# System for Linear Logic

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## Contents

# 1 Measure of derivations

- Case(s) rule **1**

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \to \quad \frac{}{\bullet\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1}$$

- Case(s) rule !

$$\frac{\mathtt{h}_1 :\vdash \mathtt{F}_2, ?\Gamma_3}{\bullet\mathtt{h}_1 :\vdash ?\Gamma_3, !\mathtt{F}_2}\ ! \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 :\vdash ?\Gamma_3, \mathtt{F}_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash ?\Gamma_3, \mathtt{F}_2}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 :\vdash ?\Gamma_3, !\mathtt{F}_2}\ !$$

- Case(s) rule $?W$

$$\frac{\mathtt{h}_1 :\vdash \Delta_3}{\bullet\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2}\ ?W \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 :\vdash \Delta_3}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_3}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2}\ ?W$$

- Case(s) rule $?C$

$$\frac{\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2, ?\mathtt{F}_2}{\bullet\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2}\ ?C \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2, ?\mathtt{F}_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2, ?\mathtt{F}_2}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2}\ ?C$$

- Case(s) rule ?

$$\frac{\mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_2}{\bullet\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2}\ ? \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_2}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2}\ ?$$

- Case(s) rule $

$$\frac{\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2\$\mathtt{F}_3}\ \$ \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2\$\mathtt{F}_3}\ \$$$

- Case(s) rule $\perp$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \perp, \Delta_2}\ \perp \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 :\vdash \Delta_2, \perp}\ \perp$$

- Case(s) rule $I$

$$\frac{}{\bullet\mathtt{h}_1 :\vdash p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I \quad \to \quad \frac{}{\bullet\bullet\mathtt{h}_1 :\vdash p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I$$

- Case(s) rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 :\vdash \Delta_5, \mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash \Delta_4, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_1 :\vdash \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_5, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet\mathtt{h}_1 :\vdash \Delta_4, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes$$

# 2   Weakening on bang: $\vdash \Gamma, !F$ implies $\Gamma, F$.

- Case(s) rule **1**

- Case(s) rule **!**

$$\frac{\mathtt{h_1} :\vdash \mathtt{F_2}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_2}, ?\Gamma_3} \; ! \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_1} :\vdash ?\Gamma_3, \mathtt{F_2}} \;\; \mathtt{ax}}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, \mathtt{F_2}} \; \mathtt{H}$$

- Case(s) rule $?W$

$$\frac{\mathtt{h_2} :\vdash \Delta_4, !\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_1}, \Delta_4, ?\mathtt{F_3}} \; ?W \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_2} :\vdash \Delta_4, !\mathtt{F_1}} \;\; \mathtt{ax}}{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}} \; \mathtt{IH}}{\bullet\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}, ?\mathtt{F_3}} \; ?W$$

- Case(s) rule $?C$

$$\frac{\mathtt{h_2} :\vdash \Delta_4, !\mathtt{F_1}, ?\mathtt{F_3}, ?\mathtt{F_3}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_1}, \Delta_4, ?\mathtt{F_3}} \; ?C \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_2} :\vdash \Delta_4, !\mathtt{F_1}, ?\mathtt{F_3}, ?\mathtt{F_3}} \;\; \mathtt{ax}}{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}, ?\mathtt{F_3}, ?\mathtt{F_3}} \; \mathtt{IH}}{\bullet\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}, ?\mathtt{F_3}} \; ?C$$

- Case(s) rule $?$

$$\frac{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_3}, !\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_1}, \Delta_4, ?\mathtt{F_3}} \; ? \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_3}, !\mathtt{F_1}} \;\; \mathtt{ax}}{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}, \mathtt{F_3}} \; \mathtt{IH}}{\bullet\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}, ?\mathtt{F_3}} \; ?$$

- Case(s) rule $\$$

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}, \mathtt{F_4}, !\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_1}, \Delta_5, \mathtt{F_3}\$\mathtt{F_4}} \; \$ \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}, \mathtt{F_4}, !\mathtt{F_1}} \;\; \mathtt{ax}}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_1}, \mathtt{F_3}, \mathtt{F_4}} \; \mathtt{IH}}{\bullet\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_1}, \mathtt{F_3}\$\mathtt{F_4}} \; \$$$

- Case(s) rule $\perp$

$$\frac{\mathtt{h_2} :\vdash \Delta_3, !\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_1}, \perp, \Delta_3} \; \perp \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_2} :\vdash \Delta_3, !\mathtt{F_1}} \;\; \mathtt{ax}}{\mathtt{h_2} :\vdash \Delta_3, \mathtt{F_1}} \; \mathtt{IH}}{\bullet\mathtt{h_2} :\vdash \Delta_3, \mathtt{F_1}, \perp} \; \perp$$

- Case(s) rule $I$

- Case(s) rule $\otimes$

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}, !\mathtt{F_1} \quad \mathtt{h_2} :\vdash \Delta_6, \mathtt{F_4}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_1}, \Delta_5, \Delta_6, \mathtt{F_3} \otimes \mathtt{F_4}} \; \otimes \qquad \rightarrow \qquad \frac{\dfrac{\overline{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}, !\mathtt{F_1}} \;\; \mathtt{ax}}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_1}, \mathtt{F_3}} \; \mathtt{IH} \quad \overline{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F_4}} \;\; \mathtt{ax}}{\bullet\mathtt{h_2} :\vdash \Delta_5, \Delta_6, \mathtt{F_1}, \mathtt{F_3} \otimes \mathtt{F_4}} \; \otimes$$

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3} \quad \mathtt{h_2} :\vdash \Delta_6, \mathtt{F_4}, !\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_1}, \Delta_5, \Delta_6, \mathtt{F_3} \otimes \mathtt{F_4}} \; \otimes \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}} \;\; \mathtt{ax} \quad \dfrac{\overline{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F_4}, !\mathtt{F_1}} \;\; \mathtt{ax}}{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F_1}, \mathtt{F_4}} \; \mathtt{IH}}{\bullet\mathtt{h_2} :\vdash \Delta_5, \Delta_6, \mathtt{F_1}, \mathtt{F_3} \otimes \mathtt{F_4}} \; \otimes$$

# 3 Invertibility of Rules

## 3.1 Status of 1: : Invertible

- Case rule **1**

$$\frac{}{\bullet h_1 \,:\vdash 1}\ \mathbf{1} \quad \rightarrow \quad \text{trivial}$$

- Case rule !

- Case rule $?W$

- Case rule $?C$

- Case rule ?

- Case rule $

- Case rule $\perp$

- Case rule $I$

- Case rule $\otimes$

## 3.2 Status of !: : Non invertible

- Case rule **1**

- Case rule !

$$\frac{h_1 \,:\vdash F_2, ?\Gamma_3}{\bullet h_1 \,:\vdash ?\Gamma_3, !F_2}\ ! \quad \rightarrow \quad \frac{\overline{h_1 \,:\vdash ?\Gamma_3, F_2}\ \mathrm{ax}}{\bullet h_1 \,:\vdash ?\Gamma_3, F_2}\ \mathrm{H}$$

- Case rule $?W$

$$\frac{h_2 \,:\vdash ?\Gamma_4, !F_1}{\bullet h_2 \,:\vdash (?\Gamma_4, !F_1), ?F_3}\ ?W \quad \rightarrow \quad \frac{\overline{h_2 \,:\vdash ?\Gamma_4, F_1}\ \mathrm{ax/ind}}{\bullet h_2 \,:\vdash ?\Gamma_4, F_1, ?F_3}\ ?W$$

- Case rule $?C$

$$\frac{h_2 \,:\vdash ?\Gamma_4, !F_1, ?F_3, ?F_3}{\bullet h_2 \,:\vdash (?\Gamma_4, !F_1), ?F_3}\ ?C \quad \rightarrow \quad \frac{\overline{h_2 \,:\vdash ?\Gamma_4, F_1, ?F_3, ?F_3}\ \mathrm{ax/ind}}{\bullet h_2 \,:\vdash ?\Gamma_4, F_1, ?F_3}\ ?C$$

- Case rule ?

$$\frac{\mathtt{h_2} : \vdash F_3, ?\Gamma_4, !F_1}{\bullet\mathtt{h_2} : \vdash (?\Gamma_4, !F_1), ?F_3} \; ? \quad \rightarrow \quad \overline{\bullet\mathtt{h_2} : \vdash F_1, ?\Gamma_4, ?F_3} \; \text{fail}$$

- Case rule $

- Case rule $\perp$

- Case rule $I$

- Case rule $\otimes$

## 3.3 Status of $?W$: : Non invertible

- Case rule $\mathbf{1}$

- Case rule !

$$\frac{\mathtt{h_2} : \vdash F_3, ?\Gamma_4, ?F_1}{\bullet\mathtt{h_2} : \vdash (?\Gamma_4, ?F_1), !F_3} \; ! \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} : \vdash ?\Gamma_4, F_3} \; \text{ax/ind}}{\bullet\mathtt{h_2} : \vdash ?\Gamma_4, !F_3} \; !$$

- Case rule $?W$

$$\frac{\mathtt{h_2} : \vdash \Delta_4, ?F_1}{\bullet\mathtt{h_2} : \vdash (\Delta_4, ?F_1), ?F_3} \; ?W \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} : \vdash \Delta_4} \; \text{ax/ind}}{\bullet\mathtt{h_2} : \vdash \Delta_4, ?F_3} \; ?W$$

$$\frac{\mathtt{h_1} : \vdash \Delta_3}{\bullet\mathtt{h_1} : \vdash \Delta_3, ?F_2} \; ?W \quad \rightarrow \quad \frac{\overline{\mathtt{h_1} : \vdash \Delta_3} \; \text{ax}}{\bullet\mathtt{h_1} : \vdash \Delta_3} \; \text{H}$$

- Case rule $?C$

$$\frac{\mathtt{h_2} : \vdash \Delta_4, ?F_1, ?F_3, ?F_3}{\bullet\mathtt{h_2} : \vdash (\Delta_4, ?F_1), ?F_3} \; ?C \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} : \vdash \Delta_4, ?F_3, ?F_3} \; \text{ax/ind}}{\bullet\mathtt{h_2} : \vdash \Delta_4, ?F_3} \; ?C$$

$$\frac{\mathtt{h_1} : \vdash \Delta_3, ?F_2, ?F_2}{\bullet\mathtt{h_1} : \vdash \Delta_3, ?F_2} \; ?C \quad \rightarrow \quad \overline{\bullet\mathtt{h_1} : \vdash \Delta_3} \; \text{fail}$$

- Case rule ?

$$\frac{\mathtt{h_2} : \vdash \Delta_4, F_3, ?F_1}{\bullet\mathtt{h_2} : \vdash (\Delta_4, ?F_1), ?F_3} \; ? \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} : \vdash \Delta_4, F_3} \; \text{ax/ind}}{\bullet\mathtt{h_2} : \vdash \Delta_4, ?F_3} \; ?$$

$$\frac{\mathtt{h_1} : \vdash \Delta_3, F_2}{\bullet\mathtt{h_1} : \vdash \Delta_3, ?F_2} \; ? \quad \rightarrow \quad \overline{\bullet\mathtt{h_1} : \vdash \Delta_3} \; \text{fail}$$

- Case rule $

$$\frac{\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_3, \texttt{F}_4, ?\texttt{F}_\mathbf{1}}{\bullet\texttt{h}_2 :\vdash (\Delta_5, ?\texttt{F}_\mathbf{1}), \texttt{F}_3\,\$\texttt{F}_4}\ \$ \quad\to\quad \frac{\dfrac{}{\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_3, \texttt{F}_4}\ \texttt{ax/ind}}{\bullet\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_3\,\$\texttt{F}_4}\ \$$$

- Case rule $\perp$

$$\frac{\texttt{h}_2 :\vdash \Delta_3, ?\texttt{F}_\mathbf{1}}{\bullet\texttt{h}_2 :\vdash \perp, \Delta_3, ?\texttt{F}_\mathbf{1}}\ \perp \quad\to\quad \frac{\dfrac{}{\texttt{h}_2 :\vdash \Delta_3}\ \texttt{ax/ind}}{\bullet\texttt{h}_2 :\vdash \Delta_3, \perp}\ \perp$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_3, ?\texttt{F}_\mathbf{1} \quad \texttt{h}_2 :\vdash \Delta_6, \texttt{F}_4}{\bullet\texttt{h}_2 :\vdash (\Delta_5, ?\texttt{F}_\mathbf{1}), \Delta_6, \texttt{F}_3 \otimes \texttt{F}_4}\ \otimes \quad\to\quad \frac{\dfrac{}{\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_3}\ \texttt{ax/ind} \quad \dfrac{}{\texttt{h}_2 :\vdash \Delta_6, \texttt{F}_4}\ \texttt{ax}}{\bullet\texttt{h}_2 :\vdash \Delta_5, \Delta_6, \texttt{F}_3 \otimes \texttt{F}_4}\ \otimes$$

$$\frac{\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_3 \quad \texttt{h}_2 :\vdash \Delta_6, \texttt{F}_4, ?\texttt{F}_\mathbf{1}}{\bullet\texttt{h}_2 :\vdash \Delta_5, (\Delta_6, ?\texttt{F}_\mathbf{1}), \texttt{F}_3 \otimes \texttt{F}_4}\ \otimes \quad\to\quad \frac{\dfrac{}{\texttt{h}_2 :\vdash \Delta_5, \texttt{F}_3}\ \texttt{ax} \quad \dfrac{}{\texttt{h}_2 :\vdash \Delta_6, \texttt{F}_4}\ \texttt{ax/ind}}{\bullet\texttt{h}_2 :\vdash \Delta_5, \Delta_6, \texttt{F}_3 \otimes \texttt{F}_4}\ \otimes$$

## 3.4   Status of $?C$:  :   Non invertible

- Case rule $\mathbf{1}$

- Case rule !

$$\frac{\texttt{h}_2 :\vdash \texttt{F}_3, ?\Gamma_4, ?\texttt{F}_\mathbf{1}}{\bullet\texttt{h}_2 :\vdash (?\Gamma_4, ?\texttt{F}_\mathbf{1}), !\texttt{F}_3}\ ! \quad\to\quad \frac{\dfrac{}{\texttt{h}_2 :\vdash ?\Gamma_4, \texttt{F}_3, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}}\ \texttt{ax/ind}}{\bullet\texttt{h}_2 :\vdash ?\Gamma_4, !\texttt{F}_3, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}}\ !$$

- Case rule $?W$

$$\frac{\texttt{h}_2 :\vdash \Delta_4, ?\texttt{F}_\mathbf{1}}{\bullet\texttt{h}_2 :\vdash (\Delta_4, ?\texttt{F}_\mathbf{1}), ?\texttt{F}_3}\ ?W \quad\to\quad \frac{\dfrac{}{\texttt{h}_2 :\vdash \Delta_4, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}}\ \texttt{ax/ind}}{\bullet\texttt{h}_2 :\vdash \Delta_4, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_3}\ ?W$$

$$\frac{\texttt{h}_1 :\vdash \Delta_3}{\bullet\texttt{h}_1 :\vdash \Delta_3, ?\texttt{F}_2}\ ?W \quad\to\quad \color{red}{\frac{}{\bullet\texttt{h}_1 :\vdash \Delta_3, ?\texttt{F}_2, ?\texttt{F}_2}\ \texttt{fail}}$$

