Modal Logic S4 (K+T+4)

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Contents

1 Height preserving admissibility of weakening on the left

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\rightarrow_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\rightarrow_R}$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\wedge_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\wedge_R}$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\vee_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\vee_R}$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\bot_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \bot, \Delta_3}\ {\bot_R}$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\top_R} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \top, \Delta_3}\ {\top_R}$$

- Case(s) rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ {A4} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2}\ {A4}$$

- Case(s) rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ {K} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2}\ {K}$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ {\rightarrow_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_W \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ {\rightarrow_L}$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_W \vdash \Delta_4}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, \mathtt{F}_W, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_W, []\mathtt{F}_2 \vdash \Delta_3}\ AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3, \mathtt{F}_W \vdash \Delta_2}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_W, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_W \vdash \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \top, \Delta_3, \mathtt{F}_W \vdash \Delta_2}\ \top_L$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_W, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R}$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\scriptstyle \wedge_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_W, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\scriptstyle \wedge_R}$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\scriptstyle \vee_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_W, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\scriptstyle \vee_R}$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\scriptstyle \bot_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \mathtt{F}_W}\ {\scriptstyle \bot_R}$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\scriptstyle \top_R} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3, \mathtt{F}_W}\ {\scriptstyle \top_R}$$

- Case(s) rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ {\scriptstyle A4} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, \mathtt{F}_W, []\mathtt{F}_2}\ {\scriptstyle A4}$$

- Case(s) rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ {\scriptstyle K} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, \mathtt{F}_W, []\mathtt{F}_2}\ {\scriptstyle K}$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \rightarrow_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \rightarrow_L}$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \wedge_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \wedge_L}$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \vee_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\ {\scriptstyle \vee_L}$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet\mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3, \mathtt{F}_W}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3, \mathtt{F}_W}\ AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2, \mathtt{F}_W}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{F}_W, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2, \mathtt{F}_W}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2, \mathtt{F}_W}\ \top_L$$

3 Measure of derivations

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\,\mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\,\mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\,\mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\,\mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \rightarrow \quad \frac{}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case(s) rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2} A4 \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}\,\mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2} A4$$

- Case(s) rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2} K \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}\,\mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2} K$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\,\mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet\mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\bullet\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet\bullet\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L$$

4 Invertibility of Rules

4.1 Status of $\rightarrow_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ {\rightarrow_R} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ {\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ {\rightarrow_R}$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\rightarrow_R} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\mathtt{H}}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ {\wedge_R} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ {\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ {\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ {\wedge_R}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ {\vee_R} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ {\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ {\vee_R}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\bot_R} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_2}\ {\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \bot, \Delta_5, \mathtt{F}_2}\ {\bot_R}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\top_R} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \top, \Delta_5, \mathtt{F}_2}\ {\top_R}$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2), []\mathtt{F}_4}\ {A4} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}\ {\mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_2, []\mathtt{F}_4}\ {A4}$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2), []\mathtt{F}_4}\ {K} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}\ {\mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_2, []\mathtt{F}_4}\ {K}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{F}_4, \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\rightarrow_L} \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}\ {\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\rightarrow_L}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2}\ \wedge_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2}\ \vee_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2}\ AT \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_2}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}\ \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vdash \Delta_4, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}\ I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}\ \top_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vdash \Delta_4, \mathtt{F}_2}\ \top_L$$

4.2 Status of $\wedge_R$: (Left Premise): `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4}\ A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_4}\ A4$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_4}\ K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{F}_4, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \top_L$$

4.3 Status of $\wedge_R$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_2}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box \Gamma_6 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4}\ A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box \Gamma_6 \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, \mathtt{F}_2, []\mathtt{F}_4}\ A4$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \Box \Gamma_6 \vdash \Delta_5, \mathtt{F}_2, []\mathtt{F}_4}\ K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{F}_4, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_2}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \top_L$$

4.4 Status of $\vee_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4}\ A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2, []\mathtt{F}_4}\ A4$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2, []\mathtt{F}_4}\ K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{F}_4, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2}\ \top_L$$

4.5 Status of $\bot_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \bot, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \bot, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \bot, \Delta_3}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash (\bot, \Delta_3), []\mathtt{F}_2}\ A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2}\ A4$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash (\bot, \Delta_3), []\mathtt{F}_2}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2}\ K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \bot, \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \bot, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \bot, \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \bot, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \bot, \Delta_4} \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \bot, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \bot, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \bot, \Delta_4} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \bot, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \bot, \Delta_3} AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3} AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \bot, \Delta_2} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_4 \vdash \mathtt{p}_3, \bot, \Delta_2} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \bot, \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \bot, \Delta_2} \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2} \top_L$$

4.6 Status of $\top_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \top, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \top, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \top, \Delta_3}\ \bot_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash (\top, \Delta_3), []\mathtt{F}_2}\ A4 \quad \rightarrow \quad \texttt{trivial}$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Delta_5 \vdash (\top, \Delta_3), []\mathtt{F}_2}\ K \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \top, \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \top, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \top, \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \top, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \top, \Delta_4}\ \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \top, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \top, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \top, \Delta_4}\ \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \top, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \top, \Delta_3}\ AT \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \top, \Delta_2}\ \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_4 \vdash \mathtt{p}_3, \top, \Delta_2}\ I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \top, \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \top, \Delta_2}\ \top_L \quad \rightarrow \quad \texttt{trivial}$$

4.7 Status of $A4$: : `Non invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \Delta_7 \vdash \mathtt{F}_4, \Delta_5, []\mathtt{F}_1}{\bullet \mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathtt{F}_1), \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \texttt{H}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_3, \Delta_5, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_4, \Delta_5, []\mathtt{F}_1}{\bullet \mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathtt{F}_1), \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \texttt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5, []\mathtt{F}_1}{\bullet \mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathtt{F}_1), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \texttt{H}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}{\bullet \mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \bot, \Delta_3, []\mathtt{F}_1}\ \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_4 \vdash \mathtt{F}_1}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Box\Gamma_4 \vdash \mathtt{F}_1}\ \texttt{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \top, \Delta_3, []\mathtt{F}_1}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Box\Gamma_4 \vdash \mathtt{F}_1}\ \texttt{fail}$$

- Case rule $A4$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \Box\Gamma_6 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []\mathtt{F}_1), []\mathtt{F}_3}\ A4 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Box\Gamma_5, \Box\Gamma_7 \vdash \mathtt{F}_1}\ \texttt{fail}$$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_5 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_3, []\mathtt{F}_2}\ A4 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_6 \vdash \mathtt{F}_2}\ \texttt{fail}$$

- Case rule K

$$\frac{\mathtt{h}_2 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []\mathtt{F}_1), []\mathtt{F}_3}\ K \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Box\Gamma_5, \Box\Gamma_7 \vdash \mathtt{F}_1}\ \texttt{fail}$$

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_5) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_3, []\mathtt{F}_2}\ K \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_6 \vdash \mathtt{F}_2}\ \texttt{fail}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_3, \Delta_5, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_4, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5, []\mathtt{F}_1} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \text{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \mathtt{F}_4, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5, []\mathtt{F}_1} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \text{H}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_4, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5, []\mathtt{F}_1} \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_6 \vdash \mathtt{F}_1}\ \text{H}$$

- Case rule AT

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \mathtt{F}_3, \Delta_6, []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_5, \Delta_6), []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Box\Gamma_5, []\mathtt{F}_3 \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_5, []\mathtt{F}_3 \vdash \mathtt{F}_1}\ \text{H}$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \mathtt{F}_3, \Delta_6, []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_5, \Delta_6), []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Box\Gamma_5 \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_5 \vdash \mathtt{F}_1}\ \text{H}$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_2 : \bot, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}\ \bot_L \quad \rightarrow \quad \overline{\bullet\mathtt{h}_2 : \Box\Gamma_4 \vdash \mathtt{F}_1}\ \text{fail}$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_2 : \mathtt{p}_4, \Box\Gamma_5, \Delta_6 \vdash \mathtt{p}_4, \Delta_3, []\mathtt{F}_1}\ I \quad \rightarrow \quad \overline{\bullet\mathtt{h}_2 : \Box\Gamma_5 \vdash \mathtt{F}_1}\ \text{fail}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \top, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1} \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Box\Gamma_4 \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Box\Gamma_4 \vdash \mathtt{F}_1}\ \text{H}$$

4.8 Status of K: : `Non invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \Delta_7 \vdash \mathtt{F}_4, \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathtt{F}_1), \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \text{H}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_3, \Delta_5, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_4, \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathtt{F}_1), \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \text{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \text{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathtt{F}_1), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}^{\ ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \bot, \Delta_3, []\mathtt{F}_1}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}^{\ ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \top, \Delta_3, []\mathtt{F}_1}\ \top_R \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \mathtt{fail}$$

- Case rule $A4$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \Box\Gamma_6 \vdash \mathtt{F}_3}{\bullet\mathtt{h}_2 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []\mathtt{F}_1), []\mathtt{F}_3}\ A4 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_7) \vdash \mathtt{F}_1}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_5 \vdash \mathtt{F}_2}{\bullet\mathtt{h}_1 : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_3, []\mathtt{F}_2}\ A4 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_6) \vdash \mathtt{F}_2}\ \mathtt{fail}$$

- Case rule K

$$\frac{\mathtt{h}_2 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash \mathtt{F}_3}{\bullet\mathtt{h}_2 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []\mathtt{F}_1), []\mathtt{F}_3}\ K \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_5), unbox(\Box\Gamma_7) \vdash \mathtt{F}_1}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_5) \vdash \mathtt{F}_2}{\bullet\mathtt{h}_1 : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_3, []\mathtt{F}_2}\ K \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_6) \vdash \mathtt{F}_2}\ \mathtt{fail}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \Delta_7 \vdash \mathtt{F}_3, \Delta_5, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_4, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5, []\mathtt{F}_1}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}^{\ ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \mathtt{F}_4, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5, []\mathtt{F}_1}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}^{\ ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_3, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1 \quad \mathtt{h}_2 : \Box\Gamma_6, \mathtt{F}_4, \Delta_7 \vdash \Delta_5, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, \Delta_7), \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5, []\mathtt{F}_1}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}^{\ ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule AT

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \mathtt{F}_3, \Delta_6, []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_5, \Delta_6), []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \mathtt{F}_3, unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : \mathtt{F}_3, unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{H}$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \mathtt{F}_3, \Delta_6, []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : (\Box\Gamma_5, \Delta_6), []\mathtt{F}_3 \vdash \Delta_4, []\mathtt{F}_1}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_2 : \bot, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \mathtt{fail}$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_2 : \mathtt{p}_4, \Box\Gamma_5, \Delta_6 \vdash \mathtt{p}_4, \Delta_3, []\mathtt{F}_1}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_1}\ \mathtt{fail}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}{\bullet\mathtt{h}_2 : \top, \Box\Gamma_4, \Delta_5 \vdash \Delta_3, []\mathtt{F}_1}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_1}\ \mathtt{H}$$

4.9 Status of $\rightarrow_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_4}{\bullet\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_5 \vdash \bot, \Delta_4, \mathtt{F}_1}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_5 \vdash \top, \Delta_4, \mathtt{F}_1}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ A4 \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet\mathtt{h}_3 : \Delta_7, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_4}\ A4$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet\mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ K \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet\mathtt{h}_3 : \Delta_7, \Box\Gamma_6 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_4}\ K$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \to_L$$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \to \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \to \quad \frac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \to \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet\mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \top_L$$

4.10 Status of $\rightarrow_L$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \top, \Delta_4} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \top, \Delta_4} \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4} A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4} A4$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4} K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4} K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L$$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {}^{\mathtt{H}}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ ax/ind \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \to \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5} AT \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, []\mathtt{F}_4 \vdash \Delta_5} AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4} \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4} I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4} \top_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ ax/ind}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4} \top_L$$

4.11 Status of $\wedge_L$: : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \to \mathtt{F}_5} \to_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \to \mathtt{F}_5} \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ ax/ind \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4}\ A4$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4}\ K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{H}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.12 Status of $\vee_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4}^{\,ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_5}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \bot, \Delta_4} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \top, \Delta_4} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \top, \Delta_4} \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4} A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}^{\,ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4} A4$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4} K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}^{\,ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4} K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4}^{\,ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6}\ ax/ind \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ ax}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{H}$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ ax/ind}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \top_L$$

4.13 Status of $\vee_L$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ ax/ind \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ ax/ind}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_6 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4}\ A4$$

- Case rule K

$$\frac{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3 : \Box\Gamma_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4}\ K \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : unbox(\Box\Gamma_6) \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \Box\Gamma_6 \vdash \Delta_5, []\mathtt{F}_4}\ K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{H}$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.14 Status of AT: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_2 : \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \Delta_4, []\mathtt{F}_1 \vdash \bot, \Delta_3} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \bot, \Delta_3} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_4, []\mathtt{F}_1 \vdash \top, \Delta_3} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \top, \Delta_3} \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, []\mathtt{F}_1 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : (\Box\Gamma_5, []\mathtt{F}_1), \Delta_6 \vdash \Delta_4, []\mathtt{F}_3} A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Box\Gamma_5, []\mathtt{F}_1 \vdash \mathtt{F}_3}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box\Gamma_5, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_3} A4$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : \Box\Gamma_5, \Delta_6, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_3} A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Box\Gamma_5 \vdash \mathtt{F}_3}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box\Gamma_5, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_3} A4$$

- Case rule K

$$\frac{\mathtt{h}_2 : \mathtt{F}_1, unbox(\Box\Gamma_5) \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : (\Box\Gamma_5, []\mathtt{F}_1), \Delta_6 \vdash \Delta_4, []\mathtt{F}_3} K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \mathtt{F}_1, unbox(\Box\Gamma_5) \vdash \mathtt{F}_3}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box\Gamma_5, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_3} K$$

$$\frac{\mathtt{h}_2 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : \Box\Gamma_5, \Delta_6, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_3} K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_3}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box\Gamma_5, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_3} K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_1 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, []\mathtt{F}_1 \vdash \Delta_5}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \mathtt{F}_4, \Delta_6, []\mathtt{F}_1 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4, []\mathtt{F}_1 \vdash \Delta_5}^{\ ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_6, []\mathtt{F}_1 \vdash \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_1 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, []\mathtt{F}_1 \vdash \Delta_5}^{\ ax/ind} \quad \overline{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, []\mathtt{F}_1 \vdash \Delta_5}^{\ ax/ind}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_5, []\mathtt{F}_1, []\mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_2 : (\Delta_5, []\mathtt{F}_1), []\mathtt{F}_3 \vdash \Delta_4}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{F}_3, []\mathtt{F}_1, []\mathtt{F}_3 \vdash \Delta_4}^{\ ax/ind}}{\bullet \mathtt{h}_2 : \Delta_5, \mathtt{F}_1, []\mathtt{F}_1, []\mathtt{F}_3 \vdash \Delta_4}\ AT$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}^{\ ax/ind}}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_2 : \mathtt{p}_4, \Delta_5, []\mathtt{F}_1 \vdash \mathtt{p}_4, \Delta_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{p}_4, []\mathtt{F}_1 \vdash \Delta_3, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \top, \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}^{\ ax/ind}}{\bullet \mathtt{h}_2 : \top, \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}\ \top_L$$

4.15 Status of $\bot_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \Delta_5 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \rightarrow \mathtt{F}_3}\ \rightarrow_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \bot, \Delta_5 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \mathtt{F}_2, \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \bot, \Delta_2}\ \bot_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \top, \Delta_2}\ \top_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \bot, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ A4 \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \bot, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ K \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \bot, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \bot, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : (\bot, \Delta_4), []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \bot, \Delta_4 \vdash \mathtt{p}_3, \Delta_2}\ I \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \bot, \Delta_3 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \texttt{trivial}$$

4.16 Status of I: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_3, \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash (\Delta_4, \mathtt{p}_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3}\ \rightarrow_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_2, \Delta_4, \mathtt{p}_5 \quad \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_3, \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash (\Delta_4, \mathtt{p}_5), \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_2, \mathtt{F}_3, \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash (\Delta_4, \mathtt{p}_5), \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \bot, \Delta_2, \mathtt{p}_3}\ \bot_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \top, \Delta_2, \mathtt{p}_3}\ \top_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \Delta_6, \mathtt{p}_4 \vdash (\Delta_3, \mathtt{p}_4), []\mathtt{F}_2}\ A4 \quad \rightarrow \quad \texttt{trivial}$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \Delta_6, \mathtt{p}_4 \vdash (\Delta_3, \mathtt{p}_4), []\mathtt{F}_2}\ K \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \mathtt{F}_2, \Delta_4, \mathtt{p}_5 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : (\Delta_6, \mathtt{p}_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4, \mathtt{p}_5}\ \rightarrow_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : (\Delta_6, \mathtt{p}_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \mathtt{p}_5} \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : (\Delta_6, \mathtt{p}_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \mathtt{p}_5} \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5, \mathtt{p}_4, []\mathtt{F}_2 \vdash \Delta_3, \mathtt{p}_4}{\bullet \mathtt{h}_1 : (\Delta_5, \mathtt{p}_4), []\mathtt{F}_2 \vdash \Delta_3, \mathtt{p}_4} AT \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3} \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_4, \Delta_5, \mathtt{p}_3 \vdash \mathtt{p}_4, \Delta_2, \mathtt{p}_3} I \quad \rightarrow \quad \texttt{trivial}$$

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_4 \vdash \mathtt{p}_3, \Delta_2} I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}{\bullet \mathtt{h}_1 : \top, \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3} \top_L \quad \rightarrow \quad \texttt{trivial}$$

4.17 Status of $\top_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \Delta_5 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \rightarrow \mathtt{F}_3} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_3}^{\,\texttt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \rightarrow \mathtt{F}_3} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \top, \Delta_5 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\,\texttt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_3}^{\,\texttt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \mathtt{F}_2, \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}^{\,\texttt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \bot, \Delta_2}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \bot, \Delta_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \top, \Delta_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top, \Delta_2}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \top, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Box\Gamma_4 \vdash \mathtt{F}_2}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2}\ A4$$

- Case rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \top, \Delta_5 \vdash \Delta_3, []\mathtt{F}_2}\ K \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : unbox(\Box\Gamma_4) \vdash \mathtt{F}_2}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_4 \vdash \Delta_3, []\mathtt{F}_2}\ K$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \top, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \top, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : (\top, \Delta_4), []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \top, \Delta_4 \vdash \mathtt{p}_3, \Delta_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{H}$$

5 Height preserving admissibility of contraction on the left

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5, \Delta_6, \Delta_6 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, \mathtt{F}_2 \to \mathtt{F}_3} \to_R \quad \to \quad \dfrac{\dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \Delta_4, \mathtt{F}_2 \to \mathtt{F}_3} \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3} \wedge_R \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \Delta_4, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \mathtt{F}_2, \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3} \vee_R \quad \to \quad \dfrac{\dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \bot, \Delta_2} \bot_R \quad \to \quad \dfrac{\dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_3, \Delta_4 \vdash \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_3, \Delta_4 \vdash \bot, \Delta_2} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \top, \Delta_2} \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_3, \Delta_4 \vdash \top, \Delta_2} \top_R$$

- Case(s) rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_5, \Box\Gamma_6 \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : (\Box\Gamma_4, \Delta_7), (\Box\Gamma_5, \Box\Gamma_6, \Delta_8), \Box\Gamma_5, \Box\Gamma_6, \Delta_8 \vdash \Delta_3, []\mathtt{F}_2} A4 \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_5, \Box\Gamma_6 \vdash \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_6 \vdash \mathtt{F}_2}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_7, \Delta_8, \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_6 \vdash \Delta_3, []\mathtt{F}_2} A4$$

