{8}: \Delta_{5}, ! \mathbf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathbf{F}_{6}) \vdash \mathbf{F}_{10}}_{\bullet \mathbf{h}_{2}: \mathsf{h}_{3}: \mathsf{h}_{4}: \mathsf{h}_{5}: \mathsf{h}_{5$$

• Case rule \multimap_R

• Case rule \oplus_{R_2}

$$\frac{\begin{array}{c} \mathbf{h}_2: \mathsf{F}_4, ! \Upsilon 3 \vdash ! \mathsf{F}_6 \\ \bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_6 \end{array} \ ! L \quad \frac{\mathbf{h}_8: \Delta_5, ! \mathsf{F}_6, contract(\mathbf{n}_7, ! \mathsf{F}_6) \vdash \mathsf{F}_{10}}{\bullet \mathbf{h}_8: contract(s\mathbf{n}_7, ! \mathsf{F}_6), \Delta_5 \vdash \mathsf{F}_9 \oplus \mathsf{F}_{10}} \quad \frac{\oplus_{R_2}}{\mathsf{Cut}} \\ \\ -: (! \Upsilon 3, ! \mathsf{F}_4), \Delta_5 \vdash \mathsf{F}_9 \oplus \mathsf{F}_{10} \\ & \longrightarrow \\ \bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathsf{F}_4 \vdash ! \mathsf{F}_6} \quad \text{ax} \quad \frac{}{\mathbf{h}_8: \Delta_5, ! \mathsf{F}_6, contract(\mathbf{n}_7, ! \mathsf{F}_6) \vdash \mathsf{F}_{10}}}{\bullet \mathbf{h}_8: \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_{10}} \quad \oplus_{R_2}} \quad \text{ax} \\ \\ \frac{-: ! \Upsilon 3, \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_{10}}{-: ! \Upsilon 3, \Delta_5, ! \mathsf{F}_4 \vdash \mathsf{F}_{10}} \quad \oplus_{R_2}} \end{array}$$

• Case rule \bigoplus_{R_1}

$$\begin{array}{c} \mathbf{h}_{2}: \mathsf{F}_{4}, ! \Upsilon 3 \vdash ! \mathsf{F}_{6} \\ \bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathsf{F}_{4} \vdash ! \mathsf{F}_{6} \\ \bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathsf{F}_{4} \vdash ! \mathsf{F}_{6} \\ \end{array} \underbrace{ \begin{array}{c} \mathbf{h}_{8}: \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{9} \\ \bullet \mathbf{h}_{8}: contract(s\mathbf{n}_{7}, ! \mathsf{F}_{6}), \Delta_{5} \vdash \mathsf{F}_{9} \oplus \mathsf{F}_{10} \\ & \hookrightarrow \\ \bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathsf{F}_{4} \vdash ! \mathsf{F}_{6} \\ \end{array} \underbrace{ \begin{array}{c} \mathsf{n}_{8}: \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{9} \\ \bullet \mathsf{h}_{2}: ! \Upsilon 3, ! \mathsf{F}_{4} \vdash ! \mathsf{F}_{6} \\ \end{array} \underbrace{ \begin{array}{c} \mathsf{n}_{8}: \Delta_{5}, ! \mathsf{F}_{6}, contract(\mathbf{n}_{7}, ! \mathsf{F}_{6}) \vdash \mathsf{F}_{9} \\ \bullet \mathsf{h}_{2}: ! \Upsilon 3, \Delta_{5}, ! \mathsf{F}_{4} \vdash \mathsf{F}_{9} \\ \hline -: ! \Upsilon 3, \Delta_{5}, ! \mathsf{F}_{4} \vdash \mathsf{F}_{9} \oplus \mathsf{F}_{10} \\ \end{array} \underbrace{ \begin{array}{c} \mathsf{n}_{1} \mathsf{n}_{2} \mathsf{n}_{3} \mathsf{n}_{4} \mathsf{n$$

