

Dyadic system for Linear Logic

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1 Weakening on the classical context

- Case(s) rule 1

$$\frac{}{\bullet h_1 : \vdash \{\Delta_2\} : 1} 1 \rightarrow \frac{}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : 1} 1$$

- Case(s) rule !

$$\frac{h_1 : \vdash \{\Delta_2\} : F_3}{\bullet h_1 : \vdash \{\Delta_2\} : !F_3} ! \rightarrow \frac{\frac{}{h_1 : \vdash \{\Delta_2\} : F_3} \text{ax}}{h_1 : \vdash \{\Delta_2, F_W\} : F_3} \text{IH}}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : !F_3} !$$

- Case(s) rule ?

$$\frac{h_1 : \vdash \{F_4, \Delta_2\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, ?F_4} ? \rightarrow \frac{\frac{}{h_1 : \vdash \{\Delta_2, F_4\} : \Delta_3} \text{ax}}{h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3} \text{IH}}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, ?F_4} ?$$

- Case(s) rule \$

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \$F_5} \$ \rightarrow \frac{\frac{}{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4, F_5} \text{ax}}{h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, F_4, F_5} \text{IH}}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, F_4 \$F_5} \$$$

- Case(s) rule &

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 : \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \&F_5} \& \rightarrow \frac{\frac{}{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ax} \quad \frac{}{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5} \text{ax}}{\frac{h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, F_4}{h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, F_5} \text{IH}}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, F_4 \&F_5} \&}$$

- Case(s) rule \oplus_B

$$\frac{h_1 : \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B \rightarrow \frac{\frac{}{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5} \text{ax}}{h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, F_5} \text{IH}}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, F_4 \oplus F_5} \oplus_B$$

- Case(s) rule \oplus_A

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A \rightarrow \frac{\frac{}{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ax}}{h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, F_4} \text{IH}}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, F_4 \oplus F_5} \oplus_A$$

- Case(s) rule \perp

$$\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \perp, \Delta_3} \perp \rightarrow \frac{\frac{}{h_1 : \vdash \{\Delta_2\} : \Delta_3} \text{ax}}{h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3} \text{IH}}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, \perp} \perp$$

- Case(s) rule \top

$$\frac{}{\bullet h_1 : \vdash \{\Delta_2\} : \top, \Delta_3} \top \rightarrow \frac{}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : \Delta_3, \top} \top$$

- Case(s) rule I_3

$$\frac{}{\bullet h_1 : \vdash \{\Delta_2\} : p(n_3), \wedge(n_3)} I_3 \rightarrow \frac{}{\bullet h_1 : \vdash \{\Delta_2, F_W\} : p(n_3), \wedge(n_3)} I_3$$

- Case(s) rule \otimes

$$\frac{\frac{h_1 \vdash \{\Delta_2\} : F_5, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : F_6, \Delta_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_4, F_5 \otimes F_6}}{\otimes} \rightarrow \frac{\frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_5} \text{IH} \quad \frac{h_1 \vdash \{\Delta_2\} : \Delta_4, F_6}{h_1 \vdash \{\Delta_2, F_W\} : \Delta_4, F_6} \text{IH}}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, \Delta_4, F_5 \otimes F_6}}{\otimes} \text{IH}$$

- Case(s) rule I_1

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, p(n_3)\} : \wedge(n_3)} I_1 \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, F_W, p(n_3)\} : \wedge(n_3)} I_1$$

- Case(s) rule I_2

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, \wedge(n_3)\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, F_W, \wedge(n_3)\} : p(n_3)} I_2$$

- Case(s) rule $?_C$

$$\frac{\frac{h_1 \vdash \{F_2, \Delta_3\} : F_2, \Delta_4}{\bullet h_1 \vdash \{F_2, \Delta_3\} : \Delta_4}}{?_C} \rightarrow \frac{\frac{\frac{h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2}{h_1 \vdash \{\Delta_3, F_2, F_W\} : \Delta_4, F_2} \text{IH}}{\bullet h_1 \vdash \{\Delta_3, F_2, F_W\} : \Delta_4}}{?_C} \text{IH}$$

2 Contraction on the classical context

- Case(s) rule 1

$$\frac{}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : 1} 1 \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : 1} 1$$

- Case(s) rule !

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : !F_4} ! \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : F_4}{h_3 \vdash \{\Delta_2, F_1\} : F_4} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : !F_4} ! \text{ax}$$

- Case(s) rule ?

$$\frac{h_3 \vdash \{F_1, F_1, F_5, \Delta_2\} : \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, ?F_5} ? \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1, F_5\} : \Delta_4}{h_3 \vdash \{\Delta_2, F_1, F_5\} : \Delta_4} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, ?F_5} ? \text{ax}$$

- Case(s) rule \$

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_5, F_6, \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5 \$F_6} \$ \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5, F_6}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5, F_6} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5 \$F_6} \$ \text{ax}$$

- Case(s) rule &

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_5, \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5 \&F_6} \& \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5 \&F_6} \& \text{ax}$$

- Case(s) rule \oplus_B

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_6, \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5 \oplus F_6} \oplus_B \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_6}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_6} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5 \oplus F_6} \oplus_B \text{ax}$$

- Case(s) rule \oplus_A

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_5, \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5 \oplus F_6} \oplus_A \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5 \oplus F_6} \oplus_A \text{ax}$$

- Case(s) rule \perp

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \perp, \Delta_4} \perp \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, \perp} \perp \text{ax}$$

- Case(s) rule \top

$$\frac{}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \top, \Delta_4} \top \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, \top} \top$$

- Case(s) rule I_3

$$\frac{}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : p(n_4), \cdot(n_4)} I_3 \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : p(n_4), \cdot(n_4)} I_3$$

- Case(s) rule \otimes

$$\frac{\frac{h_3 : \vdash \{F_1, F_1, \Delta_2\} : F_6, \Delta_4 \quad h_3 : \vdash \{F_1, F_1, \Delta_2\} : F_7, \Delta_5}{\bullet h_3 : \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, \Delta_5, F_6 \otimes F_7}}{\otimes} \rightarrow \frac{\frac{\frac{h_3 : \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_6}{h_3 : \vdash \{\Delta_2, F_1\} : \Delta_4, F_6} \text{ ax} \quad \frac{h_3 : \vdash \{\Delta_2, F_1, F_1\} : \Delta_5, F_7}{h_3 : \vdash \{\Delta_2, F_1\} : \Delta_5, F_7} \text{ IH}}{\bullet h_3 : \vdash \{\Delta_2, F_1\} : \Delta_4, \Delta_5, F_6 \otimes F_7} \otimes$$

- Case(s) rule I_1

$$\frac{}{\bullet h_2 : \vdash \{\Delta_1, p(n_3), p(n_3)\} : \wedge(n_3)} I_1 \rightarrow \frac{}{\bullet h_2 : \vdash \{\Delta_1, p(n_3)\} : \wedge(n_3)} I_1$$

$$\frac{}{\bullet h_2 : \vdash \{(\Delta_4, p(n_3)), F_1, F_1\} : \wedge(n_3)} I_1 \rightarrow \frac{}{\bullet h_2 : \vdash \{\Delta_4, F_1, p(n_3)\} : \wedge(n_3)} I_1$$

- Case(s) rule I_2

$$\frac{}{\bullet h_2 : \vdash \{\Delta_1, \wedge(n_3), \wedge(n_3)\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_2 : \vdash \{\Delta_1, \wedge(n_3)\} : p(n_3)} I_2$$

$$\frac{}{\bullet h_2 : \vdash \{(\Delta_4, \wedge(n_3)), F_1, F_1\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_2 : \vdash \{\Delta_4, F_1, \wedge(n_3)\} : p(n_3)} I_2$$

- Case(s) rule $?_C$

$$\frac{\frac{h_2 : \vdash \{F_3, F_3, \Delta_1\} : F_3, \Delta_4}{\bullet h_2 : \vdash \{\Delta_1, F_3, F_3\} : \Delta_4} ?_C}{\bullet h_2 : \vdash \{\Delta_1, F_3\} : \Delta_4} ?_C \rightarrow \frac{\frac{\frac{h_2 : \vdash \{\Delta_1, F_3, F_3\} : \Delta_4, F_3}{h_2 : \vdash \{\Delta_1, F_3\} : \Delta_4, F_3} \text{ ax} \quad \frac{h_2 : \vdash \{\Delta_1, F_3\} : \Delta_4, F_3}{\bullet h_2 : \vdash \{\Delta_1, F_3\} : \Delta_4} ?_C}{\bullet h_2 : \vdash \{\Delta_1, F_3\} : \Delta_4} ?_C$$

$$\frac{\frac{h_2 : \vdash \{F_1, F_1, F_3, \Delta_5\} : F_3, \Delta_4}{\bullet h_2 : \vdash \{(F_3, \Delta_5), F_1, F_1\} : \Delta_4} ?_C}{\bullet h_2 : \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_3} \text{ ax} \rightarrow \frac{\frac{\frac{h_2 : \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_3}{h_2 : \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_3} \text{ IH} \quad \frac{h_2 : \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_3}{\bullet h_2 : \vdash \{\Delta_5, F_1, F_3\} : \Delta_4} ?_C}{\bullet h_2 : \vdash \{\Delta_5, F_1, F_3\} : \Delta_4} ?_C$$

3 Measure of derivations

- Case(s) rule 1

$$\frac{}{\bullet h_1 : \vdash \{\Delta_2\} : 1} 1 \rightarrow \frac{}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : 1} 1$$

- Case(s) rule !

$$\frac{h_1 : \vdash \{\Delta_2\} : F_3}{\bullet h_1 : \vdash \{\Delta_2\} : !F_3} ! \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2\} : F_3}{\bullet h_1 : \vdash \{\Delta_2\} : F_3} \text{IH}}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : !F_3} \text{!}$$

- Case(s) rule ?

$$\frac{h_1 : \vdash \{F_4, \Delta_2\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, ?F_4} ? \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2, F_4\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_2, F_4\} : \Delta_3} \text{IH}}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, ?F_4} ?$$

- Case(s) rule \$

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \$F_5} \$ \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4, F_5}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4, F_5} \text{IH}}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \$F_5} \$$$

- Case(s) rule &

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 : \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \&F_5} \& \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4} \text{IH} \quad \frac{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5} \text{IH}}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \&F_5} \&$$

- Case(s) rule \oplus_B

$$\frac{h_1 : \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5} \text{IH}}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B$$

- Case(s) rule \oplus_A

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A$$

- Case(s) rule \perp

$$\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \perp, \Delta_3} \perp \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3} \text{IH}}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, \perp} \perp$$

- Case(s) rule \top

$$\frac{}{\bullet h_1 : \vdash \{\Delta_2\} : \top, \Delta_3} \top \rightarrow \frac{}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, \top} \top$$

- Case(s) rule I_3

$$\frac{}{\bullet h_1 : \vdash \{\Delta_2\} : p(n_3), \wedge(n_3)} I_3 \rightarrow \frac{}{\bullet \bullet h_1 : \vdash \{\Delta_2\} : p(n_3), \wedge(n_3)} I_3$$

- Case(s) rule \otimes

$$\frac{h_1 \vdash \{\Delta_2\} : F_5, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : F_6, \Delta_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_4, F_5 \otimes F_6} \otimes \quad \rightarrow \quad \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_5} \text{ax} \quad \frac{h_1 \vdash \{\Delta_2\} : \Delta_4, F_6}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_4, F_6} \text{ax}}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_4, F_5 \otimes F_6} \text{IH} \otimes$$

- Case(s) rule I_1

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, p(n_3)\} : \wedge(n_3)} I_1 \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 \vdash \{\Delta_2, p(n_3)\} : \wedge(n_3)} I_1$$

- Case(s) rule I_2

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, \wedge(n_3)\} : p(n_3)} I_2 \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 \vdash \{\Delta_2, \wedge(n_3)\} : p(n_3)} I_2$$

- Case(s) rule $?_C$

$$\frac{h_1 \vdash \{F_2, \Delta_3\} : F_2, \Delta_4}{\bullet h_1 \vdash \{F_2, \Delta_3\} : \Delta_4} ?_C \quad \rightarrow \quad \frac{\frac{h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2}{\bullet h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2} \text{ax}}{\bullet \bullet h_1 \vdash \{\Delta_3, F_2\} : \Delta_4} \text{IH} ?_C$$

4 Invertibility of Rules

4.1 Status of 1: : Invertible

- Case rule 1

$$\frac{}{\bullet h_1 : \vdash \{\Delta_2\} : 1} 1 \rightarrow \text{trivial}$$

- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_1 : \vdash \{F_2, \Delta_3\} : 1, F_2}{\bullet h_1 : \vdash \{F_2, \Delta_3\} : 1} ?_C \rightarrow \text{trivial}$$

4.2 Status of !: : Invertible

- Case rule 1
- Case rule !

$$\frac{h_1 : \vdash \{\Delta_2\} : F_3}{\bullet h_1 : \vdash \{\Delta_2\} : !F_3} ! \quad \rightarrow \quad \frac{\overline{h_1 : \vdash \{\Delta_2\} : F_3}^{\text{ax}}}{\bullet h_1 : \vdash \{\Delta_2\} : F_3} \text{H}$$

- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_2 : \vdash \{F_3, \Delta_4\} : F_3, !F_1}{\bullet h_2 : \vdash \{F_3, \Delta_4\} : !F_1} ?_C \quad \rightarrow \quad \frac{\overline{h_2 : \vdash \{\Delta_4, F_3\} : F_3, !F_1}^{\text{ax}}}{\bullet h_2 : \vdash \{\Delta_4, F_3\} : !F_1} ?_C \quad \frac{\bullet h_2 : \vdash \{\Delta_4, F_3\} : !F_1}{\bullet h_2 : \vdash \{\Delta_4, F_3\} : F_1} WB$$

4.3 Status of ?: : Invertible

- Case rule 1
- Case rule !

- Case rule ?

$$\frac{h_2 : \vdash \{F_4, \Delta_3\} : \Delta_5, ?F_1}{\bullet h_2 : \vdash \{\Delta_3\} : (\Delta_5, ?F_1), ?F_4} ? \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3, F_1, F_4\} : \Delta_5}}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_5, ?F_4} \text{ax/ind} ?$$

$$\frac{h_1 : \vdash \{F_4, \Delta_2\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, ?F_4} ? \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2, F_4\} : \Delta_3}}{\bullet h_1 : \vdash \{\Delta_2, F_4\} : \Delta_3} \text{ax}^H$$

- Case rule \$

$$\frac{h_2 : \vdash \{\Delta_3\} : F_4, F_5, \Delta_6, ?F_1}{\bullet h_2 : \vdash \{\Delta_3\} : (\Delta_6, ?F_1), F_4 \$F_5} \$ \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_6, F_4, F_5}}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_6, F_4 \$F_5} \text{ax/ind} \$$$

- Case rule &

$$\frac{h_2 : \vdash \{\Delta_3\} : F_4, \Delta_6, ?F_1 \quad h_2 : \vdash \{\Delta_3\} : F_5, \Delta_6, ?F_1}{\bullet h_2 : \vdash \{\Delta_3\} : (\Delta_6, ?F_1), F_4 \&F_5} \& \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_6, F_4} \text{ax/ind} \quad \overline{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_6, F_5} \text{ax/ind}}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_6, F_4 \&F_5} \&$$

- Case rule \oplus_B

$$\frac{h_2 : \vdash \{\Delta_3\} : F_5, \Delta_6, ?F_1}{\bullet h_2 : \vdash \{\Delta_3\} : (\Delta_6, ?F_1), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_6, F_5} \text{ax/ind}}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_6, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_2 : \vdash \{\Delta_3\} : F_4, \Delta_6, ?F_1}{\bullet h_2 : \vdash \{\Delta_3\} : (\Delta_6, ?F_1), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_6, F_4} \text{ax/ind}}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_6, F_4 \oplus F_5} \oplus_A$$

- Case rule \perp

$$\frac{h_2 : \vdash \{\Delta_3\} : \Delta_4, ?F_1}{\bullet h_2 : \vdash \{\Delta_3\} : \perp, \Delta_4, ?F_1} \perp \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_4} \text{ax/ind}}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_4, \perp} \perp$$

- Case rule \top

$$\frac{\overline{\bullet h_2 : \vdash \{\Delta_3\} : \top, \Delta_4, ?F_1}}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_4, \top} \top \rightarrow \frac{\overline{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_4, \top}}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_4, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_2 : \vdash \{\Delta_3\} : F_5, \Delta_7, ?F_1 \quad h_2 : \vdash \{\Delta_3\} : F_6, \Delta_4}{\bullet h_2 : \vdash \{\Delta_3\} : (\Delta_7, ?F_1), \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_7, F_5} \text{ax/ind} \quad \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_6} \text{ax}}{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_4, F_6} W}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes$$

$$\frac{h_2 : \vdash \{\Delta_3\} : F_5, \Delta_4 \quad h_2 : \vdash \{\Delta_3\} : F_6, \Delta_7, ?F_1}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_4, (\Delta_7, ?F_1), F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_5} \text{ax}}{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_4, F_5} W \quad \frac{\overline{h_2 : \vdash \{\Delta_3, F_1\} : \Delta_7, F_6} \text{ax/ind}}{\bullet h_2 : \vdash \{\Delta_3, F_1\} : \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_3 : \vdash \{F_4, \Delta_5\} : F_4, \Delta_1, ?F_2}{\bullet h_3 : \vdash \{F_4, \Delta_5\} : \Delta_1, ?F_2} ?_C \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_5, F_2, F_4\} : \Delta_1, F_4}}{\bullet h_3 : \vdash \{\Delta_5, F_2, F_4\} : \Delta_1} ?_C \text{ ax/ind}$$

4.4 Status of \$: : Invertible

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{h_3 : \vdash \{F_5, \Delta_4\} : \Delta_6, F_1 \$F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_6, F_1 \$F_2), ?F_5} ? \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4, F_5\} : \Delta_6, F_1, F_2}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_6, F_1, F_2, ?F_5} ? \text{ ax/ind}$$

- Case rule \$

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, F_6, \Delta_7, F_1 \$F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, F_1 \$F_2), F_5 \$F_6} \$ \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5, F_6}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5 \$F_6} \$ \text{ ax/ind}$$

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \$F_5} \$ \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4, F_5}}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4, F_5} \text{ ax} \text{ H}$$

- Case rule &

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \$F_2 \quad h_3 : \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \$F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, F_1 \$F_2), F_5 \&F_6} \& \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5 \&F_6} \&$$

- Case rule \oplus_B

$$\frac{h_3 : \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \$F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, F_1 \$F_2), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5 \oplus F_6} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \$F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, F_1 \$F_2), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5 \oplus F_6} \oplus_A$$

- Case rule \perp

$$\frac{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_1 \$ F_2}{\bullet h_3 : \vdash \{\Delta_4\} : \perp, \Delta_5, F_1 \$ F_2} \perp \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, F_1, F_2, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 : \vdash \{\Delta_4\} : \top, \Delta_5, F_1 \$ F_2} \top \rightarrow \frac{}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, F_1, F_2, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 : \vdash \{\Delta_4\} : F_6, \Delta_8, F_1 \$ F_2 \quad h_3 : \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_8, F_1 \$ F_2), \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_8, F_1, F_2, F_6} \text{ ax/ind} \quad \overline{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_7} \text{ ax}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_2, F_6 \otimes F_7} \otimes$$

$$\frac{h_3 : \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 : \vdash \{\Delta_4\} : F_7, \Delta_8, F_1 \$ F_2}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \$ F_2), F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_6} \text{ ax} \quad \overline{h_3 : \vdash \{\Delta_4\} : \Delta_8, F_1, F_2, F_7} \text{ ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_2, F_6 \otimes F_7} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_4 : \vdash \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \$ F_3}{\bullet h_4 : \vdash \{F_5, \Delta_6\} : \Delta_1, F_2 \$ F_3} ?_C \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_6, F_5\} : \Delta_1, F_2, F_3, F_5} \text{ ax/ind}}{\bullet h_4 : \vdash \{\Delta_6, F_5\} : \Delta_1, F_2, F_3} ?_C$$

4.5 Status of $\&$: (Left Premise): Invertible

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{h_3 : \vdash \{F_5, \Delta_4\} : \Delta_6, F_1 \& F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_6, F_1 \& F_2), ?F_5} ? \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4, F_5\} : \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_6, F_1, ?F_5} ?$$

- Case rule \$

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \$ F_6} \$ \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_5, F_6} \text{ ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \$ F_6} \$$$

- Case rule $\&$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \& F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \& F_6} \& \rightarrow \frac{\frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \& F_6} \text{ax/ind} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_6}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \& F_6} \text{ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \& F_6} \&$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \& F_5} \& \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ax}}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{H}$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_6}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \text{ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \text{ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \oplus_A$$

- Case rule \perp

$$\frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : \perp, \Delta_5, F_1 \& F_2} \perp \rightarrow \frac{\frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \perp} \text{ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \perp} \perp$$

- Case rule \top

$$\frac{\bullet h_3 \vdash \{\Delta_4\} : \top, \Delta_5, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : \top, \Delta_5, F_1, \top} \top \rightarrow \frac{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \top}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_8, F_1 \& F_2 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_8, F_1 \& F_2), \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{\frac{h_3 \vdash \{\Delta_4\} : \Delta_8, F_1, F_6}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \text{ax/ind} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_7}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \text{ax}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \otimes$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_8, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \& F_2), F_6 \otimes F_7} \otimes \rightarrow \frac{\frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_6}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_8, F_1, F_7}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \text{ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_4 \vdash \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \& F_3}{\bullet h_4 \vdash \{F_5, \Delta_6\} : \Delta_1, F_2 \& F_3} ?_C \rightarrow \frac{\frac{h_4 \vdash \{\Delta_6, F_5\} : \Delta_1, F_2, F_5}{\bullet h_4 \vdash \{\Delta_6, F_5\} : \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 \vdash \{\Delta_6, F_5\} : \Delta_1, F_2} ?_C$$

4.6 Status of & (Right Premise): : Invertible

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{h_3 : \vdash \{ \Delta_4 \} : \Delta_6, F_1 \& F_2}{\bullet h_3 : \vdash \{ \Delta_4 \} : (\Delta_6, F_1 \& F_2), ?F_5} ? \rightarrow \frac{\overline{h_3 : \vdash \{ \Delta_4, F_5 \} : \Delta_6, F_2}}{\bullet h_3 : \vdash \{ \Delta_4 \} : \Delta_6, F_2, ?F_5} \text{ax/ind} ?$$

- Case rule \$

$$\frac{h_3 : \vdash \{ \Delta_4 \} : F_5, F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 : \vdash \{ \Delta_4 \} : (\Delta_7, F_1 \& F_2), F_5 \$ F_6} \$ \rightarrow \frac{\overline{h_3 : \vdash \{ \Delta_4 \} : \Delta_7, F_2, F_5, F_6}}{\bullet h_3 : \vdash \{ \Delta_4 \} : \Delta_7, F_2, F_5 \$ F_6} \text{ax/ind} \$$$

- Case rule &

$$\frac{h_3 : \vdash \{ \Delta_4 \} : F_5, \Delta_7, F_1 \& F_2 \quad h_3 : \vdash \{ \Delta_4 \} : F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 : \vdash \{ \Delta_4 \} : (\Delta_7, F_1 \& F_2), F_5 \& F_6} \& \rightarrow \frac{\overline{h_3 : \vdash \{ \Delta_4 \} : \Delta_7, F_2, F_5} \text{ax/ind} \quad \overline{h_3 : \vdash \{ \Delta_4 \} : \Delta_7, F_2, F_6} \text{ax/ind}}{\bullet h_3 : \vdash \{ \Delta_4 \} : \Delta_7, F_2, F_5 \& F_6} \&$$

$$\frac{h_1 : \vdash \{ \Delta_2 \} : F_4, \Delta_3 \quad h_1 : \vdash \{ \Delta_2 \} : F_5, \Delta_3}{\bullet h_1 : \vdash \{ \Delta_2 \} : \Delta_3, F_4 \& F_5} \& \rightarrow \frac{\overline{h_1 : \vdash \{ \Delta_2 \} : \Delta_3, F_5} \text{ax}}{\bullet h_1 : \vdash \{ \Delta_2 \} : \Delta_3, F_5} H$$

