# Dyadic system for Linear Logic

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## 1 Weakening on the classical context

 $\bullet$  Case(s) rule 1

• Case(s) rule!

• Case(s) rule?

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_2\} : \Delta_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_4, ? \mathbf{F}_3} \end{array} ? \qquad \rightarrow \qquad \begin{array}{c} \frac{\overline{\mathbf{h}_1} : \vdash \{\Delta_2, \mathbf{F}_3\} : \Delta_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2, \mathbf{F}_3, \mathbf{F}_W\} : \Delta_4} \end{array} \overset{\mathbf{ax}}{\underset{\bullet}{\mathbf{h}_1} : \vdash \{\Delta_2, \mathbf{F}_W\} : \Delta_4, ? \mathbf{F}_3} \overset{\mathbf{IH}}{\underset{\bullet}{\mathbf{h}_1} : \vdash \{\Delta_2, \mathbf{F}_W\} : \Delta_4, ? \mathbf{F}_3} \end{aligned} ?$$

• Case(s) rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{f}_3, \mathbf{f}_4, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{f}_3 \$ \mathbf{f}_4} \quad \$ \qquad \rightarrow \qquad \frac{\frac{\overline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{f}_3, \mathbf{f}_4}}{\mathbf{h}_1 : \vdash \{\Delta_2, \mathbf{f}_W\} : \Delta_5, \mathbf{f}_3, \mathbf{f}_4}} \quad \underset{\$}{\text{III}} \quad \underset{\bullet}{\text{III}} \quad \\ \frac{\bullet \mathbf{h}_1 : \vdash \{\Delta_2, \mathbf{f}_W\} : \Delta_5, \mathbf{f}_3 \$ \mathbf{f}_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2, \mathbf{f}_W\} : \Delta_5, \mathbf{f}_3 \$ \mathbf{f}_4}} \quad \$ \end{array}$$

• Case(s) rule &

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \underbrace{\frac{\overline{\mathbf{h}_1} : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3}{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\}, \mathbf{F}_W\} : \Delta_5, \mathbf{F}_3} ^{\mathsf{ax}} \\ \underbrace{\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4}{\mathbf{h}_1 : \vdash \{\Delta_2\}, \mathbf{F}_W\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4}}_{\bullet \mathsf{b}_1 : \vdash \{\Delta_2\}, \mathbf{F}_W\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4}$$

• Case(s) rule  $\oplus_B$ 

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{h}_4, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{f}_4}}{\mathbf{h}_1 : \vdash \{\Delta_2, \mathbf{f}_W\} : \Delta_5, \mathbf{f}_4} \ _{\mathbf{1H}}^{\mathbf{ax}} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_2, \mathbf{f}_W\} : \Delta_5, \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_B$$

• Case(s) rule  $\oplus_A$ 

$$\frac{\underset{\bullet}{\mathbf{h}_1} : \vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5}{\underset{\bullet}{\mathbf{h}_1} : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4} \ \oplus_A \qquad \rightarrow \qquad \frac{\frac{\overset{\bullet}{\mathbf{h}_1} : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}{\underset{\bullet}{\mathbf{h}_1} : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3} \overset{\mathtt{ax}}{\mathsf{IH}}}{\underset{\bullet}{\mathbf{h}_1} : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}} \ \oplus_A$$

• Case(s) rule ⊥

• Case(s) rule ⊤

• Case(s) rule  $I_3$ 

$$\overline{\bullet_{\mathtt{h}_1} : \vdash \{\Delta_2\} : p(\mathtt{n}_3), \, \hat{\ }(\mathtt{n}_3)} \quad I_3 \qquad \rightarrow \qquad \overline{\bullet_{\mathtt{h}_1} : \vdash \{\Delta_2, \mathtt{F}_W\} : p(\mathtt{n}_3), \, \hat{\ }(\mathtt{n}_3)} \quad I_3$$

 $\bullet$  Case(s) rule  $\otimes$ 

$$\frac{\mathtt{h}_1 : \vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_6}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4} \quad \otimes \qquad \rightarrow \qquad \frac{\frac{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3} \quad \mathtt{IH}}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{IH}}{\Longrightarrow} \quad \frac{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{IH}}{\Longrightarrow} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{IH}}{\Longrightarrow} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{IH}}{\Longrightarrow} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4}{\mathtt{h}_1 : \vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_4,$$

• Case(s) rule  $I_1$ 

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_3, p(\mathbf{n}_2)\} : \widehat{\ } (\mathbf{n}_2)} \quad I_1 \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_3, \mathbf{F}_W, p(\mathbf{n}_2)\} : \widehat{\ } (\mathbf{n}_2)} \quad I_1$$

• Case(s) rule  $I_2$ 

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_3, \widehat{\ \ } (\mathbf{n}_2)\} : p(\mathbf{n}_2)} \quad I_2 \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_3, \mathbf{F}_W, \widehat{\ \ } (\mathbf{n}_2)\} : p(\mathbf{n}_2)} \quad I_2$$

• Case(s) rule  $?_C$ 

$$\frac{\mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : \mathbf{F}_3, \Delta_2}{\bullet \mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : \Delta_2} \ ?_C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \{\Delta_4, \mathbf{F}_3\} : \Delta_2, \mathbf{F}_3}}{\mathbf{h}_1 : \vdash \{\Delta_4, \mathbf{F}_3, \mathbf{F}_W\} : \Delta_2, \mathbf{F}_3} \overset{\mathsf{IR}}{} ?_C} ?_C$$

#### 2 Contraction on the classical context

• Case(s) rule 1

$$\frac{}{\bullet \mathtt{h}_1 : \vdash \{\Delta_3, \mathtt{F}_2, \mathtt{F}_2\} : 1} \quad 1 \qquad \rightarrow \qquad \frac{}{\bullet \mathtt{h}_1 : \vdash \{\Delta_3, \mathtt{F}_2\} : 1} \quad 1$$

• Case(s) rule!

$$\frac{\underline{h_1 :\vdash \{F_3,F_3,\Delta_4\} : F_2}}{\bullet \underline{h_1 :\vdash \{\Delta_4,F_3,F_3\} :! F_2}} \ ! \qquad \rightarrow \qquad \frac{\overline{\underline{h_1 :\vdash \{\Delta_4,F_3,F_3\} : F_2}}}{\underline{\underline{h_1 :\vdash \{\Delta_4,F_3\} :! F_2}}} \overset{ax}{\vdash \underline{H}}$$

• Case(s) rule?

$$\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\mathbf{F}_2, \mathbf{F}_4, \mathbf{F}_4, \Delta_5\} : \Delta_3} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_4, \mathbf{F}_4\} : \Delta_3, ?\mathbf{F}_2 \end{array} \ ? \qquad \rightarrow \qquad \begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_2, \mathbf{F}_4, \mathbf{F}_4\} : \Delta_3}{\mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_2, \mathbf{F}_4\} : \Delta_3} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_4\} : \Delta_3, ?\mathbf{F}_2 \end{array} \ \begin{array}{c} \mathbf{ax} \\ \mathbf{H}_1 : \vdash \{\Delta_5, \mathbf{F}_2, \mathbf{F}_4\} : \Delta_3 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_4\} : \Delta_3, ?\mathbf{F}_2 \end{array} \ ? \end{array}$$

• Case(s) rule \$

$$\frac{\mathbf{h}_1 : \vdash \{\mathtt{F}_5, \mathtt{F}_5, \Delta_6\} : \mathtt{F}_2, \mathtt{F}_3, \Delta_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3} \quad \$ \qquad \rightarrow \qquad \underbrace{\frac{\mathbf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3}{\mathbf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3} \quad \overset{ax}{\sharp} \quad \\ \underbrace{\frac{\mathbf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3}{\bullet \mathbf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3}}_{\bullet \mathsf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3} \quad \overset{ax}{\sharp} \quad \\ \underbrace{\mathsf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3}_{\bullet \mathsf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3} \quad \overset{\mathsf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3\}}{\bullet \mathsf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3\}} \quad \mathsf{h}_1 : \vdash \mathsf{h}_2 : \mathsf{h}_3 : \mathsf{h}_3$$

• Case(s) rule &

• Case(s) rule  $\oplus_B$ 

$$\frac{ \begin{smallmatrix} \mathbf{h}_1 : \vdash \{\mathtt{F}_5, \mathtt{F}_5, \Delta_6\} : \mathtt{F}_3, \Delta_4 \\ \bullet \mathtt{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3 \end{smallmatrix}}{ \begin{smallmatrix} \mathbf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_3 \\ \bullet \mathtt{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3 \end{smallmatrix}} \overset{\mathsf{ax}}{} \underbrace{ \begin{smallmatrix} \mathbf{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_3 \\ \bullet \mathtt{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3 \end{smallmatrix}}_{\oplus B}$$

• Case(s) rule  $\oplus_A$ 

$$\frac{ \mathbf{h}_1 : \vdash \{\mathbf{F}_5, \mathbf{F}_5, \Delta_6\} : \mathbf{F}_2, \Delta_4 }{ \mathbf{e} \mathbf{h}_1 : \vdash \{\Delta_6, \mathbf{F}_5, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3 } \ \oplus_A \qquad \rightarrow \qquad \frac{ \mathbf{h}_1 : \vdash \{\Delta_6, \mathbf{F}_5, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_2 }{ \mathbf{h}_1 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3 } \ \oplus_A \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3 \ \oplus_A$$

 $\bullet$  Case(s) rule  $\bot$ 

$$\begin{array}{c} \underline{h_1 :\vdash \{\mathtt{F}_3,\mathtt{F}_3,\Delta_4\} : \Delta_2} \\ \underline{\bullet h_1 :\vdash \{\Delta_4,\mathtt{F}_3,\mathtt{F}_3\} : \bot,\Delta_2} \end{array} \ \bot \qquad \rightarrow \qquad \begin{array}{c} \overline{h_1 :\vdash \{\Delta_4,\mathtt{F}_3,\mathtt{F}_3\} : \Delta_2} \\ \underline{h_1 :\vdash \{\Delta_4,\mathtt{F}_3\} : \Delta_2} \\ \underline{\bullet h_1 :\vdash \{\Delta_4,\mathtt{F}_3\} : \Delta_2} \end{array} \begin{array}{c} \mathtt{IH} \\ \underline{\bullet h_1 :\vdash \{\Delta_4,\mathtt{F}_3\} : \Delta_2} \end{array} \end{array}$$

• Case(s) rule ⊤

• Case(s) rule  $I_3$ 

• Case(s) rule  $\otimes$ 

$$\frac{\mathbf{h}_{1} : \vdash \{F_{6}, F_{6}, \Delta_{7}\} : F_{2}, \Delta_{4} \quad \mathbf{h}_{1} : \vdash \{F_{6}, F_{6}, \Delta_{7}\} : F_{3}, \Delta_{5}}{\bullet \mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}, F_{6}\} : \Delta_{4}, \Delta_{5}, F_{2} \otimes F_{3}} \otimes \\ \frac{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{4}, F_{2}}{\bullet \mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{4}, F_{2}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}, F_{6}\} : \Delta_{5}, F_{3}}_{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}}_{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}}_{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}}_{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}}_{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}, A_{5}, F_{5}\}}_{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}, A_{5}, F_{5}\}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}, A_{5}, F_{5}\}}_{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{3}, A_{5}, F_{5}\}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{5}, A_{5}, F_{5}\}}_{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{5}, A_{5}, F_{5}\}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{5}, A_{5}, F_{5}\}}_{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{5}, A_{5}, F_{5}\}} \overset{ax}{\coprod} \underbrace{\mathbf{h}_{1} : \vdash \{\Delta_{7}, F_{6}\} : \Delta_{5}, F_{5}, A_{5}, F_{5}, A_{5}, F_{5}, A_{5}, F_{5}, A_{5}, F_{5}, A_{5}, F_{5}, A_{5}, A_{5}, F_{5}, A_{5}, A_{5},$$

• Case(s) rule  $I_1$ 

$$\overline{\bullet_{h_1} : \vdash \{\Delta_3, p(\mathtt{n}_2), p(\mathtt{n}_2)\} : \widehat{\phantom{a}}(\mathtt{n}_2)} \quad I_1 \qquad \rightarrow \qquad \overline{\bullet_{h_1} : \vdash \{\Delta_3, p(\mathtt{n}_2)\} : \widehat{\phantom{a}}(\mathtt{n}_2)} \quad I_1$$

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{(\Delta_4, p(\mathbf{n}_2)), \mathbf{f}_3, \mathbf{f}_3\} : \hat{\ }(\mathbf{n}_2)} \quad I_1 \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_4, \mathbf{f}_3, p(\mathbf{n}_2)\} : \hat{\ }(\mathbf{n}_2)} \quad I_1$$

• Case(s) rule  $I_2$ 

• Case(s) rule  $?_C$ 

$$\frac{\mathtt{h}_1 : \vdash \{\mathtt{F}_3,\mathtt{F}_3,\Delta_4\} : \mathtt{F}_3,\Delta_2}{\bullet \mathtt{h}_1 : \vdash \{\Delta_4,\mathtt{F}_3,\mathtt{F}_3\} : \Delta_2} \ ?_C \qquad \rightarrow \qquad \frac{\frac{\mathtt{h}_1 : \vdash \{\Delta_4,\mathtt{F}_3,\mathtt{F}_3\} : \Delta_2,\mathtt{F}_3}{\mathtt{h}_1 : \vdash \{\Delta_4,\mathtt{F}_3\} : \Delta_2,\mathtt{F}_3}} \\ \bullet \mathtt{h}_1 : \vdash \{\Delta_4,\mathtt{F}_3\} : \Delta_2} \ ?_C$$

$$\begin{array}{l} \underline{\mathbf{h}_1 : \vdash \{\mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_4, \Delta_5\} : \mathbf{F}_3, \Delta_2}} \\ \underline{\bullet \mathbf{h}_1 : \vdash \{(\mathbf{F}_3, \Delta_5), \mathbf{F}_4, \mathbf{F}_4\} : \Delta_2}} \end{array} ? C \qquad \rightarrow \qquad \begin{array}{l} \overline{\mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_4\} : \Delta_2, \mathbf{F}_3}} \\ \underline{\mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_3, \mathbf{F}_4\} : \Delta_2, \mathbf{F}_3}} \\ \underline{\bullet \mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_3, \mathbf{F}_4\} : \Delta_2}} \end{array} ? C \\ \end{array} \end{cases} ? C$$

## 3 Measure of derivations

• Case(s) rule 1

• Case(s) rule!

• Case(s) rule?

$$\begin{array}{c} \underline{h_1 : \vdash \{F_3, \Delta_2\} : \Delta_4} \\ \bullet h_1 : \vdash \{\Delta_2\} : \Delta_4, ?F_3 \end{array} ? \qquad \rightarrow \qquad \begin{array}{c} \overline{h_1 : \vdash \{\Delta_2, F_3\} : \Delta_4} \\ \bullet h_1 : \vdash \{\Delta_2, F_3\} : \Delta_4 \end{array} \underset{1H}{\text{in}} \\ \bullet h_1 : \vdash \{\Delta_2\} : \Delta_4, ?F_3 \end{cases} ?$$

• Case(s) rule \$

$$\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \mathsf{F}_3, \mathsf{F}_4, \Delta_5} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3 \$ \mathsf{F}_4} \end{array} \ \, \$ \qquad \rightarrow \qquad \begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{ax}}{\mathsf{IH}} \\ \bullet \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3 \$ \mathsf{F}_4}} \ \, \$ \end{array} \ \, \xrightarrow{\bullet} \ \, \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3 \$ \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{ax}}{\mathsf{IH}}}_{\mathsf{IH}} \\ \bullet \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3 \$ \mathsf{F}_4}} \ \, \$ \end{array} \ \, \xrightarrow{\bullet} \ \, \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{IH}}{\mathsf{IH}}}_{\mathsf{IH}} \\ \bullet \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \times \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{IH}}{\mathsf{IH}}}_{\mathsf{IH}} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{IH}}{\mathsf{IH}}}_{\mathsf{IH}} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{IH}}{\mathsf{IH}}}_{\mathsf{IH}} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{IH}}{\mathsf{IH}}}_{\mathsf{IH}} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{IH}}{\mathsf{IH}}}_{\mathsf{IH}} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{IH}}{\mathsf{IH}}_{\mathsf{IH}} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{IH}}{\mathsf{IH}}_{\mathsf{IH}} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4, \mathsf{F}_4} \\ \bullet \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \end{array} \ \, \overset{\mathrm{IH}}{\mathsf{IH}}_{\mathsf{IH}} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4, \mathsf{F}_4} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4} \\ \bullet \underbrace{\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf$$

• Case(s) rule &

$$\frac{\mathtt{h}_1 : \vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4} \quad \& \qquad \rightarrow \qquad \underbrace{\frac{\overline{\mathtt{h}}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}}_{\bullet \bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4} \quad \underbrace{\frac{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4} \quad \underbrace{\frac{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}}_{\bullet \& \bullet \bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}$$

• Case(s) rule  $\oplus_B$ 

$$\frac{ \begin{smallmatrix} \mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{F}_4, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4 \end{smallmatrix}}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_4} \oplus_B} \quad \rightarrow \quad \underbrace{\frac{ \begin{smallmatrix} \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_4 \end{smallmatrix}}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4 \end{smallmatrix}} \oplus_B$$

• Case(s) rule  $\oplus_A$ 

$$\begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{F}_3, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4 \end{array}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \oplus_A \qquad \rightarrow \qquad \begin{array}{c} \overline{\begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \end{array}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \end{array}_{\bullet} \oplus_A$$

• Case(s) rule  $\perp$ 

• Case(s) rule ⊤

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \top, \Delta_3} \quad \top \qquad \rightarrow \qquad \frac{}{\bullet \bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_3, \top} \quad \top$$

• Case(s) rule  $I_3$ 

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : p(\mathbf{n}_3), \widehat{\phantom{a}}(\mathbf{n}_3)} \quad I_3 \qquad \rightarrow \qquad \frac{}{\bullet \bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : p(\mathbf{n}_3), \widehat{\phantom{a}}(\mathbf{n}_3)} \quad I_3$$

• Case(s) rule  $\otimes$ 

$$\frac{\mathtt{h}_1 : \vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_6}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4} \quad \otimes \quad \rightarrow \quad \underbrace{\frac{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3} \quad \overset{\mathtt{ax}}{\mathtt{l}_1} \quad \underbrace{\frac{\mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{ax}}{\mathtt{l}_1} \quad \overset{\mathtt{h}_2}{\mathtt{l}_2} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4} \quad \overset{\mathtt{ax}}{\mathtt{l}_1} \quad \overset{\mathtt{h}_2}{\mathtt{l}_2} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4} \quad \overset{\mathtt{h}_3}{\mathtt{h}_1} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4} \quad \overset{\mathtt{h}_3}{\mathtt{h}_1} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4} \quad \overset{\mathtt{h}_3}{\mathtt{h}_1} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4} \quad \overset{\mathtt{h}_3}{\mathtt{h}_1} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathtt{h}_4} \quad \overset{\mathtt{h}_3}{\mathtt{h}_1} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{h}_4}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{h}_4} \quad \overset{\mathtt{h}_3}{\mathtt{h}_1} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{h}_4}_{\bullet \mathtt{h}_1 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{h}_4} \quad \overset{\mathtt{h}_3}{\mathtt{h}_1} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{h}_4, \mathtt{h}_4}_{\bullet \mathtt{h}_1} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{h}_4, \mathtt{h}_4}_{\bullet \mathtt{h}_1} \quad \overset{\mathtt{h}_4}{\mathtt{h}_4} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{h}_4, \mathtt{h}_4}_{\bullet \mathtt{h}_4} : \underbrace{\mathtt{h}_3 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{h}_4, \mathtt{h}_4, \mathtt{h}_4}_{\bullet \mathtt{h}_4} : \underbrace{\mathtt{h}_3 : \bot \mathtt{h}_4, \mathtt{h}_4}_{\bullet \mathtt{h}_4} : \underbrace{\mathtt{h}_3 : \bot \mathtt{h}_4, \mathtt{h}_4}_{\bullet \mathtt{h}_4} : \underbrace{\mathtt{h}_3 : \bot \mathtt{h}_4, \mathtt{h}_4}_{\bullet \mathtt{h}_4} : \underbrace{\mathtt{h}_4 : \bot \mathtt{h}_4, \mathtt{h}_4, \mathtt{h}_4, \mathtt{h}_4}_{\bullet \mathtt{h}_4} : \underbrace{\mathtt{h}_4 : \bot \mathtt{h}_4, \mathtt{h}_4, \mathtt{h}_4, \mathtt{h}_4}_{\bullet \mathtt{h}_4} : \underbrace{\mathtt{h}_4 : \mathtt{h}_4, \mathtt{h}_4,$$

• Case(s) rule  $I_1$ 

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_3, p(\mathbf{n}_2)\} : \widehat{\ } (\mathbf{n}_2)} \quad I_1 \qquad \rightarrow \qquad \frac{}{\bullet \bullet \mathbf{h}_1 : \vdash \{\Delta_3, p(\mathbf{n}_2)\} : \widehat{\ } (\mathbf{n}_2)} \quad I_1$$

• Case(s) rule  $I_2$ 

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_3, \widehat{\ } (\mathbf{n}_2)\} : p(\mathbf{n}_2)} \quad I_2 \qquad \rightarrow \qquad \frac{}{\bullet \bullet \mathbf{h}_1 : \vdash \{\Delta_3, \widehat{\ } (\mathbf{n}_2)\} : p(\mathbf{n}_2)} \quad I_2$$

• Case(s) rule  $?_C$ 

$$\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : \mathbf{F}_3, \Delta_2}}_{\bullet \mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : \Delta_2} \ ?_C \\ & \xrightarrow{\bullet} \underline{\mathbf{h}_1 : \vdash \{\Delta_4, \mathbf{F}_3\} : \Delta_2, \mathbf{F}_3}}_{\bullet \bullet \mathbf{h}_1 : \vdash \{\Delta_4, \mathbf{F}_3\} : \Delta_2} \overset{\mathsf{ax}}{} \underbrace{\mathbf{h}_1 : \vdash \{\Delta_4, \mathbf{F}_3\} : \Delta_2, \mathbf{F}_3}}_{?_C} \end{array}$$

## 4 Invertibility of Rules

#### 4.1 Status of 1: : Invertible

 $\bullet \;$  Case rule  ${\bf 1}$ 

 $\frac{}{\bullet \mathbf{h}_1 \, : \vdash \, \{\Delta_2\} \, : \, \mathbf{1}} \quad \, \mathbf{1} \qquad \, \rightarrow \qquad \, \mathrm{trivial}$ 

- Case rule!
- Case rule?
- Case rule \$
- Case rule &
- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- $\bullet$  Case rule  $\bot$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$
- $\bullet\,$  Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{\mathbf{F}_2, \Delta_3\} : \mathbf{1}, \mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash \{\mathbf{F}_2, \Delta_3\} : \mathbf{1}} \end{cases} ?_C \qquad \rightarrow \qquad \text{trivial}$$

## 4.2 Status of !: : Invertible

- $\bullet \;$  Case rule  ${\bf 1}$
- Case rule!

- Case rule?
- Case rule \$
- Case rule &
- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- $\bullet$  Case rule  $\bot$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$
- Case rule  $I_1$
- Case rule  $I_2$
- $\bullet$  Case rule  $?_C$

$$\frac{\mathtt{h}_2 : \vdash \{\mathtt{F}_3, \Delta_4\} : \mathtt{F}_3, !\mathtt{F}_1}{\bullet \mathtt{h}_2 : \vdash \{\mathtt{F}_3, \Delta_4\} : !\mathtt{F}_1} ?_C \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_2 : \vdash \{\Delta_4, \mathtt{F}_3\} : \mathtt{F}_3, !\mathtt{F}_1}}{\bullet \mathtt{h}_2 : \vdash \{\Delta_4, \mathtt{F}_3\} : \mathtt{F}_1} \frac{\mathtt{ax}}{?_C} \\ \frac{\bullet \mathtt{h}_2 : \vdash \{\Delta_4, \mathtt{F}_3\} : !\mathtt{F}_1}{\bullet \mathtt{h}_2 : \vdash \{\Delta_4, \mathtt{F}_3\} : \mathtt{F}_1} \frac{\mathtt{ax}}{WB}$$

#### 4.3 Status of ?: : Invertible

- ullet Case rule 1
- Case rule!

• Case rule?

• Case rule \$

$$\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_4, \mathbf{F}_5, \Delta_6, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : (\Delta_6, ?\mathbf{F}_1), \mathbf{F}_4 \$\mathbf{F}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \mathbf{F}_4, \mathbf{F}_5}}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \mathbf{F}_4 \$\mathbf{F}_5} \quad \$$$

• Case rule &

$$\frac{ \begin{smallmatrix} \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathsf{F}_4, \Delta_6, ?\mathsf{F}_1 & \mathsf{h}_2 : \vdash \{\Delta_3\} : \mathsf{F}_5, \Delta_6, ?\mathsf{F}_1 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3\} : (\Delta_6, ?\mathsf{F}_1), \mathsf{F}_4 \& \mathsf{F}_5 \end{smallmatrix}} \quad \& \qquad \rightarrow \qquad \frac{ \begin{smallmatrix} \mathbf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \end{smallmatrix}} \quad \underset{\bullet}{\mathsf{ax/ind}} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_2} \quad & \underset{\bullet}{\mathsf{h}_2} : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_1\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{F}_5 \\ \bullet \mathsf{h}_2 : \vdash \{\Delta_3, \mathsf{F}_4\} : \Delta_6, \mathsf{F}_4 \& \mathsf{h}_2 : \bot \mathsf{h}_$$

• Case rule  $\oplus_B$ 

$$\frac{\mathtt{h}_2 : \vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_6, ?\mathtt{F}_1}{\bullet \mathtt{h}_2 : \vdash \{\Delta_3\} : (\Delta_6, ?\mathtt{F}_1), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_2 : \vdash \{\Delta_3, \mathtt{F}_1\} : \Delta_6, \mathtt{F}_5}}{\bullet \mathtt{h}_2 : \vdash \{\Delta_3, \mathtt{F}_1\} : \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_B$$

• Case rule  $\oplus_A$ 

$$\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_4, \Delta_6, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : (\Delta_6, ?\mathbf{F}_1), \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \mathbf{F}_4}}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_A$$

• Case rule  $\perp$ 

$$\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_4, ?\mathtt{F_1}}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \bot, \Delta_4, ?\mathtt{F_1}} \ \bot \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \{\Delta_3, \mathtt{F_1}\} : \Delta_4}}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathtt{F_1}\} : \Delta_4, \bot} \ \bot$$

 $\bullet$  Case rule  $\top$ 

- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$

$$\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_4, \Delta_6, ?\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_5, \Delta_7}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : (\Delta_6, ?\mathbf{F}_1), \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5} \otimes \longrightarrow \underbrace{\frac{\mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_4, \Delta_6 \quad \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_5, \Delta_7, ?\mathbf{F}_1}} \otimes \underbrace{\longrightarrow \underbrace{\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, (\Delta_7, ?\mathbf{F}_1), \mathbf{F}_4 \otimes \mathbf{F}_5}} \otimes \underbrace{\longrightarrow \underbrace{\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5}} \underbrace{\otimes}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5}}^{\bullet \mathbf{ax}} \otimes \underbrace{\longrightarrow \underbrace{\underbrace{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{F}_1\} : \Delta_6, \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5}} \underbrace{\otimes}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5}}^{\bullet \mathbf{ax}} \otimes \underbrace{\longrightarrow \underbrace{\underbrace{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{h}_4}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \mathbf{h}_4, \mathbf{h}_4}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \Delta_7, \mathbf{h}_4 \otimes \mathbf{h}_5}}^{\bullet \mathbf{ax}} \otimes \underbrace{\longrightarrow \underbrace{\mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \mathbf{h}_4}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \Delta_7, \mathbf{h}_4 \otimes \mathbf{h}_5}}^{\bullet \mathbf{ax}} \otimes \underbrace{\longrightarrow \underbrace{\mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \mathbf{h}_4}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \Delta_7, \mathbf{h}_4 \otimes \mathbf{h}_5}}^{\bullet \mathbf{ax}} \otimes \underbrace{\longrightarrow \underbrace{\mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \Delta_7, \mathbf{h}_4 \otimes \mathbf{h}_5}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \Delta_7, \mathbf{h}_4 \otimes \mathbf{h}_5}^{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \Delta_7, \mathbf{h}_4 \otimes \mathbf{h}_5}^{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \Delta_7, \mathbf{h}_4 \otimes \mathbf{h}_5}^{\bullet \mathbf{h}_2 : \vdash \{\Delta_3, \mathbf{h}_1\} : \Delta_6, \Delta_7, \mathbf{h}_4 \otimes \mathbf{h}_5}}$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{lll} \frac{\mathbf{h}_2 : \vdash \{\mathbf{F}_4, \Delta_5\} : \mathbf{F}_4, \Delta_3, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \{\mathbf{F}_4, \Delta_5\} : \Delta_3, ?\mathbf{F}_1} & ?_C & \rightarrow & & \frac{\mathbf{h}_2 : \vdash \{\Delta_5, \mathbf{F}_1, \mathbf{F}_4\} : \Delta_3, \mathbf{F}_4}{\bullet \mathbf{h}_2 : \vdash \{\Delta_5, \mathbf{F}_1, \mathbf{F}_4\} : \Delta_3} & \\ & ?_C & & \\ \end{array} \right.$$

## 4.4 Status of \$: : Invertible

- Case rule 1
- Case rule !
- Case rule?

$$\frac{\mathsf{h}_3 : \vdash \{\mathsf{F}_5, \Delta_4\} : \Delta_6, \mathsf{F}_1 \$ \mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : (\Delta_6, \mathsf{F}_1 \$ \mathsf{F}_2), ? \mathsf{F}_5} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathsf{h}_3 : \vdash \{\Delta_4, \mathsf{F}_5\} : \Delta_6, \mathsf{F}_1, \mathsf{F}_2}}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_6, \mathsf{F}_1, \mathsf{F}_2, ? \mathsf{F}_5} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \$ \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \$ \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \$ \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6}} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{Ax/ind}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{Ax/ind}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{Ax/ind}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \Delta_7, \mathbf{Ax/ind}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_3 : \Delta_7, \mathbf{Ax/ind}}^{\bullet \mathbf{ax/ind}} \quad \underbrace{\mathbf{ax/ind}}^{\bullet$$

• Case rule  $\oplus_B$ 

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{f}_6, \Delta_7, \mathbf{f}_1\$\mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{f}_1\$\mathbf{f}_2), \mathbf{f}_5 \oplus \mathbf{f}_6} \ \oplus_B \qquad \rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_6}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_5 \oplus \mathbf{f}_6} \ \oplus_B$$

• Case rule  $\oplus_A$ 

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{f}_5, \Delta_7, \mathbf{f}_1\$\mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{f}_1\$\mathbf{f}_2), \mathbf{f}_5 \oplus \mathbf{f}_6} \quad \oplus_A \qquad \rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_5}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_5 \oplus \mathbf{f}_6} \quad \oplus_A$$

• Case rule  $\perp$ 

$$\frac{\mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1 \$ \mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta, \Delta_5, \mathsf{F}_1 \$ \mathsf{F}_2} \quad \bot \qquad \rightarrow \qquad \frac{\mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1, \mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1, \mathsf{F}_2, \bot} \overset{\mathsf{ax/ind}}{\perp}$$

• Case rule  $\top$ 

$$\frac{}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \top, \Delta_5, \mathsf{F}_1 \$ \mathsf{F}_2} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1, \mathsf{F}_2, \top} \ \top$$

- Case rule  $I_3$
- Case rule  $\otimes$

$$\begin{array}{c} \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathit{F}_5, \Delta_7, \mathit{F}_1\$\mathit{F}_2 \quad \mathit{h}_3 : \vdash \{\Delta_4\} : \mathit{F}_6, \Delta_8}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_1\$\mathit{F}_2), \Delta_8, \mathit{F}_5 \otimes \mathit{F}_6} \end{array} \otimes \\ \rightarrow \begin{array}{c} \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_1, \mathit{F}_2, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}} & \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathit{F}_6}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathit{F}_1, \mathit{F}_2, \mathit{F}_5 \otimes \mathit{F}_6} \end{array} \otimes \\ \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathit{F}_5, \Delta_7, \mathit{h}_3 : \vdash \{\Delta_4\} : \mathit{F}_6, \Delta_8, \mathit{F}_1\$\mathit{F}_2}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathit{F}_1, \mathit{F}_2, \mathit{F}_5 \otimes \mathit{F}_6}} \end{array} \otimes \\ \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathit{F}_5, \Delta_7, \mathit{h}_3 : \vdash \{\Delta_4\} : \mathit{F}_6, \Delta_8, \mathit{F}_1\$\mathit{F}_2}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F}_5, \mathit{F}_5, \mathit{F}_5}} & \underline{\mathbf{ax}/\mathit{ind}}}_{\bullet \mathit{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathit{F}_5, \mathit{F$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\frac{\mathbf{h}_3 : \vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_5, \Delta_4, \mathtt{F}_1\$\mathtt{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_1\$\mathtt{F}_2} ?_C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_1, \mathtt{F}_2} \overset{\mathsf{ax/ind}}{?_C} ?_C$$

## 4.5 Status of &: (Left Premise): Invertible

- ullet Case rule 1
- Case rule!
- Case rule?

$$\frac{\mathtt{h}_3 : \vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), ?\mathtt{F}_5} ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \mathtt{F}_1}}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_1, ?\mathtt{F}_5} \overset{ax/ind}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6}} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5, \mathbf{F}_6}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \$^{\mathrm{ax/ind}} \quad \$^{\mathrm{ax/ind}}$$

• Case rule &

• Case rule  $\oplus_B$ 

$$\frac{\mathtt{h}_3 : \vdash \{\Delta_4\} : \mathtt{f}_6, \Delta_7, \mathtt{f}_1 \& \mathtt{f}_2}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathtt{f}_1 \& \mathtt{f}_2), \mathtt{f}_5 \oplus \mathtt{f}_6} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathtt{f}_1, \mathtt{f}_6}}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathtt{f}_1, \mathtt{f}_5 \oplus \mathtt{f}_6} \ \oplus_B$$

• Case rule  $\oplus_A$ 

$$\frac{\mathtt{h}_3 : \vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1, \mathtt{F}_5}}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \oplus \mathtt{F}_6} \ \oplus_A$$

 $\bullet$  Case rule  $\bot$ 

$$\frac{\mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1 \& \mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \bot, \Delta_5, \mathsf{F}_1 \& \mathsf{F}_2} \quad \bot \qquad \rightarrow \qquad \frac{\overline{\mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1}}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1, \bot} \stackrel{\mathsf{ax/ind}}{\perp}$$

 $\bullet$  Case rule  $\top$ 

$$\frac{}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \top, \Delta_5, \mathsf{F}_1 \& \mathsf{F}_2} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1, \top} \ \top$$

- Case rule  $I_3$
- Case rule  $\otimes$

$$\begin{array}{c} \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \& \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_8 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \& \mathbf{F}_2), \Delta_8, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \otimes \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5} \quad \mathbf{ax/ind} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_6} \quad \otimes \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \& \mathbf{F}_2), \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \otimes \\ \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_8, \mathbf{F}_1 \& \mathbf{F}_2} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, (\Delta_8, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \otimes \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_5} \quad \mathbf{ax} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_1, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \overset{\mathbf{ax}/ind}{\otimes} \end{array} \quad \otimes \\ \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6}} \quad \otimes \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_5} \quad \mathbf{ax} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_1, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \overset{\mathbf{ax}/ind}{\otimes} \\ \otimes \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \overset{\mathbf{ax}/ind}{\otimes} \\ \end{array}$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\frac{\mathbf{h}_3 : \vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_5, \Delta_4, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_1 \& \mathtt{F}_2} \quad ?_C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_1, \mathtt{F}_5}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_1} \quad \overset{\mathsf{ax/inc}}{?}_C$$

### 4.6 Status of & (Right Premise): : Invertible

- Case rule 1
- Case rule!
- Case rule?

$$\frac{\mathtt{h}_3 : \vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), ?\mathtt{F}_5} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \mathtt{F}_2}}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_2, ?\mathtt{F}_5} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_5 \$ \mathbf{F}_6} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \$ \mathbf{F}_6} \quad \$$$

• Case rule &

• Case rule  $\oplus_B$ 

$$\frac{\mathtt{h}_3 : \vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_6}}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6} \ \oplus_B$$

• Case rule  $\oplus_A$ 

$$\frac{\mathtt{h}_3 : \vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5}}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6} \ \oplus_A$$

 $\bullet$  Case rule  $\bot$ 

 $\bullet$  Case rule  $\top$ 

• Case rule  $I_3$ 

#### • Case rule $\otimes$

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \& \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_8}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \& \mathbf{F}_2), \Delta_8, \mathbf{F}_5 \otimes \mathbf{F}_6} \otimes \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_2, \mathbf{F}_5} \quad \mathbf{ax/ind} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_6} \quad \mathbf{ax/ind} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_6} \quad \mathbf{ax/ind} \quad \mathbf{a$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\frac{\mathbf{h}_3 : \vdash \{\mathbf{F}_5, \Delta_6\} : \mathbf{F}_5, \Delta_4, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\mathbf{F}_5, \Delta_6\} : \Delta_4, \mathbf{F}_1 \& \mathbf{F}_2} \quad ?_C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_2} \overset{\mathsf{ax/inc}}{?_C}$$

