System for Intuitionistic Linear Logic

July 6, 2022

Contents

1 Measure of derivations

- Case(s) rule $!R$

$$\frac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_3}\ !R \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_3}\ !R$$

- Case(s) rule $\mathbf{1}_R$

$$\frac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\ \mathbf{1}_R \quad \rightsquigarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\ \mathbf{1}_R$$

- Case(s) rule $\top$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top$$

- Case(s) rule $\&_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \& \mathtt{F}_4}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \& \mathtt{F}_4}\ \&_R$$

- Case(s) rule $\multimap_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \multimap \mathtt{F}_4}\ \multimap_R \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \multimap \mathtt{F}_4}\ \multimap_R$$

- Case(s) rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_2}$$

- Case(s) rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_1}$$

- Case(s) rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_3}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_3}\ \mathbf{1}_L$$

- Case(s) rule $\otimes_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R$$

- Case(s) rule W

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ W \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ W$$

- Case(s) rule C

$$\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ C \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ C$$

- Case(s) rule $!L$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ !L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ !L$$

- Case(s) rule $\&_{L2}$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L2} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L2}$$

- Case(s) rule $\&_{L1}$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L1} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L1}$$

- Case(s) rule $\otimes_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\ \otimes_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\ \otimes_L$$

- Case(s) rule $\oplus_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\ \oplus_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\ \oplus_L$$

- Case(s) rule $\multimap_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L$$

- Case(s) rule I

$$\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_2) \vdash p(\mathtt{n}_2)}\ I \quad \rightsquigarrow \quad \dfrac{}{\bullet \bullet \mathtt{h}_1 : p(\mathtt{n}_2) \vdash p(\mathtt{n}_2)}\ I$$

2 Invertibility of Rules

2.1 Status of $!R$: : `Non invertible`

- Case rule $!R$

$$\frac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_3}\ !R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W

$$\frac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_\mathbf{1}}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_\mathbf{1}}\ W \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_2 : !\Upsilon 3 \vdash \mathtt{F}_\mathbf{1}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash \mathtt{F}_\mathbf{1}}\ W$$

- Case rule C

$$\frac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_\mathbf{1}}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_\mathbf{1}}\ C \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_\mathbf{1}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash \mathtt{F}_\mathbf{1}}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_\mathbf{1}}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_\mathbf{1}}\ !L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash \mathtt{F}_\mathbf{1}}\ \mathtt{fail}$$

- Case rule $\&_{L2}$
- Case rule $\&_{L1}$

- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

2.2 Status of $\mathbf{1}_R$: : `Invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$

$$\frac{}{\bullet_{\mathtt{h}_1} : * \vdash \mathtt{1}}\ \mathtt{1}_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$

- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

2.3 Status of $\top$: : `Invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top}{\bullet \mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \top}\ \mathbf{1}_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\otimes_R$
- Case rule W

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \top}\ W \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule C

$$\frac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \top}\ C \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $!L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \top}\ !L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \top}\ \&_{L2} \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \top}\ \&_{L1} \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \top}\ \otimes_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \top \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \top}\ \oplus_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \top}{\bullet \mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \top}\ \multimap_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule I

2.4 Status of $\&_R$: (Left Premise): `Invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \& \mathtt{F}_4}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\multimap_R$

- Case rule $\oplus_{R_2}$

- Case rule $\oplus_{R_1}$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_4, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}}}\ \multimap_L$$

- Case rule I

2.5 Status of $\&_R$ (Right Premise): : `Invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \& \mathtt{F}_4}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_2}\ \mathbf{1}_L$$

- Case rule $\otimes_R$
- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_4, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ !L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \otimes_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \oplus_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_2}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \multimap_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}^{\ \mathtt{ax}} \quad \overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_2}\ \multimap_L$$

- Case rule I

2.6 Status of $\multimap_R$: : `Invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \multimap \mathtt{F}_4}\ \multimap_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\oplus_{R_2}$

- Case rule $\oplus_{R_1}$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}\ W \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_4, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}\ C \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}\ !L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}\ \otimes_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}\ \oplus_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_2}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_7 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_2}\ \multimap_L$$

- Case rule I

2.7 Status of $\oplus_{R_2}$: : Non invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{fail}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_2}\ \mathbf{1}_L$$

- Case rule $\otimes_R$
- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_4, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ !L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \otimes_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \oplus_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_2}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \multimap_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}^{\ ax} \quad \overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_2}\ \multimap_L$$

- Case rule I

2.8 Status of $\oplus_{R_1}$: : `Non invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{fail}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_4, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}}}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{\mathbf{1}}}\ \multimap_L$$

- Case rule I

2.9 Status of $\mathbf{1}_L$: : `Invertible`

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}} \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_{\mathbf{1}} \vdash \top}\ \top$$

- Case rule $\&_R$

$$\frac{\mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}} \vdash \mathtt{F}_3 \quad \mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}} \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}} \vdash \mathtt{F}_3 \& \mathtt{F}_4}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_2 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_4}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3 \& \mathtt{F}_4}\ \&_R$$

- Case rule $\multimap_R$

$$\frac{\mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}} \vdash \mathtt{F}_3 \multimap \mathtt{F}_4}\ \multimap_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_4}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3 \multimap \mathtt{F}_4}\ \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}} \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}} \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_4}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : \mathbf{1}, \Delta_{\mathbf{1}} \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_3}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ {\scriptstyle \mathtt{H}}$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_1 : \mathbf{1}, \Delta_5 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\mathbf{1}, \Delta_5), \Delta_2 \vdash \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \Delta_5 \vdash \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \mathbf{1}, \Delta_5 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2, \mathbf{1}, \Delta_5 \vdash \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_2, \Delta_5 \vdash \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes_R$$

- Case rule W

$$\frac{\mathtt{h}_1 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : (\mathbf{1}, \Delta_4), !\mathtt{F}_2 \vdash \mathtt{F}_3}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_4 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_4, !\mathtt{F}_2 \vdash \mathtt{F}_3}\ W$$

- Case rule C

$$\frac{\mathtt{h}_1 : \mathbf{1}, \Delta_4, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : (\mathbf{1}, \Delta_4), !\mathtt{F}_2 \vdash \mathtt{F}_3}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_4, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_4, !\mathtt{F}_2 \vdash \mathtt{F}_3}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_1 : \mathbf{1}, \Delta_4, \mathtt{F}_2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : (\mathbf{1}, \Delta_4), !\mathtt{F}_2 \vdash \mathtt{F}_3}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_4, !\mathtt{F}_2 \vdash \mathtt{F}_3}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_1 : \mathbf{1}, \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\mathbf{1}, \Delta_5), \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_4}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_4}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_1 : \mathbf{1}, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\mathbf{1}, \Delta_5), \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_4}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_4}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_1 : \mathbf{1}, \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\mathbf{1}, \Delta_5), \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_4}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_4}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_1 : \mathbf{1}, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \mathbf{1}, \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\mathbf{1}, \Delta_5), \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_4}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_4}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_1 : \mathbf{1}, \Delta_6 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : (\mathbf{1}, \Delta_6), \Delta_2, \mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_5}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \Delta_6, \mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_5}\ \multimap_L$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3 \quad \mathtt{h}_1 : \mathbf{1}, \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, (\mathbf{1}, \Delta_6), \mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_5}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_2, \Delta_6, \mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_5}\ \multimap_L$$

- Case rule I

2.10 Status of $\otimes_R$: (Left Premise): `Non invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : \mathbf{1}, \Delta_{\mathbf{1}}, \Delta_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \mathbf{1}, \Delta_5 \vdash \mathtt{F}_2}\ \mathbf{1}_L$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : \mathbf{1}, \Delta_{\mathbf{1}}, \Delta_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_1 : \Delta_4, \Delta_5 \vdash \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_6, \Delta_7 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : (\Delta_4, \Delta_5), \Delta_6, \Delta_7 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \Delta_6 \vdash \mathtt{F}_2}\ \text{fail}$$

- Case rule W

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_6 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_6, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ W$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule C

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_6 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_6, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ W$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $!L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_6 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_6, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ W$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L2}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L1}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2}\ \otimes_L$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_2}\ \oplus_L$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \Delta_7 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_8, \Delta_9, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}\ \multimap_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \Delta_8, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \text{fail}$$

$$\frac{\mathtt{h}_3 : \Delta_6, \Delta_7 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_8, \Delta_9, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}\ \multimap_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \Delta_8 \vdash \mathtt{F}_{\mathbf{1}}}\ \text{fail}$$

- Case rule I

2.11 Status of $\otimes_R$ (Right Premise): : `Non invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_{\mathbf{1}}, \Delta_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_{\mathbf{1}}, \Delta_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_5 \vdash \mathtt{F}_3}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_1 : \Delta_4, \Delta_5 \vdash \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_6, \Delta_7 \vdash \mathtt{F}_3}{\bullet\mathtt{h}_1 : (\Delta_4, \Delta_5), \Delta_6, \Delta_7 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes_R \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_5, \Delta_7 \vdash \mathtt{F}_3}\ \mathtt{fail}$$

- Case rule W

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet\mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet\mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_6 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_6, !\mathtt{F}_5 \vdash \mathtt{F}_3}\ W$$

- Case rule C

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet\mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, !\mathtt{F}_5, !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet\mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_6 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_6, !\mathtt{F}_5 \vdash \mathtt{F}_3}\ W$$

- Case rule $!L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_6), !\mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_6 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_6, !\mathtt{F}_5 \vdash \mathtt{F}_3}\ W$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_3}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_3}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_3}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_2 \otimes \mathtt{F}_3}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_7, \mathtt{F}_6 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_3}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \Delta_7 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_8, \Delta_9, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}\ \multimap_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_7, \Delta_9 \vdash \mathtt{F}_2}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_3 : \Delta_6, \Delta_7 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_8, \Delta_9, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}\ \multimap_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_7, \Delta_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_2}\ \mathtt{fail}$$

- Case rule I

2.12 Status of W: : `Non invertible`

- Case rule $!R$

$$\frac{\mathtt{h}_3 : !\Upsilon\mathbf{1}, !\mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : !\Upsilon\mathbf{1}, !\mathtt{F}_2 \vdash !\mathtt{F}_4}\ !R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : !\Upsilon\mathbf{1} \vdash \mathtt{F}_4}^{\ ax/ind}}{\bullet\mathtt{h}_3 : !\Upsilon\mathbf{1} \vdash !\mathtt{F}_4}\ !R$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}} \vdash \top}\ \top$$

- Case rule $\&_R$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \& \mathtt{F}_5}\ \&_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_4}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_5}^{\ ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_4 \& \mathtt{F}_5}\ \&_R$$

- Case rule $\multimap_R$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_4, !\mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \multimap \mathtt{F}_5}\ \multimap_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_4 \vdash \mathtt{F}_5}^{\ ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_4 \multimap \mathtt{F}_5}\ \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_5}^{\ ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_4}^{\ ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_2 : \Delta_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}{\bullet\mathtt{h}_2 : \mathbf{1}, \Delta_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_4 \vdash \mathtt{F}_3}^{\ ax/ind}}{\bullet\mathtt{h}_2 : \mathbf{1}, \Delta_4 \vdash \mathtt{F}_3}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \Delta_3 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{F}_4}^{\ ax/ind} \quad \overline{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5}^{\ ax}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_6 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_6, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4}^{\ ax} \quad \overline{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{F}_5}^{\ ax/ind}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_6 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R$$

- Case rule W

$$\frac{\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : (\Delta_5, !\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3 \vdash \mathtt{F}_4}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ W$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{H}$$

- Case rule C

$$\frac{\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : (\Delta_5, !\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3 \vdash \mathtt{F}_4}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ C$$

$$\frac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ C \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{fail}$$

- Case rule $!L$

$$\frac{\mathtt{h}_2 : \Delta_5, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : (\Delta_5, !\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3 \vdash \mathtt{F}_4}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ !L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ !L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\ \mathtt{fail}$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_2 : \Delta_7, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_2 : (\Delta_7, !\mathtt{F}_{\mathbf{1}}), \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_7, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_6}{\bullet \mathtt{h}_2 : \Delta_3, (\Delta_7, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_2 : \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L$$

- Case rule I

2.13 Status of C: : `Non invertible`

- Case rule $!R$

$$\frac{\mathtt{h}_3 : !\Upsilon\mathbf{1}, !\mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : !\Upsilon\mathbf{1}, !\mathtt{F}_2 \vdash !\mathtt{F}_4}\ !R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : !\Upsilon\mathbf{1}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_3 : !\Upsilon\mathbf{1}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash !\mathtt{F}_4}\ !R$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \top}\ \top$$

- Case rule $\&_R$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \& \mathtt{F}_5}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathrm{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_4 \& \mathtt{F}_5}\ \&_R$$

- Case rule $\multimap_R$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_4, !\mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \multimap \mathtt{F}_5}\ \multimap_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_4, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_4 \multimap \mathtt{F}_5}\ \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2, !\mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_2 : \Delta_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}{\bullet\mathtt{h}_2 : \mathbf{1}, \Delta_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_4, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_2 : \mathbf{1}, \Delta_4, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \Delta_3 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\ \mathrm{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5}\ \mathrm{ax}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_6, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_6, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4}\ \mathrm{ax} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_6, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R$$

- Case rule W

$$\frac{\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}{\bullet\mathtt{h}_2 : (\Delta_5, !\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3 \vdash \mathtt{F}_4}\, W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\,{\scriptstyle ax/ind}}{\bullet\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3 \vdash \mathtt{F}_4}\, W$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\, W \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,{\scriptstyle fail}$$

- Case rule C

$$\frac{\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_2 : (\Delta_5, !\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3 \vdash \mathtt{F}_4}\, C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,{\scriptstyle ax/ind}}{\bullet\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3 \vdash \mathtt{F}_4}\, C$$

$$\frac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\, C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,{\scriptstyle ax/ind}}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\, C$$

- Case rule $!L$

$$\frac{\mathtt{h}_2 : \Delta_5, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}{\bullet\mathtt{h}_2 : (\Delta_5, !\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3 \vdash \mathtt{F}_4}\, !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\,{\scriptstyle ax/ind}}{\bullet\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3 \vdash \mathtt{F}_4}\, !L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\, !L \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,{\scriptstyle fail}$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\, \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\,{\scriptstyle ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\, \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\, \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\,{\scriptstyle ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\, \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\, \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\,{\scriptstyle ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\, \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\, \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\,{\scriptstyle ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\,{\scriptstyle ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\, \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_2 : \Delta_7, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_2 : (\Delta_7, !\mathtt{F}_{\mathbf{1}}), \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\, \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_7, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\,{\scriptstyle ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\,{\scriptstyle ax}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_7, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\, \multimap_L$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_7, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_2 : \Delta_3, (\Delta_7, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\, \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4}\,{\scriptstyle ax} \quad \dfrac{}{\mathtt{h}_2 : \Delta_7, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_6}\,{\scriptstyle ax/ind}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_7, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\, \multimap_L$$

- Case rule I

2.14 Status of $!L$: : `Non invertible`

- Case rule $!R$

$$\frac{\mathtt{h}_3 : !\Upsilon\mathbf{1}, !\mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : !\Upsilon\mathbf{1}, !\mathtt{F}_2 \vdash !\mathtt{F}_4}\ !R \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h}_3 : \mathtt{F}_2, !\Upsilon\mathbf{1} \vdash !\mathtt{F}_4}\ \mathtt{fail}$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \top}\ \top$$

- Case rule $\&_R$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \& \mathtt{F}_5}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_4 \& \mathtt{F}_5}\ \&_R$$

- Case rule $\multimap_R$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_4, !\mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \multimap \mathtt{F}_5}\ \multimap_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_4 \multimap \mathtt{F}_5}\ \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, !\mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_2 : \Delta_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}{\bullet\mathtt{h}_2 : \mathbf{1}, \Delta_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : \mathbf{1}, \Delta_4, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_3}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \Delta_3 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_6, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_6, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_6, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_6, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes_R$$

- Case rule W

$$\frac{\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}{\bullet\mathtt{h}_2 : (\Delta_5, !\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3 \vdash \mathtt{F}_4}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_5, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ W$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ W \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \text{fail}$$

- Case rule C

$$\frac{\mathtt{h}_2 : \Delta_5, !\mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_2 : (\Delta_5, !\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3 \vdash \mathtt{F}_4}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_5, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ C$$

$$\frac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ C \quad \rightsquigarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \text{fail}$$

- Case rule $!L$

$$\frac{\mathtt{h}_2 : \Delta_5, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}{\bullet\mathtt{h}_2 : (\Delta_5, !\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3 \vdash \mathtt{F}_4}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_5, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ !L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_4}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \text{ax}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Delta_6, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_2 : \Delta_7, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_2 : (\Delta_7, !\mathtt{F}_{\mathbf{1}}), \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_7, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \text{ax}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_7, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_2 : \Delta_3, (\Delta_7, !\mathtt{F}_{\mathbf{1}}), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4}\ \text{ax} \quad \dfrac{}{\mathtt{h}_2 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_3, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L$$

- Case rule I

2.15 Status of $\&_{L2}$: : Non invertible

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \top}\ \top$$

- Case rule $\&_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R$$

- Case rule $\multimap_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_5, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), \Delta_4 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_6, !\mathtt{F}_4, !\mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2}$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{H}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1}$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L1} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{fail}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_8, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : (\Delta_8, \mathtt{F}_1 \& \mathtt{F}_2), \Delta_4, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_8, \mathtt{F}_6, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : \Delta_4, (\Delta_8, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_2, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

