System G3i

July 6, 2022

Contents

1 Height preserving admissibility of weakening

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \top}\ \top_R$$

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \to \mathtt{F}_4}\ \to_R \quad \to \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3 \to \mathtt{F}_4}\ \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \to \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R$$

- Case(s) rule $\vee_1$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_1 \quad \to \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_1$$

- Case(s) rule $\vee_2$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_2 \quad \to \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_2$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_5}\ \to_L \quad \to \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_5}\ \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L \quad \to \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_0, \mathtt{p}_3 \vdash \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \top, \Delta_2, \mathtt{F}_0 \vdash \mathtt{F}_3}\ \top_L$$

2 Measure of derivations

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top_R \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top_R$$

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R$$

- Case(s) rule $\vee_1$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_1 \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_1$$

- Case(s) rule $\vee_2$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_2 \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_2$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}\ \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_3}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_3}\ \top_L$$

3 Invertibility of Rules

3.1 Status of $\top_R$: : `Invertible`

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\rightarrow_R$

- Case rule $\wedge_R$

- Case rule $\vee_1$

- Case rule $\vee_2$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \top}\ \rightarrow_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \top}\ \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \top \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \top}\ \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \top}\ \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \top}\ \top_L \quad \rightarrow \quad \texttt{trivial}$$

3.2 Status of $\rightarrow_R$: : `Invertible`

- Case rule $\top_R$
- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ {\scriptstyle \mathtt{H}}$$

- Case rule $\wedge_R$
- Case rule $\vee_1$
- Case rule $\vee_2$
- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \rightarrow_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_5}\ {\scriptstyle W} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_6 \vdash \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_2}\ {\scriptstyle \rightarrow_L}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \wedge_L} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_2}\ {\scriptstyle \wedge_L}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \vee_L} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \vdash \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_6 \vdash \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_2}\ {\scriptstyle \vee_L}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \bot_L} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4, \mathtt{F}_1 \vdash \mathtt{F}_2}\ {\scriptstyle \bot_L}$$

- Case rule I
- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \top_L} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_4, \mathtt{F}_1 \vdash \mathtt{F}_2}\ {\scriptstyle \top_L}$$

3.3 Status of $\wedge_R$: (Left Premise): `Invertible`

- Case rule $\top_R$
- Case rule $\to_R$
- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\vee_1$
- Case rule $\vee_2$
- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \to_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_5}^{\ \mathtt{ax}} \quad \overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_1}\ \to_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \wedge_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_1}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \vee_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_1}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \mathtt{F}_1}\ \bot_L$$

- Case rule I
- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \mathtt{F}_1}\ \top_L$$

3.4 Status of $\wedge_R$ (Right Premise): : `Invertible`

- Case rule $\top_R$
- Case rule $\rightarrow_R$
- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\vee_1$
- Case rule $\vee_2$
- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_2}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_2}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_2}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \mathtt{F}_2}\ \bot_L$$

- Case rule I
- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \mathtt{F}_2}\ \top_L$$

3.5 Status of $\vee_1$: : Non invertible

- Case rule $\top_R$
- Case rule $\to_R$
- Case rule $\wedge_R$
- Case rule $\vee_1$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_1 \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\vee_2$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_2 \quad \to \quad \overline{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}^{\ \mathtt{fail}}$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \to_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_5}^{\ \mathtt{ax}} \quad \overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_1}\ \to_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \wedge_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_1}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \vee_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_1}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \mathtt{F}_1}\ \bot_L$$

- Case rule I
- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \top_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \mathtt{F}_1}\ \top_L$$

3.6 Status of $\vee_2$: : Non invertible

- Case rule $\top_R$
- Case rule $\rightarrow_R$
- Case rule $\wedge_R$
- Case rule $\vee_1$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_1 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{fail}$$

- Case rule $\vee_2$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_2 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \mathtt{F}_2}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_2}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_2}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \mathtt{F}_2}\ \bot_L$$

- Case rule I
- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \mathtt{F}_1 \vee \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \mathtt{F}_2}\ \top_L$$

3.7 Status of $\to_L$: (Left Premise): `Non invertible`

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \top}\ \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_2}\ \text{fail}$$

- Case rule $\to_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R \quad \to \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_2}\ \text{fail}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\vee_1$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1 \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\vee_2$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_2 \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_6}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_1}\ \mathtt{H}$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_5}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\ \text{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6}\ \wedge_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_1}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6}\ \vee_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_1}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_1}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4}\ \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_1}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_4}\ I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{p}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_1}\ \mathtt{fail}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4}\ \top_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_1}\ \top_L$$

3.8 Status of $\to_L$ (Right Premise): : `Invertible`

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \top}\ \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \top}\ \top_R$$

- Case rule $\to_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

- Case rule $\vee_1$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1 \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1$$

- Case rule $\vee_2$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_2 \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_2$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_6}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_6}\ \to_L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_5}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6}\ \wedge_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6} \vee_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6} \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4} \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_4} I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2, \mathtt{p}_4 \vdash \mathtt{p}_4} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4} \top_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4} \top_L$$

3.9 Status of $\wedge_L$: : `Invertible`

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \top} \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \top} \top_R$$

- Case rule $\to_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \to \mathtt{F}_6} \to_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \to \mathtt{F}_6} \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R$$

- Case rule $\vee_1$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \vee_1 \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \vee_1$$

- Case rule $\vee_2$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \vee_2 \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \vee_2$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_6} \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_4}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_6} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6} \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6} \wedge_L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5} \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{H}}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6} \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \mathtt{F}_4} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{p}_4} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_4 \vdash \mathtt{p}_4} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4} \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \mathtt{F}_4}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \mathtt{F}_4} \top_L$$

3.10 Status of $\vee_L$: (Left Premise): `Invertible`

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \top} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \top} \top_R$$

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R$$

- Case rule $\vee_1$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \vee_1 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \vee_1$$

- Case rule $\vee_2$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \vee_2 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6} \vee_2$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_6} \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_6} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6} \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6} \vee_L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \begin{matrix}\mathtt{ax}\\ \mathtt{H}\end{matrix}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vdash \mathtt{F}_4} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_4} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{p}_4 \vdash \mathtt{p}_4} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4} \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vdash \mathtt{F}_4} \top_L$$

3.11 Status of $\vee_L$ (Right Premise): : `Invertible`

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \top}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

- Case rule $\vee_1$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1$$

- Case rule $\vee_2$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_2 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_2$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_6}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6}\ \vee_L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{H}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_4}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \top_L$$

3.12 Status of $\bot_L$: : `Invertible`

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \top}\ \top_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_3 \quad \mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_1$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_1 \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_2$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_2 \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_2 \quad \mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_4}\ \rightarrow_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_4}\ \wedge_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_4}\ \vee_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_3}\ \bot_L \quad \to \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_2, \bot, \Delta_3 \vdash \mathtt{p}_2}\ I \quad \to \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_3 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \top, \bot, \Delta_3 \vdash \mathtt{F}_2}\ \top_L \quad \to \quad \texttt{trivial}$$

3.13 Status of I: : `Invertible`

- Case rule $\top_R$
- Case rule $\to_R$
- Case rule $\wedge_R$
- Case rule $\vee_1$
- Case rule $\vee_2$
- Case rule $\to_L$

$$\frac{\mathtt{h}_2 : \Delta_5, \mathtt{p}_1, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_2 : \Delta_5, \mathtt{F}_4, \mathtt{p}_1 \vdash \mathtt{p}_1}{\bullet \mathtt{h}_2 : (\Delta_5, \mathtt{p}_1), \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{p}_1}\ \to_L \quad \to \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{p}_1 \vdash \mathtt{p}_1}{\bullet \mathtt{h}_2 : (\Delta_5, \mathtt{p}_1), \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{p}_1}\ \wedge_L \quad \to \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Delta_5, \mathtt{F}_3, \mathtt{p}_1 \vdash \mathtt{p}_1 \quad \mathtt{h}_2 : \Delta_5, \mathtt{F}_4, \mathtt{p}_1 \vdash \mathtt{p}_1}{\bullet \mathtt{h}_2 : (\Delta_5, \mathtt{p}_1), \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{p}_1}\ \vee_L \quad \to \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_3, \mathtt{p}_1 \vdash \mathtt{p}_1}\ \bot_L \quad \to \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_2 \vdash \mathtt{p}_3}\ I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{p}_1 \vdash \mathtt{p}_1}{\bullet \mathtt{h}_2 : \top, \Delta_3, \mathtt{p}_1 \vdash \mathtt{p}_1}\ \top_L \quad \rightarrow \quad \texttt{trivial}$$