- Case rule \oplus_B

$$\frac{h_3 : \vdash \{ \Delta_4 \} : F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 : \vdash \{ \Delta_4 \} : (\Delta_7, F_1 \& F_2), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_3 : \vdash \{ \Delta_4 \} : \Delta_7, F_2, F_6} \text{ax/ind}}{\bullet h_3 : \vdash \{ \Delta_4 \} : \Delta_7, F_2, F_5 \oplus F_6} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash \{ \Delta_4 \} : F_5, \Delta_7, F_1 \& F_2}{\bullet h_3 : \vdash \{ \Delta_4 \} : (\Delta_7, F_1 \& F_2), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_3 : \vdash \{ \Delta_4 \} : \Delta_7, F_2, F_5} \text{ax/ind}}{\bullet h_3 : \vdash \{ \Delta_4 \} : \Delta_7, F_2, F_5 \oplus F_6} \oplus_A$$

- Case rule \perp

$$\frac{h_3 : \vdash \{ \Delta_4 \} : \Delta_5, F_1 \& F_2}{\bullet h_3 : \vdash \{ \Delta_4 \} : \perp, \Delta_5, F_1 \& F_2} \perp \rightarrow \frac{\overline{h_3 : \vdash \{ \Delta_4 \} : \Delta_5, F_2} \text{ax/ind}}{\bullet h_3 : \vdash \{ \Delta_4 \} : \Delta_5, F_2, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 : \vdash \{ \Delta_4 \} : \top, \Delta_5, F_1 \& F_2} \top \rightarrow \frac{}{\bullet h_3 : \vdash \{ \Delta_4 \} : \Delta_5, F_2, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 : \vdash \{\Delta_4\} : F_6, \Delta_8, F_1 \& F_2 \quad h_3 : \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_8, F_1 \& F_2), \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{\frac{h_3 : \vdash \{\Delta_4\} : \Delta_8, F_2, F_6}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \text{ax/ind} \quad \frac{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_7}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, F_7} \text{ax}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \otimes$$

$$\frac{h_3 : \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 : \vdash \{\Delta_4\} : F_7, \Delta_8, F_1 \& F_2}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \& F_2), F_6 \otimes F_7} \otimes \rightarrow \frac{\frac{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_6}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \text{ax} \quad \frac{h_3 : \vdash \{\Delta_4\} : \Delta_8, F_2, F_7}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \text{ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_4 : \vdash \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \& F_3}{\bullet h_4 : \vdash \{F_5, \Delta_6\} : \Delta_1, F_2 \& F_3} ?_C \rightarrow \frac{\frac{h_4 : \vdash \{\Delta_6, F_5\} : \Delta_1, F_3, F_5}{\bullet h_4 : \vdash \{\Delta_6, F_5\} : \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \vdash \{\Delta_6, F_5\} : \Delta_1, F_3} ?_C$$

4.7 Status of \oplus_B : Non invertible

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{h_3 : \vdash \{F_5, \Delta_4\} : \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_6, F_1 \oplus F_2), ?F_5} ? \rightarrow \frac{\frac{h_3 : \vdash \{\Delta_4, F_5\} : \Delta_6, F_2}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_6, F_2, ?F_5} \text{ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_6, F_2, ?F_5} ?$$

- Case rule \$

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \$ F_6} \$ \rightarrow \frac{\frac{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5, F_6}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \$ F_6} \text{ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \$ F_6} \$$$

- Case rule &

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2 \quad h_3 : \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \& F_6} \& \rightarrow \frac{\frac{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \& F_6} \text{ax/ind} \quad \frac{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_6}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \& F_6} \text{ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \& F_6} \&$$

- Case rule \oplus_B

$$\frac{h_3 : \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\frac{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_6}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \oplus F_6} \text{ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \oplus F_6} \oplus_B$$

$$\frac{h_1 : \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5} \text{ax}}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5} H$$

- Case rule \oplus_A

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \oplus F_6} \text{ax/ind} \oplus_A$$

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A \rightarrow \frac{\text{fail}}{\bullet h_1 : \vdash \{\Delta_2\} : F_5, \Delta_3}$$

- Case rule \perp

$$\frac{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_1 \oplus F_2}{\bullet h_3 : \vdash \{\Delta_4\} : \perp, \Delta_5, F_1 \oplus F_2} \perp \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_2}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, F_2, \perp} \text{ax/ind} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 : \vdash \{\Delta_4\} : \top, \Delta_5, F_1 \oplus F_2} \top \rightarrow \frac{}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, F_2, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 : \vdash \{\Delta_4\} : F_6, \Delta_8, F_1 \oplus F_2 \quad h_3 : \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_8, F_1 \oplus F_2), \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_8, F_2, F_6} \text{ax/ind} \quad \overline{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_7} \text{ax}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \otimes$$

$$\frac{h_3 : \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 : \vdash \{\Delta_4\} : F_7, \Delta_8, F_1 \oplus F_2}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \oplus F_2), F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4\} : \Delta_5, F_6} \text{ax} \quad \overline{h_3 : \vdash \{\Delta_4\} : \Delta_8, F_2, F_7} \text{ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_4 : \vdash \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \oplus F_3}{\bullet h_4 : \vdash \{F_5, \Delta_6\} : \Delta_1, F_2 \oplus F_3} ?_C \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_6, F_5\} : \Delta_1, F_3, F_5} \text{ax/ind}}{\bullet h_4 : \vdash \{\Delta_6, F_5\} : \Delta_1, F_3} ?_C$$

4.8 Status of \oplus_A : Non invertible

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{h_3 : \vdash \{F_5, \Delta_4\} : \Delta_6, F_1 \oplus F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_6, F_1 \oplus F_2), ?F_5} ? \rightarrow \frac{\overline{h_3 : \vdash \{\Delta_4, F_5\} : \Delta_6, F_1} \text{ax/ind}}{\bullet h_3 : \vdash \{\Delta_4\} : \Delta_6, F_1, ?F_5} ?$$

- Case rule \$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \$ F_6} \$ \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \$ F_6} \$$$

- Case rule &

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \& F_6} \& \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5} \text{ ax/ind} \quad \overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \& F_6} \&$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \oplus_B$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B \rightarrow \frac{\bullet h_1 \vdash \{\Delta_2\} : F_4, \Delta_3 \text{ fail}}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \oplus_A$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ ax}}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ H}$$

- Case rule \perp

$$\frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : \perp, \Delta_5, F_1 \oplus F_2} \perp \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 \vdash \{\Delta_4\} : \top, \Delta_5, F_1 \oplus F_2} \top \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_8, F_1 \oplus F_2 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_8, F_1 \oplus F_2), \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_8, F_1, F_6} \text{ ax/ind} \quad \overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_7} \text{ ax}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \otimes$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_8, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \oplus F_2), F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_6} \text{ ax} \quad \overline{h_3 \vdash \{\Delta_4\} : \Delta_8, F_1, F_7} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \otimes$$

- Case rule I_1

- Case rule I_2
- Case rule $?_C$

$$\frac{h_4 : \vdash \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \oplus F_3}{\bullet h_4 : \vdash \{F_5, \Delta_6\} : \Delta_1, F_2 \oplus F_3} ?_C \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_6, F_5\} : \Delta_1, F_2, F_5}}{ax/ind} ?_C$$

4.9 Status of \perp : Invertible

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{h_1 : \vdash \{F_3, \Delta_2\} : \perp, \Delta_4}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_4), ?F_3} ? \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2, F_3\} : \Delta_4}}{ax/ind} ?$$

- Case rule \$

$$\frac{h_1 : \vdash \{\Delta_2\} : \perp, F_3, F_4, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_5), F_3 \$F_4} \$ \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_3, F_4}}{ax/ind} \$$$

- Case rule &

$$\frac{h_1 : \vdash \{\Delta_2\} : \perp, F_3, \Delta_5 \quad h_1 : \vdash \{\Delta_2\} : \perp, F_4, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_5), F_3 \&F_4} \& \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_3} \quad \overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_4}}{ax/ind} \&$$

- Case rule \oplus_B

$$\frac{h_1 : \vdash \{\Delta_2\} : \perp, F_4, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_5), F_3 \oplus F_4} \oplus_B \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_4}}{ax/ind} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_1 : \vdash \{\Delta_2\} : \perp, F_3, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_5), F_3 \oplus F_4} \oplus_A \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_3}}{ax/ind} \oplus_A$$

- Case rule \perp

$$\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \perp, \Delta_3} \perp \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_3}}{ax} H$$

- Case rule \top

$$\overline{\bullet h_1 : \vdash \{\Delta_2\} : \top, \perp, \Delta_3} \top \rightarrow \overline{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{\frac{h_1 : \vdash \{\Delta_2\} : \perp, F_4, \Delta_6}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_6), \Delta_3, F_4 \otimes F_5}}{\otimes} \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2\} : \Delta_6, F_4}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5}}{\otimes} \xrightarrow{\text{ax/ind}} \frac{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5}}{\otimes} \xrightarrow{\text{ax}}$$

$$\frac{\frac{h_1 : \vdash \{\Delta_2\} : F_4, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, (\perp, \Delta_6), F_4 \otimes F_5}}{\otimes} \rightarrow \frac{\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5}}{\otimes} \xrightarrow{\text{ax/ind}} \frac{h_1 : \vdash \{\Delta_2\} : \Delta_6, F_5}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5}}{\otimes} \xrightarrow{\text{ax}}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\frac{h_2 : \vdash \{F_3, \Delta_4\} : \perp, F_3, \Delta_1}{\bullet h_2 : \vdash \{F_3, \Delta_4\} : \perp, \Delta_1}}{?_C} \rightarrow \frac{\frac{h_2 : \vdash \{\Delta_4, F_3\} : \Delta_1, F_3}{\bullet h_2 : \vdash \{\Delta_4, F_3\} : \Delta_1}}{?_C} \xrightarrow{\text{ax/ind}}$$

4.10 Status of \top : Invertible

- Case rule $\mathbf{1}$

- Case rule $!$

- Case rule $?$

$$\frac{\frac{h_1 : \vdash \{F_3, \Delta_2\} : \top, \Delta_4}{\bullet h_1 : \vdash \{\Delta_2\} : (\top, \Delta_4), ?F_3}}{?} \rightarrow \text{trivial}$$

- Case rule $\$$

$$\frac{\frac{h_1 : \vdash \{\Delta_2\} : \top, F_3, F_4, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), F_3 \$F_4}}{\$} \rightarrow \text{trivial}$$

- Case rule $\&$

$$\frac{\frac{h_1 : \vdash \{\Delta_2\} : \top, F_3, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), F_3 \&F_4}}{\&} \rightarrow \text{trivial}$$

- Case rule \oplus_B

$$\frac{\frac{h_1 : \vdash \{\Delta_2\} : \top, F_4, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), F_3 \oplus F_4}}{\oplus_B} \rightarrow \text{trivial}$$

- Case rule \oplus_A

$$\frac{h_1 : \vdash \{\Delta_2\} : \top, F_3, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), F_3 \oplus F_4} \oplus_A \rightarrow \text{trivial}$$

- Case rule \perp

$$\frac{h_1 : \vdash \{\Delta_2\} : \top, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \perp, \top, \Delta_3} \perp \rightarrow \text{trivial}$$

- Case rule \top

$$\frac{}{\bullet h_1 : \vdash \{\Delta_2\} : \top, \Delta_3} \top \rightarrow \text{trivial}$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_1 : \vdash \{\Delta_2\} : \top, F_4, \Delta_6 \quad h_1 : \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : (\top, \Delta_6), \Delta_3, F_4 \otimes F_5} \otimes \rightarrow \text{trivial}$$

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 : \vdash \{\Delta_2\} : \top, F_5, \Delta_6}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, (\top, \Delta_6), F_4 \otimes F_5} \otimes \rightarrow \text{trivial}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_2 : \vdash \{F_3, \Delta_4\} : \top, F_3, \Delta_1}{\bullet h_2 : \vdash \{F_3, \Delta_4\} : \top, \Delta_1} ?_C \rightarrow \text{trivial}$$

4.11 Status of I_3 : : Invertible

- Case rule 1

- Case rule $!$

- Case rule $?$

- Case rule $\$$

- Case rule $\&$

- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3

$$\frac{}{\bullet h_1 : \vdash \{ \Delta_2 \} : p(n_3), \wedge(n_3)} I_3 \quad \rightarrow \quad \text{trivial}$$

- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_2 : \vdash \{ F_3, \Delta_4 \} : F_3, p(n_1), \wedge(n_1)}{\bullet h_2 : \vdash \{ F_3, \Delta_4 \} : p(n_1), \wedge(n_1)} ?_C \quad \rightarrow \quad \text{trivial}$$

4.12 Status of \otimes : (Left Premise): Non invertible

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{h_4 : \vdash \{ F_6, \Delta_5 \} : \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash \{ \Delta_5 \} : (\Delta_1, \Delta_7, F_2 \otimes F_3), ?F_6} ? \quad \rightarrow \quad \frac{}{h_4 : \vdash \{ \Delta_5, F_6 \} : \Delta_7, F_2} \text{ax/ind} \quad ?$$

$$\frac{h_4 : \vdash \{ F_6, \Delta_5 \} : \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash \{ \Delta_5 \} : (\Delta_1, \Delta_7, F_2 \otimes F_3), ?F_6} ? \quad \rightarrow \quad \text{fail}$$

- Case rule \$

$$\frac{h_4 : \vdash \{ \Delta_5 \} : F_6, F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{ \Delta_5 \} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \$F_7} \$ \quad \rightarrow \quad \frac{}{h_4 : \vdash \{ \Delta_5 \} : \Delta_8, F_2, F_6, F_7} \text{ax/ind} \quad \$$$

$$\frac{h_4 : \vdash \{ \Delta_5 \} : F_6, F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{ \Delta_5 \} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \$F_7} \$ \quad \rightarrow \quad \frac{}{h_4 : \vdash \{ \Delta_5 \} : \Delta_1, F_2} \text{H}$$

- Case rule $\&$

$$\frac{h_4 : \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3 \quad h_4 : \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \& F_7} \& \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_5\} : \Delta_8, F_2, F_6} \text{ ax/ind} \quad \overline{h_4 : \vdash \{\Delta_5\} : \Delta_8, F_2, F_7} \text{ ax/ind}}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \& F_7} \&$$

$$\frac{h_4 : \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3 \quad h_4 : \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \& F_7} \& \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ H}$$

- Case rule \oplus_B

$$\frac{h_4 : \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_B \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_5\} : \Delta_8, F_2, F_7} \text{ ax/ind}}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \oplus F_7} \oplus_B$$

$$\frac{h_4 : \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_B \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ H}$$

- Case rule \oplus_A

$$\frac{h_4 : \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_A \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_5\} : \Delta_8, F_2, F_6} \text{ ax/ind}}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \oplus F_7} \oplus_A$$

$$\frac{h_4 : \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_A \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ H}$$

- Case rule \perp

$$\frac{h_4 : \vdash \{\Delta_5\} : \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : \perp, \Delta_1, \Delta_6, F_2 \otimes F_3} \perp \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_5\} : \Delta_6, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_6, F_2, \perp} \perp$$

$$\frac{h_4 : \vdash \{\Delta_5\} : \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : \perp, \Delta_1, \Delta_6, F_2 \otimes F_3} \perp \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ H}$$

- Case rule \top

$$\frac{}{\bullet h_4 : \vdash \{\Delta_5\} : \top, \Delta_1, \Delta_6, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_6, F_2, \top} \top$$

$$\frac{}{\bullet h_4 : \vdash \{\Delta_5\} : \top, \Delta_1, \Delta_6, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash \{\Delta_5\} : F_2, \Delta_1} \text{ fail}$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8, F_1 \otimes F_2 \quad h_3 : \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, F_1 \otimes F_2), (\Delta_9, \Delta_{10}), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \{\Delta_4\} : F_1, \Delta_7, \Delta_9, F_5 \otimes F_6} \text{ fail}$$

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8 \quad h_3 : \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}, F_1 \otimes F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \otimes F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \{\Delta_4\} : F_1, \Delta_7, \Delta_9, F_5 \otimes F_6} \text{ fail}$$

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8, F_1 \otimes F_2 \quad h_3 : \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, F_1 \otimes F_2), (\Delta_9, \Delta_{10}), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \{\Delta_4\} : F_1, \Delta_7, \Delta_9} \text{ fail}$$

$$\frac{h_3 : \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8 \quad h_3 : \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}, F_1 \otimes F_2}{\bullet h_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \otimes F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \{\Delta_4\} : F_1, \Delta_7, \Delta_9} \text{ fail}$$

$$\frac{h_1 : \vdash \{\Delta_2\} : F_3, \Delta_5, \Delta_6 \quad h_1 : \vdash \{\Delta_2\} : F_4, \Delta_7, \Delta_8}{\bullet h_1 : \vdash \{\Delta_2\} : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_3 \otimes F_4} \otimes \rightarrow \frac{}{\bullet h_1 : \vdash \{\Delta_2\} : F_3, \Delta_5, \Delta_7} \text{ fail}$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_5 : \vdash \{F_6, \Delta_7\} : F_6, \Delta_1, \Delta_2, F_3 \otimes F_4}{\bullet h_5 : \vdash \{F_6, \Delta_7\} : \Delta_1, \Delta_2, F_3 \otimes F_4} ?_C \rightarrow \frac{h_5 : \vdash \{\Delta_7, F_6\} : \Delta_1, F_3, F_6}{\bullet h_5 : \vdash \{\Delta_7, F_6\} : \Delta_1, F_3} \text{ ax/ind} ?_C$$

4.13 Status of \otimes (Right Premise): : Non invertible

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{h_4 : \vdash \{F_6, \Delta_5\} : \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_7, F_2 \otimes F_3), ?F_6} ? \rightarrow \frac{}{\bullet h_4 : \vdash \{\Delta_5\} : F_3, \Delta_1} \text{ fail}$$

$$\frac{h_4 : \vdash \{F_6, \Delta_5\} : \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_7, F_2 \otimes F_3), ?F_6} ? \rightarrow \frac{h_4 : \vdash \{\Delta_5, F_6\} : \Delta_7, F_3}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_7, F_3, ?F_6} \text{ ax/ind} ?$$

- Case rule \$

$$\frac{h_4 : \vdash \{\Delta_5\} : F_6, F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \$F_7} \$ \rightarrow \frac{h_4 : \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_1, F_3} \text{ ax/ind} \text{ H}$$

$$\frac{h_4 : \vdash \{\Delta_5\} : F_6, F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \$F_7} \$ \rightarrow \frac{h_4 : \vdash \{\Delta_5\} : \Delta_8, F_3, F_6, F_7}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_8, F_3, F_6 \$F_7} \text{ ax/ind} \$$$

- Case rule &

$$\frac{h_4 : \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3 \quad h_4 : \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \&F_7} \& \rightarrow \frac{h_4 : \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_1, F_3} \text{ ax/ind} \text{ H}$$

$$\frac{h_4 : \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3 \quad h_4 : \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 : \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \&F_7} \& \rightarrow \frac{h_4 : \vdash \{\Delta_5\} : \Delta_8, F_3, F_6}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_8, F_3, F_6 \&F_7} \text{ ax/ind} \& \frac{h_4 : \vdash \{\Delta_5\} : \Delta_8, F_3, F_7}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_8, F_3, F_6 \&F_7} \text{ ax/ind} \&$$

- Case rule \oplus_B

$$\frac{h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_B \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3} \text{H}}{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3} \text{H}} \text{ax/ind}$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_B \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_7}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_7} \text{H}}{\frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_7}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_7} \text{H}} \text{ax/ind}$$

- Case rule \oplus_A

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_A \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3} \text{H}}{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3} \text{H}} \text{ax/ind}$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_A \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6} \text{H}}{\frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6} \text{H}} \text{ax/ind}$$

- Case rule \perp

$$\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : \perp, \Delta_1, \Delta_6, F_2 \otimes F_3} \perp \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3} \text{H}}{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3} \text{H}} \text{ax/ind}$$

$$\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : \perp, \Delta_1, \Delta_6, F_2 \otimes F_3} \perp \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_6, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_6, F_3} \text{H}}{\frac{h_4 \vdash \{\Delta_5\} : \Delta_6, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_6, F_3} \text{H}} \text{ax/ind}$$

- Case rule \top

$$\frac{}{\bullet h_4 \vdash \{\Delta_5\} : \top, \Delta_1, \Delta_6, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 \vdash \{\Delta_5\} : F_3, \Delta_1} \text{fail}$$

$$\frac{}{\bullet h_4 \vdash \{\Delta_5\} : \top, \Delta_1, \Delta_6, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_6, F_3, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8, F_1 \otimes F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, F_1 \otimes F_2), (\Delta_9, \Delta_{10}), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_2, \Delta_8, \Delta_{10}} \text{fail}$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}, F_1 \otimes F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \otimes F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_2, \Delta_8, \Delta_{10}} \text{fail}$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8, F_1 \otimes F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, F_1 \otimes F_2), (\Delta_9, \Delta_{10}), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_2, \Delta_8, \Delta_{10}, F_5 \otimes F_6} \text{fail}$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}, F_1 \otimes F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \otimes F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_2, \Delta_8, \Delta_{10}, F_5 \otimes F_6} \text{fail}$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_3, \Delta_5, \Delta_6 \quad h_1 \vdash \{\Delta_2\} : F_4, \Delta_7, \Delta_8}{\bullet h_1 \vdash \{\Delta_2\} : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_3 \otimes F_4} \otimes \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2\} : F_4, \Delta_6, \Delta_8} \text{fail}$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_5 : \vdash \{F_6, \Delta_7\} : F_6, \Delta_1, \Delta_2, F_3 \otimes F_4}{\bullet h_5 : \vdash \{F_6, \Delta_7\} : \Delta_1, \Delta_2, F_3 \otimes F_4} ?_C \quad \rightarrow \quad \frac{\overline{h_5 : \vdash \{\Delta_7, F_6\} : \Delta_2, F_4, F_6}}{\bullet h_5 : \vdash \{\Delta_7, F_6\} : \Delta_2, F_4} \text{ax/ind} ?_C$$

4.14 Status of I_1 : : Invertible

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1

$$\overline{\bullet h_1 : \vdash \{\Delta_2, p(n_3)\} : \neg(n_3)} I_1 \quad \rightarrow \quad \text{trivial}$$

- Case rule I_2

- Case rule $?_C$

$$\frac{h_2 : \vdash \{\Delta_3, p(n_1)\} : p(n_1), \wedge(n_1)}{\bullet h_2 : \vdash \{p(n_1), \Delta_3\} : \wedge(n_1)} ?_C \rightarrow \text{trivial}$$

$$\frac{h_2 : \vdash \{F_3, \Delta_4, p(n_1)\} : F_3, \wedge(n_1)}{\bullet h_2 : \vdash \{F_3, \Delta_4, p(n_1)\} : \wedge(n_1)} ?_C \rightarrow \text{trivial}$$

4.15 Status of I_2 : : Invertible

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2

$$\frac{}{\bullet n_1 : \vdash \{\Delta_2, \wedge(n_3)\} : p(n_3)} I_2 \rightarrow \text{trivial}$$

- Case rule $?_C$

$$\frac{h_2 : \vdash \{\Delta_3, \wedge(n_1)\} : p(n_1), \wedge(n_1)}{\bullet h_2 : \vdash \{\wedge(n_1), \Delta_3\} : p(n_1)} ?_C \rightarrow \text{trivial}$$

$$\frac{h_2 : \vdash \{F_3, \Delta_4, \wedge(n_1)\} : F_3, p(n_1)}{\bullet h_2 : \vdash \{F_3, \Delta_4, \wedge(n_1)\} : p(n_1)} ?_C \rightarrow \text{trivial}$$

4.16 Status of $?_C$: : Non invertible

- Case rule 1

$$\frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : 1} 1 \rightarrow \frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : 1, F_1} \text{fail}$$

- Case rule !

