

Modal Logic S4 (K+T+4)

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1 Height preserving admissibility of weakening on the left

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \rightarrow F_4} \text{IH}}{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \rightarrow F_4} \text{ax}} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_4} \text{IH}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \wedge F_4} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \vee F_4} \text{IH}}{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_5, F_3 \vee F_4} \text{ax}} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2, F_W \vdash \perp, \Delta_3} \text{IH}}{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2, F_W \vdash \perp, \Delta_3} \text{ax}} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_W \vdash \top, \Delta_3} \top_R$$

- Case(s) rule $A4$

$$\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [\Box F_2]} A4 \rightarrow \frac{\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Delta_5, F_W, \Box \Gamma_4 \vdash \Delta_3, [\Box F_2]} \text{ax}}{\bullet h_1 : \Delta_5, F_W, \Box \Gamma_4 \vdash \Delta_3, [\Box F_2]} A4$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [\Box F_2]} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Delta_5, F_W, \Box \Gamma_4 \vdash \Delta_3, [\Box F_2]} \text{ax}}{\bullet h_1 : \Delta_5, F_W, \Box \Gamma_4 \vdash \Delta_3, [\Box F_2]} K$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5, F_W \vdash \Delta_4, F_2} \text{IH} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3, F_W \vdash \Delta_4} \text{IH}}{\bullet h_1 : \Delta_5, F_W, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{\frac{h_1 : F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4}}{\wedge_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\text{ax}}}{\frac{h_1 : \Delta_5, F_2, F_3, F_W \vdash \Delta_4}{\text{IH}}} \wedge_L}{\bullet h_1 : \Delta_5, F_W, F_2 \wedge F_3 \vdash \Delta_4}$$

- Case(s) rule \vee_L

$$\frac{\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L}{\vee_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\text{ax}}}{\frac{h_1 : \Delta_5, F_2, F_W \vdash \Delta_4}{\text{IH}}} \vee_L}{\frac{\frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\text{ax}}}{\frac{h_1 : \Delta_5, F_3, F_W \vdash \Delta_4}{\text{IH}}} \vee_L}$$

- Case(s) rule AT

$$\frac{\frac{h_1 : F_2, \Delta_4, []F_2 \vdash \Delta_3}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3}}{AT} \rightarrow \frac{\frac{\frac{h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3}{\text{ax}}}{\frac{h_1 : \Delta_4, F_2, F_W, []F_2 \vdash \Delta_3}{\text{IH}}} AT}{\bullet h_1 : \Delta_4, F_W, []F_2 \vdash \Delta_3}$$

- Case(s) rule \perp_L

$$\frac{\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2}}{\perp_L} \rightarrow \frac{\frac{h_1 : \perp, \Delta_3, F_W \vdash \Delta_2}{\bullet h_1 : \perp, \Delta_3, F_W \vdash \Delta_2}}{\perp_L}$$

- Case(s) rule I

$$\frac{\frac{h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3}}{I} \rightarrow \frac{\frac{h_1 : \Delta_4, F_W, p_3 \vdash \Delta_2, p_3}{\bullet h_1 : \Delta_4, F_W, p_3 \vdash \Delta_2, p_3}}{I}$$

- Case(s) rule \top_L

$$\frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2}}{\top_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\text{ax}}}{\frac{h_1 : \Delta_3, F_W \vdash \Delta_2}{\text{IH}}} \top_L}{\bullet h_1 : \top, \Delta_3, F_W \vdash \Delta_2}$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \rightarrow F_4} \text{ax} \quad \frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \rightarrow F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \rightarrow F_4} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \wedge F_4} \text{ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \wedge F_4} \text{ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \wedge F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \wedge F_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4, F_W}{h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \wedge F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \wedge F_4} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \vee F_4} \text{ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4, F_W}{h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \vee F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_W, F_3 \vee F_4} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{h_1 : \Delta_2 \vdash \Delta_3, F_W} \text{ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_W}{h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, F_W} \top_R$$

- Case(s) rule $A4$

$$\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [\Box F_2]} A4 \rightarrow \frac{\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, F_W, [\Box F_2]} \text{ax}}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, F_W, [\Box F_2]} A4$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [\Box F_2]} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, F_W, [\Box F_2]} \text{ax}}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, F_W, [\Box F_2]} K$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{h_1 : \Delta_5 \vdash \Delta_4, F_2, F_W} \text{ax} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W} \text{ax}}{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2, F_W}{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4, F_W} \text{IH} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W}{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4, F_W} \text{IH}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4, F_W} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4, F_W} \text{ax} \quad \frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4, F_W}{h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4, F_W} \text{IH}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4, F_W} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4, F_W} \text{ax} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W} \text{ax}}{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4, F_W}{h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4, F_W} \text{IH} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4, F_W}{h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4, F_W} \text{IH}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4, F_W} \vee_L$$

- Case(s) rule AT

$$\frac{\frac{h_1 : F_2, \Delta_4, []F_2 \vdash \Delta_3}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3} AT}{\frac{\frac{h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3}{h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3, F_W} IH}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3, F_W} AT} \rightarrow$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2, F_W} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, F_W, p_3} I$$

- Case(s) rule \top_L

$$\frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L}{\frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{h_1 : \Delta_3 \vdash \Delta_2, F_W} IH}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2, F_W} \top_L} \rightarrow$$

3 Measure of derivations

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \text{ax} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \text{ax} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \text{ax} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \text{ax} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case(s) rule $A4$

$$\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [\Box F_2]} A4 \rightarrow \frac{\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Box \Gamma_4 \vdash F_2} \text{IH}}{\bullet \bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, [\Box F_2]} \text{ax} A4$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [\Box F_2]} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2} \text{IH}}{\bullet \bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, [\Box F_2]} \text{ax} K$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{IH} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{IH}}{\bullet \bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ax} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} \text{IH}}{\bullet \bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \text{ax} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vdash \Delta_4} \text{IH} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{IH}}{\bullet \bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax} \vee_L$$

- Case(s) rule AT

$$\frac{h_1 : F_2, \Delta_4, []F_2 \vdash \Delta_3}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3} AT \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3}{\bullet h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3} IH}{\bullet \bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3} AT$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \Delta_3 \vdash \Delta_2} IH}{\bullet \bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L$$

4 Invertibility of Rules

4.1 Status of \rightarrow_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_4, F_1, F_5 \vdash \Delta_7, F_2, F_6}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{H}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \rightarrow F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \wedge F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_6}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \wedge F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5, F_6}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \vee F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \rightarrow F_2} \perp_R \rightarrow \frac{\frac{h_3 : \Delta_4, F_1 \vdash \Delta_5, F_2}{\bullet h_3 : \Delta_4, F_1 \vdash \perp, \Delta_5, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_1 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \rightarrow F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule $A4$

$$\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \rightarrow F_2), \Box F_4} A4 \rightarrow \frac{\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, F_2, \Box F_4} \text{ax}}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, F_2, \Box F_4} A4$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \rightarrow F_2), \Box F_4} K \rightarrow \frac{\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, F_2, \Box F_4} \text{ax}}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, F_2, \Box F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash F_4, \Delta_6, F_1 \rightarrow F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \rightarrow F_2} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_2, F_4}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6, F_2} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6, F_2}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6, F_2} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7 \vdash \Delta_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \rightarrow F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6, F_2} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7 \vdash \Delta_6, F_1 \rightarrow F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \rightarrow F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6, F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6, F_2} \vee_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4 \vdash \Delta_5, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_6, []F_4 \vdash \Delta_5, F_1 \rightarrow F_2} AT \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4, []F_4 \vdash \Delta_5, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_1, []F_4 \vdash \Delta_5, F_2} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \rightarrow F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4, F_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \rightarrow F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, p_5 \vdash \Delta_4, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \rightarrow F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \rightarrow F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5, F_1 \vdash \Delta_4, F_2} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4, F_2} \top_L$$

4.2 Status of \wedge_R : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5} \text{ ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1} \perp_R$$

- Case rule \top_R

$$\overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightarrow \overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1} \top_R$$

- Case rule $A4$

$$\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \wedge F_2), [] F_4} A4 \rightarrow \frac{\overline{h_3 : \Box \Gamma_6 \vdash F_4} \text{ ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, [] F_4} A4$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \wedge F_2), [] F_4} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4} \text{ ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, [] F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash F_4, \Delta_6, F_1 \wedge F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \wedge F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \wedge F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \wedge F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \vee_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, [] F_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_6, [] F_4 \vdash \Delta_5, F_1 \wedge F_2} AT \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, [] F_4 \vdash \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, [] F_4 \vdash \Delta_5, F_1} AT$$

- Case rule \perp_L

$$\overline{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \perp_L \rightarrow \overline{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1} \perp_L$$

- Case rule I

$$\overline{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \wedge F_2} I \rightarrow \overline{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash \Delta_4, F_1} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \top_L$$

4.3 Status of \wedge_R (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_4} H$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule \top_R

$$\overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightarrow \overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule $A4$

$$\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \wedge F_2), [\Box F_4]} A4 \rightarrow \frac{\overline{h_3 : \Box \Gamma_6 \vdash F_4} \text{ ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_2, [\Box F_4]} A4$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \wedge F_2), [\Box F_4]} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4} \text{ ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_2, [\Box F_4]} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash F_4, \Delta_6, F_1 \wedge F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \wedge F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash \Delta_6, F_2, F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_2} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \wedge F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_2} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \wedge F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_2} \vee_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, [\Box F_4 \vdash \Delta_5, F_1 \wedge F_2]}{\bullet h_3 : \Delta_6, [\Box F_4 \vdash \Delta_5, F_1 \wedge F_2]} AT \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, [\Box F_4 \vdash \Delta_5, F_2]} \text{ ax/ind}}{\bullet h_3 : \Delta_6, [\Box F_4 \vdash \Delta_5, F_2]} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \wedge F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \wedge F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash \Delta_4, F_2} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_2} \top_L$$

4.4 Status of \vee_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_5, \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash F_5, \Delta_7, F_1 \vee F_2 \quad h_3 : \Delta_4 \vdash F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash F_5, F_6, \Delta_7, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \vee F_6} \vee_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} H$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \vee F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1, F_2} \perp_R$$

- Case rule \top_R

$$\overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \vee F_2} \top_R \rightarrow \overline{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1, F_2} \top_R$$

- Case rule $A4$

$$\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \vee F_2), []F_4} A4 \rightarrow \frac{\overline{h_3 : \Box \Gamma_6 \vdash F_4} \text{ ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, F_2, []F_4} A4$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, F_1 \vee F_2), []F_4} K \rightarrow \frac{\overline{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4} \text{ ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, F_2, []F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7 \vdash F_4, \Delta_6, F_1 \vee F_2}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1 \vee F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_2, F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1, F_2} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7 \vdash \Delta_6, F_1 \vee F_2}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1 \vee F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1, F_2} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7 \vdash \Delta_6, F_1 \vee F_2 \quad h_3 : F_5, \Delta_7 \vdash \Delta_6, F_1 \vee F_2}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1 \vee F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1, F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1, F_2} \vee_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4 \vdash \Delta_5, F_1 \vee F_2}{\bullet h_3 : \Delta_6, []F_4 \vdash \Delta_5, F_1 \vee F_2} AT \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, []F_4 \vdash \Delta_5, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_6, []F_4 \vdash \Delta_5, F_1, F_2} AT$$

- Case rule \perp_L

$$\overline{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1 \vee F_2} \perp_L \rightarrow \overline{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1, F_2} \perp_L$$

- Case rule I

$$\overline{\bullet h_3 : p_5, \Delta_6 \vdash p_5, \Delta_4, F_1 \vee F_2} I \rightarrow \overline{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, F_2, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1 \vee F_2}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1 \vee F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash \Delta_4, F_1, F_2} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1, F_2} \top_L$$

4.5 Status of \perp_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash \perp, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \perp, F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash \perp, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ ax/ind} \quad \overline{h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \perp, F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \vee F_4} \vee_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{ H}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \perp, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case rule $A4$

$$\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash (\perp, \Delta_3), [] F_2} A4 \rightarrow \frac{\overline{h_1 : \Box \Gamma_4 \vdash F_2} \text{ ax}}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, [] F_2} A4$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash (\perp, \Delta_3), [] F_2} K \rightarrow \frac{\overline{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2} \text{ ax}}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, [] F_2} K$$

- Case rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash \perp, F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \perp, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \perp, \Delta_4} \rightarrow_L \rightarrow \frac{\overline{h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{ ax/ind} \quad \overline{h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \perp, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \perp, \Delta_4} \wedge_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \text{ax/ind} \wedge_L$$

- Case rule \vee_L

$$\frac{h_1 : F_2, \Delta_5 \vdash \perp, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \perp, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \perp, \Delta_4} \vee_L \rightarrow \frac{\overline{h_1 : \Delta_5, F_2 \vdash \Delta_4}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax/ind} \frac{\overline{h_1 : \Delta_5, F_3 \vdash \Delta_4}}{\vee_L} \text{ax/ind}$$

- Case rule AT

$$\frac{h_1 : F_2, \Delta_4, []F_2 \vdash \perp, \Delta_3}{\bullet h_1 : \Delta_4, []F_2 \vdash \perp, \Delta_3} AT \rightarrow \frac{\overline{h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3}}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3} \text{ax/ind} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \perp, \Delta_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_4 \vdash p_3, \perp, \Delta_2} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I$$

- Case rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \perp, \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \perp, \Delta_2} \top_L \rightarrow \frac{\overline{h_1 : \Delta_3 \vdash \Delta_2}}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \text{ax/ind} \top_L$$

4.6 Status of \top_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash \top, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \top, F_3, \Delta_5 \quad h_1 : \Delta_2 \vdash \top, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \wedge F_4} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \top, F_3, F_4, \Delta_5}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \vee F_4} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \top, \Delta_3} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \text{trivial}$$

- Case rule $A4$

$$\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash (\top, \Delta_3), [\Box F_2]} A4 \rightarrow \text{trivial}$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \Delta_5 \vdash (\top, \Delta_3), [\Box F_2]} K \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \Delta_5 \vdash \top, F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \top, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \top, \Delta_4} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \top, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \top, \Delta_4} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : F_2, \Delta_5 \vdash \top, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \top, \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \top, \Delta_4} \vee_L \rightarrow \text{trivial}$$

- Case rule AT

$$\frac{h_1 : F_2, \Delta_4, [\Box F_2] \vdash \top, \Delta_3}{\bullet h_1 : \Delta_4, [\Box F_2] \vdash \top, \Delta_3} AT \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \top, \Delta_2} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_4 \vdash p_3, \top, \Delta_2} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \top, \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \top, \Delta_2} \top_L \rightarrow \text{trivial}$$

4.7 Status of A4: : Non invertible

- Case rule \rightarrow_R

$$\frac{h_2 : \Box \Gamma_6, F_3, \Delta_7 \vdash F_4, \Delta_5, [F_1]}{\bullet h_2 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, [F_1]), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_2 : \Box \Gamma_6 \vdash F_1}}{\bullet h_2 : \Box \Gamma_6 \vdash F_1} \text{ax/ind} \quad \text{H}$$

- Case rule \wedge_R

$$\frac{h_2 : \Box \Gamma_6, \Delta_7 \vdash F_3, \Delta_5, [F_1] \quad h_2 : \Box \Gamma_6, \Delta_7 \vdash F_4, \Delta_5, [F_1]}{\bullet h_2 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, [F_1]), F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_2 : \Box \Gamma_6 \vdash F_1}}{\bullet h_2 : \Box \Gamma_6 \vdash F_1} \text{ax/ind} \quad \text{H}$$

- Case rule \vee_R

$$\frac{h_2 : \Box \Gamma_6, \Delta_7 \vdash F_3, F_4, \Delta_5, [F_1]}{\bullet h_2 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, [F_1]), F_3 \vee F_4} \vee_R \rightarrow \frac{\overline{h_2 : \Box \Gamma_6 \vdash F_1}}{\bullet h_2 : \Box \Gamma_6 \vdash F_1} \text{ax/ind} \quad \text{H}$$

- Case rule \perp_R

$$\frac{h_2 : \Box \Gamma_4, \Delta_5 \vdash \Delta_3, [F_1]}{\bullet h_2 : \Box \Gamma_4, \Delta_5 \vdash \perp, \Delta_3, [F_1]} \perp_R \rightarrow \frac{\overline{h_2 : \Box \Gamma_4 \vdash F_1}}{\bullet h_2 : \Box \Gamma_4 \vdash F_1} \text{ax/ind} \quad \text{H}$$

- Case rule \top_R

$$\frac{}{\bullet h_2 : \Box \Gamma_4, \Delta_5 \vdash \top, \Delta_3, [F_1]} \top_R \rightarrow \frac{}{\bullet h_2 : \Box \Gamma_4 \vdash F_1} \text{fail}$$

- Case rule A4

$$\frac{h_2 : \Box \Gamma_5, \Box \Gamma_6 \vdash F_3}{\bullet h_2 : (\Box \Gamma_5, \Box \Gamma_6), \Box \Gamma_7, \Delta_8 \vdash (\Delta_4, [F_1]), [F_3]} A4 \rightarrow \frac{}{\bullet h_2 : \Box \Gamma_5, \Box \Gamma_7 \vdash F_1} \text{fail}$$

$$\frac{h_1 : \Box \Gamma_4, \Box \Gamma_5 \vdash F_2}{\bullet h_1 : (\Box \Gamma_4, \Box \Gamma_5), \Box \Gamma_6, \Delta_7 \vdash \Delta_3, [F_2]} A4 \rightarrow \frac{}{\bullet h_1 : \Box \Gamma_4, \Box \Gamma_6 \vdash F_2} \text{fail}$$

- Case rule K

$$\frac{h_2 : \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_6) \vdash F_3}{\bullet h_2 : (\Box \Gamma_5, \Box \Gamma_6), \Box \Gamma_7, \Delta_8 \vdash (\Delta_4, [F_1]), [F_3]} K \rightarrow \frac{}{\bullet h_2 : \Box \Gamma_5, \Box \Gamma_7 \vdash F_1} \text{fail}$$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4), \text{unbox}(\Box \Gamma_5) \vdash F_2}{\bullet h_1 : (\Box \Gamma_4, \Box \Gamma_5), \Box \Gamma_6, \Delta_7 \vdash \Delta_3, [F_2]} K \rightarrow \frac{}{\bullet h_1 : \Box \Gamma_4, \Box \Gamma_6 \vdash F_2} \text{fail}$$

- Case rule \rightarrow_L

$$\frac{h_2 : \Box\Gamma_6, \Delta_7 \vdash F_3, \Delta_5, []F_1 \quad h_2 : \Box\Gamma_6, \Delta_7 \vdash \Delta_5, []F_1}{\bullet h_2 : (\Box\Gamma_6, \Delta_7), F_3 \rightarrow F_4 \vdash \Delta_5, []F_1} \rightarrow_L \rightarrow \frac{\overline{h_2 : \Box\Gamma_6 \vdash F_1}}{\bullet h_2 : \Box\Gamma_6 \vdash F_1} \text{ax/ind}_H$$

- Case rule \wedge_L

$$\frac{h_2 : \Box\Gamma_6, F_3, F_4, \Delta_7 \vdash \Delta_5, []F_1}{\bullet h_2 : (\Box\Gamma_6, \Delta_7), F_3 \wedge F_4 \vdash \Delta_5, []F_1} \wedge_L \rightarrow \frac{\overline{h_2 : \Box\Gamma_6 \vdash F_1}}{\bullet h_2 : \Box\Gamma_6 \vdash F_1} \text{ax/ind}_H$$

- Case rule \vee_L

$$\frac{h_2 : \Box\Gamma_6, F_3, \Delta_7 \vdash \Delta_5, []F_1 \quad h_2 : \Box\Gamma_6, F_4, \Delta_7 \vdash \Delta_5, []F_1}{\bullet h_2 : (\Box\Gamma_6, \Delta_7), F_3 \vee F_4 \vdash \Delta_5, []F_1} \vee_L \rightarrow \frac{\overline{h_2 : \Box\Gamma_6 \vdash F_1}}{\bullet h_2 : \Box\Gamma_6 \vdash F_1} \text{ax/ind}_H$$

- Case rule AT

$$\frac{h_2 : \Box\Gamma_5, F_3, \Delta_6, []F_3 \vdash \Delta_4, []F_1}{\bullet h_2 : (\Box\Gamma_5, \Delta_6), []F_3 \vdash \Delta_4, []F_1} AT \rightarrow \frac{\overline{h_2 : \Box\Gamma_5, []F_3 \vdash F_1}}{\bullet h_2 : \Box\Gamma_5, []F_3 \vdash F_1} \text{ax/ind}_H$$

$$\frac{h_2 : \Box\Gamma_5, F_3, \Delta_6, []F_3 \vdash \Delta_4, []F_1}{\bullet h_2 : (\Box\Gamma_5, \Delta_6), []F_3 \vdash \Delta_4, []F_1} AT \rightarrow \frac{\overline{h_2 : \Box\Gamma_5 \vdash F_1}}{\bullet h_2 : \Box\Gamma_5 \vdash F_1} \text{ax/ind}_H$$

- Case rule \perp_L

$$\frac{}{\bullet h_2 : \perp, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []F_1} \perp_L \rightarrow \frac{}{\bullet h_2 : \Box\Gamma_4 \vdash F_1} \text{fail}$$

- Case rule I

$$\frac{}{\bullet h_2 : p_4, \Box\Gamma_5, \Delta_6 \vdash p_4, \Delta_3, []F_1} I \rightarrow \frac{}{\bullet h_2 : \Box\Gamma_5 \vdash F_1} \text{fail}$$

- Case rule \top_L

$$\frac{h_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []F_1}{\bullet h_2 : \top, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []F_1} \top_L \rightarrow \frac{\overline{h_2 : \Box\Gamma_4 \vdash F_1}}{\bullet h_2 : \Box\Gamma_4 \vdash F_1} \text{ax/ind}_H$$

4.8 Status of K : : Non invertible

- Case rule \rightarrow_R

$$\frac{h_2 : \Box\Gamma_6, F_3, \Delta_7 \vdash F_4, \Delta_5, []F_1}{\bullet h_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []F_1), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_2 : unbox(\Box\Gamma_6) \vdash F_1}}{\bullet h_2 : unbox(\Box\Gamma_6) \vdash F_1} \text{ax/ind}_H$$

- Case rule \wedge_R

$$\frac{h_2 : \Box\Gamma_6, \Delta_7 \vdash F_3, \Delta_5, []F_1 \quad h_2 : \Box\Gamma_6, \Delta_7 \vdash F_4, \Delta_5, []F_1}{\bullet h_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []F_1), F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_2 : unbox(\Box\Gamma_6) \vdash F_1}}{\bullet h_2 : unbox(\Box\Gamma_6) \vdash F_1} \text{ax/ind}_H$$

- Case rule \vee_R

$$\frac{h_2 : \Box\Gamma_6, \Delta_7 \vdash F_3, F_4, \Delta_5, []F_1}{\bullet h_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []F_1), F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_2 : \text{unbox}(\Box\Gamma_6) \vdash F_1}{\bullet h_2 : \text{unbox}(\Box\Gamma_6) \vdash F_1} \text{H}}{\text{ax/ind}}$$

- Case rule \perp_R

$$\frac{h_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []F_1}{\bullet h_2 : \Box\Gamma_4, \Delta_5 \vdash \perp, \Delta_3, []F_1} \perp_R \rightarrow \frac{\frac{h_2 : \text{unbox}(\Box\Gamma_4) \vdash F_1}{\bullet h_2 : \text{unbox}(\Box\Gamma_4) \vdash F_1} \text{H}}{\text{ax/ind}}$$

- Case rule \top_R

$$\frac{}{\bullet h_2 : \Box\Gamma_4, \Delta_5 \vdash \top, \Delta_3, []F_1} \top_R \rightarrow \frac{}{\bullet h_2 : \text{unbox}(\Box\Gamma_4) \vdash F_1} \text{fail}$$

- Case rule $A4$

$$\frac{h_2 : \Box\Gamma_5, \Box\Gamma_6 \vdash F_3}{\bullet h_2 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []F_1), []F_3} A4 \rightarrow \frac{}{\bullet h_2 : \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_7) \vdash F_1} \text{fail}$$

$$\frac{h_1 : \Box\Gamma_4, \Box\Gamma_5 \vdash F_2}{\bullet h_1 : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_3, []F_2} A4 \rightarrow \frac{}{\bullet h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_6) \vdash F_2} \text{fail}$$

- Case rule K

$$\frac{h_2 : \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_6) \vdash F_3}{\bullet h_2 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []F_1), []F_3} K \rightarrow \frac{}{\bullet h_2 : \text{unbox}(\Box\Gamma_5), \text{unbox}(\Box\Gamma_7) \vdash F_1} \text{fail}$$

$$\frac{h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_5) \vdash F_2}{\bullet h_1 : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_3, []F_2} K \rightarrow \frac{}{\bullet h_1 : \text{unbox}(\Box\Gamma_4), \text{unbox}(\Box\Gamma_6) \vdash F_2} \text{fail}$$

- Case rule \rightarrow_L

$$\frac{h_2 : \Box\Gamma_6, \Delta_7 \vdash F_3, \Delta_5, []F_1 \quad h_2 : \Box\Gamma_6, F_4, \Delta_7 \vdash \Delta_5, []F_1}{\bullet h_2 : (\Box\Gamma_6, \Delta_7), F_3 \rightarrow F_4 \vdash \Delta_5, []F_1} \rightarrow_L \rightarrow \frac{\frac{h_2 : \text{unbox}(\Box\Gamma_6) \vdash F_1}{\bullet h_2 : \text{unbox}(\Box\Gamma_6) \vdash F_1} \text{H}}{\text{ax/ind}}$$

- Case rule \wedge_L

$$\frac{h_2 : \Box\Gamma_6, F_3, F_4, \Delta_7 \vdash \Delta_5, []F_1}{\bullet h_2 : (\Box\Gamma_6, \Delta_7), F_3 \wedge F_4 \vdash \Delta_5, []F_1} \wedge_L \rightarrow \frac{\frac{h_2 : \text{unbox}(\Box\Gamma_6) \vdash F_1}{\bullet h_2 : \text{unbox}(\Box\Gamma_6) \vdash F_1} \text{H}}{\text{ax/ind}}$$

- Case rule \vee_L

$$\frac{h_2 : \Box\Gamma_6, F_3, \Delta_7 \vdash \Delta_5, []F_1 \quad h_2 : \Box\Gamma_6, F_4, \Delta_7 \vdash \Delta_5, []F_1}{\bullet h_2 : (\Box\Gamma_6, \Delta_7), F_3 \vee F_4 \vdash \Delta_5, []F_1} \vee_L \rightarrow \frac{\frac{h_2 : \text{unbox}(\Box\Gamma_6) \vdash F_1}{\bullet h_2 : \text{unbox}(\Box\Gamma_6) \vdash F_1} \text{H}}{\text{ax/ind}}$$

- Case rule AT

$$\frac{h_2 : \Box\Gamma_5, F_3, \Delta_6, [\Box F_3 \vdash \Delta_4, \Box F_1]}{\bullet h_2 : (\Box\Gamma_5, \Delta_6), [\Box F_3 \vdash \Delta_4, \Box F_1]} AT \rightarrow \frac{\frac{h_2 : F_3, unbox(\Box\Gamma_5) \vdash F_1}{\bullet h_2 : F_3, unbox(\Box\Gamma_5) \vdash F_1} H}{\bullet h_2 : F_3, unbox(\Box\Gamma_5) \vdash F_1} ax/ind$$

$$\frac{h_2 : \Box\Gamma_5, F_3, \Delta_6, [\Box F_3 \vdash \Delta_4, \Box F_1]}{\bullet h_2 : (\Box\Gamma_5, \Delta_6), [\Box F_3 \vdash \Delta_4, \Box F_1]} AT \rightarrow \frac{\frac{h_2 : unbox(\Box\Gamma_5) \vdash F_1}{\bullet h_2 : unbox(\Box\Gamma_5) \vdash F_1} H}{\bullet h_2 : unbox(\Box\Gamma_5) \vdash F_1} ax/ind$$

- Case rule \perp_L

$$\frac{}{\bullet h_2 : \perp, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, [\Box F_1]} \perp_L \rightarrow \frac{}{\bullet h_2 : unbox(\Box\Gamma_4) \vdash F_1} fail$$

- Case rule I

$$\frac{}{\bullet h_2 : p_4, \Box\Gamma_5, \Delta_6 \vdash p_4, \Delta_3, [\Box F_1]} I \rightarrow \frac{}{\bullet h_2 : unbox(\Box\Gamma_5) \vdash F_1} fail$$

- Case rule \top_L

$$\frac{h_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, [\Box F_1]}{\bullet h_2 : \top, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, [\Box F_1]} \top_L \rightarrow \frac{\frac{h_2 : unbox(\Box\Gamma_4) \vdash F_1}{\bullet h_2 : unbox(\Box\Gamma_4) \vdash F_1} H}{\bullet h_2 : unbox(\Box\Gamma_4) \vdash F_1} ax/ind$$

4.9 Status of \rightarrow_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1, F_5}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \rightarrow F_5} \rightarrow_R}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \rightarrow F_5} ax/ind$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \wedge F_5} ax/ind \quad \frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_5}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \wedge F_5} ax/ind}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4, F_5}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \vee F_5} \vee_R}{\bullet h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4 \vee F_5} ax/ind$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1}{\bullet h_3 : \Delta_5 \vdash \perp, \Delta_4, F_1} \perp_R}{\bullet h_3 : \Delta_5 \vdash \perp, \Delta_4, F_1} ax/ind$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5 \vdash \top, \Delta_4, F_1} \top_R$$

- Case rule $A4$

$$\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, [] F_4} A4 \rightarrow \frac{\overline{h_3 : \Box \Gamma_6 \vdash F_4}^{ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, [] F_4} A4$$

- Case rule K

$$\frac{h_3 : unbox(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, [] F_4} K \rightarrow \frac{\overline{h_3 : unbox(\Box \Gamma_6) \vdash F_4}^{ax}}{\bullet h_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, F_1, [] F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4}^{ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}^{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \rightarrow_L$$

$$\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\overline{h_1 : \Delta_5 \vdash \Delta_4, F_2}^{ax}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2} H$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1}^{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1}^{ax/ind} \quad \overline{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}^{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \vee_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, [] F_4, F_1 \rightarrow F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \rightarrow F_2), [] F_4 \vdash \Delta_5} AT \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, [] F_4 \vdash \Delta_5, F_1}^{ax/ind}}{\bullet h_3 : \Delta_6, [] F_4 \vdash \Delta_5, F_1} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \rightarrow F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, p_5 \vdash \Delta_4, F_1, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5 \vdash \Delta_4, F_1}^{ax/ind}}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \top_L$$

4.10 Status of \rightarrow_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_2 \vdash \perp, \Delta_4} \text{ax/ind}}{\bullet h_3 : \Delta_5, F_2 \vdash \perp, \Delta_4} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_2 \vdash \top, \Delta_4} \top_R$$

- Case rule $A4$

$$\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, [\Box F_4]} A4 \rightarrow \frac{\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_6 \vdash \Delta_5, [\Box F_4]} \text{ax}}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_6 \vdash \Delta_5, [\Box F_4]} A4$$

- Case rule K

$$\frac{h_3 : unbox(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_5, [\Box F_4]} K \rightarrow \frac{\frac{h_3 : unbox(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_6 \vdash \Delta_5, [\Box F_4]} \text{ax}}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_6 \vdash \Delta_5, [\Box F_4]} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

$$\frac{h_1 : \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} H$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

$$\frac{\frac{h_3 : F_4, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6}}{\vdash_L} \rightarrow \frac{\frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6 \quad \text{ax/ind}}{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6 \quad \text{ax/ind}}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6}}{\vdash_L}$$

$$\frac{\frac{\mathbf{h}_3 : F_4, \Delta_6, [\Box F_4, F_1 \rightarrow F_2 \vdash \Delta_5]}{\bullet \mathbf{h}_3 : (\Delta_6, F_1 \rightarrow F_2), [\Box F_4 \vdash \Delta_5]} \quad AT}{\frac{\frac{\mathbf{h}_3 : \Delta_6, F_2, F_4, [\Box F_4 \vdash \Delta_5]}{\bullet \mathbf{h}_3 : \Delta_6, F_2, [\Box F_4 \vdash \Delta_5]} \quad ax/ind}{AT}} \rightarrow$$

$$\overline{\bullet \mathbf{h}_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}^{\perp L} \quad \rightarrow \quad \overline{\bullet \mathbf{h}_3 : \perp, \Delta_5, F_2 \vdash \Delta_4}^{\perp L}$$

$$\frac{\bullet \mathbf{h}_3 : \mathbf{p}_5, \Delta_6, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \mathbf{p}_5, \Delta_4}{\phantom{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_2, \mathbf{p}_5 \vdash \Delta_4, \mathbf{p}_5}} I \quad \rightarrow \quad \frac{\phantom{\bullet \mathbf{h}_3 : \mathbf{p}_5, \Delta_6, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \mathbf{p}_5, \Delta_4}}{\bullet \mathbf{h}_3 : \Delta_6, \mathbf{F}_2, \mathbf{p}_5 \vdash \Delta_4, \mathbf{p}_5} I$$

$$\frac{\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L}{\frac{h_3 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L} \text{ax/ind} \rightarrow$$

4.11 Status of \wedge_L : : Invertible

$$\frac{\frac{h_3 : F_4, \Delta_7, F_1 \wedge F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4 \rightarrow F_5}}{\rightarrow R} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \rightarrow F_5}}{\rightarrow R} \xrightarrow{ax/ind}$$

$$\frac{\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash F_4, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4 \wedge F_5}}{h_3 : \Delta_7, F_1 \wedge F_2 \vdash F_5, \Delta_6} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4 \wedge F_5}}{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_5} \text{ax/ind} \wedge_R \text{ax/ind}$$

$$\frac{\frac{\mathbf{h}_3 : \triangleleft_7, \mathbf{F}_1 \wedge \mathbf{F}_2 \vdash \mathbf{F}_4, \mathbf{F}_5, \triangleleft_6}{\bullet \mathbf{h}_3 : \triangleleft_7, \mathbf{F}_1 \wedge \mathbf{F}_2 \vdash \triangleleft_6, \mathbf{F}_4 \vee \mathbf{F}_5}}{\vee_R} \rightarrow \frac{\frac{\mathbf{h}_3 : \triangleleft_7, \mathbf{F}_1, \mathbf{F}_2 \vdash \triangleleft_6, \mathbf{F}_4, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \triangleleft_7, \mathbf{F}_1, \mathbf{F}_2 \vdash \triangleleft_6, \mathbf{F}_4 \vee \mathbf{F}_5}}{\vee_R} \text{ax/ind}$$

$$\frac{\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \wedge F_2 \vdash \perp, \Delta_4} \perp_R}{\frac{h_3 : \Delta_5, F_1, F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1, F_2 \vdash \perp, \Delta_4} \perp_R} \text{ax/ind} \rightarrow$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \wedge F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_1, F_2 \vdash \top, \Delta_4} \top_R$$

- Case rule $A4$

$$\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \wedge F_2 \vdash \Delta_5, []F_4} A4 \rightarrow \frac{\frac{}{h_3 : \Box \Gamma_6 \vdash F_4} ax}{\bullet h_3 : \Delta_7, F_1, F_2, \Box \Gamma_6 \vdash \Delta_5, []F_4} A4$$

- Case rule K

$$\frac{h_3 : unbox(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \wedge F_2 \vdash \Delta_5, []F_4} K \rightarrow \frac{\frac{}{h_3 : unbox(\Box \Gamma_6) \vdash F_4} ax}{\bullet h_3 : \Delta_7, F_1, F_2, \Box \Gamma_6 \vdash \Delta_5, []F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4} ax/ind \quad \frac{}{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} ax/ind}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash \Delta_6} ax/ind}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

$$\frac{h_1 : F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} ax}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4} H$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6} ax/ind \quad \frac{}{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} ax/ind}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \wedge F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \wedge F_2), []F_4 \vdash \Delta_5} AT \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_1, F_2, F_4, []F_4 \vdash \Delta_5} ax/ind}{\bullet h_3 : \Delta_6, F_1, F_2, []F_4 \vdash \Delta_5} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \wedge F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, F_2, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_1, F_2 \vdash \Delta_4} ax/ind}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash \Delta_4} \top_L$$

4.12 Status of \vee_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \rightarrow F_5} \text{ax/ind}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : \Delta_7, F_1 \vee F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \vee F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{h_3 : \Delta_5, F_1 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \vdash \perp, \Delta_4} \text{ax/ind}}{\bullet h_3 : \Delta_5, F_1 \vdash \perp, \Delta_4} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_1 \vdash \top, \Delta_4} \top_R$$

- Case rule $A4$

$$\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, [\Box F_4]} A4 \rightarrow \frac{\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, [\Box F_4]} \text{ax}}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, [\Box F_4]} A4$$

- Case rule K

$$\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, [\Box F_4]} K \rightarrow \frac{\frac{h_3 : \text{unbox}(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, [\Box F_4]} \text{ax}}{\bullet h_3 : \Delta_7, F_1, \Box \Gamma_6 \vdash \Delta_5, [\Box F_4]} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vdash \Delta_4} \text{ax} \quad \text{H}$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \vee F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \vee F_2), []F_4 \vdash \Delta_5} AT \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_4, []F_4 \vdash \Delta_5}{\bullet h_3 : \Delta_6, F_1, []F_4 \vdash \Delta_5} \text{ax/ind}}{AT}$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \vee F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_1 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \top_L$$

4.13 Status of \vee_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : \Delta_7, F_1 \vee F_2 \vdash F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, F_5, \Delta_6}{\bullet h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4, F_5}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4 \vee F_5} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_3 : \Delta_5, F_2 \vdash \perp, \Delta_4} \text{ax/ind}}{\bullet h_3 : \Delta_5, F_2 \vdash \perp, \Delta_4} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_5, F_1 \vee F_2 \vdash \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_2 \vdash \top, \Delta_4} \top_R$$

- Case rule $A4$

$$\frac{h_3 : \Box \Gamma_6 \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, []F_4} A4 \rightarrow \frac{\frac{}{h_3 : \Box \Gamma_6 \vdash F_4} ax}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_6 \vdash \Delta_5, []F_4} A4$$

- Case rule K

$$\frac{h_3 : unbox(\Box \Gamma_6) \vdash F_4}{\bullet h_3 : \Box \Gamma_6, \Delta_7, F_1 \vee F_2 \vdash \Delta_5, []F_4} K \rightarrow \frac{\frac{}{h_3 : unbox(\Box \Gamma_6) \vdash F_4} ax}{\bullet h_3 : \Delta_7, F_2, \Box \Gamma_6 \vdash \Delta_5, []F_4} K$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash F_4, \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} ax/ind \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} ax/ind}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : F_4, F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} ax/ind}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : F_4, \Delta_7, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : F_5, \Delta_7, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6} ax/ind \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} ax/ind}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{h_1 : F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_3 \vdash \Delta_4} ax}{\bullet h_1 : \Delta_5, F_3 \vdash \Delta_4} H$$

- Case rule AT

$$\frac{h_3 : F_4, \Delta_6, []F_4, F_1 \vee F_2 \vdash \Delta_5}{\bullet h_3 : (\Delta_6, F_1 \vee F_2), []F_4 \vdash \Delta_5} AT \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_2, F_4, []F_4 \vdash \Delta_5} ax/ind}{\bullet h_3 : \Delta_6, F_2, []F_4 \vdash \Delta_5} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_6, F_1 \vee F_2 \vdash p_5, \Delta_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_5 \vdash \Delta_4, p_5} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_2 \vdash \Delta_4} ax/ind}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

4.14 Status of AT : : Invertible

- Case rule \rightarrow_R

$$\frac{h_2 : F_3, \Delta_6, []F_1 \vdash F_4, \Delta_5}{\bullet h_2 : \Delta_6, []F_1 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3, []F_1 \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, []F_1 \vdash \Delta_5, F_3 \rightarrow F_4} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_2 : \Delta_6, []F_1 \vdash F_3, \Delta_5 \quad h_2 : \Delta_6, []F_1 \vdash F_4, \Delta_5}{\bullet h_2 : \Delta_6, []F_1 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, []F_1 \vdash \Delta_5, F_3} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_1, []F_1 \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, []F_1 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R$$

- Case rule \vee_R

$$\frac{h_2 : \Delta_6, []F_1 \vdash F_3, F_4, \Delta_5}{\bullet h_2 : \Delta_6, []F_1 \vdash \Delta_5, F_3 \vee F_4} \vee_R \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, []F_1 \vdash \Delta_5, F_3, F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, []F_1 \vdash \Delta_5, F_3 \vee F_4} \vee_R$$

- Case rule \perp_R

$$\frac{h_2 : \Delta_4, []F_1 \vdash \Delta_3}{\bullet h_2 : \Delta_4, []F_1 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\overline{h_2 : \Delta_4, F_1, []F_1 \vdash \Delta_3} \text{ ax/ind}}{\bullet h_2 : \Delta_4, F_1, []F_1 \vdash \perp, \Delta_3} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_2 : \Delta_4, []F_1 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_2 : \Delta_4, F_1, []F_1 \vdash \top, \Delta_3} \top_R$$