- Case(s) rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_5), unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash \mathtt{F}_2}{\bullet \mathtt{h}_1 : (\Box\Gamma_4, \Delta_7), (\Box\Gamma_5, \Box\Gamma_6, \Delta_8), \Box\Gamma_5, \Box\Gamma_6, \Delta_8 \vdash \Delta_3, []\mathtt{F}_2} K \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_5), unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 : unbox(\Box\Gamma_4), unbox(\Box\Gamma_5), unbox(\Box\Gamma_6) \vdash \mathtt{F}_2}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_7, \Delta_8, \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_6 \vdash \Delta_3, []\mathtt{F}_2} K$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, (\Delta_6, \mathtt{F}_2 \to \mathtt{F}_3), \Delta_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4} \to_L \quad \to \quad \dfrac{\dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_2}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_2}\ \mathtt{IH}}{\mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{IH\text{-}Mutual} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4} \to_L$$

$$\frac{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\Delta_5, \mathtt{F}_2 \to \mathtt{F}_3), \Delta_6, \Delta_6 \vdash \Delta_4} \to_L \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4} \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, (\Delta_6, \mathtt{F}_2 \wedge \mathtt{F}_3), \Delta_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_4}\ \text{inv-th/ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \text{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3), \Delta_6, \Delta_6 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \text{ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \text{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, (\Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3), \Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4}\ \text{inv-th/ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2 \vdash \Delta_4}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_4}\ \text{inv-th/ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_3 \vdash \Delta_4}\ \text{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_5, \Delta_6, \Delta_6 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3), \Delta_6, \Delta_6 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_2 \vdash \Delta_4}\ \text{ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2 \vdash \Delta_4}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3 \vdash \Delta_4}\ \text{ax}}{\mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_3 \vdash \Delta_4}\ \text{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, \Delta_5, \Delta_5, []\mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_4, (\Delta_5, []\mathtt{F}_2), \Delta_5, []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_4, \Delta_5, \Delta_5, \mathtt{F}_2, []\mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \text{ax}}{\mathtt{h}_1 : \Delta_4, \Delta_5, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \text{IH}}{\bullet \mathtt{h}_1 : \Delta_4, \Delta_5, []\mathtt{F}_2 \vdash \Delta_3}\ AT$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_4, \Delta_5, \Delta_5, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : (\Delta_4, []\mathtt{F}_2), \Delta_5, \Delta_5 \vdash \Delta_3}\ AT \quad \rightarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_4, \Delta_5, \Delta_5, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \text{ax}}{\mathtt{h}_1 : \Delta_4, \Delta_5, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}\ \text{IH}}{\bullet \mathtt{h}_1 : \Delta_4, \Delta_5, []\mathtt{F}_2 \vdash \Delta_3}\ AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_3), \Delta_4, \Delta_4 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3, \Delta_4 \vdash \Delta_2}\ \bot_L$$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_3, (\bot, \Delta_4), \bot, \Delta_4 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3, \Delta_4 \vdash \Delta_2}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : (\Delta_4, \mathtt{p}_3), \Delta_5, \Delta_5 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \Delta_5, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, (\Delta_5, \mathtt{p}_3), \Delta_5, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \Delta_5, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet \mathtt{h}_1 : (\top, \Delta_3), \Delta_4, \Delta_4 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}\ \text{ax}}{\mathtt{h}_1 : \Delta_3, \Delta_4 \vdash \Delta_2}\ \text{IH}}{\bullet \mathtt{h}_1 : \top, \Delta_3, \Delta_4 \vdash \Delta_2}\ \top_L$$

$$\frac{\mathtt{h}_1 : \top, \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \Delta_3, (\top, \Delta_4), \top, \Delta_4 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_3, \Delta_4, \Delta_4 \vdash \Delta_2}\ \text{inv-th/ax}}{\mathtt{h}_1 : \Delta_3, \Delta_4 \vdash \Delta_2}\ \text{IH}}{\bullet \mathtt{h}_1 : \top, \Delta_3, \Delta_4 \vdash \Delta_2}\ \top_L$$

6 Height preserving admissibility of contraction on the Right

- Case(s) rule $\rightarrow_R$

$$\dfrac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3 \rightarrow \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, (\Delta_6, \mathtt{F}_3 \rightarrow \mathtt{F}_4), \Delta_6, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_4, \mathtt{F}_4}\ \text{inv-th/ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_4, \mathtt{F}_4}\ \text{IH-Mutual}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \Delta_6, \mathtt{F}_4}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R$$

$$\dfrac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \mathtt{F}_4, \Delta_5, \Delta_6, \Delta_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash (\Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4), \Delta_6, \Delta_6}\ \rightarrow_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_4}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \Delta_6, \mathtt{F}_4}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R$$

- Case(s) rule $\wedge_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, (\Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3, \mathtt{F}_3}\ \text{inv-th/ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3}\ \text{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_4, \mathtt{F}_4}\ \text{inv-th/ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_4}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R$$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5, \Delta_6, \Delta_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_5, \Delta_6, \Delta_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash (\Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_6, \Delta_6}\ \wedge_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3}\ \text{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3}\ \text{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_4}\ \text{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_4}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R$$

- Case(s) rule $\vee_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, (\Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_4}\ \text{inv-th/ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3, \mathtt{F}_4}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R$$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \mathtt{F}_4, \Delta_5, \Delta_6, \Delta_6}{\bullet\mathtt{h}_1 : \Delta_2 \vdash (\Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_6, \Delta_6}\ \vee_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \Delta_6, \mathtt{F}_3, \mathtt{F}_4}\ \text{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3, \mathtt{F}_4}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R$$

- Case(s) rule $\bot_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \Delta_4, \Delta_4}{\bullet\mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_3), \Delta_4, \Delta_4}\ \bot_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \Delta_4, \Delta_4}\ \text{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \Delta_4}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \Delta_4}\ \bot_R$$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \Delta_4, \Delta_4}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, (\bot, \Delta_4), \bot, \Delta_4}\ \bot_R \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \Delta_4, \Delta_4}\ \text{inv-th/ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \Delta_4}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \Delta_4}\ \bot_R$$

- Case(s) rule $\top_R$

$$\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_3), \Delta_4, \Delta_4}\ \top_R \quad \rightarrow \quad \dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3, \Delta_4}\ \top_R$$

$$\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, (\top, \Delta_4), \top, \Delta_4}\ \top_R \quad \rightarrow \quad \dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3, \Delta_4}\ \top_R$$

- Case(s) rule $A4$

$$\dfrac{\mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2}{\bullet\mathtt{h}_1 : \Box\Gamma_5, \Delta_6 \vdash \Delta_3, (\Delta_4, []\mathtt{F}_2), \Delta_4, []\mathtt{F}_2}\ A4 \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2}\ \text{ax}}{\bullet\mathtt{h}_1 : \Delta_6, \Box\Gamma_5 \vdash \Delta_3, \Delta_4, []\mathtt{F}_2}\ A4$$

$$\dfrac{\mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2}{\bullet\mathtt{h}_1 : \Box\Gamma_5, \Delta_6 \vdash (\Delta_3, []\mathtt{F}_2), \Delta_4, \Delta_4}\ A4 \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_5 \vdash \mathtt{F}_2}\ \text{ax}}{\bullet\mathtt{h}_1 : \Delta_6, \Box\Gamma_5 \vdash \Delta_3, \Delta_4, []\mathtt{F}_2}\ A4$$

- Case(s) rule K

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_2}{\bullet\mathtt{h}_1 : \Box\Gamma_5, \Delta_6 \vdash \Delta_3, (\Delta_4, []\mathtt{F}_2), \Delta_4, []\mathtt{F}_2}\ K \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_6, \Box\Gamma_5 \vdash \Delta_3, \Delta_4, []\mathtt{F}_2}\ K$$

$$\frac{\mathtt{h}_1 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_2}{\bullet\mathtt{h}_1 : \Box\Gamma_5, \Delta_6 \vdash (\Delta_3, []\mathtt{F}_2), \Delta_4, \Delta_4}\ K \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_5) \vdash \mathtt{F}_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_6, \Box\Gamma_5 \vdash \Delta_3, \Delta_4, []\mathtt{F}_2}\ K$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_2, \Delta_4, \Delta_5, \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4, \Delta_5, \Delta_5}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_6 \vdash \Delta_4, \Delta_5, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \Delta_5, \Delta_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \Delta_5}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4, \Delta_5}\ \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \mathtt{F}_3, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \Delta_5, \Delta_5}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4, \Delta_5, \Delta_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4, \Delta_5}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \Delta_5}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_3, \Delta_6 \vdash \Delta_4, \Delta_5, \Delta_5}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \Delta_5, \Delta_5}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \vdash \Delta_4, \Delta_5, \Delta_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \vdash \Delta_4, \Delta_5}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \Delta_5, \Delta_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \Delta_5}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \Delta_5}\ \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_2, \Delta_5, []\mathtt{F}_2 \vdash \Delta_3, \Delta_4, \Delta_4}{\bullet\mathtt{h}_1 : \Delta_5, []\mathtt{F}_2 \vdash \Delta_3, \Delta_4, \Delta_4}\ AT \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3, \Delta_4, \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3, \Delta_4}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_5, []\mathtt{F}_2 \vdash \Delta_3, \Delta_4}\ AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_4 \vdash \Delta_2, \Delta_3}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet\mathtt{h}_1 : \Delta_5, \mathtt{p}_4 \vdash \Delta_2, (\Delta_3, \mathtt{p}_4), \Delta_3, \mathtt{p}_4}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_5, \mathtt{p}_4 \vdash \Delta_2, \Delta_3, \mathtt{p}_4}\ I$$

$$\frac{}{\bullet\mathtt{h}_1 : \Delta_5, \mathtt{p}_4 \vdash (\Delta_2, \mathtt{p}_4), \Delta_3, \Delta_3}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_5, \mathtt{p}_4 \vdash \Delta_2, \Delta_3, \mathtt{p}_4}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3}{\bullet\mathtt{h}_1 : \top, \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_4 \vdash \Delta_2, \Delta_3, \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_4 \vdash \Delta_2, \Delta_3}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \top, \Delta_4 \vdash \Delta_2, \Delta_3}\ \top_L$$

7 Identity-Expansion

$$\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \Box\mathtt{F}_0 \vdash \Box\mathtt{F}_0}\,K$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,\vee_L}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0 \vee \mathtt{F}_1}\,\vee_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_R}{- : \mathtt{F}_0 \wedge \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_1}\,\to_L}{- : \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_0 \to \mathtt{F}_1}\,\to_R$$

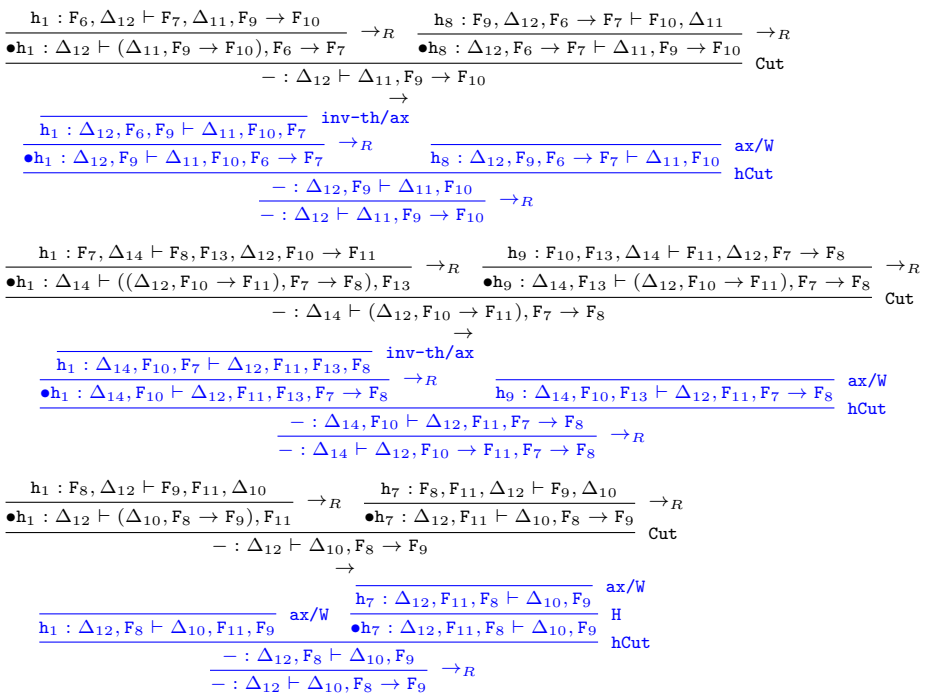
$$\dfrac{}{- : \top \vdash \top}\,\top_R$$

$$\dfrac{}{- : \bot \vdash \bot}\,\bot_L$$

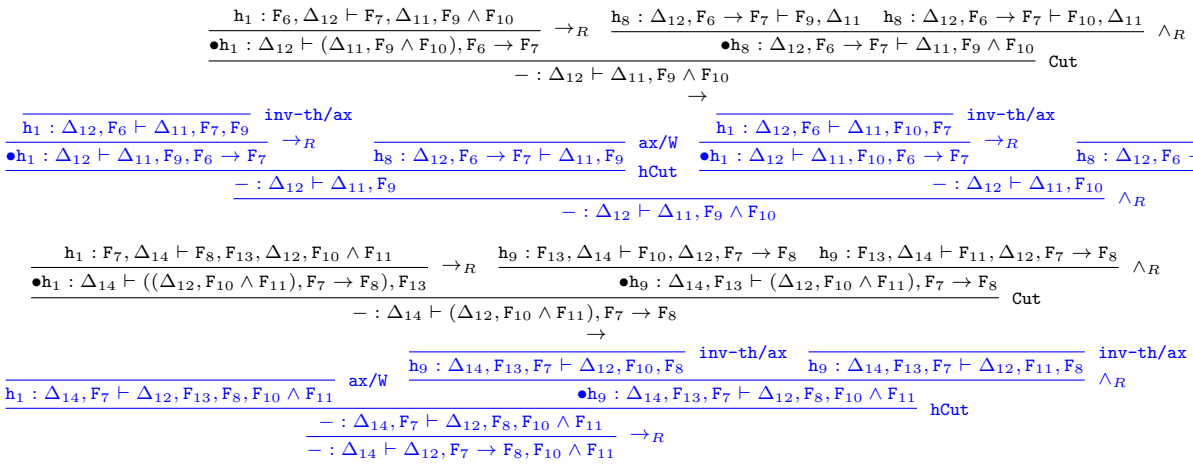
8 Cut-Elimination

8.1 Status of $\to_R$: OK

- Case rule $\to_R$



- Case rule $\wedge_R$



- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_R}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_R}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\vee_R}}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,{\vee_R}}{- : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,{\to_R}}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10} \vdash \mathtt{F}_7, \bot, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_9}\,{\bot_R}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11}}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\bot_R}}{- : \Delta_{12} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10} \vdash \mathtt{F}_7, \top, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \top, \Delta_9}\,{\top_R}}{- : \Delta_{10} \vdash \top, \Delta_9}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_9}\,{\top_R}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11}}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\top_R}}{- : \Delta_{12} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{12} \vdash \top, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,{\top_R}}$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11} \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}\,A4}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_9}\,A4}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_8, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \Box\mathtt{F}_{12}} \to_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13}, \Box\mathtt{F}_{12} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), \Box\mathtt{F}_{12} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} A4}{- : \Box\Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} \to \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Box\Gamma_{13} \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}} A4}{- : \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R \quad \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} A4}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} \to \quad \dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8} A4 \quad \mathtt{Cut}$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9} K}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_9} \to \quad \dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_9} K \quad \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_8, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \Box\mathtt{F}_{12}} \to_R \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), \Box\mathtt{F}_{12} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} K}{- : \Box\Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} \to \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : unbox(\Box\mathtt{F}_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}} K}{- : \Delta_{14}, \mathtt{F}_7, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R \quad \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} K}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8} \to \quad \dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8} K \quad \mathtt{Cut}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}} \to_L \quad \mathtt{Cut}}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}} \to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{a}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{h} \quad \to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{10} \vdash \mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7 \to \mathtt{F}_8} \to_R \quad \dfrac{\mathtt{h}_6 : \Delta_{10} \vdash \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9} \to_L \quad \mathtt{Cut}}{- : \Delta_{10} \vdash \Delta_9} \to$$

$$\dfrac{\dfrac{}{- : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{sCut}}{- : \Delta_{10} \vdash \Delta_9}\,\mathtt{sCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13} \vdash \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \to_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\to_L}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \to_L}{- : \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\to_L}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \to_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}} \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \wedge_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \wedge_L}{- : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \wedge_L}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \to_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{a}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{h}}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}} \vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13} \vdash \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \vee_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \vee_L}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8} \mathtt{hCut} \to_R}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \vee_L}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \vee_L}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \mathtt{hCut} \to_R}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_7, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}}{\bullet \mathtt{h}_8 : (\Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}}\ AT}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}} \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\ AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_8, []\mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), []\mathtt{F}_{10}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\ AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_8}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_8}\ AT}{- : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_8} \mathtt{hCut} \to_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_{12} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\ AT}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\ AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \bot, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\bullet \mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}\ \bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9} \mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\ \bot_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{11} \vdash \mathtt{F}_8, \bot, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \bot} \to_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{11}, \bot \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ \bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash \bot, \Delta_{10}, \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8}\ \bot_L}{- : \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8} \mathtt{hCut} \to_R}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \bot, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \bot, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11}} \to_R \quad \dfrac{}{\bullet \mathtt{h}_9 : (\bot, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \bot_L}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \bot_L}\ \to$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9, \mathtt{p}_{10}} I}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}} I}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{11}} \to_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8} I}{- : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}} \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}} I}{- : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}}\ \mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8} \to_R}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12}} \to_R \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8} I}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8} I}\ \to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \top, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9} \top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7} \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9} \mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\ \mathtt{hCut}}\ \to$$

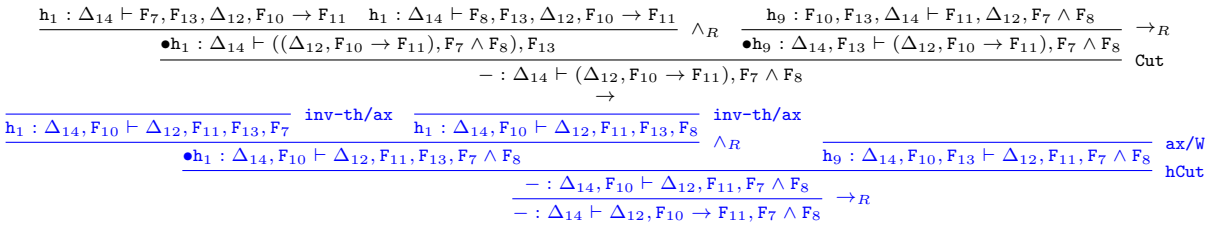
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{11} \vdash \mathtt{F}_8, \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \top} \to_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{11}, \top \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \top_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \top, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\top, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \top_L}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{hCut}}\ \to$$