### 4.7 Status of $\oplus_B$ : Non invertible

- Case rule 1
- Case rule!
- Case rule?

$$\frac{\mathsf{h}_3 : \vdash \{\mathsf{F}_5, \Delta_4\} : \Delta_6, \mathsf{F}_1 \oplus \mathsf{F}_2}{\mathsf{e}\mathsf{h}_3 : \vdash \{\Delta_4\} : (\Delta_6, \mathsf{F}_1 \oplus \mathsf{F}_2), ?\mathsf{F}_5} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathsf{h}_3 : \vdash \{\Delta_4, \mathsf{F}_5\} : \Delta_6, \mathsf{F}_2}}{\mathsf{e}\mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_6, \mathsf{F}_2, ?\mathsf{F}_5} \ ?$$

• Case rule \$

$$\frac{\mathsf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \mathsf{F}_6, \Delta_7, \mathsf{F}_1 \oplus \mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathsf{F}_1 \oplus \mathsf{F}_2), \mathsf{F}_5 \$ \mathsf{F}_6} \quad \$ \qquad \rightarrow \qquad \frac{\frac{\mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathsf{F}_2, \mathsf{F}_5, \mathsf{F}_6}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathsf{F}_2, \mathsf{F}_5, \mathsf{F}_6}} \quad \$}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathsf{F}_2, \mathsf{F}_5 \$ \mathsf{F}_6}} \quad \$$$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6} \quad \frac{\mathsf{ax/ind}}{\mathsf{ax/ind}} \quad \frac{\mathsf{a$$

• Case rule  $\oplus_B$ 

• Case rule  $\oplus_A$ 

 $\bullet$  Case rule  $\bot$ 

 $\bullet$  Case rule  $\top$ 

$$\frac{}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \top, \Delta_5, \mathsf{F}_1 \oplus \mathsf{F}_2} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_2, \top} \ \top$$

- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$

$$\begin{array}{c} \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_8} \\ \bullet \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \Delta_8, \mathbf{F}_5 \otimes \mathbf{F}_6} \end{array} \otimes \\ \end{array} \rightarrow \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_2, \mathbf{F}_5} & \underline{\mathbf{ax}/\mathbf{ind}} \\ \bullet \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \Delta_8, \mathbf{F}_5 \otimes \mathbf{F}_6} & \underline{\otimes} \\ \end{array} \\ \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_8, \mathbf{F}_1 \oplus \mathbf{F}_2} \\ \bullet \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_5} & \underline{\mathbf{ax}} & \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2, \mathbf{F}_6} \\ \underline{\bullet} \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_5} & \underline{\mathbf{ax}/\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2, \mathbf{F}_6} \\ \underline{\bullet} \underline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_2, \mathbf{F}_5 \otimes \mathbf{F}_6}} \end{array} \\ \end{array}$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\frac{\mathbf{h}_3 : \vdash \{\mathbf{F}_5, \Delta_6\} : \mathbf{F}_5, \Delta_4, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\mathbf{F}_5, \Delta_6\} : \Delta_4, \mathbf{F}_1 \oplus \mathbf{F}_2} \ ?_C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_2} \ \overset{\mathsf{ax/ind}}{?_C}$$

4.8 Status of  $\oplus_A$ : Non invertible

- Case rule 1
- Case rule!
- Case rule?

$$\frac{\mathtt{h}_3 : \vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : (\Delta_6, \mathtt{F}_1 \oplus \mathtt{F}_2), ?\mathtt{F}_5} \ ? \qquad \rightarrow \qquad \frac{\mathtt{h}_3 : \vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \mathtt{F}_1}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_1, ?\mathtt{F}_5} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{f}_5, \mathbf{f}_6, \Delta_7, \mathbf{f}_1 \oplus \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{f}_1 \oplus \mathbf{f}_2), \mathbf{f}_5\$\mathbf{f}_6} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{f}_1, \mathbf{f}_5, \mathbf{f}_6}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{f}_1, \mathbf{f}_5\$\mathbf{f}_6} \quad \$$$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \qquad \Rightarrow \mathbf{ax/ind} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6}} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6} \quad \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6} \quad$$

• Case rule  $\oplus_B$ 

• Case rule  $\oplus_A$ 

• Case rule  $\perp$ 

$$\frac{\mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1 \oplus \mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \bot, \Delta_5, \mathsf{F}_1 \oplus \mathsf{F}_2} \ \bot \qquad \rightarrow \qquad \frac{\overline{\mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1}}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathsf{F}_1, \bot} \overset{\mathsf{ax/ind}}{\perp}$$

• Case rule  $\top$ 

$$\frac{}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \top, \Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2} \quad \top \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_5, \mathbf{F}_1, \top} \quad \top$$

- Case rule  $I_3$
- Case rule  $\otimes$

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 :\vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_8}}{\bullet \mathbf{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \Delta_8, \mathbf{F}_5 \otimes \mathbf{F}_6} \end{array} \otimes \\ & \rightarrow \\ \begin{array}{c} \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5} \quad \underline{\mathbf{ax}/\mathrm{ind}} \quad \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_6}} \\ \bullet \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6} \end{array} \otimes \\ \\ \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7 \quad \mathbf{h}_3 :\vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_8, \mathbf{F}_1 \oplus \mathbf{F}_2} \\ \bullet \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \Delta_7, (\Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6} \end{array} \otimes \\ \\ \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6}} \end{array} \otimes \\ \end{array} \rightarrow \\ \begin{array}{c} \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6} \\ \bullet \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6} \end{array} \otimes \\ \\ \underline{\mathbf{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathbf{F}_1, \mathbf{F}_5 \otimes \mathbf{F}_6}} \end{array} \otimes \\ \end{array}$$

• Case rule  $I_1$ 

- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \{\mathbf{F}_5, \Delta_6\} : \mathbf{F}_5, \Delta_4, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\mathbf{F}_5, \Delta_6\} : \Delta_4, \mathbf{F}_1 \oplus \mathbf{F}_2} & ?_C & \rightarrow & & \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_1, \mathbf{F}_5}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_1} & \overset{\mathsf{ax/ind}}{?_C} \\ \end{array}$$

#### 4.9 Status of $\perp$ : Invertible

- $\bullet \;$  Case rule  ${\bf 1}$
- Case rule!
- Case rule?

$$\frac{\mathsf{h}_1 : \vdash \{\mathsf{F}_3, \Delta_2\} : \bot, \Delta_4}{\bullet \mathsf{h}_1 : \vdash \{\Delta_2\} : (\bot, \Delta_4), ?\mathsf{F}_3} \ ? \qquad \rightarrow \qquad \frac{\frac{\mathsf{h}_1 : \vdash \{\Delta_2, \mathsf{F}_3\} : \Delta_4}{\bullet \mathsf{h}_1 : \vdash \{\Delta_2\} : \Delta_4, ?\mathsf{F}_3}}{\bullet \mathsf{h}_1 : \vdash \{\Delta_2\} : \Delta_4, ?\mathsf{F}_3} \ ?$$

• Case rule \$

$$\frac{\mathsf{h}_1 : \vdash \{\Delta_2\} : \bot, \mathsf{F}_3, \mathsf{F}_4, \Delta_5}{\bullet \mathsf{h}_1 : \vdash \{\Delta_2\} : (\bot, \Delta_5), \mathsf{F}_3 \$ \mathsf{F}_4} \quad \$ \qquad \rightarrow \qquad \frac{\mathsf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_4}{\bullet \mathsf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathsf{F}_3 \$ \mathsf{F}_4} \quad \overset{\mathsf{ax/ind}}{\$}$$

• Case rule &

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \bot, \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_2\} : \bot, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : (\bot, \Delta_5), \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3} \quad \text{ax/ind}}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \underbrace{\mathsf{ax/ind}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_4 \& \mathbf{$$

• Case rule  $\oplus_B$ 

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \bot, \mathbf{f}_4, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : (\bot, \Delta_5), \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{f}_4} \ \ \mathrm{ax/ind}}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_B$$

• Case rule  $\oplus_A$ 

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \bot, \mathbf{f}_3, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : (\bot, \Delta_5), \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{f}_3} \ \ \frac{\mathbf{ax/ind}}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{f}_3 \oplus \mathbf{f}_4} \ \ \oplus_A$$

• Case rule  $\perp$ 

• Case rule  $\top$ 

- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \bot, \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{F}_4, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : (\bot, \Delta_5), \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4} \otimes \longrightarrow \frac{\overline{\mathbf{h}_1} : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4} \overset{\text{ax}}{\otimes} \otimes \mathbb{I}_4 : \vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4} \otimes \mathbb{I}_5 \otimes \mathbb{I}_5$$

- $\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_2\} : \bot, \mathbf{F}_4, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, (\bot, \Delta_6), \mathbf{F}_3 \otimes \mathbf{F}_4} \quad \otimes \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \mathbf{F}_3} \quad \overset{\mathrm{ax}}{} \quad \overline{\mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_6, \mathbf{F}_4} \quad \overset{\mathrm{ax/ind}}{\otimes} \quad \otimes \quad \xrightarrow{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4} \quad \otimes \quad \xrightarrow{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4}$
- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{lll} \underbrace{\mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : \bot, \mathbf{F}_3, \Delta_2}_{\bullet \mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : \bot, \Delta_2} & ?_C & \rightarrow & & & & \underbrace{\overline{\mathbf{h}_1 : \vdash \{\Delta_4, \mathbf{F}_3\} : \Delta_2, \mathbf{F}_3}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_4, \mathbf{F}_3\} : \Delta_2} & \overset{\mathrm{ax/ind}}{?_C} \end{array}$$

#### 4.10 Status of $\top$ : Invertible

- Case rule 1
- Case rule!
- Case rule?

$$\frac{\mathtt{h}_1 : \vdash \{\mathtt{F}_3, \Delta_2\} : \top, \Delta_4}{\bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : (\top, \Delta_4), ?\mathtt{F}_3} \ ? \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \top, \mathbf{F}_3, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), \mathbf{F}_3\$\mathbf{F}_4} \end{cases} \$ \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule &

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \top, \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_2\} : \top, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule  $\oplus_B$ 

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \top, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule  $\oplus_A$ 

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \top, \mathbf{F}_3, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A \qquad \rightarrow \qquad \mathsf{trivial}$$

 $\bullet$  Case rule  $\bot$ 

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \top, \Delta_3}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \bot, \top, \Delta_3} \ \bot \qquad \rightarrow \qquad \mathtt{trivial}$$

 $\bullet$  Case rule  $\top$ 

$$\boxed{ \bullet \mathtt{h}_1 : \vdash \{\Delta_2\} : \top, \Delta_3} \quad \top \qquad \rightarrow \qquad \mathtt{trivial}$$

- Case rule  $I_3$
- ullet Case rule  $\otimes$

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \top, \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{F}_4, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4} \quad \otimes \qquad \rightarrow \qquad \mathsf{trivial}$$

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_2\} : \top, \mathbf{F}_4, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : \Delta_5, (\top, \Delta_6), \mathbf{F}_3 \otimes \mathbf{F}_4} \quad \otimes \qquad \rightarrow \qquad \mathsf{trivial}$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{ll} \frac{\mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : \top, \mathbf{F}_3, \Delta_2}{\bullet \mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : \top, \Delta_2} \end{array} ?_C \qquad \rightarrow \qquad \mathsf{trivial} \\$$

## 4.11 Status of $I_3$ : Invertible

- $\bullet$  Case rule 1
- Case rule!
- Case rule?
- Case rule \$
- Case rule &

- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- $\bullet$  Case rule  $\bot$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_2\} : p(\mathbf{n}_3), \, \hat{\ } (\mathbf{n}_3)} \quad I_3 \qquad \to \qquad \mathsf{trivial}$$

- $\bullet$  Case rule  $\otimes$
- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{ll} \frac{\mathbf{h}_2 : \vdash \{\mathbf{F}_3, \Delta_4\} : \mathbf{F}_3, p(\mathbf{n_1}), \, \hat{\ } (\mathbf{n_1})}{\bullet \mathbf{h}_2 : \vdash \{\mathbf{F}_3, \Delta_4\} : p(\mathbf{n_1}), \, \hat{\ } (\mathbf{n_1})} \ ?_C & \rightarrow & \text{trivial} \end{array}$$

## 4.12 Status of $\otimes$ : (Left Premise): Non invertible

- Case rule 1
- Case rule!
- Case rule?

$$\frac{\mathbf{h}_3 : \vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \Delta_7, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : (\Delta_6, \Delta_7, \mathtt{F}_1 \otimes \mathtt{F}_2), ?\mathtt{F}_5} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \mathtt{F}_1}}{\bullet \mathtt{h}_3 : \vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_1, ?\mathtt{F}_5} \overset{\mathsf{ax/ind}}{?}$$

$$\begin{array}{c} \mathbf{h}_3 : \vdash \{\mathsf{F}_5, \Delta_4\} : \Delta_6, \Delta_7, \mathsf{F}_1 \otimes \mathsf{F}_2 \\ \bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : (\Delta_6, \Delta_7, \mathsf{F}_1 \otimes \mathsf{F}_2), ?\mathsf{F}_5 \end{array} ? \qquad \rightarrow \qquad \begin{array}{c} \bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_1, \Delta_6 \end{array} \text{ fail}$$

• Case rule \$

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \mathbf{F}_6, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5\$\mathbf{F}_6} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1} \quad \overset{\mathrm{ax/ind}}{\vdash} \quad \mathbb{E}_{\mathbf{h}_3} : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1} \quad \mathbb{E}_{\mathbf{h}_3} : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1 : \Delta_7, \mathbf{F}_1} = \mathbb{E}_{\mathbf{h}_3} : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1 : \Delta_7, \mathbf{F}_1$$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5} \quad \text{ax/ind}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \& \mathbf{F}_6} \quad \frac{\mathsf{ax/ind}}{\&} \\ \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6} \quad & \quad \Rightarrow \quad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{h}_1 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{h}$$

• Case rule  $\oplus_B$ 

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \oplus \mathbf{F}_6} \oplus_B \\ \\ & \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \oplus \mathbf{F}_6}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \oplus \mathbf{F}_6} \oplus_B \\ \\ & \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \oplus \mathbf{F}_6} \oplus_B \\ \end{array} \rightarrow \begin{array}{c} \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_6}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1} & \text{ax/ind} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \oplus \mathbf{F}_6 & \oplus_B \\ \end{array}$$

• Case rule  $\oplus_A$ 

$$\begin{array}{c} \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus_A \\ & \rightarrow \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5} & \mathrm{ax/ind} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus_A \\ \\ \hline \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \oplus \mathbf{F}_6} \oplus_A \\ \hline \end{array} \rightarrow \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_7, \mathbf{F}_1} & \mathrm{ax/ind} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \oplus \mathbf{F}_6 \end{array} \oplus_A \end{array}$$

• Case rule  $\perp$ 

• Case rule  $\top$ 

- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$

$$\begin{array}{c} \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_9, \Delta_{10} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), (\Delta_9, \Delta_{10}), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \Delta_8 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6 \end{array} \quad \boldsymbol{\bullet} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \Delta_9, \Delta_{10}, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \otimes \mathbf{F}_6$$

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8, \mathsf{F}_1 \otimes \mathsf{F}_2 \quad \mathsf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathsf{F}_1 \otimes \mathsf{F}_2), (\Delta_9, \Delta_{10}), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad \rightarrow \qquad \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8 \quad \mathsf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad \rightarrow \qquad \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8 \quad \mathsf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad \rightarrow \qquad \\ \frac{\mathbf{h}_1 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_6 \quad \mathsf{h}_1 : \vdash \{\Delta_2\} : \mathsf{F}_4, \Delta_7, \Delta_8}{\bullet \mathsf{h}_1 : \vdash \{\Delta_2\} : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), \mathsf{F}_3 \otimes \mathsf{F}_4} \quad \otimes \qquad \rightarrow \qquad \\ \hline \bullet \mathsf{h}_1 : \vdash \{\Delta_2\} : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), \mathsf{F}_3 \otimes \mathsf{F}_4} \quad \otimes \qquad \rightarrow \qquad \\ \hline \bullet \mathsf{h}_1 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_6 \quad \mathsf{h}_1 : \vdash \{\Delta_2\} : \mathsf{F}_4, \Delta_7, \Delta_8 \quad \mathsf{h}_3 \otimes \mathsf{F}_4} \quad \otimes \qquad \rightarrow \qquad \\ \hline \bullet \mathsf{h}_1 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_6 \quad \mathsf{h}_1 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_5, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_2\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_3, \Delta_7, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_3, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_3, \Delta_7 \quad \mathsf{fail} \quad \mathsf{h}_4 : \vdash \{\Delta_7\} : \mathsf{F}_4 : \vdash \{\Delta_$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{c} \underline{\mathbf{h}_3 : \vdash \{\mathbf{F}_6, \Delta_7\} : \mathbf{F}_6, \Delta_4, \Delta_5, \mathbf{F}_1 \otimes \mathbf{F}_2}} \\ \bullet \mathbf{h}_3 : \vdash \{\mathbf{F}_6, \Delta_7\} : \Delta_4, \Delta_5, \mathbf{F}_1 \otimes \mathbf{F}_2} \end{array} ?_C \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_7, \mathbf{F}_6\} : \Delta_4, \mathbf{F}_1, \mathbf{F}_6}} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_7, \mathbf{F}_6\} : \Delta_4, \mathbf{F}_1, \mathbf{F}_6} \end{array} ?_C$$

## 4.13 Status of $\otimes$ (Right Premise): Non invertible

- Case rule 1
- Case rule!
- Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \{ \mathbf{F}_5, \Delta_4 \} : \Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4 \} : (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), ? \mathbf{F}_5} ? \\ \\ \hline \\ \frac{\mathbf{h}_3 : \vdash \{ \mathbf{F}_5, \Delta_4 \} : \Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4 \} : (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), ? \mathbf{F}_5} ? \\ \\ \hline \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4 \} : (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), ? \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4 \} : \Delta_7, \mathbf{F}_2, ? \mathbf{F}_5} ? \\ \end{array} \right. \\ \begin{array}{c} \mathbf{h}_3 : \vdash \{ \Delta_4 \} : \Delta_7, \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{ \Delta_4 \} : \Delta_7, \mathbf{F}_2, ? \mathbf{F}_5 \\ \end{array} \right. \\ \begin{array}{c} \mathbf{ax} / \mathbf{ind} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4 \} : \Delta_7, \mathbf{F}_2, ? \mathbf{F}_5 \\ \end{array} \right. \\ \end{array}$$

• Case rule \$

$$\frac{ \begin{array}{l} \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \mathbf{F}_6, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \$ \mathbf{F}_6 \end{array}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2} \overset{\mathbf{ax/ind}}{\vdash \mathbf{h}_3} \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \mathbf{F}_6, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \$ \mathbf{F}_6}} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \$ \mathbf{F}_6} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6) \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_8, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6, \mathbf{F}_6$$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2} \overset{\text{ax/ind}}{\vdash \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_3 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_4 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_4 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_4 \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{h}_4 \\ \bullet \mathbf{h}_4 :$$

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_5, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{F}_6, \Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_5 \& \mathbf{F}_6} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{F}_2, \mathbf{F}_5 \& \mathbf{F}_6} \quad \frac{\mathsf{ax/ind}}{\mathsf{ax/ind}} \quad \frac{\mathsf{ax/ind}}{\mathsf{a$$

• Case rule  $\oplus_B$ 

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{f}_6, \Delta_7, \Delta_8, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_5 \oplus \mathbf{f}_6} \oplus_B \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3} : \vdash \{\Delta_4\} : \Delta_8, \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathbf{f}_2} \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathbf{f}_6, \Delta_7, \Delta_8, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_5 \oplus \mathbf{f}_6} \oplus_B \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3} : \vdash \{\Delta_4\} : \Delta_8, \mathbf{f}_2, \mathbf{f}_6}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_5 \oplus \mathbf{f}_6} \oplus_B$$

• Case rule  $\oplus_A$ 

$$\begin{array}{c} \begin{array}{c} \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2 \\ \hline \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6 \end{array} \oplus_{A} \end{array} \longrightarrow \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2} \\ \hline \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2 \end{array} \xrightarrow{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2} \overset{\mathbf{ax/ind}}{\mathsf{H}} \\ \\ \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2} \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6} \end{array} \oplus_{A} \end{array} \longrightarrow \begin{array}{c} \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5} \\ \overline{\mathbf{h}_3 : \vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5} \overset{\mathbf{ax/ind}}{\oplus} \\ \oplus_{A} : \vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6 \end{array} \oplus_{A} \end{array}$$

• Case rule  $\perp$ 

• Case rule  $\top$ 

- Case rule  $I_3$
- Case rule  $\otimes$

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8, \mathsf{F}_1 \otimes \mathsf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathsf{F}_1 \otimes \mathsf{F}_2), (\Delta_9, \Delta_{10}), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad & \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathsf{F}_1 \otimes \mathsf{F}_2), (\Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2)}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad & \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8, \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2)}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathsf{F}_1 \otimes \mathsf{F}_2), (\Delta_9, \Delta_{10}), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad & \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8, \quad \mathbf{h}_1 \otimes \mathsf{F}_2 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_5 \otimes \mathsf{F}_6}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathsf{F}_1 \otimes \mathsf{F}_2), (\Delta_9, \Delta_{10}), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad & \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad & \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad & \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad & \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2), \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad & \\ \frac{\mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8 \quad \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_1 \otimes \mathsf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_6, \Delta_9, \Delta_{10}, \mathsf{F}_5 \otimes \mathsf{F}_6} \quad \otimes \qquad & \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_4\} : \mathsf{F}_5, \Delta_7, \Delta_8, \Delta_{10}, \mathsf{F}_5 \otimes \mathsf{F}_6} \quad & \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_2\} : \mathsf{F}_5, \Delta_6, \Delta_6, (\Delta_7, \Delta_8), \mathsf{F}_3, \mathsf{F}_3 \otimes \mathsf{F}_4} \quad \otimes \qquad & \\ \bullet \mathbf{h}_3 : \vdash \{\Delta_2\} : \mathsf{F}_5, \Delta_6, \Delta_6, (\Delta_7, \Delta_8), \mathsf{F}_8, \mathsf{F}_8, \mathsf{F}_9, \mathsf{F}$$

- Case rule  $I_1$
- Case rule  $I_2$
- $\bullet \;$  Case rule  $?_C$

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \{\mathbf{F}_6, \Delta_7\} : \mathbf{F}_6, \Delta_4, \Delta_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \{\mathbf{F}_6, \Delta_7\} : \Delta_4, \Delta_5, \mathbf{F}_1 \otimes \mathbf{F}_2} & ?_C & \rightarrow & & & & & & & & \frac{\mathbf{h}_3 : \vdash \{\Delta_7, \mathbf{F}_6\} : \Delta_5, \mathbf{F}_2, \mathbf{F}_6}{\bullet \mathbf{h}_3 : \vdash \{\Delta_7, \mathbf{F}_6\} : \Delta_5, \mathbf{F}_2} & & & & ?_C \\ & & & & & & & & & & & & \\ \end{array}$$

## 4.14 Status of $I_1$ : : Invertible

- Case rule 1
- Case rule!
- Case rule ?
- Case rule \$
- Case rule &
- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- $\bullet$  Case rule  $\bot$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$
- Case rule  $I_1$

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_3, \, p(\mathbf{n}_2)\} \, : \, \hat{}\, (\mathbf{n}_2)} \quad I_1 \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule  $I_2$ 

• Case rule  $?_C$ 

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \{\Delta_3, p(\mathbf{n_1})\} : p(\mathbf{n_1}), \, \hat{\ }(\mathbf{n_1})}{\bullet \mathbf{h}_2 : \vdash \{p(\mathbf{n_1}), \Delta_3\} : \, \hat{\ }(\mathbf{n_1})} \end{array} ?_C \qquad \rightarrow \qquad \mathsf{trivial}$$

$$\begin{array}{l} \frac{\mathbf{h}_2 : \vdash \{\mathbf{F}_3, \Delta_4, p(\mathbf{n}_1)\} : \mathbf{F}_3, \hat{\ \ }(\mathbf{n}_1)}{\bullet \mathbf{h}_2 : \vdash \{\mathbf{F}_3, \Delta_4, p(\mathbf{n}_1)\} : \hat{\ \ }(\mathbf{n}_1)} \end{array} ?_C \qquad \rightarrow \qquad \text{trivial}$$

## 4.15 Status of $I_2$ : : Invertible

- Case rule 1
- Case rule!
- Case rule ?
- Case rule \$
- Case rule &
- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- Case rule  $\perp$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$
- Case rule  $I_1$
- Case rule  $I_2$

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\Delta_3, \, \hat{\ } (\mathbf{n}_2)\} : p(\mathbf{n}_2)} \quad I_2 \qquad \rightarrow \qquad \mathsf{trivial}$$

 $\bullet$  Case rule  $?_C$ 

$$\frac{\mathbf{h}_2 : \vdash \{\Delta_3, \hat{\ } (\mathbf{n_1})\} : p(\mathbf{n_1}), \hat{\ } (\mathbf{n_1})}{\bullet \mathbf{h}_2 : \vdash \{\hat{\ } (\mathbf{n_1}), \Delta_3\} : p(\mathbf{n_1})} \ ?_C \qquad \rightarrow \qquad \mathsf{trivial}$$

$$\begin{array}{ll} \frac{\mathbf{h}_2 : \vdash \{\mathbf{F}_3, \Delta_4, \widehat{\ } (\mathbf{n_1})\} : \mathbf{F}_3, p(\mathbf{n_1})}{\bullet \mathbf{h}_2 : \vdash \{\mathbf{F}_3, \Delta_4, \widehat{\ } (\mathbf{n_1})\} : p(\mathbf{n_1})} \end{array} ?_C \qquad \rightarrow \qquad \mathsf{trivial}$$

### 4.16 Status of $?_C$ : Non invertible

• Case rule 1

$$\frac{}{\bullet_{h_1} : \vdash \{\mathtt{F}_2, \Delta_3\} : \mathtt{1}} \quad \mathsf{1} \qquad \rightarrow \qquad \frac{}{\bullet_{h_1} : \vdash \{\mathtt{F}_2, \Delta_3\} : \mathtt{1}, \mathtt{F}_2} \quad \mathsf{fail}$$

• Case rule!

$$\begin{array}{ll} \frac{\mathbf{h}_1 : \vdash \{ \mathbf{F}_3, \Delta_4 \} : \mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_3, \Delta_4 \} : \mathbf{F}_2} & \mathbf{fail} \\ \hline \bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_3, \Delta_4 \} : \mathbf{F}_3, \mathsf{lF}_2 & \mathbf{fail} \\ \end{array}$$

• Case rule?

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_2,\mathtt{F}_4,\Delta_5\} : \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_4,\Delta_5\} : \Delta_3,?\mathtt{F}_2} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_1 :\vdash \{\Delta_5,\mathtt{F}_2,\mathtt{F}_4\} : \Delta_3,\mathtt{F}_4}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5,\mathtt{F}_4\} : \Delta_3,\mathtt{F}_4,?\mathtt{F}_2} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

$$\frac{\mathtt{h}_1 : \vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_2, \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_2\$\mathtt{F}_3} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_5}}{\bullet \mathtt{h}_1 : \vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_5, \mathtt{F}_2\$\mathtt{F}_3} \quad \$} \quad \overset{ax/ind}{\bullet}$$

• Case rule &

$$\frac{\mathbf{h}_1 : \vdash \{ \mathbf{F}_5, \Delta_6 \} : \mathbf{F}_2, \Delta_4 \quad \mathbf{h}_1 : \vdash \{ \mathbf{F}_5, \Delta_6 \} : \mathbf{F}_3, \Delta_4 \}}{\bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_5, \Delta_6 \} : \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_2, \mathbf{F}_5} \quad \overline{\mathbf{h}_1 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_2 \& \mathbf{F}_3}}{\bullet \mathbf{h}_1 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_2 \& \mathbf{F}_3} \quad \underbrace{\mathbf{ax/ind}}_{\bullet \mathbf{h}_1} \quad \underbrace{\mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_2 \& \mathbf{F}_3}_{\bullet \mathbf{h}_1} \quad \underbrace{\mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_5 \& \mathbf{F}_3}_{\bullet \mathbf{h}_1} \quad \underbrace{\mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_5 \& \mathbf{F}_3 \& \mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_5 \& \mathbf{F}_3 \& \mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_5 \& \mathbf{F}_3 \& \mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_5 \& \mathbf{F}_3 \& \mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_5 \& \mathbf{F}_3 \& \mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_5 \& \mathbf{F}_3 \& \mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_5 \& \mathbf{F}_3 \& \mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{F}_5 \& \mathbf{h}_2 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{h}_2 \& \mathbf{h}_3 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{F}_5, \mathbf{h}_2 \& \mathbf{h}_3 : \vdash \{ \Delta_6, \mathbf{F}_5 \} : \Delta_4, \mathbf{h}_3 : \vdash$$

• Case rule  $\oplus_B$ 

$$\frac{\mathbf{h}_1 : \vdash \{\mathbf{F}_5, \Delta_6\} : \mathbf{F}_3, \Delta_4}{\bullet \mathbf{h}_1 : \vdash \{\mathbf{F}_5, \Delta_6\} : \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_3, \mathbf{F}_5}}{\bullet \mathbf{h}_1 : \vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_5, \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_B$$

• Case rule  $\bigoplus_A$ 

$$\frac{\mathbf{h}_1 :\vdash \{\mathbf{F}_5, \Delta_6\} : \mathbf{F}_2, \Delta_4}{\bullet \mathbf{h}_1 :\vdash \{\mathbf{F}_5, \Delta_6\} : \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_A \qquad \rightarrow \qquad \frac{\overleftarrow{\mathbf{h}_1 :\vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_1 :\vdash \{\Delta_6, \mathbf{F}_5\} : \Delta_4, \mathbf{F}_5, \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_A$$

• Case rule  $\perp$ 

$$\frac{\mathtt{h}_1 : \vdash \{\mathtt{F}_3, \Delta_4\} : \Delta_2}{\bullet \mathtt{h}_1 : \vdash \{\mathtt{F}_3, \Delta_4\} : \bot, \Delta_2} \ \bot \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_1 : \vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3}}{\bullet \mathtt{h}_1 : \vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3, \bot} \overset{\mathsf{ax/ind}}{\bot}$$

• Case rule  $\top$ 

$$\frac{}{\bullet \mathsf{h}_1 : \vdash \{\mathsf{F}_3, \Delta_4\} : \top, \Delta_2} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_1 : \vdash \{\Delta_4, \mathsf{F}_3\} : \Delta_2, \mathsf{F}_3, \top} \ \top$$

• Case rule  $I_3$ 

$$\frac{}{\bullet \mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : p(\mathbf{n}_2), \, \hat{\ } (\mathbf{n}_2)} \quad I_3 \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1 : \vdash \{\mathbf{F}_3, \Delta_4\} : \mathbf{F}_3, \, p(\mathbf{n}_2), \, \hat{\ } (\mathbf{n}_2)} \quad \mathsf{fail}$$

 $\bullet$  Case rule  $\otimes$ 

$$\frac{\mathbf{h}_1 : \vdash \{\mathsf{F}_6, \Delta_7\} : \mathsf{F}_2, \Delta_4 \quad \mathbf{h}_1 : \vdash \{\mathsf{F}_6, \Delta_7\} : \mathsf{F}_3, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\mathsf{F}_6, \Delta_7\} : \Delta_4, \Delta_5, \mathsf{F}_2 \otimes \mathsf{F}_3} \quad \otimes \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \{\Delta_7, \mathsf{F}_6\} : \Delta_4, \mathsf{F}_2} \quad \overset{\mathsf{ax}}{\mathsf{h}_1 : \vdash \{\Delta_7, \mathsf{F}_6\} : \Delta_5, \mathsf{F}_3, \mathsf{F}_6}}{\bullet \mathbf{h}_1 : \vdash \{\Delta_7, \mathsf{F}_6\} : \Delta_4, \Delta_5, \mathsf{F}_6, \mathsf{F}_2 \otimes \mathsf{F}_3} \quad \overset{\mathsf{ax/ind}}{\otimes}$$

• Case rule  $I_1$ 

• Case rule  $I_2$ 

• Case rule  $?_C$ 

## 5 Weakening on bang: $\vdash \Gamma$ , !F implies $\Gamma$ , F.

- Case(s) rule 1
- Case(s) rule!

• Case(s) rule?

$$\frac{\mathbf{h}_2 : \vdash \{\mathtt{F}_4, \Delta_3\} : \Delta_5, \mathtt{IF}_1}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathtt{IF}_1, \Delta_5, \mathtt{?F}_4} \ ? \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_2 : \vdash \{\Delta_3, \mathtt{F}_4\} : \Delta_5, \mathtt{IF}_1}{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_5, \mathtt{F}_1} \overset{\mathsf{ax}}{=} \\ \frac{\mathbf{h}_2 : \vdash \{\Delta_3, \mathtt{F}_4\} : \Delta_5, \mathtt{F}_1}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_5, \mathtt{F}_1, \mathtt{?F}_4} \overset{\mathsf{ax}}{?}$$

• Case(s) rule \$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \mathtt{F}_4, \mathtt{F}_5, \Delta_6, \mathtt{IF}_1}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathtt{IF}_1, \Delta_6, \mathtt{F}_4 \$ \mathtt{F}_5} \quad \mathtt{s} \\ & \stackrel{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{IF}_1}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \$ \mathtt{F}_5} \quad \overset{\mathrm{ax}}{\mathtt{s}} \\ & \bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \$ \mathtt{F}_5} \\ & \bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \$ \mathtt{F}_5} \end{array}$$

• Case(s) rule &

$$\frac{ \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_4, \Delta_6, |\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_5, \Delta_6, |\mathbf{F}_1|}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : |\mathbf{F}_4, \Delta_6, \mathbf{F}_4 \& \mathbf{F}_5|} \quad \& \quad \rightarrow \quad \underbrace{\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4, |\mathbf{F}_1|}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4, |\mathbf{F}_1|}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4, \mathbf{F}_5} \underbrace{\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_5, |\mathbf{F}_1|}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4, \mathbf{F}_5}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \& \mathbf{F}_5} \underbrace{\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4, |\mathbf{F}_5|}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_1, \mathbf{F}_5, \mathbf{F}_5}}_{\bullet \& \bullet} \underbrace{\frac{\mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4, |\mathbf{F}_4|}{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_1, \mathbf{F}_5, \mathbf{$$

• Case(s) rule  $\oplus_B$ 

$$\frac{\underset{\bullet}{\mathbf{h}_2} : \vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_6, \mathtt{IF}_1}{\underset{\bullet}{\mathbf{h}_2} : \vdash \{\Delta_3\} : \mathtt{IF}_1, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_B \qquad \rightarrow \qquad \frac{\frac{\underset{\bullet}{\mathbf{h}_2} : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_5, \mathtt{IF}_1}{\underset{\bullet}{\mathbf{h}_2} : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_1, \mathtt{F}_5} \overset{\mathtt{ax}}{\underset{\mathtt{IH}}{\mathsf{IH}}}}{\underset{\bullet}{\mathsf{h}_2} : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \oplus \mathtt{F}_5} \oplus_B$$

• Case(s) rule  $\oplus_A$ 

$$\frac{\underset{\bullet}{\mathbf{h}_2} : \vdash \{\Delta_3\} : \mathtt{F}_4, \Delta_6, \mathtt{IF}_1}{\bullet \mathtt{h}_2 : \vdash \{\Delta_3\} : !\mathtt{F}_1, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5} \quad \oplus_A \qquad \rightarrow \qquad \frac{\frac{\overset{\bullet}{\mathbf{h}_2} : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_4, \mathtt{IF}_1}{\mathtt{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_1, \mathtt{F}_4} \overset{\mathtt{ax}}{}_{\mathtt{IH}}}{\bullet \mathtt{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \oplus \mathtt{F}_5} \quad \oplus_A$$

• Case(s) rule  $\perp$ 

• Case(s) rule ⊤

$$\xrightarrow{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : ! \mathsf{F}_1, \, \top, \, \Delta_4} \; \top \qquad \rightarrow \qquad \xrightarrow{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \, \Delta_4, \, \mathsf{F}_1, \, \top} \; \top$$

- Case(s) rule  $I_3$
- Case(s) rule  $\otimes$

$$\frac{ \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_4, \Delta_6 \quad \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{F}_5, \Delta_7, \mathbf{IF}_1 }{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \mathbf{IF}_1, \Delta_6, \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5 } \quad \otimes \qquad \rightarrow \qquad \underbrace{ \frac{ \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4 }{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \mathbf{F}_4 }}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \frac{\mathbf{n}_2 : \vdash \{\Delta_3\} : \Delta_6, \Delta_7, \mathbf{F}_1, \mathbf{F}_5, \mathbf{F}_1, \mathbf{F}_5 }{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \frac{\mathbf{n}_2 : \vdash \{\Delta_3\} : \Delta_6, \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5 }{\bullet \mathbf{h}_2 : \vdash \{\Delta_3\} : \Delta_6, \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5 }$$

- Case(s) rule  $I_1$
- Case(s) rule  $I_2$
- Case(s) rule  $?_C$

$$\begin{array}{c} \mathbf{h}_2 : \vdash \{ \mathbf{F}_4, \Delta_5 \} : \mathbf{F}_4, \Delta_3, \mathbf{IF}_1 \\ \bullet \mathbf{h}_2 : \vdash \{ \mathbf{F}_4, \Delta_5 \} : \mathbf{IF}_1, \Delta_3 \end{array} \ ?_C \qquad \rightarrow \qquad \begin{array}{c} \frac{\mathbf{h}_2 : \vdash \{ \Delta_5, \mathbf{F}_4 \} : \Delta_3, \mathbf{F}_4, \mathbf{IF}_1}{\mathbf{h}_2 : \vdash \{ \Delta_5, \mathbf{F}_4 \} : \Delta_3, \mathbf{F}_1, \mathbf{F}_4} \end{array} \ \underset{?_C}{\overset{\mathbf{ax}}{\bullet}} \\ \bullet \mathbf{h}_2 : \vdash \{ \Delta_5, \mathbf{F}_4 \} : \Delta_3, \mathbf{F}_1, \mathbf{F}_4 \end{array} \ ?_C$$

## 6 Identity-Expansion

#### 7 Cut-Elimination

#### 7.1 Status of 1: OK

- Case rule 1
- Case rule!
- Case rule?