• Case rule $\mathbf{1}_L$

$$\frac{\begin{array}{l} \frac{\mathbf{h}_2: \mathbf{F}_4, |\Upsilon 3 \vdash |\mathbf{F}_5}{\bullet \mathbf{h}_2: |\Upsilon 3, |\mathbf{F}_4 \vdash |\mathbf{F}_5} \end{array}{!}L \quad \frac{\mathbf{h}_7: \Delta_9, |\mathbf{F}_5, contract(\mathbf{n}_6, |\mathbf{F}_5) \vdash \mathbf{F}_8}{\bullet \mathbf{h}_7: contract(s\mathbf{n}_6, |\mathbf{F}_5), \mathbf{1}, \Delta_9 \vdash \mathbf{F}_8} \\ -: (|\Upsilon 3, |\mathbf{F}_4), \mathbf{1}, \Delta_9 \vdash \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_2: |\Upsilon 3, |\mathbf{F}_4 \vdash |\mathbf{F}_5} \quad \text{ax} \quad \xrightarrow[\mathbf{h}_7: \Delta_9, |\mathbf{F}_5, contract(\mathbf{n}_6, |\mathbf{F}_5) \vdash \mathbf{F}_8} \\ -: |\Upsilon 3, \Delta_9, |\mathbf{F}_4 \vdash \mathbf{F}_8} \quad \mathbf{1}_L \\ \hline -: \mathbf{1}, |\Upsilon 3, \Delta_9, |\mathbf{F}_4 \vdash \mathbf{F}_8} \quad \mathbf{1}_L \\ \end{array}$$

• Case rule \otimes_R

$$\frac{\frac{h_2:F_4,!\Upsilon 3 \vdash !F_5}{\bullet h_2:!\Upsilon 3,!F_4 \vdash !F_5}}{\bullet h_2:!\Upsilon 3,!F_4 \vdash !F_5} \underbrace{!L} \begin{array}{c} \frac{h_7:\Delta_{11},!F_5,contract(n_6,!F_5) \vdash F_9 - h_7:\Delta_8 \vdash F_{10}}{\bullet h_7:contract(sn_6,!F_5),\Delta_8,\Delta_{11} \vdash F_9 \otimes F_{10}} \\ \hline -:(!\Upsilon 3,!F_4),\Delta_8,\Delta_{11} \vdash F_9 \otimes F_{10} \\ \hline \bullet h_2:!\Upsilon 3,!F_4 \vdash !F_5 \end{array} \begin{array}{c} \text{ax} \\ \hline h_7:\Delta_{11},!F_5,contract(n_6,!F_5) \vdash F_9 \\ \hline -:!\Upsilon 3,\Delta_{11},!F_4 \vdash F_9 \\ \hline -:!\Upsilon 3,\Delta_{11},!F_4 \vdash F_9 \end{array} \begin{array}{c} \text{ax} \\ \hline -:!\Upsilon 3,\Delta_{11},h_4 \vdash F_9 \\ \hline \bullet h_2:!\Upsilon 3,!F_4 \vdash !F_5 \end{array} \begin{array}{c} \text{ax} \\ \hline \bullet h_7:\Delta_{10},contract(n_6,!F_5) \vdash F_8 - h_7:\Delta_{11},lF_5 \vdash F_9 \\ \hline \bullet h_7:contract(sn_6,!F_5),\Delta_{10},\Delta_{11} \vdash F_8 \otimes F_9 \end{array} \begin{array}{c} \text{Cut} \\ \hline -:(!\Upsilon 3,!F_4 \vdash !F_5) \end{array} \begin{array}{c} \text{ax} \\ \hline \bullet h_7:\Delta_{10},contract(sn_6,!F_5) \vdash F_8 - h_7:\Delta_{11},lF_5 \vdash F_9 \\ \hline \bullet h_7:Contract(sn_6,!F_5),\Delta_{10},\Delta_{11} \vdash F_8 \otimes F_9 \end{array} \begin{array}{c} \text{Cut} \\ \hline -:(!\Upsilon 3,!F_4 \vdash !F_5) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,!F_4 \vdash !F_5) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,lF_4),\Delta_{10},\Delta_{11} \vdash F_8 \otimes F_9 \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_9) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{10},\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline -:(!\Upsilon 3,\Delta_{11},lF_4 \vdash F_8) \end{array} \begin{array}{c} \text{ax} \\ \hline$$

