

System for Intuitionistic Linear Logic

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Contents

1	Measure of derivations	3
2	Invertibility of Rules	5
2.1	Status of $!R$: : Non invertible	5
2.2	Status of $\mathbf{1}_R$: : Invertible	6
2.3	Status of \top : : Invertible	7
2.4	Status of $\&_R$: (Left Premise): Invertible	8
2.5	Status of $\&_R$ (Right Premise): : Invertible	10
2.6	Status of $\neg\circ_R$: : Invertible	11
2.7	Status of \oplus_{R_2} : : Non invertible	13
2.8	Status of \oplus_{R_1} : : Non invertible	14
2.9	Status of $\mathbf{1}_L$: : Invertible	16
2.10	Status of \otimes_R : (Left Premise): Non invertible	18
2.11	Status of \otimes_R (Right Premise): : Non invertible	20
2.12	Status of W : : Non invertible	22
2.13	Status of C : : Non invertible	24
2.14	Status of $!L$: : Non invertible	26
2.15	Status of $\&_{L2}$: : Non invertible	28
2.16	Status of $\&_{L1}$: : Non invertible	30
2.17	Status of \otimes_L : : Invertible	32
2.18	Status of \oplus_L : (Left Premise): Invertible	34
2.19	Status of \oplus_L (Right Premise): : Invertible	36
2.20	Status of $\neg\circ_L$: (Left Premise): Non invertible	38
2.21	Status of $\neg\circ_L$ (Right Premise): : Non invertible	40
2.22	Status of I : : Invertible	43
3	Identity-Expansion	44
4	Weakening on bang: $\Gamma \vdash !F$ implies $\Gamma \vdash F$.	45
5	Cut-Elimination	47
5.1	Status of $!R$: OK	47
5.2	Status of $\mathbf{1}_R$: OK	50
5.3	Status of \top : OK	53
5.4	Status of $\&_R$: OK	56
5.5	Status of $\neg\circ_R$: OK	59
5.6	Status of \oplus_{R_2} : OK	62
5.7	Status of \oplus_{R_1} : OK	65
5.8	Status of $\mathbf{1}_L$: OK	68

5.9	Status of \otimes_R : OK	72
5.10	Status of W : OK	75
5.11	Status of C : OK	79
5.12	Status of $!L$: OK	83
5.13	Status of $\&_{L2}$: OK	88
5.14	Status of $\&_{L1}$: OK	92
5.15	Status of \otimes_L : OK	96
5.16	Status of \oplus_L : OK	100
5.17	Status of $\neg\circ_L$: OK	104
5.18	Status of I : OK	109
6	Cut-Elimination	112
6.1	Status of $!R$: OK	112
6.2	Status of $\mathbf{1}_R$: OK	116
6.3	Status of \top : OK	117
6.4	Status of $\&_R$: OK	118
6.5	Status of $\neg\circ_R$: OK	119
6.6	Status of \oplus_{R2} : OK	120
6.7	Status of \oplus_{R1} : OK	121
6.8	Status of $\mathbf{1}_L$: OK	122
6.9	Status of \otimes_R : OK	123
6.10	Status of W : OK	124
6.11	Status of C : OK	128
6.12	Status of $!L$: OK	132
6.13	Status of $\&_{L2}$: OK	136
6.14	Status of $\&_{L1}$: OK	137
6.15	Status of \otimes_L : OK	138
6.16	Status of \oplus_L : OK	139
6.17	Status of $\neg\circ_L$: OK	140
6.18	Status of I : OK	141

1 Measure of derivations

- Case(s) rule $!R$

$$\frac{h_1 : !\top_2 \vdash F_3}{\bullet h_1 : !\top_2 \vdash !F_3} !R \rightarrow \frac{\frac{h_1 : !\top_2 \vdash F_3}{\bullet h_1 : !\top_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : !\top_2 \vdash !F_3} !R$$

- Case(s) rule 1_R

$$\frac{}{\bullet h_1 : * \vdash 1} 1_R \rightarrow \frac{}{\bullet \bullet h_1 : * \vdash 1} 1_R$$

- Case(s) rule \top

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top} \top$$

- Case(s) rule $\&_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R$$

- Case(s) rule $\neg\circ_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \neg\circ F_4} \neg\circ_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \neg\circ F_4} \neg\circ_R$$

- Case(s) rule \oplus_{R_2}

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2}$$

- Case(s) rule \oplus_{R_1}

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1}$$

- Case(s) rule 1_L

$$\frac{h_1 : \Delta_3 \vdash F_2}{\bullet h_1 : 1, \Delta_3 \vdash F_2} 1_L \rightarrow \frac{\frac{h_1 : \Delta_3 \vdash F_2}{\bullet h_1 : \Delta_3 \vdash F_2} \text{IH}}{\bullet \bullet h_1 : 1, \Delta_3 \vdash F_2} 1_L$$

- Case(s) rule \otimes_R

$$\frac{h_1 : \Delta_4 \vdash F_2 \quad h_1 : \Delta_5 \vdash F_3}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_2 \otimes F_3} \otimes_R \rightarrow \frac{\frac{h_1 : \Delta_4 \vdash F_2}{\bullet h_1 : \Delta_4 \vdash F_2} \text{IH} \quad \frac{h_1 : \Delta_5 \vdash F_3}{\bullet h_1 : \Delta_5 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : \Delta_4, \Delta_5 \vdash F_2 \otimes F_3} \otimes_R$$

- Case(s) rule W

$$\frac{h_1 : \Delta_4 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} W \rightarrow \frac{\frac{h_1 : \Delta_4 \vdash F_3}{\bullet h_1 : \Delta_4 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : \Delta_4, !F_2 \vdash F_3} W$$

- Case(s) rule C

$$\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} C \rightarrow \frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} IH}{\bullet \bullet h_1 : \Delta_4, !F_2 \vdash F_3} C \text{ ax}$$

- Case(s) rule $!L$

$$\frac{h_1 : \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} !L \rightarrow \frac{\frac{h_1 : \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \Delta_4, F_2 \vdash F_3} IH}{\bullet \bullet h_1 : \Delta_4, !F_2 \vdash F_3} !L \text{ ax}$$

- Case(s) rule $\&_{L2}$

$$\frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L2} \rightarrow \frac{\frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_3 \vdash F_4} IH}{\bullet \bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L2} \text{ ax}$$

- Case(s) rule $\&_{L1}$

$$\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L1} \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vdash F_4} IH}{\bullet \bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L1} \text{ ax}$$

- Case(s) rule \otimes_L

$$\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_4} \otimes_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash F_4} IH}{\bullet \bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_4} \otimes_L \text{ ax}$$

- Case(s) rule \oplus_L

$$\frac{h_1 : \Delta_5, F_2 \vdash F_4 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \oplus_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vdash F_4} IH \quad \frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_3 \vdash F_4} IH}{\bullet \bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \oplus_L \text{ ax}$$

- Case(s) rule \multimap_L

$$\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_4} \multimap_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash F_2}{\bullet h_1 : \Delta_5 \vdash F_2} IH \quad \frac{h_1 : \Delta_6, F_3 \vdash F_4}{\bullet h_1 : \Delta_6, F_3 \vdash F_4} IH}{\bullet \bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_4} \multimap_L \text{ ax}$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : p(n_2) \vdash p(n_2)} I \rightarrow \frac{}{\bullet \bullet h_1 : p(n_2) \vdash p(n_2)} I$$

2 Invertibility of Rules

2.1 Status of $!R$: : Non invertible

- Case rule $!R$

$$\frac{h_1 : !\top 2 \vdash F_3}{\bullet h_1 : !\top 2 \vdash !F_3} !R \rightarrow \frac{\overline{h_1 : !\top 2 \vdash F_3} \text{ ax}}{\bullet h_1 : !\top 2 \vdash F_3} H$$

- Case rule 1_R

- Case rule \top

- Case rule $\&_R$

- Case rule $\neg\circ_R$

- Case rule \oplus_{R_2}

- Case rule \oplus_{R_1}

- Case rule 1_L

- Case rule \otimes_R

- Case rule W

$$\frac{h_2 : !\top 4 \vdash !F_1}{\bullet h_2 : !\top 4, !F_3 \vdash !F_1} W \rightarrow \frac{\overline{h_2 : !\top 4 \vdash F_1} \text{ ax/ind}}{\bullet h_2 : !\top 4, !F_3 \vdash F_1} W$$

- Case rule C

$$\frac{h_2 : !\top 4, !F_3, !F_3 \vdash !F_1}{\bullet h_2 : !\top 4, !F_3 \vdash !F_1} C \rightarrow \frac{\overline{h_2 : !\top 4, !F_3, !F_3 \vdash F_1} \text{ ax/ind}}{\bullet h_2 : !\top 4, !F_3 \vdash F_1} C$$

- Case rule $!L$

$$\frac{h_2 : F_3, !\top 4 \vdash !F_1}{\bullet h_2 : !\top 4, !F_3 \vdash !F_1} !L \rightarrow \frac{}{\bullet h_2 : !\top 4, !F_3 \vdash F_1} \text{ fail}$$

- Case rule $\&_{L2}$

- Case rule $\&_{L1}$

- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

2.2 Status of $\mathbf{1}_R$: : Invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$

$$\frac{}{\bullet \mathbf{h}_1 : * \vdash \mathbf{1}} \mathbf{1}_R \rightarrow \text{trivial}$$

- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$

- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

2.3 Status of \top : : Invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \rightarrow \text{trivial}$$

- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$

$$\frac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash \top} \mathbf{1}_L \rightarrow \text{trivial}$$

- Case rule \otimes_R
- Case rule W

$$\frac{h_1 : \Delta_3 \vdash \top}{\bullet h_1 : \Delta_3, !F_2 \vdash \top} W \rightarrow \text{trivial}$$

- Case rule C

$$\frac{h_1 : \Delta_3, !F_2, !F_2 \vdash \top}{\bullet h_1 : \Delta_3, !F_2 \vdash \top} C \rightarrow \text{trivial}$$

- Case rule $!L$

$$\frac{h_1 : \Delta_3, F_2 \vdash \top}{\bullet h_1 : \Delta_3, !F_2 \vdash \top} !L \quad \rightarrow \quad \text{trivial}$$

- Case rule $\&_{L2}$

$$\frac{h_1 : \Delta_4, F_3 \vdash \top}{\bullet h_1 : \Delta_4, F_2 \& F_3 \vdash \top} \&_{L2} \quad \rightarrow \quad \text{trivial}$$

- Case rule $\&_{L1}$

$$\frac{h_1 : \Delta_4, F_2 \vdash \top}{\bullet h_1 : \Delta_4, F_2 \& F_3 \vdash \top} \&_{L1} \quad \rightarrow \quad \text{trivial}$$

- Case rule \otimes_L

$$\frac{h_1 : \Delta_4, F_2, F_3 \vdash \top}{\bullet h_1 : \Delta_4, F_2 \otimes F_3 \vdash \top} \otimes_L \quad \rightarrow \quad \text{trivial}$$

- Case rule \oplus_L

$$\frac{h_1 : \Delta_4, F_2 \vdash \top \quad h_1 : \Delta_4, F_3 \vdash \top}{\bullet h_1 : \Delta_4, F_2 \oplus F_3 \vdash \top} \oplus_L \quad \rightarrow \quad \text{trivial}$$

- Case rule $\neg\circ_L$

$$\frac{h_1 : \Delta_4 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash \top}{\bullet h_1 : \Delta_4, \Delta_5, F_2 \neg\circ F_3 \vdash \top} \neg\circ_L \quad \rightarrow \quad \text{trivial}$$

- Case rule I

2.4 Status of $\&_R$: (Left Premise): Invertible

- Case rule $!R$

- Case rule 1_R

- Case rule \top

- Case rule $\&_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R \quad \rightarrow \quad \frac{\overline{h_1 : \Delta_2 \vdash F_3}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{ax}_H$$

- Case rule $\neg\circ_R$

- Case rule \oplus_{R_2}

- Case rule \oplus_{R_1}

- Case rule 1_L

$$\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \& F_2} 1_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : 1, \Delta_4 \vdash F_1} 1_L$$

- Case rule \otimes_R

- Case rule W

$$\frac{h_3 : \Delta_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \& F_2} W \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1} W$$

- Case rule C

$$\frac{h_3 : \Delta_5, !F_4, !F_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \& F_2} C \rightarrow \frac{\overline{h_3 : \Delta_5, !F_4, !F_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_5, F_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \& F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_5, F_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_6, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \& F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_6, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \& F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, F_4 \otimes F_5 \vdash F_1 \& F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \otimes F_5 \vdash F_1} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \& F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_1 \& F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \vdash F_1} \text{ ax/ind} \quad \overline{h_3 : \Delta_6, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_1} \oplus_L$$

- Case rule $\neg o_L$

$$\frac{\frac{h_3 : \Delta_6 \vdash F_4}{\bullet h_3 : \Delta_6, \Delta_7, F_4} \quad \frac{h_3 : \Delta_7, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, \Delta_7, F_4 \neg o F_5 \vdash F_1 \& F_2}}{\neg o_L} \rightarrow \frac{\frac{\overline{h_3 : \Delta_6 \vdash F_4}}{\bullet h_3 : \Delta_6, \Delta_7, F_4} \text{ ax} \quad \frac{\overline{h_3 : \Delta_7, F_5 \vdash F_1}}{\bullet h_3 : \Delta_6, \Delta_7, F_4 \neg o F_5 \vdash F_1} \text{ ax/ind}}{\neg o_L}$$

- Case rule I

2.5 Status of $\&_R$ (Right Premise): : Invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2} \quad \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4}}{\&_R} \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash F_4} \text{ H}$$

- Case rule $\neg o_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L

$$\frac{\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \& F_2}}{1_L} \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : 1, \Delta_4 \vdash F_2} 1_L$$

- Case rule \otimes_R
- Case rule W

$$\frac{\frac{h_3 : \Delta_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \& F_2}}{W} \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_5, !F_4 \vdash F_2} W$$

- Case rule C

$$\frac{\frac{h_3 : \Delta_5, !F_4, !F_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \& F_2}}{C} \rightarrow \frac{\overline{h_3 : \Delta_5, !F_4, !F_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_5, !F_4 \vdash F_2} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_5, F_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \& F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_5, F_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_5, !F_4 \vdash F_2} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_6, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \& F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_6, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_2} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \& F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_2} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, F_4 \otimes F_5 \vdash F_1 \& F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \otimes F_5 \vdash F_2} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \& F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_1 \& F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \vdash F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_6, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_2} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \Delta_7, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_6, \Delta_7, F_4 \multimap F_5 \vdash F_1 \& F_2} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_4} \text{ ax} \quad \overline{h_3 : \Delta_7, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, \Delta_7, F_4 \multimap F_5 \vdash F_2} \multimap_L$$

- Case rule I

2.6 Status of \multimap_R : : Invertible

- Case rule $!R$

- Case rule 1_R

- Case rule \top

- Case rule $\&_R$

- Case rule \multimap_R

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \multimap F_4} \multimap_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} H$$

- Case rule \oplus_{R_2}

- Case rule \oplus_{R_1}

- Case rule 1_L

$$\frac{h_3 : \Delta_4 \vdash F_1 \multimap F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \multimap F_2} 1_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : 1, \Delta_4, F_1 \vdash F_2} 1_L$$

- Case rule \otimes_R

- Case rule W

$$\frac{h_3 : \Delta_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \multimap F_2} W \rightarrow \frac{\overline{h_3 : \Delta_5, F_1 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_5, F_1, !F_4 \vdash F_2} W$$

- Case rule C

$$\frac{h_3 : \Delta_5, !F_4, !F_4 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \multimap F_2} C \rightarrow \frac{\overline{h_3 : \Delta_5, F_1, !F_4, !F_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_5, F_1, !F_4 \vdash F_2} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_5, F_4 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \multimap F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_5, F_1, F_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_5, F_1, !F_4 \vdash F_2} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_6, F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \multimap F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_4 \& F_5 \vdash F_2} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \multimap F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_4 \& F_5 \vdash F_2} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_6, F_4 \otimes F_5 \vdash F_1 \multimap F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_4 \otimes F_5 \vdash F_2} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \multimap F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_1 \multimap F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4 \vdash F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_6, F_1, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_4 \oplus F_5 \vdash F_2} \oplus_L$$

- Case rule $\neg\circ_L$

$$\frac{\frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \Delta_7, F_5 \vdash F_1 \neg\circ F_2}{\bullet h_3 : \Delta_6, \Delta_7, F_4 \neg\circ F_5 \vdash F_1 \neg\circ F_2} \neg\circ_L}{\frac{h_3 : \Delta_6 \vdash F_4 \quad \text{ax} \quad h_3 : \Delta_7, F_1, F_5 \vdash F_2}{\bullet h_3 : \Delta_6, \Delta_7, F_1, F_4 \neg\circ F_5 \vdash F_2} \text{ax/ind}} \neg\circ_L \rightarrow$$

- Case rule I

2.7 Status of \oplus_{R_2} : : Non invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{ax} \quad \text{H}$$

- Case rule \oplus_{R_1}

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_4} \text{fail}$$

- Case rule 1_L

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \oplus F_2} 1_L \rightarrow \frac{h_3 : \Delta_4 \vdash F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_2} \text{ax/ind} \quad 1_L$$

- Case rule \otimes_R

- Case rule W

$$\frac{h_3 : \Delta_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \oplus F_2} W \rightarrow \frac{h_3 : \Delta_5 \vdash F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_2} \text{ax/ind} \quad W$$

- Case rule C

$$\frac{h_3 : \Delta_5, !F_4, !F_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \oplus F_2} C \rightarrow \frac{h_3 : \Delta_5, !F_4, !F_4 \vdash F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_2} \text{ax/ind} \quad C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_5, F_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \oplus F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_5, F_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_5, !F_4 \vdash F_2} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_6, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \oplus F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_6, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_2} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \oplus F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_2} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, F_4 \otimes F_5 \vdash F_1 \oplus F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \otimes F_5 \vdash F_2} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \oplus F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_1 \oplus F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \vdash F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_6, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_2} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \Delta_7, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, \Delta_7, F_4 \multimap F_5 \vdash F_1 \oplus F_2} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_4} \text{ ax} \quad \overline{h_3 : \Delta_7, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, \Delta_7, F_4 \multimap F_5 \vdash F_2} \multimap_L$$

- Case rule I

2.8 Status of \oplus_{R1} : : Non invertible

- Case rule $!R$

- Case rule 1_R

- Case rule \top

- Case rule $\&_R$

- Case rule \multimap_R

- Case rule \oplus_{R2}

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R2} \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_3} \text{fail}$$

- Case rule \oplus_{R1}

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R1} \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_3} \text{ax} \quad \text{H}$$

- Case rule 1_L

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \oplus F_2} 1_L \rightarrow \frac{}{\bullet h_3 : 1, \Delta_4 \vdash F_1} \text{ax/ind} \quad 1_L$$

- Case rule \otimes_R

- Case rule W

$$\frac{h_3 : \Delta_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \oplus F_2} W \rightarrow \frac{}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1} \text{ax/ind} \quad W$$

- Case rule C

$$\frac{h_3 : \Delta_5, !F_4, !F_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \oplus F_2} C \rightarrow \frac{}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1} \text{ax/ind} \quad C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_5, F_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1 \oplus F_2} !L \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_4 \vdash F_1} \text{ax/ind} \quad !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_6, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \oplus F_2} \&_{L2} \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_5 \vdash F_1} \text{ax/ind} \quad \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1 \oplus F_2} \&_{L1} \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_4 \vdash F_1} \text{ax/ind} \quad \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, F_4 \otimes F_5 \vdash F_1 \oplus F_2} \otimes_L \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_4, F_5 \vdash F_1} \text{ax/ind} \quad \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \oplus F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_1 \oplus F_2} \oplus_L \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_4 \vdash F_1} \text{ax/ind} \quad \frac{}{h_3 : \Delta_6, F_5 \vdash F_1} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_1} \oplus_L$$

- Case rule $\neg\circ_L$

$$\frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \Delta_7, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_6, \Delta_7, F_4 \neg\circ F_5 \vdash F_1 \oplus F_2} \neg\circ_L \rightarrow \frac{\frac{}{h_3 : \Delta_6 \vdash F_4} \text{ax} \quad \frac{}{h_3 : \Delta_7, F_5 \vdash F_1} \text{ax/ind}}{\bullet h_3 : \Delta_6, \Delta_7, F_4 \neg\circ F_5 \vdash F_1} \neg\circ_L$$

- Case rule I

2.9 Status of 1_L : : Invertible

- Case rule $!R$

- Case rule 1_R

- Case rule \top

$$\frac{}{\bullet h_1 : 1, \Delta_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{h_1 : 1, \Delta_4 \vdash F_2 \quad h_1 : 1, \Delta_4 \vdash F_3}{\bullet h_1 : 1, \Delta_4 \vdash F_2 \& F_3} \&_R \rightarrow \frac{\frac{}{h_1 : \Delta_4 \vdash F_2} \text{ax/ind} \quad \frac{}{h_1 : \Delta_4 \vdash F_3} \text{ax/ind}}{\bullet h_1 : \Delta_4 \vdash F_2 \& F_3} \&_R$$

- Case rule $\neg\circ_R$

$$\frac{h_1 : 1, \Delta_4, F_2 \vdash F_3}{\bullet h_1 : 1, \Delta_4 \vdash F_2 \neg\circ F_3} \neg\circ_R \rightarrow \frac{\frac{}{h_1 : \Delta_4, F_2 \vdash F_3} \text{ax/ind}}{\bullet h_1 : \Delta_4 \vdash F_2 \neg\circ F_3} \neg\circ_R$$

- Case rule \oplus_{R_2}

$$\frac{h_1 : 1, \Delta_4 \vdash F_3}{\bullet h_1 : 1, \Delta_4 \vdash F_2 \oplus F_3} \oplus_{R_2} \rightarrow \frac{\frac{}{h_1 : \Delta_4 \vdash F_3} \text{ax/ind}}{\bullet h_1 : \Delta_4 \vdash F_2 \oplus F_3} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{h_1 : 1, \Delta_4 \vdash F_2}{\bullet h_1 : 1, \Delta_4 \vdash F_2 \oplus F_3} \oplus_{R_1} \rightarrow \frac{\frac{}{h_1 : \Delta_4 \vdash F_2} \text{ax/ind}}{\bullet h_1 : \Delta_4 \vdash F_2 \oplus F_3} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{h_1 : \Delta_3 \vdash F_2}{\bullet h_1 : 1, \Delta_3 \vdash F_2} 1_L \rightarrow \frac{\frac{}{h_1 : \Delta_3 \vdash F_2} \text{ax}}{\bullet h_1 : \Delta_3 \vdash F_2} H$$

- Case rule \otimes_R

$$\frac{h_1 : 1, \Delta_4 \vdash F_2 \quad h_1 : \Delta_5 \vdash F_3}{\bullet h_1 : (1, \Delta_4), \Delta_5 \vdash F_2 \otimes F_3} \otimes_R \rightarrow \frac{\frac{h_1 : \Delta_4 \vdash F_2}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_2 \otimes F_3} \text{ ax/ind} \quad \frac{h_1 : \Delta_5 \vdash F_3}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_2 \otimes F_3} \text{ ax}}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_2 \otimes F_3} \otimes_R$$

$$\frac{h_1 : \Delta_4 \vdash F_2 \quad h_1 : 1, \Delta_5 \vdash F_3}{\bullet h_1 : \Delta_4, 1, \Delta_5 \vdash F_2 \otimes F_3} \otimes_R \rightarrow \frac{\frac{h_1 : \Delta_4 \vdash F_2}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_2 \otimes F_3} \text{ ax} \quad \frac{h_1 : \Delta_5 \vdash F_3}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_2 \otimes F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_2 \otimes F_3} \otimes_R$$

- Case rule W

$$\frac{h_1 : 1, \Delta_4 \vdash F_3}{\bullet h_1 : (1, \Delta_4), !F_2 \vdash F_3} W \rightarrow \frac{\frac{h_1 : \Delta_4 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} W$$

- Case rule C

$$\frac{h_1 : 1, \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : (1, \Delta_4), !F_2 \vdash F_3} C \rightarrow \frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} C$$

- Case rule $!L$

$$\frac{h_1 : 1, \Delta_4, F_2 \vdash F_3}{\bullet h_1 : (1, \Delta_4), !F_2 \vdash F_3} !L \rightarrow \frac{\frac{h_1 : \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} !L$$

- Case rule $\&_{L2}$

$$\frac{h_1 : 1, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (1, \Delta_5), F_2 \& F_3 \vdash F_4} \&_{L2} \rightarrow \frac{\frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_1 : 1, \Delta_5, F_2 \vdash F_4}{\bullet h_1 : (1, \Delta_5), F_2 \& F_3 \vdash F_4} \&_{L1} \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_1 : 1, \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : (1, \Delta_5), F_2 \otimes F_3 \vdash F_4} \otimes_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_4} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_1 : 1, \Delta_5, F_2 \vdash F_4 \quad h_1 : 1, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (1, \Delta_5), F_2 \oplus F_3 \vdash F_4} \oplus_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \text{ ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_1 : 1, \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_4}{\bullet h_1 : (1, \Delta_5), \Delta_6, F_2 \multimap F_3 \vdash F_4} \multimap_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash F_2}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_4} \text{ ax/ind} \quad \frac{h_1 : \Delta_6, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_4} \multimap_L$$

$$\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : 1, \Delta_6, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, (1, \Delta_6), F_2 \multimap F_3 \vdash F_4} \multimap_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash F_2}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_4} \text{ ax} \quad \frac{h_1 : \Delta_6, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_4} \multimap_L$$

- Case rule I

2.10 Status of \otimes_R : (Left Premise): Non invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L

$$\frac{h_3 : \Delta_4, \Delta_5 \vdash F_1 \otimes F_2}{\bullet h_3 : 1, \Delta_4, \Delta_5 \vdash F_1 \otimes F_2} 1_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1}}{\bullet h_3 : 1, \Delta_4 \vdash F_1} 1_L \text{ ax/ind}$$

$$\frac{h_3 : \Delta_4, \Delta_5 \vdash F_1 \otimes F_2}{\bullet h_3 : 1, \Delta_4, \Delta_5 \vdash F_1 \otimes F_2} 1_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1}}{\bullet h_3 : \Delta_4 \vdash F_1} H \text{ ax/ind}$$

- Case rule \otimes_R

$$\frac{h_1 : \Delta_4, \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, \Delta_7 \vdash F_3}{\bullet h_1 : (\Delta_4, \Delta_5), \Delta_6, \Delta_7 \vdash F_2 \otimes F_3} \otimes_R \rightarrow \overline{\bullet h_1 : \Delta_4, \Delta_6 \vdash F_2} \text{ fail}$$

- Case rule W

$$\frac{h_3 : \Delta_5, \Delta_6 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} W \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash F_1}}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1} W \text{ ax/ind}$$

$$\frac{h_3 : \Delta_5, \Delta_6 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} W \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash F_1}}{\bullet h_3 : \Delta_5 \vdash F_1} H \text{ ax/ind}$$

- Case rule C

$$\frac{h_3 : \Delta_5, \Delta_6, !F_4, !F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} C \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash F_1}}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1} W \text{ ax/ind}$$

$$\frac{h_3 : \Delta_5, \Delta_6, !F_4, !F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} C \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash F_1}}{\bullet h_3 : \Delta_5 \vdash F_1} H \text{ ax/ind}$$

- Case rule $!L$

$$\frac{\frac{h_3 : \Delta_5, \Delta_6, F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} !L}{\frac{h_3 : \Delta_5, \Delta_6, F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} !L} \rightarrow \frac{\frac{h_3 : \Delta_5 \vdash F_1}{\bullet h_3 : \Delta_5, !F_4 \vdash F_1} \text{ax/ind}}{W}$$

- Case rule $\&_{L2}$

$$\frac{\frac{h_3 : \Delta_6, \Delta_7, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \& F_5 \vdash F_1 \otimes F_2} \& L_2 \quad \rightarrow \quad \frac{h_3 : \Delta_6, F_5 \vdash F_1}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1} \text{ax/ind} \& L_2$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \& F_5 \vdash F_1 \otimes F_2} \& L_2 \quad \rightarrow \quad \frac{h_3 : \Delta_6 \vdash F_1}{\bullet h_3 : \Delta_6 \vdash F_1} \text{ax/ind} \text{H}$$

- Case rule $\&_{L1}$

$$\frac{\frac{h_3 : \Delta_6, \Delta_7, F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \& F_5 \vdash F_1 \otimes F_2} \& L_1 \quad \rightarrow \quad \frac{h_3 : \Delta_6, F_4 \vdash F_1}{\bullet h_3 : \Delta_6, F_4 \& F_5 \vdash F_1} \text{ax/ind} \& L_1$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \& F_5 \vdash F_1 \otimes F_2} \& L_1 \quad \rightarrow \quad \frac{h_3 : \Delta_6 \vdash F_1}{\bullet h_3 : \Delta_6 \vdash F_1} \text{ax/ind}_H$$

- Case rule \otimes_L

$$\frac{h_3 : \triangle_6, \triangle_7, F_4, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\triangle_6, \triangle_7), F_4 \otimes F_5 \vdash F_1 \otimes F_2} \otimes L \quad \rightarrow \quad \frac{h_3 : \triangle_6, F_4, F_5 \vdash F_1}{\bullet h_3 : \triangle_6, F_4 \otimes F_5 \vdash F_1} \frac{\text{ax/ind}}{\otimes L}$$

$$\frac{h_3 : \triangle_6, \triangle_7, F_4, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\triangle_6, \triangle_7), F_4 \otimes F_5 \vdash F_1 \otimes F_2} \otimes L \quad \rightarrow \quad \frac{h_3 : \triangle_6 \vdash F_1}{\bullet h_3 : \triangle_6 \vdash F_1} \frac{\text{ax/ind}}{H}$$

- Case rule \oplus_L

$$\frac{\frac{h_3 : \Delta_6, \Delta_7, F_4 \vdash F_1 \otimes F_2 \quad h_3 : \Delta_6, \Delta_7, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \oplus F_5 \vdash F_1 \otimes F_2} \oplus_L \quad \rightarrow \quad \frac{h_3 : \Delta_6, F_4 \vdash F_1 \quad \text{ax/ind} \quad h_3 : \Delta_6, F_5 \vdash F_1}{\bullet h_3 : \Delta_6, F_4 \oplus F_5 \vdash F_1} \oplus_L \text{ax/ind}$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4 \vdash F_1 \otimes F_2 \quad h_3 : \Delta_6, \Delta_7, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \oplus F_5 \vdash F_1 \otimes F_2} \oplus_L \quad \rightarrow \quad \frac{h_3 : \Delta_6 \vdash F_1}{\bullet h_3 : \Delta_6 \vdash F_1} \text{H} \text{ax/ind}$$

- Case rule $\neg\circ_L$

$$\frac{\begin{array}{c} h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2 \\ \bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \multimap F_5 \vdash F_1 \otimes F_2 \end{array} \multimap_L \quad \rightarrow \quad \frac{}{\bullet h_3 : \Delta_6, \Delta_8, F_4 \multimap F_5 \vdash F_1} \text{fail}}$$

- Case rule I

2.11 Status of \otimes_R (Right Premise): : Non invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L

$$\frac{h_3 : \Delta_4, \Delta_5 \vdash F_1 \otimes F_2}{\bullet h_3 : 1, \Delta_4, \Delta_5 \vdash F_1 \otimes F_2} 1_L \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash F_2}}{\bullet h_3 : \Delta_5 \vdash F_2} \text{ax/ind}_H$$

$$\frac{h_3 : \Delta_4, \Delta_5 \vdash F_1 \otimes F_2}{\bullet h_3 : 1, \Delta_4, \Delta_5 \vdash F_1 \otimes F_2} 1_L \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash F_2}}{\bullet h_3 : 1, \Delta_5 \vdash F_2} \text{ax/ind}_{1_L}$$

- Case rule \otimes_R

$$\frac{h_1 : \Delta_4, \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, \Delta_7 \vdash F_3}{\bullet h_1 : (\Delta_4, \Delta_5), \Delta_6, \Delta_7 \vdash F_2 \otimes F_3} \otimes_R \rightarrow \frac{}{\bullet h_1 : \Delta_5, \Delta_7 \vdash F_3} \text{fail}$$

- Case rule W

$$\frac{h_3 : \Delta_5, \Delta_6 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} W \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_2}}{\bullet h_3 : \Delta_6 \vdash F_2} \text{ax/ind}_H$$

$$\frac{h_3 : \Delta_5, \Delta_6 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} W \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_2}}{\bullet h_3 : \Delta_6, !F_4 \vdash F_2} \text{ax/ind}_W$$

- Case rule C

$$\frac{h_3 : \Delta_5, \Delta_6, !F_4, !F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} C \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_2}}{\bullet h_3 : \Delta_6 \vdash F_2} \text{ax/ind}_H$$

$$\frac{h_3 : \Delta_5, \Delta_6, !F_4, !F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} C \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_2}}{\bullet h_3 : \Delta_6, !F_4 \vdash F_2} \text{ax/ind}_W$$

- Case rule $!L$

$$\frac{h_3 : \Delta_5, \Delta_6, F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6 \vdash F_2} H$$

$$\frac{h_3 : \Delta_5, \Delta_6, F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_5, \Delta_6), !F_4 \vdash F_1 \otimes F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, !F_4 \vdash F_2} W$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_6, \Delta_7, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \& F_5 \vdash F_1 \otimes F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7 \vdash F_2} H$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \& F_5 \vdash F_1 \otimes F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \& F_5 \vdash F_2} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \& F_5 \vdash F_1 \otimes F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7 \vdash F_2} H$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \& F_5 \vdash F_1 \otimes F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \& F_5 \vdash F_2} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_6, \Delta_7, F_4, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \otimes F_5 \vdash F_1 \otimes F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7 \vdash F_2} H$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \otimes F_5 \vdash F_1 \otimes F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \otimes F_5 \vdash F_2} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_6, \Delta_7, F_4 \vdash F_1 \otimes F_2 \quad h_3 : \Delta_6, \Delta_7, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \oplus F_5 \vdash F_1 \otimes F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7 \vdash F_2} H$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4 \vdash F_1 \otimes F_2 \quad h_3 : \Delta_6, \Delta_7, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), F_4 \oplus F_5 \vdash F_1 \otimes F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \oplus F_5 \vdash F_2} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \multimap F_5 \vdash F_1 \otimes F_2} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9 \vdash F_2} \text{ fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \multimap F_5 \vdash F_1 \otimes F_2} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_4 \multimap F_5 \vdash F_2} \text{ fail}$$

