System for Linear Logic

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1 Measure of derivations

• Case(s) rule 1

• Case(s) rule!

$$\frac{h_1:\vdash F_2,?\Upsilon 3}{\bullet h_1:\vdash?\Upsilon 3,!F_2} \ ! \qquad \rightarrow \qquad \frac{\frac{h_1:\vdash?\Upsilon 3,F_2}{\bullet h_1:\vdash?\Upsilon 3,F_2} \ ^{ax}}{\bullet h_1:\vdash?\Upsilon 3,!F_2} \frac{!}{!}$$

• Case(s) rule ?W

$$\frac{ \begin{smallmatrix} \mathbf{h}_1 : \vdash \Delta_3 \\ \bullet \mathbf{h}_1 : \vdash \Delta_3 , ? \mathbf{F}_2 \end{smallmatrix}}{\bullet \mathbf{h}_1 : \vdash \Delta_3 , ? \mathbf{F}_2} ? W \qquad \rightarrow \qquad \frac{ \begin{smallmatrix} \overline{\mathbf{h}_1} : \vdash \Delta_3 \\ \bullet \mathbf{h}_1 : \vdash \Delta_3 \end{smallmatrix} \overset{\mathsf{ax}}{}_{\mathsf{IH}} \\ \bullet \bullet \mathbf{h}_1 : \vdash \Delta_3 , ? \mathbf{F}_2 \end{smallmatrix} ? W$$

• Case(s) rule ?C

$$\begin{array}{cccc} \underline{\mathbf{h}_1 :\vdash \Delta_3, ?\mathsf{F}_2, ?\mathsf{F}_2} \\ \underline{\bullet} \underline{\mathbf{h}_1 :\vdash \Delta_3, ?\mathsf{F}_2} \end{array} ?C & \rightarrow & & & & & \frac{\overline{\mathbf{h}_1 :\vdash \Delta_3, ?\mathsf{F}_2, ?\mathsf{F}_2}}{\underline{\bullet} \underline{\mathbf{h}_1 :\vdash \Delta_3, ?\mathsf{F}_2, ?\mathsf{F}_2}} & \text{if} \\ \underline{\bullet} \underline{\bullet} \underline{\mathbf{h}_1 :\vdash \Delta_3, ?\mathsf{F}_2, ?\mathsf{F}_2} & ?C & & & & \\ \end{array}$$

• Case(s) rule?

$$\begin{array}{c|c} \underline{h_1 :\vdash \Delta_3, F_2} \\ \bullet \underline{h_1 :\vdash \Delta_3, ?F_2} \end{array} ? \qquad \rightarrow \qquad \begin{array}{c|c} \overline{\underline{h_1 :\vdash \Delta_3, F_2}} & \text{ax} \\ \hline \bullet \underline{h_1 :\vdash \Delta_3, F_2} & \text{IH} \\ \hline \bullet \bullet \underline{h_1 :\vdash \Delta_3, ?F_2} & ? \end{array}$$

• Case(s) rule \$

$$\begin{array}{c|c} \underline{h_1 :\vdash \Delta_4, F_2, F_3} \\ \bullet h_1 :\vdash \Delta_4, F_2\$F_3 \end{array} \ \, \$ \qquad \rightarrow \qquad \begin{array}{c|c} \underline{h_1 :\vdash \Delta_4, F_2, F_3} \\ \bullet h_1 :\vdash \Delta_4, F_2, F_3 \end{array} \ \, \overset{\text{ax}}{\text{IH}} \\ \hline \bullet \bullet h_1 :\vdash \Delta_4, F_2\$F_3 \end{array} \ \, \S$$

• Case(s) rule &

$$\frac{\mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2 \& \mathtt{F}_3} \quad \& \qquad \rightarrow \qquad \underbrace{\frac{\mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2}}_{\begin{array}{c} \bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2 \end{array}}^{ax} \quad \underbrace{\frac{\mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3}}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2 \& \mathtt{F}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{F}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2 \& \mathtt{F}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{F}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{F}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{F}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{F}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{F}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{F}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{F}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{H}_3} \quad \underbrace{\mathsf{H}_1 : \vdash \Delta_4, \mathtt{H}_3}_{\bullet \mathtt{h}_1 : \bot \Delta_4, \mathtt{H}_3}_{\mathtt$$

• Case(s) rule \oplus_B

$$\frac{\mathsf{h}_1 : \vdash \Delta_4, \mathsf{F}_3}{\bullet \mathsf{h}_1 : \vdash \Delta_4, \mathsf{F}_2 \oplus \mathsf{F}_3} \ \oplus_{\mathcal{B}} \quad \rightarrow \quad \underbrace{\frac{\frac{\mathsf{h}_1 : \vdash \Delta_4, \mathsf{F}_3}{\bullet \mathsf{h}_1 : \vdash \Delta_4, \mathsf{F}_3}}_{\bullet \bullet \mathsf{h}_1 : \vdash \Delta_4, \mathsf{F}_2 \oplus \mathsf{F}_3}^{\mathsf{ax}} \ \oplus_{\mathcal{B}}$$

• Case(s) rule \oplus_A

• Case(s) rule ⊥

 \bullet Case(s) rule \top

$$\overline{{}_{\bullet \mathbf{h}_1} :\vdash \top, \Delta_2} \ \top \qquad \rightarrow \qquad \overline{{}_{\bullet \bullet \mathbf{h}_1} :\vdash \Delta_2, \top} \ \top$$

 \bullet Case(s) rule I

 \bullet Case(s) rule \otimes

$$\frac{\mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_4, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3} \ \otimes \qquad \rightarrow \qquad \frac{\frac{\mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2}}{\bullet \mathtt{h}_1 : \vdash \Delta_4, \mathtt{F}_2} \overset{\mathtt{ax}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{ax}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_5}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_5}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_5}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \vdash \Delta_5, \mathtt{h}_5}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_1 : \bot \Delta_5, \mathtt{h}_5}} \overset{\mathtt{nx}}{\overset{\mathtt{h}_$$

2 Weakening on bang: $\vdash \Gamma$, !F implies $\vdash \Gamma$, F.

- Case(s) rule 1
- Case(s) rule!

$$\frac{h_1 : \vdash F_2, ?\Upsilon 3}{\bullet h_1 : \vdash !F_2, ?\Upsilon 3} \ ! \qquad \rightarrow \qquad \frac{\overline{h_1 : \vdash ?\Upsilon 3, F_2}}{\bullet h_1 : \vdash ?\Upsilon 3, F_2} \ ^{ax}_{H}$$

• Case(s) rule ?W

• Case(s) rule ?C

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbb{IF}_1, \mathbb{?F}_3, \mathbb{?F}_3}{\bullet \mathbf{h}_2 : \vdash \mathbb{IF}_1, \Delta_4, \mathbb{?F}_3} \ ?C \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbb{IF}_1, \mathbb{?F}_3, \mathbb{?F}_3}{\mathbf{h}_2 : \vdash \Delta_4, \mathbb{F}_1, \mathbb{?F}_3, \mathbb{?F}_3}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbb{F}_1, \mathbb{?F}_3, \mathbb{?F}_3} \ \frac{\mathbf{ax}}{\mathbb{I}} \\ \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbb{F}_1, \mathbb{?F}_3, \mathbb{?F}_3}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbb{F}_1, \mathbb{?F}_3} \ ?C$$

• Case(s) rule?

$$\begin{array}{c} \underline{\mathbf{h}_2 :\vdash \Delta_4, \mathbf{F}_3, \mathbf{!F_1}} \\ \underline{\bullet \mathbf{h}_2 :\vdash \mathbf{!F_1}, \Delta_4, \mathbf{?F_3}} \end{array} ? \qquad \rightarrow \qquad \begin{array}{c} \frac{\mathbf{h}_2 :\vdash \Delta_4, \mathbf{F}_3, \mathbf{!F_1}}{\mathbf{h}_2 :\vdash \Delta_4, \mathbf{F}_1, \mathbf{F}_3} \end{array} \overset{\mathbf{ax}}{\underset{\bullet}{\mathbf{h}_2 :\vdash \Delta_4, \mathbf{F}_1, \mathbf{?F_3}}} \end{aligned} ?$$

• Case(s) rule \$

$$\begin{array}{c} \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, !\mathbf{F}_1 \\ \bullet \mathbf{h}_2 : \vdash !\mathbf{F}_1, \Delta_5, \mathbf{F}_3 \$ \mathbf{F}_4 \end{array} \ \mathbf{s} \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, !\mathbf{F}_1} \\ \overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3, \mathbf{F}_4} \\ \hline \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \$ \mathbf{F}_4 \end{array} \ \begin{array}{c} \mathbf{n} \\ \mathbf{n} \\$$

• Case(s) rule &

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, !\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, !\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash !\mathbf{F}_1, \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \underbrace{\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{h}} \quad \underbrace{\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{h}} \quad \underbrace{\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\mathbf{h}} \quad \underbrace{\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, !\mathbf{F}_1}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4}$$

• Case(s) rule \oplus_B

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_4, !\mathbf{f}_1}{\bullet \mathbf{h}_2 :\vdash !\mathbf{f}_1, \Delta_5, \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_4, !\mathbf{f}_1}}{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_1, \mathbf{f}_4} \overset{\mathrm{ax}}{}_{\mathrm{IH}}}{}_{\oplus_B}$$

• Case(s) rule \oplus_A

$$\begin{array}{c} \underbrace{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, \mathbf{l} \mathbf{F}_1}_{\bullet \mathbf{h}_2 :\vdash \mathbf{l} \mathbf{F}_1, \ \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \\ \bullet \mathbf{h}_2 :\vdash \mathbf{l} \mathbf{F}_1, \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4 \end{array} \quad \bullet_A \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, \mathbf{l} \mathbf{F}_1} \\ \overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3} \end{array} \quad \overset{\mathbf{ax}}{\mathbf{l} \mathbf{H}}$$

• Case(s) rule \perp

• Case(s) rule \top

- \bullet Case(s) rule I
- Case(s) rule ⊗

$$\begin{array}{c} \underline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{!F_1} \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \mathbf{IF}_1, \Delta_5, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4} \quad \otimes \qquad \rightarrow \qquad \\ \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{!F_1}}}{\underline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3}} \quad \underline{\mathbf{IH}} \quad \underline{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \Delta_6, \mathbf{F}_1, \mathbf{F}_3 \otimes \mathbf{F}_4} \quad \underline{\mathbf{ax}} \quad \underline{\mathbf{ax}}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5 \otimes \mathbf{F}_4} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5 \otimes \mathbf{F}_4}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5 \otimes \mathbf{F}_4}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5 \otimes \mathbf{F}_4}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5 \otimes \mathbf{F}_4}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5 \otimes \mathbf{F}_4}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5 \otimes \mathbf{F}_4}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5 \otimes \mathbf{F}_4}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4 \otimes \mathbf{F}_4, \mathbf{h}_5 \otimes \mathbf{F}_4}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4 \otimes \mathbf{h}_5}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4 \otimes \mathbf{h}_5}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4 \otimes \mathbf{h}_5}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4 \otimes \mathbf{h}_5}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4 \otimes \mathbf{h}_5}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4 \otimes \mathbf{h}_5}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4 \otimes \mathbf{h}_5}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4 \otimes \mathbf{h}_5}} \quad \underline{\mathbf{ax}}_{\bullet \mathbf{h}_4 \otimes \mathbf{h}_5}} \quad \underline{\mathbf{ax}}_{\bullet$$

$$\begin{array}{c} \underline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{!F_1}} \\ \bullet \mathbf{h}_2 : \vdash \mathbf{!F_1}, \Delta_5, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4 \end{array} \quad \otimes \qquad \rightarrow \qquad \underbrace{\frac{\underline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3}}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, \Delta_6, \mathbf{F}_1, \mathbf{F}_3 \otimes \mathbf{F}_4} \quad \overset{\mathbf{ax}}{\underset{\bullet}{\mathsf{IH}}} \quad \underbrace{\frac{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{!F_1}}{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1}}_{\otimes} \quad \underbrace{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1}_{\bullet} \quad \underbrace{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1}_{\bullet} \otimes \underbrace{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{h}_4, \mathbf{h}_4}_{\bullet} \otimes \underbrace{\mathbf{h}_4, \mathbf{h}_4}_{\bullet} \otimes \underbrace{\mathbf{h}_4, \mathbf{h}_4}_{\bullet} \otimes \underbrace{\mathbf{h}_4, \mathbf{h}_4, \mathbf{h}_4}_{\bullet} \otimes \underbrace{\mathbf{h}_4, \mathbf{h}_4, \mathbf{h}_4, \mathbf{h}_4}_{\bullet} \otimes \underbrace{\mathbf{h}_4, \mathbf{h}_4, \mathbf{h}_4, \mathbf{h}_4}_{\bullet} \otimes \underbrace{\mathbf{h}_4, \mathbf{h}_4, \mathbf{h}_4}_{\bullet} \otimes \underbrace{\mathbf{h}_4, \mathbf{h}_4, \mathbf{h}_4}_{\bullet} \otimes \underbrace{\mathbf{h}$$

3 Invertibility of Rules

3.1 Status of 1: : Invertible

• Case rule 1

- Case rule!
- \bullet Case rule ?W
- \bullet Case rule ?C
- Case rule?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I
- Case rule \otimes

3.2 Status of !: : Non invertible

- Case rule 1
- Case rule!

• Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, \vdash \mathsf{IF}_1}{\bullet \mathbf{h}_2 : \vdash (? \Upsilon 4, \vdash \mathsf{IF}_1), ? \mathsf{F}_3} \quad ?W \qquad \rightarrow \qquad \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, \mathsf{F}_1}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \mathsf{F}_1, ? \mathsf{F}_3} \quad ?W$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{h_2:\vdash?\Upsilon 4, !F_1, ?F_3, ?F_3}{\bullet h_2:\vdash(?\Upsilon 4, !F_1), ?F_3} \ ?C \qquad \rightarrow \qquad \frac{h_2:\vdash?\Upsilon 4, F_1, ?F_3, ?F_3}{\bullet h_2:\vdash?\Upsilon 4, F_1, ?F_3} \begin{array}{c} ax/ind \\ ?C \end{array}$$

• Case rule?

$$\frac{\mathtt{h}_2 : \vdash \mathtt{F}_3, ? \Upsilon 4, ! \mathtt{F}_1}{\bullet \mathtt{h}_2 : \vdash (? \Upsilon 4, ! \mathtt{F}_1), ? \mathtt{F}_3} \ ? \qquad \rightarrow \qquad \overline{\bullet \mathtt{h}_2 : \vdash \mathtt{F}_1, ? \Upsilon 4, ? \mathtt{F}_3} \ \text{fail}$$

- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- $\bullet\,$ Case rule I
- \bullet Case rule \otimes

3.3 Status of ?W: Non invertible

- Case rule 1
- Case rule!

$$\frac{h_2 : \vdash F_3, ?\Upsilon 4, ?F_1}{\bullet h_2 : \vdash (?\Upsilon 4, ?F_1), !F_3} \ ! \qquad \rightarrow \qquad \frac{\overline{h_2 : \vdash ?\Upsilon 4, F_3}}{\bullet h_2 : \vdash ?\Upsilon 4, !F_3} \overset{ax/ind}{!}$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 :\vdash \Delta_4 \, , ? \mathbf{F_1}}{\bullet \mathbf{h}_2 :\vdash (\Delta_4 \, , ? \mathbf{F_1}) \, , ? \mathbf{F_3}} \quad ?W \qquad \rightarrow \qquad \frac{\mathbf{h}_2 :\vdash \Delta_4}{\bullet \mathbf{h}_2 :\vdash \Delta_4 \, , ? \mathbf{F_3}} \, ?W$$

• Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, ? \mathbf{F}_1, ? \mathbf{F}_3, ? \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ? \mathbf{F}_1), ? \mathbf{F}_3} \ ?C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_4, ? \mathbf{F}_3, ? \mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, ? \mathbf{F}_3} \begin{array}{c} \mathbf{ax/ind} \\ ?C \end{array}$$

• Case rule?

• Case rule \$

• Case rule &

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3} \quad \text{ax/ind} \quad \overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4} \quad \frac{\text{ax/ind}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \quad \\$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_4}{\bullet \mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4}}{\bullet \mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3 \oplus \mathbf{F}_4} \stackrel{\mathrm{ax/ind}}{\oplus_B}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_3, ?\mathbf{f}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{f}_1), \mathbf{f}_3 \oplus \mathbf{f}_4} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_3}}{\bullet \mathbf{h}_2 :\vdash \Delta_5, \mathbf{f}_3 \oplus \mathbf{f}_4} \stackrel{\mathrm{ax/ind}}{\oplus}_A$$

 \bullet Case rule \bot

 \bullet Case rule \top

- ullet Case rule I
- Case rule \otimes

$$\begin{array}{c} \frac{h_2:\vdash\Delta_5, F_3, ?F_1 \quad h_2:\vdash\Delta_6, F_4}{\bullet h_2:\vdash(\Delta_5, ?F_1), \Delta_6, F_3\otimes F_4} \ \otimes \\ \end{array} \ \rightarrow \ \ \begin{array}{c} \frac{h_2:\vdash\Delta_5, F_3}{\bullet h_2:\vdash\Delta_5, \Delta_6, F_3\otimes F_4} \end{array} \overset{ax/ind}{\xrightarrow{h_2:\vdash\Delta_6, F_4}} \overset{ax}{\otimes} \\ \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, (\Delta_6, ?\mathbf{F}_1), \mathbf{F}_3 \otimes \mathbf{F}_4} \quad \otimes \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3} \quad \overset{\mathsf{ax}}{} \quad \overline{\mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \Delta_6, \mathbf{F}_3 \otimes \mathbf{F}_4} \quad \overset{\mathsf{ax/ind}}{\otimes} \end{array}$$

3.4 Status of ?C: Non invertible

- ullet Case rule $oldsymbol{1}$
- Case rule!

$$\frac{\mathtt{h}_2 : \vdash \mathtt{F}_3, ?\Upsilon 4, ?\mathtt{F}_1}{\bullet \mathtt{h}_2 : \vdash (\Upsilon \Upsilon 4, ?\mathtt{F}_1), !\mathtt{F}_3} \ ! \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_2 : \vdash ?\Upsilon 4, \mathtt{F}_3, ?\mathtt{F}_1, ?\mathtt{F}_1}}{\bullet \mathtt{h}_2 : \vdash ?\Upsilon 4, !\mathtt{F}_3, ?\mathtt{F}_1, ?\mathtt{F}_1} \ !$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ?\mathbf{F}_1), ?\mathbf{F}_3} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_4, ?\mathbf{F}_1, ?\mathbf{F}_1}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_3} \ ?W$$

$$\frac{\mathbf{h}_1 :\vdash \Delta_3}{\bullet \mathbf{h}_1 :\vdash \Delta_3, ?\mathbf{F}_2} \ ?W \qquad \rightarrow \qquad \frac{\bullet \mathbf{h}_1 :\vdash \Delta_3, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 :\vdash \Delta_3, ?\mathbf{F}_2, ?\mathbf{F}_2} \ \mathbf{fail}$$

 \bullet Case rule ?C

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, ?\mathsf{F}_1, ?\mathsf{F}_3, ?\mathsf{F}_3}{\bullet \mathsf{h}_2 : \vdash (\Delta_4, ?\mathsf{F}_1), ?\mathsf{F}_3} \ ?C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_4, ?\mathsf{F}_1, ?\mathsf{F}_1, ?\mathsf{F}_3, ?\mathsf{F}_3}}{\bullet \mathsf{h}_2 : \vdash \Delta_4, ?\mathsf{F}_1, ?\mathsf{F}_1, ?\mathsf{F}_3} \ \frac{\mathsf{ax/ind}}{?C}$$

• Case rule?

$$\frac{\mathbf{h}_2 :\vdash \Delta_4, \mathbf{F}_3, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_4, ?\mathbf{F}_1), ?\mathbf{F}_3} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_4, \mathbf{F}_3, ?\mathbf{F}_1, ?\mathbf{F}_1}}{\bullet \mathbf{h}_2 :\vdash \Delta_4, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_3} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \$\mathbf{F}_4} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}}{\bullet \mathbf{h}_2 :\vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \$\mathbf{F}_4} \quad \$$$

• Case rule &

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad \text{ax/ind}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, ?\mathbf{F}_1} \quad & \qquad \Rightarrow \qquad \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{h}_3, ?\mathbf{F}_1, ?\mathbf{$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 :\vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1, ?\mathbf{F}_1} \ \ ^{\mathrm{ax/ind}}}{\bullet \mathbf{h}_2 :\vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1, ?\mathbf{F}_1}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A$$

 \bullet Case rule \bot

$$\frac{\mathbf{h}_2 :\vdash \Delta_3, ?\mathsf{F}_1}{\bullet \mathsf{h}_2 :\vdash \bot, \Delta_3, ?\mathsf{F}_1} \ \bot \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 :\vdash \Delta_3, ?\mathsf{F}_1, ?\mathsf{F}_1}}{\bullet \mathsf{h}_2 :\vdash \Delta_3, \bot, ?\mathsf{F}_1, ?\mathsf{F}_1} \overset{\mathsf{ax/ind}}{\bot}$$

• Case rule \top

- \bullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \frac{h_2:\vdash\Delta_5,F_3,?F_1\quad h_2:\vdash\Delta_6,F_4}{\bullet h_2:\vdash(\Delta_5,?F_1),\Delta_6,F_3\otimes F_4} \ \otimes \\ \end{array} \rightarrow \begin{array}{c} \frac{h_2:\vdash\Delta_5,F_3,?F_1,?F_1}{\bullet h_2:\vdash\Delta_5,\Delta_6,?F_1,?F_1,F_3\otimes F_4} \end{array} \stackrel{ax/ind}{\longrightarrow} \frac{h_2:\vdash\Delta_6,F_4}{\bullet h_2:\vdash\Delta_5,\Delta_6,?F_1,?F_1,F_3\otimes F_4} \end{array} \rightarrow \begin{array}{c} \frac{h_2:\vdash\Delta_5,F_3,?F_1,?F_1}{\bullet h_2:\vdash\Delta_5,K_6,?F_1,?F_1,F_3\otimes F_4} \end{array} \rightarrow \begin{array}{c} \frac{h_2:\vdash\Delta_5,F_1,F_1}{\bullet h_2:\vdash\Delta_5,K_6,F_1,?F_1,F_1} \end{array} \rightarrow \begin{array}{c} \frac{h_2:\vdash\Delta_5,F_1}{\bullet h_2:\vdash\Delta_5,K_6,F_1,?F_1,F_1} \end{array} \rightarrow \begin{array}{c} \frac{h_2:\vdash\Delta_5,K_6}{\bullet h_2:\vdash\Delta_5,K_6,F_1,F_1} \end{array} \rightarrow \begin{array}{c} \frac{h_2:\vdash\Delta_5,F_1}{\bullet h_2:\vdash\Delta_5,K_6} \end{array} \rightarrow \begin{array}{c} \frac{h_2:\vdash\Delta_5,F_1}{\bullet h_2:\vdash\Delta_5,K_6} \end{array} \rightarrow \begin{array}{c} \frac{h_2:\vdash\Delta_5,K_6}{\bullet h_2:\vdash\Delta_5,K_$$

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3 \quad \mathbf{h}_2 : \vdash \Delta_6, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash \Delta_5, (\Delta_6, ?\mathbf{F}_1), \mathbf{F}_3 \otimes \mathbf{F}_4} \ \otimes \\ \end{array} \rightarrow \\ \begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \Delta_6, ?\mathbf{F}_1, ?\mathbf{F}_1, \mathbf{F}_3 \otimes \mathbf{F}_4} \end{array} \overset{\mathbf{ax/ind}}{\otimes} \end{array}$$

3.5 Status of ?: : Non invertible

- Case rule 1
- Case rule!

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, ?_{\mathbf{f_1}}}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ?_{\mathbf{f_1}}), ?_{\mathbf{f_3}}} ?_W \qquad \rightarrow \qquad \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{f_1}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{f_1}, ?_{\mathbf{f_3}}} \overset{\mathrm{ax/ind}}{?W}$$

$$\frac{\mathbf{h}_1 : \vdash \Delta_3}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathsf{F}_2} \ ?W \qquad \rightarrow \qquad \overline{\bullet \mathbf{h}_1 : \vdash \Delta_3, \mathsf{F}_2} \ \mathsf{fail}$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_2 : \vdash \Delta_4, ?_{\mathbf{F}_1}, ?_{\mathbf{F}_3}, ?_{\mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ?_{\mathbf{F}_1}), ?_{\mathbf{F}_3}} \ ?C \qquad \rightarrow \qquad \frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{f}_1, ?_{\mathbf{F}_3}, ?_{\mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{f}_1, ?_{\mathbf{F}_3}} \begin{array}{c} \mathbf{ax/ind} \\ ?C \end{array}$$

$$\begin{array}{ccc} \underbrace{\mathbf{h}_1 : \vdash \Delta_3, ?_{F_2}, ?_{F_2}}_{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?_{F_2}} & ?C & \rightarrow & & & \\ \hline{\bullet}\mathbf{h}_1 : \vdash \Delta_3, ?_{F_2} & & \\ \end{array} \quad \text{fail}$$

• Case rule?

$$\frac{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_3, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_4, ?\mathbf{F}_1), ?\mathbf{F}_3} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_1, \mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash \Delta_4, \mathbf{F}_1, ?\mathbf{F}_3} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

• Case rule &

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1 \quad \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \& \mathbf{F}_4} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3} \quad \overset{\mathrm{ax/ind}}{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4} \quad \overset{\mathrm{ax/ind}}{\mathbb{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4} \quad & & & & & & & & \\ \bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \& \mathbf{F}_4 & & & & & & & & & \\ \end{pmatrix}$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_4, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_B$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_3, ?\mathbf{F}_1}{\bullet \mathbf{h}_2 : \vdash (\Delta_5, ?\mathbf{F}_1), \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_3 \oplus \mathbf{F}_4} \ \oplus_A$$

 \bullet Case rule \bot

 \bullet Case rule \top

- \bullet Case rule I
- Case rule \otimes

3.6 Status of \$: : Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1\$\mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_5, \mathbf{F}_1\$\mathbf{F}_2), ?\mathbf{F}_4} ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2, ?\mathbf{F}_4} ?W$$

 \bullet Case rule ?C

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, ?\mathsf{F}_4, ?\mathsf{F}_4, \mathsf{F}_1 \$\mathsf{F}_2}{\bullet \mathsf{h}_3 : \vdash (\Delta_5, \mathsf{F}_1 \$\mathsf{F}_2), ?\mathsf{F}_4} \ ?C \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathsf{F}_1, \mathsf{F}_2, ?\mathsf{F}_4, ?\mathsf{F}_4}}{\bullet \mathsf{h}_3 : \vdash \Delta_5, \mathsf{F}_1, \mathsf{F}_2, ?\mathsf{F}_4} \ \frac{\mathsf{ax/ind}}{?C}$$

• Case rule?

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, \mathbf{F}_1 \$ \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \$ \mathbf{F}_2), ?\mathbf{F}_4} \quad ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2, ?\mathbf{F}_4} \quad ?$$

• Case rule \$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \$ \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_5, \mathbf{F}_1 \$ \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{F}_1 \$ \mathbf{F}_2), \mathbf{F}_4 \& \mathbf{F}_5} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3 : \vdash \Delta_6, \mathbf{F}_$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_5, \mathbf{F}_1\$\mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{F}_1\$\mathbf{F}_2), \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_B$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_4, \mathbf{f}_1\$\mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{f}_1\$\mathbf{f}_2), \mathbf{f}_4 \oplus \mathbf{f}_5} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_4}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_4 \oplus \mathbf{f}_5} \ \oplus_A$$

• Case rule \perp

$$\frac{\mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_1\$\mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash \bot, \Delta_4, \mathbf{F}_1\$\mathbf{F}_2} \ \bot \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3 :\vdash \Delta_4, \mathbf{F}_1, \mathbf{F}_2, \bot} \overset{\mathsf{ax/ind}}{\bot}$$

• Case rule \top

- \bullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \$ \mathbf{F}_2 \quad \mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{F}_1 \$ \mathbf{F}_2), \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \ \otimes \qquad \rightarrow \qquad \begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4} \quad \mathbf{ax/ind} \quad \underline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \ \begin{array}{c} \mathbf{ax/ind} \\ \otimes \end{array}$$

3.7 Status of &: (Left Premise): Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \& \mathbf{F}_2), ?\mathbf{F}_4} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4} \ ?W$$

 \bullet Case rule ?C

$$\begin{array}{c} \underline{\mathbf{h}_3} : \vdash \Delta_5, ?\mathbf{F}_4, ?\mathbf{F}_4, \mathbf{F}_1 \& \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \& \mathbf{F}_2), ?\mathbf{F}_4 \end{array} ? C \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3} : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4, ?\mathbf{F}_4 \\ \bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4 \end{array} \begin{array}{c} \mathbf{ax/ind} \\ ?C \end{array}$$

• Case rule?