\bullet Case rule W

\bullet Case rule C

\bullet Case rule !L

$$\begin{array}{c} \frac{\mathbf{h}_2: \mathbf{F}_4, ! \Upsilon 3 \vdash ! \mathbf{F}_5}{\bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5} & !L & \frac{\mathbf{h}_7: \Delta_{10}, \mathbf{F}_8, ! \mathbf{F}_5, contract(\mathbf{n}_6, ! \mathbf{F}_5) \vdash \mathbf{F}_9}{\bullet \mathbf{h}_7: contract(s\mathbf{n}_6, ! \mathbf{F}_5), \Delta_{10}, ! \mathbf{F}_8 \vdash \mathbf{F}_9} & !L \\ & -: (! \Upsilon 3, ! \mathbf{F}_4), \Delta_{10}, ! \mathbf{F}_8 \vdash \mathbf{F}_9 & \\ \hline \bullet \mathbf{h}_2: ! \Upsilon 3, ! \mathbf{F}_4 \vdash ! \mathbf{F}_5 & \mathbf{ax} & \\ \hline & \frac{-: ! \Upsilon 3, \Delta_{10}, \mathbf{F}_8, ! \mathbf{F}_4 \vdash \mathbf{F}_9}{-: ! \Upsilon 3, \Delta_{10}, ! \mathbf{F}_4, ! \mathbf{F}_8 \vdash \mathbf{F}_9} & !L & \\ \hline \end{array}$$

$$\frac{\frac{\mathbf{h}_{2}: \mathbf{F}_{4}, ! \Upsilon 3 \vdash ! \mathbf{F}_{8}}{\bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{8}}}{\bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{8}}} : L \xrightarrow{\mathbf{h}_{7}: \Delta_{5}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9}}{\bullet \mathbf{h}_{7}: contract(\mathbf{s}_{\mathbf{n}_{6}}, ! \mathbf{F}_{8}), \Delta_{5} \vdash \mathbf{F}_{9}}} : L \\ -: (! \Upsilon 3, ! \mathbf{F}_{4}), \Delta_{5} \vdash \mathbf{F}_{9} \xrightarrow{\text{cut}} \\ -: ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{8} \xrightarrow{\text{b} Inv} \xrightarrow{\bullet \mathbf{h}_{2}: ! \Upsilon 3, ! \mathbf{F}_{4} \vdash ! \mathbf{F}_{8}} \xrightarrow{\text{ax}} \frac{\mathbf{h}_{7}: \Delta_{5}, \mathbf{F}_{8}, contract(\mathbf{n}_{6}, ! \mathbf{F}_{8}) \vdash \mathbf{F}_{9}}{\mathbf{h}_{7}: \Delta_{5}, \mathbf{F}_{8}, ! \mathbf{F}_{4} \vdash \mathbf{F}_{9}} \xrightarrow{\text{cut}} \overset{\text{ax}}{\mathbf{h}_{7}: \Delta_{5}, \mathbf{F}_{8}, ! \mathbf{F}_{4} \vdash \mathbf{F}_{9}} \xrightarrow{\text{cut}} \overset{\text{h}_{7}: \Delta_{5}, \mathbf{F}_{8}, l \mathbf{F}_{4} \vdash \mathbf{F}_{9}} \xrightarrow{\text{cut}} \overset{\text{h}_{8}}{\mathbf{h}_{8}: \mathbf{h}_{8}: \mathbf{h}_{8}:$$