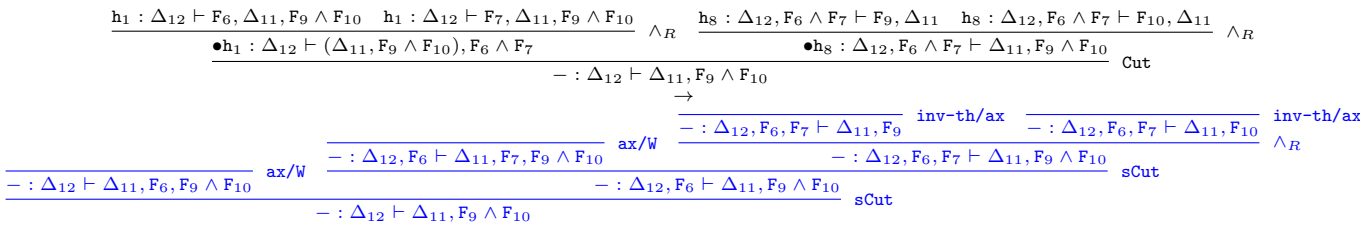
8.2 Status of $\wedge_R$: OK

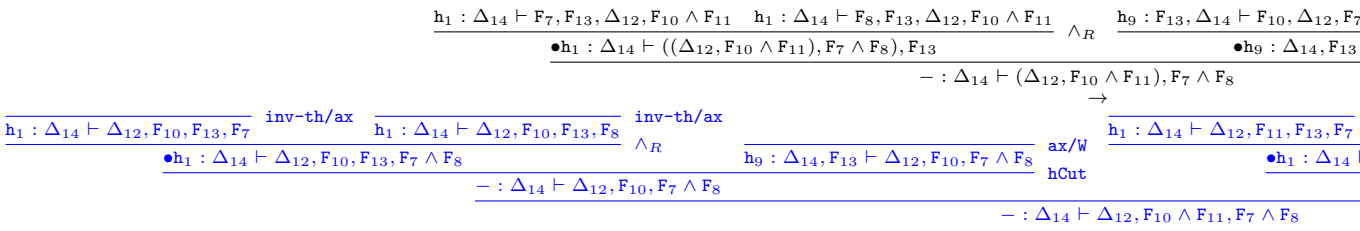
- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}} \to_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6} \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_7} \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}} \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}} \to_R}\ \to$$

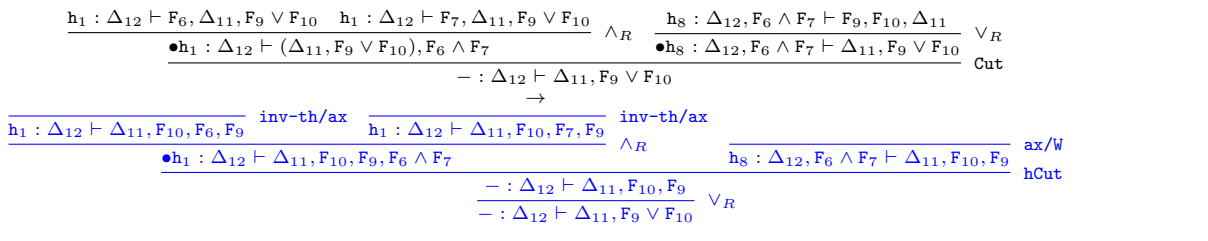


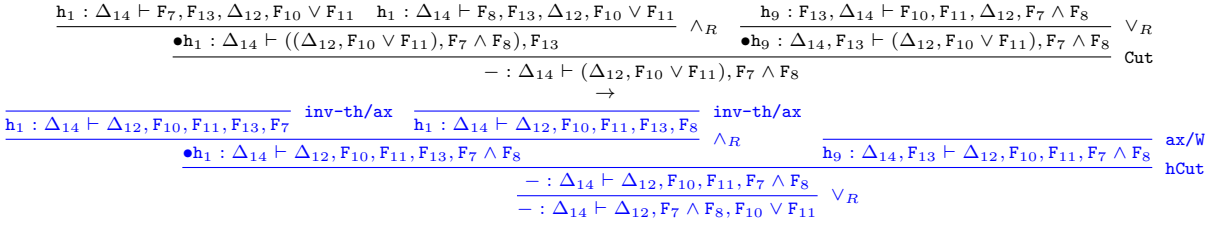
- Case rule $\wedge_R$





- Case rule $\vee_R$





- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_6, \bot, \Delta_9 \quad \mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_7, \bot, \Delta_9}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_9}\bot_R}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \bot, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash ((\bot, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\bot_R}{- : \Delta_{12} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_6, \top, \Delta_9 \quad \mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_7, \top, \Delta_9}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_9}\top_R}{- : \Delta_{10} \vdash \top, \Delta_9}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_9}\top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \top, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11}}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\top_R}{- : \Delta_{12} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{12} \vdash \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\top_R}$$

- Case rule $A4$

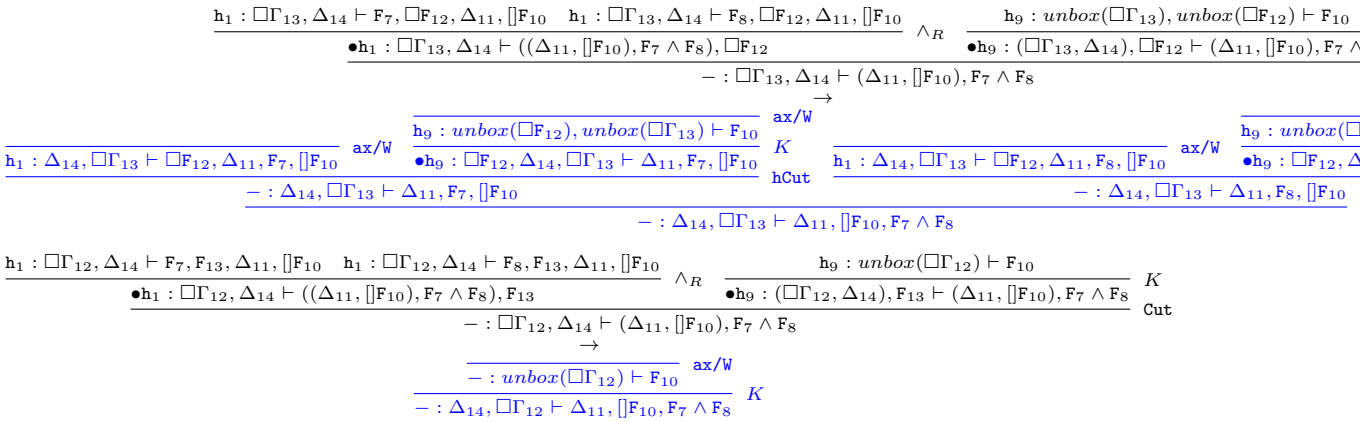
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_9 \quad \mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_9}{\bullet \mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11} \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}A4}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_9}A4}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_7, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10} \quad \mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_8, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \Box\mathtt{F}_{12}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13}, \Box\mathtt{F}_{12} \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), \Box\mathtt{F}_{12} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge}}{- : \Box\Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Box\Gamma_{13} \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}A4}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}$$

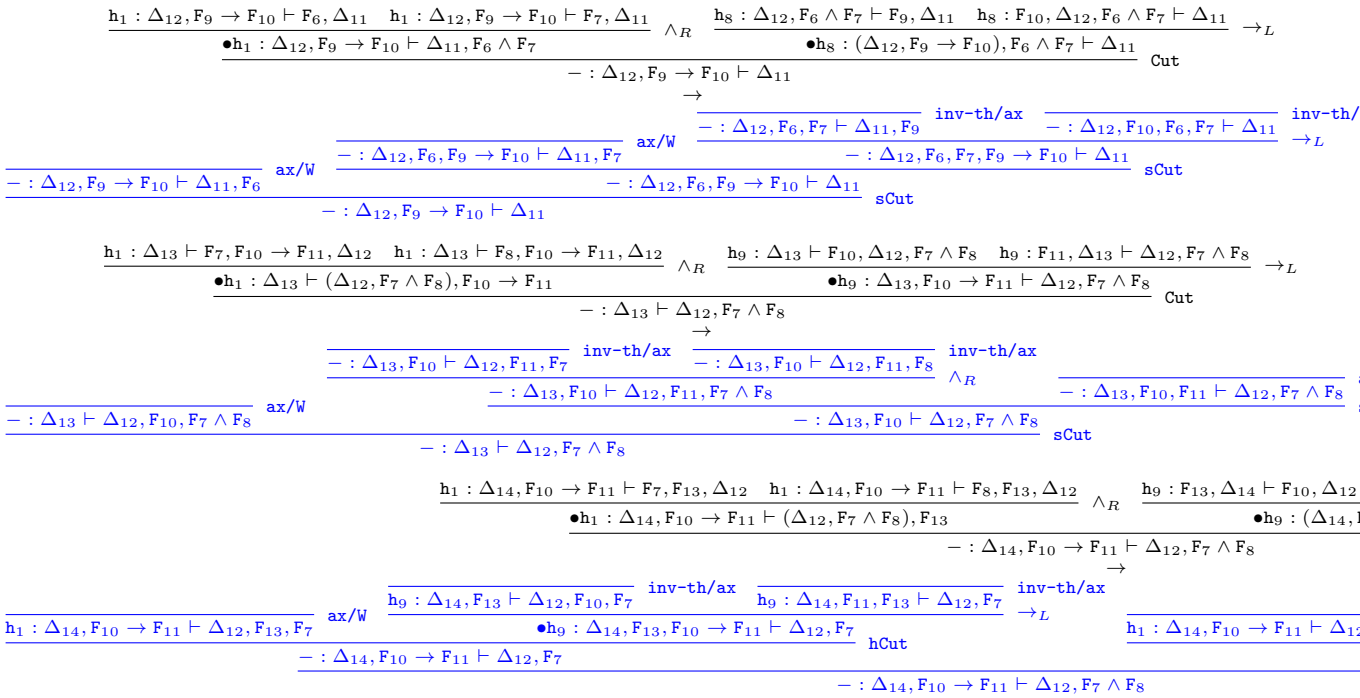
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10} \quad \mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12} \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}A4}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}A4}$$

- Case rule K

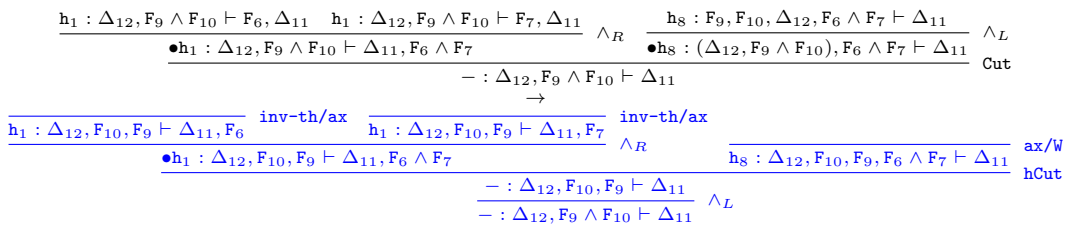
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_9 \quad \mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_9}{\bullet \mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}K}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_9}K}$$

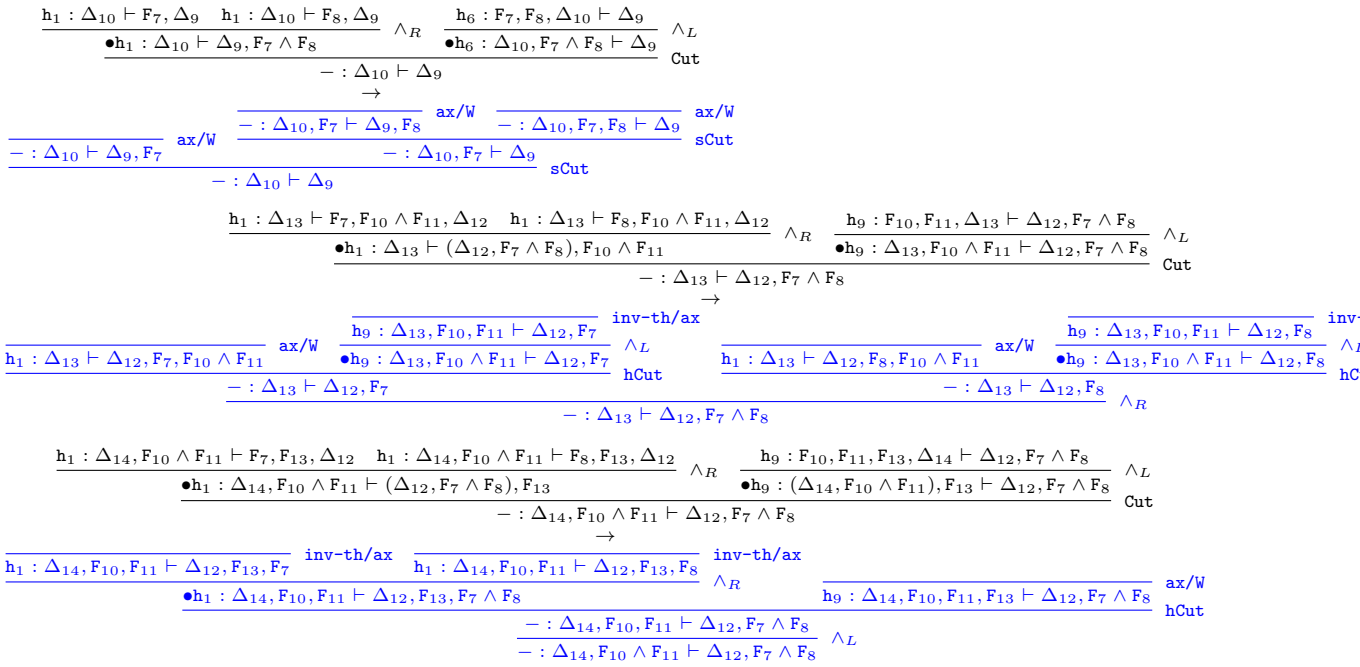


- Case rule $\rightarrow_L$

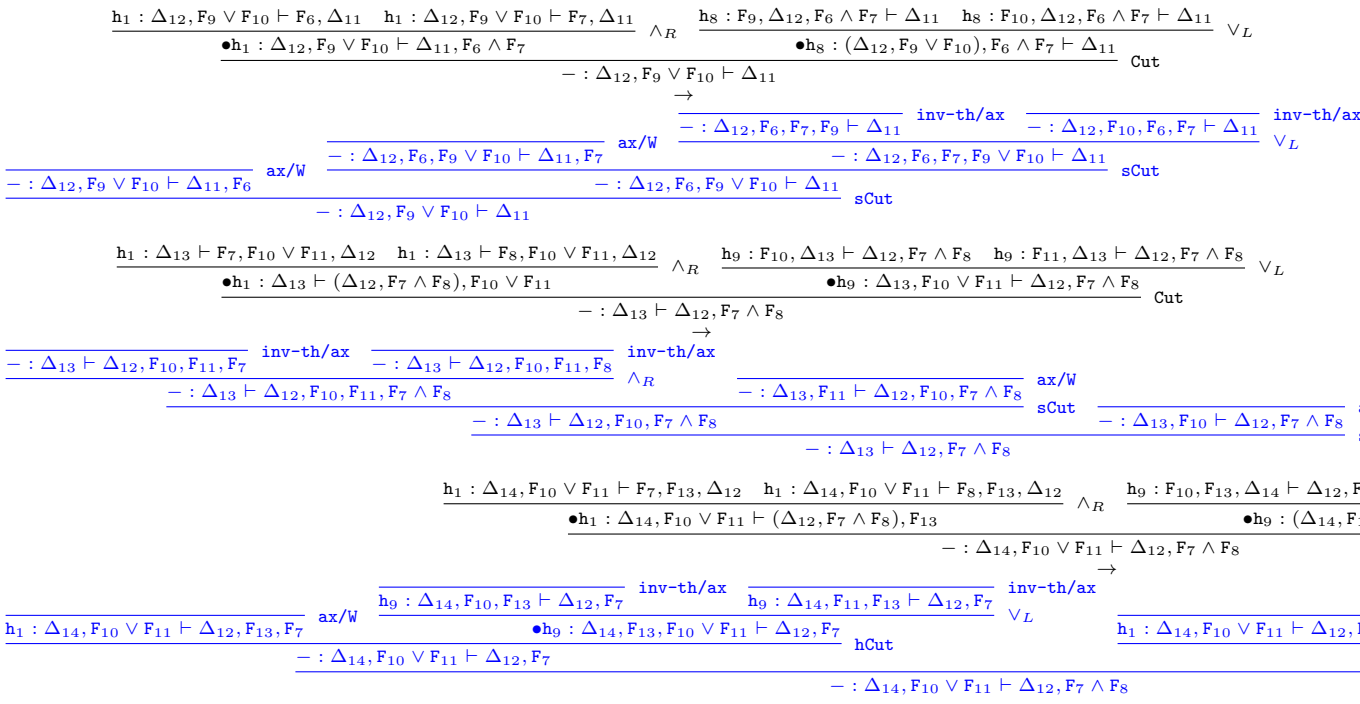


- Case rule $\wedge_L$





- Case rule $\vee_L$



- Case rule AT

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_6, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_7, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}}\ AT}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{Cut}}{\to}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}}{}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\ AT$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, []\mathtt{F}_{10}, \Delta_{11} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, []\mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), []\mathtt{F}_{10}}\ \wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}}{\to}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}{}\ \mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}\ AT}{}\ \mathtt{inv\text{-}th/ax}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{10}}{}\ \mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_8}\ AT}{}\ \mathtt{inv\text{-}th/ax}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{11} \quad \mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12}}\ \wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_{12} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ AT}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}}{\to}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}{}\ \mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}{}\ \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{10} \vdash \mathtt{F}_6, \Delta_9 \quad \mathtt{h}_1 : \bot, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\ \bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\ \mathtt{Cut}}{\to}}{\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\ \bot_L}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \bot, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_8, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \bot}\ \wedge_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{11}, \bot \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}}{\to}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\ \bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_8}{}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8}\ \bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \bot, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : \bot, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11}}\ \wedge_R \quad \dfrac{}{\bullet\mathtt{h}_9 : (\bot, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \bot_L}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}}{\to}}{\dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9, \mathtt{p}_{10}}\ I}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\ \mathtt{Cut}}{\to}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\ I}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{11}}\ \wedge_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\ I}{- : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}}{\to}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}, \mathtt{p}_{11}}{}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\ I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\ \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}{}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}}\ I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}}\ \mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12}} \wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8} I}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8} \text{Cut}}{\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8} I} \to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \mathtt{F}_6, \Delta_9 \quad \mathtt{h}_1 : \top, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9} \top_L}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7} \text{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}} \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \top, \Delta_{10} \quad \mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_8, \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \top} \wedge_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{11}, \top \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8} \top_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8} \text{Cut}}{\dfrac{}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8} \text{ax/W}} \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \top, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11}} \wedge_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\top, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8} \top_L}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8} \text{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8} \text{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8} \text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8} \text{hCut}} \to$$

8.3 Status of $\vee_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}} \to_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}} \text{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7} \text{inv-th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}} \text{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}} \text{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}} \to_R} \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}} \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \to_R}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \text{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8} \text{inv-th/ax}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8} \text{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8} \text{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8} \to_R} \to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}} \wedge_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}} \text{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9} \text{inv-th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9} \text{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9} \text{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7} \text{inv-th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}} \text{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}} \text{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}} \wedge_R} \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}} \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \wedge_R}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}} \vee_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}} \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R}{- : \Delta_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{11}} \vee_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9} \vee_R}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{ax/W}}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{H}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9} \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \bot, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_9} \bot_R}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11}} \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8} \bot_R}{- : \Delta_{12} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \top, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_9}\top_R}{- : \Delta_{10} \vdash \top, \Delta_9}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_9}\top_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11}}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\top_R}{- : \Delta_{12} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{12} \vdash \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11} \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}A4}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_9}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_9}A4$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \Box\mathtt{F}_{12}}\vee_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13}, \Box\mathtt{F}_{12} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), \Box\mathtt{F}_{12} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}A4}{- : \Box\Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Box\Gamma_{13} \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}A4}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}}\vee_R \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}A4}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}A4$$