• Case rule \$

$$\begin{array}{c|c} & \mathbf{h}_3 : \vdash \{\Delta_4\} : \bot, F_5, F_6, \Delta_7 \\ \hline \mathbf{e}_{h_1} : \vdash \{\Delta_4\} : \mathbf{1}, * & \mathbf{1} & \mathbf{e}_{h_3} : \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, F_5 \$ F_6 \\ \hline & - : \vdash \{\Delta_4\} : *, \Delta_7, F_5 \$ F_6 \\ \hline & \bullet_{h_1} : \vdash \{\Delta_4\} : \mathbf{1} & \mathrm{ax} & \mathbf{e}_{h_3} : \vdash \{\Delta_4\} : \Delta_7, F_5, F_6, \bot \\ \hline & \frac{- : \vdash \{\Delta_4\} : \Delta_7, F_5, F_6}{- : \vdash \{\Delta_4\} : \Delta_7, F_5 \$ F_6} & \$ \end{array} \right. \\ \mathbf{Cut}$$

• Case rule &

$$\frac{\underbrace{\frac{\bullet_{11} : \vdash \{\Delta_{4}\} : \mathbf{1}, *}{\bullet_{11} : \vdash \{\Delta_{4}\} : \mathbf{1}, *}} \ \mathbf{1} \ \frac{\frac{h_{3} : \vdash \{\Delta_{4}\} : \bot, F_{5}, \Delta_{7} \quad h_{3} : \vdash \{\Delta_{4}\} : \bot, F_{6}, \Delta_{7}}{\bullet_{13} : \vdash \{\Delta_{4}\} : dual(\mathbf{1}), \Delta_{7}, F_{5} \& F_{6}} \underbrace{\frac{- : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{5}, \bot}{h_{3} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{5}, \bot}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{11} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}} \underbrace{\frac{ax}{h_{13} : \vdash \{\Delta_{4}\} : \Delta_{7}, F_{6}, \bot}}_{\bullet h_{Cut}}$$

• Case rule  $\oplus_B$ 

• Case rule  $\bigoplus_A$ 

• Case rule  $\perp$ 

 $\bullet$  Case rule  $\top$ 

- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{c|c} \underline{\bullet_{h_1} : \vdash \{F_5, \Delta_6\} : 1, *} & 1 & \frac{h_3 : \vdash \{F_5, \Delta_6\} : \bot, F_5, \Delta_4}{\bullet_{h_3} : \vdash \{F_5, \Delta_6\} : dual(1), \Delta_4} & ?_{\mathcal{C}} \\ \hline & - : \vdash \{F_5, \Delta_6\} : *, \Delta_4 & \rightarrow \\ \underline{\bullet_{h_1} : \vdash \{\Delta_6, F_5\} : 1} & \text{ax} & \frac{\rightarrow}{h_3 : \vdash \{\Delta_6, F_5\} : \Delta_4, F_5, \bot} & \text{ax} \\ \hline & \frac{- : \vdash \{\Delta_6, F_5\} : \Delta_4, F_5}{- : \vdash \{\Delta_6, F_5\} : \Delta_4} & ?_{\mathcal{C}} \end{array}$$

#### 7.2 Status of !: OK

• Case rule 1

- Case rule!
- Case rule?

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \{\Delta_6\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : ! \mathbf{F}_4, *} : & \frac{\mathbf{h}_5 : \vdash \{\mathbf{F}_7, \Delta_6\} : \Delta_8, ?dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : dual(! \mathbf{F}_4), \Delta_8, ? \mathbf{F}_7} \\ & - : \vdash \{\Delta_6\} : *, \Delta_8, ? \mathbf{F}_7 \\ & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : ! \mathbf{F}_4 \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_6, \mathbf{F}_7\} : ! \mathbf{F}_4 \\ \hline & \frac{- : \vdash \{\Delta_6, \mathbf{F}_7\} : \Delta_8}{- : \vdash \{\Delta_6\} : \Delta_8, ? \mathbf{F}_7} ? \\ \hline & \frac{\mathbf{h}_1 : \vdash \{\Delta_6\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : ! \mathbf{F}_4, *} : & \frac{\mathbf{h}_5 : \vdash \{\Delta_6, dual(\mathbf{F}_4)\} : \Delta_7}{\bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : dual(! \mathbf{F}_4), \Delta_7} ? \\ \hline & \frac{- : \vdash \{\Delta_6\} : \mathbf{F}_4, *}{\bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : ! \mathbf{F}_4, *} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \{\Delta_6, dual(\mathbf{F}_4)\} : \Delta_7} \\ \hline & \frac{- : \vdash \{\Delta_6\} : \mathcal{F}_4, dual(\mathbf{F}_4)\} : \Delta_7}{- : \vdash \{\Delta_6\} : \Delta_7} & \mathbf{ax} \\ \hline & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \{\Delta_6, dual(\mathbf{F}_4)\} : \Delta_7} & \mathbf{ax} \\ \hline & - : \vdash \{\Delta_6\} : \Delta_7 & \mathbf{ax} \\ \hline \end{array}$$

• Case rule \$

$$\frac{ \begin{array}{c|c} \mathbf{h}_1 : \vdash \{\Delta_6\} : \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : \mathbf{F}_4, * \end{array} ! & \begin{array}{c|c} \mathbf{h}_5 : \vdash \{\Delta_6\} : \mathbf{F}_7, \mathbf{F}_8, \Delta_9, ?dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : dual(!\mathbf{F}_4), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : !\mathbf{F}_4 \end{array} & \begin{array}{c|c} \mathbf{f}_4 \\ \bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_7\$\mathbf{F}_8 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : !\mathbf{F}_4 \end{array} & \begin{array}{c|c} \mathbf{f}_4 \\ \bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_7, \mathbf{F}_8, ?dual(\mathbf{F}_4) \\ \hline \\ \bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_7, \mathbf{F}_8 \end{array} & \begin{array}{c|c} \mathbf{f}_4 \\ \bullet \mathbf{f}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{f}_7, \mathbf{f}_8 \end{cases} & \mathbf{f}_6 \\ \bullet \mathbf{f}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{f}_7, \mathbf{f}_8 \end{cases} & \mathbf{f}_6 \\ \bullet \mathbf{f}_7 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{f}_7, \mathbf{f}_8 \end{cases} & \mathbf{f}_7 : \mathbf{f}_8 \\ \bullet \mathbf{f}_7 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{f}_7, \mathbf{f}_8 \end{cases} & \mathbf{f}_8 \\ \bullet \mathbf{f}_7 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{f}_7, \mathbf{f}_8 \end{cases} & \mathbf{f}_8 \\ \bullet \mathbf{f}_8 : \mathbf$$

• Case rule &

$$\frac{\frac{\mathbf{h}_1 : \vdash \{\Delta_6\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : !\mathbf{F}_4, *} : \frac{\mathbf{h}_5 : \vdash \{\Delta_6\} : \mathbf{F}_7, \Delta_9, ?dual(\mathbf{F}_4) \quad \mathbf{h}_5 : \vdash \{\Delta_6\} : \mathbf{F}_8, \Delta_9, ?dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : dual(!\mathbf{F}_4), \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8} \quad \mathbf{Cut}} \\ \frac{- : \vdash \{\Delta_6\} : *, \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8}{\bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : !\mathbf{F}_4} \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_7, ?dual(\mathbf{F}_4)}^{\mathbf{ax}} \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4)}^{\mathbf{ax}} \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4)}^{\mathbf{ax}} \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4)}^{\mathbf{ax}} \underbrace{\mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4)}_{\mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4)}^{\mathbf{ax}} \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4)}^{\mathbf{ax}} \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4)}^{\mathbf{ax}} \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4)}^{\mathbf{ax}} \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4)}^{\mathbf{ax}} \underbrace{\mathbf{ax}}_{\mathbf{h}_5 : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{$$

• Case rule  $\oplus_B$ 

$$\frac{\begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_6\} : \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : ! \mathbf{F}_4, \star \end{array} ! \quad \begin{array}{c} \mathbf{h}_5 : \vdash \{\Delta_6\} : \mathbf{F}_8, \Delta_9, ?dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : dual(!\mathbf{F}_4), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 \end{array}}{ - : \vdash \{\Delta_6\} : \star, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 } \quad \begin{array}{c} \oplus_B \\ \mathsf{Cut} \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : ! \mathbf{F}_4 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \hline \mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_8, ?dual(\mathbf{F}_4) \\ \hline - : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_8 \end{array} \quad \begin{array}{c} \mathsf{ax} \\ \mathsf{hCut} \\ \hline - : \vdash \{\Delta_6\} : \Delta_9, \mathbf{F}_8 \end{array} \quad \begin{array}{c} \oplus_B \\ \mathsf{hCut} \\ \hline \end{array}$$

• Case rule  $\oplus_A$ 

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \left\{ \Delta_6 \right\} : \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_6 \right\} : ! \mathbf{F}_4 , * \end{array} ! \quad \begin{array}{c} \mathbf{h}_5 : \vdash \left\{ \Delta_6 \right\} : \mathbf{F}_7 , \Delta_9 , ?dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_6 \right\} : dual(! \mathbf{F}_4) , \Delta_9 , \mathbf{F}_7 \oplus \mathbf{F}_8 \end{array} \quad \begin{array}{c} \oplus_A \\ \text{Cut} \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_6 \right\} : ! \mathbf{F}_4 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_6 \right\} : \Delta_9 , \mathbf{F}_7 , ?dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_6 \right\} : ! \mathbf{F}_4 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_6 \right\} : \Delta_9 , \mathbf{F}_7 , ?dual(\mathbf{F}_4) \\ \hline - : \vdash \left\{ \Delta_6 \right\} : \Delta_9 , \mathbf{F}_7 \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{hCut} \end{array}$$

 $\bullet$  Case rule  $\bot$ 

$$\frac{ \begin{array}{c|c} \mathbf{h}_1 : \vdash \{\Delta_6\} : \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : ! \mathbf{F}_4 , * \end{array} ! \quad \begin{array}{c|c} \mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_7, ?dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : dual(!\mathbf{F}_4), \bot, \Delta_7 \\ \hline \\ - : \vdash \{\Delta_6\} : *, \bot, \Delta_7 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : ! \mathbf{F}_4 \end{array} \quad \begin{array}{c|c} \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : \Delta_7, ?dual(\mathbf{F}_4) \\ \hline \\ - : \vdash \{\Delta_6\} : \Delta_7, \bot \end{array} \quad \begin{array}{c|c} \mathbf{ax} \\ \mathbf{hCut} \\ \hline \\ - : \vdash \{\Delta_6\} : \Delta_7, \bot \end{array} \quad \\ \end{array}$$

 $\bullet$  Case rule  $\top$ 

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_6\} : \mathtt{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_6\} : !\mathtt{F}_4, * \end{array} ! \quad \frac{\bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \top, \Delta_7}{\bullet \mathbf{h}_5 : \vdash \{\Delta_6\} : *, \top, \Delta_7} \quad \begin{array}{c} \top \\ \mathsf{Cut} \\ & \xrightarrow{-} : \vdash \{\Delta_6\} : \Delta_7, \top \end{array} \end{array}}$$

- Case rule  $I_3$
- Case rule  $\otimes$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\frac{ \begin{array}{l} \mathbf{h}_1 : \vdash \{ \mathbf{F}_7, \Delta_8 \} : \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_7, \Delta_8 \} : ! \mathbf{F}_4, * \end{array} ! \begin{array}{l} \mathbf{h}_5 : \vdash \{ \mathbf{F}_7, \Delta_8 \} : \mathbf{F}_7, \Delta_6, ?dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 : \vdash \{ \mathbf{F}_7, \Delta_8 \} : dual(!\mathbf{F}_4), \Delta_6 \\ \hline - : \vdash \{ \mathbf{F}_7, \Delta_8 \} : *, \Delta_6 \\ \hline \bullet \mathbf{h}_1 : \vdash \{ \Delta_8, \mathbf{F}_7 \} : ! \mathbf{F}_4 \end{array} \begin{array}{l} \mathbf{ax} \\ \mathbf{h}_5 : \vdash \{ \Delta_8, \mathbf{F}_7 \} : \Delta_6, \mathbf{F}_7, ?dual(\mathbf{F}_4) \\ \hline - : \vdash \{ \Delta_8, \mathbf{F}_7 \} : \Delta_6, \mathbf{F}_7 \\ \hline - : \vdash \{ \Delta_8, \mathbf{F}_7 \} : \Delta_6 \end{array} \begin{array}{l} \mathbf{ax} \\ \mathbf{hCut} \end{array}$$

## **7.3** Status of ?: OK

- Case rule 1
- Case rule!

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \{ \mathbf{F}_5, \Delta_7 \} : \Delta_4 \\ \bullet \mathbf{h}_1 : \vdash \{ \Delta_7 \} : ? \mathbf{F}_5, \Delta_4 \end{array} ? \quad \frac{\mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5)}{\bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(?\mathbb{F}_5), *} \\ - : \vdash \{ \Delta_7 \} : \Delta_4, * \\ \\ \underline{ \begin{array}{c} \bullet \mathbf{h}_1 : \vdash \{ \Delta_7, \mathbb{F}_5 \} : \Delta_4 \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \end{array} \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ - : \vdash \{ \Delta_7 \} : \Delta_4 \end{array}} \overset{!}{\underbrace{ \begin{array}{c} \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \} : dual(\mathbb{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \Delta_7 \}$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \{\mathsf{F}_{5}, \Delta_{7}\} : \Delta_{4}}{\bullet \mathbf{h}_{1} : \vdash \{\Delta_{7}\} : ?\mathsf{F}_{5}, \Delta_{4}} ? & \frac{\mathbf{h}_{6} : \vdash \{\mathsf{F}_{8}, \Delta_{7}\} : \Delta_{9} , !dual(\mathsf{F}_{5})}{\bullet \mathbf{h}_{6} : \vdash \{\Delta_{7}\} : dual(?\mathsf{F}_{5}), \Delta_{9} , ?\mathsf{F}_{8}} ? \\ & - : \vdash \{\Delta_{7}\} : \Delta_{4}, \Delta_{9}, ?\mathsf{F}_{8} & \rightarrow \\ & \frac{\bullet \mathbf{h}_{1} : \vdash \{\Delta_{7}\} : \Delta_{4}, ?\mathsf{F}_{5}}{\bullet \mathbf{h}_{1} : \vdash \{\Delta_{7}, \mathsf{F}_{8}\} : \Delta_{4}, ?\mathsf{F}_{5}} W & \frac{\bullet \mathbf{h}_{6} : \vdash \{\Delta_{7}, \mathsf{F}_{8}\} : \Delta_{9}, !dual(\mathsf{F}_{5})}{\bullet \mathbf{h}_{6} : \vdash \{\Delta_{7}, \mathsf{F}_{8}\} : \Delta_{9}, !dual(\mathsf{F}_{5})} & \mathbf{ax} \\ & \frac{- : \vdash \{\Delta_{7}, \mathsf{F}_{8}\} : \Delta_{4}, \Delta_{9}}{- : \vdash \{\Delta_{7}\} : \Delta_{4}, \Delta_{9}, ?\mathsf{F}_{8}} ? & \frac{\bullet \mathbf{h}_{7} : \vdash \{\mathsf{F}_{9}, \Delta_{8}\} : \Delta_{10}, dual(\mathsf{F}_{6})}{\bullet \mathbf{h}_{7} : \vdash \{\Delta_{8}\} : dual(\mathsf{F}_{6}), \Delta_{10}, ?\mathsf{F}_{9}} & \mathbf{Cut} \\ & \frac{\bullet \mathbf{h}_{1} : \vdash \{\Delta_{8}\} : \mathsf{F}_{6}, \Delta_{5}, ?\mathsf{F}_{3}}{- : \vdash \{\Delta_{8}, \mathsf{F}_{3}\} : \Delta_{10}, \Delta_{5}, ?\mathsf{F}_{9}} ? & \mathbf{ax} \\ & \frac{\bullet \mathbf{h}_{1} : \vdash \{\Delta_{8}, \mathsf{F}_{3}\} : \Delta_{5}, \mathsf{F}_{6}}{- : \vdash \{\Delta_{8}\} : \Delta_{10}, \Delta_{5}, ?\mathsf{F}_{9}} ? & \mathbf{ax} \\ & \frac{- : \vdash \{\Delta_{8}\} : \Delta_{10}, \Delta_{5}, ?\mathsf{F}_{9}, dual(\mathsf{F}_{6})}{- : \vdash \{\Delta_{8}\} : \Delta_{10}, \Delta_{5}, ?\mathsf{F}_{9}} ? & \mathbf{h}_{7} : \mathbf{h}$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{ \mathbf{F}_5, \Delta_7 \} : \Delta_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : ? \mathbf{F}_5, \Delta_4} \ ? & \begin{array}{c} \mathbf{h}_6 : \vdash \{\Delta_7\} : \mathsf{F}_8, \mathsf{F}_9, \Delta_{10}, !dual(\mathsf{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : dual(?\mathsf{F}_5), \Delta_{10}, \mathsf{F}_8 \$ \mathsf{F}_9 \\ \hline - : \vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \mathsf{F}_8 \$ \mathsf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_4, ? \mathsf{F}_5 \end{array} & \begin{array}{c} \mathbf{ax} \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9, !dual(\mathsf{F}_5) \\ \hline - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathbf{ax} \\ \mathbf{h} \mathsf{Cut} \end{array} \\ \end{array} & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_8, \mathsf{F}_9 \end{array} & \\ & \begin{array}{c} \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_1,$$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \{ \mathbf{F}_3, \Delta_8 \} : \mathbf{F}_6, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \{ \Delta_8 \} : \mathbf{F}_6, \Delta_5, ?\mathbf{F}_3 \end{array} ? \begin{array}{c} \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \mathbf{F}_9, \mathbf{F}_{10}, \Delta_{11}, dual(\mathbf{F}_6) \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : dual(\mathbf{F}_6), \Delta_{11}, \mathbf{F}_9 \$ \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \{ \Delta_8 \} : \Delta_5, \mathbf{F}_6, ?\mathbf{F}_3 \end{array} \\ \begin{array}{c} \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \mathbf{F}_9 \$ \mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \{ \Delta_8 \} : \Delta_5, \mathbf{F}_6, ?\mathbf{F}_3 \end{array} \\ \begin{array}{c} \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9, dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, ?\mathbf{F}_{3}, \mathbf{F}_9 \$ \mathbf{F}_{10} \end{array} \\ \begin{array}{c} \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_3 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_8 \} : \Delta_{11}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_9, ?\mathbf{F}_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \mathbf{h}_7$$

• Case rule &

$$\frac{ \begin{array}{c} h_1 : \vdash \{F_5, \Delta_7\} : \Delta_4 \\ \bullet h_1 : \vdash \{\Delta_7\} : ?F_5, \Delta_4 \end{array}{?} \begin{array}{c} h_6 : \vdash \{\Delta_7\} : F_8, \Delta_{10}, !dual(F_5) & h_6 : \vdash \{\Delta_7\} : F_9, \Delta_{10}, !dual(F_5) \\ \bullet h_6 : \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \& F_9 \end{array}{} \\ \hline - : \vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, F_8 \& F_9 \\ \hline \bullet h_1 : \vdash \{\Delta_7\} : \Delta_4, ?F_5 \end{array}{} \begin{array}{c} \text{ax} \\ \hline - : \vdash \{\Delta_7\} : \Delta_{10}, F_8, !dual(F_5) \\ \hline - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8 \end{array}{} \begin{array}{c} \text{ax} \\ h_0 : \vdash \{\Delta_7\} : \Delta_4, ?F_5 \end{array}{} \begin{array}{c} \text{ax} \\ \hline - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8 \end{array}{} \\ \hline - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8 \end{array}{} \begin{array}{c} \text{ax} \\ h_1 : \vdash \{F_3, \Delta_8\} : F_6, \Delta_5 \\ \hline \bullet h_1 : \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_3 \end{array}{?} \begin{array}{c} h_7 : \vdash \{\Delta_8\} : F_9, \Delta_{11}, dual(F_6) & h_7 : \vdash \{\Delta_8\} : F_{10}, \Delta_{11}, dual(F_6) \\ \hline - : \vdash \{\Delta_8\} : (\Delta_5, ?F_3), \Delta_{11}, F_9 \& F_{10} \end{array}{} \begin{array}{c} \text{Cut} \\ \hline \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, dual(F_6), F_9 \& F_{10} \\ \hline \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, dual(F_6), F_9 \& F_{10} \\ \hline \bullet h_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \end{array} \begin{array}{c} \text{ax} \\ \hline \bullet h_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \bullet h_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, dual(F_6), F_9 \& F_{10} \\ \hline - : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \end{array} \begin{array}{c} \text{ax} \\ \hline \bullet h_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline - : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline - : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \hline \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_7, F_9 \& F_{10} \\ \hline \end{array} \begin{array}{c} \text{Act} \\ \bullet h_7 : \vdash \{\Delta_8\} : \Delta$$

• Case rule  $\oplus_B$ 

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{F_5, \Delta_7\} : \Delta_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : ?F_5, \Delta_4} ? & \frac{\mathbf{h}_6 : \vdash \{\Delta_7\} : F_9, \Delta_{10}, !dual(F_5)}{\bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \oplus F_9} & \oplus_B \\ \hline - : \vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, F_8 \oplus F_9 & \rightarrow_{\mathbf{h}_6} : \vdash \{\Delta_7\} : \Delta_{10}, F_9, !dual(F_5) \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_4, ?F_5 & \mathbf{ax} & \rightarrow_{\mathbf{h}_6} : \vdash \{\Delta_7\} : \Delta_{10}, F_9, !dual(F_5) \\ \hline - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_9 & \oplus_B \\ \hline \bullet \mathbf{h}_1 : \vdash \{F_3, \Delta_8\} : F_6, \Delta_5 & \mathbf{ax} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_3 & \mathbf{ax} \\ \hline - : \vdash \{\Delta_8\} : (\Delta_5, ?F_3), \Delta_{11}, F_9 \oplus F_{10} \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_8\} : \Delta_5, F_6, ?F_3 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_8\} : \Delta_5, F_6, ?F_3 & \mathbf{ax} \\ \hline - : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_{10}, ?F_3 \\ \hline - : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_3, F_9 \oplus F_{10} \\ \hline - : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_3, F_9 \oplus F_{10} \\ \hline - : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_3, F_9 \oplus F_{10} \\ \hline - : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_3, F_9 \oplus F_{10} \\ \hline - : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_3, F_9 \oplus F_{10} \\ \hline \end{array}$$

• Case rule  $\oplus_A$ 

$$\frac{\mathbf{h}_1 : \vdash \{ \mathbf{F}_5, \Delta_7 \} : \Delta_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : ? \mathbf{F}_5, \Delta_4} ? \frac{\mathbf{h}_6 : \vdash \{\Delta_7\} : \mathbf{F}_8, \Delta_{10}, !dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : dual(?\mathbf{F}_5), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \underbrace{- : \vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_4, ? \mathbf{F}_5} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathbf{F}_8}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathbf{F}_8} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathbf{F}_8}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathbf{F}_8} \underbrace{+ A}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathbf{F}_8}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9} \underbrace{+ A}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9}$$

$$\frac{\mathbf{h}_1 : \vdash \{\mathsf{F}_3, \Delta_8\} : \mathsf{F}_6, \Delta_5}{\bullet \mathsf{h}_1 : \vdash \{\Delta_8\} : \mathsf{F}_6, \Delta_5, ?\mathsf{F}_3} ? \frac{\mathbf{h}_7 : \vdash \{\Delta_8\} : \mathsf{F}_9, \Delta_{11}, dual(\mathsf{F}_6)}{\bullet \mathsf{h}_7 : \vdash \{\Delta_8\} : dual(\mathsf{F}_6), \Delta_{11}, \mathsf{F}_9 \oplus \mathsf{F}_{10}} - : \vdash \{\Delta_8\} : (\Delta_5, ?\mathsf{F}_3), \Delta_{11}, \mathsf{F}_9 \oplus \mathsf{F}_{10}}{\bullet \mathsf{h}_1 : \vdash \{\Delta_8\} : \Delta_5, \mathsf{F}_6, ?\mathsf{F}_3} \xrightarrow{\mathsf{ax}} \frac{\rightarrow}{\mathsf{h}_7 : \vdash \{\Delta_8\} : \Delta_{11}, \mathsf{F}_9, dual(\mathsf{F}_6)}} \frac{\mathsf{ax}}{\mathsf{h}\mathsf{Cut}} \\ \frac{- : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, \mathsf{F}_9, ?\mathsf{F}_3}{- : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?\mathsf{F}_3, \mathsf{F}_9 \oplus \mathsf{F}_{10}} \xrightarrow{\mathsf{d}_A} \mathsf{h}\mathsf{Cut}}$$

 $\bullet$  Case rule  $\bot$ 

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : ?\mathsf{F}_5, \Delta_4} ? & \frac{\mathbf{h}_6 : \vdash \{\Delta_7\} : \Delta_8, !dual(\mathtt{F}_5)}{\bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : dual(?\mathtt{F}_5), \bot, \Delta_8} & \bot \\ \hline - : \vdash \{\Delta_7\} : \Delta_4, \bot, \Delta_8 & \to \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5 & \text{ax} & \to \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5 & \text{ax} & \to \\ \hline - : \vdash \{\Delta_7\} : \Delta_4, \Delta_8 \\ \hline - : \vdash \{\Delta_7\} : \Delta_4, \Delta_8 , \bot & \bot \\ \hline \bullet \mathbf{h}_1 : \vdash \{\mathtt{F}_3, \Delta_8\} : \mathtt{F}_6, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_8\} : \mathtt{F}_6, \Delta_5, ?\mathtt{F}_3 & ? & \frac{\mathbf{h}_7 : \vdash \{\Delta_8\} : \Delta_9, dual(\mathtt{F}_6)}{\bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : dual(\mathtt{F}_6), \bot, \Delta_9} & \bot \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_8\} : \Delta_5, \mathtt{F}_6, ?\mathtt{F}_3 & \text{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_8\} : \Delta_5, \mathtt{F}_6, ?\mathtt{F}_3 & \bullet \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_8\} : \Delta_5, \Delta_9, ?\mathtt{F}_3 \\ \hline - : \vdash \{\Delta_8\} : \Delta_5, \Delta_9, \bot, ?\mathtt{F}_3 & \bot \\ \hline \end{array}$$

 $\bullet$  Case rule  $\top$ 

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{ \mathbf{F}_5, \Delta_7 \right\} : \Delta_4}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : ? \mathbf{F}_5, \Delta_4} ? & \frac{\bullet \mathbf{h}_6 : \vdash \left\{ \Delta_7 \right\} : dual(?\mathbf{F}_5), \top, \Delta_8}{\bullet \mathbf{h}_6 : \vdash \left\{ \Delta_7 \right\} : \Delta_4, \top, \Delta_8} & \overset{\top}{\mathsf{Cut}} \\ & \xrightarrow{-} : \vdash \left\{ \Delta_7 \right\} : \Delta_4, \Delta_8, \top & \top \\ \\ \frac{\mathbf{h}_1 : \vdash \left\{ \mathbf{F}_3, \Delta_8 \right\} : \mathbf{F}_6, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_8 \right\} : \mathbf{F}_6, \Delta_5, ? \mathbf{F}_3} ? & \underbrace{\bullet \mathbf{h}_7 : \vdash \left\{ \Delta_8 \right\} : dual(\mathbf{F}_6), \top, \Delta_9}_{-} & \overset{\top}{\mathsf{Cut}} \\ & \xrightarrow{-} : \vdash \left\{ \Delta_8 \right\} : (\Delta_5, ? \mathbf{F}_3), \top, \Delta_9} \\ & \xrightarrow{-} : \vdash \left\{ \Delta_8 \right\} : \Delta_5, \Delta_9, \top, ? \mathbf{F}_3} & \top \end{array}$$

- Case rule  $I_3$
- Case rule  $\otimes$

$$\begin{array}{c} \frac{h_1 : \vdash \{F_5, \Delta_7\} : \Delta_4}{\bullet h_1 : \vdash \{\Delta_7\} : ?F_5, \Delta_4} ? \frac{h_6 : \vdash \{\Delta_7\} : F_8, \Delta_{10}, !dual(F_5) \quad h_6 : \vdash \{\Delta_7\} : F_9, \Delta_{11}}{\bullet h_6 : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \underbrace{Cut} \\ & - : \vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \\ \hline \bullet h_1 : \vdash \{\Delta_7\} : \Delta_4, ?F_5 & ax \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, F_8, !dual(F_5) \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8 \\ \hline \bullet h_1 : \vdash \{F_5, \Delta_7\} : \Delta_4 \\ \hline \bullet h_1 : \vdash \{\Delta_7\} : ?F_5, \Delta_4 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, F_8 \otimes F_9 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, F_8 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, A_1, \Delta_4, F_8 \otimes F_9 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, A_1, \Delta_4, F_8 \otimes F_9 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, A_1, \Delta_4, F_8 \otimes F_9 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, F_8 \otimes F_9 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, F_8 \otimes F_9 \\ \hline \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \{F_3, \Delta_8\} : F_6, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_3 \end{array} ? \begin{array}{c} \mathbf{h}_7 : \vdash \{\Delta_8\} : F_9, \Delta_{11}, dual(F_6) \quad \mathbf{h}_7 : \vdash \{\Delta_8\} : F_{10}, \Delta_{12} \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : (\Delta_5, ?F_3), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10} \\ & \rightarrow \\ \hline \\ \mathbf{h}_1 : \vdash \{\Delta_8, F_3\} : \Delta_5, F_6 \end{array} \text{ax} \begin{array}{c} \bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10} \\ & \rightarrow \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_{12}, dual(F_6), F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, dual(F_6), F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \{F_3, \Delta_8\} : F_6, \Delta_5 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_8\} : (\Delta_5, ?F_3) \end{aligned} ? \begin{array}{c} \mathbf{h}_7 : \vdash \{\Delta_8\} : F_9, \Delta_{11} \quad \mathbf{h}_7 : \vdash \{\Delta_8\} : F_{10}, \Delta_{12}, dual(F_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_{12}, dual(F_6), F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8\} : \Delta_{11}, \Delta_{12}, dual(F_6), F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, dual(F_6), F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, dual(F_6), F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, dual(F_6), F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, dual(F_6), F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, dual(F_6), F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_{12}, \Delta_5, F_9 \otimes F_{10} \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{\Delta_8, F_3\} : \Delta_{11}, \Delta_$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{F_5, F_8, \Delta_9\} : \Delta_4}{\bullet \mathbf{h}_1 : \vdash \{F_8, \Delta_9\} : ?F_5, \Delta_4} ? & \frac{\mathbf{h}_6 : \vdash \{F_8, \Delta_9\} : F_8, \Delta_7, !dual(F_5)}{\bullet \mathbf{h}_6 : \vdash \{F_8, \Delta_9\} : dual(?F_5), \Delta_7} ?_C \\ \hline & - : \vdash \{F_8, \Delta_9\} : \Delta_4, \Delta_7 \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_9, F_8\} : \Delta_4, ?F_5 & \text{ax} \\ \hline & \frac{-}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9, F_8\} : \Delta_4, \Delta_7, F_8} ?_C \\ \hline & \frac{-}{\bullet \mathbf{h}_1 : \vdash \{F_3, F_9, \Delta_{10}\} : F_6, \Delta_5} ?_C \\ \hline \bullet \mathbf{h}_1 : \vdash \{F_9, \Delta_{10}\} : F_6, \Delta_5, ?F_3} ? & \frac{\mathbf{h}_7 : \vdash \{F_9, \Delta_{10}\} : F_9, \Delta_8, dual(F_6)}{\bullet \mathbf{h}_7 : \vdash \{F_9, \Delta_{10}\} : dual(F_6), \Delta_8} ?_C \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_{10}, F_9\} : \Delta_5, F_6, ?F_3} & \text{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_{10}, F_9\} : \Delta_5, F_6, ?F_3} \end{cases} & \frac{\mathbf{h}_7 : \vdash \{F_9, \Delta_{10}\} : dual(F_6), \Delta_8}{\bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} \end{cases} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_{10}, F_9\} : \Delta_5, F_6, ?F_3} & \text{ax} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_5, F_9, ?F_3} ?_C & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_6)} & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_9)} & \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_9)} & \mathbf{h}_7 : \vdash \{\Delta_{10}, F_9\} : \Delta_8, F_9, dual(F_9)} \\ \hline$$

# 7.4 Status of \$: OK

- Case rule 1
- Case rule!
- Case rule?

$$\frac{ \frac{ \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_6, \mathbf{F}_7, \Delta_5 }{ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_6 \$ \mathbf{F}_7, \Delta_5 } }{ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_6 \$ \mathbf{F}_7, \Delta_5 } } \$ \frac{ \mathbf{h}_8 : \vdash \{\mathbf{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathbf{F}_6) \otimes dual(\mathbf{F}_7) }{ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathbf{F}_6 \$ \mathbf{F}_7), \Delta_{11}, ?\mathbf{F}_{10} } } ? \\ \frac{ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathbf{F}_6 \$ \mathbf{F}_7 }{ \bullet \mathbf{h}_1 : \vdash \{\Delta_9, \mathbf{F}_{10}\} : \Delta_5, \mathbf{F}_6 \$ \mathbf{F}_7 } W \frac{ \mathbf{ax} }{ \mathbf{h}_8 : \vdash \{\Delta_9, \mathbf{F}_{10}\} : \Delta_{11}, dual(\mathbf{F}_6) \otimes dual(\mathbf{F}_7) } } \frac{ \mathbf{ax} }{ \mathbf{h}_{11} : \vdash \{\Delta_9\} : \Delta_5, \mathbf{F}_6 \$ \mathbf{F}_7 } } W \frac{ \mathbf{ax} }{ \mathbf{h}_8 : \vdash \{\Delta_9, \mathbf{F}_{10}\} : \Delta_{11}, dual(\mathbf{F}_6) \otimes dual(\mathbf{F}_7) } } \frac{ \mathbf{ax} }{ \mathbf{h}_{11} : \vdash \{\Delta_9\} : \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_7, \Delta_6 } } \frac{ \mathbf{ax} }{ \mathbf{h}_8 : \vdash \{\mathbf{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathbf{F}_7) } ? } \frac{ \mathbf{ax} }{ \mathbf{h}_{11} : \vdash \{\Delta_9\} : \mathbf{F}_7, \Delta_6, \mathbf{F}_3 \$ \mathbf{F}_4 } } \$ \frac{ \mathbf{h}_8 : \vdash \{\mathbf{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathbf{F}_7) }{ \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathbf{F}_7), \Delta_{11}, ?\mathbf{F}_{10} } ? } \frac{ \mathbf{h}_{11} : \vdash \{\Delta_9\} : \mathbf{h}_{11}, \mathbf{h}_$$