- Case rule $?C$

$$\frac{\texttt{h}_2 :\vdash \Delta_4, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_3, ?\texttt{F}_3}{\bullet\texttt{h}_2 :\vdash (\Delta_4, ?\texttt{F}_\mathbf{1}), ?\texttt{F}_3}\ ?C \quad\to\quad \frac{\dfrac{}{\texttt{h}_2 :\vdash \Delta_4, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_3, ?\texttt{F}_3}\ \texttt{ax/ind}}{\bullet\texttt{h}_2 :\vdash \Delta_4, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_3}\ ?C$$

$$\frac{\texttt{h}_1 :\vdash \Delta_3, ?\texttt{F}_2, ?\texttt{F}_2}{\bullet\texttt{h}_1 :\vdash \Delta_3, ?\texttt{F}_2}\ ?C \quad\to\quad \frac{\dfrac{}{\texttt{h}_1 :\vdash \Delta_3, ?\texttt{F}_2, ?\texttt{F}_2, ?\texttt{F}_2}\ \texttt{ax/ind}}{\bullet\texttt{h}_1 :\vdash \Delta_3, ?\texttt{F}_2, ?\texttt{F}_2}\ ?C$$

- Case rule ?

$$\dfrac{\texttt{h}_2 \; :\vdash \Delta_4, \texttt{F}_3, ?\texttt{F}_\mathbf{1}}{\bullet \texttt{h}_2 \; :\vdash (\Delta_4, ?\texttt{F}_\mathbf{1}), ?\texttt{F}_3} \; ? \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_2 \; :\vdash \Delta_4, \texttt{F}_3, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}} \; \texttt{ax/ind}}{\bullet \texttt{h}_2 \; :\vdash \Delta_4, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_3} \; ?$$

$$\dfrac{\texttt{h}_1 \; :\vdash \Delta_3, \texttt{F}_2}{\bullet \texttt{h}_1 \; :\vdash \Delta_3, ?\texttt{F}_2} \; ? \qquad \rightarrow \qquad \dfrac{}{\textcolor{red}{\bullet \texttt{h}_1 \; :\vdash \Delta_3, ?\texttt{F}_2, ?\texttt{F}_2}} \; \texttt{fail}$$

- Case rule \$

$$\dfrac{\texttt{h}_2 \; :\vdash \Delta_5, \texttt{F}_3, \texttt{F}_4, ?\texttt{F}_\mathbf{1}}{\bullet \texttt{h}_2 \; :\vdash (\Delta_5, ?\texttt{F}_\mathbf{1}), \texttt{F}_3 \$ \texttt{F}_4} \; \$ \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_2 \; :\vdash \Delta_5, \texttt{F}_3, \texttt{F}_4, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}} \; \texttt{ax/ind}}{\bullet \texttt{h}_2 \; :\vdash \Delta_5, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}, \texttt{F}_3 \$ \texttt{F}_4} \; \$$$

- Case rule $\perp$

$$\dfrac{\texttt{h}_2 \; :\vdash \Delta_3, ?\texttt{F}_\mathbf{1}}{\bullet \texttt{h}_2 \; :\vdash \perp, \Delta_3, ?\texttt{F}_\mathbf{1}} \; \perp \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_2 \; :\vdash \Delta_3, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}} \; \texttt{ax/ind}}{\bullet \texttt{h}_2 \; :\vdash \Delta_3, \perp, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}} \; \perp$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\texttt{h}_2 \; :\vdash \Delta_5, \texttt{F}_3, ?\texttt{F}_\mathbf{1} \quad \texttt{h}_2 \; :\vdash \Delta_6, \texttt{F}_4}{\bullet \texttt{h}_2 \; :\vdash (\Delta_5, ?\texttt{F}_\mathbf{1}), \Delta_6, \texttt{F}_3 \otimes \texttt{F}_4} \; \otimes \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_2 \; :\vdash \Delta_5, \texttt{F}_3, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}} \; \texttt{ax/ind} \quad \overline{\texttt{h}_2 \; :\vdash \Delta_6, \texttt{F}_4} \; \texttt{ax}}{\bullet \texttt{h}_2 \; :\vdash \Delta_5, \Delta_6, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}, \texttt{F}_3 \otimes \texttt{F}_4} \; \otimes$$

$$\dfrac{\texttt{h}_2 \; :\vdash \Delta_5, \texttt{F}_3 \quad \texttt{h}_2 \; :\vdash \Delta_6, \texttt{F}_4, ?\texttt{F}_\mathbf{1}}{\bullet \texttt{h}_2 \; :\vdash \Delta_5, (\Delta_6, ?\texttt{F}_\mathbf{1}), \texttt{F}_3 \otimes \texttt{F}_4} \; \otimes \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_2 \; :\vdash \Delta_5, \texttt{F}_3} \; \texttt{ax} \quad \overline{\texttt{h}_2 \; :\vdash \Delta_6, \texttt{F}_4, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}} \; \texttt{ax/ind}}{\bullet \texttt{h}_2 \; :\vdash \Delta_5, \Delta_6, ?\texttt{F}_\mathbf{1}, ?\texttt{F}_\mathbf{1}, \texttt{F}_3 \otimes \texttt{F}_4} \; \otimes$$

## 3.5 Status of ?: : Non invertible

- Case rule $\mathbf{1}$

- Case rule !

$$\dfrac{\texttt{h}_2 \; :\vdash \texttt{F}_3, ?\Gamma_4, ?\texttt{F}_\mathbf{1}}{\bullet \texttt{h}_2 \; :\vdash (?\Gamma_4, ?\texttt{F}_\mathbf{1}), !\texttt{F}_3} \; ! \qquad \rightarrow \qquad \dfrac{}{\textcolor{red}{\bullet \texttt{h}_2 \; :\vdash \texttt{F}_1, ?\Gamma_4, !\texttt{F}_3}} \; \texttt{fail}$$

- Case rule $?W$

$$\dfrac{\texttt{h}_2 \; :\vdash \Delta_4, ?\texttt{F}_\mathbf{1}}{\bullet \texttt{h}_2 \; :\vdash (\Delta_4, ?\texttt{F}_\mathbf{1}), ?\texttt{F}_3} \; ?W \qquad \rightarrow \qquad \dfrac{\overline{\texttt{h}_2 \; :\vdash \Delta_4, \texttt{F}_\mathbf{1}} \; \texttt{ax/ind}}{\bullet \texttt{h}_2 \; :\vdash \Delta_4, \texttt{F}_\mathbf{1}, ?\texttt{F}_3} \; ?W$$

$$\dfrac{\texttt{h}_1 \; :\vdash \Delta_3}{\bullet \texttt{h}_1 \; :\vdash \Delta_3, ?\texttt{F}_2} \; ?W \qquad \rightarrow \qquad \dfrac{}{\textcolor{red}{\bullet \texttt{h}_1 \; :\vdash \Delta_3, \texttt{F}_2}} \; \texttt{fail}$$

- Case rule ?$C$

$$\frac{\mathtt{h_2} :\vdash \Delta_4, ?\mathtt{F_1}, ?\mathtt{F_3}, ?\mathtt{F_3}}{\bullet\mathtt{h_2} :\vdash (\Delta_4, ?\mathtt{F_1}), ?\mathtt{F_3}} \; ?C \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}, ?\mathtt{F_3}, ?\mathtt{F_3}} \; \mathtt{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}, ?\mathtt{F_3}} \; ?C$$

$$\frac{\mathtt{h_1} :\vdash \Delta_3, ?\mathtt{F_2}, ?\mathtt{F_2}}{\bullet\mathtt{h_1} :\vdash \Delta_3, ?\mathtt{F_2}} \; ?C \qquad \rightarrow \qquad \frac{}{\color{red}{\bullet\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}}} \; \mathtt{fail}$$

- Case rule ?

$$\frac{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_3}, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash (\Delta_4, ?\mathtt{F_1}), ?\mathtt{F_3}} \; ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}, \mathtt{F_3}} \; \mathtt{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_4, \mathtt{F_1}, ?\mathtt{F_3}} \; ?$$

$$\frac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}}{\bullet\mathtt{h_1} :\vdash \Delta_3, ?\mathtt{F_2}} \; ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}} \; \mathtt{ax}}{\bullet\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}} \; \mathtt{H}$$

- Case rule $

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}, \mathtt{F_4}, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash (\Delta_5, ?\mathtt{F_1}), \mathtt{F_3}\$\mathtt{F_4}} \; \$ \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_1}, \mathtt{F_3}, \mathtt{F_4}} \; \mathtt{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_1}, \mathtt{F_3}\$\mathtt{F_4}} \; \$$$

- Case rule $\perp$

$$\frac{\mathtt{h_2} :\vdash \Delta_3, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash \perp, \Delta_3, ?\mathtt{F_1}} \; \perp \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_2} :\vdash \Delta_3, \mathtt{F_1}} \; \mathtt{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_3, \mathtt{F_1}, \perp} \; \perp$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}, ?\mathtt{F_1} \quad \mathtt{h_2} :\vdash \Delta_6, \mathtt{F_4}}{\bullet\mathtt{h_2} :\vdash (\Delta_5, ?\mathtt{F_1}), \Delta_6, \mathtt{F_3} \otimes \mathtt{F_4}} \; \otimes \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_1}, \mathtt{F_3}} \; \mathtt{ax/ind} \quad \overline{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F_4}} \; \mathtt{ax}}{\bullet\mathtt{h_2} :\vdash \Delta_5, \Delta_6, \mathtt{F_1}, \mathtt{F_3} \otimes \mathtt{F_4}} \; \otimes$$

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3} \quad \mathtt{h_2} :\vdash \Delta_6, \mathtt{F_4}, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash \Delta_5, (\Delta_6, ?\mathtt{F_1}), \mathtt{F_3} \otimes \mathtt{F_4}} \; \otimes \quad \rightarrow \quad \frac{\overline{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F_3}} \; \mathtt{ax} \quad \overline{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F_1}, \mathtt{F_4}} \; \mathtt{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_5, \Delta_6, \mathtt{F_1}, \mathtt{F_3} \otimes \mathtt{F_4}} \; \otimes$$

## 3.6   Status of $: :   Invertible

- Case rule $\mathbf{1}$

- Case rule !

- Case rule ?$W$

$$\frac{\mathtt{h_3} :\vdash \Delta_5, \mathtt{F_1}\$\mathtt{F_2}}{\bullet\mathtt{h_3} :\vdash (\Delta_5, \mathtt{F_1}\$\mathtt{F_2}), ?\mathtt{F_4}} \; ?W \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} :\vdash \Delta_5, \mathtt{F_1}, \mathtt{F_2}} \; \mathtt{ax/ind}}{\bullet\mathtt{h_3} :\vdash \Delta_5, \mathtt{F_1}, \mathtt{F_2}, ?\mathtt{F_4}} \; ?W$$

- Case rule $?C$

$$\frac{h_3 :\vdash \Delta_5, ?F_4, ?F_4, F_1\$F_2}{\bullet h_3 :\vdash (\Delta_5, F_1\$F_2), ?F_4} \; ?C \quad \rightarrow \quad \frac{\dfrac{}{\overline{h_3 :\vdash \Delta_5, F_1, F_2, ?F_4, ?F_4}}\; ax/ind}{\bullet h_3 :\vdash \Delta_5, F_1, F_2, ?F_4}\; ?C$$

- Case rule $?$

$$\frac{h_3 :\vdash \Delta_5, F_4, F_1\$F_2}{\bullet h_3 :\vdash (\Delta_5, F_1\$F_2), ?F_4} \; ? \quad \rightarrow \quad \frac{\dfrac{}{\overline{h_3 :\vdash \Delta_5, F_1, F_2, F_4}}\; ax/ind}{\bullet h_3 :\vdash \Delta_5, F_1, F_2, ?F_4}\; ?$$

- Case rule $\$$

$$\frac{h_3 :\vdash \Delta_6, F_4, F_5, F_1\$F_2}{\bullet h_3 :\vdash (\Delta_6, F_1\$F_2), F_4\$F_5} \; \$ \quad \rightarrow \quad \frac{\dfrac{}{\overline{h_3 :\vdash \Delta_6, F_1, F_2, F_4, F_5}}\; ax/ind}{\bullet h_3 :\vdash \Delta_6, F_1, F_2, F_4\$F_5}\; \$$$

$$\frac{h_1 :\vdash \Delta_4, F_2, F_3}{\bullet h_1 :\vdash \Delta_4, F_2\$F_3} \; \$ \quad \rightarrow \quad \frac{\dfrac{}{\overline{h_1 :\vdash \Delta_4, F_2, F_3}}\; ax}{\bullet h_1 :\vdash \Delta_4, F_2, F_3}\; H$$

- Case rule $\bot$

$$\frac{h_3 :\vdash \Delta_4, F_1\$F_2}{\bullet h_3 :\vdash \bot, \Delta_4, F_1\$F_2} \; \bot \quad \rightarrow \quad \frac{\dfrac{}{\overline{h_3 :\vdash \Delta_4, F_1, F_2}}\; ax/ind}{\bullet h_3 :\vdash \Delta_4, F_1, F_2, \bot}\; \bot$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{h_3 :\vdash \Delta_6, F_4, F_1\$F_2 \quad h_3 :\vdash \Delta_7, F_5}{\bullet h_3 :\vdash (\Delta_6, F_1\$F_2), \Delta_7, F_4 \otimes F_5} \; \otimes \quad \rightarrow \quad \frac{\dfrac{}{\overline{h_3 :\vdash \Delta_6, F_1, F_2, F_4}}\; ax/ind \quad \dfrac{}{\overline{h_3 :\vdash \Delta_7, F_5}}\; ax}{\bullet h_3 :\vdash \Delta_6, \Delta_7, F_1, F_2, F_4 \otimes F_5}\; \otimes$$

$$\frac{h_3 :\vdash \Delta_6, F_4 \quad h_3 :\vdash \Delta_7, F_5, F_1\$F_2}{\bullet h_3 :\vdash \Delta_6, (\Delta_7, F_1\$F_2), F_4 \otimes F_5} \; \otimes \quad \rightarrow \quad \frac{\dfrac{}{\overline{h_3 :\vdash \Delta_6, F_4}}\; ax \quad \dfrac{}{\overline{h_3 :\vdash \Delta_7, F_1, F_2, F_5}}\; ax/ind}{\bullet h_3 :\vdash \Delta_6, \Delta_7, F_1, F_2, F_4 \otimes F_5}\; \otimes$$