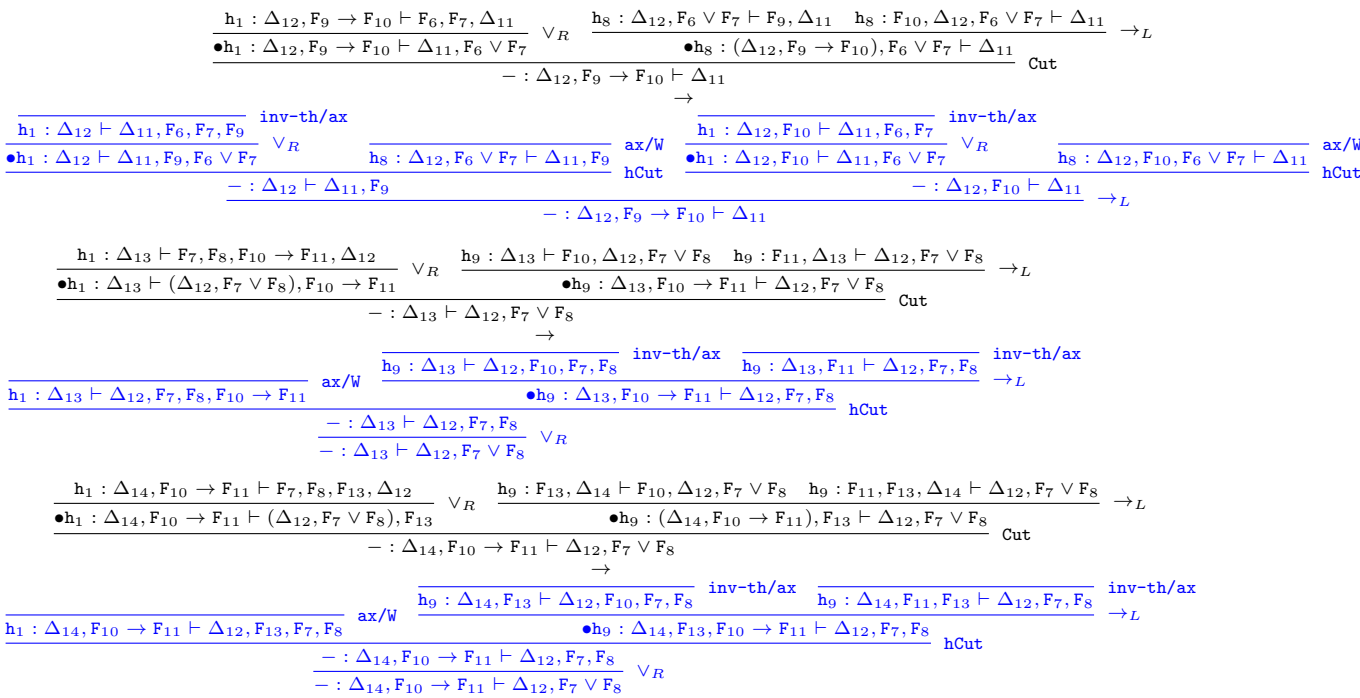
- Case rule K

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_9}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}K}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_9}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_9}K$$

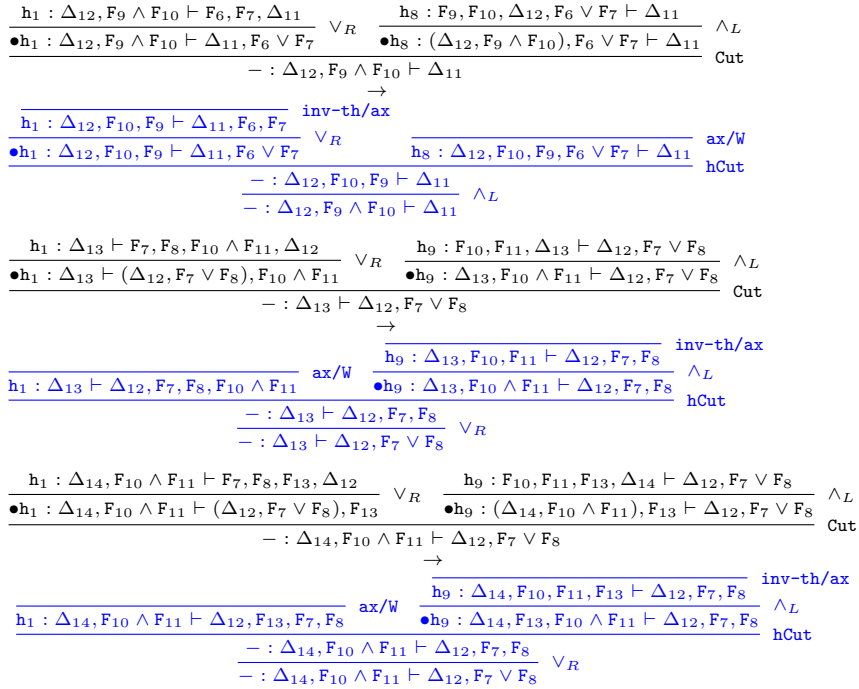
$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \Box\mathtt{F}_{12}}\vee_R \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{13}, \Delta_{14}), \Box\mathtt{F}_{12} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}K}{- : \Box\Gamma_{13}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : unbox(\Box\mathtt{F}_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}K}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}}\vee_R \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}K}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}K$$

- Case rule $\rightarrow_L$



- Case rule $\wedge_L$



- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\vee_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \Delta_{10} \vdash \Delta_9 \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\vee_L}{- : \Delta_{10} \vdash \Delta_9}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_{10}, \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7}\mathtt{sCut} \quad \dfrac{}{- : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \Delta_9}\mathtt{sCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\vee_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\mathtt{hCut}$$

$$\dfrac{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_L}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\vee_L}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\mathtt{hCut}$$

$$\dfrac{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R$$

- Case rule AT

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}}\,AT}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}}\mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\,AT$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), []\mathtt{F}_{10}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8}\,AT}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8}\mathtt{hCut}$$

$$\dfrac{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12}}\vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_{12} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,AT}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\bot_L}{\mathtt{Cut}}}{- : \bot, \Delta_{10} \vdash \Delta_9} \to \dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\bot_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \bot}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{11}, \bot \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L}{\mathtt{Cut}}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8}\bot_L}{\mathtt{hCut}}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8}\vee_R}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : \bot, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11}}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : (\bot, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L}{\mathtt{Cut}}}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9, \mathtt{p}_{10}}I}{\mathtt{Cut}}}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}} \to \dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}I$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{11}}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}I}{\mathtt{Cut}}}{- : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}I}{\mathtt{hCut}}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}\vee_R}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12}}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}I}{\mathtt{Cut}}}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\top_L}{\mathtt{Cut}}}{- : \top, \Delta_{10} \vdash \Delta_9} \to \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\mathtt{ax/W}}{\mathtt{hCut}}}{- : \top, \Delta_{10} \vdash \Delta_9}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \top}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{11}, \top \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\top_L}{\mathtt{Cut}}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \to \dfrac{}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11}} \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\top, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \top_L}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}$$

8.4 Status of $\bot_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \to \mathtt{F}_6), \bot} \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \Delta_8 \vdash \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \bot \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6} \to_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\,\mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_9} \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7} \to_R}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6), \bot} \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \bot, \Delta_8 \vdash \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \bot \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9} \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6), \bot} \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \bot \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9} \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \bot, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_5), \bot} \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_6, \bot \vdash \bot, \Delta_5} \bot_R}{- : \Delta_6 \vdash \bot, \Delta_5}\,\mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_6 \vdash \bot, \Delta_5}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_6}\ \bot_R}{- : \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_8 \vdash \bot, \Delta_6, \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \top, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_5), \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_6, \bot \vdash \top, \Delta_5}\ \top_R}{- : \Delta_6 \vdash \top, \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_6 \vdash \top, \Delta_5}\ \top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \top, \Delta_6}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\bot, \top, \Delta_6), \mathtt{F}_7}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \bot, \top, \Delta_6}\ \top_R}{- : \Delta_8 \vdash \bot, \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8 \vdash \bot, \top, \Delta_6}\ \top_R}$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_7, \Delta_8 \vdash \Delta_6, []\mathtt{F}_5}{\bullet \mathtt{h}_1 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_6, []\mathtt{F}_5), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \Box\Gamma_7 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : (\Box\Gamma_7, \Delta_8), \bot \vdash \Delta_6, []\mathtt{F}_5}\ A4}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_6, []\mathtt{F}_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_6, []\mathtt{F}_5}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}{\bullet \mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\bot, \Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_9, \Box\mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ A4}{- : \Box\Gamma_9, \Delta_{10} \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Box\mathtt{F}_8, \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash \mathtt{F}_9, \Delta_7, []\mathtt{F}_6}{\bullet \mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash (\bot, \Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ A4}{- : \Box\Gamma_8, \Delta_{10} \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_8 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_8 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ A4}$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_7, \Delta_8 \vdash \Delta_6, []\mathtt{F}_5}{\bullet \mathtt{h}_1 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_6, []\mathtt{F}_5), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \mathit{unbox}(\Box\Gamma_7) \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : (\Box\Gamma_7, \Delta_8), \bot \vdash \Delta_6, []\mathtt{F}_5}\ K}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_6, []\mathtt{F}_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_6, []\mathtt{F}_5}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}{\bullet \mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\bot, \Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathit{unbox}(\Box\Gamma_9), \mathit{unbox}(\Box\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ K}{- : \Box\Gamma_9, \Delta_{10} \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Box\mathtt{F}_8, \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash \mathtt{F}_9, \Delta_7, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash (\bot, \Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ K}{- : \Box\Gamma_8, \Delta_{10} \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_8 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ K}\ \to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \bot, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \to \mathtt{F}_6), \bot \vdash \Delta_7}\ \to_L}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \to \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8), \mathtt{F}_6 \to \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9 \vdash \bot, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9 \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \to_L}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{hCut}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash (\bot, \Delta_8), \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \bot, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_9 \vdash \bot, \Delta_8}\ \to_L}{- : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{hCut}}\ \to$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \bot \vdash \Delta_7}\ \wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8), \mathtt{F}_6 \wedge \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \Delta_9 \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \wedge_L}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{hCut}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash (\bot, \Delta_8), \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9 \vdash \bot, \Delta_8}\ \wedge_L}{- : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{hCut}}\ \to$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \Delta_8 \vdash \Delta_7 \quad \mathtt{h}_4 : \bot, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \bot \vdash \Delta_7}\ \vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \vee \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8), \mathtt{F}_6 \vee \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_9 \vdash \bot, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9 \vdash \bot, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \vee_L}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash (\bot, \Delta_8), \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9 \vdash \bot, \Delta_8}\ \vee_L}{- : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{hCut}}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}{\bullet \mathtt{h}_1 : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}{\bullet \mathtt{h}_4 : (\Delta_7, []\mathtt{F}_5), \bot \vdash \Delta_6}\ AT}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash []\mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\bot, \Delta_7), []\mathtt{F}_6}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_8, []\mathtt{F}_6 \vdash \bot, \Delta_7}{\bullet \mathtt{h}_5 : \Delta_8, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ AT}{- : \Delta_8 \vdash \bot, \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_8 \vdash \bot, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \bot, \Delta_7}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_7}{\bullet \mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash (\bot, \Delta_7), \mathtt{F}_8}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_8, \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_7}{\bullet \mathtt{h}_5 : (\Delta_9, []\mathtt{F}_6), \mathtt{F}_8 \vdash \bot, \Delta_7}\ AT}{- : \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_7, \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_8, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ \mathtt{ax/W}}{- : \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_7}\ \mathtt{hCut}}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_5, \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_6, \bot \vdash \Delta_5}\ \bot_L}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\bot, \Delta_6), \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_7, \bot \vdash \bot, \Delta_6}\ \bot_L}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \bot, \Delta_6}\ \bot_L}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash (\Delta_5, \mathtt{p}_6), \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\Delta_7, \mathtt{p}_6), \bot \vdash \Delta_5, \mathtt{p}_6}\ I}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ I}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{p}_7, \Delta_6, \mathtt{p}_7}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\bot, \Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8 \vdash \bot, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_8 \vdash \bot, \Delta_6, \mathtt{p}_7, \mathtt{p}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8 \vdash \bot, \Delta_6, \mathtt{p}_7}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{p}_7 \vdash \mathtt{F}_8, \Delta_6, \mathtt{p}_7}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_7 \vdash (\bot, \Delta_6, \mathtt{p}_7), \mathtt{F}_8}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_7), \mathtt{F}_8 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_9, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_9, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : (\top, \Delta_6), \bot \vdash \Delta_5}\ \top_L}{- : \top, \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \top, \Delta_6 \vdash \Delta_5}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \top, \Delta_6}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\bot, \Delta_6), \top}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_7 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_7, \top \vdash \bot, \Delta_6}\ \top_L}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet \mathtt{h}_1 : \top, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \bot, \Delta_6}\ \top_L}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_8 \vdash \bot, \Delta_6, \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{hCut}}$$

8.5 Status of $\top_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \to \mathtt{F}_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \Delta_8 \vdash \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \top \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vdash \top, \Delta_7, \mathtt{F}_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \top_R}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \top \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \top_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_4 : \Delta_8, \top \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_{10}, \mathtt{F}_9 \vdash \top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \top_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_6, \top \vdash \bot, \Delta_5}\ \bot_R}{- : \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \bot, \top, \Delta_5}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\top, \bot, \Delta_6), \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \top, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \top, \bot, \Delta_6}\ \bot_R}{- : \Delta_8 \vdash \top, \bot, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \bot, \top, \Delta_6}\ \top_R$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_5), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_6, \top \vdash \top, \Delta_5}\ \top_R}{- : \Delta_6 \vdash \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{F}_7 \vdash \top, \Delta_6}\ \top_R}{- : \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_6, []\mathtt{F}_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \Box\Gamma_7 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : (\Box\Gamma_7, \Delta_8), \top \vdash \Delta_6, []\mathtt{F}_5}\ A4}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_6, []\mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_6, []\mathtt{F}_5}\ A4$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\top, \Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_9, \Box\mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \top, \Delta_7, []\mathtt{F}_6}\ A4}{- : \Box\Gamma_9, \Delta_{10} \vdash \top, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_7, []\mathtt{F}_6}\ \top_R}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash (\top, \Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \top, \Delta_7, []\mathtt{F}_6}\ A4}{- : \Box\Gamma_8, \Delta_{10} \vdash \top, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \Box\Gamma_8 \vdash \top, \Delta_7, []\mathtt{F}_6}\ \top_R}\ \to$$

- Case rule K

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_6, []\mathtt{F}_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \mathit{unbox}(\Box\Gamma_7) \vdash \mathtt{F}_5}{\bullet \mathtt{h}_4 : (\Box\Gamma_7, \Delta_8), \top \vdash \Delta_6, []\mathtt{F}_5}\ K}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_6, []\mathtt{F}_5}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_7) \vdash \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_6, []\mathtt{F}_5}\ K}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\top, \Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathit{unbox}(\Box\Gamma_9), \mathit{unbox}(\Box\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \top, \Delta_7, []\mathtt{F}_6}\ K}{- : \Box\Gamma_9, \Delta_{10} \vdash \top, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_7, []\mathtt{F}_6}\ \top_R}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash (\top, \Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathit{unbox}(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \top, \Delta_7, []\mathtt{F}_6}\ K}{- : \Box\Gamma_8, \Delta_{10} \vdash \top, \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \Box\Gamma_8 \vdash \top, \Delta_7, []\mathtt{F}_6}\ \top_R}\ \to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \top, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \to \mathtt{F}_6), \top \vdash \Delta_7}\ \to_L}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \to_L}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8), \mathtt{F}_6 \to \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9 \vdash \top, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9 \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \top, \Delta_8}\ \to_L}{- : \Delta_9 \vdash \top, \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_9 \vdash \top, \Delta_8}\ \top_R}\ \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash (\top, \Delta_8), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_9, \Delta_{10} \vdash \top, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_9 \vdash \top, \Delta_8}\ \to_L}{- : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \top, \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \top, \Delta_8}\ \top_R}\ \to$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \top \vdash \Delta_7}\ \wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \wedge_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8), \mathtt{F}_6 \wedge \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \Delta_9 \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_8}\ \wedge_L}{- : \Delta_9 \vdash \top, \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_9 \vdash \top, \Delta_8}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash (\top, \Delta_8), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9 \vdash \top, \Delta_8}\ \wedge_L}{- : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \Delta_8 \vdash \Delta_7 \quad \mathtt{h}_4 : \top, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \top \vdash \Delta_7}\ \vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \vee_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8), \mathtt{F}_6 \vee \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_9 \vdash \top, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9 \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_8}\ \vee_L}{- : \Delta_9 \vdash \top, \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_9 \vdash \top, \Delta_8}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash (\top, \Delta_8), \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9 \vdash \top, \Delta_8}\ \vee_L}{- : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_8}\ \top_R$$

- Case rule AT

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}{\bullet \mathtt{h}_4 : (\Delta_7, []\mathtt{F}_5), \top \vdash \Delta_6}\ AT}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_5, []\mathtt{F}_5 \vdash \top, \Delta_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_7, \mathtt{F}_5, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_5, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{hCut}}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ AT$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\top, \Delta_7), []\mathtt{F}_6}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_8, []\mathtt{F}_6 \vdash \top, \Delta_7}{\bullet \mathtt{h}_5 : \Delta_8, []\mathtt{F}_6 \vdash \top, \Delta_7}\ AT}{- : \Delta_8 \vdash \top, \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \top, \Delta_7}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash (\top, \Delta_7), \mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_8, \Delta_9, []\mathtt{F}_6 \vdash \top, \Delta_7}{\bullet \mathtt{h}_5 : (\Delta_9, []\mathtt{F}_6), \mathtt{F}_8 \vdash \top, \Delta_7}\ AT}{- : \Delta_9, []\mathtt{F}_6 \vdash \top, \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_9, []\mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_6 \vdash \Delta_5, \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\bot, \Delta_6), \top \vdash \Delta_5}\ \bot_L}{- : \bot, \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \Delta_5}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \bot}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_7, \bot \vdash \top, \Delta_6}\ \bot_L}{- : \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \top, \Delta_6}\ \bot_L}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash (\Delta_5, \mathtt{p}_6), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\Delta_7, \mathtt{p}_6), \top \vdash \Delta_5, \mathtt{p}_6}\ I}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\top, \Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{p}_7 \vdash \top, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8 \vdash \top, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \top, \Delta_6, \mathtt{p}_7}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_7 \vdash (\top, \Delta_6, \mathtt{p}_7), \mathtt{F}_8}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_7), \mathtt{F}_8 \vdash \top, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_6, \mathtt{p}_7}\ \top_R$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_5, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_6, \top \vdash \Delta_5}\ \top_L}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_7 \vdash \top, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_7, \top \vdash \top, \Delta_6}\ \top_L}{- : \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \top, \Delta_6}{\bullet \mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \top, \Delta_6}\ \top_L}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