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : !F_4} ! \rightarrow \frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : F_1, !F_4} \text{fail}$$

- Case rule ?

$$\frac{h_3 \vdash \{F_1, F_5, \Delta_2\} : \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, ?F_5} ? \rightarrow \frac{}{h_3 \vdash \{\Delta_2, F_1, F_5\} : \Delta_4, F_1} \text{ax/ind} \quad \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, ?F_5} ?$$

- Case rule \$

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_5, F_6, \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, F_5 \$F_6} \$ \rightarrow \frac{}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5, F_6} \text{ax/ind} \quad \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5 \$F_6} \$$$

- Case rule &

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_5, \Delta_4 \quad h_3 \vdash \{F_1, \Delta_2\} : F_6, \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, F_5 \&F_6} \& \rightarrow \frac{}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5} \text{ax/ind} \quad \frac{}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_6} \text{ax/ind} \quad \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5 \&F_6} \&$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_6, \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, F_5 \oplus_B F_6} \oplus_B \rightarrow \frac{}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_6} \text{ax/ind} \quad \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5 \oplus_B F_6} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_5, \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, F_5 \oplus_A F_6} \oplus_A \rightarrow \frac{}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5} \text{ax/ind} \quad \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5 \oplus_A F_6} \oplus_A$$

- Case rule \perp

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \perp, \Delta_4} \perp \rightarrow \frac{}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1} \text{ax/ind} \quad \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \top, \Delta_4} \top \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, \top} \top$$

- Case rule I_3

$$\frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : p(n_4), \wedge(n_4)} I_3 \rightarrow \frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : F_1, p(n_4), \wedge(n_4)} \text{fail}$$

- Case rule \otimes

$$\frac{\frac{h_3 \vdash \{F_1, \Delta_2\} : F_6, \Delta_4 \quad h_3 \vdash \{F_1, \Delta_2\} : F_7, \Delta_5}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, \Delta_5, F_6 \otimes F_7} \otimes}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, \Delta_5, F_1, F_6 \otimes F_7} \otimes \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_6}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, \Delta_5, F_1, F_6 \otimes F_7} \otimes \quad \frac{h_3 \vdash \{\Delta_2, F_1\} : \Delta_5, F_1, F_7}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, \Delta_5, F_1, F_6 \otimes F_7} \otimes}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, \Delta_5, F_1, F_6 \otimes F_7} \otimes \text{ax/ind}$$

- Case rule I_1

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, p(n_3)\} : \wedge(n_3)} I_1 \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, p(n_3)\} : p(n_3), \wedge(n_3)} I_3$$

$$\frac{}{\bullet h_2 \vdash \{(F_1, \Delta_4), p(n_3)\} : \wedge(n_3)} I_1 \rightarrow \frac{}{\bullet h_2 \vdash \{F_1, \Delta_4, p(n_3)\} : F_1, \wedge(n_3)} \text{fail}$$

- Case rule I_2

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, \wedge(n_3)\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, \wedge(n_3)\} : p(n_3), \wedge(n_3)} I_3$$

$$\frac{}{\bullet h_2 \vdash \{(F_1, \Delta_4), \wedge(n_3)\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_2 \vdash \{F_1, \Delta_4, \wedge(n_3)\} : F_1, p(n_3)} \text{fail}$$

- Case rule $?_C$

$$\frac{\frac{h_1 \vdash \{F_2, \Delta_3\} : F_2, \Delta_4}{\bullet h_1 \vdash \{F_2, \Delta_3\} : \Delta_4} ?_C}{\bullet h_1 \vdash \{F_2, \Delta_3\} : \Delta_4} ?_C \rightarrow \frac{\frac{h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2, F_2}{\bullet h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2} \text{ax/ind}}{\bullet h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2} ?_C$$

$$\frac{\frac{h_2 \vdash \{F_1, F_3, \Delta_5\} : F_3, \Delta_4}{\bullet h_2 \vdash \{F_3, F_1, \Delta_5\} : \Delta_4} ?_C}{\bullet h_2 \vdash \{F_3, F_1, \Delta_5\} : \Delta_4} ?_C \rightarrow \frac{\frac{h_2 \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_1, F_3}{\bullet h_2 \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_1} \text{ax/ind}}{\bullet h_2 \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_1} ?_C$$

5 Weakening on bang: $\vdash \Gamma, !F$ implies Γ, F .

- Case(s) rule 1
- Case(s) rule !

$$\frac{h_1 : \vdash \{\Delta_2\} : F_3}{\bullet h_1 : \vdash \{\Delta_2\} : !F_3, *} ! \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : F_3}^{\text{ax}}}{\bullet h_1 : \vdash \{\Delta_2\} : F_3}^H$$

- Case(s) rule ?

$$\frac{h_2 : \vdash \{F_4, \Delta_3\} : \Delta_5, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_5, ?F_4} ? \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3, F_4\} : \Delta_5, !F_1}^{\text{ax}}}{\overline{h_2 : \vdash \{\Delta_3, F_4\} : \Delta_5, F_1}^{\text{IH}}} ?$$

- Case(s) rule \$

$$\frac{h_2 : \vdash \{\Delta_3\} : F_4, F_5, \Delta_6, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_6, F_4 \$F_5} \$ \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_4, F_5, !F_1}^{\text{ax}}}{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4, F_5}^{\text{IH}}} \$$$

- Case(s) rule &

$$\frac{h_2 : \vdash \{\Delta_3\} : F_4, \Delta_6, !F_1 \quad h_2 : \vdash \{\Delta_3\} : F_5, \Delta_6, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_6, F_4 \&F_5} \& \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_4, !F_1}^{\text{ax}}}{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4}^{\text{IH}}} \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_5, !F_1}^{\text{ax}}}{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_5}^{\text{IH}}} \&$$

- Case(s) rule \oplus_B

$$\frac{h_2 : \vdash \{\Delta_3\} : F_5, \Delta_6, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_6, F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_5, !F_1}^{\text{ax}}}{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_5}^{\text{IH}}} \oplus_B$$

- Case(s) rule \oplus_A

$$\frac{h_2 : \vdash \{\Delta_3\} : F_4, \Delta_6, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_6, F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_4, !F_1}^{\text{ax}}}{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4}^{\text{IH}}} \oplus_A$$

- Case(s) rule \perp

$$\frac{h_2 : \vdash \{\Delta_3\} : \Delta_4, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \perp, \Delta_4} \perp \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_4, !F_1}^{\text{ax}}}{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_1}^{\text{IH}}} \perp$$

- Case(s) rule \top

$$\frac{}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \top, \Delta_4} \top \rightarrow \frac{}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_4, F_1, \top} \top$$

- Case(s) rule I_3

- Case(s) rule \otimes

$$\frac{h_2 : \vdash \{\Delta_3\} : F_5, \Delta_7, !F_1 \quad h_2 : \vdash \{\Delta_3\} : F_6, \Delta_4}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_7, F_5, !F_1}^{\text{ax}}}{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_7, F_1, F_5}^{\text{IH}}} \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_6}^{\text{ax}}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes$$

$$\frac{h_2 : \vdash \{\Delta_3\} : F_5, \Delta_4 \quad h_2 : \vdash \{\Delta_3\} : F_6, \Delta_7, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_5}^{\text{ax}}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_7, F_6, !F_1}^{\text{ax}}}{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_7, F_1, F_6}^{\text{IH}}} \otimes$$

- Case(s) rule I_1
- Case(s) rule I_2
- Case(s) rule $?_C$

$$\frac{\frac{h_3 \vdash \{F_4, \Delta_5\} : F_4, \Delta_1, !F_2}{\bullet h_3 \vdash \{F_4, \Delta_5\} : !F_2, \Delta_1}}{?_C} \rightarrow \frac{\frac{\frac{h_3 \vdash \{\Delta_5, F_4\} : \Delta_1, F_4, !F_2}{h_3 \vdash \{\Delta_5, F_4\} : \Delta_1, F_2, F_4}}{\bullet h_3 \vdash \{\Delta_5, F_4\} : \Delta_1, F_2}}{\text{ax}} \text{IH} \text{ } ?_C$$

6 Identity-Expansion

$$\begin{array}{c}
\frac{}{- : \vdash \{*\} : \mathbf{0}, \top} \top \\
\\
\frac{\frac{}{- : \vdash \{*\} : \mathbf{1}}{\quad} \mathbf{1}}{- : \vdash \{*\} : \mathbf{1}, \perp} \perp \\
\\
\frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_0}, dual(\mathbf{F_0})} \text{IH}}{- : \vdash \{*\} : dual(\mathbf{F_0}), \mathbf{F_0} \oplus \mathbf{F_1}} \oplus_A \quad \frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_1}, dual(\mathbf{F_1})} \text{IH}}{- : \vdash \{*\} : dual(\mathbf{F_1}), \mathbf{F_0} \oplus \mathbf{F_1}} \oplus_B}{- : \vdash \{*\} : dual(\mathbf{F_0}) \& dual(\mathbf{F_1}), \mathbf{F_0} \oplus \mathbf{F_1}} \& \\
\\
\frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_0}, dual(\mathbf{F_0})} \text{IH}}{- : \vdash \{*\} : dual(\mathbf{F_0}), dual(\mathbf{F_1}), \mathbf{F_0} \otimes \mathbf{F_1}} \otimes \quad \frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_1}, dual(\mathbf{F_1})} \text{IH}}{- : \vdash \{*\} : dual(\mathbf{F_0}), dual(\mathbf{F_1}), \mathbf{F_0} \otimes \mathbf{F_1}} \$}{- : \vdash \{*\} : dual(\mathbf{F_0}) \$ dual(\mathbf{F_1}), \mathbf{F_0} \otimes \mathbf{F_1}} \$ \\
\\
\frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_0}, dual(\mathbf{F_0})} \text{IH}}{- : \vdash \{*\} : \mathbf{F_0}, dual(\mathbf{F_0}) \oplus dual(\mathbf{F_1})} \oplus_A \quad \frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_1}, dual(\mathbf{F_1})} \text{IH}}{- : \vdash \{*\} : \mathbf{F_1}, dual(\mathbf{F_0}) \oplus dual(\mathbf{F_1})} \oplus_B}{- : \vdash \{*\} : \mathbf{F_0} \& \mathbf{F_1}, dual(\mathbf{F_0}) \oplus dual(\mathbf{F_1})} \& \\
\\
\frac{\frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_0}, dual(\mathbf{F_0})} \text{IH}}{- : \vdash \{*\} : \mathbf{F_0}, \mathbf{F_1}, dual(\mathbf{F_0}) \otimes dual(\mathbf{F_1})} \otimes \quad \frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_1}, dual(\mathbf{F_1})} \text{IH}}{- : \vdash \{*\} : \mathbf{F_0} \$ \mathbf{F_1}, dual(\mathbf{F_0}) \otimes dual(\mathbf{F_1})} \$}{- : \vdash \{*\} : \mathbf{F_0} \$ \mathbf{F_1}, dual(\mathbf{F_0}) \otimes dual(\mathbf{F_1})} \$ \\
\\
\frac{\frac{\frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_0}, dual(\mathbf{F_0})} \text{IH}}{- : \vdash \{\mathbf{F_0}\} : \mathbf{F_0}, dual(\mathbf{F_0})} W}{- : \vdash \{\mathbf{F_0}\} : dual(\mathbf{F_0})} ?_C}{- : \vdash \{\mathbf{F_0}\} : !dual(\mathbf{F_0})} !}{- : \vdash \{*\} : !dual(\mathbf{F_0}), ?\mathbf{F_0}} ? \\
\\
\frac{\frac{}{- : \vdash \{*\} : \mathbf{1}}{\quad} \mathbf{1}}{- : \vdash \{*\} : \mathbf{1}, \perp} \perp \\
\\
\frac{}{- : \vdash \{*\} : \mathbf{0}, \top} \top \\
\\
\frac{\frac{\frac{\frac{\frac{}{- : \vdash \{*\} : \mathbf{F_0}, dual(\mathbf{F_0})} \text{IH}}{- : \vdash \{dual(\mathbf{F_0})\} : \mathbf{F_0}, dual(\mathbf{F_0})} W}{- : \vdash \{dual(\mathbf{F_0})\} : \mathbf{F_0}} ?_C}{- : \vdash \{dual(\mathbf{F_0})\} : !\mathbf{F_0}} !}{- : \vdash \{*\} : !\mathbf{F_0}, ?dual(\mathbf{F_0})} ?
\end{array}$$

7 Cut-Elimination

7.1 Status of 1: OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
 \frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, \Delta_6}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_6, ?F_5} ? \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : *, \Delta_6, ?F_5} \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{}{\bullet h_1 \vdash \{\Delta_4, F_5\} : \mathbf{1}} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4, F_5\} : \Delta_6, \perp}{\bullet h_3 \vdash \{\Delta_4, F_5\} : \Delta_6, \perp} \text{ax}}{- \vdash \{\Delta_4, F_5\} : \Delta_6} \text{hCut} \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : \Delta_6, ?F_5} ?
 \end{array}$$

- Case rule \$

$$\begin{array}{c}
 \frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, F_5, F_6, \Delta_7}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, F_5 \$ F_6} \$ \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : *, \Delta_7, F_5 \$ F_6} \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, F_6, \perp}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, F_6, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_5, F_6} \text{hCut} \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : \Delta_7, F_5 \$ F_6} \$
 \end{array}$$

- Case rule &

$$\begin{array}{c}
 \frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, F_5, \Delta_7 \quad h_3 \vdash \{\Delta_4\} : \perp, F_6, \Delta_7}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, F_5 \& F_6} \& \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : *, \Delta_7, F_5 \& F_6} \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, \perp}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_5} \text{hCut} \quad \frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_6, \perp}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_6, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_6} \text{hCut} \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : \Delta_7, F_5 \& F_6} \&
 \end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
 \frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, F_6, \Delta_7}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, F_5 \oplus F_6} \oplus_B \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : *, \Delta_7, F_5 \oplus F_6} \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_6, \perp}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_6, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_6} \text{hCut} \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : \Delta_7, F_5 \oplus F_6} \oplus_B
 \end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
 \frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, F_5, \Delta_7}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, F_5 \oplus F_6} \oplus_A \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : *, \Delta_7, F_5 \oplus F_6} \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, \perp}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_5} \text{hCut} \\
 \hline
 \frac{}{- \vdash \{\Delta_4\} : \Delta_7, F_5 \oplus F_6} \oplus_A
 \end{array}$$

- Case rule \perp

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_4\} : \mathbf{1}, *}}{\vdash : \vdash \{\Delta_4\} : *, \Delta_5} \mathbf{1} \quad \frac{h_3 : \vdash \{\Delta_4\} : \Delta_5}{\bullet h_3 : \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_5} \perp}{\vdash : \vdash \{\Delta_4\} : *, \Delta_5} \text{Cut}$$

$$\xrightarrow{\quad} \frac{}{\vdash : \vdash \{\Delta_4\} : \Delta_5} \text{ax}$$

- Case rule \top

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_4\} : \mathbf{1}, *}}{\vdash : \vdash \{\Delta_4\} : *, \top, \Delta_5} \mathbf{1} \quad \frac{}{\bullet h_3 : \vdash \{\Delta_4\} : dual(\mathbf{1}), \top, \Delta_5} \top}{\vdash : \vdash \{\Delta_4\} : *, \top, \Delta_5} \text{Cut}$$

$$\xrightarrow{\quad} \frac{}{\vdash : \vdash \{\Delta_4\} : \Delta_5, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_4\} : \mathbf{1}, *}}{\vdash : \vdash \{\Delta_4\} : *, \Delta_5, \Delta_8, F_6 \otimes F_7} \mathbf{1} \quad \frac{h_3 : \vdash \{\Delta_4\} : \perp, F_6, \Delta_8 \quad h_3 : \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 : \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes}{\vdash : \vdash \{\Delta_4\} : *, \Delta_5, \Delta_8, F_6 \otimes F_7} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_4\} : \mathbf{1}}{\vdash : \vdash \{\Delta_4\} : \Delta_8, F_6} \mathbf{1} \quad \frac{h_3 : \vdash \{\Delta_4\} : \Delta_8, F_6, \perp}{\vdash : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_6 \otimes F_7} \text{ax}}{\vdash : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_6 \otimes F_7} \text{hCut} \quad \frac{}{\vdash : \vdash \{\Delta_4\} : \Delta_5, F_7} \text{ax}}{\vdash : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes$$

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_4\} : \mathbf{1}, *}}{\vdash : \vdash \{\Delta_4\} : *, \Delta_5, \Delta_8, F_6 \otimes F_7} \mathbf{1} \quad \frac{h_3 : \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 : \vdash \{\Delta_4\} : \perp, F_7, \Delta_8}{\bullet h_3 : \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes}{\vdash : \vdash \{\Delta_4\} : *, \Delta_5, \Delta_8, F_6 \otimes F_7} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{}{\vdash : \vdash \{\Delta_4\} : \Delta_5, F_6} \text{ax} \quad \frac{\frac{}{\bullet h_1 : \vdash \{\Delta_4\} : \mathbf{1}}{\vdash : \vdash \{\Delta_4\} : \Delta_8, F_7, \perp} \mathbf{1} \quad \frac{}{\vdash : \vdash \{\Delta_4\} : \Delta_8, F_7} \text{ax}}{\vdash : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_6 \otimes F_7} \text{hCut}}{\vdash : \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \{F_5, \Delta_6\} : \mathbf{1}, *}}{\vdash : \vdash \{F_5, \Delta_6\} : *, \Delta_3} \mathbf{1} \quad \frac{h_4 : \vdash \{F_5, \Delta_6\} : \perp, F_5, \Delta_3}{\bullet h_4 : \vdash \{F_5, \Delta_6\} : dual(\mathbf{1}), \Delta_3} ?_C}{\vdash : \vdash \{F_5, \Delta_6\} : *, \Delta_3} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{}{\bullet h_1 : \vdash \{\Delta_6, F_5\} : \mathbf{1}}{\vdash : \vdash \{\Delta_6, F_5\} : \Delta_3, F_5} \mathbf{1} \quad \frac{h_4 : \vdash \{\Delta_6, F_5\} : \Delta_3, F_5, \perp}{\vdash : \vdash \{\Delta_6, F_5\} : \Delta_3} \text{ax}}{\vdash : \vdash \{\Delta_6, F_5\} : \Delta_3} \text{hCut} \quad ?_C$$

7.2 Status of !: OK

- Case rule $\mathbf{1}$

- Case rule !

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{F_7, \Delta_6\} : \Delta_8, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_8, ?F_7} \quad ?}{- : \vdash \{\Delta_6\} : *, \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6, F_7\} : !F_4} \text{ax} \quad W \quad \frac{h_5 : \vdash \{\Delta_6, F_7\} : \Delta_8, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_8, ?F_7} \text{ax}}{- : \vdash \{\Delta_6, F_7\} : \Delta_8} \text{hCut} \\
\frac{- : \vdash \{\Delta_6, F_7\} : \Delta_8}{- : \vdash \{\Delta_6\} : \Delta_8, ?F_7} ? \\
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{\Delta_6, dual(F_4)\} : \Delta_7}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_7} \quad ?}{- : \vdash \{\Delta_6\} : *, \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6, dual(F_4)\} : \Delta_7}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_7} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_7} \text{mCut}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{\Delta_6\} : F_7, F_8, \Delta_9, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \$ F_8} \quad \$}{- : \vdash \{\Delta_6\} : *, \Delta_9, F_7 \$ F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_7, F_8, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \$ F_8} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_9, F_7, F_8} \text{hCut} \\
\frac{- : \vdash \{\Delta_6\} : \Delta_9, F_7, F_8}{- : \vdash \{\Delta_6\} : \Delta_9, F_7 \$ F_8} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{\Delta_6\} : F_7, \Delta_9, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \& F_8} \quad h_5 : \vdash \{\Delta_6\} : F_8, \Delta_9, ?dual(F_4)}{\frac{- : \vdash \{\Delta_6\} : *, \Delta_9, F_7 \& F_8}{- : \vdash \{\Delta_6\} : \Delta_9, F_7 \& F_8} \text{Cut}} \& \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_7, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \& F_8} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_9, F_7} \text{hCut} \quad \frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_8, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \& F_8} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_9, F_8} \text{hCut} \\
\frac{- : \vdash \{\Delta_6\} : \Delta_9, F_7 \quad - : \vdash \{\Delta_6\} : \Delta_9, F_8}{- : \vdash \{\Delta_6\} : \Delta_9, F_7 \& F_8} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{\Delta_6\} : F_8, \Delta_9, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \oplus F_8} \quad \oplus_B}{- : \vdash \{\Delta_6\} : *, \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_8, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \oplus F_8} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_9, F_8} \text{hCut} \\
\frac{- : \vdash \{\Delta_6\} : \Delta_9, F_8}{- : \vdash \{\Delta_6\} : \Delta_9, F_7 \oplus F_8} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} ! \quad \frac{h_5 : \vdash \{\Delta_6\} : F_7, \Delta_9, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \oplus F_8} \oplus A \\
\hline
- : \vdash \{\Delta_6\} : *, \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{- : \vdash \{\Delta_6\} : \Delta_9, F_7} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_7, ?dual(F_4)}{- : \vdash \{\Delta_6\} : \Delta_9, F_7 \oplus F_8} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_9, F_7 \oplus F_8} \text{hCut} \oplus A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} ! \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_7, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \perp, \Delta_7} \perp \\
\hline
- : \vdash \{\Delta_6\} : *, \perp, \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{- : \vdash \{\Delta_6\} : \Delta_7} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_7, ?dual(F_4)}{- : \vdash \{\Delta_6\} : \Delta_7, \perp} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_7, \perp} \text{hCut} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} ! \quad \frac{}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \top, \Delta_7} \top \\
\hline
- : \vdash \{\Delta_6\} : *, \top, \Delta_7 \quad \text{Cut} \\
\rightarrow \\
- : \vdash \{\Delta_6\} : \Delta_7, \top \quad \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} ! \quad \frac{h_5 : \vdash \{\Delta_6\} : F_8, \Delta_{10}, ?dual(F_4) \quad h_5 : \vdash \{\Delta_6\} : F_9, \Delta_7}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\hline
- : \vdash \{\Delta_6\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{- : \vdash \{\Delta_6\} : \Delta_{10}, F_8} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_{10}, F_8, ?dual(F_4)}{- : \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{hCut} \otimes \\
\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} ! \quad \frac{h_5 : \vdash \{\Delta_6\} : F_8, \Delta_7 \quad h_5 : \vdash \{\Delta_6\} : F_9, \Delta_{10}, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\hline
- : \vdash \{\Delta_6\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{- : \vdash \{\Delta_6\} : \Delta_7, F_8} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_{10}, F_9, ?dual(F_4)}{- : \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{hCut} \otimes
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_7, \Delta_8\} : F_5}{\bullet h_1 \vdash \{F_7, \Delta_8\} : !F_5, *} ! \quad \frac{h_6 \vdash \{F_7, \Delta_8\} : F_7, \Delta_4, ?dual(F_5)}{\bullet h_6 \vdash \{F_7, \Delta_8\} : dual(!F_5), \Delta_4} ?_C \\
\hline
- \vdash \{F_7, \Delta_8\} : *, \Delta_4 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_8, F_7\} : !F_5}{- \vdash \{\Delta_8, F_7\} : \Delta_4, F_7} \text{ax} \quad \frac{h_6 \vdash \{\Delta_8, F_7\} : \Delta_4, F_7, ?dual(F_5)}{- \vdash \{\Delta_8, F_7\} : \Delta_4} \text{ax}}{- \vdash \{\Delta_8, F_7\} : \Delta_4} ?_C \quad \text{hCut}
\end{array}$$

7.3 Status of ?: OK

- Case rule 1
- Case rule !