• Case rule \$

• Case rule &

• Case rule \oplus_B

$$\frac{\mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{B} \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}}{\bullet \mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{B}$$

• Case rule \oplus_A

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{A} \qquad \rightarrow \qquad \frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{A}$$

• Case rule \perp

• Case rule \top

$$\frac{}{\bullet \mathsf{h}_3 : \vdash \top, \Delta_4, \mathsf{F}_1 \& \mathsf{F}_2} \quad \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3 : \vdash \Delta_4, \mathsf{F}_1, \top} \quad \top$$

- ullet Case rule I
- Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \& \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{F}_1 \& \mathbf{F}_2), \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5} \quad \otimes \qquad \rightarrow \qquad \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5} & \overset{\mathrm{ax/ind}}{\otimes} \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_4 \quad \mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \Delta_6, (\Delta_7, \mathbf{F}_1, \mathbf{F}_5), \mathbf{F}_4 \otimes \mathbf{F}_5} & \otimes \\ \frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_4 \quad \mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \Delta_6, (\Delta_7, \mathbf{F}_1, \mathbf{F}_5), \mathbf{F}_4 \otimes \mathbf{F}_5} & \otimes \\ \end{array}$$

3.8 Status of & (Right Premise): : Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_5, \mathbf{F}_1 \& \mathbf{F}_2), ?\mathbf{F}_4} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_2}}{\bullet \mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_2, ?\mathbf{F}_4} \ ?W$$

• Case rule ?C

$$\begin{array}{l} \underline{\mathbf{h}_3 :\vdash \Delta_5, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}, F_1 \& F_2}} \\ \bullet \underline{\mathbf{h}_3 :\vdash (\Delta_5, F_1 \& F_2), ?_{\mathbf{F}_4}} \end{array} ?C \qquad \rightarrow \qquad \\ \overline{\begin{array}{l} \underline{\mathbf{h}_3 :\vdash \Delta_5, F_2, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}} \\ \bullet \underline{\mathbf{h}_3 :\vdash \Delta_5, F_2, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}} \end{array}} \overset{\mathrm{ax/ind}}{?C} \end{array}$$

• Case rule?

$$\frac{\mathsf{h}_3 :\vdash \Delta_5, \mathsf{F}_4, \mathsf{F}_1 \& \mathsf{F}_2}{\bullet \mathsf{h}_3 :\vdash (\Delta_5, \mathsf{F}_1 \& \mathsf{F}_2), ?\mathsf{F}_4} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathsf{h}_3 :\vdash \Delta_5, \mathsf{F}_2, \mathsf{F}_4}}{\bullet \mathsf{h}_3 :\vdash \Delta_5, \mathsf{F}_2, ?\mathsf{F}_4} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4, \mathbf{F}_5}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$^{\mathrm{ax/ind}}$$

• Case rule &

• Case rule \oplus_B

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{B} \qquad \rightarrow \qquad \frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_5}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_{B}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \& \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4} \ \ ^{\mathrm{ax/ind}}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_A$$

• Case rule \perp

 \bullet Case rule \top

$$\frac{}{\bullet \mathsf{h}_3 : \vdash \top, \Delta_4, \mathsf{F}_1 \& \mathsf{F}_2} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3 : \vdash \Delta_4, \mathsf{F}_2, \top} \ \top$$

- ullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \& \mathbf{F}_2 \quad \mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{F}_1 \& \mathbf{F}_2), \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \ \otimes \\ \end{array} \rightarrow \quad \begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{ax/ind$$

$$\begin{array}{c} \underline{\mathbf{h}_3} : \vdash \Delta_6, \mathbf{F}_4 \quad \mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \& \mathbf{F}_2 \\ \bullet \mathbf{h}_3 : \vdash \Delta_6, (\Delta_7, \mathbf{F}_1 \& \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \quad \otimes \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3} : \vdash \Delta_6, \mathbf{F}_4 \quad \text{ax} \quad \overline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \quad \overset{\text{ax/ind}}{\otimes} \end{array}$$

3.9 Status of \oplus_B : Non invertible

- \bullet Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2), ?\mathbf{F}_4} ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_2}}{\bullet \mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_2, ?\mathbf{F}_4} \overset{\mathsf{ax/ind}}{?W}$$

 \bullet Case rule ?C

$$\begin{array}{ll} \frac{\mathbf{h}_3 : \vdash \Delta_5, ? \mathbf{f}_4, ? \mathbf{f}_4, \mathbf{f}_1 \oplus \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{f}_1 \oplus \mathbf{f}_2), ? \mathbf{f}_4} \end{array} ? C \qquad \rightarrow \qquad \\ \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{f}_2, ? \mathbf{f}_4, ? \mathbf{f}_4}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{f}_2, ? \mathbf{f}_4} \end{aligned} \stackrel{\mathsf{ax/ind}}{? C}$$

• Case rule?

$$\frac{h_3:\vdash\Delta_5,F_4,F_1\oplus F_2}{\bullet h_3:\vdash(\Delta_5,F_1\oplus F_2),?F_4}\ ? \qquad \rightarrow \qquad \frac{h_3:\vdash\Delta_5,F_2,F_4}{\bullet h_3:\vdash\Delta_5,F_2,?F_4} \stackrel{ax/ind}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{f}_4, \mathbf{f}_5, \mathbf{f}_1 \oplus \mathbf{f}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{f}_1 \oplus \mathbf{f}_2), \mathbf{f}_4 \$ \mathbf{f}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{f}_2, \mathbf{f}_4, \mathbf{f}_5}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{f}_2, \mathbf{f}_4 \$ \mathbf{f}_5} \quad \overset{\text{ax/ind}}{\$}$$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_4 \& \mathbf{F}_5} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4} \quad \text{ax/ind} \quad \overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_5}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4 \& \mathbf{F}_5} \quad & & \text{ax/ind} \quad \overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4} \\ \bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4 \& \mathbf{F}_5 \\ \end{pmatrix}$$

• Case rule \oplus_B

• Case rule \oplus_A

 \bullet Case rule \bot

 \bullet Case rule \top

- ullet Case rule I
- Case rule \otimes

$$\begin{array}{c} \underline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{F}_1 \oplus \mathbf{F}_2), \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \ \otimes \\ \end{array} \rightarrow \qquad \begin{array}{c} \underline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, \mathbf{F}_4} \quad \underline{\mathbf{a}} \times / \underline{\mathbf{n}} \overline{\mathbf{h}} \quad \underline{\mathbf{h}}_3 : \vdash \Delta_7, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \ \begin{array}{c} \underline{\mathbf{a}} \times \overline{\mathbf{h}} \overline{\mathbf{h}} \times \overline{\mathbf$$

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2} \\ \bullet \mathbf{h}_3 :\vdash \Delta_6, (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \otimes \\ \end{array} \rightarrow \\ \begin{array}{c} \overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4} \quad \overline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \otimes \\ \end{array} \end{array} \otimes \\ \begin{array}{c} \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4 \quad \overline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \otimes \\ \end{array}$$

3.10 Status of \oplus_A : Non invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2), ?\mathbf{F}_4} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4} \overset{\mathrm{ax/ind}}{?W}$$

 \bullet Case rule ?C

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_5, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2), ?_{\mathbf{F}_4}} & ?C & \rightarrow & & & & \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?_{\mathbf{F}_4}} & \frac{\mathsf{ax/ind}}{?C} \\ \end{array}$$

• Case rule?

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_4, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \mathbf{F}_1 \oplus \mathbf{F}_2), ?\mathbf{F}_4} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_4}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4} \overset{\mathsf{ax/ind}}{?}$$

• Case rule \$

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4, \mathbf{F}_5}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$'$$

• Case rule &

$$\frac{\mathbf{a}_3 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{a}_3 : \vdash \Delta_6, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2}{\bullet \mathbf{b}_3 : \vdash (\Delta_6, \mathbf{F}_1) \oplus \mathbf{F}_2), \mathbf{F}_4 \& \mathbf{F}_5} \quad \& \quad \rightarrow \quad \frac{\overline{\mathbf{a}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4} \quad \text{ax/ind}}{\bullet \mathbf{b}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \& \mathbf{F}_5} \quad \frac{\text{ax/ind}}{\&} \quad \& \quad \Rightarrow \quad \frac{\overline{\mathbf{a}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4} \quad \mathbf{ax/ind}}{\bullet \mathbf{b}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \& \mathbf{F}_5} \quad \frac{\mathbf{ax/ind}}{\&} \quad \& \quad \Rightarrow \quad \frac{\mathbf{ax/ind}}{\bullet \mathbf{b}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \& \mathbf{F}_5} \quad \frac{\mathbf{ax/ind}}{\lozenge} \quad \frac{\mathbf{ax/ind}}$$

• Case rule \oplus_B

• Case rule \oplus_A

• Case rule \perp

 \bullet Case rule \top

- \bullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \underline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \oplus \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 : \vdash (\Delta_6, \mathbf{F}_1 \oplus \mathbf{F}_2), \Delta_7, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \otimes \\ \end{array} \quad \Rightarrow \quad \begin{array}{c} \underline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5} \\ \bullet \mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_5} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{ax/ind}$$

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \oplus \mathbf{F}_2} \\ \bullet \mathbf{h}_3 :\vdash \Delta_6, (\Delta_7, \mathbf{F}_1 \oplus \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5 \end{array} \otimes \\ \end{array} \rightarrow \\ \begin{array}{c} \overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_4} \quad \text{ax} \quad \overline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5} \\ \end{array}$$

3.11 Status of \perp : Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_1 : \vdash \bot, \Delta_3}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_3), ?\mathsf{F}_2} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \Delta_3}}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathsf{F}_2} ?W$$

• Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \bot, \Delta_3, ?_{\mathsf{F}_2}, ?_{\mathsf{F}_2}}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_3), ?_{\mathsf{F}_2}} \ ?C \qquad \rightarrow \qquad \overline{\frac{\mathbf{h}_1 : \vdash \Delta_3, ?_{\mathsf{F}_2}, ?_{\mathsf{F}_2}}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?_{\mathsf{F}_2}}} \begin{array}{c} \mathbf{ax/ind} \\ ?C \end{array}$$

• Case rule?

• Case rule \$

• Case rule &

$$\frac{\mathbf{h}_1 : \vdash \bot, \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \bot, \Delta_4, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_4), \mathbf{F}_2 \& \mathbf{F}_3} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2} \quad \text{ax/ind} \quad \overline{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3}}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3} \quad & \\ & \& \qquad \qquad \end{aligned}$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_1 : \vdash \bot, \Delta_4, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_4), \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3}}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_B$$

• Case rule \bigoplus_A

$$\frac{\mathbf{h}_1 : \vdash \bot, \Delta_4, \mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash (\bot, \Delta_4), \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2}}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_A$$

 \bullet Case rule \bot

 \bullet Case rule \top

$$\overline{{}_{\bullet \mathbf{h}_1} :\vdash \top, \bot, \Delta_2} \quad \top \qquad \rightarrow \qquad \overline{{}_{\bullet \mathbf{h}_1} :\vdash \Delta_2, \top} \quad \top$$

- $\bullet\,$ Case rule I
- \bullet Case rule \otimes

3.12 Status of \top : Invertible

- Case rule 1
- Case rule!
- $\bullet \;$ Case rule ?W

$$\frac{\mathbf{h}_1 : \vdash \top, \Delta_3}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_3), ?\mathbf{F}_2} \ ?W \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule ?C

$$\begin{array}{ll} \frac{\mathbf{h}_1 : \vdash \top, \Delta_3, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_3), ?\mathbf{F}_2} \ ?C & \rightarrow & \text{trivial} \end{array}$$

• Case rule?

$$\frac{\mathtt{h}_1 : \vdash \top, \Delta_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 : \vdash (\top, \Delta_3), ?\mathtt{F}_2} \ ? \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \$

$$\begin{array}{ccc} \frac{\mathbf{h}_1 : \vdash \top, \Delta_4, \mathbf{F}_2, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_4), \mathbf{F}_2\$\mathbf{F}_3} & \$ & \rightarrow & \text{trivial} \end{array}$$

• Case rule &

$$\frac{\mathbf{h}_1 : \vdash \top, \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \top, \Delta_4, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_4), \mathbf{F}_2 \& \mathbf{F}_3} \quad \& \qquad \rightarrow \qquad \text{trivial}$$

• Case rule \oplus_B

$$\frac{\mathtt{h}_1 : \vdash \top, \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash (\top, \Delta_4), \mathtt{F}_2 \oplus \mathtt{F}_3} \ \oplus_B \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_1 : \vdash \top, \Delta_4, \mathbf{f}_2}{\bullet \mathbf{h}_1 : \vdash (\top, \Delta_4), \mathbf{f}_2 \oplus \mathbf{f}_3} \ \oplus_A \qquad \rightarrow \qquad \mathtt{trivial}$$

 \bullet Case rule \bot

$$\frac{\mathtt{h}_1 : \vdash \top, \Delta_2}{\bullet \mathtt{h}_1 : \vdash \bot, \top, \Delta_2} \ \bot \qquad \to \qquad \mathtt{trivial}$$

 \bullet Case rule \top

$$\overline{\bullet \mathtt{h}_1 : \vdash \top, \Delta_2} \ \top \qquad \rightarrow \qquad \mathtt{trivial}$$

 $\bullet\,$ Case rule I

• Case rule \otimes

$$\begin{array}{ccc} \frac{\mathtt{h}_1 : \vdash \top, \Delta_4, \mathtt{F}_2 & \mathtt{h}_1 : \vdash \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \vdash (\top, \Delta_4), \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3} & \otimes & \rightarrow & \text{trivial} \end{array}$$

$$\begin{array}{ccc} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 & \mathbf{h}_1 : \vdash \top, \Delta_5, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \Delta_4, (\top, \Delta_5), \mathbf{F}_2 \otimes \mathbf{F}_3} & \otimes & \rightarrow & \text{trivial} \end{array}$$

3.13 Status of I: Invertible

- Case rule 1
- Case rule!
- \bullet Case rule ?W
- \bullet Case rule ?C
- Case rule?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I

$$\overline{\bullet \mathbf{h}_1 : \vdash p(\mathbf{n}_2), \, \hat{\ } (\mathbf{n}_2)} \quad I \qquad \rightarrow \qquad \mathsf{trivial}$$

 \bullet Case rule \otimes

3.14 Status of \otimes : (Left Premise): Non invertible

- $\bullet \;$ Case rule ${\bf 1}$
- Case rule!
- $\bullet \;$ Case rule ?W

$$\frac{\mathbf{h}_3 :\vdash \Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2), ?\mathbf{F}_4} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1}}{\bullet \mathbf{h}_3 :\vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4} \ ?W$$

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2), ?\mathbf{F}_4} \quad ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1} \quad \mathbf{H}$$

• Case rule ?C

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_5, \Delta_6, ?\mathbf{F}_4, ?\mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2), ?\mathbf{F}_4} & ?C & \rightarrow & & \frac{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1, ?\mathbf{F}_4} & ?W \\ \end{array}$$

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_5, \Delta_6, ?\mathbf{F}_4, ?\mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2), ?\mathbf{F}_4} & ?C & \rightarrow & & \overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1} & \\ \bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2), ?\mathbf{F}_4 & ?C & \rightarrow & & \overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1} & \\ \end{array}$$

• Case rule?

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \Delta_6, \mathbf{f}_4, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{f}_1 \otimes \mathbf{f}_2), ?\mathbf{f}_4} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{f}_1} \quad \text{ax/ind}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{f}_1, ?\mathbf{f}_4} \quad ?W$$

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \Delta_6, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2), ?\mathbf{F}_4} \quad ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \Delta_5, \mathbf{F}_1} \quad \overset{\mathrm{ax/ind}}{\vdash}$$

• Case rule \$

$$\frac{\mathsf{h}_3 :\vdash \Delta_6, \Delta_7, \mathsf{F}_4, \mathsf{F}_5, \mathsf{F}_1 \otimes \mathsf{F}_2}{\mathsf{e}_{\mathsf{h}_3} :\vdash (\Delta_6, \Delta_7, \mathsf{F}_1 \otimes \mathsf{F}_2), \mathsf{F}_4 \$ \mathsf{F}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathsf{h}_3 :\vdash \Delta_6, \mathsf{F}_1, \mathsf{F}_4, \mathsf{F}_5}}{\mathsf{e}_{\mathsf{h}_3} :\vdash \Delta_6, \mathsf{F}_1, \mathsf{F}_4 \$ \mathsf{F}_5} \quad \$^{\mathsf{ax/ind}}$$

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1} \quad \overset{\mathsf{ax/ind}}{\mathsf{H}}$$

• Case rule &

$$\frac{\mathtt{h_3} : \vdash \Delta_6, \Delta_7, \mathtt{F_4}, \mathtt{F_1} \otimes \mathtt{F_2} \quad \mathtt{h_3} : \vdash \Delta_6, \Delta_7, \mathtt{F_5}, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash (\Delta_6, \Delta_7, \mathtt{F_1} \otimes \mathtt{F_2}), \mathtt{F_4} \& \mathtt{F_5}} \quad \& \qquad \rightarrow \qquad \frac{\underline{\mathtt{h_3} : \vdash \Delta_6, \mathtt{F_1}, \mathtt{F_4}} \quad \frac{\mathtt{ax/ind}}{\mathtt{h_3} : \vdash \Delta_6, \mathtt{F_1}, \mathtt{F_4}} \quad \frac{\mathtt{h_3} : \vdash \Delta_6, \mathtt{F_1}, \mathtt{F_5}}{\mathtt{\&}} \quad \& \quad \\ \bullet \mathtt{h_3} : \vdash \Delta_6, \mathtt{F_1}, \mathtt{F_4} \& \mathtt{F_5}$$

$$\frac{\mathtt{h}_3 : \vdash \Delta_6, \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \otimes \mathtt{F}_2 \quad \mathtt{h}_3 : \vdash \Delta_6, \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash (\Delta_6, \Delta_7, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1}}{\bullet \mathtt{h}_3 : \vdash \Delta_6, \mathtt{F}_1} \quad \overset{\mathsf{ax/ind}}{\mathsf{h}}$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_B \qquad \rightarrow \qquad \frac{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_B$$

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1} \ ^{\mathrm{ax/ind}}_{\mathrm{H}}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{f}_4, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_4 \oplus \mathbf{f}_5} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_6, \mathbf{f}_1, \mathbf{f}_4}}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \mathbf{f}_1, \mathbf{f}_4 \oplus \mathbf{f}_5} \ \oplus_A$$

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_1} \ \overset{\mathrm{ax/ind}}{\vdash}$$

• Case rule \perp

• Case rule \top

$$\frac{}{\bullet \mathsf{h}_3 : \vdash \top, \Delta_4, \Delta_5, \mathsf{f}_1 \otimes \mathsf{f}_2} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3 : \vdash \Delta_4, \mathsf{f}_1, \top} \ \top$$

- ullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), (\Delta_8, \Delta_9), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), (\Delta_8, \Delta_9), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \\ \\ \frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash \Delta_6, \Delta_8, \mathbf{F}_1} \quad \mathbf{fail}$$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_6, \Delta_7, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), \mathbf{F}_2 \otimes \mathbf{F}_3} \quad \otimes \qquad \rightarrow \qquad \overline{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_2} \quad \mathbf{fail} \end{array}$$

- 3.15 Status of \otimes (Right Premise): Non invertible
 - Case rule 1
 - Case rule!
 - \bullet Case rule ?W

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \Delta_6, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{f}_1 \otimes \mathbf{f}_2), ?\mathbf{f}_4} \ ?W \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_2}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_2, ?\mathbf{f}_4} \ ?W$$

• Case rule ?C

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_5, \Delta_6, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2), ?_{\mathbf{F}_4}} & ?C & \rightarrow & & & & \frac{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2} & \\ & \bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2 & & & \\ \end{array}$$

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_5, \Delta_6, ?\mathbf{F}_4, ?\mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{F}_1 \otimes \mathbf{F}_2), ?\mathbf{F}_4} & ?C & \rightarrow & & \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{F}_2, ?\mathbf{F}_4} & ?W \end{array}$$

• Case rule?

$$\frac{\mathbf{h}_3 : \vdash \Delta_5, \Delta_6, \mathbf{f}_4, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_5, \Delta_6, \mathbf{f}_1 \otimes \mathbf{f}_2), ?\mathbf{f}_4} \ ? \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_2}}{\bullet \mathbf{h}_3 : \vdash \Delta_6, \mathbf{f}_2, ?\mathbf{f}_4} \ ?W$$

• Case rule \$

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_2}}{\bullet \mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_2} \quad \mathbf{H}$$

$$\frac{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 :\vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \$ \mathbf{F}_5} \quad \$ \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_4, \mathbf{F}_5}}{\bullet \mathbf{h}_3 :\vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \$ \mathbf{F}_5} \quad \overset{\mathsf{ax/ind}}{\$}$$

• Case rule &

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \& \mathbf{F}_5} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_2}}{\bullet \mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2 \quad \mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \& \mathbf{F}_5} \quad \& \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_4} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_4} \frac{\mathsf{ax/ind}}{\mathsf{h}_3 : \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5} \quad \frac{\mathsf{ax/ind}}{\mathsf{ax/ind}} \quad \frac{\mathsf{ax/ind}}{\mathsf{ax/ind}} \frac{\mathsf{ax/ind}}{\mathsf{ax/in$$

• Case rule \oplus_B

$$\frac{\mathtt{h}_3 : \vdash \Delta_6, \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash (\Delta_6, \Delta_7, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \Delta_7, \mathtt{F}_2}}{\bullet \mathtt{h}_3 : \vdash \Delta_7, \mathtt{F}_2} \ \underset{\mathtt{H}}{\overset{\mathtt{ax/ind}}{\bullet}}$$

$$\frac{\mathtt{h}_3 : \vdash \Delta_6, \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 : \vdash (\Delta_6, \Delta_7, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_B \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3 : \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}}{\bullet \mathtt{h}_3 : \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5} \ \oplus_B$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \oplus \mathbf{F}_5} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_2}}{\bullet \mathbf{h}_3 : \vdash \Delta_7, \mathbf{F}_2} \ \underset{\mathbf{H}}{\mathsf{ax/ind}}$$

$$\frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{f}_4, \mathbf{f}_1 \otimes \mathbf{f}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{f}_1 \otimes \mathbf{f}_2), \mathbf{f}_4 \oplus \mathbf{f}_5} \ \oplus_A \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3 : \vdash \Delta_7, \mathbf{f}_2, \mathbf{f}_4}}{\bullet \mathbf{h}_3 : \vdash \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \oplus \mathbf{f}_5} \ \oplus_A$$

• Case rule \perp

 \bullet Case rule \top

$$\frac{}{\bullet \mathsf{h}_3 : \vdash \top, \Delta_4, \Delta_5, \mathsf{F}_1 \otimes \mathsf{F}_2} \ \top \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3 : \vdash \Delta_5, \mathsf{F}_2, \top} \ \top$$

- ullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2 & \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), (\Delta_8, \Delta_9), \mathbf{F}_4 \otimes \mathbf{F}_5} & \otimes & & \rightarrow & & \hline{\bullet \mathbf{h}_3 : \vdash \Delta_7, \Delta_9, \mathbf{F}_2} & \mathbf{fail} \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \ \otimes \\ \end{array} \quad \rightarrow \quad \begin{array}{c} \bullet \mathbf{h}_3 : \vdash \Delta_7, \Delta_9, \mathbf{F}_2 \end{array} \quad \mathbf{fail}$$

$$\begin{array}{lll} \frac{\mathbf{h}_3 : \vdash \Delta_6, \Delta_7, \mathbf{F}_4, \mathbf{F}_1 \otimes \mathbf{F}_2 & \mathbf{h}_3 : \vdash \Delta_8, \Delta_9, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), (\Delta_8, \Delta_9), \mathbf{F}_4 \otimes \mathbf{F}_5} & \otimes & & \rightarrow & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_6, \Delta_7, \mathbf{F}_1 \otimes \mathbf{F}_2), (\Delta_8, \Delta_9), \mathbf{F}_4 \otimes \mathbf{F}_5} & & & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{F}_4 \otimes \mathbf{F}_5) & & \\ \hline \bullet \mathbf{h}_3 : \vdash (\Delta_7, \Delta_9, \mathbf{$$

$$\begin{array}{c} \underline{\mathbf{h}_3 :\vdash \Delta_6, \Delta_7, \mathbf{F}_4 \quad \mathbf{h}_3 :\vdash \Delta_8, \Delta_9, \mathbf{F}_5, \mathbf{F}_1 \otimes \mathbf{F}_2} \\ \bullet \underline{\mathbf{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathbf{F}_1 \otimes \mathbf{F}_2), \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \otimes \\ \phantom{\mathbf{h}_3 :\vdash \Delta_7, \Delta_9, \mathbf{F}_2, \mathbf{F}_4 \otimes \mathbf{F}_5} \end{array} \mathbf{fail}$$

$$\begin{array}{c} \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_6, \Delta_7, \mathbf{F}_3 \\ \bullet \mathbf{h}_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), \mathbf{F}_2 \otimes \mathbf{F}_3 \end{array} \ \otimes \qquad \rightarrow \qquad \begin{array}{c} \bullet \mathbf{h}_1 : \vdash \Delta_5, \Delta_7, \mathbf{F}_3 \\ \bullet \mathbf{h}_1 : \vdash \Delta_5, \Delta_7, \mathbf{F}_3 \end{array} \ \mathbf{fail} \end{array}$$

4 Identity-Expansion

5 Cut-Elimination

5.1 Status of 1: OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\frac{\underbrace{\bullet_{\mathbf{h}_1} : \vdash \mathbf{1}, *}_{\bullet \mathbf{h}_1} \ \mathbf{1} \ \frac{\mathbf{h}_2 : \vdash \mathbf{1}, \Delta_4}{\bullet_{\mathbf{h}_2} : \vdash dual(\mathbf{1}), \Delta_4, ?F_3} \\ - : \vdash *, \Delta_4, ?F_3}_{\underbrace{\bullet_{\mathbf{h}_1} : \vdash \mathbf{1}}_{\bullet \mathbf{h}_1} : \vdash \mathbf{1}} \ \frac{\rightarrow}{\mathbf{h}_2 : \vdash \Delta_4, \bot}_{\bullet \mathbf{h}_2} \ \mathbf{hCut}} \\ \frac{- : \vdash \Delta_4}{- : \vdash \Delta_4, ?F_3} \ W}$$

 \bullet Case rule ?C

• Case rule?

• Case rule \$

$$\frac{\underbrace{\bullet_{\mathbf{h}_1} :\vdash \mathbf{1}, *}_{\bullet \mathbf{h}_1} \ \mathbf{1} \quad \frac{\mathbf{h}_2 :\vdash \bot, \Delta_5, \mathbf{F}_3, \mathbf{F}_4}{\bullet_{\mathbf{h}_2} :\vdash dual(\mathbf{1}), \Delta_5, \mathbf{F}_3 \$ \mathbf{F}_4} \quad \mathop{\mathsf{Cut}}_{\bullet \mathbf{h}_1} \\ - :\vdash *, \Delta_5, \mathbf{F}_3 \$ \mathbf{F}_4 \\ \underbrace{\bullet_{\mathbf{h}_1} :\vdash \mathbf{1}}_{\bullet \mathbf{h}_1} \ \overset{\mathsf{ax}}{\underset{-}{}} \quad \frac{\rightarrow}{\mathbf{h}_2 :\vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4, \bot}_{\bullet \mathbf{h}_2} \quad \underset{\mathsf{hCut}}{\underset{\bullet}{}} \quad \mathsf{hCut}}_{\bullet \mathsf{Cut}}$$

• Case rule &

$$\frac{\underbrace{\bullet \mathbf{h}_1 : \vdash \mathbf{1}, *}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}, *} \mathbf{1} \quad \underbrace{\frac{\mathbf{h}_2 : \vdash \bot, \Delta_5, F_3 \quad \mathbf{h}_2 : \vdash \bot, \Delta_5, F_4}_{\bullet \mathbf{h}_2 : \vdash dual(\mathbf{1}), \Delta_5, F_3 \& F_4}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_3, \bot} \underbrace{\frac{\mathbf{a}_3}_{h_2 : \vdash \Delta_5, F_3, \bot}}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}} \underbrace{\frac{\mathbf{a}_3}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}} \underbrace{\frac{\mathbf{a}_3}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}} \underbrace{\frac{\mathbf{a}_3}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}} \underbrace{\frac{\mathbf{a}_4}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_1 : \vdash \mathbf{1}} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_5}_{h_2 : \vdash \Delta_5, F_4, \bot}}_{\bullet \mathbf{h}_2 : \vdash \Delta_5, F_4} \underbrace{\frac{\mathbf{a}_$$

• Case rule \oplus_B

• Case rule \oplus_A

• Case rule \perp

 \bullet Case rule \top

- \bullet Case rule I
- \bullet Case rule \otimes

5.2 Status of !: OK

- Case rule 1
- Case rule!

$$\frac{ \begin{array}{c} \mathbf{h_1} : \vdash \mathbf{F_4}, ?\Upsilon3 \\ \bullet \mathbf{h_1} : \vdash ! \mathbf{F_4}, ?\Upsilon3 \end{array} \hspace{0.1cm} ! \hspace{0.1cm} \frac{\mathbf{h_5} : \vdash \mathbf{F_6}, ?\Upsilon7, ?dual(\mathbf{F_4})}{\bullet \mathbf{h_5} : \vdash dual(!\mathbf{F_4}), ?\Upsilon7, ! \mathbf{F_6}} \hspace{0.1cm} ?\mathbf{Cut} \\ \hline \\ - : \vdash ?\Upsilon3, ?\Upsilon7, ! \mathbf{F_6} \\ \hline \\ \bullet \mathbf{h_1} : \vdash ?\Upsilon3, ! \mathbf{F_4} \end{array} \hspace{0.1cm} \mathbf{ax} \hspace{0.1cm} \frac{}{\mathbf{h_5} : \vdash ?\Upsilon7, \mathbf{F_6}, ?dual(\mathbf{F_4})} \hspace{0.1cm} \mathbf{ax} \\ \hline \\ \frac{- : \vdash ?\Upsilon3, ?\Upsilon7, \mathbf{F_6}}{- : \vdash ?\Upsilon3, ?\Upsilon7, ! \mathbf{F_6}} \hspace{0.1cm} ! \end{array} \hspace{0.1cm} \mathbf{acut} \end{array}$$

 \bullet Case rule ?W

• Case rule ?C

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \mathbf{F}_2,?\Upsilon 3,5}{\bullet \mathbf{h}_1 : \vdash 5,?\Upsilon 3,! \mathbf{F}_2} : \frac{\mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_7, dual(5)}{\bullet \mathbf{h}_6 : \vdash dual(5), \Delta_8,? \mathbf{F}_7} \\ - : \vdash (?\Upsilon 3,! \mathbf{F}_2), \Delta_8,? \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1 : \vdash 5,?\Upsilon 3,! \mathbf{F}_2 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash 5,?\Upsilon 3,! \mathbf{F}_2 & \mathbf{ax} \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_7,! \mathbf{F}_2 \\ \hline - : \vdash ?\Upsilon 3, \Delta_8,! \mathbf{F}_2,? \mathbf{F}_7 & ? \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3}{\bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4,?\Upsilon 3} : \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6,? dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(! \mathbf{F}_4), \Delta_7,? \mathbf{F}_6} \\ \hline - : \vdash ?\Upsilon 3, \Delta_7,? \mathbf{F}_6 \\ \hline - : \vdash ?\Upsilon 3, \Delta_7,? \mathbf{F}_6 & ? \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3} : \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6,? dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6,? dual(\mathbf{F}_4)} \\ \hline - : \vdash ?\Upsilon 3, \Delta_7,? \mathbf{F}_6 & ? \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3} : \frac{\mathbf{h}_5 : \vdash \Delta_6, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(! \mathbf{F}_4), \Delta_6} ? \\ \hline \mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3} : \frac{\mathbf{h}_5 : \vdash \Delta_6, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(! \mathbf{F}_4), \Delta_6} ? \\ \hline - : \vdash ?\Upsilon 3, \Delta_6 & \\ \hline - : \vdash ?\Upsilon 3, \Delta_6 & \\ \hline - : \vdash ?\Upsilon 3, \Delta_6 & \\ \hline - : \vdash ?\Upsilon 3, \Delta_6 & \\ \hline - : \vdash ?\Upsilon 3, \Delta_6 & \\ \hline - : \vdash ?\Upsilon 3, \Delta_6 & \\ \hline - : \vdash ?\Upsilon 3, \Delta_6 & \\ \hline - : \vdash ?\Upsilon 3, \Delta_6 & \\ \hline \end{array}$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \mathbf{F}_2,?\Upsilon 3,5}{\bullet \mathbf{h}_1 : \vdash 5,?\Upsilon 3,! \mathbf{F}_2} : \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8, dual(5)}{\bullet \mathbf{h}_6 : \vdash dual(5), \Delta_9, \mathbf{F}_7\$ \mathbf{F}_8} \\ - : \vdash (?\Upsilon 3,! \mathbf{F}_2), \Delta_9, \mathbf{F}_7\$ \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash 5,?\Upsilon 3,! \mathbf{F}_2 & \mathbf{ax} & \xrightarrow{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8, dual(5)} \\ \hline \frac{- : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7, \mathbf{F}_8,! \mathbf{F}_2}{- : \vdash ?\Upsilon 3, \Delta_9,! \mathbf{F}_2, \mathbf{F}_7\$ \mathbf{F}_8} \$ \\ \hline \frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3}{\bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4,?\Upsilon 3} : & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7,? dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(! \mathbf{F}_4), \Delta_8, \mathbf{F}_6\$ \mathbf{F}_7} \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6\$ \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1 : \vdash ?\Upsilon 3,! \mathbf{F}_4 & \mathbf{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7,? dual(\mathbf{F}_4)} \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6\$ \mathbf{F}_7 \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6\$ \mathbf{F}_7 & \$ \\ \end{array} \qquad \mathbf{ax} \\ \mathbf{hCut} \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6\$ \mathbf{F}_7 \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6\$ \mathbf{F}_7 & \$ \\ \end{array}$$

• Case rule &

$$\frac{\frac{\mathbf{h}_1 :\vdash \mathbf{F}_2,?\Upsilon 3,5}{\bullet \mathbf{h}_1 :\vdash 5,?\Upsilon 3,! \mathbf{F}_2} : \frac{\mathbf{h}_6 :\vdash \Delta_9, \mathbf{F}_7, dual(5) \quad \mathbf{h}_6 :\vdash \Delta_9, \mathbf{F}_8, dual(5)}{\bullet \mathbf{h}_6 :\vdash \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8} \& \\ \frac{- :\vdash (?\Upsilon 3,! \mathbf{F}_2), \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8}{\bullet \mathbf{h}_1 :\vdash 5,?\Upsilon 3,! \mathbf{F}_2} \xrightarrow{\mathbf{ax}} \frac{\rightarrow}{\mathbf{h}_6 :\vdash \Delta_9, \mathbf{F}_7, dual(5)} & \\ \frac{- :\vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7, ! \mathbf{F}_2}{- :\vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7, ! \mathbf{F}_2} \xrightarrow{\mathbf{h}_1 :\vdash 5,?\Upsilon 3,! \mathbf{F}_2} \overset{\mathbf{ax}}{\mathbf{h}_6 :\vdash \Delta_9, \mathbf{F}_8, dual(5)} & \\ \frac{- :\vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7, ! \mathbf{F}_2}{- :\vdash ?\Upsilon 3, \Delta_9, ! \mathbf{F}_2, \mathbf{F}_7 \& \mathbf{F}_8} \& \\ \end{pmatrix} \text{hCut}$$

$$\frac{\frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3}{\bullet \mathbf{h}_1 : \vdash !\mathbf{F}_4,?\Upsilon 3}}{\bullet \mathbf{h}_1 : \vdash !\mathbf{F}_4,?\Upsilon 3} : \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6,?dual(\mathbf{F}_4) \quad \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7,?dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(!\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7} \underbrace{- : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \& \mathbf{F}_7}_{\bullet \mathbf{h}_1 : \vdash ?\Upsilon 3,!\mathbf{F}_4} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7,?dual(\mathbf{F}_4)} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6,?dual(\mathbf{F}_4)} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_1 : \vdash ?\Upsilon 3,!\mathbf{F}_4} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7,?dual(\mathbf{F}_4)} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7,?dual(\mathbf{F}_4)} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_5 : \vdash 2} \underbrace{\mathbf{ax}}_{\bullet \mathbf{h}_5 :$$