8.6 Status of $A4$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_6}\, A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_8, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R}}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6}\, A4 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_{13}}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7}\,{\to_R}}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10}}\, A4$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6}\, A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,{\wedge_R}}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, []\mathtt{F}_6}\, A4 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6}\, A4 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,{\wedge_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_{13}}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7}\,{\wedge_R}}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\, A4$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6}\, A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,{\vee_R}}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, []\mathtt{F}_6}\, A4 \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,{\vee_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_{13}}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7}\,{\vee_R}}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10}}\, A4$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\bot, \Delta_8), []\mathtt{F}_6}\, A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_6 \vdash \bot, \Delta_8}\,{\bot_R}}{- : \Box\Gamma_9, \Delta_{10} \vdash \bot, \Delta_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_8, []\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_6 \vdash \bot, \Delta_8}\,\mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\bot, \Delta_9), []\mathtt{F}_7), \mathtt{F}_{11}}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{10}, \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{F}_{11} \vdash (\bot, \Delta_9), []\mathtt{F}_7}\, \bot_R}{- : \Box\Gamma_{10}, \Delta_{12} \vdash (\bot, \Delta_9), []\mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{10} \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \bot, \Delta_9, []\mathtt{F}_7}\, A4$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\top, \Delta_8), []\mathtt{F}_6}\, A4 \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_6 \vdash \top, \Delta_8}\, \top_R}{- : \Box\Gamma_9, \Delta_{10} \vdash \top, \Delta_8}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_8}\, \top_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\top, \Delta_9), []\mathtt{F}_7), \mathtt{F}_{11}}\, A4 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{F}_{11} \vdash (\top, \Delta_9), []\mathtt{F}_7}\, \top_R}{- : \Box\Gamma_{10}, \Delta_{12} \vdash (\top, \Delta_9), []\mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \top, \Delta_9, []\mathtt{F}_7}\, \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\, A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_6}\, \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_8, []\mathtt{F}_6}\, A4 \quad \dfrac{}{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_8}\, \mathtt{hCut}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\, A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\, A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\, A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Box\Gamma_{14} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \Box\mathtt{F}_{12}}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\mathtt{F}_{12} \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathtt{F}_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\, A4}{- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{11}, \Box\Gamma_{14} \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\, A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{13} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{11}}\, A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\mathtt{F}_{11} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box\mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\, A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{10}, \Box\Gamma_{13} \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, []\mathtt{F}_8}\, A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Box\Gamma_{13} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \mathtt{F}_{14}}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), \mathtt{F}_{14} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\, A4}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\, \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\, A4$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{13}}\ A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_8}\ A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ A4}\ \to$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\ A4 \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_6, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6, \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{- : \mathtt{F}_6, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}\ \mathtt{sCut}}{- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_8}\ ATG}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ A4}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\ A4 \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ K}\ \to$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Box\Gamma_{14} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \Box\mathtt{F}_{12}}\ A4 \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathtt{F}_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ K}{- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11}, \Box\Gamma_{14} \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\ A4}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{13} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{11}}\ A4 \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{12}), unbox(\Box\mathtt{F}_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box\mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{10}, \Box\Gamma_{13} \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, []\mathtt{F}_8}\ A4}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, \Box\Gamma_{13} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \mathtt{F}_{14}}\ A4 \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), \mathtt{F}_{14} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ K}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11}, \Box\Gamma_{13} \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\ A4}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{13}}\ A4 \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ A4}\ \to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_{11} \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12}, F_8 \to F_9 \vdash \Delta_{10}, []F_6}\mathit{A4} \quad \dfrac{h_7 : \Box\Gamma_{11}, \Delta_{12}, []F_6 \vdash F_8, \Delta_{10} \quad h_7 : \Box\Gamma_{11}, F_9, \Delta_{12}, []F_6 \vdash \Delta_{10}}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \to F_9), []F_6 \vdash \Delta_{10}}\to_L}{- : \Box\Gamma_{11}, \Delta_{12}, F_8 \to F_9 \vdash \Delta_{10}}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_1 : \Box\Gamma_{11} \vdash F_6}\text{ax/W}}{\bullet h_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, []F_6}\mathit{A4} \quad \dfrac{}{h_7 : \Delta_{12}, \Box\Gamma_{11}, []F_6 \vdash \Delta_{10}, F_8}\text{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8}\text{hCut} \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Box\Gamma_{11} \vdash F_6}\text{ax/W}}{\bullet h_1 : \Delta_{12}, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, []F_6}\mathit{A4} \quad \dfrac{}{h_7 : \Delta_{12}, F_9, \Box\Gamma_{11}, []F_6 \vdash \Delta_{10}}\text{ax/W}}{- : \Delta_{12}, F_9, \Box\Gamma_{11} \vdash \Delta_{10}}\text{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, F_8 \to F_9 \vdash \Delta_{10}}\to_L$$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_{12} \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []F_7), F_9 \to F_{10}}\mathit{A4} \quad \dfrac{h_8 : \Box\Gamma_{12}, \Delta_{13} \vdash F_9, \Delta_{11}, []F_7 \quad h_8 : \Box\Gamma_{12}, F_{10}, \Delta_{13} \vdash \Delta_{11}, []F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{13}), F_9 \to F_{10} \vdash \Delta_{11}, []F_7}\to_L}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []F_7}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash F_7}\text{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7}\mathit{A4}$$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_{12} \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14}, F_9 \to F_{10} \vdash (\Delta_{11}, []F_7), F_{13}}\mathit{A4} \quad \dfrac{h_8 : \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, \Delta_{11}, []F_7 \quad h_8 : \Box\Gamma_{12}, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, []F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}, F_9 \to F_{10}), F_{13} \vdash \Delta_{11}, []F_7}\to_L}{- : \Box\Gamma_{12}, \Delta_{14}, F_9 \to F_{10} \vdash \Delta_{11}, []F_7}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash F_7}\text{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, F_9 \to F_{10} \vdash \Delta_{11}, []F_7}\mathit{A4}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_{11} \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}, []F_6}\mathit{A4} \quad \dfrac{h_7 : \Box\Gamma_{11}, F_8, F_9, \Delta_{12}, []F_6 \vdash \Delta_{10}}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9), []F_6 \vdash \Delta_{10}}\wedge_L}{- : \Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_1 : \Box\Gamma_{11} \vdash F_6}\text{ax/W}}{\bullet h_1 : \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, []F_6}\mathit{A4} \quad \dfrac{}{h_7 : \Delta_{12}, F_8, F_9, \Box\Gamma_{11}, []F_6 \vdash \Delta_{10}}\text{ax/W}}{- : \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}}\text{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{10}}\wedge_L$$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_{12} \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []F_7), F_9 \wedge F_{10}}\mathit{A4} \quad \dfrac{h_8 : \Box\Gamma_{12}, F_9, F_{10}, \Delta_{13} \vdash \Delta_{11}, []F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{13}), F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7}\wedge_L}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []F_7}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash F_7}\text{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7}\mathit{A4}$$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_{12} \vdash F_7}{\bullet h_1 : \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash (\Delta_{11}, []F_7), F_{13}}\mathit{A4} \quad \dfrac{h_8 : \Box\Gamma_{12}, F_9, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, []F_7}{\bullet h_8 : (\Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10}), F_{13} \vdash \Delta_{11}, []F_7}\wedge_L}{- : \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash F_7}\text{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7}\mathit{A4}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_1 : \Box\Gamma_{11} \vdash F_6}{\bullet h_1 : \Box\Gamma_{11}, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}, []F_6}\mathit{A4} \quad \dfrac{h_7 : \Box\Gamma_{11}, F_8, \Delta_{12}, []F_6 \vdash \Delta_{10} \quad h_7 : \Box\Gamma_{11}, F_9, \Delta_{12}, []F_6 \vdash \Delta_{10}}{\bullet h_7 : (\Box\Gamma_{11}, \Delta_{12}, F_8 \vee F_9), []F_6 \vdash \Delta_{10}}\vee_L}{- : \Box\Gamma_{11}, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{10}}\text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_1 : \Box\Gamma_{11} \vdash F_6}\text{ax/W}}{\bullet h_1 : \Delta_{12}, F_8, \Box\Gamma_{11} \vdash \Delta_{10}, []F_6}\mathit{A4} \quad \dfrac{}{h_7 : \Delta_{12}, F_8, \Box\Gamma_{11}, []F_6 \vdash \Delta_{10}}\text{ax/W}}{- : \Delta_{12}, F_8, \Box\Gamma_{11} \vdash \Delta_{10}}\text{hCut} \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Box\Gamma_{11} \vdash F_6}\text{ax/W}}{\bullet h_1 : \Delta_{12}, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, []F_6}\mathit{A4} \quad \dfrac{}{h_7 : \Delta_{12}, F_9, \Box\Gamma_{11}, []F_6 \vdash \Delta_{10}}\text{ax/W}}{- : \Delta_{12}, F_9, \Box\Gamma_{11} \vdash \Delta_{10}}\text{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, F_8 \vee F_9 \vdash \Delta_{10}}\vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_9 \vee \mathtt{F}_{10}}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ \vee_L}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7}\ A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{12} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_{13}}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\ \vee_L}{- : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ A4$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, []\mathtt{F}_8), \Delta_{11} \vdash \Delta_9, []\mathtt{F}_6}\ A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \mathtt{F}_8, \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, []\mathtt{F}_8), \Delta_{11}), []\mathtt{F}_6 \vdash \Delta_9}\ AT}{- : (\Box\Gamma_{10}, []\mathtt{F}_8), \Delta_{11} \vdash \Delta_9}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\ \mathtt{hCut}}{- : \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\ AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_6}\ A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \mathtt{F}_8, \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6 \vdash \Delta_9}\ AT}{- : \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\ \mathtt{hCut}}{- : \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\ AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_7}\ A4 \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_9, \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_6 : (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_7 \vdash \Delta_8}\ AT}{- : \Box\Gamma_9, \Delta_{10} \vdash \Delta_8}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8, \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7, \Box\Gamma_9 \vdash \Delta_8, []\mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \Box\Gamma_9, []\mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7, \Box\Gamma_9 \vdash \Delta_8}\ \mathtt{hCut}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8}\ \mathtt{sCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_{13} \vdash (\Delta_{10}, []\mathtt{F}_7), \mathtt{F}_{12}}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_{13}), \mathtt{F}_{12} \vdash \Delta_{10}, []\mathtt{F}_7}\ AT}{- : (\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_{13} \vdash \Delta_{10}, []\mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\ A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_7), []\mathtt{F}_9}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\ AT}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_7}\ A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9 \vdash (\Delta_{10}, []\mathtt{F}_7), \mathtt{F}_{12}}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9), \mathtt{F}_{12} \vdash \Delta_{10}, []\mathtt{F}_7}\ AT}{- : \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\ A4$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \bot, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_6}\,A4 \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \bot, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\,\bot_L}{- : \Box\Gamma_9, \bot, \Delta_{10} \vdash \Delta_8}\,\mathtt{Cut}}{- : \bot, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8}\,\bot_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, []\mathtt{F}_7), \bot}\,A4 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{11}), \bot \vdash \Delta_9, []\mathtt{F}_7}\,\bot_L}{- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{}{- : \Box\Gamma_{10} \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\,A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \bot, \Delta_{12} \vdash (\Delta_9, []\mathtt{F}_7), \mathtt{F}_{11}}\,A4 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \bot, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_7}\,\bot_L}{- : \Box\Gamma_{10}, \bot, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{Cut}}{- : \bot, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_8, \mathtt{p}_9), []\mathtt{F}_6}\,A4 \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Box\Gamma_{10}, \Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\,I}{- : \Box\Gamma_{10}, \Delta_{11}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\,\mathtt{Cut}}{- : \Delta_{11}, \Box\Gamma_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\,I$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash ((\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7), \mathtt{p}_{10}}\,A4 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\,I}{- : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_9, \mathtt{p}_{10}, []\mathtt{F}_7}\,A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7), \mathtt{F}_{12}}\,A4 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10}), \mathtt{F}_{12} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\,I}{- : \Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{- : \Delta_{13}, \Box\Gamma_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, []\mathtt{F}_7}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_6}\,A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \top, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\,\top_L}{- : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8, []\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_6 \vdash \Delta_8}\,\mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, []\mathtt{F}_7), \top}\,A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{11}), \top \vdash \Delta_9, []\mathtt{F}_7}\,\top_L}{- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{Cut}}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10} \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \top, \Delta_{12} \vdash (\Delta_9, []\mathtt{F}_7), \mathtt{F}_{11}}\,A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{10}, \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \top, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_7}\,\top_L}{- : \Box\Gamma_{10}, \top, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{}{- : \Box\Gamma_{10} \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \top, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\,A4$$

8.7 Status of K: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_8, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R}}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_{13}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7}\,{\to_R}}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10}}\,K$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,{\wedge_R}}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,{\wedge_R}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_{13}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7}\,{\wedge_R}}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,K$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,{\vee_R}}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,{\vee_R}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_{13}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_{13} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7}\,{\vee_R}}{- : \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,K$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_9) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\bot, \Delta_8), []\mathtt{F}_6}\ K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_6 \vdash \bot, \Delta_8}\ \bot_R}{- : \Box\Gamma_9, \Delta_{10} \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_8, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_6 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \bot, \Delta_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\bot, \Delta_9), []\mathtt{F}_7), \mathtt{F}_{11}}\ K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{10}, \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{F}_{11} \vdash (\bot, \Delta_9), []\mathtt{F}_7}\ \bot_R}{- : \Box\Gamma_{10}, \Delta_{12} \vdash (\bot, \Delta_9), []\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \bot, \Delta_9, []\mathtt{F}_7}\ K}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_9) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash (\top, \Delta_8), []\mathtt{F}_6}\ K \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_6 \vdash \top, \Delta_8}\ \top_R}{- : \Box\Gamma_9, \Delta_{10} \vdash \top, \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \Box\Gamma_9 \vdash \top, \Delta_8}\ \top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{12} \vdash ((\top, \Delta_9), []\mathtt{F}_7), \mathtt{F}_{11}}\ K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{F}_{11} \vdash (\top, \Delta_9), []\mathtt{F}_7}\ \top_R}{- : \Box\Gamma_{10}, \Delta_{12} \vdash (\top, \Delta_9), []\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{12}, \Box\Gamma_{10} \vdash \top, \Delta_9, []\mathtt{F}_7}\ \top_R}$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\ K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\ A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_8, []\mathtt{F}_6}\ K \quad \dfrac{}{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\ K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\ A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ A4}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}, \Box\Gamma_{14}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \Box\mathtt{F}_{12}}\ K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\mathtt{F}_{12} \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathtt{F}_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ A4}{- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{11}), \mathit{unbox}(\Box\Gamma_{14}) \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\ K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}, \Box\Gamma_{13}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{11}}\ K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\mathtt{F}_{11} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box\mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\ A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{10}), \mathit{unbox}(\Box\Gamma_{13}) \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, []\mathtt{F}_8}\ K}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \mathtt{F}_{14}}\ K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), \mathtt{F}_{14} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ A4}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\ A4}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{13}}\ K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_8}\ A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ A4}$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\ K \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_6, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_6, \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{- : \mathtt{F}_6, unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_8}\ \mathtt{sCut}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_9, []\mathtt{F}_8), []\mathtt{F}_6}\ K \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}, \Box\Gamma_{14}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \Box\mathtt{F}_{12}}\ K \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathtt{F}_{12} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ K}{- : (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\ K}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}, \Box\Gamma_{13}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{11}}\ K \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{12}), unbox(\Box\mathtt{F}_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box\mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, []\mathtt{F}_8}\ K}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7), \mathtt{F}_{14}}\ K \quad \dfrac{\mathtt{h}_8 : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_9}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), \mathtt{F}_{14} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ K}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Delta_{10}, []\mathtt{F}_9), []\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, []\mathtt{F}_7, []\mathtt{F}_9}\ K}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{13}}\ K \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_8}\ K}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_9, []\mathtt{F}_8}\ K}$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\,{\to_L}}{- : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{Cut}}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\,{\to_L}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_9 \to \mathtt{F}_{10}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \Delta_{13} \vdash \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,{\to_L}}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7}\,K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_{13}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\,{\to_L}}{- : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,K$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_8, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\,{\wedge_L}}{- : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\,{\wedge_L}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,{\wedge_L}}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7}\,K$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_{13}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\,{\wedge_L}}{- : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,K$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_8, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10} \quad \mathtt{h}_7 : \Box\Gamma_{11}, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\,{\vee_L}}{- : \Box\Gamma_{11}, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{11}) \vdash \mathtt{F}_6}\,\mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_6}\,K \quad \dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_{11} \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\,{\vee_L}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_9 \vee \mathtt{F}_{10}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\vee_L}{- : \Box\Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{12} \vdash \Delta_{11}, []\mathtt{F}_7}\,K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\Delta_{11}, []\mathtt{F}_7), \mathtt{F}_{13}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_9, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7 \quad \mathtt{h}_8 : \Box\Gamma_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_7}\vee_L}{- : \Box\Gamma_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\,K}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}, []\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, []\mathtt{F}_8), \Delta_{11} \vdash \Delta_9, []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \mathtt{F}_8, \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, []\mathtt{F}_8), \Delta_{11}), []\mathtt{F}_6 \vdash \Delta_9}\,AT}{- : (\Box\Gamma_{10}, []\mathtt{F}_8), \Delta_{11} \vdash \Delta_9}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{hCut}}{- : \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\,AT}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \mathtt{F}_8, \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : (\Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6 \vdash \Delta_9}\,AT}{- : \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\,\mathtt{hCut}}{- : \Delta_{11}, \Box\Gamma_{10}, []\mathtt{F}_8 \vdash \Delta_9}\,ATG}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_9) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_7}\,K \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_9, \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_6 : (\Box\Gamma_9, \Delta_{10}), []\mathtt{F}_7 \vdash \Delta_8}\,AT}{- : \Box\Gamma_9, \Delta_{10} \vdash \Delta_8}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{}{- : \Delta_{10}, \Box\Gamma_9, unbox(\Box\Gamma_9) \vdash \Delta_8, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7, \Box\Gamma_9, unbox(\Box\Gamma_9) \vdash \Delta_8, []\mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \Box\Gamma_9, []\mathtt{F}_7, unbox(\Box\Gamma_9) \vdash \Delta_8}\,\mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7, \Box\Gamma_9, unbox(\Box\Gamma_9) \vdash \Delta_8}\,\mathtt{hCut}}{- : \Delta_{10}, \Box\Gamma_9, unbox(\Box\Gamma_9) \vdash \Delta_8}\,\mathtt{sCut}}{- : \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8}\,ATG}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}, []\mathtt{F}_9) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_{13} \vdash (\Delta_{10}, []\mathtt{F}_7), \mathtt{F}_{12}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_{13}), \mathtt{F}_{12} \vdash \Delta_{10}, []\mathtt{F}_7}\,AT}{- : (\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_{13} \vdash \Delta_{10}, []\mathtt{F}_7}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{}{- : \mathtt{F}_9, unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\,K}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_7), []\mathtt{F}_9}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\,AT}{- : \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_7}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, []\mathtt{F}_7}\,K}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9 \vdash (\Delta_{10}, []\mathtt{F}_7), \mathtt{F}_{12}}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9), \mathtt{F}_{12} \vdash \Delta_{10}, []\mathtt{F}_7}\,AT}{- : \Box\Gamma_{11}, \Delta_{13}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\,\mathtt{Cut}}{\to}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\,K}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_9) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \bot, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_6}\,K \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \bot, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\,\bot_L}{- : \Box\Gamma_9, \bot, \Delta_{10} \vdash \Delta_8}\,\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8}\,\bot_L}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, []\mathtt{F}_7), \bot}\,K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{11}), \bot \vdash \Delta_9, []\mathtt{F}_7}\,\bot_L}{- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\,K}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \bot, \Delta_{12} \vdash (\Delta_9, []\mathtt{F}_7), \mathtt{F}_{11}}\,K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \bot, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_7}\,\bot_L}{- : \Box\Gamma_{10}, \bot, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\,\bot_L}\,\to$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_8, \mathtt{p}_9), []\mathtt{F}_6}\,K \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Box\Gamma_{10}, \Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\,I}{- : \Box\Gamma_{10}, \Delta_{11}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \Box\Gamma_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\,I}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{12} \vdash ((\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7), \mathtt{p}_{10}}\,K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\,I}{- : \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_9, \mathtt{p}_{10}, []\mathtt{F}_7}\,K}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{11}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7), \mathtt{F}_{12}}\,K \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10}), \mathtt{F}_{12} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\,I}{- : \Box\Gamma_{11}, \Delta_{13}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{13}, \Box\Gamma_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, []\mathtt{F}_7}\,I}\,\to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_9) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_6}\,K \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\Box\Gamma_9, \top, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\,\top_L}{- : \Box\Gamma_9, \top, \Delta_{10} \vdash \Delta_8}\,\mathtt{Cut}}{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8, []\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_{10}, \Box\Gamma_9, []\mathtt{F}_6 \vdash \Delta_8}\,\mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_8}\,\mathtt{hCut}}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, []\mathtt{F}_7), \top}\,K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \Delta_{11}), \top \vdash \Delta_9, []\mathtt{F}_7}\,\top_L}{- : \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{ax/W}}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \top, \Delta_{12} \vdash (\Delta_9, []\mathtt{F}_7), \mathtt{F}_{11}}\, K \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{10}, \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}{\bullet\mathtt{h}_8 : (\Box\Gamma_{10}, \top, \Delta_{12}), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_7}\, \top_L}{- : \Box\Gamma_{10}, \top, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_7}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- : \mathit{unbox}(\Box\Gamma_{10}) \vdash \mathtt{F}_7}\,\mathtt{ax/W}}{- : \top, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, []\mathtt{F}_7}\, K$$

