System G3i

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Contents

1   Height preserving admissibility of weakening

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top} \;\top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \top} \;\top_R$$

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \to \mathtt{F}_4} \;\to_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\;\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \vdash \mathtt{F}_4}\;\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3 \to \mathtt{F}_4} \;\to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4} \;\wedge_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\;\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3}\;\mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\;\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_4}\;\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4} \;\wedge_R$$

- Case(s) rule $\vee_1$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4} \;\vee_1 \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\;\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3}\;\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3 \vee \mathtt{F}_4} \;\vee_1$$

- Case(s) rule $\vee_2$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4} \;\vee_2 \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\;\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_4}\;\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3 \vee \mathtt{F}_4} \;\vee_2$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_5} \;\to_L \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\;\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\;\mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\;\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_4 \vdash \mathtt{F}_5}\;\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_5} \;\to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5} \;\wedge_L \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\;\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\;\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5} \;\wedge_L$$

- Case(s) rule $\vee_L$

$$
\cfrac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_5} \; \vee_L
\qquad \rightarrow \qquad
\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_2, F_3 \vdash F_5} \; \text{ax}}{h_1 : \Delta_2, F_0, F_3 \vdash F_5} \; \text{IH} \quad \cfrac{\cfrac{}{h_1 : \Delta_2, F_4 \vdash F_5} \; \text{ax}}{h_1 : \Delta_2, F_0, F_4 \vdash F_5} \; \text{IH}}{\bullet h_1 : \Delta_2, F_0, F_3 \vee F_4 \vdash F_5} \; \vee_L
$$

- Case(s) rule $\perp_L$

$$
\cfrac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_3} \; \perp_L
\qquad \rightarrow \qquad
\cfrac{}{\bullet h_1 : \perp, \Delta_2, F_0 \vdash F_3} \; \perp_L
$$

- Case(s) rule I

$$
\cfrac{}{\bullet h_1 : \Delta_2, p_3 \vdash p_3} \; I
\qquad \rightarrow \qquad
\cfrac{}{\bullet h_1 : \Delta_2, F_0, p_3 \vdash p_3} \; I
$$

- Case(s) rule $\top_L$

$$
\cfrac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \top, \Delta_2 \vdash F_3} \; \top_L
\qquad \rightarrow \qquad
\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_2 \vdash F_3} \; \text{ax}}{h_1 : \Delta_2, F_0 \vdash F_3} \; \text{IH}}{\bullet h_1 : \top, \Delta_2, F_0 \vdash F_3} \; \top_L
$$

2   Measure of derivations

- Case(s) rule $\top_R$

$$\dfrac{}{\bullet h_1 : \Delta_2 \vdash \top} \ \top_R \quad \rightarrow \quad \dfrac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top} \ \top_R$$

- Case(s) rule $\rightarrow_R$

$$\dfrac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \ \rightarrow_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_2, F_3 \vdash F_4} \ \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \ \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \ \rightarrow_R$$

- Case(s) rule $\wedge_R$

$$\dfrac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \ \wedge_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_3} \ \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \ \text{IH} \quad \dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_4} \ \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_4} \ \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \ \wedge_R$$

- Case(s) rule $\vee_1$

$$\dfrac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \ \vee_1 \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_3} \ \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \ \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \ \vee_1$$

- Case(s) rule $\vee_2$

$$\dfrac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \ \vee_2 \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_4} \ \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_4} \ \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \ \vee_2$$

- Case(s) rule $\rightarrow_L$

$$\dfrac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_5} \ \rightarrow_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3} \ \text{ax}}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3} \ \text{IH} \quad \dfrac{\dfrac{}{h_1 : \Delta_2, F_4 \vdash F_5} \ \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \ \text{IH}}{\bullet \bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_5} \ \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_5} \ \wedge_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_2, F_3, F_4 \vdash F_5} \ \text{ax}}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} \ \text{IH}}{\bullet \bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_5} \ \wedge_L$$

- Case(s) rule $\vee_L$

$$\dfrac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_5} \ \vee_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_2, F_3 \vdash F_5} \ \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} \ \text{IH} \quad \dfrac{\dfrac{}{h_1 : \Delta_2, F_4 \vdash F_5} \ \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \ \text{IH}}{\bullet \bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_5} \ \vee_L$$

- Case(s) rule $\bot_L$

$$\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_3} \ \bot_L \quad \rightarrow \quad \dfrac{}{\bullet \bullet h_1 : \bot, \Delta_2 \vdash F_3} \ \bot_L$$

- Case(s) rule I

$$\dfrac{}{\bullet h_1 : \Delta_2, p_3 \vdash p_3} \ I \quad \rightarrow \quad \dfrac{}{\bullet \bullet h_1 : \Delta_2, p_3 \vdash p_3} \ I$$

- Case(s) rule $\top_L$

$$\dfrac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \top, \Delta_2 \vdash F_3} \ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_3} \ \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \ \text{IH}}{\bullet \bullet h_1 : \top, \Delta_2 \vdash F_3} \ \top_L$$

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3 Invertibility of Rules

3.1 Status of $\top_R$: : Invertible

- Case rule $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \; \top_R \qquad \to \qquad \text{trivial}$$

- Case rule $\to_R$

- Case rule $\wedge_R$

- Case rule $\vee_1$

- Case rule $\vee_2$

- Case rule $\to_L$

$$\frac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash \top} \; \to_L \qquad \to \qquad \text{trivial}$$

- Case rule $\wedge_L$

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \top} \; \wedge_L \qquad \to \qquad \text{trivial}$$

- Case rule $\vee_L$

$$\frac{h_1 : \Delta_2, F_3 \vdash \top \quad h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \top} \; \vee_L \qquad \to \qquad \text{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet h_1 : \bot, \Delta_2 \vdash \top} \; \bot_L \qquad \to \qquad \text{trivial}$$

- Case rule I

- Case rule $\top_L$

$$\frac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \top, \Delta_2 \vdash \top} \; \top_L \qquad \to \qquad \text{trivial}$$

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3.2 Status of $\to_R$: : Invertible

- Case rule $\top_R$

- Case rule $\to_R$

$$\frac{\mathtt{h_1 : \Delta_2, F_3 \vdash F_4}}{\bullet\mathtt{h_1 : \Delta_2 \vdash F_3 \to F_4}} \ \to_R \qquad \to \qquad \frac{\overline{\mathtt{h_1 : \Delta_2, F_3 \vdash F_4}}\ ^{\text{ax}}}{\bullet\mathtt{h_1 : \Delta_2, F_3 \vdash F_4}}\ _{\text{H}}$$

- Case rule $\wedge_R$

- Case rule $\vee_1$

- Case rule $\vee_2$

- Case rule $\to_L$

$$\frac{\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_5} \quad \mathtt{h_3 : \Delta_4, F_6 \vdash F_1 \to F_2}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_1 \to F_2}}\ \to_L \quad \to \quad \frac{\dfrac{\overline{\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_5}}\ ^{\text{ax}}}{\mathtt{h_3 : \Delta_4, F_1, F_5 \to F_6 \vdash F_5}}\ _W \quad \overline{\mathtt{h_3 : \Delta_4, F_1, F_6 \vdash F_2}}\ ^{\text{ax/ind}}}{\bullet\mathtt{h_3 : \Delta_4, F_1, F_5 \to F_6 \vdash F_2}}\ \to_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \to F_2}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \to F_2}}\ \wedge_L \quad \to \quad \frac{\overline{\mathtt{h_3 : \Delta_4, F_1, F_5, F_6 \vdash F_2}}\ ^{\text{ax/ind}}}{\bullet\mathtt{h_3 : \Delta_4, F_1, F_5 \wedge F_6 \vdash F_2}}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h_3 : \Delta_4, F_5 \vdash F_1 \to F_2} \quad \mathtt{h_3 : \Delta_4, F_6 \vdash F_1 \to F_2}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \to F_2}}\ \vee_L \quad \to \quad \frac{\overline{\mathtt{h_3 : \Delta_4, F_1, F_5 \vdash F_2}}\ ^{\text{ax/ind}} \quad \overline{\mathtt{h_3 : \Delta_4, F_1, F_6 \vdash F_2}}\ ^{\text{ax/ind}}}{\bullet\mathtt{h_3 : \Delta_4, F_1, F_5 \vee F_6 \vdash F_2}}\ \vee_L$$

- Case rule $\perp_L$

$$\frac{}{\bullet\mathtt{h_3 : \perp, \Delta_4 \vdash F_1 \to F_2}}\ \perp_L \quad \to \quad \frac{}{\bullet\mathtt{h_3 : \perp, \Delta_4, F_1 \vdash F_2}}\ \perp_L$$

- Case rule I

- Case rule $\top_L$

$$\frac{\mathtt{h_3 : \Delta_4 \vdash F_1 \to F_2}}{\bullet\mathtt{h_3 : \top, \Delta_4 \vdash F_1 \to F_2}}\ \top_L \quad \to \quad \frac{\overline{\mathtt{h_3 : \Delta_4, F_1 \vdash F_2}}\ ^{\text{ax/ind}}}{\bullet\mathtt{h_3 : \top, \Delta_4, F_1 \vdash F_2}}\ \top_L$$

3.3 Status of $\wedge_R$: (Left Premise): `Invertible`

- Case rule $\top_R$

- Case rule $\rightarrow_R$

- Case rule $\wedge_R$

$$\frac{\texttt{h}_1 : \Delta_2 \vdash \texttt{F}_3 \quad \texttt{h}_1 : \Delta_2 \vdash \texttt{F}_4}{\bullet \texttt{h}_1 : \Delta_2 \vdash \texttt{F}_3 \wedge \texttt{F}_4} \ \wedge_R \qquad \rightarrow \qquad \frac{\overline{\texttt{h}_1 : \Delta_2 \vdash \texttt{F}_3} \ \ \text{ax}}{\bullet \texttt{h}_1 : \Delta_2 \vdash \texttt{F}_3} \ \text{H}$$

- Case rule $\vee_1$

- Case rule $\vee_2$

- Case rule $\rightarrow_L$

$$\frac{\texttt{h}_3 : \Delta_4, \texttt{F}_5 \rightarrow \texttt{F}_6 \vdash \texttt{F}_5 \quad \texttt{h}_3 : \Delta_4, \texttt{F}_6 \vdash \texttt{F}_1 \wedge \texttt{F}_2}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_5 \rightarrow \texttt{F}_6 \vdash \texttt{F}_1 \wedge \texttt{F}_2} \ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\texttt{h}_3 : \Delta_4, \texttt{F}_5 \rightarrow \texttt{F}_6 \vdash \texttt{F}_5} \ \text{ax} \quad \overline{\texttt{h}_3 : \Delta_4, \texttt{F}_6 \vdash \texttt{F}_1} \ \text{ax/ind}}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_5 \rightarrow \texttt{F}_6 \vdash \texttt{F}_1} \ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\texttt{h}_3 : \Delta_4, \texttt{F}_5, \texttt{F}_6 \vdash \texttt{F}_1 \wedge \texttt{F}_2}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_5 \wedge \texttt{F}_6 \vdash \texttt{F}_1 \wedge \texttt{F}_2} \ \wedge_L \quad \rightarrow \quad \frac{\overline{\texttt{h}_3 : \Delta_4, \texttt{F}_5, \texttt{F}_6 \vdash \texttt{F}_1} \ \text{ax/ind}}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_5 \wedge \texttt{F}_6 \vdash \texttt{F}_1} \ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\texttt{h}_3 : \Delta_4, \texttt{F}_5 \vdash \texttt{F}_1 \wedge \texttt{F}_2 \quad \texttt{h}_3 : \Delta_4, \texttt{F}_6 \vdash \texttt{F}_1 \wedge \texttt{F}_2}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_5 \vee \texttt{F}_6 \vdash \texttt{F}_1 \wedge \texttt{F}_2} \ \vee_L \quad \rightarrow \quad \frac{\overline{\texttt{h}_3 : \Delta_4, \texttt{F}_5 \vdash \texttt{F}_1} \ \text{ax/ind} \quad \overline{\texttt{h}_3 : \Delta_4, \texttt{F}_6 \vdash \texttt{F}_1} \ \text{ax/ind}}{\bullet \texttt{h}_3 : \Delta_4, \texttt{F}_5 \vee \texttt{F}_6 \vdash \texttt{F}_1} \ \vee_L$$

- Case rule $\perp_L$

$$\frac{}{\bullet \texttt{h}_3 : \perp, \Delta_4 \vdash \texttt{F}_1 \wedge \texttt{F}_2} \ \perp_L \quad \rightarrow \quad \frac{}{\bullet \texttt{h}_3 : \perp, \Delta_4 \vdash \texttt{F}_1} \ \perp_L$$

- Case rule I

- Case rule $\top_L$

$$\frac{\texttt{h}_3 : \Delta_4 \vdash \texttt{F}_1 \wedge \texttt{F}_2}{\bullet \texttt{h}_3 : \top, \Delta_4 \vdash \texttt{F}_1 \wedge \texttt{F}_2} \ \top_L \quad \rightarrow \quad \frac{\overline{\texttt{h}_3 : \Delta_4 \vdash \texttt{F}_1} \ \text{ax/ind}}{\bullet \texttt{h}_3 : \top, \Delta_4 \vdash \texttt{F}_1} \ \top_L$$

3.4 Status of $\wedge_R$ (Right Premise): : `Invertible`

- Case rule $\top_R$

- Case rule $\to_R$

- Case rule $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \wedge_R \quad \to \quad \frac{\overline{h_1 : \Delta_2 \vdash F_4} \; \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_4} \; \text{H}$$

- Case rule $\vee_1$

- Case rule $\vee_2$

- Case rule $\to_L$

$$\frac{h_3 : \Delta_4, F_5 \to F_6 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \to F_6 \vdash F_1 \wedge F_2} \to_L \quad \to \quad \frac{\overline{h_3 : \Delta_4, F_5 \to F_6 \vdash F_5} \; \text{ax} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_2} \; \text{ax/ind}}{\bullet h_3 : \Delta_4, F_5 \to F_6 \vdash F_2} \to_L$$

- Case rule $\wedge_L$

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \wedge F_2} \wedge_L \quad \to \quad \frac{\overline{h_3 : \Delta_4, F_5, F_6 \vdash F_2} \; \text{ax/ind}}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_2} \wedge_L$$

- Case rule $\vee_L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \wedge F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \wedge F_2} \vee_L \quad \to \quad \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \; \text{ax/ind} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_2} \; \text{ax/ind}}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_2} \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet h_3 : \bot, \Delta_4 \vdash F_1 \wedge F_2} \bot_L \quad \to \quad \frac{}{\bullet h_3 : \bot, \Delta_4 \vdash F_2} \bot_L$$

- Case rule I

- Case rule $\top_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \wedge F_2} \top_L \quad \to \quad \frac{\overline{h_3 : \Delta_4 \vdash F_2} \; \text{ax/ind}}{\bullet h_3 : \top, \Delta_4 \vdash F_2} \top_L$$

3.5 Status of $\vee_1$: : Non invertible

- Case rule $\top_R$

- Case rule $\to_R$

- Case rule $\wedge_R$

- Case rule $\vee_1$

$$\dfrac{\mathtt{h_1 : \Delta_2 \vdash F_3}}{\bullet\mathtt{h_1 : \Delta_2 \vdash F_3 \vee F_4}} \; \vee_1 \qquad \to \qquad \dfrac{\overline{\mathtt{h_1 : \Delta_2 \vdash F_3}} \;\; {}^{\mathtt{ax}}}{\bullet\mathtt{h_1 : \Delta_2 \vdash F_3}} \; {}_{\mathtt{H}}$$

- Case rule $\vee_2$

$$\dfrac{\mathtt{h_1 : \Delta_2 \vdash F_4}}{\bullet\mathtt{h_1 : \Delta_2 \vdash F_3 \vee F_4}} \; \vee_2 \qquad \to \qquad \overline{\textcolor{red}{\bullet\mathtt{h_1 : \Delta_2 \vdash F_3}}} \;\; {}^{\mathtt{fail}}$$

- Case rule $\to_L$

$$\dfrac{\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_5} \quad \mathtt{h_3 : \Delta_4, F_6 \vdash F_1 \vee F_2}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_1 \vee F_2}} \; \to_L \qquad \to \qquad \dfrac{\overline{\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_5}} \;\; {}^{\mathtt{ax}} \quad \overline{\mathtt{h_3 : \Delta_4, F_6 \vdash F_1}} \;\; {}^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_1}} \; \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\mathtt{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \vee F_2}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \vee F_2}} \; \wedge_L \qquad \to \qquad \dfrac{\overline{\mathtt{h_3 : \Delta_4, F_5, F_6 \vdash F_1}} \;\; {}^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1}} \; \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h_3 : \Delta_4, F_5 \vdash F_1 \vee F_2} \quad \mathtt{h_3 : \Delta_4, F_6 \vdash F_1 \vee F_2}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \vee F_2}} \; \vee_L \qquad \to \qquad \dfrac{\overline{\mathtt{h_3 : \Delta_4, F_5 \vdash F_1}} \;\; {}^{\mathtt{ax/ind}} \quad \overline{\mathtt{h_3 : \Delta_4, F_6 \vdash F_1}} \;\; {}^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1}} \; \vee_L$$

- Case rule $\bot_L$

$$\overline{\bullet\mathtt{h_3 : \bot, \Delta_4 \vdash F_1 \vee F_2}} \; \bot_L \qquad \to \qquad \overline{\bullet\mathtt{h_3 : \bot, \Delta_4 \vdash F_1}} \; \bot_L$$

- Case rule I

- Case rule $\top_L$

$$\dfrac{\mathtt{h_3 : \Delta_4 \vdash F_1 \vee F_2}}{\bullet\mathtt{h_3 : \top, \Delta_4 \vdash F_1 \vee F_2}} \; \top_L \qquad \to \qquad \dfrac{\overline{\mathtt{h_3 : \Delta_4 \vdash F_1}} \;\; {}^{\mathtt{ax/ind}}}{\bullet\mathtt{h_3 : \top, \Delta_4 \vdash F_1}} \; \top_L$$

3.6   Status of $\vee_2$: :   Non invertible

- Case rule $\top_R$

- Case rule $\to_R$

- Case rule $\wedge_R$

- Case rule $\vee_1$

$$\dfrac{\mathtt{h_1 : \Delta_2 \vdash F_3}}{\bullet\mathtt{h_1 : \Delta_2 \vdash F_3 \vee F_4}} \ \vee_1 \qquad \to \qquad \dfrac{}{\bullet\mathtt{h_1 : \Delta_2 \vdash F_4}} \ \texttt{fail}$$

- Case rule $\vee_2$

$$\dfrac{\mathtt{h_1 : \Delta_2 \vdash F_4}}{\bullet\mathtt{h_1 : \Delta_2 \vdash F_3 \vee F_4}} \ \vee_2 \qquad \to \qquad \dfrac{\dfrac{}{\mathtt{h_1 : \Delta_2 \vdash F_4}} \ \texttt{ax}}{\bullet\mathtt{h_1 : \Delta_2 \vdash F_4}} \ \texttt{H}$$

- Case rule $\to_L$

$$\dfrac{\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_5} \quad \mathtt{h_3 : \Delta_4, F_6 \vdash F_1 \vee F_2}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_1 \vee F_2}} \ \to_L \qquad \to \qquad \dfrac{\dfrac{}{\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_5}} \ \texttt{ax} \quad \dfrac{}{\mathtt{h_3 : \Delta_4, F_6 \vdash F_2}} \ \texttt{ax/ind}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \to F_6 \vdash F_2}} \ \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\mathtt{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \vee F_2}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \vee F_2}} \ \wedge_L \qquad \to \qquad \dfrac{\dfrac{}{\mathtt{h_3 : \Delta_4, F_5, F_6 \vdash F_2}} \ \texttt{ax/ind}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_2}} \ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h_3 : \Delta_4, F_5 \vdash F_1 \vee F_2} \quad \mathtt{h_3 : \Delta_4, F_6 \vdash F_1 \vee F_2}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \vee F_2}} \ \vee_L \qquad \to \qquad \dfrac{\dfrac{}{\mathtt{h_3 : \Delta_4, F_5 \vdash F_2}} \ \texttt{ax/ind} \quad \dfrac{}{\mathtt{h_3 : \Delta_4, F_6 \vdash F_2}} \ \texttt{ax/ind}}{\bullet\mathtt{h_3 : \Delta_4, F_5 \vee F_6 \vdash F_2}} \ \vee_L$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet\mathtt{h_3 : \perp, \Delta_4 \vdash F_1 \vee F_2}} \ \perp_L \qquad \to \qquad \dfrac{}{\bullet\mathtt{h_3 : \perp, \Delta_4 \vdash F_2}} \ \perp_L$$

- Case rule I

- Case rule $\top_L$

$$\dfrac{\mathtt{h_3 : \Delta_4 \vdash F_1 \vee F_2}}{\bullet\mathtt{h_3 : \top, \Delta_4 \vdash F_1 \vee F_2}} \ \top_L \qquad \to \qquad \dfrac{\dfrac{}{\mathtt{h_3 : \Delta_4 \vdash F_2}} \ \texttt{ax/ind}}{\bullet\mathtt{h_3 : \top, \Delta_4 \vdash F_2}} \ \top_L$$

3.7 Status of $\to_L$: (Left Premise): `Non invertible`

- Case rule $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \top} \ \top_R \quad \to \quad \frac{}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_2} \ \texttt{fail}$$

- Case rule $\to_R$

$$\frac{h_4 : \Delta_1, F_5, F_2 \to F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \to F_6} \ \to_R \quad \to \quad \frac{}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_2} \ \texttt{fail}$$

- Case rule $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \to F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \wedge F_6} \ \wedge_R \quad \to \quad \frac{\dfrac{}{h_4 : \Delta_1, F_2 \to F_3 \vdash F_2} \ \texttt{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_2} \ \texttt{H}$$

- Case rule $\vee_1$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \vee F_6} \ \vee_1 \quad \to \quad \frac{\dfrac{}{h_4 : \Delta_1, F_2 \to F_3 \vdash F_2} \ \texttt{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_2} \ \texttt{H}$$

- Case rule $\vee_2$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \vee F_6} \ \vee_2 \quad \to \quad \frac{\dfrac{}{h_4 : \Delta_1, F_2 \to F_3 \vdash F_2} \ \texttt{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_2} \ \texttt{H}$$

- Case rule $\to_L$

$$\frac{h_3 : \Delta_7, F_1 \to F_2, F_4 \to F_5 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \to F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \to F_2), F_4 \to F_5 \vdash F_6} \ \to_L \quad \to \quad \frac{\dfrac{}{h_3 : \Delta_7, F_1 \to F_2, F_4 \to F_5 \vdash F_1} \ \texttt{ax/ind}}{\bullet h_3 : \Delta_7, F_1 \to F_2, F_4 \to F_5 \vdash F_1} \ \texttt{H}$$

$$\frac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_5} \ \to_L \quad \to \quad \frac{\dfrac{}{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3} \ \texttt{ax/ind}}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_3} \ \texttt{H}$$

- Case rule $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \to F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \to F_2), F_4 \wedge F_5 \vdash F_6} \ \wedge_L \quad \to \quad \frac{\dfrac{}{h_3 : \Delta_7, F_4, F_5, F_1 \to F_2 \vdash F_1} \ \texttt{ax/ind}}{\bullet h_3 : \Delta_7, F_1 \to F_2, F_4 \wedge F_5 \vdash F_1} \ \wedge_L$$

- Case rule $\vee_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \to F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \to F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \to F_2), F_4 \vee F_5 \vdash F_6} \ \vee_L \quad \to \quad \frac{\dfrac{}{h_3 : \Delta_7, F_4, F_1 \to F_2 \vdash F_1} \ \texttt{ax/ind} \quad \dfrac{}{h_3 : \Delta_7, F_5, F_1 \to F_2 \vdash F_1} \ \texttt{ax/ind}}{\bullet h_3 : \Delta_7, F_1 \to F_2, F_4 \vee F_5 \vdash F_1} \ \vee_L$$

- Case rule $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \to F_2 \vdash F_4} \ \perp_L \quad \to \quad \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \to F_2 \vdash F_1} \ \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_5, F_1 \to F_2 \vdash p_4} \; I \quad \to \quad \frac{}{\bullet h_3 : \Delta_5, p_4, F_1 \to F_2 \vdash F_1} \; \text{fail}$$

- Case rule $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \to F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \to F_2 \vdash F_4} \; \top_L \quad \to \quad \frac{\overline{h_3 : \Delta_5, F_1 \to F_2 \vdash F_1} \; \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \to F_2 \vdash F_1} \; \top_L$$

3.8 Status of $\to_L$ (Right Premise): : Invertible

- Case rule $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \top} \; \top_R \quad \to \quad \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top} \; \top_R$$

- Case rule $\to_R$

$$\frac{h_4 : \Delta_1, F_5, F_2 \to F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \to F_6} \; \to_R \quad \to \quad \frac{\overline{h_4 : \Delta_1, F_3, F_5 \vdash F_6} \; \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \to F_6} \; \to_R$$

- Case rule $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \to F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \wedge F_6} \; \wedge_R \quad \to \quad \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_5} \; \text{ax/ind} \quad \overline{h_4 : \Delta_1, F_3 \vdash F_6} \; \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \wedge F_6} \; \wedge_R$$

- Case rule $\vee_1$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \vee F_6} \; \vee_1 \quad \to \quad \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_5} \; \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \vee F_6} \; \vee_1$$

- Case rule $\vee_2$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash F_5 \vee F_6} \; \vee_2 \quad \to \quad \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_6} \; \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \vee F_6} \; \vee_2$$

- Case rule $\to_L$

$$\frac{h_3 : \Delta_7, F_1 \to F_2, F_4 \to F_5 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \to F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \to F_2), F_4 \to F_5 \vdash F_6} \; \to_L \quad \to \quad \frac{\overline{h_3 : \Delta_7, F_2, F_4 \to F_5 \vdash F_4} \; \text{ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \; \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \to F_5 \vdash F_6} \; \to_L$$

$$\frac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_5} \; \to_L \quad \to \quad \frac{\overline{h_1 : \Delta_2, F_4 \vdash F_5} \; \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \; \text{H}$$

- Case rule $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \to F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \to F_2), F_4 \wedge F_5 \vdash F_6} \; \wedge_L \quad \to \quad \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6} \; \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash F_6} \; \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6} \ \vee_L \quad \to \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6} \ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6} \ \text{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6} \ \vee_L$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet\mathtt{h}_3 : \perp, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4} \ \perp_L \quad \to \quad \dfrac{}{\bullet\mathtt{h}_3 : \perp, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4} \ \perp_L$$

- Case rule I

$$\dfrac{}{\bullet\mathtt{h}_3 : \mathtt{p}_4, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_4} \ I \quad \to \quad \dfrac{}{\bullet\mathtt{h}_3 : \Delta_5, \mathtt{F}_2, \mathtt{p}_4 \vdash \mathtt{p}_4} \ I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4} \ \top_L \quad \to \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4} \ \text{ax/ind}}{\bullet\mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4} \ \top_L$$

3.9 Status of $\wedge_L$: :   Invertible

- Case rule $\top_R$

$$\dfrac{}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \top} \ \top_R \quad \to \quad \dfrac{}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \top} \ \top_R$$

- Case rule $\to_R$

$$\dfrac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \to \mathtt{F}_6} \ \to_R \quad \to \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_5 \vdash \mathtt{F}_6} \ \text{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \to \mathtt{F}_6} \ \to_R$$

- Case rule $\wedge_R$

$$\dfrac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6} \ \wedge_R \quad \to \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5} \ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_6} \ \text{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6} \ \wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \ \vee_1 \quad \to \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5} \ \text{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \ \vee_1$$

- Case rule $\vee_2$

$$\dfrac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \ \vee_2 \quad \to \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_6} \ \text{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \ \vee_2$$

- Case rule $\rightarrow_L$

$$\dfrac{\texttt{h}_3 : \Delta_7, \texttt{F}_4 \rightarrow \texttt{F}_5, \texttt{F}_1 \wedge \texttt{F}_2 \vdash \texttt{F}_4 \quad \texttt{h}_3 : \Delta_7, \texttt{F}_5, \texttt{F}_1 \wedge \texttt{F}_2 \vdash \texttt{F}_6}{\bullet\texttt{h}_3 : (\Delta_7, \texttt{F}_1 \wedge \texttt{F}_2), \texttt{F}_4 \rightarrow \texttt{F}_5 \vdash \texttt{F}_6} \rightarrow_L \quad \rightarrow \quad \dfrac{\overline{\texttt{h}_3 : \Delta_7, \texttt{F}_1, \texttt{F}_2, \texttt{F}_4 \rightarrow \texttt{F}_5 \vdash \texttt{F}_4}\ \text{ax/ind} \quad \overline{\texttt{h}_3 : \Delta_7, \texttt{F}_1, \texttt{F}_2, \texttt{F}_5 \vdash \texttt{F}_6}\ \text{ax/ind}}{\bullet\texttt{h}_3 : \Delta_7, \texttt{F}_1, \texttt{F}_2, \texttt{F}_4 \rightarrow \texttt{F}_5 \vdash \texttt{F}_6} \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\texttt{h}_3 : \Delta_7, \texttt{F}_4, \texttt{F}_5, \texttt{F}_1 \wedge \texttt{F}_2 \vdash \texttt{F}_6}{\bullet\texttt{h}_3 : (\Delta_7, \texttt{F}_1 \wedge \texttt{F}_2), \texttt{F}_4 \wedge \texttt{F}_5 \vdash \texttt{F}_6} \wedge_L \quad \rightarrow \quad \dfrac{\overline{\texttt{h}_3 : \Delta_7, \texttt{F}_1, \texttt{F}_2, \texttt{F}_4, \texttt{F}_5 \vdash \texttt{F}_6}\ \text{ax/ind}}{\bullet\texttt{h}_3 : \Delta_7, \texttt{F}_1, \texttt{F}_2, \texttt{F}_4 \wedge \texttt{F}_5 \vdash \texttt{F}_6} \wedge_L$$

$$\dfrac{\texttt{h}_1 : \Delta_2, \texttt{F}_3, \texttt{F}_4 \vdash \texttt{F}_5}{\bullet\texttt{h}_1 : \Delta_2, \texttt{F}_3 \wedge \texttt{F}_4 \vdash \texttt{F}_5} \wedge_L \quad \rightarrow \quad \dfrac{\overline{\texttt{h}_1 : \Delta_2, \texttt{F}_3, \texttt{F}_4 \vdash \texttt{F}_5}\ \text{ax}}{\bullet\texttt{h}_1 : \Delta_2, \texttt{F}_3, \texttt{F}_4 \vdash \texttt{F}_5}\ \text{H}$$

- Case rule $\vee_L$

$$\dfrac{\texttt{h}_3 : \Delta_7, \texttt{F}_4, \texttt{F}_1 \wedge \texttt{F}_2 \vdash \texttt{F}_6 \quad \texttt{h}_3 : \Delta_7, \texttt{F}_5, \texttt{F}_1 \wedge \texttt{F}_2 \vdash \texttt{F}_6}{\bullet\texttt{h}_3 : (\Delta_7, \texttt{F}_1 \wedge \texttt{F}_2), \texttt{F}_4 \vee \texttt{F}_5 \vdash \texttt{F}_6} \vee_L \quad \rightarrow \quad \dfrac{\overline{\texttt{h}_3 : \Delta_7, \texttt{F}_1, \texttt{F}_2, \texttt{F}_4 \vdash \texttt{F}_6}\ \text{ax/ind} \quad \overline{\texttt{h}_3 : \Delta_7, \texttt{F}_1, \texttt{F}_2, \texttt{F}_5 \vdash \texttt{F}_6}\ \text{ax/ind}}{\bullet\texttt{h}_3 : \Delta_7, \texttt{F}_1, \texttt{F}_2, \texttt{F}_4 \vee \texttt{F}_5 \vdash \texttt{F}_6} \vee_L$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet\texttt{h}_3 : \perp, \Delta_5, \texttt{F}_1 \wedge \texttt{F}_2 \vdash \texttt{F}_4} \perp_L \quad \rightarrow \quad \dfrac{}{\bullet\texttt{h}_3 : \perp, \Delta_5, \texttt{F}_1, \texttt{F}_2 \vdash \texttt{F}_4} \perp_L$$