• Case rule \oplus_B

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \mathbf{F}_2, ?\Upsilon 3, 5}{\bullet \mathbf{h}_1 : \vdash 5, ?\Upsilon 3, | \mathbf{F}_2} & ! & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_8, dual(5)}{\bullet \mathbf{h}_6 : \vdash dual(5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{Cut} \\ \hline & - : \vdash (?\Upsilon 3, | \mathbf{F}_2), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash 5, ?\Upsilon 3, | \mathbf{F}_2 & \mathbf{ax} & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash 5, ?\Upsilon 3, | \mathbf{F}_2 & \mathbf{ax} & \rightarrow \\ \hline & - : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_8, | \mathbf{F}_2 \\ \hline & - : \vdash ?\Upsilon 3, \Delta_9, | \mathbf{F}_2, \mathbf{F}_7 \oplus \mathbf{F}_8 & \mathbf{b} \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_4, ?\Upsilon 3 & ! & \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, ?dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_1 : \vdash | \mathbf{F}_4, ?\Upsilon 3 & ! & \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, ?dual(\mathbf{F}_4) \\ \hline & - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \hline \bullet \mathbf{h}_1 : \vdash ?\Upsilon 3, | \mathbf{F}_4 & \mathbf{ax} & \rightarrow \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, ?dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus_B \\ \hline \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \mathbf{h} \mathbf{Cut} \\ \hline \bullet \mathbf{h}_1 : \vdash ?\Upsilon 3, | \mathbf{F}_4 & \mathbf{ax} & \rightarrow \\ \hline \bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, ?dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus_B \\ \hline \end{array} \quad \mathbf{h} \mathbf{Cut}$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \mathbf{F}_2,?\Upsilon 3,5}{\bullet \mathbf{h}_1 : \vdash 5,?\Upsilon 3,! \mathbf{F}_2} : \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(5)}{\bullet \mathbf{h}_6 : \vdash dual(5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \\ \hline - : \vdash (?\Upsilon 3,! \mathbf{F}_2), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash 5,?\Upsilon 3,! \mathbf{F}_2 \\ \hline \bullet \mathbf{h}_1 : \vdash 5,?\Upsilon 3,! \mathbf{F}_2 \\ \hline - : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7, ! \mathbf{F}_2 \\ \hline - : \vdash ?\Upsilon 3, \Delta_9,! \mathbf{F}_2, \mathbf{F}_7 \oplus \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3 \\ \bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4,?\Upsilon 3 \\ \hline \bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4,?\Upsilon 3 \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \hline \bullet \mathbf{h}_1 : \vdash ?\Upsilon 3,! \mathbf{F}_4 \\ \hline \bullet \mathbf{h}_2 : \vdash 2 \mathbf{H}_3, \mathbf{H}_4 \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1 : \vdash ?\Upsilon 3,! \mathbf{F}_4 \\ \hline \bullet \mathbf{h}_2 : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_3 : \vdash 2 \mathbf{H}_3, \mathbf{H}_4 \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_4 : \vdash ?\Upsilon 3,! \mathbf{H}_4 \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_5 : \vdash 2 \mathbf{H}_3, \mathbf{H}_4 \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_5 : \vdash 2 \mathbf{H}_4, \mathbf{H}_4 \\ \hline - : \vdash ?\Upsilon 3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 \\ \hline \end{array}$$

\bullet Case rule \bot

$$\begin{array}{c} \frac{\mathbf{h}_1 :\vdash \mathbf{F}_2,?\Upsilon 3,5}{\bullet \mathbf{h}_1 :\vdash 5,?\Upsilon 3,!\mathbf{F}_2} : \frac{\mathbf{h}_6 :\vdash \Delta_7,dual(5)}{\bullet \mathbf{h}_6 :\vdash dual(5),\perp,\Delta_7} \overset{\bot}{\mathsf{Cut}} \\ \hline \\ \frac{-:\vdash (?\Upsilon 3,!\mathbf{F}_2),\perp,\Delta_7}{\bullet \mathbf{h}_1 :\vdash 5,?\Upsilon 3,!\mathbf{F}_2} \overset{\mathsf{ax}}{\overset{\mathsf{h}_6 :\vdash \Delta_7,dual(5)}{\overset{\mathsf{h}_6 :\vdash \Delta_7,dual(5)}{\overset{\mathsf{h}_6 :\vdash \Delta_7,dual(5)}{\overset{\mathsf{h}_1 :\vdash 5,?\Upsilon 3,\Delta_7,l}_{\overset{\mathsf{h}_2}{\overset{\mathsf{h}_1 :\vdash ?\Upsilon 3,\Delta_7,1}_{\overset{\mathsf{h}_2}{\overset{\mathsf{h}_1 :\vdash ?\Upsilon 3,\Delta_7,1}_{\overset{\mathsf{h}_2}{\overset{\mathsf{h}_1 :\vdash ?\Upsilon 3,\Delta_6}_{\overset{\mathsf{h}_1 :\vdash ?\Upsilon 3,1}{\overset{\mathsf{h}_5 :\vdash \Delta_6,?dual(F_4)}_{\overset{\mathsf{h}_1 :\vdash ?\Upsilon 3,1}{\overset{\mathsf{h}_5 :\vdash \Delta_6,?dual(F_4)}_{\overset{\mathsf{h}_1 :\vdash ?\Upsilon 3,1}{\overset{\mathsf{h}_5 :\vdash \Delta_6,?dual(F_4)}_{\overset{\mathsf{h}_1 :\vdash ?\Upsilon 3,\Delta_6}_{\overset{\mathsf{h}_1 :\vdash ?}}}_{\overset{\mathsf{h}_1 :\vdash ?\Upsilon 3,\Delta_6}_{\overset{\mathsf{h}_1 :\vdash ?\Upsilon 3,\Delta_6}_{\overset{\mathsf{$$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \mathbf{F}_2, ?\Upsilon 3, 5}{\bullet \mathbf{h}_1 : \vdash 5, ?\Upsilon 3, !\mathbf{F}_2} & ! & \frac{\bullet}{\bullet \mathbf{h}_6 : \vdash dual(5), \top, \Delta_7} & \top \\ \hline - : \vdash (?\Upsilon 3, !\mathbf{F}_2), \top, \Delta_7 \\ \hline - : \vdash ?\Upsilon 3, \Delta_7, \top, !\mathbf{F}_2 & \top \\ \\ \hline \frac{\mathbf{h}_1 : \vdash \mathbf{F}_4, ?\Upsilon 3}{\bullet \mathbf{h}_1 : \vdash !\mathbf{F}_4, ?\Upsilon 3} & ! & \frac{\bullet}{\bullet \mathbf{h}_5 : \vdash dual(!\mathbf{F}_4), \top, \Delta_6} & \top \\ \hline - : \vdash ?\Upsilon 3, \top, \Delta_6 \\ \hline \rightarrow \\ \hline - : \vdash ?\Upsilon 3, \Delta_6, \top & \top \\ \end{array}$$

- \bullet Case rule I
- Case rule \otimes

5.3 Status of ?W: OK

- Case rule 1
- Case rule!

$$\frac{ \begin{array}{c|c} \mathbf{h}_1 : \vdash \Delta_3 \\ \hline \bullet \mathbf{h}_1 : \vdash ? \mathbf{F}_4, \Delta_3 \end{array} ?W & \begin{array}{c} \mathbf{h}_5 : \vdash ? \Upsilon \mathbf{G}, dual(\mathbf{F}_4) \\ \hline \bullet \mathbf{h}_5 : \vdash dual(? \mathbf{F}_4), ? \Upsilon \mathbf{G} \end{array} \\ & \begin{array}{c} - : \vdash \Delta_3, ? \Upsilon \mathbf{G} \\ \hline \hline & - : \vdash \Delta_3 \end{array} \\ \hline & \begin{array}{c} - : \vdash \Delta_3 \\ \hline - : \vdash ? \Upsilon \mathbf{G}, \Delta_3 \end{array} W \end{array} \end{aligned} \mathbf{Cut}$$

\bullet Case rule ?W

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} & ?W \\ \hline & - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_8, ?\mathbf{F}_7 & \rightarrow \\ \hline \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} & \text{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5) \\ \hline & - : \vdash \Delta_3, \Delta_8, ?\mathbf{F}_2, ?\mathbf{F}_7 & ?W \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} & ?W \\ \hline & - : \vdash \Delta_3, \Delta_7, ?\mathbf{F}_6 & \rightarrow \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4 & \text{ax} & \rightarrow \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4) & \text{ax} \\ \hline & - : \vdash \Delta_3, \Delta_7, ?\mathbf{F}_6 & \rightarrow \\ \hline & \bullet \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4) & \text{ax} \\ \hline & - : \vdash \Delta_3, \Delta_7, ?\mathbf{F}_6 & W \\ \hline \end{array}$$

$\bullet \;$ Case rule ?C

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, ?\mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} & Cut \\ \hline & - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_8, ?\mathbf{F}_7 & \rightarrow \\ \hline \underbrace{\frac{\rightarrow}{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}}_{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5} & \mathbf{ax} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5) & \mathbf{hCut} \\ \hline & - : \vdash \Delta_3, \Delta_8, ?\mathbf{F}_2, ?\mathbf{F}_7 & ?W \\ \hline \\ \underbrace{\frac{\mathbf{h}_1 : \vdash \Delta_3}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3}}_{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} & ?W & \underbrace{\frac{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6}}_{\mathbf{Cut}} & Cut \\ \hline & - : \vdash \Delta_3, \Delta_7, ?\mathbf{F}_6 & \mathbf{ax} \\ \hline & \underbrace{\frac{\rightarrow}{\mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4}}_{\bullet \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4), ?\mathbf{F}_6, ?\mathbf{F}_6}}_{- : \vdash \Delta_3, \Delta_7, ?\mathbf{F}_6, ?\mathbf{F}_6} & ?C \\ \hline \end{array}$$

• Case rule?

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} ? W & \frac{\mathbf{h}_6 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} \\ \hline \\ - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_8, ?\mathbf{F}_7 \\ \hline \\ \frac{-}{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5} & \mathbf{ax} & \xrightarrow{\bullet \mathbf{h}_6 : \vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, ?\mathbf{F}_7 \\ \hline \\ - : \vdash \Delta_3, \Delta_8, ?\mathbf{F}_7, ?W \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} ? W & \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} ? \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_7, ?\mathbf{F}_6 \\ \hline \\ - : \vdash \Delta_3, \Delta_7, ?\mathbf{F}_6 ? \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_6 ? \\ \hline \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_7 : \vdash \mathbf{h}_7 : \mathbf{$$

• Case rule \$

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} : ?W & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8} & \mathbf{S} \\ \hline & - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8 \\ \hline & \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7\$\mathbf{F}_8} & \mathbf{ax} \\ \hline & \frac{- : \vdash \Delta_3, \Delta_9, \mathbf{F}_7\$\mathbf{F}_8}{- : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, \mathbf{F}_7\$\mathbf{F}_8} & ?W \\ \hline & \frac{\mathbf{h}_1 : \vdash \Delta_3}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} : ?W & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6\$\mathbf{F}_7} & \$ \\ \hline & \frac{- : \vdash \Delta_3, \Delta_8, \mathbf{F}_6\$\mathbf{F}_7}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4} & \mathbf{ax} \\ \hline & \frac{- : \vdash \Delta_3, \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline & \frac{- : \vdash \Delta_3, \Delta_8, \mathbf{F}_6, \mathbf{F}_7}{- : \vdash \Delta_3, \Delta_8, \mathbf{F}_6\$\mathbf{F}_7} & \$ \\ \hline \end{array}$$

• Case rule &

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \frac{\oplus_B}{\mathsf{Cut}} \\ \hline & - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \rightarrow & \mathbf{ax} \\ \hline \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & W & \mathsf{hCut} \\ \hline \frac{\mathbf{h}_1 : \vdash \Delta_3}{\bullet \mathbf{h}_1 : \vdash 2} & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_B \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4 & \mathbf{ax} & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4 & \mathbf{ax} & \rightarrow \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7) & \mathbf{ax} \\ \hline & - : \vdash \Delta_3, \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_7$$

• Case rule \oplus_A

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} & ?W & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathsf{Cut} \\ \hline \\ - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & \rightarrow & \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5} & \mathsf{ax} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathsf{ax} \\ \hline - : \vdash \Delta_3, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & ?W & \mathsf{hCut} \\ \hline - : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, \mathbf{F}_7 \oplus \mathbf{F}_8 & ?W & \\ \hline \\ \bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3 & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus A \\ \hline - : \vdash \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \rightarrow & \mathsf{Cut} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4 & \mathsf{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)} & \mathsf{ax} \\ \hline - : \vdash \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus A & \mathsf{hCut} \\ \hline - : \vdash \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus A & \mathsf{hCut} \\ \hline \end{array}$$

• Case rule \perp

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} \ ?W & \frac{\mathbf{h}_6 : \vdash \Delta_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \bot, \Delta_7} \\ \hline \\ \frac{- : \vdash (\Delta_3, ?\mathbf{F}_2), \bot, \Delta_7}{\bullet \mathbf{h}_6 : \vdash \Delta_7, \bot, dual(\mathbf{F}_5)} & \text{ax} \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_7, \bot}{- : \vdash \Delta_3, \Delta_7, \bot, ?\mathbf{F}_2} \ W \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_3}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} \ ?W & \frac{\mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \bot, \Delta_6} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4} & \text{ax} \\ \hline \\ \frac{\bullet \mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)} & \text{ax} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4}{\bullet \mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)} & \text{hCut} \\ \hline \end{array}$$

\bullet Case rule \top

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} & ?W & \bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \top, \Delta_7 \\ & - : \vdash (\Delta_3, ?\mathbf{F}_2), \top, \Delta_7 \\ & \rightarrow \\ \hline & - : \vdash \Delta_3, \Delta_7, \top, ?\mathbf{F}_2 & \top \\ \\ \hline \frac{\mathbf{h}_1 : \vdash \Delta_3}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} & ?W & \bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \top, \Delta_6 \\ & \rightarrow \\ \hline & - : \vdash \Delta_3, \top, \Delta_6 \\ & \rightarrow \\ \hline & - : \vdash \Delta_3, \Delta_6, \top & \top \\ \end{array}$$

- $\bullet\,$ Case rule I
- \bullet Case rule \otimes

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2 \end{array}}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} \begin{array}{c} ?W & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5) \quad \mathbf{h}_6 : \vdash \Delta_{10}, \mathbf{F}_8}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \Delta_{10}, \mathbf{F}_7 \otimes \mathbf{F}_8} \\ \hline & - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_9, \Delta_{10}, \mathbf{F}_7 \otimes \mathbf{F}_8 \\ \hline & \frac{}{\bullet \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5} \begin{array}{c} \mathbf{ax} \\ \hline \bullet \mathbf{h}_6 : \vdash \Delta_{10}, \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \otimes \mathbf{F}_8} \\ \hline & - : \vdash \Delta_{10}, \Delta_3, \Delta_9, \mathbf{F}_7 \otimes \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_3, \Delta_9, ?\mathbf{F}_2, \mathbf{F}_7 \otimes \mathbf{F}_8 \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{hCut} \end{array}$$

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_3, F_5}{\bullet \mathbf{h}_1 : \vdash F_5, \Delta_3, ?F_2} : ?W & \frac{\mathbf{h}_6 : \vdash \Delta_9, F_7 \quad \mathbf{h}_6 : \vdash \Delta_{10}, F_8, dual(F_5)}{\bullet \mathbf{h}_6 : \vdash dual(F_5), \Delta_9, \Delta_{10}, F_7 \otimes F_8} & \mathbf{Cut} \\ \hline \\ - : \vdash (\Delta_3, ?F_2), \Delta_9, \Delta_{10}, F_7 \otimes F_8 & \rightarrow & \mathbf{Aux} \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, F_5}{\bullet} & \mathbf{ax} & \xrightarrow{\bullet \mathbf{h}_6 : \vdash \Delta_{10}, \Delta_9, dual(F_5), F_7 \otimes F_8} \\ \hline \\ - : \vdash \Delta_{10}, \Delta_3, \Delta_9, F_7 \otimes F_8 & \mathbf{Aux} \\ \hline \\ \frac{- : \vdash \Delta_{10}, \Delta_3, \Delta_9, F_7 \otimes F_8}{\bullet \mathbf{h}_1 : \vdash 2F_4, \Delta_3} & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_8, F_6, !dual(F_4)}{\bullet \mathbf{h}_5 : \vdash dual(?F_4), \Delta_8, \Delta_9, F_6 \otimes F_7} & \mathbf{Cut} \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?F_4} & \mathbf{ax} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_8, F_6}{\bullet \mathbf{h}_1 : \vdash 2F_4, \Delta_3} & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_8, F_6, !dual(F_4)}{\bullet \mathbf{h}_5 : \vdash \Delta_9, F_7, !dual(F_4)} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_8, F_6}{\bullet \mathbf{h}_1 : \vdash 2F_4, \Delta_3} & ?W & \frac{\mathbf{h}_5 : \vdash \Delta_8, F_6 \quad \mathbf{h}_5 : \vdash \Delta_9, F_7, !dual(F_4)}{\bullet \mathbf{h}_5 : \vdash dual(?F_4), \Delta_8, \Delta_9, F_6 \otimes F_7} & \mathbf{Cut} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, F_6 & \mathbf{ax} & \mathbf{h}_5 : \vdash \Delta_9, F_7, !dual(F_4) \\ \hline \\ - : \vdash \Delta_3, A_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7 & \mathbf{ax} \\ \hline \\ -$$

5.4 Status of ?C: OK

- Case rule 1
- Case rule!

$$\frac{ \begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} ?C & \frac{\mathbf{h}_5 : \vdash ?\Upsilon\mathbf{6}, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), ?\Upsilon\mathbf{6}} \\ & \\ - : \vdash \Delta_3, ?\Upsilon\mathbf{6} & \\ & \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet} & \text{ax} & \frac{\mathbf{h}_5 : \vdash ?\Upsilon\mathbf{6}, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash ?\Upsilon\mathbf{6}, |dual(\mathbf{F}_4)}} \\ & \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet} & \text{ax} & \frac{\mathbf{h}_5 : \vdash ?\Upsilon\mathbf{6}, |dual(\mathbf{F}_4)}{\bullet} \\ & \\ - : \vdash ?\Upsilon\mathbf{6}, \Delta_3 & \\ \end{array} \\ \begin{array}{c} \mathbf{mCut} \\ \end{array}$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_1 :\vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2}{\underbrace{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2}_{} :\vdash (\Delta_3, ?\mathbf{F}_2), \Delta_8, ?\mathbf{F}_7} } : C \underbrace{\frac{\mathbf{h}_6 :\vdash \Delta_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 :\vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7}}_{\bullet \mathbf{h}_1 :\vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2}} \underbrace{\overset{\bullet}{\mathbf{a}x} \xrightarrow{\frac{\rightarrow}{\mathbf{h}_6 :\vdash \Delta_8, dual(\mathbf{F}_5)}}}_{\bullet \mathbf{h}_6 :\vdash \Delta_8, dual(\mathbf{F}_5)} \underbrace{\overset{\bullet}{\mathbf{h}} \mathbf{ax}}_{\bullet \mathbf{Cut}} } \underbrace{\overset{\bullet}{\mathbf{h}} \mathbf{Cut}}_{\bullet \mathbf{Cut}}$$

$$\frac{- :\vdash \Delta_3, \Delta_8, ?\mathbf{F}_2, ?\mathbf{F}_7}{- :\vdash \Delta_3, \Delta_8, ?\mathbf{F}_2, ?\mathbf{F}_7}} : W$$

$$\frac{\mathbf{h}_1 :\vdash \Delta_3, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 :\vdash ?\mathbf{F}_4, \Delta_3} : C \underbrace{\overset{\bullet}{\mathbf{h}_5} :\vdash \Delta_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 :\vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} }_{\bullet \mathbf{h}_1 :\vdash \Delta_3, ?\mathbf{F}_4} \underbrace{\overset{\bullet}{\mathbf{ax}} \xrightarrow{\frac{\rightarrow}{\mathbf{h}_5} :\vdash \Delta_7, !dual(\mathbf{F}_4)}}_{\bullet \mathbf{h}_5 :\vdash \Delta_7, !dual(\mathbf{F}_4)} \underbrace{\overset{\bullet}{\mathbf{ax}}}_{\bullet \mathbf{Cut}}$$

$$\frac{- :\vdash \Delta_3, \Delta_7}{- :\vdash \Delta_3, \Delta_7, ?\mathbf{F}_6} : W$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{5}, ?\mathbf{F}_{2}, ?\mathbf{F}_{2}}{\bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{5}, \Delta_{3}, ?\mathbf{F}_{2}} : ?C & \frac{\mathbf{h}_{6} : \vdash \Delta_{8}, ?\mathbf{F}_{7}, ?\mathbf{F}_{7}, dual(\mathbf{F}_{5})}{\bullet \mathbf{h}_{6} : \vdash dual(\mathbf{F}_{5}), \Delta_{8}, ?\mathbf{F}_{7}} & \mathbf{Cut} \\ \\ \hline \frac{\mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{5}, ?\mathbf{F}_{2}, ?\mathbf{F}_{2}}{\bullet \mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{5}, ?\mathbf{F}_{2}, ?\mathbf{F}_{2}} & \mathbf{ax} & \frac{\rightarrow}{\bullet \mathbf{h}_{6} : \vdash \Delta_{8}, ?\mathbf{F}_{7}, dual(\mathbf{F}_{5})} & \mathbf{ax} \\ \hline \frac{- : \vdash \Delta_{3}, \Delta_{8}, ?\mathbf{F}_{2}, ?\mathbf{F}_{7}, ?\mathbf{F}_{7}}{- : \vdash \Delta_{3}, \Delta_{8}, ?\mathbf{F}_{2}, ?\mathbf{F}_{7}, ?\mathbf{F}_{7}} : C \\ \hline \frac{\mathbf{h}_{1} : \vdash \Delta_{3}, ?\mathbf{F}_{4}, ?\mathbf{F}_{4}}{\bullet \mathbf{h}_{1} : \vdash ?\mathbf{F}_{4}, \Delta_{3}} : ?C & \frac{\mathbf{h}_{5} : \vdash \Delta_{7}, !dual(\mathbf{F}_{4}), ?\mathbf{F}_{6}, ?\mathbf{F}_{6}}{\bullet \mathbf{h}_{5} : \vdash dual(?\mathbf{F}_{4}), \Delta_{7}, ?\mathbf{F}_{6}} : C \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{3}, ?\mathbf{F}_{4} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_{5} : \vdash \Delta_{7}, !dual(\mathbf{F}_{4}), ?\mathbf{F}_{6}, ?\mathbf{F}_{6}}{\bullet} & \mathbf{ax} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{3}, ?\mathbf{F}_{4} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_{5} : \vdash \Delta_{7}, !dual(\mathbf{F}_{4}), ?\mathbf{F}_{6}, ?\mathbf{F}_{6}} : - \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{F}_{6}, ?\mathbf{F}_{6}} : C \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{3}, ?\mathbf{h}_{7}, ?\mathbf{h}_{6} : \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{h}_{6} : - \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{h}_{6}$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_1 :\vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} ?C & \frac{\mathbf{h}_6 :\vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 :\vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} ?C \\ \hline \\ \frac{\mathbf{h}_1 :\vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2}{-:\vdash (\Delta_3, \mathbf{F}_2, ?\mathbf{F}_2)} & \frac{\rightarrow}{\bullet \mathbf{h}_6 :\vdash \Delta_8, ?\mathbf{F}_7, dual(\mathbf{F}_5)} \\ \hline \\ \frac{-:\vdash \Delta_3, \Delta_8, ?\mathbf{F}_2, ?\mathbf{F}_2}{-:\vdash \Delta_3, \Delta_8, ?\mathbf{F}_2, ?\mathbf{F}_7} ?C \\ \hline \\ \frac{\mathbf{h}_1 :\vdash \Delta_3, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 :\vdash ?\mathbf{F}_4, \Delta_3} ?C & \frac{\mathbf{h}_5 :\vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 :\vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} ?\\ \hline \\ \frac{\bullet \mathbf{h}_1 :\vdash \Delta_3, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 :\vdash \Delta_3, ?\mathbf{F}_4} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 :\vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)} \\ \hline \\ \frac{\bullet \mathbf{h}_1 :\vdash \Delta_3, ?\mathbf{F}_4}{\bullet \mathbf{h}_5 :\vdash \Delta_7, \mathbf{F}_6, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ \frac{-:\vdash \Delta_3, \Delta_7, ?\mathbf{F}_6}{-:\vdash \Delta_3, \Delta_7, ?\mathbf{F}_6} ? \end{cases} ? \end{array}$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} &?C & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8} & \underbrace{\mathbf{Cut}} \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2} & \mathbf{ax} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7\$\mathbf{F}_8} \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, ?\mathbf{F}_2, \mathbf{F}_7\$\mathbf{F}_8}{- : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, \mathbf{F}_7\$\mathbf{F}_8} &?C \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} &?C & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6\$\mathbf{F}_7} & \\ \hline \\ \frac{\bullet}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} \\ \hline \\ \frac{\bullet}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} \\ \hline \\ \frac{\bullet}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, \mathbf{F}_7, !dual(\mathbf{F}_4)} \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_8, \mathbf{F}_6\$\mathbf{F}_7}{- : \vdash \Delta_3, \Delta_8, \mathbf{F}_6\$\mathbf{F}_7} & \$ \\ \hline \end{array}$$

• Case rule &

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} &?C & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5) \quad \mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8} & \mathbf{Cut} \\ & & - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_9, \mathbf{F}_7 \& \mathbf{F}_8 \\ & & \frac{}{\underbrace{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2}} & \mathbf{ax} & \frac{}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \& \mathbf{F}_8}} & \mathbf{ax} \\ & & \frac{}{- : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, ?\mathbf{F}_2, \mathbf{F}_7 \& \mathbf{F}_8}} & ?C & \\ & & & \frac{}{- : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, \mathbf{F}_7 \& \mathbf{F}_8}} & ?C & \end{array}$$

$$\frac{\frac{\mathbf{h}_1 :\vdash \Delta_3, ?_{\mathbf{F}_4}, ?_{\mathbf{F}_4}}{\bullet \mathbf{h}_1 :\vdash ?_{\mathbf{F}_4}, \Delta_3}}{\bullet \mathbf{h}_1 :\vdash ?_{\mathbf{F}_4}, \Delta_3}} ?C \xrightarrow{\begin{array}{c} \mathbf{h}_5 :\vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) & \mathbf{h}_5 :\vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 :\vdash dual(?_{\mathbf{F}_4}), \Delta_8, \mathbf{F}_6 \&_{\mathbf{F}_7} \\ & \rightarrow \\ \hline \\ \mathbf{h}_5 :\vdash \Delta_3, \Delta_8, \mathbf{F}_6 \&_{\mathbf{F}_7} \\ & \rightarrow \\ \hline \\ \bullet \mathbf{h}_5 :\vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) \end{array}} \underbrace{\begin{array}{c} \mathbf{ax} \\ \mathbf{h}_5 :\vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 :\vdash \Delta_8, !dual(\mathbf{F}_4), \mathbf{F}_6 \&_{\mathbf{F}_7} \\ \bullet \mathbf{h}_5 :\vdash \Delta_8, \mathbf{h}_6 \&_{\mathbf{F}_7} \\ \hline \\ \bullet \mathbf{h}_5 :\vdash \Delta_8, \mathbf{h}_6 \&_{\mathbf{F}_7} \\ \bullet \mathbf{h}_6 :\vdash \Delta_8, \mathbf{h}_6 &\vdash \Delta_8, \mathbf{h}_6 \&_{\mathbf{F}_7} \\ \bullet \mathbf{h}_6 :\vdash \Delta_8, \mathbf{h}_6 &\vdash \Delta_8, \mathbf{h}_6 &\vdash \Delta_8, \mathbf{h}_6 \\ \bullet \mathbf{h}_6 :\vdash \Delta_8, \mathbf{h}_6 &\vdash \Delta_8, \mathbf{h}_6 &\vdash \Delta_8, \mathbf{h}_6 \\ \bullet \mathbf{h}_6 :\vdash \Delta_8,$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} ?C & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \\ \underline{- : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{Cut} \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, ?\mathbf{F}_2, \mathbf{F}_7 \oplus \mathbf{F}_8} & ?C \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, \mathbf{F}_7 \oplus \mathbf{F}_8}{- : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, \mathbf{F}_7 \oplus \mathbf{F}_8} ?C \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} ?C & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_B \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4} & \mathbf{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7}{- : \vdash \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_B \\ \hline \end{array}$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathsf{F}_5, ?\mathsf{F}_2, ?\mathsf{F}_2}{\bullet \mathbf{h}_1 : \vdash \mathsf{F}_5, \Delta_3, ?\mathsf{F}_2} ?C & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathsf{F}_7, dual(\mathsf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathsf{F}_5), \Delta_9, \mathsf{F}_7 \oplus \mathsf{F}_8} & \mathsf{Cut} \\ \hline \\ - : \vdash (\Delta_3, ?\mathsf{F}_2), \Delta_9, \mathsf{F}_7 \oplus \mathsf{F}_8 & \rightarrow \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathsf{F}_5, ?\mathsf{F}_2, ?\mathsf{F}_2}{\bullet \mathbf{h}_1 : \vdash \Delta_3, \Delta_9, ?\mathsf{F}_2, ?\mathsf{F}_2, \mathsf{F}_7 \oplus \mathsf{F}_8} & ?C \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_9, ?\mathsf{F}_2, \mathsf{F}_7 \oplus \mathsf{F}_8}{- : \vdash \Delta_3, \Delta_9, ?\mathsf{F}_2, \mathsf{F}_7 \oplus \mathsf{F}_8} ?C \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, ?\mathsf{F}_4, ?\mathsf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathsf{F}_4, \Delta_3} ?C & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathsf{F}_6, !dual(\mathsf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathsf{F}_4), \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7} & \oplus_{\mathsf{A}} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathsf{F}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathsf{F}_4} & \mathbf{ax} & \xrightarrow{\mathsf{h}_5 : \vdash \Delta_8, \mathsf{F}_6, !dual(\mathsf{F}_4)} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_8, \mathsf{F}_6}{- : \vdash \Delta_3, \Delta_8, \mathsf{F}_6 \oplus \mathsf{F}_7} & \oplus_{\mathsf{A}} \\ \hline \end{array}$$

\bullet Case rule \bot

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2, ?\mathbf{F}_2}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} ?C \quad & \underbrace{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \top, \Delta_7}_{-: \vdash (\Delta_3, ?\mathbf{F}_2), \top, \Delta_7} \quad & \mathsf{Cut} \\ \\ \frac{}{-: \vdash \Delta_3, \Delta_7, \top, ?\mathbf{F}_2} \top \\ \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} ?C \quad & \underbrace{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \top, \Delta_6}_{-: \vdash \Delta_3, \top, \Delta_6} \top \\ \\ \frac{}{-: \vdash \Delta_3, \top, \Delta_6, \top} \top \end{array}$$

- \bullet Case rule I
- Case rule \otimes

5.5 Status of ?: OK

- Case rule 1
- Case rule!