8.8 Status of $\rightarrow_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_{13}}\,\rightarrow_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\,\rightarrow_R}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\,\rightarrow_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\,\rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{13}}\,\rightarrow_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \mathtt{F}_1}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{13}}\,\rightarrow_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{14},}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{13}}\,\rightarrow_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\vee_R}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}}\,\rightarrow_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\vee_R$$

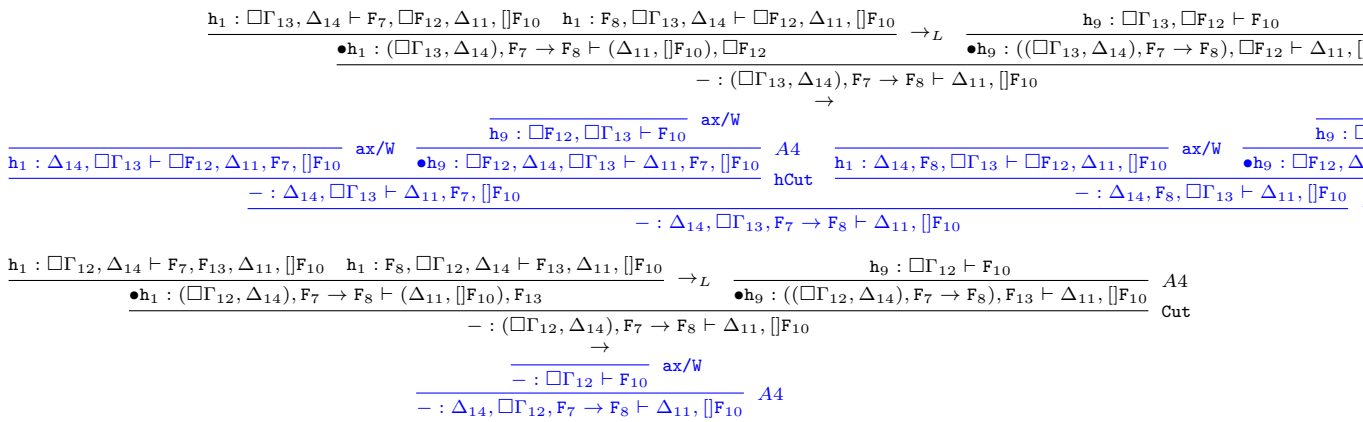
- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \bot, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{12} \vdash \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{11}}\,\rightarrow_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{F}_{11} \vdash \bot, \Delta_{10}}\,\bot_R}{- : \Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \bot, \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{hCut}$$

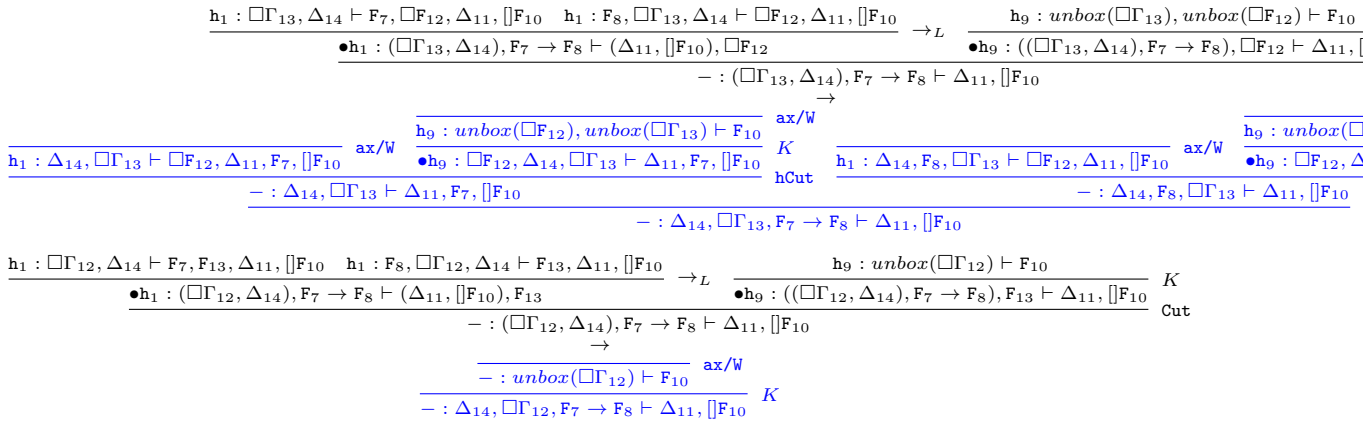
- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \top, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{12} \vdash \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{11}}\,\rightarrow_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{F}_{11} \vdash \top, \Delta_{10}}\,\top_R}{- : \Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \top, \Delta_{10}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \top, \Delta_{10}}\,\top_R$$

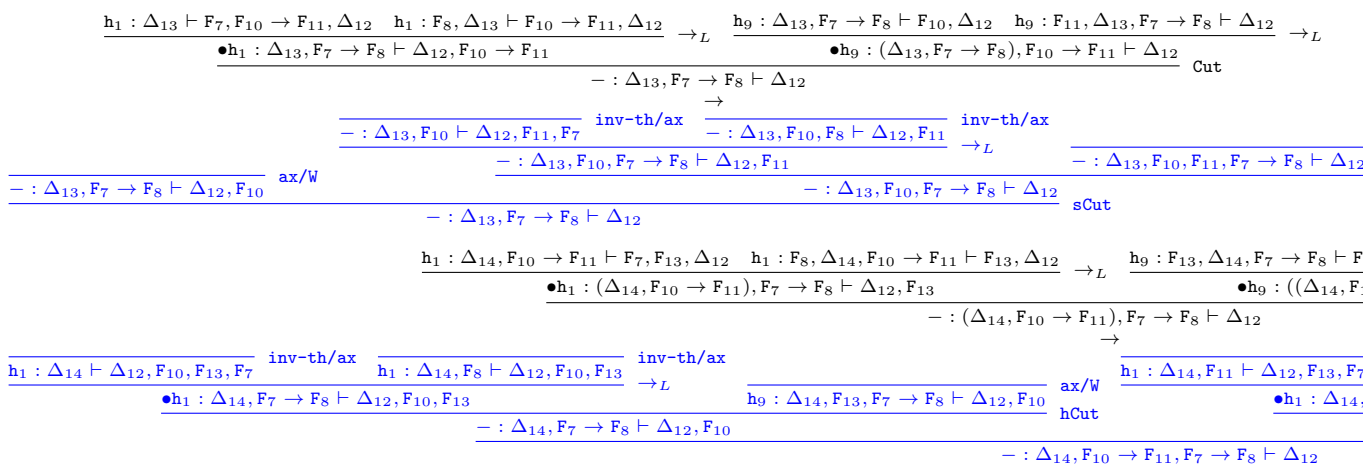
- Case rule $A4$



- Case rule K



- Case rule $\rightarrow_L$



$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_9, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}} \to_L \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12} \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}}{\bullet \mathtt{h}_7 : (\Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_{11} \vdash \Delta_{10}} \to_L}{- : \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}} \text{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8}\ \text{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8}{\bullet \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8}\ \text{H}}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8}\ \text{ax/W}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8}\ \text{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \text{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_9 \vdash \Delta_{10}}{\bullet \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_9 \vdash \Delta_{10}}\ \text{H}}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_9 \vdash \Delta_{10}}\ \text{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{10}}\ \text{hCut}}{- : \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}} \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \Delta_{12} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \to_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}} \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}} \text{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ \text{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7}\ \wedge_L}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7}\ \text{inv-th/ax}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7}\ \text{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ \text{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\ \wedge_L}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\ \text{inv-th/ax}}{- : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}}\ \text{hC}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}} \to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}} \to_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}} \wedge_L}{- : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}} \text{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7}\ \text{inv-th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\ \text{inv-th/ax} \to_L}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}}\ \text{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}}\ \text{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \to \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}} \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \to_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}} \vee_L}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}} \text{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7}\ \text{inv-th/ax} \quad \dfrac{- : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}{- : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\ \text{inv-th/ax} \to_L}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}} \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\ \text{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\ \text{sCut} \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}} \to_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_1}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{12}}\ \text{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7}\ \text{hCut}}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7}\ \text{inv-th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_7}\ \text{inv-th/ax} \vee_L}{- : \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta}$$

$$- : \Delta_{14}, \mathtt{F}_7 \to \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, []\mathtt{F}_{10}, \Delta_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{12} \vdash []\mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}} \to_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), []\mathtt{F}_{10} \vdash \Delta_{11}}\ AT}{- : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}} \text{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_{10}}\ \text{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}{\bullet \mathtt{h}_9 : \Delta_{12}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}\ AT}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, []\mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}\ \text{inv-th/ax}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7}\ \text{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\ \text{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8, []\mathtt{F}_{10} \vdash \Delta_{11}}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_8, []\mathtt{F}_{10} \vdash \Delta_{11}}\ AT}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8, []\mathtt{F}_{10} \vdash \Delta_{11}}\ \text{inv-th/ax}}{- : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{11}}\ \text{hCut}}{- : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}} \to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_{12}, \Delta_{11}}{\bullet\mathtt{h}_1 : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}, \mathtt{F}_{12}} \to_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}}{\bullet\mathtt{h}_9 : ((\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{11}}\ AT}{- : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}, \mathtt{F}_{12}}{\to}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}}\ \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}}\ \mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{11}}\ ATG$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \bot, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{11} \vdash \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \bot} \to_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \bot \vdash \Delta_{10}}\ \bot_L}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7}{\to}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\ \bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \bot, \Delta_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}}\ \bot_L}{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\ \to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \bot, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\bot, \Delta_{12}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}} \to_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\bot, \Delta_{12}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11} \vdash \Delta_{10}}\ \bot_L}{- : (\bot, \Delta_{12}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{Cut}}{- : \bot, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \mathtt{F}_7, \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{12} \vdash \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}} \to_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\ I}{- : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}, \mathtt{p}_{11}}{\to}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\ I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\ I}{- : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ \mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ \to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{12}} \to_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{p}_{11}}\ I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ \mathtt{Cut}}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \mathtt{F}_7, \top, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \top} \to_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \top \vdash \Delta_{10}}\ \top_L}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{Cut}}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \top, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\top, \Delta_{12}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}} \to_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : ((\top, \Delta_{12}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{11} \vdash \Delta_{10}}\ \top_L}{- : (\top, \Delta_{12}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 : \top, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}{\to}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{hCut}$$

8.9 Status of $\wedge_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{13}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}} \to_R}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}} \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}} \wedge_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}} \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}} \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}} \to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{13}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \wedge_R}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}} \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}} \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \wedge_R}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \wedge_L$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{13}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_R}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}} \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}} \wedge_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}} \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}} \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{12} \vdash \mathtt{F}_{11}, \bot, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{11}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11} \vdash \bot, \Delta_{10}} \bot_R}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}, \mathtt{F}_{11}} \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{12} \vdash \mathtt{F}_{11}, \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{11}} \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11} \vdash \top, \Delta_{10}} \top_R}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \top, \Delta_{10}} \mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \top, \Delta_{10}} \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_1 : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \Box\mathtt{F}_{12}} \wedge_L \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{13}, \Box\mathtt{F}_{12} \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : ((\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8), \Box\mathtt{F}_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}} A4}{- : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}} \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}} \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Box\Gamma_{13} \vdash \mathtt{F}_{10}} \mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}} A4}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}} \mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}} \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_{13}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}}\ A4}{- : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12} \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\ A4$$

- Case rule K

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \Box\mathtt{F}_{12}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{13}), unbox(\Box\mathtt{F}_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8), \Box\mathtt{F}_{12} \vdash \Delta_{11}, []\mathtt{F}_{10}}\ K}{- : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Box\mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : unbox(\Box\mathtt{F}_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Box\mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}}\ K}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}}\ \mathtt{hCut}$$

$$\dfrac{}{- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\ \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Box\Gamma_{12}, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{11}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_1 : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{F}_{13}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{11}, []\mathtt{F}_{10}}\ K}{- : (\Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{12}) \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\ K$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}}\ \rightarrow_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}}\ \rightarrow_L}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{hCut}$$

$$\dfrac{}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet\mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}}\ \rightarrow_L}{- : (\Delta_{14}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}}\ \rightarrow_L}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}}\ \mathtt{hCut}$$

$$\dfrac{}{- : \Delta_{14}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \wedge_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\ \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\ \wedge_L}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\ \mathtt{hCut}$$

$$\dfrac{}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\ \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}}\wedge_L}{- : (\Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\wedge_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_8, \mathtt{F}_9, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\wedge_L \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{12} \vdash \Delta_{10}}{\bullet \mathtt{h}_7 : (\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11} \vdash \Delta_{10}}\wedge_L}{- : \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{ax/W}}{\bullet \mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{H}}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\wedge_L}{- : \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13} \vdash \mathtt{F}_{10} \vee \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\vee_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\vee_L}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_1 : (\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12} \quad \mathtt{h}_9 : \mathtt{F}_{11}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : ((\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}}\vee_L}{- : (\Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\vee_L}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{13}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\wedge_L}{- : \Delta_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}$$

- Case rule AT

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{12} \vdash []\mathtt{F}_{10}, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), []\mathtt{F}_{10} \vdash \Delta_{11}}\,AT}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, []\mathtt{F}_{10} \vdash \Delta_{11}}\,AT}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{11}, []\mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{11}}\wedge_L}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13}, []\mathtt{F}_{10} \vdash \mathtt{F}_{12}, \Delta_{11}}{\bullet \mathtt{h}_1 : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, \mathtt{F}_{12}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}{\bullet \mathtt{h}_9 : ((\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{11}}\,AT}{- : (\Delta_{13}, []\mathtt{F}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}, \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{12}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}\,\mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{11}}\,AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{11} \vdash \bot, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \bot}\ \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \bot \vdash \Delta_{10}}\ \bot_L}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \bot, \Delta_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}}\ \bot_L}{- : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\ \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \bot, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : (\bot, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : ((\bot, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11} \vdash \Delta_{10}}\ \bot_L}{- : (\bot, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{Cut}}{- : \bot, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{12} \vdash \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}\ \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\ I}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\ I}{- : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ \mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{12}}\ \wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{p}_{11}}\ I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ \mathtt{Cut}}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \top}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \top \vdash \Delta_{10}}\ \top_L}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{Cut}}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{ax/W}$$

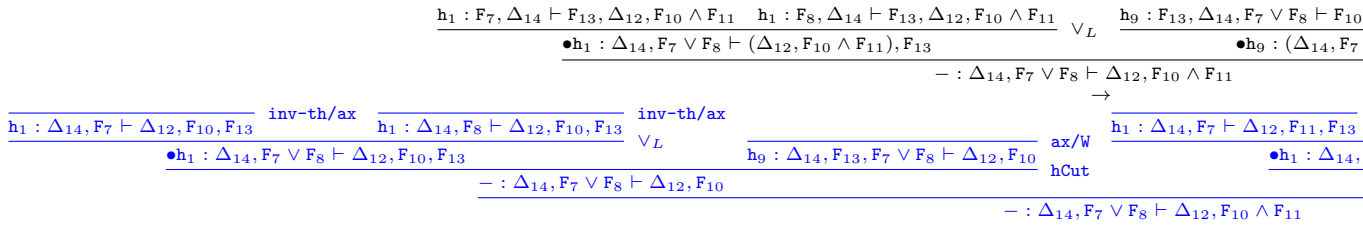
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \mathtt{F}_8, \top, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet \mathtt{h}_1 : (\top, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : ((\top, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{11} \vdash \Delta_{10}}\ \top_L}{- : (\top, \Delta_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\ \mathtt{hCut}}$$

8.10 Status of $\vee_L$: OK

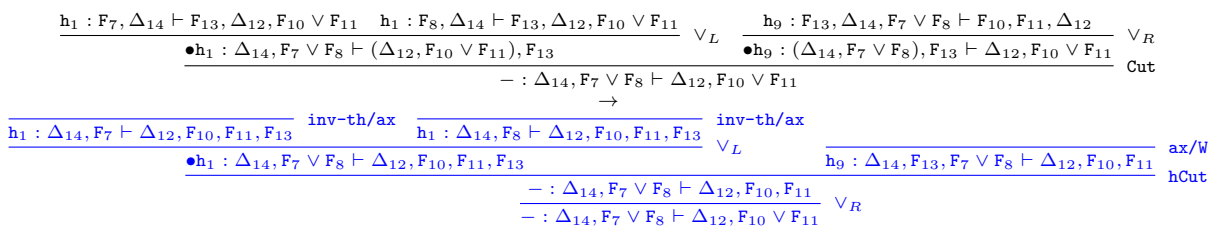
- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{13}, \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{13}}\ \vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\ \to_R}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{13}}\ \vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\ \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\ \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\ \to_R$$