- Case rule I

$$\dfrac{}{\bullet\texttt{h}_3 : \texttt{p}_4, \Delta_5, \texttt{F}_1 \wedge \texttt{F}_2 \vdash \texttt{p}_4} I \quad \rightarrow \quad \dfrac{}{\bullet\texttt{h}_3 : \Delta_5, \texttt{F}_1, \texttt{F}_2, \texttt{p}_4 \vdash \texttt{p}_4} I$$

- Case rule $\top_L$

$$\dfrac{\texttt{h}_3 : \Delta_5, \texttt{F}_1 \wedge \texttt{F}_2 \vdash \texttt{F}_4}{\bullet\texttt{h}_3 : \top, \Delta_5, \texttt{F}_1 \wedge \texttt{F}_2 \vdash \texttt{F}_4} \top_L \quad \rightarrow \quad \dfrac{\overline{\texttt{h}_3 : \Delta_5, \texttt{F}_1, \texttt{F}_2 \vdash \texttt{F}_4}\ \text{ax/ind}}{\bullet\texttt{h}_3 : \top, \Delta_5, \texttt{F}_1, \texttt{F}_2 \vdash \texttt{F}_4} \top_L$$

3.10 Status of $\vee_L$: (Left Premise): `Invertible`

- Case rule $\top_R$

$$\dfrac{}{\bullet\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \top} \top_R \quad \rightarrow \quad \dfrac{}{\bullet\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vdash \top} \top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\texttt{h}_4 : \Delta_1, \texttt{F}_5, \texttt{F}_2 \vee \texttt{F}_3 \vdash \texttt{F}_6}{\bullet\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \texttt{F}_5 \rightarrow \texttt{F}_6} \rightarrow_R \quad \rightarrow \quad \dfrac{\overline{\texttt{h}_4 : \Delta_1, \texttt{F}_2, \texttt{F}_5 \vdash \texttt{F}_6}\ \text{ax/ind}}{\bullet\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vdash \texttt{F}_5 \rightarrow \texttt{F}_6} \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \texttt{F}_5 \quad \texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \texttt{F}_6}{\bullet\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \texttt{F}_5 \wedge \texttt{F}_6} \wedge_R \quad \rightarrow \quad \dfrac{\overline{\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vdash \texttt{F}_5}\ \text{ax/ind} \quad \overline{\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vdash \texttt{F}_6}\ \text{ax/ind}}{\bullet\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vdash \texttt{F}_5 \wedge \texttt{F}_6} \wedge_R$$

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- Case rule $\vee_1$

$$\dfrac{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_5}}{\bullet \mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_5} \vee \mathtt{F_6}} \ \vee_1 \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \mathtt{F_5}} \ \mathrm{ax/ind}}{\bullet \mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \mathtt{F_5} \vee \mathtt{F_6}} \ \vee_1$$

- Case rule $\vee_2$

$$\dfrac{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_6}}{\bullet \mathtt{h_4} : \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_5} \vee \mathtt{F_6}} \ \vee_2 \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \mathtt{F_6}} \ \mathrm{ax/ind}}{\bullet \mathtt{h_4} : \Delta_1, \mathtt{F_2} \vdash \mathtt{F_5} \vee \mathtt{F_6}} \ \vee_2$$

- Case rule $\rightarrow_L$

$$\dfrac{\mathtt{h_3} : \Delta_7, \mathtt{F_4} \rightarrow \mathtt{F_5}, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{F_4} \quad \mathtt{h_3} : \Delta_7, \mathtt{F_5}, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{F_6}}{\bullet \mathtt{h_3} : (\Delta_7, \mathtt{F_1} \vee \mathtt{F_2}), \mathtt{F_4} \rightarrow \mathtt{F_5} \vdash \mathtt{F_6}} \ \rightarrow_L \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_3} : \Delta_7, \mathtt{F_1}, \mathtt{F_4} \rightarrow \mathtt{F_5} \vdash \mathtt{F_4}} \ \mathrm{ax/ind} \quad \overline{\mathtt{h_3} : \Delta_7, \mathtt{F_1}, \mathtt{F_5} \vdash \mathtt{F_6}} \ \mathrm{ax/ind}}{\bullet \mathtt{h_3} : \Delta_7, \mathtt{F_1}, \mathtt{F_4} \rightarrow \mathtt{F_5} \vdash \mathtt{F_6}} \ \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\mathtt{h_3} : \Delta_7, \mathtt{F_4}, \mathtt{F_5}, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{F_6}}{\bullet \mathtt{h_3} : (\Delta_7, \mathtt{F_1} \vee \mathtt{F_2}), \mathtt{F_4} \wedge \mathtt{F_5} \vdash \mathtt{F_6}} \ \wedge_L \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_3} : \Delta_7, \mathtt{F_1}, \mathtt{F_4}, \mathtt{F_5} \vdash \mathtt{F_6}} \ \mathrm{ax/ind}}{\bullet \mathtt{h_3} : \Delta_7, \mathtt{F_1}, \mathtt{F_4} \wedge \mathtt{F_5} \vdash \mathtt{F_6}} \ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h_3} : \Delta_7, \mathtt{F_4}, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{F_6} \quad \mathtt{h_3} : \Delta_7, \mathtt{F_5}, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{F_6}}{\bullet \mathtt{h_3} : (\Delta_7, \mathtt{F_1} \vee \mathtt{F_2}), \mathtt{F_4} \vee \mathtt{F_5} \vdash \mathtt{F_6}} \ \vee_L \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_3} : \Delta_7, \mathtt{F_1}, \mathtt{F_4} \vdash \mathtt{F_6}} \ \mathrm{ax/ind} \quad \overline{\mathtt{h_3} : \Delta_7, \mathtt{F_1}, \mathtt{F_5} \vdash \mathtt{F_6}} \ \mathrm{ax/ind}}{\bullet \mathtt{h_3} : \Delta_7, \mathtt{F_1}, \mathtt{F_4} \vee \mathtt{F_5} \vdash \mathtt{F_6}} \ \vee_L$$

$$\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vdash \mathtt{F_5} \quad \mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \mathtt{F_5}}{\bullet \mathtt{h_1} : \Delta_2, \mathtt{F_3} \vee \mathtt{F_4} \vdash \mathtt{F_5}} \ \vee_L \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vdash \mathtt{F_5}} \ \mathrm{ax}}{\bullet \mathtt{h_1} : \Delta_2, \mathtt{F_3} \vdash \mathtt{F_5}} \ \mathtt{H}$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet \mathtt{h_3} : \perp, \Delta_5, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{F_4}} \ \perp_L \qquad \rightarrow \qquad \dfrac{}{\bullet \mathtt{h_3} : \perp, \Delta_5, \mathtt{F_1} \vdash \mathtt{F_4}} \ \perp_L$$

- Case rule I

$$\dfrac{}{\bullet \mathtt{h_3} : \mathtt{p_4}, \Delta_5, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{p_4}} \ I \qquad \rightarrow \qquad \dfrac{}{\bullet \mathtt{h_3} : \Delta_5, \mathtt{F_1}, \mathtt{p_4} \vdash \mathtt{p_4}} \ I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h_3} : \Delta_5, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{F_4}}{\bullet \mathtt{h_3} : \top, \Delta_5, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{F_4}} \ \top_L \qquad \rightarrow \qquad \dfrac{\overline{\mathtt{h_3} : \Delta_5, \mathtt{F_1} \vdash \mathtt{F_4}} \ \mathrm{ax/ind}}{\bullet \mathtt{h_3} : \top, \Delta_5, \mathtt{F_1} \vdash \mathtt{F_4}} \ \top_L$$

3.11   Status of $\vee_L$ (Right Premise): :   Invertible

- Case rule $\top_R$

$$\cfrac{}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \top} \ \top_R \quad \rightarrow \quad \cfrac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top} \ \top_R$$

- Case rule $\rightarrow_R$

$$\cfrac{h_4 : \Delta_1, F_5, F_2 \vee F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \rightarrow F_6} \ \rightarrow_R \quad \rightarrow \quad \cfrac{\cfrac{}{h_4 : \Delta_1, F_3, F_5 \vdash F_6} \ \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \rightarrow F_6} \ \rightarrow_R$$

- Case rule $\wedge_R$

$$\cfrac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \wedge F_6} \ \wedge_R \quad \rightarrow \quad \cfrac{\cfrac{}{h_4 : \Delta_1, F_3 \vdash F_5} \ \text{ax/ind} \quad \cfrac{}{h_4 : \Delta_1, F_3 \vdash F_6} \ \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \wedge F_6} \ \wedge_R$$

- Case rule $\vee_1$

$$\cfrac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \vee F_6} \ \vee_1 \quad \rightarrow \quad \cfrac{\cfrac{}{h_4 : \Delta_1, F_3 \vdash F_5} \ \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \vee F_6} \ \vee_1$$

- Case rule $\vee_2$

$$\cfrac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \vee F_6} \ \vee_2 \quad \rightarrow \quad \cfrac{\cfrac{}{h_4 : \Delta_1, F_3 \vdash F_6} \ \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \vee F_6} \ \vee_2$$

- Case rule $\rightarrow_L$

$$\cfrac{h_3 : \Delta_7, F_4 \rightarrow F_5, F_1 \vee F_2 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash F_6} \ \rightarrow_L \quad \rightarrow \quad \cfrac{\cfrac{}{h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_4} \ \text{ax/ind} \quad \cfrac{}{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_6} \ \rightarrow_L$$

- Case rule $\wedge_L$

$$\cfrac{h_3 : \Delta_7, F_4, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash F_6} \ \wedge_L \quad \rightarrow \quad \cfrac{\cfrac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash F_6} \ \wedge_L$$

- Case rule $\vee_L$

$$\cfrac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash F_6} \ \vee_L \quad \rightarrow \quad \cfrac{\cfrac{}{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \ \text{ax/ind} \quad \cfrac{}{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash F_6} \ \vee_L$$

$$\cfrac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_5} \ \vee_L \quad \rightarrow \quad \cfrac{\cfrac{}{h_1 : \Delta_2, F_4 \vdash F_5} \ \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \ \text{H}$$

- Case rule $\bot_L$

$$\cfrac{}{\bullet h_3 : \bot, \Delta_5, F_1 \vee F_2 \vdash F_4} \ \bot_L \quad \rightarrow \quad \cfrac{}{\bullet h_3 : \bot, \Delta_5, F_2 \vdash F_4} \ \bot_L$$

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- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_5, F_1 \vee F_2 \vdash p_4} \; I \quad \rightarrow \quad \frac{}{\bullet h_3 : \Delta_5, F_2, p_4 \vdash p_4} \; I$$

- Case rule $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash F_4} \; \top_L \quad \rightarrow \quad \frac{\overline{h_3 : \Delta_5, F_2 \vdash F_4} \; \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash F_4} \; \top_L$$

3.12   Status of $\bot_L$: :   Invertible

- Case rule $\top_R$

$$\frac{}{\bullet h_2 : \bot, \Delta_1 \vdash \top} \; \top_R \quad \rightarrow \quad \text{trivial}$$

- Case rule $\rightarrow_R$

$$\frac{h_2 : \bot, \Delta_1, F_3 \vdash F_4}{\bullet h_2 : \bot, \Delta_1 \vdash F_3 \rightarrow F_4} \; \rightarrow_R \quad \rightarrow \quad \text{trivial}$$

- Case rule $\wedge_R$

$$\frac{h_2 : \bot, \Delta_1 \vdash F_3 \quad h_2 : \bot, \Delta_1 \vdash F_4}{\bullet h_2 : \bot, \Delta_1 \vdash F_3 \wedge F_4} \; \wedge_R \quad \rightarrow \quad \text{trivial}$$

- Case rule $\vee_1$

$$\frac{h_2 : \bot, \Delta_1 \vdash F_3}{\bullet h_2 : \bot, \Delta_1 \vdash F_3 \vee F_4} \; \vee_1 \quad \rightarrow \quad \text{trivial}$$

- Case rule $\vee_2$

$$\frac{h_2 : \bot, \Delta_1 \vdash F_4}{\bullet h_2 : \bot, \Delta_1 \vdash F_3 \vee F_4} \; \vee_2 \quad \rightarrow \quad \text{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{h_1 : \bot, \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \bot, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\bot, \Delta_5), F_2 \rightarrow F_3 \vdash F_4} \; \rightarrow_L \quad \rightarrow \quad \text{trivial}$$

- Case rule $\wedge_L$

$$\frac{h_1 : \bot, \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : (\bot, \Delta_5), F_2 \wedge F_3 \vdash F_4} \; \wedge_L \quad \rightarrow \quad \text{trivial}$$

- Case rule $\vee_L$

$$\frac{h_1 : \bot, \Delta_5, F_2 \vdash F_4 \quad h_1 : \bot, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\bot, \Delta_5), F_2 \vee F_3 \vdash F_4} \; \vee_L \quad \rightarrow \quad \text{trivial}$$

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- Case rule $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_3} \ \perp_L \qquad \rightarrow \qquad \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_2, \perp, \Delta_3 \vdash p_2} \ I \qquad \rightarrow \qquad \text{trivial}$$

- Case rule $\top_L$

$$\frac{h_1 : \perp, \Delta_3 \vdash F_2}{\bullet h_1 : \top, \perp, \Delta_3 \vdash F_2} \ \top_L \qquad \rightarrow \qquad \text{trivial}$$

3.13 Status of I: : Invertible

- Case rule $\top_R$

- Case rule $\rightarrow_R$

- Case rule $\wedge_R$

- Case rule $\vee_1$

- Case rule $\vee_2$

- Case rule $\rightarrow_L$

$$\frac{h_2 : \Delta_5, p_1, F_3 \rightarrow F_4 \vdash F_3 \quad h_2 : \Delta_5, F_4, p_1 \vdash p_1}{\bullet h_2 : (\Delta_5, p_1), F_3 \rightarrow F_4 \vdash p_1} \ \rightarrow_L \qquad \rightarrow \qquad \text{trivial}$$

- Case rule $\wedge_L$

$$\frac{h_2 : \Delta_5, F_3, F_4, p_1 \vdash p_1}{\bullet h_2 : (\Delta_5, p_1), F_3 \wedge F_4 \vdash p_1} \ \wedge_L \qquad \rightarrow \qquad \text{trivial}$$

- Case rule $\vee_L$

$$\frac{h_2 : \Delta_5, F_3, p_1 \vdash p_1 \quad h_2 : \Delta_5, F_4, p_1 \vdash p_1}{\bullet h_2 : (\Delta_5, p_1), F_3 \vee F_4 \vdash p_1} \ \vee_L \qquad \rightarrow \qquad \text{trivial}$$

- Case rule $\perp_L$

$$\frac{}{\bullet h_2 : \perp, \Delta_3, p_1 \vdash p_1} \ \perp_L \qquad \rightarrow \qquad \text{trivial}$$

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- Case rule I

$$\frac{}{\bullet \mathsf{h}_1 : \mathsf{p}_3, \Delta_2 \vdash \mathsf{p}_3} \; I \qquad \rightarrow \qquad \mathtt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathsf{h}_2 : \Delta_3, \mathsf{p}_1 \vdash \mathsf{p}_1}{\bullet \mathsf{h}_2 : \top, \Delta_3, \mathsf{p}_1 \vdash \mathsf{p}_1} \; \top_L \qquad \rightarrow \qquad \mathtt{trivial}$$

3.14 Status of $\top_L$: : Invertible

- Case rule $\top_R$

$$\frac{}{\bullet \mathsf{h}_2 : \top, \Delta_1 \vdash \top} \; \top_R \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_2 : \Delta_1 \vdash \top} \; \top_R$$

- Case rule $\rightarrow_R$

$$\frac{\mathsf{h}_2 : \top, \Delta_1, \mathsf{F}_3 \vdash \mathsf{F}_4}{\bullet \mathsf{h}_2 : \top, \Delta_1 \vdash \mathsf{F}_3 \rightarrow \mathsf{F}_4} \; \rightarrow_R \qquad \rightarrow \qquad \frac{\dfrac{}{\mathsf{h}_2 : \Delta_1, \mathsf{F}_3 \vdash \mathsf{F}_4} \; \mathtt{ax/ind}}{\bullet \mathsf{h}_2 : \Delta_1 \vdash \mathsf{F}_3 \rightarrow \mathsf{F}_4} \; \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathsf{h}_2 : \top, \Delta_1 \vdash \mathsf{F}_3 \quad \mathsf{h}_2 : \top, \Delta_1 \vdash \mathsf{F}_4}{\bullet \mathsf{h}_2 : \top, \Delta_1 \vdash \mathsf{F}_3 \wedge \mathsf{F}_4} \; \wedge_R \qquad \rightarrow \qquad \frac{\dfrac{}{\mathsf{h}_2 : \Delta_1 \vdash \mathsf{F}_3} \; \mathtt{ax/ind} \quad \dfrac{}{\mathsf{h}_2 : \Delta_1 \vdash \mathsf{F}_4} \; \mathtt{ax/ind}}{\bullet \mathsf{h}_2 : \Delta_1 \vdash \mathsf{F}_3 \wedge \mathsf{F}_4} \; \wedge_R$$

- Case rule $\vee_1$

$$\frac{\mathsf{h}_2 : \top, \Delta_1 \vdash \mathsf{F}_3}{\bullet \mathsf{h}_2 : \top, \Delta_1 \vdash \mathsf{F}_3 \vee \mathsf{F}_4} \; \vee_1 \qquad \rightarrow \qquad \frac{\dfrac{}{\mathsf{h}_2 : \Delta_1 \vdash \mathsf{F}_3} \; \mathtt{ax/ind}}{\bullet \mathsf{h}_2 : \Delta_1 \vdash \mathsf{F}_3 \vee \mathsf{F}_4} \; \vee_1$$

- Case rule $\vee_2$

$$\frac{\mathsf{h}_2 : \top, \Delta_1 \vdash \mathsf{F}_4}{\bullet \mathsf{h}_2 : \top, \Delta_1 \vdash \mathsf{F}_3 \vee \mathsf{F}_4} \; \vee_2 \qquad \rightarrow \qquad \frac{\dfrac{}{\mathsf{h}_2 : \Delta_1 \vdash \mathsf{F}_4} \; \mathtt{ax/ind}}{\bullet \mathsf{h}_2 : \Delta_1 \vdash \mathsf{F}_3 \vee \mathsf{F}_4} \; \vee_2$$

- Case rule $\rightarrow_L$

$$\frac{\mathsf{h}_1 : \top, \Delta_5, \mathsf{F}_2 \rightarrow \mathsf{F}_3 \vdash \mathsf{F}_2 \quad \mathsf{h}_1 : \top, \Delta_5, \mathsf{F}_3 \vdash \mathsf{F}_4}{\bullet \mathsf{h}_1 : (\top, \Delta_5), \mathsf{F}_2 \rightarrow \mathsf{F}_3 \vdash \mathsf{F}_4} \; \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathsf{h}_1 : \Delta_5, \mathsf{F}_2 \rightarrow \mathsf{F}_3 \vdash \mathsf{F}_2} \; \mathtt{ax/ind} \quad \dfrac{}{\mathsf{h}_1 : \Delta_5, \mathsf{F}_3 \vdash \mathsf{F}_4} \; \mathtt{ax/ind}}{\bullet \mathsf{h}_1 : \Delta_5, \mathsf{F}_2 \rightarrow \mathsf{F}_3 \vdash \mathsf{F}_4} \; \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathsf{h}_1 : \top, \Delta_5, \mathsf{F}_2, \mathsf{F}_3 \vdash \mathsf{F}_4}{\bullet \mathsf{h}_1 : (\top, \Delta_5), \mathsf{F}_2 \wedge \mathsf{F}_3 \vdash \mathsf{F}_4} \; \wedge_L \qquad \rightarrow \qquad \frac{\dfrac{}{\mathsf{h}_1 : \Delta_5, \mathsf{F}_2, \mathsf{F}_3 \vdash \mathsf{F}_4} \; \mathtt{ax/ind}}{\bullet \mathsf{h}_1 : \Delta_5, \mathsf{F}_2 \wedge \mathsf{F}_3 \vdash \mathsf{F}_4} \; \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h_1} : \top, \Delta_5, \mathtt{F_2} \vdash \mathtt{F_4} \quad \mathtt{h_1} : \top, \Delta_5, \mathtt{F_3} \vdash \mathtt{F_4}}{\bullet \mathtt{h_1} : (\top, \Delta_5), \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_4}} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_1} : \Delta_5, \mathtt{F_2} \vdash \mathtt{F_4}} \text{ ax/ind} \quad \dfrac{}{\mathtt{h_1} : \Delta_5, \mathtt{F_3} \vdash \mathtt{F_4}} \text{ ax/ind}}{\bullet \mathtt{h_1} : \Delta_5, \mathtt{F_2} \vee \mathtt{F_3} \vdash \mathtt{F_4}} \vee_L$$

- Case rule $\perp_L$

$$\frac{}{\bullet \mathtt{h_1} : \perp, \top, \Delta_3 \vdash \mathtt{F_2}} \perp_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h_1} : \perp, \Delta_3 \vdash \mathtt{F_2}} \perp_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h_1} : \mathtt{p_2}, \top, \Delta_3 \vdash \mathtt{p_2}} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h_1} : \Delta_3, \mathtt{p_2} \vdash \mathtt{p_2}} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h_1} : \Delta_2 \vdash \mathtt{F_3}}{\bullet \mathtt{h_1} : \top, \Delta_2 \vdash \mathtt{F_3}} \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_1} : \Delta_2 \vdash \mathtt{F_3}} \text{ ax}}{\bullet \mathtt{h_1} : \Delta_2 \vdash \mathtt{F_3}} H$$

4 Height preserving admissibility of contraction

- Case(s) rule $\top_R$

$$\frac{}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \top}\ \top_R \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \top}\ \top_R$$

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \text{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case(s) rule $\vee_1$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_1 \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \text{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \text{IH}}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_1$$

- Case(s) rule $\vee_2$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_2 \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_2$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \text{ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{inv-th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}\ \rightarrow_L$$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \rightarrow \mathtt{F}_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_5}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \text{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}\ \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \wedge \mathtt{F}_4, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{\overline{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{inv-th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{IH}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L$$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_5}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vee \mathtt{F}_4, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{inv-th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{inv-th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L$$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_5}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L$$

- Case(s) rule $\perp_L$

$$\frac{}{\bullet h_2 : \Delta_1, \perp, \perp \vdash F_3} \perp_L \qquad \rightarrow \qquad \frac{}{\bullet h_2 : \perp, \Delta_1 \vdash F_3} \perp_L$$

$$\frac{}{\bullet h_2 : (\perp, \Delta_4), F_1, F_1 \vdash F_3} \perp_L \qquad \rightarrow \qquad \frac{}{\bullet h_2 : \perp, \Delta_4, F_1 \vdash F_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_2 : \Delta_1, p_3, p_3 \vdash p_3} I \qquad \rightarrow \qquad \frac{}{\bullet h_2 : \Delta_1, p_3 \vdash p_3} I$$

$$\frac{}{\bullet h_2 : (\Delta_4, p_3), F_1, F_1 \vdash p_3} I \qquad \rightarrow \qquad \frac{}{\bullet h_2 : \Delta_4, F_1, p_3 \vdash p_3} I$$

- Case(s) rule $\top_L$

$$\frac{h_2 : \top, \Delta_1 \vdash F_3}{\bullet h_2 : \Delta_1, \top, \top \vdash F_3} \top_L \qquad \rightarrow \qquad \frac{\dfrac{}{h_2 : \Delta_1 \vdash F_3} \text{inv-th/ax}}{\bullet h_2 : \top, \Delta_1 \vdash F_3} \top_L$$

$$\frac{h_2 : \Delta_4, F_1, F_1 \vdash F_3}{\bullet h_2 : (\top, \Delta_4), F_1, F_1 \vdash F_3} \top_L \qquad \rightarrow \qquad \frac{\dfrac{\dfrac{}{h_2 : \Delta_4, F_1, F_1 \vdash F_3} \text{ax}}{h_2 : \Delta_4, F_1 \vdash F_3} \text{IH}}{\bullet h_2 : \top, \Delta_4, F_1 \vdash F_3} \top_L$$

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5   Identity-Expansion

$$\dfrac{\dfrac{\overline{\,-:\mathsf{F}_0\vdash\mathsf{F}_0\,}\;\texttt{IH}}{-:\mathsf{F}_0\vdash\mathsf{F}_0\vee\mathsf{F}_1}\vee_1 \qquad \dfrac{\overline{\,-:\mathsf{F}_1\vdash\mathsf{F}_1\,}\;\texttt{IH}}{-:\mathsf{F}_1\vdash\mathsf{F}_0\vee\mathsf{F}_1}\vee_2}{-:\mathsf{F}_0\vee\mathsf{F}_1\vdash\mathsf{F}_0\vee\mathsf{F}_1}\vee_L$$

$$\dfrac{\dfrac{\overline{\,-:\mathsf{F}_0\vdash\mathsf{F}_0\,}\;\texttt{IH}}{-:\mathsf{F}_0,\mathsf{F}_1\vdash\mathsf{F}_0}\,W \qquad \dfrac{\overline{\,-:\mathsf{F}_1\vdash\mathsf{F}_1\,}\;\texttt{IH}}{-:\mathsf{F}_0,\mathsf{F}_1\vdash\mathsf{F}_1}\,W}{\dfrac{-:\mathsf{F}_0,\mathsf{F}_1\vdash\mathsf{F}_0\wedge\mathsf{F}_1}{-:\mathsf{F}_0\wedge\mathsf{F}_1\vdash\mathsf{F}_0\wedge\mathsf{F}_1}\wedge_L}\wedge_R$$

$$\dfrac{\dfrac{\overline{\,-:\mathsf{F}_0\vdash\mathsf{F}_0\,}\;\texttt{IH}}{-:\mathsf{F}_0,\mathsf{F}_0\to\mathsf{F}_1\vdash\mathsf{F}_0}\,W \qquad \dfrac{\overline{\,-:\mathsf{F}_1\vdash\mathsf{F}_1\,}\;\texttt{IH}}{-:\mathsf{F}_0,\mathsf{F}_1\vdash\mathsf{F}_1}\,W}{\dfrac{-:\mathsf{F}_0,\mathsf{F}_0\to\mathsf{F}_1\vdash\mathsf{F}_1}{-:\mathsf{F}_0\to\mathsf{F}_1\vdash\mathsf{F}_0\to\mathsf{F}_1}\to_R}\to_L$$

$$\overline{\,-:\top\vdash\top\,}\;\top_R$$

$$\overline{\,-:\bot\vdash\bot\,}\;\bot_L$$

6 Cut-Elimination

6.1 Status of $\top_R$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_3 \vdash \top}\ \top_R \quad \overline{\bullet h_4 : \Delta_3, \top \vdash \top}\ \top_R}{- : \Delta_3 \vdash \top}\ \text{Cut}}{\dfrac{\rightarrow}{\overline{- : \Delta_3 \vdash \top}\ \top_R}}$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_3 \vdash \top}\ \top_R \quad \dfrac{h_4 : \top, \Delta_3, F_5 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \rightarrow F_6}\ \rightarrow_R}{- : \Delta_3 \vdash F_5 \rightarrow F_6}\ \text{Cut}}{\dfrac{\rightarrow}{\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_3, F_5 \vdash \top}\ \top_R \quad \overline{h_4 : \top, \Delta_3, F_5 \vdash F_6}\ \text{ax/W}}{- : \Delta_3, F_5 \vdash F_6}\ \text{hCut}}{- : \Delta_3 \vdash F_5 \rightarrow F_6}\ \rightarrow_R}}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_3 \vdash \top}\ \top_R \quad \dfrac{h_4 : \top, \Delta_3 \vdash F_5 \quad h_4 : \top, \Delta_3 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \wedge F_6}\ \wedge_R}{- : \Delta_3 \vdash F_5 \wedge F_6}\ \text{Cut}}{\dfrac{\rightarrow}{\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_3 \vdash \top}\ \text{ax/W} \quad \overline{h_4 : \top, \Delta_3 \vdash F_5}\ \text{ax/W}}{- : \Delta_3 \vdash F_5}\ \text{hCut} \quad \dfrac{\overline{\bullet h_1 : \Delta_3 \vdash \top}\ \text{ax/W} \quad \overline{h_4 : \top, \Delta_3 \vdash F_6}\ \text{ax/W}}{- : \Delta_3 \vdash F_6}\ \text{hCut}}{- : \Delta_3 \vdash F_5 \wedge F_6}\ \wedge_R}}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_3 \vdash \top}\ \top_R \quad \dfrac{h_4 : \top, \Delta_3 \vdash F_5}{\bullet h_4 : \Delta_3, \top \vdash F_5 \vee F_6}\ \vee_1}{- : \Delta_3 \vdash F_5 \vee F_6}\ \text{Cut}}{\dfrac{\rightarrow}{\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_3 \vdash \top}\ \text{ax/W} \quad \overline{h_4 : \top, \Delta_3 \vdash F_5}\ \text{ax/W}}{- : \Delta_3 \vdash F_5}\ \text{hCut}}{- : \Delta_3 \vdash F_5 \vee F_6}\ \vee_1}}$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_3 \vdash \top}\ \top_R \quad \dfrac{h_4 : \top, \Delta_3 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \vee F_6}\ \vee_2}{- : \Delta_3 \vdash F_5 \vee F_6}\ \text{Cut}}{\dfrac{\rightarrow}{\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_3 \vdash \top}\ \top_R \quad \overline{h_4 : \top, \Delta_3 \vdash F_6}\ \text{ax/W}}{- : \Delta_3 \vdash F_6}\ \text{hCut}}{- : \Delta_3 \vdash F_5 \vee F_6}\ \vee_2}}$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_7, F_4 \rightarrow F_5 \vdash \top}\ \top_R \quad \dfrac{h_3 : \top, \Delta_7, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \rightarrow F_5), \top \vdash F_6}\ \rightarrow_L}{- : \Delta_7, F_4 \rightarrow F_5 \vdash F_6}\ \text{Cut}}{\dfrac{\rightarrow}{\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_7, F_4 \rightarrow F_5 \vdash \top}\ \text{ax/W} \quad \overline{h_3 : \top, \Delta_7, F_4 \rightarrow F_5 \vdash F_4}\ \text{ax/W}}{- : \Delta_7, F_4 \rightarrow F_5 \vdash F_4}\ \text{hCut} \quad \dfrac{\overline{\bullet h_1 : \Delta_7, F_5 \vdash \top}\ \top_R \quad \overline{h_3 : \top, \Delta_7, F_5 \vdash F_6}\ \text{ax/W}}{- : \Delta_7, F_5 \vdash F_6}\ \text{hCut}}{- : \Delta_7, F_4 \rightarrow F_5 \vdash F_6}\ \rightarrow_L}}$$