- Case rule $A4$

$$\frac{h_2 : \Box \Gamma_5, []F_1 \vdash F_3}{\bullet h_2 : (\Box \Gamma_5, []F_1), \Delta_6 \vdash \Delta_4, []F_3} A4 \rightarrow \frac{\overline{h_2 : \Box \Gamma_5, []F_1 \vdash F_3} \text{ ax}}{\bullet h_2 : \Delta_6, F_1, \Box \Gamma_5, []F_1 \vdash \Delta_4, []F_3} A4$$

$$\frac{h_2 : \Box \Gamma_5 \vdash F_3}{\bullet h_2 : \Box \Gamma_5, \Delta_6, []F_1 \vdash \Delta_4, []F_3} A4 \rightarrow \frac{\overline{h_2 : \Box \Gamma_5 \vdash F_3} \text{ ax}}{\bullet h_2 : \Delta_6, F_1, \Box \Gamma_5, []F_1 \vdash \Delta_4, []F_3} A4$$

- Case rule K

$$\frac{h_2 : F_1, unbox(\Box \Gamma_5) \vdash F_3}{\bullet h_2 : (\Box \Gamma_5, []F_1), \Delta_6 \vdash \Delta_4, []F_3} K \rightarrow \frac{\overline{h_2 : F_1, unbox(\Box \Gamma_5) \vdash F_3} \text{ ax}}{\bullet h_2 : \Delta_6, F_1, \Box \Gamma_5, []F_1 \vdash \Delta_4, []F_3} K$$

$$\frac{h_2 : unbox(\Box \Gamma_5) \vdash F_3}{\bullet h_2 : \Box \Gamma_5, \Delta_6, []F_1 \vdash \Delta_4, []F_3} K \rightarrow \frac{\overline{h_2 : unbox(\Box \Gamma_5) \vdash F_3} \text{ ax}}{\bullet h_2 : \Delta_6, F_1, \Box \Gamma_5, []F_1 \vdash \Delta_4, []F_3} K$$

- Case rule \rightarrow_L

$$\frac{h_2 : \Delta_6, []F_1 \vdash F_3, \Delta_5 \quad h_2 : F_4, \Delta_6, []F_1 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, []F_1), F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, []F_1 \vdash \Delta_5, F_3} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_1, F_4, []F_1 \vdash \Delta_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_2 : F_3, F_4, \Delta_6, []F_1 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, []F_1), F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightarrow \frac{\frac{h_2 : \Delta_6, F_1, F_3, F_4, []F_1 \vdash \Delta_5}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \wedge F_4 \vdash \Delta_5} \text{ax/ind}}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L$$

- Case rule \vee_L

$$\frac{h_2 : F_3, \Delta_6, []F_1 \vdash \Delta_5 \quad h_2 : F_4, \Delta_6, []F_1 \vdash \Delta_5}{\bullet h_2 : (\Delta_6, []F_1), F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightarrow \frac{\frac{h_2 : \Delta_6, F_1, F_3, []F_1 \vdash \Delta_5}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \vee F_4 \vdash \Delta_5} \text{ax/ind} \quad \frac{h_2 : \Delta_6, F_1, F_4, []F_1 \vdash \Delta_5}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \vee F_4 \vdash \Delta_5} \text{ax/ind}}{\bullet h_2 : \Delta_6, F_1, []F_1, F_3 \vee F_4 \vdash \Delta_5} \vee_L$$

- Case rule AT

$$\frac{h_2 : F_3, \Delta_5, []F_1, []F_3 \vdash \Delta_4}{\bullet h_2 : (\Delta_5, []F_1), []F_3 \vdash \Delta_4} AT \rightarrow \frac{\frac{h_2 : \Delta_5, F_1, F_3, []F_1, []F_3 \vdash \Delta_4}{\bullet h_2 : \Delta_5, F_1, []F_1, []F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_2 : \Delta_5, F_1, []F_1, []F_3 \vdash \Delta_4} AT$$

$$\frac{h_1 : F_2, \Delta_4, []F_2 \vdash \Delta_3}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3} AT \rightarrow \frac{\frac{h_1 : \Delta_4, F_2, F_2, []F_2 \vdash \Delta_3}{\bullet h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3} \text{ax/ind}}{\bullet h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_2 : \perp, \Delta_4, []F_1 \vdash \Delta_3} \perp_L \rightarrow \frac{}{\bullet h_2 : \perp, \Delta_4, F_1, []F_1 \vdash \Delta_3} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_2 : p_4, \Delta_5, []F_1 \vdash p_4, \Delta_3} I \rightarrow \frac{}{\bullet h_2 : \Delta_5, F_1, p_4, []F_1 \vdash \Delta_3, p_4} I$$

- Case rule \top_L

$$\frac{h_2 : \Delta_4, []F_1 \vdash \Delta_3}{\bullet h_2 : \top, \Delta_4, []F_1 \vdash \Delta_3} \top_L \rightarrow \frac{\frac{h_2 : \Delta_4, F_1, []F_1 \vdash \Delta_3}{\bullet h_2 : \top, \Delta_4, F_1, []F_1 \vdash \Delta_3} \text{ax/ind}}{\bullet h_2 : \top, \Delta_4, F_1, []F_1 \vdash \Delta_3} \top_L$$

4.15 Status of \perp_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \perp, F_2, \Delta_5 \vdash F_3, \Delta_4}{\bullet h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \perp, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \perp, \Delta_5 \vdash F_3, \Delta_4}{\bullet h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \perp, \Delta_5 \vdash F_2, F_3, \Delta_4}{\bullet h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \perp, \Delta_3 \vdash \perp, \Delta_2} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \top, \Delta_2} \top_R \rightarrow \text{trivial}$$

- Case rule $A4$

$$\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \perp, \Delta_5 \vdash \Delta_3, []F_2} A4 \rightarrow \text{trivial}$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \perp, \Delta_5 \vdash \Delta_3, []F_2} K \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \perp, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \perp, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : \perp, F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : \perp, F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : \perp, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \text{trivial}$$

- Case rule AT

$$\frac{h_1 : \perp, F_2, \Delta_4, []F_2 \vdash \Delta_3}{\bullet h_1 : (\perp, \Delta_4), []F_2 \vdash \Delta_3} AT \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \perp, \Delta_4 \vdash p_3, \Delta_2} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \perp, \Delta_3 \vdash \Delta_2} \top_L \rightarrow \text{trivial}$$

4.16 Status of I : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : F_2, \Delta_6, p_5 \vdash F_3, \Delta_4, p_5}{\bullet h_1 : \Delta_6, p_5 \vdash (\Delta_4, p_5), F_2 \rightarrow F_3} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_6, p_5 \vdash F_2, \Delta_4, p_5 \quad h_1 : \Delta_6, p_5 \vdash F_3, \Delta_4, p_5}{\bullet h_1 : \Delta_6, p_5 \vdash (\Delta_4, p_5), F_2 \wedge F_3} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_6, p_5 \vdash F_2, F_3, \Delta_4, p_5}{\bullet h_1 : \Delta_6, p_5 \vdash (\Delta_4, p_5), F_2 \vee F_3} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3}{\bullet h_1 : \Delta_4, p_3 \vdash \perp, \Delta_2, p_3} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \top, \Delta_2, p_3} \top_R \rightarrow \text{trivial}$$

- Case rule $A4$

$$\frac{h_1 : \Box \Gamma_5 \vdash F_2}{\bullet h_1 : \Box \Gamma_5, \Delta_6, p_4 \vdash (\Delta_3, p_4), \Box F_2} A4 \rightarrow \text{trivial}$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_5) \vdash F_2}{\bullet h_1 : \Box \Gamma_5, \Delta_6, p_4 \vdash (\Delta_3, p_4), \Box F_2} K \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \Delta_6, p_5 \vdash F_2, \Delta_4, p_5 \quad h_1 : F_3, \Delta_6, p_5 \vdash \Delta_4, p_5}{\bullet h_1 : (\Delta_6, p_5), F_2 \rightarrow F_3 \vdash \Delta_4, p_5} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_6, P_5 \vdash \Delta_4, P_5}{\bullet h_1 : (\Delta_6, P_5), F_2 \wedge F_3 \vdash \Delta_4, P_5} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : F_2, \Delta_6, P_5 \vdash \Delta_4, P_5 \quad h_1 : F_3, \Delta_6, P_5 \vdash \Delta_4, P_5}{\bullet h_1 : (\Delta_6, P_5), F_2 \vee F_3 \vdash \Delta_4, P_5} \vee_L \rightarrow \text{trivial}$$

- Case rule AT

$$\frac{h_1 : F_2, \Delta_5, P_4, []F_2 \vdash \Delta_3, P_4}{\bullet h_1 : (\Delta_5, P_4), []F_2 \vdash \Delta_3, P_4} AT \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_4, P_3 \vdash \Delta_2, P_3} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : P_4, \Delta_5, P_3 \vdash P_4, \Delta_2, P_3} I \rightarrow \text{trivial}$$

$$\frac{}{\bullet h_1 : P_3, \Delta_4 \vdash P_3, \Delta_2} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \Delta_4, P_3 \vdash \Delta_2, P_3}{\bullet h_1 : \top, \Delta_4, P_3 \vdash \Delta_2, P_3} \top_L \rightarrow \text{trivial}$$

4.17 Status of \top_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \top, F_2, \Delta_5 \vdash F_3, \Delta_4}{\bullet h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_2 \vdash \Delta_4, F_3} \text{ax/ind}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_1 : \top, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \top, \Delta_5 \vdash F_3, \Delta_4}{\bullet h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R \rightarrow \frac{\frac{}{h_1 : \Delta_5 \vdash \Delta_4, F_2} \text{ax/ind} \quad \frac{}{h_1 : \Delta_5 \vdash \Delta_4, F_3} \text{ax/ind}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R$$

- Case rule \vee_R

$$\frac{h_1 : \top, \Delta_5 \vdash F_2, F_3, \Delta_4}{\bullet h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \vee_R \rightarrow \frac{\frac{}{h_1 : \Delta_5 \vdash \Delta_4, F_2, F_3} \text{ax/ind}}{\bullet h_1 : \Delta_5 \vdash \Delta_4, F_2 \vee F_3} \vee_R$$

- Case rule \perp_R

$$\frac{h_1 : \top, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \perp, \Delta_2} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \Delta_3 \vdash \perp, \Delta_2} \text{ax/ind}}{\bullet h_1 : \Delta_3 \vdash \perp, \Delta_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \top, \Delta_3 \vdash \top, \Delta_2} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_3 \vdash \top, \Delta_2} \top_R$$

- Case rule $A4$

$$\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \top, \Delta_5 \vdash \Delta_3, []F_2} A4 \rightarrow \frac{\frac{h_1 : \Box \Gamma_4 \vdash F_2}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, []F_2} \text{ax}}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, []F_2} A4$$

- Case rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Box \Gamma_4, \top, \Delta_5 \vdash \Delta_3, []F_2} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4) \vdash F_2}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, []F_2} \text{ax}}{\bullet h_1 : \Delta_5, \Box \Gamma_4 \vdash \Delta_3, []F_2} K$$

- Case rule \rightarrow_L

$$\frac{h_1 : \top, \Delta_5 \vdash F_2, \Delta_4 \quad h_1 : \top, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_1 : \top, F_2, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \wedge_L$$

- Case rule \vee_L

$$\frac{h_1 : \top, F_2, \Delta_5 \vdash \Delta_4 \quad h_1 : \top, F_3, \Delta_5 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \vee_L$$

- Case rule AT

$$\frac{h_1 : \top, F_2, \Delta_4, []F_2 \vdash \Delta_3}{\bullet h_1 : (\top, \Delta_4), []F_2 \vdash \Delta_3} AT \rightarrow \frac{\frac{h_1 : \Delta_4, F_2, []F_2 \vdash \Delta_3}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3} \text{ax/ind}}{\bullet h_1 : \Delta_4, []F_2 \vdash \Delta_3} AT$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \top, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \top, \Delta_4 \vdash p_3, \Delta_2} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, p_3 \vdash \Delta_2, p_3} I$$

- Case rule \top_L

$$\frac{h_1 : \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \quad \rightarrow \quad \frac{\overline{h_1 : \Delta_3 \vdash \Delta_2}^{\text{ax}}}{\bullet h_1 : \Delta_3 \vdash \Delta_2} \text{H}$$

5 Height preserving admissibility of contraction on the left

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_2, \Delta_5, \Delta_6, \Delta_6 \vdash F_3, \Delta_4}{\bullet h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2 \vdash \Delta_4, F_3}{h_1 : \Delta_5, \Delta_6, F_2 \vdash \Delta_4, F_3} \text{ax} \quad \frac{h_1 : \Delta_5, \Delta_6, F_2 \vdash \Delta_4, F_3}{\bullet h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \rightarrow F_3} \text{IH}}{\bullet h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \rightarrow F_3} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash F_2, \Delta_4 \quad h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash F_3, \Delta_4}{\bullet h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2 \wedge F_3} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2} \text{ax} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_3}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_3} \text{ax}}{\frac{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \wedge F_3}{\bullet h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \wedge F_3} \text{IH}} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash F_2, F_3, \Delta_4}{\bullet h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2 \vee F_3} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2, F_3}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \vee F_3} \text{ax} \quad \frac{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \vee F_3}{\bullet h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \vee F_3} \text{IH}}{\bullet h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2 \vee F_3} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \perp, \Delta_2} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{h_1 : \Delta_3, \Delta_4 \vdash \Delta_2} \text{ax} \quad \frac{h_1 : \Delta_3, \Delta_4 \vdash \Delta_2}{\bullet h_1 : \Delta_3, \Delta_4 \vdash \perp, \Delta_2} \text{IH}}{\bullet h_1 : \Delta_3, \Delta_4 \vdash \perp, \Delta_2} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \top, \Delta_2} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_3, \Delta_4 \vdash \top, \Delta_2} \top_R$$

- Case(s) rule $A4$

$$\frac{h_1 : \Box \Gamma_4, \Box \Gamma_5, \Box \Gamma_5, \Box \Gamma_6 \vdash F_2}{\bullet h_1 : (\Box \Gamma_4, \Delta_7), (\Box \Gamma_5, \Box \Gamma_6, \Delta_8), \Box \Gamma_5, \Box \Gamma_6, \Delta_8 \vdash \Delta_3, [F_2]} A4 \rightarrow \frac{\frac{h_1 : \Box \Gamma_4, \Box \Gamma_5, \Box \Gamma_5, \Box \Gamma_6 \vdash F_2}{h_1 : \Box \Gamma_4, \Box \Gamma_5, \Box \Gamma_6 \vdash F_2} \text{ax} \quad \frac{h_1 : \Box \Gamma_4, \Box \Gamma_5, \Box \Gamma_6 \vdash F_2}{\bullet h_1 : \Delta_7, \Delta_8, \Box \Gamma_4, \Box \Gamma_5, \Box \Gamma_6 \vdash \Delta_3, [F_2]} \text{IH}}{\bullet h_1 : \Delta_7, \Delta_8, \Box \Gamma_4, \Box \Gamma_5, \Box \Gamma_6 \vdash \Delta_3, [F_2]} A4$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_4), \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_6) \vdash F_2}{\bullet h_1 : (\Box \Gamma_4, \Delta_7), (\Box \Gamma_5, \Box \Gamma_6, \Delta_8), \Box \Gamma_5, \Box \Gamma_6, \Delta_8 \vdash \Delta_3, [F_2]} K \rightarrow \frac{\frac{h_1 : \text{unbox}(\Box \Gamma_4), \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_6) \vdash F_2}{h_1 : \text{unbox}(\Box \Gamma_4), \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_6) \vdash F_2} \text{ax} \quad \frac{h_1 : \text{unbox}(\Box \Gamma_4), \text{unbox}(\Box \Gamma_5), \text{unbox}(\Box \Gamma_6) \vdash F_2}{\bullet h_1 : \Delta_7, \Delta_8, \Box \Gamma_4, \Box \Gamma_5, \Box \Gamma_6 \vdash \Delta_3, [F_2]} \text{IH}}{\bullet h_1 : \Delta_7, \Delta_8, \Box \Gamma_4, \Box \Gamma_5, \Box \Gamma_6 \vdash \Delta_3, [F_2]} K$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2 \rightarrow F_3 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5, \Delta_6, \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, (\Delta_6, F_2 \rightarrow F_3), \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2, F_2}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2, F_2} \text{inv-th/ax} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_3, F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_3 \vdash \Delta_4} \text{inv-th/ax}}{\frac{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2, F_2}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2} \text{IH-Mutual} \quad \frac{h_1 : \Delta_5, \Delta_6, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4} \text{IH}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L$$

$$\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash F_2, \Delta_4 \quad h_1 : F_3, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4}{\bullet h_1 : (\Delta_5, F_2 \rightarrow F_3), \Delta_6, \Delta_6 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, F_2}{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2} \text{ax} \quad \frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_3 \vdash \Delta_4} \text{ax}}{\frac{h_1 : \Delta_5, \Delta_6 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4} \text{IH}} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{\frac{h_1 : F_2, F_3, \Delta_5, \Delta_6, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, (\Delta_6, F_2 \wedge F_3), \Delta_6, F_2 \wedge F_3 \vdash \Delta_4}}{\wedge_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2, F_2, F_3, F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2, F_3 \vdash \Delta_4} \text{inv-th/ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4} \text{IH}}{\wedge_L}$$

$$\frac{\frac{h_1 : F_2, F_3, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4}{\bullet h_1 : (\Delta_5, F_2 \wedge F_3), \Delta_6, \Delta_6 \vdash \Delta_4}}{\wedge_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2, F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2, F_3 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \wedge F_3 \vdash \Delta_4} \text{IH}}{\wedge_L}$$

- Case(s) rule \vee_L

$$\frac{\frac{h_1 : F_2, \Delta_5, \Delta_6, \Delta_6, F_2 \vee F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, (\Delta_6, F_2 \vee F_3), \Delta_6, F_2 \vee F_3 \vdash \Delta_4}}{\vee_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2, F_2 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2 \vdash \Delta_4} \text{inv-th/ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \vee F_3 \vdash \Delta_4} \text{IH} \quad \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_3, F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_3 \vdash \Delta_4} \text{inv-th/ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \vee F_3 \vdash \Delta_4} \text{IH}}{\vee_L}$$

$$\frac{\frac{h_1 : F_2, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4}{\bullet h_1 : (\Delta_5, F_2 \vee F_3), \Delta_6, \Delta_6 \vdash \Delta_4}}{\vee_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_2 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_2 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \vee F_3 \vdash \Delta_4} \text{IH} \quad \frac{\frac{h_1 : \Delta_5, \Delta_6, \Delta_6, F_3 \vdash \Delta_4}{h_1 : \Delta_5, \Delta_6, F_3 \vdash \Delta_4} \text{ax}}{\bullet h_1 : \Delta_5, \Delta_6, F_2 \vee F_3 \vdash \Delta_4} \text{IH}}{\vee_L}$$

- Case(s) rule AT

$$\frac{\frac{h_1 : F_2, \Delta_4, \Delta_5, \Delta_5, []F_2, []F_2 \vdash \Delta_3}{\bullet h_1 : \Delta_4, (\Delta_5, []F_2), \Delta_5, []F_2 \vdash \Delta_3}}{AT} \rightarrow \frac{\frac{\frac{h_1 : \Delta_4, \Delta_5, \Delta_5, F_2, []F_2, []F_2 \vdash \Delta_3}{h_1 : \Delta_4, \Delta_5, F_2, []F_2 \vdash \Delta_3} \text{ax}}{\bullet h_1 : \Delta_4, \Delta_5, []F_2 \vdash \Delta_3} \text{IH}}{AT}$$

$$\frac{\frac{h_1 : F_2, \Delta_4, \Delta_5, \Delta_5, []F_2 \vdash \Delta_3}{\bullet h_1 : (\Delta_4, []F_2), \Delta_5, \Delta_5 \vdash \Delta_3}}{AT} \rightarrow \frac{\frac{\frac{h_1 : \Delta_4, \Delta_5, \Delta_5, F_2, []F_2 \vdash \Delta_3}{h_1 : \Delta_4, \Delta_5, F_2, []F_2 \vdash \Delta_3} \text{ax}}{\bullet h_1 : \Delta_4, \Delta_5, []F_2 \vdash \Delta_3} \text{IH}}{AT}$$

- Case(s) rule \perp_L

$$\frac{\frac{h_1 : (\perp, \Delta_3), \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet h_1 : (\perp, \Delta_3), \Delta_4, \Delta_4 \vdash \Delta_2}}{\perp_L} \rightarrow \frac{\frac{h_1 : \perp, \Delta_3, \Delta_4 \vdash \Delta_2}{\bullet h_1 : \perp, \Delta_3, \Delta_4 \vdash \Delta_2}}{\perp_L}$$

$$\frac{\frac{h_1 : \Delta_3, (\perp, \Delta_4), \perp, \Delta_4 \vdash \Delta_2}{\bullet h_1 : \Delta_3, (\perp, \Delta_4), \perp, \Delta_4 \vdash \Delta_2}}{\perp_L} \rightarrow \frac{\frac{h_1 : \perp, \Delta_3, \Delta_4 \vdash \Delta_2}{\bullet h_1 : \perp, \Delta_3, \Delta_4 \vdash \Delta_2}}{\perp_L}$$

- Case(s) rule I

$$\frac{\frac{h_1 : (\Delta_4, p_3), \Delta_5, \Delta_5 \vdash \Delta_2, p_3}{\bullet h_1 : (\Delta_4, p_3), \Delta_5, \Delta_5 \vdash \Delta_2, p_3}}{I} \rightarrow \frac{\frac{h_1 : \Delta_4, \Delta_5, p_3 \vdash \Delta_2, p_3}{\bullet h_1 : \Delta_4, \Delta_5, p_3 \vdash \Delta_2, p_3}}{I}$$

$$\frac{\frac{h_1 : \Delta_4, (\Delta_5, p_3), \Delta_5, p_3 \vdash \Delta_2, p_3}{\bullet h_1 : \Delta_4, (\Delta_5, p_3), \Delta_5, p_3 \vdash \Delta_2, p_3}}{I} \rightarrow \frac{\frac{h_1 : \Delta_4, \Delta_5, p_3 \vdash \Delta_2, p_3}{\bullet h_1 : \Delta_4, \Delta_5, p_3 \vdash \Delta_2, p_3}}{I}$$

- Case(s) rule \top_L

$$\frac{\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet h_1 : (\top, \Delta_3), \Delta_4, \Delta_4 \vdash \Delta_2}}{\top_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{h_1 : \Delta_3, \Delta_4 \vdash \Delta_2} \text{ax}}{\bullet h_1 : \top, \Delta_3, \Delta_4 \vdash \Delta_2} \text{IH}}{\top_L}$$

$$\frac{\frac{h_1 : \top, \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet h_1 : \Delta_3, (\top, \Delta_4), \top, \Delta_4 \vdash \Delta_2}}{\top_L} \rightarrow \frac{\frac{\frac{h_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{h_1 : \Delta_3, \Delta_4 \vdash \Delta_2} \text{inv-th/ax}}{\bullet h_1 : \top, \Delta_3, \Delta_4 \vdash \Delta_2} \text{IH}}{\top_L}$$

6 Height preserving admissibility of contraction on the Right

- Case(s) rule \rightarrow_R

$$\frac{h_1 : F_3, \Delta_2 \vdash F_4, \Delta_5, \Delta_6, \Delta_6, F_3 \rightarrow F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, (\Delta_6, F_3 \rightarrow F_4), \Delta_6, F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_3 \vdash \Delta_5, \Delta_6, \Delta_6, F_4, F_4}{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, \Delta_6, F_4, F_4} \text{IH-Mutual} \quad \frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \rightarrow F_4} \rightarrow_R}{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, \Delta_6} \text{ax} \quad \frac{h_1 : \Delta_2, F_3 \vdash \Delta_5, \Delta_6, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \rightarrow F_4} \rightarrow_R \text{IH}$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5, \Delta_6, \Delta_6, F_3 \wedge F_4 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5, \Delta_6, \Delta_6, F_3 \wedge F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, (\Delta_6, F_3 \wedge F_4), \Delta_6, F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_3}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_4, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \wedge F_4} \wedge_R \text{inv-th/ax}$$

$$\frac{h_1 : \Delta_2 \vdash F_3, \Delta_5, \Delta_6, \Delta_6 \quad h_1 : \Delta_2 \vdash F_4, \Delta_5, \Delta_6, \Delta_6}{\bullet h_1 : \Delta_2 \vdash (\Delta_5, F_3 \wedge F_4), \Delta_6, \Delta_6} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_4} \text{IH}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \wedge F_4} \wedge_R \text{ax}$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5, \Delta_6, \Delta_6, F_3 \vee F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_5, (\Delta_6, F_3 \vee F_4), \Delta_6, F_3 \vee F_4} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_3, F_4, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3, F_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_4, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \vee F_4} \vee_R}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \vee F_4} \vee_R \text{inv-th/ax}$$

$$\frac{h_1 : \Delta_2 \vdash F_3, F_4, \Delta_5, \Delta_6, \Delta_6}{\bullet h_1 : \Delta_2 \vdash (\Delta_5, F_3 \vee F_4), \Delta_6, \Delta_6} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3, F_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, F_3, F_4}{h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \vee F_4} \vee_R}{\bullet h_1 : \Delta_2 \vdash \Delta_5, \Delta_6, F_3 \vee F_4} \vee_R \text{ax}$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4, \Delta_4}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_3), \Delta_4, \Delta_4} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4, \Delta_4}{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, \Delta_4} \perp_R}{\bullet h_1 : \Delta_2 \vdash \Delta_3, \Delta_4} \perp_R \text{ax}$$

$$\frac{h_1 : \Delta_2 \vdash \perp, \Delta_3, \Delta_4, \Delta_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, (\perp, \Delta_4), \perp, \Delta_4} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4, \Delta_4}{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, \Delta_4}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, \Delta_4} \perp_R}{\bullet h_1 : \Delta_2 \vdash \Delta_3, \Delta_4} \perp_R \text{inv-th/ax}$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_3), \Delta_4, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, \Delta_4} \top_R$$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \Delta_3, (\top, \Delta_4), \top, \Delta_4} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, \Delta_4} \top_R$$

- Case(s) rule $A4$

$$\frac{h_1 : \Box \Gamma_5 \vdash F_2}{\bullet h_1 : \Box \Gamma_5, \Delta_6 \vdash \Delta_3, (\Delta_4, [\Box F_2]), \Delta_4, [\Box F_2]} A4 \rightarrow \frac{h_1 : \Box \Gamma_5 \vdash F_2}{\bullet h_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_3, \Delta_4, [\Box F_2]} \text{ax} A4$$

$$\frac{h_1 : \Box \Gamma_5 \vdash F_2}{\bullet h_1 : \Box \Gamma_5, \Delta_6 \vdash (\Delta_3, [\Box F_2]), \Delta_4, \Delta_4} A4 \rightarrow \frac{h_1 : \Box \Gamma_5 \vdash F_2}{\bullet h_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_3, \Delta_4, [\Box F_2]} \text{ax} A4$$

- Case(s) rule K

$$\frac{h_1 : \text{unbox}(\Box \Gamma_5) \vdash F_2}{\bullet h_1 : \Box \Gamma_5, \Delta_6 \vdash \Delta_3, (\Delta_4, [\Box F_2]), \Delta_4, [\Box F_2]} K \rightarrow \frac{\overline{h_1 : \text{unbox}(\Box \Gamma_5) \vdash F_2}^{\text{ax}}}{\bullet h_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_3, \Delta_4, [\Box F_2]} K$$

$$\frac{h_1 : \text{unbox}(\Box \Gamma_5) \vdash F_2}{\bullet h_1 : \Box \Gamma_5, \Delta_6 \vdash (\Delta_3, [\Box F_2]), \Delta_4, \Delta_4} K \rightarrow \frac{\overline{h_1 : \text{unbox}(\Box \Gamma_5) \vdash F_2}^{\text{ax}}}{\bullet h_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_3, \Delta_4, [\Box F_2]} K$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_6 \vdash F_2, \Delta_4, \Delta_5, \Delta_5 \quad h_1 : F_3, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5}{\bullet h_1 : \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4, \Delta_5, \Delta_5} \rightarrow_L \rightarrow \frac{\frac{\overline{h_1 : \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5, F_2}^{\text{ax}}}{h_1 : \Delta_6 \vdash \Delta_4, \Delta_5, F_2} \text{IH} \quad \frac{\overline{h_1 : \Delta_6, F_3 \vdash \Delta_4, \Delta_5, \Delta_5}^{\text{ax}}}{h_1 : \Delta_6, F_3 \vdash \Delta_4, \Delta_5} \text{IH}}{\bullet h_1 : \Delta_6, F_2 \rightarrow F_3 \vdash \Delta_4, \Delta_5} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : F_2, F_3, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5}{\bullet h_1 : \Delta_6, F_2 \wedge F_3 \vdash \Delta_4, \Delta_5, \Delta_5} \wedge_L \rightarrow \frac{\frac{\overline{h_1 : \Delta_6, F_2, F_3 \vdash \Delta_4, \Delta_5, \Delta_5}^{\text{ax}}}{h_1 : \Delta_6, F_2, F_3 \vdash \Delta_4, \Delta_5} \text{IH}}{\bullet h_1 : \Delta_6, F_2 \wedge F_3 \vdash \Delta_4, \Delta_5} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : F_2, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5 \quad h_1 : F_3, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5}{\bullet h_1 : \Delta_6, F_2 \vee F_3 \vdash \Delta_4, \Delta_5, \Delta_5} \vee_L \rightarrow \frac{\frac{\overline{h_1 : \Delta_6, F_2 \vdash \Delta_4, \Delta_5, \Delta_5}^{\text{ax}}}{h_1 : \Delta_6, F_2 \vdash \Delta_4, \Delta_5} \text{IH} \quad \frac{\overline{h_1 : \Delta_6, F_3 \vdash \Delta_4, \Delta_5, \Delta_5}^{\text{ax}}}{h_1 : \Delta_6, F_3 \vdash \Delta_4, \Delta_5} \text{IH}}{\bullet h_1 : \Delta_6, F_2 \vee F_3 \vdash \Delta_4, \Delta_5} \vee_L$$

- Case(s) rule AT

$$\frac{h_1 : F_2, \Delta_5, [\Box F_2] \vdash \Delta_3, \Delta_4, \Delta_4}{\bullet h_1 : \Delta_5, [\Box F_2] \vdash \Delta_3, \Delta_4, \Delta_4} AT \rightarrow \frac{\frac{\overline{h_1 : \Delta_5, F_2, [\Box F_2] \vdash \Delta_3, \Delta_4, \Delta_4}^{\text{ax}}}{h_1 : \Delta_5, F_2, [\Box F_2] \vdash \Delta_3, \Delta_4} \text{IH}}{\bullet h_1 : \Delta_5, [\Box F_2] \vdash \Delta_3, \Delta_4} AT$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_4 \vdash \Delta_2, \Delta_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_5, p_4 \vdash \Delta_2, (\Delta_3, p_4), \Delta_3, p_4} I \rightarrow \frac{}{\bullet h_1 : \Delta_5, p_4 \vdash \Delta_2, \Delta_3, p_4} I$$

$$\frac{}{\bullet h_1 : \Delta_5, p_4 \vdash (\Delta_2, p_4), \Delta_3, \Delta_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_5, p_4 \vdash \Delta_2, \Delta_3, p_4} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3}{\bullet h_1 : \top, \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3} \top_L \rightarrow \frac{\frac{\overline{h_1 : \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3}^{\text{ax}}}{h_1 : \Delta_4 \vdash \Delta_2, \Delta_3} \text{IH}}{\bullet h_1 : \top, \Delta_4 \vdash \Delta_2, \Delta_3} \top_L$$

7 Identity-Expansion

$$\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : \Box F_0 \vdash \Box F_0} K$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0, F_1} W}{\frac{- : F_0 \vee F_1 \vdash F_0, F_1}{- : F_0 \vee F_1 \vdash F_0 \vee F_1} \vee_R} \vee_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_1 \vdash F_0} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{\frac{- : F_0, F_1 \vdash F_0 \wedge F_1}{- : F_0 \wedge F_1 \vdash F_0 \wedge F_1} \wedge_L} \wedge_R$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{\frac{- : F_0, F_0 \rightarrow F_1 \vdash F_1}{- : F_0 \rightarrow F_1 \vdash F_0 \rightarrow F_1} \rightarrow_R} \rightarrow_L$$

$$\overline{- : \top \vdash \top} \top_R$$

$$\overline{- : \perp \vdash \perp} \perp_L$$

8 Cut-Elimination

8.1 Status of \rightarrow_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{h_1 : F_6, \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \rightarrow F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_{10}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}}{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}} \text{ax/W}}{\frac{- : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}}{\rightarrow_R} \text{hCut}} \\
\frac{\frac{h_1 : F_7, \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8} \rightarrow_R}{\frac{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_7 \vdash \Delta_{12}, F_{11}, F_{13}, F_8}{\bullet h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8}{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8} \text{ax/W}}{\frac{- : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \rightarrow F_8}{\rightarrow_R} \text{hCut}} \\
\frac{\frac{h_1 : F_8, \Delta_{12} \vdash F_9, F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{10}, F_8 \rightarrow F_9), F_{11}} \rightarrow_R \quad \frac{h_7 : F_8, F_{11}, \Delta_{12} \vdash F_9, \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R}{\frac{- : \Delta_{12} \vdash \Delta_{10}, F_8 \rightarrow F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, F_{11}, F_9}{\bullet h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, F_{11}, F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9}{\bullet h_7 : \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} \text{ax/W}}{\frac{- : \Delta_{12}, F_8 \vdash \Delta_{10}, F_9}{\rightarrow_R} \text{hCut}}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{h_1 : F_6, \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \wedge F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9, \Delta_{11} \quad h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \rightarrow_R}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9} \text{ax/W}}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9}{\rightarrow_R} \text{hCut}} \\
\frac{\frac{h_1 : F_7, \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8} \rightarrow_R}{\frac{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{13}, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{13}, F_8, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}}{\rightarrow_R} \text{hCut}}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \vee F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9, F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_{10}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9, F_6 \rightarrow F_7} \text{inv-th/ax}}{\rightarrow_R} \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_{10}, F_9}{\text{ax/W}} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9 \quad \vee_R \\
- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \quad \text{hCut} \\
\hline
\frac{h_1 : F_7, \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \vee F_{11}), F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, F_{11}, \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \rightarrow F_8} \vee_R \\
\hline
- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{13}, F_8, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{13}, F_8, F_{10} \vee F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_{11}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11}} \text{inv-th/ax} \\
\hline
- : \Delta_{14}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11} \quad \vee_R \\
- : \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8, F_{10} \vee F_{11} \quad \text{hCut} \\
\rightarrow_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{10} \vdash F_7, \perp, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_9} \perp_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_9 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \rightarrow F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_9 \quad \text{hCut} \\
\hline
\frac{h_1 : F_7, \Delta_{12} \vdash F_8, F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\perp, \Delta_{10}), F_7 \rightarrow F_8), F_{11}} \rightarrow_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\perp, \Delta_{10}), F_7 \rightarrow F_8} \perp_R \\
\hline
- : \Delta_{12} \vdash (\perp, \Delta_{10}), F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{12} \vdash \perp, \Delta_{10}, F_{11}, F_7 \rightarrow F_8}{\text{ax/W}} \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8} \text{ax/W} \\
\hline
- : \Delta_{12} \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8 \quad \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{10} \vdash F_7, \top, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_9} \top_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_9 \\
\rightarrow \\
- : \Delta_{10} \vdash \top, \Delta_9 \quad \top_R \\
\hline
\frac{h_1 : F_7, \Delta_{12} \vdash F_8, F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), F_7 \rightarrow F_8), F_{11}} \rightarrow_R \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8} \top_R \\
\hline
- : \Delta_{12} \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8 \\
\rightarrow \\
- : \Delta_{12} \vdash \top, \Delta_{10}, F_7 \rightarrow F_8 \quad \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{h_1 : F_6, \Box \Gamma_{11}, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_9}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \Box F_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Box \Gamma_{11} \vdash F_9}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{12}), F_6 \rightarrow F_7 \vdash \Delta_{10}, \Box F_9} A4 \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \Box F_9 \\
\rightarrow \\
- : \Box \Gamma_{11} \vdash F_9 \quad \text{ax/W} \\
\hline
- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_9 \quad A4
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Box \Gamma_{13}, \Delta_{14} \vdash F_8, \Box F_{12}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8), \Box F_{12}} \rightarrow_R \quad \frac{h_9 : \Box \Gamma_{13}, \Box F_{12} \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{13}, \Delta_{14}), \Box F_{12} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8} \begin{array}{l} A4 \\ \text{Cut} \end{array} \\
\hline
- : \Box \Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_8, \Box F_{10}}{- : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}} \text{ ax/W} \quad \frac{h_9 : \Box F_{12}, \Box \Gamma_{13} \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}} \begin{array}{l} A4 \\ \text{hCut} \end{array}}{- : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}} \rightarrow_R \\
\hline
- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, \Box F_{10}, F_7 \rightarrow F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Box \Gamma_{12}, \Delta_{14} \vdash F_8, F_{13}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : \Box \Gamma_{12} \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8} \begin{array}{l} A4 \\ \text{Cut} \end{array} \\
\hline
- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_{10}}{- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_{10}, F_7 \rightarrow F_8} \begin{array}{l} \text{ax/W} \\ A4 \end{array}
\end{array}$$

• Case rule K

$$\begin{array}{c}
\frac{h_1 : F_6, \Box \Gamma_{11}, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_9}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \Box F_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_{11}) \vdash F_9}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{12}), F_6 \rightarrow F_7 \vdash \Delta_{10}, \Box F_9} \begin{array}{l} K \\ \text{Cut} \end{array} \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \Box F_9 \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_9}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_9} \begin{array}{l} \text{ax/W} \\ K \end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Box \Gamma_{13}, \Delta_{14} \vdash F_8, \Box F_{12}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8), \Box F_{12}} \rightarrow_R \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{13}), \text{unbox}(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{13}, \Delta_{14}), \Box F_{12} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8} \begin{array}{l} K \\ \text{Cut} \end{array} \\
\hline
- : \Box \Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_8, \Box F_{10}}{- : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}} \text{ ax/W} \quad \frac{h_9 : \text{unbox}(\Box F_{12}), \text{unbox}(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}} \begin{array}{l} K \\ \text{hCut} \end{array}}{- : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}} \rightarrow_R \\
\hline
- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, \Box F_{10}, F_7 \rightarrow F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Box \Gamma_{12}, \Delta_{14} \vdash F_8, F_{13}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : \text{unbox}(\Box \Gamma_{12}) \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8} \begin{array}{l} K \\ \text{Cut} \end{array} \\
\hline
- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_{12}) \vdash F_{10}}{- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_{10}, F_7 \rightarrow F_8} \begin{array}{l} \text{ax/W} \\ K \end{array}
\end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \begin{array}{l} \text{Cut} \\ \rightarrow_L \end{array} \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \text{ inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \text{ ax/W} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{ inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \text{ a}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{10} \vdash F_8, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_6 : \Delta_{10} \vdash F_7, \Delta_9 \quad h_6 : F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \begin{array}{l} \text{Cut} \\ \rightarrow_L \end{array} \\
\hline
- : \Delta_{10} \vdash \Delta_9 \\
\rightarrow \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7}{- : \Delta_{10} \vdash \Delta_9} \text{ ax/W} \quad \frac{- : \Delta_{10}, F_7 \vdash \Delta_9, F_8}{- : \Delta_{10}, F_7 \vdash \Delta_9} \text{ ax/W} \quad \frac{- : \Delta_{10}, F_7, F_8 \vdash \Delta_9}{- : \Delta_{10} \vdash \Delta_9} \text{ sCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{13} \vdash F_8, F_{10} \rightarrow F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{10} \rightarrow F_{11}} \rightarrow_R \quad \frac{h_9 : \Delta_{13} \vdash F_{10}, \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \rightarrow F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \\
\hline
- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8 \quad \text{hCut} \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \rightarrow_R
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \rightarrow_L \\
\hline
- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}, F_8}{\bullet h_1 : \Delta_{14}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \\
\hline
- : \Delta_{14}, F_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_8 \quad \text{hCut} \\
\hline
- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \rightarrow_R
\end{array}$$

• Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \wedge_L \\
\hline
- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_{10}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \\
\hline
- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11} \quad \text{hCut} \\
\hline
- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \quad \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{13} \vdash F_8, F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{10} \wedge F_{11}} \rightarrow_R \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \wedge_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \\
\hline
- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8 \quad \wedge_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{14}, F_{10} \wedge F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \wedge F_{11}), F_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \wedge_L \\
\hline
- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_{13}, F_8}{\bullet h_1 : \Delta_{14}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_{13}, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \text{inv-th/ax} \\
\hline
- : \Delta_{14}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8 \quad \wedge_L \\
\hline
- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{hCut}
\end{array}$$

• Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \vee_L \\
\hline
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7} \text{inv-th/ax} \\
\hline
- : \Delta_{12}, F_9 \vdash \Delta_{11} \quad \text{hCut} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7} \text{inv-th/ax} \\
\hline
- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11} \quad \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{13} \vdash F_8, F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{10} \vee F_{11}} \rightarrow_R \quad \frac{h_9 : F_{10}, \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8} \vee_L \\
\hline
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \vee_L \\
\hline
- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_8 \quad \text{hCut} \\
\rightarrow_R \\
- : \Delta_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8 \\
\hline
\frac{h_1 : F_7, \Delta_{14}, F_{10} \vee F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash (\Delta_{12}, F_7 \rightarrow F_8), F_{13}} \rightarrow_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad h_9 : F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \rightarrow F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \vee F_{11}), F_{13} \vdash \Delta_{12}, F_7 \rightarrow F_8} \vee_L \\
\hline
- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_8}{\bullet h_1 : \Delta_{14}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax} \vee_L \\
\hline
- : \Delta_{14}, F_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8 \quad \text{hCut} \\
\rightarrow_R \\
- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \rightarrow F_8
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{11}, [F_9 \vdash F_7, \Delta_{10}]}{\bullet h_1 : \Delta_{11}, [F_9 \vdash \Delta_{10}, F_6 \rightarrow F_7]} \rightarrow_R \quad \frac{h_8 : F_9, \Delta_{11}, [F_9, F_6 \rightarrow F_7 \vdash \Delta_{10}]}{\bullet h_8 : (\Delta_{11}, [F_9]), F_6 \rightarrow F_7 \vdash \Delta_{10}} AT \\
\hline
- : \Delta_{11}, [F_9 \vdash \Delta_{10}] \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}, F_6 \rightarrow F_7]}{\bullet h_1 : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}]} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_9, [F_9, F_6 \rightarrow F_7 \vdash \Delta_{10}]}{\bullet h_8 : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}]} \text{ax/W} \\
\hline
- : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}] \quad \text{hCut} \\
\rightarrow_R \\
- : \Delta_{11}, [F_9 \vdash \Delta_{10}] \quad AT \\
\hline
\frac{h_1 : F_7, \Delta_{12} \vdash F_8, [F_{10}, \Delta_{11}]}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_7 \rightarrow F_8), [F_{10}]} \rightarrow_R \quad \frac{h_9 : F_{10}, \Delta_{12}, [F_{10} \vdash \Delta_{11}, F_7 \rightarrow F_8]}{\bullet h_9 : \Delta_{12}, [F_{10} \vdash \Delta_{11}, F_7 \rightarrow F_8]} AT \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_7 \vdash \Delta_{11}, F_8, [F_{10}]}{\bullet h_1 : \Delta_{12}, F_7 \vdash \Delta_{11}, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_{10}, F_7, [F_{10} \vdash \Delta_{11}, F_8]}{\bullet h_9 : \Delta_{12}, F_7, [F_{10} \vdash \Delta_{11}, F_8]} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{12}, F_{10}, F_7, [F_{10} \vdash \Delta_{11}, F_8]}{\bullet h_9 : \Delta_{12}, F_7, [F_{10} \vdash \Delta_{11}, F_8]} AT \\
\hline
- : \Delta_{12}, F_7 \vdash \Delta_{11}, F_8 \quad \text{hCut} \\
\rightarrow_R \\
- : \Delta_{12} \vdash \Delta_{11}, F_7 \rightarrow F_8 \\
\hline
\frac{h_1 : F_7, \Delta_{13}, [F_{10} \vdash F_8, F_{12}, \Delta_{11}]}{\bullet h_1 : \Delta_{13}, [F_{10} \vdash (\Delta_{11}, F_7 \rightarrow F_8), F_{12}]} \rightarrow_R \quad \frac{h_9 : F_{10}, F_{12}, \Delta_{13}, [F_{10} \vdash \Delta_{11}, F_7 \rightarrow F_8]}{\bullet h_9 : (\Delta_{13}, [F_{10}], F_{12} \vdash \Delta_{11}, F_7 \rightarrow F_8)} AT \\
\hline
- : \Delta_{13}, [F_{10} \vdash \Delta_{11}, F_7 \rightarrow F_8] \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{13}, F_{10}, [F_{10} \vdash \Delta_{11}, F_{12}, F_7 \rightarrow F_8]}{\bullet h_1 : \Delta_{13}, F_{10}, [F_{10} \vdash \Delta_{11}, F_{12}, F_7 \rightarrow F_8]} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{12}, [F_{10} \vdash \Delta_{11}, F_7 \rightarrow F_8]}{\bullet h_9 : \Delta_{13}, F_{10}, F_{12}, [F_{10} \vdash \Delta_{11}, F_7 \rightarrow F_8]} \text{ax/W} \\
\hline
- : \Delta_{13}, F_{10}, [F_{10} \vdash \Delta_{11}, F_7 \rightarrow F_8] \quad \text{hCut} \\
\rightarrow_R \\
- : \Delta_{13}, [F_{10} \vdash \Delta_{11}, F_7 \rightarrow F_8] \quad AT
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : F_6, \perp, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{\bullet h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \perp_L \\
\hline
- : \perp, \Delta_{10} \vdash \Delta_9 \quad \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_{10} \vdash \Delta_9 \quad \perp_L \\
\hline
\frac{h_1 : F_7, \Delta_{11} \vdash F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \rightarrow F_8), \perp} \rightarrow_R \quad \frac{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \rightarrow F_8} \perp_L \\
\hline
- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}, F_8}{\bullet h_1 : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}, F_8} \text{ax/W} \quad \frac{\bullet h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}, F_8}{\bullet h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}, F_8} \perp_L \\
\hline
- : \Delta_{11}, F_7 \vdash \Delta_{10}, F_8 \quad \text{hCut} \\
\rightarrow_R \\
- : \Delta_{11} \vdash \Delta_{10}, F_7 \rightarrow F_8
\end{array}$$

- Case rule I

- Case rule \top_L

8.2 Status of \wedge_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11} \quad h_1 : \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8} \rightarrow_R \\
\frac{}{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_8}{\bullet h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_8} \text{inv-th/ax}}{\frac{}{- : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8} \wedge_R} \text{ax/W} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8} \rightarrow_R \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8} \text{ax/W} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8} \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, \Delta_{11}, F_9 \wedge F_{10} \quad h_1 : \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \wedge F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9, \Delta_{11} \quad h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \wedge_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_6, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9} \text{inv-th/ax} \quad \frac{}{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_{10}} \text{inv-th/ax} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_6, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6 \vdash \Delta_{11}, F_9 \wedge F_{10}} \text{sCut} \quad \frac{}{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \wedge_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}} \text{sCut} \\
\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \wedge F_{11} \quad h_1 : \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_7}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_8}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_8} \text{inv-th/ax}}{\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8} \wedge_R} \text{ax/W} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8} \text{hCut} \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8} \text{ax/W} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \wedge F_8} \text{hCut} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \wedge F_8} \wedge_R \\
\frac{h_1 : \Delta_{12} \vdash F_8, F_{11}, \Delta_{10} \quad h_1 : \Delta_{12} \vdash F_9, F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{10}, F_8 \wedge F_9), F_{11}} \wedge_R \quad \frac{h_7 : F_{11}, \Delta_{12} \vdash F_8, \Delta_{10} \quad h_7 : F_{11}, \Delta_{12} \vdash F_9, \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_8}{\bullet h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8} \text{H} \quad \frac{h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_9}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_9} \text{ax/W} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_8} \text{hCut} \quad \frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_9} \text{hCut} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, \Delta_{11}, F_9 \vee F_{10} \quad h_1 : \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \vee F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9, F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6, F_9} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_7, F_9} \text{inv-th/ax}}{\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9} \wedge_R} \text{ax/W} \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9} \text{hCut} \quad \frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R \\
\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \vee F_{11} \quad h_1 : \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, F_{11}, \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8} \vee_R \\
\frac{}{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_8}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_8} \text{inv-th/ax}}{\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8} \wedge_R} \text{ax/W} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8} \text{hCut} \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8} \text{ax/W} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8} \text{hCut} \\
\frac{}{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8} \vee_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_6, \perp, \Delta_9 \quad h_1 : \Delta_{10} \vdash F_7, \perp, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_9} \perp_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_9 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \wedge F_7}{- : \Delta_{10} \vdash \perp, \Delta_9} \text{ax/W} \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_9}{- : \Delta_{10} \vdash \perp, \Delta_9} \text{hCut} \\
\hline
\frac{h_1 : \Delta_{12} \vdash F_7, F_{11}, \perp, \Delta_{10} \quad h_1 : \Delta_{12} \vdash F_8, F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\perp, \Delta_{10}), F_7 \wedge F_8), F_{11}} \wedge_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\perp, \Delta_{10}), F_7 \wedge F_8} \perp_R \\
\hline
- : \Delta_{12} \vdash (\perp, \Delta_{10}), F_7 \wedge F_8 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{12} \vdash \perp, \Delta_{10}, F_{11}, F_7 \wedge F_8}{- : \Delta_{12} \vdash \perp, \Delta_{10}, F_7 \wedge F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \wedge F_8}{- : \Delta_{12} \vdash \perp, \Delta_{10}, F_7 \wedge F_8} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_6, \top, \Delta_9 \quad h_1 : \Delta_{10} \vdash F_7, \top, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_9), F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \top, \Delta_9} \top_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_9 \\
\rightarrow \\
\frac{}{- : \Delta_{10} \vdash \top, \Delta_9} \top_R \\
\hline
\frac{h_1 : \Delta_{12} \vdash F_7, F_{11}, \top, \Delta_{10} \quad h_1 : \Delta_{12} \vdash F_8, F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), F_7 \wedge F_8), F_{11}} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\top, \Delta_{10}), F_7 \wedge F_8} \top_R \\
\hline
- : \Delta_{12} \vdash (\top, \Delta_{10}), F_7 \wedge F_8 \\
\rightarrow \\
\frac{}{- : \Delta_{12} \vdash \top, \Delta_{10}, F_7 \wedge F_8} \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash F_6, \Delta_{10}, \Box F_9 \quad h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_9}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \Box F_9), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Box \Gamma_{11} \vdash F_9}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{12}), F_6 \wedge F_7 \vdash \Delta_{10}, \Box F_9} A4 \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \Box F_9 \\
\rightarrow \\
\frac{}{- : \Box \Gamma_{11} \vdash F_9} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_9} A4 \\
\hline
\frac{h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash F_7, \Box F_{12}, \Delta_{11}, \Box F_{10} \quad h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash F_8, \Box F_{12}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Box \Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \wedge F_8), \Box F_{12}} \wedge_R \quad \frac{h_9 : \Box \Gamma_{13}, \Box F_{12} \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{13}, \Delta_{14}), \Box F_{12} \vdash (\Delta_{11}, \Box F_{10}), F_7 \wedge F_8} A4 \\
\hline
- : \Box \Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \wedge F_8 \\
\rightarrow \\
\frac{h_9 : \Box F_{12}, \Box \Gamma_{13} \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} \text{ax/W} \quad \frac{}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} A4 \\
\frac{h_1 : \Delta_{14}, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_7, \Box F_{10}}{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} \text{ax/W} \quad \frac{h_1 : \Delta_{14}, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_8, \Box F_{10}}{- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_8, \Box F_{10}} \text{ax/W} \\
\hline
- : \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, \Box F_{10}, F_7 \wedge F_8 \\
\hline
\frac{h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash F_7, F_{13}, \Delta_{11}, \Box F_{10} \quad h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash F_8, F_{13}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \Box F_{10}), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : \Box \Gamma_{12} \vdash F_{10}}{\bullet h_9 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, \Box F_{10}), F_7 \wedge F_8} A4 \\
\hline
- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \Box F_{10}), F_7 \wedge F_8 \\
\rightarrow \\
\frac{}{- : \Box \Gamma_{12} \vdash F_{10}} \text{ax/W} \\
\frac{}{- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_{10}, F_7 \wedge F_8} A4
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash F_6, \Delta_{10}, \Box F_9 \quad h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash F_7, \Delta_{10}, \Box F_9}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \Box F_9), F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \text{unbox}(\Box \Gamma_{11}) \vdash F_9}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{12}), F_6 \wedge F_7 \vdash \Delta_{10}, \Box F_9} K \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \Box F_9 \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box \Gamma_{11}) \vdash F_9} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_9} K
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_{13}, \Delta_{14} \vdash F_7, \Box F_{12}, \Delta_{11}, \llbracket F_{10} \rrbracket \quad h_1 : \Box\Gamma_{13}, \Delta_{14} \vdash F_8, \Box F_{12}, \Delta_{11}, \llbracket F_{10} \rrbracket}{\bullet h_1 : \Box\Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, \llbracket F_{10} \rrbracket), F_7 \wedge F_8), \Box F_{12}} \wedge_R \quad \frac{h_9 : unbox(\Box\Gamma_{13}), unbox(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : (\Box\Gamma_{13}, \Delta_{14}), \Box F_{12} \vdash (\Delta_{11}, \llbracket F_{10} \rrbracket), F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Box\Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, \llbracket F_{10} \rrbracket), F_7 \wedge F_8} \rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_7, \llbracket F_{10} \rrbracket}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \llbracket F_{10} \rrbracket} \text{ax/W} \quad \frac{\frac{h_9 : unbox(\Box F_{12}), unbox(\Box\Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \llbracket F_{10} \rrbracket} \text{ax/W} \quad \frac{}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \llbracket F_{10} \rrbracket} K}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \llbracket F_{10} \rrbracket} hCut \\
\frac{}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \llbracket F_{10}, F_7 \wedge F_8 \rrbracket} \rightarrow \\
\frac{h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash F_7, F_{13}, \Delta_{11}, \llbracket F_{10} \rrbracket \quad h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash F_8, F_{13}, \Delta_{11}, \llbracket F_{10} \rrbracket}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \llbracket F_{10} \rrbracket), F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : unbox(\Box\Gamma_{12}) \vdash F_{10}}{\bullet h_9 : (\Box\Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, \llbracket F_{10} \rrbracket), F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \llbracket F_{10} \rrbracket), F_7 \wedge F_8} \rightarrow \\
\frac{}{- : unbox(\Box\Gamma_{12}) \vdash F_{10}} \text{ax/W} \quad \frac{}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \llbracket F_{10}, F_7 \wedge F_8 \rrbracket} K
\end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_R \\
\frac{}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow \\
\frac{}{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9} \text{inv-th/ax} \quad \frac{}{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}} \text{inv-th/ax} \\
\frac{}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_7} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut} \\
\frac{}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_7} \text{ax/W} \quad \frac{}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut} \\
\frac{}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{sCut} \\
\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \rightarrow F_{11}, \Delta_{12} \quad h_1 : \Delta_{13} \vdash F_8, F_{10} \rightarrow F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \rightarrow F_{11}} \wedge_R \quad \frac{h_9 : \Delta_{13} \vdash F_{10}, \Delta_{12}, F_7 \wedge F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \rightarrow \\
\frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7} \text{inv-th/ax} \quad \frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_8} \text{inv-th/ax} \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8} \text{ax/W} \quad \frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8} \wedge_R \quad \frac{}{- : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8} \text{ax/W} \quad \frac{}{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \wedge F_8} \wedge_R \quad \frac{}{- : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \text{sCut} \\
\frac{h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_{10} \rightarrow F_{11}) \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \rightarrow \\
\frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7 \quad h_9 : \Delta_{14}, F_{11}, F_{13} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{14}, F_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax} \quad \frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax} \\
\frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}, F_7} \text{ax/W} \quad \frac{}{- : \Delta_{14}, F_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \text{hCut} \quad \frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \text{hCut} \\
\frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7} \text{hCut} \quad \frac{}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L
\end{array}$$

• Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_R \\
\frac{}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \rightarrow \\
\frac{}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6} \text{inv-th/ax} \quad \frac{}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7} \text{inv-th/ax} \\
\frac{}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{}{- : \Delta_{12}, F_{10}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}} \text{ax/W} \\
\frac{}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \wedge_L \quad \frac{}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{10} \vdash F_7, \Delta_9 \quad h_1 : \Delta_{10} \vdash F_8, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \wedge F_8} \wedge_R \quad \frac{h_6 : F_7, F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \wedge_L}{\frac{- : \Delta_{10} \vdash \Delta_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{10} \vdash \Delta_9, F_7}{\text{ax/W}} \quad \frac{\frac{- : \Delta_{10}, F_7 \vdash \Delta_9, F_8}{\text{ax/W}} \quad \frac{- : \Delta_{10}, F_7, F_8 \vdash \Delta_9}{\text{sCut}}}{- : \Delta_{10} \vdash \Delta_9} \text{sCut} \\
\frac{\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \wedge F_{11}, \Delta_{12} \quad h_1 : \Delta_{13} \vdash F_8, F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \wedge F_{11}} \wedge_R \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \wedge F_{11}}{\text{ax/W}} \quad \frac{\frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax}}{\text{hCut}} \wedge_L \quad \frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_8, F_{10} \wedge F_{11}}{\text{ax/W}} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \text{inv-th/ax}}{\text{hCut}} \wedge_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_R \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \wedge F_{11}), F_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_{13}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_{13}, F_8}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_{13}, F_7 \wedge F_8} \text{inv-th/ax}}{\text{hCut}} \wedge_R \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\text{ax/W}} \text{hCut} \\
\frac{- : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L}{\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6}{\text{ax/W}} \quad \frac{\frac{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7}{\text{ax/W}} \quad \frac{- : \Delta_{12}, F_6, F_7, F_9 \vdash \Delta_{11}}{\text{inv-th/ax}} \quad \frac{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}}{\text{inv-th/ax}}}{\text{sCut}} \wedge_L \\
\frac{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{sCut} \\
\frac{\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \vee F_{11}, \Delta_{12} \quad h_1 : \Delta_{13} \vdash F_8, F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{10} \vee F_{11}} \wedge_R \quad \frac{h_9 : F_{10}, \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7}{\text{inv-th/ax}} \quad \frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_8}{\text{inv-th/ax}}}{\text{hCut}} \wedge_R \quad \frac{- : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8}{\text{ax/W}} \text{sCut} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \wedge F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7 \wedge F_8} \wedge_R \quad \frac{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash (\Delta_{12}, F_7 \wedge F_8), F_{13}} \wedge_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \wedge F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \vee F_{11}), F_{13} \vdash \Delta_{12}, F_7 \wedge F_8} \wedge_L}{\frac{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_7}{\text{ax/W}} \quad \frac{\frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax}}{\text{hCut}} \wedge_L \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13} \vdash \Delta_{12}, F_7}{\text{inv-th/ax}} \wedge_L}{\frac{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7}{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8}}
\end{array}$$

- Case rule $\wedge T$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, []F_9 \vdash F_6, \Delta_{10} \quad h_1 : \Delta_{11}, []F_9 \vdash F_7, \Delta_{10}}{\bullet h_1 : \Delta_{11}, []F_9 \vdash \Delta_{10}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : F_9, \Delta_{11}, []F_9, F_6 \wedge F_7 \vdash \Delta_{10}}{\bullet h_8 : (\Delta_{11}, []F_9), F_6 \wedge F_7 \vdash \Delta_{10}} AT \\
\frac{}{- : \Delta_{11}, []F_9 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}, F_6 \wedge F_7}{- : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}} ax/W \quad \frac{h_8 : \Delta_{11}, F_9, []F_9, F_6 \wedge F_7 \vdash \Delta_{10}}{h_8 : (\Delta_{11}, F_9), F_6 \wedge F_7 \vdash \Delta_{10}} ax/W}{- : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}} hCut \\
\frac{}{- : \Delta_{11}, []F_9 \vdash \Delta_{10}} AT \\
\frac{h_1 : \Delta_{12} \vdash F_7, []F_{10}, \Delta_{11} \quad h_1 : \Delta_{12} \vdash F_8, []F_{10}, \Delta_{11}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_7 \wedge F_8), []F_{10}} \wedge_R \quad \frac{h_9 : F_{10}, \Delta_{12}, []F_{10} \vdash \Delta_{11}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{12}, []F_{10} \vdash \Delta_{11}, F_7 \wedge F_8} AT \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_7, []F_{10}}{h_1 : \Delta_{12} \vdash \Delta_{11}, F_7, []F_{10}} ax/W \quad \frac{h_9 : \Delta_{12}, F_{10}, []F_{10} \vdash \Delta_{11}, F_7}{\bullet h_9 : \Delta_{12}, []F_{10} \vdash \Delta_{11}, F_7} inv-th/ax \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_7} hCut \quad \frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_8, []F_{10}}{h_1 : \Delta_{12} \vdash \Delta_{11}, F_8, []F_{10}} ax/W \quad \frac{h_9 : \Delta_{12}, F_{10}, []F_{10} \vdash \Delta_{11}, F_8}{\bullet h_9 : \Delta_{12}, []F_{10} \vdash \Delta_{11}, F_8} inv-th/ax \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_8} hCut \quad \frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{11}, F_7 \wedge F_8} AT \\
\frac{h_1 : \Delta_{13}, []F_{10} \vdash F_7, F_{12}, \Delta_{11} \quad h_1 : \Delta_{13}, []F_{10} \vdash F_8, F_{12}, \Delta_{11}}{\bullet h_1 : \Delta_{13}, []F_{10} \vdash (\Delta_{11}, F_7 \wedge F_8), F_{12}} \wedge_R \quad \frac{h_9 : F_{10}, F_{12}, \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \wedge F_8}{\bullet h_9 : (\Delta_{13}, []F_{10}), F_{12} \vdash \Delta_{11}, F_7 \wedge F_8} AT \\
\frac{}{- : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{13}, F_{10}, []F_{10} \vdash \Delta_{11}, F_{12}, F_7 \wedge F_8}{- : \Delta_{13}, F_{10}, []F_{10} \vdash \Delta_{11}, F_7 \wedge F_8} ax/W \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{12}, []F_{10} \vdash \Delta_{11}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{13}, F_{10}, F_{12} \vdash \Delta_{11}, F_7 \wedge F_8} ax/W \\
\frac{}{- : \Delta_{13}, F_{10}, []F_{10} \vdash \Delta_{11}, F_7 \wedge F_8} hCut \quad \frac{}{- : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \wedge F_8} AT
\end{array}$$

• Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \perp, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{\bullet h_8 : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \perp_L \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L \\
\frac{h_1 : \Delta_{11} \vdash F_7, \perp, \Delta_{10} \quad h_1 : \Delta_{11} \vdash F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \wedge F_8), \perp} \wedge_R \quad \frac{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \wedge F_8} \perp_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7}{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7} ax/W \quad \frac{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7}{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7} \perp_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7} hCut \quad \frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_8}{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_8} ax/W \quad \frac{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_8}{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_8} \perp_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_8} hCut \quad \frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \wedge_R \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8} AT \\
\frac{h_1 : \perp, \Delta_{12} \vdash F_7, F_{11}, \Delta_{10} \quad h_1 : \perp, \Delta_{12} \vdash F_8, F_{11}, \Delta_{10}}{\bullet h_1 : \perp, \Delta_{12} \vdash (\Delta_{10}, F_7 \wedge F_8), F_{11}} \wedge_R \quad \frac{\bullet h_9 : (\perp, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : (\perp, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \perp_L \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_{10}, F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_{10}, F_7 \wedge F_8} \perp_L
\end{array}$$

• Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, p_{10} \vdash F_6, \Delta_9, p_{10} \quad h_1 : \Delta_{11}, p_{10} \vdash F_7, \Delta_9, p_{10}}{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6 \wedge F_7} \wedge_R \quad \frac{\bullet h_8 : (\Delta_{11}, p_{10}), F_6 \wedge F_7 \vdash \Delta_9, p_{10}}{\bullet h_8 : (\Delta_{11}, p_{10}), F_6 \wedge F_7 \vdash \Delta_9, p_{10}} I \\
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I \\
\frac{h_1 : \Delta_{12} \vdash F_7, p_{11}, \Delta_{10}, p_{11} \quad h_1 : \Delta_{12} \vdash F_8, p_{11}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{12} \vdash ((\Delta_{10}, p_{11}), F_7 \wedge F_8), p_{11}} \wedge_R \quad \frac{\bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8}{\bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8} I \\
\frac{}{- : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}, p_{11}}{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}, p_{11}} ax/W \quad \frac{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, p_{11}}{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, p_{11}} I \\
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}} hCut \quad \frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_8, p_{11}, p_{11}}{h_1 : \Delta_{12} \vdash \Delta_{10}, F_8, p_{11}, p_{11}} ax/W \quad \frac{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_8, p_{11}}{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_8, p_{11}} I \\
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_8, p_{11}} hCut \quad \frac{}{- : \Delta_{12} \vdash \Delta_{10}, p_{11}, F_7 \wedge F_8} \wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, p_{11} \vdash F_7, F_{12}, \Delta_{10}, p_{11} \quad h_1 : \Delta_{13}, p_{11} \vdash F_8, F_{12}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{13}, p_{11} \vdash ((\Delta_{10}, p_{11}), F_7 \wedge F_8), F_{12}} \wedge_R \quad \frac{}{\bullet h_9 : (\Delta_{13}, p_{11}), F_{12} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8} I \\
\hline
- : \Delta_{13}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \wedge F_8 \\
\hline
\rightarrow \\
\hline
- : \Delta_{13}, p_{11} \vdash \Delta_{10}, p_{11}, F_7 \wedge F_8 \quad I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \top, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \top_L \\
\hline
- : \top, \Delta_{10} \vdash \Delta_9 \\
\hline
\rightarrow \\
\hline
\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/w} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9} \text{hCut} \\
\hline
\frac{h_1 : \Delta_{11} \vdash F_7, \top, \Delta_{10} \quad h_1 : \Delta_{11} \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \wedge F_8), \top} \wedge_R \quad \frac{h_9 : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : \Delta_{11}, \top \vdash \Delta_{10}, F_7 \wedge F_8} \top_L \\
\hline
- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8 \\
\hline
\rightarrow \\
\hline
- : \Delta_{11} \vdash \Delta_{10}, F_7 \wedge F_8 \quad \text{ax/w} \\
\hline
\frac{h_1 : \top, \Delta_{12} \vdash F_7, F_{11}, \Delta_{10} \quad h_1 : \top, \Delta_{12} \vdash F_8, F_{11}, \Delta_{10}}{\bullet h_1 : \top, \Delta_{12} \vdash (\Delta_{10}, F_7 \wedge F_8), F_{11}} \wedge_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \wedge F_8}{\bullet h_9 : (\top, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \top_L \\
\hline
- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \wedge F_8 \\
\hline
\rightarrow \\
\hline
\frac{\bullet h_1 : \top, \Delta_{12} \vdash \Delta_{10}, F_{11}, F_7 \wedge F_8}{- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \wedge F_8} \text{ax/w} \quad \frac{h_9 : \top, \Delta_{12}, F_{11} \vdash \Delta_{10}, F_7 \wedge F_8}{h_9 : \top, \Delta_{12}, F_{11} \vdash \Delta_{10}, F_7 \wedge F_8} \text{hCut}
\end{array}$$

8.3 Status of \vee_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, F_7, \Delta_{11}, F_9 \rightarrow F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \vee F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \rightarrow F_{10}} \rightarrow_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10} \\
\hline
\rightarrow \\
\hline
\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}} \text{ax/w} \\
\hline
- : \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10} \quad \rightarrow_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \rightarrow F_{10} \\
\hline
\frac{h_1 : \Delta_{14} \vdash F_7, F_8, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \vee F_8} \rightarrow_R \\
\hline
- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \\
\hline
\rightarrow \\
\hline
\frac{h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7, F_8}{\bullet h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8}{\bullet h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8} \text{ax/w} \\
\hline
- : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \vee F_8 \quad \rightarrow_R \\
\hline
- : \Delta_{14} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}, F_7 \vee F_8
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_6, F_7, \Delta_{11}, F_9 \wedge F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \wedge F_{10}), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_9, \Delta_{11} \quad h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \wedge F_{10}} \wedge_R \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10} \\
\hline
\rightarrow \\
\hline
\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9} \text{ax/w} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \\
\hline
\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}} \text{ax/w} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_{10} \\
\hline
- : \Delta_{12} \vdash \Delta_{11}, F_9 \wedge F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{14} \vdash F_7, F_8, F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \vee F_8} \wedge_R}{\frac{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_7 \vee F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{13}, F_7, F_8, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{13}, F_7, F_8, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \wedge F_{11}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{11}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{11}, F_7, F_8} \text{inv-th/ax}}{\frac{- : \Delta_{14} \vdash \Delta_{12}, F_7, F_8, F_{10} \wedge F_{11}}{\rightarrow} \text{hCut}} \vee_R \\
\frac{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \wedge F_{11}, F_7 \vee F_8}{\rightarrow} \vee_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12} \vdash F_6, F_7, \Delta_{11}, F_9 \vee F_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_9, F_{10}, \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \vee_R}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}, F_9}{\bullet h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}, F_9} \text{ax/W}}{\frac{- : \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9}{\rightarrow} \text{hCut}} \vee_R \\
\frac{- : \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10}}{\rightarrow} \vee_R \\
\frac{\frac{h_1 : \Delta_{14} \vdash F_7, F_8, F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, F_{11}, \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8} \vee_R}{\frac{- : \Delta_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7, F_8}{\bullet h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8}{\bullet h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8} \text{ax/W}}{\frac{- : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_7 \vee F_8}{\rightarrow} \text{hCut}} \vee_R \\
\frac{- : \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8}{\rightarrow} \vee_R \\
\frac{\frac{h_1 : \Delta_{12} \vdash F_8, F_9, F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{10}, F_8 \vee F_9), F_{11}} \vee_R \quad \frac{h_7 : F_{11}, \Delta_{12} \vdash F_8, F_9, \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8 \vee F_9} \vee_R}{\frac{- : \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_8, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_8, F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8, F_9}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8, F_9} \text{H}}{\frac{- : \Delta_{12} \vdash \Delta_{10}, F_8, F_9}{\rightarrow} \text{hCut}} \vee_R \\
\frac{- : \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9}{\rightarrow} \vee_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{10} \vdash F_6, F_7, \perp, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_9), F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_9} \perp_R}{\frac{- : \Delta_{10} \vdash \perp, \Delta_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \vee F_7}{\bullet h_1 : \Delta_{10} \vdash \perp, \Delta_9, F_6 \vee F_7} \text{ax/W} \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_9}{\bullet h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_9} \text{H}}{\frac{- : \Delta_{10} \vdash \perp, \Delta_9}{\rightarrow} \text{hCut}} \vee_R \\
\frac{- : \Delta_{10} \vdash \perp, \Delta_9}{\rightarrow} \vee_R \\
\frac{\frac{h_1 : \Delta_{12} \vdash F_7, F_8, F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12} \vdash ((\perp, \Delta_{10}), F_7 \vee F_8), F_{11}} \vee_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash (\perp, \Delta_{10}), F_7 \vee F_8} \perp_R}{\frac{- : \Delta_{12} \vdash (\perp, \Delta_{10}), F_7 \vee F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{12} \vdash \perp, \Delta_{10}, F_{11}, F_7 \vee F_8}{\bullet h_1 : \Delta_{12} \vdash \perp, \Delta_{10}, F_{11}, F_7 \vee F_8} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{12}, F_{11} \vdash \perp, \Delta_{10}, F_7 \vee F_8} \text{H}}{\frac{- : \Delta_{12} \vdash \perp, \Delta_{10}, F_7 \vee F_8}{\rightarrow} \text{hCut}} \vee_R \\
\frac{- : \Delta_{12} \vdash \perp, \Delta_{10}, F_7 \vee F_8}{\rightarrow} \vee_R
\end{array}$$

- Case rule \top_R

- Case rule $A4$

- Case rule K

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- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash F_9, \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \rightarrow_L}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \vee_R}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{hCut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13} \vdash F_7, F_8, F_{10} \rightarrow F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \rightarrow F_{11}} \vee_R \quad \frac{h_9 : \Delta_{13} \vdash F_{10}, \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \rightarrow_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{h_9 : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \quad \frac{h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax}}{- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \text{hCut} \\
\vee_R \\
\frac{h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_7, F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_{13} \vdash \Delta_{12}, F_7 \vee F_8} \rightarrow_L}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{h_9 : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \quad \frac{h_9 : F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8}{- : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax}}{- : \Delta_{14}, F_{13}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \text{hCut} \\
\text{ax/W} \\
\frac{h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}, F_7, F_8}{- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7, F_8} \vee_R \\
- : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_7 \vee F_8
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \text{hCut} \\
\wedge_L \\
\frac{h_1 : \Delta_{13} \vdash F_7, F_8, F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \wedge F_{11}} \vee_R \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \wedge_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \wedge_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \text{hCut} \\
\vee_R \\
\frac{h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash F_7, F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \wedge F_{11}), F_{13} \vdash \Delta_{12}, F_7 \vee F_8} \wedge_L}{- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8}{- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} \wedge_L}{- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \text{hCut} \\
\vee_R \\
- : \Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : F_9, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11} \quad h_8 : F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \vee_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{h_8 : \Delta_{12}, F_9 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_9 \vdash \Delta_{11}} \text{hCut} \\
\frac{- : \Delta_{12}, F_9 \vdash \Delta_{11} \quad - : \Delta_{12}, F_{10} \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L \\
\frac{\frac{h_1 : \Delta_{10} \vdash F_7, F_8, \Delta_9}{\bullet h_1 : \Delta_{10} \vdash \Delta_9, F_7 \vee F_8} \vee_R \quad \frac{h_6 : F_7, \Delta_{10} \vdash \Delta_9 \quad h_6 : F_8, \Delta_{10} \vdash \Delta_9}{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \vee_L}{- : \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_{10} \vdash \Delta_9, F_7, F_8}{- : \Delta_{10} \vdash \Delta_9, F_7} \text{ax/W} \quad \frac{- : \Delta_{10}, F_8 \vdash \Delta_9, F_7}{- : \Delta_{10}, F_8 \vdash \Delta_9} \text{sCut} \quad \frac{- : \Delta_{10}, F_7 \vdash \Delta_9}{- : \Delta_{10}, F_7 \vdash \Delta_9} \text{sCut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13} \vdash F_7, F_8, F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \vee F_{11}} \vee_R \quad \frac{h_9 : F_{10}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \vee_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \vee F_{11}}{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{11} \vdash \Delta_{12}, F_7, F_8}{h_9 : \Delta_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax}}{- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8} \text{hCut} \\
\frac{- : \Delta_{13} \vdash \Delta_{12}, F_7, F_8}{- : \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8} \vee_R \\
\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash F_7, F_8, F_{13}, \Delta_{12}}{\bullet h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{13}} \vee_R \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8 \quad h_9 : F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet h_9 : (\Delta_{14}, F_{10} \vee F_{11}), F_{13} \vdash \Delta_{12}, F_7 \vee F_8} \vee_L}{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_7, F_8}{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_7, F_8} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_7, F_8}{\bullet h_9 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax} \vee_L \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13} \vdash \Delta_{12}, F_7, F_8}{h_9 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{inv-th/ax}}{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8} \text{hCut} \\
\frac{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7, F_8}{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8} \vee_R
\end{array}$$

• Case rule AT

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, [F_9 \vdash F_6, F_7, \Delta_{10}]}{\bullet h_1 : \Delta_{11}, [F_9 \vdash \Delta_{10}, F_6 \vee F_7]} \vee_R \quad \frac{h_8 : F_9, \Delta_{11}, [F_9, F_6 \vee F_7 \vdash \Delta_{10}]}{\bullet h_8 : (\Delta_{11}, [F_9]), F_6 \vee F_7 \vdash \Delta_{10}} AT}{- : \Delta_{11}, [F_9 \vdash \Delta_{10}] \text{Cut}} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}, F_6 \vee F_7]}{- : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}]} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_9, [F_9, F_6 \vee F_7 \vdash \Delta_{10}]}{- : \Delta_{11}, [F_9 \vdash \Delta_{10}]} \text{hCut} \\
\frac{- : \Delta_{11}, F_9, [F_9 \vdash \Delta_{10}]}{- : \Delta_{11}, [F_9 \vdash \Delta_{10}]} AT \\
\frac{h_1 : \Delta_{12} \vdash F_7, F_8, [F_{10}, \Delta_{11}]}{\bullet h_1 : \Delta_{12} \vdash (\Delta_{11}, F_7 \vee F_8), [F_{10}]} \vee_R \quad \frac{h_9 : F_{10}, \Delta_{12}, [F_{10} \vdash \Delta_{11}, F_7 \vee F_8]}{\bullet h_9 : \Delta_{12}, [F_{10} \vdash \Delta_{11}, F_7 \vee F_8]} AT}{- : \Delta_{12} \vdash \Delta_{11}, F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_7, F_8, [F_{10}]}{h_1 : \Delta_{12} \vdash \Delta_{11}, F_7, F_8} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{12}, F_{10}, [F_{10} \vdash \Delta_{11}, F_7, F_8]}{\bullet h_9 : \Delta_{12}, [F_{10} \vdash \Delta_{11}, F_7, F_8]} \text{inv-th/ax} AT}{- : \Delta_{12} \vdash \Delta_{11}, F_7, F_8} \text{hCut} \\
\frac{- : \Delta_{12} \vdash \Delta_{11}, F_7, F_8}{- : \Delta_{12} \vdash \Delta_{11}, F_7 \vee F_8} \vee_R \\
\frac{h_1 : \Delta_{13}, [F_{10} \vdash F_7, F_8, F_{12}, \Delta_{11}]}{\bullet h_1 : \Delta_{13}, [F_{10} \vdash (\Delta_{11}, F_7 \vee F_8), F_{12}]} \vee_R \quad \frac{h_9 : F_{10}, F_{12}, \Delta_{13}, [F_{10} \vdash \Delta_{11}, F_7 \vee F_8]}{\bullet h_9 : (\Delta_{13}, [F_{10}]), F_{12} \vdash \Delta_{11}, F_7 \vee F_8} AT}{- : \Delta_{13}, [F_{10} \vdash \Delta_{11}, F_7 \vee F_8]} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{13}, F_{10}, [F_{10} \vdash \Delta_{11}, F_{12}, F_7 \vee F_8]}{- : \Delta_{13}, F_{10}, [F_{10} \vdash \Delta_{11}, F_7 \vee F_8]} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{12}, [F_{10} \vdash \Delta_{11}, F_7 \vee F_8]}{- : \Delta_{13}, F_{10}, [F_{10} \vdash \Delta_{11}, F_7 \vee F_8]} \text{hCut} \\
\frac{- : \Delta_{13}, F_{10}, [F_{10} \vdash \Delta_{11}, F_7 \vee F_8]}{- : \Delta_{13}, [F_{10} \vdash \Delta_{11}, F_7 \vee F_8]} AT
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \perp, \Delta_{10} \vdash F_6, F_7, \Delta_9}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \perp_L}{\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut}} \xrightarrow{\quad} \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L \\
\\
\frac{\frac{h_1 : \Delta_{11} \vdash F_7, F_8, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \vee F_8), \perp} \vee_R \quad \frac{}{\bullet h_9 : \Delta_{11}, \perp \vdash \Delta_{10}, F_7 \vee F_8} \perp_L}{\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{Cut}} \xrightarrow{\quad} \\
\frac{\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7, F_8}{\bullet h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7, F_8} \text{ax/W} \quad \frac{}{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7, F_8} \perp_L}{\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7, F_8} \text{hCut}} \vee_R \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \perp_L \\
\\
\frac{\frac{h_1 : \perp, \Delta_{12} \vdash F_7, F_8, F_{11}, \Delta_{10}}{\bullet h_1 : \perp, \Delta_{12} \vdash (\Delta_{10}, F_7 \vee F_8), F_{11}} \vee_R \quad \frac{}{\bullet h_9 : (\perp, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \vee F_8} \perp_L}{\frac{}{- : \perp, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8} \text{Cut}} \xrightarrow{\quad} \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, p_{10} \vdash F_6, F_7, \Delta_9, p_{10}}{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_6 \vee F_7} \vee_R \quad \frac{}{\bullet h_8 : (\Delta_{11}, p_{10}), F_6 \vee F_7 \vdash \Delta_9, p_{10}} I}{\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \text{Cut}} \xrightarrow{\quad} \\
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I \\
\\
\frac{\frac{h_1 : \Delta_{12} \vdash F_7, F_8, p_{11}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{12} \vdash ((\Delta_{10}, p_{11}), F_7 \vee F_8), p_{11}} \vee_R \quad \frac{}{\bullet h_9 : \Delta_{12}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} I}{\frac{}{- : \Delta_{12} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} \text{Cut}} \xrightarrow{\quad} \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11}, p_{11}}{\bullet h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11}, p_{11}} \text{ax/W} \quad \frac{}{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, F_8, p_{11}} I}{\frac{}{- : \Delta_{12} \vdash \Delta_{10}, F_7, F_8, p_{11}} \text{hCut}} \vee_R \\
\frac{}{- : \Delta_{12} \vdash \Delta_{10}, p_{11}, F_7 \vee F_8} \perp_L \\
\\
\frac{\frac{h_1 : \Delta_{13}, p_{11} \vdash F_7, F_8, F_{12}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{13}, p_{11} \vdash ((\Delta_{10}, p_{11}), F_7 \vee F_8), F_{12}} \vee_R \quad \frac{}{\bullet h_9 : (\Delta_{13}, p_{11}), F_{12} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} I}{\frac{}{- : \Delta_{13}, p_{11} \vdash (\Delta_{10}, p_{11}), F_7 \vee F_8} \text{Cut}} \xrightarrow{\quad} \\
\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{10}, p_{11}, F_7 \vee F_8} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : \top, \Delta_{10} \vdash F_6, F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \top_L}{\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut}} \xrightarrow{\quad} \\
\frac{\frac{}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9} \text{ax/W}}{\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}} \\
\\
\frac{\frac{h_1 : \Delta_{11} \vdash F_7, F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_{11} \vdash (\Delta_{10}, F_7 \vee F_8), \top} \vee_R \quad \frac{h_9 : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : \Delta_{11}, \top \vdash \Delta_{10}, F_7 \vee F_8} \top_L}{\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{Cut}} \xrightarrow{\quad} \\
\frac{}{- : \Delta_{11} \vdash \Delta_{10}, F_7 \vee F_8} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_{12} \vdash F_7, F_8, F_{11}, \Delta_{10}}{\bullet h_1 : \top, \Delta_{12} \vdash (\Delta_{10}, F_7 \vee F_8), F_{11}} \vee_R \quad \frac{h_9 : F_{11}, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8}{\bullet h_9 : (\top, \Delta_{12}), F_{11} \vdash \Delta_{10}, F_7 \vee F_8} \top_L \\
\hline
- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \top, \Delta_{12} \vdash \Delta_{10}, F_{11}, F_7 \vee F_8}{- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8} \text{ax/W} \quad \frac{h_9 : \top, \Delta_{12}, F_{11} \vdash \Delta_{10}, F_7 \vee F_8}{- : \top, \Delta_{12} \vdash \Delta_{10}, F_7 \vee F_8} \text{ax/W} \\
\hline
\text{hCut}
\end{array}$$