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), *} ! \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, * \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 \vdash \{\Delta_7, F_5\} : \Delta_3}{- \vdash \{\Delta_7\} : \Delta_3} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : !dual(F_5)} \text{ax}}{- \vdash \{\Delta_7\} : \Delta_3} ! \quad \text{mCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{F_8, \Delta_7\} : \Delta_9, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_9, ?F_8} ? \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{\bullet h_1 \vdash \{\Delta_7, F_8\} : \Delta_3, ?F_5} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7, F_8\} : \Delta_9, !dual(F_5)}{- \vdash \{\Delta_7, F_8\} : \Delta_3, \Delta_9} \text{ax}}{- \vdash \{\Delta_7\} : \Delta_3, \Delta_9, ?F_8} ? \quad \text{hCut} \\
\hline
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{F_9, \Delta_8\} : \Delta_{10}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_{10}, ?F_9} ? \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{10}, ?F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_8, F_4\} : \Delta_5, F_6}{- \vdash \{\Delta_8, F_4\} : \Delta_{10}, \Delta_5, ?F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \{\Delta_8\} : \Delta_{10}, ?F_9, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8, F_4\} : \Delta_{10}, ?F_9, dual(F_6)} \text{ax}}{- \vdash \{\Delta_8\} : \Delta_{10}, \Delta_5, ?F_4, ?F_9} ? \quad \text{hCut} \\
\hline
\frac{- \vdash \{\Delta_8, F_4\} : \Delta_{10}, \Delta_5, ?F_9}{- \vdash \{\Delta_8\} : \Delta_{10}, \Delta_5, ?F_4, ?F_9} ? \quad \text{W}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, F_9, \Delta_{10}, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \$F_9} \$ \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_{10}, F_8 \$F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8, F_9} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_8, F_9, !dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \$F_9} \text{ax}}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \$F_9} \$ \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_9, F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, F_9 \& F_{10}} \$ \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{11}, F_9 \& F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_8\} : \Delta_5, F_6, ?F_4}{- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_{10}, F_9, ?F_4} \text{ax} \quad \frac{h_7 \vdash \{\Delta_8\} : \Delta_{11}, F_{10}, F_9, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \& F_{10}} \text{ax} \\
\hline
- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \& F_{10} \quad \text{hCut} \\
\hline
- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \& F_{10} \quad \$
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, \Delta_{10}, !dual(F_5) \quad h_6 \vdash \{\Delta_7\} : F_9, \Delta_{10}, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_8, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_9, !dual(F_5)} \text{ax} \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \quad - \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_9 \quad \text{hCut} \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \& F_9 \quad \&
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_9, \Delta_{11}, dual(F_6) \quad h_7 \vdash \{\Delta_8\} : F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, F_9 \& F_{10}} \& \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{11}, F_9 \& F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_2 \vdash \{\Delta_8, F_4\} : \Delta_5, F_6}{\bullet h_2 \vdash \{\Delta_8, F_4\} : \Delta_{11}, dual(F_6), F_9 \& F_{10}} \text{ax} \quad \frac{\bullet h_7 \vdash \{\Delta_8\} : \Delta_{11}, dual(F_6), F_9 \& F_{10}}{\bullet h_7 \vdash \{\Delta_8, F_4\} : \Delta_{11}, dual(F_6), F_9 \& F_{10}} W \\
\hline
- \vdash \{\Delta_8, F_4\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \quad - \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \& F_{10} \quad ? \\
\hline
- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \& F_{10} \quad \text{hCut}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : F_9, \Delta_{10}, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_9} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_9, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_9} \text{ax} \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_9 \quad - \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \oplus F_9 \quad \oplus_B \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \oplus F_9
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, F_9 \oplus F_{10}} \oplus_B \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{11}, F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_8\} : \Delta_5, F_6, ?F_4}{- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_{10}, ?F_4} \text{ax} \quad \frac{h_7 \vdash \{\Delta_8\} : \Delta_{11}, F_{10}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \oplus F_{10}} \text{ax} \\
\hline
- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \oplus F_{10} \quad \oplus_B \\
\hline
- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \oplus F_{10}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, \Delta_{10}, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_{10}, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_8, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8} \text{ax} \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \quad - \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \oplus F_9 \quad \oplus_A \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \oplus F_9
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_9, \Delta_{11}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, F_9 \oplus F_{10}} \oplus_A \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{11}, F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_8\} : \Delta_5, F_6, ?F_4}{- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9, ?F_4} \text{ax} \quad \frac{h_7 \vdash \{\Delta_8\} : \Delta_{11}, F_9, dual(F_6)}{- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \oplus F_{10}} \text{ax} \\
\hline
- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \oplus F_{10} \quad \text{hCut} \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_8, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \perp, \Delta_8} \perp \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \perp, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_3, \Delta_8} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_8, !dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_3, \Delta_8, \perp} \text{ax} \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_8, \perp \quad \text{hCut} \quad \perp \\
\hline
\rightarrow \\
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : \Delta_9, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \perp, \Delta_9} \perp \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \perp, \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_8\} : \Delta_5, F_6, ?F_4}{- \vdash \{\Delta_8\} : \Delta_5, \Delta_9, ?F_4} \text{ax} \quad \frac{h_7 \vdash \{\Delta_8\} : \Delta_9, dual(F_6)}{- \vdash \{\Delta_8\} : \Delta_5, \Delta_9, \perp, ?F_4} \text{ax} \\
\hline
- \vdash \{\Delta_8\} : \Delta_5, \Delta_9, \perp, ?F_4 \quad \text{hCut} \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \top, \Delta_8} \top \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \top, \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
- \vdash \{\Delta_7\} : \Delta_3, \Delta_8, \top \quad \top \\
\hline
\rightarrow \\
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \top, \Delta_9} \top \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \top, \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
- \vdash \{\Delta_8\} : \Delta_5, \Delta_9, \top, ?F_4 \quad \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : F_9, \Delta_{11}, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_3, F_9} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{11}, F_9, !dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_3, \Delta_8, F_9 \otimes F_{10}} \text{ax} \\
\hline
- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_3, F_9 \quad \text{hCut} \quad - \vdash \{\Delta_7\} : \Delta_8, F_{10} \quad \otimes \\
\hline
\rightarrow \\
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : F_9, \Delta_8}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_8, F_9} \text{ax} \quad \frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_3, F_{10}} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{11}, F_{10}, !dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_3, F_{10}} \text{ax} \\
\hline
- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_3, \Delta_8, F_9 \otimes F_{10} \quad \text{hCut} \quad \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_{10}, \Delta_{12}, dual(F_6) \quad h_7 \vdash \{\Delta_8\} : F_{11}, \Delta_9}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_8, F_4\} : \Delta_5, F_6}{- \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_5, \Delta_9, F_{10} \otimes F_{11}} ax \quad \frac{\bullet h_7 \vdash \{\Delta_8\} : \Delta_{12}, \Delta_9, dual(F_6), F_{10} \otimes F_{11}}{\bullet h_7 \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_9, dual(F_6), F_{10} \otimes F_{11}} ax}{- \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_5, \Delta_9, ?F_4, F_{10} \otimes F_{11}} W \quad hCut \\
\rightarrow \\
- \vdash \{\Delta_8\} : \Delta_{12}, \Delta_5, \Delta_9, ?F_4, F_{10} \otimes F_{11} ? \\
\hline
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_{10}, \Delta_9 \quad h_7 \vdash \{\Delta_8\} : F_{11}, \Delta_{12}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_8, F_4\} : \Delta_5, F_6}{- \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_5, \Delta_9, F_{10} \otimes F_{11}} ax \quad \frac{\bullet h_7 \vdash \{\Delta_8\} : \Delta_{12}, \Delta_9, dual(F_6), F_{10} \otimes F_{11}}{\bullet h_7 \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_9, dual(F_6), F_{10} \otimes F_{11}} W}{- \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_5, \Delta_9, ?F_4, F_{10} \otimes F_{11}} hCut \\
\rightarrow \\
- \vdash \{\Delta_8\} : \Delta_{12}, \Delta_5, \Delta_9, ?F_4, F_{10} \otimes F_{11} ?
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_6, F_8, \Delta_9\} : \Delta_3}{\bullet h_1 \vdash \{F_8, \Delta_9\} : ?F_6, \Delta_3} ? \quad \frac{h_7 \vdash \{F_8, \Delta_9\} : F_8, \Delta_5, !dual(F_6)}{\bullet h_7 \vdash \{F_8, \Delta_9\} : dual(?F_6), \Delta_5} ?_C \\
\hline
- \vdash \{F_8, \Delta_9\} : \Delta_3, \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9, F_8\} : \Delta_3, ?F_6}{- \vdash \{\Delta_9, F_8\} : \Delta_3, \Delta_5, F_8} ax \quad \frac{h_7 \vdash \{\Delta_9, F_8\} : \Delta_5, F_8, !dual(F_6)}{\bullet h_7 \vdash \{\Delta_9, F_8\} : \Delta_3, \Delta_5, F_8} ax}{- \vdash \{\Delta_9, F_8\} : \Delta_3, \Delta_5} hCut \quad ?_C \\
\rightarrow \\
- \vdash \{\Delta_9, F_8\} : \Delta_3, \Delta_5 \\
\hline
\frac{h_2 \vdash \{F_4, F_9, \Delta_{10}\} : F_7, \Delta_5}{\bullet h_2 \vdash \{F_9, \Delta_{10}\} : F_7, \Delta_5, ?F_4} ? \quad \frac{h_8 \vdash \{F_9, \Delta_{10}\} : F_9, \Delta_6, dual(F_7)}{\bullet h_8 \vdash \{F_9, \Delta_{10}\} : dual(F_7), \Delta_6} ?_C \\
\hline
- \vdash \{F_9, \Delta_{10}\} : (\Delta_5, ?F_4), \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_{10}, F_9\} : \Delta_5, F_7, ?F_4}{- \vdash \{\Delta_{10}, F_9\} : \Delta_5, \Delta_6, F_9, ?F_4} ax \quad \frac{h_8 \vdash \{\Delta_{10}, F_9\} : \Delta_6, F_9, dual(F_7)}{\bullet h_8 \vdash \{\Delta_{10}, F_9\} : \Delta_5, \Delta_6, F_9, ?F_4} ax}{- \vdash \{\Delta_{10}, F_9\} : \Delta_5, \Delta_6, ?F_4} hCut \quad ?_C \\
\rightarrow \\
- \vdash \{\Delta_{10}, F_9\} : \Delta_5, \Delta_6, ?F_4
\end{array}$$

7.4 Status of \$: OK

- Case rule 1

- Case rule !

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \$ F_7, \Delta_3} \$ \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_6) \otimes dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \$ F_7), \Delta_{11}, ?F_{10}} ?}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{11}, ?F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \$ F_7}{\bullet h_1 \vdash \{\Delta_9, F_{10}\} : \Delta_3, F_6 \$ F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, dual(F_6) \otimes dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \$ F_7), \Delta_{11}, ?F_{10}} \text{ax}}{- \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3} ? \text{hCut} \\
- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_3, ?F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \$ F_5} \$ \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{11}, ?F_{10}} ?}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \Delta_{11}, ?F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{11}, ?F_{10}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{11}, ?F_{10}, dual(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, F_4, F_5, ?F_{10}} ? \text{hCut} \\
- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \$ F_5 \$
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \$ F_7, \Delta_3} \$ \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_6) \otimes dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \$ F_7), \Delta_{12}, F_{10} \$ F_{11}} \$}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \$ F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \$ F_7}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \$ F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_6) \otimes dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_6) \otimes dual(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} ? \text{hCut} \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \$ F_{11} \$
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \$ F_5} \$ \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \$ F_{11}} \$}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \Delta_{12}, F_{10} \$ F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \$ F_5}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \$ F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \$ F_5} ? \text{hCut} \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \$ F_{11}, F_4 \$ F_5 \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \$ F_7, \Delta_3} \$ \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \otimes dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \$ F_7), \Delta_{12}, F_{10} \& F_{11}} \&}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \& F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \$ F_7}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \$ F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \otimes dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \otimes dual(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} ? \text{hCut} \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \& F_{11} \&
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \$ F_5} \$ \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \& F_{11}} \&}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \Delta_{12}, F_{10} \& F_{11}} \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, dual(F_7), F_{10} \& F_{11}}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, dual(F_7), F_{10} \& F_{11}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4, F_5, F_{10} \& F_{11}} ? \text{hCut} \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \$ F_5, F_{10} \& F_{11} \$
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3 \quad h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_6) \otimes dual(F_7)}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \mathbb{S} F_7, \Delta_3 \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \mathbb{S} F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_B \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \mathbb{S} F_7 \quad ax \quad h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \otimes dual(F_7) \quad ax}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}} hCut \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} \oplus_B \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6 \quad h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \mathbb{S} F_5 \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_B \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \mathbb{S} F_5), \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7 \quad ax \quad \bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \quad ax}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4, F_5, F_{10} \oplus F_{11}} hCut \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \mathbb{S} F_5, F_{10} \oplus F_{11}} \mathbb{S}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3 \quad \bullet h_1 \vdash \{\Delta_9\} : F_6 \$ F_7, \Delta_3}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \oplus F_{11}} \$ \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \otimes dual(F_7) \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \$ F_7), \Delta_{12}, F_{10} \oplus F_{11}}{\oplus A} \\
\text{Cut} \\
\frac{\quad}{\rightarrow} \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \$ F_7 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \otimes dual(F_7)}{\text{hCut}} \quad \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} \\
\frac{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \quad \oplus A}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6 \quad \bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \$ F_5}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \Delta_{12}, F_{10} \oplus F_{11}} \$ \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7) \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}}{\oplus A} \\
\text{Cut} \\
\frac{\quad}{\rightarrow} \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7 \quad \text{ax} \quad \bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \quad \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4, F_5, F_{10} \oplus F_{11}} \quad \text{hCut} \\
\frac{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4, F_5, F_{10} \oplus F_{11} \quad \$}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \$ F_5, F_{10} \oplus F_{11}}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3 \quad \bullet h_1 \vdash \{\Delta_9\} : F_6 \$ F_7, \Delta_3}{- \vdash \{\Delta_9\} : \Delta_3, \perp, \Delta_{10}} \$ \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \otimes dual(F_7) \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \$ F_7), \perp, \Delta_{10}}{\vdash} \perp}{- \vdash \{\Delta_9\} : \Delta_3, \perp, \Delta_{10}} \text{Cut} \\
\downarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \$ F_7 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \otimes dual(F_7) \quad \text{ax}}{h_8 \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} \text{hCut}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \perp \\
\downarrow \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6 \quad \bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \$ F_5}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \perp, \Delta_{10}} \$ \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7) \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \perp, \Delta_{10}}{\vdash} \perp}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \perp, \Delta_{10}} \text{Cut} \\
\downarrow \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7 \quad \text{ax} \quad \bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \perp, dual(F_7) \quad \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, F_4, F_5, \perp} \text{hCut} \\
\downarrow \\
- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \$ F_5 \$
\end{array}$$

- Case rule \top

$$\frac{\frac{\frac{\mathbf{h}_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\mathbf{h}_1 \vdash \{\Delta_9\} : F_6 \S F_7, \Delta_3} \quad \S \quad \frac{}{\mathbf{h}_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \S F_7), \top, \Delta_{10}} \top}{- \vdash \{\Delta_9\} : \Delta_3, \top, \Delta_{10}} \text{Cut} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_3, \top, \Delta_{10}} \rightarrow \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \top} \top$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \$ F_5} \$ \frac{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \top, \Delta_{10}}{\top} \text{Cut} \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \top, \Delta_{10}} \rightarrow \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \$ F_5} \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \$ F_7, \Delta_3} \$ \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_6) \otimes dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \$ F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \$ F_7}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, dual(F_6) \otimes dual(F_7)}{hCut} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{12}} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes \\
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \$ F_7, \Delta_3} \$ \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_6) \otimes dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \$ F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \rightarrow \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{11}} \text{ax} \quad \frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \$ F_7}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{12}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, dual(F_6) \otimes dual(F_7)}{hCut} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \rightarrow \\
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \$ F_7, \Delta_3} \$ \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \quad h_8 \vdash \{\Delta_9\} : \Delta_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \$ F_7), \Delta_{10}, \Delta_{11}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{11}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{11}} \rightarrow \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, F_6, F_7} \text{ax} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{11}, dual(F_7)} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_3, F_6} \text{sCut} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6)} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{11}, \Delta_3} \text{sCut} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \$ F_5} \$ \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \rightarrow \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_5, F_{11} \otimes F_{12}} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12}}{hCut} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_5, F_{11} \otimes F_{12}} \$ \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \$ F_5, F_{11} \otimes F_{12}} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \$ F_5} \$ \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \$ F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \rightarrow \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_5, F_{11} \otimes F_{12}} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12}}{hCut} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_5, F_{11} \otimes F_{12}} \$ \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \$ F_5, F_{11} \otimes F_{12}}
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7, F_8, \Delta_3}{\bullet h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7 \$ F_8, \Delta_3} \$ \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_6, dual(F_7) \otimes dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_7 \$ F_8), \Delta_6} ?_C}{- \vdash \{F_{10}, \Delta_{11}\} : \Delta_3, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, F_7 \$ F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6, F_{10}} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_{10}, dual(F_7) \otimes dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \text{ax}}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \text{hCut} \\
\text{?}_C \\
\frac{h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_6, F_4 \$ F_5} \$ \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_8), \Delta_7} ?_C}{- \vdash \{F_{10}, \Delta_{11}\} : (\Delta_6, F_4 \$ F_5), \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_4, F_5, F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4, F_5} \text{ax} \quad \frac{\bullet h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_7, dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \$ F_5} \text{ax}}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \$ F_5} \text{hCut} \\
\$
\end{array}$$

7.5 Status of &: OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{11}, ?F_{10}} ?}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{11}, ?F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7}{\bullet h_1 \vdash \{\Delta_9, F_{10}\} : \Delta_3, F_6 \& F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3} \text{ax}}{- \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3, ?F_{10}} \text{hCut} \\
W \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{11}, ?F_{10}} ?}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{11}, ?F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5}{\bullet h_2 \vdash \{\Delta_9, F_{10}\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_6, F_4 \& F_5} \text{ax}}{- \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \& F_5} \text{hCut} \\
W \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \& F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, \Delta_{12}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{12}, F_{10} \$ F_{11}} \$}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \$ F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, \Delta_{12}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{hCut} \\
\$
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{12}, F_{10} \$ F_{11}} \$}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \$ F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, \Delta_{12}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{hCut} \\
\$
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \$ F_{11}} \$ \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{12}, F_{10} \$ F_{11}} \text{Cut} \\
\frac{}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_7)} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \& F_5} \text{hCut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \$ F_{11}, F_4 \& F_5} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \oplus dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{12}, F_{10} \& F_{11}} \& \\
\frac{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \& F_{11}}{\rightarrow} \text{Cut} \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \oplus dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \oplus dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \& F_{11}} \& \quad \frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \oplus dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \& F_{11}} \& \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \& F_{11}} \& \\
\frac{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{12}, F_{10} \& F_{11}}{\rightarrow} \text{Cut} \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_4 \& F_5} \quad \text{ax} \quad \frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{11}, F_4 \& F_5}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \& F_{11}, F_4 \& F_5} \& \quad \frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{11}, F_4 \& F_5}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \& F_{11}, F_4 \& F_5} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus B \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \oplus F_{11}} \rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \oplus dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}} \text{hCut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} \oplus B \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} \oplus B \\
\\
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{10}} \oplus B \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}} \rightarrow \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, F_7} \text{ax} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\text{sCut}} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \\
\\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus B \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{12}, F_{10} \oplus F_{11}} \rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{11}, F_4 \& F_5} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{11}, F_4 \& F_5} \text{hCut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \& F_5, F_{10} \oplus F_{11}} \oplus B \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \& F_5, F_{10} \oplus F_{11}} \oplus B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\mathbf{h}_1 \vdash \{\Delta_9\} : \mathbf{F}_6, \Delta_3 \quad \mathbf{h}_1 \vdash \{\Delta_9\} : \mathbf{F}_7, \Delta_3}{\bullet \mathbf{h}_1 \vdash \{\Delta_9\} : \mathbf{F}_6 \& \mathbf{F}_7, \Delta_3} \quad \& \quad \frac{\mathbf{h}_8 \vdash \{\Delta_9\} : \mathbf{F}_{10}, \Delta_{12}, \mathit{dual}(\mathbf{F}_6) \oplus \mathit{dual}(\mathbf{F}_7)}{\bullet \mathbf{h}_8 \vdash \{\Delta_9\} : \mathit{dual}(\mathbf{F}_6 \& \mathbf{F}_7), \Delta_{12}, \mathbf{F}_{10} \oplus \mathbf{F}_{11}} \quad \oplus_A \\
\hline
- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\bullet \mathbf{h}_1 \vdash \{\Delta_9\} : \Delta_3, \mathbf{F}_6 \& \mathbf{F}_7 \quad \text{ax} \quad \mathbf{h}_8 \vdash \{\Delta_9\} : \Delta_{12}, \mathbf{F}_{10}, \mathit{dual}(\mathbf{F}_6) \oplus \mathit{dual}(\mathbf{F}_7) \quad \text{ax} \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, \mathbf{F}_{10} \quad \text{hCut} \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \quad \oplus_A
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{10}} \oplus_A \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\frac{}{- : \vdash \{\Delta_9\} : \Delta_3, F_6} \text{ax} \quad \frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6)} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{sCut} \\
\\
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \frac{h_8 : \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_A \\
\frac{}{- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\frac{}{\bullet h_2 : \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{}{h_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_7)} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_4 \& F_5} \text{hCut} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \& F_5, F_{10} \oplus F_{11}} \oplus_A \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \& F_5, F_{10} \oplus F_{11}} \oplus_A
\end{array}$$

• Case rule \perp

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \& F_7), \perp, \Delta_{10}} \perp \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_3, \perp, \Delta_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7} \text{ax} \quad \frac{}{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \oplus dual(F_7)} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{hCut} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} \perp \\
\\
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \perp, \Delta_{10}} \perp \\
\frac{}{- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \perp, \Delta_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\frac{}{\bullet h_2 : \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{}{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, F_4 \& F_5} \text{hCut} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \& F_5} \perp
\end{array}$$

• Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \frac{}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \& F_7), \top, \Delta_{10}} \top \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_3, \top, \Delta_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \top} \top \\
\\
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \frac{}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \top, \Delta_{10}} \top \\
\frac{}{- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \top, \Delta_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \& F_5} \top
\end{array}$$

• Case rule I_3

• Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_6) \oplus dual(F_7) \quad h_8 : \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7} \text{ax} \quad \frac{}{h_8 : \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, dual(F_6) \oplus dual(F_7)} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{11}} \text{hCut} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, F_{12}} \text{ax} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_6) \oplus dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{11}} \text{ax} \quad \frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, dual(F_6) \oplus dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, dual(F_6) \oplus dual(F_7)} \text{ax}}{\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes} \text{hCut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, dual(F_7)} \text{ax}}{\frac{}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_6, F_{11}, F_4 \& F_5} \text{hCut} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{12}} \text{ax}}{\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \& F_5, F_{11} \otimes F_{12}} \otimes} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \& F_5, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \& F_5, F_{11} \otimes F_{12}} \otimes \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{11}} \text{ax} \quad \frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, dual(F_7)} \text{ax}}{\frac{}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_6, F_{12}, F_4 \& F_5} \text{hCut} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \& F_5, F_{11} \otimes F_{12}} \otimes} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \& F_5, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \& F_5, F_{11} \otimes F_{12}} \otimes
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7, \Delta_3 \quad h_1 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_3}{\bullet h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7 \& F_8, \Delta_3} \& \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_6, dual(F_7) \oplus dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_7 \& F_8), \Delta_6} \otimes \\
\frac{}{- \vdash \{F_{10}, \Delta_{11}\} : \Delta_3, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, F_7 \& F_8} \text{ax} \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : \Delta_6, F_{10}, dual(F_7) \oplus dual(F_8)}{h_9 \vdash \{F_{10}, \Delta_{11}\} : \Delta_6, F_{10}, dual(F_7) \oplus dual(F_8)} \text{ax}}{\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6, F_{10}} \text{hCut} \quad \frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} ?_C} \otimes \\
\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} ?_C \\
\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} ?_C \\
\frac{h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_4, \Delta_6 \quad h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_5, \Delta_6}{\bullet h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_6, F_4 \& F_5} \& \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_8), \Delta_7} \otimes \\
\frac{}{- \vdash \{F_{10}, \Delta_{11}\} : (\Delta_6, F_4 \& F_5), \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_8, F_4 \& F_5} \text{ax} \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : \Delta_7, F_{10}, dual(F_8)}{h_9 \vdash \{F_{10}, \Delta_{11}\} : \Delta_7, F_{10}, dual(F_8)} \text{ax}}{\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_{10}, F_4 \& F_5} \text{hCut} \quad \frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \& F_5} ?_C} \otimes \\
\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_{10}, F_4 \& F_5} ?_C \\
\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \& F_5} ?_C
\end{array}$$

7.6 Status of \oplus_B : OK

- Case rule 1

- Case rule !

- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{11}, ?F_{10}} ? \\
\hline
- \vdash \{\Delta_9\} : \Delta_3, \Delta_{11}, ?F_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{\bullet h_1 \vdash \{\Delta_9, F_{10}\} : \Delta_3, F_6 \oplus F_7} \text{ax} \\
\frac{h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3} \text{ax} \\
\frac{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_3, ?F_{10}}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_3, ?F_{10}} ? \\
\hline
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{11}, ?F_{10}} ? \\
\hline
- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{11}, ?F_{10} \quad \text{Cut} \\
\hline
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_5, F_7}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{11}, ?F_{10}, dual(F_7)} \text{ax} \\
\frac{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, F_5, ?F_{10}}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \oplus F_5} \text{hCut} \\
\hline
\frac{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \oplus F_5}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \oplus F_5} \oplus_B
\end{array}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \$ F_{11}} \$ \\
\hline
- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \$ F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \$ F_{11}} \text{ax} \\
\hline
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11} \quad \$ \\
\hline
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \$ F_{11} \quad \text{hCut} \\
\hline
\rightarrow \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \$ F_{11}} \$ \\
\hline
- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \$ F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \oplus F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, F_4 \oplus F_5} \text{ax} \\
\hline
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \oplus F_5 \quad \$ \\
\hline
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \$ F_{11}, F_4 \oplus F_5 \quad \text{hCut}
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \& dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \& F_{11}} \&}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \& F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad ax \quad h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} \quad \frac{ax \quad h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad hCut \quad h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}} \&}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \& F_{11}} \\
\\
\frac{\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \quad h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}} \&}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{\Delta_9\} : \Delta_3, F_7 \quad ax \quad - \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{sCut} \\
\\
\frac{\frac{h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \& F_{11}} \&}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \& F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_5, F_7 \quad ax \quad \bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, dual(F_7), F_{10} \& F_{11}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_5, F_{10} \& F_{11}} \quad \frac{ax \quad hCut}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \& F_{11}, F_4 \oplus F_5} \oplus_B}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \& F_{11}, F_4 \oplus F_5}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_B \\
\hline
- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \oplus F_{11} \\
\rightarrow \\
\frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}} ax \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \& dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} ax \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11} \oplus_B \\
\oplus_B \quad \frac{h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_B \\
\hline
- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \oplus F_{11} \\
\rightarrow \\
\frac{\bullet h_2 : \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{11}, F_4 \oplus F_5} ax \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5} ax \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5 \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_A \\
\hline
- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \oplus F_{11} \\
\rightarrow \\
\frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} ax \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \& dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} ax \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11} \oplus_A \\
\oplus_B \quad \frac{h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_A \\
\hline
- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \oplus F_{11} \\
\rightarrow \\
\frac{h_2 : \vdash \{\Delta_9\} : \Delta_6, F_5, F_7}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_5, F_{10} \oplus F_{11}} ax \quad \frac{\bullet h_8 : \vdash \{\Delta_9\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11}}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5} ax \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5 \oplus_B
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \perp, \Delta_{10}} \perp \\
\hline
- : \vdash \{\Delta_9\} : \Delta_3, \perp, \Delta_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} ax \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \& dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} ax \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp \perp \\
\oplus_B \quad \frac{h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \perp, \Delta_{10}} \perp \\
\hline
- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \perp, \Delta_{10} \\
\rightarrow \\
\frac{h_2 : \vdash \{\Delta_9\} : \Delta_6, F_5, F_7}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, F_5, \perp} ax \quad \frac{\bullet h_8 : \vdash \{\Delta_9\} : \Delta_{10}, \perp, dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \oplus F_5} ax \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \oplus F_5 \oplus_B
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \top, \Delta_{10}} \top \\
\hline
- : \vdash \{\Delta_9\} : \Delta_3, \top, \Delta_{10} \\
\rightarrow \\
- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \top \quad \top \\
\hline
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \top, \Delta_{10}} \top \\
\hline
- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \top, \Delta_{10} \\
\rightarrow \\
- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \oplus F_5 \quad \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_6) \& dual(F_7) \quad h_8 : \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad ax \quad h_8 : \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, dual(F_6) \& dual(F_7) \quad ax}{- : \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{11}} hCut \quad \frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, F_{12}} ax \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12} \quad \otimes \\
\hline
\frac{h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 : \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, F_{11}} ax \quad \frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad ax \quad h_8 : \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, dual(F_6) \& dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{12}} hCut \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12} \quad \otimes \\
\hline
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_7) \quad h_8 : \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{h_2 : \vdash \{\Delta_9\} : \Delta_6, F_5, F_7 \quad ax \quad \bullet h_8 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12} \quad ax}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_5, F_{11} \otimes F_{12}} hCut \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_{11} \otimes F_{12}, F_4 \oplus F_5 \quad \oplus_B \\
\hline
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 : \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{h_2 : \vdash \{\Delta_9\} : \Delta_6, F_5, F_7 \quad ax \quad \bullet h_8 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12} \quad ax}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_5, F_{11} \otimes F_{12}} hCut \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_{11} \otimes F_{12}, F_4 \oplus F_5 \quad \oplus_B
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_3}{\bullet h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7 \oplus F_8, \Delta_3} \oplus_B \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_6, dual(F_7) \& dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_7 \oplus F_8), \Delta_6} ?_C \\
\hline
- \vdash \{F_{10}, \Delta_{11}\} : \Delta_3, \Delta_6 \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, F_7 \oplus F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6, F_{10}} ax \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_{10}, dual(F_7) \& dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} ?_C \\
\hline
- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6 \quad hCut
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_5, \Delta_6}{\bullet h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_8), \Delta_7} ?_C \\
\hline
- \vdash \{F_{10}, \Delta_{11}\} : (\Delta_6, F_4 \oplus F_5), \Delta_7 \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_5, F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_5} ax \quad \frac{\bullet h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_7, dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \oplus F_5} ax \\
\hline
- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \oplus F_5 \quad hCut \quad \oplus_B
\end{array}$$

7.7 Status of \oplus_A : OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{11}, ?F_{10}} ? \\
\hline
- \vdash \{\Delta_9\} : \Delta_3, \Delta_{11}, ?F_{10} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{\bullet h_1 \vdash \{\Delta_9, F_{10}\} : \Delta_3, F_6 \oplus F_7} ax \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_6) \& dual(F_7)}{h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, dual(F_6) \& dual(F_7)} ax \\
\hline
- \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3 \quad ? \\
- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_3, ?F_{10} \quad hCut
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{11}, ?F_{10}} ? \\
\hline
- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{11}, ?F_{10} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_7}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_7} ax \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{11}, ?F_{10}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{11}, ?F_{10}, dual(F_7)} ax \\
\hline
- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, F_4, ?F_{10} \quad hCut \\
- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \oplus F_5 \quad \oplus_A
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \$ F_{11}} \$ \\
\hline
- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \$ F_{11} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} ax \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_6) \& dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} ax \\
\hline
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11} \quad \$ \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \$ F_{11} \quad hCut
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \$ F_{11}} \$ \\
\hline
- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \$ F_{11} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \oplus F_5} ax \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \oplus F_5} ax \\
\hline
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \oplus F_5 \quad \$ \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \$ F_{11}, F_4 \oplus F_5 \quad hCut
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{\text{h}_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet \text{h}_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus A \quad \frac{\text{h}_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, \text{dual}(F_6) \& \text{dual}(F_7) \quad \text{h}_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, \text{dual}(F_6) \& \text{dual}(F_7)}{\bullet \text{h}_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \oplus F_7), \Delta_{12}, F_{10} \& F_{11}} \&}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \& F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\text{h}_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad \text{ax} \quad \text{h}_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, \text{dual}(F_6) \& \text{dual}(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} \quad \frac{\text{hCut} \quad \frac{\text{h}_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad \text{ax} \quad \text{h}_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, \text{dual}(F_6) \& \text{dual}(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}} \&}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \& F_{11}} \& \\
\\
\frac{\frac{\text{h}_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet \text{h}_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus A \quad \frac{\text{h}_8 \vdash \{\Delta_9\} : \Delta_{10}, \text{dual}(F_6) \quad \text{h}_8 \vdash \{\Delta_9\} : \Delta_{10}, \text{dual}(F_7)}{\bullet \text{h}_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \oplus F_7), \Delta_{10}} \&}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- \vdash \{\Delta_9\} : \Delta_3, F_6 \quad \text{ax} \quad - \vdash \{\Delta_9\} : \Delta_{10}, \text{dual}(F_6)}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{sCut}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \\
\\
\frac{\frac{\text{h}_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet \text{h}_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus A \quad \frac{\text{h}_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, \text{dual}(F_7) \quad \text{h}_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, \text{dual}(F_7)}{\bullet \text{h}_8 \vdash \{\Delta_9\} : \text{dual}(F_7), \Delta_{12}, F_{10} \& F_{11}} \&}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \& F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\text{h}_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_7 \quad \text{ax} \quad \bullet \text{h}_8 \vdash \{\Delta_9\} : \Delta_{12}, \text{dual}(F_7), F_{10} \& F_{11}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4, F_{10} \& F_{11}} \text{hCut} \quad \oplus A}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \& F_{11}, F_4 \oplus F_5} \oplus A
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_6}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_B \\
\hline
- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \oplus F_{11} \quad \text{Cut} \\
\downarrow \\
\frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad ax \quad h_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \& dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}} \quad ax \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11} \quad \oplus_B \quad \text{hCut} \\
\downarrow \\
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_B \\
\hline
- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \oplus F_{11} \quad \text{Cut} \\
\downarrow \\
\frac{\bullet h_2 : \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5 \quad ax \quad h_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{11}, F_4 \oplus F_5} \quad ax \\
\hline
- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5 \quad \oplus_B \quad \text{hCut} \\
\downarrow \\
- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5 \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus A \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} \oplus A \quad \text{hCut} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus A \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5 \quad h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_4 \oplus F_5} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5} \oplus A \quad \text{hCut}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \perp, \Delta_{10}} \perp}{- : \vdash \{\Delta_9\} : \Delta_3, \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{ax} \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \& dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} \text{hCut} \\
\perp \\
\frac{\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \perp, \Delta_{10}} \perp}{- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \{\Delta_9\} : \Delta_6, F_4, F_7}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, F_4, \perp} \text{ax} \quad \frac{\bullet h_8 : \vdash \{\Delta_9\} : \Delta_{10}, \perp, dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, F_4, \perp} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \oplus F_5} \text{hCut} \\
\oplus_A
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \top, \Delta_{10}}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \top, \Delta_{10}} \top}{- : \vdash \{\Delta_9\} : \Delta_3, \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \top}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \top} \top \\
\frac{\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 : \vdash \{\Delta_9\} : dual(F_7), \top, \Delta_{10}}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \top, \Delta_{10}} \top}{- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \oplus F_5}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \oplus F_5} \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes}{- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- : \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{11}} \text{ax} \quad \frac{h_8 : \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, dual(F_6) \& dual(F_7)}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \text{hCut} \\
\otimes \\
\frac{\frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{10}}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes}{- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \vdash \{\Delta_9\} : \Delta_{10}, F_{11}}{- : \vdash \{\Delta_9\} : \Delta_{10}, F_{11}} \text{ax} \quad \frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- : \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{12}} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \text{hCut} \\
\otimes \\
\frac{\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes}{- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \{\Delta_9\} : \Delta_6, F_4, F_7}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_{11} \otimes F_{12}} \text{ax} \quad \frac{\bullet h_8 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_{11} \otimes F_{12}} \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_{11} \otimes F_{12}} \text{hCut} \\
\oplus_A
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_{11} \otimes F_{12}} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_{11} \otimes F_{12}, F_4 \oplus F_5} \text{hCut}}{\rightarrow} \oplus_A
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7, \Delta_3}{\bullet h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7 \oplus F_8, \Delta_3} \oplus_A \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_6, dual(F_7) \& dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_7 \oplus F_8), \Delta_6} ?_C \\
\hline
- \vdash \{F_{10}, \Delta_{11}\} : \Delta_3, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, F_7 \oplus F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6, F_{10}} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_{10}, dual(F_7) \& dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \text{hCut}}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} ?_C \\
\rightarrow \\
\frac{h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_4, \Delta_6}{\bullet h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_8), \Delta_7} ?_C \\
\hline
- \vdash \{F_{10}, \Delta_{11}\} : (\Delta_6, F_4 \oplus F_5), \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_4, F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4} \text{ax} \quad \frac{\bullet h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_7, dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \oplus F_5} \text{hCut}}{\rightarrow} \oplus_A
\end{array}$$

7.8 Status of \perp : OK

- Case rule 1

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), *}{- \vdash \{\Delta_5\} : \Delta_3, *} \mathbf{1} \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, * \quad \text{Cut} \\
\rightarrow \\
- \vdash \{\Delta_5\} : \Delta_3 \quad \text{ax}
\end{array}$$

- Case rule !

- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{F_6, \Delta_5\} : \mathbf{1}, \Delta_7}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_7, ?F_6} ? \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5, F_6\} : \Delta_3, \Delta_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5, F_6\} : \mathbf{1}, \Delta_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6} ?
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 \vdash \{F_8, \Delta_7\} : \Delta_9, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_9, ?F_8} ? \\
\hline
- \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_9, ?F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_7\} : \Delta_4, F_5}{\bullet h_2 \vdash \{\Delta_7\} : \Delta_4, \Delta_9, ?F_8} \text{ax} \quad \frac{\bullet h_6 \vdash \{\Delta_7\} : \Delta_9, ?F_8, dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_4, \Delta_9, ?F_8} \text{ax}}{- \vdash \{\Delta_7\} : \Delta_4, \Delta_9, \perp, ?F_8} \text{hCut} \\
\hline
- \vdash \{\Delta_7\} : \Delta_4, \Delta_9, \perp, ?F_8 \quad \perp
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_6, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_8, F_6 \$ F_7} \$ \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \$ F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6, F_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_6, F_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \$ F_7} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \$ F_7} \text{hCut} \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \$ F_7 \quad \$ \\
\hline
\frac{h_2 \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8 \$ F_9} \$ \\
\hline
- \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_{10}, F_8 \$ F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_7\} : \Delta_4, F_5, \perp}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8, F_9, \perp} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_8, F_9, dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \$ F_9} \text{ax}}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \$ F_9} \text{hCut} \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \$ F_9 \quad \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_6, \Delta_8 \quad h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_8, F_6 \& F_7} \& \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \& F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_6}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{hCut} \\
\hline
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \& F_7} \text{hCut} \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \& F_7 \quad \& \\
\hline
\frac{h_2 \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, \Delta_{10}, dual(F_5) \quad h_6 \vdash \{\Delta_7\} : F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8 \& F_9} \& \\
\hline
- \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_{10}, F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_7\} : \Delta_4, F_5}{\bullet h_2 \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_6 \vdash \{\Delta_7\} : \Delta_{10}, dual(F_5), F_8 \& F_9}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8 \& F_9} \text{ax}}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8 \& F_9} \text{hCut} \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \& F_9 \quad \perp
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_8, F_6 \oplus F_7} \oplus_B \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \text{hCut} \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7 \quad \oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 : \vdash \{\Delta_7\} : F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_{10}, F_8 \oplus F_9 \\
\rightarrow \\
\frac{\bullet h_2 : \vdash \{\Delta_7\} : \Delta_4, F_5, \perp}{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_9, \perp} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_7\} : \Delta_{10}, F_9, dual(F_5)}{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \oplus F_9} \text{ax} \\
\hline
- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \oplus F_9 \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, F_6, \Delta_8}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\perp), \Delta_8, F_6 \oplus F_7} \oplus_A \\
\hline
- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7 \\
\rightarrow \\
\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \perp}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{ax} \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_6}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \text{ax} \\
\hline
- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7 \oplus_A
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 : \vdash \{\Delta_7\} : F_8, \Delta_{10}, dual(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_{10}, F_8 \oplus F_9 \\
\rightarrow \\
\frac{\bullet h_2 : \vdash \{\Delta_7\} : \Delta_4, F_5, \perp}{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8, \perp} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_7\} : \Delta_{10}, F_8, dual(F_5)}{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \oplus F_9} \text{ax} \\
\hline
- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \oplus F_9 \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_6}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\perp), \perp, \Delta_6} \perp \\
\hline
- : \vdash \{\Delta_5\} : \Delta_3, \perp, \Delta_6 \\
\rightarrow \\
\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \perp}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6} \text{ax} \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_6}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \perp} \text{ax} \\
\hline
- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \perp \perp
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 : \vdash \{\Delta_7\} : \Delta_8, dual(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \perp, \Delta_8} \perp \\
\hline
- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \perp, \Delta_8 \\
\rightarrow \\
\frac{h_2 : \vdash \{\Delta_7\} : \Delta_4, F_5}{- : \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \perp} \text{ax} \quad \frac{\bullet h_6 : \vdash \{\Delta_7\} : \Delta_8, \perp, dual(F_5)}{- : \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \perp, \perp} \text{ax} \\
\hline
- : \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \perp, \perp \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\perp), \top, \Delta_6} \top \\
\hline
- : \vdash \{\Delta_5\} : \Delta_3, \top, \Delta_6 \\
\rightarrow \\
- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \top \top
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \top, \Delta_8} \top \\
\hline
- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \top, \Delta_8 \\
\rightarrow \\
- : \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \perp, \top \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, F_7, \Delta_9 \quad h_4 : \vdash \{\Delta_5\} : F_8, \Delta_6}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\perp), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \perp}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_7} \text{ax} \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_9, F_7}{hCut} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\frac{\frac{h_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 : \vdash \{\Delta_5\} : F_7, \Delta_6 \quad h_4 : \vdash \{\Delta_5\} : \mathbf{1}, F_8, \Delta_9}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\perp), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \vdash \{\Delta_5\} : \Delta_6, F_7}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax} \quad \frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \perp \quad h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_9, F_8}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_8} \text{hCut} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\frac{\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 : \vdash \{\Delta_7\} : F_9, \Delta_{11}, dual(F_5) \quad h_6 : \vdash \{\Delta_7\} : F_{10}, \Delta_8}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \{\Delta_7\} : \Delta_4, F_5}{- : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_6 : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_8, dual(F_5), F_9 \otimes F_{10}}{hCut} \text{ax}}{- : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, \perp, F_9 \otimes F_{10}} \perp \\
\frac{\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 : \vdash \{\Delta_7\} : F_9, \Delta_8 \quad h_6 : \vdash \{\Delta_7\} : F_{10}, \Delta_{11}, dual(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \{\Delta_7\} : \Delta_4, F_5}{- : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_6 : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_8, dual(F_5), F_9 \otimes F_{10}}{hCut} \text{ax}}{- : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, \perp, F_9 \otimes F_{10}} \perp
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 : \vdash \{F_6, \Delta_7\} : \Delta_3}{\bullet h_1 : \vdash \{F_6, \Delta_7\} : \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \{F_6, \Delta_7\} : \mathbf{1}, F_6, \Delta_4}{\bullet h_5 : \vdash \{F_6, \Delta_7\} : dual(\perp), \Delta_4} ?_C}{- : \vdash \{F_6, \Delta_7\} : \Delta_3, \Delta_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_7, F_6\} : \Delta_3, \perp}{- : \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4, F_6} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_7, F_6\} : \mathbf{1}, \Delta_4, F_6}{hCut} \text{ax}}{- : \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4} ?_C \\
\frac{h_2 : \vdash \{F_8, \Delta_9\} : F_6, \Delta_4}{\bullet h_2 : \vdash \{F_8, \Delta_9\} : F_6, \perp, \Delta_4} \perp \quad \frac{h_7 : \vdash \{F_8, \Delta_9\} : F_8, \Delta_5, dual(F_6)}{\bullet h_7 : \vdash \{F_8, \Delta_9\} : dual(F_6), \Delta_5} ?_C}{- : \vdash \{F_8, \Delta_9\} : (\perp, \Delta_4), \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \{\Delta_9, F_8\} : \Delta_4, F_6}{- : \vdash \{\Delta_9, F_8\} : \Delta_4, \Delta_5} \text{ax} \quad \frac{\bullet h_7 : \vdash \{\Delta_9, F_8\} : \Delta_5, dual(F_6)}{hCut} \text{ax}}{- : \vdash \{\Delta_9, F_8\} : \Delta_4, \Delta_5, \perp} \perp
\end{array}$$

7.9 Status of \top : OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 : \vdash \{F_6, \Delta_5\} : \mathbf{0}, \Delta_7}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\top), \Delta_7, ?F_6} ?}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_5, F_6\} : \Delta_3, \top}{- : \vdash \{\Delta_5, F_6\} : \Delta_3, \Delta_7} \top \quad \frac{h_4 : \vdash \{\Delta_5, F_6\} : \mathbf{0}, \Delta_7}{\bullet h_4 : \vdash \{\Delta_5, F_6\} : \Delta_3, \Delta_7, ?F_6} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6} \text{hCut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 : \vdash \{F_8, \Delta_7\} : \Delta_9, dual(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_9, ?F_8} ?}{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
- : \vdash \{\Delta_7\} : \Delta_4, \Delta_9, \top, ?F_8 \top
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{0}, F_6, F_7, \Delta_8}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\top), \Delta_8, F_6\$F_7} \$}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\$F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \top}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6, F_7} \text{ax} \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_6, F_7}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\$F_7} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\$F_7} \text{hCut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 : \vdash \{\Delta_7\} : F_8, F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8\$F_9} \$}{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8\$F_9} \text{Cut} \\
\rightarrow \\
- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8\$F_9 \top
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{0}, F_6, \Delta_8 \quad h_4 : \vdash \{\Delta_5\} : \mathbf{0}, F_7, \Delta_8}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\top), \Delta_8, F_6\&F_7} \&}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \top}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_6}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{hCut} \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \top}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_7}{\bullet h_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{hCut} \\
\& \\
- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7 \& \\
\rightarrow \\
\frac{\frac{}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 : \vdash \{\Delta_7\} : F_8, \Delta_{10}, dual(F_5) \quad h_6 : \vdash \{\Delta_7\} : F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8\&F_9} \&}{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8\&F_9} \text{Cut} \\
\rightarrow \\
- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8\&F_9 \top
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\top), \Delta_8, F_6 \oplus F_7} \oplus_B}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \top}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \oplus_B \\
\frac{\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{\Delta_7\} : F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \oplus F_9} \top
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, F_6, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\top), \Delta_8, F_6 \oplus F_7} \oplus_A}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \top}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_6}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \oplus_A \\
\frac{\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, \Delta_{10}, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_A}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \oplus F_9} \top
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_6}{\bullet h_4 \vdash \{\Delta_5\} : dual(\top), \perp, \Delta_6} \perp}{- \vdash \{\Delta_5\} : \Delta_3, \perp, \Delta_6} \text{Cut} \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp, \top}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \perp} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_6}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \perp} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \perp} \text{hCut} \\
\frac{\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_8, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \perp, \Delta_8} \perp}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \perp, \Delta_8} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \perp, \top} \top
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{}{\bullet h_4 \vdash \{\Delta_5\} : dual(\top), \top, \Delta_6} \top}{- \vdash \{\Delta_5\} : \Delta_3, \top, \Delta_6} \text{Cut} \\
\frac{}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \top} \top \\
\frac{\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \top, \Delta_8} \top}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \top, \Delta_8} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \top, \top} \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{0}, F_7, \Delta_9 \quad h_4 : \vdash \{\Delta_5\} : F_8, \Delta_6}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\top), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \top}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_7} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{0}, \Delta_9, F_7}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\frac{}{\bullet h_1 : \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : F_7, \Delta_6 \quad h_4 : \vdash \{\Delta_5\} : \mathbf{0}, F_8, \Delta_9}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\top), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \vdash \{\Delta_5\} : \Delta_6, F_7} \text{ax} \quad \frac{\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \top}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_7} \text{ax} \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{0}, \Delta_9, F_8}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_8} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{hCut}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\frac{}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 : \vdash \{\Delta_7\} : F_9, \Delta_{11}, dual(F_5) \quad h_6 : \vdash \{\Delta_7\} : F_{10}, \Delta_8}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, \top, F_9 \otimes F_{10}} \top \\
\frac{}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 : \vdash \{\Delta_7\} : F_9, \Delta_8 \quad h_6 : \vdash \{\Delta_7\} : F_{10}, \Delta_{11}, dual(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, \top, F_9 \otimes F_{10}} \top
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \{F_6, \Delta_7\} : \top, \Delta_3} \top \quad \frac{h_5 : \vdash \{F_6, \Delta_7\} : \mathbf{0}, F_6, \Delta_4}{\bullet h_5 : \vdash \{F_6, \Delta_7\} : dual(\top), \Delta_4} ?_C}{- : \vdash \{F_6, \Delta_7\} : \Delta_3, \Delta_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_7, F_6\} : \Delta_3, \top}{- : \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4, F_6} \top \quad \frac{h_5 : \vdash \{\Delta_7, F_6\} : \mathbf{0}, \Delta_4, F_6}{- : \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4} \text{ax}}{- : \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4} \text{hCut} \\
?_C \\
\frac{}{\bullet h_2 : \vdash \{F_8, \Delta_9\} : F_6, \top, \Delta_4} \top \quad \frac{h_7 : \vdash \{F_8, \Delta_9\} : F_8, \Delta_5, dual(F_6)}{\bullet h_7 : \vdash \{F_8, \Delta_9\} : dual(F_6), \Delta_5} ?_C}{- : \vdash \{F_8, \Delta_9\} : (\top, \Delta_4), \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_9, F_8\} : \Delta_4, \Delta_5, \top} \top
\end{array}$$

7.10 Status of I_3 : OK

- Case rule 1

- Case rule !