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- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h_1} : \Delta_7, \mathtt{F_4} \wedge \mathtt{F_5} \vdash \top}\ \top_R \qquad \dfrac{\mathtt{h_3} : \top, \Delta_7, \mathtt{F_4}, \mathtt{F_5} \vdash \mathtt{F_6}}{\bullet \mathtt{h_3} : (\Delta_7, \mathtt{F_4} \wedge \mathtt{F_5}), \top \vdash \mathtt{F_6}}\ \wedge_L}{- : \Delta_7, \mathtt{F_4} \wedge \mathtt{F_5} \vdash \mathtt{F_6}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h_1} : \Delta_7, \mathtt{F_4}, \mathtt{F_5} \vdash \top}\ \top_R \quad \dfrac{}{\mathtt{h_3} : \top, \Delta_7, \mathtt{F_4}, \mathtt{F_5} \vdash \mathtt{F_6}}\ \text{ax/W}}{- : \Delta_7, \mathtt{F_4}, \mathtt{F_5} \vdash \mathtt{F_6}}\ \text{hCut}}{- : \Delta_7, \mathtt{F_4} \wedge \mathtt{F_5} \vdash \mathtt{F_6}}\ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h_1} : \Delta_7, \mathtt{F_4} \vee \mathtt{F_5} \vdash \top}\ \top_R \qquad \dfrac{\mathtt{h_3} : \top, \Delta_7, \mathtt{F_4} \vdash \mathtt{F_6} \quad \mathtt{h_3} : \top, \Delta_7, \mathtt{F_5} \vdash \mathtt{F_6}}{\bullet \mathtt{h_3} : (\Delta_7, \mathtt{F_4} \vee \mathtt{F_5}), \top \vdash \mathtt{F_6}}\ \vee_L}{- : \Delta_7, \mathtt{F_4} \vee \mathtt{F_5} \vdash \mathtt{F_6}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h_1} : \Delta_7, \mathtt{F_4} \vdash \top}\ \top_R \quad \dfrac{}{\mathtt{h_3} : \top, \Delta_7, \mathtt{F_4} \vdash \mathtt{F_6}}\ \text{ax/W}}{- : \Delta_7, \mathtt{F_4} \vdash \mathtt{F_6}}\ \text{hCut} \qquad \dfrac{\dfrac{}{\bullet \mathtt{h_1} : \Delta_7, \mathtt{F_5} \vdash \top}\ \top_R \quad \dfrac{}{\mathtt{h_3} : \top, \Delta_7, \mathtt{F_5} \vdash \mathtt{F_6}}\ \text{ax/W}}{- : \Delta_7, \mathtt{F_5} \vdash \mathtt{F_6}}\ \text{hCut}}{- : \Delta_7, \mathtt{F_4} \vee \mathtt{F_5} \vdash \mathtt{F_6}}\ \vee_L$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h_1} : \perp, \Delta_5 \vdash \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h_3} : (\perp, \Delta_5), \top \vdash \mathtt{F_4}}\ \perp_L}{- : \perp, \Delta_5 \vdash \mathtt{F_4}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \perp, \Delta_5 \vdash \mathtt{F_4}}\ \perp_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h_1} : \Delta_5, \mathtt{p_4} \vdash \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h_3} : (\Delta_5, \mathtt{p_4}), \top \vdash \mathtt{p_4}}\ I}{- : \Delta_5, \mathtt{p_4} \vdash \mathtt{p_4}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_5, \mathtt{p_4} \vdash \mathtt{p_4}}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h_1} : \Delta_4 \vdash \top}\ \top_R \quad \dfrac{\mathtt{h_3} : \Delta_4 \vdash \mathtt{F_5}}{\bullet \mathtt{h_3} : \Delta_4, \top \vdash \mathtt{F_5}}\ \top_L}{- : \Delta_4 \vdash \mathtt{F_5}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \mathtt{F_5}}\ \text{ax/W}$$

6.2   Status of $\rightarrow_R$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h_1} : \Delta_5, \mathtt{F_6} \vdash \mathtt{F_7}}{\bullet \mathtt{h_1} : \Delta_5 \vdash \mathtt{F_6} \rightarrow \mathtt{F_7}}\ \rightarrow_R \quad \dfrac{}{\bullet \mathtt{h_8} : \Delta_5, \mathtt{F_6} \rightarrow \mathtt{F_7} \vdash \top}\ \top_R}{- : \Delta_5 \vdash \top}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_5 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

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$$\dfrac{\dfrac{h_1 : \Delta_5, F_6 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_5, F_9, F_6 \to F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \to F_{10}} \to_R}{- : \Delta_5 \vdash F_9 \to F_{10}} \ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5, F_9 \vdash F_6 \to F_7} \ \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_9, F_6 \to F_7 \vdash F_{10}} \ \text{ax/W}}{- : \Delta_5, F_9 \vdash F_{10}} \ \text{hCut}}{- : \Delta_5 \vdash F_9 \to F_{10}} \to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_1 : \Delta_5, F_6 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \to F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \wedge F_{10}} \ \wedge_R}{- : \Delta_5 \vdash F_9 \wedge F_{10}} \ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \to F_7} \ \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \to F_7 \vdash F_9} \ \text{ax/W}}{- : \Delta_5 \vdash F_9} \ \text{hCut} \quad \dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \to F_7} \ \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \to F_7 \vdash F_{10}} \ \text{ax/W}}{- : \Delta_5 \vdash F_{10}} \ \text{hCut}}{- : \Delta_5 \vdash F_9 \wedge F_{10}} \ \wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{h_1 : \Delta_5, F_6 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_5, F_6 \to F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \vee F_{10}} \ \vee_1}{- : \Delta_5 \vdash F_9 \vee F_{10}} \ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \to F_7} \ \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \to F_7 \vdash F_9} \ \text{ax/W}}{- : \Delta_5 \vdash F_9} \ \text{hCut}}{- : \Delta_5 \vdash F_9 \vee F_{10}} \ \vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{h_1 : \Delta_5, F_6 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_5, F_6 \to F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \vee F_{10}} \ \vee_2}{- : \Delta_5 \vdash F_9 \vee F_{10}} \ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \to F_7} \ \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \to F_7 \vdash F_{10}} \ \text{ax/W}}{- : \Delta_5 \vdash F_{10}} \ \text{hCut}}{- : \Delta_5 \vdash F_9 \vee F_{10}} \ \vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_1 : (\Delta_{11}, F_8 \to F_9), F_5 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5 \to F_6} \to_R \quad \dfrac{h_7 : \Delta_{11}, F_5 \to F_6, F_8 \to F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \to F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \to F_9), F_5 \to F_6 \vdash F_{10}} \to_L}{- : \Delta_{11}, F_8 \to F_9 \vdash F_{10}} \ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5 \to F_6} \ \text{ax/W} \quad \dfrac{}{h_7 : \Delta_{11}, F_5 \to F_6, F_8 \to F_9 \vdash F_8} \ \text{ax/W}}{- : \Delta_{11}, F_8 \to F_9 \vdash F_8} \ \text{hCut} \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_{11}, F_5, F_9 \vdash F_6} \ \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \to F_6} \to_R \quad \dfrac{}{h_7 : \Delta_{11}, F_9, F_5 \to F_6 \vdash F_{10}} \ \text{ax/W}}{- : \Delta_{11}, F_9 \vdash F_{10}} \ \text{hCut}}{- : \Delta_{11}, F_8 \to F_9 \vdash F_{10}} \ \to_L$$

$$\dfrac{\dfrac{h_1 : \Delta_6, F_7 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash F_7 \to F_8} \to_R \quad \dfrac{h_5 : \Delta_6, F_7 \to F_8 \vdash F_7 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \to F_8 \vdash F_9} \to_L}{- : \Delta_6 \vdash F_9} \ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_6 \vdash F_7 \to F_8} \ \text{ax/W} \quad \dfrac{}{h_5 : \Delta_6, F_7 \to F_8 \vdash F_7} \ \text{ax/W}}{- : \Delta_6 \vdash F_7} \ \text{hCut} \quad \dfrac{\dfrac{}{- : \Delta_6, F_7 \vdash F_8} \ \text{ax/W} \quad \dfrac{}{- : \Delta_6, F_7, F_8 \vdash F_9} \ \text{ax/W}}{- : \Delta_6, F_7 \vdash F_9} \ \text{sCut}}{- : \Delta_6 \vdash F_9} \ \text{sCut}$$

- Case rule $\wedge_L$

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$$\dfrac{\dfrac{h_1:(\Delta_{11},F_8\wedge F_9),F_5\vdash F_6}{\bullet h_1:\Delta_{11},F_8\wedge F_9\vdash F_5\to F_6}\to_R \qquad \dfrac{h_7:\Delta_{11},F_8,F_9,F_5\to F_6\vdash F_{10}}{\bullet h_7:(\Delta_{11},F_8\wedge F_9),F_5\to F_6\vdash F_{10}}\wedge_L}{-:\Delta_{11},F_8\wedge F_9\vdash F_{10}}\;\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\overline{h_1:\Delta_{11},F_5,F_8,F_9\vdash F_6}\ \text{inv-th/ax}}{\bullet h_1:\Delta_{11},F_8,F_9\vdash F_5\to F_6}\to_R \qquad \dfrac{\overline{h_7:\Delta_{11},F_8,F_9,F_5\to F_6\vdash F_{10}}\ \text{ax/W}}{}}{-:\Delta_{11},F_8,F_9\vdash F_{10}}\;\text{hCut}}{-:\Delta_{11},F_8\wedge F_9\vdash F_{10}}\;\wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_1:(\Delta_{11},F_8\vee F_9),F_5\vdash F_6}{\bullet h_1:\Delta_{11},F_8\vee F_9\vdash F_5\to F_6}\to_R \qquad \dfrac{h_7:\Delta_{11},F_8,F_5\to F_6\vdash F_{10}\quad h_7:\Delta_{11},F_9,F_5\to F_6\vdash F_{10}}{\bullet h_7:(\Delta_{11},F_8\vee F_9),F_5\to F_6\vdash F_{10}}\vee_L}{-:\Delta_{11},F_8\vee F_9\vdash F_{10}}\;\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\overline{h_1:\Delta_{11},F_5,F_8\vdash F_6}\ \text{inv-th/ax}}{\bullet h_1:\Delta_{11},F_8\vdash F_5\to F_6}\to_R\quad \dfrac{\overline{h_7:\Delta_{11},F_8,F_5\to F_6\vdash F_{10}}\ \text{ax/W}}{}}{-:\Delta_{11},F_8\vdash F_{10}}\text{hCut}\quad \dfrac{\dfrac{\overline{h_1:\Delta_{11},F_5,F_9\vdash F_6}\ \text{inv-th/ax}}{\bullet h_1:\Delta_{11},F_9\vdash F_5\to F_6}\to_R\quad \dfrac{\overline{h_7:\Delta_{11},F_9,F_5\to F_6\vdash F_{10}}\ \text{ax/W}}{}}{-:\Delta_{11},F_9\vdash F_{10}}\text{hCut}}{-:\Delta_{11},F_8\vee F_9\vdash F_{10}}\;\vee_L$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{h_1:(\perp,\Delta_9),F_5\vdash F_6}{\bullet h_1:\perp,\Delta_9\vdash F_5\to F_6}\to_R \qquad \dfrac{}{\bullet h_7:(\perp,\Delta_9),F_5\to F_6\vdash F_8}\perp_L}{-:\perp,\Delta_9\vdash F_8}\;\text{Cut}$$

$$\to$$

$$\dfrac{}{-:\perp,\Delta_9\vdash F_8}\;\perp_L$$

- Case rule I

$$\dfrac{\dfrac{h_1:(\Delta_9,p_8),F_5\vdash F_6}{\bullet h_1:\Delta_9,p_8\vdash F_5\to F_6}\to_R \qquad \dfrac{}{\bullet h_7:(\Delta_9,p_8),F_5\to F_6\vdash p_8}I}{-:\Delta_9,p_8\vdash p_8}\;\text{Cut}$$

$$\to$$

$$\dfrac{}{-:\Delta_9,p_8\vdash p_8}\;I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{h_1:(\top,\Delta_9),F_5\vdash F_6}{\bullet h_1:\top,\Delta_9\vdash F_5\to F_6}\to_R \qquad \dfrac{h_7:\Delta_9,F_5\to F_6\vdash F_8}{\bullet h_7:(\top,\Delta_9),F_5\to F_6\vdash F_8}\top_L}{-:\top,\Delta_9\vdash F_8}\;\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet h_1:\top,\Delta_9\vdash F_5\to F_6}\text{ax/W}\quad \dfrac{}{h_7:\top,\Delta_9,F_5\to F_6\vdash F_8}\text{ax/W}}{-:\top,\Delta_9\vdash F_8}\;\text{hCut}$$

6.3 Status of $\wedge_R$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{h_1:\Delta_5\vdash F_6\quad h_1:\Delta_5\vdash F_7}{\bullet h_1:\Delta_5\vdash F_6\wedge F_7}\wedge_R \qquad \dfrac{}{\bullet h_8:\Delta_5,F_6\wedge F_7\vdash\top}\top_R}{-:\Delta_5\vdash\top}\;\text{Cut}$$

$$\to$$

$$\dfrac{}{-:\Delta_5\vdash\top}\;\top_R$$

- Case rule $\to_R$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \Delta_5 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7} \wedge_R \quad
\cfrac{h_8 : \Delta_5, F_9, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \to F_{10}} \to_R
}{- : \Delta_5 \vdash F_9 \to F_{10}}
\;\text{Cut}
}{
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_5, F_9 \vdash F_6 \wedge F_7}\;\text{ax/W} \quad
\cfrac{}{h_8 : \Delta_5, F_9, F_6 \wedge F_7 \vdash F_{10}}\;\text{ax/W}
}{- : \Delta_5, F_9 \vdash F_{10}}\;\text{hCut}
}{- : \Delta_5 \vdash F_9 \to F_{10}} \to_R
}\;\to
$$

- Case rule $\wedge_R$

$$
\cfrac{
\cfrac{h_1 : \Delta_5 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7} \wedge_R \quad
\cfrac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \wedge F_{10}} \wedge_R
}{- : \Delta_5 \vdash F_9 \wedge F_{10}}
\;\text{Cut}
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7}\text{ax/W}\quad\cfrac{}{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9}\text{ax/W}
}{- : \Delta_5 \vdash F_9}\text{hCut}\quad
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7}\text{ax/W}\quad\cfrac{}{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10}}\text{ax/W}
}{- : \Delta_5 \vdash F_{10}}\text{hCut}
}{- : \Delta_5 \vdash F_9 \wedge F_{10}}\wedge_R
$$

- Case rule $\vee_1$

$$
\cfrac{
\cfrac{h_1 : \Delta_5 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7} \wedge_R \quad
\cfrac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \vee F_{10}} \vee_1
}{- : \Delta_5 \vdash F_9 \vee F_{10}}
\;\text{Cut}
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7}\text{ax/W}\quad\cfrac{}{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9}\text{ax/W}
}{- : \Delta_5 \vdash F_9}\text{hCut}
}{- : \Delta_5 \vdash F_9 \vee F_{10}}\vee_1
$$

- Case rule $\vee_2$

$$
\cfrac{
\cfrac{h_1 : \Delta_5 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7} \wedge_R \quad
\cfrac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \vee F_{10}} \vee_2
}{- : \Delta_5 \vdash F_9 \vee F_{10}}
\;\text{Cut}
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7}\text{ax/W}\quad\cfrac{}{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10}}\text{ax/W}
}{- : \Delta_5 \vdash F_{10}}\text{hCut}
}{- : \Delta_5 \vdash F_9 \vee F_{10}}\vee_2
$$

- Case rule $\to_L$

$$
\cfrac{
\cfrac{h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5 \wedge F_6} \wedge_R \quad
\cfrac{h_7 : \Delta_{11}, F_8 \to F_9, F_5 \wedge F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \to F_9), F_5 \wedge F_6 \vdash F_{10}} \to_L
}{- : \Delta_{11}, F_8 \to F_9 \vdash F_{10}}
\;\text{Cut}
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5 \wedge F_6}\text{ax/W}\; \cfrac{}{h_7 : \Delta_{11}, F_8 \to F_9, F_5 \wedge F_6 \vdash F_8}\text{ax/W}
}{- : \Delta_{11}, F_8 \to F_9 \vdash F_8}\text{hCut}\quad
\cfrac{
\cfrac{\cfrac{h_1 : \Delta_{11}, F_9 \vdash F_5}{}\text{inv-th/ax}\; \cfrac{h_1 : \Delta_{11}, F_9 \vdash F_6}{}\text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6}\wedge_R\quad h_7 : \Delta_{11}, F_9, \ldots
}{- : \Delta_{11}, F_9 \vdash F_{10}}\to_L
}{- : \Delta_{11}, F_8 \to F_9 \vdash F_{10}}\;\to
$$

- Case rule $\wedge_L$

$$
\cfrac{
\cfrac{h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \wedge F_6} \wedge_R \quad
\cfrac{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \wedge F_6 \vdash F_{10}} \wedge_L
}{- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}}
\;\text{Cut}
$$

$$
\cfrac{
\cfrac{
\cfrac{\cfrac{h_1 : \Delta_{11}, F_8, F_9 \vdash F_5}{}\text{inv-th/ax}\;\cfrac{h_1 : \Delta_{11}, F_8, F_9 \vdash F_6}{}\text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \wedge F_6}\wedge_R\quad
\cfrac{}{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}\text{ax/W}
}{- : \Delta_{11}, F_8, F_9 \vdash F_{10}}\text{hCut}
}{- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}}\wedge_L
$$

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$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_6 \vdash F_7 \quad h_1 : \Delta_6 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash F_7 \wedge F_8} \wedge_R \quad \cfrac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \wedge F_8 \vdash F_9} \wedge_L}{- : \Delta_6 \vdash F_9} \text{Cut}}{\xrightarrow{}}$$

$$\cfrac{\cfrac{}{- : \Delta_6 \vdash F_7} \text{ax/W} \quad \cfrac{\cfrac{}{- : \Delta_6, F_7 \vdash F_8} \text{ax/W} \quad \cfrac{}{- : \Delta_6, F_7, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_6, F_7 \vdash F_9} \text{sCut}}{- : \Delta_6 \vdash F_9} \text{sCut}$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \wedge F_6} \wedge_R \quad \cfrac{\cfrac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \wedge F_6 \vdash F_{10}} \vee_L}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut}}{\xrightarrow{}}$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_{11}, F_8 \vdash F_5}{} \text{inv-th/ax} \quad \cfrac{h_1 : \Delta_{11}, F_8 \vdash F_6}{} \text{inv-th/ax}}{\cfrac{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \wedge F_6}{} \wedge_R \quad \cfrac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10}}{} \text{ax/W}}{- : \Delta_{11}, F_8 \vdash F_{10}} \text{hCut} \quad \cfrac{\cfrac{h_1 : \Delta_{11}, F_9 \vdash F_5}{} \text{inv-th/ax} \quad \cfrac{h_1 : \Delta_{11}, F_9 \vdash F_6}{} \text{inv-th}}{\cfrac{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6}{} \wedge_R}{- : \Delta_{11}, F_9 \vdash 1}}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}}$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{\cfrac{h_1 : \perp, \Delta_9 \vdash F_5 \quad h_1 : \perp, \Delta_9 \vdash F_6}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \wedge F_6} \wedge_R \quad \cfrac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \wedge F_6 \vdash F_8} \perp_L}{- : \perp, \Delta_9 \vdash F_8} \text{Cut}}{\cfrac{\xrightarrow{}}{\cfrac{}{- : \perp, \Delta_9 \vdash F_8} \perp_L}}$$

- Case rule I

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_9, p_8 \vdash F_5 \quad h_1 : \Delta_9, p_8 \vdash F_6}{\bullet h_1 : \Delta_9, p_8 \vdash F_5 \wedge F_6} \wedge_R \quad \cfrac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \wedge F_6 \vdash p_8} I}{- : \Delta_9, p_8 \vdash p_8} \text{Cut}}{\cfrac{\xrightarrow{}}{\cfrac{}{- : \Delta_9, p_8 \vdash p_8} I}}$$

- Case rule $\top_L$

$$\cfrac{\cfrac{\cfrac{h_1 : \top, \Delta_9 \vdash F_5 \quad h_1 : \top, \Delta_9 \vdash F_6}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \wedge F_6} \wedge_R \quad \cfrac{h_7 : \Delta_9, F_5 \wedge F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \wedge F_6 \vdash F_8} \top_L}{- : \top, \Delta_9 \vdash F_8} \text{Cut}}{\cfrac{\xrightarrow{}}{\cfrac{\cfrac{}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \wedge F_6} \text{ax/W} \quad \cfrac{}{h_7 : \top, \Delta_9, F_5 \wedge F_6 \vdash F_8} \text{ax/W}}{- : \top, \Delta_9 \vdash F_8} \text{hCut}}}$$

6.4 Status of $\vee_1$: OK

- Case rule $\top_R$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_1 \quad \cfrac{}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top} \top_R}{- : \Delta_5 \vdash \top} \text{Cut}}{\cfrac{\xrightarrow{}}{\cfrac{}{- : \Delta_5 \vdash \top} \top_R}}$$

- Case rule $\rightarrow_R$

$$
\dfrac{\dfrac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}\ \vee_1 \qquad \dfrac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \to F_{10}}\ \to_R}{- : \Delta_5 \vdash F_9 \to F_{10}}\ \text{Cut}
$$

$$\to$$

$$
\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5, F_9 \vdash F_6 \vee F_7}\ \text{ax/W} \qquad \dfrac{}{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}\ \text{ax/W}}{- : \Delta_5, F_9 \vdash F_{10}}\ \text{hCut}}{- : \Delta_5 \vdash F_9 \to F_{10}}\ \to_R
$$

- Case rule $\wedge_R$

$$
\dfrac{\dfrac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}\ \vee_1 \qquad \dfrac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \wedge F_{10}}\ \wedge_R}{- : \Delta_5 \vdash F_9 \wedge F_{10}}\ \text{Cut}
$$

$$\to$$

$$
\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}\ \text{ax/W}\ \dfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}\ \text{ax/W}}{- : \Delta_5 \vdash F_9}\ \text{hCut}\quad \dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}\ \text{ax/W}\ \dfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}\ \text{ax/W}}{- : \Delta_5 \vdash F_{10}}\ \text{hCut}}{- : \Delta_5 \vdash F_9 \wedge F_{10}}\ \wedge_R
$$

- Case rule $\vee_1$

$$
\dfrac{\dfrac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}\ \vee_1 \qquad \dfrac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}}\ \vee_1}{- : \Delta_5 \vdash F_9 \vee F_{10}}\ \text{Cut}
$$

$$\to$$

$$
\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}\ \text{ax/W}\ \dfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}\ \text{ax/W}}{- : \Delta_5 \vdash F_9}\ \text{hCut}}{- : \Delta_5 \vdash F_9 \vee F_{10}}\ \vee_1
$$

- Case rule $\vee_2$

$$
\dfrac{\dfrac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}\ \vee_1 \qquad \dfrac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}}\ \vee_2}{- : \Delta_5 \vdash F_9 \vee F_{10}}\ \text{Cut}
$$

$$\to$$

$$
\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}\ \text{ax/W}\ \dfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}\ \text{ax/W}}{- : \Delta_5 \vdash F_{10}}\ \text{hCut}}{- : \Delta_5 \vdash F_9 \vee F_{10}}\ \vee_2
$$

- Case rule $\to_L$

$$
\dfrac{\dfrac{h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5 \vee F_6}\ \vee_1 \quad \dfrac{h_7 : \Delta_{11}, F_8 \to F_9, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \to F_9), F_5 \vee F_6 \vdash F_{10}}\ \to_L}{- : \Delta_{11}, F_8 \to F_9 \vdash F_{10}}\ \text{Cut}
$$

$$\to$$

$$
\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5 \vee F_6}\ \text{ax/W}\ \dfrac{}{h_7 : \Delta_{11}, F_8 \to F_9, F_5 \vee F_6 \vdash F_8}\ \text{ax/W}}{- : \Delta_{11}, F_8 \to F_9 \vdash F_8}\ \text{hCut}\quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_{11}, F_9 \vdash F_5}\ \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6}\ \vee_1\ \dfrac{}{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}\ \text{ax/W}}{- : \Delta_{11}, F_9 \vdash F_{10}}\ \text{hCut}}{- : \Delta_{11}, F_8 \to F_9 \vdash F_{10}}\ \to_L
$$

- Case rule $\wedge_L$

$$
\dfrac{\dfrac{h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \vee F_6}\ \vee_1 \quad \dfrac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}}\ \wedge_L}{- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}}\ \text{Cut}
$$

$$\to$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{h_1 : \Delta_{11}, F_8, F_9 \vdash F_5}\ \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \vee F_6}\ \vee_1\ \dfrac{}{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}\ \text{ax/W}}{- : \Delta_{11}, F_8, F_9 \vdash F_{10}}\ \text{hCut}}{- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}}\ \wedge_L
$$

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- Case rule $\vee_L$

$$\dfrac{\dfrac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \vee F_6} \vee_1 \quad \dfrac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\overline{h_1 : \Delta_{11}, F_8 \vdash F_5}}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \vee F_6} \vee_1 \quad \dfrac{}{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_8 \vdash F_{10}} \text{hCut} \quad \dfrac{\dfrac{\overline{h_1 : \Delta_{11}, F_9 \vdash F_5}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \vee_1 \quad \dfrac{}{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \vee_L$$

(with inv-th/ax on the $\overline{h_1 : \Delta_{11}, F_8 \vdash F_5}$ and $\overline{h_1 : \Delta_{11}, F_9 \vdash F_5}$ leaves)

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7}{\bullet h_1 : \Delta_6 \vdash F_7 \vee F_8} \vee_1 \quad \dfrac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9} \vee_L}{- : \Delta_6 \vdash F_9} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Delta_6 \vdash F_7} \text{ax/W} \quad \dfrac{}{- : \Delta_6, F_7 \vdash F_9} \text{ax/W}}{- : \Delta_6 \vdash F_9} \text{sCut}$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{h_1 : \perp, \Delta_9 \vdash F_5}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \vee F_6} \vee_1 \quad \dfrac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8} \perp_L}{- : \perp, \Delta_9 \vdash F_8} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \perp, \Delta_9 \vdash F_8} \perp_L$$

- Case rule I

$$\dfrac{\dfrac{h_1 : \Delta_9, p_8 \vdash F_5}{\bullet h_1 : \Delta_9, p_8 \vdash F_5 \vee F_6} \vee_1 \quad \dfrac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8} I}{- : \Delta_9, p_8 \vdash p_8} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_9, p_8 \vdash p_8} I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{h_1 : \top, \Delta_9 \vdash F_5}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6} \vee_1 \quad \dfrac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L}{- : \top, \Delta_9 \vdash F_8} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6} \text{ax/W} \quad \dfrac{}{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8} \text{ax/W}}{- : \top, \Delta_9 \vdash F_8} \text{hCut}$$

6.5 Status of $\vee_2$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \dfrac{}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top} \top_R}{- : \Delta_5 \vdash \top} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_5 \vdash \top} \top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \dfrac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \to F_{10}} \to_R}{- : \Delta_5 \vdash F_9 \to F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5, F_9 \vdash F_6 \vee F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_5, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_5 \vdash F_9 \to F_{10}} \to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \dfrac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \wedge F_{10}} \wedge_R}{- : \Delta_5 \vdash F_9 \wedge F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \text{ax/W}}{- : \Delta_5 \vdash F_9} \text{hCut} \quad \dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_5 \vdash F_9 \wedge F_{10}} \wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \dfrac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_1}{- : \Delta_5 \vdash F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \text{ax/W}}{- : \Delta_5 \vdash F_9} \text{hCut}}{- : \Delta_5 \vdash F_9 \vee F_{10}} \vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \dfrac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_2}{- : \Delta_5 \vdash F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_5 \vdash F_9 \vee F_{10}} \vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5 \vee F_6} \vee_2 \quad \dfrac{h_7 : \Delta_{11}, F_8 \to F_9, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \to F_9), F_5 \vee F_6 \vdash F_{10}} \to_L}{- : \Delta_{11}, F_8 \to F_9 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_{11}, F_8 \to F_9 \vdash F_5 \vee F_6} \text{ax/W} \quad \dfrac{}{h_7 : \Delta_{11}, F_8 \to F_9, F_5 \vee F_6 \vdash F_8} \text{ax/W}}{- : \Delta_{11}, F_8 \to F_9 \vdash F_8} \text{hCut} \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_{11}, F_9 \vdash F_6} \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \vee_2 \quad \dfrac{}{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, F_8 \to F_9 \vdash F_{10}} \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \vee F_6} \vee_2 \quad \dfrac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L}{- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_1 : \Delta_{11}, F_8, F_9 \vdash F_6} \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \vee F_6} \vee_2 \quad \dfrac{}{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_8, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \wedge_L$$

- Case rule $\vee_L$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \vee F_6} \vee_2
\quad
\cfrac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L
}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut}
}{}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{
\cfrac{
\cfrac{\cfrac{h_1 : \Delta_{11}, F_8 \vdash F_6}{} \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \vee F_6} \vee_2
\quad
\cfrac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{} \text{ax/W}
}{- : \Delta_{11}, F_8 \vdash F_{10}} \text{hCut}
\quad
\cfrac{
\cfrac{\cfrac{h_1 : \Delta_{11}, F_9 \vdash F_6}{} \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \vee_2
\quad
\cfrac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{} \text{ax/W}
}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{hCut}
}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \vee_L
}{}
$$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \Delta_6 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash F_7 \vee F_8} \vee_2
\quad
\cfrac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9} \vee_L
}{- : \Delta_6 \vdash F_9} \text{Cut}
}{}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{- : \Delta_6 \vdash F_8}{} \text{ax/W}
\quad
\cfrac{- : \Delta_6, F_8 \vdash F_9}{} \text{ax/W}
}{- : \Delta_6 \vdash F_9} \text{sCut}
$$

- Case rule $\perp_L$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \perp, \Delta_9 \vdash F_6}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \vee F_6} \vee_2
\quad
\cfrac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8} \perp_L
}{- : \perp, \Delta_9 \vdash F_8} \text{Cut}
}{}
$$

$$\rightarrow$$

$$
\cfrac{}{- : \perp, \Delta_9 \vdash F_8} \perp_L
$$

- Case rule I

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \Delta_9, p_8 \vdash F_6}{\bullet h_1 : \Delta_9, p_8 \vdash F_5 \vee F_6} \vee_2
\quad
\cfrac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8} I
}{- : \Delta_9, p_8 \vdash p_8} \text{Cut}
}{}
$$

$$\rightarrow$$

$$
\cfrac{}{- : \Delta_9, p_8 \vdash p_8} I
$$

- Case rule $\top_L$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \top, \Delta_9 \vdash F_6}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6} \vee_2
\quad
\cfrac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L
}{- : \top, \Delta_9 \vdash F_8} \text{Cut}
}{}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6}{} \text{ax/W}
\quad
\cfrac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{} \text{ax/W}
}{- : \top, \Delta_9 \vdash F_8} \text{hCut}
$$