- Case rule I

2.12 Status of W : : Non invertible

- Case rule $!R$

$$\frac{h_2 : !\top_4, !F_1 \vdash F_3}{\bullet h_2 : !\top_4, !F_1 \vdash !F_3} !R \rightarrow \frac{\overline{h_2 : !\top_4 \vdash F_3} \text{ ax/ind}}{\bullet h_2 : !\top_4 \vdash !F_3} !R$$

- Case rule 1_R

- Case rule \top

$$\overline{\bullet h_2 : \Delta_3, !F_1 \vdash \top} \top \rightarrow \overline{\bullet h_2 : \Delta_3 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_3 \quad h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \& F_4} \&_R \rightarrow \frac{\overline{h_2 : \Delta_5 \vdash F_3} \text{ ax/ind} \quad \overline{h_2 : \Delta_5 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5 \vdash F_3 \& F_4} \&_R$$

- Case rule $\neg o_R$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \neg o F_4} \neg o_R \rightarrow \frac{\overline{h_2 : \Delta_5, F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5 \vdash F_3 \neg o F_4} \neg o_R$$

- Case rule \oplus_{R_2}

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{\overline{h_2 : \Delta_5 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5 \vdash F_3 \oplus F_4} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_3}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{\overline{h_2 : \Delta_5 \vdash F_3} \text{ ax/ind}}{\bullet h_2 : \Delta_5 \vdash F_3 \oplus F_4} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{h_2 : \Delta_4, !F_1 \vdash F_3}{\bullet h_2 : 1, \Delta_4, !F_1 \vdash F_3} 1_L \rightarrow \frac{\overline{h_2 : \Delta_4 \vdash F_3} \text{ ax/ind}}{\bullet h_2 : 1, \Delta_4 \vdash F_3} 1_L$$

- Case rule \otimes_R

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_3 \quad h_2 : \Delta_6 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), \Delta_6 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_5 \vdash F_3} \text{ ax/ind} \quad \overline{h_2 : \Delta_6 \vdash F_4} \text{ ax}}{\bullet h_2 : \Delta_5, \Delta_6 \vdash F_3 \otimes F_4} \otimes_R$$

$$\frac{h_2 : \Delta_5 \vdash F_3 \quad h_2 : \Delta_6, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, \Delta_6, !F_1 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_5 \vdash F_3} \text{ ax} \quad \overline{h_2 : \Delta_6 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, \Delta_6 \vdash F_3 \otimes F_4} \otimes_R$$

- Case rule W

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_2 : \Delta_5 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_3 \vdash F_4} W$$

$$\frac{h_1 : \Delta_4 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} W \rightarrow \frac{\overline{h_1 : \Delta_4 \vdash F_3} \text{ ax}}{\bullet h_1 : \Delta_4 \vdash F_3} H$$

- Case rule C

$$\frac{h_2 : \Delta_5, !F_1, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_2 : \Delta_5, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_3 \vdash F_4} C$$

$$\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} C \rightarrow \frac{}{\bullet h_1 : \Delta_4 \vdash F_3} \text{ fail}$$

- Case rule $!L$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_2 : \Delta_5, F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_3 \vdash F_4} !L$$

$$\frac{h_1 : \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} !L \rightarrow \frac{}{\bullet h_1 : \Delta_4 \vdash F_3} \text{ fail}$$

- Case rule $\&_{L2}$

$$\frac{h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_6, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_6, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_2 : \Delta_6, F_3, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5 \quad h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3 \vdash F_5} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule $\neg\circ_L$

$$\frac{h_2 : \Delta_6, !F_1 \vdash F_3 \quad h_2 : \Delta_7, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), \Delta_7, F_3 \neg\circ F_4 \vdash F_5} \neg\circ_L \rightarrow \frac{\overline{h_2 : \Delta_6 \vdash F_3} \text{ ax/ind} \quad \overline{h_2 : \Delta_7, F_4 \vdash F_5} \text{ ax}}{\bullet h_2 : \Delta_6, \Delta_7, F_3 \neg\circ F_4 \vdash F_5} \neg\circ_L$$

$$\frac{h_2 : \Delta_6 \vdash F_3 \quad h_2 : \Delta_7, F_4, !F_1 \vdash F_5}{\bullet h_2 : \Delta_6, (\Delta_7, !F_1), F_3 \neg\circ F_4 \vdash F_5} \neg\circ_L \rightarrow \frac{\overline{h_2 : \Delta_6 \vdash F_3} \text{ ax} \quad \overline{h_2 : \Delta_7, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, \Delta_7, F_3 \neg\circ F_4 \vdash F_5} \neg\circ_L$$

- Case rule I

2.13 Status of C : : Non invertible

- Case rule $!R$

$$\frac{h_2 : !\top 4, !F_1 \vdash F_3}{\bullet h_2 : !\top 4, !F_1 \vdash !F_3} !R \rightarrow \frac{\overline{h_2 : !\top 4, !F_1, !F_1 \vdash F_3}}{\bullet h_2 : !\top 4, !F_1, !F_1 \vdash !F_3} !R \text{ ax/ind}$$

- Case rule 1_R

- Case rule \top

$$\overline{\bullet h_2 : \Delta_3, !F_1 \vdash \top} \top \rightarrow \overline{\bullet h_2 : \Delta_3, !F_1, !F_1 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_3 \quad h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \& F_4} \&_R \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1 \vdash F_3} \text{ ax/ind} \quad \overline{h_2 : \Delta_5, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1 \vdash F_3 \& F_4} \&_R$$

- Case rule $\neg o_R$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \neg o F_4} \neg o_R \rightarrow \frac{\overline{h_2 : \Delta_5, F_3, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1 \vdash F_3 \neg o F_4} \neg o_R$$

- Case rule \oplus_{R_2}

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1 \vdash F_3 \oplus F_4} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_3}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1 \vdash F_3} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1 \vdash F_3 \oplus F_4} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{h_2 : \Delta_4, !F_1 \vdash F_3}{\bullet h_2 : 1, \Delta_4, !F_1 \vdash F_3} 1_L \rightarrow \frac{\overline{h_2 : \Delta_4, !F_1, !F_1 \vdash F_3} \text{ ax/ind}}{\bullet h_2 : 1, \Delta_4, !F_1, !F_1 \vdash F_3} 1_L$$

- Case rule \otimes_R

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_3 \quad h_2 : \Delta_6 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), \Delta_6 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1 \vdash F_3} \text{ ax/ind} \quad \overline{h_2 : \Delta_6 \vdash F_4} \text{ ax}}{\bullet h_2 : \Delta_5, \Delta_6, !F_1, !F_1 \vdash F_3 \otimes F_4} \otimes_R$$

$$\frac{h_2 : \Delta_5 \vdash F_3 \quad h_2 : \Delta_6, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, \Delta_6, !F_1 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_5 \vdash F_3} \text{ ax} \quad \overline{h_2 : \Delta_6, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, \Delta_6, !F_1, !F_1 \vdash F_3 \otimes F_4} \otimes_R$$

- Case rule W

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1, !F_3 \vdash F_4} W$$

$$\frac{h_1 : \Delta_4 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} W \rightarrow \frac{}{\bullet h_1 : \Delta_4, !F_2, !F_2 \vdash F_3} \text{ fail}$$

- Case rule C

$$\frac{h_2 : \Delta_5, !F_1, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1, !F_3 \vdash F_4} C$$

$$\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} C \rightarrow \frac{\overline{h_1 : \Delta_4, !F_2, !F_2, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2, !F_2 \vdash F_3} C$$

- Case rule $!L$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_2 : \Delta_5, F_3, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1, !F_3 \vdash F_4} !L$$

$$\frac{h_1 : \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} !L \rightarrow \frac{}{\bullet h_1 : \Delta_4, !F_2, !F_2 \vdash F_3} \text{ fail}$$

- Case rule $\&_{L2}$

$$\frac{h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_6, F_4, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_2 : \Delta_6, F_3, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, F_4, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, !F_1, !F_1 \vdash F_5} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_4, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_2 : \Delta_6, !F_1 \vdash F_3 \quad h_2 : \Delta_7, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), \Delta_7, F_3 \multimap F_4 \vdash F_5} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_6, !F_1, !F_1 \vdash F_3} \text{ ax/ind} \quad \overline{h_2 : \Delta_7, F_4 \vdash F_5} \text{ ax}}{\bullet h_2 : \Delta_6, \Delta_7, !F_1, !F_1, F_3 \multimap F_4 \vdash F_5} \multimap_L$$

$$\frac{h_2 : \Delta_6 \vdash F_3 \quad h_2 : \Delta_7, F_4, !F_1 \vdash F_5}{\bullet h_2 : \Delta_6, (\Delta_7, !F_1), F_3 \multimap F_4 \vdash F_5} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_6 \vdash F_3} \text{ ax} \quad \overline{h_2 : \Delta_7, F_4, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, \Delta_7, !F_1, !F_1, F_3 \multimap F_4 \vdash F_5} \multimap_L$$

- Case rule I

2.14 Status of $!L$: : Non invertible

- Case rule $!R$

$$\frac{h_2 : !\top_4, !F_1 \vdash F_3}{\bullet h_2 : !\top_4, !F_1 \vdash !F_3} !R \rightarrow \frac{}{\bullet h_2 : F_1, !\top_4 \vdash !F_3} \text{fail}$$

- Case rule 1_R

- Case rule \top

$$\frac{}{\bullet h_2 : \Delta_3, !F_1 \vdash \top} \top \rightarrow \frac{}{\bullet h_2 : \Delta_3, F_1 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_2 : \Delta_5, !F_1 \vdash F_3}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \& F_4} \&_R \quad \frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \& F_4} \&_R}{\bullet h_2 : \Delta_5, F_1 \vdash F_3 \& F_4} \text{ax/ind} \rightarrow \frac{\frac{}{h_2 : \Delta_5, F_1 \vdash F_3} \text{ax/ind} \quad \frac{}{h_2 : \Delta_5, F_1 \vdash F_4} \text{ax/ind}}{\bullet h_2 : \Delta_5, F_1 \vdash F_3 \& F_4} \&_R$$

- Case rule $\neg\circ_R$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \neg\circ F_4} \neg\circ_R \rightarrow \frac{\frac{}{h_2 : \Delta_5, F_1, F_3 \vdash F_4} \text{ax/ind}}{\bullet h_2 : \Delta_5, F_1 \vdash F_3 \neg\circ F_4} \neg\circ_R$$

- Case rule \oplus_{R_2}

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{\frac{}{h_2 : \Delta_5, F_1 \vdash F_4} \text{ax/ind}}{\bullet h_2 : \Delta_5, F_1 \vdash F_3 \oplus F_4} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_3}{\bullet h_2 : \Delta_5, !F_1 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{\frac{}{h_2 : \Delta_5, F_1 \vdash F_3} \text{ax/ind}}{\bullet h_2 : \Delta_5, F_1 \vdash F_3 \oplus F_4} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{h_2 : \Delta_4, !F_1 \vdash F_3}{\bullet h_2 : 1, \Delta_4, !F_1 \vdash F_3} 1_L \rightarrow \frac{\frac{}{h_2 : \Delta_4, F_1 \vdash F_3} \text{ax/ind}}{\bullet h_2 : 1, \Delta_4, F_1 \vdash F_3} 1_L$$

- Case rule \otimes_R

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_3 \quad h_2 : \Delta_6 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), \Delta_6 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\frac{}{h_2 : \Delta_5, F_1 \vdash F_3} \text{ax/ind} \quad \frac{}{h_2 : \Delta_6 \vdash F_4} \text{ax}}{\bullet h_2 : \Delta_5, \Delta_6, F_1 \vdash F_3 \otimes F_4} \otimes_R$$

$$\frac{h_2 : \Delta_5 \vdash F_3 \quad h_2 : \Delta_6, !F_1 \vdash F_4}{\bullet h_2 : \Delta_5, \Delta_6, !F_1 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\frac{}{h_2 : \Delta_5 \vdash F_3} \text{ax} \quad \frac{}{h_2 : \Delta_6, F_1 \vdash F_4} \text{ax/ind}}{\bullet h_2 : \Delta_5, \Delta_6, F_1 \vdash F_3 \otimes F_4} \otimes_R$$

- Case rule W

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_2 : \Delta_5, F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, F_1, !F_3 \vdash F_4} W$$

$$\frac{h_1 : \Delta_4 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} W \rightarrow \frac{\bullet h_1 : \Delta_4, F_2 \vdash F_3 \text{ fail}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} W$$

- Case rule C

$$\frac{h_2 : \Delta_5, !F_1, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_2 : \Delta_5, F_1, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, F_1, !F_3 \vdash F_4} C$$

$$\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} C \rightarrow \frac{\bullet h_1 : \Delta_4, F_2 \vdash F_3 \text{ fail}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} C$$

- Case rule $!L$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_2 : \Delta_5, F_1, F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, F_1, !F_3 \vdash F_4} !L$$

$$\frac{h_1 : \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} !L \rightarrow \frac{\overline{h_1 : \Delta_4, F_2 \vdash F_3} \text{ ax}}{\bullet h_1 : \Delta_4, F_2 \vdash F_3} H$$

- Case rule $\&_{L2}$

$$\frac{h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_2 : \Delta_6, F_3, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5 \quad h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3 \vdash F_5} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_1, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_2 : \Delta_6, !F_1 \vdash F_3 \quad h_2 : \Delta_7, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), \Delta_7, F_3 \multimap F_4 \vdash F_5} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_1 \vdash F_3} \text{ ax/ind} \quad \overline{h_2 : \Delta_7, F_4 \vdash F_5} \text{ ax}}{\bullet h_2 : \Delta_6, \Delta_7, F_1, F_3 \multimap F_4 \vdash F_5} \multimap_L$$

$$\frac{h_2 : \Delta_6 \vdash F_3 \quad h_2 : \Delta_7, F_4, !F_1 \vdash F_5}{\bullet h_2 : \Delta_6, (\Delta_7, !F_1), F_3 \multimap F_4 \vdash F_5} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_6 \vdash F_3} \text{ ax} \quad \overline{h_2 : \Delta_7, F_1, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, \Delta_7, F_1, F_3 \multimap F_4 \vdash F_5} \multimap_L$$

- Case rule I

2.15 Status of $\&_{L2}$: : Non invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \& F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \& F_5} \&_R}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \& F_5} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_6, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \& F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_6, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \& F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \& F_5} \&_R$$

- Case rule $\neg o_R$

$$\frac{\frac{h_3 : \Delta_6, F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \neg o F_5} \neg o_R}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \neg o F_5} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_6, F_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \neg o F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \neg o F_5} \neg o_R$$

- Case rule \oplus_{R2}

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \oplus F_5} \oplus_{R2}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_6, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \oplus_{R2}$$

- Case rule \oplus_{R1}

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \oplus F_5} \oplus_{R1}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_6, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \oplus_{R1}$$

- Case rule 1_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \& F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \& F_2 \vdash F_4} 1_L}{\bullet h_3 : 1, \Delta_5, F_2 \vdash F_4} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : 1, \Delta_5, F_2 \vdash F_4} 1_L$$

- Case rule \otimes_R

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \quad h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), \Delta_7 \vdash F_4 \otimes F_5} \otimes_R}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \otimes F_5} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_6, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \text{ax}}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \otimes_R$$

$$\frac{\frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \Delta_7, F_1 \& F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \& F_2 \vdash F_4 \otimes F_5} \otimes_R}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_6 \vdash F_4}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \text{ax} \quad \frac{h_3 : \Delta_7, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \otimes_R$$

- Case rule W

$$\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} W \rightarrow \frac{\overline{h_3 : \Delta_6, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} W$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, !F_4, !F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \&_{L2}$$

$$\frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L2} \rightarrow \frac{\overline{h_1 : \Delta_5, F_3 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_5, F_3 \vdash F_4} H$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \&_{L1}$$

$$\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L1} \rightarrow \frac{\overline{\bullet h_1 : \Delta_5, F_3 \vdash F_4} \text{ fail}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \otimes F_5 \vdash F_6} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \oplus F_5 \vdash F_6} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_7, F_1 \& F_2 \vdash F_4 \quad h_3 : \Delta_8, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), \Delta_8, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_8, F_5 \vdash F_6} \text{ ax}}{\bullet h_3 : \Delta_7, \Delta_8, F_2, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

$$\frac{h_3 : \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : \Delta_7, (\Delta_8, F_1 \& F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_4} \text{ ax} \quad \overline{h_3 : \Delta_8, F_2, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, \Delta_8, F_2, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

- Case rule I

2.16 Status of $\&_{L1}$: Non invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \& F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \& F_5} \&_R}{\frac{h_3 : \Delta_6, F_1 \vdash F_4 \quad \text{ax/ind} \quad \frac{h_3 : \Delta_6, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \& F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \& F_5} \&_R} \rightarrow$$

- Case rule $\neg o_R$

$$\frac{\frac{h_3 : \Delta_6, F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \neg o F_5} \neg o_R}{\frac{h_3 : \Delta_6, F_1 \vdash F_4 \quad \text{ax/ind} \quad \frac{h_3 : \Delta_6, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \neg o F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \neg o F_5} \neg o_R} \rightarrow$$

- Case rule \oplus_{R2}

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \oplus F_5} \oplus_{R2}}{\frac{h_3 : \Delta_6, F_1 \vdash F_5 \quad \text{ax/ind} \quad \frac{h_3 : \Delta_6, F_1 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \oplus F_5} \oplus_{R2}}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \oplus F_5} \oplus_{R2}} \rightarrow$$

- Case rule \oplus_{R1}

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \oplus F_5} \oplus_{R1}}{\frac{h_3 : \Delta_6, F_1 \vdash F_4 \quad \text{ax/ind} \quad \frac{h_3 : \Delta_6, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \oplus F_5} \oplus_{R1}}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \oplus F_5} \oplus_{R1}} \rightarrow$$

- Case rule 1_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \& F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \& F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_1 \vdash F_4 \quad \text{ax/ind} \quad \frac{h_3 : \Delta_5, F_1 \vdash F_5}{\bullet h_3 : 1, \Delta_5, F_1 \vdash F_4} 1_L}} \rightarrow$$

- Case rule \otimes_R

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_4 \quad h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), \Delta_7 \vdash F_4 \otimes F_5} \otimes_R}{\frac{h_3 : \Delta_6, F_1 \vdash F_4 \quad \text{ax/ind} \quad \frac{h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \vdash F_4 \otimes F_5} \text{ax}}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \vdash F_4 \otimes F_5} \otimes_R} \rightarrow$$

$$\frac{\frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \Delta_7, F_1 \& F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \& F_2 \vdash F_4 \otimes F_5} \otimes_R}{\frac{h_3 : \Delta_6 \vdash F_4 \quad \text{ax} \quad \frac{h_3 : \Delta_7, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \vdash F_4 \otimes F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \vdash F_4 \otimes F_5} \otimes_R} \rightarrow$$

- Case rule W

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_1 \vdash F_5 \quad \text{ax/ind} \quad \frac{h_3 : \Delta_6, F_1 \vdash F_4}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} W}} \rightarrow$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, !F_4, !F_4 \vdash F_5}^{ax/ind}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4 \vdash F_5}^{ax/ind}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \&_{L2}$$

$$\frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L2} \rightarrow \frac{\overline{\bullet h_1 : \Delta_5, F_2 \vdash F_4}^{fail}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \&_{L1}$$

$$\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L1} \rightarrow \frac{\overline{h_1 : \Delta_5, F_2 \vdash F_4}^{ax}}{\bullet h_1 : \Delta_5, F_2 \vdash F_4} H$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash F_6}^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \otimes F_5 \vdash F_6} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}^{ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}^{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \oplus F_5 \vdash F_6} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_7, F_1 \& F_2 \vdash F_4 \quad h_3 : \Delta_8, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), \Delta_8, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1 \vdash F_4}^{ax/ind} \quad \overline{h_3 : \Delta_8, F_5 \vdash F_6}^{ax}}{\bullet h_3 : \Delta_7, \Delta_8, F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

$$\frac{h_3 : \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : \Delta_7, (\Delta_8, F_1 \& F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_4}^{ax} \quad \overline{h_3 : \Delta_8, F_1, F_5 \vdash F_6}^{ax/ind}}{\bullet h_3 : \Delta_7, \Delta_8, F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

- Case rule I

2.17 Status of \otimes_L : : Invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \otimes F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1, F_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_4 \& F_5} \&_R}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \& F_5} \&_R} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4} \text{ax/ind} \quad \frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \& F_5} \&_R$$

- Case rule $\neg\circ_R$

$$\frac{\frac{h_3 : \Delta_6, F_4, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R} \rightarrow$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}} \rightarrow$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}} \rightarrow$$

- Case rule 1_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \otimes F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \otimes F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1, F_2 \vdash F_4} 1_L} \rightarrow \frac{\frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1, F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1, F_2 \vdash F_4} 1_L} \rightarrow$$

- Case rule \otimes_R

$$\frac{\frac{h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_4 \quad h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), \Delta_7 \vdash F_4 \otimes F_5} \otimes_R}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4 \quad h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1, F_2 \vdash F_4 \otimes F_5} \otimes_R} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4} \text{ax/ind} \quad \frac{h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : \Delta_7 \vdash F_5} \text{ax}}{\bullet h_3 : \Delta_6, \Delta_7, F_1, F_2 \vdash F_4 \otimes F_5} \otimes_R$$

$$\frac{\frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \Delta_7, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \otimes F_2 \vdash F_4 \otimes F_5} \otimes_R}{\frac{h_3 : \Delta_6 \vdash F_4}{\bullet h_3 : \Delta_6 \vdash F_4} \text{ax} \quad \frac{h_3 : \Delta_7, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, \Delta_7, F_1, F_2 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{h_3 : \Delta_6 \vdash F_4}{\bullet h_3 : \Delta_6 \vdash F_4} \text{ax} \quad \frac{h_3 : \Delta_7, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, \Delta_7, F_1, F_2 \vdash F_4 \otimes F_5} \otimes_R$$

- Case rule W

$$\frac{\frac{h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_5} \text{ax/ind}} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_5} \text{ax/ind}}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_5} \text{ax/ind}} W$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_2, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} \frac{ax/ind}{C}$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_2, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} \frac{ax/ind}{!L}$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \& F_5 \vdash F_6} \frac{ax/ind}{\&_{L2}}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \& F_5 \vdash F_6} \frac{ax/ind}{\&_{L1}}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \otimes F_5 \vdash F_6} \frac{ax/ind}{\otimes_L}$$

$$\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_4} \otimes_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2, F_3 \vdash F_4}}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash F_4} \frac{ax}{H}$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \otimes F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_2, F_4 \vdash F_6} \quad \overline{h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \oplus F_5 \vdash F_6} \frac{ax/ind}{\oplus_L}$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_7, F_1 \otimes F_2 \vdash F_4 \quad h_3 : \Delta_8, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), \Delta_8, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_2 \vdash F_4} \quad \overline{h_3 : \Delta_8, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, \Delta_8, F_1, F_2, F_4 \multimap F_5 \vdash F_6} \frac{ax/ind}{\multimap_L}$$

$$\frac{h_3 : \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, F_5, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : \Delta_7, (\Delta_8, F_1 \otimes F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_4} \quad \overline{h_3 : \Delta_8, F_1, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, \Delta_8, F_1, F_2, F_4 \multimap F_5 \vdash F_6} \frac{ax/ind}{\multimap_L}$$

- Case rule I

2.18 Status of \oplus_L : (Left Premise): Invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \oplus F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \& F_5} \&_R}{\frac{h_3 : \Delta_6, F_1 \vdash F_4 \quad \text{ax/ind} \quad h_3 : \Delta_6, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \& F_5} \&_R} \text{ax/ind}$$

- Case rule $\neg o_R$

$$\frac{\frac{h_3 : \Delta_6, F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \neg o F_5} \neg o_R}{\frac{h_3 : \Delta_6, F_1 \vdash F_4 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \neg o F_5} \neg o_R} \text{ax/ind}$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}}{\frac{h_3 : \Delta_6, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \oplus F_5} \oplus_{R_2}} \text{ax/ind}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}}{\frac{h_3 : \Delta_6, F_1 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \oplus F_5} \oplus_{R_1}} \text{ax/ind}$$

- Case rule 1_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \oplus F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \oplus F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_1 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \vdash F_4} 1_L} \text{ax/ind}$$

- Case rule \otimes_R

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \quad h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), \Delta_7 \vdash F_4 \otimes F_5} \otimes_R}{\frac{h_3 : \Delta_6, F_1 \vdash F_4 \quad \text{ax/ind} \quad h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \vdash F_4 \otimes F_5} \otimes_R} \text{ax}$$

$$\frac{\frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \oplus F_2 \vdash F_4 \otimes F_5} \otimes_R}{\frac{h_3 : \Delta_6 \vdash F_4 \quad \text{ax} \quad h_3 : \Delta_7, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \vdash F_4 \otimes F_5} \otimes_R} \text{ax/ind}$$

- Case rule W

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} W} \text{ax/ind}$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \frac{ax/ind}{C}$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \frac{ax/ind}{!L}$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \frac{ax/ind}{\&_{L2}}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \frac{ax/ind}{\&_{L1}}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \otimes F_5 \vdash F_6} \frac{ax/ind}{\otimes_L}$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \oplus F_5 \vdash F_6} \frac{ax/ind}{\oplus_L} \frac{\overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}}{\oplus_L} \frac{ax/ind}{\oplus_L}$$

$$\frac{h_1 : \Delta_5, F_2 \vdash F_4 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \oplus_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2 \vdash F_4}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \frac{ax}{H}$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_4 \quad h_3 : \Delta_8, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), \Delta_8, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1 \vdash F_4}}{\bullet h_3 : \Delta_7, \Delta_8, F_1, F_4 \multimap F_5 \vdash F_6} \frac{ax/ind}{\multimap_L} \frac{\overline{h_3 : \Delta_8, F_5 \vdash F_6}}{\multimap_L} \frac{ax}{\multimap_L}$$

$$\frac{h_3 : \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : \Delta_7, (\Delta_8, F_1 \oplus F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_4}}{\bullet h_3 : \Delta_7, \Delta_8, F_1, F_4 \multimap F_5 \vdash F_6} \frac{ax}{\multimap_L} \frac{\overline{h_3 : \Delta_8, F_1, F_5 \vdash F_6}}{\multimap_L} \frac{ax/ind}{\multimap_L}$$

- Case rule I

2.19 Status of \oplus_L (Right Premise): : Invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \oplus F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \& F_5} \&_R}{\frac{h_3 : \Delta_6, F_2 \vdash F_4 \quad ax/ind \quad h_3 : \Delta_6, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \& F_5} \&_R} \rightarrow \frac{ax/ind}{\&_R}$$

- Case rule $\neg o_R$

$$\frac{h_3 : \Delta_6, F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \neg o F_5} \neg o_R \rightarrow \frac{\frac{h_3 : \Delta_6, F_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \neg o F_5} \neg o_R}{\frac{h_3 : \Delta_6, F_2 \vdash F_4 \vdash F_5}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \neg o F_5} \neg o_R} \rightarrow \frac{ax/ind}{\neg o_R}$$

- Case rule \oplus_{R_2}

$$\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \oplus F_5} \oplus_{R_2} \rightarrow \frac{\frac{h_3 : \Delta_6, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}}{\frac{h_3 : \Delta_6, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}} \rightarrow \frac{ax/ind}{\oplus_{R_2}}$$

- Case rule \oplus_{R_1}

$$\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \oplus F_5} \oplus_{R_1} \rightarrow \frac{\frac{h_3 : \Delta_6, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}}{\frac{h_3 : \Delta_6, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}} \rightarrow \frac{ax/ind}{\oplus_{R_1}}$$

- Case rule 1_L

$$\frac{h_3 : \Delta_5, F_1 \oplus F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \oplus F_2 \vdash F_4} 1_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_2 \vdash F_4} 1_L} \rightarrow \frac{ax/ind}{1_L}$$

- Case rule \otimes_R

$$\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_4 \quad h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), \Delta_7 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{h_3 : \Delta_6, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \otimes_R}{\frac{h_3 : \Delta_6, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \otimes_R} \rightarrow \frac{ax/ind}{\otimes_R}$$

$$\frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \oplus F_2 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{h_3 : \Delta_6 \vdash F_4}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \otimes_R}{\frac{h_3 : \Delta_6 \vdash F_4}{\bullet h_3 : \Delta_6, \Delta_7, F_2 \vdash F_4 \otimes F_5} \otimes_R} \rightarrow \frac{ax}{\otimes_R}$$

- Case rule W

$$\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} W \rightarrow \frac{\frac{h_3 : \Delta_6, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} W} \rightarrow \frac{ax/ind}{W}$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} \text{ax/ind} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} \text{ax/ind} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \otimes F_5 \vdash F_6} \text{ax/ind} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \text{ax/ind} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \oplus F_5 \vdash F_6} \oplus_L$$

$$\frac{h_1 : \Delta_5, F_2 \vdash F_4 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \oplus_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_3 \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_5, F_3 \vdash F_4} H$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_4 \quad h_3 : \Delta_8, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), \Delta_8, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_4} \text{ax/ind} \quad \overline{h_3 : \Delta_8, F_5 \vdash F_6} \text{ax}}{\bullet h_3 : \Delta_7, \Delta_8, F_2, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

$$\frac{h_3 : \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : \Delta_7, (\Delta_8, F_1 \oplus F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_4} \text{ax} \quad \overline{h_3 : \Delta_8, F_2, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, \Delta_8, F_2, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

- Case rule I

2.20 Status of \neg_L : (Left Premise): Non invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_3 : \Delta_4, \Delta_5, F_1 \neg F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_4 \vdash F_1} \text{fail}$$

- Case rule $\&_R$

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \neg F_2 \vdash F_4 \quad h_3 : \Delta_6, \Delta_7, F_1 \neg F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \neg F_2 \vdash F_4 \& F_5} \&_R \rightarrow \frac{h_3 : \Delta_6 \vdash F_1}{\bullet h_3 : \Delta_6 \vdash F_1} \frac{\text{ax/ind}}{H}$$

- Case rule \neg_R

$$\frac{h_3 : \Delta_6, \Delta_7, F_4, F_1 \neg F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \neg F_2 \vdash F_4 \neg F_5} \neg_R \rightarrow \frac{h_3 : \Delta_6 \vdash F_1}{\bullet h_3 : \Delta_6 \vdash F_1} \frac{\text{ax/ind}}{H}$$

- Case rule \oplus_{R_2}

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \neg F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \neg F_2 \vdash F_4 \oplus F_5} \oplus_{R_2} \rightarrow \frac{h_3 : \Delta_6 \vdash F_1}{\bullet h_3 : \Delta_6 \vdash F_1} \frac{\text{ax/ind}}{H}$$

- Case rule \oplus_{R_1}

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \neg F_2 \vdash F_4}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \neg F_2 \vdash F_4 \oplus F_5} \oplus_{R_1} \rightarrow \frac{h_3 : \Delta_6 \vdash F_1}{\bullet h_3 : \Delta_6 \vdash F_1} \frac{\text{ax/ind}}{H}$$

- Case rule 1_L

$$\frac{h_3 : \Delta_5, \Delta_6, F_1 \neg F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, \Delta_6, F_1 \neg F_2 \vdash F_4} 1_L \rightarrow \frac{h_3 : \Delta_5 \vdash F_1}{\bullet h_3 : 1, \Delta_5 \vdash F_1} \frac{\text{ax/ind}}{1_L}$$

$$\frac{h_3 : \Delta_5, \Delta_6, F_1 \neg F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, \Delta_6, F_1 \neg F_2 \vdash F_4} 1_L \rightarrow \frac{h_3 : \Delta_5 \vdash F_1}{\bullet h_3 : \Delta_5 \vdash F_1} \frac{\text{ax/ind}}{H}$$

- Case rule \otimes_R

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \neg F_2 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \neg F_2), \Delta_8, \Delta_9 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_1 \neg F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7), \Delta_8, \Delta_9, F_1 \neg F_2 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_1} \text{fail}$$

- Case rule W

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} W \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, !F_4 \vdash F_1} W$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} W \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6 \vdash F_1} H$$

- Case rule C

$$\frac{h_3 : \Delta_6, \Delta_7, !F_4, !F_4, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, !F_4 \vdash F_1} W$$

$$\frac{h_3 : \Delta_6, \Delta_7, !F_4, !F_4, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6 \vdash F_1} H$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, !F_4 \vdash F_1} W$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6 \vdash F_1} H$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, \Delta_8, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \& F_5 \vdash F_1} \&_{L2}$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7 \vdash F_1} H$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \& F_5 \vdash F_1} \&_{L1}$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7 \vdash F_1} H$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \otimes F_5 \vdash F_1} \otimes_L$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7 \vdash F_1} H$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_1 \multimap F_2 \vdash F_6 \quad h_3 : \Delta_7, \Delta_8, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_4 \vdash F_1}{\bullet h_3 : \Delta_7, F_4 \oplus F_5 \vdash F_1} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash F_1}{\bullet h_3 : \Delta_7, F_4 \oplus F_5 \vdash F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \oplus F_5 \vdash F_1} \oplus_L$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_1 \multimap F_2 \vdash F_6 \quad h_3 : \Delta_7, \Delta_8, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\frac{h_3 : \Delta_7 \vdash F_1}{\bullet h_3 : \Delta_7 \vdash F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7 \vdash F_1} \text{H}$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_4 \multimap F_5 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_4 \multimap F_5 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9 \vdash F_1} \text{fail}$$

$$\frac{h_1 : \Delta_5, \Delta_6 \vdash F_2 \quad h_1 : \Delta_7, \Delta_8, F_3 \vdash F_4}{\bullet h_1 : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_2 \multimap F_3 \vdash F_4} \multimap_L \rightarrow \frac{}{\bullet h_1 : \Delta_5, \Delta_7 \vdash F_2} \text{fail}$$

- Case rule I

2.21 Status of \multimap_L (Right Premise): : Non invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_3 : \Delta_4, \Delta_5, F_1 \multimap F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_4 \& F_5} \&_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash F_4}{\bullet h_3 : \Delta_7, F_2 \vdash F_4 \& F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2 \vdash F_5}{\bullet h_3 : \Delta_7, F_2 \vdash F_4 \& F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash F_4 \& F_5} \&_R$$

- Case rule \multimap_R

$$\frac{h_3 : \Delta_6, \Delta_7, F_4, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_4 \multimap F_5} \multimap_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_7, F_2 \vdash F_4 \multimap F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash F_4 \multimap F_5} \multimap_R$$

- Case rule \oplus_{R_2}

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_4 \oplus F_5} \oplus_{R_2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_4}{\bullet h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_4 \oplus F_5} \oplus_{R_1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{h_3 : \Delta_5, \Delta_6, F_1 \multimap F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, \Delta_6, F_1 \multimap F_2 \vdash F_4} 1_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_2 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4} H$$

$$\frac{h_3 : \Delta_5, \Delta_6, F_1 \multimap F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, \Delta_6, F_1 \multimap F_2 \vdash F_4} 1_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_2 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : 1, \Delta_6, F_2 \vdash F_4} 1_L$$