3.14 Status of $\top_L$: : `Invertible`

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \top}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \top, \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_3 \quad \mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \mathtt{F}_3}\ \texttt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \mathtt{F}_4}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R$$

- Case rule $\vee_1$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_1 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \mathtt{F}_3}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_1$$

- Case rule $\vee_2$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_2 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \mathtt{F}_4}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_2$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_2 \quad \mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_4}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_2}\ \texttt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \texttt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_4}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_4}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \texttt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_4}\ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_4}\ \vee_L}{} \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_4}\ \vee_L$$

- Case rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_3 \vdash \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\dfrac{}{\bullet \mathtt{h}_1 : \mathtt{p}_2, \top, \Delta_3 \vdash \mathtt{p}_2}\ I \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_1 : \Delta_3, \mathtt{p}_2 \vdash \mathtt{p}_2}\ I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_3}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{H}$$

4 Height preserving admissibility of contraction

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \top}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \top}\ \top_R$$

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case(s) rule $\vee_1$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_1 \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_1$$

- Case(s) rule $\vee_2$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_2 \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_2$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}\ \rightarrow_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}\ \rightarrow_L$$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \rightarrow \mathtt{F}_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_5}\ \rightarrow_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \mathtt{F}_5}\ \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \wedge \mathtt{F}_4, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L$$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_5}\ \wedge_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_5}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vee \mathtt{F}_4, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L$$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_5}\ \vee_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_5}\ \vee_L$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_1, \bot, \bot \vdash \mathtt{F}_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_3}\ \bot_L$$

$$\frac{}{\bullet \mathtt{h}_2 : (\bot, \Delta_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_4, \mathtt{F}_1 \vdash \mathtt{F}_3}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{p}_3, \mathtt{p}_3 \vdash \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{p}_3 \vdash \mathtt{p}_3}\ I$$

$$\frac{}{\bullet \mathtt{h}_2 : (\Delta_4, \mathtt{p}_3), \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_4, \mathtt{F}_1, \mathtt{p}_3 \vdash \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : \Delta_1, \top, \top \vdash \mathtt{F}_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \mathtt{F}_3}\ \texttt{inv-th/ax}}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_3}\ \top_L$$

$$\frac{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : (\top, \Delta_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, \mathtt{F}_1 \vdash \mathtt{F}_3}\ \texttt{ax}}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1 \vdash \mathtt{F}_3}\ \texttt{IH}}{\bullet \mathtt{h}_2 : \top, \Delta_4, \mathtt{F}_1 \vdash \mathtt{F}_3}\ \top_L$$

5 Identity-Expansion

$$\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0 \vee \mathtt{F}_1}\,\vee_1 \quad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_1 \vdash \mathtt{F}_0 \vee \mathtt{F}_1}\,\vee_2}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0 \vee \mathtt{F}_1}\,\vee_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0}\,W \quad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_R}{- : \mathtt{F}_0 \wedge \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_0}\,W \quad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_1}\,\to_L}{- : \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_0 \to \mathtt{F}_1}\,\to_R$$

$$\dfrac{}{- : \top \vdash \top}\,\top_R$$

$$\dfrac{}{- : \bot \vdash \bot}\,\bot_L$$

6 Cut-Elimination

6.1 Status of $\top_R$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top}\,\top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \top}\,\top_R}{- : \Delta_3 \vdash \top}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{}{- : \Delta_3 \vdash \top}\,\top_R$$

- Case rule $\to_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,\to_R}{- : \Delta_3 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_3 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\,\wedge_R}{- : \Delta_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_3 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\,\wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\vee_1}{- : \Delta_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{hCut}}{- : \Delta_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\vee_2}{- : \Delta_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_3 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \to \mathtt{F}_5), \top \vdash \mathtt{F}_6}\,\to_L}{- : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_4}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_4}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_5 \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \mathtt{F}_6}\,\to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \top}\ \top_R \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5), \top \vdash \mathtt{F}_6}\ \wedge_L}{- : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \top}\ \top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \top}\ \top_R \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5), \top \vdash \mathtt{F}_6}\ \vee_L}{- : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_4 \vdash \top}\ \top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_5 \vdash \top}\ \top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6}\ \vee_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_3 : (\bot, \Delta_5), \top \vdash \mathtt{F}_4}\ \bot_L}{- : \bot, \Delta_5 \vdash \mathtt{F}_4}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \mathtt{F}_4}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_4 \vdash \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_3 : (\Delta_5, \mathtt{p}_4), \top \vdash \mathtt{p}_4}\ I}{- : \Delta_5, \mathtt{p}_4 \vdash \mathtt{p}_4}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_5, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash \top}\ \top_R \quad \dfrac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_4, \top \vdash \mathtt{F}_5}\ \top_L}{- : \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{ax/W}$$

6.2 Status of $\rightarrow_R$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \top}\ \top_R}{- : \Delta_5 \vdash \top}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_5 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_R}}{- : \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_R}}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\wedge_R}}{- : \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\wedge_R}}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_1}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_1}}$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_2}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_2}}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \to \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,{\to_L}}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \to \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_5, \mathtt{F}_9 \vdash \mathtt{F}_6}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,{\to_L}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_9}\,{\to_L}}{- : \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{- : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{sCut}}{- : \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{sCut}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,{\wedge_L}}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_5, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_6}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,{\wedge_L}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,{\vee_L}}{- : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_5, \mathtt{F}_8 \vdash \mathtt{F}_6}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_5, \mathtt{F}_9 \vdash \mathtt{F}_6}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,{\vee_L}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\bot, \Delta_9), \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_8}\,{\bot_L}}{- : \bot, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \bot, \Delta_9 \vdash \mathtt{F}_8}\,{\bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{p}_8}\,I}{- : \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\top, \Delta_9), \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \top, \Delta_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_8}\,{\top_L}}{- : \top, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_9 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

6.3 Status of $\wedge_R$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\,{\wedge_R} \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top}\,{\top_R}}{- : \Delta_5 \vdash \top}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_5 \vdash \top}\,{\top_R}$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}{- : \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1$$

- Case rule $\vee_2$

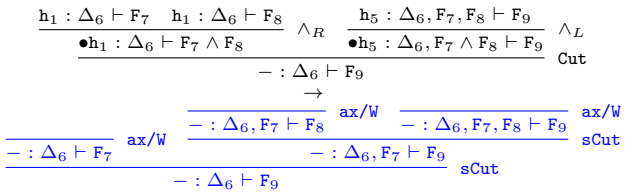
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2$$

- Case rule $\to_L$

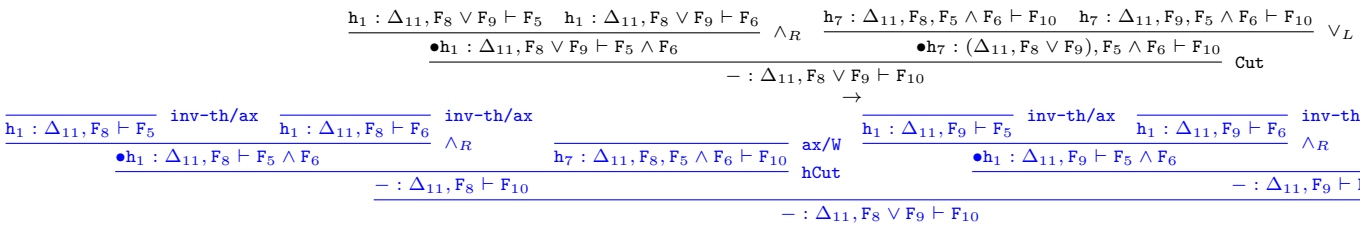
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}\to_L}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_8}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5}{}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_6}{}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9,}}{- : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}$$