• Case rule $\&_{L2}$

$$\frac{\begin{array}{c} \mathbf{h}_2: \mathsf{F}_4, | \Upsilon 3 \vdash \! | \mathsf{F}_5 \\ \bullet \mathbf{h}_2: | \Upsilon 3, | \mathsf{F}_4 \vdash \! | \mathsf{F}_5 \end{array}}{\bullet \mathbf{h}_7: | \Upsilon 3, | \mathsf{F}_4 \vdash \! | \mathsf{F}_5 \end{array}} \underbrace{\begin{array}{c} \mathbf{h}_7: \Delta_{11}, \mathsf{F}_9, | \mathsf{F}_5, contract(\mathsf{n}_6, | \mathsf{F}_5) \vdash \mathsf{F}_{10} \\ \bullet \mathbf{h}_7: contract(\mathsf{s}_{\mathsf{n}_6}, | \mathsf{F}_5), \Delta_{11}, \mathsf{F}_8 \& \mathsf{F}_9 \vdash \mathsf{F}_{10} \end{array}}_{\bullet \mathsf{h}_7: \Delta_{11}, \mathsf{F}_8 \& \mathsf{F}_9 \vdash \mathsf{F}_{10}} \underbrace{\begin{array}{c} \mathcal{L}_2 \\ \bullet \mathsf{h}_2: | \Upsilon 3, | \mathsf{F}_4 \vdash \! | \mathsf{F}_5 \end{array}}_{\bullet \mathsf{h}_7: \Delta_{11}, \mathsf{F}_9, | \mathsf{F}_5, contract(\mathsf{n}_6, | \mathsf{F}_5) \vdash \mathsf{F}_{10} \\ \bullet \mathsf{h}_7: \Delta_{11}, \mathsf{F}_9, | \mathsf{F}_5, contract(\mathsf{n}_6, | \mathsf{F}_5) \vdash \mathsf{F}_{10} \\ \bullet \mathsf{h}_7: \Delta_{11}, \mathsf{F}_9, | \mathsf{F}_4 \vdash \mathsf{F}_{10} \\ \bullet \mathsf{h}_7: \Delta_{11}, \mathsf{F}_9, | \mathsf{F}_8 \& \mathsf{F}_9 \vdash \mathsf{F}_{10} \end{array}}_{\bullet \mathsf{h}_2} \underbrace{\begin{array}{c} \mathsf{h}_2 \\ \mathsf{h}_3 \\ \mathsf{h}_4 \\ \mathsf{h}_5 \\ \mathsf{h}$$

• Case rule $\&_{L1}$

• Case rule \otimes_L

• Case rule \oplus_L

$$\frac{\underbrace{\frac{\mathbf{h}_2: F_4, ! \Upsilon 3 \vdash ! F_5}{\bullet \mathbf{h}_2 :! \Upsilon 3, ! F_4 \vdash ! F_5}}_{\bullet \mathbf{h}_2 :! \Upsilon 3, ! F_4 \vdash ! F_5} \underbrace{!L} \underbrace{\frac{\mathbf{h}_7: \Delta_{11}, F_8, ! F_5, contract(\mathbf{n}_6, ! F_5) \vdash F_{10}}{\bullet \mathbf{h}_7 : contract(s\mathbf{n}_6, ! F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}}}_{\bullet \mathbf{h}_7 : contract(s\mathbf{n}_6, ! F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}}}_{\bullet \mathbf{t}_7 : \Delta_{11}, F_8, ! F_5, contract(\mathbf{n}_6, ! F_5) \vdash F_{10}}} \underbrace{\frac{\mathbf{c}}{\mathbf{c}}}_{\bullet \mathbf{h}_2 :! \Upsilon 3, ! F_4 \vdash ! F_5}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8, ! F_5, contract(\mathbf{n}_6, ! F_5) \vdash F_{10}}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8, ! F_8, contract(\mathbf{n}_6, ! F_5) \vdash F_{10}}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8, ! F_8, contract(\mathbf{n}_6, ! F_5) \vdash F_{10}}} \underbrace{\frac{\mathbf{c}}{\mathbf{h}_7 : \Delta_{11}, F_9, ! F_8, contract(\mathbf{n}_6, ! F_5) \vdash F_{10}}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8, ! F_8 \vdash F_{10}}} \underbrace{\frac{\mathbf{c}}{\mathbf{h}_7 : \Delta_{11}, F_8, ! F_8 \vdash F_{10}}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8 \vdash F_{10}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8 \vdash F_{10}}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8 \vdash F_{10}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8 \vdash F_{10}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8 \vdash F_{10}}}_{\bullet \mathbf{h}_7 : \Delta_{11}, F_8 \vdash F_{10}}_{\bullet \mathbf{h}_7 : \Delta_{11},$$