## 3.7 Status of $\bot$: : Invertible

- Case rule $\mathbf{1}$

- Case rule $!$

- Case rule $?W$

$$\frac{h_1 :\vdash \bot, \Delta_3}{\bullet h_1 :\vdash (\bot, \Delta_3), ?F_2} \; ?W \quad \rightarrow \quad \frac{\dfrac{}{\overline{h_1 :\vdash \Delta_3}}\; ax/ind}{\bullet h_1 :\vdash \Delta_3, ?F_2}\; ?W$$

- Case rule $?C$

$$\frac{h_1 :\vdash \bot, \Delta_3, ?F_2, ?F_2}{\bullet h_1 :\vdash (\bot, \Delta_3), ?F_2} \; ?C \quad \rightarrow \quad \frac{\dfrac{}{\overline{h_1 :\vdash \Delta_3, ?F_2, ?F_2}}\; ax/ind}{\bullet h_1 :\vdash \Delta_3, ?F_2}\; ?C$$

- Case rule ?

$$\frac{\mathtt{h_1} :\vdash \perp, \Delta_3, \mathtt{F_2}}{\bullet\mathtt{h_1} :\vdash (\perp, \Delta_3), ?\mathtt{F_2}}\ ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}}\ \mathtt{ax/ind}}{\bullet\mathtt{h_1} :\vdash \Delta_3, ?\mathtt{F_2}}\ ?$$

- Case rule \$

$$\frac{\mathtt{h_1} :\vdash \perp, \Delta_4, \mathtt{F_2}, \mathtt{F_3}}{\bullet\mathtt{h_1} :\vdash (\perp, \Delta_4), \mathtt{F_2}\$\mathtt{F_3}}\ \$ \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_3}}\ \mathtt{ax/ind}}{\bullet\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}\$\mathtt{F_3}}\ \$$$

- Case rule $\perp$

$$\frac{\mathtt{h_1} :\vdash \Delta_2}{\bullet\mathtt{h_1} :\vdash \perp, \Delta_2}\ \perp \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_2}\ \mathtt{ax}}{\bullet\mathtt{h_1} :\vdash \Delta_2}\ \mathtt{H}$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h_1} :\vdash \perp, \Delta_4, \mathtt{F_2} \quad \mathtt{h_1} :\vdash \Delta_5, \mathtt{F_3}}{\bullet\mathtt{h_1} :\vdash (\perp, \Delta_4), \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}}\ \otimes \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}}\ \mathtt{ax/ind} \quad \overline{\mathtt{h_1} :\vdash \Delta_5, \mathtt{F_3}}\ \mathtt{ax}}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}}\ \otimes$$

$$\frac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2} \quad \mathtt{h_1} :\vdash \perp, \Delta_5, \mathtt{F_3}}{\bullet\mathtt{h_1} :\vdash \Delta_4, (\perp, \Delta_5), \mathtt{F_2} \otimes \mathtt{F_3}}\ \otimes \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}}\ \mathtt{ax} \quad \overline{\mathtt{h_1} :\vdash \Delta_5, \mathtt{F_3}}\ \mathtt{ax/ind}}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}}\ \otimes$$

## 3.8 Status of $I$: : Invertible

- Case rule $\mathbf{1}$

- Case rule !

- Case rule $?W$

- Case rule $?C$

- Case rule ?

- Case rule \$

- Case rule $\perp$

- Case rule $I$

$$\frac{}{\bullet\mathtt{h_1} :\vdash p(\mathtt{n_2}), \hat{}(\mathtt{n_2})}\ I \qquad \rightarrow \qquad \mathtt{trivial}$$

- Case rule $\otimes$

## 3.9 Status of ⊗: (Left Premise): `Non invertible`

- Case rule **1**


- Case rule **!**


- Case rule $?W$


$$\frac{\mathtt{h_3} : \vdash \Delta_5, \Delta_6, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash (\Delta_5, \Delta_6, \mathtt{F_1} \otimes \mathtt{F_2}), ?\mathtt{F_4}} \, ?W \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}, ?\mathtt{F_4}} \, ?W$$

$$\frac{\mathtt{h_3} : \vdash \Delta_5, \Delta_6, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash (\Delta_5, \Delta_6, \mathtt{F_1} \otimes \mathtt{F_2}), ?\mathtt{F_4}} \, ?W \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}} \, {}^{H}$$

- Case rule $?C$


$$\frac{\mathtt{h_3} : \vdash \Delta_5, \Delta_6, ?\mathtt{F_4}, ?\mathtt{F_4}, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash (\Delta_5, \Delta_6, \mathtt{F_1} \otimes \mathtt{F_2}), ?\mathtt{F_4}} \, ?C \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}, ?\mathtt{F_4}} \, ?W$$

$$\frac{\mathtt{h_3} : \vdash \Delta_5, \Delta_6, ?\mathtt{F_4}, ?\mathtt{F_4}, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash (\Delta_5, \Delta_6, \mathtt{F_1} \otimes \mathtt{F_2}), ?\mathtt{F_4}} \, ?C \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}} \, {}^{H}$$

- Case rule ?


$$\frac{\mathtt{h_3} : \vdash \Delta_5, \Delta_6, \mathtt{F_4}, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash (\Delta_5, \Delta_6, \mathtt{F_1} \otimes \mathtt{F_2}), ?\mathtt{F_4}} \, ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}, ?\mathtt{F_4}} \, ?W$$

$$\frac{\mathtt{h_3} : \vdash \Delta_5, \Delta_6, \mathtt{F_4}, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash (\Delta_5, \Delta_6, \mathtt{F_1} \otimes \mathtt{F_2}), ?\mathtt{F_4}} \, ? \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_5, \mathtt{F_1}} \, {}^{H}$$

- Case rule $

$$\frac{\mathtt{h_3} : \vdash \Delta_6, \Delta_7, \mathtt{F_4}, \mathtt{F_5}, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash (\Delta_6, \Delta_7, \mathtt{F_1} \otimes \mathtt{F_2}), \mathtt{F_4}\$\mathtt{F_5}} \, \$ \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_6, \mathtt{F_1}, \mathtt{F_4}, \mathtt{F_5}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_6, \mathtt{F_1}, \mathtt{F_4}\$\mathtt{F_5}} \, \$$$

$$\frac{\mathtt{h_3} : \vdash \Delta_6, \Delta_7, \mathtt{F_4}, \mathtt{F_5}, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash (\Delta_6, \Delta_7, \mathtt{F_1} \otimes \mathtt{F_2}), \mathtt{F_4}\$\mathtt{F_5}} \, \$ \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_6, \mathtt{F_1}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_6, \mathtt{F_1}} \, {}^{H}$$

- Case rule ⊥


$$\frac{\mathtt{h_3} : \vdash \Delta_4, \Delta_5, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash \perp, \Delta_4, \Delta_5, \mathtt{F_1} \otimes \mathtt{F_2}} \, \perp \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_4, \mathtt{F_1}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_4, \mathtt{F_1}, \perp} \, \perp$$

$$\frac{\mathtt{h_3} : \vdash \Delta_4, \Delta_5, \mathtt{F_1} \otimes \mathtt{F_2}}{\bullet \mathtt{h_3} : \vdash \perp, \Delta_4, \Delta_5, \mathtt{F_1} \otimes \mathtt{F_2}} \, \perp \qquad \rightarrow \qquad \frac{\overline{\mathtt{h_3} : \vdash \Delta_4, \mathtt{F_1}} \; {}^{ax/ind}}{\bullet \mathtt{h_3} : \vdash \Delta_4, \mathtt{F_1}} \, {}^{H}$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{h_3 :\vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 :\vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 :\vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \quad \rightarrow \quad \dfrac{}{\color{red}\bullet h_3 :\vdash \Delta_6, \Delta_8, F_1, F_4 \otimes F_5} \text{ fail}$$

$$\dfrac{h_3 :\vdash \Delta_6, \Delta_7, F_4 \quad h_3 :\vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \quad \rightarrow \quad \dfrac{}{\color{red}\bullet h_3 :\vdash \Delta_6, \Delta_8, F_1, F_4 \otimes F_5} \text{ fail}$$

$$\dfrac{h_3 :\vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 :\vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 :\vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \quad \rightarrow \quad \dfrac{}{\color{red}\bullet h_3 :\vdash \Delta_6, \Delta_8, F_1} \text{ fail}$$

$$\dfrac{h_3 :\vdash \Delta_6, \Delta_7, F_4 \quad h_3 :\vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \quad \rightarrow \quad \dfrac{}{\color{red}\bullet h_3 :\vdash \Delta_6, \Delta_8, F_1} \text{ fail}$$

$$\dfrac{h_1 :\vdash \Delta_4, \Delta_5, F_2 \quad h_1 :\vdash \Delta_6, \Delta_7, F_3}{\bullet h_1 :\vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), F_2 \otimes F_3} \otimes \quad \rightarrow \quad \dfrac{}{\color{red}\bullet h_1 :\vdash \Delta_4, \Delta_6, F_2} \text{ fail}$$

## 3.10   Status of $\otimes$ (Right Premise): :   Non invertible

- Case rule $\mathbf{1}$


- Case rule $!$


- Case rule $?W$

$$\dfrac{h_3 :\vdash \Delta_5, \Delta_6, F_1 \otimes F_2}{\bullet h_3 :\vdash (\Delta_5, \Delta_6, F_1 \otimes F_2), ?F_4} \ ?W \quad \rightarrow \quad \dfrac{\dfrac{}{h_3 :\vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_2} \text{ H}$$

$$\dfrac{h_3 :\vdash \Delta_5, \Delta_6, F_1 \otimes F_2}{\bullet h_3 :\vdash (\Delta_5, \Delta_6, F_1 \otimes F_2), ?F_4} \ ?W \quad \rightarrow \quad \dfrac{\dfrac{}{h_3 :\vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_2, ?F_4} \ ?W$$

- Case rule $?C$

$$\dfrac{h_3 :\vdash \Delta_5, \Delta_6, ?F_4, ?F_4, F_1 \otimes F_2}{\bullet h_3 :\vdash (\Delta_5, \Delta_6, F_1 \otimes F_2), ?F_4} \ ?C \quad \rightarrow \quad \dfrac{\dfrac{}{h_3 :\vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_2} \text{ H}$$

$$\dfrac{h_3 :\vdash \Delta_5, \Delta_6, ?F_4, ?F_4, F_1 \otimes F_2}{\bullet h_3 :\vdash (\Delta_5, \Delta_6, F_1 \otimes F_2), ?F_4} \ ?C \quad \rightarrow \quad \dfrac{\dfrac{}{h_3 :\vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_2, ?F_4} \ ?W$$

- Case rule $?$

$$\dfrac{h_3 :\vdash \Delta_5, \Delta_6, F_4, F_1 \otimes F_2}{\bullet h_3 :\vdash (\Delta_5, \Delta_6, F_1 \otimes F_2), ?F_4} \ ? \quad \rightarrow \quad \dfrac{\dfrac{}{h_3 :\vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_2} \text{ H}$$

$$\dfrac{h_3 :\vdash \Delta_5, \Delta_6, F_4, F_1 \otimes F_2}{\bullet h_3 :\vdash (\Delta_5, \Delta_6, F_1 \otimes F_2), ?F_4} \ ? \quad \rightarrow \quad \dfrac{\dfrac{}{h_3 :\vdash \Delta_6, F_2} \text{ ax/ind}}{\bullet h_3 :\vdash \Delta_6, F_2, ?F_4} \ ?W$$

- Case rule $

$$\frac{\text{h}_3 : \vdash \Delta_6, \Delta_7, \text{F}_4, \text{F}_5, \text{F}_\mathbf{1} \otimes \text{F}_2}{\bullet\text{h}_3 : \vdash (\Delta_6, \Delta_7, \text{F}_\mathbf{1} \otimes \text{F}_2), \text{F}_4\,\$\text{F}_5} \;\$ \qquad \rightarrow \qquad \frac{\overline{\text{h}_3 : \vdash \Delta_7, \text{F}_2}\;^{\text{ax/ind}}}{\bullet\text{h}_3 : \vdash \Delta_7, \text{F}_2}\;^{\text{H}}$$

$$\frac{\text{h}_3 : \vdash \Delta_6, \Delta_7, \text{F}_4, \text{F}_5, \text{F}_\mathbf{1} \otimes \text{F}_2}{\bullet\text{h}_3 : \vdash (\Delta_6, \Delta_7, \text{F}_\mathbf{1} \otimes \text{F}_2), \text{F}_4\,\$\text{F}_5} \;\$ \qquad \rightarrow \qquad \frac{\overline{\text{h}_3 : \vdash \Delta_7, \text{F}_2, \text{F}_4, \text{F}_5}\;^{\text{ax/ind}}}{\bullet\text{h}_3 : \vdash \Delta_7, \text{F}_2, \text{F}_4\,\$\text{F}_5}\;^{\$}$$

- Case rule $\perp$

$$\frac{\text{h}_3 : \vdash \Delta_4, \Delta_5, \text{F}_\mathbf{1} \otimes \text{F}_2}{\bullet\text{h}_3 : \vdash \perp, \Delta_4, \Delta_5, \text{F}_\mathbf{1} \otimes \text{F}_2} \;\perp \qquad \rightarrow \qquad \frac{\overline{\text{h}_3 : \vdash \Delta_5, \text{F}_2}\;^{\text{ax/ind}}}{\bullet\text{h}_3 : \vdash \Delta_5, \text{F}_2}\;^{\text{H}}$$

$$\frac{\text{h}_3 : \vdash \Delta_4, \Delta_5, \text{F}_\mathbf{1} \otimes \text{F}_2}{\bullet\text{h}_3 : \vdash \perp, \Delta_4, \Delta_5, \text{F}_\mathbf{1} \otimes \text{F}_2} \;\perp \qquad \rightarrow \qquad \frac{\overline{\text{h}_3 : \vdash \Delta_5, \text{F}_2}\;^{\text{ax/ind}}}{\bullet\text{h}_3 : \vdash \Delta_5, \text{F}_2, \perp}\;^{\perp}$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\text{h}_3 : \vdash \Delta_6, \Delta_7, \text{F}_4, \text{F}_\mathbf{1} \otimes \text{F}_2 \quad \text{h}_3 : \vdash \Delta_8, \Delta_9, \text{F}_5}{\bullet\text{h}_3 : \vdash (\Delta_6, \Delta_7, \text{F}_\mathbf{1} \otimes \text{F}_2), (\Delta_8, \Delta_9), \text{F}_4 \otimes \text{F}_5} \;\otimes \qquad \rightarrow \qquad {\color{red}\frac{}{\bullet\text{h}_3 : \vdash \Delta_7, \Delta_9, \text{F}_2}\;^{\text{fail}}}$$

$$\frac{\text{h}_3 : \vdash \Delta_6, \Delta_7, \text{F}_4 \quad \text{h}_3 : \vdash \Delta_8, \Delta_9, \text{F}_5, \text{F}_\mathbf{1} \otimes \text{F}_2}{\bullet\text{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \text{F}_\mathbf{1} \otimes \text{F}_2), \text{F}_4 \otimes \text{F}_5} \;\otimes \qquad \rightarrow \qquad {\color{red}\frac{}{\bullet\text{h}_3 : \vdash \Delta_7, \Delta_9, \text{F}_2}\;^{\text{fail}}}$$

$$\frac{\text{h}_3 : \vdash \Delta_6, \Delta_7, \text{F}_4, \text{F}_\mathbf{1} \otimes \text{F}_2 \quad \text{h}_3 : \vdash \Delta_8, \Delta_9, \text{F}_5}{\bullet\text{h}_3 : \vdash (\Delta_6, \Delta_7, \text{F}_\mathbf{1} \otimes \text{F}_2), (\Delta_8, \Delta_9), \text{F}_4 \otimes \text{F}_5} \;\otimes \qquad \rightarrow \qquad {\color{red}\frac{}{\bullet\text{h}_3 : \vdash \Delta_7, \Delta_9, \text{F}_2, \text{F}_4 \otimes \text{F}_5}\;^{\text{fail}}}$$

$$\frac{\text{h}_3 : \vdash \Delta_6, \Delta_7, \text{F}_4 \quad \text{h}_3 : \vdash \Delta_8, \Delta_9, \text{F}_5, \text{F}_\mathbf{1} \otimes \text{F}_2}{\bullet\text{h}_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \text{F}_\mathbf{1} \otimes \text{F}_2), \text{F}_4 \otimes \text{F}_5} \;\otimes \qquad \rightarrow \qquad {\color{red}\frac{}{\bullet\text{h}_3 : \vdash \Delta_7, \Delta_9, \text{F}_2, \text{F}_4 \otimes \text{F}_5}\;^{\text{fail}}}$$

$$\frac{\text{h}_1 : \vdash \Delta_4, \Delta_5, \text{F}_2 \quad \text{h}_1 : \vdash \Delta_6, \Delta_7, \text{F}_3}{\bullet\text{h}_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), \text{F}_2 \otimes \text{F}_3} \;\otimes \qquad \rightarrow \qquad {\color{red}\frac{}{\bullet\text{h}_1 : \vdash \Delta_5, \Delta_7, \text{F}_3}\;^{\text{fail}}}$$

