# Modal Logic K+T+4+5

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#### Abstract

This system does not have cut-elimination.

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## 1 Height preserving admissibility of weakening on the left

• Case(s) rule  $\rightarrow_R$ 

$$\begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_1: \mathbf{F}_4, \Delta_2 \vdash \mathbf{F}_5, \Delta_3 \\ \bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_4 \rightarrow \mathbf{F}_5 \end{array}}_{\bullet \mathbf{h}_1: \Delta_2 \vdash \mathbf{h}_3, \mathbf{F}_4 \rightarrow \mathbf{F}_5} \xrightarrow{\mathbf{ax}} \underbrace{\begin{array}{c} \mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_3, \mathbf{F}_5 \\ \mathbf{h}_1: \Delta_2, \mathbf{F}_4, \mathbf{F}_W \vdash \Delta_3, \mathbf{F}_5 \end{array}}_{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_W \vdash \Delta_3, \mathbf{F}_4 \rightarrow \mathbf{F}_5} \xrightarrow{\mathbf{ax}} \mathbf{H}$$

• Case(s) rule  $\wedge_R$ 

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4, \Delta_3 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5, \Delta_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_4 \land \mathbf{F}_5} \quad \wedge_R \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_4}{\mathbf{h}_1: \Delta_2, \mathbf{F}_W \vdash \Delta_3, \mathbf{F}_4} \quad \mathbf{IH} \quad \frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_5}{\mathbf{h}_1: \Delta_2, \mathbf{F}_W \vdash \Delta_3, \mathbf{F}_5} \quad \mathbf{IH}}_{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_W \vdash \Delta_3, \mathbf{F}_4 \land \mathbf{F}_5} \quad \mathbf{IH}} \quad \wedge_R \quad \wedge_$$

• Case(s) rule  $\vee_R$ 

• Case(s) rule  $\perp_R$ 

• Case(s) rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3} \ ^{\top}R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{F}_W \vdash \top, \Delta_3} \ ^{\top}R$$

• Case(s) rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_2) \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \Box \Gamma_2, \Delta_3 \vdash \Delta_4, []\mathbf{F}_5} \quad K \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1: unbox(\Box \Gamma_2) \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_1: \Delta_3, \mathbf{F}_W, \Box \Gamma_2 \vdash \Delta_4, []\mathbf{F}_5} \quad K$$

• Case(s) rule A45

$$\frac{\mathbf{h}_1: \Box \mathbf{r}_2 \vdash \Box \mathbf{r}_4, \mathbf{f}_6}{\bullet \mathbf{h}_1: \Box \mathbf{r}_2, \Delta_3 \vdash \Box \mathbf{r}_4, \Delta_5, []\mathbf{f}_6} \quad \text{a45} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1: \Box \mathbf{r}_2 \vdash \mathbf{f}_6, \Box \mathbf{r}_4}}{\bullet \mathbf{h}_1: \Delta_3, \mathbf{f}_W, \Box \mathbf{r}_2 \vdash \Delta_5, \Box \mathbf{r}_4, []\mathbf{f}_6} \quad \text{a45}$$

• Case(s) rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3,\Delta_5 \quad \mathbf{h}_1:\mathbf{F}_4,\Delta_2 \vdash \Delta_5}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5} \rightarrow_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1:\Delta_2 \vdash \Delta_5,\mathbf{F}_3}{\mathbf{h}_1:\Delta_2,\mathbf{F}_W \vdash \Delta_5,\mathbf{F}_3} \quad \mathbf{IH}}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_W,\mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5} \quad \mathbf{IH} \\ \bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_W,\mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5} \rightarrow_L$$

• Case(s) rule  $\wedge_L$ 

$$\frac{\underset{\bullet}{\mathbf{h}_1: \mathsf{F}_3, \mathsf{F}_4, \Delta_2 \vdash \Delta_5}{\bullet}}{\underset{\bullet}{\mathbf{h}_1: \Delta_2, \mathsf{F}_3 \land \mathsf{F}_4 \vdash \Delta_5}}{\bullet} \wedge_L \qquad \leadsto \qquad \frac{\frac{\overset{\bullet}{\mathbf{h}_1: \Delta_2, \mathsf{F}_3, \mathsf{F}_4 \vdash \Delta_5}}{\bullet} \underset{\bullet}{\mathsf{h}_1: \Delta_2, \mathsf{F}_3, \mathsf{F}_4 \vdash \Delta_5}}{\bullet} \underset{\bullet}{\mathsf{IH}} \\ \frac{\mathsf{h}_1: \Delta_2, \mathsf{F}_3, \mathsf{F}_4 \vdash \Delta_5}{\bullet} \wedge_L$$

• Case(s) rule  $\vee_L$ 

$$\frac{\mathbf{h}_{1}: \mathbf{F}_{3}, \Delta_{2} \vdash \Delta_{5} \quad \mathbf{h}_{1}: \mathbf{F}_{4}, \Delta_{2} \vdash \Delta_{5}}{\bullet \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \lor \mathbf{F}_{4} \vdash \Delta_{5}} \quad \vee_{L} \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \vdash \Delta_{5}}{\mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{3} \vdash \Delta_{5}} \quad \mathbf{IH}}{\bullet \mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{4} \lor \mathbf{h}_{2}} \quad \frac{\mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{4} \vdash \Delta_{5}}{\mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{4} \lor \mathbf{h}_{2}} \quad \mathbf{IH}} \quad \frac{\mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{4} \vdash \Delta_{5}}{\mathbf{h}_{1}: \Delta_{2}, \mathbf{F}_{4} \lor \mathbf{h}_{2}} \quad \mathbf{IH}} \quad \vee_{L}$$

 $\bullet$  Case(s) rule AT

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2, (|\mathbf{F}_3| \vdash \Delta_4)}{\bullet \mathbf{h}_1: \Delta_2, (|\mathbf{F}_3| \vdash \Delta_4)} \quad AT \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3, (|\mathbf{F}_3| \vdash \Delta_4)}{\mathbf{h}_1: \Delta_2, \mathbf{F}_3, \mathbf{F}_W, (|\mathbf{F}_3| \vdash \Delta_4)}}_{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_W, (|\mathbf{F}_3| \vdash \Delta_4)} \quad _{AT}^{\mathrm{BH}}$$

• Case(s) rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_1:\bot,\Delta_2\vdash \Delta_3} \ ^\bot_L \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_1:\bot,\Delta_2,\mathbf{F}_W\vdash \Delta_3} \ ^\bot_L$$

 $\bullet$  Case(s) rule I

$$\frac{}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{p}_3 \vdash \Delta_4, \mathbf{p}_3} \quad I \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{f}_W, \mathbf{p}_3 \vdash \Delta_4, \mathbf{p}_3} \quad I$$

• Case(s) rule  $\top_L$ 

## 2 Height preserving admissibility of weakening on the right

• Case(s) rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_1: \mathbf{F}_4, \Delta_2 \vdash \mathbf{F}_5, \Delta_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_4 \to \mathbf{F}_5} \ \rightarrow_R \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_3, \mathbf{F}_5}{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_3, \mathbf{F}_5, \mathbf{F}_W}}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_W, \mathbf{F}_4 \to \mathbf{F}_5} \xrightarrow{\bullet} \mathcal{H}$$

• Case(s) rule  $\wedge_R$ 

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4, \Delta_3 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_5, \Delta_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_4 \land \mathbf{F}_5} \quad \wedge_R \qquad \leadsto \qquad \frac{\frac{1}{\mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_4} \quad \mathbf{m}}{\underbrace{\mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_4, \mathbf{F}_W}} \quad \mathbf{m} \quad \frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_5}{\mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_5, \mathbf{F}_W} \quad \mathbf{m} \quad \mathbf{m}$$

• Case(s) rule  $\vee_R$ 

$$\begin{array}{c} \frac{\mathsf{h}_1:\Delta_2 \vdash \mathsf{F}_4,\mathsf{F}_5,\Delta_3}{\bullet \mathsf{h}_1:\Delta_2 \vdash \Delta_3,\mathsf{F}_4 \lor \mathsf{F}_5} \ \vee_R \qquad \leadsto \qquad \frac{\frac{\overline{\mathsf{h}_1:\Delta_2 \vdash \Delta_3,\mathsf{F}_4,\mathsf{F}_5}}{\mathsf{h}_1:\Delta_2 \vdash \Delta_3,\mathsf{F}_4,\mathsf{F}_5,\mathsf{F}_W}}{\bullet \mathsf{h}_1:\Delta_2 \vdash \Delta_3,\mathsf{F}_W,\mathsf{F}_4 \lor \mathsf{F}_5} \overset{\mathsf{ax}}{\mathsf{IH}} \\ & \frac{\bullet \mathsf{h}_1:\Delta_2 \vdash \Delta_3,\mathsf{F}_W,\mathsf{F}_4 \lor \mathsf{F}_5}{\mathsf{h}_1:\Delta_2 \vdash \Delta_3,\mathsf{F}_W,\mathsf{F}_4 \lor \mathsf{F}_5} \end{array}$$

• Case(s) rule  $\perp_R$ 

• Case(s) rule  $\top_R$ 

• Case(s) rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_2) \vdash \mathbf{F}_5}{\bullet \mathbf{h}_1: \Box \Gamma_2, \Delta_3 \vdash \Delta_4, []\mathbf{F}_5} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_1: unbox(\Box \Gamma_2) \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_1: \Delta_3, \Box \Gamma_2 \vdash \Delta_4, \mathbf{F}_W, []\mathbf{F}_5} \quad K$$

• Case(s) rule A45

$$\frac{\mathbf{h}_1: \Box \Gamma_2 \vdash \Box \Gamma_4, \mathbf{f}_6}{\bullet \mathbf{h}_1: \Box \Gamma_2, \Delta_3 \vdash \Box \Gamma_4, \Delta_5, []\mathbf{f}_6} \quad {}^{A45} \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1: \Box \Gamma_2 \vdash \mathbf{f}_6, \Box \Gamma_4}}{\bullet \mathbf{h}_1: \Delta_3, \Box \Gamma_2 \vdash \Delta_5, \mathbf{f}_W, \Box \Gamma_4, []\mathbf{f}_6} \quad {}^{A45}$$

• Case(s) rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3,\Delta_5 \quad \mathbf{h}_1:\mathbf{F}_4,\Delta_2 \vdash \Delta_5}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3 \to \mathbf{F}_4 \vdash \Delta_5} \to_L \qquad \leadsto \qquad \frac{\frac{\overline{\mathbf{h}}_1:\Delta_2 \vdash \Delta_5,\mathbf{F}_3}{\bullet \mathbf{h}_1:\Delta_2 \vdash \Delta_5,\mathbf{F}_3,\mathbf{F}_W} \quad \mathbf{IH} \quad \frac{\overline{\mathbf{h}}_1:\Delta_2,\mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_4 \vdash \Delta_5,\mathbf{F}_W} \quad \mathbf{IH} \quad \frac{\overline{\mathbf{h}}_1:\Delta_2,\mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_4 \vdash \Delta_5,\mathbf{F}_W} \quad \mathbf{IH} \quad \mathbf{h}_1:\Delta_2,\mathbf{F}_4 \vdash \Delta_5,\mathbf{F}_W} \to_L$$

• Case(s) rule  $\wedge_L$ 

$$\frac{\mathbf{h}_1: \mathbf{f}_3, \mathbf{f}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{f}_3 \land \mathbf{f}_4 \vdash \Delta_5} \ \land_L \qquad \leadsto \qquad \frac{\frac{\overline{\mathbf{h}_1: \Delta_2, \mathbf{f}_3, \mathbf{f}_4 \vdash \Delta_5}}{\mathbf{h}_1: \Delta_2, \mathbf{f}_3, \mathbf{f}_4 \vdash \Delta_5, \mathbf{f}_W}} \overset{\mathrm{ax}}{\mathbf{IH}} \\ \frac{\mathbf{h}_1: \Delta_2, \mathbf{f}_3, \mathbf{f}_4 \vdash \Delta_5, \mathbf{f}_W}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{f}_3 \land \mathbf{f}_4 \vdash \Delta_5, \mathbf{f}_W}} \land_L$$

• Case(s) rule  $\vee_L$ 

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2 \vdash \Delta_5 \quad \mathbf{h}_1: \mathbf{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \lor \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \Delta_5}{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_W} \quad \mathbf{IH} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5}{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_W} \quad \mathbf{IH} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5}{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_W} \quad \mathbf{IH} \quad \vee_L \quad \mathbf{h}_1: \Delta_2, \mathbf{h}_2 \vdash \Delta_5 \mid \mathbf{h}_2 \mid \mathbf{h}_$$

 $\bullet$  Case(s) rule AT

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2, []\mathbf{F}_3 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_2, []\mathbf{F}_3 \vdash \Delta_4} \quad AT \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3, []\mathbf{F}_3 \vdash \Delta_4}{\mathbf{h}_1: \Delta_2, \mathbf{F}_3, []\mathbf{F}_3 \vdash \Delta_4, \mathbf{F}_W}}{\bullet \mathbf{h}_1: \Delta_2, []\mathbf{F}_3 \vdash \Delta_4, \mathbf{F}_W} \quad \frac{\mathbf{H}_1: \Delta_2, \mathbf{H}_2 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3 \vdash \Delta_4, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac{\mathbf{H}_3: \Delta_3, \mathbf{H}_3}{\mathbf{H}_3: \Delta_3, \mathbf{H}_3} \quad \frac$$

• Case(s) rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_1:\bot,\Delta_2\vdash \Delta_3} \ ^\bot_L \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_1:\bot,\Delta_2\vdash \Delta_3,\mathbf{f}_W} \ ^\bot_L$$

 $\bullet$  Case(s) rule I

• Case(s) rule  $\top_L$ 

#### 3 Measure of derivations

• Case(s) rule  $\rightarrow_R$ 

$$\frac{ \begin{smallmatrix} h_1: F_4, \, \Delta_2 \vdash F_5, \, \Delta_3 \\ \bullet h_1: \, \Delta_2 \vdash \Delta_3, F_4 \to F_5 \end{smallmatrix}}{\bullet h_1: \, \Delta_2 \vdash \Delta_3, F_4 \to F_5} \xrightarrow{lH} \underbrace{ \begin{smallmatrix} h_1: \, \Delta_2, F_4 \vdash \Delta_3, F_5 \\ \bullet h_1: \, \Delta_2 \vdash \Delta_3, F_4 \to F_5 \end{smallmatrix}}_{\bullet \bullet h_1: \, \Delta_2 \vdash \Delta_3, F_4 \to F_5} \xrightarrow{lH} \to_R$$

• Case(s) rule  $\wedge_R$ 

$$\frac{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_4,\Delta_3\quad \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_5,\Delta_3}{\bullet \mathbf{h}_1:\Delta_2\vdash \Delta_3, \mathbf{F}_4\wedge \mathbf{F}_5} \quad \wedge_R \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{F}_4}{\bullet \mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{F}_4} \quad \underset{\bullet}{\mathsf{In}} \quad \frac{\mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{F}_4\wedge \mathbf{F}_5} \quad \underset{\bullet}{\mathsf{In}} \quad \underset{\bullet}{\mathsf{In}} \quad \frac{\mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{F}_5}{\bullet \mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{F}_4\wedge \mathbf{F}_5} \quad \underset{\bullet}{\mathsf{In}} \quad \underset{\bullet$$

• Case(s) rule  $\vee_R$ 

• Case(s) rule  $\perp_R$ 

• Case(s) rule  $\top_R$ 

• Case(s) rule K

$$\underbrace{ \begin{array}{c} \mathbf{h}_1 : unbox(\Box \Gamma_2) \vdash \mathbf{F}_5 \\ \bullet \mathbf{h}_1 : \Box \Gamma_2, \Delta_3 \vdash \Delta_4, []\mathbf{F}_5 \end{array}}_{\bullet \mathbf{h}_1 : \mathbf{h}_2} K \qquad \leadsto \qquad \underbrace{ \begin{array}{c} \overline{\mathbf{h}_1 : unbox(\Box \Gamma_2) \vdash \mathbf{F}_5} \\ \bullet \mathbf{h}_1 : unbox(\Box \Gamma_2) \vdash \mathbf{F}_5 \end{array}}_{\bullet \bullet \mathbf{h}_1 : \Delta_3, \, \Box \Gamma_2 \vdash \Delta_4, \, []\mathbf{F}_5} \overset{\mathrm{ax}}{}_{\mathrm{H}}$$

• Case(s) rule A45

$$\frac{ \begin{smallmatrix} \mathbf{h}_1 : \Box \Gamma_2 \vdash \Box \Gamma_4, F_6 \\ \bullet \mathbf{h}_1 : \Box \Gamma_2, \Delta_3 \vdash \Box \Gamma_4, \Delta_5, [] F_6 \end{smallmatrix}}{\bullet \mathbf{h}_1 : \Box \Gamma_2, \Delta_3 \vdash \Box \Gamma_4, \Delta_5, [] F_6} \overset{A45}{} \xrightarrow{\bullet \bullet} \frac{ \begin{smallmatrix} \mathbf{h}_1 : \Box \Gamma_2 \vdash F_6, \Box \Gamma_4 \\ \bullet \mathbf{h}_1 : \Box \Gamma_2 \vdash F_6, \Box \Gamma_4 \end{smallmatrix}}{\bullet \bullet \mathbf{h}_1 : \Delta_3, \Box \Gamma_2 \vdash \Delta_5, \Box \Gamma_4, [] F_6} \overset{A45}{}$$

• Case(s) rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1: \mathbf{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5} \rightarrow_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3} \quad \frac{\mathbf{ax}}{\mathbf{IH}} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5} \quad \frac{\mathbf{ax}}{\mathbf{IH}} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5} \quad \frac{\mathbf{ax}}{\mathbf{IH}} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5} \quad \frac{\mathbf{ax}}{\mathbf{IH}} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5} \quad \frac{\mathbf{ax}}{\mathbf{IH}} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{ax}}{\bullet} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{ax}}{\bullet} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{ax}}{\bullet} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_4: \Delta_4, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_4: \Delta_4, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_4: \Delta_4, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_1: \Delta_4, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_4: \Delta_4, \mathbf{h}_4 \vdash \Delta_5} \quad \frac{\mathbf{h}_4: \Delta_4, \mathbf{h}_4 \vdash \Delta_5}{\bullet \mathbf{h}_4: \Delta_4, \mathbf{h}_$$

• Case(s) rule  $\wedge_L$ 

• Case(s) rule  $\vee_L$ 

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2 \vdash \Delta_5 \quad \mathbf{h}_1: \mathbf{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \lor \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \Delta_5} \quad \mathbf{IH}}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \Delta_5} \quad \mathbf{IH} \quad \frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5} \quad \mathbf{IH}}_{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \lor \mathbf{h}_4} \quad \vee_L$$

 $\bullet$  Case(s) rule AT

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2, []\mathbf{F}_3 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_2, []\mathbf{F}_3 \vdash \Delta_4} \quad AT \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3, []\mathbf{F}_3 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3, []\mathbf{F}_3 \vdash \Delta_4}}{\bullet \bullet \mathbf{h}_1: \Delta_2, []\mathbf{F}_3 \vdash \Delta_4} \quad \frac{\mathbf{ax}}{\mathbf{H}} \quad AT$$

• Case(s) rule  $\perp_L$ 

 $\bullet$  Case(s) rule I

• Case(s) rule  $\top_L$ 

$$\begin{array}{c} \mathbf{h}_1: \Delta_2 \vdash \Delta_3 \\ \hline \bullet \mathbf{h}_1: \top, \Delta_2 \vdash \Delta_3 \end{array} \ \top_L \qquad \leadsto \qquad \begin{array}{c} \overline{\mathbf{h}_1: \Delta_2 \vdash \Delta_3} \\ \hline \bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_3 \end{array} \overset{\mathrm{ax}}{} \\ \hline \bullet \bullet \mathbf{h}_1: \top, \Delta_2 \vdash \Delta_3 \end{array} \ \top_L$$

## 4 Invertibility of Rules

#### 4.1 Status of $\rightarrow_R$ : : Invertible

• Case rule  $\rightarrow_R$ 

$$\begin{array}{c} \frac{\mathbf{h}_3: \mathbf{F}_5, \Delta_4 \vdash \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \to \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \to \mathbf{F}_2), \mathbf{F}_5 \to \mathbf{F}_6} & \rightarrow_R & \sim & \frac{\overline{\mathbf{h}_3: \Delta_4, \mathbf{F}_1, \mathbf{F}_5 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_4, \mathbf{F}_1 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \to \mathbf{F}_6} & \rightarrow_R \\ \\ \frac{\mathbf{h}_1: \mathbf{F}_4, \Delta_2 \vdash \mathbf{F}_5, \Delta_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_4 \to \mathbf{F}_5} & \rightarrow_R & \sim & \frac{\overline{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_3, \mathbf{F}_5}}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_3, \mathbf{F}_5} & \mathbf{H} \end{array}$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5,\Delta_7,\mathbf{F}_1\rightarrow \mathbf{F}_2\quad \mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6,\Delta_7,\mathbf{F}_1\rightarrow \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4\vdash (\Delta_7,\mathbf{F}_1\rightarrow \mathbf{F}_2),\mathbf{F}_5\land \mathbf{F}_6} \quad \wedge_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5} \quad \text{ax/ind} \quad \overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5} \quad \frac{\mathsf{ax/ind}}{\land R} \quad \wedge_R \quad \wedge$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5,\mathbf{F}_6,\Delta_7,\mathbf{F}_1\rightarrow \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4\vdash (\Delta_7,\mathbf{F}_1\rightarrow \mathbf{F}_2),\mathbf{F}_5\vee \mathbf{F}_6} \ \vee_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5,\mathbf{F}_6}}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5\vee \mathbf{F}_6} \ \vee_R$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{F}_1\rightarrow\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{F}_1\rightarrow\mathbf{F}_2}\ \perp_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash\Delta_5,\mathbf{F}_2}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash\bot,\Delta_5,\mathbf{F}_2}\ ^{\perp}_R$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{f}_1 \rightarrow \mathbf{f}_2} \ \top_R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{f}_1 \vdash \top,\Delta_5,\mathbf{f}_2} \ \top_R$$

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2), []\mathbf{F}_6} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_5, \mathbf{F}_1, \Box \Gamma_4 \vdash \Delta_7, \mathbf{F}_2, []\mathbf{F}_6} \quad K \sim \mathbf{h}_3: \Delta_5 = \mathbf{h}_3 = \mathbf{h}_3$$

• Case rule A45

$$\frac{\mathtt{h}_3: \Box \Gamma_4 \vdash \Box \Gamma_6, \mathtt{F}_7}{\bullet \mathtt{h}_3: \Box \Gamma_4, \Delta_5 \vdash \Box \Gamma_6, (\Delta_8, \mathtt{F}_1 \to \mathtt{F}_2), []\mathtt{F}_7} \quad \text{A45} \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_3: \Box \Gamma_4 \vdash \mathtt{F}_7, \Box \Gamma_6}}{\bullet \mathtt{h}_3: \Delta_5, \mathtt{F}_1, \Box \Gamma_4 \vdash \Delta_8, \mathtt{F}_2, \Box \Gamma_6, []\mathtt{F}_7} \quad \text{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_4:\Delta_5\vdash \mathbf{F}_6,\Delta_1,\mathbf{F}_2\to \mathbf{F}_3\quad \mathbf{h}_4:\mathbf{F}_7,\Delta_5\vdash \Delta_1,\mathbf{F}_2\to \mathbf{F}_3}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{F}_7\vdash \Delta_1,\mathbf{F}_2\to \mathbf{F}_3} \quad \to_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_5,\mathbf{F}_2\vdash \Delta_1,\mathbf{F}_3,\mathbf{F}_6}\quad \overline{\mathbf{h}_4:\Delta_5,\mathbf{F}_2,\mathbf{F}_7\vdash \Delta_1,\mathbf{F}_3}\quad \overline{\mathbf{h}_4:\Delta_5,\mathbf{F}_2\vdash \Delta_1,\mathbf{F}_3}\quad \overline{\mathbf{h}_4:\Delta_5,\mathbf{F}$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_4:\mathbf{h}_6,\mathbf{h}_7,\Delta_5\vdash\Delta_1,\mathbf{h}_2\rightarrow\mathbf{h}_3}{\bullet\mathbf{h}_4:\Delta_5,\mathbf{h}_6\wedge\mathbf{h}_7\vdash\Delta_1,\mathbf{h}_2\rightarrow\mathbf{h}_3} \ \land_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_5,\mathbf{h}_2,\mathbf{h}_6,\mathbf{h}_7\vdash\Delta_1,\mathbf{h}_3}}{\bullet\mathbf{h}_4:\Delta_5,\mathbf{h}_2,\mathbf{h}_6\wedge\mathbf{h}_7\vdash\Delta_1,\mathbf{h}_3} \ \stackrel{\mathrm{ax/ind}}{\wedge}_L$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_4: \mathbf{F}_6, \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \quad \mathbf{h}_4: \mathbf{F}_7, \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \rightarrow \mathbf{F}_3}{\bullet \mathbf{h}_4: \Delta_5, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \Delta_1, \mathbf{F}_2 \rightarrow \mathbf{F}_3} \quad \vee_L \qquad \rightsquigarrow \qquad \frac{\mathbf{h}_4: \Delta_5, \mathbf{F}_2, \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_3}{\bullet \mathbf{h}_4: \Delta_5, \mathbf{F}_2, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \Delta_1, \mathbf{F}_3} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_4: \Delta_5, \mathbf{F}_2, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \Delta_1, \mathbf{F}_3} \quad \vee_L$$

 $\bullet$  Case rule AT

$$\begin{array}{c} \underline{\mathbf{h}_4: \mathbf{F}_6, \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2 \rightarrow \mathbf{F}_3}}{\bullet \mathbf{h}_4: \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2 \rightarrow \mathbf{F}_3} \end{array} \ \, AT \qquad \rightsquigarrow \qquad \begin{array}{c} \overline{\underline{\mathbf{h}_4: \Delta_5, \mathbf{F}_2, \mathbf{F}_6, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_3}}\\ \bullet \mathbf{h}_4: \Delta_5, \mathbf{F}_2, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_3 \end{array} \ \, \xrightarrow{\mathrm{ax/ind}} \end{array} \ \, AT$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_4:\bot,\Delta_5\vdash \Delta_1,\mathbf{F}_2\to\mathbf{F}_3} \ ^\perp L \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_4:\bot,\Delta_5,\mathbf{F}_2\vdash \Delta_1,\mathbf{F}_3} \ ^\perp L$$

 $\bullet$  Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_5, \Delta_4 \vdash \mathsf{p}_5, \Delta_6, \mathsf{F}_1 \to \mathsf{F}_2} \quad I \qquad \rightsquigarrow \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_4, \mathsf{F}_1, \mathsf{p}_5 \vdash \Delta_6, \mathsf{F}_2, \mathsf{p}_5} \quad I$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_4: \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \rightarrow \mathbf{F}_3}{\bullet \mathbf{h}_4: \top, \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \rightarrow \mathbf{F}_3} \ \top_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_4: \Delta_5, \mathbf{F}_2 \vdash \Delta_1, \mathbf{F}_3}}{\bullet \mathbf{h}_4: \top, \Delta_5, \mathbf{F}_2 \vdash \Delta_1, \mathbf{F}_3} \ ^{\mathrm{ax/ind}}$$

#### 4.2 Status of $\wedge_R$ : (Left Premise): Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_5, \Delta_4 \vdash \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \to \mathbf{F}_6} \ \rightarrow_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: \Delta_4, \mathbf{F}_5 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \to \mathbf{F}_6} \overset{\mathrm{ax/ind}}{\to}_R$$

• Case rule  $\wedge_R$ 

$$\begin{array}{c} \frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_5,\Delta_7,\mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4 \vdash (\Delta_7,\mathbf{F}_1 \land \mathbf{F}_2),\mathbf{F}_5 \land \mathbf{F}_6} & \wedge_R \\ \\ \bullet \mathbf{h}_3:\Delta_4 \vdash (\Delta_7,\mathbf{F}_1,\mathbf{F}_5) & \frac{\mathbf{h}_3:\Delta_4 \vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_5}{\bullet \mathbf{h}_3:\Delta_4 \vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_5 \land \mathbf{F}_6} & \wedge_R \\ \\ \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_4,\Delta_3 \quad \mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_5,\Delta_3}{\bullet \mathbf{h}_1:\Delta_2 \vdash \Delta_3,\mathbf{F}_4 \land \mathbf{F}_5} & \wedge_R \\ \end{array} \quad \stackrel{\mathbf{h}_1:\Delta_2 \vdash \Delta_3,\mathbf{F}_4}{\bullet \mathbf{h}_1:\Delta_2 \vdash \Delta_3,\mathbf{F}_4} \overset{\mathbf{ax}}{\mathsf{h}}_{\mathsf{h}} \\ \end{array}$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_5, \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \lor \mathbf{F}_6} \ \lor_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5, \mathbf{F}_6}}{\bullet \mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \lor \mathbf{F}_6} \ \lor_R$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{f}_1\land\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{f}_1\land\mathbf{f}_2}\ \bot_R \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{f}_1}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{f}_1}\ \frac{\mathbf{ax/ind}}{\bot_R}$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top, \Delta_5, \mathbf{F}_1 \wedge \mathbf{F}_2} \ \top_R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top, \Delta_5, \mathbf{F}_1} \ \top_R$$

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), []\mathbf{F}_6} \quad K \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_5, \Box \Gamma_4 \vdash \Delta_7, \mathbf{F}_1, []\mathbf{F}_6} \quad K$$

• Case rule A45

$$\frac{h_3: \Box\Gamma_4 \vdash \Box\Gamma_6, F_7}{\bullet h_3: \Box\Gamma_4, \Delta_5 \vdash \Box\Gamma_6, (\Delta_8, F_1 \land F_2), []F_7} \ \ \text{A45} \qquad \rightsquigarrow \qquad \frac{\overline{h_3: \Box\Gamma_4 \vdash F_7, \Box\Gamma_6}}{\bullet h_3: \Delta_5, \Box\Gamma_4 \vdash \Delta_8, F_1, \Box\Gamma_6, []F_7} \ \ \text{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_4:\Delta_5\vdash \mathbf{F}_6,\Delta_1,\mathbf{F}_2\land \mathbf{F}_3\quad \mathbf{h}_4:\mathbf{F}_7,\Delta_5\vdash \Delta_1,\mathbf{F}_2\land \mathbf{F}_3}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{F}_7\vdash \Delta_1,\mathbf{F}_2\land \mathbf{F}_3} \quad \rightarrow_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_4:\Delta_5\vdash \Delta_1,\mathbf{F}_2,\mathbf{F}_6}\quad \mathbf{ax/ind}}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{F}_7\vdash \Delta_1,\mathbf{F}_2} \quad \xrightarrow{\mathbf{ax/ind}} \quad \xrightarrow{\mathbf{h}_4:\Delta_5\vdash \Delta_1,\mathbf{F}_2,\mathbf{F}_6}\quad \mathbf{ax/ind}} \quad \xrightarrow{\mathbf{h}_4:\Delta_5\vdash \Delta_1,\mathbf{F}_2\land \mathbf{F}_3} \quad \xrightarrow{\mathbf{h}_4:\Delta_5\vdash \Delta_1,\mathbf{F}_3\vdash \Delta_1,\mathbf{F}_3} \quad \xrightarrow{\mathbf{h}_4:\Delta_5\vdash \Delta_1,\mathbf{F}_3\vdash \Delta_1,\mathbf$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_4:\mathbf{F}_6,\mathbf{F}_7,\Delta_5\vdash\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3}{\bullet\mathbf{h}_4:\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7\vdash\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3} \ \land L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_5,\mathbf{F}_6,\mathbf{F}_7\vdash\Delta_1,\mathbf{F}_2}}{\bullet\mathbf{h}_4:\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7\vdash\Delta_1,\mathbf{F}_2} \overset{\mathrm{ax/ind}}{\wedge} L$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_{4}: \mathbf{F}_{6}, \Delta_{5} \vdash \Delta_{1}, \mathbf{F}_{2} \wedge \mathbf{F}_{3} \quad \mathbf{h}_{4}: \mathbf{F}_{7}, \Delta_{5} \vdash \Delta_{1}, \mathbf{F}_{2} \wedge \mathbf{F}_{3}}{\bullet \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2} \wedge \mathbf{F}_{3}} \quad \vee_{L} \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \overline{\mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \overline{\mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \vee_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{h}_{7} \vdash \Delta_{1}, \mathbf{F}_{2}} \quad \nabla_{L} \wedge \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{h}_{7} \vdash \Delta_{1}, \mathbf{F}_{2} \vee \mathbf{h}_{7} \vdash \Delta_{1}, \mathbf{F}_{2} \vee \mathbf{h}_{7} \vee \mathbf{$$

ullet Case rule AT

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_4:\bot,\Delta_5\vdash \Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3} \ ^\bot L \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_4:\bot,\Delta_5\vdash \Delta_1,\mathbf{F}_2} \ ^\bot L$$

ullet Case rule I

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_4: \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \land \mathbf{F}_3}{\bullet \mathbf{h}_4: \top, \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \land \mathbf{F}_3} \ \top_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: \Delta_5 \vdash \Delta_1, \mathbf{F}_2}}{\bullet \mathbf{h}_4: \top, \Delta_5 \vdash \Delta_1, \mathbf{F}_2} \overset{\mathrm{ax/ind}}{\top_L}$$

#### 4.3 Status of $\wedge_R$ (Right Premise): : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_5, \Delta_4 \vdash \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \to \mathbf{F}_6} \xrightarrow{\rightarrow_R} \xrightarrow{\bullet} \frac{\mathbf{h}_3: \Delta_4, \mathbf{F}_5 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_6}{\bullet} \xrightarrow{\bullet} \frac{\mathbf{a} \times \mathbf{f} \cdot \mathbf{h}_3}{\bullet} \xrightarrow{\bullet} \frac{\mathbf{h}_3: \Delta_4 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \to \mathbf{F}_6}{\bullet}$$

• Case rule  $\wedge_R$ 

$$\begin{array}{c} \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5,\Delta_7,\mathbf{F}_1\land \mathbf{F}_2\quad \mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6,\Delta_7,\mathbf{F}_1\land \mathbf{F}_2}} \\ \bullet \mathbf{h}_3:\Delta_4\vdash (\Delta_7,\mathbf{F}_1\land \mathbf{F}_2),\mathbf{F}_5\land \mathbf{F}_6 \end{array} \quad \wedge_R \qquad \leadsto \qquad \begin{array}{c} \underline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5} & \mathrm{ax/ind} \\ \bullet \mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5\land \mathbf{F}_6 \end{array}} \\ \underline{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_4,\Delta_3\quad \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_5,\Delta_3} \\ \bullet \mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{F}_4\land \mathbf{F}_5} \end{array} \quad \wedge_R \qquad \leadsto \qquad \begin{array}{c} \underline{\mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{F}_5} \\ \bullet \mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{F}_5 \end{array} \quad \mathbf{h} \\ \mathbf{h}_1:\Delta_2\vdash \Delta_3,\mathbf{h}_5 \end{array} \quad \mathbf{h} \\ \mathbf{h}_1:\Delta_1\vdash \Delta_1,\mathbf{h}_1 \vdash \Delta_2\vdash \Delta_3,\mathbf{h}_2 \vdash \Delta_3,\mathbf{h}_5 \vdash \Delta_3 \vdash \Delta_3,\mathbf{h}_5 \vdash \Delta_3,\mathbf{h}$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_5, \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \lor \mathbf{F}_6} \quad \vee_R \qquad \leadsto \qquad \frac{\mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6}{\bullet \mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \lor \mathbf{F}_6} \quad \vee_R$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{f}_1\land\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{f}_1\land\mathbf{f}_2}\ \bot_R \qquad \leadsto \qquad \frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{f}_2} \overset{\mathrm{ax/ind}}{\bot_R}$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top, \Delta_5, \mathbf{F}_1 \wedge \mathbf{F}_2} \ \top_R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top, \Delta_5, \mathbf{F}_2} \ \top_R$$

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), []\mathbf{F}_6} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_5, \Box \Gamma_4 \vdash \Delta_7, \mathbf{F}_2, []\mathbf{F}_6} \quad K$$

 $\bullet$  Case rule A45

$$\frac{h_3: \Box \Gamma_4 \vdash \Box \Gamma_6, F_7}{\bullet h_3: \Box \Gamma_4, \Delta_5 \vdash \Box \Gamma_6, (\Delta_8, F_1 \land F_2), []F_7} \quad \text{A45} \qquad \rightsquigarrow \qquad \frac{\overline{h_3: \Box \Gamma_4 \vdash F_7, \Box \Gamma_6}}{\bullet h_3: \Delta_5, \Box \Gamma_4 \vdash \Delta_8, F_2, \Box \Gamma_6, []F_7} \quad \text{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_4:\Delta_5\vdash \mathbf{F}_6,\Delta_1,\mathbf{F}_2\land \mathbf{F}_3\quad \mathbf{h}_4:\mathbf{F}_7,\Delta_5\vdash \Delta_1,\mathbf{F}_2\land \mathbf{F}_3}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{F}_7\vdash \Delta_1,\mathbf{F}_2\land \mathbf{F}_3} \quad \rightarrow_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_5\vdash \Delta_1,\mathbf{F}_3,\mathbf{F}_6}\quad \text{ax/ind}}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{F}_7\vdash \Delta_1,\mathbf{F}_3} \quad \xrightarrow{\bullet \mathbf{h}_4:\Delta_5\vdash \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{F}_7\vdash \Delta_1,\mathbf{F}_3} \quad \xrightarrow{\bullet \mathbf{h}_4:\Delta_5\vdash \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{h}_7\vdash \Delta_1,\mathbf{F}_3} \quad \xrightarrow{\bullet \mathbf{h}_4:\Delta_5\vdash \mathbf{h$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_4:\mathbf{F}_6,\mathbf{F}_7,\Delta_5\vdash\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3}{\bullet\mathbf{h}_4:\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7\vdash\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3} \ \wedge_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_5,\mathbf{F}_6,\mathbf{F}_7\vdash\Delta_1,\mathbf{F}_3}}{\bullet\mathbf{h}_4:\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7\vdash\Delta_1,\mathbf{F}_3} \ \stackrel{\mathrm{ax/ind}}{\wedge_L}$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_{4}: \mathbf{F}_{6}, \Delta_{5} \vdash \Delta_{1}, \mathbf{F}_{2} \wedge \mathbf{F}_{3} \quad \mathbf{h}_{4}: \mathbf{F}_{7}, \Delta_{5} \vdash \Delta_{1}, \mathbf{F}_{2} \wedge \mathbf{F}_{3}}{\bullet \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{2} \wedge \mathbf{F}_{3}} \quad \vee_{L} \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vdash \Delta_{1}, \mathbf{F}_{3}} \quad \overline{\mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{3}}}{\bullet \mathbf{h}_{4}: \Delta_{5}, \mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{1}, \mathbf{F}_{3}} \quad \vee_{L}$$

 $\bullet$  Case rule AT

$$\begin{array}{ll} \frac{\mathbf{h}_4: \mathbf{F}_6, \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2 \wedge \mathbf{F}_3}{\bullet \mathbf{h}_4: \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2 \wedge \mathbf{F}_3} & AT \end{array} \quad \overset{\mathbf{h}_4: \Delta_5, \mathbf{F}_6, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_3}{\bullet \mathbf{h}_4: \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_3} \quad \overset{\mathrm{ax/ind}}{AT}$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_4: \bot, \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \wedge \mathbf{F}_3} \ ^{\bot}L \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_4: \bot, \Delta_5 \vdash \Delta_1, \mathbf{F}_3} \ ^{\bot}L$$

 $\bullet$  Case rule I

$$\overline{\bullet \mathsf{h}_3 : \mathsf{p}_5, \Delta_4 \vdash \mathsf{p}_5, \Delta_6, \mathsf{F}_1 \wedge \mathsf{F}_2} \quad I \qquad \rightsquigarrow \qquad \overline{\bullet \mathsf{h}_3 : \Delta_4, \mathsf{p}_5 \vdash \Delta_6, \mathsf{F}_2, \mathsf{p}_5} \quad I \\$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_4: \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \land \mathbf{F}_3}{\bullet \mathbf{h}_4: \top, \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \land \mathbf{F}_3} \ \top_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: \Delta_5 \vdash \Delta_1, \mathbf{F}_3}}{\bullet \mathbf{h}_4: \top, \Delta_5 \vdash \Delta_1, \mathbf{F}_3} \overset{\mathrm{ax/ind}}{\top}_L$$

#### 4.4 Status of $\vee_R$ : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_5, \Delta_4 \vdash \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2), \mathbf{F}_5 \to \mathbf{F}_6} \ \rightarrow_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: \Delta_4, \mathbf{F}_5 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \to \mathbf{F}_6} \xrightarrow{\mathbf{ax}/\mathbf{ind}} \rightarrow_R$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5,\Delta_7,\mathbf{F}_1\vee \mathbf{F}_2\quad \mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6,\Delta_7,\mathbf{F}_1\vee \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4\vdash (\Delta_7,\mathbf{F}_1\vee \mathbf{F}_2),\mathbf{F}_5\wedge \mathbf{F}_6}\quad \wedge_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_5}\quad ^{\mathrm{ax/ind}}\quad \overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_6}\quad ^{\mathrm{ax/ind}}\quad \overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_6}\quad \wedge_R \quad \wedge_R$$

• Case rule  $\vee_R$ 

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_3: \Delta_4 \vdash \Delta_5, \mathbf{f}_1 \vee \mathbf{f}_2}{\bullet \mathbf{h}_3: \Delta_4 \vdash \bot, \Delta_5, \mathbf{f}_1 \vee \mathbf{f}_2} \ \bot_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: \Delta_4 \vdash \Delta_5, \mathbf{f}_1, \mathbf{f}_2}}{\bullet \mathbf{h}_3: \Delta_4 \vdash \bot, \Delta_5, \mathbf{f}_1, \mathbf{f}_2} \overset{\mathsf{ax/ind}}{}{} \bot_R$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{F}_1 \vee \mathbf{F}_2} \ \top_R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{F}_1,\mathbf{F}_2} \ \top_R$$

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Box \Gamma_4, \Delta_5 \vdash (\Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2), []\mathbf{F}_6} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6} \quad \mathbf{ax}}{\bullet \mathbf{h}_3: \Delta_5, \Box \Gamma_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_2, []\mathbf{F}_6} \quad K$$

• Case rule A45

$$\frac{\mathtt{h}_3: \Box \Gamma_4 \vdash \Box \Gamma_6, \mathtt{F}_7}{\bullet \mathtt{h}_3: \Box \Gamma_4, \Delta_5 \vdash \Box \Gamma_6, (\Delta_8, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_7} \quad \text{A45} \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_3: \Box \Gamma_4 \vdash \mathtt{F}_7, \Box \Gamma_6}}{\bullet \mathtt{h}_3: \Delta_5, \Box \Gamma_4 \vdash \Delta_8, \mathtt{F}_1, \mathtt{F}_2, \Box \Gamma_6, []\mathtt{F}_7} \quad \text{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_4:\Delta_5\vdash \mathbf{F}_6,\Delta_1,\mathbf{F}_2\vee \mathbf{F}_3\quad \mathbf{h}_4:\mathbf{F}_7,\Delta_5\vdash \Delta_1,\mathbf{F}_2\vee \mathbf{F}_3}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{F}_7\vdash \Delta_1,\mathbf{F}_2\vee \mathbf{F}_3} \quad \rightarrow_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_4:\Delta_5\vdash \Delta_1,\mathbf{F}_2,\mathbf{F}_3,\mathbf{F}_6}\quad \text{ax/ind} \quad \overline{\mathbf{h}_4:\Delta_5,\mathbf{F}_7\vdash \Delta_1,\mathbf{F}_2,\mathbf{F}_3}\quad \frac{\text{ax/ind}}{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{F}_7\vdash \Delta_1,\mathbf{F}_2,\mathbf{F}_3}\quad \xrightarrow{\bullet \mathbf{h}_4:\Delta_5,\mathbf{F}_6\to \mathbf{h}_7\vdash \Delta_1,\mathbf{F}_2,\mathbf{F}_3}\quad \xrightarrow{\bullet \mathbf{h}_4:$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_4:\mathbf{F}_6,\mathbf{F}_7,\Delta_5\vdash\Delta_1,\mathbf{F}_2\vee\mathbf{F}_3}{\bullet\mathbf{h}_4:\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7\vdash\Delta_1,\mathbf{F}_2\vee\mathbf{F}_3} \ \land L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_5,\mathbf{F}_6,\mathbf{F}_7\vdash\Delta_1,\mathbf{F}_2,\mathbf{F}_3}}{\bullet\mathbf{h}_4:\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7\vdash\Delta_1,\mathbf{F}_2,\mathbf{F}_3} \ \overset{\mathsf{ax/ind}}{\wedge} L$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_4: \mathbf{F}_6, \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \lor \mathbf{F}_3 \quad \mathbf{h}_4: \mathbf{F}_7, \Delta_5 \vdash \Delta_1, \mathbf{F}_2 \lor \mathbf{F}_3}{\bullet \mathbf{h}_4: \Delta_5, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \Delta_1, \mathbf{F}_2 \lor \mathbf{F}_3} \quad \forall_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_4: \Delta_5, \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2, \mathbf{F}_3} \quad \text{ax/ind}}{\bullet \mathbf{h}_4: \Delta_5, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \Delta_1, \mathbf{F}_2, \mathbf{F}_3} \quad \forall_L \quad$$

 $\bullet$  Case rule AT

$$\begin{array}{ll} \frac{\mathbf{h}_4: \mathbf{F}_6, \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2 \vee \mathbf{F}_3}{\bullet \mathbf{h}_4: \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2 \vee \mathbf{F}_3} & AT \end{array} \quad \rightsquigarrow \quad \\ \frac{\frac{\mathbf{h}_4: \Delta_5, \mathbf{F}_6, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2, \mathbf{F}_3}{\bullet \mathbf{h}_4: \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2, \mathbf{F}_3} & AT \end{array} \quad \\ \frac{\mathbf{h}_4: \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2, \mathbf{F}_3}{\bullet \mathbf{h}_4: \Delta_5, [] \mathbf{F}_6 \vdash \Delta_1, \mathbf{F}_2, \mathbf{F}_3} & AT \end{array}$$

• Case rule  $\perp_L$ 

ullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_5, \Delta_4 \vdash \mathsf{p}_5, \Delta_6, \mathsf{F}_1 \vee \mathsf{F}_2} \quad I \qquad \leadsto \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_4, \mathsf{p}_5 \vdash \Delta_6, \mathsf{F}_1, \mathsf{F}_2, \mathsf{p}_5} \quad I$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_4: \Delta_5 \vdash \Delta_1, \mathbf{f}_2 \vee \mathbf{f}_3}{\bullet \mathbf{h}_4: \top, \Delta_5 \vdash \Delta_1, \mathbf{f}_2 \vee \mathbf{f}_3} \ \top_L \qquad \leadsto \qquad \frac{\mathbf{h}_4: \Delta_5 \vdash \Delta_1, \mathbf{f}_2, \mathbf{f}_3}{\bullet \mathbf{h}_4: \top, \Delta_5 \vdash \Delta_1, \mathbf{f}_2, \mathbf{f}_3} \ ^{\mathrm{ax/ind}} \top_L$$

## 4.5 Status of $\perp_R$ : : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2 \vdash \bot, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash (\bot, \Delta_5), \mathbf{F}_3 \rightarrow \mathbf{F}_4} \quad \rightarrow_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_4}}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3 \rightarrow \mathbf{F}_4} \xrightarrow{\mathsf{ax/ind}} \rightarrow_R$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_1:\Delta_2\vdash\bot,\mathbf{F}_3,\Delta_5\quad\mathbf{h}_1:\Delta_2\vdash\bot,\mathbf{F}_4,\Delta_5}{\bullet\mathbf{h}_1:\Delta_2\vdash(\bot,\Delta_5),\mathbf{F}_3\land\mathbf{F}_4}\quad\wedge_R\qquad\rightsquigarrow\qquad \frac{\overline{\mathbf{h}_1:\Delta_2\vdash\Delta_5,\mathbf{F}_3}\quad\text{ax/ind}\quad \overline{\mathbf{h}_1:\Delta_2\vdash\Delta_5,\mathbf{F}_4}\quad \frac{\mathbf{ax/ind}}{\land_R}\quad\wedge_R$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_1:\Delta_2\vdash\bot,\mathbf{F}_3,\mathbf{F}_4,\Delta_5}{\bullet\mathbf{h}_1:\Delta_2\vdash(\bot,\Delta_5),\mathbf{F}_3\vee\mathbf{F}_4}\ \vee_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1:\Delta_2\vdash\Delta_5,\mathbf{F}_3,\mathbf{F}_4}}{\bullet\mathbf{h}_1:\Delta_2\vdash\Delta_5,\mathbf{F}_3\vee\mathbf{F}_4} \overset{\mathrm{ax/ind}}{\vee_R}$$

• Case rule  $\perp_R$ 

$$\begin{array}{cccc} \frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \bot, \Delta_3} & \bot_R & & \leadsto & & \frac{\overline{\mathbf{h}_1: \Delta_2 \vdash \Delta_3}}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_3} & \mathbf{H} \end{array}$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \bot, \Delta_3} \ ^\top R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3} \ ^\top R$$

 $\bullet$  Case rule K

$$\frac{h_1: unbox(\Box \Gamma_2) \vdash \mathtt{F_4}}{\bullet h_1: \Box \Gamma_2, \Delta_3 \vdash (\bot, \Delta_5), []\mathtt{F_4}} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{h_1: unbox(\Box \Gamma_2) \vdash \mathtt{F_4}}}{\bullet h_1: \Delta_3, \Box \Gamma_2 \vdash \Delta_5, []\mathtt{F_4}} \quad K$$

• Case rule A45

$$\frac{\mathtt{h}_1: \Box \mathtt{\Gamma}_2 \vdash \Box \mathtt{\Gamma}_4, \mathtt{F}_5}{\bullet \mathtt{h}_1: \Box \mathtt{\Gamma}_2, \Delta_3 \vdash \Box \mathtt{\Gamma}_4, (\bot, \Delta_6), []\mathtt{F}_5} \quad {}^{A45} \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_1: \Box \mathtt{\Gamma}_2 \vdash \mathtt{F}_5, \Box \mathtt{\Gamma}_4} \quad \mathtt{ax}}{\bullet \mathtt{h}_1: \Delta_3, \Box \mathtt{\Gamma}_2 \vdash \Delta_6, \Box \mathtt{\Gamma}_4, []\mathtt{F}_5} \quad {}^{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_2:\Delta_3\vdash\bot,\mathbf{f}_4,\Delta_1\quad \mathbf{h}_2:\mathbf{f}_5,\Delta_3\vdash\bot,\Delta_1}{\bullet\mathbf{h}_2:\Delta_3,\mathbf{f}_4\to\mathbf{f}_5\vdash\bot,\Delta_1} \ \to_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_3\vdash\Delta_1,\mathbf{f}_4} \ \ \text{ax/ind}}{\bullet\mathbf{h}_2:\Delta_3,\mathbf{f}_4\to\mathbf{f}_5\vdash\Delta_1} \ \ \frac{\mathbf{ax/ind}}{\to_L}$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_2: \mathbf{F}_4, \mathbf{F}_5, \Delta_3 \vdash \bot, \Delta_1}{\bullet \mathbf{h}_2: \Delta_3, \mathbf{F}_4 \land \mathbf{F}_5 \vdash \bot, \Delta_1} \ \land_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2: \Delta_3, \mathbf{F}_4, \mathbf{F}_5 \vdash \Delta_1}}{\bullet \mathbf{h}_2: \Delta_3, \mathbf{F}_4 \land \mathbf{F}_5 \vdash \Delta_1} \ \land_L$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_2: \mathbf{F}_4, \Delta_3 \vdash \bot, \Delta_1 \quad \mathbf{h}_2: \mathbf{F}_5, \Delta_3 \vdash \bot, \Delta_1}{\bullet \mathbf{h}_2: \Delta_3, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \bot, \Delta_1} \quad \vee_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2: \Delta_3, \mathbf{F}_4 \vdash \Delta_1} \quad \text{ax/ind}}{\bullet \mathbf{h}_2: \Delta_3, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_1} \quad \overset{\text{ax/ind}}{\vee_L}$$

 $\bullet$  Case rule AT

$$\frac{\mathbf{h}_2: \mathbf{f}_4, \Delta_3, []\mathbf{f}_4 \vdash \bot, \Delta_1}{\bullet \mathbf{h}_2: \Delta_3, []\mathbf{f}_4 \vdash \bot, \Delta_1} \quad AT \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2: \Delta_3, \mathbf{f}_4, []\mathbf{f}_4 \vdash \Delta_1}}{\bullet \mathbf{h}_2: \Delta_3, []\mathbf{f}_4 \vdash \Delta_1} \quad \frac{\mathbf{ax/ind}}{AT}$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_2:\bot,\Delta_3\vdash\bot,\Delta_1} \ ^\bot L \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_2:\bot,\Delta_3\vdash\Delta_1} \ ^\bot L$$

ullet Case rule I

$$\frac{}{\bullet \mathbf{h}_1: \mathbf{p}_3, \Delta_2 \vdash \mathbf{p}_3, \bot, \Delta_4} \quad I \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{p}_3 \vdash \Delta_4, \mathbf{p}_3} \quad I$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_2:\Delta_3\vdash\bot,\Delta_1}{\bullet\mathbf{h}_2:\top,\Delta_3\vdash\bot,\Delta_1}\ \top_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_3\vdash\Delta_1}}{\bullet\mathbf{h}_2:\top,\Delta_3\vdash\Delta_1} \overset{\mathrm{ax/ind}}{\top_L}$$

## 4.6 Status of $\top_R$ : : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2 \vdash \top, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash (\top, \Delta_5), \mathbf{F}_3 \rightarrow \mathbf{F}_4} \ \rightarrow_{R} \qquad \rightsquigarrow \qquad \mathsf{trivial}$$

• Case rule  $\wedge_R$ 

$$\begin{array}{ccc} \frac{\mathbf{h}_1:\Delta_2 \vdash \top, \mathbf{F}_3, \Delta_5 & \mathbf{h}_1:\Delta_2 \vdash \top, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1:\Delta_2 \vdash (\top, \Delta_5), \mathbf{F}_3 \wedge \mathbf{F}_4} & \wedge_R & \longrightarrow & \text{trivial} \end{array}$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \top, \mathbf{F}_3, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash (\top, \Delta_5), \mathbf{F}_3 \vee \mathbf{F}_4} \ \lor_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \top, \Delta_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \bot, \top, \Delta_3} \ \bot_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3} \ \top_R \qquad \leadsto \qquad \mathsf{trivial}$$

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_2) \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1: \Box \Gamma_2, \Delta_3 \vdash (\top, \Delta_5), []\mathbf{F}_4} \quad K \qquad \leadsto \qquad \mathsf{trivial}$$

 $\bullet$  Case rule A45

$$\frac{\mathbf{h}_1:\Box\Gamma_2\vdash\Box\Gamma_4,\mathbf{F}_5}{\bullet\mathbf{h}_1:\Box\Gamma_2,\Delta_3\vdash\Box\Gamma_4,(\top,\Delta_6),[]\mathbf{F}_5}\quad A45\qquad \leadsto \qquad \text{trivial}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_2:\Delta_3\vdash \top, \mathbf{F}_4,\Delta_1\quad \mathbf{h}_2:\mathbf{F}_5,\Delta_3\vdash \top,\Delta_1}{\bullet \mathbf{h}_2:\Delta_3,\mathbf{F}_4\to \mathbf{F}_5\vdash \top,\Delta_1} \ \to_L \qquad \leadsto \qquad \text{trivial}$$

• Case rule  $\wedge_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_2: \mathbf{F}_4, \mathbf{F}_5, \Delta_3 \vdash \top, \Delta_1}{\bullet \mathbf{h}_2: \Delta_3, \mathbf{F}_4 \land \mathbf{F}_5 \vdash \top, \Delta_1} \ \land_L & \longrightarrow & \text{trivial} \end{array}$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_2: \mathbf{F}_4, \Delta_3 \vdash \top, \Delta_1 \quad \mathbf{h}_2: \mathbf{F}_5, \Delta_3 \vdash \top, \Delta_1}{\bullet \mathbf{h}_2: \Delta_3, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \top, \Delta_1} \quad \vee_L \qquad \leadsto \qquad \mathsf{trivial}$$

 $\bullet$  Case rule AT

$$\begin{array}{ll} \mathbf{h}_2: \mathbf{F}_4, \Delta_3, []\mathbf{F}_4 \vdash \top, \Delta_1 \\ & \bullet \mathbf{h}_2: \Delta_3, []\mathbf{F}_4 \vdash \top, \Delta_1 \end{array} \quad AT \qquad \leadsto \qquad \mathrm{trivial}$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_2:\bot,\Delta_3\vdash\top,\Delta_1}\ ^{\bot}L \qquad \leadsto \qquad \mathsf{trivial}$$

 $\bullet$  Case rule I

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_2: \Delta_3 \vdash \top, \Delta_1}{\bullet \mathbf{h}_2: \top, \Delta_3 \vdash \top, \Delta_1} \ \top_L \qquad \leadsto \qquad \mathsf{trivial}$$

#### 4.7 Status of K: Non invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_4: \Box \Gamma_1, \mathbf{F}_5, \Delta_2 \vdash \mathbf{F}_6, \Delta_7, []\mathbf{F}_3}{\bullet \mathbf{h}_4: \Box \Gamma_1, \Delta_2 \vdash (\Delta_7, []\mathbf{F}_3), \mathbf{F}_5 \to \mathbf{F}_6} \ \to_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3} \ \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_4: \Box \Gamma_1, \Delta_2 \vdash \mathbf{F}_5, \Delta_7, []\mathbf{F}_3 \quad \mathbf{h}_4: \Box \Gamma_1, \Delta_2 \vdash \mathbf{F}_6, \Delta_7, []\mathbf{F}_3}{\bullet \mathbf{h}_4: \Box \Gamma_1, \Delta_2 \vdash (\Delta_7, []\mathbf{F}_3), \mathbf{F}_5 \land \mathbf{F}_6} \quad \land R \quad \quad \leadsto \quad \quad \frac{\overline{\mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3} \quad \overset{\mathrm{ax/ind}}{\mathbf{H}}$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_4: \Box \Gamma_1, \Delta_2 \vdash \mathbf{F}_5, \mathbf{F}_6, \Delta_7, []\mathbf{F}_3}{\bullet \mathbf{h}_4: \Box \Gamma_1, \Delta_2 \vdash (\Delta_7, []\mathbf{F}_3), \mathbf{F}_5 \lor \mathbf{F}_6} \quad \vee_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3} \overset{\mathrm{ax/ind}}{\vdash}$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_4: \Box \Gamma_1, \Delta_2 \vdash \Delta_5, []\mathbf{F}_3}{\bullet \mathbf{h}_4: \Box \Gamma_1, \Delta_2 \vdash \bot, \Delta_5, []\mathbf{F}_3} \quad \bot_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3} \quad \underset{\mathbb{H}}{\text{ax/ind}}$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_4: \Box \Gamma_1, \Delta_2 \vdash \top, \Delta_5, []\mathbf{F}_3} \ ^\top R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3} \ ^\mathsf{fail}$$

 $\bullet$  Case rule K

• Case rule A45

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_4:\square\Gamma_1,\Delta_7 \vdash \mathbf{F}_5,\Delta_2, []\mathbf{F}_3 \quad \mathbf{h}_4:\square\Gamma_1,\mathbf{F}_6,\Delta_7 \vdash \Delta_2, []\mathbf{F}_3}{\bullet \mathbf{h}_4:(\square\Gamma_1,\Delta_7),\mathbf{F}_5 \to \mathbf{F}_6 \vdash \Delta_2, []\mathbf{F}_3} \quad \to L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:unbox(\square\Gamma_1) \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4:unbox(\square\Gamma_1) \vdash \mathbf{F}_3} \quad \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_4: \Box \Gamma_1, \mathbf{F}_5, \mathbf{F}_6, \Delta_7 \vdash \Delta_2, []\mathbf{F}_3}{\bullet \mathbf{h}_4: (\Box \Gamma_1, \Delta_7), \mathbf{F}_5 \land \mathbf{F}_6 \vdash \Delta_2, []\mathbf{F}_3} \quad \wedge_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3} \quad \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_4:\square\Gamma_1,\mathbf{F}_5,\Delta_7\vdash\Delta_2,[]\mathbf{F}_3\quad\mathbf{h}_4:\square\Gamma_1,\mathbf{F}_6,\Delta_7\vdash\Delta_2,[]\mathbf{F}_3}{\bullet\mathbf{h}_4:(\square\Gamma_1,\Delta_7),\mathbf{F}_5\vee\mathbf{F}_6\vdash\Delta_2,[]\mathbf{F}_3}\quad\vee_L\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_4:unbox(\square\Gamma_1)\vdash\mathbf{F}_3}}{\bullet\mathbf{h}_4:unbox(\square\Gamma_1)\vdash\mathbf{F}_3} \overset{\mathrm{ax/ind}}{\bullet}$$

 $\bullet$  Case rule AT

$$\frac{\mathbf{h}_4: \Box \Gamma_6, \mathbf{F}_5, \Delta_1, []\mathbf{F}_5 \vdash \Delta_2, []\mathbf{F}_3}{\bullet \mathbf{h}_4: (\Box \Gamma_6, \Delta_1), []\mathbf{F}_5 \vdash \Delta_2, []\mathbf{F}_3} \quad AT \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: \mathbf{F}_5, unbox(\Box \Gamma_6) \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4: \mathbf{F}_5, unbox(\Box \Gamma_6) \vdash \mathbf{F}_3} \quad \overset{\mathrm{ax/ind}}{\mathbf{H}}$$

$$\frac{\mathbf{h}_4: \Box \Gamma_1, \mathbf{F}_5, \Delta_6, []\mathbf{F}_5 \vdash \Delta_2, []\mathbf{F}_3}{\bullet \mathbf{h}_4: (\Box \Gamma_1, \Delta_6), []\mathbf{F}_5 \vdash \Delta_2, []\mathbf{F}_3} \quad AT \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3} \quad \mathbf{H}$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathsf{h}_4: \bot, \Box \Gamma_1, \Delta_5 \vdash \Delta_2, []\mathsf{F}_3} \ \bot_L \qquad \leadsto \qquad \frac{}{\bullet \mathsf{h}_4: \mathit{unbox}(\Box \Gamma_1) \vdash \mathsf{F}_3} \ \mathsf{fail}$$

 $\bullet$  Case rule I

$$\frac{}{\bullet \mathtt{h}_3:\mathtt{p}_4,\Box\Gamma_1,\Delta_6 \vdash \mathtt{p}_4,\Delta_5,[]\mathtt{F}_2} \quad I \qquad \rightsquigarrow \qquad \frac{}{\bullet \mathtt{h}_3:\mathit{unbox}(\Box\Gamma_1) \vdash \mathtt{F}_2} \quad \mathtt{fail}$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_4: \Box \Gamma_1, \Delta_5 \vdash \Delta_2, []\mathbf{F}_3}{\bullet \mathbf{h}_4: \top, \Box \Gamma_1, \Delta_5 \vdash \Delta_2, []\mathbf{F}_3} \ \ \top_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3}}{\bullet \mathbf{h}_4: unbox(\Box \Gamma_1) \vdash \mathbf{F}_3} \ \ \frac{\mathbf{ax/ind}}{\mathbf{H}}$$

#### 4.8 Status of A45: Non invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathtt{h}_5: \Box \Gamma_1, \mathtt{F}_6, \Delta_2 \vdash \Box \Gamma_3, \mathtt{F}_7, \Delta_8, []\mathtt{F}_4}{\bullet \mathtt{h}_5: \Box \Gamma_1, \Delta_2 \vdash (\Box \Gamma_3, \Delta_8, []\mathtt{F}_4), \mathtt{F}_6 \to \mathtt{F}_7} \to_R \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_5: \Box \Gamma_1 \vdash \mathtt{F}_4, \Box \Gamma_3}}{\bullet \mathtt{h}_5: \Box \Gamma_1 \vdash \mathtt{F}_4, \Box \Gamma_3} \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_5: \Box \Gamma_1, \Delta_2 \vdash \Box \Gamma_3, \mathbf{F}_6, \Delta_8, ( \Box \mathbf{F}_4 \quad \mathbf{h}_5: \Box \Gamma_1, \Delta_2 \vdash \Box \Gamma_3, \mathbf{F}_7, \Delta_8, ( \Box \mathbf{F}_4)}{\bullet \mathbf{h}_5: \Box \Gamma_1, \Delta_2 \vdash (\Box \Gamma_3, \Delta_8, ( \Box \mathbf{F}_4), \mathbf{F}_6 \land \mathbf{F}_7} \quad \land_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_3}}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_3} \quad \mathbf{H}$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_5: \Box \Gamma_1, \Delta_2 \vdash \Box \Gamma_3, \mathbf{F}_6, \mathbf{F}_7, \Delta_8, []\mathbf{F}_4}{\bullet \mathbf{h}_5: \Box \Gamma_1, \Delta_2 \vdash (\Box \Gamma_3, \Delta_8, []\mathbf{F}_4), \mathbf{F}_6 \vee \mathbf{F}_7} \quad \vee_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_3}}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_3} \stackrel{\mathrm{ax/ind}}{\to} \mathbf{H}$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_5: \Box \mathbf{r}_1, \Delta_2 \vdash \Box \mathbf{r}_3, \Delta_6, []\mathbf{F}_4}{\bullet \mathbf{h}_5: \Box \mathbf{r}_1, \Delta_2 \vdash \bot, \Box \mathbf{r}_3, \Delta_6, []\mathbf{F}_4} \ \bot_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5: \Box \mathbf{r}_1 \vdash \mathbf{F}_4, \Box \mathbf{r}_3}}{\bullet \mathbf{h}_5: \Box \mathbf{r}_1 \vdash \mathbf{F}_4, \Box \mathbf{r}_3} \ _{\mathbf{H}}^{\mathrm{ax/ind}}$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathsf{h}_5:\Box \Gamma_1,\Delta_2 \vdash \top,\Box \Gamma_3,\Delta_6,[]\mathsf{F}_4} \ \ ^\top \mathit{R} \qquad \leadsto \qquad \frac{}{\bullet \mathsf{h}_5:\Box \Gamma_1 \vdash \Box \Gamma_3,\mathsf{F}_4} \ \ \mathsf{fail}$$

ullet Case rule K

• Case rule A45

$$\begin{array}{c} h_2: \square_{\Gamma_8}, \square_{\Gamma_9} \vdash \square_{\Gamma_4}, \square_{\Gamma_5}, F_3, [F_1] \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, [F_1), (\square_{\Gamma_6}, \Delta_7), [F_3] \\ \hline \\ h_2: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3) \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3) \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3, [F_1], (\square_{\Gamma_6}, \Delta_7), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3, [F_1], (\square_{\Gamma_6}, \Delta_7), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3) \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3) \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_1], [F_$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_5: \Box \Gamma_1, \Delta_8 \vdash \Box \Gamma_2, \mathbf{F}_6, \Delta_3, []\mathbf{F}_4 \quad \mathbf{h}_5: \Box \Gamma_1, \mathbf{F}_7, \Delta_8 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4}{\bullet \mathbf{h}_5: (\Box \Gamma_1, \Delta_8), \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4} \\ \rightarrow_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \overset{\mathrm{ax/ind}}{\vdash \mathbf{h}_5: \Box \Gamma_1} \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \leftarrow \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_5: \Box \Gamma_1, \mathbf{F}_6, \mathbf{F}_7, \Delta_8 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4}{\bullet \mathbf{h}_5: (\Box \Gamma_1, \Delta_8), \mathbf{F}_6 \land \mathbf{F}_7 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4} \ \land_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \ \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_5: \Box \Gamma_1, \mathbf{F}_6, \Delta_8 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4 \quad \mathbf{h}_5: \Box \Gamma_1, \mathbf{F}_7, \Delta_8 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4}{\bullet \mathbf{h}_5: (\Box \Gamma_1, \Delta_8), \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4} \quad \vee_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \overset{\mathrm{ax/ind}}{\vdash \mathbf{h}_5: \Box \Gamma_1} = \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2} \times \frac{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma_2}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{h}_4, \Box \Gamma$$

 $\bullet$  Case rule AT

$$\frac{\mathbf{h}_5: \Box \Gamma_7, \mathbf{f}_6, \Delta_1, ([\mathbf{f}_6 \vdash \Box \Gamma_2, \Delta_3, ([\mathbf{f}_4 \\ \bullet \mathbf{h}_5: (\Box \Gamma_7, \Delta_1), ([\mathbf{f}_6 \vdash \Box \Gamma_2, \Delta_3, ([\mathbf{f}_4 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{f}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{h}_4, \Box \Gamma_2 \\ \bullet \mathbf{h}_5: \Box \Gamma_7, ([\mathbf{f}_6 \vdash \mathbf{$$

$$\begin{array}{lll} \underbrace{\mathbf{h}_5: \Box \Gamma_1, \mathbf{F}_6, \Delta_7, []\mathbf{F}_6 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4}_{\bullet \mathbf{h}_5: (\Box \Gamma_1, \Delta_7), []\mathbf{F}_6 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4} & AT \\ \end{array} \\ & \xrightarrow{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \\ \underbrace{\phantom{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}_{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}}^{\bullet \mathbf{ax/ind}} \\ \end{array}$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathsf{h}_5: \bot, \Box \Gamma_1, \Delta_6 \vdash \Box \Gamma_2, \Delta_3, []\mathsf{F}_4} \ \bot_L \qquad \leadsto \qquad \frac{}{\bullet \mathsf{h}_5: \Box \Gamma_1 \vdash \Box \Gamma_2, \mathsf{F}_4} \ \mathsf{fail}$$

 $\bullet$  Case rule I

$$\overbrace{\bullet_{h_4:\,p_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,p_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,p_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,p_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,p_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,p_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,p_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,D_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,D_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,D_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,D_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,D_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,D_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,D_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,D_5,\,\Box\Gamma_2,\,\Delta_6,\,[]\mathbb{F}_3}^{\bullet_{h_4:\,D_5,\,\Box\Gamma_1,\,\Delta_7\,\vdash\,D_5,\,\Box\Gamma_2,\,\Delta_6,\,\Box\Gamma_2,\,\Box\Gamma_2,\,\Delta_6,\,\Box\Gamma_2,\,\Box$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_5: \Box \Gamma_1, \Delta_6 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4}{\bullet \mathbf{h}_5: \top, \Box \Gamma_1, \Delta_6 \vdash \Box \Gamma_2, \Delta_3, []\mathbf{F}_4} \ \, \top_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2}}{\bullet \mathbf{h}_5: \Box \Gamma_1 \vdash \mathbf{F}_4, \Box \Gamma_2} \ \, \overset{\mathrm{ax/ind}}{\mathrm{H}}$$

#### 4.9 Status of $\rightarrow_L$ : (Left Premise): Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_4: \mathbf{F}_6, \Delta_1, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \mathbf{F}_7, \Delta_5}{\bullet \mathbf{h}_4: \Delta_1, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_6 \rightarrow \mathbf{F}_7} \rightarrow_R \qquad \rightsquigarrow \qquad \frac{\mathbf{h}_4: \Delta_1, \mathbf{F}_6 \vdash \Delta_5, \mathbf{F}_2, \mathbf{F}_7}{\bullet \mathbf{h}_4: \Delta_1 \vdash \Delta_5, \mathbf{F}_2, \mathbf{F}_6 \rightarrow \mathbf{F}_7} \xrightarrow{ax/ind}$$

• Case rule  $\wedge_R$ 

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_6,\mathbf{F}_7,\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\vee\mathbf{F}_7}\ \vee_R \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_4:\Delta_1\vdash\Delta_5,\mathbf{F}_2,\mathbf{F}_6,\mathbf{F}_7}{\bullet\mathbf{h}_4:\Delta_1\vdash\Delta_5,\mathbf{F}_2,\mathbf{F}_6\vee\mathbf{F}_7}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_4:\Delta_1\vdash\Delta_5,\mathbf{F}_2,\mathbf{F}_6\vee\mathbf{F}_7}\ ^{\mathrm{ax/ind}}}$$

• Case rule  $\perp_R$ 

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_4:\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\top,\Delta_5} \ \top_R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_4:\Delta_1\vdash\top,\Delta_5,\mathbf{F}_2} \ \top_R$$

ullet Case rule K

$$\frac{\mathtt{h}_3: unbox(\Box \Gamma_4) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3: \Box \Gamma_4, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_6} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_3: unbox(\Box \Gamma_4) \vdash \mathtt{F}_6}}{\bullet \mathtt{h}_3: \Delta_7, \Box \Gamma_4 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_6} \quad K$$

 $\bullet$  Case rule A45

$$\frac{h_3:\Box\Gamma_4\vdash\Box\Gamma_5,F_7}{\bullet h_3:\Box\Gamma_4,\Delta_8,F_1\to F_2\vdash\Box\Gamma_5,\Delta_6,[]F_7} \ \ _{A45} \qquad \rightsquigarrow \qquad \frac{\overline{h_3:\Box\Gamma_4\vdash F_7,\Box\Gamma_5}}{\bullet h_3:\Delta_8,\Box\Gamma_4\vdash\Delta_6,F_1,\Box\Gamma_5,[]F_7} \ \ _{A45}$$

• Case rule  $\rightarrow_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_3:\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \mathbf{F}_4, \Delta_6 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2), \mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \Delta_6} \\ & \bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2), \mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \Delta_6 \end{array} \rightarrow_L \\ & \sim \begin{array}{c} \frac{\mathbf{h}_3:\Delta_7 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1} \\ \bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \end{array} \xrightarrow{\mathbf{ax/ind}} \begin{array}{c} \mathbf{ax/ind} \\ \bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \rightarrow \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \end{array} \rightarrow_L \\ & \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5} \\ \rightarrow_L \end{array} \rightarrow_L \begin{array}{c} \frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3} \end{array} \times_{\mathbf{H}} \end{array}$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_3:\mathbf{F}_4,\mathbf{F}_5,\Delta_7,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\rightarrow\mathbf{F}_2),\mathbf{F}_4\wedge\mathbf{F}_5\vdash\Delta_6} \ \wedge_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_5\vdash\Delta_6,\mathbf{F}_1}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_4\wedge\mathbf{F}_5\vdash\Delta_6,\mathbf{F}_1} \overset{\mathrm{ax/ind}}{\wedge_L}$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_6 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2), \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \leadsto \qquad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_1}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1} \quad \overset{\mathrm{ax/ind}}{\vee_L} \quad \vee_L \quad \overset{\mathrm{ax/ind}}{\vee_L} \quad \overset{\mathrm{ax/ind}}{$$

 $\bullet$  Case rule AT

$$\begin{array}{lll} \frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_6, ([\mathbf{F}_4, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_5}{\bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_4 \rightarrow \mathbf{F}_2), ([\mathbf{F}_4 \vdash \Delta_5]} & AT \end{array} & \sim & & & & \\ \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1]}{\bullet \mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1]} & AT \end{array} & \overset{\mathrm{av}/\mathrm{ind}}{\bullet} \\ \frac{\mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1])}{\bullet} & & & & \\ \frac{\mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1])}{\bullet} & & & \\ \mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1]) & & \\ \mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1]) & & \\ \mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1]) & & \\ \mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1]) & & \\ \mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1]) & & \\ \mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1]) & & \\ \mathbf{h}_3: \Delta_6 & &$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_4}\ ^\bot_L\qquad \leadsto\qquad \frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5\vdash\Delta_4,\mathbf{F}_1}\ ^\bot_L$$

ullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_4, \Delta_6, \mathsf{F}_1 \to \mathsf{F}_2 \vdash \mathsf{p}_4, \Delta_5} \quad I \qquad \leadsto \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_6, \mathsf{p}_4 \vdash \Delta_5, \mathsf{F}_1, \mathsf{p}_4} \quad I$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_4}\ \top_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_5\vdash\Delta_4,\mathbf{F}_1}}{\bullet\mathbf{h}_3:\top,\Delta_5\vdash\Delta_4,\mathbf{F}_1} \stackrel{\mathrm{ax/ind}}{\top_L}$$

## 4.10 Status of $\rightarrow_L$ (Right Premise): : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_4:\mathbf{F}_6,\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_7,\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\to\mathbf{F}_7}\to_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3,\mathbf{F}_6\vdash\Delta_5,\mathbf{F}_7}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\to\mathbf{F}_7}\xrightarrow{\mathrm{ax/ind}}$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_6,\Delta_5\quad\mathbf{h}_4:\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_7,\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7}\quad \wedge_R\qquad \leadsto\qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad \text{ax/ind}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7}\quad \frac{\mathbf{ax/ind}}{\wedge_R}\quad \wedge_R$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\mathbf{F}_6,\mathbf{F}_7,\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\to\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\vee\mathbf{F}_7} \ \vee_R \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6,\mathbf{F}_7}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\vee\mathbf{F}_7}} \overset{\mathrm{ax/ind}}{\vee_R}$$

• Case rule  $\perp_R$ 

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_4:\Delta_1,\mathbf{f}_2\to\mathbf{f}_3\vdash \top,\Delta_5} \ \top_R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_4:\Delta_1,\mathbf{f}_3\vdash \top,\Delta_5} \ \top_R$$

 $\bullet$  Case rule K

$$\frac{\mathtt{h}_3: unbox(\Box \Gamma_4) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3: \Box \Gamma_4, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_6} \quad K \qquad \leadsto \qquad \frac{\overline{\mathtt{h}_3: unbox(\Box \Gamma_4) \vdash \mathtt{F}_6}}{\bullet \mathtt{h}_3: \Delta_7, \mathtt{F}_2, \Box \Gamma_4 \vdash \Delta_5, []\mathtt{F}_6} \quad K$$