8.4 Status of \perp_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \rightarrow F_6), \perp} \perp_R \quad \frac{h_4 : \perp, F_5, \Delta_8 \vdash F_6, \Delta_7}{\bullet h_4 : \Delta_8, \perp \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow_R \\
\hline
- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \rightarrow F_7}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_8, F_6 \rightarrow F_7), F_9} \perp_R \quad \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \perp, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \rightarrow_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \rightarrow F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash \perp, \Delta_8, F_9, F_6 \rightarrow F_7}{- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7}{- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \\
\hline
\text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \wedge F_6), \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \perp, \Delta_8 \vdash F_6, \Delta_7}{\bullet h_4 : \Delta_8, \perp \vdash \Delta_7, F_5 \wedge F_6} \wedge_R \\
\hline
- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6}{- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \wedge F_7}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_8, F_6 \wedge F_7), F_9} \perp_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \perp, F_6, \Delta_8 \quad h_5 : F_9, \Delta_{10} \vdash \perp, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_6 \wedge F_7} \wedge_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \wedge F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash \perp, \Delta_8, F_9, F_6 \wedge F_7}{- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \wedge F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_6 \wedge F_7}{- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \wedge F_7} \text{ax/W} \\
\hline
\text{hCut}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash \Delta_7, F_5 \vee F_6}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \vee F_6), \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash F_5, F_6, \Delta_7}{\bullet h_4 : \Delta_8, \perp \vdash \Delta_7, F_5 \vee F_6} \vee_R \\
\hline
- : \Delta_8 \vdash \Delta_7, F_5 \vee F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_8 \vdash \Delta_7, F_5 \vee F_6}{- : \Delta_8 \vdash \Delta_7, F_5 \vee F_6} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \vee F_7}{\bullet h_1 : \Delta_{10} \vdash (\perp, \Delta_8, F_6 \vee F_7), F_9} \perp_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \perp, F_6, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_6 \vee F_7} \vee_R \\
\hline
- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \vee F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{10} \vdash \perp, \Delta_8, F_9, F_6 \vee F_7}{- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \vee F_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, F_9 \vdash \perp, \Delta_8, F_6 \vee F_7}{- : \Delta_{10} \vdash \perp, \Delta_8, F_6 \vee F_7} \text{ax/W} \\
\hline
\text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash \perp, \Delta_5}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_5), \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_6 \vdash \Delta_5}{\bullet h_4 : \Delta_6, \perp \vdash \perp, \Delta_5} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_6 \vdash \perp, \Delta_5}{- : \Delta_6 \vdash \perp, \Delta_5} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \Delta_8 \vdash (\perp, \Delta_6), F_7} \perp_R \quad \frac{h_5 : F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : \Delta_8, F_7 \vdash \perp, \Delta_6} \perp_R \\
\hline
- : \Delta_8 \vdash \perp, \Delta_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_8 \vdash \perp, \Delta_6, F_7}{- : \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, F_7 \vdash \perp, \Delta_6}{- : \Delta_8 \vdash \perp, \Delta_6} \text{ax/W}}{- : \Delta_8 \vdash \perp, \Delta_6} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash \top, \Delta_5}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_5), \perp} \perp_R \quad \frac{}{\bullet h_4 : \Delta_6, \perp \vdash \top, \Delta_5} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_5 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_5} \top_R \\
\hline
\frac{h_1 : \Delta_8 \vdash F_7, \top, \Delta_6}{\bullet h_1 : \Delta_8 \vdash (\perp, \top, \Delta_6), F_7} \perp_R \quad \frac{}{\bullet h_5 : \Delta_8, F_7 \vdash \perp, \top, \Delta_6} \top_R \\
\hline
- : \Delta_8 \vdash \perp, \top, \Delta_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_8 \vdash \perp, \top, \Delta_6} \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_7, \Delta_8 \vdash \Delta_6, \Box F_5}{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Delta_6, \Box F_5), \perp} \perp_R \quad \frac{h_4 : \Box \Gamma_7 \vdash F_5}{\bullet h_4 : (\Box \Gamma_7, \Delta_8), \perp \vdash \Delta_6, \Box F_5} A4 \\
\hline
- : \Box \Gamma_7, \Delta_8 \vdash \Delta_6, \Box F_5 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_6, \Box F_5} \text{ax/W} \\
\hline
\frac{h_1 : \Box \Gamma_9, \Delta_{10} \vdash \Box F_8, \Delta_7, \Box F_6}{\bullet h_1 : \Box \Gamma_9, \Delta_{10} \vdash (\perp, \Delta_7, \Box F_6), \Box F_8} \perp_R \quad \frac{h_5 : \Box \Gamma_9, \Box F_8 \vdash F_6}{\bullet h_5 : (\Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \perp, \Delta_7, \Box F_6} A4 \\
\hline
- : \Box \Gamma_9, \Delta_{10} \vdash \perp, \Delta_7, \Box F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Box F_8, \Delta_7, \Box F_6}{- : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, \Box F_6} \text{ax/W} \quad \frac{\bullet h_5 : \Box F_8, \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, \Box F_6}{- : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, \Box F_6} \text{ax/W}}{- : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, \Box F_6} \text{hCut} \\
\hline
\frac{h_1 : \Box \Gamma_8, \Delta_{10} \vdash F_9, \Delta_7, \Box F_6}{\bullet h_1 : \Box \Gamma_8, \Delta_{10} \vdash (\perp, \Delta_7, \Box F_6), F_9} \perp_R \quad \frac{h_5 : \Box \Gamma_8 \vdash F_6}{\bullet h_5 : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \perp, \Delta_7, \Box F_6} A4 \\
\hline
- : \Box \Gamma_8, \Delta_{10} \vdash \perp, \Delta_7, \Box F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Box \Gamma_8 \vdash F_6} \text{ax/W} \\
\hline
\frac{}{- : \Delta_{10}, \Box \Gamma_8 \vdash \perp, \Delta_7, \Box F_6} A4
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_7, \Delta_8 \vdash \Delta_6, \Box F_5}{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Delta_6, \Box F_5), \perp} \perp_R \quad \frac{h_4 : \text{unbox}(\Box \Gamma_7) \vdash F_5}{\bullet h_4 : (\Box \Gamma_7, \Delta_8), \perp \vdash \Delta_6, \Box F_5} K \\
\hline
- : \Box \Gamma_7, \Delta_8 \vdash \Delta_6, \Box F_5 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_6, \Box F_5} \text{ax/W} \\
\hline
\frac{h_1 : \Box \Gamma_9, \Delta_{10} \vdash \Box F_8, \Delta_7, \Box F_6}{\bullet h_1 : \Box \Gamma_9, \Delta_{10} \vdash (\perp, \Delta_7, \Box F_6), \Box F_8} \perp_R \quad \frac{h_5 : \text{unbox}(\Box \Gamma_9), \text{unbox}(\Box F_8) \vdash F_6}{\bullet h_5 : (\Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \perp, \Delta_7, \Box F_6} K \\
\hline
- : \Box \Gamma_9, \Delta_{10} \vdash \perp, \Delta_7, \Box F_6 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Box F_8, \Delta_7, \Box F_6}{- : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, \Box F_6} \text{ax/W} \quad \frac{\bullet h_5 : \Box F_8, \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, \Box F_6}{- : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, \Box F_6} \text{ax/W}}{- : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_7, \Box F_6} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_8, \Delta_{10} \vdash F_9, \Delta_7, \Box F_6}{\bullet h_1 : \Box\Gamma_8, \Delta_{10} \vdash (\perp, \Delta_7, \Box F_6), F_9} \perp_R \quad \frac{h_5 : \text{unbox}(\Box\Gamma_8) \vdash F_6}{\bullet h_5 : (\Box\Gamma_8, \Delta_{10}), F_9 \vdash \perp, \Delta_7, \Box F_6} K \\
\hline
- : \Box\Gamma_8, \Delta_{10} \vdash \perp, \Delta_7, \Box F_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_8) \vdash F_6} \text{ax/W} \\
\hline
- : \Delta_{10}, \Box\Gamma_8 \vdash \perp, \Delta_7, \Box F_6 \quad K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \perp, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \perp \vdash \Delta_7} \rightarrow_L \\
\hline
- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_9 \vdash F_6 \rightarrow F_7, \Delta_8}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8), F_6 \rightarrow F_7} \perp_R \quad \frac{h_5 : \Delta_9 \vdash \perp, F_6, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8} \rightarrow_L \\
\hline
- : \Delta_9 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7}{\bullet h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8} \text{ax/W} \\
\hline
- : \Delta_9 \vdash \perp, \Delta_8 \quad \text{hCut} \\
\hline
\frac{h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash (\perp, \Delta_8), F_9} \perp_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \perp, F_6, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \perp, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \perp, \Delta_8} \rightarrow_L \\
\hline
- : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_8, F_9}{\bullet h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_8, F_9} \text{ax/W} \quad \frac{h_5 : \Delta_{10}, F_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9, F_6 \rightarrow F_7 \vdash \perp, \Delta_8} \text{ax/W} \\
\hline
- : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp, \Delta_8 \quad \text{hCut}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \perp \vdash \Delta_7} \wedge_L \\
\hline
- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_9 \vdash F_6 \wedge F_7, \Delta_8}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8), F_6 \wedge F_7} \perp_R \quad \frac{h_5 : F_6, F_7, \Delta_9 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8} \wedge_L \\
\hline
- : \Delta_9 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \wedge F_7}{\bullet h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \wedge F_7} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash \perp, \Delta_8} \text{ax/W} \\
\hline
- : \Delta_9 \vdash \perp, \Delta_8 \quad \text{hCut} \\
\hline
\frac{h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash (\perp, \Delta_8), F_9} \perp_R \quad \frac{h_5 : F_6, F_7, F_9, \Delta_{10} \vdash \perp, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \wedge F_7), F_9 \vdash \perp, \Delta_8} \wedge_L \\
\hline
- : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8, F_9}{\bullet h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8, F_9} \text{ax/W} \quad \frac{h_5 : \Delta_{10}, F_9, F_6 \wedge F_7 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9, F_6 \wedge F_7 \vdash \perp, \Delta_8} \text{ax/W} \\
\hline
- : \Delta_{10}, F_6 \wedge F_7 \vdash \perp, \Delta_8 \quad \text{hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \perp, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \perp \vdash \Delta_7} \vee_L \\
\hline
- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9 \vdash F_6 \vee F_7, \Delta_8}{\bullet h_1 : \Delta_9 \vdash (\perp, \Delta_8), F_6 \vee F_7} \perp_R \quad \frac{h_5 : F_6, \Delta_9 \vdash \perp, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \perp, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8} \vee_L}{- : \Delta_9 \vdash \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_9 \vdash \perp, \Delta_8, F_6 \vee F_7}{- : \Delta_9 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \perp, \Delta_8}{\text{hCut}} \text{ax/W}}{- : \Delta_9 \vdash \perp, \Delta_8} \\
\\
\frac{\frac{h_1 : \Delta_{10}, F_6 \vee F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \Delta_{10}, F_6 \vee F_7 \vdash (\perp, \Delta_8), F_9} \perp_R \quad \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \perp, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \perp, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \vee F_7), F_9 \vdash \perp, \Delta_8} \vee_L}{- : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8, F_9}{- : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_{10}, F_9, F_6 \vee F_7 \vdash \perp, \Delta_8}{\text{hCut}} \text{ax/W}}{- : \Delta_{10}, F_6 \vee F_7 \vdash \perp, \Delta_8}
\end{array}$$

• Case rule AT

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_7, []F_5 \vdash \Delta_6}{\bullet h_1 : \Delta_7, []F_5 \vdash \Delta_6, \perp} \perp_R \quad \frac{h_4 : \perp, F_5, \Delta_7, []F_5 \vdash \Delta_6}{\bullet h_4 : (\Delta_7, []F_5), \perp \vdash \Delta_6} AT}{- : \Delta_7, []F_5 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7, []F_5 \vdash \Delta_6}{- : \Delta_7, []F_5 \vdash \Delta_6} \text{ax/W} \\
\\
\frac{\frac{h_1 : \Delta_8 \vdash []F_6, \Delta_7}{\bullet h_1 : \Delta_8 \vdash (\perp, \Delta_7), []F_6} \perp_R \quad \frac{h_5 : F_6, \Delta_8, []F_6 \vdash \perp, \Delta_7}{\bullet h_5 : \Delta_8, []F_6 \vdash \perp, \Delta_7} AT}{- : \Delta_8 \vdash \perp, \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_8 \vdash \perp, \Delta_7, []F_6}{- : \Delta_8 \vdash \perp, \Delta_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, []F_6 \vdash \perp, \Delta_7}{\text{hCut}} \text{ax/W}}{- : \Delta_8 \vdash \perp, \Delta_7} \\
\\
\frac{\frac{h_1 : \Delta_9, []F_6 \vdash F_8, \Delta_7}{\bullet h_1 : \Delta_9, []F_6 \vdash (\perp, \Delta_7), F_8} \perp_R \quad \frac{h_5 : F_6, F_8, \Delta_9, []F_6 \vdash \perp, \Delta_7}{\bullet h_5 : (\Delta_9, []F_6), F_8 \vdash \perp, \Delta_7} AT}{- : \Delta_9, []F_6 \vdash \perp, \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_9, []F_6 \vdash \perp, \Delta_7, F_8}{- : \Delta_9, []F_6 \vdash \perp, \Delta_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_9, F_8, []F_6 \vdash \perp, \Delta_7}{\text{hCut}} \text{ax/W}}{- : \Delta_9, []F_6 \vdash \perp, \Delta_7}
\end{array}$$

• Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash \Delta_5}{\bullet h_1 : \Delta_6 \vdash \Delta_5, \perp} \perp_R \quad \frac{}{\bullet h_4 : \Delta_6, \perp \vdash \Delta_5} \perp_L}{- : \Delta_6 \vdash \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_6 \vdash \Delta_5}{- : \Delta_6 \vdash \Delta_5} \text{ax/W} \\
\\
\frac{\frac{h_1 : \Delta_7 \vdash \perp, \Delta_6}{\bullet h_1 : \Delta_7 \vdash (\perp, \Delta_6), \perp} \perp_R \quad \frac{}{\bullet h_5 : \Delta_7, \perp \vdash \perp, \Delta_6} \perp_L}{- : \Delta_7 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7 \vdash \perp, \Delta_6}{- : \Delta_7 \vdash \perp, \Delta_6} \text{ax/W} \\
\\
\frac{\frac{h_1 : \perp, \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \perp, \Delta_8 \vdash (\perp, \Delta_6), F_7} \perp_R \quad \frac{}{\bullet h_5 : (\perp, \Delta_8), F_7 \vdash \perp, \Delta_6} \perp_L}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{- : \perp, \Delta_8 \vdash \perp, \Delta_6}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \perp_L
\end{array}$$

• Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_7, p_6 \vdash \Delta_5, p_6}{\bullet h_1 : \Delta_7, p_6 \vdash (\Delta_5, p_6), \perp} \perp_R \quad \frac{}{\bullet h_4 : (\Delta_7, p_6), \perp \vdash \Delta_5, p_6} I}{- : \Delta_7, p_6 \vdash \Delta_5, p_6} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7, p_6 \vdash \Delta_5, p_6}{- : \Delta_7, p_6 \vdash \Delta_5, p_6} I
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash p_7, \Delta_6, p_7}{\bullet h_1 : \Delta_8 \vdash (\perp, \Delta_6, p_7), p_7} \perp_R \quad \frac{}{\bullet h_5 : \Delta_8, p_7 \vdash \perp, \Delta_6, p_7} I \\
\hline
\frac{}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \Delta_8 \vdash \perp, \Delta_6, p_7, p_7}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_8, p_7 \vdash \perp, \Delta_6, p_7}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} I \\
\hline
\frac{}{- : \Delta_8 \vdash \perp, \Delta_6, p_7} \text{hCut} \\
\hline
\frac{h_1 : \Delta_9, p_7 \vdash F_8, \Delta_6, p_7}{\bullet h_1 : \Delta_9, p_7 \vdash (\perp, \Delta_6, p_7), F_8} \perp_R \quad \frac{}{\bullet h_5 : (\Delta_9, p_7), F_8 \vdash \perp, \Delta_6, p_7} I \\
\hline
\frac{}{- : \Delta_9, p_7 \vdash \perp, \Delta_6, p_7} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_9, p_7 \vdash \perp, \Delta_6, p_7} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet h_1 : \top, \Delta_6 \vdash \Delta_5, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_6 \vdash \Delta_5}{\bullet h_4 : (\top, \Delta_6), \perp \vdash \Delta_5} \top_L \\
\hline
\frac{}{- : \top, \Delta_6 \vdash \Delta_5} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \top, \Delta_6 \vdash \Delta_5} \text{ax/W} \\
\hline
\frac{h_1 : \Delta_7 \vdash \top, \Delta_6}{\bullet h_1 : \Delta_7 \vdash (\perp, \Delta_6), \top} \perp_R \quad \frac{h_5 : \Delta_7 \vdash \perp, \Delta_6}{\bullet h_5 : \Delta_7, \top \vdash \perp, \Delta_6} \top_L \\
\hline
\frac{}{- : \Delta_7 \vdash \perp, \Delta_6} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_7 \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
\frac{h_1 : \top, \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \top, \Delta_8 \vdash (\perp, \Delta_6), F_7} \perp_R \quad \frac{h_5 : F_7, \Delta_8 \vdash \perp, \Delta_6}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \perp, \Delta_6} \top_L \\
\hline
\frac{}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \top, \Delta_8 \vdash \perp, \Delta_6, F_7}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, F_7 \vdash \perp, \Delta_6}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{hCut}
\end{array}$$

8.5 Status of \top_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \rightarrow F_6), \top} \top_R \quad \frac{h_4 : \top, F_5, \Delta_8 \vdash F_6, \Delta_7}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow_R \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7, F_6}{- : \Delta_8, F_5 \vdash \Delta_7, F_6} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7, F_6}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{ax/W} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \text{hCut} \\
\hline
\frac{}{\rightarrow_R} \\
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \rightarrow F_6} \rightarrow_R \\
\hline
\frac{h_1 : \Delta_{10} \vdash (\top, \Delta_8, F_6 \rightarrow F_7), F_9}{\bullet h_1 : \Delta_{10} \vdash \top, \Delta_8, F_6 \rightarrow F_7} \top_R \quad \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \top, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \top, \Delta_8, F_6 \rightarrow F_7} \rightarrow_R \\
\hline
\frac{}{- : \Delta_{10} \vdash \top, \Delta_8, F_6 \rightarrow F_7} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{10} \vdash \top, \Delta_8, F_6 \rightarrow F_7} \top_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \wedge F_6), \top} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \top, \Delta_8 \vdash F_6, \Delta_7}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \wedge F_6} \wedge_R \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5}{- : \Delta_8 \vdash \Delta_7, F_5} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5}{- : \Delta_8 \vdash \Delta_7, F_5} \text{ax/W} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5} \text{hCut} \\
\hline
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_6}{- : \Delta_8 \vdash \Delta_7, F_6} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_6}{- : \Delta_8 \vdash \Delta_7, F_6} \text{ax/W} \\
\hline
\frac{}{- : \Delta_8 \vdash \Delta_7, F_6} \text{hCut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6} \wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_8, F_6 \wedge F_7), F_9} \top_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \top, F_6, \Delta_8 \quad h_5 : F_9, \Delta_{10} \vdash \top, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \top, \Delta_8, F_6 \wedge F_7} \wedge_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_8, F_6 \wedge F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{10} \vdash \top, \Delta_8, F_6 \wedge F_7} \top_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\Delta_7, F_5 \vee F_6), \top} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash F_5, F_6, \Delta_7}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \vee F_6} \vee_R \\
\hline
- : \Delta_8 \vdash \Delta_7, F_5 \vee F_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5, F_6} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5, F_6}{\bullet h_4 : \Delta_8, \top \vdash \Delta_7, F_5 \vee F_6} \text{ax/W} \\
\hline
- : \Delta_8 \vdash \top, \Delta_7, F_5, F_6 \quad \text{hCut} \\
\rightarrow \\
- : \Delta_8 \vdash \Delta_7, F_5 \vee F_6 \quad \vee_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_{10} \vdash (\top, \Delta_8, F_6 \vee F_7), F_9} \top_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \top, F_6, F_7, \Delta_8}{\bullet h_5 : \Delta_{10}, F_9 \vdash \top, \Delta_8, F_6 \vee F_7} \vee_R \\
\hline
- : \Delta_{10} \vdash \top, \Delta_8, F_6 \vee F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{10} \vdash \top, \Delta_8, F_6 \vee F_7} \top_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_5), \top} \top_R \quad \frac{h_4 : \top, \Delta_6 \vdash \Delta_5}{\bullet h_4 : \Delta_6, \top \vdash \perp, \Delta_5} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : \Delta_6 \vdash \perp, \top, \Delta_5} \top_R \quad \frac{h_4 : \top, \Delta_6 \vdash \perp, \Delta_5}{\bullet h_4 : \Delta_6, \top \vdash \perp, \Delta_5} \text{ax/W} \\
\hline
- : \Delta_6 \vdash \perp, \Delta_5 \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\top, \perp, \Delta_6), F_7} \top_R \quad \frac{h_5 : F_7, \Delta_8 \vdash \top, \Delta_6}{\bullet h_5 : \Delta_8, F_7 \vdash \top, \perp, \Delta_6} \perp_R \\
\hline
- : \Delta_8 \vdash \top, \perp, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8 \vdash \perp, \top, \Delta_6} \top_R
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_5), \top} \top_R \quad \frac{}{\bullet h_4 : \Delta_6, \top \vdash \top, \Delta_5} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6 \vdash \top, \Delta_5} \top_R
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_8 \vdash (\top, \Delta_6), F_7} \top_R \quad \frac{}{\bullet h_5 : \Delta_8, F_7 \vdash \top, \Delta_6} \top_R \\
\hline
- : \Delta_8 \vdash \top, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8 \vdash \top, \Delta_6} \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Delta_6, []F_5), \top} \top_R \quad \frac{h_4 : \Box \Gamma_7 \vdash F_5}{\bullet h_4 : (\Box \Gamma_7, \Delta_8), \top \vdash \Delta_6, []F_5} A4 \\
\hline
- : \Box \Gamma_7, \Delta_8 \vdash \Delta_6, []F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Box \Gamma_7 \vdash F_5} \text{ax/W} \\
\hline
- : \Delta_8, \Box \Gamma_7 \vdash \Delta_6, []F_5 \quad A4
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Box \Gamma_9, \Delta_{10} \vdash (\top, \Delta_7, \llbracket F_6 \rrbracket), \Box F_8}{- : \Box \Gamma_9, \Delta_{10} \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} \top_R \quad \frac{h_5 : \Box \Gamma_9, \Box F_8 \vdash F_6}{\bullet h_5 : (\Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} A4}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, \Box \Gamma_9 \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Box \Gamma_8, \Delta_{10} \vdash (\top, \Delta_7, \llbracket F_6 \rrbracket), F_9}{- : \Box \Gamma_8, \Delta_{10} \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} \top_R \quad \frac{h_5 : \Box \Gamma_8 \vdash F_6}{\bullet h_5 : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} A4}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, \Box \Gamma_8 \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} \top_R
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Box \Gamma_7, \Delta_8 \vdash (\Delta_6, \llbracket F_5 \rrbracket), \top}{- : \Box \Gamma_7, \Delta_8 \vdash \Delta_6, \llbracket F_5 \rrbracket} \top_R \quad \frac{h_4 : \text{unbox}(\Box \Gamma_7) \vdash F_5}{\bullet h_4 : (\Box \Gamma_7, \Delta_8), \top \vdash \Delta_6, \llbracket F_5 \rrbracket} K}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_8, \Box \Gamma_7 \vdash \Delta_6, \llbracket F_5 \rrbracket} \text{ax/w} \quad K
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Box \Gamma_9, \Delta_{10} \vdash (\top, \Delta_7, \llbracket F_6 \rrbracket), \Box F_8}{- : \Box \Gamma_9, \Delta_{10} \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} \top_R \quad \frac{h_5 : \text{unbox}(\Box \Gamma_9), \text{unbox}(\Box F_8) \vdash F_6}{\bullet h_5 : (\Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} K}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, \Box \Gamma_9 \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Box \Gamma_8, \Delta_{10} \vdash (\top, \Delta_7, \llbracket F_6 \rrbracket), F_9}{- : \Box \Gamma_8, \Delta_{10} \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} \top_R \quad \frac{h_5 : \text{unbox}(\Box \Gamma_8) \vdash F_6}{\bullet h_5 : (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} K}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, \Box \Gamma_8 \vdash \top, \Delta_7, \llbracket F_6 \rrbracket} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash F_5, \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \top \vdash \Delta_7} \rightarrow_L}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{ax/w} \quad \text{hCut} \quad \frac{\frac{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7}{- : \Delta_8, F_6 \vdash \top, \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{- : \Delta_8, F_6 \vdash \Delta_7} \text{ax/w}}{\rightarrow_L} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \rightarrow F_7}{- : \Delta_9 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : \Delta_9 \vdash \top, F_6, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \rightarrow_L}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_9 \vdash \top, \Delta_8} \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash (\top, \Delta_8), F_9}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : F_9, \Delta_{10} \vdash \top, F_6, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \top, \Delta_8} \rightarrow_L}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \top}{\vdash : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, F_5, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \top \vdash \Delta_7} \wedge_L}{\vdash : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5, F_6 \vdash \top, \Delta_7}{\vdash : \Delta_8, F_5, F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}{\vdash : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{ax/W}}{\vdash : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{hCut} \wedge_L \\
\\
\frac{\frac{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \wedge F_7}{\vdash : \Delta_9 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : F_6, F_7, \Delta_9 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash \top, \Delta_8} \wedge_L}{\vdash : \Delta_9 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_9 \vdash \top, \Delta_8 \top_R \\
\\
\frac{\frac{\bullet h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash (\top, \Delta_8), F_9}{\vdash : \Delta_{10}, F_6 \wedge F_7 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : F_6, F_7, F_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \wedge F_7), F_9 \vdash \top, \Delta_8} \wedge_L}{\vdash : \Delta_{10}, F_6 \wedge F_7 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{10}, F_6 \wedge F_7 \vdash \top, \Delta_8 \top_R
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \top}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \top \vdash \Delta_7} \vee_L}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7}{\vdash : \Delta_8, F_5 \vdash \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{ax/W}}{\vdash : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{hCut} \vee_L \\
\\
\frac{\frac{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \vee F_7}{\vdash : \Delta_9 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : F_6, \Delta_9 \vdash \top, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \top, \Delta_8} \vee_L}{\vdash : \Delta_9 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_9 \vdash \top, \Delta_8 \top_R \\
\\
\frac{\frac{\bullet h_1 : \Delta_{10}, F_6 \vee F_7 \vdash (\top, \Delta_8), F_9}{\vdash : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_8} \top_R \quad \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \top, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet h_5 : (\Delta_{10}, F_6 \vee F_7), F_9 \vdash \top, \Delta_8} \vee_L}{\vdash : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_8 \top_R
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, \boxed{F_5} \vdash \Delta_6, \top}{\vdash : \Delta_7, \boxed{F_5} \vdash \Delta_6} \top_R \quad \frac{h_4 : \top, F_5, \Delta_7, \boxed{F_5} \vdash \Delta_6}{\bullet h_4 : (\Delta_7, \boxed{F_5}), \top \vdash \Delta_6} AT}{\vdash : \Delta_7, \boxed{F_5} \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_7, F_5, \boxed{F_5} \vdash \top, \Delta_6}{\vdash : \Delta_7, F_5, \boxed{F_5} \vdash \Delta_6} \top_R \quad \frac{h_4 : \top, \Delta_7, F_5, \boxed{F_5} \vdash \Delta_6}{\vdash : \Delta_7, \boxed{F_5} \vdash \Delta_6} \text{ax/W}}{\vdash : \Delta_7, \boxed{F_5} \vdash \Delta_6} \text{hCut} AT \\
\\
\frac{\frac{\bullet h_1 : \Delta_8 \vdash (\top, \Delta_7), \boxed{F_6}}{\vdash : \Delta_8 \vdash \top, \Delta_7} \top_R \quad \frac{h_5 : F_6, \Delta_8, \boxed{F_6} \vdash \top, \Delta_7}{\bullet h_5 : \Delta_8, \boxed{F_6} \vdash \top, \Delta_7} AT}{\vdash : \Delta_8 \vdash \top, \Delta_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_8 \vdash \top, \Delta_7 \top_R \\
\\
\frac{\frac{\bullet h_1 : \Delta_9, \boxed{F_6} \vdash (\top, \Delta_7), F_8}{\vdash : \Delta_9, \boxed{F_6} \vdash \top, \Delta_7} \top_R \quad \frac{h_5 : F_6, F_8, \Delta_9, \boxed{F_6} \vdash \top, \Delta_7}{\bullet h_5 : (\Delta_9, \boxed{F_6}), F_8 \vdash \top, \Delta_7} AT}{\vdash : \Delta_9, \boxed{F_6} \vdash \top, \Delta_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_9, \boxed{F_6} \vdash \top, \Delta_7 \top_R
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_6 \vdash \Delta_5, \top}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \top_R \quad \frac{\bullet h_4 : (\perp, \Delta_6), \top \vdash \Delta_5}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \perp_L}{\vdash : \perp, \Delta_6 \vdash \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_6 \vdash \Delta_5 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7 \vdash (\top, \Delta_6), \perp}{\vdash : \Delta_7 \vdash \top, \Delta_6} \top_R \quad \frac{\bullet h_5 : \Delta_7, \perp \vdash \top, \Delta_6}{\vdash : \Delta_7 \vdash \top, \Delta_6} \perp_L}{\vdash : \Delta_7 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_8 \vdash (\top, \Delta_6), F_7}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \top_R \quad \frac{\bullet h_5 : (\perp, \Delta_8), F_7 \vdash \top, \Delta_6}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \perp_L}{\vdash : \perp, \Delta_8 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_8 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_6 \vdash (\Delta_5, p_6), \top}{\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6} \top_R \quad \frac{\bullet h_4 : (\Delta_7, p_6), \top \vdash \Delta_5, p_6}{\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6} I}{\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7, p_6 \vdash \Delta_5, p_6 \quad I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8 \vdash (\top, \Delta_6, p_7), p_7}{\vdash : \Delta_8 \vdash \top, \Delta_6, p_7} \top_R \quad \frac{\bullet h_5 : \Delta_8, p_7 \vdash \top, \Delta_6, p_7}{\vdash : \Delta_8 \vdash \top, \Delta_6, p_7} I}{\vdash : \Delta_8 \vdash \top, \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_8 \vdash \top, \Delta_6, p_7 \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_7 \vdash (\top, \Delta_6, p_7), F_8}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_6, p_7} \top_R \quad \frac{\bullet h_5 : (\Delta_9, p_7), F_8 \vdash \top, \Delta_6, p_7}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_6, p_7} I}{\vdash : \Delta_9, p_7 \vdash \top, \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_9, p_7 \vdash \top, \Delta_6, p_7 \quad \top_R
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_6 \vdash \Delta_5, \top}{\vdash : \Delta_6 \vdash \Delta_5} \top_R \quad \frac{\frac{h_4 : \Delta_6 \vdash \Delta_5}{\bullet h_4 : \Delta_6, \top \vdash \Delta_5} \top_L}{\vdash : \Delta_6 \vdash \Delta_5} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_6 \vdash \Delta_5 \quad \text{ax/w}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7 \vdash (\top, \Delta_6), \top}{\vdash : \Delta_7 \vdash \top, \Delta_6} \top_R \quad \frac{\frac{h_5 : \Delta_7 \vdash \top, \Delta_6}{\bullet h_5 : \Delta_7, \top \vdash \top, \Delta_6} \top_L}{\vdash : \Delta_7 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_7 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \top, \Delta_8 \vdash (\top, \Delta_6), F_7}{\vdash : \top, \Delta_8 \vdash \top, \Delta_6} \top_R \quad \frac{\frac{h_5 : F_7, \Delta_8 \vdash \top, \Delta_6}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \top, \Delta_6} \top_L}{\vdash : \top, \Delta_8 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \top, \Delta_8 \vdash \top, \Delta_6 \quad \top_R
\end{array}$$

8.6 Status of A4: OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \rightarrow F_9), [\Box F_6]} A4 \quad \frac{h_7 : \Box \Gamma_{11}, F_8, \Delta_{12}, [\Box F_6 \vdash F_9, \Delta_{10}]}{\bullet h_7 : (\Box \Gamma_{11}, \Delta_{12}), [\Box F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9]} \rightarrow_R \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \rightarrow F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Delta_{12}, F_8, \Box \Gamma_{11} \vdash \Delta_{10}, F_9, [\Box F_6]} ax/W \quad \frac{h_7 : \Delta_{12}, F_8, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_{10}, F_9]}{\bullet h_7 : \Delta_{12}, F_8, \Box \Gamma_{11} \vdash \Delta_{10}, F_9} A4}{- : \Delta_{12}, F_8, \Box \Gamma_{11} \vdash \Delta_{10}, F_9} hCut \\
\hline
- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8 \rightarrow F_9 \quad \rightarrow_R \\
\hline
\frac{h_1 : \Box \Gamma_{12} \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \rightarrow F_{10}), [\Box F_7], F_{13})} A4 \quad \frac{h_8 : \Box \Gamma_{12}, F_9, F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{11}, [\Box F_7]}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), [\Box F_7]} \rightarrow_R \\
\hline
- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), [\Box F_7] \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_7}{- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, [\Box F_7, F_9 \rightarrow F_{10}]} ax/W \quad A4
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \wedge F_9), [\Box F_6]} A4 \quad \frac{h_7 : \Box \Gamma_{11}, \Delta_{12}, [\Box F_6 \vdash F_8, \Delta_{10}] \quad h_7 : \Box \Gamma_{11}, \Delta_{12}, [\Box F_6 \vdash F_9, \Delta_{10}]}{\bullet h_7 : (\Box \Gamma_{11}, \Delta_{12}), [\Box F_6 \vdash \Delta_{10}, F_8 \wedge F_9]} \wedge_R \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \wedge F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, [\Box F_6]} ax/W \quad \frac{h_7 : \Delta_{12}, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_{10}, F_8]}{\bullet h_7 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8} A4}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8} hCut \\
\hline
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, [\Box F_6]} ax/W \quad \frac{h_7 : \Delta_{12}, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_{10}, F_8]}{\bullet h_7 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8} A4}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8} hCut \\
\hline
- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8 \wedge F_9 \quad \wedge_R \\
\hline
\frac{h_1 : \Box \Gamma_{12} \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \wedge F_{10}), [\Box F_7], F_{13})} A4 \quad \frac{h_8 : \Box \Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, \Delta_{11}, [\Box F_7] \quad h_8 : \Box \Gamma_{12}, F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{11}, [\Box F_7]}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, F_9 \wedge F_{10}), [\Box F_7]} \wedge_R \\
\hline
- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \wedge F_{10}), [\Box F_7] \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_7}{- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, [\Box F_7, F_9 \wedge F_{10}]} ax/W \quad A4
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \vee F_9), [\Box F_6]} A4 \quad \frac{h_7 : \Box \Gamma_{11}, \Delta_{12}, [\Box F_6 \vdash F_8, F_9, \Delta_{10}]}{\bullet h_7 : (\Box \Gamma_{11}, \Delta_{12}), [\Box F_6 \vdash \Delta_{10}, F_8 \vee F_9]} \vee_R \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, F_9, [\Box F_6]} ax/W \quad \frac{h_7 : \Delta_{12}, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_{10}, F_8, F_9]}{\bullet h_7 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, F_9} A4}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, F_9} hCut \\
\hline
- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8 \vee F_9 \quad \vee_R \\
\hline
\frac{h_1 : \Box \Gamma_{12} \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \vee F_{10}), [\Box F_7], F_{13})} A4 \quad \frac{h_8 : \Box \Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, F_{10}, \Delta_{11}, [\Box F_7]}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, F_9 \vee F_{10}), [\Box F_7]} \vee_R \\
\hline
- : \Box \Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \vee F_{10}), [\Box F_7] \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_7}{- : \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, [\Box F_7, F_9 \vee F_{10}]} ax/W \quad A4
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_9 \vdash F_6}{\bullet h_1 : \Box \Gamma_9, \Delta_{10} \vdash (\perp, \Delta_8), [\Box F_6]} A4 \quad \frac{h_7 : \Box \Gamma_9, \Delta_{10}, [\Box F_6 \vdash \Delta_8]}{\bullet h_7 : (\Box \Gamma_9, \Delta_{10}), [\Box F_6 \vdash \perp, \Delta_8]} \perp_R \\
\hline
- : \Box \Gamma_9, \Delta_{10} \vdash \perp, \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Box \Gamma_9 \vdash F_6}{\bullet h_1 : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_8, [\Box F_6]} ax/W \quad \frac{h_7 : \Delta_{10}, \Box \Gamma_9, [\Box F_6 \vdash \perp, \Delta_8]}{\bullet h_7 : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_8} A4}{- : \Delta_{10}, \Box \Gamma_9 \vdash \perp, \Delta_8} hCut
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_{10} \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\perp, \Delta_9), [F_7]), F_{11}} \quad A4 \quad \frac{h_8 : \Box\Gamma_{10}, F_{11}, \Delta_{12} \vdash \Delta_9, [F_7]}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{12}), F_{11} \vdash (\perp, \Delta_9), [F_7]} \quad \begin{array}{l} \perp_R \\ \text{Cut} \end{array} \\
\hline
- : \Box\Gamma_{10}, \Delta_{12} \vdash (\perp, \Delta_9), [F_7] \\
\rightarrow \\
\frac{\text{ax/w}}{- : \Box\Gamma_{10} \vdash F_7} \quad A4 \\
\hline
- : \Delta_{12}, \Box\Gamma_{10} \vdash \perp, \Delta_9, [F_7]
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_9 \vdash F_6}{\bullet h_1 : \Box\Gamma_9, \Delta_{10} \vdash (\top, \Delta_8), [F_6]} \quad A4 \quad \frac{}{\bullet h_7 : (\Box\Gamma_9, \Delta_{10}), [F_6] \vdash \top, \Delta_8} \quad \begin{array}{l} \top_R \\ \text{Cut} \end{array} \\
\hline
- : \Box\Gamma_9, \Delta_{10} \vdash \top, \Delta_8 \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_8} \quad \top_R \\
\hline
\frac{h_1 : \Box\Gamma_{10} \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\top, \Delta_9), [F_7]), F_{11}} \quad A4 \quad \frac{}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{12}), F_{11} \vdash (\top, \Delta_9), [F_7]} \quad \begin{array}{l} \top_R \\ \text{Cut} \end{array} \\
\hline
- : \Box\Gamma_{10}, \Delta_{12} \vdash (\top, \Delta_9), [F_7] \\
\rightarrow \\
\frac{}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \top, \Delta_9, [F_7]} \quad \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, [F_8]), [F_6]} \quad A4 \quad \frac{h_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, [F_6] \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), [F_6] \vdash \Delta_9, [F_8]} \quad \begin{array}{l} A4 \\ \text{Cut} \end{array} \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, [F_8] \\
\rightarrow \\
\frac{\text{ax/w}}{h_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash F_6} \quad A4 \quad \frac{}{h_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, [F_6] \vdash F_8} \quad \begin{array}{l} \text{ax/w} \\ \text{hCut} \end{array} \\
\hline
- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash F_8 \\
\hline
- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, [F_8] \quad A4 \\
\hline
\frac{h_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, [F_8]), [F_6]} \quad A4 \quad \frac{h_7 : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), [F_6] \vdash \Delta_9, [F_8]} \quad \begin{array}{l} A4 \\ \text{Cut} \end{array} \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, [F_8] \\
\rightarrow \\
\frac{\text{ax/w}}{- : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash F_8} \quad A4 \\
\hline
- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, [F_8] \\
\hline
\frac{h_1 : \Box\Gamma_{11}, \Box\Gamma_{14} \vdash F_7}{\bullet h_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{10}, [F_9]), [F_7]), \Box F_{12}} \quad A4 \quad \frac{h_8 : \Box\Gamma_{11}, \Box\Gamma_{13}, \Box F_{12} \vdash F_9}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box F_{12} \vdash (\Delta_{10}, [F_9]), [F_7]} \quad \begin{array}{l} A4 \\ \text{Cut} \end{array} \\
\hline
- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{10}, [F_9]), [F_7] \\
\rightarrow \\
\frac{\text{ax/w}}{- : \Box\Gamma_{11}, \Box\Gamma_{14} \vdash F_7} \quad A4 \\
\hline
- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, [F_7], [F_9] \\
\hline
\frac{h_1 : \Box\Gamma_{10}, \Box\Gamma_{13} \vdash F_8}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, [F_8]), \Box F_{11}} \quad A4 \quad \frac{h_7 : \Box\Gamma_{10}, \Box\Gamma_{12}, \Box F_{11} \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box F_{11} \vdash \Delta_9, [F_8]} \quad \begin{array}{l} A4 \\ \text{Cut} \end{array} \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, [F_8] \\
\rightarrow \\
\frac{\text{ax/w}}{- : \Box\Gamma_{10}, \Box\Gamma_{13} \vdash F_8} \quad A4 \\
\hline
- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, [F_8] \\
\hline
\frac{h_1 : \Box\Gamma_{11}, \Box\Gamma_{13} \vdash F_7}{\bullet h_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Delta_{10}, [F_9]), [F_7]), F_{14}} \quad A4 \quad \frac{h_8 : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash F_9}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), F_{14} \vdash (\Delta_{10}, [F_9]), [F_7]} \quad \begin{array}{l} A4 \\ \text{Cut} \end{array} \\
\hline
- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Delta_{10}, [F_9]), [F_7] \\
\rightarrow \\
\frac{\text{ax/w}}{- : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash F_9} \quad A4 \\
\hline
- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, [F_7], [F_9]
\end{array}$$