- Case rule I

2.16 Status of $\&_{L1}$: : `Non invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \top}\ \top$$

- Case rule $\&_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R$$

- Case rule $\multimap_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_5, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \& \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), \Delta_4 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}\ \text{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ \text{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_6, !\mathtt{F}_4, !\mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2}$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L2} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{fail}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1}$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{H}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_8, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : (\Delta_8, \mathtt{F}_1 \& \mathtt{F}_2), \Delta_4, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_1 \vdash \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \text{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_1, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_8, \mathtt{F}_6, \mathtt{F}_1 \& \mathtt{F}_2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : \Delta_4, (\Delta_8, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ \text{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_1, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_1, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

- Case rule I

2.17 Status of $\otimes_L$: : `Invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3 \vdash \top}\ \top$$

- Case rule $\&_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R$$

- Case rule $\multimap_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \otimes \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \Delta_4 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}\ \text{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ \text{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_6, !\mathtt{F}_4, !\mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{H}$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\Delta_8, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \Delta_4, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax}}{\bullet\mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_8, \mathtt{F}_6, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_3 : \Delta_4, (\Delta_8, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

- Case rule I

2.18 Status of $\oplus_L$: (Left Premise): `Invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \top}\ \top$$

- Case rule $\&_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R$$

- Case rule $\multimap_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_5, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_4}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \Delta_4 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}\ ^{\mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ ^{\mathtt{ax}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_6, !\mathtt{F}_4, !\mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}}, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \vdash \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{H}$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : (\Delta_8, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \Delta_4, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_8, \mathtt{F}_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : \Delta_4, (\Delta_8, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6 \vdash \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

- Case rule I

2.19 Status of $\oplus_L$ (Right Premise): : `Invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \top}\ \top$$

- Case rule $\&_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\ \&_R$$

- Case rule $\multimap_R$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_5, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\ \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_2 \oplus \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \mathbf{1}, \Delta_5, \mathtt{F}_2 \vdash \mathtt{F}_4}\ \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \Delta_4 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_7, \mathtt{F}_2 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

- Case rule W

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ W$$

- Case rule C

$$\frac{\mathtt{h}_3 : \Delta_6, !\mathtt{F}_4, !\mathtt{F}_4, \mathtt{F}_1 \oplus \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \oplus \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ C$$

- Case rule $!L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \oplus \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \oplus \mathtt{F}_2), !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ !L$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \oplus \mathtt{F}_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \oplus \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\ \oplus_L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\ \oplus_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{H}$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_8, \mathtt{F}_1 \oplus \mathtt{F}_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : (\Delta_8, \mathtt{F}_1 \oplus \mathtt{F}_2), \Delta_4, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_2 \vdash \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_8, \mathtt{F}_6, \mathtt{F}_1 \oplus \mathtt{F}_2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : \Delta_4, (\Delta_8, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_2, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\ \multimap_L$$

- Case rule I

2.20 Status of $\multimap_L$: (Left Premise): `Non invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$

$$\frac{}{\bullet h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_3 \multimap F_4 \vdash \top}\ \top \quad \rightsquigarrow \quad \frac{}{\bullet h_5 : \Delta_{\mathbf{1}} \vdash F_3}\ \text{fail}$$

- Case rule $\&_R$

$$\frac{h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_3 \multimap F_4 \vdash F_6 \quad h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_3 \multimap F_4 \vdash F_7}{\bullet h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_3 \multimap F_4 \vdash F_6 \& F_7}\ \&_R \quad \rightsquigarrow \quad \frac{\frac{}{h_5 : \Delta_{\mathbf{1}} \vdash F_3}\ \text{ax/ind}}{\bullet h_5 : \Delta_{\mathbf{1}} \vdash F_3}\ \text{H}$$

- Case rule $\multimap_R$

$$\frac{h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_6, F_3 \multimap F_4 \vdash F_7}{\bullet h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_3 \multimap F_4 \vdash F_6 \multimap F_7}\ \multimap_R \quad \rightsquigarrow \quad \frac{\frac{}{h_5 : \Delta_{\mathbf{1}} \vdash F_3}\ \text{ax/ind}}{\bullet h_5 : \Delta_{\mathbf{1}} \vdash F_3}\ \text{H}$$

- Case rule $\oplus_{R_2}$

$$\frac{h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_3 \multimap F_4 \vdash F_7}{\bullet h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_3 \multimap F_4 \vdash F_6 \oplus F_7}\ \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\frac{}{h_5 : \Delta_{\mathbf{1}} \vdash F_3}\ \text{ax/ind}}{\bullet h_5 : \Delta_{\mathbf{1}} \vdash F_3}\ \text{H}$$

- Case rule $\oplus_{R_1}$

$$\frac{h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_3 \multimap F_4 \vdash F_6}{\bullet h_5 : \Delta_{\mathbf{1}}, \Delta_2, F_3 \multimap F_4 \vdash F_6 \oplus F_7}\ \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\frac{}{h_5 : \Delta_{\mathbf{1}} \vdash F_3}\ \text{ax/ind}}{\bullet h_5 : \Delta_{\mathbf{1}} \vdash F_3}\ \text{H}$$

- Case rule $\mathbf{1}_L$

$$\frac{h_4 : \Delta_{\mathbf{1}}, \Delta_6, F_2 \multimap F_3 \vdash F_5}{\bullet h_4 : \mathbf{1}, \Delta_{\mathbf{1}}, \Delta_6, F_2 \multimap F_3 \vdash F_5}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\frac{}{h_4 : \Delta_6 \vdash F_2}\ \text{ax/ind}}{\bullet h_4 : \mathbf{1}, \Delta_6 \vdash F_2}\ \mathbf{1}_L$$

$$\frac{h_4 : \Delta_{\mathbf{1}}, \Delta_6, F_2 \multimap F_3 \vdash F_5}{\bullet h_4 : \mathbf{1}, \Delta_{\mathbf{1}}, \Delta_6, F_2 \multimap F_3 \vdash F_5}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\frac{}{h_4 : \Delta_{\mathbf{1}} \vdash F_2}\ \text{ax/ind}}{\bullet h_4 : \Delta_{\mathbf{1}} \vdash F_2}\ \text{H}$$

- Case rule $\otimes_R$

$$\frac{h_3 : \Delta_6, \Delta_7, F_{\mathbf{1}} \multimap F_2 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_{\mathbf{1}} \multimap F_2), \Delta_8, \Delta_9 \vdash F_4 \otimes F_5}\ \otimes_R \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_{\mathbf{1}}}\ \text{fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_{\mathbf{1}} \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7), \Delta_8, \Delta_9, F_{\mathbf{1}} \multimap F_2 \vdash F_4 \otimes F_5}\ \otimes_R \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_{\mathbf{1}}}\ \text{fail}$$

- Case rule W

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6}\ W \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_7 \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_7, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ W$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6}\ W \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule C

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, !\mathtt{F}_5, !\mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6}\ C \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_7 \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_7, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ W$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, !\mathtt{F}_5, !\mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6}\ C \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $!L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6}\ !L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_7 \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_7, !\mathtt{F}_5 \vdash \mathtt{F}_2}\ W$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6}\ !L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L2}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_2}\ \&_{L1}$$

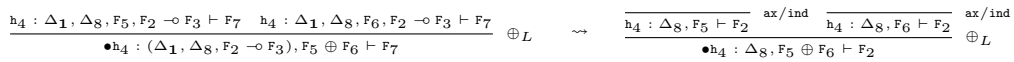
$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

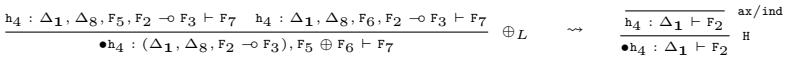
- Case rule $\otimes_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_7}\ \otimes_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_2}\ \otimes_L$$

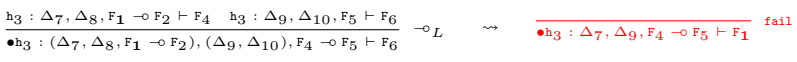
$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_7}\ \otimes_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}} \vdash \mathtt{F}_2}\ \mathtt{H}$$

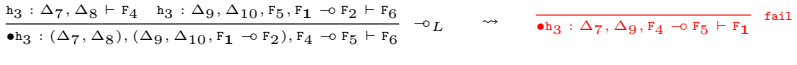
- Case rule $\oplus_L$

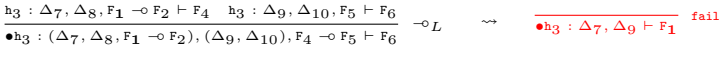


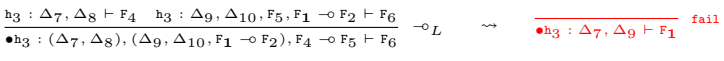


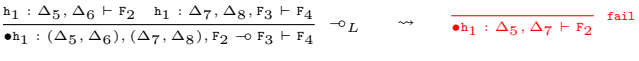
- Case rule $\multimap_L$











- Case rule I

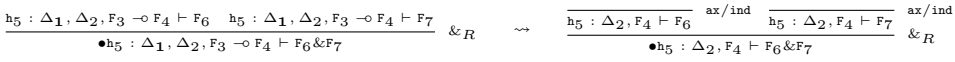
2.21 Status of $\multimap_L$ (Right Premise): : `Non invertible`

- Case rule $!R$

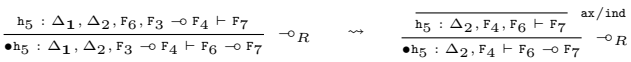
- Case rule $\mathbf{1}_R$

- Case rule $\top$

- Case rule $\&_R$



- Case rule $\multimap_R$



- Case rule $\oplus_{R_2}$

$$\frac{\mathtt{h}_5 : \Delta_{\mathbf{1}}, \Delta_2, \mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_5 : \Delta_{\mathbf{1}}, \Delta_2, \mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7} \oplus_{R_2} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_5 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_5 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7} \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\frac{\mathtt{h}_5 : \Delta_{\mathbf{1}}, \Delta_2, \mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : \Delta_{\mathbf{1}}, \Delta_2, \mathtt{F}_3 \multimap \mathtt{F}_4 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7} \oplus_{R_1} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_5 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_5 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7} \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \mathtt{1}, \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_5} \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_5} \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \mathtt{1}, \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_5} \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \mathtt{1}, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5} \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\frac{\mathtt{h}_3 : \Delta_6, \Delta_{\mathbf{7}}, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_8, \Delta_9 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \Delta_{\mathbf{7}}, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2), \Delta_8, \Delta_9 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_{\mathbf{7}}, \Delta_9, \mathtt{F}_2 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_3 : \Delta_6, \Delta_{\mathbf{7}} \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_8, \Delta_9, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, \Delta_{\mathbf{7}}), \Delta_8, \Delta_9, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_{\mathbf{7}}, \Delta_9, \mathtt{F}_2 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{fail}$$

- Case rule W

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6} W \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6} \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6} W \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_3, !\mathtt{F}_5 \vdash \mathtt{F}_6} W$$

- Case rule C

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, !\mathtt{F}_5, !\mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6} C \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6} \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, !\mathtt{F}_5, !\mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6} C \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_3, !\mathtt{F}_5 \vdash \mathtt{F}_6} W$$

- Case rule $!L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6} !L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_6} \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \multimap \mathtt{F}_3), !\mathtt{F}_5 \vdash \mathtt{F}_6} !L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_7, \mathtt{F}_3, !\mathtt{F}_5 \vdash \mathtt{F}_6} W$$

- Case rule $\&_{L2}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_7}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_7}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_8, \mathtt{F}_3, \mathtt{F}_6 \vdash \mathtt{F}_7}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_8, \mathtt{F}_3, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_7}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_7}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_8, \mathtt{F}_3, \mathtt{F}_5 \vdash \mathtt{F}_7}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_8, \mathtt{F}_3, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_7}\ \otimes_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_7}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_7}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_7}\ \otimes_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_8, \mathtt{F}_3, \mathtt{F}_5, \mathtt{F}_6 \vdash \mathtt{F}_7}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_8, \mathtt{F}_3, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_7}\ \otimes_L$$

- Case rule $\oplus_L$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_7}\ \oplus_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_7}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_{\mathbf{1}}, \mathtt{F}_3 \vdash \mathtt{F}_7}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_5, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_4 : \Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_6, \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_{\mathbf{1}}, \Delta_8, \mathtt{F}_2 \multimap \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_7}\ \oplus_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_8, \mathtt{F}_3, \mathtt{F}_5 \vdash \mathtt{F}_7}^{\ \mathrm{ax/ind}} \quad \overline{\mathtt{h}_4 : \Delta_8, \mathtt{F}_3, \mathtt{F}_6 \vdash \mathtt{F}_7}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_8, \mathtt{F}_3, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_7}\ \oplus_L$$

- Case rule $\multimap_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_9, \Delta_{10}, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2), (\Delta_9, \Delta_{10}), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_8, \Delta_{10}, \mathtt{F}_2 \vdash \mathtt{F}_6}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_3 : \Delta_7, \Delta_8 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_9, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_8, \Delta_{10}, \mathtt{F}_2 \vdash \mathtt{F}_6}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_3 : \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_9, \Delta_{10}, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2), (\Delta_9, \Delta_{10}), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_8, \Delta_{10}, \mathtt{F}_2, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_3 : \Delta_7, \Delta_8 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_9, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathtt{F}_{\mathbf{1}} \multimap \mathtt{F}_2), \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \multimap_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_8, \Delta_{10}, \mathtt{F}_2, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_7, \Delta_8, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), \mathtt{F}_2 \multimap \mathtt{F}_3 \vdash \mathtt{F}_4}\ \multimap_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_6, \Delta_8, \mathtt{F}_3 \vdash \mathtt{F}_4}\ \mathtt{fail}$$

- Case rule I

2.22 Status of I: : `Invertible`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_2) \vdash p(\mathtt{n}_2)}\ I \quad \rightsquigarrow \quad \texttt{trivial}$$

3 Identity-Expansion

$$\frac{}{- : \top \vdash \top}\ \top$$

$$\frac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\ \mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0 \oplus \mathtt{F}_{\mathbf{1}}}\ \oplus_{R_1} \quad \dfrac{\dfrac{}{- : \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{- : \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_0 \oplus \mathtt{F}_{\mathbf{1}}}\ \oplus_{R_2}}{- : \mathtt{F}_0 \oplus \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_0 \oplus \mathtt{F}_{\mathbf{1}}}\ \oplus_L$$

$$\frac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\ \mathtt{IH} \quad \dfrac{}{- : \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_0 \multimap \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_{\mathbf{1}}}\ \multimap_L}{- : \mathtt{F}_0 \multimap \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_0 \multimap \mathtt{F}_{\mathbf{1}}}\ \multimap_R$$

$$\frac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\ \mathtt{IH} \quad \dfrac{}{- : \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_0 \otimes \mathtt{F}_{\mathbf{1}}}\ \otimes_R}{- : \mathtt{F}_0 \otimes \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_0 \otimes \mathtt{F}_{\mathbf{1}}}\ \otimes_L$$

$$\frac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\ \mathtt{IH}}{- : \mathtt{F}_0 \& \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_0}\ \&_{L1} \quad \dfrac{\dfrac{}{- : \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{- : \mathtt{F}_0 \& \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_{\mathbf{1}}}\ \&_{L2}}{- : \mathtt{F}_0 \& \mathtt{F}_{\mathbf{1}} \vdash \mathtt{F}_0 \& \mathtt{F}_{\mathbf{1}}}\ \&_R$$

$$\frac{\dfrac{}{- : * \vdash \mathbf{1}}\ \mathbf{1}_R}{- : \mathbf{1} \vdash \mathbf{1}}\ \mathbf{1}_L$$

4 Weakening on bang: $\Gamma \vdash !F$ implies $\Gamma \vdash F$.

- Case(s) rule $!R$

$$\frac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_3}\ !R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case(s) rule $\mathbf{1}_R$
- Case(s) rule $\top$
- Case(s) rule $\&_R$
- Case(s) rule $\multimap_R$
- Case(s) rule $\oplus_{R_2}$
- Case(s) rule $\oplus_{R_1}$
- Case(s) rule $\mathbf{1}_L$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 : \mathbf{1}, \Delta_3 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathbf{1}_L \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \mathbf{1}, \Delta_3 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathbf{1}_L$$

- Case(s) rule $\otimes_R$
- Case(s) rule W

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 : \Delta_3, !\mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}\ W \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_3, !\mathtt{F}_4 \vdash \mathtt{F}_{\mathbf{1}}}\ W$$

- Case(s) rule C

$$\frac{\mathtt{h}_2 : \Delta_3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 : \Delta_3, !\mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}\ C \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_3, !\mathtt{F}_4 \vdash \mathtt{F}_{\mathbf{1}}}\ C$$

- Case(s) rule $!L$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 : \Delta_3, !\mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}\ !L \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_3, !\mathtt{F}_4 \vdash \mathtt{F}_{\mathbf{1}}}\ !L$$

- Case(s) rule $\&_{L2}$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \& \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}\ \&_{L2} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \&_{L2}$$

- Case(s) rule $\&_{L1}$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \& \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}\ \&_{L1} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \&_{L1}$$

- Case(s) rule $\otimes_L$

$$\dfrac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}\ \otimes_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \otimes_L$$

- Case(s) rule $\oplus_L$

$$\dfrac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}\ \oplus_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_{\mathbf{1}}}\ \oplus_L$$

- Case(s) rule $\multimap_L$

$$\dfrac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_4, \mathtt{F}_6 \vdash !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 : \Delta_3, \Delta_4, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash !\mathtt{F}_{\mathbf{1}}}\ \multimap_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_6 \vdash !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_3, \Delta_4, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{\mathbf{1}}}\ \multimap_L$$