- Case rule \otimes_R

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), \Delta_8, \Delta_9 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_2 \vdash F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7), \Delta_8, \Delta_9, F_1 \multimap F_2 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_2 \vdash F_4 \otimes F_5} \text{ fail}$$

- Case rule W

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} W \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash F_5} H$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} W \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, !F_4 \vdash F_5} W$$

- Case rule C

$$\frac{h_3 : \Delta_6, \Delta_7, !F_4, !F_4, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash F_5} H$$

$$\frac{h_3 : \Delta_6, \Delta_7, !F_4, !F_4, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, !F_4 \vdash F_5} W$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash F_5} H$$

$$\frac{h_3 : \Delta_6, \Delta_7, F_4, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_2, !F_4 \vdash F_5} W$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, \Delta_8, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\frac{h_3 : \Delta_8, F_2 \vdash F_6}{\bullet h_3 : \Delta_8, F_2 \vdash F_6} \text{ax/ind}}{H} \&_{L2}$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\frac{h_3 : \Delta_8, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_8, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind}}{\&_{L2}}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\frac{h_3 : \Delta_8, F_2 \vdash F_6}{\bullet h_3 : \Delta_8, F_2 \vdash F_6} \text{ax/ind}}{H} \&_{L1}$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\frac{h_3 : \Delta_8, F_2, F_4 \vdash F_6}{\bullet h_3 : \Delta_8, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind}}{\&_{L1}}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\frac{h_3 : \Delta_8, F_2 \vdash F_6}{\bullet h_3 : \Delta_8, F_2 \vdash F_6} \text{ax/ind}}{H} \otimes_L$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\frac{h_3 : \Delta_8, F_2, F_4, F_5 \vdash F_6}{\bullet h_3 : \Delta_8, F_2, F_4 \otimes F_5 \vdash F_6} \text{ax/ind}}{\otimes_L}$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_1 \multimap F_2 \vdash F_6 \quad h_3 : \Delta_7, \Delta_8, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\frac{h_3 : \Delta_8, F_2 \vdash F_6}{\bullet h_3 : \Delta_8, F_2 \vdash F_6} \text{ax/ind}}{H} \oplus_L$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_4, F_1 \multimap F_2 \vdash F_6 \quad h_3 : \Delta_7, \Delta_8, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\frac{h_3 : \Delta_8, F_2, F_4 \vdash F_6}{\bullet h_3 : \Delta_8, F_2, F_4 \oplus F_5 \vdash F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_8, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_8, F_2, F_4 \oplus F_5 \vdash F_6} \text{ax/ind}}{\oplus_L}$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\bullet h_3 : \Delta_8, \Delta_{10}, F_2 \vdash F_6}{\text{fail}}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\bullet h_3 : \Delta_8, \Delta_{10}, F_2 \vdash F_6}{\text{fail}}$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\bullet h_3 : \Delta_8, \Delta_{10}, F_2, F_4 \multimap F_5 \vdash F_6}{\text{fail}}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\bullet h_3 : \Delta_8, \Delta_{10}, F_2, F_4 \multimap F_5 \vdash F_6}{\text{fail}}$$

$$\frac{h_1 : \Delta_5, \Delta_6 \vdash F_2 \quad h_1 : \Delta_7, \Delta_8, F_3 \vdash F_4}{\bullet h_1 : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_2 \multimap F_3 \vdash F_4} \multimap_L \rightarrow \frac{\bullet h_1 : \Delta_6, \Delta_8, F_3 \vdash F_4}{\text{fail}}$$

- Case rule I

2.22 Status of I : : Invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- Case rule I

$$\frac{}{\bullet h_1 : p(n_2) \vdash p(n_2)} I \quad \rightarrow \quad \text{trivial}$$

3 Identity-Expansion

$$\begin{array}{c}
\overline{- : \top \vdash \top} \top \\
\\
\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0 \oplus F_1} \oplus_{R_1} \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0 \oplus F_1} \oplus_{R_2} \\
\hline
- : F_0 \oplus F_1 \vdash F_0 \oplus F_1 \quad \oplus_L \\
\\
\frac{\overline{- : F_0 \vdash F_0} \text{ IH} \quad \overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_0 \multimap F_1 \vdash F_1} \multimap_L \\
\hline
- : F_0 \multimap F_1 \vdash F_0 \multimap F_1 \quad \multimap_R \\
\\
\frac{\overline{- : F_0 \vdash F_0} \text{ IH} \quad \overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_0 \otimes F_1} \otimes_R \\
\hline
- : F_0 \otimes F_1 \vdash F_0 \otimes F_1 \quad \otimes_L \\
\\
\frac{\overline{- : F_0 \vdash F_0} \text{ IH} \quad \overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0 \& F_1 \vdash F_0} \&_{L1} \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0 \& F_1 \vdash F_1} \&_{L2} \\
\hline
- : F_0 \& F_1 \vdash F_0 \& F_1 \quad \&_R \\
\\
\frac{\overline{- : * \vdash \mathbf{1}} \mathbf{1}_R}{- : \mathbf{1} \vdash \mathbf{1}} \mathbf{1}_L
\end{array}$$

4 Weakening on bang: $\Gamma \vdash !F$ implies $\Gamma \vdash F$.

- Case(s) rule $!R$

$$\frac{h_1 : !\top \vdash F_3}{\bullet h_1 : !\top \vdash !F_3} !R \rightarrow \frac{\overline{h_1 : !\top \vdash F_3}^{\text{ax}}}{\bullet h_1 : !\top \vdash F_3} \text{H}$$

- Case(s) rule $\mathbf{1}_R$
- Case(s) rule \top
- Case(s) rule $\&_R$
- Case(s) rule $\neg\circ_R$
- Case(s) rule \oplus_{R_2}
- Case(s) rule \oplus_{R_1}
- Case(s) rule $\mathbf{1}_L$

$$\frac{h_2 : \Delta_3 \vdash !F_1}{\bullet h_2 : \mathbf{1}, \Delta_3 \vdash !F_1} \mathbf{1}_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \mathbf{1}, \Delta_3 \vdash F_1} \text{IH} \mathbf{1}_L$$

- Case(s) rule \otimes_R
- Case(s) rule W

$$\frac{h_2 : \Delta_4 \vdash !F_1}{\bullet h_2 : \Delta_4, !F_3 \vdash !F_1} W \rightarrow \frac{\overline{h_2 : \Delta_4 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_4, !F_3 \vdash F_1} \text{IH} W$$

- Case(s) rule C

$$\frac{h_2 : \Delta_4, !F_3, !F_3 \vdash !F_1}{\bullet h_2 : \Delta_4, !F_3 \vdash !F_1} C \rightarrow \frac{\overline{h_2 : \Delta_4, !F_3, !F_3 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_4, !F_3 \vdash F_1} \text{IH} C$$

- Case(s) rule $!L$

$$\frac{h_2 : \Delta_4, F_3 \vdash !F_1}{\bullet h_2 : \Delta_4, !F_3 \vdash !F_1} !L \rightarrow \frac{\overline{h_2 : \Delta_4, F_3 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_4, !F_3 \vdash F_1} \text{IH} !L$$

- Case(s) rule $\&_{L2}$

$$\frac{h_2 : \Delta_5, F_4 \vdash !F_1}{\bullet h_2 : \Delta_5, F_3 \& F_4 \vdash !F_1} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_5, F_4 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_5, F_3 \& F_4 \vdash F_1} \text{IH} \&_{L2}$$

- Case(s) rule $\&_{L1}$

$$\frac{h_2 : \Delta_5, F_3 \vdash !F_1}{\bullet h_2 : \Delta_5, F_3 \& F_4 \vdash !F_1} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_5, F_3 \vdash !F_1}^{\text{ax}}}{\bullet h_2 : \Delta_5, F_3 \& F_4 \vdash F_1} \text{IH} \&_{L1}$$

- Case(s) rule \otimes_L

$$\frac{h_2 : \Delta_5, F_3, F_4 \vdash !F_1}{\bullet h_2 : \Delta_5, F_3 \otimes F_4 \vdash !F_1} \otimes_L \rightarrow \frac{\frac{h_2 : \Delta_5, F_3, F_4 \vdash !F_1}{h_2 : \Delta_5, F_3, F_4 \vdash F_1} \text{IH} \quad \frac{}{h_2 : \Delta_5, F_3, F_4 \vdash !F_1} \text{ax}}{\bullet h_2 : \Delta_5, F_3 \otimes F_4 \vdash F_1} \otimes_L$$

- Case(s) rule \oplus_L

$$\frac{h_2 : \Delta_5, F_3 \vdash !F_1 \quad h_2 : \Delta_5, F_4 \vdash !F_1}{\bullet h_2 : \Delta_5, F_3 \oplus F_4 \vdash !F_1} \oplus_L \rightarrow \frac{\frac{h_2 : \Delta_5, F_3 \vdash !F_1}{h_2 : \Delta_5, F_3 \vdash F_1} \text{IH} \quad \frac{h_2 : \Delta_5, F_4 \vdash !F_1}{h_2 : \Delta_5, F_4 \vdash F_1} \text{IH} \quad \frac{}{h_2 : \Delta_5, F_3 \oplus F_4 \vdash !F_1} \text{ax}}{\bullet h_2 : \Delta_5, F_3 \oplus F_4 \vdash F_1} \oplus_L$$

- Case(s) rule $\neg\circ_L$

$$\frac{h_2 : \Delta_5 \vdash F_3 \quad h_2 : \Delta_6, F_4 \vdash !F_1}{\bullet h_2 : \Delta_5, \Delta_6, F_3 \neg\circ F_4 \vdash !F_1} \neg\circ_L \rightarrow \frac{\frac{h_2 : \Delta_5 \vdash F_3}{h_2 : \Delta_5 \vdash F_3} \text{ax} \quad \frac{h_2 : \Delta_6, F_4 \vdash !F_1}{h_2 : \Delta_6, F_4 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_5, \Delta_6, F_3 \neg\circ F_4 \vdash F_1} \neg\circ_L$$

- Case(s) rule I

5 Cut-Elimination

5.1 Status of $!R$: OK

- Case rule $!R$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : !\Upsilon 7, !F_4 \vdash F_6}{\bullet h_5 : !\Upsilon 7, !F_4 \vdash !F_6} !R \\
 \hline
 - : !\Upsilon 2, !\Upsilon 7 \vdash F_6 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, !\Upsilon 7 \vdash F_6} \text{ax/W} \quad \frac{h_5 : !\Upsilon 7, !F_4 \vdash F_6}{- : !\Upsilon 2, !\Upsilon 7 \vdash !F_6} \text{ax/W} \\
 \hline
 - : !\Upsilon 2, !\Upsilon 7 \vdash F_6 \quad \text{hCut} \\
 \hline
 - : !\Upsilon 2, !\Upsilon 7 \vdash !F_6 \quad !R
 \end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{}{\bullet h_5 : \Delta_6, !F_4 \vdash \top} \top \\
 \hline
 - : !\Upsilon 2, \Delta_6 \vdash \top \quad \text{Cut} \\
 \hline
 \rightarrow \\
 - : !\Upsilon 2, \Delta_6 \vdash \top \quad \top
 \end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_6 \quad h_5 : \Delta_8, !F_4 \vdash F_7}{\bullet h_5 : \Delta_8, !F_4 \vdash F_6 \& F_7} \&_R \\
 \hline
 - : !\Upsilon 2, \Delta_8 \vdash F_6 \& F_7 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8 \vdash F_6} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_6}{- : !\Upsilon 2, \Delta_8 \vdash F_6} \text{ax/W} \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_7}{- : !\Upsilon 2, \Delta_8 \vdash F_7} \text{ax/W} \\
 \hline
 - : !\Upsilon 2, \Delta_8 \vdash F_6 \quad \text{hCut} \quad - : !\Upsilon 2, \Delta_8 \vdash F_7 \quad \text{hCut} \\
 \hline
 - : !\Upsilon 2, \Delta_8 \vdash F_6 \& F_7 \quad \&_R
 \end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, F_6, !F_4 \vdash F_7}{\bullet h_5 : \Delta_8, !F_4 \vdash F_6 \neg\circ F_7} \neg\circ_R \\
 \hline
 - : !\Upsilon 2, \Delta_8 \vdash F_6 \neg\circ F_7 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8, F_6 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_6, !F_4 \vdash F_7}{- : !\Upsilon 2, \Delta_8, F_6 \vdash F_7} \text{ax/W} \\
 \hline
 - : !\Upsilon 2, \Delta_8, F_6 \vdash F_7 \quad \text{hCut} \\
 \hline
 - : !\Upsilon 2, \Delta_8 \vdash F_6 \neg\circ F_7 \quad \neg\circ_R
 \end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_7}{\bullet h_5 : \Delta_8, !F_4 \vdash F_6 \oplus F_7} \oplus_{R_2} \\
 \hline
 - : !\Upsilon 2, \Delta_8 \vdash F_6 \oplus F_7 \quad \text{Cut} \\
 \hline
 \rightarrow \\
 \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_7}{- : !\Upsilon 2, \Delta_8 \vdash F_7} \text{ax/W} \\
 \hline
 - : !\Upsilon 2, \Delta_8 \vdash F_7 \quad \text{hCut} \\
 \hline
 - : !\Upsilon 2, \Delta_8 \vdash F_6 \oplus F_7 \quad \oplus_{R_2}
 \end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_6}{\bullet h_5 : \Delta_8, !F_4 \vdash F_6 \oplus F_7} \oplus_{R_1} \\
\hline
- : !\Upsilon 2, \Delta_8 \vdash F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8 \vdash F_6} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_6}{- : !\Upsilon 2, \Delta_8 \vdash F_6 \oplus F_7} \text{ax/W}}{- : !\Upsilon 2, \Delta_8 \vdash F_6 \oplus F_7} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_7, !F_4 \vdash F_6}{\bullet h_5 : (1, \Delta_7), !F_4 \vdash F_6} 1_L \\
\hline
- : !\Upsilon 2, 1, \Delta_7 \vdash F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : 1, !\Upsilon 2, \Delta_7 \vdash F_6} \text{ax/W} \quad \frac{h_5 : 1, \Delta_7, !F_4 \vdash F_6}{- : 1, !\Upsilon 2, \Delta_7 \vdash F_6} \text{ax/W}}{- : 1, !\Upsilon 2, \Delta_7 \vdash F_6} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_6 \quad h_5 : \Delta_9 \vdash F_7}{\bullet h_5 : (\Delta_8, \Delta_9), !F_4 \vdash F_6 \otimes F_7} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_8, \Delta_9 \vdash F_6 \otimes F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8 \vdash F_6} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_6}{- : !\Upsilon 2, \Delta_8, \Delta_9 \vdash F_6 \otimes F_7} \text{hCut} \quad \frac{h_5 : \Delta_9 \vdash F_7}{- : \Delta_9 \vdash F_7} \text{ax/W}}{- : !\Upsilon 2, \Delta_8, \Delta_9 \vdash F_6 \otimes F_7} \otimes_R \\
\\
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8 \vdash F_6 \quad h_5 : \Delta_9, !F_4 \vdash F_7}{\bullet h_5 : (\Delta_8, \Delta_9), !F_4 \vdash F_6 \otimes F_7} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_8, \Delta_9 \vdash F_6 \otimes F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_6}{- : !\Upsilon 2, \Delta_8 \vdash F_6} \text{ax/W} \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_9, !F_4 \vdash F_7}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{hCut}}{- : !\Upsilon 2, \Delta_8, \Delta_9 \vdash F_6 \otimes F_7} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), !F_4 \vdash F_7} W \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{hCut}}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \\
\\
\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_4 : \Delta_7 \vdash F_6}{\bullet h_4 : \Delta_7, !F_5 \vdash F_6} W \\
\hline
- : !\Upsilon 2, \Delta_7 \vdash F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : !\Upsilon 2, \Delta_7 \vdash F_6}{- : !\Upsilon 2, \Delta_7 \vdash F_6} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), !F_4 \vdash F_7} C \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{hCut}}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_4 : \Delta_7, !F_5, !F_5 \vdash F_6}{\bullet h_4 : \Delta_7, !F_5 \vdash F_6} C \\
\hline
- : !\Upsilon 2, \Delta_7 \vdash F_6 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2 \vdash !F_5} \text{ax/W} \quad \frac{h_4 : \Delta_7, !F_5, !F_5 \vdash F_6}{- : \Delta_7, !F_5 \vdash F_6} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_7 \vdash F_6 \quad \text{mCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, F_6, !F_4 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), !F_4 \vdash F_7} !L \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_6, !F_4 \vdash F_7}{- : \Delta_8, F_6 \vdash F_7} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_8, F_6 \vdash F_7 \quad !L \\
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_4 : \Delta_7, F_5 \vdash F_6}{\bullet h_4 : \Delta_7, !F_5 \vdash F_6} !L \\
\hline
- : !\Upsilon 2, \Delta_7 \vdash F_6 \\
\rightarrow \\
\frac{- : !\Upsilon 2 \vdash F_5}{- : !\Upsilon 2 \vdash F_5} \text{ax/W} \quad \frac{- : \Delta_7, F_5 \vdash F_6}{- : \Delta_7, !F_5 \vdash F_6} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_7 \vdash F_6 \quad \text{sCut}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), !F_4 \vdash F_8} \&_{L2} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{- : \Delta_9, F_7 \vdash F_8} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8 \quad \text{hCut} \\
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \quad \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), !F_4 \vdash F_8} \&_{L1} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8}{- : \Delta_9, F_6 \vdash F_8} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \vdash F_8 \quad \text{hCut} \\
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \quad \&_{L1}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_6, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \otimes F_7), !F_4 \vdash F_8} \otimes_L \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \otimes F_7 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6, F_7, !F_4 \vdash F_8}{- : \Delta_9, F_6, F_7 \vdash F_8} \text{ax/W} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6, F_7 \vdash F_8 \quad \text{hCut} \\
- : !\Upsilon 2, \Delta_9, F_6 \otimes F_7 \vdash F_8 \quad \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8 \quad h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), !F_4 \vdash F_8} \oplus_L}{\frac{}{- : !\Upsilon 2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{Cut}} \rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_9, F_6 \vdash F_8} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), !F_4 \vdash F_8} \text{ax/W}}{\frac{}{- : !\Upsilon 2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{hCut}} \rightarrow \\
\frac{\frac{}{- : !\Upsilon 2, \Delta_9, F_6 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8} \text{ax/W}}{\frac{}{- : !\Upsilon 2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{hCut}} \oplus_L
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, !F_4 \vdash F_6 \quad h_5 : \Delta_{10}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}, F_6 \neg\circ F_7), !F_4 \vdash F_8} \neg\circ_L}{\frac{}{- : !\Upsilon 2, \Delta_9, \Delta_{10}, F_6 \neg\circ F_7 \vdash F_8} \text{Cut}} \rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_9 \vdash F_6} \text{ax/W} \quad \frac{h_5 : \Delta_9, !F_4 \vdash F_6}{\bullet h_5 : (\Delta_9, \Delta_{10}, F_6 \neg\circ F_7), !F_4 \vdash F_8} \text{ax/W}}{\frac{}{- : !\Upsilon 2, \Delta_{10}, \Delta_9, F_6 \neg\circ F_7 \vdash F_8} \text{hCut}} \rightarrow \\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9 \vdash F_6 \quad h_5 : \Delta_{10}, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}, F_6 \neg\circ F_7), !F_4 \vdash F_8} \neg\circ_L}{\frac{}{- : !\Upsilon 2, \Delta_9, \Delta_{10}, F_6 \neg\circ F_7 \vdash F_8} \text{Cut}} \rightarrow \\
\frac{\frac{}{- : \Delta_9 \vdash F_6} \text{ax/W} \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_{10}, F_7 \vdash F_8} \text{ax/W}}{\frac{}{- : !\Upsilon 2, \Delta_{10}, \Delta_9, F_6 \neg\circ F_7 \vdash F_8} \text{hCut}} \neg\circ_L
\end{array}$$

- Case rule I

5.2 Status of 1_R : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : * \vdash 1}{- : *, \Delta_3 \vdash \top} 1_R \quad \frac{\bullet h_2 : \Delta_3, 1 \vdash \top}{- : \Delta_3 \vdash \top} \top}{\frac{}{- : \Delta_3 \vdash \top} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_3 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_5 \vdash F_3 \quad h_2 : 1, \Delta_5 \vdash F_4}{\bullet h_2 : \Delta_5, 1 \vdash F_3 \& F_4} \&_R}{\frac{}{- : *, \Delta_5 \vdash F_3 \& F_4} \text{Cut}} \rightarrow \\
\frac{\frac{\bullet h_1 : * \vdash 1}{- : \Delta_5 \vdash F_3} \text{ax/W} \quad \frac{h_2 : 1, \Delta_5 \vdash F_3}{\bullet h_2 : \Delta_5, 1 \vdash F_3 \& F_4} \text{ax/W}}{\frac{}{- : \Delta_5 \vdash F_3} \text{hCut}} \rightarrow \\
\frac{\frac{}{- : \Delta_5 \vdash F_3} \text{ax/W} \quad \frac{\bullet h_1 : * \vdash 1}{- : \Delta_5 \vdash F_4} \text{ax/W}}{\frac{}{- : \Delta_5 \vdash F_3 \& F_4} \text{hCut}} \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_5, F_3 \vdash F_4}{\bullet h_2 : \Delta_5, 1 \vdash F_3 \multimap F_4} \multimap_R}{- : *, \Delta_5 \vdash F_3 \multimap F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_2 : 1, \Delta_5, F_3 \vdash F_4}{\bullet h_2 : \Delta_5, 1 \vdash F_3 \multimap F_4} \text{ax/W}}{- : \Delta_5, F_3 \vdash F_4} \text{hCut} \\
\frac{}{- : \Delta_5 \vdash F_3 \multimap F_4} \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_5 \vdash F_4}{\bullet h_2 : \Delta_5, 1 \vdash F_3 \oplus F_4} \oplus_{R_2}}{- : *, \Delta_5 \vdash F_3 \oplus F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_2 : 1, \Delta_5 \vdash F_4}{\bullet h_2 : \Delta_5, 1 \vdash F_3 \oplus F_4} \text{ax/W}}{- : \Delta_5 \vdash F_4} \text{hCut} \\
\frac{}{- : \Delta_5 \vdash F_3 \oplus F_4} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_5 \vdash F_3}{\bullet h_2 : \Delta_5, 1 \vdash F_3 \oplus F_4} \oplus_{R_1}}{- : *, \Delta_5 \vdash F_3 \oplus F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_5 \vdash F_3}{\bullet h_2 : \Delta_5, 1 \vdash F_3 \oplus F_4} \text{ax/W}}{- : \Delta_5 \vdash F_3} \text{hCut} \\
\frac{}{- : \Delta_5 \vdash F_3 \oplus F_4} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : \Delta_4 \vdash F_3}{\bullet h_2 : \Delta_4, 1 \vdash F_3} 1_L}{- : *, \Delta_4 \vdash F_3} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_4 \vdash F_3} \text{ax/W}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_5 \vdash F_3 \quad h_2 : \Delta_6 \vdash F_4}{\bullet h_2 : (\Delta_5, \Delta_6), 1 \vdash F_3 \otimes F_4} \otimes_R}{- : *, \Delta_5, \Delta_6 \vdash F_3 \otimes F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_2 : 1, \Delta_5 \vdash F_3}{\bullet h_2 : \Delta_5, 1 \vdash F_3 \otimes F_4} \text{ax/W}}{- : \Delta_5 \vdash F_3} \text{hCut} \quad \frac{}{- : \Delta_6 \vdash F_4} \text{ax/W} \\
\frac{}{- : \Delta_5, \Delta_6 \vdash F_3 \otimes F_4} \otimes_R \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : \Delta_5 \vdash F_3 \quad h_2 : 1, \Delta_6 \vdash F_4}{\bullet h_2 : (\Delta_5, \Delta_6), 1 \vdash F_3 \otimes F_4} \otimes_R}{- : *, \Delta_5, \Delta_6 \vdash F_3 \otimes F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- : \Delta_5 \vdash F_3} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_6 \vdash F_4}{\bullet h_2 : 1, \Delta_6 \vdash F_4} \text{ax/W}}{- : \Delta_6 \vdash F_4} \text{hCut} \\
\frac{}{- : \Delta_5, \Delta_6 \vdash F_3 \otimes F_4} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_5 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), 1 \vdash F_4} W}{- : *, \Delta_5, !F_3 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_5, !F_3 \vdash F_4}{\bullet h_2 : 1, \Delta_5, !F_3 \vdash F_4} \text{ax/W}}{- : \Delta_5, !F_3 \vdash F_4} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_5, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), 1 \vdash F_4} C}{- : *, \Delta_5, !F_3 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_5, !F_3, !F_3 \vdash F_4}{- : \Delta_5, !F_3, !F_3 \vdash F_4} \text{ax/W}}{- : \Delta_5, !F_3 \vdash F_4} \text{hCut} \quad C
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_5, F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), 1 \vdash F_4} !L}{- : *, \Delta_5, !F_3 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_5, F_3 \vdash F_4}{- : \Delta_5, F_3 \vdash F_4} \text{ax/W}}{- : \Delta_5, !F_3 \vdash F_4} \text{hCut} \quad !L
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_6, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \& F_4), 1 \vdash F_5} \&_{L2}}{- : *, \Delta_6, F_3 \& F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_6, F_4 \vdash F_5}{- : \Delta_6, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \& F_4 \vdash F_5} \text{hCut} \quad \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_6, F_3 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \& F_4), 1 \vdash F_5} \&_{L1}}{- : *, \Delta_6, F_3 \& F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_6, F_3 \vdash F_5}{- : \Delta_6, F_3 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \& F_4 \vdash F_5} \text{hCut} \quad \&_{L1}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_6, F_3, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \otimes F_4), 1 \vdash F_5} \otimes_L}{- : *, \Delta_6, F_3 \otimes F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{\text{ax/W} \quad \frac{h_2 : 1, \Delta_6, F_3, F_4 \vdash F_5}{- : \Delta_6, F_3, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \otimes F_4 \vdash F_5} \text{hCut} \quad \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{1_R \quad \frac{h_2 : 1, \Delta_6, F_3 \vdash F_5 \quad h_2 : 1, \Delta_6, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \oplus F_4), 1 \vdash F_5} \oplus_L}{- : *, \Delta_6, F_3 \oplus F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : * \vdash 1} \quad \frac{\text{ax/W} \quad \frac{h_2 : 1, \Delta_6, F_3 \vdash F_5}{- : \Delta_6, F_3 \vdash F_5} \text{ax/W} \quad \frac{\text{ax/W} \quad \frac{h_2 : 1, \Delta_6, F_4 \vdash F_5}{- : \Delta_6, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \oplus F_4 \vdash F_5} \text{hCut} \quad \oplus_L
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : 1, \Delta_6 \vdash F_3 \quad h_2 : \Delta_7, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, \Delta_7, F_3 \neg\circ F_4), 1 \vdash F_5} \neg\circ_L}{- : *, \Delta_6, \Delta_7, F_3 \neg\circ F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_2 : 1, \Delta_6 \vdash F_3}{- : \Delta_6 \vdash F_3} \text{ax/W}}{- : \Delta_6, \Delta_7, F_3 \neg\circ F_4 \vdash F_5} \text{hCut} \quad \frac{- : \Delta_7, F_4 \vdash F_5}{- : \Delta_6, \Delta_7, F_3 \neg\circ F_4 \vdash F_5} \text{ax/W} \neg\circ_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} 1_R \quad \frac{h_2 : \Delta_6 \vdash F_3 \quad h_2 : 1, \Delta_7, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, \Delta_7, F_3 \neg\circ F_4), 1 \vdash F_5} \neg\circ_L}{- : *, \Delta_6, \Delta_7, F_3 \neg\circ F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_6 \vdash F_3}{- : \Delta_6, \Delta_7, F_3 \neg\circ F_4 \vdash F_5} \text{ax/W} \quad \frac{\frac{\frac{}{\bullet h_1 : * \vdash 1} \text{ax/W} \quad \frac{h_2 : 1, \Delta_7, F_4 \vdash F_5}{- : \Delta_7, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, \Delta_7, F_3 \neg\circ F_4 \vdash F_5} \text{hCut} \neg\circ_L
\end{array}$$

- Case rule I

5.3 Status of \top : OK

- Case rule $!R$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{}{\bullet h_3 : \Delta_4, \top \vdash \top} \top}{- : \Delta_2, \Delta_4 \vdash \top} \text{Cut} \\
\rightarrow \\
- : \Delta_2, \Delta_4 \vdash \top \quad \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6 \vdash F_4 \quad h_3 : \top, \Delta_6 \vdash F_5}{\bullet h_3 : \Delta_6, \top \vdash F_4 \& F_5} \&_R}{- : \Delta_2, \Delta_6 \vdash F_4 \& F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top \vdash F_4}{- : \Delta_2, \Delta_6 \vdash F_4} \text{ax/W}}{- : \Delta_2, \Delta_6 \vdash F_4 \& F_5} \text{hCut} \quad \frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top \vdash F_5}{- : \Delta_2, \Delta_6 \vdash F_5} \text{ax/W}}{- : \Delta_2, \Delta_6 \vdash F_4 \& F_5} \text{hCut} \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6, F_4 \vdash F_5}{\bullet h_3 : \Delta_6, \top \vdash F_4 \neg\circ F_5} \neg\circ_R}{- : \Delta_2, \Delta_6 \vdash F_4 \neg\circ F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, F_4, \top \vdash F_5}{- : \Delta_2, \Delta_6, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_2, \Delta_6 \vdash F_4 \neg\circ F_5} \text{hCut} \neg\circ_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6 \vdash F_5}{\bullet h_3 : \Delta_6, \top \vdash F_4 \oplus F_5} \oplus_{R_2}}{\neg : \Delta_2, \Delta_6 \vdash F_4 \oplus F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top \vdash F_5}{\neg : \Delta_2, \Delta_6 \vdash F_5} \text{ax/W}}{\neg : \Delta_2, \Delta_6 \vdash F_4 \oplus F_5} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6 \vdash F_4}{\bullet h_3 : \Delta_6, \top \vdash F_4 \oplus F_5} \oplus_{R_1}}{\neg : \Delta_2, \Delta_6 \vdash F_4 \oplus F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top \vdash F_4}{\neg : \Delta_2, \Delta_6 \vdash F_4} \text{ax/W}}{\neg : \Delta_2, \Delta_6 \vdash F_4 \oplus F_5} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_5 \vdash F_4}{\bullet h_3 : (1, \Delta_5), \top \vdash F_4} 1_L}}{\neg : \Delta_2, 1, \Delta_5 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : 1, \Delta_2, \Delta_5, \top \vdash F_4}{\neg : 1, \Delta_2, \Delta_5 \vdash F_4} \text{ax/W}}{\neg : 1, \Delta_2, \Delta_5 \vdash F_4} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6 \vdash F_4 \quad h_3 : \Delta_7 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7), \top \vdash F_4 \otimes F_5} \otimes_R}}{\neg : \Delta_2, \Delta_6, \Delta_7 \vdash F_4 \otimes F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_6, \top \vdash F_4}{\neg : \Delta_6 \vdash F_4} \text{ax/W}}{\neg : \Delta_2, \Delta_6, \Delta_7 \vdash F_4 \otimes F_5} \text{hCut} \frac{\neg : \Delta_2, \Delta_7 \vdash F_5}{\neg : \Delta_2, \Delta_6, \Delta_7 \vdash F_4 \otimes F_5} \otimes_R \\
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \Delta_6 \vdash F_4 \quad h_3 : \top, \Delta_7 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7), \top \vdash F_4 \otimes F_5} \otimes_R}}{\neg : \Delta_2, \Delta_6, \Delta_7 \vdash F_4 \otimes F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\neg : \Delta_6 \vdash F_4} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, \top \vdash F_5}{\neg : \Delta_2, \Delta_7 \vdash F_5} \text{ax/W}}{\neg : \Delta_2, \Delta_6, \Delta_7 \vdash F_4 \otimes F_5} \text{hCut}}{\neg : \Delta_2, \Delta_6, \Delta_7 \vdash F_4 \otimes F_5} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6 \vdash F_5}{\bullet h_3 : (\Delta_6, !F_4), \top \vdash F_5} W}}{\neg : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top, !F_4 \vdash F_5}{\neg : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{ax/W}}{\neg : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6, !F_4, !F_4 \vdash F_5}{\bullet h_3 : (\Delta_6, !F_4), \top \vdash F_5} C}{\frac{}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top, !F_4, !F_4 \vdash F_5}{\frac{}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} C} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{hCut}}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6, F_4 \vdash F_5}{\bullet h_3 : (\Delta_6, !F_4), \top \vdash F_5} !L}{\frac{}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, F_4, \top \vdash F_5}{\frac{}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} !L} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{hCut}}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \& F_5), \top \vdash F_6} \&_{L2}}{\frac{}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_5, \top \vdash F_6}{\frac{}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \&_{L2}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{hCut}}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \& F_5), \top \vdash F_6} \&_{L1}}{\frac{}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_4, \top \vdash F_6}{\frac{}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \&_{L1}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{hCut}}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \otimes F_5), \top \vdash F_6} \otimes_L}}{\frac{}{- : \Delta_2, \Delta_7, F_4 \otimes F_5 \vdash F_6} \rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_4, F_5, \top \vdash F_6}{\frac{}{- : \Delta_2, \Delta_7, F_4 \otimes F_5 \vdash F_6} \otimes_L} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_7, F_4 \otimes F_5 \vdash F_6} \text{hCut}}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \oplus F_5), \top \vdash F_6} \oplus_L}}{\frac{}{- : \Delta_2, \Delta_7, F_4 \oplus F_5 \vdash F_6} \rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_4, \top \vdash F_6}{\frac{}{- : \Delta_2, \Delta_7, F_4 \oplus F_5 \vdash F_6} \oplus_L} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_5, \top \vdash F_6}{\frac{}{- : \Delta_2, \Delta_7, F_5 \vdash F_6} \oplus_L} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_7, F_4 \oplus F_5 \vdash F_6} \text{hCut}}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\top} \quad \frac{\frac{h_3 : \top, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_4 \multimap F_5), \top \vdash F_6} \multimap_L}{\multimap : \Delta_2, \Delta_7, \Delta_8, F_4 \multimap F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : * \vdash \top}{\top} \quad \frac{h_3 : \Delta_7, \top \vdash F_4}{\text{ax/W}} \quad \frac{\text{hCut}}{\multimap : \Delta_2, \Delta_7, \Delta_8, F_4 \multimap F_5 \vdash F_6} \quad \frac{\multimap : \Delta_2, \Delta_8, F_5 \vdash F_6}{\text{ax/W}}}{\multimap : \Delta_2, \Delta_7, \Delta_8, F_4 \multimap F_5 \vdash F_6} \multimap_L \\
\\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\top} \quad \frac{\frac{h_3 : \Delta_7 \vdash F_4 \quad h_3 : \top, \Delta_8, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_4 \multimap F_5), \top \vdash F_6} \multimap_L}{\multimap : \Delta_2, \Delta_7, \Delta_8, F_4 \multimap F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\multimap : \Delta_7 \vdash F_4}{\text{ax/W}} \quad \frac{\frac{\bullet h_1 : * \vdash \top}{\top} \quad \frac{h_3 : \Delta_2, \Delta_8, F_5, \top \vdash F_6}{\text{ax/W}}}{\multimap : \Delta_2, \Delta_8, F_5 \vdash F_6} \text{hCut}}{\multimap : \Delta_2, \Delta_7, \Delta_8, F_4 \multimap F_5 \vdash F_6} \multimap_L
\end{array}$$