• Case rule \$

$$\begin{array}{c} \frac{h_1 : \vdash \left\{ \Delta_9 \right\} : F_6, F_7, \Delta_5}{\bullet h_1 : \vdash \left\{ \Delta_9 \right\} : F_6 \$ F_7, \Delta_5} \quad \$ \quad \frac{h_8 : \vdash \left\{ \Delta_9 \right\} : F_{10}, F_{11}, \Delta_{12}, dual(F_6) \otimes dual(F_7)}{\bullet h_8 : \vdash \left\{ \Delta_9 \right\} : dual(F_6 \$ F_7), \Delta_{12}, F_{10} \$ F_{11}} \quad Cut} \quad \$ \\ \hline - : \vdash \left\{ \Delta_9 \right\} : \Delta_5, \Delta_{12}, F_{10} \$ F_{11} \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_5, F_6 \$ F_7} \quad \text{ax} \quad \frac{\rightarrow h_8 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, F_{10}, F_{11}, dual(F_6) \otimes dual(F_7)}{h_8 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{10}, F_{11}} \quad \$ \\ \hline \frac{- : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{10}, F_{11}}{- : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{10} \$ F_{11}} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : F_7, \Delta_6, F_3 \$ F_4} \quad \$ \quad \frac{h_8 : \vdash \left\{ \Delta_9 \right\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 : \vdash \left\{ \Delta_9 \right\} : dual(F_7), \Delta_{12}, F_{10} \$ F_{11}} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \left\{ \Delta_6, F_3 \$ F_4 \right\} \quad \Rightarrow \quad \frac{\rightarrow h_8 : \vdash \left\{ \Delta_9 \right\} : dual(F_7), \Delta_{12}, F_{10} \$ F_{11}}{\bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{11}, dual(F_7)} \quad \text{ax} \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_6, F_7, F_3 \$ F_4} \quad \Rightarrow \quad \frac{\rightarrow h_8 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, F_{10}, F_{11}, dual(F_7)}{h_8 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4}} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \$ F_4} \quad \$ \\ \hline \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3}, F_{4}, F_{40}, F_{40}, F_{40}, F_{40}, F_{40}, F_{40}, F_{40}, F_{40}, F_{40}, F_{40},$$

• Case rule &

$$\frac{ \begin{array}{c} h_1 : \vdash \left\{ \Delta_9 \right\} : F_6, F_7, \Delta_5 \\ \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : F_6 \$ F_7, \Delta_5 \\ \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : F_6 \$ F_7, \Delta_5 \\ \end{array}{} \$ \begin{array}{c} h_8 : \vdash \left\{ \Delta_9 \right\} : F_{10}, \Delta_{12}, dual(F_6) \otimes dual(F_7) \\ \bullet h_8 : \vdash \left\{ \Delta_9 \right\} : dual(F_6 \$ F_7), \Delta_{12}, F_{10} \& F_{11} \\ \hline \\ - : \vdash \left\{ \Delta_9 \right\} : \Delta_5, \Delta_{12}, F_{10} \& F_{11} \\ \hline \\ - : \vdash \left\{ \Delta_9 \right\} : \Delta_5, F_6 \$ F_7 \end{array} \begin{array}{c} \text{ax} \\ \hline \\ h_8 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, F_{10}, dual(F_6) \otimes dual(F_7) \\ \hline \\ - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{10} \\ \hline \\ - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{10} \\ \hline \\ - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{10} \\ \hline \\ - : \vdash \left\{ \Delta_9 \right\} : F_{11}, \Delta_{12}, dual(F_7) \\ \hline \\ \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : F_{11}, A_{12}, dual(F_7) \\ \hline \\ \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : F_{11}, A_{12}, dual(F_7) \\ \hline \\ \bullet h_2 : \vdash \left\{ \Delta_9 \right\} : A_{12}, A_{12}, F_{11}, dual(F_7) \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{10} \\ \hline \\ \bullet h_1 : \vdash \left\{ \Delta_9 \right\} : A_{12}, A_{12}, dual(F_7) \\ \hline \\ \bullet h_2 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{10} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{3} \$ F_4, A_{12}, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_{3}, F_{4}, F_{7} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3, F_4, F_{10} \& F_{11} \\ \hline \\ \bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_6, F_3$$

• Case rule  $\oplus_B$ 

 $\frac{}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_3 \$ F_4, F_{10} \& F_{11}}$ 

$$\frac{ \frac{h_1 : \vdash \left\{ \Delta_9 \right\} : F_6, F_7, \Delta_5}{\bullet h_1 : \vdash \left\{ \Delta_9 \right\} : F_6 \$ F_7, \Delta_5} }{ \bullet h_2 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, F_{10} \oplus F_{11} } \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, F_{10} \oplus F_{11} } \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, F_{10} \oplus F_{11} } \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, F_{10} \oplus F_{11} } \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, F_{11}, dual(F_6) \otimes dual(F_7)} \xrightarrow{h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{11} \oplus B} \xrightarrow{h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : F_{11}, \Delta_{12}, dual(F_7) \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_{12}, F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11} \oplus B} \xrightarrow{\bullet h_3 : \vdash \left\{ \Delta_$$

• Case rule  $\oplus_A$ 

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{6}, \mathsf{F}_{7}, \Delta_{5}}{\bullet \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{6} \$\mathsf{F}_{7}, \Delta_{5}} \quad \$ \quad \frac{\mathbf{h}_{8} : \vdash \{\Delta_{9}\} : \mathsf{F}_{10}, \Delta_{12}, dual(\mathsf{F}_{6}) \otimes dual(\mathsf{F}_{7})}{\bullet \mathsf{h}_{8} : \vdash \{\Delta_{9}\} : dual(\mathsf{F}_{6} \$\mathsf{F}_{7}), \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11}} \\ \hline - : \vdash \{\Delta_{9}\} : \Delta_{5}, \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \\ \hline \bullet_{\mathsf{h}_{1}} : \vdash \{\Delta_{9}\} : \Delta_{5}, \mathsf{F}_{6} \$\mathsf{F}_{7}} \quad \text{ax} \quad \frac{\rightarrow}{\mathsf{h}_{8} : \vdash \{\Delta_{9}\} : \Delta_{12}, \mathsf{F}_{10}, dual(\mathsf{F}_{6}) \otimes dual(\mathsf{F}_{7})} \\ \hline - : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{5}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \\ \hline - : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{5}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \\ \hline \bullet_{\mathsf{h}_{1}} : \vdash \{\Delta_{9}\} : \mathsf{F}_{3}, \mathsf{F}_{4}, \mathsf{F}_{7}, \Delta_{6} \\ \bullet_{\mathsf{h}_{1}} : \vdash \{\Delta_{9}\} : \mathsf{F}_{7}, \Delta_{6}, \mathsf{F}_{3} \$\mathsf{F}_{4} \\ \hline - : \vdash \{\Delta_{9}\} : (\Delta_{6}, \mathsf{F}_{3} \$\mathsf{F}_{4}), \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \\ \hline - : \vdash \{\Delta_{9}\} : (\Delta_{6}, \mathsf{F}_{3} \$\mathsf{F}_{4}), \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \\ \hline \bullet_{\mathsf{h}_{3}} : \vdash \{\Delta_{9}\} : \Delta_{\mathsf{h}_{5}}, \mathsf{F}_{\mathsf{h}_{7}}, \mathsf{F}_{\mathsf{h}_{7}} \end{array} \qquad \bullet_{\mathsf{h}_{8}} : \vdash \{\Delta_{\mathsf{h}_{9}}\} : \Delta_{\mathsf{h}_{2}}, dual(\mathsf{F}_{\mathsf{h}_{7}}), \mathsf{F}_{\mathsf{h}_{0}} \oplus \mathsf{F}_{\mathsf{h}_{1}} \\ \hline \bullet_{\mathsf{h}_{3}} : \vdash \{\Delta_{\mathsf{h}_{9}}\} : \Delta_{\mathsf{h}_{5}}, \mathsf{F}_{\mathsf{h}_{7}}, \mathsf{F}_{\mathsf{h}_{7}} \end{array} \qquad \bullet_{\mathsf{h}_{8}} : \vdash \{\Delta_{\mathsf{h}_{9}}\} : \Delta_{\mathsf{h}_{2}}, dual(\mathsf{F}_{\mathsf{h}_{7}}), \mathsf{F}_{\mathsf{h}_{0}} \oplus \mathsf{F}_{\mathsf{h}_{1}} \\ \hline \bullet_{\mathsf{h}_{8}} : \vdash \{\Delta_{\mathsf{h}_{9}}\} : \Delta_{\mathsf{h}_{2}}, \mathsf{H}_{\mathsf{h}_{\mathsf{$$

• Case rule  $\perp$ 

• Case rule  $\top$ 

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_6, \mathbf{F}_7, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_6\$\mathbf{F}_7, \Delta_5} \quad \$ \quad & \frac{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathbf{F}_6\$\mathbf{F}_7), \top, \Delta_{10}}{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : \Delta_5, \top, \Delta_{10}} \quad \top \\ & \frac{- : \vdash \left\{\Delta_9\right\} : \Delta_5, \top, \Delta_{10}}{- : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_5, \top} \quad \top \end{array}$$

$$\frac{ \begin{array}{l} \frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_3, \mathbf{F}_4, \mathbf{F}_7, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_7, \Delta_6, \mathbf{F}_3 \$ \mathbf{F}_4} & \$ & \frac{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathbf{F}_7), \top, \Delta_{10}}{- : \vdash \{\Delta_9\} : (\Delta_6, \mathbf{F}_3 \$ \mathbf{F}_4), \top, \Delta_{10}} & \top \\ & \frac{- : \vdash \{\Delta_9\} : (\Delta_6, \mathbf{F}_3 \$ \mathbf{F}_4), \top, \Delta_{10}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, \mathbf{F}_3 \$ \mathbf{F}_4} & \top \end{array} }$$

- Case rule  $I_3$
- Case rule  $\otimes$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{c} \frac{h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_6, F_7, \Delta_5}{\bullet h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_6 \$ F_7, \Delta_5} \quad \$ \quad \begin{array}{c} h_8 : \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_9, dual(F_6) \otimes dual(F_7)}{\bullet h_8 : \vdash \{F_{10}, \Delta_{11}\} : dual(F_6 \$ F_7), \Delta_9} \\ & - : \vdash \{F_{10}, \Delta_{11}\} : \Delta_5, \Delta_9 \\ & \rightarrow \\ \bullet h_1 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, F_6 \$ F_7 \quad \text{ax} \quad \begin{array}{c} \rightarrow \\ h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, F_{10}, dual(F_6) \otimes dual(F_7)} \\ \hline & - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, \Delta_9, F_{10} \\ & - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, \Delta_9, F_{10} \\ \hline & - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, \Delta_9 \end{array} \quad ?_C \\ \hline \\ \frac{h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_3, F_4, F_7, \Delta_6}{\bullet h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_7, \Delta_6, F_3 \$ F_4} \quad \$ \quad \begin{array}{c} h_8 : \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_9, dual(F_7) \\ \bullet h_8 : \vdash \{F_{10}, \Delta_{11}\} : dual(F_7), \Delta_9 \\ \hline & \bullet h_8 : \vdash \{F_{10}, \Delta_{11}\} : dual(F_7), \Delta_9 \end{array} \quad ?_C \\ \hline \\ \frac{h_1 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_3, \$ F_4, F_7}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, Hall(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7)} \quad \text{ax} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, \Delta_9, F_3, F_4}{\bullet h_8 : \vdash \{\Delta_{11},$$

# 7.5 Status of &: OK

- Case rule 1
- Case rule!
- Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{6}, \Delta_{5} \quad \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{7}, \Delta_{5}}{\bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{6} \& \mathsf{F}_{7}, \Delta_{5}} \quad \& \quad \frac{\mathsf{h}_{8} : \vdash \left\{ \mathsf{F}_{10}, \Delta_{9} \right\} : \Delta_{11}, dual(\mathsf{F}_{6}) \oplus dual(\mathsf{F}_{7})}{\bullet \mathsf{h}_{8} : \vdash \left\{ \Delta_{9} \right\} : dual(\mathsf{F}_{6} \& \mathsf{F}_{7}), \Delta_{11}, ?\mathsf{F}_{10}} \quad ?} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{5}, \Delta_{11}, ?\mathsf{F}_{10} \\ & \rightarrow \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \Delta_{5}, \mathsf{F}_{6} \& \mathsf{F}_{7}} \quad \mathsf{ax} \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \Delta_{5}, \mathsf{F}_{6} \& \mathsf{F}_{7}} \quad W \quad \frac{\mathsf{h}_{8} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, dual(\mathsf{F}_{6}) \oplus dual(\mathsf{F}_{7})}{\mathsf{h}_{Cut}} \quad \mathsf{hCut} \\ & - : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, \Delta_{5}, ?\mathsf{F}_{10}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{3}, \mathsf{F}_{7}, \Delta_{6} \quad \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{4}, \mathsf{F}_{7}, \Delta_{6}} \quad \& \quad \frac{\mathsf{h}_{8} : \vdash \left\{ \mathsf{F}_{10}, \Delta_{9} \right\} : \Delta_{11}, dual(\mathsf{F}_{7})}{\bullet \mathsf{h}_{8} : \vdash \left\{ \Delta_{9} \right\} : dual(\mathsf{F}_{7}), \Delta_{11}, ?\mathsf{F}_{10}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{7}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad \Rightarrow \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad \mathsf{ax} \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad W \quad \mathsf{h}_{8} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, dual(\mathsf{F}_{7}) \quad \mathsf{hCut} \\ & - : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, \Delta_{6}, \mathsf{F}_{7} \& \mathsf{F}_{4}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4}} \quad ? \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9}, \mathsf{F}_{10} \right\} : \Delta_{11}, \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{10} \quad ? \\ & \bullet \mathsf{h}_{1}$$

• Case rule \$

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6, \Delta_5 \quad \mathsf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_5}{\underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6 \& \mathsf{F}_7, \Delta_5}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_{11}, \Delta_{12}, dual(\mathsf{F}_6) \oplus dual(\mathsf{F}_7)}} \\ \underbrace{- : \vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathsf{F}_{10} \$ \mathsf{F}_{11}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \& \mathsf{F}_7} \xrightarrow{\mathsf{ax}} \underbrace{- : \vdash \{\Delta_9\} : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_{11}, dual(\mathsf{F}_6) \oplus dual(\mathsf{F}_7)}_{\mathsf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathsf{F}_{10}, \mathsf{F}_{11}} \xrightarrow{\mathsf{ax}}_{\mathsf{hCut}} \\ \underbrace{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathsf{F}_{10}, \mathsf{F}_{11}}_{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathsf{F}_{10}, \mathsf{F}_{11}} \$$$

$$\frac{\mathbf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{3}, \mathsf{F}_{7}, \Delta_{6} \quad \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{4}, \mathsf{F}_{7}, \Delta_{6}}{\bullet \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{10}, \mathsf{F}_{11}, \Delta_{12}, dual(\mathsf{F}_{7})} \underbrace{\bullet \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{7}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}}_{\bullet \mathsf{h}_{3} : \vdash \{\Delta_{9}\} : dual(\mathsf{F}_{7}), \Delta_{12}, \mathsf{F}_{10} \$ \mathsf{F}_{11}}} \circ \mathsf{Cut}}_{\bullet \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4}\}} \underbrace{\bullet \mathsf{h}_{2} : \Delta_{6}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_{10} \$ \mathsf{F}_{11}}_{\bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_{11}, \mathsf{dual}(\mathsf{F}_{7})} \circ \mathsf{hCut}} \underbrace{\bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_{3} \& \mathsf{F}_{4}}_{\bullet \mathsf{hCut}}}_{\bullet \mathsf{hCut}}$$

### • Case rule &

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6 \& \mathsf{F}_7, \Delta_5} \quad \& \quad \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : \mathsf{F}_{10}, \Delta_{12}, dual(\mathsf{F}_6) \oplus dual(\mathsf{F}_7) \quad \mathbf{h}_8 : \vdash \{\Delta_9\} : \mathsf{F}_{11}, \Delta_{12}, dual(\mathsf{F}_6) \oplus dual(\mathsf{F}_7)}{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathsf{F}_{10} \& \mathsf{F}_{11}} \quad \mathsf{Cut} \\ & - : \vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathsf{F}_{10} \& \mathsf{F}_{11} \\ & - : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \& \mathsf{F}_7 \quad \mathsf{ax} \quad \frac{\mathsf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \mathsf{F}_{10}, dual(\mathsf{F}_6) \oplus dual(\mathsf{F}_7)}{\bullet \mathsf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_{10} \& \mathsf{F}_{11}} \quad \mathsf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathsf{F}_{10} & - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathsf{F}_{11} \\ & - : \vdash \{\Delta_9\} : \mathsf{F}_{11}, \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{11}, \mathsf{F}_{12}, \mathsf{F}_{12}, \mathsf{F}_{11}, \mathsf{F}_{12}, \mathsf{F}_{12}, \mathsf{F}_{12}, \mathsf{F}_{11}, \mathsf{F}_{12}, \mathsf{F}_$$

#### • Case rule $\oplus_B$

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{6}, \Delta_{5} \quad \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{7}, \Delta_{5}}{\bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{6} \& \mathsf{F}_{7}, \Delta_{5}} \quad \& \quad \frac{\mathsf{h}_{8} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{dual}(\mathsf{F}_{6} \& \mathsf{F}_{7}), \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11}}{\bullet \mathsf{h}_{3} : \vdash \left\{ \Delta_{9} \right\} : \Delta_{5}, \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11}} \quad \Box \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{5}, \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{5}, \mathsf{F}_{11} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{5}, \mathsf{F}_{11} \right] \quad \oplus B \\ \\ \frac{\mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{6}, \Delta_{5} \quad \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{7}, \Delta_{5} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{5}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \end{array} \oplus B \\ \\ \frac{\mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{6}, \Delta_{5} \quad \mathsf{h}_{1} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{7}, \Delta_{5} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{10}, \mathsf{dual}(\mathsf{F}_{7}) \end{array} \oplus \frac{\mathsf{dual}(\mathsf{F}_{7})}{\mathsf{dual}(\mathsf{F}_{6} \& \mathsf{F}_{7}), \Delta_{10}} \quad \Box \\ \\ - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{5}, \Delta_{10} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{5}, \Delta_{10} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{5}, \Delta_{10} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{10}, \mathsf{dual}(\mathsf{F}_{7}) \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{10}, \mathsf{dual}(\mathsf{F}_{7}) \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{10}, \mathsf{dual}(\mathsf{F}_{7}) \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{10}, \mathsf{dual}(\mathsf{F}_{7}) \\ & \bullet \mathsf{h}_{8} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{F}_{11}, \Delta_{12}, \mathsf{dual}(\mathsf{F}_{7}) \\ & - : \vdash \left\{ \Delta_{9} \right\} : (\Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}) \\ & - : \vdash \left\{ \Delta_{9} \right\} : (\Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}) \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{10}, \mathsf{dual}(\mathsf{F}_{7}) \\ & \bullet \mathsf{h}_{8} : \vdash \left\{ \Delta_{9} \right\} : \mathsf{dual}(\mathsf{F}_{7}), \Delta_{12}, \mathsf{f}_{10} \oplus \mathsf{F}_{11} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{11}, \mathsf{F}_{3} \& \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{11}, \mathsf{F}_{3} \& \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{11}, \mathsf{F}_{3} \& \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{11}, \mathsf{F}_{3} \& \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{11}, \mathsf{F}_{3} \& \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{11}, \mathsf{F}_{3} \& \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{9} \right\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{11}, \mathsf{F}_{3} \& \mathsf{F}_{4} \\ & - : \vdash \left$$

### • Case rule $\oplus_A$

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_5}{\underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6 \& \mathsf{F}_7, \Delta_5}_{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathsf{F}_6 \& \mathsf{F}_7), \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11}}}{\underbrace{- : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \& \mathsf{F}_7}_{\bullet \mathbf{k} : \vdash \{\Delta_9\} : \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11}}}^{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_{10} \oplus \mathsf{F}_{11}} \underbrace{- : \vdash \{\Delta_9\} : \Delta_{12}, \mathsf{F}_{10}, dual(\mathsf{F}_6) \oplus dual(\mathsf{F}_7)}_{\bullet \mathbf{k} : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathsf{F}_{10}}_{\bullet \mathsf{Cut}}}_{\bullet \mathsf{Cut}}$$

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{6}, \Delta_{5} \quad \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{7}, \Delta_{5}}{\bullet \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{6} \& \mathsf{F}_{7}, \Delta_{5}} \quad \& \quad \frac{\mathbf{h}_{8} : \vdash \{\Delta_{9}\} : \Delta_{10}, dual(\mathsf{F}_{6})}{\bullet \mathsf{h}_{8} : \vdash \{\Delta_{9}\} : dual(\mathsf{F}_{6} \& \mathsf{F}_{7}), \Delta_{10}} \quad \bigoplus_{\mathsf{Cut}} \\ & - : \vdash \{\Delta_{9}\} : \Delta_{5}, \Delta_{10} \\ & - : \vdash \{\Delta_{9}\} : \Delta_{5}, \mathsf{F}_{6} \quad \text{ax} \quad \xrightarrow{- : \vdash \{\Delta_{9}\} : \Delta_{10}, dual(\mathsf{F}_{6})} \quad \mathsf{sCut}} \\ \frac{\mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \Delta_{5}, \mathsf{F}_{6} \quad \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \Delta_{10}, \Delta_{5}} \\ \bullet \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{3}, \mathsf{F}_{7}, \Delta_{6} \quad \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \mathsf{F}_{4}, \mathsf{F}_{7}, \Delta_{6} \quad & \quad \mathsf{h}_{8} : \vdash \{\Delta_{9}\} : \mathsf{F}_{10}, \Delta_{12}, dual(\mathsf{F}_{7}) \\ \bullet \mathsf{h}_{8} : \vdash \{\Delta_{9}\} : dual(\mathsf{F}_{7}), \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \\ & - : \vdash \{\Delta_{9}\} : (\Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}), \Delta_{12}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \quad & \bullet \\ \bullet \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \Delta_{6}, \mathsf{F}_{7}, \mathsf{F}_{3} \& \mathsf{F}_{4} \quad & \bullet \\ \bullet \mathsf{h}_{1} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{10}, \mathsf{F}_{3} \& \mathsf{F}_{4} \\ & - : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{10}, \mathsf{F}_{3} \& \mathsf{F}_{4} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{10}, \mathsf{F}_{3} \& \mathsf{F}_{4} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{F}_{4}, \mathsf{F}_{10} \oplus \mathsf{F}_{11} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \vdash \{\Delta_{9}\} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{h}_{2} : \Delta_{12}, \Delta_{6}, \mathsf{F}_{3} \& \mathsf{h}_{2} \quad & \bullet \\ \bullet \mathsf{h}_{2} : \Delta_{12}, \Delta_{12}$$

### • Case rule $\perp$

$$\begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_6, \Delta_5 \quad \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_7, \Delta_5}_{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_6 \& \mathbf{F}_7, \Delta_5} \quad \& \quad & \underbrace{\begin{array}{c} \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, dual(\mathbf{F}_6) \oplus dual(\mathbf{F}_7)}_{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathbf{F}_6 \& \mathbf{F}_7), \bot, \Delta_{10}} \\ & - : \vdash \left\{\Delta_9\right\} : \Delta_5, \bot, \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_5, \mathbf{F}_6 \& \mathbf{F}_7 & \mathbf{ax} \\ \hline \\ - : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_5, \bot \\ \hline \\ - : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_5, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_3, \mathbf{F}_7, \Delta_6 \quad \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_4, \mathbf{F}_7, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_3, \mathbf{F}_7, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline \\ - : \vdash \left\{\Delta_9\right\} : \left(\Delta_6, \mathbf{F}_3 \& \mathbf{F}_4\right), \bot, \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4, \bot, \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline \\ - : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline \\ - : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \bot, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline \\ - : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \bot, \mathbf{F}_3 \& \mathbf{F}_4 \\ \hline \end{array}$$

## $\bullet$ Case rule $\top$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_6, \Delta_5 \quad \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_7, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_6 \& \mathsf{F}_7, \Delta_5} \quad \& \quad \frac{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathsf{F}_6 \& \mathsf{F}_7), \top, \Delta_{10}}{- : \vdash \left\{\Delta_9\right\} : \Delta_5, \top, \Delta_{10}} \quad \mathsf{Cut} \\ & \xrightarrow{- : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_5, \top} \quad \top \\ \\ \frac{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_3, \mathsf{F}_7, \Delta_6 \quad \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_4, \mathsf{F}_7, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_7, \Delta_6, \mathsf{F}_3 \& \mathsf{F}_4} \quad & \bullet_{\mathsf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathsf{F}_7), \top, \Delta_{10}} \\ & \xrightarrow{- : \vdash \left\{\Delta_9\right\} : \left(\Delta_6, \mathsf{F}_3 \& \mathsf{F}_4\right), \top, \Delta_{10}} \quad & \to \\ & \xrightarrow{- : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \top, \mathsf{F}_3 \& \mathsf{F}_4} \quad \top \\ \end{array}$$

- Case rule  $I_3$
- Case rule  $\otimes$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : F_6, \Delta_5 \quad \mathbf{h}_1 : \vdash \{\Delta_9\} : F_7, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_5} & \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : F_{10}, \Delta_{12} \quad \mathbf{h}_8 : \vdash \{\Delta_9\} : f_{11}, \Delta_{13}, dual(F_6) \oplus dual(F_7)}{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(F_6 \& F_7), \Delta_{12}, \Delta_{13}, F_{10} \otimes F_{11}} & \mathbf{Cut} \\ & - : \vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \Delta_{13}, F_{10} \otimes F_{11} \\ & - : \vdash \{\Delta_9\} : \Delta_{12}, F_{10} & \mathbf{ax} & \frac{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, F_6 \& F_7}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_{13}, \Delta_5, F_{11}} \otimes \\ & - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_5, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_{12}, A_{13}, \Delta_5, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : F_7, A_6 \quad \mathbf{h}_1 : \vdash \{\Delta_9\} : F_4, F_7, \Delta_6 & \mathbf{h}_8 : \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7) \quad \mathbf{h}_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{13} \\ & \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : A_6, F_7, F_3 \& F_4 & \mathbf{ax} & \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, \Delta_{13}, F_{10} \otimes F_{11} & \mathbf{cut} \\ & - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_3 \& F_4 & \mathbf{ax} & \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, A_{13}, \Delta_6, F_3 \& F_4, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_3 \& F_4 & \mathbf{h}_8 : \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{13}, F_{11} \otimes \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_3 \& F_4 & \mathbf{h}_8 : \vdash \{\Delta_9\} : A_{13}, F_{11} \otimes \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, F_3 \& F_4, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : A_{12}, A_{13}, A_6, F_3 \& F_4, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : A_{13}, A_{13}, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_6, F_7, F_3 \& F_4 & \mathbf{h}_8 : \vdash \{\Delta_9\} : A_{13}, F_{11}, \Delta_{13}, dual(F_7) & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_1, A_1, A_1, A_2, A_1, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : A_1, A_1, A_1, A_2, A_1, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_1, F_{10}, A_1, A_2, A_1, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_1, F_{10}, A_1, A_2, A_1, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_1, F_{10}, A_1, A_2, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_1, F_{10}, A_1, A_2, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_1, A_1, A_2, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash \{\Delta_9\} : \Delta_1, A_2, F_{10} \otimes F_{11} & \mathbf{ax} \\ & - : \vdash$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\frac{h_{1} : \vdash \{F_{10}, \Delta_{11}\} : F_{6}, \Delta_{5} \quad h_{1} : \vdash \{F_{10}, \Delta_{11}\} : F_{7}, \Delta_{5}}{\bullet h_{1} : \vdash \{F_{10}, \Delta_{11}\} : F_{6}\&F_{7}, \Delta_{5}} \quad \& \quad \frac{h_{8} : \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_{9}, dual(F_{6}) \oplus dual(F_{7})}{\bullet h_{8} : \vdash \{F_{10}, \Delta_{11}\} : dual(F_{6}\&F_{7}), \Delta_{9}} \quad Cut} \quad ?_{C} \\ \hline - : \vdash \{F_{10}, \Delta_{11}\} : \Delta_{5}, \Delta_{9} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{5}, F_{6}\&F_{7}} \quad \text{ax} \quad \frac{\bullet h_{1} : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{9}, F_{10}, dual(F_{6}) \oplus dual(F_{7})}{\bullet h_{1} : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{5}, \Delta_{9}} \quad ?_{C} \\ \hline \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{5}, \Delta_{9} \quad P_{10}}{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{5}, \Delta_{9}} \quad ?_{C} \\ \hline \frac{\bullet h_{1} : \vdash \{F_{10}, \Delta_{11}\} : F_{7}, \Delta_{6} \quad h_{1} : \vdash \{F_{10}, \Delta_{11}\} : F_{7}, \Delta_{6}}{\bullet h_{1} : \vdash \{F_{10}, \Delta_{11}\} : F_{7}, \Delta_{6}} \quad \& \quad \frac{h_{8} : \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_{9}, dual(F_{7})}{\bullet h_{8} : \vdash \{F_{10}, \Delta_{11}\} : dual(F_{7}), \Delta_{9}} \quad Cut} \\ \hline - : \vdash \{F_{10}, \Delta_{11}\} : (\Delta_{6}, F_{3}\&F_{4}), \Delta_{9}} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{10}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{10}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{6}, \Delta_{9}, F_{3}\&F_{4}} \quad ?_{C} \\ \hline - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{11}, F_{10}\} : \Delta_{11}, F_{11},$$

## 7.6 Status of $\oplus_B$ : OK

- Case rule 1
- Case rule!
- Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6 \oplus \mathsf{F}_7, \Delta_5} & \oplus_B & \frac{\mathbf{h}_8 : \vdash \{\mathsf{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathsf{F}_6) \& dual(\mathsf{F}_7)}{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathsf{F}_6 \oplus \mathsf{F}_7), \Delta_{11}, ?\mathsf{F}_{10}} & ?\\ \hline & - : \vdash \{\Delta_9\} : \Delta_5, \Delta_{11}, ?\mathsf{F}_{10} & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 & \mathbf{w} & \frac{\mathbf{a}_8 : \vdash \{\Delta_9, \mathsf{F}_{10}\} : \Delta_{11}, dual(\mathsf{F}_6) \& dual(\mathsf{F}_7)}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9, \mathsf{F}_{10}\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7} & W & \frac{- : \vdash \{\Delta_9, \mathsf{F}_{10}\} : \Delta_{11}, \Delta_5}{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_5, ?\mathsf{F}_{10}} ?\\ \hline & \frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_4, \mathsf{F}_7, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_6, \mathsf{F}_3 \oplus \mathsf{F}_4} & \oplus_B & \frac{\mathbf{h}_8 : \vdash \{\mathsf{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathsf{F}_7)}{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathsf{F}_7), \Delta_{11}, ?\mathsf{F}_{10}} & ?\\ \hline & \frac{- : \vdash \{\Delta_9\} : (\Delta_6, \mathsf{F}_3 \oplus \mathsf{F}_4), \Delta_{11}, ?\mathsf{F}_{10}}{\bullet \mathsf{h}_8 : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}} & \overset{\mathsf{ax}}{\mathsf{hCut}} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}}{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}} & \oplus_B & \overset{\mathsf{hCut}}{\mathsf{hCut}} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}}{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}} & \oplus_B & \overset{\mathsf{hCut}}{\mathsf{hCut}} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}}{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}} & \oplus_B & \overset{\mathsf{hCut}}{\mathsf{hCut}} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}}{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}} & \oplus_B & \overset{\mathsf{hCut}}{\mathsf{hCut}} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}}{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}} & \oplus_B & \overset{\mathsf{hCut}}{\mathsf{hCut}} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}}{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}} & \oplus_B & \overset{\mathsf{hCut}}{\mathsf{hCut}} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}}{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}} & \oplus_B & \overset{\mathsf{hCut}}{\mathsf{hCut}} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}}{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}} & \overset{\mathsf{hCut}}{\mathsf{hCut}} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathsf{F}_4, ?\mathsf{F}_{10}, \mathsf{F}_4, \mathsf{F}_4, \mathsf{F}_{10}, \mathsf{F}_4, \mathsf{F}_4, \mathsf{F}_{10}, \mathsf{F}$$

# • Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_7, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_6 \oplus \mathbf{F}_7, \Delta_5} \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_6 \oplus \mathbf{F}_7, \Delta_5} \\ & \bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathbf{F}_6 \oplus \mathbf{F}_7), \Delta_{12}, \mathbf{F}_{10} \$ \mathbf{F}_{11} \\ & - : \vdash \left\{\Delta_9\right\} : \Delta_5, \Delta_{12}, \mathbf{F}_{10} \$ \mathbf{F}_{11} \\ & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7} \\ & \mathbf{ax} \\ & \frac{- : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_{11}, dual(\mathbf{F}_6) \& dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_{11}, dual(\mathbf{F}_6) \& dual(\mathbf{F}_7)} \\ & \frac{- : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_5, \mathbf{F}_{10}, \mathbf{F}_{11}}{- : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_5, \mathbf{F}_{10} \$ \mathbf{F}_{11}} \\ & \frac{\mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_4, \mathbf{F}_7, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \left(\Delta_6, \mathbf{F}_3 \oplus \mathbf{F}_4\right), \Delta_{12}, \mathbf{F}_{10} \$ \mathbf{F}_{11}} \\ & - : \vdash \left\{\Delta_9\right\} : \left(\Delta_6, \mathbf{F}_3 \oplus \mathbf{F}_4\right), \Delta_{12}, \mathbf{F}_{10} \$ \mathbf{F}_{11} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_6, \mathbf{F}_7, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_6, \mathbf{F}_7, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_6, \mathbf{F}_7, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_6, \mathbf{F}_7, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ & \frac{\rightarrow}{\bullet \mathbf{h}_2 : \vdash \left\{\Delta_9\right\} : \Delta_{12}, \Delta_6, \mathbf{F}_{10$$

### • Case rule &

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : F_7, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_5} \oplus_B \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \& F_{11}} \xrightarrow{\mathbf{Cut}} \mathbb{C}$$

$$\frac{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, F_6 \oplus F_7}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_1, F_{10}, dual(F_6) \& dual(F_7)} \xrightarrow{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_1, F_{10} \& F_{11}} \xrightarrow{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, F_6 \oplus F_7} \mathbb{A} \times \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \& dual(F_7)}{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_1, \Delta_5, F_{10}} \xrightarrow{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_1, \Delta_5, F_{10}} \xrightarrow{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_1, \Delta_5, F_{10}} \mathbb{A} \times \frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_1, \Delta_5, F_{10} \& F_{11}}{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, F_7, \Delta_5} \oplus_B \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6)}{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(F_6) \oplus F_7, \Delta_{10}} \xrightarrow{\mathbf{Cut}} \mathbb{A} \times \frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, F_7}{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \Delta_{10}} \times \frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, F_7}{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_1, \Delta_5, F_7} \xrightarrow{\mathbf{a}} \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_5} \times \mathbb{A} \times \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7)}{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_1, \Delta_5, F_{10} \& F_{11}} \times \mathbb{A} \times \mathbb{A}$$

• Case rule  $\oplus_B$ 

• Case rule  $\oplus_A$ 

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_7, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_6 \oplus \mathbf{F}_7, \Delta_5 \end{array} \oplus_B & \begin{array}{c} \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : \mathbf{F}_{10}, \Delta_{12}, dual(\mathbf{F}_6) \& dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathbf{F}_6 \oplus \mathbf{F}_7), \Delta_{12}, \mathbf{F}_{10} \oplus \mathbf{F}_{11}} \\ \hline - : \vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathbf{F}_{10} \oplus \mathbf{F}_{11} & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} & \begin{array}{c} \mathbf{ax} \\ \hline \bullet \mathbf{h}_2 : \vdash \{\Delta_9\} : \Delta_{12}, \mathbf{F}_{10}, dual(\mathbf{F}_6) \& dual(\mathbf{F}_7)} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathbf{F}_{10} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \end{array} & \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_C \mathbf{ut} \end{array} \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_4, \mathbf{F}_7, \Delta_6 \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_7, \Delta_6, \mathbf{F}_3 \oplus \mathbf{F}_4 \end{array} & \begin{array}{c} \mathbf{h}_8 : \vdash \{\Delta_9\} : \mathbf{F}_{10}, \Delta_{12}, dual(\mathbf{F}_7) \\ \hline \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathbf{F}_7), \Delta_{12}, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \end{array} & \begin{array}{c} \oplus A \\ \mathbf{Cut} \end{array} \\ \hline - : \vdash \{\Delta_9\} : (\Delta_6, \mathbf{F}_3 \oplus \mathbf{F}_4), \Delta_{12}, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline \bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{12}, dual(\mathbf{F}_7), \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{11} \\ \hline - : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathbf{F}_4, \mathbf{F}_{10} \oplus \mathbf{F}_{$$

 $\bullet$  Case rule  $\bot$ 

$$\begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_5}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6 \oplus \mathsf{F}_7, \Delta_5} \end{array} \oplus_B & \underbrace{\begin{array}{c} \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(\mathsf{F}_6) \& dual(\mathsf{F}_7)}_{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathsf{F}_6 \oplus \mathsf{F}_7), \bot, \Delta_{10}} \end{array}}_{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(\mathsf{F}_6) \& dual(\mathsf{F}_7)} \end{array}} \\ \underbrace{\begin{array}{c} - : \vdash \{\Delta_9\} : \Delta_5, \bot, \Delta_{10} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array}}_{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{10}, dual(\mathsf{F}_6) \& dual(\mathsf{F}_7)} \end{array}}_{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \bot} \\ \underbrace{\begin{array}{c} - : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_5 \\ - : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \bot \end{array}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_4, \mathsf{F}_7, \Delta_6} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_4, \mathsf{F}_7, \Delta_6 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_6, \mathsf{F}_3 \oplus \mathsf{F}_4 \end{array}}_{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathsf{F}_7), \bot, \Delta_{10} \\ \underbrace{\begin{array}{c} - : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{10}, \bot, dual(\mathsf{F}_7) \end{array}}_{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{10}, \bot, dual(\mathsf{F}_7) \end{array}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \bot, dual(\mathsf{F}_7) \end{array}}_{\bullet \mathbf{h}_2 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathsf{F}_4, \bot}_{\bullet \mathbf{h}_3 : \bot}_{\bullet \mathbf$$