- Case(s) rule I

5 Cut-Elimination

5.1 Status of $!R$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,!R \quad \dfrac{\mathtt{h}_6 :!\Upsilon 4, !\mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 :!\Upsilon 4, !\mathtt{F}_5 \vdash !\mathtt{F}_7}\,!R}{- :!\Upsilon 2, !\Upsilon 4 \vdash !\mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 :!\Upsilon 4, !\mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- :!\Upsilon 2, !\Upsilon 4 \vdash \mathtt{F}_7}\,\mathtt{hCut}}{- :!\Upsilon 2, !\Upsilon 4 \vdash !\mathtt{F}_7}\,!R$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,!R \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \top}\,\top}{- :!\Upsilon 2, \Delta_4 \vdash \top}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- :!\Upsilon 2, \Delta_4 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,!R \quad \dfrac{\mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\,\&_R}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\,\&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,!R \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_7, !\mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}\,\multimap_R}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, \mathtt{F}_7, !\mathtt{F}_5 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- :!\Upsilon 2, \Delta_4, \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}\,\multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,!R \quad \dfrac{\mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_{R_2}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_5}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\, \oplus_{R_1}}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, !\mathtt{F}_5 \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7}\, \mathtt{hCut}$$

$$\dfrac{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\, \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_5 : \Delta_7, !\mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\mathbf{1}, \Delta_7), !\mathtt{F}_4 \vdash \mathtt{F}_6}\, \mathbf{1}_L}{- : !\Upsilon 2, \mathbf{1}, \Delta_7 \vdash \mathtt{F}_6}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \mathbf{1}, \Delta_7, !\mathtt{F}_4 \vdash \mathtt{F}_6}\, \mathtt{ax/W}}{- : \mathbf{1}, !\Upsilon 2, \Delta_7 \vdash \mathtt{F}_6}\, \mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_5 : \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_6, \Delta_9), !\mathtt{F}_4 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes_R}{- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_9 \vdash \mathtt{F}_7}\, \mathtt{hCut} \quad \dfrac{}{- : \Delta_6 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_6, \Delta_9), !\mathtt{F}_4 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes_R}{- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_6 \vdash \mathtt{F}_7}\, \mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_9 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_5 : \Delta_8, !\mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_5 : (\Delta_8, !\mathtt{F}_6), !\mathtt{F}_4 \vdash \mathtt{F}_7}\, W}{- : !\Upsilon 2, \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_8, !\mathtt{F}_4, !\mathtt{F}_6 \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_6}\, !R \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}\, W}{- : !\Upsilon 2, \Delta_5 \vdash \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : !\Upsilon 2, \Delta_5 \vdash \mathtt{F}_7}\, \mathtt{ax/W}$$

- Case rule C

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_5 : \Delta_8, !\mathtt{F}_4, !\mathtt{F}_6, !\mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_5 : (\Delta_8, !\mathtt{F}_6), !\mathtt{F}_4 \vdash \mathtt{F}_7}\, C}{- : !\Upsilon 2, \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_8, !\mathtt{F}_4, !\mathtt{F}_6, !\mathtt{F}_6 \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_8, !\mathtt{F}_6, !\mathtt{F}_6 \vdash \mathtt{F}_7}\, \mathtt{hCut}$$

$$\dfrac{- : !\Upsilon 2, \Delta_8, !\mathtt{F}_6, !\mathtt{F}_6 \vdash \mathtt{F}_7}{- : !\Upsilon 2, \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\, C$$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_6}\ !R \quad \dfrac{\mathtt{h}_4 : \Delta_5, !\mathtt{F}_6, !\mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}\ C}{- : !\Upsilon 2, \Delta_5 \vdash \mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, !\mathtt{F}_6, !\mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_5 \vdash \mathtt{F}_7}\ \mathtt{mCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\ !R \quad \dfrac{\mathtt{h}_5 : \Delta_8, \mathtt{F}_6, !\mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_5 : (\Delta_8, !\mathtt{F}_6), !\mathtt{F}_4 \vdash \mathtt{F}_7}\ !L}{- : !\Upsilon 2, \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_8, \mathtt{F}_6, !\mathtt{F}_4 \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{hCut}}{- : !\Upsilon 2, \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\ !L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_6}\ !R \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}\ !L}{- : !\Upsilon 2, \Delta_5 \vdash \mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : !\Upsilon 2 \vdash \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{- : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_5 \vdash \mathtt{F}_7}\ \mathtt{sCut}$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\ !R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, !\mathtt{F}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \& \mathtt{F}_7), !\mathtt{F}_4 \vdash \mathtt{F}_8}\ \&_{L2}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, !\mathtt{F}_4 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{hCut}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\ !R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, !\mathtt{F}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \& \mathtt{F}_7), !\mathtt{F}_4 \vdash \mathtt{F}_8}\ \&_{L1}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, !\mathtt{F}_4 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}\ \mathtt{hCut}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\ !R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, !\mathtt{F}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7), !\mathtt{F}_4 \vdash \mathtt{F}_8}\ \otimes_L}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, !\mathtt{F}_4 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{hCut}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_8}\ \otimes_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\,!R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, !\mathtt{F}_4 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, !\mathtt{F}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \oplus \mathtt{F}_7), !\mathtt{F}_4 \vdash \mathtt{F}_8}\,\oplus_L}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, !\mathtt{F}_4 \vdash \mathtt{F}_8}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, !\mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, !\mathtt{F}_4 \vdash \mathtt{F}_8}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, !\mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_8}\,\oplus_L$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\,!R \quad \dfrac{\mathtt{h}_5 : \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_5 : (\Delta_6, \Delta_{10}, \mathtt{F}_7 \multimap \mathtt{F}_8), !\mathtt{F}_4 \vdash \mathtt{F}_9}\,\multimap_L}{- : !\Upsilon 2, \Delta_6, \Delta_{10}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_5 : \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_7}{\mathtt{h}_5 : \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : !\Upsilon 2, \Delta_{10} \vdash \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : !\Upsilon 2, \Delta_{10}, \Delta_6, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\,\multimap_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\,!R \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_5 : (\Delta_6, \Delta_{10}, \mathtt{F}_7 \multimap \mathtt{F}_8), !\mathtt{F}_4 \vdash \mathtt{F}_9}\,\multimap_L}{- : !\Upsilon 2, \Delta_6, \Delta_{10}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_6 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_5 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}{\mathtt{h}_5 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : !\Upsilon 2, \Delta_{10}, \Delta_6, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\,\multimap_L$$

- Case rule I

5.2 Status of $\mathbf{1}_R$: OK

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{}{\bullet \mathtt{h}_3 : \Delta_2, \mathbf{1} \vdash \top}\,\top}{- : *, \Delta_2 \vdash \top}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_2 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_2, \mathbf{1} \vdash \mathtt{F}_4 \& \mathtt{F}_5}\,\&_R}{- : *, \Delta_2 \vdash \mathtt{F}_4 \& \mathtt{F}_5}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}{\mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W}}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}{\mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{hCut}}{- : \Delta_2 \vdash \mathtt{F}_4 \& \mathtt{F}_5}\,\&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_3 : \mathbf{1}, \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_2, \mathbf{1} \vdash \mathtt{F}_4 \multimap \mathtt{F}_5}\,\multimap_R}{- : *, \Delta_2 \vdash \mathtt{F}_4 \multimap \mathtt{F}_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_3 : \mathbf{1}, \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{hCut}$$
$$\dfrac{- : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{- : \Delta_2 \vdash \mathtt{F}_4 \multimap \mathtt{F}_5}\,\multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_2, \mathbf{1} \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\,\oplus_{R_2}}{- : *, \Delta_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{}{\mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{hCut}$$
$$\dfrac{- : \Delta_2 \vdash \mathtt{F}_5}{- : \Delta_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\,\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Delta_2, \mathbf{1} \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\,\oplus_{R_1}}{- : *, \Delta_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{}{\mathtt{h}_3 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W}}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{hCut}$$
$$\dfrac{- : \Delta_2 \vdash \mathtt{F}_4}{- : \Delta_2 \vdash \mathtt{F}_4 \oplus \mathtt{F}_5}\,\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \Delta_3, \mathbf{1} \vdash \mathtt{F}_4}\,\mathbf{1}_L}{- : *, \Delta_3 \vdash \mathtt{F}_4}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_3 \vdash \mathtt{F}_4}\,\mathtt{ax/W}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \mathbf{1}, \Delta_6 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_3, \Delta_6), \mathbf{1} \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\,\otimes_R}{- : *, \Delta_3, \Delta_6 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_6 \vdash \mathtt{F}_4}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \mathtt{F}_4}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_6 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\,\otimes_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \mathbf{1}, \Delta_6 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_3, \Delta_6), \mathbf{1} \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\,\otimes_R}{- : *, \Delta_3, \Delta_6 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_3 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_6 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \mathtt{F}_5}\,\mathtt{hCut}}{- : \Delta_3, \Delta_6 \vdash \mathtt{F}_4 \otimes \mathtt{F}_5}\,\otimes_R$$

- Case rule W

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \mathbf{1}, \Delta_5 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : (\Delta_5, !\mathtt{F}_3), \mathbf{1} \vdash \mathtt{F}_4}\,W}{- : *, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,\mathtt{ax/W}}{- : \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,\mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \mathbf{1}, \Delta_5, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : (\Delta_5, !\mathtt{F}_3), \mathbf{1} \vdash \mathtt{F}_4}\,C}{- : *, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,\mathtt{Cut} \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_5, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,\mathtt{ax/W}}{- : \Delta_5, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,\mathtt{hCut}}{- : \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,C$$

- Case rule $!L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \mathbf{1}, \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : (\Delta_5, !\mathtt{F}_3), \mathbf{1} \vdash \mathtt{F}_4}\,!L}{- : *, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,\mathtt{Cut} \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}\,\mathtt{ax/W}}{- : \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_4}\,\mathtt{hCut}}{- : \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_4}\,!L$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \& \mathtt{F}_4), \mathbf{1} \vdash \mathtt{F}_5}\,\&_{L2}}{- : *, \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{Cut} \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\,\&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \& \mathtt{F}_4), \mathbf{1} \vdash \mathtt{F}_5}\,\&_{L1}}{- : *, \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{Cut} \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_5}\,\&_{L1}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4), \mathbf{1} \vdash \mathtt{F}_5}\,\otimes_L}{- : *, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{Cut} \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_5}\,\otimes_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \mathbf{1} \vdash \mathtt{F}_5}\,\oplus_L}{- : *, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{Cut} \quad \rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_5}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{hCut}}{- : \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_5}\,\oplus_L$$

- Case rule $-\circ_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \mathbf{1}, \Delta_7 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_2 : (\Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5), \mathbf{1} \vdash \mathtt{F}_6}\,-\circ_L}{- : *, \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_7 \vdash \mathtt{F}_4}\,\mathtt{ax/W}}{- : \Delta_7 \vdash \mathtt{F}_4}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\,-\circ_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4 \quad \mathtt{h}_2 : \mathbf{1}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_2 : (\Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5), \mathbf{1} \vdash \mathtt{F}_6}\,-\circ_L}{- : *, \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_3 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \mathbf{1}}\,\mathbf{1}_R \quad \dfrac{}{\mathtt{h}_2 : \mathbf{1}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_6}\,-\circ_L$$

- Case rule I

5.3 Status of $\top$: OK

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \top}\,\top}{- : \Delta_2, \Delta_3 \vdash \top}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_2, \Delta_3 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_5 \quad \mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_4 : \Delta_2, \Delta_3, \top \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_4 : \Delta_2, \Delta_3, \top \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R$$

- Case rule $-\circ_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,-\circ_R}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_4 : \Delta_2, \Delta_3, \mathtt{F}_5, \top \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,-\circ_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\ \top \quad \dfrac{}{\mathtt{h}_4 : \Delta_2, \Delta_3, \top \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top \quad \dfrac{\mathtt{h}_4 : \top, \Delta_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : \Delta_3, \top \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\ \top \quad \dfrac{}{\mathtt{h}_4 : \Delta_2, \Delta_3, \top \vdash \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5}\ \mathtt{hCut}}{- : \Delta_2, \Delta_3 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : (\mathbf{1}, \Delta_5), \top \vdash \mathtt{F}_4}\ \mathbf{1}_L}{- : \Delta_2, \mathbf{1}, \Delta_5 \vdash \mathtt{F}_4}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\ \top \quad \dfrac{}{\mathtt{h}_3 : \mathbf{1}, \Delta_2, \Delta_5, \top \vdash \mathtt{F}_4}\ \mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathtt{F}_4}\ \mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_4, \Delta_7), \top \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R}{- : \Delta_2, \Delta_4, \Delta_7 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\ \top \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \top \vdash \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_7 \vdash \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{}{- : \Delta_2, \Delta_4 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_2, \Delta_4, \Delta_7 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top \quad \dfrac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \top, \Delta_7 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_4, \Delta_7), \top \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R}{- : \Delta_2, \Delta_4, \Delta_7 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_4 \vdash \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\ \top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_7, \top \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_2, \Delta_7 \vdash \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_2, \Delta_4, \Delta_7 \vdash \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes_R$$

- Case rule W

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\ \top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_6 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, !\mathtt{F}_4), \top \vdash \mathtt{F}_5}\ W}{- : \Delta_2, \Delta_6, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\ \top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_6, \top, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_6, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, !\mathtt{F}_4), \top \vdash \mathtt{F}_5}\,C}{- : \Delta_2, \Delta_6, !\mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_6, \top, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{hCut} \quad C$$

$$- : \Delta_2, \Delta_6, !\mathtt{F}_4 \vdash \mathtt{F}_5$$

- Case rule $!L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_3 : (\Delta_6, !\mathtt{F}_4), \top \vdash \mathtt{F}_5}\,!L}{- : \Delta_2, \Delta_6, !\mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_6, \mathtt{F}_4, \top \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_5}\,\mathtt{hCut} \quad !L$$

$$- : \Delta_2, \Delta_6, !\mathtt{F}_4 \vdash \mathtt{F}_5$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \& \mathtt{F}_5), \top \vdash \mathtt{F}_6}\,\&_{L2}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_7, \mathtt{F}_5, \top \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{hCut} \quad \&_{L2}$$

$$- : \Delta_2, \Delta_7, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \& \mathtt{F}_5), \top \vdash \mathtt{F}_6}\,\&_{L1}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_7, \mathtt{F}_4, \top \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{hCut} \quad \&_{L1}$$

$$- : \Delta_2, \Delta_7, \mathtt{F}_4 \& \mathtt{F}_5 \vdash \mathtt{F}_6$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \otimes \mathtt{F}_5), \top \vdash \mathtt{F}_6}\,\otimes_L}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \top \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{hCut} \quad \otimes_L$$

$$- : \Delta_2, \Delta_7, \mathtt{F}_4 \otimes \mathtt{F}_5 \vdash \mathtt{F}_6$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6 \quad \mathtt{h}_3 : \top, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_4 \oplus \mathtt{F}_5), \top \vdash \mathtt{F}_6}\,\oplus_L}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_7, \mathtt{F}_4, \top \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_7, \mathtt{F}_5, \top \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_6}\,\mathtt{hCut}$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \oplus \mathtt{F}_5 \vdash \mathtt{F}_6}\,\oplus_L$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_3 : \top, \Delta_8 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : (\Delta_4, \Delta_8, \mathtt{F}_5 \multimap \mathtt{F}_6), \top \vdash \mathtt{F}_7}\,{\multimap_L}}{- : \Delta_2, \Delta_4, \Delta_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \top \vdash \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_8 \vdash \mathtt{F}_5}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_2, \Delta_4, \mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_4, \Delta_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\,{\multimap_L}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top}\,\top \quad \dfrac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5 \quad \mathtt{h}_3 : \top, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_3 : (\Delta_4, \Delta_8, \mathtt{F}_5 \multimap \mathtt{F}_6), \top \vdash \mathtt{F}_7}\,{\multimap_L}}{- : \Delta_2, \Delta_4, \Delta_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_4 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : * \vdash \top}\,\top \quad \dfrac{}{\mathtt{h}_3 : \Delta_2, \Delta_8, \mathtt{F}_6, \top \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{hCut}}{- : \Delta_2, \Delta_4, \Delta_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_7}\,{\multimap_L}$$

- Case rule I

5.4 Status of $\&_R$: OK

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\&_R \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \top}\,\top}{- : \Delta_2, \Delta_5 \vdash \top}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\&_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\&_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\&_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,{\multimap_R}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,{\multimap_R}$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\&_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_2}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_2}}\,\mathtt{hCut}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\&_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_1}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_1}}\,\mathtt{hCut}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : (\mathbf{1}, \Delta_9), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathbf{1}_L}{- : \Delta_2, \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_8 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}\,W}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}\,C}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,C$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}\,!L}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,!L$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\&_{L2}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L2}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\,\&_R \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\,\&_{L2}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{- : \Delta_2 \vdash \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{- : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{sCut}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\&_{L1}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L1}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\,\&_R \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\,\&_{L1}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{- : \Delta_2 \vdash \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{- : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{sCut}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\otimes_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\otimes_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\oplus_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\oplus_L$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_9}{\mathtt{ax/W}}}{- : \Delta_{12}, \Delta_2 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\multimap_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}\,\&_R \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \& \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_{11}}{\mathtt{ax/W}}}{- : \Delta_{12}, \Delta_2, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\multimap_L$$

