

# System for Intuitionistic Linear Logic

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## Contents

<b>1</b>	<b>Measure of derivations</b>	<b>3</b>
<b>2</b>	<b>Invertibility of Rules</b>	<b>5</b>
2.1	Status of $!R$ : : Non invertible . . . . .	5
2.2	Status of $\mathbf{1}_R$ : : Invertible . . . . .	6
2.3	Status of $\top$ : : Invertible . . . . .	7
2.4	Status of $\&_R$ : (Left Premise): Invertible . . . . .	8
2.5	Status of $\&_R$ (Right Premise): : Invertible . . . . .	10
2.6	Status of $\neg\circ_R$ : : Invertible . . . . .	11
2.7	Status of $\oplus_{R_2}$ : : Non invertible . . . . .	13
2.8	Status of $\oplus_{R_1}$ : : Non invertible . . . . .	14
2.9	Status of $\mathbf{1}_L$ : : Invertible . . . . .	16
2.10	Status of $\otimes_R$ : (Left Premise): Non invertible . . . . .	18
2.11	Status of $\otimes_R$ (Right Premise): : Non invertible . . . . .	20
2.12	Status of $W$ : : Non invertible . . . . .	22
2.13	Status of $C$ : : Non invertible . . . . .	24
2.14	Status of $!L$ : : Non invertible . . . . .	26
2.15	Status of $\&_{L2}$ : : Non invertible . . . . .	28
2.16	Status of $\&_{L1}$ : : Non invertible . . . . .	30
2.17	Status of $\otimes_L$ : : Invertible . . . . .	32
2.18	Status of $\oplus_L$ : (Left Premise): Invertible . . . . .	34
2.19	Status of $\oplus_L$ (Right Premise): : Invertible . . . . .	36
2.20	Status of $\neg\circ_L$ : (Left Premise): Non invertible . . . . .	38
2.21	Status of $\neg\circ_L$ (Right Premise): : Non invertible . . . . .	40
2.22	Status of $I$ : : Invertible . . . . .	43
<b>3</b>	<b>Identity-Expansion</b>	<b>44</b>
<b>4</b>	<b>Weakening on bang: <math>\Gamma \vdash !F</math> implies <math>\Gamma \vdash F</math>.</b>	<b>45</b>
<b>5</b>	<b>Cut-Elimination</b>	<b>47</b>
5.1	Status of $!R$ : OK . . . . .	47
5.2	Status of $\mathbf{1}_R$ : OK . . . . .	50
5.3	Status of $\top$ : OK . . . . .	53
5.4	Status of $\&_R$ : OK . . . . .	56
5.5	Status of $\neg\circ_R$ : OK . . . . .	59
5.6	Status of $\oplus_{R_2}$ : OK . . . . .	62
5.7	Status of $\oplus_{R_1}$ : OK . . . . .	65
5.8	Status of $\mathbf{1}_L$ : OK . . . . .	68

5.9	Status of $\otimes_R$ : OK . . . . .	72
5.10	Status of $W$ : OK . . . . .	75
5.11	Status of $C$ : OK . . . . .	79
5.12	Status of $!L$ : OK . . . . .	83
5.13	Status of $\&_{L2}$ : OK . . . . .	88
5.14	Status of $\&_{L1}$ : OK . . . . .	92
5.15	Status of $\otimes_L$ : OK . . . . .	96
5.16	Status of $\oplus_L$ : OK . . . . .	100
5.17	Status of $\neg\circ_L$ : OK . . . . .	104
5.18	Status of $I$ : OK . . . . .	109
<b>6</b>	<b>Cut-Elimination</b> . . . . .	<b>112</b>
6.1	Status of $!R$ : OK . . . . .	112
6.2	Status of $\mathbf{1}_R$ : OK . . . . .	116
6.3	Status of $\top$ : OK . . . . .	117
6.4	Status of $\&_R$ : OK . . . . .	118
6.5	Status of $\neg\circ_R$ : OK . . . . .	119
6.6	Status of $\oplus_{R2}$ : OK . . . . .	120
6.7	Status of $\oplus_{R1}$ : OK . . . . .	121
6.8	Status of $\mathbf{1}_L$ : OK . . . . .	122
6.9	Status of $\otimes_R$ : OK . . . . .	123
6.10	Status of $W$ : OK . . . . .	124
6.11	Status of $C$ : OK . . . . .	128
6.12	Status of $!L$ : OK . . . . .	132
6.13	Status of $\&_{L2}$ : OK . . . . .	136
6.14	Status of $\&_{L1}$ : OK . . . . .	137
6.15	Status of $\otimes_L$ : OK . . . . .	138
6.16	Status of $\oplus_L$ : OK . . . . .	139
6.17	Status of $\neg\circ_L$ : OK . . . . .	140
6.18	Status of $I$ : OK . . . . .	141

# 1 Measure of derivations

- Case(s) rule  $!R$

$$\frac{h_1 : !\top_2 \vdash F_3}{\bullet h_1 : !\top_2 \vdash !F_3} !R \rightarrow \frac{\frac{h_1 : !\top_2 \vdash F_3}{\bullet h_1 : !\top_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : !\top_2 \vdash !F_3} !R$$

- Case(s) rule  $1_R$

$$\frac{}{\bullet h_1 : * \vdash 1} 1_R \rightarrow \frac{}{\bullet \bullet h_1 : * \vdash 1} 1_R$$

- Case(s) rule  $\top$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top} \top$$

- Case(s) rule  $\&_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R$$

- Case(s) rule  $\neg\circ_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \neg\circ F_4} \neg\circ_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \neg\circ F_4} \neg\circ_R$$

- Case(s) rule  $\oplus_{R_2}$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2}$$

- Case(s) rule  $\oplus_{R_1}$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1}$$

- Case(s) rule  $1_L$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : 1, \Delta_2 \vdash F_3} 1_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : 1, \Delta_2 \vdash F_3} 1_L$$

- Case(s) rule  $\otimes_R$

$$\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3 \vdash F_5}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH} \quad \frac{h_1 : \Delta_3 \vdash F_5}{\bullet h_1 : \Delta_3 \vdash F_5} \text{IH}}{\bullet \bullet h_1 : \Delta_2, \Delta_3 \vdash F_4 \otimes F_5} \otimes_R$$

- Case(s) rule  $W$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} W \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2, !F_3 \vdash F_4} W$$

- Case(s) rule  $C$

$$\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C \rightarrow \frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} IH}{\bullet \bullet h_1 : \Delta_2, !F_3 \vdash F_4} C \text{ ax}$$

- Case(s) rule  $!L$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} IH}{\bullet \bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L \text{ ax}$$

- Case(s) rule  $\&_{L2}$

$$\frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} IH}{\bullet \bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L2} \text{ ax}$$

- Case(s) rule  $\&_{L1}$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} IH}{\bullet \bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L1} \text{ ax}$$

- Case(s) rule  $\otimes_L$

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} IH}{\bullet \bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_5} \otimes_L \text{ ax}$$

- Case(s) rule  $\oplus_L$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} IH \quad \frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} IH}{\bullet \bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_5} \oplus_L \text{ ax}$$

- Case(s) rule  $\multimap_L$

$$\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_6}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} IH \quad \frac{h_1 : \Delta_3, F_5 \vdash F_6}{\bullet h_1 : \Delta_3, F_5 \vdash F_6} IH}{\bullet \bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_6} \multimap_L \text{ ax}$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_1 : p(n_2) \vdash p(n_2)} I \rightarrow \frac{}{\bullet \bullet h_1 : p(n_2) \vdash p(n_2)} I$$

## 2 Invertibility of Rules

### 2.1 Status of $!R$ : : Non invertible

- Case rule  $!R$

$$\frac{h_1 : !\top 2 \vdash F_3}{\bullet h_1 : !\top 2 \vdash !F_3} !R \quad \rightarrow \quad \frac{\overline{h_1 : !\top 2 \vdash F_3} \text{ ax}}{\bullet h_1 : !\top 2 \vdash F_3} H$$

- Case rule  $1_R$

- Case rule  $\top$

- Case rule  $\&_R$

- Case rule  $\neg\circ_R$

- Case rule  $\oplus_{R_2}$

- Case rule  $\oplus_{R_1}$

- Case rule  $1_L$

- Case rule  $\otimes_R$

- Case rule  $W$

$$\frac{h_2 : !\top 3 \vdash !F_1}{\bullet h_2 : !\top 3, !F_4 \vdash !F_1} W \quad \rightarrow \quad \frac{\overline{h_2 : !\top 3 \vdash F_1} \text{ ax/ind}}{\bullet h_2 : !\top 3, !F_4 \vdash F_1} W$$

- Case rule  $C$

$$\frac{h_2 : !\top 3, !F_4, !F_4 \vdash !F_1}{\bullet h_2 : !\top 3, !F_4 \vdash !F_1} C \quad \rightarrow \quad \frac{\overline{h_2 : !\top 3, !F_4, !F_4 \vdash F_1} \text{ ax/ind}}{\bullet h_2 : !\top 3, !F_4 \vdash F_1} C$$

- Case rule  $!L$

$$\frac{h_2 : F_4, !\top 3 \vdash !F_1}{\bullet h_2 : !\top 3, !F_4 \vdash !F_1} !L \quad \rightarrow \quad \frac{}{\bullet h_2 : !\top 3, !F_4 \vdash F_1} \text{ fail}$$

- Case rule  $\&_{L2}$

- Case rule  $\&_{L1}$

- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

## 2.2 Status of $1_R$ : : Invertible

- Case rule  $!R$
- Case rule  $1_R$

$$\frac{}{\bullet h_1 : * \vdash 1} 1_R \rightarrow \text{trivial}$$

- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$

- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

### 2.3 Status of $\top$ : : Invertible

- Case rule  $!R$
- Case rule  $\mathbf{1}_R$
- Case rule  $\top$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \rightarrow \text{trivial}$$

- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $\mathbf{1}_L$

$$\frac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash \top} \mathbf{1}_L \rightarrow \text{trivial}$$

- Case rule  $\otimes_R$
- Case rule  $W$

$$\frac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \Delta_2, !F_3 \vdash \top} W \rightarrow \text{trivial}$$

- Case rule  $C$

$$\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash \top}{\bullet h_1 : \Delta_2, !F_3 \vdash \top} C \rightarrow \text{trivial}$$

- Case rule  $!L$

$$\frac{h_1 : \Delta_2, F_3 \vdash \top}{\bullet h_1 : \Delta_2, !F_3 \vdash \top} !L \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\&_{L2}$

$$\frac{h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash \top} \&_{L2} \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\&_{L1}$

$$\frac{h_1 : \Delta_2, F_3 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash \top} \&_{L1} \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\otimes_L$

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash \top} \otimes_L \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\oplus_L$

$$\frac{h_1 : \Delta_2, F_3 \vdash \top \quad h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash \top} \oplus_L \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\neg\circ_L$

$$\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash \top}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \neg\circ F_5 \vdash \top} \neg\circ_L \quad \rightarrow \quad \text{trivial}$$

- Case rule  $I$

## 2.4 Status of $\&_R$ : (Left Premise): Invertible

- Case rule  $!R$

- Case rule  $1_R$

- Case rule  $\top$

- Case rule  $\&_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R \quad \rightarrow \quad \frac{\overline{h_1 : \Delta_2 \vdash F_3}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{ax}_H$$

- Case rule  $\neg\circ_R$



- Case rule  $\oplus_{R_2}$

- Case rule  $\oplus_{R_1}$

- Case rule  $1_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \& F_2} 1_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : 1, \Delta_4 \vdash F_1} 1_L$$

- Case rule  $\otimes_R$

- Case rule  $W$

$$\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} W \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} W$$

- Case rule  $C$

$$\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} C \rightarrow \frac{\overline{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} C$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} !L$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \& F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_4, F_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \& F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \& F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5, F_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \& F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_1} \text{ ax/ind} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1} \oplus_L$$

- Case rule  $\neg o_L$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_5, F_6} \quad \frac{h_3 : \Delta_5, F_7 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg o F_7 \vdash F_1 \& F_2}}{\neg o_L} \rightarrow \frac{\frac{\overline{h_3 : \Delta_4 \vdash F_6}}{\bullet h_3 : \Delta_4, \Delta_5, F_6} \text{ ax} \quad \frac{\overline{h_3 : \Delta_5, F_7 \vdash F_1}}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg o F_7 \vdash F_1} \text{ ax/ind}}{\neg o_L}$$

- Case rule  $I$

## 2.5 Status of $\&_R$ (Right Premise): : Invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \quad \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4}}{\&_R} \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash F_4} \text{ H}$$

- Case rule  $\neg o_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \& F_2}}{1_L} \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : 1, \Delta_4 \vdash F_2} 1_L$$

- Case rule  $\otimes_R$
- Case rule  $W$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2}}{W} \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} W$$

- Case rule  $C$

$$\frac{\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2}}{C} \rightarrow \frac{\overline{h_3 : \Delta_4, !F_5, !F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} C$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} !L$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \& F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_4, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_2} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \& F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_2} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \& F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_2} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \& F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_2} \oplus_L$$

- Case rule  $\multimap_L$

$$\frac{h_3 : \Delta_4 \vdash F_6 \quad h_3 : \Delta_5, F_7 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \multimap F_7 \vdash F_1 \& F_2} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_6} \text{ ax} \quad \overline{h_3 : \Delta_5, F_7 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \multimap F_7 \vdash F_2} \multimap_L$$

- Case rule  $I$

## 2.6 Status of $\multimap_R$ : : Invertible

- Case rule  $!R$

- Case rule  $1_R$

- Case rule  $\top$

- Case rule  $\&_R$

- Case rule  $\multimap_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \multimap F_4} \multimap_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} H$$

- Case rule  $\oplus_{R_2}$

- Case rule  $\oplus_{R_1}$

- Case rule  $1_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \multimap F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \multimap F_2} 1_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : 1, \Delta_4, F_1 \vdash F_2} 1_L$$

- Case rule  $\otimes_R$

- Case rule  $W$

$$\frac{h_3 : \Delta_4 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \multimap F_2} W \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, !F_5 \vdash F_2} W$$

- Case rule  $C$

$$\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \multimap F_2} C \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, !F_5, !F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, !F_5 \vdash F_2} C$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \multimap F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, !F_5 \vdash F_2} !L$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \multimap F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, F_5 \& F_6 \vdash F_2} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \multimap F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, F_5 \& F_6 \vdash F_2} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \multimap F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, F_5 \otimes F_6 \vdash F_2} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \multimap F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \multimap F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_4, F_1, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, F_5 \oplus F_6 \vdash F_2} \oplus_L$$

- Case rule  $\neg\circ_L$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_6 \quad h_3 : \Delta_5, F_7 \vdash F_1 \neg\circ F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_1 \neg\circ F_2} \neg\circ_L}{\frac{h_3 : \Delta_4 \vdash F_6 \quad \text{ax} \quad h_3 : \Delta_5, F_1, F_7 \vdash F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_1, F_6 \neg\circ F_7 \vdash F_2} \text{ax/ind}} \rightarrow \neg\circ_L$$

- Case rule  $I$

## 2.7 Status of $\oplus_{R_2}$ : : Non invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \begin{matrix} \text{ax} \\ \text{H} \end{matrix}$$

- Case rule  $\oplus_{R_1}$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_4} \text{fail}$$

- Case rule  $1_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \oplus F_2} 1_L \rightarrow \frac{h_3 : \Delta_4 \vdash F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_2} \begin{matrix} \text{ax/ind} \\ 1_L \end{matrix}$$

- Case rule  $\otimes_R$

- Case rule  $W$

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} W \rightarrow \frac{h_3 : \Delta_4 \vdash F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} \begin{matrix} \text{ax/ind} \\ W \end{matrix}$$

- Case rule  $C$

$$\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} C \rightarrow \frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} \begin{matrix} \text{ax/ind} \\ C \end{matrix}$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} !L$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \oplus F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_4, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_2} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \oplus F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_2} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \oplus F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_2} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \oplus F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_2} \oplus_L$$

- Case rule  $\neg\circ_L$

$$\frac{h_3 : \Delta_4 \vdash F_6 \quad h_3 : \Delta_5, F_7 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_1 \oplus F_2} \neg\circ_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_6} \text{ ax} \quad \overline{h_3 : \Delta_5, F_7 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_2} \neg\circ_L$$

- Case rule  $I$

## 2.8 Status of $\oplus_{R1}$ : : Non invertible

- Case rule  $!R$

- Case rule  $1_R$

- Case rule  $\top$

- Case rule  $\&_R$

- Case rule  $\neg\circ_R$

- Case rule  $\oplus_{R_2}$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_3} \text{fail}$$

- Case rule  $\oplus_{R_1}$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_3} \begin{matrix} \text{ax} \\ \text{H} \end{matrix}$$

- Case rule  $1_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \oplus F_2} 1_L \rightarrow \frac{}{\bullet h_3 : 1, \Delta_4 \vdash F_1} \begin{matrix} \text{ax/ind} \\ 1_L \end{matrix}$$

- Case rule  $\otimes_R$

- Case rule  $W$

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} W \rightarrow \frac{}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} \begin{matrix} \text{ax/ind} \\ W \end{matrix}$$

- Case rule  $C$

$$\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} C \rightarrow \frac{}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} \begin{matrix} \text{ax/ind} \\ C \end{matrix}$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} !L \rightarrow \frac{}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} \begin{matrix} \text{ax/ind} \\ !L \end{matrix}$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \oplus F_2} \&_{L2} \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1} \begin{matrix} \text{ax/ind} \\ \&_{L2} \end{matrix}$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \oplus F_2} \&_{L1} \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1} \begin{matrix} \text{ax/ind} \\ \&_{L1} \end{matrix}$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \oplus F_2} \otimes_L \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1} \begin{matrix} \text{ax/ind} \\ \otimes_L \end{matrix}$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \oplus F_2} \oplus_L \rightarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1} \text{ax/ind} \quad \frac{h_3 : \Delta_4, F_6 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1} \oplus_L$$

- Case rule  $\neg o_L$

$$\frac{h_3 : \Delta_4 \vdash F_6 \quad h_3 : \Delta_5, F_7 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg o F_7 \vdash F_1 \oplus F_2} \neg o_L \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg o F_7 \vdash F_1} \text{ax} \quad \frac{h_3 : \Delta_5, F_7 \vdash F_1}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg o F_7 \vdash F_1} \text{ax/ind}}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg o F_7 \vdash F_1} \neg o_L$$

- Case rule  $I$

## 2.9 Status of $1_L$ : : Invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\frac{}{\bullet h_2 : 1, \Delta_1 \vdash \top} \top \rightarrow \frac{}{\bullet h_2 : \Delta_1 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{h_2 : 1, \Delta_1 \vdash F_3 \quad h_2 : 1, \Delta_1 \vdash F_4}{\bullet h_2 : 1, \Delta_1 \vdash F_3 \& F_4} \&_R \rightarrow \frac{\frac{h_2 : \Delta_1 \vdash F_3}{\bullet h_2 : \Delta_1 \vdash F_3 \& F_4} \text{ax/ind} \quad \frac{h_2 : \Delta_1 \vdash F_4}{\bullet h_2 : \Delta_1 \vdash F_3 \& F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \& F_4} \&_R$$

- Case rule  $\neg o_R$

$$\frac{h_2 : 1, \Delta_1, F_3 \vdash F_4}{\bullet h_2 : 1, \Delta_1 \vdash F_3 \neg o F_4} \neg o_R \rightarrow \frac{\frac{h_2 : \Delta_1, F_3 \vdash F_4}{\bullet h_2 : \Delta_1 \vdash F_3 \neg o F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \neg o F_4} \neg o_R$$

- Case rule  $\oplus_{R_2}$

$$\frac{h_2 : 1, \Delta_1 \vdash F_4}{\bullet h_2 : 1, \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{\frac{h_2 : \Delta_1 \vdash F_4}{\bullet h_2 : \Delta_1 \vdash F_3 \oplus F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_2}$$

- Case rule  $\oplus_{R_1}$

$$\frac{h_2 : 1, \Delta_1 \vdash F_3}{\bullet h_2 : 1, \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{\frac{h_2 : \Delta_1 \vdash F_3}{\bullet h_2 : \Delta_1 \vdash F_3 \oplus F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_1}$$

- Case rule  $1_L$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : 1, \Delta_2 \vdash F_3} 1_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} H$$



- Case rule  $\otimes_R$

$$\frac{h_1 : 1, \Delta_5 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : (1, \Delta_5), \Delta_2 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash F_3}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \text{ ax/ind} \quad \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \text{ ax}}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \otimes_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : 1, \Delta_5 \vdash F_4}{\bullet h_1 : \Delta_2, 1, \Delta_5 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \text{ ax} \quad \frac{h_1 : \Delta_5 \vdash F_4}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \otimes_R$$

- Case rule  $W$

$$\frac{h_1 : 1, \Delta_4 \vdash F_3}{\bullet h_1 : (1, \Delta_4), !F_2 \vdash F_3} W \rightarrow \frac{\frac{h_1 : \Delta_4 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} W$$

- Case rule  $C$

$$\frac{h_1 : 1, \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : (1, \Delta_4), !F_2 \vdash F_3} C \rightarrow \frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} C$$

- Case rule  $!L$

$$\frac{h_1 : 1, \Delta_4, F_2 \vdash F_3}{\bullet h_1 : (1, \Delta_4), !F_2 \vdash F_3} !L \rightarrow \frac{\frac{h_1 : \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} !L$$

- Case rule  $\&_{L2}$

$$\frac{h_1 : 1, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (1, \Delta_5), F_2 \& F_3 \vdash F_4} \&_{L2} \rightarrow \frac{\frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_1 : 1, \Delta_5, F_2 \vdash F_4}{\bullet h_1 : (1, \Delta_5), F_2 \& F_3 \vdash F_4} \&_{L1} \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_1 : 1, \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : (1, \Delta_5), F_2 \otimes F_3 \vdash F_4} \otimes_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_4} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_1 : 1, \Delta_5, F_2 \vdash F_4 \quad h_1 : 1, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (1, \Delta_5), F_2 \oplus F_3 \vdash F_4} \oplus_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \text{ ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \oplus_L$$

- Case rule  $\multimap_L$

$$\frac{h_1 : 1, \Delta_6 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : (1, \Delta_6), \Delta_2, F_3 \multimap F_4 \vdash F_5} \multimap_L \rightarrow \frac{\frac{h_1 : \Delta_6 \vdash F_3}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \text{ ax/ind} \quad \frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \text{ ax}}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \multimap_L$$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : 1, \Delta_6, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, (1, \Delta_6), F_3 \multimap F_4 \vdash F_5} \multimap_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \text{ ax} \quad \frac{h_1 : \Delta_6, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \multimap_L$$

- Case rule  $I$

## 2.10 Status of $\otimes_R$ : (Left Premise): Non invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$

$$\frac{h_4 : \Delta_1, \Delta_5 \vdash F_2 \otimes F_3}{\bullet h_4 : 1, \Delta_1, \Delta_5 \vdash F_2 \otimes F_3} 1_L \rightarrow \frac{}{\bullet h_4 : 1, \Delta_5 \vdash F_2} \frac{\text{ax/ind}}{1_L}$$

$$\frac{h_4 : \Delta_1, \Delta_5 \vdash F_2 \otimes F_3}{\bullet h_4 : 1, \Delta_1, \Delta_5 \vdash F_2 \otimes F_3} 1_L \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \frac{\text{ax/ind}}{H}$$

- Case rule  $\otimes_R$

$$\frac{h_1 : \Delta_4, \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, \Delta_7 \vdash F_3}{\bullet h_1 : (\Delta_4, \Delta_5), \Delta_6, \Delta_7 \vdash F_2 \otimes F_3} \otimes_R \rightarrow \frac{}{\bullet h_1 : \Delta_4, \Delta_6 \vdash F_2} \text{fail}$$

- Case rule  $W$

$$\frac{h_4 : \Delta_1, \Delta_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} W \rightarrow \frac{h_4 : \Delta_6 \vdash F_2}{\bullet h_4 : \Delta_6, !F_5 \vdash F_2} \frac{\text{ax/ind}}{W}$$

$$\frac{h_4 : \Delta_1, \Delta_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} W \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \frac{\text{ax/ind}}{H}$$

- Case rule  $C$

$$\frac{h_4 : \Delta_1, \Delta_6, !F_5, !F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} C \rightarrow \frac{h_4 : \Delta_6 \vdash F_2}{\bullet h_4 : \Delta_6, !F_5 \vdash F_2} \frac{\text{ax/ind}}{W}$$

$$\frac{h_4 : \Delta_1, \Delta_6, !F_5, !F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} C \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \frac{\text{ax/ind}}{H}$$

- Case rule  $!L$

$$\frac{h_4 : \Delta_1, \Delta_6, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} !L \rightarrow \frac{\overline{h_4 : \Delta_6 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_6, !F_5 \vdash F_2} W$$

$$\frac{h_4 : \Delta_1, \Delta_6, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} !L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\&_{L2}$

$$\frac{h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_7, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_5 \& F_6 \vdash F_2} \&_{L2}$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\&_{L1}$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_7, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_5 \& F_6 \vdash F_2} \&_{L1}$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\otimes_L$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \otimes F_6 \vdash F_2 \otimes F_3} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_7, F_5, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_5 \otimes F_6 \vdash F_2} \otimes_L$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \otimes F_6 \vdash F_2 \otimes F_3} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\oplus_L$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3 \quad h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \oplus F_6 \vdash F_2 \otimes F_3} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_7, F_5 \vdash F_2} \text{ ax/ind} \quad \overline{h_4 : \Delta_7, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_5 \oplus F_6 \vdash F_2} \oplus_L$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3 \quad h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \oplus F_6 \vdash F_2 \otimes F_3} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\multimap_L$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \multimap F_5 \vdash F_1 \otimes F_2} \multimap_L \rightarrow \frac{\overline{\bullet h_3 : \Delta_6, \Delta_8, F_4 \multimap F_5 \vdash F_1} \text{ fail}}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \multimap F_5 \vdash F_1 \otimes F_2} \multimap_L$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \multimap F_5 \vdash F_1 \otimes F_2} \multimap_L \rightarrow \frac{\overline{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_1} \text{ fail}}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \multimap F_5 \vdash F_1 \otimes F_2} \multimap_L$$

- Case rule  $I$

## 2.11 Status of $\otimes_R$ (Right Premise): : Non invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$

$$\frac{h_4 : \Delta_1, \Delta_5 \vdash F_2 \otimes F_3}{\bullet h_4 : 1, \Delta_1, \Delta_5 \vdash F_2 \otimes F_3} 1_L \rightarrow \frac{}{\bullet h_4 : \Delta_1 \vdash F_3} \frac{\text{ax/ind}}{H}$$

$$\frac{h_4 : \Delta_1, \Delta_5 \vdash F_2 \otimes F_3}{\bullet h_4 : 1, \Delta_1, \Delta_5 \vdash F_2 \otimes F_3} 1_L \rightarrow \frac{h_4 : \Delta_5 \vdash F_3}{\bullet h_4 : 1, \Delta_5 \vdash F_3} \frac{\text{ax/ind}}{1_L}$$