- Case rule ?

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_6\} : \wedge(n_4), p(n_4)} I_3 \quad \frac{h_5 \vdash \{F_7, \Delta_6\} : \Delta_8, p(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\wedge(n_4)), \Delta_8, ?F_7} ? \\
\frac{}{- \vdash \{\Delta_6\} : p(n_4), \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_6, F_7\} : \Delta_8, p(n_4)} \text{ax} \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_8, ?F_7, p(n_4)} ?
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_6\} : p(n_4), \wedge(n_4)} I_3 \quad \frac{h_5 \vdash \{F_7, \Delta_6\} : \Delta_8, \wedge(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_8, ?F_7} ? \\
\frac{}{- \vdash \{\Delta_6\} : \wedge(n_4), \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_6, F_7\} : \Delta_8, \wedge(n_4)} \text{ax} \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_8, ?F_7, \wedge(n_4)} ?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_6\} : \wedge(n_4), p(n_4)} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, F_8, \Delta_9, p(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\wedge(n_4)), \Delta_9, F_7\$F_8} \$ \\
\frac{}{- \vdash \{\Delta_6\} : p(n_4), \Delta_9, F_7\$F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, F_7, F_8, p(n_4)} \text{ax} \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7\$F_8} \$
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_6\} : p(n_4), \wedge(n_4)} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, F_8, \Delta_9, \wedge(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_9, F_7\$F_8} \$ \\
\frac{}{- \vdash \{\Delta_6\} : \wedge(n_4), \Delta_9, F_7\$F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, F_7, F_8, \wedge(n_4)} \text{ax} \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, \wedge(n_4), F_7\$F_8} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_6\} : \wedge(n_4), p(n_4)} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, \Delta_9, p(n_4) \quad h_5 \vdash \{\Delta_6\} : F_8, \Delta_9, p(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\wedge(n_4)), \Delta_9, F_7\&F_8} \& \\
\frac{}{- \vdash \{\Delta_6\} : p(n_4), \Delta_9, F_7\&F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, F_7, p(n_4)} \text{ax} \quad \frac{}{- \vdash \{\Delta_6\} : \Delta_9, F_8, p(n_4)} \text{ax} \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7\&F_8} \&
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_6\} : p(n_4), \wedge(n_4)} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, \Delta_9, \wedge(n_4) \quad h_5 \vdash \{\Delta_6\} : F_8, \Delta_9, \wedge(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_9, F_7\&F_8} \& \\
\frac{}{- \vdash \{\Delta_6\} : \wedge(n_4), \Delta_9, F_7\&F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, F_7, \wedge(n_4)} \text{ax} \quad \frac{}{- \vdash \{\Delta_6\} : \Delta_9, F_8, \wedge(n_4)} \text{ax} \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, \wedge(n_4), F_7\&F_8} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_6\} : \wedge(n_4), p(n_4)} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_9, p(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\wedge(n_4)), \Delta_9, F_7 \oplus F_8} \oplus_B \\
\frac{}{- \vdash \{\Delta_6\} : p(n_4), \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, F_8, p(n_4)} \text{ax} \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7 \oplus F_8} \oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \{\Delta_6\} : p(n_4), \wedge(n_4)} I_3 \quad \frac{h_5 : \vdash \{\Delta_6\} : F_8, \Delta_9, \wedge(n_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_9, F_7 \oplus F_8} \oplus_B \\
\hline
- : \vdash \{\Delta_6\} : \wedge(n_4), \Delta_9, F_7 \oplus F_8 \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_6\} : \Delta_9, F_8, \wedge(n_4)} \text{ax} \\
\hline
- : \vdash \{\Delta_6\} : \Delta_9, \wedge(n_4), F_7 \oplus F_8 \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \{\Delta_6\} : \wedge(n_4), p(n_4)} I_3 \quad \frac{h_5 : \vdash \{\Delta_6\} : F_7, \Delta_9, p(n_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(\wedge(n_4)), \Delta_9, F_7 \oplus F_8} \oplus_A \\
\hline
- : \vdash \{\Delta_6\} : p(n_4), \Delta_9, F_7 \oplus F_8 \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_6\} : \Delta_9, F_7, p(n_4)} \text{ax} \\
\hline
- : \vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7 \oplus F_8 \oplus_A
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \{\Delta_6\} : p(n_4), \wedge(n_4)} I_3 \quad \frac{h_5 : \vdash \{\Delta_6\} : F_7, \Delta_9, \wedge(n_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_9, F_7 \oplus F_8} \oplus_A \\
\hline
- : \vdash \{\Delta_6\} : \wedge(n_4), \Delta_9, F_7 \oplus F_8 \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_6\} : \Delta_9, F_7, \wedge(n_4)} \text{ax} \\
\hline
- : \vdash \{\Delta_6\} : \Delta_9, \wedge(n_4), F_7 \oplus F_8 \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \{\Delta_6\} : \wedge(n_4), p(n_4)} I_3 \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_7, p(n_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(\wedge(n_4)), \perp, \Delta_7} \perp \\
\hline
- : \vdash \{\Delta_6\} : p(n_4), \perp, \Delta_7 \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_6\} : \Delta_7, p(n_4)} \text{ax} \\
\hline
- : \vdash \{\Delta_6\} : \Delta_7, \perp, p(n_4) \perp
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \{\Delta_6\} : p(n_4), \wedge(n_4)} I_3 \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_7, \wedge(n_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(p(n_4)), \perp, \Delta_7} \perp \\
\hline
- : \vdash \{\Delta_6\} : \wedge(n_4), \perp, \Delta_7 \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_6\} : \Delta_7, \wedge(n_4)} \text{ax} \\
\hline
- : \vdash \{\Delta_6\} : \Delta_7, \perp, \wedge(n_4) \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \{\Delta_6\} : \wedge(n_4), p(n_4)} I_3 \quad \frac{}{\bullet h_5 : \vdash \{\Delta_6\} : dual(\wedge(n_4)), \top, \Delta_7} \top \\
\hline
- : \vdash \{\Delta_6\} : p(n_4), \top, \Delta_7 \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_6\} : \Delta_7, \top, p(n_4)} \top
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \{\Delta_6\} : p(n_4), \wedge(n_4)} I_3 \quad \frac{}{\bullet h_5 : \vdash \{\Delta_6\} : dual(p(n_4)), \top, \Delta_7} \top \\
\hline
- : \vdash \{\Delta_6\} : \wedge(n_4), \top, \Delta_7 \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_6\} : \Delta_7, \top, \wedge(n_4)} \top
\end{array}$$

- Case rule I_3

$$\begin{array}{c}
\frac{}{\bullet h_1 : \vdash \{\Delta_5\} : \wedge(n_6), p(n_6)} I_3 \quad \frac{}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\wedge(n_6)), \wedge(n_6)} I_3 \\
\hline
- : \vdash \{\Delta_5\} : p(n_6), \wedge(n_6) \\
\rightarrow \\
- : \vdash \{\Delta_5\} : p(n_6), \wedge(n_6) I_3
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_5\} : p(n_6), \wedge(n_6)}{- \vdash \{\Delta_5\} : \wedge(n_6), p(n_6)} I_3 \quad \frac{\bullet h_4 \vdash \{\Delta_5\} : dual(p(n_6)), p(n_6)}{- \vdash \{\Delta_5\} : \wedge(n_6), p(n_6)} I_3 \\
\rightarrow \\
- \vdash \{\Delta_5\} : p(n_6), \wedge(n_6) I_3
\end{array}$$

- Case rule \otimes

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_6\} : \wedge(n_4), p(n_4)}{- \vdash \{\Delta_6\} : p(n_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_{10}, p(n_4) \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_7}{\bullet h_5 \vdash \{\Delta_6\} : dual(\wedge(n_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_{10}, F_8, p(n_4) \quad ax \quad - \vdash \{\Delta_6\} : \Delta_7, F_9 \quad ax}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, p(n_4), F_8 \otimes F_9} \otimes \\
\\
\frac{\bullet h_1 \vdash \{\Delta_6\} : \wedge(n_4), p(n_4)}{- \vdash \{\Delta_6\} : p(n_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_7 \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_{10}, p(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\wedge(n_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_7, F_8 \quad ax \quad - \vdash \{\Delta_6\} : \Delta_{10}, F_9, p(n_4) \quad ax}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, p(n_4), F_8 \otimes F_9} \otimes \\
\\
\frac{\bullet h_1 \vdash \{\Delta_6\} : p(n_4), \wedge(n_4)}{- \vdash \{\Delta_6\} : \wedge(n_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_{10}, \wedge(n_4) \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_7}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_{10}, F_8, \wedge(n_4) \quad ax \quad - \vdash \{\Delta_6\} : \Delta_7, F_9 \quad ax}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, \wedge(n_4), F_8 \otimes F_9} \otimes \\
\\
\frac{\bullet h_1 \vdash \{\Delta_6\} : p(n_4), \wedge(n_4)}{- \vdash \{\Delta_6\} : \wedge(n_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_7 \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_{10}, \wedge(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_7, F_8 \quad ax \quad - \vdash \{\Delta_6\} : \Delta_{10}, F_9, \wedge(n_4) \quad ax}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, \wedge(n_4), F_8 \otimes F_9} \otimes
\end{array}$$

- Case rule I_1

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_5, p(n_6)\} : p(n_6), \wedge(n_6)}{- \vdash \{\Delta_5, p(n_6)\} : \wedge(n_6), *} I_3 \quad \frac{\bullet h_4 \vdash \{\Delta_5, p(n_6)\} : dual(p(n_6)), *}{- \vdash \{\Delta_5, p(n_6)\} : \wedge(n_6), *} I_1 \\
\rightarrow \\
- \vdash \{\Delta_5, p(n_6)\} : \wedge(n_6) I_1
\end{array}$$

- Case rule I_2

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_5, \wedge(n_6)\} : \wedge(n_6), p(n_6)}{- \vdash \{\Delta_5, \wedge(n_6)\} : p(n_6), *} I_3 \quad \frac{\bullet h_4 \vdash \{\Delta_5, \wedge(n_6)\} : dual(\wedge(n_6)), *}{- \vdash \{\Delta_5, \wedge(n_6)\} : p(n_6), *} I_2 \\
\rightarrow \\
- \vdash \{\Delta_5, \wedge(n_6)\} : p(n_6) I_2
\end{array}$$

- Case rule $?_C$

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{F_7, \Delta_8\} : \wedge(n_5), p(n_5)}{- \vdash \{F_7, \Delta_8\} : p(n_5), \Delta_4} I_3 \quad \frac{h_6 \vdash \{F_7, \Delta_8\} : F_7, \Delta_4, p(n_5)}{\bullet h_6 \vdash \{F_7, \Delta_8\} : dual(\wedge(n_5)), \Delta_4} ?_C \\
\rightarrow \\
\frac{- \vdash \{\Delta_8, F_7\} : \Delta_4, F_7, p(n_5) \quad ax}{- \vdash \{\Delta_8, F_7\} : \Delta_4, p(n_5)} ?_C
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{F_7, \Delta_8\} : p(n_5), \wedge(n_5)}{- \vdash \{F_7, \Delta_8\} : \wedge(n_5), \Delta_4} I_3 \quad \frac{h_6 \vdash \{F_7, \Delta_8\} : F_7, \Delta_4, \wedge(n_5)}{\bullet h_6 \vdash \{F_7, \Delta_8\} : dual(p(n_5)), \Delta_4} ?_C \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{- \vdash \{\Delta_8, F_7\} : \Delta_4, F_7, \wedge(n_5)}{- \vdash \{\Delta_8, F_7\} : \Delta_4, \wedge(n_5)} ax}{- \vdash \{\Delta_8, F_7\} : \Delta_4, \wedge(n_5)} ?_C
\end{array}$$

7.11 Status of \otimes : OK

- Case rule 1

- Case rule !

- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \quad \frac{h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_7) \$ dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{12}, ?F_{11}} ? \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8}{\bullet h_1 \vdash \{\Delta_{10}, F_{11}\} : \Delta_3, \Delta_4, F_7 \otimes F_8} ax \quad W \quad \frac{h_9 \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, dual(F_7) \$ dual(F_8)}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_3, \Delta_4} ax}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_3, \Delta_4, ?F_{11}} hCut \\
? \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_{10}\} : F_8, F_5, \Delta_7 \quad h_2 \vdash \{\Delta_{10}\} : F_6, \Delta_4}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{12}, ?F_{11}} ? \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{\bullet h_2 \vdash \{\Delta_{10}, F_{11}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6} ax \quad W \quad \frac{h_9 \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, dual(F_8)}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_4, \Delta_7, F_5 \otimes F_6} ax}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_4, \Delta_7, ?F_{11}, F_5 \otimes F_6} hCut \\
? \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{12}, ?F_{11}} ? \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{\bullet h_2 \vdash \{\Delta_{10}, F_{11}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6} ax \quad W \quad \frac{h_9 \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, dual(F_8)}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_4, \Delta_7, F_5 \otimes F_6} ax}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_4, \Delta_7, ?F_{11}, F_5 \otimes F_6} hCut \\
?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{11}, F_{12}, \Delta_{13}, dual(F_7) \$ dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{13}, F_{11} \$ F_{12}} \$ \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8}{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_{13}, F_{11}, F_{12}, dual(F_7) \$ dual(F_8)} ax \quad W \quad \frac{h_9 \vdash \{\Delta_{10}\} : \Delta_{13}, F_{11}, F_{12}, dual(F_7) \$ dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{11}, F_{12}} ax}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{11} \$ F_{12}} hCut \\
\$
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{11}, \Delta_{13}, dual(F_7), dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{11}} \text{Cut} \\
\frac{- \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \Delta_{11}}{\rightarrow} \\
\frac{- \vdash \{\Delta_{10}\} : \Delta_4, F_8 \quad - \vdash \{\Delta_{10}\} : \Delta_{11}, dual(F_7), dual(F_8)}{\rightarrow} \text{ax} \quad \text{sCut} \\
\frac{- \vdash \{\Delta_{10}\} : \Delta_3, F_7 \quad - \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, dual(F_7)}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_3, \Delta_4} \text{sCut} \\
\\
\frac{h_2 \vdash \{\Delta_{10}\} : F_8, F_5, \Delta_7 \quad h_2 \vdash \{\Delta_{10}\} : F_6, \Delta_4}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{11}, F_{12}, \Delta_{13}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{13}, F_{11} \$ F_{12}} \$ \\
\frac{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{13}, F_{11} \$ F_{12}}{\rightarrow} \\
\frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6 \quad h_9 \vdash \{\Delta_{10}\} : \Delta_{13}, F_{11}, F_{12}, dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_4, \Delta_7, F_{11}, F_{12}, F_5 \otimes F_6} \text{ax} \quad \text{hCut} \\
\frac{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_4, \Delta_7, F_{11}, F_{12}, F_5 \otimes F_6}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_4, \Delta_7, F_{11} \$ F_{12}, F_5 \otimes F_6} \$ \\
\\
\frac{h_2 \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{11}, F_{12}, \Delta_{13}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{13}, F_{11} \$ F_{12}} \$ \\
\frac{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{13}, F_{11} \$ F_{12}}{\rightarrow} \\
\frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6 \quad h_9 \vdash \{\Delta_{10}\} : \Delta_{13}, F_{11}, F_{12}, dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_4, \Delta_7, F_{11}, F_{12}, F_5 \otimes F_6} \text{ax} \quad \text{hCut} \\
\frac{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_4, \Delta_7, F_{11}, F_{12}, F_5 \otimes F_6}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_4, \Delta_7, F_{11} \$ F_{12}, F_5 \otimes F_6} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{11}, \Delta_{13}, dual(F_7) \$ dual(F_8) \quad h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{13}, dual(F_7) \$ dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{13}, F_{11} \& F_{12}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \Delta_{13}, F_{11} \& F_{12}} \rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8 \quad \text{ax} \quad h_9 \vdash \{\Delta_{10}\} : \Delta_{13}, F_{11}, dual(F_7) \$ dual(F_8) \quad \text{ax}}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{11}} \text{hCut} \quad \frac{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8 \quad \text{ax} \quad h_9 \vdash \{\Delta_{10}\} : \Delta_{13}, F_{12}, dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{12}} \& \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{11} \& F_{12}} \otimes \\
\frac{h_2 \vdash \{\Delta_{10}\} : F_8, F_5, \Delta_7 \quad h_2 \vdash \{\Delta_{10}\} : F_6, \Delta_4}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{11}, \Delta_{13}, dual(F_8) \quad h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{13}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{13}, F_{11} \& F_{12}} \& \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{13}, F_{11} \& F_{12}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_2 \vdash \{\Delta_{10}\} : \Delta_7, F_5, F_8 \quad \text{ax} \quad \bullet h_9 \vdash \{\Delta_{10}\} : \Delta_{13}, dual(F_8), F_{11} \& F_{12}}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_7, F_5, F_{11} \& F_{12}} \text{hCut} \quad \frac{}{- \vdash \{\Delta_{10}\} : \Delta_4, F_6} \text{ax} \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_4, \Delta_7, F_{11} \& F_{12}, F_5 \otimes F_6} \otimes \\
\frac{h_2 \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{11}, \Delta_{13}, dual(F_8) \quad h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{13}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{13}, F_{11} \& F_{12}} \& \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{13}, F_{11} \& F_{12}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{\text{ax}} \quad \frac{h_2 \vdash \{\Delta_{10}\} : \Delta_7, F_6, F_8 \quad \text{ax} \quad \bullet h_9 \vdash \{\Delta_{10}\} : \Delta_{13}, dual(F_8), F_{11} \& F_{12}}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_7, F_6, F_{11} \& F_{12}} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_4, F_5} \text{ax} \quad \frac{}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_4, \Delta_7, F_{11} \& F_{12}, F_5 \otimes F_6} \otimes
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{13}, \text{dual}(F_7) \$ \text{dual}(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : \text{dual}(F_7 \otimes F_8), \Delta_{13}, F_{11} \oplus F_{12}} \oplus_B \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \Delta_{13}, F_{11} \oplus F_{12}} \text{Cut} \\
\frac{}{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8} \text{ax} \quad \frac{}{h_9 \vdash \{\Delta_{10}\} : \Delta_{13}, F_{12}, \text{dual}(F_7) \$ \text{dual}(F_8)} \text{ax} \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{12}} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{11} \oplus F_{12}} \oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : \Delta_{11}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \perp, \Delta_{11}} \perp \\
\hline
- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \perp, \Delta_{11} \quad \text{Cut} \\
\hline
\frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6 \quad \text{ax} \quad h_9 \vdash \{\Delta_{10}\} : \Delta_{11}, dual(F_8) \quad \text{ax}}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{hCut} \\
\hline
- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, \Delta_7, \perp, F_5 \otimes F_6 \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \frac{}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \top, \Delta_{11}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \top, \Delta_{11}} \rightarrow \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_3, \Delta_4, \top} \top \\
\\
\frac{h_2 \vdash \{\Delta_{10}\} : F_8, F_5, \Delta_7 \quad h_2 \vdash \{\Delta_{10}\} : F_6, \Delta_4}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \frac{}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \top, \Delta_{11}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \top, \Delta_{11}} \rightarrow \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, \Delta_7, \top, F_5 \otimes F_6} \top \\
\\
\frac{h_2 \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \frac{}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \top, \Delta_{11}} \text{Cut} \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \top, \Delta_{11}} \rightarrow \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, \Delta_7, \top, F_5 \otimes F_6} \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{11}, dual(F_7) \$ dual(F_8) \quad h_9 \vdash \{\Delta_{10}\} : F_{13}, \Delta_{11}}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \text{Cut} \\
\frac{- \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}}{\rightarrow} \\
\frac{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8}{- \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_3, \Delta_4, F_{12}} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{10}\} : \Delta_{14}, F_{12}, dual(F_7) \$ dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_3, \Delta_4, F_{12} \otimes F_{13}} \text{hCut} \quad \frac{- \vdash \{\Delta_{10}\} : \Delta_{11}, F_{13}}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_3, \Delta_4, F_{12} \otimes F_{13}} \text{ax} \otimes \\
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{11} \quad h_9 \vdash \{\Delta_{10}\} : F_{13}, \Delta_{14}, dual(F_7) \$ dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \text{Cut} \\
\frac{- \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}}{\rightarrow} \\
\frac{- \vdash \{\Delta_{10}\} : \Delta_{11}, F_{12}}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_3, \Delta_4, F_{12} \otimes F_{13}} \text{ax} \quad \frac{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8}{- \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_3, \Delta_4, F_{13}} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{10}\} : \Delta_{14}, F_{13}, dual(F_7) \$ dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_3, \Delta_4, F_{13}} \text{hCut} \otimes \\
\frac{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_3, \Delta_4, F_{12} \otimes F_{13}}{\rightarrow} \\
\frac{h_2 \vdash \{\Delta_{10}\} : F_8, F_5, \Delta_7 \quad h_2 \vdash \{\Delta_{10}\} : F_6, \Delta_4}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{14}, dual(F_8) \quad h_9 \vdash \{\Delta_{10}\} : F_{13}, \Delta_{11}}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \text{Cut} \\
\frac{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}}{\rightarrow} \\
\frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{- \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_4, \Delta_7, F_{12}, F_5 \otimes F_6} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{10}\} : \Delta_{14}, F_{12}, dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_4, \Delta_7, F_{12} \otimes F_{13}, F_5 \otimes F_6} \text{hCut} \quad \frac{- \vdash \{\Delta_{10}\} : \Delta_{11}, F_{13}}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_4, \Delta_7, F_{12} \otimes F_{13}, F_5 \otimes F_6} \text{ax} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_{10}\} : F_8, F_5, \Delta_7 \quad h_2 \vdash \{\Delta_{10}\} : F_6, \Delta_4}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{11} \quad h_9 \vdash \{\Delta_{10}\} : F_{13}, \Delta_{14}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \otimes \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{11}, F_{12}} \text{ax} \quad \frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{- \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_4, \Delta_7, F_{13}, F_5 \otimes F_6} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{10}\} : \Delta_{14}, F_{13}, dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_4, \Delta_7, F_{12} \otimes F_{13}, F_5 \otimes F_6} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_4, \Delta_7, F_{12} \otimes F_{13}, F_5 \otimes F_6} \otimes \\
\frac{h_2 \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{14}, dual(F_8) \quad h_9 \vdash \{\Delta_{10}\} : F_{13}, \Delta_{11}}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \otimes \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{- \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_4, \Delta_7, F_{12}, F_5 \otimes F_6} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{10}\} : \Delta_{14}, F_{12}, dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{11}, F_{13}} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_4, \Delta_7, F_{12} \otimes F_{13}, F_5 \otimes F_6} \otimes \\
\frac{h_2 \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_9 \vdash \{\Delta_{10}\} : F_{12}, \Delta_{11} \quad h_9 \vdash \{\Delta_{10}\} : F_{13}, \Delta_{14}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \otimes \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{11}, F_{12}} \text{ax} \quad \frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{- \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_4, \Delta_7, F_{13}, F_5 \otimes F_6} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{10}\} : \Delta_{14}, F_{13}, dual(F_8)}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_4, \Delta_7, F_{12} \otimes F_{13}, F_5 \otimes F_6} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_4, \Delta_7, F_{12} \otimes F_{13}, F_5 \otimes F_6} \otimes
\end{array}$$