- Case rule I

5.5 Status of $\multimap_R$: OK

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\,\multimap_R \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \top}\,\top}{- : \Delta_2, \Delta_5 \vdash \top}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\,\multimap_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\&_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{ax/W}}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\,\multimap_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\multimap_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\,\multimap_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_2}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\,\multimap_R \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_1}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,\multimap_R \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : (\mathbf{1}, \Delta_9), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathbf{1}_L}{- : \Delta_2, \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,\multimap_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_8 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,\multimap_R \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\multimap_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}W}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}{}\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\multimap_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}C}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}{}\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}C$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\multimap_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}!L}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}{}\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}!L$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\multimap_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}\&_{L2}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\multimap_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}\&_{L1}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\&_{L1}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\multimap_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}\otimes_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\otimes_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,{\multimap_R} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,{\oplus_L}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,{\oplus_L}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,{\multimap_R} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,{\multimap_L}}{- : \Delta_2, \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{- : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,{\multimap_L}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}\,{\multimap_R} \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,{\multimap_L}}{- : \Delta_2, \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{- : \Delta_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \multimap \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_5 \multimap \mathtt{F}_6 \vdash \mathtt{F}_{11}}{}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,{\multimap_L}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\,{\multimap_R} \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_5 : (\Delta_6, \Delta_7), \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,{\multimap_L}}{- : \Delta_2, \Delta_6, \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{- : \Delta_6 \vdash \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{- : \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{- : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{sCut}}{- : \Delta_2, \Delta_6, \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{sCut}$$

- Case rule I

5.6 Status of $\oplus_{R_2}$: OK

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\,{\oplus_{R_2}} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \top}\,\top}{- : \Delta_2, \Delta_5 \vdash \top}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\oplus_{R_2} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\&_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\oplus_{R_2} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\multimap_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\oplus_{R_2} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_2}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\oplus_{R_2} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_1}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : (\mathbf{1}, \Delta_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_8}\mathbf{1}_L}{- : \Delta_2, \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_8 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}W}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}C}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}C$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}!L}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}!L$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\&_{L2}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\&_{L1}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\&_{L1}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\otimes_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\otimes_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\oplus_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\oplus_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_{R_2} \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\,\oplus_L}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{sCut}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\multimap_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_{R_2} \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\multimap_L$$

- Case rule I

5.7 Status of $\oplus_{R_1}$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_{R_1} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \top}\,\top}{- : \Delta_2, \Delta_5 \vdash \top}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_5 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\oplus_{R_1} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\&_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\oplus_{R_1} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\multimap_R}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\oplus_{R_1} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_2}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\oplus_{R_1} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_1}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_2, \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : (\mathbf{1}, \Delta_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_8}\mathbf{1}_L}{- : \Delta_2, \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_8}{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash \mathtt{F}_8}\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash \mathtt{F}_9}\mathtt{hCut} \quad \dfrac{}{- : \Delta_8 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}W}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}C}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}C$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}!L}{- : \Delta_2, \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8 \vdash \mathtt{F}_9}!L$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\&_{L2}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\&_{L1}}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\&_{L1}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\otimes_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\otimes_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\oplus_L}{- : \Delta_2, \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\oplus_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_{R_1} \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\,\oplus_L}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{sCut}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\multimap_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_{R_1} \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\multimap_L$$

- Case rule I

5.8 Status of $\mathbf{1}_L$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash 5}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash 5}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_6 : !\Upsilon 4, 5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_6 : !\Upsilon 4, 5 \vdash !\mathtt{F}_7}\,!R}{- : (\mathbf{1}, \Delta_2), !\Upsilon 4 \vdash !\mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash 5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \mathbf{1}, 5, !\Upsilon 4 \vdash !\mathtt{F}_7}\,\mathtt{ax/W}}{- : \mathbf{1}, !\Upsilon 4, \Delta_2 \vdash !\mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}\,\mathbf{1}_L \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \top}\,\top}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash \top}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\,\&_R}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \mathbf{1}, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}\,\multimap_R}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \mathbf{1}, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_{R_2}}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \mathbf{1}, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_5}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_6 : \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_{R_1}}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \mathbf{1}, \Delta_4, \mathtt{F}_5 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathbf{1}}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathbf{1}}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_4 : \Delta_5, \mathbf{1} \vdash \mathtt{F}_6}\,\mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\mathbf{1}, \Delta_7), \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \mathbf{1}, \Delta_7 \vdash \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \mathbf{1}, \mathbf{1}, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{- : \mathbf{1}, \mathbf{1}, \Delta_2, \Delta_7 \vdash \mathtt{F}_6}\,\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\Delta_6, \Delta_9), \mathtt{F}_4 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\otimes_R}{- : (\mathbf{1}, \Delta_2), \Delta_6, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_6, \Delta_9, \mathtt{F}_4 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_6, \Delta_9, \mathtt{F}_4 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_6, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_4 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\Delta_6, \Delta_9), \mathtt{F}_4 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\otimes_R}{- : (\mathbf{1}, \Delta_2), \Delta_6, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_6, \Delta_9, \mathtt{F}_4 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_6, \Delta_9, \mathtt{F}_4 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_6, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_8, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : (\Delta_8, !\mathtt{F}_6), \mathtt{F}_4 \vdash \mathtt{F}_7}\,W}{- : (\mathbf{1}, \Delta_2), \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_8, \mathtt{F}_4, !\mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_8, \mathtt{F}_4, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash !\mathtt{F}_6}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash !\mathtt{F}_6}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_4 : \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,W}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash !\mathtt{F}_6}{\mathtt{h}_1 : \Delta_2 \vdash !\mathtt{F}_6}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathtt{F}_7}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_8, \mathtt{F}_4, !\mathtt{F}_6, !\mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : (\Delta_8, !\mathtt{F}_6), \mathtt{F}_4 \vdash \mathtt{F}_7}\,C}{- : (\mathbf{1}, \Delta_2), \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_8, \mathtt{F}_4, !\mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_8, \mathtt{F}_4, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash !\mathtt{F}_6}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash !\mathtt{F}_6}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, !\mathtt{F}_6, !\mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_4 : \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,C}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash !\mathtt{F}_6}{\mathtt{h}_1 : \Delta_2 \vdash !\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_8, \mathtt{F}_4, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : (\Delta_8, !\mathtt{F}_6), \mathtt{F}_4 \vdash \mathtt{F}_7}\,!L}{- : (\mathbf{1}, \Delta_2), \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_8, \mathtt{F}_4, !\mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_8, \mathtt{F}_4, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash !\mathtt{F}_6}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash !\mathtt{F}_6}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_4 : \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,!L}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash !\mathtt{F}_6}{\mathtt{h}_1 : \Delta_2 \vdash !\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_5, !\mathtt{F}_6 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \& \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\&_{L2}}{- : (\mathbf{1}, \Delta_2), \Delta_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\&_{L2}}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \& \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\&_{L1}}{- : (\mathbf{1}, \Delta_2), \Delta_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\&_{L1}}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_5, \mathtt{F}_6 \& \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\otimes_L}{- : (\mathbf{1}, \Delta_2), \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_8}\,\otimes_L}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_4 : \mathbf{1}, \Delta_5, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_4, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \oplus \mathtt{F}_7), \mathtt{F}_4 \vdash \mathtt{F}_8}\,\oplus_L}{- : (\mathbf{1}, \Delta_2), \Delta_9, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \mathbf{1}, \Delta_9, \mathtt{F}_4, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}{\bullet \mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_8}\,\oplus_L}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_4 : \mathbf{1}, \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7 \vdash \mathtt{F}_8}{}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_{10}, \mathtt{F}_4 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_5 : (\Delta_6, \Delta_{10}, \mathtt{F}_7 \multimap \mathtt{F}_8), \mathtt{F}_4 \vdash \mathtt{F}_9}\,\multimap_L}{- : (\mathbf{1}, \Delta_2), \Delta_6, \Delta_{10}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{}\,\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_5 : \mathbf{1}, \Delta_{10}, \Delta_6, \mathtt{F}_4, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_4}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_7 \quad \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_4, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_5 : (\Delta_6, \Delta_{10}, \mathtt{F}_7 \multimap \mathtt{F}_8), \mathtt{F}_4 \vdash \mathtt{F}_9}\,\multimap_L}{- : (\mathbf{1}, \Delta_2), \Delta_6, \Delta_{10}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4}{}\,\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_5 : \mathbf{1}, \Delta_{10}, \Delta_6, \mathtt{F}_4, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}{\bullet \mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}\,\mathbf{1}_L \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_7 \quad \mathtt{h}_4 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_4 : (\Delta_5, \Delta_6), \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\,\multimap_L}{- : (\mathbf{1}, \Delta_2), \Delta_5, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \multimap \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_4 : \mathbf{1}, \Delta_5, \Delta_6, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5, \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash p(\mathtt{n}_5)}{\bullet \mathtt{h}_1 : \mathbf{1}, \Delta_2 \vdash p(\mathtt{n}_5)}\,\mathbf{1}_L \quad \dfrac{}{\bullet \mathtt{h}_4 : *, p(\mathtt{n}_5) \vdash p(\mathtt{n}_5)}\,I}{- : (\mathbf{1}, \Delta_2), * \vdash p(\mathtt{n}_5)}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \mathbf{1}, \Delta_2 \vdash p(\mathtt{n}_5)}\,\mathtt{ax/W}$$

5.9 Status of $\otimes_R$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\otimes_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \top}\,\top}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash \top}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\otimes_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{10}\&\mathtt{F}_{11}}\&_R}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\otimes_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{10} \multimap \mathtt{F}_{11}}\multimap_R}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash \mathtt{F}_{10} \multimap \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \mathtt{F}_{10} \multimap \mathtt{F}_{11}}\multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\otimes_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_{R_2}}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\otimes_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_{R_1}}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\mathbf{1}, \Delta_{10}), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_9}\mathbf{1}_L}{- : (\Delta_2, \Delta_3), \mathbf{1}, \Delta_{10} \vdash \mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \mathbf{1}, \Delta_{10}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_3 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10} \quad \mathtt{h}_8 : \Delta_9 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_9, \Delta_{12}), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes_R}{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{12} \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_9 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9 \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_9 \vdash \mathtt{F}_{10} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_9, \Delta_{12}), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes_R}{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{12} \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9 \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, !\mathtt{F}_9), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}W}{- : (\Delta_2, \Delta_3), \Delta_{11}, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, !\mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, !\mathtt{F}_9, !\mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, !\mathtt{F}_9), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}C}{- : (\Delta_2, \Delta_3), \Delta_{11}, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, !\mathtt{F}_9, !\mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, !\mathtt{F}_9, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_3, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,C$$

- Case rule $!L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, !\mathtt{F}_9), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}!L}{- : (\Delta_2, \Delta_3), \Delta_{11}, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_3, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,!L$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \& \mathtt{F}_{10}), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\&_{L2}}{- : (\Delta_2, \Delta_3), \Delta_{12}, \mathtt{F}_9 \& \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9 \& \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \& \mathtt{F}_{10}), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\&_{L1}}{- : (\Delta_2, \Delta_3), \Delta_{12}, \mathtt{F}_9 \& \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9 \& \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\&_{L1}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_{10}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \otimes \mathtt{F}_{10}), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\otimes_L}{- : (\Delta_2, \Delta_3), \Delta_{12}, \mathtt{F}_9 \otimes \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}$$
$$\dfrac{}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9 \otimes \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\otimes_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_8 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\otimes_R \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\otimes_L}{- : (\Delta_2, \Delta_3), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_3 \vdash \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\,\mathtt{sCut}}{- : \Delta_2, \Delta_3, \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{sCut}$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \oplus \mathtt{F}_{10}), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\oplus_L}{- : (\Delta_2, \Delta_3), \Delta_{12}, \mathtt{F}_9 \oplus \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9 \oplus \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\oplus_L$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_{13}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10} \quad \mathtt{h}_8 : \Delta_9, \mathtt{F}_{11} \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_8 : (\Delta_9, \Delta_{13}, \mathtt{F}_{10} \multimap \mathtt{F}_{11}), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{12}}\multimap_L}{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{13}, \mathtt{F}_{10} \multimap \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{13}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_9, \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, \mathtt{F}_{10} \multimap \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\multimap_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\otimes_R \quad \dfrac{\mathtt{h}_8 : \Delta_9 \vdash \mathtt{F}_{10} \quad \mathtt{h}_8 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_8 : (\Delta_9, \Delta_{13}, \mathtt{F}_{10} \multimap \mathtt{F}_{11}), \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{12}}\multimap_L}{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{13}, \mathtt{F}_{10} \multimap \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_6 \otimes \mathtt{F}_7 \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3, \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, \mathtt{F}_{10} \multimap \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\multimap_L$$

- Case rule I

5.10 Status of W: OK

- Case rule $!R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash 6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash 6}\,W \quad \dfrac{\mathtt{h}_7 : !\Upsilon 5, 6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : !\Upsilon 5, 6 \vdash !\mathtt{F}_8}\,!R}{- : (\Delta_2, !\mathtt{F}_3), !\Upsilon 5 \vdash !\mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash 6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : 6, !\Upsilon 5, !\mathtt{F}_3 \vdash !\mathtt{F}_8}\,\mathtt{ax/W}}{- : !\Upsilon 5, \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, W \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top}\, \top}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \top}\, \mathtt{Cut}}{\rightsquigarrow}$$
$$\dfrac{}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \top}\, \top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \&_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, !\mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \mathtt{hCut}$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \multimap_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, !\mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \mathtt{hCut}$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_2}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, !\mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{hCut}$$

- Case rule $\oplus_{R_1}$

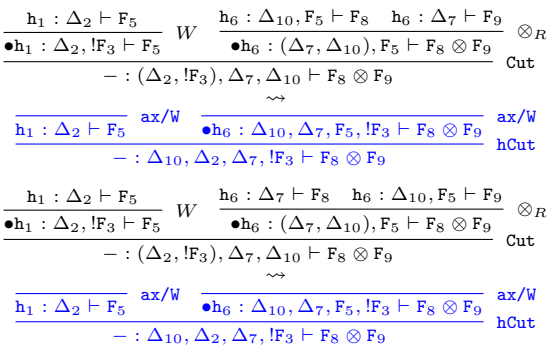
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_1}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, !\mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{hCut}$$

- Case rule $\mathbf{1}_L$

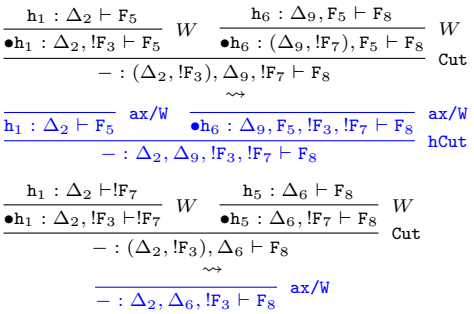
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathbf{1}}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathbf{1}}\, W \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : \Delta_6, \mathbf{1} \vdash \mathtt{F}_7}\, \mathbf{1}_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_7}\, \mathtt{Cut}}{\rightsquigarrow}$$
$$\dfrac{}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_7}\, \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_6 : \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_6 : (\mathbf{1}, \Delta_8), \mathtt{F}_5 \vdash \mathtt{F}_7}\, \mathbf{1}_L}{- : (\Delta_2, !\mathtt{F}_3), \mathbf{1}, \Delta_8 \vdash \mathtt{F}_7}\, \mathtt{Cut}}{\rightsquigarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \mathbf{1}, \Delta_8, \mathtt{F}_5, !\mathtt{F}_3 \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !\mathtt{F}_3 \vdash \mathtt{F}_7}\, \mathtt{hCut}$$

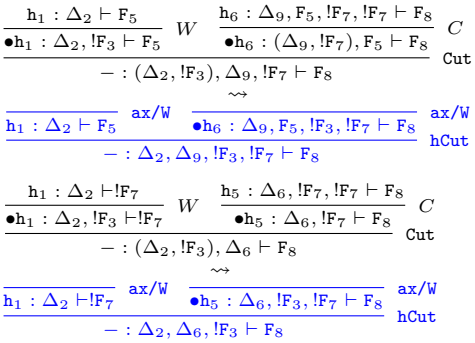
- Case rule $\otimes_R$



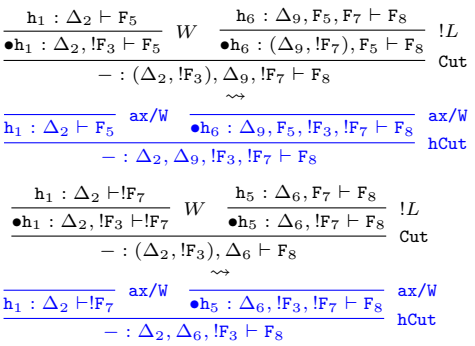
- Case rule W



- Case rule C



- Case rule $!L$



- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\, \&_{L2}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, !\mathtt{F}_3, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \& \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\, W \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \&_{L2}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_3, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, \mathtt{hCut}}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\, \&_{L1}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, !\mathtt{F}_3, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \& \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\, W \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \&_{L1}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \& \mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_3, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, \mathtt{hCut}}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\, \otimes_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, !\mathtt{F}_3, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, W \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \otimes_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_3, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, \mathtt{hCut}}$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7 \oplus \mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\, \oplus_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, !\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\, W \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \oplus_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_5 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9), \mathtt{F}_5 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_{11}, \Delta_7, \mathtt{F}_5, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9), \mathtt{F}_5 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_{11}, \Delta_7, \mathtt{F}_5, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, W \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_5 : (\Delta_6, \Delta_7), \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6, \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \Delta_6, \Delta_7, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \Delta_7, !\mathtt{F}_3 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash p(\mathtt{n}_6)}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash p(\mathtt{n}_6)}\, W \quad \dfrac{}{\bullet\mathtt{h}_5 : *, p(\mathtt{n}_6) \vdash p(\mathtt{n}_6)}\, I}{- : (\Delta_2, !\mathtt{F}_3), * \vdash p(\mathtt{n}_6)}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, !\mathtt{F}_3 \vdash p(\mathtt{n}_6)}\,\mathtt{ax/W}$$