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3 \end{array} ? \quad \begin{array}{c} \mathbf{h}_5 : \vdash ?\Upsilon 6, dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), ?\Upsilon 6 \end{array} ? \\ \hline \begin{array}{c} - : \vdash \Delta_3, ?\Upsilon 6 \\ \hline - : \vdash \Delta_3, \mathbf{F}_4 \end{array} \begin{array}{c} - : \vdash ?\Upsilon 6, dual(\mathbf{F}_4) \\ - : \vdash ?\Upsilon 6, \Delta_3 \end{array} \end{array} \begin{array}{c} \mathbf{ax} \\ \mathbf{sCut} \end{array}$$

\bullet Case rule ?W

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} ? & \frac{\mathbf{h}_6 : \vdash \Delta_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_8, ?\mathbf{F}_7} ?W \\ \hline \\ & - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_8, ?\mathbf{F}_7 \\ \hline \\ & \bullet \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_5, ?\mathbf{F}_2 & \text{ax} & \xrightarrow{\mathbf{h}_6 : \vdash \Delta_8, dual(\mathbf{F}_5)} \\ \hline \\ & - : \vdash \Delta_3, \Delta_8, ?\mathbf{F}_2, ?\mathbf{F}_7 & W \\ \hline \\ & \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} ? & \frac{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} & ?W \\ \hline \\ & - : \vdash \Delta_3, \Delta_7, ?\mathbf{F}_6 \\ \hline \\ & \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4 & \text{ax} & \xrightarrow{\mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4)} \\ \hline \\ & \bullet \mathbf{h}_5 : \vdash \Delta_3, \Delta_7 \\ \hline \\ & \bullet \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4) & \text{ax} \\ \hline \\ & \bullet \mathbf{h}_5 : \vdash \Delta_7, !dual(\mathbf{F}_4) & \text{hCut} \\ \hline \\ & \bullet \mathbf{h}_5 : \vdash \Delta_3, \Delta_7 \\ \hline \\ & - : \vdash \Delta_3, \Delta_7 \\ \hline \\ & - : \vdash \Delta_3, \Delta_7, ?\mathbf{F}_6 & \text{hCut} \\ \hline \end{array}$$

\bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{2}, \mathbf{F}_{5}}{\bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{5}, \Delta_{3}, ?\mathbf{F}_{2}} ? & \frac{\mathbf{h}_{6} : \vdash \Delta_{8}, ?\mathbf{F}_{7}, ?\mathbf{F}_{7}, dual(\mathbf{F}_{5})}{\bullet \mathbf{h}_{6} : \vdash dual(\mathbf{F}_{5}), \Delta_{8}, ?\mathbf{F}_{7}} & ?C \\ \hline & - : \vdash (\Delta_{3}, ?\mathbf{F}_{2}), \Delta_{8}, ?\mathbf{F}_{7} & \rightarrow \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{5}, ?\mathbf{F}_{2} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_{6} : \vdash \Delta_{8}, ?\mathbf{F}_{7}, ?\mathbf{F}_{7}, dual(\mathbf{F}_{5})} & \mathbf{ax} \\ \hline & \frac{- : \vdash \Delta_{3}, \Delta_{8}, ?\mathbf{F}_{2}, ?\mathbf{F}_{7}, ?\mathbf{F}_{7}}{- : \vdash \Delta_{3}, \Delta_{8}, ?\mathbf{F}_{2}, ?\mathbf{F}_{7}, ?\mathbf{F}_{7}} & ?C \\ \hline & \frac{\mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{4}}{\bullet \mathbf{h}_{1} : \vdash ?\mathbf{F}_{4}, \Delta_{3}} ? & \frac{\mathbf{h}_{5} : \vdash \Delta_{7}, !dual(\mathbf{F}_{4}), ?\mathbf{F}_{6}, ?\mathbf{F}_{6}}{\bullet \mathbf{h}_{5} : \vdash dual(?\mathbf{F}_{4}), \Delta_{7}, ?\mathbf{F}_{6}} & ?C \\ \hline & \frac{\bullet}{\mathbf{h}_{1}} : \vdash \Delta_{3}, ?\mathbf{F}_{4} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_{5}} : \vdash \Delta_{7}, !dual(\mathbf{F}_{4}), ?\mathbf{F}_{6}, ?\mathbf{F}_{6}} & \mathbf{ax} \\ \hline & \frac{- : \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{F}_{6}, ?\mathbf{F}_{6}}{- : \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{F}_{6}} & ?C \\ \hline & \frac{- : \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{F}_{6}, ?\mathbf{F}_{6}}{- : \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{F}_{6}} & ?C \\ \hline \end{array} \right.$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{2}, \mathbf{F}_{5}}{\bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{5}, \Delta_{3}, ?\mathbf{F}_{2}} ? & \frac{\mathbf{h}_{6} : \vdash \Delta_{8}, \mathbf{F}_{7}, dual(\mathbf{F}_{5})}{\bullet \mathbf{h}_{6} : \vdash dual(\mathbf{F}_{5}), \Delta_{8}, ?\mathbf{F}_{7}} ? \\ \hline - : \vdash (\Delta_{3}, ?\mathbf{F}_{2}), \Delta_{8}, ?\mathbf{F}_{7} & \rightarrow \\ \hline \frac{\mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{2}, \mathbf{F}_{5}}{\bullet \mathbf{h}_{1} : \vdash \Delta_{3}, \Delta_{8}, \mathbf{F}_{2}, ?\mathbf{F}_{7}} ? \\ \hline \\ \frac{- : \vdash \Delta_{3}, \Delta_{8}, \mathbf{F}_{2}, ?\mathbf{F}_{7}}{- : \vdash \Delta_{3}, \Delta_{8}, ?\mathbf{F}_{2}, ?\mathbf{F}_{7}} ? \\ \hline \\ \frac{\mathbf{h}_{1} : \vdash \Delta_{3}, \mathbf{F}_{4}}{\bullet \mathbf{h}_{1} : \vdash ?\mathbf{F}_{4}, \Delta_{3}} ? & \frac{\mathbf{h}_{5} : \vdash \Delta_{7}, \mathbf{F}_{6}, !dual(\mathbf{F}_{4})}{\bullet \mathbf{h}_{5} : \vdash dual(?\mathbf{F}_{4}), \Delta_{7}, ?\mathbf{F}_{6}} ? \\ \hline \\ \frac{\bullet \mathbf{h}_{1} : \vdash \Delta_{3}, ?\mathbf{F}_{4}}{\bullet \mathbf{h}_{1} : \vdash \Delta_{3}, ?\mathbf{F}_{4}} & \mathbf{ax} & \frac{- : \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{F}_{6}}{\bullet : \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{F}_{6}} ? \\ \hline \\ \frac{- : \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{F}_{6}}{- : \vdash \Delta_{3}, \Delta_{7}, ?\mathbf{F}_{6}} ? \end{cases} \end{cases}$$

• Case rule \$

• Case rule &

• Case rule \oplus_B

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} ? & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{Cut} \\ \hline \\ - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 & \rightarrow \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5}{\bullet} & \mathbf{ax} & \frac{\rightarrow}{\bullet \mathbf{h}_6 : \vdash \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \oplus \mathbf{F}_8} \\ \hline \\ - : \vdash \Delta_3, \Delta_9, ?\mathbf{F}_2, \mathbf{F}_7 \oplus \mathbf{F}_8 & ? \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} ? & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \mathbf{Cut} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \rightarrow \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4 & \mathbf{ax} & \rightarrow \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus \mathbf{A} \\ \hline \\ - : \vdash \Delta_3, \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus \mathbf{A} \\ \hline \end{array}$$

• Case rule \perp

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} ? & \frac{\mathbf{h}_6 : \vdash \Delta_7, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \bot, \Delta_7} \\ \hline \\ - : \vdash (\Delta_3, ?\mathbf{F}_2), \bot, \Delta_7 \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5}{\bullet} & \mathbf{ax} & \frac{\bullet}{\bullet \mathbf{h}_6 : \vdash \Delta_7, \bot, dual(\mathbf{F}_5)} \\ \hline \\ \frac{- : \vdash \Delta_3, \Delta_7, \mathbf{F}_2, \bot}{- : \vdash \Delta_3, \Delta_7, \bot, ?\mathbf{F}_2} ? \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash ?\mathbf{F}_4, \Delta_3} ? & \frac{\mathbf{h}_5 : \vdash \Delta_6, !dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \bot, \Delta_6} \\ \hline \\ - : \vdash \Delta_3, \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4 & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4 & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_3, \Delta_6, \bot \\ \hline \\ - : \vdash \Delta_3, \Delta_6, \bot \\ \hline \\ & \mathbf{h}_{\mathbf{Cut}} \\ \hline \end{array}$$

\bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 :\vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5}{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} ? & \frac{\bullet \mathbf{h}_6 :\vdash dual(\mathbf{F}_5), \top, \Delta_7}{\bullet \mathbf{h}_6 :\vdash dual(\mathbf{F}_5), \top, \Delta_7} & \mathsf{Cut} \\ & \xrightarrow{-:\vdash (\Delta_3, ?\mathbf{F}_2), \top, \Delta_7} & \xrightarrow{-} \\ & \xrightarrow{-:\vdash \Delta_3, \Delta_7, \top, ?\mathbf{F}_2} & \top \\ \\ \frac{\mathbf{h}_1 :\vdash \Delta_3, \mathbf{F}_4}{\bullet \mathbf{h}_1 :\vdash ?\mathbf{F}_4, \Delta_3} ? & \xrightarrow{\bullet \mathbf{h}_5 :\vdash dual(?\mathbf{F}_4), \top, \Delta_6} & \top \\ & \xrightarrow{-:\vdash \Delta_3, \top, \Delta_6} & \xrightarrow{-} \\ & \xrightarrow{-:\vdash \Delta_3, \Delta_6, \top} & \top \\ \end{array}$$

- ullet Case rule I
- Case rule \otimes

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2 \end{array} }{ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2 } \ ? \ \begin{array}{c} \mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, dual(\mathbf{F}_5) \quad \mathbf{h}_6 : \vdash \Delta_{10}, \mathbf{F}_8 \\ \bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \Delta_{10}, \mathbf{F}_7 \otimes \mathbf{F}_8 \\ \hline \\ - : \vdash (\Delta_3, ?\mathbf{F}_2), \Delta_9, \Delta_{10}, \mathbf{F}_7 \otimes \mathbf{F}_8 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5 \end{array} \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_6 : \vdash \Delta_{10}, \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \otimes \mathbf{F}_8 \\ \hline \\ - : \vdash \Delta_{10}, \Delta_3, \Delta_9, ?\mathbf{F}_2, \mathbf{F}_7 \otimes \mathbf{F}_8 \end{array} \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{hCut} \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5, \Delta_3, ?\mathbf{F}_2} ? & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7 \quad \mathbf{h}_6 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5)}{\bullet \mathbf{h}_6 : \vdash dual(\mathbf{F}_5), \Delta_9, \Delta_{10}, \mathbf{F}_7 \otimes \mathbf{F}_8} & \mathbf{Cut} \\ \hline \\ \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5} & \mathbf{ax} & \xrightarrow{} & \xrightarrow{} & \underbrace{\mathbf{h}_6 : \vdash \Delta_{10}, \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \otimes \mathbf{F}_8}_{\bullet \mathbf{h}_6 : \vdash \Delta_{10}, \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \otimes \mathbf{F}_8} & \mathbf{ax} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_2, \mathbf{F}_5} & \mathbf{ax} & \xrightarrow{} & \underbrace{\mathbf{h}_6 : \vdash \Delta_{10}, \Delta_9, dual(\mathbf{F}_5), \mathbf{F}_7 \otimes \mathbf{F}_8}_{\bullet \mathbf{h}_6 : \vdash \Delta_{10}, \Delta_3, \Delta_9, \mathbf{F}_2, \mathbf{F}_7 \otimes \mathbf{F}_8} ? \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_4} & ? & \underbrace{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4) \quad \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7}_{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \Delta_9, \mathbf{F}_6 \otimes \mathbf{F}_7} & \mathbf{Cut} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, ?\mathbf{F}_4} & \mathbf{ax} & \underbrace{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{A}_8, \Delta_8, \mathbf{A}_9, \mathbf{F}_6 \otimes \mathbf{F}_7} & \mathbf{ax} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_4} & ? & \underbrace{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_4} & ? & \underbrace{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6 \quad \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash dual(?\mathbf{F}_4), \Delta_8, \Delta_9, \mathbf{F}_6 \otimes \mathbf{F}_7} & \mathbf{Cut} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_4} & ? & \underbrace{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_6 \quad \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{F}_6} & \mathbf{ax} & \underbrace{\mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \mathbf{A}_8, \Delta_9, \mathbf{F}_6 \otimes \mathbf{F}_7} & \mathbf{ax} & \underbrace{\mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \Delta_8, \Delta_9, \mathbf{F}_6 \otimes \mathbf{F}_7} & \mathbf{ax} & \underbrace{\mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \underline{\mathbf{h}_1 : \vdash \Delta_3, \Delta_8, \Delta_9, \mathbf{F}_6 \otimes \mathbf{F}_7} & \mathbf{ax} & \underbrace{\mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf{h}_5 : \vdash \Delta_9, \mathbf{F}_7, !dual(\mathbf{F}_4)}_{\bullet \mathbf$$

5.6 Status of \$: OK

- ullet Case rule $oldsymbol{1}$
- Case rule!
- Case rule ?W

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \$ \mathbf{F}_3} & \$ & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ? \mathbf{F}_8} & ?W \\ \hline \\ - : \vdash (\Delta_4, \mathbf{F}_2 \$ \mathbf{F}_3), \Delta_9, ? \mathbf{F}_8 & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \$ \mathbf{F}_3} & \text{ax} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6)} & \text{ax} \\ \hline \\ - : \vdash \Delta_4, \Delta_9, \mathbf{F}_2 \$ \mathbf{F}_3 & W & \text{hCut} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_6 & \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \$ \mathbf{F}_6, \Delta_4 & \$ & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \$ \mathbf{F}_6), \Delta_9, ? \mathbf{F}_8} & ?W \\ \hline \\ - : \vdash \Delta_4, \Delta_9, ? \mathbf{F}_8 & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)} & \text{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \$ \mathbf{F}_6 & \text{ax} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)} & \text{ax} \\ \hline \\ - : \vdash \Delta_4, \Delta_9, ? \mathbf{F}_8 & ?W & \text{hCut} \\ \hline \\ \hline \end{array}$$

 \bullet Case rule ?C

$$\frac{ \begin{array}{c} \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{2}, \mathbf{F}_{3}, \mathbf{F}_{6} \\ \bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{6}, \Delta_{4}, \mathbf{F}_{2} \$ \mathbf{F}_{3} \end{array} \$ \begin{array}{c} \mathbf{h}_{7} : \vdash \Delta_{9}, ? \mathbf{F}_{8}, ? \mathbf{F}_{8}, dual(\mathbf{F}_{6}) \\ \bullet \mathbf{h}_{7} : \vdash dual(\mathbf{F}_{6}), \Delta_{9}, ? \mathbf{F}_{8} \end{array}}{ - : \vdash (\Delta_{4}, \mathbf{F}_{2} \$ \mathbf{F}_{3}), \Delta_{9}, ? \mathbf{F}_{8} } \xrightarrow{\bullet} \\ \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{6}, \mathbf{F}_{2} \$ \mathbf{F}_{3} \end{array}} \begin{array}{c} ?C \\ \mathsf{Cut} \\ \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{6}, \mathbf{F}_{2} \$ \mathbf{F}_{3} \\ \hline - : \vdash \Delta_{4}, \Delta_{9}, ? \mathbf{F}_{8}, ? \mathbf{F}_{8}, ? \mathbf{F}_{8}, dual(\mathbf{F}_{6}) \\ \hline - : \vdash \Delta_{4}, \Delta_{9}, ? \mathbf{F}_{8}, \mathbf{F}_{2} \$ \mathbf{F}_{3} \\ \hline - : \vdash \Delta_{4}, \Delta_{9}, ? \mathbf{F}_{8}, \mathbf{F}_{2} \$ \mathbf{F}_{3} \end{array}} \begin{array}{c} ?C \\ \bullet \mathbf{hCut} \\ \bullet \mathbf{h$$

$$\frac{ \begin{array}{l} \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \$ \mathbf{F}_6, \Delta_4 \end{array}}{ \bullet \mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6) } \\ \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \$ \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8 \\ & - : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_8 \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \$ \mathbf{F}_6 \end{array}} \xrightarrow{\mathbf{ax}} \begin{array}{l} - : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6) \\ \hline - : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6) \\ \hline - : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_8 \end{cases} \xrightarrow{?C} \\ \mathbf{hCut}$$

• Case rule?

• Case rule \$

\bullet Case rule &

$$\frac{ \frac{h_1 : \vdash \Delta_4, F_2, F_3, F_6}{\bullet h_1 : \vdash F_6, \Delta_4, F_2\$F_3} }{ - : \vdash (\Delta_4, F_2\$F_3), \Delta_{10}, F_8\&F_9} \underbrace{ \begin{array}{c} - : \vdash (\Delta_4, F_2\$F_3), \Delta_{10}, F_8\&F_9 \\ \hline - : \vdash (\Delta_4, F_2\$F_3), \Delta_{10}, F_8\&F_9 \\ \hline - : \vdash (\Delta_4, F_2\$F_3), \Delta_{10}, F_8\&F_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, F_2, F_3, F_6 \\ \hline - : \vdash \Delta_{10}, \Delta_4, F_2, F_3, F_8\&F_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, F_2\$F_3, F_8\&F_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, F_2\$F_3, F_8\&F_9 \\ \hline - : \vdash \Delta_4, F_5, F_6 \\ \hline \bullet h_1 : \vdash F_5\$F_6, \Delta_4 \\ \hline \end{array} \underbrace{ \begin{array}{c} h_1 : \vdash \Delta_4, F_5, F_6 \\ \bullet h_1 : \vdash F_5\$F_6, \Delta_4 \\ \hline - : \vdash \Delta_4, \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6), h_7 : \vdash \Delta_{10}, F_9, dual(F_5) \otimes dual(F_6) \\ \hline - : \vdash \Delta_4, \Delta_{10}, F_8\&F_9 \\ \hline - : \vdash \Delta_4, \Delta_{10}, F_8\&F_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, F_8 \\ \hline - : \vdash \Delta_{10}, \Delta_4, F_8 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_1 : \vdash \Delta_4, F_5\$F_6 \\ h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6) \\ \hline - : \vdash \Delta_{10}, \Delta_4, F_8 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_1 : \vdash \Delta_4, F_5\$F_6 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_8 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_1 : \vdash \Delta_4, F_5\$F_6 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_8 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_1 : \vdash \Delta_4, F_5\$F_6 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_8 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_1 : \vdash \Delta_4, F_5\$F_6 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_8 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_1 : \vdash \Delta_4, F_5\$F_6 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, A_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_{10}, \Delta_4, F_9 \\ \hline \end{array} \underbrace{ \begin{array}{c} \bullet h_7 : \vdash \Delta_1, \Delta_2, F_9 \\ \hline \end{array}$$

• Case rule \oplus_B

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \$ \mathbf{F}_3} & \$ & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \oplus B \\ \hline - : \vdash (\Delta_4, \mathbf{F}_2 \$ \mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \to \\ \hline \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_6}{\bullet} & \text{ax} & \frac{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, dual(\mathbf{F}_6), \mathbf{F}_8 \oplus \mathbf{F}_9}{\bullet} & \text{hCut}} & \text{hCut} \\ \hline \frac{- : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \$ \mathbf{F}_6, \Delta_4} \$ & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \$ \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \oplus B \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \$ \mathbf{F}_6 & \text{ax} & \frac{\rightarrow}{\bullet} \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \$ \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} & \oplus B \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \$ \mathbf{F}_6 & \text{ax} & \frac{\rightarrow}{\bullet} \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)} & \text{ax} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_9 & \oplus B \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_9 & \oplus B \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 & \oplus B \\ \hline \end{array}$$

• Case rule \oplus_A

• Case rule \perp

$$\begin{array}{c|c} \frac{\mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \$ \mathbf{F}_3} \$ & \frac{\mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_6), \bot, \Delta_8} & \bot \\ \hline & - :\vdash (\Delta_4, \mathbf{F}_2 \$ \mathbf{F}_3), \bot, \Delta_8 & \to \\ \hline \frac{\mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_8, \mathbf{F}_2, \mathbf{F}_3, \bot} & \mathbf{ax} \\ \hline & \frac{- :\vdash \Delta_4, \Delta_8, \mathbf{F}_2, \mathbf{F}_3, \bot}{- :\vdash \Delta_4, \Delta_8, \bot, \mathbf{F}_2 \$ \mathbf{F}_3} \$ \\ \hline \\ \frac{\mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5, \mathbf{F}_6}{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_5 \$ \mathbf{F}_6, \Delta_4} \$ & \frac{\mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_5 \$ \mathbf{F}_6), \bot, \Delta_8} & \bot \\ \hline & - :\vdash \Delta_4, \bot, \Delta_8 \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \$ \mathbf{F}_6} & \mathbf{ax} & \to \\ \hline \\ \frac{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \$ \mathbf{F}_6}{\bullet \mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_5) \otimes dual(\mathbf{F}_6)} & \mathbf{ax} \\ \hline \\ & - :\vdash \Delta_4, \Delta_8 \\ \hline \\ - :\vdash \Delta_4, \Delta_8, \bot & \bot \\ \hline \end{array}$$

\bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \$ \mathbf{F}_3} & \$ & \frac{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \top, \Delta_8}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \top, \Delta_8} & \mathsf{Cut} \\ & & \xrightarrow{-} : \vdash (\Delta_4, \mathbf{F}_2 \$ \mathbf{F}_3), \top, \Delta_8 & \xrightarrow{-} & \vdash (-1)^{-1} + (-1)^{-1}$$

- \bullet Case rule I
- Case rule \otimes

$$\begin{array}{c|c|c|c} \frac{\mathbf{h}_1 : \vdash \Delta_4, F_2, F_3, F_6}{\mathbf{e}_{11} : \vdash F_6, \Delta_4, F_2 \$ F_3} & \mathbf{s} & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, F_8, dual(F_6)}{\mathbf{e}_{17} : \vdash dual(F_6), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} & \mathbf{cut} \\ \hline & - : \vdash (\Delta_4, F_2 \$ F_3), \Delta_{10}, \Delta_{11}, dual(F_6), F_8 \otimes F_9} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_10, \Delta_{11}, \Delta_4, F_2, F_3, F_8} & \mathbf{ax} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, F_3, F_8 \otimes F_9} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, F_3, F_8 \otimes F_9} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, \$ F_3, F_8 \otimes F_9} \\ \hline & \frac{\mathbf{h}_1 : \vdash \Delta_4, F_2, F_3, F_6}{-} & \mathbf{ax} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, \$ F_3, F_8 \otimes F_9} \\ \hline & \mathbf{h}_1 : \vdash F_6, \Delta_4, F_2, F_3, F_6} & \mathbf{ax} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, A_4, F_2, \$ F_3, F_8 \otimes F_9} \\ \hline & \mathbf{h}_1 : \vdash \Delta_4, F_2, F_3, F_6} & \mathbf{ax} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, \$ F_3, F_8 \otimes F_9} \\ \hline & \mathbf{h}_1 : \vdash \Delta_4, F_2, F_3, F_6} & \mathbf{ax} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, \$ F_3, F_8 \otimes F_9} \\ \hline & \mathbf{h}_1 : \vdash \Delta_4, F_2, F_3, F_6} & \mathbf{ax} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, \$ F_3, F_8 \otimes F_9} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, \$ F_3, F_8 \otimes F_9} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_4, F_2, F_3, F_6} & \mathbf{ax} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, \$ F_3, F_8 \otimes F_9} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_4, F_2, F_3, F_6} & \mathbf{ax} \\ \hline & \frac{-}{\mathbf{e}_{17} : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, \$ F_3, F_8 \otimes F_9} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_4, F_2, F_8, \Phi_9} & \mathbf{e} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_4, F_2, F_8, \Phi_9} & \mathbf{e} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_4, F_2, F_8, \Phi_9} & \mathbf{e} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_4, F_3, F_6} & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_1, F_8, \Phi_9 \\ \hline & \mathbf{e}_{17} : \vdash \Delta_1, \Delta_4, F_8 \otimes F_9} & \mathbf{e} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_4, F_3, F_6} & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_1, F_8, \Phi_9 \\ \hline & \mathbf{e}_{17} : \vdash \Delta_1, \Delta_4, F_8 \otimes F_9} & \mathbf{e} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_1, F_8, \Phi_9 \\ \hline & \mathbf{e}_{17} : \vdash \Delta_1, \Delta_1, F_8 \otimes F_9 \\ \hline & \mathbf{e}_{17} : \vdash \Delta_1, \Delta_1, F_8 \otimes F_9} & \mathbf{e} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_1, \Delta_1, F_8 \otimes F_9} & \mathbf{e} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_1, \Delta_1, F_8 \otimes F_9} & \mathbf{e} \\ \hline & \mathbf{e}_{17} : \vdash \Delta_1, \Delta_1, F_8$$

5.7 Status of &: OK

• Case rule 1

- Case rule!
- \bullet Case rule ?W

$$\frac{\mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6 \quad \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\underbrace{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3}} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \underbrace{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3}_{-:\vdash (\Delta_4, \mathbf{F}_2 \& \mathbf{F}_3), \Delta_9, ?\mathbf{F}_8} \underbrace{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \& \mathbf{F}_3}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_6)} \underbrace{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \& \mathbf{F}_3}_{\bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, \mathbf{F}_2 \& \mathbf{F}_3} ?W$$

$$\underbrace{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \quad \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6}_{\bullet \mathbf{h}_1 :\vdash \mathbf{h}_4, \mathbf{h}_6} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} ?W}_{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \& \mathbf{F}_6} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \& \mathbf{F}_6}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)} \underbrace{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)}_{\bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{h}_7)} \underbrace{\bullet$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{2}, \mathbf{F}_{6} \quad \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{3}, \mathbf{F}_{6}}{\bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{6}, \Delta_{4}, \mathbf{F}_{2} \& \mathbf{F}_{3}} \quad \& \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{6})}{\bullet \mathbf{h}_{7} : \vdash dual(\mathbf{F}_{6}), \Delta_{9}, ?\mathbf{F}_{8}} \\ \hline - : \vdash (\Delta_{4}, \mathbf{F}_{2} \& \mathbf{F}_{3}), \Delta_{9}, ?\mathbf{F}_{8} \\ \hline - : \vdash (\Delta_{4}, \mathbf{F}_{2} \& \mathbf{F}_{3}), \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{6})} \\ \hline - : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, qual(\mathbf{F}_{6})} \\ \hline - : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, \mathbf{F}_{2} \& \mathbf{F}_{3} \\ \hline - : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{5} \& \mathbf{F}_{6}, \Delta_{4} \\ \hline \bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{5} \& \mathbf{F}_{6}, \Delta_{4} \\ \hline \bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{5} \& \mathbf{F}_{6}, \Delta_{4} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{5} \& \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8} \\ \hline - : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{5} \& \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{h}_{5}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{h}_{5}, \mathbf{F}_{6} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{9}, ?\mathbf{F}_{8}, ?\mathbf{F}_{8}, dual(\mathbf{F}_{6}) \\ \hline \bullet \mathbf{h}_{1} : \vdash \Delta_{4},$$

• Case rule?

$$\begin{array}{c|c} \underline{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}_{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3} \\ \underline{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3}_{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{F}_6, \mathbf{F}_8, \mathbf{h}_4} \\ \underline{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{F}_6, \mathbf{F}_2 \& \mathbf{F}_3}_{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{F}_2 \& \mathbf{F}_3} \\ \underline{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \& \mathbf{F}_3}_{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{F}_2 \& \mathbf{F}_3} \\ \underline{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_6}_{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_4} & \underbrace{\bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, \mathbf{h}_4 ual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6, \mathbf{h}_6} \\ \underline{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_5 \& \mathbf{F}_6, \mathbf{h}_4}_{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_6} & \underbrace{\bullet \mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, \mathbf{h}_4 ual(\mathbf{F}_5) \oplus \mathbf{h}_4 ual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6} \\ \underline{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5, \mathbf{h}_6}_{\bullet \mathbf{h}_7} & \underbrace{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6} \\ \underbrace{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6} \\ \underbrace{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6} \\ \underbrace{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6} \\ \underbrace{\bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}_6}_{\bullet \mathbf{h}_7 : \vdash \mathbf{h}_6, \mathbf{h}$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{2}, \mathbf{F}_{6} \quad \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{3}, \mathbf{F}_{6}}{\bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{6}, \Delta_{4}, \mathbf{F}_{2} \& \mathbf{F}_{3}} & \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{6})}{\bullet \mathbf{h}_{7} : \vdash dual(\mathbf{F}_{6}), \Delta_{10}, \mathbf{F}_{8} \& \mathbf{F}_{9}} & \mathbf{Cut} \\ \hline & - : \vdash (\Delta_{4}, \mathbf{F}_{2} \& \mathbf{F}_{3}), \Delta_{10}, \mathbf{F}_{8} \& \mathbf{F}_{9} \\ \hline & \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{6}, \mathbf{F}_{2} \& \mathbf{F}_{3} & \mathbf{ax} & \rightarrow \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, \mathbf{F}_{2} \& \mathbf{F}_{3} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8} \& \mathbf{F}_{9}, \mathbf{F}_{2} \& \mathbf{F}_{3} & \mathbf{s} \\ \hline & \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{5} & \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{6} \\ \hline & \bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{5} \& \mathbf{F}_{6}, \Delta_{4} & & \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})}{\bullet \mathbf{h}_{7} : \vdash dual(\mathbf{F}_{5} \& \mathbf{F}_{6}), \Delta_{10}, \mathbf{F}_{8} \& \mathbf{F}_{9}} & \mathbf{Cut} \\ \hline & \bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{5} \& \mathbf{F}_{6} & \mathbf{ax} & & \frac{- : \vdash \Delta_{4}, \Delta_{10}, \mathbf{F}_{8} \& \mathbf{F}_{9}}{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6})} & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5}) \oplus dual(\mathbf{F}_{6}) & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_{4}, \mathbf{F}_{8}, \mathbf{F}_{9}, dual(\mathbf{F}_{5})$$

• Case rule &

$$\frac{ \underbrace{ \begin{array}{c} \underbrace{ \begin{array}{c} \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3 \end{array} }_{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3} \underbrace{ \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_6) \quad \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_6) \\ \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \& \mathbf{F}_9 \end{array} }_{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, dual(\mathbf{F}_6), \mathbf{F}_8 \& \mathbf{F}_9} \underbrace{ \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6 \end{array} }_{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, dual(\mathbf{F}_6), \mathbf{F}_8 \& \mathbf{F}_9} \underbrace{ \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6 \end{array} }_{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, dual(\mathbf{F}_6), \mathbf{F}_8 \& \mathbf{F}_9} \underbrace{ \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_8 \& \mathbf{F}_9 \end{array} }_{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_2, \mathbf{F}_8 \& \mathbf{F}_9} \underbrace{ \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{h}_4 \end{array} }_{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6) \underbrace{ \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_6) \\ \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5, \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_1, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_1, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_1, \mathbf{h}_2 : \vdash \Delta_{10}, \mathbf{h}_2, \mathbf{h}_2 : \vdash$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3} \quad \& \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline - : \vdash (\Delta_4, \mathbf{F}_2 \& \mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline - : \vdash (\Delta_4, \mathbf{F}_2 \& \mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_9, \mathbf{F}_2 \& \mathbf{F}_3} \quad \text{ax} \quad & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_6)} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_9, \mathbf{F}_2 \& \mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_2 & \bullet \mathbf{h}_3 : \vdash \mathbf{h}_4, \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5 & \bullet \mathbf{h}_4 : \vdash \mathbf{h}_4, \mathbf{h}_6 \\ \hline - : \vdash \Delta_4, \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \& \mathbf{F}_6, \Delta_4 & \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5 & \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5 & \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_6 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_6 & \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_6) \\ \hline - : \vdash \Delta_4, \Delta_8 \\$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3} \quad \& \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \& \mathbf{F}_3} \quad \text{ax} \quad & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_6)} \quad \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \& \mathbf{F}_3} \quad \text{ax} \quad & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_6)} \quad \mathbf{hCut} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5, \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6} \quad & \underbrace{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \quad \oplus \mathbf{A} \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5 & \mathbf{h}_4 : \vdash \mathbf{h}_4, \mathbf{h}_6} \quad & \underbrace{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{h}_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \quad \oplus \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 & \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_6} \quad & \underbrace{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{h}_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}_{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9} \quad \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5)} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 & \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_6} \quad & \underbrace{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \Delta_8} \quad \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5)} \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_8 & \underbrace{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5)}_{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \Delta_8} \quad \mathbf{h}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_8 & \underbrace{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)}_{\bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)} \quad \mathbf{h}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_8 & \underbrace{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)}_{\bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)} \quad \mathbf{h}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_8 & \underbrace{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)}_{\bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)} \quad \mathbf{h}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_8 & \underbrace{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)}_{\bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)} \quad \mathbf{h}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_8 & \underbrace{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)}_{\bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)} \quad \mathbf{h}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_8 & \underbrace{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_5)}_{\bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_8)} \quad \mathbf{h}_8 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_8 & \underbrace{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{h}_8)}_{\bullet \mathbf{h}_7 : \vdash \Delta$$