# 4 Identity-Expansion

$$\cfrac{\cfrac{}{-\,:\vdash \mathbf{1}}\ \mathbf{1}}{-\,:\vdash \mathbf{1},\bot}\ \bot$$

$$\cfrac{\cfrac{\cfrac{}{-\,:\vdash \mathtt{F}_0, dual(\mathtt{F}_0)}\ \mathtt{IH}\qquad \cfrac{}{-\,:\vdash \mathtt{F_1}, dual(\mathtt{F_1})}\ \mathtt{IH}}{-\,:\vdash dual(\mathtt{F}_0), dual(\mathtt{F_1}), \mathtt{F}_0 \otimes \mathtt{F_1}}\ \otimes}{-\,:\vdash dual(\mathtt{F}_0)\$dual(\mathtt{F_1}), \mathtt{F}_0 \otimes \mathtt{F_1}}\ \$$$

$$\cfrac{\cfrac{\cfrac{}{-\,:\vdash \mathtt{F}_0, dual(\mathtt{F}_0)}\ \mathtt{IH}\qquad \cfrac{}{-\,:\vdash \mathtt{F_1}, dual(\mathtt{F_1})}\ \mathtt{IH}}{-\,:\vdash \mathtt{F}_0, \mathtt{F_1}, dual(\mathtt{F}_0) \otimes dual(\mathtt{F_1})}\ \otimes}{-\,:\vdash \mathtt{F}_0\$\mathtt{F_1}, dual(\mathtt{F}_0) \otimes dual(\mathtt{F_1})}\ \$$$

$$\cfrac{\cfrac{\cfrac{}{-\,:\vdash \mathtt{F}_0, dual(\mathtt{F}_0)}\ \mathtt{IH}}{-\,:\vdash ?\mathtt{F}_0, dual(\mathtt{F}_0)}\ ?}{-\,:\vdash !dual(\mathtt{F}_0), ?\mathtt{F}_0}\ !$$

$$\cfrac{\cfrac{}{-\,:\vdash \mathbf{1}}\ \mathbf{1}}{-\,:\vdash \mathbf{1},\bot}\ \bot$$

$$\cfrac{\cfrac{\cfrac{}{-\,:\vdash \mathtt{F}_0, dual(\mathtt{F}_0)}\ \mathtt{IH}}{-\,:\vdash \mathtt{F}_0, ?dual(\mathtt{F}_0)}\ ?}{-\,:\vdash !\mathtt{F}_0, ?dual(\mathtt{F}_0)}\ !$$

# 5 Cut-Elimination

## 5.1 Status of 1: `OK`

- Case rule **1**


- Case rule !


- Case rule $?W$

$$\cfrac{\cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \cfrac{\mathtt{h}_2 :\vdash \bot, \Delta_4}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_4, ?\mathtt{F}_3}\ ?W}{- :\vdash *, \Delta_4, ?\mathtt{F}_3}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \text{ax} \quad \cfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \bot}\ \text{ax}}{- :\vdash \Delta_4}\ \text{hCut}}{- :\vdash \Delta_4, ?\mathtt{F}_3}\ ?W$$

- Case rule $?C$

$$\cfrac{\cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \cfrac{\mathtt{h}_2 :\vdash \bot, \Delta_4, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_4, ?\mathtt{F}_3}\ ?C}{- :\vdash *, \Delta_4, ?\mathtt{F}_3}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \text{ax} \quad \cfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \bot, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \text{ax}}{- :\vdash \Delta_4, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \text{hCut}}{- :\vdash \Delta_4, ?\mathtt{F}_3}\ ?C$$

- Case rule ?

$$\cfrac{\cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \cfrac{\mathtt{h}_2 :\vdash \bot, \Delta_4, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_4, ?\mathtt{F}_3}\ ?}{- :\vdash *, \Delta_4, ?\mathtt{F}_3}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \text{ax} \quad \cfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \bot}\ \text{ax}}{- :\vdash \Delta_4, \mathtt{F}_3}\ \text{hCut}}{- :\vdash \Delta_4, ?\mathtt{F}_3}\ ?$$

- Case rule $

$$\cfrac{\cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \cfrac{\mathtt{h}_2 :\vdash \bot, \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$}{- :\vdash *, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}}\ \text{ax} \quad \cfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \bot}\ \text{ax}}{- :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \text{hCut}}{- :\vdash \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$$$

- Case rule $\bot$

$$\cfrac{\cfrac{}{\bullet\mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \cfrac{\mathtt{h}_2 :\vdash \Delta_3}{\bullet\mathtt{h}_2 :\vdash dual(\mathbf{1}), \Delta_3}\ \bot}{- :\vdash *, \Delta_3}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{}{- :\vdash \Delta_3}\ \text{ax}$$

- Case rule $I$

- Case rule $\otimes$

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash \mathbf{1}, *}\ \mathbf{1}
\qquad
\cfrac{h_2 :\vdash \bot, \Delta_5, F_3 \quad h_2 :\vdash \Delta_6, F_4}{\bullet h_2 :\vdash dual(\mathbf{1}), \Delta_5, \Delta_6, F_3 \otimes F_4}\ \otimes
}{- :\vdash *, \Delta_5, \Delta_6, F_3 \otimes F_4}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 :\vdash \mathbf{1}}\ \text{ax}
\quad
\cfrac{}{h_2 :\vdash \Delta_5, F_3, \bot}\ \text{ax}
}{- :\vdash \Delta_5, F_3}\ \text{hCut}
\qquad
\cfrac{}{- :\vdash \Delta_6, F_4}\ \text{ax}
}{- :\vdash \Delta_5, \Delta_6, F_3 \otimes F_4}\ \otimes
$$

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash \mathbf{1}, *}\ \mathbf{1}
\qquad
\cfrac{h_2 :\vdash \Delta_5, F_3 \quad h_2 :\vdash \bot, \Delta_6, F_4}{\bullet h_2 :\vdash dual(\mathbf{1}), \Delta_5, \Delta_6, F_3 \otimes F_4}\ \otimes
}{- :\vdash *, \Delta_5, \Delta_6, F_3 \otimes F_4}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{}{- :\vdash \Delta_5, F_3}\ \text{ax}
\qquad
\cfrac{
\cfrac{}{\bullet h_1 :\vdash \mathbf{1}}\ \text{ax}
\quad
\cfrac{}{h_2 :\vdash \Delta_6, F_4, \bot}\ \text{ax}
}{- :\vdash \Delta_6, F_4}\ \text{hCut}
}{- :\vdash \Delta_5, \Delta_6, F_3 \otimes F_4}\ \otimes
$$

## 5.2 Status of !: OK

- Case rule $\mathbf{1}$

- Case rule !

$$
\cfrac{
\cfrac{h_1 :\vdash F_4, ?\Gamma_3}{\bullet h_1 :\vdash !F_4, ?\Gamma_3}\ !
\qquad
\cfrac{h_5 :\vdash F_6, ?\Gamma_7, ?dual(F_4)}{\bullet h_5 :\vdash dual(!F_4), ?\Gamma_7, !F_6}\ !
}{- :\vdash ?\Gamma_3, ?\Gamma_7, !F_6}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 :\vdash ?\Gamma_3, !F_4}\ \text{ax}
\qquad
\cfrac{}{h_5 :\vdash ?\Gamma_7, F_6, ?dual(F_4)}\ \text{ax}
}{- :\vdash ?\Gamma_3, ?\Gamma_7, F_6}\ \text{hCut}
}{- :\vdash ?\Gamma_3, ?\Gamma_7, !F_6}\ !
$$

- Case rule $?W$

$$
\cfrac{
\cfrac{h_1 :\vdash F_2, ?\Gamma_3, 5}{\bullet h_1 :\vdash 5, ?\Gamma_3, !F_2}\ !
\qquad
\cfrac{h_6 :\vdash \Delta_8, dual(5)}{\bullet h_6 :\vdash dual(5), \Delta_8, ?F_7}\ ?W
}{- :\vdash (?\Gamma_3, !F_2), \Delta_8, ?F_7}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 :\vdash 5, ?\Gamma_3, !F_2}\ \text{ax}
\qquad
\cfrac{}{h_6 :\vdash \Delta_8, dual(5)}\ \text{ax}
}{- :\vdash ?\Gamma_3, \Delta_8, !F_2}\ \text{hCut}
}{- :\vdash ?\Gamma_3, \Delta_8, !F_2, ?F_7}\ ?W
$$

$$
\cfrac{
\cfrac{h_1 :\vdash F_4, ?\Gamma_3}{\bullet h_1 :\vdash !F_4, ?\Gamma_3}\ !
\qquad
\cfrac{h_5 :\vdash \Delta_7, ?dual(F_4)}{\bullet h_5 :\vdash dual(!F_4), \Delta_7, ?F_6}\ ?W
}{- :\vdash ?\Gamma_3, \Delta_7, ?F_6}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{
\cfrac{}{\bullet h_1 :\vdash ?\Gamma_3, !F_4}\ \text{ax}
\qquad
\cfrac{}{h_5 :\vdash \Delta_7, ?dual(F_4)}\ \text{ax}
}{- :\vdash ?\Gamma_3, \Delta_7}\ \text{hCut}
}{- :\vdash ?\Gamma_3, \Delta_7, ?F_6}\ ?W
$$

$$
\cfrac{
\cfrac{h_1 :\vdash F_4, ?\Gamma_3}{\bullet h_1 :\vdash !F_4, ?\Gamma_3}\ !
\qquad
\cfrac{h_5 :\vdash \Delta_6}{\bullet h_5 :\vdash dual(!F_4), \Delta_6}\ ?W
}{- :\vdash ?\Gamma_3, \Delta_6}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
\cfrac{}{- :\vdash \Delta_6}\ \text{ax}
}{- :\vdash ?\Gamma_3, \Delta_6}\ W
$$

- Case rule $?C$

$$
\cfrac{
  \cfrac{\mathtt{h_1} :\vdash \mathtt{F_2}, ?\Gamma_3, 5}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}} \; ! \qquad
  \cfrac{\mathtt{h_6} :\vdash \Delta_8, ?\mathtt{F_7}, ?\mathtt{F_7}, dual(5)}{\bullet\mathtt{h_6} :\vdash dual(5), \Delta_8, ?\mathtt{F_7}} \; ?C
}{- :\vdash (?\Gamma_3, !\mathtt{F_2}), \Delta_8, ?\mathtt{F_7}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}} \; \text{ax} \qquad
    \cfrac{}{\mathtt{h_6} :\vdash \Delta_8, ?\mathtt{F_7}, ?\mathtt{F_7}, dual(5)} \; \text{ax}
  }{- :\vdash ?\Gamma_3, \Delta_8, !\mathtt{F_2}, ?\mathtt{F_7}, ?\mathtt{F_7}} \; \text{hCut}
}{- :\vdash ?\Gamma_3, \Delta_8, !\mathtt{F_2}, ?\mathtt{F_7}} \; ?C
$$

$$
\cfrac{
  \cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \qquad
  \cfrac{\mathtt{h_5} :\vdash \Delta_7, ?\mathtt{F_6}, ?\mathtt{F_6}, ?dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(!\mathtt{F_4}), \Delta_7, ?\mathtt{F_6}} \; ?C
}{- :\vdash ?\Gamma_3, \Delta_7, ?\mathtt{F_6}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \qquad
    \cfrac{}{\mathtt{h_5} :\vdash \Delta_7, ?\mathtt{F_6}, ?\mathtt{F_6}, ?dual(\mathtt{F_4})} \; \text{ax}
  }{- :\vdash ?\Gamma_3, \Delta_7, ?\mathtt{F_6}, ?\mathtt{F_6}} \; \text{hCut}
}{- :\vdash ?\Gamma_3, \Delta_7, ?\mathtt{F_6}} \; ?C
$$

$$
\cfrac{
  \cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \qquad
  \cfrac{\mathtt{h_5} :\vdash \Delta_6, ?dual(\mathtt{F_4}), ?dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(!\mathtt{F_4}), \Delta_6} \; ?C
}{- :\vdash ?\Gamma_3, \Delta_6} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \qquad
  \cfrac{}{\mathtt{h_5} :\vdash \Delta_6, ?dual(\mathtt{F_4}), ?dual(\mathtt{F_4})} \; \text{ax}
}{- :\vdash ?\Gamma_3, \Delta_6} \; \text{mCut}
$$