- Case rule  $\otimes_R$

$$\frac{h_1 : \Delta_4, \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, \Delta_7 \vdash F_3}{\bullet h_1 : (\Delta_4, \Delta_5), \Delta_6, \Delta_7 \vdash F_2 \otimes F_3} \otimes_R \rightarrow \frac{}{\bullet h_1 : \Delta_5, \Delta_7 \vdash F_3} \text{fail}$$

- Case rule  $W$

$$\frac{h_4 : \Delta_1, \Delta_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} W \rightarrow \frac{h_4 : \Delta_1 \vdash F_3}{\bullet h_4 : \Delta_1 \vdash F_3} \frac{\text{ax/ind}}{H}$$

$$\frac{h_4 : \Delta_1, \Delta_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} W \rightarrow \frac{h_4 : \Delta_6 \vdash F_3}{\bullet h_4 : \Delta_6, !F_5 \vdash F_3} \frac{\text{ax/ind}}{W}$$

- Case rule  $C$

$$\frac{h_4 : \Delta_1, \Delta_6, !F_5, !F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} C \rightarrow \frac{h_4 : \Delta_1 \vdash F_3}{\bullet h_4 : \Delta_1 \vdash F_3} \frac{\text{ax/ind}}{H}$$

$$\frac{h_4 : \Delta_1, \Delta_6, !F_5, !F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} C \rightarrow \frac{h_4 : \Delta_6 \vdash F_3}{\bullet h_4 : \Delta_6, !F_5 \vdash F_3} \frac{\text{ax/ind}}{W}$$

- Case rule  $!L$

$$\frac{h_4 : \Delta_1, \Delta_6, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} !L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_3} H$$

$$\frac{h_4 : \Delta_1, \Delta_6, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} !L \rightarrow \frac{\overline{h_4 : \Delta_6 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_6, !F_5 \vdash F_3} W$$

- Case rule  $\&_{L2}$

$$\frac{h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_3} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_7, F_6 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_5 \& F_6 \vdash F_3} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_3} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_7, F_5 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_5 \& F_6 \vdash F_3} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \otimes F_6 \vdash F_2 \otimes F_3} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_3} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \otimes F_6 \vdash F_2 \otimes F_3} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_7, F_5, F_6 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_5 \otimes F_6 \vdash F_3} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3 \quad h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \oplus F_6 \vdash F_2 \otimes F_3} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_3} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3 \quad h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \oplus F_6 \vdash F_2 \otimes F_3} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_7, F_5 \vdash F_3} \text{ ax/ind} \quad \overline{h_4 : \Delta_7, F_6 \vdash F_3} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_5 \oplus F_6 \vdash F_3} \oplus_L$$

- Case rule  $\multimap_L$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \multimap F_5 \vdash F_1 \otimes F_2} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9 \vdash F_2} \text{ fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \multimap F_5 \vdash F_1 \otimes F_2} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_4 \multimap F_5 \vdash F_2} \text{ fail}$$

- Case rule  $I$

## 2.12 Status of $W$ : : Non invertible

- Case rule  $!R$

$$\frac{h_3 : !\top 1, !F_2 \vdash F_4}{\bullet h_3 : !\top 1, !F_2 \vdash !F_4} !R \rightarrow \frac{\frac{h_3 : !\top 1 \vdash F_4}{\bullet h_3 : !\top 1 \vdash !F_4} \text{ax/ind}}{!R}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\frac{}{\bullet h_3 : \Delta_1, !F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_1 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4 \quad h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \& F_5} \&_R \rightarrow \frac{\frac{h_3 : \Delta_1 \vdash F_4}{\bullet h_3 : \Delta_1 \vdash F_4 \& F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_1 \vdash F_5}{\bullet h_3 : \Delta_1 \vdash F_4 \& F_5} \text{ax/ind}}{\&_R}$$

- Case rule  $\neg o_R$

$$\frac{h_3 : \Delta_1, F_4, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \neg o F_5} \neg o_R \rightarrow \frac{\frac{h_3 : \Delta_1, F_4 \vdash F_5}{\bullet h_3 : \Delta_1 \vdash F_4 \neg o F_5} \text{ax/ind}}{\neg o_R}$$

- Case rule  $\oplus_{R_2}$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_2} \rightarrow \frac{\frac{h_3 : \Delta_1 \vdash F_5}{\bullet h_3 : \Delta_1 \vdash F_4 \oplus F_5} \text{ax/ind}}{\oplus_{R_2}}$$

- Case rule  $\oplus_{R_1}$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_1} \rightarrow \frac{\frac{h_3 : \Delta_1 \vdash F_4}{\bullet h_3 : \Delta_1 \vdash F_4 \oplus F_5} \text{ax/ind}}{\oplus_{R_1}}$$

- Case rule  $1_L$

$$\frac{h_2 : \Delta_4, !F_1 \vdash F_3}{\bullet h_2 : 1, \Delta_4, !F_1 \vdash F_3} 1_L \rightarrow \frac{\frac{h_2 : \Delta_4 \vdash F_3}{\bullet h_2 : 1, \Delta_4 \vdash F_3} \text{ax/ind}}{1_L}$$

- Case rule  $\otimes_R$

$$\frac{h_2 : \Delta_6, !F_1 \vdash F_4 \quad h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), \Delta_3 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{h_2 : \Delta_6 \vdash F_4}{\bullet h_2 : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \text{ax/ind} \quad \frac{h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \text{ax}}{\otimes_R}$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_6, !F_1 \vdash F_5}{\bullet h_2 : \Delta_3, \Delta_6, !F_1 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash F_4}{\bullet h_2 : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \text{ax} \quad \frac{h_2 : \Delta_6 \vdash F_5}{\bullet h_2 : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \text{ax/ind}}{\otimes_R}$$

- Case rule  $W$

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_2 : \Delta_5 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_3 \vdash F_4} W$$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash F_4} H$$

- Case rule  $C$

$$\frac{h_2 : \Delta_5, !F_1, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_2 : \Delta_5, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_3 \vdash F_4} C$$

$$\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C \rightarrow \frac{\overline{\bullet h_1 : \Delta_2 \vdash F_4} \text{ fail}}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C$$

- Case rule  $!L$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_2 : \Delta_5, F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_3 \vdash F_4} !L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L \rightarrow \frac{\overline{\bullet h_1 : \Delta_2 \vdash F_4} \text{ fail}}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L$$

- Case rule  $\&_{L2}$

$$\frac{h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_6, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_6, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_2 : \Delta_6, F_3, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5 \quad h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3 \vdash F_5} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule  $\multimap_L$

$$\frac{h_2 : \Delta_7, !F_1 \vdash F_4 \quad h_2 : \Delta_3, F_5 \vdash F_6}{\bullet h_2 : (\Delta_7, !F_1), \Delta_3, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_7 \vdash F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash F_6} \text{ ax}}{\bullet h_2 : \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_7, F_5, !F_1 \vdash F_6}{\bullet h_2 : \Delta_3, (\Delta_7, !F_1), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash F_4} \text{ ax} \quad \overline{h_2 : \Delta_7, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_2 : \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

- Case rule  $I$

### 2.13 Status of $C$ : : Non invertible

- Case rule  $!R$

$$\frac{h_3 : !\top 1, !F_2 \vdash F_4}{\bullet h_3 : !\top 1, !F_2 \vdash !F_4} !R \rightarrow \frac{\overline{h_3 : !\top 1, !F_2, !F_2 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : !\top 1, !F_2, !F_2 \vdash !F_4} !R$$

- Case rule  $1_R$

- Case rule  $\top$

$$\overline{\bullet h_3 : \Delta_1, !F_2 \vdash \top} \top \rightarrow \overline{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4 \quad h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \& F_5} \&_R \rightarrow \frac{\overline{h_3 : \Delta_1, !F_2, !F_2 \vdash F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_1, !F_2, !F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash F_4 \& F_5} \&_R$$

- Case rule  $\neg o_R$

$$\frac{h_3 : \Delta_1, F_4, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \neg o F_5} \neg o_R \rightarrow \frac{\overline{h_3 : \Delta_1, F_4, !F_2, !F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash F_4 \neg o F_5} \neg o_R$$

- Case rule  $\oplus_{R_2}$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_2} \rightarrow \frac{\overline{h_3 : \Delta_1, !F_2, !F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}$$

- Case rule  $\oplus_{R_1}$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_1} \rightarrow \frac{\overline{h_3 : \Delta_1, !F_2, !F_2 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}$$

- Case rule  $1_L$

$$\frac{h_2 : \Delta_4, !F_1 \vdash F_3}{\bullet h_2 : 1, \Delta_4, !F_1 \vdash F_3} 1_L \rightarrow \frac{\overline{h_2 : \Delta_4, !F_1, !F_1 \vdash F_3} \text{ ax/ind}}{\bullet h_2 : 1, \Delta_4, !F_1, !F_1 \vdash F_3} 1_L$$

- Case rule  $\otimes_R$

$$\frac{h_2 : \Delta_6, !F_1 \vdash F_4 \quad h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), \Delta_3 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_6, !F_1, !F_1 \vdash F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3 \vdash F_5} \text{ ax}}{\bullet h_2 : \Delta_3, \Delta_6, !F_1, !F_1 \vdash F_4 \otimes F_5} \otimes_R$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_6, !F_1 \vdash F_5}{\bullet h_2 : \Delta_3, \Delta_6, !F_1 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash F_4} \text{ ax} \quad \overline{h_2 : \Delta_6, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_3, \Delta_6, !F_1, !F_1 \vdash F_4 \otimes F_5} \otimes_R$$



- Case rule  $W$

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1, !F_3 \vdash F_4} W$$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} W \rightarrow \frac{}{\bullet h_1 : \Delta_2, !F_3, !F_3 \vdash F_4} \text{ fail}$$

- Case rule  $C$

$$\frac{h_2 : \Delta_5, !F_1, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1, !F_3 \vdash F_4} C$$

$$\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_1 : \Delta_2, !F_3, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2, !F_3, !F_3 \vdash F_4} C$$

- Case rule  $!L$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_2 : \Delta_5, F_3, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1, !F_3 \vdash F_4} !L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L \rightarrow \frac{}{\bullet h_1 : \Delta_2, !F_3, !F_3 \vdash F_4} \text{ fail}$$

- Case rule  $\&_{L2}$

$$\frac{h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_6, F_4, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_2 : \Delta_6, F_3, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, F_4, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, !F_1, !F_1 \vdash F_5} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_4, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule  $\multimap_L$

$$\frac{h_2 : \Delta_7, !F_1 \vdash F_4 \quad h_2 : \Delta_3, F_5 \vdash F_6}{\bullet h_2 : (\Delta_7, !F_1), \Delta_3, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_7, !F_1, !F_1 \vdash F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash F_6} \text{ ax}}{\bullet h_2 : \Delta_3, \Delta_7, !F_1, !F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_7, F_5, !F_1 \vdash F_6}{\bullet h_2 : \Delta_3, (\Delta_7, !F_1), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash F_4} \text{ ax} \quad \overline{h_2 : \Delta_7, F_5, !F_1, !F_1 \vdash F_6} \text{ ax/ind}}{\bullet h_2 : \Delta_3, \Delta_7, !F_1, !F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

- Case rule  $I$

## 2.14 Status of $!L$ : : Non invertible

- Case rule  $!R$

$$\frac{h_3 : !\top 1, !F_2 \vdash F_4}{\bullet h_3 : !\top 1, !F_2 \vdash !F_4} !R \rightarrow \frac{}{\bullet h_3 : F_2, !\top 1 \vdash !F_4} \text{fail}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\frac{}{\bullet h_3 : \Delta_1, !F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_1, F_2 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4 \quad h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \& F_5} \&_R \rightarrow \frac{\frac{}{h_3 : \Delta_1, F_2 \vdash F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_1, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2 \vdash F_4 \& F_5} \&_R$$

- Case rule  $\neg\circ_R$

$$\frac{h_3 : \Delta_1, F_4, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R \rightarrow \frac{\frac{}{h_3 : \Delta_1, F_2, F_4 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R$$

- Case rule  $\oplus_{R_2}$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_2} \rightarrow \frac{\frac{}{h_3 : \Delta_1, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}$$

- Case rule  $\oplus_{R_1}$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_1} \rightarrow \frac{\frac{}{h_3 : \Delta_1, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}$$

- Case rule  $1_L$

$$\frac{h_2 : \Delta_4, !F_1 \vdash F_3}{\bullet h_2 : 1, \Delta_4, !F_1 \vdash F_3} 1_L \rightarrow \frac{\frac{}{h_2 : \Delta_4, F_1 \vdash F_3} \text{ax/ind}}{\bullet h_2 : 1, \Delta_4, F_1 \vdash F_3} 1_L$$

- Case rule  $\otimes_R$

$$\frac{h_2 : \Delta_6, !F_1 \vdash F_4 \quad h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), \Delta_3 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{}{h_2 : \Delta_6, F_1 \vdash F_4} \text{ax/ind} \quad \frac{}{h_2 : \Delta_3 \vdash F_5} \text{ax}}{\bullet h_2 : \Delta_3, \Delta_6, F_1 \vdash F_4 \otimes F_5} \otimes_R$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_6, !F_1 \vdash F_5}{\bullet h_2 : \Delta_3, \Delta_6, !F_1 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{}{h_2 : \Delta_3 \vdash F_4} \text{ax} \quad \frac{}{h_2 : \Delta_6, F_1 \vdash F_5} \text{ax/ind}}{\bullet h_2 : \Delta_3, \Delta_6, F_1 \vdash F_4 \otimes F_5} \otimes_R$$

- Case rule  $W$

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_2 : \Delta_5, F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, F_1, !F_3 \vdash F_4} W$$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} W \rightarrow \frac{\overline{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{ fail}}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} W$$

- Case rule  $C$

$$\frac{h_2 : \Delta_5, !F_1, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_2 : \Delta_5, F_1, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, F_1, !F_3 \vdash F_4} C$$

$$\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C \rightarrow \frac{\overline{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{ fail}}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C$$

- Case rule  $!L$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_2 : \Delta_5, F_1, F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, F_1, !F_3 \vdash F_4} !L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} H$$

- Case rule  $\&_{L2}$

$$\frac{h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_2 : \Delta_6, F_3, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5 \quad h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3 \vdash F_5} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_1, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule  $\multimap_L$

$$\frac{h_2 : \Delta_7, !F_1 \vdash F_4 \quad h_2 : \Delta_3, F_5 \vdash F_6}{\bullet h_2 : (\Delta_7, !F_1), \Delta_3, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_7, F_1 \vdash F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash F_6} \text{ ax}}{\bullet h_2 : \Delta_3, \Delta_7, F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_7, F_5, !F_1 \vdash F_6}{\bullet h_2 : \Delta_3, (\Delta_7, !F_1), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash F_4} \text{ ax} \quad \overline{h_2 : \Delta_7, F_1, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_2 : \Delta_3, \Delta_7, F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

- Case rule  $I$

## 2.15 Status of $\&_{L2}$ : : Non invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \& F_6} \&_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash F_5} \text{ax/ind} \quad \frac{}{h_4 : \Delta_1, F_3 \vdash F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \& F_6} \&_R$$

- Case rule  $\neg o_R$

$$\frac{h_4 : \Delta_1, F_5, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \neg o F_6} \neg o_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \neg o F_6} \neg o_R$$

- Case rule  $\oplus_{R2}$

$$\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \oplus F_6} \oplus_{R2} \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R2}$$

- Case rule  $\oplus_{R1}$

$$\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \oplus F_6} \oplus_{R1} \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash F_5} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R1}$$

- Case rule  $1_L$

$$\frac{h_3 : \Delta_5, F_1 \& F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \& F_2 \vdash F_4} 1_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : 1, \Delta_5, F_2 \vdash F_4} 1_L$$

- Case rule  $\otimes_R$

$$\frac{h_3 : \Delta_7, F_1 \& F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2 \vdash F_5} \text{ax/ind} \quad \frac{}{h_3 : \Delta_4 \vdash F_6} \text{ax}}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \& F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \& F_2 \vdash F_5 \otimes F_6} \otimes_R \rightarrow \frac{\frac{}{h_3 : \Delta_4 \vdash F_5} \text{ax} \quad \frac{}{h_3 : \Delta_7, F_2 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R$$

- Case rule  $W$

$$\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} W \rightarrow \frac{\overline{h_3 : \Delta_6, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} W$$

- Case rule  $C$

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, !F_4, !F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} C$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} !L$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \&_{L2}$$

$$\frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_1 : \Delta_2, F_4 \vdash F_5} \text{ ax}}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} H$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \&_{L1}$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \text{ fail}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \otimes F_5 \vdash F_6} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \oplus F_5 \vdash F_6} \oplus_L$$

- Case rule  $\multimap_L$

$$\frac{h_3 : \Delta_8, F_1 \& F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_8, F_1 \& F_2), \Delta_4, F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_8, F_2 \vdash F_5} \text{ ax/ind} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_7} \text{ ax}}{\bullet h_3 : \Delta_4, \Delta_8, F_2, F_5 \multimap F_6 \vdash F_7} \multimap_L$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_6, F_1 \& F_2 \vdash F_7}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \& F_2), F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_5} \text{ ax} \quad \overline{h_3 : \Delta_8, F_2, F_6 \vdash F_7} \text{ ax/ind}}{\bullet h_3 : \Delta_4, \Delta_8, F_2, F_5 \multimap F_6 \vdash F_7} \multimap_L$$

- Case rule  $I$

## 2.16 Status of $\&_{L1}$ : : Non invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \& F_6} \&_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_2 \vdash F_5} \text{ax/ind} \quad \frac{}{h_4 : \Delta_1, F_2 \vdash F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \& F_6} \&_R$$

- Case rule  $\neg\circ_R$

$$\frac{h_4 : \Delta_1, F_5, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_2 \vdash F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \neg\circ F_6} \neg\circ_R$$

- Case rule  $\oplus_{R_2}$

$$\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \oplus F_6} \oplus_{R_2} \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_2 \vdash F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \oplus_{R_2}$$

- Case rule  $\oplus_{R_1}$

$$\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \oplus F_6} \oplus_{R_1} \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \oplus_{R_1}$$

- Case rule  $1_L$

$$\frac{h_3 : \Delta_5, F_1 \& F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \& F_2 \vdash F_4} 1_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_1 \vdash F_4} \text{ax/ind}}{\bullet h_3 : 1, \Delta_5, F_1 \vdash F_4} 1_L$$

- Case rule  $\otimes_R$

$$\frac{h_3 : \Delta_7, F_1 \& F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_1 \vdash F_5} \text{ax/ind} \quad \frac{}{h_3 : \Delta_4 \vdash F_6} \text{ax}}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \& F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \& F_2 \vdash F_5 \otimes F_6} \otimes_R \rightarrow \frac{\frac{}{h_3 : \Delta_4 \vdash F_5} \text{ax} \quad \frac{}{h_3 : \Delta_7, F_1 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R$$

- Case rule  $W$

$$\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} W \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_1 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} W$$

- Case rule  $C$

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, !F_4, !F_4 \vdash F_5}^{ax/ind}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} C$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4 \vdash F_5}^{ax/ind}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} !L$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \&_{L2}$$

$$\frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \textcolor{red}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} \textcolor{red}{fail}$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \&_{L1}$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_5}^{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} H$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash F_6}^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \otimes F_5 \vdash F_6} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}^{ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \oplus F_5 \vdash F_6} \oplus_L$$

- Case rule  $\multimap_L$

$$\frac{h_3 : \Delta_8, F_1 \& F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_8, F_1 \& F_2), \Delta_4, F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_8, F_1 \vdash F_5}^{ax/ind} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_7}^{ax}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \multimap_L$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_6, F_1 \& F_2 \vdash F_7}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \& F_2), F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_5}^{ax} \quad \overline{h_3 : \Delta_8, F_1, F_6 \vdash F_7}^{ax/ind}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \multimap_L$$

- Case rule  $I$

## 2.17 Status of $\otimes_L$ : : Invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \& F_6} \&_R}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \quad ax/ind \quad h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \& F_6} \&_R} \rightarrow$$

- Case rule  $\neg\circ_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \otimes F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R} \rightarrow$$

- Case rule  $\oplus_{R_2}$

$$\frac{\frac{h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}} \rightarrow$$

- Case rule  $\oplus_{R_1}$

$$\frac{\frac{h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}} \rightarrow$$

- Case rule  $1_L$

$$\frac{\frac{h_3 : \Delta_5, F_1 \otimes F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \otimes F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1, F_2 \vdash F_4} 1_L} \rightarrow \frac{\frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1, F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1, F_2 \vdash F_4} 1_L} \rightarrow$$

- Case rule  $\otimes_R$

$$\frac{\frac{h_3 : \Delta_7, F_1 \otimes F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_7, F_1, F_2 \vdash F_5 \quad ax/ind \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2 \vdash F_5 \quad ax/ind \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_7, F_1, F_2 \vdash F_5 \quad ax/ind \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow$$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \otimes F_2 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_4 \vdash F_5 \quad ax \quad h_3 : \Delta_7, F_1, F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad ax \quad h_3 : \Delta_7, F_1, F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_4 \vdash F_5 \quad ax \quad h_3 : \Delta_7, F_1, F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow$$

- Case rule  $W$

$$\frac{\frac{h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} W} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} W} \rightarrow$$



- Case rule  $C$

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_2, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} \frac{ax/ind}{C}$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_2, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} \frac{ax/ind}{!L}$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \& F_5 \vdash F_6} \frac{ax/ind}{\&_{L2}}$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \& F_5 \vdash F_6} \frac{ax/ind}{\&_{L1}}$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \otimes F_5 \vdash F_6} \frac{ax/ind}{\otimes_L}$$

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_1 : \Delta_2, F_3, F_4 \vdash F_5}}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} \frac{ax}{H}$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_4 \vdash F_6} \quad \overline{h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \oplus F_5 \vdash F_6} \frac{ax/ind}{\oplus_L}$$

- Case rule  $\neg\circ_L$

$$\frac{h_3 : \Delta_8, F_1 \otimes F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_8, F_1 \otimes F_2), \Delta_4, F_5 \neg\circ F_6 \vdash F_7} \neg\circ_L \rightarrow \frac{\overline{h_3 : \Delta_8, F_1, F_2 \vdash F_5} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_2, F_5 \neg\circ F_6 \vdash F_7} \frac{ax/ind}{\neg\circ_L}$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_6, F_1 \otimes F_2 \vdash F_7}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \otimes F_2), F_5 \neg\circ F_6 \vdash F_7} \neg\circ_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_5} \quad \overline{h_3 : \Delta_8, F_1, F_2, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_2, F_5 \neg\circ F_6 \vdash F_7} \frac{ax/ind}{\neg\circ_L}$$

- Case rule  $I$

## 2.18 Status of $\oplus_L$ : (Left Premise): Invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \& F_6} \&_R}{\frac{h_4 : \Delta_1, F_2 \vdash F_5 \quad ax/ind \quad \frac{h_4 : \Delta_1, F_2 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \& F_6} \&_R}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \& F_6} \&_R} \rightarrow$$

- Case rule  $\neg o_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \neg o F_6} \neg o_R}{\frac{h_4 : \Delta_1, F_2, F_5 \vdash F_6 \quad ax/ind}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \neg o F_6} \neg o_R} \rightarrow$$

- Case rule  $\oplus_{R_2}$

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\frac{h_4 : \Delta_1, F_2 \vdash F_6 \quad ax/ind}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \oplus_{R_2}} \rightarrow$$

- Case rule  $\oplus_{R_1}$

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\frac{h_4 : \Delta_1, F_2 \vdash F_5 \quad ax/ind}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \oplus_{R_1}} \rightarrow$$

- Case rule  $1_L$

$$\frac{\frac{h_3 : \Delta_5, F_1 \oplus F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \oplus F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_1 \vdash F_4 \quad ax/ind}{\bullet h_3 : 1, \Delta_5, F_1 \vdash F_4} 1_L} \rightarrow$$

- Case rule  $\otimes_R$

$$\frac{\frac{h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_7, F_1 \vdash F_5 \quad ax/ind \quad \frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow$$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \oplus F_2 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_4 \vdash F_5 \quad ax \quad \frac{h_3 : \Delta_7, F_1 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow$$

- Case rule  $W$

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_1 \vdash F_5 \quad ax/ind}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} W} \rightarrow$$

- Case rule  $C$

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \frac{ax/ind}{C}$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \frac{ax/ind}{!L}$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \frac{ax/ind}{\&_{L2}}$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \frac{ax/ind}{\&_{L1}}$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \otimes F_5 \vdash F_6} \frac{ax/ind}{\otimes_L}$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \oplus F_5 \vdash F_6} \frac{ax/ind}{\oplus_L} \frac{\overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_5 \oplus F_4 \vdash F_6} \frac{ax/ind}{\oplus_L}$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_5}}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} \frac{ax}{H}$$

- Case rule  $\multimap_L$

$$\frac{h_3 : \Delta_8, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_8, F_1 \oplus F_2), \Delta_4, F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_8, F_1 \vdash F_5}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \frac{ax/ind}{\multimap_L} \frac{\overline{h_3 : \Delta_4, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \frac{ax}{\multimap_L}$$

$$\frac{h_3 : \Delta_4 \vdash F_5}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \oplus F_2), F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_5}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \frac{ax}{\multimap_L} \frac{\overline{h_3 : \Delta_8, F_1, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \frac{ax/ind}{\multimap_L}$$

- Case rule  $I$

## 2.19 Status of $\oplus_L$ (Right Premise): : Invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \& F_6} \&_R}{\frac{h_4 : \Delta_1, F_3 \vdash F_5 \quad ax/ind \quad h_4 : \Delta_1, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \& F_6} \&_R} \rightarrow$$

- Case rule  $\neg o_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \neg o F_6} \neg o_R}{\frac{h_4 : \Delta_1, F_3, F_5 \vdash F_6 \quad ax/ind}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \neg o F_6} \neg o_R} \rightarrow$$

- Case rule  $\oplus_{R_2}$

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\frac{h_4 : \Delta_1, F_3 \vdash F_6 \quad ax/ind}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}} \rightarrow$$

- Case rule  $\oplus_{R_1}$

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\frac{h_4 : \Delta_1, F_3 \vdash F_5 \quad ax/ind}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}} \rightarrow$$

- Case rule  $1_L$

$$\frac{\frac{h_3 : \Delta_5, F_1 \oplus F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \oplus F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_2 \vdash F_4 \quad ax/ind}{\bullet h_3 : 1, \Delta_5, F_2 \vdash F_4} 1_L} \rightarrow$$