 $\bullet$  Case rule  $\top$ 

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_7, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_6 \oplus \mathbf{F}_7, \Delta_5} \overset{\oplus B}{} & \frac{}{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathbf{F}_6 \oplus \mathbf{F}_7), \top, \Delta_{10}} \\ & - : \vdash \left\{\Delta_9\right\} : \Delta_5, \top, \Delta_{10} \\ & - : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_5, \top \end{array} \overset{\top}{} \\ \frac{\mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_4, \mathbf{F}_7, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_7, \Delta_6, \mathbf{F}_3 \oplus \mathbf{F}_4} \overset{\oplus B}{} & \frac{}{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathbf{F}_7), \top, \Delta_{10}} \\ & - : \vdash \left\{\Delta_9\right\} : (\Delta_6, \mathbf{F}_3 \oplus \mathbf{F}_4), \top, \Delta_{10} \\ & - : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \top, \mathbf{F}_3 \oplus \mathbf{F}_4} \end{aligned} \overset{\top}{} \\ \text{Cut}$$

- Case rule  $I_3$
- Case rule  $\otimes$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \mathsf{F}_7, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \mathsf{F}_6 \oplus \mathsf{F}_7, \Delta_5} \quad \oplus_B \quad \frac{\mathbf{h}_8 : \vdash \left\{ \Delta_9 \right\} : \mathsf{f}_{10}, \Delta_{12}, \mathsf{dual}(\mathsf{F}_6) \& \mathsf{dual}(\mathsf{F}_7) \quad \mathbf{h}_8 : \vdash \left\{ \Delta_9 \right\} : \mathsf{F}_{11}, \Delta_{13}}{\bullet \mathbf{h}_8 : \vdash \left\{ \Delta_9 \right\} : \mathsf{dual}(\mathsf{F}_6 \oplus \mathsf{F}_7), \Delta_{12}, \Delta_{13}, \mathsf{F}_{10} \otimes \mathsf{F}_{11}} \\ \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_5, \Delta_{12}, \Delta_{13}, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \right) \quad \mathbf{ax} \\ \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_5, \mathsf{F}_{10} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \mathsf{F}_7, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \mathsf{F}_7, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \mathsf{F}_7, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_{13}, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \mathsf{F}_{10} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_{13}, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \mathsf{F}_7, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \mathsf{F}_{10} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_{13}, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \mathsf{F}_{10} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_{13}, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \mathsf{F}_{10} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \mathsf{F}_{10} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \mathsf{F}_7, \Delta_6, \mathsf{F}_3 \oplus \mathsf{F}_4 \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_6, \mathsf{F}_4, \mathsf{F}_7 & \mathsf{ax} \\ \hline \quad \bullet \mathbf{h}_2 : \vdash \left\{ \Delta_9 \right\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_12, \Delta_{13}, \Delta_6, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_12, \Delta_{13}, \Delta_6, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_12, \Delta_{13}, \Delta_6, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_6, \mathsf{F}_4, \mathsf{F}_7 & \mathsf{ax} \\ \hline \quad \bullet \mathbf{h}_2 : \vdash \left\{ \Delta_9 \right\} : \Delta_12, \Delta_{13}, \Delta_6, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \quad - : \vdash \left\{ \Delta_9 \right\} : \Delta_6, \mathsf{F}_4, \mathsf{F}_7 & \mathsf{ax} \\ \hline \quad \bullet \mathbf{h}_2 : \vdash \left\{ \Delta_9 \right\} : \Delta_12, \Delta_13, \Delta_6, \mathsf{F}_{10} \otimes \mathsf{F}_{11} \\ \hline \quad \bullet \mathbf{h}_2 : \Delta_1$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\frac{ \begin{array}{c} h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_7, \Delta_5 \\ \bullet h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_6 \oplus F_7, \Delta_5 \end{array} \oplus_B \begin{array}{c} h_8 : \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_9, dual(F_6) \& dual(F_7) \\ \bullet h_8 : \vdash \{F_{10}, \Delta_{11}\} : dual(F_6 \oplus F_7), \Delta_9 \end{array} \\ \hline - : \vdash \{F_{10}, \Delta_{11}\} : \Delta_5, \Delta_9 \\ \hline \\ \bullet h_1 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, F_6 \oplus F_7 \end{array} \begin{array}{c} ax \\ \hline \\ h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, F_{10}, dual(F_6) \& dual(F_7) \\ \hline \\ \hline \\ - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, \Delta_9, F_{10} \\ \hline \\ - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, \Delta_9 \end{array} \begin{array}{c} c \\ \hline \\ h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_4, F_7, \Delta_6 \\ \hline \\ \bullet h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_7, \Delta_6, F_3 \oplus F_4 \end{array} \oplus_B \begin{array}{c} h_8 : \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_9, dual(F_7) \\ \hline \\ \bullet h_8 : \vdash \{F_{10}, \Delta_{11}\} : dual(F_7), \Delta_9 \\ \hline \\ - : \vdash \{F_{10}, \Delta_{11}\} : (\Delta_6, F_3 \oplus F_4), \Delta_9 \end{array} \begin{array}{c} c \\ \hline \\ \bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7) \\ \hline \\ - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_4 \\ \hline \\ - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_4 \end{array} \oplus_B \end{array} \begin{array}{c} h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7) \\ h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7) \\ \hline \\ - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_4 \\ \hline \\ - : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_4 \end{array} \oplus_B \end{array} \begin{array}{c} h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7) \\ h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7) \\ h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7) \\ \hline \end{array} \begin{array}{c} ax \\ h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7) \\ h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, dual(F_7) \\ \hline \end{array}$$

## 7.7 Status of $\oplus_A$ : OK

- $\bullet$  Case rule 1
- Case rule!
- Case rule?

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6, \Delta_5 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_6 \oplus \mathsf{F}_7, \Delta_5 \end{array} \oplus \mathbf{A} \\ \bullet \mathbf{h}_8 : \vdash \{\mathsf{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathsf{F}_6) \& dual(\mathsf{F}_7) \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathsf{F}_6 \oplus \mathsf{F}_7), \Delta_{11}, ?\mathsf{F}_{10} \\ & \rightarrow \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \xrightarrow{\mathbf{ax}} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \xrightarrow{\mathbf{ax}} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \xrightarrow{\mathbf{ax}} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \xrightarrow{\mathbf{ax}} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \xrightarrow{\mathbf{ax}} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \xrightarrow{\mathbf{ax}} \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \xrightarrow{\mathbf{ax}} \xrightarrow{\mathbf{h}_8 : \vdash \{\Delta_9, \mathsf{F}_{10}\} : \Delta_{11}, \Delta_5} ?$$

$$\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_3, \mathsf{F}_7, \Delta_6 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_6, \mathsf{F}_3 \oplus \mathsf{F}_4 \end{array} \xrightarrow{\mathbf{ax}} \xrightarrow{\mathbf{h}_8 : \vdash \{\mathsf{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathsf{F}_7)} ?$$

$$\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_6, \mathsf{F}_3 \oplus \mathsf{F}_4 \end{array} \xrightarrow{\mathbf{h}_8 : \vdash \{\mathsf{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathsf{F}_7)} ?$$

$$\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \Delta_6, \mathsf{F}_3 \oplus \mathsf{F}_4 \end{array} \xrightarrow{\mathbf{h}_8 : \vdash \{\mathsf{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathsf{F}_7)} ?$$

$$\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathsf{F}_7, \mathsf{F}_7$$

• Case rule \$

$$\begin{array}{c} \frac{h_1 : \vdash \{\Delta_9\} : F_6, \Delta_5}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_5} \ \oplus_A \ & \frac{h_8 : \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \$ F_{11}} \ & Cut \\ \hline & - : \vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, F_{10} \$ F_{11} \\ & \rightarrow \\ \hline \bullet h_1 : \vdash \{\Delta_9\} : \Delta_5, F_6 \oplus F_7 \end{array} \ & \text{ax} \ & \frac{\rightarrow}{h_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_6) \& dual(F_7)} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, F_{10}, F_{11}}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, F_{10}, F_{11}} \ & \$ \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, F_{10}, F_{11}}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, F_{10}, F_{11}, \Delta_{12}, dual(F_7)} \\ \hline \bullet h_1 : \vdash \{\Delta_9\} : F_7, A_6, F_3 \oplus F_4 \end{array} \ & \frac{h_8 : \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10}, F_{11}} \ & Cut \\ \hline & \frac{- : \vdash \{\Delta_9\} : (\Delta_6, F_3 \oplus F_4), \Delta_{12}, F_{10}, F_{11}}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_{3} \oplus F_4} \ & \text{hCut} \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_3 \oplus F_4}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_3 \oplus F_4} \ & \$ \\ \hline & \frac{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_3 \oplus F_4}{- : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_3 \oplus F_4} \ & \$ \\ \hline \end{array}$$

• Case rule &

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : F_6, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_5} \oplus_A \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \& F_{11}} \xrightarrow{\mathbf{Cut}} \mathbb{C}$$

$$\frac{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, F_6 \oplus F_7}{\bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \& dual(F_7)} \xrightarrow{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_5, F_6 \oplus F_7} \mathbb{A} \times \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \& dual(F_7)}{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_5, F_6 \oplus F_7} \xrightarrow{\mathbf{ax}} \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \& dual(F_7)}{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_1, \Delta_5, F_{10}} \xrightarrow{\mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_5, F_6 \oplus F_7} \mathbb{A} \times \frac{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \& dual(F_7)}{\mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_1, \Delta_5, F_{10} \& F_{11}} & \mathbb{A} \times \mathbb{A} \times$$

• Case rule  $\oplus_B$ 

• Case rule  $\oplus_A$ 

#### • Case rule $\perp$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_6, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_6 \oplus \mathsf{F}_7, \Delta_5} \quad \oplus_A \quad \frac{\mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, dual(\mathsf{F}_6) \& dual(\mathsf{F}_7)}{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathsf{F}_6 \oplus \mathsf{F}_7), \bot, \Delta_{10}} \quad \text{Cut} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \quad \text{ax} \quad & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \quad \text{ax} \quad & \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, dual(\mathsf{F}_6) \& dual(\mathsf{F}_7)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_5, \mathsf{F}_6 \oplus \mathsf{F}_7 \quad \text{ax} \quad & \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, dual(\mathsf{F}_6) \& dual(\mathsf{F}_7)} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_3, \mathsf{F}_7, \Delta_6 \quad & \bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, dual(\mathsf{F}_7)} \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathsf{F}_7, \Delta_6, \mathsf{F}_3 \oplus \mathsf{F}_4 \quad & \bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathsf{F}_7), \bot, \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \left(\Delta_6, \mathsf{F}_3 \oplus \mathsf{F}_4\right), \bot, \Delta_{10} \quad & \to \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_6, \mathsf{F}_3, \mathsf{F}_7 \quad \text{ax} \quad & \bullet \\ \hline \bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \bot, dual(\mathsf{F}_7) \quad & \mathsf{hCut} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{F}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{F}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{F}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{F}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{F}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{F}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{F}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{F}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{F}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \Delta_{10}, \Delta_6, \mathsf{h}_3, \bot \\ \hline \\ \bullet \mathsf{h}_3 : \Delta_{10}, \Delta_6, \Delta_6, \bot \\ \hline$$

### • Case rule $\top$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_6, \Delta_5}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_6 \oplus \mathbf{F}_7, \Delta_5} \oplus_A & \frac{\bullet}{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathbf{F}_6 \oplus \mathbf{F}_7), \top, \Delta_{10}} & \mathsf{Cut} \\ & \xrightarrow{-} : \vdash \left\{\Delta_9\right\} : \Delta_5, \top, \Delta_{10} \\ & \xrightarrow{-} : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_5, \top} & \top \\ \\ \frac{\mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_3, \mathbf{F}_7, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_9\right\} : \mathbf{F}_7, \Delta_6, \mathbf{F}_3 \oplus \mathbf{F}_4} & \oplus_A & \frac{\bullet}{\bullet \mathbf{h}_8 : \vdash \left\{\Delta_9\right\} : dual(\mathbf{F}_7), \top, \Delta_{10}} & \top \\ & \xrightarrow{-} : \vdash \left\{\Delta_9\right\} : \left(\Delta_6, \mathbf{F}_3 \oplus \mathbf{F}_4\right), \top, \Delta_{10}} & \xrightarrow{-} : \vdash \left\{\Delta_9\right\} : \Delta_{10}, \Delta_6, \top, \mathbf{F}_3 \oplus \mathbf{F}_4} & \top \end{array}$$

- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_3, \mathbf{F}_7, \Delta_6 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_7, \Delta_6, \mathbf{F}_3 \oplus \mathbf{F}_4 \end{array} \oplus_{A} \begin{array}{c} \mathbf{h}_8 : \vdash \{\Delta_9\} : \mathbf{F}_{10}, \Delta_{12} \quad \mathbf{h}_8 : \vdash \{\Delta_9\} : \mathbf{F}_{11}, \Delta_{13}, dual(\mathbf{F}_7) \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathbf{F}_7), \Delta_{12}, \Delta_{13}, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_6, \mathbf{F}_3, \mathbf{F}_7 \end{array} \xrightarrow{\mathbf{ax}} \begin{array}{c} \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : dual(\mathbf{F}_7), \Delta_{12}, \Delta_{13}, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, dual(\mathbf{F}_7), \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, dual(\mathbf{F}_7), \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{10} \otimes \mathbf{F}_{11} \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{F}_{11} \otimes \mathbf{h}_8 \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{h}_{11} \otimes \mathbf{h}_8 \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{h}_{11} \otimes \mathbf{h}_8 \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{h}_{11} \otimes \mathbf{h}_8 \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathbf{h}_{11} \otimes \mathbf{h}_8 \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_{13} \otimes \mathbf{h}_8 \\ \hline \\ \bullet \mathbf{h}_8 : \vdash \{\Delta_9\} : \Delta_$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{c} \frac{h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_6, \Delta_5}{\bullet h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_6 \oplus F_7, \Delta_5} \ \oplus A & \frac{h_8 : \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_9, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{F_{10}, \Delta_{11}\} : dual(F_6 \oplus F_7), \Delta_9} \ Cut \\ \hline \\ - : \vdash \{F_{10}, \Delta_{11}\} : \Delta_5, \Delta_9 \\ \hline \\ \bullet h_1 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, F_6 \oplus F_7 \end{array} & \underset{h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_9, F_{10}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, \Delta_9} \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, \Delta_9 F_{10}}{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_5, \Delta_9} ?_C \\ \hline \\ \frac{h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_3, F_7, \Delta_6}{\bullet h_1 : \vdash \{F_{10}, \Delta_{11}\} : F_7, \Delta_6, F_3 \oplus F_4} \oplus A & \underset{h_8 : \vdash \{F_{10}, \Delta_{11}\} : dual(F_7)}{\bullet h_8 : \vdash \{F_{10}, \Delta_{11}\} : dual(F_7)} ?_C \\ \hline \\ \frac{h_1 : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_3 \oplus F_4}{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3} \oplus A \\ \hline \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3}{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_9, F_3} \oplus A} \oplus A \\ \hline \end{array}$$

# 7.8 Status of $\perp$ : OK

• Case rule 1

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \bot, \Delta_3} \ \bot \quad & \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : dual(\bot), * \\ & - : \vdash \{\Delta_5\} : \Delta_3, * \\ & - : \vdash \{\Delta_5\} : \Delta_3 \end{array} \text{ ax} \\ \hline \\ \begin{array}{c} \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3 \\ \hline \\ - : \vdash \{\Delta_5\} : \Delta_3 \end{array} \text{ ax} \end{array}$$

- Case rule!
- Case rule?

$$\begin{array}{c|c} \mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \bot, \Delta_3 \end{array} \perp \begin{array}{c} \mathbf{h}_4 : \vdash \{\mathbf{F}_6, \Delta_5\} : \mathbf{1}, \Delta_7 \\ \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : dual(\bot), \Delta_7, ?\mathbf{F}_6 \\ & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3, \bot \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3, \bot \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_4 : \vdash \{\Delta_5, \mathbf{F}_6\} : \Delta_3, \bot \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3, \bot \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_4 : \vdash \{\Delta_5, \mathbf{F}_6\} : \mathbf{1}, \Delta_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5, \mathbf{F}_6\} : \Delta_3, \bot \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_4 : \vdash \{\Delta_5, \mathbf{F}_6\} : \mathbf{1}, \Delta_7 \\ \hline - : \vdash \{\Delta_5, \mathbf{F}_6\} : \Delta_3, \Delta_7, ?\mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_5 : \mathbf{1}, \mathbf$$

$$\begin{array}{c} \underline{\mathbf{h}_1 : \vdash \{\Delta_7\} : \mathsf{F}_5, \Delta_4}}{ \underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \mathsf{F}_5, \bot, \Delta_4}}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \mathsf{F}_5, \bot, \Delta_4}} \perp & \underline{\mathbf{h}_6 : \vdash \{\mathsf{F}_8, \Delta_7\} : \Delta_9, dual(\mathsf{F}_5)}}_{\bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : dual(\mathsf{F}_5), \Delta_9, ?\mathsf{F}_8}} ? \\ \underline{- : \vdash \{\Delta_7\} : (\bot, \Delta_4), \Delta_9, ?\mathsf{F}_8}}_{\bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : \Delta_9, ?\mathsf{F}_8, dual(\mathsf{F}_5)}} \\ \underline{\mathbf{h}_1 : \vdash \{\Delta_7\} : \Delta_4, \mathsf{F}_5}}_{\bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : \Delta_4, \Delta_9, ?\mathsf{F}_8}} \perp \\ \underline{- : \vdash \{\Delta_7\} : \Delta_4, \Delta_9, \bot, ?\mathsf{F}_8}}_{\bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : \Delta_4, \Delta_9, \bot, ?\mathsf{F}_8}} \perp \\ \end{array}$$

## • Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{ \Delta_5 \right\} : \Delta_3}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_5 \right\} : \bot, \Delta_3} \ \bot \ \frac{\mathbf{h}_4 : \vdash \left\{ \Delta_5 \right\} : \mathbf{1}, \mathbf{F}_6, \mathbf{F}_7, \Delta_8}{\bullet \mathbf{h}_4 : \vdash \left\{ \Delta_5 \right\} : dual(\bot), \Delta_8, \mathbf{F}_6 \$ \mathbf{F}_7} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_5 \right\} : \Delta_3, \bot}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_5 \right\} : \Delta_3, \Delta_8, \mathbf{F}_6 \$ \mathbf{F}_7} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_5 \right\} : \Delta_3, \bot}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_5 \right\} : \Delta_3, \Delta_8, \mathbf{F}_6, \mathbf{F}_7} \\ \hline \\ - : \vdash \left\{ \Delta_5 \right\} : \Delta_3, \Delta_8, \mathbf{F}_6, \mathbf{F}_7} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \mathbf{F}_5, \Delta_4}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \mathbf{F}_5, \bot, \Delta_4} \ \bot \ \frac{\mathbf{h}_6 : \vdash \left\{ \Delta_7 \right\} : \mathbf{F}_8, \mathbf{F}_9, \Delta_{10}, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash \left\{ \Delta_7 \right\} : dual(\mathbf{F}_5), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \left\{ \Delta_7 \right\} : \left\{ (\bot, \Delta_4), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9 \right\} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \bot \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{h}_1, \mathbf{h}_3, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{h}_1, \mathbf{h}_3, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \mathbf{h}_1, \mathbf{h}_3, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \Delta_1, \mathbf{h}_3, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \Delta_1, \mathbf{h}_3, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_{10}, \Delta_4, \Delta_1, \Delta_1, \Delta_2, \Delta_3, \Delta_3, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_1, \Delta_2, \Delta_3, \Delta_3, \Delta_3, \mathbf{h}_3 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \Delta_1,$$

#### • Case rule &

$$\frac{ \frac{h_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \bot, \Delta_3} \ \bot \ \frac{h_4 : \vdash \{\Delta_5\} : 1, F_6, \Delta_8 \ h_4 : \vdash \{\Delta_5\} : 1, F_7, \Delta_8}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\bot), \Delta_8, F_6\&F_7} \ \text{Cut} } \\ \frac{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7}{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \bot} \ \frac{\bot}{h_4 : \vdash \{\Delta_5\} : 1, \Delta_8, F_6} \ \frac{\bot}{h_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7} \ \frac{\bot}{h_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \ \frac{\bot}{h_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \ \frac{\bot}{h_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \ \& \\ \frac{\bot}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \ \frac{\bot}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7} \ \& \\ \frac{\bot}{- : \vdash \{\Delta_7\} : F_5, \bot, \Delta_4} \ \bot \ \frac{\bot}{- : \vdash \{\Delta_7\} : A_1, A_2, A_1, F_8\&F_9} \ \frac{\bot}{h_6 : \vdash \{\Delta_7\} : \Delta_1, \Delta_4, A_1, A_1, F_8\&F_9} \ \frac{\bot}{h_6 : \vdash \{\Delta_7\} : \Delta_1, \Delta_4, F_8\&F_9} \ \frac{\bot}{- : \vdash \{\Delta_7\} : \Delta_1, \Delta_4, F_8\&F_9} \ \bot$$

### • Case rule $\oplus_B$

$$\begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \bot, \Delta_3 \end{array} \bot \quad \begin{array}{c} \mathbf{h}_4 : \vdash \{\Delta_5\} : 1, \mathbf{F}_7, \Delta_8 \\ \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : dual(\bot), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \quad \begin{array}{c} \oplus_B \\ - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \quad \begin{array}{c} \oplus_B \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3, \bot \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : 1, \Delta_8, \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \bot \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : 1, \Delta_8, \mathbf{F}_7 \\ \hline - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathbf{F}_7 \\ \hline - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \quad \begin{array}{c} \oplus_B \end{array} \quad \begin{array}{c} \oplus_B \end{array}$$

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_7\} : \mathsf{F}_5, \Delta_4}{\bullet \mathsf{h}_1 : \vdash \{\Delta_7\} : \mathsf{F}_5, \bot, \Delta_4} \perp \underbrace{\begin{array}{c} \mathbf{h}_6 : \vdash \{\Delta_7\} : \mathsf{F}_9, \Delta_{10}, dual(\mathsf{F}_5) \\ \bullet \mathsf{h}_6 : \vdash \{\Delta_7\} : dual(\mathsf{F}_5), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline - : \vdash \{\Delta_7\} : (\bot, \Delta_4), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline \bullet \mathsf{h}_1 : \vdash \{\Delta_7\} : \Delta_4, \mathsf{F}_5, \bot \end{array} \begin{array}{c} \mathsf{ax} \\ \bullet \mathsf{h}_6 : \vdash \{\Delta_7\} : \Delta_{10}, \mathsf{F}_9, dual(\mathsf{F}_5) \\ \hline - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_9, \bot \end{array} \begin{array}{c} \mathsf{ax} \\ \mathsf{hCut} \\ \hline - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathsf{F}_9, \bot \end{array} \begin{array}{c} \mathsf{dual}(\mathsf{F}_5) \\ \bullet \mathsf{hCut} \\ \hline \end{array}$$

• Case rule  $\oplus_A$ 

• Case rule  $\perp$ 

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_5\right\} : \Delta_3}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_5\right\} : \bot, \Delta_3} & \bot & \frac{\mathbf{h}_4 : \vdash \left\{\Delta_5\right\} : \mathbf{1}, \Delta_6}{\bullet \mathbf{h}_4 : \vdash \left\{\Delta_5\right\} : \mathbf{dual}(\bot), \bot, \Delta_6} & \bot \\ \hline & - : \vdash \left\{\Delta_5\right\} : \Delta_3, \bot, \Delta_6 & \mathbf{Cut} \\ \hline & \bullet \mathbf{h}_1 : \vdash \left\{\Delta_5\right\} : \Delta_3, \bot & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_4 : \vdash \left\{\Delta_5\right\} : \mathbf{1}, \Delta_6} & \mathbf{ax} \\ \hline & \bullet \mathbf{h}_1 : \vdash \left\{\Delta_5\right\} : \Delta_3, \bot & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_4 : \vdash \left\{\Delta_5\right\} : \mathbf{1}, \Delta_6} & \mathbf{hCut} \\ \hline & - : \vdash \left\{\Delta_5\right\} : \Delta_3, \Delta_6, \bot & \bot \\ \hline & \bullet \mathbf{h}_1 : \vdash \left\{\Delta_7\right\} : \mathbf{F}_5, \Delta_4 & \bot & \frac{\mathbf{h}_6 : \vdash \left\{\Delta_7\right\} : \Delta_8, \mathbf{dual}(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash \left\{\Delta_7\right\} : \mathbf{dual}(\mathbf{F}_5), \bot, \Delta_8} & \bot \\ \hline & - : \vdash \left\{\Delta_7\right\} : \left(\bot, \Delta_4\right), \bot, \Delta_8 & \bot \\ \hline & \bullet \mathbf{h}_1 : \vdash \left\{\Delta_7\right\} : \Delta_4, \mathbf{F}_5 & \mathbf{ax} & \bullet \\ \hline & \bullet \mathbf{h}_6 : \vdash \left\{\Delta_7\right\} : \Delta_8, \bot, \mathbf{dual}(\mathbf{F}_5) & \mathbf{ax} \\ \hline & - : \vdash \left\{\Delta_7\right\} : \Delta_4, \Delta_8, \bot, \bot & \bot \\ \hline & - : \vdash \left\{\Delta_7\right\} : \Delta_4, \Delta_8, \bot, \bot & \bot \\ \hline & - : \vdash \left\{\Delta_7\right\} : \Delta_4, \Delta_8, \bot, \bot & \bot \\ \hline \end{array}$$

 $\bullet$  Case rule  $\top$ 

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_5\right\} : \Delta_3}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_5\right\} : \bot, \Delta_3} \ \bot \ & \frac{\bullet \mathbf{h}_4 : \vdash \left\{\Delta_5\right\} : dual(\bot), \top, \Delta_6}{\bullet \mathbf{h}_4 : \vdash \left\{\Delta_5\right\} : \Delta_3, \top, \Delta_6} \\ & \xrightarrow{-} : \vdash \left\{\Delta_5\right\} : \Delta_3, \top, \Delta_6 \\ & \xrightarrow{-} : \vdash \left\{\Delta_5\right\} : \Delta_3, \Delta_6, \top \end{array} \top \\ \\ \frac{\mathbf{h}_1 : \vdash \left\{\Delta_7\right\} : \mathbf{F}_5, \Delta_4}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_7\right\} : \mathbf{F}_5, \bot, \Delta_4} \ \bot \ & \frac{\bullet \mathbf{h}_6 : \vdash \left\{\Delta_7\right\} : dual(\mathbf{F}_5), \top, \Delta_8}{\bullet \mathbf{h}_6 : \vdash \left\{\Delta_7\right\} : \left(\bot, \Delta_4\right), \top, \Delta_8} \\ & \xrightarrow{-} : \vdash \left\{\Delta_7\right\} : \Delta_4, \Delta_8, \bot, \top} \top \\ \end{array}$$

• Case rule  $I_3$ 

#### • Case rule $\otimes$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{ \mathsf{F}_6, \Delta_7 \right\} : \Delta_3}{\bullet \mathbf{h}_1 : \vdash \left\{ \mathsf{F}_6, \Delta_7 \right\} : \bot, \Delta_3} \perp & \frac{\mathbf{h}_4 : \vdash \left\{ \mathsf{F}_6, \Delta_7 \right\} : 1, \mathsf{F}_6, \Delta_5}{\bullet \mathbf{h}_4 : \vdash \left\{ \mathsf{F}_6, \Delta_7 \right\} : dual(\bot), \Delta_5} \\ & \qquad \qquad \cdot \vdash \left\{ \mathsf{F}_6, \Delta_7 \right\} : \Delta_3, \Delta_5 \\ \hline \bullet \underbrace{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7, \mathsf{F}_6 \right\} : \Delta_3, \bot}_{\bullet \mathbf{h}_4 : \vdash \left\{ \Delta_7, \mathsf{F}_6 \right\} : 1, \Delta_5, \mathsf{F}_6} & \text{ax} \\ & \qquad \qquad \cdot \vdash \left\{ \Delta_7, \mathsf{F}_6 \right\} : \Delta_3, \Delta_5, \mathsf{F}_6 \\ \hline & \qquad \qquad \cdot \vdash \left\{ \Delta_7, \mathsf{F}_6 \right\} : \Delta_3, \Delta_5, \mathsf{F}_6 \\ \hline & \qquad \qquad \cdot \vdash \left\{ \Delta_7, \mathsf{F}_6 \right\} : \Delta_3, \Delta_5, \mathsf{F}_6 \\ \hline & \qquad \qquad \cdot \vdash \left\{ \Delta_7, \mathsf{F}_6 \right\} : \Delta_3, \Delta_5 & ?_C \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{ \mathsf{F}_8, \Delta_9 \right\} : \mathsf{F}_5, \Delta_4 & \bot & \underbrace{ \begin{array}{c} \mathsf{h}_6 : \vdash \left\{ \mathsf{F}_8, \Delta_9 \right\} : \mathsf{F}_8, \Delta_7, dual(\mathsf{F}_5) \\ \bullet \mathsf{h}_6 : \vdash \left\{ \mathsf{F}_8, \Delta_9 \right\} : dual(\mathsf{F}_5), \Delta_7 \\ \hline & \qquad \qquad \cdot \vdash \left\{ \mathsf{F}_8, \Delta_9 \right\} : \Delta_4, \Delta_7 \\ \hline & \qquad \qquad \bullet \mathsf{h}_6 : \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : \Delta_7, dual(\mathsf{F}_5) \\ \hline & \qquad \qquad \bullet \mathsf{h}_6 : \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : \Delta_7, dual(\mathsf{F}_5) \\ \hline & \qquad \qquad \cdot \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : \Delta_4, \Delta_7, \bot \\ \hline & \qquad \qquad \cdot \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : \Delta_4, \Delta_7, \bot \\ \hline & \qquad \qquad \cdot \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : \Delta_4, \Delta_7, \bot \\ \hline & \qquad \qquad \cdot \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : \Delta_4, \Delta_7, \bot \\ \hline \end{array}$$

# 7.9 Status of $\top$ : OK

- Case rule 1
- Case rule!
- Case rule?

$$\begin{array}{c|c} & \underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \top, \Delta_3} \quad \top \quad & \underbrace{\bullet \mathbf{h}_4 : \vdash \{\mathbf{F}_6, \Delta_5\} : \mathbf{0}, \Delta_7}_{\bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : dual(\top), \Delta_7, ?\mathbf{F}_6} \quad ?}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?\mathbf{F}_6} \\ \hline \\ & \underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_6\} : \Delta_3, \top}_{\bullet \mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_6\} : \Delta_3, \Delta_7} \quad \top \\ \hline \\ & \underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_6\} : \Delta_3, \top}_{\bullet \mathbf{h}_4 : \vdash \{\Delta_5, \mathbf{F}_6\} : \mathbf{0}, \Delta_7} \quad \text{ax} \\ \hline \\ & \underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_5, \mathbf{F}_6\} : \Delta_3, \Delta_7, ?\mathbf{F}_6}_{\bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : \Delta_9, dual(\mathbf{F}_5)} \\ \hline \\ & \underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : \mathbf{F}_5, \top, \Delta_4}_{\bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : dual(\mathbf{F}_5), \Delta_9, ?\mathbf{F}_8} \\ \hline \\ & \underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_9, ?\mathbf{F}_8}_{\vdash \mathbf{h}_6 : \vdash \{\Delta_7\} : \Delta_4, \Delta_9, \top, ?\mathbf{F}_8} \quad \top \end{array} \right. \quad ? \quad \text{Cut}$$

• Case rule \$

$$\begin{array}{c} \frac{\bullet h_1 : \vdash \{\Delta_5\} : \top, \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \$ F_7} & \$ \\ \hline - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \$ F_7 \\ \hline \bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \top & \top & \frac{\rightarrow}{h_4 : \vdash \{\Delta_5\} : 0, \Delta_8, F_6, F_7} & \text{Cut} \\ \hline \\ \frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \top}{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6, F_7} & * & \text{hCut} \\ \hline & \frac{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6, F_7}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6, F_7} & \$ \\ \hline \\ \frac{\bullet h_1 : \vdash \{\Delta_7\} : F_5, \top, \Delta_4}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8 \$ F_9} \\ \hline & \frac{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8 \$ F_9}{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \$ F_9} & \top \\ \end{array}$$

• Case rule &

$$\frac{\underbrace{\bullet_{h_1} : \vdash \{\Delta_5\} : \top, \Delta_3}_{\bullet h_1} \; \top \; \frac{h_4 : \vdash \{\Delta_5\} : 0, F_6, \Delta_8 \quad h_4 : \vdash \{\Delta_5\} : 0, F_7, \Delta_8}_{\bullet h_4 : \vdash \{\Delta_5\} : dual(\top), \Delta_8, F_6 \& F_7} \underbrace{Cut} \\ - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \& F_7 \\ \xrightarrow{\bullet_{h_1} : \vdash \{\Delta_5\} : \Delta_3, \top} \underbrace{ax}_{h_4 : \vdash \{\Delta_5\} : 0, \Delta_8, F_6} \underbrace{ax}_{hCut} \underbrace{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7}_{\bullet Lut} \underbrace{ax}_{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \underbrace{ax}_{hCut} \underbrace{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7}_{\bullet Lut} \underbrace{ax}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7}_{\bullet Lut} \underbrace{ax}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7}_{\bullet Lut} \underbrace{ax}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7}_{\bullet Lut} \underbrace{ax}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : F_8, \Delta_{10}, dual(F_5), h_6 : \vdash \{\Delta_7\} : F_9, \Delta_{10}, dual(F_5)}_{\bullet Lut} \underbrace{ax}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \& F_9}_{\bullet Lut} \underbrace{- :$$

• Case rule  $\oplus_B$ 

$$\begin{array}{c} \underbrace{\begin{array}{c} \bullet_{h_1} : \vdash \{\Delta_5\} : \mathsf{T}, \Delta_8 \\ \bullet_{h_4} : \vdash \{\Delta_5\} : dual(\top), \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \\ - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \\ \bullet_{h_1} : \vdash \{\Delta_5\} : \Delta_3, \top \end{array} \xrightarrow{\bullet} \begin{array}{c} \bullet_{h_4} : \vdash \{\Delta_5\} : dual(\top), \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \\ \bullet_{h_1} : \vdash \{\Delta_5\} : \Delta_3, \top \end{array} \xrightarrow{\bullet} \begin{array}{c} \to \\ \bullet_{h_2} : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \hline \bullet_{h_2} : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathsf{F}_7 \\ \hline - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \xrightarrow{\bullet} \begin{array}{c} \bullet_B \\ \bullet \mathsf{h}_6 : \vdash \{\Delta_7\} : \mathsf{F}_9, \Delta_{10}, dual(\mathsf{F}_5) \\ \hline \bullet_{h_6} : \vdash \{\Delta_7\} : dual(\mathsf{F}_5), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline - : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \end{array} \xrightarrow{\bullet} \begin{array}{c} \oplus_B \\ \mathsf{Cut} \end{array}$$

• Case rule  $\oplus_A$ 

$$\begin{array}{c} \underbrace{ \begin{array}{c} \bullet_{h_1} : \vdash \{\Delta_5\} : \mathsf{T}, \Delta_3 \\ \bullet_{h_4} : \vdash \{\Delta_5\} : dual(\top), \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \hline \\ - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \hline \\ \bullet_{h_1} : \vdash \{\Delta_5\} : \Delta_3, \top \\ \hline \\ \hline \\ \bullet_{h_1} : \vdash \{\Delta_5\} : \Delta_3, \top \\ \hline \\ \hline \\ - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \hline \\ \hline \\ - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \hline \\ - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \hline \\ \bullet_{h_6} : \vdash \{\Delta_7\} : \mathsf{F}_8, \Delta_{10}, dual(\mathsf{F}_5) \\ \hline \\ \bullet_{h_6} : \vdash \{\Delta_7\} : dual(\mathsf{F}_5), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline \\ - : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline \\ - : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \hline \end{array} \right. \end{array} \begin{array}{c} \oplus_A \\ \text{Cut} \\ \end{array}$$

 $\bullet$  Case rule  $\bot$ 

 $\bullet$  Case rule  $\top$ 

$$\begin{array}{c|c} \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \top, \Delta_3 & \top & \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : dual(\top), \top, \Delta_6 \\ \hline & - : \vdash \{\Delta_5\} : \Delta_3, \top, \Delta_6 \\ \hline & - : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \top \\ \hline \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : F_5, \top, \Delta_4 & \top & \bullet \mathbf{h}_6 : \vdash \{\Delta_7\} : dual(F_5), \top, \Delta_8 \\ \hline & - : \vdash \{\Delta_7\} : (\top, \Delta_4), \top, \Delta_8 \\ \hline & - : \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \top, \top \\ \hline \end{array} \right. \\ \hline \end{array}$$

• Case rule  $I_3$ 

• Case rule  $\otimes$ 

$$\begin{array}{c} \underbrace{\bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \top, \Delta_3}_{} = \top & \frac{\mathbf{h}_4 : \vdash \{\Delta_5\} : 0, F_6, \Delta_8 \quad \mathbf{h}_4 : \vdash \{\Delta_5\} : F_7, \Delta_9}{\bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7} \\ - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \Delta_3, \top & \top & \frac{\rightarrow}{\mathbf{h}_4 : \vdash \{\Delta_5\} : 0, \Delta_8, F_6} \quad \mathbf{hCut} \\ \hline - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \\ \hline - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \\ \hline - : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, A_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_5\} : \top, \Delta_3 \\ \hline \end{array} & \begin{array}{c} \mathbf{h}_4 : \vdash \{\Delta_5\} : 0, \Delta_8, F_6 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{\Delta_5\} : \Delta_3, \Delta_3, \Delta_9, F_6 \otimes F_7 \\ \hline \bullet \mathbf{h}_4 : \vdash \{$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{c} \underbrace{\bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_6, \Delta_7 \} : \mathsf{T}, \Delta_3}_{\quad \bullet \mathbf{h}_4 : \vdash \{ \mathbf{F}_6, \Delta_7 \} : \mathbf{d}ual(\mathsf{T}), \Delta_5} \\ - : \vdash \{ \mathbf{F}_6, \Delta_7 \} : \Delta_3, \Delta_5 \\ \\ \underbrace{\bullet \mathbf{h}_1 : \vdash \{ \Delta_7, \mathbf{F}_6 \} : \Delta_3, \mathsf{T}}_{\quad \bullet \mathbf{h}_4 : \vdash \{ \Delta_7, \mathbf{F}_6 \} : \mathbf{0}, \Delta_5, \mathbf{F}_6} \\ \bullet \mathbf{h}_1 : \vdash \{ \Delta_7, \mathbf{F}_6 \} : \Delta_3, \mathsf{T} \\ \hline - : \vdash \{ \Delta_7, \mathbf{F}_6 \} : \Delta_3, \Delta_5, \mathbf{F}_6 \\ - : \vdash \{ \Delta_7, \mathbf{F}_6 \} : \Delta_3, \Delta_5 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_8, \Delta_9 \} : \mathbf{F}_8, \mathsf{T}, \mathsf{T} \\ \hline \bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_8, \Delta_9 \} : \mathbf{F}_8, \mathsf{T}, \mathsf{T} \\ \hline \bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_8, \Delta_9 \} : \mathbf{F}_8, \mathsf{T}, \mathsf{T} \\ \hline - : \vdash \{ \mathbf{F}_8, \Delta_9 \} : (\mathsf{T}, \Delta_4), \Delta_7 \\ \hline - : \vdash \{ \Delta_9, \mathbf{F}_8 \} : \Delta_4, \Delta_7, \mathsf{T} \\ \hline \end{array} \begin{array}{c} \mathbf{h}_4 : \vdash \{ \mathbf{F}_6, \Delta_7 \} : \mathbf{0}, \Delta_5, \mathbf{F}_6 \\ \bullet \mathbf{h}_4 : \vdash \{ \mathbf{F}_8, \Delta_9 \} : \mathbf{F}_8, \mathsf{T}, \mathsf{d}ual(\mathbf{F}_5) \\ \bullet \mathbf{h}_6 : \vdash \{ \mathbf{F}_8, \Delta_9 \} : \mathsf{d}ual(\mathbf{F}_5), \Delta_7 \\ \mathsf{Cut} \\ \hline \end{array} \begin{array}{c} \mathbf{e}_{\mathbf{h}_1} : \vdash \{ \mathbf{F}_8, \Delta_9 \} : (\mathsf{T}, \Delta_4), \Delta_7 \\ \hline - : \vdash \{ \Delta_9, \mathbf{F}_8 \} : \Delta_4, \Delta_7, \mathsf{T} \\ \hline \end{array}$$

# 7.10 Status of $I_3$ : OK

- Case rule 1
- Case rule!