- Case rule $?$

$$
\cfrac{
  \cfrac{\mathtt{h_1} :\vdash \mathtt{F_2}, ?\Gamma_3, 5}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}} \; ! \qquad
  \cfrac{\mathtt{h_6} :\vdash \Delta_8, \mathtt{F_7}, dual(5)}{\bullet\mathtt{h_6} :\vdash dual(5), \Delta_8, ?\mathtt{F_7}} \; ?
}{- :\vdash (?\Gamma_3, !\mathtt{F_2}), \Delta_8, ?\mathtt{F_7}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}} \; \text{ax} \qquad
    \cfrac{}{\mathtt{h_6} :\vdash \Delta_8, \mathtt{F_7}, dual(5)} \; \text{ax}
  }{- :\vdash ?\Gamma_3, \Delta_8, \mathtt{F_7}, !\mathtt{F_2}} \; \text{hCut}
}{- :\vdash ?\Gamma_3, \Delta_8, !\mathtt{F_2}, ?\mathtt{F_7}} \; ?
$$

$$
\cfrac{
  \cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \qquad
  \cfrac{\mathtt{h_5} :\vdash \Delta_7, \mathtt{F_6}, ?dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(!\mathtt{F_4}), \Delta_7, ?\mathtt{F_6}} \; ?
}{- :\vdash ?\Gamma_3, \Delta_7, ?\mathtt{F_6}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \qquad
    \cfrac{}{\mathtt{h_5} :\vdash \Delta_7, \mathtt{F_6}, ?dual(\mathtt{F_4})} \; \text{ax}
  }{- :\vdash ?\Gamma_3, \Delta_7, \mathtt{F_6}} \; \text{hCut}
}{- :\vdash ?\Gamma_3, \Delta_7, ?\mathtt{F_6}} \; ?
$$

$$
\cfrac{
  \cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \qquad
  \cfrac{\mathtt{h_5} :\vdash \Delta_6, dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(!\mathtt{F_4}), \Delta_6} \; ?
}{- :\vdash ?\Gamma_3, \Delta_6} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{}{- :\vdash ?\Gamma_3, \mathtt{F_4}} \; \text{ax} \qquad
  \cfrac{}{- :\vdash \Delta_6, dual(\mathtt{F_4})} \; \text{ax}
}{- :\vdash ?\Gamma_3, \Delta_6} \; \text{sCut}
$$

- Case rule $\$$

$$
\cfrac{
  \cfrac{\mathtt{h_1} :\vdash \mathtt{F_2}, ?\Gamma_3, 5}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}} \; ! \qquad
  \cfrac{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7}, \mathtt{F_8}, dual(5)}{\bullet\mathtt{h_6} :\vdash dual(5), \Delta_9, \mathtt{F_7}\$\mathtt{F_8}} \; \$
}{- :\vdash (?\Gamma_3, !\mathtt{F_2}), \Delta_9, \mathtt{F_7}\$\mathtt{F_8}} \; \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}} \; \text{ax} \qquad
    \cfrac{}{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7}, \mathtt{F_8}, dual(5)} \; \text{ax}
  }{- :\vdash ?\Gamma_3, \Delta_9, \mathtt{F_7}, \mathtt{F_8}, !\mathtt{F_2}} \; \text{hCut}
}{- :\vdash ?\Gamma_3, \Delta_9, !\mathtt{F_2}, \mathtt{F_7}\$\mathtt{F_8}} \; \$
$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3}\ ! \quad \dfrac{\mathtt{h_5} :\vdash \Delta_8, \mathtt{F_6}, \mathtt{F_7}, ?dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(!\mathtt{F_4}), \Delta_8, \mathtt{F_6}\$\mathtt{F_7}}\ \$}{-\ :\vdash ?\Gamma_3, \Delta_8, \mathtt{F_6}\$\mathtt{F_7}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}}\ \text{ax} \quad \dfrac{}{\mathtt{h_5} :\vdash \Delta_8, \mathtt{F_6}, \mathtt{F_7}, ?dual(\mathtt{F_4})}\ \text{ax}}{-\ :\vdash ?\Gamma_3, \Delta_8, \mathtt{F_6}, \mathtt{F_7}}\ \text{hCut}}{-\ :\vdash ?\Gamma_3, \Delta_8, \mathtt{F_6}\$\mathtt{F_7}}\ \$$$

- Case rule $\bot$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_2}, ?\Gamma_3, 5}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}}\ ! \quad \dfrac{\mathtt{h_6} :\vdash \Delta_7, dual(5)}{\bullet\mathtt{h_6} :\vdash dual(5), \bot, \Delta_7}\ \bot}{-\ :\vdash (?\Gamma_3, !\mathtt{F_2}), \bot, \Delta_7}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}}\ \text{ax} \quad \dfrac{}{\mathtt{h_6} :\vdash \Delta_7, dual(5)}\ \text{ax}}{-\ :\vdash ?\Gamma_3, \Delta_7, !\mathtt{F_2}}\ \text{hCut}}{-\ :\vdash ?\Gamma_3, \Delta_7, \bot, !\mathtt{F_2}}\ \bot$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3}\ ! \quad \dfrac{\mathtt{h_5} :\vdash \Delta_6, ?dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(!\mathtt{F_4}), \bot, \Delta_6}\ \bot}{-\ :\vdash ?\Gamma_3, \bot, \Delta_6}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}}\ \text{ax} \quad \dfrac{}{\mathtt{h_5} :\vdash \Delta_6, ?dual(\mathtt{F_4})}\ \text{ax}}{-\ :\vdash ?\Gamma_3, \Delta_6}\ \text{hCut}}{-\ :\vdash ?\Gamma_3, \Delta_6, \bot}\ \bot$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_2}, ?\Gamma_3, 5}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}}\ ! \quad \dfrac{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7}, dual(5) \quad \mathtt{h_6} :\vdash \Delta_{10}, \mathtt{F_8}}{\bullet\mathtt{h_6} :\vdash dual(5), \Delta_9, \Delta_{10}, \mathtt{F_7} \otimes \mathtt{F_8}}\ \otimes}{-\ :\vdash (?\Gamma_3, !\mathtt{F_2}), \Delta_9, \Delta_{10}, \mathtt{F_7} \otimes \mathtt{F_8}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}}\ \text{ax} \quad \dfrac{}{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7}, dual(5)}\ \text{ax}}{-\ :\vdash ?\Gamma_3, \Delta_9, \mathtt{F_7}, !\mathtt{F_2}}\ \text{hCut} \quad \dfrac{}{-\ :\vdash \Delta_{10}, \mathtt{F_8}}\ \text{ax}}{-\ :\vdash ?\Gamma_3, \Delta_{10}, \Delta_9, !\mathtt{F_2}, \mathtt{F_7} \otimes \mathtt{F_8}}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_2}, ?\Gamma_3, 5}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}}\ ! \quad \dfrac{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7} \quad \mathtt{h_6} :\vdash \Delta_{10}, \mathtt{F_8}, dual(5)}{\bullet\mathtt{h_6} :\vdash dual(5), \Delta_9, \Delta_{10}, \mathtt{F_7} \otimes \mathtt{F_8}}\ \otimes}{-\ :\vdash (?\Gamma_3, !\mathtt{F_2}), \Delta_9, \Delta_{10}, \mathtt{F_7} \otimes \mathtt{F_8}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{-\ :\vdash \Delta_9, \mathtt{F_7}}\ \text{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash 5, ?\Gamma_3, !\mathtt{F_2}}\ \text{ax} \quad \dfrac{}{\mathtt{h_6} :\vdash \Delta_{10}, \mathtt{F_8}, dual(5)}\ \text{ax}}{-\ :\vdash ?\Gamma_3, \Delta_{10}, \mathtt{F_8}, !\mathtt{F_2}}\ \text{hCut}}{-\ :\vdash ?\Gamma_3, \Delta_{10}, \Delta_9, !\mathtt{F_2}, \mathtt{F_7} \otimes \mathtt{F_8}}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3}\ ! \quad \dfrac{\mathtt{h_5} :\vdash \Delta_8, \mathtt{F_6}, ?dual(\mathtt{F_4}) \quad \mathtt{h_5} :\vdash \Delta_9, \mathtt{F_7}}{\bullet\mathtt{h_5} :\vdash dual(!\mathtt{F_4}), \Delta_8, \Delta_9, \mathtt{F_6} \otimes \mathtt{F_7}}\ \otimes}{-\ :\vdash ?\Gamma_3, \Delta_8, \Delta_9, \mathtt{F_6} \otimes \mathtt{F_7}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}}\ \text{ax} \quad \dfrac{}{\mathtt{h_5} :\vdash \Delta_8, \mathtt{F_6}, ?dual(\mathtt{F_4})}\ \text{ax}}{-\ :\vdash ?\Gamma_3, \Delta_8, \mathtt{F_6}}\ \text{hCut} \quad \dfrac{}{-\ :\vdash \Delta_9, \mathtt{F_7}}\ \text{ax}}{-\ :\vdash ?\Gamma_3, \Delta_8, \Delta_9, \mathtt{F_6} \otimes \mathtt{F_7}}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3}\ ! \quad \dfrac{\mathtt{h_5} :\vdash \Delta_8, \mathtt{F_6} \quad \mathtt{h_5} :\vdash \Delta_9, \mathtt{F_7}, ?dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(!\mathtt{F_4}), \Delta_8, \Delta_9, \mathtt{F_6} \otimes \mathtt{F_7}}\ \otimes}{-\ :\vdash ?\Gamma_3, \Delta_8, \Delta_9, \mathtt{F_6} \otimes \mathtt{F_7}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{-\ :\vdash \Delta_8, \mathtt{F_6}}\ \text{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}}\ \text{ax} \quad \dfrac{}{\mathtt{h_5} :\vdash \Delta_9, \mathtt{F_7}, ?dual(\mathtt{F_4})}\ \text{ax}}{-\ :\vdash ?\Gamma_3, \Delta_9, \mathtt{F_7}}\ \text{hCut}}{-\ :\vdash ?\Gamma_3, \Delta_8, \Delta_9, \mathtt{F_6} \otimes \mathtt{F_7}}\ \otimes$$

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## 5.3 Status of $?W$: OK

- Case rule $\mathbf{1}$

- Case rule $!$

$$
\cfrac{
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \Delta_3}{\bullet\mathtt{h_1} :\vdash ?\mathtt{F_4}, \Delta_3}\ ?W
\quad
\cfrac{\mathtt{h_5} :\vdash ?\Gamma_6, dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(?\mathtt{F_4}), ?\Gamma_6}\ !
}{- :\vdash \Delta_3, ?\Gamma_6}\ \mathtt{Cut}
}{
\cfrac{\cfrac{}{- :\vdash \Delta_3}\ \mathtt{ax}}{- :\vdash ?\Gamma_6, \Delta_3}\ W
}\ \rightarrow
$$

- Case rule $?W$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_5}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_3, ?\mathtt{F_2}}\ ?W
\quad
\cfrac{\mathtt{h_6} :\vdash \Delta_8, dual(\mathtt{F_5})}{\bullet\mathtt{h_6} :\vdash dual(\mathtt{F_5}), \Delta_8, ?\mathtt{F_7}}\ ?W
}{- :\vdash (\Delta_3, ?\mathtt{F_2}), \Delta_8, ?\mathtt{F_7}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{
\cfrac{\cfrac{}{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_5}}\ \mathtt{ax}
\quad
\cfrac{}{\bullet\mathtt{h_6} :\vdash \Delta_8, ?\mathtt{F_7}, dual(\mathtt{F_5})}\ \mathtt{ax}
}{- :\vdash \Delta_3, \Delta_8, ?\mathtt{F_7}}\ \mathtt{hCut}
}{- :\vdash \Delta_3, \Delta_8, ?\mathtt{F_2}, ?\mathtt{F_7}}\ ?W
$$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \Delta_3}{\bullet\mathtt{h_1} :\vdash ?\mathtt{F_4}, \Delta_3}\ ?W
\quad
\cfrac{\mathtt{h_5} :\vdash \Delta_7, !dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(?\mathtt{F_4}), \Delta_7, ?\mathtt{F_6}}\ ?W
}{- :\vdash \Delta_3, \Delta_7, ?\mathtt{F_6}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{
\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_3, ?\mathtt{F_4}}\ \mathtt{ax}
\quad
\cfrac{}{\mathtt{h_5} :\vdash \Delta_7, !dual(\mathtt{F_4})}\ \mathtt{ax}
}{- :\vdash \Delta_3, \Delta_7}\ \mathtt{hCut}
}{- :\vdash \Delta_3, \Delta_7, ?\mathtt{F_6}}\ ?W
$$

- Case rule $?C$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_5}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_3, ?\mathtt{F_2}}\ ?W
\quad
\cfrac{\mathtt{h_6} :\vdash \Delta_8, ?\mathtt{F_7}, ?\mathtt{F_7}, dual(\mathtt{F_5})}{\bullet\mathtt{h_6} :\vdash dual(\mathtt{F_5}), \Delta_8, ?\mathtt{F_7}}\ ?C
}{- :\vdash (\Delta_3, ?\mathtt{F_2}), \Delta_8, ?\mathtt{F_7}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{
\cfrac{\cfrac{}{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_5}}\ \mathtt{ax}
\quad
\cfrac{}{\bullet\mathtt{h_6} :\vdash \Delta_8, ?\mathtt{F_7}, dual(\mathtt{F_5})}\ \mathtt{ax}
}{- :\vdash \Delta_3, \Delta_8, ?\mathtt{F_7}}\ \mathtt{hCut}
}{- :\vdash \Delta_3, \Delta_8, ?\mathtt{F_2}, ?\mathtt{F_7}}\ ?W
$$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \Delta_3}{\bullet\mathtt{h_1} :\vdash ?\mathtt{F_4}, \Delta_3}\ ?W
\quad
\cfrac{\mathtt{h_5} :\vdash \Delta_7, !dual(\mathtt{F_4}), ?\mathtt{F_6}, ?\mathtt{F_6}}{\bullet\mathtt{h_5} :\vdash dual(?\mathtt{F_4}), \Delta_7, ?\mathtt{F_6}}\ ?C
}{- :\vdash \Delta_3, \Delta_7, ?\mathtt{F_6}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_3, ?\mathtt{F_4}}\ \mathtt{ax}
\quad
\cfrac{}{\mathtt{h_5} :\vdash \Delta_7, !dual(\mathtt{F_4}), ?\mathtt{F_6}, ?\mathtt{F_6}}\ \mathtt{ax}
}{- :\vdash \Delta_3, \Delta_7, ?\mathtt{F_6}, ?\mathtt{F_6}}\ \mathtt{hCut}
}{- :\vdash \Delta_3, \Delta_7, ?\mathtt{F_6}}\ ?C
}{}
$$

- Case rule $?$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_5}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_3, ?\mathtt{F_2}}\ ?W
\quad
\cfrac{\mathtt{h_6} :\vdash \Delta_8, \mathtt{F_7}, dual(\mathtt{F_5})}{\bullet\mathtt{h_6} :\vdash dual(\mathtt{F_5}), \Delta_8, ?\mathtt{F_7}}\ ?
}{- :\vdash (\Delta_3, ?\mathtt{F_2}), \Delta_8, ?\mathtt{F_7}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_5}}\ \mathtt{ax}
\quad
\cfrac{}{\bullet\mathtt{h_6} :\vdash \Delta_8, ?\mathtt{F_7}, dual(\mathtt{F_5})}\ \mathtt{ax}
}{- :\vdash \Delta_3, \Delta_8, ?\mathtt{F_7}}\ \mathtt{hCut}
}{- :\vdash \Delta_3, \Delta_8, ?\mathtt{F_2}, ?\mathtt{F_7}}\ W
}{}
$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?W \qquad \cfrac{h_5 :\vdash \Delta_7, F_6, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_7, ?F_6}\ ?}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\ \text{ax} \qquad \cfrac{}{h_5 :\vdash \Delta_7, F_6, !dual(F_4)}\ \text{ax}}{\cfrac{- :\vdash \Delta_3, \Delta_7, F_6}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ ?}\ \text{hCut}$$