• Case rule A45

$$\frac{h_3:\Box\Gamma_4\vdash\Box\Gamma_5,F_7}{\bullet h_3:\Box\Gamma_4,\Delta_8,F_1\to F_2\vdash\Box\Gamma_5,\Delta_6,[]F_7} \ \ _{A45} \qquad \leadsto \qquad \frac{\overline{h_3:\Box\Gamma_4\vdash F_7,\Box\Gamma_5} \ \ _{ax}}{\bullet h_3:\Delta_8,F_2,\Box\Gamma_4\vdash\Delta_6,\Box\Gamma_5,[]F_7} \ \ _{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6\quad\mathbf{h}_3:\mathbf{F}_5,\Delta_7,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\to\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6}\to_L\qquad \leadsto\qquad \frac{\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6}\xrightarrow{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}\frac{\mathbf{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2\to\Delta_6}\xrightarrow{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6}\to_L$$

$$\frac{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_3,\Delta_5\quad \mathbf{h}_1:\mathbf{F}_4,\Delta_2\vdash \Delta_5}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_3\to \mathbf{F}_4\vdash \Delta_5} \ \to_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \Delta_5}}{\bullet \mathbf{h}_1:\Delta_2,\mathbf{F}_4\vdash \Delta_5} \ ^{\mathrm{ax}}$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_3: \mathbf{f}_4, \mathbf{f}_5, \Delta_7, \mathbf{f}_1 \rightarrow \mathbf{f}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{f}_1 \rightarrow \mathbf{f}_2), \mathbf{f}_4 \wedge \mathbf{f}_5 \vdash \Delta_6} \ \land L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4, \mathbf{f}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \wedge \mathbf{f}_5 \vdash \Delta_6} \ \land L$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_3: \mathbf{f}_4, \Delta_7, \mathbf{f}_1 \rightarrow \mathbf{f}_2 \vdash \Delta_6 \quad \mathbf{h}_3: \mathbf{f}_5, \Delta_7, \mathbf{f}_1 \rightarrow \mathbf{f}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{f}_1 \rightarrow \mathbf{f}_2), \mathbf{f}_4 \vee \mathbf{f}_5 \vdash \Delta_6}} \quad \vee_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vee \mathbf{f}_5 \vdash \Delta_6}} \quad \vee_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vee \mathbf{f}_5 \vdash \Delta_6}} \quad \vee_L \qquad \vee_L$$

 $\bullet$  Case rule AT

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_4}\ ^{\bot}L\qquad \leadsto\qquad \frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_2\vdash\Delta_4}\ ^{\bot}L$$

ullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_4, \Delta_6, \mathsf{F}_1 \to \mathsf{F}_2 \vdash \mathsf{p}_4, \Delta_5} \quad I \qquad \rightsquigarrow \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_6, \mathsf{F}_2, \mathsf{p}_4 \vdash \Delta_5, \mathsf{p}_4} \quad I$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_4}\ \top_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_2\vdash\Delta_4}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_2\vdash\Delta_4}\ \top_L$$

#### 4.11 Status of $\wedge_L$ : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_4: \mathbf{F}_6, \Delta_1, \mathbf{F}_2 \wedge \mathbf{F}_3 \vdash \mathbf{F}_7, \Delta_5}{\bullet \mathbf{h}_4: \Delta_1, \mathbf{F}_2 \wedge \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_6 \rightarrow \mathbf{F}_7} \rightarrow_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: \Delta_1, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_6 \vdash \Delta_5, \mathbf{F}_7}}{\bullet \mathbf{h}_4: \Delta_1, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_6 \rightarrow \mathbf{F}_7} \rightarrow_R$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3\vdash\mathbf{F}_6,\Delta_5\quad\mathbf{h}_4:\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3\vdash\mathbf{F}_7,\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7}\quad\wedge_R\quad\quad \leadsto\quad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad\text{ax/ind}\quad\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_7}\quad\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_7}\quad \overset{\text{ax/ind}}{\wedge_R}\quad \wedge_R\quad\quad \Leftrightarrow\quad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad\mathbf{x}_1\wedge\mathbf{x}_2\wedge\mathbf$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{f}_2\wedge\mathbf{f}_3\vdash\mathbf{f}_6,\mathbf{f}_7,\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{f}_2\wedge\mathbf{f}_3\vdash\Delta_5,\mathbf{f}_6\vee\mathbf{f}_7} \ \vee_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{f}_2,\mathbf{f}_3\vdash\Delta_5,\mathbf{f}_6,\mathbf{f}_7}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{f}_2,\mathbf{f}_3\vdash\Delta_5,\mathbf{f}_6\vee\mathbf{f}_7} \ \frac{\mathsf{ax/ind}}{\vee_R}$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3\vdash\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3\vdash\bot,\Delta_5}\ \bot_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\Delta_5}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash\bot,\Delta_5}\ \underline{\bot_R}$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_4:\Delta_1,\mathbf{F}_2\wedge\mathbf{F}_3\vdash \top,\Delta_5} \ \top_R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_4:\Delta_1,\mathbf{F}_2,\mathbf{F}_3\vdash \top,\Delta_5} \ \top_R$$

ullet Case rule K

$$\frac{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Box \Gamma_4, \Delta_7, \mathbf{F}_1 \land \mathbf{F}_2 \vdash \Delta_5, []\mathbf{F}_6} \quad K \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \Box \Gamma_4 \vdash \Delta_5, []\mathbf{F}_6} \quad K$$

 $\bullet$  Case rule A45

$$\frac{h_3: \Box \Gamma_4 \vdash \Box \Gamma_5, F_7}{\bullet h_3: \Box \Gamma_4, \Delta_8, F_1 \land F_2 \vdash \Box \Gamma_5, \Delta_6, []F_7} \ \ ^{A45} \qquad \leadsto \qquad \frac{\overline{h_3: \Box \Gamma_4 \vdash F_7, \Box \Gamma_5}}{\bullet h_3: \Delta_8, F_1, F_2, \Box \Gamma_4 \vdash \Delta_6, \Box \Gamma_5, []F_7} \ \ ^{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6-\mathbf{h}_3:\mathbf{F}_5,\Delta_7,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\wedge\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \to_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \xrightarrow{\mathrm{ax/ind}} \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \xrightarrow{\mathrm{ax/ind}} \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6}$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7, \mathbf{F}_1 \wedge \mathbf{F}_2 \vdash \Delta_6 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \wedge \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \wedge \mathbf{F}_2), \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \mathbf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_6} \quad \vee_L \qquad \Rightarrow \qquad \frac{\overline$$

 $\bullet$  Case rule AT

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_6, ([\mathbf{F}_4, \mathbf{F}_1 \wedge \mathbf{F}_2 \vdash \Delta_5]}{\bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2), ([\mathbf{F}_4 \vdash \Delta_5]} \quad AT \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5]}}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5]} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6,$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\Delta_4} \ \bot_L \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_4} \ \bot_L$$

 $\bullet$  Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_4, \Delta_6, \mathsf{F}_1 \wedge \mathsf{F}_2 \vdash \mathsf{p}_4, \Delta_5} \quad I \qquad \rightsquigarrow \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_6, \mathsf{F}_1, \mathsf{F}_2, \mathsf{p}_4 \vdash \Delta_5, \mathsf{p}_4} \quad I$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\Delta_4}\ \top_L\qquad\rightsquigarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_4}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_4}\ \top_L$$

#### 4.12 Status of $\vee_L$ : (Left Premise): Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_4: \mathbf{F}_6, \Delta_1, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \mathbf{F}_7, \Delta_5}{\bullet \mathbf{h}_4: \Delta_1, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_6 \rightarrow \mathbf{F}_7} \rightarrow_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4: \Delta_1, \mathbf{F}_2, \mathbf{F}_6 \vdash \Delta_5, \mathbf{F}_7}}{\bullet \mathbf{h}_4: \Delta_1, \mathbf{F}_2 \vdash \Delta_5, \mathbf{F}_6 \rightarrow \mathbf{F}_7} \xrightarrow{\mathrm{ax/ind}}$$

• Case rule  $\wedge_R$ 

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{f}_2\vee\mathbf{f}_3\vdash\mathbf{f}_6,\mathbf{f}_7,\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{f}_2\vee\mathbf{f}_3\vdash\Delta_5,\mathbf{f}_6\vee\mathbf{f}_7} \ \vee_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{f}_2\vdash\Delta_5,\mathbf{f}_6,\mathbf{f}_7}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{f}_2\vdash\Delta_5,\mathbf{f}_6\vee\mathbf{f}_7} \overset{\mathrm{ax/ind}}{\vee_R}$$

• Case rule  $\perp_R$ 

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_4 : \Delta_1, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \top, \Delta_5} \ \top_R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_4 : \Delta_1, \mathbf{F}_2 \vdash \top, \Delta_5} \ \top_R$$

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Box \Gamma_4, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_5, []\mathbf{F}_6} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \Box \Gamma_4 \vdash \Delta_5, []\mathbf{F}_6} \quad K$$

• Case rule A45

$$\frac{h_3:\Box\Gamma_4\vdash\Box\Gamma_5,F_7}{\bullet h_3:\Box\Gamma_4,\Delta_8,F_1\vee F_2\vdash\Box\Gamma_5,\Delta_6,[]F_7} \ _{A45} \qquad \rightsquigarrow \qquad \frac{\overline{h_3:\Box\Gamma_4\vdash F_7,\Box\Gamma_5}}{\bullet h_3:\Delta_8,F_1,\Box\Gamma_4\vdash\Delta_6,\Box\Gamma_5,[]F_7} \ _{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6\quad\mathbf{a}_3:\mathbf{F}_5,\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \to_L \qquad \Leftrightarrow \qquad \frac{\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4}\quad\text{ax/ind}\quad \overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_5\vdash\Delta_6}\quad \overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_5\vdash\Delta_6} \to_L \\ \bullet \mathbf{b}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \to_L \\ \bullet \mathbf{b}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\to\mathbf{b}_6} \to_L \\ \bullet \mathbf{b}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\to\mathbf{b}_6$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2), \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6} \ \, \wedge_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4, \mathbf{F}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6} \ \, \wedge_L$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_6 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2), \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \Longrightarrow \qquad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}} \quad \times_L \quad \Longrightarrow \quad \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_3 \vee \mathbf{F}_4 \vee \mathbf{F}_5 \vee \mathbf{F}_5 \vee \mathbf{F}_6}$$

 $\bullet$  Case rule AT

$$\begin{array}{lll} \frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_6, []\mathbf{F}_4, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_5}{\bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2), []\mathbf{F}_4 \vdash \Delta_5} & AT & & \\ & & \bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_1, []\mathbf{F}_4 \vdash \Delta_5} & & AT & \\ \end{array} \quad \xrightarrow{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, []\mathbf{F}_4 \vdash \Delta_5} \quad \overset{\mathrm{ax/ind}}{\bullet} AT \\ \end{array}$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}^{} \ ^{\bot}L \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_1\vdash\Delta_4}^{} \ ^{\bot}L$$

ullet Case rule I

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}\ \top_L\qquad \leadsto\qquad \frac{\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\vdash\Delta_4}\ ^{\mathrm{ax/ind}}}{\bullet}_{\mathbf{L}}$$

#### 4.13 Status of $\vee_L$ (Right Premise): : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_4: \mathbf{F}_6, \Delta_1, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \mathbf{F}_7, \Delta_5}{\bullet \mathbf{h}_4: \Delta_1, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_6 \rightarrow \mathbf{F}_7} \rightarrow_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_4: \Delta_1, \mathbf{F}_3, \mathbf{F}_6 \vdash \Delta_5, \mathbf{F}_7} \quad \frac{\mathsf{ax/ind}}{\bullet \mathbf{h}_4: \Delta_1, \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_6 \rightarrow \mathbf{F}_7} \rightarrow_R$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_6,\Delta_5\quad\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_7,\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vee\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\wedge\mathbf{F}_7}\quad\wedge_R\quad\quad \leadsto\quad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad\text{ax/ind}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_7}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_7}\quad \overline{\wedge}_R\quad \wedge_R\quad \wedge_R\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_7}\quad \overline{\wedge}_R\quad \overline{\wedge}_R\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_7}\quad \overline{\wedge}_R\quad \overline{\wedge}_R\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad \overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6}\quad \overline{\wedge}_R\quad \overline$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vee\mathbf{F}_3\vdash\mathbf{F}_6,\mathbf{F}_7,\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vee\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\vee\mathbf{F}_7} \ \vee_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6,\mathbf{F}_7}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5,\mathbf{F}_6\vee\mathbf{F}_7} \overset{\mathrm{ax/ind}}{\vee_R}$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vee\mathbf{F}_3\vdash\Delta_5}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_2\vee\mathbf{F}_3\vdash\bot,\Delta_5}\ \bot_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\Delta_5}}{\bullet\mathbf{h}_4:\Delta_1,\mathbf{F}_3\vdash\bot,\Delta_5}\ ^{\mathrm{ax/ind}}$$

• Case rule  $\top_R$ 

ullet Case rule K

$$\frac{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_3: \Box \Gamma_4, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_5, []\mathbf{F}_6} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: unbox(\Box \Gamma_4) \vdash \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \Box \Gamma_4 \vdash \Delta_5, []\mathbf{F}_6} \quad K$$

• Case rule A45

$$\frac{h_3:\Box\Gamma_4\vdash\Box\Gamma_5,F_7}{\bullet h_3:\Box\Gamma_4,\Delta_8,F_1\vee F_2\vdash\Box\Gamma_5,\Delta_6,[]F_7} \ _{A45} \qquad \rightsquigarrow \qquad \frac{\overline{h_3:\Box\Gamma_4\vdash F_7,\Box\Gamma_5} \ _{ax}}{\bullet h_3:\Delta_8,F_2,\Box\Gamma_4\vdash\Delta_6,\Box\Gamma_5,[]F_7} \ _{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6\quad\mathbf{h}_3:\mathbf{F}_5,\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \to_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}\quad \text{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \xrightarrow{\mathbf{ax/ind}} \to_L$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_3: \mathbf{f}_4, \mathbf{f}_5, \Delta_7, \mathbf{f}_1 \vee \mathbf{f}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{f}_1 \vee \mathbf{f}_2), \mathbf{f}_4 \wedge \mathbf{f}_5 \vdash \Delta_6} \ \wedge_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4, \mathbf{f}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \wedge \mathbf{f}_5 \vdash \Delta_6} \ \wedge_L$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_6 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2), \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \text{ax/ind} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vee \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vee \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vee \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_4 \vee \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{h}_4 \vee \Delta_6} \quad \times_L = \frac{\mathbf{h}_3: \Delta_7, \mathbf{h}_4 \vee \Delta_6}{\bullet \mathbf{h}_4 \vee \Delta_6}$$

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2 \vdash \Delta_5 \quad \mathbf{h}_1: \mathbf{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3 \lor \mathbf{F}_4 \vdash \Delta_5} \quad \lor_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5}}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_4 \vdash \Delta_5} \quad \mathbf{H}$$

 $\bullet$  Case rule AT

$$\begin{array}{lll} \frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_6, ([\mathbf{F}_4, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_5}{\bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2), ([\mathbf{F}_4 \vdash \Delta_5})} & AT & & & & & & & \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5]}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5]} & AT & & & & & & & \\ \end{array}$$

• Case rule  $\perp_L$ 

$$\underbrace{ }_{\bullet \mathbf{h}_3 \ : \ \bot, \ \Delta_5, \ \mathbf{F}_1 \ \lor \ \mathbf{F}_2 \ \vdash \ \Delta_4 } \quad \bot_L \qquad \leadsto \qquad \underbrace{ \bullet_{\mathbf{h}_3 \ : \ \bot, \ \Delta_5, \ \mathbf{F}_2 \ \vdash \ \Delta_4 } }_{\bullet \mathbf{h}_3 \ : \ \bot, \ \Delta_5, \ \mathbf{F}_2 \ \vdash \ \Delta_4 } \quad \bot_L$$

 $\bullet$  Case rule I

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}\ \top_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_2\vdash\Delta_4}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_2\vdash\Delta_4}\ \top_L$$

#### 4.14 Status of AT: : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_3: \mathbf{f}_5, \Delta_1, []\mathbf{f}_2 \vdash \mathbf{f}_6, \Delta_4}{\bullet \mathbf{h}_3: \Delta_1, []\mathbf{f}_2 \vdash \Delta_4, \mathbf{f}_5 \rightarrow \mathbf{f}_6} \ \rightarrow_{R} \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_1, \mathbf{f}_2, \mathbf{f}_5, []\mathbf{f}_2 \vdash \Delta_4, \mathbf{f}_6} \ ^{\mathrm{ax/ind}}}{\bullet \mathbf{h}_3: \Delta_1, \mathbf{f}_2, []\mathbf{f}_2 \vdash \Delta_4, \mathbf{f}_5 \rightarrow \mathbf{f}_6} \xrightarrow{\mathrm{ax/ind}}$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{a}_3:\Delta_1, [[\mathsf{F}_2 \vdash \mathsf{F}_5, \Delta_4 \quad \mathsf{h}_3:\Delta_1, []\mathsf{F}_2 \vdash \mathsf{F}_6, \Delta_4}{\bullet \mathsf{h}_3:\Delta_1, []\mathsf{F}_2 \vdash \Delta_4, \mathsf{F}_5 \land \mathsf{F}_6} \quad \wedge_R \quad \Rightarrow \quad \frac{\overline{\mathsf{h}_3:\Delta_1, \mathsf{F}_2, []\mathsf{F}_2 \vdash \Delta_4, \mathsf{F}_5} \quad \frac{\mathsf{ax/ind}}{\bullet \mathsf{h}_3:\Delta_1, \mathsf{F}_2, []\mathsf{F}_2 \vdash \Delta_4, \mathsf{F}_5 \land \mathsf{F}_6} \quad \frac{\mathsf{ax/ind}}{\land_R} \quad \wedge_R \quad$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_3:\Delta_1, []\mathbf{F}_2 \vdash \mathbf{F}_5, \mathbf{F}_6, \Delta_4}{\bullet \mathbf{h}_3:\Delta_1, []\mathbf{F}_2 \vdash \Delta_4, \mathbf{F}_5 \vee \mathbf{F}_6} \quad \vee_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_1, \mathbf{F}_2, []\mathbf{F}_2 \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_6}}{\bullet \mathbf{h}_3:\Delta_1, \mathbf{F}_2, []\mathbf{F}_2 \vdash \Delta_4, \mathbf{F}_5 \vee \mathbf{F}_6} \quad \vee_R} \overset{\mathrm{ax/ind}}{} \vee_R$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_3:\Delta_1, []\mathbf{F}_2 \vdash \Delta_4}{\bullet \mathbf{h}_3:\Delta_1, []\mathbf{F}_2 \vdash \bot, \Delta_4} \ \bot_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_1, \mathbf{F}_2, []\mathbf{F}_2 \vdash \Delta_4}}{\bullet \mathbf{h}_3:\Delta_1, \mathbf{F}_2, []\mathbf{F}_2 \vdash \bot, \Delta_4} \ \bot_R$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_3:\Delta_1,\, []\mathbf{F}_2\vdash \top,\, \Delta_4} \ \ \, \top_R \qquad \rightsquigarrow \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_1,\, \mathbf{F}_2,\, []\mathbf{F}_2\vdash \top,\, \Delta_4} \ \ \, \top_R}$$

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_2: \mathbf{F}_1, unbox(\Box \Gamma_6) \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2: (\Box \Gamma_6, []\mathbf{F}_1), \Delta_3 \vdash \Delta_4, []\mathbf{F}_5} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_2: \mathbf{F}_1, unbox(\Box \Gamma_6) \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_2: \Delta_3, \mathbf{F}_1, \Box \Gamma_6, []\mathbf{F}_1 \vdash \Delta_4, []\mathbf{F}_5} \quad K$$

$$\frac{\mathbf{h}_2: unbox(\Box \Gamma_3) \vdash \mathbf{F}_5}{\bullet \mathbf{h}_2: \Box \Gamma_3, \Delta_6, []\mathbf{F}_1 \vdash \Delta_4, []\mathbf{F}_5} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_2: unbox(\Box \Gamma_3) \vdash \mathbf{F}_5}}{\bullet \mathbf{h}_2: \Delta_6, \mathbf{F}_1, \Box \Gamma_3, []\mathbf{F}_1 \vdash \Delta_4, []\mathbf{F}_5} \quad K$$

• Case rule A45

$$\frac{\mathtt{h}_2:\Box\Gamma_7, []\mathtt{F}_1\vdash\Box\Gamma_4, \mathtt{F}_6}{\bullet\mathtt{h}_2:(\Box\Gamma_7, []\mathtt{F}_1), \Delta_3\vdash\Box\Gamma_4, \Delta_5, []\mathtt{F}_6} \ \ A45 \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_2:\Box\Gamma_7, []\mathtt{F}_1\vdash\mathtt{F}_6, \Box\Gamma_4}}{\bullet\mathtt{h}_2:\Delta_3, \mathtt{F}_1, \Box\Gamma_7, []\mathtt{F}_1\vdash\Delta_5, \Box\Gamma_4, []\mathtt{F}_6} \ \ A45}$$

$$\frac{\mathtt{h}_2: \Box \Gamma_3 + \Box \Gamma_4, \mathtt{f}_6}{\bullet \mathtt{h}_2: \Box \Gamma_3, \Delta_7, []\mathtt{f}_1 + \Box \Gamma_4, \Delta_5, []\mathtt{f}_6} \quad ^{A45} \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_2: \Box \Gamma_3 + \mathtt{f}_6, \Box \Gamma_4} \quad ^{\mathtt{ax}}}{\bullet \mathtt{h}_2: \Delta_7, \mathtt{f}_1, \Box \Gamma_3, []\mathtt{f}_1 + \Delta_5, \Box \Gamma_4, []\mathtt{f}_6} \quad ^{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_2:\Delta_6,[]\mathbf{F}_1\vdash\mathbf{F}_3,\Delta_5\quad\mathbf{h}_2:\mathbf{F}_4,\Delta_6,[]\mathbf{F}_1\vdash\Delta_5}{\bullet\mathbf{h}_2:(\Delta_6,[]\mathbf{F}_1),\mathbf{F}_3\rightarrow\mathbf{F}_4\vdash\Delta_5}\rightarrow_L \qquad \leadsto \qquad \frac{\mathbf{h}_2:\Delta_6,\mathbf{F}_1,[]\mathbf{F}_1\vdash\Delta_5,\mathbf{F}_3}{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_1,[]\mathbf{F}_1,\mathbf{F}_3\rightarrow\mathbf{F}_4\vdash\Delta_5} \xrightarrow{\mathbf{h}_2:\Delta_6,\mathbf{F}_1,[]\mathbf{F}_1,\mathbf{F}_3\rightarrow\mathbf{F}_4\vdash\Delta_5} \xrightarrow{\mathbf{h}_2:\Delta_6,\mathbf{F}_1,[]\mathbf{F}_1,\mathbf{F}_3\rightarrow\mathbf{F}_4\vdash\Delta_5} \xrightarrow{\mathbf{h}_2:\Delta_6,\mathbf{F}_1,\mathbf{F}_4,\mathbf{$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_2: \mathbf{F}_3, \mathbf{F}_4, \Delta_6, []\mathbf{F}_1 \vdash \Delta_5}{\bullet \mathbf{h}_2: (\Delta_6, []\mathbf{F}_1), \mathbf{F}_3 \land \mathbf{F}_4 \vdash \Delta_5} \ \land_L \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, \mathbf{F}_3, \mathbf{F}_4, []\mathbf{F}_1 \vdash \Delta_5}{\bullet \mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \land \mathbf{F}_4 \vdash \Delta_5} \ \land_L}$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_2: \mathbf{F}_3, \Delta_6, []\mathbf{F}_1 \vdash \Delta_5 \quad \mathbf{h}_2: \mathbf{F}_4, \Delta_6, []\mathbf{F}_1 \vdash \Delta_5}{\bullet \mathbf{h}_2: (\Delta_6, []\mathbf{F}_1), \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, \mathbf{F}_3, []\mathbf{F}_1 \vdash \Delta_5} \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \Longrightarrow \quad \frac{\overline{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, \mathbf{F}_3, []\mathbf{F}_1 \vdash \Delta_5} \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \times_L \quad \Longrightarrow \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{H}_1, \mathbf{H}_2, \mathbf{$$

 $\bullet$  Case rule AT

$$\frac{\mathbf{h}_2: \mathbf{F}_3, \Delta_5, []\mathbf{F}_1, []\mathbf{F}_3 \vdash \Delta_4}{\bullet \mathbf{h}_2: (\Delta_5, []\mathbf{F}_1), []\mathbf{F}_3 \vdash \Delta_4} \quad AT \qquad \rightsquigarrow \qquad \frac{\mathbf{h}_2: \Delta_5, \mathbf{F}_1, \mathbf{F}_3, []\mathbf{F}_1, []\mathbf{F}_3 \vdash \Delta_4}{\bullet \mathbf{h}_2: \Delta_5, \mathbf{F}_1, []\mathbf{F}_1, []\mathbf{F}_3 \vdash \Delta_4} \quad AT \qquad AT$$

$$\begin{array}{ll} \underline{\mathbf{h}_1: \mathbf{F}_3, \Delta_2, []\mathbf{F}_3 \vdash \Delta_4} \\ \bullet \mathbf{h}_1: \Delta_2, []\mathbf{F}_3 \vdash \Delta_4 \end{array} \quad AT \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1: \Delta_2, \mathbf{F}_3, \mathbf{F}_3, []\mathbf{F}_3 \vdash \Delta_4}} {\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_3, []\mathbf{F}_3 \vdash \Delta_4} \end{array} \quad \overset{\mathrm{ax/ind}}{AT}$$

• Case rule  $\perp_L$ 

$$\overbrace{\bullet \mathbf{h}_2 : \bot, \Delta_4, [] \mathbf{f}_1 \vdash \Delta_3}^{} \ ^\bot L \qquad \leadsto \qquad \overline{\bullet \mathbf{h}_2 : \bot, \Delta_4, \mathbf{f}_1, [] \mathbf{f}_1 \vdash \Delta_3}^{} \ ^\bot L$$

ullet Case rule I

$$\overline{\bullet \mathtt{h}_2 : \mathtt{p}_3 \,, \Delta_5, \, []\mathtt{F}_1 \vdash \mathtt{p}_3, \Delta_4 } \quad I \qquad \rightsquigarrow \qquad \overline{\bullet \mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{p}_3, \, []\mathtt{F}_1 \vdash \Delta_4, \mathtt{p}_3 } \quad I$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_2:\Delta_4, []\mathbf{F}_1 \vdash \Delta_3}{\bullet \mathbf{h}_2:\top, \Delta_4, []\mathbf{F}_1 \vdash \Delta_3} \ \, \top_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2:\Delta_4, \mathbf{F}_1, []\mathbf{F}_1 \vdash \Delta_3}}{\bullet \mathbf{h}_2:\top, \Delta_4, \mathbf{F}_1, []\mathbf{F}_1 \vdash \Delta_3} \ \, \top_L$$

#### 4.15 Status of $\perp_L$ : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_2:\bot,\mathbf{F}_4,\Delta_1\vdash\mathbf{F}_5,\Delta_3}{\bullet\mathbf{h}_2:\bot,\Delta_1\vdash\Delta_3,\mathbf{F}_4\to\mathbf{F}_5}\ \to_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_2:\bot,\Delta_1 \vdash \mathbf{F}_4,\Delta_3 \quad \mathbf{h}_2:\bot,\Delta_1 \vdash \mathbf{F}_5,\Delta_3}{\bullet \mathbf{h}_2:\bot,\Delta_1 \vdash \Delta_3,\mathbf{F}_4 \land \mathbf{F}_5} \quad \land_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_2:\bot,\Delta_1\vdash\mathbf{F}_4,\mathbf{F}_5,\Delta_3}{\bullet\mathbf{h}_2:\bot,\Delta_1\vdash\Delta_3,\mathbf{F}_4\vee\mathbf{F}_5}\ \vee_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_2:\bot,\Delta_1\vdash\Delta_3}{\bullet\mathbf{h}_2:\bot,\Delta_1\vdash\bot,\Delta_3}\ \bot_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_2 : \bot, \Delta_1 \vdash \top, \Delta_3} \ \top_R \qquad \leadsto \qquad \mathsf{trivial}$$

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_2) \vdash \mathbf{F}_4}{\bullet \mathbf{h}_1: \Box \Gamma_2, \bot, \Delta_5 \vdash \Delta_3, []\mathbf{F}_4} \quad K \qquad \leadsto \qquad \mathsf{trivial}$$

 $\bullet$  Case rule A45

$$\frac{\mathbf{h}_1: \Box \Gamma_2 \vdash \Box \Gamma_3, \mathbf{F}_5}{\bullet \mathbf{h}_1: \Box \Gamma_2, \bot, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, []\mathbf{F}_5} \ A45 \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_1:\bot,\Delta_5\vdash\mathbf{F}_2,\Delta_4\quad\mathbf{h}_1:\bot,\mathbf{F}_3,\Delta_5\vdash\Delta_4}{\bullet\mathbf{h}_1:(\bot,\Delta_5),\mathbf{F}_2\to\mathbf{F}_3\vdash\Delta_4}\ \to_L \qquad \leadsto \qquad \text{trivial}$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_1: \bot, \mathbf{F}_2, \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: (\bot, \Delta_5), \mathbf{F}_2 \land \mathbf{F}_3 \vdash \Delta_4} \ \land_L \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_1:\bot,\mathbf{F}_2,\Delta_5\vdash\Delta_4\quad \mathbf{h}_1:\bot,\mathbf{F}_3,\Delta_5\vdash\Delta_4}{\bullet\mathbf{h}_1:(\bot,\Delta_5),\mathbf{F}_2\vee\mathbf{F}_3\vdash\Delta_4} \ \lor_L \qquad \leadsto \qquad \mathsf{trivial}$$

ullet Case rule AT

$$\begin{array}{ll} \frac{\mathbf{h}_1:\bot,\mathbf{F}_2,\Delta_4,[]\mathbf{F}_2\vdash\Delta_3}{\bullet\mathbf{h}_1:(\bot,\Delta_4),[]\mathbf{F}_2\vdash\Delta_3} & AT & \longrightarrow & \text{trivial} \end{array}$$

• Case rule  $\perp_L$ 

ullet Case rule I

$$\frac{}{ \bullet \mathbf{h}_1 : \mathbf{p}_2, \perp, \Delta_4 \vdash \mathbf{p}_2, \Delta_3} \quad I \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_1:\bot,\Delta_3\vdash\Delta_2}{\bullet\mathbf{h}_1:\top,\bot,\Delta_3\vdash\Delta_2}\ \top_L \qquad \leadsto \qquad \mathsf{trivial}$$

#### 4.16 Status of *I*:: Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_1, \mathbf{p}_2 \vdash \mathbf{F}_5, \Delta_6, \mathbf{p}_2}{\bullet \mathbf{h}_3: \Delta_1, \mathbf{p}_2 \vdash (\Delta_6, \mathbf{p}_2), \mathbf{F}_4 \to \mathbf{F}_5} \ \to_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{p}_2\vdash\mathbf{F}_4,\Delta_6,\mathbf{p}_2\quad \mathbf{h}_3:\Delta_1,\mathbf{p}_2\vdash\mathbf{F}_5,\Delta_6,\mathbf{p}_2}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{p}_2\vdash(\Delta_6,\mathbf{p}_2),\mathbf{F}_4\wedge\mathbf{F}_5} \quad \wedge_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{p}_2 \vdash \mathbf{F}_4,\mathbf{F}_5,\Delta_6,\mathbf{p}_2}{\bullet \mathbf{h}_3:\Delta_1,\mathbf{p}_2 \vdash (\Delta_6,\mathbf{p}_2),\mathbf{F}_4 \vee \mathbf{F}_5} \quad \vee_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_3:\Delta_1,\mathbf{p}_2\vdash\Delta_4,\mathbf{p}_2}{\bullet\mathbf{h}_3:\Delta_1,\mathbf{p}_2\vdash\bot,\Delta_4,\mathbf{p}_2}\ \bot_R \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\top_R$ 

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_2: unbox(\Box \Gamma_3) \vdash \mathbf{F}_4}{\bullet \mathbf{h}_2: \Box \Gamma_3, \Delta_6, \mathbf{p}_1 \vdash (\Delta_5, \mathbf{p}_1), []\mathbf{F}_4} \quad \leftarrow \quad \text{trivial}$$

• Case rule A45

$$\frac{\mathbf{h}_2: \Box \Gamma_3 \vdash \Box \Gamma_4, \mathbf{F}_5}{\bullet \mathbf{h}_2: \Box \Gamma_3, \Delta_7, \mathbf{p}_1 \vdash \Box \Gamma_4, (\Delta_6, \mathbf{p}_1), []\mathbf{F}_5} \quad \textit{A45} \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_3:\Delta_6,\mathbf{p}_1\vdash \mathbf{F}_4,\Delta_2,\mathbf{p}_1\quad \mathbf{h}_3:\mathbf{F}_5,\Delta_6,\mathbf{p}_1\vdash \Delta_2,\mathbf{p}_1}{\bullet \mathbf{h}_3:(\Delta_6,\mathbf{p}_1),\mathbf{F}_4\to \mathbf{F}_5\vdash \Delta_2,\mathbf{p}_1}\ \to_L \qquad \leadsto \qquad \text{trivial}$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \mathbf{F}_5, \Delta_6, \mathbf{p}_1 \vdash \Delta_2, \mathbf{p}_1}{\bullet \mathbf{h}_3: (\Delta_6, \mathbf{p}_1), \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_2, \mathbf{p}_1} \ \wedge_L \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_6, \mathbf{p}_1 \vdash \Delta_2, \mathbf{p}_1 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_6, \mathbf{p}_1 \vdash \Delta_2, \mathbf{p}_1}{\bullet \mathbf{h}_3: (\Delta_6, \mathbf{p}_1), \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_2, \mathbf{p}_1} \quad \vee_L \qquad \leadsto \qquad \mathsf{trivial}$$

 $\bullet$  Case rule AT

$$\begin{array}{ll} \frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_5, \mathbf{p}_1, []\mathbf{F}_4 \vdash \Delta_2, \mathbf{p}_1}{\bullet \mathbf{h}_3: (\Delta_5, \mathbf{p}_1), []\mathbf{F}_4 \vdash \Delta_2, \mathbf{p}_1} & AT & \longrightarrow & \text{trivial} \end{array}$$

• Case rule  $\perp_L$ 

$$\overbrace{ \bullet \mathbf{h}_3 : \bot, \Delta_4, \mathbf{p}_1 \vdash \Delta_2, \mathbf{p}_1 }^{} \ \ \, \bot_L \qquad \leadsto \qquad \mathbf{trivial}$$

 $\bullet$  Case rule I

$$\frac{}{\bullet \mathbf{h}_2: \mathbf{p}_3, \Delta_5, \mathbf{p}_1 \vdash \mathbf{p}_3, \Delta_4, \mathbf{p}_1} \quad I \qquad \leadsto \qquad \mathsf{trivial}$$

• Case rule  $\top_L$ 

$$\frac{\mathbf{h}_3:\Delta_4,\mathbf{p}_1\vdash\Delta_2,\mathbf{p}_1}{\bullet\mathbf{h}_3:\top,\Delta_4,\mathbf{p}_1\vdash\Delta_2,\mathbf{p}_1}\ \top_L \qquad \leadsto \qquad \mathsf{trivial}$$

### 4.17 Status of $\top_L$ : Invertible

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_2: \top, \mathbf{F}_4, \Delta_1 \vdash \mathbf{F}_5, \Delta_3}{\bullet \mathbf{h}_2: \top, \Delta_1 \vdash \Delta_3, \mathbf{F}_4 \to \mathbf{F}_5} \ \rightarrow_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_2: \Delta_1, \mathbf{F}_4 \vdash \Delta_3, \mathbf{F}_5}}{\bullet \mathbf{h}_2: \Delta_1 \vdash \Delta_3, \mathbf{F}_4 \to \mathbf{F}_5} \overset{\mathsf{ax/ind}}{\to}_R$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_2: \top, \Delta_1 \vdash \mathbf{F}_4, \Delta_3 \quad \mathbf{h}_2: \top, \Delta_1 \vdash \mathbf{F}_5, \Delta_3}{\bullet \mathbf{h}_2: \top, \Delta_1 \vdash \Delta_3, \mathbf{F}_4 \land \mathbf{F}_5} \quad \wedge_R \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_2: \Delta_1 \vdash \Delta_3, \mathbf{F}_4} \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_1 \vdash \Delta_3, \mathbf{F}_4 \land \mathbf{F}_5} \quad \frac{\mathbf{ax/ind}}{\wedge_R} \quad \wedge_R \quad$$

• Case rule  $\vee_R$ 

$$\frac{\mathbf{h}_2: \top, \Delta_1 \vdash \mathbf{F}_4, \mathbf{F}_5, \Delta_3}{\bullet \mathbf{h}_2: \top, \Delta_1 \vdash \Delta_3, \mathbf{F}_4 \vee \mathbf{F}_5} \quad \vee_R \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_2: \Delta_1 \vdash \Delta_3, \mathbf{F}_4, \mathbf{F}_5}{\bullet \mathbf{h}_2: \Delta_1 \vdash \Delta_3, \mathbf{F}_4 \vee \mathbf{F}_5}}{\bullet \mathbf{h}_2: \Delta_1 \vdash \Delta_3, \mathbf{F}_4 \vee \mathbf{F}_5} \overset{\mathsf{ax/ind}}{\vee}_R$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_2: \top, \Delta_1 \vdash \Delta_3}{\bullet \mathbf{h}_2: \top, \Delta_1 \vdash \bot, \Delta_3} \ \bot_R \qquad \leadsto \qquad \frac{\frac{\mathbf{h}_2: \Delta_1 \vdash \Delta_3}{\bullet \mathbf{h}_2: \Delta_1 \vdash \bot, \Delta_3} \ \mathbf{ax/ind}}{\bullet \mathbf{h}_2: \Delta_1 \vdash \bot, \Delta_3} \ \bot_R$$

• Case rule  $\top_R$ 

$$\frac{}{\bullet \mathbf{h}_2 : \top, \Delta_1 \vdash \top, \Delta_3} \ \top_R \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_2 : \Delta_1 \vdash \top, \Delta_3} \ \top_R$$

 $\bullet$  Case rule K

$$\frac{\mathsf{h}_1: unbox(\Box \Gamma_2) \vdash \mathsf{F}_4}{\bullet \mathsf{h}_1: \Box \Gamma_2, \top, \Delta_5 \vdash \Delta_3, []\mathsf{F}_4} \quad K \qquad \rightsquigarrow \qquad \frac{\overline{\mathsf{h}_1: unbox(\Box \Gamma_2) \vdash \mathsf{F}_4}}{\bullet \mathsf{h}_1: \Delta_5, \Box \Gamma_2 \vdash \Delta_3, []\mathsf{F}_4} \stackrel{\mathsf{ax}}{} K$$

 $\bullet$  Case rule A45

$$\frac{\mathtt{h}_1: \Box \mathtt{r}_2 \vdash \Box \mathtt{r}_3, \mathtt{f}_5}{\bullet \mathtt{h}_1: \Box \mathtt{r}_2, \top, \Delta_6 \vdash \Box \mathtt{r}_3, \Delta_4, []\mathtt{f}_5} \quad \textit{A45} \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_1: \Box \mathtt{r}_2 \vdash \mathtt{f}_5, \Box \mathtt{r}_3} \quad ^{\mathsf{ax}}}{\bullet \mathtt{h}_1: \Delta_6, \Box \mathtt{r}_2 \vdash \Delta_4, \Box \mathtt{r}_3, []\mathtt{f}_5} \quad \textit{A45}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\mathbf{h}_1: \top, \Delta_5 \vdash \mathbf{F}_2, \Delta_4 \quad \mathbf{h}_1: \top, \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: (\top, \Delta_5), \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \ \rightarrow_L \qquad \rightsquigarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5 \vdash \Delta_4, \mathbf{F}_2} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{ax/ind}} \quad \xrightarrow{\Delta_1: \Delta_2 \vdash \Delta_2, \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_2 \vdash \Delta_3, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_3 \vdash \Delta_4, \mathbf{F}_2} \quad \xrightarrow{\mathbf{h}_1: \Delta_3 \vdash \Delta_4, \mathbf{F}_2} \quad \xrightarrow{\mathbf{h}_1: \Delta_3 \vdash \Delta_4, \mathbf{F}_2} \quad \xrightarrow{\mathbf{h}_1: \Delta_5 \vdash \Delta_4, \mathbf{h}_2} \quad \xrightarrow{\mathbf{h}_1: \Delta$$

• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_1: \top, \mathbf{f}_2, \mathbf{f}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: (\top, \Delta_5), \mathbf{f}_2 \wedge \mathbf{f}_3 \vdash \Delta_4} \ \, \wedge_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{f}_2, \mathbf{f}_3 \vdash \Delta_4}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{f}_2 \wedge \mathbf{f}_3 \vdash \Delta_4} \overset{\mathrm{ax/ind}}{\wedge}_L$$

• Case rule  $\vee_L$ 

$$\frac{\mathbf{h}_1: \top, \mathbf{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathbf{h}_1: \top, \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: (\top, \Delta_5), \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \leadsto \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \\$$

 $\bullet$  Case rule AT

$$\begin{array}{lll} \frac{\mathbf{h}_1: \top, \mathbf{F}_2, \Delta_4, ( \| \mathbf{F}_2 \vdash \Delta_3 \|}{\bullet \mathbf{h}_1: (\top, \Delta_4), ( \| \mathbf{F}_2 \vdash \Delta_3 \|} & AT & & \\ & & & & \\ \hline \bullet \mathbf{h}_1: \Delta_4, \mathbf{F}_2, ( \| \mathbf{F}_2 \vdash \Delta_3 \|} & AT & & \\ \end{array} \qquad \xrightarrow{\bullet \mathbf{h}_1: \Delta_4, \mathbf{F}_2, ( \| \mathbf{F}_2 \vdash \Delta_3 \|} & \frac{\mathsf{ax/ind}}{AT} \\ \end{array}$$

• Case rule  $\perp_L$ 

$$\frac{}{\bullet \mathbf{h}_1:\bot,\top,\Delta_3\vdash \Delta_2} \ ^\bot L \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_1:\bot,\Delta_3\vdash \Delta_2} \ ^\bot L$$

ullet Case rule I

$$\frac{}{\bullet \mathbf{h}_1: \mathbf{p}_2, \top, \Delta_4 \vdash \mathbf{p}_2, \Delta_3} \quad I \qquad \leadsto \qquad \frac{}{\bullet \mathbf{h}_1: \Delta_4, \mathbf{p}_2 \vdash \Delta_3, \mathbf{p}_2} \quad I$$

• Case rule  $\top_L$ 

## 5 Identity-Expansion

$$\frac{\frac{-: F_0 \vdash F_0}{-: []F_0 \vdash []F_0} \ IH}{\frac{-: F_0 \vdash F_0}{-: F_0 \vdash F_0} \ W} \frac{\frac{-: F_1 \vdash F_1}{-: F_1 \vdash F_0, F_1} \ W}{\frac{-: F_0 \vdash F_0, F_1}{-: F_0 \lor F_1 \vdash F_0, F_1}} \bigvee_{L} W$$

$$\frac{\frac{-: F_0 \vdash F_0}{-: F_0 \lor F_1 \vdash F_0, F_1} \lor_{R}}{\frac{-: F_0 \vdash F_0}{-: F_0, F_1 \vdash F_0} \ W} \frac{\frac{-: F_1 \vdash F_1}{-: F_0, F_1 \vdash F_1} \ W}{\frac{-: F_0, F_1 \vdash F_0 \land F_1}{-: F_0 \land F_1} \land_{L}} \bigvee_{L} W$$

$$\frac{\frac{-: F_0 \vdash F_0}{-: F_0 \vdash F_0, F_1} \ IH}{\frac{-: F_0 \vdash F_0, F_1}{-: F_0 \vdash F_0, F_1 \vdash F_1} \xrightarrow{L} W} \bigvee_{L} W$$

$$\frac{-: F_0 \vdash F_0, F_1}{-: F_0 \vdash F_0, F_1 \vdash F_0 \to F_1} \xrightarrow{L} W$$

$$\frac{-: F_0 \vdash F_0, F_1 \vdash F_0 \to F_1}{-: F_0 \to F_1 \vdash F_0 \to F_1} \xrightarrow{L} W$$

#### 6 Cut-Elimination

#### 6.1 Status of $\rightarrow_R$ : OK

• Case rule  $\rightarrow_R$ 

$$\begin{array}{c} \frac{h_1:F_7,\Delta_6 \vdash F_8,\Delta_{10},F_{11} \to F_{12}}{\bullet h_1:\Delta_6 \vdash (\Delta_{10},F_{11} \to F_{12}),F_7 \to F_8} \to_R & \frac{h_9:F_{11},\Delta_6,F_7 \to F_8 \vdash F_{12},\Delta_{10}}{\bullet h_9:\Delta_6,F_7 \to F_8 \vdash \Delta_{10},F_{11} \to F_{12}} \\ \hline -:\Delta_6 \vdash \Delta_{10},F_{11} \to F_{12} \\ \hline \frac{h_1:\Delta_6,F_{11},F_7 \vdash \Delta_{10},F_{12},F_8}{\bullet h_1:\Delta_6,F_{11} \vdash \Delta_{10},F_{12},F_8} & \frac{inv - th/ax}{h_9:\Delta_6,F_{11},F_7 \to F_8 \vdash \Delta_{10},F_{12}} \\ \hline \frac{h_1:\Delta_6,F_{11},F_7 \vdash \Delta_{10},F_{12},F_8}{\bullet h_1:\Delta_6,F_{11} \vdash \Delta_{10},F_{12},F_7 \to F_8} & \frac{h_9:\Delta_6,F_{11},F_7 \to F_8 \vdash \Delta_{10},F_{12}}{h_9:\Delta_6,F_{11},F_7 \to F_8 \vdash \Delta_{10},F_{12}} & \frac{ax/W}{hCut} \\ \hline -:\Delta_6 \vdash \Delta_{10},F_{11} \to F_{12} \\ \hline \hline h_2:F_9,\Delta_8 \vdash F_7,F_{10},\Delta_{14},F_{12} \to F_{13} \\ \hline -:\Delta_8 \vdash (\Delta_{14},F_{12} \to F_{13}),F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash (\Delta_{14},F_{12} \to F_{13}),F_9 \to F_{10} \\ \hline \hline -:\Delta_8 \vdash (\Delta_{14},F_{12} \to F_{13}),F_9 \to F_{10} \\ \hline \hline -:\Delta_8 \vdash \Delta_{14},F_{13},F_7,F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{14},F_{12} \to F_{13},F_9 \to F_{10} \\ \hline -:\Delta_8 \vdash \Delta_{10},F_{11} \to F_{12} \\ \hline -:\Delta_8 \vdash \Delta_{10},F_{11} \to F_{12}$$

• Case rule  $\wedge_R$ 

$$\frac{ \begin{array}{c} h_1 : F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \land F_{12} \\ \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \land F_{12}), F_7 \to F_8 \\ \hline \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \land F_{12}), F_7 \to F_8 \\ \hline \\ - : \Delta_6 \vdash \Delta_{10}, F_{11} \land F_{12} \\ \hline \\ \bullet h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_8 \\ \hline \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \land F_{12}), F_7 \to F_8 \\ \hline \\ \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11}, F_8) \\ \hline \\ \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11}, F_8) \\ \hline \\ \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11}, F_8) \\ \hline \\ \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11}, F_8) \\ \hline \\ \bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11}, F_8) \\ \hline \\ \bullet h_2 : \Delta_6 \vdash (\Delta_{10}, F_{11}, F_8) \\ \hline \\ \bullet h_2 : F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \land F_{13} \\ \hline \\ \bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \land F_{13}), F_9 \to F_{10}), F_7 \\ \hline \\ \bullet h_1 : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \land F_{13}), F_9 \to F_{10} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F_{13} \\ \hline \\ \bullet h_1 : \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \land F$$

• Case rule  $\vee_R$ 

$$\begin{array}{c} \frac{h_1: F_7, \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \lor F_{12}}{\bullet h_1: \Delta_6 \vdash (\Delta_{10}, F_{11} \lor F_{12}), F_7 \to F_8} & \frac{h_9: \Delta_6, F_7 \to F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9: \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11} \lor F_{12}} & \vee_R \\ \hline & -: \Delta_6 \vdash \Delta_{10}, F_{11} \lor F_{12} & \text{Cut} \\ \hline \frac{h_1: \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}{\bullet h_1: \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_8} & \frac{\text{inv-th/ax}}{\bullet h_1: \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \to F_8} & h_9: \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11}, F_{12} \\ \hline & \frac{-: \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}}{-: \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}} & \vee_R \\ \hline & \frac{h_2: F_9, \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \lor F_{13}}{-: \Delta_8 \vdash (\Delta_{14}, F_{12} \lor F_{13}), F_9 \to F_{10}} & \rightarrow_R & \frac{h_{11}: F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \to F_{10}}{\bullet h_{11}: \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \lor F_{13}), F_9 \to F_{10}} & \vee_R \\ \hline & \frac{h_2: \Delta_8 \vdash ((\Delta_{14}, F_{12} \lor F_{13}), F_9 \to F_{10}), F_7}{-: \Delta_8 \vdash (\Delta_{14}, F_{12} \lor F_{13})} & ax/W & \frac{h_{11}: \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \lor F_{13}}{\bullet h_{11}: \Delta_8, F_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \lor F_{13}} & \frac{\text{inv-th/ax}}{\vee_R} \\ \hline & \frac{-: \Delta_8 \vdash \Delta_{14}, F_9 \to F_{10}, F_{12} \lor F_{13}}{-: \Delta_8 \vdash \Delta_{14}, F_9 \to F_{10}, F_{12} \lor F_{13}} & \rightarrow_R \\ \hline \end{array}$$

#### • Case rule $\perp_R$

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_7, \Delta_6 \vdash \mathbf{F}_8, \bot, \Delta_{10}}{\bullet \mathbf{h}_1: \Delta_6 \vdash (\bot, \Delta_{10}), \mathbf{F}_7 \to \mathbf{F}_8} \xrightarrow{} \frac{\mathbf{h}_9: \Delta_6, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_{10}}{\bullet \mathbf{h}_9: \Delta_6, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \bot, \Delta_{10}} \xrightarrow{} \underset{\mathbf{Cut}}{\bot_R} \\ \hline -: \Delta_6 \vdash \bot, \Delta_{10} & \\ \hline \bullet \mathbf{h}_1: \Delta_6 \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & \mathbf{ax/W} \xrightarrow{} \frac{\mathbf{h}_9: \Delta_6, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \bot, \Delta_{10}}{\bullet \mathbf{h}_2: \Delta_6 \vdash \bot, \Delta_{10}} \xrightarrow{\mathbf{ax/W}} \underbrace{\mathbf{hCut}} \\ \hline \bullet \mathbf{h}_2: \mathbf{F}_9, \Delta_8 \vdash \mathbf{F}_7, \mathbf{F}_{10}, \bot, \Delta_{12} & \rightarrow_R & \underbrace{\mathbf{h}_{11}: \mathbf{F}_7, \Delta_8 \vdash \Delta_{12}, \mathbf{F}_9 \to \mathbf{F}_{10}}_{\bullet \mathbf{h}_{11}: \Delta_8, \mathbf{F}_7 \vdash (\bot, \Delta_{12}), \mathbf{F}_9 \to \mathbf{F}_{10}} \xrightarrow{\mathbf{Cut}} \\ \hline \bullet \mathbf{h}_2: \Delta_8 \vdash ((\bot, \Delta_{12}), \mathbf{F}_9 \to \mathbf{F}_{10}) & \xrightarrow{\bullet} \underbrace{\mathbf{h}_{11}: \Delta_8, \mathbf{F}_7 \vdash (\bot, \Delta_{12}), \mathbf{F}_9 \to \mathbf{F}_{10}}_{\bullet \mathbf{h}_{21}: \Delta_8 \vdash \bot, \Delta_{12}, \mathbf{F}_9 \to \mathbf{F}_{10}} \xrightarrow{\mathbf{ax/W}} \underbrace{\mathbf{hCut}} \\ \hline \bullet \mathbf{h}_2: \Delta_8 \vdash \bot, \Delta_{12}, \mathbf{F}_7, \mathbf{F}_9 \to \mathbf{F}_{10} & \underbrace{\mathbf{ax/W}}_{\mathbf{h}_{11}: \Delta_8, \mathbf{F}_7 \vdash \bot, \Delta_{12}, \mathbf{F}_9 \to \mathbf{F}_{10}}_{\bullet \mathbf{hCut}} \xrightarrow{\mathbf{ax/W}} \underbrace{\mathbf{hCut}}_{\mathbf{hCut}} \\ \hline \bullet \mathbf{h}_2: \Delta_8 \vdash \bot, \Delta_{12}, \mathbf{F}_7, \mathbf{F}_9 \to \mathbf{F}_{10} & \underbrace{\mathbf{ax/W}}_{\mathbf{hCut}} & \underbrace{\mathbf{hCut}}_{\mathbf{hCut}} \\ -: \Delta_8 \vdash \bot, \Delta_{12}, \mathbf{F}_9 \to \mathbf{F}_{10} & \underbrace{\mathbf{hCut}}_{\mathbf{hCut}} & \underbrace{\mathbf{hCut}}_{\mathbf{hCut}} \\ -: \Delta_8 \vdash \bot, \Delta_{12}, \mathbf{F}_9 \to \mathbf{F}_{10} & \underbrace{\mathbf{hCut}}_{\mathbf{hCut}} & \underbrace{\mathbf{hCut}}_{\mathbf{hCut}} \\ -: \Delta_8 \vdash \bot, \Delta_{12}, \mathbf{F}_9 \to \mathbf{F}_{10} & \underbrace{\mathbf{hCut}}_{\mathbf{hCut}} & \underbrace{\mathbf{hCut}}_{\mathbf{hCut}} \\ -: \Delta_8 \vdash \bot, \Delta_{12}, \mathbf{F}_9 \to \mathbf{F}_{10} & \underbrace{\mathbf{hCut}}_{\mathbf{hCut}} & \underbrace{\mathbf{hCut$$

# • Case rule $\top_R$

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_7, \Delta_6 \vdash \mathbf{F}_8, \top, \Delta_{10}}{\bullet \mathbf{h}_1: \Delta_6 \vdash (\top, \Delta_{10}), \mathbf{F}_7 \to \mathbf{F}_8} \xrightarrow{\bullet_R} \begin{array}{c} \bullet \mathbf{h}_9: \Delta_6, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \top, \Delta_{10} \\ & -: \Delta_6 \vdash \top, \Delta_{10} \\ & & -: \Delta_6 \vdash \top, \Delta_{10} \end{array} \begin{array}{c} \top_R \\ \text{Cut} \end{array}$$

#### $\bullet$ Case rule K

$$\frac{\begin{array}{l} \mathbf{h}_1: \mathbf{F}_6, \Box \Gamma_9, \Delta_{12} \vdash \mathbf{F}_7, \Delta_{10}, []\mathbf{F}_{11} \\ \bullet \mathbf{h}_1: \Box \Gamma_9, \Delta_{12} \vdash (\Delta_{10}, []\mathbf{F}_{11}), \mathbf{F}_6 \to \mathbf{F}_7 \end{array} \xrightarrow{\bullet}_R \begin{array}{l} \mathbf{h}_8: unbox(\Box \Gamma_9) \vdash \mathbf{F}_{11} \\ \bullet \mathbf{h}_8: (\Box \Gamma_9, \Delta_{12}), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \Delta_{10}, []\mathbf{F}_{11} \\ -: \Box \Gamma_9, \Delta_{12} \vdash \Delta_{10}, []\mathbf{F}_{11} \\ \hline -: unbox(\Box \Gamma_9) \vdash \mathbf{F}_{11} \\ \hline -: \Delta_{12}, \Box \Gamma_9 \vdash \Delta_{10}, []\mathbf{F}_{11} \end{array} \begin{array}{l} K \\ \text{Cut} \end{array}$$

#### • Case rule A45

$$\begin{array}{c} \underline{\mathbf{h}_1: \mathsf{F}_6, \Box \mathsf{P}_9, \Delta_{13} \vdash \mathsf{F}_7, \Box \mathsf{P}_{10}, \Delta_{11}, [\mathsf{F}_{12})}_{\bullet \mathsf{h}_1: \Box \mathsf{P}_9, \Delta_{13} \vdash (\Box \mathsf{P}_{10}, \Delta_{11}, [\mathsf{F}_{12}), \mathsf{F}_6 \to \mathsf{F}_7} \\ -: \Box \mathsf{P}_9, \Delta_{13} \vdash (\Box \mathsf{P}_{10}, \Delta_{11}, [\mathsf{F}_{12}), \mathsf{F}_6 \to \mathsf{F}_7} \\ -: \Box \mathsf{P}_9, \Delta_{13} \vdash (\Box \mathsf{P}_{10}, \Delta_{11}, [\mathsf{F}_{12}), \mathsf{F}_6 \to \mathsf{F}_7 \vdash (\Box \mathsf{P}_{10}, \Delta_{11}, [\mathsf{F}_{12}), \mathsf{F}_6 \to \mathsf{F}_7 \vdash (\Box \mathsf{P}_{10}, \Delta_{11}, [\mathsf{F}_{12}), \mathsf{F}_6 \to \mathsf{F}_7 \vdash (\Box \mathsf{P}_{10}, \Delta_{11}, [\mathsf{F}_{12}), \mathsf{F}_6 \to \mathsf{F}_7), \mathsf{F}_7 \vdash (\Box \mathsf{P}_{12}, \mathsf{P}_{13}, \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Delta_{13}, \Box \mathsf{P}_9 \vdash \Delta_{11}, \Box \mathsf{P}_{10}, [\mathsf{F}_{12}] \\ -: \Delta_{13}, \Box \mathsf{P}_9 \vdash \Delta_{11}, \Box \mathsf{P}_{10}, [\mathsf{F}_{12}] \\ -: \Delta_{13}, \Box \mathsf{P}_9 \vdash \Delta_{11}, \Box \mathsf{P}_{10}, [\mathsf{F}_{12}] \\ -: \Delta_{13}, \Box \mathsf{P}_9 \vdash \Delta_{11}, \Box \mathsf{P}_{10}, [\mathsf{F}_{12}] \\ -: \Delta_{13}, \Box \mathsf{P}_9 \vdash \Delta_{11}, \Box \mathsf{P}_{10}, [\mathsf{F}_{12}] \\ -: \Delta_{13}, \Box \mathsf{P}_9 \vdash \Delta_{11}, \Box \mathsf{P}_{10}, [\mathsf{F}_{12}] \\ -: \Delta_{14}, [\mathsf{F}_{13}), \mathsf{F}_8 \to \mathsf{F}_9), \Box \mathsf{F}_7 \\ -: \Delta_{15}, \Delta_{11} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{F}_{13}), \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{15}, \Delta_{11} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{F}_{13}), \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{15}, \Delta_{11} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{F}_{13}), \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Delta_{14}, \mathsf{F}_8, \Box \mathsf{F}_7, \Delta_{14}, \mathsf{F}_9, \Box \mathsf{F}_{12}, [\mathsf{F}_{13}] \\ -: \Delta_{14}, \mathsf{F}_9, \Box \mathsf{F}_{12}, [\mathsf{F}_{13}] \\ -: \Delta_{14}, \mathsf{F}_9, \Box \mathsf{F}_{12}, \Delta_{14}, [\mathsf{F}_{13}) \\ -: \Delta_{14}, \Box \mathsf{F}_{12}, [\mathsf{F}_{13}, \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{11}, \Delta_{15} \vdash \mathsf{F}_7, \mathsf{F}_9, \Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{F}_{13}) \\ -: \Box \mathsf{P}_{11}, \Delta_{15} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{F}_{13}), \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{11}, \Delta_{15} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{F}_{13}), \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{11}, \Delta_{15} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{F}_{13}), \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{11}, \Delta_{15} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{F}_{13}), \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{11}, \Delta_{15} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{F}_{13}), \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{11}, \Delta_{15} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{P}_{13}, \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{11}, \Delta_{15} \vdash (\Box \mathsf{P}_{12}, \Delta_{14}, [\mathsf{P}_{13}, \mathsf{F}_8 \to \mathsf{F}_9) \\ -: \Box \mathsf{P}_{11}, \Delta_{15} \vdash (\Box \mathsf{P$$

• Case rule  $\rightarrow_L$ 

$$\frac{\frac{\mathbf{h}_1: \mathbf{f}_6, \Delta_{12}, \mathbf{F}_9 \rightarrow \mathbf{f}_{10} \vdash \mathbf{F}_7, \Delta_{11}}{\bullet \mathbf{h}_1: \Delta_{12}, \mathbf{f}_9 \rightarrow \mathbf{f}_{10} \vdash \Delta_{11}, \mathbf{f}_6 \rightarrow \mathbf{F}_7} \rightarrow_R \frac{\mathbf{h}_8: \Delta_{12}, \mathbf{f}_6 \rightarrow \mathbf{F}_7 \vdash \mathbf{F}_9, \Delta_{11}}{\bullet \mathbf{h}_8: (\Delta_{12}, \mathbf{F}_9 \rightarrow \mathbf{f}_{10}), \mathbf{f}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_{11}}}{\bullet \mathbf{h}_8: (\Delta_{12}, \mathbf{F}_9 \rightarrow \mathbf{f}_{10}), \mathbf{f}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_{11}} \underbrace{\mathbf{Cut}} \rightarrow_L \underbrace{\frac{\mathbf{h}_1: \Delta_{12}, \mathbf{f}_6 \vdash \Delta_{11}, \mathbf{F}_7, \mathbf{F}_9}{\bullet \mathbf{h}_1: \Delta_{12}, \mathbf{f}_6 \vdash \Delta_{11}, \mathbf{F}_7, \mathbf{F}_9}}_{\bullet \mathbf{h}_1: \Delta_{12}, \mathbf{f}_6 \vdash \Delta_{11}, \mathbf{f}_9, \mathbf{f}_6 \rightarrow \mathbf{F}_7} \rightarrow_R \underbrace{\frac{\mathbf{ax}/\mathbf{W}}{\mathbf{h}_8: \Delta_{12}, \mathbf{f}_6 \rightarrow \mathbf{f}_7 \vdash \Delta_{11}, \mathbf{f}_9}}_{\bullet \mathbf{h}_8: \Delta_{12}, \mathbf{f}_6 \rightarrow \mathbf{f}_7 \vdash \Delta_{11}, \mathbf{f}_9} \underbrace{\frac{\mathbf{ax}/\mathbf{W}}{\mathbf{h}_{\mathbf{Cut}}}} \underbrace{\frac{\mathbf{h}_1: \Delta_{12}, \mathbf{f}_{10}, \mathbf{f}_6 \vdash \Delta_{11}, \mathbf{f}_7}{\bullet \mathbf{h}_1: \Delta_{12}, \mathbf{f}_{10} \vdash \Delta_{11}, \mathbf{f}_6 \rightarrow \mathbf{f}_7} \rightarrow_R \underbrace{\frac{\mathbf{h}_8: \Delta_{12}, \mathbf{f}_{10}, \mathbf{f}_6 \rightarrow \mathbf{f}_7 \vdash \Delta_{11}}_{\bullet \mathbf{h}_6: \mathbf{f}_7, \mathbf{f}_8 \rightarrow \mathbf{f}_9 \rightarrow \mathbf{f}_{10} \vdash \Delta_{11}}} \underbrace{-: \Delta_{12}, \mathbf{f}_{10} \vdash \Delta_{11}, \mathbf{f}_6 \rightarrow \mathbf{f}_7} \rightarrow_R \underbrace{\frac{\mathbf{h}_8: \Delta_{12}, \mathbf{f}_{10}, \mathbf{f}_6 \rightarrow \mathbf{f}_7 \vdash \Delta_{10}}_{\bullet \mathbf{h}_6: \mathbf{f}_7, \mathbf{f}_8 \rightarrow \mathbf{f}_9 \rightarrow \mathbf{f}_{10} \vdash \Delta_{11}}} \rightarrow_L \underbrace{-: \Delta_{12}, \mathbf{f}_{10} \vdash \Delta_{11}}_{\bullet \mathbf{h}_6: \mathbf{f}_7, \mathbf{f}_8 \rightarrow \mathbf{f}_9 \rightarrow \mathbf{f}_{10} \vdash \Delta_{11}}} \rightarrow_L \underbrace{-: \Delta_7, \mathbf{f}_8 \vdash \Delta_{10}, \mathbf{f}_9}_{\bullet \mathbf{h}_6: \Delta_7, \mathbf{f}_8 \rightarrow \mathbf{f}_9 \rightarrow \mathbf{h}_{10}}} \underbrace{-: \Delta_7, \mathbf{f}_8 \vdash \Delta_{10}}_{\bullet \mathbf{h}_6: \mathbf{f}_7, \mathbf{f}_8 \rightarrow \mathbf{f}_9 \rightarrow \mathbf{h}_{10}}}_{\bullet \mathbf{Cut}} \underbrace{-: \Delta_7, \mathbf{f}_8 \vdash \Delta_{10}, \mathbf{f}_9}_{\bullet \mathbf{f}_9 \rightarrow \mathbf{f}_{10} \rightarrow \mathbf{h}_{10}}} \underbrace{-: \Delta_7, \mathbf{f}_8 \vdash \Delta_{10}}_{\bullet \mathbf{f}_9 \rightarrow \mathbf{f}_{10} \rightarrow \mathbf{h}_{10}}}_{\bullet \mathbf{Cut}}$$

 $-:\Delta_7 \vdash \Delta_{10}$ 

$$\frac{\frac{h_2: F_8, \Delta_{11} \vdash F_{12} \to F_{13}, F_9, \Delta_7}{\bullet h_2: \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), F_{12} \to F_{13}}}{\bullet h_2: \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), F_{12} \to F_{13}}} \xrightarrow{P_R} \frac{h_{10}: \Delta_{11} \vdash F_{12}, \Delta_7, F_8 \to F_9}{\bullet h_{10}: \Delta_{11}, F_{12} \to F_{13} \vdash \Delta_7, F_8 \to F_9}} \underbrace{Cut} \xrightarrow{-: \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \underbrace{-: \Delta_{11} \vdash \Delta_7, F_8 \to F_9}}_{h_{10}: \Delta_{11}, F_8 \vdash \Delta_7, F_{12}, F_9}} \xrightarrow{h_{10}: \Delta_{11}, F_8 \vdash \Delta_7, F_9} \xrightarrow{h_{10}: \Delta_{11}, F_8, F_{12} \to F_{13} \vdash \Delta_7, F_9}}_{h_{10}: \Delta_{11}, F_8, F_{12} \to F_{13} \vdash \Delta_7, F_9}} \xrightarrow{h_{10}: \Delta_{11}, F_8, F_{12} \to F_{13} \vdash \Delta_7, F_9}}_{h_{10}: \Delta_{11}, F_8, F_{12} \to F_{13} \vdash \Delta_7, F_9}} \xrightarrow{h_{10}: \Delta_{11}, F_8, F_{12} \to F_{13} \vdash \Delta_7, F_9}}_{h_{10}: \Delta_{11}, F_8, F_{12} \to F_{13} \vdash \Delta_8, F_9 \to F_{10}}}_{h_{11}: (\Delta_{14}, F_{12} \to F_{13}), F_7 \vdash \Delta_8, F_9 \to F_{10}}} \xrightarrow{h_{11}: \Delta_{14}, F_{12} \to F_{13}}_{h_{11}: \Delta_{14}, F_{12} \to F_{13}}}_{h_{11}: \Delta_{14}, F_{12} \to F_{13}}_{h_{11}: \Delta_8, F_9 \to F_{10}}}_{h_{11}: \Delta_{14}, F_{12} \to F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}_{h_{11}: \Delta_{14}, F_7, F_9, F_{12} \to F_{13} \vdash \Delta_8, F_{10}}_{h_{11}: \Delta_{$$

### • Case rule $\wedge_L$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \mathsf{F}_6, \Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10} \vdash \mathsf{F}_7, \Delta_{11}}{\bullet \mathsf{h}_1 : \Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10} \vdash \Delta_{11}, \mathsf{F}_6 \to \mathsf{F}_7} \to_R & \frac{\mathbf{h}_8 : \mathsf{F}_9, \mathsf{F}_{10}, \Delta_{12}, \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_{11}}{\bullet \mathsf{h}_8 : (\Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10}), \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_{11}} & \wedge_L \\ & - : \Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10} \vdash \Delta_{11} & \\ & \frac{\mathsf{h}_1 : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_6, \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_7}{\bullet \mathsf{h}_1 : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_7} & \to_R \\ & \frac{\mathsf{h}_8 : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_9, \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_{11}}{\bullet \mathsf{h}_2 : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_9 \vdash \Delta_{11}} \wedge_L & \mathsf{ax/W} \\ & \frac{- : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_9 \vdash \Delta_{11}}{- : \Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10} \vdash \Delta_{11}} \wedge_L & \mathsf{ax/W} \\ & \frac{- : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_9 \vdash \Delta_{11}}{\bullet \mathsf{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathsf{F}_8 \to \mathsf{F}_9), \mathsf{F}_{12} \wedge \mathsf{F}_{13}} \to_R & \frac{\mathsf{h}_{10} : \mathsf{F}_{12}, \mathsf{F}_{13}, \Delta_{11} \vdash \Delta_7, \mathsf{F}_8 \to \mathsf{F}_9}{\bullet \mathsf{h}_{10} : \Delta_{11}, \mathsf{F}_{12} \wedge \mathsf{F}_{13} \vdash \Delta_7, \mathsf{F}_8 \to \mathsf{F}_9} & \wedge_L \\ & \frac{\mathsf{cut}}{- : \Delta_{11} \vdash \Delta_7, \mathsf{F}_8 \to \mathsf{F}_9} & \wedge_R & \frac{\mathsf{h}_{10} : \mathsf{F}_{12}, \mathsf{F}_{13}, \Delta_{11} \vdash \Delta_7, \mathsf{F}_8 \to \mathsf{F}_9}{\bullet \mathsf{h}_{10} : \Delta_{11}, \mathsf{F}_{12}, \mathsf{F}_{13}, \mathsf{F}_8 \vdash \Delta_7, \mathsf{F}_9} & \wedge_L \\ & \frac{\mathsf{cut}}{- : \Delta_{11} \vdash \Delta_7, \mathsf{F}_8 \to \mathsf{F}_9} & \wedge_R & \frac{\mathsf{h}_{10} : \mathsf{F}_{12}, \mathsf{F}_{13} \vdash \Delta_7, \mathsf{F}_9}{\bullet \mathsf{h}_{10} : \Delta_{11}, \mathsf{F}_{12}, \mathsf{F}_{13} \vdash \Delta_7, \mathsf{F}_9} & \wedge_L \\ & \frac{\mathsf{cut}}{- : \Delta_{11}, \mathsf{F}_8 \vdash \Delta_7, \mathsf{F}_9} & \to_R & \frac{\mathsf{h}_{10} : \Delta_{11}, \mathsf{F}_{12}, \mathsf{F}_{13}, \mathsf{F}_8 \vdash \Delta_7, \mathsf{F}_9}{\bullet \mathsf{h}_{10} : \Delta_{11}, \mathsf{F}_{13}, \mathsf{F}_{13} \vdash \Delta_7, \mathsf{F}_9} & \wedge_L \\ & \frac{\mathsf{cut}}{- : \Delta_{11}, \mathsf{F}_8 \vdash \Delta_7, \mathsf{F}_9} & \to_R & \frac{\mathsf{h}_{10} : \Delta_{11}, \mathsf{F}_{12}, \mathsf{F}_{13}, \Delta_{14} \vdash \Delta_8, \mathsf{F}_9 \to \mathsf{F}_{10}}{\bullet \mathsf{h}_{11} : (\Delta_{14}, \mathsf{F}_{12}, \mathsf{F}_{13}, \Delta_14 \vdash \Delta_8, \mathsf{F}_9 \to \mathsf{F}_{10}} & \wedge_L \\ & \frac{\mathsf{cut}}{- : \Delta_{14}, \mathsf{F}_{12} \wedge \mathsf{F}_{13} \vdash \Delta_8, \mathsf{F}_{10}, \mathsf{F}_7} & \frac{\mathsf{h}_{11} : \Delta_{14}, \mathsf{F}_{12}, \mathsf{F}_{13}, \mathsf{F}_7, \mathsf{F}_9 \vdash \Delta_8, \mathsf{F}_{10}}{\bullet \mathsf{h}_{11} : \Delta_{14}, \mathsf{F}_{12}, \mathsf{F}_{13} \vdash \Delta_8, \mathsf{F}_{10}} & \wedge_L \\ & \frac{\mathsf{h}_{11} : \Delta_{14}, \mathsf{F}_{12}, \mathsf{F}_{13} \vdash \Delta_8, \mathsf{F}_{10}}{\bullet \mathsf{h}_{11}, \mathsf{F}_{13}, \mathsf{F}_9 \to \mathsf{F}_{10}} & \wedge_$$

#### • Case rule $\vee_L$

$$\frac{\frac{h_{1}:F_{6},\Delta_{12},F_{9}\vee F_{10}\vdash F_{7},\Delta_{11}}{\bullet h_{1}:\Delta_{12},F_{9}\vee F_{10}\vdash \Delta_{11},F_{6}\to F_{7}}}{\bullet h_{1}:\Delta_{12},F_{9}\vee F_{10}\vdash \Delta_{11},F_{6}\to F_{7}}} \xrightarrow{A_{R}} \frac{h_{8}:F_{9},\Delta_{12},F_{6}\to F_{7}\vdash \Delta_{11}}{\bullet h_{8}:(\Delta_{12},F_{9}\vee F_{10}),F_{6}\to F_{7}\vdash \Delta_{11}}} \text{Cut}}{-:\Delta_{12},F_{9}\vee F_{10}\vdash \Delta_{11}} \xrightarrow{h_{1}:\Delta_{12},F_{6}\to F_{7}\vdash \Delta_{11}} \frac{\text{inv-th/ax}}{\bullet h_{1}:\Delta_{12},F_{9}\vdash \Delta_{11},F_{6}\to F_{7}} \xrightarrow{A_{R}} \frac{\text{ax/W}}{h_{1}:\Delta_{12},F_{10}\vdash \Delta_{11},F_{10}\vdash \Delta_{11},F_{10}\vdash \Delta_{11},F_{10}\vdash \Delta_{11}}} \xrightarrow{-:\Delta_{12},F_{9}\vdash \Delta_{11}} \vee_{L}$$

# $\bullet$ Case rule AT

$$\frac{ \frac{h_1 : F_6, \Delta_{11}, []F_9 \vdash F_7, \Delta_{10}}{\bullet h_1 : \Delta_{11}, []F_9 \vdash \Delta_{10}, F_6 \to F_7} \to_R \quad \frac{h_8 : F_9, \Delta_{11}, []F_9, F_6 \to F_7 \vdash \Delta_{10}}{\bullet h_8 : (\Delta_{11}, []F_9), F_6 \to F_7 \vdash \Delta_{10}} \quad AT \\ - : \Delta_{11}, []F_9 \vdash \Delta_{10} \\ \hline \bullet h_1 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}, F_6 \to F_7 \\ \hline - : \Delta_{11}, []F_9 \vdash \Delta_{10} \quad ATG \\ \hline - : \Delta_{11}, []F_9 \vdash \Delta_{10} \quad ATG \\ \hline \frac{h_2 : F_8, \Delta_{11} \vdash []F_{12}, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), []F_1} \to_R \quad \frac{h_{10} : F_{12}, \Delta_{11}, []F_{12} \vdash \Delta_7, F_8 \to F_9}{\bullet h_{10} : \Delta_{11}, []F_{12} \vdash \Delta_7, F_8 \to F_9} \quad AT \\ \hline - : \Delta_{11} \vdash \Delta_7, F_8 \to F_9 \\ \hline - : \Delta_{11} \vdash \Delta_7, F_8 \to F_9 \\ \hline - : \Delta_{11}, F_8 \vdash \Delta_7, F_9 \\ \hline - : \Delta_{11}, F_8 \to \Delta_7, F_9 \\ \hline - : \Delta_{11}, F_8 \to \Phi_7, F_9 \to F_{10} \\ \hline - : \Delta_{12}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to \Delta_8, F_9 \to F_{10} \\ \hline - : \Delta_{13}, F_{12} \to$$

# • Case rule $\perp_L$

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_6, \bot, \Delta_{10} \vdash \mathbf{F}_7, \Delta_9}{\bullet \mathbf{h}_1: \bot, \Delta_{10} \vdash \Delta_9, \mathbf{F}_6 \to \mathbf{F}_7} \to_R & \frac{}{\bullet \mathbf{h}_8: (\bot, \Delta_{10}), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \Delta_9} & \overset{\bot_L}{\subset} \\ \hline & -: \bot, \Delta_{10} \vdash \Delta_9 & \overset{}{\sim} \\ \hline & -: \bot, \Delta_{10} \vdash \Delta_9 & \overset{}{\searrow} \\ \hline & \frac{}{-: \bot, \Delta_{10} \vdash \Delta_9} & \bot_L \\ \\ \hline \frac{\mathbf{h}_2: \mathbf{F}_8, \Delta_{11} \vdash \bot, \mathbf{F}_9, \Delta_7}{\bullet \mathbf{h}_2: \Delta_{11} \vdash (\Delta_7, \mathbf{F}_8 \to \mathbf{F}_9), \bot} \to_R & \frac{}{\bullet \mathbf{h}_{10}: \Delta_{11}, \bot \vdash \Delta_7, \mathbf{F}_8 \to \mathbf{F}_9} & \overset{\bot_L}{\subset} \\ \hline & -: \Delta_{11} \vdash \Delta_7, \mathbf{F}_8 \to \mathbf{F}_9 \\ \hline & \frac{}{\bullet \mathbf{h}_2: \Delta_{11}, \mathbf{F}_8 \vdash \bot, \Delta_7, \mathbf{F}_9} & \mathbf{ax/W} & \overset{}{\bullet} \mathbf{h}_{10}: \bot, \Delta_{11}, \mathbf{F}_8 \vdash \Delta_7, \mathbf{F}_9} & \overset{\bot_L}{\to} \\ \hline & \frac{}{-: \Delta_{11}, \mathbf{F}_8 \vdash \Delta_7, \mathbf{F}_9} & \to_R \\ \hline \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_2: \mathbf{F}_9, \bot, \Delta_{12} \vdash \mathbf{F}_7, \mathbf{F}_{10}, \Delta_8 \\ \hline \bullet \mathbf{h}_2: \bot, \Delta_{12} \vdash (\Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10}), \mathbf{F}_7 \end{array} \rightarrow_R \frac{ }{\bullet \mathbf{h}_{11}: (\bot, \Delta_{12}), \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} }{ -: \bot, \Delta_{12} \vdash \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10}} \begin{array}{c} \bot_L \\ \hline \\ -: \bot, \Delta_{12} \vdash \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \end{array} \end{array}} \begin{array}{c} \bot_L \\ \text{Cut} \end{array}$$

• Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1 : \mathbf{F}_6, \Delta_{11}, \mathbf{p}_9 \vdash \mathbf{F}_7, \Delta_{10}, \mathbf{p}_9}{\bullet \mathbf{h}_1 : \Delta_{11}, \mathbf{p}_9 \vdash (\Delta_{10}, \mathbf{p}_9), \mathbf{F}_6 \to \mathbf{F}_7} \xrightarrow{\bullet_R} \frac{\mathbf{h}_8 : (\Delta_{11}, \mathbf{p}_9), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \Delta_{10}, \mathbf{p}_9}{\bullet \mathbf{h}_8 : (\Delta_{11}, \mathbf{p}_9), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \Delta_{10}, \mathbf{p}_9} \xrightarrow{Cut} \\ & -: \Delta_{11}, \mathbf{p}_9 \vdash \Delta_{10}, \mathbf{p}_9 & I \\ \hline \frac{\mathbf{h}_2 : \mathbf{F}_7, \Delta_{10} \vdash \mathbf{p}_{11}, \mathbf{F}_8, \Delta_{12}, \mathbf{p}_{11}}{\bullet \mathbf{h}_2 : \Delta_{10} \vdash ((\Delta_{12}, \mathbf{p}_{11}), \mathbf{F}_7 \to \mathbf{F}_8), \mathbf{p}_{11}} \xrightarrow{\bullet_R} \frac{\mathbf{h}_9 : \Delta_{10}, \mathbf{p}_{11} \vdash (\Delta_{12}, \mathbf{p}_{11}), \mathbf{F}_7 \to \mathbf{F}_8}{\bullet \mathbf{h}_9 : \Delta_{10}, \mathbf{F}_7, \mathbf{p}_{11} \vdash \Delta_{12}, \mathbf{F}_8, \mathbf{p}_{11}} \xrightarrow{I} \text{Cut}} \\ \hline \frac{\mathbf{h}_2 : \Delta_{10} \vdash ((\Delta_{12}, \mathbf{F}_{11}), \mathbf{p}_{11}}{\bullet (\mathbf{h}_2 : \Delta_{10}, \mathbf{F}_7 \vdash \Delta_{12}, \mathbf{F}_8, \mathbf{p}_{11}, \mathbf{p}_{11}} \xrightarrow{\bullet_R} \xrightarrow{\bullet_{10} : (\Delta_{10}, \mathbf{F}_7, \mathbf{p}_{11} \vdash \Delta_{12}, \mathbf{F}_8, \mathbf{p}_{11})} \xrightarrow{I} \text{hCut}} \\ \hline \frac{-: \Delta_{10}, \mathbf{F}_7 \vdash \Delta_{12}, \mathbf{F}_8, \mathbf{p}_{11}, \mathbf{p}_{11}}{\bullet (\mathbf{h}_2 : \Delta_{10}, \mathbf{F}_7, \mathbf{F}_9, \Delta_{12}, \mathbf{p}_{11}, \mathbf{F}_7 \to \mathbf{F}_8}} \xrightarrow{\bullet_{10} : (\Delta_{13}, \mathbf{p}_{11}), \mathbf{F}_7 \vdash (\Delta_{12}, \mathbf{p}_{11}), \mathbf{F}_8 \to \mathbf{F}_9}} \xrightarrow{I} \text{Cut}} \\ \hline \bullet \mathbf{h}_2 : \mathbf{F}_8, \Delta_{13}, \mathbf{p}_{11} \vdash ((\Delta_{12}, \mathbf{p}_{11}), \mathbf{F}_8 \to \mathbf{F}_9), \mathbf{F}_7} \xrightarrow{\bullet} \mathbf{h}_{10} : (\Delta_{13}, \mathbf{p}_{11}), \mathbf{F}_7 \vdash (\Delta_{12}, \mathbf{p}_{11}), \mathbf{F}_8 \to \mathbf{F}_9}} \xrightarrow{I} \text{Cut}} \\ \hline \bullet \mathbf{h}_2 : \Delta_{13}, \mathbf{h}_{11} \vdash ((\Delta_{12}, \mathbf{h}_{11}), \mathbf{h}_8 \to \mathbf{h}_9), \mathbf{h}_7} \xrightarrow{\bullet} \mathbf{h}_{10} : (\Delta_{13}, \mathbf{h}_{11}), \mathbf{h}_7 \vdash (\Delta_{12}, \mathbf{h}_{11}), \mathbf{h}_8 \to \mathbf{h}_9}} \xrightarrow{I} \text{Cut}}$$

• Case rule  $\top_L$ 

$$\begin{array}{c} \frac{h_1:F_6,\top,\Delta_{10}\vdash F_7,\Delta_9}{\bullet h_1:\top,\Delta_{10}\vdash \Delta_9,F_6\to F_7}\to R & \frac{h_8:\Delta_{10},F_6\to F_7\vdash \Delta_9}{\bullet h_8:(\top,\Delta_{10}),F_6\to F_7\vdash \Delta_9} & \top_L \\ \hline -:\top,\Delta_{10}\vdash \Delta_9 & \text{cut} \\ \hline \\ \bullet h_1:\top,\Delta_{10}\vdash \Delta_9,F_6\to F_7 & \text{ax/W} & h_8:\top,\Delta_{10},F_6\to F_7\vdash \Delta_9 \\ \hline -:\top,\Delta_{10}\vdash \Delta_9 & \text{hcut} \\ \hline \\ \bullet h_2:F_8,\Delta_{11}\vdash \top,F_9,\Delta_7 & h_{10}:\Delta_{11}\vdash \Delta_7,F_8\to F_9 \\ \hline \bullet h_2:\Delta_{11}\vdash (\Delta_7,F_8\to F_9),\top & \bullet h_{10}:\Delta_{11}\vdash \Delta_7,F_8\to F_9 \\ \hline \\ -:\Delta_{11}\vdash \Delta_7,F_8\to F_9 & \text{ax/W} \\ \hline \\ \bullet h_2:F_9,\top,\Delta_{12}\vdash F_7,F_{10},\Delta_8 & h_{11}:F_7,\Delta_{12}\vdash \Delta_8,F_9\to F_{10} \\ \hline \bullet h_2:\top,\Delta_{12}\vdash (\Delta_8,F_9\to F_{10}),F_7 & \bullet h_{11}:(\top,\Delta_{12}),F_7\vdash \Delta_8,F_9\to F_{10} \\ \hline \\ \bullet h_2:\top,\Delta_{12}\vdash \Delta_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_1:\top,\Delta_{12}\vdash F_7,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash \Delta_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash \Delta_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash \Delta_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash \Delta_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash \Delta_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline \\ \bullet h_2:T,\Delta_{12}\vdash A_8,F_9\to F_{10} & \text{ax/W} \\ \hline$$

# 6.2 Status of $\wedge_R$ : OK

• Case rule  $\rightarrow_R$ 

$$\frac{\frac{h_1: \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \to F_{12} \quad h_1: \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \to F_{12}}{\bullet h_1: \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7 \land F_8} \land_R \quad \frac{h_9: F_{11}, \Delta_6, F_7 \land F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9: \Delta_6, F_7 \land F_8 \vdash \Delta_{10}, F_{11} \to F_{12}} \\ \hline \\ \frac{-: \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}}{\bullet} \\ \hline \frac{h_1: \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7}{\bullet h_1: \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_8} \quad \frac{\text{inv-th/ax}}{\land_R} \\ \hline \\ \frac{\bullet h_1: \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \land F_8}{\bullet h_1: \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_{12}} \to_R} \\ \hline \\ \frac{-: \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{-: \Delta_6 \vdash \Delta_{10}, F_{11}} \to_{R}} \\ \hline$$

$$\frac{\frac{h_2:\Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \to F_{13} \quad h_2:\Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \to F_{13}}{\bullet h_2:\Delta_8 \vdash ((\Delta_{14}, F_{12} \to F_{13}), F_9 \land F_{10}), F_7} \quad \wedge_R \quad \frac{h_{11}:F_7, F_{12},\Delta_8 \vdash F_{13}, \Delta_{14}, F_9 \land F_{10}}{\bullet h_{11}:\Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_9 \land F_{10}} \quad \to_R \quad \text{Cut}}{-:\Delta_8 \vdash ((\Delta_{14}, F_{12} \to F_{13}), F_9 \land F_{10}} \quad \frac{h_2:\Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7}{\bullet h_2:\Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_7} \quad \frac{\text{inv-th/ax}}{\land_R} \quad \frac{h_{11}:\Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \land F_{10}}{\bullet h_{11}:\Delta_8, F_{12}, F_7 \vdash \Delta_{14}, F_{13}, F_9 \land F_{10}} \quad \frac{\text{ax/W}}{\land_{Cut}} \quad \frac{-:\Delta_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \land F_{10}}{-:\Delta_8 \vdash \Delta_{14}, F_{12} \to F_{13}, F_9 \land F_{10}} \quad \to_R}$$

### • Case rule $\wedge_R$

$$\frac{\mathbf{h}_{1}:\Delta_{6} \vdash F_{7},\Delta_{10},F_{11} \land F_{12} \quad \mathbf{h}_{1}:\Delta_{6} \vdash F_{8},\Delta_{10},F_{11} \land F_{12}}{\bullet \mathbf{h}_{1}:\Delta_{6} \vdash (\Delta_{10},F_{11} \land F_{12}),F_{7} \land F_{8}} } \wedge_{R} \frac{\mathbf{h}_{9}:\Delta_{6},F_{7} \land F_{8} \vdash F_{11},\Delta_{10} \quad \mathbf{h}_{9}:\Delta_{6},F_{7} \land F_{8} \vdash F_{12},\Delta_{10}}{\bullet \mathbf{h}_{9}:\Delta_{6},F_{7} \land F_{8} \vdash \Delta_{10},F_{11} \land F_{12}} } \wedge_{R} \frac{\mathbf{h}_{11}:\Delta_{6} \vdash (\Delta_{10},F_{11} \land F_{12}),F_{7} \land F_{8}} }{-:\Delta_{6} \vdash \Delta_{10},F_{11} \land F_{12}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{6} \vdash F_{11},\Delta_{10} \quad \mathbf{h}_{9}:\Delta_{6},F_{7} \land F_{8} \vdash \Delta_{10},F_{11} \land F_{12}} }{-:\Delta_{6} \vdash \Delta_{10},F_{11} \land F_{12}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{6} \vdash F_{7},F_{8} \vdash \Delta_{10},F_{11}} {-:\Delta_{6},F_{7},F_{8} \vdash \Delta_{10},F_{11} \land F_{12}} }{-:\Delta_{6} \vdash \Delta_{10},F_{11} \land F_{12}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{6} \vdash F_{7},F_{8} \vdash \Delta_{10},F_{11} \land F_{12}} {-:\Delta_{6},F_{7},F_{8} \vdash \Delta_{10},F_{11} \land F_{12}}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{6} \vdash F_{7},F_{10},\Delta_{14},F_{12} \land F_{13}} {-:\Delta_{6},F_{7},F_{8} \vdash \Delta_{10},F_{11} \land F_{12}}} {-:\Delta_{6},F_{7},F_{8} \vdash \Delta_{10},F_{11} \land F_{12}}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{6} \vdash F_{7},F_{10},\Delta_{14},F_{12} \land F_{13}} {-:\Delta_{6},F_{7},F_{8} \vdash \Delta_{10},F_{11} \land F_{12}}} {-:\Delta_{6},F_{7},F_{8} \vdash \Delta_{10},F_{11} \land F_{12}}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{6} \vdash F_{7},F_{10},\Delta_{14},F_{12} \land F_{13}} {-:\Delta_{6},F_{7},F_{8} \vdash \Delta_{10},F_{11} \land F_{12}} } {-:\Delta_{6},F_{7},F_{8} \vdash \Delta_{10},F_{11} \land F_{12}}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{6} \vdash F_{7},F_{10},\Delta_{14},F_{12} \land F_{13}} {-:\Delta_{8} \vdash (\Delta_{10},F_{11} \land F_{12},F_{7},F_{9} \land F_{10})} }{-:\Delta_{8} \vdash \Delta_{14},F_{12},F_{7},F_{9} \land F_{10}}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{8} \vdash \Delta_{14},F_{12},F_{7},F_{9} \land F_{10}} {-:\Delta_{8} \vdash \Delta_{14},F_{12},F_{9} \land F_{10}}} {-:\Delta_{8} \vdash \Delta_{14},F_{12},F_{9} \land F_{10}}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{8} \vdash \Delta_{14},F_{12},F_{7},F_{9} \land F_{10}} }{-:\Delta_{8} \vdash \Delta_{14},F_{12},F_{7},F_{9} \land F_{10}}} \mathbf{x}^{\mathsf{W}} \frac{-:\Delta_{8} \vdash \Delta_{14},F_{12},F_{13},F$$

### • Case rule $\vee_R$

$$\frac{h_1: \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \lor F_{12} \quad h_1: \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \lor F_{12}}{\bullet h_1: \Delta_6 \vdash (\Delta_{10}, F_{11} \lor F_{12}), F_7 \land F_8} \land_R \quad \frac{h_9: \Delta_6, F_7 \land F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9: \Delta_6, F_7 \land F_8 \vdash \Delta_{10}, F_{11} \lor F_{12}} \lor_R \\ \hline -: \Delta_6 \vdash \Delta_{10}, F_{11} \lor F_{12} \\ \hline \frac{h_1: \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7}{\bullet h_1: \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_8} \quad \frac{inv - th/ax}{\land_R} \\ \hline \frac{\bullet h_1: \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \land F_8}{\bullet h_1: \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \land F_8} \quad \frac{inv - th/ax}{\land_R} \\ \hline \frac{-: \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}}{-: \Delta_6 \vdash \Delta_{10}, F_{11}, V_{12}} \lor_R \\ \hline \frac{h_2: \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \lor F_{13} \quad h_2: \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \lor F_{13}}{-: \Delta_8 \vdash (\Delta_{14}, F_{12} \lor F_{13}), F_9 \land F_{10}} \\ \hline \frac{\bullet h_2: \Delta_8 \vdash (\Delta_{14}, F_{12} \lor F_{13}), F_9 \land F_{10}}{\bullet h_2: \Delta_8 \vdash (\Delta_{14}, F_{12} \lor F_{13}), F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash (\Delta_{14}, F_{12} \lor F_{13}), F_9 \land F_{10}} \\ \hline \frac{\bullet h_2: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \land F_{10}}{-: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}}} \\ \hline \frac{\bullet h_2: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \land F_{10}}{-: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}}} \\ \hline \frac{\bullet h_2: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \land F_{10}}{-: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \land F_{10}} \\ \hline -: \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_$$

#### • Case rule $\perp_R$

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{6}\vdash F_{7},\bot,\Delta_{10}\quad \mathbf{h}_{1}:\Delta_{6}\vdash F_{8},\bot,\Delta_{10}}{\bullet \mathbf{h}_{1}:\Delta_{6}\vdash (\bot,\Delta_{10}),F_{7}\wedge F_{8}} & \wedge_{R} & \frac{\mathbf{h}_{9}:\Delta_{6},F_{7}\wedge F_{8}\vdash \Delta_{10}}{\bullet \mathbf{h}_{9}:\Delta_{6},F_{7}\wedge F_{8}\vdash \bot,\Delta_{10}} & \bot_{R} \\ \hline & -:\Delta_{6}\vdash \bot,\Delta_{10} \\ \hline & \frac{\bullet \mathbf{h}_{1}:\Delta_{6}\vdash \bot,\Delta_{10},F_{7}\wedge F_{8}}{\bullet \mathbf{h}_{1}:\Delta_{6}\vdash \bot,\Delta_{10},F_{7}\wedge F_{8}} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{9}:\Delta_{6},F_{7}\wedge F_{8}\vdash \bot,\Delta_{10}} & \frac{\mathbf{ax/W}}{\bullet \mathbf{hCut}} \\ \hline & \frac{\mathbf{h}_{2}:\Delta_{8}\vdash F_{7},F_{9},\bot,\Delta_{12}\quad \mathbf{h}_{2}:\Delta_{8}\vdash F_{7},F_{10},\bot,\Delta_{12}}{\bullet \mathbf{h}_{2}:\Delta_{8}\vdash ((\bot,\Delta_{12}),F_{9}\wedge F_{10}),F_{7}} & \wedge_{R} & \frac{\mathbf{h}_{11}:F_{7},\Delta_{8}\vdash \Delta_{12},F_{9}\wedge F_{10}}{\bullet \mathbf{h}_{11}:\Delta_{8},F_{7}\vdash (\bot,\Delta_{12}),F_{9}\wedge F_{10}} & \bot_{R} \\ \hline & \frac{\bullet \mathbf{h}_{2}:\Delta_{8}\vdash ((\bot,\Delta_{12}),F_{9}\wedge F_{10}),F_{7}}{\bullet \mathbf{h}_{2}:\Delta_{8}\vdash (\bot,\Delta_{12}),F_{9}\wedge F_{10}} & \mathbf{ax/W} \\ \hline & \frac{\bullet \mathbf{h}_{2}:\Delta_{8}\vdash \bot,\Delta_{12},F_{7},F_{9}\wedge F_{10}}{\bullet \mathbf{h}_{11}:\Delta_{8},F_{7}\vdash \bot,\Delta_{12},F_{9}\wedge F_{10}} & \mathbf{ax/W} \\ \hline & -:\Delta_{8}\vdash \bot,\Delta_{12},F_{9}\wedge F_{10} & \mathbf{ax/W} \\ \hline & -:\Delta_{8}\vdash \bot,\Delta_{12},F_{9}\wedge F_{10} & \mathbf{ax/W} \\ \hline \end{pmatrix}_{\mathbf{hCut}} \end{array}$$

## • Case rule $\top_R$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_6 \vdash \mathbf{F}_7, \top, \Delta_{10} \quad \mathbf{h}_1:\Delta_6 \vdash \mathbf{F}_8, \top, \Delta_{10}}{\bullet \mathbf{h}_1:\Delta_6 \vdash (\top, \Delta_{10}), \mathbf{F}_7 \land \mathbf{F}_8} \quad \wedge_R \quad \frac{\bullet \mathbf{h}_9:\Delta_6, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \top, \Delta_{10}}{\bullet \mathbf{h}_9:\Delta_6, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \top, \Delta_{10}} \quad \nabla_R \\ \hline \\ -:\Delta_6 \vdash \top, \Delta_{10} \\ \hline \\ -:\Delta_6 \vdash \top, \Delta_{10} \quad & \\ \hline \\ \frac{\bullet \mathbf{h}_2:\Delta_8 \vdash \mathbf{F}_7, \mathbf{F}_9, \top, \Delta_{12} \quad \mathbf{h}_2:\Delta_8 \vdash \mathbf{F}_7, \mathbf{F}_{10}, \top, \Delta_{12}}{\bullet \cdot \mathbf{h}_2:\Delta_8 \vdash ((\top, \Delta_{12}), \mathbf{F}_9 \land \mathbf{F}_{10}), \mathbf{F}_7} \quad \wedge_R \quad & \\ \hline \\ \bullet \mathbf{h}_1:\Delta_8, \mathbf{F}_7 \vdash (\top, \Delta_{12}), \mathbf{F}_9 \land \mathbf{F}_{10}} \quad & \\ \hline \\ -:\Delta_8 \vdash (\top, \Delta_{12}), \mathbf{F}_9 \land \mathbf{F}_{10}} \quad & \\ \hline \\ -:\Delta_8 \vdash \top, \Delta_{12}, \mathbf{F}_9 \land \mathbf{F}_{10}} \quad & \\ \hline \\ \hline \\ -:\Delta_8 \vdash \top, \Delta_{12}, \mathbf{F}_9 \land \mathbf{F}_{10}} \end{array}$$

# $\bullet$ Case rule K

#### • Case rule A45

$$\frac{\mathbf{h}_1: \Box \Gamma_9, \Delta_{13} \vdash \mathbf{F}_6, \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_{12} \quad \mathbf{h}_1: \Box \Gamma_9, \Delta_{13} \vdash \mathbf{F}_7, \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_{12}}{\bullet \mathbf{h}_1: \Box \Gamma_9, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_{12}), \mathbf{F}_6 \land \mathbf{F}_7} \wedge_R \quad \frac{\mathbf{h}_8: \Box \Gamma_9 \vdash \Box \Gamma_{10}, \mathbf{F}_{12}}{\bullet \mathbf{h}_8: (\Box \Gamma_9, \Delta_{13}), \mathbf{F}_6 \land \mathbf{F}_7 \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_{12}} \\ -: \Box \Gamma_9, \Delta_{13} \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_{12} \\ -: \Box \Gamma_9 \vdash \mathbf{F}_{12}, \Box \Gamma_{10} \quad \text{ax/W} \\ \hline -: \Delta_{13}, \Box \Gamma_9 \vdash \Delta_{11}, \Box \Gamma_{10}, []\mathbf{F}_{12} \quad A45 \\ \end{pmatrix}} \quad A45$$

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\frac{\mathbf{h}_2:\square\Gamma_{15},\Delta_{11}\vdash\square\mathbf{F}_7,\mathbf{F}_8,\square\Gamma_{12},\Delta_{14},[[\mathbf{F}_{13}\quad\mathbf{h}_2:\square\Gamma_{15},\Delta_{11}\vdash\square\mathbf{F}_7,\mathbf{F}_9,\square\Gamma_{12},\Delta_{14},[[\mathbf{F}_{13}\quad \boldsymbol{h}_2:\square\Gamma_{15},\square\mathbf{F}_7\vdash\square\mathbf{F}_7,\boldsymbol{h}_9]),\square\mathbf{F}_7}{\bullet\mathbf{h}_2:\square\Gamma_{15},\Delta_{11}\vdash((\square\Gamma_{12},\Delta_{14},[[\mathbf{F}_{13}),\mathbf{F}_8\wedge\mathbf{F}_9),\square\mathbf{F}_7} \quad \wedge_R \quad \frac{\mathbf{h}_{10}:\square\Gamma_{15},\square\mathbf{F}_7\vdash\square\mathbf{F}_7\vdash\square\mathbf{F}_7}{\bullet\mathbf{h}_{10}:(\square\Gamma_{15},\Delta_{11}),\square\mathbf{F}_7\vdash(\square\mathbf{F}_7,\mathbf{F}_8),\square\mathbf{F}_7\vdash(\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7,\mathbf{F}_8,\square\mathbf{F}_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -:\Box\Gamma_{15},\Delta_{11}\vdash(\Box\Gamma_{12},\Delta_{14},[]\mathtt{F}_{13}),\mathtt{F}_{8}\wedge\mathtt{F}_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   h_{10}:\Box \mathtt{F}_{7},\Box \Gamma_{15} \vdash \mathtt{F}_{13},\Box \Gamma_{12} ax/W
   \frac{\mathsf{h}_{10}: \Box \mathsf{F}_{7}, \Box \Gamma_{15} \vdash \mathsf{F}_{13}, \Box \Gamma_{12}}{\mathsf{h}_{2}: \Delta_{11}, \Box \Gamma_{15} \vdash \Box \mathsf{F}_{7}, \Delta_{14}, \mathsf{F}_{8}, \Box \Gamma_{12}, []\mathsf{F}_{13}} \ \mathsf{ax/W} \ \frac{\mathsf{h}_{10}: \Box \mathsf{F}_{7}, \Delta_{11}, \Box \Gamma_{15} \vdash \mathsf{F}_{13}, \Box \Gamma_{12}}{\bullet \mathsf{h}_{10}: \Box \mathsf{F}_{7}, \Delta_{11}, \Box \Gamma_{15} \vdash \Delta_{14}, \mathsf{F}_{8}, \Box \Gamma_{12}, []\mathsf{F}_{13}} \ \mathsf{hCut} \ \frac{\mathsf{A45}}{\mathsf{h}_{2}: \Delta_{11}, \Box \Gamma_{15} \vdash \Box \mathsf{F}_{7}, \Delta_{14}, \mathsf{F}_{9}, \Box \Gamma_{12}, []\mathsf{F}_{13}}{-: \Delta_{11}, \Box \Gamma_{15} \vdash \Box \mathsf{F}_{7}, \Delta_{14}, \mathsf{F}_{9}, \Box \Gamma_{12}, []\mathsf{F}_{13}} \ \mathsf{ax}
                                                                                                                                                                                                                                                                                                                                                                                                                                                       -:\Delta_{11},\Box\Gamma_{15}\vdash\Delta_{14},\mathsf{F}_8,\Box\Gamma_{12},[]\mathsf{F}_{13}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        -:\Delta_{11},\Box\Gamma_{15}\vdash\Delta_{14},\Box\Gamma_{12},[]\mathsf{F}_{13},\mathsf{F}_8\wedge\mathsf{F}_9
\frac{\mathbf{h}_2: \square\Gamma_{11}, \Delta_{15} \vdash \mathbf{F}_7, \mathbf{F}_8, \square\Gamma_{12}, \Delta_{14}, []\mathbf{F}_{13} \quad \mathbf{h}_2: \square\Gamma_{11}, \Delta_{15} \vdash \mathbf{F}_7, \mathbf{F}_9, \square\Gamma_{12}, \Delta_{14}, []\mathbf{F}_{13}}{\mathbf{h}_2: \square\Gamma_{11}, \Delta_{15} \vdash ((\square\Gamma_{12}, \Delta_{14}, []\mathbf{F}_{13}), \mathbf{F}_8 \land \mathbf{F}_9), \mathbf{F}_7} \quad \wedge_R \quad \frac{\mathbf{h}_{10}: \square\Gamma_{11} \vdash \square\Gamma_{12}, \mathbf{F}_{13}}{\mathbf{h}_{10}: (\square\Gamma_{11}, \Delta_{15}), \mathbf{F}_7 \vdash (\square\Gamma_{12}, \Delta_{14}, []\mathbf{F}_{13}), \mathbf{F}_8 \land \mathbf{F}_9} \\ \mathbf{Cut} \quad \mathbf{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     -: \Box\Gamma_{11}, \Delta_{15} \vdash (\Box\Gamma_{12}, \Delta_{14}, []\mathtt{F}_{13}), \mathtt{F}_8 \wedge \mathtt{F}_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             \frac{ \begin{array}{c} -: \square\Gamma_{11} \vdash \mathtt{F}_{13}, \square\Gamma_{12} \end{array}}{-: \Delta_{15}, \square\Gamma_{11} \vdash \Delta_{14}, \square\Gamma_{12}, []\mathtt{F}_{13}, \mathtt{F}_8 \wedge \mathtt{F}_9} \ A45 \\ \end{array}
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## • Case rule $\rightarrow_L$

$$\frac{h_1: \Delta_{12}, F_9 \to F_{10} \vdash F_6, \Delta_{11}}{eh_1: \Delta_{12}, F_9 \to F_{10} \vdash F_7, \Delta_{11}}{eh_1: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge R \xrightarrow{h_8: (\Delta_{12}, F_9 \to F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}}{eh_8: (\Delta_{12}, F_9 \to F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \cap Cut} \xrightarrow{-: \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}}$$

### • Case rule $\wedge_L$

$$\frac{\begin{array}{c} \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \mathbf{F}_{6}, \Delta_{11} \quad \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \mathbf{F}_{7}, \Delta_{11} \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{6} \wedge \mathbf{F}_{7} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11}, \mathbf{F}_{6} \wedge \mathbf{F}_{7} \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11}, \mathbf{F}_{6} \wedge \mathbf{F}_{7} \\ \hline \\ \bullet \mathbf{h}_{2}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11}$$

$$\frac{\mathbf{h}_{1}:\Delta_{7} \vdash F_{8},\Delta_{10} \quad \mathbf{h}_{1}:\Delta_{7} \vdash F_{9},\Delta_{10}}{\bullet \mathbf{h}_{1}:\Delta_{7} \vdash \Delta_{10}} \wedge_{\mathbf{k}} = \frac{\mathbf{h}_{0}:F_{8},F_{9},\Delta_{7} \vdash \Delta_{10}}{\bullet \mathbf{h}_{0}:\Delta_{7},F_{8} \land F_{9} \vdash \Delta_{10}} \wedge_{\mathbf{k}} \\ - : \Delta_{7} \vdash \Delta_{10} \\ - : \Delta_{7} \vdash F_{8} \vdash \Delta_{10},F_{9} = \frac{\mathbf{a} \times \mathbb{W}}{\bullet \mathbf{h}_{1}:\Delta_{1},F_{12} \land F_{13},F_{8} \vdash \Delta_{10}} \otimes_{\mathbf{k}} = \frac{\mathbf{a} \times \mathbb{W}}{\bullet \mathbf{h}_{10}:\Delta_{11},F_{12} \land F_{13},F_{9} \vdash \Delta_{10}} \otimes_{\mathbf{k}} \\ - : \Delta_{7} \vdash \Delta_{10} = \frac{\mathbf{h}_{2}:\Delta_{11} \vdash F_{12} \land F_{13},F_{8},\Delta_{7} \quad \mathbf{h}_{2}:\Delta_{11} \vdash F_{12} \land F_{13},F_{9},\Delta_{7}}{\bullet \mathbf{h}_{2}:\Delta_{11} \vdash (\Delta_{7},F_{8} \land F_{9}),F_{12} \land F_{13}} \wedge_{\mathbf{k}} = \frac{\mathbf{h}_{10}:F_{12},F_{13},\Delta_{11} \vdash \Delta_{7},F_{8} \land F_{9}}{\bullet \mathbf{h}_{2}:\Delta_{11} \vdash (\Delta_{7},F_{8} \land F_{9}),F_{12} \land F_{13}} \wedge_{\mathbf{k}} \\ - : \Delta_{11} \vdash \Delta_{7},F_{8} \land F_{9} + \Delta_{10} = \frac{\mathbf{h}_{10}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{8}}{\bullet \mathbf{h}_{10}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{8}} \wedge_{\mathbf{k}} \\ - : \Delta_{11} \vdash \Delta_{7},F_{8} \land F_{9} + \Delta_{10} = \frac{\mathbf{h}_{10}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{8}}{\bullet \mathbf{h}_{10}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{9}} \wedge_{\mathbf{k}} \\ - : \Delta_{11} \vdash \Delta_{7},F_{8} \land F_{9} + \Delta_{10} = \frac{\mathbf{h}_{10}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{9}}{\bullet \mathbf{h}_{10}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{9}} \wedge_{\mathbf{k}} \\ - : \Delta_{11} \vdash \Delta_{7},F_{8} \land F_{9} + \Delta_{10} = \frac{\mathbf{h}_{10}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{9}}{\bullet \mathbf{h}_{10}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{9}} \wedge_{\mathbf{k}} \\ - : \Delta_{11} \vdash \Delta_{7},F_{8} \land F_{9} + \Delta_{10} = \frac{\mathbf{h}_{11}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{9}}{\bullet \mathbf{h}_{10}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{7},F_{9}} \wedge_{\mathbf{k}} \\ - : \Delta_{11} \vdash \Delta_{7},F_{8} \land F_{9} \\ - : \Delta_{11} \vdash \Delta_{7},F_{8} \land F_{9} + A_{10} = \frac{\mathbf{h}_{11}:\Delta_{11},F_{12},F_{13} \vdash \Delta_{8},F_{9} \land F_{10}}{\bullet \mathbf{h}_{11}:\Delta_{11},F_{12},F_{13},\Delta_{14} \vdash \Delta_{8},F_{9} \land F_{10}} \wedge_{\mathbf{k}} \\ - : \Delta_{11},F_{12},F_{13} \vdash \Delta_{8},F_{7},F_{9} \land F_{10} \\ - : \Delta_{14},F_{12},F_{13} \vdash \Delta_{8},F_{9} \land F_{10} \\ - : \Delta_$$

#### • Case rule $\vee_L$

$$\frac{\mathbf{h}_{1}:\Delta_{12}, \mathbf{F}_{9} \vee \mathbf{F}_{10} \vdash \mathbf{F}_{6}, \Delta_{11} \quad \mathbf{h}_{1}:\Delta_{12}, \mathbf{F}_{9} \vee \mathbf{F}_{10} \vdash \mathbf{F}_{7}, \Delta_{11}}{\bullet \mathbf{h}_{1}:\Delta_{12}, \mathbf{F}_{9} \vee \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{6} \wedge \mathbf{F}_{7}} \wedge \mathbf{h}_{8}: (\Delta_{12}, \mathbf{F}_{9} \vee \mathbf{F}_{10}), \mathbf{F}_{6} \wedge \mathbf{F}_{7} \vdash \Delta_{11}} \\ -:\Delta_{12}, \mathbf{F}_{9} \vee \mathbf{F}_{10} \vdash \Delta_{11} \\ -:\Delta_{11}, \mathbf{F}_{12} \vee \mathbf{F}_{13} \vdash \Delta_{12} \vee \mathbf{F}_{13} \vdash \Delta$$

### $\bullet$ Case rule AT

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{11}, \| \mathbf{F}_{9} \vdash \mathbf{F}_{6}, \Delta_{10} \quad \mathbf{h}_{1}:\Delta_{11}, \| \mathbf{F}_{9} \vdash \mathbf{F}_{7}, \Delta_{10}}{\bullet \mathbf{h}_{1}:\Delta_{11}, \| \mathbf{F}_{9} \vdash \mathbf{F}_{7}, \Delta_{10}} \wedge_{R} & \frac{\mathbf{h}_{8}: \mathbf{F}_{9}, \Delta_{11}, \| \mathbf{F}_{9}, \mathbf{F}_{6} \wedge \mathbf{F}_{7} \vdash \Delta_{10}}{\bullet \mathbf{h}_{8}: (\Delta_{11}, \| \mathbf{F}_{9}), \mathbf{F}_{6} \wedge \mathbf{F}_{7} \vdash \Delta_{10}} & \mathbf{AT} \\ & -: \Delta_{11}, \| \mathbf{F}_{9} \vdash \mathbf{F}_{0} \wedge_{R} \wedge_$$

#### • Case rule $\perp_L$

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\bot,\Delta_{10}\vdash\mathsf{F}_{6},\Delta_{9}\quad \mathbf{h}_{1}:\bot,\Delta_{10}\vdash\mathsf{F}_{7},\Delta_{9}}{\bullet \mathbf{h}_{1}:\bot,\Delta_{10}\vdash\Delta_{9},\mathsf{F}_{6}\land\mathsf{F}_{7}} & \wedge_{R} & \frac{\bullet \mathbf{h}_{8}:(\bot,\Delta_{10}),\mathsf{F}_{6}\land\mathsf{F}_{7}\vdash\Delta_{9}}{\bullet \mathbf{h}_{8}:(\bot,\Delta_{10}),\mathsf{F}_{6}\land\mathsf{F}_{7}\vdash\Delta_{9}} & \bot_{L} \\ & & -:\bot,\Delta_{10}\vdash\Delta_{9} & \bot_{L} \\ & \frac{\mathbf{h}_{2}:\Delta_{11}\vdash\bot,\mathsf{F}_{8},\Delta_{7}\quad \mathbf{h}_{2}:\Delta_{11}\vdash\bot,\mathsf{F}_{9},\Delta_{7}}{\bullet \mathbf{h}_{2}:\Delta_{11}\vdash(\Delta_{7},\mathsf{F}_{8}\land\mathsf{F}_{9}),\bot} & \wedge_{R} & \frac{\bullet \mathbf{h}_{10}:\Delta_{11},\bot\vdash\Delta_{7},\mathsf{F}_{8}\land\mathsf{F}_{9}}{\bullet \mathbf{h}_{10}:\Delta_{11},\bot\vdash\Delta_{7},\mathsf{F}_{8}\land\mathsf{F}_{9}} & \bot_{L} \\ & -:\Delta_{11}\vdash\Delta_{7},\mathsf{F}_{8}\land\mathsf{F}_{9} & \times & \bullet \\ & \frac{\bullet_{2}:\Delta_{11}\vdash\bot,\Delta_{7},\mathsf{F}_{8}}{\bullet \mathbf{h}_{10}:\bot,\Delta_{11}\vdash\Delta_{7},\mathsf{F}_{8}} & \bot_{L} \\ & -:\Delta_{11}\vdash\Delta_{7},\mathsf{F}_{8} & \wedge_{F}_{9} & \times & \bullet \\ & -:\Delta_{11}\vdash\Delta_{7},\mathsf{F}_{8} & \wedge_{F}_{9} & \times \\ & -:\Delta_{11}\vdash\Delta_{7},\mathsf{F}_{9},\Delta_{8}\quad \mathbf{h}_{2}:\bot,\Delta_{12}\vdash\mathsf{F}_{7},\mathsf{F}_{10},\Delta_{8} \\ & \bullet_{11}:(\bot,\Delta_{12}),\mathsf{F}_{7}\vdash\Delta_{8},\mathsf{F}_{9}\land\mathsf{F}_{10}} & \bot_{L} \\ & \bullet_{11}:(\bot,\Delta_{12}),\mathsf{F}_{7}\vdash\Delta_{8},\mathsf{F}_{9}\land\mathsf{F}_{10} & \bot_{L} \\ & -:\bot,\Delta_{12}\vdash\Delta_{8},\mathsf{F}_{9}\land\mathsf{F}_{10} & \bot_{L} \\ & -:\bot,\Delta_{12}\vdash\Delta$$

# $\bullet$ Case rule I

$$\frac{\mathbf{h}_{2}:\Delta_{13},\mathbf{p}_{11}\vdash \mathbf{F}_{7},\mathbf{F}_{8},\Delta_{12},\mathbf{p}_{11}\quad \mathbf{h}_{2}:\Delta_{13},\mathbf{p}_{11}\vdash \mathbf{F}_{7},\mathbf{F}_{9},\Delta_{12},\mathbf{p}_{11}}{\bullet \mathbf{h}_{2}:\Delta_{13},\mathbf{p}_{11}\vdash ((\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{8}\land \mathbf{F}_{9}),\mathbf{F}_{7}} \quad \wedge_{R} \quad \frac{\bullet \mathbf{h}_{10}:(\Delta_{13},\mathbf{p}_{11}),\mathbf{F}_{7}\vdash (\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{8}\land \mathbf{F}_{9}}{-:\Delta_{13},\mathbf{p}_{11}\vdash (\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{8}\land \mathbf{F}_{9}} \quad I \quad \text{Cut}$$

• Case rule  $\top_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_{1}: \top, \Delta_{10} \vdash \mathbf{F}_{6}, \Delta_{9} \quad \mathbf{h}_{1}: \top, \Delta_{10} \vdash \mathbf{F}_{7}, \Delta_{9}}{\bullet \mathbf{h}_{1}: \top, \Delta_{10} \vdash \Delta_{9}, \mathbf{F}_{6} \land \mathbf{F}_{7}} \quad \wedge_{R} \quad \frac{\mathbf{h}_{8}: \Delta_{10}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \Delta_{9}}{\bullet \mathbf{h}_{8}: (\top, \Delta_{10}), \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \Delta_{9}} \quad \top_{L} \\ \hline -: \top, \Delta_{10} \vdash \Delta_{9} \\ \hline \bullet \mathbf{h}_{1}: \top, \Delta_{10} \vdash \Delta_{9}, \mathbf{F}_{6} \land \mathbf{F}_{7} \quad \mathbf{ax/W} \quad \frac{\mathbf{h}_{8}: \top, \Delta_{10}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \Delta_{9}}{\bullet \mathbf{h}_{1}: \top, \Delta_{10} \vdash \Delta_{9}, \mathbf{F}_{6} \land \mathbf{F}_{7}} \quad \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{2}: \Delta_{11} \vdash \top, \mathbf{F}_{8}, \Delta_{7} \quad \mathbf{h}_{2}: \Delta_{11} \vdash \top, \mathbf{F}_{9}, \Delta_{7} \quad \wedge_{R} \quad \frac{\mathbf{h}_{10}: \Delta_{11} \vdash \Delta_{7}, \mathbf{F}_{8} \land \mathbf{F}_{9}}{\bullet \mathbf{h}_{10}: \Delta_{11}, \top \vdash \Delta_{7}, \mathbf{F}_{8} \land \mathbf{F}_{9}} \quad \top_{L} \\ \hline \bullet \mathbf{h}_{2}: \Delta_{11} \vdash (\Delta_{7}, \mathbf{F}_{8} \land \mathbf{F}_{9}), \top \quad \wedge_{R} \quad \frac{\mathbf{h}_{10}: \Delta_{11} \vdash \Delta_{7}, \mathbf{F}_{8} \land \mathbf{F}_{9}}{\bullet \mathbf{h}_{10}: \Delta_{11}, \top \vdash \Delta_{7}, \mathbf{F}_{8} \land \mathbf{F}_{9}} \quad \nabla_{L} \\ \hline \bullet \mathbf{h}_{2}: \top, \Delta_{12} \vdash (\Delta_{7}, \mathbf{F}_{8} \land \mathbf{F}_{9}), \top \quad \wedge_{R} \quad \frac{\mathbf{h}_{10}: \Delta_{11} \vdash \Delta_{7}, \mathbf{F}_{8} \land \mathbf{F}_{9}}{\bullet \mathbf{h}_{10}: \Delta_{11}, \top \vdash \Delta_{7}, \mathbf{F}_{8} \land \mathbf{F}_{9}} \quad \nabla_{L} \\ \hline \bullet \mathbf{h}_{2}: \top, \Delta_{12} \vdash \mathbf{F}_{7}, \mathbf{F}_{9}, \Delta_{8} \quad \mathbf{h}_{2}: \top, \Delta_{12} \vdash \mathbf{F}_{7}, \mathbf{F}_{10}, \Delta_{8} \quad \bullet_{R} \quad \frac{\mathbf{h}_{11}: \mathbf{F}_{7}, \Delta_{12} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10}}{\bullet \mathbf{h}_{11}: (\top, \Delta_{12}), \mathbf{F}_{7} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \quad \nabla_{L} \\ \hline \bullet \mathbf{h}_{2}: \top, \Delta_{12} \vdash (\Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10}), \mathbf{F}_{7} \quad \mathbf{h}_{R} \quad \frac{\mathbf{h}_{11}: \mathbf{F}_{7}, \Delta_{12} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10}}{\bullet \mathbf{h}_{11}: (\top, \Delta_{12}), \mathbf{F}_{7} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \quad \nabla_{L} \\ \hline \bullet \mathbf{h}_{2}: \top, \Delta_{12} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10} \quad \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{2}: \top, \Delta_{12} \vdash \Delta_{8}, \mathbf{F}_{7}, \mathbf{F}_{9} \land \mathbf{F}_{10} \quad \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{11}: \top, \Delta_{12}, \mathbf{F}_{7} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{2}: \top, \Delta_{12} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10} \quad \mathbf{h}_{11}: \top, \Delta_{12}, \mathbf{F}_{7} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{2}: \top, \Delta_{12} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10} \quad \mathbf{h}_{11}: \top, \Delta_{12}, \mathbf{F}_{7} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{11}: \top, \Delta_{12}, \mathbf{F}_{7} \vdash \Delta_{8}, \mathbf{F}_{9} \land \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{2}: \top, \Delta_{12} \vdash \Delta_{8},$$

# 6.3 Status of $\vee_R$ : OK

• Case rule  $\rightarrow_R$ 

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_6 \vdash \mathbf{F}_7, \mathbf{F}_8, \Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12}}{\bullet \mathbf{h}_1: \Delta_6 \vdash (\Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12}), \mathbf{F}_7 \vee \mathbf{F}_8} & \vee_R & \frac{\mathbf{h}_9: \mathbf{F}_{11}, \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8 \vdash \mathbf{F}_{12}, \Delta_{10}}{\bullet \mathbf{h}_9: \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12}} & \rightarrow_R \\ \hline & -: \Delta_6 \vdash \Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12} & \text{Cut} \\ \hline & \frac{\mathbf{h}_1: \Delta_6, \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_{12}, \mathbf{F}_7, \mathbf{F}_8}{\bullet \mathbf{h}_1: \Delta_6, \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_{12}, \mathbf{F}_7 \vee \mathbf{F}_8} & \text{inv-th/ax} \\ \hline & \frac{-: \Delta_6, \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_{12}, \mathbf{F}_7 \vee \mathbf{F}_8 \lor \Delta_{10}, \mathbf{F}_{12}}{-: \Delta_6 \vdash \Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12}} & \rightarrow_R \\ \hline & \frac{\mathbf{h}_2: \Delta_8 \vdash \mathbf{F}_7, \mathbf{F}_9, \mathbf{F}_{10}, \Delta_{14}, \mathbf{F}_{12} \to \mathbf{F}_{13}}{-: \Delta_8 \vdash (\Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12})} & \rightarrow_R \\ \hline & \frac{\mathbf{h}_2: \Delta_8 \vdash \mathbf{F}_7, \mathbf{F}_9, \mathbf{F}_{10}, \Delta_{14}, \mathbf{F}_{12} \to \mathbf{F}_{13}}{-: \Delta_8 \vdash (\Delta_{14}, \mathbf{F}_{12} \to \mathbf{F}_{13}), \mathbf{F}_9 \vee \mathbf{F}_{10}} & \bullet_{\mathbf{h}_{11}}: \mathbf{F}_7, \mathbf{F}_{12}, \Delta_8 \vdash \mathbf{F}_{13}, \Delta_{14}, \mathbf{F}_9 \vee \mathbf{F}_{10}} \\ \hline & -: \Delta_8 \vdash (\Delta_{14}, \mathbf{F}_{12} \to \mathbf{F}_{13}), \mathbf{F}_9 \vee \mathbf{F}_{10} & \bullet_{\mathbf{h}_{11}}: \Delta_8, \mathbf{F}_7 \vdash (\Delta_{14}, \mathbf{F}_{12} \to \mathbf{F}_{13}), \mathbf{F}_9 \vee \mathbf{F}_{10}} \\ \hline & \bullet_{\mathbf{h}_2}: \Delta_8, \mathbf{F}_{12} \vdash \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{13}, \mathbf{F}_7, \mathbf{F}_9} & \text{inv-th/ax} \\ \hline & \bullet_{\mathbf{h}_2}: \Delta_8, \mathbf{F}_{12} \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_7, \mathbf{F}_9 \vee \mathbf{F}_{10}} & \bullet_{\mathbf{h}_{11}}: \Delta_8, \mathbf{F}_{12}, \mathbf{F}_7 \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_9 \vee \mathbf{F}_{10} \\ \hline & \bullet_{\mathbf{h}_2}: \Delta_8, \mathbf{F}_{12} \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_7, \mathbf{F}_9 \vee \mathbf{F}_{10} & \bullet_{\mathbf{h}_{11}}: \Delta_8, \mathbf{F}_{12}, \mathbf{F}_7 \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_9 \vee \mathbf{F}_{10} \\ \hline & \bullet_{\mathbf{h}_2}: \Delta_8, \mathbf{F}_{12} \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_7, \mathbf{F}_9 \vee \mathbf{F}_{10} & \bullet_{\mathbf{h}_{11}}: \Delta_8, \mathbf{F}_{12}, \mathbf{F}_7 \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_9 \vee \mathbf{F}_{10} \\ \hline & \bullet_{\mathbf{h}_2}: \Delta_8, \mathbf{F}_{12} \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_7, \mathbf{F}_9 \vee \mathbf{F}_{10} & \bullet_{\mathbf{h}_{11}}: \Delta_8, \mathbf{F}_{12}, \mathbf{F}_7 \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_9 \vee \mathbf{F}_{10} \\ \hline & \bullet_{\mathbf{h}_2}: \Delta_8, \mathbf{F}_{12} \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_9 \vee \mathbf{F}_{10} & \bullet_{\mathbf{h}_{11}}: \Delta_8, \mathbf{F}_{12}, \mathbf{F}_7 \vdash \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_9 \vee \mathbf{F}_{10} \\ \hline & \bullet_{\mathbf{h}_2}: \Delta_8, \mathbf{F}_{12} \vdash \Delta_{14},$$

• Case rule  $\wedge_R$ 

$$\frac{\frac{h_{1}: \Delta_{6} \vdash F_{7}, F_{8}, \Delta_{10}, F_{11} \land F_{12}}{\bullet_{h_{1}}: \Delta_{6} \vdash (\Delta_{10}, F_{11} \land F_{12}), F_{7} \lor F_{8}} \lor_{R} \xrightarrow{h_{9}: \Delta_{6}, F_{7} \lor F_{8} \vdash F_{11}, \Delta_{10} \quad h_{9}: \Delta_{6}, F_{7} \lor F_{8} \vdash F_{12}, \Delta_{10}}{\bullet_{h_{9}: \Delta_{6}, F_{7} \lor F_{8} \vdash \Delta_{10}, F_{11} \land F_{12}}} \land_{R}} \xrightarrow{-: \Delta_{6} \vdash \Delta_{10}, F_{11}, F_{7}, F_{8}} \lor_{R} \xrightarrow{h_{9}: \Delta_{6}, F_{7} \lor F_{8} \vdash \Delta_{10}, F_{11}} \land_{R}} \xrightarrow{h_{1}: \Delta_{6} \vdash \Delta_{10}, F_{11}, F_{7}, F_{8}} \lor_{R} \xrightarrow{h_{1}: \Delta_{6} \vdash \Delta_{10}, F_{12}, F_{7}, F_{8}} \lor_{R}} \xrightarrow{h_{1}: \Delta_{6} \vdash \Delta_{10}, F_{12}, F_{7} \lor F_{8}} \lor_{R} \xrightarrow{h_{9}: \Delta_{6}, F_{7} \lor F_{8} \vdash \Delta_{10}, F_{12}} \land_{R}} \xrightarrow{-: \Delta_{6} \vdash \Delta_{10}, F_{11}} \land_{R}$$

$$\frac{ \begin{array}{c} \frac{h_2: \Delta_8 \vdash F_7, F_9, F_{10}, \Delta_{14}, F_{12} \land F_{13}}{\bullet h_2: \Delta_8 \vdash ((\Delta_{14}, F_{12} \land F_{13}), F_9 \lor F_{10}), F_7} \lor_R & \begin{array}{c} \frac{h_{11}: F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \lor F_{10}}{\bullet h_{11}: \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \land F_{13}), F_9 \lor F_{10}} & \wedge_R \\ & -: \Delta_8 \vdash (\Delta_{14}, F_{12} \land F_{13}), F_9 \lor F_{10} & \\ & & \\ \hline \\ \frac{h_2: \Delta_8 \vdash \Delta_{14}, F_{10}, F_7, F_9, F_{12} \land F_{13}}{\bullet} & \text{ax/W} & \begin{array}{c} \frac{h_{11}: F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \lor F_{10}}{\bullet h_{11}: \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \land F_{13}), F_9 \lor F_{10}} & \wedge_R \\ \\ \hline \\ \frac{h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_{12}, F_9}{\bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}} & \text{inv-th/ax} \\ \hline \\ \frac{-: \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}}{-: \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}} & \vee_R \\ \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}} \\ \hline \\ -: \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}} & \vee_R \\ \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}} \\ \hline \\ -: \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}} \\ \hline \\ -: \Delta_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13}} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13} \\ \hline \end{array} & \begin{array}{c} \text{inv-th/ax} \\ \bullet h_{11}: \Delta_8, F_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \land F_{13} \\ \hline$$

#### • Case rule $\vee_R$

### • Case rule $\perp_R$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_6\vdash F_7,F_8,\bot,\Delta_{10}}{\bullet \mathbf{h}_1:\Delta_6\vdash (\bot,\Delta_{10}),F_7\vee F_8} \vee_R & \frac{\mathbf{h}_9:\Delta_6,F_7\vee F_8\vdash \Delta_{10}}{\bullet \mathbf{h}_9:\Delta_6,F_7\vee F_8\vdash \bot,\Delta_{10}} & \bot_R \\ \hline -:\Delta_6\vdash \bot,\Delta_{10} & \mathbf{cut} \\ \hline \\ \hline \bullet \mathbf{h}_1:\Delta_6\vdash \bot,\Delta_{10},F_7\vee F_8 & \mathbf{ax/W} & \mathbf{h}_9:\Delta_6,F_7\vee F_8\vdash \bot,\Delta_{10} \\ \hline \bullet \mathbf{h}_1:\Delta_6\vdash \bot,\Delta_{10},F_7\vee F_8 & \mathbf{ax/W} & \mathbf{h}_9:\Delta_6,F_7\vee F_8\vdash \bot,\Delta_{10} \\ \hline -:\Delta_6\vdash \bot,\Delta_{10} & \mathbf{h}_0:\Delta_6\vdash \bot,\Delta_{10} \\ \hline \bullet \mathbf{h}_2:\Delta_8\vdash F_7,F_9,F_{10},\bot,\Delta_{12} & \vee_R & \mathbf{h}_{11}:F_7,\Delta_8\vdash \Delta_{12},F_9\vee F_{10} \\ \hline \bullet \mathbf{h}_2:\Delta_8\vdash ((\bot,\Delta_{12}),F_9\vee F_{10}),F_7 & \mathbf{ax/W} & \mathbf{h}_{11}:\Delta_8,F_7\vdash (\bot,\Delta_{12}),F_9\vee F_{10} \\ \hline -:\Delta_8\vdash (\bot,\Delta_{12}),F_9\vee F_{10} & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_2:\Delta_8\vdash \bot,\Delta_{12},F_7,F_9\vee F_{10} & \mathbf{ax/W} \\ \hline -:\Delta_8\vdash \bot,\Delta_{12},F_9\vee F_{10} & \mathbf{ax/W} \\ \hline \end{pmatrix} \mathbf{h}_{Cut} \\ \hline \end{array}$$

#### • Case rule $\top_R$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_6 \vdash \mathbf{F}_7, \mathbf{F}_8, \top, \Delta_{10}}{\bullet \mathbf{h}_1: \Delta_6 \vdash (\top, \Delta_{10}), \mathbf{F}_7 \vee \mathbf{F}_8} \quad \vee_R \quad & \bullet_{\mathbf{h}_9}: \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8 \vdash \top, \Delta_{10}}{\bullet \mathbf{h}_9: \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8 \vdash \top, \Delta_{10}} \quad & \top_R \\ & -: \Delta_6 \vdash \top, \Delta_{10} \\ & -: \Delta_6 \vdash \top, \Delta_{10} \\ & & -: \Delta_6 \vdash \top, \Delta_{10} \end{array} \quad \top_R \\ \\ \frac{\mathbf{h}_2: \Delta_8 \vdash \mathbf{F}_7, \mathbf{F}_9, \mathbf{F}_{10}, \top, \Delta_{12}}{\bullet \mathbf{h}_2: \Delta_8 \vdash ((\top, \Delta_{12}), \mathbf{F}_9 \vee \mathbf{F}_{10}), \mathbf{F}_7} \quad \vee_R \quad & \bullet_{\mathbf{h}_{11}: \Delta_8, \mathbf{F}_7 \vdash (\top, \Delta_{12}), \mathbf{F}_9 \vee \mathbf{F}_{10}} \\ & -: \Delta_8 \vdash (\top, \Delta_{12}), \mathbf{F}_9 \vee \mathbf{F}_{10} \\ & & \longrightarrow \\ & -: \Delta_8 \vdash \top, \Delta_{12}, \mathbf{F}_9 \vee \mathbf{F}_{10} \end{array} \quad \top_R \\ \end{array}$$

#### $\bullet$ Case rule K

$$\begin{array}{c} \frac{h_1: \Box \Gamma_9, \Delta_{12} \vdash F_6, F_7, \Delta_{10}, []F_{11}}{\bullet h_1: \Box \Gamma_9, \Delta_{12} \vdash (\Delta_{10}, []F_{11}), F_6 \lor F_7} \lor_R & \frac{h_8: unbox(\Box \Gamma_9) \vdash F_{11}}{\bullet h_8: (\Box \Gamma_9, \Delta_{12}), F_6 \lor F_7 \vdash \Delta_{10}, []F_{11}} & K \\ \hline -: \Box \Gamma_9, \Delta_{12} \vdash \Delta_{10}, []F_{11} & ax/W \\ \hline -: unbox(\Box \Gamma_9) \vdash F_{11} & ax/W \\ \hline -: \Delta_{12}, \Box \Gamma_9 \vdash \Delta_{10}, []F_{11} & K \\ \hline \\ \frac{h_2: \Box \Gamma_{14}, \Delta_{11} \vdash \Box F_7, F_8, F_9, \Delta_{13}, []F_{12}}{\bullet h_2: \Box \Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, []F_{12}), F_8 \lor F_9), \Box F_7} \lor_R & \frac{h_{10}: unbox(\Box \Gamma_{14}), unbox(\Box F_7) \vdash F_{12}}{\bullet h_{10}: (\Box \Gamma_{14}, \Delta_{11}), \Box F_7 \vdash (\Delta_{13}, []F_{12}), F_8 \lor F_9} & K \\ \hline -: \Box \Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, []F_{12}), F_8 \lor F_9 & \frac{h_{10}: unbox(\Box F_7), unbox(\Box \Gamma_{14}) \vdash F_{12}}{\bullet h_{10}: unbox(\Box F_7), unbox(\Box \Gamma_{14}) \vdash F_{12}} & ax/W \\ \hline h_{10}: unbox(\Box F_7), unbox(\Box \Gamma_{14}) \vdash F_{12} & ax/W \\ \hline -: \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, F_8, F_9, []F_{12} & \vee_R \\ \hline -: \Delta_{11}, \Box \Gamma_{14} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: \Delta_{11}, \Box \Gamma_{14} \vdash (\Delta_{13}, []F_{12}), F_8 \lor F_9 \\ \hline \bullet_{10}: (\Box \Gamma_{11}, \Delta_{14}), F_7 \vdash (\Delta_{13}, []F_{12}), F_8 \lor F_9 \\ \hline -: unbox(\Box \Gamma_{11}) \vdash F_{12} & ax/W \\ \hline -: unbox(\Box \Gamma_{11}) \vdash F_{12} & ax/W \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: unbox(\Box \Gamma_{11}) \vdash F_{12} & ax/W \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: unbox(\Box \Gamma_{11}) \vdash F_{12} & ax/W \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: unbox(\Box \Gamma_{11}) \vdash F_{12} & ax/W \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: unbox(\Box \Gamma_{11}) \vdash F_{12} & ax/W \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: unbox(\Box \Gamma_{11}) \vdash F_{12} & ax/W \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: \Delta_{14}, \Box \Gamma_{11} \vdash \Delta_{13}, []F_{12}, F_8 \lor F_9 \\ \hline -: \Delta_{14}, \Box$$

## $\bullet$ Case rule A45

• Case rule  $\rightarrow_L$ 

• Case rule  $\wedge_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{12},\mathbf{F}_{9}\wedge\mathbf{F}_{10}\vdash\mathbf{F}_{6},\mathbf{F}_{7},\Delta_{11}}{\mathbf{e}_{11}:\Delta_{12},\mathbf{F}_{9}\wedge\mathbf{F}_{10}\vdash\Delta_{11},\mathbf{F}_{6}\vee\mathbf{F}_{7}} \quad \vee_{R} \quad \frac{\mathbf{h}_{8}:\mathbf{F}_{9},\mathbf{F}_{10},\Delta_{12},\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\Delta_{11}}{\mathbf{e}_{18}:(\Delta_{12},\mathbf{F}_{9}\wedge\mathbf{F}_{10}),\mathbf{F}_{6}\vee\mathbf{F}_{7}\vdash\Delta_{11}} \quad \wedge_{L} \\ -:\Delta_{12},\mathbf{F}_{9}\wedge\mathbf{F}_{10}\vdash\Delta_{11} \\ \hline -:\Delta_{12},\mathbf{F}_{9}\wedge\mathbf{F}_{10}\vdash\Delta_{11} \\ \hline \mathbf{e}_{11}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{9}\vdash\Delta_{11},\mathbf{F}_{6}\vee\mathbf{F}_{7} \quad \mathbf{e}_{10}\vee\mathbf{F}_{10}\vee\mathbf{F}_{10} \\ \hline \mathbf{e}_{11}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{9}\vdash\Delta_{11},\mathbf{F}_{6}\vee\mathbf{F}_{7} \quad \mathbf{e}_{10}\vee\mathbf{F}_{10}\vee\mathbf{F}_{10} \\ \hline \mathbf{e}_{11}:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_{9}\vdash\Delta_{11},\mathbf{F}_{6}\vee\mathbf{F}_{7} \quad \mathbf{e}_{10}\vee\mathbf{F}_{10}\vee\mathbf{F}_{10} \\ \hline -:\Delta_{12},\mathbf{F}_{9}\wedge\mathbf{F}_{10}\vdash\Delta_{11} \quad \wedge_{L} \\ \hline -:\Delta_{12},\mathbf{F}_{9}\wedge\mathbf{F}_{10}\vdash\Delta_{11} \quad \wedge_{L} \\ \hline \mathbf{h}_{2}:\Delta_{11}\vdash\mathbf{F}_{12}\wedge\mathbf{F}_{13},\mathbf{F}_{8},\mathbf{F}_{9},\Delta_{7} \\ \hline \mathbf{e}_{12}:\Delta_{11}\vdash(\Delta_{7},\mathbf{F}_{8}\vee\mathbf{F}_{9}),\mathbf{F}_{12}\wedge\mathbf{F}_{13} \quad \mathbf{e}_{10}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{13},\Delta_{11}\vdash\Delta_{7},\mathbf{F}_{8}\vee\mathbf{F}_{9} \\ \hline -:\Delta_{11}\vdash\Delta_{7},\mathbf{F}_{8}\vee\mathbf{F}_{9} \\ \hline \mathbf{e}_{10}:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{13}\vdash\Delta_{7},\mathbf{F}_{8},\mathbf{F}_{9} \\ \hline -:\Delta_{11}\vdash\Delta_{7},\mathbf{F}_{8},\mathbf{F}_{9} \\ \hline -:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{13}\vdash\Delta_{8},\mathbf{F}_{10},\mathbf{F}_{9} \\ \hline -:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{13}\vdash\Delta_{8},\mathbf{F}_{10},\mathbf{F}_{9} \\ \hline -:\Delta_{11},\mathbf{F}_{12},\mathbf{F}_{13}\vdash\Delta_{8},\mathbf{F}_{10},\mathbf{F}_{9} \\ \hline -:\Delta_{14},\mathbf{F}_{12},\mathbf{F}_{13}\vdash\Delta_{8},\mathbf{F}_{10},\mathbf{F}_{9} \\ \hline -:\Delta_$$