\bullet Case rule \bot

$$\begin{array}{c} \frac{\mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6 \quad \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3} \quad \& \quad \frac{\mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_6), \perp, \Delta_8} \quad \frac{\bot}{\mathsf{Cut}} \\ & \xrightarrow{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \& \mathbf{F}_3} \quad \Rightarrow \quad \frac{\to}{\mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_6)} \quad \frac{\mathsf{ax}}{\mathsf{hCut}} \\ & \xrightarrow{-: \vdash \Delta_4, \Delta_8, \mathbf{F}_2 \& \mathbf{F}_3} \quad \bot \\ & \xrightarrow{-: \vdash \Delta_4, \Delta_8, \mathbf{F}_2 \& \mathbf{F}_3} \quad \bot \\ & \xrightarrow{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \quad \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6} \quad \& \quad \frac{\mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \perp, \Delta_8} \quad \bot} \\ & \xrightarrow{-: \vdash \Delta_4, \Delta_8, \perp, \Delta_8} \quad \to \\ & \xrightarrow{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \& \mathbf{F}_6} \quad \mathbf{ax} \quad \xrightarrow{\bullet} \quad \frac{\to}{\mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)} \\ & \xrightarrow{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \& \mathbf{F}_6} \quad \mathbf{ax} \quad \xrightarrow{\bullet} \quad \frac{\to}{\mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)} \\ & \xrightarrow{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \& \mathbf{F}_6} \quad \mathbf{ax} \quad \xrightarrow{\bullet} \quad \frac{\to}{\mathbf{h}_7 :\vdash \Delta_8, dual(\mathbf{F}_5) \oplus dual(\mathbf{F}_6)} \\ & \xrightarrow{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \& \mathbf{F}_6} \quad \mathbf{ax} \quad \xrightarrow{\bullet} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad \mathbf{hCut} \\ & \xrightarrow{\bullet} \quad \mathbf{hCut} \quad \xrightarrow{\bullet} \quad$$

\bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \& \mathbf{F}_3} \quad \& \quad \frac{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \top, \Delta_8}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \top, \Delta_8} \quad \frac{\vdash \mathsf{Cut}}{\mathsf{Cut}} \\ & \qquad \qquad - : \vdash (\Delta_4, \mathbf{F}_2 \& \mathbf{F}_3), \top, \Delta_8 \\ & \qquad \qquad - : \vdash \Delta_4, \Delta_8, \top, \mathbf{F}_2 \& \mathbf{F}_3} \quad \top \\ \\ \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \quad \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \& \mathbf{F}_6, \Delta_4} \quad \& \quad \frac{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \top, \Delta_8}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \& \mathbf{F}_6), \top, \Delta_8} \quad \top \\ & \qquad \qquad - : \vdash \Delta_4, \top, \Delta_8 \\ & \qquad \qquad - : \vdash \Delta_4, \Delta_8, \top \end{array} \quad \top$$

- \bullet Case rule I
- \bullet Case rule \otimes

5.8 Status of \oplus_B : OK

- Case rule 1
- Case rule!
- ullet Case rule ?W

$$\begin{array}{c} \frac{\mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \oplus_B & \frac{\mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \\ \hline \bullet \mathbf{h}_1 :\vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3 & \rightarrow \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3 & \rightarrow \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3 & \rightarrow \\ \hline - :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, \mathbf{F}_2 \oplus \mathbf{F}_3 & W \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4 & \oplus_B & \frac{\mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} & ?W \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4 & \oplus_B & \frac{\mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} & Cut \\ \hline \\ \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 :\vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8 & \land \mathbf{hCut} \\ \hline \\ \hline \\ - :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8 & ?W \\ \hline \end{array}$$

 \bullet Case rule ?C

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \\ \hline - : \vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \Delta_9, ?\mathbf{F}_8 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_9, \mathbf{F}_8, \mathbf{F}_2 \oplus \mathbf{F}_3} & ? & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_6 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9} & \\ \hline - : \vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9 & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3 & \text{ax} & \rightarrow \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_2 \oplus \mathbf{F}_3 & \$ \\ \hline \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \$ \mathbf{F}_9, \mathbf{F}_2 \oplus \mathbf{F}_3 & \$ \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4 & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9} & \$ \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \$ \mathbf{F}_9} & \$ \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9} & \$ \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9} & \$ \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & \mathbf{h}_{20} & \mathbf{$$

• Case rule &

• Case rule \oplus_B

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline - : \vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_3, \mathbf{F}_6 & \mathbf{e} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4 & \oplus_B & \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_4, \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline - : \vdash \Delta_1, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \Delta_4, \Delta_$$

• Case rule \perp

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3 \end{array} \oplus_B \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_6) \\ \bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \bot, \Delta_8 \end{array} \\ - : \vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \bot, \Delta_8 \\ \hline \\ \underline{ \begin{array}{c} \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6 \end{array}} \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, \bot, dual(\mathbf{F}_6) \end{array} \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, \bot, dual(\mathbf{F}_6) \end{array} \\ \hline \\ \underline{ \begin{array}{c} - : \vdash \Delta_4, \Delta_8, \mathbf{F}_3, \bot \\ - : \vdash \Delta_4, \Delta_8, \bot, \mathbf{F}_2 \oplus \mathbf{F}_3 \end{array}} \begin{array}{c} \mathbf{ax} \\ \bullet \mathbf{hCut} \end{array} \end{array}$$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \ \oplus_B \quad \overline{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \top, \Delta_8} \\ - : \vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \top, \Delta_8 \\ \hline - : \vdash \Delta_4, \Delta_8, \top, \mathbf{F}_2 \oplus \mathbf{F}_3 \end{array} \top \\ \\ \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4} \ \oplus_B \quad \overline{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \top, \Delta_8} \\ \hline - : \vdash \Delta_4, \top, \Delta_8 \\ \hline - : \vdash \Delta_4, \Delta_8, \top \end{array} \top \\ \\ \mathbf{Cut}$$

- $\bullet\,$ Case rule I
- Case rule \otimes

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} & \oplus_B & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, dual(\mathbf{F}_6), \Delta_{10}, \Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \Delta_{11}, \mathbf{F}_8 \otimes \mathbf{F}_9} & \mathbf{Cut} \\ \hline \\ \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6 & \mathbf{ax} & & & & & & & & \\ \hline \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6 & \mathbf{ax} & & & & & & \\ \hline \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6 & \mathbf{ax} & & & & & & \\ \hline - : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & \oplus_B & \oplus_B & & & \\ \hline - : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, \mathbf{F}_8 \otimes \mathbf{F}_9, \mathbf{F}_2 \oplus \mathbf{F}_3 & \oplus_B & & & \\ \hline - : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6 & \mathbf{ax} & & & & & & \\ \hline - : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6 & \mathbf{ax} & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_3, \mathbf{F}_6 & \mathbf{ax} & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_3, \mathbf{F}_6 & \mathbf{ax} & & & & & \\ \hline - : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, \mathbf{F}_8 \otimes \mathbf{F}_9, \mathbf{F}_2 \oplus \mathbf{F}_3 & & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{ax} & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & \mathbf{ax} & & & & & \\ \hline - : \vdash \Delta_{10}, \Delta_{11}, \Delta_4, \mathbf{F}_8 \otimes \mathbf{F}_9, \mathbf{F}_2 \oplus \mathbf{F}_3 & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 & & \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 &$$

5.9 Status of \bigoplus_A : OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} & \oplus_A & \frac{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} & ?W \\ \hline \\ - : \vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \Delta_9, ?\mathbf{F}_8 & \rightarrow\\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_6)} & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_2 \oplus \mathbf{F}_3 & W & \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4 & \oplus_{\mathbf{h}_7} : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4 & \bullet_{\mathbf{h}_7} : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_9, ?\mathbf{F}_8 \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \rightarrow\\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \rightarrow\\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \mathbf{ax} & \rightarrow\\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_9, ?\mathbf{F}_8 & \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_7) \& dual(\mathbf{F}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_7) \& dual(\mathbf{F}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_7) \& dual(\mathbf{F}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_7) \& dual(\mathbf{F}_8) \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_9, dual(\mathbf{F}_8) \\ \hline \\ \bullet \mathbf{h$$

 \bullet Case rule ?C

$$\begin{array}{c} \frac{\mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6}{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} & \oplus_A & \frac{\mathbf{h}_7 :\vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} & \mathsf{Cut} \\ \hline - :\vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \Delta_9, ?\mathbf{F}_8 & \to\\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3} & \mathsf{ax} & \to\\ \hline \frac{\bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3}{- :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_6)} & \mathsf{ax} \\ \hline - :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, \mathbf{F}_2 \oplus \mathbf{F}_3} & ?C \\ \hline \bullet \mathbf{h}_1 :\vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4 & \bullet_A & \bullet_{17} :\vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \bullet \mathbf{h}_1 :\vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4 & \bullet_{17} :\vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \bullet_{17} :\vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \bullet_{17} :\vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \bullet_{17} :\vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{F}_6 & \bullet_{17} :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{F}_6 & \bullet_{17} :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{F}_6 & \bullet_{17} :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6) \\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, ?\mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \oplus_{\mathbf{A}} & \frac{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_9, ?\mathbf{F}_8} \\ \hline \\ \frac{- : \vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \Delta_9, ?\mathbf{F}_8}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_6)} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_9, \mathbf{F}_8, \mathbf{F}_2 \oplus \mathbf{F}_3}{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_8, \mathbf{F}_2 \oplus \mathbf{F}_3} ? \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4} & \oplus_{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_8}{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_8} ? \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_7 : \vdash \Delta_9, \mathbf{F}_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_9, \mathbf{F}_8}{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_8} ? \end{cases} ? \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{F}_6} & \mathbf{ax} & \mathbf{h}_{\mathbf{Cut}} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_9, \mathbf{F}_8}{- : \vdash \Delta_4, \Delta_9, ?\mathbf{F}_8} ? \end{cases} ? \\ \hline \\ \bullet \mathbf{h}_{\mathbf{Cut}} & \mathbf{h}_{\mathbf{Cut}} & \mathbf{h}_{\mathbf{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathbf{Cut}} & \mathbf{h}_{\mathbf{Cut}} \\ \hline \\ \bullet \mathbf{h}_{\mathbf{Cut}} & \mathbf{$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6}{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} & \oplus_A & \frac{\mathbf{h}_7 :\vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8\$\mathbf{F}_9} & \$\\ \hline - :\vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \Delta_{10}, \mathbf{F}_8\$\mathbf{F}_9 & \to\\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3} & \overset{\mathbf{ax}}{\longrightarrow} & \overset{\mathbf{h}_7 :\vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9, \mathbf{F}_2 \oplus \mathbf{F}_3} & \$\\ \hline \frac{\mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5}{- :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8\$\mathbf{F}_9, \mathbf{F}_2 \oplus \mathbf{F}_3} & \$\\ \hline \bullet \mathbf{h}_1 :\vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4 & \overset{\mathbf{h}_7 :\vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \Delta_{10}, \mathbf{F}_8\$\mathbf{F}_9} & \$\\ \hline - :\vdash \Delta_4, \Delta_{10}, \mathbf{F}_8\$\mathbf{F}_9 & \to\\ \hline \bullet \mathbf{h}_1 :\vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6 & \overset{\mathbf{h}_7 :\vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 :\vdash \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} & \text{ax}\\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & \text{ax}\\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9 & & & & & & & & & & \text{hCut} \\ \hline - :\vdash \Delta_{10}, \Delta_4, \mathbf{F}_8, \mathbf{F}_9$$

• Case rule &

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} \oplus_A & \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \\ \hline - : \vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6, \mathbf{F}_2 \oplus \mathbf{F}_3} & \text{ax} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_6)} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_9, \mathbf{F}_2 \oplus \mathbf{F}_3 \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_5 \oplus \mathbf{h}_6, \Delta_4} & \xrightarrow{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{h}_9, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_4, \Delta_{10}, \mathbf{h}_8 \oplus \mathbf{h}_9 \\ \hline - : \vdash \Delta_4, \Delta_{10}, \mathbf{h}_8 \oplus \mathbf{h}_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \Delta_4, \mathbf{h}_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 \oplus \mathbf{h}_6} & \text{ax} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \mathbf{h}_9 \\ \hline - : \vdash$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{h_1 : \vdash \Delta_4, F_2, F_6}{\bullet h_1 : \vdash F_6, \Delta_4, F_2 \oplus F_3} \oplus_A & \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_{10}, F_8 \oplus F_9} \\ \hline \\ \frac{- : \vdash (\Delta_4, F_2 \oplus F_3), \Delta_{10}, F_8 \oplus F_9}{\bullet h_7 : \vdash \Delta_{10}, dual(F_6), F_8 \oplus F_9} & \text{ax} \\ \hline \\ \frac{- : \vdash \Delta_{10}, \Delta_4, F_2, F_8 \oplus F_9}{- : \vdash \Delta_{10}, \Delta_4, F_2 \oplus F_3, F_8 \oplus F_9} \oplus_A \\ \hline \\ \frac{h_1 : \vdash \Delta_4, F_5}{\bullet h_1 : \vdash F_5 \oplus F_6, \Delta_4} \oplus_A & \frac{h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \\ \hline \\ \frac{\bullet h_1 : \vdash \Delta_4, F_5 \oplus F_6}{\bullet h_1 : \vdash F_8, \Phi_8, \Phi_8} & \text{ax} \\ \hline \\ \frac{- : \vdash \Delta_1, \Delta_1, F_8, dual(F_5) \& dual(F_6)}{\bullet h_7 : \vdash \Delta_{10}, F_8, dual(F_5) \& dual(F_6)} & \text{ax} \\ \hline \\ \frac{- : \vdash \Delta_{10}, \Delta_4, F_8}{- : \vdash \Delta_{10}, \Delta_4, F_8} \oplus_F_9 & \oplus_A \\ \hline \end{array}$$

• Case rule \perp

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_6}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6, \Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3} & \oplus_A & \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_6), \bot, \Delta_8} & \bot \\ \hline \\ \frac{- : \vdash (\Delta_4, \mathbf{F}_2 \oplus \mathbf{F}_3), \bot, \Delta_8}{\bullet \mathbf{h}_7 : \vdash \Delta_8, \bot, dual(\mathbf{F}_6)} & \text{ax} \\ \hline \\ \frac{- : \vdash \Delta_4, \Delta_8, \mathbf{F}_2, \bot}{- : \vdash \Delta_4, \Delta_8, \bot, \mathbf{F}_2 \oplus \mathbf{F}_3} & \oplus_A \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_5 \oplus \mathbf{F}_6, \Delta_4} & \oplus_A & \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)}{\bullet \mathbf{h}_7 : \vdash dual(\mathbf{F}_5 \oplus \mathbf{F}_6), \bot, \Delta_8} & \bot \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_5 \oplus \mathbf{F}_6}{\bullet} & \mathbf{ax} & \xrightarrow{\bullet} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_8 & \to \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_8 & \to \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_8 & \to \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_8, \bot & \bot \\ \hline \end{array} \quad \begin{array}{c} \mathbf{ax} \\ \mathbf{h}_6 \\ \hline \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_5) \& dual(\mathbf{F}_6)} \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8)} \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8)} \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_7 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \& dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual(\mathbf{F}_8) \\ \bullet \mathbf{h}_8 : \vdash \Delta_8, dual($$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, F_2, F_6}{\bullet \mathbf{h}_1 : \vdash F_6, \Delta_4, F_2 \oplus F_3} \oplus_A & \overline{\bullet \mathbf{h}_7 : \vdash dual(F_6), \top, \Delta_8} & \top \\ \hline - : \vdash (\Delta_4, F_2 \oplus F_3), \top, \Delta_8 & \rightarrow \\ \hline - : \vdash \Delta_4, \Delta_8, \top, F_2 \oplus F_3 & \top \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_4, F_5}{\bullet \mathbf{h}_1 : \vdash F_5 \oplus F_6, \Delta_4} \oplus_A & \overline{\bullet \mathbf{h}_7 : \vdash dual(F_5 \oplus F_6), \top, \Delta_8} & \top \\ \hline - : \vdash \Delta_4, \top, \Delta_8 & \rightarrow \\ \hline - : \vdash \Delta_4, \Delta_8, \top & \top \\ \hline \end{array}$$

- \bullet Case rule I
- \bullet Case rule \otimes

5.10 Status of \perp : OK

 \bullet Case rule 1

$$\begin{array}{ccc} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \ \bot & \frac{\bullet}{\bullet \mathbf{h}_3 : \vdash dual(\bot), *} \\ & \frac{- : \vdash \Delta_2, *}{- : \vdash \Delta_2} \ \mathbf{ax} \end{array} \mathbf{Cut}$$

- Case rule!
- Case rule ?W

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \perp & \frac{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_5}{\bullet \mathbf{h}_3 : \vdash dual(\bot), \Delta_5, ?F_4} \\ & - : \vdash \Delta_2, \Delta_5, ?F_4 \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \bot}{\bullet \mathbf{h}_1 : \vdash \Delta_2, \bot} \overset{\mathsf{ax}}{\underset{\mathsf{h}_3}} \overset{\mathsf{ax}}{\underset{\mathsf{h}_3}} \overset{\mathsf{ax}}{\underset{\mathsf{h}_{\mathsf{Cut}}}} \\ & - : \vdash \Delta_2, \Delta_5 \\ \hline & - : \vdash \Delta_2, \Delta_5 \\ \hline & - : \vdash \Delta_2, \Delta_5, ?F_4 \end{array} ?W \\ \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, F_4}{\bullet \mathbf{h}_1 : \vdash F_4, \bot, \Delta_2} \perp & \frac{\mathbf{h}_5 : \vdash \Delta_7, dual(F_4)}{\bullet \mathbf{h}_5 : \vdash dual(F_4), \Delta_7, ?F_6} & ?W \\ \hline & - : \vdash (\bot, \Delta_2), \Delta_7, ?F_6 \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_4, \bot} \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_4, \bot} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_4, \bot} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_1 : \vdash \Delta_2, F_4, \bot} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_2 : \vdash \Delta_2, \Delta_7, \bot} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_{\mathsf{Cut}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_{\mathsf{Cut}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_{\mathsf{Cut}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_{\mathsf{Cut}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_{\mathsf{h}_5} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ \hline & \bullet \mathbf{h}_{\mathsf{h}_5} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} \\ & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}_5}} & \overset{\mathsf{ax}}{\underset{\mathsf{h}$$

• Case rule ?C

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \perp & \frac{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_5, \mathbf{F}_4}{\bullet \mathbf{h}_3 : \vdash dual(\bot), \Delta_5, ?\mathbf{F}_4} ? \\ \hline \\ \frac{- : \vdash \Delta_2, \Delta_5, ?\mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \Delta_2, \bot} & \mathbf{ax} & \frac{\rightarrow}{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_5, \mathbf{F}_4} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_2, \Delta_5, \mathbf{F}_4}{- : \vdash \Delta_2, \Delta_5, ?\mathbf{F}_4} ? & \mathbf{hCut} \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{- : \vdash \Delta_2, \Delta_5, ?\mathbf{F}_4} ? & \frac{\mathbf{h}_5 : \vdash \Delta_7, \mathbf{F}_6, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} ? \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4, \bot}{- : \vdash \Delta_2, \Delta_7, ?\mathbf{F}_6, dual(\mathbf{F}_4)} & \mathbf{ax} \\ \hline \\ \frac{- : \vdash \Delta_2, \Delta_7, \mathbf{F}_6, \bot}{- : \vdash \Delta_2, \Delta_7, \mathbf{F}_6, \bot} ? & \mathbf{hCut} \\ \hline \end{array}$$

• Case rule \$

• Case rule &

$$\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \bot, \Delta_2} \ \bot \ \frac{h_3 : \vdash 1, \Delta_6, F_4 \ h_3 : \vdash 1, \Delta_6, F_5}{\bullet h_3 : \vdash dual(\bot), \Delta_6, F_4\&F_5} \ \&}{- : \vdash \Delta_2, \Delta_6, F_4\&F_5} \ \text{Cut}} \xrightarrow{\bullet h_1 : \vdash \Delta_2, \bot} \frac{\text{ax}}{h_3 : \vdash 1, \Delta_6, F_4} \ \frac{\rightarrow}{h_1 : \vdash \Delta_2, \bot} \ \frac{\text{ax}}{h_3 : \vdash 1, \Delta_6, F_5} \ \frac{\rightarrow}{h_1 : \vdash \Delta_2, \Delta_6, F_4} \ \frac{- : \vdash \Delta_2, \Delta_6, F_4}{- : \vdash \Delta_2, \Delta_6, F_4\&F_5} \ \&} \ \frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash F_4, \bot, \Delta_2} \ \bot \ \frac{h_5 : \vdash \Delta_8, F_6, dual(F_4) \ h_5 : \vdash \Delta_8, F_7, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6\&F_7} \ \text{Cut}}{- : \vdash (\bot, \Delta_2), \Delta_8, F_6\&F_7} \ \frac{\rightarrow}{h_1 : \vdash \Delta_2, F_4} \ \frac{\rightarrow}{ax} \ \frac{\rightarrow}{\bullet h_5 : \vdash \Delta_8, dual(F_4), F_6\&F_7} \ \frac{\rightarrow}{- : \vdash \Delta_2, \Delta_8, F_6\&F_7} \ \frac{\rightarrow}{- : \vdash \Delta_2, \Delta_8, F_6\&F_7} \ \bot$$

• Case rule \oplus_B

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \perp & \frac{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_6, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash dual(\bot), \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5} \\ \hline - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \\ \hline \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \bot}{\bullet \mathbf{h}_1 : \vdash \Delta_2, \bot} \overset{\mathrm{ax}}{\underset{\bullet}{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_6, \mathbf{F}_5}} & \frac{\mathbf{ax}}{\mathsf{hCut}} \\ \hline - : \vdash \Delta_2, \Delta_6, \mathbf{F}_5 & \oplus_B \\ \hline \\ \frac{- : \vdash \Delta_2, \Delta_6, \mathbf{F}_5}{- : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5} & \oplus_B \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_4, \bot, \Delta_2} \perp & \frac{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \mathbf{Cut} \\ \hline \\ - : \vdash (\bot, \Delta_2), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \oplus_B \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4, \bot} & \overset{\mathrm{ax}}{\underset{\bullet}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4)}} & \overset{\mathrm{dx}}{\underset{\bullet}{\mathbf{h}_5 : \vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4)}} & \mathbf{ax} \\ \hline \\ - : \vdash \Delta_2, \Delta_8, \mathbf{F}_7, \bot} & & & & \mathbf{hCut} \\ \hline \\ - : \vdash \Delta_2, \Delta_8, \bot, \mathbf{F}_6 \oplus \mathbf{F}_7 & & & & \mathbf{hCut} \\ \hline \end{array}$$

• Case rule \oplus_A

• Case rule \perp

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \ \bot \ \frac{\mathbf{h}_3 : \vdash \mathbf{1}, \Delta_4}{\bullet \mathbf{h}_3 : \vdash dual(\bot), \bot, \Delta_4} \ \\ \hline - : \vdash \Delta_2, \bot, \Delta_4 \\ \hline \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \bot}{\bullet \mathbf{h}_3 : \vdash \mathbf{1}, \Delta_4} \ \frac{\mathsf{ax}}{\mathsf{h}_3 : \vdash \mathbf{1}, \Delta_4} \ \frac{\mathsf{ax}}{\mathsf{hCut}} \\ \hline \frac{- : \vdash \Delta_2, \Delta_4}{- : \vdash \Delta_2, \Delta_4, \bot} \ \bot \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_4, \bot, \Delta_2 \end{array} \bot \begin{array}{c} \mathbf{h}_5 : \vdash \Delta_6, dual(\mathbf{F}_4) \\ \bullet \mathbf{h}_5 : \vdash dual(\mathbf{F}_4), \bot, \Delta_6 \end{array} }{ \begin{array}{c} \vdash (\bot, \Delta_2), \bot, \Delta_6 \\ \bullet \mathbf{h}_5 : \vdash \Delta_6, \bot, dual(\mathbf{F}_4) \end{array} } \begin{array}{c} \bot \\ \mathsf{Cut} \\ \bullet \mathsf{h}_1 : \vdash \Delta_2, \mathbf{F}_4 \end{array} \begin{array}{c} \mathsf{ax} \\ \bullet \mathsf{h}_5 : \vdash \Delta_6, \bot, dual(\mathbf{F}_4) \end{array} \\ \frac{\vdash : \vdash \Delta_2, \Delta_6, \bot}{\vdash : \vdash \Delta_2, \Delta_6, \bot, \bot} \begin{array}{c} \bot \\ \mathsf{hCut} \end{array}$$

\bullet Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_2}{\bullet \mathbf{h}_1 : \vdash \bot, \Delta_2} \ \bot \ \hline \bullet \mathbf{h}_3 : \vdash dual(\bot), \top, \Delta_4 \\ \hline - : \vdash \Delta_2, \top, \Delta_4 \\ \hline - : \vdash \Delta_2, \Delta_4, \top \end{array} \\ \hline \frac{\bullet}{- : \vdash \Delta_2, \Delta_4, \top} \ \top \\ \\ \frac{\mathbf{h}_1 : \vdash \Delta_2, \mathbf{F}_4}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_4, \bot, \Delta_2} \ \bot \ \hline \bullet \mathbf{h}_5 : \vdash dual(\mathbf{F}_4), \top, \Delta_6 \\ \hline - : \vdash (\bot, \Delta_2), \top, \Delta_6 \\ \hline - : \vdash (\bot, \Delta_2, \Delta_6, \bot, \top \end{array} \\ \hline \end{array} \\ \begin{array}{c} \top \\ \mathsf{Cut} \end{array}$$

- ullet Case rule I
- \bullet Case rule \otimes

5.11 Status of \top : OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\begin{array}{c|c} & \mathbf{h_3} : \vdash \mathbf{0}, \Delta_5 \\ \hline \bullet \mathbf{h_1} : \vdash \top, \Delta_2 & \top & \bullet \mathbf{h_3} : \vdash dual(\top), \Delta_5, ?F_4 \\ \hline & -: \vdash \Delta_2, \Delta_5, ?F_4 \\ \hline \bullet \mathbf{h_1} : \vdash \Delta_2, \top, ?F_4 & \top & \mathbf{h_3} : \vdash \mathbf{0}, \Delta_5 \\ \hline & \bullet \mathbf{h_1} : \vdash \Delta_2, \top, ?F_4 & \top \\ \hline & \bullet \mathbf{h_1} : \vdash \Delta_2, \top, ?F_4 & \top \\ \hline \hline \bullet \mathbf{h_1} : \vdash F_4, \top, \Delta_2 & \top & \bullet \mathbf{h_5} : \vdash \Delta_7, dual(F_4) \\ \hline & \bullet \mathbf{h_5} : \vdash dual(F_4), \Delta_7, ?F_6 \\ \hline & -: \vdash (\top, \Delta_2), \Delta_7, ?F_6 \\ \hline & -: \vdash \Delta_2, \Delta_7, \top, ?F_6 \\ \hline & -: \vdash \Delta_2, \Delta_7, \top, ?F_6 \end{array} \begin{array}{c} ?W \\ \text{Cut} \\ \hline \end{array}$$

 \bullet Case rule ?C

$$\begin{array}{c|c} \frac{\bullet \mathbf{h}_1 : \vdash \top, \Delta_2}{\bullet \mathbf{h}_1 : \vdash \top, \Delta_2} \top & \frac{\mathbf{h}_3 : \vdash \mathbf{0}, \Delta_5, ?\mathbf{F}_4, ?\mathbf{F}_4}{\bullet \mathbf{h}_3 : \vdash dual(\top), \Delta_5, ?\mathbf{F}_4} & ?C \\ \hline & - : \vdash \Delta_2, \Delta_5, ?\mathbf{F}_4 \\ \hline \bullet \underline{\bullet} \mathbf{h}_1 : \vdash \Delta_2, \top & \top & \frac{\rightarrow}{\mathbf{h}_3 : \vdash \mathbf{0}, \Delta_5, ?\mathbf{F}_4, ?\mathbf{F}_4} & \mathbf{ax} \\ \hline & \frac{- : \vdash \Delta_2, \Delta_5, ?\mathbf{F}_4, ?\mathbf{F}_4}{- : \vdash \Delta_2, \Delta_5, ?\mathbf{F}_4, ?\mathbf{F}_4} & ?C \\ \hline \\ \hline \bullet \underline{\bullet} \mathbf{h}_1 : \vdash \mathbf{F}_4, \top, \Delta_2} & \top & \frac{\mathbf{h}_5 : \vdash \Delta_7, ?\mathbf{F}_6, ?\mathbf{F}_6, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 : \vdash dual(\mathbf{F}_4), \Delta_7, ?\mathbf{F}_6} & ?C \\ \hline & - : \vdash (\top, \Delta_2), \Delta_7, ?\mathbf{F}_6 \\ \hline & \rightarrow \\ \hline & - : \vdash \Delta_2, \Delta_7, \top, ?\mathbf{F}_6 & \top \\ \hline \end{array}$$

• Case rule ?

$$\begin{array}{c} \frac{\bullet_{h_1} : \vdash \top, \Delta_2}{\bullet_{h_1} : \vdash \Delta_2, \Delta_5, ?F_4} & ? \\ \frac{- : \vdash \Delta_2, \Delta_5, ?F_4}{\bullet_{h_1} : \vdash \Delta_2, \top} & \frac{\rightarrow}{h_3} : \vdash dual(\top), \Delta_5, ?F_4} \\ \frac{\bullet_{h_1} : \vdash \Delta_2, \top}{\bullet_{h_1} : \vdash \Delta_2, \top} & \frac{\rightarrow}{h_3} : \vdash 0, \Delta_5, F_4} & \text{ax} \\ \frac{- : \vdash \Delta_2, \Delta_5, F_4}{- : \vdash \Delta_2, \Delta_5, ?F_4} & ? \\ \\ \frac{\bullet_{h_1} : \vdash F_4, \top, \Delta_2}{- : \vdash (\top, \Delta_2), \Delta_7, ?F_6} & \frac{\uparrow}{h_5} : \vdash dual(F_4), \Delta_7, ?F_6} \\ \frac{\rightarrow}{- : \vdash (\top, \Delta_2), \Delta_7, ?F_6} & \top \\ \end{array} \begin{array}{c} ? \\ \text{Cut} \\ \frac{\rightarrow}{- : \vdash \Delta_2, \Delta_7, \top, ?F_6} & \top \\ \end{array}$$

• Case rule \$

\bullet Case rule &

$$\frac{\underbrace{\bullet \mathbf{h}_1 : \vdash \top, \Delta_2} \ \top \ \frac{\mathbf{h}_3 : \vdash \mathbf{0}, \Delta_6, \mathbf{F}_4 \ \mathbf{h}_3 : \vdash \mathbf{0}, \Delta_6, \mathbf{F}_5}{\bullet \mathbf{h}_3 : \vdash dual(\top), \Delta_6, \mathbf{F}_4 \& \mathbf{F}_5} \ \underbrace{\bullet}_{\mathbf{Cut}} \\ - : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \& \mathbf{F}_5} \\ \underbrace{\bullet \mathbf{h}_1 : \vdash \Delta_2, \top} \ \top \ \frac{\rightarrow}{\mathbf{h}_3 : \vdash \mathbf{0}, \Delta_6, \mathbf{F}_4} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_6, \mathbf{F}_5} \ \top \ \frac{\mathbf{h}_3 : \vdash \mathbf{0}, \Delta_6, \mathbf{F}_5}{- : \vdash \Delta_2, \Delta_6, \mathbf{F}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_6, \mathbf{F}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_2, \Delta_6, \mathbf{F}_5}{- : \vdash \Delta_2, \Delta_6, \mathbf{F}_4 \& \mathbf{F}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \mathbf{h}_3 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_2\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_3\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5} \ \underbrace{\bullet}_{\mathbf{h}_5\mathbf{Ut}} \ \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5, \mathbf{h}_5 : \vdash \Delta_3, \mathbf{h}_5$$