- Case rule I

5.4 Status of $\&_R$: OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{\bullet h_7 : \Delta_8, F_5 \& F_6 \vdash \top}{\top}}{\multimap : \Delta_2, \Delta_8 \vdash \top} \text{Cut} \\
\rightarrow \\
\multimap : \Delta_2, \Delta_8 \vdash \top \quad \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{\frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8 \& F_9} \&_R}{\multimap : \Delta_2, \Delta_{10} \vdash F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\text{ax/W}} \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8}{\text{ax/W}} \quad \frac{\text{hCut}}{\multimap : \Delta_{10}, \Delta_2 \vdash F_8} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\text{ax/W}} \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_9}{\text{ax/W}}}{\multimap : \Delta_{10}, \Delta_2 \vdash F_9} \text{hCut}}{\multimap : \Delta_{10}, \Delta_2 \vdash F_8 \& F_9} \&_R
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{\frac{h_7 : \Delta_{10}, F_8, F_5 \& F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8 \multimap F_9} \multimap_R}{\multimap : \Delta_2, \Delta_{10} \vdash F_8 \multimap F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\text{ax/W}} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \& F_6 \vdash F_9}{\text{ax/W}}}{\multimap : \Delta_{10}, \Delta_2 \vdash F_8 \vdash F_9} \text{hCut} \\
\multimap : \Delta_{10}, \Delta_2 \vdash F_8 \multimap F_9 \quad \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8 \oplus F_9} \oplus_{R_2} \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \oplus F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8 \oplus F_9} \oplus_{R_1} \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \oplus F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_9, F_5 \& F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_5 \& F_6 \vdash F_8} 1_L \\
\hline
- : \Delta_2, 1, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_5 \& F_6 \vdash F_8}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}}{- : 1, \Delta_2, \Delta_9 \vdash F_8}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8 \quad h_7 : \Delta_{11} \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{11}), F_5 \& F_6 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : \Delta_2, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_8}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \text{hCut}}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{- : \Delta_{11} \vdash F_9}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \otimes_R}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10} \vdash F_8 \quad h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{11}), F_5 \& F_6 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : \Delta_2, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{- : \Delta_{10} \vdash F_8}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_9}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \otimes_R}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \& F_6 \vdash F_9} W \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \& F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \& F_6 \vdash F_9} C \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \& F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} C
\end{array}$$

- Case rule !L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \& F_6 \vdash F_9} !L \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \& F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} !L
\end{array}$$

- Case rule &L2

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \& F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\\
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L2} \\
\hline
- : \Delta_2, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_7}{- : \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{- : \Delta_9, F_7 \vdash F_8}{- : \Delta_2, \Delta_9 \vdash F_8} \text{sCut}}{- : \Delta_2, \Delta_9 \vdash F_8} \text{sCut}
\end{array}$$

- Case rule &L1

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \& F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\\
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L1} \\
\hline
- : \Delta_2, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_6}{- : \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{- : \Delta_9, F_6 \vdash F_8}{- : \Delta_2, \Delta_9 \vdash F_8} \text{sCut}}{- : \Delta_2, \Delta_9 \vdash F_8} \text{sCut}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \& F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \& F_6 \vdash F_{10}} \oplus_L}{- : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \oplus_L
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_8 \quad h_7 : \Delta_{12}, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \neg\circ F_9), F_5 \& F_6 \vdash F_{10}} \neg\circ_L}{- : \Delta_2, \Delta_{11}, \Delta_{12}, F_8 \neg\circ F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_8}{- : \Delta_{11}, \Delta_2 \vdash F_8} \text{hCut}}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \neg\circ F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{h_7 : \Delta_{12}, F_9 \vdash F_{10}}{- : \Delta_{12}, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \neg\circ F_9 \vdash F_{10}} \neg\circ_L \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11} \vdash F_8 \quad h_7 : \Delta_{12}, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \neg\circ F_9), F_5 \& F_6 \vdash F_{10}} \neg\circ_L}{- : \Delta_2, \Delta_{11}, \Delta_{12}, F_8 \neg\circ F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{11} \vdash F_8}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \neg\circ F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{12}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_9, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{12}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \neg\circ F_9 \vdash F_{10}} \neg\circ_L
\end{array}$$

- Case rule I

5.5 Status of $\neg\circ_R$: OK

- Case rule $!R$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \neg\circ F_6} \neg\circ_R \quad \frac{}{\bullet h_7 : \Delta_8, F_5 \neg\circ F_6 \vdash \top} \top}{- : \Delta_2, \Delta_8 \vdash \top} \text{Cut} \\
\rightarrow \\
- : \Delta_2, \Delta_8 \vdash \top \quad \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \neg\circ F_6} \neg\circ_R \quad \frac{h_7 : \Delta_{10}, F_5 \neg\circ F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \neg\circ F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \neg\circ F_6 \vdash F_8 \& F_9}}{- : \Delta_2, \Delta_{10} \vdash F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \neg\circ F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \neg\circ F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \& F_9} \text{ax/W} \quad \frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \neg\circ F_6}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \neg\circ F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \& F_9} \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_8 \multimap F_9} \multimap_R \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \multimap F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_8 \multimap F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \multimap F_9 \quad \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_8 \oplus F_9} \oplus_{R_2} \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9 \quad \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_8 \oplus F_9} \oplus_{R_1} \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9 \quad \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_9, F_5 \multimap F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_5 \multimap F_6 \vdash F_8} 1_L \\
\hline
- : \Delta_2, 1, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_5 \multimap F_6 \vdash F_8}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_8 \quad h_7 : \Delta_{11} \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{11}), F_5 \multimap F_6 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : \Delta_2, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut} \quad \frac{- : \Delta_{11} \vdash F_9}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10} \vdash F_8 \quad h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{11}), F_5 \multimap F_6 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : \Delta_2, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_{10} \vdash F_8}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{hCut}}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9 \quad \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \multimap F_6 \vdash F_9} W \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \multimap F_6 \vdash F_9} C \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad C
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \multimap F_6 \vdash F_9} !L \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, F_8 \vdash F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad !L
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \multimap F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10} \quad \&_{L2} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \multimap F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10} \quad \&_{L1} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \multimap F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10} \quad \otimes_L \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{\frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \multimap F_6 \vdash F_{10}} \oplus_L}{\multimap : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \multimap F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \multimap F_6 \vdash F_{10}} \text{ax/W}}{\multimap : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{hCut} \quad \frac{\frac{h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \multimap F_6 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \multimap F_6 \vdash F_{10}} \text{ax/W}}{\multimap : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut} \\
\oplus_L \\
\multimap : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{\frac{h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_8 \quad h_7 : \Delta_{12}, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \multimap F_6 \vdash F_{10}} \multimap_L}{\multimap : \Delta_2, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \multimap F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_8}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \multimap F_6 \vdash F_{10}} \text{ax/W}}{\multimap : \Delta_{11}, \Delta_2 \vdash F_8} \text{hCut} \quad \frac{\frac{h_7 : \Delta_{12}, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \multimap F_6 \vdash F_{10}} \text{ax/W}}{\multimap : \Delta_{12}, F_9 \vdash F_{10}} \text{ax/W} \\
\multimap_L \\
\multimap : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \multimap F_9 \vdash F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{\frac{h_7 : \Delta_{11} \vdash F_8 \quad h_7 : \Delta_{12}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \multimap F_6 \vdash F_{10}} \multimap_L}{\multimap : \Delta_2, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \multimap F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \multimap F_6 \vdash F_{10}} \text{ax/W}}{\multimap : \Delta_{12}, \Delta_2, F_9 \vdash F_{10}} \text{hCut} \\
\multimap_L \\
\multimap : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \multimap F_9 \vdash F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7} \multimap_R \quad \frac{\frac{h_5 : \Delta_9 \vdash F_6 \quad h_5 : \Delta_{10}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}), F_6 \multimap F_7 \vdash F_8} \multimap_L}{\multimap : \Delta_2, \Delta_9, \Delta_{10} \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7} \text{ax/W} \quad \frac{h_5 : \Delta_9 \vdash F_6 \quad h_5 : \Delta_{10}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}), F_6 \multimap F_7 \vdash F_8} \text{ax/W}}{\multimap : \Delta_2, F_6 \vdash F_7} \text{ax/W} \quad \frac{\frac{h_5 : \Delta_{10}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}), F_6 \multimap F_7 \vdash F_8} \text{ax/W}}{\multimap : \Delta_{10}, F_7 \vdash F_8} \text{ax/W} \\
\text{sCut} \\
\multimap : \Delta_{10}, \Delta_2, \Delta_9 \vdash F_8
\end{array}$$

- Case rule I

5.6 Status of \oplus_{R2} : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{\bullet h_7 : \Delta_8, F_5 \oplus F_6 \vdash \top}{\top} \top}{\multimap : \Delta_2, \Delta_8 \vdash \top} \text{Cut} \\
\rightarrow \\
\multimap : \Delta_2, \Delta_8 \vdash \top \quad \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \& F_9} \&_R \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut} \quad \frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \& F_9 \quad \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \neg\circ F_9} \neg\circ_R \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \neg\circ F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \neg\circ F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \neg\circ F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \neg\circ F_9 \quad \neg\circ_R
\end{array}$$

- Case rule \oplus_{R2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \oplus F_9} \oplus_{R2} \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9 \quad \oplus_{R2}
\end{array}$$

- Case rule \oplus_{R1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \oplus F_9} \oplus_{R1} \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9 \quad \oplus_{R1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_9, F_5 \oplus F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_5 \oplus F_6 \vdash F_8} 1_L \\
\hline
- : \Delta_2, 1, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_5 \oplus F_6 \vdash F_8}{h_7 : 1, \Delta_9, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \quad h_7 : \Delta_{11} \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{11}), F_5 \oplus F_6 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : \Delta_2, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut} \quad \frac{- : \Delta_{11} \vdash F_9}{- : \Delta_{11} \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9 \quad \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{\frac{h_7 : \Delta_{10} \vdash F_8 \quad h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{11}), F_5 \oplus F_6 \vdash F_8 \otimes F_9} \otimes_R}{- : \Delta_2, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{10} \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{\frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} W}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} C}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut} C
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} !L}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut} !L
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L2}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut} \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L1}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut} \&_{L1}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \oplus F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ hCut} \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10}} \oplus_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ hCut} \oplus_L \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R_2} \frac{h_5 : \Delta_9, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \oplus F_7 \vdash F_8} \oplus_L}{- : \Delta_2, \Delta_9 \vdash F_8} \text{ Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_7}{- : \Delta_2, \Delta_9 \vdash F_8} \text{ ax/W} \quad \frac{- : \Delta_9, F_7 \vdash F_8}{- : \Delta_2, \Delta_9 \vdash F_8} \text{ ax/W}}{- : \Delta_2, \Delta_9 \vdash F_8} \text{ sCut}
\end{array}$$

- Case rule $\neg \circ_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_8 \quad h_7 : \Delta_{12}, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \neg \circ F_9), F_5 \oplus F_6 \vdash F_{10}} \neg \circ_L \\
\hline
- : \Delta_2, \Delta_{11}, \Delta_{12}, F_8 \neg \circ F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_8}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \neg \circ F_9 \vdash F_{10}} \text{ hCut} \quad \frac{- : \Delta_{12}, F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \neg \circ F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \neg \circ F_9 \vdash F_{10}} \neg \circ_L \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \frac{h_7 : \Delta_{11} \vdash F_8 \quad h_7 : \Delta_{12}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \neg \circ F_9), F_5 \oplus F_6 \vdash F_{10}} \neg \circ_L}{- : \Delta_2, \Delta_{11}, \Delta_{12}, F_8 \neg \circ F_9 \vdash F_{10}} \text{ Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{11} \vdash F_8}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \neg \circ F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{12}, \Delta_2, F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \neg \circ F_9 \vdash F_{10}} \text{ hCut} \neg \circ_L
\end{array}$$

- Case rule I

5.7 Status of \oplus_{R_1} : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_1} \frac{}{\bullet h_7 : \Delta_8, F_5 \oplus F_6 \vdash \top} \top \\
\hline
- : \Delta_2, \Delta_8 \vdash \top \\
\rightarrow \\
- : \Delta_2, \Delta_8 \vdash \top \quad \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \& F_9} \&_R}{- : \Delta_2, \Delta_{10} \vdash F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut} \quad \frac{\frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \& F_9} \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \neg\circ F_9} \neg\circ_R}{- : \Delta_2, \Delta_{10} \vdash F_8 \neg\circ F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \neg\circ F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \neg\circ F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \neg\circ F_9} \neg\circ_R
\end{array}$$

- Case rule \oplus_{R2}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \oplus F_9} \oplus_{R2}}{- : \Delta_2, \Delta_{10} \vdash F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9} \oplus_{R2}
\end{array}$$

- Case rule \oplus_{R1}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \oplus F_9} \oplus_{R1}}{- : \Delta_2, \Delta_{10} \vdash F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \oplus F_9} \oplus_{R1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_9, F_5 \oplus F_6 \vdash F_8}{\bullet h_7 : (1, \Delta_9), F_5 \oplus F_6 \vdash F_8} 1_L}{- : \Delta_2, 1, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : 1, \Delta_9, F_5 \oplus F_6 \vdash F_8}{h_7 : 1, \Delta_9, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : 1, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}}{- : 1, \Delta_2, \Delta_9 \vdash F_8}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8 \quad h_7 : \Delta_{11} \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{11}), F_5 \oplus F_6 \vdash F_8 \otimes F_9} \otimes_R}{- : \Delta_2, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8}{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut} \quad \frac{h_7 : \Delta_{11} \vdash F_9}{h_7 : \Delta_{11} \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10} \vdash F_8 \quad h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{11}), F_5 \oplus F_6 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : \Delta_2, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_{11}, \Delta_2 \vdash F_8 \otimes F_9 \quad \otimes_R \quad \text{hCut}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} W \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} C \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad C \quad \text{hCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} !L \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, F_8 \vdash F_9 \quad !L \\
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10} \quad \&_{L2} \quad \text{hCut}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10} \quad \&_{L1} \quad \text{hCut} \\
- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \oplus F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \oplus F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10} \quad \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10}} \oplus_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{hCut} \quad \frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10} \quad \oplus_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \oplus F_7 \vdash F_8} \oplus_L \\
\hline
- : \Delta_2, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_2 \vdash F_6 \quad \text{ax/W} \quad - : \Delta_9, F_6 \vdash F_8}{- : \Delta_2, \Delta_9 \vdash F_8} \text{sCut}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_8 \quad h_7 : \Delta_{12}, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \oplus F_6 \vdash F_{10}} \multimap_L \\
\hline
- : \Delta_2, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_8}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \oplus F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash F_8} \text{hCut} \quad \frac{- : \Delta_{12}, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \oplus F_6 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \multimap F_9 \vdash F_{10} \quad \multimap_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11} \vdash F_8 \quad h_7 : \Delta_{12}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \oplus F_6 \vdash F_{10}} \multimap_L \\
\hline
- : \Delta_2, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_{11} \vdash F_8 \quad \text{ax/W} \quad \frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_5 \oplus F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_{12}, \Delta_2, F_8 \multimap F_9 \vdash F_{10} \quad \multimap_L
\end{array}$$

- Case rule I

5.8 Status of 1_L : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash 7}{\bullet h_1 : 1, \Delta_3 \vdash 7} 1_L \quad \frac{h_4 : !\Upsilon 6, 7 \vdash F_5}{\bullet h_4 : !\Upsilon 6, 7 \vdash !F_5} !R \\
\hline
- : (1, \Delta_3), !\Upsilon 6 \vdash F_5 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_3 \vdash 7 \quad \text{ax/W} \quad \bullet h_4 : 1, 7, !\Upsilon 6 \vdash F_5}{- : 1, !\Upsilon 6, \Delta_3 \vdash !F_5} \text{hCut}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{\frac{h_1 : \Delta_3 \vdash F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_6} \mathbf{1}_L \quad \frac{\bullet h_4 : \Delta_5, F_6 \vdash \top}{\vdash : (\mathbf{1}, \Delta_3), \Delta_5 \vdash \top} \top}{\vdash : (\mathbf{1}, \Delta_3), \Delta_5 \vdash \top} \text{Cut} \quad \rightarrow \quad \frac{}{\vdash : \mathbf{1}, \Delta_3, \Delta_5 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_8} \mathbf{1}_L \quad \frac{\frac{h_4 : \Delta_7, F_8 \vdash F_5 \quad h_4 : \Delta_7, F_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \& F_6} \&_R}{\vdash : (\mathbf{1}, \Delta_3), \Delta_7 \vdash F_5 \& F_6} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash F_8}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \& F_6} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_7, F_8 \vdash F_5 \& F_6}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \& F_6} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \& F_6} \text{hCut}$$

- Case rule $\neg\circ_R$

$$\frac{\frac{\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_8} \mathbf{1}_L \quad \frac{h_4 : \Delta_7, F_5, F_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\vdash : (\mathbf{1}, \Delta_3), \Delta_7 \vdash F_5 \neg\circ F_6} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash F_8}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \neg\circ F_6} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_7, F_8 \vdash F_5 \neg\circ F_6}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \neg\circ F_6} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \neg\circ F_6} \text{hCut}$$

- Case rule \oplus_{R_2}

$$\frac{\frac{\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_8} \mathbf{1}_L \quad \frac{h_4 : \Delta_7, F_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\vdash : (\mathbf{1}, \Delta_3), \Delta_7 \vdash F_5 \oplus F_6} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash F_8}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_7, F_8 \vdash F_5 \oplus F_6}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \oplus F_6} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \oplus F_6} \text{hCut}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_8} \mathbf{1}_L \quad \frac{h_4 : \Delta_7, F_8 \vdash F_5}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\vdash : (\mathbf{1}, \Delta_3), \Delta_7 \vdash F_5 \oplus F_6} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash F_8}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_7, F_8 \vdash F_5 \oplus F_6}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \oplus F_6} \text{ax/W}}{\vdash : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_5 \oplus F_6} \text{hCut}$$

- Case rule $\mathbf{1}_L$

$$\frac{\frac{\frac{h_1 : \Delta_3 \vdash \mathbf{1}}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash \mathbf{1}} \mathbf{1}_L \quad \frac{\frac{h_4 : \Delta_6 \vdash F_5}{\bullet h_4 : \Delta_6, \mathbf{1} \vdash F_5} \mathbf{1}_L}{\vdash : (\mathbf{1}, \Delta_3), \Delta_6 \vdash F_5} \text{Cut} \quad \rightarrow \quad \frac{}{\vdash : \mathbf{1}, \Delta_3, \Delta_6 \vdash F_5} \text{ax/W} \quad \frac{\frac{h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_7} \mathbf{1}_L \quad \frac{\frac{h_4 : \Delta_6, F_7 \vdash F_5}{\bullet h_4 : (\mathbf{1}, \Delta_6), F_7 \vdash F_5} \mathbf{1}_L}{\vdash : (\mathbf{1}, \Delta_3), \mathbf{1}, \Delta_6 \vdash F_5} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash F_7}{\vdash : \mathbf{1}, \mathbf{1}, \Delta_3, \Delta_6 \vdash F_5} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \mathbf{1}, \Delta_6, F_7 \vdash F_5}{\vdash : \mathbf{1}, \mathbf{1}, \Delta_3, \Delta_6 \vdash F_5} \text{ax/W}}{\vdash : \mathbf{1}, \mathbf{1}, \Delta_3, \Delta_6 \vdash F_5} \text{hCut}$$

- Case rule \otimes_R

[illegible]

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : 1, \Delta_3 \vdash F_8} \text{ 1L} \quad \frac{h_4 : \Delta_7, F_8 \vdash F_6}{\bullet h_4 : (\Delta_7, !F_5), F_8 \vdash F_6} \text{ W}}{\vdash : (1, \Delta_3), \Delta_7, !F_5 \vdash F_6} \text{ Cut} \\
\downarrow \text{ax/W} \\
\frac{h_1 : \Delta_3 \vdash F_8}{\vdash : 1, \Delta_3, \Delta_7, !F_5 \vdash F_6} \text{ ax/W} \quad \frac{\bullet h_4 : 1, \Delta_7, F_8, !F_5 \vdash F_6}{\vdash : 1, \Delta_3, \Delta_7, !F_5 \vdash F_6} \text{ hCut} \\
\downarrow \text{1L} \\
\frac{\frac{h_1 : \Delta_3 \vdash !F_5}{\bullet h_1 : 1, \Delta_3 \vdash !F_5} \text{ 1L} \quad \frac{h_4 : \Delta_7 \vdash F_6}{\bullet h_4 : \Delta_7, !F_5 \vdash F_6} \text{ W}}{\vdash : (1, \Delta_3), \Delta_7 \vdash F_6} \text{ Cut} \\
\downarrow \text{ax/W} \\
\vdash : 1, \Delta_3, \Delta_7 \vdash F_6
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_8} \quad \mathbf{1}_L \quad \frac{h_4 : \Delta_7, F_8, !F_5, !F_5 \vdash F_6}{\bullet h_4 : (\Delta_7, !F_5), F_8 \vdash F_6} \quad C}{- : (\mathbf{1}, \Delta_3), \Delta_7, !F_5 \vdash F_6} \quad \text{Cut} \\
\downarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \mathbf{1}, \Delta_7, F_8, !F_5 \vdash F_6} \quad \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_7, F_8, !F_5 \vdash F_6}{- : \mathbf{1}, \Delta_3, \Delta_7, !F_5 \vdash F_6} \quad \text{hCut}}{} \\
\downarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash !F_5}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash !F_5} \quad \mathbf{1}_L \quad \frac{h_4 : \Delta_7, !F_5, !F_5 \vdash F_6}{\bullet h_4 : \Delta_7, !F_5 \vdash F_6} \quad C}{- : (\mathbf{1}, \Delta_3), \Delta_7 \vdash F_6} \quad \text{Cut} \\
\downarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash !F_5}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash !F_5} \quad \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_7, !F_5 \vdash F_6}{- : \mathbf{1}, \Delta_3, \Delta_7 \vdash F_6} \quad \text{hCut}}{}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : 1, \Delta_3 \vdash F_8} \quad 1L \quad \frac{h_4 : \Delta_7, F_5, F_8 \vdash F_6}{\bullet h_4 : (\Delta_7, !F_5), F_8 \vdash F_6} \quad !L}{- : (1, \Delta_3), \Delta_7, !F_5 \vdash F_6} \quad \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{h_1 : \Delta_3 \vdash F_8}{- : 1, \Delta_3, \Delta_7, !F_5 \vdash F_6} \text{ax/W} \quad \frac{\bullet h_4 : 1, \Delta_7, F_8, !F_5 \vdash F_6}{- : 1, \Delta_3, \Delta_7, !F_5 \vdash F_6} \text{ax/W} \\
\quad \quad \quad \text{hCut} \\
\frac{\frac{h_1 : \Delta_3 \vdash !F_5}{\bullet h_1 : 1, \Delta_3 \vdash !F_5} \quad 1L \quad \frac{h_4 : \Delta_7, F_5 \vdash F_6}{\bullet h_4 : \Delta_7, !F_5 \vdash F_6} \quad !L}{- : (1, \Delta_3), \Delta_7 \vdash F_6} \quad \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{h_1 : \Delta_3 \vdash !F_5}{- : 1, \Delta_3, \Delta_7 \vdash F_6} \text{ax/W} \quad \frac{\bullet h_4 : 1, \Delta_7, !F_5 \vdash F_6}{- : 1, \Delta_3, \Delta_7 \vdash F_6} \text{ax/W} \\
\quad \quad \quad \text{hCut}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \quad \mathbf{1}_L \quad \frac{h_4 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \& F_6), F_9 \vdash F_7}}{\vdash : (\mathbf{1}, \Delta_3), \Delta_8, F_5 \& F_6 \vdash F_7} \&_{L2} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \quad \frac{h_4 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_8, F_9, F_5 \& F_6 \vdash F_7}}{\vdash : \mathbf{1}, \Delta_3, \Delta_8, F_5 \& F_6 \vdash F_7} \text{ ax/W hCut} \\
\\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \& F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \& F_6} \quad \mathbf{1}_L \quad \frac{h_4 : \Delta_8, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \& F_6 \vdash F_7}}{\vdash : (\mathbf{1}, \Delta_3), \Delta_8 \vdash F_7} \&_{L2} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \& F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \& F_6} \quad \frac{h_4 : \Delta_8, F_6 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_8, F_5 \& F_6 \vdash F_7}}{\vdash : \mathbf{1}, \Delta_3, \Delta_8 \vdash F_7} \text{ ax/W hCut}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \quad \mathbf{1}_L \quad \frac{h_4 : \Delta_8, F_5, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \& F_6), F_9 \vdash F_7}}{\vdash : (\mathbf{1}, \Delta_3), \Delta_8, F_5 \& F_6 \vdash F_7} \&_{L1} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \quad \frac{h_4 : \Delta_8, F_5, F_9 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_8, F_9, F_5 \& F_6 \vdash F_7}}{\vdash : \mathbf{1}, \Delta_3, \Delta_8, F_5 \& F_6 \vdash F_7} \text{ ax/W hCut} \\
\\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \& F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \& F_6} \quad \mathbf{1}_L \quad \frac{h_4 : \Delta_8, F_5 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \& F_6 \vdash F_7}}{\vdash : (\mathbf{1}, \Delta_3), \Delta_8 \vdash F_7} \&_{L1} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \& F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \& F_6} \quad \frac{h_4 : \Delta_8, F_5 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_8, F_5 \& F_6 \vdash F_7}}{\vdash : \mathbf{1}, \Delta_3, \Delta_8 \vdash F_7} \text{ ax/W hCut}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \quad \mathbf{1}_L \quad \frac{h_4 : \Delta_8, F_5, F_6, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \otimes F_6), F_9 \vdash F_7}}{\vdash : (\mathbf{1}, \Delta_3), \Delta_8, F_5 \otimes F_6 \vdash F_7} \otimes_L \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \quad \frac{h_4 : \Delta_8, F_5, F_6, F_9 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_8, F_9, F_5 \otimes F_6 \vdash F_7}}{\vdash : \mathbf{1}, \Delta_3, \Delta_8, F_5 \otimes F_6 \vdash F_7} \text{ ax/W hCut} \\
\\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \otimes F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \otimes F_6} \quad \mathbf{1}_L \quad \frac{h_4 : \Delta_8, F_5, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \otimes F_6 \vdash F_7}}{\vdash : (\mathbf{1}, \Delta_3), \Delta_8 \vdash F_7} \otimes_L \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \otimes F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \otimes F_6} \quad \frac{h_4 : \Delta_8, F_5, F_6 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_8, F_5 \otimes F_6 \vdash F_7}}{\vdash : \mathbf{1}, \Delta_3, \Delta_8 \vdash F_7} \text{ ax/W hCut}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \quad \mathbf{1}_L \quad \frac{h_4 : \Delta_8, F_5, F_9 \vdash F_7 \quad h_4 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \oplus F_6), F_9 \vdash F_7}}{\vdash : (\mathbf{1}, \Delta_3), \Delta_8, F_5 \oplus F_6 \vdash F_7} \oplus_L \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \quad \frac{h_4 : \Delta_8, F_5, F_9 \vdash F_7 \quad h_4 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_8, F_9, F_5 \oplus F_6 \vdash F_7}}{\vdash : \mathbf{1}, \Delta_3, \Delta_8, F_5 \oplus F_6 \vdash F_7} \text{ ax/W hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash F_5 \oplus F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \oplus F_6} \mathbf{1}_L \quad \frac{h_4 : \Delta_8, F_5 \vdash F_7 \quad h_4 : \Delta_8, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \oplus F_6 \vdash F_7} \oplus_L \\
\hline
- : (\mathbf{1}, \Delta_3), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \oplus F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_8, F_5 \oplus F_6 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_8, F_5 \oplus F_6 \vdash F_7} \text{ax/W}}{- : \mathbf{1}, \Delta_3, \Delta_8 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \mathbf{1}_L \quad \frac{h_4 : \Delta_8, F_9 \vdash F_5 \quad h_4 : \Delta_{10}, F_6 \vdash F_7}{\bullet h_4 : (\Delta_8, \Delta_{10}, F_5 \multimap F_6), F_9 \vdash F_7} \multimap_L \\
\hline
- : (\mathbf{1}, \Delta_3), \Delta_8, \Delta_{10}, F_5 \multimap F_6 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_{10}, \Delta_8, F_9, F_5 \multimap F_6 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_{10}, \Delta_3, \Delta_8, F_5 \multimap F_6 \vdash F_7} \text{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_3, \Delta_8, F_5 \multimap F_6 \vdash F_7} \text{hCut} \\
\\
\frac{h_1 : \Delta_3 \vdash F_{10}}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_{10}} \mathbf{1}_L \quad \frac{h_4 : \Delta_8 \vdash F_5 \quad h_4 : \Delta_9, F_6, F_{10} \vdash F_7}{\bullet h_4 : (\Delta_8, \Delta_9, F_5 \multimap F_6), F_{10} \vdash F_7} \multimap_L \\
\hline
- : (\mathbf{1}, \Delta_3), \Delta_8, \Delta_9, F_5 \multimap F_6 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_{10}}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_8, \Delta_9, F_{10}, F_5 \multimap F_6 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_3, \Delta_8, \Delta_9, F_5 \multimap F_6 \vdash F_7} \text{ax/W}}{- : \mathbf{1}, \Delta_3, \Delta_8, \Delta_9, F_5 \multimap F_6 \vdash F_7} \text{hCut} \\
\\
\frac{h_1 : \Delta_3 \vdash F_5 \multimap F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \multimap F_6} \mathbf{1}_L \quad \frac{h_4 : \Delta_8 \vdash F_5 \quad h_4 : \Delta_9, F_6 \vdash F_7}{\bullet h_4 : (\Delta_8, \Delta_9), F_5 \multimap F_6 \vdash F_7} \multimap_L \\
\hline
- : (\mathbf{1}, \Delta_3), \Delta_8, \Delta_9 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \multimap F_6}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash F_5 \multimap F_6} \text{ax/W} \quad \frac{\bullet h_4 : \mathbf{1}, \Delta_8, \Delta_9, F_5 \multimap F_6 \vdash F_7}{\bullet h_4 : \mathbf{1}, \Delta_3, \Delta_8, \Delta_9 \vdash F_7} \text{ax/W}}{- : \mathbf{1}, \Delta_3, \Delta_8, \Delta_9 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash p(n_5)}{\bullet h_1 : \mathbf{1}, \Delta_3 \vdash p(n_5)} \mathbf{1}_L \quad \frac{}{\bullet h_4 : *, p(n_5) \vdash p(n_5)} I \\
\hline
- : (\mathbf{1}, \Delta_3), * \vdash p(n_5) \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \mathbf{1}, \Delta_3 \vdash p(n_5)} \text{ax/W}
\end{array}$$