• Case rule  $\vee_L$ 

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 \frac{\mathbf{h}_1 : \Delta_{12}, \mathbf{F}_9 \vee \mathbf{F}_{10} \vdash \mathbf{F}_6, \mathbf{F}_7, \Delta_{11}}{\bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{F}_9 \vee \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_6 \vee \mathbf{F}_7} \quad \vee_R \quad \frac{\mathbf{h}_8 : \mathbf{F}_9, \Delta_{12}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_{11} \quad \mathbf{h}_8 : \mathbf{F}_{10}, \Delta_{12}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_{11}}{\bullet \mathbf{h}_8 : (\Delta_{12}, \mathbf{F}_9 \vee \mathbf{F}_{10}), \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_{11}} \quad \vee_L 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              -: \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}
  \frac{}{\mathbf{h}_8:\Delta_{12},\mathbf{F}_{10},\mathbf{F}_6\vee\mathbf{F}_7\vdash\Delta_{11}} \overset{\text{ax,v}}{\overset{\text{hCut}}{\overset{\text{h}}{\overset{\text{cut}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{\text{const}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}{\overset{c}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}{\overset{c}}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     \frac{-:\Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}{-:\Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}} \vee_{L}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            -:\Delta_{12},\mathtt{F}_{9}\vee\mathtt{F}_{10}\vdash\Delta_{11}
                                                                 \begin{array}{c} \mathbf{h}_1: \Delta_7 \vdash \mathbf{F}_8, \mathbf{F}_9, \Delta_{10} \\ \bullet \mathbf{h}_1: \Delta_7 \vdash \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9 \end{array} \vee_R \quad \frac{\mathbf{h}_6: \mathbf{F}_8, \Delta_7 \vdash \Delta_{10} \quad \mathbf{h}_6: \mathbf{F}_9, \Delta_7 \vdash \Delta_{10}}{\bullet \mathbf{h}_6: \Delta_7, \mathbf{F}_8 \vee \mathbf{F}_9 \vdash \Delta_{10}} \quad \mathbf{Cut} \end{array}
                                                        \bulleth<sub>1</sub> : \Delta_7 \vdash \Delta_{10}, F_8 \lor F_9
                                                                                                                                                                                                                                                                                                                                        -: \Delta_7 \vdash \Delta_{10}
          \frac{ \frac{-:\Delta_7 \vdash \Delta_{10}, F_8, F_9}{-:\Delta_7 \vdash \Delta_{10}, F_8} \xrightarrow{\text{ax/W}} \xrightarrow{-:\Delta_7, F_9 \vdash \Delta_{10}, F_8} \text{sCut}}{ \frac{-:\Delta_7, F_8 \vdash \Delta_{10}}{\circ} \xrightarrow{\text{sCut}}} \xrightarrow{\text{ax/W}} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        -: \Delta_7 \vdash \Delta_{10}
                                                                                                 \frac{\mathbf{h}_2 : \Delta_{11} \vdash \mathbf{F}_{12} \lor \mathbf{F}_{13}, \mathbf{F}_{8}, \mathbf{F}_{9}, \Delta_{7}}{\mathbf{h}_2 : \Delta_{11} \vdash (\Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9}), \mathbf{F}_{12} \lor \mathbf{F}_{13}} \ \lor_{R} \ \frac{\mathbf{h}_{10} : \mathbf{F}_{12}, \Delta_{11} \vdash \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \quad \mathbf{h}_{10} : \mathbf{F}_{13}, \Delta_{11} \vdash \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9}}{\bullet \mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \lor \mathbf{F}_{13} \vdash \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9}} \ \mathsf{Cut}
                                                                                 \bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \lor F_9), F_{12} \lor F_{13} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           -:\Delta_{11}\vdash\Delta_{7},\mathtt{F}_{8}\vee\mathtt{F}_{9}
                                                                                                                                                                                                                                                                                                                                                                                                               \underbrace{ \frac{\mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9} \quad \text{inv-th/ax}}_{\bullet \mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{13} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9}} }_{\bullet \mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \vee \mathbf{F}_{13} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9}}_{\bullet \mathbf{h}_{0tt}} \quad \underbrace{\mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \vee \mathbf{F}_{13} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9}}_{\bullet \mathbf{h}_{0tt}} \quad \underbrace{\mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \vee \mathbf{F}_{13} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9}}_{\bullet \mathbf{h}_{0tt}} \quad \underbrace{\mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9}}_{\bullet \mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9}} \quad \underbrace{\mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9}}_{\bullet \mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9}}_{\bullet \mathbf{h}_{10} : \Delta_{11}, \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9}} \quad \underbrace{\mathbf{h}_{10} : \Delta_{11}, \mathbf{h}_{12} \vdash \Delta_{
                                                                                                                                                                                                                                                                                                                                                                                                                                                    -:\Delta_{11}\vdash\Delta_7,\mathsf{F}_8,\mathsf{F}_9 \lor_R
                                                                                                                                                                                                                                                                                                                                                                                                                                    \overline{-:\Delta_{11}\vdash\Delta_{7},\mathtt{F}_{8}\vee\mathtt{F}_{9}}
                                                                         -:\Delta_{14},\mathtt{F}_{12}\vee\mathtt{F}_{13}\vdash\Delta_{8},\mathtt{F}_{9}\vee\mathtt{F}_{10}
\frac{\mathbf{h}_{1}:\Delta_{14},\mathbf{F}_{12}\vee\mathbf{F}_{13}\vdash\Delta_{8},\mathbf{F}_{10},\mathbf{F}_{7},\mathbf{F}_{9}}{\mathbf{h}_{2}:\Delta_{14},\mathbf{F}_{12}\vee\mathbf{F}_{13}\vdash\Delta_{8},\mathbf{F}_{10},\mathbf{F}_{9}} \\ \mathbf{ax/W} \\ \frac{\mathbf{h}_{11}:\Delta_{14},\mathbf{F}_{12},\mathbf{F}_{7}\vdash\Delta_{8},\mathbf{F}_{10},\mathbf{F}_{9}}{\bullet\mathbf{h}_{11}:\Delta_{14},\mathbf{F}_{7},\mathbf{F}_{12}\vee\mathbf{F}_{13}\vdash\Delta_{8},\mathbf{F}_{10},\mathbf{F}_{9}} \\ \mathbf{h}_{11}:\Delta_{14},\mathbf{F}_{13},\mathbf{F}_{7}\vdash\Delta_{8},\mathbf{F}_{10},\mathbf{F}_{9} \\ \vee_{L} \\ \mathbf{h}_{11}:\Delta_{14},\mathbf{F}_{13},\mathbf{F}_{14}\vdash\Delta_{14},\mathbf{F}_{13},\mathbf{F}_{14}\vdash\Delta_{14},\mathbf{F}_{14},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15},\mathbf{F}_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash\Delta_{15}\vdash
                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{\mathsf{F}_9}{-:\Delta_{14},\mathsf{F}_{12}\vee\mathsf{F}_{13}\vdash\Delta_8,\mathsf{F}_{10},\mathsf{F}_9}\vee_R
                                                                                                                                                                                                                                                                                                                                                                                                                               -: \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_{8}, F_{9} \vee F_{10}
```

### $\bullet$ Case rule AT

$$\frac{ \begin{array}{c} \mathbf{h}_{1} : \Delta_{11}, [] \mathbf{F}_{9} \vdash \mathbf{F}_{6}, \mathbf{F}_{7}, \Delta_{10} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{11}, [] \mathbf{F}_{9} \vdash \Delta_{10}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \end{array} }{ \bullet \mathbf{h}_{8} : (\Delta_{11}, [] \mathbf{F}_{9}), \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] \mathbf{F}_{9} \vdash \Delta_{10} \end{array} } \begin{array}{c} AT \\ \hline - : \Delta_{11}, [] \mathbf{F}_{9} \vdash \Delta_{10} \end{array} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{11}, \mathbf{F}_{9}, [] \mathbf{F}_{9} \vdash \Delta_{10}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \end{array} } \begin{array}{c} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{11}, \mathbf{F}_{9}, [] \mathbf{F}_{9} \vdash \Delta_{10} \end{array} } \begin{array}{c} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{11}, \mathbf{F}_{9}, [] \mathbf{F}_{9} \vdash \Delta_{10} \end{array} } \begin{array}{c} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{11}, \mathbf{F}_{9}, [] \mathbf{F}_{9} \vdash \Delta_{10} \end{array} } \begin{array}{c} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{2} : \Delta_{11} \vdash [] \mathbf{F}_{12}, \mathbf{F}_{8}, \mathbf{F}_{9}, \Delta_{7} \\ \hline \bullet \mathbf{h}_{2} : \Delta_{11} \vdash (\Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9}), [] \mathbf{F}_{12} \end{array} } \begin{array}{c} \mathbf{AT} \\ \hline \bullet \mathbf{h}_{2} : \Delta_{11} \vdash (\Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9}), [] \mathbf{F}_{12} \end{array} \\ \hline \bullet \mathbf{h}_{2} : \Delta_{11} \vdash (\Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9}), [] \mathbf{F}_{12} \end{array} } \begin{array}{c} \mathbf{AT} \\ \hline \bullet \mathbf{h}_{10} : \mathbf{h}_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{10} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{10} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{10} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{10} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{10} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{10} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{10} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{10} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{11} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{7}, \mathbf{F}_{8}, \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{11} : \Delta_{11}, [] \mathbf{F}_{12} \vdash \Delta_{8}, \mathbf{F}_{9} \lor \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{11} : \Delta_{13}, [] \mathbf{F}_{12} \vdash \Delta_{8}, \mathbf{F}_{9} \lor \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{2} : \Delta_{13}, [] \mathbf{F}_{12} \vdash \Delta_{8}, \mathbf{F}_{9} \lor \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{2} : \Delta_{13}, [] \mathbf{F}_{12} \vdash \Delta_{8}, \mathbf{F}_{9} \lor \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{2} : \Delta_{13}, [] \mathbf{F}_{12} \vdash \Delta_{8}, \mathbf{F}_{7} \lor \mathbf{F}_{9} \lor \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{2} : \Delta_{13}, [] \mathbf{F}_{12} \vdash \Delta_{8}, \mathbf{F}_{7} \lor \mathbf{F}_{9} \lor \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_{2} : \Delta_{13},$$

#### • Case rule $\perp_L$

$$\begin{array}{c|c} \frac{h_1: \bot, \Delta_{10} \vdash F_6, F_7, \Delta_9}{\bullet h_1: \bot, \Delta_{10} \vdash \Delta_9, F_6 \lor F_7} \lor_R & \frac{}{\bullet h_8: (\bot, \Delta_{10}), F_6 \lor F_7 \vdash \Delta_9} & \bot_L \\ \hline \\ -: \bot, \Delta_{10} \vdash \Delta_9 & \\ \hline \\ -: \bot, \Delta_{10} \vdash \Delta_9 & \\ \hline \\ -: \bot, \Delta_{10} \vdash \Delta_9 & \bot_L \\ \hline \\ \frac{h_2: \Delta_{11} \vdash \bot, F_8, F_9, \Delta_7}{\bullet h_2: \Delta_{11} \vdash (\Delta_7, F_8 \lor F_9), \bot} & \sqrt{R} & \frac{}{\bullet h_{10}: \Delta_{11}, \bot \vdash \Delta_7, F_8 \lor F_9} & \bot_L \\ \hline \\ \frac{h_2: \Delta_{11} \vdash \bot, \Delta_7, F_8, F_9 & \text{ax/W}}{\bullet h_{10}: \bot, \Delta_{11} \vdash \Delta_7, F_8, F_9} & \bot_L \\ \hline \\ \frac{-: \Delta_{11} \vdash \Delta_7, F_8, F_9}{-: \Delta_{11} \vdash \Delta_7, F_8, F_9} & \vee_R \\ \hline \\ \frac{h_2: \Delta_{11} \vdash \bot, \Delta_7, F_8, F_9}{-: \Delta_{11} \vdash \Delta_7, F_8, F_9} & \vee_R \\ \hline \\ \frac{\bullet h_2: \bot, \Delta_{12} \vdash F_7, F_9, F_{10}, \Delta_8}{-: \bot, \Delta_{12} \vdash (\Delta_8, F_9 \lor F_{10}), F_7} & \sqrt{R} & \frac{\bullet_{h_{11}: (\bot, \Delta_{12}), F_7} \vdash \Delta_8, F_9 \lor F_{10}}{\bullet h_{21}: \bot, \Delta_{12} \vdash (\Delta_8, F_9 \lor F_{10})} & \bot_L \\ \hline \\ -: \bot, \Delta_{12} \vdash \Delta_8, F_9 \lor F_{10} & \bot_L \\ \hline \\ -: \bot, \Delta_{12} \vdash \Delta_8, F_9 \lor F_{10} & \bot_L \\ \hline \end{array}$$

## ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{p}_{9} \vdash \mathbf{F}_{6},\mathbf{F}_{7},\Delta_{10},\mathbf{p}_{9}}{\bullet \mathbf{h}_{1}:\Delta_{11},\mathbf{p}_{9} \vdash (\Delta_{10},\mathbf{p}_{9}),\mathbf{F}_{6} \vee \mathbf{F}_{7}} \quad \vee_{R} \quad & \bullet \mathbf{h}_{8}:(\Delta_{11},\mathbf{p}_{9}),\mathbf{F}_{6} \vee \mathbf{F}_{7} \vdash \Delta_{10},\mathbf{p}_{9}} \\ & -:\Delta_{11},\mathbf{p}_{9} \vdash \Delta_{10},\mathbf{p}_{9} \\ & -:\Delta_{11},\mathbf{p}_{9} \vdash \Delta_{10},\mathbf{p}_{9} \end{array} I \\ \\ \frac{\mathbf{h}_{2}:\Delta_{10} \vdash \mathbf{p}_{11},\mathbf{F}_{7},\mathbf{F}_{8},\Delta_{12},\mathbf{p}_{11}}{\bullet \mathbf{h}_{2}:\Delta_{10} \vdash ((\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{7} \vee \mathbf{F}_{8}),\mathbf{p}_{11}} \quad \vee_{R} \quad & \bullet \mathbf{h}_{9}:\Delta_{10},\mathbf{p}_{11} \vdash (\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{7} \vee \mathbf{F}_{8} \\ \hline & -:\Delta_{10} \vdash (\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{7} \vee \mathbf{F}_{8} \\ \hline & \bullet \mathbf{h}_{9}:\Delta_{10},\mathbf{p}_{11} \vdash (\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{7} \vee \mathbf{F}_{8} \\ \hline & -:\Delta_{10} \vdash (\Delta_{12},\mathbf{F}_{7},\mathbf{F}_{8},\mathbf{p}_{11} \\ \hline & -:\Delta_{10} \vdash \Delta_{12},\mathbf{F}_{7},\mathbf{F}_{8},\mathbf{p}_{11} \\ \hline & -:\Delta_{10} \vdash \Delta_{12},\mathbf{F}_{7},\mathbf{F}_{8},\mathbf{p}_{11} \\ \hline & -:\Delta_{10} \vdash \Delta_{12},\mathbf{p}_{11},\mathbf{F}_{7} \vee \mathbf{F}_{8} \\ \hline & \bullet \mathbf{h}_{2}:\Delta_{13},\mathbf{p}_{11} \vdash \mathbf{F}_{7},\mathbf{F}_{8},\mathbf{F}_{9},\Delta_{12},\mathbf{p}_{11} \\ \hline \bullet \mathbf{h}_{2}:\Delta_{13},\mathbf{p}_{11} \vdash ((\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{8} \vee \mathbf{F}_{9}),\mathbf{F}_{7} \\ \hline & \bullet \mathbf{h}_{10}:(\Delta_{13},\mathbf{p}_{11}),\mathbf{F}_{7} \vdash (\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{8} \vee \mathbf{F}_{9} \\ \hline & -:\Delta_{13},\mathbf{p}_{11} \vdash (\Delta_{12},\mathbf{p}_{11}),\mathbf{F}_{8} \vee \mathbf{F}_{9} \\ \hline & -:\Delta_{13},\mathbf{p}_{11} \vdash \Delta_{12},\mathbf{p}_{11},\mathbf{F}_{8} \vee \mathbf{F}_{9} \\ \hline & -:\Delta_{13},\mathbf{p}_{11} \vdash \Delta_$$

### • Case rule $\top_L$

$$\begin{array}{c} \frac{\mathbf{h}_1: \top, \Delta_{10} \vdash \mathbf{F}_6, \mathbf{F}_7, \Delta_9}{\bullet \mathbf{h}_1: \top, \Delta_{10} \vdash \Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7} \lor_R & \frac{\mathbf{h}_8: \Delta_{10}, \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \Delta_9}{\bullet \mathbf{h}_8: (\top, \Delta_{10}), \mathbf{F}_6 \lor \mathbf{F}_7 \vdash \Delta_9} & \top_L \\ \hline -: \top, \Delta_{10} \vdash \Delta_9 & \mathbf{ax/W} & \\ \hline \bullet \mathbf{h}_1: \top, \Delta_{10} \vdash \Delta_9, \mathbf{F}_6 \lor \mathbf{F}_7 & \mathbf{ax/W} & \\ \hline \bullet \mathbf{h}_2: \Delta_{11} \vdash \top, \mathbf{F}_8, \mathbf{F}_9, \Delta_7 & \mathbf{ax/W} & \mathbf{h}_{10}: \Delta_{11} \vdash \Delta_7, \mathbf{F}_8 \lor \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_2: \Delta_{11} \vdash (\Delta_7, \mathbf{F}_8 \lor \mathbf{F}_9), \top & \mathbf{ax/W} & \mathbf{h}_{10}: \Delta_{11}, \top \vdash \Delta_7, \mathbf{F}_8 \lor \mathbf{F}_9 \\ \hline -: \Delta_{11} \vdash \Delta_7, \mathbf{F}_8 \lor \mathbf{F}_9 & \mathbf{ax/W} & \\ \hline -: \Delta_{11} \vdash \Delta_7, \mathbf{F}_8 \lor \mathbf{F}_9 & \mathbf{ax/W} & \\ \hline \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_2: \top, \Delta_{12} \vdash \mathbf{F}_7, \mathbf{F}_9, \mathbf{F}_{10}, \Delta_8 \\ \bullet \mathbf{h}_2: \top, \Delta_{12} \vdash (\Delta_8, \mathbf{F}_9 \vee \mathbf{F}_{10}), \mathbf{F}_7 \end{array} \vee_R \quad \frac{\mathbf{h}_{11}: \mathbf{F}_7, \Delta_{12} \vdash \Delta_8, \mathbf{F}_9 \vee \mathbf{F}_{10}}{\bullet \mathbf{h}_{11}: (\top, \Delta_{12}), \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9 \vee \mathbf{F}_{10}} \quad \frac{\top_L}{\mathsf{Cut}} \\ \hline \\ \bullet \mathbf{h}_2: \top, \Delta_{12} \vdash \Delta_8, \mathbf{F}_7, \mathbf{F}_9 \vee \mathbf{F}_{10} \end{array} \quad \frac{\mathsf{ax}/\mathsf{W}}{\mathsf{h}_{11}: \top, \Delta_{12}, \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9 \vee \mathbf{F}_{10}} \quad \frac{\mathsf{ax}/\mathsf{W}}{\mathsf{h}_{\mathrm{Cut}}} \\ \hline \\ \bullet \mathbf{h}_2: \top, \Delta_{12} \vdash \Delta_8, \mathbf{F}_7, \mathbf{F}_9 \vee \mathbf{F}_{10}} \quad \frac{\mathsf{ax}/\mathsf{W}}{\mathsf{h}_{\mathrm{Cut}}} \\ \hline \\ \bullet \mathbf{h}_2: \top, \Delta_{12} \vdash \Delta_8, \mathbf{F}_7, \mathbf{F}_9 \vee \mathbf{F}_{10}} \quad \mathsf{ax}/\mathsf{W} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{h}_{\mathrm{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathrm{Cut}} \vdash \mathbf{$$

# 6.4 Status of $\perp_R$ : OK

• Case rule  $\rightarrow_R$ 

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_4 \vdash \Delta_6, \mathsf{F}_7 \to \mathsf{F}_8}{\bullet \mathbf{h}_1: \Delta_4 \vdash (\Delta_6, \mathsf{F}_7 \to \mathsf{F}_8), \bot} \ \bot_R \ \frac{\mathbf{h}_5: \bot, \mathsf{F}_7, \Delta_4 \vdash \mathsf{F}_8, \Delta_6}{\bullet \mathbf{h}_5: \Delta_4, \bot \vdash \Delta_6, \mathsf{F}_7 \to \mathsf{F}_8} \ \xrightarrow{\bullet}_R \ \\ \hline -: \Delta_4 \vdash \Delta_6, \mathsf{F}_7 \to \mathsf{F}_8 \ \hline \\ \hline -: \Delta_4 \vdash \Delta_6, \mathsf{F}_7 \to \mathsf{F}_8 \ \hline \\ \hline -: \Delta_4 \vdash \Delta_6, \mathsf{F}_7 \to \mathsf{F}_8 \ \hline \\ \bullet \\ \hline \bullet \mathbf{h}_2: \Delta_6 \vdash \mathsf{F}_5, \Delta_{10}, \mathsf{F}_8 \to \mathsf{F}_9 \ \\ \hline \bullet \mathbf{h}_2: \Delta_6 \vdash (\bot, \Delta_{10}, \mathsf{F}_8 \to \mathsf{F}_9), \mathsf{F}_5 \ \hline \\ -: \Delta_6 \vdash \bot, \Delta_{10}, \mathsf{F}_8 \to \mathsf{F}_9 \ \hline \\ \hline -: \Delta_6 \vdash \bot, \Delta_{10}, \mathsf{F}_8 \to \mathsf{F}_9 \ \hline \\ \hline \bullet \mathbf{h}_7: \Delta_6, \mathsf{F}_5 \vdash \bot, \Delta_{10}, \mathsf{F}_8 \to \mathsf{F}_9 \ \hline \\ \hline \bullet \mathbf{h}_7: \Delta_6, \mathsf{F}_5 \vdash \bot, \Delta_{10}, \mathsf{F}_8 \to \mathsf{F}_9 \ \hline \\ -: \Delta_6 \vdash \bot, \Delta_{10}, \mathsf{F}_8 \to \mathsf{F}_9 \ \hline \\ \hline -: \Delta_6 \vdash \bot, \Delta_{10}, \mathsf{F}_8 \to \mathsf{F}_9 \ \hline \end{array} \right. \xrightarrow{\bullet}_R \ \mathsf{Cut}$$

• Case rule  $\wedge_R$ 

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \\ \bullet \mathbf{h}_1 : \Delta_4 \vdash (\Delta_6, F_7 \wedge F_8), \bot \end{array} \begin{array}{c} \bot_R \end{array} \begin{array}{c} \mathbf{h}_5 : \bot, \Delta_4 \vdash F_7, \Delta_6 & \mathbf{h}_5 : \bot, \Delta_4 \vdash F_8, \Delta_6 \\ \bullet \mathbf{h}_5 : \Delta_4, \bot \vdash \Delta_6, F_7 \wedge F_8 \end{array} \\ \hline -: \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \\ \hline -: \Delta_4 \vdash \Delta_6, F_7 \wedge F_8 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_2 : \Delta_6 \vdash F_5, \Delta_{10}, F_8 \wedge F_9 \\ \bullet \mathbf{h}_2 : \Delta_6 \vdash (\bot, \Delta_{10}, F_8 \wedge F_9), F_5 \end{array} \begin{array}{c} \bot_R \end{array} \begin{array}{c} \mathbf{h}_7 : F_5, \Delta_6 \vdash \bot, F_8, \Delta_{10} & \mathbf{h}_7 : F_5, \Delta_6 \vdash \bot, F_9, \Delta_{10} \\ \bullet \mathbf{h}_7 : \Delta_6, F_5 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \Delta_R \\ \hline -: \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{cut} \\ \hline -: \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_7 : \Delta_6, F_5 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ -: \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_7 : \Delta_6, F_5 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_6 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_8 \vdash \bot, \Delta_8 \vdash \bot, \Delta_{10}, F_8 \wedge F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_7 : \Delta_8 \vdash \bot, \Delta_8 \vdash \bot, \Delta_8 \vdash \bot, \Delta_8 \vdash \bot$$

• Case rule  $\vee_R$ 

$$\begin{array}{c|c} \frac{h_1:\Delta_4\vdash\Delta_6,F_7\vee F_8}{\bullet h_1:\Delta_4\vdash(\Delta_6,F_7\vee F_8),\bot} & \bot_R & \frac{h_5:\bot,\Delta_4\vdash F_7,F_8,\Delta_6}{\bullet h_5:\Delta_4,\bot\vdash\Delta_6,F_7\vee F_8} & \lor_R \\ \hline & -:\Delta_4\vdash\Delta_6,F_7\vee F_8 & \text{Cut} \\ \hline & -:\Delta_4\vdash\Delta_6,F_7\vee F_8 & \text{ax/W} \\ \hline & \frac{h_2:\Delta_6\vdash F_5,\Delta_{10},F_8\vee F_9}{\bullet h_2:\Delta_6\vdash(\bot,\Delta_{10},F_8\vee F_9),F_5} & \bot_R & \frac{h_7:F_5,\Delta_6\vdash\bot,F_8,F_9,\Delta_{10}}{\bullet h_7:\Delta_6,F_5\vdash\bot,\Delta_{10},F_8\vee F_9} & \lor_R \\ \hline & \frac{-:\Delta_6\vdash\bot,\Delta_{10},F_8\vee F_9}{\bullet h_2:\Delta_6\vdash\bot,\Delta_{10},F_8\vee F_9} & \text{ax/W} \\ \hline & \frac{h_2:\Delta_6\vdash\bot,\Delta_{10},F_5,F_8\vee F_9}{\bullet h_2:\Delta_6\vdash\bot,\Delta_{10},F_8\vee F_9} & \text{ax/W} \\ \hline \end{array}$$

• Case rule  $\perp_R$ 

$$\frac{\begin{array}{c} \mathbf{h}_1: \Delta_4 \vdash \bot, \Delta_6 \\ \bullet \mathbf{h}_1: \Delta_4 \vdash (\bot, \Delta_6), \bot \end{array} \perp_R \quad \frac{\mathbf{h}_5: \bot, \Delta_4 \vdash \Delta_6}{\bullet \mathbf{h}_5: \Delta_4, \bot \vdash \bot, \Delta_6} \quad \frac{\bot_R}{\mathsf{Cut}} \\ -: \Delta_4 \vdash \bot, \Delta_6 \\ \hline -: \Delta_4 \vdash \bot, \Delta_6 \quad \mathsf{ax/W} \end{array}}$$

$$\frac{\begin{array}{c} \mathbf{h}_2: \Delta_6 \vdash \mathbf{F}_5, \Delta_8 \\ \bullet \mathbf{h}_2: \Delta_6 \vdash (\bot, \Delta_8), \mathbf{F}_5 \end{array} \perp_R \quad \frac{\mathbf{h}_7: \mathbf{F}_5, \Delta_6 \vdash \Delta_8}{\bullet \mathbf{h}_7: \Delta_6, \mathbf{F}_5 \vdash \bot, \Delta_8} \quad \frac{\bot_R}{\mathsf{Cut}} \\ \hline -: \Delta_6 \vdash \bot, \Delta_8 \\ \hline \\ \frac{\mathbf{h}_2: \Delta_6 \vdash \bot, \Delta_8, \mathbf{F}_5}{\bullet \mathbf{h}_7: \Delta_6, \mathbf{F}_5 \vdash \bot, \Delta_8} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \hline \\ -: \Delta_6 \vdash \bot, \Delta_8 \end{array}$$

# • Case rule $\top_R$

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_4 \vdash \top,\Delta_6}{\bullet \mathbf{h}_1:\Delta_4 \vdash (\top,\Delta_6),\bot} \quad \bot_R \quad \frac{\bullet \mathbf{h}_5:\Delta_4,\bot \vdash \top,\Delta_6}{\bullet \mathbf{h}_5:\Delta_4,\bot \vdash \top,\Delta_6} \quad \overset{\top_R}{\mathsf{Cut}} \\ & \xrightarrow{-:\Delta_4 \vdash \top,\Delta_6} \quad \top_R \\ \\ \frac{\mathbf{h}_2:\Delta_6 \vdash \mathbf{F}_5,\top,\Delta_8}{\bullet \mathbf{h}_2:\Delta_6 \vdash (\bot,\top,\Delta_8),\mathbf{F}_5} \quad \bot_R \quad \frac{\bullet \mathbf{h}_7:\Delta_6,\mathbf{F}_5 \vdash \bot,\top,\Delta_8}{\bullet \mathbf{h}_7:\Delta_6,\mathbf{F}_5 \vdash \bot,\top,\Delta_8} \quad \overset{\top_R}{\mathsf{Cut}} \\ & \xrightarrow{-:\Delta_6 \vdash \bot,\top,\Delta_8} \quad \top_R \end{array}$$

### $\bullet$ Case rule K

$$\begin{array}{c} \begin{array}{c} \begin{array}{c} \mathbf{h}_1: \square\Gamma_5, \Delta_8 \vdash \Delta_6, []F_7 \\ \hline \bullet \mathbf{h}_1: \square\Gamma_5, \Delta_8 \vdash (\Delta_6, []F_7), \bot \end{array} & \begin{array}{c} \mathbf{h}_4: unbox(\square\Gamma_5) \vdash F_7 \\ \hline \bullet \mathbf{h}_4: (\square\Gamma_5, \Delta_8), \bot \vdash \Delta_6, []F_7 \end{array} & K \\ \hline \\ -: \square\Gamma_5, \Delta_8 \vdash \Delta_6, []F_7 \\ \hline \hline \\ -: \Delta_8, \square\Gamma_5 \vdash \Delta_6, []F_7 \end{array} & \mathbf{ax/W} \\ \hline \\ \begin{array}{c} \mathbf{h}_2: \square\Gamma_{10}, \Delta_7 \vdash \square F_5, \Delta_9, []F_8 \\ \hline \bullet \mathbf{h}_2: \square\Gamma_{10}, \Delta_7 \vdash (\bot, \Delta_9, []F_8), \square F_5 \end{array} & \bot_R & \begin{array}{c} \mathbf{h}_6: unbox(\square\Gamma_{10}), unbox(\square F_5) \vdash F_8 \\ \hline \bullet \mathbf{h}_6: (\square\Gamma_{10}, \Delta_7), \square F_5 \vdash \bot, \Delta_9, []F_8 \end{array} & K \\ \hline \\ -: \square\Gamma_{10}, \Delta_7 \vdash \bot, \Delta_9, []F_8 \end{array} & \mathbf{Cut} \\ \hline \\ \begin{array}{c} \mathbf{h}_2: \Delta_7, \square\Gamma_{10} \vdash \bot, \square F_5, \Delta_9, []F_8 \end{array} & \mathbf{ax/W} \\ \hline \\ \begin{array}{c} \mathbf{h}_2: \Delta_7, \square\Gamma_{10} \vdash \bot, \square F_5, \Delta_9, []F_8 \end{array} & \mathbf{ax/W} \\ \hline \\ \begin{array}{c} \mathbf{h}_2: \square\Gamma_7, \Delta_{10} \vdash \bot, \square F_5, \Delta_9, []F_8 \end{array} & \mathbf{ax/W} \\ \hline \\ \begin{array}{c} \mathbf{h}_2: \square\Gamma_7, \Delta_{10} \vdash \bot, \Delta_9, []F_8 \end{array} & \mathbf{A}_6: unbox(\square\Gamma_7) \vdash F_8 \\ \hline \\ \bullet \mathbf{h}_6: \square\Gamma_7, \Delta_{10}, \square\Gamma_7, \Delta_{10} \vdash \bot, \Delta_9, []F_8 \end{array} & K \\ \hline \\ \begin{array}{c} \mathbf{h}_2: \square\Gamma_7, \Delta_{10} \vdash (\bot, \Delta_9, []F_8), F_5 \end{array} & \mathbf{A}_R & \begin{array}{c} \mathbf{h}_6: unbox(\square\Gamma_7) \vdash F_8 \\ \hline \\ \bullet \mathbf{h}_6: (\square\Gamma_7, \Delta_{10}), F_5 \vdash \bot, \Delta_9, []F_8 \end{array} & K \\ \hline \\ -: \square\Gamma_7, \Delta_{10} \vdash \bot, \Delta_9, []F_8 \end{array} & \mathbf{A}_R & \begin{array}{c} \mathbf{h}_6: unbox(\square\Gamma_7) \vdash F_8 \\ \hline \\ \bullet \mathbf{h}_6: (\square\Gamma_7, \Delta_{10}), F_5 \vdash \bot, \Delta_9, []F_8 \end{array} & K \\ \hline \\ -: \square\Gamma_7, \Delta_{10} \vdash \bot, \Delta_9, []F_8 \end{array} & \mathbf{A}_R & \mathbf{A}_R$$

# • Case rule A45

$$\frac{\begin{array}{c} \mathbf{h}_2: \square\Gamma_7, \Delta_{11} \vdash \mathbf{F}_5, \square\Gamma_8, \Delta_{10}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_2: \square\Gamma_7, \Delta_{11} \vdash (\bot, \square\Gamma_8, \Delta_{10}, []\mathbf{F}_9), \mathbf{F}_5 \end{array} \perp_R \begin{array}{c} \mathbf{h}_6: \square\Gamma_7 \vdash \square\Gamma_8, \mathbf{F}_9 \\ \bullet \mathbf{h}_6: (\square\Gamma_7, \Delta_{11}), \mathbf{F}_5 \vdash \bot, \square\Gamma_8, \Delta_{10}, []\mathbf{F}_9 \\ \hline -: \square\Gamma_7, \Delta_{11} \vdash \bot, \square\Gamma_8, \Delta_{10}, []\mathbf{F}_9 \\ \hline \sim \\ \hline -: \square\Gamma_7 \vdash \mathbf{F}_9, \square\Gamma_8 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline -: \Delta_{11}, \square\Gamma_7 \vdash \bot, \Delta_{10}, \square\Gamma_8, []\mathbf{F}_9 \end{array} A45$$

• Case rule  $\rightarrow_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \Delta_7}{\bullet \mathbf{h}_1:\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \Delta_7, \bot} \quad \bot_R \quad \frac{\mathbf{h}_4:\bot,\Delta_8 \vdash \mathbf{F}_5,\Delta_7 \quad \mathbf{h}_4:\bot,\mathbf{F}_6,\Delta_8 \vdash \Delta_7}{\bullet \mathbf{h}_4:(\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6),\bot \vdash \Delta_7} \quad \bot_L \\ \hline -:\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \Delta_7 \\ \hline -:\Delta_8, \mathbf{F}_5 \to \mathbf{F}_6 \vdash \Delta_7 \\ \hline \bullet \mathbf{h}_2:\Delta_7 \vdash \mathbf{F}_8 \to \mathbf{F}_9,\Delta_5 \\ \bullet \mathbf{h}_2:\Delta_7 \vdash (\bot,\Delta_5), \mathbf{F}_8 \to \mathbf{F}_9 \quad \bot_R \quad \frac{\mathbf{h}_6:\Delta_7 \vdash \bot,\mathbf{F}_8,\Delta_5 \quad \mathbf{h}_6:\mathbf{F}_9,\Delta_7 \vdash \bot,\Delta_5}{\bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_5} \quad \mathtt{Cut} \\ \hline -:\Delta_7 \vdash \bot,\Delta_5 \\ \hline \bullet \mathbf{h}_2:\Delta_7 \vdash \bot,\Delta_5, \mathbf{F}_8 \to \mathbf{F}_9 \quad \underbrace{\mathsf{ax/W}}_{\bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_5}^{\bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_5} \quad \mathtt{cut} \\ \hline -:\Delta_7 \vdash \bot,\Delta_5 \\ \hline \bullet \mathbf{h}_2:\Delta_{10}, \mathbf{F}_8 \to \mathbf{F}_9 \vdash \mathbf{F}_5,\Delta_6 \quad \bullet_{17}:\mathbf{F}_5,\Delta_{10} \vdash \bot,\mathbf{F}_8,\Delta_6 \quad \mathbf{h}_7:\mathbf{F}_5,\mathbf{F}_9,\Delta_{10} \vdash \bot,\Delta_6}^{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_6} \quad \mathtt{Cut} \\ \hline -:\Delta_{10},\mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_6 \\ \hline \bullet \mathbf{h}_2:\Delta_{10},\mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_6,\mathbf{F}_5} \quad \underbrace{\mathbf{ax/W}}_{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5,\mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_6}^{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5,\mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_6}^{\bullet \mathbf{ax/W}} \quad \mathtt{hCut} \\ \hline -:\Delta_{10},\mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_6 \\ \hline \bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_6}^{\bullet \mathbf{h}_7:\Delta_{10},\mathbf{F}_5,\mathbf{F}_8 \to \mathbf{F}_9 \vdash \bot,\Delta_6}^{\bullet \mathbf{ax/W}} \quad \mathtt{hCut} \\ \hline \end{array}$$

• Case rule  $\wedge_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \Delta_7}{\bullet \mathbf{h}_1:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \Delta_7, \bot} \quad \bot_R \quad \frac{\mathbf{h}_4:\bot, \mathbf{F}_5, \mathbf{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathbf{h}_4:(\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6), \bot \vdash \Delta_7} \quad \bigwedge_L \\ \hline -:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \Delta_7 \\ \hline -:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \Delta_7 \\ \hline \bullet \mathbf{h}_2:\Delta_7 \vdash \mathbf{F}_8 \wedge \mathbf{F}_9, \Delta_5 \\ \bullet \mathbf{h}_2:\Delta_7 \vdash (\bot, \Delta_5), \mathbf{F}_8 \wedge \mathbf{F}_9 \quad \bot_R \quad \frac{\mathbf{h}_6:\mathbf{F}_8, \mathbf{F}_9, \Delta_7 \vdash \bot, \Delta_5}{\bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_5} \quad \bigwedge_L \\ \hline -:\Delta_7 \vdash \bot, \Delta_5 \\ \hline \hline \bullet \mathbf{h}_2:\Delta_7 \vdash \bot, \Delta_5, \mathbf{F}_8 \wedge \mathbf{F}_9 \quad \mathbf{ax/W} \quad \bullet \mathbf{h}_6:\Delta_7, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_5} \quad \mathbf{ax/W} \\ \hline -:\Delta_7 \vdash \bot, \Delta_5 \\ \hline \bullet \mathbf{h}_2:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \mathbf{F}_5, \Delta_6 \\ \bullet \mathbf{h}_2:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash (\bot, \Delta_6), \mathbf{F}_5 \quad \bot_R \quad \bullet \mathbf{h}_7:\mathbf{F}_5, \mathbf{F}_8, \mathbf{F}_9, \Delta_{10} \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline \bullet \mathbf{h}_2:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6, \mathbf{F}_5 \quad \bullet \mathbf{h}_7:\Delta_{10}, \mathbf{F}_5, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline -:\Delta_{10}, \mathbf{F}_8 \wedge \mathbf{F}_9 \vdash \bot, \Delta_6 \\ \hline$$

• Case rule  $\vee_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \Delta_7}{\bullet \mathbf{h}_1: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \Delta_7, \bot} \ \bot_R & \frac{\mathbf{h}_4: \bot, \mathbf{F}_5, \Delta_8 \vdash \Delta_7 \quad \mathbf{h}_4: \bot, \mathbf{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathbf{h}_4: (\Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6), \bot \vdash \Delta_7} \ \mathsf{Cut} \\ & -: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \Delta_7 \\ & & -: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \Delta_7 \end{array} \ \mathbf{ax/W}$$

$$\frac{ \begin{array}{c} \mathbf{h}_2: \Delta_7 \vdash F_8 \lor F_9, \Delta_5 \\ \bullet \mathbf{h}_2: \Delta_7 \vdash (\bot, \Delta_5), F_8 \lor F_9 \end{array} \bot_R \begin{array}{c} \mathbf{h}_6: F_8, \Delta_7 \vdash \bot, \Delta_5 & \mathbf{h}_6: F_9, \Delta_7 \vdash \bot, \Delta_5 \\ \hline \bullet \mathbf{h}_6: \Delta_7, F_8 \lor F_9 \vdash \bot, \Delta_5 & \mathbf{cut} \\ \hline \\ \hline \\ -: \Delta_7 \vdash \bot, \Delta_5 \\ \hline \\ \hline \mathbf{h}_2: \Delta_7 \vdash \bot, \Delta_5, F_8 \lor F_9 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_6: \Delta_7, F_8 \lor F_9 \vdash \bot, \Delta_5 & \mathbf{ax/W} \\ \hline \\ -: \Delta_7 \vdash \bot, \Delta_5 & \mathbf{hCut} \\ \hline \\ \hline \\ \bullet \mathbf{h}_2: \Delta_{10}, F_8 \lor F_9 \vdash F_5, \Delta_6 & \mathbf{h}_7: F_5, F_8, \Delta_{10} \vdash \bot, \Delta_6 & \mathbf{h}_7: F_5, F_9, \Delta_{10} \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_2: \Delta_{10}, F_8 \lor F_9 \vdash (\bot, \Delta_6), F_5 & \mathbf{h}_7: (\Delta_{10}, F_8 \lor F_9), F_5 \vdash \bot, \Delta_6 \\ \hline \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_5, F_8 \lor F_9 \vdash \bot, \Delta_6 \\ \hline \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_5, F_8 \lor F_9 \vdash \bot, \Delta_6 \\ \hline \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_5, F_8 \lor F_9 \vdash \bot, \Delta_6 \\ \hline \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_5, F_8 \lor F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_5, F_8 \lor F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_5, F_8 \lor F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_8 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7: \Delta_{10}, F_9 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \lor F_9 \vdash \bot, \Delta_6 & \mathbf{h}_7 \lor \Delta_7 \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \lor \mathbf{h}_7 \to \Delta_7 \\ \hline$$

#### $\bullet$ Case rule AT

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_7, []\mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_1:\Delta_7, []\mathbf{F}_5 \vdash \Delta_6, \bot} \quad \bot_R \quad \frac{\mathbf{h}_4:\bot, \mathbf{F}_5, \Delta_7, []\mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_4:(\Delta_7, []\mathbf{F}_5), \bot \vdash \Delta_6} \quad AT \\ \hline -:\Delta_7, []\mathbf{F}_5 \vdash \Delta_6 \quad \text{ax/W} \\ \hline -:\Delta_7, []\mathbf{F}_5 \vdash \Delta_6 \quad \text{ax/W} \\ \hline \frac{\mathbf{h}_2:\Delta_7 \vdash []\mathbf{F}_8, \Delta_5}{\bullet \mathbf{h}_2:\Delta_7 \vdash (\bot, \Delta_5), []\mathbf{F}_8} \quad \bot_R \quad \frac{\mathbf{h}_6:\mathbf{F}_8, \Delta_7, []\mathbf{F}_8 \vdash \bot, \Delta_5}{\bullet \mathbf{h}_6:\Delta_7, []\mathbf{F}_8 \vdash \bot, \Delta_5} \quad AT \\ \hline -:\Delta_7 \vdash \bot, \Delta_5 \quad & \\ \hline \frac{\mathbf{h}_2:\Delta_7 \vdash \bot, \Delta_5, []\mathbf{F}_8}{\bullet \mathbf{h}_2:\Delta_7 \vdash \bot, \Delta_5, []\mathbf{F}_8} \quad \text{ax/W} \quad & \\ \hline -:\Delta_7 \vdash \bot, \Delta_5 \quad & \\ \hline \frac{\mathbf{h}_2:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6}{\bullet \mathbf{h}_2:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6} \quad \mathbf{AT} \\ \hline -:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline -:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_2:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6, \mathbf{F}_5 \quad \text{ax/W} \\ \hline -:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, \mathbf{F}_5, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, \mathbf{F}_5, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, \mathbf{F}_5, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{F}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_8 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_9 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_9 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_9 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}_9 \vdash \bot, \Delta_6 \quad & \\ \hline \mathbf{h}_7:\Delta_9, []\mathbf{h}$$

# • Case rule $\perp_L$

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_5\vdash\Delta_6}{\bullet\mathbf{h}_1:\Delta_5\vdash\Delta_6,\bot} & \bot_R & \frac{}{\bullet\mathbf{h}_4:\Delta_5,\bot\vdash\Delta_6} & \bot_L \\ \hline -:\Delta_5\vdash\Delta_6 & \mathbf{ax/W} \\ \hline \\ \hline -:\Delta_5\vdash\Delta_6 & \mathbf{ax/W} \\ \hline \\ \frac{\mathbf{h}_2:\Delta_7\vdash\bot,\Delta_5}{\bullet\mathbf{h}_2:\Delta_7\vdash(\bot,\Delta_5),\bot} & \bot_R & \frac{}{\bullet\mathbf{h}_6:\Delta_7,\bot\vdash\bot,\Delta_5} & \bot_L \\ \hline -:\Delta_7\vdash\bot,\Delta_5 & \mathbf{cut} \\ \hline \hline \\ \hline -:\Delta_7\vdash\bot,\Delta_5 & \mathbf{ax/W} \\ \hline \\ \frac{\mathbf{h}_2:\bot,\Delta_8\vdash\mathsf{F}_5,\Delta_6}{\bullet\mathbf{h}_2:\bot,\Delta_8\vdash(\bot,\Delta_6),\mathsf{F}_5} & \bot_R & \frac{}{\bullet\mathbf{h}_7:(\bot,\Delta_8),\mathsf{F}_5\vdash\bot,\Delta_6} & \bot_L \\ \hline \\ \hline -:\bot,\Delta_8\vdash\bot,\Delta_6 & \bot_C \\ \hline \hline \\ \hline -:\bot,\Delta_8\vdash\bot,\Delta_6 & \bot_L \\ \hline \end{array}$$

## ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_7, \mathbf{p}_5 \vdash \Delta_6, \mathbf{p}_5}{\bullet \mathbf{h}_1:\Delta_7, \mathbf{p}_5 \vdash (\Delta_6, \mathbf{p}_5), \bot} \ \bot_R & \frac{\bullet \mathbf{h}_4:(\Delta_7, \mathbf{p}_5), \bot \vdash \Delta_6, \mathbf{p}_5}{\bullet} \ \mathbf{Cut} \\ & -:\Delta_7, \mathbf{p}_5 \vdash \Delta_6, \mathbf{p}_5 \\ & \stackrel{\smile}{\longrightarrow} \ I \end{array}$$

$$\begin{array}{c|c} \frac{\mathbf{h}_2:\Delta_6\vdash \mathbf{p}_7,\Delta_8,\mathbf{p}_7}{\bullet \mathbf{h}_2:\Delta_6\vdash (\bot,\Delta_8,\mathbf{p}_7),\mathbf{p}_7} \stackrel{\bot_R}{\to} \frac{\bullet \mathbf{h}_5:\Delta_6,\mathbf{p}_7\vdash \bot,\Delta_8,\mathbf{p}_7} \stackrel{I}{\subset} \mathbf{cut} \\ \hline -:\Delta_6\vdash \bot,\Delta_8,\mathbf{p}_7 & \mathbf{ax/W} \stackrel{\bullet}{\to} \mathbf{h}_5:\Delta_6,\mathbf{p}_7\vdash \bot,\Delta_8,\mathbf{p}_7} \stackrel{I}{\to} \mathbf{hcut} \\ \hline \frac{\mathbf{h}_2:\Delta_6\vdash \bot,\Delta_8,\mathbf{p}_7,\mathbf{p}_7}{\to:\Delta_6\vdash \bot,\Delta_8,\mathbf{p}_7} \stackrel{\mathbf{ax/W}}{\to} \frac{\bullet \mathbf{h}_5:\Delta_6,\mathbf{p}_7\vdash \bot,\Delta_8,\mathbf{p}_7} \stackrel{I}{\to} \mathbf{hcut} \\ \hline \frac{\mathbf{h}_2:\Delta_9,\mathbf{p}_7\vdash F_5,\Delta_8,\mathbf{p}_7}{\bullet \mathbf{h}_2:\Delta_9,\mathbf{p}_7\vdash (\bot,\Delta_8,\mathbf{p}_7),F_5} \stackrel{\bot_R}{\to} \frac{\bullet \mathbf{h}_6:(\Delta_9,\mathbf{p}_7),F_5\vdash \bot,\Delta_8,\mathbf{p}_7} \stackrel{I}{\subset} \mathbf{cut} \\ \hline -:\Delta_9,\mathbf{p}_7\vdash \bot,\Delta_8,\mathbf{p}_7} \stackrel{I}{\to} \mathbf{cut} \\ \hline -:\Delta_9,\mathbf{p}_7\vdash \bot,\Delta_8,\mathbf{p}_7} \stackrel{I}{\to} \mathbf{cut} \\ \hline \end{array}$$

• Case rule  $\top_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_1: \top, \Delta_6 \vdash \Delta_5}{\bullet \mathbf{h}_1: \top, \Delta_6 \vdash \Delta_5, \bot} \perp_R & \frac{\mathbf{h}_4: \bot, \Delta_6 \vdash \Delta_5}{\bullet \mathbf{h}_4: (\top, \Delta_6), \bot \vdash \Delta_5} & \top_L \\ \hline \\ -: \top, \Delta_6 \vdash \Delta_5 & \\ \hline \\ \hline \\ -: \top, \Delta_6 \vdash \Delta_5 & \\ \hline \\ \hline \\ \bullet \mathbf{h}_2: \Delta_7 \vdash \top, \Delta_5 & \mathbf{ax/W} \\ \hline \\ \frac{\mathbf{h}_2: \Delta_7 \vdash (\bot, \Delta_5), \top}{\bullet \mathbf{h}_2: \Delta_7 \vdash (\bot, \Delta_5), \top} \perp_R & \frac{\mathbf{h}_6: \Delta_7 \vdash \bot, \Delta_5}{\bullet \mathbf{h}_6: \Delta_7, \top \vdash \bot, \Delta_5} & \top_L \\ \hline \\ \hline \\ -: \Delta_7 \vdash \bot, \Delta_5 & \\ \hline \\ \hline \\ -: \Delta_7 \vdash \bot, \Delta_5 & \\ \hline \\ \bullet \mathbf{h}_2: \top, \Delta_8 \vdash \mathbf{f}_5, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_2: \top, \Delta_8 \vdash (\bot, \Delta_6), \mathbf{f}_5 & \mathbf{h}_7: \mathbf{f}_5, \Delta_8 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8 \vdash \bot, \Delta_6 & \\ \hline \\ \hline \\ -: \top, \Delta_8 \vdash \bot, \Delta_6 & \\ \hline \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \hline \\ -: \top, \Delta_8 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ -: \top, \Delta_8 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{f}_5 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_6 & \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_8, \mathbf{h}_7 \vdash \bot, \Delta_8 \vdash \bot, \Delta_$$

# 6.5 Status of $\top_R$ : OK

• Case rule  $\rightarrow_R$ 

$$\begin{array}{c|c} \frac{\bullet h_1: \Delta_4 \vdash (\Delta_6, F_7 \to F_8), \top}{-: \Delta_4 \vdash \Delta_6, F_7 \to F_8} & \xrightarrow{h_5: \top, F_7, \Delta_4 \vdash F_8, \Delta_6} \\ \hline -: \Delta_4 \vdash \Delta_6, F_7 \to F_8 & \text{Cut} \\ \hline \\ \bullet h_1: \Delta_4, F_7 \vdash \top, \Delta_6, F_8 & \xrightarrow{T_R} \\ \hline \bullet h_1: \Delta_4, F_7 \vdash \top, \Delta_6, F_8 & \xrightarrow{h_5: \top, \Delta_4, F_7 \vdash \Delta_6, F_8} \\ \hline -: \Delta_4 \vdash \Delta_6, F_8 & \xrightarrow{h_5: \top, \Delta_4, F_7 \vdash \Delta_6, F_8} \\ \hline -: \Delta_4 \vdash \Delta_6, F_7 \to F_8 & \xrightarrow{\bullet} \\ \hline \bullet h_2: \Delta_6 \vdash (\top, \Delta_{10}, F_8 \to F_9), F_5 & \xrightarrow{\bullet} \\ \hline -: \Delta_6 \vdash \top, \Delta_{10}, F_8 \to F_9 & \xrightarrow{T_R} \\ \hline \hline -: \Delta_6 \vdash \top, \Delta_{10}, F_8 \to F_9 & \xrightarrow{T_R} \\ \hline -: \Delta_6 \vdash \top, \Delta_{10}, F_8 \to F_9 & \xrightarrow{T_R} \\ \hline \end{array}$$

• Case rule  $\wedge_R$ 

$$\frac{\underbrace{\bullet \mathbf{h}_1 : \Delta_4 \vdash (\Delta_6, F_7 \land F_8), \top}_{\bullet \mathbf{h}_1 : \Delta_4 \vdash (\Delta_6, F_7 \land F_8), \top}_{\bullet \mathbf{h}_5 : \top, \Delta_4 \vdash F_7, \Delta_6} \underbrace{\phantom{\bullet} \mathbf{h}_5 : \top, \Delta_4 \vdash F_8, \Delta_6}_{\bullet \mathbf{h}_5 : \Delta_4, \top \vdash \Delta_6, F_7 \land F_8} \underbrace{\phantom{\bullet} \mathbf{h}_5 : -\mathbf{h}_5 \vdash \mathbf{h}_5 \vdash \mathbf{h}_5$$

$$\frac{\bullet \mathbf{h}_2 : \Delta_6 \vdash (\top, \Delta_{10}, \mathbf{F}_8 \land \mathbf{F}_9), \mathbf{F}_5}{-: \Delta_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \land \mathbf{F}_9} \xrightarrow{\bullet}_R \frac{\mathbf{h}_7 : \mathbf{F}_5, \Delta_6 \vdash \top, \mathbf{F}_8, \Delta_{10} \quad \mathbf{h}_7 : \mathbf{F}_5, \Delta_6 \vdash \top, \mathbf{F}_9, \Delta_{10}}{\bullet \mathbf{h}_7 : \Delta_6, \mathbf{F}_5 \vdash \top, \Delta_{10}, \mathbf{F}_8 \land \mathbf{F}_9} \xrightarrow{\bullet}_{Cut} \xrightarrow{\bullet}_{-: \Delta_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \land \mathbf{F}_9} \top_R$$

• Case rule  $\vee_R$ 

$$\begin{array}{c|c} & \frac{\mathbf{h}_5 : \top, \Delta_4 \vdash \mathbf{F}_7, \mathbf{F}_8, \Delta_6}{\bullet \mathbf{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8} & \vee_R \\ & -: \Delta_4 \vdash \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8 & \mathsf{Cut} \\ \hline & \frac{\bullet \mathbf{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathbf{F}_7, \mathbf{F}_8}{\bullet \mathbf{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_8} & \mathsf{ax/W} \\ \hline & \frac{\bullet \mathbf{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathbf{F}_7, \mathbf{F}_8}{\bullet \mathbf{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_8} & \mathsf{hCut} \\ \hline & \frac{-: \Delta_4 \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_8}{-: \Delta_4 \vdash \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8} \vee_R \\ \hline \hline & \bullet \mathbf{h}_2 : \Delta_6 \vdash (\top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9), \mathbf{F}_5 & \frac{\mathbf{h}_7 : \mathbf{F}_5, \Delta_6 \vdash \top, \mathbf{F}_8, \mathbf{F}_9, \Delta_{10}}{\bullet \mathbf{h}_7 : \Delta_6, \mathbf{F}_5 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9} & \mathsf{Cut} \\ \hline & -: \Delta_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9 & \\ \hline & -: \Delta_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9 & \\ \hline & -: \Delta_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9 & \\ \hline \end{array}$$

• Case rule  $\perp_R$ 

• Case rule  $\top_R$ 

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1} : \Delta_4 \vdash (\top, \Delta_6), \top & \overline{}^\intercal_R & \overline{}_{\mathbf{h}_5} : \Delta_4, \top \vdash \top, \Delta_6 \\ \hline & -: \Delta_4 \vdash \top, \Delta_6 & \\ \hline & \overline{}^\intercal_R & \\ \hline & -: \Delta_4 \vdash \top, \Delta_6 & \overline{}^\intercal_R \\ \hline \hline \bullet_{\mathbf{h}_2} : \Delta_6 \vdash (\top, \Delta_8), \mathbf{F}_5 & \overline{}^\intercal_R & \overline{}_{\mathbf{h}_7} : \Delta_6, \mathbf{F}_5 \vdash \top, \Delta_8 & \overline{}^\intercal_R \\ \hline & -: \Delta_6 \vdash \top, \Delta_8 & \overline{}^\intercal_R & \\ \hline & \overline{}^\intercal_R & \overline{}^\intercal_R & \overline{}^\intercal_R & \overline{}^\intercal_R & \overline{}^\intercal_R \\ \hline \end{array}$$

ullet Case rule K

$$\frac{\bullet_{\mathbf{h}_1:\; \Box\Gamma_5,\, \Delta_8\; \vdash\; (\Delta_6,\, []\mathsf{F}_7),\, \top}}{-:\Box\Gamma_5,\, \Delta_8\; \vdash\; \Delta_6,\, []\mathsf{F}_7} \xrightarrow{\bullet_{\mathbf{h}_4:\; (\Box\Gamma_5,\, \Delta_8),\, \top\; \vdash\; \Delta_6,\, []\mathsf{F}_7}} \underbrace{\begin{array}{c} K\\ \mathsf{Cut} \\ \hline\\ -:unbox(\Box\Gamma_5)\; \vdash\; \mathsf{F}_7 \\ \hline\\ -:\Delta_8,\, \Box\Gamma_5\; \vdash\; \Delta_6,\, []\mathsf{F}_7 \end{array}}_{\mathsf{ax/W}} K$$

$$\frac{ \begin{array}{c} \bullet_{h_2}: \square\Gamma_{10}, \Delta_7 \vdash (\top, \Delta_9, [\![\mathsf{F}_8), \square\mathsf{F}_5] \end{array}}{ -: \square\Gamma_{10}, \Delta_7 \vdash (\top, \Delta_9, [\![\mathsf{F}_8], \square\mathsf{F}_5]} } \ \top_R \ \begin{array}{c} h_6: unbox(\square\Gamma_{10}), unbox(\square\mathsf{F}_5) \vdash \mathsf{F}_8 \\ \bullet_{h_6}: (\square\Gamma_{10}, \Delta_7), \square\mathsf{F}_5 \vdash \top, \Delta_9, [\![\mathsf{F}_8] \end{array}} \\ \hline -: \square\Gamma_{10}, \Delta_7 \vdash \top, \Delta_9, [\![\mathsf{F}_8] \end{array}} \ \begin{array}{c} \Gamma_R \\ \hline -: \Delta_7, \square\Gamma_{10} \vdash \top, \Delta_9, [\![\mathsf{F}_8] \end{array}} \ \top_R \\ \hline \\ \bullet_{h_2}: \square\Gamma_7, \Delta_{10} \vdash (\top, \Delta_9, [\![\mathsf{F}_8], \mathsf{F}_5] \end{array}} \ \top_R \ \begin{array}{c} h_6: unbox(\square\Gamma_7) \vdash \mathsf{F}_8 \\ \bullet_{h_6}: (\square\Gamma_7, \Delta_{10}), \mathsf{F}_5 \vdash \top, \Delta_9, [\![\mathsf{F}_8] \end{array}} \\ \hline -: \square\Gamma_7, \Delta_{10} \vdash \top, \Delta_9, [\![\mathsf{F}_8] \end{array}} \ \begin{array}{c} K \\ \text{Cut} \\ \hline \\ -: \square\Gamma_7, \Delta_{10} \vdash \top, \Delta_9, [\![\mathsf{F}_8] \end{array}} \end{array}$$

• Case rule A45

$$\begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_{4} : \square \Gamma_{5} + \square \Gamma_{6}, \mathbf{F}_{8} \\ \bullet \mathbf{h}_{4} : (\square \Gamma_{5}, \Delta_{9}), \top \vdash \square \Gamma_{6}, \Delta_{7}, || \mathbf{F}_{8} \\ \hline \\ - : \square \Gamma_{5}, \Delta_{9} \vdash (\square \Gamma_{6}, \Delta_{7}, || \mathbf{F}_{8} \\ \hline \\ - : \square \Gamma_{5}, \Delta_{9} \vdash \square \Gamma_{6}, \Delta_{7}, || \mathbf{F}_{8} \\ \hline \\ - : \square \Gamma_{5} \vdash \mathbf{F}_{8}, \square \Gamma_{6} \\ \hline \\ - : \Delta_{9}, \square \Gamma_{5} \vdash \Delta_{7}, \square \Gamma_{6}, || \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{2} : \square \Gamma_{11}, \Delta_{7} \vdash (\top, \square \Gamma_{8}, \Delta_{10}, || \mathbf{F}_{9}), \square \Gamma_{5} \\ \hline \\ - : \square \Gamma_{11}, \Delta_{7} \vdash \top, \square \Gamma_{8}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \\ - : \Delta_{7}, \square \Gamma_{11} \vdash \top, \Delta_{10}, \square \Gamma_{8}, || \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{2} : \square \Gamma_{7}, \Delta_{11} \vdash (\top, \square \Gamma_{8}, \Delta_{10}, || \mathbf{F}_{9}), \mathbf{F}_{5} \\ \hline \\ \bullet \mathbf{h}_{6} : (\square \Gamma_{7}, \Delta_{11}), || \mathbf{F}_{5} \vdash \nabla_{7}, \mathbf{F}_{8}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{6} : \square \Gamma_{7}, \Delta_{11}, || \mathbf{F}_{7}, \Delta_{10}, || \mathbf{F}_{9}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{6} : (\square \Gamma_{7}, \Delta_{11}), || \mathbf{F}_{5} \vdash \nabla, \square \Gamma_{8}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{6} : \square \Gamma_{7}, \Delta_{11}, || \mathbf{F}_{7}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \\ - : \square \Gamma_{7}, \Delta_{11} \vdash \nabla, \square \Gamma_{8}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \\ \bullet \mathbf{h}_{6} : \square \Gamma_{7}, \Delta_{11}, || \mathbf{F}_{5} \vdash \nabla, \square \Gamma_{8}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \\ - : \square \Gamma_{7}, \Delta_{11} \vdash \nabla, \square \Gamma_{8}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \\ - : \square \Gamma_{7}, \Delta_{11} \vdash \nabla, \square \Gamma_{8}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \\ - : \square \Gamma_{7}, \Delta_{11} \vdash \nabla, \square \Gamma_{8}, \Delta_{10}, || \mathbf{F}_{9} \\ \hline \end{array}$$

• Case rule  $\rightarrow_L$ 

$$\frac{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top} \xrightarrow{\bullet h_4 : \top, \Delta_8 \vdash F_5, \Delta_7} \underbrace{h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}_{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \top \vdash \Delta_7} \underbrace{cut} \xrightarrow{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5} \xrightarrow{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5} \underbrace{ax/W}_{hCut} \xrightarrow{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \xrightarrow{T_R} \underbrace{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}_{h_2 : \Delta_8, F_6 \vdash \Delta_7} \xrightarrow{-: \Delta_8, F_6 \vdash \Delta_7} \xrightarrow{\bullet h_6 : \Delta_7 \vdash \top, F_8, \Delta_5} \underbrace{h_6 : F_9, \Delta_7 \vdash \top, \Delta_5}_{cut} \xrightarrow{\bullet h_2 : \Delta_7 \vdash \top, \Delta_5} \xrightarrow{\bullet} \underbrace{-: \Delta_7 \vdash \top, \Delta_5}_{-: \Delta_7 \vdash \top, \Delta_5} \xrightarrow{\bullet} \underbrace{-: \Delta_7 \vdash \top, \Delta_5}_{\bullet h_7 : F_5, \Delta_{10} \vdash \top, F_8, \Delta_6} \xrightarrow{h_7 : F_5, F_9, \Delta_{10} \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_9, F_7, \Delta_6} \xrightarrow{\bullet} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6} \xrightarrow{\top_R} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6} \xrightarrow{\top_R} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_5, F_9, F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_7, A_8} \xrightarrow{\bullet} \underbrace{-: \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6}_{\bullet h_7 : F_7, A_8} \xrightarrow{$$

• Case rule  $\wedge_L$ 

$$\begin{array}{c} \frac{ \mathbf{h}_{1}:\Delta_{8}, \mathbf{F}_{5} \wedge \mathbf{F}_{6} \vdash \Delta_{7}, \top}{-:\Delta_{8}, \mathbf{F}_{5} \wedge \mathbf{F}_{6} \vdash \Delta_{7}} & \wedge_{\mathbf{h}_{1}}:(\Delta_{8}, \mathbf{F}_{5} \wedge \mathbf{F}_{6}), \top \vdash \Delta_{7}}{-:\Delta_{8}, \mathbf{F}_{5} \wedge \mathbf{F}_{6} \vdash \Delta_{7}} & \wedge_{\mathbf{L}} \\ \hline -:\Delta_{8}, \mathbf{F}_{5} \wedge \mathbf{F}_{6} \vdash \Delta_{7} & & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{1}:\Delta_{8}, \mathbf{F}_{5}, \mathbf{F}_{6} \vdash \top, \Delta_{7} & \top_{R} & & \mathbf{ax/W} \\ \hline -:\Delta_{8}, \mathbf{F}_{5}, \mathbf{F}_{6} \vdash \Delta_{7} & \wedge_{\mathbf{L}} \\ \hline -:\Delta_{8}, \mathbf{F}_{5} \wedge \mathbf{F}_{6} \vdash \Delta_{7} & \wedge_{\mathbf{L}} \\ \hline \bullet \mathbf{h}_{2}:\Delta_{7} \vdash (\top,\Delta_{5}), \mathbf{F}_{8} \wedge \mathbf{F}_{9} & \top_{R} & & \mathbf{h}_{6}:\mathbf{F}_{8}, \mathbf{F}_{9}, \Delta_{7} \vdash \top, \Delta_{5} \\ \hline -:\Delta_{7} \vdash \top, \Delta_{5} & \top_{R} \\ \hline \hline \bullet \mathbf{h}_{2}:\Delta_{7} \vdash (\top,\Delta_{6}), \mathbf{F}_{8} \wedge \mathbf{F}_{9} \vdash (\top,\Delta_{6}), \mathbf{F}_{5} & \top_{R} & & \mathbf{h}_{7}:\mathbf{F}_{5}, \mathbf{F}_{8}, \mathbf{F}_{9}, \Delta_{10} \vdash \top, \Delta_{6} \\ \hline -:\Delta_{10}, \mathbf{F}_{8} \wedge \mathbf{F}_{9} \vdash \top, \Delta_{6} & & \wedge_{\mathbf{L}} \\ \hline -:\Delta_{10}, \mathbf{F}_{8} \wedge \mathbf{F}_{9} \vdash \top, \Delta_{6} & & \\ \hline -:\Delta_{10}, \mathbf{F}_{8} \wedge \mathbf{F}_{9} \vdash \top, \Delta_{6} & & \\ \hline -:\Delta_{10}, \mathbf{F}_{8} \wedge \mathbf{F}_{9} \vdash \top, \Delta_{6} & & \\ \hline -:\Delta_{10}, \mathbf{F}_{8} \wedge \mathbf{F}_{9} \vdash \top, \Delta_{6} & & \\ \hline -:\Delta_{10}, \mathbf{F}_{8} \wedge \mathbf{F}_{9} \vdash \top, \Delta_{6} & & \\ \hline \end{array}$$

## • Case rule $\vee_L$

$$\frac{\underbrace{\bullet_{h_1}:\Delta_8,F_5\vee F_6\vdash \Delta_7,\top}_{\bullet h_1}\top_R \quad \frac{h_4:\top,F_5,\Delta_8\vdash \Delta_7 \quad h_4:\top,F_6,\Delta_8\vdash \Delta_7}_{\bullet h_4:(\Delta_8,F_5\vee F_6),\top\vdash \Delta_7} }_{-:\Delta_8,F_5\vee F_6\vdash \Delta_7} \quad \text{Cut}} \\ \frac{\bullet_{h_1}:\Delta_8,F_5\vdash \top,\Delta_7}_{-:\Delta_8,F_5\vdash \Delta_7} \quad \frac{ax/W}{h^2} \quad \frac{\bullet_{h_1}:\Delta_8,F_6\vdash \top,\Delta_7}_{-:\Delta_8,F_6\vdash \Delta_7} \quad \frac{h_4:\top,\Delta_8,F_6\vdash \Delta_7}{-:\Delta_8,F_6\vdash \Delta_7} \\ \frac{-:\Delta_8,F_5\vdash \Delta_7}{-:\Delta_8,F_5\vdash \Delta_7} \quad \frac{-:\Delta_8,F_5\vdash T,\Delta_5}_{\bullet h_6:F_9,\Delta_7\vdash \top,\Delta_5} \quad \vee_L \\ \\ \frac{\bullet_{h_2}:\Delta_7\vdash (\top,\Delta_5),F_8\vee F_9}{-:\Delta_7\vdash \top,\Delta_5} \quad \frac{h_6:F_8,\Delta_7\vdash \top,\Delta_5}_{\bullet h_6:\Delta_7,F_8\vee F_9\vdash \top,\Delta_5} \\ \frac{-:\Delta_7\vdash \top,\Delta_5}{-:\Delta_7\vdash \top,\Delta_5} \quad \top_R \\ \\ \frac{\bullet_{h_2}:\Delta_{10},F_8\vee F_9\vdash (\top,\Delta_6),F_5}{-:\Delta_{10},F_8\vee F_9\vdash \top,\Delta_6} \quad \frac{\bullet_{h_7}:F_5,F_8,\Delta_{10}\vdash \top,\Delta_6}_{\bullet h_7:F_5,F_9,\Delta_{10}\vdash \top,\Delta_6} }{-:\Delta_{10},F_8\vee F_9\vdash \top,\Delta_6} \quad \text{Cut} \\ \\ \frac{-:\Delta_{10},F_8\vee F_9\vdash \top,\Delta_6}{-:\Delta_{10},F_8\vee F_9\vdash \top,\Delta_6} \quad \top_R \\ \\ \frac{\bullet_{10}:\Delta_{10},F_8\vee F_9\vdash \top,\Delta_6}{-:\Delta_{10},F_8\vee F_9\vdash \top,\Delta_6} \quad \top_R \\ \\ \end{array}$$

## $\bullet$ Case rule AT

$$\begin{array}{c} \frac{\mathbf{h}_{4}: \boldsymbol{\Delta}_{7}, []\mathbf{F}_{5} \vdash \boldsymbol{\Delta}_{6}, \boldsymbol{\top}}{\mathbf{e}\mathbf{h}_{1}: \boldsymbol{\Delta}_{7}, []\mathbf{F}_{5} \vdash \boldsymbol{\Delta}_{6}} \\ -: \boldsymbol{\Delta}_{7}, []\mathbf{F}_{5} \vdash \boldsymbol{\Delta}_{6} \\ \hline \\ -: \boldsymbol{\Delta}_{7}, []\mathbf{F}_{5} \vdash \boldsymbol{\Delta}_{6} \\ \hline \\ \bullet \mathbf{h}_{1}: \boldsymbol{\Delta}_{7}, \mathbf{F}_{5}, []\mathbf{F}_{5} \vdash \boldsymbol{\top}, \boldsymbol{\Delta}_{6} \\ \hline \\ \bullet \mathbf{h}_{1}: \boldsymbol{\Delta}_{7}, \mathbf{F}_{5}, []\mathbf{F}_{5} \vdash \boldsymbol{\nabla}, \boldsymbol{\Delta}_{6} \\ \hline \\ -: \boldsymbol{\Delta}_{7}, []\mathbf{F}_{5} \vdash \boldsymbol{\Delta}_{6} \\ \hline \\ \bullet \mathbf{h}_{2}: \boldsymbol{\Delta}_{7} \vdash (\boldsymbol{\top}, \boldsymbol{\Delta}_{5}), []\mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{2}: \boldsymbol{\Delta}_{7} \vdash (\boldsymbol{\top}, \boldsymbol{\Delta}_{5}), []\mathbf{F}_{8} \\ \hline \\ -: \boldsymbol{\Delta}_{7} \vdash \boldsymbol{\top}, \boldsymbol{\Delta}_{5} \\ \hline \\ -: \boldsymbol{\Delta}_{7} \vdash \boldsymbol{\top}, \boldsymbol{\Delta}_{5} \\ \hline \\ \bullet \mathbf{h}_{2}: \boldsymbol{\Delta}_{9}, []\mathbf{F}_{8} \vdash (\boldsymbol{\top}, \boldsymbol{\Delta}_{6}), \mathbf{F}_{5} \\ \hline \\ \bullet \mathbf{h}_{7}: (\boldsymbol{\Delta}_{9}, []\mathbf{F}_{8}), \mathbf{F}_{5} \vdash \boldsymbol{\top}, \boldsymbol{\Delta}_{6} \\ \hline \\ \bullet \mathbf{h}_{7}: \boldsymbol{\Delta}_{9}, []\mathbf{F}_{8} \vdash \boldsymbol{\top}, \boldsymbol{\Delta}_{6} \\ \hline \\ -: \boldsymbol{\Delta}_{9}, []\mathbf{F}_{8} \vdash \boldsymbol{\top}, \boldsymbol{\Delta}_{6} \\ \hline \\ -: \boldsymbol{\Delta}_{9}, []\mathbf{F}_{8} \vdash \boldsymbol{\top}, \boldsymbol{\Delta}_{6} \\ \hline \\ -: \boldsymbol{\Delta}_{9}, []\mathbf{F}_{8} \vdash \boldsymbol{\top}, \boldsymbol{\Delta}_{6} \\ \hline \\ -: \boldsymbol{\Delta}_{9}, []\mathbf{F}_{8} \vdash \boldsymbol{\top}, \boldsymbol{\Delta}_{6} \\ \hline \\ \hline \end{array} \right]$$

• Case rule  $\perp_L$ 

$$\begin{array}{c} \underbrace{\bullet_{\mathbf{h}_1}: \bot, \Delta_6 \vdash \Delta_5, \top}_{} \quad T_R \quad \underbrace{\bullet_{\mathbf{h}_4}: (\bot, \Delta_6), \top \vdash \Delta_5}_{} \quad \bot_L \\ \\ \underline{-: \bot, \Delta_6 \vdash \Delta_5}_{} \quad \underbrace{-: \bot, \Delta_6 \vdash \Delta_5}_{} \quad \bot_L \\ \\ \\ \underbrace{\bullet_{\mathbf{h}_2}: \Delta_7 \vdash (\top, \Delta_5), \bot}_{} \quad T_R \quad \underbrace{\bullet_{\mathbf{h}_6}: \Delta_7, \bot \vdash \top, \Delta_5}_{} \quad \bot_L \\ \\ \underline{-: \Delta_7 \vdash \top, \Delta_5}_{} \quad \underbrace{-: \Delta_7 \vdash \top, \Delta_5}_{} \quad T_R \\ \\ \underbrace{-: \Delta_7 \vdash \top, \Delta_5}_{} \quad T_R \\ \\ \underbrace{\bullet_{\mathbf{h}_2}: \bot, \Delta_8 \vdash (\top, \Delta_6), F_5}_{} \quad T_R \quad \underbrace{\bullet_{\mathbf{h}_7}: (\bot, \Delta_8), F_5 \vdash \top, \Delta_6}_{} \quad \underbrace{\downarrow_L}_{} \\ \underbrace{\bullet_{\mathbf{h}_2}: \bot, \Delta_8 \vdash (\top, \Delta_6), F_5}_{} \quad \underbrace{\neg: \bot, \Delta_8 \vdash \top, \Delta_6}_{} \quad \underbrace{\neg: \bot, \Delta_8 \vdash \top, \Delta_6}_{} \\ \underbrace{\neg: \bot, \Delta_8 \vdash \top, \Delta_6}_{} \quad T_R \\ \end{array}$$

ullet Case rule I

$$\begin{array}{c} \begin{array}{c} \bullet \mathbf{h}_1 : \Delta_7, \mathbf{p}_5 \vdash (\Delta_6, \mathbf{p}_5), \top & \overline{\phantom{A}}_R & \overline{\phantom{A}}_{\Phi \mathbf{h}_4} : (\Delta_7, \mathbf{p}_5), \top \vdash \Delta_6, \mathbf{p}_5 \\ \hline -: \Delta_7, \mathbf{p}_5 \vdash \Delta_6, \mathbf{p}_5 & \overline{\phantom{A}}_{Cut} \\ \hline \hline -: \Delta_7, \mathbf{p}_5 \vdash \Delta_6, \mathbf{p}_5 & I \\ \hline \\ \hline \bullet \mathbf{h}_2 : \Delta_6 \vdash (\top, \Delta_8, \mathbf{p}_7), \mathbf{p}_7 & \overline{\phantom{A}}_{\Phi \mathbf{h}_5} : \Delta_6, \mathbf{p}_7 \vdash \top, \Delta_8, \mathbf{p}_7 \\ \hline -: \Delta_6 \vdash \top, \Delta_8, \mathbf{p}_7 & \overline{\phantom{A}}_R \\ \hline \hline -: \Delta_6 \vdash \top, \Delta_8, \mathbf{p}_7 & \overline{\phantom{A}}_R \\ \hline \hline \bullet \mathbf{h}_2 : \Delta_9, \mathbf{p}_7 \vdash (\top, \Delta_8, \mathbf{p}_7), \mathbf{F}_5 & \overline{\phantom{A}}_R & \bullet \mathbf{h}_6 : (\Delta_9, \mathbf{p}_7), \mathbf{F}_5 \vdash \top, \Delta_8, \mathbf{p}_7 \\ \hline -: \Delta_9, \mathbf{p}_7 \vdash \top, \Delta_8, \mathbf{p}_7 & \overline{\phantom{A}}_R \\ \hline \hline -: \Delta_9, \mathbf{p}_7 \vdash \top, \Delta_8, \mathbf{p}_7 & \overline{\phantom{A}}_R \\ \hline -: \Delta_9, \mathbf{p}_7 \vdash \top, \Delta_8, \mathbf{p}_7 & \overline{\phantom{A}}_R \end{array}$$

• Case rule  $\top_L$ 

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1} : \Delta_5 \vdash \Delta_6, \top & \top_R & \frac{\mathbf{h}_4 : \Delta_5 \vdash \Delta_6}{\bullet \mathbf{h}_4 : \Delta_5, \top \vdash \Delta_6} & \top_L \\ \hline -: \Delta_5 \vdash \Delta_6 & \\ \hline \hline -: \Delta_5 \vdash \Delta_6 & \\ \hline \hline \bullet_{\mathbf{h}_2} : \Delta_7 \vdash (\top, \Delta_5), \top & \top_R & \frac{\mathbf{h}_6 : \Delta_7 \vdash \top, \Delta_5}{\bullet \mathbf{h}_6 : \Delta_7, \top \vdash \top, \Delta_5} & \top_L \\ \hline -: \Delta_7 \vdash \top, \Delta_5 & \\ \hline \hline -: \Delta_7 \vdash \top, \Delta_5 & \\ \hline \hline -: \Delta_7 \vdash \top, \Delta_5 & \\ \hline \hline -: \Delta_7 \vdash \top, \Delta_6 & \\ \hline \hline \bullet_{\mathbf{h}_2} : \top, \Delta_8 \vdash (\top, \Delta_6), F_5 & T_R & \frac{\mathbf{h}_7 : F_5, \Delta_8 \vdash \top, \Delta_6}{\bullet \mathbf{h}_7 : (\top, \Delta_8), F_5 \vdash \top, \Delta_6} & \top_L \\ \hline \hline -: \top, \Delta_8 \vdash \top, \Delta_6 & \\ \hline \hline -: \top, \Delta_8 \vdash \top, \Delta_6 & \\ \hline \hline -: \top, \Delta_8 \vdash \top, \Delta_6 & \\ \hline \hline -: \top, \Delta_8 \vdash \top, \Delta_6 & \\ \hline \hline \end{array}$$

# 6.6 Status of K: OK

• Case rule  $\rightarrow_R$ 

$$\begin{array}{c} \begin{array}{c} h_1: unbox(\Box\Gamma_6) \vdash F_8 \\ \hline \bullet h_1: \Box\Gamma_6, \Delta_7 \vdash (\Delta_{10}, F_{11} \to F_{12}), []F_8 \end{array} K & \begin{array}{c} h_9: \Box\Gamma_6, F_{11}, \Delta_7, []F_8 \vdash F_{12}, \Delta_{10} \\ \hline \bullet h_9: (\Box\Gamma_6, \Delta_7), []F_8 \vdash \Delta_{10}, F_{11} \to F_{12} \end{array} \end{array} \xrightarrow{\bullet}_{R} \\ \hline \begin{array}{c} -: \Box\Gamma_6, \Delta_7 \vdash \Delta_{10}, F_{11} \to F_{12} \\ \hline \hline \\ \hline & h_1: unbox(\Box\Gamma_6) \vdash F_8 \end{array} \text{ax/W} \\ \hline \begin{array}{c} \bullet h_1: unbox(\Box\Gamma_6) \vdash F_8 \\ \hline \hline \bullet h_1: \Delta_7, F_{11}, \Box\Gamma_6 \vdash \Delta_{10}, F_{12}, []F_8 \end{array} K & \begin{array}{c} h_9: \Delta_7, F_{11}, \Box\Gamma_6, []F_8 \vdash \Delta_{10}, F_{12} \\ \hline \hline \\ \bullet h_1: \Delta_7, F_{11}, \Box\Gamma_6 \vdash \Delta_{10}, F_{12} \\ \hline \\ \hline & -: \Delta_7, F_{11}, \Box\Gamma_6 \vdash \Delta_{10}, F_{12} \\ \hline \\ -: \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, F_{11} \to F_{12} \end{array} \xrightarrow{\bullet}_{R} \\ \hline \begin{array}{c} h_2: unbox(\Box\Gamma_7) \vdash F_{10} \\ \hline \\ \bullet h_2: \Box\Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \to F_{13}), []F_{10}), F_8 \end{array} K & \begin{array}{c} h_{11}: \Box\Gamma_7, F_8, F_{12}, \Delta_9 \vdash F_{13}, \Delta_{14}, []F_{10} \\ \hline \\ \bullet h_{11}: (\Box\Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \to F_{13}), []F_{10} \end{array} \xrightarrow{\bullet}_{R} \\ \hline \begin{array}{c} -: unbox(\Box\Gamma_7) \vdash F_{10} \\ \hline \\ -: unbox(\Box\Gamma_7) \vdash F_{10} \end{array} \end{array} \xrightarrow{\bullet}_{R} \end{array} Cut$$

• Case rule  $\wedge_R$ 

$$\frac{ \begin{array}{c} h_1: unbox(\Box \Gamma_6) \vdash F_8 \\ \hline \bullet h_1: \Box \Gamma_6, \Delta_7 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), []F_8 \end{array}}{ } K \begin{array}{c} h_9: \Box \Gamma_6, \Delta_7, []F_8 \vdash F_{11}, \Delta_{10} \quad h_9: \Box \Gamma_6, \Delta_7, []F_8 \vdash F_{12}, \Delta_{10} \\ \hline \bullet h_9: (\Box \Gamma_6, \Delta_7), []F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12} \end{array}}{ \\ \hline Cut \\ \hline \\ \hline h_1: unbox(\Box \Gamma_6) \vdash F_8 \end{array} \begin{array}{c} ax/W \\ \hline \bullet h_1: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, []F_8 \end{array} K \begin{array}{c} h_9: \Delta_7, \Box \Gamma_6, []F_8 \vdash \Delta_{10}, F_{11} \\ \hline h_1: unbox(\Box \Gamma_6) \vdash F_8 \end{array} \begin{array}{c} ax/W \\ \hline \bullet h_1: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, []F_8 \end{array} K \begin{array}{c} h_1: unbox(\Box \Gamma_6) \vdash F_8 \\ \hline \bullet h_1: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{12}, []F_8 \end{array} K \begin{array}{c} h_1: unbox(\Box \Gamma_6) \vdash F_8 \end{array} \begin{array}{c} ax/W \\ \hline \bullet h_1: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{12}, []F_8 \end{array} K \begin{array}{c} h_1: unbox(\Box \Gamma_6) \vdash F_8 \end{array} K \begin{array}{c} h_9: \Delta_7, \Box \Gamma_6, []F_8 \vdash \Delta_{10}, F_{12} \\ \hline \bullet h_1: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11} \wedge F_{12} \end{array} \\ \hline \begin{array}{c} h_2: unbox(\Box \Gamma_7) \vdash F_{10} \\ \hline \bullet h_2: \Box \Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), []F_{10}), F_8 \end{array} K \begin{array}{c} h_{11}: \Box \Gamma_7, F_8, \Delta_9 \vdash F_{12}, \Delta_{14}, []F_{10} \quad h_{11}: \Box \Gamma_7, F_8, \Delta_9 \vdash F_{13}, \Delta_{14}, []F_{10} \\ \hline \bullet h_{11}: (\Box \Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), []F_{10} \end{array} \\ \hline \begin{array}{c} \bullet h_{11}: unbox(\Box \Gamma_7) \vdash F_{10} \\ \hline \bullet h_2: \Box \Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), []F_{10} \\ \hline \bullet h_{11}: (\Box \Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), []F_{10} \end{array} \\ \hline \begin{array}{c} \bullet h_{11}: unbox(\Box \Gamma_7) \vdash F_{10} \\ \hline \bullet h_{11}: (\Box \Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), []F_{10} \\ \hline -: unbox(\Box \Gamma_7) \vdash F_{10} \end{array} \end{array} \\ \hline \begin{array}{c} \bullet h_{11}: (\Box \Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), []F_{10} \\ \hline -: unbox(\Box \Gamma_7) \vdash F_{10} \end{array} \end{array}$$

• Case rule  $\vee_R$ 

$$\begin{array}{c} h_1: unbox(\Box \Gamma_6) \vdash F_8 \\ \hline \bullet h_1: \Box \Gamma_6, \Delta_7 \vdash (\Delta_{10}, F_{11} \vee F_{12}), []F_8 \\ \hline \\ \bullet h_2: (\Box \Gamma_6, \Delta_7), []F_8 \vdash \Delta_{10}, F_{11} \vee F_{12} \\ \hline \\ -: \Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, F_{11} \vee F_{12} \\ \hline \\ \hline h_1: unbox(\Box \Gamma_6) \vdash F_8 \\ \hline \\ \bullet h_1: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, F_{12}, []F_8 \\ \hline \\ \bullet h_1: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, F_{12}, []F_8 \\ \hline \\ \hline \\ \bullet h_2: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, F_{12} \\ \hline \\ -: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, F_{12} \\ \hline \\ -: \Delta_7, \Box \Gamma_6 \vdash \Delta_{10}, F_{11}, F_{12} \\ \hline \\ \bullet h_2: unbox(\Box \Gamma_7) \vdash F_{10} \\ \hline \\ \bullet h_2: \Box \Gamma_7, \Delta_9 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), []F_{10}), F_8 \\ \hline \\ \bullet h_1: (\Box \Gamma_7, \Delta_9), F_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), []F_{10} \\ \hline \\ -: unbox(\Box \Gamma_7) \vdash F_{10} \\ \hline \\ -: unbox(\Box \Gamma_7) \vdash F_{10$$

• Case rule  $\perp_R$ 

$$\frac{ \begin{array}{c} \mathbf{h}_1: unbox(\Box \Gamma_6) \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_1: \Box \Gamma_6, \Delta_7 \vdash (\bot, \Delta_{10}), [] \mathbf{F}_8 \end{array} K \quad \frac{\mathbf{h}_9: \Box \Gamma_6, \Delta_7, [] \mathbf{F}_8 \vdash \Delta_{10}}{\bullet \mathbf{h}_9: (\Box \Gamma_6, \Delta_7), [] \mathbf{F}_8 \vdash \bot, \Delta_{10}} \quad \frac{\bot_R}{\mathsf{Cut}} \\ \hline \\ -: \Box \Gamma_6, \Delta_7 \vdash \bot, \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_7, \Box \Gamma_6 \vdash \bot, \Delta_{10}, [] \mathbf{F}_8 \end{array} \overset{\mathsf{ax/W}}{\longrightarrow} \frac{\mathsf{ax/W}}{\mathsf{h}_9: \Delta_7, \Box \Gamma_6, [] \mathbf{F}_8 \vdash \bot, \Delta_{10}} \overset{\mathsf{ax/W}}{\longleftarrow} \\ \hline \\ -: \Delta_7, \Box \Gamma_6 \vdash \bot, \Delta_{10} \end{array} \overset{\mathsf{h}_{Cut}}{\longrightarrow} \frac{\mathsf{h}_{Cut}}{\mathsf{h}_{Cut}} \\ \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_2 : unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \\ \\ \underline{\bullet \mathbf{h}_2 : \Box \Gamma_7, \Delta_9 \vdash ((\bot, \Delta_{12}), [] \mathbf{F}_{10}), \mathbf{F}_8} \quad K \quad \begin{array}{c} \mathbf{h}_{11} : \Box \Gamma_7, \mathbf{F}_8, \Delta_9 \vdash \Delta_{12}, [] \mathbf{F}_{10} \\ \\ \underline{\bullet \mathbf{h}_{11} : (\Box \Gamma_7, \Delta_9), \mathbf{F}_8 \vdash (\bot, \Delta_{12}), [] \mathbf{F}_{10}} \\ \\ - : \Box \Gamma_7, \Delta_9 \vdash (\bot, \Delta_{12}), [] \mathbf{F}_{10} \\ \\ \hline \\ \underline{- : unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10}} \quad \mathbf{ax/W} \\ \\ \hline \\ - : \Delta_9, \Box \Gamma_7 \vdash \bot, \Delta_{12}, [] \mathbf{F}_{10} \end{array} \quad \begin{array}{c} \bot_R \\ \mathbf{cut} \end{array}$$

• Case rule  $\top_R$ 

$$\frac{ \begin{array}{c} \mathbf{h}_1 : unbox(\Box \Gamma_6) \vdash \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1 : \Box \Gamma_6, \Delta_7 \vdash (\top, \Delta_{10}), [] \mathbf{F}_8 \end{array} K \begin{array}{c} \overline{\bullet} \mathbf{h}_9 : (\Box \Gamma_6, \Delta_7), [] \mathbf{F}_8 \vdash \top, \Delta_{10} \\ \hline -: \Box \Gamma_6, \Delta_7 \vdash \top, \Delta_{10} \\ \hline \hline -: \Delta_7, \Box \Gamma_6 \vdash \top, \Delta_{10} \end{array} } \begin{array}{c} \top_R \\ \mathbf{cut} \\ \hline \\ \hline \bullet \mathbf{h}_2 : unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \\ \hline \bullet \mathbf{h}_2 : \Box \Gamma_7, \Delta_9 \vdash ((\top, \Delta_{12}), [] \mathbf{F}_{10}), \mathbf{F}_8 \end{array} K \begin{array}{c} \overline{\bullet} \mathbf{h}_{11} : (\Box \Gamma_7, \Delta_9), \mathbf{F}_8 \vdash (\top, \Delta_{12}), [] \mathbf{F}_{10} \\ \hline -: \Box \Gamma_7, \Delta_9 \vdash (\top, \Delta_{12}), [] \mathbf{F}_{10} \\ \hline \hline -: \Delta_9, \Box \Gamma_7 \vdash \top, \Delta_{12}, [] \mathbf{F}_{10} \end{array} \begin{array}{c} \top_R \\ \mathbf{cut} \\ \hline \end{array}$$

 $\bullet$  Case rule K

$$\begin{array}{c} \frac{\mathbf{h}_{1}: unbox(\Box \Gamma_{10}, \Box \Gamma_{12}) \vdash F_{6}}{\bullet \mathbf{h}_{1}: (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash (\Delta_{8}, [F_{9}), [F_{6}]} \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash (\Delta_{8}, [F_{9}), [F_{6}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash (\Delta_{8}, [F_{9}] \\ - : unbox(\Box \Gamma_{10}), unbox(\Box \Gamma_{11}), unbox(\Box \Gamma_{12}) \vdash F_{6}, F_{9} \\ - : unbox(\Box \Gamma_{10}), unbox(\Box \Gamma_{11}), unbox(\Box \Gamma_{11}), unbox(\Box \Gamma_{12}) \vdash F_{9} \\ - : unbox(\Box \Gamma_{10}), unbox(\Box \Gamma_{11}), unbox(\Box \Gamma_{11}), unbox(\Box \Gamma_{12}) \vdash F_{9} \\ - : unbox(\Box \Gamma_{10}, \Box \Gamma_{12}) \vdash F_{6} \\ \bullet \mathbf{h}_{1}: unbox(\Box \Gamma_{10}, \Box \Gamma_{12}) \vdash F_{6} \\ \bullet \mathbf{h}_{1}: unbox(\Box \Gamma_{10}, \Box \Gamma_{12}) \vdash F_{6} \\ - : unbox(\Box \Gamma_{10}, \Box \Gamma_{12}) \vdash F_{6} \\ \bullet \mathbf{h}_{1}: (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash (\Delta_{8}, [F_{9}), [F_{6}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{11}, \Box \Gamma_{13}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{8}, [F_{9}] \\ - : (\Box \Gamma_{11}, \Box \Gamma_{13}), \Box \Gamma_{11}, \Delta_{13} \vdash \Delta_{11}, [F_{10}, \Box \Gamma_{13}), \Box \Gamma_{12}, \Delta_{11}, [F_{10}, \Box F_{11}] \\ - : (\Box \Gamma_{11}, \Box \Gamma_{13}), \Box \Gamma_{12}, \Delta_{14} \vdash \Delta_{11}, [F_{10}, \Box F_{13}] \\ - : (\Box \Gamma_{11}, \Box \Gamma_{13}), \Box \Gamma_{12}, \Delta_{14} \vdash \Delta_{11}, [F_{10}, \Box F_{13}] \\ - : (\Box \Gamma_{11}, \Box \Gamma_{13}), \Box \Gamma_{12}, \Delta_{14} \vdash \Delta_{11}, [F_{10}, \Box F_{13}] \\ - : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash (\Delta_{11}, [F_{10}, \Box F_{13}] \\ - : (\Box \Gamma_$$

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\frac{\mathbf{h}_2: unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash F_{10}}{\underbrace{\bullet \mathbf{h}_2: (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []F_{10}), F_7}}_{\bullet \mathbf{h}_2: (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []F_{10}), F_7} K \underbrace{\frac{\mathbf{h}_8: unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash F_{10}}{\bullet \mathbf{h}_8: ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), F_7 \vdash \Delta_9, []F_{10}}}_{\leftarrow} \underbrace{\frac{K}{\mathbf{Cut}}}_{-: unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash F_{10}}_{\leftarrow} \underbrace{\frac{\mathbf{ax/W}}{K}}_{K}}_{\bullet \mathbf{k}}
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### $\bullet$ Case rule A45

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\frac{\mathbf{h}_1: unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: (\Box\Gamma_{11}, \Box\Gamma_{12}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Box\Gamma_8, \Delta_9, []\mathbf{F}_{10}), []\mathbf{F}_6} \quad K \quad \frac{\mathbf{h}_7: \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathbf{F}_6 \vdash \Box\Gamma_8, \mathbf{F}_{10}}{\bullet \mathbf{h}_7: ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), []\mathbf{F}_6 \vdash \Box\Gamma_8, \Delta_9, []\mathbf{F}_{10}} \quad \underbrace{A45}_{C...}
                                                                                                                                                                                                                                               -: (\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14} \vdash \Box\Gamma_{8},\Delta_{9},[]\mathtt{F}_{10}
                                                                                                    \frac{\overbrace{\mathbf{h}_1: unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{13}) \vdash \mathbf{F}_6}^{} \text{ax/W}}{\underbrace{\bullet \mathbf{h}_1: \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \mathbf{F}_{10}, \Box\Gamma_8, []\mathbf{F}_6}^{} K} \frac{\mathbf{ax/W}}{\mathbf{h}_7: \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathbf{F}_6 \vdash \mathbf{F}_{10}, \Box\Gamma_8}^{} \underbrace{\mathbf{ax/W}}_{\mathbf{h}Cnt}}^{\mathbf{ax/W}}
                                                                                                                                                                                                                                                                                             -: \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \mathfrak{f}_{10}, \Box\Gamma_{8}
                                                                                                                                                                                                                                                     \frac{}{-:\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13}\vdash\Delta_{9},\Box\Gamma_{8},[]\mathsf{F}_{10}}\ A45
                                                                                          h_1: unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash F_6
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   \mathtt{h}_7: \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Box\Gamma_{8}, \mathtt{F}_{10}
   \frac{\mathbf{h}_1: unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Box\Gamma_8, \Delta_9, []\mathbf{F}_{10}), []\mathbf{F}_6} \quad K \quad \frac{\mathbf{h}_7: (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), []\mathbf{F}_6 \vdash \Box\Gamma_8, \Delta_9, []\mathbf{F}_{10}}{\bullet \mathbf{h}_7: ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), []\mathbf{F}_6 \vdash \Box\Gamma_8, \Delta_9, []\mathbf{F}_{10}} \quad Cut
                                                                                                                                                                                                                                               -: (\Box\Gamma_{11},\Box\Gamma_{13}),\Box\Gamma_{12},\Delta_{14} \vdash \Box\Gamma_{8},\Delta_{9},[]\mathtt{F}_{10}
                                                                                                                                                                                                                                                       \frac{ -: \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathbf{F}_{10}, \Box\Gamma_{8} \quad \mathsf{ax/W} }{ -: \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{9}, \Box\Gamma_{8}, []\mathbf{F}_{10} } \quad A45
                                                                                                                       h_2: unbox(\Box\Gamma_{13}, \Box\Gamma_{15}) \vdash F_8
  \underbrace{ \begin{array}{c} \mathbf{h}_2 : unbox(\Box \Gamma_{13}, \Box \Gamma_{15}) \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_2 : (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash ((\Box \Gamma_{12}, \Delta_{10}, []\mathbf{F}_{11}), []\mathbf{F}_8), \Box \mathbf{F}_7 \\ - : (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash (\Box \Gamma_{12}, \Delta_{10}, []\mathbf{F}_{11}), []\mathbf{F}_8 \\ \longrightarrow \end{array}} K \underbrace{ \begin{array}{c} \mathbf{h}_9 : \Box \Gamma_{13}, \Box \Gamma_{14}, \Delta_{16}, \Box \Gamma_{14}, \Delta_{16}, \Box \Gamma_{17}, \Delta_{17}, \Box \Gamma_{17}, \Delta_
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \mathtt{h}_9: \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\mathtt{F}_7 \vdash \Box\Gamma_{12}, \mathtt{F}_{11}, []\mathtt{F}_8
                                                                                                                                                                                                                                                                                                \frac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}) \vdash \mathbb{F}_8}{-: \Delta_{16}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{10}, \Box\Gamma_{12}, []\mathbb{F}_{11}, []\mathbb{F}_8} \ K
 \frac{\mathtt{h}_2: unbox(\Box\Gamma_{13}, \Box\Gamma_{15}) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_2: (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash ((\Box\Gamma_{10}, \Delta_{12}, []\mathtt{F}_{11}), []\mathtt{F}_8), \Box\mathtt{F}_7} \quad K \quad \frac{\mathtt{h}_9: \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{14}, \Box\Gamma_{10}, \mathtt{F}_1}{\bullet \mathtt{h}_9: ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \Box\mathtt{F}_7 \vdash (\Box\Gamma_{10}, \Delta_{12}, []\mathtt{F}_{11}), []\mathtt{F}_8} \quad \mathcal{A}45 \quad \mathsf{Cut}
                                                                                                                                                                                                                                                                            -: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash (\Box\Gamma_{10},\Delta_{12},[]F_{11}),[]F_8
                                                                                                                                                                                                                                                                                                 \frac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F_8} \quad \mathsf{ax/W}}{-: \Delta_{16}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{12}, \Box\Gamma_{10}, []\mathtt{F}_{11}, []\mathtt{F}_8} \quad K
                                                                                              \mathtt{h}_2: unbox(\Box\Gamma_{12},\Box\Gamma_{14}) \vdash \mathtt{F}_{11}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           \mathtt{h}_8: \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\mathtt{F}_7 \vdash \Box\Gamma_{9}, \mathtt{F}_{11}
  \begin{array}{c} \mathbf{n}_2: unoox(\Box 1 12, \Box 1 14) \vdash \mathbf{F} 11 \\ \bullet \mathbf{h}_2: (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash ((\Box \Gamma_{9}, \Delta_{10}), [[\mathbf{F}_{11}), \Box \mathbf{F}_7 \\ \bullet \mathbf{h}_8: ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), \Box \mathbf{F}_7 \vdash (\Box \Gamma_{9}, \Delta_{10}), [[\mathbf{F}_{11}, \Box \Gamma_{14}, \Box \Gamma_{14}, \Box \Gamma_{14}, \Box \Gamma_{14}, \Box \Gamma_{15}, \Delta_{15}), \Box \mathbf{F}_7 \vdash (\Box \Gamma_{9}, \Delta_{10}), [[\mathbf{F}_{11}, \Box \Gamma_{14}, \Box \Gamma_{14}, \Box \Gamma_{14}, \Box \Gamma_{14}, \Box \Gamma_{15}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{17}, \Delta_{
                                                                                                                                                                                                                                                           -: (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{9}, \Delta_{10}), []\mathtt{F}_{11}
                                                                                                                                                                                                                                                                               \frac{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{14}) \vdash \mathbf{F}_{11}}{-: \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, \Box\Gamma_{9}, []\mathbf{F}_{11}} \ K
\frac{\mathbf{h}_2: unbox(\Box\Gamma_{13}, \Box\Gamma_{15}) \vdash \mathbf{F}_8}{\bullet \mathbf{h}_2: (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash ((\Box\Gamma_{12}, \Delta_{10}, []\mathbf{F}_{11}), []\mathbf{F}_8), \mathbf{F}_7} \quad K \quad \frac{\mathbf{h}_9: \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Box\Gamma_{12}, \mathbf{F}_{11}, []\mathbf{F}_8}{\bullet \mathbf{h}_9: ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \mathbf{F}_7 \vdash (\Box\Gamma_{12}, \Delta_{10}, []\mathbf{F}_{11}), []\mathbf{F}_8} \\ -: (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_{12}, \Delta_{10}, []\mathbf{F}_{11}), []\mathbf{F}_8
                                                                                                                                                                                                                                                                                  \frac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}) \vdash F_8}{-: \Delta_{16}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{10}, \Box\Gamma_{12}, []F_{11}, []F_8} \ K
                                                                                                             h_2: unbox(\Box\Gamma_{13}, \Box\Gamma_{15}) \vdash F_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \mathtt{h}_9: \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Box\Gamma_{10}, \mathtt{F}_{11}
\frac{ \begin{array}{c} h_2 : unbox(\sqcup \Gamma_{13}, \sqcup \Gamma_{15}) \vdash F_8 \\ \hline \bullet h_2 : (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash ((\Box \Gamma_{10}, \Delta_{12}, []F_{11}), []F_8), F_7 \end{array} K \begin{array}{c} n_9 : \sqcup I_{13}, \sqcup I_{14} \vdash \sqcup I_{10}, F_{11} \\ \hline \bullet h_9 : ((\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16}), F_7 \vdash (\Box \Gamma_{10}, \Delta_{12}, []F_{11}), []F_8 \\ \hline - : (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash (\Box \Gamma_{10}, \Delta_{12}, []F_{11}), []F_8 \end{array}
                                                                                                                                                                                                                                                                                                                                   \frac{}{-:unbox(\Box\Gamma_{13}),unbox(\Box\Gamma_{15})\vdash \mathtt{F}_{8}} \text{ ax/W}
                                                                                                                                                                                                                                                                                     \frac{1}{-:\Delta_{16},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{15}\vdash\Delta_{12},\Box\Gamma_{10},[]\mathsf{F}_{11},[]\mathsf{F}_{8}}\ K
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$$\frac{\mathbf{h}_{2}: unbox(\Box\Gamma_{12}, \Box\Gamma_{14}) \vdash F_{11}}{\bullet \mathbf{h}_{2}: (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Box\Gamma_{9}, \Delta_{10}), []F_{11}), F_{7}} K \quad \frac{\mathbf{h}_{8}: \Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{9}, \Delta_{10}), []F_{11}}{\bullet \mathbf{h}_{8}: ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), F_{7} \vdash (\Box\Gamma_{9}, \Delta_{10}), []F_{11}} \quad Cut} \quad \frac{-: (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{9}, \Delta_{10}), []F_{11}}{\circ}}{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{14}) \vdash F_{11}} \quad ax/\mathsf{W}}{-: \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, \Box\Gamma_{9}, []F_{11}} \quad K}$$

• Case rule  $\rightarrow_L$ 

$$\frac{h_1: unbox(\Box \Gamma_6) \vdash F_7}{\bullet h_1: \Box \Gamma_6, \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, []F_7} K \xrightarrow{h_8: \Box \Gamma_6, \Delta_{12}, []F_7 \vdash F_9, \Delta_{11}} \bullet h_8: \Box \Gamma_6, F_{10}, \Delta_{12}, []F_7 \vdash \Delta_{11}} \to L$$

$$\frac{\bullet h_1: \Box \Gamma_6, \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}}{-: \Box \Gamma_6, \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \xrightarrow{\bullet h_8: (\Box \Gamma_6, \Delta_{12}, F_9 \to F_{10}), []F_7 \vdash \Delta_{11}}} tout$$

$$\frac{h_1: unbox(\Box \Gamma_6) \vdash F_7}{\bullet h_1: \Delta_{12}, \Box \Gamma_6 \vdash \Delta_{11}, F_9, []F_7} K \xrightarrow{h_8: \Delta_{12}, \Box \Gamma_6, []F_7 \vdash \Delta_{11}, F_9} x \xrightarrow{\bullet h_1: \Delta_{12}, F_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []F_7} K \xrightarrow{\bullet h_1: unbox(\Box \Gamma_6) \vdash F_7} x \xrightarrow{\bullet h_1: \Delta_{12}, F_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []F_7} K \xrightarrow{\bullet h_8: \Delta_{12}, F_{10}, \Box \Gamma_6, []F_7 \vdash \Delta_{11}} \xrightarrow{\bullet h_2: unbox(\Box \Gamma_7) \vdash F_{10}} x \xrightarrow{\bullet h_1: \Box \Gamma_7, \Delta_8 \vdash F_{12}, \Delta_9, []F_{10}, h_{11}: \Box \Gamma_7, F_{13}, \Delta_8 \vdash \Delta_9, []F_{10}} \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_8), F_{12} \to F_{13} \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_8), F_{12} \to F_{13} \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13} \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13} \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box \Gamma_7, \Delta_14, F_{12} \to F_{13}), F_8 \vdash \Delta_9, []F_{10}} x \xrightarrow{\bullet h_1: (\Box$$

• Case rule  $\wedge_L$ 

• Case rule  $\vee_L$ 

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\frac{\mathbf{h}_1: unbox(\Box \Gamma_6) \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Box \Gamma_6, \Delta_{12}, \mathbf{F}_9 \vee \mathbf{F}_{\underline{10}} \vdash \Delta_{\underline{11}}, \underline{[]} \mathbf{F}_7} \quad K \quad \frac{\mathbf{h}_8: \Box \Gamma_6, \mathbf{F}_9, \Delta_{12}, \underline{[]} \mathbf{F}_7 \vdash \Delta_{11} \quad \mathbf{h}_8: \Box \Gamma_6, \mathbf{F}_{\underline{10}}, \Delta_{\underline{12}}, \underline{[]} \mathbf{F}_7 \vdash \Delta_{\underline{11}}}{\bullet \mathbf{h}_8: (\Box \Gamma_6, \Delta_{\underline{12}}, \mathbf{F}_9 \vee \mathbf{F}_{\underline{10}}), \underline{[]} \mathbf{F}_7 \vdash \Delta_{\underline{11}}} \quad \mathbf{Cut}} \quad \vee_L
                                                                                                                                                                                                                                                                                                                                                                            -:\Box\Gamma_{6},\Delta_{12},\mathsf{F}_{9}\vee\mathsf{F}_{10}\vdash\Delta_{11}
                           \boxed{ {\tt h}_1: unbox(\Box \Gamma_6) \vdash {\tt F}_7} \quad {\tt ax/W}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     \frac{}{\mathtt{h}_1: unbox}(\Box\Gamma_6) \vdash \mathtt{F}_7 \quad \mathtt{ax/W}
 \underbrace{ \begin{array}{c} \textbf{n}_1 : unoox(\Box \textbf{1}_6) \vdash \textbf{F}_7 \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_9, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_9, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}, []\textbf{F}_7 \end{array}}_{\textbf{h}_8 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}, []\textbf{F}_7 \end{bmatrix}}_{\textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{F}_7 \vdash \Delta_{11}} \underbrace{ \begin{array}{c} \textbf{ax/W} \\ \bullet \textbf{h}_1 : \Delta_{12}, \textbf{F}_{10}, \Box \Gamma_6, []\textbf{h}_1 : \Delta_{12}, []\textbf{h}_2 : \Delta_{12}, []\textbf{h}_2 : \Delta_{12}, []\textbf{h}_1 : \Delta_{12}, []\textbf{h}_2 : \Delta_{12}, []\textbf{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      -:\Delta_{12}, \mathsf{F}_{10}, \Box \Gamma_6 \vdash \Delta_{11}
                                                                                                                                                                \overline{-:\Delta_{12},\mathtt{F}_{9},\Box\Gamma_{6}}\vdash\Delta_{11}
                                                                                                                                                                                                                                                                                                                                                                                                                                    -:\Delta_{12},\Box\Gamma_6,\mathsf{F}_9\vee\mathsf{F}_{10}\vdash\Delta_{11}
                                                                                                                                                                                                                                                                                \frac{1}{\mathsf{F}_{13}} \ K \ \frac{\mathsf{h}_{11}: \Box \Gamma_7, \mathsf{F}_{12}, \Delta_8 \vdash \Delta_9, []\mathsf{F}_{10} \quad \mathsf{h}_{11}: \Box \Gamma_7, \mathsf{F}_{13}, \Delta_8 \vdash \Delta_9, []\mathsf{F}_{10}}{\bullet \mathsf{h}_{11}: (\Box \Gamma_7, \Delta_8), \mathsf{F}_{12} \vee \mathsf{F}_{13} \vdash \Delta_9, []\mathsf{F}_{10}} \ \mathsf{Cut}
                                                        \mathtt{h}_2: unbox(\Box \Gamma_7) \vdash \mathtt{F}_{10}
 \bullet \mathtt{h}_2: \Box \Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_{12} \vee \mathtt{F}_{13}
                                                                                                                                                                                                                                                                              -: \Box \Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}
                                                                                                                                                                                                                                                                                                                              \frac{-: unbox(\Box \Gamma_7) \vdash F_{10}}{-: \Delta_2 \Box \Gamma_7 \vdash \Delta_2 \Box F_{10}} \times K
                                                                                                                                                                                                                                                                                                                            -:\Delta_8,\Box\Gamma_7\vdash\Delta_9,[]\mathsf{F}_{10}
 \frac{\mathbf{h}_{2} : unbox(\Box \Gamma_{7}) \vdash \mathbf{F}_{10}}{\underbrace{\bullet \mathbf{h}_{2} : \Box \Gamma_{7}, \Delta_{14}, \mathbf{F}_{12} \vee \mathbf{F}_{13} \vdash (\Delta_{9}, []\mathbf{F}_{10}), \mathbf{F}_{8}}_{K} \quad \underbrace{\frac{\mathbf{h}_{11} : \Box \Gamma_{7}, \mathbf{F}_{8}, \mathbf{F}_{12}, \Delta_{14} \vdash \Delta_{9}, []\mathbf{F}_{10}}{\bullet \mathbf{h}_{11} : (\Box \Gamma_{7}, \Delta_{14}, \mathbf{F}_{12} \vee \mathbf{F}_{13}), \mathbf{F}_{8} \vdash \Delta_{9}, []\mathbf{F}_{10}}_{Cut}} \quad \vee_{L} \quad \underbrace{\bullet \mathbf{h}_{11} : \Box \Gamma_{7}, \mathbf{h}_{14}, \mathbf{h}_{12} \vee \mathbf{F}_{13}, \mathbf{h}_{14} \vdash \Delta_{9}, []\mathbf{F}_{10}}_{Cut}}_{Cut}
                                                                                                                                                                                                                                                                             -: \Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}
                                                                                                                                                                                                                                                                                                                                                                                      \frac{}{-:unbox(\Box\Gamma_7)\vdash \mathtt{F}_{10}} \text{ ax/W}
                                                                                                                                                                                                                                                                                                                                          -:\Delta_{14},\Box\Gamma_7,\mathsf{F}_{12}\vee\mathsf{F}_{13}\vdash\Delta_9,[]\mathsf{F}_{10}
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### $\bullet$ Case rule AT

$$\begin{array}{c} \frac{\mathbf{h}_1 : unbox(\Box \Gamma_{11}, \| \mathbf{F}_9) + \mathbf{F}_7}{\bullet \mathbf{h}_1 : (\Box \Gamma_{11}, \| \mathbf{F}_9), \Delta_6 \vdash \Delta_{10}, \| \mathbf{F}_7} K & \frac{\mathbf{h}_8 : \Box \Gamma_{11}, \mathbf{F}_9, \Delta_6, \| \mathbf{F}_7, \| \mathbf{F}_9 \vdash \Delta_{10}}{\bullet \mathbf{h}_8 : ((\Box \Gamma_{11}, \| \mathbf{F}_9), \Delta_6), \| \mathbf{F}_7 \vdash \Delta_{10}} & \mathbf{AT} \\ - : (\Box \Gamma_{11}, \| \mathbf{F}_9), \Delta_6 \vdash \Delta_{10}, \| \mathbf{F}_7 & \mathbf{ax}^{/\vee} \\ \mathbf{h}_8 : \Delta_6, \mathbf{F}_9, \Box \Gamma_{11}, \| \mathbf{F}_9 \vdash \Delta_{10} \\ - : \Delta_6, \mathbf{F}_9, \Box \Gamma_{11}, \| \mathbf{F}_9 \vdash \Delta_{10} \\ - : \Delta_6, \mathbf{F}_9, \Box \Gamma_{11}, \| \mathbf{F}_9 \vdash \Delta_{10} \\ - : \Delta_6, \Box \Gamma_{11}, \| \mathbf{F}_9 \vdash \Delta_{10} \\ \mathbf{h}_1 : unbox(\Box \Gamma_6) \vdash \mathbf{F}_7 & \mathbf{h}_8 : \Box \Gamma_6, \mathbf{F}_9, \Delta_{11}, \| \mathbf{F}_7, \| \mathbf{F}_9 \vdash \Delta_{10} \\ \bullet \mathbf{h}_8 : \Box \Gamma_6, \Delta_{11}, \| \mathbf{F}_9 \vdash \Delta_{10} \\ - : \Box \Gamma_6, \Delta_{11}, \| \mathbf{F}_9 \vdash \Delta_{10} \\ \mathbf{h}_1 : unbox(\Box \Gamma_6) \vdash \mathbf{F}_7 & \mathbf{ax}^{/\vee} \\ \bullet \mathbf{h}_8 : (\Box \Gamma_6, \Delta_{11}, \| \mathbf{F}_9), \Box \Gamma_6, \| \mathbf{F}_9 \vdash \Delta_{10} \\ - : \Box \Gamma_6, \Delta_{11}, \| \mathbf{F}_9 \vdash \Delta_{10} \\ \mathbf{h}_1 : unbox(\Box \Gamma_6) \vdash \mathbf{F}_9 & \mathbf{Ax}^{/\vee} \\ \bullet \mathbf{h}_1 : \Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, \| \mathbf{F}_7 & \mathbf{Ax}^{/\vee} \\ \bullet \mathbf{h}_1 : \Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, \| \mathbf{F}_9 \vdash \Delta_{10} \\ - : \Delta_{11}, \Box \Gamma_6, \| \mathbf{F}_9 \vdash \Delta_{10} \\ \bullet \mathbf{h}_1 : unbox(\Box \Gamma_6) \vdash \mathbf{F}_9 & \mathbf{Ax}^{/\vee} \\ \bullet \mathbf{h}_1 : \Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, \| \mathbf{F}_9 \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, \mathbf{F}_9 \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, \mathbf{F}_9 \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, \mathbf{F}_9 \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, \mathbf{F}_9 \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, \mathbf{F}_9 \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_6, \Delta_7 \vdash \Delta_{10}, \mathbf{F}_9 \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_6, \Delta_7 \vdash \Delta_1, \mathbf{F}_9 \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_6, \Delta_7 \vdash \Delta_1, \mathbf{F}_9 \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_6, \Delta_7 \vdash \Delta_9, \| \mathbf{F}_{10} \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_7, \Delta_8, (\Delta_9, \| \mathbf{F}_{10}), \mathbf{F}_7 \end{pmatrix} \begin{pmatrix} \mathbf{a} \mathbf{x} / \mathbf{a} \\ \bullet \mathbf{h}_1 : (\Box \Gamma_7, \Delta_8 \vdash \Delta_9, \| \mathbf{F}_{10} \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_7, \Delta_8 \vdash \Delta_9, \| \mathbf{F}_{10} \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_7, \Delta_8, | \mathbf{F}_{12} \vdash \Delta_9, \| \mathbf{F}_{10} \vdash K \\ \bullet \mathbf{h}_1 : (\Box \Gamma_7, \Delta_8, | \mathbf{F}_{12} \vdash \Delta_9, | \mathbf{F}_{10} \vdash K \\ \end{pmatrix} \begin{pmatrix} \mathbf{A} T \\ \mathbf{Cut} \\ - : \mathbf{U} \mathbf{A} \mathbf{A} \mathbf{A} \\ - : \mathbf{A} \mathbf{A} \mathbf{A} \mathbf{A} \mathbf{A} \\ - : \mathbf{A} \mathbf{A} \mathbf{A} \mathbf{A} \mathbf{A} \\ - : \mathbf{A} \mathbf{A} \mathbf{A} \mathbf{A} \mathbf{A} \mathbf{A} \end{pmatrix} \begin{pmatrix} \mathbf{A}$$

$$\frac{ \begin{array}{c} \mathbf{h}_{2}: unbox(\Box \Gamma_{7}) \vdash \mathbf{F}_{10} \\ \\ \underline{\bullet \mathbf{h}_{2}: \Box \Gamma_{7}, \Delta_{13}, [] \mathbf{F}_{12} \vdash (\Delta_{9}, [] \mathbf{F}_{10}), \mathbf{F}_{8} \end{array} K \begin{array}{c} \mathbf{h}_{11}: \Box \Gamma_{7}, \mathbf{F}_{8}, \mathbf{F}_{12}, \Delta_{13}, [] \mathbf{F}_{12} \vdash \Delta_{9}, [] \mathbf{F}_{10} \\ \\ \underline{\bullet \mathbf{h}_{11}: (\Box \Gamma_{7}, \Delta_{13}, [] \mathbf{F}_{12}), \mathbf{F}_{8} \vdash \Delta_{9}, [] \mathbf{F}_{10} \\ \\ \underline{-}: \Box \Gamma_{7}, \Delta_{13}, [] \mathbf{F}_{12} \vdash \Delta_{9}, [] \mathbf{F}_{10} \\ \\ \underline{-}: unbox(\Box \Gamma_{7}) \vdash \mathbf{F}_{10} \end{array} \begin{array}{c} AT \\ \mathbf{Cut} \end{array}$$

• Case rule  $\perp_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_1: unbox(\Box \Gamma_6) \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Box \Gamma_6, \bot, \Delta_{10} \vdash \Delta_9, []\mathbf{F}_7} \quad K \quad & \bullet \mathbf{h}_8: (\Box \Gamma_6, \bot, \Delta_{10}), []\mathbf{F}_7 \vdash \Delta_9} \\ & -: \Box \Gamma_6, \bot, \Delta_{10} \vdash \Delta_9 \\ & -: \bot, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \\ \hline & -: \bot, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_2: unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \\ & \bullet \mathbf{h}_2: \Box \Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathbf{F}_{10}), \bot \quad K \\ \hline & -: \Box \Gamma_7, \Delta_8 \vdash \Delta_9, []\mathbf{F}_{10} \\ & -: \Box \Gamma_7, \Delta_8 \vdash \Delta_9, []\mathbf{F}_{10} \\ & -: unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \\ \hline & -: unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \\ \hline & \bullet \mathbf{h}_2: unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \\ & -: \Delta_8, \Box \Gamma_7 \vdash \Delta_9, []\mathbf{F}_{10} \\ \hline & \bullet \mathbf{h}_2: \Box \Gamma_7, \bot, \Delta_{12} \vdash (\Delta_9, []\mathbf{F}_{10}), \mathbf{F}_8 \\ \hline & \bullet \mathbf{h}_{11}: (\Box \Gamma_7, \bot, \Delta_{12}), \mathbf{F}_8 \vdash \Delta_9, []\mathbf{F}_{10} \\ \hline & -: \Box \Gamma_7, \bot, \Delta_{12} \vdash \Delta_9, []\mathbf{F}_{10} \\ \hline & -: \Box \Gamma_7, \bot, \Delta_{12} \vdash \Delta_9, []\mathbf{F}_{10} \\ \hline & -: \bot_L \Delta_{12}, \Box \Gamma_7 \vdash \Delta_9, []\mathbf{F}_{10} \\ \hline & -: \bot_L \Delta_{12}, \Box \Gamma_7 \vdash \Delta_9, []\mathbf{F}_{10} \\ \hline \end{array}$$

• Case rule I

$$\begin{array}{c} \frac{h_1: unbox(\Box \Gamma_6) \vdash F_7}{\bullet h_1: \Box \Gamma_6, \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), []F_7} \quad K \quad \hline \bullet h_8: (\Box \Gamma_6, \Delta_{11}, p_9), []F_7 \vdash \Delta_{10}, p_9} \\ \hline -: \Box \Gamma_6, \Delta_{11}, p_9 \vdash \Delta_{10}, p_9 \\ \hline -: \Delta_{11}, \Box \Gamma_6, p_9 \vdash \Delta_{10}, p_9 \\ \hline \hline \bullet h_2: unbox(\Box \Gamma_7) \vdash F_9 \\ \hline \bullet h_2: \Box \Gamma_7, \Delta_8 \vdash ((\Delta_{12}, p_{11}), []F_9), p_{11} \\ \hline -: \Box \Gamma_7, \Delta_8 \vdash (\Delta_{12}, p_{11}), []F_9 \\ \hline -: unbox(\Box \Gamma_7) \vdash F_9 \\ \hline \bullet h_2: unbox(\Box \Gamma_7) \vdash F_9 \\ \hline \bullet h_2: unbox(\Box \Gamma_7) \vdash F_9 \\ \hline \bullet h_2: unbox(\Box \Gamma_7) \vdash F_9 \\ \hline -: \Delta_8, \Box \Gamma_7 \vdash \Delta_{12}, p_{11}, []F_9 \\ \hline \hline \bullet h_2: \Box \Gamma_7, \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), []F_9), F_8 \\ \hline \bullet h_2: \Box \Gamma_7, \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), []F_9), F_8 \\ \hline \hline -: \Box \Gamma_7, \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), []F_9 \\ \hline -: \Box \Gamma_7, \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), []F_9 \\ \hline \hline -: \Delta_{13}, \Box \Gamma_7, p_{11} \vdash \Delta_{12}, p_{11}, []F_9 \\ \hline \hline \end{array}$$

• Case rule  $\top_L$ 

$$\frac{ \begin{array}{l} \mathbf{h}_1: unbox(\Box \Gamma_6) \vdash \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_6, \top, \Delta_{10} \vdash \Delta_9, []\mathbf{F}_7 \end{array} K \quad \begin{array}{l} \mathbf{h}_8: \Box \Gamma_6, \Delta_{10}, []\mathbf{F}_7 \vdash \Delta_9 \\ \hline \bullet \mathbf{h}_8: (\Box \Gamma_6, \top, \Delta_{10}), []\mathbf{F}_7 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \top_L \\ \mathrm{cut} \\ \hline \hline \bullet \mathbf{h}_1: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9, []\mathbf{F}_7 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_8: \top, \Delta_{10}, \Box \Gamma_6, []\mathbf{F}_7 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_1: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9, []\mathbf{F}_7 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_1: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9, []\mathbf{F}_7 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_6 \vdash \Delta_9 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}: \Delta_1 \to \Delta_1 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_1 \to \Delta_1 \end{array} \quad \begin{array}{l} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_1 \to \Delta_1 \end{array} \quad \begin{array}{l} \mathbf{h}_7: \Delta_1 \to \Delta_2 \\ \hline \bullet \mathbf{h}_7: \Delta_1 \to \Delta_2 \end{array} \quad \begin{array}{l} \mathbf{h}_7: \Delta_1 \to \Delta_1 \to \Delta_2 \\ \hline \bullet \mathbf{h}_7: \Delta_1 \to \Delta_2 \end{array} \quad \begin{array}{l} \mathbf{h}_7: \Delta_1 \to \Delta_2 \\ \hline \bullet \mathbf{h}_7: \Delta_1 \to \Delta_2 \end{array} \quad$$

$$\frac{ \begin{array}{c} \mathbf{h}_2 : unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_2 : \Box \Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathbf{F}_{10}), \top \end{array} K \quad \frac{\mathbf{h}_{11} : \Box \Gamma_7, \Delta_8 \vdash \Delta_9, []\mathbf{F}_{10} \\ \bullet \mathbf{h}_{11} : (\Box \Gamma_7, \Delta_8), \top \vdash \Delta_9, []\mathbf{F}_{10} \end{array}}{ - : \Box \Gamma_7, \Delta_8 \vdash \Delta_9, []\mathbf{F}_{10} } \quad \mathbf{T}_L \\ \mathbf{Cut} \\ \hline \begin{array}{c} \mathbf{h}_2 : unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \\ \bullet \mathbf{h}_2 : unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \end{array} K \quad \frac{\mathbf{h}_{11} : \Box \Gamma_7, \mathbf{F}_8, \Delta_{12} \vdash \Delta_9, []\mathbf{F}_{10} \\ \bullet \mathbf{h}_{11} : (\Box \Gamma_7, \top, \Delta_{12} \vdash \Delta_9, []\mathbf{F}_{10} \end{array}}{ - : \Box \Gamma_7, \top, \Delta_{12} \vdash \Delta_9, []\mathbf{F}_{10} \end{array} \quad \mathbf{T}_L \\ \mathbf{Cut} \\ \hline \begin{array}{c} \mathbf{h}_2 : unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \\ \hline - : unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \end{array} K \\ \hline \begin{array}{c} \mathbf{h}_{11} : \Box \Gamma_7, \mathbf{F}_8, \Delta_{12} \vdash \Delta_9, []\mathbf{F}_{10} \\ \hline - : \Box \Gamma_7, \top, \Delta_{12} \vdash \Delta_9, []\mathbf{F}_{10} \end{array} K \\ \hline \begin{array}{c} \mathbf{h}_{11} : (\Box \Gamma_7, \top, \Delta_{12}), \mathbf{F}_8 \vdash \Delta_9, []\mathbf{F}_{10} \\ \hline - : unbox(\Box \Gamma_7) \vdash \mathbf{F}_{10} \end{array} K \\ \hline \end{array}$$

# 6.7 Status of A45: fail

• Case rule  $\rightarrow_R$ 

• Case rule  $\wedge_R$ 

$$\frac{h_1: \Box \Gamma_7 \vdash \Box \Gamma_{10}, F_9}{\bullet h_1: \Box \Gamma_7, \Delta_8 \vdash (\Box \Gamma_{10}, \Delta_{14}, F_{12} \land F_{13}), []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Box \Gamma_7, \Delta_8, []F_9 \vdash \Box \Gamma_{10}, F_{12}, \Delta_1_4 \quad h_{11}: \Box \Gamma_7, \Delta_8, []F_9 \vdash \Box \Gamma_{10}, F_{13}, \Delta_{14}]} \xrightarrow{\bullet h_{11}: (\Box \Gamma_7, \Delta_8), []F_9 \vdash \Box \Gamma_{10}, \Delta_{14}, F_{12} \land F_{13}]} \xrightarrow{Cut} \xrightarrow{\bullet h_1: \Box \Gamma_7 \vdash F_9, \Box \Gamma_{10}} \xrightarrow{ax/W} \xrightarrow{h_1: \Box \Gamma_7 \vdash F_9, \Box \Gamma_{10}} \xrightarrow{ax/W} \xrightarrow{h_1: \Box \Gamma_7 \vdash \Delta_{14}, F_{12}, \Box \Gamma_{10}} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{12}, \Box \Gamma_{10}} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_{14}, F_{13}, \Box \Gamma_{10}, []F_9} \xrightarrow{A45} \xrightarrow{h_{11}: \Delta_8, \Box \Gamma_7 \vdash \Delta_14, F_{13}, \Box \Gamma_14, F_{13}, \Box \Gamma_14, []F_9}$$

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 \underbrace{ \begin{array}{c} h_2 : \Box \Gamma_8 \vdash \Box \Gamma_{11}, F_{12}, \Box F_{10} \\ \bullet h_2 : \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{11}, (\Delta_{16}, F_{14} \land F_{15}), [F_{12}), \Box F_{10} \\ \bullet h_2 : \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{11}, (\Delta_{16}, F_{14} \land F_{15}), [F_{12}), \Box F_{10} \\ & - : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \land F_{15}), [F_{12} \\ & - : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, \Box \Gamma_{11}, [F_{12} \\ & - : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, \Box \Gamma_{11}, [F_{12} \\ & - : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, \Box \Gamma_{11}, [F_{12} \\ & - : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, \Box \Gamma_{11}, [F_{12} \\ & - : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, \Box \Gamma_{11}, [F_{12} \\ & - : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, F_{14}, \Box \Gamma_{11}, [F_{12} \\ & - : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, [F_{12} \\ & - : \Delta_9, \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12} \\ & - : \Box \Gamma_8, \Delta_{10} \vdash (\Box \Gamma_{11}, (\Delta_{16}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8, \Delta_{10} \vdash \Box \Gamma_{11}, (\Delta_{16}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \\ & - : \Box \Gamma_8 \vdash \Delta_{16}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12} \vdash \Delta_{12}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}), [F_{12}, F_{14} \land F_{15}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}, \Box \Gamma_{11}, [F_{12}, F_{14} \land F_{15}, \Box \Gamma_{11}, \Box \Gamma_{11}, [F_{12}, F_{14} \land
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### • Case rule $\vee_R$

$$\begin{array}{c} h_1: \Box r_7 \vdash \Box r_{10}, r_9 \\ \bullet h_1: \Box r_7, \Delta_8 \vdash (\Box r_{10}, \Delta_{14}, r_{12} \vee r_{13}), \Vert r_9 \\ \bullet h_1: \Box r_7, \Delta_8 \vdash (\Box r_{10}, \Delta_{14}, r_{12} \vee r_{13}), \Vert r_9 \\ & -: \Box r_7, \Delta_8 \vdash \Box r_{10}, \Delta_{14}, r_{12} \vee r_{13} \\ \hline & -: \Box r_7, \Delta_8 \vdash \Box r_{10}, \Delta_{14}, r_{12} \vee r_{13} \\ \hline & -: \Box r_8, \Box r_7 \vdash \Delta_{14}, r_{12}, r_{13}, \Box r_{10}, \Vert r_9 \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, r_{12}, r_{13}, \Box r_{10} \\ & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, r_{12}, r_{13}, \Box r_{10} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, r_{12}, r_{13}, \Box r_{10} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Delta_8, \Box r_7 \vdash \Delta_{14}, \Box r_{10}, r_{12} \vee r_{13} \\ \hline & -: \Box r_8, \Delta_9 \vdash \Box r_{11}, (\Delta_{16}, r_{14} \vee r_{15}), \Vert r_{12} \vee r_{13} \\ \hline & -: \Box r_8, \Delta_9 \vdash \Box r_{11}, (\Delta_{16}, r_{14} \vee r_{15}), \Vert r_{12} \vee r_{13} \\ \hline & -: \Delta_9, \Box r_8 \vdash \Delta_{16}, \Box r_{11}, \Vert r_{12} \vee r_{14} \vee r_{15} \\ \hline & -: \Delta_9, \Box r_8 \vdash \Delta_{16}, \Box r_{11}, \Vert r_{12} \vee r_{14} \vee r_{15} \\ \hline & -: \Delta_9, \Box r_8 \vdash \Delta_{16}, \Box r_{11}, \Vert r_{12} \vee r_{14} \vee r_{15} \\ \hline & -: \Delta_9, \Box r_8 \vdash \Delta_{16}, \Box r_{11}, \Vert r_{12} \vee r_{14} \vee r_{15} \\ \hline & -: \Delta_8, \Box r_{11}, (\Delta_{16}, r_{14} \vee r_{15}), \Vert r_{12} \vee r_{14} \vee r_{15} \\ \hline & -: \Delta_9, \Box r_8 \vdash \Delta_{16}, \Box r_{11}, (\Delta_{16}, r_{14} \vee r_{15}), \Vert r_{12} \vee r_{14} \vee r_{15} \\ \hline & -: \Box r_8, \Delta_{10} \vdash (\Box r_{11}, (\Delta_{16}, r_{14} \vee r_{15}), \Vert r_{12} \vee r_{14} \vee r_{15} \\ \hline & -: \Box r_8 \vdash r_{12}, \Box r_{11} \\ \hline & -: \Delta_9, \Box r_8 \vdash \Delta_{16}, \Box r_{11}, (\Delta_{16}, r_{14} \vee r_{15}), \Vert r_{12} \vee r_{14} \vee r_{15} \\ \hline &$$

### • Case rule $\perp_R$

$$\begin{array}{c} h_1: \square\Gamma_7 \vdash \square\Gamma_{10}, F_9 \\ \bullet h_1: \square\Gamma_7, \Delta_8 \vdash (\square\Gamma_{10}, \bot, \Delta_{12}), []F_9 \\ \hline \bullet h_1: \square\Gamma_7, \Delta_8 \vdash (\square\Gamma_{10}, \bot, \Delta_{12}), []F_9 \\ \hline -: \square\Gamma_7, \Delta_8 \vdash \square\Gamma_{10}, \bot, \Delta_{12} \\ \hline \bullet h_1: \square\Gamma_7, \Delta_8 \vdash \square\Gamma_{10}, \bot, \Delta_{12} \\ \hline \bullet h_1: \Delta_8, \square\Gamma_7 \vdash \bot, \Delta_{12}, \square\Gamma_{10}, []F_9 \\ \hline \hline \bullet h_1: \Delta_8, \square\Gamma_7 \vdash \bot, \Delta_{12}, \square\Gamma_{10}, []F_9 \\ \hline \hline \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{11}, F_{12}, \squareF_{10} \\ \hline \bullet h_2: \square\Gamma_8, \Delta_9 \vdash (\square\Gamma_{11}, (\bot, \Delta_{14}), []F_{12}), \squareF_{10} \\ \hline \hline \bullet h_2: \square\Gamma_8, \Delta_9 \vdash (\square\Gamma_{11}, (\bot, \Delta_{14}), []F_{12}), \squareF_{10} \\ \hline \bullet h_2: \square\Gamma_8, \Delta_9 \vdash \square\Gamma_{11}, (\bot, \Delta_{14}), []F_{12} \\ \hline \hline \bullet h_2: \square\Gamma_8, \Delta_9 \vdash \square\Gamma_{11}, (\bot, \Delta_{14}), []F_{12} \\ \hline \hline \bullet h_2: \square\Gamma_8, \Delta_9 \vdash \square\Gamma_{11}, (\bot, \Delta_{14}), []F_{12} \\ \hline \hline \bullet h_2: \square\Gamma_8, \Delta_9 \vdash \square\Gamma_{11}, (\bot, \Delta_{14}), []F_{12} \\ \hline \hline \bullet h_2: \square\Gamma_8, \Delta_9 \vdash \square\Gamma_{11}, (\bot, \Delta_{14}), []F_{12} \\ \hline \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_1, \Delta_1, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2: \Delta_9, \square\Gamma_8 \vdash \bot, \Delta_1, \Delta_1, \square\Gamma_{11}, []F_{12} \\ \hline \bullet h_2$$

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 \begin{array}{c} \mathbf{h}_{2}: \square\Gamma_{8} \vdash \square\Gamma_{11}, \mathbf{F}_{12} \\ \hline \bullet \mathbf{h}_{2}: \square\Gamma_{8}, \Delta_{10} \vdash (\square\Gamma_{11}, (\bot, \Delta_{14}), []\mathbf{F}_{12}), \mathbf{F}_{9} \end{array} \begin{array}{c} \mathbf{h}_{13}: \square\Gamma_{8}, \mathbf{F}_{9}, \Delta_{10} \vdash \square\Gamma_{11}, \Delta_{14}, []\mathbf{F}_{12} \\ \hline \bullet \mathbf{h}_{13}: (\square\Gamma_{8}, \Delta_{10}), \mathbf{F}_{9} \vdash \square\Gamma_{11}, (\bot, \Delta_{14}), []\mathbf{F}_{12} \end{array} \begin{array}{c} \bot_{R} \\ \mathbf{Cut} \\ \hline \\ -: \square\Gamma_{8}, \Delta_{10} \vdash \square\Gamma_{11}, (\bot, \Delta_{14}), []\mathbf{F}_{12} \\ \hline \\ -: \square\Gamma_{8} \vdash \mathbf{F}_{12}, \square\Gamma_{11} \\ \hline \\ -: \Delta_{10}, \square\Gamma_{8} \vdash \bot, \Delta_{14}, \square\Gamma_{11}, []\mathbf{F}_{12} \end{array} \begin{array}{c} A45 \end{array}
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• Case rule  $\top_R$ 

$$\begin{array}{c} h_1: \square\Gamma_7 \vdash \square\Gamma_{10}, F_9 \\ \hline \bullet h_1: \square\Gamma_7, \Delta_8 \vdash (\square\Gamma_{10}, \top, \Delta_{12}), [F_9] & A45 \\ \hline \bullet h_1: \square\Gamma_7, \Delta_8 \vdash (\square\Gamma_{10}, \top, \Delta_{12}), [F_9] & \Gamma_{10}, \top, \Delta_{12} \\ \hline & -: \square\Gamma_7, \Delta_8 \vdash \square\Gamma_{10}, \top, \Delta_{12} \\ \hline & -: \Delta_8, \square\Gamma_7 \vdash \top, \Delta_{12}, \square\Gamma_{10} \\ \hline \hline \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{11}, F_{12}, \squareF_{10} \\ \hline & \bullet h_2: \square\Gamma_8, \Delta_9 \vdash (\square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}), \squareF_{10} \\ \hline & -: \square\Gamma_8, \Delta_9 \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_9 \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{11}, F_{12} \\ \hline & \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{11}, F_{12} \\ \hline & \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{11}, F_{12} \\ \hline & \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{11}, F_{12} \\ \hline & \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{11}, F_{12} \\ \hline & \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}), F_9 \\ \hline & \bullet h_1: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & \bullet h_2: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & \bullet h_1: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, (\top, \Delta_{14}), [F_{12}] \\ \hline & -: \square\Gamma_8, \Delta_{10} \vdash \square\Gamma_{11}, [\Gamma, \Delta_{14}], [\Gamma, \Delta_{1$$

 $\bullet$  Case rule K

$$\frac{\mathbf{h}_1: \Box_{\Gamma_{12}}, \Box_{\Gamma_{14}} \vdash (\Box_{\Gamma_{11}}, [\mathbb{F}_{10}), \mathbb{F}_7}{\bullet \mathbf{h}_1: (\Box_{\Gamma_{12}}, \Box_{\Gamma_{14}}), \Box_{\Gamma_{13}}, \Delta_{15} \vdash ((\Box_{\Gamma_{11}}, [\mathbb{F}_{10}), \Delta_8), [\mathbb{F}_7]} \xrightarrow{A45} \frac{\mathbf{h}_9: \mathbb{F}_7, unbox(\Box_{\Gamma_{12}}, unbox(\Box_{\Gamma_{12}}), unbox(\Box_{\Gamma_{13}}) \vdash \mathbb{F}_{10}}{\bullet \mathbf{h}_9: ((\Box_{\Gamma_{12}}, \Box_{\Gamma_{14}}), \Box_{\Gamma_{13}}, \Delta_{15}), (\mathbb{F}_7 \vdash (\Box_{\Gamma_{11}}, \mathbb{F}_{10}), \Delta_8} \xrightarrow{K} \xrightarrow{Cut} -: (\Box_{\Gamma_{12}}, \Box_{\Gamma_{14}}), \Box_{\Gamma_{13}}, \Delta_{15} \vdash (\Box_{\Gamma_{11}}, \mathbb{F}_{10}), \Delta_8$$

### Axioms assumed:

```
inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula | -- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
                                  \mathtt{h}_1: \Box \Gamma_{12}, \Box \Gamma_{14} \vdash \Box \Gamma_{8}, \mathtt{F}_{7}
                                                                                                                                                      h_9: F_7, unbox(\Box \Gamma_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10}
    \frac{\mathbf{h}_{1}: \sqcup \mathbf{1}_{12}, \sqcup \mathbf{1}_{14} \sqcap \sqcup \mathbf{1}_{8}, \mathbf{r}_{7}}{\bullet \mathbf{h}_{1}: (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{8}, \Delta_{11}, []\mathbf{F}_{10}), []\mathbf{F}_{7}} \quad A45 \quad \frac{\mathbf{e}_{3} \cdot \mathbf{r}_{7}, \ldots \mathbf{r}_{12}}{\bullet \mathbf{h}_{9}: ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), []\mathbf{F}_{7} \vdash \Box \Gamma_{8}, \Delta_{11}, []\mathbf{F}_{10}} \quad K \quad \text{Cut}
                                                                                  -: (\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15} \vdash \Box\Gamma_{8},\Delta_{11},[]\mathtt{F}_{10}
          \frac{-: \Gamma_{12}, \square\Gamma_{13}, \square\Gamma_{14} \vdash F_{10}, F_{7}, \square\Gamma_{8}}{-: \square\Gamma_{12}, \square\Gamma_{13}, \square\Gamma_{14} \vdash F_{10}, \square\Gamma_{8}} \xrightarrow{\text{ax/W}} \frac{-: F_{7}, \square\Gamma_{12}, \square\Gamma_{13}, \square\Gamma_{14}, unbox(\square\Gamma_{12}), unbox(\square\Gamma_{13}) \vdash F_{10}, \square\Gamma_{8}}{-: F_{7}, \square\Gamma_{12}, \square\Gamma_{12}, \square\Gamma_{13}, \square\Gamma_{14} \vdash F_{10}, \square\Gamma_{8}} \xrightarrow{\text{sCut}} \xrightarrow{\text{ax/W}} ATG
                                                                    -:\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash \mathfrak{f}_{10},\Box\Gamma_{8}
                                                                 \overline{-:\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_{11},\Box\Gamma_{8},[]\mathtt{F}_{10}}
                           \mathtt{h}_1:\Box\Gamma_{12},\Box\Gamma_{14}\vdash(\Box\Gamma_{11},[]\mathtt{F}_{10}),\mathtt{F}_7
                                                                                                                                                                   h_9: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash F_{10}
    \begin{array}{c} \mathbf{h}_{1}: \sqcup \Gamma_{12}, \sqcup \Gamma_{14} \vdash (\sqcup \Gamma_{11}, || \mathbf{F}_{10}), \mathbf{F}_{7} \\ \bullet \mathbf{h}_{1}: (\square \Gamma_{12}, \square \Gamma_{14}), \square \Gamma_{13}, \Delta_{15} \vdash ((\square \Gamma_{11}, || \mathbf{F}_{10}), \Delta_{8}), || \mathbf{F}_{7} \end{array} \begin{array}{c} A45 \end{array} \begin{array}{c} \mathbf{h}_{9}: ((\square \Gamma_{12}, \square \Gamma_{14}), \square \Gamma_{13}, \Delta_{15}), || \mathbf{F}_{7} \vdash (\square \Gamma_{11}, || \mathbf{F}_{10}), \Delta_{8} \\ \mathbf{Cut} \end{array} 
                                                                                   -: (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{11}, []\mathtt{F}_{10}), \Delta_{8}
                                                                                               \overline{-:\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_{8},\Box\Gamma_{11},[]\mathsf{F}_{10}}
```

```
h_9: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash F_{10}
                                                                                                          \mathtt{h}_1:\Box\Gamma_{12},\Box\Gamma_{14}\vdash\Box\Gamma_{8},\mathtt{F}_{7}
                 \frac{1}{\bullet h_1 : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{8}, \Delta_{11}, []F_{10}), []F_7} A45 }{\bullet h_9 : ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), []F_7 \vdash \Box \Gamma_{8}, \Delta_{11}, []F_{10} } 
                                                                                                                                                                                                                                                    -: (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash \Box \Gamma_{8}, \Delta_{11}, []\mathtt{F}_{10}
                                                                                                                                                                                                                                                                                \frac{}{-:unbox(\Box\Gamma_{12}),unbox(\Box\Gamma_{13})\vdash \mathtt{F}_{10}}\text{ ax/W}
                                                                                                                                                                                                                                                            \frac{}{-:\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_{11},\Box\Gamma_{8},[]\mathsf{F}_{10}}\ K
                                                                                    \mathtt{h}_2: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{13}, []\mathtt{F}_{12}), \mathtt{F}_{10}, \Box\mathtt{F}_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \mathtt{h}_{11}: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}), unbox(\Box\mathtt{F}_{8}) \vdash \mathtt{F}_{12}
            \frac{\mathbf{h}_{2}: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{13}, []\mathsf{F}_{12}), \mathsf{F}_{10}, \Box\mathsf{F}_{8}}{\bullet \mathbf{h}_{2}: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{13}, []\mathsf{F}_{12}), \Delta_{9}, []\mathsf{F}_{10}), \Box\mathsf{F}_{8}} \quad A45 \quad \frac{\mathbf{h}_{11}: \mathit{unbox}(\Box_{14}), \mathit{unbox}(\Box_{14}), \mathit{unbox}(\Box_{15}), \mathit{u
                                                                                                                                                                                                                                                                                                                                                                                                            \underbrace{ \begin{array}{c} \mathbf{ax/W} \\ \mathbf{ax/W} \end{array} }_{ \begin{array}{c} \mathbf{h}_{11} : unbox(\Box \mathbf{F}_8), unbox(\Box \Gamma_{14}), unbox(\Box \Gamma_{15}) \vdash \mathbf{F}_{12} \\ \mathbf{\bullet} \mathbf{h}_{11} : \Box \mathbf{F}_8, \Box \Gamma_{14}, \Box \Gamma_{15}, \Box \Gamma_{16} \vdash \mathbf{F}_{10}, \Box \Gamma_{13}, []\mathbf{F}_{12} \\ \mathbf{h}_{Cut} \end{array} }_{ \begin{array}{c} \mathbf{ax/W} \\ K \end{array} } 
                                                                                                     \overline{h_2:\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Box F_8,F_{10},\Box\Gamma_{13},[]F_{12}} \ \ ^{c}
                                                                                                                                                                                                                                                                                                            \begin{array}{l} \mathsf{F}_{10},\sqcup_{13},\sqcup_{\mathsf{F}_{12}} \\ -: \square\Gamma_{14},\square\Gamma_{15}, \underline{\square\Gamma_{16} \vdash \mathsf{F}_{10}, \square\Gamma_{13}, \underline{\square\mathsf{F}_{12}}} \\ \end{array} \quad A45 
                                                                                                                                                                                                                                                                                   -: \Delta_{17}, \Box \Gamma_{14}, \Box \Gamma_{15}, \Box \Gamma_{16} \vdash \Delta_{9}, \Box \Gamma_{13}, []F_{10}, []F_{12}
              \frac{\mathbf{h}_2: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \Box\Gamma_{9}, \mathbf{f}_{10}, \Box\mathbf{f}_{8}}{\bullet \mathbf{h}_2: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{9}, (\Delta_{13}, []\mathbf{f}_{12}), []\mathbf{f}_{10}), \Box\mathbf{f}_{8}} \quad A45 \quad \frac{\mathbf{h}_{11}: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}), unbox(\Box\mathbf{f}_{8}) \vdash \mathbf{f}_{12}}{\bullet \mathbf{h}_{11}: ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box\mathbf{f}_{8} \vdash \Box\Gamma_{9}, (\Delta_{13}, []\mathbf{f}_{12}), []\mathbf{f}_{10}} \quad K \in \mathbf{Cut}
                                                                                                                                                                                                                                                                              -: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash \Box\Gamma_{9}, (\Delta_{13}, []\mathtt{F}_{12}), []\mathtt{F}_{10}
                                                                                                      \frac{\mathbf{a}_{11} : unbox(\Box F_8), unbox(\Box \Gamma_{14}), unbox(\Box \Gamma_{15}) \vdash F_{12}}{\mathbf{b}_{12} : \Box \Gamma_{14}, \Box \Gamma_{15}, \Box \Gamma_{16} \vdash \Box F_8, F_{10}, \Box \Gamma_9, []F_{12}} \quad \mathbf{ax/W} \quad \frac{\mathbf{b}_{11} : unbox(\Box F_8), unbox(\Box \Gamma_{14}), unbox(\Box \Gamma_{15}) \vdash F_{12}}{\mathbf{b}_{11} : \Box F_8, \Box \Gamma_{14}, \Box \Gamma_{15}, \Box \Gamma_{16} \vdash F_{10}, \Box \Gamma_9, []F_{12}} \quad K
                                                                                                                                                                                                                                                                                                                  -: \square\Gamma_{14}, \square\Gamma_{15}, \square\Gamma_{16} \vdash \mathtt{F}_{10}, \square\Gamma_{9}, []\mathtt{F}_{12}
                                                                                                                                                                                                                                                                              \frac{-1}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_{13},\Box\Gamma_{9},[]\mathsf{F}_{10},[]\mathsf{F}_{12}}\ A45
\frac{\mathtt{h}_2: \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \Box\Gamma_{9}, \mathtt{f}_{12}, \Box\mathsf{f}_{8}}{\bullet \mathtt{h}_2: (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_{9}, \Delta_{10}, []\mathtt{f}_{12}), \Box\mathsf{f}_{8}} \quad A45 \quad \frac{\mathtt{h}_{11}: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{8}) \vdash \mathtt{f}_{12}}{\bullet \mathtt{h}_{11}: ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \Box\mathsf{f}_{8} \vdash \Box\Gamma_{9}, \Delta_{10}, []\mathtt{f}_{12}} \quad \underset{\mathtt{Cut}}{K} \quad \overset{K}{\sim} \quad \frac{-: (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_{15}, \Box\Gamma_{1
                                                                                                                                                                                                                 -: (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash \Box \Gamma_{9}, \Delta_{10}, []F_{12}
  Axioms assumed:
  inf : C:MSFormula |-- True ; C':MSFormula
  inf : False ; C:MSFormula | -- C':MSFormula
  inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
  suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
  suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
  suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
                                                                                  \mathtt{h}_2:\Box\Gamma_{14},\Box\Gamma_{16}\vdash(\Box\Gamma_{13},[]\mathtt{F}_{12}),\mathtt{F}_{10},\Box\mathtt{F}_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \mathtt{h}_{11}: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_{12}
               \begin{array}{c} \bullet_{\mathbf{h}_2}: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{13}, []\mathsf{F}_{12}), \Delta_{9}, []\mathsf{F}_{10}), \Box\mathsf{F}_{8} \end{array} \begin{array}{c} A45 \\ \bullet_{\mathbf{h}_{11}}: ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box\mathsf{F}_{8} \vdash (\Box\Gamma_{13}, []\mathsf{F}_{12}), \Delta_{9}, []\mathsf{F}_{10} \\ \bullet_{\mathbf{cut}} \end{array} \begin{array}{c} K \\ \mathsf{cut} \end{array}
                                                                                                                                                                                                                                                                              -: (\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17} \vdash (\Box\Gamma_{13},[]F_{12}),\Delta_{9},[]F_{10}
                                                                                                                                                                                                                                                                                              \frac{-: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_{12}}{-: \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{9}, \Box\Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_{12}} \ \ K
                                                                                                             \mathtt{h}_2:\Box\Gamma_{14},\Box\Gamma_{16}\vdash\Box\Gamma_{9},\mathtt{F}_{10},\Box\mathtt{F}_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               h_{11}: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash F_{12}
               \underbrace{ \overset{\bullet}{\bullet} \mathbf{h}_2 : (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash (\Box \Gamma_{9}, (\Delta_{13}, []F_{12}), []F_{10}), \Box F_8}_{\bullet} \underbrace{ \overset{\bullet}{\bullet} \mathbf{h}_{11} : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), \Box F_8 \vdash \Box \Gamma_{9}, (\Delta_{13}, []F_{12}), []F_{10}}_{\bullet} \underbrace{ K}_{\mathsf{Cut}} \underbrace{ K}_{\mathsf{Cut}}
                                                                                                                                                                                                                                                                              -: (\Box\Gamma_{14},\Box\Gamma_{16}),\Box\Gamma_{15},\Delta_{17} \vdash \Box\Gamma_{9},(\Delta_{13},[]\textbf{F}_{12}),[]\textbf{F}_{10}
                                                                                                                                                                                                                                                                                              \frac{-: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_{12}}{-: \Delta_{17}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Delta_{13}, \Box\Gamma_{9}, []\mathtt{F}_{10}, []\mathtt{F}_{12}}
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\mathtt{h}_2: \Box \Gamma_{13}, \Box \Gamma_{15} \vdash \Box \Gamma_{9}, \mathtt{F}_{12}, \Box \mathtt{F}_{8}
                                                                                                                                                                                                                                                                                                                                                     \mathtt{h}_{11}: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{12}
   \frac{\mathbf{A}_{2} \cdot \Box \mathbf{1}_{13}, \Box \mathbf{1}_{15} + \Box \mathbf{1}_{9}, \mathbf{1}_{12}, \Box \mathbf{8}}{\bullet \mathbf{h}_{2} : (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash (\Box \Gamma_{9}, \Delta_{10}, []\mathbf{F}_{12}), \Box \mathbf{F}_{8}} 
 \frac{\mathbf{A}_{45}}{\bullet \mathbf{h}_{11} : ((\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16}), \Box \mathbf{F}_{8} \vdash \Box \Gamma_{9}, \Delta_{10}, []\mathbf{F}_{12}}{\bullet \mathbf{h}_{11} : ((\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16}), \Box \mathbf{F}_{8} \vdash \Box \Gamma_{9}, \Delta_{10}, []\mathbf{F}_{12}} 
                                                                                                                                                                                     \overline{-: (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash \Box \Gamma_{9}, \Delta_{10}, []F_{12}}
                                                                                                                                                                                                         \frac{}{-:unbox(\Box\Gamma_{13}),unbox(\Box\Gamma_{14})\vdash F_{12}} \text{ ax/W}
                                                                                                                                                                                            \frac{}{-:\Delta_{16},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{15}\vdash\Delta_{10},\Box\Gamma_{9},[]F_{12}}\ K
                                                                  \mathtt{h}_2:\Box\Gamma_{14},\Box\Gamma_{16}\vdash(\Box\Gamma_{13},[]\mathtt{F}_{12}),\mathtt{F}_{10}
                                                                                                                                                                                                                                                                                                                                                                                 \mathtt{h}_{11}: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}), unbox(\Box\mathtt{F}_{8}) \vdash \mathtt{F}_{12}
\frac{\mathbf{h}_{2}: (\square\Gamma_{14}, \square\Gamma_{16}), \square\Gamma_{15}, \Delta_{17} \vdash ((\square\Gamma_{13}, []\mathbf{F}_{12}), \mathbf{F}_{10}}{\bullet \mathbf{h}_{2}: (\square\Gamma_{14}, \square\Gamma_{16}), \square\Gamma_{15}, \Delta_{17} \vdash ((\square\Gamma_{13}, []\mathbf{F}_{12}), \Delta_{9}, []\mathbf{F}_{10}), \square\mathbf{F}_{8}} \quad A45 \quad \frac{\mathbf{n}_{11}: \mathit{unova}(\square_{14}), \mathit{unova}(\square_{15}), \mathit{unova}(\square_{
                                                                                                                                                                                                            \frac{-:\Box\Gamma_{14},\Box\Gamma_{16}\vdash F_{10},\Box\Gamma_{13},\left[\![F_{12}\right]^a ax/W}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_9,\Box\Gamma_{13},\left[\![F_{10},\left[\![F_{12}\right]^a 445\right]^a]^a}
                                                                                                                                                                                                                                                                                                                                                                                 h_{11}: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}), unbox(\Box\mathsf{F}_8) \vdash \mathsf{F}_{12}
                                                                                      h_2: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \Box\Gamma_9, F_{10}
 \frac{\mathbf{h}_{2}: \sqcup \Gamma_{14}, \sqcup \Gamma_{16} \vdash \sqcup \Gamma_{9}, \mathbf{f}_{10}}{\bullet \mathbf{h}_{2}: (\square \Gamma_{14}, \square \Gamma_{16}), \square \Gamma_{15}, \Delta_{17} \vdash (\square \Gamma_{9}, (\underline{\Delta_{13}, []\mathbf{f}_{12}), []\mathbf{f}_{10}), \square \mathbf{f}_{8}} \quad A45 \quad \frac{\mathbf{n}_{11}: \mathit{unoce}(\square \Gamma_{14}, \square \Gamma_{16}), \square \Gamma_{15}, \Delta_{17}), \square \mathbf{f}_{8} \vdash \square \Gamma_{9}, (\underline{\Delta_{13}, []\mathbf{f}_{12}), []\mathbf{f}_{10}}{\bullet \mathbf{h}_{11}: ((\square \Gamma_{14}, \square \Gamma_{16}), \square \Gamma_{15}, \Delta_{17}), \square \mathbf{f}_{8} \vdash \square \Gamma_{9}, (\underline{\Delta_{13}, []\mathbf{f}_{12}), []\mathbf{f}_{10}} \quad \mathsf{Cut}} \quad K
                                                                                                                                                                                                      -: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash \Box \Gamma_{9}, (\Delta_{13}, []F_{12}), []F_{10}
                                                                                                                                                                                                             \frac{-:\Box\Gamma_{14},\Box\Gamma_{16} \vdash \mathsf{F}_{10},\Box\Gamma_{9}}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16} \vdash \Delta_{13},\Box\Gamma_{9},[[\mathsf{F}_{10},[]\mathsf{F}_{12}]} \ A45
  \frac{\mathbf{h}_2: \Box \Gamma_{13}, \Box \Gamma_{15} \vdash \Box \Gamma_{9}, \mathbf{F}_{12}}{\bullet \mathbf{h}_2: (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash (\Box \Gamma_{9}, \Delta_{10}, []\mathbf{F}_{12}), \Box \mathbf{F}_{8}} \quad A45 \quad \frac{\mathbf{h}_{11}: unbox(\Box \Gamma_{13}), unbox(\Box \Gamma_{14}), unbox(\Box \Gamma_{8}) \vdash \mathbf{F}_{12}}{\bullet \mathbf{h}_{11}: ((\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16}), \Box \mathbf{F}_{8} \vdash \Box \Gamma_{9}, \Delta_{10}, []\mathbf{F}_{12}} \quad K \in \mathbf{Cut}
                                                                                                                                                                                      -: (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_{9}, \Delta_{10}, []F_{12}
                                                                                                                                                                                       \frac{-\square \Gamma_{13},\square \Gamma_{15} \vdash \mathtt{F}_{12},\square \Gamma_{9}}{-:\Delta_{16},\square \Gamma_{13},\square \Gamma_{14},\square \Gamma_{15} \vdash \Delta_{10},\square \Gamma_{9},[]\mathtt{F}_{12}} \ A45
                                                              h_2: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{13}, []F_{12}), F_{10}
                                                                                                                                                                                                                                                                                                                                                                                                       h_{11}: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash F_{12}
  \frac{\mathbf{n}_2 : \sqcup_{114}, \sqcup_{16} : (\sqcup_{13}, \sqcup_{F12}), \mathbf{r}_{10}}{\bullet \mathbf{h}_2 : (\square\Gamma_{14}, \square\Gamma_{16}), \underline{\square\Gamma_{15}, \Delta_{17}} \vdash ((\square\Gamma_{13}, []\mathbf{F}_{12}), \Delta_{9}, []\mathbf{F}_{10}), \mathbf{F}_8} \quad A45 \quad \frac{\mathbf{n}_{11} : ((\square\Gamma_{14}, \square\Gamma_{16}), \square\Gamma_{15}, \Delta_{17}), \mathbf{F}_8 \vdash (\square\Gamma_{13}, []\mathbf{F}_{12}), \Delta_{9}, []\mathbf{F}_{10}}{\bullet \mathbf{h}_{11} : ((\square\Gamma_{14}, \square\Gamma_{16}), \square\Gamma_{15}, \Delta_{17}), \mathbf{F}_8 \vdash (\square\Gamma_{13}, []\mathbf{F}_{12}), \Delta_{9}, []\mathbf{F}_{10}} \quad K \in \mathbb{C}_{\mathbf{n}} 
                                                                                                                                                                                            -: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash (\Box \Gamma_{13}, []F_{12}), \Delta_{9}, []F_{10}
                                                                                                                                                                                                       \frac{- : \mathit{unbox}(\Box \Gamma_{14}), \mathit{unbox}(\Box \Gamma_{15}) \vdash F_{12} \ \mathsf{ax/W}}{- : \Delta_{17}, \Box \Gamma_{14}, \Box \Gamma_{15}, \Box \Gamma_{16} \vdash \Delta_{9}, \Box \Gamma_{13}, []F_{10}, []F_{12}} \ \mathit{K}
                                                                                  \mathtt{h}_2:\Box\Gamma_{14},\Box\Gamma_{16}\vdash\Box\Gamma_{9},\mathtt{F}_{10}
                                                                                                                                                                                                                                                                                                                                                                                                 \mathtt{h}_{11}: unbox(\Box\Gamma_{14}), unbox(\Box\Gamma_{15}) \vdash \mathtt{F}_{12}
  \frac{\mathbf{h}_{2}: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash \Box\Gamma_{9}, F_{10}}{\bullet \mathbf{h}_{2}: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{9}, (\Delta_{13}, []F_{12}), []F_{10}), F_{8}} \quad \frac{\mathbf{h}_{11}: unoox(\Box 1_{14}), unoox(\Box 1_{15}) \vdash F_{12}}{\bullet \mathbf{h}_{11}: ((\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), F_{8} \vdash \Box\Gamma_{9}, (\Delta_{13}, []F_{12}), []F_{10}} \quad K \subset \mathbf{ut} 
                                                                                                                                                                                            -: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash \Box\Gamma_{9}, (\Delta_{13}, []\mathtt{F}_{12}), []\mathtt{F}_{10}
                                                                                                                                                                                                                                    \frac{}{-:unbox(\Box\Gamma_{14}),unbox(\Box\Gamma_{15})\vdash \mathtt{F}_{12}} \ \ \mathtt{ax/W}
                                                                                                                                                                                                        \frac{}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_{13},\Box\Gamma_{9},[]\mathsf{F}_{10},[]\mathsf{F}_{12}}\ K
  \frac{\mathbf{h}_2: \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \Box\Gamma_{9}, \mathbf{F}_{12}}{\bullet \mathbf{h}_2: (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash (\Box\Gamma_{9}, \Delta_{10}, []\mathbf{F}_{12}), \mathbf{F}_{8}} \quad A45 \quad \frac{\mathbf{h}_{11}: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathbf{F}_{12}}{\bullet \mathbf{h}_{11}: ((\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16}), \mathbf{F}_{8} \vdash \Box\Gamma_{9}, \Delta_{10}, []\mathbf{F}_{12}} \quad K \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                          \mathtt{h}_{11}: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{12}
                                                                                                                                                                            -: (\Box\Gamma_{13}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_{9}, \Delta_{10}, []F_{12}
                                                                                                                                                                                                    \frac{}{-:unbox(\Box\Gamma_{13}),unbox(\Box\Gamma_{14})\vdash F_{12}} \text{ ax/W}
                                                                                                                                                                                    \frac{}{-:\Delta_{16},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{15}\vdash\Delta_{10},\Box\Gamma_{9},[]\mathsf{F}_{12}}\ K
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 $\bullet$  Case rule A45

$$\frac{\mathbf{h}_{1}: \Box \Gamma_{14}, \Box \Gamma_{16} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \mathbf{F}_{7}}{\bullet \mathbf{h}_{1}: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \Box \Gamma_{11}, \Delta_{13}), [] \mathbf{F}_{7}}{\bullet \mathbf{h}_{8}: ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), [] \mathbf{F}_{7} \vdash (\Box \Gamma_{10}, \Box \Gamma_{11}, \mathbf{F}_{9})} \underbrace{A45}_{\bullet \mathbf{h}_{8}: ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), [] \mathbf{F}_{7} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \Box \Gamma_{11}, \Delta_{13}} \\ -: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \Box \Gamma_{11}, \Delta_{13}} \underbrace{A45}_{\bullet \mathbf{h}_{8}: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}, [] \mathbf{F}_{7} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \Box \Gamma_{11}, \Delta_{13}} \underbrace{A45}_{\bullet \mathbf{h}_{8}: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}, [] \mathbf{F}_{7} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \Box \Gamma_{11}, \Delta_{13}} \underbrace{A45}_{\bullet \mathbf{h}_{8}: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}, [] \mathbf{F}_{7} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \Box \Gamma_{11}, \Delta_{13}} \underbrace{A45}_{\bullet \mathbf{h}_{8}: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}, [] \mathbf{F}_{7} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \Box \Gamma_{11}, \Delta_{13}} \underbrace{A45}_{\bullet \mathbf{h}_{8}: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}, [] \mathbf{F}_{7} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \Box \Gamma_{11}, \Delta_{13}} \underbrace{A45}_{\bullet \mathbf{h}_{8}: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}, [] \mathbf{F}_{7} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, [] \mathbf{F}_{9}), \Box \Gamma_{11}, \Delta_{13}} \underbrace{A45}_{\bullet \mathbf{h}_{8}: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}, [] \mathbf{F}_{17}, \Delta_{17}, \Box \Gamma_{17}, \Delta_{17}, \Delta_{17},$$

Axioms assumed:

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inf : C:MSFormula |-- True ; C':MSFormula
  inf : False ; C:MSFormula |-- C':MSFormula
  inf : P:Prop ; C:MSFormula | -- P:Prop ; C':MSFormula
  suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
  suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
  suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
                                                                                                                h_1: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), F_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \mathtt{h}_8: \Box\Gamma_{14}, \Box\Gamma_{15}, []\mathtt{F}_7 \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9
              \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : (\Box \Gamma_{14}, \Box \Gamma_{15}, \Delta_{17} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9), []\mathbf{F}_7 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{15}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \end{array} }_{\bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{16}, \Box \Gamma_{16}, \Box
                                                                                                                                                                                                                                                                              -: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []F_9
                                                                                                                                                                              h_1:\Box\Gamma_{14},\Box\Gamma_{16}\vdash F_7,\Box\Gamma_{10},\Box\Gamma_{12} ax/W
                                                                                                                     \begin{array}{c} \bullet h_1 : \Box \Gamma_{14}, \Box \Gamma_{16}, \Box \Gamma_{16}, \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, \Box \Gamma_{1
                                                                                                                                                                                                                                                                                        \frac{-:\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash F_{9},\Box\Gamma_{10},\Box\Gamma_{11},\Box\Gamma_{12}}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash \Delta_{13},\Box\Gamma_{10},\Box\Gamma_{11},\Box\Gamma_{12},[]F_{9}} \ A45
                                                                                                   \mathtt{h}_1:\Box\Gamma_{14},\Box\Gamma_{16}\vdash(\Box\Gamma_{10},\Box\Gamma_{12},[]\mathtt{F}_9),\mathtt{F}_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \mathtt{h}_8:\Box\Gamma_{14},\Box\Gamma_{15}\vdash\Box\Gamma_{10},\Box\Gamma_{11},\mathtt{F}_9
              \begin{array}{c} \bullet_{h_1} : (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}, []F_9), \Box \Gamma_{11}, \Delta_{13}), []F_7 \end{array} \\ \bullet_{h_8} : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, []F_9), \Box \Gamma_{11}, \Delta_{13} \\ & - : (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, []F_9), \Box \Gamma_{11}, \Delta_{13} \end{array} 
                                                                                                                                                                                                                                                                                       \mathtt{h}_1:\Box\Gamma_{14},\Box\Gamma_{16}\vdash(\Box\Gamma_{10},\Box\Gamma_{12}),\mathtt{f}_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \mathtt{h}_8: \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9
              \underbrace{ \begin{array}{c} 1 \\ \bullet \mathbf{h}_1 : (\Box \Gamma_{14}, \Box \Gamma_{15}, \Delta_{17} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9), []\mathbf{F}_7 \end{array}}_{\boldsymbol{\Phi}\mathbf{h}_3 : ((\Box \Gamma_{14}, \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9) 
                                                                                                                                                                                                                                                                              -: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, []F_9
                                                                                                                                                                                                                                                                                       \frac{ \overbrace{-:\Box\Gamma_{14},\Box\Gamma_{15}\vdash \mathtt{F}_{9},\Box\Gamma_{10},\Box\Gamma_{11}}^{\quad \text{ax/W}}}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_{13},\Box\Gamma_{10},\Box\Gamma_{11},\Box\Gamma_{12},[]\mathtt{F}_{9}} \ A45
                                                                                   \mathtt{h}_2: \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []\mathtt{F}_{11}), \mathtt{F}_9, \Box\mathtt{F}_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \mathtt{h}_{10}: \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\mathtt{f}_{8} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, \mathtt{f}_{11}, []\mathtt{f}_{9}
\frac{\mathbb{A}_{2} : (\square_{16}, \square_{18}) , (\square_{17}, \square_{18}) , (\square_{17}, \square_{18}) , (\square_{17}, \square_{18}, \square_{17}, \square_{18}) , (\square_{18}, \square_{18}, \square_{18}, \square_{18}, \square_{18}) , (\square_{18}, \square_{18}, \square_{18}, \square_{18}, \square_{18}) , (\square_{18}, \square_{18}, \square_{18}, \square_{18}, \square_{18}, \square_{18}) , (\square_{18}, \square_{18}, \square_{18}, \square_{18}, \square_{18}, \square_{18}) , (\square_{18}, \square_{18}, \square_{18}, \square_{18}, \square_{18}, \square_{18}, \square_{18}) , (\square_{18}, \square_{18}, \square_{18},
                                                                                                                                                                                                                                                                              : (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, []F_{11}), (\Box \Gamma_{13}, \Delta_{15}), []F_{9}
 Axioms assumed:
  inf : C:MSFormula |-- True ; C':MSFormula
  inf : False ; C:MSFormula |-- C':MSFormula
  inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
  suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
  suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
  suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
                                                                                                          \mathtt{h}_2: \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []\mathtt{F}_{11}), \mathtt{F}_9, \Box\mathtt{F}_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \mathtt{h}_{10}: \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\mathtt{f}_{8} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, \mathtt{f}_{11}
             \bullet \mathbf{h}_2: (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}, []\mathbf{F}_{11}), (\Box \Gamma_{13}, \Delta_{15}), []\mathbf{F}_{9}), \Box \mathbf{F}_{8}
\bullet \mathbf{h}_1: ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}, []\mathbf{F}_{11}), (\Box \Gamma_{13}, \Delta_{15}), []\mathbf{F}_{9}), \Box \mathbf{F}_{8}
\bullet \mathbf{h}_{10}: ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, []\mathbf{F}_{11}), (\Box \Gamma_{18}, \Box \Gamma
                                                                                                                                                                                                                                                                                                         -: (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             h_{10}: \Box F_8, \Box \Gamma_{16}, \Box \Gamma_{17} \vdash F_{11}, \Box \Gamma_{12}, \Box \Gamma_{13} ax/W
                                                                                                                               \frac{1}{\mathbf{h}_2:\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Box F_8,F_9,\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11}}}{\bullet \mathbf{h}_{10}:\Box F_8,\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash F_9,\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11}}}
                                                                                                                                                                                                                                                                                                                                         -: \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash F_9, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}
                                                                                                                                                                                                                                                                                                                         \frac{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11},[]F_{9}}{A45}
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\mathtt{h}_2:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),\mathtt{F}_9,\Box\mathtt{F}_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \mathtt{h}_{10}:\Box\Gamma_{16},\Box\Gamma_{17},\Box\mathtt{F}_{8}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F}_{11},[]\mathtt{F}_{9}
-: (\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19} \vdash (\Box\Gamma_{12},\Box\Gamma_{14}), (\Box\Gamma_{13},\Delta_{15},[]\mathtt{F}_{11}),[]\mathtt{F}_{9}
                                                                                                                                                                                                                                                                                                                          \mathtt{h}_2:\Box\Gamma_{16},\Box\Gamma_{18}\vdash\Box\mathtt{f}_8,\mathtt{f}_9,\Box\Gamma_{12},\Box\Gamma_{14} ax/W
                                                                                                                                                                                                               \begin{array}{c} \mathbf{h}_2: \Box \Gamma_{16}, \Box \Gamma_{18} \vdash \Box F_8, \mathbf{f}_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, [\mathbf{f}_9] \\ \bullet \mathbf{h}_2: \Box \Gamma_{16}, \Box \Gamma_{17}, \Box \Gamma_{18} \vdash \Box F_8, \mathbf{f}_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, [\mathbf{f}_9] \\ \bullet \mathbf{h}_2: \Box \Gamma_{16}, \Box \Gamma_{17}, \Box \Gamma_{18} \vdash \mathbf{f}_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, [\mathbf{f}_9] \\ \bullet \mathbf{h}_2: \Box \Gamma_{16}, \Box \Gamma_{17}, \Box \Gamma_{18} \vdash \mathbf{f}_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, [\mathbf{f}_9] \\ \bullet \mathbf{h}_2: \Box \Gamma_{16}, \Box \Gamma_{17}, \Box \Gamma_{18} \vdash \Box \Gamma_{17}, \Box \Gamma_{18}, \Box \Gamma
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                -: \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash F_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            -: \Delta_{19}, \Box \overline{\Gamma_{16}, \Box} \overline{\Gamma_{17}, \Box \Gamma_{18} \vdash \Delta_{15}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, ||F_{11}, ||F_{9}|} \quad A45
                                                                                                                                                                                                     \mathtt{h}_2: \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), \mathtt{F}_9, \Box\mathtt{F}_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \underline{h_{10}:\Box\Gamma_{16},\Box\Gamma_{17},\Box\digamma_{8}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\digamma_{11}}
 \underbrace{- \bullet_{12} : (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []F_{11}), []F_{9}), \Box F_{8}}_{\bullet h_{10}} A_{45} \underbrace{- \bullet_{h_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []F_{11}), \Box F_{18} \underbrace{- \bullet_{h_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Box \Gamma_{17}, \Delta_{19}, \Box \Gamma_{17}, \Delta_{19}), (\Box \Gamma_{17}, \Delta_{19}, \Box \Gamma_{17}, \Delta_{19}, \Box \Gamma_{17}, \Delta_{19}), (\Box \Gamma_{18}, \Box \Gamma_{17}, \Delta_{19}, \Box \Gamma_{17}, \Delta_{19}, \Box \Gamma_{17}, \Delta_{19}), (\Box \Gamma_{18}, \Box \Gamma_{17}, \Delta_{19}, \Box \Gamma
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             -: (\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19} \vdash (\Box\Gamma_{12},\Box\Gamma_{14}), (\Box\Gamma_{13},\Delta_{15},[]F_{11}),[]F_{9}
                                                                                                                                                                                                                                                                                                                      \underline{\mathbf{h}_2}: \Box \Gamma_{16}, \Box \Gamma_{18} \vdash \Box \mathbf{f}_8, \mathbf{f}_9, \Box \Gamma_{12}, \Box \Gamma_{14} \quad \text{ax/W}
                                                                                                                                                                                                             \begin{array}{c} \mathbf{a}_2 : \Box \mathbf{1}_{16}, \Box \mathbf{1}_{18} \vdash \Box \mathbf{a}_{3}, \neg g, \Box \mathbf{1}_{2}, \Box \mathbf{1}_{14} \\ \bullet \mathbf{h}_2 : \Box \Gamma_{16}, \Box \Gamma_{17}, \Box \Gamma_{18} \vdash \Box \mathbf{F}_{8}, \mathbf{F}_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, []\mathbf{F}_9 \end{array} \begin{array}{c} \mathbf{a}_{10} : \Box \mathbf{F}_{8}, \Box \Gamma_{16}, \Box \Gamma_{17}, \Box \Gamma_{18} \vdash \mathbf{F}_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, []\mathbf{F}_9 \\ \mathbf{h}_{10} : \Box \mathbf{F}_{16}, \Box \mathbf{F}_{17}, \Box \mathbf{F}_{18} \vdash \mathbf{F}_{11}, \Box \mathbf{F}_{12}, \Box \mathbf{F}_{13}, \Box \mathbf{F}_{14}, []\mathbf{F}_9 \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          -: \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \mathsf{F}_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []\mathsf{F}_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \overline{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box}\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathtt{F}_{11},[]\mathtt{F}_{9}
                                                                                                                                                                      h_2: \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), F_{10}, \Box F_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \mathtt{h}_9: \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\mathtt{F}_8 \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}
 \underbrace{ \begin{array}{c} 2 \cdot \square 13, \square 17 \cdot (\square 11, \square 13), \square 10, \square 18 \\ \bullet \mathbf{h}_2 : (\square \Gamma_{15}, \square \Gamma_{17}), \square \Gamma_{16}, \Delta_{18} \vdash ((\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), []\mathbf{F}_{10}), \square \mathbf{F}_8 \end{array}} }_{\bullet \mathbf{h}_9 : ((\square \Gamma_{15}, \square \Gamma_{17}), \square \Gamma_{16}, \Delta_{18}), \square \mathbf{F}_8 \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), (\square \Gamma_{12}, \Delta_{14}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square \mathbf{F}_{18} \vdash (\square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}), \square
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                                                                                                                                                                                                           \frac{\mathbf{h}_9: \Box \mathbf{F}_8, \Box \mathbf{F}_{15}, \Box \mathbf{F}_{16}, \Box \mathbf{F}_{17} \vdash \Box \mathbf{F}_8, \mathbf{F}_{10}, \Box \mathbf{F}_{11}, \Box \mathbf{F}_{12}, \Box \mathbf{F}_{13}}{\mathbf{e}x^{1/2}} \mathbf{a}x^{1/2} \mathbf{e} 
 \frac{\mathbf{h}_9: \Box \mathbf{F}_8, \Box \mathbf{F}_{15}, \Box \mathbf{F}_{16}, \Box \mathbf{F}_{17} \vdash \mathbf{F}_{10}, \Box \mathbf{F}_{11}, \Box \mathbf{F}_{12}, \Box \mathbf{F}_{13}}{\mathbf{e}x^{1/2}} \mathbf{e} 
 \mathbf{e} \mathbf{h}_9: \Box \mathbf{F}_8, \Box \mathbf{F}_{15}, \Box \mathbf{F}_{16}, \Box \mathbf{F}_{17} \vdash \mathbf{F}_{10}, \Box \mathbf{F}_{11}, \Box \mathbf{F}_{12}, \Box \mathbf{F}_{13}}{\mathbf{e}x^{1/2}} \mathbf{e} 
 \mathbf{h}_{10}: \Box \mathbf{F}_{10}, \Box \mathbf{F}_{10}, \Box \mathbf{F}_{11}, \Box \mathbf{F}_{12}, \Box \mathbf{F}_{13}, \Box \mathbf{F}_{13}, \Box \mathbf{F}_{14}, \Box \mathbf{F}_{15}, \Box \mathbf{F}_{16}, \Box \mathbf{F}_{17} \vdash \mathbf{F}_{10}, \Box \mathbf{F}_{11}, \Box \mathbf{F}_{12}, \Box \mathbf{F}_{13}, \Box \mathbf{F}_{14}, \Box \mathbf{F}_{15}, \Box \mathbf{F}_{16}, \Box \mathbf{F}_{17}, \Box \mathbf{F}_{16}, \Box \mathbf{F}_{17}, 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 -: \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{17} \vdash F_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \frac{1}{-:\Delta_{18},\Box\Gamma_{15},\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]F_{10}} A45 
                                                                                                                                                                      \mathtt{h}_2: \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []\mathtt{F}_{11}), \mathtt{F}_9, \Box\mathtt{F}_8
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 \underbrace{- \bullet_{h_2} : (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}, []F_{11}), (\Box \Gamma_{13}, \Delta_{15}), []F_{9}), \Box F_{8}}_{\bullet h_{10}} \xrightarrow{A45} \underbrace{- \bullet_{h_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, []F_{11}), (\Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, []F_{11}), (\Box \Gamma_{18}, \Box \Gamma_
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      \frac{}{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash F_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]F_{9}}\text{ ax/W}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            -: \Delta_{19}, \Box \Gamma_{16}, \Box \Gamma_{17}, \Box \Gamma_{18} \vdash \Delta_{15}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, []F_{11}, []F_{9}  A45
                                                                                                                                                                      \mathtt{h}_2:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F}_{11}),\mathtt{F}_9,\Box\mathtt{F}_8
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 \underbrace{ \begin{array}{c} \square_{2} \square_{10} \square_{10}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           -: (\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19} \vdash (\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F}_{11}),(\Box\Gamma_{13},\Delta_{15}),[]\mathtt{F}_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \frac{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash F_{11},\Box\Gamma_{12},\Box\Gamma_{13}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11},[]F_{9}}\ A45
                                                                                                                                                                                                  h_2: \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), F_9, \Box F_8
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\bullet \mathbf{h}_2: (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), []\mathbf{F}_{9}), \Box \mathbf{F}_{8} \xrightarrow{\bullet} \mathbf{h}_{10}: ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), \Box \mathbf{F}_{18} \xrightarrow{\bullet} \mathbf{h}_{10}: ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \xrightarrow{\bullet} \mathbf{h}_{10}: ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \Gamma_{17}, \Delta_{19}, \Box \Gamma_{17}, \Delta_{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             -: (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \frac{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash F_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]F_9}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11},[]F_9}\ A45
                                                                                                                                                                                                  \mathtt{h}_2: \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), \mathtt{F}_9, \Box\mathtt{F}_8
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 \underbrace{\bullet_{\mathbf{h}_2} : (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), []\mathbf{F}_{9}), \Box \mathbf{F}_{8} }_{\bullet \mathbf{h}_{10}} \ \underbrace{\bullet_{\mathbf{h}_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), \Box \mathbf{F}_{18} }_{\bullet \mathbf{h}_{10}} \ \underbrace{\bullet_{\mathbf{h}_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), \Box \mathbf{F}_{18} }_{\bullet \mathbf{h}_{10}} \ \underbrace{\bullet_{\mathbf{h}_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), \Box \mathbf{F}_{18} }_{\bullet \mathbf{h}_{10}} \ \underbrace{\bullet_{\mathbf{h}_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), \Box \mathbf{F}_{18} }_{\bullet \mathbf{h}_{10}} \ \underbrace{\bullet_{\mathbf{h}_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), \Box \mathbf{F}_{18} }_{\bullet \mathbf{h}_{10}} \ \underbrace{\bullet_{\mathbf{h}_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), \Box \mathbf{F}_{18} }_{\bullet \mathbf{h}_{10}} \ \underbrace{\bullet_{\mathbf{h}_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{14}, \Box \Gamma_{14}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             -: (\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19} \vdash (\Box\Gamma_{12},\Box\Gamma_{14}), (\Box\Gamma_{13},\Delta_{15},[]F_{11}),[]F_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \frac{\overbrace{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash F_{11},\Box\Gamma_{12},\Box\Gamma_{13}}^{\text{ax/W}}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11},[]F_{9}}
A45
                                                                                                                                                                      \mathtt{h}_2: \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{F}_{10}, \Box\mathtt{F}_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   h_9: \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, F_{10}
 \underbrace{- \bullet_{h_2} : (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}), []F_{10}), \Box F_8}_{\bullet h_9} \underbrace{A45}_{\bullet h_9} : ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_8 \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}), \Box F_8 \vdash (\Box \Gamma_{11}, \Box \Gamma_{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 -: (\Box\Gamma_{15},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{18} \vdash (\Box\Gamma_{11},\Box\Gamma_{13}),(\Box\Gamma_{12},\Delta_{14}),[]\mathtt{F}_{10}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{}{-:\Delta_{18},\Box\Gamma_{15},\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathsf{F}_{10}}
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\mathtt{h}_2: \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []\mathtt{F}_{11}), \mathtt{F}_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \mathtt{h}_{10}: \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\mathtt{F}_{8} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, \mathtt{F}_{11}, []\mathtt{F}_{9}
 \underbrace{-10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot (\Box \Gamma_{12}, \Box \Gamma_{14}, \Box \Gamma_{17}, \Delta_{15}, \Box \Gamma_{18}, \Delta_{15}, \Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}, []F_{11}), (\Box \Gamma_{13}, \Delta_{15}), []F_{9}), \Box F_{8} }_{\bullet h_{10} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, []F_{11}), (\Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, []F_{11}), (\Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, \Box \Gamma_{18}, \Box \Gamma_{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      -: (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{\phantom{a}}{-:\Box\Gamma_{16},\Box\Gamma_{18}\vdash F_{9},\Box\Gamma_{12},\Box\Gamma_{14},[]F_{11}}\text{ ax/W}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   \frac{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11},[]F_{9}}{A45}
                                                                                                                                                                                          \mathtt{h}_2:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F}_{11}),\mathtt{F}_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \mathtt{h}_{10}: \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\mathtt{F}_{8} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, \mathtt{F}_{11}
  \bullet_{h_2}: (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}, [F_{11}), (\Box\Gamma_{13}, \Delta_{15}), [F_{9}), \Box F_{8}) 
\bullet_{h_2}: (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{12}, \Box\Gamma_{14}, [F_{11}), (\Box\Gamma_{13}, \Delta_{15}), [F_{9}), \Box F_{8}) 
\bullet_{h_{10}}: ((\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_{8} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, [F_{11}), (\Box\Gamma_{18}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, [F_{11}), (\Box\Gamma_{18}, \Box\Gamma_{18}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box F_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, [F_{11}), (\Box\Gamma_{18}, \Box\Gamma_{18}, \Box\Gamma_{18
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  -: (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{ \overline{-: \Box\Gamma_{16}, \Box\Gamma_{18} \vdash F_9, \Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}} \quad \text{ax/W} }{ -: \Delta_{19}, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}, []F_9} \quad A45
                                                                                                                                                                                                                       \mathtt{h}_2:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),\mathtt{F}_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \mathtt{h}_{10}: \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\mathtt{F}_{8} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, \mathtt{F}_{11}, []\mathtt{F}_{9}
 \bullet \mathbf{h}_2: (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, [] \mathbf{F}_{11}), [] \mathbf{F}_{9}), \Box \mathbf{F}_{8}   \bullet \mathbf{h}_{10}: ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, [] \mathbf{F}_{11}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{17}, \Delta_{19}), \Box \Gamma_{17}, \Delta_{19}, 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \overline{-: (\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19}\vdash (\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15},[]F_{11}),[]F_{9}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{-:\Box\Gamma_{16},\Box\Gamma_{18}\vdash F_9,\Box\Gamma_{12},\Box\Gamma_{14}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11},[]F_9}\ A45
                                                                                                                                                                                                                     \mathtt{h}_2:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),\mathtt{F}_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \mathtt{h}_{10}: \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\mathtt{F}_{8} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, \mathtt{F}_{11}
 \underbrace{ \begin{array}{c} \mathbf{a}_{12} \cdot \Box \mathbf{a}_{16}, \Box \mathbf{a}_{18} + (\Box \mathbf{a}_{12}, \Box \mathbf{a}_{14}), \mathbf{r}_{9} \\ \bullet \mathbf{h}_{2} : (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), []\mathbf{F}_{9}), \Box \mathbf{F}_{8} \end{array}}_{\bullet \mathbf{h}_{10} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), (\Box \Gamma_{16}, \Box \Gamma_{18}), (\Box \Gamma_{17}, \Delta_{19}), (\Box \Gamma_{18}, \Box \Gamma_{18}, (\Box \Gamma_{18}, \Box \Gamma_{1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \overline{-: (\Box\Gamma_{16},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19} \vdash (\Box\Gamma_{12},\Box\Gamma_{14}), (\Box\Gamma_{13},\Delta_{15},[]F_{11}),[]F_{9}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{-:\Box\Gamma_{16},\Box\Gamma_{18}\vdash F_9,\Box\Gamma_{12},\Box\Gamma_{14}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11},[]F_9}\ A45
                                                                                                                                                                                       h_2: \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), F_{10}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               h_9: \Box\Gamma_{15}, \Box\Gamma_{16}, \Box F_8 \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, F_{10}
 \underbrace{- \bullet_{h_2} : (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}), []F_{10}), \Box F_8}_{\bullet h_9} \underbrace{A45}_{\bullet h_9} : ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_8 \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}), \Box F_8 \vdash (\Box \Gamma_{11}, \Box \Gamma_{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                           -: (\Box\Gamma_{15},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{18} \vdash (\Box\Gamma_{11},\Box\Gamma_{13}), (\Box\Gamma_{12},\Delta_{14}), []F_{10}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \frac{\neg}{-:\Box\Gamma_{15},\Box\Gamma_{17}\vdash \mathsf{F}_{10},\Box\Gamma_{11},\Box\Gamma_{13}} \ \mathsf{ax/W} \\ \frac{\neg}{-:\Delta_{18},\Box\Gamma_{15},\Box\Gamma_{16},\Box\Gamma_{17}\vdash \Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathsf{F}_{10}} \ A45
                                                                                                                                                                              \mathtt{h}_2:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14},[]\mathtt{F}_{11}),\mathtt{F}_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \mathtt{h}_{10}: \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, \mathtt{F}_{11}, []\mathtt{F}_{9}
 \underbrace{ \begin{array}{c} \mathbf{a}_{12} : \exists 1_{16}, \exists 1_{18} : (\exists 1_{12}, \exists 1_{14}, [\exists 1_{11}), \exists 9 \\ \bullet \mathbf{h}_{2} : (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}, [\exists 1_{1}), (\Box \Gamma_{13}, \Delta_{15}), [\exists F_{9}), F_{8} \\ \end{array}}_{\bullet \mathbf{h}_{10} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, [\exists F_{11}), (\Box \Gamma_{18}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, [\exists F_{11}), (\Box \Gamma_{18}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, [\exists F_{11}), (\Box \Gamma_{18}, \Box \Gamma_{18}), \Box \Gamma_{18}, \Box \Gamma_{18}), \Box \Gamma_{18}, \Box \Gamma_{18},
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                -: (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []F_{11}), (\Box\Gamma_{13}, \Delta_{15}), []F_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash F_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]F_9}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11},[]F_9}\ A45
                                                                                                                                                                              \mathtt{h}_2: \Box\Gamma_{16}, \Box\Gamma_{18} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}, []\mathtt{F}_{11}), \mathtt{F}_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \mathtt{h}_{10}:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F}_{11}
-: (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}, []\mathtt{F}_{11}), (\Box \Gamma_{13}, \Delta_{15}), []\mathtt{F}_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             \frac{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash \mathbf{F}_{11},\Box\Gamma_{12},\Box\Gamma_{13}}{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]\mathbf{F}_{11},[]\mathbf{F}_{9}}\ A45
                                                                                                                                                                                                         \mathtt{h}_2:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),\mathtt{F}_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \mathtt{h}_{10}: \Box\Gamma_{16}, \Box\Gamma_{17} \vdash \Box\Gamma_{12}, \Box\Gamma_{13}, \mathtt{F}_{11}, []\mathtt{F}_{9}
   \underbrace{ \begin{array}{c} \mathbf{a}_2 : \Box \mathbf{1}_{16}, \Box \mathbf{1}_{18} : (\Box \mathbf{1}_{12}, \Box \mathbf{1}_{14}), \mathbf{r}_9 \\ \bullet \mathbf{h}_2 : (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), []\mathbf{F}_9), \mathbf{F}_8 \end{array}}_{\bullet \mathbf{h}_{10} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \mathbf{F}_8 \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []\mathbf{F}_{11}), (\Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{18}, \Box \Gamma_{17}, \Delta_{19}), \mathbf{F}_8 \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, \Box \Gamma_{18}, \Box 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       -: (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{-}{-:\Box\Gamma_{16},\Box\Gamma_{17}\vdash F_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]F_9} \text{ ax/W}}\\ \frac{-:\Delta_{19},\Box\Gamma_{16},\Box\Gamma_{17},\Box\Gamma_{18}\vdash \Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14},[]F_{11},[]F_9}{A45}
                                                                                                                                                                                                            \mathtt{h}_2:\Box\Gamma_{16},\Box\Gamma_{18}\vdash(\Box\Gamma_{12},\Box\Gamma_{14}),\mathtt{F}_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \mathtt{h}_{10}:\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Box\Gamma_{12},\Box\Gamma_{13},\mathtt{F}_{11}
 \underbrace{- \bullet_{h_2} : (\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{15}, []F_{11}), []F_{9}), F_{8}}_{\bullet h_{10}} \ \underbrace{- \bullet_{h_{10}} : ((\Box \Gamma_{16}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{18}, \Box \Gamma_{18}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), F_{8} \vdash (\Box \Gamma_{12}, \Box \Gamma_{14}), (\Box \Gamma_{13}, \Delta_{18}, \Box \Gamma_{18}, \Box \Gamma_{18}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         -: (\Box\Gamma_{16}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{12}, \Box\Gamma_{14}), (\Box\Gamma_{13}, \Delta_{15}, []F_{11}), []F_{9}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               -: \overline{\Delta_{19}, \Box\Gamma_{16}, \Box\Gamma_{17}, \Box\Gamma_{18} \vdash \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, []F_{11}, []F_{9}} \quad A45
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\begin{array}{c} h_2: \square\Gamma_{15}, \square\Gamma_{17} \vdash (\square\Gamma_{11}, \square\Gamma_{13}), F_{10} \\ \hline \bullet h_2: (\square\Gamma_{15}, \square\Gamma_{17}), \square\Gamma_{16}, \Delta_{18} \vdash ((\square\Gamma_{11}, \square\Gamma_{13}), (\square\Gamma_{12}, \Delta_{14}), []F_{10}), F_8 \\ \hline & -: (\square\Gamma_{15}, \square\Gamma_{17}), \square\Gamma_{16}, \Delta_{18} \vdash ((\square\Gamma_{11}, \square\Gamma_{12}), (\square\Gamma_{12}, \Delta_{14}), []F_{10} \\ \hline & -: (\square\Gamma_{15}, \square\Gamma_{17}), \square\Gamma_{16}, \Delta_{18} \vdash (\square\Gamma_{11}, \square\Gamma_{12}), (\square\Gamma_{12}, \Delta_{14}), []F_{10} \\ \hline & -: \square\Gamma_{15}, \square\Gamma_{16} \vdash F_{10}, \square\Gamma_{11}, \square\Gamma_{12} \\ \hline & -: \square\Gamma_{15}, \square\Gamma_{16}, \square\Gamma_{17} \vdash \Delta_{14}, \square\Gamma_{11}, \square\Gamma_{12}, \square\Gamma_{13}, []F_{10} \end{array} \quad A45 \\ \end{array}
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• Case rule  $\wedge_L$ 

$$\frac{\mathbf{h}_{1}: \Box \Gamma_{7} \vdash \Box \Gamma_{9}, F_{8}}{\bullet \mathbf{h}_{1}: \Box \Gamma_{7}, \Delta_{14}, F_{12} \land F_{13} \vdash (\Box \Gamma_{9}, \Delta_{10}), []_{F_{8}}} A_{45} \xrightarrow{\bullet \mathbf{h}_{11}: (\Box \Gamma_{7}, \Delta_{14}, F_{12} \land F_{13}), []_{F_{8}} \vdash \Box \Gamma_{9}, \Delta_{10}} \\ -: \Box \Gamma_{7}, \Delta_{14}, F_{12} \land F_{13} \vdash \Box \Gamma_{9}, \Delta_{10} \xrightarrow{\bullet} \\ \hline \frac{-: \Box \Gamma_{7}, \Delta_{14}, F_{12} \land F_{13} \vdash \Box \Gamma_{9}, \Delta_{10}}{\bullet \mathbf{h}_{1}: \Box \Gamma_{7} \vdash F_{8}, \Box \Gamma_{9}} \xrightarrow{\bullet \mathbf{x}/\mathsf{W}} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{14}, F_{12}, F_{13}, \Box \Gamma_{7} \vdash \Delta_{10}, \Box \Gamma_{9}, []_{F_{8}}} A_{45} \xrightarrow{\bullet} \\ \hline \frac{-: \Delta_{14}, F_{12}, F_{13}, \Box \Gamma_{7} \vdash \Delta_{10}, \Box \Gamma_{9}}{-: \Delta_{14}, \Box \Gamma_{7}, F_{12} \land F_{13} \vdash \Delta_{10}, \Box \Gamma_{9}} \land_{L} \\ \hline \bullet \mathbf{h}_{11}: \Delta_{14}, C_{17}, C_{1$$

• Case rule  $\vee_L$ 

$$\frac{h_1: \Box r_1 + \Box r_2 + \Box r_3}{h_1: \Box r_1 \wedge h_1 + r_2 \vee r_{13} + (\Box r_2 \wedge h_1) | F_8} A_{45} \begin{cases} h_1: \Box r_1 \wedge h_1 + r_2 \wedge h_1 | F_8 + \Box r_2 \wedge h_2 | F_8 \wedge h_2 | F_8 \wedge h_2 \wedge h_2 \wedge h_2 \wedge h_2 \wedge h_2 | F_8 \wedge h_2 \wedge h$$