• Case rule I_1

• Case rule I_2

• Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_{11}, \Delta_{12}\} : F_8, \Delta_3 \quad h_1 \vdash \{F_{11}, \Delta_{12}\} : F_9, \Delta_4}{\bullet h_1 \vdash \{F_{11}, \Delta_{12}\} : F_8 \otimes F_9, \Delta_3, \Delta_4} \otimes \quad \frac{h_{10} \vdash \{F_{11}, \Delta_{12}\} : F_{11}, \Delta_7, dual(F_8) \$ dual(F_9)}{\bullet h_{10} \vdash \{F_{11}, \Delta_{12}\} : dual(F_8 \otimes F_9), \Delta_7} \text{Cut} \\
\frac{}{- \vdash \{F_{11}, \Delta_{12}\} : (\Delta_3, \Delta_4), \Delta_7} \rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_{12}, F_{11}\} : \Delta_3, \Delta_4, F_8 \otimes F_9}{- \vdash \{\Delta_{12}, F_{11}\} : \Delta_3, \Delta_4, \Delta_7, F_{11}} \text{ax} \quad \frac{h_{10} \vdash \{\Delta_{12}, F_{11}\} : \Delta_7, F_{11}, dual(F_8) \$ dual(F_9)}{- \vdash \{\Delta_{12}, F_{11}\} : \Delta_3, \Delta_4, \Delta_7} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{12}, F_{11}\} : \Delta_3, \Delta_4, \Delta_7} \text{?}_C \\
\frac{h_2 \vdash \{F_{11}, \Delta_{12}\} : F_9, F_5, \Delta_7 \quad h_2 \vdash \{F_{11}, \Delta_{12}\} : F_6, \Delta_4}{\bullet h_2 \vdash \{F_{11}, \Delta_{12}\} : F_9, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_{10} \vdash \{F_{11}, \Delta_{12}\} : F_{11}, \Delta_8, dual(F_9)}{\bullet h_{10} \vdash \{F_{11}, \Delta_{12}\} : dual(F_9), \Delta_8} \text{Cut} \\
\frac{}{- \vdash \{F_{11}, \Delta_{12}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_8} \rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_{12}, F_{11}\} : \Delta_4, \Delta_7, F_9, F_5 \otimes F_6}{- \vdash \{\Delta_{12}, F_{11}\} : \Delta_4, \Delta_7, \Delta_8, F_{11}, F_5 \otimes F_6} \text{ax} \quad \frac{h_{10} \vdash \{\Delta_{12}, F_{11}\} : \Delta_8, F_{11}, dual(F_9)}{- \vdash \{\Delta_{12}, F_{11}\} : \Delta_4, \Delta_7, \Delta_8, F_5 \otimes F_6} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{12}, F_{11}\} : \Delta_4, \Delta_7, \Delta_8, F_5 \otimes F_6} \text{?}_C \\
\frac{h_2 \vdash \{F_{11}, \Delta_{12}\} : F_5, \Delta_4 \quad h_2 \vdash \{F_{11}, \Delta_{12}\} : F_9, F_6, \Delta_7}{\bullet h_2 \vdash \{F_{11}, \Delta_{12}\} : F_9, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \quad \frac{h_{10} \vdash \{F_{11}, \Delta_{12}\} : F_{11}, \Delta_8, dual(F_9)}{\bullet h_{10} \vdash \{F_{11}, \Delta_{12}\} : dual(F_9), \Delta_8} \text{Cut} \\
\frac{}{- \vdash \{F_{11}, \Delta_{12}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_8} \rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_{12}, F_{11}\} : \Delta_4, \Delta_7, F_9, F_5 \otimes F_6}{- \vdash \{\Delta_{12}, F_{11}\} : \Delta_4, \Delta_7, \Delta_8, F_{11}, F_5 \otimes F_6} \text{ax} \quad \frac{h_{10} \vdash \{\Delta_{12}, F_{11}\} : \Delta_8, F_{11}, dual(F_9)}{- \vdash \{\Delta_{12}, F_{11}\} : \Delta_4, \Delta_7, \Delta_8, F_5 \otimes F_6} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{12}, F_{11}\} : \Delta_4, \Delta_7, \Delta_8, F_5 \otimes F_6} \text{?}_C
\end{array}$$

7.12 Status of I_1 : OK

• Case rule 1

- Case rule !

- Case rule ?

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_5)\} : \wedge(n_5), *} I_1 \quad \frac{h_6 \vdash \{F_7, \Delta_4, p(n_5)\} : \Delta_8, p(n_5)}{\bullet h_6 \vdash \{\Delta_4, p(n_5)\} : dual(\wedge(n_5)), \Delta_8, ?F_7} ? \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : *, \Delta_8, ?F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{\Delta_4, F_7, p(n_5)\} : \Delta_8, p(n_5)}{- \vdash \{\Delta_4, F_7, p(n_5)\} : \Delta_8} \text{ax} \\
\frac{- \vdash \{\Delta_4, F_7, p(n_5)\} : \Delta_8}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_8, ?F_7} ?_C \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_8, ?F_7}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_8, ?F_7} ?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_5)\} : \wedge(n_5), *} I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(n_5)\} : F_7, F_8, \Delta_9, p(n_5)}{\bullet h_6 \vdash \{\Delta_4, p(n_5)\} : dual(\wedge(n_5)), \Delta_9, F_7 \$ F_8} \$ \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : *, \Delta_9, F_7 \$ F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7, F_8, p(n_5)}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7, F_8} \text{ax} \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7, F_8}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7 \$ F_8} ?_C \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7 \$ F_8}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7 \$ F_8} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_5)\} : \wedge(n_5), *} I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(n_5)\} : F_7, \Delta_9, p(n_5) \quad h_6 \vdash \{\Delta_4, p(n_5)\} : F_8, \Delta_9, p(n_5)}{\bullet h_6 \vdash \{\Delta_4, p(n_5)\} : dual(\wedge(n_5)), \Delta_9, F_7 \& F_8} \& \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : *, \Delta_9, F_7 \& F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7, p(n_5)}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7, p(n_5)} \text{ax} \quad \frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_8, p(n_5)}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_8, p(n_5)} \text{ax} \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7, p(n_5) \quad - \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_8, p(n_5)}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, p(n_5), F_7 \& F_8} \& \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, p(n_5), F_7 \& F_8}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7 \& F_8} ?_C
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_5)\} : \wedge(n_5), *} I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(n_5)\} : F_8, \Delta_9, p(n_5)}{\bullet h_6 \vdash \{\Delta_4, p(n_5)\} : dual(\wedge(n_5)), \Delta_9, F_7 \oplus F_8} \oplus_B \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : *, \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_8, p(n_5)}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_8, p(n_5)} \text{ax} \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_8, p(n_5)}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_8} ?_C \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_8}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7 \oplus F_8} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_5)\} : \wedge(n_5), *} I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(n_5)\} : F_7, \Delta_9, p(n_5)}{\bullet h_6 \vdash \{\Delta_4, p(n_5)\} : dual(\wedge(n_5)), \Delta_9, F_7 \oplus F_8} \oplus_A \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : *, \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7, p(n_5)}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7, p(n_5)} \text{ax} \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7, p(n_5)}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7} ?_C \\
\frac{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_9, F_7 \oplus F_8} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_5)\} : \wedge(n_5), *} \quad I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(n_5)\} : \Delta_7, p(n_5)}{\bullet h_6 \vdash \{\Delta_4, p(n_5)\} : dual(\wedge(n_5)), \perp, \Delta_7} \quad \perp \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : *, \perp, \Delta_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_7, p(n_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_7} ?_C \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : \Delta_7, \perp \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_5)\} : \wedge(n_5), *} \quad I_1 \quad \frac{}{\bullet h_6 \vdash \{\Delta_4, p(n_5)\} : dual(\wedge(n_5)), \top, \Delta_7} \quad \top \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : *, \top, \Delta_7 \quad \text{Cut} \\
\hline
\rightarrow \\
- \vdash \{\Delta_4, p(n_5)\} : \Delta_7, \top \quad \top
\end{array}$$

- Case rule I_3

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_6)\} : \wedge(n_6), *} \quad I_1 \quad \frac{}{\bullet h_5 \vdash \{\Delta_4, p(n_6)\} : dual(\wedge(n_6)), \wedge(n_6)} \quad I_3 \\
\hline
- \vdash \{\Delta_4, p(n_6)\} : *, \wedge(n_6) \quad \text{Cut} \\
\hline
\rightarrow \\
- \vdash \{\Delta_4, p(n_6)\} : \wedge(n_6) \quad I_1
\end{array}$$

- Case rule \otimes

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_5)\} : \wedge(n_5), *} \quad I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(n_5)\} : F_8, \Delta_{10}, p(n_5) \quad h_6 \vdash \{\Delta_4, p(n_5)\} : F_9, \Delta_7}{\bullet h_6 \vdash \{\Delta_4, p(n_5)\} : dual(\wedge(n_5)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \quad \otimes \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_{10}, F_8, p(n_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_{10}, F_8} ?_C \quad \frac{}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_7, F_9} \text{ax} \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9 \quad \otimes \\
\\
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(n_5)\} : \wedge(n_5), *} \quad I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(n_5)\} : F_8, \Delta_7 \quad h_6 \vdash \{\Delta_4, p(n_5)\} : F_9, \Delta_{10}, p(n_5)}{\bullet h_6 \vdash \{\Delta_4, p(n_5)\} : dual(\wedge(n_5)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \quad \otimes \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_7, F_8} \text{ax} \quad \frac{}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_{10}, F_9, p(n_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_7, F_8} ?_C \quad \frac{}{- \vdash \{\Delta_4, p(n_5)\} : \Delta_{10}, F_9} ?_C \\
\hline
- \vdash \{\Delta_4, p(n_5)\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9 \quad \otimes
\end{array}$$

- Case rule I_1

- Case rule I_2

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{(\Delta_6, \wedge(n_5)), p(n_5)\} : \wedge(n_5), *} \quad I_1 \quad \frac{}{\bullet h_4 \vdash \{(\Delta_6, \wedge(n_5)), p(n_5)\} : dual(\wedge(n_5)), *} \quad I_2 \\
\hline
- \vdash \{(\Delta_6, \wedge(n_5)), p(n_5)\} : *, * \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_6, p(n_5), \wedge(n_5)\} : p(n_5)} \quad I_2 \\
\hline
- \vdash \{\Delta_6, p(n_5), \wedge(n_5)\} : * \quad ?_C
\end{array}$$

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_7, p(n_4)\} : \wedge(n_4), *}}{I_1} \quad \frac{\frac{h_6 \vdash \{\Delta_7, p(n_4)\} : \Delta_5, p(n_4), p(n_4)}{\bullet h_6 \vdash \{\Delta_7, p(n_4)\} : dual(\wedge(n_4)), \Delta_5}}{?_C}}{\frac{}{- \vdash \{\Delta_7, p(n_4)\} : *, \Delta_5}}{Cut}} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_7, p(n_4)\} : \Delta_5, p(n_4), p(n_4)}}{ax}}{\frac{}{- \vdash \{\Delta_7, p(n_4)\} : \Delta_5, p(n_4)}}{?_C}} \\
\frac{}{- \vdash \{\Delta_7, p(n_4)\} : \Delta_5} \quad ?_C \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_7, p(n_4)\} : \Delta_5}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{(F_7, \Delta_8), p(n_4)\} : \wedge(n_4), *}}{I_1} \quad \frac{\frac{h_6 \vdash \{F_7, \Delta_8, p(n_4)\} : F_7, \Delta_5, p(n_4)}{\bullet h_6 \vdash \{(F_7, \Delta_8), p(n_4)\} : dual(\wedge(n_4)), \Delta_5}}{?_C}}{\frac{}{- \vdash \{(F_7, \Delta_8), p(n_4)\} : *, \Delta_5}}{Cut}} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_8, F_7, p(n_4)\} : \Delta_5, F_7, p(n_4)}}{ax}}{\frac{}{- \vdash \{\Delta_8, F_7, p(n_4)\} : \Delta_5, F_7}}{?_C}} \\
\frac{}{- \vdash \{\Delta_8, F_7, p(n_4)\} : \Delta_5} \quad ?_C \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_8, F_7, p(n_4)\} : \Delta_5}
\end{array}$$

7.13 Status of I_2 : OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4, \wedge(n_5)\} : p(n_5), *}}{I_2} \quad \frac{\frac{h_6 \vdash \{F_7, \Delta_4, \wedge(n_5)\} : \Delta_8, \wedge(n_5)}{\bullet h_6 \vdash \{\Delta_4, \wedge(n_5)\} : dual(p(n_5)), \Delta_8, ?F_7}}{?}}{\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : *, \Delta_8, ?F_7}}{Cut}} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_4, F_7, \wedge(n_5)\} : \Delta_8, \wedge(n_5)}}{ax}}{\frac{}{- \vdash \{\Delta_4, F_7, \wedge(n_5)\} : \Delta_8}}{?_C}} \\
\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_8, ?F_7} \quad ? \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_8, ?F_7}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4, \wedge(n_5)\} : p(n_5), *}}{I_2} \quad \frac{\frac{h_6 \vdash \{\Delta_4, \wedge(n_5)\} : F_7, F_8, \Delta_9, \wedge(n_5)}{\bullet h_6 \vdash \{\Delta_4, \wedge(n_5)\} : dual(p(n_5)), \Delta_9, F_7\$F_8}}{\$}}{\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : *, \Delta_9, F_7\$F_8}}{Cut}} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_9, F_7, F_8, \wedge(n_5)}}{ax}}{\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_9, F_7, F_8}}{?_C}} \\
\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_9, F_7\$F_8} \quad \$ \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_9, F_7\$F_8}
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4, \wedge(n_5)\} : p(n_5), *}}{I_2} \quad \frac{\frac{h_6 \vdash \{\Delta_4, \wedge(n_5)\} : F_7, \Delta_9, \wedge(n_5)}{\bullet h_6 \vdash \{\Delta_4, \wedge(n_5)\} : dual(p(n_5)), \Delta_9, F_7\&F_8}}{\&}}{\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : *, \Delta_9, F_7\&F_8}}{Cut}} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_9, F_7, \wedge(n_5)}}{ax}}{\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_9, F_7, \wedge(n_5)}}{?_C}} \\
\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_9, F_7\&F_8} \quad \& \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \wedge(n_5)\} : \Delta_9, F_7\&F_8}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : p(n_5), *} \quad I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : F_8, \Delta_9, \hat{\neg}(n_5)}{\bullet h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : dual(p(n_5)), \Delta_9, F_7 \oplus F_8} \quad \oplus_B \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : *, \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_9, F_8, \hat{\neg}(n_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_9, F_8} ?_C \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_9, F_7 \oplus F_8 \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : p(n_5), *} \quad I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : F_7, \Delta_9, \hat{\neg}(n_5)}{\bullet h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : dual(p(n_5)), \Delta_9, F_7 \oplus F_8} \quad \oplus_A \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : *, \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_9, F_7, \hat{\neg}(n_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_9, F_7} ?_C \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_9, F_7 \oplus F_8 \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : p(n_5), *} \quad I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_7, \hat{\neg}(n_5)}{\bullet h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : dual(p(n_5)), \perp, \Delta_7} \quad \perp \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : *, \perp, \Delta_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_7, \hat{\neg}(n_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_7} ?_C \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_7, \perp \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : p(n_5), *} \quad I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : dual(p(n_5)), \top, \Delta_7}{\bullet h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : *, \top, \Delta_7} \quad \top \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : *, \top, \Delta_7 \quad \text{Cut} \\
\hline
\rightarrow \\
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_7, \top \quad \top
\end{array}$$

- Case rule I_3

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\neg}(n_6)\} : p(n_6), *} \quad I_2 \quad \frac{}{\bullet h_5 \vdash \{\Delta_4, \hat{\neg}(n_6)\} : dual(p(n_6)), p(n_6)} \quad I_3 \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_6)\} : *, p(n_6) \quad \text{Cut} \\
\hline
\rightarrow \\
- \vdash \{\Delta_4, \hat{\neg}(n_6)\} : p(n_6) \quad I_2
\end{array}$$

- Case rule \otimes

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : p(n_5), *} \quad I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : F_8, \Delta_{10}, \hat{\neg}(n_5) \quad h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : F_9, \Delta_7}{\bullet h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : dual(p(n_5)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \quad \otimes \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_{10}, F_8, \hat{\neg}(n_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_{10}, F_8} ?_C \quad \frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_7, F_9} \text{ax} \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9 \quad \otimes \\
\\
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : p(n_5), *} \quad I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : F_8, \Delta_7 \quad h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : F_9, \Delta_{10}, \hat{\neg}(n_5)}{\bullet h_6 \vdash \{\Delta_4, \hat{\neg}(n_5)\} : dual(p(n_5)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \quad \otimes \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_7, F_8} \text{ax} \quad \frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_{10}, F_9, \hat{\neg}(n_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_7, F_8} ?_C \quad \frac{}{- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_{10}, F_9} ?_C \\
\hline
- \vdash \{\Delta_4, \hat{\neg}(n_5)\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9 \quad \otimes
\end{array}$$

- Case rule I_1

$$\frac{\frac{\frac{\bullet h_1 \vdash \{(\Delta_6, p(n_5)), \wedge(n_5)\} : p(n_5), *}{- \vdash \{(\Delta_6, p(n_5)), \wedge(n_5)\} : *, *} \quad I_2 \quad \frac{\bullet h_4 \vdash \{(\Delta_6, p(n_5)), \wedge(n_5)\} : dual(p(n_5)), *}{- \vdash \{(\Delta_6, p(n_5)), \wedge(n_5)\} : *, *} \quad I_1}{\rightarrow} \quad \text{Cut}$$

$$\frac{\frac{- \vdash \{(\Delta_6, p(n_5)), \wedge(n_5)\} : p(n_5)}{- \vdash \{(\Delta_6, p(n_5)), \wedge(n_5)\} : *} \quad I_2}{?_C}$$

- Case rule I_2

- Case rule $?_C$

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_7, \wedge(n_4)\} : p(n_4), *}{- \vdash \{\Delta_7, \wedge(n_4)\} : *, \Delta_5} \quad I_2 \quad \frac{\frac{h_6 \vdash \{\Delta_7, \wedge(n_4)\} : \Delta_5, \wedge(n_4), \wedge(n_4)}{\bullet h_6 \vdash \{\Delta_7, \wedge(n_4)\} : dual(p(n_4)), \Delta_5} \quad ?_C}{\rightarrow} \quad \text{Cut}$$

$$\frac{\frac{- \vdash \{\Delta_7, \wedge(n_4)\} : \Delta_5, \wedge(n_4), \wedge(n_4)}{- \vdash \{\Delta_7, \wedge(n_4)\} : \Delta_5, \wedge(n_4)} \quad \text{ax}}{?_C}$$

$$\frac{\frac{- \vdash \{\Delta_7, \wedge(n_4)\} : \Delta_5, \wedge(n_4)}{- \vdash \{\Delta_7, \wedge(n_4)\} : \Delta_5} \quad ?_C}{- \vdash \{\Delta_7, \wedge(n_4)\} : \Delta_5}$$

$$\frac{\frac{\bullet h_1 \vdash \{(F_7, \Delta_8), \wedge(n_4)\} : p(n_4), *}{- \vdash \{(F_7, \Delta_8), \wedge(n_4)\} : *, \Delta_5} \quad I_2 \quad \frac{\frac{h_6 \vdash \{F_7, \Delta_8, \wedge(n_4)\} : F_7, \Delta_5, \wedge(n_4)}{\bullet h_6 \vdash \{(F_7, \Delta_8), \wedge(n_4)\} : dual(p(n_4)), \Delta_5} \quad ?_C}{\rightarrow} \quad \text{Cut}$$

$$\frac{\frac{- \vdash \{\Delta_8, F_7, \wedge(n_4)\} : \Delta_5, F_7, \wedge(n_4)}{- \vdash \{\Delta_8, F_7, \wedge(n_4)\} : \Delta_5, F_7} \quad \text{ax}}{?_C}$$

$$\frac{- \vdash \{\Delta_8, F_7, \wedge(n_4)\} : \Delta_5, F_7}{- \vdash \{\Delta_8, F_7, \wedge(n_4)\} : \Delta_5} \quad ?_C$$

7.14 Status of $?_C$: OK

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \quad ?_C \quad \frac{h_9 \vdash \{F_6, F_{10}, \Delta_7\} : \Delta_{11}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{11}, ?F_{10}} \quad ?}{\rightarrow} \quad \text{Cut}$$

$$\frac{\frac{h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_6, F_8}{- \vdash \{\Delta_7, F_6\} : \Delta_{11}, \Delta_2, F_6, ?F_{10}} \quad \text{ax}}{?_C} \quad \frac{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{11}, ?F_{10}, dual(F_8)}{\text{hCut}}$$

- Case rule \$

$$\frac{\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \quad ?_C \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{10}, F_{11}, \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{12}, F_{10} \$ F_{11}} \quad \$}{\rightarrow} \quad \text{Cut}$$

$$\frac{\frac{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_8}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10}, F_{11}} \quad \text{ax}}{?_C} \quad \frac{h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, F_{10}, F_{11}, dual(F_8)}{\text{hCut}}$$

$$\frac{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10}, F_{11}}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \$ F_{11}} \quad \$$$