5.11 Status of C: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash 6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash 6}\, C \quad \dfrac{\mathtt{h}_7 : !\Upsilon 5, 6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : !\Upsilon 5, 6 \vdash !\mathtt{F}_8}\, !R}{- : (\Delta_2, !\mathtt{F}_3), !\Upsilon 5 \vdash !\mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash 6}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : 6, !\Upsilon 5 \vdash !\mathtt{F}_8}{}\,\mathtt{ax/W}}{- : !\Upsilon 5, \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash !\mathtt{F}_8}\,\mathtt{hCut}}{- : !\Upsilon 5, \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_8}\, C$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, C \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top}\, \top}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \top}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\,\&_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \& \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, C$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\,\multimap_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_8, !\mathtt{F}_3 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\,\multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_{R_2}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_9}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_{R_1}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathbf{1}}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathbf{1}}\, C \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : \Delta_6, \mathbf{1} \vdash \mathtt{F}_7}\,\mathbf{1}_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_7}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_6 : (\mathbf{1}, \Delta_8), \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathbf{1}_L}{- : (\Delta_2, !\mathtt{F}_3), \mathbf{1}, \Delta_8 \vdash \mathtt{F}_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_6 : \mathbf{1}, \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_7}{\mathtt{h}_6 : \mathbf{1}, \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !\mathtt{F}_3 \vdash \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{10}), \mathtt{F}_5 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{10} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{hCut} \qquad \dfrac{}{- : \Delta_{10}, \Delta_2, \Delta_7, !\mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{10}), \mathtt{F}_5 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{10} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{hCut} \qquad \dfrac{}{- : \Delta_{10}, \Delta_2, \Delta_7, !\mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, C$$

- Case rule W

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Delta_9, !\mathtt{F}_7), \mathtt{F}_5 \vdash \mathtt{F}_8}\, W}{- : (\Delta_2, !\mathtt{F}_3), \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_9, !\mathtt{F}_3, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash !\mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_7}\, C \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, W}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_8}\, \mathtt{ax/W}$$

- Case rule C

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, !\mathtt{F}_7, !\mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Delta_9, !\mathtt{F}_7), \mathtt{F}_5 \vdash \mathtt{F}_8}\, C}{- : (\Delta_2, !\mathtt{F}_3), \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_9, !\mathtt{F}_3, !\mathtt{F}_3, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{hCut} \qquad \dfrac{}{- : \Delta_2, \Delta_9, !\mathtt{F}_3, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, C$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash !\mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_7}\, C \quad \dfrac{\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7, !\mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, C}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_7}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_8}\, \mathtt{mCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Delta_9, !\mathtt{F}_7), \mathtt{F}_5 \vdash \mathtt{F}_8}\, !L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_9, !\mathtt{F}_3, !\mathtt{F}_3, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{hCut} \qquad \dfrac{}{- : \Delta_2, \Delta_9, !\mathtt{F}_3, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash !\mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_7}\, C \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, !L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash !\mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_8}\, C$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7\&\mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\,\&_{L2}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}\, C \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\&_{L2}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, C$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7\&\mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\,\&_{L1}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}\, C \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\&_{L1}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, C$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\,\otimes_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, C \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\,\otimes_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, C$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7 \oplus \mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\, \oplus_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{\dfrac{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, C}\, \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\, C \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \oplus_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}{}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_6, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_9}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, C}\, \mathtt{hCut}}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_5 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9), \mathtt{F}_5 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{11}, \Delta_7, \mathtt{F}_5, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{\dfrac{- : \Delta_{11}, \Delta_2, \Delta_7, !\mathtt{F}_3, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}{- : \Delta_{11}, \Delta_2, \Delta_7, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, C}\, \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9), \mathtt{F}_5 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_5}{}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{11}, \Delta_7, \mathtt{F}_5, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{\dfrac{- : \Delta_{11}, \Delta_2, \Delta_7, !\mathtt{F}_3, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}{- : \Delta_{11}, \Delta_2, \Delta_7, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, C}\, \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, C \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_5 : (\Delta_6, \Delta_7), \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6, \Delta_7 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}{}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \Delta_7, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_6, \Delta_7, !\mathtt{F}_3, !\mathtt{F}_3 \vdash \mathtt{F}_{10}}{- : \Delta_2, \Delta_6, \Delta_7, !\mathtt{F}_3 \vdash \mathtt{F}_{10}}\, C}\, \mathtt{hCut}}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3, !\mathtt{F}_3 \vdash p(\mathtt{n}_6)}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash p(\mathtt{n}_6)}\, C \quad \dfrac{}{\bullet\mathtt{h}_5 : *, p(\mathtt{n}_6) \vdash p(\mathtt{n}_6)}\, I}{- : (\Delta_2, !\mathtt{F}_3), * \vdash p(\mathtt{n}_6)}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_2, !\mathtt{F}_3 \vdash p(\mathtt{n}_6)}\, \mathtt{ax/W}}$$

5.12 Status of $!L$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash 6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash 6}\, !L \quad \dfrac{\mathtt{h}_7 : !\Upsilon 5, 6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : !\Upsilon 5, 6 \vdash !\mathtt{F}_8}\, !R}{- : (\Delta_2, !\mathtt{F}_3), !\Upsilon 5 \vdash !\mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash 6}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : 6, !\Upsilon 5 \vdash !\mathtt{F}_8}\, \mathtt{ax/W}}{- : !\Upsilon 5, \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_8}\, \mathtt{hCut}}{- : !\Upsilon 5, \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_8}\, !L$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, !L \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top}\, \top}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \top}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \top}\, \top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \&_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, !L$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \multimap_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_5, \mathtt{F}_8, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_2}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_1}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_5 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : \Delta_2, \Delta_5, !\mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathbf{1}}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathbf{1}}\, !L \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : \Delta_6, \mathbf{1} \vdash \mathtt{F}_7}\, \mathbf{1}_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_7}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_7}\, \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_6 : \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_6 : (\mathbf{1}, \Delta_8), \mathtt{F}_5 \vdash \mathtt{F}_7}\, \mathbf{1}_L}{- : (\Delta_2, !\mathtt{F}_3), \mathbf{1}, \Delta_8 \vdash \mathtt{F}_7}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \mathbf{1}, \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !\mathtt{F}_3 \vdash \mathtt{F}_7}\, \mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{10}), \mathtt{F}_5 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{10} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_{10}, \Delta_2, \Delta_7, !\mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, !L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{10}), \mathtt{F}_5 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{10} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_{10}, \Delta_2, \Delta_7, !\mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, !L$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Delta_9, !\mathtt{F}_7), \mathtt{F}_5 \vdash \mathtt{F}_8}\, W}{- : (\Delta_2, !\mathtt{F}_3), \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_9, !\mathtt{F}_3, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_7}\, !L \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, W}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_8}\, \mathtt{ax/W}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, !\mathtt{F}_7, !\mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Delta_9, !\mathtt{F}_7), \mathtt{F}_5 \vdash \mathtt{F}_8}\, C}{- : (\Delta_2, !\mathtt{F}_3), \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, !\mathtt{F}_7, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_9, !\mathtt{F}_3, !\mathtt{F}_7, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{hCut}$$

$$\dfrac{}{- : \Delta_2, \Delta_9, !\mathtt{F}_3, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, C$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_7}\, !L \qquad \dfrac{\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7, !\mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, C}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_7}\, \mathtt{ax/W} \qquad \dfrac{}{\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_8}\, \mathtt{mCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \qquad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : (\Delta_9, !\mathtt{F}_7), \mathtt{F}_5 \vdash \mathtt{F}_8}\, !L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \qquad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_9, \mathtt{F}_5, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_9, \mathtt{F}_3, !\mathtt{F}_7 \vdash \mathtt{F}_8}{- : \Delta_2, \Delta_9, !\mathtt{F}_3, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, !L}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash !\mathtt{F}_7}\, !L \qquad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, !L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_8}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_7}\, \mathtt{ax/W} \qquad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_8}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_8}\, !L}\, \mathtt{hCut}$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \qquad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7\&\mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\, \&_{L2}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \qquad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{\dfrac{- : \Delta_{10}, \Delta_2, \mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, !L}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}\, !L \qquad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, \&_{L2}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}\, \mathtt{ax/W} \qquad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_9}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, !L}\, \mathtt{hCut}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \qquad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7\&\mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\, \&_{L1}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \qquad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{\dfrac{- : \Delta_{10}, \Delta_2, \mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, !L}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}\, !L \qquad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, \&_{L1}}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7\&\mathtt{F}_8}\, \mathtt{ax/W} \qquad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7\&\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_9}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, !L}\, \mathtt{hCut}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\, \otimes_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_3, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, !L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, !L \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \otimes_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, !L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : (\Delta_{10}, \mathtt{F}_7 \oplus \mathtt{F}_8), \mathtt{F}_5 \vdash \mathtt{F}_9}\, \oplus_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_{10}, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, !L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\, !L \quad \dfrac{\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_5 : \Delta_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \oplus_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \oplus \mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_2, \Delta_6, !\mathtt{F}_3 \vdash \mathtt{F}_9}\, !L$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_5 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9), \mathtt{F}_5 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_{11}, \Delta_7, \mathtt{F}_5, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_{11}, \Delta_7, \mathtt{F}_5, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_7, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, !L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_5, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9), \mathtt{F}_5 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_5}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_{11}, \Delta_7, \mathtt{F}_5, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_{11}, \Delta_7, \mathtt{F}_5, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_7, !\mathtt{F}_3, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, !L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, !L \quad \dfrac{\mathtt{h}_5 : \Delta_6 \vdash \mathtt{F}_8 \quad \mathtt{h}_5 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_5 : (\Delta_6, \Delta_7), \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (\Delta_2, !\mathtt{F}_3), \Delta_6, \Delta_7 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \Delta_6, \Delta_7, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_5 : \Delta_6, \Delta_7, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : \Delta_2, \Delta_6, \Delta_7, !\mathtt{F}_3 \vdash \mathtt{F}_{10}}\, !L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash p(\mathtt{n}_6)}{\bullet \mathtt{h}_1 : \Delta_2, !\mathtt{F}_3 \vdash p(\mathtt{n}_6)}\, !L \quad \dfrac{}{\bullet \mathtt{h}_5 : *, p(\mathtt{n}_6) \vdash p(\mathtt{n}_6)}\, I}{- : (\Delta_2, !\mathtt{F}_3), * \vdash p(\mathtt{n}_6)}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, !\mathtt{F}_3 \vdash p(\mathtt{n}_6)}\, \mathtt{ax/W}$$

5.13 Status of $\&_{L2}$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash 7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash 7}\, \&_{L2} \quad \dfrac{\mathtt{h}_8 : !\Upsilon 6, 7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : !\Upsilon 6, 7 \vdash !\mathtt{F}_9}\, !R}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), !\Upsilon 6 \vdash !\mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash 7}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_8 : 7, !\Upsilon 6 \vdash !\mathtt{F}_9}\, \mathtt{ax/W}}{- : !\Upsilon 6, \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 6, \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash !\mathtt{F}_9}\, \&_{L2}$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_7}\, \&_{L2} \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \top}\, \top}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_6 \vdash \top}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \top}\, \top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_7}\, \&_{L2} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \&_R}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_4 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \&_{L2}$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_7}\, \&_{L2} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \multimap_R}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_7}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_9, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_7}\, \&_{L2} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_2}}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_7}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_7}\,\&_{L2} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_1}}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_7}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{ax/W}}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathbf{1}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathbf{1}}\,\&_{L2} \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_7, \mathbf{1} \vdash \mathtt{F}_8}\,\mathbf{1}_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : (\mathbf{1}, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathbf{1}_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\mathtt{ax/W}}}{- : \mathbf{1}, \Delta_2, \Delta_9, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\bullet \mathtt{h}_7 : \Delta_{11}, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}{\mathtt{ax/W}}}{\mathtt{hCut}}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\&_{L2}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\bullet \mathtt{h}_7 : \Delta_{11}, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}{\mathtt{ax/W}}}{\mathtt{hCut}}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\&_{L2}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}\,W}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8 \vdash \mathtt{F}_9}{\mathtt{ax/W}}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash !\mathtt{F}_8}\,\&_{L2} \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,W}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{ax/W}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}\,C}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9}\,C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash !\mathtt{F}_8}\,\&_{L2} \quad \dfrac{\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,C}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash !\mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{mCut}}$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}\,!L}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9}\,!L}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash !\mathtt{F}_8}\,\&_{L2} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,!L}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9}\,\&_{L2}$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\&_{L2}}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_4, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3\&\mathtt{F}_4, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L2}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8\&\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_8\&\mathtt{F}_9}\,\&_{L2} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L2}}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8\&\mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\&_{L1}}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_4, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3\&\mathtt{F}_4, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L2}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\,\&_{L2} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L1}}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\&_{L2}}\,\mathtt{hCut}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\otimes_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\dfrac{- : \Delta_{11}, \Delta_2, \mathtt{F}_4, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L2}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\,\&_{L2} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\otimes_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\&_{L2}}\,\mathtt{hCut}$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\oplus_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\dfrac{- : \Delta_{11}, \Delta_2, \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L2}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\,\&_{L2} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\oplus_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\&_{L2}}\,\mathtt{hCut}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\multimap_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{12}, \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{\dfrac{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\&_{L2}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L2} \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\multimap_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{12}, \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{\dfrac{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\&_{L2}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\&_{L2} \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_6 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_8), \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\multimap_L}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_7, \Delta_8 \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \Delta_8, \mathtt{F}_4 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \Delta_8, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_{11}}\,\&_{L2}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash p(\mathtt{n}_7)}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash p(\mathtt{n}_7)}\,\&_{L2} \quad \dfrac{}{\bullet\mathtt{h}_6 : *, p(\mathtt{n}_7) \vdash p(\mathtt{n}_7)}\,I}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), * \vdash p(\mathtt{n}_7)}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash p(\mathtt{n}_7)}\,\mathtt{ax/W}$$

5.14 Status of $\&_{L1}$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash 7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash 7}\,\&_{L1} \quad \dfrac{\mathtt{h}_8 : !\Upsilon 6, 7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : !\Upsilon 6, 7 \vdash !\mathtt{F}_9}\,!R}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), !\Upsilon 6 \vdash !\mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash 7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : 7, !\Upsilon 6 \vdash !\mathtt{F}_9}\,\mathtt{ax/W}}{- : !\Upsilon 6, \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_9}\,\mathtt{hCut}}{- : !\Upsilon 6, \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash !\mathtt{F}_9}\,\&_{L1}$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_7}\,\&_{L1} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \top}\,\top}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_6 \vdash \top}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_7}\,\&_{L1} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}\,\&_R}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}\,\&_{L1}$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_7}\,\&_{L1} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\multimap_R}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_9, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_7}\,\&_{L1} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_2}}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_7}\,\&_{L1} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_1}}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathbf{1}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathbf{1}}\,\&_{L1} \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : \Delta_7, \mathbf{1} \vdash \mathtt{F}_8}\,\mathbf{1}_L}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : (\mathbf{1}, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathbf{1}_L}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\&_{L1}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes_R}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\&_{L1}$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}\,W}{- : (\Delta_2, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash !\mathtt{F}_8}\,\&_{L1} \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,W}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{ax/W}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}\,C}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash !\mathtt{F}_8}\,\&_{L1} \quad \dfrac{\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,C}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash !\mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{mCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}\,!L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_8, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,!L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash !\mathtt{F}_8}\,\&_{L1} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,!L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9}\,\&_{L1}$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\&_{L2}}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L2}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\,\&_{L1} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L2}}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\&_{L1}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\ \&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\ \&_{L1}}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{\dfrac{- : \Delta_{11}, \Delta_2, \mathtt{F}_3, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \&_{L1}}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\ \&_{L1} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \&_{L1}}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\ \&_{L1}}\ \mathtt{hCut}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\ \&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\ \otimes_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{\dfrac{- : \Delta_{11}, \Delta_2, \mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \&_{L1}}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\ \&_{L1} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \otimes_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\ \&_{L1}}\ \mathtt{hCut}$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\ \&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\ \oplus_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{\dfrac{- : \Delta_{11}, \Delta_2, \mathtt{F}_3, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \&_{L1}}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\ \&_{L1} \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \oplus_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{\dfrac{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{10}}\ \&_{L1}}\ \mathtt{hCut}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\ \&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_6 \vdash \mathtt{F}_{11}}\ \multimap_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\ \mathtt{ax/W}}{\dfrac{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\ \&_{L1}}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_6}\,\&_{L1} \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_6 \vdash \mathtt{F}_{11}}\,\multimap_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \Delta_{12}, \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_7 : \Delta_{12}, \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\&_{L1}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\&_{L1} \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_6 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_6 : (\Delta_7, \Delta_8), \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\multimap_L}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_7, \Delta_8 \vdash \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \Delta_7, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_6 : \Delta_7, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \Delta_8, \mathtt{F}_3 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4 \vdash \mathtt{F}_{11}}\,\&_{L1}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash p(\mathtt{n}_7)}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash p(\mathtt{n}_7)}\,\&_{L1} \quad \dfrac{}{\bullet \mathtt{h}_6 : *, p(\mathtt{n}_7) \vdash p(\mathtt{n}_7)}\,I}{- : (\Delta_2, \mathtt{F}_3 \& \mathtt{F}_4), * \vdash p(\mathtt{n}_7)}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4 \vdash p(\mathtt{n}_7)}\,\mathtt{ax/W}$$