• Case rule \oplus_B

$$\begin{array}{c|c} & \frac{\mathbf{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathbf{F}_5}{\bullet \mathbf{h}_3 :\vdash dual(\top), \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5} \\ & - :\vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5 \\ \hline & \frac{\bullet \mathbf{h}_1 :\vdash \Delta_2, \top}{\bullet \mathbf{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathbf{F}_5} & \frac{\mathsf{ax}}{\mathsf{hCut}} \\ \hline & \frac{- :\vdash \Delta_2, \Delta_6, \mathbf{F}_5}{- :\vdash \Delta_2, \Delta_6, \mathbf{F}_5} \oplus_B \\ \hline & \frac{- :\vdash \Delta_2, \Delta_6, \mathbf{F}_5}{- :\vdash \Delta_2, \Delta_6, \mathbf{F}_4 \oplus \mathbf{F}_5} & \oplus_B \\ \hline \\ \hline & \frac{\bullet \mathbf{h}_1 :\vdash \mathbf{F}_4, \top, \Delta_2}{\bullet \mathbf{h}_5 :\vdash \Delta_2, \Delta_6, \mathbf{F}_6 \oplus \mathbf{F}_7} & \frac{\mathsf{h}_5 :\vdash \Delta_8, \mathbf{F}_7, dual(\mathbf{F}_4)}{\bullet \mathbf{h}_5 :\vdash dual(\mathbf{F}_4), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7} & \oplus_B \\ \hline & - :\vdash (\top, \Delta_2), \Delta_8, \mathbf{F}_6 \oplus \mathbf{F}_7 & \\ \hline & - :\vdash \Delta_2, \Delta_8, \top, \mathbf{F}_6 \oplus \mathbf{F}_7 & \\ \hline \end{array}$$

• Case rule \oplus_A

$$\begin{array}{c} \underbrace{\frac{\mathbf{h}_{1} : \vdash \mathsf{T}, \Delta_{2}}{- : \vdash \Delta_{2}, \Delta_{6}, \mathsf{F}_{4} \oplus \mathsf{F}_{5}}}_{\bullet \mathsf{h}_{3} : \vdash dual(\mathsf{T}), \Delta_{6}, \mathsf{F}_{4} \oplus \mathsf{F}_{5}} \underbrace{\frac{\oplus_{A}}{- : \vdash \Delta_{2}, \Delta_{6}, \mathsf{F}_{4} \oplus \mathsf{F}_{5}}}_{\mathsf{Cut}} \\ \\ \underbrace{\frac{- : \vdash \Delta_{2}, \mathsf{T}}{\bullet_{1} : \vdash \Delta_{2}, \mathsf{T}}}_{\bullet \mathsf{h}_{3} : \vdash \mathsf{D}, \Delta_{6}, \mathsf{F}_{4}} \underbrace{\frac{\mathsf{ax}}{\mathsf{hCut}}}_{\bullet \mathsf{Cut}} \\ \underbrace{\frac{- : \vdash \Delta_{2}, \Delta_{6}, \mathsf{F}_{4}}{- : \vdash \Delta_{2}, \Delta_{6}, \mathsf{F}_{4} \oplus \mathsf{F}_{5}}}_{\bullet \mathsf{hCut}} \underbrace{\frac{\mathsf{ax}}{\mathsf{hCut}}}_{\bullet \mathsf{hCut}} \\ \underbrace{\frac{- : \vdash \Delta_{2}, \Delta_{6}, \mathsf{F}_{4} \oplus \mathsf{F}_{5}}{\bullet_{\mathsf{h}_{5}} : \vdash dual(\mathsf{F}_{4}), \Delta_{8}, \mathsf{F}_{6} \oplus \mathsf{F}_{7}}}_{\mathsf{Cut}} \underbrace{\frac{\oplus_{A}}{\mathsf{Cut}}}_{- : \vdash (\mathsf{T}, \Delta_{2}), \Delta_{8}, \mathsf{F}_{6} \oplus \mathsf{F}_{7}}}_{- : \vdash \Delta_{2}, \Delta_{8}, \mathsf{T}, \mathsf{F}_{6} \oplus \mathsf{F}_{7}} \underbrace{\mathsf{T}}_{\bullet} \end{array}$$

• Case rule \perp

$$\begin{array}{c} \underbrace{\begin{array}{c} \bullet_{h_1} : \vdash \top, \Delta_2 \end{array} \top \begin{array}{c} h_3 : \vdash \mathbf{0}, \Delta_4 \\ \bullet_{h_3} : \vdash dual(\top), \bot, \Delta_4 \end{array}}_{\bullet h_3 : \vdash dual(\top), \bot, \Delta_4} \begin{array}{c} \bot \\ \hline - : \vdash \Delta_2, \bot, \Delta_4 \\ \hline \\ \bullet_{h_1} : \vdash \Delta_2, \bot, \top \end{array} \begin{array}{c} \rightarrow \\ h_3 : \vdash \mathbf{0}, \Delta_4 \end{array} \begin{array}{c} \mathbf{ax} \\ h \mathbf{Cut} \\ \hline - : \vdash \Delta_2, \Delta_4, \bot \end{array} \\ \hline \\ \underbrace{\begin{array}{c} \bullet_{h_1} : \vdash F_4, \top, \Delta_2 \end{array}}_{\bullet h_3 : \vdash \Delta_6, dual(F_4)} \begin{array}{c} \bot \\ \bullet_{h_5} : \vdash dual(F_4), \bot, \Delta_6 \\ \hline - : \vdash (\top, \Delta_2), \bot, \Delta_6 \\ \hline - : \vdash \Delta_2, \Delta_6, \bot, \top \end{array} \begin{array}{c} \bot \\ \mathbf{Cut} \end{array}$$

\bullet Case rule \top

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1} : \vdash \top, \Delta_2 & \top & \hline \bullet_{\mathbf{h}_3} : \vdash dual(\top), \top, \Delta_4 & \top \\ \hline - : \vdash \Delta_2, \top, \Delta_4 & \rightarrow \\ \hline - : \vdash \Delta_2, \Delta_4, \top & \top \\ \hline \hline \bullet_{\mathbf{h}_1} : \vdash \mathsf{F}_4, \top, \Delta_2 & \top & \hline \bullet_{\mathbf{h}_5} : \vdash dual(\mathsf{F}_4), \top, \Delta_6 & \top \\ \hline - : \vdash (\top, \Delta_2), \top, \Delta_6 & \rightarrow \\ \hline - : \vdash \Delta_2, \Delta_6, \top, \top & \top \\ \hline \end{array}$$

ullet Case rule I

$\bullet \;$ Case rule \otimes

5.12 Status of I: OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W

 \bullet Case rule ?C

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \vdash p(\mathbf{n}_3), \, \hat{\ \ }(\mathbf{n}_3) \\ \bullet \\ & \vdash \\ I \end{array} \begin{array}{c} I \end{array} \begin{array}{c} \frac{\mathbf{h}_4 : \vdash \Delta_6, \, ?\mathbf{F}_5, \, ?\mathbf{F}_5, \, \hat{\ \ }(\mathbf{n}_3) \\ \bullet_{\mathbf{h}_4} : \vdash dual(p(\mathbf{n}_3)), \, \Delta_6, \, ?\mathbf{F}_5 \\ & \vdash \\ & \vdash \\ I \end{array} \begin{array}{c} -: \vdash \hat{\ \ }(\mathbf{n}_3), \, \Delta_6, \, ?\mathbf{F}_5 \\ & \vdash \\ I \end{array} \begin{array}{c} \mathbf{n}_4 : \vdash \Delta_6, \, ?\mathbf{F}_5, \, \hat{\ \ }(\mathbf{n}_3) \\ \hline -: \vdash \Delta_6, \, ?\mathbf{F}_5, \, \hat{\ \ }(\mathbf{n}_3) \\ \bullet_{\mathbf{h}_4} : \vdash dual(\hat{\ \ }(\mathbf{n}_3)), \, \Delta_6, \, ?\mathbf{F}_5 \\ \hline -: \vdash p(\mathbf{n}_3), \, \Delta_6, \, ?\mathbf{F}_5 \\ & \vdash \\ I \end{array} \begin{array}{c} \mathbf{n}_4 : \vdash \Delta_6, \, ?\mathbf{n}_5 \\ \hline -: \vdash p(\mathbf{n}_3), \, \Delta_6, \, ?\mathbf{n}_5 \\ \hline -: \vdash \Delta_6, \, ?\mathbf{n}_5, \, p(\mathbf{n}_3) \\ \hline -: \vdash \Delta_6, \, ?\mathbf{n}_5, \, p(\mathbf{n}_3) \\ \hline -: \vdash \Delta_6, \, ?\mathbf{n}_5, \, p(\mathbf{n}_3) \\ \hline -: \vdash \Delta_6, \, ?\mathbf{n}_5, \, p(\mathbf{n}_3) \\ \hline -: \vdash \Delta_6, \, ?\mathbf{n}_5, \, p(\mathbf{n}_3) \\ \hline -: \vdash \Delta_6, \, ?\mathbf{n}_5, \, p(\mathbf{n}_3) \\ \hline \end{array} \begin{array}{c} \mathbf{n}_4 : \vdash \mathbf{n}_5, \, \mathbf{n}_5 \\ \hline -: \vdash \mathbf{n}_6, \, \mathbf{n}_5, \, \mathbf{n}_5,$$

• Case rule?

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \vdash p(n_3), \, \hat{\ \ }(n_3) \\ - : \vdash \hat{\ \ }(n_3), \, \Delta_6, \, ?F_5 \\ - : \vdash \hat{\ \ }(n_3), \, \Delta_6, \, ?F_5 \\ - : \vdash \Delta_6, \, F_5, \, \hat{\ \ }(n_3) \\ - : \vdash \Delta_6, \, ?F_5, \, \hat{\ \ }(n_3) \\ - : \vdash \Delta_6, \, ?F_5, \, \hat{\ \ }(n_3) \end{array} }{ \begin{array}{c} \bullet_{h_4} : \vdash dual(p(n_3)), \, \Delta_6, \, ?F_5 \\ - : \vdash \Delta_6, \, F_5, \, \hat{\ \ }(n_3) \\ \hline \bullet_{h_4} : \vdash \Delta_6, \, F_5, \, p(n_3) \\ - : \vdash p(n_3), \, \Delta_6, \, ?F_5 \\ - \vdots \vdash \Delta_6, \, F_5, \, p(n_3) \\ \hline - : \vdash \Delta_6, \, F_5, \, p(n_3) \\ - : \vdash \Delta_6, \, ?F_5, \, p(n_3) \end{array} } \stackrel{?}{\text{Cut}}$$

• Case rule \$

• Case rule &

• Case rule \oplus_B

$$\frac{ \underbrace{\bullet_{h_1} :\vdash p(n_3), \, \hat{\ }_{(n_3)} } }_{\bullet h_1 :\vdash p(n_3), \, \hat{\ }_{(n_3)} } I \underbrace{ \begin{array}{c} h_4 :\vdash \Delta_7, F_6, \, \hat{\ }_{(n_3)} \\ \bullet_{h_4} :\vdash dual(p(n_3)), \Delta_7, F_5 \oplus F_6 \\ \hline \\ - :\vdash \hat{\ }_{(n_3)}, \Delta_7, F_5 \oplus F_6 \\ \hline \\ - :\vdash \Delta_7, F_6, \, \hat{\ }_{(n_3)} \\ \hline \\ - :\vdash \Delta_7, \, \hat{\ }_{(n_3)}, F_5 \oplus F_6 \\ \hline \\ \bullet_{h_1} :\vdash \hat{\ }_{(n_3)}, p(n_3) \end{array} }_{\bullet h_4 :\vdash \Delta_7, F_6, \, p(n_3)} \underbrace{ \begin{array}{c} \oplus_B \\ \\ \oplus_{h_1} :\vdash \hat{\ }_{(n_3)}, p(n_3) \\ \hline \\ - :\vdash p(n_3), \Delta_7, F_5 \oplus F_6 \\ \hline \\ - :\vdash \Delta_7, F_6, p(n_3) \\ \hline \\ - :\vdash \Delta_7, F_6, p(n_3) \\ \hline \\ - :\vdash \Delta_7, p(n_3), F_5 \oplus F_6 \\ \hline \end{array} }_{\bullet B}$$

• Case rule \oplus_A

• Case rule \perp

$$\begin{array}{c|c} \bullet_{\mathbf{h}_1} : \vdash p(\mathbf{n}_3), \, \hat{\ } (\mathbf{n}_3) & I & \frac{\mathbf{h}_4 : \vdash \Delta_5, \, \hat{\ } (\mathbf{n}_3)}{\bullet \mathbf{h}_4 : \vdash dual(p(\mathbf{n}_3)), \, \bot, \, \Delta_5} & \bot \\ & & & \\ & & & \\ & & & \\ \hline & & & - : \vdash \hat{\ } (\mathbf{n}_3), \, \bot, \, \Delta_5} & \\ & & & & \\ \hline & & & - : \vdash \Delta_5, \, \hat{\ } (\mathbf{n}_3)} & \bot & \\ \hline & & & & - : \vdash \Delta_5, \, \bot, \, \hat{\ } (\mathbf{n}_3)} & \bot & \\ \hline & & & & \\ \hline & & \bullet_{\mathbf{h}_1} : \vdash \hat{\ } (\mathbf{n}_3), \, p(\mathbf{n}_3) & I & \frac{\mathbf{h}_4 : \vdash \Delta_5, \, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 : \vdash dual(\hat{\ } (\mathbf{n}_3)), \, \bot, \, \Delta_5} & \bot \\ & & & & \\ \hline & & & & - : \vdash p(\mathbf{n}_3), \, \bot, \, \Delta_5} & \\ & & & & & \\ \hline & & & & - : \vdash \Delta_5, \, p(\mathbf{n}_3) & \bot \\ \hline & & & & - : \vdash \Delta_5, \, L, \, p(\mathbf{n}_3) & \bot \\ \hline & & & & - : \vdash \Delta_5, \, \bot, \, p(\mathbf{n}_3) & \bot \\ \hline \end{array}$$

\bullet Case rule \top

$$\begin{array}{c|c} \hline { \bullet \mathbf{h}_1 : \vdash p(\mathbf{n}_3), \hat{\ } (\mathbf{n}_3) } & I & \bullet \mathbf{h}_4 : \vdash dual(p(\mathbf{n}_3)), \top, \Delta_5 \\ \hline & - : \vdash \hat{\ } (\mathbf{n}_3), \top, \Delta_5 \\ \hline & - : \vdash \Delta_5, \top, \hat{\ } (\mathbf{n}_3) \\ \hline \hline { \bullet \mathbf{h}_1 : \vdash \hat{\ } (\mathbf{n}_3), p(\mathbf{n}_3) } & I & \bullet \mathbf{h}_4 : \vdash dual(\hat{\ } (\mathbf{n}_3)), \top, \Delta_5 \\ \hline & - : \vdash p(\mathbf{n}_3), \top, \Delta_5 \\ \hline & - : \vdash p(\mathbf{n}_3), \top, \Delta_5 \\ \hline & - : \vdash \Delta_5, \top, p(\mathbf{n}_3) \\ \hline \end{array} \right. \end{array}$$

\bullet Case rule I

$$\begin{array}{c|c} \hline { \bullet \mathbf{h}_1 : \vdash p(\mathbf{n}_4), \widehat{}(\mathbf{n}_4) } & I & \hline {\bullet \mathbf{h}_3 : \vdash dual(p(\mathbf{n}_4)), p(\mathbf{n}_4) } \\ & & - : \vdash \widehat{}(\mathbf{n}_4), p(\mathbf{n}_4) \\ \hline & & - : \vdash p(\mathbf{n}_4), \widehat{}(\mathbf{n}_4) \\ \hline {\bullet \mathbf{h}_1 : \vdash \widehat{}(\mathbf{n}_4), p(\mathbf{n}_4) } & I & \hline {\bullet \mathbf{h}_3 : \vdash dual(\widehat{}(\mathbf{n}_4)), \widehat{}(\mathbf{n}_4) } \\ \hline {\bullet \mathbf{h}_1 : \vdash \widehat{}(\mathbf{n}_4), p(\mathbf{n}_4) } & I & \hline {\bullet \mathbf{h}_3 : \vdash dual(\widehat{}(\mathbf{n}_4)), \widehat{}(\mathbf{n}_4) } & Cut \\ \hline & - : \vdash p(\mathbf{n}_4), \widehat{}(\mathbf{n}_4) & I \\ \hline & - : \vdash p(\mathbf{n}_4), \widehat{}(\mathbf{n}_4) & I \\ \hline \end{array}$$

\bullet Case rule \otimes

$$\begin{array}{c|c} & \underline{\bullet_{h_1} : \vdash \hat{\ } (n_3), p(n_3)} & I & \frac{h_4 : \vdash \Delta_7, F_5, p(n_3) & h_4 : \vdash \Delta_8, F_6}{\bullet_{h_4} : \vdash dual(\hat{\ } (n_3)), \Delta_7, \Delta_8, F_5 \otimes F_6} & \otimes \\ & \underline{- : \vdash p(n_3), \Delta_7, \Delta_8, F_5 \otimes F_6} & \underline{- : \vdash \Delta_7, F_5, p(n_3)} & \underline{- : \vdash \Delta_8, F_6} & \underline{ax} \\ & \underline{- : \vdash \Delta_7, F_5, p(n_3)} & \underline{- : \vdash \Delta_8, F_6} & \underline{ax} \\ & \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \\ \underline{\bullet_{h_1} : \vdash \hat{\ } (n_3), p(n_3)} & I & \underline{\bullet_{h_4} : \vdash \Delta_7, F_5, h_4 : \vdash \Delta_8, F_6, p(n_3)} \\ \underline{\bullet_{h_4} : \vdash dual(\hat{\ } (n_3)), \Delta_7, \Delta_8, F_5 \otimes F_6} & \underline{cut} \\ \\ \underline{- : \vdash \Delta_7, F_5} & \underline{ax} & \underline{- : \vdash \Delta_8, F_6, p(n_3)} \\ \underline{- : \vdash \Delta_7, F_5} & \underline{ax} & \underline{- : \vdash \Delta_8, F_6, p(n_3)} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{ax} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} & \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F_6} \\ \underline{- : \vdash \Delta_7, \Delta_8, p(n_3), F_5 \otimes F$$

5.13 Status of \otimes : OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_7 \quad \mathbf{h}_1 : \vdash \Delta_6, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} \\ \hline - : \vdash (\Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), \Delta_{10}, ?\mathbf{F}_9 & \rightarrow \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_7)} & \mathbf{ax} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3} & W & \\ \hline \\ \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_7 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3} & \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} & ?W \\ \hline \\ - : \vdash (\Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3) & & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3 & & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_7)} & \mathbf{ax} \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5 & & \frac{\mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_7}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} & ?W \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_6 & & \mathbf{h}_1 : \vdash \Delta_5, \mathbf{h}_7 \\ \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \mathbf{h}_5 & & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} & ?W \\ \hline \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7 & & \mathbf{ax} \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7 & & \mathbf{ax} \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7 & & \mathbf{ax} \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7 & & \mathbf{ax} \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7 & & \mathbf{ax} \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7 & & \mathbf{ax} \\ \bullet \mathbf{h}_3 : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9 & ?W \\ \hline \end{array}$$

• Case rule ?C

$$\frac{ \begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_7 \quad \mathbf{h}_1 : \vdash \Delta_6, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3} \otimes \begin{array}{c} \frac{\mathbf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} \\ - : \vdash (\Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), \Delta_{10}, ?\mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} \xrightarrow{\mathbf{ax}} \begin{array}{c} \rightarrow \\ \mathbf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_7)} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \Delta_6, ?\mathbf{F}_9, ?\mathbf{F}_9, \mathbf{F}_2 \otimes \mathbf{F}_3} & ?C \end{array} \end{array} \right. \\ \begin{array}{c} \mathbf{ax} \\ \mathbf{hCut} \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_7}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} \\ & - : \vdash (\Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3), \Delta_{10}, ?\mathbf{F}_9 \\ \hline & & \rightarrow \\ & & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} & \mathbf{ax} & \rightarrow \\ \hline & \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3}{- : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_7)} & \mathbf{ax} \\ \hline & & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, \mathbf{F}_2 \otimes \mathbf{F}_3} & ?C \\ \hline \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_4, \Delta_5} & \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{10}, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6)\$dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} & \mathbf{Cut} \\ \hline \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline & \rightarrow \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7 & \mathbf{ax} \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9, dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9, ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9 & ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9 & ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9 & ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9 & ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9 & ?\mathbf{F}_9 & dual(\mathbf{F}_6)\$dual(\mathbf{F}_7) \\ \hline & - : \vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathbf{F}_9 & ?\mathbf{F}_9 & dua$$

• Case rule?

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_7 \quad \mathbf{h}_1 : \vdash \Delta_6, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{10}, ?\mathbf{F}_9} ? \\ \hline - : \vdash (\Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), \Delta_{10}, ?\mathbf{F}_9 & \rightarrow \\ \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} & \xrightarrow{\mathbf{ax}} & \mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)} \\ \hline - : \vdash \Delta_{10}, \Delta_4, \Delta_6, \mathbf{F}_9, \mathbf{F}_2 \otimes \mathbf{F}_3} ? & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)} \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_7} & \otimes & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)} ? \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3} & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)} ? \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{h}_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)} ? \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)} \end{cases} & \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{10}, \mathbf{F}_9, dual(\mathbf{F}_7)} ? \\ \hline \bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_7, \mathbf{h}_2 \otimes \mathbf{h}_3, \mathbf{h}_3$$

• Case rule \$

$$\begin{array}{c} \frac{\mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{2}, \mathbf{F}_{7} \quad \mathbf{h}_{1} : \vdash \Delta_{6}, \mathbf{F}_{3}}{\bullet \mathbf{h}_{1} : \vdash \mathbf{F}_{7}, \Delta_{4}, \Delta_{6}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}} \otimes \frac{\mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{9}, \mathbf{F}_{10}, dual(\mathbf{F}_{7})}{\bullet \mathbf{h}_{8} : \vdash dual(\mathbf{F}_{7}), \Delta_{11}, \mathbf{F}_{9} \$ \mathbf{F}_{10}} & \mathsf{Cut} \\ \hline & - : \vdash (\Delta_{4}, \Delta_{6}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}), \Delta_{11}, \mathbf{F}_{9} \$ \mathbf{F}_{10}} & \mathsf{ax} \\ \hline & \frac{\bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{6}, \mathbf{F}_{7}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}}{- : \vdash \Delta_{11}, \Delta_{4}, \Delta_{6}, \mathbf{F}_{10}, \mathbf{F}_{9}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}} & \mathsf{hCut} \\ \hline & \frac{- : \vdash \Delta_{11}, \Delta_{4}, \Delta_{6}, \mathbf{F}_{10}, \mathbf{F}_{9}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}}{- : \vdash \Delta_{11}, \Delta_{4}, \Delta_{6}, \mathbf{F}_{9} \$ \mathbf{F}_{10}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}} & \$ \\ \hline & \frac{\mathbf{h}_{1} : \vdash \Delta_{4}, \mathbf{F}_{2} \quad \mathbf{h}_{1} : \vdash \Delta_{5}, \mathbf{F}_{3}, \mathbf{F}_{7}}{- : \vdash (\Delta_{4}, \Delta_{5}, \mathbf{F}_{2} \otimes \mathbf{F}_{3})} \otimes & \frac{\mathbf{h}_{8} : \vdash \Delta_{11}, \mathbf{F}_{9}, \mathbf{F}_{10}, dual(\mathbf{F}_{7})}{\bullet \mathbf{h}_{8} : \vdash dual(\mathbf{F}_{7}), \Delta_{11}, \mathbf{F}_{9} \$ \mathbf{F}_{10}} & \$ \\ \hline & \frac{\bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{5}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}}{- : \vdash (\Delta_{11}, \Delta_{4}, \Delta_{5}, \mathbf{F}_{10}, \mathbf{F}_{9}, \mathbf{F}_{2} \otimes \mathbf{F}_{3})} & \mathbf{ax} \\ \hline & \frac{\bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{5}, \mathbf{F}_{7}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}}{- : \vdash \Delta_{11}, \Delta_{4}, \Delta_{5}, \mathbf{F}_{10}, \mathbf{F}_{9}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}} & \$ \\ \hline & \frac{\bullet \mathbf{h}_{1} : \vdash \Delta_{4}, \Delta_{5}, \mathbf{F}_{7}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}}{- : \vdash \Delta_{11}, \Delta_{4}, \Delta_{5}, \mathbf{F}_{10}, \mathbf{F}_{9}, \mathbf{F}_{2} \otimes \mathbf{F}_{3}} & \$ \\ \hline \end{array}$$

• Case rule &

$$\frac{\mathbf{h}_1 : \vdash \Delta_4, F_2, F_7 \quad \mathbf{h}_1 : \vdash \Delta_6, F_3}{\bullet \mathbf{h}_1 : \vdash F_7, \Delta_4, \Delta_6, F_2 \otimes F_3} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_{11}, F_9, dual(F_7) \quad \mathbf{h}_8 : \vdash \Delta_{11}, F_{10}, dual(F_7)}{\bullet \mathbf{h}_8 : \vdash dual(F_7), \Delta_{11}, F_9 \& F_{10}} \otimes \mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{h}_{11}, \mathbf{h}_{12}, \mathbf{h}_{13}, \mathbf{h}_{14}, \mathbf{h}_$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_7 \quad \mathbf{h}_1 : \vdash \Delta_6, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10}} \xrightarrow{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} \xrightarrow{\mathrm{ax}} \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7)}{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7)} \xrightarrow{\mathrm{ax}} \frac{- : \vdash \Delta_{11}, \Delta_4, \Delta_6, \mathbf{F}_{10}, \mathbf{F}_2 \otimes \mathbf{F}_3}{- : \vdash \Delta_{11}, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3, \mathbf{F}_9 \oplus \mathbf{F}_{10}} \oplus B} \xrightarrow{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_7} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10}} \xrightarrow{\mathbf{H}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_7)} \xrightarrow{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{h}_9 : \mathbf{h}_9 : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 : \mathbf{h}_9 : \vdash \Delta_1, \Delta_2, \Delta_3, \mathbf{h}_9 : \vdash \Delta_1, \Delta_$$

$$\frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_7}{\underbrace{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_4, \Delta_5}} \otimes \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_6)\$dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10}} \\ - : \vdash (\Delta_4, \Delta_5), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10} \\ \xrightarrow{\bullet} \underbrace{\mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7} \quad \text{ax} \quad \frac{\rightarrow}{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_{10}, dual(\mathbf{F}_6)\$dual(\mathbf{F}_7)} \\ \underbrace{- : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_{10}}_{- : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_{9} \oplus \mathbf{F}_{10}} \oplus_B \quad \oplus_B$$

• Case rule \oplus_A

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_7 \quad \mathbf{h}_1 : \vdash \Delta_6, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \overset{\oplus A}{\mathsf{Cut}} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} & \overset{\to}{\mathsf{ax}} & \frac{\mathsf{ax}}{\mathsf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, dual(\mathbf{F}_7)} & \mathsf{ax} \\ \hline \\ \frac{- : \vdash \Delta_{11}, \Delta_4, \Delta_6, \mathbf{F}_9, \mathbf{F}_2 \otimes \mathbf{F}_3}{- : \vdash \Delta_{11}, \Delta_4, \Delta_6, \mathbf{F}_9, \mathbf{F}_2 \otimes \mathbf{F}_3} & \oplus \mathsf{A} \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_7}{- : \vdash \Delta_1, \Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3} & \otimes & \frac{\mathsf{h}_8 : \vdash \Delta_{11}, \mathbf{F}_9, dual(\mathbf{F}_7)}{\bullet \mathsf{h}_8 : \vdash dual(\mathbf{F}_7), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \oplus \mathsf{A} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3}{- : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9, \mathbf{F}_2 \otimes \mathbf{F}_3} & \oplus \mathsf{A} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3}{- : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9, \mathbf{F}_2 \otimes \mathbf{F}_3} & \oplus \mathsf{A} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 \quad \mathsf{h}_1 : \vdash \Delta_5, \mathbf{F}_7}{- : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9, \mathbf{F}_2 \otimes \mathbf{F}_3, \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \oplus \mathsf{A} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 \quad \mathsf{h}_1 : \vdash \Delta_5, \mathbf{F}_7}{- : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \oplus \mathsf{A} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 \quad \mathsf{h}_1 : \vdash \Delta_5, \mathbf{F}_7}{- : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 \oplus \mathbf{F}_{10}} & \oplus \mathsf{A} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 \quad \mathsf{h}_1 : \vdash \Delta_5, \mathbf{F}_7}{- : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9} & \oplus \mathbf{F}_{10} \\ \hline \\ - : \vdash (\Delta_4, \Delta_5), \Delta_{11}, \mathbf{F}_9 \oplus \mathbf{F}_{10} & \oplus \mathsf{A} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_4, \Delta_5}{- : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9} & \oplus \mathbf{F}_{10} \\ \hline \\ - : \vdash (\Delta_1, \Delta_4, \Delta_5, \mathbf{F}_9) & \oplus \mathbf{F}_{10} \\ \hline \\ - : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 & \oplus \mathbf{F}_{10} \\ \hline \\ - : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 & \oplus \mathbf{F}_{10} \\ \hline \\ - : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 & \oplus \mathbf{F}_{10} \\ \hline \\ - : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 & \oplus \mathbf{F}_{10} \\ \hline \\ - : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 & \oplus \mathbf{F}_{10} \\ \hline \\ - : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 & \oplus \mathbf{F}_{10} \\ \hline \\ - : \vdash \Delta_{11}, \Delta_4, \Delta_5, \mathbf{F}_9 & \oplus \mathbf{F}_{10} \\ \hline \\ \end{array}$$

• Case rule \perp

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_7 \quad \mathbf{h}_1 : \vdash \Delta_6, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \bot, \Delta_9} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} & \xrightarrow{\mathbf{dx}} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_6, \Delta_9, \mathbf{F}_2 \otimes \mathbf{F}_3} & \bot \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_7}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3} & \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \bot, \Delta_9} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3} & \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_7), \bot, \Delta_9} \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_7, \mathbf{F}_2 \otimes \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \Delta_9, \bot, \mathbf{F}_2 \otimes \mathbf{F}_3} & \bot \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_7}{\bullet \mathbf{h}_4 : \vdash \Delta_5, \mathbf{h}_7} \otimes & \frac{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7)}{\bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \bot, \Delta_9} & \bot \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \mathbf{h}_5, \mathbf{h}_7}{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{h}_9, \bot, \Delta_9} & \to \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7} \text{ ax} & \frac{\rightarrow}{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7)} & \Delta_9 \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7} \text{ ax} & \frac{\rightarrow}{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7)} & \Delta_9 \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{F}_6 \otimes \mathbf{F}_7} \text{ ax} & \frac{\rightarrow}{\mathbf{h}_8 : \vdash \Delta_9, dual(\mathbf{F}_6) \$ dual(\mathbf{F}_7)} & \Delta_9 \\ \hline \\ \frac{\bullet \mathbf{h}_1 : \vdash \Delta_4, \Delta_5, \mathbf{h}_9, \bot}{\bullet \mathbf{h}_4, \Delta_5, \Delta_9, \bot} & \bot \\ \hline \end{array}$$