- Case rule $\&$

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} ?_C \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{10}, \Delta_{12}, dual(F_8) \quad h_9 \vdash \{F_6, \Delta_7\} : F_{11}, \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{12}, F_{10} \& F_{11}} \& \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{12}, F_{10} \& F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_6, F_8}{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_{12}, dual(F_8), F_{10} \& F_{11}} \text{ax} \quad \frac{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, dual(F_8), F_{10} \& F_{11}}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_6, F_{10} \& F_{11}} \text{ax} \\
\hline
- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \& F_{11} \quad ?_C \\
\hline
- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \& F_{11}
\end{array}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} ?_C \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{11}, \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_B \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{12}, F_{10} \oplus F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_8}{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{11}} \text{ax} \quad \frac{h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, F_{11}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{11}} \text{ax} \\
\hline
- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{11} \quad \oplus_B \\
\hline
- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \oplus F_{11}
\end{array}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} ?_C \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{10}, \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_A \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{12}, F_{10} \oplus F_{11} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_8}{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10}} \text{ax} \quad \frac{h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, F_{10}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10}} \text{ax} \\
\hline
- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \quad \oplus_A \\
\hline
- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \oplus F_{11}
\end{array}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} ?_C \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : \Delta_{10}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \perp, \Delta_{10}} \perp \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \perp, \Delta_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_8}{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \perp} \text{ax} \quad \frac{h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \perp} \text{ax} \\
\hline
- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \perp \quad \perp \\
\hline
- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \perp
\end{array}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} ?_C \quad \frac{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \top, \Delta_{10}}{- \vdash \{F_6, \Delta_7\} : \Delta_2, \top, \Delta_{10}} \top \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \top, \Delta_{10} \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \top}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \top} \top \\
\hline
- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \top
\end{array}
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \quad ?_C \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{11}, \Delta_{13}, dual(F_8) \quad h_9 \vdash \{F_6, \Delta_7\} : F_{12}, \Delta_{10}}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \quad \otimes}{- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_6, F_8}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_6, F_{11} \otimes F_{12}} \quad \text{ax} \quad \frac{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, dual(F_8), F_{11} \otimes F_{12}}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}} \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}} \quad \text{hCut} \\
?_C \\
- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \quad ?_C \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{11}, \Delta_{10} \quad h_9 \vdash \{F_6, \Delta_7\} : F_{12}, \Delta_{13}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \quad \otimes}{- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_6, F_8}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_6, F_{11} \otimes F_{12}} \quad \text{ax} \quad \frac{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, dual(F_8), F_{11} \otimes F_{12}}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}} \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}} \quad \text{hCut} \\
?_C \\
- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}
\end{array}$$

• Case rule I_1

• Case rule I_2

• Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{h_3 \vdash \{F_9, \Delta_{10}\} : F_7, F_9, \Delta_2}{\bullet h_3 \vdash \{F_9, \Delta_{10}\} : F_7, \Delta_2} \quad ?_C \quad \frac{h_8 \vdash \{F_9, \Delta_{10}\} : F_9, \Delta_6, dual(F_7)}{\bullet h_8 \vdash \{F_9, \Delta_{10}\} : dual(F_7), \Delta_6} \quad ?_C}{- \vdash \{F_9, \Delta_{10}\} : \Delta_2, \Delta_6} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 \vdash \{\Delta_{10}, F_9\} : \Delta_2, F_7, F_9}{- \vdash \{\Delta_{10}, F_9\} : \Delta_2, \Delta_6, F_9} \quad \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_{10}, F_9\} : \Delta_6, dual(F_7)}{- \vdash \{\Delta_{10}, F_9\} : \Delta_2, \Delta_6} \quad \text{ax}}{- \vdash \{\Delta_{10}, F_9\} : \Delta_2, \Delta_6} \quad \text{hCut} \\
?_C \\
- \vdash \{\Delta_{10}, F_9\} : \Delta_2, \Delta_6
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_3 \vdash \{F_6, F_{10}, \Delta_{11}\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, F_{10}, \Delta_{11}\} : F_8, \Delta_2} \quad ?_C \quad \frac{h_9 \vdash \{F_6, F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, dual(F_8)}{\bullet h_9 \vdash \{F_6, F_{10}, \Delta_{11}\} : dual(F_8), \Delta_7} \quad ?_C}{- \vdash \{F_6, F_{10}, \Delta_{11}\} : \Delta_2, \Delta_7} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_3 \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_2, F_8}{- \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_2, \Delta_7, F_{10}} \quad \text{ax} \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_7, F_{10}, dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_2, \Delta_7} \quad \text{ax}}{- \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_2, \Delta_7} \quad \text{hCut} \\
?_C \\
- \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_2, \Delta_7
\end{array}$$

8 Cut-Elimination

8.1 Status of 1: OK

- Case rule 1
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.2 Status of !: OK

- Case rule 1

$$\frac{\frac{\frac{\mathbf{h}_1 \vdash \{\Delta_4\} : \mathbf{F}_5}{\bullet \mathbf{h}_1 \vdash \{\Delta_4\} : !\mathbf{F}_5} \quad ! \quad \frac{\bullet \mathbf{h}_6 \vdash \{dual(\mathbf{F}_5), \Delta_4\} : \mathbf{1}}{\mathbf{1}}}{\frac{- \vdash \{\Delta_4\} : \mathbf{1}}{\rightarrow} \quad \text{Cut}} \quad \mathbf{1}$$

$$\frac{}{- \vdash \{\Delta_4\} : \mathbf{1}} \quad \mathbf{1}$$

- Case rule !

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_4\} : F_5}{\bullet h_1 : \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 : \vdash \{\Delta_4, dual(F_5)\} : F_7}{\bullet h_6 : \vdash \{dual(F_5), \Delta_4\} : !F_7} !}{- : \vdash \{\Delta_4\} : !F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_4\} : !F_5}{- : \vdash \{\Delta_4\} : F_7} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_4, dual(F_5)\} : F_7}{- : \vdash \{\Delta_4\} : !F_7} \text{ax}}{- : \vdash \{\Delta_4\} : !F_7} ! \text{hCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_4\} : F_5}{\bullet h_1 : \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 : \vdash \{F_8, \Delta_4, dual(F_5)\} : \Delta_7}{\bullet h_6 : \vdash \{dual(F_5), \Delta_4\} : \Delta_7, ?F_8} ?}{- : \vdash \{\Delta_4\} : \Delta_7, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_4\} : !F_5}{\bullet h_1 : \vdash \{\Delta_4, F_8\} : !F_5} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_4, F_8, dual(F_5)\} : \Delta_7}{- : \vdash \{\Delta_4, F_8\} : \Delta_7} \text{ax}}{- : \vdash \{\Delta_4\} : \Delta_7, ?F_8} ? \text{hCut}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_4\} : F_5}{\bullet h_1 : \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 : \vdash \{\Delta_4, dual(F_5)\} : F_8, F_9, \Delta_7}{\bullet h_6 : \vdash \{dual(F_5), \Delta_4\} : \Delta_7, F_8 \$F_9} \$}{- : \vdash \{\Delta_4\} : \Delta_7, F_8 \$F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_4\} : !F_5}{- : \vdash \{\Delta_4\} : \Delta_7, F_8, F_9} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_8, F_9}{- : \vdash \{\Delta_4\} : \Delta_7, F_8 \$F_9} \text{ax}}{- : \vdash \{\Delta_4\} : \Delta_7, F_8 \$F_9} \$ \text{hCut}
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_4\} : F_5}{\bullet h_1 : \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 : \vdash \{\Delta_4, dual(F_5)\} : F_8, \Delta_7 \quad h_6 : \vdash \{\Delta_4, dual(F_5)\} : F_9, \Delta_7}{\bullet h_6 : \vdash \{dual(F_5), \Delta_4\} : \Delta_7, F_8 \&F_9} \&}{- : \vdash \{\Delta_4\} : \Delta_7, F_8 \&F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_4\} : !F_5}{- : \vdash \{\Delta_4\} : \Delta_7, F_8} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_8}{- : \vdash \{\Delta_4\} : \Delta_7, F_8} \text{ax}}{- : \vdash \{\Delta_4\} : \Delta_7, F_8} \text{hCut} \quad \frac{\frac{\bullet h_1 : \vdash \{\Delta_4\} : !F_5}{- : \vdash \{\Delta_4\} : \Delta_7, F_9} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_9}{- : \vdash \{\Delta_4\} : \Delta_7, F_9} \text{ax}}{- : \vdash \{\Delta_4\} : \Delta_7, F_8 \&F_9} \& \text{hCut}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_4\} : F_5}{\bullet h_1 : \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 : \vdash \{\Delta_4, dual(F_5)\} : F_9, \Delta_7}{\bullet h_6 : \vdash \{dual(F_5), \Delta_4\} : \Delta_7, F_8 \oplus F_9} \oplus_B}{- : \vdash \{\Delta_4\} : \Delta_7, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_4\} : !F_5}{- : \vdash \{\Delta_4\} : \Delta_7, F_9} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_9}{- : \vdash \{\Delta_4\} : \Delta_7, F_8 \oplus F_9} \text{ax}}{- : \vdash \{\Delta_4\} : \Delta_7, F_8 \oplus F_9} \oplus_B \text{hCut}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : F_8, \Delta_7}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \Delta_7, F_8 \oplus F_9} \oplus A \\
\hline
- \vdash \{\Delta_4\} : \Delta_7, F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_4\} : !F_5 \quad \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_8}{- \vdash \{\Delta_4\} : \Delta_7, F_8} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_8 \oplus F_9} \text{hCut} \oplus A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \perp, \Delta_7} \perp \\
\hline
- \vdash \{\Delta_4\} : \perp, \Delta_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_4\} : !F_5 \quad \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7}{- \vdash \{\Delta_4\} : \Delta_7} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, \perp} \text{hCut} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \top, \Delta_7} \top \\
\hline
- \vdash \{\Delta_4\} : \top, \Delta_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4\} : \Delta_7, \top} \top
\end{array}$$

- Case rule I_3

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : p(n_7), \wedge(n_7)} I_3 \\
\hline
- \vdash \{\Delta_4\} : p(n_7), \wedge(n_7) \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_4\} : p(n_7), \wedge(n_7)} I_3
\end{array}$$

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : F_9, \Delta_7 \quad h_6 \vdash \{\Delta_4, dual(F_5)\} : F_{10}, \Delta_8}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \Delta_7, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash \{\Delta_4\} : \Delta_7, \Delta_8, F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_4\} : !F_5 \quad \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_9}{- \vdash \{\Delta_4\} : \Delta_7, F_9} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, \Delta_8, F_9 \otimes F_{10}} \text{hCut} \otimes
\end{array}$$

- Case rule I_1

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_7, p(n_6)\} : F_4}{\bullet h_1 \vdash \{\Delta_7, p(n_6)\} : !F_4} ! \quad \frac{}{\bullet h_5 \vdash \{dual(F_4), \Delta_7, p(n_6)\} : \wedge(n_6)} I_1 \\
\hline
- \vdash \{\Delta_7, p(n_6)\} : \wedge(n_6) \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_7, p(n_6)\} : \wedge(n_6)} I_1
\end{array}$$

- Case rule I_2

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_7, \wedge(n_6)\} : F_4}{\bullet h_1 \vdash \{\Delta_7, \wedge(n_6)\} : !F_4} ! \quad \frac{}{\bullet h_5 \vdash \{dual(F_4), \Delta_7, \wedge(n_6)\} : p(n_6)} I_2 \\
\hline
- \vdash \{\Delta_7, \wedge(n_6)\} : p(n_6) \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- \vdash \{\Delta_7, \wedge(n_6)\} : p(n_6)} I_2
\end{array}$$

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} ! \quad \frac{h_5 : \vdash \{\Delta_6, dual(F_4)\} : \Delta_7, dual(F_4)}{\bullet h_5 : \vdash \{dual(F_4), \Delta_6\} : \Delta_7} ?_C}{- : \vdash \{\Delta_6\} : \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \vdash \{\Delta_6\} : F_4}{- : \vdash \{\Delta_6\} : \Delta_7} \text{ax} \quad \frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{- : \vdash \{\Delta_6\} : \Delta_7, dual(F_4)} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6, dual(F_4)\} : \Delta_7, dual(F_4)}{- : \vdash \{\Delta_6\} : \Delta_7, dual(F_4)} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_7} \text{mCut} \\
\frac{\frac{h_1 : \vdash \{F_6, \Delta_8\} : F_4}{\bullet h_1 : \vdash \{F_6, \Delta_8\} : !F_4} ! \quad \frac{h_5 : \vdash \{F_6, \Delta_8, dual(F_4)\} : F_6, \Delta_7}{\bullet h_5 : \vdash \{dual(F_4), F_6, \Delta_8\} : \Delta_7} ?_C}{- : \vdash \{F_6, \Delta_8\} : \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_8, F_6\} : !F_4}{- : \vdash \{\Delta_8, F_6\} : \Delta_7, F_6} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_8, F_6, dual(F_4)\} : \Delta_7, F_6}{- : \vdash \{\Delta_8, F_6\} : \Delta_7, F_6} \text{ax}}{- : \vdash \{\Delta_8, F_6\} : \Delta_7} \text{hCut} \\
\rightarrow \\
- : \vdash \{\Delta_8, F_6\} : \Delta_7 \quad ?_C
\end{array}$$

8.3 Status of $?_C$: OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2

- Case rule $?_C$

8.4 Status of $\$$: OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.5 Status of $\&$: OK

- Case rule **1**
- Case rule **!**
- Case rule **?**

- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.6 Status of \oplus_B : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp

- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.7 Status of \oplus_A : OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.8 Status of \perp : OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.9 Status of \top : OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**

- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.10 Status of I_3 : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top

- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.11 Status of \otimes : OK

- Case rule 1
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.12 Status of I_1 : OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.13 Status of I_2 : OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**

- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.14 Status of $?_C$: OK

- Case rule **1**

$$\frac{\frac{h_2 : \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 : \vdash \{F_5, \Delta_6\} : !F_7} \quad ?_C \quad \frac{}{\bullet h_8 : \vdash \{dual(F_7), F_5, \Delta_6\} : \mathbf{1}} \quad \mathbf{1}}{\frac{}{- : \vdash \{F_5, \Delta_6\} : \mathbf{1}} \quad \rightarrow} \quad \frac{}{- : \vdash \{\Delta_6, F_5\} : \mathbf{1}} \quad \mathbf{1} \quad \text{Cut}$$

- Case rule **!**

$$\frac{\frac{h_2 : \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 : \vdash \{F_5, \Delta_6\} : !F_7} \quad ?_C \quad \frac{h_8 : \vdash \{F_5, \Delta_6, dual(F_7)\} : F_9}{\bullet h_8 : \vdash \{dual(F_7), F_5, \Delta_6\} : !F_9} \quad !}{\frac{}{- : \vdash \{F_5, \Delta_6\} : !F_9} \quad \rightarrow} \quad \frac{}{- : \vdash \{\Delta_6, F_5\} : F_9} \quad \text{ax} \quad \frac{}{- : \vdash \{\Delta_6, F_5, dual(F_7)\} : F_9} \quad \text{ax}}{\frac{}{- : \vdash \{\Delta_6, F_5\} : F_9} \quad !} \quad \text{hCut}$$

- Case rule **?**

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, F_{10}, \Delta_6, dual(F_7)\} : \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, ?F_{10}} ? \\
\hline
- \vdash \{F_5, \Delta_6\} : \Delta_9, ?F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_{10}, F_5\} : !F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_{10}, F_5, dual(F_7)\} : \Delta_9}{h_8 \vdash \{\Delta_6, F_{10}, F_5\} : \Delta_9} \text{ax} \\
\hline
- \vdash \{\Delta_6, F_{10}, F_5\} : \Delta_9 \quad \text{hCut} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, ?F_{10} \quad ?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{10}, F_{11}, \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, F_{10} \$ F_{11}} \$ \\
\hline
- \vdash \{F_5, \Delta_6\} : \Delta_9, F_{10} \$ F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{10}, F_{11}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}, F_{11}} \text{ax} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}, F_{11} \quad \text{hCut} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \$ F_{11} \quad \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{10}, \Delta_9 \quad h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{11}, \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, F_{10} \& F_{11}} \& \\
\hline
- \vdash \{F_5, \Delta_6\} : \Delta_9, F_{10} \& F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{10}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}} \text{ax} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \quad \text{hCut} \\
\hline
\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{11}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11} \quad \text{hCut} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \& F_{11} \quad \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{11}, \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, F_{10} \oplus F_{11}} \oplus_B \\
\hline
- \vdash \{F_5, \Delta_6\} : \Delta_9, F_{10} \oplus F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{11}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11} \quad \text{hCut} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \oplus F_{11} \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{10}, \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, F_{10} \oplus F_{11}} \oplus_A \\
\hline
- \vdash \{F_5, \Delta_6\} : \Delta_9, F_{10} \oplus F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{10}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}} \text{ax} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \quad \text{hCut} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \oplus F_{11} \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \perp, \Delta_9} \perp \\
\hline
- \vdash \{F_5, \Delta_6\} : \perp, \Delta_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9} \text{ax} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9 \quad \text{hCut} \\
\hline
- \vdash \{\Delta_6, F_5\} : \Delta_9, \perp \quad \perp
\end{array}$$

- Case rule \top

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \top, \Delta_9} \top}{\frac{}{- \vdash \{F_5, \Delta_6\} : \top, \Delta_9} \text{Cut}} \xrightarrow{\quad} \frac{}{- \vdash \{\Delta_6, F_5\} : \Delta_9, \top} \top$$

- Case rule I_3

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : p(n_9), \wedge(n_9)} I_3}{\frac{}{- \vdash \{F_5, \Delta_6\} : p(n_9), \wedge(n_9)} \text{Cut}} \xrightarrow{\quad} \frac{}{- \vdash \{\Delta_6, F_5\} : p(n_9), \wedge(n_9)} I_3$$

- Case rule \otimes

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{11}, \Delta_9 \quad h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, \Delta_{10}, F_{11} \otimes F_{12}} \text{Cut}}{\frac{}{- \vdash \{F_5, \Delta_6\} : \Delta_9, \Delta_{10}, F_{11} \otimes F_{12}} \text{Cut}} \xrightarrow{\quad} \frac{\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{11}}{hCut} \quad \frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{- \vdash \{\Delta_6, F_5\} : \Delta_{10}, F_{12}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_{10}, F_{12}}{hCut}}{\frac{}{- \vdash \{\Delta_6, F_5\} : \Delta_{10}, \Delta_9, F_{11} \otimes F_{12}} \otimes} \otimes$$

- Case rule I_1

$$\frac{\frac{h_2 \vdash \{p(n_8), \Delta_5\} : p(n_8), !F_6}{\bullet h_2 \vdash \{p(n_8), \Delta_5\} : !F_6} ?_C \quad \frac{}{\bullet h_7 \vdash \{dual(F_6), p(n_8), \Delta_5\} : \wedge(n_8)} I_1}{\frac{}{- \vdash \{p(n_8), \Delta_5\} : \wedge(n_8)} \text{Cut}} \xrightarrow{\quad} \frac{}{- \vdash \{\Delta_5, p(n_8)\} : \wedge(n_8)} I_1$$

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_9, p(n_8)\} : F_5, !F_6}{\bullet h_2 \vdash \{F_5, \Delta_9, p(n_8)\} : !F_6} ?_C \quad \frac{}{\bullet h_7 \vdash \{dual(F_6), F_5, \Delta_9, p(n_8)\} : \wedge(n_8)} I_1}{\frac{}{- \vdash \{F_5, \Delta_9, p(n_8)\} : \wedge(n_8)} \text{Cut}} \xrightarrow{\quad} \frac{}{- \vdash \{\Delta_9, F_5, p(n_8)\} : \wedge(n_8)} I_1$$

- Case rule I_2

$$\frac{\frac{h_2 \vdash \{\wedge(n_8), \Delta_5\} : \wedge(n_8), !F_6}{\bullet h_2 \vdash \{\wedge(n_8), \Delta_5\} : !F_6} ?_C \quad \frac{}{\bullet h_7 \vdash \{dual(F_6), \wedge(n_8), \Delta_5\} : p(n_8)} I_2}{\frac{}{- \vdash \{\wedge(n_8), \Delta_5\} : p(n_8)} \text{Cut}} \xrightarrow{\quad} \frac{}{- \vdash \{\Delta_5, \wedge(n_8)\} : p(n_8)} I_2$$

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_9, \wedge(n_8)\} : F_5, !F_6}{\bullet h_2 \vdash \{F_5, \Delta_9, \wedge(n_8)\} : !F_6} ?_C \quad \frac{}{\bullet h_7 \vdash \{dual(F_6), F_5, \Delta_9, \wedge(n_8)\} : p(n_8)} I_2}{\frac{}{- \vdash \{F_5, \Delta_9, \wedge(n_8)\} : p(n_8)} \text{Cut}} \xrightarrow{\quad} \frac{}{- \vdash \{\Delta_9, F_5, \wedge(n_8)\} : p(n_8)} I_2$$

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 : \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 : \vdash \{F_5, \Delta_6, dual(F_7)\} : \Delta_9, dual(F_7)}{\bullet h_8 : \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9} ?_C}{- : \vdash \{F_5, \Delta_6\} : \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \vdash \{\Delta_6, F_5\} : !F_7}{- : \vdash \{\Delta_6, F_5\} : F_7} \text{ax} \quad \frac{\frac{\bullet h_2 : \vdash \{\Delta_6, F_5\} : !F_7}{- : \vdash \{\Delta_6, F_5\} : \Delta_9, dual(F_7)} \text{ax} \quad \frac{h_8 : \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, dual(F_7)}{- : \vdash \{\Delta_6, F_5\} : \Delta_9, dual(F_7)} \text{ax}}{- : \vdash \{\Delta_6, F_5\} : \Delta_9} \text{mCut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \{F_8, \Delta_5\} : F_8, !F_6}{\bullet h_2 : \vdash \{F_8, \Delta_5\} : !F_6} ?_C \quad \frac{h_7 : \vdash \{F_8, \Delta_5, dual(F_6)\} : F_8, \Delta_9}{\bullet h_7 : \vdash \{dual(F_6), F_8, \Delta_5\} : \Delta_9} ?_C}{- : \vdash \{F_8, \Delta_5\} : \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \{\Delta_5, F_8\} : !F_6}{- : \vdash \{\Delta_5, F_8\} : \Delta_9, F_8} \text{ax} \quad \frac{h_7 : \vdash \{\Delta_5, F_8, dual(F_6)\} : \Delta_9, F_8}{- : \vdash \{\Delta_5, F_8\} : \Delta_9, F_8} \text{ax}}{- : \vdash \{\Delta_5, F_8\} : \Delta_9} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \{F_5, F_8, \Delta_{10}\} : F_5, !F_6}{\bullet h_2 : \vdash \{F_5, F_8, \Delta_{10}\} : !F_6} ?_C \quad \frac{h_7 : \vdash \{F_5, F_8, \Delta_{10}, dual(F_6)\} : F_8, \Delta_9}{\bullet h_7 : \vdash \{dual(F_6), F_5, F_8, \Delta_{10}\} : \Delta_9} ?_C}{- : \vdash \{F_5, F_8, \Delta_{10}\} : \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \{\Delta_{10}, F_5, F_8\} : !F_6}{- : \vdash \{\Delta_{10}, F_5, F_8\} : \Delta_9, F_8} \text{ax} \quad \frac{h_7 : \vdash \{\Delta_{10}, F_5, F_8, dual(F_6)\} : \Delta_9, F_8}{- : \vdash \{\Delta_{10}, F_5, F_8\} : \Delta_9, F_8} \text{ax}}{- : \vdash \{\Delta_{10}, F_5, F_8\} : \Delta_9} \text{hCut} \\
\rightarrow \\
\frac{- : \vdash \{\Delta_{10}, F_5, F_8\} : \Delta_9, F_8}{- : \vdash \{\Delta_{10}, F_5, F_8\} : \Delta_9} ?_C
\end{array}$$