5.15 Status of $\otimes_L$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash 7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash 7}\,\otimes_L \quad \dfrac{\mathtt{h}_8 : !\Upsilon 6, 7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : !\Upsilon 6, 7 \vdash !\mathtt{F}_9}\,!R}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), !\Upsilon 6 \vdash !\mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash 7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_8 : 7, !\Upsilon 6 \vdash !\mathtt{F}_9}\,\mathtt{ax/W}}{- : !\Upsilon 6, \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash !\mathtt{F}_9}\,\mathtt{hCut}}{- : !\Upsilon 6, \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash !\mathtt{F}_9}\,\otimes_L$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_7}\,\otimes_L \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \top}\,\top}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_6 \vdash \top}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_7}\,\otimes_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\&_R}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\otimes_L$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_7}\otimes_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}} \multimap_R}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_7}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_9, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}} \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_7}\otimes_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}} \oplus_{R_2}}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_7}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}} \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_7}\otimes_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}} \oplus_{R_1}}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_7}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}} \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathbf{1}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathbf{1}}\otimes_L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \Delta_7, \mathbf{1} \vdash \mathtt{F}_8}\mathbf{1}_L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_8}\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : (\mathbf{1}, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\mathbf{1}_L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_8}\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \Delta_{11}, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \Delta_{11}, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \Delta_{11}, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \Delta_{11}, \Delta_8, \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_L$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}W}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9}\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash !\mathtt{F}_8}\otimes_L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}W}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\mathtt{Cut}}{\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9}\mathtt{ax/W}}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}C}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9}C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash !\mathtt{F}_8}\otimes_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}C}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\mathtt{Cut}}{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash !\mathtt{F}_8}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9}\mathtt{mCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}!L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9}!L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash !\mathtt{F}_8}\otimes_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}!L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash !\mathtt{F}_8}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9}\otimes_L$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\&_{L2}}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \& \mathtt{F}_9, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}\&_{L2}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\otimes_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\&_{L2}}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{hCut}}}{- : \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\otimes_L}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\&_{L1}}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{hCut}}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}\&_{L1}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \& \mathtt{F}_9, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\otimes_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\&_{L1}}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{hCut}}}{- : \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\otimes_L}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\otimes_L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{hCut}}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\otimes_L}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\otimes_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\otimes_L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{hCut}}}{- : \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\otimes_L}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\oplus_L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{hCut}}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\otimes_L}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\otimes_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\oplus_L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\mathtt{ax/W}}{\mathtt{hCut}}}{- : \Delta_2, \Delta_7, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\otimes_L}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{10}}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_6 \vdash \mathtt{F}_{11}}\multimap_L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : \Delta_{12}, \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : \Delta_{12}, \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3 \otimes \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\otimes_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_6}\otimes_L \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10}), \mathtt{F}_6 \vdash \mathtt{F}_{11}}\multimap_L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_7 : \Delta_{12}, \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : \Delta_{12}, \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, \mathtt{F}_3 \otimes \mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\otimes_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\otimes_L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_6 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_6 : (\Delta_7, \Delta_8), \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\multimap_L}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_7, \Delta_8 \vdash \mathtt{F}_{11}}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_6 : \Delta_7, \Delta_8, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \Delta_8, \mathtt{F}_3, \mathtt{F}_4 \vdash \mathtt{F}_{11}}\mathtt{hCut}}{- : \Delta_2, \Delta_7, \Delta_8, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash \mathtt{F}_{11}}\otimes_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash p(\mathtt{n}_7)}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash p(\mathtt{n}_7)}\otimes_L \quad \dfrac{}{\bullet\mathtt{h}_6 : *, p(\mathtt{n}_7) \vdash p(\mathtt{n}_7)}I}{- : (\Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4), * \vdash p(\mathtt{n}_7)}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4 \vdash p(\mathtt{n}_7)}\mathtt{ax/W}$$

5.16 Status of $\oplus_L$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash 7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash 7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash 7}\oplus_L \quad \dfrac{\mathtt{h}_8 : !\Upsilon 6, 7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : !\Upsilon 6, 7 \vdash !\mathtt{F}_9}!R}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), !\Upsilon 6 \vdash !\mathtt{F}_9}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash 7}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash 7}\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_8 : 7, !\Upsilon 6 \vdash !\mathtt{F}_9}{\bullet\mathtt{h}_8 : 7, !\Upsilon 6 \vdash !\mathtt{F}_9}\mathtt{ax/W}}{- : !\Upsilon 6, \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_9}\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash 7}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash 7}\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_8 : 7, !\Upsilon 6 \vdash !\mathtt{F}_9}{\bullet\mathtt{h}_8 : 7, !\Upsilon 6 \vdash !\mathtt{F}_9}\mathtt{ax/W}}{- : !\Upsilon 6, \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_9}\mathtt{hCut}}{- : !\Upsilon 6, \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash !\mathtt{F}_9}\oplus_L$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}\oplus_L \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \top}\top}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_6 \vdash \top}\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \top}\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}\oplus_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\&_R}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\,\&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}\oplus_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\multimap_R}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\multimap_R}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}\oplus_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_2}}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}\oplus_L \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_1}}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_6 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_6, \mathtt{F}_7 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathbf{1} \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathbf{1}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathbf{1}}\oplus_L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_6 : \Delta_7, \mathbf{1} \vdash \mathtt{F}_8}\mathbf{1}_L}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_8}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\oplus_L \quad \dfrac{\mathtt{h}_7 : \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : (\mathbf{1}, \Delta_9), \mathtt{F}_6 \vdash \mathtt{F}_8}\mathbf{1}_L}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \mathbf{1}, \Delta_9, \mathtt{F}_6 \vdash \mathtt{F}_8}{}\,\mathtt{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\oplus_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_8 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\oplus_L \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_{11}), \mathtt{F}_6 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\oplus_L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}W}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_8 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash !\mathtt{F}_8}\oplus_L \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}W}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{ax/W}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\oplus_L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}C}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, !\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_8 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash !\mathtt{F}_8}\oplus_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}C}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash !\mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{mCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\oplus_L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, !\mathtt{F}_8), \mathtt{F}_6 \vdash \mathtt{F}_9}!L}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \Delta_2, \mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{10}, \Delta_2, !\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}!L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_8 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash !\mathtt{F}_8}\oplus_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,!L}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash !\mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash !\mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, !\mathtt{F}_8 \vdash \mathtt{F}_9}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_9}\,\oplus_L$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\oplus_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\&_{L2}}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8\&\mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\&_{L2}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8\&\mathtt{F}_9 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8\&\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_8\&\mathtt{F}_9}\oplus_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L2}}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8\&\mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8\&\mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\oplus_L$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\oplus_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\&_{L1}}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8\&\mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\&_{L1}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8\&\mathtt{F}_9 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8\&\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_8\&\mathtt{F}_9}\oplus_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\&_{L1}}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8\&\mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8\&\mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8\&\mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\oplus_L$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}\oplus_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9), \mathtt{F}_6 \vdash \mathtt{F}_{10}}\,\otimes_L}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

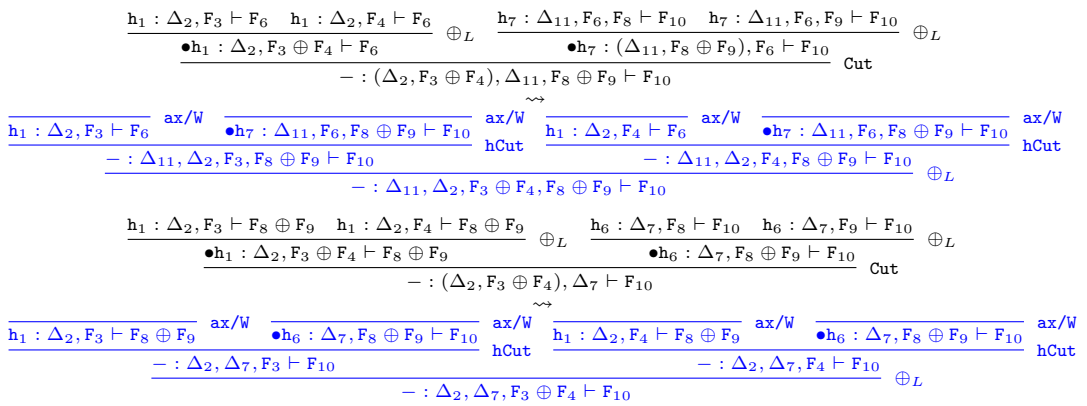
$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_6}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \mathtt{F}_8 \otimes \mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\otimes_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\oplus_L \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\otimes_L}{- : (\Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_7 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}$$

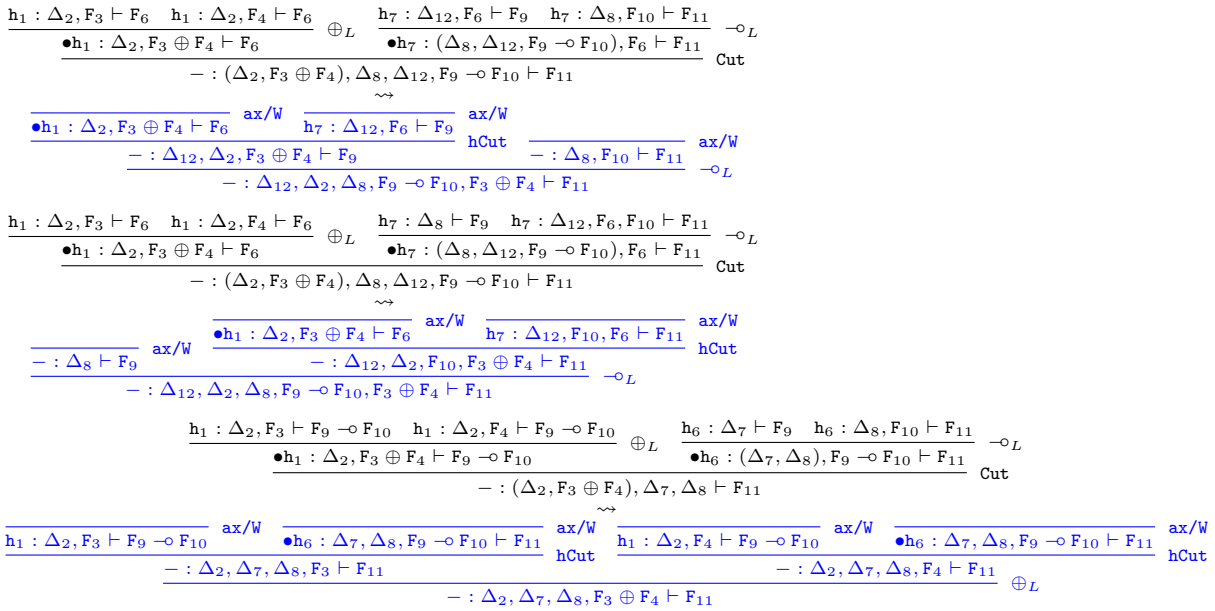
$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_7, \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_7, \mathtt{F}_3 \oplus \mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\oplus_L$$

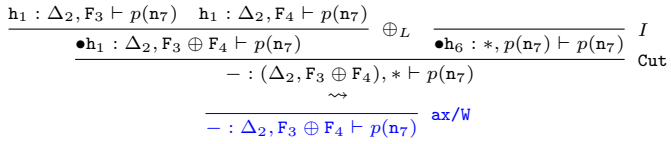
- Case rule $\oplus_L$



- Case rule $\multimap_L$



- Case rule I



5.17 Status of $\multimap_L$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash 8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash 8}\multimap_L \quad \dfrac{\mathtt{h}_9 : !\Upsilon 7, 8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : !\Upsilon 7, 8 \vdash !\mathtt{F}_{10}}\,!R}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), !\Upsilon 7 \vdash !\mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash 8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : 8, !\Upsilon 7 \vdash !\mathtt{F}_{10}}\,\mathtt{ax/W}}{- : !\Upsilon 7, \Delta_3, \mathtt{F}_5 \vdash !\mathtt{F}_{10}}\,\mathtt{hCut}}{- : !\Upsilon 7, \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash !\mathtt{F}_{10}}\multimap_L$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_8}\multimap_L \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \top}\,\top}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_7 \vdash \top}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_2, \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \top}\,\top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_8}\multimap_L \quad \dfrac{\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\&_R}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_7 \vdash \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_7, \mathtt{F}_5 \vdash \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}\&\mathtt{F}_{11}}\multimap_L$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_8}\multimap_L \quad \dfrac{\mathtt{h}_9 : \Delta_7, \mathtt{F}_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \multimap \mathtt{F}_{11}}\multimap_R}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_7 \vdash \mathtt{F}_{10} \multimap \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \mathtt{F}_{11}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \Delta_7, \mathtt{F}_{10}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10} \multimap \mathtt{F}_{11}}\multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_8}\multimap_L \quad \dfrac{\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_{R_2}}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_7 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{11}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_8}\multimap_L \quad \dfrac{\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_{R_1}}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_7 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \mathtt{F}_{10}}{}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_7, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathbf{1}}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathbf{1}} \multimap_L \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_8, \mathbf{1} \vdash \mathtt{F}_9}\,\mathbf{1}_L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{- : \Delta_2, \Delta_3, \Delta_8, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_9}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7} \multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\mathbf{1}, \Delta_{10}), \mathtt{F}_7 \vdash \mathtt{F}_9}\,\mathbf{1}_L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \mathbf{1}, \Delta_{10} \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_8 : \mathbf{1}, \Delta_{10}, \mathtt{F}_7 \vdash \mathtt{F}_9}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_9}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7} \multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7 \vdash \mathtt{F}_{10} \quad \mathtt{h}_8 : \Delta_9 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_9, \Delta_{12}), \mathtt{F}_7 \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes_R}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_9, \Delta_{12} \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}} \quad \dfrac{- : \Delta_9 \vdash \mathtt{F}_{11}}{\mathtt{ax/W}}\,\otimes_R}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7} \multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_9 \vdash \mathtt{F}_{10} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_9, \Delta_{12}), \mathtt{F}_7 \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes_R}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_9, \Delta_{12} \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7 \vdash \mathtt{F}_{11}}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}} \quad \dfrac{- : \Delta_9 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}\,\otimes_R}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10} \otimes \mathtt{F}_{11}}$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7} \multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, !\mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,W}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_{11}, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_7, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_3, !\mathtt{F}_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash !\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash !\mathtt{F}_9} \multimap_L \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : \Delta_8, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,W}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{- : \Delta_2, \Delta_3, \Delta_8, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7} \multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_7, !\mathtt{F}_9, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, !\mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,C}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_{11}, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_7, !\mathtt{F}_9, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_3, !\mathtt{F}_9, !\mathtt{F}_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}\,C}{- : \Delta_{11}, \Delta_2, \Delta_3, !\mathtt{F}_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash !\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash !\mathtt{F}_9}\multimap_L \quad \dfrac{\mathtt{h}_7 : \Delta_8, !\mathtt{F}_9, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : \Delta_8, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\, C}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash !\mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_8, !\mathtt{F}_9, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_2, \Delta_3, \Delta_8, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}\,\mathtt{mCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, !\mathtt{F}_9), \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,!L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_{11}, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_3, !\mathtt{F}_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}\,!L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash !\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash !\mathtt{F}_9}\multimap_L \quad \dfrac{\mathtt{h}_7 : \Delta_8, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : \Delta_8, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,!L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_8 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash !\mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_8, !\mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_8, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}\multimap_L$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9\&\mathtt{F}_{10}), \mathtt{F}_7 \vdash \mathtt{F}_{11}}\,\&_{L2}}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_{12}, \mathtt{F}_9\&\mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_{10}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9\&\mathtt{F}_{10}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\&_{L2}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}\multimap_L \quad \dfrac{\mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : \Delta_8, \mathtt{F}_9\&\mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\&_{L2}}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_8 \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_8, \mathtt{F}_9\&\mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_8, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\multimap_L$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9\&\mathtt{F}_{10}), \mathtt{F}_7 \vdash \mathtt{F}_{11}}\,\&_{L1}}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_{12}, \mathtt{F}_9\&\mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9\&\mathtt{F}_{10}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\&_{L1}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}\multimap_L \quad \dfrac{\mathtt{h}_7 : \Delta_8, \mathtt{F}_9 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : \Delta_8, \mathtt{F}_9\&\mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\&_{L1}}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_8 \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_9\&\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_8, \mathtt{F}_9\&\mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_8, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\multimap_L$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \otimes \mathtt{F}_{10}), \mathtt{F}_7 \vdash \mathtt{F}_{11}}\otimes_L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_{12}, \mathtt{F}_9 \otimes \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_9 \otimes \mathtt{F}_{10}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\otimes_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\multimap_L \quad \dfrac{\mathtt{h}_7 : \Delta_8, \mathtt{F}_9, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\otimes_L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_8 \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_8, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\multimap_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_9 \vdash \mathtt{F}_{11} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \oplus \mathtt{F}_{10}), \mathtt{F}_7 \vdash \mathtt{F}_{11}}\oplus_L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_{12}, \mathtt{F}_9 \oplus \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_9 \oplus \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \Delta_3, \mathtt{F}_5, \mathtt{F}_9 \oplus \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5, \mathtt{F}_9 \oplus \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\multimap_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\multimap_L \quad \dfrac{\mathtt{h}_7 : \Delta_8, \mathtt{F}_9 \vdash \mathtt{F}_{11} \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_7 : \Delta_8, \mathtt{F}_9 \oplus \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\oplus_L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_8 \vdash \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_8, \mathtt{F}_9 \oplus \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_8, \mathtt{F}_5 \vdash \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_8, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{11}}\multimap_L$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_{13}, \mathtt{F}_7 \vdash \mathtt{F}_{10} \quad \mathtt{h}_8 : \Delta_9, \mathtt{F}_{11} \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_8 : (\Delta_9, \Delta_{13}, \mathtt{F}_{10} \multimap \mathtt{F}_{11}), \mathtt{F}_7 \vdash \mathtt{F}_{12}}\multimap_L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_9, \Delta_{13}, \mathtt{F}_{10} \multimap \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{13}, \mathtt{F}_7 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{}{- : \Delta_9, \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, \mathtt{F}_{10} \multimap \mathtt{F}_{11}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{12}}\multimap_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\multimap_L \quad \dfrac{\mathtt{h}_8 : \Delta_9 \vdash \mathtt{F}_{10} \quad \mathtt{h}_8 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{11} \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_8 : (\Delta_9, \Delta_{13}, \mathtt{F}_{10} \multimap \mathtt{F}_{11}), \mathtt{F}_7 \vdash \mathtt{F}_{12}}\multimap_L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_9, \Delta_{13}, \mathtt{F}_{10} \multimap \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_9 \vdash \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3, \mathtt{F}_{11}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, \mathtt{F}_{10} \multimap \mathtt{F}_{11}, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{12}}\multimap_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_{10} \multimap \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{10} \multimap \mathtt{F}_{11}}\multimap_L \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_9, \mathtt{F}_{11} \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_7 : (\Delta_8, \Delta_9), \mathtt{F}_{10} \multimap \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\multimap_L}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), \Delta_8, \Delta_9 \vdash \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_2 \vdash \mathtt{F}_4}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash \mathtt{F}_{10} \multimap \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_8, \Delta_9, \mathtt{F}_{10} \multimap \mathtt{F}_{11} \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_5 \vdash \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_2, \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash \mathtt{F}_{12}}\multimap_L$$