- Case rule $\wedge_L$

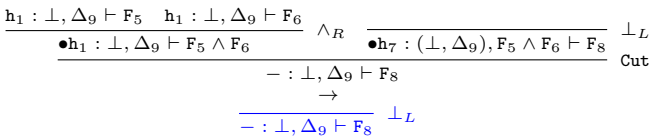
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}\wedge_L}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_5}{}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_6}{}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L$$



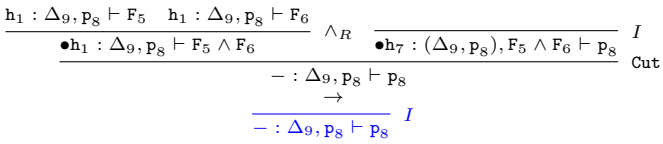
- Case rule $\vee_L$



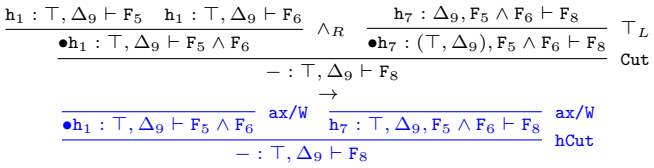
- Case rule $\bot_L$



- Case rule I

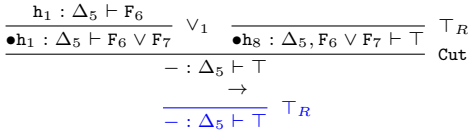


- Case rule $\top_L$



6.4 Status of $\vee_1$: OK

- Case rule $\top_R$



- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_1 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}{- : \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_1 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_1 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_1 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\to_L}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5}\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\wedge_L}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_5}\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\ \vee_L}{- : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_5}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\ \mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_1 \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_9}\ \vee_L}{- : \Delta_6 \vdash \mathtt{F}_9}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_6 \vdash \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \mathtt{F}_9}\ \mathtt{sCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_9 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1 \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\ \bot_L}{- : \bot, \Delta_9 \vdash \mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_9 \vdash \mathtt{F}_8}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1 \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{p}_8}\ I}{- : \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_1 \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\ \top_L}{- : \top, \Delta_9 \vdash \mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \mathtt{F}_8}\ \mathtt{hCut}$$

6.5 Status of $\vee_2$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_2 \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top}\ \top_R}{- : \Delta_5 \vdash \top}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_5 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_2 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}{- : \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_2 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_2 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_2 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\to_L}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_6}\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\wedge_L}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_6}\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\vee_L}{- : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_6}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_6}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\vee_2 \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_9}\vee_L}{- : \Delta_6 \vdash \mathtt{F}_9}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \mathtt{F}_9}\mathtt{sCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_9 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \bot, \Delta_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{}{\bullet\mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\bot_L}{- : \bot, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_9 \vdash \mathtt{F}_8}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{p}_8}\,I}{- : \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\top_L}{- : \top, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_9 \vdash \mathtt{F}_8}\mathtt{hCut}$$

6.6 Status of $\rightarrow_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \mathtt{F}_7}\rightarrow_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9), \mathtt{F}_7 \vdash \top}\top_R}{- : \Delta_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \top}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \top}\,\top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_7}\to_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \to \mathtt{F}_{12}}\to_R}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11} \to \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{12}}\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{12}}\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11} \to \mathtt{F}_{12}}\to_R}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_7}\to_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11} \quad \mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11}}\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11}}\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{12}}\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{12}}\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_7}\to_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_1}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11}}\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11}}\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_1$$

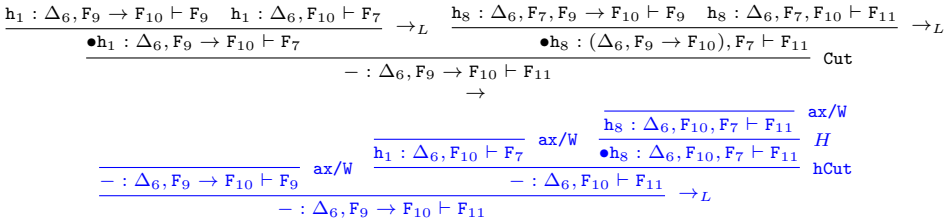
- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_7}\to_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_2}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{12}}\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{12}}\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_2$$

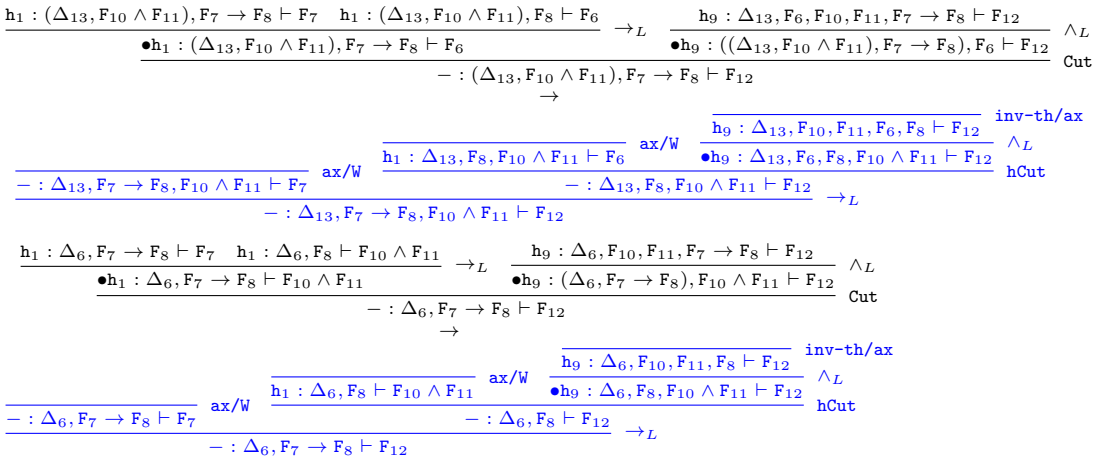
- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_6}\to_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_7 \to \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_1}{\bullet\mathtt{h}_9 : ((\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_{12}}}{- : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{10}}\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{12}}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\to_L}{- : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{12}}$$

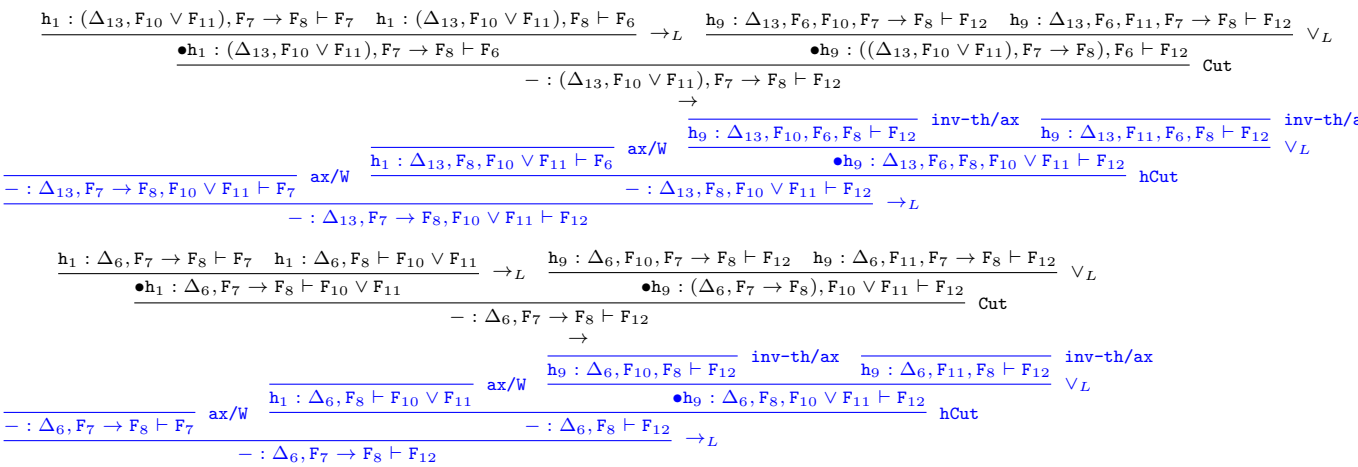
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}}\to_L \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\to_L}{- : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}}\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{10}}\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\to_L}{- : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{12}}\to_L$$