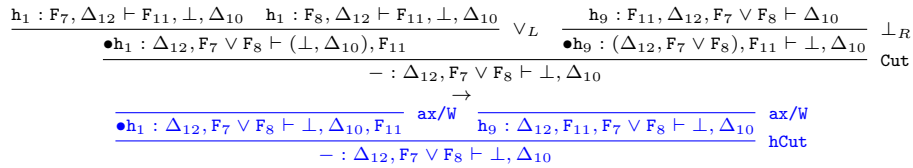
- Case rule $\wedge_R$



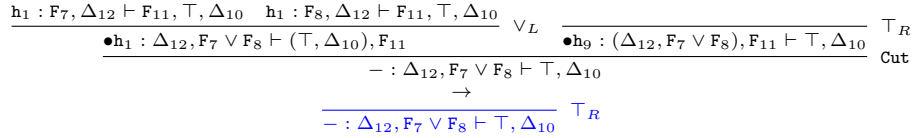
- Case rule $\vee_R$



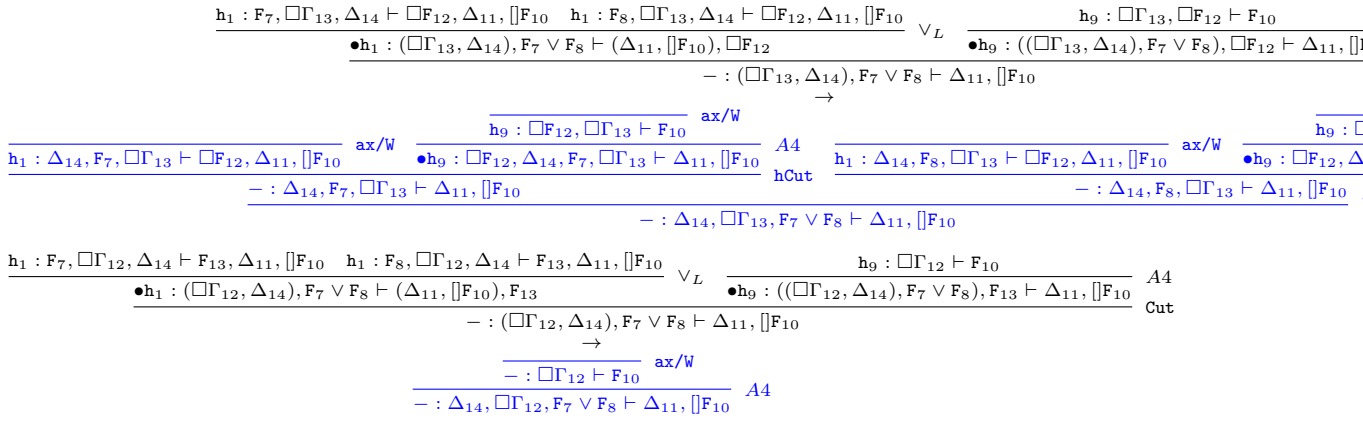
- Case rule $\bot_R$



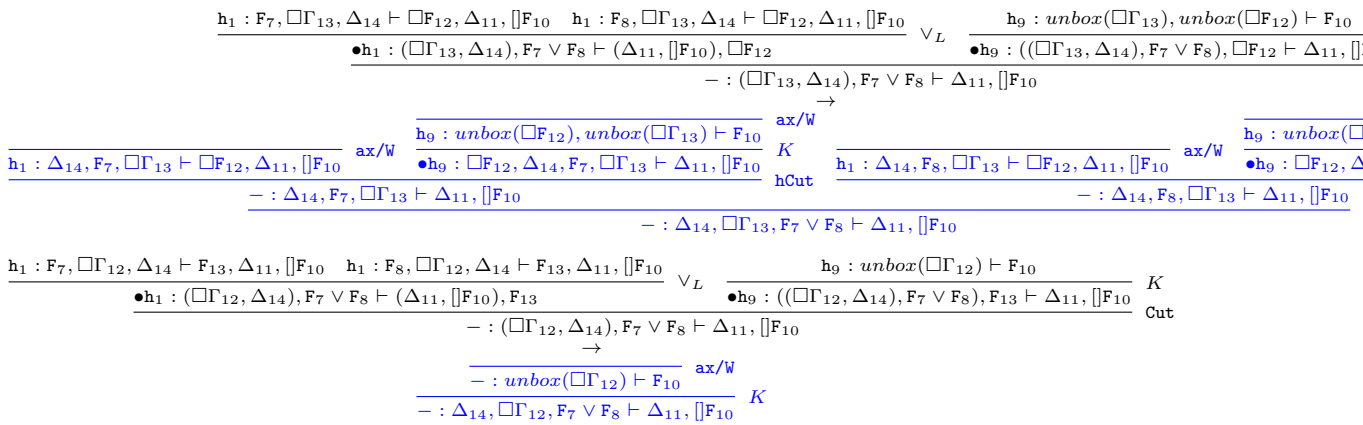
- Case rule $\top_R$



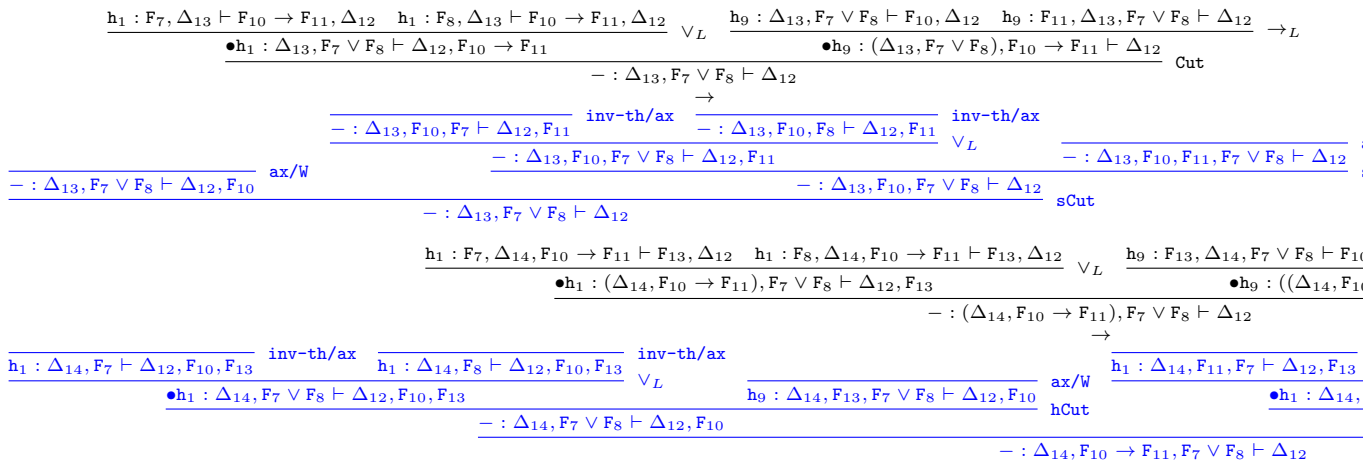
- Case rule $A4$



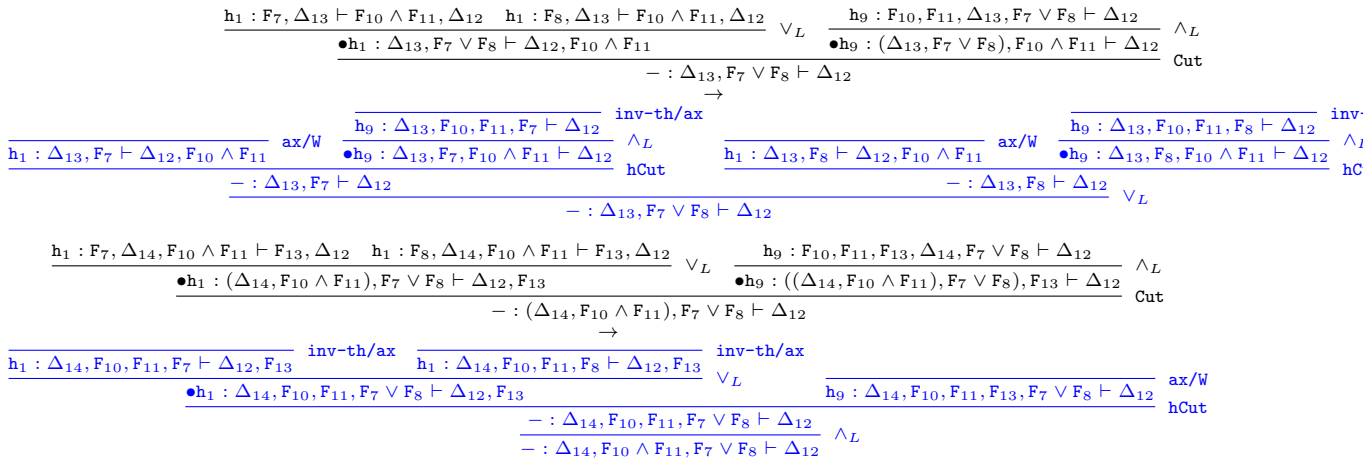
- Case rule K



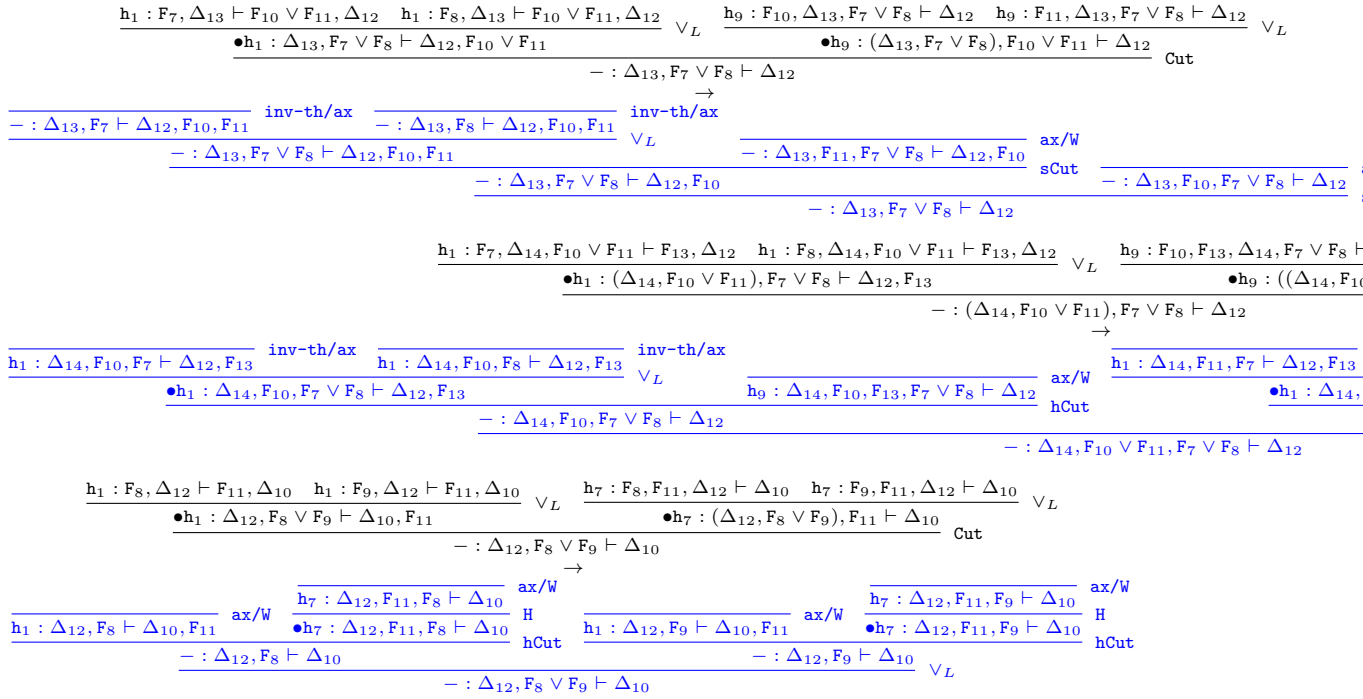
- Case rule $\rightarrow_L$



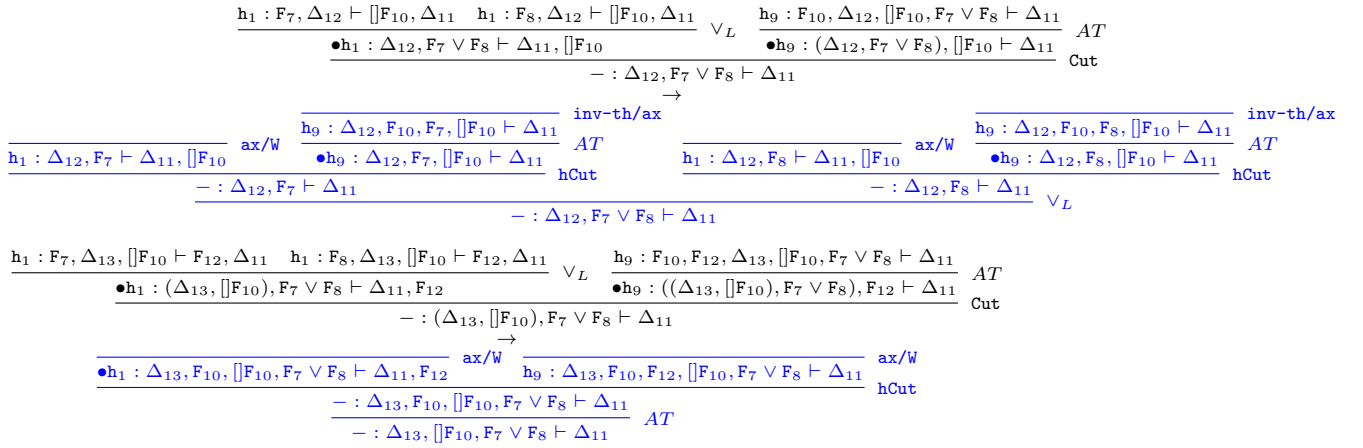
- Case rule $\wedge_L$



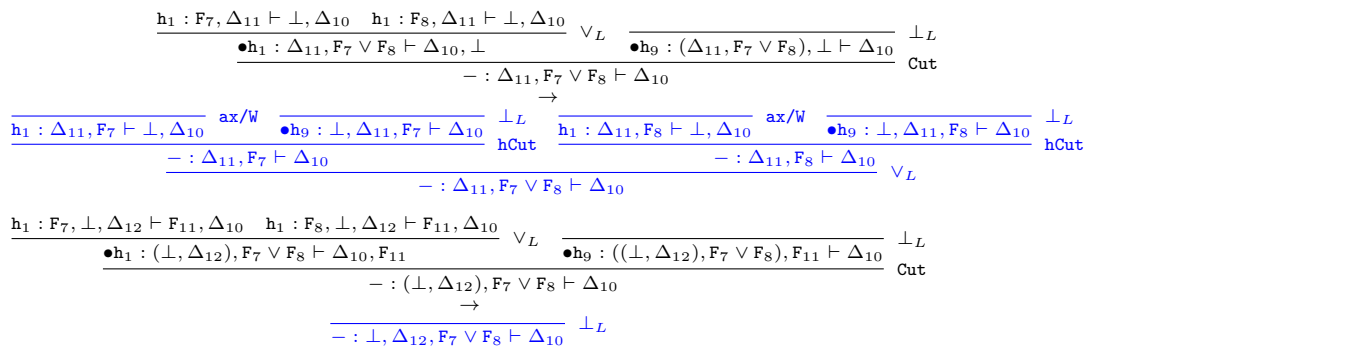
- Case rule $\vee_L$



- Case rule AT



- Case rule $\bot_L$



- Case rule I

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{12} \vdash \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{12} \vdash \mathtt{p}_{11}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}\vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}{- : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}{- : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\vee_L}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_{12}, \Delta_{10}, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{12}}\vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{11} \vdash \top, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \top}\vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \top \vdash \Delta_{10}}\,\top_L}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \top, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_1 : \mathtt{F}_8, \top, \Delta_{12} \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\top, \Delta_{12}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\vee_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : ((\top, \Delta_{12}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{11} \vdash \Delta_{10}}\,\top_L}{- : (\top, \Delta_{12}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\bullet\mathtt{h}_1 : \top, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}{- : \top, \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \top, \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{}\,\mathtt{ax/W}}{}\,\mathtt{hCut}}$$

8.11 Status of AT: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12}, []\mathtt{F}_6 \vdash (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{12}, []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\to_R}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{ax/W}}{}\,\mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\, ATG$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12}, []\mathtt{F}_6 \vdash (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{12}, []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\wedge_R}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W}}{}\,\mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\, AT$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12}, []\mathtt{F}_6 \vdash (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{12}, []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\, \vee_R}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9}\, \mathtt{ax/W} \quad \dfrac{\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\, AT$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \bot, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash (\bot, \Delta_8), \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\Delta_{10}, []\mathtt{F}_6), \mathtt{F}_9 \vdash \bot, \Delta_8}\, \bot_R}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \bot, \Delta_8}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash \bot, \Delta_8, \mathtt{F}_9}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6 \vdash \bot, \Delta_8}\, \mathtt{ax/W}}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \bot, \Delta_8}\, \mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_6 \vdash \mathtt{F}_9, \top, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash (\top, \Delta_8), \mathtt{F}_9}\, AT \quad \dfrac{}{\bullet\mathtt{h}_7 : (\Delta_{10}, []\mathtt{F}_6), \mathtt{F}_9 \vdash \top, \Delta_8}\, \top_R}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \top, \Delta_8}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \top, \Delta_8}\, \top_R}$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{10}}\, AT \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Box\mathtt{F}_{10}, []\mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6), \Box\mathtt{F}_{10} \vdash \Delta_9, []\mathtt{F}_8}\, A4}{- : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Box\mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, ATG$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, []\mathtt{F}_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\, A4}{- : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{10}, []\mathtt{F}_6 \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, A4}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{10}}\, AT \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{11}, \Box\mathtt{F}_{10} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6), \Box\mathtt{F}_{10} \vdash \Delta_9, []\mathtt{F}_8}\, A4}{- : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Box\mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, AT$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\, A4}{- : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{10} \vdash \mathtt{F}_8}\, \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, A4}$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{10}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_6, unbox(\Box\Gamma_{11}), unbox(\Box\mathtt{F}_{10}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6), \Box\mathtt{F}_{10} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Box\mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, ATG}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_6, unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : \mathtt{F}_6, unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, K}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_{10}}\, AT \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{11}), unbox(\Box\mathtt{F}_{10}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6), \Box\mathtt{F}_{10} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Box\mathtt{F}_{10}, \Delta_9, []\mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Box\mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_9, []\mathtt{F}_8}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash (\Delta_9, []\mathtt{F}_8), \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_8}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_9, []\mathtt{F}_8}\, K}{- : (\Box\Gamma_{10}, \Delta_{12}), []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{10}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, K}$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8 \rightarrow \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, []\mathtt{F}_6), \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \Delta_{10}}\,\rightarrow_L}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\, ATG}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{12}, \mathtt{F}_8 \rightarrow \mathtt{F}_9), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_8 \rightarrow \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : ((\Delta_{12}, \mathtt{F}_8 \rightarrow \mathtt{F}_9), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}}\,\rightarrow_L}{- : (\Delta_{12}, \mathtt{F}_8 \rightarrow \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6, \mathtt{F}_8 \rightarrow \mathtt{F}_9 \vdash \Delta_{10}}\, AT}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, []\mathtt{F}_6), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\,\wedge_L}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\, AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : ((\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}}\, \wedge_L}{- : (\Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\, \mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\, AT$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_8 \vee \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, []\mathtt{F}_6), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\, \vee_L}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\, \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_{10}}\, \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_6 \vdash \Delta_{10}}\, AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6 \vdash \mathtt{F}_{11}, \Delta_{10}}{\bullet\mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_9, \mathtt{F}_{11}, \Delta_{12}, []\mathtt{F}_6 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : ((\Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6), \mathtt{F}_{11} \vdash \Delta_{10}}\, \vee_L}{- : (\Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_6 \vdash \Delta_{10}}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{11}}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_6, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\, \mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}}\, AT$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_6 \vdash []\mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \Delta_{10}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : (\Delta_{10}, []\mathtt{F}_6), []\mathtt{F}_8 \vdash \Delta_9}\, AT}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_9}\, ATG$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6 \vdash \mathtt{F}_{10}, \Delta_9}{\bullet\mathtt{h}_1 : (\Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6 \vdash \Delta_9, \mathtt{F}_{10}}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_8, \mathtt{F}_{10}, \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_7 : ((\Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6), \mathtt{F}_{10} \vdash \Delta_9}\, AT}{- : (\Delta_{11}, []\mathtt{F}_8), []\mathtt{F}_6 \vdash \Delta_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_{10}}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_6, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_6, []\mathtt{F}_8 \vdash \Delta_9}\, ATG$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_9, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_6 : (\Delta_{10}, []\mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\, AT}{- : \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_9, []\mathtt{F}_7 \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_8}\, AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_9, []\mathtt{F}_6 \vdash \bot, \Delta_8}{\bullet \mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8, \bot}\, AT \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_9, []\mathtt{F}_6), \bot \vdash \Delta_8}\, \bot_L}{- : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6, []\mathtt{F}_6 \vdash \bot, \Delta_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \bot, \Delta_9, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_8}\, \bot_L}{- : \Delta_9, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}\, AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\bot, \Delta_{10}), []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_8}{\bullet \mathtt{h}_1 : (\bot, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9}\, AT \quad \dfrac{}{\bullet \mathtt{h}_7 : ((\bot, \Delta_{10}), []\mathtt{F}_6), \mathtt{F}_9 \vdash \Delta_8}\, \bot_L}{- : (\bot, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \bot, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}\, \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_6 \vdash \mathtt{p}_9, \Delta_8, \mathtt{p}_9}{\bullet \mathtt{h}_1 : \Delta_{10}, []\mathtt{F}_6 \vdash (\Delta_8, \mathtt{p}_9), \mathtt{p}_9}\, AT \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_{10}, []\mathtt{F}_6), \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, I}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9, \mathtt{p}_9}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{10}, \mathtt{F}_6, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, I}{- : \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, \mathtt{hCut}}{- : \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6 \vdash \mathtt{F}_{10}, \Delta_8, \mathtt{p}_9}{\bullet \mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6 \vdash (\Delta_8, \mathtt{p}_9), \mathtt{F}_{10}}\, AT \quad \dfrac{}{\bullet \mathtt{h}_7 : ((\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{p}_9}\, I}{- : (\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, \mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_9}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_9, []\mathtt{F}_6 \vdash \top, \Delta_8}{\bullet \mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8, \top}\, AT \quad \dfrac{\mathtt{h}_7 : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : (\Delta_9, []\mathtt{F}_6), \top \vdash \Delta_8}\, \top_L}{- : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_9, []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, (\top, \Delta_{10}), []\mathtt{F}_6 \vdash \mathtt{F}_9, \Delta_8}{\bullet \mathtt{h}_1 : (\top, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9}\, AT \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_9, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : ((\top, \Delta_{10}), []\mathtt{F}_6), \mathtt{F}_9 \vdash \Delta_8}\, \top_L}{- : (\top, \Delta_{10}), []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \top, \Delta_{10}, \mathtt{F}_9, []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{ax/W}}{- : \top, \Delta_{10}, []\mathtt{F}_6 \vdash \Delta_8}\, \mathtt{hCut}$$