- Case rule I

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_3, \mathtt{F}_5 \vdash p(\mathtt{n}_8)}{\bullet\mathtt{h}_1 : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash p(\mathtt{n}_8)}\ \multimap_L \quad \dfrac{}{\bullet\mathtt{h}_7 : *, p(\mathtt{n}_8) \vdash p(\mathtt{n}_8)}\ I}{- : (\Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5), * \vdash p(\mathtt{n}_8)}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_2, \Delta_3, \mathtt{F}_4 \multimap \mathtt{F}_5 \vdash p(\mathtt{n}_8)}\ \mathtt{ax/W}$$

5.18 Status of I: OK

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : p(\mathtt{n}_4) \vdash p(\mathtt{n}_4)}\ I \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_3, p(\mathtt{n}_4) \vdash \top}\ \top}{- : p(\mathtt{n}_4), \Delta_3 \vdash \top}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_3, p(\mathtt{n}_4) \vdash \top}\ \top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : p(\mathtt{n}_4) \vdash p(\mathtt{n}_4)}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6 \& \mathtt{F}_7}\ \&_R}{- : p(\mathtt{n}_4), \Delta_3 \vdash \mathtt{F}_6 \& \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6 \& \mathtt{F}_7}\ \mathtt{ax/W}$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : p(\mathtt{n}_4) \vdash p(\mathtt{n}_4)}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_3, \mathtt{F}_6, p(\mathtt{n}_4) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\ \multimap_R}{- : p(\mathtt{n}_4), \Delta_3 \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6 \multimap \mathtt{F}_7}\ \mathtt{ax/W}$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : p(\mathtt{n}_4) \vdash p(\mathtt{n}_4)}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_5 : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_{R_2}}{- : p(\mathtt{n}_4), \Delta_3 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{ax/W}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : p(\mathtt{n}_4) \vdash p(\mathtt{n}_4)}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_{R_1}}{- : p(\mathtt{n}_4), \Delta_3 \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_3, p(\mathtt{n}_4) \vdash \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{ax/W}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_6, p(\mathtt{n}_3) \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : (\mathbf{1}, \Delta_6), p(\mathtt{n}_3) \vdash \mathtt{F}_5}\ \mathbf{1}_L}{- : p(\mathtt{n}_3), \mathbf{1}, \Delta_6 \vdash \mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \mathbf{1}, \Delta_6, p(\mathtt{n}_3) \vdash \mathtt{F}_5}\ \mathtt{ax/W}$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_8, p(\mathtt{n}_3) \vdash \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_5, \Delta_8), p(\mathtt{n}_3) \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes_R}{- : p(\mathtt{n}_3), \Delta_5, \Delta_8 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_5, \Delta_8, p(\mathtt{n}_3) \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_8, p(\mathtt{n}_3) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_5, \Delta_8), p(\mathtt{n}_3) \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes_R}{- : p(\mathtt{n}_3), \Delta_5, \Delta_8 \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_5, \Delta_8, p(\mathtt{n}_3) \vdash \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{ax/W}$$

- Case rule W

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_7, p(\mathtt{n}_3) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_7, !\mathtt{F}_5), p(\mathtt{n}_3) \vdash \mathtt{F}_6}\ W}{- : p(\mathtt{n}_3), \Delta_7, !\mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_7, !\mathtt{F}_5, p(\mathtt{n}_3) \vdash \mathtt{F}_6}\ \mathtt{ax/W}$$

- Case rule C

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_7, !\mathtt{F}_5, !\mathtt{F}_5, p(\mathtt{n}_3) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_7, !\mathtt{F}_5), p(\mathtt{n}_3) \vdash \mathtt{F}_6}\ C}{- : p(\mathtt{n}_3), \Delta_7, !\mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_7, !\mathtt{F}_5, p(\mathtt{n}_3) \vdash \mathtt{F}_6}\ \mathtt{ax/W}$$

- Case rule $!L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_7, \mathtt{F}_5, p(\mathtt{n}_3) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_4 : (\Delta_7, !\mathtt{F}_5), p(\mathtt{n}_3) \vdash \mathtt{F}_6}\ !L}{- : p(\mathtt{n}_3), \Delta_7, !\mathtt{F}_5 \vdash \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_7, !\mathtt{F}_5, p(\mathtt{n}_3) \vdash \mathtt{F}_6}\ \mathtt{ax/W}$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_8, \mathtt{F}_6, p(\mathtt{n}_3) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \& \mathtt{F}_6), p(\mathtt{n}_3) \vdash \mathtt{F}_7}\ \&_{L2}}{- : p(\mathtt{n}_3), \Delta_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_8, p(\mathtt{n}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax/W}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_8, \mathtt{F}_5, p(\mathtt{n}_3) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \& \mathtt{F}_6), p(\mathtt{n}_3) \vdash \mathtt{F}_7}\ \&_{L1}}{- : p(\mathtt{n}_3), \Delta_8, \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_8, p(\mathtt{n}_3), \mathtt{F}_5 \& \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax/W}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_8, \mathtt{F}_5, \mathtt{F}_6, p(\mathtt{n}_3) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6), p(\mathtt{n}_3) \vdash \mathtt{F}_7}\ \otimes_L}{- : p(\mathtt{n}_3), \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_8, p(\mathtt{n}_3), \mathtt{F}_5 \otimes \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax/W}$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_8, \mathtt{F}_5, p(\mathtt{n}_3) \vdash \mathtt{F}_7 \quad \mathtt{h}_4 : \Delta_8, \mathtt{F}_6, p(\mathtt{n}_3) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \oplus \mathtt{F}_6), p(\mathtt{n}_3) \vdash \mathtt{F}_7}\ \oplus_L}{- : p(\mathtt{n}_3), \Delta_8, \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_8, p(\mathtt{n}_3), \mathtt{F}_5 \oplus \mathtt{F}_6 \vdash \mathtt{F}_7}\ \mathtt{ax/W}$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_9, p(\mathtt{n}_3) \vdash \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_4 : (\Delta_5, \Delta_9, \mathtt{F}_6 \multimap \mathtt{F}_7), p(\mathtt{n}_3) \vdash \mathtt{F}_8}\ \multimap_L}{- : p(\mathtt{n}_3), \Delta_5, \Delta_9, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_5, \Delta_9, p(\mathtt{n}_3), \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_3) \vdash p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_9, \mathtt{F}_7, p(\mathtt{n}_3) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_4 : (\Delta_5, \Delta_9, \mathtt{F}_6 \multimap \mathtt{F}_7), p(\mathtt{n}_3) \vdash \mathtt{F}_8}\ \multimap_L}{- : p(\mathtt{n}_3), \Delta_5, \Delta_9, \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_5, \Delta_9, p(\mathtt{n}_3), \mathtt{F}_6 \multimap \mathtt{F}_7 \vdash \mathtt{F}_8}\ \mathtt{ax/W}$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : p(\mathtt{n}_4) \vdash p(\mathtt{n}_4)}\ I \quad \dfrac{}{\bullet \mathtt{h}_3 : *, p(\mathtt{n}_4) \vdash p(\mathtt{n}_4)}\ I}{- : p(\mathtt{n}_4), * \vdash p(\mathtt{n}_4)}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : p(\mathtt{n}_4) \vdash p(\mathtt{n}_4)}\ I$$

6 Cut-Elimination

6.1 Status of $!R$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, !R \quad \dfrac{\mathtt{h}_7 :!\Upsilon 4, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), !\Upsilon 4 \vdash !\mathtt{F}_8}\, !R}{- :!\Upsilon 2, !\Upsilon 4 \vdash !\mathtt{F}_8}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :!\Upsilon 4, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- :!\Upsilon 2, !\Upsilon 4 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- :!\Upsilon 2, !\Upsilon 4 \vdash !\mathtt{F}_8}\, !R$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, !R \quad \dfrac{}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_4 \vdash \top}\, \top}{- :!\Upsilon 2, \Delta_4 \vdash \top}\, \mathtt{Cut}}{\dfrac{}{- :!\Upsilon 2, \Delta_4 \vdash \top}\, \top}$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, !R \quad \dfrac{\mathtt{h}_7 : \Delta_4, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_4, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \&_R}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_4, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_4, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8 \& \mathtt{F}_9}\, \&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, !R \quad \dfrac{\mathtt{h}_7 : \Delta_4, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_4 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \multimap_R}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_4, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- :!\Upsilon 2, \Delta_4, \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8 \multimap \mathtt{F}_9}\, \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :!\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, !R \quad \dfrac{\mathtt{h}_7 : \Delta_4, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_2}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :!\Upsilon 2 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_4, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- :!\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_5}\, !R \quad \dfrac{\mathtt{h}_7 : \Delta_4, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : contract(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_1}}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \mathtt{Cut}}{}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_5}\, ax \quad \dfrac{}{\mathtt{h}_7 : \Delta_4, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, ax}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8 \oplus \mathtt{F}_9}\, \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_8, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \mathbf{1}, \Delta_8 \vdash \mathtt{F}_7}\, \mathbf{1}_L}{- : !\Upsilon 2, \mathbf{1}, \Delta_8 \vdash \mathtt{F}_7}\, \mathtt{Cut}}{}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, ax \quad \dfrac{}{\mathtt{h}_6 : \Delta_8, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_7}\, ax}{- : !\Upsilon 2, \Delta_8 \vdash \mathtt{F}_7}\, \mathtt{hCut}}{- : \mathbf{1}, !\Upsilon 2, \Delta_8 \vdash \mathtt{F}_7}\, \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_7, \Delta_{10} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : !\Upsilon 2, \Delta_7, \Delta_{10} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}{}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, ax \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}\, ax}{- : !\Upsilon 2, \Delta_{10} \vdash \mathtt{F}_8}\, \mathtt{hCut} \quad \dfrac{}{- : \Delta_7 \vdash \mathtt{F}_9}\, ax}{- : !\Upsilon 2, \Delta_{10}, \Delta_7 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_9, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_9, \Delta_{10} \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes_R}{- : !\Upsilon 2, \Delta_9, \Delta_{10} \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{Cut}}{}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, ax \quad \dfrac{}{\mathtt{h}_6 : \Delta_9, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_7}\, ax}{- : !\Upsilon 2, \Delta_9 \vdash \mathtt{F}_7}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, ax \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, ax}{- : !\Upsilon 2, \Delta_{10} \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : !\Upsilon 2, !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes_R}{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_{10}, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_9, \Delta_{10} \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes_R}{- : !\Upsilon 2, \Delta_9, \Delta_{10} \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{Cut}}{}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, ax \quad \dfrac{}{\mathtt{h}_6 : \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_7}\, ax}{- : !\Upsilon 2, \Delta_9 \vdash \mathtt{F}_7}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, ax \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}\, ax}{- : !\Upsilon 2, \Delta_{10} \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : !\Upsilon 2, !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes_R}{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash \mathtt{F}_7 \otimes \mathtt{F}_8}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_{10}, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_7, \Delta_{10} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : !\Upsilon 2, \Delta_7, \Delta_{10} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}{}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_7 \vdash \mathtt{F}_8}\, ax \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, ax \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}\, ax}{- : !\Upsilon 2, \Delta_{10} \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_{10}, \Delta_7 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_9, !\mathtt{F}_4, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \mathit{contract}(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, W}{- : !\Upsilon 2, \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 : \Delta_9, !\mathtt{F}_4, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_9 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_7}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_7) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \mathit{contract}(s\mathtt{n}_5, !\mathtt{F}_7), \Delta_4 \vdash \mathtt{F}_8}\, W}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_7}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_7) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\, \mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_9, !\mathtt{F}_4, !\mathtt{F}_7, !\mathtt{F}_7, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \mathit{contract}(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, C}{- : !\Upsilon 2, \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 : \Delta_9, !\mathtt{F}_4, !\mathtt{F}_7, !\mathtt{F}_7, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_9, !\mathtt{F}_7, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_7}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_4, !\mathtt{F}_7, !\mathtt{F}_7, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_7) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \mathit{contract}(s\mathtt{n}_5, !\mathtt{F}_7), \Delta_4 \vdash \mathtt{F}_8}\, C}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_7}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, !\mathtt{F}_7, !\mathtt{F}_7, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_7) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\, \mathtt{hCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{F}_7, !\mathtt{F}_4, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \mathit{contract}(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, !L}{- : !\Upsilon 2, \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 : \Delta_9, \mathtt{F}_7, !\mathtt{F}_4, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_9, \mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_9, !\mathtt{F}_7 \vdash \mathtt{F}_8}\, !L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_7}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_4, \mathtt{F}_7, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_7) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : \mathit{contract}(s\mathtt{n}_5, !\mathtt{F}_7), \Delta_4 \vdash \mathtt{F}_8}\, !L}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{- : !\Upsilon 2 \vdash \mathtt{F}_7}\, \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_7}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 : \Delta_4, \mathtt{F}_7, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_7) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_4, \mathtt{F}_7 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_4 \vdash \mathtt{F}_8}\, \mathtt{Cut}$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : \mathit{contract}(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \&_{L2}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4, \mathit{contract}(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \&_{L1}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \& \mathtt{F}_8 \vdash \mathtt{F}_9}\, \&_{L1}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \otimes_L}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8 \vdash \mathtt{F}_9}\, \otimes_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_{10}, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \oplus_L}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \vdash \mathtt{F}_9}\, \mathtt{hCut} \quad \dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 2, \Delta_{10}, \mathtt{F}_7 \oplus \mathtt{F}_8 \vdash \mathtt{F}_9}\, \oplus_L$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : !\Upsilon 2, \Delta_7, \Delta_{11}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

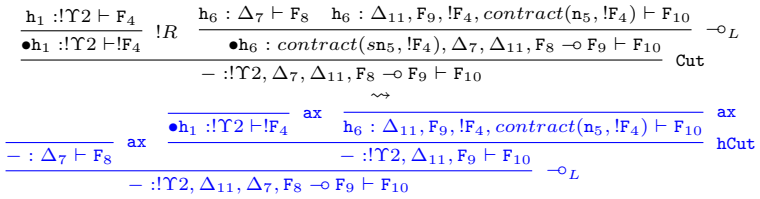
$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}{\mathtt{h}_6 : \Delta_{11}, !\mathtt{F}_4, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{11} \vdash \mathtt{F}_8}\, \mathtt{hCut} \quad \dfrac{}{- : \Delta_7, \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{11}, \Delta_7, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \multimap_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_{10}, \Delta_{11}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\, \multimap_L}{- : !\Upsilon 2, \Delta_{10}, \Delta_{11}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_7}{\mathtt{h}_6 : \Delta_{10}, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_7}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{10} \vdash \mathtt{F}_7}\, \mathtt{hCut} \quad \dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 2, !\Upsilon 2, \Delta_{10}, \Delta_{11}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\, \multimap_L}{- : !\Upsilon 2, \Delta_{10}, \Delta_{11}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : !\Upsilon 2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, !R \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : contract(s\mathtt{n}_5, !\mathtt{F}_4), \Delta_{10}, \Delta_{11}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\, \multimap_L}{- : !\Upsilon 2, \Delta_{10}, \Delta_{11}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_7}{\mathtt{h}_6 : \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_7}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{10} \vdash \mathtt{F}_7}\, \mathtt{hCut} \quad \dfrac{\dfrac{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}{\bullet \mathtt{h}_1 : !\Upsilon 2 \vdash !\mathtt{F}_4}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, contract(\mathtt{n}_5, !\mathtt{F}_4) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 2, \Delta_{11}, \mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 2, !\Upsilon 2, \Delta_{10}, \Delta_{11}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\, \multimap_L}{- : !\Upsilon 2, \Delta_{10}, \Delta_{11}, \mathtt{F}_7 \multimap \mathtt{F}_8 \vdash \mathtt{F}_9}\, C$$



- Case rule I

6.2 Status of $\mathbf{1}_R$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$

- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.3 Status of $\top$: `OK`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$