- Case rule \$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_5}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?W \qquad \cfrac{h_6 :\vdash \Delta_9, F_7, F_8, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_9, F_7\$F_8}\ \$}{- :\vdash (\Delta_3, ?F_2), \Delta_9, F_7\$F_8}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 :\vdash \Delta_3, F_5}\ \text{ax} \qquad \cfrac{}{\bullet h_6 :\vdash \Delta_9, dual(F_5), F_7\$F_8}\ \text{ax}}{\cfrac{- :\vdash \Delta_3, \Delta_9, F_7\$F_8}{- :\vdash \Delta_3, \Delta_9, ?F_2, F_7\$F_8}\ ?W}\ \text{hCut}$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?W \qquad \cfrac{h_5 :\vdash \Delta_8, F_6, F_7, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_8, F_6\$F_7}\ \$}{- :\vdash \Delta_3, \Delta_8, F_6\$F_7}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\ \text{ax} \qquad \cfrac{}{h_5 :\vdash \Delta_8, F_6, F_7, !dual(F_4)}\ \text{ax}}{\cfrac{- :\vdash \Delta_3, \Delta_8, F_6, F_7}{- :\vdash \Delta_3, \Delta_8, F_6\$F_7}\ \$}\ \text{hCut}$$

- Case rule $\perp$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_5}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?W \qquad \cfrac{h_6 :\vdash \Delta_7, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \perp, \Delta_7}\ \perp}{- :\vdash (\Delta_3, ?F_2), \perp, \Delta_7}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 :\vdash \Delta_3, F_5}\ \text{ax} \qquad \cfrac{}{\bullet h_6 :\vdash \Delta_7, \perp, dual(F_5)}\ \text{ax}}{\cfrac{- :\vdash \Delta_3, \Delta_7, \perp}{- :\vdash \Delta_3, \Delta_7, \perp, ?F_2}\ ?W}\ \text{hCut}$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?W \qquad \cfrac{h_5 :\vdash \Delta_6, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \perp, \Delta_6}\ \perp}{- :\vdash \Delta_3, \perp, \Delta_6}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\ \text{ax} \qquad \cfrac{}{h_5 :\vdash \Delta_6, !dual(F_4)}\ \text{ax}}{\cfrac{- :\vdash \Delta_3, \Delta_6}{- :\vdash \Delta_3, \Delta_6, \perp}\ \perp}\ \text{hCut}$$

- Case rule $I$

- Case rule $\otimes$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_5}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?W \qquad \cfrac{h_6 :\vdash \Delta_9, F_7, dual(F_5) \quad h_6 :\vdash \Delta_{10}, F_8}{\bullet h_6 :\vdash dual(F_5), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \otimes}{- :\vdash (\Delta_3, ?F_2), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 :\vdash \Delta_3, F_5}\ \text{ax} \qquad \cfrac{}{\bullet h_6 :\vdash \Delta_{10}, \Delta_9, dual(F_5), F_7 \otimes F_8}\ \text{ax}}{\cfrac{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, F_7 \otimes F_8}{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, ?F_2, F_7 \otimes F_8}\ ?W}\ \text{hCut}$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_3, F_5}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?W \qquad \dfrac{h_6 :\vdash \Delta_9, F_7 \quad h_6 :\vdash \Delta_{10}, F_8, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \otimes}{- :\vdash (\Delta_3, ?F_2), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{h_1 :\vdash \Delta_3, F_5}\ ax \qquad \dfrac{}{\bullet h_6 :\vdash \Delta_{10}, \Delta_9, dual(F_5), F_7 \otimes F_8}\ ax}{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, F_7 \otimes F_8}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, ?F_2, F_7 \otimes F_8}\ ?W$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_3}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?W \qquad \dfrac{h_5 :\vdash \Delta_8, F_6, !dual(F_4) \quad h_5 :\vdash \Delta_9, F_7}{\bullet h_5 :\vdash dual(?F_4), \Delta_8, \Delta_9, F_6 \otimes F_7}\ \otimes}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\ ax \quad \dfrac{}{h_5 :\vdash \Delta_8, F_6, !dual(F_4)}\ ax}{- :\vdash \Delta_3, \Delta_8, F_6}\ \text{hCut} \qquad \dfrac{}{- :\vdash \Delta_9, F_7}\ ax}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\ \otimes$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_3}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?W \qquad \dfrac{h_5 :\vdash \Delta_8, F_6 \quad h_5 :\vdash \Delta_9, F_7, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_8, \Delta_9, F_6 \otimes F_7}\ \otimes}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_8, F_6}\ ax \qquad \dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\ ax \quad \dfrac{}{h_5 :\vdash \Delta_9, F_7, !dual(F_4)}\ ax}{- :\vdash \Delta_3, \Delta_9, F_7}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\ \otimes$$

## 5.4   Status of $?C$:   OK

- Case rule **1**


- Case rule **!**

$$\dfrac{\dfrac{h_1 :\vdash \Delta_3, ?F_4, ?F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?C \qquad \dfrac{h_5 :\vdash ?\Gamma_6, dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), ?\Gamma_6}\ !}{- :\vdash \Delta_3, ?\Gamma_6}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{h_1 :\vdash \Delta_3, ?F_4, ?F_4}\ ax \qquad \dfrac{\dfrac{}{h_5 :\vdash ?\Gamma_6, dual(F_4)}\ ax}{\bullet h_5 :\vdash ?\Gamma_6, !dual(F_4)}\ !}{- :\vdash ?\Gamma_6, \Delta_3}\ \text{mCut}$$

- Case rule $?W$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?C \qquad \dfrac{h_6 :\vdash \Delta_8, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_8, ?F_7}\ ?W}{- :\vdash (\Delta_3, ?F_2), \Delta_8, ?F_7}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_3, F_5, ?F_2}\ ax \qquad \dfrac{}{h_6 :\vdash \Delta_8, dual(F_5)}\ ax}{- :\vdash \Delta_3, \Delta_8, ?F_2}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_8, ?F_2, ?F_7}\ ?W$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_3, ?F_4, ?F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?C \qquad \dfrac{h_5 :\vdash \Delta_7, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_7, ?F_6}\ ?W}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\ ax \qquad \dfrac{}{h_5 :\vdash \Delta_7, !dual(F_4)}\ ax}{- :\vdash \Delta_3, \Delta_7}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ W$$

- Case rule $?C$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2} \ ?C \qquad \cfrac{h_6 :\vdash \Delta_8, ?F_7, ?F_7, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_8, ?F_7} \ ?C}{- :\vdash (\Delta_3, ?F_2), \Delta_8, ?F_7} \ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2} \ \text{ax} \qquad \overline{\bullet h_6 :\vdash \Delta_8, ?F_7, dual(F_5)} \ \text{ax}}{- :\vdash \Delta_3, \Delta_8, ?F_2, ?F_2, ?F_7} \ \text{hCut}}{- :\vdash \Delta_3, \Delta_8, ?F_2, ?F_7} \ ?C$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, ?F_4, ?F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3} \ ?C \qquad \cfrac{h_5 :\vdash \Delta_7, !dual(F_4), ?F_6, ?F_6}{\bullet h_5 :\vdash dual(?F_4), \Delta_7, ?F_6} \ ?C}{- :\vdash \Delta_3, \Delta_7, ?F_6} \ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_3, ?F_4} \ \text{ax} \qquad \overline{h_5 :\vdash \Delta_7, !dual(F_4), ?F_6, ?F_6} \ \text{ax}}{- :\vdash \Delta_3, \Delta_7, ?F_6, ?F_6} \ \text{hCut}}{- :\vdash \Delta_3, \Delta_7, ?F_6} \ ?C$$

- Case rule ?

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2} \ ?C \qquad \cfrac{h_6 :\vdash \Delta_8, F_7, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_8, ?F_7} \ ?}{- :\vdash (\Delta_3, ?F_2), \Delta_8, ?F_7} \ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2} \ \text{ax} \qquad \overline{\bullet h_6 :\vdash \Delta_8, ?F_7, dual(F_5)} \ \text{ax}}{- :\vdash \Delta_3, \Delta_8, ?F_2, ?F_2, ?F_7} \ \text{hCut}}{- :\vdash \Delta_3, \Delta_8, ?F_2, ?F_7} \ ?C$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, ?F_4, ?F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3} \ ?C \qquad \cfrac{h_5 :\vdash \Delta_7, F_6, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_7, ?F_6} \ ?}{- :\vdash \Delta_3, \Delta_7, ?F_6} \ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_3, ?F_4} \ \text{ax} \qquad \overline{h_5 :\vdash \Delta_7, F_6, !dual(F_4)} \ \text{ax}}{- :\vdash \Delta_3, \Delta_7, F_6} \ \text{hCut}}{- :\vdash \Delta_3, \Delta_7, ?F_6} \ ?$$

- Case rule $

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2} \ ?C \qquad \cfrac{h_6 :\vdash \Delta_9, F_7, F_8, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_9, F_7\$F_8} \ \$}{- :\vdash (\Delta_3, ?F_2), \Delta_9, F_7\$F_8} \ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2} \ \text{ax} \qquad \overline{\bullet h_6 :\vdash \Delta_9, dual(F_5), F_7\$F_8} \ \text{ax}}{- :\vdash \Delta_3, \Delta_9, ?F_2, ?F_2, F_7\$F_8} \ \text{hCut}}{- :\vdash \Delta_3, \Delta_9, ?F_2, F_7\$F_8} \ ?C$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, ?F_4, ?F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3} \ ?C \qquad \cfrac{h_5 :\vdash \Delta_8, F_6, F_7, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_8, F_6\$F_7} \ \$}{- :\vdash \Delta_3, \Delta_8, F_6\$F_7} \ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_3, ?F_4} \ \text{ax} \qquad \overline{h_5 :\vdash \Delta_8, F_6, F_7, !dual(F_4)} \ \text{ax}}{- :\vdash \Delta_3, \Delta_8, F_6, F_7} \ \text{hCut}}{- :\vdash \Delta_3, \Delta_8, F_6\$F_7} \ \$$$

- Case rule $\bot$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2} \ ?C \qquad \cfrac{h_6 :\vdash \Delta_7, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \bot, \Delta_7} \ \bot}{- :\vdash (\Delta_3, ?F_2), \bot, \Delta_7} \ \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2} \ \text{ax} \qquad \overline{\bullet h_6 :\vdash \Delta_7, \bot, dual(F_5)} \ \text{ax}}{- :\vdash \Delta_3, \Delta_7, \bot, ?F_2, ?F_2} \ \text{hCut}}{- :\vdash \Delta_3, \Delta_7, \bot, ?F_2} \ ?C$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, ?F_4, ?F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?C \qquad \cfrac{h_5 :\vdash \Delta_6, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \bot, \Delta_6}\ \bot}{- :\vdash \Delta_3, \bot, \Delta_6}\ \mathtt{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{h_1 :\vdash \Delta_3, ?F_4, ?F_4}\ \mathtt{ax} \qquad \cfrac{\cfrac{}{h_5 :\vdash \Delta_6, !dual(F_4)}\ \mathtt{ax}}{\bullet h_5 :\vdash \Delta_6, \bot, !dual(F_4)}\ \bot}{- :\vdash \Delta_3, \Delta_6, \bot}\ \mathtt{mCut}$$

- Case rule $I$

- Case rule $\otimes$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?C \qquad \cfrac{h_6 :\vdash \Delta_9, F_7, dual(F_5) \quad h_6 :\vdash \Delta_{10}, F_8}{\bullet h_6 :\vdash dual(F_5), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \otimes}{- :\vdash (\Delta_3, ?F_2), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \mathtt{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2}\ \mathtt{ax} \qquad \cfrac{}{\bullet h_6 :\vdash \Delta_{10}, \Delta_9, dual(F_5), F_7 \otimes F_8}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, ?F_2, ?F_2, F_7 \otimes F_8}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, ?F_2, F_7 \otimes F_8}\ ?C$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?C \qquad \cfrac{h_6 :\vdash \Delta_9, F_7 \quad h_6 :\vdash \Delta_{10}, F_8, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \otimes}{- :\vdash (\Delta_3, ?F_2), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \mathtt{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{}{h_1 :\vdash \Delta_3, F_5, ?F_2, ?F_2}\ \mathtt{ax} \qquad \cfrac{}{\bullet h_6 :\vdash \Delta_{10}, \Delta_9, dual(F_5), F_7 \otimes F_8}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, ?F_2, ?F_2, F_7 \otimes F_8}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, ?F_2, F_7 \otimes F_8}\ ?C$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, ?F_4, ?F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?C \qquad \cfrac{h_5 :\vdash \Delta_8, F_6, !dual(F_4) \quad h_5 :\vdash \Delta_9, F_7}{\bullet h_5 :\vdash dual(?F_4), \Delta_8, \Delta_9, F_6 \otimes F_7}\ \otimes}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\ \mathtt{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{h_1 :\vdash \Delta_3, ?F_4, ?F_4}\ \mathtt{ax} \qquad \cfrac{\cfrac{}{h_5 :\vdash \Delta_8, F_6, !dual(F_4)}\ \mathtt{ax} \qquad \cfrac{}{h_5 :\vdash \Delta_9, F_7}\ \mathtt{ax}}{\bullet h_5 :\vdash \Delta_8, \Delta_9, !dual(F_4), F_6 \otimes F_7}\ \otimes}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\ \mathtt{mCut}$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, ?F_4, ?F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?C \qquad \cfrac{h_5 :\vdash \Delta_8, F_6 \quad h_5 :\vdash \Delta_9, F_7, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_8, \Delta_9, F_6 \otimes F_7}\ \otimes}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\ \mathtt{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{h_1 :\vdash \Delta_3, ?F_4, ?F_4}\ \mathtt{ax} \qquad \cfrac{\cfrac{}{h_5 :\vdash \Delta_8, F_6}\ \mathtt{ax} \qquad \cfrac{}{h_5 :\vdash \Delta_9, F_7, !dual(F_4)}\ \mathtt{ax}}{\bullet h_5 :\vdash \Delta_8, \Delta_9, !dual(F_4), F_6 \otimes F_7}\ \otimes}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\ \mathtt{mCut}$$

## 5.5 Status of $?$: OK

- Case rule $\mathbf{1}$

- Case rule !