5.9 Status of \otimes_R : OK

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{}{\bullet h_8 : \Delta_9, F_6 \otimes F_7 \vdash \top} \top \\
\hline
- : (\Delta_4, \Delta_5), \Delta_9 \vdash \top \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_4, \Delta_5, \Delta_9 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9 \quad h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9 \& F_{10}} \&_R}{- : (\Delta_4, \Delta_5), \Delta_{11} \vdash F_9 \& F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9} \text{hCut}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{\frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_{10}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_{10}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9 \& F_{10}} \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_9, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9 \neg\circ F_{10}} \neg\circ_R}{- : (\Delta_4, \Delta_5), \Delta_{11} \vdash F_9 \neg\circ F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9 \neg\circ F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_9, F_6 \otimes F_7 \vdash F_{10}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9 \neg\circ F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9 \neg\circ F_{10}} \neg\circ_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2}}{- : (\Delta_4, \Delta_5), \Delta_{11} \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_{10}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_{10}} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9}{\bullet h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1}}{- : (\Delta_4, \Delta_5), \Delta_{11} \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9} \text{hCut}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{10}, F_6 \otimes F_7 \vdash F_9}{\bullet h_8 : (\mathbf{1}, \Delta_{10}), F_6 \otimes F_7 \vdash F_9} \mathbf{1}_L}{- : (\Delta_4, \Delta_5), \mathbf{1}, \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \mathbf{1}, \Delta_{10}, \Delta_4, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \mathbf{1}, \Delta_{10}, F_6 \otimes F_7 \vdash F_9}{- : \mathbf{1}, \Delta_{10}, \Delta_4, \Delta_5 \vdash F_9} \text{hCut}}{- : \mathbf{1}, \Delta_{10}, \Delta_4, \Delta_5 \vdash F_9} \text{ax/W}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9 \quad h_8 : \Delta_{12} \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, \Delta_{12}), F_6 \otimes F_7 \vdash F_9 \otimes F_{10}} \otimes_R}{- : (\Delta_4, \Delta_5), \Delta_{11}, \Delta_{12} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_9}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9} \text{hCut}}{- : \Delta_{11}, \Delta_4, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{\frac{h_8 : \Delta_{12} \vdash F_{10}}{- : \Delta_{12} \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_{12}, \Delta_4, \Delta_5 \vdash F_9 \otimes F_{10}} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{\frac{h_8 : \Delta_{11} \vdash F_9 \quad h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, \Delta_{12}), F_6 \otimes F_7 \vdash F_9 \otimes F_{10}} \otimes_R}{\frac{- : (\Delta_4, \Delta_5), \Delta_{11}, \Delta_{12} \vdash F_9 \otimes F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{11} \vdash F_9}{\text{ax/W}} \quad \frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{10}}{\text{ax/W}}}{\frac{- : \Delta_{12}, \Delta_4, \Delta_5 \vdash F_{10}}{\otimes_R} \text{hCut}} \\
\frac{}{- : \Delta_{11}, \Delta_{12}, \Delta_4, \Delta_5 \vdash F_9 \otimes F_{10}}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_6 \otimes F_7 \vdash F_{10}} W}{\frac{- : (\Delta_4, \Delta_5), \Delta_{11}, !F_9 \vdash F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_{11}, !F_9, F_6 \otimes F_7 \vdash F_{10}}{\text{ax/W}}}{\frac{- : \Delta_{11}, \Delta_4, \Delta_5, !F_9 \vdash F_{10}}{\text{hCut}}}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, !F_9, !F_9, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_6 \otimes F_7 \vdash F_{10}} C}{\frac{- : (\Delta_4, \Delta_5), \Delta_{11}, !F_9 \vdash F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_{11}, !F_9, !F_9, F_6 \otimes F_7 \vdash F_{10}}{\text{ax/W}}}{\frac{- : \Delta_{11}, \Delta_4, \Delta_5, !F_9, !F_9 \vdash F_{10}}{\text{hCut}} C} \\
\frac{}{- : \Delta_{11}, \Delta_4, \Delta_5, !F_9 \vdash F_{10}}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_9, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_6 \otimes F_7 \vdash F_{10}} !L}{\frac{- : (\Delta_4, \Delta_5), \Delta_{11}, !F_9 \vdash F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_{11}, F_9, F_6 \otimes F_7 \vdash F_{10}}{\text{ax/W}}}{\frac{- : \Delta_{11}, \Delta_4, \Delta_5, F_9 \vdash F_{10}}{\text{hCut}} !L} \\
\frac{}{- : \Delta_{11}, \Delta_4, \Delta_5, !F_9 \vdash F_{10}}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_6 \otimes F_7 \vdash F_{11}} \&_{L2}}{\frac{- : (\Delta_4, \Delta_5), \Delta_{12}, F_9 \& F_{10} \vdash F_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\text{ax/W}}}{\frac{- : \Delta_{12}, \Delta_4, \Delta_5, F_{10} \vdash F_{11}}{\&_{L2}} \text{hCut}} \\
\frac{}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \& F_{10} \vdash F_{11}}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_6 \otimes F_7 \vdash F_{11}} \&_{L1}}{\frac{- : (\Delta_4, \Delta_5), \Delta_{12}, F_9 \& F_{10} \vdash F_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11}}{\text{ax/W}}}{\frac{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \vdash F_{11}}{\&_{L1}} \text{hCut}} \\
\frac{}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \& F_{10} \vdash F_{11}}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \otimes F_{10}), F_6 \otimes F_7 \vdash F_{11}} \otimes_L \\
\hline
- : (\Delta_4, \Delta_5), \Delta_{12}, F_9 \otimes F_{10} \vdash F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_4, \Delta_5, F_{10}, F_9 \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \otimes F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \otimes F_{10} \vdash F_{11}} \text{hCut} \otimes_L \\
\hline
\frac{h_1 : \Delta_4 \vdash F_7 \quad h_1 : \Delta_5 \vdash F_8}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_6 : \Delta_{10}, F_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L \\
\hline
- : (\Delta_4, \Delta_5), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_4 \vdash F_7}{- : \Delta_{10}, \Delta_4, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{- : \Delta_5 \vdash F_8}{- : \Delta_{10}, F_7, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_4, \Delta_5 \vdash F_9} \text{sCut} \\
\hline
- : \Delta_{10}, \Delta_4, \Delta_5 \vdash F_9 \quad \text{sCut}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \oplus F_{10}), F_6 \otimes F_7 \vdash F_{11}} \oplus_L \\
\hline
- : (\Delta_4, \Delta_5), \Delta_{12}, F_9 \oplus F_{10} \vdash F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \vdash F_{11}} \text{hCut} \oplus_L \\
\hline
\frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_4, \Delta_5, F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \oplus F_{10} \vdash F_{11}} \text{hCut} \oplus_L
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_9 \quad h_8 : \Delta_{13}, F_{10} \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, \Delta_{13}, F_9 \multimap F_{10}), F_6 \otimes F_7 \vdash F_{11}} \multimap_L \\
\hline
- : (\Delta_4, \Delta_5), \Delta_{12}, \Delta_{13}, F_9 \multimap F_{10} \vdash F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_4, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_9}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_4, \Delta_5, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut} \multimap_L \\
\hline
\frac{h_1 : \Delta_4 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12} \vdash F_9 \quad h_8 : \Delta_{13}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, \Delta_{13}, F_9 \multimap F_{10}), F_6 \otimes F_7 \vdash F_{11}} \multimap_L \\
\hline
- : (\Delta_4, \Delta_5), \Delta_{12}, \Delta_{13}, F_9 \multimap F_{10} \vdash F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{12} \vdash F_9}{- : \Delta_{12}, \Delta_{13}, \Delta_4, \Delta_5, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_4, \Delta_5 \vdash F_6 \otimes F_7}{- : \Delta_{13}, \Delta_4, \Delta_5, F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_{13}, \Delta_4, \Delta_5, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut} \multimap_L
\end{array}$$

- Case rule I

5.10 Status of W : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash 8}{\bullet h_1 : \Delta_4, !F_2 \vdash 8} W \quad \frac{h_5 : !\Upsilon 7, 8 \vdash F_6}{\bullet h_5 : !\Upsilon 7, 8 \vdash !F_6} !R \\
\hline
- : (\Delta_4, !F_2), !\Upsilon 7 \vdash !F_6 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_4 \vdash 8}{- : !\Upsilon 7, \Delta_4, !F_2 \vdash !F_6} \text{ax/W} \quad \frac{\bullet h_5 : 8, !\Upsilon 7, !F_2 \vdash !F_6}{- : !\Upsilon 7, \Delta_4, !F_2 \vdash !F_6} \text{hCut}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{h_1 : \Delta_4 \vdash F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_7} W \quad \frac{\bullet h_5 : \Delta_6, F_7 \vdash \top}{\top} \top}{\frac{- : (\Delta_4, !F_2), \Delta_6 \vdash \top}{\rightarrow} \text{Cut}} \frac{}{- : \Delta_4, \Delta_6, !F_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} W \quad \frac{\frac{h_5 : \Delta_8, F_9 \vdash F_6 \quad h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \& F_7} \&_R}{\frac{- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \& F_7}{\rightarrow} \text{Cut}} \frac{\frac{h_1 : \Delta_4 \vdash F_9}{\text{ax/W}} \quad \frac{\bullet h_5 : \Delta_8, F_9, !F_2 \vdash F_6 \& F_7}{\text{ax/W}}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \& F_7} \text{hCut}$$

- Case rule \multimap_R

$$\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} W \quad \frac{h_5 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \multimap F_7} \multimap_R}{\frac{- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \multimap F_7}{\rightarrow} \text{Cut}} \frac{\frac{h_1 : \Delta_4 \vdash F_9}{\text{ax/W}} \quad \frac{\bullet h_5 : \Delta_8, F_9, !F_2 \vdash F_6 \multimap F_7}{\text{ax/W}}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \multimap F_7} \text{hCut}$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} W \quad \frac{h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \oplus F_7} \oplus_{R_2}}{\frac{- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \oplus F_7}{\rightarrow} \text{Cut}} \frac{\frac{h_1 : \Delta_4 \vdash F_9}{\text{ax/W}} \quad \frac{\bullet h_5 : \Delta_8, F_9, !F_2 \vdash F_6 \oplus F_7}{\text{ax/W}}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7} \text{hCut}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} W \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \oplus F_7} \oplus_{R_1}}{\frac{- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \oplus F_7}{\rightarrow} \text{Cut}} \frac{\frac{h_1 : \Delta_4 \vdash F_9}{\text{ax/W}} \quad \frac{\bullet h_5 : \Delta_8, F_9, !F_2 \vdash F_6 \oplus F_7}{\text{ax/W}}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7} \text{hCut}$$

- Case rule $\mathbf{1}_L$

$$\frac{\frac{h_1 : \Delta_4 \vdash \mathbf{1}}{\bullet h_1 : \Delta_4, !F_2 \vdash \mathbf{1}} W \quad \frac{h_5 : \Delta_7 \vdash F_6}{\bullet h_5 : \Delta_7, \mathbf{1} \vdash F_6} \mathbf{1}_L}{\frac{- : (\Delta_4, !F_2), \Delta_7 \vdash F_6}{\rightarrow} \text{Cut}} \frac{}{- : \Delta_4, \Delta_7, !F_2 \vdash F_6} \text{ax/W}$$

$$\frac{\frac{h_1 : \Delta_4 \vdash F_8}{\bullet h_1 : \Delta_4, !F_2 \vdash F_8} W \quad \frac{h_5 : \Delta_7, F_8 \vdash F_6}{\bullet h_5 : (\mathbf{1}, \Delta_7), F_8 \vdash F_6} \mathbf{1}_L}{\frac{- : (\Delta_4, !F_2), \mathbf{1}, \Delta_7 \vdash F_6}{\rightarrow} \text{Cut}} \frac{\frac{h_1 : \Delta_4 \vdash F_8}{\text{ax/W}} \quad \frac{\bullet h_5 : \mathbf{1}, \Delta_7, F_8, !F_2 \vdash F_6}{\text{ax/W}}}{- : \mathbf{1}, \Delta_4, \Delta_7, !F_2 \vdash F_6} \text{hCut}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} W \quad \frac{\frac{h_5 : \Delta_8, F_9 \vdash F_6 \quad h_5 : \Delta_{10} \vdash F_7}{\bullet h_5 : (\Delta_8, \Delta_{10}), F_9 \vdash F_6 \otimes F_7} \otimes_R}{\vdash : (\Delta_4, !F_2), \Delta_8, \Delta_{10} \vdash F_6 \otimes F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\vdash : \Delta_{10}, \Delta_4, \Delta_8, !F_2 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, \Delta_8, F_9, !F_2 \vdash F_6 \otimes F_7}{\vdash : \Delta_{10}, \Delta_4, \Delta_8, !F_2 \vdash F_6 \otimes F_7} \text{hCut}}{\vdash : \Delta_{10}, \Delta_4, \Delta_8, !F_2 \vdash F_6 \otimes F_7} \\
\\
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} W \quad \frac{\frac{h_5 : \Delta_8 \vdash F_6 \quad h_5 : \Delta_9, F_{10} \vdash F_7}{\bullet h_5 : (\Delta_8, \Delta_9), F_{10} \vdash F_6 \otimes F_7} \otimes_R}{\vdash : (\Delta_4, !F_2), \Delta_8, \Delta_9 \vdash F_6 \otimes F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\vdash : \Delta_4, \Delta_8, \Delta_9, !F_2 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, \Delta_9, F_{10}, !F_2 \vdash F_6 \otimes F_7}{\vdash : \Delta_4, \Delta_8, \Delta_9, !F_2 \vdash F_6 \otimes F_7} \text{hCut}}{\vdash : \Delta_4, \Delta_8, \Delta_9, !F_2 \vdash F_6 \otimes F_7}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} W \quad \frac{\frac{h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_9 \vdash F_7} W}{\vdash : (\Delta_4, !F_2), \Delta_8, !F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\vdash : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, F_9, !F_2, !F_6 \vdash F_7}{\vdash : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{hCut}}{\vdash : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \\
\\
\frac{\frac{h_1 : \Delta_4 \vdash !F_6}{\bullet h_1 : \Delta_4, !F_2 \vdash !F_6} W \quad \frac{\frac{h_5 : \Delta_8 \vdash F_7}{\bullet h_5 : \Delta_8, !F_6 \vdash F_7} W}{\vdash : (\Delta_4, !F_2), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\vdash : \Delta_4, \Delta_8, !F_2 \vdash F_7}{\vdash : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} W \quad \frac{\frac{h_5 : \Delta_8, F_9, !F_6, !F_6 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_9 \vdash F_7} C}{\vdash : (\Delta_4, !F_2), \Delta_8, !F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\vdash : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, F_9, !F_2, !F_6 \vdash F_7}{\vdash : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{hCut}}{\vdash : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \\
\\
\frac{\frac{h_1 : \Delta_4 \vdash !F_6}{\bullet h_1 : \Delta_4, !F_2 \vdash !F_6} W \quad \frac{\frac{h_5 : \Delta_8, !F_6, !F_6 \vdash F_7}{\bullet h_5 : \Delta_8, !F_6 \vdash F_7} C}{\vdash : (\Delta_4, !F_2), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash !F_6}{\vdash : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, !F_2, !F_6 \vdash F_7}{\vdash : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{hCut}}{\vdash : \Delta_4, \Delta_8, !F_2 \vdash F_7}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} W \quad \frac{\frac{h_5 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_9 \vdash F_7} !L}{\vdash : (\Delta_4, !F_2), \Delta_8, !F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_9}{\vdash : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, F_9, !F_2, !F_6 \vdash F_7}{\vdash : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{hCut}}{\vdash : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \\
\\
\frac{\frac{h_1 : \Delta_4 \vdash !F_6}{\bullet h_1 : \Delta_4, !F_2 \vdash !F_6} W \quad \frac{\frac{h_5 : \Delta_8, F_6 \vdash F_7}{\bullet h_5 : \Delta_8, !F_6 \vdash F_7} !L}{\vdash : (\Delta_4, !F_2), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash !F_6}{\vdash : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, !F_2, !F_6 \vdash F_7}{\vdash : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{hCut}}{\vdash : \Delta_4, \Delta_8, !F_2 \vdash F_7}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} W \quad \frac{h_5 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_{10} \vdash F_8}}{\vdash : (\Delta_4, !F_2), \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L2} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_{10}, !F_2, F_6 \& F_7 \vdash F_8}{\vdash : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{ ax/W}}{\vdash : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{ hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \& F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \& F_7} W \quad \frac{h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8}}{\vdash : (\Delta_4, !F_2), \Delta_9 \vdash F_8} \&_{L2} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \& F_7}{\bullet h_1 : \Delta_4 \vdash F_6 \& F_7} \text{ ax/W} \quad \frac{\bullet h_5 : \Delta_9, !F_2, F_6 \& F_7 \vdash F_8}{\vdash : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ ax/W}}{\vdash : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ hCut}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} W \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_{10} \vdash F_8}}{\vdash : (\Delta_4, !F_2), \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L1} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_{10}, !F_2, F_6 \& F_7 \vdash F_8}{\vdash : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{ ax/W}}{\vdash : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{ hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \& F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \& F_7} W \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8}}{\vdash : (\Delta_4, !F_2), \Delta_9 \vdash F_8} \&_{L1} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \& F_7}{\bullet h_1 : \Delta_4 \vdash F_6 \& F_7} \text{ ax/W} \quad \frac{\bullet h_5 : \Delta_9, !F_2, F_6 \& F_7 \vdash F_8}{\vdash : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ ax/W}}{\vdash : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ hCut}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} W \quad \frac{h_5 : \Delta_9, F_6, F_7, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \otimes F_7), F_{10} \vdash F_8}}{\vdash : (\Delta_4, !F_2), \Delta_9, F_6 \otimes F_7 \vdash F_8} \otimes_L \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_{10}, !F_2, F_6 \otimes F_7 \vdash F_8}{\vdash : \Delta_4, \Delta_9, !F_2, F_6 \otimes F_7 \vdash F_8} \text{ ax/W}}{\vdash : \Delta_4, \Delta_9, !F_2, F_6 \otimes F_7 \vdash F_8} \text{ hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \otimes F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \otimes F_7} W \quad \frac{h_5 : \Delta_9, F_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \otimes F_7 \vdash F_8}}{\vdash : (\Delta_4, !F_2), \Delta_9 \vdash F_8} \otimes_L \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \otimes F_7}{\bullet h_1 : \Delta_4 \vdash F_6 \otimes F_7} \text{ ax/W} \quad \frac{\bullet h_5 : \Delta_9, !F_2, F_6 \otimes F_7 \vdash F_8}{\vdash : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ ax/W}}{\vdash : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ hCut}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} W \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash F_8 \quad h_5 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), F_{10} \vdash F_8}}{\vdash : (\Delta_4, !F_2), \Delta_9, F_6 \oplus F_7 \vdash F_8} \oplus_L \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_{10}, !F_2, F_6 \oplus F_7 \vdash F_8}{\vdash : \Delta_4, \Delta_9, !F_2, F_6 \oplus F_7 \vdash F_8} \text{ ax/W}}{\vdash : \Delta_4, \Delta_9, !F_2, F_6 \oplus F_7 \vdash F_8} \text{ hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \oplus F_7} W \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \oplus F_7 \vdash F_8} \oplus_L \\
\hline
\frac{}{- : (\Delta_4, !F_2), \Delta_9 \vdash F_8} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\hline
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, !F_2, F_6 \oplus F_7 \vdash F_8}{\bullet h_5 : \Delta_9, !F_2 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} W \quad \frac{h_5 : \Delta_9, F_{10} \vdash F_6 \quad h_5 : \Delta_{11}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{11}, F_6 \neg\circ F_7), F_{10} \vdash F_8} \neg\circ_L \\
\hline
\frac{}{- : (\Delta_4, !F_2), \Delta_9, \Delta_{11}, F_6 \neg\circ F_7 \vdash F_8} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\hline
\frac{\frac{h_1 : \Delta_4 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{11}, \Delta_9, F_{10}, !F_2, F_6 \neg\circ F_7 \vdash F_8}{\bullet h_5 : \Delta_{11}, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{ax/W}}{- : \Delta_{11}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{hCut} \\
\\
\frac{h_1 : \Delta_4 \vdash F_{11}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{11}} W \quad \frac{h_5 : \Delta_9 \vdash F_6 \quad h_5 : \Delta_{10}, F_7, F_{11} \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}, F_6 \neg\circ F_7), F_{11} \vdash F_8} \neg\circ_L \\
\hline
\frac{}{- : (\Delta_4, !F_2), \Delta_9, \Delta_{10}, F_6 \neg\circ F_7 \vdash F_8} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\hline
\frac{\frac{h_1 : \Delta_4 \vdash F_{11}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, \Delta_9, F_{11}, !F_2, F_6 \neg\circ F_7 \vdash F_8}{\bullet h_5 : \Delta_{10}, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{hCut} \\
\\
\frac{h_1 : \Delta_4 \vdash F_6 \neg\circ F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \neg\circ F_7} W \quad \frac{h_5 : \Delta_9 \vdash F_6 \quad h_5 : \Delta_{10}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}), F_6 \neg\circ F_7 \vdash F_8} \neg\circ_L \\
\hline
\frac{}{- : (\Delta_4, !F_2), \Delta_9, \Delta_{10} \vdash F_8} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\hline
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \neg\circ F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \neg\circ F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8}{\bullet h_5 : \Delta_{10}, \Delta_9, !F_2 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash p(n_6)}{\bullet h_1 : \Delta_4, !F_2 \vdash p(n_6)} W \quad \frac{}{\bullet h_5 : *, p(n_6) \vdash p(n_6)} I \\
\hline
\frac{}{- : (\Delta_4, !F_2), * \vdash p(n_6)} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\hline
\frac{}{- : \Delta_4, !F_2 \vdash p(n_6)} \text{ax/W}
\end{array}$$

5.11 Status of C : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash 8}{\bullet h_1 : \Delta_4, !F_2 \vdash 8} C \quad \frac{h_5 : !\Upsilon 7, 8 \vdash F_6}{\bullet h_5 : !\Upsilon 7, 8 \vdash !F_6} !R \\
\hline
\frac{}{- : (\Delta_4, !F_2), !\Upsilon 7 \vdash !F_6} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\hline
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash 8}{\bullet h_1 : \Delta_4, !F_2 \vdash 8} \text{ax/W} \quad \frac{\bullet h_5 : 8, !\Upsilon 7 \vdash !F_6}{\bullet h_5 : 8, !\Upsilon 7 \vdash !F_6} \text{ax/W}}{- : !\Upsilon 7, \Delta_4, !F_2, !F_2 \vdash !F_6} \text{hCut} \\
\hline
\frac{}{- : !\Upsilon 7, \Delta_4, !F_2 \vdash !F_6} C
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_7} C \quad \frac{}{\bullet h_5 : \Delta_6, F_7 \vdash \top} \top \\
\hline
\frac{}{- : (\Delta_4, !F_2), \Delta_6 \vdash \top} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\hline
\frac{}{- : \Delta_4, \Delta_6, !F_2 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} C \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6 \quad h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \& F_7} \&_R \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \& F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{- : \Delta_4, \Delta_8, !F_2, !F_2 \vdash F_6 \& F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \& F_7}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \& F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \& F_7} \text{hCut} \\
\hline
- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \& F_7 \quad C
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} C \quad \frac{h_5 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \neg\circ F_7} \neg\circ_R \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \neg\circ F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, !F_2 \vdash F_9}{- : \Delta_4, \Delta_8, F_6, !F_2 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_6, F_9 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \neg\circ F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \neg\circ F_7} \text{hCut} \\
\hline
- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \neg\circ F_7 \quad \neg\circ_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} C \quad \frac{h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \oplus F_7} \oplus_{R_2} \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, !F_2 \vdash F_9}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_9 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7} \text{hCut} \\
\hline
- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7 \quad \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} C \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \oplus F_7} \oplus_{R_1} \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, !F_2 \vdash F_9}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7} \text{hCut} \\
\hline
- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7 \quad \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash \mathbf{1}}{\bullet h_1 : \Delta_4, !F_2 \vdash \mathbf{1}} C \quad \frac{h_5 : \Delta_7 \vdash F_6}{\bullet h_5 : \Delta_7, \mathbf{1} \vdash F_6} 1_L \\
\hline
- : (\Delta_4, !F_2), \Delta_7 \vdash F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_4, \Delta_7, !F_2 \vdash F_6} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_8}{\bullet h_1 : \Delta_4, !F_2 \vdash F_8} C \quad \frac{h_5 : \Delta_7, F_8 \vdash F_6}{\bullet h_5 : (\mathbf{1}, \Delta_7), F_8 \vdash F_6} 1_L \\
\hline
- : (\Delta_4, !F_2), \mathbf{1}, \Delta_7 \vdash F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, !F_2 \vdash F_8}{- : \mathbf{1}, \Delta_4, \Delta_7, !F_2 \vdash F_6} \text{ax/W} \quad \frac{h_5 : \mathbf{1}, \Delta_7, F_8 \vdash F_6}{- : \mathbf{1}, \Delta_4, \Delta_7, !F_2 \vdash F_6} \text{ax/W}}{- : \mathbf{1}, \Delta_4, \Delta_7, !F_2 \vdash F_6} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} C \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6 \quad h_5 : \Delta_{10} \vdash F_7}{\bullet h_5 : (\Delta_8, \Delta_{10}), F_9 \vdash F_6 \otimes F_7} \otimes_R \\
\hline
- : (\Delta_4, !F_2), \Delta_8, \Delta_{10} \vdash F_6 \otimes F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{- : \Delta_{10}, \Delta_4, \Delta_8, !F_2, !F_2 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, \Delta_8, F_9 \vdash F_6 \otimes F_7}{- : \Delta_{10}, \Delta_4, \Delta_8, !F_2 \vdash F_6 \otimes F_7} \text{hCut}}{- : \Delta_{10}, \Delta_4, \Delta_8, !F_2 \vdash F_6 \otimes F_7} C \\
\hline
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} C \quad \frac{h_5 : \Delta_8 \vdash F_6 \quad h_5 : \Delta_9, F_{10} \vdash F_7}{\bullet h_5 : (\Delta_8, \Delta_9), F_{10} \vdash F_6 \otimes F_7} \otimes_R \\
\hline
- : (\Delta_4, !F_2), \Delta_8, \Delta_9 \vdash F_6 \otimes F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{- : \Delta_4, \Delta_8, \Delta_9, !F_2, !F_2 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, \Delta_9, F_{10} \vdash F_6 \otimes F_7}{- : \Delta_4, \Delta_8, \Delta_9, !F_2 \vdash F_6 \otimes F_7} \text{hCut}}{- : \Delta_4, \Delta_8, \Delta_9, !F_2 \vdash F_6 \otimes F_7} C
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} C \quad \frac{h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_9 \vdash F_7} W \\
\hline
- : (\Delta_4, !F_2), \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, !F_2 \vdash F_9}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_9, !F_6 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{hCut}}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} C \\
\hline
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash !F_6}{\bullet h_1 : \Delta_4, !F_2 \vdash !F_6} C \quad \frac{h_5 : \Delta_8 \vdash F_7}{\bullet h_5 : \Delta_8, !F_6 \vdash F_7} W \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_4, \Delta_8, !F_2 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} C \quad \frac{h_5 : \Delta_8, F_9, !F_6 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_9 \vdash F_7} C \\
\hline
- : (\Delta_4, !F_2), \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{- : \Delta_4, \Delta_8, !F_2, !F_2, !F_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, F_9, !F_6 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{hCut}}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} C \\
\hline
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash !F_6}{\bullet h_1 : \Delta_4, !F_2 \vdash !F_6} C \quad \frac{h_5 : \Delta_8, !F_6 \vdash F_7}{\bullet h_5 : \Delta_8, !F_6 \vdash F_7} C \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, !F_2 \vdash !F_6}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_6 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{mCut}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} C \quad \frac{h_5 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_9 \vdash F_7} !L \\
\hline
- : (\Delta_4, !F_2), \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_9}{- : \Delta_4, \Delta_8, !F_2, !F_2, !F_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, F_9, !F_6 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{hCut}}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash !F_6}{\bullet h_1 : \Delta_4, !F_2 \vdash !F_6} C \quad \frac{h_5 : \Delta_8, F_6 \vdash F_7}{\bullet h_5 : \Delta_8, !F_6 \vdash F_7} !L \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash !F_6}{- : \Delta_4, \Delta_8, !F_2, !F_2 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, !F_6 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{hCut} \\
\hline
C
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} C \quad \frac{h_5 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_{10} \vdash F_8} \&_{L2} \\
\hline
- : (\Delta_4, !F_2), \Delta_9, F_6 \& F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{- : \Delta_4, \Delta_9, !F_2, !F_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_{10}, F_6 \& F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{hCut} \\
\hline
C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \& F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \& F_7} C \quad \frac{h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L2} \\
\hline
- : (\Delta_4, !F_2), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \& F_7}{- : \Delta_4, \Delta_9, !F_2, !F_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{hCut} \\
\hline
C
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} C \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_{10} \vdash F_8} \&_{L1} \\
\hline
- : (\Delta_4, !F_2), \Delta_9, F_6 \& F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{- : \Delta_4, \Delta_9, !F_2, !F_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_{10}, F_6 \& F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{hCut} \\
\hline
C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \& F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \& F_7} C \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L1} \\
\hline
- : (\Delta_4, !F_2), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \& F_7}{- : \Delta_4, \Delta_9, !F_2, !F_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{hCut} \\
\hline
C
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} C \quad \frac{h_5 : \Delta_9, F_6, F_7, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \otimes F_7), F_{10} \vdash F_8} \otimes_L \\
\hline
- : (\Delta_4, !F_2), \Delta_9, F_6 \otimes F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{- : \Delta_4, \Delta_9, !F_2, !F_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_{10}, F_6 \otimes F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2, F_6 \otimes F_7 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, !F_2, F_6 \otimes F_7 \vdash F_8} \text{hCut} \\
\hline
C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \otimes F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \otimes F_7} C \quad \frac{h_5 : \Delta_9, F_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \otimes F_7 \vdash F_8} \otimes_L \\
\hline
- : (\Delta_4, !F_2), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \otimes F_7}{- : \Delta_4, \Delta_9, !F_2, !F_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_6 \otimes F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{hCut} \\
\hline
C
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10} \quad \bullet h_1 : \Delta_4, !F_2 \vdash F_{10} \quad C}{- : (\Delta_4, !F_2), \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{Cut} \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash F_8 \quad h_5 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), F_{10} \vdash F_8} \oplus_L \\
\downarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{- : \Delta_4, \Delta_9, !F_2, !F_2, F_6 \oplus F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_{10}, F_6 \oplus F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2, F_6 \oplus F_7 \vdash F_8} \text{hCut}}{- : \Delta_4, \Delta_9, !F_2, F_6 \oplus F_7 \vdash F_8} \text{hCut} \\
\downarrow \\
\frac{- : \Delta_4, \Delta_9, !F_2, F_6 \oplus F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2, F_6 \oplus F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \oplus F_7 \quad \bullet h_1 : \Delta_4, !F_2 \vdash F_6 \oplus F_7 \quad C}{- : (\Delta_4, !F_2), \Delta_9 \vdash F_8} \text{Cut} \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \oplus F_7 \vdash F_8} \oplus_L \\
\downarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \oplus F_7}{- : \Delta_4, \Delta_9, !F_2, !F_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_6 \oplus F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{hCut}}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{hCut} \\
\downarrow \\
\frac{- : \Delta_4, \Delta_9, !F_2 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} C
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10} \quad \bullet h_1 : \Delta_4, !F_2 \vdash F_{10} \quad C}{- : (\Delta_4, !F_2), \Delta_9, \Delta_{11}, F_6 \neg\circ F_7 \vdash F_8} \text{Cut} \quad \frac{h_5 : \Delta_9, F_{10} \vdash F_6 \quad h_5 : \Delta_{11}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{11}, F_6 \neg\circ F_7), F_{10} \vdash F_8} \neg\circ_L \\
\downarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{10}}{- : \Delta_{11}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{11}, \Delta_9, F_{10}, F_6 \neg\circ F_7 \vdash F_8}{- : \Delta_{11}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{hCut}}{- : \Delta_{11}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{hCut} \\
\downarrow \\
\frac{- : \Delta_{11}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8}{- : \Delta_{11}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{11} \quad \bullet h_1 : \Delta_4, !F_2 \vdash F_{11} \quad C}{- : (\Delta_4, !F_2), \Delta_9, \Delta_{10}, F_6 \neg\circ F_7 \vdash F_8} \text{Cut} \quad \frac{h_5 : \Delta_9 \vdash F_6 \quad h_5 : \Delta_{10}, F_7, F_{11} \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}, F_6 \neg\circ F_7), F_{11} \vdash F_8} \neg\circ_L \\
\downarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_{11}}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, \Delta_9, F_{11}, F_6 \neg\circ F_7 \vdash F_8}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{hCut}}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} \text{hCut} \\
\downarrow \\
\frac{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2, F_6 \neg\circ F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \neg\circ F_7 \quad \bullet h_1 : \Delta_4, !F_2 \vdash F_6 \neg\circ F_7 \quad C}{- : (\Delta_4, !F_2), \Delta_9, \Delta_{10} \vdash F_8} \text{Cut} \quad \frac{h_5 : \Delta_9 \vdash F_6 \quad h_5 : \Delta_{10}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}), F_6 \neg\circ F_7 \vdash F_8} \neg\circ_L \\
\downarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_6 \neg\circ F_7}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2, !F_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, \Delta_9, F_6 \neg\circ F_7 \vdash F_8}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{hCut}}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{hCut} \\
\downarrow \\
\frac{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2 \vdash F_8}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2 \vdash F_8} C
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash p(n_6) \quad \bullet h_1 : \Delta_4, !F_2 \vdash p(n_6) \quad C}{- : (\Delta_4, !F_2), * \vdash p(n_6)} \text{Cut} \quad \frac{}{\bullet h_5 : *, p(n_6) \vdash p(n_6)} I \\
\downarrow \\
\frac{- : (\Delta_4, !F_2), * \vdash p(n_6)}{- : \Delta_4, !F_2 \vdash p(n_6)} \text{ax/W}
\end{array}$$

5.12 Status of $!L$: OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash 8}{\bullet h_1 : \Delta_4, !F_2 \vdash 8} !L \quad \frac{h_5 : !\Upsilon 7, 8 \vdash F_6}{\bullet h_5 : !\Upsilon 7, 8 \vdash !F_6} !R \\
\hline
- : (\Delta_4, !F_2), !\Upsilon 7 \vdash !F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash 8}{\bullet h_1 : \Delta_4, !F_2 \vdash 8} \text{ax/W} \quad \frac{h_5 : 8, !\Upsilon 7 \vdash !F_6}{\bullet h_5 : 8, !\Upsilon 7 \vdash !F_6} \text{ax/W}}{- : !\Upsilon 7, \Delta_4, F_2 \vdash !F_6} \text{hCut} \\
\hline
- : !\Upsilon 7, \Delta_4, !F_2 \vdash !F_6 \quad !L
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_7} !L \quad \frac{}{\bullet h_5 : \Delta_6, F_7 \vdash \top} \top \\
\hline
- : (\Delta_4, !F_2), \Delta_6 \vdash \top \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_4, \Delta_6, !F_2 \vdash \top \quad \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} !L \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6 \quad h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \& F_7} \&_R \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \& F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6 \& F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \& F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, F_2 \vdash F_6 \& F_7} \text{hCut} \\
\hline
- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \& F_7 \quad !L
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} !L \quad \frac{h_5 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \multimap F_7} \multimap_R \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \multimap F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \multimap F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, F_6, !F_2 \vdash F_7} \text{hCut} \\
\hline
- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \multimap F_7 \quad \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} !L \quad \frac{h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \oplus F_7} \oplus_{R_2} \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \oplus F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{hCut} \\
\hline
- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7 \quad \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} !L \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \oplus F_7} \oplus_{R_1} \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_6 \oplus F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4, !F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6}{\bullet h_5 : \Delta_8, F_9 \vdash F_6 \oplus F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_6} \text{hCut} \\
\hline
- : \Delta_4, \Delta_8, !F_2 \vdash F_6 \oplus F_7 \quad \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash \mathbf{1} \quad !L \quad \frac{h_5 : \Delta_7 \vdash F_6}{\bullet h_5 : \Delta_7, \mathbf{1} \vdash F_6} 1_L}{- : (\Delta_4, !F_2), \Delta_7 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_4, \Delta_7, !F_2 \vdash F_6} \text{ax/W} \\
\\
\frac{h_1 : \Delta_4, F_2 \vdash F_8 \quad !L \quad \frac{h_5 : \Delta_7, F_8 \vdash F_6}{\bullet h_5 : (\mathbf{1}, \Delta_7), F_8 \vdash F_6} 1_L}{- : (\Delta_4, !F_2), \mathbf{1}, \Delta_7 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_4, !F_2 \vdash F_8} \text{ax/W} \quad \frac{h_5 : \mathbf{1}, \Delta_7, F_8 \vdash F_6}{\bullet h_5 : \mathbf{1}, \Delta_7, F_8 \vdash F_6} \text{ax/W}}{- : \mathbf{1}, \Delta_4, \Delta_7, !F_2 \vdash F_6} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_9 \quad !L \quad \frac{h_5 : \Delta_8, F_9 \vdash F_6 \quad h_5 : \Delta_{10} \vdash F_7}{\bullet h_5 : (\Delta_8, \Delta_{10}), F_9 \vdash F_6 \otimes F_7} \otimes_R}{- : (\Delta_4, !F_2), \Delta_8, \Delta_{10} \vdash F_6 \otimes F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{h_1 : \Delta_4, F_2 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, \Delta_8, F_9 \vdash F_6 \otimes F_7}{\bullet h_5 : \Delta_{10}, \Delta_8, F_9 \vdash F_6 \otimes F_7} \text{ax/W}}{- : \Delta_{10}, \Delta_4, \Delta_8, F_2 \vdash F_6 \otimes F_7} \text{hCut} \\
\frac{}{- : \Delta_{10}, \Delta_4, \Delta_8, !F_2 \vdash F_6 \otimes F_7} !L \\
\\
\frac{h_1 : \Delta_4, F_2 \vdash F_{10} \quad !L \quad \frac{h_5 : \Delta_8 \vdash F_6 \quad h_5 : \Delta_9, F_{10} \vdash F_7}{\bullet h_5 : (\Delta_8, \Delta_9), F_{10} \vdash F_6 \otimes F_7} \otimes_R}{- : (\Delta_4, !F_2), \Delta_8, \Delta_9 \vdash F_6 \otimes F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{h_1 : \Delta_4, F_2 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, \Delta_9, F_{10} \vdash F_6 \otimes F_7}{\bullet h_5 : \Delta_8, \Delta_9, F_{10} \vdash F_6 \otimes F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, \Delta_9, F_2 \vdash F_6 \otimes F_7} \text{hCut} \\
\frac{}{- : \Delta_4, \Delta_8, \Delta_9, !F_2 \vdash F_6 \otimes F_7} !L
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_9 \quad !L \quad \frac{h_5 : \Delta_8, F_9 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_9 \vdash F_7} W}{- : (\Delta_4, !F_2), \Delta_8, !F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_9, !F_6 \vdash F_7}{\bullet h_5 : \Delta_8, F_9, !F_6 \vdash F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{hCut} \\
\\
\frac{h_1 : \Delta_4, F_2 \vdash !F_6 \quad !L \quad \frac{h_5 : \Delta_8 \vdash F_7}{\bullet h_5 : \Delta_8, !F_6 \vdash F_7} W}{- : (\Delta_4, !F_2), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_9 \quad !L \quad \frac{h_5 : \Delta_8, F_9, !F_6, !F_6 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_9 \vdash F_7} C}{- : (\Delta_4, !F_2), \Delta_8, !F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_9, !F_6, !F_6 \vdash F_7}{\bullet h_5 : \Delta_8, F_9, !F_6, !F_6 \vdash F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2, !F_6, !F_6 \vdash F_7} \text{hCut} \\
\frac{}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash !F_6}{\bullet h_1 : \Delta_4, !F_2 \vdash !F_6} !L \quad \frac{h_5 : \Delta_8, !F_6, !F_6 \vdash F_7}{\bullet h_5 : \Delta_8, !F_6 \vdash F_7} C \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, !F_2 \vdash !F_6}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_6, !F_6 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{mCut}
\end{array}$$