8.12 Status of $\bot_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_9}\, \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\, \to_R}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\, \mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\, \bot_L$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \bot_L$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \bot_L$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \bot, \Delta_6}\ \bot_R}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \bot_L$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \top, \Delta_6}\ \top_R}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_9, \Box\mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \Delta_7, []\mathtt{F}_6}\ A4}{- : \bot, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Box\Gamma_8, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_7, []\mathtt{F}_6}\ A4}{- : \bot, \Box\Gamma_8, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_8 \vdash \Delta_7, []\mathtt{F}_6}\ \bot_L$$

- Case rule K

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_9), unbox(\Box\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \Delta_7, []\mathtt{F}_6}\ K}{- : \bot, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Box\Gamma_8, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : (\bot, \Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_7, []\mathtt{F}_6}\ K}{- : \bot, \Box\Gamma_8, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_8 \vdash \Delta_7, []\mathtt{F}_6}\ \bot_L$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9 \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\ \to_L}{- : \bot, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8}\ \bot_L}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \to_L}{- : \bot, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\ \bot_L}\ \to$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \wedge_L}{- : \bot, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8}\ \bot_L}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \wedge_L}{- : \bot, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \bot_L}\ \to$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \Delta_9 \vdash \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \vee_L}{- : \bot, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8}\ \bot_L}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8 \quad \mathtt{h}_5 : \bot, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \vee_L}{- : \bot, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \bot_L}\ \to$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash \Delta_7, []\mathtt{F}_6}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \Delta_8, []\mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), []\mathtt{F}_6 \vdash \Delta_7}\ AT}{- : \bot, \Delta_8 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_8 \vdash \Delta_7}\ \bot_L}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7, \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_6, \mathtt{F}_8, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, []\mathtt{F}_6), \mathtt{F}_8 \vdash \Delta_7}\ AT}{- : \bot, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}\ \bot_L}\ \to$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{F}_7 \vdash \Delta_6}\ \bot_L}{- : \bot, \Delta_8 \vdash \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_8 \vdash \Delta_6}\ \bot_L}\ \to$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ I}{- : \bot, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{F}_8}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, \mathtt{p}_7), \mathtt{F}_8 \vdash \Delta_6, \mathtt{p}_7}\ I}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ \bot_L$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_7 \vdash \Delta_6, \top}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_7 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \top \vdash \Delta_6}\ \top_L}{- : \bot, \Delta_7 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_7 \vdash \Delta_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\bot, \top, \Delta_8), \mathtt{F}_7 \vdash \Delta_6}\ \top_L}{- : \bot, \top, \Delta_8 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \top, \Delta_8 \vdash \Delta_6}\ \bot_L$$

8.13 Status of I: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash (\Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{p}_{10}}\ I \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}\ \begin{array}{l}\mathtt{ax/W}\\ \mathtt{hCut}\end{array}}{- : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11}}\ I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_8, \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{12}, \mathtt{p}_{10}), \mathtt{F}_{11} \vdash (\Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{p}_{10}}\ \rightarrow_R}{- : \Delta_{12}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{p}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ I$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10}}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\ \begin{array}{l}\mathtt{ax/W}\\ \mathtt{hCut}\end{array}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}\ \begin{array}{l}\mathtt{ax/W}\\ \mathtt{hCut}\end{array}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11}}\ I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_8, \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{12}, \mathtt{p}_{10}), \mathtt{F}_{11} \vdash (\Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}}\ \wedge_R}{- : \Delta_{12}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ I$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\, \vee_R}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7}{}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\, \vee_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{12}, \mathtt{p}_{10}), \mathtt{F}_{11} \vdash (\Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}}\, \vee_R}{- : \Delta_{12}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\, I$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_8 \vdash (\bot, \Delta_6), \mathtt{p}_8}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_8), \mathtt{p}_8 \vdash \bot, \Delta_6}\, \bot_R}{- : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_6, \mathtt{p}_8}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \bot, \Delta_6}{}\, \mathtt{ax/W}}{- : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_6}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_8 \vdash ((\bot, \Delta_7), \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_9, \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_8), \mathtt{F}_9 \vdash (\bot, \Delta_7), \mathtt{p}_8}\, \bot_R}{- : \Delta_{10}, \mathtt{p}_8 \vdash (\bot, \Delta_7), \mathtt{p}_8}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8 \vdash \bot, \Delta_7, \mathtt{p}_8}\, I$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_8 \vdash (\top, \Delta_6), \mathtt{p}_8}\, I \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_8), \mathtt{p}_8 \vdash \top, \Delta_6}\, \top_R}{- : \Delta_7, \mathtt{p}_8 \vdash \top, \Delta_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_7, \mathtt{p}_8 \vdash \top, \Delta_6}\, \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_8 \vdash ((\top, \Delta_7), \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_8), \mathtt{F}_9 \vdash (\top, \Delta_7), \mathtt{p}_8}\, \top_R}{- : \Delta_{10}, \mathtt{p}_8 \vdash (\top, \Delta_7), \mathtt{p}_8}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8 \vdash \top, \Delta_7, \mathtt{p}_8}\, \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box \Gamma_8, \Delta_9), \mathtt{p}_{10} \vdash (\Delta_7, []\mathtt{F}_6), \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \Box \Gamma_8 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : ((\Box \Gamma_8, \Delta_9), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_7, []\mathtt{F}_6}\, A4}{- : (\Box \Gamma_8, \Delta_9), \mathtt{p}_{10} \vdash \Delta_7, []\mathtt{F}_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box \Gamma_8 \vdash \mathtt{F}_6}\, \mathtt{ax/W}}{- : \Delta_9, \Box \Gamma_8, \mathtt{p}_{10} \vdash \Delta_7, []\mathtt{F}_6}\, A4$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box \Gamma_{11}, \Delta_{12}), \mathtt{p}_9 \vdash ((\Delta_8, []\mathtt{F}_7), \mathtt{p}_9), \Box \mathtt{F}_{10}}\, I \quad \dfrac{\mathtt{h}_6 : \Box \Gamma_{11}, \Box \mathtt{F}_{10} \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : ((\Box \Gamma_{11}, \Delta_{12}), \mathtt{p}_9), \Box \mathtt{F}_{10} \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, A4}{- : (\Box \Gamma_{11}, \Delta_{12}), \mathtt{p}_9 \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{12}, \Box \Gamma_{11}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9, []\mathtt{F}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{p}_9 \vdash ((\Delta_8, []\mathtt{F}_7), \mathtt{p}_9), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_{10} \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : ((\Box\Gamma_{10}, \Delta_{12}), \mathtt{p}_9), \mathtt{F}_{11} \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, A4}{- : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{p}_9 \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{12}, \Box\Gamma_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9, []\mathtt{F}_7}\, I$$

- Case rule K

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_8, \Delta_9), \mathtt{p}_{10} \vdash (\Delta_7, []\mathtt{F}_6), \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet \mathtt{h}_5 : ((\Box\Gamma_8, \Delta_9), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_7, []\mathtt{F}_6}\, K}{- : (\Box\Gamma_8, \Delta_9), \mathtt{p}_{10} \vdash \Delta_7, []\mathtt{F}_6}\, \mathtt{Cut}$$
$$\dfrac{\dfrac{}{- : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}\, \mathtt{ax/W}}{- : \Delta_9, \Box\Gamma_8, \mathtt{p}_{10} \vdash \Delta_7, []\mathtt{F}_6}\, K$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{p}_9 \vdash ((\Delta_8, []\mathtt{F}_7), \mathtt{p}_9), \Box\mathtt{F}_{10}}\, I \quad \dfrac{\mathtt{h}_6 : unbox(\Box\Gamma_{11}), unbox(\Box\mathtt{F}_{10}) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : ((\Box\Gamma_{11}, \Delta_{12}), \mathtt{p}_9), \Box\mathtt{F}_{10} \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, K}{- : (\Box\Gamma_{11}, \Delta_{12}), \mathtt{p}_9 \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{12}, \Box\Gamma_{11}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9, []\mathtt{F}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{p}_9 \vdash ((\Delta_8, []\mathtt{F}_7), \mathtt{p}_9), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : unbox(\Box\Gamma_{10}) \vdash \mathtt{F}_7}{\bullet \mathtt{h}_6 : ((\Box\Gamma_{10}, \Delta_{12}), \mathtt{p}_9), \mathtt{F}_{11} \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, K}{- : (\Box\Gamma_{10}, \Delta_{12}), \mathtt{p}_9 \vdash (\Delta_8, []\mathtt{F}_7), \mathtt{p}_9}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{12}, \Box\Gamma_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9, []\mathtt{F}_7}\, I$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8}\, \rightarrow_L}{- : (\Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{Cut}$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\, \rightarrow_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7 \rightarrow \mathtt{F}_8}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, \rightarrow_L}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \mathtt{F}_7, \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : ((\Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11} \vdash \Delta_9, \mathtt{p}_{10}}\, \rightarrow_L}{- : (\Delta_{12}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8}\, \wedge_L}{- : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{Cut}$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\, \wedge_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, \wedge_L}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : ((\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11} \vdash \Delta_9, \mathtt{p}_{10}}\, \wedge_L}{- : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8 \quad \mathtt{h}_5 : \mathtt{F}_7, \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8}\, \vee_L}{- : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{Cut}$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\, \vee_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, \vee_L}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \mathtt{F}_8, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : ((\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{11} \vdash \Delta_9, \mathtt{p}_{10}}\, \vee_L}{- : (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

- Case rule AT

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_8, []\mathtt{F}_6), \mathtt{p}_9 \vdash \Delta_7, \mathtt{p}_9}\, I \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_6, \Delta_8, \mathtt{p}_9, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_5 : ((\Delta_8, []\mathtt{F}_6), \mathtt{p}_9), \mathtt{p}_9 \vdash \Delta_7}\, AT}{- : (\Delta_8, []\mathtt{F}_6), \mathtt{p}_9 \vdash \Delta_7}\, \mathtt{Cut}$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7, \mathtt{p}_9}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \Delta_8, \mathtt{F}_6, \mathtt{p}_9, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7}\, \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7}\, \mathtt{hCut}$$
$$\dfrac{}{- : \Delta_8, \mathtt{p}_9, []\mathtt{F}_6 \vdash \Delta_7}\, ATG$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_9 \vdash (\Delta_8, \mathtt{p}_9), []\mathtt{F}_7}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \Delta_{10}, \mathtt{p}_9, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{p}_9}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_9), []\mathtt{F}_7 \vdash \Delta_8, \mathtt{p}_9}\, AT}{- : \Delta_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{11}, []\mathtt{F}_7), \mathtt{p}_9 \vdash (\Delta_8, \mathtt{p}_9), \mathtt{F}_{10}}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_{10}, \Delta_{11}, \mathtt{p}_9, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{p}_9}{\bullet \mathtt{h}_6 : ((\Delta_{11}, []\mathtt{F}_7), \mathtt{p}_9), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{p}_9}\, AT}{- : (\Delta_{11}, []\mathtt{F}_7), \mathtt{p}_9 \vdash \Delta_8, \mathtt{p}_9}\, \mathtt{Cut}$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_9, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{p}_9}\, I$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_7), \mathtt{p}_8 \vdash \Delta_6, \mathtt{p}_8}\, I \quad \dfrac{}{\bullet \mathtt{h}_5 : ((\bot, \Delta_7), \mathtt{p}_8), \mathtt{p}_8 \vdash \Delta_6}\, \bot_L}{- : (\bot, \Delta_7), \mathtt{p}_8 \vdash \Delta_6}\, \mathtt{Cut}$$
$$\dfrac{}{- : \bot, \Delta_7, \mathtt{p}_8 \vdash \Delta_6}\, \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \bot}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_8), \bot \vdash \Delta_7, \mathtt{p}_8}\, \bot_L}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_{10}), \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : ((\bot, \Delta_{10}), \mathtt{p}_8), \mathtt{F}_9 \vdash \Delta_7, \mathtt{p}_8}\, \bot_L}{- : (\bot, \Delta_{10}), \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}\, I \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_8, \mathtt{p}_7), \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\, I}{- : \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_8, \mathtt{p}_7), \mathtt{p}_9 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_5 : ((\Delta_8, \mathtt{p}_7), \mathtt{p}_9), \mathtt{p}_9 \vdash \Delta_6, \mathtt{p}_7}\, I}{- : (\Delta_8, \mathtt{p}_7), \mathtt{p}_9 \vdash \Delta_6, \mathtt{p}_7}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8, \mathtt{p}_7, \mathtt{p}_9 \vdash \Delta_6, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_8 \vdash ((\Delta_7, \mathtt{p}_9), \mathtt{p}_8), \mathtt{p}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_8), \mathtt{p}_9 \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\, I}{- : \Delta_{10}, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8, \mathtt{p}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_9), \mathtt{p}_8 \vdash ((\Delta_7, \mathtt{p}_9), \mathtt{p}_8), \mathtt{F}_{10}}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : ((\Delta_{11}, \mathtt{p}_9), \mathtt{p}_8), \mathtt{F}_{10} \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\, I}{- : (\Delta_{11}, \mathtt{p}_9), \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_8, \mathtt{p}_9 \vdash \Delta_7, \mathtt{p}_8, \mathtt{p}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_8), \mathtt{F}_9 \vdash \Delta_7, \mathtt{p}_8}\, I}{- : \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\top, \Delta_7), \mathtt{p}_8 \vdash \Delta_6, \mathtt{p}_8}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : ((\top, \Delta_7), \mathtt{p}_8), \mathtt{p}_8 \vdash \Delta_6}\, \top_L}{- : (\top, \Delta_7), \mathtt{p}_8 \vdash \Delta_6}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_7, \mathtt{p}_8 \vdash \Delta_6, \mathtt{p}_8}\, I \quad \dfrac{}{\mathtt{h}_5 : \top, \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \Delta_6}\, \mathtt{ax/W}}{- : \top, \Delta_7, \mathtt{p}_8 \vdash \Delta_6}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \top}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_8), \top \vdash \Delta_7, \mathtt{p}_8}\, \top_L}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\top, \Delta_{10}), \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \mathtt{F}_9}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_9, \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}{\bullet \mathtt{h}_6 : ((\top, \Delta_{10}), \mathtt{p}_8), \mathtt{F}_9 \vdash \Delta_7, \mathtt{p}_8}\, \top_L}{- : (\top, \Delta_{10}), \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \top, \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, I$$

8.14 Status of $\top_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{}\ \mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{}\ \mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \mathtt{F}_9, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{}\ \mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \bot, \Delta_6}{\bullet\mathtt{h}_1 : \top, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_7}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet\mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \bot, \Delta_6}\ \bot_R}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_8 \vdash \bot, \Delta_6, \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_6}{}\ \mathtt{ax/W}}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \top, \Delta_6}{\bullet\mathtt{h}_1 : \top, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_7}\ \top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \top, \Delta_6}\ \top_R}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \top, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \top_L \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_9, \Box\mathtt{F}_8 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\top, \Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \Delta_7, []\mathtt{F}_6}\ A4}{- : \top, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Box\mathtt{F}_8, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_6}{}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash \mathtt{F}_9, \Delta_7, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \top, \Box\Gamma_8, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \Box\Gamma_8 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\top, \Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_7, []\mathtt{F}_6}\ A4}{- : \top, \Box\Gamma_8, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_8 \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_8 \vdash \Delta_7, []\mathtt{F}_6}\ A4$$

- Case rule K

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{10} \vdash \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \top, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \Box\mathtt{F}_8}\ \top_L \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_9), unbox(\Box\mathtt{F}_8) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\top, \Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_8 \vdash \Delta_7, []\mathtt{F}_6}\ K}{- : \top, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Box\mathtt{F}_8, \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Box\mathtt{F}_8, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_8, \Delta_{10} \vdash \mathtt{F}_9, \Delta_7, []\mathtt{F}_6}{\bullet\mathtt{h}_1 : \top, \Box\Gamma_8, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_6), \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}{\bullet\mathtt{h}_5 : (\top, \Box\Gamma_8, \Delta_{10}), \mathtt{F}_9 \vdash \Delta_7, []\mathtt{F}_6}\ K}{- : \top, \Box\Gamma_8, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : unbox(\Box\Gamma_8) \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_8 \vdash \Delta_7, []\mathtt{F}_6}\ K}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \to \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9 \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\ \to_L}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_9, \Delta_{10} \vdash \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \to_L}{- : \top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{hCut}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \wedge \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \wedge_L}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\ \wedge_L}{- : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{hCut}}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \mathtt{F}_6 \vee \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \Delta_9 \vdash \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_7, \Delta_9 \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \vee_L}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_8}{\bullet \mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8 \quad \mathtt{h}_5 : \top, \mathtt{F}_7, \mathtt{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_9 \vdash \Delta_8}\,\vee_L}{- : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\,\mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\,\mathtt{hCut}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash []\mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_7, []\mathtt{F}_6}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \Delta_8, []\mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_5 : (\top, \Delta_8), []\mathtt{F}_6 \vdash \Delta_7}\,AT}{- : \top, \Delta_8 \vdash \Delta_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_7, []\mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_8, []\mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}}{- : \top, \Delta_8 \vdash \Delta_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, []\mathtt{F}_6 \vdash \mathtt{F}_8, \Delta_7}{\bullet \mathtt{h}_1 : \top, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7, \mathtt{F}_8}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_6, \mathtt{F}_8, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_5 : (\top, \Delta_9, []\mathtt{F}_6), \mathtt{F}_8 \vdash \Delta_7}\,AT}{- : \top, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_8, []\mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}}{- : \top, \Delta_9, []\mathtt{F}_6 \vdash \Delta_7}\,\mathtt{hCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_1 : \top, \Delta_7 \vdash \Delta_6, \bot}\,\top_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\top, \Delta_7), \bot \vdash \Delta_6}\,\bot_L}{- : \top, \Delta_7 \vdash \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_7 \vdash \bot, \Delta_6}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \bot, \top, \Delta_7 \vdash \Delta_6}\,\bot_L}{- : \top, \Delta_7 \vdash \Delta_6}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet \mathtt{h}_1 : \top, \bot, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\,\top_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\top, \bot, \Delta_8), \mathtt{F}_7 \vdash \Delta_6}\,\bot_L}{- : \top, \bot, \Delta_8 \vdash \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \top, \Delta_8 \vdash \Delta_6}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{p}_7, \Delta_6, \mathtt{p}_7}{\bullet \mathtt{h}_1 : \top, \Delta_8 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}\,\top_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\top, \Delta_8), \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,I}{- : \top, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_6, \mathtt{p}_7, \mathtt{p}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,I}{- : \top, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{p}_7 \vdash \mathtt{F}_8, \Delta_6, \mathtt{p}_7}{\bullet \mathtt{h}_1 : \top, \Delta_9, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{F}_8}\,\top_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\top, \Delta_9, \mathtt{p}_7), \mathtt{F}_8 \vdash \Delta_6, \mathtt{p}_7}\,I}{- : \top, \Delta_9, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \top, \Delta_9, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \mathtt{F}_7, \Delta_6}{\bullet \mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_5 : \mathtt{F}_7, \Delta_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\top, \Delta_8), \mathtt{F}_7 \vdash \Delta_6}\,\top_L}{- : \top, \Delta_8 \vdash \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_6, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_8, \mathtt{F}_7 \vdash \Delta_6}\,\mathtt{ax/W}}{- : \top, \Delta_8 \vdash \Delta_6}\,\mathtt{hCut}$$