- Case rule  $\otimes_R$

$$\frac{\frac{h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_7, F_2 \vdash F_5 \quad ax/ind \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow$$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \oplus F_2 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_4 \vdash F_5 \quad ax \quad h_3 : \Delta_7, F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow$$

- Case rule  $W$

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_2 \vdash F_5 \quad ax/ind}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} W} \rightarrow$$

- Case rule  $C$

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} \text{ax/ind} C$$

- Case rule  $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} \text{ax/ind} !L$$

- Case rule  $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&_{L2}$$

- Case rule  $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&_{L1}$$

- Case rule  $\otimes_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \otimes F_5 \vdash F_6} \text{ax/ind} \otimes_L$$

- Case rule  $\oplus_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \oplus F_5 \vdash F_6} \text{ax/ind} \oplus_L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_1 : \Delta_2, F_4 \vdash F_5}}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \text{ax} H$$

- Case rule  $\multimap_L$

$$\frac{h_3 : \Delta_8, F_1 \oplus F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_8, F_1 \oplus F_2), \Delta_4, F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_8, F_2 \vdash F_5} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_2, F_5 \multimap F_6 \vdash F_7} \text{ax/ind} \multimap_L$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_6, F_1 \oplus F_2 \vdash F_7}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \oplus F_2), F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_5} \quad \overline{h_3 : \Delta_8, F_2, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_2, F_5 \multimap F_6 \vdash F_7} \text{ax/ind} \multimap_L$$

- Case rule  $I$

## 2.20 Status of $\neg_L$ : (Left Premise): Non invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\frac{}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg F_4 \vdash \top} \top \rightarrow \frac{}{\bullet h_5 : \Delta_1 \vdash F_3} \text{fail}$$

- Case rule  $\&_R$

$$\frac{h_5 : \Delta_1, \Delta_2, F_3, \neg F_4 \vdash F_6 \quad h_5 : \Delta_1, \Delta_2, F_3, \neg F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg F_4 \vdash F_6 \& F_7} \&_R \rightarrow \frac{h_5 : \Delta_1 \vdash F_3}{\bullet h_5 : \Delta_1 \vdash F_3} \text{ax/ind}$$

- Case rule  $\neg_R$

$$\frac{h_5 : \Delta_1, \Delta_2, F_6, F_3, \neg F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg F_4 \vdash F_6 \neg F_7} \neg_R \rightarrow \frac{h_5 : \Delta_1 \vdash F_3}{\bullet h_5 : \Delta_1 \vdash F_3} \text{ax/ind}$$

- Case rule  $\oplus_{R_2}$

$$\frac{h_5 : \Delta_1, \Delta_2, F_3, \neg F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg F_4 \vdash F_6 \oplus F_7} \oplus_{R_2} \rightarrow \frac{h_5 : \Delta_1 \vdash F_3}{\bullet h_5 : \Delta_1 \vdash F_3} \text{ax/ind}$$

- Case rule  $\oplus_{R_1}$

$$\frac{h_5 : \Delta_1, \Delta_2, F_3, \neg F_4 \vdash F_6}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg F_4 \vdash F_6 \oplus F_7} \oplus_{R_1} \rightarrow \frac{h_5 : \Delta_1 \vdash F_3}{\bullet h_5 : \Delta_1 \vdash F_3} \text{ax/ind}$$

- Case rule  $1_L$

$$\frac{h_4 : \Delta_1, \Delta_6, F_2, \neg F_3 \vdash F_5}{\bullet h_4 : 1, \Delta_1, \Delta_6, F_2, \neg F_3 \vdash F_5} 1_L \rightarrow \frac{h_4 : \Delta_6 \vdash F_2}{\bullet h_4 : 1, \Delta_6 \vdash F_2} \text{ax/ind}$$

$$\frac{h_4 : \Delta_1, \Delta_6, F_2, \neg F_3 \vdash F_5}{\bullet h_4 : 1, \Delta_1, \Delta_6, F_2, \neg F_3 \vdash F_5} 1_L \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind}$$

- Case rule  $\otimes_R$

$$\frac{h_3 : \Delta_6, \Delta_7, F_1, \neg F_2 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1, \neg F_2), \Delta_8, \Delta_9 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_1, \neg F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7), \Delta_8, \Delta_9, F_1, \neg F_2 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_1} \text{fail}$$

- Case rule  $W$

$$\frac{h_4 : \Delta_1, \Delta_7, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} W \rightarrow \frac{\overline{h_4 : \Delta_7 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_7, !F_5 \vdash F_2} W$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} W \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $C$

$$\frac{h_4 : \Delta_1, \Delta_7, !F_5, !F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} C \rightarrow \frac{\overline{h_4 : \Delta_7 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_7, !F_5 \vdash F_2} W$$

$$\frac{h_4 : \Delta_1, \Delta_7, !F_5, !F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} C \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $!L$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} !L \rightarrow \frac{\overline{h_4 : \Delta_7 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_7, !F_5 \vdash F_2} W$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} !L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\&_{L2}$

$$\frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_8, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_8, F_5 \& F_6 \vdash F_2} \&_{L2}$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\&_{L1}$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_8, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_8, F_5 \& F_6 \vdash F_2} \&_{L1}$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\otimes_L$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \otimes F_6 \vdash F_7} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_8, F_5, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_8, F_5 \otimes F_6 \vdash F_2} \otimes_L$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \otimes F_6 \vdash F_7} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\oplus_L$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7 \quad h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_8, F_5 \vdash F_2} \text{ ax/ind} \quad \overline{h_4 : \Delta_8, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_8, F_5 \oplus F_6 \vdash F_2} \oplus_L$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7 \quad h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} H$$

- Case rule  $\multimap_L$

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_4 \multimap F_5 \vdash F_1} \text{ fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_4 \multimap F_5 \vdash F_1} \text{ fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9 \vdash F_1} \text{ fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9 \vdash F_1} \text{ fail}$$

$$\frac{h_1 : \Delta_5, \Delta_6 \vdash F_2 \quad h_1 : \Delta_7, \Delta_8, F_3 \vdash F_4}{\bullet h_1 : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_2 \multimap F_3 \vdash F_4} \multimap_L \rightarrow \frac{}{\bullet h_1 : \Delta_5, \Delta_7 \vdash F_2} \text{ fail}$$

- Case rule  $I$

## 2.21 Status of $\multimap_L$ (Right Premise): : Non invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\frac{}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash \top} \top \rightarrow \frac{}{\bullet h_5 : \Delta_2, F_4 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \quad h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \& F_7} \&_R \rightarrow \frac{\overline{h_5 : \Delta_2, F_4 \vdash F_6} \text{ ax/ind} \quad \overline{h_5 : \Delta_2, F_4 \vdash F_7} \text{ ax/ind}}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \& F_7} \&_R$$

- Case rule  $\multimap_R$

$$\frac{h_5 : \Delta_1, \Delta_2, F_6, F_3 \multimap F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \multimap F_7} \multimap_R \rightarrow \frac{\overline{h_5 : \Delta_2, F_4, F_6 \vdash F_7} \text{ ax/ind}}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \multimap F_7} \multimap_R$$



- Case rule  $\oplus_{R_2}$

$$\frac{h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \oplus F_7} \oplus_{R_2} \rightarrow \frac{\overline{h_5 : \Delta_2, F_4 \vdash F_7} \text{ ax/ind}}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \oplus F_7} \oplus_{R_2}$$

- Case rule  $\oplus_{R_1}$

$$\frac{h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \oplus F_7} \oplus_{R_1} \rightarrow \frac{\overline{h_5 : \Delta_2, F_4 \vdash F_6} \text{ ax/ind}}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \oplus F_7} \oplus_{R_1}$$

- Case rule  $1_L$

$$\frac{h_4 : \Delta_1, \Delta_6, F_2 \multimap F_3 \vdash F_5}{\bullet h_4 : 1, \Delta_1, \Delta_6, F_2 \multimap F_3 \vdash F_5} 1_L \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5} H$$

$$\frac{h_4 : \Delta_1, \Delta_6, F_2 \multimap F_3 \vdash F_5}{\bullet h_4 : 1, \Delta_1, \Delta_6, F_2 \multimap F_3 \vdash F_5} 1_L \rightarrow \frac{\overline{h_4 : \Delta_6, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_4 : 1, \Delta_6, F_3 \vdash F_5} 1_L$$

- Case rule  $\otimes_R$

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), \Delta_8, \Delta_9 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_2 \vdash F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7), \Delta_8, \Delta_9, F_1 \multimap F_2 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_2 \vdash F_4 \otimes F_5} \text{ fail}$$

- Case rule  $W$

$$\frac{h_4 : \Delta_1, \Delta_7, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} W \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_6} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} W \rightarrow \frac{\overline{h_4 : \Delta_7, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_3, !F_5 \vdash F_6} W$$

- Case rule  $C$

$$\frac{h_4 : \Delta_1, \Delta_7, !F_5, !F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} C \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_6} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, !F_5, !F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} C \rightarrow \frac{\overline{h_4 : \Delta_7, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_3, !F_5 \vdash F_6} W$$

- Case rule  $!L$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} !L \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_6} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} !L \rightarrow \frac{\overline{h_4 : \Delta_7, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_3, !F_5 \vdash F_6} W$$

- Case rule  $\&_{L2}$

$$\frac{\frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7}}{\&_{L2}} \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash F_7}{\bullet h_4 : \Delta_1, F_3 \vdash F_7}}{\text{H}} \text{ax/ind}$$

$$\frac{\frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7}}{\&_{L2}} \rightarrow \frac{\frac{h_4 : \Delta_8, F_3, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_3, F_5 \& F_6 \vdash F_7}}{\&_{L2}} \text{ax/ind}$$

- Case rule  $\&_{L1}$

$$\frac{\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7}}{\&_{L1}} \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash F_7}{\bullet h_4 : \Delta_1, F_3 \vdash F_7}}{\text{H}} \text{ax/ind}$$

$$\frac{\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7}}{\&_{L1}} \rightarrow \frac{\frac{h_4 : \Delta_8, F_3, F_5 \vdash F_7}{\bullet h_4 : \Delta_8, F_3, F_5 \& F_6 \vdash F_7}}{\&_{L1}} \text{ax/ind}$$

- Case rule  $\otimes_L$

$$\frac{\frac{h_4 : \Delta_1, \Delta_8, F_5, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \otimes F_6 \vdash F_7}}{\otimes_L} \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash F_7}{\bullet h_4 : \Delta_1, F_3 \vdash F_7}}{\text{H}} \text{ax/ind}$$

$$\frac{\frac{h_4 : \Delta_1, \Delta_8, F_5, F_6, F_2 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \otimes F_6 \vdash F_7}}{\otimes_L} \rightarrow \frac{\frac{h_4 : \Delta_8, F_3, F_5, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_3, F_5 \otimes F_6 \vdash F_7}}{\otimes_L} \text{ax/ind}$$

- Case rule  $\oplus_L$

$$\frac{\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7} \quad \frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7}}{\oplus_L} \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash F_7}{\bullet h_4 : \Delta_1, F_3 \vdash F_7}}{\text{H}} \text{ax/ind}$$

$$\frac{\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7} \quad \frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7}}{\oplus_L} \rightarrow \frac{\frac{h_4 : \Delta_8, F_3, F_5 \vdash F_7}{\bullet h_4 : \Delta_8, F_3, F_5 \oplus F_6 \vdash F_7} \quad \frac{h_4 : \Delta_8, F_3, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_3, F_5 \oplus F_6 \vdash F_7}}{\oplus_L} \text{ax/ind}$$

- Case rule  $\multimap_L$

$$\frac{\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6}}{\multimap_L} \rightarrow \frac{\bullet h_3 : \Delta_8, \Delta_{10}, F_2 \vdash F_6}{\text{fail}}$$

$$\frac{\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6}}{\multimap_L} \rightarrow \frac{\bullet h_3 : \Delta_8, \Delta_{10}, F_2 \vdash F_6}{\text{fail}}$$

$$\frac{\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6}}{\multimap_L} \rightarrow \frac{\bullet h_3 : \Delta_8, \Delta_{10}, F_2, F_4 \multimap F_5 \vdash F_6}{\text{fail}}$$

$$\frac{\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6}}{\multimap_L} \rightarrow \frac{\bullet h_3 : \Delta_8, \Delta_{10}, F_2, F_4 \multimap F_5 \vdash F_6}{\text{fail}}$$

$$\frac{\frac{h_1 : \Delta_5, \Delta_6 \vdash F_2 \quad h_1 : \Delta_7, \Delta_8, F_3 \vdash F_4}{\bullet h_1 : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_2 \multimap F_3 \vdash F_4}}{\multimap_L} \rightarrow \frac{\bullet h_1 : \Delta_6, \Delta_8, F_3 \vdash F_4}{\text{fail}}$$

- Case rule  $I$

## 2.22 Status of $I$ : : Invertible

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\multimap_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\multimap_L$
- Case rule  $I$

$$\frac{\bullet h_1 : p(n_2) \vdash p(n_2)}{I} \rightarrow \text{trivial}$$

### 3 Identity-Expansion

$$\begin{array}{c}
\overline{- : \top \vdash \top} \top \\
\\
\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0 \oplus F_1} \oplus_{R_1} \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0 \oplus F_1} \oplus_{R_2} \\
\hline
- : F_0 \oplus F_1 \vdash F_0 \oplus F_1 \quad \oplus_L \\
\\
\frac{\overline{- : F_0 \vdash F_0} \text{ IH} \quad \overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_0 \multimap F_1 \vdash F_1} \multimap_L \\
\hline
- : F_0 \multimap F_1 \vdash F_0 \multimap F_1 \quad \multimap_R \\
\\
\frac{\overline{- : F_0 \vdash F_0} \text{ IH} \quad \overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_0 \otimes F_1} \otimes_R \\
\hline
- : F_0 \otimes F_1 \vdash F_0 \otimes F_1 \quad \otimes_L \\
\\
\frac{\overline{- : F_0 \vdash F_0} \text{ IH} \quad \overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0 \& F_1 \vdash F_0} \&_{L1} \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0 \& F_1 \vdash F_1} \&_{L2} \\
\hline
- : F_0 \& F_1 \vdash F_0 \& F_1 \quad \&_R \\
\\
\frac{\overline{- : * \vdash \mathbf{1}} \mathbf{1}_R}{- : \mathbf{1} \vdash \mathbf{1}} \mathbf{1}_L
\end{array}$$

## 4 Weakening on bang: $\Gamma \vdash !F$ implies $\Gamma \vdash F$ .

- Case(s) rule  $!R$

$$\frac{h_1 : !\top \vdash F_3}{\bullet h_1 : !\top \vdash F_3} !R \rightarrow \frac{\overline{h_1 : !\top \vdash F_3}^{\text{ax}}}{\bullet h_1 : !\top \vdash F_3} \text{H}$$

- Case(s) rule  $\mathbf{1}_R$
- Case(s) rule  $\top$
- Case(s) rule  $\&_R$
- Case(s) rule  $\neg\circ_R$
- Case(s) rule  $\oplus_{R_2}$
- Case(s) rule  $\oplus_{R_1}$
- Case(s) rule  $\mathbf{1}_L$

$$\frac{h_2 : \Delta_3 \vdash !F_1}{\bullet h_2 : \mathbf{1}, \Delta_3 \vdash !F_1} \mathbf{1}_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \mathbf{1}, \Delta_3 \vdash F_1} \text{IH} \mathbf{1}_L$$

- Case(s) rule  $\otimes_R$
- Case(s) rule  $W$

$$\frac{h_2 : \Delta_3 \vdash !F_1}{\bullet h_2 : \Delta_3, !F_4 \vdash !F_1} W \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_3, !F_4 \vdash F_1} \text{IH} W$$

- Case(s) rule  $C$

$$\frac{h_2 : \Delta_3, !F_4, !F_4 \vdash !F_1}{\bullet h_2 : \Delta_3, !F_4 \vdash !F_1} C \rightarrow \frac{\overline{h_2 : \Delta_3, !F_4, !F_4 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_3, !F_4 \vdash F_1} \text{IH} C$$

- Case(s) rule  $!L$

$$\frac{h_2 : \Delta_3, F_4 \vdash !F_1}{\bullet h_2 : \Delta_3, !F_4 \vdash !F_1} !L \rightarrow \frac{\overline{h_2 : \Delta_3, F_4 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_3, !F_4 \vdash F_1} \text{IH} !L$$

- Case(s) rule  $\&_{L2}$

$$\frac{h_2 : \Delta_3, F_5 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \& F_5 \vdash !F_1} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_3, F_5 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_3, F_4 \& F_5 \vdash F_1} \text{IH} \&_{L2}$$

- Case(s) rule  $\&_{L1}$

$$\frac{h_2 : \Delta_3, F_4 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \& F_5 \vdash !F_1} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_3, F_4 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_3, F_4 \& F_5 \vdash F_1} \text{IH} \&_{L1}$$

- Case(s) rule  $\otimes_L$

$$\frac{h_2 : \Delta_3, F_4, F_5 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \otimes F_5 \vdash !F_1} \otimes_L \rightarrow \frac{\frac{h_2 : \Delta_3, F_4, F_5 \vdash !F_1}{h_2 : \Delta_3, F_4, F_5 \vdash F_1} \text{IH} \quad \frac{}{h_2 : \Delta_3, F_4, F_5 \vdash !F_1} \text{ax}}{\bullet h_2 : \Delta_3, F_4 \otimes F_5 \vdash F_1} \otimes_L$$

- Case(s) rule  $\oplus_L$

$$\frac{h_2 : \Delta_3, F_4 \vdash !F_1 \quad h_2 : \Delta_3, F_5 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \oplus F_5 \vdash !F_1} \oplus_L \rightarrow \frac{\frac{h_2 : \Delta_3, F_4 \vdash !F_1}{h_2 : \Delta_3, F_4 \vdash F_1} \text{IH} \quad \frac{h_2 : \Delta_3, F_5 \vdash !F_1}{h_2 : \Delta_3, F_5 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, F_4 \oplus F_5 \vdash F_1} \oplus_L$$

- Case(s) rule  $\neg\circ_L$

$$\frac{h_2 : \Delta_3 \vdash F_5 \quad h_2 : \Delta_4, F_6 \vdash !F_1}{\bullet h_2 : \Delta_3, \Delta_4, F_5 \neg\circ F_6 \vdash !F_1} \neg\circ_L \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash F_5}{h_2 : \Delta_3 \vdash F_5} \text{ax} \quad \frac{h_2 : \Delta_4, F_6 \vdash !F_1}{h_2 : \Delta_4, F_6 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, \Delta_4, F_5 \neg\circ F_6 \vdash F_1} \neg\circ_L$$

- Case(s) rule  $I$

## 5 Cut-Elimination

### 5.1 Status of $!R$ : OK

- Case rule  $!R$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : !\Upsilon 4, !F_5 \vdash F_7}{\bullet h_6 : !\Upsilon 4, !F_5 \vdash !F_7} !R \\
 \hline
 - : !\Upsilon 2, !\Upsilon 4 \vdash F_7 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, !\Upsilon 4 \vdash F_7} \text{ax/W} \quad \frac{h_6 : !\Upsilon 4, !F_5 \vdash F_7}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_7} \text{ax/W}}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_7} \text{hCut} \\
 \hline
 - : !\Upsilon 2, !\Upsilon 4 \vdash !F_7 \quad !R
 \end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{}{\bullet h_6 : \Delta_4, !F_5 \vdash \top} \top \\
 \hline
 - : !\Upsilon 2, \Delta_4 \vdash \top \quad \text{Cut} \\
 \hline
 \rightarrow \\
 - : !\Upsilon 2, \Delta_4 \vdash \top \quad \top
 \end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_7 \quad h_6 : \Delta_4, !F_5 \vdash F_8}{\bullet h_6 : \Delta_4, !F_5 \vdash F_7 \& F_8} \&_R \\
 \hline
 - : !\Upsilon 2, \Delta_4 \vdash F_7 \& F_8 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_7}{- : !\Upsilon 2, \Delta_4 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \& F_8} \text{hCut} \quad \&_R
 \end{array}$$

- Case rule  $\neg\circ_R$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : \Delta_4, F_7, !F_5 \vdash F_8}{\bullet h_6 : \Delta_4, !F_5 \vdash F_7 \neg\circ F_8} \neg\circ_R \\
 \hline
 - : !\Upsilon 2, \Delta_4 \vdash F_7 \neg\circ F_8 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_4, F_7, !F_5 \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \neg\circ F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \neg\circ F_8} \text{hCut} \quad \neg\circ_R
 \end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_8}{\bullet h_6 : \Delta_4, !F_5 \vdash F_7 \oplus F_8} \oplus_{R_2} \\
 \hline
 - : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8} \text{hCut} \quad \oplus_{R_2}
 \end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_7}{\bullet h_6 : \Delta_4, !F_5 \vdash F_7 \oplus F_8} \oplus_{R_1} \\
\hline
- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_7}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_7, !F_4 \vdash F_6}{\bullet h_5 : (1, \Delta_7), !F_4 \vdash F_6} 1_L \\
\hline
- : !\Upsilon 2, 1, \Delta_7 \vdash F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : 1, !\Upsilon 2, \Delta_7 \vdash F_6} \text{ax/W} \quad \frac{h_5 : 1, \Delta_7, !F_4 \vdash F_6}{- : 1, !\Upsilon 2, \Delta_7 \vdash F_6} \text{ax/W}}{- : 1, !\Upsilon 2, \Delta_7 \vdash F_6} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, !F_4 \vdash F_7 \quad h_5 : \Delta_6 \vdash F_8}{\bullet h_5 : (\Delta_6, \Delta_9), !F_4 \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_9, !F_4 \vdash F_7}{- : \Delta_6 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{hCut} \otimes_R \\
\hline
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_6 \vdash F_7 \quad h_5 : \Delta_9, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_6, \Delta_9), !F_4 \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \Delta_6 \vdash F_7}{- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_9 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{hCut} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), !F_4 \vdash F_7} W \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{ax/W}}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{hCut} \\
\hline
\frac{h_1 : !\Upsilon 2 \vdash F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} !R \quad \frac{h_4 : \Delta_5 \vdash F_7}{\bullet h_4 : \Delta_5, !F_6 \vdash F_7} W \\
\hline
- : !\Upsilon 2, \Delta_5 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
- : !\Upsilon 2, \Delta_5 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), !F_4 \vdash F_7} C \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{ax/W}}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{hCut} C
\end{array}$$



$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} !R \quad \frac{h_4 : \Delta_5, !F_6, !F_6 \vdash F_7}{\bullet h_4 : \Delta_5, !F_6 \vdash F_7} C \\
\hline
- : !\Upsilon 2, \Delta_5 \vdash F_7 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_6}{- : !\Upsilon 2 \vdash !F_6} \text{ax/W} \quad \frac{h_4 : \Delta_5, !F_6, !F_6 \vdash F_7}{h_4 : \Delta_5, !F_6 \vdash F_7} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_5 \vdash F_7 \quad \text{mCut}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, F_6, !F_4 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), !F_4 \vdash F_7} !L \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_6, !F_4 \vdash F_7}{h_5 : \Delta_8, F_6, !F_4 \vdash F_7} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_8, F_6 \vdash F_7 \quad !L \\
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} !R \quad \frac{h_4 : \Delta_5, F_6 \vdash F_7}{\bullet h_4 : \Delta_5, !F_6 \vdash F_7} !L \\
\hline
- : !\Upsilon 2, \Delta_5 \vdash F_7 \\
\rightarrow \\
\frac{- : !\Upsilon 2 \vdash F_6}{- : !\Upsilon 2 \vdash F_6} \text{ax/W} \quad \frac{- : \Delta_5, F_6 \vdash F_7}{- : \Delta_5, F_6 \vdash F_7} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_5 \vdash F_7 \quad \text{sCut}
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), !F_4 \vdash F_8} \&_{L2} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{h_5 : \Delta_9, F_7, !F_4 \vdash F_8} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8 \quad \&_{L2} \\
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), !F_4 \vdash F_8} \&_{L1} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8}{h_5 : \Delta_9, F_6, !F_4 \vdash F_8} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \vdash F_8 \quad \&_{L1} \\
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_6, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \otimes F_7), !F_4 \vdash F_8} \otimes_L \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \otimes F_7 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6, F_7, !F_4 \vdash F_8}{h_5 : \Delta_9, F_6, F_7, !F_4 \vdash F_8} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6, F_7 \vdash F_8 \quad \otimes_L \\
- : !\Upsilon 2, \Delta_9, F_6 \otimes F_7 \vdash F_8
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8 \quad h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), !F_4 \vdash F_8} \oplus_L}{- : !\Upsilon 2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), !F_4 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_9, F_6 \vdash F_8} \text{hCut} \quad \frac{\frac{h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), !F_4 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8} \text{hCut}}{- : !\Upsilon 2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \oplus_L
\end{array}$$

- Case rule  $\neg o_L$

$$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_{10}, !F_4 \vdash F_7 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg o F_8), !F_4 \vdash F_9} \neg o_L}{- : !\Upsilon 2, \Delta_6, \Delta_{10}, F_7 \neg o F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_{10}, !F_4 \vdash F_7}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg o F_8), !F_4 \vdash F_9} \text{ax/W}}{- : !\Upsilon 2, \Delta_{10} \vdash F_7} \text{hCut} \quad \frac{- : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg o F_8), !F_4 \vdash F_9} \text{ax/W}}{- : !\Upsilon 2, \Delta_{10}, \Delta_6, F_7 \neg o F_8 \vdash F_9} \neg o_L \\
\\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_6 \vdash F_7 \quad h_5 : \Delta_{10}, F_8, !F_4 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg o F_8), !F_4 \vdash F_9} \neg o_L}{- : !\Upsilon 2, \Delta_6, \Delta_{10}, F_7 \neg o F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_6 \vdash F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{\frac{h_5 : \Delta_{10}, F_8, !F_4 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg o F_8), !F_4 \vdash F_9} \text{ax/W}}{- : !\Upsilon 2, \Delta_{10}, F_8 \vdash F_9} \text{hCut}}{- : !\Upsilon 2, \Delta_{10}, \Delta_6, F_7 \neg o F_8 \vdash F_9} \neg o_L
\end{array}$$

- Case rule  $I$

## 5.2 Status of $1_R$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\begin{array}{c}
\frac{\frac{h_1 : * \vdash 1}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_3 : \Delta_2, 1 \vdash \top}{\bullet h_3 : \Delta_2, 1 \vdash \top} \top}{- : *, \Delta_2 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2 \vdash \top} \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_3 : 1, \Delta_2 \vdash F_4 \quad h_3 : 1, \Delta_2 \vdash F_5}{\bullet h_3 : \Delta_2, 1 \vdash F_4 \& F_5} \&_R}{- : *, \Delta_2 \vdash F_4 \& F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : * \vdash 1}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_3 : 1, \Delta_2 \vdash F_4}{\bullet h_3 : \Delta_2, 1 \vdash F_4 \& F_5} \text{ax/W}}{- : \Delta_2 \vdash F_4} \text{hCut} \quad \frac{\frac{h_1 : * \vdash 1}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_3 : 1, \Delta_2 \vdash F_5}{\bullet h_3 : \Delta_2, 1 \vdash F_4 \& F_5} \text{ax/W}}{- : \Delta_2 \vdash F_5} \text{hCut}}{- : \Delta_2 \vdash F_4 \& F_5} \&_R
\end{array}$$