 $\bullet$  Case rule AT

```
\frac{\mathbf{h}_{1}:\square\Gamma_{13}, []\mathbf{F}_{12}\vdash\square\Gamma_{9}, \mathbf{F}_{8}}{\bullet\mathbf{h}_{1}:(\square\Gamma_{13}, []\mathbf{F}_{12}), \underline{\Delta_{7}\vdash(\square\Gamma_{9}, \Delta_{10}), []\mathbf{F}_{8}}} \quad A45 \quad \frac{\mathbf{h}_{11}:\square\Gamma_{13}, \mathbf{F}_{12}, \Delta_{7}, []\mathbf{F}_{8}, []\mathbf{F}_{12}\vdash\square\Gamma_{9}, \Delta_{10}}{\bullet\mathbf{h}_{11}:((\square\Gamma_{13}, []\mathbf{F}_{12}), \Delta_{7}), []\mathbf{F}_{8}\vdash\square\Gamma_{9}, \Delta_{10}} \quad AT \quad Cut
                                                                                                                                                                                                                                                                             -: (\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_7 \vdash \Box\Gamma_9, \Delta_{10}
                  \underbrace{\bullet_{\mathbf{h}_1}:\Delta_7,\mathsf{F}_{12},\Box\Gamma_{13},[]\mathsf{F}_{12}\vdash\Delta_{10},\Box\Gamma_9,[]\mathsf{F}_8}_{\mathsf{h}_{\mathsf{Cut}}} \underbrace{\mathsf{ax/W}}^{\mathsf{w}} \frac{1}{\mathsf{h}_{11}:\Delta_7,\mathsf{F}_{12},\Box\Gamma_{13},[]\mathsf{F}_{12},[]\mathsf{F}_8\vdash\Delta_{10},\Box\Gamma_9}_{\mathsf{h}_{\mathsf{Cut}}} \underbrace{\mathsf{ax/W}}_{\mathsf{h}_{\mathsf{Cut}}}
                                                                                                                                                                                                                                                                  -: \Delta_7, \mathtt{F}_{12}, \Box \Gamma_{13}, []\mathtt{F}_{12} \vdash \Delta_{10}, \Box \Gamma_9
                                                                                                                                                                                                                                                                                     -:\Delta_7,\Box\Gamma_{13},[]\mathsf{F}_{12}\vdash\Delta_{10},\Box\Gamma_9
                               \frac{\mathbf{h}_1: \Box \Gamma_7 \vdash \Box \Gamma_9, \mathbf{f}_8}{\bullet \mathbf{h}_1: \Box \Gamma_7, \Delta_{13}, []\mathbf{f}_{12} \vdash (\Box \Gamma_9, \Delta_{10}), []\mathbf{f}_8} \quad A45 \quad \frac{\mathbf{h}_{11}: \Box \Gamma_7, \mathbf{f}_{12}, \Delta_{13}, []\mathbf{f}_8, []\mathbf{f}_{12} \vdash \Box \Gamma_9, \Delta_{10}}{\bullet \mathbf{h}_{11}: (\Box \Gamma_7, \Delta_{13}, []\mathbf{f}_{12}), []\mathbf{f}_8 \vdash \Box \Gamma_9, \Delta_{10}} \quad AT \quad \mathbf{f}_{11}: (\Box \Gamma_7, \Delta_{13}, []\mathbf{f}_{12}), (]\mathbf{f}_8 \vdash \Box \Gamma_9, \Delta_{10}
                                                                                                                                                                                                                                                                                   -:\Box\Gamma_7,\Delta_{13},[]F_{12}\vdash\Box\Gamma_9,\Delta_{10}
                 \frac{\bullet \mathbf{h}_1 : \Delta_{13}, \mathbf{F}_{12}, \Box \Gamma_7, []\mathbf{F}_{12} \vdash \Delta_{10}, \Box \Gamma_9, []\mathbf{F}_8}{-: \Delta_{13}, \mathbf{F}_{12}, \Box \Gamma_7, []\mathbf{F}_{12} \vdash \Delta_{10}, \Box \Gamma_9} \underbrace{-: \Delta_{13}, \mathbf{F}_{12}, \Box \Gamma_7, []\mathbf{F}_{12} \vdash \Delta_{10}, \Box \Gamma_9}_{ATG} ATG} ATG 
                                                                                                                                                                                                                                                                                         -:\Delta_{13},\Box\Gamma_{7},[]\mathsf{F}_{12}\vdash\Delta_{10},\Box\Gamma_{9}
                                                                                                                                                                                               \frac{\mathbf{h}_1:\Box\Gamma_7\vdash\Box\Gamma_9,\mathbf{F}_{12}}{\bullet\mathbf{h}_1:\Box\Gamma_7,\Delta_8\vdash(\Box\Gamma_9,\Delta_{10}),[]\mathbf{F}_{12}} \quad A45 \quad \frac{\mathbf{h}_{11}:\Box\Gamma_7,\mathbf{F}_{12},\Delta_8,[]\mathbf{F}_{12}\vdash\Box\Gamma_9,\Delta_{10}}{\bullet\mathbf{h}_{11}:(\Box\Gamma_7,\Delta_8),[]\mathbf{F}_{12}\vdash\Box\Gamma_9,\Delta_{10}} \quad AT \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                              -:\Box\Gamma_7,\Delta_8\vdash\Box\Gamma_9,\Delta_{10}
                  \frac{-:\Delta_8, \square\Gamma_7 \vdash \Delta_{10}, F_{12}, \square\Gamma_9}{-:\Delta_8, \square\Gamma_7 \vdash \Delta_{10}, F_{12}, \square\Gamma_9} \text{ ax/W } \frac{\begin{bmatrix}\bullet h_1 : \Delta_8, F_{12}, \square\Gamma_7 \vdash \Delta_{10}, \square\Gamma_9, []F_{12} & \text{ax/W} \\ & -:\Delta_8, F_{12}, \square\Gamma_7 \vdash \Delta_{10}, \square\Gamma_9 \\ & & \text{sCut}\end{bmatrix}}{-:\Delta_8, F_{12}, \square\Gamma_7 \vdash \Delta_{10}, \square\Gamma_9} \text{ acade in the second of t
                                                                                                                                                                                                                                                                                                               -: \Delta_8, \Box \Gamma_7 \vdash \Delta_{10}, \Box \Gamma_9
                      \frac{\mathbf{h}_{2}:\square\Gamma_{15},[]\mathbf{F}_{14}\vdash\square\Gamma_{10},\mathbf{F}_{12},\square\mathbf{F}_{9}}{\bullet\mathbf{h}_{2}:(\square\Gamma_{15},[]\mathbf{F}_{14}),\Delta_{8}\vdash(\square\Gamma_{10},\Delta_{11},[]\mathbf{F}_{12}),\square\mathbf{F}_{9}} \ A45 \ \frac{\mathbf{h}_{13}:\square\Gamma_{15},\mathbf{F}_{14},\Delta_{8},\square\mathbf{F}_{9},[]\mathbf{F}_{14}\vdash\square\Gamma_{10},\Delta_{11},[]\mathbf{F}_{12}}{\bullet\mathbf{h}_{13}:((\square\Gamma_{15},[]\mathbf{F}_{14}),\Delta_{8}),\square\mathbf{F}_{9}\vdash\square\Gamma_{10},\Delta_{11},[]\mathbf{F}_{12}} \ Cut
                                                                                      \mathtt{h}_2:\Box\Gamma_{15},[]\mathtt{F}_{14}\vdash\Box\Gamma_{10},\mathtt{F}_{12},\Box\mathtt{F}_{9}
                    \bullet_{h_2}:\Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Box\mathsf{F}_9, \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_2}:\Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{12} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_{14} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Delta_8, \mathsf{F}_{14}, \Box\Gamma_{10}, []\mathsf{F}_9, \Box\Gamma_{10}, []\mathsf{F}_{14} \\ \bullet_{h_{13}}: \Box\mathsf{F}_9, \Box\Gamma_{10}, []\mathsf{F}_{14}, \Box\Gamma_{10}, []\mathsf{F}_{14} \\ \bullet_{h_
                                                                                                                                                                                                                                                                                            -: \Delta_{8}, \mathsf{F}_{14}, \Box \Gamma_{15}, []\mathsf{F}_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, []\mathsf{F}_{12}  ATG
                                                                                                                                                                                                                                                                                                                       -: \Delta_8, \Box \Gamma_{15}, []F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, []F_{12}
                               \frac{\mathbf{h}_2: \Box \Gamma_8 \vdash \Box \Gamma_{10}, \mathbf{f}_{12}, \Box \mathbf{f}_9}{\bullet \mathbf{h}_2: \Box \Gamma_8, \Delta_{15}, \underline{[]} \mathbf{f}_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, \underline{[]} \mathbf{f}_{12}), \Box \mathbf{f}_9} \quad A45 \quad \frac{\mathbf{h}_{13}: \Box \Gamma_8, \mathbf{f}_{14}, \Delta_{15}, \Box \mathbf{f}_9, \underline{[]} \mathbf{f}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, \underline{[]} \mathbf{f}_{12}}{\bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_{15}, \underline{[]} \mathbf{f}_{14}), \Box \mathbf{f}_9 \vdash \Box \Gamma_{10}, \Delta_{11}, \underline{[]} \mathbf{f}_{12}} \quad AT \quad \mathbf{f}_{15} = \mathbf{f}_{15} \mathbf{f
                                                                                                                                                                                                                                                                                                                   -: \Box \Gamma_{8}, \Delta_{15}, []\mathtt{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathtt{F}_{12}
                  \frac{-:\Delta_{15},\mathsf{F}_{14},\Box\Gamma_{8},[]\mathsf{F}_{14}\vdash\Delta_{11},\Box\Gamma_{10},[]\mathsf{F}_{12}}{\Box} \quad ATG
                                                                                                                                                                                                                                                                                                                   -: \Delta_{15}, \Box \Gamma_8, []F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, []F_{12}
 \underbrace{ \begin{array}{c} \mathbf{h}_2: \Box \Gamma_8 \vdash \Box \Gamma_{10}, \mathbf{F}_{12}, [] \mathbf{F}_{14} \\ \bullet \mathbf{h}_2: \underline{\Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12}), [] \mathbf{F}_{14} \end{array}}_{\bullet \mathbf{h}_13: \Box \Gamma_8, \mathbf{F}_{14}, \Delta_9, [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{12} \\ \bullet \mathbf{h}_{13}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{12} \\ \bullet \mathbf{h}_{14}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{14} \\ \bullet \mathbf{h}_{14}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{14} \\ \bullet \mathbf{h}_{14}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{14} \\ \bullet \mathbf{h}_{14}: (\Box \Gamma_8, \Delta_9), [] \mathbf{h}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{h}_{14} \\ \bullet \mathbf{h}_{14}: (\Box \Gamma_8, \Delta_9)
                                                                                                                                                                                                                              -:\Box\Gamma_8,\Delta_9\vdash\Box\Gamma_{10},\Delta_{11},[]F_{12}
Axioms assumed:
inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula | -- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
               \underbrace{ \begin{array}{c} \mathbf{h}_2: \Box \Gamma_{15}, [[\mathbf{f}_{14} \vdash \Box \Gamma_{10}, \mathbf{f}_{12} \\ \hline \bullet \mathbf{h}_2: (\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9 \vdash (\Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12}), \mathbf{f}_8 \\ \end{array}) }_{A45} \underbrace{ \begin{array}{c} \mathbf{h}_{13}: \Box \Gamma_{15}, \mathbf{f}_8, \mathbf{f}_{14}, \Delta_9, [[\mathbf{f}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12} \\ \bullet \mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12} \\ \bullet \mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12} \\ \bullet \mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12} \\ \bullet \mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12} \\ \bullet \mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12} \\ \bullet \mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12} \\ \bullet \mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12} \\ \bullet \mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12}], \mathbf{f}_8) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12}], \mathbf{f}_8) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12}], \mathbf{f}_8) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12}], \mathbf{f}_8) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}), \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12}], \mathbf{f}_8) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}], \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12}], \mathbf{f}_8) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}], \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12}], \mathbf{f}_8) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}], \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_{11}, [[\mathbf{f}_{12}], \Delta_9) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}], \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_9) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}], \Delta_9), \mathbf{f}_8), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_9) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}], \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_9), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_9) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}], \Delta_9), \mathbf{f}_8), \mathbf{f}_8 \vdash \Box \Gamma_{10}, \Delta_9) \\ (\mathbf{h}_{13}: ((\Box \Gamma_{15}, [[\mathbf{f}_{14}], \Delta_9), \mathbf{f}_8), \Delta_9) \\ (\mathbf{h}_{13}: ((\Box \Gamma_
                                                                                                                                                                                                                                                                                   -: (\Box\Gamma_{15}, []F_{14}), \Delta_9 \vdash \Box\Gamma_{10}, \Delta_{11}, []F_{12}
                                                                                                                                                                                                                                                                                                                     \overline{-:\Box\Gamma_{15},[]\mathtt{F}_{14}\vdash \mathtt{F}_{12},\Box\Gamma_{10}} ax/W
                                                                                                                                                                                                                                                                                              \frac{107 \, \square \, 14 + 212, \square \, 210}{-: \Delta_9, \square \Gamma_{15}, []F_{14} \vdash \Delta_{11}, \square \Gamma_{10}, []F_{12}} A45
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$$\begin{array}{c} h_2: \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12} \\ \hline \bullet h_2: \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{10}, \Delta_{11}, []F_{12}), []F_{14} \\ \hline & \bullet h_3: (\Box \Gamma_8, \Delta_9), []F_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, []F_{12} \\ \hline & -: \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{10}, \Delta_{11}, []F_{12} \\ \hline & -: \Box \Gamma_8 \vdash F_{12}, \Box \Gamma_{10} \\ \hline & -: \Delta_9, \Box \Gamma_8 \vdash \Delta_{11}, \Box \Gamma_{10}, []F_{12} \\ \hline & \bullet h_3: (\Box \Gamma_8, \Delta_9), []F_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, []F_{12} \\ \hline & -: \Delta_9, \Box \Gamma_8 \vdash F_{12}, \Box \Gamma_{10} \\ \hline & \bullet h_2: \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12} \\ \hline & \bullet h_2: \Box \Gamma_8, \Delta_{15}, []F_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, []F_{12}), F_9 \\ \hline & -: \Box \Gamma_8, \Delta_{15}, []F_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, []F_{12}), F_9 \\ \hline & -: \Box \Gamma_8, \Delta_{15}, []F_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, []F_{12} \\ \hline & -: \Box \Gamma_8 \vdash F_{12}, \Box \Gamma_{10} \\ \hline & -: \Box \Gamma_8 \vdash F_{12}, \Box \Gamma_{10} \\ \hline & -: \Box \Gamma_8 \vdash F_{12}, \Box \Gamma_{10} \\ \hline & -: \Box \Gamma_8, []F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, []F_{12} \\ \hline & -: \Delta_{15}, \Box \Gamma_8, []F_{14} \vdash \Delta_{11}, \Box \Gamma_{10}, []F_{12} \\ \hline \end{array} \right. A45 \\ \hline$$

## • Case rule $\perp_L$

$$\begin{array}{c} \begin{array}{c} h_1: \ \Box \Gamma_7 \vdash \Box \Gamma_9, F_8 \\ \hline \bullet h_1: \ \Box \Gamma_7, \bot, \Delta_{12} \vdash (\Box \Gamma_9, \Delta_{10}), \ []F_8 \end{array} \end{array} \begin{array}{c} A45 \\ \hline \bullet h_{11}: \ \Box \Gamma_7, \bot, \Delta_{12} \vdash (\Box \Gamma_9, \Delta_{10}) \\ \hline -: \ \Box \Gamma_7, \bot, \Delta_{12} \vdash \Box \Gamma_9, \Delta_{10} \\ \hline \hline -: \bot, \Delta_{12}, \ \Box \Gamma_7 \vdash \Delta_{10}, \Box \Gamma_9 \end{array} \end{array} \begin{array}{c} \bot_L \\ \hline h_2: \ \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12}, \Box F_9 \\ \hline \bullet h_2: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, \ []F_{12}), \Box F_9 \end{array} \begin{array}{c} A45 \\ \hline \bullet h_{13}: \ (\Box \Gamma_8, \bot, \Delta_{14}), \ \Box F_9 \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline -: \bot, \Delta_{14}, \ \Box \Gamma_8 \vdash \Delta_{11}, \ \Box \Gamma_{10}, \ []F_{12} \end{array} \begin{array}{c} \bot_L \\ \hline Cut \end{array} \\ \hline \begin{array}{c} h_2: \ \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12} \\ \hline \bullet h_2: \ \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12} \end{array} \begin{array}{c} A45 \\ \hline \bullet h_{13}: \ (\Box \Gamma_8, \Delta_9), \bot \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline \bullet h_2: \ \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{10}, \Delta_{11}, \ []F_{12}), \bot \end{array} \begin{array}{c} \Delta_L \\ \hline \bullet h_{13}: \ (\Box \Gamma_8, \Delta_9), \bot \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline \bullet h_2: \ \Box \Gamma_8, \Delta_9 \vdash (\Box \Gamma_{10}, \Delta_{11}, \ []F_{12}), \bot \end{array} \begin{array}{c} \Delta_L \\ \hline -: \ \Box \Gamma_8, \Delta_9 \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline \bullet h_2: \ \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12} \end{array} \begin{array}{c} \Delta_L \\ \hline -: \ \Box \Gamma_8 \vdash F_{12}, \ \Box \Gamma_{10} \\ \hline \bullet h_{2}: \ \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12} \end{array} \begin{array}{c} \Delta_L \\ \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, \ []F_{12}), F_9 \end{array} \begin{array}{c} \Delta_L \\ \hline \bullet h_{13}: \ (\Box \Gamma_8, \bot, \Delta_{14}), F_9 \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \bullet h_{2}: \ \Box \Gamma_8 \vdash \Box \Gamma_{10}, F_{12} \end{array} \begin{array}{c} \Delta_L \\ \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \bullet h_{2}: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, \ []F_{12}), F_9 \end{array} \begin{array}{c} \Delta_L \\ \hline \bullet h_{13}: \ (\Box \Gamma_8, \bot, \Delta_{14}), F_9 \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline \bullet h_{2}: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline -: \ \Box \Gamma_8, \bot, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, \ []F_{12} \\ \hline \hline -: \ \Box$$

# $\bullet\,$ Case rule I

$$\begin{array}{c} \begin{array}{c} h_1: \square\Gamma_7 \vdash \square\Gamma_9, F_8 \\ \hline \bullet h_1: \square\Gamma_7, \Delta_{13}, p_{11} \vdash (\square\Gamma_9, \Delta_{12}, p_{11}), ||F_8| & A45 \\ \hline \bullet h_{10}: (\square\Gamma_7, \Delta_{13}, p_{11}), ||F_8| \vdash \square\Gamma_9, \Delta_{12}, p_{11} \\ \hline \\ -: \square\Gamma_7, \Delta_{13}, p_{11} \vdash \square\Gamma_9, \Delta_{12}, p_{11} \\ \hline \\ \hline -: \Delta_{13}, \square\Gamma_7, p_{11} \vdash \Delta_{12}, \square\Gamma_9, p_{11} \\ \hline \end{array} \begin{array}{c} I \\ \hline \\ h_2: \square\Gamma_8 \vdash \square\Gamma_{10}, F_{11}, \square F_9 \\ \hline \bullet h_2: \square\Gamma_8, \Delta_{15}, p_{13} \vdash (\square\Gamma_{10}, (\Delta_{14}, p_{13}), ||F_{11}), \square F_9 \\ \hline \\ \bullet h_2: \square\Gamma_8, \Delta_{15}, p_{13} \vdash (\square\Gamma_{10}, (\Delta_{14}, p_{13}), ||F_{11}), \square F_9 \\ \hline \\ -: \square\Gamma_8, \Delta_{15}, p_{13} \vdash \square\Gamma_{10}, (\Delta_{14}, p_{13}), ||F_{11} \\ \hline \\ \hline \\ \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{10}, F_{11} \\ \hline \\ \bullet h_2: \square\Gamma_8 \vdash \square\Gamma_{10}, F_{11} \\ \hline \\ \bullet h_2: \square\Gamma_8, \Delta_9 \vdash (\square\Gamma_{10}, (\Delta_{14}, p_{13}), ||F_{11}), p_{13} \\ \hline \\ \bullet h_{12}: (\square\Gamma_8, \Delta_9), p_{13} \vdash \square\Gamma_{10}, (\Delta_{14}, p_{13}), ||F_{11} \\ \hline \\ -: \square\Gamma_8, \Delta_9 \vdash \square\Gamma_{10}, (\Delta_{14}, p_{13}), ||F_{11} \\ \hline \\ -: \square\Gamma_8, \Delta_9 \vdash \square\Gamma_{10}, (\Delta_{14}, p_{13}), ||F_{11} \\ \hline \\ -: \square\Gamma_8 \vdash F_{11}, \square\Gamma_{10} \\ \hline \\ -: \square\Gamma_8 \vdash F_{11}, \square\Gamma_{10} \\ \hline \\ -: \square\Gamma_8 \vdash \Delta_{14}, \square\Gamma_{10}, p_{13}, ||F_{11} \\ \hline \end{array} \begin{array}{c} I \\ \text{Cut} \\ \hline \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_{2}: \Box \Gamma_{8} \vdash \Box \Gamma_{10}, F_{11} \\ \hline \bullet \mathbf{h}_{2}: \Box \Gamma_{8}, \Delta_{15}, \mathbf{p}_{13} \vdash (\Box \Gamma_{10}, (\Delta_{14}, \mathbf{p}_{13}), [] F_{11}), F_{9} \end{array} \begin{array}{c} A45 \\ \hline \bullet \mathbf{h}_{12}: (\Box \Gamma_{8}, \Delta_{15}, \mathbf{p}_{13}), F_{9} \vdash \Box \Gamma_{10}, (\Delta_{14}, \mathbf{p}_{13}), [] F_{11} \\ \hline -: \Box \Gamma_{8}, \Delta_{15}, \mathbf{p}_{13} \vdash \Box \Gamma_{10}, (\Delta_{14}, \mathbf{p}_{13}), [] F_{11} \\ \hline \hline -: \Delta_{15}, \Box \Gamma_{8}, \mathbf{p}_{13} \vdash \Delta_{14}, \Box \Gamma_{10}, \mathbf{p}_{13}, [] F_{11} \end{array} \begin{array}{c} I \\ \hline \end{array}$$

$$\begin{array}{c} h_1: \Box r_7 \vdash \Box r_9, r_8 \\ \hline \bullet h_1: \Box r_7, \top \Delta_{12} \vdash (\Box r_9, \Delta_{10}), []r_8 \\ \hline \bullet h_1: \Box r_7, \top \Delta_{12} \vdash (\Box r_9, \Delta_{10}), []r_8 \\ \hline -: \Box r_7, \top \Delta_{12} \vdash \Box r_9, \Delta_{10} \\ \hline \\ \bullet h_1: T, \Delta_{12}, \Box r_7 \vdash \Delta_{10}, \Box r_9, []r_8 \\ \hline \bullet h_1: T, \Delta_{12}, \Box r_7 \vdash \Delta_{10}, \Box r_9, []r_8 \\ \hline \\ \bullet h_1: T, \Delta_{12}, \Box r_7 \vdash \Delta_{10}, \Box r_9, []r_8 \\ \hline \\ \bullet h_2: \Box r_8 \vdash \Box r_{10}, r_{12}, \Box r_9 \\ \hline \\ \bullet h_2: \Box r_8, T, \Delta_{14} \vdash (\Box r_{10}, \Delta_{11}, []r_{12}), \Box r_9 \\ \hline \\ \bullet h_2: \Box r_8, T, \Delta_{14} \vdash (\Box r_{10}, \Delta_{11}, []r_{12}), \Box r_9 \\ \hline \\ \bullet h_2: T, \Delta_{14}, \Box r_8 \vdash \Box r_{10}, r_{12}, \Box r_9 \\ \hline \\ \bullet h_2: T, \Delta_{14}, \Box r_8 \vdash \Box r_{10}, r_{10}, []r_{12} \\ \hline \\ \bullet h_2: T, \Delta_{14}, \Box r_8 \vdash \Box r_{10}, \Delta_{11}, \Box r_{10}, []r_{12} \\ \hline \\ \bullet h_2: T, \Delta_{14}, \Box r_8 \vdash \Box r_{10}, \Delta_{11}, \Box r_{10}, []r_{12} \\ \hline \\ \bullet h_2: T, \Delta_{14}, \Box r_8 \vdash \Delta_{11}, \Box r_{10}, []r_{12} \\ \hline \\ \bullet h_2: T, \Delta_{14}, \Box r_8 \vdash \Delta_{11}, \Box r_{10}, []r_{12} \\ \hline \\ \bullet h_2: T, \Delta_{14}, \Box r_8 \vdash \Delta_{11}, \Box r_{10}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, \Delta_9 \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: T, \Box r_9, \Delta_{14}, \Box r_8 \vdash \Delta_{11}, \Box r_{10}, []r_{12} \\ \hline \\ \bullet h_{13}: T, \Box r_9, \Delta_{14}, \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: T, \Box r_9, \Delta_{14}, \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, \Delta_9 \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, \Delta_9 \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, \Delta_9 \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, \Delta_9 \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, \Delta_9 \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, \Delta_9 \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, \Delta_9 \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, \Delta_9 \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, r_9, \Delta_{14} \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet h_{13}: \Box r_8, r_9, \Delta_{14} \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet r_{13}: \Box r_8, r_9, \Delta_{14} \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet r_{13}: \Box r_8, r_9, \Delta_{14} \vdash \Box r_{10}, \Delta_{11}, []r_{12} \\ \hline \\ \bullet r_{13}: \Box r_{13}: \Box r_{13}: \Delta r_{13$$

## 6.8 Status of $\rightarrow_L$ : OK

• Case rule  $\rightarrow_R$ 

$$\frac{\mathbf{h}_{3}:\Delta_{8}\vdash \mathsf{F}_{7},\mathsf{F}_{9},\Delta_{12},\mathsf{F}_{13}\to\mathsf{F}_{14}\quad \mathsf{h}_{3}:\mathsf{F}_{10},\Delta_{8}\vdash \mathsf{F}_{7},\Delta_{12},\mathsf{F}_{13}\to\mathsf{F}_{14}}{\bullet_{\mathbf{h}_{3}}:\Delta_{8},\mathsf{F}_{9}\to\mathsf{F}_{10}\vdash (\Delta_{12},\mathsf{F}_{13}\to\mathsf{F}_{14}),\mathsf{F}_{7}}\to L \quad \frac{\mathsf{h}_{11}:\mathsf{F}_{7},\mathsf{F}_{13},\Delta_{8},\mathsf{F}_{9}\to\mathsf{F}_{10}\vdash\mathsf{F}_{14},\Delta_{12}}{\bullet_{\mathbf{h}_{11}}:(\Delta_{8},\mathsf{F}_{9}\to\mathsf{F}_{10}),\mathsf{F}_{7}\vdash \Delta_{12},\mathsf{F}_{13}\to\mathsf{F}_{14}} \\ & -:\Delta_{8},\mathsf{F}_{9}\to\mathsf{F}_{10}\vdash \Delta_{12},\mathsf{F}_{13}\to\mathsf{F}_{14} \\ \hline \bullet_{\mathbf{h}_{3}}:\Delta_{8},\mathsf{F}_{13}\vdash \Delta_{12},\mathsf{F}_{14},\mathsf{F}_{7}\to L \\ \hline \bullet_{\mathbf{h}_{3}}:\Delta_{8},\mathsf{F}_{13},\mathsf{F}_{9}\to\mathsf{F}_{10}\vdash \Delta_{12},\mathsf{F}_{14},\mathsf{F}_{7} \\ \hline \bullet_{\mathbf{h}_{3}}:\Delta_{8},\mathsf{F}_{13},\mathsf{F}_{9}\to\mathsf{F}_{10}\vdash \Delta_{12},\mathsf{F}_{14},\mathsf{F}_{7} \\ \hline \bullet_{\mathbf{h}_{3}}:\Delta_{8},\mathsf{F}_{13},\mathsf{F}_{9}\to\mathsf{F}_{10}\vdash \Delta_{12},\mathsf{F}_{14} \to R \\ \hline \bullet_{\mathbf{h}_{3}}:\Delta_{8},\mathsf{F}_{13},\mathsf{F}_{9}\to\mathsf{F}_{10}\vdash \Delta_{12},\mathsf{F}_{14} \\ \hline -:\Delta_{8},\mathsf{F}_{9}\to\mathsf{F}_{10}\vdash \Delta_{12},\mathsf{F}_{14} \\ \hline -:\Delta_{8},\mathsf{F}_{9}\to\mathsf{F}_{10} \\$$

• Case rule  $\wedge_R$ 

$$\frac{\frac{h_{3}:\Delta_{8}\vdash F_{7},F_{9},\Delta_{12},F_{13}\wedge F_{14}}{\bullet h_{3}:\Delta_{8},F_{9}\to F_{10}\vdash (\Delta_{12},F_{13}\wedge F_{14}),F_{7}}}{\bullet h_{3}:\Delta_{8},F_{9}\to F_{10}\vdash (\Delta_{12},F_{13}\wedge F_{14}),F_{7}}}\to L \xrightarrow{\frac{h_{11}:F_{7},\Delta_{8},F_{9}\to F_{10}\vdash F_{13},\Delta_{8}}{\bullet h_{11}:(\Delta_{8},F_{9}\to F_{10}\vdash \Delta_{12},F_{13}\wedge F_{14}),F_{7}}}}{\vdots \text{inv-th/ax}} \xrightarrow{\frac{\bullet h_{3}:\Delta_{8}\vdash A_{12},F_{13},F_{7}}{\bullet h_{3}:\Delta_{8},F_{9}\to F_{10}\vdash \Delta_{12},F_{13},F_{7}}}} \xrightarrow{\frac{\text{inv-th/ax}}{h_{11}:\Delta_{8},F_{7},F_{9}\to F_{10}\vdash \Delta_{12},F_{13}}}} \xrightarrow{\frac{\bullet x/W}{hCut}} \xrightarrow{\frac{\bullet h_{3}:\Delta_{8}\vdash \Delta_{12},F_{14},F_{7},F_{9}\to F_{10}\vdash \Delta_{12},F_{13}}{\bullet h_{3}:\Delta_{8},F_{9}\to F_{10}\vdash \Delta_{12},F_{13}\wedge F_{14}}}} \xrightarrow{\frac{\bullet h_{3}:\Delta_{8}\vdash F_{10}\vdash \Delta_{12},F_{13}\wedge F_{14}}{\bullet h_{3}:\Delta_{8},F_{9}\to F_{10}\vdash \Delta_{12},F_{13}\wedge F_{14}}}} \xrightarrow{\frac{\bullet h_{3}:\Delta_{8}\vdash F_{10}\vdash A_{12},F_{13}\wedge F_{14}}{\bullet h_{3}:\Delta_{8}\vdash F_{9}\to F_{10}\vdash \Delta_{12},F_{13}\wedge F_{14}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},\Delta_{8},F_{9}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}{\bullet h_{11}:A_{8},F_{9}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},\Delta_{8},F_{9}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}{\bullet h_{11}:A_{8},F_{9}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},\Delta_{8},F_{9}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}{\bullet h_{11}:A_{8},F_{7}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},\Delta_{8},F_{9}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}{\bullet h_{11}:A_{8},F_{7}\to A_{11},F_{12},F_{13}\wedge F_{14}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},\Delta_{8},F_{9}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}{\bullet h_{11}:A_{8},F_{7}\to A_{12},F_{13}\wedge F_{14}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},\Delta_{8},F_{9}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}{\bullet h_{11}:A_{8},F_{7}\to A_{12},F_{13}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},\Delta_{8},F_{9}\to F_{10}\vdash A_{12},F_{13}\wedge F_{14}}{\bullet h_{11}:A_{8},F_{7}\to A_{12},F_{13},F_{14}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},A_{11},F_{7}\to A_{12},F_{13}\wedge F_{14}}}{\bullet h_{11}:A_{11}:A_{11},A_{12},F_{13}\to A_{14}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},A_{11},F_{7}\to A_{12},F_{13},A_{14}}}{\bullet h_{11}:A_{11}:A_{11},A_{12}\to A_{12},F_{13}\to A_{14}}}} \xrightarrow{\frac{\bullet h_{11}:F_{7},A_{11},A_{11},A_{11},A_{11},A_{12},A$$

• Case rule  $\vee_R$ 

$$\frac{\frac{\mathbf{h}_{3}:\Delta_{8}\vdash \mathbf{F}_{7},\mathbf{F}_{9},\Delta_{12},\mathbf{F}_{13}\vee \mathbf{F}_{14}}{\bullet \mathbf{h}_{3}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash (\Delta_{12},\mathbf{F}_{13}\vee \mathbf{F}_{14}),\mathbf{F}_{7}}} \xrightarrow{\mathbf{h}_{11}:\mathbf{F}_{7},\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \mathbf{F}_{13},\mathbf{F}_{14},\Delta_{12}}} \underbrace{\begin{array}{c} \vee_{R} \\ \vee_{R} \\ \bullet \mathbf{h}_{11}:(\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \mathbf{F}_{13},\mathbf{F}_{14},\mathbf{F}_{14},\mathbf{F}_{14}) \\ \bullet \mathbf{h}_{11}:(\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \mathbf{F}_{12},\mathbf{F}_{13}\vee \mathbf{F}_{14}) \\ \bullet \mathbf{h}_{11}:(\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}),\mathbf{F}_{7}\vdash \Delta_{12},\mathbf{F}_{13}\vee \mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:(\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}),\mathbf{F}_{7}\vdash \Delta_{12},\mathbf{F}_{13}\vee \mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}),\mathbf{F}_{7}\vdash \Delta_{12},\mathbf{F}_{13}\vee \mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10},\mathbf{F}_{12},\mathbf{F}_{13}\vee \mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{7},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{7},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{7},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{7},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{7},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{7},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{7},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{7},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{7},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{h}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{h}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{F}_{9}\to \mathbf{h}_{10}\vdash \Delta_{12},\mathbf{F}_{13},\mathbf{F}_{14} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{h}_{11}\to \mathbf{h}_{12},\mathbf{h}_{12}\to \mathbf{h}_{12},\mathbf{h}_{12}\to \mathbf{h}_{12},\mathbf{h}_{13} \\ \bullet \mathbf{h}_{11}:\Delta_{8},\mathbf{h}_{12}\to \mathbf{h}_{12},\mathbf{h}_{13}\to \mathbf{h}_{13} \\ \bullet \mathbf{h}_$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_3:\Delta_8 \vdash \mathbf{F}_7, \mathbf{F}_9, \bot, \Delta_{12} \quad \mathbf{h}_3: \mathbf{F}_{10}, \Delta_8 \vdash \mathbf{F}_7, \bot, \Delta_{12}}{\bullet \mathbf{h}_3:\Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \vdash (\bot, \Delta_{12}), \mathbf{F}_7} \xrightarrow{\bullet} \underbrace{\frac{\mathbf{h}_{11}: \mathbf{F}_7, \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \vdash \Delta_{12}}{\bullet \mathbf{h}_{11}: (\Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10}), \mathbf{F}_7 \vdash \bot, \Delta_{12}}}_{\bullet \mathbf{h}_3:\Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \vdash \bot, \Delta_{12}, \mathbf{F}_7}} \xrightarrow{\mathbf{ax/W}} \underbrace{\frac{\mathbf{h}_{11}:\Delta_8, \mathbf{F}_7, \mathbf{F}_9 \to \mathbf{F}_{10} \vdash \bot, \Delta_{12}}{\bullet \mathbf{h}_{11}:\Delta_8, \mathbf{F}_7, \mathbf{F}_9 \to \mathbf{F}_{10} \vdash \bot, \Delta_{12}}}_{\bullet \mathbf{h}Cut}}_{\bullet \mathbf{h}Cut}$$

• Case rule  $\top_R$ 

$$\frac{\mathbf{h}_3:\Delta_8 \vdash \mathbf{F}_7, \mathbf{F}_9, \top, \Delta_{12} \quad \mathbf{h}_3: \mathbf{F}_{10}, \Delta_8 \vdash \mathbf{F}_7, \top, \Delta_{12}}{\underbrace{\bullet \mathbf{h}_3:\Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \vdash (\top, \Delta_{12}), \mathbf{F}_7}_{} \quad \rightarrow_L \quad \underbrace{\bullet \mathbf{h}_{11}:(\Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10}), \mathbf{F}_7 \vdash \top, \Delta_{12}}_{} \quad \leftarrow_{\mathbf{Cut}}$$

 $\bullet$  Case rule K

$$\frac{ \begin{array}{c} \underline{\mathbf{h}}_3: \Box \Gamma_{13}, \Delta_{14} \vdash \Box \mathbf{F}_7, \mathbf{F}_8, \Delta_{11}, []\mathbf{F}_{12} \quad \mathbf{h}_3: \mathbf{F}_9, \Box \Gamma_{13}, \Delta_{14} \vdash \Box \mathbf{F}_7, \Delta_{11}, []\mathbf{F}_{12} \\ \underline{\bullet}\mathbf{h}_3: (\Box \Gamma_{13}, \Delta_{14}), \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash (\Delta_{11}, []\mathbf{F}_{12}), \Box \mathbf{F}_7 \\ \underline{\bullet}\mathbf{h}_3: (\Box \Gamma_{13}, \Delta_{14}), \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash (\Delta_{11}, []\mathbf{F}_{12}), \Box \mathbf{F}_7 \\ \underline{\bullet}\mathbf{h}_{10}: (\Box \Gamma_{13}, \Delta_{14}), \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash \Delta_{11}, []\mathbf{F}_{12} \\ \underline{\bullet}\mathbf{h}_{10}: (\Box \Gamma_{13}, \Delta_{14}), \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash \Delta_{11}, []\mathbf{F}_{12} \\ \underline{\bullet}\mathbf{h}_{10}: (\Box \Gamma_{13}, \Delta_{14}), \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash \Delta_{11}, []\mathbf{F}_{12} \\ \underline{\bullet}\mathbf{h}_{10}: \mathbf{unbox}(\Box \mathbf{F}_7), \mathbf{unbox}(\Box \Gamma_{13}) \vdash \mathbf{F}_{12} \\ \underline{\bullet}\mathbf{h}_{10}: \Box \mathbf{h}_{13}: \Delta_{11}, \mathbf{F}_8, []\mathbf{F}_{12} \\ \underline{\bullet}\mathbf{h}_{10}: \Box \mathbf{h}_{10}: \Box \mathbf{h}_{13}: \Delta_{11}, \mathbf{F}_8, []\mathbf{F}_{12} \\ \underline{\bullet}\mathbf{h}_{10}: \Box \mathbf{h}_{13}: \Delta_{11}, \mathbf{h}_{13}: \mathbf{h}_{1$$

• Case rule A45

$$\frac{h_3: \Box\Gamma_{14}, \Delta_{15} \vdash \Box F_7, F_8, \Box\Gamma_{11}, \Delta_{12}, []F_{13} \quad h_3: F_9, \Box\Gamma_{14}, \Delta_{15} \vdash \Box F_7, \Box\Gamma_{11}, \Delta_{12}, []F_{13}}{\bullet h_3: (\Box\Gamma_{14}, \Delta_{15}), F_8 \rightarrow F_9 \vdash (\Box\Gamma_{11}, \Delta_{12}, []F_{13}), \Box F_7} \rightarrow_L \frac{h_{10}: \Box\Gamma_{14}, \Box F_7 \vdash \Box F_7, \Box\Gamma_{14}, \Box F_7 \vdash \Box F_7, \Box\Gamma_{14}, \Delta_{15}), F_8 \rightarrow F_9 \vdash \Box\Gamma_{11}, \Delta_{12}, []F_{13}}{\bullet h_{10}: (\Box\Gamma_{14}, \Delta_{15}), F_8 \rightarrow F_9 \vdash \Box\Gamma_{11}, \Delta_{12}, []F_{13}} \xrightarrow{\bullet \bullet} \frac{h_{10}: (\Box\Gamma_{14}, \Delta_{15}), F_8 \rightarrow F_9 \vdash \Box\Gamma_{11}, \Delta_{12}, []F_{13}}{\bullet h_{10}: \Box F_7, \Box\Gamma_{14} \vdash F_{13}, \Box\Gamma_{11}} \xrightarrow{\bullet \bullet} \frac{A45}{h_{Cut}} \xrightarrow{h_3: \Delta_{15}, F_9, \Box\Gamma_{14} \vdash \Box F_7, \Delta_{12}, \Box\Gamma_{11}, []F_{13}} \xrightarrow{\bullet} \frac{h_{10}: \Box\Gamma_{14}, \Box\Gamma_{$$

$$\frac{\mathbf{h}_{3}: \Box\Gamma_{11}, \Delta_{15} \vdash \mathbf{F}_{7}, \mathbf{F}_{8}, \Box\Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14} \quad \mathbf{h}_{3}: \mathbf{F}_{9}, \Box\Gamma_{11}, \Delta_{15} \vdash \mathbf{F}_{7}, \Box\Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14}}{\bullet \mathbf{h}_{3}: (\Box\Gamma_{11}, \Delta_{15}), \mathbf{F}_{8} \to \mathbf{F}_{9} \vdash (\Box\Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14}), \mathbf{F}_{7}} \xrightarrow{\bullet \mathbf{h}_{10}: ((\Box\Gamma_{11}, \Delta_{15}), \mathbf{F}_{8} \to \mathbf{F}_{9}), \mathbf{F}_{7} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14}} \xrightarrow{-: (\Box\Gamma_{11}, \Delta_{15}), \mathbf{F}_{8} \to \mathbf{F}_{9} \vdash \Box\Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14}} \xrightarrow{\bullet \mathbf{x}/\mathbf{W}} \xrightarrow{-: \Box\Gamma_{11} \vdash \mathbf{F}_{14}, \Box\Gamma_{12}} \underbrace{\mathbf{ax}/\mathbf{W}}_{-: \Delta_{15}, \Box\Gamma_{11}, \mathbf{F}_{8} \to \mathbf{F}_{9} \vdash \Delta_{13}, \Box\Gamma_{12}, []\mathbf{F}_{14}}^{\mathbf{A}45}$$

$$\frac{\mathbf{h}_{3} : \Delta_{7} \vdash F_{11} \to F_{12}, F_{8}, \Delta_{13}}{\bullet \mathbf{h}_{3} : \Delta_{7}, F_{8} \to F_{9} \vdash \Delta_{13}, F_{11} \to F_{12}} \to L \underbrace{\frac{\mathbf{h}_{10} : \Delta_{7}, F_{8} \to F_{9} \vdash F_{11}, \Delta_{13}}{\bullet \mathbf{h}_{10} : (\Delta_{7}, F_{8} \to F_{9} \vdash \Delta_{13})}_{\text{Cut}} \to L \underbrace{\frac{-: \Delta_{7}, F_{8} \to F_{9} \vdash \Delta_{13}}{\bullet \mathbf{h}_{10} : (\Delta_{7}, F_{8} \to F_{9}), F_{11} \to F_{12} \vdash \Delta_{13}}_{\text{Cut}}}_{-: \Delta_{7}, F_{11} \vdash \Delta_{13}, F_{12}, F_{8}} \underbrace{\frac{-: \Delta_{7}, F_{11} \vdash A_{13}, F_{12}, F_{8}}{\bullet \mathbf{h}_{10} : \Delta_{7}, F_{11}, F_{9} \vdash \Delta_{13}, F_{12}}}_{-: \Delta_{7}, F_{11}, F_{11}, F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}, F_{12}}} \underbrace{\frac{-: \Delta_{7}, F_{11}, F_{9} \vdash \Delta_{13}, F_{12}}{\bullet \mathbf{h}_{3} : \Delta_{14}, F_{11} \to F_{12} \vdash F_{7}, F_{8}, \Delta_{13}}_{-: \Delta_{7}, F_{11}, F_{8} \to F_{9} \vdash \Delta_{13}} \underbrace{\frac{-: \Delta_{7}, F_{11}, F_{9} \vdash \Delta_{13}, F_{12}}{\bullet \mathbf{h}_{3} : \Delta_{14}, F_{11} \to F_{12} \vdash F_{7}, F_{8}, \Delta_{13}}_{-: \Delta_{7}, F_{11}, F_{8} \to F_{9} \vdash \Delta_{13}} \underbrace{\frac{-: \Delta_{7}, F_{11}, F_{9} \vdash \Delta_{13}, F_{12}}{\bullet \mathbf{h}_{3} : \Delta_{14}, F_{11} \to F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}, F_{11}}}_{-: \Delta_{14}, F_{11} \to F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}, F_{11}}} \underbrace{\frac{-: \Delta_{7}, F_{11}, F_{9} \vdash \Delta_{13}, F_{11} \vdash F_{7}, F_{8} \vdash \Delta_{13}, F_{11} \vdash \Delta_{12}}{\bullet \mathbf{h}_{10} : \Delta_{14}, F_{11} \to F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}}}}_{-: \Delta_{7}, F_{11}, F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}}}_{-: \Delta_{7}, F_{11}, F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}}} \underbrace{\frac{-: \Delta_{7}, F_{11}, F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}}{\bullet \mathbf{h}_{13} : \Delta_{14}, F_{11} \to F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}, F_{11}}}_{-: \Delta_{14}, F_{11} \to F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}, F_{11}}}_{-: \Delta_{14}, F_{11} \to F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}, F_{11}}}} \underbrace{\frac{\mathbf{h}_{10} : A_{14}, F_{11} \to F_{12} \vdash A_{14}, F_{11} \to F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}}_{\bullet \mathbf{h}_{13} : \Delta_{14}, F_{11} \to F_{12}, F_{8} \to F_{9} \vdash \Delta_{13}, F_{11}}}_{\bullet \mathbf{h}_{10} : \Delta_{14}, F_{11} \to F_{12}, F_{13}, F_{14}}}_{\bullet \mathbf{h}_{10} : \Delta_{14}, F_{11} \to \Delta_{12}, F_{11}}}_{\bullet \mathbf{h}_{10} : \Delta_{14}, F_{11} \to A_{12}, F_{11}}}_{\bullet \mathbf{h}_{11} : \Delta_{12}, F_{11} \to \Delta_{12}, F_{11}}}_{\bullet \mathbf{h}_{11} : \Delta_{12}, F_{11} \to \Delta_{12}, F_{11}}}_{\bullet \mathbf{h}_{11} : \Delta_{12}, F_{11} \to \Delta_{12}, F_{11}}}_{\bullet \mathbf{h}_{11} : \Delta_{12}, F_{11} \to \Delta_{1$$

• Case rule  $\wedge_L$ 

$$\frac{h_3: \Delta_7 \vdash F_{11} \land F_{12}, F_8, \Delta_{13} \quad h_3: F_9, \Delta_7 \vdash F_{11} \land F_{12}, \Delta_{13}}{\bullet h_3: \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \land F_{12}} \rightarrow_L \quad \frac{h_{10}: F_{11}, F_{12}, \Delta_7, F_8 \to F_9 \vdash \Delta_{13}}{\bullet h_{10}: (\Delta_7, F_8 \to F_9), F_{11} \land F_{12} \vdash \Delta_{13}} \\ -: \Delta_7, F_8 \to F_9 \vdash \Delta_{13}$$

$$\frac{h_3: \Delta_7 \vdash \Delta_{13}, F_8, F_{11} \land F_{12}}{\bullet h_{10}: \Delta_7, F_{11}, F_{12} \vdash \Delta_{13}, F_8} \land_L \\ -: \Delta_7, F_{11} \land F_{12} \vdash \Delta_{13}, F_8} \land_L \\ -: \Delta_7, F_8 \to F_9 \vdash \Delta_{13}$$

$$\frac{h_3: \Delta_7 \vdash \Delta_{13}, F_8, F_{11} \land F_{12}}{\bullet h_{10}: \Delta_7, F_{11} \land F_{12} \vdash \Delta_{13}, F_8} \rightarrow_L \\ -: \Delta_7, F_8 \to F_9 \vdash \Delta_{13}$$

$$-: \Delta_7, F_9 \vdash \Delta_{13} \rightarrow_L \\ -: \Delta_7, F_9 \vdash \Delta_{13} \rightarrow_L$$

$$\frac{h_3: \Delta_{14}, F_{11} \land F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3: F_9, \Delta_{14}, F_{11} \land F_{12} \vdash F_7, \Delta_{13}}{\bullet h_{10}: (\Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13})} \rightarrow_L$$

$$\frac{h_3: \Delta_{14}, F_{11} \land F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3: F_9, \Delta_{14}, F_{11} \land F_{12} \vdash F_7, \Delta_{13}}{\bullet h_{10}: (\Delta_{14}, F_{11}, F_{12}, F_8 \to F_9), F_7 \vdash \Delta_{13}} \land_L$$

$$\frac{\bullet h_3: \Delta_{14}, F_{11}, F_{12} \vdash \Delta_{13}, F_7, F_8} \quad \text{inv-th/ax}}{\bullet h_{10}: \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_7}} \rightarrow_L$$

$$\frac{\bullet h_3: \Delta_{14}, F_{11}, F_{12} \vdash F_7, F_8, \Delta_{13} \quad h_3: F_9, \Delta_{14}, F_{11}, F_{12}, F_9 \vdash \Delta_{13}, F_7}}{\bullet h_{10}: (\Delta_{14}, F_{11}, F_{12}, F_8 \to F_9), F_7 \vdash \Delta_{13}}} \land_L$$

$$\frac{\bullet h_3: \Delta_{14}, F_{11}, F_{12} \vdash F_7, F_8 \to F_9 \vdash \Delta_{13}, F_7}}{\bullet h_{10}: \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}}} \land_L$$

$$\frac{\bullet h_3: \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_7}}{\bullet h_{10}: \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}}} \land_L$$

$$\frac{\bullet h_3: \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_7}}{\bullet h_{10}: \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}}} \land_L$$

$$\frac{\bullet h_3: \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_7}}{\bullet h_{10}: \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}}} \land_L$$

• Case rule  $\vee_L$ 

## $\bullet\,$ Case rule AT

$$\frac{\frac{\mathbf{h}_{3}:\Delta_{7} \vdash []\mathsf{F}_{11},\mathsf{F}_{8},\Delta_{12} \quad \mathbf{h}_{3}:\mathsf{F}_{9},\Delta_{7} \vdash []\mathsf{F}_{11},\Delta_{12}}{\bullet \mathbf{h}_{3}:\Delta_{7},\mathsf{F}_{8} \to \mathsf{F}_{9} \vdash \Delta_{12},[]\mathsf{F}_{11}}} \rightarrow_{L} \frac{\mathbf{h}_{10}:\mathsf{F}_{11},\Delta_{7},[]\mathsf{F}_{11},\mathsf{F}_{8} \to \mathsf{F}_{9} \vdash \Delta_{12}}{\bullet \mathbf{h}_{10}:(\Delta_{7},\mathsf{F}_{8} \to \mathsf{F}_{9}),[]\mathsf{F}_{11} \vdash \Delta_{12}}} AT \\ -:\Delta_{7},\mathsf{F}_{8} \to \mathsf{F}_{9} \vdash \Delta_{12}$$

$$\frac{\mathbf{h}_{3}:\Delta_{7} \vdash \Delta_{12},\mathsf{F}_{8},[]\mathsf{F}_{11}}{\bullet \mathbf{h}_{10}:\Delta_{7},\mathsf{F}_{11},[]\mathsf{F}_{11} \vdash \Delta_{12},\mathsf{F}_{8}} \underbrace{AT}_{\bullet \mathsf{h}\mathsf{Cut}} \underbrace{\frac{\mathbf{h}_{10}:\Delta_{7},\mathsf{F}_{11},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}{\bullet \mathbf{h}_{10}:\Delta_{7},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}}_{\bullet \mathsf{h}\mathsf{Cut}} \underbrace{\frac{\mathbf{h}_{10}:\Delta_{7},\mathsf{F}_{11},[]\mathsf{F}_{11}}_{\bullet \mathsf{h}\mathsf{Cut}} + \underbrace{\frac{\mathbf{h}_{10}:\Delta_{7},\mathsf{F}_{9} \vdash \Delta_{12}}{\bullet \mathbf{h}_{10}:\Delta_{7},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}}_{\bullet \mathsf{h}\mathsf{Cut}} \underbrace{\frac{\mathsf{h}_{10}:\Delta_{7},\mathsf{F}_{9} \vdash \Delta_{12}}{\bullet \mathbf{h}_{10}:\Delta_{7},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}}_{\bullet \mathsf{h}\mathsf{Cut}} \underbrace{\frac{\mathsf{h}_{10}:\Delta_{7},\mathsf{F}_{9} \vdash \Delta_{12}}{\bullet \mathbf{h}_{10}:\Delta_{7},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}}_{\bullet \mathsf{h}\mathsf{Cut}} \underbrace{\frac{\mathsf{h}_{10}:\Delta_{7},\mathsf{F}_{11},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}{\bullet \mathsf{h}_{10}:\Delta_{7},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}}}_{\bullet \mathsf{h}\mathsf{Cut}} \underbrace{\frac{\mathsf{h}_{10}:\Delta_{7},\mathsf{F}_{9} \vdash \Delta_{12}}{\bullet \mathsf{h}_{10}:\Delta_{7},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}}_{\bullet \mathsf{h}\mathsf{Cut}} \underbrace{\frac{\mathsf{h}_{10}:\Delta_{7},\mathsf{F}_{11},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}{\bullet \mathsf{h}_{10}:\Delta_{7},\mathsf{F}_{9},[]\mathsf{F}_{11} \vdash \Delta_{12}}}_{\bullet \mathsf{h}\mathsf{Cut}} \underbrace{\frac{\mathsf{h}_{10}:\Delta_{7},\mathsf{F}_{9} \vdash \Delta_{12}}_{\bullet \mathsf{h}\mathsf{L}}}_{\bullet \mathsf{h}\mathsf{L}\mathsf{L}}}_{\bullet \mathsf{L}}$$

## • Case rule $\perp_L$

$$\frac{\frac{h_3:\Delta_7\vdash\bot,F_8,\Delta_{11}\quad h_3:F_9,\Delta_7\vdash\bot,\Delta_{11}}{\bullet h_3:\Delta_7,F_8\to F_9\vdash\Delta_{11},\bot}}{-:\Delta_7,F_8\to F_9\vdash\Delta_{11}} \to_L \frac{h_3:\Delta_7\vdash\bot,\Delta_{11}\quad h_3:F_9,\Delta_7\vdash\bot,\Delta_{11}}{\bullet h_{10}:(\Delta_7,F_8\to F_9),\bot\vdash\Delta_{11}} \xrightarrow{L_L} \frac{\bot_L}{\cot}$$

$$\frac{h_3:\Delta_7\vdash\bot,\Delta_{11},F_8}{-:\Delta_7\vdash\Delta_{11},F_8} \xrightarrow{\bullet h_{10}:\bot,\Delta_7\vdash\Delta_{11},F_8} \frac{\bot_L}{h_{Cut}} \xrightarrow{h_3:\Delta_7,F_9\vdash\bot,\Delta_{11}} \xrightarrow{\bullet k_1} \frac{ax/W}{\bullet h_{10}:\bot,\Delta_7,F_9\vdash\Delta_{11}} \to_L$$

$$\frac{-:\Delta_7\vdash\Delta_{11},F_8}{-:\Delta_7,F_8\to F_9\vdash\Delta_{11}} \to_L$$

$$\frac{h_3:\bot,\Delta_{12}\vdash F_7,F_8,\Delta_{11}\quad h_3:F_9,\bot,\Delta_{12}\vdash F_7,\Delta_{11}}{-:(\bot,\Delta_{12}),F_8\to F_9\vdash\Delta_{11}} \to_L$$

$$\frac{\bullet h_3:(\bot,\Delta_{12}),F_8\to F_9\vdash\Delta_{11},F_7}{-:(\bot,\Delta_{12}),F_8\to F_9\vdash\Delta_{11}} \xrightarrow{L_L} \xrightarrow{\bullet h_{10}:((\bot,\Delta_{12}),F_8\to F_9),F_7\vdash\Delta_{11}} \xrightarrow{L_L} \xrightarrow{\bullet h_{10}:((\bot,\Delta_{12}),F_8\to F_9\vdash\Delta_{11}} \xrightarrow{L_L} \xrightarrow{\bullet h_{10}:((\bot,\Delta_{12}),F_8\to F_9),F_7\vdash\Delta_{11}} \xrightarrow{L_L} \xrightarrow{\bullet h_{10}:(\bot,\Delta_{12}),F_8\to F_9\vdash\Delta_{11}} \xrightarrow{\bullet h_{10}:(\bot,\Delta_{12}),F_8\to F_9\to\Delta_{11}} \xrightarrow{\to h_{10}:(\bot,\Delta_{12}),F_8\to F_9\to\Delta_{11}} \xrightarrow{\to h_{10}:(\bot,\Delta_{12}),$$

#### ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_3:\Delta_7\vdash \top, \mathbf{F}_8, \Delta_{11}\quad \mathbf{h}_3:\mathbf{F}_9, \Delta_7\vdash \top, \Delta_{11}}{\bullet \mathbf{h}_3:\Delta_7, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}, \top} \to_L & \frac{\mathbf{h}_{10}:\Delta_7, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}}{\bullet \mathbf{h}_{10}:(\Delta_7, \mathbf{F}_8\to \mathbf{F}_9), \top\vdash \Delta_{11}} & \top_L \\ & -:\Delta_7, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11} & \text{cut} \\ \hline & -:\Delta_7, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11} & \text{ax/W} \\ \hline & -:\Delta_7, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11} & \text{ax/W} \\ \hline & \frac{\mathbf{h}_3:\top,\Delta_{12}\vdash \mathbf{F}_7, \mathbf{F}_8, \Delta_{11}\quad \mathbf{h}_3:\mathbf{F}_9, \top, \Delta_{12}\vdash \mathbf{F}_7, \Delta_{11}}{\bullet \mathbf{h}_3:(\top,\Delta_{12}), \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}} \to_L & \frac{\mathbf{h}_{10}:\mathbf{F}_7,\Delta_{12}, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}}{\bullet \mathbf{h}_{10}:((\top,\Delta_{12}), \mathbf{F}_8\to \mathbf{F}_9), \mathbf{F}_7\vdash \Delta_{11}} & \top_L \\ \hline & -:(\top,\Delta_{12}), \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11} & \text{cut} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12}, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}, \mathbf{F}_7 & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{10}:\top,\Delta_{12}, \mathbf{F}_7, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}} & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{\text{Cut}}} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12}, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}, \mathbf{F}_7 & \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_{10}:\top,\Delta_{12}, \mathbf{F}_7, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}} & \mathbf{Ax/W} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12}, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}, \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12}, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}, \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12}, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}, \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12}, \mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}, \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12},\mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}, \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12},\mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11}, \mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12},\mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11},\mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12},\mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11},\mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_3:\top,\Delta_{12},\mathbf{F}_8\to \mathbf{F}_9\vdash \Delta_{11},\mathbf{F}_7 & \mathbf{ax/W} \\ \hline & \bullet \mathbf{h}_3:$$

# 6.9 Status of $\wedge_L$ : OK

• Case rule  $\rightarrow_R$ 

$$\frac{\begin{array}{l} h_3: F_9, F_{10}, \Delta_8 \vdash F_7, \Delta_{12}, F_{13} \to F_{14} \\ \bullet h_3: \Delta_8, F_9 \land F_{10} \vdash (\Delta_{12}, F_{13} \to F_{14}), F_7 \end{array} \land_L \begin{array}{l} h_{11}: F_7, F_{13}, \Delta_8, F_9 \land F_{10} \vdash F_{14}, \Delta_{12} \\ \bullet h_{11}: (\Delta_8, F_9 \land F_{10}), F_7 \vdash \Delta_{12}, F_{13} \to F_{14} \end{array} \xrightarrow{\bullet}_{Cut} \\ \hline \\ (Cut) \\ \hline \begin{array}{l} h_3: \Delta_8, F_9 \land F_{10} \vdash (\Delta_{12}, F_{14}, F_7) \\ \hline \\ \bullet h_3: \Delta_8, F_{10}, F_{13}, F_9 \vdash \Delta_{12}, F_{14}, F_7 \\ \hline \\ \bullet h_3: \Delta_8, F_{13}, F_9 \land F_{10} \vdash \Delta_{12}, F_{14}, F_7 \\ \hline \\ -: \Delta_8, F_{13}, F_9 \land F_{10} \vdash \Delta_{12}, F_{14} \\ \hline \\ -: \Delta_8, F_9 \land F_{10} \vdash \Delta_{12}, F_{14} \\ \hline \\ -: \Delta_8, F_9 \land F_{10} \vdash \Delta_{12}, F_{14} \\ \hline \end{array} \xrightarrow{\bullet}_{R} \end{array} \xrightarrow{\bullet}_{Cut}$$

• Case rule  $\wedge_R$ 

$$\frac{\frac{\mathbf{h}_{3}: F_{9}, F_{10}, \Delta_{8} \vdash F_{7}, \Delta_{12}, F_{13} \land F_{14}}{\bullet \mathbf{h}_{3}: \Delta_{8}, F_{9} \land F_{10} \vdash (\Delta_{12}, F_{13} \land F_{14}), F_{7}} \land_{L} \frac{\mathbf{h}_{11}: F_{7}, \Delta_{8}, F_{9} \land F_{10} \vdash F_{13}, \Delta_{12} \quad \mathbf{h}_{11}: F_{7}, \Delta_{8}, F_{9} \land F_{10} \vdash F_{14}, \Delta_{12}}{\bullet \mathbf{h}_{11}: (\Delta_{8}, F_{9} \land F_{10}), F_{7} \vdash \Delta_{12}, F_{13} \land F_{14}} \text{ Cut}} \land_{R} \\ \frac{-: \Delta_{8}, F_{9} \land F_{10} \vdash \Delta_{12}, F_{13} \land F_{14}}{\bullet} \xrightarrow{\bullet \mathbf{h}_{11}: \Delta_{8}, F_{10}, F_{7}, F_{9} \vdash \Delta_{12}, F_{13}} \frac{\mathsf{inv-th/ax}}{\mathsf{h}_{11}: \Delta_{8}, F_{10}, F_{7}, F_{9} \vdash \Delta_{12}, F_{14}}} \xrightarrow{\bullet \mathbf{h}_{11}: \Delta_{8}, F_{10}, F_{7}, F_{9} \vdash \Delta_{12}, F_{13} \land F_{14}}} \overset{\mathsf{inv-th/ax}}{\bullet} \\ \frac{-: \Delta_{8}, F_{10}, F_{9} \vdash \Delta_{12}, F_{13} \land F_{14}}{-: \Delta_{8}, F_{9} \land F_{10} \vdash \Delta_{12}, F_{13} \land F_{14}}} \land_{L} \\ \bullet \mathsf{hCut}$$

• Case rule  $\vee_R$ 

$$\frac{\begin{array}{l} \mathbf{h}_{3}: \mathbf{F}_{9}, \mathbf{F}_{10}, \Delta_{8} \vdash \mathbf{F}_{7}, \Delta_{12}, \mathbf{F}_{13} \vee \mathbf{F}_{14} \\ \bullet \mathbf{h}_{3}: \Delta_{8}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash (\Delta_{12}, \mathbf{F}_{13} \vee \mathbf{F}_{14}), \mathbf{F}_{7} \end{array} \wedge_{L} \quad \frac{\mathbf{h}_{11}: \mathbf{F}_{7}, \Delta_{8}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \mathbf{F}_{13}, \mathbf{F}_{14}, \Delta_{12}}{\bullet \mathbf{h}_{11}: (\Delta_{8}, \mathbf{F}_{9} \wedge \mathbf{F}_{10}), \mathbf{F}_{7} \vdash \Delta_{12}, \mathbf{F}_{13} \vee \mathbf{F}_{14}} \quad \mathbf{Cut} \\ \\ \frac{\mathbf{h}_{3}: \Delta_{8}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \Delta_{12}, \mathbf{F}_{13}, \mathbf{F}_{14}, \mathbf{F}_{7}}{\bullet \mathbf{h}_{3}: \Delta_{8}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \Delta_{12}, \mathbf{F}_{13}, \mathbf{F}_{14}, \mathbf{F}_{7}} \quad \mathbf{h}_{11}: \Delta_{8}, \mathbf{F}_{7}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \Delta_{12}, \mathbf{F}_{13}, \mathbf{F}_{14}} \\ \\ \frac{-: \Delta_{8}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \Delta_{12}, \mathbf{F}_{13}, \mathbf{F}_{14}}{-: \Delta_{8}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \Delta_{12}, \mathbf{F}_{13}, \mathbf{F}_{14}} \quad \mathbf{h}_{R} \end{array}} \quad \mathbf{A}_{R}$$

• Case rule  $\perp_R$ 

$$\frac{ \begin{array}{c} \mathbf{h}_3: F_9, F_{10}, \Delta_8 \vdash F_7, \bot, \Delta_{12} \\ \bullet \mathbf{h}_3: \Delta_8, F_9 \land F_{10} \vdash (\bot, \Delta_{12}), F_7 \end{array} \land_L \quad \begin{array}{c} \mathbf{h}_{11}: F_7, \Delta_8, F_9 \land F_{10} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{11}: (\Delta_8, F_9 \land F_{10}), F_7 \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bot_R \\ \text{Cut} \\ \hline \\ \bullet \mathbf{h}_3: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12}, F_7 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_3: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12}, F_7 \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_7, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_7, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_7, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \end{array} \quad \begin{array}{c} \bullet \mathbf{h}_{11}: \Delta_8, F_9 \land F_{10} \vdash \bot, \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{$$

• Case rule  $\top_R$ 

 $\bullet$  Case rule K

 $\bullet$  Case rule A45

$$\frac{\mathbf{h}_3: \mathbf{F}_8, \mathbf{F}_9, \Box \Gamma_{14}, \Delta_{15} \vdash \Box \mathbf{F}_7, \Box \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{13}}{\bullet \mathbf{h}_3: (\Box \Gamma_{14}, \Delta_{15}), \mathbf{F}_8 \land \mathbf{F}_9 \vdash (\Box \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{13}), \Box \mathbf{F}_7} \land_L \frac{\mathbf{h}_{10}: (\Box \Gamma_{14}, \Box \mathbf{F}_7 \vdash \Box \Gamma_{11}, \mathbf{F}_{13})}{\bullet \mathbf{h}_{10}: ((\Box \Gamma_{14}, \Delta_{15}), \mathbf{F}_8 \land \mathbf{F}_9), \Box \mathbf{F}_7 \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{13}} \land_{L} \\ -: (\Box \Gamma_{14}, \Delta_{15}), \mathbf{F}_8 \land \mathbf{F}_9 \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{13}} & \text{ax/W} \\ \frac{\mathbf{h}_{10}: \Box \mathbf{F}_7, \Box \Gamma_{14} \vdash \mathbf{F}_{13}, \Box \Gamma_{11}}{\bullet \mathbf{h}_{10}: \Box \mathbf{F}_7, \Delta_{15}, \mathbf{F}_8, \mathbf{F}_9, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, []\mathbf{F}_{13}}} \\ \frac{-: \Delta_{15}, \mathbf{F}_8, \mathbf{F}_9, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, []\mathbf{F}_{13}}{\bullet \mathbf{h}_{10}: \Box \mathbf{F}_7, \Delta_{15}, \mathbf{F}_8, \mathbf{F}_9, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, []\mathbf{F}_{13}}} \land_{L} \\ \end{pmatrix} A45} \\ \frac{-: \Delta_{15}, \mathbf{F}_8, \mathbf{F}_9, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, []\mathbf{F}_{13}}{\bullet \mathbf{h}_{10}: \Box \mathbf{F}_7, \Delta_{15}, \mathbf{F}_8, \mathbf{F}_9, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, []\mathbf{F}_{13}}} \land_{L}}$$

$$\frac{ \begin{array}{c} \mathbf{h}_{3}: \mathbf{F}_{8}, \mathbf{F}_{9}, \Box \Gamma_{11}, \Delta_{15} \vdash \mathbf{F}_{7}, \Box \Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14} \\ \bullet \mathbf{h}_{3}: (\Box \Gamma_{11}, \Delta_{15}), \mathbf{F}_{8} \land \mathbf{F}_{9} \vdash (\Box \Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14}), \mathbf{F}_{7} \end{array} \wedge_{L} \quad \begin{array}{c} \mathbf{h}_{10}: \Box \Gamma_{11} \vdash \Box \Gamma_{12}, \mathbf{F}_{14} \\ \bullet \mathbf{h}_{10}: ((\Box \Gamma_{11}, \Delta_{15}), \mathbf{F}_{8} \land \mathbf{F}_{9}), \mathbf{F}_{7} \vdash \Box \Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14} \\ -: (\Box \Gamma_{11}, \Delta_{15}), \mathbf{F}_{8} \land \mathbf{F}_{9} \vdash \Box \Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14} \\ \hline -: \Box \Gamma_{11} \vdash \mathbf{F}_{14}, \Box \Gamma_{12} \\ \hline -: \Delta_{15}, \Box \Gamma_{11}, \mathbf{F}_{8} \land \mathbf{F}_{9} \vdash \Delta_{13}, \Box \Gamma_{12}, []\mathbf{F}_{14} \end{array} \quad A45 \end{array}$$

$$\frac{\frac{h_{3}:F_{8},F_{9},\Delta_{7}\vdash F_{11}\to F_{12},\Delta_{13}}{\bullet h_{3}:\Delta_{7},F_{8}\land F_{9}\vdash \Delta_{13},F_{11}\to F_{12}}}{\circ h_{13}:\Delta_{7},F_{8}\land F_{9}\vdash \Delta_{13},F_{11}\to F_{12}\vdash \Delta_{13}}} \xrightarrow{\bullet h_{10}:(\Delta_{7},F_{8}\land F_{9}),F_{11}\to F_{12}\vdash \Delta_{13}} Cut} \xrightarrow{-:\Delta_{7},F_{8}\land F_{9}\vdash \Delta_{13}} \xrightarrow{\bullet h_{10}:(\Delta_{7},F_{8}\land F_{9}),F_{11}\to F_{12}\vdash \Delta_{13}} Cut} \xrightarrow{-:\Delta_{7},F_{8}\land F_{9}\vdash \Delta_{13},F_{11}} \xrightarrow{\bullet h_{10}:\Delta_{7},F_{8},F_{9}\vdash \Delta_{13}} \xrightarrow{h_{10}:\Delta_{7},F_{8},F_{9}\vdash \Delta_{13}} \xrightarrow{h_{10}:A_{7},F_{8},F_{9}\vdash \Delta_{13}} \xrightarrow{h_{10}:A_{7},F_{8},F_{9}\vdash$$

• Case rule  $\wedge_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_{3}: F_{8}, F_{9}, \Delta_{7} \vdash F_{11} \land F_{12}, \Delta_{13}}{\bullet \mathbf{h}_{3}: \Delta_{7}, F_{8} \land F_{9} \vdash \Delta_{13}, F_{11} \land F_{12}} \\ \bullet \mathbf{h}_{3}: \Delta_{7}, F_{8} \land F_{9} \vdash \Delta_{13}, F_{11} \land F_{12}} \\ & -: \Delta_{7}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \frac{\mathbf{h}_{3}: \Delta_{7}, F_{8}, F_{9} \vdash \Delta_{13}, F_{11} \land F_{12}}{\bullet \mathbf{h}_{10}: \Delta_{7}, F_{11}, F_{12}, F_{8}, F_{9} \vdash \Delta_{13}} \\ \bullet \mathbf{h}_{10}: \Delta_{7}, F_{11}, F_{12}, F_{8}, F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{3}: \Delta_{7}, F_{8}, F_{9} \vdash \Delta_{13}, F_{11} \land F_{12} \\ \hline \\ \frac{-: \Delta_{7}, F_{8}, F_{9} \vdash \Delta_{13}}{\bullet \mathbf{h}_{10}: \Delta_{7}, F_{8}, F_{9} \vdash A_{13}} \land L \\ \hline \\ \mathbf{h}_{3}: F_{8}, F_{9}, \Delta_{14}, F_{11} \land F_{12} \vdash F_{7}, \Delta_{13} \\ \bullet \mathbf{h}_{3}: (\Delta_{14}, F_{11} \land F_{12}), F_{8} \land F_{9} \vdash \Delta_{13}, F_{7} \\ \hline \\ \mathbf{h}_{3}: (\Delta_{14}, F_{11}, F_{12}, F_{8}, F_{9} \vdash \Delta_{13}, F_{7} \\ \hline \\ \mathbf{h}_{3}: \Delta_{14}, F_{11}, F_{12}, F_{8}, F_{9} \vdash \Delta_{13}, F_{7} \\ \hline \\ \mathbf{h}_{3}: \Delta_{14}, F_{11}, F_{12}, F_{8}, F_{9} \vdash \Delta_{13}, F_{7} \\ \hline \\ \mathbf{h}_{3}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13}, F_{7} \\ \hline \\ \mathbf{h}_{3}: \Delta_{14}, F_{11}, F_{12}, F_{8}, F_{9} \vdash \Delta_{13}, F_{7} \\ \hline \\ \mathbf{h}_{3}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13}, F_{7} \\ \hline \\ \mathbf{h}_{3}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13}, F_{7} \\ \hline \\ \mathbf{h}_{3}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13}, F_{7} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{7}, F_{8} \land F_{9} \vdash \Delta_{13} \\ \hline \\ \mathbf{h}_{10}: \Delta_{14}, F_{11}, F_{12}, F_{13}, F_{14}, F_{14$$

• Case rule  $\vee_L$ 

$$\frac{h_3: F_8, F_9, \Delta_7 \vdash F_{11} \lor F_{12}, \Delta_{13}}{\bullet h_3: \Delta_7, F_8 \land F_9 \vdash \Delta_{13}, F_{11} \lor F_{12}} \land_L \frac{h_{10}: F_{11}, \Delta_7, F_8 \land F_9 \vdash \Delta_{13} \quad h_{10}: F_{12}, \Delta_7, F_8 \land F_9 \vdash \Delta_{13}}{\bullet h_{10}: (\Delta_7, F_8 \land F_9), F_{11} \lor F_{12} \vdash \Delta_{13}} \underbrace{\text{Cut}} \\ -: \Delta_7, F_8 \land F_9 \vdash \Delta_{13} \\ \hline \\ h_3: \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \lor F_{12} \\ \hline \\ h_3: \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \lor F_{12} \\ \hline \\ h_3: F_8, F_9 \land_{13}, F_{11} \lor F_{12} \\ \hline \\ h_3: A_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \lor F_{12} \\ \hline \\ h_3: A_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \lor F_{12} \vdash F_7, A_{13} \\ \hline \\ h_3: A_7, F_8, F_9 \vdash \Delta_{13}, F_7 \\ \hline \\ h_3: A_7, F_8, F_9 \vdash A_{13}, F_7 \\ \hline \\ h_1: A_7, F_8, F_9 \vdash A_{13} \\ \hline \\ h_1: A_7, F_8, F_9 \vdash A_{13} \\ \hline \\ h_1: A_7, F_8, F_9 \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9 \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9 \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9 \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9 \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9 \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9 \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A_{13} \\ \hline \\ \bullet h_1: A_7, F_8, F_9, F_{11} \lor F_{12} \vdash A$$

#### $\bullet$ Case rule AT

$$\frac{\frac{h_{3}:F_{8},F_{9},\Delta_{7}\vdash [[F_{11},\Delta_{12}]}{\bullet h_{3}:\Delta_{7},F_{8}\land F_{9}\vdash \Delta_{12},[]F_{11}}}{-:\Delta_{7},F_{8}\land F_{9}\vdash \Delta_{12}} \wedge_{L} \frac{\frac{h_{10}:F_{11},\Delta_{7},[[F_{11},F_{8}\land F_{9}\vdash \Delta_{12}]}{\bullet h_{10}:(\Delta_{7},F_{8}\land F_{9}),[]F_{11}\vdash \Delta_{12}}}{-:\Delta_{7},F_{8}\land F_{9}\vdash \Delta_{12}} \wedge_{L} \frac{AT}{h_{10}:\Delta_{7},F_{11},F_{8},F_{9},[]F_{11}\vdash \Delta_{12}}}{\frac{h_{10}:\Delta_{7},F_{11},F_{8},F_{9},[]F_{11}\vdash \Delta_{12}}{\bullet h_{10}:\Delta_{7},F_{8},F_{9},[]F_{11}\vdash \Delta_{12}}} \wedge_{L} \frac{inv-th/ax}{AT} \\ \frac{-:\Delta_{7},F_{8},F_{9}\vdash \Delta_{12}}{-:\Delta_{7},F_{8}\land F_{9}\vdash \Delta_{12}} \wedge_{L} \\ \frac{h_{3}:F_{8},F_{9},\Delta_{13},[]F_{11}\vdash F_{7},\Delta_{12}}{-:\Delta_{7},F_{8}\land F_{9}\vdash \Delta_{12}} \wedge_{L} \frac{h_{10}:F_{7},F_{11},\Delta_{13},[]F_{11},F_{8}\land F_{9}\vdash \Delta_{12}}{\bullet h_{10}:((\Delta_{13},[]F_{11}),F_{8}\land F_{9}\vdash \Delta_{12}} \wedge_{L} \\ \frac{\bullet_{13}:(\Delta_{13},[]F_{11}),F_{8}\land F_{9}\vdash \Delta_{12},F_{7}}{-:(\Delta_{13},[]F_{11}),F_{8}\land F_{9}\vdash \Delta_{12}} ATG \\ \frac{-:\Delta_{13},F_{11},[]F_{11},F_{8}\land F_{9}\vdash \Delta_{12}}{-:\Delta_{13},[]F_{11},F_{8}\land F_{9}\vdash \Delta_{12}} ATG \\ \frac{-:\Delta_{13},F_{11},[]F_{11},F_{8}\land F_{9}\vdash \Delta_{12}}{-:\Delta_{13},[]F_{11},F_{8}\land F_{9}\vdash \Delta_{12}} ATG \\ \end{pmatrix} ATG$$

## • Case rule $\perp_L$

$$\begin{array}{c} \frac{\mathbf{h}_{3}: \mathsf{F}_{8}, \mathsf{F}_{9}, \Delta_{7} \vdash \bot, \Delta_{11}}{\bullet \mathbf{h}_{3}: \Delta_{7}, \mathsf{F}_{8} \land \mathsf{F}_{9} \vdash \Delta_{11}, \bot} \ \land L \\ \hline \\ \bullet \mathbf{h}_{3}: \Delta_{7}, \mathsf{F}_{8} \land \mathsf{F}_{9} \vdash \Delta_{11}, \bot \\ \hline \\ \frac{\mathbf{h}_{3}: \Delta_{7}, \mathsf{F}_{8}, \mathsf{F}_{9} \vdash \bot, \Delta_{11}}{\bullet \mathbf{h}_{3}: \Delta_{7}, \mathsf{F}_{8}, \mathsf{F}_{9} \vdash \bot, \Delta_{11}} \ \mathbf{ax/W} & \bullet \mathbf{h}_{10}: \bot, \Delta_{7}, \mathsf{F}_{8}, \mathsf{F}_{9} \vdash \Delta_{11} \\ \hline \\ \frac{-: \Delta_{7}, \mathsf{F}_{8}, \mathsf{F}_{9} \vdash \Delta_{11}}{-: \Delta_{7}, \mathsf{F}_{8} \land \mathsf{F}_{9} \vdash \Delta_{11}} \ \land L \\ \hline \\ \bullet \mathbf{h}_{3}: \mathsf{F}_{8}, \mathsf{F}_{9}, \bot, \Delta_{12} \vdash \mathsf{F}_{7}, \Delta_{11} \\ \hline \\ \bullet \mathbf{h}_{3}: (\bot, \Delta_{12}), \mathsf{F}_{8} \land \mathsf{F}_{9} \vdash \Delta_{11}, \mathsf{F}_{7} \end{array} & \wedge L \\ \hline \\ \bullet \mathbf{h}_{10}: (\bot, \Delta_{12}), \mathsf{F}_{8} \land \mathsf{F}_{9}), \mathsf{F}_{7} \vdash \Delta_{11} \\ \hline \\ -: (\bot, \Delta_{12}), \mathsf{F}_{8} \land \mathsf{F}_{9} \vdash \Delta_{11} \\ \hline \\ -: \bot, \Delta_{12}, \mathsf{F}_{8} \land \mathsf{F}_{9} \vdash \Delta_{11} \end{array} & \bot_{L} \end{array} & \mathsf{Cut}$$

#### ullet Case rule I

$$\begin{array}{c} \frac{h_{3}:F_{8},F_{9},\Delta_{7}\vdash p_{11},\Delta_{12},p_{11}}{\bullet h_{3}:\Delta_{7},F_{8}\wedge F_{9}\vdash (\Delta_{12},p_{11}),p_{11}} & \wedge_{L} & \\ \hline \bullet h_{10}:(\Delta_{7},F_{8}\wedge F_{9}),p_{11}\vdash \Delta_{12},p_{11} & \\ \hline & -:\Delta_{7},F_{8}\wedge F_{9}\vdash \Delta_{12},p_{11} & \\ \hline & \frac{h_{3}:\Delta_{7},F_{8},F_{9}\vdash \Delta_{12},p_{11},p_{11}}{\bullet h_{10}:\Delta_{7},F_{8},F_{9},p_{11}\vdash \Delta_{12},p_{11}} & I \\ \hline & \frac{-:\Delta_{7},F_{8},F_{9}\vdash \Delta_{12},p_{11}}{-:\Delta_{7},F_{8}\wedge F_{9}\vdash \Delta_{12},p_{11}} & \wedge_{L} \\ \hline & \frac{h_{3}:F_{8},F_{9},\Delta_{13},p_{11}\vdash F_{7},\Delta_{12},p_{11}}{-:\Delta_{7},F_{8}\wedge F_{9}\vdash \Delta_{12},p_{11}} & \wedge_{L} \\ \hline & \frac{h_{3}:F_{8},F_{9},\Delta_{13},p_{11}\vdash F_{7},\Delta_{12},p_{11}}{-:\Delta_{13},p_{11}),F_{8}\wedge F_{9}\vdash \Delta_{12},p_{11}} & I \\ \hline & -:(\Delta_{13},p_{11}),F_{8}\wedge F_{9}\vdash \Delta_{12},p_{11} & \\ \hline & -:(\Delta_{13},p_{11}),F_{8}\wedge F_{9}\vdash \Delta_{12},p_{11} & I \\ \hline \end{array}$$

$$\frac{\mathbf{h}_3: \mathsf{F}_8, \mathsf{F}_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet \mathsf{h}_3: \Delta_7, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11}} \uparrow_L \frac{\mathsf{h}_{10}: \Delta_7, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11}}{\bullet \mathsf{h}_{10}: (\Delta_7, \mathsf{F}_8 \land \mathsf{F}_9), \top \vdash \Delta_{11}} \uparrow_L \\ -: \Delta_7, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11} \\ \hline -: \Delta_7, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11}} \text{ ax/W} \\ \\ \frac{\mathsf{h}_3: \mathsf{F}_8, \mathsf{F}_9, \top, \Delta_{12} \vdash \mathsf{F}_7, \Delta_{11}}{-: \Delta_7, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11}} \wedge_L \frac{\mathsf{h}_{10}: \mathsf{F}_7, \Delta_{12}, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11}}{\bullet \mathsf{h}_{10}: ((\top, \Delta_{12}), \mathsf{F}_8 \land \mathsf{F}_9), \mathsf{F}_7 \vdash \Delta_{11}} \wedge_L \\ -: (\top, \Delta_{12}), \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11} \\ \hline -: (\top, \Delta_{12}), \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11} \\ \hline \bullet \mathsf{h}_3: \top, \Delta_{12}, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_7} \xrightarrow{\mathsf{ax/W}} \frac{\mathsf{ax/W}}{\mathsf{h}_{10}: \top, \Delta_{12}, \mathsf{F}_7, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11}} \wedge_\mathsf{hCut} \\ \hline \bullet \mathsf{h}_3: \top, \Delta_{12}, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11} \\ \hline -: \top, \Delta_{12}, \mathsf{F}_8 \land \mathsf{F}_9 \vdash \Delta_{11}} & \mathsf{ax/W} \\ \hline \mathsf{hCut} \\ \hline \end{pmatrix}$$

# 6.10 Status of $\vee_L$ : OK

• Case rule  $\rightarrow_R$ 

$$\frac{\frac{h_{3}: F_{9}, \Delta_{8} \vdash F_{7}, \Delta_{12}, F_{13} \to F_{14}}{\bullet h_{3}: F_{10}, \Delta_{8} \vdash F_{7}, \Delta_{12}, F_{13} \to F_{14}}}{\bullet h_{3}: F_{10} \vdash (\Delta_{12}, F_{13} \to F_{14}), F_{7}} \lor_{L} \frac{\frac{h_{11}: F_{7}, F_{13}, \Delta_{8}, F_{9} \vee F_{10} \vdash F_{14}, \Delta_{12}}{\bullet h_{11}: (\Delta_{8}, F_{9} \vee F_{10}), F_{7} \vdash \Delta_{12}, F_{13} \to F_{14}}} \xrightarrow{P_{R}} Cut} \frac{-: \Delta_{8}, F_{9} \vee F_{10} \vdash \Delta_{12}, F_{13} \to F_{14}}}{\frac{h_{3}: \Delta_{8}, F_{13}, F_{9} \vdash \Delta_{12}, F_{14}, F_{7}}{\bullet h_{3}: \Delta_{8}, F_{10}, F_{13} \vdash \Delta_{12}, F_{14}, F_{7}}}} \xrightarrow{inv-th/ax} \frac{h_{11}: \Delta_{8}, F_{13}, F_{7}, F_{9} \vee F_{10} \vdash \Delta_{12}, F_{14}}}{\frac{-: \Delta_{8}, F_{13}, F_{9} \vee F_{10} \vdash \Delta_{12}, F_{14}}{-: \Delta_{8}, F_{9} \vee F_{10} \vdash \Delta_{12}, F_{14}}} \xrightarrow{P_{R}} ax/W} hCut}$$

• Case rule  $\wedge_R$ 

$$\frac{ \underbrace{ \begin{array}{c} \mathbf{h}_{3}: F_{9}, \Delta_{8} \vdash F_{7}, \Delta_{12}, F_{13} \land F_{14} \\ \bullet h_{3}: \Delta_{8}, F_{9} \lor F_{10} \vdash (\Delta_{12}, F_{13} \land F_{14}), F_{7} \end{array}}_{\bullet h_{3}: \Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13}, F_{7}} \quad \forall L \quad \underbrace{ \begin{array}{c} \mathbf{h}_{11}: F_{7}, \Delta_{8}, F_{9} \lor F_{10} \vdash F_{13}, \Delta_{12}, F_{13} \land F_{14} \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \mathbf{h}_{3}: \Delta_{8}, F_{9} \vdash \Delta_{12}, F_{13}, F_{7} \\ \bullet h_{3}: \Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13}, F_{7} \\ \hline \\ \bullet h_{3}: \Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13}, F_{7} \\ \hline \\ -: \Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \\ \hline \\ -: \Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \end{array}}_{\bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14})} \quad \underbrace{ \begin{array}{c} \mathbf{h}_{11}: F_{7}, \Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta_{12}, F_{13} \land F_{14} \\ \hline \\ \bullet h_{11}: (\Delta_{8}, F_{9} \lor F_{10} \vdash \Delta$$

• Case rule  $\vee_R$ 

$$\frac{\begin{array}{l} \frac{h_{3}:F_{9},\Delta_{8}\vdash F_{7},\Delta_{12},F_{13}\vee F_{14}\quad h_{3}:F_{10},\Delta_{8}\vdash F_{7},\Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:(\Delta_{8},F_{9}\vee F_{10}\vdash F_{13},F_{14},\Delta_{12})} \\ & \frac{\bullet h_{3}:\Delta_{8},F_{9}\vee F_{10}\vdash (\Delta_{12},F_{13}\vee F_{14}),F_{7}}{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}} \\ & \frac{h_{11}:F_{7},\Delta_{8},F_{9}\vee F_{10}\vdash F_{13},F_{14},\Delta_{12}}{\bullet h_{11}:(\Delta_{8},F_{9}\vee F_{10}),F_{7}\vdash \Delta_{12},F_{13}\vee F_{14}} \\ & \frac{h_{11}:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}} \\ & \frac{h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}}{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13},F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_{10}\vdash \Delta_{12},F_{13}\vee F_{14}}{\bullet h_{11}:\Delta_{8},F_{7}\vee F_{10}} \\ & \frac{-:\Delta_{8},F_{9}\vee F_$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_{3}: \mathsf{F}_{9}, \Delta_{8} \vdash \mathsf{F}_{7}, \bot, \Delta_{12} \quad \mathsf{h}_{3}: \mathsf{F}_{10}, \Delta_{8} \vdash \mathsf{F}_{7}, \bot, \Delta_{12}}{\bullet \mathsf{h}_{3}: \Delta_{8}, \mathsf{F}_{9} \lor \mathsf{F}_{10} \vdash (\bot, \Delta_{12}), \mathsf{F}_{7}} \quad \lor_{L} \quad \frac{\mathbf{h}_{11}: \mathsf{F}_{7}, \Delta_{8}, \mathsf{F}_{9} \lor \mathsf{F}_{10} \vdash \Delta_{12}}{\bullet \mathsf{h}_{11}: (\Delta_{8}, \mathsf{F}_{9} \lor \mathsf{F}_{10}), \mathsf{F}_{7} \vdash \bot, \Delta_{12}} \quad \underbrace{\mathsf{Cut}}_{\bullet \mathsf{h}_{3}: \Delta_{8}, \mathsf{F}_{9} \lor \mathsf{F}_{10} \vdash \bot, \Delta_{12}, \mathsf{F}_{7}}^{\bullet \mathsf{h}_{3}: \Delta_{8}, \mathsf{F}_{9} \lor \mathsf{F}_{10} \vdash \bot, \Delta_{12}, \mathsf{F}_{7}}_{\bullet \mathsf{h}_{11}: \Delta_{8}, \mathsf{F}_{7}, \mathsf{F}_{9} \lor \mathsf{F}_{10} \vdash \bot, \Delta_{12}}^{\bullet \mathsf{h}_{21}: \Delta_{8}, \mathsf{F}_{9} \lor \mathsf{F}_{10} \vdash \bot, \Delta_{12}}_{\bullet \mathsf{Cut}}$$

• Case rule  $\top_R$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_9, \Delta_8 \vdash \mathbf{F}_7, \top, \Delta_{12} \quad \mathbf{h}_3: \mathbf{F}_{10}, \Delta_8 \vdash \mathbf{F}_7, \top, \Delta_{12}}{\bullet \mathbf{h}_3: \Delta_8, \mathbf{F}_9 \vee \mathbf{F}_{10} \vdash (\top, \Delta_{12}), \mathbf{F}_7} \quad \vee_L \quad \frac{\bullet \mathbf{h}_{11}: (\Delta_8, \mathbf{F}_9 \vee \mathbf{F}_{10}), \mathbf{F}_7 \vdash \top, \Delta_{12}}{-: \Delta_8, \mathbf{F}_9 \vee \mathbf{F}_{10} \vdash \top, \Delta_{12}} \quad \mathsf{Cut} \\ & \xrightarrow{-: \Delta_8, \mathbf{F}_9 \vee \mathbf{F}_{10} \vdash \top, \Delta_{12}} \quad \top_R$$

 $\bullet$  Case rule K

$$\frac{h_3: F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, []F_{12} \quad h_3: F_9, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_7, \Delta_{11}, []F_{12}}{\bullet h_3: (\Box \Gamma_{13}, \Delta_{14}), F_8 \lor F_9 \vdash (\Delta_{11}, []F_{12}), \Box F_7} \lor_L \frac{h_{10}: unbox(\Box \Gamma_{13}), unbox(\Box F_7) \vdash F_{12}}{\bullet h_{10}: ((\Box \Gamma_{13}, \Delta_{14}), F_8 \lor F_9), \Box F_7 \vdash \Delta_{11}, []F_{12}} \lor_L \frac{h_{10}: unbox(\Box F_7), unbox(\Box F_7), unbox(\Box \Gamma_{13}) \vdash F_{12}}{\bullet h_{10}: unbox(\Box F_7), unbox(\Box F_7), unbox(\Box \Gamma_{13}) \vdash F_{12}} \frac{ax/W}{A_{10}: unbox(\Box F_7), unbox(\Box F_7), unbox(\Box \Gamma_{13}) \vdash F_{12}} \underbrace{\begin{pmatrix} h_{10}: unbox(\Box F_7), unbo$$

 $\bullet$  Case rule A45

$$\frac{\underline{h_3:F_8,\Box\Gamma_{14},\Delta_{15}\vdash\Box F_7,\Box\Gamma_{11},\Delta_{12},[]F_{13}\quad h_3:F_9,\Box\Gamma_{14},\Delta_{15}\vdash\Box F_7,\Box\Gamma_{11},\Delta_{12},[]F_{13}}{\bullet h_3:(\Box\Gamma_{14},\Delta_{15}),F_8\vee F_9\vdash(\Box\Gamma_{11},\Delta_{12},[]F_{13}),\Box F_7}\vee_L \frac{h_{10}:\Box\Gamma_{14},\Box\Gamma_{14},\Box\Gamma_{17},\Box\Gamma_{14},\Box\Gamma_{17},\Box\Gamma_{14},\Box\Gamma_{17},\Box\Gamma_{14},\Delta_{15}),F_8\vee F_9\vdash\Box\Gamma_{11},\Delta_{12},[]F_{13}}{\bullet h_{10}:(\Box\Gamma_{14},\Delta_{15}),F_8\vee F_9\vdash\Box\Gamma_{11},\Delta_{12},[]F_{13}} \\ \underline{-:(\Box\Gamma_{14},\Delta_{15}),F_8\vee F_9\vdash\Box\Gamma_{11},\Delta_{12},[]F_{13}} \\ \underline{h_{10}:\Box F_7,\Box\Gamma_{14}\vdash F_{13},\Box\Gamma_{11}} \\ \bullet h_{10}:\Box F_7,\Box\Gamma_{14}\vdash F_{13},\Box\Gamma_{11}} \\ \bullet h_{10}:\Box F_7,\Delta_{15},F_8,\Box\Gamma_{14}\vdash\Delta_{12},\Box\Gamma_{11},[]F_{13}} \\ \underline{-:\Delta_{15},F_8,\Box\Gamma_{14}\vdash\Delta_{12},\Box\Gamma_{11},[]F_{13}} \\ \underline{-:\Delta_{15},F_8,\Box\Gamma_{14}\vdash\Delta_{12},\Box\Gamma_{11},[]F_{13}} \\ \underline{-:\Delta_{15},F_9,\Box\Gamma_{14},[]F_{13}} \\ \underline{-:\Delta_{15},\Box\Gamma_{14},F_8\vee F_9\vdash\Delta_{12},\Box\Gamma_{11},[]F_{13}} \\ \underline{-:\Delta_{15},F_9,\Box\Gamma_{14},\Box\Gamma_{11},[]F_{13}} \\ \underline{-:\Delta_{15},G_{14},F_8\vee F_9\vdash\Delta_{12},\Box\Gamma_{11},[]F_{13}} \\ \underline{-:\Delta_{15},F_9,\Box\Gamma_{14},\Box\Gamma_{11},[]F_{13}} \\ \underline{-:\Delta_{15},G_{14},F_8\vee F_9\vdash\Delta_{12},\Box\Gamma_{11},[]F_{13}} \\ \underline{-:\Delta_{15},G_{14},G_{14},G_{15$$

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\frac{\mathbf{h}_{3}: \mathbf{F}_{8}, \Box \Gamma_{11}, \Delta_{15} \vdash \mathbf{F}_{7}, \Box \Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14} \quad \mathbf{h}_{3}: \mathbf{F}_{9}, \Box \Gamma_{11}, \Delta_{15} \vdash \mathbf{F}_{7}, \Box \Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14}}{\bullet \mathbf{h}_{3}: (\Box \Gamma_{11}, \Delta_{15}), \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash (\Box \Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14}), \mathbf{F}_{7}} \quad \lor_{L} \quad \frac{\mathbf{h}_{10}: (\Box \Gamma_{11} \vdash \Box \Gamma_{12}, \mathbf{F}_{14})}{\bullet \mathbf{h}_{10}: ((\Box \Gamma_{11}, \Delta_{15}), \mathbf{F}_{8} \lor \mathbf{F}_{9}), \mathbf{F}_{7} \vdash \Box \Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14}} \quad Cut
-: (\Box \Gamma_{11}, \Delta_{15}), \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Box \Gamma_{12}, \Delta_{13}, []\mathbf{F}_{14}
-: (\Box \Gamma_{11} \vdash \mathbf{F}_{14}, \Box \Gamma_{12}) \quad \mathbf{ax/W}
-: \Box \Gamma_{11} \vdash \mathbf{F}_{14}, \Box \Gamma_{12}, []\mathbf{F}_{14} \quad A45
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$$\frac{\mathbf{h}_{3} : \mathsf{F}_{8}, \Delta_{7} \vdash \mathsf{F}_{11} \to \mathsf{F}_{12}, \Delta_{13} \quad \mathsf{h}_{3} : \mathsf{F}_{9}, \Delta_{7} \vdash \mathsf{F}_{11} \to \mathsf{F}_{12}, \Delta_{13}}{\bullet \mathsf{h}_{3} : \Delta_{7}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13}, \mathsf{F}_{11} \to \mathsf{F}_{12}} \bigvee_{\mathsf{e}} \frac{\mathbf{h}_{10} : \Delta_{7}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \mathsf{F}_{11}, \Delta_{13} \quad \mathsf{h}_{10} : \mathsf{F}_{12}, \Delta_{7}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13}}{\bullet \mathsf{h}_{10} : (\Delta_{7}, \mathsf{F}_{8} \lor \mathsf{F}_{9}), \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \Delta_{13}} \underbrace{\mathsf{Cut}} \to_{\mathsf{L}} \\ -: \Delta_{7}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13} \\ -: \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{8} \vdash \Delta_{13}, \mathsf{F}_{12} & \mathsf{inv}\text{-th/ax} \\ -: \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13}, \mathsf{F}_{12} \\ -: \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13}, \mathsf{F}_{12} \\ -: \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13}, \mathsf{F}_{12} \\ -: \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13} \\ -: \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13}, \mathsf{F}_{12} \\ -: \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13} \\ -: \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13} \\ -: \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{13} \\ \bullet \mathsf{h}_{3} : \mathsf{F}_{11}, \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \mathsf{F}_{11}, \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \mathsf{F}_{11}, \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{3} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{13} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{13} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{13} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{F}_{7}, \Delta_{13} \\ \bullet \mathsf{h}_{13} : \Delta_{14}, \mathsf{F}_{11} \to \mathsf{F}_{12} \vdash \mathsf{$$

• Case rule  $\wedge_L$ 

$$\frac{h_3: F_8, \Delta_7 \vdash F_{11} \land F_{12}, \Delta_{13} \quad h_3: F_9, \Delta_7 \vdash F_{11} \land F_{12}, \Delta_{13}}{\bullet_{h_3}: \Delta_7, F_8 \lor F_9 \vdash \Delta_{13}, F_{11} \land F_{12}} \lor_L \quad \frac{h_{10}: F_{11}, F_{12}, \Delta_7, F_8 \lor F_9 \vdash \Delta_{13}}{\bullet_{h_{10}}: (\Delta_7, F_8 \lor F_9), F_{11} \land F_{12} \vdash \Delta_{13}} \quad \land_L \\ \hline -: \Delta_7, F_8 \lor F_9 \vdash \Delta_{13} \\ \hline \frac{h_3: \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \land F_{12}}{\bullet_{h_{10}}: \Delta_7, F_8, F_{11} \land F_{12} \vdash \Delta_{13}} \quad \frac{h_{10}: \Delta_7, F_{11}, F_{12}, F_8 \vdash \Delta_{13}}{\bullet_{h_{10}}: \Delta_7, F_8, F_{11} \land F_{12} \vdash \Delta_{13}} \quad \land_L \\ \hline -: \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \land F_{12} \vdash F_7, F_8, F_{11} \land F_{12} \vdash F_7, A_{13} \quad h_{13}: F_9, F_{11} \land F_{12} \vdash F_7, A_{13} \quad h_{13}: F_9, F_{11} \land F_{12} \vdash F_7, A_{13} \quad \lor_L \\ \hline -: (\Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}) \quad \lor_L \\ \hline -: (\Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}) \quad \bullet_{h_{10}}: ((\Delta_1, F_{11}, F_{12}, F_8 \lor F_9), F_7 \vdash \Delta_{13} \quad \land_L \\ \hline -: (\Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}) \quad \lor_L \\ \hline \hline h_3: \Delta_{14}, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}, F_7 \quad \text{inv-th/ax}} \quad \bullet_{h_{10}: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}} \quad \land_L \\ \hline \hline h_3: \Delta_{14}, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}, F_7 \quad \text{inv-th/ax}} \quad \bullet_{h_{10}: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}} \quad \land_L \\ \hline \hline -: (\Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}, F_7 \quad \text{inv-th/ax}} \quad \bullet_{h_{10}: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}} \quad \land_L \\ \hline \hline -: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}, F_7 \quad \text{inv-th/ax}} \quad \bullet_{h_{10}: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}} \quad \land_L \\ \hline \hline -: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}, F_7 \quad \text{inv-th/ax}} \quad \bullet_{h_{10}: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}} \quad \land_L \\ \hline \hline -: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}, F_7 \quad \text{inv-th/ax}} \quad \bullet_{h_{10}: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}} \quad \land_L \\ \hline \hline -: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}, F_7 \quad \text{inv-th/ax}} \quad \bullet_{h_{10}: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}} \quad \land_L \\ \hline \hline -: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}, F_7 \quad \text{inv-th/ax}} \quad \bullet_{h_{10}: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}} \quad \land_L \\ \hline \hline -: \Delta_1, F_{11}, F_{12}, F_8 \lor F_9 \vdash \Delta_{13}, F_7 \quad \text{inv-th/ax}} \quad \bullet_{h_{10}: \Delta_1, F_{11}, F_{12},$$