- Case rule $\multimap_L$
- Case rule I

6.4 Status of $\&_R$: `OK`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.5 Status of $\multimap_R$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.6 Status of $\oplus_{R_2}$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.7 Status of $\oplus_{R_1}$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.8 Status of $\mathbf{1}_L$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.9 Status of $\otimes_R$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.10 Status of W: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, W \quad \dfrac{\mathtt{h}_8 : !\Upsilon 5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), !\Upsilon 5 \vdash !\mathtt{F}_9}\, !R}{- : (!\Upsilon 3, !\mathtt{F}_4), !\Upsilon 5 \vdash !\mathtt{F}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : !\Upsilon 5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, !\Upsilon 5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, !\Upsilon 5, !\mathtt{F}_4 \vdash !\mathtt{F}_9}\, !R$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, W \quad \dfrac{}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \top}\, \top}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \top}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \top}\, \top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, W \quad \dfrac{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \&_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{\bullet \mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \&_R}{- : !\Upsilon 3, \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, W$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, W \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \multimap_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, \mathtt{F}_9, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, W \quad \dfrac{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_2}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, W \quad \dfrac{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_1}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\, \mathbf{1}_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : \mathbf{1}, !\Upsilon 3, \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut} \quad \dfrac{}{- : \Delta_8 \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !\mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5 \vdash \mathtt{F}_9}\, \mathtt{ax}}{\bullet \mathtt{h}_7 : \Delta_{10}, \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5 \vdash \mathtt{F}_8}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{\bullet \mathtt{h}_7 : \Delta_{10}, \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\, \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !\mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, W}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\,\mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_8), \Delta_5 \vdash \mathtt{F}_9}\, W}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}\,\mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, C}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\,\mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_8), \Delta_5 \vdash \mathtt{F}_9}\, C}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}\,\mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, !L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\,\mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, !L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_8), \Delta_5 \vdash \mathtt{F}_9}\, !L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

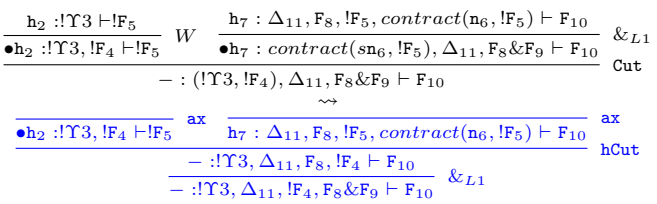
$$\dfrac{\dfrac{}{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}\,\mathtt{ax}}{\bullet \mathtt{h}_7 : \Delta_5, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}\, !L}{- : !\Upsilon 3, \Delta_5 \vdash \mathtt{F}_9}\,\mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, W$$

- Case rule $\&_{L2}$

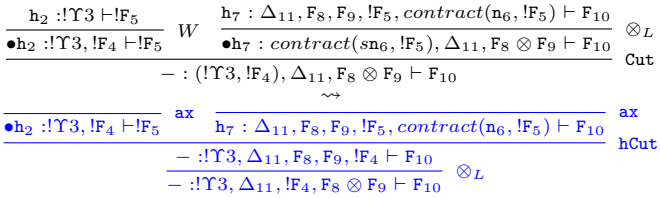
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \&_{L2}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\,\mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\,\mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \&_{L2}$$

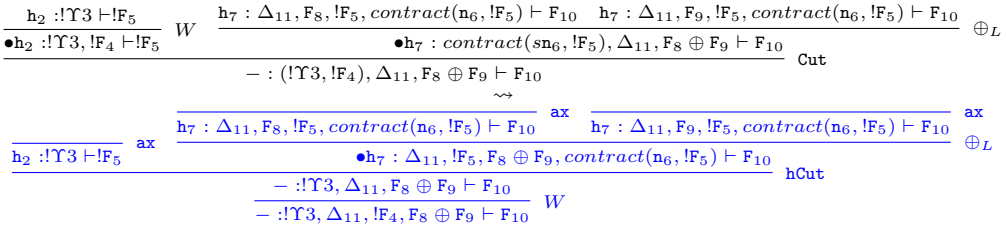
- Case rule $\&_{L1}$



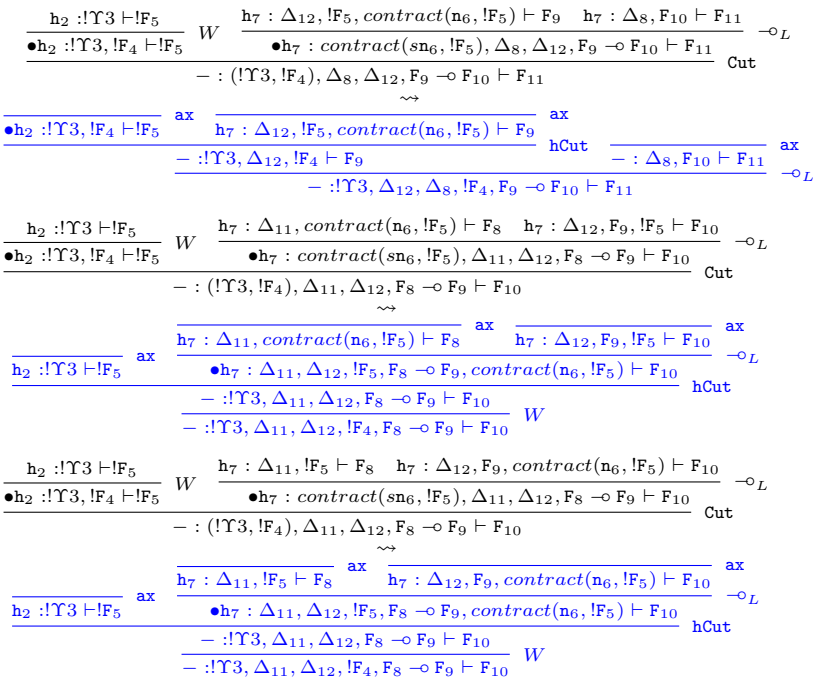
- Case rule $\otimes_L$



- Case rule $\oplus_L$



- Case rule $\multimap_L$



$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, W \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \multimap_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\, \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{11}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{12}, \mathtt{F}_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_{11}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !\mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \multimap_L$$

- Case rule I

6.11 Status of C: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, C \quad \dfrac{\mathtt{h}_8 : !\Upsilon 5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), !\Upsilon 5 \vdash !\mathtt{F}_9}\, !R}{- : (!\Upsilon 3, !\mathtt{F}_4), !\Upsilon 5 \vdash !\mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : !\Upsilon 5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, !\Upsilon 5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, !\Upsilon 5, !\mathtt{F}_4 \vdash !\mathtt{F}_9}\, !R$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, C \quad \dfrac{}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \top}\, \top}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \top}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \top}\, \top$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, C \quad \dfrac{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \&_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{\bullet \mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \&_R}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, C$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, C \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \multimap_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, \mathtt{F}_9, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, C \quad \dfrac{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_2}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}{}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, C \quad \dfrac{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_1}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}{}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\, \mathbf{1}_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : \mathbf{1}, !\Upsilon 3, \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut} \quad \dfrac{}{- : \Delta_8 \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !\mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5 \vdash \mathtt{F}_9}\, \mathtt{ax}}{\bullet \mathtt{h}_7 : \Delta_{10}, \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5 \vdash \mathtt{F}_8}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{\bullet \mathtt{h}_7 : \Delta_{10}, \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\, \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !\mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, W}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_8), \Delta_5 \vdash \mathtt{F}_9}\, W}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, C}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_8), \Delta_5 \vdash \mathtt{F}_9}\, C}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, !L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, !L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_8), \Delta_5 \vdash \mathtt{F}_9}\, !L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}\, \mathtt{ax}}{\bullet \mathtt{h}_7 : \Delta_5, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}\, !L}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, C$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : contract(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \&_{L2}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}}{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : contract(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \&_{L1}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}}{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \&_{L1}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : contract(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \otimes_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}}{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \otimes_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : contract(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \oplus_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}}{\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathtt{F}_8 \oplus \mathtt{F}_9, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{\bullet \mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathtt{F}_8 \oplus \mathtt{F}_9, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \oplus_L}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, !\mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, C$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_7 : contract(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \multimap_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \mathtt{Cut}}{\rightsquigarrow}}{\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\mathtt{h}_7 : \Delta_{12}, !\mathtt{F}_5, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{12}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut} \quad \dfrac{- : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{- : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !\mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \multimap_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_9, !\mathtt{F}_5 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : contract(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \Delta_{12}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \Delta_{12}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}}{\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}{\mathtt{h}_7 : \Delta_{11}, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_9, !\mathtt{F}_5 \vdash \mathtt{F}_{10}}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_9, !\mathtt{F}_5 \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{\bullet \mathtt{h}_7 : \Delta_{11}, \Delta_{12}, !\mathtt{F}_5, \mathtt{F}_8 \multimap \mathtt{F}_9, contract(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \multimap_L}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !\mathtt{F}_4, !\mathtt{F}_4, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !\mathtt{F}_4, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_9, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \Delta_{12}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \multimap_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \Delta_{12}, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{\dfrac{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : \Delta_{11}, \Delta_{12}, !\mathtt{F}_5, \mathtt{F}_8 \multimap \mathtt{F}_9, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\, \multimap_L}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !\mathtt{F}_4, !\mathtt{F}_4, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !\mathtt{F}_4, !\mathtt{F}_4, \mathtt{F}_8 \multimap \mathtt{F}_9 \vdash \mathtt{F}_{10}}\, C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, C \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \multimap_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\, \mathtt{ax} \quad \dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{11}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{12}, \mathtt{F}_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_{11}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !\mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\, \multimap_L}$$

- Case rule I

6.12 Status of $!L$: OK

- Case rule $!R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, !L \quad \dfrac{\mathtt{h}_8 : !\Upsilon 5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), !\Upsilon 5 \vdash !\mathtt{F}_9}\, !R}{- : (!\Upsilon 3, !\mathtt{F}_4), !\Upsilon 5 \vdash !\mathtt{F}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : !\Upsilon 5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, !\Upsilon 5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, !\Upsilon 5, !\mathtt{F}_4 \vdash !\mathtt{F}_9}\, !R$$

- Case rule $\mathbf{1}_R$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, !L \quad \dfrac{}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \top}\, \top}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \top}\, \mathtt{Cut}}{\dfrac{}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \top}\, \top}$$

- Case rule $\&_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, !L \quad \dfrac{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \&_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \& \mathtt{F}_{10}}\, \&_R$$

- Case rule $\multimap_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, !L \quad \dfrac{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \multimap_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, \mathtt{F}_9, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, \mathtt{F}_9, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \multimap \mathtt{F}_{10}}\, \multimap_R$$

- Case rule $\oplus_{R_2}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, !L \quad \dfrac{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_2}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_2}$$

- Case rule $\oplus_{R_1}$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_6}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, !L \quad \dfrac{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : \mathit{contract}(s\mathtt{n}_7, !\mathtt{F}_6), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_1}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_5, !\mathtt{F}_6, \mathit{contract}(\mathtt{n}_7, !\mathtt{F}_6) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9 \oplus \mathtt{F}_{10}}\, \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\, \mathbf{1}_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \mathbf{1}, \Delta_9 \vdash \mathtt{F}_8}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : \mathbf{1}, !\Upsilon 3, \Delta_9, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, \mathbf{1}_L$$

- Case rule $\otimes_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut} \quad \dfrac{}{- : \Delta_8 \vdash \mathtt{F}_{10}}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !\mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_8}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5 \vdash \mathtt{F}_9}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5 \vdash \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{11}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, \Delta_{11} \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5 \vdash \mathtt{F}_8}{}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_8}\, \mathtt{hCut} \quad \dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, \otimes_R}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_8 \otimes \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_8 \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{11} \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_8 \vdash \mathtt{F}_9}\, \mathtt{ax} \quad \dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !\mathtt{F}_4 \vdash \mathtt{F}_9 \otimes \mathtt{F}_{10}}\, \otimes_R$$

- Case rule W

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, W}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_8), \Delta_5 \vdash \mathtt{F}_9}\, W}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}{}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}$$

- Case rule C

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, C}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, !\mathtt{F}_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_8), \Delta_5 \vdash \mathtt{F}_9}\, C}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}{}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_5, !\mathtt{F}_8, !\mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}$$

- Case rule $!L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\, !L \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, !L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{10}, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, \mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}{}\, \mathtt{ax} \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}{}\, \mathtt{ax}}{- : !\Upsilon 3, \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}\, \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{10}, !\mathtt{F}_4, !\mathtt{F}_8 \vdash \mathtt{F}_9}\, !L$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_8}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\ !L \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_8), \Delta_5 \vdash \mathtt{F}_9}\ !L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_5 \vdash \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{- : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\ \mathtt{ax}}{- : !\Upsilon 3, !\mathtt{F}_4 \vdash \mathtt{F}_8}\ bInv \quad \dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_8}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_8, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_8) \vdash \mathtt{F}_9}\ \mathtt{ax}}{- : !\Upsilon 3, \Delta_5, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_9}\ \mathtt{hCut}}{- : !\Upsilon 3, \Delta_5, !\mathtt{F}_4 \vdash \mathtt{F}_9}\ \mathtt{Cut}$$

- Case rule $\&_{L2}$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ !L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \&_{L2}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\ \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\ \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \&_{L2}$$

- Case rule $\&_{L1}$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ !L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \&_{L1}}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\ \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\ \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, \mathtt{F}_8 \& \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \&_{L1}$$

- Case rule $\otimes_L$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ !L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \otimes_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\ \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\ \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, \mathtt{F}_8 \otimes \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \otimes_L$$

- Case rule $\oplus_L$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ !L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \oplus_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_{11}, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

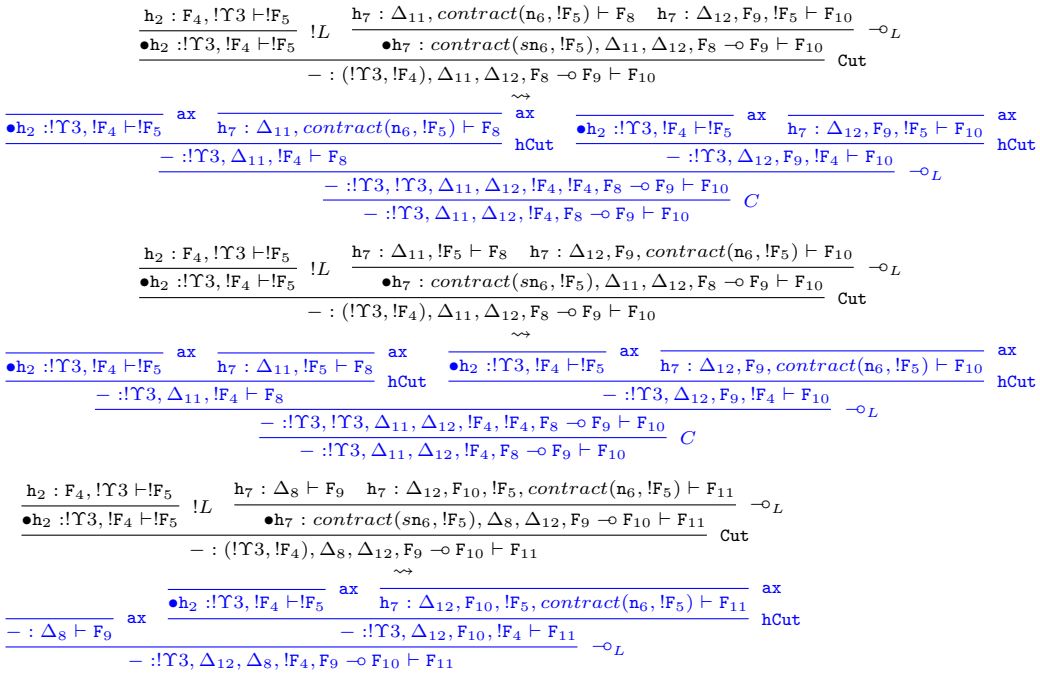
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\ \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \mathtt{F}_8, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_{10}}\ \mathtt{ax}}{- : !\Upsilon 3, \Delta_{11}, \mathtt{F}_9, !\mathtt{F}_4 \vdash \mathtt{F}_{10}}\ \mathtt{hCut}}{- : !\Upsilon 3, \Delta_{11}, !\mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9 \vdash \mathtt{F}_{10}}\ \oplus_L$$

- Case rule $\multimap_L$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_4, !\Upsilon 3 \vdash !\mathtt{F}_5}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ !L \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_7 : \mathit{contract}(s\mathtt{n}_6, !\mathtt{F}_5), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\ \multimap_L}{- : (!\Upsilon 3, !\mathtt{F}_4), \Delta_8, \Delta_{12}, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : !\Upsilon 3, !\mathtt{F}_4 \vdash !\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, !\mathtt{F}_5, \mathit{contract}(\mathtt{n}_6, !\mathtt{F}_5) \vdash \mathtt{F}_9}\ \mathtt{ax}}{- : !\Upsilon 3, \Delta_{12}, !\mathtt{F}_4 \vdash \mathtt{F}_9}\ \mathtt{hCut} \quad \dfrac{}{- : \Delta_8, \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\ \mathtt{ax}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !\mathtt{F}_4, \mathtt{F}_9 \multimap \mathtt{F}_{10} \vdash \mathtt{F}_{11}}\ \multimap_L$$



- Case rule I

6.13 Status of $\&_{L2}$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W

- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.14 Status of $\&_{L1}$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C

- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.15 Status of $\otimes_L$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$

- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.16 Status of $\oplus_L$: `OK`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$

- Case rule $\&_{L1}$
- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.17 Status of $\multimap_L$: `OK`

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$

- Case rule $\otimes_L$
- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I

6.18 Status of I: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule $\top$
- Case rule $\&_R$
- Case rule $\multimap_R$
- Case rule $\oplus_{R_2}$
- Case rule $\oplus_{R_1}$
- Case rule $\mathbf{1}_L$
- Case rule $\otimes_R$
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule $\otimes_L$

- Case rule $\oplus_L$
- Case rule $\multimap_L$
- Case rule I