6.6 Status of $\rightarrow_L$: OK

- Case rule $\top_R$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7} \rightarrow_L
\quad
\cfrac{}{\bullet h_{10} : (\Delta_6, F_8 \rightarrow F_9), F_7 \vdash \top} \top_R
}{- : \Delta_6, F_8 \rightarrow F_9 \vdash \top} \text{Cut}
}{}
$$

$$\rightarrow$$

$$
\cfrac{}{- : \Delta_6, F_8 \rightarrow F_9 \vdash \top} \top_R
$$

- Case rule $\rightarrow_R$

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$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_6, F_8 \to F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \to F_9 \vdash F_7} \to_L \quad \cfrac{h_{10} : \Delta_6, F_7, F_{11}, F_8 \to F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \to F_9), F_7 \vdash F_{11} \to F_{12}} \to_R}{- : \Delta_6, F_8 \to F_9 \vdash F_{11} \to F_{12}} \text{Cut}}{\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_6, F_{11}, F_8 \to F_9 \vdash F_7} \text{ax/W} \quad \cfrac{}{h_{10} : \Delta_6, F_{11}, F_7, F_8 \to F_9 \vdash F_{12}} \text{ax/W}}{- : \Delta_6, F_{11}, F_8 \to F_9 \vdash F_{12}} \text{hCut}}{- : \Delta_6, F_8 \to F_9 \vdash F_{11} \to F_{12}} \to_R}$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_6, F_8 \to F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \to F_9 \vdash F_7} \to_L \quad \cfrac{h_{10} : \Delta_6, F_7, F_8 \to F_9 \vdash F_{11} \quad h_{10} : \Delta_6, F_7, F_8 \to F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \to F_9), F_7 \vdash F_{11} \wedge F_{12}} \wedge_R}{- : \Delta_6, F_8 \to F_9 \vdash F_{11} \wedge F_{12}} \text{Cut}}{\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_6, F_8 \to F_9 \vdash F_7} \text{ax/W} \quad \cfrac{}{h_{10} : \Delta_6, F_7, F_8 \to F_9 \vdash F_{11}} \text{ax/W}}{- : \Delta_6, F_8 \to F_9 \vdash F_{11}} \text{hCut} \quad \cfrac{\cfrac{}{\bullet h_1 : \Delta_6, F_8 \to F_9 \vdash F_7} \text{ax/W} \quad \cfrac{}{h_{10} : \Delta_6, F_7, F_8 \to F_9 \vdash F_{12}} \text{ax/W}}{- : \Delta_6, F_8 \to F_9 \vdash F_{12}} \text{hCut}}{- : \Delta_6, F_8 \to F_9 \vdash F_{11} \wedge F_{12}} \wedge_R}$$

- Case rule $\vee_1$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_6, F_8 \to F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \to F_9 \vdash F_7} \to_L \quad \cfrac{h_{10} : \Delta_6, F_7, F_8 \to F_9 \vdash F_{11}}{\bullet h_{10} : (\Delta_6, F_8 \to F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_1}{- : \Delta_6, F_8 \to F_9 \vdash F_{11} \vee F_{12}} \text{Cut}}{\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_6, F_8 \to F_9 \vdash F_7} \text{ax/W} \quad \cfrac{}{h_{10} : \Delta_6, F_7, F_8 \to F_9 \vdash F_{11}} \text{ax/W}}{- : \Delta_6, F_8 \to F_9 \vdash F_{11}} \text{hCut}}{- : \Delta_6, F_8 \to F_9 \vdash F_{11} \vee F_{12}} \vee_1}$$

- Case rule $\vee_2$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_6, F_8 \to F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \to F_9 \vdash F_7} \to_L \quad \cfrac{h_{10} : \Delta_6, F_7, F_8 \to F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \to F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_2}{- : \Delta_6, F_8 \to F_9 \vdash F_{11} \vee F_{12}} \text{Cut}}{\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_6, F_8 \to F_9 \vdash F_7} \text{ax/W} \quad \cfrac{}{h_{10} : \Delta_6, F_7, F_8 \to F_9 \vdash F_{12}} \text{ax/W}}{- : \Delta_6, F_8 \to F_9 \vdash F_{12}} \text{hCut}}{- : \Delta_6, F_8 \to F_9 \vdash F_{11} \vee F_{12}} \vee_2}$$

- Case rule $\to_L$

$$\cfrac{\cfrac{h_1 : (\Delta_{13}, F_{10} \to F_{11}), F_7 \to F_8 \vdash F_7 \quad h_1 : (\Delta_{13}, F_{10} \to F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \to F_{11}), F_7 \to F_8 \vdash F_6} \to_L \quad \cfrac{h_9 : \Delta_{13}, F_6, F_7 \to F_8, F_{10} \to F_{11} \vdash F_{10} \quad h_9 : \Delta_{13}, F_6, F_{11}, F_7 \to F_8 \vdash F_{1}}{\bullet h_9 : ((\Delta_{13}, F_{10} \to F_{11}), F_7 \to F_8), F_6 \vdash F_{12}} \text{Cut}}{- : (\Delta_{13}, F_{10} \to F_{11}), F_7 \to F_8 \vdash F_{12}}$$

$$\cfrac{\cfrac{}{- : \Delta_{13}, F_{10} \to F_{11}, F_7 \to F_8 \vdash F_7} \text{ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_{13}, F_8, F_{10} \to F_{11} \vdash F_6} \text{ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_{13}, F_6, F_8, F_{10} \to F_{11} \vdash F_{10}} \text{inv-th/ax} \quad \cfrac{}{h_9 : \Delta_{13}, F_{11}, F_6, F_8 \vdash F_{12}}}{\bullet h_9 : \Delta_{13}, F_6, F_8, F_{10} \to F_{11} \vdash F_{12}} \text{hCut}}{- : \Delta_{13}, F_8, F_{10} \to F_{11} \vdash F_{12}} \to_L}{- : \Delta_{13}, F_{10} \to F_{11}, F_7 \to F_8 \vdash F_{12}}$$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_7 \to F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash F_{10} \to F_{11}}{\bullet h_1 : \Delta_6, F_7 \to F_8 \vdash F_{10} \to F_{11}} \to_L \quad \cfrac{h_9 : \Delta_6, F_7 \to F_8, F_{10} \to F_{11} \vdash F_{10} \quad h_9 : \Delta_6, F_{11}, F_7 \to F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \to F_8), F_{10} \to F_{11} \vdash F_{12}} \to_L}{- : \Delta_6, F_7 \to F_8 \vdash F_{12}} \text{Cut}$$

$$\cfrac{\cfrac{}{- : \Delta_6, F_7 \to F_8 \vdash F_7} \text{ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_8 \vdash F_{10} \to F_{11}} \text{ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_6, F_8, F_{10} \to F_{11} \vdash F_{10}} \text{inv-th/ax} \quad \cfrac{}{h_9 : \Delta_6, F_{11}, F_8 \vdash F_{12}} \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_8, F_{10} \to F_{11} \vdash F_{12}} \to_L}{- : \Delta_6, F_8 \vdash F_{12}} \text{hCut}}{- : \Delta_6, F_7 \to F_8 \vdash F_{12}} \to_L$$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_9 \to F_{10} \vdash F_9 \quad h_1 : \Delta_6, F_{10} \vdash F_7}{\bullet h_1 : \Delta_6, F_9 \to F_{10} \vdash F_7} \to_L \quad \cfrac{h_8 : \Delta_6, F_7, F_9 \to F_{10} \vdash F_9 \quad h_8 : \Delta_6, F_7, F_{10} \vdash F_{11}}{\bullet h_8 : (\Delta_6, F_9 \to F_{10}), F_7 \vdash F_{11}} \to_L}{- : \Delta_6, F_9 \to F_{10} \vdash F_{11}} \text{ Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_6, F_9 \to F_{10} \vdash F_9} \text{ ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_{10} \vdash F_7} \text{ ax/W} \quad \cfrac{\cfrac{}{h_8 : \Delta_6, F_{10}, F_7 \vdash F_{11}} \text{ ax/W}}{\bullet h_8 : \Delta_6, F_{10}, F_7 \vdash F_{11}} \begin{matrix}\text{ax/W}\\H\end{matrix}}{- : \Delta_6, F_{10} \vdash F_{11}} \text{ hCut}}{- : \Delta_6, F_9 \to F_{10} \vdash F_{11}} \to_L$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \to F_8 \vdash F_7 \quad h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \to F_8 \vdash F_6} \to_L \quad \cfrac{h_9 : \Delta_{13}, F_6, F_{10}, F_{11}, F_7 \to F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \to F_8), F_6 \vdash F_{12}} \wedge_L}{- : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \to F_8 \vdash F_{12}} \text{ Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_{13}, F_7 \to F_8, F_{10} \wedge F_{11} \vdash F_7} \text{ ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_{13}, F_8, F_{10} \wedge F_{11} \vdash F_6} \text{ ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_{13}, F_{10}, F_{11}, F_6, F_8 \vdash F_{12}} \begin{matrix}\text{inv-th/ax}\\\wedge_L\end{matrix}}{\bullet h_9 : \Delta_{13}, F_6, F_8, F_{10} \wedge F_{11} \vdash F_{12}}}{- : \Delta_{13}, F_8, F_{10} \wedge F_{11} \vdash F_{12}} \text{ hCut}}{- : \Delta_{13}, F_7 \to F_8, F_{10} \wedge F_{11} \vdash F_{12}} \to_L$$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_7 \to F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_6, F_7 \to F_8 \vdash F_{10} \wedge F_{11}} \to_L \quad \cfrac{h_9 : \Delta_6, F_{10}, F_{11}, F_7 \to F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \to F_8), F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L}{- : \Delta_6, F_7 \to F_8 \vdash F_{12}} \text{ Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_6, F_7 \to F_8 \vdash F_7} \text{ ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_8 \vdash F_{10} \wedge F_{11}} \text{ ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_6, F_{10}, F_{11}, F_8 \vdash F_{12}} \begin{matrix}\text{inv-th/ax}\\\wedge_L\end{matrix}}{\bullet h_9 : \Delta_6, F_8, F_{10} \wedge F_{11} \vdash F_{12}}}{- : \Delta_6, F_8 \vdash F_{12}} \text{ hCut}}{- : \Delta_6, F_7 \to F_8 \vdash F_{12}} \to_L$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \to F_8 \vdash F_7 \quad h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \to F_8 \vdash F_6} \to_L \quad \cfrac{h_9 : \Delta_{13}, F_6, F_{10}, F_7 \to F_8 \vdash F_{12} \quad h_9 : \Delta_{13}, F_6, F_{11}, F_7 \to F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \vee F_{11}), F_7 \to F_8), F_6 \vdash F_{12}} \vee_L}{- : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \to F_8 \vdash F_{12}} \text{ Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_{13}, F_7 \to F_8, F_{10} \vee F_{11} \vdash F_7} \text{ ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_{13}, F_8, F_{10} \vee F_{11} \vdash F_6} \text{ ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_{13}, F_{10}, F_6, F_8 \vdash F_{12}} \begin{matrix}\text{inv-th/ax}\end{matrix} \quad \cfrac{}{h_9 : \Delta_{13}, F_{11}, F_6, F_8 \vdash F_{12}} \begin{matrix}\text{inv-th/a}\\\vee_L\end{matrix}}{\bullet h_9 : \Delta_{13}, F_6, F_8, F_{10} \vee F_{11} \vdash F_{12}}}{- : \Delta_{13}, F_8, F_{10} \vee F_{11} \vdash F_{12}} \text{ hCut}}{- : \Delta_{13}, F_7 \to F_8, F_{10} \vee F_{11} \vdash F_{12}} \to_L$$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_7 \to F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash F_{10} \vee F_{11}}{\bullet h_1 : \Delta_6, F_7 \to F_8 \vdash F_{10} \vee F_{11}} \to_L \quad \cfrac{h_9 : \Delta_6, F_{10}, F_7 \to F_8 \vdash F_{12} \quad h_9 : \Delta_6, F_{11}, F_7 \to F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \to F_8), F_{10} \vee F_{11} \vdash F_{12}} \vee_L}{- : \Delta_6, F_7 \to F_8 \vdash F_{12}} \text{ Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_6, F_7 \to F_8 \vdash F_7} \text{ ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_8 \vdash F_{10} \vee F_{11}} \text{ ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_6, F_{10}, F_8 \vdash F_{12}} \begin{matrix}\text{inv-th/ax}\end{matrix} \quad \cfrac{}{h_9 : \Delta_6, F_{11}, F_8 \vdash F_{12}} \begin{matrix}\text{inv-th/ax}\\\vee_L\end{matrix}}{\bullet h_9 : \Delta_6, F_8, F_{10} \vee F_{11} \vdash F_{12}}}{- : \Delta_6, F_8 \vdash F_{12}} \text{ hCut}}{- : \Delta_6, F_7 \to F_8 \vdash F_{12}} \to_L$$

- Case rule $\bot_L$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_7 \to F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash \bot}{\bullet h_1 : \Delta_6, F_7 \to F_8 \vdash \bot} \to_L \quad \cfrac{}{\bullet h_9 : (\Delta_6, F_7 \to F_8), \bot \vdash F_{10}} \bot_L}{- : \Delta_6, F_7 \to F_8 \vdash F_{10}} \text{ Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_6, F_7 \to F_8 \vdash F_7} \text{ ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_8 \vdash \bot} \text{ ax/W} \quad \cfrac{}{\bullet h_9 : \bot, \Delta_6, F_8 \vdash F_{10}} \bot_L}{- : \Delta_6, F_8 \vdash F_{10}} \text{ hCut}}{- : \Delta_6, F_7 \to F_8 \vdash F_{10}} \to_L$$

$$\dfrac{\dfrac{\dfrac{h_1 : (\bot, \Delta_{11}), F_7 \to F_8 \vdash F_7 \quad h_1 : (\bot, \Delta_{11}), F_8 \vdash F_6}{\bullet h_1 : (\bot, \Delta_{11}), F_7 \to F_8 \vdash F_6} \to_L \quad \dfrac{}{\bullet h_9 : ((\bot, \Delta_{11}), F_7 \to F_8), F_6 \vdash F_{10}} \bot_L}{\overset{\displaystyle - : (\bot, \Delta_{11}), F_7 \to F_8 \vdash F_{10}}{\to}} \text{Cut}}{- : \bot, \Delta_{11}, F_7 \to F_8 \vdash F_{10}} \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{h_1 : (\Delta_{11}, p_{10}), F_7 \to F_8 \vdash F_7 \quad h_1 : (\Delta_{11}, p_{10}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{11}, p_{10}), F_7 \to F_8 \vdash F_6} \to_L \quad \dfrac{}{\bullet h_9 : ((\Delta_{11}, p_{10}), F_7 \to F_8), F_6 \vdash p_{10}} I}{\overset{\displaystyle - : (\Delta_{11}, p_{10}), F_7 \to F_8 \vdash p_{10}}{\to}} \text{Cut}}{- : \Delta_{11}, p_{10}, F_7 \to F_8 \vdash p_{10}} I$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_6, F_7 \to F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash p_{10}}{\bullet h_1 : \Delta_6, F_7 \to F_8 \vdash p_{10}} \to_L \quad \dfrac{}{\bullet h_9 : (\Delta_6, F_7 \to F_8), p_{10} \vdash p_{10}} I}{\overset{\displaystyle - : \Delta_6, F_7 \to F_8 \vdash p_{10}}{\to}} \text{Cut}}{- : \Delta_6, F_7 \to F_8 \vdash p_{10}} \text{ax/W}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_6, F_7 \to F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash \top}{\bullet h_1 : \Delta_6, F_7 \to F_8 \vdash \top} \to_L \quad \dfrac{h_9 : \Delta_6, F_7 \to F_8 \vdash F_{10}}{\bullet h_9 : (\Delta_6, F_7 \to F_8), \top \vdash F_{10}} \top_L}{\overset{\displaystyle - : \Delta_6, F_7 \to F_8 \vdash F_{10}}{\to}} \text{Cut}}{- : \Delta_6, F_7 \to F_8 \vdash F_{10}} \text{ax/W}$$

$$\dfrac{\dfrac{\dfrac{h_1 : (\top, \Delta_{11}), F_7 \to F_8 \vdash F_7 \quad h_1 : (\top, \Delta_{11}), F_8 \vdash F_6}{\bullet h_1 : (\top, \Delta_{11}), F_7 \to F_8 \vdash F_6} \to_L \quad \dfrac{h_9 : \Delta_{11}, F_6, F_7 \to F_8 \vdash F_{10}}{\bullet h_9 : ((\top, \Delta_{11}), F_7 \to F_8), F_6 \vdash F_{10}} \top_L}{\overset{\displaystyle - : (\top, \Delta_{11}), F_7 \to F_8 \vdash F_{10}}{\to}} \text{Cut}}{\dfrac{\dfrac{}{\bullet h_1 : \top, \Delta_{11}, F_7 \to F_8 \vdash F_6} \text{ax/W} \quad \dfrac{}{h_9 : \top, \Delta_{11}, F_6, F_7 \to F_8 \vdash F_{10}} \text{ax/W}}{- : \top, \Delta_{11}, F_7 \to F_8 \vdash F_{10}} \text{hCut}}$$

6.7   Status of $\wedge_L$:   OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \quad \dfrac{}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash \top} \top_R}{\overset{\displaystyle - : \Delta_6, F_8 \wedge F_9 \vdash \top}{\to}} \text{Cut}}{- : \Delta_6, F_8 \wedge F_9 \vdash \top} \top_R$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \quad \dfrac{h_{10} : \Delta_6, F_7, F_{11}, F_8 \wedge F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash F_{11} \to F_{12}} \to_R}{\overset{\displaystyle - : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \to F_{12}}{\to}} \text{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_6, F_{11}, F_8 \wedge F_9 \vdash F_7} \text{ax/W} \quad \dfrac{}{h_{10} : \Delta_6, F_{11}, F_7, F_8 \wedge F_9 \vdash F_{12}} \text{ax/W}}{- : \Delta_6, F_{11}, F_8 \wedge F_9 \vdash F_{12}} \text{hCut}}{- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \to F_{12}} \to_R}$$

- Case rule $\wedge_R$

$$\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \qquad \frac{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{11} \quad h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash F_{11} \wedge F_{12}} \wedge_R$$
$$\overline{\quad - : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \wedge F_{12} \quad} \; \text{Cut}$$

$$\rightarrow$$

$$\frac{\overline{h_1 : \Delta_6, F_8, F_9 \vdash F_7} \; \text{ax/W} \qquad \dfrac{\dfrac{\overline{h_{10} : \Delta_6, F_7, F_8, F_9 \vdash F_{11}}}{} \text{inv-th/ax} \quad \dfrac{\overline{h_{10} : \Delta_6, F_7, F_8, F_9 \vdash F_{12}}}{} \text{inv-th/ax}}{\bullet h_{10} : \Delta_6, F_7, F_8, F_9 \vdash F_{11} \wedge F_{12}} \wedge_R}{\dfrac{- : \Delta_6, F_8, F_9 \vdash F_{11} \wedge F_{12}}{- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \wedge F_{12}} \wedge_L} \; \text{hCut}$$

- Case rule $\vee_1$

$$\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \qquad \frac{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{11}}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_1$$
$$\overline{\quad - : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \vee F_{12} \quad} \; \text{Cut}$$

$$\rightarrow$$

$$\frac{\overline{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \; \text{ax/W} \qquad \overline{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{11}} \; \text{ax/W}}{\dfrac{- : \Delta_6, F_8 \wedge F_9 \vdash F_{11}}{- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \vee F_{12}} \vee_1} \; \text{hCut}$$

- Case rule $\vee_2$

$$\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \qquad \frac{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_2$$
$$\overline{\quad - : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \vee F_{12} \quad} \; \text{Cut}$$

$$\rightarrow$$

$$\frac{\overline{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \; \text{ax/W} \qquad \overline{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{12}} \; \text{ax/W}}{\dfrac{- : \Delta_6, F_8 \wedge F_9 \vdash F_{12}}{- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \vee F_{12}} \vee_2} \; \text{hCut}$$

- Case rule $\rightarrow_L$

$$\frac{h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7, F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash F_6} \wedge_L \qquad \frac{h_9 : \Delta_{13}, F_6, F_{10} \rightarrow F_{11}, F_7 \wedge F_8 \vdash F_{10} \quad h_9 : \Delta_{13}, F_6, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8), F_6 \vdash F_{12}} \rightarrow_L$$
$$\overline{\quad - : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash F_{12} \quad} \; \text{Cut}$$

$$\rightarrow$$

$$\frac{\overline{h_1 : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_6} \; \text{ax/W} \qquad \dfrac{\dfrac{\overline{h_9 : \Delta_{13}, F_6, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{10}}}{} \text{inv-th/ax} \quad \dfrac{\overline{h_9 : \Delta_{13}, F_{11}, F_6, F_7, F_8 \vdash F_{12}}}{} \text{inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_6, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{12}} \rightarrow_L}{\dfrac{- : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{12}}{- : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8 \vdash F_{12}} \wedge_L} \; \text{hCut}$$

$$\frac{h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_6, F_7 \wedge F_8 \vdash F_{10} \rightarrow F_{11}} \wedge_L \qquad \frac{h_9 : \Delta_6, F_{10} \rightarrow F_{11}, F_7 \wedge F_8 \vdash F_{10} \quad h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), F_{10} \rightarrow F_{11} \vdash F_{12}} \rightarrow_L$$
$$\overline{\quad - : \Delta_6, F_7 \wedge F_8 \vdash F_{12} \quad} \; \text{Cut}$$

$$\rightarrow$$

$$\frac{\overline{h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \rightarrow F_{11}} \; \text{ax/W} \qquad \dfrac{\dfrac{\overline{h_9 : \Delta_6, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{10}}}{} \text{inv-th/ax} \quad \dfrac{\overline{h_9 : \Delta_6, F_{11}, F_7, F_8 \vdash F_{12}}}{} \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{12}} \rightarrow_L}{\dfrac{- : \Delta_6, F_7, F_8 \vdash F_{12}}{- : \Delta_6, F_7 \wedge F_8 \vdash F_{12}} \wedge_L} \; \text{hCut}$$

- Case rule $\wedge_L$

$$\frac{h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7, F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash F_6} \wedge_L \qquad \frac{h_9 : \Delta_{13}, F_6, F_{10}, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8), F_6 \vdash F_{12}} \wedge_L$$
$$\overline{\quad - : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash F_{12} \quad} \; \text{Cut}$$

$$\rightarrow$$

$$\frac{\dfrac{\dfrac{\overline{h_1 : \Delta_{13}, F_{10}, F_{11}, F_7, F_8 \vdash F_6}}{} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash F_6} \wedge_L \qquad \overline{h_9 : \Delta_{13}, F_{10}, F_{11}, F_6, F_7 \wedge F_8 \vdash F_{12}} \; \text{ax/W}}{\dfrac{- : \Delta_{13}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{- : \Delta_{13}, F_{10} \wedge F_{11}, F_7 \wedge F_8 \vdash F_{12}} \wedge_L} \; \text{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{10}}\wedge \mathtt{F_{11}}}{\bullet\mathtt{h_1}:\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{10}}\wedge \mathtt{F_{11}}}\wedge_L \qquad \dfrac{\mathtt{h_9}:\Delta_6,\mathtt{F_{10}},\mathtt{F_{11}},\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}}{\bullet\mathtt{h_9}:(\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}),\mathtt{F_{10}}\wedge \mathtt{F_{11}}\vdash \mathtt{F_{12}}}\wedge_L}{-:\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_1}:\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{10}}\wedge \mathtt{F_{11}}}\mathtt{ax/W}\quad \dfrac{\dfrac{\mathtt{h_9}:\Delta_6,\mathtt{F_{10}},\mathtt{F_{11}},\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{12}}}{\bullet\mathtt{h_9}:\Delta_6,\mathtt{F_7},\mathtt{F_8},\mathtt{F_{10}}\wedge \mathtt{F_{11}}\vdash \mathtt{F_{12}}}\mathtt{inv\text{-}th/ax}}{}\wedge_L}{-:\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{12}}}\mathtt{hCut}}{-:\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}}\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_6,\mathtt{F_9},\mathtt{F_{10}}\vdash \mathtt{F_7}}{\bullet\mathtt{h_1}:\Delta_6,\mathtt{F_9}\wedge \mathtt{F_{10}}\vdash \mathtt{F_7}}\wedge_L \qquad \dfrac{\mathtt{h_8}:\Delta_6,\mathtt{F_7},\mathtt{F_9},\mathtt{F_{10}}\vdash \mathtt{F_{11}}}{\bullet\mathtt{h_8}:(\Delta_6,\mathtt{F_9}\wedge \mathtt{F_{10}}),\mathtt{F_7}\vdash \mathtt{F_{11}}}\wedge_L}{-:\Delta_6,\mathtt{F_9}\wedge \mathtt{F_{10}}\vdash \mathtt{F_{11}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_1}:\Delta_6,\mathtt{F_{10}},\mathtt{F_9}\vdash \mathtt{F_7}}\mathtt{ax/W}\quad \dfrac{\dfrac{\mathtt{h_8}:\Delta_6,\mathtt{F_{10}},\mathtt{F_7},\mathtt{F_9}\vdash \mathtt{F_{11}}}{\bullet\mathtt{h_8}:\Delta_6,\mathtt{F_{10}},\mathtt{F_7},\mathtt{F_9}\vdash \mathtt{F_{11}}}\mathtt{ax/W}}{}H}{-:\Delta_6,\mathtt{F_{10}},\mathtt{F_9}\vdash \mathtt{F_{11}}}\mathtt{hCut}}{-:\Delta_6,\mathtt{F_9}\wedge \mathtt{F_{10}}\vdash \mathtt{F_{11}}}\wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h_1}:(\Delta_{13},\mathtt{F_{10}}\vee \mathtt{F_{11}}),\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_6}}{\bullet\mathtt{h_1}:(\Delta_{13},\mathtt{F_{10}}\vee \mathtt{F_{11}}),\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_6}}\wedge_L \quad \dfrac{\mathtt{h_9}:\Delta_{13},\mathtt{F_6},\mathtt{F_{10}},\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}\quad \mathtt{h_9}:\Delta_{13},\mathtt{F_6},\mathtt{F_{11}},\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}}{\bullet\mathtt{h_9}:((\Delta_{13},\mathtt{F_{10}}\vee \mathtt{F_{11}}),\mathtt{F_7}\wedge \mathtt{F_8}),\mathtt{F_6}\vdash \mathtt{F_{12}}}\vee_L}{-:(\Delta_{13},\mathtt{F_{10}}\vee \mathtt{F_{11}}),\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_1}:\Delta_{13},\mathtt{F_7},\mathtt{F_8},\mathtt{F_{10}}\vee \mathtt{F_{11}}\vdash \mathtt{F_6}}\mathtt{ax/W}\quad \dfrac{\dfrac{\mathtt{h_9}:\Delta_{13},\mathtt{F_{10}},\mathtt{F_6},\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{12}}}{}\mathtt{inv\text{-}th/ax}\quad \dfrac{\mathtt{h_9}:\Delta_{13},\mathtt{F_{11}},\mathtt{F_6},\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{12}}}{}\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h_9}:\Delta_{13},\mathtt{F_6},\mathtt{F_7},\mathtt{F_8},\mathtt{F_{10}}\vee \mathtt{F_{11}}\vdash \mathtt{F_{12}}}\vee_L}{-:\Delta_{13},\mathtt{F_7},\mathtt{F_8},\mathtt{F_{10}}\vee \mathtt{F_{11}}\vdash \mathtt{F_{12}}}\mathtt{hCut}}{-:\Delta_{13},\mathtt{F_7}\wedge \mathtt{F_8},\mathtt{F_{10}}\vee \mathtt{F_{11}}\vdash \mathtt{F_{12}}}\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{10}}\vee \mathtt{F_{11}}}{\bullet\mathtt{h_1}:\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{10}}\vee \mathtt{F_{11}}}\wedge_L \quad \dfrac{\mathtt{h_9}:\Delta_6,\mathtt{F_{10}},\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}\quad \mathtt{h_9}:\Delta_6,\mathtt{F_{11}},\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}}{\bullet\mathtt{h_9}:(\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}),\mathtt{F_{10}}\vee \mathtt{F_{11}}\vdash \mathtt{F_{12}}}\vee_L}{-:\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_1}:\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{10}}\vee \mathtt{F_{11}}}\mathtt{ax/W}\quad \dfrac{\dfrac{\mathtt{h_9}:\Delta_6,\mathtt{F_{10}},\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{12}}}{}\mathtt{inv\text{-}th/ax}\quad \dfrac{\mathtt{h_9}:\Delta_6,\mathtt{F_{11}},\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{12}}}{}\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h_9}:\Delta_6,\mathtt{F_7},\mathtt{F_8},\mathtt{F_{10}}\vee \mathtt{F_{11}}\vdash \mathtt{F_{12}}}\vee_L}{-:\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{12}}}\mathtt{hCut}}{-:\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{12}}}\wedge_L$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \perp}{\bullet\mathtt{h_1}:\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}\vdash \perp}\wedge_L \quad \dfrac{}{\bullet\mathtt{h_9}:(\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}),\perp\vdash \mathtt{F_{10}}}\perp_L}{-:\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{10}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_1}:\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \perp}\mathtt{ax/W}\quad \dfrac{}{\bullet\mathtt{h_9}:\perp,\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{10}}}\perp_L}{-:\Delta_6,\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_{10}}}\mathtt{hCut}}{-:\Delta_6,\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{10}}}\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h_1}:(\perp,\Delta_{11}),\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_6}}{\bullet\mathtt{h_1}:(\perp,\Delta_{11}),\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_6}}\wedge_L \quad \dfrac{}{\bullet\mathtt{h_9}:((\perp,\Delta_{11}),\mathtt{F_7}\wedge \mathtt{F_8}),\mathtt{F_6}\vdash \mathtt{F_{10}}}\perp_L}{-:(\perp,\Delta_{11}),\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{10}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{-:\perp,\Delta_{11},\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_{10}}}\perp_L$$

- Case rule I

$$\dfrac{\dfrac{\mathtt{h_1}:(\Delta_{11},\mathtt{p_{10}}),\mathtt{F_7},\mathtt{F_8}\vdash \mathtt{F_6}}{\bullet\mathtt{h_1}:(\Delta_{11},\mathtt{p_{10}}),\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{F_6}}\wedge_L \quad \dfrac{}{\bullet\mathtt{h_9}:((\Delta_{11},\mathtt{p_{10}}),\mathtt{F_7}\wedge \mathtt{F_8}),\mathtt{F_6}\vdash \mathtt{p_{10}}}I}{-:(\Delta_{11},\mathtt{p_{10}}),\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{p_{10}}}\;\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{-:\Delta_{11},\mathtt{p_{10}},\mathtt{F_7}\wedge \mathtt{F_8}\vdash \mathtt{p_{10}}}I$$

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$$\dfrac{\dfrac{h_1 : \Delta_6, F_7, F_8 \vdash p_{10}}{\bullet h_1 : \Delta_6, F_7 \wedge F_8 \vdash p_{10}} \wedge_L \quad \dfrac{}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), p_{10} \vdash p_{10}} \ I}{- : \Delta_6, F_7 \wedge F_8 \vdash p_{10}} \ \text{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6, F_7 \wedge F_8 \vdash p_{10}} \ \text{ax/W}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{h_1 : \Delta_6, F_7, F_8 \vdash \top}{\bullet h_1 : \Delta_6, F_7 \wedge F_8 \vdash \top} \wedge_L \quad \dfrac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{10}}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), \top \vdash F_{10}} \ \top_L}{- : \Delta_6, F_7 \wedge F_8 \vdash F_{10}} \ \text{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6, F_7 \wedge F_8 \vdash F_{10}} \ \text{ax/W}$$

$$\dfrac{\dfrac{h_1 : (\top, \Delta_{11}), F_7, F_8 \vdash F_6}{\bullet h_1 : (\top, \Delta_{11}), F_7 \wedge F_8 \vdash F_6} \wedge_L \quad \dfrac{h_9 : \Delta_{11}, F_6, F_7 \wedge F_8 \vdash F_{10}}{\bullet h_9 : ((\top, \Delta_{11}), F_7 \wedge F_8), F_6 \vdash F_{10}} \ \top_L}{- : (\top, \Delta_{11}), F_7 \wedge F_8 \vdash F_{10}} \ \text{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet h_1 : \top, \Delta_{11}, F_7 \wedge F_8 \vdash F_6} \ \text{ax/W} \quad \dfrac{}{h_9 : \top, \Delta_{11}, F_6, F_7 \wedge F_8 \vdash F_{10}} \ \text{ax/W}}{- : \top, \Delta_{11}, F_7 \wedge F_8 \vdash F_{10}} \ \text{hCut}$$