#### • Case rule?

# • Case rule \$

## • Case rule &

$$\begin{array}{c} \underbrace{\begin{array}{c} \bullet_{h_1} : \vdash \{\Delta_6\} : \hat{\cap}(\mathbf{n}_4), p(\mathbf{n}_4) \\ \bullet_{h_1} : \vdash \{\Delta_6\} : \hat{\cap}(\mathbf{n}_4), p(\mathbf{n}_4) \\ \end{array}}_{ \begin{array}{c} \bullet_{h_2} : \vdash \{\Delta_6\} : F_7, \Delta_9, p(\mathbf{n}_4) \\ \bullet_{h_3} : \vdash \{\Delta_6\} : dual(\hat{\cap}(\mathbf{n}_4)), \Delta_9, F_7 \& F_8 \\ \end{array} \\ \begin{array}{c} - : \vdash \{\Delta_6\} : p(\mathbf{n}_4), \Delta_9, F_7 \& F_8 \\ \hline \\ - : \vdash \{\Delta_6\} : \Delta_9, F_7, p(\mathbf{n}_4) \\ \hline \\ - : \vdash \{\Delta_6\} : \Delta_9, p(\mathbf{n}_4), F_7 \& F_8 \\ \hline \\ \bullet_{h_1} : \vdash \{\Delta_6\} : p(\mathbf{n}_4), \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_2} : \vdash \{\Delta_6\} : F_7, \Delta_9, \hat{\cap}(\mathbf{n}_4) \\ \hline \\ \bullet_{h_3} : \vdash \{\Delta_6\} : f_7, \Delta_9, \hat{\cap}(\mathbf{n}_4) \\ \hline \bullet_{h_3} : \vdash \{\Delta_6\} : f_8, \Delta_9, \hat{\cap}(\mathbf{n}_4) \\ \hline \\ \bullet_{h_3} : \vdash \{\Delta_6\} : dual(p(\mathbf{n}_4)), \Delta_9, F_7 \& F_8 \\ \hline \\ - : \vdash \{\Delta_6\} : \hat{\cap}(\mathbf{n}_4), \Delta_9, F_7 \& F_8 \\ \hline \\ - : \vdash \{\Delta_6\} : \Delta_9, F_7, \hat{\cap}(\mathbf{n}_4) \\ \hline \\ - : \vdash \{\Delta_6\} : \Delta_9, F_7, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \\ - : \vdash \{\Delta_6\} : \Delta_9, F_7, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\cap}(\mathbf{n}_4) \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \bullet_{h_3} : \bullet_{h_3} \\ \hline \end{array} \\ \begin{array}{c} \bullet_{h_3} : \bullet_{h_3}$$

## • Case rule $\oplus_B$

$$\frac{ \underbrace{ \begin{array}{c} \mathbf{h}_{5} : \vdash \{\Delta_{6}\} : \mathbf{F}_{8}, \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}) \\ \\ \bullet \mathbf{h}_{1} : \vdash \{\Delta_{6}\} : p(\mathbf{n}_{4}), \widehat{\phantom{a}}(\mathbf{n}_{4}) \end{array} I_{3} \quad \frac{\mathbf{h}_{5} : \vdash \{\Delta_{6}\} : \mathbf{F}_{8}, \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}) \\ \\ \bullet \mathbf{h}_{5} : \vdash \{\Delta_{6}\} : \operatorname{dual}(p(\mathbf{n}_{4})), \Delta_{9}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \\ \\ - : \vdash \{\Delta_{6}\} : \widehat{\phantom{a}}(\mathbf{n}_{4}), \Delta_{9}, \mathbf{F}_{7} \oplus \mathbf{F}_{8} \\ \\ \hline - : \vdash \{\Delta_{6}\} : \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}) \\ \hline - : \vdash \{\Delta_{6}\} : \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}), \mathbf{F}_{7} \oplus \mathbf{F}_{8} \end{array} \oplus_{B}$$

• Case rule  $\oplus_A$ 

 $\bullet$  Case rule  $\bot$ 

 $\bullet$  Case rule  $\top$ 

• Case rule  $I_3$ 

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \vdash \{\Delta_5\} : \, \hat{\ } (\mathbf{n}_6), p(\mathbf{n}_6) \\ \\ - : \vdash \{\Delta_5\} : p(\mathbf{n}_6), \hat{\ } (\mathbf{n}_6) \\ \\ \hline \\ - : \vdash \{\Delta_5\} : p(\mathbf{n}_6), \hat{\ } (\mathbf{n}_6) \\ \\ \hline \\ - : \vdash \{\Delta_5\} : p(\mathbf{n}_6), \hat{\ } (\mathbf{n}_6) \\ \\ \end{array} } \underbrace{ \begin{array}{c} I_3 \\ \text{Cut} \\ \\ I_3 \\ \end{array} }$$

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1} : \vdash \{\Delta_5\} : p(\mathbf{n}_6), \hat{\ } (\mathbf{n}_6) & I_3 & \hline \bullet_{\mathbf{h}_4} : \vdash \{\Delta_5\} : dual(p(\mathbf{n}_6)), p(\mathbf{n}_6) \\ \hline & - : \vdash \{\Delta_5\} : \hat{\ } (\mathbf{n}_6), p(\mathbf{n}_6) \\ \hline & - : \vdash \{\Delta_5\} : p(\mathbf{n}_6), \hat{\ } (\mathbf{n}_6) & I_3 \\ \hline \end{array}$$

 $\bullet$  Case rule  $\otimes$ 

• Case rule  $I_1$ 

• Case rule  $I_2$ 

• Case rule  $?_C$ 

$$\begin{array}{c} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : \hat{\mathsf{r}}_{(\mathbf{n}_4)}, p(\mathbf{n}_4) \end{array} I_3 & \frac{\mathsf{h}_5 : \vdash \{ \mathsf{F}_7, \Delta_8 \} : \mathsf{F}_7, \Delta_6, p(\mathbf{n}_4)}{\bullet \mathsf{h}_5 : \vdash \{ \mathsf{F}_7, \Delta_8 \} : dual(\hat{\mathsf{r}}(\mathbf{n}_4)), \Delta_6} & \underbrace{\phantom{} ?_C \\ - : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ & \qquad \qquad \qquad \\ \hline \phantom{} - : \vdash \{ \Delta_8, \mathsf{F}_7 \} : \Delta_6, \mathsf{F}_7, p(\mathbf{n}_4) \\ - : \vdash \{ \Delta_8, \mathsf{F}_7 \} : \Delta_6, p(\mathbf{n}_4) & \underbrace{\phantom{} ?_C \\ \phantom{} ?_C \end{array} } \end{array} } \\ \phantom{\bullet \bullet_{\mathbf{h}_1} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : \hat{\mathsf{r}}_{(\mathbf{n}_4)}, p(\mathbf{n}_4) \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet_{\mathbf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathbf{n}_4), \Delta_6 \\ \phantom{\bullet \bullet} : p(\mathbf{n}_4) : p(\mathbf{$$

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathsf{n}_4), \, \hat{\phantom{a}}(\mathsf{n}_4) \\ \bullet_{\mathbf{h}_1} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : p(\mathsf{n}_4), \, \hat{\phantom{a}}(\mathsf{n}_4) \end{array} }_{\bullet \mathsf{h}_5} I_3} \begin{array}{c} \bullet_{\mathsf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : \mathsf{F}_7, \Delta_6, \, \hat{\phantom{a}}(\mathsf{n}_4) \\ \bullet_{\mathsf{h}_5} : \vdash \{ \mathsf{F}_7, \Delta_8 \} : dual(p(\mathsf{n}_4)), \Delta_6 \\ & \rightarrow \\ \hline - : \vdash \{ \mathsf{F}_7, \Delta_8 \} : \hat{\phantom{a}}(\mathsf{n}_4), \Delta_6 \\ & \rightarrow \\ \hline - : \vdash \{ \Delta_8, \mathsf{F}_7 \} : \Delta_6, \mathsf{F}_7, \, \hat{\phantom{a}}(\mathsf{n}_4) \\ & - : \vdash \{ \Delta_8, \mathsf{F}_7 \} : \Delta_6, \, \hat{\phantom{a}}(\mathsf{n}_4) \end{array} }_{?C} \end{array} \right.$$

## 7.11 Status of $\otimes$ : OK

- Case rule 1
- Case rule!
- Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{7}, \Delta_{5} \quad \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{8}, \Delta_{6}}{\bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{7} \otimes F_{8}, \Delta_{5}, \Delta_{6}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{7} \otimes F_{8}, \Delta_{5}, \Delta_{6}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{7} \otimes F_{8}, \Delta_{5}, \Delta_{6}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{5}, \Delta_{6}, F_{7} \otimes F_{8}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{5}, \Delta_{6}, F_{7} \otimes F_{8}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10}, F_{11} \right\} : \Delta_{5}, \Delta_{6}, F_{7} \otimes F_{8}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10}, F_{11} \right\} : \Delta_{5}, \Delta_{6}, F_{7} \otimes F_{8}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10}, F_{11} \right\} : \Delta_{5}, \Delta_{6}, F_{7} \otimes F_{8}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10}, F_{11} \right\} : \Delta_{12}, \Delta_{5}, \Delta_{6}} \\ & & \vdots & \vdots & \vdots & \vdots \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10}, F_{11} \right\} : \Delta_{12}, \Delta_{5}, \Delta_{6}, F_{7} \otimes F_{11}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{8}, \Delta_{6}, \Delta_{7}, F_{8} \otimes F_{4}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{8}, \Delta_{6}, \Delta_{7}, F_{8} \otimes F_{4}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{12}, \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4}} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{8}, \Delta_{5}, \Delta_{7}, F_{8} \otimes F_{4} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{12}, \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4} \\ & \bullet \mathbf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{12}, \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4} \\ & \bullet \mathbf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{12}, \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4} \\ & \bullet \mathbf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{12}, \Delta_{5}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4} \\ & \bullet \mathbf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{12}, \Delta_{5}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{5}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{5}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{12}, \Delta_{5}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4} \\ & \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10$$

• Case rule \$

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_{10}\} : \mathsf{F}_7, \Delta_5 \quad \mathsf{h}_1 : \vdash \{\Delta_{10}\} : \mathsf{F}_8, \Delta_6}{\underbrace{\bullet \mathsf{h}_1 : \vdash \{\Delta_{10}\} : \mathsf{F}_8, \Delta_5, \Delta_6}}_{\bullet \mathsf{h}_1 : \vdash \{\Delta_{10}\} : \mathsf{F}_7 \otimes \mathsf{F}_8, \Delta_5, \Delta_6}} \otimes \frac{\mathbf{h}_9 : \vdash \{\Delta_{10}\} : \mathsf{F}_{11}, \mathsf{F}_{12}, \Delta_{13}, dual(\mathsf{F}_7)\$dual(\mathsf{F}_8)}}{\bullet \mathsf{h}_9 : \vdash \{\Delta_{10}\} : dual(\mathsf{F}_7 \otimes \mathsf{F}_8), \Delta_{13}, \mathsf{F}_{11}\$\mathsf{F}_{12}}} \\ \underbrace{- : \vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{13}, \mathsf{F}_{11}\$\mathsf{F}_{12}}_{\bullet \mathsf{h}_1 : \vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathsf{F}_7 \otimes \mathsf{F}_8} \underbrace{\mathsf{ax}}_{\mathsf{h}_9 : \vdash \{\Delta_{10}\} : \Delta_{13}, \mathsf{F}_{11}, \mathsf{F}_{12}, dual(\mathsf{F}_7)\$dual(\mathsf{F}_8)}}_{\bullet \mathsf{h}_2 : \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathsf{F}_{11}, \mathsf{F}_{12}}} \underbrace{\mathsf{ax}}_{\mathsf{h}_{\mathsf{Cut}}}$$

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\frac{ \begin{array}{c} \frac{h_1 : \vdash \left\{ \Delta_{10} \right\} : F_7, \Delta_5 \quad h_1 : \vdash \left\{ \Delta_{10} \right\} : F_8, \Delta_6}{\bullet^{h_1} : \vdash \left\{ \Delta_{10} \right\} : F_7 \otimes F_8, \Delta_5, \Delta_6} \\ & & \begin{array}{c} \frac{h_1 : \vdash \left\{ \Delta_{10} \right\} : F_7, \Delta_5 \quad h_1 : \vdash \left\{ \Delta_{10} \right\} : F_8, \Delta_6}{\bullet^{h_2} : \vdash \left\{ \Delta_{10} \right\} : \left\{ \Delta_{11} \right\} : \left\{ \Delta_{10} \right\} : \left\{ \Delta_{11} \right\} : \left\{ \Delta_{10} \right\} : \left\{ \Delta_{10} \right\} : \left\{ \Delta_{11} \right\} : \left\{ \Delta_{10} \right\}
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#### • Case rule &

$$\frac{\mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{7}, \Delta_{5} \quad \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{8}, \Delta_{6}}{\bullet} \otimes \begin{array}{c} \mathbf{h}_{9} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{11}, \Delta_{13}, dual(\mathsf{F}_{7}) \$ dual(\mathsf{F}_{8}) \quad \mathbf{h}_{9} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{12}, \Delta_{13}, dual(\mathsf{F}_{7}) \$ dual(\mathsf{F}_{8}) \\ \bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{8} \times \mathsf{F}_{8}, \Delta_{5}, \Delta_{6} \\ & \bullet \mathbf{h}_{9} : \vdash \left\{ \Delta_{10} \right\} : \mathcal{A}_{13}, \mathsf{F}_{11} \& \mathsf{F}_{12} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{13}, \mathcal{F}_{11}, \mathcal{F}_{12} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{13}, \mathcal{F}_{14}, \mathcal{F}_{12} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{13}, \mathcal{F}_{14}, \mathcal{F}_{12} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{14}, \mathcal{F}_{14} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{15}, \mathcal{F}_{15} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathcal{F}_{15}, \mathcal{F}_{$$

• Case rule  $\oplus_B$ 

#### • Case rule $\oplus_A$

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{7}, \Delta_{5} \quad \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{8}, \Delta_{6}}{\bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{7}, \Delta_{5} \quad \mathsf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{8}, \Delta_{5}, \Delta_{6}} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \left( \Delta_{5}, \Delta_{6}, \Delta_{13}, \mathsf{F}_{11} \oplus \mathsf{F}_{12} \right. \\ & - : \vdash \left\{ \Delta_{10} \right\} : \left( \Delta_{5}, \Delta_{6}, \Delta_{13}, \mathsf{F}_{11} \oplus \mathsf{F}_{12} \right. \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11} \oplus \mathsf{F}_{12} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11} \oplus \mathsf{F}_{12} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11} \oplus \mathsf{F}_{12} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11} \oplus \mathsf{F}_{12} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{8}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \left( \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \right) \otimes \underbrace{\begin{array}{c} \mathsf{h}_{9} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{11}, \Delta_{13}, \mathsf{dual}(\mathsf{F}_{8}) \\ \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{8}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \left( \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \right) \otimes \underbrace{\begin{array}{c} \mathsf{h}_{9} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{11}, \Delta_{13}, \mathsf{dual}(\mathsf{F}_{8}) \\ \bullet \mathsf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \mathsf{F}_{11} \oplus \mathsf{F}_{12} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{11}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \\ & \bullet \mathsf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{3}, \Delta_{5}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \\ & \bullet \mathsf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \mathsf{F}_{3}, \Delta_{5}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4} \\ & \bullet \mathsf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \mathsf{F}_{11}, \Delta_{13}, \Delta_{14} \mathsf{ud}(\mathsf{F}_{8}) \\ & \bullet \mathsf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{13}, \Delta_{11}, \Delta_{13}, \Delta_{14} \mathsf{ud}(\mathsf{F}_{8}) \\ & \bullet \mathsf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{13}, \Delta_{13}, \Delta_{11}, \Delta_{11}, \Delta_{13}, \Delta_{14} \mathsf{ud}(\mathsf{F}_{8}) \\ & \bullet \mathsf{h}_{2} : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, \Delta_{13}, \Delta_{13}, \Delta_{11}, \Delta_{13}, \Delta_{13}, \Delta_{11}, \Delta_{13}, \Delta_{13}, \Delta_{11}, \Delta_{14}, \Delta_{14}, \Delta_{14}, \Delta_{14}, \Delta_{14}, \Delta$$

### • Case rule $\perp$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_7, \Delta_5 \quad \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_8, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_7 \otimes \mathsf{F}_8, \Delta_5, \Delta_6} \qquad \otimes \begin{array}{c} \frac{\mathbf{h}_9 : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, dual(\mathsf{F}_7) \$ dual(\mathsf{F}_8)}{\bullet \mathbf{h}_9 : \vdash \left\{\Delta_{10}\right\} : dual(\mathsf{F}_7 \otimes \mathsf{F}_8), \bot, \Delta_{11}} \\ \hline - : \vdash \left\{\Delta_{10}\right\} : (\Delta_5, \Delta_6), \bot, \Delta_{11} \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \Delta_5, \Delta_6, \mathsf{F}_7 \otimes \mathsf{F}_8} & \text{ax} & \frac{\rightarrow}{\mathbf{h}_9 : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, dual(\mathsf{F}_7) \$ dual(\mathsf{F}_8)} \\ \hline - : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_6 \\ \hline - : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_6, \bot \end{array} \\ \frac{- : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_6, \bot}{- : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_6, \bot} \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_3, \mathsf{F}_8, \Delta_6, \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_4, \Delta_7} \otimes \begin{array}{c} \mathbf{h}_9 : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, dual(\mathsf{F}_8) \\ \hline \bullet \mathbf{h}_9 : \vdash \left\{\Delta_{10}\right\} : dual(\mathsf{F}_8), \bot, \Delta_{11} \\ \hline - : \vdash \left\{\Delta_{10}\right\} : (\Delta_6, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4), \bot, \Delta_{11} \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \Delta_6, \Delta_7, \mathsf{F}_8, \mathsf{F}_3 \otimes \mathsf{F}_4 \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_9 : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, dual(\mathsf{F}_8) \\ \hline - : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_6, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4 \\ \hline - : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_6, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4 \\ \hline - : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_6, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4 \\ \hline - : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_6, \Delta_7, \bot, \mathsf{F}_3 \otimes \mathsf{F}_4 \end{array} \right] \bot$$

$$\frac{\mathbf{h}_1 : \vdash \{\Delta_{10}\} : \mathsf{F}_3, \Delta_5 \quad \mathsf{h}_1 : \vdash \{\Delta_{10}\} : \mathsf{F}_4, \mathsf{F}_8, \Delta_7}{\underbrace{\bullet \mathsf{h}_1 : \vdash \{\Delta_{10}\} : \mathsf{F}_8, \Delta_5, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4}}_{= : \vdash \{\Delta_{10}\} : (\Delta_5, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4)} \otimes \underbrace{\frac{\mathsf{h}_9 : \vdash \{\Delta_{10}\} : \Delta_{11}, \mathit{dual}(\mathsf{F}_8)}{\bullet \mathsf{h}_9 : \vdash \{\Delta_{10}\} : \mathit{dual}(\mathsf{F}_8), \bot, \Delta_{11}}}_{= : \vdash \{\Delta_{10}\} : \Delta_5, \Delta_7, \mathsf{F}_8, \mathsf{F}_3 \otimes \mathsf{F}_4}} \xrightarrow{\overset{\rightarrow}{\mathsf{ax}}}_{= : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4}}_{= : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_7, \bot, \mathsf{F}_3 \otimes \mathsf{F}_4}}_{= : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_7, \bot, \mathsf{F}_3 \otimes \mathsf{F}_4}} \bot$$

## • Case rule $\top$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_7, \Delta_5 \quad \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_8, \Delta_6}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_7 \otimes \mathsf{F}_8, \Delta_5, \Delta_6} \quad \otimes \quad \frac{\bullet \mathbf{h}_9 : \vdash \left\{\Delta_{10}\right\} : \mathit{dual}(\mathsf{F}_7 \otimes \mathsf{F}_8), \top, \Delta_{11}}{- : \vdash \left\{\Delta_{10}\right\} : \left(\Delta_5, \Delta_6, \right), \top, \Delta_{11}} \quad \overset{\vdash}{\leftarrow} \mathsf{Cut}} \\ \frac{- : \vdash \left\{\Delta_{10}\right\} : \left(\Delta_5, \Delta_6, \right), \top, \Delta_{11}}{- : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_6, \top} \quad \top \\ \\ \frac{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_3, \mathsf{F}_8, \Delta_6, \, \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_4, \Delta_7}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \left(\Delta_6, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4\right), \top, \Delta_{11}} \quad \overset{\vdash}{\leftarrow} \mathsf{Cut}} \\ \frac{- : \vdash \left\{\Delta_{10}\right\} : \left(\Delta_6, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4\right), \top, \Delta_{11}}{- : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_8, \Delta_5, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4} \quad \top \\ \frac{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_3, \Delta_5, \, \, \, \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_4, \mathsf{F}_8, \Delta_7}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_{10}\right\} : \mathsf{F}_8, \Delta_5, \Delta_7, \mathsf{F}_3 \otimes \mathsf{F}_4} \quad \otimes \quad \bullet \mathsf{h}_9 : \vdash \left\{\Delta_{10}\right\} : \mathit{dual}(\mathsf{F}_8), \top, \Delta_{11}} \quad \overset{\vdash}{\leftarrow} \mathsf{Cut}} \\ \frac{- : \vdash \left\{\Delta_{10}\right\} : \left(\Delta_5, \Delta_7, \mathsf{F}_7, \mathsf{F}_3 \otimes \mathsf{F}_4\right), \top, \Delta_{11}}{- : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_7, \top, \mathsf{F}_3 \otimes \mathsf{F}_4}} \quad \top \\ \mathsf{Cut} \\ \frac{- : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_7, \top, \mathsf{F}_3 \otimes \mathsf{F}_4}}{- : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_7, \top, \mathsf{F}_3 \otimes \mathsf{F}_4}} \quad \top \\ \mathsf{Cut} \\ \frac{- : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_7, \top, \mathsf{F}_3 \otimes \mathsf{F}_4}}{- : \vdash \left\{\Delta_{10}\right\} : \Delta_{11}, \Delta_5, \Delta_7, \top, \mathsf{F}_3 \otimes \mathsf{F}_4}} \quad \top \\ \mathsf{Cut} \\ \mathsf{Cut$$

### • Case rule $I_3$

## • Case rule $\otimes$

$$\frac{ h_1 : \vdash \{\Delta_{10}\} : F_7, \Delta_5 \quad h_1 : \vdash \{\Delta_{10}\} : F_8, \Delta_6 }{ \bullet h_1 : \vdash \{\Delta_{10}\} : F_8, \Delta_6 } \otimes \frac{ h_9 : \vdash \{\Delta_{10}\} : F_{11}, \Delta_{13}, dual(F_7)\$dual(F_8) \quad h_9 : \vdash \{\Delta_{10}\} : F_{12}, \Delta_{14} }{ \bullet h_9 : \vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{13}, \Delta_{14}, F_{11} \otimes F_{12} } \quad Cut } \otimes \\ \frac{ \bullet h_1 : \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_5, \Delta_6 }{ - : \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_{15}, \Delta_{14}, F_{11} \otimes F_{12} } \otimes \frac{ ax }{ h_9 : \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_{14}, \Delta_5, \Delta_6, F_{11} \otimes F_{12} } \otimes \frac{ ax }{ - : \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, F_{11} } \otimes \frac{ ax }{ - : \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_{14}, \Delta_5, \Delta_6, F_{11} \otimes F_{12} } \otimes \frac{ ax }{ - : \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_{15}, \Delta_6, F_{11} \otimes F_{12} } \otimes \frac{ ax }{ - : \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_{14}, \Delta_5, \Delta_6, F_{11} \otimes F_{12} } \otimes \frac{ ax }{ - : \vdash \{\Delta_{10}\} : A_{15}, A_{15}$$

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{3}, F_{8}, \Delta_{6} \quad h_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{4}, \Delta_{7}}{\bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{8}, \Delta_{6}, \Delta_{7}, F_{3} \otimes F_{4}} \\ & \otimes \frac{\bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{8}, \Delta_{6}, \Delta_{7}, F_{3} \otimes F_{4}}{\bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{10} \right\} : \left( \Delta_{6}, \Delta_{7}, F_{3} \otimes F_{4} \right), \Delta_{13}, \Delta_{14}, F_{11} \otimes F_{12}} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \left( \Delta_{6}, \Delta_{7}, F_{8} \otimes F_{4} \right), \Delta_{13}, \Delta_{14}, F_{11} \otimes F_{12}} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, F_{11} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \Delta_{13}, A_{14}, \Delta_{6}, \Delta_{7}, F_{8}, F_{3} \otimes F_{4}} \\ & - : \vdash \left\{ \Delta_{10} \right\} : F_{3}, \Delta_{5} \quad h_{1} : \vdash \left\{ \Delta_{10} \right\} : F_{4}, F_{8}, \Delta_{7}} \\ & - : \vdash \left\{ \Delta_{10} \right\} : \left\{ \Delta_{10}$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\frac{\mathbf{h}_{1} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{7}, \Delta_{5} \quad \mathsf{h}_{1} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{8}, \Delta_{6}}{\bullet \mathsf{h}_{1} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{7} \otimes \mathsf{F}_{8}, \Delta_{5}, \Delta_{6}}} \otimes \frac{\mathbf{h}_{9} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{11}, \Delta_{10}, dual(\mathsf{F}_{7}) \otimes dual(\mathsf{F}_{8})}{\bullet \mathsf{h}_{9} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : dual(\mathsf{F}_{7} \otimes \mathsf{F}_{8}), \Delta_{10}}} \underbrace{\mathsf{Cut}}^{?}C$$

$$- : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : (\Delta_{5}, \Delta_{6}), \Delta_{10}} \\ - : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{5}, \Delta_{6}, \mathsf{F}_{7} \otimes \mathsf{F}_{8}} \xrightarrow{\mathsf{ax}} \frac{\mathsf{h}_{9} : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \mathsf{F}_{11}, dual(\mathsf{F}_{7}) \otimes dual(\mathsf{F}_{8})}} \underbrace{\mathsf{ax}}_{\mathsf{hCut}} \\ - : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11}} \\ - : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_{5}, \Delta_{6}, \mathsf{F}_{11}} \\ - : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{8}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4}} \\ - : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{8}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4}} \xrightarrow{\mathsf{h}_{9} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{11}, \Delta_{10}, dual(\mathsf{F}_{8})} \underbrace{\mathsf{h}_{1} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{8}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4}} \\ - : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{8}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4}} \xrightarrow{\mathsf{h}_{9} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : dual(\mathsf{F}_{8}), \Delta_{10}} \underbrace{\mathsf{Cut}} \underbrace{\mathsf{cut}}$$

$$\underbrace{\mathsf{h}_{1} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{3}, \mathsf{F}_{3}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4}} \\ - : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4}} \underbrace{\mathsf{h}_{9} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{dual}(\mathsf{F}_{8}), \Delta_{10}} \\ - : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_{6}, \Delta_{7}, \mathsf{F}_{3} \otimes \mathsf{F}_{4}} \underbrace{\mathsf{h}_{9} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{11}, \Delta_{10}, \mathsf{dual}(\mathsf{F}_{8})} \underbrace{\mathsf{h}_{10} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{11}, \Delta_{10}, \mathsf{dual}(\mathsf{F}_{8})} \underbrace{\mathsf{h}_{10} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{11}, \Delta_{10}, \mathsf{dual}(\mathsf{F}_{8}), \Delta_{10}} \underbrace{\mathsf{h}_{11} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_{11}, \Delta_{10}, \mathsf{dual}(\mathsf{F}_{8})} \underbrace{\mathsf{h}_{10} : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{h}_{11}, \mathsf$$

# 7.12 Status of $I_1$ : OK

• Case rule 1

- Case rule!
- Case rule?

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \vdash \{\Delta_8, p(\mathbf{n}_4)\} : \hat{\ } (\mathbf{n}_4), * \\ \\ \bullet_{\mathbf{h}_1} : \vdash \{\Delta_8, p(\mathbf{n}_4)\} : \hat{\ } (\mathbf{n}_4), * \\ \end{array}}_{\bullet \mathbf{h}_5} I_1 \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_5} : \vdash \{F_6, \Delta_8, p(\mathbf{n}_4)\} : \Delta_7, p(\mathbf{n}_4) \\ \\ \bullet_{\mathbf{h}_5} : \vdash \{\Delta_8, p(\mathbf{n}_4)\} : dual(\hat{\ } (\mathbf{n}_4)), \Delta_7, ?F_6 \\ \\ \to \\ \hline \\ - : \vdash \{\Delta_8, p(\mathbf{n}_4)\} : \Delta_7, p(\mathbf{n}_4) \\ \hline \\ - : \vdash \{\Delta_8, F_6, p(\mathbf{n}_4)\} : \Delta_7 \\ \hline \\ - : \vdash \{\Delta_8, F_6, p(\mathbf{n}_4)\} : \Delta_7, ?F_6 \\ \end{array}}_{?C} \underbrace{ \begin{array}{c} ? \\ ? \\ ? \\ ? \end{array}}_{?C}$$

• Case rule \$

• Case rule &

$$\frac{\bullet \mathbf{h}_1 : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \hat{\ } (\mathbf{n}_4), *}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \hat{\ } (\mathbf{h}_4), *} I_1 \xrightarrow{\bullet \mathbf{h}_5 : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \mathcal{H}_{\Delta_9, p(\mathbf{n}_4)} : dual(\hat{\ } (\mathbf{n}_4)), \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7} \underbrace{- : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathbf{F}_6, p(\mathbf{n}_4)\} : \lambda_8, \mathbf{F}_6 \& \mathbf{F}_7}_{\bullet : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathbf{F}_6, p(\mathbf{n}_4)\} : \Delta_8, p(\mathbf{n}_4)\} : \Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)} \underbrace{\bullet \mathbf{x}}_{\bullet : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, p(\mathbf{n}_4), \mathbf{F}_6 \& \mathbf{F}_7}_{\bullet : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7}_{?C} ?_C}_{\bullet : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7}$$

• Case rule  $\oplus_B$ 

$$\begin{array}{c} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Gamma_7, \Delta_8, p(\mathbf{n}_4) \\ \bullet_{\mathbf{h}_1} : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \widehat{\phantom{a}}(\mathbf{n}_4), * \end{array} I_1 \quad \begin{array}{c} \bullet_{\mathbf{h}_5} : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \mathsf{F}_7, \Delta_8, p(\mathbf{n}_4) \\ \bullet_{\mathbf{h}_5} : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : dual(\widehat{\phantom{a}}(\mathbf{n}_4)), \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ & \rightarrow \\ \hline - : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ \hline - : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathsf{F}_7 \\ \hline - : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathsf{F}_7 \\ \hline - : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \begin{array}{c} \oplus_B \end{array} } \end{array}$$

• Case rule  $\oplus_A$ 

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \hat{\mathsf{r}}_{(\mathbf{h}_4), *} & I_1 & \frac{\mathsf{h}_5 : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \mathsf{r}_{6}, \Delta_8, p(\mathbf{n}_4)}{\bullet_{\mathbf{h}_5} : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : dual(\hat{\mathsf{r}}(\mathbf{n}_4)), \Delta_8, \mathsf{r}_6 \oplus \mathsf{F}_7} \\ & - : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : *, \Delta_8, \mathsf{r}_6 \oplus \mathsf{F}_7 \\ & - : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathsf{r}_{6}, p(\mathbf{n}_4)} & \mathsf{ax} \\ & - : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathsf{r}_6 \\ & - : \vdash \{\Delta_9, p(\mathbf{n}_4)\} : \Delta_8, \mathsf{r}_6 \oplus \mathsf{F}_7 \end{array} } \xrightarrow{\bullet_A}$$

• Case rule  $\perp$ 

 $\bullet$  Case rule  $\top$ 

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1} : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : \widehat{\phantom{a}}(\mathbf{n}_4), * & I_1 & \hline \bullet_{\mathbf{h}_5} : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : dual(\widehat{\phantom{a}}(\mathbf{n}_4)), \top, \Delta_6 \\ & - : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : *, \top, \Delta_6 \\ & - & \hline - : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : \Delta_6, \top & \top \end{array}$$

• Case rule  $I_3$ 

$$\frac{ \underbrace{ \bullet_{\mathbf{h}_1} : \vdash \{\Delta_6, p(\mathbf{n}_5)\} : \hat{\ }(\mathbf{n}_5), *}_{ } I_1 }_{ - : \vdash \{\Delta_6, p(\mathbf{n}_5)\} : *, \hat{\ }(\mathbf{n}_5)} : *, \hat{\ }(\mathbf{n}_5)}_{ - : \vdash \{\Delta_6, p(\mathbf{n}_5)\} : \hat{\ }(\mathbf{n}_5)} : t_1} \underbrace{ I_3 \atop - : \vdash \{\Delta_6, p(\mathbf{n}_5)\} : \hat{\ }(\mathbf{n}_5) \atop - : \vdash \{\Delta_6, p(\mathbf{n}_5)\} : \hat{\ }(\mathbf{n}_5) }_{ } I_1}$$

 $\bullet$  Case rule  $\otimes$ 

$$\begin{array}{c} \underbrace{\bullet h_1 : \vdash \{\Delta_{10}, p(n_4)\} : \hat{\ }_{(n_4), *} } \quad I_1 \quad \underbrace{\frac{h_5 : \vdash \{\Delta_{10}, p(n_4)\} : F_6, \Delta_8, p(n_4) \quad h_5 : \vdash \{\Delta_{10}, p(n_4)\} : F_7, \Delta_9}{\bullet h_5 : \vdash \{\Delta_{10}, p(n_4)\} : dual(\hat{\ }_{(n_4)}), \Delta_8, \Delta_9, F_6 \otimes F_7}}_{\quad \ - : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_8, F_6, p(n_4)} \\ \underbrace{\frac{- : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_8, F_6, p(n_4)}{- : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_8, F_6}}_{\quad \ - : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_8, F_6 \otimes F_7} \\ \underbrace{\frac{- : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_8, F_6}{- : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_8, \Delta_9, F_6 \otimes F_7}}_{\quad \ - : \vdash \{\Delta_{10}, p(n_4)\} : A_8, \Delta_9, F_6 \otimes F_7} \\ \underbrace{\frac{- : \vdash \{\Delta_{10}, p(n_4)\} : \hat{\ }_{(n_4), k_1} }_{\quad \ - : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_8, F_6 \otimes F_7}}_{\quad \ - : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_8, F_6 \otimes F_7}_{\quad \ - : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_9, F_7, p(n_4)}_{\quad \ - : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_8, F_6 \otimes F_7}_{\quad \ - : \vdash \{\Delta_{10}, p(n_4)\} : \Delta_9, F_7, p(n_4), A_9, F_9, P(n_4), A_$$

- Case rule  $I_1$
- Case rule  $I_2$

$$\frac{ \underbrace{ \bullet_{h_1} : \vdash \{(\Delta_6, \, \, \, \hat{}\, (n_5)), \, p(n_5)\} : \, \, \, \hat{}\, \, (n_5), \, *}_{\quad \, \cap_{h_4} : \vdash \{(\Delta_6, \, \, \, \, \, \hat{}\, (n_5)), \, p(n_5)\} : \, dual(\, \, \, \, \hat{}\, (n_5)), \, *}_{\quad \, - : \vdash \{(\Delta_6, \, \, \, \, \, \, \hat{}\, (n_5)), \, \, \, \, \, (n_5)\} : \, *, \, *}_{\quad \, - : \vdash \{\Delta_6, \, p(n_5), \, \, \, \, \, \, \, (n_5)\} : \, *}_{\quad \, - : \vdash \{\Delta_6, \, p(n_5), \, \, \, \, \, \, \, \, (n_5)\} : \, *}_{\quad \, ?_C}$$

• Case rule  $?_C$ 

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : \hat{\wedge}_{(h_4), *} & I_1 \\ \bullet_{h_5} : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : dual(\hat{\wedge}(\mathbf{n}_4), p(\mathbf{n}_4) \\ & - : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : *, \Delta_6 \\ \hline \\ & - : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : \Delta_6, p(\mathbf{n}_4), p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : \Delta_6, p(\mathbf{n}_4), p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : \Delta_6, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_7, p(\mathbf{n}_4)\} : \Delta_6 \end{array}} \, \stackrel{\mathbf{ax}}{?_C} \\ \\ & \underbrace{ \begin{array}{c} \bullet_{h_1} : \vdash \{(\mathbf{F}_7, \Delta_8), p(\mathbf{n}_4)\} : \hat{\wedge}(\mathbf{n}_4), * \\ \hline \\ \bullet_{h_1} : \vdash \{(\mathbf{F}_7, \Delta_8), p(\mathbf{n}_4)\} : \hat{\wedge}(\mathbf{n}_4), * \\ \hline \\ & - : \vdash \{(\mathbf{F}_7, \Delta_8), p(\mathbf{n}_4)\} : *, \Delta_6 \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ & - : \vdash \{\Delta_8, \mathbf{F}_7, p(\mathbf{n}_4)\} : \Delta_6, \mathbf{F}_7, p(\mathbf{n}_4) \\ \hline \\ \end{array} \, \begin{array}{c} ?_C \\ ?_C \\$$

# 7.13 Status of $I_2$ : OK

- Case rule 1
- Case rule!
- Case rule?