• Case rule \top

$$\begin{array}{c} \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_7 \quad \mathbf{h}_1 : \vdash \Delta_6, \mathbf{F}_3}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3} \otimes \\ \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3} & \rightarrow \\ \hline - : \vdash (\Delta_4, \Delta_6, \mathbf{F}_2 \otimes \mathbf{F}_3), \top, \Delta_9 \\ \hline - : \vdash \Delta_4, \Delta_6, \Delta_9, \top, \mathbf{F}_2 \otimes \mathbf{F}_3} & \top \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_2 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_7}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3} \otimes \\ \hline \hline \bullet \mathbf{h}_1 : \vdash \mathbf{F}_7, \Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3} & \rightarrow \\ \hline - : \vdash (\Delta_4, \Delta_5, \mathbf{F}_2 \otimes \mathbf{F}_3), \top, \Delta_9 \\ \hline - : \vdash \Delta_4, \Delta_5, \Delta_9, \top, \mathbf{F}_2 \otimes \mathbf{F}_3} & \top \\ \hline \\ \frac{\mathbf{h}_1 : \vdash \Delta_4, \mathbf{F}_6 \quad \mathbf{h}_1 : \vdash \Delta_5, \mathbf{F}_7}{\bullet \mathbf{h}_1 : \vdash \mathbf{F}_6 \otimes \mathbf{F}_7, \Delta_4, \Delta_5} \otimes \\ \hline \bullet \mathbf{h}_8 : \vdash dual(\mathbf{F}_6 \otimes \mathbf{F}_7), \top, \Delta_9 \\ \hline - : \vdash (\Delta_4, \Delta_5), \top, \Delta_9 \\ \hline - : \vdash (\Delta_4, \Delta_5), \top, \Delta_9 \\ \hline - : \vdash \Delta_4, \Delta_5, \Delta_9, \top & \top \\ \hline \end{array}$$

ullet Case rule I

• Case rule ⊗

```
\frac{\underset{\bullet}{\underbrace{\mathbf{h}_1 :\vdash \Delta_4, F_6 \quad \mathbf{h}_1 :\vdash \Delta_5, F_7}}{\underbrace{\bullet \mathbf{h}_1 :\vdash F_6 \otimes F_7, \Delta_4, \Delta_5}} \otimes \frac{\underset{\bullet}{\underbrace{\mathbf{h}_8 :\vdash \Delta_{11}, F_9, dual(F_6) \$ dual(F_7) \quad \mathbf{h}_8 :\vdash \Delta_{12}, F_{10}}}{\underbrace{\bullet \mathbf{h}_8 :\vdash dual(F_6 \otimes F_7), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}}_{\underbrace{\phantom{\bullet}}{\phantom{\bullet}} \underbrace{\phantom{\bullet}} \underbrace{\phantom
```

6 Cut-Elimination

6.1 Status of 1: OK

- ullet Case rule $oldsymbol{1}$
- Case rule!
- ullet Case rule ?W
- \bullet Case rule ?C
- Case rule?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I
- \bullet Case rule \otimes

6.2 Status of !: OK

- $\bullet \;$ Case rule ${\bf 1}$
- Case rule!

$$\frac{ \begin{array}{c} \frac{h_1 : \vdash F_4,?\Upsilon3}{\bullet h_1 : \vdash !F_4,?\Upsilon3} \ ! & \frac{h_6 : \vdash F_7,?\Upsilon8,?dual(F_4),contract(n_5,?dual(F_4))}{\bullet h_6 : \vdash contract(sn_5,?dual(F_4)),?\Upsilon8,!F_7} \ \\ \hline \\ \frac{- : \vdash ?\Upsilon3,?\Upsilon8,!F_7}{\bullet h_1 : \vdash ?\Upsilon3,!F_4} \ \text{ax} & \frac{- : \vdash ?\Upsilon3,?\Upsilon8,F_7}{- : \vdash ?\Upsilon3,?\Upsilon8,!F_7} \ ! \end{array} \\ \\ \frac{\bullet h_1 : \vdash ?T3,?\Upsilon8,F_7}{- : \vdash ?\Upsilon3,?\Upsilon8,!F_7} \ ! \\ \end{array}$$

\bullet Case rule ?W

$$\frac{\frac{\mathbf{h}_1 :\vdash \mathsf{F}_4,?\Upsilon 3}{\bullet \mathbf{h}_1 :\vdash !\mathsf{F}_4,?\Upsilon 3} : \frac{\mathbf{h}_6 :\vdash \Delta_8,?dual(\mathsf{F}_4),contract(\mathsf{n}_5,?dual(\mathsf{F}_4))}{\bullet \mathsf{h}_6 :\vdash contract(s\mathsf{n}_5,?dual(\mathsf{F}_4)),\Delta_8,?\mathsf{F}_7} \underbrace{?W}_{-:\vdash?\Upsilon 3,\Delta_8,?\mathsf{F}_7} \\ \frac{\bullet \mathbf{h}_1 :\vdash ?\Upsilon 3,!\mathsf{F}_4}{\bullet \mathbf{h}_6 :\vdash \Delta_8,?dual(\mathsf{F}_4),contract(\mathsf{n}_5,?dual(\mathsf{F}_4))} \\ \frac{-:\vdash?\Upsilon 3,\Delta_8}{\bullet \mathbf{h}_1 :\vdash !\mathsf{F}_4,?\Upsilon 3} \underbrace{\cdot \frac{\vdash \cdot \vdash ?\Upsilon 3,\Delta_8}{\bullet \mathbf{h}_1 :\vdash !\mathsf{F}_4,?\Upsilon 3} !}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{F}_4))} \underbrace{\cdot W}_{\bullet \mathbf{h}_6 :\vdash contract(s\mathsf{n}_5,?dual(\mathsf{F}_4)),\Delta_7}_{\bullet \mathbf{h}_6 :\vdash contract(s\mathsf{n}_5,?dual(\mathsf{F}_4))} \underbrace{\cdot W}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{F}_4))}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{F}_4))} \underbrace{\cdot W}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{F}_4))}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{F}_4))} \underbrace{\cdot W}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{F}_4))}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{F}_4))} \underbrace{\cdot W}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{F}_4))}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{n}_4))} \underbrace{\cdot W}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{n}_4))}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf{n}_5,?dual(\mathsf{n}_4))} \underbrace{\cdot W}_{\bullet \mathbf{h}_6 :\vdash \Delta_7,contract(\mathsf$$

\bullet Case rule ?C

• Case rule?

$$\frac{\underbrace{\frac{h_1 :\vdash F_4,?\Upsilon 3}{\bullet h_1 :\vdash !F_4,?\Upsilon 3}}_{\bullet h_1 :\vdash !F_4,?\Upsilon 3} : \underbrace{\frac{h_6 :\vdash \Delta_8, F_7,?dual(F_4),contract(n_5,?dual(F_4))}{\bullet h_6 :\vdash contract(sn_5,?dual(F_4)),\Delta_8,?F_7}}_{\bullet h_1 :\vdash ?\Upsilon 3,!F_4} \underbrace{\frac{-:\vdash ?\Upsilon 3,\Delta_8,F_7}{h_6 :\vdash \Delta_8, F_7,?dual(F_4),contract(n_5,?dual(F_4))}}_{\bullet h_6 :\vdash \Delta_8, F_7,?dual(F_4),contract(n_5,?dual(F_4))}} \underbrace{\frac{-:\vdash ?\Upsilon 3,\Delta_8,F_7}{-:\vdash ?\Upsilon 3,\Delta_8,?F_7}?}_{\bullet h_1 :\vdash !F_4,?\Upsilon 3} : \underbrace{\frac{h_1 :\vdash F_4,?\Upsilon 3}{\bullet h_6 :\vdash contract(sn_5,?dual(F_4)),\Delta_7}}_{\bullet h_6 :\vdash contract(sn_5,?dual(F_4)),\Delta_7} \underbrace{\frac{h_1 :\vdash ?\Upsilon 3,!F_4}{\bullet h_6 :\vdash \Delta_7,dual(F_4),contract(n_5,?dual(F_4))}}_{\bullet h_6 :\vdash \Delta_7,dual(F_4),contract(n_5,?dual(F_4))} \underbrace{\frac{ax}{hCut}}_{\bullet Lut} \underbrace{\frac{-:\vdash ?\Upsilon 3,F_4}{\bullet h_1 :\vdash ?\Upsilon 3,IF_4}}_{\bullet Lut} \underbrace{\frac{ax}{hCut}}_{\bullet Lut} \underbrace{\frac{-:\vdash ?\Upsilon 3,\Delta_7,dual(F_4)}{\bullet hCut}}_{\bullet Lut} \underbrace{\frac{ax}{hCut}}_{\bullet Lut}$$

• Case rule \$

$$\frac{\frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3}{\bullet \mathbf{h}_1 : \vdash !\mathbf{F}_4,?\Upsilon 3}}{\bullet \mathbf{h}_1 : \vdash !\mathbf{F}_4,?\Upsilon 3} : \frac{\frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8,?dual(\mathbf{F}_4), contract(\mathbf{n}_5,?dual(\mathbf{F}_4))}{\bullet \mathbf{h}_6 : \vdash contract(\mathbf{s}\mathbf{n}_5,?dual(\mathbf{F}_4)), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8}}\underbrace{\frac{- : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7\$\mathbf{F}_8}{\bullet \mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8,?dual(\mathbf{F}_4), contract(\mathbf{n}_5,?dual(\mathbf{F}_4))}_{\bullet \mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7, \mathbf{F}_8,?dual(\mathbf{F}_4), contract(\mathbf{n}_5,?dual(\mathbf{F}_4))}_{\bullet \mathbf{h}_6 : \vdash 2}} \overset{\mathrm{ax}}{\underbrace{\frac{- : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7, \mathbf{F}_8}{\bullet : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7\$\mathbf{F}_8}}}} \$$$

• Case rule &

$$\frac{\underbrace{\frac{h_1 :\vdash F_4,?\Upsilon 3}{\bullet h_1 :\vdash !F_4,?\Upsilon 3} : \frac{h_6 :\vdash \Delta_9,F_7,?dual(F_4),contract(n_5,?dual(F_4))}{\bullet h_6 :\vdash contract(sn_5,?dual(F_4)),\Delta_9,F_7\&F_8}}_{\bullet h_1 :\vdash ?\Upsilon 3,!F_4} \underbrace{\frac{cut}{h_6 :\vdash \Delta_9,F_7,?dual(F_4),contract(n_5,?dual(F_4))}}_{\bullet h_6 :\vdash \Delta_9,F_7,?dual(F_4),contract(n_5,?dual(F_4))}}_{\bullet h_1 :\vdash ?\Upsilon 3,\Delta_9,F_7} \underbrace{\frac{cut}{h_6 :\vdash \Delta_9,F_7,?dual(F_4),contract(n_5,?dual(F_4))}}_{\bullet h_1 :\vdash ?\Upsilon 3,\Delta_9,F_7} \underbrace{\frac{cut}{h_6 :\vdash \Delta_9,F_8,?dual(F_4),contract(n_5,?dual(F_4))}}_{\bullet h_1 :\vdash ?\Upsilon 3,\Delta_9,F_8}}_{\bullet h_1 :\vdash ?\Upsilon 3,\Delta_9,F_8} \underbrace{\frac{cut}{h_6 :\vdash \Delta_9,F_8,?dual(F_4),contract(n_5,?dual(F_4))}}_{\bullet h_1 :\vdash ?\Upsilon 3,\Delta_9,F_8}}_{\bullet h_1 :\vdash ?\Upsilon 3,\Delta_9,F_8}}_{\bullet h_1 :\vdash ?\Upsilon 3,\Delta_9,F_8}$$

• Case rule \oplus_B

• Case rule \oplus_A

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3}{\bullet \mathbf{h}_1 : \vdash ! \mathbf{F}_4,?\Upsilon 3} : & \frac{\mathbf{h}_6 : \vdash \Delta_9, \mathbf{F}_7,?dual(\mathbf{F}_4), contract(\mathbf{n}_5,?dual(\mathbf{F}_4))}{\bullet \mathbf{h}_6 : \vdash contract(\mathbf{s}_{\mathbf{n}_5},?dual(\mathbf{F}_4)), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \mathbf{Cut} \\ & - : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8 \\ & \rightarrow \\ \bullet \mathbf{h}_1 : \vdash ?\Upsilon 3, ! \mathbf{F}_4 & \mathbf{ax} & \frac{- : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7}{\bullet : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \oplus_A \\ & & \frac{- : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7}{- : \vdash ?\Upsilon 3, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} & \oplus_A \end{array}$$

 \bullet Case rule \bot

$$\frac{\underbrace{\frac{h_1 :\vdash F_4,?\Upsilon 3}{\bullet h_1 :\vdash !F_4,?\Upsilon 3} : \begin{array}{c} \frac{h_6 :\vdash \Delta_7,?dual(F_4),contract(n_5,?dual(F_4))}{\bullet h_6 :\vdash contract(sn_5,?dual(F_4)),\bot,\Delta_7} \\ - :\vdash ?\Upsilon 3,\bot,\Delta_7 \\ \hline \bullet h_1 :\vdash ?\Upsilon 3,!F_4 \end{array}}_{\bullet h_6 :\vdash \Delta_7,?dual(F_4),contract(n_5,?dual(F_4))} \begin{array}{c} \bot \\ \text{out} \\ \hline - :\vdash ?\Upsilon 3,\bot,\Delta_7 \\ \hline - :\vdash ?\Upsilon 3,\Delta_7 \\ \hline - :\vdash ?\Upsilon 3,\Delta_7 \\ \hline - :\vdash ?\Upsilon 3,\Delta_7,\bot \end{array}}_{\bullet hCut}$$

 \bullet Case rule \top

$$\begin{array}{c|c} \frac{\mathbf{h}_1 : \vdash \mathbf{F}_4,?\Upsilon 3}{\bullet \mathbf{h}_1 : \vdash !\mathbf{F}_4,?\Upsilon 3} & ! & \hline\\ \bullet \mathbf{h}_6 : \vdash contract(s\mathbf{n}_5,?dual(\mathbf{F}_4)),\top,\Delta_7} \\ & & - : \vdash ?\Upsilon 3,\top,\Delta_7 \\ \hline & & - : \vdash ?\Upsilon 3,\Delta_7,\top \end{array} \top$$

- \bullet Case rule I
- \bullet Case rule \otimes

$$\begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 3}_{\bullet \mathbf{h}_{1} : \vdash \vdash F_{4}, ? \Upsilon 3} : \begin{array}{c} \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, ? dual(F_{4}), contract(\mathbf{n}_{5}, ? dual(F_{4})), \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8} \\ - : \vdash ? \Upsilon 3, \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, ? dual(F_{4}), contract(\mathbf{n}_{5}, ? dual(F_{4}))} \underbrace{\begin{array}{c} \mathbf{a} \mathbf{x} \\ - : \vdash ? \Upsilon 3, \Delta_{9}, F_{7}, \neg \mathbf{x} \\ \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, ? dual(F_{4}), contract(\mathbf{n}_{5}, ? dual(F_{4}))} \\ - : \vdash ? \Upsilon 3, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash \vdash F_{4}, ? \Upsilon 3} \underbrace{\begin{array}{c} \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, ? dual(F_{4}), \Delta_{9}, F_{7} \otimes F_{8} \\ - : \vdash ? \Upsilon 3, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash \vdash F_{4}, ? \Upsilon 3} \underbrace{\begin{array}{c} \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ? dual(F_{4})), \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8} \\ - : \vdash ? \Upsilon 3, \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash \vdash F_{4}, ? \Upsilon 3} \underbrace{\begin{array}{c} \mathbf{h}_{6} : \vdash \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ? dual(F_{4})), \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8} \\ - : \vdash ? \Upsilon 3, \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ? dual(F_{4})), \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ? dual(F_{4})), \Delta_{9}, \Delta_{10}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ? dual(F_{4})), \Delta_{9}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ? dual(F_{4})), \Delta_{9}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ? dual(F_{4})), \Delta_{9}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{9}, F_{7}, contract(\mathbf{n}_{5}, ? dual(F_{4})), \Delta_{9}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 3} \underbrace{\begin{array}{c} \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 3 \\ \bullet \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 3 \end{array}}_{\bullet \mathbf{h}_{1} : \vdash F_{4}, ? \Upsilon 3} \underbrace{\begin{array}{c} \mathbf{h}_{1} : \vdash \Delta_{9}, F_{7}, ? dual(F_{4}) \\ \bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{9}, A_{10}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{9}, A_{10}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{9}, A_{10}, \bullet \mathbf{h}_{9}, F_{7}, \otimes \mathbf{h}_{10} : \bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{8} \end{array}}_{\bullet \mathbf{h}_{1} : \vdash ? \Upsilon 3, \Delta_{10}, \Delta_{9}, F_{7} \otimes F_{$$

6.3 Status of ?W: OK

- \bullet Case rule 1
- Case rule!

$$\frac{\mathbf{h}_2 : \vdash ?\Upsilon 4, !\mathsf{F}_5}{\underbrace{\bullet \mathbf{h}_2 : \vdash !\mathsf{F}_5, ?\Upsilon 4, ?\mathsf{F}_3}} ?W \quad \frac{\mathbf{h}_7 : \vdash \mathsf{F}_8, ?\Upsilon 9, ?dual(\mathsf{F}_5), contract(\mathsf{n}_6, ?dual(\mathsf{F}_5))}{\underbrace{\bullet \mathsf{h}_7 : \vdash contract(\mathsf{sn}_6, ?dual(\mathsf{F}_5)), ?\Upsilon 9, !\mathsf{F}_8}} Cut \\ \underbrace{- : \vdash (?\Upsilon 4, ?\mathsf{F}_3), ?\Upsilon 9, !\mathsf{F}_8} \\ \underbrace{- : \vdash (?\Upsilon 4, ?\mathsf{F}_3), ?\Upsilon 9, !\mathsf{F}_8} \\ \underbrace{- : \vdash ?\Upsilon 4, !\mathsf{F}_5, ?\mathsf{F}_3} \\ \underbrace{- : \vdash ?\Upsilon 4, ?\Upsilon 9, \mathsf{F}_8, ?\mathsf{F}_3} \\ \underbrace{- : \vdash ?\Upsilon 4, ?\Upsilon 9, !\mathsf{F}_8, ?\mathsf{F}_3} !$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, \mathsf{!F}_5}{\bullet \mathbf{h}_2 : \vdash \mathsf{!F}_5, ? \Upsilon 4, ? \mathsf{F}_3} ?W \quad \frac{\mathbf{h}_7 : \vdash \Delta_8, contract(\mathbf{n}_6, ?dual(\mathsf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ?dual(\mathsf{F}_5)), \Delta_8} ?W \\ - : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_8 \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \mathsf{!F}_5, ? \mathsf{F}_3} \quad \text{ax} \quad \frac{\rightarrow}{\mathbf{h}_7 : \vdash \Delta_8, contract(\mathbf{n}_6, ?dual(\mathsf{F}_5))}} \quad \text{ax} \\ \hline - : \vdash ? \Upsilon 4, \Delta_8, ? \mathsf{F}_3} \quad \text{ax} \quad \frac{\rightarrow}{\mathbf{h}^7 : \vdash \Delta_8, contract(\mathbf{n}_6, ?dual(\mathsf{F}_5))}} \quad \text{ax} \\ \leftarrow : \vdash ? \Upsilon 4, \Delta_8, ? \mathsf{F}_3}$$

\bullet Case rule ?C

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} ? W \quad \frac{\mathbf{h}_7 : \vdash \Delta_9, ? F_8, ? F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_{\mathbf{n}_6}, ? dual(F_5)), \Delta_9, ? F_8} \end{aligned} ? Cut \\ - : \vdash (? \Upsilon 4, ? F_3), \Delta_9, ? F_8 \\ - : \vdash (? \Upsilon 4, ? F_8, ? F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\bullet \mathbf{h}_7 : \vdash \Delta_9, ? F_8, ? F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \end{aligned} \xrightarrow{\mathbf{ax}} \mathbf{hCut} \\ \frac{- : \vdash ? \Upsilon 4, \Delta_9, ? F_8, ? F_8, ? F_8}{- : \vdash ? \Upsilon 4, \Delta_9, ? F_3, ? F_8} ? C \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} ? W \xrightarrow{\mathbf{h}_7 : \vdash \Delta_8, ? dual(F_5), ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \\ \bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_{\mathbf{n}_6}, ? dual(F_5)), \Delta_8} \\ - : \vdash (? \Upsilon 4, ? F_3), \Delta_8 \\ - : \vdash (? \Upsilon 4, ? F_3), \Delta_8 \\ - : \vdash ? \Upsilon 4, \Delta_8, ? dual(F_5), ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \\ - : \vdash ? \Upsilon 4, \Delta_8, ? F_3 \end{aligned} \xrightarrow{\mathbf{hCut}} \overset{\mathbf{hCut}}{\mathbf{hCut}}$$

• Case rule?

$$\frac{ \begin{array}{c} \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} \end{array}{?W} \quad \frac{ \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_9, F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s} \mathbf{n}_6, ? dual(F_5)), \Delta_9, ? F_8} \end{array}{} \underbrace{ \begin{array}{c} ? \\ \mathsf{Cut} \\ } \\ - : \vdash (? \Upsilon 4, ? F_3), \Delta_9, ? F_8 \\ & \rightarrow \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3 \end{array}{} \begin{array}{c} \mathsf{ax} \\ \hline \\ \frac{-}{} : \vdash ? \Upsilon 4, \Delta_9, F_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{-} \\ \hline \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} \end{array}{} ? W \quad \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s} \mathbf{n}_6, ? dual(F_5)), \Delta_8} \\ \hline \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5} \end{array}{} \mathbf{ax} \quad \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(F_5), contract(\mathbf{s} \mathbf{n}_6, ? dual(F_5)), \Delta_8}{\bullet \mathbf{h}_7 : \vdash \Delta_8, dual(F_5), contract(\mathbf{s} \mathbf{n}_6, ? dual(F_5))} \end{array}{} ? \\ \hline \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5} \end{array}{} \mathbf{ax} \quad \frac{\mathbf{h}_7 : \vdash \Delta_8, dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\bullet \mathbf{h}_7 : \vdash \Delta_8, dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \end{array}{} ? \\ \hline \\ \frac{-}{\bullet \mathbf{h}_7 : \vdash \Delta_8, dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))}{\bullet \mathbf{h}_7 : \vdash \Delta_8, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \end{array}{} ? \\ \mathbf{h}_{\mathsf{Cut}} \\ \hline \\ \frac{-}{\bullet \mathbf{h}_7 : \vdash \Delta_8, r_4 \mathbf{h}_8, r_5}{\bullet \mathbf{h}_7 : \vdash \Delta_8, r_5, r_5} ? W}$$

• Case rule \$

$$\frac{\begin{array}{c} \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5 \\ \bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3 \end{array}}{\bullet \mathbf{h}_2 : \vdash \mathsf{P}_5, ? \Upsilon 4, ? F_3} \ ?W \ \begin{array}{c} \mathbf{h}_7 : \vdash \Delta_{10}, F_8, F_9, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \\ \bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ? dual(F_5)), \Delta_{10}, F_8 \$ F_9 \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3 \end{array}} \ \mathbf{ax} \ \begin{array}{c} - : \vdash (? \Upsilon 4, ? F_3), \Delta_{10}, F_8 \$ F_9 \\ \hline \bullet \mathbf{h}_7 : \vdash \Delta_{10}, F_8, F_9, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \\ \hline \\ & \frac{- : \vdash ? \Upsilon 4, \Delta_{10}, F_8, F_9, ? f_3}{- : \vdash ? \Upsilon 4, \Delta_{10}, ? F_3, F_8 \$ F_9} \ \$ \end{array}$$

• Case rule &

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\bullet \mathbf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3} \quad ? W \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5)) \quad \mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathsf{s}_\mathsf{n}_6, ? dual(\mathsf{F}_5)), \Delta_{10}, \mathsf{F}_8 \& \mathsf{F}_9} \quad \mathsf{Cut} \\ \\ \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5} \quad \mathsf{ax} \quad \frac{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_{10}, \mathsf{F}_8 \& \mathsf{F}_9}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \quad \mathsf{ax} \quad \frac{\mathsf{ax}}{\mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \quad \mathsf{ax} \\ & \frac{- : \vdash ? \Upsilon 4, \Delta_{10}, ? \mathsf{F}_8 \& \mathsf{F}_9}{- : \vdash ? \Upsilon 4, \Delta_{10}, ? \mathsf{F}_3, \mathsf{F}_8 \& \mathsf{F}_9} \quad ? W \\ \\ \end{array} \quad \qquad \qquad \mathsf{hCut}$$

• Case rule \oplus_B

$$\frac{ \begin{array}{c} \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5 \\ \bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3 \end{array}}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, F_9, ? dual(F_5), contract(\mathbf{n}_6, ? dual(F_5))} \\ \hline \\ \frac{\bullet \mathbf{h}_7 : \vdash \mathsf{contract}(\mathbf{s} \mathbf{n}_6, ? dual(F_5)), \Delta_{10}, F_8 \oplus F_9}{\bullet \mathbf{h}_7 : \vdash (? \Upsilon 4, ? F_3), \Delta_{10}, F_8 \oplus F_9} \\ \hline \\ \frac{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3}{\bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3} \end{array} \begin{array}{c} \mathbf{ax} \\ - : \vdash ? \Upsilon 4, \Delta_{10}, F_9, ? F_3 \\ \hline - : \vdash ? \Upsilon 4, \Delta_{10}, ? F_3, F_8 \oplus F_9 \end{array} \begin{array}{c} \oplus_B \\ \oplus_B \end{array}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5}{\bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3} \quad ?W \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, F_8, ?dual(F_5), contract(\mathbf{n}_6, ?dual(F_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s} \mathbf{n}_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \quad \frac{- : \vdash (? \Upsilon 4, ? F_3), \Delta_{10}, F_8 \oplus F_9}{\bullet} \quad \frac{-}{\mathbf{h}_7 : \vdash \Delta_{10}, F_8, ?dual(F_5), contract(\mathbf{n}_6, ?dual(F_5))}}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, F_8, ?dual(F_5), contract(\mathbf{n}_6, ?dual(F_5))} \quad \frac{\mathsf{ax}}{\mathsf{hCut}} \quad \frac{- : \vdash ? \Upsilon 4, \Delta_{10}, F_8, ? F_3}{- : \vdash ? \Upsilon 4, \Delta_{10}, ? F_8, F_8} \quad \oplus_A$$

• Case rule \perp

• Case rule \top

$$\begin{array}{c|c} \frac{\mathbf{h}_2 : \vdash : \Upsilon 4, \mathsf{IF}_5}{\bullet \mathbf{h}_2 : \vdash : \mathsf{IF}_5, : \Upsilon 4, : \mathsf{F}_3} & ?W & \hline \bullet \mathbf{h}_7 : \vdash contract(s\mathbf{n}_6, ?dual(\mathbf{F}_5)), \top, \Delta_8} \\ & - : \vdash (: \Upsilon 4, : \mathsf{F}_3), \top, \Delta_8 \\ \hline & - : \vdash : \Upsilon 4, \Delta_8, \top, : \mathsf{F}_3} & \top \end{array}$$
 Cut

- ullet Case rule I
- Case rule \otimes

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}}{\bullet \mathbf{h}_{2} : \vdash ! F_{5}, ? \Upsilon 4, ? F_{3}} \quad ?W \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, contract(\mathbf{n}_{6}, ?dual(F_{5})), \quad \mathbf{h}_{7} : \vdash \Delta_{11}, F_{9}, ?dual(F_{5})}{\bullet \mathbf{h}_{7} : \vdash contract(s\mathbf{n}_{6}, ?dual(F_{5})), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}} \quad \mathbf{Cut}$$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}}{\bullet \mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}} \quad \mathbf{ax} \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, contract(\mathbf{n}_{6}, ?dual(F_{5}))}{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \Delta_{11}, ?dual(F_{5}), F_{8} \otimes F_{9}, contract(\mathbf{n}_{6}, ?dual(F_{5}))} \quad \mathbf{ax}} \quad \mathbf{h}_{7} : \vdash \Delta_{11}, F_{9}, ?dual(F_{5})} \quad \mathbf{ax}$$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}}{\bullet \mathbf{h}_{2} : \vdash ! F_{5}, ? \Upsilon 4, ? F_{3}} \quad ?W \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, ?dual(F_{5})}{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, ?dual(F_{5})} \quad \mathbf{h}_{7} : \vdash \Delta_{11}, F_{9}, contract(\mathbf{n}_{6}, ?dual(F_{5}))} \quad \mathbf{ax}$$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}}{\bullet \mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}} \quad \mathbf{ax} \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, ?dual(F_{5})}{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, ?dual(F_{5})} \quad \mathbf{ax}$$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}}{\bullet \mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}} \quad \mathbf{ax} \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, ?dual(F_{5})}{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}} \quad \mathbf{Cut}$$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}}{\bullet \mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}} \quad \mathbf{ax} \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, ?dual(F_{5})}{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}} \quad ?W$$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}}{\bullet \mathbf{h}_{2} : \vdash ! F_{5}, ? \Upsilon 4, ? F_{3}} \quad ?W \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, h_{7} : \vdash \Delta_{11}, F_{9}, ?dual(F_{5}), contract(\mathbf{n}_{6}, ?dual(F_{5}))}{\bullet \mathbf{h}_{7} : \vdash Contract(\mathbf{s}_{10}, ?dual(F_{5})), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}} \quad \mathbf{cut}$$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}}{\bullet \mathbf{h}_{2} : \vdash ! F_{5}, ? \Upsilon 4, ? F_{3}} \quad \mathbf{ax} \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, h_{7} : \vdash \Delta_{11}, F_{9}, ?dual(F_{5}), contract(\mathbf{n}_{6}, ?dual(F_{5}))}{\bullet \mathbf{h}_{7} : \vdash contract(\mathbf{s}_{10}, ?dual(F_{5}), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}} \quad \mathbf{cut}$$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}, ? F_{3}}{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, ? F_{3}} \quad \mathbf{ax} \quad \frac{\mathbf{h}_{7} : \vdash \Delta_{11}, F_{9}, ?dual(F_{5}), contract(\mathbf{n}_{6}, ?dual(F_{5}))}{\bullet \mathbf{h}$$

6.4 Status of ?*C*: OK

- Case rule 1
- Case rule!

$$\frac{\begin{array}{c} \frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3, ? \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3} \end{array} ? C \quad \frac{\begin{array}{c} \mathbf{h}_7 : \vdash \mathbf{F}_8, ? \Upsilon 9, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ? dual(\mathbf{F}_5)), ? \Upsilon 9, ! \mathbf{F}_8} \\ \\ \underline{\begin{array}{c} - : \vdash (? \Upsilon 4, ? \mathbf{F}_3), ? \Upsilon 9, ! \mathbf{F}_8 \\ \rightarrow \\ \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3 \end{array}} \quad \underset{\mathbf{ax}}{\mathbf{ax}} \quad \frac{- : \vdash ? \Upsilon 9, \mathbf{F}_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash ? \Upsilon 9, \mathbf{F}_8, ? \mathbf{f}_3} \\ \underline{- : \vdash ? \Upsilon 4, ? \Upsilon 9, ! \mathbf{F}_8, ? \mathbf{F}_3} \end{array} \quad ! \\ \\ \begin{array}{c} \mathbf{h}_7 : \vdash \mathbf{h}_7 \mathbf{h}_7 : \vdash \mathbf{h}_7 \mathbf{h}_7 \cdot \mathbf{h}$$

 \bullet Case rule ?W

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, |\mathbf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3|}{\underbrace{\bullet \mathbf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}}_{\underbrace{\bullet \mathbf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}}_{} ?C \underbrace{\frac{\mathbf{h}_7 : \vdash \Delta_9, ? dual(\mathsf{F}_5), contract(\mathbf{n}_6, ? dual(\mathsf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_{\mathbf{n}_6}, ? dual(\mathsf{F}_5)), \Delta_9, ? \mathsf{F}_8}}_{\underbrace{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_9, ? \mathsf{F}_8}_{}}_{} Cut}_{\underbrace{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_9, ? \mathsf{F}_8}_{} ?W}_{}_{\underbrace{- : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3}_{}}_{\underbrace{- : \vdash ? \Upsilon 4, \Delta_9, ? \mathsf{F}_3, ? \mathsf{F}_8}_{}}_{} ?W}$$

• Case rule ?C

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3}{\underbrace{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}} ? C \quad \frac{\mathbf{h}_7 : \vdash \Delta_9, ? \mathsf{F}_8, ? \mathsf{f}_8, ? \mathsf{f}_0 : \vdash \mathsf{contract}(\mathsf{n}_6, ? \mathsf{dual}(\mathsf{F}_5)), \Delta_9, ? \mathsf{F}_8}{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5)), \Delta_9, ? \mathsf{F}_8} \underbrace{\mathsf{Cut}} ? C \\ \\ \underbrace{\bullet \mathsf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3} \quad \mathsf{ax} \quad \frac{\vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_9, ? \mathsf{F}_8}{\bullet \mathsf{h}_7 : \vdash \Delta_9, ? \mathsf{F}_8, ? \mathsf{f}_8, ? \mathsf{f}_0 : \vdash \mathsf{contract}(\mathsf{n}_6, ? \mathsf{dual}(\mathsf{F}_5))} \quad \mathsf{ax} \\ \\ \underbrace{\vdash : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3}_{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \mathsf{F}_4, ? \mathsf{F}_3} ? C \quad \frac{\mathsf{h}_7 : \vdash \Delta_8, ? \mathsf{dual}(\mathsf{F}_5), ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{n}_6, ? \mathsf{dual}(\mathsf{F}_5))}{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5)), \Delta_8} \underbrace{\mathsf{Cut}}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5)), \Delta_8} C \\ \underbrace{\vdash \cdot \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_8}_{\bullet \mathsf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3} \quad \mathsf{ax} \quad \underbrace{\vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_8}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{n}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{n}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5), \mathsf{contract}(\mathsf{sn}_6, ? \mathsf{dual}(\mathsf{F}_5$$

• Case rule?