6.8 Status of $\vee_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{h_1 : \Delta_6, F_8 \vdash F_7 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \dfrac{}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash \top} \ \top_R}{- : \Delta_6, F_8 \vee F_9 \vdash \top} \ \text{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6, F_8 \vee F_9 \vdash \top} \ \top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{h_1 : \Delta_6, F_8 \vdash F_7 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \dfrac{h_{10} : \Delta_6, F_7, F_{11}, F_8 \vee F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash F_{11} \rightarrow F_{12}} \ \rightarrow_R}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \rightarrow F_{12}} \ \text{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_6, F_{11}, F_8 \vee F_9 \vdash F_7} \ \text{ax/W} \quad \dfrac{}{h_{10} : \Delta_6, F_{11}, F_7, F_8 \vee F_9 \vdash F_{12}} \ \text{ax/W}}{- : \Delta_6, F_{11}, F_8 \vee F_9 \vdash F_{12}} \ \text{hCut}}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \rightarrow F_{12}} \ \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_1 : \Delta_6, F_8 \vdash F_7 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \dfrac{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11} \quad h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash F_{11} \wedge F_{12}} \ \wedge_R}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \wedge F_{12}} \ \text{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \ \text{ax/W} \quad \dfrac{}{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11}} \ \text{ax/W}}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11}} \ \text{hCut} \quad \dfrac{\dfrac{}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \ \text{ax/W} \quad \dfrac{}{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{12}} \ \text{ax/W}}{- : \Delta_6, F_8 \vee F_9 \vdash F_{12}} \ \text{hCut}}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \wedge F_{12}} \ \wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{h_1 : \Delta_6, F_8 \vdash F_7 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \dfrac{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11}}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash F_{11} \vee F_{12}} \ \vee_1}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \vee F_{12}} \ \text{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \ \text{ax/W} \quad \dfrac{}{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11}} \ \text{ax/W}}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11}} \ \text{hCut}}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \vee F_{12}} \ \vee_1$$

- Case rule $\vee_2$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_8 \vdash F_7 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \cfrac{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_2}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \vee F_{12}} \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \text{ax/W} \quad \cfrac{}{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{12}} \text{ax/W}}{- : \Delta_6, F_8 \vee F_9 \vdash F_{12}} \text{hCut}}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \vee F_{12}} \vee_2$$

- Case rule $\to_L$

$$\cfrac{\cfrac{h_1 : (\Delta_{13}, F_{10} \to F_{11}), F_7 \vdash F_6 \quad h_1 : (\Delta_{13}, F_{10} \to F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \to F_{11}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \cfrac{h_9 : \Delta_{13}, F_6, F_{10} \to F_{11}, F_7 \vee F_8 \vdash F_{10} \quad h_9 : \Delta_{13}, F_6, F_{11},}{\bullet h_9 : ((\Delta_{13}, F_{10} \to F_{11}), F_7 \vee F_8), F_6 \vdash F_{12}}}{- : (\Delta_{13}, F_{10} \to F_{11}), F_7 \vee F_8 \vdash F_{12}}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_{13}, F_{10} \to F_{11}, F_7 \vee F_8 \vdash F_6} \text{ax/W} \quad \cfrac{}{h_9 : \Delta_{13}, F_6, F_{10} \to F_{11}, F_7 \vee F_8 \vdash F_{10}} \text{ax/W}}{- : \Delta_{13}, F_{10} \to F_{11}, F_7 \vee F_8 \vdash F_{10}} \text{hCut}}{- : \Delta_{13}, F_{10} \to F_{11}, F_7 \vee F_8 \vdash F_{12}} \quad \cfrac{\cfrac{\cfrac{}{h_1 : \Delta_{13}, F_{11}, F_7 \vdash F_6} \text{inv-th/ax} \quad \cfrac{}{h_1 : \Delta_{13}, F_{11}, F_8 \vdash F_6} \text{inv-t}}{\bullet h_1 : \Delta_{13}, F_{11}, F_7 \vee F_8 \vdash F_6} \vee_L}{- : \Delta_{13}, F_{11}, F_7 \vee}$$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_7 \vdash F_{10} \to F_{11} \quad h_1 : \Delta_6, F_8 \vdash F_{10} \to F_{11}}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash F_{10} \to F_{11}} \vee_L \quad \cfrac{h_9 : \Delta_6, F_{10} \to F_{11}, F_7 \vee F_8 \vdash F_{10} \quad h_9 : \Delta_6, F_{11},}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), F_{10} \to F_{11} \vdash F_{12}}}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_6, F_7 \vdash F_{10} \to F_{11}} \text{ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_6, F_7, F_{10} \to F_{11} \vdash F_{10}} \text{inv-th/ax} \quad \cfrac{}{h_9 : \Delta_6, F_{11}, F_7 \vdash F_{12}} \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_7, F_{10} \to F_{11} \vdash F_{12}} \to_L}{- : \Delta_6, F_7 \vdash F_{12}} \text{hCut} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_8 \vdash F_{10} \to F_{11}} \text{ax/W} \quad \cfrac{}{h_9 : \Delta_6, F_8}}{- : \Delta_6, F_8}}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}}$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vdash F_6 \quad h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \cfrac{h_9 : \Delta_{13}, F_6, F_{10}, F_{11}, F_7 \vee F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8), F_6 \vdash F_{12}} \wedge_L}{- : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash F_{12}} \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \vdash F_6} \text{inv-th/ax} \quad \cfrac{}{h_1 : \Delta_{13}, F_{10}, F_{11}, F_8 \vdash F_6} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash F_6} \vee_L \quad \cfrac{}{h_9 : \Delta_{13}, F_{10}, F_{11}, F_6, F_7 \vee F_8 \vdash F_{12}} \text{ax/W}}{- : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash F_{12}} \text{hCut}}{- : \Delta_{13}, F_{10} \wedge F_{11}, F_7 \vee F_8 \vdash F_{12}} \wedge_L$$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_7 \vdash F_{10} \wedge F_{11} \quad h_1 : \Delta_6, F_8 \vdash F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash F_{10} \wedge F_{11}} \vee_L \quad \cfrac{h_9 : \Delta_6, F_{10}, F_{11}, F_7 \vee F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}} \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_6, F_7 \vdash F_{10} \wedge F_{11}} \text{ax/W} \quad \cfrac{}{h_9 : \Delta_6, F_{10}, F_{11}, F_7 \vdash F_{12}} \text{inv-th/ax}}{- : \Delta_6, F_7 \vdash F_{12}} \text{hCut} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_8 \vdash F_{10} \wedge F_{11}} \text{ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_6, F_{10}, F_{11}, F_8 \vdash F_{12}} \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_8, F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L}{- : \Delta_6, F_8 \vdash F_{12}} \text{hCut}}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}} \vee_L$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vdash F_6 \quad h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \cfrac{h_9 : \Delta_{13}, F_6, F_{10}, F_7 \vee F_8 \vdash F_{12} \quad h_9 : \Delta_{13}, F_{6}}{\bullet h_9 : ((\Delta_{13}, F_{10} \vee F_{11}), F_7 \vee F_8),}}{- : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vee F_8 \vdash F_{12}} \to$$

$$\cfrac{\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_{13}, F_{10}, F_7 \vdash F_6} \text{inv-th/ax} \quad \cfrac{}{h_1 : \Delta_{13}, F_{10}, F_8 \vdash F_6} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash F_6} \vee_L \quad \cfrac{}{h_9 : \Delta_{13}, F_{10}, F_6, F_7 \vee F_8 \vdash F_{12}} \text{ax/W}}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash F_{12}} \text{hCut} \quad \cfrac{\cfrac{}{h_1 : \Delta_{13}, F_{11}, F_7 \vdash F_6} \text{inv-th/ax} \quad \cfrac{}{h_1 : \Delta_{1}}{\bullet h_1 : \Delta_{13}, F_{11}, F_7 \vee F_8 \vdash F_6}}{- : \Delta_{13}, F_{10} \vee F_{11}, F_7 \vee F_8 \vdash F_{12}}$$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_7 \vdash F_{10} \vee F_{11} \quad h_1 : \Delta_6, F_8 \vdash F_{10} \vee F_{11}}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash F_{10} \vee F_{11}} \vee_L \quad \cfrac{h_9 : \Delta_6, F_{10}, F_7 \vee F_8 \vdash F_{12} \quad h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash F_{1}}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), F_{10} \vee F_{11} \vdash F_{12}} \text{Cut}}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}} \to$$

$$\cfrac{\cfrac{}{h_1 : \Delta_6, F_7 \vdash F_{10} \vee F_{11}} \text{ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_6, F_{10}, F_7 \vdash F_{12}} \text{inv-th/ax} \quad \cfrac{}{h_9 : \Delta_6, F_{11}, F_7 \vdash F_{12}} \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_7, F_{10} \vee F_{11} \vdash F_{12}} \vee_L}{- : \Delta_6, F_7 \vdash F_{12}} \text{hCut} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_8 \vdash F_{10} \vee F_{11}} \text{ax/W} \quad \cfrac{}{h_9 : \Delta_6, F_{10}, F_8 \vdash F}}{\bullet h_9 :} \cfrac{}{- : \Delta_6, F_8 \vdash F_{12}}$$
$$- : \Delta_6, F_7 \vee F_8 \vdash F_{12}$$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_9 \vdash F_7 \quad h_1 : \Delta_6, F_{10} \vdash F_7}{\bullet h_1 : \Delta_6, F_9 \vee F_{10} \vdash F_7} \vee_L \quad \cfrac{h_8 : \Delta_6, F_7, F_9 \vdash F_{11} \quad h_8 : \Delta_6, F_7, F_{10} \vdash F_{11}}{\bullet h_8 : (\Delta_6, F_9 \vee F_{10}), F_7 \vdash F_{11}} \vee_L}{- : \Delta_6, F_9 \vee F_{10} \vdash F_{11}} \text{Cut} \to$$

$$\cfrac{\cfrac{}{h_1 : \Delta_6, F_9 \vdash F_7} \text{ax/W} \quad \cfrac{\cfrac{}{h_8 : \Delta_6, F_7, F_9 \vdash F_{11}} \text{ax/W}}{\bullet h_8 : \Delta_6, F_7, F_9 \vdash F_{11}} H}{- : \Delta_6, F_9 \vdash F_{11}} \text{hCut} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_{10} \vdash F_7} \text{ax/W} \quad \cfrac{\cfrac{}{h_8 : \Delta_6, F_{10}, F_7 \vdash F_{11}} \text{ax/W}}{\bullet h_8 : \Delta_6, F_{10}, F_7 \vdash F_{11}} H}{- : \Delta_6, F_{10} \vdash F_{11}} \text{hCut}}{- : \Delta_6, F_9 \vee F_{10} \vdash F_{11}} \vee_L$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_7 \vdash \perp \quad h_1 : \Delta_6, F_8 \vdash \perp}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash \perp} \vee_L \quad \cfrac{}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), \perp \vdash F_{10}} \perp_L}{- : \Delta_6, F_7 \vee F_8 \vdash F_{10}} \text{Cut} \to$$

$$\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_6, F_7 \vdash \perp} \text{ax/W} \quad \cfrac{}{\bullet h_9 : \perp, \Delta_6, F_7 \vdash F_{10}} \perp_L}{- : \Delta_6, F_7 \vdash F_{10}} \text{hCut} \quad \cfrac{\cfrac{}{h_1 : \Delta_6, F_8 \vdash \perp} \text{ax/W} \quad \cfrac{}{\bullet h_9 : \perp, \Delta_6, F_8 \vdash F_{10}} \perp_L}{- : \Delta_6, F_8 \vdash F_{10}} \text{hCut}}{- : \Delta_6, F_7 \vee F_8 \vdash F_{10}} \vee_L$$

$$\cfrac{\cfrac{h_1 : (\perp, \Delta_{11}), F_7 \vdash F_6 \quad h_1 : (\perp, \Delta_{11}), F_8 \vdash F_6}{\bullet h_1 : (\perp, \Delta_{11}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \cfrac{}{\bullet h_9 : ((\perp, \Delta_{11}), F_7 \vee F_8), F_6 \vdash F_{10}} \perp_L}{- : (\perp, \Delta_{11}), F_7 \vee F_8 \vdash F_{10}} \text{Cut} \to$$
$$\cfrac{}{- : \perp, \Delta_{11}, F_7 \vee F_8 \vdash F_{10}} \perp_L$$

- Case rule I

$$\cfrac{\cfrac{h_1 : (\Delta_{11}, p_{10}), F_7 \vdash F_6 \quad h_1 : (\Delta_{11}, p_{10}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{11}, p_{10}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \cfrac{}{\bullet h_9 : ((\Delta_{11}, p_{10}), F_7 \vee F_8), F_6 \vdash p_{10}} I}{- : (\Delta_{11}, p_{10}), F_7 \vee F_8 \vdash p_{10}} \text{Cut} \to$$
$$\cfrac{}{- : \Delta_{11}, p_{10}, F_7 \vee F_8 \vdash p_{10}} I$$

$$\cfrac{\cfrac{h_1 : \Delta_6, F_7 \vdash p_{10} \quad h_1 : \Delta_6, F_8 \vdash p_{10}}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash p_{10}} \vee_L \quad \cfrac{}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), p_{10} \vdash p_{10}} I}{- : \Delta_6, F_7 \vee F_8 \vdash p_{10}} \text{Cut} \to$$
$$\cfrac{}{- : \Delta_6, F_7 \vee F_8 \vdash p_{10}} \text{ax/W}$$

- Case rule $\top_L$

$$
\cfrac{
  \cfrac{
    \cfrac{h_1 : \Delta_6, F_7 \vdash \top \quad h_1 : \Delta_6, F_8 \vdash \top}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash \top}\ \vee_L
    \qquad
    \cfrac{h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{10}}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), \top \vdash F_{10}}\ \top_L
  }{- : \Delta_6, F_7 \vee F_8 \vdash F_{10}}\ \text{Cut}
}{- : \Delta_6, F_7 \vee F_8 \vdash F_{10}}\ \begin{array}{c}\to\\[-2pt]\text{ax/W}\end{array}
$$

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{h_1 : (\top, \Delta_{11}), F_7 \vdash F_6 \quad h_1 : (\top, \Delta_{11}), F_8 \vdash F_6}{\bullet h_1 : (\top, \Delta_{11}), F_7 \vee F_8 \vdash F_6}\ \vee_L
      \qquad
      \cfrac{h_9 : \Delta_{11}, F_6, F_7 \vee F_8 \vdash F_{10}}{\bullet h_9 : ((\top, \Delta_{11}), F_7 \vee F_8), F_6 \vdash F_{10}}\ \top_L
    }{- : (\top, \Delta_{11}), F_7 \vee F_8 \vdash F_{10}}\ \begin{array}{c}\text{Cut}\\[-2pt]\to\end{array}
  }{
    \cfrac{}{\bullet h_1 : \top, \Delta_{11}, F_7 \vee F_8 \vdash F_6}\ \text{ax/W}
    \qquad
    \cfrac{}{h_9 : \top, \Delta_{11}, F_6, F_7 \vee F_8 \vdash F_{10}}\ \text{ax/W}
  }\ \text{hCut}
}{- : \top, \Delta_{11}, F_7 \vee F_8 \vdash F_{10}}
$$

6.9 Status of $\perp_L$: OK

- Case rule $\top_R$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5}\ \perp_L
    \qquad
    \cfrac{}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash \top}\ \top_R
  }{- : \perp, \Delta_4 \vdash \top}\ \begin{array}{c}\text{Cut}\\[-2pt]\to\end{array}
}{- : \perp, \Delta_4 \vdash \top}\ \top_R
$$

- Case rule $\to_R$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5}\ \perp_L
    \qquad
    \cfrac{h_6 : \perp, \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash F_7 \to F_8}\ \to_R
  }{- : \perp, \Delta_4 \vdash F_7 \to F_8}\ \begin{array}{c}\text{Cut}\\[-2pt]\to\end{array}
}{- : \perp, \Delta_4 \vdash F_7 \to F_8}\ \perp_L
$$

- Case rule $\wedge_R$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5}\ \perp_L
    \qquad
    \cfrac{h_6 : \perp, \Delta_4, F_5 \vdash F_7 \quad h_6 : \perp, \Delta_4, F_5 \vdash F_8}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash F_7 \wedge F_8}\ \wedge_R
  }{- : \perp, \Delta_4 \vdash F_7 \wedge F_8}\ \begin{array}{c}\text{Cut}\\[-2pt]\to\end{array}
}{- : \perp, \Delta_4 \vdash F_7 \wedge F_8}\ \perp_L
$$

- Case rule $\vee_1$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5}\ \perp_L
    \qquad
    \cfrac{h_6 : \perp, \Delta_4, F_5 \vdash F_7}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash F_7 \vee F_8}\ \vee_1
  }{- : \perp, \Delta_4 \vdash F_7 \vee F_8}\ \begin{array}{c}\text{Cut}\\[-2pt]\to\end{array}
}{- : \perp, \Delta_4 \vdash F_7 \vee F_8}\ \perp_L
$$

- Case rule $\vee_2$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5}\ \perp_L
    \qquad
    \cfrac{h_6 : \perp, \Delta_4, F_5 \vdash F_8}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash F_7 \vee F_8}\ \vee_2
  }{- : \perp, \Delta_4 \vdash F_7 \vee F_8}\ \begin{array}{c}\text{Cut}\\[-2pt]\to\end{array}
}{- : \perp, \Delta_4 \vdash F_7 \vee F_8}\ \perp_L
$$

- Case rule $\to_L$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_4} \ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8} \ \to_L}{- : \bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8} \ \text{Cut}$$

$$\dfrac{\to}{\dfrac{}{- : \bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8} \ \bot_L}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_6 \to \mathtt{F}_7} \ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\bot, \Delta_4), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8} \ \to_L}{- : \bot, \Delta_4 \vdash \mathtt{F}_8} \ \text{Cut}$$

$$\dfrac{\to}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_8} \ \bot_L}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_4} \ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8} \ \wedge_L}{- : \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8} \ \text{Cut}$$

$$\dfrac{\to}{\dfrac{}{- : \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8} \ \bot_L}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7} \ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\bot, \Delta_4), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8} \ \wedge_L}{- : \bot, \Delta_4 \vdash \mathtt{F}_8} \ \text{Cut}$$

$$\dfrac{\to}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_8} \ \bot_L}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_4} \ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8} \ \vee_L}{- : \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8} \ \text{Cut}$$

$$\dfrac{\to}{\dfrac{}{- : \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8} \ \bot_L}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_6 \vee \mathtt{F}_7} \ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\bot, \Delta_4), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8} \ \vee_L}{- : \bot, \Delta_4 \vdash \mathtt{F}_8} \ \text{Cut}$$

$$\dfrac{\to}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_8} \ \bot_L}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_5} \ \bot_L \quad \dfrac{}{\bullet\mathtt{h}_6 : (\bot, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7} \ \bot_L}{- : \bot, \Delta_4 \vdash \mathtt{F}_7} \ \text{Cut}$$

$$\dfrac{\to}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_7} \ \bot_L}$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_7, \mathtt{p}_6 \vdash \mathtt{F}_4} \ \bot_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\bot, \Delta_7, \mathtt{p}_6), \mathtt{F}_4 \vdash \mathtt{p}_6} \ I}{- : \bot, \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6} \ \text{Cut}$$

$$\dfrac{\to}{\dfrac{}{- : \bot, \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6} \ \bot_L}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{p}_6} \ \bot_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\bot, \Delta_4), \mathtt{p}_6 \vdash \mathtt{p}_6} \ I}{- : \bot, \Delta_4 \vdash \mathtt{p}_6} \ \text{Cut}$$

$$\dfrac{\to}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{p}_6} \ \bot_L}$$

- Case rule $\top_L$

$$\frac{\dfrac{}{\bullet h_1 : \bot, \Delta_4 \vdash \top}\ \bot_L \qquad \dfrac{h_5 : \bot, \Delta_4 \vdash F_6}{\bullet h_5 : (\bot, \Delta_4), \top \vdash F_6}\ \top_L}{-\ :\ \bot, \Delta_4 \vdash F_6}\ \text{Cut}$$
$$\rightarrow$$
$$\frac{}{-\ :\ \bot, \Delta_4 \vdash F_6}\ \bot_L$$

$$\frac{\dfrac{}{\bullet h_1 : \bot, \top, \Delta_7 \vdash F_4}\ \bot_L \qquad \dfrac{h_5 : \bot, \Delta_7, F_4 \vdash F_6}{\bullet h_5 : (\bot, \top, \Delta_7), F_4 \vdash F_6}\ \top_L}{-\ :\ \bot, \top, \Delta_7 \vdash F_6}\ \text{Cut}$$
$$\rightarrow$$
$$\frac{}{-\ :\ \bot, \top, \Delta_7 \vdash F_6}\ \bot_L$$

6.10 Status of I: `OK`

- Case rule $\top_R$

$$\frac{\dfrac{}{\bullet h_1 : \Delta_4, p_5 \vdash p_5}\ I \qquad \dfrac{}{\bullet h_6 : (\Delta_4, p_5), p_5 \vdash \top}\ \top_R}{-\ :\ \Delta_4, p_5 \vdash \top}\ \text{Cut}$$
$$\rightarrow$$
$$\frac{}{-\ :\ \Delta_4, p_5 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

$$\frac{\dfrac{}{\bullet h_1 : \Delta_4, p_5 \vdash p_5}\ I \qquad \dfrac{h_6 : \Delta_4, F_7, p_5, p_5 \vdash F_8}{\bullet h_6 : (\Delta_4, p_5), p_5 \vdash F_7 \rightarrow F_8}\ \rightarrow_R}{-\ :\ \Delta_4, p_5 \vdash F_7 \rightarrow F_8}\ \text{Cut}$$
$$\rightarrow$$
$$\frac{\dfrac{}{\bullet h_1 : \Delta_4, F_7, p_5 \vdash p_5}\ I \qquad \dfrac{}{h_6 : \Delta_4, F_7, p_5, p_5 \vdash F_8}\ \text{ax/W}}{\dfrac{-\ :\ \Delta_4, F_7, p_5 \vdash F_8}{-\ :\ \Delta_4, p_5 \vdash F_7 \rightarrow F_8}\ \rightarrow_R}\ \text{hCut}$$

- Case rule $\wedge_R$

$$\frac{\dfrac{}{\bullet h_1 : \Delta_4, p_5 \vdash p_5}\ I \qquad \dfrac{h_6 : \Delta_4, p_5, p_5 \vdash F_7 \quad h_6 : \Delta_4, p_5, p_5 \vdash F_8}{\bullet h_6 : (\Delta_4, p_5), p_5 \vdash F_7 \wedge F_8}\ \wedge_R}{-\ :\ \Delta_4, p_5 \vdash F_7 \wedge F_8}\ \text{Cut}$$
$$\rightarrow$$
$$\frac{\dfrac{\frac{}{\bullet h_1 : \Delta_4, p_5 \vdash p_5}\ \text{ax/W} \quad \frac{}{h_6 : \Delta_4, p_5, p_5 \vdash F_7}\ \text{ax/W}}{-\ :\ \Delta_4, p_5 \vdash F_7}\ \text{hCut} \qquad \dfrac{\frac{}{\bullet h_1 : \Delta_4, p_5 \vdash p_5}\ \text{ax/W} \quad \frac{}{h_6 : \Delta_4, p_5, p_5 \vdash F_8}\ \text{ax/W}}{-\ :\ \Delta_4, p_5 \vdash F_8}\ \text{hCut}}{-\ :\ \Delta_4, p_5 \vdash F_7 \wedge F_8}\ \wedge_R$$

- Case rule $\vee_1$

$$\frac{\dfrac{}{\bullet h_1 : \Delta_4, p_5 \vdash p_5}\ I \qquad \dfrac{h_6 : \Delta_4, p_5, p_5 \vdash F_7}{\bullet h_6 : (\Delta_4, p_5), p_5 \vdash F_7 \vee F_8}\ \vee_1}{-\ :\ \Delta_4, p_5 \vdash F_7 \vee F_8}\ \text{Cut}$$
$$\rightarrow$$
$$\frac{\dfrac{\frac{}{\bullet h_1 : \Delta_4, p_5 \vdash p_5}\ \text{ax/W} \quad \frac{}{h_6 : \Delta_4, p_5, p_5 \vdash F_7}\ \text{ax/W}}{-\ :\ \Delta_4, p_5 \vdash F_7}\ \text{hCut}}{-\ :\ \Delta_4, p_5 \vdash F_7 \vee F_8}\ \vee_1$$

- Case rule $\vee_2$

$$
\cfrac{
  \cfrac{}{\bullet h_1 : \Delta_4, p_5 \vdash p_5}\ I
  \qquad
  \cfrac{h_6 : \Delta_4, p_5, p_5 \vdash F_8}{\bullet h_6 : (\Delta_4, p_5), p_5 \vdash F_7 \vee F_8}\ \vee_2
}{- : \Delta_4, p_5 \vdash F_7 \vee F_8}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{\bullet h_1 : \Delta_4, p_5 \vdash p_5}{}\ ax/W
    \qquad
    \cfrac{h_6 : \Delta_4, p_5, p_5 \vdash F_8}{}\ ax/W
  }{- : \Delta_4, p_5 \vdash F_8}\ \text{hCut}
}{- : \Delta_4, p_5 \vdash F_7 \vee F_8}\ \vee_2
$$

- Case rule $\rightarrow_L$

$$
\cfrac{
  \cfrac{}{\bullet h_1 : (\Delta_9, F_6 \rightarrow F_7), p_4 \vdash p_4}\ I
  \qquad
  \cfrac{h_5 : \Delta_9, p_4, p_4, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \Delta_9, F_7, p_4, p_4 \vdash F_8}{\bullet h_5 : ((\Delta_9, F_6 \rightarrow F_7), p_4), p_4 \vdash F_8}\ \rightarrow_L
}{- : (\Delta_9, F_6 \rightarrow F_7), p_4 \vdash F_8}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{\bullet h_1 : \Delta_9, p_4, F_6 \rightarrow F_7 \vdash p_4}{}\ ax/W
    \quad
    \cfrac{h_5 : \Delta_9, p_4, p_4, F_6 \rightarrow F_7 \vdash F_6}{}\ ax/W
  }{- : \Delta_9, p_4, F_6 \rightarrow F_7 \vdash F_6}\ \text{hCut}
  \quad
  \cfrac{
    \cfrac{}{\bullet h_1 : \Delta_9, F_7, p_4 \vdash p_4}\ I
    \quad
    \cfrac{h_5 : \Delta_9, F_7, p_4, p_4 \vdash F_8}{}\ ax/W
  }{- : \Delta_9, F_7, p_4 \vdash F_8}\ \text{hCut}
}{- : \Delta_9, p_4, F_6 \rightarrow F_7 \vdash F_8}\ \rightarrow_L
$$

- Case rule $\wedge_L$

$$
\cfrac{
  \cfrac{}{\bullet h_1 : (\Delta_9, F_6 \wedge F_7), p_4 \vdash p_4}\ I
  \qquad
  \cfrac{h_5 : \Delta_9, F_6, F_7, p_4, p_4 \vdash F_8}{\bullet h_5 : ((\Delta_9, F_6 \wedge F_7), p_4), p_4 \vdash F_8}\ \wedge_L
}{- : (\Delta_9, F_6 \wedge F_7), p_4 \vdash F_8}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{\bullet h_1 : \Delta_9, F_6, F_7, p_4 \vdash p_4}{}\ I
    \qquad
    \cfrac{h_5 : \Delta_9, F_6, F_7, p_4, p_4 \vdash F_8}{}\ ax/W
  }{- : \Delta_9, F_6, F_7, p_4 \vdash F_8}\ \text{hCut}
}{- : \Delta_9, p_4, F_6 \wedge F_7 \vdash F_8}\ \wedge_L
$$

- Case rule $\vee_L$

$$
\cfrac{
  \cfrac{}{\bullet h_1 : (\Delta_9, F_6 \vee F_7), p_4 \vdash p_4}\ I
  \qquad
  \cfrac{h_5 : \Delta_9, F_6, p_4, p_4 \vdash F_8 \quad h_5 : \Delta_9, F_7, p_4, p_4 \vdash F_8}{\bullet h_5 : ((\Delta_9, F_6 \vee F_7), p_4), p_4 \vdash F_8}\ \vee_L
}{- : (\Delta_9, F_6 \vee F_7), p_4 \vdash F_8}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{\bullet h_1 : \Delta_9, F_6, p_4 \vdash p_4}{}\ I
    \quad
    \cfrac{h_5 : \Delta_9, F_6, p_4, p_4 \vdash F_8}{}\ ax/W
  }{- : \Delta_9, F_6, p_4 \vdash F_8}\ \text{hCut}
  \qquad
  \cfrac{
    \cfrac{\bullet h_1 : \Delta_9, F_7, p_4 \vdash p_4}{}\ I
    \quad
    \cfrac{h_5 : \Delta_9, F_7, p_4, p_4 \vdash F_8}{}\ ax/W
  }{- : \Delta_9, F_7, p_4 \vdash F_8}\ \text{hCut}
}{- : \Delta_9, p_4, F_6 \vee F_7 \vdash F_8}\ \vee_L
$$

- Case rule $\perp_L$

$$
\cfrac{
  \cfrac{}{\bullet h_1 : (\perp, \Delta_7), p_4 \vdash p_4}\ I
  \qquad
  \cfrac{}{\bullet h_5 : ((\perp, \Delta_7), p_4), p_4 \vdash F_6}\ \perp_L
}{- : (\perp, \Delta_7), p_4 \vdash F_6}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{}{- : \perp, \Delta_7, p_4 \vdash F_6}\ \perp_L
$$

- Case rule I

$$
\cfrac{
  \cfrac{}{\bullet h_1 : \Delta_4, p_6 \vdash p_6}\ I
  \qquad
  \cfrac{}{\bullet h_5 : (\Delta_4, p_6), p_6 \vdash p_6}\ I
}{- : \Delta_4, p_6 \vdash p_6}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{}{- : \Delta_4, p_6 \vdash p_6}\ I
$$

$$
\cfrac{
  \cfrac{}{\bullet h_1 : (\Delta_7, p_6), p_4 \vdash p_4}\ I
  \qquad
  \cfrac{}{\bullet h_5 : ((\Delta_7, p_6), p_4), p_4 \vdash p_6}\ I
}{- : (\Delta_7, p_6), p_4 \vdash p_6}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{}{- : \Delta_7, p_4, p_6 \vdash p_6}\ I
$$

- Case rule $\top_L$

$$\cfrac{\cfrac{}{\bullet h_1 : (\top, \Delta_7), p_4 \vdash p_4} \ I \quad \cfrac{h_5 : \Delta_7, p_4, p_4 \vdash F_6}{\bullet h_5 : ((\top, \Delta_7), p_4), p_4 \vdash F_6} \ \top_L}{- : (\top, \Delta_7), p_4 \vdash F_6} \ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 : \top, \Delta_7, p_4 \vdash p_4} \ \text{ax/W} \quad \cfrac{}{h_5 : \top, \Delta_7, p_4, p_4 \vdash F_6} \ \text{ax/W}}{- : \top, \Delta_7, p_4 \vdash F_6} \ \text{hCut}$$

6.11 Status of $\top_L$: OK

- Case rule $\top_R$

$$\cfrac{\cfrac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \ \top_L \quad \cfrac{}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash \top} \ \top_R}{- : \top, \Delta_4 \vdash \top} \ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \top, \Delta_4 \vdash \top} \ \top_R$$