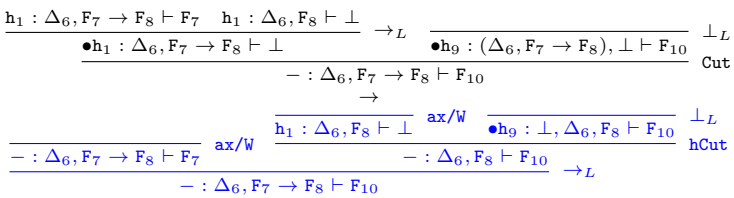
- Case rule $\wedge_L$



- Case rule $\vee_L$



- Case rule $\bot_L$



$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_6}\to_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\bot, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\bot_L}{- : (\bot, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{- : \bot, \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10}}{}\bot_L}\to$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_6}\to_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{p}_{10}}I}{- : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{p}_{10}}\,\mathtt{Cut}}{\dfrac{- : \Delta_{11}, \mathtt{p}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{p}_{10}}{}I}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{p}_{10}}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{p}_{10}}\to_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10} \vdash \mathtt{p}_{10}}I}{- : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{p}_{10}}\,\mathtt{Cut}}{\dfrac{- : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{p}_{10}}{}\,\mathtt{ax/W}}\to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vdash \top}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \top}\to_L \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \to \mathtt{F}_8), \top \vdash \mathtt{F}_{10}}\top_L}{- : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{- : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_6}\to_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\top, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\top_L}{- : (\top, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\bullet\mathtt{h}_1 : \top, \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}\to$$

6.7 Status of $\wedge_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \top}\top_R}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \top}\,\mathtt{Cut}}{\dfrac{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \top}{}\top_R}\to$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \to \mathtt{F}_{12}}\to_R}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11} \to \mathtt{F}_{12}}\,\mathtt{Cut}}{\dfrac{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{12}}\,\mathtt{hCut}}{\dfrac{- : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{12}}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11} \to \mathtt{F}_{12}}\to_R}\to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11} \quad \mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_7}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\mathtt{inv\text{-}th/ax}\wedge_R}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_7}\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_L}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_L$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_1}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11}}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11}}\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11}}\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_2}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{12}}\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{12}}\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_9 : ((\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_{12}}\to_L}{- : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_6}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{inv\text{-}th/ax}\to_L}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\wedge_L}{- : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\to_L}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}}\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{inv\text{-}th/ax}\to_L}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \to \mathtt{F}_{11}}\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\wedge_L}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\wedge_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_9 : ((\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_{12}}\wedge_L}{- : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{ax/W}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\wedge_L}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_{12}} \wedge_L}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_{12}} \wedge_L}{- : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}} \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_9, \mathtt{F}_{10} \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_7} \wedge_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_8 : (\Delta_6, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7 \vdash \mathtt{F}_{11}} \wedge_L}{- : \Delta_6, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,H}{- : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_{11}} \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_6} \wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_9 : ((\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_{12}} \vee_L}{- : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_{12}} \vee_L}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_{12}} \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10} \vee \mathtt{F}_{11}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12} \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_{12}} \vee_L}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_{12}} \vee_L}{- : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}} \wedge_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \bot}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot} \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \bot \vdash \mathtt{F}_{10}} \bot_L}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \bot}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \bot, \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}} \bot_L}{- : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}} \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_6} \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : ((\bot, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_{10}} \bot_L}{- : (\bot, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}} \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_6} \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : ((\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{p}_{10}} I}{- : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{p}_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{p}_{10}}\,I$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{p}_{10}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{p}_{10}}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10} \vdash \mathtt{p}_{10}} I}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{p}_{10}}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{p}_{10}}\,\mathtt{ax/W}}\,\rightarrow$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \top}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \top \vdash \mathtt{F}_{10}}\top_L}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}\,\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : ((\top, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\top_L}{- : (\top, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}\,\rightarrow$$

6.8 Status of $\vee_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_7}\vee_L \quad \dfrac{}{\bullet \mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \top}\top_R}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \top}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \top}\top_R}\,\rightarrow$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\rightarrow_R}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\rightarrow_R}\,\rightarrow$$

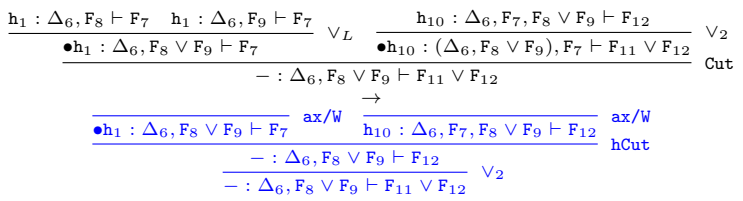
- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11} \quad \mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R}\,\rightarrow$$

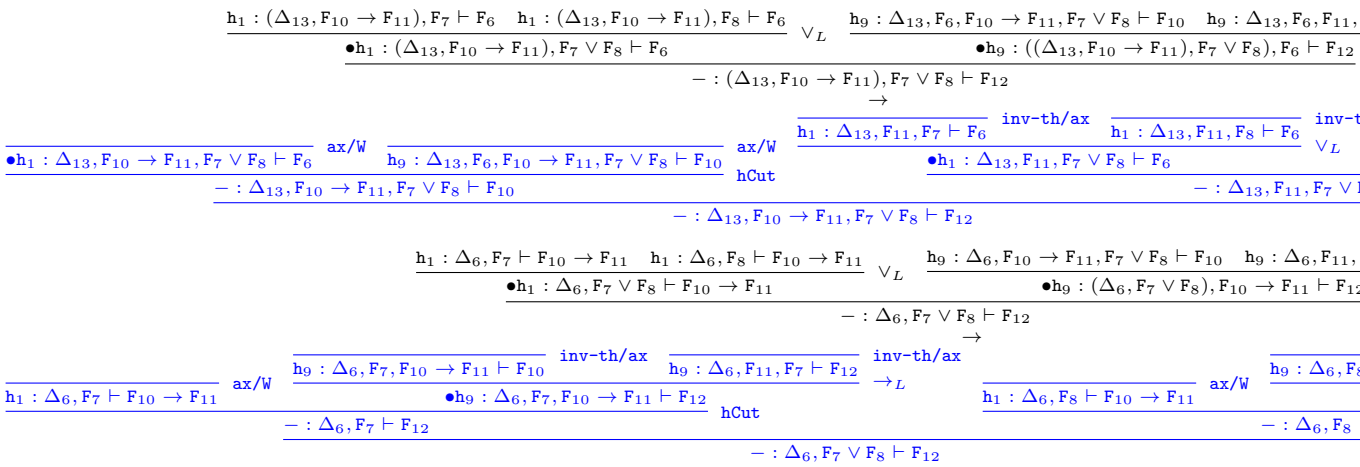
- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_{10} : (\Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_1}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_1}\,\rightarrow$$