- Case rule  $\neg o_R$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_3 : 1, \Delta_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_2, 1 \vdash F_4 \multimap F_5} \multimap_R}{- : *, \Delta_2 \vdash F_4 \multimap F_5} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_3 : 1, \Delta_2, F_4 \vdash F_5}{\text{ax/W}}}{- : \Delta_2, F_4 \vdash F_5} \text{hCut}}{- : \Delta_2 \vdash F_4 \multimap F_5} \multimap_R
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_3 : 1, \Delta_2 \vdash F_5}{\bullet h_3 : \Delta_2, 1 \vdash F_4 \oplus F_5} \oplus_{R_2}}{- : *, \Delta_2 \vdash F_4 \oplus F_5} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_3 : 1, \Delta_2 \vdash F_5}{\text{ax/W}}}{- : \Delta_2 \vdash F_5} \text{hCut}}{- : \Delta_2 \vdash F_4 \oplus F_5} \oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_3 : 1, \Delta_2 \vdash F_4}{\bullet h_3 : \Delta_2, 1 \vdash F_4 \oplus F_5} \oplus_{R_1}}{- : *, \Delta_2 \vdash F_4 \oplus F_5} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_3 : 1, \Delta_2 \vdash F_4}{\text{ax/W}}}{- : \Delta_2 \vdash F_4} \text{hCut}}{- : \Delta_2 \vdash F_4 \oplus F_5} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : \Delta_3 \vdash F_4}{\bullet h_2 : \Delta_3, 1 \vdash F_4} 1_L}{- : *, \Delta_3 \vdash F_4} \text{Cut}}{\rightarrow} \\
- : \Delta_3 \vdash F_4 \quad \text{ax/W}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_6 \vdash F_4 \quad h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : (\Delta_3, \Delta_6), 1 \vdash F_4 \otimes F_5} \otimes_R}{- : *, \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_6 \vdash F_4}{\text{ax/W}}}{- : \Delta_6 \vdash F_4} \text{hCut} \quad \frac{- : \Delta_3 \vdash F_5}{\text{ax/W}}}{- : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \otimes_R \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : 1, \Delta_6 \vdash F_5}{\bullet h_2 : (\Delta_3, \Delta_6), 1 \vdash F_4 \otimes F_5} \otimes_R}{- : *, \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_3 \vdash F_4}{\text{ax/W}} \quad \frac{\frac{}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_2 : 1, \Delta_6 \vdash F_5}{\text{ax/W}}}{- : \Delta_6 \vdash F_5} \text{hCut}}{- : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_5 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), 1 \vdash F_4} W}{- : *, \Delta_5, !F_3 \vdash F_4} \text{Cut}}{\rightarrow} \\
\frac{\frac{}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_2 : 1, \Delta_5, !F_3 \vdash F_4}{\text{ax/W}}}{- : \Delta_5, !F_3 \vdash F_4} \text{hCut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad 1_R \quad \frac{h_2 : 1, \Delta_5, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), 1 \vdash F_4} C}{- : *, \Delta_5, !F_3 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad \text{ax/W} \quad \frac{h_2 : 1, \Delta_5, !F_3, !F_3 \vdash F_4}{- : \Delta_5, !F_3, !F_3 \vdash F_4} \text{ax/W}}{- : \Delta_5, !F_3 \vdash F_4} \text{hCut} \quad C
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad 1_R \quad \frac{h_2 : 1, \Delta_5, F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), 1 \vdash F_4} !L}{- : *, \Delta_5, !F_3 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad 1_R \quad \frac{h_2 : 1, \Delta_5, F_3 \vdash F_4}{- : \Delta_5, F_3 \vdash F_4} \text{ax/W}}{- : \Delta_5, !F_3 \vdash F_4} \text{hCut} \quad !L
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad 1_R \quad \frac{h_2 : 1, \Delta_6, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \& F_4), 1 \vdash F_5} \&_{L2}}{- : *, \Delta_6, F_3 \& F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad 1_R \quad \frac{h_2 : 1, \Delta_6, F_4 \vdash F_5}{- : \Delta_6, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \& F_4 \vdash F_5} \text{hCut} \quad \&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad 1_R \quad \frac{h_2 : 1, \Delta_6, F_3 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \& F_4), 1 \vdash F_5} \&_{L1}}{- : *, \Delta_6, F_3 \& F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad \text{ax/W} \quad \frac{h_2 : 1, \Delta_6, F_3 \vdash F_5}{- : \Delta_6, F_3 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \& F_4 \vdash F_5} \text{hCut} \quad \&_{L1}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad 1_R \quad \frac{h_2 : 1, \Delta_6, F_3, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \otimes F_4), 1 \vdash F_5} \otimes_L}{- : *, \Delta_6, F_3 \otimes F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad \text{ax/W} \quad \frac{h_2 : 1, \Delta_6, F_3, F_4 \vdash F_5}{- : \Delta_6, F_3, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \otimes F_4 \vdash F_5} \text{hCut} \quad \otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad 1_R \quad \frac{h_2 : 1, \Delta_6, F_3 \vdash F_5 \quad h_2 : 1, \Delta_6, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \oplus F_4), 1 \vdash F_5} \oplus_L}{- : *, \Delta_6, F_3 \oplus F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash 1} \quad \text{ax/W} \quad \frac{h_2 : 1, \Delta_6, F_3 \vdash F_5}{- : \Delta_6, F_3 \vdash F_5} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : * \vdash 1} \quad \text{ax/W} \quad \frac{h_2 : 1, \Delta_6, F_4 \vdash F_5}{- : \Delta_6, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \oplus F_4 \vdash F_5} \text{hCut} \quad \oplus_L
\end{array}$$

- Case rule  $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_7 \vdash F_4 \quad h_2 : \Delta_3, F_5 \vdash F_6}{\bullet h_2 : (\Delta_3, \Delta_7, F_4 \neg\circ F_5), 1 \vdash F_6} \neg\circ_L}{- : *, \Delta_3, \Delta_7, F_4 \neg\circ F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_7 \vdash F_4}{\neg : \Delta_7 \vdash F_4} \text{ax/W} \quad \frac{}{\neg : \Delta_3, F_5 \vdash F_6} \text{ax/W}}{\neg : \Delta_3, \Delta_7, F_4 \neg\circ F_5 \vdash F_6} \text{hCut} \neg\circ_L \\
\\
\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : 1, \Delta_7, F_5 \vdash F_6}{\bullet h_2 : (\Delta_3, \Delta_7, F_4 \neg\circ F_5), 1 \vdash F_6} \neg\circ_L}{- : *, \Delta_3, \Delta_7, F_4 \neg\circ F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\neg : \Delta_3 \vdash F_4} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_7, F_5 \vdash F_6}{\neg : \Delta_7, F_5 \vdash F_6} \text{ax/W}}{\neg : \Delta_3, \Delta_7, F_4 \neg\circ F_5 \vdash F_6} \text{hCut} \neg\circ_L
\end{array}$$

- Case rule  $I$

### 5.3 Status of $\top$ : OK

- Case rule  $!R$

- Case rule  $1_R$

- Case rule  $\top$

$$\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{}{\bullet h_4 : \Delta_3, \top \vdash \top} \top}{- : \Delta_2, \Delta_3 \vdash \top} \text{Cut} \\
\rightarrow \\
- : \Delta_2, \Delta_3 \vdash \top \quad \top$$

- Case rule  $\&_R$

$$\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_4 : \top, \Delta_3 \vdash F_5 \quad h_4 : \top, \Delta_3 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \& F_6} \&_R}{- : \Delta_2, \Delta_3 \vdash F_5 \& F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_4 : \Delta_2, \Delta_3, \top \vdash F_5}{\neg : \Delta_2, \Delta_3 \vdash F_5} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_4 : \Delta_2, \Delta_3, \top \vdash F_6}{\neg : \Delta_2, \Delta_3 \vdash F_6} \text{ax/W}}{\neg : \Delta_2, \Delta_3 \vdash F_5 \& F_6} \text{hCut} \&_R$$

- Case rule  $\neg\circ_R$

$$\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_4 : \top, \Delta_3, F_5 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \neg\circ F_6} \neg\circ_R}{- : \Delta_2, \Delta_3 \vdash F_5 \neg\circ F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_4 : \Delta_2, \Delta_3, F_5, \top \vdash F_6}{\neg : \Delta_2, \Delta_3, F_5 \vdash F_6} \text{ax/W}}{\neg : \Delta_2, \Delta_3 \vdash F_5 \neg\circ F_6} \text{hCut} \neg\circ_R$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_4 : \top, \Delta_3 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \oplus F_6} \oplus_{R_2}}{\neg : \Delta_2, \Delta_3 \vdash F_5 \oplus F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_4 : \Delta_2, \Delta_3, \top \vdash F_6}{\neg : \Delta_2, \Delta_3 \vdash F_6} \text{ax/W}}{\neg : \Delta_2, \Delta_3 \vdash F_5 \oplus F_6} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_4 : \top, \Delta_3 \vdash F_5}{\bullet h_4 : \Delta_3, \top \vdash F_5 \oplus F_6} \oplus_{R_1}}{\neg : \Delta_2, \Delta_3 \vdash F_5 \oplus F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_4 : \Delta_2, \Delta_3, \top \vdash F_5}{\neg : \Delta_2, \Delta_3 \vdash F_5} \text{ax/W}}{\neg : \Delta_2, \Delta_3 \vdash F_5 \oplus F_6} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_5 \vdash F_4}{\bullet h_3 : (1, \Delta_5), \top \vdash F_4} 1_L}{\neg : \Delta_2, 1, \Delta_5 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : 1, \Delta_2, \Delta_5, \top \vdash F_4}{\neg : 1, \Delta_2, \Delta_5 \vdash F_4} \text{ax/W}}{\neg : 1, \Delta_2, \Delta_5 \vdash F_4} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_4, \Delta_7), \top \vdash F_5 \otimes F_6} \otimes_R}{\neg : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_7, \top \vdash F_5}{\neg : \Delta_7 \vdash F_5} \text{ax/W}}{\neg : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6} \text{hCut} \frac{\neg : \Delta_2, \Delta_4 \vdash F_6}{\neg : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6} \otimes_R \\
\\
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \top, \Delta_7 \vdash F_6}{\bullet h_3 : (\Delta_4, \Delta_7), \top \vdash F_5 \otimes F_6} \otimes_R}{\neg : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\neg : \Delta_4 \vdash F_5} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, \top \vdash F_6}{\neg : \Delta_2, \Delta_7 \vdash F_6} \text{ax/W}}{\neg : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6} \text{hCut} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6 \vdash F_5}{\bullet h_3 : (\Delta_6, !F_4), \top \vdash F_5} W}{\neg : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top, !F_4 \vdash F_5}{\neg : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{ax/W}}{\neg : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{hCut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6, !F_4, !F_4 \vdash F_5}{\bullet h_3 : (\Delta_6, !F_4), \top \vdash F_5} C}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : * \vdash \top}{- : \Delta_2, \Delta_6, !F_4, !F_4 \vdash F_5} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top, !F_4, !F_4 \vdash F_5}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{hCut} \\
C
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6, F_4 \vdash F_5}{\bullet h_3 : (\Delta_6, !F_4), \top \vdash F_5} !L}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : * \vdash \top}{- : \Delta_2, \Delta_6, F_4 \vdash F_5} \top \quad \frac{h_3 : \Delta_2, \Delta_6, F_4, \top \vdash F_5}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{hCut} \\
!L
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \& F_5), \top \vdash F_6} \&_{L2}}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : * \vdash \top}{- : \Delta_2, \Delta_7, F_5 \vdash F_6} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_5, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{hCut} \\
\&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \& F_5), \top \vdash F_6} \&_{L1}}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : * \vdash \top}{- : \Delta_2, \Delta_7, F_4 \vdash F_6} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_4, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{hCut} \\
\&_{L1}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \otimes F_5), \top \vdash F_6} \otimes_L}{- : \Delta_2, \Delta_7, F_4 \otimes F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : * \vdash \top}{- : \Delta_2, \Delta_7, F_4, F_5 \vdash F_6} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_4, F_5, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_4 \otimes F_5 \vdash F_6} \text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \otimes F_5 \vdash F_6} \text{hCut} \\
\otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \oplus F_5), \top \vdash F_6} \oplus_L}{- : \Delta_2, \Delta_7, F_4 \oplus F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : * \vdash \top}{- : \Delta_2, \Delta_7, F_4 \vdash F_6} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_4, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_4 \oplus F_5 \vdash F_6} \text{ax/W} \quad \frac{\bullet h_1 : * \vdash \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_5, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_5 \vdash F_6} \text{ax/W}}{- : \Delta_2, \Delta_7, F_5 \vdash F_6} \text{hCut}}{- : \Delta_2, \Delta_7, F_4 \oplus F_5 \vdash F_6} \oplus_L
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\top} \quad \frac{h_3 : \top, \Delta_8 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_4, \Delta_8, F_5 \multimap F_6), \top \vdash F_7} \multimap_L}{\frac{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : * \vdash \top}{\top} \quad \frac{h_3 : \Delta_8, \top \vdash F_5}{\text{ax/W}} \quad \frac{\text{ax/W}}{h\text{Cut}} \quad \frac{- : \Delta_2, \Delta_4, F_6 \vdash F_7}{\text{ax/W}}}{\frac{- : \Delta_8 \vdash F_5}{\text{ax/W}} \quad \frac{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7}{\multimap_L}} \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\top} \quad \frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \top, \Delta_8, F_6 \vdash F_7}{\bullet h_3 : (\Delta_4, \Delta_8, F_5 \multimap F_6), \top \vdash F_7} \multimap_L}{\frac{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_4 \vdash F_5}{\text{ax/W}} \quad \frac{\bullet h_1 : * \vdash \top}{\top} \quad \frac{h_3 : \Delta_2, \Delta_8, F_6, \top \vdash F_7}{\text{ax/W}} \quad \frac{\text{ax/W}}{h\text{Cut}}}{\frac{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7}{\multimap_L}}
\end{array}$$

- Case rule  $I$

## 5.4 Status of $\&_R$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash \top}{\top} \text{Cut}}{\frac{- : \Delta_2, \Delta_5 \vdash \top}{\rightarrow} \top}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \& F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \& F_{10}} \&_R}{\frac{- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9}{\text{ax/W}}}{\frac{- : \Delta_2, \Delta_5 \vdash F_9}{h\text{Cut}}} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_{10}}{\text{ax/W}}}{\frac{- : \Delta_2, \Delta_5 \vdash F_{10}}{h\text{Cut}}} \&_R
\end{array}$$

- Case rule  $\multimap_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_8 : \Delta_5, F_9, F_6 \& F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \multimap F_{10}} \multimap_R}{\frac{- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_5, F_9, F_6 \& F_7 \vdash F_{10}}{\text{ax/W}}}{\frac{- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10}}{h\text{Cut}}} \multimap_R
\end{array}$$

- Case rule  $\oplus_{R_2}$



$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{hCut} \\
\oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{hCut} \\
\oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_9, F_5 \& F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_5 \& F_6 \vdash F_8} 1_L \\
\hline
- : \Delta_2, 1, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_5 \& F_6 \vdash F_8}{h_7 : 1, \Delta_9, F_5 \& F_6 \vdash F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \& F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_9}{h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{hCut} \\
\otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \& F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{hCut} \\
\otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \& F_6 \vdash F_9} W \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \& F_6 \vdash F_9}{h_7 : \Delta_{10}, !F_8, F_5 \& F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \& F_6 \vdash F_9} C \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \& F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad C
\end{array}$$

- Case rule !L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \& F_6 \vdash F_9} !L \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \& F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad !L
\end{array}$$

- Case rule &L2

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \& F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10} \quad \&_{L2}
\end{array}$$
  

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \& F_8} \&_R \quad \frac{h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&_{L2} \\
\hline
- : \Delta_2, \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_7}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ax/W} \quad \frac{- : \Delta_6, F_8 \vdash F_9}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6 \vdash F_9} \text{sCut}
\end{array}$$

- Case rule &L1

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \& F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10} \quad \&_{L1}
\end{array}$$
  

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \& F_8} \&_R \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&_{L1} \\
\hline
- : \Delta_2, \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_7}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ax/W} \quad \frac{- : \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6 \vdash F_9} \text{sCut}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \& F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10} \quad \otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \& F_6 \vdash F_{10}} \oplus_L}{\neg : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\neg : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10}}{\neg : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{hCut}}{\neg : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\neg : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{\neg : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{\neg : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \oplus_L
\end{array}$$

- Case rule  $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{12}, F_5 \& F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_5 \& F_6 \vdash F_{11}} \neg\circ_L}{\neg : \Delta_2, \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\neg : \Delta_{12}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_5 \& F_6 \vdash F_9}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \text{hCut}}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\frac{\neg : \Delta_8, F_{10} \vdash F_{11}}{\neg : \Delta_8, F_{10} \vdash F_{11}} \text{ax/W}}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, F_5 \& F_6 \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_5 \& F_6 \vdash F_{11}} \neg\circ_L}{\neg : \Delta_2, \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\neg : \Delta_8 \vdash F_9}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\neg : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{10}, F_5 \& F_6 \vdash F_{11}}{\neg : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{hCut}}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L
\end{array}$$

- Case rule  $I$

## 5.5 Status of $\neg\circ_R$ : OK

- Case rule  $!R$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \neg\circ F_7} \neg\circ_R \quad \frac{}{\bullet h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash \top} \top}{\neg : \Delta_2, \Delta_5 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{\neg : \Delta_2, \Delta_5 \vdash \top} \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \neg\circ F_7} \neg\circ_R \quad \frac{h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_9 \& F_{10}} \&_R}{\neg : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \neg\circ F_7}{\neg : \Delta_2, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_9}{\neg : \Delta_2, \Delta_5 \vdash F_9} \text{hCut}}{\neg : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{ax/W} \quad \frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \neg\circ F_7}{\neg : \Delta_2, \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_{10}}{\neg : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{\neg : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \&_R
\end{array}$$

- Case rule  $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7} \multimap_R \quad \frac{h_8 : \Delta_5, F_9, F_6 \multimap F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9 \multimap F_{10}} \multimap_R \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7}{- : \Delta_2, \Delta_5 \vdash F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \multimap F_7 \vdash F_{10}}{- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10}} \text{hCut} \multimap_R
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7} \multimap_R \quad \frac{h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_{10}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7} \multimap_R \quad \frac{h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_9, F_5 \multimap F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_5 \multimap F_6 \vdash F_8} 1_L \\
\hline
- : \Delta_2, 1, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_5 \multimap F_6 \vdash F_8}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \multimap F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R \\
\hline
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \multimap F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W} \quad h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \multimap F_6 \vdash F_9} W \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad \text{hCut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \multimap F_6 \vdash F_9} C \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad C
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \multimap F_6 \vdash F_9} !L \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, F_8 \vdash F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad !L
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \multimap F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10} \quad \&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \multimap F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10} \quad \&_{L1}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \multimap F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10} \quad \otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{\frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \multimap F_6 \vdash F_{10}} \oplus_L}{\multimap : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \multimap F_6}{\multimap : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10}}{\multimap : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W}}{\multimap : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \oplus_L
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{\frac{h_7 : \Delta_{12}, F_5 \multimap F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_5 \multimap F_6 \vdash F_{11}} \multimap_L}{\multimap : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \multimap F_6}{\multimap : \Delta_{12}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_5 \multimap F_6 \vdash F_9}{\multimap : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W}}{\multimap : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut} \multimap_L \\
\\
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{\frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, F_5 \multimap F_6 \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_5 \multimap F_6 \vdash F_{11}} \multimap_L}{\multimap : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \multimap F_6}{\multimap : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{10}, F_5 \multimap F_6 \vdash F_{11}}{\multimap : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{ax/W}}{\multimap : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut} \multimap_L \\
\\
\frac{\frac{h_1 : \Delta_2, F_8 \vdash F_9}{\bullet h_1 : \Delta_2 \vdash F_8 \multimap F_9} \multimap_R \quad \frac{\frac{h_5 : \Delta_6 \vdash F_8 \quad h_5 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_5 : (\Delta_6, \Delta_7), F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{\multimap : \Delta_2, \Delta_6, \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_8 \multimap F_9}{\multimap : \Delta_2, \Delta_6, \Delta_7 \vdash F_{10}} \text{ax/W} \quad \frac{h_5 : \Delta_6 \vdash F_8}{\multimap : \Delta_2, \Delta_7, F_8 \vdash F_{10}} \text{ax/W}}{\multimap : \Delta_2, \Delta_6, \Delta_7 \vdash F_{10}} \text{sCut}
\end{array}$$

- Case rule  $I$

## 5.6 Status of $\oplus_{R2}$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R2} \quad \frac{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash \top}{\multimap : \Delta_2, \Delta_5 \vdash \top} \top}{\multimap : \Delta_2, \Delta_5 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{\multimap : \Delta_2, \Delta_5 \vdash \top}{\multimap : \Delta_2, \Delta_5 \vdash \top} \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R2} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \& F_{10}} \&_R \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \quad \frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10} \quad \&_R
\end{array}$$

- Case rule  $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R2} \quad \frac{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \neg\circ F_{10}} \neg\circ_R \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \neg\circ F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \neg\circ F_{10} \quad \neg\circ_R
\end{array}$$

- Case rule  $\oplus_{R2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R2} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \oplus F_{10}} \oplus_{R2} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \oplus_{R2}
\end{array}$$

- Case rule  $\oplus_{R1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R2} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \oplus F_{10}} \oplus_{R1} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \oplus_{R1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_9, F_5 \oplus F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_5 \oplus F_6 \vdash F_8} 1_L \\
\hline
- : \Delta_2, 1, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_5 \oplus F_6 \vdash F_8}{h_7 : 1, \Delta_9, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \oplus F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{hCut} \quad \frac{- : \Delta_8 \vdash F_{10}}{- : \Delta_8 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10} \quad \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{\frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \oplus F_6 \vdash F_9 \otimes F_{10}} \otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} W}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} C}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut} C
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} !L}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut} !L
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L2}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut} \&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L1}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut} \&_{L1}
\end{array}$$

- Case rule  $\otimes_L$



$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \oplus F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ hCut} \otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10}} \oplus_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ hCut} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_9 \oplus F_8 \vdash F_{10}} \text{ hCut}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \oplus_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \oplus F_8} \oplus_{R_2} \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus_L \\
\hline
- : \Delta_2, \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_8}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ ax/W} \quad \frac{- : \Delta_6, F_8 \vdash F_9}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ sCut}}{- : \Delta_2, \Delta_6 \vdash F_9}
\end{array}$$

- Case rule  $\neg \circ_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \frac{h_7 : \Delta_{12}, F_5 \oplus F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg \circ F_{10}), F_5 \oplus F_6 \vdash F_{11}} \neg \circ_L \\
\hline
- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \neg \circ F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{12}, \Delta_2 \vdash F_9} \text{ ax/W} \quad \frac{h_7 : \Delta_{12}, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg \circ F_{10} \vdash F_{11}} \text{ hCut} \quad \frac{- : \Delta_8, F_{10} \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg \circ F_{10} \vdash F_{11}} \text{ ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg \circ F_{10} \vdash F_{11}} \neg \circ_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, F_5 \oplus F_6 \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg \circ F_{10}), F_5 \oplus F_6 \vdash F_{11}} \neg \circ_L \\
\hline
- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \neg \circ F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{- : \Delta_8 \vdash F_9 \text{ ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{ ax/W} \quad \frac{h_7 : \Delta_{12}, F_{10}, F_5 \oplus F_6 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{ hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg \circ F_{10} \vdash F_{11}} \neg \circ_L
\end{array}$$

- Case rule  $I$

## 5.7 Status of $\oplus_{R_1}$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R_1} \frac{}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash \top} \top \\
\hline
- : \Delta_2, \Delta_5 \vdash \top \\
\rightarrow \\
- : \Delta_2, \Delta_5 \vdash \top \quad \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \& F_{10}} \&_R}{- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \quad \frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \&_R
\end{array}$$

- Case rule  $\neg\circ_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \neg\circ F_{10}} \neg\circ_R}{- : \Delta_2, \Delta_5 \vdash F_9 \neg\circ F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \neg\circ F_{10}} \neg\circ_R
\end{array}$$

- Case rule  $\oplus_{R2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \oplus F_{10}} \oplus_{R2}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R2}
\end{array}$$

- Case rule  $\oplus_{R1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \oplus F_{10}} \oplus_{R1}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_9, F_5 \oplus F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_5 \oplus F_6 \vdash F_8} 1_L}{- : \Delta_2, 1, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_5 \oplus F_6 \vdash F_8}{h_7 : 1, \Delta_9, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \oplus F_6 \vdash F_9 \otimes F_{10}} \otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{hCut} \quad \frac{- : \Delta_8 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{\frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \oplus F_6 \vdash F_9 \otimes F_{10}} \otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \otimes_R \text{hCut}
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} W}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} C}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut} \quad C
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} !L}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut} \quad !L
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L2}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut} \quad \&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L1}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut} \quad \&_{L1}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \oplus F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ hCut} \otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10}} \oplus_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ hCut} \oplus_L \\
\hline
\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_7 \oplus F_8} \oplus_{R1} \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus_L \\
\hline
- : \Delta_2, \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{- : \Delta_2 \vdash F_7 \quad \frac{\text{ax/W}}{- : \Delta_6, F_7 \vdash F_9} \text{ ax/W}}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ sCut}
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{12}, F_5 \oplus F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_5 \oplus F_6 \vdash F_{11}} \multimap_L \\
\hline
- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{12}, \Delta_2 \vdash F_9} \text{ ax/W} \quad \frac{h_7 : \Delta_{12}, F_5 \oplus F_6 \vdash F_9}{- : \Delta_8, F_{10} \vdash F_{11}} \text{ ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ hCut} \multimap_L \\
\hline
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, F_5 \oplus F_6 \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_5 \oplus F_6 \vdash F_{11}} \multimap_L \\
\hline
- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{- : \Delta_8 \vdash F_9 \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{ ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ hCut} \multimap_L
\end{array}$$