- Case rule $\rightarrow_R$

$$\cfrac{\cfrac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \ \top_L \quad \cfrac{h_6 : \top, \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7 \rightarrow F_8} \ \rightarrow_R}{- : \top, \Delta_4 \vdash F_7 \rightarrow F_8} \ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 : \top, \Delta_4 \vdash F_5} \ \text{ax/W} \quad \cfrac{}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \rightarrow F_8} \ \text{ax/W}}{- : \top, \Delta_4 \vdash F_7 \rightarrow F_8} \ \text{hCut}$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \ \top_L \quad \cfrac{h_6 : \top, \Delta_4, F_5 \vdash F_7 \quad h_6 : \top, \Delta_4, F_5 \vdash F_8}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7 \wedge F_8} \ \wedge_R}{- : \top, \Delta_4 \vdash F_7 \wedge F_8} \ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 : \top, \Delta_4 \vdash F_5} \ \text{ax/W} \quad \cfrac{}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \wedge F_8} \ \text{ax/W}}{- : \top, \Delta_4 \vdash F_7 \wedge F_8} \ \text{hCut}$$

- Case rule $\vee_1$

$$\cfrac{\cfrac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \ \top_L \quad \cfrac{h_6 : \top, \Delta_4, F_5 \vdash F_7}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7 \vee F_8} \ \vee_1}{- : \top, \Delta_4 \vdash F_7 \vee F_8} \ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 : \top, \Delta_4 \vdash F_5} \ \text{ax/W} \quad \cfrac{}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \vee F_8} \ \text{ax/W}}{- : \top, \Delta_4 \vdash F_7 \vee F_8} \ \text{hCut}$$

- Case rule $\vee_2$

$$\cfrac{\cfrac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \ \top_L \quad \cfrac{h_6 : \top, \Delta_4, F_5 \vdash F_8}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7 \vee F_8} \ \vee_2}{- : \top, \Delta_4 \vdash F_7 \vee F_8} \ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 : \top, \Delta_4 \vdash F_5} \ \text{ax/W} \quad \cfrac{}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \vee F_8} \ \text{ax/W}}{- : \top, \Delta_4 \vdash F_7 \vee F_8} \ \text{hCut}$$

- Case rule $\rightarrow_L$

$$
\cfrac{
\cfrac{h_1 : \Delta_9, F_6 \to F_7 \vdash F_4}{\bullet h_1 : \top, \Delta_9, F_6 \to F_7 \vdash F_4}\ {\scriptstyle \top_L}
\qquad
\cfrac{h_5 : \top, \Delta_9, F_4, F_6 \to F_7 \vdash F_6 \qquad h_5 : \top, \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_9, F_6 \to F_7), F_4 \vdash F_8}\ {\scriptstyle \to_L}
}{- : \top, \Delta_9, F_6 \to F_7 \vdash F_8}\ {\scriptstyle \text{Cut}}
$$

$$\to$$

$$
\cfrac{
\cfrac{}{h_1 : \top, \Delta_9, F_6 \to F_7 \vdash F_4}\ {\scriptstyle \text{ax/W}}
\qquad
\cfrac{}{\bullet h_5 : \top, \Delta_9, F_4, F_6 \to F_7 \vdash F_8}\ {\scriptstyle \text{ax/W}}
}{- : \top, \Delta_9, F_6 \to F_7 \vdash F_8}\ {\scriptstyle \text{hCut}}
$$

$$
\cfrac{
\cfrac{h_1 : \Delta_4 \vdash F_6 \to F_7}{\bullet h_1 : \top, \Delta_4 \vdash F_6 \to F_7}\ {\scriptstyle \top_L}
\qquad
\cfrac{h_5 : \top, \Delta_4, F_6 \to F_7 \vdash F_6 \qquad h_5 : \top, \Delta_4, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_4), F_6 \to F_7 \vdash F_8}\ {\scriptstyle \to_L}
}{- : \top, \Delta_4 \vdash F_8}\ {\scriptstyle \text{Cut}}
$$

$$\to$$

$$
\cfrac{
\cfrac{}{h_1 : \top, \Delta_4 \vdash F_6 \to F_7}\ {\scriptstyle \text{ax/W}}
\qquad
\cfrac{}{\bullet h_5 : \top, \Delta_4, F_6 \to F_7 \vdash F_8}\ {\scriptstyle \text{ax/W}}
}{- : \top, \Delta_4 \vdash F_8}\ {\scriptstyle \text{hCut}}
$$

- Case rule $\wedge_L$

$$
\cfrac{
\cfrac{h_1 : \Delta_9, F_6 \wedge F_7 \vdash F_4}{\bullet h_1 : \top, \Delta_9, F_6 \wedge F_7 \vdash F_4}\ {\scriptstyle \top_L}
\qquad
\cfrac{h_5 : \top, \Delta_9, F_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_9, F_6 \wedge F_7), F_4 \vdash F_8}\ {\scriptstyle \wedge_L}
}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash F_8}\ {\scriptstyle \text{Cut}}
$$

$$\to$$

$$
\cfrac{
\cfrac{}{h_1 : \top, \Delta_9, F_6 \wedge F_7 \vdash F_4}\ {\scriptstyle \text{ax/W}}
\qquad
\cfrac{}{\bullet h_5 : \top, \Delta_9, F_4, F_6 \wedge F_7 \vdash F_8}\ {\scriptstyle \text{ax/W}}
}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash F_8}\ {\scriptstyle \text{hCut}}
$$

$$
\cfrac{
\cfrac{h_1 : \Delta_4 \vdash F_6 \wedge F_7}{\bullet h_1 : \top, \Delta_4 \vdash F_6 \wedge F_7}\ {\scriptstyle \top_L}
\qquad
\cfrac{h_5 : \top, \Delta_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_4), F_6 \wedge F_7 \vdash F_8}\ {\scriptstyle \wedge_L}
}{- : \top, \Delta_4 \vdash F_8}\ {\scriptstyle \text{Cut}}
$$

$$\to$$

$$
\cfrac{
\cfrac{}{h_1 : \top, \Delta_4 \vdash F_6 \wedge F_7}\ {\scriptstyle \text{ax/W}}
\qquad
\cfrac{}{\bullet h_5 : \top, \Delta_4, F_6 \wedge F_7 \vdash F_8}\ {\scriptstyle \text{ax/W}}
}{- : \top, \Delta_4 \vdash F_8}\ {\scriptstyle \text{hCut}}
$$

- Case rule $\vee_L$

$$
\cfrac{
\cfrac{h_1 : \Delta_9, F_6 \vee F_7 \vdash F_4}{\bullet h_1 : \top, \Delta_9, F_6 \vee F_7 \vdash F_4}\ {\scriptstyle \top_L}
\qquad
\cfrac{h_5 : \top, \Delta_9, F_4, F_6 \vdash F_8 \qquad h_5 : \top, \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_9, F_6 \vee F_7), F_4 \vdash F_8}\ {\scriptstyle \vee_L}
}{- : \top, \Delta_9, F_6 \vee F_7 \vdash F_8}\ {\scriptstyle \text{Cut}}
$$

$$\to$$

$$
\cfrac{
\cfrac{}{h_1 : \top, \Delta_9, F_6 \vee F_7 \vdash F_4}\ {\scriptstyle \text{ax/W}}
\qquad
\cfrac{}{\bullet h_5 : \top, \Delta_9, F_4, F_6 \vee F_7 \vdash F_8}\ {\scriptstyle \text{ax/W}}
}{- : \top, \Delta_9, F_6 \vee F_7 \vdash F_8}\ {\scriptstyle \text{hCut}}
$$

$$
\cfrac{
\cfrac{h_1 : \Delta_4 \vdash F_6 \vee F_7}{\bullet h_1 : \top, \Delta_4 \vdash F_6 \vee F_7}\ {\scriptstyle \top_L}
\qquad
\cfrac{h_5 : \top, \Delta_4, F_6 \vdash F_8 \qquad h_5 : \top, \Delta_4, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_4), F_6 \vee F_7 \vdash F_8}\ {\scriptstyle \vee_L}
}{- : \top, \Delta_4 \vdash F_8}\ {\scriptstyle \text{Cut}}
$$

$$\to$$

$$
\cfrac{
\cfrac{}{h_1 : \top, \Delta_4 \vdash F_6 \vee F_7}\ {\scriptstyle \text{ax/W}}
\qquad
\cfrac{}{\bullet h_5 : \top, \Delta_4, F_6 \vee F_7 \vdash F_8}\ {\scriptstyle \text{ax/W}}
}{- : \top, \Delta_4 \vdash F_8}\ {\scriptstyle \text{hCut}}
$$

- Case rule $\bot_L$

$$
\cfrac{
\cfrac{h_1 : \Delta_4 \vdash \bot}{\bullet h_1 : \top, \Delta_4 \vdash \bot}\ {\scriptstyle \top_L}
\qquad
\cfrac{}{\bullet h_5 : (\top, \Delta_4), \bot \vdash F_6}\ {\scriptstyle \bot_L}
}{- : \top, \Delta_4 \vdash F_6}\ {\scriptstyle \text{Cut}}
$$

$$\to$$

$$
\cfrac{
\cfrac{}{h_1 : \top, \Delta_4 \vdash \bot}\ {\scriptstyle \text{ax/W}}
\qquad
\cfrac{}{\bullet h_5 : \bot, \top, \Delta_4 \vdash F_6}\ {\scriptstyle \bot_L}
}{- : \top, \Delta_4 \vdash F_6}\ {\scriptstyle \text{hCut}}
$$

$$
\cfrac{
\cfrac{h_1 : \bot, \Delta_7 \vdash F_4}{\bullet h_1 : \top, \bot, \Delta_7 \vdash F_4}\ {\scriptstyle \top_L}
\qquad
\cfrac{}{\bullet h_5 : (\top, \bot, \Delta_7), F_4 \vdash F_6}\ {\scriptstyle \bot_L}
}{- : \top, \bot, \Delta_7 \vdash F_6}\ {\scriptstyle \text{Cut}}
$$

$$\to$$

$$
\cfrac{}{- : \bot, \top, \Delta_7 \vdash F_6}\ {\scriptstyle \bot_L}
$$

- Case rule I

$$
\cfrac{
  \cfrac{
    \cfrac{h_1 : \Delta_7, p_6 \vdash F_4}{\bullet h_1 : \top, \Delta_7, p_6 \vdash F_4} \ \top_L
    \quad
    \overline{\bullet h_5 : (\top, \Delta_7, p_6), F_4 \vdash p_6} \ I
  }{- : \top, \Delta_7, p_6 \vdash p_6} \ \text{Cut}
}{- : \top, \Delta_7, p_6 \vdash p_6} \ I
$$

$$
\cfrac{
  \cfrac{
    \cfrac{h_1 : \Delta_4 \vdash p_6}{\bullet h_1 : \top, \Delta_4 \vdash p_6} \ \top_L
    \quad
    \overline{\bullet h_5 : (\top, \Delta_4), p_6 \vdash p_6} \ I
  }{- : \top, \Delta_4 \vdash p_6} \ \text{Cut}
}{- : \top, \Delta_4 \vdash p_6} \ \text{ax/W}
$$

- Case rule $\top_L$

$$
\cfrac{
  \cfrac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \ \top_L
  \quad
  \cfrac{h_6 : \Delta_4, F_5 \vdash F_7}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7} \ \top_L
}{- : \top, \Delta_4 \vdash F_7} \ \text{Cut}
$$

$$
\cfrac{
  \overline{h_1 : \top, \Delta_4 \vdash F_5} \ \text{ax/W}
  \quad
  \overline{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7} \ \text{ax/W}
}{- : \top, \Delta_4 \vdash F_7} \ \text{hCut}
$$

7 Cut-Elimination

7.1 Status of $\top_R$: OK

- Case rule $\top_R$

$$
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{}{\bullet h_4 : \Delta_3, \top \vdash \top}\ \top_R
}{- : \Delta_2, \Delta_3 \vdash \top}\ \text{Cut}
$$

$$\to$$

$$
\cfrac{}{- : \Delta_2, \Delta_3 \vdash \top}\ \top_R
$$

- Case rule $\to_R$

$$
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{h_4 : \top, \Delta_3, F_5 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \to F_6}\ \to_R
}{- : \Delta_2, \Delta_3 \vdash F_5 \to F_6}\ \text{Cut}
$$

$$\to$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \quad \cfrac{}{h_4 : \top, \Delta_2, \Delta_3, F_5 \vdash F_6}\ \text{ax/W}
}{- : \Delta_2, \Delta_3, F_5 \vdash F_6}\ \text{hCut}
}{- : \Delta_2, \Delta_3 \vdash F_5 \to F_6}\ \to_R
$$

- Case rule $\wedge_R$

$$
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{h_4 : \top, \Delta_3 \vdash F_5 \quad h_4 : \top, \Delta_3 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \wedge F_6}\ \wedge_R
}{- : \Delta_2, \Delta_3 \vdash F_5 \wedge F_6}\ \text{Cut}
$$

$$\to$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \ \cfrac{}{h_4 : \top, \Delta_2, \Delta_3 \vdash F_5}\ \text{ax/W}
}{- : \Delta_2, \Delta_3 \vdash F_5}\ \text{hCut} \quad
\cfrac{
\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \ \cfrac{}{h_4 : \top, \Delta_2, \Delta_3 \vdash F_6}\ \text{ax/W}
}{- : \Delta_2, \Delta_3 \vdash F_6}\ \text{hCut}
}{- : \Delta_2, \Delta_3 \vdash F_5 \wedge F_6}\ \wedge_R
$$

- Case rule $\vee_1$

$$
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{h_4 : \top, \Delta_3 \vdash F_5}{\bullet h_4 : \Delta_3, \top \vdash F_5 \vee F_6}\ \vee_1
}{- : \Delta_2, \Delta_3 \vdash F_5 \vee F_6}\ \text{Cut}
$$

$$\to$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \ \cfrac{}{h_4 : \top, \Delta_2, \Delta_3 \vdash F_5}\ \text{ax/W}
}{- : \Delta_2, \Delta_3 \vdash F_5}\ \text{hCut}
}{- : \Delta_2, \Delta_3 \vdash F_5 \vee F_6}\ \vee_1
$$

- Case rule $\vee_2$

$$
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{h_4 : \top, \Delta_3 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \vee F_6}\ \vee_2
}{- : \Delta_2, \Delta_3 \vdash F_5 \vee F_6}\ \text{Cut}
$$

$$\to$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \ \cfrac{}{h_4 : \top, \Delta_2, \Delta_3 \vdash F_6}\ \text{ax/W}
}{- : \Delta_2, \Delta_3 \vdash F_6}\ \text{hCut}
}{- : \Delta_2, \Delta_3 \vdash F_5 \vee F_6}\ \vee_2
$$

- Case rule $\to_L$

$$
\cfrac{
\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{h_3 : \top, \Delta_7, F_4 \to F_5 \vdash F_4 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \to F_5), \top \vdash F_6}\ \to_L
}{- : \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_6}\ \text{Cut}
$$

$$\to$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \ \cfrac{}{h_3 : \top, \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_4}\ \text{ax/W}
}{- : \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_4}\ \text{hCut} \quad
\cfrac{
\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \ \cfrac{}{h_3 : \top, \Delta_2, \Delta_7, F_5 \vdash F_6}\ \text{ax/W}
}{- : \Delta_2, \Delta_7, F_5 \vdash F_6}\ \text{hCut}
}{- : \Delta_2, \Delta_7, F_4 \to F_5 \vdash F_6}\ \to_L
$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \wedge F_5), \top \vdash F_6}\ \wedge_L}{- : \Delta_2, \Delta_7, F_4 \wedge F_5 \vdash F_6}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \quad h_3 : \top, \Delta_2, \Delta_7, F_4, F_5 \vdash F_6}{- : \Delta_2, \Delta_7, F_4, F_5 \vdash F_6}\ \text{ax/W hCut}}{- : \Delta_2, \Delta_7, F_4 \wedge F_5 \vdash F_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{h_3 : \top, \Delta_7, F_4 \vdash F_6 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \vee F_5), \top \vdash F_6}\ \vee_L}{- : \Delta_2, \Delta_7, F_4 \vee F_5 \vdash F_6}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \quad h_3 : \top, \Delta_2, \Delta_7, F_4 \vdash F_6}{- : \Delta_2, \Delta_7, F_4 \vdash F_6}\ \text{ax/W hCut} \quad \cfrac{\cfrac{}{\bullet h_1 : * \vdash \top}\ \top_R \quad h_3 : \top, \Delta_2, \Delta_7, F_5 \vdash F_6}{- : \Delta_2, \Delta_7, F_5 \vdash F_6}\ \text{ax/W hCut}}{- : \Delta_2, \Delta_7, F_4 \vee F_5 \vdash F_6}\ \vee_L$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{}{\bullet h_3 : (\perp, \Delta_5), \top \vdash F_4}\ \perp_L}{- : \Delta_2, \perp, \Delta_5 \vdash F_4}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \perp, \Delta_2, \Delta_5 \vdash F_4}\ \perp_L$$

- Case rule I

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{}{\bullet h_3 : (\Delta_5, p_4), \top \vdash p_4}\ I}{- : \Delta_2, \Delta_5, p_4 \vdash p_4}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_2, \Delta_5, p_4 \vdash p_4}\ I$$

- Case rule $\top_L$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash \top}\ \top_R \quad \cfrac{h_3 : \Delta_4 \vdash F_5}{\bullet h_3 : \Delta_4, \top \vdash F_5}\ \top_L}{- : \Delta_2, \Delta_4 \vdash F_5}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_2, \Delta_4 \vdash F_5}\ \text{ax/W}$$

7.2 Status of $\rightarrow_R$: OK

- Case rule $\top_R$

$$\cfrac{\cfrac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7}\ \rightarrow_R \quad \cfrac{}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash \top}\ \top_R}{- : \Delta_2, \Delta_5 \vdash \top}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_5, F_9, F_6 \to F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \to F_{10}} \to_R}{- : \Delta_2, \Delta_5 \vdash F_9 \to F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \to F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_9, F_6 \to F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \to F_{10}} \to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \to F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \wedge F_{10}} \wedge_R}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \to F_7} \text{ax/W} \ \dfrac{}{h_8 : \Delta_5, F_6 \to F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \quad \dfrac{\dfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \to F_7} \text{ax/W} \ \dfrac{}{h_8 : \Delta_5, F_6 \to F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_5, F_6 \to F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \vee F_{10}} \vee_1}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \to F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \to F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_5, F_6 \to F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \to F_7 \vdash F_9 \vee F_{10}} \vee_2}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \to F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_5, F_6 \to F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6} \to_R \quad \dfrac{h_7 : \Delta_{11}, F_5 \to F_6, F_8 \to F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \to F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \to F_9), F_5 \to F_6 \vdash F_{10}} \to_L}{- : \Delta_2, \Delta_{11}, F_8 \to F_9 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6} \text{ax/W} \ \dfrac{}{h_7 : \Delta_{11}, F_5 \to F_6, F_8 \to F_9 \vdash F_8} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \to F_9 \vdash F_8} \text{hCut} \quad \dfrac{\dfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6} \text{ax/W} \ \dfrac{}{h_7 : \Delta_{11}, F_9, F_5 \to F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \to F_9 \vdash F_{10}} \to_L$$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_7 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \to F_8} \to_R \quad \dfrac{h_5 : \Delta_6, F_7 \to F_8 \vdash F_7 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \to F_8 \vdash F_9} \to_L}{- : \Delta_2, \Delta_6 \vdash F_9} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_2 \vdash F_7 \to F_8} \text{ax/W} \ \dfrac{}{h_5 : \Delta_6, F_7 \to F_8 \vdash F_7} \text{ax/W}}{- : \Delta_2, \Delta_6 \vdash F_7} \text{hCut} \ \dfrac{\dfrac{}{- : \Delta_2, F_7 \vdash F_8} \text{ax/W} \ \dfrac{}{- : \Delta_6, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_7 \vdash F_9} \text{sCut}}{- : \Delta_2, \Delta_2, \Delta_6, \Delta_6 \vdash F_9} \text{sCut}}{- : \Delta_2, \Delta_6 \vdash F_9} C$$

- Case rule $\wedge_L$

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$$\cfrac{\cfrac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6}\ {\to_R} \quad \cfrac{h_7 : \Delta_{11}, F_8, F_9, F_5 \to F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \to F_6 \vdash F_{10}}\ {\wedge_L}}{- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6}\ \text{ax/W} \quad \cfrac{}{h_7 : \Delta_{11}, F_8, F_9, F_5 \to F_6 \vdash F_{10}}\ \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}}\ \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10}}\ {\wedge_L}$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6}\ {\to_R} \quad \cfrac{h_7 : \Delta_{11}, F_8, F_5 \to F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \to F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \to F_6 \vdash F_{10}}\ {\vee_L}}{- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10}}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6}\ \text{ax/W} \quad \cfrac{}{h_7 : \Delta_{11}, F_8, F_5 \to F_6 \vdash F_{10}}\ \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}}\ \text{hCut} \qquad \cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6}\ \text{ax/W} \quad \cfrac{}{h_7 : \Delta_{11}, F_9, F_5 \to F_6 \vdash F_{10}}\ \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}}\ \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}}\ {\vee_L}$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6}\ {\to_R} \quad \cfrac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \to F_6 \vdash F_8}\ {\perp_L}}{- : \Delta_2, \perp, \Delta_9 \vdash F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \perp, \Delta_2, \Delta_9 \vdash F_8}\ {\perp_L}$$

- Case rule I

$$\cfrac{\cfrac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6}\ {\to_R} \quad \cfrac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \to F_6 \vdash p_8}\ {I}}{- : \Delta_2, \Delta_9, p_8 \vdash p_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_2, \Delta_9, p_8 \vdash p_8}\ {I}$$

- Case rule $\top_L$

$$\cfrac{\cfrac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6}\ {\to_R} \quad \cfrac{h_7 : \Delta_9, F_5 \to F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \to F_6 \vdash F_8}\ {\top_L}}{- : \Delta_2, \top, \Delta_9 \vdash F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \to F_6}\ \text{ax/W} \quad \cfrac{}{h_7 : \top, \Delta_9, F_5 \to F_6 \vdash F_8}\ \text{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash F_8}\ \text{hCut}$$

7.3   Status of $\wedge_R$: OK

- Case rule $\top_R$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7}\ {\wedge_R} \quad \cfrac{}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash \top}\ {\top_R}}{- : \Delta_2, \Delta_5 \vdash \top}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\ {\top_R}$$

- Case rule $\to_R$

$$\frac{\dfrac{h_1:\Delta_2\vdash F_6 \quad h_1:\Delta_2\vdash F_7}{\bullet h_1:\Delta_2\vdash F_6\wedge F_7}\wedge_R \quad \dfrac{h_8:\Delta_5,F_9,F_6\wedge F_7\vdash F_{10}}{\bullet h_8:\Delta_5,F_6\wedge F_7\vdash F_9\to F_{10}}\to_R}{-:\Delta_2,\Delta_5\vdash F_9\to F_{10}}\text{Cut}$$

$$\to$$

$$\frac{\dfrac{}{\bullet h_1:\Delta_2\vdash F_6\wedge F_7}ax/W \quad \dfrac{}{h_8:\Delta_5,F_9,F_6\wedge F_7\vdash F_{10}}ax/W}{\dfrac{-:\Delta_2,\Delta_5,F_9\vdash F_{10}}{-:\Delta_2,\Delta_5\vdash F_9\to F_{10}}\to_R}hCut$$

- Case rule $\wedge_R$

$$\frac{\dfrac{h_1:\Delta_2\vdash F_6 \quad h_1:\Delta_2\vdash F_7}{\bullet h_1:\Delta_2\vdash F_6\wedge F_7}\wedge_R \quad \dfrac{h_8:\Delta_5,F_6\wedge F_7\vdash F_9 \quad h_8:\Delta_5,F_6\wedge F_7\vdash F_{10}}{\bullet h_8:\Delta_5,F_6\wedge F_7\vdash F_9\wedge F_{10}}\wedge_R}{-:\Delta_2,\Delta_5\vdash F_9\wedge F_{10}}\text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{}{\bullet h_1:\Delta_2\vdash F_6\wedge F_7}ax/W \quad \dfrac{}{h_8:\Delta_5,F_6\wedge F_7\vdash F_9}ax/W}{-:\Delta_2,\Delta_5\vdash F_9}hCut \quad \dfrac{\dfrac{}{\bullet h_1:\Delta_2\vdash F_6\wedge F_7}ax/W \quad \dfrac{}{h_8:\Delta_5,F_6\wedge F_7\vdash F_{10}}ax/W}{-:\Delta_2,\Delta_5\vdash F_{10}}hCut}{-:\Delta_2,\Delta_5\vdash F_9\wedge F_{10}}\wedge_R$$

- Case rule $\vee_1$

$$\frac{\dfrac{h_1:\Delta_2\vdash F_6 \quad h_1:\Delta_2\vdash F_7}{\bullet h_1:\Delta_2\vdash F_6\wedge F_7}\wedge_R \quad \dfrac{h_8:\Delta_5,F_6\wedge F_7\vdash F_9}{\bullet h_8:\Delta_5,F_6\wedge F_7\vdash F_9\vee F_{10}}\vee_1}{-:\Delta_2,\Delta_5\vdash F_9\vee F_{10}}\text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{}{\bullet h_1:\Delta_2\vdash F_6\wedge F_7}ax/W \quad \dfrac{}{h_8:\Delta_5,F_6\wedge F_7\vdash F_9}ax/W}{\dfrac{-:\Delta_2,\Delta_5\vdash F_9}{-:\Delta_2,\Delta_5\vdash F_9\vee F_{10}}\vee_1}hCut}{}$$

- Case rule $\vee_2$

$$\frac{\dfrac{h_1:\Delta_2\vdash F_6 \quad h_1:\Delta_2\vdash F_7}{\bullet h_1:\Delta_2\vdash F_6\wedge F_7}\wedge_R \quad \dfrac{h_8:\Delta_5,F_6\wedge F_7\vdash F_{10}}{\bullet h_8:\Delta_5,F_6\wedge F_7\vdash F_9\vee F_{10}}\vee_2}{-:\Delta_2,\Delta_5\vdash F_9\vee F_{10}}\text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{}{\bullet h_1:\Delta_2\vdash F_6\wedge F_7}ax/W \quad \dfrac{}{h_8:\Delta_5,F_6\wedge F_7\vdash F_{10}}ax/W}{\dfrac{-:\Delta_2,\Delta_5\vdash F_{10}}{-:\Delta_2,\Delta_5\vdash F_9\vee F_{10}}\vee_2}hCut}{}$$

- Case rule $\to_L$

$$\frac{\dfrac{h_1:\Delta_2\vdash F_5 \quad h_1:\Delta_2\vdash F_6}{\bullet h_1:\Delta_2\vdash F_5\wedge F_6}\wedge_R \quad \dfrac{h_7:\Delta_{11},F_8\to F_9,F_5\wedge F_6\vdash F_8 \quad h_7:\Delta_{11},F_9,F_5\wedge F_6\vdash F_{10}}{\bullet h_7:(\Delta_{11},F_8\to F_9),F_5\wedge F_6\vdash F_{10}}\to_L}{-:\Delta_2,\Delta_{11},F_8\to F_9\vdash F_{10}}\text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{}{\bullet h_1:\Delta_2\vdash F_5\wedge F_6}ax/W \quad \dfrac{}{h_7:\Delta_{11},F_8\to F_9,F_5\wedge F_6\vdash F_8}ax/W}{-:\Delta_{11},\Delta_2,F_8\to F_9\vdash F_8}hCut \quad \dfrac{\dfrac{}{\bullet h_1:\Delta_2\vdash F_5\wedge F_6}ax/W \quad \dfrac{}{h_7:\Delta_{11},F_9,F_5\wedge F_6\vdash F_{10}}ax/W}{-:\Delta_{11},\Delta_2,F_9\vdash F_{10}}hCut}{-:\Delta_{11},\Delta_2,F_8\to F_9\vdash F_{10}}\to_L$$

- Case rule $\wedge_L$

$$\frac{\dfrac{h_1:\Delta_2\vdash F_5 \quad h_1:\Delta_2\vdash F_6}{\bullet h_1:\Delta_2\vdash F_5\wedge F_6}\wedge_R \quad \dfrac{h_7:\Delta_{11},F_8,F_9,F_5\wedge F_6\vdash F_{10}}{\bullet h_7:(\Delta_{11},F_8\wedge F_9),F_5\wedge F_6\vdash F_{10}}\wedge_L}{-:\Delta_2,\Delta_{11},F_8\wedge F_9\vdash F_{10}}\text{Cut}$$

$$\to$$

$$\frac{\dfrac{\dfrac{}{\bullet h_1:\Delta_2\vdash F_5\wedge F_6}ax/W \quad \dfrac{}{h_7:\Delta_{11},F_8,F_9,F_5\wedge F_6\vdash F_{10}}ax/W}{\dfrac{-:\Delta_{11},\Delta_2,F_8,F_9\vdash F_{10}}{-:\Delta_{11},\Delta_2,F_8\wedge F_9\vdash F_{10}}\wedge_L}hCut}{}$$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \wedge F_8}\wedge_R \quad \cfrac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \wedge F_8 \vdash F_9}\wedge_L}{- : \Delta_2, \Delta_6 \vdash F_9}\text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{- : \Delta_2 \vdash F_7}\text{ax/W} \quad \cfrac{\cfrac{}{- : \Delta_2 \vdash F_8}\text{ax/W} \quad \cfrac{}{- : \Delta_6, \Delta_6, F_7, F_8 \vdash F_9}\text{ax/W}}{- : \Delta_2, \Delta_6, \Delta_6, F_7 \vdash F_9}\text{sCut}}{\cfrac{- : \Delta_2, \Delta_2, \Delta_6, \Delta_6 \vdash F_9}{- : \Delta_2, \Delta_6 \vdash F_9}C}}{}\text{sCut}$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}\wedge_R \quad \cfrac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \wedge F_6 \vdash F_{10}}\vee_L}{- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10}}\text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}\text{ax/W} \quad \cfrac{}{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10}}\text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}}\text{hCut} \quad \cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}\text{ax/W} \quad \cfrac{}{h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}\text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}}\text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}}\vee_L$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}\wedge_R \quad \cfrac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \wedge F_6 \vdash F_8}\perp_L}{- : \Delta_2, \perp, \Delta_9 \vdash F_8}\text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \perp, \Delta_2, \Delta_9 \vdash F_8}\perp_L$$

- Case rule I

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}\wedge_R \quad \cfrac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \wedge F_6 \vdash p_8}I}{- : \Delta_2, \Delta_9, p_8 \vdash p_8}\text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_2, \Delta_9, p_8 \vdash p_8}I$$

- Case rule $\top_L$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}\wedge_R \quad \cfrac{h_7 : \Delta_9, F_5 \wedge F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \wedge F_6 \vdash F_8}\top_L}{- : \Delta_2, \top, \Delta_9 \vdash F_8}\text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}\text{ax/W} \quad \cfrac{}{h_7 : \top, \Delta_9, F_5 \wedge F_6 \vdash F_8}\text{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash F_8}\text{hCut}$$

7.4 Status of $\vee_1$: OK

- Case rule $\top_R$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7}\vee_1 \quad \cfrac{}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top}\top_R}{- : \Delta_2, \Delta_5 \vdash \top}\text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\top_R$$

- Case rule $\rightarrow_R$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \qquad \cfrac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \to F_{10}} \to_R}{- : \Delta_2, \Delta_5 \vdash F_9 \to F_{10}} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \text{ax/W} \qquad \cfrac{}{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \; \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \to F_{10}} \to_R$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \qquad \cfrac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \qquad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \wedge F_{10}} \wedge_R}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \text{ax/W} \quad \cfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \quad \cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \text{ax/W} \quad \cfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \wedge_R$$

- Case rule $\vee_1$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \qquad \cfrac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_1}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \text{ax/W} \qquad \cfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \; \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \vee_1$$

- Case rule $\vee_2$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \qquad \cfrac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_2}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \text{ax/W} \qquad \cfrac{}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \; \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \vee_2$$