- Case rule !L

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_9}{\bullet h_1 : \Delta_4, !F_2 \vdash F_9} !L \quad \frac{h_5 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), F_9 \vdash F_7} !L \\
\hline
- : (\Delta_4, !F_2), \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_9}{- : \Delta_4, \Delta_8, F_2, !F_6 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, F_9, !F_6 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2, !F_6 \vdash F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, F_2, !F_6 \vdash F_7} \text{hCut} \\
\hline
\frac{h_1 : \Delta_4, F_2 \vdash !F_6}{\bullet h_1 : \Delta_4, !F_2 \vdash !F_6} !L \quad \frac{h_5 : \Delta_8, F_6 \vdash F_7}{\bullet h_5 : \Delta_8, !F_6 \vdash F_7} !L \\
\hline
- : (\Delta_4, !F_2), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash !F_6}{- : \Delta_4, \Delta_8, F_2 \vdash F_7} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_6 \vdash F_7}{- : \Delta_4, \Delta_8, !F_2 \vdash F_7} \text{ax/W}}{- : \Delta_4, \Delta_8, F_2 \vdash F_7} \text{hCut} \\
\hline
- : \Delta_4, \Delta_8, !F_2 \vdash F_7 \quad !L
\end{array}$$

- Case rule &_{L2}

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} !L \quad \frac{h_5 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_{10} \vdash F_8} \&_{L2} \\
\hline
- : (\Delta_4, !F_2), \Delta_9, F_6 \& F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{- : \Delta_4, \Delta_9, F_2, F_6 \& F_7 \vdash F_8} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_{10}, F_6 \& F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, F_2, F_6 \& F_7 \vdash F_8} \text{hCut} \\
\hline
\frac{h_1 : \Delta_4, F_2 \vdash F_6 \& F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \& F_7} !L \quad \frac{h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L2} \\
\hline
- : (\Delta_4, !F_2), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_6 \& F_7}{- : \Delta_4, \Delta_9, F_2 \vdash F_8} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6 \& F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, F_2 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_4, \Delta_9, !F_2 \vdash F_8 \quad !L
\end{array}$$

- Case rule &_{L1}

$$\begin{array}{c}
\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} !L \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_{10} \vdash F_8} \&_{L1} \\
\hline
- : (\Delta_4, !F_2), \Delta_9, F_6 \& F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{- : \Delta_4, \Delta_9, F_2, F_6 \& F_7 \vdash F_8} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_{10}, F_6 \& F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2, F_6 \& F_7 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, F_2, F_6 \& F_7 \vdash F_8} \text{hCut} \\
\hline
\frac{h_1 : \Delta_4, F_2 \vdash F_6 \& F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \& F_7} !L \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L1} \\
\hline
- : (\Delta_4, !F_2), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_6 \& F_7}{- : \Delta_4, \Delta_9, F_2 \vdash F_8} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6 \& F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, F_2 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_4, \Delta_9, !F_2 \vdash F_8 \quad !L
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} !L \quad \frac{h_5 : \Delta_9, F_6, F_7, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \otimes F_7), F_{10} \vdash F_8} \otimes_L}{- : (\Delta_4, !F_2), \Delta_9, F_6 \otimes F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_{10}, F_6 \otimes F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_{10}, F_6 \otimes F_7 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, F_2, F_6 \otimes F_7 \vdash F_8} \text{hCut} \\
\frac{- : \Delta_4, \Delta_9, F_2, F_6 \otimes F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2, F_6 \otimes F_7 \vdash F_8} !L \\
\frac{h_1 : \Delta_4, F_2 \vdash F_6 \otimes F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \otimes F_7} !L \quad \frac{h_5 : \Delta_9, F_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \otimes F_7 \vdash F_8} \otimes_L}{- : (\Delta_4, !F_2), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_6 \otimes F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6 \otimes F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \otimes F_7 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, F_2 \vdash F_8} \text{hCut} \\
\frac{- : \Delta_4, \Delta_9, F_2 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} !L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} !L \quad \frac{h_5 : \Delta_9, F_6, F_{10} \vdash F_8 \quad h_5 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), F_{10} \vdash F_8} \oplus_L}{- : (\Delta_4, !F_2), \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_{10}, F_6 \oplus F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_{10}, F_6 \oplus F_7 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, F_2, F_6 \oplus F_7 \vdash F_8} \text{hCut} \\
\frac{- : \Delta_4, \Delta_9, F_2, F_6 \oplus F_7 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2, F_6 \oplus F_7 \vdash F_8} !L \\
\frac{h_1 : \Delta_4, F_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \oplus F_7} !L \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \oplus F_7 \vdash F_8} \oplus_L}{- : (\Delta_4, !F_2), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6 \oplus F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{ax/W}}{- : \Delta_4, \Delta_9, F_2 \vdash F_8} \text{hCut} \\
\frac{- : \Delta_4, \Delta_9, F_2 \vdash F_8}{- : \Delta_4, \Delta_9, !F_2 \vdash F_8} !L
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} !L \quad \frac{h_5 : \Delta_9, F_{10} \vdash F_6 \quad h_5 : \Delta_{11}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{11}, F_6 \multimap F_7), F_{10} \vdash F_8} \multimap_L}{- : (\Delta_4, !F_2), \Delta_9, \Delta_{11}, F_6 \multimap F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{10}} \text{ax/W} \quad \frac{h_5 : \Delta_{11}, \Delta_9, F_{10}, F_6 \multimap F_7 \vdash F_8}{\bullet h_5 : \Delta_{11}, \Delta_9, F_{10}, F_6 \multimap F_7 \vdash F_8} \text{ax/W}}{- : \Delta_{11}, \Delta_4, \Delta_9, F_2, F_6 \multimap F_7 \vdash F_8} \text{hCut} \\
\frac{- : \Delta_{11}, \Delta_4, \Delta_9, F_2, F_6 \multimap F_7 \vdash F_8}{- : \Delta_{11}, \Delta_4, \Delta_9, !F_2, F_6 \multimap F_7 \vdash F_8} !L \\
\frac{h_1 : \Delta_4, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{11}} !L \quad \frac{h_5 : \Delta_9 \vdash F_6 \quad h_5 : \Delta_{10}, F_7, F_{11} \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}, F_6 \multimap F_7), F_{11} \vdash F_8} \multimap_L}{- : (\Delta_4, !F_2), \Delta_9, \Delta_{10}, F_6 \multimap F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_{11}} \text{ax/W} \quad \frac{h_5 : \Delta_{10}, \Delta_9, F_{11}, F_6 \multimap F_7 \vdash F_8}{\bullet h_5 : \Delta_{10}, \Delta_9, F_{11}, F_6 \multimap F_7 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_4, \Delta_9, F_2, F_6 \multimap F_7 \vdash F_8} \text{hCut} \\
\frac{- : \Delta_{10}, \Delta_4, \Delta_9, F_2, F_6 \multimap F_7 \vdash F_8}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2, F_6 \multimap F_7 \vdash F_8} !L \\
\frac{h_1 : \Delta_4, F_2 \vdash F_6 \multimap F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \multimap F_7} !L \quad \frac{h_5 : \Delta_9 \vdash F_6 \quad h_5 : \Delta_{10}, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, \Delta_{10}), F_6 \multimap F_7 \vdash F_8} \multimap_L}{- : (\Delta_4, !F_2), \Delta_9, \Delta_{10} \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_4, F_2 \vdash F_6 \multimap F_7}{\bullet h_1 : \Delta_4, !F_2 \vdash F_6 \multimap F_7} \text{ax/W} \quad \frac{h_5 : \Delta_{10}, \Delta_9, F_6 \multimap F_7 \vdash F_8}{\bullet h_5 : \Delta_{10}, \Delta_9, F_6 \multimap F_7 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_4, \Delta_9, F_2 \vdash F_8} \text{hCut} \\
\frac{- : \Delta_{10}, \Delta_4, \Delta_9, F_2 \vdash F_8}{- : \Delta_{10}, \Delta_4, \Delta_9, !F_2 \vdash F_8} !L
\end{array}$$

- Case rule I

$$\frac{\frac{h_1 : \Delta_4, F_2 \vdash p(n_6)}{\bullet h_1 : \Delta_4, !F_2 \vdash p(n_6)} \text{!}L \quad \frac{}{\bullet h_5 : *, p(n_6) \vdash p(n_6)} I}{\frac{}{- : (\Delta_4, !F_2), * \vdash p(n_6)} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{}{- : \Delta_4, !F_2 \vdash p(n_6)}$$

5.13 Status of $\&_{L2}$: OK

- Case rule $!R$

$$\frac{\frac{h_1 : \Delta_5, F_3 \vdash 9}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash 9} \&_{L2} \quad \frac{h_6 : !\Upsilon 8, 9 \vdash F_7}{\bullet h_6 : !\Upsilon 8, 9 \vdash !F_7} \text{!}R}{\frac{}{- : (\Delta_5, F_2 \& F_3), !\Upsilon 8 \vdash !F_7} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{\frac{h_1 : \Delta_5, F_3 \vdash 9}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash 9} \text{ax/W} \quad \frac{h_6 : !\Upsilon 8, 9 \vdash !F_7}{\bullet h_6 : !\Upsilon 8, 9 \vdash !F_7} \text{ax/W}}{\frac{}{- : !\Upsilon 8, \Delta_5, F_2 \& F_3 \vdash !F_7} \text{hCut}} \&_{L2} \frac{}{- : !\Upsilon 8, \Delta_5, F_2 \& F_3 \vdash !F_7}$$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_8} \&_{L2} \quad \frac{}{\bullet h_6 : \Delta_7, F_8 \vdash \top} \top}{\frac{}{- : (\Delta_5, F_2 \& F_3), \Delta_7 \vdash \top} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{}{- : \Delta_5, \Delta_7, F_2 \& F_3 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8} \&_R}{\frac{}{- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_7 \& F_8} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8} \text{ax/W}}{\frac{}{- : \Delta_5, \Delta_9, F_3 \vdash F_7 \& F_8} \text{hCut}} \&_{L2} \frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \& F_8}$$

- Case rule $\neg \circ_R$

$$\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \neg \circ F_8} \neg \circ_R}{\frac{}{- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_7 \neg \circ F_8} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{\frac{h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8}{\bullet h_6 : \Delta_9, F_{10}, F_7 \vdash F_8} \text{ax/W}}{\frac{}{- : \Delta_5, \Delta_9, F_7, F_2 \& F_3 \vdash F_8} \text{hCut}} \neg \circ_R \frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \neg \circ F_8}$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \oplus_{R_2}}{\frac{}{- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_7 \oplus F_8} \text{Cut}} \xrightarrow{\text{ax/W}} \frac{\frac{h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \text{ax/W}}{\frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_8} \text{hCut}} \oplus_{R_2} \frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \oplus F_8}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \oplus_{R_1} \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \oplus F_8} \text{hCut} \\
\hline
- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \oplus F_8 \quad \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash 1}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash 1} \&_{L2} \quad \frac{h_6 : \Delta_8 \vdash F_7}{\bullet h_6 : \Delta_8, 1 \vdash F_7} 1_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_5, \Delta_8, F_2 \& F_3 \vdash F_7 \quad \text{ax/W} \\
\hline
\frac{h_1 : \Delta_5, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L2} \quad \frac{h_6 : \Delta_8, F_9 \vdash F_7}{\bullet h_6 : (1, \Delta_8), F_9 \vdash F_7} 1_L \\
\hline
- : (\Delta_5, F_2 \& F_3), 1, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_9}{- : 1, \Delta_5, \Delta_8, F_2 \& F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : 1, \Delta_8, F_9 \vdash F_7}{h_6 : 1, \Delta_8, F_9 \vdash F_7} \text{ax/W}}{- : 1, \Delta_5, \Delta_8, F_2 \& F_3 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_{11} \vdash F_8}{\bullet h_6 : (\Delta_9, \Delta_{11}), F_{10} \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9, \Delta_{11} \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{- : \Delta_{11}, \Delta_5, \Delta_9, F_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{11}, \Delta_9, F_{10} \vdash F_7 \otimes F_8}{h_6 : \Delta_{11}, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8} \text{ax/W}}{- : \Delta_{11}, \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8} \&_{L2} \\
\hline
\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L2} \quad \frac{h_6 : \Delta_9 \vdash F_7 \quad h_6 : \Delta_{10}, F_{11} \vdash F_8}{\bullet h_6 : (\Delta_9, \Delta_{10}), F_{11} \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, \Delta_9, F_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_9, F_{11} \vdash F_7 \otimes F_8}{h_6 : \Delta_{10}, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8} \&_{L2}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} W \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, !F_7 \vdash F_8}{h_6 : \Delta_9, F_{10}, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} \text{hCut} \\
\hline
\frac{h_1 : \Delta_5, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash !F_7} \&_{L2} \quad \frac{h_6 : \Delta_9 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} W \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_8 \quad \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_9, F_{10}, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} C}{- : (\Delta_5, F_2 \& F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, !F_7, !F_7, F_2 \& F_3 \vdash F_8} \text{hCut}}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} C \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash !F_7} \&_{L2} \quad \frac{h_6 : \Delta_9, !F_7, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} C}{- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash !F_7} \text{ax/W} \quad \frac{h_6 : \Delta_9, !F_7, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_8} \text{mCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} !L}{- : (\Delta_5, F_2 \& F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_7, F_2 \& F_3 \vdash F_8} \text{hCut}}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} !L \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash !F_7} \&_{L2} \quad \frac{h_6 : \Delta_9, F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} !L}{- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_3 \vdash !F_7} \text{ax/W} \quad \frac{h_6 : \Delta_9, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_3 \vdash F_8} \text{hCut}}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_8} \&_{L2}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L2} \quad \frac{h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \&_{L2}}{- : (\Delta_5, F_2 \& F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_3 \vdash F_{11}} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_3, F_7 \& F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \& F_8 \vdash F_9} \&_{L2} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \& F_8} \&_{L2} \quad \frac{h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2}}{- : (\Delta_5, F_2 \& F_3), \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_5, F_3 \vdash F_7 \& F_8} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L2} \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \&_{L1}}{- : (\Delta_5, F_2 \& F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_3 \vdash F_{11}} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_3, F_7 \& F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \& F_8 \vdash F_9} \&_{L2}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \& F_8} \&_{L2} \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L1} \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_7 \& F_8}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L2}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L2} \quad \frac{h_6 : \Delta_{10}, F_7, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_{11} \vdash F_9} \otimes_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \otimes F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \otimes F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \otimes F_8 \vdash F_9} \&_{L2} \\
\\
\frac{h_1 : \Delta_5, F_3 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \otimes F_8} \&_{L2} \quad \frac{h_6 : \Delta_{10}, F_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_7 \otimes F_8}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \otimes F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L2}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L2} \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_{11} \vdash F_9} \oplus_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_3, F_7 \oplus F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \oplus F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \oplus F_8 \vdash F_9} \&_{L2} \\
\\
\frac{h_1 : \Delta_5, F_3 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \oplus F_8} \&_{L2} \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \oplus_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_7 \oplus F_8}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \oplus F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L2}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L2} \quad \frac{h_6 : \Delta_{10}, F_{11} \vdash F_7 \quad h_6 : \Delta_{12}, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{12}, F_7 \multimap F_8), F_{11} \vdash F_9} \multimap_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10}, \Delta_{12}, F_7 \multimap F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_{12}, \Delta_5, F_3, F_7 \multimap F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_{12}, F_{11}, F_7 \multimap F_8 \vdash F_9}{- : \Delta_{10}, \Delta_{12}, \Delta_5, F_2 \& F_3, F_7 \multimap F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_{12}, \Delta_5, F_2 \& F_3, F_7 \multimap F_8 \vdash F_9} \&_{L2} \\
\\
\frac{h_1 : \Delta_5, F_3 \vdash F_{12}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{12}} \&_{L2} \quad \frac{h_6 : \Delta_{10} \vdash F_7 \quad h_6 : \Delta_{11}, F_8, F_{12} \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}, F_7 \multimap F_8), F_{12} \vdash F_9} \multimap_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{12}}{- : \Delta_{10}, \Delta_{11}, \Delta_5, F_3, F_7 \multimap F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_{11}, F_{12}, F_7 \multimap F_8 \vdash F_9}{- : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \& F_3, F_7 \multimap F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \& F_3, F_7 \multimap F_8 \vdash F_9} \&_{L2}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash F_7 \multimap F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \multimap F_8} \&_{L2} \quad \frac{h_6 : \Delta_{10} \vdash F_7 \quad h_6 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}), F_7 \multimap F_8 \vdash F_9} \multimap_L \\
\hline
\frac{}{- : (\Delta_5, F_2 \& F_3), \Delta_{10}, \Delta_{11} \vdash F_9} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_7 \multimap F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \multimap F_8} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \text{ax/W}}{\frac{}{- : \Delta_{10}, \Delta_{11}, \Delta_5, F_3 \vdash F_9} \text{hCut}}{\frac{}{- : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L2}}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_3 \vdash p(n_7)}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash p(n_7)} \&_{L2} \quad \frac{}{\bullet h_6 : *, p(n_7) \vdash p(n_7)} I \\
\hline
\frac{}{- : (\Delta_5, F_2 \& F_3), * \vdash p(n_7)} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_5, F_2 \& F_3 \vdash p(n_7)} \text{ax/W}
\end{array}$$

5.14 Status of $\&_{L1}$: OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash 9}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash 9} \&_{L1} \quad \frac{h_6 : !\Upsilon 8, 9 \vdash F_7}{\bullet h_6 : !\Upsilon 8, 9 \vdash !F_7} !R \\
\hline
\frac{}{- : (\Delta_5, F_2 \& F_3), !\Upsilon 8 \vdash !F_7} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash 9}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash 9} \text{ax/W} \quad \frac{h_6 : 9, !\Upsilon 8 \vdash !F_7}{\bullet h_6 : 9, !\Upsilon 8 \vdash !F_7} \text{ax/W}}{\frac{}{- : !\Upsilon 8, \Delta_5, F_2 \vdash !F_7} \text{hCut}}{\frac{}{- : !\Upsilon 8, \Delta_5, F_2 \& F_3 \vdash !F_7} \&_{L1}}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_8} \&_{L1} \quad \frac{}{\bullet h_6 : \Delta_7, F_8 \vdash \top} \top \\
\hline
\frac{}{- : (\Delta_5, F_2 \& F_3), \Delta_7 \vdash \top} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_5, \Delta_7, F_2 \& F_3 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L1} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8} \&_R \\
\hline
\frac{}{- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_7 \& F_8} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8} \text{ax/W}}{\frac{}{- : \Delta_5, \Delta_9, F_2 \vdash F_7 \& F_8} \text{hCut}}{\frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \& F_8} \&_{L1}}
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L1} \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \multimap F_8} \multimap_R \\
\hline
\frac{}{- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_7 \multimap F_8} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8}{\bullet h_6 : \Delta_9, F_{10}, F_7 \vdash F_8} \text{ax/W}}{\frac{}{- : \Delta_5, \Delta_9, F_7, F_2 \& F_3 \vdash F_8} \text{hCut}}{\frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \multimap F_8} \multimap_R}
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L1} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \oplus_{R2} \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \oplus F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \oplus F_8 \quad \oplus_{R2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L1} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \oplus_{R1} \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \oplus F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7} \text{hCut} \\
\hline
- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \oplus F_8 \quad \oplus_{R1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash 1}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash 1} \&_{L1} \quad \frac{h_6 : \Delta_8 \vdash F_7}{\bullet h_6 : \Delta_8, 1 \vdash F_7} 1_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_5, \Delta_8, F_2 \& F_3 \vdash F_7 \quad \text{ax/W} \\
\hline
\frac{h_1 : \Delta_5, F_2 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L1} \quad \frac{h_6 : \Delta_8, F_9 \vdash F_7}{\bullet h_6 : (1, \Delta_8), F_9 \vdash F_7} 1_L \\
\hline
- : (\Delta_5, F_2 \& F_3), 1, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_9}{- : 1, \Delta_5, \Delta_8, F_2 \& F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : 1, \Delta_8, F_9 \vdash F_7}{- : 1, \Delta_5, \Delta_8, F_2 \& F_3 \vdash F_7} \text{ax/W}}{- : 1, \Delta_5, \Delta_8, F_2 \& F_3 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L1} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_{11} \vdash F_8}{\bullet h_6 : (\Delta_9, \Delta_{11}), F_{10} \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9, \Delta_{11} \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{- : \Delta_{11}, \Delta_5, \Delta_9, F_2 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{11}, \Delta_9, F_{10} \vdash F_7 \otimes F_8}{- : \Delta_{11}, \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8} \text{ax/W}}{- : \Delta_{11}, \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8 \quad \&_{L1} \\
\hline
\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L1} \quad \frac{h_6 : \Delta_9 \vdash F_7 \quad h_6 : \Delta_{10}, F_{11} \vdash F_8}{\bullet h_6 : (\Delta_9, \Delta_{10}), F_{11} \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, \Delta_9, F_2 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_9, F_{11} \vdash F_7 \otimes F_8}{- : \Delta_{10}, \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_7 \otimes F_8 \quad \&_{L1}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L1} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} W \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, !F_7 \vdash F_8}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash !F_7} \&_{L1} \quad \frac{h_6 : \Delta_9 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} W \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_8} \text{ax/W} \\
\text{Cut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L1} \quad \frac{h_6 : \Delta_9, F_{10}, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} C \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9, !F_7 \vdash F_8 \\
\rightarrow \\
\frac{}{- : \Delta_5, \Delta_9, !F_7, !F_7, F_2 \& F_3 \vdash F_8} \text{ax/W} \quad \frac{}{- : \Delta_5, \Delta_9, !F_7, !F_7 \vdash F_8} \text{ax/W} \\
\frac{}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} C \quad \text{hCut} \\
\hline
\frac{h_1 : \Delta_5, F_2 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash !F_7} \&_{L1} \quad \frac{h_6 : \Delta_9, !F_7, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} C \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash !F_7} \text{ax/W} \quad \frac{}{- : \Delta_5, \Delta_9, !F_7, !F_7 \vdash F_8} \text{ax/W} \\
\frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_8} \text{mCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{10}} \&_{L1} \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} !L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9, !F_7 \vdash F_8 \\
\rightarrow \\
\frac{}{- : \Delta_5, \Delta_9, F_7, F_2 \& F_3 \vdash F_8} \text{ax/W} \quad \frac{}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} !L \\
\frac{}{- : \Delta_5, \Delta_9, !F_7, F_2 \& F_3 \vdash F_8} \text{hCut} \\
\hline
\frac{h_1 : \Delta_5, F_2 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash !F_7} \&_{L1} \quad \frac{h_6 : \Delta_9, F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} !L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{}{- : \Delta_5, \Delta_9, F_2 \vdash !F_7} \text{ax/W} \quad \frac{}{- : \Delta_5, \Delta_9, !F_7 \vdash F_8} \text{ax/W} \\
\frac{}{- : \Delta_5, \Delta_9, F_2 \vdash F_8} \text{hCut} \\
\frac{}{- : \Delta_5, \Delta_9, F_2 \& F_3 \vdash F_8} \&_{L1}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L1} \quad \frac{h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \&_{L2} \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_5, F_8, F_2 \& F_3 \vdash F_9} \text{ax/W} \quad \frac{}{- : \Delta_{10}, \Delta_5, F_8, F_2 \& F_3 \vdash F_9} \text{ax/W} \\
\frac{}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \& F_8 \vdash F_9} \&_{L2} \quad \text{hCut} \\
\hline
\frac{h_1 : \Delta_5, F_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \& F_8} \&_{L1} \quad \frac{h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2} \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10} \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_7 \& F_8} \text{ax/W} \quad \frac{}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_7 \& F_8} \text{ax/W} \\
\frac{}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L1} \quad \text{hCut}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L1} \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \&_{L1} \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_2, F_7 \& F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \& F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \& F_8 \vdash F_9} \&_{L1}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \& F_8} \&_{L1} \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L1} \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \& F_8}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L1}
\end{array}$$

• Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L1} \quad \frac{h_6 : \Delta_{10}, F_7, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_{11} \vdash F_9} \otimes_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_2, F_7 \otimes F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \otimes F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \otimes F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \otimes F_8 \vdash F_9} \&_{L1}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \otimes F_8} \&_{L1} \quad \frac{h_6 : \Delta_{10}, F_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \otimes F_8}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \otimes F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L1}
\end{array}$$

• Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L1} \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_{11} \vdash F_9} \oplus_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_2, F_7 \oplus F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \oplus F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3, F_7 \oplus F_8 \vdash F_9} \&_{L1}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \oplus F_8} \&_{L1} \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \oplus_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \oplus F_8}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \oplus F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_2 \& F_3 \vdash F_9} \&_{L1}
\end{array}$$

• Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{11}} \&_{L1} \quad \frac{h_6 : \Delta_{10}, F_{11} \vdash F_7 \quad h_6 : \Delta_{12}, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{12}, F_7 \multimap F_8), F_{11} \vdash F_9} \multimap_L \\
\hline
- : (\Delta_5, F_2 \& F_3), \Delta_{10}, \Delta_{12}, F_7 \multimap F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{- : \Delta_{10}, \Delta_{12}, \Delta_5, F_2, F_7 \multimap F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_{12}, F_{11}, F_7 \multimap F_8 \vdash F_9}{- : \Delta_{10}, \Delta_{12}, \Delta_5, F_2 \& F_3, F_7 \multimap F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_{12}, \Delta_5, F_2 \& F_3, F_7 \multimap F_8 \vdash F_9} \&_{L1}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{12}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_{12}} \&_{L1} \quad \frac{\frac{h_6 : \Delta_{10} \vdash F_7 \quad h_6 : \Delta_{11}, F_8, F_{12} \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}, F_7 \multimap F_8), F_{12} \vdash F_9} \multimap_L}{\frac{- : (\Delta_5, F_2 \& F_3), \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{12}}{\bullet h_1 : \Delta_5, F_2 \vdash F_{12}} \text{ax/W} \quad \frac{\frac{h_6 : \Delta_{10}, \Delta_{11}, F_{12}, F_7 \multimap F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}, F_7 \multimap F_8), F_{12} \vdash F_9} \text{ax/W}}{\frac{- : \Delta_{10}, \Delta_{11}, \Delta_5, F_2, F_7 \multimap F_8 \vdash F_9}{\rightarrow} \text{hCut}} \&_{L1} \\
\frac{- : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \& F_3, F_7 \multimap F_8 \vdash F_9}{\rightarrow} \&_{L1} \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \multimap F_8}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_7 \multimap F_8} \&_{L1} \quad \frac{\frac{h_6 : \Delta_{10} \vdash F_7 \quad h_6 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}), F_7 \multimap F_8 \vdash F_9} \multimap_L}{\frac{- : (\Delta_5, F_2 \& F_3), \Delta_{10}, \Delta_{11} \vdash F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \multimap F_8}{\bullet h_1 : \Delta_5, F_2 \vdash F_7 \multimap F_8} \text{ax/W} \quad \frac{\frac{h_6 : \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}, F_7 \multimap F_8), F_7 \multimap F_8 \vdash F_9} \text{ax/W}}{\frac{- : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \vdash F_9}{\rightarrow} \text{hCut}} \&_{L1} \\
\frac{- : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \& F_3 \vdash F_9}{\rightarrow} \&_{L1}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash p(n_7)}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash p(n_7)} \&_{L1} \quad \frac{}{\bullet h_6 : *, p(n_7) \vdash p(n_7)} I}{\frac{- : (\Delta_5, F_2 \& F_3), * \vdash p(n_7)}{\rightarrow} \text{Cut}} \\
\frac{}{\rightarrow} \text{ax/W} \\
\frac{}{- : \Delta_5, F_2 \& F_3 \vdash p(n_7)} \text{ax/W}
\end{array}$$