- Case rule  $I$

## 5.8 Status of $1_L$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash 5}{\bullet h_1 : 1, \Delta_2 \vdash 5} 1_L \quad \frac{h_6 : !\Upsilon 4, 5 \vdash F_7}{\bullet h_6 : !\Upsilon 4, 5 \vdash !F_7} !R \\
\hline
- : (1, \Delta_2), !\Upsilon 4 \vdash !F_7 \\
\rightarrow \\
\frac{h_1 : \Delta_2 \vdash 5}{- : 1, !\Upsilon 4, \Delta_2 \vdash !F_7} \text{ ax/W} \quad \frac{\bullet h_6 : 1, 5, !\Upsilon 4 \vdash !F_7}{- : 1, !\Upsilon 4, \Delta_2 \vdash !F_7} \text{ hCut}
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L \quad \frac{}{\bullet h_6 : \Delta_4, F_5 \vdash \top} \top}{\vdash : (\mathbf{1}, \Delta_2), \Delta_4 \vdash \top} \text{Cut} \quad \rightarrow \quad \frac{}{\vdash : \mathbf{1}, \Delta_2, \Delta_4 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7 \quad h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \& F_8} \&_R}{\vdash : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \& F_8} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{\bullet h_6 : \mathbf{1}, \Delta_4, F_5 \vdash F_7 \& F_8}{\bullet h_6 : \mathbf{1}, \Delta_4, F_5 \vdash F_7 \& F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \& F_8} \text{hCut}$$

- Case rule  $\neg\circ_R$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L \quad \frac{h_6 : \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \neg\circ F_8} \neg\circ_R}{\vdash : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \neg\circ F_8} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{\bullet h_6 : \mathbf{1}, \Delta_4, F_5 \vdash F_7 \neg\circ F_8}{\bullet h_6 : \mathbf{1}, \Delta_4, F_5 \vdash F_7 \neg\circ F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \neg\circ F_8} \text{hCut}$$

- Case rule  $\oplus_{R_2}$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \oplus F_8} \oplus_{R_2}}{\vdash : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \oplus F_8} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{\bullet h_6 : \mathbf{1}, \Delta_4, F_5 \vdash F_7 \oplus F_8}{\bullet h_6 : \mathbf{1}, \Delta_4, F_5 \vdash F_7 \oplus F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \oplus F_8} \text{hCut}$$

- Case rule  $\oplus_{R_1}$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \oplus F_8} \oplus_{R_1}}{\vdash : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \oplus F_8} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{\bullet h_6 : \mathbf{1}, \Delta_4, F_5 \vdash F_7 \oplus F_8}{\bullet h_6 : \mathbf{1}, \Delta_4, F_5 \vdash F_7 \oplus F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \oplus F_8} \text{hCut}$$

- Case rule  $\mathbf{1}_L$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash \mathbf{1}}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash \mathbf{1}} \mathbf{1}_L \quad \frac{h_4 : \Delta_5 \vdash F_6}{\bullet h_4 : \Delta_5, \mathbf{1} \vdash F_6} \mathbf{1}_L}{\vdash : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_6} \text{Cut} \quad \rightarrow \quad \frac{}{\vdash : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_6} \text{ax/W}$$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_7, F_4 \vdash F_6}{\bullet h_5 : (\mathbf{1}, \Delta_7), F_4 \vdash F_6} \mathbf{1}_L}{\vdash : (\mathbf{1}, \Delta_2), \mathbf{1}, \Delta_7 \vdash F_6} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{\bullet h_5 : \mathbf{1}, \mathbf{1}, \Delta_7, F_4 \vdash F_6}{\bullet h_5 : \mathbf{1}, \mathbf{1}, \Delta_7, F_4 \vdash F_6} \text{ax/W}}{\vdash : \mathbf{1}, \mathbf{1}, \Delta_2, \Delta_7 \vdash F_6} \text{hCut}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : 1, \Delta_2 \vdash F_4} 1_L \quad \frac{h_5 : \Delta_9, F_4 \vdash F_7 \quad h_5 : \Delta_6 \vdash F_8}{\bullet h_5 : (\Delta_6, \Delta_9), F_4 \vdash F_7 \otimes F_8} \otimes_R}{- : (1, \Delta_2), \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{- : 1, \Delta_2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{\bullet h_5 : 1, \Delta_6, \Delta_9, F_4 \vdash F_7 \otimes F_8}{- : 1, \Delta_2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : 1, \Delta_2 \vdash F_4} 1_L \quad \frac{h_5 : \Delta_6 \vdash F_7 \quad h_5 : \Delta_9, F_4 \vdash F_8}{\bullet h_5 : (\Delta_6, \Delta_9), F_4 \vdash F_7 \otimes F_8} \otimes_R}{- : (1, \Delta_2), \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{- : 1, \Delta_2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{\bullet h_5 : 1, \Delta_6, \Delta_9, F_4 \vdash F_7 \otimes F_8}{- : 1, \Delta_2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8} \text{hCut}
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : 1, \Delta_2 \vdash F_4} \text{ 1L} \quad \frac{h_5 : \Delta_8, F_4 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_4 \vdash F_7} W}{- : (1, \Delta_2), \Delta_8, !F_6 \vdash F_7} \text{ Cut} \\
\downarrow \rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : 1, \Delta_2, \Delta_8, !F_6 \vdash F_7} \text{ ax/W} \quad \frac{\bullet h_5 : 1, \Delta_8, F_4, !F_6 \vdash F_7}{- : 1, \Delta_2, \Delta_8, !F_6 \vdash F_7} \text{ ax/W}}{- : 1, \Delta_2, \Delta_8, !F_6 \vdash F_7} \text{ hCut} \\
\downarrow \rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash !F_6}{\bullet h_1 : 1, \Delta_2 \vdash !F_6} \text{ 1L} \quad \frac{h_4 : \Delta_5 \vdash F_7}{\bullet h_4 : \Delta_5, !F_6 \vdash F_7} W}{- : (1, \Delta_2), \Delta_5 \vdash F_7} \text{ Cut} \\
\downarrow \rightarrow \\
- : 1, \Delta_2, \Delta_5 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : 1, \Delta_2 \vdash F_4} \quad 1_L \quad \frac{h_5 : \Delta_8, F_4, !F_6, !F_6 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_4 \vdash F_7} \quad C \\
\hline
- : (1, \Delta_2), \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_5 : 1, \Delta_8, F_4, !F_6 \vdash F_7} \text{ax/W} \quad \frac{\text{ax/W}}{\text{hCut}} \\
\hline
- : 1, \Delta_2, \Delta_8, !F_6 \vdash F_7 \\
\hline
\frac{h_1 : \Delta_2 \vdash !F_6}{\bullet h_1 : 1, \Delta_2 \vdash !F_6} \quad 1_L \quad \frac{h_4 : \Delta_5, !F_6, !F_6 \vdash F_7}{\bullet h_4 : \Delta_5, !F_6 \vdash F_7} \quad C \\
\hline
- : (1, \Delta_2), \Delta_5 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_2 \vdash !F_6}{\bullet h_4 : 1, \Delta_5, !F_6 \vdash F_7} \text{ax/W} \quad \frac{\text{ax/W}}{\text{hCut}} \\
\hline
- : 1, \Delta_2, \Delta_5 \vdash F_7
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : 1, \Delta_2 \vdash F_4} \text{1L} \quad \frac{h_5 : \Delta_8, F_4, F_6 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_4 \vdash F_7} \text{!L}}{\quad \rightarrow \quad} \text{Cut} \\
\frac{\quad}{- : (1, \Delta_2), \Delta_8, !F_6 \vdash F_7} \\
\frac{h_1 : \Delta_2 \vdash F_4}{\quad} \text{ax/W} \quad \frac{\bullet h_5 : 1, \Delta_8, F_4, !F_6 \vdash F_7}{\quad} \text{ax/W} \\
\frac{\quad}{- : 1, \Delta_2, \Delta_8, !F_6 \vdash F_7} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash !F_6}{\bullet h_1 : 1, \Delta_2 \vdash !F_6} \text{1L} \quad \frac{h_4 : \Delta_5, F_6 \vdash F_7}{\bullet h_4 : \Delta_5, !F_6 \vdash F_7} \text{!L}}{\quad \rightarrow \quad} \text{Cut} \\
\frac{\quad}{- : (1, \Delta_2), \Delta_5 \vdash F_7} \\
\frac{h_1 : \Delta_2 \vdash !F_6}{\quad} \text{ax/W} \quad \frac{\bullet h_4 : 1, \Delta_5, !F_6 \vdash F_7}{\quad} \text{ax/W} \\
\frac{\quad}{- : 1, \Delta_2, \Delta_5 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_4 \vdash F_8} \&_{L2}}{\vdash : (\mathbf{1}, \Delta_2), \Delta_9, F_6 \& F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_4, F_6 \& F_7 \vdash F_8}{\bullet h_5 : \mathbf{1}, \Delta_9, F_4, F_6 \& F_7 \vdash F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_9, F_6 \& F_7 \vdash F_8} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \& F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \& F_7} \mathbf{1}_L \quad \frac{h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \& F_7 \vdash F_8} \&_{L2}}{\vdash : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \& F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \& F_7} \text{ax/W} \quad \frac{h_4 : \Delta_5, F_6 \& F_7 \vdash F_8}{\bullet h_4 : \mathbf{1}, \Delta_5, F_6 \& F_7 \vdash F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_9, F_4, F_6 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_4 \vdash F_8} \&_{L1}}{\vdash : (\mathbf{1}, \Delta_2), \Delta_9, F_6 \& F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_4, F_6 \& F_7 \vdash F_8}{\bullet h_5 : \mathbf{1}, \Delta_9, F_4, F_6 \& F_7 \vdash F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_9, F_6 \& F_7 \vdash F_8} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \& F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \& F_7} \mathbf{1}_L \quad \frac{h_4 : \Delta_5, F_6 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \& F_7 \vdash F_8} \&_{L1}}{\vdash : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \& F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \& F_7} \text{ax/W} \quad \frac{h_4 : \Delta_5, F_6 \& F_7 \vdash F_8}{\bullet h_4 : \mathbf{1}, \Delta_5, F_6 \& F_7 \vdash F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_9, F_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \otimes F_7), F_4 \vdash F_8} \otimes_L}}{\vdash : (\mathbf{1}, \Delta_2), \Delta_9, F_6 \otimes F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_4, F_6 \otimes F_7 \vdash F_8}{\bullet h_5 : \mathbf{1}, \Delta_9, F_4, F_6 \otimes F_7 \vdash F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_9, F_6 \otimes F_7 \vdash F_8} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \otimes F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \otimes F_7} \mathbf{1}_L \quad \frac{h_4 : \Delta_5, F_6, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \otimes F_7 \vdash F_8} \otimes_L}}{\vdash : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \otimes F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{h_4 : \Delta_5, F_6 \otimes F_7 \vdash F_8}{\bullet h_4 : \mathbf{1}, \Delta_5, F_6 \otimes F_7 \vdash F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_9, F_4, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), F_4 \vdash F_8} \oplus_L}}{\vdash : (\mathbf{1}, \Delta_2), \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_4, F_6 \oplus F_7 \vdash F_8}{\bullet h_5 : \mathbf{1}, \Delta_9, F_4, F_6 \oplus F_7 \vdash F_8} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \oplus F_7} \mathbf{1}_L \quad \frac{h_4 : \Delta_5, F_6 \vdash F_8 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \oplus F_7 \vdash F_8} \oplus_L \\
\hline
\frac{}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_5, F_6 \oplus F_7 \vdash F_8}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\neg\circ_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_{10}, F_4 \vdash F_7 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg\circ F_8), F_4 \vdash F_9} \neg\circ_L \\
\hline
\frac{}{- : (\mathbf{1}, \Delta_2), \Delta_6, \Delta_{10}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_5 : \mathbf{1}, \Delta_{10}, \Delta_6, F_4, F_7 \neg\circ F_8 \vdash F_9}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{hCut} \\
\\
\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_6 \vdash F_7 \quad h_5 : \Delta_{10}, F_4, F_8 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg\circ F_8), F_4 \vdash F_9} \neg\circ_L \\
\hline
\frac{}{- : (\mathbf{1}, \Delta_2), \Delta_6, \Delta_{10}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_5 : \mathbf{1}, \Delta_{10}, \Delta_6, F_4, F_7 \neg\circ F_8 \vdash F_9}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{hCut} \\
\\
\frac{h_1 : \Delta_2 \vdash F_7 \neg\circ F_8}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_7 \neg\circ F_8} \mathbf{1}_L \quad \frac{h_4 : \Delta_5 \vdash F_7 \quad h_4 : \Delta_6, F_8 \vdash F_9}{\bullet h_4 : (\Delta_5, \Delta_6), F_7 \neg\circ F_8 \vdash F_9} \neg\circ_L \\
\hline
\frac{}{- : (\mathbf{1}, \Delta_2), \Delta_5, \Delta_6 \vdash F_9} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \neg\circ F_8}{- : \mathbf{1}, \Delta_2, \Delta_5, \Delta_6 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_5, \Delta_6, F_7 \neg\circ F_8 \vdash F_9}{- : \mathbf{1}, \Delta_2, \Delta_5, \Delta_6 \vdash F_9} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5, \Delta_6 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash p(n_5)}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash p(n_5)} \mathbf{1}_L \quad \frac{}{\bullet h_4 : *, p(n_5) \vdash p(n_5)} I \\
\hline
\frac{}{- : (\mathbf{1}, \Delta_2), * \vdash p(n_5)} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \mathbf{1}, \Delta_2 \vdash p(n_5)} \text{ax/W}
\end{array}$$

## 5.9 Status of $\otimes_R$ : OK

- Case rule  $!R$

- Case rule  $\mathbf{1}_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash \top} \top \\
\hline
\frac{}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash \top} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \top} \top
\end{array}$$

- Case rule  $\&_R$



$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \quad h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{11}}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \& F_{11}} \&_R}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash F_{10} \& F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10}}{\text{hCut}} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{11}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \& F_{11}} \&_R}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \& F_{11}} \text{hCut}
\end{array}$$

- Case rule  $\neg\circ_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_9 : \Delta_6, F_{10}, F_7 \otimes F_8 \vdash F_{11}}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \neg\circ F_{11}} \neg\circ_R}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash F_{10} \neg\circ F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_{10}, F_7 \otimes F_8 \vdash F_{11}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \neg\circ F_{11}} \neg\circ_R}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \neg\circ F_{11}}
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{11}}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \oplus F_{11}} \oplus_{R_2}}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{11}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{11}} \oplus_{R_2}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \oplus F_{11}}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10}}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \oplus F_{11}} \oplus_{R_1}}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10}} \oplus_{R_1}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \oplus F_{11}}
\end{array}$$

- Case rule  $\mathbf{1}_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{10}, F_6 \otimes F_7 \vdash F_9}{\bullet h_8 : (\mathbf{1}, \Delta_{10}), F_6 \otimes F_7 \vdash F_9} \mathbf{1}_L}{- : (\Delta_2, \Delta_3), \mathbf{1}, \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{h_8 : \Delta_{10}, F_6 \otimes F_7 \vdash F_9}{\text{hCut}} \text{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_3 \vdash F_9}}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{10} \quad h_8 : \Delta_9 \vdash F_{11}}{\bullet h_8 : (\Delta_9, \Delta_{12}), F_6 \otimes F_7 \vdash F_{10} \otimes F_{11}} \otimes_R}{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{12} \vdash F_{10} \otimes F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{10}}{\text{hCut}} \text{ax/W} \quad \frac{- : \Delta_9 \vdash F_{11}}{\otimes_R} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3 \vdash F_{10}} \otimes_R}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9 \vdash F_{10} \otimes F_{11}}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{\frac{h_8 : \Delta_9 \vdash F_{10} \quad h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_9, \Delta_{12}), F_6 \otimes F_7 \vdash F_{10} \otimes F_{11}} \otimes_R}{\frac{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{12} \vdash F_{10} \otimes F_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_9 \vdash F_{10} \quad \frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \frac{h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{11}}{\rightarrow} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3 \vdash F_{11}} \otimes_R}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9 \vdash F_{10} \otimes F_{11}} \text{hCut}
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_6 \otimes F_7 \vdash F_{10}} W}{\frac{- : (\Delta_2, \Delta_3), \Delta_{11}, !F_9 \vdash F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9 \vdash F_{10}} \quad \frac{h_8 : \Delta_{11}, !F_9, F_6 \otimes F_7 \vdash F_{10}}{\text{hCut}}}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9 \vdash F_{10}}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, !F_9, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_6 \otimes F_7 \vdash F_{10}} C}{\frac{- : (\Delta_2, \Delta_3), \Delta_{11}, !F_9 \vdash F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9 \vdash F_{10}} \quad \frac{h_8 : \Delta_{11}, !F_9, F_6 \otimes F_7 \vdash F_{10}}{\text{hCut}}}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9 \vdash F_{10}} C
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_9, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_6 \otimes F_7 \vdash F_{10}} !L}{\frac{- : (\Delta_2, \Delta_3), \Delta_{11}, !F_9 \vdash F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, F_9 \vdash F_{10}} \quad \frac{h_8 : \Delta_{11}, F_9, F_6 \otimes F_7 \vdash F_{10}}{\text{hCut}}}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9 \vdash F_{10}} !L
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_6 \otimes F_7 \vdash F_{11}} \&_{L2}}{\frac{- : (\Delta_2, \Delta_3), \Delta_{12}, F_9 \& F_{10} \vdash F_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_{10} \vdash F_{11}} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\text{hCut}}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \& F_{10} \vdash F_{11}} \&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_6 \otimes F_7 \vdash F_{11}} \&_{L1}}{\frac{- : (\Delta_2, \Delta_3), \Delta_{12}, F_9 \& F_{10} \vdash F_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \vdash F_{11}} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11}}{\text{hCut}}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \& F_{10} \vdash F_{11}} \&_{L1}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \otimes F_{10}), F_6 \otimes F_7 \vdash F_{11}} \otimes_L \\
\hline
- : (\Delta_2, \Delta_3), \Delta_{12}, F_9 \otimes F_{10} \vdash F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_2, \Delta_3, F_{10}, F_9 \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \otimes F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \otimes F_{10} \vdash F_{11}} \text{hCut} \\
\hline
\frac{h_1 : \Delta_2 \vdash F_8 \quad h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_8 \otimes F_9} \otimes_R \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, \Delta_3), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_8}{- : \Delta_2, \Delta_3, \Delta_7 \vdash F_{10}} \text{ax/W} \quad \frac{- : \Delta_3 \vdash F_9}{- : \Delta_3, \Delta_7, F_8 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_7 \vdash F_{10}} \text{sCut} \\
\hline
\frac{- : \Delta_2 \vdash F_8}{- : \Delta_2, \Delta_3, \Delta_7 \vdash F_{10}} \text{sCut}
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \oplus F_{10}), F_6 \otimes F_7 \vdash F_{11}} \oplus_L \\
\hline
- : (\Delta_2, \Delta_3), \Delta_{12}, F_9 \oplus F_{10} \vdash F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \oplus F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \oplus F_{10} \vdash F_{11}} \text{hCut} \\
\hline
\frac{\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \oplus F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \oplus F_{10} \vdash F_{11}} \text{hCut}
\end{array}$$

- Case rule  $\neg\circ_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{13}, F_6 \otimes F_7 \vdash F_{10} \quad h_8 : \Delta_9, F_{11} \vdash F_{12}}{\bullet h_8 : (\Delta_9, \Delta_{13}, F_{10} \neg\circ F_{11}), F_6 \otimes F_7 \vdash F_{12}} \neg\circ_L \\
\hline
- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{13}, F_{10} \neg\circ F_{11} \vdash F_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{13}, \Delta_2, \Delta_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_{13}, F_6 \otimes F_7 \vdash F_{10}}{- : \Delta_9, F_{11} \vdash F_{12}} \text{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \neg\circ F_{11} \vdash F_{12}} \text{hCut} \\
\hline
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_9 \vdash F_{10} \quad h_8 : \Delta_{13}, F_{11}, F_6 \otimes F_7 \vdash F_{12}}{\bullet h_8 : (\Delta_9, \Delta_{13}, F_{10} \neg\circ F_{11}), F_6 \otimes F_7 \vdash F_{12}} \neg\circ_L \\
\hline
- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{13}, F_{10} \neg\circ F_{11} \vdash F_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \Delta_9 \vdash F_{10}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \neg\circ F_{11} \vdash F_{12}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{13}, \Delta_2, \Delta_3, F_{11} \vdash F_{12}} \text{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \neg\circ F_{11} \vdash F_{12}} \text{hCut}
\end{array}$$

- Case rule  $I$

## 5.10 Status of $W$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash 6}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} W \quad \frac{h_7 : !\Upsilon 5, 6 \vdash F_8}{\bullet h_7 : !\Upsilon 5, 6 \vdash !F_8} !R \\
\hline
- : (\Delta_2, !F_3), !\Upsilon 5 \vdash !F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_2 \vdash 6}{- : !\Upsilon 5, \Delta_2, !F_3 \vdash !F_8} \text{ax/W} \quad \frac{\bullet h_7 : 6, !\Upsilon 5, !F_3 \vdash !F_8}{- : !\Upsilon 5, \Delta_2, !F_3 \vdash !F_8} \text{hCut}
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{}{\bullet h_7 : \Delta_5, F_6 \vdash \top} \top}{\frac{}{- : (\Delta_2, !F_3), \Delta_5 \vdash \top} \text{Cut}} \rightarrow \frac{}{- : \Delta_2, \Delta_5, !F_3 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8 \quad h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9} \&_R}{\frac{}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \& F_9} \text{Cut}} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \& F_9} \text{ax/W} \quad \frac{}{\bullet h_7 : \Delta_5, F_6, !F_3 \vdash F_8 \& F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \& F_9} \text{hCut}$$

- Case rule  $\multimap_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \multimap F_9} \multimap_R}{\frac{}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \multimap F_9} \text{Cut}} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} \text{ax/W} \quad \frac{}{\bullet h_7 : \Delta_5, F_6, !F_3 \vdash F_8 \multimap F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \multimap F_9} \text{hCut}$$

- Case rule  $\oplus_{R_2}$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_2}}{\frac{}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9} \text{Cut}} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \oplus F_9} \text{ax/W} \quad \frac{}{\bullet h_7 : \Delta_5, F_6, !F_3 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \text{hCut}$$

- Case rule  $\oplus_{R_1}$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_1}}{\frac{}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9} \text{Cut}} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \oplus F_9} \text{ax/W} \quad \frac{}{\bullet h_7 : \Delta_5, F_6, !F_3 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \text{hCut}$$

- Case rule  $\mathbf{1}_L$

$$\frac{\frac{h_1 : \Delta_2 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, !F_3 \vdash \mathbf{1}} W \quad \frac{h_5 : \Delta_6 \vdash F_7}{\bullet h_5 : \Delta_6, \mathbf{1} \vdash F_7} \mathbf{1}_L}{\frac{}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_7} \text{Cut}} \rightarrow \frac{}{- : \Delta_2, \Delta_6, !F_3 \vdash F_7} \text{ax/W}$$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_8, F_5 \vdash F_7}{\bullet h_6 : (\mathbf{1}, \Delta_8), F_5 \vdash F_7} \mathbf{1}_L}{\frac{}{- : (\Delta_2, !F_3), \mathbf{1}, \Delta_8 \vdash F_7} \text{Cut}} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7} \text{ax/W} \quad \frac{}{\bullet h_6 : \mathbf{1}, \Delta_8, F_5, !F_3 \vdash F_7} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{hCut}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{\frac{h_6 : \Delta_{10}, F_5 \vdash F_8 \quad h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_7, F_5, !F_3 \vdash F_8 \otimes F_9}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{\frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{10}, F_5 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_7, F_5, !F_3 \vdash F_8 \otimes F_9}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9}
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{\frac{h_6 : \Delta_9, F_5 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} W}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, F_5, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} W \quad \frac{\frac{h_5 : \Delta_6 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} W}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_2, \Delta_6, !F_3 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{\frac{h_6 : \Delta_9, F_5, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} C}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, F_5, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} W \quad \frac{\frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} C}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash !F_7}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{hCut}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{\frac{h_6 : \Delta_9, F_5, F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} !L}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, F_5, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} W \quad \frac{\frac{h_5 : \Delta_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} !L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash !F_7}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{hCut}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8}
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9}}{\vdash : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, !F_3, F_7 \& F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_5, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{\vdash : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} W \quad \frac{h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}}{\vdash : (\Delta_2, !F_3), \Delta_6 \vdash F_9} \&_{L2} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_3, F_7 \& F_8 \vdash F_9}{\bullet h_5 : \Delta_6, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{\vdash : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9}}{\vdash : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L1} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, !F_3, F_7 \& F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_5, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{\vdash : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} W \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}}{\vdash : (\Delta_2, !F_3), \Delta_6 \vdash F_9} \&_{L1} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_3, F_7 \& F_8 \vdash F_9}{\bullet h_5 : \Delta_6, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{\vdash : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{10}, F_5, F_7, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_5 \vdash F_9}}{\vdash : (\Delta_2, !F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, !F_3, F_7 \otimes F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_5, !F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{\vdash : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \otimes F_8} W \quad \frac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9}}{\vdash : (\Delta_2, !F_3), \Delta_6 \vdash F_9} \otimes_L \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_3, F_7 \otimes F_8 \vdash F_9}{\bullet h_5 : \Delta_6, !F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{\vdash : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_5 \vdash F_9}}{\vdash : (\Delta_2, !F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \oplus_L \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, !F_3, F_7 \oplus F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_5, !F_3, F_7 \oplus F_8 \vdash F_9} \text{ax/W}}{\vdash : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \oplus F_8} W \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus_L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \oplus F_8} \text{ ax/W} \quad \frac{\bullet h_5 : \Delta_6, !F_3, F_7 \oplus F_8 \vdash F_9}{\bullet h_5 : \Delta_6, !F_3 \vdash F_9} \text{ ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{ hCut}
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{11}, F_5 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, !F_3, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_6 : \Delta_{11}, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{ hCut} \\
\\
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{11}, F_5, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, !F_3, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_6 : \Delta_{11}, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{ hCut} \\
\\
\frac{h_1 : \Delta_2 \vdash F_8 \multimap F_9}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} W \quad \frac{h_5 : \Delta_6 \vdash F_8 \quad h_5 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_5 : (\Delta_6, \Delta_7), F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (\Delta_2, !F_3), \Delta_6, \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_8 \multimap F_9}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} \text{ ax/W} \quad \frac{\bullet h_5 : \Delta_6, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_5 : \Delta_6, \Delta_7, !F_3 \vdash F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_6, \Delta_7, !F_3 \vdash F_{10}} \text{ hCut}
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash p(n_6)}{\bullet h_1 : \Delta_2, !F_3 \vdash p(n_6)} W \quad \frac{}{\bullet h_5 : *, p(n_6) \vdash p(n_6)} I \\
\hline
- : (\Delta_2, !F_3), * \vdash p(n_6) \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_2, !F_3 \vdash p(n_6)} \text{ ax/W}
\end{array}$$