$$\frac{\frac{\frac{h_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash F_8}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_9, \Box F_8), F_{13}} \quad A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash \Delta_9, \Box F_8} \quad A4}{\frac{- : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash F_8 \quad \text{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, \Box F_8} \quad A4} \quad \text{Cut}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, [F_8]), [F_6]} A4 \quad \frac{h_7 : F_6, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), [F_6] \vdash \Delta_9, [F_8]} K \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, [F_8] \quad \text{Cut} \\
\hline
\frac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash F_6, F_8} ax/W \quad \frac{}{- : F_6, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash F_8} ax/W \\
\hline
\frac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash F_8} ATG \quad \frac{}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, [F_8]} A4 \\
\hline
\frac{h_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, [F_8]), [F_6]} A4 \quad \frac{h_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), [F_6] \vdash \Delta_9, [F_8]} K \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, [F_8] \quad \text{Cut} \\
\hline
\frac{}{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash F_8} ax/W \quad \frac{}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, [F_8]} K \\
\hline
\frac{h_1 : \Box\Gamma_{11}, \Box\Gamma_{14} \vdash F_7}{\bullet h_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{10}, [F_9]), [F_7]), \Box F_{12}} A4 \quad \frac{h_8 : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{13}), unbox(\Box F_{12}) \vdash F_9}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box F_{12} \vdash (\Delta_{10}, [F_9]), [F_7]} K \\
\hline
- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{10}, [F_9]), [F_7] \quad \text{Cut} \\
\hline
\frac{}{- : \Box\Gamma_{11}, \Box\Gamma_{14} \vdash F_7} ax/W \quad \frac{}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, [F_7], [F_9]} A4 \\
\hline
\frac{h_1 : \Box\Gamma_{10}, \Box\Gamma_{13} \vdash F_8}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, [F_8]), \Box F_{11}} A4 \quad \frac{h_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{12}), unbox(\Box F_{11}) \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box F_{11} \vdash \Delta_9, [F_8]} K \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, [F_8] \quad \text{Cut} \\
\hline
\frac{}{- : \Box\Gamma_{10}, \Box\Gamma_{13} \vdash F_8} ax/W \quad \frac{}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, [F_8]} A4 \\
\hline
\frac{h_1 : \Box\Gamma_{11}, \Box\Gamma_{13} \vdash F_7}{\bullet h_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Delta_{10}, [F_9]), [F_7]), F_{14}} A4 \quad \frac{h_8 : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash F_9}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), F_{14} \vdash (\Delta_{10}, [F_9]), [F_7]} K \\
\hline
- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Delta_{10}, [F_9]), [F_7] \quad \text{Cut} \\
\hline
\frac{}{- : \Box\Gamma_{11}, \Box\Gamma_{13} \vdash F_7} ax/W \quad \frac{}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, [F_7], [F_9]} A4 \\
\hline
\frac{h_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash F_8}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_9, [F_8]), F_{13}} A4 \quad \frac{h_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), F_{13} \vdash \Delta_9, [F_8]} K \\
\hline
- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash \Delta_9, [F_8] \quad \text{Cut} \\
\hline
\frac{}{- : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash F_8} ax/W \quad \frac{}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, [F_8]} A4
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10}, \Box F_6} A4 \quad \frac{h_7 : \Box \Gamma_{11}, \Delta_{12}, \Box F_6 \vdash F_8, \Delta_{10} \quad h_7 : \Box \Gamma_{11}, F_9, \Delta_{12}, \Box F_6 \vdash \Delta_{10}}{\bullet h_7 : (\Box \Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9), \Box F_6 \vdash \Delta_{10}} \rightarrow_L \\
\hline
- : \Box \Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, \Box F_6} ax/W \quad \frac{h_7 : \Delta_{12}, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}, F_8}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8} A4 \quad \frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Delta_{12}, F_9, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_6} ax/W \quad \frac{h_7 : \Delta_{12}, F_9, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}}{h_7 : \Delta_{12}, F_9, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}} A4 \quad \frac{h_7 : \Delta_{12}, F_9, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}}{h_7 : \Delta_{12}, F_9, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}} ax/W}{- : \Delta_{12}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10}} hCut \rightarrow_L \\
\rightarrow \\
\frac{h_1 : \Box \Gamma_{12} \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, \Box F_7), F_9 \rightarrow F_{10}} A4 \quad \frac{h_8 : \Box \Gamma_{12}, \Delta_{13} \vdash F_9, \Delta_{11}, \Box F_7 \quad h_8 : \Box \Gamma_{12}, F_{10}, \Delta_{13} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{13}), F_9 \rightarrow F_{10} \vdash \Delta_{11}, \Box F_7} \rightarrow_L \\
\hline
- : \Box \Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_7}{- : \Delta_{13}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_7} ax/W \quad A4 \\
\rightarrow \\
\frac{h_1 : \Box \Gamma_{12} \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \vdash (\Delta_{11}, \Box F_7), F_{13}} A4 \quad \frac{h_8 : \Box \Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, \Delta_{11}, \Box F_7 \quad h_8 : \Box \Gamma_{12}, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10}), F_{13} \vdash \Delta_{11}, \Box F_7} \rightarrow_L \\
\hline
- : \Box \Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_7}{- : \Delta_{14}, \Box \Gamma_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, \Box F_7} ax/W \quad A4
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}, \Box F_6} A4 \quad \frac{h_7 : \Box \Gamma_{11}, F_8, F_9, \Delta_{12}, \Box F_6 \vdash \Delta_{10}}{\bullet h_7 : (\Box \Gamma_{11}, \Delta_{12}, F_8 \wedge F_9), \Box F_6 \vdash \Delta_{10}} \wedge_L \\
\hline
- : \Box \Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Delta_{12}, F_8, F_9, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_6} ax/W \quad \frac{h_7 : \Delta_{12}, F_8, F_9, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}}{h_7 : \Delta_{12}, F_8, F_9, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}} ax/W}{- : \Delta_{12}, F_8, F_9, \Box \Gamma_{11} \vdash \Delta_{10}} hCut \quad \wedge_L \\
\rightarrow \\
\frac{h_1 : \Box \Gamma_{12} \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, \Box F_7), F_9 \wedge F_{10}} A4 \quad \frac{h_8 : \Box \Gamma_{12}, F_9, F_{10}, \Delta_{13} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{13}), F_9 \wedge F_{10} \vdash \Delta_{11}, \Box F_7} \wedge_L \\
\hline
- : \Box \Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_7}{- : \Delta_{13}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_7} ax/W \quad A4 \\
\rightarrow \\
\frac{h_1 : \Box \Gamma_{12} \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash (\Delta_{11}, \Box F_7), F_{13}} A4 \quad \frac{h_8 : \Box \Gamma_{12}, F_9, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10}), F_{13} \vdash \Delta_{11}, \Box F_7} \wedge_L \\
\hline
- : \Box \Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_7}{- : \Delta_{14}, \Box \Gamma_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, \Box F_7} ax/W \quad A4
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}, \Box F_6} A4 \quad \frac{h_7 : \Box \Gamma_{11}, F_8, \Delta_{12}, \Box F_6 \vdash \Delta_{10} \quad h_7 : \Box \Gamma_{11}, F_9, \Delta_{12}, \Box F_6 \vdash \Delta_{10}}{\bullet h_7 : (\Box \Gamma_{11}, \Delta_{12}, F_8 \vee F_9), \Box F_6 \vdash \Delta_{10}} \vee_L \\
\hline
- : \Box \Gamma_{11}, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Delta_{12}, F_8, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_6} ax/W \quad \frac{h_7 : \Delta_{12}, F_8, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}}{h_7 : \Delta_{12}, F_8, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}} ax/W}{- : \Delta_{12}, F_8, \Box \Gamma_{11} \vdash \Delta_{10}} hCut \quad \vee_L \\
\rightarrow \\
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_6}{\bullet h_1 : \Delta_{12}, F_9, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_6} ax/W \quad \frac{h_7 : \Delta_{12}, F_9, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}}{h_7 : \Delta_{12}, F_9, \Box \Gamma_{11}, \Box F_6 \vdash \Delta_{10}} ax/W}{- : \Delta_{12}, F_9, \Box \Gamma_{11} \vdash \Delta_{10}} hCut \quad \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{12} \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, \Box F_7), F_9 \vee F_{10}} A4 \quad \frac{h_8 : \Box \Gamma_{12}, F_9, \Delta_{13} \vdash \Delta_{11}, \Box F_7 \quad h_8 : \Box \Gamma_{12}, F_{10}, \Delta_{13} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{13}), F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7} \vee_L \\
\hline
- : \Box \Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_7}{- : \Delta_{13}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box F_7} \text{ax/W} \quad A4
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{12} \vdash F_7}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{14}, F_9 \vee F_{10} \vdash (\Delta_{11}, \Box F_7), F_{13}} A4 \quad \frac{h_8 : \Box \Gamma_{12}, F_9, F_{13}, \Delta_{14} \vdash \Delta_{11}, \Box F_7 \quad h_8 : \Box \Gamma_{12}, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{14}, F_9 \vee F_{10}), F_{13} \vdash \Delta_{11}, \Box F_7} \vee_L \\
\hline
- : \Box \Gamma_{12}, \Delta_{14}, F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Box \Gamma_{12} \vdash F_7}{- : \Delta_{14}, \Box \Gamma_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7} \text{ax/W} \quad A4
\end{array}$$

• Case rule AT

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{10}, \Box F_8 \vdash F_6}{\bullet h_1 : (\Box \Gamma_{10}, \Box F_8), \Delta_{11} \vdash \Delta_9, \Box F_6} A4 \quad \frac{h_7 : \Box \Gamma_{10}, F_8, \Delta_{11}, \Box F_6 \vdash \Delta_9}{\bullet h_7 : ((\Box \Gamma_{10}, \Box F_8), \Delta_{11}), \Box F_6 \vdash \Delta_9} AT \\
\hline
- : (\Box \Gamma_{10}, \Box F_8), \Delta_{11} \vdash \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_{11}, F_8, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9, \Box F_6}{\bullet h_1 : \Delta_{11}, F_8, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9}{\bullet h_7 : ((\Delta_{11}, F_8), \Box \Gamma_{10}), \Box F_6 \vdash \Delta_9} \text{ax/W} \\
\hline
- : \Delta_{11}, F_8, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9 \quad \text{hCut} \\
\hline
- : \Delta_{11}, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9 \quad AT
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{10} \vdash F_6}{\bullet h_1 : \Box \Gamma_{10}, \Delta_{11}, \Box F_8 \vdash \Delta_9, \Box F_6} A4 \quad \frac{h_7 : \Box \Gamma_{10}, F_8, \Delta_{11}, \Box F_6 \vdash \Delta_9}{\bullet h_7 : (\Box \Gamma_{10}, \Delta_{11}, \Box F_8), \Box F_6 \vdash \Delta_9} AT \\
\hline
- : \Box \Gamma_{10}, \Delta_{11}, \Box F_8 \vdash \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_{11}, F_8, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9, \Box F_6}{\bullet h_1 : \Delta_{11}, F_8, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9}{\bullet h_7 : ((\Delta_{11}, F_8), \Box \Gamma_{10}), \Box F_6 \vdash \Delta_9} \text{ax/W} \\
\hline
- : \Delta_{11}, F_8, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9 \quad \text{hCut} \\
\hline
- : \Delta_{11}, \Box \Gamma_{10}, \Box F_6 \vdash \Delta_9 \quad AT
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_9 \vdash F_7}{\bullet h_1 : \Box \Gamma_9, \Delta_{10} \vdash \Delta_8, \Box F_7} A4 \quad \frac{h_6 : \Box \Gamma_9, F_7, \Delta_{10}, \Box F_7 \vdash \Delta_8}{\bullet h_6 : (\Box \Gamma_9, \Delta_{10}), \Box F_7 \vdash \Delta_8} AT \\
\hline
- : \Box \Gamma_9, \Delta_{10} \vdash \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8, F_7}{\bullet h_1 : \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8, F_7} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_7, \Box \Gamma_9, \Box F_7 \vdash \Delta_8}{\bullet h_6 : ((\Delta_{10}, F_7), \Box \Gamma_9), \Box F_7 \vdash \Delta_8} \text{ax/W} \\
\hline
- : \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8, F_7 \quad \text{hCut} \\
\hline
- : \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8 \quad \text{sCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11}, \Box F_9 \vdash F_7}{\bullet h_1 : (\Box \Gamma_{11}, \Box F_9), \Delta_{13} \vdash (\Delta_{10}, \Box F_7), F_{12}} A4 \quad \frac{h_8 : \Box \Gamma_{11}, F_9, F_{12}, \Delta_{13}, \Box F_9 \vdash \Delta_{10}, \Box F_7}{\bullet h_8 : ((\Box \Gamma_{11}, \Box F_9), \Delta_{13}), F_{12} \vdash \Delta_{10}, \Box F_7} AT \\
\hline
- : (\Box \Gamma_{11}, \Box F_9), \Delta_{13} \vdash \Delta_{10}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Box \Gamma_{11}, \Box F_9 \vdash F_7}{- : \Delta_{13}, \Box \Gamma_{11}, \Box F_9 \vdash \Delta_{10}, \Box F_7} \text{ax/W} \quad A4
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11} \vdash F_7}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \Box F_7), \Box F_9} A4 \quad \frac{h_8 : \Box \Gamma_{11}, F_9, \Delta_{12}, \Box F_9 \vdash \Delta_{10}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{12}), \Box F_9 \vdash \Delta_{10}, \Box F_7} AT \\
\hline
- : \Box \Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Box \Gamma_{11} \vdash F_7}{- : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box F_7} \text{ax/W} \quad A4
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Box \Gamma_{11} \vdash F_7}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{13}, \Box F_9 \vdash (\Delta_{10}, \Box F_7), F_{12}} A4 \quad \frac{h_8 : \Box \Gamma_{11}, F_9, F_{12}, \Delta_{13}, \Box F_9 \vdash \Delta_{10}, \Box F_7}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{13}, \Box F_9), F_{12} \vdash \Delta_{10}, \Box F_7} AT \\
\hline
- : \Box \Gamma_{11}, \Delta_{13}, \Box F_9 \vdash \Delta_{10}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{- : \Box \Gamma_{11} \vdash F_7}{- : \Delta_{13}, \Box \Gamma_{11}, \Box F_9 \vdash \Delta_{10}, \Box F_7} \text{ax/W} \quad A4
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Box \Gamma_9 \vdash F_6}{\bullet h_1 : \Box \Gamma_9, \perp, \Delta_{10} \vdash \Delta_8, [\Box F_6]} A4 \quad \frac{}{\bullet h_7 : (\Box \Gamma_9, \perp, \Delta_{10}), [\Box F_6] \vdash \Delta_8} \perp_L}{\frac{}{\vdash : \Box \Gamma_9, \perp, \Delta_{10} \vdash \Delta_8} \text{Cut}} \\
\frac{}{\vdash : \perp, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8} \perp_L \\
\frac{}{\vdash : \perp, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8} \perp_L \\
\frac{\frac{h_1 : \Box \Gamma_{10} \vdash F_7}{\bullet h_1 : \Box \Gamma_{10}, \Delta_{11} \vdash (\Delta_9, [\Box F_7]), \perp} A4 \quad \frac{}{\bullet h_8 : (\Box \Gamma_{10}, \Delta_{11}), \perp \vdash \Delta_9, [\Box F_7]} \perp_L}{\frac{}{\vdash : \Box \Gamma_{10}, \Delta_{11} \vdash \Delta_9, [\Box F_7]} \text{Cut}} \\
\frac{}{\vdash : \Box \Gamma_{10} \vdash F_7} \text{ax/w} \\
\frac{}{\vdash : \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_9, [\Box F_7]} A4 \\
\frac{\frac{h_1 : \Box \Gamma_{10} \vdash F_7}{\bullet h_1 : \Box \Gamma_{10}, \perp, \Delta_{12} \vdash (\Delta_9, [\Box F_7]), F_{11}} A4 \quad \frac{}{\bullet h_8 : (\Box \Gamma_{10}, \perp, \Delta_{12}), F_{11} \vdash \Delta_9, [\Box F_7]} \perp_L}{\frac{}{\vdash : \Box \Gamma_{10}, \perp, \Delta_{12} \vdash \Delta_9, [\Box F_7]} \text{Cut}} \\
\frac{}{\vdash : \perp, \Delta_{12}, \Box \Gamma_{10} \vdash \Delta_9, [\Box F_7]} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Box \Gamma_{10} \vdash F_6}{\bullet h_1 : \Box \Gamma_{10}, \Delta_{11}, p_9 \vdash (\Delta_8, p_9), [\Box F_6]} A4 \quad \frac{}{\bullet h_7 : (\Box \Gamma_{10}, \Delta_{11}, p_9), [\Box F_6] \vdash \Delta_8, p_9} I}{\frac{}{\vdash : \Box \Gamma_{10}, \Delta_{11}, p_9 \vdash \Delta_8, p_9} \text{Cut}} \\
\frac{}{\vdash : \Delta_{11}, \Box \Gamma_{10}, p_9 \vdash \Delta_8, p_9} I \\
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_7}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{12} \vdash ((\Delta_9, p_{10}), [\Box F_7]), p_{10}} A4 \quad \frac{}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{12}), p_{10} \vdash (\Delta_9, p_{10}), [\Box F_7]} I}{\frac{}{\vdash : \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_9, p_{10}), [\Box F_7]} \text{Cut}} \\
\frac{}{\vdash : \Box \Gamma_{11} \vdash F_7} \text{ax/w} \\
\frac{}{\vdash : \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_9, p_{10}, [\Box F_7]} A4 \\
\frac{\frac{h_1 : \Box \Gamma_{11} \vdash F_7}{\bullet h_1 : \Box \Gamma_{11}, \Delta_{13}, p_{10} \vdash ((\Delta_9, p_{10}), [\Box F_7]), F_{12}} A4 \quad \frac{}{\bullet h_8 : (\Box \Gamma_{11}, \Delta_{13}, p_{10}), F_{12} \vdash (\Delta_9, p_{10}), [\Box F_7]} I}{\frac{}{\vdash : \Box \Gamma_{11}, \Delta_{13}, p_{10} \vdash (\Delta_9, p_{10}), [\Box F_7]} \text{Cut}} \\
\frac{}{\vdash : \Delta_{13}, \Box \Gamma_{11}, p_{10} \vdash \Delta_9, p_{10}, [\Box F_7]} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Box \Gamma_9 \vdash F_6}{\bullet h_1 : \Box \Gamma_9, \top, \Delta_{10} \vdash \Delta_8, [\Box F_6]} A4 \quad \frac{h_7 : \Box \Gamma_9, \Delta_{10}, [\Box F_6] \vdash \Delta_8}{\bullet h_7 : (\Box \Gamma_9, \top, \Delta_{10}), [\Box F_6] \vdash \Delta_8} \top_L}{\frac{}{\vdash : \Box \Gamma_9, \top, \Delta_{10} \vdash \Delta_8} \text{Cut}} \\
\frac{}{\vdash : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8, [\Box F_6]} \text{ax/w} \\
\frac{}{\vdash : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8} \text{hCut} \\
\frac{\frac{h_1 : \Box \Gamma_{10} \vdash F_7}{\bullet h_1 : \Box \Gamma_{10}, \Delta_{11} \vdash (\Delta_9, [\Box F_7]), \top} A4 \quad \frac{h_8 : \Box \Gamma_{10}, \Delta_{11} \vdash \Delta_9, [\Box F_7]}{\bullet h_8 : (\Box \Gamma_{10}, \Delta_{11}), \top \vdash \Delta_9, [\Box F_7]} \top_L}{\frac{}{\vdash : \Box \Gamma_{10}, \Delta_{11} \vdash \Delta_9, [\Box F_7]} \text{Cut}} \\
\frac{}{\vdash : \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_9, [\Box F_7]} \text{ax/w} \\
\frac{\frac{h_1 : \Box \Gamma_{10} \vdash F_7}{\bullet h_1 : \Box \Gamma_{10}, \top, \Delta_{12} \vdash (\Delta_9, [\Box F_7]), F_{11}} A4 \quad \frac{h_8 : \Box \Gamma_{10}, F_{11}, \Delta_{12} \vdash \Delta_9, [\Box F_7]}{\bullet h_8 : (\Box \Gamma_{10}, \top, \Delta_{12}), F_{11} \vdash \Delta_9, [\Box F_7]} \top_L}{\frac{}{\vdash : \Box \Gamma_{10}, \top, \Delta_{12} \vdash \Delta_9, [\Box F_7]} \text{Cut}} \\
\frac{}{\vdash : \top, \Delta_{12}, \Box \Gamma_{10} \vdash \Delta_9, [\Box F_7]} \text{ax/w} \\
\frac{}{\vdash : \top, \Delta_{12}, \Box \Gamma_{10} \vdash \Delta_9, [\Box F_7]} A4
\end{array}$$

8.7 Status of K : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \rightarrow F_9), \Box F_6} K \quad \frac{h_7 : \Box\Gamma_{11}, F_8, \Delta_{12}, \Box F_6 \vdash F_9, \Delta_{10}}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \rightarrow F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, F_8, \Box\Gamma_{11} \vdash \Delta_{10}, F_9, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_8, \Box\Gamma_{11}, \Box F_6 \vdash \Delta_{10}, F_9}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9} \text{ax/W}}{- : \Delta_{12}, F_8, \Box\Gamma_{11} \vdash \Delta_{10}, F_9} \text{hCut} \\
\hline
- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8 \rightarrow F_9 \quad \rightarrow_R \\
\rightarrow \\
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \rightarrow F_{10}), \Box F_7), F_{13}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_9, F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), \Box F_7} \rightarrow_R \\
\hline
- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \rightarrow F_{10}), \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7, F_9 \rightarrow F_{10}} \text{ax/W} \\
\hline
- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7, F_9 \rightarrow F_{10} \quad K
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \wedge F_9), \Box F_6} K \quad \frac{h_7 : \Box\Gamma_{11}, \Delta_{12}, \Box F_6 \vdash F_8, \Delta_{10} \quad h_7 : \Box\Gamma_{11}, \Delta_{12}, \Box F_6 \vdash F_9, \Delta_{10}}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R \\
\hline
- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \wedge F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, \Box\Gamma_{11}, \Box F_6 \vdash \Delta_{10}, F_8}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \wedge F_9} \text{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8} \text{hCut} \\
\hline
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_9, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, \Box\Gamma_{11}, \Box F_6 \vdash \Delta_{10}, F_9}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \wedge F_9} \text{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_9} \text{hCut} \\
\hline
- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8 \wedge F_9 \quad \wedge_R \\
\rightarrow \\
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \wedge F_{10}), \Box F_7), F_{13}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, \Delta_{11}, \Box F_7 \quad h_8 : \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, F_9 \wedge F_{10}), \Box F_7} \wedge_R \\
\hline
- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \wedge F_{10}), \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7, F_9 \wedge F_{10}} \text{ax/W} \\
\hline
- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7, F_9 \wedge F_{10} \quad K
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \vee F_9), \Box F_6} K \quad \frac{h_7 : \Box\Gamma_{11}, \Delta_{12}, \Box F_6 \vdash F_8, F_9, \Delta_{10}}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \vee F_9} \vee_R \\
\hline
- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, F_9, \Box F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, \Box\Gamma_{11}, \Box F_6 \vdash \Delta_{10}, F_8, F_9}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}), \Box F_6 \vdash \Delta_{10}, F_8 \vee F_9} \text{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, F_9} \text{hCut} \\
\hline
- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8 \vee F_9 \quad \vee_R \\
\rightarrow \\
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \vee F_{10}), \Box F_7), F_{13}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, F_{10}, \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}), F_{13} \vdash (\Delta_{11}, F_9 \vee F_{10}), \Box F_7} \vee_R \\
\hline
- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \vee F_{10}), \Box F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7, F_9 \vee F_{10}} \text{ax/W} \\
\hline
- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7, F_9 \vee F_{10} \quad K
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_9) \vdash F_6}{\bullet h_1 : \Box\Gamma_9, \Delta_{10} \vdash (\perp, \Delta_8), \llbracket F_6 \rrbracket} K \quad \frac{h_7 : \Box\Gamma_9, \Delta_{10}, \llbracket F_6 \rrbracket \vdash \Delta_8}{\bullet h_7 : (\Box\Gamma_9, \Delta_{10}), \llbracket F_6 \rrbracket \vdash \perp, \Delta_8} \perp_R \\
\hline
\frac{}{- : \Box\Gamma_9, \Delta_{10} \vdash \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10}, \Box\Gamma_9 \vdash \perp, \Delta_8, \llbracket F_6 \rrbracket}{- : \Delta_{10}, \Box\Gamma_9 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, \Box\Gamma_9, \llbracket F_6 \rrbracket \vdash \perp, \Delta_8}{h_7 : \Delta_{10}, \Box\Gamma_9 \vdash \perp, \Delta_8} \text{hCut} \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\perp, \Delta_9), \llbracket F_7 \rrbracket), F_{11}} K \quad \frac{h_8 : \Box\Gamma_{10}, F_{11}, \Delta_{12} \vdash \Delta_9, \llbracket F_7 \rrbracket}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{12}), F_{11} \vdash (\perp, \Delta_9), \llbracket F_7 \rrbracket} \perp_R \\
\hline
\frac{}{- : \Box\Gamma_{10}, \Delta_{12} \vdash (\perp, \Delta_9), \llbracket F_7 \rrbracket} \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{10}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \perp, \Delta_9, \llbracket F_7 \rrbracket} K
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_9) \vdash F_6}{\bullet h_1 : \Box\Gamma_9, \Delta_{10} \vdash (\top, \Delta_8), \llbracket F_6 \rrbracket} K \quad \frac{}{\bullet h_7 : (\Box\Gamma_9, \Delta_{10}), \llbracket F_6 \rrbracket \vdash \top, \Delta_8} \top_R \\
\hline
\frac{}{- : \Box\Gamma_9, \Delta_{10} \vdash \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_8} \top_R \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\top, \Delta_9), \llbracket F_7 \rrbracket), F_{11}} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{12}), F_{11} \vdash (\top, \Delta_9), \llbracket F_7 \rrbracket} \top_R \\
\hline
\frac{}{- : \Box\Gamma_{10}, \Delta_{12} \vdash (\top, \Delta_9), \llbracket F_7 \rrbracket} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \top, \Delta_9, \llbracket F_7 \rrbracket} \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, \llbracket F_8 \rrbracket), \llbracket F_6 \rrbracket} K \quad \frac{h_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, \llbracket F_6 \rrbracket \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), \llbracket F_6 \rrbracket \vdash \Delta_9, \llbracket F_8 \rrbracket} A4 \\
\hline
\frac{}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, \llbracket F_8 \rrbracket} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{12}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash F_8, \llbracket F_6 \rrbracket} \text{ax/W} \quad \frac{h_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \llbracket F_6 \rrbracket \vdash F_8}{h_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash F_8} \text{hCut}}{\frac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash F_8} A4} \\
\frac{}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, \llbracket F_8 \rrbracket} A4 \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, \llbracket F_8 \rrbracket), \llbracket F_6 \rrbracket} K \quad \frac{h_7 : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), \llbracket F_6 \rrbracket \vdash \Delta_9, \llbracket F_8 \rrbracket} A4 \\
\hline
\frac{}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, \llbracket F_8 \rrbracket} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash F_8} \text{ax/W} \\
\frac{}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, \llbracket F_8 \rrbracket} A4 \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}, \Box\Gamma_{14}) \vdash F_7}{\bullet h_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{10}, \llbracket F_9 \rrbracket), \llbracket F_7 \rrbracket), \Box F_{12}} K \quad \frac{h_8 : \Box\Gamma_{11}, \Box\Gamma_{13}, \Box F_{12} \vdash F_9}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box F_{12} \vdash (\Delta_{10}, \llbracket F_9 \rrbracket), \llbracket F_7 \rrbracket} A4 \\
\hline
\frac{}{- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{10}, \llbracket F_9 \rrbracket), \llbracket F_7 \rrbracket} \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{11}), \text{unbox}(\Box\Gamma_{14}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, \llbracket F_7 \rrbracket, \llbracket F_9 \rrbracket} K \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{13}) \vdash F_8}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, \llbracket F_8 \rrbracket), \Box F_{11}} K \quad \frac{h_7 : \Box\Gamma_{10}, \Box\Gamma_{12}, \Box F_{11} \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box F_{11} \vdash \Delta_9, \llbracket F_8 \rrbracket} A4 \\
\hline
\frac{}{- : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, \llbracket F_8 \rrbracket} \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{10}), \text{unbox}(\Box\Gamma_{13}) \vdash F_8} \text{ax/W} \\
\frac{}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, \llbracket F_8 \rrbracket} K
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash F_7}{\bullet h_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Delta_{10}, [\![F_9]\!]), [\![F_7]\!]), F_{14}} K \quad \frac{h_8 : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash F_9}{\bullet h_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), F_{14} \vdash (\Delta_{10}, [\![F_9]\!]), [\![F_7]\!]} A4 \\
\hline
\frac{}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Delta_{10}, [\![F_9]\!]), [\![F_7]\!]} \rightarrow \text{Cut} \\
\frac{}{- : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash F_9} \text{ax/W} \\
\frac{}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, [\![F_7]\!], [\![F_9]\!]} A4
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash F_8}{\bullet h_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_9, [\![F_8]\!]), F_{13}} K \quad \frac{h_7 : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash F_8}{\bullet h_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), F_{13} \vdash \Delta_9, [\![F_8]\!]} A4 \\
\hline
\frac{}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash \Delta_9, [\![F_8]\!]} \rightarrow \text{Cut} \\
\frac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash F_8} \text{ax/W} \\
\frac{}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, [\![F_8]\!]} A4
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10}, [\Box F_6]} K \quad \frac{h_7 : \Box\Gamma_{11}, \Delta_{12}, [\Box F_6 \vdash F_8, \Delta_{10}] \quad h_7 : \Box\Gamma_{11}, F_9, \Delta_{12}, [\Box F_6 \vdash \Delta_{10}] \rightarrow_L}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9), [\Box F_6 \vdash \Delta_{10}] \text{ Cut}} \\
\hline
- : \Box\Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, [\Box F_6]} K \quad \frac{h_7 : \Delta_{12}, \Box\Gamma_{11}, [\Box F_6 \vdash \Delta_{10}, F_8]}{h_7 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8} \text{ ax/W} \quad \frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, [\Box F_6]} K \quad \frac{h_7 : \Delta_{12}, F_9, \Box\Gamma_{11}, [\Box F_6 \vdash \Delta_{10}]}{h_7 : \Delta_{12}, F_9, \Box\Gamma_{11} \vdash \Delta_{10}} \text{ ax/W}}{\frac{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8}{- : \Delta_{12}, \Box\Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10}} \text{ hCut}} \\
\rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, [\Box F_7]), F_9 \rightarrow F_{10}} K \quad \frac{h_8 : \Box\Gamma_{12}, \Delta_{13} \vdash F_9, \Delta_{11}, [\Box F_7] \quad h_8 : \Box\Gamma_{12}, F_{10}, \Delta_{13} \vdash \Delta_{11}, [\Box F_7] \rightarrow_L}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{13}), F_9 \rightarrow F_{10} \vdash \Delta_{11}, [\Box F_7]} \text{ Cut} \\
\hline
- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, [\Box F_7] \\
\rightarrow \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, [\Box F_7]} \text{ ax/W}}{K} \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \vdash (\Delta_{11}, [\Box F_7]), F_{13}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, \Delta_{11}, [\Box F_7] \quad h_8 : \Box\Gamma_{12}, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, [\Box F_7] \rightarrow_L}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10}), F_{13} \vdash \Delta_{11}, [\Box F_7]} \text{ Cut} \\
\hline
- : \Box\Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, [\Box F_7] \\
\rightarrow \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{14}, \Box\Gamma_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, [\Box F_7]} \text{ ax/W}}{K}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}, [\Box F_6]} K \quad \frac{h_7 : \Box\Gamma_{11}, F_8, F_9, \Delta_{12}, [\Box F_6 \vdash \Delta_{10}] \quad \bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9), [\Box F_6 \vdash \Delta_{10}] \wedge_L}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9), [\Box F_6 \vdash \Delta_{10}] \text{ Cut}} \\
\hline
- : \Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, [\Box F_6]} K \quad \frac{h_7 : \Delta_{12}, F_8, F_9, \Box\Gamma_{11}, [\Box F_6 \vdash \Delta_{10}]}{h_7 : \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}} \text{ ax/W}}{\frac{- : \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}}{- : \Delta_{12}, \Box\Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{10}} \text{ hCut}} \\
\rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, [\Box F_7]), F_9 \wedge F_{10}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_9, F_{10}, \Delta_{13} \vdash \Delta_{11}, [\Box F_7] \quad \bullet h_8 : (\Box\Gamma_{12}, \Delta_{13}), F_9 \wedge F_{10} \vdash \Delta_{11}, [\Box F_7] \wedge_L}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{13}), F_9 \wedge F_{10} \vdash \Delta_{11}, [\Box F_7]} \text{ Cut} \\
\hline
- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, [\Box F_7] \\
\rightarrow \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, [\Box F_7]} \text{ ax/W}}{K} \\
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash (\Delta_{11}, [\Box F_7]), F_{13}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_9, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, [\Box F_7] \quad \bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10}), F_{13} \vdash \Delta_{11}, [\Box F_7] \wedge_L}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10}), F_{13} \vdash \Delta_{11}, [\Box F_7]} \text{ Cut} \\
\hline
- : \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, [\Box F_7] \\
\rightarrow \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{14}, \Box\Gamma_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, [\Box F_7]} \text{ ax/W}}{K}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}, [\Box F_6]} K \quad \frac{h_7 : \Box\Gamma_{11}, F_8, \Delta_{12}, [\Box F_6 \vdash \Delta_{10}] \quad h_7 : \Box\Gamma_{11}, F_9, \Delta_{12}, [\Box F_6 \vdash \Delta_{10}] \vee_L}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \vee F_9), [\Box F_6 \vdash \Delta_{10}] \text{ Cut}} \\
\hline
- : \Box\Gamma_{11}, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1 : \Delta_{12}, F_8, \Box\Gamma_{11} \vdash \Delta_{10}, [\Box F_6]} K \quad \frac{h_7 : \Delta_{12}, F_8, \Box\Gamma_{11}, [\Box F_6 \vdash \Delta_{10}]}{h_7 : \Delta_{12}, F_8, \Box\Gamma_{11} \vdash \Delta_{10}} \text{ ax/W}}{\frac{- : \Delta_{12}, F_8, \Box\Gamma_{11} \vdash \Delta_{10}}{- : \Delta_{12}, \Box\Gamma_{11}, F_8 \vee F_9 \vdash \Delta_{10}} \text{ hCut}} \\
\rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, \Box F_7), F_9 \vee F_{10}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_9, \Delta_{13} \vdash \Delta_{11}, \Box F_7 \quad h_8 : \Box\Gamma_{12}, F_{10}, \Delta_{13} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{13}), F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7} \vee_L \\
\hline
- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7} \text{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, \Box F_7} K \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14}, F_9 \vee F_{10} \vdash (\Delta_{11}, \Box F_7), F_{13}} K \quad \frac{h_8 : \Box\Gamma_{12}, F_9, F_{13}, \Delta_{14} \vdash \Delta_{11}, \Box F_7 \quad h_8 : \Box\Gamma_{12}, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}, F_9 \vee F_{10}), F_{13} \vdash \Delta_{11}, \Box F_7} \vee_L \\
\hline
- : \Box\Gamma_{12}, \Delta_{14}, F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_7}{- : \Delta_{14}, \Box\Gamma_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7} \text{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, \Box F_7} K
\end{array}$$

• Case rule AT

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}, \Box F_8) \vdash F_6}{\bullet h_1 : (\Box\Gamma_{10}, \Box F_8), \Delta_{11} \vdash \Delta_9, \Box F_6} K \quad \frac{h_7 : \Box\Gamma_{10}, F_8, \Delta_{11}, \Box F_6, \Box F_8 \vdash \Delta_9}{\bullet h_7 : ((\Box\Gamma_{10}, \Box F_8), \Delta_{11}), \Box F_6 \vdash \Delta_9} AT \\
\hline
- : (\Box\Gamma_{10}, \Box F_8), \Delta_{11} \vdash \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \Delta_{11}, F_8, \Box\Gamma_{10}, \Box F_8 \vdash \Delta_9, \Box F_6}{- : \Delta_{11}, F_8, \Box\Gamma_{10}, \Box F_8 \vdash \Delta_9} \text{ax/W}}{\frac{- : \Delta_{11}, F_8, \Box\Gamma_{10}, \Box F_8 \vdash \Delta_9}{- : \Delta_{11}, \Box\Gamma_{10}, \Box F_8 \vdash \Delta_9} AT} \text{hCut} \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{11}, \Box F_8 \vdash \Delta_9, \Box F_6} K \quad \frac{h_7 : \Box\Gamma_{10}, F_8, \Delta_{11}, \Box F_6, \Box F_8 \vdash \Delta_9}{\bullet h_7 : (\Box\Gamma_{10}, \Delta_{11}, \Box F_8), \Box F_6 \vdash \Delta_9} AT \\
\hline
- : \Box\Gamma_{10}, \Delta_{11}, \Box F_8 \vdash \Delta_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \Delta_{11}, F_8, \Box\Gamma_{10}, \Box F_8 \vdash \Delta_9, \Box F_6}{- : \Delta_{11}, F_8, \Box\Gamma_{10}, \Box F_8 \vdash \Delta_9} \text{ax/W}}{\frac{- : \Delta_{11}, F_8, \Box\Gamma_{10}, \Box F_8 \vdash \Delta_9}{- : \Delta_{11}, \Box\Gamma_{10}, \Box F_8 \vdash \Delta_9} ATG} \text{hCut} \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_9) \vdash F_7}{\bullet h_1 : \Box\Gamma_9, \Delta_{10} \vdash \Delta_8, \Box F_7} K \quad \frac{h_6 : \Box\Gamma_9, F_7, \Delta_{10}, \Box F_7 \vdash \Delta_8}{\bullet h_6 : (\Box\Gamma_9, \Delta_{10}), \Box F_7 \vdash \Delta_8} AT \\
\hline
- : \Box\Gamma_9, \Delta_{10} \vdash \Delta_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \Delta_{10}, \Box\Gamma_9, \text{unbox}(\Box\Gamma_9) \vdash \Delta_8, F_7}{- : \Delta_{10}, \Box\Gamma_9, \text{unbox}(\Box\Gamma_9) \vdash \Delta_8, F_7} \text{ax/W}}{\frac{\frac{- : \Delta_{10}, \Box\Gamma_9, \text{unbox}(\Box\Gamma_9) \vdash \Delta_8, F_7}{- : \Delta_{10}, \Box\Gamma_9, \text{unbox}(\Box\Gamma_9) \vdash \Delta_8} \text{ax/W}}{- : \Delta_{10}, \Box\Gamma_9, \text{unbox}(\Box\Gamma_9) \vdash \Delta_8} \text{sCut}} \text{ATG} \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}, \Box F_9) \vdash F_7}{\bullet h_1 : (\Box\Gamma_{11}, \Box F_9), \Delta_{13} \vdash (\Delta_{10}, \Box F_7), F_{12}} K \quad \frac{h_8 : \Box\Gamma_{11}, F_9, F_{12}, \Delta_{13}, \Box F_9 \vdash \Delta_{10}, \Box F_7}{\bullet h_8 : ((\Box\Gamma_{11}, \Box F_9), \Delta_{13}), F_{12} \vdash \Delta_{10}, \Box F_7} AT \\
\hline
- : (\Box\Gamma_{11}, \Box F_9), \Delta_{13} \vdash \Delta_{10}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : F_9, \text{unbox}(\Box\Gamma_{11}) \vdash F_7}{- : \Delta_{13}, \Box\Gamma_{11}, \Box F_9 \vdash \Delta_{10}, \Box F_7} \text{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, \Box F_9 \vdash \Delta_{10}, \Box F_7} K \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \Box F_7), \Box F_9} K \quad \frac{h_8 : \Box\Gamma_{11}, F_9, \Delta_{12}, \Box F_9 \vdash \Delta_{10}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{11}, \Delta_{12}), \Box F_9 \vdash \Delta_{10}, \Box F_7} AT \\
\hline
- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{11}) \vdash F_7}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \Box F_7} \text{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \Box F_7} K \\
\hline
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{13}, \Box F_9 \vdash (\Delta_{10}, \Box F_7), F_{12}} K \quad \frac{h_8 : \Box\Gamma_{11}, F_9, F_{12}, \Delta_{13}, \Box F_9 \vdash \Delta_{10}, \Box F_7}{\bullet h_8 : (\Box\Gamma_{11}, \Delta_{13}, \Box F_9), F_{12} \vdash \Delta_{10}, \Box F_7} AT \\
\hline
- : \Box\Gamma_{11}, \Delta_{13}, \Box F_9 \vdash \Delta_{10}, \Box F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{- : \text{unbox}(\Box\Gamma_{11}) \vdash F_7}{- : \Delta_{13}, \Box\Gamma_{11}, \Box F_9 \vdash \Delta_{10}, \Box F_7} \text{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, \Box F_9 \vdash \Delta_{10}, \Box F_7} K
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_9) \vdash F_6}{\bullet h_1 : \Box\Gamma_9, \perp, \Delta_{10} \vdash \Delta_8, [\Box F_6]} K \quad \frac{}{\bullet h_7 : (\Box\Gamma_9, \perp, \Delta_{10}), [\Box F_6] \vdash \Delta_8} \perp_L \\
\hline
- : \Box\Gamma_9, \perp, \Delta_{10} \vdash \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8} \perp_L \\
\\
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, [\Box F_7]), \perp} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{11}), \perp \vdash \Delta_9, [\Box F_7]} \perp_L \\
\hline
- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, [\Box F_7] \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{10}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, [\Box F_7]} K \\
\\
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \perp, \Delta_{12} \vdash (\Delta_9, [\Box F_7]), F_{11}} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_{10}, \perp, \Delta_{12}), F_{11} \vdash \Delta_9, [\Box F_7]} \perp_L \\
\hline
- : \Box\Gamma_{10}, \perp, \Delta_{12} \vdash \Delta_9, [\Box F_7] \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, [\Box F_7]} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_6}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{11}, p_9 \vdash (\Delta_8, p_9), [\Box F_6]} K \quad \frac{}{\bullet h_7 : (\Box\Gamma_{10}, \Delta_{11}, p_9), [\Box F_6] \vdash \Delta_8, p_9} I \\
\hline
- : \Box\Gamma_{10}, \Delta_{11}, p_9 \vdash \Delta_8, p_9 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, \Box\Gamma_{10}, p_9 \vdash \Delta_8, p_9} I \\
\\
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12} \vdash ((\Delta_9, p_{10}), [\Box F_7]), p_{10}} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_{11}, \Delta_{12}), p_{10} \vdash (\Delta_9, p_{10}), [\Box F_7]} I \\
\hline
- : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_9, p_{10}), [\Box F_7] \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{11}) \vdash F_7} \text{ax/W} \\
\frac{}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_9, p_{10}, [\Box F_7]} K \\
\\
\frac{h_1 : \text{unbox}(\Box\Gamma_{11}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{13}, p_{10} \vdash ((\Delta_9, p_{10}), [\Box F_7]), F_{12}} K \quad \frac{}{\bullet h_8 : (\Box\Gamma_{11}, \Delta_{13}, p_{10}), F_{12} \vdash (\Delta_9, p_{10}), [\Box F_7]} I \\
\hline
- : \Box\Gamma_{11}, \Delta_{13}, p_{10} \vdash (\Delta_9, p_{10}), [\Box F_7] \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{13}, \Box\Gamma_{11}, p_{10} \vdash \Delta_9, p_{10}, [\Box F_7]} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_9) \vdash F_6}{\bullet h_1 : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8, [\Box F_6]} K \quad \frac{h_7 : \Box\Gamma_9, \Delta_{10}, [\Box F_6] \vdash \Delta_8}{\bullet h_7 : (\Box\Gamma_9, \top, \Delta_{10}), [\Box F_6] \vdash \Delta_8} \top_L \\
\hline
- : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8 \quad \text{Cut} \\
\rightarrow \\
\frac{}{\bullet h_1 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8, [\Box F_6]} \text{ax/W} \quad \frac{h_7 : \top, \Delta_{10}, \Box\Gamma_9, [\Box F_6] \vdash \Delta_8}{\bullet h_7 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8} \text{ax/W} \\
\hline
- : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8 \quad \text{hCut} \\
\\
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, [\Box F_7]), \top} K \quad \frac{h_8 : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, [\Box F_7]}{\bullet h_8 : (\Box\Gamma_{10}, \Delta_{11}), \top \vdash \Delta_9, [\Box F_7]} \top_L \\
\hline
- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, [\Box F_7] \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, [\Box F_7]} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{\bullet h_1 : \Box\Gamma_{10}, \top, \Delta_{12} \vdash (\Delta_9, \Box F_7), F_{11}} \quad K \quad \frac{h_8 : \Box\Gamma_{10}, F_{11}, \Delta_{12} \vdash \Delta_9, \Box F_7}{\bullet h_8 : (\Box\Gamma_{10}, \top, \Delta_{12}), F_{11} \vdash \Delta_9, \Box F_7} \quad \top_L \\
\hline
- : \Box\Gamma_{10}, \top, \Delta_{12} \vdash \Delta_9, \Box F_7 \\
\rightarrow \\
\frac{- : \text{unbox}(\Box\Gamma_{10}) \vdash F_7}{- : \top, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, \Box F_7} \quad \text{ax/w} \\
\hline
- : \top, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, \Box F_7 \quad K
\end{array}$$