5.15 Status of \otimes_L : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash 9}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash 9} \otimes_L \quad \frac{\frac{h_6 : !\Upsilon 8, 9 \vdash F_7}{\bullet h_6 : !\Upsilon 8, 9 \vdash !F_7} !R}{\frac{- : (\Delta_5, F_2 \otimes F_3), !\Upsilon 8 \vdash !F_7}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash 9}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash 9} \text{ax/W} \quad \frac{\frac{h_6 : 9, !\Upsilon 8 \vdash !F_7}{\bullet h_6 : 9, !\Upsilon 8 \vdash !F_7} \text{ax/W}}{\frac{- : !\Upsilon 8, \Delta_5, F_2, F_3 \vdash !F_7}{\rightarrow} \text{hCut}} \otimes_L \\
\frac{- : !\Upsilon 8, \Delta_5, F_2 \otimes F_3 \vdash !F_7}{\rightarrow} \otimes_L
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_8} \otimes_L \quad \frac{}{\bullet h_6 : \Delta_7, F_8 \vdash \top} \top}{\frac{- : (\Delta_5, F_2 \otimes F_3), \Delta_7 \vdash \top}{\rightarrow} \text{Cut}} \\
\frac{}{\rightarrow} \top \\
\frac{}{- : \Delta_5, \Delta_7, F_2 \otimes F_3 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}} \otimes_L \quad \frac{\frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8} \&_R}{\frac{- : (\Delta_5, F_2 \otimes F_3), \Delta_9 \vdash F_7 \& F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8} \text{ax/W}}{\frac{- : \Delta_5, \Delta_9, F_2, F_3 \vdash F_7 \& F_8}{\rightarrow} \text{hCut}} \otimes_L \\
\frac{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \& F_8}{\rightarrow} \otimes_L
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}} \otimes_L \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \multimap F_8} \multimap_R \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_9 \vdash F_7 \multimap F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_7, F_2 \otimes F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \multimap F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \multimap F_8} \text{hCut} \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}} \otimes_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \oplus_{R_2} \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_9 \vdash F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \oplus F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \oplus F_8} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}} \otimes_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \oplus_{R_1} \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_9 \vdash F_7 \oplus F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \oplus F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \oplus F_8} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash 1}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash 1} \otimes_L \quad \frac{h_6 : \Delta_8 \vdash F_7}{\bullet h_6 : \Delta_8, 1 \vdash F_7} 1_L \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_5, \Delta_8, F_2 \otimes F_3 \vdash F_7 \quad \text{ax/W} \\
\\
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_9} \otimes_L \quad \frac{h_6 : \Delta_8, F_9 \vdash F_7}{\bullet h_6 : (1, \Delta_8), F_9 \vdash F_7} 1_L \\
\hline
- : (\Delta_5, F_2 \otimes F_3), 1, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_9}{- : 1, \Delta_5, \Delta_8, F_2 \otimes F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : 1, \Delta_8, F_9 \vdash F_7}{- : 1, \Delta_5, \Delta_8, F_2 \otimes F_3 \vdash F_7} \text{ax/W}}{- : 1, \Delta_5, \Delta_8, F_2 \otimes F_3 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}} \otimes_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_{11} \vdash F_8}{\bullet h_6 : (\Delta_9, \Delta_{11}), F_{10} \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_9, \Delta_{11} \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{- : \Delta_{11}, \Delta_5, \Delta_9, F_2, F_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{11}, \Delta_9, F_{10} \vdash F_7 \otimes F_8}{- : \Delta_{11}, \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \otimes F_8} \text{ax/W}}{- : \Delta_{11}, \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \otimes F_8} \text{hCut} \otimes_L \\
\\
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}} \otimes_L \quad \frac{h_6 : \Delta_9 \vdash F_7 \quad h_6 : \Delta_{10}, F_{11} \vdash F_8}{\bullet h_6 : (\Delta_9, \Delta_{10}), F_{11} \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, \Delta_9, F_2, F_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_9, F_{11} \vdash F_7 \otimes F_8}{- : \Delta_{10}, \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \otimes F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_7 \otimes F_8} \text{hCut} \otimes_L
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}} \otimes_L \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} W}{- : (\Delta_5, F_2 \otimes F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, !F_7, F_2 \otimes F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, !F_7 \vdash F_8}{- : \Delta_5, \Delta_9, !F_7, F_2 \otimes F_3 \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, !F_7, F_2 \otimes F_3 \vdash F_8} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash !F_7} \otimes_L \frac{h_6 : \Delta_9 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} W}{- : (\Delta_5, F_2 \otimes F_3), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_8}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_8} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}} \otimes_L \frac{h_6 : \Delta_9, F_{10}, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} C}{- : (\Delta_5, F_2 \otimes F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, !F_7, !F_7, F_2 \otimes F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, !F_7, !F_7 \vdash F_8}{- : \Delta_5, \Delta_9, !F_7, F_2 \otimes F_3 \vdash F_8} \text{hCut}}{- : \Delta_5, \Delta_9, !F_7, F_2 \otimes F_3 \vdash F_8} C \\
\\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash !F_7} \otimes_L \frac{h_6 : \Delta_9, !F_7, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} C}{- : (\Delta_5, F_2 \otimes F_3), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash !F_7}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, !F_7, !F_7 \vdash F_8}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_8} \text{mCut}}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_8}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}} \otimes_L \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} !L}{- : (\Delta_5, F_2 \otimes F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_7, F_2 \otimes F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8}{- : \Delta_5, \Delta_9, !F_7, F_2 \otimes F_3 \vdash F_8} \text{hCut}}{- : \Delta_5, \Delta_9, !F_7, F_2 \otimes F_3 \vdash F_8} !L \\
\\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash !F_7} \otimes_L \frac{h_6 : \Delta_9, F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} !L}{- : (\Delta_5, F_2 \otimes F_3), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash !F_7}{- : \Delta_5, \Delta_9, F_2, F_3 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, !F_7 \vdash F_8}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_8} \text{hCut}}{- : \Delta_5, \Delta_9, F_2 \otimes F_3 \vdash F_8} \otimes_L
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}} \otimes_L \frac{h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \&_{L2}}{- : (\Delta_5, F_2 \otimes F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_8, F_2 \otimes F_3 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_{11}, F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_7 \& F_8, F_2 \otimes F_3 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_7 \& F_8, F_2 \otimes F_3 \vdash F_9} \&_{L2}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_7 \& F_8} \otimes_L \quad \frac{h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2} \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \& F_8}{- : \Delta_{10}, \Delta_5, F_2, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \text{hCut} \otimes_L
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}} \otimes_L \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \&_{L1} \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_{11}, F_7 \vdash F_9}{h_6 : \Delta_{10}, F_{11}, F_7 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_7, F_2 \otimes F_3 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_5, F_7 \& F_8, F_2 \otimes F_3 \vdash F_9} \&_{L1} \\
\hline
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_7 \& F_8} \otimes_L \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L1} \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \& F_8}{- : \Delta_{10}, \Delta_5, F_2, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \text{hCut} \otimes_L
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}} \otimes_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_{11} \vdash F_9} \otimes_L \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \otimes F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2, F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3, F_7 \otimes F_8 \vdash F_9} \text{hCut} \otimes_L \\
\hline
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_7 \otimes F_8} \otimes_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \otimes F_8}{- : \Delta_{10}, \Delta_5, F_2, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \otimes F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \text{hCut} \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}} \otimes_L \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_{11} \vdash F_9} \oplus_L \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \oplus F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2, F_3, F_7 \oplus F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut} \otimes_L \\
\hline
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_7 \oplus F_8} \otimes_L \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \oplus_L \\
\hline
- : (\Delta_5, F_2 \otimes F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \oplus F_8}{- : \Delta_{10}, \Delta_5, F_2, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \oplus F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \text{hCut} \otimes_L
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{11}} \otimes_L \quad \frac{\frac{h_6 : \Delta_{10}, F_{11} \vdash F_7 \quad h_6 : \Delta_{12}, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{12}, F_7 \neg\circ F_8), F_{11} \vdash F_9} \neg\circ_L}{\neg : (\Delta_5, F_2 \otimes F_3), \Delta_{10}, \Delta_{12}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\neg : \Delta_{10}, \Delta_{12}, \Delta_5, F_2, F_3, F_7 \neg\circ F_8 \vdash F_9} \text{ax/W} \quad \frac{\frac{\bullet h_6 : \Delta_{10}, \Delta_{12}, F_{11}, F_7 \neg\circ F_8 \vdash F_9}{\neg : \Delta_{10}, \Delta_{12}, \Delta_5, F_2 \otimes F_3, F_7 \neg\circ F_8 \vdash F_9} \text{hCut}}{\neg : \Delta_{10}, \Delta_{12}, \Delta_5, F_2 \otimes F_3, F_7 \neg\circ F_8 \vdash F_9} \otimes_L \\
\\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{12}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_{12}} \otimes_L \quad \frac{\frac{h_6 : \Delta_{10} \vdash F_7 \quad h_6 : \Delta_{11}, F_8, F_{12} \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}, F_7 \neg\circ F_8), F_{12} \vdash F_9} \neg\circ_L}{\neg : (\Delta_5, F_2 \otimes F_3), \Delta_{10}, \Delta_{11}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{12}}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_2, F_3, F_7 \neg\circ F_8 \vdash F_9} \text{ax/W} \quad \frac{\frac{\bullet h_6 : \Delta_{10}, \Delta_{11}, F_{12}, F_7 \neg\circ F_8 \vdash F_9}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \otimes F_3, F_7 \neg\circ F_8 \vdash F_9} \text{hCut}}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \otimes F_3, F_7 \neg\circ F_8 \vdash F_9} \otimes_L \\
\\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \neg\circ F_8}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_7 \neg\circ F_8} \otimes_L \quad \frac{\frac{h_6 : \Delta_{10} \vdash F_7 \quad h_6 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}), F_7 \neg\circ F_8 \vdash F_9} \neg\circ_L}{\neg : (\Delta_5, F_2 \otimes F_3), \Delta_{10}, \Delta_{11} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \neg\circ F_8}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_2, F_3 \vdash F_9} \text{ax/W} \quad \frac{\frac{\bullet h_6 : \Delta_{10}, \Delta_{11}, F_7 \neg\circ F_8 \vdash F_9}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \text{hCut}}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \otimes F_3 \vdash F_9} \otimes_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash p(n_7)}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash p(n_7)} \otimes_L \quad \frac{\bullet h_6 : *, p(n_7) \vdash p(n_7)}{I} \\
\frac{\neg : (\Delta_5, F_2 \otimes F_3), * \vdash p(n_7)}{\rightarrow} \text{Cut} \\
\frac{\neg : \Delta_5, F_2 \otimes F_3 \vdash p(n_7)}{\text{ax/W}}
\end{array}$$

5.16 Status of \oplus_L : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash 9 \quad h_1 : \Delta_5, F_3 \vdash 9}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash 9} \oplus_L \quad \frac{\frac{h_6 : !\Upsilon 8, 9 \vdash F_7}{\bullet h_6 : !\Upsilon 8, 9 \vdash !F_7} !R}{\neg : (\Delta_5, F_2 \oplus F_3), !\Upsilon 8 \vdash !F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash 9}{\neg : !\Upsilon 8, \Delta_5, F_2 \vdash !F_7} \text{ax/W} \quad \frac{\frac{\bullet h_6 : 9, !\Upsilon 8 \vdash !F_7}{h_6 : 9, !\Upsilon 8 \vdash !F_7} \text{hCut} \quad \frac{h_1 : \Delta_5, F_3 \vdash 9}{\neg : !\Upsilon 8, \Delta_5, F_3 \vdash !F_7} \text{ax/W} \quad \frac{\frac{\bullet h_6 : 9, !\Upsilon 8 \vdash !F_7}{h_6 : 9, !\Upsilon 8 \vdash !F_7} \text{hCut}}{\neg : !\Upsilon 8, \Delta_5, F_2 \oplus F_3 \vdash !F_7} \oplus_L
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_8 \quad h_1 : \Delta_5, F_3 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_8} \oplus_L \quad \frac{\bullet h_6 : \Delta_7, F_8 \vdash \top}{\top} \\
\frac{\neg : (\Delta_5, F_2 \oplus F_3), \Delta_7 \vdash \top}{\rightarrow} \text{Cut} \\
\frac{\neg : \Delta_5, \Delta_7, F_2 \oplus F_3 \vdash \top}{\top}
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \oplus_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \& F_8} \&_R}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9 \vdash F_7 \& F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{h_6 : \Delta_9, F_{10} \vdash F_7} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_7} \text{hCut} \quad \frac{\frac{h_6 : \Delta_9, F_{10} \vdash F_8}{h_6 : \Delta_9, F_{10} \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{h_6 : \Delta_9, F_{10} \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_8} \text{hCut}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_7 \& F_8} \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \oplus_L \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \neg\circ F_8} \neg\circ_R}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9 \vdash F_7 \neg\circ F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8}{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_7, F_2 \oplus F_3 \vdash F_8} \text{hCut}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_7 \neg\circ F_8} \neg\circ_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \oplus_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \oplus_{R_2}}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9 \vdash F_7 \oplus F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{h_6 : \Delta_9, F_{10} \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_8} \text{hCut}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_7 \oplus F_8} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \oplus_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \oplus F_8} \oplus_{R_1}}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9 \vdash F_7 \oplus F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{h_6 : \Delta_9, F_{10} \vdash F_7} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_7} \text{hCut}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_7 \oplus F_8} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash 1 \quad h_1 : \Delta_5, F_3 \vdash 1}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash 1} \oplus_L \quad \frac{h_6 : \Delta_8 \vdash F_7}{\bullet h_6 : \Delta_8, 1 \vdash F_7} 1_L}{- : (\Delta_5, F_2 \oplus F_3), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_5, \Delta_8, F_2 \oplus F_3 \vdash F_7}{- : \Delta_5, \Delta_8, F_2 \oplus F_3 \vdash F_7} \text{ax/W} \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_9 \quad h_1 : \Delta_5, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_9} \oplus_L \quad \frac{h_6 : \Delta_8, F_9 \vdash F_7}{\bullet h_6 : (1, \Delta_8), F_9 \vdash F_7} 1_L}{- : (\Delta_5, F_2 \oplus F_3), 1, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_9} \text{ax/W} \quad \frac{h_6 : 1, \Delta_8, F_9 \vdash F_7}{h_6 : 1, \Delta_8, F_9 \vdash F_7} \text{ax/W}}{- : 1, \Delta_5, \Delta_8, F_2 \oplus F_3 \vdash F_7} \text{hCut}}{- : 1, \Delta_5, \Delta_8, F_2 \oplus F_3 \vdash F_7}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \oplus_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_{11} \vdash F_8}{\bullet h_6 : (\Delta_9, \Delta_{11}), F_{10} \vdash F_7 \otimes F_8} \otimes_R}{\frac{}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9, \Delta_{11} \vdash F_7 \otimes F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8} \text{ax/W}}{\frac{}{- : \Delta_{11}, \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_7 \otimes F_8} \text{hCut}} \otimes_R \\
\frac{}{- : \Delta_{11} \vdash F_8} \text{ax/W} \\
\frac{}{- : \Delta_5, F_2 \vdash F_{11} \quad h_1 : \Delta_5, F_3 \vdash F_{11}} \oplus_L \quad \frac{h_6 : \Delta_9 \vdash F_7 \quad h_6 : \Delta_{10}, F_{11} \vdash F_8}{\bullet h_6 : (\Delta_9, \Delta_{10}), F_{11} \vdash F_7 \otimes F_8} \otimes_R}{\frac{}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{}{- : \Delta_9 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_2 \oplus F_3 \vdash F_8} \text{ax/W}}{\frac{}{- : \Delta_{10}, \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_7 \otimes F_8} \text{hCut}} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \oplus_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} W}{\frac{}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, !F_7, F_2 \oplus F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, !F_7 \vdash F_8}{h_6 : \Delta_9, F_{10}, !F_7, F_8 \vdash F_8} \text{ax/W}}{\frac{}{- : \Delta_5, \Delta_9, !F_7, F_2 \oplus F_3 \vdash F_8} \text{hCut}} \\
\frac{h_1 : \Delta_5, F_2 \vdash !F_7 \quad h_1 : \Delta_5, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash !F_7} \oplus_L \quad \frac{h_6 : \Delta_9 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} W}{\frac{}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9 \vdash F_8} \text{Cut}} \\
\rightarrow \\
\frac{}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_8} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \oplus_L \quad \frac{h_6 : \Delta_9, F_{10}, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} C}{\frac{}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, !F_7, !F_7, F_2 \oplus F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, !F_7, !F_7 \vdash F_8}{h_6 : \Delta_9, F_{10}, !F_7, F_8 \vdash F_8} \text{ax/W}}{\frac{}{- : \Delta_5, \Delta_9, !F_7, F_2 \oplus F_3 \vdash F_8} \text{hCut}} C \\
\frac{h_1 : \Delta_5, F_2 \vdash !F_7 \quad h_1 : \Delta_5, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash !F_7} \oplus_L \quad \frac{h_6 : \Delta_9, !F_7, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} C}{\frac{}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9 \vdash F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash !F_7}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, !F_7, !F_7 \vdash F_8}{h_6 : \Delta_9, !F_7, F_8 \vdash F_8} \text{ax/W}}{\frac{}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_8} \text{mCut}}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}} \oplus_L \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_{10} \vdash F_8} !L}{\frac{}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_7, F_2 \oplus F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8}{h_6 : \Delta_9, F_{10}, F_7, F_8 \vdash F_8} \text{ax/W}}{\frac{}{- : \Delta_5, \Delta_9, F_7, F_2 \oplus F_3 \vdash F_8} \text{hCut}} !L \\
\frac{}{- : \Delta_5, \Delta_9, !F_7, F_2 \oplus F_3 \vdash F_8} !L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash !F_7 \quad h_1 : \Delta_5, F_3 \vdash !F_7}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash !F_7} \oplus_L \quad \frac{h_6 : \Delta_9, F_7 \vdash F_8}{\bullet h_6 : \Delta_9, !F_7 \vdash F_8} !L}{- : (\Delta_5, F_2 \oplus F_3), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash !F_7}{- : \Delta_5, \Delta_9, F_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, !F_7 \vdash F_8}{- : \Delta_5, \Delta_9, F_3 \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \oplus F_3 \vdash F_8} \text{hCut} \oplus_L
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11} \quad h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}} \oplus_L \quad \frac{h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \&_{L2}}{- : (\Delta_5, F_2 \oplus F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_8, F_2 \oplus F_3 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_{11}, F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_7 \& F_8, F_2 \oplus F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_7 \& F_8, F_2 \oplus F_3 \vdash F_9} \text{hCut} \&_{L2} \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \& F_8 \quad h_1 : \Delta_5, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_7 \& F_8} \oplus_L \quad \frac{h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2}}{- : (\Delta_5, F_2 \oplus F_3), \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \& F_8}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \oplus F_3 \vdash F_9} \text{hCut} \oplus_L
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11} \quad h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}} \oplus_L \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_{11} \vdash F_9} \&_{L1}}{- : (\Delta_5, F_2 \oplus F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_7, F_2 \oplus F_3 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_{11}, F_7 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_7 \& F_8, F_2 \oplus F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_7 \& F_8, F_2 \oplus F_3 \vdash F_9} \text{hCut} \&_{L1} \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \& F_8 \quad h_1 : \Delta_5, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_7 \& F_8} \oplus_L \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L1}}{- : (\Delta_5, F_2 \oplus F_3), \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \& F_8}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \oplus F_3 \vdash F_9} \text{hCut} \oplus_L
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11} \quad h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}} \oplus_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_{11} \vdash F_9} \otimes_L}}{- : (\Delta_5, F_2 \oplus F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_7, F_8, F_2 \oplus F_3 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_{11}, F_7, F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_7 \otimes F_8, F_2 \oplus F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_7 \otimes F_8, F_2 \oplus F_3 \vdash F_9} \text{hCut} \otimes_L \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \otimes F_8 \quad h_1 : \Delta_5, F_3 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_7 \otimes F_8} \oplus_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L}}{- : (\Delta_5, F_2 \oplus F_3), \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \otimes F_8}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \otimes F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \oplus F_3 \vdash F_9} \text{hCut} \oplus_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11} \quad h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}} \oplus_L \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_{11} \vdash F_9} \oplus_L}{\neg : (\Delta_5, F_2 \oplus F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{\neg : \Delta_{10}, \Delta_5, F_2 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \oplus F_8 \vdash F_9}{\neg : \Delta_{10}, \Delta_5, F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut}}{\neg : \Delta_{10}, \Delta_5, F_2 \oplus F_3, F_7 \oplus F_8 \vdash F_9} \oplus_L \\
\rightarrow \\
\frac{h_1 : \Delta_5, F_2 \vdash F_7 \oplus F_8 \quad h_1 : \Delta_5, F_3 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_7 \oplus F_8} \oplus_L \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \oplus_L}{\neg : (\Delta_5, F_2 \oplus F_3), \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \oplus F_8}{\neg : \Delta_{10}, \Delta_5, F_2 \vdash F_7 \oplus F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \oplus F_8 \vdash F_9}{\neg : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{hCut}}{\neg : \Delta_{10}, \Delta_5, F_2 \oplus F_3 \vdash F_9} \oplus_L
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11} \quad h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}} \oplus_L \quad \frac{h_6 : \Delta_{10}, F_{11} \vdash F_7 \quad h_6 : \Delta_{12}, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{12}, F_7 \neg\circ F_8), F_{11} \vdash F_9} \neg\circ_L}{\neg : (\Delta_5, F_2 \oplus F_3), \Delta_{10}, \Delta_{12}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{11}}{\neg : \Delta_{10}, \Delta_5, F_2 \oplus F_3 \vdash F_{11}} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_{11} \vdash F_7}{\neg : \Delta_{12}, F_8 \vdash F_9} \text{hCut}}{\neg : \Delta_{10}, \Delta_{12}, \Delta_5, F_7 \neg\circ F_8, F_2 \oplus F_3 \vdash F_9} \neg\circ_L \\
\rightarrow \\
\frac{h_1 : \Delta_5, F_2 \vdash F_{12} \quad h_1 : \Delta_5, F_3 \vdash F_{12}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{12}} \oplus_L \quad \frac{h_6 : \Delta_{10} \vdash F_7 \quad h_6 : \Delta_{11}, F_8, F_{12} \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}, F_7 \neg\circ F_8), F_{12} \vdash F_9} \neg\circ_L}{\neg : (\Delta_5, F_2 \oplus F_3), \Delta_{10}, \Delta_{11}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\neg : \Delta_{10} \vdash F_7}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_7 \neg\circ F_8, F_2 \oplus F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_{12}}{\neg : \Delta_{11}, \Delta_5, F_8, F_2 \oplus F_3 \vdash F_9} \text{hCut}}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_7 \neg\circ F_8, F_2 \oplus F_3 \vdash F_9} \neg\circ_L \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \neg\circ F_8 \quad h_1 : \Delta_5, F_3 \vdash F_7 \neg\circ F_8}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_7 \neg\circ F_8} \oplus_L \quad \frac{h_6 : \Delta_{10} \vdash F_7 \quad h_6 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, \Delta_{11}), F_7 \neg\circ F_8 \vdash F_9} \neg\circ_L}{\neg : (\Delta_5, F_2 \oplus F_3), \Delta_{10}, \Delta_{11} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \neg\circ F_8}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \vdash F_7 \neg\circ F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_{11}, F_7 \neg\circ F_8 \vdash F_9}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_3 \vdash F_9} \text{hCut}}{\neg : \Delta_{10}, \Delta_{11}, \Delta_5, F_2 \oplus F_3 \vdash F_9} \oplus_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash p(n_7) \quad h_1 : \Delta_5, F_3 \vdash p(n_7)}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash p(n_7)} \oplus_L \quad \frac{\bullet h_6 : *, p(n_7) \vdash p(n_7)}{\neg : (\Delta_5, F_2 \oplus F_3), * \vdash p(n_7)} I}{\neg : \Delta_5, F_2 \oplus F_3 \vdash p(n_7)} \text{Cut}
\end{array}$$

5.17 Status of $\neg\circ_L$: OK

- Case rule $!R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash 10}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash 10} \multimap_L \quad \frac{h_7 : !\Upsilon 9, 10 \vdash F_8}{\bullet h_7 : !\Upsilon 9, 10 \vdash !F_8} !R}{\frac{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), !\Upsilon 9 \vdash !F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_5 \vdash F_2}{\text{ax/W}} \quad \frac{\frac{h_1 : \Delta_6, F_3 \vdash 10}{\text{ax/W}} \quad \frac{\bullet h_7 : 10, !\Upsilon 9 \vdash !F_8}{\text{ax/W}}}{\frac{- : !\Upsilon 9, \Delta_6, F_3 \vdash !F_8}{\text{hCut}}} \multimap_L \\
\frac{}{- : !\Upsilon 9, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash !F_8}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_9} \multimap_L \quad \frac{}{\bullet h_7 : \Delta_8, F_9 \vdash \top} \top}{\frac{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_8 \vdash \top}{\rightarrow} \text{Cut}} \\
\frac{}{- : \Delta_5, \Delta_6, \Delta_8, F_2 \multimap F_3 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}} \multimap_L \quad \frac{h_7 : \Delta_{10}, F_{11} \vdash F_8 \quad h_7 : \Delta_{10}, F_{11} \vdash F_9}{\bullet h_7 : \Delta_{10}, F_{11} \vdash F_8 \& F_9} \&_R}{\frac{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{10} \vdash F_8 \& F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_5 \vdash F_2}{\text{ax/W}} \quad \frac{\frac{h_1 : \Delta_6, F_3 \vdash F_{11}}{\text{ax/W}} \quad \frac{\bullet h_7 : \Delta_{10}, F_{11} \vdash F_8 \& F_9}{\text{ax/W}}}{\frac{- : \Delta_{10}, \Delta_6, F_3 \vdash F_8 \& F_9}{\text{hCut}}} \multimap_L \\
\frac{}{- : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8 \& F_9}
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}} \multimap_L \quad \frac{h_7 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_7 : \Delta_{10}, F_{11} \vdash F_8 \multimap F_9} \multimap_R}{\frac{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{10} \vdash F_8 \multimap F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}}{\text{ax/W}} \quad \frac{h_7 : \Delta_{10}, F_{11}, F_8 \vdash F_9}{\text{ax/W}}}{\frac{- : \Delta_{10}, \Delta_5, \Delta_6, F_8, F_2 \multimap F_3 \vdash F_9}{\text{hCut}}} \multimap_R \\
\frac{}{- : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8 \multimap F_9}
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}} \multimap_L \quad \frac{h_7 : \Delta_{10}, F_{11} \vdash F_9}{\bullet h_7 : \Delta_{10}, F_{11} \vdash F_8 \oplus F_9} \oplus_{R_2}}{\frac{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{10} \vdash F_8 \oplus F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}}{\text{ax/W}} \quad \frac{h_7 : \Delta_{10}, F_{11} \vdash F_9}{\text{ax/W}}}{\frac{- : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_9}{\text{hCut}}} \oplus_{R_2} \\
\frac{}{- : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8 \oplus F_9}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}} \multimap_L \quad \frac{h_7 : \Delta_{10}, F_{11} \vdash F_8}{\bullet h_7 : \Delta_{10}, F_{11} \vdash F_8 \oplus F_9} \oplus_{R_1}}{\frac{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{10} \vdash F_8 \oplus F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}}{\text{ax/W}} \quad \frac{h_7 : \Delta_{10}, F_{11} \vdash F_8}{\text{ax/W}}}{\frac{- : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8}{\text{hCut}}} \oplus_{R_1} \\
\frac{}{- : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8 \oplus F_9}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash \mathbf{1}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash \mathbf{1}} \multimap_L \quad \frac{h_7 : \Delta_9 \vdash F_8}{\bullet h_7 : \Delta_9, \mathbf{1} \vdash F_8} 1_L \\
\hline
\frac{}{\vdash : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_9 \vdash F_8} \text{Cut} \\
\hline
\frac{}{\vdash : \Delta_5, \Delta_6, \Delta_9, F_2 \multimap F_3 \vdash F_8} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{10}} \multimap_L \quad \frac{h_7 : \Delta_9, F_{10} \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_{10} \vdash F_8} 1_L \\
\hline
\frac{}{\vdash : (\Delta_5, \Delta_6, F_2 \multimap F_3), \mathbf{1}, \Delta_9 \vdash F_8} \text{Cut} \\
\hline
\frac{}{\vdash : \mathbf{1}, \Delta_5, \Delta_6, \Delta_9, F_2 \multimap F_3 \vdash F_8} \text{ax/W} \quad \frac{}{\vdash : \mathbf{1}, \Delta_9, F_{10} \vdash F_8} \text{ax/W} \\
\hline
\frac{}{\vdash : \mathbf{1}, \Delta_5, \Delta_6, \Delta_9, F_2 \multimap F_3 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}} \multimap_L \quad \frac{h_7 : \Delta_{10}, F_{11} \vdash F_8 \quad h_7 : \Delta_{12} \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{12}), F_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
\frac{}{\vdash : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{10}, \Delta_{12} \vdash F_8 \otimes F_9} \text{Cut} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_{11} \vdash F_8}{\bullet h_7 : (\Delta_{10}, F_{11}), F_{11} \vdash F_8} \text{ax/W} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8} \text{hCut} \quad \frac{}{\vdash : \Delta_{12} \vdash F_9} \text{ax/W} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_{12}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{12}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{12}} \multimap_L \quad \frac{h_7 : \Delta_{10} \vdash F_8 \quad h_7 : \Delta_{11}, F_{12} \vdash F_9}{\bullet h_7 : (\Delta_{10}, \Delta_{11}), F_{12} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
\frac{}{\vdash : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{Cut} \\
\hline
\frac{}{\vdash : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{}{\vdash : \Delta_{11}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_9} \text{ax/W} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_{11}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8 \otimes F_9} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}} \multimap_L \quad \frac{h_7 : \Delta_{10}, F_{11} \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_{11} \vdash F_9} W \\
\hline
\frac{}{\vdash : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_{11}, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, F_{11}), !F_8 \vdash F_9} \text{ax/W} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_9} \text{hCut} \\
\hline
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash !F_8}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash !F_8} \multimap_L \quad \frac{h_7 : \Delta_{10} \vdash F_9}{\bullet h_7 : \Delta_{10}, !F_8 \vdash F_9} W \\
\hline
\frac{}{\vdash : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{10} \vdash F_9} \text{Cut} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_9} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}} \multimap_L \quad \frac{h_7 : \Delta_{10}, F_{11}, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_{11} \vdash F_9} C \\
\hline
\frac{}{\vdash : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_{11}, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, F_{11}), !F_8, !F_8 \vdash F_9} \text{ax/W} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_5, \Delta_6, !F_8, !F_8, F_2 \multimap F_3 \vdash F_9} \text{hCut} \\
\hline
\frac{}{\vdash : \Delta_{10}, \Delta_5, \Delta_6, !F_8, F_2 \multimap F_3 \vdash F_9} C
\end{array}$$

- Case rule $!L$

- Case rule $\&_{L2}$

- Case rule $\&_{L1}$

107

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{12}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{12}} \multimap_L \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_{12} \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_{12} \vdash F_{10}} \otimes_L \\
\text{Cut} \\
\frac{}{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \rightarrow \\
\frac{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{12} \quad \frac{h_7 : \Delta_{11}, F_{12}, F_8, F_9 \vdash F_{10}}{\text{hCut}}}{- : \Delta_{11}, \Delta_5, \Delta_6, F_8, F_9, F_2 \multimap F_3 \vdash F_{10}} \text{ax/W} \\
\frac{}{- : \Delta_{11}, \Delta_5, \Delta_6, F_8 \otimes F_9, F_2 \multimap F_3 \vdash F_{10}} \otimes_L \\
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_8 \otimes F_9}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8 \otimes F_9} \multimap_L \quad \frac{h_7 : \Delta_{11}, F_8, F_9 \vdash F_{10}}{\bullet h_7 : \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\text{Cut} \\
\frac{}{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{11} \vdash F_{10}} \rightarrow \\
\frac{}{- : \Delta_5 \vdash F_2} \text{ax/W} \quad \frac{h_1 : \Delta_6, F_3 \vdash F_8 \otimes F_9 \quad \bullet h_7 : \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_6, F_3 \vdash F_{10}} \text{ax/W} \\
\text{hCut} \\
\frac{}{- : \Delta_{11}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{10}} \multimap_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{12} \quad \frac{h_7 : \Delta_{11}, F_8, F_{12} \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_{12} \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_{12} \vdash F_{10}} \oplus_L}{\frac{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{12} \quad \frac{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}}{\rightarrow} \text{Cut}}{\frac{- : \Delta_5 \vdash F_2 \quad \frac{h_1 : \Delta_6, F_3 \vdash F_{12} \quad \frac{\bullet h_7 : \Delta_{11}, F_{12}, F_8 \oplus F_9 \vdash F_{10}}{\text{hCut}}}{\rightarrow} \text{ax/W}}{\frac{- : \Delta_{11}, \Delta_5, \Delta_6, F_2 \multimap F_3, F_8 \oplus F_9 \vdash F_{10}}{\multimap_L} \text{ax/W}} \oplus_L} \\
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_8 \oplus F_9 \quad \frac{h_7 : \Delta_{11}, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9 \vdash F_{10}}{\bullet h_7 : \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus_L}{\frac{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8 \oplus F_9 \quad \frac{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{11} \vdash F_{10}}{\rightarrow} \text{Cut}}{\frac{- : \Delta_5 \vdash F_2 \quad \frac{h_1 : \Delta_6, F_3 \vdash F_8 \oplus F_9 \quad \frac{\bullet h_7 : \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}}{\text{hCut}}}{\rightarrow} \text{ax/W}}{\frac{- : \Delta_{11}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{10}}{\multimap_L} \text{ax/W}} \oplus_L}
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{12} \quad \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_{12} \vdash F_8 \quad h_7 : \Delta_{13}, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{13}, F_8 \multimap F_9), F_{12} \vdash F_{10}} \multimap_L}{\frac{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{12}}{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{11}, \Delta_{13}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut}} \\
\frac{\quad}{\rightarrow} \\
\frac{\frac{- : \Delta_5 \vdash F_2 \quad \text{ax/W} \quad \frac{h_1 : \Delta_6, F_3 \vdash F_{12} \quad \text{ax/W} \quad \bullet h_7 : \Delta_{11}, \Delta_{13}, F_{12}, F_8 \multimap F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_{13}, \Delta_6, F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{\frac{- : \Delta_{11}, \Delta_{13}, \Delta_5, \Delta_6, F_2 \multimap F_3, F_8 \multimap F_9 \vdash F_{10}}{\multimap_L}} \\
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_{13} \quad \text{ax/W} \quad \frac{h_7 : \Delta_{11} \vdash F_8 \quad h_7 : \Delta_{12}, F_9, F_{13} \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}, F_8 \multimap F_9), F_{13} \vdash F_{10}} \multimap_L}{\frac{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{13}}{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut}} \\
\frac{\quad}{\rightarrow} \\
\frac{\frac{- : \Delta_5 \vdash F_2 \quad \text{ax/W} \quad \frac{h_1 : \Delta_6, F_3 \vdash F_{13} \quad \text{ax/W} \quad \bullet h_7 : \Delta_{11}, \Delta_{12}, F_{13}, F_8 \multimap F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_{12}, \Delta_6, F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{\frac{- : \Delta_{11}, \Delta_{12}, \Delta_5, \Delta_6, F_2 \multimap F_3, F_8 \multimap F_9 \vdash F_{10}}{\multimap_L}} \\
\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash F_8 \multimap F_9 \quad \text{ax/W} \quad \frac{h_7 : \Delta_{11} \vdash F_8 \quad h_7 : \Delta_{12}, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, \Delta_{12}), F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{\frac{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_8 \multimap F_9}{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), \Delta_{11}, \Delta_{12} \vdash F_{10}} \text{Cut}} \\
\frac{\quad}{\rightarrow} \\
\frac{\frac{- : \Delta_5 \vdash F_2 \quad \text{ax/W} \quad \frac{h_1 : \Delta_6, F_3 \vdash F_8 \multimap F_9 \quad \text{ax/W} \quad \bullet h_7 : \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_{12}, \Delta_6, F_3 \vdash F_{10}} \text{hCut}}{\frac{- : \Delta_{11}, \Delta_{12}, \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash F_{10}}{\multimap_L}}
\end{array}$$

- Case rule I

$$\frac{\frac{\frac{h_1 : \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, F_3 \vdash p(n_8)}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash p(n_8)} \multimap_L \quad \frac{}{\bullet h_7 : *, p(n_8) \vdash p(n_8)} I}{\frac{}{- : (\Delta_5, \Delta_6, F_2 \multimap F_3), * \vdash p(n_8)} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_5, \Delta_6, F_2 \multimap F_3 \vdash p(n_8)} \text{ax/w}$$

5.18 Status of I : OK

- Case rule $!R$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{}{\bullet h_4 : \Delta_5, p(n_3) \vdash \top} \top}{\frac{}{- : p(n_3), \Delta_5 \vdash \top} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_5, p(n_3) \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{\frac{h_4 : \Delta_7, p(n_3) \vdash F_5 \quad h_4 : \Delta_7, p(n_3) \vdash F_6}{\bullet h_4 : \Delta_7, p(n_3) \vdash F_5 \& F_6} \&_R}{\frac{}{- : p(n_3), \Delta_7 \vdash F_5 \& F_6} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_7, p(n_3) \vdash F_5 \& F_6} \text{ax/w}$$

- Case rule \multimap_R

$$\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{\frac{h_4 : \Delta_7, F_5, p(n_3) \vdash F_6}{\bullet h_4 : \Delta_7, p(n_3) \vdash F_5 \multimap F_6} \multimap_R}{\frac{}{- : p(n_3), \Delta_7 \vdash F_5 \multimap F_6} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_7, p(n_3) \vdash F_5 \multimap F_6} \text{ax/w}$$

- Case rule \oplus_{R_2}

$$\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{\frac{h_4 : \Delta_7, p(n_3) \vdash F_6}{\bullet h_4 : \Delta_7, p(n_3) \vdash F_5 \oplus F_6} \oplus_{R_2}}{\frac{}{- : p(n_3), \Delta_7 \vdash F_5 \oplus F_6} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_7, p(n_3) \vdash F_5 \oplus F_6} \text{ax/w}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{\frac{h_4 : \Delta_7, p(n_3) \vdash F_5}{\bullet h_4 : \Delta_7, p(n_3) \vdash F_5 \oplus F_6} \oplus_{R_1}}{\frac{}{- : p(n_3), \Delta_7 \vdash F_5 \oplus F_6} \text{Cut}} \xrightarrow{\quad} \frac{}{- : \Delta_7, p(n_3) \vdash F_5 \oplus F_6} \text{ax/w}$$