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ? \qquad \cfrac{h_5 :\vdash ?\Gamma_6, dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), ?\Gamma_6}\ !}{- :\vdash \Delta_3, ?\Gamma_6}\ \mathtt{Cut}$$

$$\to$$

$$\cfrac{\cfrac{}{- :\vdash \Delta_3, F_4}\ \mathtt{ax} \qquad \cfrac{}{- :\vdash ?\Gamma_6, dual(F_4)}\ \mathtt{ax}}{- :\vdash ?\Gamma_6, \Delta_3}\ \mathtt{sCut}$$

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- Case rule $?W$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_2, F_5}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?\quad \cfrac{h_6 :\vdash \Delta_8, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_8, ?F_7}\ ?W}{- :\vdash (\Delta_3, ?F_2), \Delta_8, ?F_7}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_3, F_5, ?F_2}\ \text{ax}\quad \cfrac{}{h_6 :\vdash \Delta_8, dual(F_5)}\ \text{ax}}{- :\vdash \Delta_3, \Delta_8, ?F_2}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_8, ?F_2, ?F_7}\ W$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?\quad \cfrac{h_5 :\vdash \Delta_7, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_7, ?F_6}\ ?W}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\ \text{ax}\quad \cfrac{}{h_5 :\vdash \Delta_7, !dual(F_4)}\ \text{ax}}{- :\vdash \Delta_3, \Delta_7}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ ?W$$

- Case rule $?C$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_2, F_5}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?\quad \cfrac{h_6 :\vdash \Delta_8, ?F_7, ?F_7, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_8, ?F_7}\ ?C}{- :\vdash (\Delta_3, ?F_2), \Delta_8, ?F_7}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_3, F_5, ?F_2}\ \text{ax}\quad \cfrac{}{h_6 :\vdash \Delta_8, ?F_7, ?F_7, dual(F_5)}\ \text{ax}}{- :\vdash \Delta_3, \Delta_8, ?F_2, ?F_7, ?F_7}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_8, ?F_2, ?F_7}\ ?C$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?\quad \cfrac{h_5 :\vdash \Delta_7, !dual(F_4), ?F_6, ?F_6}{\bullet h_5 :\vdash dual(?F_4), \Delta_7, ?F_6}\ ?C}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\ \text{ax}\quad \cfrac{}{h_5 :\vdash \Delta_7, !dual(F_4), ?F_6, ?F_6}\ \text{ax}}{- :\vdash \Delta_3, \Delta_7, ?F_6, ?F_6}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ ?C$$

- Case rule $?$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_2, F_5}{\bullet h_1 :\vdash F_5, \Delta_3, ?F_2}\ ?\quad \cfrac{h_6 :\vdash \Delta_8, F_7, dual(F_5)}{\bullet h_6 :\vdash dual(F_5), \Delta_8, ?F_7}\ ?}{- :\vdash (\Delta_3, ?F_2), \Delta_8, ?F_7}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_1 :\vdash \Delta_3, F_2, F_5}\ \text{ax}\quad \cfrac{}{\bullet h_6 :\vdash \Delta_8, ?F_7, dual(F_5)}\ \text{ax}}{- :\vdash \Delta_3, \Delta_8, F_2, ?F_7}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_8, ?F_2, ?F_7}\ ?$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_4}{\bullet h_1 :\vdash ?F_4, \Delta_3}\ ?\quad \cfrac{h_5 :\vdash \Delta_7, F_6, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_7, ?F_6}\ ?}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\ \text{ax}\quad \cfrac{}{h_5 :\vdash \Delta_7, F_6, !dual(F_4)}\ \text{ax}}{- :\vdash \Delta_3, \Delta_7, F_6}\ \text{hCut}}{- :\vdash \Delta_3, \Delta_7, ?F_6}\ ?$$

- Case rule $\$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}, \mathtt{F_5}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_3, ?\mathtt{F_2}} \; ? \qquad \dfrac{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7}, \mathtt{F_8}, dual(\mathtt{F_5})}{\bullet\mathtt{h_6} :\vdash dual(\mathtt{F_5}), \Delta_9, \mathtt{F_7}\$\mathtt{F_8}} \; \$}{- :\vdash (\Delta_3, ?\mathtt{F_2}), \Delta_9, \mathtt{F_7}\$\mathtt{F_8}} \; Cut$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}, \mathtt{F_5}} \; ax \qquad \dfrac{}{\bullet\mathtt{h_6} :\vdash \Delta_9, dual(\mathtt{F_5}), \mathtt{F_7}\$\mathtt{F_8}} \; ax}{\dfrac{- :\vdash \Delta_3, \Delta_9, \mathtt{F_2}, \mathtt{F_7}\$\mathtt{F_8}}{- :\vdash \Delta_3, \Delta_9, ?\mathtt{F_2}, \mathtt{F_7}\$\mathtt{F_8}} \; ?} \; hCut$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_4}}{\bullet\mathtt{h_1} :\vdash ?\mathtt{F_4}, \Delta_3} \; ? \qquad \dfrac{\mathtt{h_5} :\vdash \Delta_8, \mathtt{F_6}, \mathtt{F_7}, !dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(?\mathtt{F_4}), \Delta_8, \mathtt{F_6}\$\mathtt{F_7}} \; \$}{- :\vdash \Delta_3, \Delta_8, \mathtt{F_6}\$\mathtt{F_7}} \; Cut$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_3, ?\mathtt{F_4}} \; ax \qquad \dfrac{}{\mathtt{h_5} :\vdash \Delta_8, \mathtt{F_6}, \mathtt{F_7}, !dual(\mathtt{F_4})} \; ax}{\dfrac{- :\vdash \Delta_3, \Delta_8, \mathtt{F_6}, \mathtt{F_7}}{- :\vdash \Delta_3, \Delta_8, \mathtt{F_6}\$\mathtt{F_7}} \; \$} \; hCut$$

- Case rule $\perp$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}, \mathtt{F_5}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_3, ?\mathtt{F_2}} \; ? \qquad \dfrac{\mathtt{h_6} :\vdash \Delta_7, dual(\mathtt{F_5})}{\bullet\mathtt{h_6} :\vdash dual(\mathtt{F_5}), \perp, \Delta_7} \; \perp}{- :\vdash (\Delta_3, ?\mathtt{F_2}), \perp, \Delta_7} \; Cut$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}, \mathtt{F_5}} \; ax \qquad \dfrac{}{\bullet\mathtt{h_6} :\vdash \Delta_7, \perp, dual(\mathtt{F_5})} \; ax}{\dfrac{- :\vdash \Delta_3, \Delta_7, \mathtt{F_2}, \perp}{- :\vdash \Delta_3, \Delta_7, \perp, ?\mathtt{F_2}} \; ?} \; hCut$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_4}}{\bullet\mathtt{h_1} :\vdash ?\mathtt{F_4}, \Delta_3} \; ? \qquad \dfrac{\mathtt{h_5} :\vdash \Delta_6, !dual(\mathtt{F_4})}{\bullet\mathtt{h_5} :\vdash dual(?\mathtt{F_4}), \perp, \Delta_6} \; \perp}{- :\vdash \Delta_3, \perp, \Delta_6} \; Cut$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_3, ?\mathtt{F_4}} \; ax \qquad \dfrac{}{\mathtt{h_5} :\vdash \Delta_6, !dual(\mathtt{F_4})} \; ax}{\dfrac{- :\vdash \Delta_3, \Delta_6}{- :\vdash \Delta_3, \Delta_6, \perp} \; \perp} \; hCut$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}, \mathtt{F_5}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_3, ?\mathtt{F_2}} \; ? \qquad \dfrac{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7}, dual(\mathtt{F_5}) \quad \mathtt{h_6} :\vdash \Delta_{10}, \mathtt{F_8}}{\bullet\mathtt{h_6} :\vdash dual(\mathtt{F_5}), \Delta_9, \Delta_{10}, \mathtt{F_7} \otimes \mathtt{F_8}} \; \otimes}{- :\vdash (\Delta_3, ?\mathtt{F_2}), \Delta_9, \Delta_{10}, \mathtt{F_7} \otimes \mathtt{F_8}} \; Cut$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}, \mathtt{F_5}} \; ax \qquad \dfrac{}{\bullet\mathtt{h_6} :\vdash \Delta_{10}, \Delta_9, dual(\mathtt{F_5}), \mathtt{F_7} \otimes \mathtt{F_8}} \; ax}{\dfrac{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, \mathtt{F_2}, \mathtt{F_7} \otimes \mathtt{F_8}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, ?\mathtt{F_2}, \mathtt{F_7} \otimes \mathtt{F_8}} \; ?} \; hCut$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}, \mathtt{F_5}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}, \Delta_3, ?\mathtt{F_2}} \; ? \qquad \dfrac{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7} \quad \mathtt{h_6} :\vdash \Delta_{10}, \mathtt{F_8}, dual(\mathtt{F_5})}{\bullet\mathtt{h_6} :\vdash dual(\mathtt{F_5}), \Delta_9, \Delta_{10}, \mathtt{F_7} \otimes \mathtt{F_8}} \; \otimes}{- :\vdash (\Delta_3, ?\mathtt{F_2}), \Delta_9, \Delta_{10}, \mathtt{F_7} \otimes \mathtt{F_8}} \; Cut$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_2}, \mathtt{F_5}} \; ax \qquad \dfrac{}{\bullet\mathtt{h_6} :\vdash \Delta_{10}, \Delta_9, dual(\mathtt{F_5}), \mathtt{F_7} \otimes \mathtt{F_8}} \; ax}{\dfrac{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, \mathtt{F_2}, \mathtt{F_7} \otimes \mathtt{F_8}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_9, ?\mathtt{F_2}, \mathtt{F_7} \otimes \mathtt{F_8}} \; ?} \; hCut$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_3, \mathtt{F_4}}{\bullet\mathtt{h_1} :\vdash ?\mathtt{F_4}, \Delta_3} \; ? \qquad \dfrac{\mathtt{h_5} :\vdash \Delta_8, \mathtt{F_6}, !dual(\mathtt{F_4}) \quad \mathtt{h_5} :\vdash \Delta_9, \mathtt{F_7}}{\bullet\mathtt{h_5} :\vdash dual(?\mathtt{F_4}), \Delta_8, \Delta_9, \mathtt{F_6} \otimes \mathtt{F_7}} \; \otimes}{- :\vdash \Delta_3, \Delta_8, \Delta_9, \mathtt{F_6} \otimes \mathtt{F_7}} \; Cut$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_3, ?\mathtt{F_4}} \; ax \quad \dfrac{}{\mathtt{h_5} :\vdash \Delta_8, \mathtt{F_6}, !dual(\mathtt{F_4})} \; ax}{- :\vdash \Delta_3, \Delta_8, \mathtt{F_6}} \; hCut \qquad \dfrac{}{- :\vdash \Delta_9, \mathtt{F_7}} \; ax}{- :\vdash \Delta_3, \Delta_8, \Delta_9, \mathtt{F_6} \otimes \mathtt{F_7}} \; \otimes$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_3, F_4}{\bullet h_1 :\vdash\, ?F_4, \Delta_3}\; ? \quad \cfrac{h_5 :\vdash \Delta_8, F_6 \quad h_5 :\vdash \Delta_9, F_7, !dual(F_4)}{\bullet h_5 :\vdash dual(?F_4), \Delta_8, \Delta_9, F_6 \otimes F_7}\; \otimes}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\; \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{- :\vdash \Delta_8, F_6}\; \text{ax} \quad \cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_3, ?F_4}\; \text{ax} \quad \cfrac{}{h_5 :\vdash \Delta_9, F_7, !dual(F_4)}\; \text{ax}}{- :\vdash \Delta_3, \Delta_9, F_7}\; \text{hCut}}{- :\vdash \Delta_3, \Delta_8, \Delta_9, F_6 \otimes F_7}\; \otimes$$

## 5.6 Status of $: OK

- Case rule **1**


- Case rule !


- Case rule ?$W$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_4, F_2, F_3, F_6}{\bullet h_1 :\vdash F_6, \Delta_4, F_2\$F_3}\; \$ \quad \cfrac{h_7 :\vdash \Delta_9, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_9, ?F_8}\; ?W}{- :\vdash (\Delta_4, F_2\$F_3), \Delta_9, ?F_8}\; \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_4, F_6, F_2\$F_3}\; \text{ax} \quad \cfrac{}{h_7 :\vdash \Delta_9, dual(F_6)}\; \text{ax}}{- :\vdash \Delta_4, \Delta_9, F_2\$F_3}\; \text{hCut}}{- :\vdash \Delta_4, \Delta_9, ?F_8, F_2\$F_3}\; ?W$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_4, F_5, F_6}{\bullet h_1 :\vdash F_5\$F_6, \Delta_4}\; \$ \quad \cfrac{h_7 :\vdash \Delta_9, dual(F_5) \otimes dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_9, ?F_8}\; ?W}{- :\vdash \Delta_4, \Delta_9, ?F_8}\; \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_4, F_5\$F_6}\; \text{ax} \quad \cfrac{}{h_7 :\vdash \Delta_9, dual(F_5) \otimes dual(F_6)}\; \text{ax}}{\cfrac{- :\vdash \Delta_4, \Delta_9}{- :\vdash \Delta_4, \Delta_9, ?F_8}\; W}\; \text{hCut}}{}$$

- Case rule ?$C$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_4, F_2, F_3, F_6}{\bullet h_1 :\vdash F_6, \Delta_4, F_2\$F_3}\; \$ \quad \cfrac{h_7 :\vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_9, ?F_8}\; ?C}{- :\vdash (\Delta_4, F_2\$F_3), \Delta_9, ?F_8}\; \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_4, F_6, F_2\$F_3}\; \text{ax} \quad \cfrac{}{h_7 :\vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}\; \text{ax}}{\cfrac{- :\vdash \Delta_4, \Delta_9, ?F_8, ?F_8, F_2\$F_3}{- :\vdash \Delta_4, \Delta_9, ?F_8, F_2\$F_3}\; ?C}\; \text{hCut}}{}$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_4, F_5, F_6}{\bullet h_1 :\vdash F_5\$F_6, \Delta_4}\; \$ \quad \cfrac{h_7 :\vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \otimes dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_9, ?F_8}\; ?C}{- :\vdash \Delta_4, \Delta_9, ?F_8}\; \text{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_4, F_5\$F_6}\; \text{ax} \quad \cfrac{}{h_7 :\vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \otimes dual(F_6)}\; \text{ax}}{\cfrac{- :\vdash \Delta_4, \Delta_9, ?F_8, ?F_8}{- :\vdash \Delta_4, \Delta_9, ?F_8}\; ?C}\; \text{hCut}}{}$$

- Case rule ?