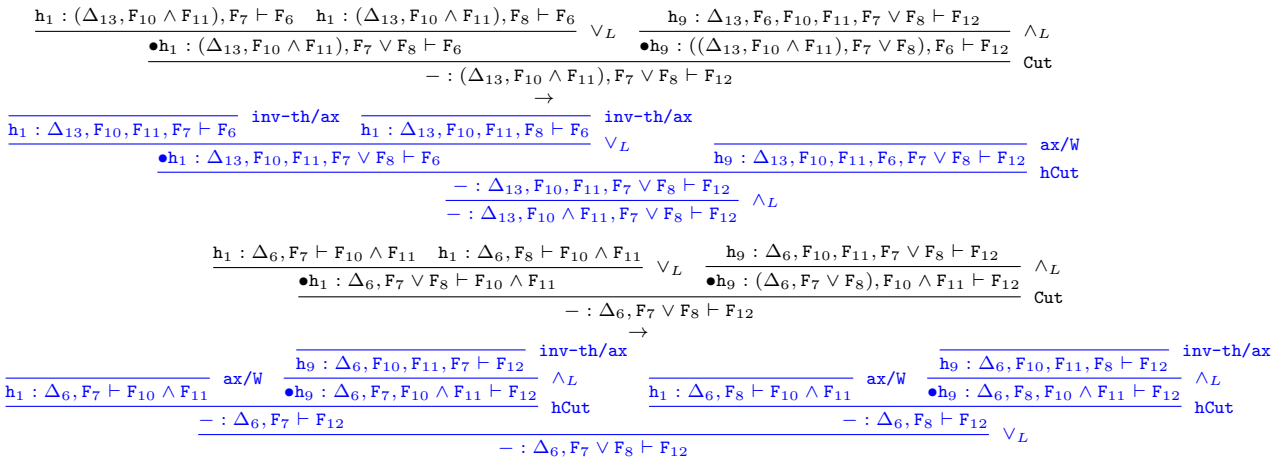
- Case rule $\vee_2$



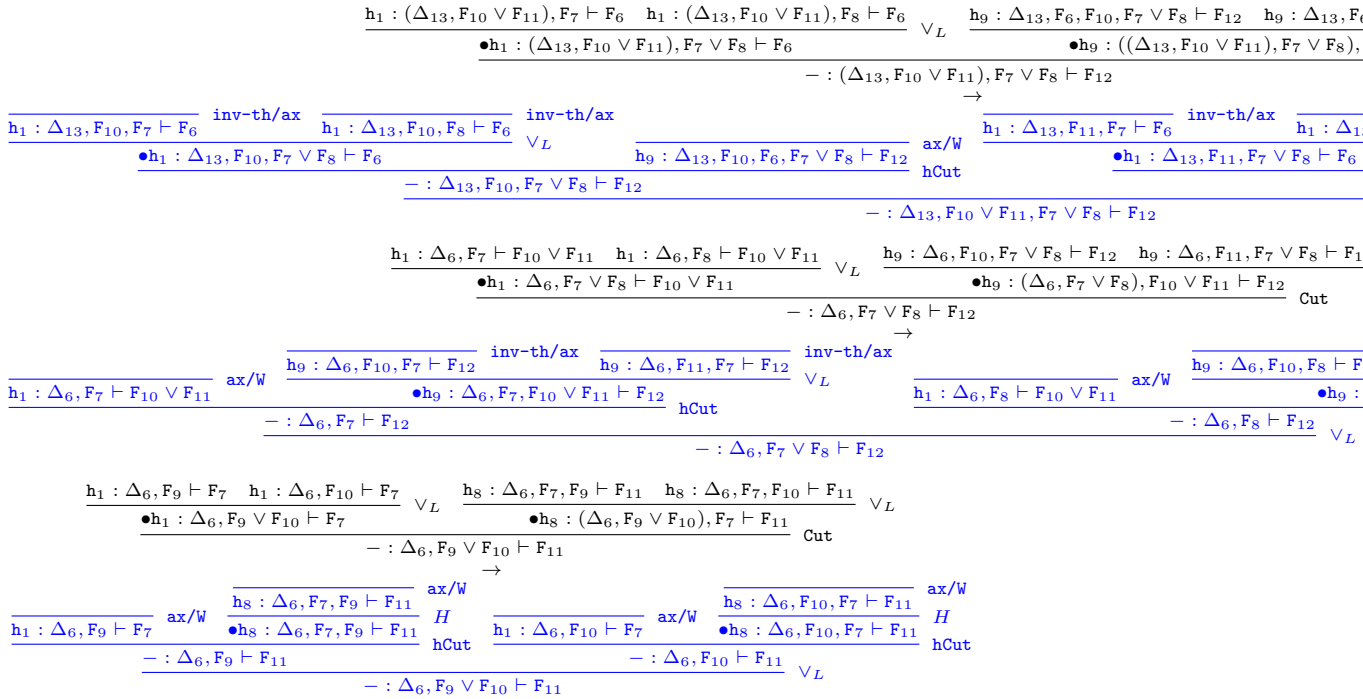
- Case rule $\to_L$



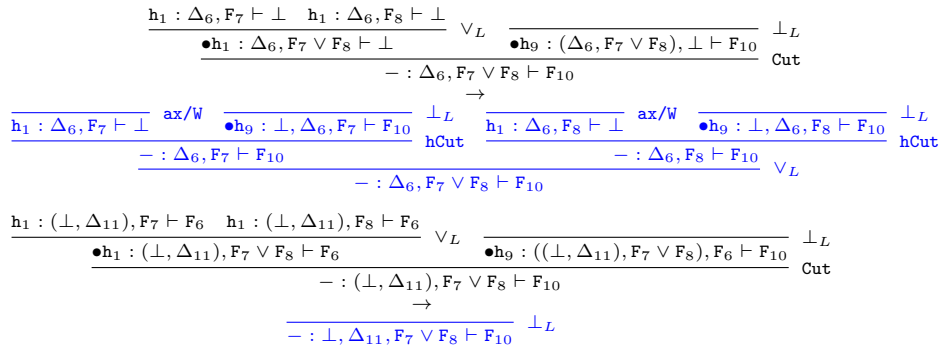
- Case rule $\wedge_L$



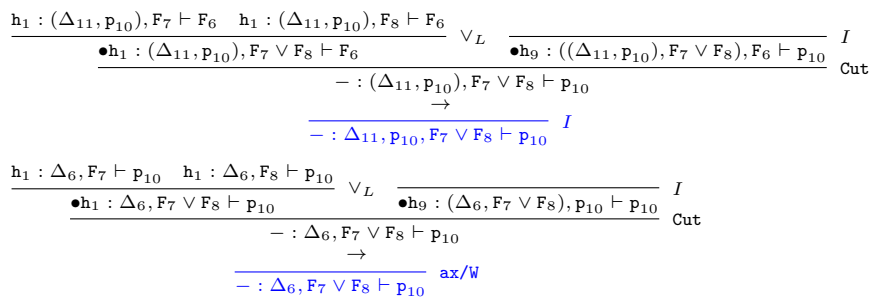
- Case rule $\vee_L$



- Case rule $\bot_L$



- Case rule I



- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vdash \top \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_8 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \top}\vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8), \top \vdash \mathtt{F}_{10}}\top_L}{- : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{}{- : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_6}\vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : ((\top, \Delta_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\top_L}{- : (\top, \Delta_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \top, \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{hCut}}\to$$

6.9 Status of $\bot_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{}{\bullet \mathtt{h}_6 : (\bot, \Delta_4), \mathtt{F}_5 \vdash \top}\top_R}{- : \bot, \Delta_4 \vdash \top}\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \top}\top_R}\to$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_4, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\bot, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\to_R}{- : \bot, \Delta_4 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\bot_L}\to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \bot, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\bot, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R}{- : \bot, \Delta_4 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\bot_L}\to$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : (\bot, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\vee_1}{- : \bot, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L}\to$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\bot, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\vee_2}{- : \bot, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L}\to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\ \to_L}{- : \bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\ \bot_L}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_4), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\ \to_L}{- : \bot, \Delta_4 \vdash \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_8}\ \bot_L}\ \to$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\ \wedge_L}{- : \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\ \bot_L}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_4), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\ \wedge_L}{- : \bot, \Delta_4 \vdash \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_8}\ \bot_L}\ \to$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\ \vee_L}{- : \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\ \bot_L}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_4), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\ \vee_L}{- : \bot, \Delta_4 \vdash \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_8}\ \bot_L}\ \to$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{F}_5}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_6 : (\bot, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7}\ \bot_L}{- : \bot, \Delta_4 \vdash \mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_7}\ \bot_L}\ \to$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_7, \mathtt{p}_6 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_7, \mathtt{p}_6), \mathtt{F}_4 \vdash \mathtt{p}_6}\ I}{- : \bot, \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6}\ \bot_L}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \mathtt{p}_6}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_4), \mathtt{p}_6 \vdash \mathtt{p}_6}\ I}{- : \bot, \Delta_4 \vdash \mathtt{p}_6}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{p}_6}\ \bot_L}\ \to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \top}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \Delta_4), \top \vdash \mathtt{F}_6}\ \top_L}{- : \bot, \Delta_4 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_4 \vdash \mathtt{F}_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_7 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \top, \Delta_7), \mathtt{F}_4 \vdash \mathtt{F}_6}\ \top_L}{- : \bot, \top, \Delta_7 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \top, \Delta_7 \vdash \mathtt{F}_6}\ \bot_L$$