- Case rule 1_L

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_6, p(n_3) \vdash F_5}{\bullet h_4 : (1, \Delta_6), p(n_3) \vdash F_5} 1_L}{- : p(n_3), 1, \Delta_6 \vdash F_5} \text{Cut}}{\rightarrow} \text{ax/W}$$

$$\frac{}{- : 1, \Delta_6, p(n_3) \vdash F_5}$$

- Case rule \otimes_R

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_7, p(n_3) \vdash F_5 \quad h_4 : \Delta_8 \vdash F_6}{\bullet h_4 : (\Delta_7, \Delta_8), p(n_3) \vdash F_5 \otimes F_6} \otimes_R}{- : p(n_3), \Delta_7, \Delta_8 \vdash F_5 \otimes F_6} \text{Cut}}{\rightarrow} \text{ax/W}$$

$$\frac{}{- : \Delta_7, \Delta_8, p(n_3) \vdash F_5 \otimes F_6}$$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_7 \vdash F_5 \quad h_4 : \Delta_8, p(n_3) \vdash F_6}{\bullet h_4 : (\Delta_7, \Delta_8), p(n_3) \vdash F_5 \otimes F_6} \otimes_R}{- : p(n_3), \Delta_7, \Delta_8 \vdash F_5 \otimes F_6} \text{Cut}}{\rightarrow} \text{ax/W}$$

$$\frac{}{- : \Delta_7, \Delta_8, p(n_3) \vdash F_5 \otimes F_6}$$

- Case rule W

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_7, p(n_3) \vdash F_6}{\bullet h_4 : (\Delta_7, !F_5), p(n_3) \vdash F_6} W}{- : p(n_3), \Delta_7, !F_5 \vdash F_6} \text{Cut}}{\rightarrow} \text{ax/W}$$

$$\frac{}{- : \Delta_7, !F_5, p(n_3) \vdash F_6}$$

- Case rule C

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_7, !F_5, !F_5, p(n_3) \vdash F_6}{\bullet h_4 : (\Delta_7, !F_5), p(n_3) \vdash F_6} C}{- : p(n_3), \Delta_7, !F_5 \vdash F_6} \text{Cut}}{\rightarrow} \text{ax/W}$$

$$\frac{}{- : \Delta_7, !F_5, p(n_3) \vdash F_6}$$

- Case rule $!L$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_7, F_5, p(n_3) \vdash F_6}{\bullet h_4 : (\Delta_7, !F_5), p(n_3) \vdash F_6} !L}{- : p(n_3), \Delta_7, !F_5 \vdash F_6} \text{Cut}}{\rightarrow} \text{ax/W}$$

$$\frac{}{- : \Delta_7, !F_5, p(n_3) \vdash F_6}$$

- Case rule $\&_{L2}$

$$\frac{\frac{\frac{}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_8, F_6, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \& F_6), p(n_3) \vdash F_7} \&_{L2}}{- : p(n_3), \Delta_8, F_5 \& F_6 \vdash F_7} \text{Cut}}{\rightarrow} \text{ax/W}$$

$$\frac{}{- : \Delta_8, p(n_3), F_5 \& F_6 \vdash F_7}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_8, F_5, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \& F_6), p(n_3) \vdash F_7} \&_{L1} \\
\hline
- : p(n_3), \Delta_8, F_5 \& F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
- : \Delta_8, p(n_3), F_5 \& F_6 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_8, F_5, F_6, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \otimes F_6), p(n_3) \vdash F_7} \otimes_L \\
\hline
- : p(n_3), \Delta_8, F_5 \otimes F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
- : \Delta_8, p(n_3), F_5 \otimes F_6 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_8, F_5, p(n_3) \vdash F_7 \quad h_4 : \Delta_8, F_6, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \oplus F_6), p(n_3) \vdash F_7} \oplus_L \\
\hline
- : p(n_3), \Delta_8, F_5 \oplus F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
- : \Delta_8, p(n_3), F_5 \oplus F_6 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_8, p(n_3) \vdash F_5 \quad h_4 : \Delta_9, F_6 \vdash F_7}{\bullet h_4 : (\Delta_8, \Delta_9, F_5 \multimap F_6), p(n_3) \vdash F_7} \multimap_L \\
\hline
- : p(n_3), \Delta_8, \Delta_9, F_5 \multimap F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
- : \Delta_8, \Delta_9, p(n_3), F_5 \multimap F_6 \vdash F_7 \quad \text{ax/W} \\
\\
\frac{\bullet h_1 : p(n_3) \vdash p(n_3)}{\bullet h_1 : p(n_3) \vdash p(n_3)} I \quad \frac{h_4 : \Delta_8 \vdash F_5 \quad h_4 : \Delta_9, F_6, p(n_3) \vdash F_7}{\bullet h_4 : (\Delta_8, \Delta_9, F_5 \multimap F_6), p(n_3) \vdash F_7} \multimap_L \\
\hline
- : p(n_3), \Delta_8, \Delta_9, F_5 \multimap F_6 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
- : \Delta_8, \Delta_9, p(n_3), F_5 \multimap F_6 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\bullet h_1 : p(n_4) \vdash p(n_4)}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{\bullet h_3 : *, p(n_4) \vdash p(n_4)}{\bullet h_3 : *, p(n_4) \vdash p(n_4)} I \\
\hline
- : p(n_4), * \vdash p(n_4) \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
- : p(n_4) \vdash p(n_4) \quad I
\end{array}$$

6 Cut-Elimination

6.1 Status of $!R$: OK

- Case rule $!R$

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : !\Upsilon 8, !F_4, \text{contract}(n_5, !F_4) \vdash F_7}{\bullet h_6 : \text{contract}(sn_5, !F_4), !\Upsilon 8 \vdash !F_7} !R}{- : !\Upsilon 2, !\Upsilon 8 \vdash !F_7} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : !\Upsilon 8, !F_4, \text{contract}(n_5, !F_4) \vdash F_7}{- : !\Upsilon 8, !F_4, \text{contract}(n_5, !F_4) \vdash F_7} \text{ax}}{- : !\Upsilon 2, !\Upsilon 8 \vdash F_7} \text{hCut} \\
 \frac{- : !\Upsilon 2, !\Upsilon 8 \vdash F_7}{- : !\Upsilon 2, !\Upsilon 8 \vdash !F_7} !R
 \end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_7 \vdash \top} \top}{- : !\Upsilon 2, \Delta_7 \vdash \top} \text{Cut} \\
 \rightarrow \\
 \frac{}{- : !\Upsilon 2, \Delta_7 \vdash \top} \top
 \end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_7 \quad h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9 \vdash F_7 \& F_8} \&_R}{- : !\Upsilon 2, \Delta_9 \vdash F_7 \& F_8} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_7}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{ax}}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{hCut} \\
 \frac{- : !\Upsilon 2, \Delta_9 \vdash F_7 \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9 \vdash F_7 \& F_8} \&_R
 \end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9 \vdash F_7 \neg\circ F_8} \neg\circ_R}{- : !\Upsilon 2, \Delta_9 \vdash F_7 \neg\circ F_8} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8} \text{hCut} \\
 \frac{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8}{- : !\Upsilon 2, \Delta_9 \vdash F_7 \neg\circ F_8} \neg\circ_R
 \end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
 \frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9 \vdash F_7 \oplus F_8} \oplus_{R_2}}{- : !\Upsilon 2, \Delta_9 \vdash F_7 \oplus F_8} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9 \vdash F_8} \text{hCut} \\
 \frac{- : !\Upsilon 2, \Delta_9 \vdash F_8}{- : !\Upsilon 2, \Delta_9 \vdash F_7 \oplus F_8} \oplus_{R_2}
 \end{array}$$

- Case rule \oplus_{R_1}

- Case rule $\mathbf{1}_L$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, !F_7 \vdash F_8} W \\
\hline
- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} h\text{Cut} \\
\hline
- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8 \quad W
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} !R \quad \frac{h_5 : \Delta_8, \text{contract}(n_4, !F_6) \vdash F_7}{\bullet h_5 : \text{contract}(sn_4, !F_6), \Delta_8 \vdash F_7} W \\
\hline
- : !\Upsilon 2, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} \text{ax} \quad \frac{h_5 : \Delta_8, \text{contract}(n_4, !F_6) \vdash F_7}{- : !\Upsilon 2, \Delta_8 \vdash F_7} \text{ax}}{- : !\Upsilon 2, \Delta_8 \vdash F_7} h\text{Cut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4, !F_7, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, !F_7 \vdash F_8} C \\
\hline
- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4, !F_7, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9, !F_7, !F_7 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} h\text{Cut} \\
\hline
- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8 \quad C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} !R \quad \frac{h_5 : \Delta_8, !F_6, !F_7, \text{contract}(n_4, !F_6) \vdash F_7}{\bullet h_5 : \text{contract}(sn_4, !F_6), \Delta_8 \vdash F_7} C \\
\hline
- : !\Upsilon 2, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} \text{ax} \quad \frac{h_5 : \Delta_8, !F_6, !F_7, \text{contract}(n_4, !F_6) \vdash F_7}{- : !\Upsilon 2, \Delta_8 \vdash F_7} \text{ax}}{- : !\Upsilon 2, \Delta_8 \vdash F_7} h\text{Cut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, !F_7 \vdash F_8} !L \\
\hline
- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} h\text{Cut} \\
\hline
- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8 \quad !L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} !R \quad \frac{h_5 : \Delta_8, F_6, \text{contract}(n_4, !F_6) \vdash F_7}{\bullet h_5 : \text{contract}(sn_4, !F_6), \Delta_8 \vdash F_7} !L \\
\hline
- : !\Upsilon 2, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} \text{ax} \quad \frac{h_5 : \Delta_8, F_6, \text{contract}(n_4, !F_6) \vdash F_7}{- : !\Upsilon 2, \Delta_8, F_6 \vdash F_7} \text{ax}}{- : !\Upsilon 2, \Delta_8 \vdash F_7} h\text{Cut} \\
\hline
- : !\Upsilon 2 \vdash F_6 \quad \text{Cut} \\
\hline
- : !\Upsilon 2, \Delta_8 \vdash F_7
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2} \\
\hline
- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{- : !\Upsilon 2, \Delta_{10}, F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9} h\text{Cut} \\
\hline
- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \&_{L2}
\end{array}$$

- $$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{!R} \quad \frac{h_6 : \Delta_{10}, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L1} \\
\hline
- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_{10}, F_7 \vdash F_9} \text{ax} \quad \frac{h_6 : \Delta_{10}, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{- : !\Upsilon 2, \Delta_{10}, F_7 \vdash F_9} \text{ax} \\
\hline
- : !\Upsilon 2, \Delta_{10}, F_7 \vdash F_9 \quad \text{hCut} \\
\hline
- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9 \quad \&_{L1}
\end{array}$$

- $$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} \text{!R} \quad \frac{h_6 : \Delta_{10}, F_7, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \otimes_L}{- : !\Upsilon 2, \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \text{Cut} \\
\frac{\quad}{\quad} \rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash F_4 \quad \text{ax} \quad \frac{h_6 : \Delta_{10}, F_7, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{- : !\Upsilon 2, \Delta_{10}, F_7, F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \text{hCut} \quad \otimes_L
\end{array}$$

- $$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash F_4} \text{!R} \quad \frac{h_6 : \Delta_{10}, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_9 \quad h_8 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \oplus_L}{- : !\Upsilon 2, \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash F_4}{- : !\Upsilon 2, \Delta_{10}, F_7 \vdash F_9} \text{ax} \quad \frac{h_6 : \Delta_{10}, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\text{hCut}} \quad \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash F_4}{- : !\Upsilon 2, \Delta_{10}, F_8 \vdash F_9} \text{ax} \quad \frac{h_8 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\text{hCut}}}{- : !\Upsilon 2, \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \oplus_L
\end{array}$$

- $$\begin{array}{c}
\frac{\frac{h_1 : !Y2 \vdash F_4}{\bullet h_1 : !Y2 \vdash F_4} IR}{\quad} \quad \frac{h_6 : \Delta_{10}, !F_4, contract(n_5, !F_4) \vdash F_7 \quad h_8 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_6 : contract(sn_5, !F_4), \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \multimap L \\
\hline
- : !Y2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9 \quad Cut \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !Y2 \vdash F_4}{ax} \quad \frac{h_6 : \Delta_{10}, !F_4, contract(n_5, !F_4) \vdash F_7}{- : !Y2, \Delta_{10} \vdash F_7} ax}{\quad} \quad \frac{hCut}{- : \Delta_{11}, F_8 \vdash F_9} ax \quad \multimap L \\
\hline
- : !Y2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9 \\
\\
\frac{\frac{h_1 : !Y2 \vdash F_4}{\bullet h_1 : !Y2 \vdash F_4} IR}{\quad} \quad \frac{h_6 : \Delta_{10}, contract(n_5, !F_4) \vdash F_7 \quad h_8 : \Delta_{11}, F_8, !F_4 \vdash F_9}{\bullet h_6 : contract(sn_5, !F_4), \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \multimap L \\
\hline
- : !Y2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9 \quad Cut \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !Y2 \vdash F_4}{ax} \quad \frac{h_6 : \Delta_{10}, contract(n_5, !F_4) \vdash F_7}{- : !Y2, \Delta_{10} \vdash F_7} ax}{\quad} \quad \frac{\frac{\bullet h_1 : !Y2 \vdash F_4}{ax} \quad h_8 : \Delta_{11}, F_8, !F_4 \vdash F_9}{- : !Y2, \Delta_{11}, F_8 \vdash F_9} ax \quad hCut \\
\hline
- : !Y2, !Y2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9 \quad C \\
\hline
- : !Y2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9 \\
\\
\frac{\frac{h_1 : !Y2 \vdash F_4}{\bullet h_1 : !Y2 \vdash F_4} IR}{\quad} \quad \frac{h_6 : \Delta_{10}, !F_4 \vdash F_7 \quad h_8 : \Delta_{11}, F_8, contract(n_5, !F_4) \vdash F_9}{\bullet h_6 : contract(sn_5, !F_4), \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \multimap L \\
\hline
- : !Y2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9 \quad Cut \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : !Y2 \vdash F_4}{ax} \quad \frac{h_6 : \Delta_{10}, !F_4 \vdash F_7}{- : !Y2, \Delta_{10} \vdash F_7} ax}{\quad} \quad \frac{\frac{\bullet h_1 : !Y2 \vdash F_4}{ax} \quad h_8 : \Delta_{11}, F_8, contract(n_5, !F_4) \vdash F_9}{- : !Y2, \Delta_{11}, F_8 \vdash F_9} ax \quad hCut \\
\hline
- : !Y2, !Y2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9 \quad C \\
\hline
- : !Y2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9
\end{array}$$

$$\frac{\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{!R} \quad \frac{\frac{h_6 : \Delta_{10} \vdash F_7 \quad h_6 : \Delta_{11}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \multimap_L}{- : !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \text{Cut} \quad \frac{}{\rightarrow} \quad \frac{\frac{}{- : \Delta_{10} \vdash F_7} \text{ax} \quad \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{h_6 : \Delta_{11}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_{11}, F_8 \vdash F_9} \text{hCut}}{- : !\Upsilon 2, \Delta_{10}, \Delta_{11}, F_7 \multimap F_8 \vdash F_9} \multimap_L$$

- Case rule I

6.2 Status of 1_R : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L

- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.3 Status of \top : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L

- Case rule $\neg\circ_L$
- Case rule I

6.4 Status of $\&_R$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.5 Status of $\neg\circ_R$: OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.6 Status of \oplus_{R_2} : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- Case rule I

6.7 Status of \oplus_{R_1} : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- Case rule I

6.8 Status of 1_L : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- Case rule I

6.9 Status of \otimes_R : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- Case rule I

6.10 Status of W : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : !\Upsilon 9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), !\Upsilon 9 \vdash !F_8} !R}{- : (!\Upsilon 4, !F_3), !\Upsilon 9 \vdash !F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : !\Upsilon 9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 9, !F_5 \vdash F_8} \text{ax}}{- : !\Upsilon 4, !\Upsilon 9, !F_3 \vdash F_8} \text{hCut} \\
\rightarrow \\
- : !\Upsilon 4, !\Upsilon 9, !F_3 \vdash !F_8 \quad !R
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8 \vdash \top} \top}{- : (!\Upsilon 4, !F_3), \Delta_8 \vdash \top} \text{Cut} \\
\rightarrow \\
- : !\Upsilon 4, \Delta_8, !F_3 \vdash \top \quad \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \& F_9} \&_R}{- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} \text{ax} \quad \frac{\frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \& F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \& F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10} \vdash F_8 \& F_9} \text{hCut} \\
\rightarrow \\
- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \& F_9 \quad W
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \multimap F_9} \multimap_R}{- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \multimap F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, F_8, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \multimap F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \multimap F_9} \text{hCut} \\
\rightarrow \\
- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \multimap F_9 \quad \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \oplus F_9} \oplus_{R_2}}{- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \oplus F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \oplus F_9} \text{hCut} \\
\rightarrow \\
- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \oplus F_9 \quad \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \oplus F_9} \oplus_{R_1} \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \oplus F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \oplus F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \oplus F_9} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), \mathbf{1}, \Delta_9 \vdash F_8} 1_L \\
\hline
- : (!\Upsilon 4, !F_3), \mathbf{1}, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax}}{- : \mathbf{1}, !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{hCut} 1_L
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11} \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{hCut} \quad \frac{- : \Delta_{11} \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{ax} \otimes_R \\
\\
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11}, !F_5 \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{- : !\Upsilon 4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \Delta_{10}, \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{hCut} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{ax} \otimes_R \\
\\
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8 \quad h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{- : !\Upsilon 4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8}{\bullet h_7 : \Delta_{10}, \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{hCut} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{ax} \otimes_R \\
\\
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10} \vdash F_8 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{- : \Delta_{10} \vdash F_8}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{ax} \quad \frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} W \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_9} \text{ax} \\
\hline
- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9 \quad W \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 : !\Upsilon 4 \vdash !F_7}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7} W \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_9 \vdash F_8} W \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \\
\hline
- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8 \quad \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} C \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} \text{ax} \\
\hline
- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9 \quad C \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 : !\Upsilon 4 \vdash !F_7}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7} W \quad \frac{h_6 : \Delta_9, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_9 \vdash F_8} C \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \\
\hline
- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8 \quad \text{hCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} !L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, F_8, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, F_8, !F_3 \vdash F_9} \text{ax} \\
\hline
- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9 \quad !L \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 : !\Upsilon 4 \vdash !F_7}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7} W \quad \frac{h_6 : \Delta_9, F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_9 \vdash F_8} !L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_2 : !\Upsilon 4 \vdash !F_7}{- : !\Upsilon 4, \Delta_9 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \Delta_9, !F_7, \text{contract}(n_5, !F_7) \vdash F_8} !L \\
\hline
- : !\Upsilon 4, \Delta_9 \vdash F_8 \quad \text{hCut} \\
\rightarrow \\
- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8 \quad W
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, F_9, !F_3 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 4, \Delta_{11}, F_9, !F_3 \vdash F_{10}} \text{ax} \\
\hline
- : !\Upsilon 4, \Delta_{11}, F_9, !F_3 \vdash F_{10} \quad \&_{L2} \quad \text{hCut} \\
\rightarrow \\
- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \& F_9 \vdash F_{10}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, F_8, !F_3 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \& F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \& F_9 \vdash F_{10}} \text{hCut} \&_{L1}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, F_8, F_9, !F_3 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \otimes F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \otimes F_9 \vdash F_{10}} \text{hCut} \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, F_8, !F_3 \vdash F_{10}} \text{ax} \quad \frac{\frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \Delta_{11}, !F_5, F_8 \oplus F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 4, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \oplus_L \\
\rightarrow \\
- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \oplus F_9 \vdash F_{10} W
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_8} \text{hCut} \quad \frac{h_7 : \Delta_{12}, F_9 \vdash F_{10}}{- : \Delta_{12}, F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\rightarrow \\
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{- : \Delta_{12}, F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \multimap_L \\
\rightarrow \\
- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10} W \\
\rightarrow \\
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 4 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8}{\bullet h_7 : \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : \Delta_{12}, F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \multimap_L \\
\rightarrow \\
- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10} W
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11} \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{}{- : \Delta_{11} \vdash F_8} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5 \quad \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\text{ax}}}{- : !\Upsilon 4, \Delta_{12}, F_9, !F_3 \vdash F_{10}} \text{hCut}}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}} \multimap_L
\end{array}$$

- Case rule I

6.11 Status of C : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : !\Upsilon 9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), !\Upsilon 9 \vdash !F_8} !R \\
\hline
- : (!\Upsilon 4, !F_3), !\Upsilon 9 \vdash !F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\text{ax}} \quad \frac{h_7 : !\Upsilon 9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, !\Upsilon 9, !F_3 \vdash F_8} \text{ax}}{- : !\Upsilon 4, !\Upsilon 9, !F_3 \vdash !F_8} \text{hCut} \\
\hline
!R
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8 \vdash \top} \top \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_8 \vdash \top \quad \text{Cut} \\
\hline
\rightarrow \\
- : !\Upsilon 4, \Delta_8, !F_3 \vdash \top \quad \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \& F_9} \&_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \& F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\text{ax}} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \quad \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\text{ax}}}{\bullet h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \& F_9} \&_R} \\
\hline
- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \& F_9 \quad \text{hCut} \\
\hline
- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \& F_9 \quad C
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \multimap F_9} \multimap_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \multimap F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\text{ax}} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, F_8, !F_3 \vdash F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \multimap F_9} \text{hCut} \\
\hline
\multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

- Case rule \oplus_{R_1}

- Case rule $\mathbf{1}_L$

- Case rule \otimes_R

129

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10} \vdash F_8 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{}{- : \Delta_{10} \vdash F_8} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \otimes_R \quad \text{hCut}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} W \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} \text{hCut} \quad W \\
\hline
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_7}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7} C \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_9 \vdash F_8} W \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax}}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} C \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8, !F_8 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} \text{hCut} \quad C \\
\hline
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_7}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7} C \quad \frac{h_6 : \Delta_9, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_9 \vdash F_8} C \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax}}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} !L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, F_8, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} \text{hCut} \quad !L \\
\hline
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_7}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7} C \quad \frac{h_6 : \Delta_9, F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_9 \vdash F_8} !L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_7}{- : !\Upsilon 4, \Delta_9, !F_3, !F_3 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \Delta_9, !F_7, \text{contract}(n_5, !F_7) \vdash F_8} !L}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{hCut} \quad C
\end{array}$$

- $$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L2}}{\vdash : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\vdash : !\Upsilon 4, !F_3 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\vdash : \Delta_{11}, F_9, !F_5 \vdash F_{10}} \text{ax}}{\vdash : !\Upsilon 4, \Delta_{11}, F_9, !F_3 \vdash F_{10}} \text{hCut} \\
\vdash : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \& F_9 \vdash F_{10} \quad \&_{L2}
\end{array}$$

- $$\begin{array}{c}
\frac{\frac{\mathbf{h}_2 : !\Upsilon 4, !\mathbf{F}_3, !\mathbf{F}_3 \vdash !\mathbf{F}_5}{\bullet \mathbf{h}_2 : !\Upsilon 4, !\mathbf{F}_3 \vdash !\mathbf{F}_5} C \quad \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, !\mathbf{F}_5, \text{contract}(\mathbf{n}_6, !\mathbf{F}_5) \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : \text{contract}(\mathbf{s}\mathbf{n}_6, !\mathbf{F}_5), \Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10}} \&_{L1}}{\vdash : (!\Upsilon 4, !\mathbf{F}_3), \Delta_{11}, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10}} \text{Cut} \\
\downarrow \\
\frac{\frac{\bullet \mathbf{h}_2 : !\Upsilon 4, !\mathbf{F}_3 \vdash !\mathbf{F}_5}{\vdash : !\Upsilon 4, !\mathbf{F}_3 \vdash !\mathbf{F}_5} \text{ax} \quad \frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8, !\mathbf{F}_5, \text{contract}(\mathbf{n}_6, !\mathbf{F}_5) \vdash \mathbf{F}_{10}}{\vdash : \Delta_{11}, \mathbf{F}_8, !\mathbf{F}_5 \vdash \mathbf{F}_{10}} \text{ax}}{\vdash : !\Upsilon 4, \Delta_{11}, \mathbf{F}_8, !\mathbf{F}_3 \vdash \mathbf{F}_{10}} \text{hCut} \\
\vdash : !\Upsilon 4, \Delta_{11}, !\mathbf{F}_3, \mathbf{F}_8 \& \mathbf{F}_9 \vdash \mathbf{F}_{10} \quad \&_{L1}
\end{array}$$

- $$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes L}{\vdash : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\vdash : !\Upsilon 4, \Delta_{11}, F_8, F_9, !F_3 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\vdash : !\Upsilon 4, \Delta_{11}, F_3, F_8 \otimes F_9 \vdash F_{10}} \text{ax}}{\vdash : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \otimes F_9 \vdash F_{10}} \text{hCut} \otimes L
\end{array}$$

- $$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus L}{- : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\downarrow \\
\frac{\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5} \text{ax} \quad \frac{\frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \Delta_{11}, !F_5, F_8 \oplus F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \Delta_{11}, !F_5, F_8 \oplus F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax}}{\frac{- : !\Upsilon 4, \Delta_{11}, !F_3, !F_3, F_8 \oplus F_9 \vdash F_{10}}{- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \oplus F_9 \vdash F_{10}} \text{hCut}} \oplus L \\
\downarrow \\
\frac{- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \oplus F_9 \vdash F_{10}}{- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \oplus F_9 \vdash F_{10}} C
\end{array}$$

- $$\begin{array}{c}
\frac{h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, !F_5, contract(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap L \\
\frac{- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}}{\rightarrow} Cut \\
\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} ax \quad \frac{h_7 : \Delta_{11}, !F_5, contract(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_8} ax \quad \frac{h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{- : \Delta_{12}, F_9 \vdash F_{10}} ax \\
\frac{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_8 \quad - : \Delta_{12}, F_9 \vdash F_{10}}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}} \multimap L \\
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, contract(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap L \\
\frac{- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}}{\rightarrow} Cut \\
\frac{h_7 : \Delta_{11}, contract(n_6, !F_5) \vdash F_8}{h_7 : \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, contract(n_6, !F_5) \vdash F_{10}} ax \quad \frac{h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}} ax \\
\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} ax \quad \frac{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, !F_3, F_8 \multimap F_9 \vdash F_{10}}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}} hCut \\
\frac{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}} C
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{\frac{h_7 : \Delta_{11}, !F_5 \vdash F_8 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{\rightarrow} \text{Cut} \\
\frac{}{- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \\
\frac{\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} \text{ax} \quad \frac{\frac{h_7 : \Delta_{11}, !F_5 \vdash F_8}{\bullet h_7 : \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax}}{\rightarrow} \multimap_L \\
\frac{}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \\
\frac{}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}} C
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 4, !F_3, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} C \quad \frac{\frac{h_7 : \Delta_{11} \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{\rightarrow} \text{Cut} \\
\frac{}{- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \\
\frac{\frac{}{- : \Delta_{11} \vdash F_8} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{ax}}{\rightarrow} \multimap_L \\
\frac{}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}
\end{array}$$

- Case rule I

6.12 Status of $!L$: OK

- Case rule $!R$

$$\begin{array}{c}
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{\frac{h_7 : !\Upsilon 9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), !\Upsilon 9 \vdash !F_8} !R}{\rightarrow} \text{Cut} \\
\frac{}{- : (!\Upsilon 4, !F_3), !\Upsilon 9 \vdash !F_8} \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} \text{ax} \quad \frac{h_7 : !\Upsilon 9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), !\Upsilon 9 \vdash !F_8} \text{ax}}{\rightarrow} \text{hCut} \\
\frac{}{- : !\Upsilon 4, !\Upsilon 9, !F_3 \vdash F_8} !R \\
\frac{}{- : !\Upsilon 4, !\Upsilon 9, !F_3 \vdash !F_8}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8 \vdash \top} \top}{\rightarrow} \text{Cut} \\
\frac{}{- : (!\Upsilon 4, !F_3), \Delta_8 \vdash \top} \\
\frac{}{- : !\Upsilon 4, \Delta_8, !F_3 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{\frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \& F_9} \&_R}{\rightarrow} \text{Cut} \\
\frac{}{- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \& F_9} \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \& F_9} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \& F_9} \text{ax}}{\rightarrow} \text{hCut} \\
\frac{}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \&_R \\
\frac{}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \& F_9}
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \multimap F_9} \multimap_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \multimap F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, F_8, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \multimap F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \multimap F_9} \text{hCut} \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \oplus F_9} \oplus_{R_2} \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \oplus F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \oplus F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \oplus F_9} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10} \vdash F_8 \oplus F_9} \oplus_{R_1} \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10} \vdash F_8 \oplus F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \oplus F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8 \oplus F_9} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), 1, \Delta_9 \vdash F_8} 1_L \\
\hline
- : (!\Upsilon 4, !F_3), 1, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : 1, !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax}}{- : 1, !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{hCut} 1_L
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11} \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R \\
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11}, !F_5 \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_9}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R \\
\frac{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} C
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8 \quad h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R}{- : (!\Upsilon 4, !F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \text{hCut} \quad \frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \otimes_R \\
\frac{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} C \\
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10} \vdash F_8 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R}{- : (!\Upsilon 4, !F_3), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{10} \vdash F_8}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{ax} \quad \frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_9} \text{ax}}{- : !\Upsilon 4, \Delta_{10}, \Delta_{11}, !F_3 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} W}{- : (!\Upsilon 4, !F_3), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3 \vdash F_9} \text{hCut}}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} W \\
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_7}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7} !L \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_9 \vdash F_8} W}{- : (!\Upsilon 4, !F_3), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{hCut}}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} C}{- : (!\Upsilon 4, !F_3), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} \text{hCut}}{- : !\Upsilon 4, \Delta_{10}, !F_3, !F_8 \vdash F_9} C \\
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_7}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7} !L \quad \frac{h_6 : \Delta_9, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_9 \vdash F_8} C}{- : (!\Upsilon 4, !F_3), \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{hCut}}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} !L}{- : (!\Upsilon 4, !F_3), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{10}, F_8, !F_3 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 4, \Delta_{10}, F_8, !F_3 \vdash F_9} \text{hCut}}{- : !\Upsilon 4, \Delta_{10}, F_8, !F_3 \vdash F_9} !L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_7}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7} !L \quad \frac{h_6 : \Delta_9, F_7, contract(n_5, !F_7) \vdash F_8}{\bullet h_6 : contract(sn_5, !F_7), \Delta_9 \vdash F_8} !L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : !\Upsilon 4, !F_3 \vdash !F_7}{- : !\Upsilon 4, !F_3 \vdash F_7} ax \quad \frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_7}{- : !\Upsilon 4, !F_3 \vdash F_7} bInv \quad \frac{\frac{h_6 : \Delta_9, F_7, contract(n_5, !F_7) \vdash F_8}{\bullet h_6 : contract(sn_5, !F_7), \Delta_9 \vdash F_8} !L}{- : !\Upsilon 4, \Delta_9, F_7, !F_3 \vdash F_8} ax \quad \frac{- : !\Upsilon 4, \Delta_9, F_7, !F_3 \vdash F_8}{- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8} hCut \\
\hline
- : !\Upsilon 4, \Delta_9, !F_3 \vdash F_8 \quad \text{Cut}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, contract(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, !F_3 \vdash F_5} ax \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, contract(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} ax}{- : !\Upsilon 4, \Delta_{11}, F_9, !F_3 \vdash F_{10}} hCut \\
\hline
- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \& F_9 \vdash F_{10} \quad \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, contract(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, !F_3 \vdash F_5} ax \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, contract(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} ax}{- : !\Upsilon 4, \Delta_{11}, F_8, !F_3 \vdash F_{10}} hCut \\
\hline
- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \& F_9 \vdash F_{10} \quad \&_{L1}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, contract(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, !F_3 \vdash F_5} ax \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, contract(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} ax}{- : !\Upsilon 4, \Delta_{11}, F_8, F_9, !F_3 \vdash F_{10}} hCut \\
\hline
- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \otimes F_9 \vdash F_{10} \quad \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, contract(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, contract(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, !F_3 \vdash F_5} ax \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, contract(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} ax}{- : !\Upsilon 4, \Delta_{11}, F_8, !F_3 \vdash F_{10}} hCut \quad \frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, !F_3 \vdash F_5} ax \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, contract(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} ax}{- : !\Upsilon 4, \Delta_{11}, F_9, !F_3 \vdash F_{10}} hCut \\
\hline
- : !\Upsilon 4, \Delta_{11}, !F_3, F_8 \oplus F_9 \vdash F_{10} \quad \oplus_L
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, !F_5, contract(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9 \vdash F_{10}}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, !F_3 \vdash F_5} ax \quad \frac{h_7 : \Delta_{11}, !F_5, contract(n_6, !F_5) \vdash F_8}{\bullet h_7 : contract(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} ax}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_8} hCut \quad \frac{- : \Delta_{12}, F_9 \vdash F_{10}}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}} ax \\
\hline
- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10} \quad \multimap_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8}{h\text{Cut}} \quad \frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{12}, F_9, !F_3 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{h\text{Cut}}}{- : !\Upsilon 4, \Delta_{12}, F_9, !F_3 \vdash F_{10}} \multimap_L}{- : !\Upsilon 4, !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, !F_3, F_8 \multimap F_9 \vdash F_{10}} C \\
- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10} \\
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{11}, !F_3 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8}{h\text{Cut}} \quad \frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{12}, F_9, !F_3 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{h\text{Cut}}}{- : !\Upsilon 4, \Delta_{12}, F_9, !F_3 \vdash F_{10}} \multimap_L}{- : !\Upsilon 4, !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, !F_3, F_8 \multimap F_9 \vdash F_{10}} C \\
- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10} \\
\frac{\frac{h_2 : F_3, !\Upsilon 4 \vdash !F_5}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11} \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : (!\Upsilon 4, !F_3), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{11} \vdash F_8}{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5} \text{ax} \quad \frac{\frac{\bullet h_2 : !\Upsilon 4, !F_3 \vdash !F_5}{- : !\Upsilon 4, \Delta_{12}, F_9, !F_3 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{h\text{Cut}}}{- : !\Upsilon 4, \Delta_{12}, F_9, !F_3 \vdash F_{10}} \multimap_L}{- : !\Upsilon 4, \Delta_{11}, \Delta_{12}, !F_3, F_8 \multimap F_9 \vdash F_{10}}
\end{array}$$

- Case rule I

6.13 Status of $\&_{L2}$: OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L
- Case rule \otimes_R
- Case rule W

- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.14 Status of $\&_{L1}$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R2}
- Case rule \oplus_{R1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C

- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.15 Status of \otimes_L : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R2}
- Case rule \oplus_{R1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$

- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.16 Status of \oplus_L : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R2}
- Case rule \oplus_{R1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$

- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.17 Status of $\neg\circ_L$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$

- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.18 Status of I : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L

- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I