• Case rule \multimap_L

$$\frac{\underbrace{\frac{h_2:F_4,!\Upsilon 3 \vdash !F_5}{\bullet h_2:!\Upsilon 3,!F_4 \vdash !F_5}}_{\bullet h_2:!\Upsilon 3,!F_4 \vdash !F_5} : L \quad \underbrace{\frac{h_7:\Delta_{12},!F_5,contract(n_6,!F_5) \vdash F_9 \quad h_7:\Delta_8,F_{10} \vdash F_{11}}{\bullet h_7:contract(sn_6,!F_5),\Delta_8,\Delta_{12},F_9 \multimap F_{10} \vdash F_{11}}}_{\bullet h_2:!\Upsilon 3,!F_4 \vdash !F_5} \underbrace{-:(!\Upsilon 3,!F_4),\Delta_8,\Delta_{12},F_9 \multimap F_{10} \vdash F_{11}}_{\bullet h_2:!\Upsilon 3,\Delta_{12},!F_4 \vdash F_9} \underbrace{-:!\Upsilon 3,\Delta_{12},!F_5,contract(n_6,!F_5) \vdash F_9}_{h_2:!\Upsilon 3,\Delta_{12},LF_4 \vdash F_9} \underbrace{\frac{ax}{hCut}}_{-:\Delta_8,F_{10} \vdash F_{11}} \underbrace{\frac{ax}{-\circ_L}}_{\bullet L}$$

$$\frac{h_2: F_4, |\Upsilon 3 \vdash |F_5}{\bullet h_2: |\Upsilon 3, |F_4 \vdash |F_5}}{\iota 2} : L \quad \frac{h_7: \Delta_{11}, contract(n_6, |F_5) \vdash F_8}{\bullet h_7: \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}}}{\iota 2} \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}}{\iota 2} \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}}{\iota 2} \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |T_3, \Delta_{11}, |F_4 \vdash F_8) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |T_3, \Delta_{11}, \Delta_{12}, |F_4, |F_4, F_8 \multimap F_9 \vdash F_{10}} \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F_5) \quad \mathcal{O}_L \\ - : (|\Upsilon 3, |F_4 \vdash |F$$

 \bullet Case rule I

6.13 Status of $\&_{L2}$: OK

- Case rule !R
- Case rule $\mathbf{1}_{R}$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- \bullet Case rule W

• Case rule C• Case rule L• Case rule L

6.14 Status of $\&_{L1}$: OK

- Case rule !R
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule \otimes_R
- $\bullet \;$ Case rule W
- ullet Case rule C

- Case rule !L• Case rule $\&_{L2}$
- Case rule \otimes_L
- Case rule \oplus_L
- $\bullet\,$ Case rule I

6.15 Status of \otimes_L : OK

- Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule \otimes_R
- $\bullet \;$ Case rule W
- $\bullet\,$ Case rule C
- \bullet Case rule !L

- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- ullet Case rule I

6.16 Status of \oplus_L : OK

- $\bullet \;$ Case rule !R
- \bullet Case rule \top
- \bullet Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet\,$ Case rule W
- $\bullet\,$ Case rule C
- $\bullet \;$ Case rule !L
- \bullet Case rule $\&_{L2}$

- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- $\bullet\,$ Case rule I

6.17 Status of \multimap_L : OK

- Case rule !R
- \bullet Case rule \top
- Case rule $\&_R$
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- $\bullet \;$ Case rule W
- $\bullet\,$ Case rule C
- $\bullet \;$ Case rule !L
- \bullet Case rule $\&_{L1}$

- Case rule \otimes_L
- Case rule \oplus_L
- $\bullet\,$ Case rule I

6.18 Status of I: OK

- \bullet Case rule !R
- Case rule $\mathbf{1}_R$
- \bullet Case rule \top
- \bullet Case rule $\&_R$
- Case rule \multimap_R
- Case rule \bigoplus_{R_2}
- Case rule \bigoplus_{R_1}
- Case rule \otimes_R
- $\bullet\,$ Case rule W
- $\bullet\,$ Case rule C
- \bullet Case rule !L

- Case rule \otimes_L

- Case rule \oplus_L
- $\bullet \;$ Case rule I