6.10 Status of I: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_4, \mathtt{p}_5), \mathtt{p}_5 \vdash \top}\ \top_R}{- : \Delta_4, \mathtt{p}_5 \vdash \top}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_4, \mathtt{p}_5 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_7, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\Delta_4, \mathtt{p}_5), \mathtt{p}_5 \vdash \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \rightarrow_R}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_7, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, \mathtt{F}_7, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_4, \mathtt{F}_7, \mathtt{p}_5 \vdash \mathtt{F}_8}\ \mathtt{hCut}$$
$$\dfrac{}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_4, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\Delta_4, \mathtt{p}_5), \mathtt{p}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{p}_5}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{p}_5}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : (\Delta_4, \mathtt{p}_5), \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_1}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{p}_5}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7}\ \mathtt{hCut}$$
$$\dfrac{}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\Delta_4, \mathtt{p}_5), \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_2}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{Cut}$$

$$\Rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_8}{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{\dfrac{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_8}{- : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_2}\ \mathtt{hCut}$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_4, \mathtt{p}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_4), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \to_L}{- : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$

$$\Rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{p}_4}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_4, \mathtt{p}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\ \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_4), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \wedge_L}{- : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$

$$\Rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{\dfrac{- : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{F}_8}{- : \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\ \wedge_L}\ \mathtt{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_4), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \vee_L}{- : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$

$$\Rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6, \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\ \vee_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_7), \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{}{\bullet \mathtt{h}_5 : ((\bot, \Delta_7), \mathtt{p}_4), \mathtt{p}_4 \vdash \mathtt{F}_6}\ \bot_L}{- : (\bot, \Delta_7), \mathtt{p}_4 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$

$$\Rightarrow$$

$$\dfrac{}{- : \bot, \Delta_7, \mathtt{p}_4 \vdash \mathtt{F}_6}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_4, \mathtt{p}_6), \mathtt{p}_6 \vdash \mathtt{p}_6}\ I}{- : \Delta_4, \mathtt{p}_6 \vdash \mathtt{p}_6}\ \mathtt{Cut}$$

$$\Rightarrow$$

$$\dfrac{}{- : \Delta_4, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_7, \mathtt{p}_6), \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{}{\bullet \mathtt{h}_5 : ((\Delta_7, \mathtt{p}_6), \mathtt{p}_4), \mathtt{p}_4 \vdash \mathtt{p}_6}\ I}{- : (\Delta_7, \mathtt{p}_6), \mathtt{p}_4 \vdash \mathtt{p}_6}\ \mathtt{Cut}$$

$$\Rightarrow$$

$$\dfrac{}{- : \Delta_7, \mathtt{p}_4, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\top, \Delta_7), \mathtt{p}_4 \vdash \mathtt{p}_4}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : ((\top, \Delta_7), \mathtt{p}_4), \mathtt{p}_4 \vdash \mathtt{F}_6}\, \top_L}{- : (\top, \Delta_7), \mathtt{p}_4 \vdash \mathtt{F}_6}\, \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_7, \mathtt{p}_4 \vdash \mathtt{p}_4}\, I \quad \dfrac{\mathtt{h}_5 : \top, \Delta_7, \mathtt{p}_4, \mathtt{p}_4 \vdash \mathtt{F}_6}{}\, \mathtt{ax/W}}{- : \top, \Delta_7, \mathtt{p}_4 \vdash \mathtt{F}_6}\, \mathtt{hCut}$$

6.11 Status of $\top_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\, \top_L \quad \dfrac{}{\bullet \mathtt{h}_6 : (\top, \Delta_4), \mathtt{F}_5 \vdash \top}\, \top_R}{- : \top, \Delta_4 \vdash \top}\, \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{}{- : \top, \Delta_4 \vdash \top}\, \top_R$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\, \top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\top, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\, \to_R}{- : \top, \Delta_4 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\, \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\, \mathtt{ax/W}}{- : \top, \Delta_4 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\, \mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\, \top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\top, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\, \wedge_R}{- : \top, \Delta_4 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\, \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\, \mathtt{ax/W}}{- : \top, \Delta_4 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\, \mathtt{hCut}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\, \top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : (\top, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\, \vee_1}{- : \top, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\, \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\, \mathtt{ax/W}}{- : \top, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\, \mathtt{hCut}$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\, \top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\top, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\, \vee_2}{- : \top, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\, \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\, \mathtt{ax/W}}{- : \top, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\, \mathtt{hCut}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\to_L}{- : \top, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_6 \to \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\top, \Delta_4), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\to_L}{- : \top, \Delta_4 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\wedge_L}{- : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\top, \Delta_4), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\wedge_L}{- : \top, \Delta_4 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\vee_L}{- : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\top, \Delta_4), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\vee_L}{- : \top, \Delta_4 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \bot}{\bullet\mathtt{h}_1 : \top, \Delta_4 \vdash \bot}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_4), \bot \vdash \mathtt{F}_6}\,\bot_L}{- : \top, \Delta_4 \vdash \mathtt{F}_6}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_4 \vdash \bot}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \bot, \top, \Delta_4 \vdash \mathtt{F}_6}\,\bot_L}{- : \top, \Delta_4 \vdash \mathtt{F}_6}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_7 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \top, \bot, \Delta_7 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \bot, \Delta_7), \mathtt{F}_4 \vdash \mathtt{F}_6}\,\bot_L}{- : \top, \bot, \Delta_7 \vdash \mathtt{F}_6}\,\mathtt{Cut}}{\dfrac{}{- : \bot, \top, \Delta_7 \vdash \mathtt{F}_6}\,\bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \top, \Delta_7, \mathtt{p}_6 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_7, \mathtt{p}_6), \mathtt{F}_4 \vdash \mathtt{p}_6}\,I}{- : \top, \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6}\,\mathtt{Cut}}{\dfrac{}{- : \top, \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6}\,I}\,{\scriptstyle\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{p}_6}{\bullet\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{p}_6}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_4), \mathtt{p}_6 \vdash \mathtt{p}_6}\,I}{- : \top, \Delta_4 \vdash \mathtt{p}_6}\,\mathtt{Cut}}{\dfrac{}{- : \top, \Delta_4 \vdash \mathtt{p}_6}\,\mathtt{ax/W}}\,{\scriptstyle\rightarrow}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_6 : (\top, \Delta_4), \mathtt{F}_5 \vdash \mathtt{F}_7}\,\top_L}{- : \top, \Delta_4 \vdash \mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_4 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \top, \Delta_4 \vdash \mathtt{F}_7}\,\mathtt{hCut}}\,{\scriptstyle\rightarrow}$$