## 5.11 Status of $C$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash 6}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} C \quad \frac{h_7 : !\Upsilon 5, 6 \vdash F_8}{\bullet h_7 : !\Upsilon 5, 6 \vdash !F_8} !R \\
\hline
- : (\Delta_2, !F_3), !\Upsilon 5 \vdash !F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash 6}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} \text{ ax/W} \quad \frac{\bullet h_7 : 6, !\Upsilon 5 \vdash !F_8}{\bullet h_7 : 6, !\Upsilon 5 \vdash !F_8} \text{ ax/W}}{- : !\Upsilon 5, \Delta_2, !F_3, !F_3 \vdash !F_8} \text{ hCut} \\
\hline
- : !\Upsilon 5, \Delta_2, !F_3 \vdash !F_8 \quad C
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} C \quad \frac{}{\bullet h_7 : \Delta_5, F_6 \vdash \top} \top \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash \top \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_5, !F_3 \vdash \top} \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} C \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8 \quad h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9} \&_R \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{- : \Delta_2, \Delta_5, !F_3, !F_3 \vdash F_8 \& F_9} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \& F_9} \text{hCut}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \& F_9} C
\end{array}$$

- Case rule  $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} C \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \neg\circ F_9} \neg\circ_R \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \neg\circ F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_6}{- : \Delta_2, \Delta_5, F_8, !F_3 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \neg\circ F_9} \text{hCut}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \neg\circ F_9} \neg\circ_R
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} C \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_2} \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_6}{- : \Delta_2, \Delta_5, !F_3 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \text{hCut}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} C \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_1} \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_6}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \text{hCut}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, !F_3 \vdash \mathbf{1}} C \quad \frac{h_5 : \Delta_6 \vdash F_7}{\bullet h_5 : \Delta_6, \mathbf{1} \vdash F_7} 1_L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, !F_3 \vdash F_7} \text{ax/W} \\
\\
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_8, F_5 \vdash F_7}{\bullet h_6 : (\mathbf{1}, \Delta_8), F_5 \vdash F_7} 1_L \\
\hline
- : (\Delta_2, !F_3), \mathbf{1}, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_5}{- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \mathbf{1}, \Delta_8, F_5 \vdash F_7}{- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{hCut}}{- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$



$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5 \vdash F_8 \quad h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3, !F_3 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{10}, F_5 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3, !F_3 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} C
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_9, F_5 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} W \\
\hline
- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} C \quad \frac{h_5 : \Delta_6 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} W \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_2, \Delta_6, !F_3 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_9, F_5, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} C \\
\hline
- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} C \quad \frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} C \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{mCut}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_9, F_5, F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} !L \\
\hline
- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} C \quad \frac{h_5 : \Delta_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} !L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_8 \quad C
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9} \&_{L2} \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, !F_3, F_7 \& F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9 \quad C \\
\\
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} C \quad \frac{h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&_{L2} \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_9 \quad C
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9} \&_{L1} \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, !F_3, F_7 \& F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9 \quad C \\
\\
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} C \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&_{L1} \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_9 \quad C
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5, F_7, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_5 \vdash F_9} \otimes_L \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9 \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \otimes F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_5, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, !F_3, F_7 \otimes F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9 \quad C \\
\\
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \otimes F_8} C \quad \frac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9} \otimes_L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_9 \quad C
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_5 \vdash F_9} \oplus_L \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, !F_3, !F_3 \vdash F_5} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_5, F_7 \oplus F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9} C \\
\hline
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \oplus F_8} C \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus_L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \oplus F_8}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} C
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{11}, F_5 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, !F_3 \vdash F_5} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} C \\
\hline
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{11}, F_5, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, !F_3 \vdash F_5} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} C \\
\hline
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_8 \multimap F_9}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} C \quad \frac{h_5 : \Delta_6 \vdash F_8 \quad h_5 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_5 : (\Delta_6, \Delta_7), F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (\Delta_2, !F_3), \Delta_6, \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_8 \multimap F_9}{- : \Delta_2, \Delta_6, \Delta_7, !F_3, !F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, \Delta_7, F_8 \multimap F_9 \vdash F_{10}}{- : \Delta_2, \Delta_6, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_6, \Delta_7, !F_3 \vdash F_{10}} C
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash p(n_6)}{\bullet h_1 : \Delta_2, !F_3 \vdash p(n_6)} C \quad \frac{}{\bullet h_5 : *, p(n_6) \vdash p(n_6)} I \\
\hline
- : (\Delta_2, !F_3), * \vdash p(n_6) \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_2, !F_3 \vdash p(n_6)} \text{ax/W}
\end{array}$$

## 5.12 Status of $!L$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash 6}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} !L \quad \frac{h_7 : !\Upsilon 5, 6 \vdash F_8}{\bullet h_7 : !\Upsilon 5, 6 \vdash !F_8} !R \\
\hline
- : (\Delta_2, !F_3), !\Upsilon 5 \vdash !F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash 6}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} \text{ax/W} \quad \frac{\bullet h_7 : 6, !\Upsilon 5 \vdash !F_8}{\bullet h_7 : 6, !\Upsilon 5 \vdash !F_8} \text{ax/W}}{\frac{- : !\Upsilon 5, \Delta_2, F_3 \vdash !F_8}{- : !\Upsilon 5, \Delta_2, !F_3 \vdash !F_8} !L} \text{hCut}
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{}{\bullet h_7 : \Delta_5, F_6 \vdash \top} \top \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash \top \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_2, \Delta_5, !F_3 \vdash \top \quad \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8 \quad h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9} \&_R \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9} \text{ax/W}}{\frac{- : \Delta_2, \Delta_5, F_3 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \& F_9} !L} \text{hCut}
\end{array}$$

- Case rule  $\multimap_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \multimap F_9} \multimap_R \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \multimap F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \multimap F_9} \text{ax/W}}{\frac{- : \Delta_2, \Delta_5, F_8, !F_3 \vdash F_9}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \multimap F_9} \multimap_R} \text{hCut}
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_2} \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \text{ax/W}}{\frac{- : \Delta_2, \Delta_5, !F_3 \vdash F_9}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \oplus_{R_2}} \text{hCut}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_1} \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \text{ax/W}}{\frac{- : \Delta_2, \Delta_5, !F_3 \vdash F_8}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \oplus_{R_1}} \text{hCut}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, !F_3 \vdash \mathbf{1}} !L \quad \frac{h_5 : \Delta_6 \vdash F_7}{\bullet h_5 : \Delta_6, \mathbf{1} \vdash F_7} 1_L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, !F_3 \vdash F_7} \text{ax/W} \\
\\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_8, F_5 \vdash F_7}{\bullet h_6 : (\mathbf{1}, \Delta_8), F_5 \vdash F_7} 1_L}{- : (\Delta_2, !F_3), \mathbf{1}, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_5}{- : \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \mathbf{1}, \Delta_8, F_5 \vdash F_7}{- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5 \vdash F_8 \quad h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, \Delta_7, F_3 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, F_3 \vdash F_8 \otimes F_9} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{10}, F_5 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, \Delta_7, F_3 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, F_3 \vdash F_8 \otimes F_9} \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} !L
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_9, F_5 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} W}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} !L \quad \frac{h_5 : \Delta_6 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} W}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_9, F_5, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} C}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, !F_3 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_7, !F_7 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} !L \quad \frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} C \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3 \vdash !F_7}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{mCut}
\end{array}$$

- Case rule !L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_9, F_5, F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} !L \\
\hline
- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{- : \Delta_2, \Delta_9, F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut} \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} !L \quad \frac{h_5 : \Delta_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} !L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash !F_7}{- : \Delta_2, \Delta_6, F_3 \vdash F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_7 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_8 \quad !L
\end{array}$$

- Case rule &L<sub>2</sub>

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9} \&L_2 \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, F_3, F_7 \& F_8 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{hCut} \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} !L \quad \frac{h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&L_2 \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \& F_8}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_9 \quad !L
\end{array}$$

- Case rule &L<sub>1</sub>

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9} \&L_1 \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, F_3, F_7 \& F_8 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{hCut} \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} !L \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&L_1 \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \& F_8}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_9 \quad !L
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5, F_7, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_5 \vdash F_9} \otimes_L}{- : (\Delta_2, !F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \otimes F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_5 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, F_3, F_7 \otimes F_8 \vdash F_9} \text{hCut} \\
\frac{}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9} !L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \otimes F_8} !L \quad \frac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9} \otimes_L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} !L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_5 \vdash F_9} \oplus_L}{- : (\Delta_2, !F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \oplus F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_5 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut} \\
\frac{}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9} !L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \oplus F_8} !L \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus_L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \oplus F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} !L
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{11}, F_5 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10})} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} !L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{11}, F_5, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10})} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} !L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \multimap F_9}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} !L \quad \frac{h_5 : \Delta_6 \vdash F_8 \quad h_5 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_5 : (\Delta_6, \Delta_7), F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : (\Delta_2, !F_3), \Delta_6, \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \multimap F_9}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} \text{ax/W} \quad \frac{h_5 : \Delta_6, \Delta_7, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_5 : (\Delta_6, \Delta_7, F_8 \multimap F_9 \vdash F_{10})} \text{ax/W}}{- : \Delta_2, \Delta_6, \Delta_7, F_3 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_6, \Delta_7, !F_3 \vdash F_{10}} !L
\end{array}$$

- Case rule  $I$

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash p(n_6)}{\bullet h_1 : \Delta_2, !F_3 \vdash p(n_6)} \text{!}L \quad \frac{}{\bullet h_5 : *, p(n_6) \vdash p(n_6)} I}{\frac{}{- : (\Delta_2, !F_3), * \vdash p(n_6)} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{}{- : \Delta_2, !F_3 \vdash p(n_6)}$$

### 5.13 Status of $\&_{L2}$ : OK

- Case rule  $!R$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 7} \&_{L2} \quad \frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : !\Upsilon 6, 7 \vdash !F_9} \text{!}R}{\frac{}{- : (\Delta_2, F_3 \& F_4), !\Upsilon 6 \vdash !F_9} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{\frac{h_1 : \Delta_2, F_4 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 7} \text{ax/W} \quad \frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : !\Upsilon 6, 7 \vdash !F_9} \text{ax/W}}{\frac{}{- : !\Upsilon 6, \Delta_2, F_4 \vdash !F_9} \text{hCut}} \&_{L2} \frac{}{- : !\Upsilon 6, \Delta_2, F_3 \& F_4 \vdash !F_9}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \quad \frac{}{\bullet h_8 : \Delta_6, F_7 \vdash \top} \top}{\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash \top} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \&_R}{\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \& F_{10}} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, F_4 \vdash F_9 \& F_{10}} \text{hCut}} \&_{L2} \frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \& F_{10}}$$

- Case rule  $\multimap_R$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \multimap F_{10}} \multimap_R}{\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \multimap F_{10}} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{\frac{h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \multimap F_{10}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, F_9, F_3 \& F_4 \vdash F_{10}} \text{hCut}} \multimap_R \frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \multimap F_{10}}$$

- Case rule  $\oplus_{R_2}$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2}}{\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \oplus F_{10}} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{\frac{h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_{10}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_{10}} \text{hCut}} \oplus_{R_2} \frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}}$$



- Case rule  $\oplus_{R1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R1} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \oplus F_{10} \\
\hline
\text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{\text{ax/W}} \xrightarrow{\quad} \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\text{ax/W}} \\
\hline
- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \\
\hline
- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10} \quad \oplus_{R1} \\
\hline
\text{hCut}
\end{array}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash 1}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 1} \&_{L2} \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, 1 \vdash F_8} 1_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_8 \\
\hline
\text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 1}{\text{ax/W}} \xrightarrow{\quad} \frac{h_6 : \Delta_7 \vdash F_8}{\text{ax/W}} \\
\hline
- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_8 \\
\hline
\text{hCut}
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_6 \vdash F_8} 1_L \\
\hline
- : (\Delta_2, F_3 \& F_4), 1, \Delta_9 \vdash F_8 \\
\hline
\text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6}{\text{ax/W}} \xrightarrow{\quad} \frac{h_7 : 1, \Delta_9, F_6 \vdash F_8}{\text{ax/W}} \\
\hline
- : 1, \Delta_2, \Delta_9, F_3 \& F_4 \vdash F_8 \\
\hline
\text{hCut}
\end{array}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \frac{h_7 : \Delta_{11}, F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\hline
\text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\text{ax/W}} \xrightarrow{\quad} \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{\text{ax/W}} \\
\hline
- : \Delta_{11}, \Delta_2, \Delta_8, F_4 \vdash F_9 \otimes F_{10} \\
\hline
- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10} \quad \&_{L2} \\
\hline
\text{hCut}
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\hline
\text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\text{ax/W}} \xrightarrow{\quad} \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{\text{ax/W}} \\
\hline
- : \Delta_{11}, \Delta_2, \Delta_8, F_4 \vdash F_9 \otimes F_{10} \\
\hline
- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10} \quad \&_{L2} \\
\hline
\text{hCut}
\end{array}
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} W \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9 \\
\hline
\text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6}{\text{ax/W}} \xrightarrow{\quad} \frac{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9}{\text{ax/W}} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9 \\
\hline
\text{hCut}
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L2} \frac{h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} W \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9 \\
\hline
\text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8}{\text{ax/W}} \xrightarrow{\quad} \frac{h_6 : \Delta_7 \vdash F_9}{\text{ax/W}} \\
\hline
- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9 \\
\hline
\text{hCut}
\end{array}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} C}{- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, !F_8, F_3 \& F_4 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} C \\
\\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L2} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} C}{- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \text{ax/W} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \text{mCut}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} !L}{- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, F_8, F_3 \& F_4 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} !L \\
\\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L2} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} !L}{- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \&_{L2}
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L2}}{- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \& F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \&_{L2} \quad \frac{h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L2}}{- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L1}}{- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \& F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10}} \&_{L2}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \&_{L2} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L2}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_6 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \otimes F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \otimes F_9 \vdash F_{10}} \&_{L2} \\
\\
\frac{h_1 : \Delta_2, F_4 \vdash F_8 \otimes F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \otimes F_9} \&_{L2} \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \otimes F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L2}
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_6 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \oplus F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \oplus F_9 \vdash F_{10}} \&_{L2} \\
\\
\frac{h_1 : \Delta_2, F_4 \vdash F_8 \oplus F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \oplus F_9} \&_{L2} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \oplus F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L2}
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{12}, F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}} \multimap_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{12}, \Delta_2, \Delta_8, F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \multimap F_{10} \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \&_{L2} \\
\\
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_6, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}} \multimap_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{12}, \Delta_2, \Delta_8, F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \multimap F_{10} \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \&_{L2}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_9 \multimap F_{10}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_7 \vdash F_9 \quad h_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8), F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\hline
\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_7, \Delta_8 \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_9 \multimap F_{10}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_7, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}}{\bullet h_6 : \Delta_7, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{11}} \text{hCut}} \&_{L2} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \& F_4 \vdash F_{11}} \&_{L2}
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash p(n_7)}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash p(n_7)} \&_{L2} \quad \frac{}{\bullet h_6 : *, p(n_7) \vdash p(n_7)} I \\
\hline
\frac{}{- : (\Delta_2, F_3 \& F_4), * \vdash p(n_7)} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, F_3 \& F_4 \vdash p(n_7)} \text{ax/W}
\end{array}$$

## 5.14 Status of $\&_{L1}$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 7} \&_{L1} \quad \frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : !\Upsilon 6, 7 \vdash !F_9} !R \\
\hline
\frac{}{- : (\Delta_2, F_3 \& F_4), !\Upsilon 6 \vdash !F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 7} \text{ax/W} \quad \frac{h_8 : 7, !\Upsilon 6 \vdash !F_9}{\bullet h_8 : 7, !\Upsilon 6 \vdash !F_9} \text{ax/W}}{\frac{}{- : !\Upsilon 6, \Delta_2, F_3 \vdash !F_9} \text{hCut}} \&_{L1} \\
\rightarrow \\
\frac{}{- : !\Upsilon 6, \Delta_2, F_3 \& F_4 \vdash !F_9} \&_{L1}
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \quad \frac{}{\bullet h_8 : \Delta_6, F_7 \vdash \top} \top \\
\hline
\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash \top} \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \&_R \\
\hline
\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \& F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, F_3 \vdash F_9 \& F_{10}} \text{hCut}} \&_{L1} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \& F_{10}} \&_{L1}
\end{array}$$

- Case rule  $\multimap_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \multimap F_{10}} \multimap_R \\
\hline
\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \multimap F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \multimap F_{10}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, F_9, F_3 \& F_4 \vdash F_{10}} \text{hCut}} \multimap_R \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \multimap_R
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \oplus F_{10} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{- : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \oplus F_{10} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{- : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash 1}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 1} \&_{L1} \quad \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, 1 \vdash F_8} 1_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_8 \\
\hline
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_8} \text{ax/W} \\
\\
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_6 \vdash F_8} 1_L \\
\hline
- : (\Delta_2, F_3 \& F_4), 1, \Delta_9 \vdash F_8 \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6}{- : 1, \Delta_2, \Delta_9, F_3 \& F_4 \vdash F_8} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_6 \vdash F_8}{- : 1, \Delta_9, F_6 \vdash F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_9, F_3 \& F_4 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{11}, F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \&_{L1} \\
\\
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \&_{L1}
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} W \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9 \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{- : \Delta_{10}, F_6, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L1} \quad \frac{h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} W \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \text{ax/W} \\
\text{Cut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} C \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_2, !F_8, !F_8, F_3 \& F_4 \vdash F_9} \text{ax/W} \quad \frac{}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} C \\
\text{hCut} \\
\frac{h_1 : \Delta_2, F_3 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L1} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} C \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \text{ax/W} \quad \frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \text{mCut} \\
\text{Cut}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} !L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_2, F_8, F_3 \& F_4 \vdash F_9} \text{ax/W} \quad \frac{}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} !L \\
\text{hCut} \\
\frac{h_1 : \Delta_2, F_3 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L1} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} !L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \vdash F_9} \text{ax/W} \quad \frac{}{- : \Delta_2, \Delta_7, !F_8 \vdash F_9} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \&_{L1} \\
\text{Cut}
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, \Delta_2, F_9, F_3 \& F_4 \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\text{hCut} \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \&_{L1} \quad \frac{h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_2, \Delta_7, F_8 \& F_9 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L1} \\
\text{Cut}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \& F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_3, F_8 \& F_9 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10} \quad \&_{L1}
\end{array} \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \&_{L1} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_2, \Delta_7, F_3 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10} \quad \&_{L1}
\end{array}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_6 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_8 \otimes F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_3, F_8 \otimes F_9 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \otimes F_9 \vdash F_{10} \quad \&_{L1}
\end{array} \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \otimes F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \otimes F_9} \&_{L1} \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \otimes F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_2, \Delta_7, F_3 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10} \quad \&_{L1}
\end{array}
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_6 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_3, F_8 \oplus F_9 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \oplus F_9 \vdash F_{10} \quad \&_{L1}
\end{array} \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \oplus F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \oplus F_9} \&_{L1} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \oplus F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_2, \Delta_7, F_3 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10} \quad \&_{L1}
\end{array}
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{12}, F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}} \multimap_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \multimap F_{10} \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W} \\
\hline
- : \Delta_{12}, \Delta_2, \Delta_8, F_3, F_9 \multimap F_{10} \vdash F_{11} \quad \text{hCut} \\
\hline
- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11} \quad \&_{L1}
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{\frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_6, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}} \multimap_L}{\multimap_L} \text{Cut} \\
\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \text{ax/W} \quad \frac{\frac{h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \multimap F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_{12}, \Delta_8, F_6, F_9 \multimap F_{10}) \vdash F_{11}} \text{ax/W}}{\frac{}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut}} \&_{L1} \\
\frac{}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_9 \multimap F_{10}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \&_{L1} \quad \frac{\frac{h_6 : \Delta_7 \vdash F_9 \quad h_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8), F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}{\multimap_L} \text{Cut} \\
\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_7, \Delta_8 \vdash F_{11}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_9 \multimap F_{10}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \text{ax/W} \quad \frac{\frac{h_6 : \Delta_7, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8, F_9 \multimap F_{10}) \vdash F_{11}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \vdash F_{11}} \text{hCut}} \&_{L1} \\
\frac{}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \& F_4 \vdash F_{11}}{\rightarrow}
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash p(n_7)}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash p(n_7)} \&_{L1} \quad \frac{}{\bullet h_6 : *, p(n_7) \vdash p(n_7)} I}{\frac{}{- : (\Delta_2, F_3 \& F_4), * \vdash p(n_7)} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_2, F_3 \& F_4 \vdash p(n_7)} \text{ax/W}
\end{array}$$

## 5.15 Status of $\otimes_L$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash 7} \otimes_L \quad \frac{\frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : !\Upsilon 6, 7 \vdash !F_9} !R}{\frac{}{- : (\Delta_2, F_3 \otimes F_4), !\Upsilon 6 \vdash !F_9} \text{Cut}} \rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash 7} \text{ax/W} \quad \frac{\frac{h_8 : 7, !\Upsilon 6 \vdash !F_9}{\bullet h_8 : 7, !\Upsilon 6 \vdash !F_9} \text{ax/W}}{\frac{}{- : !\Upsilon 6, \Delta_2, F_3, F_4 \vdash !F_9} \text{hCut}} \otimes_L \\
\frac{}{- : !\Upsilon 6, \Delta_2, F_3 \otimes F_4 \vdash !F_9}
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{}{\bullet h_8 : \Delta_6, F_7 \vdash \top} \top}{\frac{}{- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash \top} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash \top} \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{\frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \&_R}{\frac{}{- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash F_9 \& F_{10}} \text{Cut}} \rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \text{ax/W} \quad \frac{\frac{h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, F_3, F_4 \vdash F_9 \& F_{10}} \text{hCut}} \otimes_L \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \& F_{10}}
\end{array}$$

- Case rule  $\multimap_R$



$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \multimap F_{10}} \multimap_R \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash F_9 \multimap F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_9, F_3 \otimes F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \multimap F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \multimap F_{10}} \text{hCut} \multimap_R
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash 1}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash 1} \otimes_L \quad \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, 1 \vdash F_8} 1_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_8 \quad \text{ax/W} \\
\\
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_6 \vdash F_8} 1_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), 1, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : 1, \Delta_2, \Delta_9, F_3 \otimes F_4 \vdash F_8} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_6 \vdash F_8}{- : 1, \Delta_2, \Delta_9, F_3 \otimes F_4 \vdash F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_9, F_3 \otimes F_4 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_{11}, F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3, F_4 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \otimes F_4 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \otimes F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_L \\
\\
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3, F_4 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \otimes F_4 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \otimes F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_L
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} \frac{W}{\text{Cut}}}{- : (\Delta_2, F_3 \otimes F_4), \Delta_{10}, !F_8 \vdash F_9} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9}{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash !F_8} \otimes_L \frac{h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} \frac{W}{\text{Cut}}}{- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_9} \\
\rightarrow \\
\frac{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9} \text{ax/W}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} \frac{C}{\text{Cut}}}{- : (\Delta_2, F_3 \otimes F_4), \Delta_{10}, !F_8 \vdash F_9} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8, F_3 \otimes F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} \text{hCut} \\
\frac{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} C \\
\\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash !F_8} \otimes_L \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} \frac{C}{\text{Cut}}}{- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_9} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash !F_8}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{h_6 : \Delta_7, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9} \text{mCut}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} \frac{!L}{\text{Cut}}}{- : (\Delta_2, F_3 \otimes F_4), \Delta_{10}, !F_8 \vdash F_9} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, F_8, F_3 \otimes F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} \text{hCut} \\
\frac{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} !L \\
\\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash !F_8} \otimes_L \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} \frac{!L}{\text{Cut}}}{- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_9} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash !F_8}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_7, !F_8 \vdash F_9}{h_6 : \Delta_7, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_9} \text{hCut} \\
\frac{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_9}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9} \otimes_L
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \frac{\&_{L2}}{\text{Cut}}}{- : (\Delta_2, F_3 \otimes F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_9, F_3 \otimes F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9, F_3 \otimes F_4 \vdash F_{10}} \text{hCut} \\
\frac{- : \Delta_{11}, \Delta_2, F_9, F_3 \otimes F_4 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \otimes F_4 \vdash F_{10}} \&_{L2}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_8 \& F_9} \otimes_L \quad \frac{h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut} \otimes_L
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_8, F_3 \otimes F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \otimes F_4 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut} \&_{L1} \\
\hline
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_8 \& F_9} \otimes_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut} \otimes_L
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_6 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \otimes F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_3 \otimes F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ hCut} \otimes_L \\
\hline
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \otimes F_9}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_8 \otimes F_9} \otimes_L \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \otimes F_9}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut} \otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_6 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \otimes F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_3 \otimes F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ hCut} \otimes_L \\
\hline
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \oplus F_9}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_8 \oplus F_9} \otimes_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \oplus F_9}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut} \otimes_L
\end{array}$$

- Case rule  $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_{12}, F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_6 \vdash F_{11}} \neg\circ_L}{\neg : (\Delta_2, F_3 \otimes F_4), \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3, F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \neg\circ F_{10} \vdash F_{11}}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3 \otimes F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \text{hCut}}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3 \otimes F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \otimes_L \\
\\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_6, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_6 \vdash F_{11}} \neg\circ_L}{\neg : (\Delta_2, F_3 \otimes F_4), \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3, F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \neg\circ F_{10} \vdash F_{11}}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3 \otimes F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \text{hCut}}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3 \otimes F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \otimes_L \\
\\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_9 \neg\circ F_{10}}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_9 \neg\circ F_{10}} \otimes_L \quad \frac{h_6 : \Delta_7 \vdash F_9 \quad h_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8), F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L}{\neg : (\Delta_2, F_3 \otimes F_4), \Delta_7, \Delta_8 \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_9 \neg\circ F_{10}}{\neg : \Delta_2, \Delta_7, \Delta_8, F_3, F_4 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}}{\neg : \Delta_2, \Delta_7, \Delta_8, F_3 \otimes F_4 \vdash F_{11}} \text{hCut}}{\neg : \Delta_2, \Delta_7, \Delta_8, F_3 \otimes F_4 \vdash F_{11}} \otimes_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash p(n_7)}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash p(n_7)} \otimes_L \quad \frac{\bullet h_6 : *, p(n_7) \vdash p(n_7)}{\neg : (\Delta_2, F_3 \otimes F_4), * \vdash p(n_7)} I}{\neg : (\Delta_2, F_3 \otimes F_4), * \vdash p(n_7)} \text{Cut} \\
\rightarrow \\
\frac{}{\neg : \Delta_2, F_3 \otimes F_4 \vdash p(n_7)} \text{ax/W}
\end{array}$$