- Case rule $\to_L$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \qquad \cfrac{h_7 : \Delta_{11}, F_8 \to F_9, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \to F_9), F_5 \vee F_6 \vdash F_{10}} \to_L}{- : \Delta_2, \Delta_{11}, F_8 \to F_9 \vdash F_{10}} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{ax/W} \quad \cfrac{}{h_7 : \Delta_{11}, F_8 \to F_9, F_5 \vee F_6 \vdash F_8} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \to F_9 \vdash F_8} \text{hCut} \quad \cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{ax/W} \quad \cfrac{}{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \to F_9 \vdash F_{10}} \to_L$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \qquad \cfrac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L}{- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{ax/W} \qquad \cfrac{}{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \; \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10}} \wedge_L$$

- Case rule $\vee_L$

$$
\cfrac{
  \cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}\ \vee_1
  \qquad
  \cfrac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}}\ \vee_L
}{- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10}}\ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{
  \cfrac{
    \cfrac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{}\ \mathtt{ax/W} \quad \cfrac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{}\ \mathtt{ax/W}
  }{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}}\ \mathtt{hCut}
  \qquad
  \cfrac{
    \cfrac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{}\ \mathtt{ax/W} \quad \cfrac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{}\ \mathtt{ax/W}
  }{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}}\ \mathtt{hCut}
}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}}\ \vee_L
$$

$$
\cfrac{
  \cfrac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_7 \vee F_8}\ \vee_1
  \qquad
  \cfrac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9}\ \vee_L
}{- : \Delta_2, \Delta_6 \vdash F_9}\ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{
  \cfrac{- : \Delta_2 \vdash F_7}{}\ \mathtt{ax/W} \quad \cfrac{- : \Delta_6, F_7 \vdash F_9}{}\ \mathtt{ax/W}
}{- : \Delta_2, \Delta_6 \vdash F_9}\ \mathtt{sCut}
$$

- Case rule $\perp_L$

$$
\cfrac{
  \cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}\ \vee_1
  \qquad
  \cfrac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8}\ \perp_L
}{- : \Delta_2, \perp, \Delta_9 \vdash F_8}\ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{}{- : \perp, \Delta_2, \Delta_9 \vdash F_8}\ \perp_L
$$

- Case rule I

$$
\cfrac{
  \cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}\ \vee_1
  \qquad
  \cfrac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8}\ I
}{- : \Delta_2, \Delta_9, p_8 \vdash p_8}\ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{}{- : \Delta_2, \Delta_9, p_8 \vdash p_8}\ I
$$

- Case rule $\top_L$

$$
\cfrac{
  \cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}\ \vee_1
  \qquad
  \cfrac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8}\ \top_L
}{- : \Delta_2, \top, \Delta_9 \vdash F_8}\ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{
  \cfrac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{}\ \mathtt{ax/W} \quad \cfrac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{}\ \mathtt{ax/W}
}{- : \top, \Delta_2, \Delta_9 \vdash F_8}\ \mathtt{hCut}
$$

7.5   Status of $\vee_2$: `OK`

- Case rule $\top_R$

$$
\cfrac{
  \cfrac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7}\ \vee_2
  \qquad
  \cfrac{}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top}\ \top_R
}{- : \Delta_2, \Delta_5 \vdash \top}\ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\ \top_R
$$

- Case rule $\rightarrow_R$

$$
\cfrac{
  \cfrac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7}\ \vee_2
  \qquad
  \cfrac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \rightarrow F_{10}}\ \rightarrow_R
}{- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}}\ \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\cfrac{
  \cfrac{
    \cfrac{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7}{}\ \mathtt{ax/W} \quad \cfrac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{}\ \mathtt{ax/W}
  }{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}}\ \mathtt{hCut}
}{- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}}\ \rightarrow_R
$$

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- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathbf{h}_1 : \Delta_2 \vdash F_7}{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_2 \quad \dfrac{\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \quad \mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \wedge F_{10}} \wedge_R}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_6 \vee F_7 \; \text{ax/W} \quad \overline{\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \; \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \quad \dfrac{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_6 \vee F_7 \; \text{ax/W} \quad \overline{\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \; \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}}} \wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\mathbf{h}_1 : \Delta_2 \vdash F_7}{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_2 \quad \dfrac{\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{\bullet\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_1}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_6 \vee F_7 \; \text{ax/W} \quad \overline{\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \; \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}}} \vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\mathbf{h}_1 : \Delta_2 \vdash F_7}{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_2 \quad \dfrac{\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_2}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_6 \vee F_7 \; \text{ax/W} \quad \overline{\mathbf{h}_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \; \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}}} \vee_2$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathbf{h}_1 : \Delta_2 \vdash F_6}{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \dfrac{\mathbf{h}_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8 \quad \mathbf{h}_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet\mathbf{h}_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \vee F_6 \vdash F_{10}} \rightarrow_L}{- : \Delta_2, \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_5 \vee F_6 \; \text{ax/W} \quad \overline{\mathbf{h}_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8} \; \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{hCut} \quad \dfrac{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_5 \vee F_6 \; \text{ax/W} \quad \overline{\mathbf{h}_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \; \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10}}} \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathbf{h}_1 : \Delta_2 \vdash F_6}{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \dfrac{\mathbf{h}_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet\mathbf{h}_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L}{- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_5 \vee F_6 \; \text{ax/W} \quad \overline{\mathbf{h}_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}} \; \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10}}} \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathbf{h}_1 : \Delta_2 \vdash F_6}{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \dfrac{\mathbf{h}_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \quad \mathbf{h}_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet\mathbf{h}_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L}{- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_5 \vee F_6 \; \text{ax/W} \quad \overline{\mathbf{h}_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}} \; \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{hCut} \quad \dfrac{\bullet\mathbf{h}_1 : \Delta_2 \vdash F_5 \vee F_6 \; \text{ax/W} \quad \overline{\mathbf{h}_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \; \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}}} \vee_L$$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \vee F_8}\ \vee_2 \qquad \cfrac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9}\ \vee_L}{- : \Delta_2, \Delta_6 \vdash F_9}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_2 \vdash F_8}\ \text{ax/W} \qquad \cfrac{}{- : \Delta_6, F_8 \vdash F_9}\ \text{ax/W}}{- : \Delta_2, \Delta_6 \vdash F_9}\ \text{sCut}$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}\ \vee_2 \qquad \cfrac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8}\ \perp_L}{- : \Delta_2, \perp, \Delta_9 \vdash F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \perp, \Delta_2, \Delta_9 \vdash F_8}\ \perp_L$$

- Case rule I

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}\ \vee_2 \qquad \cfrac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8}\ I}{- : \Delta_2, \Delta_9, p_8 \vdash p_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_2, \Delta_9, p_8 \vdash p_8}\ I$$

- Case rule $\top_L$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}\ \vee_2 \qquad \cfrac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8}\ \top_L}{- : \Delta_2, \top, \Delta_9 \vdash F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}\ \text{ax/W} \qquad \cfrac{}{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}\ \text{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash F_8}\ \text{hCut}$$

7.6 Status of $\to_L$: OK

- Case rule $\top_R$

$$\cfrac{\cfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_7}\ \to_L \qquad \cfrac{}{\bullet h_8 : \Delta_6, F_7 \vdash \top}\ \top_R}{- : (\Delta_2, F_3 \to F_4), \Delta_6 \vdash \top}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \Delta_2, \Delta_6, F_3 \to F_4 \vdash \top}\ \top_R$$

- Case rule $\to_R$

$$\cfrac{\cfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_7}\ \to_L \qquad \cfrac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \to F_{10}}\ \to_R}{- : (\Delta_2, F_3 \to F_4), \Delta_6 \vdash F_9 \to F_{10}}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_7}\ \text{ax/W} \qquad \cfrac{}{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}\ \text{ax/W}}{- : \Delta_2, \Delta_6, F_9, F_3 \to F_4 \vdash F_{10}}\ \text{hCut}}{- : \Delta_2, \Delta_6, F_3 \to F_4 \vdash F_9 \to F_{10}}\ \to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_7} \to_L \quad \dfrac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}} \wedge_R}{- : (\Delta_2, F_3 \to F_4), \Delta_6 \vdash F_9 \wedge F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Delta_2, \Delta_6, F_3 \to F_4 \vdash F_3} \text{ax/W} \quad \dfrac{\dfrac{}{h_1 : \Delta_2, F_4 \vdash F_7} \text{ax/W} \quad \dfrac{}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_4 \vdash F_9 \wedge F_{10}} \text{hCut}}{- : \Delta_2, \Delta_6, F_3 \to F_4 \vdash F_9 \wedge F_{10}} \to_L$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_7} \to_L \quad \dfrac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_1}{- : (\Delta_2, F_3 \to F_4), \Delta_6 \vdash F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_6, F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \to F_4 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_6, F_3 \to F_4 \vdash F_9 \vee F_{10}} \vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_7} \to_L \quad \dfrac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_2}{- : (\Delta_2, F_3 \to F_4), \Delta_6 \vdash F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_7} \text{ax/W} \quad \dfrac{}{h_8 : \Delta_6, F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \to F_4 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_6, F_3 \to F_4 \vdash F_9 \vee F_{10}} \vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_6} \to_L \quad \dfrac{h_7 : \Delta_{11}, F_6, F_8 \to F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \to F_9), F_6 \vdash F_{10}} \to_L}{- : (\Delta_2, F_3 \to F_4), \Delta_{11}, F_8 \to F_9 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Delta_{11}, \Delta_2, F_3 \to F_4, F_8 \to F_9 \vdash F_3} \text{ax/W} \quad \dfrac{\dfrac{}{h_1 : \Delta_2, F_4 \vdash F_6} \text{ax/W} \quad \dfrac{}{\bullet h_7 : \Delta_{11}, F_6, F_8 \to F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \to F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_3 \to F_4, F_8 \to F_9 \vdash F_{10}} \to_L$$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_8 \to F_9}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_8 \to F_9} \to_L \quad \dfrac{h_6 : \Delta_7, F_8 \to F_9 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash F_{10}} \to_L}{- : (\Delta_2, F_3 \to F_4), \Delta_7 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Delta_2, \Delta_7, F_3 \to F_4 \vdash F_3} \text{ax/W} \quad \dfrac{\dfrac{}{h_1 : \Delta_2, F_4 \vdash F_8 \to F_9} \text{ax/W} \quad \dfrac{}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_7, F_3 \to F_4 \vdash F_{10}} \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_6} \to_L \quad \dfrac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_6 \vdash F_{10}} \wedge_L}{- : (\Delta_2, F_3 \to F_4), \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_6} \text{ax/W} \quad \dfrac{}{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8, F_9, F_3 \to F_4 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_3 \to F_4, F_8 \wedge F_9 \vdash F_{10}} \wedge_L$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_8 \wedge F_9}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_8 \wedge F_9}\,{\to_L} \quad \cfrac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash F_{10}}\,{\wedge_L}}{- : (\Delta_2, F_3 \to F_4), \Delta_7 \vdash F_{10}}\,\text{Cut}}{}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_2, \Delta_7, F_3 \to F_4 \vdash F_3}\,\text{ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_2, F_4 \vdash F_8 \wedge F_9}\,\text{ax/W} \quad \cfrac{}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash F_{10}}\,\text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}}\,\text{hCut}}{- : \Delta_2, \Delta_7, F_3 \to F_4 \vdash F_{10}}\,{\to_L}$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_6}\,{\to_L} \quad \cfrac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_6 \vdash F_{10}}\,{\vee_L}}{- : (\Delta_2, F_3 \to F_4), \Delta_{11}, F_8 \vee F_9 \vdash F_{10}}\,\text{Cut}}{}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_{11}, \Delta_2, F_3 \to F_4, F_8 \vee F_9 \vdash F_3}\,\text{ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_2, F_4 \vdash F_6}\,\text{ax/W} \quad \cfrac{}{\bullet h_7 : \Delta_{11}, F_6, F_8 \vee F_9 \vdash F_{10}}\,\text{ax/W}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \vee F_9 \vdash F_{10}}\,\text{hCut}}{- : \Delta_{11}, \Delta_2, F_3 \to F_4, F_8 \vee F_9 \vdash F_{10}}\,{\to_L}$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_8 \vee F_9}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_8 \vee F_9}\,{\to_L} \quad \cfrac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash F_{10}}\,{\vee_L}}{- : (\Delta_2, F_3 \to F_4), \Delta_7 \vdash F_{10}}\,\text{Cut}}{}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_2, \Delta_7, F_3 \to F_4 \vdash F_3}\,\text{ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_2, F_4 \vdash F_8 \vee F_9}\,\text{ax/W} \quad \cfrac{}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash F_{10}}\,\text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}}\,\text{hCut}}{- : \Delta_2, \Delta_7, F_3 \to F_4 \vdash F_{10}}\,{\to_L}$$

- Case rule $\bot_L$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash \bot}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash \bot}\,{\to_L} \quad \cfrac{}{\bullet h_6 : \Delta_7, \bot \vdash F_8}\,{\bot_L}}{- : (\Delta_2, F_3 \to F_4), \Delta_7 \vdash F_8}\,\text{Cut}}{}$$

$$\to$$

$$\cfrac{\cfrac{}{- : \Delta_2, \Delta_7, F_3 \to F_4 \vdash F_3}\,\text{ax/W} \quad \cfrac{\cfrac{}{h_1 : \Delta_2, F_4 \vdash \bot}\,\text{ax/W} \quad \cfrac{}{\bullet h_6 : \bot, \Delta_7 \vdash F_8}\,{\bot_L}}{- : \Delta_2, \Delta_7, F_4 \vdash F_8}\,\text{hCut}}{- : \Delta_2, \Delta_7, F_3 \to F_4 \vdash F_8}\,{\to_L}$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_6}\,{\to_L} \quad \cfrac{}{\bullet h_7 : (\bot, \Delta_9), F_6 \vdash F_8}\,{\bot_L}}{- : (\Delta_2, F_3 \to F_4), \bot, \Delta_9 \vdash F_8}\,\text{Cut}}{}$$

$$\to$$

$$\cfrac{}{- : \bot, \Delta_2, \Delta_9, F_3 \to F_4 \vdash F_8}\,{\bot_L}$$

- Case rule I

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash F_6}\,{\to_L} \quad \cfrac{}{\bullet h_7 : (\Delta_9, p_8), F_6 \vdash p_8}\,{I}}{- : (\Delta_2, F_3 \to F_4), \Delta_9, p_8 \vdash p_8}\,\text{Cut}}{}$$

$$\to$$

$$\cfrac{}{- : \Delta_2, \Delta_9, p_8, F_3 \to F_4 \vdash p_8}\,{I}$$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_2, F_3 \to F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash p_8}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash p_8}\,{\to_L} \quad \cfrac{}{\bullet h_6 : \Delta_7, p_8 \vdash p_8}\,{I}}{- : (\Delta_2, F_3 \to F_4), \Delta_7 \vdash p_8}\,\text{Cut}}{}$$

$$\to$$

$$\cfrac{}{- : \Delta_2, \Delta_7, F_3 \to F_4 \vdash p_8}\,\text{ax/W}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_2,\mathtt{F_3}\rightarrow\mathtt{F_4}\vdash\mathtt{F_3}\quad\mathtt{h_1}:\Delta_2,\mathtt{F_4}\vdash\top}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\rightarrow\mathtt{F_4}\vdash\top}\rightarrow_L\quad\dfrac{\mathtt{h_6}:\Delta_7\vdash\mathtt{F_8}}{\bullet\mathtt{h_6}:\Delta_7,\top\vdash\mathtt{F_8}}\top_L}{\dfrac{-:(\Delta_2,\mathtt{F_3}\rightarrow\mathtt{F_4}),\Delta_7\vdash\mathtt{F_8}}{\dfrac{\rightarrow}{-:\Delta_2,\Delta_7,\mathtt{F_3}\rightarrow\mathtt{F_4}\vdash\mathtt{F_8}}\mathtt{ax/W}}\mathtt{Cut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_1}:\Delta_2,\mathtt{F_3}\rightarrow\mathtt{F_4}\vdash\mathtt{F_3}\quad\mathtt{h_1}:\Delta_2,\mathtt{F_4}\vdash\mathtt{F_6}}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\rightarrow\mathtt{F_4}\vdash\mathtt{F_6}}\rightarrow_L\quad\dfrac{\mathtt{h_7}:\Delta_9,\mathtt{F_6}\vdash\mathtt{F_8}}{\bullet\mathtt{h_7}:(\top,\Delta_9),\mathtt{F_6}\vdash\mathtt{F_8}}\top_L}{\dfrac{-:(\Delta_2,\mathtt{F_3}\rightarrow\mathtt{F_4}),\top,\Delta_9\vdash\mathtt{F_8}}{\rightarrow}}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\rightarrow\mathtt{F_4}\vdash\mathtt{F_6}}\mathtt{ax/W}\quad\dfrac{}{\mathtt{h_7}:\top,\Delta_9,\mathtt{F_6}\vdash\mathtt{F_8}}\mathtt{ax/W}}{-:\top,\Delta_2,\Delta_9,\mathtt{F_3}\rightarrow\mathtt{F_4}\vdash\mathtt{F_8}}\mathtt{hCut}}$$

7.7 Status of $\wedge_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_2,\mathtt{F_3},\mathtt{F_4}\vdash\mathtt{F_7}}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_7}}\wedge_L\quad\dfrac{}{\bullet\mathtt{h_8}:\Delta_6,\mathtt{F_7}\vdash\top}\top_R}{\dfrac{-:(\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}),\Delta_6\vdash\top}{\dfrac{\rightarrow}{-:\Delta_2,\Delta_6,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\top}\top_R}}\mathtt{Cut}}$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_2,\mathtt{F_3},\mathtt{F_4}\vdash\mathtt{F_7}}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_7}}\wedge_L\quad\dfrac{\mathtt{h_8}:\Delta_6,\mathtt{F_7},\mathtt{F_9}\vdash\mathtt{F_{10}}}{\bullet\mathtt{h_8}:\Delta_6,\mathtt{F_7}\vdash\mathtt{F_9}\rightarrow\mathtt{F_{10}}}\rightarrow_R}{\dfrac{-:(\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}),\Delta_6\vdash\mathtt{F_9}\rightarrow\mathtt{F_{10}}}{\dfrac{\rightarrow}{\dfrac{\dfrac{}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_7}}\mathtt{ax/W}\quad\dfrac{}{\mathtt{h_8}:\Delta_6,\mathtt{F_7},\mathtt{F_9}\vdash\mathtt{F_{10}}}\mathtt{ax/W}}{\dfrac{-:\Delta_2,\Delta_6,\mathtt{F_9},\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_{10}}}{-:\Delta_2,\Delta_6,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_9}\rightarrow\mathtt{F_{10}}}\rightarrow_R}\mathtt{hCut}}}\mathtt{Cut}}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_2,\mathtt{F_3},\mathtt{F_4}\vdash\mathtt{F_7}}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_7}}\wedge_L\quad\dfrac{\mathtt{h_8}:\Delta_6,\mathtt{F_7}\vdash\mathtt{F_9}\quad\mathtt{h_8}:\Delta_6,\mathtt{F_7}\vdash\mathtt{F_{10}}}{\bullet\mathtt{h_8}:\Delta_6,\mathtt{F_7}\vdash\mathtt{F_9}\wedge\mathtt{F_{10}}}\wedge_R}{\dfrac{-:(\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}),\Delta_6\vdash\mathtt{F_9}\wedge\mathtt{F_{10}}}{\dfrac{\rightarrow}{\dfrac{\dfrac{}{\mathtt{h_1}:\Delta_2,\mathtt{F_3},\mathtt{F_4}\vdash\mathtt{F_7}}\mathtt{ax/W}\quad\dfrac{}{\bullet\mathtt{h_8}:\Delta_6,\mathtt{F_7}\vdash\mathtt{F_9}\wedge\mathtt{F_{10}}}\mathtt{ax/W}}{\dfrac{-:\Delta_2,\Delta_6,\mathtt{F_3},\mathtt{F_4}\vdash\mathtt{F_9}\wedge\mathtt{F_{10}}}{-:\Delta_2,\Delta_6,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_9}\wedge\mathtt{F_{10}}}\wedge_L}\mathtt{hCut}}}\mathtt{Cut}}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_2,\mathtt{F_3},\mathtt{F_4}\vdash\mathtt{F_7}}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_7}}\wedge_L\quad\dfrac{\mathtt{h_8}:\Delta_6,\mathtt{F_7}\vdash\mathtt{F_9}}{\bullet\mathtt{h_8}:\Delta_6,\mathtt{F_7}\vdash\mathtt{F_9}\vee\mathtt{F_{10}}}\vee_1}{\dfrac{-:(\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}),\Delta_6\vdash\mathtt{F_9}\vee\mathtt{F_{10}}}{\dfrac{\rightarrow}{\dfrac{\dfrac{}{\bullet\mathtt{h_1}:\Delta_2,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_7}}\mathtt{ax/W}\quad\dfrac{}{\mathtt{h_8}:\Delta_6,\mathtt{F_7}\vdash\mathtt{F_9}}\mathtt{ax/W}}{\dfrac{-:\Delta_2,\Delta_6,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_9}}{-:\Delta_2,\Delta_6,\mathtt{F_3}\wedge\mathtt{F_4}\vdash\mathtt{F_9}\vee\mathtt{F_{10}}}\vee_1}\mathtt{hCut}}}\mathtt{Cut}}$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_7} \wedge_L \quad \dfrac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_2}{- : (\Delta_2, F_3 \wedge F_4), \Delta_6 \vdash F_9 \vee F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_7}{} ax/W \quad \dfrac{h_8 : \Delta_6, F_7 \vdash F_{10}}{} ax/W}{- : \Delta_2, \Delta_6, F_3 \wedge F_4 \vdash F_{10}} hCut}{- : \Delta_2, \Delta_6, F_3 \wedge F_4 \vdash F_9 \vee F_{10}} \vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_6} \wedge_L \quad \dfrac{h_7 : \Delta_{11}, F_6, F_8 \to F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \to F_9), F_6 \vdash F_{10}} \to_L}{- : (\Delta_2, F_3 \wedge F_4), \Delta_{11}, F_8 \to F_9 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{} ax/W \quad \dfrac{\bullet h_7 : \Delta_{11}, F_6, F_8 \to F_9 \vdash F_{10}}{} ax/W}{- : \Delta_{11}, \Delta_2, F_3, F_4, F_8 \to F_9 \vdash F_{10}} hCut}{- : \Delta_{11}, \Delta_2, F_8 \to F_9, F_3 \wedge F_4 \vdash F_{10}} \wedge_L$$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \to F_9}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_8 \to F_9} \wedge_L \quad \dfrac{h_6 : \Delta_7, F_8 \to F_9 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash F_{10}} \to_L}{- : (\Delta_2, F_3 \wedge F_4), \Delta_7 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \to F_9}{} ax/W \quad \dfrac{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash F_{10}}{} ax/W}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} hCut}{- : \Delta_2, \Delta_7, F_3 \wedge F_4 \vdash F_{10}} \wedge_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_6} \wedge_L \quad \dfrac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_6 \vdash F_{10}} \wedge_L}{- : (\Delta_2, F_3 \wedge F_4), \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{} ax/W \quad \dfrac{\bullet h_7 : \Delta_{11}, F_6, F_8 \wedge F_9 \vdash F_{10}}{} ax/W}{- : \Delta_{11}, \Delta_2, F_3, F_4, F_8 \wedge F_9 \vdash F_{10}} hCut}{- : \Delta_{11}, \Delta_2, F_3 \wedge F_4, F_8 \wedge F_9 \vdash F_{10}} \wedge_L$$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \wedge F_9}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_8 \wedge F_9} \wedge_L \quad \dfrac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash F_{10}} \wedge_L}{- : (\Delta_2, F_3 \wedge F_4), \Delta_7 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \wedge F_9}{} ax/W \quad \dfrac{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash F_{10}}{} ax/W}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} hCut}{- : \Delta_2, \Delta_7, F_3 \wedge F_4 \vdash F_{10}} \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_6} \wedge_L \quad \dfrac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_6 \vdash F_{10}} \vee_L}{- : (\Delta_2, F_3 \wedge F_4), \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{} ax/W \quad \dfrac{\bullet h_7 : \Delta_{11}, F_6, F_8 \vee F_9 \vdash F_{10}}{} ax/W}{- : \Delta_{11}, \Delta_2, F_3, F_4, F_8 \vee F_9 \vdash F_{10}} hCut}{- : \Delta_{11}, \Delta_2, F_3 \wedge F_4, F_8 \vee F_9 \vdash F_{10}} \wedge_L$$

$$\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \vee F_9}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_8 \vee F_9} \wedge_L \quad \dfrac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash F_{10}} \vee_L}{- : (\Delta_2, F_3 \wedge F_4), \Delta_7 \vdash F_{10}} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \vee F_9}{} ax/W \quad \dfrac{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash F_{10}}{} ax/W}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} hCut}{- : \Delta_2, \Delta_7, F_3 \wedge F_4 \vdash F_{10}} \wedge_L$$

- Case rule $\perp_L$

$$
\dfrac{\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \perp}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \perp} \wedge_L \quad \dfrac{}{\bullet\mathtt{h_6} : \Delta_7, \perp \vdash \mathtt{F_8}} \perp_L}{- : (\Delta_2, \mathtt{F_3} \wedge \mathtt{F_4}), \Delta_7 \vdash \mathtt{F_8}} \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\dfrac{\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \perp}{} \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_6} : \perp, \Delta_7 \vdash \mathtt{F_8}} \perp_L}{\dfrac{- : \Delta_2, \Delta_7, \mathtt{F_3}, \mathtt{F_4} \vdash \mathtt{F_8}}{- : \Delta_2, \Delta_7, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{F_8}} \wedge_L} \mathtt{hCut}
$$

$$
\dfrac{\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \mathtt{F_6}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{F_6}} \wedge_L \quad \dfrac{}{\bullet\mathtt{h_7} : (\perp, \Delta_9), \mathtt{F_6} \vdash \mathtt{F_8}} \perp_L}{- : (\Delta_2, \mathtt{F_3} \wedge \mathtt{F_4}), \perp, \Delta_9 \vdash \mathtt{F_8}} \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\dfrac{}{- : \perp, \Delta_2, \Delta_9, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{F_8}} \perp_L
$$

- Case rule I

$$
\dfrac{\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \mathtt{F_6}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{F_6}} \wedge_L \quad \dfrac{}{\bullet\mathtt{h_7} : (\Delta_9, \mathtt{p_8}), \mathtt{F_6} \vdash \mathtt{p_8}} I}{- : (\Delta_2, \mathtt{F_3} \wedge \mathtt{F_4}), \Delta_9, \mathtt{p_8} \vdash \mathtt{p_8}} \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p_8}, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{p_8}} I
$$

$$
\dfrac{\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \mathtt{p_8}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{p_8}} \wedge_L \quad \dfrac{}{\bullet\mathtt{h_6} : \Delta_7, \mathtt{p_8} \vdash \mathtt{p_8}} I}{- : (\Delta_2, \mathtt{F_3} \wedge \mathtt{F_4}), \Delta_7 \vdash \mathtt{p_8}} \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{p_8}} \mathtt{ax/W}
$$

- Case rule $\top_L$

$$
\dfrac{\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \top}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \top} \wedge_L \quad \dfrac{\mathtt{h_6} : \Delta_7 \vdash \mathtt{F_8}}{\bullet\mathtt{h_6} : \Delta_7, \top \vdash \mathtt{F_8}} \top_L}{- : (\Delta_2, \mathtt{F_3} \wedge \mathtt{F_4}), \Delta_7 \vdash \mathtt{F_8}} \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{F_8}} \mathtt{ax/W}
$$

$$
\dfrac{\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \mathtt{F_6}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{F_6}} \wedge_L \quad \dfrac{\mathtt{h_7} : \Delta_9, \mathtt{F_6} \vdash \mathtt{F_8}}{\bullet\mathtt{h_7} : (\top, \Delta_9), \mathtt{F_6} \vdash \mathtt{F_8}} \top_L}{- : (\Delta_2, \mathtt{F_3} \wedge \mathtt{F_4}), \top, \Delta_9 \vdash \mathtt{F_8}} \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\dfrac{\dfrac{}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{F_6}} \mathtt{ax/W} \quad \dfrac{}{\mathtt{h_7} : \top, \Delta_9, \mathtt{F_6} \vdash \mathtt{F_8}} \mathtt{ax/W}}{- : \top, \Delta_2, \Delta_9, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \mathtt{F_8}} \mathtt{hCut}
$$

7.8 Status of $\vee_L$: OK

- Case rule $\top_R$

$$
\dfrac{\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vdash \mathtt{F_7} \quad \mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \mathtt{F_7}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vee \mathtt{F_4} \vdash \mathtt{F_7}} \vee_L \quad \dfrac{}{\bullet\mathtt{h_8} : \Delta_6, \mathtt{F_7} \vdash \top} \top_R}{- : (\Delta_2, \mathtt{F_3} \vee \mathtt{F_4}), \Delta_6 \vdash \top} \mathtt{Cut}
$$

$$
\rightarrow
$$

$$
\dfrac{}{- : \Delta_2, \Delta_6, \mathtt{F_3} \vee \mathtt{F_4} \vdash \top} \top_R
$$

- Case rule $\rightarrow_R$

$$
\frac{
  \dfrac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7} \vee_L
  \quad
  \dfrac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \to F_{10}} \to_R
}{
  - : (\Delta_2, F_3 \vee F_4), \Delta_6 \vdash F_9 \to F_{10}
} \; \text{Cut}
$$

$$\to$$

$$
\frac{
  \dfrac{\ }{\color{blue}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}} \text{ax/W}
  \quad
  \dfrac{\ }{\color{blue}{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}} \text{ax/W}
}{
  \dfrac{\color{blue}{- : \Delta_2, \Delta_6, F_9, F_3 \vee F_4 \vdash F_{10}}}{\color{blue}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \to F_{10}}} \to_R
} \; \text{hCut}
$$

- Case rule $\wedge_R$

$$
\frac{
  \dfrac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7} \vee_L
  \quad
  \dfrac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}} \wedge_R
}{
  - : (\Delta_2, F_3 \vee F_4), \Delta_6 \vdash F_9 \wedge F_{10}
} \; \text{Cut}
$$

$$\to$$

$$
\frac{
  \dfrac{\dfrac{\ }{\color{blue}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}}\text{ax/W} \; \dfrac{\ }{\color{blue}{h_8 : \Delta_6, F_7 \vdash F_9}}\text{ax/W}}{\color{blue}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9}}\text{hCut}
  \quad
  \dfrac{\dfrac{\ }{\color{blue}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}}\text{ax/W} \; \dfrac{\ }{\color{blue}{h_8 : \Delta_6, F_7 \vdash F_{10}}}\text{ax/W}}{\color{blue}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_{10}}}\text{hCut}
}{
  \color{blue}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \wedge F_{10}}
} \; \wedge_R
$$

- Case rule $\vee_1$

$$
\frac{
  \dfrac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7} \vee_L
  \quad
  \dfrac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_1
}{
  - : (\Delta_2, F_3 \vee F_4), \Delta_6 \vdash F_9 \vee F_{10}
} \; \text{Cut}
$$

$$\to$$

$$
\frac{
  \dfrac{\dfrac{\ }{\color{blue}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}}\text{ax/W} \; \dfrac{\ }{\color{blue}{h_8 : \Delta_6, F_7 \vdash F_9}}\text{ax/W}}{\color{blue}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9}}\text{hCut}
}{
  \color{blue}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \vee F_{10}}
} \; \vee_1
$$