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \vdash \{\Delta_8, \, {}^{\smallfrown}(\mathbf{n}_4)\} : p(\mathbf{n}_4), * \\ \bullet \\ - : \vdash \{\Delta_8, \, {}^{\smallfrown}(\mathbf{n}_4)\} : p(\mathbf{n}_4), * \end{array} } I_2 \quad \frac{ \begin{array}{c} \bullet_{\mathbf{h}_5} : \vdash \{\mathsf{F}_6, \Delta_8, \, {}^{\smallfrown}(\mathbf{n}_4)\} : \Delta_7, \, {}^{\smallfrown}(\mathbf{n}_4) \\ \bullet \\ \bullet_{\mathbf{h}_5} : \vdash \{\Delta_8, \, {}^{\smallfrown}(\mathbf{n}_4)\} : dual(p(\mathbf{n}_4)), \Delta_7, \, {}^{\backprime}\mathsf{F}_6 \\ \bullet \\ \hline \\ - : \vdash \{\Delta_8, \, {}^{\smallfrown}(\mathbf{n}_4)\} : \Delta_7, \, {}^{\backprime}(\mathbf{n}_4) \\ \hline \\ - : \vdash \{\Delta_8, \, \mathsf{F}_6, \, {}^{\smallfrown}(\mathbf{n}_4)\} : \Delta_7, \, {}^{\backprime}\mathsf{F}_6 \\ \hline \\ - : \vdash \{\Delta_8, \, {}^{\smallfrown}(\mathbf{n}_4)\} : \Delta_7, \, {}^{\backprime}\mathsf{F}_6 \end{array} } \stackrel{\mathbf{ax}}{?}$$

• Case rule \$

$$\begin{array}{c} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_{1}} : \vdash \{\Delta_{9}, \, {}^{\hat{}}(\mathbf{n}_{4})\} : p(\mathbf{n}_{4}), *} & I_{2} & \underbrace{ \begin{array}{c} \mathbf{h}_{5} : \vdash \{\Delta_{9}, \, {}^{\hat{}}(\mathbf{n}_{4})\} : F_{6}, F_{7}, \Delta_{8}, \, {}^{\hat{}}(\mathbf{n}_{4})}_{\bullet \mathbf{h}_{5} : \vdash \{\Delta_{9}, \, {}^{\hat{}}(\mathbf{n}_{4})\} : dual(p(\mathbf{n}_{4})), \Delta_{8}, F_{6}\$F_{7}} \\ & - : \vdash \{\Delta_{9}, \, {}^{\hat{}}(\mathbf{n}_{4})\} : *, \Delta_{8}, F_{6}\$F_{7} \\ & - : \vdash \{\Delta_{9}, \, {}^{\hat{}}(\mathbf{n}_{4})\} : \Delta_{8}, F_{6}, F_{7}, \, {}^{\hat{}}(\mathbf{n}_{4})}_{\bullet} \\ & - : \vdash \{\Delta_{9}, \, {}^{\hat{}}(\mathbf{n}_{4})\} : \Delta_{8}, F_{6}, F_{7}, \, {}^{\hat{}}(\mathbf{n}_{4})}_{\bullet} \\ & - : \vdash \{\Delta_{9}, \, {}^{\hat{}}(\mathbf{n}_{4})\} : \Delta_{8}, F_{6}, F_{7}, \, {}^{\hat{}}(\mathbf{n}_{4})}_{\bullet} \end{array} \right. \end{array}$$

• Case rule &

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \vdash \{\Delta_9, \, \hat{}(n_4)\} : p(n_4), * \\ \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : p(n_4), * \end{array}} I_2 \begin{array}{c} \underbrace{ \begin{array}{c} h_5 : \vdash \{\Delta_9, \, \hat{}(n_4)\} : F_6, \, \Delta_8, \, \hat{}(n_4) & h_5 : \vdash \{\Delta_9, \, \hat{}(n_4)\} : F_7, \, \Delta_8, \, \hat{}(n_4) \\ \\ \hline \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : *, \, \Delta_8, F_6 \& F_7 \\ \hline \\ \hline \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, F_6, \, \hat{}(n_4) & \underbrace{ \begin{array}{c} \rightarrow \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, \, \hat{}(n_4), \, F_6 \& F_7 \\ \hline \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, \, \hat{}(n_4), \, F_6 \& F_7 \\ \hline \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, \, F_6, \, \hat{}(n_4) & \underbrace{ \begin{array}{c} \rightarrow \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, \, \hat{}(n_4), \, F_6 \& F_7 \\ \hline \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, \, F_6, \, \hat{}(n_4) & \underbrace{ \begin{array}{c} \rightarrow \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, \, F_6, \, \hat{}(n_4), \, F_6 \& F_7 \\ \hline \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, \, F_6, \, \hat{}(n_4) & \underbrace{ \begin{array}{c} \rightarrow \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, \, F_6, \, \hat{}(n_4), \, F_6 \& F_7 \\ \hline \\ - : \vdash \{\Delta_9, \, \hat{}(n_4)\} : \Delta_8, \, F_6, \, \hat{}(n_4), \, F_6 \& F_7 \\ \hline \end{array} } }_{CU}$$

• Case rule  $\oplus_B$ 

$$\begin{array}{c} \frac{ \text{$\mathsf{h}_5 : \vdash \{\Delta_9, \char`(\mathsf{n}_4)\} : \mathsf{F}_7, \Delta_8, \char`(\mathsf{n}_4)$}}{ \bullet \mathsf{h}_5 : \vdash \{\Delta_9, \char`(\mathsf{n}_4)\} : dual(p(\mathsf{n}_4)), \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 }} & \oplus_B \\ & - : \vdash \{\Delta_9, \char`(\mathsf{n}_4)\} : *, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ & \to \\ & - : \vdash \{\Delta_9, \char`(\mathsf{n}_4)\} : \times, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \\ & \to \\ & - : \vdash \{\Delta_9, \char`(\mathsf{n}_4)\} : \Delta_8, \mathsf{F}_7, \char`(\mathsf{n}_4) \\ & - : \vdash \{\Delta_9, \char`(\mathsf{n}_4)\} : \Delta_8, \mathsf{F}_7 \\ & - : \vdash \{\Delta_9, \char`(\mathsf{n}_4)\} : \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7 \end{array} \end{array} \text{ax}$$

• Case rule  $\oplus_A$ 

$$\begin{array}{c|c} & \underbrace{ \begin{array}{c} \mathbf{h}_{5} : \vdash \left\{ \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}) \right\} : \mathbf{F}_{6}, \Delta_{8}, \widehat{\phantom{a}}(\mathbf{n}_{4})}_{\bullet \mathbf{h}_{5} : \vdash \left\{ \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}) \right\} : dual(p(\mathbf{n}_{4})), \Delta_{8}, \mathbf{F}_{6} \oplus \mathbf{F}_{7}}_{\bullet \mathbf{h}_{5} : \vdash \left\{ \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}) \right\} : A_{8}, \mathbf{F}_{6} \oplus \mathbf{F}_{7}} \\ & \underbrace{ \begin{array}{c} - : \vdash \left\{ \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}) \right\} : \Delta_{8}, \mathbf{F}_{6}, \widehat{\phantom{a}}(\mathbf{n}_{4})}_{\circ C} \\ - : \vdash \left\{ \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}) \right\} : \Delta_{8}, \mathbf{F}_{6}, \widehat{\phantom{a}}(\mathbf{n}_{4})}_{\circ C} \\ - : \vdash \left\{ \Delta_{9}, \widehat{\phantom{a}}(\mathbf{n}_{4}) \right\} : \Delta_{8}, \mathbf{F}_{6} \oplus \mathbf{F}_{7} \end{array}} \begin{array}{c} \oplus_{A} \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{array}$$

• Case rule  $\perp$ 

 $\bullet$  Case rule  $\top$ 

$$\frac{ \underbrace{ \bullet \mathbf{h}_1 : \vdash \{\Delta_7, \widehat{\phantom{a}}(\mathbf{n}_4)\} : p(\mathbf{n}_4), *}_{ = 12} } \underbrace{ I_2 }_{ \bullet \mathbf{h}_5 : \vdash \{\Delta_7, \widehat{\phantom{a}}(\mathbf{n}_4)\} : dual(p(\mathbf{n}_4)), \top, \Delta_6 }_{ = 12} }_{ = 122 } \underbrace{ \Box \mathbf{h}_5 : \vdash \{\Delta_7, \widehat{\phantom{a}}(\mathbf{n}_4)\} : *, \top, \Delta_6 }_{ \to 122 } }_{ = 122 } \underbrace{ \Box \mathbf{h}_5 : \vdash \{\Delta_7, \widehat{\phantom{a}}(\mathbf{n}_4)\} : *, \top, \Delta_6 }_{ \to 122 } }_{ \to 122 }$$

• Case rule  $I_3$ 

$$\cfrac{ \cfrac{\bullet_{\mathbf{h}_1} : \vdash \{\Delta_6, \, \widehat{\ } (\mathbf{n}_5)\} : p(\mathbf{n}_5), *}{- : \vdash \{\Delta_6, \, \widehat{\ } (\mathbf{n}_5)\} : *, p(\mathbf{n}_5)} \cfrac{I_3}{\bullet_{\mathbf{h}_4} : \vdash \{\Delta_6, \, \widehat{\ } (\mathbf{n}_5)\} : *, p(\mathbf{n}_5)} } \quad \mathbf{Cut} \\ \cfrac{- : \vdash \{\Delta_6, \, \widehat{\ } (\mathbf{n}_5)\} : p(\mathbf{n}_5)}{- : \vdash \{\Delta_6, \, \widehat{\ } (\mathbf{n}_5)\} : p(\mathbf{n}_5)} \quad I_2 }$$

 $\bullet \;$  Case rule  $\otimes$ 

• Case rule  $I_1$ 

$$\frac{ \underbrace{ \bullet_{h_1} : \vdash \{(\Delta_6, p(\mathbf{n}_5)), \, \hat{\ } (\mathbf{n}_5)\} : p(\mathbf{n}_5), *}_{\quad \bullet_{h_4} : \vdash \{(\Delta_6, p(\mathbf{n}_5)), \, \hat{\ } (\mathbf{n}_5)\} : dual(p(\mathbf{n}_5)), *}_{\quad \bullet_{h_4} : \vdash \{(\Delta_6, p(\mathbf{n}_5)), \, \hat{\ } (\mathbf{n}_5)\} : *, *} \\ \underbrace{ \begin{matrix} - : \vdash \{(\Delta_6, p(\mathbf{n}_5)), \, \hat{\ } (\mathbf{n}_5)\} : p(\mathbf{n}_5) \\ \hline - : \vdash \{\Delta_6, p(\mathbf{n}_5), \, \hat{\ } (\mathbf{n}_5)\} : * \end{matrix}}_{\quad ?_C} \underbrace{ \begin{matrix} I_2 \\ ?_C \end{matrix}}_{\quad ?_C}$$

- Case rule  $I_2$
- Case rule  $?_C$

#### 7.14 Status of $?_C$ : OK

- Case rule 1
- Case rule!
- Case rule?

$$\frac{\mathbf{h}_{1} : \vdash \left\{ \mathbf{F}_{10}, \Delta_{11} \right\} : \mathbf{F}_{6}, \mathbf{F}_{10}, \Delta_{3}}{\bullet \mathbf{h}_{1} : \vdash \left\{ \mathbf{F}_{10}, \Delta_{11} \right\} : \mathbf{F}_{6}, \Delta_{3}} ?_{C} \quad \frac{\mathbf{h}_{7} : \vdash \left\{ \mathbf{F}_{8}, \mathbf{F}_{10}, \Delta_{11} \right\} : \Delta_{9}, dual(\mathbf{F}_{6})}{\bullet \mathbf{h}_{7} : \vdash \left\{ \mathbf{F}_{10}, \Delta_{11} \right\} : dual(\mathbf{F}_{6}), \Delta_{9}, ?\mathbf{F}_{8}} ?_{Cut}} ?_{Cut} \\ \frac{\mathbf{h}_{1} : \vdash \left\{ \Delta_{11}, \mathbf{F}_{10} \right\} : \Delta_{3}, \mathbf{F}_{10}, \mathbf{F}_{6}}{\bullet \mathbf{h}_{1} : \vdash \left\{ \Delta_{11}, \mathbf{F}_{10} \right\} : \Delta_{9}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{6})} } \overset{\mathbf{ax}}{\bullet \mathbf{h}_{7} : \vdash \left\{ \Delta_{11}, \mathbf{F}_{10} \right\} : \Delta_{9}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{6})}} ?_{Cut} \\ \frac{- : \vdash \left\{ \Delta_{11}, \mathbf{F}_{10} \right\} : \Delta_{3}, \Delta_{9}, \mathbf{F}_{10}, ?\mathbf{F}_{8}}{- : \vdash \left\{ \Delta_{11}, \mathbf{F}_{10} \right\} : \Delta_{3}, \Delta_{9}, ?\mathbf{F}_{8}} ?_{C}} ?_{Cut}$$

• Case rule \$

$$\frac{\mathbf{h}_1 : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_6, \mathsf{F}_{11}, \Delta_3}{\bullet \mathsf{h}_1 : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_6, \Delta_3} ?_C \quad \frac{\mathsf{h}_7 : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_8, \mathsf{F}_9, \Delta_{10}, dual(\mathsf{F}_6)}{\bullet \mathsf{h}_7 : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : dual(\mathsf{F}_6), \Delta_{10}, \mathsf{F}_8 \$ \mathsf{F}_9} \\ - : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \Delta_3, \Delta_{10}, \mathsf{F}_8 \$ \mathsf{F}_9 \\ - : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_3, \mathsf{F}_6} \quad \frac{\to}{\mathsf{h}_7 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \mathsf{F}_8, \mathsf{F}_9, dual(\mathsf{F}_6)}}{\bullet \mathsf{h}_7 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8, \mathsf{F}_9, dual(\mathsf{F}_6)}} \quad \frac{\mathsf{ax}}{\mathsf{h}_7 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8, \mathsf{F}_9}} \\ \frac{- : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8, \mathsf{F}_9}{- : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8, \mathsf{F}_9}} \quad \$$$

• Case rule &

$$\frac{ \begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{ \mathbf{F}_{11}, \Delta_{12} \} : \mathbf{F}_6, \mathbf{F}_{11}, \Delta_3}{\bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_{11}, \Delta_{12} \} : \mathbf{F}_6, \Delta_3} \end{array} ?_C & \begin{array}{c} \frac{\mathbf{h}_7 : \vdash \{ \mathbf{F}_{11}, \Delta_{12} \} : \mathbf{F}_8, \Delta_{10}, dual(\mathbf{F}_6) & \mathbf{h}_7 : \vdash \{ \mathbf{F}_{11}, \Delta_{12} \} : \mathbf{F}_9, \Delta_{10}, dual(\mathbf{F}_6)} \\ \bullet \mathbf{h}_7 : \vdash \{ \mathbf{F}_{11}, \Delta_{12} \} : dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \\ - : \vdash \{ \mathbf{F}_{11}, \Delta_{12} \} : \Delta_3, \Delta_{10}, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_{12}, \mathbf{F}_{11} \} : \Delta_{10}, \mathbf{h}_{10}, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \{ \Delta_{12}, \mathbf{F}_{11} \} : \Delta_{10}, dual(\mathbf{F}_6), \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \\ - : \vdash \{ \Delta_{12}, \mathbf{F}_{11} \} : \Delta_{10}, \Delta_3, \mathbf{F}_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \\ - : \vdash \{ \Delta_{12}, \mathbf{F}_{11} \} : \Delta_{10}, \Delta_3, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \\ - : \vdash \{ \Delta_{12}, \mathbf{F}_{11} \} : \Delta_{10}, \Delta_3, \mathbf{F}_8 \& \mathbf{F}_9 \\ \hline \end{array} ?_C \end{array} & \begin{array}{c} \mathbf{ax} \\ \mathbf{hCut} \\ \end{array}$$

• Case rule  $\oplus_B$ 

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \{ \mathsf{F}_{11}, \Delta_{12} \} : \mathsf{F}_6, \mathsf{F}_{11}, \Delta_3 \\ \\ \underline{\bullet \mathsf{h}_1 : \vdash \{ \mathsf{F}_{11}, \Delta_{12} \} : \mathsf{F}_6, \Delta_3} \end{array} ?_C \quad \frac{ \begin{array}{c} \mathsf{h}_7 : \vdash \{ \mathsf{F}_{11}, \Delta_{12} \} : \mathsf{F}_9, \Delta_{10}, dual(\mathsf{F}_6) \\ \\ \underline{\bullet \mathsf{h}_1 : \vdash \{ \mathsf{F}_{11}, \Delta_{12} \} : \mathsf{F}_6, \Delta_3} \end{array} ?_C \quad \frac{ \begin{array}{c} \mathsf{h}_7 : \vdash \{ \mathsf{F}_{11}, \Delta_{12} \} : \mathsf{F}_9, \Delta_{10}, dual(\mathsf{F}_6) \\ \\ \underline{\bullet \mathsf{h}_7 : \vdash \{ \mathsf{F}_{11}, \Delta_{12} \} : \Delta_3, \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9} \\ \\ \underline{\bullet \mathsf{h}_1 : \vdash \{ \Delta_{12}, \mathsf{F}_{11} \} : \Delta_3, \mathsf{F}_6} \quad \text{ax} \quad \frac{}{\mathsf{h}_7 : \vdash \{ \Delta_{12}, \mathsf{F}_{11} \} : \Delta_{10}, \mathsf{F}_9, dual(\mathsf{F}_6) \\ \\ \underline{- : \vdash \{ \Delta_{12}, \mathsf{F}_{11} \} : \Delta_{10}, \Delta_3, \mathsf{F}_9 \\ \\ \underline{- : \vdash \{ \Delta_{12}, \mathsf{F}_{11} \} : \Delta_{10}, \Delta_3, \mathsf{F}_8 \oplus \mathsf{F}_9} \quad \oplus_B \end{array}} \quad \begin{array}{c} \oplus_B \\ \text{Cut} \\ \\ \mathsf{hCut} \\ \end{array}$$

• Case rule  $\oplus_A$ 

$$\frac{\mathbf{h}_1 : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_6, \mathsf{F}_{11}, \Delta_3}{\bullet \mathsf{h}_1 : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_6, \Delta_3} ?_C \quad \frac{\mathbf{h}_7 : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \mathsf{F}_8, \Delta_{10}, dual(\mathsf{F}_6)}{\bullet \mathsf{h}_7 : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : dual(\mathsf{F}_6), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9} \quad \underbrace{- : \vdash \{\mathsf{F}_{11}, \Delta_{12}\} : \Delta_3, \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9}_{\bullet \mathsf{h}_1 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_3, \mathsf{F}_6} \quad \underset{\mathsf{h}_7 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8}{\bullet \mathsf{h}_7 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8} \quad \underbrace{+ \mathsf{h}_4}_{\bullet \mathsf{Lut}} \quad \underbrace{+ \mathsf{h}_4 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8}_{\bullet \mathsf{Lut}} \quad \underbrace{+ \mathsf{h}_4 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8}_{\bullet \mathsf{Lut}} \quad \underbrace{+ \mathsf{h}_4 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8}_{\bullet \mathsf{Lut}} \quad \underbrace{+ \mathsf{h}_4 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8 \oplus \mathsf{F}_9}_{\bullet \mathsf{Lut}} \quad \underbrace{+ \mathsf{h}_4 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8 \oplus \mathsf{F}_9}_{\bullet \mathsf{Lut}} \quad \underbrace{+ \mathsf{h}_4 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8 \oplus \mathsf{F}_9}_{\bullet \mathsf{Lut}} \quad \underbrace{+ \mathsf{h}_4 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8 \oplus \mathsf{F}_9}_{\bullet \mathsf{Lut}}}_{\bullet \mathsf{Lut}} \quad \underbrace{+ \mathsf{h}_4 : \vdash \{\Delta_{12}, \mathsf{F}_{11}\} : \Delta_{10}, \Delta_3, \mathsf{F}_8 \oplus \mathsf{F}_9}_{\bullet \mathsf{Lut}}}_{\bullet \mathsf{Lut}}$$

 $\bullet$  Case rule  $\bot$ 

$$\frac{\begin{array}{l} \underline{\mathbf{h}}_1 : \vdash \left\{ \mathbf{F}_9, \Delta_{10} \right\} : \mathbf{F}_6, \mathbf{F}_9, \Delta_3}{\bullet \mathbf{h}_1 : \vdash \left\{ \mathbf{F}_9, \Delta_{10} \right\} : \mathbf{F}_6, \Delta_3} ?_C & \frac{\mathbf{h}_7 : \vdash \left\{ \mathbf{F}_9, \Delta_{10} \right\} : \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash \left\{ \mathbf{F}_9, \Delta_{10} \right\} : dual(\mathbf{F}_6), \bot, \Delta_8} \\ \hline \\ \underline{- : \vdash \left\{ \mathbf{F}_9, \Delta_{10} \right\} : \Delta_3, \bot, \Delta_8} & \xrightarrow{\bullet} \\ \underline{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_{10}, \mathbf{F}_9 \right\} : \Delta_3, \mathbf{F}_6} & \text{ax} & \xrightarrow{\bullet} \\ \hline \\ \underline{- : \vdash \left\{ \Delta_{10}, \mathbf{F}_9 \right\} : \Delta_3, \Delta_8} \\ \underline{- : \vdash \left\{ \Delta_{10}, \mathbf{F}_9 \right\} : \Delta_3, \Delta_8, \bot} & \bot \\ \end{array} \quad \begin{array}{l} L \\ \text{Cut} \\ \text{hCut} \\ \end{array}$$

• Case rule  $\top$ 

$$\frac{\underset{\bullet}{\mathbf{h}_1} : \vdash \{ \mathbf{F}_9, \Delta_{10} \} : \mathbf{F}_6, \mathbf{F}_9, \Delta_3}{\underset{\bullet}{\bullet} \mathbf{h}_1 : \vdash \{ \mathbf{F}_9, \Delta_{10} \} : \mathbf{F}_6, \Delta_3} ?_C \quad \underbrace{\underset{\bullet}{\bullet} \mathbf{h}_7 : \vdash \{ \mathbf{F}_9, \Delta_{10} \} : \mathit{dual}(\mathbf{F}_6), \top, \Delta_8}_{- : \vdash \{ \mathbf{F}_9, \Delta_{10} \} : \Delta_3, \top, \Delta_8} \quad \top}_{- : \vdash \{ \Delta_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8, \top} \quad \top$$

- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \{F_{12}, \Delta_{13}\} : F_{6}, F_{12}, \Delta_{3}}{\bullet \mathbf{h}_{1} : \vdash \{F_{12}, \Delta_{13}\} : F_{6}, \Delta_{3}} &?_{C} & \frac{\mathbf{h}_{7} : \vdash \{F_{12}, \Delta_{13}\} : F_{8}, \Delta_{10}, dual(F_{6}) \quad \mathbf{h}_{7} : \vdash \{F_{12}, \Delta_{13}\} : F_{9}, \Delta_{11}}{\bullet \mathbf{h}_{7} : \vdash \{F_{12}, \Delta_{13}\} : dual(F_{6}), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}} \\ & & - : \vdash \{F_{12}, \Delta_{13}\} : \Delta_{3}, \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9} \\ & & \rightarrow \\ \hline & \frac{\mathbf{h}_{1} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{3}, F_{12}, F_{6}}{\bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, dual(F_{6}), F_{8} \otimes F_{9}} \\ & & - : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_{3}, F_{12}, F_{8} \otimes F_{9} \\ & - : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_{3}, F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{F_{12}, \Delta_{13}\} : F_{9}, \Delta_{11}, dual(F_{6}) \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{F_{12}, \Delta_{13}\} : F_{8}, \Delta_{10} \quad \mathbf{h}_{7} : \vdash \{F_{12}, \Delta_{13}\} : F_{9}, \Delta_{11}, dual(F_{6}) \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{F_{12}, \Delta_{13}\} : dual(F_{6}), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{F_{12}, \Delta_{13}\} : dual(F_{6}), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, dual(F_{6}), F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, dual(F_{6}), F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, dual(F_{6}), F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, dual(F_{6}), F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, dual(F_{6}), F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, dual(F_{6}), F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, dual(F_{6}), F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, dual(F_{6}), F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_{3}, F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_{3}, F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_{3}, F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_{3}, F_{8} \otimes F_{9} \\ \hline & \bullet \mathbf{h}_{7} : \vdash \{\Delta_{13}, F_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_{3$$

- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \{ \mathbf{F}_9, \Delta_{10} \} : \mathbf{F}_6, \mathbf{F}_9, \Delta_3}{\bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_9, \Delta_{10} \} : \mathbf{F}_6, \Delta_3} ?_C & \frac{\mathbf{h}_7 : \vdash \{ \mathbf{F}_9, \Delta_{10} \} : \mathbf{F}_9, \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash \{ \mathbf{F}_9, \Delta_{10} \} : dual(\mathbf{F}_6), \Delta_8} ?_C \\ \hline \\ \frac{- : \vdash \{ \mathbf{F}_9, \Delta_{10} \} : \Delta_3, \Delta_8}{\bullet \mathbf{h}_1 : \vdash \{ \Delta_{10}, \mathbf{F}_9 \} : \Delta_3, \mathbf{F}_6, \mathbf{F}_9} & \frac{\rightarrow}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} \\ \hline \\ \frac{- : \vdash \{ \Delta_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8, \mathbf{F}_9}{- : \vdash \{ \Delta_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8} ?_C \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \{ \mathbf{F}_{10}, \mathbf{F}_9, \Delta_{11} \} : \mathbf{F}_6, \mathbf{F}_{10}, \Delta_3}{\bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_{10}, \mathbf{F}_9, \Delta_{11} \} : \mathbf{F}_6, \mathbf{F}_{10}, \Delta_3} ?_C & \frac{\mathbf{h}_7 : \vdash \{ \mathbf{F}_9, \mathbf{F}_{10}, \Delta_{11} \} : \mathbf{F}_9, \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash \{ \mathbf{F}_{10}, \mathbf{F}_9, \Delta_{11} \} : dual(\mathbf{F}_6), \Delta_8} ?_C \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \{ \mathbf{F}_{10}, \mathbf{F}_9, \Delta_{11} \} : \mathbf{F}_6, \mathbf{h}_3}{\bullet \mathbf{h}_7 : \vdash \{ \mathbf{F}_{10}, \mathbf{F}_9, \Delta_{11} \} : dual(\mathbf{F}_6), \Delta_8} ?_C \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} ?_C \\ \hline \\ \frac{- : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} ?_C \\ \hline \\ \frac{- : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} ?_C \\ \hline \\ \frac{- : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} ?_C \\ \hline \\ \frac{- : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} ?_C \\ \hline \\ \frac{- : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} ?_C \\ \hline \\ \frac{- : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} ?_C \\ \hline \\ \frac{- : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_3, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} ?_C \\ \hline \\ \frac{- : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, \Delta_8, \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : \vdash \{ \Delta_{11}, \mathbf{F}_{10}, \mathbf{F}_9 \} : \Delta_8, dual(\mathbf{F}_6)} ?_$$

# 8 Cut-Elimination

### 8.1 Status of 1: OK

- Case rule 1
- Case rule!
- Case rule?
- Case rule \$
- Case rule &
- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- $\bullet$  Case rule  $\bot$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$
- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

### 8.2 Status of !: OK

 $\bullet \;$  Case rule 1

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_6\right\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_6\right\} : ! \mathbf{F}_4} \ ! & \frac{\bullet \mathbf{h}_5 : \vdash \left\{dual(\mathbf{F}_4), \Delta_6\right\} : \mathbf{1}}{\bullet \mathbf{h}_5 : \vdash \left\{\Delta_6\right\} : \mathbf{1}} & \mathbf{Cut} \\ & \xrightarrow{- : \vdash \left\{\Delta_6\right\} : \mathbf{1}} & \mathbf{1} \end{array}$$

• Case rule!

$$\frac{ \begin{array}{l} \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : ! \mathbf{F}_4 \end{array} ! \quad \begin{array}{l} \mathbf{h}_5 : \vdash \left\{ \Delta_7, dual(\mathbf{F}_4) \right\} : \mathbf{F}_6 \\ \bullet \mathbf{h}_5 : \vdash \left\{ dual(\mathbf{F}_4), \Delta_7 \right\} : ! \mathbf{F}_6 \end{array} }{ \begin{array}{l} - : \vdash \left\{ \Delta_7 \right\} : ! \mathbf{F}_6 \\ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_7 \right\} : ! \mathbf{F}_6 \end{array} } \quad \begin{array}{l} \mathbf{Cut} \\ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_7 \right\} : ! \mathbf{F}_6 \\ \hline - : \vdash \left\{ \Delta_7 \right\} : \mathbf{F}_6 \end{array} } \\ \bullet \mathbf{hCut} \\ \hline \\ - : \vdash \left\{ \Delta_7 \right\} : ! \mathbf{F}_6 \end{array} !$$

• Case rule ?

$$\frac{ \begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{ \Delta_8 \right\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_8 \right\} : ! \mathbf{F}_4} : \begin{array}{c} \mathbf{h}_5 : \vdash \left\{ \mathbf{F}_6, \Delta_8, dual(\mathbf{F}_4) \right\} : \Delta_7}{\bullet \mathbf{h}_5 : \vdash \left\{ dual(\mathbf{F}_4), \Delta_8 \right\} : \Delta_7, ? \mathbf{F}_6} \\ \hline - : \vdash \left\{ \Delta_8 \right\} : \Delta_7, ? \mathbf{F}_6} \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_8 \right\} : ! \mathbf{F}_4} \end{array} \begin{array}{c} ? \\ \mathsf{cut} \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_8 \right\} : ! \mathbf{F}_4} \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_8, \mathbf{F}_6 \right\} : ! \mathbf{F}_4} \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_5 : \vdash \left\{ \Delta_8, \mathbf{F}_6, dual(\mathbf{F}_4) \right\} : \Delta_7} \\ \hline - : \vdash \left\{ \Delta_8, \mathbf{F}_6 \right\} : \Delta_7, ? \mathbf{F}_6} \end{array} \begin{array}{c} \mathsf{ax} \\ \mathsf{hCut} \\ \hline \end{array}$$

• Case rule \$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \{\Delta_9\} : F_4 \\ \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : !F_4 \end{array} : \begin{array}{c} \frac{\mathbf{h}_5 : \vdash \{\Delta_9, dual(F_4)\} : F_6, F_7, \Delta_8}{\bullet \mathbf{h}_5 : \vdash \{dual(F_4), \Delta_9\} : \Delta_8, F_6\$F_7} \\ \hline - : \vdash \{\Delta_9\} : \Delta_8, F_6\$F_7 \\ \hline \bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : !F_4 \end{array} \xrightarrow{\mathbf{ax}} \begin{array}{c} \mathbf{ax} \\ \frac{- : \vdash \{\Delta_9\} : \Delta_8, F_6, F_7}{- : \vdash \{\Delta_9\} : \Delta_8, F_6\$F_7} \\ \hline \end{array} \xrightarrow{\mathbf{hCut}} \mathbf{hCut}$$

• Case rule &

$$\frac{\frac{\mathbf{h}_1 : \vdash \{\Delta_9\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : !\mathbf{F}_4} \; ! \; \frac{\mathbf{h}_5 : \vdash \{\Delta_9, dual(\mathbf{F}_4)\} : \mathbf{F}_6, \Delta_8 \quad \mathbf{h}_5 : \vdash \{\Delta_9, dual(\mathbf{F}_4)\} : \mathbf{F}_7, \Delta_8}{\bullet \mathbf{h}_5 : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7} \quad \mathbf{Cut}} \; \& \\ \frac{- : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7}{\bullet \mathbf{h}_1 : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_6} \overset{\mathbf{ax}}{\underset{\mathbf{h}_5 : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_6}{\bullet \mathbf{h}_1} : \vdash \{\Delta_9\} : \mathcal{A}_8, \mathbf{F}_6}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_5 : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_6}{\bullet \mathbf{h}_1}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_5 : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_7}{\bullet \mathbf{h}_1}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_5 : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_7}{\bullet \mathbf{h}_1}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_5 : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_7}{\bullet \mathbf{h}_1}}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_5 : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_7}{\bullet \mathbf{h}_1}} \overset{\mathbf{ax}}{\underset{\mathbf{h}_5 : \vdash \{\Delta_9\} : \Delta_8, \mathbf{F}_7}{\bullet \mathbf{h}_1}} \&$$