• Case rule \$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}, ? F_{3}, ? F_{3}}{\underbrace{\bullet \mathbf{h}_{2} : \vdash ! F_{5}, ? \Upsilon 4, ? F_{3}}_{\bullet \mathbf{h}_{2} : \vdash ? \Upsilon 4, ! F_{5}, ? F_{3}}} ? C \xrightarrow{\mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, F_{9}, ? dual(F_{5}), contract(\mathbf{n}_{6}, ? dual(F_{5}))}_{\bullet \mathbf{h}_{7} : \vdash contract(\mathbf{s}\mathbf{n}_{6}, ? dual(F_{5})), \Delta_{10}, F_{8} \$ F_{9}} Cut} \$ \\ \underbrace{- : \vdash (? \Upsilon 4, ? F_{3}), \Delta_{10}, F_{8} \$ F_{9}}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, F_{9}, ? dual(F_{5}), contract(\mathbf{n}_{6}, ? dual(F_{5}))}_{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, F_{8}, F_{9}, ? dual(F_{5}), contract(\mathbf{n}_{6}, ? dual(F_{5}))}} * \underbrace{- : \vdash ? \Upsilon 4, \Delta_{10}, F_{8}, F_{9}, ? F_{3}}_{\bullet \mathbf{h} \mathsf{Cut}} \$ }$$

• Case rule &

$$\frac{\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3}{\bullet \mathbf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}}{\circ \mathbf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}} ?C \xrightarrow{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \bullet \mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \& \\ \frac{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_{10}, \mathsf{F}_8 \& \mathsf{F}_9}{\bullet \mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \bullet \mathsf{ax} \xrightarrow{\mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))} \bullet \mathsf{h}_7 : \vdash \Delta_{10}, ? \mathsf{h}_7 : \vdash \Delta_$$

• Case rule \oplus_B

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3}{\underbrace{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}}_{\bullet \mathsf{h}_2 : \vdash : \vdash \mathsf{F}_5, ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3} ? C \quad \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash contract(\mathsf{s}_\mathsf{n}_6, ? dual(\mathsf{F}_5)), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9} \\ - : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ \to \\ - : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9 \\ - : \vdash ? \Upsilon 4, \Delta_{10}, \mathsf{F}_9, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}_{\mathsf{h}_2} \xrightarrow{\mathsf{h}_2}_{\mathsf{h}_2}$$

• Case rule \oplus_A

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3, ? \mathsf{F}_3}{\underbrace{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ? \mathsf{F}_3}_{\bullet \mathsf{h}_2 : \vdash ! \mathsf{F}_5, ? \Upsilon 4, ! \mathsf{F}_5, ? \mathsf{F}_3}} \ ?C \ \frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}_{\bullet \mathsf{h}_7 : \vdash contract(\mathsf{s}_{\mathsf{n}_6}, ? dual(\mathsf{F}_5)), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9} \ Cut} \\ = \underbrace{\frac{- : \vdash (? \Upsilon 4, ? \mathsf{F}_3), \Delta_{10}, \mathsf{F}_8 \oplus \mathsf{F}_9}_{\mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}_{\mathsf{h}_7 : \vdash \Delta_{10}, \mathsf{F}_8, ? dual(\mathsf{F}_5), contract(\mathsf{n}_6, ? dual(\mathsf{F}_5))}} \ \frac{\mathsf{ax}}{\mathsf{hCut}} \\ \frac{- : \vdash ? \Upsilon 4, \Delta_{10}, \mathsf{F}_8, ? \mathsf{F}_3}{- : \vdash ? \Upsilon 4, \Delta_{10}, ? \mathsf{F}_3, \mathsf{F}_8 \oplus \mathsf{F}_9} \ \oplus_{A}$$

• Case rule \perp

$$\frac{\mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3, ? \mathbf{F}_3}{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3} ? C \xrightarrow{\begin{array}{c} \mathbf{h}_7 : \vdash \Delta_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ? dual(\mathbf{F}_5)), \bot, \Delta_8} & \mathsf{Cut} \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3 & \mathsf{ax} & \xrightarrow{} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \mathsf{cut} \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, ! \mathbf{F}_5, ? \mathbf{F}_3 & \mathsf{ax} & \xrightarrow{} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \overset{}{\rightarrow} & \mathsf{cut} \\ \hline & & & & & & & & & & \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \Delta_8, ? \mathbf{F}_3 & \bot & & & & & & \\ \hline \bullet \mathbf{h}_2 : \vdash ? \Upsilon 4, \Delta_8, ? \mathbf{F}_3 & \bot & & & & & \\ \hline \end{array}$$

• Case rule \top

$$\frac{ \begin{array}{c} \mathbf{h}_2 : \vdash ? \Upsilon 4, ! F_5, ? F_3, ? F_3 \\ \hline \bullet \mathbf{h}_2 : \vdash ! F_5, ? \Upsilon 4, ? F_3 \end{array} ? C \quad \overline{ \begin{array}{c} \bullet \mathbf{h}_7 : \vdash contract(sn_6, ?dual(F_5)), \top, \Delta_8 \\ \hline \\ - : \vdash (? \Upsilon 4, ? F_3), \top, \Delta_8 \\ \hline \\ - : \vdash ? \Upsilon 4, \Delta_8, \top, ? F_3 \end{array} \top }$$

- ullet Case rule I
- Case rule \otimes

$$\frac{h_{2}:\vdash ?\Upsilon 4, !F_{5}, ?F_{3}, ?F_{3}}{\bullet h_{2}:\vdash !F_{5}, ?\Upsilon 4, ?F_{3}} \quad ?C \quad \frac{h_{7}:\vdash \Delta_{10}, F_{8}, ?dual(F_{5}), contract(n_{6}, ?dual(F_{5})), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}}{\bullet h_{7}:\vdash contract(sn_{6}, ?dual(F_{5})), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}} \quad Cut \\ \hline \bullet h_{2}:\vdash ?\Upsilon 4, !F_{5}, ?F_{3} \quad ax \quad & -:\vdash (?\Upsilon 4, ?F_{3}), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9} \\ \hline \bullet h_{2}:\vdash ?\Upsilon 4, !F_{5}, ?F_{3} \quad ax \quad & -:\vdash ?\Upsilon 4, \Delta_{10}, F_{8}, ?dual(F_{5}), contract(n_{6}, ?dual(F_{5})) \\ \hline \bullet h_{2}:\vdash ?\Upsilon 4, !F_{5}, ?F_{3}, ?F_{3} \quad ?C \quad & \frac{h_{7}:\vdash \Delta_{10}, F_{8}, contract(n_{6}, ?dual(F_{5})) \quad h_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5})}{\bullet h_{7}:\vdash contract(sn_{6}, ?dual(F_{5})), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}} \quad Cut \\ \hline \bullet h_{2}:\vdash ?\Upsilon 4, !F_{5}, ?\Upsilon 4, ?F_{3} \quad ?C \quad & \frac{h_{7}:\vdash \Delta_{10}, F_{8}, contract(n_{6}, ?dual(F_{5}))}{\bullet h_{7}:\vdash contract(sn_{6}, ?dual(F_{5})), \Delta_{10}, \Delta_{11}, F_{8} \otimes F_{9}} \quad Cut \\ \hline \bullet h_{7}:\vdash Contract(sn_{6}, ?dual(F_{5})) \quad ax \quad & h_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5})}{\bullet h_{7}:\vdash \Delta_{10}, A_{11}, ?dual(F_{5}), F_{8} \otimes F_{9}, contract(n_{6}, ?dual(F_{5}))} \quad Ar_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5})} \quad Ar_{7}:\vdash Contract(sn_{6}, ?dual(F_{5})) \quad Ar_{7}:\vdash \Delta_{11}, F_{9}, ?dual(F_{5})) \quad Ar_{7}:\vdash Contract(sn_{6}, ?$$

$$\frac{\mathbf{h}_{2} : \vdash ? \Upsilon 4, ! \mathsf{F}_{5}, ? \mathsf{F}_{3}, ? \mathsf{F}_{3}}{\bullet \mathsf{h}_{2} : \vdash ! \mathsf{F}_{5}, ? \Upsilon 4, ? \mathsf{F}_{3}} ? C \xrightarrow{\mathbf{h}_{7} : \vdash \Delta_{10}, \mathsf{F}_{8}, ? dual(\mathsf{F}_{5})} \bullet \mathsf{h}_{7} : \vdash \Delta_{11}, \mathsf{F}_{9}, contract(\mathsf{n}_{6}, ? dual(\mathsf{F}_{5}))} \otimes \\ - : \vdash (? \Upsilon 4, ? \mathsf{F}_{3}), \Delta_{10}, \Delta_{11}, \mathsf{F}_{8} \otimes \mathsf{F}_{9} \\ - : \vdash (? \Upsilon 4, ? \mathsf{F}_{3}), \Delta_{10}, \Delta_{11}, \mathsf{F}_{8} \otimes \mathsf{F}_{9} \\ \hline - : \vdash (? \Upsilon 4, ? \mathsf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathsf{F}_{8} \otimes \mathsf{F}_{9} \\ - : \vdash (? \Upsilon 4, ? \mathsf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathsf{F}_{8} \otimes \mathsf{F}_{9} \\ \hline - : \vdash ? \Upsilon 4, \mathsf{F}_{5}, ? \mathsf{F}_{3}, ? \mathsf{F}_{3} & \mathsf{ax} & \frac{\mathsf{h}_{7} : \vdash \Delta_{10}, \mathsf{F}_{8}, ? dual(\mathsf{F}_{5})}{\bullet \mathsf{h}_{7} : \vdash \Delta_{10}, \Delta_{11}, ? \mathsf{f}_{3}, ? \mathsf{F}_{3}, \mathsf{F}_{8} \otimes \mathsf{F}_{9}, contract(\mathsf{n}_{6}, ? dual(\mathsf{F}_{5}))} & \mathsf{hCut} \\ \hline - : \vdash ? \Upsilon 4, \Delta_{10}, \Delta_{11}, ? \mathsf{F}_{3}, ? \mathsf{F}_{3}, \mathsf{F}_{8} \otimes \mathsf{F}_{9} & ? C \\ \hline \bullet \mathsf{h}_{2} : \vdash ? \Upsilon 4, ! \mathsf{F}_{5}, ? \mathsf{F}_{3}, ? \mathsf{F}_{3} & ? C & \frac{\mathsf{h}_{7} : \vdash \Delta_{10}, \mathsf{F}_{8} & \mathsf{h}_{7} : \vdash \Delta_{11}, \mathsf{F}_{9}, ? dual(\mathsf{F}_{5}), contract(\mathsf{n}_{6}, ? dual(\mathsf{F}_{5}))}{\bullet \mathsf{h}_{7} : \vdash contract(\mathsf{sn}_{6}, ? dual(\mathsf{F}_{5})), \Delta_{10}, \Delta_{11}, \mathsf{F}_{8} \otimes \mathsf{F}_{9}} & \mathsf{Cut} \\ \hline - : \vdash (? \Upsilon 4, ? \mathsf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathsf{F}_{8} \otimes \mathsf{F}_{9} & \to \mathsf{h}_{7} : \vdash \Delta_{11}, \mathsf{F}_{9}, ? dual(\mathsf{F}_{5}), contract(\mathsf{n}_{6}, ? dual(\mathsf{F}_{5}))} & \mathsf{ax} \\ \hline - : \vdash (? \Upsilon 4, ? \mathsf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathsf{F}_{8} \otimes \mathsf{F}_{9} & \to \mathsf{h}_{7} : \vdash \Delta_{11}, \mathsf{F}_{9}, ? dual(\mathsf{F}_{5}), contract(\mathsf{n}_{6}, ? dual(\mathsf{F}_{5}))} & \mathsf{ax} \\ \hline - : \vdash (? \Upsilon 4, ! \mathsf{F}_{5}, ? \mathsf{F}_{3}) & \mathsf{ax} & \to \mathsf{h}_{7} : \vdash \Delta_{11}, \mathsf{F}_{9}, ? dual(\mathsf{F}_{5}), contract(\mathsf{n}_{6}, ? dual(\mathsf{F}_{5})) & \mathsf{ax} \\ \hline - : \vdash ? \Upsilon 4, \mathsf{h}_{11}, ? \mathsf{h}_{11$$

6.5 Status of ?: OK

- Case rule 1
- Case rule!

$$\frac{\frac{\mathbf{h}_2 :\vdash F_3,?\Upsilon 4,!F_5}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3}}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3}? \frac{\frac{\mathbf{h}_7 :\vdash F_8,?\Upsilon 9,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}{\bullet \mathbf{h}_7 :\vdash contract(\mathbf{s}\mathbf{n}_6,?dual(F_5)),?\Upsilon 9,!F_8}}{\bullet} \underbrace{\overset{-}{\cdot \vdash (\Upsilon 4,?F_3),?\Upsilon 9,!F_8}}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,!F_5,?F_3}} \underbrace{\overset{-}{\cdot \vdash (\Upsilon 4,?\Upsilon 9,F_8,?F_3}}_{\bullet \mathbf{h}_7 :\vdash ?\Upsilon 4,?\Upsilon 9,!F_8,?F_3} !}$$

 \bullet Case rule ?W

• Case rule ?C

$$\frac{\underbrace{\frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_5}{\mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, ?\mathbf{F}_3}}_{\mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, !\mathbf{F}_5}? \frac{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{f}_8, ?\mathbf{dual}(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}{\mathbf{o}_{\mathbf{h}_7} : \vdash contract(\mathbf{s}_{\mathbf{n}_6}, ?dual(\mathbf{F}_5)), \Delta_9, ?\mathbf{F}_8} \underbrace{\mathbf{Cut}}_{\mathbf{c}_{\mathbf{h}_7} : \vdash (?\Upsilon 4, ?\mathbf{F}_3), \Delta_9, ?\mathbf{F}_8} \underbrace{\mathbf{Cut}}_{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{f}_8, ?\mathbf{dual}(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}_{\mathbf{h}_7 : \vdash \Delta_9, ?\mathbf{F}_8, ?\mathbf{F}_8, ?\mathbf{f}_8, ?\mathbf{F}_8} \underbrace{\mathbf{c}_{\mathbf{h}_7} : \vdash ?\Upsilon 4, \Delta_9, ?\mathbf{F}_3, ?\mathbf{F}_8}_{\mathbf{h}_7} ?\mathbf{c}_{\mathbf{h}_7} \underbrace{\mathbf{c}_{\mathbf{h}_7} : \vdash \mathcal{C}_{\mathbf{h}_7} \mathcal{C}_{\mathbf{h}_7} \mathcal{C}_{\mathbf{h}_7} \mathcal{C}_{\mathbf{h}_7}}_{\mathbf{h}_7} \underbrace{\mathbf{c}_{\mathbf{h}_7} : \vdash \mathcal{C}_{\mathbf{h}_7} \mathcal{C}_{\mathbf{h}_7$$

$$\frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, ?\mathbf{F}_3} ? \frac{\mathbf{h}_7 : \vdash \Delta_8, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_{\mathbf{n}_6}, ?dual(\mathbf{F}_5)), \Delta_8} \underbrace{\mathbf{Cut}}_{- : \vdash (?\Upsilon 4, ?\mathbf{F}_3), \Delta_8} \underbrace{- : \vdash (?\Upsilon 4, ?\mathbf{F}_3), \Delta_8}_{\mathbf{h}_7 : \vdash \Delta_8, ?dual(\mathbf{F}_5), ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}_{- : \vdash ?\Upsilon 4, \Delta_8, ?\mathbf{F}_3} \underbrace{\mathbf{ax}}_{- : \vdash ?\Upsilon 4, \Delta_8, ?\mathbf{F}_3} \underbrace{\mathbf{h}_{\mathbf{Cur}}}_{- : \vdash ?\Upsilon 4, \Delta_8, ?\mathbf{F}_3}$$

• Case rule?

$$\frac{\underbrace{\frac{h_2 :\vdash F_3,?\Upsilon 4,!F_5}{\bullet h_2 :\vdash !F_5,?\Upsilon 4,?F_3}}_{\bullet h_2 :\vdash ?\Upsilon 4,!F_5,?F_3}? \underbrace{\frac{h_7 :\vdash \Delta_9,F_8,?dual(F_5),contract(n_6,?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6,?dual(F_5)),\Delta_9,?F_8}}_{\bullet h_2 :\vdash ?\Upsilon 4,!F_5,?F_3} \underbrace{\frac{-:\vdash (?\Upsilon 4,2F_3),\Delta_9,F_8,?f_3}{h_7 :\vdash \Delta_9,F_8,?dual(F_5),contract(n_6,?dual(F_5))}}_{\bullet h_7 :\vdash \Delta_9,F_8,?dual(F_5),contract(n_6,?dual(F_5))}} \underbrace{\frac{-:\vdash ?\Upsilon 4,\Delta_9,F_8,?F_3}{-:\vdash ?\Upsilon 4,\Delta_9,?F_3,?F_8}?}_{\bullet h_2 :\vdash !F_5,?\Upsilon 4,?F_3}? \underbrace{\frac{h_7 :\vdash \Delta_8,dual(F_5),contract(n_6,?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6,?dual(F_5)),\Delta_8}}_{\bullet h_7 :\vdash contract(sn_6,?dual(F_5)),\Delta_8} \underbrace{\frac{-:\vdash ?\Upsilon 4,!F_5,?F_3}{\bullet h_7 :\vdash \Delta_8,dual(F_5),contract(n_6,?dual(F_5))}}_{-:\vdash ?\Upsilon 4,F_5,?F_3} \underbrace{\frac{a_X}{h_7 :\vdash \Delta_8,dual(F_5),contract(n_6,?dual(F_5))}}_{-:\vdash ?\Upsilon 4,\Delta_8,?F_3,dual(F_5)} \underbrace{\frac{a_X}{h_{Cut}}}_{h_{Cut}}$$

• Case rule \$

$$\frac{\underbrace{\frac{\mathbf{h}_2 :\vdash F_3,?\Upsilon 4,!F_5}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3}}_{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,!F_5}? \cdot \frac{\frac{\mathbf{h}_7 :\vdash \Delta_{10},F_8,F_9,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}{\bullet \mathbf{h}_7 :\vdash contract(\mathbf{s}_\mathbf{n}_6,?dual(F_5)),\Delta_{10},F_8\$F_9} \cdot \underbrace{\frac{-:\vdash (\Upsilon 4,?F_3),\Delta_{10},F_8\$F_9}{\to}}_{\mathsf{h}_7 :\vdash \Delta_{10},F_8,F_9,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}}_{\mathsf{h}_{\mathsf{Cut}}} \underbrace{\frac{-:\vdash ?\Upsilon 4,\Delta_{10},F_8,F_9,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}{-:\vdash ?\Upsilon 4,\Delta_{10},?F_3,F_8\$F_9}}_{\mathsf{h}_{\mathsf{Cut}}}$$

 \bullet Case rule &

$$\frac{\frac{\mathbf{h}_2 :\vdash F_3,?\Upsilon 4,!F_5}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3}}{\bullet \mathbf{h}_2 :\vdash !F_5,?\Upsilon 4,?F_3} ? \frac{\mathbf{h}_7 :\vdash \Delta_{10},F_8,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}{\bullet \mathbf{h}_7 :\vdash contract(\mathbf{s}\mathbf{n}_6,?dual(F_5)),\Delta_{10},F_8\&F_9} \underbrace{-:\vdash (?\Upsilon 4,?F_3),\Delta_{10},F_8\&F_9}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,!F_5,?F_3} \underbrace{-:\vdash ?\Upsilon 4,\Delta_{10},F_8,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}_{\bullet \mathbf{h}\mathbf{Cut}} \underbrace{-:\vdash ?\Upsilon 4,\Delta_{10},F_8,?dual(F_5),contract(\mathbf{n}_6,?dual(F_5))}_{\bullet \mathbf{h}\mathbf{Cut}} \underbrace{-:\vdash ?\Upsilon 4,\Delta_{10},F_9,?f_3}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,\Delta_{10},F_9,?F_3} \underbrace{\&}$$

• Case rule \oplus_B

$$\frac{\frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, ?\mathbf{F}_3}}{\bullet \mathbf{h}_2 : \vdash ?\Upsilon 4, !\mathbf{F}_5, ?\mathbf{F}_3} ? \frac{\frac{\mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_9, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ?dual(\mathbf{F}_5)), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9} \underbrace{\frac{-}{\mathsf{cut}} \cdot (?\Upsilon 4, ?\mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9}_{\bullet \mathbf{h}_2 : \vdash ?\Upsilon 4, !\mathbf{F}_5, ?\mathbf{F}_3} \underbrace{\frac{-}{\mathsf{c}} \vdash ?\Upsilon 4, \Delta_{10}, \mathbf{F}_9, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))}_{\bullet \mathbf{h}_2 : \vdash ?\Upsilon 4, \Delta_{10}, ?\mathbf{F}_3, \mathbf{F}_8 \oplus \mathbf{F}_9} \underbrace{\theta_B}}_{\bullet B}$$

• Case rule \oplus_A

$$\frac{\begin{array}{c} \mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_5 \\ \bullet \mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, ?\mathbf{F}_3 \end{array}}{\bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5))} \\ \bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}\mathbf{n}_6, ?dual(\mathbf{F}_5)), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, ?\mathbf{F}_3), \Delta_{10}, \mathbf{F}_8 \oplus \mathbf{F}_9 \\ \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \bullet \mathbf{h}_7 : \vdash \Delta_{10}, \mathbf{F}_8, ?dual(\mathbf{F}_5), contract(\mathbf{n}_6, ?dual(\mathbf{F}_5)) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{F}_3) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{F}_8, ?\mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 : \vdash (?\Upsilon 4, \Delta_{10}, \mathbf{h}_7, \mathbf{h}_7) \\ \bullet \mathbf{h}_7 :$$

 \bullet Case rule \bot

$$\frac{ \frac{\mathbf{h}_2 : \vdash \mathbf{F}_3, ? \Upsilon 4, ! \mathbf{F}_5}{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3} }{\bullet \mathbf{h}_2 : \vdash ! \mathbf{F}_5, ? \Upsilon 4, ? \mathbf{F}_3} ? \frac{\mathbf{h}_7 : \vdash \Delta_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 : \vdash contract(\mathbf{s}_6, ? dual(\mathbf{F}_5)), \bot, \Delta_8} \underbrace{\mathsf{Cut}}_{} \\ - : \vdash (? \Upsilon 4, ? \mathbf{F}_3), \bot, \Delta_8 \\ \xrightarrow{} \\ \mathbf{h}_7 : \vdash \Delta_8, ? dual(\mathbf{F}_5), contract(\mathbf{n}_6, ? dual(\mathbf{F}_5))}_{} \underbrace{\mathsf{hCut}}_{} \\ \frac{- : \vdash ? \Upsilon 4, \Delta_8, ? \mathbf{F}_3}{- : \vdash ? \Upsilon 4, \Delta_8, \bot, ? \mathbf{F}_3} \bot$$

• Case rule \top

$$\frac{ \begin{array}{c} \mathbf{h}_2 : \vdash \mathbf{F}_3, ?\Upsilon 4, !\mathbf{F}_5 \\ \bullet \mathbf{h}_2 : \vdash !\mathbf{F}_5, ?\Upsilon 4, ?\mathbf{F}_3 \end{array} ? \quad \overline{\bullet \mathbf{h}_7 : \vdash contract(s\mathbf{n}_6, ?dual(\mathbf{F}_5)), \top, \Delta_8} \quad \overline{\top} \\ - : \vdash (?\Upsilon 4, ?\mathbf{F}_3), \top, \Delta_8 \\ \hline - : \vdash ?\Upsilon 4, \Delta_8, \top, ?\mathbf{F}_3 \end{array} \top \\ \end{array}} \quad \overline{\top}$$

- \bullet Case rule I
- Case rule \otimes

$$\frac{\frac{\mathbf{h}_{2} : \vdash \mathbf{F}_{3}, ? \Upsilon \mathbf{4}, ! \mathbf{F}_{5}}{\bullet \mathbf{h}_{2} : \vdash ! \mathbf{F}_{5}, ? \Upsilon \mathbf{4}, ? \mathbf{F}_{3}}}{\bullet \mathbf{h}_{7} : \vdash \Delta_{10}, \mathbf{F}_{8}, ? dual(\mathbf{F}_{5}), contract(\mathbf{n}_{6}, ? dual(\mathbf{F}_{5})), \Delta_{10}, \Delta_{11}, \mathbf{F}_{8} \otimes \mathbf{F}_{9}}{\bullet \mathbf{h}_{7} : \vdash contract(\mathbf{s} \mathbf{n}_{6}, ? dual(\mathbf{F}_{5})), \Delta_{10}, \Delta_{11}, \mathbf{F}_{8} \otimes \mathbf{F}_{9}} \\ - : \vdash (? \Upsilon \mathbf{4}, ? \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash ? \Upsilon \mathbf{4}, \mathbf{1}_{10}, \mathbf{F}_{8}, ? \mathbf{5}_{10}, \Delta_{10}, \mathbf{5}_{8}, ? dual(\mathbf{F}_{5}), contract(\mathbf{n}_{6}, ? dual(\mathbf{F}_{5}))} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \mathbf{F}_{8}, ? \mathbf{F}_{3} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \mathbf{F}_{8}, ? \mathbf{F}_{3} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \mathbf{F}_{8}, ? \mathbf{F}_{3} \\ - : \vdash ? \Upsilon \mathbf{4}, \mathbf{1}_{10}, \mathbf{F}_{8}, ? \mathbf{F}_{4}, \mathbf{F}_{5} \\ \bullet \mathbf{h}_{2} : \vdash ! \mathbf{F}_{5}, ? \Upsilon \mathbf{4}, ? \mathbf{F}_{3} \\ - : \vdash (? \Upsilon \mathbf{4}, ? \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, ? \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, ? \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, ? \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \mathbf{F}_{8}, ? \mathbf{7}_{3} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \mathbf{F}_{8}, ? \mathbf{7}_{3} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \mathbf{F}_{8}, ? \mathbf{7}_{3} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \mathbf{F}_{8}, ? \mathbf{7}_{3} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{3} \otimes \mathbf{F}_{9} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash ? \Upsilon \mathbf{4}, \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, ? \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, ? \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, 2 \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, 2 \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, \mathbf{F}_{8} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, 2 \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{3} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, 2 \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{3} \otimes \mathbf{F}_{9} \\ - : \vdash (? \Upsilon \mathbf{4}, 2 \mathbf{F}_{3}), \Delta_{10}, \Delta_{11}, ? \mathbf{F}_{3}, ? \mathbf{F}_{3$$

$$\frac{\frac{\mathbf{h}_2 :\vdash \mathbf{F}_3,?\Upsilon 4,!\mathbf{F}_5}{\bullet \mathbf{h}_2 :\vdash !\mathbf{F}_5,?\Upsilon 4,?\mathbf{F}_3}}{\bullet \mathbf{h}_2 :\vdash !\mathbf{F}_5,?\Upsilon 4,?\mathbf{F}_3}} ? \frac{\mathbf{h}_7 :\vdash \Delta_{10},\mathbf{F}_8 \quad \mathbf{h}_7 :\vdash \Delta_{11},\mathbf{F}_9,?dual(\mathbf{F}_5),contract(\mathbf{n}_6,?dual(\mathbf{F}_5))}{\bullet \mathbf{h}_7 :\vdash contract(\mathbf{s}_{\mathbf{n}_6},?dual(\mathbf{F}_5)),\Delta_{10},\Delta_{11},\mathbf{F}_8 \otimes \mathbf{F}_9}}{\bullet \mathbf{h}_2 :\vdash !\mathbf{F}_3,?\mathbf{F}_3} \underbrace{- :\vdash (?\Upsilon 4,!\mathbf{F}_5,?\mathbf{F}_3)}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,!\mathbf{F}_5,?\mathbf{F}_3} \underbrace{\times}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,!\mathbf{F}_5,?\mathbf{F}_3} \underbrace{\times}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,\Delta_{11},\mathbf{F}_9,?dual(\mathbf{F}_5),contract(\mathbf{n}_6,?dual(\mathbf{F}_5))}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,\Delta_{11},\Upsilon 7,\mathbf{F}_3,\mathbf{F}_8 \otimes \mathbf{F}_9} \underbrace{\times}_{\bullet \mathbf{h}_2 :\vdash ?\Upsilon 4,\Delta_{11},\Upsilon 7,\mathbf{F}_3,\mathbf{F}_8 \otimes \mathbf{F}_9}$$

6.6 Status of \$: OK

- Case rule 1
- Case rule!
- \bullet Case rule ?W
- \bullet Case rule ?C
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- \bullet Case rule I
- \bullet Case rule \otimes

6.7 Status of &: OK

- Case rule 1
- Case rule!

•	Case ru	le	?W	
•	Case ru	le	?C	
•	Case ru	le	?	
•	Case ru	le	\$	
•	Case ru	le	&	
•	Case ru	le	\oplus_B	
•	Case ru	le	\oplus_A	
•	Case ru	le	1	
•	Case ru	le	Т	
•	Case ru	le	I	
•	Case ru	le	\otimes	
			\otimes of \oplus_B :	OK
6.8		ıs	of \oplus_B :	OK
6.8	Stati	ıs le	of \oplus_B :	OK
6.8	Stat ı Case ru	ıs le	of \oplus_B : 1	OK
6.8	Statu Case ru Case ru	ıs le le	of \bigoplus_B : 1 ! ? W	OK
6.8	Statu Case ru Case ru	le le le	of ⊕ _B : 1 ! ?W ?C	OK
•	Statu Case ru Case ru Case ru Case ru	le le le	of ⊕ _B : 1 ! ?W ?C	OK
6.8	Statu Case ru Case ru Case ru Case ru Case ru	le le le le	of ⊕ _B : 1 ! ?W ?C ?	OK

• Case rule \oplus_A
\bullet Case rule \top
Case rule I

6.9 Status of \oplus_A : OK

ullet Case rule $oldsymbol{1}$

 $\bullet \;$ Case rule \otimes

- Case rule !
- $\bullet \;$ Case rule ?W
- ullet Case rule ?C
- Case rule?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- $\bullet\,$ Case rule I
- \bullet Case rule \otimes

6.10 Status of \perp : OK

- ullet Case rule $oldsymbol{1}$
- Case rule!
- Case rule ?W
- $\bullet \;$ Case rule ?C
- Case rule?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I
- \bullet Case rule \otimes

6.11 Status of \top : OK

- ullet Case rule $oldsymbol{1}$
- Case rule!
- ullet Case rule ?W
- Case rule ?C
- Case rule ?

\bullet Case rule \top		
\bullet Case rule I		
\bullet Case rule \otimes		
6.12 Status of <i>I</i> : • Case rule 1	ОК	
• Case rule!		
• Case rule $?W$		
• Case rule $?C$		
• Case rule ?		
• Case rule \$		
• Case rule &		
• Case rule \oplus_B		
• Case rule \oplus_A		
\bullet Case rule \bot		
• Case rule \top		

• Case rule \$

• Case rule &

• Case rule \oplus_B

• Case rule \oplus_A

 \bullet Case rule \bot

- ullet Case rule I
- \bullet Case rule \otimes

6.13 Status of \otimes : OK

- $\bullet \;$ Case rule ${\bf 1}$
- Case rule!
- $\bullet \;$ Case rule ?W
- \bullet Case rule ?C
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- \bullet Case rule \bot
- \bullet Case rule \top
- ullet Case rule I
- \bullet Case rule \otimes