7 Cut-Elimination

7.1 Status of $\top_R$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \top}\,\top_R}{- : \Delta_2, \Delta_3 \vdash \top}\,\mathtt{Cut}$$

$$\Downarrow \rightarrow$$

$$\dfrac{}{- : \Delta_2, \Delta_3 \vdash \top}\,\top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \rightarrow \mathtt{F}_6}\,\rightarrow_R}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \rightarrow \mathtt{F}_6}\,\mathtt{Cut}$$

$$\Downarrow \rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_2, \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \rightarrow \mathtt{F}_6}\,\rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\,\wedge_R}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{Cut}$$

$$\Downarrow \rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_2, \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_2, \Delta_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\,\wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\vee_1}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{Cut}$$

$$\Downarrow \rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_2, \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\vee_2}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{Cut}$$

$$\Downarrow \rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_2, \Delta_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\vee_2$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top_R \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5), \top \vdash \mathtt{F}_6}\,\rightarrow_L}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\Downarrow \rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_2, \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_4}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_4}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_2, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \mathtt{F}_6}\,\rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top_R \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5), \top \vdash \mathtt{F}_6}\ \wedge_L}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\ \top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_2, \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \mathtt{F}_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top_R \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5), \top \vdash \mathtt{F}_6}\ \vee_L}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\ \top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\ \top_R \quad \dfrac{}{\mathtt{h}_3 : \top, \Delta_2, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \mathtt{F}_6}\ \vee_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_3 : (\bot, \Delta_5), \top \vdash \mathtt{F}_4}\ \bot_L}{- : \Delta_2, \bot, \Delta_5 \vdash \mathtt{F}_4}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_2, \Delta_5 \vdash \mathtt{F}_4}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_3 : (\Delta_5, \mathtt{p}_4), \top \vdash \mathtt{p}_4}\ I}{- : \Delta_2, \Delta_5, \mathtt{p}_4 \vdash \mathtt{p}_4}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_5, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top_R \quad \dfrac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_4, \top \vdash \mathtt{F}_5}\ \top_L}{- : \Delta_2, \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{ax/W}$$

7.2 Status of $\rightarrow_R$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \top}\ \top_R}{- : \Delta_2, \Delta_5 \vdash \top}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_R}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_R}}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\wedge_R}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\wedge_R}}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_1}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_1}}$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_2}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_2}}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \to \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,{\to_L}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \to \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,{\to_L}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_9}\,{\to_L}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{- : \Delta_2, \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{sCut}}{- : \Delta_2, \Delta_2, \Delta_6, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{sCut}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,C$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,{\wedge_L}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,{\wedge_L}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,{\vee_L}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}\,\mathtt{hCut} \quad \dfrac{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,{\vee_L}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_8}\,{\bot_L}}{- : \Delta_2, \bot, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\,{\bot_L}}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{p}_8}\,I}{- : \Delta_2, \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,I}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}\,{\to_R} \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_8}\,{\top_L}}{- : \Delta_2, \top, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \to \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \mathtt{F}_8}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \top, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}$$

7.3 Status of $\wedge_R$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\,{\wedge_R} \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top}\,{\top_R}}{- : \Delta_2, \Delta_5 \vdash \top}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\,{\top_R}}$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}\to_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_8}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}\wedge_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_9}\wedge_L}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_6, \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{sCut}}{- : \Delta_2, \Delta_2, \Delta_6, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{sCut}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,C}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}\vee_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\vee_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_8}\bot_L}{- : \Delta_2, \bot, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\,\bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{p}_8}I}{- : \Delta_2, \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,I}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_8}\top_L}{- : \Delta_2, \top, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

7.4 Status of $\vee_1$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_1 \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top}\top_R}{- : \Delta_2, \Delta_5 \vdash \top}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\,\top_R}$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_1 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_1 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_1 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_1 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\to_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\wedge_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\vee_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\vee_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\vee_1 \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_9}\vee_L}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\mathtt{sCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{}{\bullet\mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\bot_L}{- : \Delta_2, \bot, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{p}_8}\,I}{- : \Delta_2, \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_1 \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\top_L}{- : \Delta_2, \top, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\mathtt{hCut}$$

7.5 Status of $\vee_2$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_2 \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top}\top_R}{- : \Delta_2, \Delta_5 \vdash \top}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\,\top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_2 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\rightarrow_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_2 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}{\mathtt{ax/W} \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}{\mathtt{ax/W} \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_2 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}{\mathtt{ax/W} \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\vee_2 \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}{\mathtt{ax/W} \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \rightarrow \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \rightarrow \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\rightarrow_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}{\mathtt{ax/W} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8 \rightarrow \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \mathtt{F}_8}\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}{\mathtt{ax/W} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \mathtt{F}_{10}}\rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\wedge_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}{\mathtt{ax/W} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\vee_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}{\mathtt{ax/W} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}{\mathtt{ax/W} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\vee_2 \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_9}\vee_L}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{sCut}}\;\to$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{}{\bullet\mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\bot_L}{- : \Delta_2, \bot, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\bot_L}\;\to$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{p}_8}I}{- : \Delta_2, \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}I}\;\to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\vee_2 \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\top_L}{- : \Delta_2, \top, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{hCut}}\;\to$$

7.6 Status of $\to_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_7}\to_L \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \top}\top_R}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_6 \vdash \top}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \top}\top_R}\;\to$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_7}\to_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_9, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}\;\to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_7}\to_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_2, \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\to_L}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_7}\to_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_7}\to_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_6}\to_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\to_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_3}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_4, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}\to_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_{10}}\to_L}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_6}\to_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\wedge_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}\to_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_{10}}\to_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_6}\to_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\vee_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_3}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_4, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}\to_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\vee_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_{10}}\to_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \bot}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \bot}\to_L \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \bot \vdash \mathtt{F}_8}\bot_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \bot}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \bot, \Delta_7 \vdash \mathtt{F}_8}\bot_L}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_8}\to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_6}\to_L \quad \dfrac{}{\bullet\mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\bot_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \bot, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \bot, \Delta_2, \Delta_9, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_8}\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_6}\to_L \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_6 \vdash \mathtt{p}_8}I}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p}_8, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{p}_8}I$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{p}_8}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{p}_8}\to_L \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{p}_8 \vdash \mathtt{p}_8}I}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_7 \vdash \mathtt{p}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{p}_8}\,\mathtt{ax/W}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \top}\to_L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_7, \top \vdash \mathtt{F}_8}\top_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_6}\to_L \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\top_L}{- : (\Delta_2, \mathtt{F}_3 \to \mathtt{F}_4), \top, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_9, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}}\to$$

7.7 Status of $\wedge_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \top}\top_R}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_6 \vdash \top}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \top}\top_R}\to$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_9, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}\to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_L}\to$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}\to$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_7 \to \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_{10}}\to}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\to_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6 \to \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_{10}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}\wedge_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9 \to \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\to}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_{10}}\wedge_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\wedge_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6 \to \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}\wedge_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9 \to \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\to}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_{10}}\wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\vee_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6 \to \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}\wedge_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\vee_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9 \to \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\to}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_{10}}\wedge_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \bot}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \bot}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \bot \vdash \mathtt{F}_8}\bot_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \bot}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \bot, \Delta_7 \vdash \mathtt{F}_8}\bot_L}{- : \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_8}\wedge_L}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\bot_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \bot, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{- : \bot, \Delta_2, \Delta_9, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_8}\bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_6 \vdash \mathtt{p}_8}I}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\mathtt{Cut}}{- : \Delta_2, \Delta_9, \mathtt{p}_8, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{p}_8}I}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{p}_8}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{p}_8}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{p}_8 \vdash \mathtt{p}_8}I}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_7 \vdash \mathtt{p}_8}\mathtt{Cut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{p}_8}\mathtt{ax/W}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \top}\wedge_L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_7, \top \vdash \mathtt{F}_8}\top_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\mathtt{Cut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_8}\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_6}\wedge_L \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\top_L}{- : (\Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4), \top, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_9, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \mathtt{F}_8}\mathtt{hCut}}$$

7.8 Status of $\vee_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\vee_L \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \top}\top_R}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_6 \vdash \top}\mathtt{Cut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \top}\top_R$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_9, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_9}\mathtt{hCut} \quad \dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_1$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_2$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}\vee_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\to_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_8}\mathtt{hCut} \quad \dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \to \mathtt{F}_9, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \to \mathtt{F}_9 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}\vee_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\to_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\to}$$
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \to \mathtt{F}_9}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \to \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \to \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\vee_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}\vee_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\wedge_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \wedge \mathtt{F}_9, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}\vee_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\wedge_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\vee_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}\vee_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\vee_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_4, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\vee_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \vee \mathtt{F}_9 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}\vee_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\vee_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \vee \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_{10}}\vee_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \bot \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \bot}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \bot}\vee_L \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \bot \vdash \mathtt{F}_8}\bot_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \bot}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \bot}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \bot, \Delta_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \bot, \Delta_7 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_8}\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \bot}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \bot}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \bot, \Delta_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \bot, \Delta_7 \vdash \mathtt{F}_8}\bot_L}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_8}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_8}\vee_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}\vee_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\bot_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \bot, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_2, \Delta_9, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_8}\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}\vee_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_6 \vdash \mathtt{p}_8}I}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_9, \mathtt{p}_8 \vdash \mathtt{p}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p}_8, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{p}_8}I$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{p}_8 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{p}_8}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{p}_8}\vee_L \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{p}_8 \vdash \mathtt{p}_8}I}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_7 \vdash \mathtt{p}_8}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{p}_8}\mathtt{ax/W}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \top \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \top}\vee_L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_7, \top \vdash \mathtt{F}_8}\top_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\mathtt{Cut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_8}\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}\vee_L \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : (\top, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\top_L}{- : (\Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4), \top, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \top, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\mathtt{ax/W}}}{- : \top, \Delta_2, \Delta_9, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \mathtt{F}_8}\mathtt{hCut}$$