8.8 Status of \rightarrow_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{13}} \rightarrow_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \rightarrow F_8 \vdash F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R \\
\hline
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \\
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \quad \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \rightarrow_L \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}}{\bullet h_9 : (\Delta_{14}, F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \text{ax/w} \\
\hline
- : \Delta_{14}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11} \quad \rightarrow_R \\
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \quad \text{hCut}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \wedge F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{13}} \rightarrow_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \rightarrow F_8 \vdash F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}, F_{10} \wedge F_{11}} \rightarrow_R \\
\hline
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11} \\
\rightarrow \\
\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \quad \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \rightarrow_L \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}}{\bullet h_9 : (\Delta_{14}, F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{ax/w} \\
\hline
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \quad \text{hCut} \\
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{14} \vdash F_7, F_{13}, \Delta_{12}, F_{10} \vee F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{13}} \rightarrow_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \rightarrow F_8 \vdash F_{10}, F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R \\
\hline
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \vee F_{11} \\
\rightarrow \\
\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}} \quad \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}} \rightarrow_L \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\bullet h_9 : (\Delta_{14}, F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{ax/w} \\
\hline
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{11} \quad \vee_R \\
- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \vee F_{11} \quad \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_7, F_{11}, \perp, \Delta_{10} \quad h_1 : F_8, \Delta_{12} \vdash F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash (\perp, \Delta_{10}), F_{11}} \rightarrow_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \rightarrow F_8), F_{11} \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}, F_{11}}{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}} \quad \text{ax/w} \quad \frac{h_9 : \Delta_{12}, F_{11}, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \rightarrow F_8), F_{11} \vdash \perp, \Delta_{10}} \text{ax/w} \\
\hline
- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10} \quad \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_7, F_{11}, \top, \Delta_{10} \quad h_1 : F_8, \Delta_{12} \vdash F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash (\top, \Delta_{10}), F_{11}} \rightarrow_L \quad \frac{h_9 : (\Delta_{12}, F_7 \rightarrow F_8), F_{11} \vdash \top, \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \rightarrow F_8), F_{11} \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \top, \Delta_{10} \\
\rightarrow \\
- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \top, \Delta_{10} \quad \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_{13}, \Delta_{14} \vdash F_7, \Box F_{12}, \Delta_{11}, \Box F_{10} \quad h_1 : F_8, \Box\Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10} \rightarrow_L \quad \frac{h_9 : \Box\Gamma_{13}, \Box F_{12} \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{13}, \Delta_{14}), F_7 \rightarrow F_8), \Box F_{12} \vdash \Delta_{11}, \Box F_{10}}}{\frac{\bullet h_1 : (\Box\Gamma_{13}, \Delta_{14}), F_7 \rightarrow F_8 \vdash (\Delta_{11}, \Box F_{10}), \Box F_{12}}{- : (\Box\Gamma_{13}, \Delta_{14}), F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \rightarrow} \\
\frac{\frac{h_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_7, \Box F_{10} \quad \frac{h_9 : \Box F_{12}, \Box\Gamma_{13} \vdash F_{10} \quad ax/W}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} \quad A4}{\frac{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}}{- : \Delta_{14}, \Box\Gamma_{13}, F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \quad hCut} \quad \frac{h_1 : \Delta_{14}, F_8, \Box\Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10} \quad \frac{h_9 : \Box F_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} \quad ax/W}{\frac{- : \Delta_{14}, F_8, \Box\Gamma_{13} \vdash \Delta_{11}, \Box F_{10}}{- : \Delta_{14}, \Box\Gamma_{13}, F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \quad hCut} \\
\frac{h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash F_7, F_{13}, \Delta_{11}, \Box F_{10} \quad h_1 : F_8, \Box\Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, \Box F_{10} \rightarrow_L \quad \frac{h_9 : \Box\Gamma_{12} \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{12}, \Delta_{14}), F_7 \rightarrow F_8), F_{13} \vdash \Delta_{11}, \Box F_{10}} \quad A4}{\frac{\bullet h_1 : (\Box\Gamma_{12}, \Delta_{14}), F_7 \rightarrow F_8 \vdash (\Delta_{11}, \Box F_{10}), F_{13}}{- : (\Box\Gamma_{12}, \Delta_{14}), F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \rightarrow} \quad \frac{- : \Box\Gamma_{12} \vdash F_{10} \quad ax/W}{- : \Delta_{14}, \Box\Gamma_{12}, F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \quad A4
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : \Box\Gamma_{13}, \Delta_{14} \vdash F_7, \Box F_{12}, \Delta_{11}, \Box F_{10} \quad h_1 : F_8, \Box\Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10} \rightarrow_L \quad \frac{h_9 : unbox(\Box\Gamma_{13}), unbox(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{13}, \Delta_{14}), F_7 \rightarrow F_8), \Box F_{12} \vdash \Delta_{11}, \Box F_{10}}}{\frac{\bullet h_1 : (\Box\Gamma_{13}, \Delta_{14}), F_7 \rightarrow F_8 \vdash (\Delta_{11}, \Box F_{10}), \Box F_{12}}{- : (\Box\Gamma_{13}, \Delta_{14}), F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \rightarrow} \\
\frac{\frac{h_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_7, \Box F_{10} \quad \frac{h_9 : unbox(\Box F_{12}), unbox(\Box\Gamma_{13}) \vdash F_{10} \quad ax/W}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} \quad K}{\frac{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}}{- : \Delta_{14}, \Box\Gamma_{13}, F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \quad hCut} \quad \frac{h_1 : \Delta_{14}, F_8, \Box\Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10} \quad \frac{h_9 : unbox(\Box F_{12}), unbox(\Box\Gamma_{13}) \vdash F_{10} \quad ax/W}{\bullet h_9 : \Box F_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, F_7, \Box F_{10}} \quad K}{\frac{- : \Delta_{14}, F_8, \Box\Gamma_{13} \vdash \Delta_{11}, \Box F_{10}}{- : \Delta_{14}, \Box\Gamma_{13}, F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \quad hCut} \\
\frac{h_1 : \Box\Gamma_{12}, \Delta_{14} \vdash F_7, F_{13}, \Delta_{11}, \Box F_{10} \quad h_1 : F_8, \Box\Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, \Box F_{10} \rightarrow_L \quad \frac{h_9 : unbox(\Box\Gamma_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{12}, \Delta_{14}), F_7 \rightarrow F_8), F_{13} \vdash \Delta_{11}, \Box F_{10}} \quad K}{\frac{\bullet h_1 : (\Box\Gamma_{12}, \Delta_{14}), F_7 \rightarrow F_8 \vdash (\Delta_{11}, \Box F_{10}), F_{13}}{- : (\Box\Gamma_{12}, \Delta_{14}), F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \rightarrow} \quad \frac{- : unbox(\Box\Gamma_{12}) \vdash F_{10} \quad ax/W}{- : \Delta_{14}, \Box\Gamma_{12}, F_7 \rightarrow F_8 \vdash \Delta_{11}, \Box F_{10}} \quad K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \rightarrow F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \rightarrow F_{11}, \Delta_{12} \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_7 \rightarrow F_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \rightarrow F_8), F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \rightarrow_L}{\frac{\bullet h_1 : (\Delta_{13}, F_7 \rightarrow F_8 \vdash F_{10}, \Delta_{12}), F_{10} \rightarrow F_{11}}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \rightarrow} \quad \frac{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \quad Cut \\
\frac{\frac{- : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \quad inv-th/ax}{- : \Delta_{13}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}} \quad \frac{- : \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11} \quad inv-th/ax}{- : \Delta_{13}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}} \rightarrow_L}{\frac{- : \Delta_{13}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{11}}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \quad sCut} \\
\frac{h_1 : \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_{13}, \Delta_{12} \rightarrow_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \rightarrow F_8 \vdash F_{10}}{\bullet h_9 : ((\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}), F_{13}}}{\frac{\bullet h_1 : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{13}}{- : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}} \rightarrow} \\
\frac{h_1 : \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{13}, F_7 \quad inv-th/ax \quad \frac{h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \rightarrow_L \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}}{\bullet h_9 : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \quad ax/W}{\frac{- : \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}}{- : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \quad hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12} \vdash F_8, F_{11}, \Delta_{10} \quad h_1 : F_9, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10}, F_{11}} \rightarrow_L \quad \frac{h_7 : F_{11}, \Delta_{12} \vdash F_8, \Delta_{10} \quad h_7 : F_9, F_{11}, \Delta_{12} \vdash \Delta_{10}}{\bullet h_7 : (\Delta_{12}, F_8 \rightarrow F_9), F_{11} \vdash \Delta_{10}} \rightarrow_L}{- : \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_{11}, F_8}{- : \Delta_{12} \vdash \Delta_{10}, F_8} \text{ax/W} \quad \frac{\frac{h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8}{\bullet h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8} \text{ax/W} \quad H}{h_7 : \Delta_{12}, F_{11} \vdash \Delta_{10}, F_8} \text{H}}{- : \Delta_{12} \vdash \Delta_{10}, F_8} \text{hCut} \quad \frac{\frac{h_7 : \Delta_{12}, F_{11}, F_9 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11}, F_9 \vdash \Delta_{10}} \text{ax/W} \quad H}{h_7 : \Delta_{12}, F_{11}, F_9 \vdash \Delta_{10}} \text{H}}{- : \Delta_{12}, F_9 \vdash \Delta_{10}} \text{hCut} \\
- : \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10} \rightarrow_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \wedge F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \rightarrow_L \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \rightarrow F_8), F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13} \vdash \Delta_{12}, F_7, F_{10} \wedge F_{11}}{- : \Delta_{13} \vdash \Delta_{12}, F_7} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax} \quad \wedge_L}{h_9 : \Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7} \wedge_L}{- : \Delta_{13} \vdash \Delta_{12}, F_7} \text{hCut} \quad \frac{\frac{h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{- : \Delta_{13}, F_8 \vdash \Delta_{12}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_8, F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L}{- : \Delta_{13}, F_8 \vdash \Delta_{12}} \wedge_L \\
- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12} \rightarrow_L \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \wedge F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \wedge F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{13}} \rightarrow_L \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}} \wedge_L}{- : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_{11} \vdash \Delta_{12}, F_{13}, F_7}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{13}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_{11}, F_8 \vdash \Delta_{12}, F_{13}}{- : \Delta_{14}, F_{10}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \text{inv-th/ax}}{- : \Delta_{14}, F_{10}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{13}} \rightarrow_L \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{- : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \text{ax/W} \\
- : \Delta_{14}, F_7 \rightarrow F_8, F_{10} \wedge F_{11} \vdash \Delta_{12} \wedge_L \text{hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \vee F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \rightarrow_L \quad \frac{h_9 : F_{10}, \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \rightarrow F_8), F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{13} \vdash \Delta_{12}, F_{10}, F_{11}, F_7}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{inv-th/ax} \quad \frac{- : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{- : \Delta_{13}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}} \text{inv-th/ax}}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}} \rightarrow_L \quad \frac{- : \Delta_{13}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}}{- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10}} \text{ax/W} \\
- : \Delta_{13}, F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{10} \text{sCut} \quad - : \Delta_{13}, F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{12} \\
\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \vee F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}, F_{13}} \rightarrow_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \rightarrow F_8}{\bullet h_9 : ((\Delta_{14}, F_{10} \vee F_{11}), F_7 \rightarrow F_8), F_{13} \vdash \Delta_{12}} \wedge_L}{- : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}, F_7}{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13} \vdash \Delta_{12}, F_7}{\bullet h_9 : \Delta_{14}, F_{13}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{inv-th/ax}}{- : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7} \text{hCut} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13} \vdash \Delta_{12}, F_7}{- : \Delta_{14}, F_{11}, F_{13} \vdash \Delta_{12}, F_7} \text{inv-th/ax} \\
- : \Delta_{14}, F_7 \rightarrow F_8, F_{10} \vee F_{11} \vdash \Delta_{12} \vee_L \text{hCut}
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{12} \vdash F_7, \llbracket F_{10}, \Delta_{11} \rrbracket \quad h_1 : F_8, \Delta_{12} \vdash \llbracket F_{10}, \Delta_{11} \rrbracket}{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{11}, \llbracket F_{10} \rrbracket} \rightarrow_L \quad \frac{h_9 : F_{10}, \Delta_{12}, \llbracket F_{10}, F_7 \rightarrow F_8 \rrbracket \vdash \Delta_{11}}{\bullet h_9 : (\Delta_{12}, F_7 \rightarrow F_8), \llbracket F_{10} \rrbracket \vdash \Delta_{11}} AT}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_7, \llbracket F_{10} \rrbracket}{- : \Delta_{12} \vdash \Delta_{11}, F_7} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{12}, F_{10}, \llbracket F_{10} \rrbracket \vdash \Delta_{11}, F_7}{\bullet h_9 : \Delta_{12}, \llbracket F_{10} \rrbracket \vdash \Delta_{11}, F_7} \text{inv-th/ax} \quad AT}{h_9 : \Delta_{12}, \llbracket F_{10} \rrbracket \vdash \Delta_{11}, F_7} AT}{- : \Delta_{12} \vdash \Delta_{11}, F_7} \text{hCut} \quad \frac{\frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{11}, \llbracket F_{10} \rrbracket}{- : \Delta_{12}, F_8 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_{10}, F_8, \llbracket F_{10} \rrbracket \vdash \Delta_{11}}{\bullet h_9 : \Delta_{12}, F_8, \llbracket F_{10} \rrbracket \vdash \Delta_{11}} AT}{- : \Delta_{12}, F_8 \vdash \Delta_{11}} AT \\
- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{11} \rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, [\Box F_{10} \vdash F_7, F_{12}, \Delta_{11} \quad h_1 : F_8, \Delta_{13}, [\Box F_{10} \vdash F_{12}, \Delta_{11}]}{\bullet h_1 : (\Delta_{13}, [\Box F_{10}]), F_7 \rightarrow F_8 \vdash \Delta_{11}, F_{12}} \rightarrow_L \quad \frac{h_9 : F_{10}, F_{12}, \Delta_{13}, [\Box F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{11}]}{\bullet h_9 : ((\Delta_{13}, [\Box F_{10}]), F_7 \rightarrow F_8), F_{12} \vdash \Delta_{11}} \text{AT} \\
\frac{\quad}{- : (\Delta_{13}, [\Box F_{10}]), F_7 \rightarrow F_8 \vdash \Delta_{11}} \text{Cut} \\
\frac{\quad}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{13}, F_{10}, [\Box F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{11}, F_{12}]}{\bullet h_1 : \Delta_{13}, F_{10}, [\Box F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{11}]} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{12}, [\Box F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{11}]}{\bullet h_9 : \Delta_{13}, F_{10}, F_{12}, [\Box F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{11}]} \text{ax/W} \\
\frac{\quad}{- : \Delta_{13}, F_{10}, [\Box F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{11}]} \text{hCut} \quad \frac{\quad}{- : \Delta_{13}, [\Box F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{11}]} \text{ATG} \\
\frac{\quad}{- : \Delta_{13}, [\Box F_{10}, F_7 \rightarrow F_8 \vdash \Delta_{11}]}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash F_7, \perp, \Delta_{10} \quad h_1 : F_8, \Delta_{11} \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, \perp} \rightarrow_L \quad \frac{\quad}{\bullet h_9 : (\Delta_{11}, F_7 \rightarrow F_8), \perp \vdash \Delta_{10}} \perp_L \\
\frac{\quad}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{Cut} \\
\frac{\quad}{\rightarrow} \\
\frac{h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7}{\bullet h_1 : \Delta_{11} \vdash \perp, \Delta_{10}, F_7} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7}{\bullet h_9 : \perp, \Delta_{11} \vdash \Delta_{10}, F_7} \perp_L \\
\frac{\quad}{- : \Delta_{11} \vdash \Delta_{10}, F_7} \text{hCut} \quad \frac{h_1 : \Delta_{11}, F_8 \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_8 \vdash \perp, \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}}{\bullet h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}} \perp_L \\
\frac{\quad}{- : \Delta_{11}, F_8 \vdash \Delta_{10}} \text{hCut} \quad \frac{\quad}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \rightarrow_L \\
\frac{\quad}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_{12} \vdash F_7, F_{11}, \Delta_{10} \quad h_1 : F_8, \perp, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\perp, \Delta_{12}), F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}} \rightarrow_L \quad \frac{\quad}{\bullet h_9 : ((\perp, \Delta_{12}), F_7 \rightarrow F_8), F_{11} \vdash \Delta_{10}} \perp_L \\
\frac{\quad}{- : (\perp, \Delta_{12}), F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{Cut} \\
\frac{\quad}{\rightarrow} \\
\frac{\quad}{- : \perp, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_{12} \vdash F_7, p_{11}, \Delta_{10}, p_{11} \quad h_1 : F_8, \Delta_{12} \vdash p_{11}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{12}, F_7 \rightarrow F_8 \vdash (\Delta_{10}, p_{11}), p_{11}} \rightarrow_L \quad \frac{\quad}{\bullet h_9 : (\Delta_{12}, F_7 \rightarrow F_8), p_{11} \vdash \Delta_{10}, p_{11}} I \\
\frac{\quad}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \text{Cut} \\
\frac{\quad}{\rightarrow} \\
\frac{h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}, p_{11}}{\bullet h_1 : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}, p_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, p_{11}}{\bullet h_9 : \Delta_{12}, p_{11} \vdash \Delta_{10}, F_7, p_{11}} I \\
\frac{\quad}{- : \Delta_{12} \vdash \Delta_{10}, F_7, p_{11}} \text{hCut} \quad \frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}, p_{11}}{\bullet h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}, p_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_8, p_{11} \vdash \Delta_{10}, p_{11}}{\bullet h_9 : \Delta_{12}, F_8, p_{11} \vdash \Delta_{10}, p_{11}} I \\
\frac{\quad}{- : \Delta_{12}, F_8 \vdash \Delta_{10}, p_{11}} \text{hCut} \quad \frac{\quad}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \rightarrow_L \\
\frac{\quad}{- : \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{13}, p_{11} \vdash F_7, F_{12}, \Delta_{10}, p_{11} \quad h_1 : F_8, \Delta_{13}, p_{11} \vdash F_{12}, \Delta_{10}, p_{11}}{\bullet h_1 : (\Delta_{13}, p_{11}), F_7 \rightarrow F_8 \vdash (\Delta_{10}, p_{11}), F_{12}} \rightarrow_L \quad \frac{\quad}{\bullet h_9 : ((\Delta_{13}, p_{11}), F_7 \rightarrow F_8), F_{12} \vdash \Delta_{10}, p_{11}} I \\
\frac{\quad}{- : (\Delta_{13}, p_{11}), F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} \text{Cut} \\
\frac{\quad}{\rightarrow} \\
\frac{\quad}{- : \Delta_{13}, p_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, p_{11}} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Delta_{11} \vdash F_7, \top, \Delta_{10} \quad h_1 : F_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, \top} \rightarrow_L \quad \frac{h_9 : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \rightarrow F_8), \top \vdash \Delta_{10}} \top_L \\
\frac{\quad}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{Cut} \\
\frac{\quad}{\rightarrow} \\
\frac{\quad}{- : \Delta_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ax/W} \\
\frac{h_1 : \top, \Delta_{12} \vdash F_7, F_{11}, \Delta_{10} \quad h_1 : F_8, \top, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\top, \Delta_{12}), F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}} \rightarrow_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : ((\top, \Delta_{12}), F_7 \rightarrow F_8), F_{11} \vdash \Delta_{10}} \top_L \\
\frac{\quad}{- : (\top, \Delta_{12}), F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{Cut} \\
\frac{\quad}{\rightarrow} \\
\frac{h_1 : \top, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}}{\bullet h_1 : \top, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}} \text{ax/W} \quad \frac{h_9 : \top, \Delta_{12}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : \top, \Delta_{12}, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{ax/W} \\
\frac{\quad}{- : \top, \Delta_{12}, F_7 \rightarrow F_8 \vdash \Delta_{10}} \text{hCut}
\end{array}$$

8.9 Status of \wedge_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \wedge F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{13}} \wedge_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \wedge F_8), F_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R \\
\hline
- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_7, F_8 \vdash \Delta_{12}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \wedge_L \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}}{\bullet h_9 : (\Delta_{14}, F_{10}, F_{13}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}} \text{ax/W}}{- : \Delta_{14}, F_{10}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{11}} \text{hCut} \\
\rightarrow_R \\
- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \wedge F_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), F_{13}} \wedge_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \wedge F_8), F_{13} \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_R \\
\hline
- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, F_8 \vdash \Delta_{12}, F_{13}, F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_{14}, F_7, F_8 \vdash \Delta_{12}, F_{13}, F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10}}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{inv-th/ax}}{- : \Delta_{14}, F_7, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \text{hCut} \\
\wedge_L \\
- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \wedge F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{13}} \wedge_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{10}, F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \wedge F_8), F_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R \\
\hline
- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\bullet h_9 : \Delta_{14}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{ax/W}}{- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{hCut} \\
\vee_R \\
- : \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{12} \vdash F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash (\perp, \Delta_{10}), F_{11}} \wedge_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \wedge F_8), F_{11} \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}, F_{11}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}, F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{12}, F_{11}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : \Delta_{12}, F_{11}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}} \text{hCut} \\
\rightarrow \\
- : \Delta_{12}, F_7 \wedge F_8 \vdash \perp, \Delta_{10}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{12} \vdash F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash (\top, \Delta_{10}), F_{11}} \wedge_L \quad \frac{h_9 : (\Delta_{12}, F_7 \wedge F_8), F_{11} \vdash \top, \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \wedge F_8), F_{11} \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_{12}, F_7 \wedge F_8 \vdash \top, \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
- : \Delta_{12}, F_7 \wedge F_8 \vdash \top, \Delta_{10} \quad \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : (\Box \Gamma_{13}, \Delta_{14}), F_7 \wedge F_8 \vdash (\Delta_{11}, \Box F_{10}), \Box F_{12}} \wedge_L \quad \frac{h_9 : \Box \Gamma_{13}, \Box F_{12} \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{13}, \Delta_{14}), F_7 \wedge F_8), \Box F_{12} \vdash \Delta_{11}, \Box F_{10}} A4 \\
\hline
- : (\Box \Gamma_{13}, \Delta_{14}), F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, F_8, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : \Delta_{14}, F_7, F_8, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10}} \text{ax/W} \quad \frac{h_9 : \Box F_{12}, \Box \Gamma_{13} \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_7, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, \Box F_{10}} \text{ax/W}}{- : \Delta_{14}, F_7, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, \Box F_{10}} \text{hCut} \\
\wedge_L \\
- : \Delta_{14}, \Box \Gamma_{13}, F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Box\Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : (\Box\Gamma_{12}, \Delta_{14}), F_7 \wedge F_8 \vdash (\Delta_{11}, \Box F_{10}), F_{13}} \wedge_L \quad \frac{h_9 : \Box\Gamma_{12} \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{12}, \Delta_{14}), F_7 \wedge F_8), F_{13} \vdash \Delta_{11}, \Box F_{10}} \text{A4} \\
\hline
- : (\Box\Gamma_{12}, \Delta_{14}), F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Box\Gamma_{12} \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{14}, \Box\Gamma_{12}, F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10} \quad \text{A4}
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Box\Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : (\Box\Gamma_{13}, \Delta_{14}), F_7 \wedge F_8 \vdash (\Delta_{11}, \Box F_{10}), \Box F_{12}} \wedge_L \quad \frac{h_9 : \text{unbox}(\Box\Gamma_{13}), \text{unbox}(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{13}, \Delta_{14}), F_7 \wedge F_8), \Box F_{12} \vdash \Delta_{11}, \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{13}, \Delta_{14}), F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{14}, F_7, F_8, \Box\Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, \Box F_{10}} \text{ax/W} \quad \frac{h_9 : \text{unbox}(\Box F_{12}), \text{unbox}(\Box\Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_7, F_8, \Box\Gamma_{13} \vdash \Delta_{11}, \Box F_{10}} \text{K} \\
\hline
- : \Delta_{14}, F_7, F_8, \Box\Gamma_{13} \vdash \Delta_{11}, \Box F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{14}, \Box\Gamma_{13}, F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10} \quad \wedge_L \\
\hline
\frac{h_1 : F_7, F_8, \Box\Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, \Box F_{10}}{\bullet h_1 : (\Box\Gamma_{12}, \Delta_{14}), F_7 \wedge F_8 \vdash (\Delta_{11}, \Box F_{10}), F_{13}} \wedge_L \quad \frac{h_9 : \text{unbox}(\Box\Gamma_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box\Gamma_{12}, \Delta_{14}), F_7 \wedge F_8), F_{13} \vdash \Delta_{11}, \Box F_{10}} K \\
\hline
- : (\Box\Gamma_{12}, \Delta_{14}), F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \text{unbox}(\Box\Gamma_{12}) \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{14}, \Box\Gamma_{12}, F_7 \wedge F_8 \vdash \Delta_{11}, \Box F_{10} \quad K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{13} \vdash F_{10} \rightarrow F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_7 \wedge F_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \rightarrow_L \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \quad \text{inv-th/ax} \quad h_9 : \Delta_{13}, F_{11}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \text{inv-th/ax} \\
\hline
- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \quad \text{hCut} \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \quad \wedge_L \\
\hline
\frac{h_1 : F_7, F_8, \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{13}} \wedge_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_{11}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8), F_{13} \vdash \Delta_{12}} \rightarrow_L \\
\hline
- : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash \Delta_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{14}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}, F_{13}} \text{ax/W} \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \quad \text{inv-th/ax} \quad h_9 : \Delta_{14}, F_{11}, F_{13}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \text{inv-th/ax} \\
\hline
- : \Delta_{14}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash \Delta_{12} \quad \text{hCut} \\
\hline
- : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8 \vdash \Delta_{12} \quad \wedge_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{13} \vdash F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \wedge_L \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{13}, F_{10}, F_{11}, F_7, F_8 \vdash \Delta_{12}} \text{inv-th/ax} \\
\hline
\frac{}{- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7, F_8 \vdash \Delta_{12} \quad \wedge_L \quad h_9 : \Delta_{13}, F_7, F_8, F_{10} \wedge F_{11} \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L \\
\hline
- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \quad \text{hCut} \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \quad \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{14}, F_{10} \wedge F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{13}} \wedge_L \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \wedge F_{11}), F_7 \wedge F_8), F_{13} \vdash \Delta_{12}} \wedge_L \\
\hline
- : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash \Delta_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_{11}, F_7, F_8 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{13}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \wedge F_8 \vdash \Delta_{12}} \text{ax/W}}{\frac{- : \Delta_{14}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}}{- : \Delta_{14}, F_{10} \wedge F_{11}, F_7 \wedge F_8 \vdash \Delta_{12}} \wedge_L} \text{hCut} \\
\hline
\frac{h_1 : F_8, F_9, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}, F_{11}} \wedge_L \quad \frac{h_7 : F_8, F_9, F_{11}, \Delta_{12} \vdash \Delta_{10}}{\bullet h_7 : (\Delta_{12}, F_8 \wedge F_9), F_{11} \vdash \Delta_{10}} \wedge_L \\
\hline
- : \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_7 : \Delta_{12}, F_{11}, F_8, F_9 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11}, F_8, F_9 \vdash \Delta_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{11}, F_8, F_9 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11}, F_8, F_9 \vdash \Delta_{10}} \text{H}}{\frac{- : \Delta_{12}, F_8, F_9 \vdash \Delta_{10}}{- : \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}} \wedge_L} \text{hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{13} \vdash F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \wedge_L \quad \frac{h_9 : F_{10}, \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \wedge F_8), F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L \\
\hline
- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{13}, F_{10}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{11}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{inv-th/ax}}{\frac{- : \Delta_{13}, F_7, F_8 \vdash \Delta_{12}}{- : \Delta_{13}, F_7 \wedge F_8 \vdash \Delta_{12}} \wedge_L} \text{hCut} \\
\hline
\frac{h_1 : F_7, F_8, \Delta_{14}, F_{10} \vee F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \wedge F_8 \vdash \Delta_{12}, F_{13}} \wedge_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12} \quad h_9 : F_{11}, F_{13}, \Delta_{14}, F_7 \wedge F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \vee F_{11}), F_7 \wedge F_8), F_{13} \vdash \Delta_{12}} \vee_L \\
\hline
- : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \wedge F_8 \vdash \Delta_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{13}} \text{ax/W} \quad \frac{\frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{14}, F_{11}, F_{13}, F_7, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{14}, F_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{inv-th/ax}}{\frac{- : \Delta_{14}, F_7, F_8, F_{10} \vee F_{11} \vdash \Delta_{12}}{- : \Delta_{14}, F_7 \wedge F_8, F_{10} \vee F_{11} \vdash \Delta_{12}} \wedge_L} \text{hCut}
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{12} \vdash \boxed{F_{10}}, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{11}, \boxed{F_{10}}} \wedge_L \quad \frac{h_9 : F_{10}, \Delta_{12}, \boxed{F_{10}}, F_7 \wedge F_8 \vdash \Delta_{11}}{\bullet h_9 : (\Delta_{12}, F_7 \wedge F_8), \boxed{F_{10}} \vdash \Delta_{11}} AT \\
\hline
- : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_9 : \Delta_{12}, F_{10}, F_7, F_8, \boxed{F_{10}} \vdash \Delta_{11}}{\bullet h_9 : \Delta_{12}, F_7, F_8, \boxed{F_{10}} \vdash \Delta_{11}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{12}, F_7, F_8, \boxed{F_{10}} \vdash \Delta_{11}}{\bullet h_9 : \Delta_{12}, F_7, F_8, \boxed{F_{10}} \vdash \Delta_{11}} AT}{\frac{- : \Delta_{12}, F_7, F_8 \vdash \Delta_{11}}{- : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{11}} \wedge_L} \text{hCut} \\
\hline
\frac{h_1 : F_7, F_8, \Delta_{13}, \boxed{F_{10}} \vdash F_{12}, \Delta_{11}}{\bullet h_1 : (\Delta_{13}, \boxed{F_{10}}), F_7 \wedge F_8 \vdash \Delta_{11}, F_{12}} \wedge_L \quad \frac{h_9 : F_{10}, F_{12}, \Delta_{13}, \boxed{F_{10}}, F_7 \wedge F_8 \vdash \Delta_{11}}{\bullet h_9 : ((\Delta_{13}, \boxed{F_{10}}), F_7 \wedge F_8), F_{12} \vdash \Delta_{11}} AT \\
\hline
- : (\Delta_{13}, \boxed{F_{10}}), F_7 \wedge F_8 \vdash \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_9 : \Delta_{13}, F_{10}, F_{12}, \boxed{F_{10}}, F_7 \wedge F_8 \vdash \Delta_{11}}{\bullet h_9 : \Delta_{13}, F_{10}, F_{12}, \boxed{F_{10}}, F_7 \wedge F_8 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{12}, \boxed{F_{10}}, F_7 \wedge F_8 \vdash \Delta_{11}}{\bullet h_9 : \Delta_{13}, F_{10}, F_{12}, \boxed{F_{10}}, F_7 \wedge F_8 \vdash \Delta_{11}} AT}{\frac{- : \Delta_{13}, F_{10}, \boxed{F_{10}}, F_7 \wedge F_8 \vdash \Delta_{11}}{- : \Delta_{13}, \boxed{F_{10}}, F_7 \wedge F_8 \vdash \Delta_{11}} AT} \text{hCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{11} \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, \perp} \wedge_L \quad \frac{}{\bullet h_9 : (\Delta_{11}, F_7 \wedge F_8), \perp \vdash \Delta_{10}} \perp_L \\
\hline
- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_7, F_8 \vdash \perp, \Delta_{10}}{- : \Delta_{11}, F_7, F_8 \vdash \Delta_{10}} \text{ax/W} \quad \frac{\bullet h_9 : \perp, \Delta_{11}, F_7, F_8 \vdash \Delta_{10}}{- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}} \perp_L}{- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{hCut} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \perp, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\perp, \Delta_{12}), F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}} \wedge_L \quad \frac{}{\bullet h_9 : ((\perp, \Delta_{12}), F_7 \wedge F_8), F_{11} \vdash \Delta_{10}} \perp_L \\
\hline
- : (\perp, \Delta_{12}), F_7 \wedge F_8 \vdash \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \perp, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{12} \vdash p_{11}, \Delta_{10}, p_{11}}{\bullet h_1 : \Delta_{12}, F_7 \wedge F_8 \vdash (\Delta_{10}, p_{11}), p_{11}} \wedge_L \quad \frac{}{\bullet h_9 : (\Delta_{12}, F_7 \wedge F_8), p_{11} \vdash \Delta_{10}, p_{11}} I \\
\hline
- : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}, p_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_7, F_8 \vdash \Delta_{10}, p_{11}, p_{11}}{- : \Delta_{12}, F_7, F_8 \vdash \Delta_{10}, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{12}, F_7, F_8, p_{11} \vdash \Delta_{10}, p_{11}}{- : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}, p_{11}} I}{- : \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}, p_{11}} \text{hCut} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{13}, p_{11} \vdash F_{12}, \Delta_{10}, p_{11}}{\bullet h_1 : (\Delta_{13}, p_{11}), F_7 \wedge F_8 \vdash (\Delta_{10}, p_{11}), F_{12}} \wedge_L \quad \frac{}{\bullet h_9 : ((\Delta_{13}, p_{11}), F_7 \wedge F_8), F_{12} \vdash \Delta_{10}, p_{11}} I \\
\hline
- : (\Delta_{13}, p_{11}), F_7 \wedge F_8 \vdash \Delta_{10}, p_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{13}, p_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, p_{11}} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, \top} \wedge_L \quad \frac{h_9 : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \wedge F_8), \top \vdash \Delta_{10}} \top_L \\
\hline
- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{}{- : \Delta_{11}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, F_8, \top, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\top, \Delta_{12}), F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}} \wedge_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : ((\top, \Delta_{12}), F_7 \wedge F_8), F_{11} \vdash \Delta_{10}} \top_L \\
\hline
- : (\top, \Delta_{12}), F_7 \wedge F_8 \vdash \Delta_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}}{- : \top, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{ax/W} \quad \frac{h_9 : \top, \Delta_{12}, F_{11}, F_7 \wedge F_8 \vdash \Delta_{10}}{- : \top, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{ax/W}}{- : \top, \Delta_{12}, F_7 \wedge F_8 \vdash \Delta_{10}} \text{hCut}
\end{array}$$

8.10 Status of \vee_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), F_{13}} \vee_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \vee F_8), F_{13} \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R \\
\hline
- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_7 \vdash \Delta_{12}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_8 \vee F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \text{inv-th/ax}}{- : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}, F_{13}} \vee_L \\
\frac{\frac{}{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}} \text{ax/W}}{- : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}} \text{hCut} \rightarrow_R \\
\frac{}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \rightarrow_R
\end{array}$$

- Case rule \wedge_R

[illegible]

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \vee F_{11} \quad h_1 : F_8, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \vee F_{11}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{13}} \vee_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash F_{10}, F_{11}, \Delta_{12}}{\bullet h_9 : (\Delta_{14}, F_7 \vee F_8), F_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R \\
\frac{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_{13} \quad \bullet h_9 : (\Delta_{14}, F_7 \vee F_8), F_{13} \vdash \Delta_{12}, F_{10} \vee F_{11}}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \text{Cut} \\
\downarrow \rightarrow \\
\frac{h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13} \quad \text{inv-th/ax} \quad h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13} \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}} \vee_L \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{h_9 : \Delta_{14}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \text{ax/W} \\
\frac{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}, F_{13} \quad h_9 : \Delta_{14}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}} \vee_R \\
\frac{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_R \quad \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{12} \vdash F_{11}, \perp, \Delta_{10} \quad h_1 : F_8, \Delta_{12} \vdash F_{11}, \perp, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash (\perp, \Delta_{10}), F_{11}} \vee_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{12}, F_7 \vee F_8), F_{11} \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_{12}, F_7 \vee F_8 \vdash \perp, \Delta_{10} \quad \text{Cut} \\
\hline
\frac{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash \perp, \Delta_{10}, F_{11} \quad \text{ax/W} \quad h_9 : \Delta_{12}, F_{11}, F_7 \vee F_8 \vdash \perp, \Delta_{10} \quad \text{ax/W}}{- : \Delta_{12}, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\frac{\frac{h_1 : F_7, \Delta_{12} \vdash F_{11}, \top, \Delta_{10} \quad h_1 : F_8, \Delta_{12} \vdash F_{11}, \top, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash (\top, \Delta_{10}), F_{11}} \vee_L \quad \frac{}{\bullet h_9 : (\Delta_{12}, F_7 \vee F_8), F_{11} \vdash \top, \Delta_{10}} \text{Cut}}{\frac{- : \Delta_{12}, F_7 \vee F_8 \vdash \top, \Delta_{10}}{\rightarrow}} \text{Cut}$$
$$\frac{}{- : \Delta_{12}, F_7 \vee F_8 \vdash \top, \Delta_{10}} \top_R$$