- Case rule $\vee_2$

$$
\frac{
  \dfrac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7} \vee_L
  \quad
  \dfrac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_2
}{
  - : (\Delta_2, F_3 \vee F_4), \Delta_6 \vdash F_9 \vee F_{10}
} \; \text{Cut}
$$

$$\to$$

$$
\frac{
  \dfrac{\dfrac{\ }{\color{blue}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}}\text{ax/W} \; \dfrac{\ }{\color{blue}{h_8 : \Delta_6, F_7 \vdash F_{10}}}\text{ax/W}}{\color{blue}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_{10}}}\text{hCut}
}{
  \color{blue}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \vee F_{10}}
} \; \vee_2
$$

- Case rule $\to_L$

$$
\frac{
  \dfrac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6} \vee_L
  \quad
  \dfrac{h_7 : \Delta_{11}, F_6, F_8 \to F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \to F_9), F_6 \vdash F_{10}} \to_L
}{
  - : (\Delta_2, F_3 \vee F_4), \Delta_{11}, F_8 \to F_9 \vdash F_{10}
} \; \text{Cut}
$$

$$\to$$

$$
\frac{
  \dfrac{\dfrac{\ }{\color{blue}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6}}\text{ax/W} \; \dfrac{\ }{\color{blue}{h_7 : \Delta_{11}, F_6, F_8 \to F_9 \vdash F_8}}\text{ax/W}}{\color{blue}{- : \Delta_{11}, \Delta_2, F_8 \to F_9, F_3 \vee F_4 \vdash F_8}}\text{hCut}
  \quad
  \dfrac{\dfrac{\ }{\color{blue}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6}}\text{ax/W} \; \dfrac{\ }{\color{blue}{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}}\text{ax/W}}{\color{blue}{- : \Delta_{11}, \Delta_2, F_9, F_3 \vee F_4 \vdash F_{10}}}\text{hCut}
}{
  \color{blue}{- : \Delta_{11}, \Delta_2, F_8 \to F_9, F_3 \vee F_4 \vdash F_{10}}
} \; \to_L
$$

$$
\frac{
  \dfrac{h_1 : \Delta_2, F_3 \vdash F_8 \to F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \to F_9}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_8 \to F_9} \vee_L
  \quad
  \dfrac{h_6 : \Delta_7, F_8 \to F_9 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash F_{10}} \to_L
}{
  - : (\Delta_2, F_3 \vee F_4), \Delta_7 \vdash F_{10}
} \; \text{Cut}
$$

$$\to$$

$$
\frac{
  \dfrac{\dfrac{\ }{\color{blue}{h_1 : \Delta_2, F_3 \vdash F_8 \to F_9}}\text{ax/W} \; \dfrac{\ }{\color{blue}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash F_{10}}}\text{ax/W}}{\color{blue}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}}}\text{hCut}
  \quad
  \dfrac{\dfrac{\ }{\color{blue}{h_1 : \Delta_2, F_4 \vdash F_8 \to F_9}}\text{ax/W} \; \dfrac{\ }{\color{blue}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash F_{10}}}\text{ax/W}}{\color{blue}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}}}\text{hCut}
}{
  \color{blue}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}}
} \; \vee_L
$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash F_6 \quad \mathbf{h_1}:\Delta_2,F_4\vdash F_6}{\bullet\mathbf{h_1}:\Delta_2,F_3\vee F_4\vdash F_6}\ \vee_L \quad \cfrac{\mathbf{h_7}:\Delta_{11},F_6,F_8,F_9\vdash F_{10}}{\bullet\mathbf{h_7}:(\Delta_{11},F_8\wedge F_9),F_6\vdash F_{10}}\ \wedge_L}{-:(\Delta_2,F_3\vee F_4),\Delta_{11},F_8\wedge F_9\vdash F_{10}}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{\bullet\mathbf{h_1}:\Delta_2,F_3\vee F_4\vdash F_6}{}\ \text{ax/W}\quad \cfrac{\mathbf{h_7}:\Delta_{11},F_6,F_8,F_9\vdash F_{10}}{}\ \text{ax/W}}{-:\Delta_{11},\Delta_2,F_8,F_9,F_3\vee F_4\vdash F_{10}}\ \text{hCut}}{-:\Delta_{11},\Delta_2,F_8\wedge F_9,F_3\vee F_4\vdash F_{10}}\ \wedge_L$$

$$\cfrac{\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash F_8\wedge F_9\quad \mathbf{h_1}:\Delta_2,F_4\vdash F_8\wedge F_9}{\bullet\mathbf{h_1}:\Delta_2,F_3\vee F_4\vdash F_8\wedge F_9}\ \vee_L\quad \cfrac{\mathbf{h_6}:\Delta_7,F_8,F_9\vdash F_{10}}{\bullet\mathbf{h_6}:\Delta_7,F_8\wedge F_9\vdash F_{10}}\ \wedge_L}{-:(\Delta_2,F_3\vee F_4),\Delta_7\vdash F_{10}}}{}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash F_8\wedge F_9}{}\ \text{ax/W}\quad \cfrac{\bullet\mathbf{h_6}:\Delta_7,F_8\wedge F_9\vdash F_{10}}{}\ \text{ax/W}}{-:\Delta_2,\Delta_7,F_3\vdash F_{10}}\ \text{hCut}\quad \cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_4\vdash F_8\wedge F_9}{}\ \text{ax/W}\quad \cfrac{\bullet\mathbf{h_6}:\Delta_7,F_8\wedge F_9\vdash F_{10}}{}\ \text{ax/W}}{-:\Delta_2,\Delta_7,F_4\vdash F_{10}}\ \text{hCut}}{-:\Delta_2,\Delta_7,F_3\vee F_4\vdash F_{10}}\ \vee_L$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash F_6\quad \mathbf{h_1}:\Delta_2,F_4\vdash F_6}{\bullet\mathbf{h_1}:\Delta_2,F_3\vee F_4\vdash F_6}\ \vee_L\quad \cfrac{\mathbf{h_7}:\Delta_{11},F_6,F_8\vdash F_{10}\quad \mathbf{h_7}:\Delta_{11},F_6,F_9\vdash F_{10}}{\bullet\mathbf{h_7}:(\Delta_{11},F_8\vee F_9),F_6\vdash F_{10}}\ \vee_L}{-:(\Delta_2,F_3\vee F_4),\Delta_{11},F_8\vee F_9\vdash F_{10}}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash F_6}{}\ \text{ax/W}\quad \cfrac{\bullet\mathbf{h_7}:\Delta_{11},F_6,F_8\vee F_9\vdash F_{10}}{}\ \text{ax/W}}{-:\Delta_{11},\Delta_2,F_3,F_8\vee F_9\vdash F_{10}}\ \text{hCut}\quad \cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_4\vdash F_6}{}\ \text{ax/W}\quad \cfrac{\bullet\mathbf{h_7}:\Delta_{11},F_6,F_8\vee F_9\vdash F_{10}}{}\ \text{ax/W}}{-:\Delta_{11},\Delta_2,F_4,F_8\vee F_9\vdash F_{10}}\ \text{hCut}}{-:\Delta_{11},\Delta_2,F_3\vee F_4,F_8\vee F_9\vdash F_{10}}\ \vee_L$$

$$\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash F_8\vee F_9\quad \mathbf{h_1}:\Delta_2,F_4\vdash F_8\vee F_9}{\bullet\mathbf{h_1}:\Delta_2,F_3\vee F_4\vdash F_8\vee F_9}\ \vee_L\quad \cfrac{\mathbf{h_6}:\Delta_7,F_8\vdash F_{10}\quad \mathbf{h_6}:\Delta_7,F_9\vdash F_{10}}{\bullet\mathbf{h_6}:\Delta_7,F_8\vee F_9\vdash F_{10}}\ \vee_L}{-:(\Delta_2,F_3\vee F_4),\Delta_7\vdash F_{10}}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash F_8\vee F_9}{}\ \text{ax/W}\quad \cfrac{\bullet\mathbf{h_6}:\Delta_7,F_8\vee F_9\vdash F_{10}}{}\ \text{ax/W}}{-:\Delta_2,\Delta_7,F_3\vdash F_{10}}\ \text{hCut}\quad \cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_4\vdash F_8\vee F_9}{}\ \text{ax/W}\quad \cfrac{\bullet\mathbf{h_6}:\Delta_7,F_8\vee F_9\vdash F_{10}}{}\ \text{ax/W}}{-:\Delta_2,\Delta_7,F_4\vdash F_{10}}\ \text{hCut}}{-:\Delta_2,\Delta_7,F_3\vee F_4\vdash F_{10}}\ \vee_L$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash\perp\quad \mathbf{h_1}:\Delta_2,F_4\vdash\perp}{\bullet\mathbf{h_1}:\Delta_2,F_3\vee F_4\vdash\perp}\ \vee_L\quad \cfrac{}{\bullet\mathbf{h_6}:\Delta_7,\perp\vdash F_8}\ \perp_L}{-:(\Delta_2,F_3\vee F_4),\Delta_7\vdash F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash\perp}{}\ \text{ax/W}\quad \cfrac{\bullet\mathbf{h_6}:\perp,\Delta_7\vdash F_8}{}\ \text{ax/W}}{-:\Delta_2,\Delta_7,F_3\vdash F_8}\ \text{hCut}\quad \cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_4\vdash\perp}{}\ \text{ax/W}\quad \cfrac{\bullet\mathbf{h_6}:\perp,\Delta_7\vdash F_8}{}\ \text{ax/W}}{-:\Delta_2,\Delta_7,F_4\vdash F_8}\ \text{hCut}}{-:\Delta_2,\Delta_7,F_3\vee F_4\vdash F_8}\ \vee_L$$

$$\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash F_6\quad \mathbf{h_1}:\Delta_2,F_4\vdash F_6}{\bullet\mathbf{h_1}:\Delta_2,F_3\vee F_4\vdash F_6}\ \vee_L\quad \cfrac{}{\bullet\mathbf{h_7}:(\perp,\Delta_9),F_6\vdash F_8}\ \perp_L}{-:(\Delta_2,F_3\vee F_4),\perp,\Delta_9\vdash F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{-:\perp,\Delta_2,\Delta_9,F_3\vee F_4\vdash F_8}\ \perp_L$$

- Case rule I

$$\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash F_6\quad \mathbf{h_1}:\Delta_2,F_4\vdash F_6}{\bullet\mathbf{h_1}:\Delta_2,F_3\vee F_4\vdash F_6}\ \vee_L\quad \cfrac{}{\bullet\mathbf{h_7}:(\Delta_9,p_8),F_6\vdash p_8}\ I}{-:(\Delta_2,F_3\vee F_4),\Delta_9,p_8\vdash p_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{-:\Delta_2,\Delta_9,p_8,F_3\vee F_4\vdash p_8}\ I$$

$$\cfrac{\cfrac{\mathbf{h_1}:\Delta_2,F_3\vdash p_8\quad \mathbf{h_1}:\Delta_2,F_4\vdash p_8}{\bullet\mathbf{h_1}:\Delta_2,F_3\vee F_4\vdash p_8}\ \vee_L\quad \cfrac{}{\bullet\mathbf{h_6}:\Delta_7,p_8\vdash p_8}\ I}{-:(\Delta_2,F_3\vee F_4),\Delta_7\vdash p_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{-:\Delta_2,\Delta_7,F_3\vee F_4\vdash p_8}\ \text{ax/W}$$

- Case rule $\top_L$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \Delta_2, F_3 \vdash \top \quad h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \top} \vee_L \quad \cfrac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, \top \vdash F_8} \top_L
}{- : (\Delta_2, F_3 \vee F_4), \Delta_7 \vdash F_8} \text{Cut}
}{
\cfrac{\rightarrow}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_8}
} \text{ax/W}
$$

$$
\cfrac{
\cfrac{
\cfrac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6} \vee_L \quad \cfrac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_6 \vdash F_8} \top_L
}{- : (\Delta_2, F_3 \vee F_4), \top, \Delta_9 \vdash F_8} \text{Cut}
}{
\cfrac{\rightarrow}{\cfrac{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6}{} \text{ax/W} \quad \cfrac{h_7 : \top, \Delta_9, F_6 \vdash F_8}{} \text{ax/W}}{- : \top, \Delta_2, \Delta_9, F_3 \vee F_4 \vdash F_8} \text{hCut}
}
$$

7.9 Status of $\perp_L$: OK

- Case rule $\top_R$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \cfrac{}{\bullet h_6 : \Delta_4, F_5 \vdash \top} \top_R
}{- : (\perp, \Delta_2), \Delta_4 \vdash \top} \text{Cut}
}{
\cfrac{\rightarrow}{- : \perp, \Delta_2, \Delta_4 \vdash \top} \top_R
}
$$

- Case rule $\rightarrow_R$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \cfrac{h_6 : \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \rightarrow F_8} \rightarrow_R
}{- : (\perp, \Delta_2), \Delta_4 \vdash F_7 \rightarrow F_8} \text{Cut}
}{
\cfrac{\rightarrow}{- : \perp, \Delta_2, \Delta_4 \vdash F_7 \rightarrow F_8} \perp_L
}
$$

- Case rule $\wedge_R$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \cfrac{h_6 : \Delta_4, F_5 \vdash F_7 \quad h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \wedge F_8} \wedge_R
}{- : (\perp, \Delta_2), \Delta_4 \vdash F_7 \wedge F_8} \text{Cut}
}{
\cfrac{\rightarrow}{- : \perp, \Delta_2, \Delta_4 \vdash F_7 \wedge F_8} \perp_L
}
$$

- Case rule $\vee_1$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \cfrac{h_6 : \Delta_4, F_5 \vdash F_7}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8} \vee_1
}{- : (\perp, \Delta_2), \Delta_4 \vdash F_7 \vee F_8} \text{Cut}
}{
\cfrac{\rightarrow}{- : \perp, \Delta_2, \Delta_4 \vdash F_7 \vee F_8} \perp_L
}
$$

- Case rule $\vee_2$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \cfrac{h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8} \vee_2
}{- : (\perp, \Delta_2), \Delta_4 \vdash F_7 \vee F_8} \text{Cut}
}{
\cfrac{\rightarrow}{- : \perp, \Delta_2, \Delta_4 \vdash F_7 \vee F_8} \perp_L
}
$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_4} \ {\bot_L} \quad \dfrac{h_5 : \Delta_9, F_4, F_6 \to F_7 \vdash F_6 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \to F_7), F_4 \vdash F_8} \ {\to_L}}{\dfrac{- : (\bot, \Delta_2), \Delta_9, F_6 \to F_7 \vdash F_8}{\dfrac{\to}{- : \bot, \Delta_2, \Delta_9, F_6 \to F_7 \vdash F_8} \ {\bot_L}} \ \text{Cut}}$$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_6 \to F_7} \ {\bot_L} \quad \dfrac{h_4 : \Delta_5, F_6 \to F_7 \vdash F_6 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \to F_7 \vdash F_8} \ {\to_L}}{\dfrac{- : (\bot, \Delta_2), \Delta_5 \vdash F_8}{\dfrac{\to}{- : \bot, \Delta_2, \Delta_5 \vdash F_8} \ {\bot_L}} \ \text{Cut}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_4} \ {\bot_L} \quad \dfrac{h_5 : \Delta_9, F_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \wedge F_7), F_4 \vdash F_8} \ {\wedge_L}}{\dfrac{- : (\bot, \Delta_2), \Delta_9, F_6 \wedge F_7 \vdash F_8}{\dfrac{\to}{- : \bot, \Delta_2, \Delta_9, F_6 \wedge F_7 \vdash F_8} \ {\bot_L}} \ \text{Cut}}$$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_6 \wedge F_7} \ {\bot_L} \quad \dfrac{h_4 : \Delta_5, F_6, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash F_8} \ {\wedge_L}}{\dfrac{- : (\bot, \Delta_2), \Delta_5 \vdash F_8}{\dfrac{\to}{- : \bot, \Delta_2, \Delta_5 \vdash F_8} \ {\bot_L}} \ \text{Cut}}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_4} \ {\bot_L} \quad \dfrac{h_5 : \Delta_9, F_4, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \vee F_7), F_4 \vdash F_8} \ {\vee_L}}{\dfrac{- : (\bot, \Delta_2), \Delta_9, F_6 \vee F_7 \vdash F_8}{\dfrac{\to}{- : \bot, \Delta_2, \Delta_9, F_6 \vee F_7 \vdash F_8} \ {\bot_L}} \ \text{Cut}}$$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_6 \vee F_7} \ {\bot_L} \quad \dfrac{h_4 : \Delta_5, F_6 \vdash F_8 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash F_8} \ {\vee_L}}{\dfrac{- : (\bot, \Delta_2), \Delta_5 \vdash F_8}{\dfrac{\to}{- : \bot, \Delta_2, \Delta_5 \vdash F_8} \ {\bot_L}} \ \text{Cut}}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash \bot} \ {\bot_L} \quad \dfrac{}{\bullet h_4 : \Delta_5, \bot \vdash F_6} \ {\bot_L}}{\dfrac{- : (\bot, \Delta_2), \Delta_5 \vdash F_6}{\dfrac{\to}{- : \bot, \Delta_2, \Delta_5 \vdash F_6} \ {\bot_L}} \ \text{Cut}}$$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_4} \ {\bot_L} \quad \dfrac{}{\bullet h_5 : (\bot, \Delta_7), F_4 \vdash F_6} \ {\bot_L}}{\dfrac{- : (\bot, \Delta_2), \bot, \Delta_7 \vdash F_6}{\dfrac{\to}{- : \bot, \bot, \Delta_2, \Delta_7 \vdash F_6} \ {\bot_L}} \ \text{Cut}}$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_4} \ {\bot_L} \quad \dfrac{}{\bullet h_5 : (\Delta_7, p_6), F_4 \vdash p_6} \ {I}}{\dfrac{- : (\bot, \Delta_2), \Delta_7, p_6 \vdash p_6}{\dfrac{\to}{- : \bot, \Delta_2, \Delta_7, p_6 \vdash p_6} \ {\bot_L}} \ \text{Cut}}$$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash p_6} \ {}^{\bot_L} \quad \dfrac{}{\bullet h_4 : \Delta_5, p_6 \vdash p_6} \ {}^{I}}{-: (\bot, \Delta_2), \Delta_5 \vdash p_6} \ {}^{Cut}$$
$$\to$$
$$\dfrac{}{-: \bot, \Delta_2, \Delta_5 \vdash p_6} \ {}^{\bot_L}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash \top} \ {}^{\bot_L} \quad \dfrac{h_4 : \Delta_5 \vdash F_6}{\bullet h_4 : \Delta_5, \top \vdash F_6} \ {}^{\top_L}}{-: (\bot, \Delta_2), \Delta_5 \vdash F_6} \ {}^{Cut}$$
$$\to$$
$$\dfrac{}{-: \bot, \Delta_2, \Delta_5 \vdash F_6} \ {}^{\bot_L}$$

$$\dfrac{\dfrac{}{\bullet h_1 : \bot, \Delta_2 \vdash F_4} \ {}^{\bot_L} \quad \dfrac{h_5 : \Delta_7, F_4 \vdash F_6}{\bullet h_5 : (\top, \Delta_7), F_4 \vdash F_6} \ {}^{\top_L}}{-: (\bot, \Delta_2), \top, \Delta_7 \vdash F_6} \ {}^{Cut}$$
$$\to$$
$$\dfrac{}{-: \bot, \top, \Delta_2, \Delta_7 \vdash F_6} \ {}^{\bot_L}$$

7.10   Status of I: `OK`

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} \ {}^{I} \quad \dfrac{}{\bullet h_6 : \Delta_4, p_5 \vdash \top} \ {}^{\top_R}}{-: (\Delta_2, p_5), \Delta_4 \vdash \top} \ {}^{Cut}$$
$$\to$$
$$\dfrac{}{-: \Delta_2, \Delta_4, p_5 \vdash \top} \ {}^{\top_R}$$

- Case rule $\to_R$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} \ {}^{I} \quad \dfrac{h_6 : \Delta_4, F_7, p_5 \vdash F_8}{\bullet h_6 : \Delta_4, p_5 \vdash F_7 \to F_8} \ {}^{\to_R}}{-: (\Delta_2, p_5), \Delta_4 \vdash F_7 \to F_8} \ {}^{Cut}$$
$$\to$$
$$\dfrac{}{-: \Delta_2, \Delta_4, p_5 \vdash F_7 \to F_8} \ {}^{ax/W}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} \ {}^{I} \quad \dfrac{h_6 : \Delta_4, p_5 \vdash F_7 \quad h_6 : \Delta_4, p_5 \vdash F_8}{\bullet h_6 : \Delta_4, p_5 \vdash F_7 \wedge F_8} \ {}^{\wedge_R}}{-: (\Delta_2, p_5), \Delta_4 \vdash F_7 \wedge F_8} \ {}^{Cut}$$
$$\to$$
$$\dfrac{}{-: \Delta_2, \Delta_4, p_5 \vdash F_7 \wedge F_8} \ {}^{ax/W}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} \ {}^{I} \quad \dfrac{h_6 : \Delta_4, p_5 \vdash F_7}{\bullet h_6 : \Delta_4, p_5 \vdash F_7 \vee F_8} \ {}^{\vee_1}}{-: (\Delta_2, p_5), \Delta_4 \vdash F_7 \vee F_8} \ {}^{Cut}$$
$$\to$$
$$\dfrac{}{-: \Delta_2, \Delta_4, p_5 \vdash F_7 \vee F_8} \ {}^{ax/W}$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} \ {}^{I} \quad \dfrac{h_6 : \Delta_4, p_5 \vdash F_8}{\bullet h_6 : \Delta_4, p_5 \vdash F_7 \vee F_8} \ {}^{\vee_2}}{-: (\Delta_2, p_5), \Delta_4 \vdash F_7 \vee F_8} \ {}^{Cut}$$
$$\to$$
$$\dfrac{}{-: \Delta_2, \Delta_4, p_5 \vdash F_7 \vee F_8} \ {}^{ax/W}$$

- Case rule $\to_L$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I
\qquad
\cfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \to_L
}{
- : (\Delta_2, \mathtt{p}_4), \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8
}\ \mathtt{Cut}
}{
- : \Delta_2, \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8
}\ \mathtt{ax/W}
$$

- Case rule $\wedge_L$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I
\qquad
\cfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \wedge_L
}{
- : (\Delta_2, \mathtt{p}_4), \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8
}\ \mathtt{Cut}
}{
- : \Delta_2, \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8
}\ \mathtt{ax/W}
$$

- Case rule $\vee_L$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I
\qquad
\cfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_4 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \vee_L
}{
- : (\Delta_2, \mathtt{p}_4), \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8
}\ \mathtt{Cut}
}{
- : \Delta_2, \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8
}\ \mathtt{ax/W}
$$

- Case rule $\perp_L$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I
\qquad
\cfrac{}{\bullet \mathtt{h}_5 : (\perp, \Delta_7), \mathtt{p}_4 \vdash \mathtt{F}_6}\ \perp_L
}{
- : (\Delta_2, \mathtt{p}_4), \perp, \Delta_7 \vdash \mathtt{F}_6
}\ \mathtt{Cut}
}{
- : \perp, \Delta_2, \Delta_7, \mathtt{p}_4 \vdash \mathtt{F}_6
}\ \perp_L
$$

- Case rule I

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I
\qquad
\cfrac{}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_6), \mathtt{p}_4 \vdash \mathtt{p}_6}\ I
}{
- : (\Delta_2, \mathtt{p}_4), \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6
}\ \mathtt{Cut}
}{
- : \Delta_2, \Delta_7, \mathtt{p}_4, \mathtt{p}_6 \vdash \mathtt{p}_6
}\ I
$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I
\qquad
\cfrac{}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I
}{
- : (\Delta_2, \mathtt{p}_6), \Delta_5 \vdash \mathtt{p}_6
}\ \mathtt{Cut}
}{
- : \Delta_2, \Delta_5, \mathtt{p}_6 \vdash \mathtt{p}_6
}\ I
$$

- Case rule $\top_L$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I
\qquad
\cfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\top, \Delta_7), \mathtt{p}_4 \vdash \mathtt{F}_6}\ \top_L
}{
- : (\Delta_2, \mathtt{p}_4), \top, \Delta_7 \vdash \mathtt{F}_6
}\ \mathtt{Cut}
}{
- : \top, \Delta_2, \Delta_7, \mathtt{p}_4 \vdash \mathtt{F}_6
}\ \mathtt{ax/W}
$$

7.11   Status of $\top_L$: `OK`

- Case rule $\top_R$

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$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5}\ \top_L \qquad \cfrac{}{\bullet h_6 : \Delta_4, F_5 \vdash \top}\ \top_R}{- : (\top, \Delta_2), \Delta_4 \vdash \top}\ \text{Cut}$$

$$\to$$

$$\cfrac{}{- : \top, \Delta_2, \Delta_4 \vdash \top}\ \top_R$$

- Case rule $\to_R$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5}\ \top_L \qquad \cfrac{h_6 : \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \to F_8}\ \to_R}{- : (\top, \Delta_2), \Delta_4 \vdash F_7 \to F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{h_1 : \Delta_2 \vdash F_5}\ \text{ax/W} \qquad \cfrac{}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \to F_8}\ \text{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash F_7 \to F_8}\ \text{hCut}$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5}\ \top_L \qquad \cfrac{h_6 : \Delta_4, F_5 \vdash F_7 \quad h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \wedge F_8}\ \wedge_R}{- : (\top, \Delta_2), \Delta_4 \vdash F_7 \wedge F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{h_1 : \Delta_2 \vdash F_5}\ \text{ax/W} \qquad \cfrac{}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \wedge F_8}\ \text{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash F_7 \wedge F_8}\ \text{hCut}$$

- Case rule $\vee_1$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5}\ \top_L \qquad \cfrac{h_6 : \Delta_4, F_5 \vdash F_7}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8}\ \vee_1}{- : (\top, \Delta_2), \Delta_4 \vdash F_7 \vee F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{h_1 : \Delta_2 \vdash F_5}\ \text{ax/W} \qquad \cfrac{}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \vee F_8}\ \text{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash F_7 \vee F_8}\ \text{hCut}$$

- Case rule $\vee_2$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5}\ \top_L \qquad \cfrac{h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8}\ \vee_2}{- : (\top, \Delta_2), \Delta_4 \vdash F_7 \vee F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{h_1 : \Delta_2 \vdash F_5}\ \text{ax/W} \qquad \cfrac{}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \vee F_8}\ \text{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash F_7 \vee F_8}\ \text{hCut}$$

- Case rule $\to_L$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4}\ \top_L \qquad \cfrac{h_5 : \Delta_9, F_4, F_6 \to F_7 \vdash F_6 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \to F_7), F_4 \vdash F_8}\ \to_L}{- : (\top, \Delta_2), \Delta_9, F_6 \to F_7 \vdash F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{h_1 : \Delta_2 \vdash F_4}\ \text{ax/W} \qquad \cfrac{}{\bullet h_5 : \top, \Delta_9, F_4, F_6 \to F_7 \vdash F_8}\ \text{ax/W}}{- : \top, \Delta_2, \Delta_9, F_6 \to F_7 \vdash F_8}\ \text{hCut}$$

$$\cfrac{\cfrac{h_1 : \Delta_2 \vdash F_6 \to F_7}{\bullet h_1 : \top, \Delta_2 \vdash F_6 \to F_7}\ \top_L \qquad \cfrac{h_4 : \Delta_5, F_6 \to F_7 \vdash F_6 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \to F_7 \vdash F_8}\ \to_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_8}\ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{h_1 : \Delta_2 \vdash F_6 \to F_7}\ \text{ax/W} \qquad \cfrac{}{\bullet h_4 : \top, \Delta_5, F_6 \to F_7 \vdash F_8}\ \text{ax/W}}{- : \top, \Delta_2, \Delta_5 \vdash F_8}\ \text{hCut}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \dfrac{h_5 : \Delta_9, F_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \wedge F_7), F_4 \vdash F_8} \wedge_L}{- : (\top, \Delta_2), \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_4} \text{ax/W} \quad \dfrac{}{\bullet h_5 : \top, \Delta_9, F_4, F_6 \wedge F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{hCut}$$

$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash F_6 \wedge F_7}{\bullet h_1 : \top, \Delta_2 \vdash F_6 \wedge F_7} \top_L \quad \dfrac{h_4 : \Delta_5, F_6, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash F_8} \wedge_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_8} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_6 \wedge F_7} \text{ax/W} \quad \dfrac{}{\bullet h_4 : \top, \Delta_5, F_6 \wedge F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \dfrac{h_5 : \Delta_9, F_4, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \vee F_7), F_4 \vdash F_8} \vee_L}{- : (\top, \Delta_2), \Delta_9, F_6 \vee F_7 \vdash F_8} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_4} \text{ax/W} \quad \dfrac{}{\bullet h_5 : \top, \Delta_9, F_4, F_6 \vee F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_9, F_6 \vee F_7 \vdash F_8} \text{hCut}$$

$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash F_6 \vee F_7}{\bullet h_1 : \top, \Delta_2 \vdash F_6 \vee F_7} \top_L \quad \dfrac{h_4 : \Delta_5, F_6 \vdash F_8 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash F_8} \vee_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_8} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_6 \vee F_7} \text{ax/W} \quad \dfrac{}{\bullet h_4 : \top, \Delta_5, F_6 \vee F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash \perp}{\bullet h_1 : \top, \Delta_2 \vdash \perp} \top_L \quad \dfrac{}{\bullet h_4 : \Delta_5, \perp \vdash F_6} \perp_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_6} \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash \perp} \text{ax/W} \quad \dfrac{}{\bullet h_4 : \perp, \top, \Delta_5 \vdash F_6} \perp_L}{- : \top, \Delta_2, \Delta_5 \vdash F_6} \text{hCut}$$

$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \dfrac{}{\bullet h_5 : (\perp, \Delta_7), F_4 \vdash F_6} \perp_L}{- : (\top, \Delta_2), \perp, \Delta_7 \vdash F_6} \text{Cut}$$

$$\to$$

$$\dfrac{}{- : \perp, \top, \Delta_2, \Delta_7 \vdash F_6} \perp_L$$

- Case rule I

$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \dfrac{}{\bullet h_5 : (\Delta_7, p_6), F_4 \vdash p_6} I}{- : (\top, \Delta_2), \Delta_7, p_6 \vdash p_6} \text{Cut}$$

$$\to$$

$$\dfrac{}{- : \top, \Delta_2, \Delta_7, p_6 \vdash p_6} I$$

$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash p_6}{\bullet h_1 : \top, \Delta_2 \vdash p_6} \top_L \quad \dfrac{}{\bullet h_4 : \Delta_5, p_6 \vdash p_6} I}{- : (\top, \Delta_2), \Delta_5 \vdash p_6} \text{Cut}$$

$$\to$$

$$\dfrac{}{- : \top, \Delta_2, \Delta_5 \vdash p_6} \text{ax/W}$$

- Case rule $\top_L$

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$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \top, \Delta_2 \vdash \top} \top_L \qquad \dfrac{h_4 : \Delta_5 \vdash F_6}{\bullet h_4 : \Delta_5, \top \vdash F_6} \top_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_6} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \top, \Delta_2, \Delta_5 \vdash F_6} \text{ax/W}$$

$$\dfrac{\dfrac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \qquad \dfrac{h_5 : \Delta_7, F_4 \vdash F_6}{\bullet h_5 : (\top, \Delta_7), F_4 \vdash F_6} \top_L}{- : (\top, \Delta_2), \top, \Delta_7 \vdash F_6} \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{h_1 : \Delta_2 \vdash F_4} \text{ax/W} \qquad \dfrac{}{\bullet h_5 : \top, \top, \Delta_7, F_4 \vdash F_6} \text{ax/W}}{- : \top, \top, \Delta_2, \Delta_7 \vdash F_6} \text{hCut}$$