7.9 Status of $\bot_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \top}\top_R}{- : (\bot, \Delta_2), \Delta_4 \vdash \top}\mathtt{Cut}}{- : \bot, \Delta_2, \Delta_4 \vdash \top}\top_R$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\to_R}{- : (\bot, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\mathtt{Cut}}{- : \bot, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\bot_L$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R}{- : (\bot, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\mathtt{Cut}}{- : \bot, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\bot_L$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\vee_1}{- : (\bot, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}}{- : \bot, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_5}\bot_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\vee_2}{- : (\bot, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}}{- : \bot, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L$$

- Case rule $\to_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\ \to_L}{- : (\bot, \Delta_2), \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\ \to_L}{- : (\bot, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\ \bot_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\ \wedge_L}{- : (\bot, \Delta_2), \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\ \wedge_L}{- : (\bot, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\ \bot_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\ \vee_L}{- : (\bot, \Delta_2), \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\ \vee_L}{- : (\bot, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\ \bot_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \bot}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_5, \bot \vdash \mathtt{F}_6}\ \bot_L}{- : (\bot, \Delta_2), \Delta_5 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_5 \vdash \mathtt{F}_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \mathtt{F}_4 \vdash \mathtt{F}_6}\ \bot_L}{- : (\bot, \Delta_2), \bot, \Delta_7 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \bot, \Delta_2, \Delta_7 \vdash \mathtt{F}_6}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_6), \mathtt{F}_4 \vdash \mathtt{p}_6}\ I}{- : (\bot, \Delta_2), \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6}\ \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{p}_6}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I}{- : (\bot, \Delta_2), \Delta_5 \vdash \mathtt{p}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_5 \vdash \mathtt{p}_6}\ \bot_L$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \top}\ \bot_L \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_5, \top \vdash \mathtt{F}_6}\ \top_L}{- : (\bot, \Delta_2), \Delta_5 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_5 \vdash \mathtt{F}_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \mathtt{F}_4}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\top, \Delta_7), \mathtt{F}_4 \vdash \mathtt{F}_6}\ \top_L}{- : (\bot, \Delta_2), \top, \Delta_7 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \top, \Delta_2, \Delta_7 \vdash \mathtt{F}_6}\ \bot_L$$

7.10 Status of I: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{p}_5 \vdash \top}\ \top_R}{- : (\Delta_2, \mathtt{p}_5), \Delta_4 \vdash \top}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_4, \mathtt{p}_5 \vdash \top}\ \top_R$$

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_7, \mathtt{p}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \rightarrow_R}{- : (\Delta_2, \mathtt{p}_5), \Delta_4 \vdash \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \mathtt{ax/W}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R}{- : (\Delta_2, \mathtt{p}_5), \Delta_4 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{ax/W}$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_1}{- : (\Delta_2, \mathtt{p}_5), \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{ax/W}$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_5 \vdash \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_2}{- : (\Delta_2, \mathtt{p}_5), \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_4, \mathtt{p}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{ax/W}$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \rightarrow_L}{- : (\Delta_2, \mathtt{p}_4), \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{ax/W}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \wedge_L}{- : (\Delta_2, \mathtt{p}_4), \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{ax/W}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_4 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_4 \vdash \mathtt{F}_8}\ \vee_L}{- : (\Delta_2, \mathtt{p}_4), \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_9, \mathtt{p}_4, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{ax/W}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \mathtt{p}_4 \vdash \mathtt{F}_6}\ \bot_L}{- : (\Delta_2, \mathtt{p}_4), \bot, \Delta_7 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_2, \Delta_7, \mathtt{p}_4 \vdash \mathtt{F}_6}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_6), \mathtt{p}_4 \vdash \mathtt{p}_6}\ I}{- : (\Delta_2, \mathtt{p}_4), \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{p}_4, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I}{- : (\Delta_2, \mathtt{p}_6), \Delta_5 \vdash \mathtt{p}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_5, \mathtt{p}_6 \vdash \mathtt{p}_6}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_4 \vdash \mathtt{p}_4}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\top, \Delta_7), \mathtt{p}_4 \vdash \mathtt{F}_6}\ \top_L}{- : (\Delta_2, \mathtt{p}_4), \top, \Delta_7 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \top, \Delta_2, \Delta_7, \mathtt{p}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/W}$$

7.11 Status of $\top_L$: OK

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_5}\,\top_L \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \top}\,\top_R}{- : (\top, \Delta_2), \Delta_4 \vdash \top}\,\mathtt{Cut}}{\dfrac{}{- : \top, \Delta_2, \Delta_4 \vdash \top}\,\top_R}\,\to$$

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\,\to_R}{- : (\top, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{hCut}}\,\to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\,\wedge_R}{- : (\top, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut}}\,\to$$

- Case rule $\vee_1$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\,\vee_1}{- : (\top, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}}\,\to$$

- Case rule $\vee_2$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\,\vee_2}{- : (\top, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}}\,\to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\to_L}{- : (\top, \Delta_2), \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}\,\to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\to_L}{- : (\top, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_4 : \top, \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{hCut}}\,\to$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\wedge_L}{- : (\top, \Delta_2), \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\wedge_L}{- : (\top, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_4 : \top, \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\vee_L}{- : (\top, \Delta_2), \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\vee_L}{- : (\top, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_4 : \top, \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \bot}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \bot}\,\top_L \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_5, \bot \vdash \mathtt{F}_6}\,\bot_L}{- : (\top, \Delta_2), \Delta_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \bot}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_4 : \bot, \top, \Delta_5 \vdash \mathtt{F}_6}\,\bot_L}{- : \top, \Delta_2, \Delta_5 \vdash \mathtt{F}_6}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \mathtt{F}_4 \vdash \mathtt{F}_6}\,\bot_L}{- : (\top, \Delta_2), \bot, \Delta_7 \vdash \mathtt{F}_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \top, \Delta_2, \Delta_7 \vdash \mathtt{F}_6}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_6), \mathtt{F}_4 \vdash \mathtt{p}_6}\,I}{- : (\top, \Delta_2), \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \top, \Delta_2, \Delta_7, \mathtt{p}_6 \vdash \mathtt{p}_6}\,I$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{p}_6}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{p}_6}\,\top_L \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{p}_6 \vdash \mathtt{p}_6}\,I}{- : (\top, \Delta_2), \Delta_5 \vdash \mathtt{p}_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \top, \Delta_2, \Delta_5 \vdash \mathtt{p}_6}\,\mathtt{ax/W}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \top}{\bullet\mathtt{h}_1 : \top, \Delta_2 \vdash \top}\,\top_L \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_4 : \Delta_5, \top \vdash \mathtt{F}_6}\,\top_L}{- : (\top, \Delta_2), \Delta_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \top, \Delta_2, \Delta_5 \vdash \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \top, \Delta_2 \vdash \mathtt{F}_4}\,\top_L \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\top, \Delta_7), \mathtt{F}_4 \vdash \mathtt{F}_6}\,\top_L}{- : (\top, \Delta_2), \top, \Delta_7 \vdash \mathtt{F}_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \top, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \top, \top, \Delta_2, \Delta_7 \vdash \mathtt{F}_6}\,\mathtt{hCut}$$