• Case rule  $\vee_L$ 

$$\frac{h_3: F_8, \Delta_{14}, F_{11} \vee F_{12} \vdash F_7, \Delta_{13}}{\bullet h_3: (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \frac{h_{10}: F_7, F_{11}, \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10}: ((\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7)} \vee_L \frac{h_{10}: A_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10}: \Delta_{14}, F_{11}, F_8 \vdash \Delta_{13}, F_7} \text{ inv-th/ax} \\ \frac{\bullet_{13}: \Delta_{14}, F_{11}, F_8 \vdash \Delta_{13}, F_7}{\bullet h_3: \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \frac{\bullet_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}}{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}} \frac{\bullet_{13}: \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_3: \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}} \times_{h_{10}: \Delta_{14},$$

 $\bullet$  Case rule AT

$$\frac{ \underbrace{ \begin{array}{c} \mathbf{h}_{3} : \mathbf{F}_{8}, \Delta_{7} \vdash [] \mathbf{F}_{11}, \Delta_{12} \quad \mathbf{h}_{3} : \mathbf{F}_{9}, \Delta_{7} \vdash [] \mathbf{F}_{11}, \Delta_{12}}_{\bullet \mathbf{h}_{3} : \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12}, [] \mathbf{F}_{11}} \\ \underline{ \begin{array}{c} \bullet_{\mathbf{h}_{3}} : \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12}, [] \mathbf{F}_{11}}_{\bullet \mathbf{h}_{10} : (\Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9}), [] \mathbf{F}_{11} \vdash \Delta_{12}} \\ \underline{ \begin{array}{c} \bullet_{\mathbf{h}_{3}} : \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12} \\ \hline \\ - : \Delta_{7}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12} \\ \hline \\ \bullet_{\mathbf{h}_{10}} : \Delta_{7}, \mathbf{F}_{11}, \mathbf{F}_{8}, [] \mathbf{F}_{11} \vdash \Delta_{12}}_{\bullet \mathbf{h}_{10}} \\ \underline{ \begin{array}{c} \bullet_{\mathbf{h}_{10}} : \Delta_{7}, \mathbf{F}_{11}, \mathbf{F}_{8}, [] \mathbf{F}_{11} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{11}, \mathbf{F}_{8}, [] \mathbf{F}_{11} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{9} \vdash \Delta_{12}, [] \mathbf{F}_{11} \\ \hline \end{array}} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_{10}} : \Delta_{7}, \mathbf{F}_{11}, \mathbf{F}_{9}, [] \mathbf{F}_{11} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{11}, \mathbf{F}_{9}, [] \mathbf{F}_{11} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{9}, [] \mathbf{F}_{11} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{9} \vdash \Delta_{12}, [] \mathbf{F}_{11} \\ \hline \end{array}} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_{10}} : \Delta_{7}, \mathbf{F}_{9} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{9}, [] \mathbf{F}_{11} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{9}, [] \mathbf{F}_{11} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{9}, [] \mathbf{F}_{11} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{9}, [] \mathbf{F}_{11} \vdash \Delta_{12} \\ \hline \end{array}} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_{10}} : \Delta_{7}, \mathbf{F}_{9} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{9}, [] \mathbf{F}_{11}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : \Delta_{7}, \mathbf{F}_{9} \vdash \Delta_{12} \\ \hline \end{array}} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_{10}} : \Delta_{7}, \mathbf{F}_{9} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : (\Delta_{13}, [] \mathbf{F}_{11}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : (\Delta_{13}, [] \mathbf{F}_{11}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12} \\ \hline \end{array}} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_{10}} : \Delta_{7}, \mathbf{F}_{9} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : (\Delta_{13}, [] \mathbf{F}_{11}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12} \\ \bullet \mathbf{h}_{10} : (\Delta_{13}, [] \mathbf{F}_{11}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{10} : \Delta_{13}, [] \mathbf{F}_{11}, \mathbf{F}_{8} \lor \mathbf{F}_{9} \vdash \Delta_{12} \\ \hline \\ \bullet \mathbf{h}_{10} : \Delta_{13}, [] \mathbf{H}_{11}, \mathbf{F}_{11}, \mathbf{F}_{11}, \mathbf{F}_{12}, \mathbf{F}_{11}, \mathbf{F}_{11}, \mathbf{F}_{12}, \mathbf{F}_{13} \\ \hline \\ \bullet \mathbf{h}_{10} : \Delta_{13}, [] \mathbf{H}_{11}, \mathbf{F}_{11}$$

• Case rule  $\perp_L$ 

$$\frac{\frac{h_{3}:F_{8},\Delta_{7}\vdash\bot,\Delta_{11}}{\bullet h_{3}:\Delta_{7},F_{8}\vee F_{9}\vdash\Delta_{11},\bot}}{\bullet h_{3}:\Delta_{7},F_{8}\vee F_{9}\vdash\Delta_{11},\bot}} \vee_{L} \frac{\bullet h_{10}:(\Delta_{7},F_{8}\vee F_{9}),\bot\vdash\Delta_{11}}{\bullet h_{10}:(\Delta_{7},F_{8}\vee F_{9}),\bot\vdash\Delta_{11}}} \xrightarrow{\bot_{L}}_{Cut} \frac{\bot_{L}}{\bullet h_{10}:\Delta_{7},F_{8}\vee F_{9}\vdash\Delta_{11}}} \frac{\bot_{L}}{\bullet h_{10}:\bot\Delta_{7},F_{8}\vdash\Delta_{11}} \frac{\bot_{L}}{\bullet h_{10}:\bot\Delta_{7},F_{9}\vdash\Delta_{11}} \frac{\bot_{L}}{\bullet h_{10}:\bot\Delta_{7},F_{9}\vdash\Delta_{11}} \frac{\bot_{L}}{\bullet h_{10}:\bot\Delta_{7},F_{9}\vdash\Delta_{11}} \frac{\bot_{L}}{\bullet h_{10}:\Delta_{7},F_{9}\vdash\Delta_{11}} \frac{\bot_{L}}{\bullet h_{10}:(\bot,\Delta_{7},F_{9}\vdash\Delta_{11})} \frac{\bot_{L}}{\bullet h_{10}:(\bot,\Delta_{12}),F_{8}\vee F_{9}\vdash\Delta_{11}} \frac{\bot_{L}}{\bullet h_{10}:(\bot,\Delta_{12}),F_{11}} \frac{\bot_{L}}{\bullet h_{10}:(\bot,\Delta_{12}),F_{11}} \frac{\bot_{L}}{\bullet h_{10}:(\bot,\Delta_{12$$

 $\bullet$  Case rule I

$$\frac{\frac{\mathbf{h}_{3}: \mathbf{F}_{8}, \Delta_{7} \vdash \mathbf{p}_{11}, \Delta_{12}, \mathbf{p}_{11} \quad \mathbf{h}_{3}: \mathbf{F}_{9}, \Delta_{7} \vdash \mathbf{p}_{11}, \Delta_{12}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{3}: \Delta_{7}, \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash (\Delta_{12}, \mathbf{p}_{11}), \mathbf{p}_{11}} \quad \vee_{L} \quad \frac{\bullet \mathbf{h}_{10}: (\Delta_{7}, \mathbf{F}_{8} \vee \mathbf{F}_{9}), \mathbf{p}_{11} \vdash \Delta_{12}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{10}: \Delta_{12}, \mathbf{p}_{11}} \quad I_{\text{Cut}} \quad \frac{-: \Delta_{7}, \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash \Delta_{12}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{10}: \Delta_{7}, \mathbf{F}_{9}, \mathbf{p}_{11} \vdash \Delta_{12}, \mathbf{p}_{11}} \quad I_{\text{hCut}} \quad \frac{\bullet \mathbf{h}_{10}: \Delta_{7}, \mathbf{F}_{9} \vdash \Delta_{12}, \mathbf{p}_{11}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{10}: \Delta_{7}, \mathbf{F}_{9}, \mathbf{p}_{11} \vdash \Delta_{12}, \mathbf{p}_{11}} \quad V_{L} \quad \frac{\bullet \mathbf{h}_{3}: \mathbf{F}_{8}, \Delta_{12}, \mathbf{p}_{11} \vdash \mathbf{h}_{3}: \mathbf{F}_{9}, \Delta_{13}, \mathbf{p}_{11} \vdash \mathbf{F}_{7}, \Delta_{12}, \mathbf{p}_{11}}{-: \Delta_{7}, \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash \Delta_{12}, \mathbf{p}_{11}} \quad V_{L} \quad \frac{\bullet \mathbf{h}_{3}: \mathbf{F}_{8}, \Delta_{13}, \mathbf{p}_{11} \vdash \mathbf{F}_{7}, \Delta_{12}, \mathbf{p}_{11} \vdash \mathbf{h}_{3}: \mathbf{F}_{9}, \Delta_{13}, \mathbf{p}_{11} \vdash \mathbf{F}_{7}, \Delta_{12}, \mathbf{p}_{11}}{\bullet} \quad \vee_{L} \quad \frac{\bullet \mathbf{h}_{10}: ((\Delta_{13}, \mathbf{p}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9}), \mathbf{F}_{7} \vdash \Delta_{12}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{10}: ((\Delta_{13}, \mathbf{p}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9}), \mathbf{F}_{7} \vdash \Delta_{12}, \mathbf{p}_{11}} \quad I_{\mathbf{Cut}} \quad \frac{\bullet \mathbf{h}_{10}: (\Delta_{13}, \mathbf{p}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash \Delta_{12}, \mathbf{p}_{11}}{-: (\Delta_{13}, \mathbf{p}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash \Delta_{12}, \mathbf{p}_{11}} \quad I_{\mathbf{Cut}} \quad \frac{\bullet \mathbf{h}_{10}: ((\Delta_{13}, \mathbf{h}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9}), \mathbf{F}_{7} \vdash \Delta_{12}, \mathbf{h}_{11}}{-: (\Delta_{13}, \mathbf{h}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash \Delta_{12}, \mathbf{h}_{11}} \quad I_{\mathbf{Cut}} \quad \frac{\bullet \mathbf{h}_{10}: ((\Delta_{13}, \mathbf{h}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9}), \mathbf{F}_{7} \vdash \Delta_{12}, \mathbf{h}_{11}}{-: (\Delta_{13}, \mathbf{h}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash \Delta_{12}, \mathbf{h}_{11}} \quad I_{\mathbf{Cut}} \quad \frac{\bullet \mathbf{h}_{10}: ((\Delta_{13}, \mathbf{h}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9}), \mathbf{F}_{7} \vdash \Delta_{12}, \mathbf{h}_{11}}{-: (\Delta_{13}, \mathbf{h}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash \Delta_{12}, \mathbf{h}_{11}} \quad I_{\mathbf{Cut}} \quad \frac{\bullet \mathbf{h}_{10}: ((\Delta_{13}, \mathbf{h}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9}), \mathbf{F}_{7} \vdash \Delta_{12}, \mathbf{h}_{11}}{-: (\Delta_{13}, \mathbf{h}_{11}), \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash \Delta_{12}, \mathbf{h}_{11}} \quad I_{\mathbf{Cut}} \quad \frac{\bullet \mathbf{h}_{10}: (\Delta_{13}, \mathbf{h}_{11}), \mathbf{h}_{11} \vee \mathbf{h}_{12} \vee \mathbf{h}_{11}}{-: (\Delta_{13}, \mathbf{h}_{11}), \mathbf{h}$$

$$\frac{\mathbf{h}_{3}: \mathsf{F}_{8}, \Delta_{7} \vdash \top, \Delta_{11} \quad \mathbf{h}_{3}: \mathsf{F}_{9}, \Delta_{7} \vdash \top, \Delta_{11}}{\bullet_{\mathbf{h}_{10}}: \Delta_{7}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{11}} \quad \vee_{L} \quad \frac{\mathbf{h}_{10}: \Delta_{7}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{11}}{\bullet_{\mathbf{h}_{10}}: (\Delta_{7}, \mathsf{F}_{8} \lor \mathsf{F}_{9}), \top \vdash \Delta_{11}} \quad \top_{L} \quad \mathsf{cut}} \\ -: \Delta_{7}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{11} \quad \mathsf{ax/W}} \\ \frac{\mathbf{h}_{3}: \mathsf{F}_{8}, \top, \Delta_{12} \vdash \mathsf{F}_{7}, \Delta_{11} \quad \mathbf{h}_{3}: \mathsf{F}_{9}, \top, \Delta_{12} \vdash \mathsf{F}_{7}, \Delta_{11}}{\bullet_{11}} \quad \mathsf{ax/W}}{\bullet_{\mathbf{h}_{3}}: (\top, \Delta_{12}), \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{11}} \quad \vee_{L} \quad \frac{\mathbf{h}_{10}: \mathsf{F}_{7}, \Delta_{12}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{11}}{\bullet_{\mathbf{h}_{10}}: ((\top, \Delta_{12}), \mathsf{F}_{8} \lor \mathsf{F}_{9}), \mathsf{F}_{7} \vdash \Delta_{11}} \quad \top_{L} \\ -: (\top, \Delta_{12}), \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{11} \\ & -: (\top, \Delta_{12}), \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{11} \\ \hline \bullet_{\mathbf{h}_{3}}: \top, \Delta_{12}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{11}, \mathsf{F}_{7} \quad \mathsf{ax/W} \\ & -: \top, \Delta_{12}, \mathsf{F}_{8} \lor \mathsf{F}_{9} \vdash \Delta_{11} \\ \hline \end{pmatrix} \quad \mathsf{hCut}$$

## 6.11 Status of AT: OK

• Case rule  $\rightarrow_R$ 

$$\frac{\frac{\mathbf{h}_{3} : \mathbf{F}_{8}, \Delta_{7}, []\mathbf{F}_{8} \vdash \mathbf{F}_{6}, \Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12}}{\mathbf{h}_{3} : \Delta_{7}, []\mathbf{F}_{8} \vdash (\Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12}), \mathbf{F}_{6}} \quad AT \quad \frac{\mathbf{h}_{9} : \mathbf{F}_{6}, \mathbf{F}_{11}, \Delta_{7}, []\mathbf{F}_{8} \vdash \mathbf{F}_{12}, \Delta_{10}}{\mathbf{h}_{9} : (\Delta_{7}, []\mathbf{F}_{8}), \mathbf{F}_{6} \vdash \Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12}} \quad Cut} \\ - : \Delta_{7}, []\mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12} \quad \\ \frac{\mathbf{h}_{3} : \Delta_{7}, \mathbf{F}_{8}, []\mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{F}_{6}, \mathbf{F}_{8}, []\mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12}}{\mathbf{h}_{7} : \Delta_{7}, \mathbf{F}_{8}, []\mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{F}_{11} \to \mathbf{F}_{12}} \quad AT} \quad \frac{\mathbf{ax/W}}{\mathbf{h}_{7}}$$

• Case rule  $\wedge_R$ 

$$\frac{\mathbf{h}_{3}: \mathsf{F}_{8}, \Delta_{7}, []\mathsf{F}_{8} \vdash \mathsf{F}_{6}, \Delta_{10}, \mathsf{F}_{11} \wedge \mathsf{F}_{12}]}{\bullet \mathsf{h}_{3}: \Delta_{7}, []\mathsf{F}_{8} \vdash (\Delta_{10}, \mathsf{F}_{11} \wedge \mathsf{F}_{12}), \mathsf{F}_{6}} \quad AT \quad \frac{\mathbf{h}_{9}: \mathsf{F}_{6}, \Delta_{7}, []\mathsf{F}_{8} \vdash \mathsf{F}_{11}, \Delta_{10} \quad \mathsf{h}_{9}: \mathsf{F}_{6}, \Delta_{7}, []\mathsf{F}_{8} \vdash \mathsf{F}_{12}, \Delta_{10}}{\bullet \mathsf{h}_{9}: (\Delta_{7}, []\mathsf{F}_{8}), \mathsf{F}_{6} \vdash \Delta_{10}, \mathsf{F}_{11} \wedge \mathsf{F}_{12}} \quad \mathsf{Cut} \\ \frac{-: \Delta_{7}, []\mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{F}_{11} \wedge \mathsf{F}_{12}}{\bullet \mathsf{h}_{9}: \Delta_{7}, \mathsf{F}_{6}, \mathsf{F}_{8}, []\mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{F}_{11} \wedge \mathsf{F}_{12}} \quad \mathsf{ax/W} \\ \frac{-: \Delta_{7}, \mathsf{F}_{8}, []\mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{F}_{11} \wedge \mathsf{F}_{12}}{-: \Delta_{7}, []\mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{F}_{11} \wedge \mathsf{F}_{12}} \quad ATG \\ \bullet \mathsf{h}_{9}: \mathsf{h}_{11} \wedge \mathsf{h}_{12} \quad \mathsf{h}_{12} \wedge \mathsf{h}_{13} \wedge \mathsf{h}_{14} \wedge \mathsf{h}_{14} \wedge \mathsf{h}_{14} \wedge \mathsf{h}_{14} \wedge \mathsf{h}_{14} \wedge \mathsf{h}_{14}} \quad \mathsf{h}_{14} \wedge \mathsf{h}_{15} \wedge$$

• Case rule  $\vee_R$ 

$$\frac{\frac{\mathbf{h}_{3}: \mathbf{F}_{8}, \Delta_{7}, []\mathbf{F}_{8} \vdash \mathbf{F}_{6}, \Delta_{10}, \mathbf{F}_{11} \vee \mathbf{F}_{12}}{\bullet \mathbf{h}_{3}: \Delta_{7}, []\mathbf{F}_{8} \vdash (\Delta_{10}, \mathbf{F}_{11} \vee \mathbf{F}_{12}), \mathbf{F}_{6}} \quad AT \quad \frac{\mathbf{h}_{9}: \mathbf{F}_{6}, \Delta_{7}, []\mathbf{F}_{8} \vdash \mathbf{F}_{11}, \mathbf{F}_{12}, \Delta_{10}}{\bullet \mathbf{h}_{9}: (\Delta_{7}, []\mathbf{F}_{8}), \mathbf{F}_{6} \vdash \Delta_{10}, \mathbf{F}_{11} \vee \mathbf{F}_{12}} \quad Cut} \\ \frac{-: \Delta_{7}, []\mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{F}_{11} \vee \mathbf{F}_{12}}{\bullet \mathbf{h}_{9}: \Delta_{7}, \mathbf{F}_{6}, \mathbf{F}_{8}, []\mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{F}_{11} \vee \mathbf{F}_{12}} \quad AT} \\ \frac{-: \Delta_{7}, \mathbf{F}_{8}, []\mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{F}_{11} \vee \mathbf{F}_{12}}{-: \Delta_{7}, []\mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{F}_{11} \vee \mathbf{F}_{12}} \quad AT}$$

• Case rule  $\perp_R$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_8, \Delta_7, \left[\!\left[\mathbf{F}_8 \vdash \mathbf{F}_6, \bot, \Delta_{10}\right.\right]}{\bullet \mathbf{h}_3: \Delta_7, \left[\!\left[\mathbf{F}_8 \vdash (\bot, \Delta_{10}), \mathbf{F}_6\right.\right]} AT \quad \frac{\mathbf{h}_9: \mathbf{F}_6, \Delta_7, \left[\!\left[\mathbf{F}_8 \vdash \Delta_{10}\right.\right.\right]}{\bullet \mathbf{h}_9: \left(\Delta_7, \left[\!\left[\mathbf{F}_8\right), \mathbf{F}_6 \vdash \bot, \Delta_{10}\right.\right]} \quad \frac{\bot_R}{\mathsf{Cut}} \\ \frac{-: \Delta_7, \left[\!\left[\mathbf{F}_8 \vdash \bot, \Delta_{10}\right.\right]}{\bullet \mathbf{h}_3: \Delta_7, \left[\!\left[\mathbf{F}_8 \vdash \bot, \Delta_{10}, \mathbf{F}_6\right.\right]} \quad \frac{\mathsf{ax/W}}{\mathsf{h}_9: \Delta_7, \mathbf{F}_6, \left[\!\left[\mathbf{F}_8 \vdash \bot, \Delta_{10}\right.\right]} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}}$$

• Case rule  $\top_R$ 

$$\frac{\mathbf{h}_3: \mathbf{F}_8, \Delta_7, []\mathbf{F}_8 \vdash \mathbf{F}_6, \top, \Delta_{10}}{\bullet \mathbf{h}_3: \Delta_7, []\mathbf{F}_8 \vdash (\top, \Delta_{10}), \mathbf{F}_6} \quad AT \quad \xrightarrow{\bullet \mathbf{h}_9: (\Delta_7, []\mathbf{F}_8), \mathbf{F}_6 \vdash \top, \Delta_{10}} \quad \xrightarrow{\frown} \quad \mathbf{Cut} \\ \frac{-: \Delta_7, []\mathbf{F}_8 \vdash \top, \Delta_{10}}{-: \Delta_7, []\mathbf{F}_8 \vdash \top, \Delta_{10}} \quad \top_R$$

 $\bullet$  Case rule K

$$\begin{array}{c} \frac{h_{3}:F_{7},(\square\Gamma_{12},\Delta_{9}),[|F_{7}|\vdash\square F_{6},\Delta_{10},[|F_{11}}{\bullet h_{3}:(\square\Gamma_{12},\Delta_{9}),[|F_{7}|\vdash(\Delta_{10},[|F_{11}),\square F_{6}]} AT & \frac{h_{8}:F_{7},unbox(\square\Gamma_{12}),unbox(\square F_{6})\vdash F_{11}}{\bullet h_{3}:(\square\Gamma_{12},\Delta_{9}),[|F_{7}|\vdash\square F_{6},\Delta_{10},[|F_{11}]} X \\ & -:(\square\Gamma_{12},\Delta_{9}),[|F_{7}|\vdash\Delta_{10},[|F_{11}] \\ \hline \\ \frac{h_{3}:\Delta_{9},F_{7},\square\Gamma_{12},[|F_{7}|\vdash\square F_{6},\Delta_{10},[|F_{11}]}{\bullet h_{3}:F_{7},\square\Gamma_{12},[|F_{7}|\vdash\Delta_{10},[|F_{11}]} ATG \\ \hline \\ \frac{h_{3}:F_{7},(\square\Gamma_{11},\Delta_{12}),[|F_{7}|\vdash\square F_{6},\Delta_{9},[|F_{10}]}{\bullet h_{3}:(\square\Gamma_{11},\Delta_{12}),[|F_{7}|\vdash\Delta_{10},[|F_{11}]} ATG \\ \hline \\ \frac{h_{3}:F_{7},(\square\Gamma_{11},\Delta_{12}),[|F_{7}|\vdash\alpha_{9},Q|,|F_{10}]}{\bullet h_{3}:(\square\Gamma_{11},\Delta_{12}),[|F_{7}|\vdash\square F_{6},\Delta_{9},[|F_{10}]]} X \\ \hline \\ \frac{h_{3}:F_{7},(\square\Gamma_{11},\Delta_{12}),[|F_{7}|\vdash\alpha_{9},Q|,|F_{10}]}{\bullet h_{3}:(\square\Gamma_{11},\Delta_{12}),[|F_{7}|\vdash\alpha_{9},Q|,|F_{10}]} X \\ \hline \\ \frac{h_{3}:F_{7},(\square\Gamma_{11},[|F_{7}|\vdash\alpha_{9},Q|,|F_{10}]}{\bullet h_{3}:(\square\Gamma_{11},\Delta_{12}),[|F_{7}|\vdash\alpha_{9},Q|,|F_{10}]} X \\ \hline \\ \frac{h_{3}:F_{7},(\square\Gamma_{11},[|F_{7}|\vdash\alpha_{9},Q|,|F_{10}]}{\bullet h_{3}:(\square\Gamma_{11},[|F_{7}|\vdash\alpha_{9},Q|,|F_{10}]} X \\ \hline \\ \frac{h_{3}:F_{7},(\square\Gamma_{11},[|F_{7}|\vdash\alpha_{9},Q|,|F_{11}|,|F_{7}|\vdash\alpha_{9},|F_{10}|,|F_{11}|,|F_{7}|\vdash\alpha_{9},|F_{10}|,|F_{11}|,|F_{7}|\vdash\alpha_{9},|F_{1$$

• Case rule A45

$$\frac{\mathbf{h}_3: \mathsf{F}_7, (\Box \Gamma_{13}, \Delta_9), (]\mathsf{F}_7 \vdash \Box \mathsf{F}_6, \Box \Gamma_{10}, \Delta_{11}, []\mathsf{F}_{12}}{\bullet \mathsf{h}_3: (\Box \Gamma_{13}, \Delta_9), (]\mathsf{F}_7 \vdash (\Box \Gamma_{10}, \Delta_{11}, []\mathsf{F}_{12}), \Box \mathsf{F}_6} \ AT \ \frac{\mathbf{h}_8: \Box \Gamma_{13}, \Box \mathsf{F}_6, (]\mathsf{F}_7 \vdash \Box \Gamma_{10}, \mathsf{F}_{12}}{\bullet \mathsf{h}_8: ((\Box \Gamma_{13}, \Delta_9), []\mathsf{F}_7), \Box \mathsf{F}_6 \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathsf{F}_{12}} \ Cut \\ -: (\Box \Gamma_{13}, \Delta_9), (]\mathsf{F}_7 \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathsf{F}_{12} \\ \frac{\mathsf{h}_3: \Delta_9, \mathsf{F}_7, \Box \Gamma_{13}, []\mathsf{F}_7 \vdash \Box \mathsf{F}_6, \Delta_{11}, \Box \Gamma_{10}, []\mathsf{F}_{12}}{\bullet \mathsf{h}_8: \Box \mathsf{F}_6, \Delta_9, \mathsf{F}_7, \Box \Gamma_{13}, []\mathsf{F}_7 \vdash \Delta_{11}, \Box \Gamma_{10}, []\mathsf{F}_{12}} \ ATG \\ \frac{-: \Delta_9, \Gamma_7, \Box \Gamma_{13}, []\mathsf{F}_7 \vdash \Delta_{11}, \Box \Gamma_{10}, []\mathsf{F}_{12}}{-: \Delta_9, \Box \Gamma_{13}, []\mathsf{F}_7 \vdash \Delta_{11}, \Box \Gamma_{10}, []\mathsf{F}_{12}} \ ATG \\ \end{array}$$

$$\begin{array}{c} \frac{h_{3}:F_{7},(\Box\Gamma_{12},\Delta_{13}),[]F_{7}\vdash F_{6},\Box\Gamma_{9},\Delta_{10},[]F_{11}]}{\bullet h_{3}:(\Box\Gamma_{12},\Delta_{13}),[]F_{7}\vdash(\Box\Gamma_{9},\Delta_{10},[]F_{11}),F_{6}} & AT & \frac{h_{8}:(\Box\Gamma_{12},\Box_{13}),[]F_{7}\vdash\Box\Gamma_{9},F_{11}}{\bullet h_{8}:((\Box\Gamma_{12},\Delta_{13}),[]F_{7}),F_{6}\vdash\Box\Gamma_{9},\Delta_{10},[]F_{11}} & A45 \\ & -:(\Box\Gamma_{12},\Delta_{13}),[]F_{7}\vdash\Box\Gamma_{9},\Delta_{10},[]F_{11} & A45 \\ & -:\Box\Gamma_{12},[]F_{7}\vdashF_{11},\Box\Gamma_{9} & ax/W \\ & -:\Delta_{13},\Box\Gamma_{12},[]F_{7}\vdash\Delta_{10},\Box\Gamma_{9},[]F_{11} & A45 \\ & \frac{h_{3}:F_{7},(\Box\Gamma_{12},\Delta_{13}),[]F_{7}\vdash\BoxF_{6},\Box\Gamma_{9},\Delta_{10},[]F_{11}}{\bullet h_{3}:(\Box\Gamma_{12},\Delta_{13}),[]F_{7}\vdash(\Box\Gamma_{9},\Delta_{10},[]F_{11}),\Box F_{6}} & AT & \frac{h_{8}:\Box\Gamma_{12},\Box F_{6}\vdash\Box\Gamma_{9},F_{11}}{\bullet h_{8}:((\Box\Gamma_{12},\Delta_{13}),[]F_{7}\vdash\Box F_{9},\Delta_{10},[]F_{11}} & A45 \\ & -:(\Box\Gamma_{12},\Delta_{13}),[]F_{7}\vdash\Box\Gamma_{9},\Delta_{10},[]F_{11} & ax/W & \bullet_{h_{8}:((\Box\Gamma_{12},\Delta_{13}),[]F_{7}\vdash\Delta_{10},\Box\Gamma_{9},[]F_{11}} \\ & \frac{-:\Delta_{13},F_{7},\Box\Gamma_{12},[]F_{7}\vdash\Delta_{10},\Box\Gamma_{9},[]F_{11}}{\bullet h_{3}:(\Box\Gamma_{9},\Delta_{13}),[]F_{7}\vdash F_{6},\Box\Gamma_{10},\Delta_{11},[]F_{12}} & AT & \frac{h_{8}:\Box\Gamma_{9}\vdash\Box\Gamma_{10},F_{12}}{\bullet h_{3}:(\Box\Gamma_{9},\Delta_{13}),[]F_{7}\vdash F_{6},\Box\Gamma_{10},\Delta_{11},[]F_{12}} & AT & \frac{h_{8}:\Box\Gamma_{9}\vdash\Box\Gamma_{10},F_{12}}{\bullet h_{3}:(\Box\Gamma_{9},\Delta_{13}),[]F_{7}\vdash G_{10},\Delta_{11},[]F_{12}} & A45 \\ & \frac{-:\Delta_{13},\Box\Gamma_{12},\Box\Gamma_{10}}{\bullet h_{3}:(\Box\Gamma_{9},\Delta_{13}),[]F_{7}\vdash G_{10},\Delta_{11},[]F_{12}} & A45 \\ & \frac{-:\Box\Gamma_{9}\vdash F_{12},\Box\Gamma_{10}}{\bullet h_{3}:(\Box\Gamma_{9},\Delta_{13}),[]F$$

$$\frac{\mathbf{h}_{3}: \mathsf{F}_{7}, \Delta_{6}, []\mathsf{F}_{7} \vdash \mathsf{F}_{9} \to \mathsf{F}_{10}, \Delta_{11}}{\bullet_{11}, \mathsf{F}_{9} \to \mathsf{F}_{10}} \quad AT \quad \frac{\mathbf{h}_{8}: \Delta_{6}, []\mathsf{F}_{7} \vdash \mathsf{F}_{9}, \Delta_{11} \quad \mathsf{h}_{8}: \mathsf{F}_{10}, \Delta_{6}, []\mathsf{F}_{7} \vdash \Delta_{11}}{\bullet_{18}: (\Delta_{6}, []\mathsf{F}_{7}), \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11}} \quad \mathsf{Cut} \\ \\ -: \Delta_{6}, []\mathsf{F}_{7} \vdash \Delta_{11} \\ \hline \frac{\mathbf{h}_{3}: \Delta_{6}, \mathsf{F}_{7}, []\mathsf{F}_{7} \vdash \Delta_{11}, \mathsf{F}_{9} \to \mathsf{F}_{10}}{\bullet_{10}} \quad \mathsf{ax}/\mathsf{W}} \quad \frac{\mathsf{ax}/\mathsf{W}}{\bullet_{18}: \Delta_{6}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11}} \quad \mathsf{ax}/\mathsf{W}}{\bullet_{10}\mathsf{L}} \quad \mathsf{hCut} \\ \hline \frac{-: \Delta_{6}, \mathsf{F}_{7}, []\mathsf{F}_{7} \vdash \Delta_{11}}{-: \Delta_{6}, []\mathsf{F}_{7} \vdash \Delta_{11}} \quad AT \\ \hline \bullet_{13}: \mathsf{F}_{7}, (\Delta_{12}, \mathsf{F}_{9} \to \mathsf{F}_{10}), []\mathsf{F}_{7} \vdash \mathsf{F}_{6}, \Delta_{11}}{\bullet_{11}, \mathsf{F}_{6}} \quad AT \quad \frac{\mathsf{h}_{8}: \mathsf{F}_{6}, \Delta_{12}, []\mathsf{F}_{7} \vdash \mathsf{F}_{9}, \Delta_{11} \quad \mathsf{h}_{8}: \mathsf{F}_{6}, \mathsf{F}_{10}, \Delta_{12}, []\mathsf{F}_{7} \vdash \Delta_{11}}{\bullet_{11}} \quad \mathsf{Cut} \\ \hline -: (\Delta_{12}, \mathsf{F}_{9} \to \mathsf{F}_{10}), []\mathsf{F}_{7} \vdash \Delta_{11}} \quad \mathsf{Cut} \\ \hline \bullet_{13}: \Delta_{12}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11}, \mathsf{F}_{6}} \quad \mathsf{ax}/\mathsf{W} \quad \bullet_{18}: \Delta_{12}, \mathsf{F}_{6}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11}} \quad \mathsf{ATG} \\ \hline \bullet_{13}: \Delta_{12}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11}, \mathsf{F}_{6}} \quad \mathsf{ax}/\mathsf{W} \quad \bullet_{18}: \Delta_{12}, \mathsf{F}_{6}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11}} \quad \mathsf{ATG} \\ \hline \bullet_{13}: \Delta_{12}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11} \quad \mathsf{ATG} \\ \hline \bullet_{13}: \Delta_{12}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11} \quad \mathsf{ATG} \\ \hline \bullet_{13}: \Delta_{12}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11} \quad \mathsf{ATG} \\ \hline \bullet_{13}: \Delta_{12}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11} \quad \mathsf{ATG} \\ \hline \bullet_{13}: \Delta_{12}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11} \quad \mathsf{ATG} \\ \hline \bullet_{13}: \Delta_{12}, \mathsf{F}_{7}, []\mathsf{F}_{7}, \mathsf{F}_{9} \to \mathsf{F}_{10} \vdash \Delta_{11} \quad \mathsf{ATG} \\ \hline \bullet_{13}: \Delta_{12}: \mathsf{F}_{13}: \Delta_{12}: \mathsf{F}_{13}: \Delta_{13}: \Delta_{13$$

• Case rule  $\wedge_L$ 

$$\frac{\begin{array}{l} \frac{h_3:F_7,\Delta_6, \left[|F_7\vdash F_9\wedge F_{10},\Delta_{11}\right]}{\bullet h_3:\Delta_6, \left[|F_7\vdash \Delta_{11},F_9\wedge F_{10}\right]} AT & \frac{h_8:F_9,F_{10},\Delta_6, \left[|F_7\vdash \Delta_{11}\right]}{\bullet h_8:(\Delta_6, \left[|F_7),F_9\wedge F_{10}\vdash \Delta_{11}\right]} \\ & \frac{h_3:\Delta_6, \left[|F_7\vdash \Delta_{11},F_9\wedge F_{10}\right]}{\bullet h_8:\Delta_6,F_7, \left[|F_7\vdash \Delta_{11}\right]} & \frac{\lambda_L}{\cot} \\ \hline \\ \frac{h_3:\Delta_6,F_7, \left[|F_7\vdash \Delta_{11},F_9\wedge F_{10}\right]}{\bullet h_8:\Delta_6,F_7, \left[|F_7\vdash \Delta_{11}\right]} ATG \\ \hline \\ \frac{h_3:F_7, (\Delta_{12},F_9\wedge F_{10}), \left[|F_7\vdash F_6,\Delta_{11}\right]}{\bullet h_8:(\Delta_{12},F_9\wedge F_{10}), \left[|F_7\vdash A_{11}\right]} ATG \\ \hline \\ \frac{h_3:F_7, (\Delta_{12},F_9\wedge F_{10}), \left[|F_7\vdash A_{11},F_6\right]}{\bullet h_8:(\Delta_{12},F_9\wedge F_{10}), \left[|F_7\vdash A_{11}\right]} \\ -:(\Delta_{12},F_9\wedge F_{10}), \left[|F_7\vdash A_{11}\right]} \\ \hline \\ \frac{h_8:F_6,F_9,F_{10},\Delta_{12}, \left[|F_7\vdash A_{11}\right]}{\bullet h_8:((\Delta_{12},F_9\wedge F_{10}), \left[|F_7\vdash A_{11}\right]} \\ \hline \\ \frac{h_8:\Delta_{12},F_7, \left[|F_7,F_9\wedge F_{10}\vdash A_{11}\right]}{\bullet h_8:\Delta_{12},F_6,F_7, \left[|F_7,F_9\wedge F_{10}\vdash A_{11}\right]} \\ \hline \\ \frac{-:\Delta_{12},F_7, \left[|F_7,F_9\wedge F_{10}\vdash A_{11}\right]}{-:\Delta_{12}, \left[|F_7,F_9\wedge F_{10}\vdash A_{11}\right]} ATG \\ \hline \end{array}$$

• Case rule  $\vee_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_{3}: F_{7}, \Delta_{6}, []F_{7} \vdash F_{9} \vee F_{10}, \Delta_{11}]}{\bullet \mathbf{h}_{3}: \Delta_{6}, []F_{7} \vdash \Delta_{11} & \mathbf{h}_{8}: F_{10}, \Delta_{6}, []F_{7} \vdash \Delta_{11}} & \vee_{L} \\ \hline \bullet \mathbf{h}_{3}: \Delta_{6}, []F_{7} \vdash \Delta_{11}, F_{9} \vee F_{10} & \mathbf{a}_{1} \\ \hline & -: \Delta_{6}, []F_{7} \vdash \Delta_{11} \\ \hline & \frac{\mathbf{h}_{3}: \Delta_{6}, F_{7}, []F_{7} \vdash \Delta_{11}, F_{9} \vee F_{10}} \\ \hline & \mathbf{a}_{1} \vee \mathbf{a}_{1} \vee \mathbf{a}_{1} \vee \mathbf{a}_{1} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{6}, F_{7}, []F_{7} \vdash \Delta_{11}, F_{9} \vee F_{10}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{6}, F_{7}, []F_{7} \vdash \Delta_{11}, F_{9} \vee F_{10} \vdash \Delta_{11}} \\ \hline & \frac{-: \Delta_{6}, F_{7}, []F_{7} \vdash \Delta_{11}}{-: \Delta_{6}, []F_{7} \vdash \Delta_{11}} & AT \\ \hline & \frac{\mathbf{h}_{3}: F_{7}, (\Delta_{12}, F_{9} \vee F_{10}), []F_{7} \vdash F_{6}, \Delta_{11}} {-: \Delta_{11}, F_{6}} & AT \\ \hline & \frac{\mathbf{h}_{8}: F_{6}, F_{9}, \Delta_{12}, []F_{7} \vdash \Delta_{11}} {\bullet \mathbf{h}_{8}: ((\Delta_{12}, F_{9} \vee F_{10}), []F_{7}), F_{6} \vdash \Delta_{11}} \\ \hline & -: (\Delta_{12}, F_{9} \vee F_{10}), []F_{7} \vdash \Delta_{11} & \bullet \\ \hline & -: (\Delta_{12}, F_{9} \vee F_{10}), []F_{7} \vdash \Delta_{11} & \bullet \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, []F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{8}: \Delta_{12}, F_{6}, F_{7}, []F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, []F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{8}: \Delta_{12}, F_{6}, F_{7}, []F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, []F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{9}: \Delta_{12}, F_{6}, F_{7}, []F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, []F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{9}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, []F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{9}: \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{9}: \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{9}: \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{9}: \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{9}: \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{7}, F_{9} \vee F_{10} \vdash \Delta_{11}} {\bullet \mathbf{h}_{9}: \Delta_{11}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{13} \vee \mathbf{h}_{13}} {\bullet \mathbf{h}_{13}} \\ \hline & \frac{\mathbf{a}_{1}: \Delta_{12}, F_{13} \vee \mathbf{h}_{13}} {\bullet \mathbf{$$

#### $\bullet$ Case rule AT

$$\frac{ \frac{\mathbf{h}_{3} : \mathbf{F}_{7}, \Delta_{6}, []\mathbf{F}_{7} \vdash []\mathbf{F}_{9}, \Delta_{10}}{\bullet \mathbf{h}_{3} : \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{10}, []\mathbf{F}_{9}} \quad AT \quad \frac{\mathbf{h}_{8} : \mathbf{F}_{9}, \Delta_{6}, []\mathbf{F}_{7}, []\mathbf{F}_{9} \vdash \Delta_{10}}{\bullet \mathbf{h}_{8} : (\Delta_{6}, []\mathbf{F}_{7}), []\mathbf{F}_{9} \vdash \Delta_{10}} \quad AT \quad \mathbf{Cut} \\ \hline - : \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{10} \\ \hline \frac{\mathbf{h}_{3} : \Delta_{6}, \mathbf{F}_{7}, []\mathbf{F}_{7} \vdash \Delta_{10}, []\mathbf{F}_{9}}{\bullet \mathbf{h}_{8} : \Delta_{6}, \mathbf{F}_{7}, []\mathbf{F}_{7} \vdash \Delta_{10}} \quad \mathbf{AT} \\ \hline - : \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{10} \quad AT \\ \hline - : \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{10} \quad AT \\ \hline - : \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{10} \quad AT \\ \hline - : \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7} \vdash \mathbf{F}_{6}, \Delta_{10} \quad \mathbf{AT} \\ \hline - : (\Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : (\Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{11}, []\mathbf{F}_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{11}, []\mathbf{F}_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{ATG} \\ \hline - : \Delta_{11}, []\mathbf{F}_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{ATG} \\ \hline - : \Delta_{11}, []\mathbf{F}_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{ATG} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \quad \mathbf{h}_{8} : \mathbf{F}_{6}, \mathbf{F}_{9}, \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \quad \mathbf{h}_{8} : \Delta_{7}, \mathbf{F}_{6}, \mathbf{F}_{9}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \quad \mathbf{h}_{8} : \Delta_{7}, \mathbf{F}_{6}, \mathbf{F}_{9}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \quad \mathbf{h}_{8} : \Delta_{7}, \mathbf{F}_{6}, \mathbf{F}_{9}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \quad \mathbf{h}_{8} : \Delta_{7}, \mathbf{F}_{6}, \mathbf{F}_{9}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \quad \mathbf{h}_{8} : \Delta_{7}, \mathbf{F}_{6}, \mathbf{F}_{9}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{7}, []\mathbf{F}_{9} \vdash \Delta_{10} \quad \mathbf{AT} \\ \hline - : \Delta_{1}, []\mathbf{F}_{9} \vdash \Delta_{10}$$

## • Case rule $\perp_L$

$$\begin{array}{c} \frac{\mathbf{h}_3: \mathbf{F}_7, \Delta_6, \left[ \left[ \mathbf{F}_7 \vdash \bot, \Delta_9 \right] }{\bullet \mathbf{h}_3: \Delta_6, \left[ \left[ \mathbf{F}_7 \vdash \Delta_9, \bot \right] } \ AT & \frac{\bullet \mathbf{h}_8: \left( \Delta_6, \left[ \left[ \mathbf{F}_7 \right), \bot \vdash \Delta_9 \right] }{\bullet \mathbf{h}_8: \Delta_6, \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \\ \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_7, \left[ \left[ \mathbf{F}_7 \vdash \bot, \Delta_9 \right] }{\bullet \mathbf{h}_8: \Delta_6, \mathbf{F}_7, \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \\ \frac{-: \Delta_6, \mathbf{F}_7, \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] }{-: \Delta_6, \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{AT}{\bullet \mathbf{h}_8: \left( \left( \bot, \Delta_{10} \right), \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \\ \frac{\bullet \mathbf{h}_3: \mathbf{F}_7, \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9, \mathbf{F}_6 \right] \right] }{\bullet \mathbf{h}_3: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9, \mathbf{F}_6 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_3: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9, \mathbf{F}_6 \right] \right] } \\ \frac{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] }{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \\ \frac{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] }{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \\ \frac{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] }{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \\ \frac{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] }{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \\ \frac{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] }{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \\ \frac{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] }{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \\ \frac{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] }{-: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \bot, \Delta_{10} \right), \left[ \left[ \mathbf{F}_7 \vdash \Delta_9 \right] \right] } \frac{\bot_L}{\bullet \mathbf{h}_5: \left( \bot, \Delta$$

#### $\bullet$ Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_{3}: \mathbf{F}_{7}, \Delta_{6}, []\mathbf{F}_{7} \vdash \mathbf{p}_{9}, \Delta_{10}, \mathbf{p}_{9}}{\bullet \mathbf{h}_{3}: \Delta_{6}, []\mathbf{F}_{7} \vdash (\Delta_{10}, \mathbf{p}_{9}), \mathbf{p}_{9}} \quad AT \quad & \bullet \mathbf{h}_{8}: (\Delta_{6}, []\mathbf{F}_{7}), \mathbf{p}_{9} \vdash \Delta_{10}, \mathbf{p}_{9} \quad I \\ \hline & -: \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{10}, \mathbf{p}_{9} \quad \text{Cut} \\ \hline \\ \frac{\mathbf{h}_{3}: \Delta_{6}, \mathbf{F}_{7}, []\mathbf{F}_{7} \vdash \Delta_{10}, \mathbf{p}_{9}, \mathbf{p}_{9} \quad \text{ax/W}}{\bullet \mathbf{h}_{8}: \Delta_{6}, \mathbf{F}_{7}, \mathbf{p}_{9}, []\mathbf{F}_{7} \vdash \Delta_{10}, \mathbf{p}_{9} \quad \mathbf{h}_{Cut} \\ \hline & \frac{-: \Delta_{6}, \mathbf{F}_{7}, []\mathbf{F}_{7} \vdash \Delta_{10}, \mathbf{p}_{9} \quad ATG}{\bullet \mathbf{h}_{8}: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{7} \vdash \mathbf{F}_{6}, \Delta_{10}, \mathbf{p}_{9} \quad ATG} \\ \hline & \bullet \mathbf{h}_{3}: \mathbf{F}_{7}, (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{7} \vdash \mathbf{F}_{6}, \Delta_{10}, \mathbf{p}_{9} \quad ATG \quad & \bullet \mathbf{h}_{8}: ((\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{7}), \mathbf{F}_{6} \vdash \Delta_{10}, \mathbf{p}_{9} \\ \hline & \bullet \mathbf{h}_{3}: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{7} \vdash (\Delta_{10}, \mathbf{p}_{9}), []\mathbf{F}_{7} \vdash \Delta_{10}, \mathbf{p}_{9} \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{7} \vdash \Delta_{10}, \mathbf{p}_{9} \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{7} \vdash \Delta_{10}, \mathbf{p}_{9} \\ \hline & -: \Delta_{11}, \mathbf{p}_{9}, []\mathbf{F}_{7} \vdash \Delta_{10}, \mathbf{p}_{9} \\ \hline \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_{3}: \mathbf{F}_{7}, \Delta_{6}, []\mathbf{F}_{7} \vdash \top, \Delta_{9}}{\bullet \mathbf{h}_{3}: \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{9}, \top} \quad AT \quad \frac{\mathbf{h}_{8}: \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{9}}{\bullet \mathbf{h}_{8}: (\Delta_{6}, []\mathbf{F}_{7}), \top \vdash \Delta_{9}} \quad \mathbf{T}_{L} \\ \hline -: \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{9} \\ \hline -: \Delta_{6}, []\mathbf{F}_{7} \vdash \Delta_{9} \\ \hline \bullet \mathbf{h}_{3}: \mathbf{F}_{7}, (\top, \Delta_{10}), []\mathbf{F}_{7} \vdash \mathbf{F}_{6}, \Delta_{9} \\ \bullet \mathbf{h}_{3}: (\top, \Delta_{10}), []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline -: (\top, \Delta_{10}), []\mathbf{F}_{7} \vdash \Delta_{9} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \top, \Delta_{10}, []\mathbf{F}_{7} \vdash \Delta_{9}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{3}: \mathbf$$

# 6.12 Status of $\perp_L$ : OK

• Case rule  $\rightarrow_R$ 

$$\frac{ \underbrace{\bullet \mathbf{h}_3 : \bot, \Delta_6 \vdash (\Delta_8, \mathbf{F}_9 \rightarrow \mathbf{F}_{10}), \mathbf{F}_5}_{\bullet \mathbf{h}_3 : \bot, \Delta_6 \vdash (\Delta_8, \mathbf{F}_9 \rightarrow \mathbf{F}_{10}), \mathbf{F}_5} \ \bot_L \ \frac{ \mathbf{h}_7 : \bot, \mathbf{F}_5, \mathbf{F}_9, \Delta_6 \vdash \mathbf{F}_{10}, \Delta_8 }{\bullet \mathbf{h}_7 : (\bot, \Delta_6), \mathbf{F}_5 \vdash \Delta_8, \mathbf{F}_9 \rightarrow \mathbf{F}_{10}} \ \mathsf{Cut} }{ - : \bot, \Delta_6 \vdash \Delta_8, \mathbf{F}_9 \rightarrow \mathbf{F}_{10} } \ \bot_L$$

• Case rule  $\wedge_R$ 

$$\frac{\bullet_{\mathbf{h}_3}: \bot, \Delta_6 \vdash (\Delta_8, \mathsf{F}_9 \land \mathsf{F}_{10}), \mathsf{F}_5}{-: \bot, \Delta_6 \vdash \Delta_8, \mathsf{F}_9 \land \mathsf{F}_{10}} \ \bot_L \ \frac{\mathsf{h}_7: \bot, \mathsf{F}_5, \Delta_6 \vdash \mathsf{F}_9, \Delta_8 \quad \mathsf{h}_7: \bot, \mathsf{F}_5, \Delta_6 \vdash \mathsf{F}_{10}, \Delta_8}{\bullet \mathsf{h}_7: (\bot, \Delta_6), \mathsf{F}_5 \vdash \Delta_8, \mathsf{F}_9 \land \mathsf{F}_{10}} \ \mathsf{Cut} \\ -: \bot, \Delta_6 \vdash \Delta_8, \mathsf{F}_9 \land \mathsf{F}_{10} \ \\ -: \bot, \Delta_6 \vdash \Delta_8, \mathsf{F}_9 \land \mathsf{F}_{10} \ \bot_L$$

• Case rule  $\vee_R$ 

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h3} : \bot, \Delta_6 \vdash (\Delta_8, F_9 \vee F_{10}), F_5 \end{array}}_{\bullet h_7 : \bot, F_5, \Delta_6 \vdash F_9, F_{10}, \Delta_8} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : (\bot, \Delta_6), F_5 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : (\bot, \Delta_6), F_5 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{-: \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10}} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_{h7} : \bot, \Delta_6 \vdash \Delta_8, F_9 \vee F_{10} \end{array}}_{\bullet L} \underbrace{ \begin{array}{c} \vee_R \\ \bullet_R$$

• Case rule  $\perp_R$ 

$$\begin{array}{c|c} \bullet_{\textbf{h}_3}: \bot, \Delta_6 \vdash (\bot, \Delta_8), F_5 & \bot_L & \frac{\textbf{h}_7: \bot, F_5, \Delta_6 \vdash \Delta_8}{\bullet \textbf{h}_7: (\bot, \Delta_6), F_5 \vdash \bot, \Delta_8} & \bot_R \\ \hline -: \bot, \Delta_6 \vdash \bot, \Delta_8 & & \\ \hline -: \bot, \Delta_6 \vdash \bot, \Delta_8 & & \bot_L \end{array}$$

$$\begin{array}{c|c} \bullet_{\mathbf{h}_3}: \bot, \Delta_6 \vdash (\top, \Delta_8), \mathsf{F}_5 & \bot_L & \hline \bullet_{\mathbf{h}_7}: (\bot, \Delta_6), \mathsf{F}_5 \vdash \top, \Delta_8 \\ \hline -: \bot, \Delta_6 \vdash \top, \Delta_8 & & \\ \hline -: \bot, \Delta_6 \vdash \top, \Delta_8 & & \\ \hline -: \bot, \Delta_6 \vdash \top, \Delta_8 & & \\ \hline \end{array}$$

ullet Case rule K

$$\begin{array}{c|c} \underline{\bullet_{h_3}:\bot,\Box\Gamma_9,\Delta_{10}\vdash(\Delta_7,[]F_8),\Box F_5} & \bot_L & \frac{h_6:unbox(\Box\Gamma_9),unbox(\Box F_5)\vdash F_8}{\bullet h_6:(\bot,\Box\Gamma_9,\Delta_{10}),\Box F_5\vdash\Delta_7,[]F_8} & K\\ \underline{-:\bot,\Box\Gamma_9,\Delta_{10},\Box\Gamma_9\vdash\Delta_7,[]F_8} & \bot_L \\ \hline \\ \underline{-:\bot,\Delta_{10},\Box\Gamma_9\vdash\Delta_7,[]F_8} & \bot_L \\ \hline \\ \underline{\bullet_{h_3}:\bot,\Box\Gamma_7,\Delta_{10}\vdash(\Delta_8,[]F_9),F_5} & \bot_L & \frac{h_6:unbox(\Box\Gamma_7)\vdash F_9}{\bullet h_6:(\bot,\Box\Gamma_7,\Delta_{10}),F_5\vdash\Delta_8,[]F_9} & K\\ \underline{-:\bot,\Box\Gamma_7,\Delta_{10}\vdash\Delta_8,[]F_9} & \underline{\smile}\\ \underline{-:\bot,\Delta_{10},\Box\Gamma_7\vdash\Delta_8,[]F_9} & \bot_L \end{array}$$

 $\bullet$  Case rule A45

• Case rule  $\rightarrow_L$ 

• Case rule  $\wedge_L$ 

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_3} : \bot, \Delta_5 \vdash \Delta_9, F_7 \land F_8 \end{array}}_{\bullet h_6} \ \bot_L \ \begin{array}{c} \frac{h_6 : \bot, F_7, F_8, \Delta_5 \vdash \Delta_9}{\bullet h_6 : (\bot, \Delta_5), F_7 \land F_8 \vdash \Delta_9} \\ - : \bot, \Delta_5 \vdash \Delta_9 \\ \hline - : \bot, \Delta_5 \vdash \Delta_9 \end{array} \ \begin{array}{c} \wedge_L \\ \text{Cut} \end{array}$$

$$\frac{ \bullet_{h_3}: \bot, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5}{-: \bot, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \stackrel{\bot_L}{\bullet_{h_6}: (\bot, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \\ \stackrel{\longleftarrow}{-: \bot, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \stackrel{\bot_L}{-: \bot, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \stackrel{\bot_L}{}$$

 $\bullet$  Case rule  $\vee_L$ 

ullet Case rule AT

$$\begin{array}{c|c} \bullet_{\mathbf{h}_3}: \bot, \Delta_5 \vdash \Delta_8, []{\mathsf{F}_7} & \bot_L & \frac{\mathsf{h}_6: \bot, \mathsf{F}_7, \Delta_5, []{\mathsf{F}_7} \vdash \Delta_8}{\bullet \mathsf{h}_6: (\bot, \Delta_5), []{\mathsf{F}_7} \vdash \Delta_8} & AT \\ \hline & -: \bot, \Delta_5 \vdash \Delta_8 & \mathsf{Cut} \\ \hline & & -: \bot, \Delta_5 \vdash \Delta_8 & \bot_L \\ \hline \\ \bullet_{\mathbf{h}_3}: \bot, \Delta_9, []{\mathsf{F}_7} \vdash \Delta_8, \mathsf{F}_5 & \bot_L & \frac{\mathsf{h}_6: \bot, \mathsf{F}_5, \mathsf{F}_7, \Delta_9, []{\mathsf{F}_7} \vdash \Delta_8}{\bullet \mathsf{h}_6: (\bot, \Delta_9, []{\mathsf{F}_7}), \mathsf{F}_5 \vdash \Delta_8} & AT \\ \hline & -: \bot, \Delta_9, []{\mathsf{F}_7} \vdash \Delta_8 & \mathsf{Cut} \\ \hline & & -: \bot, \Delta_9, []{\mathsf{F}_7} \vdash \Delta_8 & \bot_L \\ \hline \hline & & -: \bot, \Delta_9, []{\mathsf{F}_7} \vdash \Delta_8 & \bot_L \\ \hline \end{array}$$

• Case rule  $\perp_L$ 

ullet Case rule I

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_3}: \bot, \Delta_5 \vdash (\Delta_8, \mathbf{p}_7), \mathbf{p}_7 & \bot_L & \hline \bullet_{\mathbf{h}_6}: (\bot, \Delta_5), \mathbf{p}_7 \vdash \Delta_8, \mathbf{p}_7 & \mathbf{Cut} \\ \hline & -: \bot, \Delta_5 \vdash \Delta_8, \mathbf{p}_7 & \\ \hline & \hline & -: \bot, \Delta_5 \vdash \Delta_8, \mathbf{p}_7 & \bot_L \\ \hline \hline \bullet_{\mathbf{h}_3}: \bot, \Delta_9, \mathbf{p}_7 \vdash (\Delta_8, \mathbf{p}_7), \mathbf{F}_5 & \bot_L & \bullet_{\mathbf{h}_6}: (\bot, \Delta_9, \mathbf{p}_7), \mathbf{F}_5 \vdash \Delta_8, \mathbf{p}_7 & \\ \hline & -: \bot, \Delta_9, \mathbf{p}_7 \vdash \Delta_8, \mathbf{p}_7 & \\ \hline & -: \bot, \Delta_9, \mathbf{p}_7 \vdash \Delta_8, \mathbf{p}_7 & \bot_L & \\ \hline \hline & -: \bot, \Delta_9, \mathbf{p}_7 \vdash \Delta_8, \mathbf{p}_7 & \bot_L & \\ \hline \end{array}$$

• Case rule  $\top_L$ 

$$\begin{array}{c|c} \underline{\bullet \mathbf{h}_{3}: \bot, \top, \Delta_{8} \vdash \Delta_{7}, F_{5}} & \bot_{L} & \frac{\mathbf{h}_{6}: \bot, F_{5}, \Delta_{8} \vdash \Delta_{7}}{\bullet \mathbf{h}_{6}: (\bot, \top, \Delta_{8}), F_{5} \vdash \Delta_{7}} & \top_{L} \\ \hline & -: \bot, \top, \Delta_{8} \vdash \Delta_{7} & \\ \hline & \overline{-: \bot, \top, \Delta_{8} \vdash \Delta_{7}} & \bot_{L} \end{array}$$

# 6.13 Status of I: OK

• Case rule  $\rightarrow_R$ 

$$\begin{array}{c} \underbrace{ \begin{array}{c} \bullet_{h_1} : \Delta_5, p_6 \vdash (\Delta_8, F_9 \to F_{10}), p_6 \\ \hline \\ \bullet_{h_2} : \Delta_5, p_6 \vdash (\Delta_8, F_9 \to F_{10}), p_6 \\ \hline \\ - : \Delta_5, p_6 \vdash \Delta_8, F_9 \to F_{10} \\ \hline \end{array}}_{\bullet h_7} \underbrace{ \begin{array}{c} \bullet_{h_7} : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \to F_{10} \\ \hline \\ \bullet_{h_1} : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10}, p_6 \\ \hline \\ \hline \\ - : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10} \\ \hline \\ - : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10} \\ \hline \\ - : \Delta_5, p_6 \vdash \Delta_8, F_{9} \to F_{10} \\ \hline \end{array}}_{\bullet h_2} \underbrace{ \begin{array}{c} \bullet_{h_1} : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10} \\ \hline \\ - : \Delta_5, p_6 \vdash \Delta_8, F_{9} \to F_{10} \\ \hline \\ - : \Delta_5, p_6 \vdash \Delta_8, F_{9} \to F_{10} \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_6, F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8 \\ \hline \\ \bullet_{h_2} : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \to F_{11}), p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \to F_{11}), p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \to F_{11} \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_6, F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8 \\ \hline \\ \bullet_{h_9} : (\Delta_7, p_8), F_6 \vdash (\Delta_{12}, F_{10} \to F_{11}), p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \to F_{11}), p_8 \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_6, F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \to F_{11}), p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \to F_{11} \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_6, F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \to F_{11} \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_6, F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \to F_{11} \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_6, F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_{12}, p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash \Delta_{12}, p_8, F_{10} \to F_{11} \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_6, F_{10}, \Delta_8, F_{10} \\ \hline \\ \bullet_{h_9} : F_{10}, \Delta_7, p_8 \vdash F_{10}, \Delta_7, p_8 \vdash F_{11}, \Delta_7, p_8 \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_{10}, \Delta_7, p_8 \vdash A_1, \Delta_7, p_8 \\ \hline \\ \bullet \underbrace{ \begin{array}{c} \bullet_{h_9} : F_{10}, \Delta_7, p_8 \vdash A_1, \Delta_7, p_8 \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_{10}, \Delta_7, p_8 \vdash A_1, \Delta_7, p_8 \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_{10}, \Delta_7, p_8 \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_{10}, \Delta_7, p_8 \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_{10}, \Delta_7, p_8 \\ \hline \end{array}_{\bullet h_9} \underbrace{ \begin{array}{c} \bullet_{h_9} : F_{10}, \Delta_7, p_8 \\ \hline \end{array}_{\bullet h_9$$

• Case rule  $\wedge_R$ 

$$\frac{\frac{\bullet_{h_1}:\Delta_5,p_6 \vdash (\Delta_8,F_9 \land F_{10}),p_6}{\bullet_{h_1}:\Delta_5,p_6 \vdash (\Delta_8,F_9 \land F_{10}),p_6}}{I} \frac{I \xrightarrow{h_7:\Delta_5,p_6,p_6 \vdash F_9,\Delta_8 \quad h_7:\Delta_5,p_6,p_6 \vdash F_{10},\Delta_8} \bullet_{h_7}:\Delta_5,p_6,p_6 \vdash F_{10},\Delta_8} \land_R \\ \frac{-:\Delta_5,p_6 \vdash \Delta_8,F_9 \land F_{10}}{\bullet_{h_7}:\Delta_5,p_6 \vdash \Delta_8,F_9 \land F_{10}} \cot \\ \frac{-:\Delta_5,p_6 \vdash \Delta_8,F_9,p_6}{\bullet_{h_7}:\Delta_5,p_6 \vdash \Delta_8,F_9} \xrightarrow{\text{ax/W}} \underbrace{\bullet_{h_1}:\Delta_5,p_6 \vdash \Delta_8,F_{10},p_6}_{\bullet_{h_1}:\Delta_5,p_6 \vdash \Delta_8,F_{10}} I \xrightarrow{h_7:\Delta_5,p_6,p_6 \vdash \Delta_8,F_{10}} \land_R \\ \frac{-:\Delta_5,p_6 \vdash \Delta_8,F_9}{\bullet_{h_7}:\Delta_5,p_6 \vdash \Delta_8,F_9 \land F_{10}} \land_R \\ \frac{-:\Delta_5,p_6 \vdash \Delta_8,F_9 \land F_{10}}{\bullet_{h_9}:(\Delta_7,p_8 \vdash F_{10},\Delta_{12},p_8 \quad h_9:F_6,\Delta_7,p_8 \vdash F_{11},\Delta_{12},p_8} \land_R \\ \frac{\bullet_{h_2}:\Delta_7,p_8 \vdash ((\Delta_{12},F_{10} \land F_{11}),p_8)}{\bullet_{h_9}:(\Delta_7,p_8) \vdash (\Delta_{12},F_{10} \land F_{11}),p_8} \cot \\ \frac{\bullet_{h_7}:\Delta_7,p_8 \vdash (\Delta_{12},F_{10} \land F_{11}),p_8}{\bullet_{h_7}:\Delta_7,p_8 \vdash \Delta_{12},p_8,F_{10} \land F_{11}} I$$

• Case rule  $\vee_R$ 

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \Delta_5, p_6 \vdash (\Delta_8, F_9 \vee F_{10}), p_6 \\ - : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10} \\ \hline \\ - : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10} \\ \hline \end{array} }_{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \vee F_{10}} \begin{array}{c} \vee_R \\ \text{Cut} \\ \hline \\ \bullet_{h_1} : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_{9}, p_6 \\ \hline \\ \hline \\ - : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9 \\ \hline \\ - : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9 \\ \hline \\ \hline \\ - : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9 \\ \hline \\ \hline \\ - : \Delta_5, p_6 \vdash \Delta_8, F_{9} \vee F_{10} \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_1} : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9 \\ \hline \\ \bullet_{h_2} : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \vee F_{11}), p_8), F_6 \\ \hline \\ \bullet_{h_2} : \Delta_7, p_8 \vdash ((\Delta_{12}, F_{10} \vee F_{11}), p_8), F_6 \\ \hline \\ - : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8 \\ \hline \\ \hline \\ - : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8 \\ \hline \\ - : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_{12}, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \Delta_7, p_8 \vdash (\Delta_1, p_8, F_{10} \vee F_{11}), p_8 \\ \hline \end{array}$$

• Case rule  $\perp_R$ 

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \Delta_5, \mathbf{p}_6 \vdash (\top, \Delta_8), \mathbf{p}_6 & I & \hline \bullet_{\mathbf{h}_7}: (\Delta_5, \mathbf{p}_6), \mathbf{p}_6 \vdash \top, \Delta_8 & \mathsf{Cut} \\ \hline & -: \Delta_5, \mathbf{p}_6 \vdash \top, \Delta_8 & \mathsf{Cut} \\ \hline & \overline{\phantom{a}}: \Delta_5, \mathbf{p}_6 \vdash \top, \Delta_8 & \top_R \\ \hline \hline \bullet_{\mathbf{h}_2}: \Delta_7, \mathbf{p}_8 \vdash ((\top, \Delta_{10}), \mathbf{p}_8), \mathbf{F}_6 & I & \bullet_{\mathbf{h}_9}: (\Delta_7, \mathbf{p}_8), \mathbf{F}_6 \vdash (\top, \Delta_{10}), \mathbf{p}_8 & \mathsf{T}_R \\ \hline & -: \Delta_7, \mathbf{p}_8 \vdash (\top, \Delta_{10}), \mathbf{p}_8 & \mathsf{Cut} \\ \hline & \overline{\phantom{a}}: \Delta_7, \mathbf{p}_8 \vdash \top, \Delta_{10}, \mathbf{p}_8 & \top_R \\ \hline \end{array}$$

 $\bullet$  Case rule K

$$\begin{array}{c} \underbrace{\bullet h_1 : (\Box \Gamma_7, \Delta_{10}), p_5 \vdash (\Delta_8, []F_9), p_5}_{\bullet h_6 : ((\Box \Gamma_7, \Delta_{10}), p_5), p_5 \vdash \Delta_8, []F_9} \\ - : (\Box \Gamma_7, \Delta_{10}), p_5 \vdash \Delta_8, []F_9 \\ \hline \\ - : unbox(\Box \Gamma_7) \vdash F_9 \\ \hline \\ - : \Delta_{10}, \Box \Gamma_7, p_5 \vdash \Delta_8, []F_9 \\ \hline \\ \bullet h_2 : (\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash ((\Delta_{10}, []F_9), p_7), \Box F_6 \\ \hline \\ - : (\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash (\Delta_{10}, []F_9), p_7 \\ \hline \\ \bullet h_2 : (\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash ((\Delta_{10}, []F_9), p_7), \Box F_6 \\ \hline \\ \bullet h_8 : (U\Box \Gamma_{11}, \Delta_{12}), p_7 \vdash (\Delta_{10}, []F_9), p_7 \\ \hline \\ - : \Delta_{12}, \Box \Gamma_{11}, p_7 \vdash \Delta_{10}, p_7, []F_9 \\ \hline \\ \bullet h_8 : unbox(\Box \Gamma_9) \vdash F_{10} \\ \hline \\ \bullet h_8 : (\Box \Gamma_9, \Delta_{12}), p_7 \vdash (\Delta_{11}, []F_{10}), p_7 \\ \hline \\ - : (\Box \Gamma_9, \Delta_{12}), p_7 \vdash (\Delta_{11}, []F_{10}), p_7 \\ \hline \\ - : (\Box \Gamma_9, \Delta_{12}), p_7 \vdash (\Delta_{11}, []F_{10}), p_7 \\ \hline \\ - : \Delta_{12}, \Box \Gamma_9, p_7 \vdash \Delta_{11}, p_7, []F_{10} \\ \hline \end{array}$$

• Case rule A45

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : (\Box \Gamma_7, \Delta_{11}), p_5 \vdash (\Box \Gamma_8, \Delta_9, [] F_{10}), p_5}_{\bullet h_6 : ((\Box \Gamma_7, \Delta_{11}), p_5), p_5 \vdash \Box \Gamma_8, \Delta_9, [] F_{10}} \\ - : (\Box \Gamma_7, \Delta_{11}), p_5 \vdash \Box \Gamma_8, \Delta_9, [] F_{10} \\ \\ \hline \\ - : \Box \Gamma_7 \vdash F_{10}, \Box \Gamma_8 \\ \hline \\ - : \Delta_{11}, \Box \Gamma_7, p_5 \vdash \Delta_9, \Box \Gamma_8, [] F_{10} \\ \hline \\ \bullet_{h_2} : (\Box \Gamma_{12}, \Delta_{13}), p_7 \vdash ((\Box \Gamma_9, \Delta_{11}, [] F_{10}), p_7), \Box F_6 \\ \hline \\ - : (\Box \Gamma_{12}, \Delta_{13}), p_7 \vdash (\Box \Gamma_9, \Delta_{11}, [] F_{10}), p_7 \\ \hline \\ - : (\Box \Gamma_{12}, \Delta_{13}), p_7 \vdash (\Box \Gamma_9, \Delta_{11}, [] F_{10}), p_7 \\ \hline \\ - : (\Box \Gamma_{12}, \Delta_{13}), p_7 \vdash (\Box \Gamma_9, \Delta_{11}, [] F_{10}), p_7 \\ \hline \\ - : (\Box \Gamma_{12}, \Delta_{13}), D_7 \vdash (\Box \Gamma_9, \Delta_{11}, [] F_{10}), p_7 \\ \hline \\ - : (\Box \Gamma_{12}, \Delta_{13}), D_7 \vdash (\Box \Gamma_9, \Delta_{11}, [] F_{10}), p_7 \\ \hline \\ - : (\Box \Gamma_{12}, \Delta_{13}), D_7 \vdash (\Box \Gamma_9, \Delta_{11}, [] F_{10}), p_7 \\ \hline \\ \hline \end{array}$$

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_2} : (\Box \Gamma_9, \Delta_{13}), p_7 \vdash ((\Box \Gamma_{10}, \Delta_{12}, []F_{11}), p_7), F_6 \\ - : (\Box \Gamma_9, \Delta_{13}), p_7 \vdash ((\Box \Gamma_{10}, \Delta_{12}, []F_{11}), p_7 \\ \hline \\ - : (\Delta_{13}, \Box \Gamma_9, p_7 \vdash \Delta_{12}, \Box \Gamma_{10}, p_7, []F_{11} \\ \hline \end{array} } \begin{array}{c} h_8 : (\Box \Gamma_9 \vdash \Box \Gamma_{10}, F_{11}) \\ \bullet_{h_8} : ((\Box \Gamma_9, \Delta_{13}), p_7), F_6 \vdash (\Box \Gamma_{10}, \Delta_{12}, []F_{11}), p_7 \\ \hline \\ - : \Delta_{13}, \Box \Gamma_9, p_7 \vdash \Delta_{12}, \Box \Gamma_{10}, p_7, []F_{11} \\ \end{array} \begin{array}{c} A45 \\ \text{Cut} \\ \hline \end{array}$$

$$\frac{\underbrace{\bullet_{h_1}: (\Delta_{10}, F_7 \to F_8), p_5 \vdash \Delta_9, p_5}_{\bullet h_1: (\Delta_{10}, F_7 \to F_8), p_5 \vdash \Delta_9, p_5} I \xrightarrow{\bullet_{h_6}: (\Delta_{10}, F_7 \to F_8), p_5 \vdash F_7, \Delta_9}_{\bullet h_6: ((\Delta_{10}, F_7 \to F_8), p_5), p_5 \vdash \Delta_9} Cut} \xrightarrow{\bullet_{h_1}: \Delta_{10}, p_5 \vdash \Delta_9, F_7} I \xrightarrow{h_6: \Delta_{10}, p_5, p_5 \vdash \Delta_9, F_7}_{\bullet_{h_6}: \Delta_{10}, p_5, p_5 \vdash \Delta_9, F_7} \underbrace{\bullet_{h_1}: \Delta_{10}, p_5 \vdash \Delta_9, p_5}_{\bullet h_0: ut} I \xrightarrow{\bullet_{h_1}: \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5} I \xrightarrow{h_6: \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9}_{\bullet_{h_2}: \Delta_{10}, p_5 \vdash \Delta_9, F_7} \to L$$

$$\underbrace{\bullet_{h_1}: \Delta_{10}, p_5 \vdash \Delta_9, F_7}_{\bullet L_{10}, p_5 \vdash \Delta_9, F_7} I \xrightarrow{\bullet_{h_1}: \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5}_{\bullet L_{10}, p_5 \vdash \Delta_9, p_5} I \xrightarrow{h_6: \Delta_{10}, F_8, p_5 \vdash \Delta_9}_{\bullet L_{10}} \to L$$

$$\underbrace{\bullet_{h_1}: \Delta_{10}, p_5 \vdash \Delta_9, F_7}_{\bullet L_{10}, p_5 \vdash \Delta_9, F_7} I \xrightarrow{\bullet_{h_1}: \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5}_{\bullet L_{10}: \Delta_1, F_8, p_5 \vdash \Delta_9} \to L$$

$$\underbrace{\bullet_{h_1}: \Delta_{10}, p_5 \vdash \Delta_9, F_7}_{\bullet L_{10}, p_5 \vdash \Delta_9, F_7} I \xrightarrow{\bullet_{h_1}: \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5}_{\bullet L_{10}: \Delta_1, F_8, p_5 \vdash \Delta_9} \to L$$

$$\underbrace{\bullet_{h_1}: \Delta_{10}, p_5 \vdash \Delta_9, F_7}_{\bullet L_{10}: \Delta_1, p_5 \vdash \Delta_8, p_7} I \xrightarrow{\bullet_{h_2}: \Delta_1, p_7 \vdash \Delta_8, p_7}_{\bullet L_{10}: \Delta_1, p_7 \vdash \Delta_8, p_7}_{\bullet L_{10}: \Delta_1, p_7 \vdash \Delta_8, p_7}} Cut$$

$$\underbrace{\bullet_{h_2}: \Delta_{10}, p_5 \vdash \Delta_9, p_7}_{\bullet L_{10}: \Delta_1, p_7 \vdash \Delta_8, p_7} I \xrightarrow{\bullet_{h_2}: \Delta_1, p_7 \vdash \Delta_8, p_7}_{\bullet L_{10}: \Delta_1, p_7 \vdash \Delta_8, p_7}}_{\bullet L_{10}: \Delta_1, p_7 \vdash \Delta_8, p_7}_{\bullet L_{10}: \Delta_1, p_7 \vdash \Delta_8, p_$$

• Case rule  $\wedge_L$ 

$$\begin{array}{c} \frac{\bullet \mathbf{h}_1 : (\Delta_{10}, \mathbf{F}_7 \wedge \mathbf{F}_8), \mathbf{p}_5 \vdash \Delta_9, \mathbf{p}_5}{\bullet \mathbf{h}_6 : ((\Delta_{10}, \mathbf{F}_7 \wedge \mathbf{F}_8), \mathbf{p}_5), \mathbf{p}_5 \vdash \Delta_9} \\ - : (\Delta_{10}, \mathbf{F}_7 \wedge \mathbf{F}_8), \mathbf{p}_5 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_1 : \Delta_{10}, \mathbf{F}_7, \mathbf{F}_8, \mathbf{p}_5 \vdash \Delta_9, \mathbf{p}_5} & I & \overset{\sim}{\mathbf{h}_6} : ((\Delta_{10}, \mathbf{F}_7 \wedge \mathbf{F}_8), \mathbf{p}_5), \mathbf{p}_5 \vdash \Delta_9} \\ \hline \bullet \mathbf{h}_1 : \Delta_{10}, \mathbf{F}_7, \mathbf{F}_8, \mathbf{p}_5 \vdash \Delta_9, \mathbf{p}_5} & I & \overset{\sim}{\mathbf{h}_6} : \Delta_{10}, \mathbf{F}_7, \mathbf{F}_8, \mathbf{p}_5, \mathbf{p}_5 \vdash \Delta_9} \\ \hline \bullet \mathbf{h}_1 : \Delta_{10}, \mathbf{F}_7, \mathbf{F}_8, \mathbf{p}_5 \vdash \Delta_9, \mathbf{p}_5} & I & \overset{\leftarrow}{\mathbf{h}_6} : \Delta_{10}, \mathbf{F}_7, \mathbf{F}_8, \mathbf{p}_5, \mathbf{p}_5 \vdash \Delta_9} \\ \hline \bullet \mathbf{h}_2 : \Delta_{10}, \mathbf{p}_5, \mathbf{F}_7 \wedge \mathbf{F}_8, \mathbf{p}_5 \vdash \Delta_9} & \wedge_L \\ \hline \bullet \mathbf{h}_2 : \Delta_6, \mathbf{p}_7 \vdash (\Delta_8, \mathbf{p}_7), \mathbf{F}_{10} \wedge \mathbf{F}_{11} & I & \overset{\bullet}{\mathbf{h}_9} : \mathbf{F}_{10}, \mathbf{F}_{11}, \Delta_6, \mathbf{p}_7 \vdash \Delta_8, \mathbf{p}_7} \\ \hline \bullet \mathbf{h}_2 : \Delta_6, \mathbf{p}_7 \vdash (\Delta_8, \mathbf{p}_7), \mathbf{F}_{10} \wedge \mathbf{F}_{11} & \bullet \mathbf{h}_9 : \mathbf{F}_{6}, \mathbf{F}_{10}, \mathbf{F}_{11} \vdash \Delta_8, \mathbf{p}_7} & \Delta_L \\ \hline \bullet \mathbf{h}_2 : (\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{p}_7 \vdash (\Delta_8, \mathbf{p}_7), \mathbf{F}_6} & I & \overset{\bullet}{\mathbf{h}_9} : \mathbf{F}_{6}, \mathbf{F}_{10}, \mathbf{F}_{11}, \Delta_{12}, \mathbf{p}_7 \vdash \Delta_8, \mathbf{p}_7} \\ \hline \bullet \mathbf{h}_2 : (\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{p}_7 \vdash (\Delta_8, \mathbf{p}_7), \mathbf{F}_6} & I & \overset{\bullet}{\mathbf{h}_9} : ((\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{p}_7), \mathbf{F}_6 \vdash \Delta_8, \mathbf{p}_7} \\ \hline \bullet \mathbf{h}_2 : (\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{p}_7 \vdash \Delta_8, \mathbf{p}_7} & I & \overset{\bullet}{\mathbf{h}_9} : (\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{p}_7), \mathbf{F}_6 \vdash \Delta_8, \mathbf{p}_7} & \Delta_L \\ \hline \bullet \mathbf{h}_2 : (\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{p}_7 \vdash \Delta_8, \mathbf{p}_7} & I & \overset{\bullet}{\mathbf{h}_9} : (\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{p}_7), \mathbf{F}_6 \vdash \Delta_8, \mathbf{p}_7} & \Delta_L \\ \hline \bullet \mathbf{h}_2 : (\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{h}_7 \vdash \Delta_8, \mathbf{p}_7} & I & \overset{\bullet}{\mathbf{h}_9} : (\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{h}_7, \mathbf{h}_7 \vdash \Delta_8, \mathbf{h}_7} & I & \overset{\bullet}{\mathbf{h}_9} : (\Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}), \mathbf{h}_7, \mathbf{h}_7 \vdash \Delta_8, \mathbf{h}_7 & \Delta_1, \mathbf{h}_7 \vdash \Delta_8, \mathbf{h}_7 &$$

• Case rule  $\vee_L$ 

$$\frac{\bullet \mathbf{h}_1 : (\Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8), \mathbf{p}_5 \vdash \Delta_9, \mathbf{p}_5}{I} \xrightarrow{\begin{array}{c} \mathbf{h}_6 : \mathbf{F}_7, \Delta_{10}, \mathbf{p}_5, \mathbf{p}_5 \vdash \Delta_9 & \mathbf{h}_6 : \mathbf{F}_8, \Delta_{10}, \mathbf{p}_5, \mathbf{p}_5 \vdash \Delta_9 \\ \bullet \mathbf{h}_6 : ((\Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8), \mathbf{p}_5), \mathbf{p}_5 \vdash \Delta_9 & \mathbf{Cut} \\ \\ \hline -: (\Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8), \mathbf{p}_5 \vdash \Delta_9 & \\ \hline \bullet \mathbf{h}_1 : \Delta_{10}, \mathbf{F}_7, \mathbf{p}_5 \vdash \Delta_9, \mathbf{p}_5 & I & \\ \hline -: \Delta_{10}, \mathbf{F}_7, \mathbf{p}_5 \vdash \Delta_9 & \mathbf{ax/W} \\ \hline -: \Delta_{10}, \mathbf{F}_7, \mathbf{p}_5 \vdash \Delta_9 & \\ \hline -: \Delta_{10}, \mathbf{F}_7, \mathbf{p}_5 \vdash \Delta_9 & \\ \hline -: \Delta_{10}, \mathbf{F}_8, \mathbf{p}_5 \vdash \Delta_9 & \\ \hline -: \Delta_{10}, \mathbf{F}_8, \mathbf{p}_5 \vdash \Delta_9 & \\ \hline -: \Delta_{10}, \mathbf{F}_8, \mathbf{p}_5 \vdash \Delta_9 & \\ \hline \end{array} \\ \downarrow_L$$

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_2} : \Delta_6, \mathsf{p}_7 \vdash (\Delta_8, \mathsf{p}_7), \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \bullet \mathsf{h}_9 : \mathsf{F}_{10}, \Delta_6, \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 & \mathsf{h}_9 : \mathsf{F}_{11}, \Delta_6, \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \bullet \mathsf{h}_9 : (\Delta_6, \mathsf{p}_7), \mathsf{F}_{10} \vee \mathsf{F}_{11} \vdash \Delta_8, \mathsf{p}_7 \\ & - : \Delta_6, \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_2 : (\Delta_6, \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7) & I \\ \hline \\ \bullet \mathsf{h}_2 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash (\Delta_8, \mathsf{p}_7), \mathsf{F}_6 & I \\ \hline \\ \bullet \mathsf{h}_2 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash (\Delta_8, \mathsf{p}_7), \mathsf{F}_6 & I \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{p}_7 \vdash \Delta_8, \mathsf{p}_7 \\ \hline \\ \bullet \mathsf{h}_3 : (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{h}_3 : \mathsf{h}_3$$

 $\bullet$  Case rule AT

$$\begin{array}{c} \frac{\bullet \mathbf{h}_1 : (\Delta_9, [] \mathbf{F}_7), \mathbf{p}_5 \vdash \Delta_8, \mathbf{p}_5}{\bullet \mathbf{h}_1 : (\Delta_9, [] \mathbf{F}_7), \mathbf{p}_5 \vdash \Delta_8, \mathbf{p}_5} & I & \frac{\mathbf{h}_6 : \mathbf{F}_7, \Delta_9, \mathbf{p}_5, \mathbf{p}_5, [] \mathbf{F}_7 \vdash \Delta_8}{\bullet \mathbf{h}_6 : ((\Delta_9, [] \mathbf{F}_7), \mathbf{p}_5), \mathbf{p}_5 \vdash \Delta_8} & AT \\ & - : (\Delta_9, [] \mathbf{F}_7), \mathbf{p}_5 \vdash \Delta_8 & \\ \hline \bullet \mathbf{h}_1 : \Delta_9, \mathbf{F}_7, \mathbf{p}_5, [] \mathbf{F}_7 \vdash \Delta_8, \mathbf{p}_5} & \mathbf{ax/W} & \mathbf{h}_6 : \Delta_9, \mathbf{F}_7, \mathbf{p}_5, \mathbf{p}_5, [] \mathbf{F}_7 \vdash \Delta_8} \\ \hline \bullet \mathbf{h}_1 : \Delta_9, \mathbf{F}_7, \mathbf{p}_5, [] \mathbf{F}_7 \vdash \Delta_8, \mathbf{p}_7 & \mathbf{h}_7, \mathbf{p}_7, \mathbf{p}_7$$

• Case rule  $\perp_L$ 

$$\begin{array}{c|c} \hline \bullet_{h_1}: (\bot, \Delta_8), p_5 \vdash \Delta_7, p_5 & I & \hline \bullet_{h_6}: ((\bot, \Delta_8), p_5), p_5 \vdash \Delta_7 \\ \hline & -: (\bot, \Delta_8), p_5 \vdash \Delta_7 \\ \hline & \hline & -: \bot, \Delta_8, p_5 \vdash \Delta_7 \\ \hline \hline \bullet_{h_2}: \Delta_6, p_7 \vdash (\Delta_8, p_7), \bot & I & \hline \bullet_{h_9}: (\Delta_6, p_7), \bot \vdash \Delta_8, p_7 \\ \hline & -: \Delta_6, p_7 \vdash \Delta_8, p_7 \\ \hline & \hline & -: \Delta_6, p_7 \vdash \Delta_8, p_7 \\ \hline \hline & \bullet_{h_2}: (\bot, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6 & I & \hline \bullet_{h_9}: ((\bot, \Delta_{10}), p_7), F_6 \vdash \Delta_8, p_7 \\ \hline & -: (\bot, \Delta_{10}), p_7 \vdash \Delta_8, p_7 \\ \hline & \hline & -: \bot, \Delta_{10}, p_7 \vdash \Delta_8, p_7 \\ \hline & \hline & -: \bot, \Delta_{10}, p_7 \vdash \Delta_8, p_7 \\ \hline \end{array}$$

ullet Case rule I

# 6.14 Status of $\top_L$ : OK

• Case rule  $\rightarrow_R$ 

$$\frac{\begin{array}{l} \mathbf{h}_3: \Delta_6 \vdash \mathbf{F}_5, \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \\ \bullet \mathbf{h}_3: \top, \Delta_6 \vdash (\Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10}), \mathbf{F}_5 \end{array} \top_L \begin{array}{l} \mathbf{h}_7: \top, \mathbf{F}_5, \mathbf{F}_9, \Delta_6 \vdash \mathbf{F}_{10}, \Delta_8 \\ \bullet \mathbf{h}_7: (\top, \Delta_6), \mathbf{F}_5 \vdash \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \end{array}}{-: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10}} \begin{array}{l} \rightarrow_R \\ \mathsf{cut} \\ \bullet \mathbf{h}_7: \top, \Delta_6, \mathbf{F}_5 \vdash \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \end{array} \begin{array}{l} \rightarrow_R \\ \mathsf{cut} \\ \bullet \mathbf{h}_7: \top, \Delta_6, \mathbf{F}_5 \vdash \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \\ \bullet \mathbf{h}_7: \top, \Delta_6, \mathbf{F}_5 \vdash \Delta_8, \mathbf{F}_9 \to \mathbf{F}_{10} \end{array} \begin{array}{l} \mathsf{ax/W} \\ \mathsf{hCut} \end{array}$$

• Case rule  $\wedge_R$ 

$$\frac{ \begin{array}{c} \mathbf{h}_3: \Delta_6 \vdash \mathbf{F}_5, \Delta_8, \mathbf{F}_9 \land \mathbf{F}_{10} \\ \bullet \mathbf{h}_3: \top, \Delta_6 \vdash (\Delta_8, \mathbf{F}_9 \land \mathbf{F}_{10}), \mathbf{F}_5 \end{array} }{ -: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_9 \land \mathbf{F}_{10}} \ \, \mathbf{Cut} \\ \\ \hline -: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_9 \land \mathbf{F}_{10} \\ \hline \\ \frac{\mathbf{h}_3: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_5, \mathbf{F}_9 \land \mathbf{F}_{10}}{ \bullet \mathbf{h}_7: \top, \Delta_6, \mathbf{F}_5 \vdash \Delta_8, \mathbf{F}_9 \land \mathbf{F}_{10}} \ \, \mathbf{ax/W} \\ \hline \\ -: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_9 \land \mathbf{F}_{10} \end{array} } \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_7: \top, \Delta_6, \mathbf{F}_5 \vdash \Delta_8, \mathbf{F}_9 \land \mathbf{F}_{10} \end{array} \end{array} \\ \bullet \mathbf{h}_7: \nabla, \Delta_6 \vdash \Delta_8, \mathbf{F}_9 \land \mathbf{F}_{10} \end{array}$$

• Case rule  $\vee_R$ 

$$\frac{ \begin{array}{c} \mathbf{h}_3: \Delta_6 \vdash \mathbf{F}_5, \Delta_8, \mathbf{F}_9 \lor \mathbf{F}_{10} \\ \bullet \mathbf{h}_3: \top, \Delta_6 \vdash (\Delta_8, \mathbf{F}_9 \lor \mathbf{F}_{10}), \mathbf{F}_5 \end{array} }{ -: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_9 \lor \mathbf{F}_{10}} \begin{array}{c} \mathsf{L} \\ \bullet \mathbf{h}_7: \top, \mathbf{F}_5, \Delta_6 \vdash \mathbf{F}_9, \mathbf{F}_{10}, \Delta_8 \\ \bullet \mathbf{h}_7: (\top, \Delta_6), \mathbf{F}_5 \vdash \Delta_8, \mathbf{F}_9 \lor \mathbf{F}_{10} \end{array} } \begin{array}{c} \lor_R \\ \mathsf{Cut} \\ \\ \bullet \mathbf{h}_3: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_5, \mathbf{F}_9 \lor \mathbf{F}_{10} \end{array} \\ & \overset{\bullet}{\bullet} \mathbf{h}_7: \top, \Delta_6, \mathbf{F}_5 \vdash \Delta_8, \mathbf{F}_9 \lor \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_7: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_9 \lor \mathbf{F}_{10} \end{array} \\ \begin{array}{c} \mathsf{L} \mathsf{L} \\ \bullet \mathsf{L} \\ \mathsf{L} \\ \mathsf{L} \\ \bullet \mathsf{$$

• Case rule  $\perp_R$ 

$$\frac{\begin{array}{l} \mathbf{h}_3: \Delta_6 \vdash \mathbf{F}_5, \bot, \Delta_8 \\ \bullet \mathbf{h}_3: \top, \Delta_6 \vdash (\bot, \Delta_8), \mathbf{F}_5 \end{array} \top_L \quad \frac{\mathbf{h}_7: \top, \mathbf{F}_5, \Delta_6 \vdash \Delta_8}{\bullet \mathbf{h}_7: (\top, \Delta_6), \mathbf{F}_5 \vdash \bot, \Delta_8} \quad \frac{\bot_R}{\mathsf{Cut}} \\ \hline \\ -: \top, \Delta_6 \vdash \bot, \Delta_8 \\ \hline \\ \underline{\mathbf{h}_3: \top, \Delta_6 \vdash \bot, \Delta_8, \mathbf{F}_5} \quad \text{ax/W} \quad \frac{\bullet}{\bullet \mathbf{h}_7: \top, \Delta_6, \mathbf{F}_5 \vdash \bot, \Delta_8} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \hline \\ -: \top, \Delta_6 \vdash \bot, \Delta_8 \end{array}$$

• Case rule  $\top_R$ 

$$\frac{ \begin{array}{c} \mathbf{h}_3: \Delta_6 \vdash \mathbf{F}_5, \top, \Delta_8 \\ \bullet \mathbf{h}_3: \top, \Delta_6 \vdash (\top, \Delta_8), \mathbf{F}_5 \end{array} \top_L \quad \begin{array}{c} \bullet \mathbf{h}_7: (\top, \Delta_6), \mathbf{F}_5 \vdash \top, \Delta_8 \\ -: \top, \Delta_6 \vdash \top, \Delta_8 \\ \hline \\ \hline -: \top, \Delta_6 \vdash \top, \Delta_8 \end{array} \quad \begin{array}{c} \top_R \\ \text{Cut} \end{array} }$$

 $\bullet$  Case rule K

$$\frac{\begin{array}{c} \mathbf{h}_3: \Box \Gamma_9, \Delta_{10} \vdash \Box \mathbf{F}_5, \Delta_7, [] \mathbf{F}_8 \\ \bullet \mathbf{h}_3: \top, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, [] \mathbf{F}_8), \Box \mathbf{F}_5 \end{array}}{-: \top, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, [] \mathbf{F}_8} \xrightarrow{\mathbf{A}_K / \mathbf{F}_5} \begin{array}{c} \mathbf{A}_6: unbox(\Box \Gamma_9), unbox(\Box \mathbf{F}_5) \vdash \mathbf{F}_8 \\ \bullet \mathbf{h}_6: (\top, \Box \Gamma_9, \Delta_{10}), \Box \mathbf{F}_5 \vdash \Delta_7, [] \mathbf{F}_8 \end{array}} \xrightarrow{\mathbf{A}_K / \mathbf{K}} \\ \underline{\mathbf{h}_3: \top, \Delta_{10}, \Box \Gamma_9 \vdash \Box \mathbf{F}_5, \Delta_7, [] \mathbf{F}_8} \xrightarrow{\mathbf{A}_K / \mathbf{K}} \xrightarrow{\mathbf{A}_6: T, \Box \mathbf{F}_5, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [] \mathbf{F}_8} \xrightarrow{\mathbf{A}_K / \mathbf{K}} \\ -: \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [] \mathbf{F}_8 \end{array}} \xrightarrow{\mathbf{A}_K / \mathbf{K}} \\ \underline{\mathbf{h}_3: \Box \Gamma_7, \Delta_{10} \vdash \mathbf{F}_5, \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{F}_5} \begin{array}{c} \mathbf{A}_6: unbox(\Box \Gamma_7) \vdash \mathbf{F}_9 \\ \bullet \mathbf{h}_6: (\top, \Box \Gamma_7, \Delta_{10}), \mathbf{F}_5 \vdash \Delta_8, [] \mathbf{F}_9 \end{array}} \xrightarrow{\mathbf{K}} \\ \underline{\mathbf{h}_3: \top, \Box \Gamma_7, \Delta_{10} \vdash (\Delta_8, [] \mathbf{F}_9), \mathbf{F}_5} \xrightarrow{\mathbf{K}} \mathbf{K}} \\ \underline{\mathbf{Cut}} \\ \underline{-: T, \Box \Gamma_7, \Delta_{10} \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: unbox(\Box \Gamma_7) \vdash \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: unbox(\Box \Gamma_7) \vdash \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: unbox(\Box \Gamma_7) \vdash \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{F}_9} \xrightarrow{\mathbf{K}_K} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{K}_8} \xrightarrow{\mathbf{K}_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{K}_8} \\ \underline{-: T, \Delta_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{K}_8} \xrightarrow{\mathbf{K}_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{K}_8} \xrightarrow{\mathbf{K}_{10}, \Box \Gamma_7 \vdash \Delta_8, [] \mathbf{K}_7} \xrightarrow$$

 $\bullet$  Case rule A45

$$\begin{array}{c} \begin{array}{c} h_3: \square\Gamma_{10}, \Delta_{11} \vdash \square F_5, \square\Gamma_7, \Delta_8, []F_9 \\ \hline \bullet h_3: \top, \square\Gamma_{10}, \Delta_{11} \vdash (\square\Gamma_7, \Delta_8, []F_9), \square F_5 \end{array} \\ \hline -: \top, \square\Gamma_{10}, \Delta_{11} \vdash \square\Gamma_7, \Delta_8, []F_9 \\ \hline \\ h_3: \top, \Delta_{11}, \square\Gamma_{10} \vdash \square F_5, \Delta_8, \square\Gamma_7, []F_9 \end{array} \\ \hline \begin{array}{c} ax/w \\ \hline \bullet h_6: (\top, \square\Gamma_{10}, \Delta_{11}), \square\Gamma_{10} \vdash \Delta_8, \square\Gamma_7, []F_9 \end{array} \\ \hline h_3: \top, \Delta_{11}, \square\Gamma_{10} \vdash \square F_5, \Delta_8, \square\Gamma_7, []F_9 \end{array} \\ \hline \begin{array}{c} ax/w \\ \hline \bullet h_6: \top, \square F_5, \Delta_{11}, \square\Gamma_{10} \vdash \Delta_8, \square\Gamma_7, []F_9 \end{array} \\ \hline \\ h_3: \square\Gamma_7, \Delta_{11} \vdash F_5, \square\Gamma_8, \Delta_9, []F_{10} \\ \hline -: \top, \Delta_{11}, \square\Gamma_{10} \vdash \Delta_8, \square\Gamma_7, []F_9 \end{array} \\ \hline \begin{array}{c} h_6: \square\Gamma_7 \vdash \square\Gamma_8, F_{10} \\ \hline \bullet h_6: (\top, \square\Gamma_7, \Delta_{11}), F_5 \vdash \square\Gamma_8, \Delta_9, []F_{10} \\ \hline -: \top, \square\Gamma_7, \Delta_{11} \vdash \square F_8, \Delta_9, []F_{10} \end{array} \\ \hline \\ -: \top, \square\Gamma_7, \Delta_{11} \vdash \square F_8, \Delta_9, []F_{10} \end{array} \\ \hline \begin{array}{c} \Delta 45 \\ \text{Cut} \end{array} \\ \hline -: \square\Gamma_7 \vdash F_{10}, \square\Gamma_8 \end{array} \\ \hline \begin{array}{c} A45 \\ \text{Cut} \end{array} \\ \hline \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_3:\Delta_5 \vdash \mathbf{F}_7 \to \mathbf{F}_8, \Delta_9}{\bullet \mathbf{h}_3: \top, \Delta_5 \vdash \Delta_9, \mathbf{F}_7 \to \mathbf{F}_8} & \top_L & \frac{\mathbf{h}_6: \top, \Delta_5 \vdash \mathbf{F}_7, \Delta_9 \quad \mathbf{h}_6: \top, \mathbf{F}_8, \Delta_5 \vdash \Delta_9}{\bullet \mathbf{h}_6: (\top, \Delta_5), \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9} & \mathbf{Cut} \\ \hline & -: \top, \Delta_5 \vdash \Delta_9 \\ \hline & \frac{\mathbf{h}_3: \top, \Delta_5 \vdash \Delta_9, \mathbf{F}_7 \to \mathbf{F}_8}{\bullet \mathbf{h}_6: \top, \Delta_5, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9} & \mathbf{ax/W} \\ \hline & -: \top, \Delta_5 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_5, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & -: \top, \Delta_5 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \mathbf{F}_7, \Delta_9 \quad \mathbf{h}_6: \top, \mathbf{F}_7, \mathbf{F}_8, \Delta_{10} \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_3: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9, \mathbf{F}_5 & \mathbf{h}_6: (\top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8), \mathbf{F}_5 \vdash \Delta_9 \\ \hline & -: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{h}_7 \to \mathbf{h}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{h}_7 \to \mathbf{h}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{h}_7 \to \mathbf{h}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{h}_7 \to \mathbf{h}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{h}_7 \to \mathbf{h}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathbf{h}_7 \to \mathbf{h}_8 \vdash \Delta_9 \\ \hline & \bullet \mathbf{h}_8 \to \mathbf{h}_8 \to \mathbf{h}_$$

• Case rule  $\wedge_L$ 

$$\begin{array}{c} \begin{array}{c} \mathbf{h}_3: \Delta_5 \vdash \mathsf{F}_7 \wedge \mathsf{F}_8, \Delta_9 \\ \bullet \mathbf{h}_3: \top, \Delta_5 \vdash \Delta_9, \mathsf{F}_7 \wedge \mathsf{F}_8 \end{array} \\ \top_L & \begin{array}{c} \mathbf{h}_6: \top, \mathsf{F}_7, \mathsf{F}_8, \Delta_5 \vdash \Delta_9 \\ \bullet \mathbf{h}_6: (\top, \Delta_5), \mathsf{F}_7 \wedge \mathsf{F}_8 \vdash \Delta_9 \end{array} \end{array} \\ \begin{array}{c} \wedge_L \\ \text{Cut} \end{array} \\ \\ \hline \begin{array}{c} \mathbf{h}_3: \top, \Delta_5 \vdash \Delta_9, \mathsf{F}_7 \wedge \mathsf{F}_8 \end{array} \\ \hline \mathbf{h}_3: \top, \Delta_5 \vdash \Delta_9, \mathsf{F}_7 \wedge \mathsf{F}_8 \end{array} \\ \begin{array}{c} \mathsf{ax/W} \\ \bullet \mathbf{h}_6: \top, \Delta_5, \mathsf{F}_7 \wedge \mathsf{F}_8 \vdash \Delta_9 \end{array} \\ \hline \begin{array}{c} \mathbf{h}_3: \Delta_{10}, \mathsf{F}_7 \wedge \mathsf{F}_8 \vdash \mathsf{F}_5, \Delta_9 \\ \bullet \mathbf{h}_3: \top, \Delta_{10}, \mathsf{F}_7 \wedge \mathsf{F}_8 \vdash \Delta_9, \mathsf{F}_5 \end{array} \\ \hline \begin{array}{c} \mathsf{h}_6: \top, \mathsf{F}_5, \mathsf{F}_7, \mathsf{F}_8, \Delta_{10} \vdash \Delta_9 \\ \bullet \mathbf{h}_6: (\top, \Delta_{10}, \mathsf{F}_7 \wedge \mathsf{F}_8), \mathsf{F}_5 \vdash \Delta_9 \end{array} \\ \hline \begin{array}{c} \wedge_L \\ \bullet \mathbf{h}_6: \top, \Delta_{10}, \mathsf{F}_7 \wedge \mathsf{F}_8 \vdash \Delta_9, \mathsf{F}_5 \end{array} \\ \hline \begin{array}{c} \mathsf{h}_6: \top, \Delta_{10}, \mathsf{F}_7 \wedge \mathsf{F}_8 \vdash \Delta_9 \end{array} \\ \hline \begin{array}{c} \mathsf{h}_6: \top, \Delta_{10}, \mathsf{F}_7 \wedge \mathsf{F}_8 \vdash \Delta_9 \end{array} \\ \hline \begin{array}{c} \wedge_L \\ \mathsf{Cut} \end{array} \\ \hline \begin{array}{c} \mathsf{h}_3: \top, \Delta_{10}, \mathsf{F}_7 \wedge \mathsf{F}_8 \vdash \Delta_9, \mathsf{F}_5 \end{array} \\ \hline \begin{array}{c} \mathsf{ax/W} \\ \bullet \mathsf{h}_6: \top, \Delta_{10}, \mathsf{F}_5, \mathsf{F}_7 \wedge \mathsf{F}_8 \vdash \Delta_9 \end{array} \end{array} \\ \begin{array}{c} \mathsf{Ax/W} \\ \mathsf{h}\mathsf{Cut} \end{array} \\ \hline \end{array}$$

• Case rule  $\vee_L$ 

$$\begin{array}{c} \frac{\mathbf{h}_3:\Delta_5 \vdash \mathbf{F}_7 \vee \mathbf{F}_8,\Delta_9}{\bullet \mathbf{h}_3:\top,\Delta_5 \vdash \Delta_9, \mathbf{F}_7 \vee \mathbf{F}_8} \; \top_L \; \begin{array}{c} \mathbf{h}_6:\top,\mathbf{F}_7,\Delta_5 \vdash \Delta_9 \; \mathbf{h}_6:\top,\mathbf{F}_8,\Delta_5 \vdash \Delta_9 \\ \bullet \mathbf{h}_6:(\top,\Delta_5),\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_9 \end{array} \; \mathbf{Cut} \\ \\ \hline -:\top,\Delta_5 \vdash \Delta_9 \\ \hline \\ \underline{\mathbf{h}_3:\top,\Delta_5 \vdash \Delta_9,\mathbf{F}_7 \vee \mathbf{F}_8} \; \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_6:\top,\Delta_5,\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_9 \end{array} \; \mathbf{ax/W} \\ -:\top,\Delta_5 \vdash \Delta_9 \\ \hline \bullet \mathbf{h}_3:\Delta_{10},\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \mathbf{F}_5,\Delta_9 \\ \bullet \mathbf{h}_3:\top,\Delta_{10},\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_9,\mathbf{F}_5 \end{array} \; T_L \; \begin{array}{c} \mathbf{h}_6:\top,\mathbf{F}_5,\mathbf{F}_7,\Delta_{10} \vdash \Delta_9 \; \mathbf{h}_6:\top,\mathbf{F}_5,\mathbf{F}_8,\Delta_{10} \vdash \Delta_9 \\ \bullet \mathbf{h}_6:(\top,\Delta_{10},\mathbf{F}_7 \vee \mathbf{F}_8),\mathbf{F}_5 \vdash \Delta_9 \end{array} \; \mathbf{Cut} \\ \hline -:\top,\Delta_{10},\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_9 \qquad \mathbf{Cut} \\ \hline -:\top,\Delta_{10},\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_9 \end{array} \; \mathbf{ax/W} \; \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_6:\top,\Delta_{10},\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_9 \end{array} \; \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_6:\top,\Delta_{10},\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_9 \end{array} \; \mathbf{ax/W} \; \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_6:\top,\Delta_{10},\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_9 \end{array} \; \mathbf{ax/W} \\ \hline -:\top,\Delta_{10},\mathbf{F}_7 \vee \mathbf{F}_8 \vdash \Delta_9 \end{array} \; \mathbf{ax/W} \; \mathbf{hCut} \end{array} \; \mathbf{ax/W} \; \mathbf{hCut}$$

 $\bullet$  Case rule AT

$$\frac{\mathbf{h}_3: \Delta_5 \vdash [[\mathsf{F}_7, \Delta_8}{\bullet \mathsf{h}_3: \top, \Delta_5 \vdash \Delta_8, []\mathsf{F}_7} \; \top_L \; \frac{\mathsf{h}_6: \top, \mathsf{F}_7, \Delta_5, []\mathsf{F}_7 \vdash \Delta_8}{\bullet \mathsf{h}_6: (\top, \Delta_5), []\mathsf{F}_7 \vdash \Delta_8} \; \frac{AT}{\mathsf{Cut}} \\ \frac{-: \top, \Delta_5 \vdash \Delta_8}{\bullet \mathsf{h}_6: \top, \Delta_5, []\mathsf{F}_7 \vdash \Delta_8} \; \frac{\mathsf{ax/W}}{\bullet \mathsf{h}_6: \top, \Delta_5, []\mathsf{F}_7 \vdash \Delta_8} \\ \frac{-: \top, \Delta_5 \vdash \Delta_8}{\bullet, \mathsf{Cut}} \; \frac{\mathsf{ax/W}}{\bullet, \mathsf{Cut}}$$

$$\frac{\mathbf{h}_3:\Delta_9, []\mathbf{F}_7 \vdash \mathbf{F}_5, \Delta_8}{\bullet \mathbf{h}_3:\top, \Delta_9, []\mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_5} \ \top_L \ \frac{\mathbf{h}_6:\top, \mathbf{F}_5, \mathbf{F}_7, \Delta_9, []\mathbf{F}_7 \vdash \Delta_8}{\bullet \mathbf{h}_6:(\top, \Delta_9, []\mathbf{F}_7), \mathbf{F}_5 \vdash \Delta_8} \ \frac{AT}{\mathsf{Cut}} \\ \hline -:\top, \Delta_9, []\mathbf{F}_7 \vdash \Delta_8 \\ \hline \frac{\mathbf{h}_3:\top, \Delta_9, []\mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_5}{\bullet \mathbf{h}_6:\top, \Delta_9, \mathbf{F}_5, []\mathbf{F}_7 \vdash \Delta_8} \ \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \hline -:\top, \Delta_9, []\mathbf{F}_7 \vdash \Delta_8$$

## • Case rule $\perp_L$

#### ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_3:\Delta_5 \vdash \mathbf{p}_7,\Delta_8,\mathbf{p}_7}{\bullet \mathbf{h}_3:\top,\Delta_5 \vdash (\Delta_8,\mathbf{p}_7),\mathbf{p}_7} \; \top_L \quad \\ \hline \bullet \mathbf{h}_6:(\top,\Delta_5),\mathbf{p}_7 \vdash \Delta_8,\mathbf{p}_7} \\ \hline \\ \frac{\mathbf{h}_3:\top,\Delta_5 \vdash (\Delta_8,\mathbf{p}_7),\mathbf{p}_7}{\bullet \mathbf{h}_3:\top,\Delta_5 \vdash \Delta_8,\mathbf{p}_7,\mathbf{p}_7} \quad \mathbf{ax/W} \quad \\ \hline \bullet \mathbf{h}_6:(\top,\Delta_5,\mathbf{p}_7 \vdash \Delta_8,\mathbf{p}_7)} \quad I \\ \hline \\ \bullet \mathbf{h}_3:\top,\Delta_5 \vdash \Delta_8,\mathbf{p}_7,\mathbf{p}_7} \quad \mathbf{ax/W} \quad \\ \hline \bullet \mathbf{h}_6:(\top,\Delta_5,\mathbf{p}_7 \vdash \Delta_8,\mathbf{p}_7)} \quad I \\ \hline \\ \bullet \mathbf{h}_3:\Delta_9,\mathbf{p}_7 \vdash F_5,\Delta_8,\mathbf{p}_7} \quad \top_L \quad \\ \hline \bullet \mathbf{h}_6:(\top,\Delta_9,\mathbf{p}_7),F_5 \vdash \Delta_8,\mathbf{p}_7} \quad I \\ \hline \\ \bullet \mathbf{h}_3:\top,\Delta_9,\mathbf{p}_7 \vdash (\Delta_8,\mathbf{p}_7),F_5} \quad \\ \hline \\ \bullet \mathbf{h}_6:(\top,\Delta_9,\mathbf{p}_7),F_5 \vdash \Delta_8,\mathbf{p}_7} \quad I \\ \hline \\ \hline \\ \bullet \mathbf{h}_6:(\top,\Delta_9,\mathbf{p}_7),F_5 \vdash \Delta_8,\mathbf{p}_7} \quad I \\ \hline \\ \hline \\ \bullet \mathbf{h}_7:\top,\Delta_9,\mathbf{p}_7 \vdash \Delta_8,\mathbf{p}_7} \quad I \\ \hline \end{array}$$

## • Case rule $\top_L$

$$\frac{ \begin{array}{c} \mathbf{h}_3: \Delta_6 \vdash \mathbf{F}_5, \Delta_8 \\ \hline \bullet \mathbf{h}_3: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_5 \end{array} \top_L \quad \begin{array}{c} \mathbf{h}_7: \mathbf{F}_5, \Delta_6 \vdash \Delta_8 \\ \hline \bullet \mathbf{h}_7: (\top, \Delta_6), \mathbf{F}_5 \vdash \Delta_8 \end{array} \begin{array}{c} \top_L \\ \hline -: \top, \Delta_6 \vdash \Delta_8 \\ \hline \\ \hline \mathbf{h}_3: \top, \Delta_6 \vdash \Delta_8, \mathbf{F}_5 \end{array} \quad \begin{array}{c} \mathbf{a}\mathbf{x}/\mathbb{W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_6, \mathbf{F}_5 \vdash \Delta_8 \end{array} \begin{array}{c} \mathbf{a}\mathbf{x}/\mathbb{W} \\ \mathbf{h}\mathbf{C}\mathbf{u}\mathbf{t} \end{array}$$