## 5.16 Status of $\oplus_L$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash 7 \quad h_1 : \Delta_2, F_4 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash 7} \oplus_L \quad \frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : !\Upsilon 6, 7 \vdash !F_9} !R}{\neg : (\Delta_2, F_3 \oplus F_4), !\Upsilon 6 \vdash !F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash 7}{\neg : !\Upsilon 6, \Delta_2, F_3 \vdash !F_9} \text{ax/W} \quad \frac{\bullet h_8 : 7, !\Upsilon 6 \vdash !F_9}{\neg : !\Upsilon 6, \Delta_2, F_3 \oplus F_4 \vdash !F_9} \text{hCut}}{\neg : !\Upsilon 6, \Delta_2, F_3 \oplus F_4 \vdash !F_9} \oplus_L
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{\bullet h_8 : \Delta_6, F_7 \vdash \top}{\neg : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash \top} \top}{\neg : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{\neg : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash \top} \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \&_R}{- : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash F_9 \& F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{hCut} \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9 \& F_{10}} \text{hCut} \&_R
\end{array}$$

- Case rule  $\neg\circ_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \neg\circ F_{10}} \neg\circ_R}{- : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash F_9 \neg\circ F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \\
\frac{- : \Delta_2, \Delta_6, F_9, F_3 \oplus F_4 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9 \neg\circ F_{10}} \neg\circ_R
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \\
\frac{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9 \oplus F_{10}} \oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{hCut} \\
\frac{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9 \oplus F_{10}} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash \mathbf{1} \quad h_1 : \Delta_2, F_4 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash \mathbf{1}} \oplus_L \quad \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, \mathbf{1} \vdash F_8} \mathbf{1}_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_8}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_8} \text{ax/W} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_6 \vdash F_8} \mathbf{1}_L}{- : (\Delta_2, F_3 \oplus F_4), \mathbf{1}, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \oplus F_4 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \mathbf{1}, \Delta_9, F_6 \vdash F_8}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \oplus F_4 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \oplus F_4 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{11}, F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R}{- : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6 \vdash F_9}{- : \Delta_8 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \oplus F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R \\
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R}{- : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \oplus F_4 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \oplus F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{- : \Delta_{10}, F_6, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} \text{hCut} \\
\frac{h_1 : \Delta_2, F_3 \vdash !F_8 \quad h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash !F_8} \oplus_L \quad \frac{h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9} \text{ax/W}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9}{- : \Delta_{10}, F_6, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} \text{hCut} \\
\frac{h_1 : \Delta_2, F_3 \vdash !F_8 \quad h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash !F_8} \oplus_L \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash !F_8}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9} \text{mCut}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, F_8, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{- : \Delta_{10}, F_6, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, F_8, F_3 \oplus F_4 \vdash F_9} \text{hCut} \\
\rightarrow \\
\frac{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} !L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash !F_8 \quad h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash !F_8} \oplus_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} !L \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash !F_8}{- : \Delta_2, \Delta_7, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, !F_8 \vdash F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9} \text{hCut} \oplus_L
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \&_{L2} \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_8 \& F_9} \oplus_L \quad \frac{h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \oplus_L
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_8, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \&_{L1} \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_8 \& F_9} \oplus_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \oplus_L
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_6 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \otimes_L \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \otimes F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \otimes F_9}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_8 \otimes F_9} \oplus_L \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \otimes F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \oplus_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_6 \vdash F_{10}} \oplus_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_3 \oplus F_4, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \oplus_L \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \oplus F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \oplus F_9}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_8 \oplus F_9} \oplus_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}} \oplus_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \oplus F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \oplus_L
\end{array}$$

- Case rule  $\neg \circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{12}, F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg \circ F_{10}), F_6 \vdash F_{11}} \neg \circ_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{12}, F_9 \neg \circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{12}, \Delta_2, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_6 \vdash F_9}{- : \Delta_8, F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg \circ F_{10}, F_3 \oplus F_4 \vdash F_{11}} \text{hCut} \neg \circ_L \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_6, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg \circ F_{10}), F_6 \vdash F_{11}} \neg \circ_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{12}, F_9 \neg \circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg \circ F_{10}, F_3 \oplus F_4 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{12}, \Delta_2, F_{10}, F_6 \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg \circ F_{10}, F_3 \oplus F_4 \vdash F_{11}} \text{hCut} \neg \circ_L \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_9 \neg \circ F_{10} \quad h_1 : \Delta_2, F_4 \vdash F_9 \neg \circ F_{10}}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_9 \neg \circ F_{10}} \oplus_L \quad \frac{h_6 : \Delta_7 \vdash F_9 \quad h_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8), F_9 \neg \circ F_{10} \vdash F_{11}} \neg \circ_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7, \Delta_8 \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_9 \neg \circ F_{10}}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, \Delta_8, F_9 \neg \circ F_{10} \vdash F_{11}}{- : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{11}} \text{ax/W}}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \oplus F_4 \vdash F_{11}} \text{hCut} \oplus_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash p(n_7) \quad h_1 : \Delta_2, F_4 \vdash p(n_7)}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash p(n_7)} \oplus_L \quad \frac{\bullet h_6 : *, p(n_7) \vdash p(n_7)}{- : (\Delta_2, F_3 \oplus F_4), * \vdash p(n_7)} I}{- : \Delta_2, F_3 \oplus F_4 \vdash p(n_7)} \text{Cut} \\
\rightarrow \\
- : \Delta_2, F_3 \oplus F_4 \vdash p(n_7) \text{ ax/W}
\end{array}$$

## 5.17 Status of $\neg \circ_L$ : OK

- Case rule  $!R$



$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash 8}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash 8} \multimap_L \quad \frac{h_9 : !\Upsilon 7, 8 \vdash F_{10}}{\bullet h_9 : !\Upsilon 7, 8 \vdash !F_{10}} !R \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), !\Upsilon 7 \vdash !F_{10}} \text{Cut} \\
\hline
\frac{}{} \rightarrow \\
\frac{}{- : \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_1 : \Delta_3, F_5 \vdash 8}{- : !\Upsilon 7, \Delta_3, F_5 \vdash !F_{10}} \text{ax/W} \quad \frac{\bullet h_9 : 8, !\Upsilon 7 \vdash !F_{10}}{- : !\Upsilon 7, \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash !F_{10}} \text{hCut} \\
\hline
- : !\Upsilon 7, \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash !F_{10} \multimap_L
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_8} \multimap_L \quad \frac{}{\bullet h_9 : \Delta_7, F_8 \vdash \top} \top \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_7 \vdash \top} \text{Cut} \\
\hline
\frac{}{} \rightarrow \\
- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash \top \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_8} \multimap_L \quad \frac{h_9 : \Delta_7, F_8 \vdash F_{10} \quad h_9 : \Delta_7, F_8 \vdash F_{11}}{\bullet h_9 : \Delta_7, F_8 \vdash F_{10} \& F_{11}} \&_R \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_7 \vdash F_{10} \& F_{11}} \text{Cut} \\
\hline
\frac{}{} \rightarrow \\
\frac{}{- : \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_1 : \Delta_3, F_5 \vdash F_8}{- : \Delta_3, \Delta_7, F_5 \vdash F_{10} \& F_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_7, F_8 \vdash F_{10} \& F_{11}}{- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{10} \& F_{11}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{10} \& F_{11} \multimap_L
\end{array}$$

- Case rule  $\multimap_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_8} \multimap_L \quad \frac{h_9 : \Delta_7, F_8, F_{10} \vdash F_{11}}{\bullet h_9 : \Delta_7, F_8 \vdash F_{10} \multimap F_{11}} \multimap_R \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_7 \vdash F_{10} \multimap F_{11}} \text{Cut} \\
\hline
\frac{}{} \rightarrow \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_8}{- : \Delta_2, \Delta_3, \Delta_7, F_{10}, F_4 \multimap F_5 \vdash F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_7, F_{10}, F_8 \vdash F_{11}}{- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{10} \multimap F_{11}} \text{ax/W} \\
\hline
- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{10} \multimap F_{11} \multimap_R
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_8} \multimap_L \quad \frac{h_9 : \Delta_7, F_8 \vdash F_{11}}{\bullet h_9 : \Delta_7, F_8 \vdash F_{10} \oplus F_{11}} \oplus_{R_2} \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_7 \vdash F_{10} \oplus F_{11}} \text{Cut} \\
\hline
\frac{}{} \rightarrow \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_8}{- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_7, F_8 \vdash F_{11}}{- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{10} \oplus F_{11}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{10} \oplus F_{11} \oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_8} \multimap_L \quad \frac{h_9 : \Delta_7, F_8 \vdash F_{10}}{\bullet h_9 : \Delta_7, F_8 \vdash F_{10} \oplus F_{11}} \oplus_{R_1} \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_7 \vdash F_{10} \oplus F_{11}} \text{Cut} \\
\hline
\frac{}{} \rightarrow \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_8}{- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_9 : \Delta_7, F_8 \vdash F_{10}}{- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{10} \oplus F_{11}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_{10} \oplus F_{11} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash \mathbf{1}} \multimap_L \quad \frac{h_7 : \Delta_8 \vdash F_9}{\bullet h_7 : \Delta_8, \mathbf{1} \vdash F_9} 1_L \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_8 \vdash F_9} \text{Cut} \\
\hline
\frac{}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \multimap F_5 \vdash F_9} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \multimap_L \quad \frac{h_8 : \Delta_{10}, F_7 \vdash F_9}{\bullet h_8 : (\mathbf{1}, \Delta_{10}), F_7 \vdash F_9} 1_L \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \mathbf{1}, \Delta_{10} \vdash F_9} \text{Cut} \\
\hline
\frac{}{- : \Delta_2, \Delta_3, \Delta_{10}, F_4 \multimap F_5 \vdash F_9} \text{ax/W} \quad \frac{}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \multimap_L \quad \frac{h_8 : \Delta_{12}, F_7 \vdash F_{10} \quad h_8 : \Delta_9 \vdash F_{11}}{\bullet h_8 : (\Delta_9, \Delta_{12}), F_7 \vdash F_{10} \otimes F_{11}} \otimes_R \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_9, \Delta_{12} \vdash F_{10} \otimes F_{11}} \text{Cut} \\
\hline
\frac{}{- : \Delta_{12}, \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_4 \multimap F_5 \vdash F_{10} \otimes F_{11}} \text{hCut} \quad \frac{}{- : \Delta_9 \vdash F_{11}} \text{ax/W} \\
\hline
\frac{}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_4 \multimap F_5 \vdash F_{10} \otimes F_{11}} \otimes_R \\
\hline
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \multimap_L \quad \frac{h_8 : \Delta_9 \vdash F_{10} \quad h_8 : \Delta_{12}, F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_9, \Delta_{12}), F_7 \vdash F_{10} \otimes F_{11}} \otimes_R \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_9, \Delta_{12} \vdash F_{10} \otimes F_{11}} \text{Cut} \\
\hline
\frac{}{- : \Delta_9 \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_{11}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_4 \multimap F_5 \vdash F_{10} \otimes F_{11}} \text{hCut} \\
\hline
\frac{}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_4 \multimap F_5 \vdash F_{10} \otimes F_{11}} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \multimap_L \quad \frac{h_8 : \Delta_{11}, F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_7 \vdash F_{10}} W \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_{11}, !F_9 \vdash F_{10}} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9, F_4 \multimap F_5 \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{11}, F_7, !F_9 \vdash F_{10}} \text{hCut} \\
\hline
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash !F_9}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash !F_9} \multimap_L \quad \frac{h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : \Delta_8, !F_9 \vdash F_{10}} W \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_8 \vdash F_{10}} \text{Cut} \\
\hline
\frac{}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \multimap F_5 \vdash F_{10}} \text{ax/W}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \multimap_L \quad \frac{h_8 : \Delta_{11}, F_7, !F_9, !F_9 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_7 \vdash F_{10}} C \\
\hline
\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_{11}, !F_9 \vdash F_{10}} \text{Cut} \\
\hline
\frac{}{- : \Delta_{11}, \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \text{ax/W} \quad \frac{}{- : \Delta_{11}, F_7, !F_9, !F_9 \vdash F_{10}} \text{hCut} \\
\hline
\frac{}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9, !F_9, F_4 \multimap F_5 \vdash F_{10}} C \\
\hline
\frac{}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9, F_4 \multimap F_5 \vdash F_{10}} C
\end{array}$$

- Case rule  $!L$

- Case rule  $\&_{L2}$

- Case rule  $\&_{L1}$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \multimap_L \quad \frac{h_8 : \Delta_{12}, F_7, F_9, F_{10} \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \otimes F_{10}), F_7 \vdash F_{11}} \otimes_L \\
\hline
- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_{12}, F_9 \otimes F_{10} \vdash F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7}{- : \Delta_{12}, \Delta_2, \Delta_3, F_{10}, F_9, F_4 \multimap F_5 \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_7, F_9 \vdash F_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_7, F_9 \vdash F_{11}} \text{ax/W} \\
\hline
- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \otimes F_{10}, F_4 \multimap F_5 \vdash F_{11} \quad \otimes_L \quad \text{hCut}
\end{array} \\
\hline
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_9 \otimes F_{10}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_9 \otimes F_{10}} \multimap_L \quad \frac{h_7 : \Delta_8, F_9, F_{10} \vdash F_{11}}{\bullet h_7 : \Delta_8, F_9 \otimes F_{10} \vdash F_{11}} \otimes_L \\
\hline
- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_8 \vdash F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4}{- : \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_1 : \Delta_3, F_5 \vdash F_9 \otimes F_{10}}{- : \Delta_3, \Delta_8, F_5 \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_8, F_9 \otimes F_{10} \vdash F_{11}}{\bullet h_7 : \Delta_8, F_9 \otimes F_{10} \vdash F_{11}} \text{ax/W} \\
\hline
- : \Delta_2, \Delta_3, \Delta_8, F_4 \multimap F_5 \vdash F_{11} \quad \multimap_L \quad \text{hCut}
\end{array}
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \multimap_L \quad \frac{h_8 : \Delta_{12}, F_7, F_9 \vdash F_{11} \quad h_8 : \Delta_{12}, F_7, F_{10} \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \oplus F_{10}), F_7 \vdash F_{11}} \oplus_L \\
\hline
- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_{12}, F_9 \oplus F_{10} \vdash F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{- : \Delta_2 \vdash F_4}{- : \Delta_{12}, \Delta_2, \Delta_3, F_4 \multimap F_5, F_9 \oplus F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_1 : \Delta_3, F_5 \vdash F_7}{- : \Delta_{12}, \Delta_3, F_5, F_9 \oplus F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_7, F_9 \oplus F_{10} \vdash F_{11}}{\bullet h_8 : \Delta_{12}, F_7, F_9 \oplus F_{10} \vdash F_{11}} \text{ax/W} \\
\hline
- : \Delta_{12}, \Delta_2, \Delta_3, F_4 \multimap F_5, F_9 \oplus F_{10} \vdash F_{11} \quad \multimap_L \quad \text{hCut}
\end{array} \\
\hline
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_9 \oplus F_{10}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_9 \oplus F_{10}} \multimap_L \quad \frac{h_7 : \Delta_8, F_9 \vdash F_{11} \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : \Delta_8, F_9 \oplus F_{10} \vdash F_{11}} \oplus_L \\
\hline
- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_8 \vdash F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{- : \Delta_2 \vdash F_4}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \multimap F_5 \vdash F_{11}} \text{ax/W} \quad \frac{h_1 : \Delta_3, F_5 \vdash F_9 \oplus F_{10}}{- : \Delta_3, \Delta_8, F_5 \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_8, F_9 \oplus F_{10} \vdash F_{11}}{\bullet h_7 : \Delta_8, F_9 \oplus F_{10} \vdash F_{11}} \text{ax/W} \\
\hline
- : \Delta_2, \Delta_3, \Delta_8, F_4 \multimap F_5 \vdash F_{11} \quad \multimap_L \quad \text{hCut}
\end{array}
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \multimap_L \quad \frac{h_8 : \Delta_{13}, F_7 \vdash F_{10} \quad h_8 : \Delta_9, F_{11} \vdash F_{12}}{\bullet h_8 : (\Delta_9, \Delta_{13}, F_{10} \multimap F_{11}), F_7 \vdash F_{12}} \multimap_L \\
\hline
- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_9, \Delta_{13}, F_{10} \multimap F_{11} \vdash F_{12} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7}{- : \Delta_{13}, \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_{13}, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_{13}, F_7 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11}, F_4 \multimap F_5 \vdash F_{12} \quad \multimap_L \quad \text{hCut}
\end{array} \\
\hline
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7} \multimap_L \quad \frac{h_8 : \Delta_9 \vdash F_{10} \quad h_8 : \Delta_{13}, F_7, F_{11} \vdash F_{12}}{\bullet h_8 : (\Delta_9, \Delta_{13}, F_{10} \multimap F_{11}), F_7 \vdash F_{12}} \multimap_L \\
\hline
- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_9, \Delta_{13}, F_{10} \multimap F_{11} \vdash F_{12} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{- : \Delta_9 \vdash F_{10}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11}, F_4 \multimap F_5 \vdash F_{12}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_7}{- : \Delta_{13}, \Delta_2, \Delta_3, F_{11}, F_4 \multimap F_5 \vdash F_{12}} \text{ax/W} \quad \frac{h_8 : \Delta_{13}, F_{11}, F_7 \vdash F_{12}}{\bullet h_8 : \Delta_{13}, F_{11}, F_7 \vdash F_{12}} \text{ax/W} \\
\hline
- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11}, F_4 \multimap F_5 \vdash F_{12} \quad \multimap_L \quad \text{hCut}
\end{array} \\
\hline
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_{10} \multimap F_{11}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash F_{10} \multimap F_{11}} \multimap_L \quad \frac{h_7 : \Delta_8 \vdash F_{10} \quad h_7 : \Delta_9, F_{11} \vdash F_{12}}{\bullet h_7 : (\Delta_8, \Delta_9), F_{10} \multimap F_{11} \vdash F_{12}} \multimap_L \\
\hline
- : (\Delta_2, \Delta_3, F_4 \multimap F_5), \Delta_8, \Delta_9 \vdash F_{12} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{- : \Delta_2 \vdash F_4}{- : \Delta_2, \Delta_3, \Delta_8, \Delta_9, F_4 \multimap F_5 \vdash F_{12}} \text{ax/W} \quad \frac{h_1 : \Delta_3, F_5 \vdash F_{10} \multimap F_{11}}{- : \Delta_3, \Delta_8, \Delta_9, F_5 \vdash F_{12}} \text{ax/W} \quad \frac{h_7 : \Delta_8, \Delta_9, F_{10} \multimap F_{11} \vdash F_{12}}{\bullet h_7 : \Delta_8, \Delta_9, F_{10} \multimap F_{11} \vdash F_{12}} \text{ax/W} \\
\hline
- : \Delta_2, \Delta_3, \Delta_8, \Delta_9, F_4 \multimap F_5 \vdash F_{12} \quad \multimap_L \quad \text{hCut}
\end{array}
\end{array}$$

- Case rule  $I$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash p(n_8)}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash p(n_8)} \multimap_L \quad \frac{}{\bullet h_7 : *, p(n_8) \vdash p(n_8)} I}{\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), * \vdash p(n_8)} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash p(n_8)} \text{ax/w}$$

## 5.18 Status of $I$ : OK

- Case rule  $!R$

- Case rule  $1_R$

- Case rule  $\top$

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{}{\bullet h_5 : \Delta_3, p(n_4) \vdash \top} \top}{\frac{}{- : p(n_4), \Delta_3 \vdash \top} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_3, p(n_4) \vdash \top} \top$$

- Case rule  $\&_R$

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{\frac{h_5 : \Delta_3, p(n_4) \vdash F_6 \quad h_5 : \Delta_3, p(n_4) \vdash F_7}{\bullet h_5 : \Delta_3, p(n_4) \vdash F_6 \& F_7} \&_R}{\frac{}{- : p(n_4), \Delta_3 \vdash F_6 \& F_7} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_3, p(n_4) \vdash F_6 \& F_7} \text{ax/w}$$

- Case rule  $\multimap_R$

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{\frac{h_5 : \Delta_3, F_6, p(n_4) \vdash F_7}{\bullet h_5 : \Delta_3, p(n_4) \vdash F_6 \multimap F_7} \multimap_R}{\frac{}{- : p(n_4), \Delta_3 \vdash F_6 \multimap F_7} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_3, p(n_4) \vdash F_6 \multimap F_7} \text{ax/w}$$

- Case rule  $\oplus_{R_2}$

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{\frac{h_5 : \Delta_3, p(n_4) \vdash F_7}{\bullet h_5 : \Delta_3, p(n_4) \vdash F_6 \oplus F_7} \oplus_{R_2}}{\frac{}{- : p(n_4), \Delta_3 \vdash F_6 \oplus F_7} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_3, p(n_4) \vdash F_6 \oplus F_7} \text{ax/w}$$

- Case rule  $\oplus_{R_1}$

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{\frac{h_5 : \Delta_3, p(n_4) \vdash F_6}{\bullet h_5 : \Delta_3, p(n_4) \vdash F_6 \oplus F_7} \oplus_{R_1}}{\frac{}{- : p(n_4), \Delta_3 \vdash F_6 \oplus F_7} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_3, p(n_4) \vdash F_6 \oplus F_7} \text{ax/w}$$

- Case rule  $1_L$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_6, p(n_3) \vdash F_5}{\bullet h_4 : (1, \Delta_6), p(n_3) \vdash F_5} 1_L}{- : p(n_3), 1, \Delta_6 \vdash F_5} \text{Cut}}{\rightarrow} \frac{}{- : 1, \Delta_6, p(n_3) \vdash F_5} \text{ax/W}$$

- Case rule  $\otimes_R$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_8, p(n_3) \vdash F_6 \quad h_4 : \Delta_5 \vdash F_7}{\bullet h_4 : (\Delta_5, \Delta_8), p(n_3) \vdash F_6 \otimes F_7} \otimes_R}{- : p(n_3), \Delta_5, \Delta_8 \vdash F_6 \otimes F_7} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_5, \Delta_8, p(n_3) \vdash F_6 \otimes F_7} \text{ax/W}$$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_5 \vdash F_6 \quad h_4 : \Delta_8, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_5, \Delta_8), p(n_3) \vdash F_6 \otimes F_7} \otimes_R}{- : p(n_3), \Delta_5, \Delta_8 \vdash F_6 \otimes F_7} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_5, \Delta_8, p(n_3) \vdash F_6 \otimes F_7} \text{ax/W}$$

- Case rule  $W$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_7, p(n_3) \vdash F_6}{\bullet h_4 : (\Delta_7, !F_5), p(n_3) \vdash F_6} W}{- : p(n_3), \Delta_7, !F_5 \vdash F_6} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_7, !F_5, p(n_3) \vdash F_6} \text{ax/W}$$

- Case rule  $C$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_7, !F_5, !F_5, p(n_3) \vdash F_6}{\bullet h_4 : (\Delta_7, !F_5), p(n_3) \vdash F_6} C}{- : p(n_3), \Delta_7, !F_5 \vdash F_6} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_7, !F_5, p(n_3) \vdash F_6} \text{ax/W}$$

- Case rule  $!L$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_7, F_5, p(n_3) \vdash F_6}{\bullet h_4 : (\Delta_7, !F_5), p(n_3) \vdash F_6} !L}{- : p(n_3), \Delta_7, !F_5 \vdash F_6} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_7, !F_5, p(n_3) \vdash F_6} \text{ax/W}$$

- Case rule  $\&_{L2}$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_8, F_6, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \& F_6), p(n_3) \vdash F_7} \&_{L2}}{- : p(n_3), \Delta_8, F_5 \& F_6 \vdash F_7} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_8, p(n_3), F_5 \& F_6 \vdash F_7} \text{ax/W}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\vdash : p(n_3), \Delta_8, F_5 \& F_6 \vdash F_7} I \quad \frac{h_4 : \Delta_8, F_5, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \& F_6), p(n_3) \vdash F_7} \&_{L1} \\
\vdash : p(n_3), \Delta_8, F_5 \& F_6 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\vdash : \Delta_8, p(n_3), F_5 \& F_6 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\vdash : p(n_3), \Delta_8, F_5 \otimes F_6 \vdash F_7} I \quad \frac{h_4 : \Delta_8, F_5, F_6, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \otimes F_6), p(n_3) \vdash F_7} \otimes_L \\
\vdash : p(n_3), \Delta_8, F_5 \otimes F_6 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\vdash : \Delta_8, p(n_3), F_5 \otimes F_6 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\vdash : p(n_3), \Delta_8, F_5 \oplus F_6 \vdash F_7} I \quad \frac{h_4 : \Delta_8, F_5, p(n_3) \vdash F_7 \quad h_4 : \Delta_8, F_6, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \oplus F_6), p(n_3) \vdash F_7} \oplus_L \\
\vdash : p(n_3), \Delta_8, F_5 \oplus F_6 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\vdash : \Delta_8, p(n_3), F_5 \oplus F_6 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\vdash : p(n_3), \Delta_5, \Delta_9, F_6 \multimap F_7 \vdash F_8} I \quad \frac{h_4 : \Delta_9, p(n_3) \vdash F_6 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : (\Delta_5, \Delta_9, F_6 \multimap F_7), p(n_3) \vdash F_8} \multimap_L \\
\vdash : p(n_3), \Delta_5, \Delta_9, F_6 \multimap F_7 \vdash F_8 \quad \text{Cut} \\
\rightarrow \\
\vdash : \Delta_5, \Delta_9, p(n_3), F_6 \multimap F_7 \vdash F_8 \quad \text{ax/W} \\
\\
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\vdash : p(n_3), \Delta_5, \Delta_9, F_6 \multimap F_7 \vdash F_8} I \quad \frac{h_4 : \Delta_5 \vdash F_6 \quad h_4 : \Delta_9, F_7, p(n_3) \vdash F_8}{\bullet h_4 : (\Delta_5, \Delta_9, F_6 \multimap F_7), p(n_3) \vdash F_8} \multimap_L \\
\vdash : p(n_3), \Delta_5, \Delta_9, F_6 \multimap F_7 \vdash F_8 \quad \text{Cut} \\
\rightarrow \\
\vdash : \Delta_5, \Delta_9, p(n_3), F_6 \multimap F_7 \vdash F_8 \quad \text{ax/W}
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_4) \vdash p(n_4)}{\vdash : p(n_4), * \vdash p(n_4)} I \quad \frac{\bullet h_3 : *, p(n_4) \vdash p(n_4)}{\vdash : p(n_4) \vdash p(n_4)} I \\
\vdash : p(n_4), * \vdash p(n_4) \quad \text{Cut} \\
\rightarrow \\
\vdash : p(n_4) \vdash p(n_4) \quad I
\end{array}$$

## 6 Cut-Elimination

### 6.1 Status of $!R$ : OK

- Case rule  $!R$

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : !\Upsilon 4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), !\Upsilon 4 \vdash !F_8} !R}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_8} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2 \vdash !F_5} \text{ax} \quad \frac{h_7 : !\Upsilon 4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8} \text{ax}}{- : !\Upsilon 2, !\Upsilon 4 \vdash F_8} \text{hCut} \\
 \frac{- : !\Upsilon 2, !\Upsilon 4 \vdash F_8}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_8} !R
 \end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash \top} \top}{- : !\Upsilon 2, \Delta_4 \vdash \top} \text{Cut} \\
 \rightarrow \\
 - : !\Upsilon 2, \Delta_4 \vdash \top \quad \top
 \end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash F_8 \& F_9} \text{Cut}}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \& F_9} \&_R \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{hCut} \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 2, \Delta_4 \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \& F_9} \text{hCut} \&_R
 \end{array}$$