- Case rule *A4*

$$\begin{array}{c}
\frac{h_1 : F_7, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10} \quad h_1 : F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}}{\bullet h_1 : (\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, [\Box F_{10}), \Box F_{12}} \vee L \quad \frac{h_9 : \Box \Gamma_{13}, \Box F_{12} \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8), \Box F_{12} \vdash \Delta_{11}, [\Box F_{10}} \\
\frac{\bullet h_1 : (\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, [\Box F_{10}), \Box F_{12}}{\quad \rightarrow \quad} \frac{- : (\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8 \vdash \Delta_{11}, [\Box F_{10}}{\quad} \\
\frac{h_1 : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}}{\quad} \text{ax/W} \quad \frac{h_9 : \Box F_{12}, \Box \Gamma_{13} \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}} \text{ax/W} \quad \frac{A4}{hCut} \quad \frac{h_1 : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, [\Box F_{10}}{\quad} \text{ax/W} \quad \frac{h_9 : \Box F_{12}, \Delta_{14}, F_8, \Box \Gamma_{13} \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}} \\
\frac{- : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}}{\quad} \quad \frac{- : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [\Box F_{10}}{\quad} \\
\frac{\quad}{- : \Delta_{14}, \Box \Gamma_{13}, F_7 \vee F_8 \vdash \Delta_{11}, [\Box F_{10}} \\
\frac{h_1 : F_7, \Box \Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, [\Box F_{10} \quad h_1 : F_8, \Box \Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, [\Box F_{10}}{\bullet h_1 : (\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, [\Box F_{10}), F_{13}} \vee L \quad \frac{h_9 : \Box \Gamma_{12} \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8), F_{13} \vdash \Delta_{11}, [\Box F_{10}} A4 \\
\frac{\bullet h_1 : (\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, [\Box F_{10}), F_{13}}{\quad \rightarrow \quad} \frac{- : (\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8 \vdash \Delta_{11}, [\Box F_{10}}{\quad} \text{Cut} \\
\frac{\quad}{- : \Delta_{14}, \Box \Gamma_{12}, F_7 \vee F_8 \vdash \Delta_{11}, [\Box F_{10}} \\
\frac{- : \Box \Gamma_{12} \vdash F_{10}}{\quad} \text{ax/W} \quad \frac{A4}{\quad} \\
\frac{- : \Delta_{14}, \Box \Gamma_{12}, F_7 \vee F_8 \vdash \Delta_{11}, [\Box F_{10}}{\quad}
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : F_7, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, [] F_{10} \quad h_1 : F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, [] F_{10}}{\bullet h_1 : (\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, [] F_{10}), \Box F_{12}} \vee_L \quad \frac{h_9 : unbox(\Box \Gamma_{13}), unbox(\Box F_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8), \Box F_{12} \vdash \Delta_{11}, [] F_{10}} \\
\hline
- : (\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8 \vdash \Delta_{11}, [] F_{10} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, [] F_{10}}{- : \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, [] F_{10}} \text{ ax/w} \quad \frac{\frac{h_9 : unbox(\Box F_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10}}{\bullet h_9 : \Box F_{12}, \Delta_{14}, F_7, \Box \Gamma_{13} \vdash \Delta_{11}, [] F_{10}} \text{ ax/w} \quad \frac{K}{hCut}}{\frac{h_1 : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, [] F_{10}}{- : \Delta_{14}, F_8, \Box \Gamma_{13} \vdash \Delta_{11}, [] F_{10}} \text{ ax/w}} \vee_L \\
\hline
- : \Delta_{14}, \Box \Gamma_{13}, F_7 \vee F_8 \vdash \Delta_{11}, [] F_{10} \\
\hline
\frac{h_1 : F_7, \Box \Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, [] F_{10} \quad h_1 : F_8, \Box \Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, [] F_{10}}{\bullet h_1 : (\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, [] F_{10}), F_{13}} \vee_L \quad \frac{h_9 : unbox(\Box \Gamma_{12}) \vdash F_{10}}{\bullet h_9 : ((\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8), F_{13} \vdash \Delta_{11}, [] F_{10}} K \\
\hline
- : (\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8 \vdash \Delta_{11}, [] F_{10} \\
\hline
\rightarrow \\
\frac{- : unbox(\Box \Gamma_{12}) \vdash F_{10}}{- : \Delta_{14}, \Box \Gamma_{12}, F_7 \vee F_8 \vdash \Delta_{11}, [] F_{10}} \text{ ax/w} \quad K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{13} \vdash F_{10} \rightarrow F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \rightarrow F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \rightarrow F_{11}} \vee_L \quad \frac{h_9 : \Delta_{13}, F_7 \vee F_8 \vdash F_{10}, \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \rightarrow_L \\
\hline
- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12} \\
\hline
\rightarrow \\
\frac{- : \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_{11}}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}} \text{ inv-th/ax} \quad \frac{- : \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}} \text{ inv-th/ax} \\
\hline
\frac{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ ax/w} \quad \frac{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{11}}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}} \vee_L \quad \frac{- : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ sCut} \\
\hline
\frac{h_1 : F_7, \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \rightarrow F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \vee_L \quad \frac{h_9 : F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash F_{10}}{\bullet h_9 : ((\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \vee F_8) \vdash \Delta_{12}} \\
\hline
- : (\Delta_{14}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \vdash \Delta_{12} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_7 \vdash \Delta_{12}, F_{10}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_8 \vdash \Delta_{12}, F_{10}, F_{13}}{\bullet h_1 : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{13}} \text{ inv-th/ax} \\
\hline
\frac{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{- : \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}} \vee_L \quad \frac{h_9 : \Delta_{14}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{- : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ ax/w} \quad \frac{h_1 : \Delta_{14}, F_{11}, F_7 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ hCut}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{13} \vdash F_{10} \wedge F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \wedge F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}} \vee_L \quad \frac{h_9 : F_{10}, F_{11}, \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L \\
\hline
- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{- : \Delta_{13}, F_7 \vdash \Delta_{12}} \text{ ax/w} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_7 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_7, F_{10} \wedge F_{11} \vdash \Delta_{12}} \text{ inv-th/ax} \\
\hline
\frac{- : \Delta_{13}, F_7 \vdash \Delta_{12}}{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}} \wedge_L \quad \frac{h_1 : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10} \wedge F_{11}}{- : \Delta_{13}, F_8 \vdash \Delta_{12}} \text{ ax/w} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_{13}, F_8, F_{10} \wedge F_{11} \vdash \Delta_{12}} \text{ inv-th/ax} \\
\hline
- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12} \\
\hline
\rightarrow \\
\frac{h_1 : F_7, \Delta_{14}, F_{10} \wedge F_{11} \vdash F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \wedge F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \vee_L \quad \frac{h_9 : F_{10}, F_{11}, F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \wedge F_{11}), F_7 \vee F_8), F_{13} \vdash \Delta_{12}} \wedge_L \\
\hline
- : (\Delta_{14}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash \Delta_{12} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \text{ inv-th/ax} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_{11}, F_8 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \text{ inv-th/ax} \\
\hline
\frac{- : \Delta_{14}, F_{10}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}}{- : \Delta_{14}, F_{10} \wedge F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \wedge_L \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{- : \Delta_{14}, F_{10} \wedge F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ ax/w} \quad \frac{- : \Delta_{14}, F_{10}, F_{11}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{- : \Delta_{14}, F_{10} \wedge F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{ hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, \Delta_{13} \vdash F_{10} \vee F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \vee F_{11}, \Delta_{12}}{\bullet h_1 : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10} \vee F_{11}} \vee_L \quad \frac{h_9 : F_{10}, \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12} \quad h_9 : F_{11}, \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : (\Delta_{13}, F_7 \vee F_8), F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L}{\frac{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\rightarrow} \text{Cut}} \\
\frac{\frac{- : \Delta_{13}, F_7 \vdash \Delta_{12}, F_{10}, F_{11}}{\text{inv-th/ax}} \quad \frac{- : \Delta_{13}, F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\text{inv-th/ax}}}{\frac{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}, F_{11}}{\vee_L}} \quad \frac{- : \Delta_{13}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{\text{ax/W}}}{\frac{- : \Delta_{13}, F_7 \vee F_8 \vdash \Delta_{12}, F_{10}}{\text{sCut}}} \quad \frac{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}}{\rightarrow} \\
\frac{h_1 : F_7, \Delta_{14}, F_{10} \vee F_{11} \vdash F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \vee F_{11} \vdash F_{13}, \Delta_{12}}{\bullet h_1 : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \vee F_8 \vdash \Delta_{12}, F_{13}} \vee_L \quad \frac{h_9 : F_{10}, F_{13}, \Delta_{14}, F_7 \vee F_8 \vdash \Delta_{12}}{\bullet h_9 : ((\Delta_{14}, F_{10} \vee F_{11}), F_7 \vee F_8) \vdash \Delta_{12}} \vee_L}{\frac{- : (\Delta_{14}, F_{10} \vee F_{11}), F_7 \vee F_8 \vdash \Delta_{12}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{14}, F_{10}, F_7 \vdash \Delta_{12}, F_{13}}{\text{inv-th/ax}} \quad \frac{h_1 : \Delta_{14}, F_{10}, F_8 \vdash \Delta_{12}, F_{13}}{\text{inv-th/ax}}}{\frac{\bullet h_1 : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}, F_{13}}{\vee_L}} \quad \frac{h_9 : \Delta_{14}, F_{10}, F_{13}, F_7 \vee F_8 \vdash \Delta_{12}}{\text{ax/W}}}{\frac{- : \Delta_{14}, F_{10}, F_7 \vee F_8 \vdash \Delta_{12}}{\text{hCut}}} \quad \frac{h_1 : \Delta_{14}, F_{11}, F_7 \vdash \Delta_{12}, F_{13}}{\bullet h_1 : \Delta_{14}, F_{11}, F_7 \vee F_8 \vdash \Delta_{12}} \text{hCut} \\
\frac{- : \Delta_{14}, F_{10} \vee F_{11}, F_7 \vee F_8 \vdash \Delta_{12}}{\rightarrow} \\
\frac{h_1 : F_8, \Delta_{12} \vdash F_{11}, \Delta_{10} \quad h_1 : F_9, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}, F_{11}} \vee_L \quad \frac{h_7 : F_8, F_{11}, \Delta_{12} \vdash \Delta_{10} \quad h_7 : F_9, F_{11}, \Delta_{12} \vdash \Delta_{10}}{\bullet h_7 : (\Delta_{12}, F_8 \vee F_9), F_{11} \vdash \Delta_{10}} \vee_L}{\frac{- : \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{10}, F_{11}}{\text{ax/W}} \quad \frac{h_7 : \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}} \text{H}}{\frac{- : \Delta_{12}, F_8 \vdash \Delta_{10}}{\text{hCut}}} \quad \frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{10}, F_{11}}{\text{ax/W}} \quad \frac{h_7 : \Delta_{12}, F_{11}, F_9 \vdash \Delta_{10}}{\bullet h_7 : \Delta_{12}, F_{11}, F_9 \vdash \Delta_{10}} \text{H}}{\frac{- : \Delta_{12}, F_9 \vdash \Delta_{10}}{\text{hCut}}} \quad \frac{- : \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}}{\vee_L}
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{\frac{h_1 : F_7, \Delta_{12} \vdash \boxed{F_{10}}, \Delta_{11} \quad h_1 : F_8, \Delta_{12} \vdash \boxed{F_{10}}, \Delta_{11}}{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{11}, \boxed{F_{10}}} \vee_L \quad \frac{h_9 : F_{10}, \Delta_{12}, \boxed{F_{10}}, F_7 \vee F_8 \vdash \Delta_{11}}{\bullet h_9 : (\Delta_{12}, F_7 \vee F_8), \boxed{F_{10}} \vdash \Delta_{11}} \text{AT}}{\frac{- : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{12}, F_7 \vdash \Delta_{11}, \boxed{F_{10}}}{\text{ax/W}} \quad \frac{h_9 : \Delta_{12}, F_{10}, F_7, \boxed{F_{10}} \vdash \Delta_{11}}{\bullet h_9 : \Delta_{12}, F_7, \boxed{F_{10}} \vdash \Delta_{11}} \text{AT}}{\frac{- : \Delta_{12}, F_7 \vdash \Delta_{11}}{\text{hCut}}} \quad \frac{\frac{h_1 : \Delta_{12}, F_8 \vdash \Delta_{11}, \boxed{F_{10}}}{\text{ax/W}} \quad \frac{h_9 : \Delta_{12}, F_{10}, F_8, \boxed{F_{10}} \vdash \Delta_{11}}{\bullet h_9 : \Delta_{12}, F_8, \boxed{F_{10}} \vdash \Delta_{11}} \text{AT}}{\frac{- : \Delta_{12}, F_8 \vdash \Delta_{11}}{\text{hCut}}} \quad \frac{- : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{11}}{\vee_L} \\
\frac{h_1 : F_7, \Delta_{13}, \boxed{F_{10}} \vdash F_{12}, \Delta_{11} \quad h_1 : F_8, \Delta_{13}, \boxed{F_{10}} \vdash F_{12}, \Delta_{11}}{\bullet h_1 : (\Delta_{13}, \boxed{F_{10}}), F_7 \vee F_8 \vdash \Delta_{11}, F_{12}} \vee_L \quad \frac{h_9 : F_{10}, F_{12}, \Delta_{13}, \boxed{F_{10}}, F_7 \vee F_8 \vdash \Delta_{11}}{\bullet h_9 : ((\Delta_{13}, \boxed{F_{10}}), F_7 \vee F_8), F_{12} \vdash \Delta_{11}} \text{AT}}{\frac{- : (\Delta_{13}, \boxed{F_{10}}), F_7 \vee F_8 \vdash \Delta_{11}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_{13}, F_{10}, \boxed{F_{10}}, F_7 \vee F_8 \vdash \Delta_{11}, F_{12}}{\text{ax/W}} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{12}, \boxed{F_{10}}, F_7 \vee F_8 \vdash \Delta_{11}}{\text{hCut}}}{\frac{- : \Delta_{13}, F_{10}, \boxed{F_{10}}, F_7 \vee F_8 \vdash \Delta_{11}}{\text{AT}}}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{11} \vdash \perp, \Delta_{10} \quad h_1 : F_8, \Delta_{11} \vdash \perp, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}, \perp} \vee_L \quad \frac{h_9 : (\Delta_{11}, F_7 \vee F_8), \perp \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \vee F_8), \perp \vdash \Delta_{10}} \perp_L}{\frac{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_{11}, F_7 \vdash \perp, \Delta_{10}}{\text{ax/W}} \quad \frac{h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}}{\bullet h_9 : \perp, \Delta_{11}, F_7 \vdash \Delta_{10}} \perp_L}{\frac{- : \Delta_{11}, F_7 \vdash \Delta_{10}}{\text{hCut}}} \quad \frac{\frac{h_1 : \Delta_{11}, F_8 \vdash \perp, \Delta_{10}}{\text{ax/W}} \quad \frac{h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}}{\bullet h_9 : \perp, \Delta_{11}, F_8 \vdash \Delta_{10}} \perp_L}{\frac{- : \Delta_{11}, F_8 \vdash \Delta_{10}}{\text{hCut}}} \quad \frac{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}}{\vee_L} \\
\frac{h_1 : F_7, \perp, \Delta_{12} \vdash F_{11}, \Delta_{10} \quad h_1 : F_8, \perp, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\perp, \Delta_{12}), F_7 \vee F_8 \vdash \Delta_{10}, F_{11}} \vee_L \quad \frac{h_9 : ((\perp, \Delta_{12}), F_7 \vee F_8), F_{11} \vdash \Delta_{10}}{\bullet h_9 : ((\perp, \Delta_{12}), F_7 \vee F_8), F_{11} \vdash \Delta_{10}} \perp_L}{\frac{- : (\perp, \Delta_{12}), F_7 \vee F_8 \vdash \Delta_{10}}{\rightarrow} \text{Cut}} \\
\frac{- : \perp, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{\perp_L}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{12} \vdash P_{11}, \Delta_{10}, P_{11} \quad h_1 : F_8, \Delta_{12} \vdash P_{11}, \Delta_{10}, P_{11}}{\bullet h_1 : \Delta_{12}, F_7 \vee F_8 \vdash (\Delta_{10}, P_{11}), P_{11}} \vee_L \quad \frac{}{\bullet h_9 : (\Delta_{12}, F_7 \vee F_8), P_{11} \vdash \Delta_{10}, P_{11}} I \\
\frac{}{- : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}, P_{11}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_1 : \Delta_{12}, F_7 \vdash \Delta_{10}, P_{11}, P_{11} \quad \bullet h_9 : \Delta_{12}, F_7, P_{11} \vdash \Delta_{10}, P_{11}}{- : \Delta_{12}, F_7 \vdash \Delta_{10}, P_{11}} \text{ax/W} \quad \frac{}{\bullet h_9 : \Delta_{12}, F_8, P_{11} \vdash \Delta_{10}, P_{11}} I \\
\frac{}{- : \Delta_{12}, F_8 \vdash \Delta_{10}, P_{11}} \text{hCut} \quad \frac{}{- : \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}, P_{11}} \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{13}, P_{11} \vdash F_{12}, \Delta_{10}, P_{11} \quad h_1 : F_8, \Delta_{13}, P_{11} \vdash F_{12}, \Delta_{10}, P_{11}}{\bullet h_1 : (\Delta_{13}, P_{11}), F_7 \vee F_8 \vdash (\Delta_{10}, P_{11}), F_{12}} \vee_L \quad \frac{}{\bullet h_9 : ((\Delta_{13}, P_{11}), F_7 \vee F_8), F_{12} \vdash \Delta_{10}, P_{11}} I \\
\frac{}{- : (\Delta_{13}, P_{11}), F_7 \vee F_8 \vdash \Delta_{10}, P_{11}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{13}, P_{11}, F_7 \vee F_8 \vdash \Delta_{10}, P_{11}} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : F_7, \Delta_{11} \vdash \top, \Delta_{10} \quad h_1 : F_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}, \top} \vee_L \quad \frac{h_9 : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : (\Delta_{11}, F_7 \vee F_8), \top \vdash \Delta_{10}} \top_L \\
\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{}{- : \Delta_{11}, F_7 \vee F_8 \vdash \Delta_{10}} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : F_7, \top, \Delta_{12} \vdash F_{11}, \Delta_{10} \quad h_1 : F_8, \top, \Delta_{12} \vdash F_{11}, \Delta_{10}}{\bullet h_1 : (\top, \Delta_{12}), F_7 \vee F_8 \vdash \Delta_{10}, F_{11}} \vee_L \quad \frac{h_9 : F_{11}, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : ((\top, \Delta_{12}), F_7 \vee F_8), F_{11} \vdash \Delta_{10}} \top_L \\
\frac{}{- : (\top, \Delta_{12}), F_7 \vee F_8 \vdash \Delta_{10}} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{\bullet h_1 : \top, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \quad \text{ax/W} \quad h_9 : \top, \Delta_{12}, F_{11}, F_7 \vee F_8 \vdash \Delta_{10} \quad \text{ax/W}}{- : \top, \Delta_{12}, F_7 \vee F_8 \vdash \Delta_{10}} \text{hCut}
\end{array}$$

8.11 Status of AT : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, [F_6 \vdash F_{11}, \Delta_{10}, F_8 \rightarrow F_9] \quad \bullet h_1 : \Delta_{12}, [F_6 \vdash (\Delta_{10}, F_8 \rightarrow F_9), F_{11}]}{\bullet h_1 : \Delta_{12}, [F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9]} AT \quad \frac{h_7 : F_8, F_{11}, \Delta_{12}, [F_6 \vdash F_9, \Delta_{10}]}{\bullet h_7 : (\Delta_{12}, [F_6], F_{11} \vdash \Delta_{10}, F_8 \rightarrow F_9)} \rightarrow_R \\
\frac{}{- : \Delta_{12}, [F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9]} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_1 : \Delta_{12}, F_6, [F_6 \vdash \Delta_{10}, F_{11}, F_8 \rightarrow F_9] \quad \bullet h_7 : \Delta_{12}, F_{11}, F_6, [F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9]}{- : \Delta_{12}, F_6, [F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9]} \text{ax/W} \quad \frac{}{- : \Delta_{12}, [F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9]} \text{hCut} \\
\frac{}{- : \Delta_{12}, [F_6 \vdash \Delta_{10}, F_8 \rightarrow F_9]} ATG
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, [F_6 \vdash F_{11}, \Delta_{10}, F_8 \wedge F_9] \quad \bullet h_1 : \Delta_{12}, [F_6 \vdash (\Delta_{10}, F_8 \wedge F_9), F_{11}]}{\bullet h_1 : \Delta_{12}, [F_6 \vdash \Delta_{10}, F_8 \wedge F_9]} AT \quad \frac{h_7 : F_{11}, \Delta_{12}, [F_6 \vdash F_8, \Delta_{10}] \quad h_7 : F_{11}, \Delta_{12}, [F_6 \vdash F_9, \Delta_{10}]}{\bullet h_7 : (\Delta_{12}, [F_6], F_{11} \vdash \Delta_{10}, F_8 \wedge F_9)} \wedge_R \\
\frac{}{- : \Delta_{12}, [F_6 \vdash \Delta_{10}, F_8 \wedge F_9]} \text{Cut} \\
\frac{}{\rightarrow} \\
\frac{h_1 : \Delta_{12}, F_6, [F_6 \vdash \Delta_{10}, F_{11}, F_8 \wedge F_9] \quad \bullet h_7 : \Delta_{12}, F_{11}, F_6, [F_6 \vdash \Delta_{10}, F_8 \wedge F_9]}{- : \Delta_{12}, F_6, [F_6 \vdash \Delta_{10}, F_8 \wedge F_9]} \text{ax/W} \quad \frac{}{- : \Delta_{12}, [F_6 \vdash \Delta_{10}, F_8 \wedge F_9]} \text{hCut} \\
\frac{}{- : \Delta_{12}, [F_6 \vdash \Delta_{10}, F_8 \wedge F_9]} AT
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{12}, [\Box F_6 \vdash F_{11}, \Delta_{10}, F_8 \vee F_9 \quad \bullet h_1 : \Delta_{12}, [\Box F_6 \vdash (\Delta_{10}, F_8 \vee F_9), F_{11}] \quad AT \quad \frac{h_7 : F_{11}, \Delta_{12}, [\Box F_6 \vdash F_8, F_9, \Delta_{10} \quad \bullet h_7 : (\Delta_{12}, [\Box F_6], F_{11} \vdash \Delta_{10}, F_8 \vee F_9 \quad \vee_R}{\vdash : \Delta_{12}, [\Box F_6 \vdash \Delta_{10}, F_8 \vee F_9} \quad \text{Cut}} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6, [\Box F_6 \vdash \Delta_{10}, F_{11}, F_8 \vee F_9 \quad \bullet h_1 : \Delta_{12}, F_{11}, F_6, [\Box F_6 \vdash \Delta_{10}, F_8 \vee F_9 \quad ax/W}{\vdash : \Delta_{12}, F_6, [\Box F_6 \vdash \Delta_{10}, F_8 \vee F_9} \quad hCut \\
\vdash : \Delta_{12}, [\Box F_6 \vdash \Delta_{10}, F_8 \vee F_9 \quad AT
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{10}, [\Box F_6 \vdash F_9, \perp, \Delta_8 \quad \bullet h_1 : \Delta_{10}, [\Box F_6 \vdash (\perp, \Delta_8), F_9] \quad AT \quad \frac{h_7 : F_9, \Delta_{10}, [\Box F_6 \vdash \Delta_8 \quad \bullet h_7 : (\Delta_{10}, [\Box F_6], F_9 \vdash \perp, \Delta_8}{\vdash : \Delta_{10}, [\Box F_6 \vdash \perp, \Delta_8} \quad \perp_R}{\vdash : \Delta_{10}, [\Box F_6 \vdash \perp, \Delta_8} \quad \text{Cut}} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10}, [\Box F_6 \vdash \perp, \Delta_8, F_9 \quad h_7 : \Delta_{10}, F_9, [\Box F_6 \vdash \perp, \Delta_8 \quad ax/W}{\vdash : \Delta_{10}, [\Box F_6 \vdash \perp, \Delta_8} \quad hCut
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{10}, [\Box F_6 \vdash F_9, \top, \Delta_8 \quad \bullet h_1 : \Delta_{10}, [\Box F_6 \vdash (\top, \Delta_8), F_9] \quad AT \quad \frac{h_7 : (\Delta_{10}, [\Box F_6], F_9 \vdash \top, \Delta_8}{\vdash : \Delta_{10}, [\Box F_6 \vdash \top, \Delta_8} \quad \top_R}{\vdash : \Delta_{10}, [\Box F_6 \vdash \top, \Delta_8} \quad \text{Cut}} \\
\rightarrow \\
\vdash : \Delta_{10}, [\Box F_6 \vdash \top, \Delta_8 \quad \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{h_1 : F_6, (\Box \Gamma_{11}, \Delta_{12}), [\Box F_6 \vdash \Box F_{10}, \Delta_9, [\Box F_8 \quad \bullet h_1 : (\Box \Gamma_{11}, \Delta_{12}), [\Box F_6 \vdash (\Delta_9, [\Box F_8]), \Box F_{10}] \quad AT \quad \frac{h_7 : \Box \Gamma_{11}, \Box F_{10}, [\Box F_6 \vdash F_8 \quad \bullet h_7 : ((\Box \Gamma_{11}, \Delta_{12}), [\Box F_6], \Box F_{10} \vdash \Delta_9, [\Box F_8 \quad A4}{\vdash : (\Box \Gamma_{11}, \Delta_{12}), [\Box F_6 \vdash \Delta_9, [\Box F_8} \quad \text{Cut}} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6, \Box \Gamma_{11}, [\Box F_6 \vdash \Box F_{10}, \Delta_9, [\Box F_8 \quad \bullet h_1 : \Delta_{12}, F_6, \Box \Gamma_{11}, [\Box F_6 \vdash \Box F_{10}, \Delta_9, [\Box F_8 \quad ax/W}{\vdash : \Delta_{12}, F_6, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_9, [\Box F_8} \quad hCut \\
\vdash : \Delta_{12}, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_9, [\Box F_8 \quad ATG \\
\vdash : \Delta_{12}, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_9, [\Box F_8 \\
\frac{h_1 : F_6, (\Box \Gamma_{10}, \Delta_{12}), [\Box F_6 \vdash F_{11}, \Delta_9, [\Box F_8 \quad \bullet h_1 : (\Box \Gamma_{10}, \Delta_{12}), [\Box F_6 \vdash (\Delta_9, [\Box F_8]), F_{11}] \quad AT \quad \frac{h_7 : \Box \Gamma_{10}, [\Box F_6 \vdash F_8 \quad \bullet h_7 : ((\Box \Gamma_{10}, \Delta_{12}), [\Box F_6], F_{11} \vdash \Delta_9, [\Box F_8 \quad A4}{\vdash : (\Box \Gamma_{10}, \Delta_{12}), [\Box F_6 \vdash \Delta_9, [\Box F_8} \quad \text{Cut}} \\
\rightarrow \\
\frac{\vdash : \Box \Gamma_{10}, [\Box F_6 \vdash F_8 \quad ax/W}{\vdash : \Delta_{12}, \Box \Gamma_{10}, [\Box F_6 \vdash \Delta_9, [\Box F_8} \quad A4 \\
\vdash : \Delta_{12}, \Box \Gamma_{10}, [\Box F_6 \vdash \Delta_9, [\Box F_8 \\
\frac{h_1 : F_6, (\Box \Gamma_{11}, \Delta_{12}), [\Box F_6 \vdash \Box F_{10}, \Delta_9, [\Box F_8 \quad \bullet h_1 : (\Box \Gamma_{11}, \Delta_{12}), [\Box F_6 \vdash (\Delta_9, [\Box F_8]), \Box F_{10}] \quad AT \quad \frac{h_7 : \Box \Gamma_{11}, \Box F_{10} \vdash F_8 \quad \bullet h_7 : ((\Box \Gamma_{11}, \Delta_{12}), [\Box F_6], \Box F_{10} \vdash \Delta_9, [\Box F_8 \quad A4}{\vdash : (\Box \Gamma_{11}, \Delta_{12}), [\Box F_6 \vdash \Delta_9, [\Box F_8} \quad \text{Cut}} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6, \Box \Gamma_{11}, [\Box F_6 \vdash \Box F_{10}, \Delta_9, [\Box F_8 \quad \bullet h_1 : \Delta_{12}, F_6, \Box \Gamma_{11}, [\Box F_6 \vdash \Box F_{10}, \Delta_9, [\Box F_8 \quad ax/W}{\vdash : \Delta_{12}, F_6, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_9, [\Box F_8} \quad hCut \\
\vdash : \Delta_{12}, F_6, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_9, [\Box F_8 \quad AT \\
\vdash : \Delta_{12}, \Box \Gamma_{11}, [\Box F_6 \vdash \Delta_9, [\Box F_8 \\
\frac{h_1 : F_6, (\Box \Gamma_{10}, \Delta_{12}), [\Box F_6 \vdash F_{11}, \Delta_9, [\Box F_8 \quad \bullet h_1 : (\Box \Gamma_{10}, \Delta_{12}), [\Box F_6 \vdash (\Delta_9, [\Box F_8]), F_{11}] \quad AT \quad \frac{h_7 : \Box \Gamma_{10} \vdash F_8 \quad \bullet h_7 : ((\Box \Gamma_{10}, \Delta_{12}), [\Box F_6], F_{11} \vdash \Delta_9, [\Box F_8 \quad A4}{\vdash : (\Box \Gamma_{10}, \Delta_{12}), [\Box F_6 \vdash \Delta_9, [\Box F_8} \quad \text{Cut}} \\
\rightarrow \\
\frac{\vdash : \Box \Gamma_{10} \vdash F_8 \quad ax/W}{\vdash : \Delta_{12}, \Box \Gamma_{10}, [\Box F_6 \vdash \Delta_9, [\Box F_8} \quad A4 \\
\vdash : \Delta_{12}, \Box \Gamma_{10}, [\Box F_6 \vdash \Delta_9, [\Box F_8
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{h_1 : F_6, (\Delta_{12}, F_8 \wedge F_9), [F_6 \vdash F_{11}, \Delta_{10}] \quad AT \quad \frac{h_7 : F_8, F_9, F_{11}, \Delta_{12}, [F_6 \vdash \Delta_{10}] \quad \wedge_L}{\bullet h_7 : ((\Delta_{12}, F_8 \wedge F_9), [F_6]), F_{11} \vdash \Delta_{10}} \quad Cut}{- : (\Delta_{12}, F_8 \wedge F_9), [F_6 \vdash \Delta_{10}]} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6, [F_6, F_8 \wedge F_9 \vdash \Delta_{10}, F_{11}] \quad ax/W \quad \bullet h_7 : \Delta_{12}, F_{11}, F_6, [F_6, F_8 \wedge F_9 \vdash \Delta_{10}] \quad ax/W}{- : \Delta_{12}, F_6, [F_6, F_8 \wedge F_9 \vdash \Delta_{10}] \quad hCut} \quad AT}{- : \Delta_{12}, [F_6, F_8 \wedge F_9 \vdash \Delta_{10}]}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{11}, [F_6 \vdash F_8 \vee F_9, \Delta_{10}] \quad AT \quad \frac{h_7 : F_8, \Delta_{11}, [F_6 \vdash \Delta_{10}] \quad h_7 : F_9, \Delta_{11}, [F_6 \vdash \Delta_{10}] \quad \vee_L}{\bullet h_7 : (\Delta_{11}, [F_6]), F_8 \vee F_9 \vdash \Delta_{10}} \quad Cut}{- : \Delta_{11}, [F_6 \vdash \Delta_{10}]} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_6, [F_6 \vdash \Delta_{10}, F_8 \vee F_9] \quad ax/W \quad \bullet h_7 : \Delta_{11}, F_6, [F_6, F_8 \vee F_9 \vdash \Delta_{10}] \quad ax/W}{- : \Delta_{11}, F_6, [F_6 \vdash \Delta_{10}] \quad hCut} \quad AT}{- : \Delta_{11}, [F_6 \vdash \Delta_{10}]} \\
\\
\frac{h_1 : F_6, (\Delta_{12}, F_8 \vee F_9), [F_6 \vdash F_{11}, \Delta_{10}] \quad AT \quad \frac{h_7 : F_8, F_{11}, \Delta_{12}, [F_6 \vdash \Delta_{10}] \quad h_7 : F_9, F_{11}, \Delta_{12}, [F_6 \vdash \Delta_{10}] \quad \vee_L}{\bullet h_7 : ((\Delta_{12}, F_8 \vee F_9), [F_6]), F_{11} \vdash \Delta_{10}} \quad Cut}{- : (\Delta_{12}, F_8 \vee F_9), [F_6 \vdash \Delta_{10}]} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6, [F_6, F_8 \vee F_9 \vdash \Delta_{10}, F_{11}] \quad ax/W \quad \bullet h_7 : \Delta_{12}, F_{11}, F_6, [F_6, F_8 \vee F_9 \vdash \Delta_{10}] \quad ax/W}{- : \Delta_{12}, F_6, [F_6, F_8 \vee F_9 \vdash \Delta_{10}] \quad hCut} \quad AT}{- : \Delta_{12}, [F_6, F_8 \vee F_9 \vdash \Delta_{10}]}
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{h_1 : F_6, \Delta_{10}, [F_6 \vdash [F_8, \Delta_9]] \quad AT \quad \frac{h_7 : F_8, \Delta_{10}, [F_6, [F_8 \vdash \Delta_9]] \quad AT}{\bullet h_7 : (\Delta_{10}, [F_6]), [F_8 \vdash \Delta_9]} \quad Cut}{- : \Delta_{10}, [F_6 \vdash \Delta_9]} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, F_6, [F_6 \vdash \Delta_9, [F_8]] \quad ax/W \quad \bullet h_7 : \Delta_{10}, F_6, [F_6, [F_8 \vdash \Delta_9]] \quad ax/W}{- : \Delta_{10}, F_6, [F_6 \vdash \Delta_9] \quad hCut} \quad ATG}{- : \Delta_{10}, [F_6 \vdash \Delta_9]} \\
\\
\frac{h_1 : F_6, (\Delta_{11}, [F_8]), [F_6 \vdash F_{10}, \Delta_9] \quad AT \quad \frac{h_7 : F_8, F_{10}, \Delta_{11}, [F_6, [F_8 \vdash \Delta_9]] \quad AT}{\bullet h_7 : ((\Delta_{11}, [F_8]), [F_6]), F_{10} \vdash \Delta_9} \quad Cut}{- : (\Delta_{11}, [F_8]), [F_6 \vdash \Delta_9]} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_6, [F_6, [F_8 \vdash \Delta_9, F_{10}]] \quad ax/W \quad \bullet h_7 : \Delta_{11}, F_{10}, F_6, [F_6, [F_8 \vdash \Delta_9]] \quad ax/W}{- : \Delta_{11}, F_6, [F_6, [F_8 \vdash \Delta_9]] \quad hCut} \quad ATG}{- : \Delta_{11}, [F_6, [F_8 \vdash \Delta_9]]} \\
\\
\frac{h_1 : F_7, \Delta_{10}, [F_7 \vdash F_9, \Delta_8] \quad AT \quad \frac{h_6 : F_7, F_9, \Delta_{10}, [F_7 \vdash \Delta_8] \quad AT}{\bullet h_6 : (\Delta_{10}, [F_7]), F_9 \vdash \Delta_8} \quad Cut}{- : \Delta_{10}, [F_7 \vdash \Delta_8]} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, F_7, [F_7 \vdash \Delta_8, F_9] \quad ax/W \quad \bullet h_6 : \Delta_{10}, F_7, F_9, [F_7 \vdash \Delta_8] \quad ax/W}{- : \Delta_{10}, F_7, [F_7 \vdash \Delta_8] \quad hCut} \quad AT}{- : \Delta_{10}, [F_7 \vdash \Delta_8]}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : F_6, \Delta_9, \llbracket F_6 \vdash \perp, \Delta_8 \rrbracket}{\bullet h_1 : \Delta_9, \llbracket F_6 \vdash \Delta_8, \perp \rrbracket} AT \quad \frac{}{\bullet h_7 : (\Delta_9, \llbracket F_6 \rrbracket, \perp \vdash \Delta_8)} \perp_L}{\frac{}{- : \Delta_9, \llbracket F_6 \vdash \Delta_8 \rrbracket} \text{Cut}} \rightarrow \\
\frac{\frac{h_1 : \Delta_9, F_6, \llbracket F_6 \vdash \perp, \Delta_8 \rrbracket}{\frac{}{- : \Delta_9, F_6, \llbracket F_6 \vdash \Delta_8 \rrbracket} AT} \text{ax/W} \quad \frac{\bullet h_7 : \perp, \Delta_9, F_6, \llbracket F_6 \vdash \Delta_8 \rrbracket}{\frac{}{- : \Delta_9, \llbracket F_6 \vdash \Delta_8 \rrbracket} AT} \perp_L}{\frac{}{- : \Delta_9, \llbracket F_6 \vdash \Delta_8 \rrbracket} \text{hCut}} \\
\\
\frac{\frac{h_1 : F_6, (\perp, \Delta_{10}), \llbracket F_6 \vdash F_9, \Delta_8 \rrbracket}{\bullet h_1 : (\perp, \Delta_{10}), \llbracket F_6 \vdash \Delta_8, F_9 \rrbracket} AT \quad \frac{}{\bullet h_7 : ((\perp, \Delta_{10}), \llbracket F_6 \rrbracket, F_9 \vdash \Delta_8)} \perp_L}{\frac{}{- : (\perp, \Delta_{10}), \llbracket F_6 \vdash \Delta_8 \rrbracket} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_{10}, \llbracket F_6 \vdash \Delta_8 \rrbracket} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : F_6, \Delta_{10}, \llbracket F_6 \vdash p_9, \Delta_8, p_9 \rrbracket}{\bullet h_1 : \Delta_{10}, \llbracket F_6 \vdash (\Delta_8, p_9), p_9 \rrbracket} AT \quad \frac{}{\bullet h_7 : (\Delta_{10}, \llbracket F_6 \rrbracket, p_9 \vdash \Delta_8, p_9)} I}{\frac{}{- : \Delta_{10}, \llbracket F_6 \vdash \Delta_8, p_9 \rrbracket} \text{Cut}} \rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, F_6, \llbracket F_6 \vdash \Delta_8, p_9, p_9 \rrbracket}{\frac{}{- : \Delta_{10}, F_6, \llbracket F_6 \vdash \Delta_8, p_9 \rrbracket} AT} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{10}, F_6, p_9, \llbracket F_6 \vdash \Delta_8, p_9 \rrbracket}{\frac{}{- : \Delta_{10}, \llbracket F_6 \vdash \Delta_8, p_9 \rrbracket} AT} I}{\frac{}{- : \Delta_{10}, \llbracket F_6 \vdash \Delta_8, p_9 \rrbracket} \text{hCut}} \\
\\
\frac{\frac{h_1 : F_6, (\Delta_{11}, p_9), \llbracket F_6 \vdash F_{10}, \Delta_8, p_9 \rrbracket}{\bullet h_1 : (\Delta_{11}, p_9), \llbracket F_6 \vdash (\Delta_8, p_9), F_{10} \rrbracket} AT \quad \frac{}{\bullet h_7 : ((\Delta_{11}, p_9), \llbracket F_6 \rrbracket, F_{10} \vdash \Delta_8, p_9)} I}{\frac{}{- : (\Delta_{11}, p_9), \llbracket F_6 \vdash \Delta_8, p_9 \rrbracket} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_{11}, p_9, \llbracket F_6 \vdash \Delta_8, p_9 \rrbracket} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_1 : F_6, \Delta_9, \llbracket F_6 \vdash \top, \Delta_8 \rrbracket}{\bullet h_1 : \Delta_9, \llbracket F_6 \vdash \Delta_8, \top \rrbracket} AT \quad \frac{h_7 : \Delta_9, \llbracket F_6 \vdash \Delta_8 \rrbracket}{\bullet h_7 : (\Delta_9, \llbracket F_6 \rrbracket, \top \vdash \Delta_8)} \top_L}{\frac{}{- : \Delta_9, \llbracket F_6 \vdash \Delta_8 \rrbracket} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_9, \llbracket F_6 \vdash \Delta_8 \rrbracket} \text{ax/W} \\
\\
\frac{\frac{h_1 : F_6, (\top, \Delta_{10}), \llbracket F_6 \vdash F_9, \Delta_8 \rrbracket}{\bullet h_1 : (\top, \Delta_{10}), \llbracket F_6 \vdash \Delta_8, F_9 \rrbracket} AT \quad \frac{h_7 : F_9, \Delta_{10}, \llbracket F_6 \vdash \Delta_8 \rrbracket}{\bullet h_7 : ((\top, \Delta_{10}), \llbracket F_6 \rrbracket, F_9 \vdash \Delta_8)} \top_L}{\frac{}{- : (\top, \Delta_{10}), \llbracket F_6 \vdash \Delta_8 \rrbracket} \text{Cut}} \rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_{10}, \llbracket F_6 \vdash \Delta_8, F_9 \rrbracket}{\frac{}{- : \top, \Delta_{10}, \llbracket F_6 \vdash \Delta_8 \rrbracket} \text{ax/W}} \text{ax/W} \quad \frac{h_7 : \top, \Delta_{10}, F_9, \llbracket F_6 \vdash \Delta_8 \rrbracket}{\frac{}{- : \top, \Delta_{10}, \llbracket F_6 \vdash \Delta_8 \rrbracket} \text{hCut}} \text{hCut}
\end{array}$$

8.12 Status of \perp_L : OK

- Case rule \rightarrow_R

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_{10} \vdash (\Delta_8, F_6 \rightarrow F_7), F_9} \perp_L \quad \frac{h_5 : \perp, F_6, F_9, \Delta_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\perp, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \rightarrow F_7}}{\frac{}{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \perp_L$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_{10} \vdash (\Delta_8, F_6 \wedge F_7), F_9} \perp_L \quad \frac{h_5 : \perp, F_9, \Delta_{10} \vdash F_6, \Delta_8 \quad h_5 : \perp, F_9, \Delta_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\perp, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \wedge F_7} \wedge_R \\
\hline
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7} \perp_L
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_{10} \vdash (\Delta_8, F_6 \vee F_7), F_9} \perp_L \quad \frac{h_5 : \perp, F_9, \Delta_{10} \vdash F_6, F_7, \Delta_8}{\bullet h_5 : (\perp, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \vee F_7} \vee_R \\
\hline
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7} \perp_L
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_8 \vdash (\perp, \Delta_6), F_7} \perp_L \quad \frac{h_5 : \perp, F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : (\perp, \Delta_8), F_7 \vdash \perp, \Delta_6} \perp_R \\
\hline
\frac{}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \perp_L
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_8 \vdash (\top, \Delta_6), F_7} \perp_L \quad \frac{}{\bullet h_5 : (\perp, \Delta_8), F_7 \vdash \top, \Delta_6} \top_R \\
\hline
\frac{}{- : \perp, \Delta_8 \vdash \top, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_8 \vdash \top, \Delta_6} \top_R
\end{array}$$