• Case rule  $\oplus_B$ 

$$\begin{array}{c} \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : ! \mathbf{F}_4 \end{array} : \begin{array}{c} \mathbf{h}_5 : \vdash \left\{ \Delta_9, dual(\mathbf{F}_4) \right\} : \mathbf{F}_7, \Delta_8 \\ \bullet \mathbf{h}_5 : \vdash \left\{ dual(\mathbf{F}_4), \Delta_9 \right\} : \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ - : \vdash \left\{ \Delta_9 \right\} : \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : ! \mathbf{F}_4 \end{array} \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_9, dual(\mathbf{F}_4) \right\} : \Delta_8, \mathbf{F}_7 \\ \hline - : \vdash \left\{ \Delta_9 \right\} : \Delta_8, \mathbf{F}_7 \\ - : \vdash \left\{ \Delta_9 \right\} : \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} \begin{array}{c} \mathbf{ax} \\ \mathsf{hCut} \end{array}$$

• Case rule  $\oplus_A$ 

$$\frac{ \begin{array}{l} \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_9 \right\} : ! \mathbf{F}_4 \end{array} }{ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_9, dual(\mathbf{F}_4) \right\} : \mathbf{F}_6, \Delta_8 \\ } \underbrace{ \begin{array}{l} \bullet \mathbf{h}_5 : \vdash \left\{ dual(\mathbf{F}_4), \Delta_9 \right\} : \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_9 \right\} : \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_9 \right\} : \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} }_{ \begin{array}{l} \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_9 \right\} : \Delta_8, \mathbf{F}_6 \\ \bullet \mathbf{h}_5 : \vdash \left\{ \Delta_9 \right\} : \Delta_8, \mathbf{F}_6 \\ \hline - : \vdash \left\{ \Delta_9 \right\} : \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \end{array} }_{ \begin{array}{l} \bullet \mathbf{h} \mathbf{Cut} \\ \bullet \mathbf{h} \mathbf{Cut} \end{array}$$

• Case rule  $\perp$ 

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : ! \mathbf{F}_4} \ \, ! \quad \begin{array}{c} \mathbf{h}_5 : \vdash \left\{ \Delta_7, dual(\mathbf{F}_4) \right\} : \Delta_6 \\ \bullet \mathbf{h}_5 : \vdash \left\{ dual(\mathbf{F}_4), \Delta_7 \right\} : \bot, \Delta_6 \end{array} \\ \\ - : \vdash \left\{ \Delta_7 \right\} : \bot, \Delta_6 \\ \longrightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7 \right\} : ! \mathbf{F}_4 \end{array} \quad \begin{array}{c} \mathbf{h}_5 : \vdash \left\{ \Delta_7, dual(\mathbf{F}_4) \right\} : \bot, \Delta_6 \end{array} \\ \leftarrow : \vdash \left\{ \Delta_7 \right\} : \Delta_6 \\ \longrightarrow \vdots \vdash \left\{ \Delta_7 \right\} : \Delta_6 \\ \longrightarrow \vdots \vdash \left\{ \Delta_7 \right\} : \Delta_6 , \bot \end{array} \quad \mathbf{hCut} \end{array}$$

 $\bullet$  Case rule  $\top$ 

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \left\{\Delta_7\right\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \left\{\Delta_7\right\} : ! \mathbf{F}_4} \ \, ! \quad & \\ \hline - : \vdash \left\{\Delta_7\right\} : \top, \Delta_6 \\ \hline - : \vdash \left\{\Delta_7\right\} : \top, \Delta_6 \\ \hline - : \vdash \left\{\Delta_7\right\} : \Delta_6, \top \end{array} \ \, \top \\ \end{array}$$

• Case rule  $I_3$ 

$$\frac{ \frac{\mathbf{h}_1 : \vdash \{\Delta_7\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : ! \mathbf{F}_4} \ ! \quad \frac{\bullet \mathbf{h}_5 : \vdash \{dual(\mathbf{F}_4), \Delta_7\} : p(\mathbf{n}_6), \hat{\ }(\mathbf{n}_6)}{- : \vdash \{\Delta_7\} : p(\mathbf{n}_6), \hat{\ }(\mathbf{n}_6)} \quad \frac{I_3}{- : \vdash \{\Delta_7\} : p(\mathbf{n}_6), \hat{\ }(\mathbf{n}_6)} \quad I_3}$$

• Case rule  $\otimes$ 

$$\frac{\underbrace{\frac{h_1 : \vdash \left\{\Delta_{10}\right\} : F_4}{\bullet h_1 : \vdash \left\{\Delta_{10}\right\} : !F_4}}_{\bullet h_1 : \vdash \left\{\Delta_{10}\right\} : !F_4} : \underbrace{\frac{h_5 : \vdash \left\{\Delta_{10}, dual(F_4)\right\} : F_6, \Delta_8 \quad h_5 : \vdash \left\{\Delta_{10}, dual(F_4)\right\} : F_7, \Delta_9}{\bullet h_5 : \vdash \left\{dual(F_4), \Delta_{10}\right\} : \Delta_8, \Delta_9, F_6 \otimes F_7} \underbrace{Cut} \\ - : \vdash \left\{\Delta_{10}\right\} : \Delta_8, \Delta_9, F_6 \otimes F_7 \\ \hline \underbrace{\frac{\bullet h_1 : \vdash \left\{\Delta_{10}\right\} : !F_4}{\bullet h_5 : \vdash \left\{\Delta_{10}, dual(F_4)\right\} : \Delta_8, F_6}}_{- : \vdash \left\{\Delta_{10}\right\} : \Delta_8, F_6} \underbrace{\frac{a_1}{\bullet h_1} : \vdash \left\{\Delta_{10}\right\} : !F_4}_{\bullet h_1} \underbrace{\frac{a_2}{\bullet h_1} : \vdash \left\{\Delta_{10}\right\} : \Delta_9, F_7}_{\bullet h_2} \underbrace{\delta_{10}}_{\bullet h_1} : \vdash \left\{\Delta_{10}\right\} : \Delta_9, F_7}_{\bullet h_2} \underbrace{\delta_{10}}_{\bullet h_1} : \vdash \left\{\Delta_{10}\right\} : \Delta_9, F_7}_{\bullet h_2} \underbrace{\delta_{10}}_{\bullet h_1} : \vdash \left\{\Delta_{10}\right\} : \Delta_9, F_7}_{\bullet h_2} \underbrace{\delta_{10}}_{\bullet h_1} : \vdash \left\{\Delta_{10}\right\} : \Delta_9, F_7}_{\bullet h_2} \underbrace{\delta_{10}}_{\bullet h_1} : \vdash \left\{\Delta_{10}\right\} : \Delta_9, F_7}_{\bullet h_2} \underbrace{\delta_{10}}_{\bullet h_2} : \Delta_9, F_8 \otimes F_8 \otimes F_7}_{\bullet h_2} \underbrace$$

• Case rule  $I_1$ 

$$\frac{ \frac{\mathbf{h}_1 : \vdash \{\Delta_7, p(\mathbf{n}_6)\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_7, p(\mathbf{n}_6)\} : ! \mathbf{F}_4} }{\bullet \mathbf{h}_1 : \vdash \{\Delta_7, p(\mathbf{n}_6)\} : ! \mathbf{F}_4} } : \frac{\bullet \mathbf{h}_5 : \vdash \{dual(\mathbf{F}_4), \Delta_7, p(\mathbf{n}_6)\} : \hat{\ } (\mathbf{n}_6)}}{- : \vdash \{\Delta_7, p(\mathbf{n}_6)\} : \hat{\ } (\mathbf{n}_6)} } \frac{I_1}{- : \vdash \{\Delta_7, p(\mathbf{n}_6)\} : \hat{\ } (\mathbf{n}_6)}}$$
 Cut

• Case rule  $I_2$ 

$$\frac{ \underset{\bullet}{\mathbf{h}_1 : \vdash \left\{ \Delta_7, \widehat{\phantom{A}}(\mathbf{n}_6) \right\} : \mathcal{F}_4}{\bullet \mathbf{h}_1 : \vdash \left\{ \Delta_7, \widehat{\phantom{A}}(\mathbf{n}_6) \right\} : \mathcal{F}_4} \ !} \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_5 : \vdash \left\{ dual(\mathcal{F}_4), \Delta_7, \widehat{\phantom{A}}(\mathbf{n}_6) \right\} : p(\mathbf{n}_6) \\ \hline \\ - : \vdash \left\{ \Delta_7, \widehat{\phantom{A}}(\mathbf{n}_6) \right\} : p(\mathbf{n}_6) \\ \hline \\ - : \vdash \left\{ \Delta_7, \widehat{\phantom{A}}(\mathbf{n}_6) \right\} : p(\mathbf{n}_6) \end{array}} I_2$$
 Cut

• Case rule  $?_C$ 

$$\frac{\frac{\mathbf{h}_1 : \vdash \{\Delta_7\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : ! \mathbf{F}_4} \ ! \ \frac{\mathbf{h}_5 : \vdash \{\Delta_7, dual(\mathbf{F}_4)\} : \Delta_6, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \{dual(\mathbf{F}_4), \Delta_7\} : \Delta_6} \ \mathbf{Cut}} ?_C}{- : \vdash \{\Delta_7\} : \Delta_6} \\ \frac{- : \vdash \{\Delta_7\} : \Delta_6}{\bullet \mathbf{h}_1 : \vdash \{\Delta_7\} : ! \mathbf{F}_4} \ \mathbf{ax} \xrightarrow{\mathbf{h}_5 : \vdash \{\Delta_7, dual(\mathbf{F}_4)\} : \Delta_6, dual(\mathbf{F}_4)}} \ \mathbf{ax}}{- : \vdash \{\Delta_7\} : \Delta_6, dual(\mathbf{F}_4)\}} \ \mathbf{mCut}} \\ \frac{\mathbf{h}_1 : \vdash \{\mathbf{F}_7, \Delta_8\} : \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \{\mathbf{F}_7, \Delta_8\} : ! \mathbf{F}_4} \ ! \ \frac{\mathbf{h}_5 : \vdash \{\mathbf{F}_7, \Delta_8, dual(\mathbf{F}_4)\} : \mathbf{F}_7, \Delta_6}{\bullet \mathbf{h}_5 : \vdash \{dual(\mathbf{F}_4), \mathbf{F}_7, \Delta_8\} : \Delta_6}} ?_C}{- : \vdash \{\mathbf{F}_7, \Delta_8\} : \Delta_6} \\ \frac{\bullet \mathbf{h}_1 : \vdash \{\Delta_8, \mathbf{F}_7\} : ! \mathbf{F}_4}{\bullet \mathbf{h}_5 : \vdash \{\Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4)\} : \Delta_6, \mathbf{F}_7}} \xrightarrow{\mathbf{ax}} \mathbf{hCut}} \\ \frac{\bullet \mathbf{h}_1 : \vdash \{\Delta_8, \mathbf{F}_7\} : ! \mathbf{h}_6}{- : \vdash \{\Delta_8, \mathbf{F}_7\} : \Delta_6, \mathbf{F}_7}} ?_C}$$

### **8.3** Status of ?: OK

- ullet Case rule 1
- Case rule!
- Case rule?
- Case rule \$
- $\bullet$  Case rule &
- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- $\bullet$  Case rule  $\bot$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$
- Case rule  $I_1$
- Case rule  $I_2$

 Case rule  $?_C$ 

 $\bullet \;$  Case rule  ${\bf 1}$ 

Status of \$: OK

• Case rule!
• Case rule ?
• Case rule \$
• Case rule &
• Case rule $\oplus_B$
• Case rule $\oplus_A$
$\bullet$ Case rule $\bot$
• Case rule ⊤

- **8.5** Status of &: OK
  - $\bullet \;$  Case rule  ${\bf 1}$

• Case rule  $I_3$ 

 $\bullet \;$  Case rule  $\otimes$ 

 $\bullet\,$  Case rule  $I_1$ 

 $\bullet$  Case rule  $I_2$ 

• Case rule  $?_C$ 

- Case rule!
- Case rule ?

•	Case rule \$	
•	Case rule &	
•	Case rule $\oplus_B$	
•	Case rule $\bigoplus_A$	
•	Case rule $\perp$	
•	Case rule $\top$	
•	Case rule $I_3$	
•	Case rule $\otimes$	
•	Case rule $I_1$	
•	Case rule $I_2$	
•	Case rule $?_C$	
		$E\oplus_B$ : OK
8.6	Case rule $?_C$ Status of Case rule 1	: ⊕ <sub>B</sub> : OK
8.6	Status of	÷⊕ <sub>B</sub> : OK
8.6	Status of Case rule 1	÷⊕ <sub>B</sub> : OK
8.6	Status of Case rule 1 Case rule!	÷⊕ <sub>B</sub> : OK
8.6	Status of Case rule 1 Case rule! Case rule?	÷⊕ <sub>B</sub> : OK
8.6	Status of Case rule 1 Case rule ! Case rule ? Case rule \$	
8.6	Status of Case rule 1 Case rule ! Case rule ? Case rule \$ Case rule &	

•	Case rule $\top$	
•	Case rule $I_3$	
•	Case rule $\otimes$	
•	Case rule $I_1$	
•	Case rule $I_2$	
•	Case rule $?_C$	
8.7	Status of $\oplus_A$ :	OK
•	Case rule 1	
•	Case rule!	
•	Case rule ?	
•	Case rule \$	

• Case rule &

• Case rule  $\oplus_B$ 

• Case rule  $\oplus_A$ 

 $\bullet$  Case rule  $\bot$ 

 $\bullet \;$  Case rule  $\top$ 

• Case rule  $I_3$ 

 $\bullet$  Case rule  $\otimes$ 

• Case rule  $I_1$ 

 $\bullet\,$  Case rule  $I_2$ 

### 8.8 Status of $\perp$ : OK

- ullet Case rule  $oldsymbol{1}$
- Case rule!
- Case rule?
- $\bullet$  Case rule \$
- Case rule &
- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- $\bullet$  Case rule  $\bot$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$
- Case rule  $I_1$
- Case rule  $I_2$
- $\bullet \;$  Case rule  $?_C$

### 8.9 Status of $\top$ : OK

- ullet Case rule  $oldsymbol{1}$
- Case rule!
- Case rule?
- $\bullet$  Case rule \$

• (	Case	rule	&	
• (	Case	rule	$\oplus_B$	
• (	Case	rule	$\oplus_A$	
• (	Case	rule	1	
• (	Case	rule	Т	
• (	Case	rule	$I_3$	
• (	Case	rule	$\otimes$	
• (	Case	rule	$I_1$	
• (	Case	rule	$I_2$	
• (	Case	rule	$?_C$	
8.10				OK
		tatu	us of $I_3$ :	OK
• (	St	atu rule	as of $I_3$ :	OK
• (	<b>St</b> Case	t <b>atu</b> rule rule	as of $I_3$ :  1	OK
• (	<b>St</b> Case Case	c <b>at</b> urule	us of <i>I</i> <sub>3</sub> :  1  !	ОК
• (	St Case Case	rule rule rule rule	s of I <sub>3</sub> :  1  !  ?	ОК
• (	St Case Case Case	rule rule rule rule rule	s of I <sub>3</sub> :  1  !  ?  \$	OK
• (	St Case Case Case	rule rule rule rule rule	as of $I_3$ :  1  ! $?$ \$  & $\oplus_B$	OK
	St Case Case Case Case	rule rule rule rule rule rule	as of $I_3$ :  1  ! $?$ $\&$ $\oplus_B$ $\oplus_A$	OK

•	Case	rule	$I_3$	
•	Case	rule	$\otimes$	
•	Case	rule	$I_1$	
•	Case	rule	$I_2$	
•	Case	rule	$?_C$	
8.11	St	tatu	ıs of ⊗:	OK
•	Case	rule	1	
•	Case	rule	!	
•	Case	rule	?	
•	Case	rule	\$	
•	Case	rule	&	
•	Case	rule	$\oplus_B$	
•	Case	rule	$\oplus_A$	
•	Case	rule	Т	
•	Case	rule	Т	
•	Case	rule	$I_3$	
•	Case	rule	$\otimes$	
•	Case	rule	$I_1$	
•	Case	rule	$I_2$	

• Case rule  $?_C$ 

# 8.12 Status of $I_1$ : OK

- ullet Case rule  $oldsymbol{1}$
- Case rule!
- Case rule?
- Case rule \$
- Case rule &
- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- $\bullet$  Case rule  $\bot$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$
- $\bullet$  Case rule  $I_1$
- $\bullet\,$  Case rule  $I_2$
- Case rule  $?_C$

# 8.13 Status of $I_2$ : OK

- $\bullet \;$  Case rule  ${\bf 1}$
- Case rule!
- Case rule ?
- $\bullet$  Case rule \$

- Case rule &
- Case rule  $\oplus_B$
- Case rule  $\oplus_A$
- $\bullet$  Case rule  $\bot$
- $\bullet$  Case rule  $\top$
- Case rule  $I_3$
- $\bullet$  Case rule  $\otimes$
- Case rule  $I_1$
- Case rule  $I_2$
- Case rule  $?_C$

### 8.14 Status of $?_C$ : OK

 $\bullet \;$  Case rule  ${\bf 1}$ 

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \left\{ \mathbf{F}_7, \Delta_8 \right\} : \mathbf{F}_7, !\mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \left\{ \mathbf{F}_7, \Delta_8 \right\} : !\mathbf{F}_5} &?_C & \frac{}{\bullet \mathbf{h}_6 : \vdash \left\{ dual(\mathbf{F}_5), \mathbf{F}_7, \Delta_8 \right\} : \mathbf{1}} & \mathbf{Cut} \\ & \frac{- : \vdash \left\{ \mathbf{F}_7, \Delta_8 \right\} : \mathbf{1}}{- : \vdash \left\{ \Delta_8, \mathbf{F}_7 \right\} : \mathbf{1}} & \mathbf{1} \end{array}$$

• Case rule!

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \left\{ \mathsf{F}_8, \Delta_9 \right\} : \mathsf{F}_8, |\mathsf{F}_5|}{\bullet \mathsf{h}_2 : \vdash \left\{ \mathsf{F}_8, \Delta_9 \right\} : !\mathsf{F}_5} &?_C & \frac{\mathbf{h}_6 : \vdash \left\{ \mathsf{F}_8, \Delta_9, dual(\mathsf{F}_5) \right\} : \mathsf{F}_7}{\bullet \mathsf{h}_6 : \vdash \left\{ dual(\mathsf{F}_5), \mathsf{F}_8, \Delta_9 \right\} : !\mathsf{F}_7} & \mathsf{Cut} \\ \hline & - : \vdash \left\{ \mathsf{F}_8, \Delta_9 \right\} : !\mathsf{F}_7} & \rightarrow \\ & \frac{\bullet \mathsf{h}_2 : \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : !\mathsf{F}_5}{\bullet \mathsf{ax}} & \frac{\rightarrow}{\mathsf{h}_6 : \vdash \left\{ \Delta_9, \mathsf{F}_8, dual(\mathsf{F}_5) \right\} : \mathsf{F}_7}}{\mathsf{h}_6 : \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : !\mathsf{F}_7} & \mathsf{h}_6 \mathsf{Cut} \\ \hline & \frac{- : \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : !\mathsf{F}_7}{- : \vdash \left\{ \Delta_9, \mathsf{F}_8 \right\} : !\mathsf{F}_7} & ! \end{array}$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \left\{ \mathbf{F}_9, \Delta_{10} \right\} : \mathbf{F}_9, |\mathbf{F}_5|}{\bullet \mathbf{h}_2 : \vdash \left\{ \mathbf{F}_9, \Delta_{10} \right\} : |\mathbf{F}_5|} ?_C & \frac{\mathbf{h}_6 : \vdash \left\{ \mathbf{F}_7, \mathbf{F}_9, \Delta_{10}, dual(\mathbf{F}_5) \right\} : \Delta_8}{\bullet \mathbf{h}_6 : \vdash \left\{ dual(\mathbf{F}_5), \mathbf{F}_9, \Delta_{10} \right\} : \Delta_8, ?\mathbf{F}_7} \\ & \xrightarrow{\bullet \mathbf{h}_2 : \vdash \left\{ \Delta_{10}, \mathbf{F}_9 \right\} : |\mathbf{F}_5|} \mathbf{ax} \\ \bullet \mathbf{h}_2 : \vdash \left\{ \Delta_{10}, \mathbf{F}_7, \mathbf{F}_9 \right\} : |\mathbf{F}_5|} W & \frac{\mathbf{h}_6 : \vdash \left\{ \Delta_{10}, \mathbf{F}_7, \mathbf{F}_9, dual(\mathbf{F}_5) \right\} : \Delta_8}{\bullet \mathbf{h}_6 : \vdash \left\{ \Delta_{10}, \mathbf{F}_7, \mathbf{F}_9 \right\} : \Delta_8} \\ & \xrightarrow{- : \vdash \left\{ \Delta_{10}, \mathbf{F}_9 \right\} : \Delta_8, ?\mathbf{F}_7} ? \end{array} \\ \mathbf{hCut} \end{array}$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \left\{ \mathsf{F}_{10}, \Delta_{11} \right\} : \mathsf{F}_{10} : \vdash \mathsf{F}_5}{\bullet \mathbf{h}_2 : \vdash \left\{ \mathsf{F}_{10}, \Delta_{11} \right\} : \vdash \mathsf{F}_5} &?_C & \frac{\mathbf{h}_6 : \vdash \left\{ \mathsf{F}_{10}, \Delta_{11}, dual(\mathsf{F}_5) \right\} : \mathsf{F}_7, \mathsf{F}_8, \Delta_9}{\bullet \mathbf{h}_6 : \vdash \left\{ dual(\mathsf{F}_5), \mathsf{F}_{10}, \Delta_{11} \right\} : \Delta_9, \mathsf{F}_7 \$ \mathsf{F}_8} & \mathsf{Cut} \\ & & - : \vdash \left\{ \mathsf{F}_{10}, \Delta_{11} \right\} : \Delta_9, \mathsf{F}_7 \$ \mathsf{F}_8} & \rightarrow \\ & & \bullet \mathbf{h}_2 : \vdash \left\{ \Delta_{11}, \mathsf{F}_{10} \right\} : \vdash \mathsf{F}_5} & \mathsf{ax} & & - : \vdash \left\{ \Delta_{11}, \mathsf{F}_{10}, dual(\mathsf{F}_5) \right\} : \Delta_9, \mathsf{F}_7, \mathsf{F}_8} \\ & & - : \vdash \left\{ \Delta_{11}, \mathsf{F}_{10} \right\} : \Delta_9, \mathsf{F}_7 \$ \mathsf{F}_8} & \$ & \mathsf{hCut} \\ & & & - : \vdash \left\{ \Delta_{11}, \mathsf{F}_{10} \right\} : \Delta_9, \mathsf{F}_7 \$ \mathsf{F}_8} & \$ & & \mathsf{hCut} \\ \end{array}$$

• Case rule &

$$\frac{\frac{h_{2} : \vdash \{F_{10}, \Delta_{11}\} : F_{10}, !F_{5}}{\bullet h_{2} : \vdash \{F_{10}, \Delta_{11}\} : !F_{5}} ?_{C} }{\frac{\bullet h_{6} : \vdash \{F_{10}, \Delta_{11}, dual(F_{5})\} : F_{7}, \Delta_{9} }{\bullet h_{6} : \vdash \{dual(F_{5}), F_{10}, \Delta_{11}, dual(F_{5})\} : F_{8}, \Delta_{9}}} }{ \\ \frac{- : \vdash \{F_{10}, \Delta_{11}\} : \Delta_{9}, F_{7} \& F_{8}}{h_{6} : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{9}, F_{7}} } \underbrace{cut} \\ \\ \frac{- : \vdash \{\Delta_{11}, F_{10}\} : !F_{5}}{h_{6} : \vdash \{\Delta_{11}, F_{10}, dual(F_{5})\} : \Delta_{9}, F_{7}} } \underbrace{ax}_{h_{Cut}} \underbrace{- : \vdash \{\Delta_{11}, F_{10}\} : !F_{5}}_{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{9}, F_{8}} \underbrace{k}} \underbrace{ax}_{h_{Cut}} \underbrace{- : \vdash \{\Delta_{11}, F_{10}\} : \Delta_{9}, F_{8}}} \underbrace{k}$$

• Case rule  $\oplus_B$ 

$$\frac{\mathbf{h}_2 : \vdash \{\mathsf{F}_{10}, \Delta_{11}\} : \mathsf{F}_{10}, !\mathsf{F}_5}{\underbrace{\bullet \mathsf{h}_2 : \vdash \{\mathsf{F}_{10}, \Delta_{11}\} : !\mathsf{F}_5}_{\bullet \mathsf{h}_2 : \vdash \{\mathsf{F}_{10}, \Delta_{11}\} : !\mathsf{F}_5}}?_C \quad \frac{\mathbf{h}_6 : \vdash \{\mathsf{F}_{10}, \Delta_{11}, dual(\mathsf{F}_5)\} : \mathsf{F}_8, \Delta_9}{\bullet \mathsf{h}_6 : \vdash \{dual(\mathsf{F}_5), \mathsf{F}_{10}, \Delta_{11}\} : \Delta_9, \mathsf{F}_7 \oplus \mathsf{F}_8} \\ - : \vdash \{\mathsf{F}_{10}, \Delta_{11}\} : \Delta_9, \mathsf{F}_7 \oplus \mathsf{F}_8} \\ \underbrace{\bullet \mathsf{h}_2 : \vdash \{\Delta_{11}, \mathsf{F}_{10}\} : !\mathsf{F}_5}_{\bullet \mathsf{h}_2 : \vdash \{\Delta_{11}, \mathsf{F}_{10}\} : \Delta_9, \mathsf{F}_8} \underbrace{\to \mathsf{h}_6 : \vdash \{\Delta_{11}, \mathsf{F}_{10}, dual(\mathsf{F}_5)\} : \Delta_9, \mathsf{F}_8}_{hCut}}_{\bullet \mathsf{h}Cut}$$

• Case rule  $\oplus_A$ 

$$\frac{\mathbf{h}_{2} : \vdash \left\{ \mathsf{F}_{10}, \Delta_{11} \right\} : \mathsf{F}_{10}, !\mathsf{F}_{5}}{\underbrace{\bullet \mathsf{h}_{2} : \vdash \left\{ \mathsf{F}_{10}, \Delta_{11} \right\} : !\mathsf{F}_{5}}_{\bullet \mathsf{h}_{2} : \vdash \left\{ \mathsf{F}_{10}, \Delta_{11} \right\} : !\mathsf{F}_{5}} ?_{C} \quad \frac{\mathbf{h}_{6} : \vdash \left\{ \mathsf{F}_{10}, \Delta_{11}, dual(\mathsf{F}_{5}) \right\} : \mathsf{F}_{7}, \Delta_{9}}{\bullet \mathsf{h}_{6} : \vdash \left\{ dual(\mathsf{F}_{5}), \mathsf{F}_{10}, \Delta_{11} \right\} : \Delta_{9}, \mathsf{F}_{7} \oplus \mathsf{F}_{8}} \\ - : \vdash \left\{ \mathsf{F}_{10}, \Delta_{11} \right\} : \Delta_{9}, \mathsf{F}_{7} \oplus \mathsf{F}_{8} \\ \xrightarrow{\bullet \mathsf{h}_{2} : \vdash \left\{ \Delta_{11}, \mathsf{F}_{10} \right\} : !\mathsf{F}_{5}} \underbrace{\mathsf{ax}}_{\mathsf{h}_{6} : \vdash \left\{ \Delta_{11}, \mathsf{F}_{10}, dual(\mathsf{F}_{5}) \right\} : \Delta_{9}, \mathsf{F}_{7}}_{\mathsf{h}_{6} : \vdash \left\{ \Delta_{11}, \mathsf{F}_{10} \right\} : \Delta_{9}, \mathsf{F}_{7}} \underbrace{\mathsf{ax}}_{\mathsf{h}_{Cut}}$$

 $\bullet$  Case rule  $\bot$ 

$$\frac{\mathbf{h}_2 : \vdash \left\{ \mathbf{F}_8, \Delta_9 \right\} : \mathbf{F}_8, \mathsf{IF}_5}{\underbrace{\bullet \mathbf{h}_2 : \vdash \left\{ \mathbf{F}_8, \Delta_9 \right\} : !\mathbf{F}_5}}_{\bullet \mathbf{h}_2 : \vdash \left\{ \mathbf{F}_8, \Delta_9 \right\} : !\mathbf{F}_5} ?_C \quad \frac{\mathbf{h}_6 : \vdash \left\{ \mathbf{F}_8, \Delta_9, dual(\mathbf{F}_5) \right\} : \Delta_7}{\bullet \mathbf{h}_6 : \vdash \left\{ dual(\mathbf{F}_5), \mathbf{F}_8, \Delta_9 \right\} : \bot, \Delta_7}_{\bullet \mathbf{h}_2 : \vdash \left\{ \Delta_9, \mathbf{F}_8 \right\} : !\mathbf{F}_5} \overset{\mathbf{ax}}{\underset{\bullet}{\to}} \frac{\rightarrow}{\mathbf{h}_6 : \vdash \left\{ \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \right\} : \Delta_7}_{\bullet \mathbf{h}_6 : \vdash \left\{ \Delta_9, \mathbf{F}_8 \right\} : \Delta_7}_{\bullet \mathbf{c} \vdash \left\{ \Delta_9, \mathbf{F}_8 \right\} : \Delta_7, \bot} \overset{\mathbf{ax}}{\vdash} \overset{\mathbf{h}_C \mathbf{u}}{\underset{\bullet}{\to}} \overset{\mathbf{ax}}{\vdash} \overset{\mathbf{h}_C \mathbf{u}}{\vdash} \overset{\mathbf{h}_C \mathbf{u}}{\vdash}}_{\bullet \mathbf{h}_C \mathbf{u}}$$

• Case rule  $\top$ 

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \{\mathbf{F}_8, \Delta_9\} : \mathbf{F}_8, !\mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash \{\mathbf{F}_8, \Delta_9\} : !\mathbf{F}_5} &?_C & \frac{}{\bullet \mathbf{h}_6 : \vdash \{\mathit{dual}(\mathbf{F}_5), \mathbf{F}_8, \Delta_9\} : \top, \Delta_7} & \top \\ & & \frac{}{- : \vdash \{\mathbf{F}_8, \Delta_9\} : \top, \Delta_7} & \xrightarrow{} & \mathsf{Cut} \\ & & \frac{}{- : \vdash \{\Delta_9, \mathbf{F}_8\} : \Delta_7, \top} & \top \end{array}$$

• Case rule  $I_3$ 

$$\begin{array}{c} \frac{\mathbf{h}_{2} : \vdash \{\mathbf{F}_{8}, \Delta_{9}\} : \mathbf{F}_{8}, !\mathbf{F}_{5}}{\bullet \mathbf{h}_{2} : \vdash \{\mathbf{F}_{8}, \Delta_{9}\} : !\mathbf{F}_{5}} ?_{C} & \frac{}{\bullet \mathbf{h}_{6} : \vdash \{dual(\mathbf{F}_{5}), \mathbf{F}_{8}, \Delta_{9}\} : p(\mathbf{n}_{7}), \hat{\ \ } (\mathbf{n}_{7})} \\ & - : \vdash \{\mathbf{F}_{8}, \Delta_{9}\} : p(\mathbf{n}_{7}), \hat{\ \ } (\mathbf{n}_{7}) \\ & - : \vdash \{\Delta_{9}, \mathbf{F}_{8}\} : p(\mathbf{n}_{7}), \hat{\ \ } (\mathbf{n}_{7})} & I_{3} \\ \end{array}$$

• Case rule  $\otimes$ 

$$\frac{\underset{\bullet h_{2} : \vdash \{F_{11}, \Delta_{12}\} : F_{1}, |F_{5}|}{\bullet h_{2} : \vdash \{F_{11}, \Delta_{12}\} : F_{5}|} ?_{C} \quad \underset{\bullet h_{6} : \vdash \{F_{11}, \Delta_{12}, dual(F_{5})\} : F_{7}, \Delta_{9} \quad h_{6} : \vdash \{F_{11}, \Delta_{12}, dual(F_{5})\} : F_{8}, \Delta_{10}}{\bullet h_{6} : \vdash \{dual(F_{5}), F_{11}, \Delta_{12}\} : \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8}} \underbrace{\underset{\bullet h_{2} : \vdash \{\Delta_{12}, F_{11}\} : |F_{5}|}{- : \vdash \{\Delta_{12}, F_{11}\} : \Delta_{9}, F_{7}}} \underset{h_{6} : \vdash \{\Delta_{12}, F_{11}\} : \Delta_{9}, F_{7}}{\underbrace{\underset{\bullet h_{2} : \vdash \{\Delta_{12}, F_{11}\} : |F_{5}|}{\bullet \text{cut}}} \underset{h_{6} : \vdash \{\Delta_{12}, F_{11}, dual(F_{5})\} : \Delta_{10}, F_{8}}{\bullet \text{hCut}}} \underset{h_{6} : \vdash \{\Delta_{12}, F_{11}\} : \Delta_{10}, F_{8}}{\bullet \text{hCut}}$$

• Case rule  $I_1$ 

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \{p(\mathbf{n}_7), \Delta_8\} : p(\mathbf{n}_7), !\mathsf{F}_5}{\bullet \mathbf{h}_2 : \vdash \{p(\mathbf{n}_7), \Delta_8\} : !\mathsf{F}_5} ?_C & \frac{\bullet \mathbf{h}_6 : \vdash \{dual(\mathsf{F}_5), p(\mathbf{n}_7), \Delta_8\} : \hat{\ \ } (\mathbf{n}_7)}{- : \vdash \{p(\mathbf{n}_7), \Delta_8\} : \hat{\ \ } (\mathbf{n}_7)} & \frac{I_1}{\mathsf{Cut}} \\ & \frac{- : \vdash \{p(\mathbf{n}_7), \Delta_8\} : \hat{\ \ } (\mathbf{n}_7)}{- : \vdash \{\Delta_8, p(\mathbf{n}_7)\} : \hat{\ \ } (\mathbf{n}_7)} & I_1 \\ \\ \frac{\mathbf{h}_2 : \vdash \{\mathsf{F}_8, \Delta_9, p(\mathbf{n}_7)\} : \mathsf{F}_8, !\mathsf{F}_5}{\bullet \mathbf{h}_2 : \vdash \{\mathsf{F}_8, \Delta_9, p(\mathbf{n}_7)\} : !\mathsf{F}_5} & ?_C & \frac{\bullet \mathbf{h}_6 : \vdash \{dual(\mathsf{F}_5), \mathsf{F}_8, \Delta_9, p(\mathbf{n}_7)\} : \hat{\ \ } (\mathbf{n}_7)}{- : \vdash \{\mathsf{F}_8, \Delta_9, p(\mathbf{n}_7)\} : \hat{\ \ } (\mathbf{n}_7)} & I_1 \\ & \frac{- : \vdash \{\mathsf{F}_8, \Delta_9, p(\mathbf{n}_7)\} : \hat{\ \ } (\mathbf{n}_7)}{- : \vdash \{\Delta_9, \mathsf{F}_8, p(\mathbf{n}_7)\} : \hat{\ \ } (\mathbf{n}_7)} & I_1 \\ \end{array}$$

 $\bullet\,$  Case rule  $I_2$ 

• Case rule  $?_C$ 

$$\frac{\frac{h_2 : \vdash \{F_8, \Delta_9\} : F_8, \frac{!F_5}{\bullet}}{\bullet h_2 : \vdash \{F_8, \Delta_9\} : !F_5}}{\bullet h_2 : \vdash \{F_8, \Delta_9\} : !F_5} ?_C \frac{h_6 : \vdash \{F_8, \Delta_9, dual(F_5)\} : \Delta_7, dual(F_5)}{\bullet h_6 : \vdash \{dual(F_5), F_8, \Delta_9\} : \Delta_7} ?_C \\ - : \vdash \{F_8, \Delta_9\} : \Delta_7 \\ \hline - : \vdash \{\Delta_9, F_8\} : !F_5 \\ \hline - : \vdash \{\Delta_9, F_8\} : F_5 \\ \hline - : \vdash \{\Delta_9, F_8\} : F_5 \\ \hline - : \vdash \{\Delta_9, F_8\} : F_5 \\ \hline - : \vdash \{\Delta_9, F_8\} : \Delta_7 \\ \hline - : \vdash \{\Delta_9, F_8\} : \Delta_7 \\ \hline - : \vdash \{F_8, \Delta_9\} : F_5 \\ \hline - : \vdash \{F_8, \Delta_9\} : F_5 \\ \hline - : \vdash \{F_8, \Delta_9\} : A_7 \\ \hline - : \vdash \{F_8, \Delta_9\} : A_7 \\ \hline - : \vdash \{F_8, \Delta_9\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{\Delta_9, F_8\} : A_7 \\ \hline - : \vdash \{\Delta_9, F_8\} : A_7 \\ \hline - : \vdash \{\Delta_9, F_8\} : A_7 \\ \hline - : \vdash \{\Delta_9, F_8\} : A_7 \\ \hline - : \vdash \{\Delta_9, F_8\} : A_7 \\ \hline - : \vdash \{\Delta_9, F_8\} : A_7 \\ \hline - : \vdash \{\Delta_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_9, F_8\} : A_7 \\ \hline - : \vdash \{A_{10}, F_8, F_9\} : A_7 \\ \hline - :$$