- Case rule  $\multimap_R$

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : \Delta_4, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash F_8 \multimap F_9} \multimap_R}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \multimap F_9} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_4, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 2, \Delta_4, F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_4, F_8 \vdash F_9} \text{hCut} \\
 \frac{- : !\Upsilon 2, \Delta_4, F_8 \vdash F_9}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \multimap F_9} \multimap_R
 \end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash F_8 \oplus F_9} \oplus_{R_2}}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 2, \Delta_4 \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_9} \text{hCut} \\
 \frac{- : !\Upsilon 2, \Delta_4 \vdash F_9}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9} \oplus_{R_2}
 \end{array}$$

- Case rule  $\oplus_{R_1}$



$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash F_8 \oplus F_9} \oplus_{R_1} \\
\hline
- : !\Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{hCut} \\
\hline
- : !\Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9 \quad \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_7}{\bullet h_6 : \text{contract}(sn_5, !F_4), 1, \Delta_8 \vdash F_7} 1_L \\
\hline
- : !\Upsilon 2, 1, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_7}{- : !\Upsilon 2, \Delta_8 \vdash F_7} \text{ax}}{- : !\Upsilon 2, \Delta_8 \vdash F_7} \text{hCut} \\
\hline
- : 1, !\Upsilon 2, \Delta_8 \vdash F_7 \quad 1_L
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_{10}, !F_4, \text{contract}(n_5, !F_4) \vdash F_8 \quad h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_{10} \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_{10} \vdash F_8} \text{hCut} \\
\hline
\frac{- : !\Upsilon 2, \Delta_{10} \vdash F_8 \quad - : \Delta_7 \vdash F_9}{- : !\Upsilon 2, \Delta_{10}, \Delta_7 \vdash F_8 \otimes F_9} \otimes_R
\end{array}$$
  

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_4) \vdash F_7 \quad h_6 : \Delta_{10}, !F_4 \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_4) \vdash F_7}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{ax}}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{hCut} \\
\hline
\frac{- : !\Upsilon 2, \Delta_9 \vdash F_7 \quad - : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8}{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8} \otimes_R
\end{array}$$
  

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4 \vdash F_7 \quad h_6 : \Delta_{10}, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4 \vdash F_7}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{ax}}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{hCut} \\
\hline
\frac{- : !\Upsilon 2, \Delta_9 \vdash F_7 \quad - : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8}{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8} \otimes_R
\end{array}$$
  

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{10}, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_7 \vdash F_8 \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax}}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{- : !\Upsilon 2, \Delta_{10} \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, \Delta_7 \vdash F_8 \otimes F_9} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, !F_7 \vdash F_8} W \\
\hline
- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} h\text{Cut} \\
\hline
\frac{}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} W
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_7} !R \quad \frac{h_6 : \Delta_4, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_4 \vdash F_8} W \\
\hline
- : !\Upsilon 2, \Delta_4 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_7} \text{ax} \quad \frac{h_6 : \Delta_4, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8} h\text{Cut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4, !F_7, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, !F_7 \vdash F_8} C \\
\hline
- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4, !F_7, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9, !F_7, !F_7 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} h\text{Cut} \\
\hline
\frac{}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_7} !R \quad \frac{h_6 : \Delta_4, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_4 \vdash F_8} C \\
\hline
- : !\Upsilon 2, \Delta_4 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_7} \text{ax} \quad \frac{h_6 : \Delta_4, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8} h\text{Cut}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, !F_7 \vdash F_8} !L \\
\hline
- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} h\text{Cut} \\
\hline
\frac{}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} !L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_7} !R \quad \frac{h_6 : \Delta_4, F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_4 \vdash F_8} !L \\
\hline
- : !\Upsilon 2, \Delta_4 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_7} \text{ax} \quad \frac{h_6 : \Delta_4, F_7, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 2, \Delta_4, F_7 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8} h\text{Cut} \\
\hline
\frac{}{- : !\Upsilon 2 \vdash F_7} \text{ax} \quad \frac{}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{Cut}
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2} \\
\hline
- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{- : !\Upsilon 2, \Delta_{10}, F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9} h\text{Cut} \\
\hline
\frac{}{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2}
\end{array}$$

- $$\begin{array}{c}
\frac{\begin{array}{c} h_1 : !\Upsilon 2 \vdash F_4 \\ \bullet h_1 : !\Upsilon 2 \vdash F_4 \end{array} \quad !R \quad \frac{\begin{array}{c} h_6 : \Delta_{10}, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_9 \\ \bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, F_7 \& F_8 \vdash F_9 \end{array} \quad \&_{L1}}{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9} \quad \text{Cut} \\
\rightarrow \\
\frac{\begin{array}{c} \bullet h_1 : !\Upsilon 2 \vdash F_4 \quad \text{ax} \\ \bullet h_1 : !\Upsilon 2 \vdash F_4 \end{array} \quad \frac{\begin{array}{c} h_6 : \Delta_{10}, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_9 \\ - : !\Upsilon 2, \Delta_{10}, F_7 \vdash F_9 \end{array} \quad \text{ax}}{- : !\Upsilon 2, \Delta_{10}, F_7 \vdash F_9} \quad \text{hCut} \\
\frac{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \&_{L1}}{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9}
\end{array}$$

- $$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{!R} \quad \frac{h_6 : \Delta_{10}, F_7, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L \\
\\
\frac{}{- : !\Upsilon 2, \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \text{Cut} \\
\\
\frac{}{\rightarrow} \\
\\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{h_6 : \Delta_{10}, F_7, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9} \text{ax} \quad \frac{}{- : !\Upsilon 2, \Delta_{10}, F_7, F_8 \vdash F_9} \text{ax} \\
\\
\frac{}{- : !\Upsilon 2, \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L \quad \text{hCut}
\end{array}$$

- $$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} \text{!R} \quad \frac{h_6 : \Delta_{10}, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_9 \quad h_6 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \oplus_L}{- : !\Upsilon 2, \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{- : !\Upsilon 2, \Delta_{10}, F_7 \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, F_7 \vdash F_9} \text{hCut} \quad \frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{- : !\Upsilon 2, \Delta_{10}, F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \oplus_L \text{hCut}
\end{array}$$

- $$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} !R \quad \frac{h_6 : \Delta_{11}, !F_4, contract(n_5, !F_4) \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10} \quad \neg O_L}{\bullet h_6 : contract(sn_5, !F_4), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \multimap L}{\neg : !\Upsilon 2, \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} Cut \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} ax \quad \frac{h_6 : \Delta_{11}, !F_4, contract(n_5, !F_4) \vdash F_8}{\neg : !\Upsilon 2, \Delta_{11} \vdash F_8} ax}{\neg : !\Upsilon 2, \Delta_{11} \vdash F_8} hCut \quad \frac{h_6 : \Delta_{11}, F_8, !F_4 \vdash F_9}{\neg : \Delta_7, F_9 \vdash F_{10}} ax}{\neg : !\Upsilon 2, \Delta_{11}, \Delta_7, F_8 \multimap F_9 \vdash F_{10}} \neg O_L \\
\\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} !R \quad \frac{h_6 : \Delta_{10}, contract(n_5, !F_4) \vdash F_7 \quad h_6 : \Delta_{11}, F_8, !F_4 \vdash F_9}{\bullet h_6 : contract(sn_5, !F_4), \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \neg O_L}{\neg : !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} Cut \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} ax \quad \frac{h_6 : \Delta_{10}, contract(n_5, !F_4) \vdash F_7}{\neg : !\Upsilon 2, \Delta_{10} \vdash F_7} ax}{\neg : !\Upsilon 2, \Delta_{10} \vdash F_7} hCut \quad \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} ax \quad \frac{h_6 : \Delta_{11}, F_8, !F_4 \vdash F_9}{\neg : !\Upsilon 2, \Delta_{11}, F_8 \vdash F_9} ax}{\neg : !\Upsilon 2, \Delta_{11}, F_8 \vdash F_9} hCut}{\neg : !\Upsilon 2, !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \neg O_L \\
\rightarrow \\
\frac{\neg : !\Upsilon 2, !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9}{\neg : !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} C \\
\\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} !R \quad \frac{h_6 : \Delta_{10}, !F_4 \vdash F_7 \quad h_6 : \Delta_{11}, F_8, contract(n_5, !F_4) \vdash F_9}{\bullet h_6 : contract(sn_5, !F_4), \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \neg O_L}{\neg : !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} Cut \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} ax \quad \frac{h_6 : \Delta_{10}, !F_4 \vdash F_7}{\neg : !\Upsilon 2, \Delta_{10} \vdash F_7} ax}{\neg : !\Upsilon 2, \Delta_{10} \vdash F_7} hCut \quad \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} ax \quad \frac{h_6 : \Delta_{11}, F_8, contract(n_5, !F_4) \vdash F_9}{\neg : !\Upsilon 2, \Delta_{11}, F_8 \vdash F_9} ax}{\neg : !\Upsilon 2, \Delta_{11}, F_8 \vdash F_9} hCut}{\neg : !\Upsilon 2, !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \neg O_L \\
\rightarrow \\
\frac{\neg : !\Upsilon 2, !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9}{\neg : !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} C
\end{array}$$

$$\frac{\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{!R} \quad \frac{\frac{h_6 : \Delta 7 \vdash F_8 \quad h_5 : \Delta_{11}, F_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_{10}}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta 7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : !\Upsilon 2, \Delta 7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut}}{\frac{\frac{\frac{- : \Delta 7 \vdash F_8}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_{11}, F_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_{10}}{- : !\Upsilon 2, \Delta_{11}, F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 2, \Delta_{11}, \Delta 7, F_8 \multimap F_9 \vdash F_{10}} \multimap_L} \text{hCut}$$

- Case rule  $I$

6.2 Status of  $1_R$ : OK

- Case rule  $!R$
- Case rule  $\mathbf{1}_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $\mathbf{1}_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L_2}$
- Case rule  $\&_{L_1}$
- Case rule  $\otimes_L$

- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

### 6.3 Status of $\top$ : OK

- Case rule  $!R$
- Case rule  $\mathbf{1}_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $\mathbf{1}_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$

- Case rule  $\neg\circ_L$
- Case rule  $I$

#### 6.4 Status of $\&_R$ : OK

- Case rule  $!R$
- Case rule  $\mathbf{1}_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $\mathbf{1}_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

## 6.5 Status of $\neg\circ_R$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

## 6.6 Status of $\oplus_{R_2}$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\multimap_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\multimap_L$
- Case rule  $I$



## 6.7 Status of $\oplus_{R_1}$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\multimap_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\multimap_L$
- Case rule  $I$

## 6.8 Status of $1_L$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\multimap_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\multimap_L$
- Case rule  $I$

## 6.9 Status of $\otimes_R$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\multimap_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\multimap_L$
- Case rule  $I$

## 6.10 Status of $W$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} W \quad \frac{h_8 : !\Upsilon 5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), !\Upsilon 5 \vdash !F_9} !R \\
\hline
- : (!\Upsilon 3, !F_4), !\Upsilon 5 \vdash !F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : !\Upsilon 5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), !\Upsilon 5 \vdash !F_9} \text{ax}}{- : !\Upsilon 3, !\Upsilon 5, !F_4 \vdash F_9} \text{hCut} \\
\hline
- : !\Upsilon 3, !\Upsilon 5, !F_4 \vdash F_9 \quad !R \\
\hline
- : !\Upsilon 3, !\Upsilon 5, !F_4 \vdash !F_9
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} W \quad \frac{}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash \top} \top \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash \top \quad \text{Cut} \\
\hline
\rightarrow \\
- : !\Upsilon 3, \Delta_5, !F_4 \vdash \top \quad \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} W \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \& F_{10}} \&_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \& F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \& F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_5 \vdash F_9 \& F_{10}} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \& F_{10} \quad W \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \& F_{10}
\end{array}$$

- Case rule  $\multimap_R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} W \quad \frac{h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \multimap F_{10}} \multimap_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \multimap F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \multimap F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_5, F_9, !F_4 \vdash F_{10}} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \multimap F_{10} \quad \multimap_R \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \multimap F_{10}
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} W \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_{10}} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10} \quad \oplus_{R_2} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} W \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{- : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), \mathbf{1}, \Delta_9 \vdash F_8} 1_L \\
\hline
- : (!\Upsilon 3, !F_4), \mathbf{1}, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 3, \Delta_9, !F_4 \vdash F_8} \text{ax}}{- : \mathbf{1}, !\Upsilon 3, \Delta_9, !F_4 \vdash F_8} \text{hCut} 1_L
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \quad \frac{- : \Delta_8 \vdash F_{10}}{- : \Delta_8 \vdash F_{10}} \text{ax} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11}, !F_5 \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \Delta_{10}, \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{hCut} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{ax} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8 \quad h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8}{\bullet h_7 : \Delta_{10}, \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{hCut} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{ax} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} W \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax} \\
\hline
- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9 \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 : !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} W \quad \frac{h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} W \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \quad \text{hCut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} C \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8, !F_8 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax} \\
\hline
- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9 \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 : !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} W \quad \frac{h_7 : \Delta_5, !F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} C \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, !F_8, \text{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \quad \text{hCut}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} !L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax} \\
\hline
- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9 \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 : !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} W \quad \frac{h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} !L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{h_2 : !\Upsilon 3 \vdash !F_8}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \Delta_5, !F_8, \text{contract}(n_6, !F_8) \vdash F_9} !L \\
\hline
- : !\Upsilon 3, \Delta_5 \vdash F_9 \quad \text{hCut} \\
\rightarrow \\
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \quad W
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax} \\
\hline
- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10} \quad \text{hCut} \\
\rightarrow \\
- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10} \quad \&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \&_{L1}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10}} \text{hCut} \otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \oplus_L \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \quad \frac{\bullet h_7 : \Delta_{11}, !F_5, F_8 \oplus F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}} \text{hCut}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}} W
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{12}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ax}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut} \multimap_L \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \multimap_L \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, !F_5 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{hCut} \multimap_L \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{ax} \quad \frac{\bullet h_7 : \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} W
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5 \quad \text{ax} \quad h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11} \quad \text{ax}}{- : \Delta_8 \vdash F_9 \quad \text{ax} \quad - : !\Upsilon 3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11} \quad \text{hCut}} \multimap_L \\
\hline
- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}
\end{array}$$

- Case rule  $I$

## 6.11 Status of $C$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} C \quad \frac{h_8 : !\Upsilon 5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), !\Upsilon 5 \vdash !F_9} !R \\
\hline
- : (!\Upsilon 3, !F_4), !\Upsilon 5 \vdash !F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6 \quad \text{ax} \quad h_8 : !\Upsilon 5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad \text{ax}}{- : !\Upsilon 3, !\Upsilon 5, !F_4 \vdash F_9 \quad \text{hCut}} \\
\hline
- : !\Upsilon 3, !\Upsilon 5, !F_4 \vdash !F_9 \quad !R
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} C \quad \frac{}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash \top} \top \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash \top \quad \text{Cut} \\
\hline
\rightarrow \\
- : !\Upsilon 3, \Delta_5, !F_4 \vdash \top \quad \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} C \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \& F_{10}} \&_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \& F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad \text{ax} \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10} \quad \text{ax}}{\bullet h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \& F_{10}} \&_R \\
\hline
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_6 \quad \text{ax} \quad \bullet h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \& F_{10} \quad \text{hCut}}{- : !\Upsilon 3, \Delta_5, !F_4, !F_4 \vdash F_9 \& F_{10} \quad C} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \& F_{10}
\end{array}$$

- Case rule  $\multimap_R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} C \quad \frac{h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \multimap F_{10}} \multimap_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \multimap F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6 \quad \text{ax} \quad h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10} \quad \text{ax}}{- : !\Upsilon 3, \Delta_5, F_9, !F_4 \vdash F_{10} \quad \text{hCut}} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \multimap F_{10} \quad \multimap_R
\end{array}$$

- Case rule  $\oplus_{R_2}$



- Case rule  $\oplus_{R_1}$

- Case rule  $1_L$

- Case rule  $\otimes_R$

129

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \quad \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} W \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut} \quad W
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} C \quad \frac{h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} W \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} C \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8, !F_8 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut} \quad C
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} C \quad \frac{h_7 : \Delta_5, !F_8, !F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} C \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, !F_8, !F_8, \text{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} !L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, F_8, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut} \quad !L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} C \quad \frac{h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} !L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \Delta_5, !F_8, \text{contract}(n_6, !F_8) \vdash F_9} \text{ax} \quad \frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_8}{\bullet h_7 : \Delta_5, !F_8, \text{contract}(n_6, !F_8) \vdash F_9} !L}{- : !\Upsilon 3, \Delta_5, !F_4, !F_8 \vdash F_9} \text{hCut} \quad C
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \&_{L1}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_8, F_9, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10}} \text{hCut} \otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_5, F_8 \oplus F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \oplus_L \\
\frac{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}} C
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{12}, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut} \multimap_L \\
\frac{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} C
\end{array}$$
  

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \multimap_L \\
\frac{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} C
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\frac{}{- : (!\Upsilon 3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8}{\bullet h_7 : \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \\
\frac{}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \\
\frac{}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} C \\
\frac{}{\rightarrow} \\
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\frac{}{- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_8 \vdash F_9} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11}} \text{ax} \\
\frac{}{- : !\Upsilon 3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11}} \text{hCut} \\
\frac{}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L
\end{array}$$

- Case rule  $I$

## 6.12 Status of $!L$ : OK

- Case rule  $!R$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : !\Upsilon 5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), !\Upsilon 5 \vdash !F_9} !R \\
\frac{}{- : (!\Upsilon 3, !F_4), !\Upsilon 5 \vdash !F_9} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : !\Upsilon 5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : !\Upsilon 5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9} \text{ax} \\
\frac{}{- : !\Upsilon 3, !\Upsilon 5, !F_4 \vdash F_9} \text{hCut} \\
\frac{}{- : !\Upsilon 3, !\Upsilon 5, !F_4 \vdash F_9} !R
\end{array}$$

- Case rule  $1_R$

- Case rule  $\top$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} !L \quad \frac{}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash \top} \top \\
\frac{}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash \top} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash \top} \top
\end{array}$$

- Case rule  $\&_R$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \& F_{10}} \&_R \\
\frac{}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \& F_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}} \text{ax} \\
\frac{}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut} \quad \frac{}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_{10}} \text{hCut} \\
\frac{}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \& F_{10}} \&_R
\end{array}$$

- Case rule  $\multimap_R$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \multimap F_{10}} \multimap_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \multimap F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{- : !\Upsilon 3, \Delta_5, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \multimap F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \multimap F_{10}} \text{hCut} \multimap_R
\end{array}$$

- Case rule  $\oplus_{R_2}$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule  $\oplus_{R_1}$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule  $1_L$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), 1, \Delta_9 \vdash F_8} 1_L \\
\hline
- : (!\Upsilon 3, !F_4), 1, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_9, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : 1, !\Upsilon 3, \Delta_9, !F_4 \vdash F_8} \text{ax}}{- : 1, !\Upsilon 3, \Delta_9, !F_4 \vdash F_8} \text{hCut} 1_L
\end{array}$$

- Case rule  $\otimes_R$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_9} \text{hCut} \otimes_R \\
\hline
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11}, !F_5 \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4 \vdash F_8} \text{hCut} \otimes_R \\
\hline
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R \\
\hline
\frac{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} C
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : F_4, !Y3 \vdash !F_5}{\bullet h_2 : !Y3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8 \quad h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R}{- : (!Y3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !Y3, !F_4 \vdash !F_5}{- : !Y3, \Delta_{10}, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8}{- : !Y3, !Y3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{hCut} \quad \frac{\bullet h_2 : !Y3, !F_4 \vdash !F_5}{- : !Y3, \Delta_{11}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{- : !Y3, \Delta_{11}, !F_4 \vdash F_9} \text{hCut}}{- : !Y3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \otimes_R \\
\rightarrow \\
\frac{- : !Y3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9}{- : !Y3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} C \\
\rightarrow \\
\frac{\frac{h_2 : F_4, !Y3 \vdash !F_5}{\bullet h_2 : !Y3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R}{- : (!Y3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_8 \vdash F_9}{- : !Y3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_2 : !Y3, !F_4 \vdash !F_5}{- : !Y3, \Delta_{11}, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !Y3, \Delta_{11}, !F_4 \vdash F_{10}} \text{hCut}}{- : !Y3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \otimes_R
\end{array}$$

- Case rule  $W$

$$\begin{array}{c}
\frac{\frac{h_2 : F_4, !Y3 \vdash !F_5}{\bullet h_2 : !Y3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} W}{- : (!Y3, !F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !Y3, !F_4 \vdash !F_5}{- : !Y3, \Delta_{10}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !Y3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut}}{- : !Y3, \Delta_{10}, !F_4, !F_8 \vdash F_9} W \\
\rightarrow \\
\frac{\frac{h_2 : F_4, !Y3 \vdash !F_8}{\bullet h_2 : !Y3, !F_4 \vdash !F_8} !L \quad \frac{h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} W}{- : (!Y3, !F_4), \Delta_5 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !Y3, !F_4 \vdash !F_8}{- : !Y3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9}{- : !Y3, \Delta_5, !F_4 \vdash F_9} \text{hCut}}{- : !Y3, \Delta_5, !F_4 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $C$

$$\begin{array}{c}
\frac{\frac{h_2 : F_4, !Y3 \vdash !F_5}{\bullet h_2 : !Y3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} C}{- : (!Y3, !F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !Y3, !F_4 \vdash !F_5}{- : !Y3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{- : !Y3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut}}{- : !Y3, \Delta_{10}, !F_4, !F_8 \vdash F_9} C \\
\rightarrow \\
\frac{\frac{h_2 : F_4, !Y3 \vdash !F_8}{\bullet h_2 : !Y3, !F_4 \vdash !F_8} !L \quad \frac{h_7 : \Delta_5, !F_8, !F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} C}{- : (!Y3, !F_4), \Delta_5 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !Y3, !F_4 \vdash !F_8}{- : !Y3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, !F_8, !F_8, \text{contract}(n_6, !F_8) \vdash F_9}{- : !Y3, \Delta_5, !F_4 \vdash F_9} \text{hCut}}{- : !Y3, \Delta_5, !F_4 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $!L$

$$\begin{array}{c}
\frac{\frac{h_2 : F_4, !Y3 \vdash !F_5}{\bullet h_2 : !Y3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} !L}{- : (!Y3, !F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !Y3, !F_4 \vdash !F_5}{- : !Y3, \Delta_{10}, F_8, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !Y3, \Delta_{10}, F_8, !F_4 \vdash F_9} \text{hCut}}{- : !Y3, \Delta_{10}, !F_4, !F_8 \vdash F_9} !L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} !L \quad \frac{h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} !L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, !F_4 \vdash F_8} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, \Delta_5, F_8, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, F_8, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \quad \text{Cut}
\end{array}$$

- Case rule  $\&_{L2}$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10} \quad \&_{L2}
\end{array}$$

- Case rule  $\&_{L1}$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10} \quad \&_{L1}
\end{array}$$

- Case rule  $\otimes_L$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10} \quad \otimes_L
\end{array}$$

- Case rule  $\oplus_L$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{hCut} \quad \frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10} \quad \oplus_L
\end{array}$$

- Case rule  $\multimap_L$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{12}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{12}, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{12}, !F_4 \vdash F_9} \text{hCut} \quad \frac{- : \Delta_8, F_{10} \vdash F_{11}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ax}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10} \quad \multimap_L}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{\multimap : (!\Upsilon 3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{\multimap : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8}{\multimap : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\multimap : !\Upsilon 3, \Delta_{12}, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{\multimap : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_8 \quad \multimap : !\Upsilon 3, \Delta_{12}, F_9, !F_4 \vdash F_{10}}{\multimap : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{\multimap : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{C} \\
\rightarrow \\
\frac{\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10} \quad \multimap_L}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{\multimap : (!\Upsilon 3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{\multimap : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8}{\multimap : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\multimap : !\Upsilon 3, \Delta_{12}, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{\multimap : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_8 \quad \multimap : !\Upsilon 3, \Delta_{12}, F_9, !F_4 \vdash F_{10}}{\multimap : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{\multimap : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{C} \\
\rightarrow \\
\frac{\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11} \quad \multimap_L}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}{\multimap : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\multimap : \Delta_8 \vdash F_9 \quad \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{\multimap : !\Upsilon 3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11}}{\multimap : !\Upsilon 3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11}} \text{ax} \quad \frac{\multimap : \Delta_8 \vdash F_9 \quad \multimap : !\Upsilon 3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11}}{\multimap : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut}}{\multimap : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{C}
\end{array}$$

- Case rule  $I$

### 6.13 Status of $\&_{L2}$ : OK

- Case rule  $!R$
- Case rule  $1_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\multimap_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $1_L$
- Case rule  $\otimes_R$
- Case rule  $W$



- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

#### 6.14 Status of $\&_{L1}$ : OK

- Case rule  $!R$
- Case rule  $\mathbf{1}_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R2}$
- Case rule  $\oplus_{R1}$
- Case rule  $\mathbf{1}_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$

- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

### 6.15 Status of $\otimes_L$ : OK

- Case rule  $!R$
- Case rule  $\mathbf{1}_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R2}$
- Case rule  $\oplus_{R1}$
- Case rule  $\mathbf{1}_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$

- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

### 6.16 Status of $\oplus_L$ : OK

- Case rule  $!R$
- Case rule  $\mathbf{1}_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R2}$
- Case rule  $\oplus_{R1}$
- Case rule  $\mathbf{1}_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$

- Case rule  $\&_{L1}$
- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

### 6.17 Status of $\neg\circ_L$ : OK

- Case rule  $!R$
- Case rule  $\mathbf{1}_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $\mathbf{1}_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L2}$
- Case rule  $\&_{L1}$

- Case rule  $\otimes_L$
- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$

### 6.18 Status of $I$ : OK

- Case rule  $!R$
- Case rule  $\mathbf{1}_R$
- Case rule  $\top$
- Case rule  $\&_R$
- Case rule  $\neg\circ_R$
- Case rule  $\oplus_{R_2}$
- Case rule  $\oplus_{R_1}$
- Case rule  $\mathbf{1}_L$
- Case rule  $\otimes_R$
- Case rule  $W$
- Case rule  $C$
- Case rule  $!L$
- Case rule  $\&_{L_2}$
- Case rule  $\&_{L_1}$
- Case rule  $\otimes_L$

- Case rule  $\oplus_L$
- Case rule  $\neg\circ_L$
- Case rule  $I$