- Case rule $A4$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, \Box F_6), \Box F_8} \perp_L \quad \frac{h_5 : \Box \Gamma_9, \Box F_8 \vdash F_6}{\bullet h_5 : (\perp, \Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \Delta_7, \Box F_6} A4 \\
\hline
\frac{}{- : \perp, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, \Box F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_6} \perp_L \\
\\
\frac{}{\bullet h_1 : \perp, \Box \Gamma_8, \Delta_{10} \vdash (\Delta_7, \Box F_6), F_9} \perp_L \quad \frac{h_5 : \Box \Gamma_8 \vdash F_6}{\bullet h_5 : (\perp, \Box \Gamma_8, \Delta_{10}), F_9 \vdash \Delta_7, \Box F_6} A4 \\
\hline
\frac{}{- : \perp, \Box \Gamma_8, \Delta_{10} \vdash \Delta_7, \Box F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10}, \Box \Gamma_8 \vdash \Delta_7, \Box F_6} \perp_L
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, \Box F_6), \Box F_8} \perp_L \quad \frac{h_5 : \text{unbox}(\Box \Gamma_9), \text{unbox}(\Box F_8) \vdash F_6}{\bullet h_5 : (\perp, \Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \Delta_7, \Box F_6} K \\
\hline
\frac{}{- : \perp, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, \Box F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_6} \perp_L \\
\\
\frac{}{\bullet h_1 : \perp, \Box \Gamma_8, \Delta_{10} \vdash (\Delta_7, \Box F_6), F_9} \perp_L \quad \frac{h_5 : \text{unbox}(\Box \Gamma_8) \vdash F_6}{\bullet h_5 : (\perp, \Box \Gamma_8, \Delta_{10}), F_9 \vdash \Delta_7, \Box F_6} K \\
\hline
\frac{}{- : \perp, \Box \Gamma_8, \Delta_{10} \vdash \Delta_7, \Box F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10}, \Box \Gamma_8 \vdash \Delta_7, \Box F_6} \perp_L
\end{array}$$

- Case rule \rightarrow_L

- Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_8 \vdash (\Delta_6, p_7), p_7}{\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7} \perp_L \quad \frac{\bullet h_5 : (\perp, \Delta_8), p_7 \vdash \Delta_6, p_7}{\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7} I}{\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_8 \vdash \Delta_6, p_7 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_9, p_7 \vdash (\Delta_6, p_7), F_8}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_6, p_7} \perp_L \quad \frac{\bullet h_5 : (\perp, \Delta_9, p_7), F_8 \vdash \Delta_6, p_7}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_6, p_7} I}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_6, p_7} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_9, p_7 \vdash \Delta_6, p_7 \quad \perp_L
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_7 \vdash \Delta_6, \top}{\vdash : \perp, \Delta_7 \vdash \Delta_6} \perp_L \quad \frac{\frac{h_5 : \perp, \Delta_7 \vdash \Delta_6}{\bullet h_5 : (\perp, \Delta_7), \top \vdash \Delta_6} \top_L}{\vdash : \perp, \Delta_7 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \Delta_7 \vdash \Delta_6 \quad \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \top, \Delta_8 \vdash \Delta_6, F_7}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_6} \perp_L \quad \frac{\frac{h_5 : \perp, F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : (\perp, \top, \Delta_8), F_7 \vdash \Delta_6} \top_L}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\vdash : \perp, \top, \Delta_8 \vdash \Delta_6 \quad \perp_L
\end{array}$$

8.13 Status of I : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \rightarrow F_7), p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} I \quad \frac{\frac{h_5 : F_6, \Delta_9, p_{10}, p_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \rightarrow_R}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, F_6, p_{10} \vdash \Delta_8, F_7, p_{10}}{\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8, F_7} I \quad \frac{h_5 : \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8, F_7}{\vdash : \Delta_9, F_6, p_{10} \vdash \Delta_8, F_7} \text{ax/w}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{hCut} \\
\rightarrow_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{12}, p_{10} \vdash ((\Delta_9, F_7 \rightarrow F_8), p_{10}), F_{11}}{\vdash : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \rightarrow F_8), p_{10}} I \quad \frac{\frac{h_6 : F_7, F_{11}, \Delta_{12}, p_{10} \vdash F_8, \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{12}, p_{10}), F_{11} \vdash (\Delta_9, F_7 \rightarrow F_8), p_{10}} \rightarrow_R}{\vdash : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \rightarrow F_8), p_{10}} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}, F_7 \rightarrow F_8 \quad I
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \wedge F_7), p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \wedge F_7} I \quad \frac{\frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash F_6, \Delta_8}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \wedge F_7} \wedge_R}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} \text{ax/w}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} \text{hCut} \\
\wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_7, p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_7} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_7}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_7} \text{ax/w}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_7} \text{hCut} \\
\wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{12}, p_{10} \vdash ((\Delta_9, F_7 \wedge F_8), p_{10}), F_{11}}{\vdash : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \wedge F_8), p_{10}} I \quad \frac{\frac{h_6 : F_{11}, \Delta_{12}, p_{10} \vdash F_7, \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{12}, p_{10}), F_{11} \vdash (\Delta_9, F_7 \wedge F_8), p_{10}} \wedge_R}{\vdash : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \wedge F_8), p_{10}} \text{Cut} \\
\rightarrow \\
\vdash : \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}, F_7 \wedge F_8 \quad I
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \vee F_7), p_{10}}{- : \Delta_9, p_{10} \vdash \Delta_8, F_6 \vee F_7} I \quad \frac{\frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash F_6, F_7, \Delta_8}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \vee F_7} \vee_R}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7, p_{10}}{- : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6, F_7}{\bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \vee F_7} \text{ax/W}}{- : \Delta_9, p_{10} \vdash \Delta_8, F_6 \vee F_7} \text{hCut} \\
\vee_R \\
\frac{\frac{\bullet h_1 : \Delta_{12}, p_{10} \vdash ((\Delta_9, F_7 \vee F_8), p_{10}), F_{11}}{- : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10}} I \quad \frac{\frac{h_6 : F_{11}, \Delta_{12}, p_{10} \vdash F_7, F_8, \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{12}, p_{10}), F_{11} \vdash (\Delta_9, F_7 \vee F_8), p_{10}} \vee_R}{\text{Cut}} \\
\rightarrow \\
\frac{- : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10}}{- : \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}, F_7 \vee F_8} I
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash (\perp, \Delta_6), p_8}{- : \Delta_7, p_8 \vdash \perp, \Delta_6} I \quad \frac{\frac{h_5 : \Delta_7, p_8, p_8 \vdash \Delta_6}{\bullet h_5 : (\Delta_7, p_8), p_8 \vdash \perp, \Delta_6} \perp_R}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash \perp, \Delta_6, p_8}{- : \Delta_7, p_8 \vdash \perp, \Delta_6} I \quad \frac{h_5 : \Delta_7, p_8, p_8 \vdash \perp, \Delta_6}{\bullet h_5 : (\Delta_7, p_8), p_8 \vdash \perp, \Delta_6} \text{ax/W}}{- : \Delta_7, p_8 \vdash \perp, \Delta_6} \text{hCut} \\
\perp_R \\
\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash ((\perp, \Delta_7), p_8), F_9}{- : \Delta_{10}, p_8 \vdash (\perp, \Delta_7), p_8} I \quad \frac{\frac{h_6 : F_9, \Delta_{10}, p_8 \vdash \Delta_7, p_8}{\bullet h_6 : (\Delta_{10}, p_8), F_9 \vdash (\perp, \Delta_7), p_8} \perp_R}{\text{Cut}} \\
\rightarrow \\
\frac{- : \Delta_{10}, p_8 \vdash (\perp, \Delta_7), p_8}{- : \Delta_{10}, p_8 \vdash \perp, \Delta_7, p_8} I
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash (\top, \Delta_6), p_8}{- : \Delta_7, p_8 \vdash \top, \Delta_6} I \quad \frac{\frac{h_5 : (\Delta_7, p_8), p_8 \vdash \top, \Delta_6}{\bullet h_5 : (\Delta_7, p_8), p_8 \vdash \top, \Delta_6} \top_R}{\text{Cut}} \\
\rightarrow \\
\frac{- : \Delta_7, p_8 \vdash \top, \Delta_6}{- : \Delta_7, p_8 \vdash \top, \Delta_6} \top_R \\
\top_R \\
\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash ((\top, \Delta_7), p_8), F_9}{- : \Delta_{10}, p_8 \vdash (\top, \Delta_7), p_8} I \quad \frac{\frac{h_6 : (\Delta_{10}, p_8), F_9 \vdash (\top, \Delta_7), p_8}{\bullet h_6 : (\Delta_{10}, p_8), F_9 \vdash (\top, \Delta_7), p_8} \top_R}{\text{Cut}} \\
\rightarrow \\
\frac{- : \Delta_{10}, p_8 \vdash (\top, \Delta_7), p_8}{- : \Delta_{10}, p_8 \vdash \top, \Delta_7, p_8} \top_R
\end{array}$$

- Case rule A4

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Box \Gamma_8, \Delta_9), p_{10} \vdash (\Delta_7, \Box F_6), p_{10}}{- : (\Box \Gamma_8, \Delta_9), p_{10} \vdash \Delta_7, \Box F_6} I \quad \frac{\frac{h_5 : \Box \Gamma_8 \vdash F_6}{\bullet h_5 : ((\Box \Gamma_8, \Delta_9), p_{10}), p_{10} \vdash \Delta_7, \Box F_6} A4}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{- : \Box \Gamma_8 \vdash F_6}{- : \Delta_9, \Box \Gamma_8, p_{10} \vdash \Delta_7, \Box F_6} \text{ax/W}}{- : \Delta_9, \Box \Gamma_8, p_{10} \vdash \Delta_7, \Box F_6} A4 \\
\Box \\
\frac{\frac{\bullet h_1 : (\Box \Gamma_{11}, \Delta_{12}), p_9 \vdash ((\Delta_8, \Box F_7), p_9), \Box F_{10}}{- : (\Box \Gamma_{11}, \Delta_{12}), p_9 \vdash (\Delta_8, \Box F_7), p_9} I \quad \frac{\frac{h_6 : \Box \Gamma_{11}, \Box F_{10} \vdash F_7}{\bullet h_6 : ((\Box \Gamma_{11}, \Delta_{12}), p_9), \Box F_{10} \vdash (\Delta_8, \Box F_7), p_9} A4}{\text{Cut}} \\
\rightarrow \\
\frac{- : (\Box \Gamma_{11}, \Delta_{12}), p_9 \vdash (\Delta_8, \Box F_7), p_9}{- : \Delta_{12}, \Box \Gamma_{11}, p_9 \vdash \Delta_8, p_9, \Box F_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \llbracket F_7 \rrbracket), p_9), F_{11}}{\vdash : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \llbracket F_7 \rrbracket), p_9)} I \quad \frac{h_6 : \Box \Gamma_{10} \vdash F_7}{\bullet h_6 : ((\Box \Gamma_{10}, \Delta_{12}), p_9), F_{11} \vdash (\Delta_8, \llbracket F_7 \rrbracket), p_9} A4 \\
\vdash : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \llbracket F_7 \rrbracket), p_9) \xrightarrow{\text{Cut}} \\
\vdash : \Delta_{12}, \Box \Gamma_{10}, p_9 \vdash \Delta_8, p_9, \llbracket F_7 \rrbracket I
\end{array}$$

- Case rule K

$$\begin{array}{c}
\frac{\bullet h_1 : (\Box \Gamma_8, \Delta_9), p_{10} \vdash (\Delta_7, \llbracket F_6 \rrbracket), p_{10}}{\vdash : (\Box \Gamma_8, \Delta_9), p_{10} \vdash (\Delta_7, \llbracket F_6 \rrbracket)} I \quad \frac{h_5 : unbox(\Box \Gamma_8) \vdash F_6}{\bullet h_5 : ((\Box \Gamma_8, \Delta_9), p_{10}), p_{10} \vdash \Delta_7, \llbracket F_6 \rrbracket} K \\
\vdash : (\Box \Gamma_8, \Delta_9), p_{10} \vdash (\Delta_7, \llbracket F_6 \rrbracket) \xrightarrow{\text{Cut}} \\
\vdash : unbox(\Box \Gamma_8) \vdash F_6 \text{ ax/W} \\
\vdash : \Delta_9, \Box \Gamma_8, p_{10} \vdash \Delta_7, \llbracket F_6 \rrbracket K \\
\\
\frac{\bullet h_1 : (\Box \Gamma_{11}, \Delta_{12}), p_9 \vdash ((\Delta_8, \llbracket F_7 \rrbracket), p_9), \Box F_{10}}{\vdash : (\Box \Gamma_{11}, \Delta_{12}), p_9 \vdash ((\Delta_8, \llbracket F_7 \rrbracket), p_9), \Box F_{10}} I \quad \frac{h_6 : unbox(\Box \Gamma_{11}), unbox(\Box F_{10}) \vdash F_7}{\bullet h_6 : ((\Box \Gamma_{11}, \Delta_{12}), p_9), \Box F_{10} \vdash (\Delta_8, \llbracket F_7 \rrbracket), p_9} K \\
\vdash : (\Box \Gamma_{11}, \Delta_{12}), p_9 \vdash ((\Delta_8, \llbracket F_7 \rrbracket), p_9), \Box F_{10} \xrightarrow{\text{Cut}} \\
\vdash : \Delta_{12}, \Box \Gamma_{11}, p_9 \vdash \Delta_8, p_9, \llbracket F_7 \rrbracket I \\
\\
\frac{\bullet h_1 : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \llbracket F_7 \rrbracket), p_9), F_{11}}{\vdash : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \llbracket F_7 \rrbracket), p_9), F_{11}} I \quad \frac{h_6 : unbox(\Box \Gamma_{10}) \vdash F_7}{\bullet h_6 : ((\Box \Gamma_{10}, \Delta_{12}), p_9), F_{11} \vdash (\Delta_8, \llbracket F_7 \rrbracket), p_9} K \\
\vdash : (\Box \Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, \llbracket F_7 \rrbracket), p_9), F_{11} \xrightarrow{\text{Cut}} \\
\vdash : \Delta_{12}, \Box \Gamma_{10}, p_9 \vdash \Delta_8, p_9, \llbracket F_7 \rrbracket I
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8, p_{10}}{\vdash : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8, p_{10}} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash F_6, \Delta_8 \quad h_5 : F_7, \Delta_9, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_5 : ((\Delta_9, F_6 \rightarrow F_7), p_{10}), p_{10} \vdash \Delta_8} \rightarrow_L \\
\vdash : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash \Delta_8, p_{10} \xrightarrow{\text{Cut}} \\
\vdash : \Delta_9, p_{10}, F_6 \rightarrow F_7 \vdash \Delta_8 \\
\\
\frac{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, p_{10}}{\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6} I \quad \frac{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6}{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, p_{10}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10}}{\vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10}} I \quad \frac{h_5 : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8}{\vdash : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8} \text{hCut} \\
\vdash : \Delta_9, p_{10} \vdash \Delta_8, F_6 \xrightarrow{\text{hCut}} \vdash : \Delta_9, F_7, p_{10} \vdash \Delta_8 \rightarrow_L \\
\vdash : \Delta_9, p_{10}, F_6 \rightarrow F_7 \vdash \Delta_8 \\
\\
\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \rightarrow F_8}{\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \rightarrow F_8} I \quad \frac{h_6 : \Delta_{11}, p_{10} \vdash F_7, \Delta_9, p_{10} \quad h_6 : F_8, \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \rightarrow F_8 \vdash \Delta_9, p_{10}} \rightarrow_L \\
\vdash : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \rightarrow F_8 \xrightarrow{\text{Cut}} \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \\
\vdash : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} I \\
\\
\frac{\bullet h_1 : (\Delta_{12}, F_7 \rightarrow F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}}{\vdash : (\Delta_{12}, F_7 \rightarrow F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}} I \quad \frac{h_6 : F_{11}, \Delta_{12}, p_{10} \vdash F_7, \Delta_9, p_{10} \quad h_6 : F_8, F_{11}, \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : ((\Delta_{12}, F_7 \rightarrow F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}} \rightarrow_L \\
\vdash : (\Delta_{12}, F_7 \rightarrow F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11} \xrightarrow{\text{Cut}} \\
\vdash : \Delta_{12}, p_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, p_{10} I
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, F_6 \wedge F_7), p_{10} \vdash \Delta_8, p_{10}}{\vdash : (\Delta_9, F_6 \wedge F_7), p_{10} \vdash \Delta_8, p_{10}} I \quad \frac{h_5 : F_6, F_7, \Delta_9, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_5 : ((\Delta_9, F_6 \wedge F_7), p_{10}), p_{10} \vdash \Delta_8} \wedge_L \\
\vdash : (\Delta_9, F_6 \wedge F_7), p_{10} \vdash \Delta_8, p_{10} \xrightarrow{\text{Cut}} \\
\vdash : \Delta_9, p_{10}, F_6 \wedge F_7 \vdash \Delta_8 \\
\\
\frac{\bullet h_1 : \Delta_9, F_6, F_7, p_{10} \vdash \Delta_8, p_{10}}{\vdash : \Delta_9, F_6, F_7, p_{10} \vdash \Delta_8, p_{10}} I \quad \frac{h_5 : \Delta_9, F_6, F_7, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_1 : \Delta_9, F_6, F_7, p_{10} \vdash \Delta_8, p_{10}} \text{ax/W} \\
\vdash : \Delta_9, F_6, F_7, p_{10} \vdash \Delta_8, p_{10} \xrightarrow{\text{hCut}} \vdash : \Delta_9, p_{10}, F_6 \wedge F_7 \vdash \Delta_8 \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \wedge F_8}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I \quad \frac{h_6 : F_7, F_8, \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \wedge F_8 \vdash \Delta_9, p_{10}} \wedge_L}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_{12}, F_7 \wedge F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}}{- : (\Delta_{12}, F_7 \wedge F_8), p_{10} \vdash \Delta_9, p_{10}} I \quad \frac{h_6 : F_7, F_8, F_{11}, \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : ((\Delta_{12}, F_7 \wedge F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}} \wedge_L}{- : (\Delta_{12}, F_7 \wedge F_8), p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{12}, p_{10}, F_7 \wedge F_8 \vdash \Delta_9, p_{10}} I
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8, p_{10}}{- : (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8} I \quad \frac{h_5 : F_6, \Delta_9, p_{10}, p_{10} \vdash \Delta_8 \quad h_5 : F_7, \Delta_9, p_{10}, p_{10} \vdash \Delta_8}{\bullet h_5 : ((\Delta_9, F_6 \vee F_7), p_{10}), p_{10} \vdash \Delta_8} \vee_L}{- : (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_9, F_6, p_{10} \vdash \Delta_8, p_{10}}{- : \Delta_9, F_6, p_{10} \vdash \Delta_8} I \quad \frac{h_5 : \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8}{- : \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8} \text{ax/w}}{- : \Delta_9, F_6, p_{10} \vdash \Delta_8} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10}}{- : \Delta_9, F_7, p_{10} \vdash \Delta_8} I \quad \frac{h_5 : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8}{- : \Delta_9, F_7, p_{10}, p_{10} \vdash \Delta_8} \text{ax/w}}{- : \Delta_9, F_7, p_{10} \vdash \Delta_8} \text{hCut} \\
\vee_L \\
- : \Delta_9, p_{10}, F_6 \vee F_7 \vdash \Delta_8
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \vee F_8}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I \quad \frac{h_6 : F_7, \Delta_{11}, p_{10} \vdash \Delta_9, p_{10} \quad h_6 : F_8, \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \vee F_8 \vdash \Delta_9, p_{10}} \vee_L}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_{12}, F_7 \vee F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}}{- : (\Delta_{12}, F_7 \vee F_8), p_{10} \vdash \Delta_9, p_{10}} I \quad \frac{h_6 : F_7, F_{11}, \Delta_{12}, p_{10} \vdash \Delta_9, p_{10} \quad h_6 : F_8, F_{11}, \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}}{\bullet h_6 : ((\Delta_{12}, F_7 \vee F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}} \vee_L}{- : (\Delta_{12}, F_7 \vee F_8), p_{10} \vdash \Delta_9, p_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{12}, p_{10}, F_7 \vee F_8 \vdash \Delta_9, p_{10}} I
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_8, \llbracket F_6 \rrbracket, p_9 \vdash \Delta_7, p_9}{- : (\Delta_8, \llbracket F_6 \rrbracket, p_9 \vdash \Delta_7} I \quad \frac{h_5 : F_6, \Delta_8, p_9, p_9, \llbracket F_6 \rrbracket \vdash \Delta_7}{\bullet h_5 : ((\Delta_8, \llbracket F_6 \rrbracket, p_9), p_9) \vdash \Delta_7} AT}{- : (\Delta_8, \llbracket F_6 \rrbracket, p_9 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_6, p_9, \llbracket F_6 \rrbracket \vdash \Delta_7, p_9}{- : \Delta_8, F_6, p_9, \llbracket F_6 \rrbracket \vdash \Delta_7} \text{ax/w} \quad \frac{h_5 : \Delta_8, F_6, p_9, p_9, \llbracket F_6 \rrbracket \vdash \Delta_7}{- : \Delta_8, F_6, p_9, p_9, \llbracket F_6 \rrbracket \vdash \Delta_7} \text{ax/w}}{- : \Delta_8, F_6, p_9, \llbracket F_6 \rrbracket \vdash \Delta_7} \text{hCut} \\
ATG \\
- : \Delta_8, p_9, \llbracket F_6 \rrbracket \vdash \Delta_7
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_{10}, p_9 \vdash (\Delta_8, p_9), \llbracket F_7 \rrbracket}{- : \Delta_{10}, p_9 \vdash \Delta_8, p_9} I \quad \frac{h_6 : F_7, \Delta_{10}, p_9, \llbracket F_7 \rrbracket \vdash \Delta_8, p_9}{\bullet h_6 : (\Delta_{10}, p_9), \llbracket F_7 \rrbracket \vdash \Delta_8, p_9} AT}{- : \Delta_{10}, p_9 \vdash \Delta_8, p_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{10}, p_9 \vdash \Delta_8, p_9} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\Delta_{11}, \llbracket F_7 \rrbracket, p_9 \vdash (\Delta_8, p_9), F_{10}}{- : (\Delta_{11}, \llbracket F_7 \rrbracket, p_9 \vdash \Delta_8, p_9} I \quad \frac{h_6 : F_7, F_{10}, \Delta_{11}, p_9, \llbracket F_7 \rrbracket \vdash \Delta_8, p_9}{\bullet h_6 : ((\Delta_{11}, \llbracket F_7 \rrbracket, p_9), F_{10}) \vdash \Delta_8, p_9} AT}{- : (\Delta_{11}, \llbracket F_7 \rrbracket, p_9 \vdash \Delta_8, p_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_9, \llbracket F_7 \rrbracket \vdash \Delta_8, p_9} I
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\perp, \Delta_7), p_8 \vdash \Delta_6, p_8}{- : (\perp, \Delta_7), p_8 \vdash \Delta_6} I \quad \frac{h_5 : ((\perp, \Delta_7), p_8), p_8 \vdash \Delta_6}{\bullet h_5 : ((\perp, \Delta_7), p_8), p_8 \vdash \Delta_6} \perp_L}{- : (\perp, \Delta_7), p_8 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_7, p_8 \vdash \Delta_6} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_9, p_8 \vdash (\Delta_7, p_8), \perp}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I \quad \frac{\bullet h_6 : (\Delta_9, p_8), \perp \vdash \Delta_7, p_8}{\perp_L} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I \\
\frac{\frac{\bullet h_1 : (\perp, \Delta_{10}), p_8 \vdash (\Delta_7, p_8), F_9}{- : (\perp, \Delta_{10}), p_8 \vdash \Delta_7, p_8} I \quad \frac{\bullet h_6 : ((\perp, \Delta_{10}), p_8), F_9 \vdash \Delta_7, p_8}{\perp_L} \text{Cut}}{\rightarrow} \\
\frac{}{- : \perp, \Delta_{10}, p_8 \vdash \Delta_7, p_8} \perp_L
\end{array}$$

• Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_8, p_7 \vdash (\Delta_6, p_7), p_7}{- : \Delta_8, p_7 \vdash \Delta_6, p_7} I \quad \frac{\bullet h_5 : (\Delta_8, p_7), p_7 \vdash \Delta_6, p_7}{\text{Cut}} I}{\rightarrow} \\
\frac{}{- : \Delta_8, p_7 \vdash \Delta_6, p_7} I \\
\frac{\frac{\bullet h_1 : (\Delta_8, p_7), p_9 \vdash (\Delta_6, p_7), p_9}{- : (\Delta_8, p_7), p_9 \vdash \Delta_6, p_7} I \quad \frac{\bullet h_5 : ((\Delta_8, p_7), p_9), p_9 \vdash \Delta_6, p_7}{\text{Cut}} I}{\rightarrow} \\
\frac{}{- : \Delta_8, p_7, p_9 \vdash \Delta_6, p_7} I \\
\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash ((\Delta_7, p_9), p_8), p_9}{- : \Delta_{10}, p_8 \vdash (\Delta_7, p_9), p_8} I \quad \frac{\bullet h_6 : (\Delta_{10}, p_8), p_9 \vdash (\Delta_7, p_9), p_8}{\text{Cut}} I}{\rightarrow} \\
\frac{}{- : \Delta_{10}, p_8 \vdash \Delta_7, p_8, p_9} I \\
\frac{\frac{\bullet h_1 : (\Delta_{11}, p_9), p_8 \vdash ((\Delta_7, p_9), p_8), F_{10}}{- : (\Delta_{11}, p_9), p_8 \vdash (\Delta_7, p_9), p_8} I \quad \frac{\bullet h_6 : ((\Delta_{11}, p_9), p_8), F_{10} \vdash (\Delta_7, p_9), p_8}{\text{Cut}} I}{\rightarrow} \\
\frac{}{- : \Delta_{11}, p_8, p_9 \vdash \Delta_7, p_8, p_9} I \\
\frac{\frac{\bullet h_1 : \Delta_{10}, p_8 \vdash (\Delta_7, p_8), F_9}{- : \Delta_{10}, p_8 \vdash \Delta_7, p_8} I \quad \frac{\bullet h_6 : (\Delta_{10}, p_8), F_9 \vdash \Delta_7, p_8}{\text{Cut}} I}{\rightarrow} \\
\frac{}{- : \Delta_{10}, p_8 \vdash \Delta_7, p_8} I
\end{array}$$

• Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : (\top, \Delta_7), p_8 \vdash \Delta_6, p_8}{- : (\top, \Delta_7), p_8 \vdash \Delta_6} I \quad \frac{\frac{h_5 : \Delta_7, p_8, p_8 \vdash \Delta_6}{\bullet h_5 : ((\top, \Delta_7), p_8), p_8 \vdash \Delta_6} \top_L}{\text{Cut}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \top, \Delta_7, p_8 \vdash \Delta_6, p_8}{- : \top, \Delta_7, p_8 \vdash \Delta_6} I \quad \frac{h_5 : \top, \Delta_7, p_8, p_8 \vdash \Delta_6}{\text{hCut}} \text{ax/w}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \Delta_9, p_8 \vdash (\Delta_7, p_8), \top}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I \quad \frac{\frac{h_6 : \Delta_9, p_8 \vdash \Delta_7, p_8}{\bullet h_6 : (\Delta_9, p_8), \top \vdash \Delta_7, p_8} \top_L}{\text{Cut}} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_9, p_8 \vdash \Delta_7, p_8} I \\
\frac{\frac{\bullet h_1 : (\top, \Delta_{10}), p_8 \vdash (\Delta_7, p_8), F_9}{- : (\top, \Delta_{10}), p_8 \vdash \Delta_7, p_8} I \quad \frac{\frac{h_6 : F_9, \Delta_{10}, p_8 \vdash \Delta_7, p_8}{\bullet h_6 : ((\top, \Delta_{10}), p_8), F_9 \vdash \Delta_7, p_8} \top_L}{\text{Cut}} \text{Cut}}{\rightarrow} \\
\frac{}{- : \top, \Delta_{10}, p_8 \vdash \Delta_7, p_8} I
\end{array}$$

8.14 Status of \top_L : OK

- Case rule \rightarrow_R

$$\frac{\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \rightarrow F_7}{\bullet h_1 : \top, \Delta_{10} \vdash (\Delta_8, F_6 \rightarrow F_7), F_9} \top_L \quad \frac{h_5 : \top, F_6, F_9, \Delta_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\top, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \rightarrow F_7} \rightarrow_R}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_{10} \vdash \Delta_8, F_9, F_6 \rightarrow F_7}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9 \vdash \Delta_8, F_6 \rightarrow F_7}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \text{hCut}$$

- Case rule \wedge_R

$$\frac{\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \wedge F_7}{\bullet h_1 : \top, \Delta_{10} \vdash (\Delta_8, F_6 \wedge F_7), F_9} \top_L \quad \frac{h_5 : \top, F_9, \Delta_{10} \vdash F_6, \Delta_8 \quad h_5 : \top, F_9, \Delta_{10} \vdash F_7, \Delta_8}{\bullet h_5 : (\top, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \wedge F_7} \wedge_R}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_{10} \vdash \Delta_8, F_9, F_6 \wedge F_7}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9 \vdash \Delta_8, F_6 \wedge F_7}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \wedge F_7} \text{hCut}$$

- Case rule \vee_R

$$\frac{\frac{h_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \vee F_7}{\bullet h_1 : \top, \Delta_{10} \vdash (\Delta_8, F_6 \vee F_7), F_9} \top_L \quad \frac{h_5 : \top, F_9, \Delta_{10} \vdash F_6, F_7, \Delta_8}{\bullet h_5 : (\top, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \vee F_7} \vee_R}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_{10} \vdash \Delta_8, F_9, F_6 \vee F_7}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9 \vdash \Delta_8, F_6 \vee F_7}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7} \text{hCut}$$

- Case rule \perp_R

$$\frac{\frac{h_1 : \Delta_8 \vdash F_7, \perp, \Delta_6}{\bullet h_1 : \top, \Delta_8 \vdash (\perp, \Delta_6), F_7} \top_L \quad \frac{h_5 : \top, F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \perp, \Delta_6} \perp_R}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_8 \vdash \perp, \Delta_6, F_7}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, F_7 \vdash \perp, \Delta_6}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{ax/W}}{- : \top, \Delta_8 \vdash \perp, \Delta_6} \text{hCut}$$

- Case rule \top_R

$$\frac{\frac{h_1 : \Delta_8 \vdash F_7, \top, \Delta_6}{\bullet h_1 : \top, \Delta_8 \vdash (\top, \Delta_6), F_7} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \top, \Delta_6} \top_R}{- : \top, \Delta_8 \vdash \top, \Delta_6} \text{Cut}$$

$$\xrightarrow{\quad} \frac{}{- : \top, \Delta_8 \vdash \top, \Delta_6} \top_R$$

- Case rule A4

$$\frac{\frac{h_1 : \Box \Gamma_9, \Delta_{10} \vdash \Box F_8, \Delta_7, \Box F_6}{\bullet h_1 : \top, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, \Box F_6), \Box F_8} \top_L \quad \frac{h_5 : \Box \Gamma_9, \Box F_8 \vdash F_6}{\bullet h_5 : (\top, \Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \Delta_7, \Box F_6} A4}{- : \top, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, \Box F_6} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{h_1 : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Box F_8, \Delta_7, \Box F_6}{- : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Box \Gamma_8, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_6}{- : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_6} \text{ax/W}}{- : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, \Box F_6} \text{hCut}$$

$$\frac{\frac{h_1 : \Box \Gamma_8, \Delta_{10} \vdash F_9, \Delta_7, \Box F_6}{\bullet h_1 : \top, \Box \Gamma_8, \Delta_{10} \vdash (\Delta_7, \Box F_6), F_9} \top_L \quad \frac{h_5 : \Box \Gamma_8 \vdash F_6}{\bullet h_5 : (\top, \Box \Gamma_8, \Delta_{10}), F_9 \vdash \Delta_7, \Box F_6} A4}{- : \top, \Box \Gamma_8, \Delta_{10} \vdash \Delta_7, \Box F_6} \text{Cut}$$

$$\xrightarrow{\quad} \frac{}{- : \Box \Gamma_8 \vdash F_6} \text{ax/W}$$

$$\frac{}{- : \top, \Delta_{10}, \Box \Gamma_8 \vdash \Delta_7, \Box F_6} A4$$

- Case rule K

$$\begin{array}{c}
\frac{\frac{h_1 : \Box \Gamma_9, \Delta_{10} \vdash \Box F_8, \Delta_7, [] F_6}{\bullet h_1 : \top, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, [] F_6), \Box F_8} \top_L \quad \frac{h_5 : \text{unbox}(\Box \Gamma_9), \text{unbox}(\Box F_8) \vdash F_6}{\bullet h_5 : (\top, \Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \Delta_7, [] F_6} K}{- : \top, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, [] F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Box F_8, \Delta_7, [] F_6}{- : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [] F_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Box F_8, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [] F_6}{- : \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [] F_6} \text{hCut}}{\rightarrow} \\
\frac{\frac{h_1 : \Box \Gamma_8, \Delta_{10} \vdash F_9, \Delta_7, [] F_6}{\bullet h_1 : \top, \Box \Gamma_8, \Delta_{10} \vdash (\Delta_7, [] F_6), F_9} \top_L \quad \frac{h_5 : \text{unbox}(\Box \Gamma_8) \vdash F_6}{\bullet h_5 : (\top, \Box \Gamma_8, \Delta_{10}), F_9 \vdash \Delta_7, [] F_6} K}{- : \top, \Box \Gamma_8, \Delta_{10} \vdash \Delta_7, [] F_6} \text{Cut} \\
\rightarrow \\
\frac{- : \text{unbox}(\Box \Gamma_8) \vdash F_6}{- : \top, \Delta_{10}, \Box \Gamma_8 \vdash \Delta_7, [] F_6} \text{ax/W} \quad K
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9 \vdash F_6 \rightarrow F_7, \Delta_8}{\bullet h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7} \top_L \quad \frac{h_5 : \top, \Delta_9 \vdash F_6, \Delta_8 \quad h_5 : \top, F_7, \Delta_9 \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9), F_6 \rightarrow F_7 \vdash \Delta_8} \rightarrow_L}{- : \top, \Delta_9 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \rightarrow F_7}{- : \top, \Delta_9 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_6 \rightarrow F_7 \vdash \Delta_8}{- : \top, \Delta_9 \vdash \Delta_8} \text{hCut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8, F_9} \top_L \quad \frac{h_5 : \top, F_9, \Delta_{10} \vdash F_6, \Delta_8 \quad h_5 : \top, F_7, F_9, \Delta_{10} \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \Delta_8} \rightarrow_L}{- : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8, F_9}{- : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_8}{- : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_8} \text{hCut}}{\rightarrow}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9 \vdash F_6 \wedge F_7, \Delta_8}{\bullet h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7} \top_L \quad \frac{h_5 : \top, F_6, F_7, \Delta_9 \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9), F_6 \wedge F_7 \vdash \Delta_8} \wedge_L}{- : \top, \Delta_9 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \wedge F_7}{- : \top, \Delta_9 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_6 \wedge F_7 \vdash \Delta_8}{- : \top, \Delta_9 \vdash \Delta_8} \text{hCut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8, F_9} \top_L \quad \frac{h_5 : \top, F_6, F_7, F_9, \Delta_{10} \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_{10}, F_6 \wedge F_7), F_9 \vdash \Delta_8} \wedge_L}{- : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8, F_9}{- : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9, F_6 \wedge F_7 \vdash \Delta_8}{- : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8} \text{hCut}}{\rightarrow}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9 \vdash F_6 \vee F_7, \Delta_8}{\bullet h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \vee F_7} \top_L \quad \frac{h_5 : \top, F_6, \Delta_9 \vdash \Delta_8 \quad h_5 : \top, F_7, \Delta_9 \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_9), F_6 \vee F_7 \vdash \Delta_8} \vee_L}{- : \top, \Delta_9 \vdash \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9 \vdash \Delta_8, F_6 \vee F_7}{- : \top, \Delta_9 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_6 \vee F_7 \vdash \Delta_8}{- : \top, \Delta_9 \vdash \Delta_8} \text{hCut}}{\rightarrow}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_6 \vee F_7 \vdash F_9, \Delta_8}{\bullet h_1 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8, F_9} \top_L \quad \frac{h_5 : \top, F_6, F_9, \Delta_{10} \vdash \Delta_8 \quad h_5 : \top, F_7, F_9, \Delta_{10} \vdash \Delta_8}{\bullet h_5 : (\top, \Delta_{10}, F_6 \vee F_7), F_9 \vdash \Delta_8} \vee_L \\
\hline
- : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8, F_9}{- : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_{10}, F_9, F_6 \vee F_7 \vdash \Delta_8}{- : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_8} \text{hCut}
\end{array}$$

- Case rule AT

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash \boxed{F_6}, \Delta_7}{\bullet h_1 : \top, \Delta_8 \vdash \Delta_7, \boxed{F_6}} \top_L \quad \frac{h_5 : \top, F_6, \Delta_8, \boxed{F_6} \vdash \Delta_7}{\bullet h_5 : (\top, \Delta_8), \boxed{F_6} \vdash \Delta_7} AT \\
\hline
- : \top, \Delta_8 \vdash \Delta_7 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \top, \Delta_8 \vdash \Delta_7, \boxed{F_6}}{- : \top, \Delta_8 \vdash \Delta_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, \boxed{F_6} \vdash \Delta_7}{- : \top, \Delta_8 \vdash \Delta_7} \text{hCut} \\
\hline
\frac{h_1 : \Delta_9, \boxed{F_6} \vdash F_8, \Delta_7}{\bullet h_1 : \top, \Delta_9, \boxed{F_6} \vdash \Delta_7, F_8} \top_L \quad \frac{h_5 : \top, F_6, F_8, \Delta_9, \boxed{F_6} \vdash \Delta_7}{\bullet h_5 : (\top, \Delta_9, \boxed{F_6}), F_8 \vdash \Delta_7} AT \\
\hline
- : \top, \Delta_9, \boxed{F_6} \vdash \Delta_7 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \top, \Delta_9, \boxed{F_6} \vdash \Delta_7, F_8}{- : \top, \Delta_9, \boxed{F_6} \vdash \Delta_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_8, \boxed{F_6} \vdash \Delta_7}{- : \top, \Delta_9, \boxed{F_6} \vdash \Delta_7} \text{hCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_1 : \Delta_7 \vdash \perp, \Delta_6}{\bullet h_1 : \top, \Delta_7 \vdash \Delta_6, \perp} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_7), \perp \vdash \Delta_6} \perp_L \\
\hline
- : \top, \Delta_7 \vdash \Delta_6 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \top, \Delta_7 \vdash \perp, \Delta_6}{- : \top, \Delta_7 \vdash \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \perp, \top, \Delta_7 \vdash \Delta_6}{- : \top, \Delta_7 \vdash \Delta_6} \perp_L \\
\hline
\frac{h_1 : \perp, \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \top, \perp, \Delta_8 \vdash \Delta_6, F_7} \top_L \quad \frac{}{\bullet h_5 : (\top, \perp, \Delta_8), F_7 \vdash \Delta_6} \perp_L \\
\hline
- : \top, \perp, \Delta_8 \vdash \Delta_6 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \perp, \top, \Delta_8 \vdash \Delta_6} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash p_7, \Delta_6, p_7}{\bullet h_1 : \top, \Delta_8 \vdash (\Delta_6, p_7), p_7} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_8), p_7 \vdash \Delta_6, p_7} I \\
\hline
- : \top, \Delta_8 \vdash \Delta_6, p_7 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \top, \Delta_8 \vdash \Delta_6, p_7, p_7}{- : \top, \Delta_8 \vdash \Delta_6, p_7} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, p_7 \vdash \Delta_6, p_7}{- : \top, \Delta_8 \vdash \Delta_6, p_7} I \\
\hline
\frac{h_1 : \Delta_9, p_7 \vdash F_8, \Delta_6, p_7}{\bullet h_1 : \top, \Delta_9, p_7 \vdash (\Delta_6, p_7), F_8} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_9, p_7), F_8 \vdash \Delta_6, p_7} I \\
\hline
- : \top, \Delta_9, p_7 \vdash \Delta_6, p_7 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{}{- : \top, \Delta_9, p_7 \vdash \Delta_6, p_7} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash F_7, \Delta_6}{\bullet h_1 : \top, \Delta_8 \vdash \Delta_6, F_7} \top_L \quad \frac{h_5 : F_7, \Delta_8 \vdash \Delta_6}{\bullet h_5 : (\top, \Delta_8), F_7 \vdash \Delta_6} \top_L \\
\hline
- : \top, \Delta_8 \vdash \Delta_6 \quad \text{Cut} \\
\hline
\frac{}{\rightarrow} \\
\frac{h_1 : \top, \Delta_8 \vdash \Delta_6, F_7}{- : \top, \Delta_8 \vdash \Delta_6} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_8, F_7 \vdash \Delta_6}{- : \top, \Delta_8 \vdash \Delta_6} \text{hCut}
\end{array}$$