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_3}, \mathtt{F_6}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_4, \mathtt{F_2}\$\mathtt{F_3}}\ \$ \quad \dfrac{\mathtt{h_7} :\vdash \Delta_9, \mathtt{F_8}, dual(\mathtt{F_6})}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_6}), \Delta_9, ?\mathtt{F_8}}\ ?}{- :\vdash (\Delta_4, \mathtt{F_2}\$\mathtt{F_3}), \Delta_9, ?\mathtt{F_8}}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\rule{0pt}{1em}}{\bullet\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_6}, \mathtt{F_2}\$\mathtt{F_3}}\ \mathtt{ax} \quad \dfrac{\rightarrow}{\mathtt{h_7} :\vdash \Delta_9, \mathtt{F_8}, dual(\mathtt{F_6})}\ \mathtt{ax}}{\dfrac{- :\vdash \Delta_4, \Delta_9, \mathtt{F_8}, \mathtt{F_2}\$\mathtt{F_3}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F_8}, \mathtt{F_2}\$\mathtt{F_3}}\ ?}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_5}, \mathtt{F_6}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}\$\mathtt{F_6}, \Delta_4}\ \$ \quad \dfrac{\mathtt{h_7} :\vdash \Delta_9, \mathtt{F_8}, dual(\mathtt{F_5}) \otimes dual(\mathtt{F_6})}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_5}\$\mathtt{F_6}), \Delta_9, ?\mathtt{F_8}}\ ?}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F_8}}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\rule{0pt}{1em}}{\bullet\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_5}\$\mathtt{F_6}}\ \mathtt{ax} \quad \dfrac{\rightarrow}{\mathtt{h_7} :\vdash \Delta_9, \mathtt{F_8}, dual(\mathtt{F_5}) \otimes dual(\mathtt{F_6})}\ \mathtt{ax}}{\dfrac{- :\vdash \Delta_4, \Delta_9, \mathtt{F_8}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F_8}}\ ?}\ \mathtt{hCut}$$

- Case rule $

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_3}, \mathtt{F_6}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_4, \mathtt{F_2}\$\mathtt{F_3}}\ \$ \quad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, \mathtt{F_9}, dual(\mathtt{F_6})}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_6}), \Delta_{10}, \mathtt{F_8}\$\mathtt{F_9}}\ \$}{- :\vdash (\Delta_4, \mathtt{F_2}\$\mathtt{F_3}), \Delta_{10}, \mathtt{F_8}\$\mathtt{F_9}}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\rule{0pt}{1em}}{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_3}, \mathtt{F_6}}\ \mathtt{ax} \quad \dfrac{\rightarrow}{\bullet\mathtt{h_7} :\vdash \Delta_{10}, dual(\mathtt{F_6}), \mathtt{F_8}\$\mathtt{F_9}}\ \mathtt{ax}}{\dfrac{- :\vdash \Delta_{10}, \Delta_4, \mathtt{F_2}, \mathtt{F_3}, \mathtt{F_8}\$\mathtt{F_9}}{- :\vdash \Delta_{10}, \Delta_4, \mathtt{F_2}\$\mathtt{F_3}, \mathtt{F_8}\$\mathtt{F_9}}\ \$}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_5}, \mathtt{F_6}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}\$\mathtt{F_6}, \Delta_4}\ \$ \quad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, \mathtt{F_9}, dual(\mathtt{F_5}) \otimes dual(\mathtt{F_6})}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_5}\$\mathtt{F_6}), \Delta_{10}, \mathtt{F_8}\$\mathtt{F_9}}\ \$}{- :\vdash \Delta_4, \Delta_{10}, \mathtt{F_8}\$\mathtt{F_9}}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\rule{0pt}{1em}}{\bullet\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_5}\$\mathtt{F_6}}\ \mathtt{ax} \quad \dfrac{\rightarrow}{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, \mathtt{F_9}, dual(\mathtt{F_5}) \otimes dual(\mathtt{F_6})}\ \mathtt{ax}}{\dfrac{- :\vdash \Delta_{10}, \Delta_4, \mathtt{F_8}, \mathtt{F_9}}{- :\vdash \Delta_{10}, \Delta_4, \mathtt{F_8}\$\mathtt{F_9}}\ \$}\ \mathtt{hCut}$$

- Case rule $\perp$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_3}, \mathtt{F_6}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_6}, \Delta_4, \mathtt{F_2}\$\mathtt{F_3}}\ \$ \quad \dfrac{\mathtt{h_7} :\vdash \Delta_8, dual(\mathtt{F_6})}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_6}), \perp, \Delta_8}\ \perp}{- :\vdash (\Delta_4, \mathtt{F_2}\$\mathtt{F_3}), \perp, \Delta_8}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\rule{0pt}{1em}}{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_3}, \mathtt{F_6}}\ \mathtt{ax} \quad \dfrac{\rightarrow}{\bullet\mathtt{h_7} :\vdash \Delta_8, \perp, dual(\mathtt{F_6})}\ \mathtt{ax}}{\dfrac{- :\vdash \Delta_4, \Delta_8, \mathtt{F_2}, \mathtt{F_3}, \perp}{- :\vdash \Delta_4, \Delta_8, \perp, \mathtt{F_2}\$\mathtt{F_3}}\ \$}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_5}, \mathtt{F_6}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_5}\$\mathtt{F_6}, \Delta_4}\ \$ \quad \dfrac{\mathtt{h_7} :\vdash \Delta_8, dual(\mathtt{F_5}) \otimes dual(\mathtt{F_6})}{\bullet\mathtt{h_7} :\vdash dual(\mathtt{F_5}\$\mathtt{F_6}), \perp, \Delta_8}\ \perp}{- :\vdash \Delta_4, \perp, \Delta_8}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\rule{0pt}{1em}}{\bullet\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_5}\$\mathtt{F_6}}\ \mathtt{ax} \quad \dfrac{\rightarrow}{\mathtt{h_7} :\vdash \Delta_8, dual(\mathtt{F_5}) \otimes dual(\mathtt{F_6})}\ \mathtt{ax}}{\dfrac{- :\vdash \Delta_4, \Delta_8}{- :\vdash \Delta_4, \Delta_8, \perp}\ \perp}\ \mathtt{hCut}$$

- Case rule $I$


- Case rule $\otimes$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_2, F_3, F_6}{\bullet h_1 :\vdash F_6, \Delta_4, F_2\$F_3}\ \$ \quad \dfrac{h_7 :\vdash \Delta_{10}, F_8, dual(F_6) \quad h_7 :\vdash \Delta_{11}, F_9}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \otimes}{\dfrac{- :\vdash (\Delta_4, F_2\$F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}{}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_1 :\vdash \Delta_4, F_2, F_3, F_6}\ \text{ax} \quad \dfrac{}{\bullet h_7 :\vdash \Delta_{10}, \Delta_{11}, dual(F_6), F_8 \otimes F_9}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, F_3, F_8 \otimes F_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2\$F_3, F_8 \otimes F_9}\ \$$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_2, F_3, F_6}{\bullet h_1 :\vdash F_6, \Delta_4, F_2\$F_3}\ \$ \quad \dfrac{h_7 :\vdash \Delta_{10}, F_8 \quad h_7 :\vdash \Delta_{11}, F_9, dual(F_6)}{\bullet h_7 :\vdash dual(F_6), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \otimes}{\dfrac{- :\vdash (\Delta_4, F_2\$F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}{}}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_1 :\vdash \Delta_4, F_2, F_3, F_6}\ \text{ax} \quad \dfrac{}{\bullet h_7 :\vdash \Delta_{10}, \Delta_{11}, dual(F_6), F_8 \otimes F_9}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2, F_3, F_8 \otimes F_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_2\$F_3, F_8 \otimes F_9}\ \$$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_5, F_6}{\bullet h_1 :\vdash F_5\$F_6, \Delta_4}\ \$ \quad \dfrac{h_7 :\vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6) \quad h_7 :\vdash \Delta_{11}, F_9}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \otimes}{- :\vdash \Delta_4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_4, F_5\$F_6}\ \text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_{10}, F_8, dual(F_5) \otimes dual(F_6)}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_4, F_8}\ \text{hCut} \quad \dfrac{}{- :\vdash \Delta_{11}, F_9}\ \text{ax}}{- :\vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_8 \otimes F_9}}{}\ \otimes$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_5, F_6}{\bullet h_1 :\vdash F_5\$F_6, \Delta_4}\ \$ \quad \dfrac{h_7 :\vdash \Delta_{10}, F_8 \quad h_7 :\vdash \Delta_{11}, F_9, dual(F_5) \otimes dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \otimes}{- :\vdash \Delta_4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_{10}, F_8}\ \text{ax} \quad \dfrac{\dfrac{}{\bullet h_1 :\vdash \Delta_4, F_5\$F_6}\ \text{ax} \quad \dfrac{}{h_7 :\vdash \Delta_{11}, F_9, dual(F_5) \otimes dual(F_6)}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_4, F_9}\ \text{hCut}}{- :\vdash \Delta_{10}, \Delta_{11}, \Delta_4, F_8 \otimes F_9}\ \otimes$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_5, F_6}{\bullet h_1 :\vdash F_5\$F_6, \Delta_4}\ \$ \quad \dfrac{h_7 :\vdash \Delta_8, dual(F_5) \quad h_7 :\vdash \Delta_9, dual(F_6)}{\bullet h_7 :\vdash dual(F_5\$F_6), \Delta_8, \Delta_9}\ \otimes}{- :\vdash \Delta_4, \Delta_8, \Delta_9}\ \text{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \Delta_4, F_5, F_6}\ \text{ax} \quad \dfrac{}{- :\vdash \Delta_9, dual(F_6)}\ \text{ax}}{- :\vdash \Delta_4, \Delta_9, F_5}\ \text{sCut} \quad \dfrac{}{- :\vdash \Delta_8, dual(F_5)}\ \text{ax}}{- :\vdash \Delta_4, \Delta_8, \Delta_9}\ \text{sCut}$$

## 5.7   Status of $\perp$:  OK

- Case rule **1**

$$\dfrac{\dfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \perp, \Delta_2}\ \perp \quad \dfrac{}{\bullet h_3 :\vdash dual(\perp), *}\ \mathbf{1}}{\dfrac{- :\vdash \Delta_2, *}{\dfrac{\rightarrow}{- :\vdash \Delta_2}\ \text{ax}}}\ \text{Cut}$$

- Case rule !

- Case rule ?$W$

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$$\cfrac{\cfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \qquad \cfrac{h_3 :\vdash \mathbf{1}, \Delta_5}{\bullet h_3 :\vdash dual(\bot), \Delta_5, ?F_4}\ ?W}{- :\vdash \Delta_2, \Delta_5, ?F_4}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_2, \bot}\ ax \qquad \overline{h_3 :\vdash \mathbf{1}, \Delta_5}\ ax}{- :\vdash \Delta_2, \Delta_5}\ hCut}{- :\vdash \Delta_2, \Delta_5, ?F_4}\ W$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2, F_4}{\bullet h_1 :\vdash F_4, \bot, \Delta_2}\ \bot \qquad \cfrac{h_5 :\vdash \Delta_7, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_7, ?F_6}\ ?W}{- :\vdash (\bot, \Delta_2), \Delta_7, ?F_6}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_2, F_4, \bot}\ ax \qquad \overline{h_5 :\vdash \Delta_7, dual(F_4)}\ ax}{- :\vdash \Delta_2, \Delta_7, \bot}\ hCut}{- :\vdash \Delta_2, \Delta_7, \bot, ?F_6}\ W$$

- Case rule $?C$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \qquad \cfrac{h_3 :\vdash \mathbf{1}, \Delta_5, ?F_4, ?F_4}{\bullet h_3 :\vdash dual(\bot), \Delta_5, ?F_4}\ ?C}{- :\vdash \Delta_2, \Delta_5, ?F_4}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_2, \bot}\ ax \qquad \overline{h_3 :\vdash \mathbf{1}, \Delta_5, ?F_4, ?F_4}\ ax}{- :\vdash \Delta_2, \Delta_5, ?F_4, ?F_4}\ hCut}{- :\vdash \Delta_2, \Delta_5, ?F_4}}{}\ ?C$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2, F_4}{\bullet h_1 :\vdash F_4, \bot, \Delta_2}\ \bot \qquad \cfrac{h_5 :\vdash \Delta_7, ?F_6, ?F_6, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_7, ?F_6}\ ?C}{- :\vdash (\bot, \Delta_2), \Delta_7, ?F_6}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_2, F_4, \bot}\ ax \qquad \overline{h_5 :\vdash \Delta_7, ?F_6, ?F_6, dual(F_4)}\ ax}{- :\vdash \Delta_2, \Delta_7, \bot, ?F_6, ?F_6}\ hCut}{- :\vdash \Delta_2, \Delta_7, \bot, ?F_6}\ ?C$$

- Case rule ?

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \qquad \cfrac{h_3 :\vdash \mathbf{1}, \Delta_5, F_4}{\bullet h_3 :\vdash dual(\bot), \Delta_5, ?F_4}\ ?}{- :\vdash \Delta_2, \Delta_5, ?F_4}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_2, \bot}\ ax \qquad \overline{h_3 :\vdash \mathbf{1}, \Delta_5, F_4}\ ax}{- :\vdash \Delta_2, \Delta_5, F_4}\ hCut}{- :\vdash \Delta_2, \Delta_5, ?F_4}\ ?$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2, F_4}{\bullet h_1 :\vdash F_4, \bot, \Delta_2}\ \bot \qquad \cfrac{h_5 :\vdash \Delta_7, F_6, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_7, ?F_6}\ ?}{- :\vdash (\bot, \Delta_2), \Delta_7, ?F_6}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_2, F_4, \bot}\ ax \qquad \overline{h_5 :\vdash \Delta_7, F_6, dual(F_4)}\ ax}{- :\vdash \Delta_2, \Delta_7, F_6, \bot}\ hCut}{- :\vdash \Delta_2, \Delta_7, \bot, ?F_6}\ ?$$

- Case rule $

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \qquad \cfrac{h_3 :\vdash \mathbf{1}, \Delta_6, F_4, F_5}{\bullet h_3 :\vdash dual(\bot), \Delta_6, F_4\$F_5}\ \$}{- :\vdash \Delta_2, \Delta_6, F_4\$F_5}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{\overline{\bullet h_1 :\vdash \Delta_2, \bot}\ ax \qquad \overline{h_3 :\vdash \mathbf{1}, \Delta_6, F_4, F_5}\ ax}{- :\vdash \Delta_2, \Delta_6, F_4, F_5}\ hCut}{- :\vdash \Delta_2, \Delta_6, F_4\$F_5}\ \$$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2, F_4}{\bullet h_1 :\vdash F_4, \bot, \Delta_2}\ \bot \qquad \cfrac{h_5 :\vdash \Delta_8, F_6, F_7, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, F_6 \$ F_7}\ \$}{- :\vdash (\bot, \Delta_2), \Delta_8, F_6 \$ F_7}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, F_4, \bot}\ ax \qquad \cfrac{}{h_5 :\vdash \Delta_8, F_6, F_7, dual(F_4)}\ ax}{\cfrac{\cfrac{- :\vdash \Delta_2, \Delta_8, F_6, F_7, \bot}{- :\vdash \Delta_2, \Delta_8, \bot, F_6 \$ F_7}\ \$}{}}\ hCut$$

- Case rule $\bot$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \qquad \cfrac{h_3 :\vdash \mathbf{1}, \Delta_4}{\bullet h_3 :\vdash dual(\bot), \bot, \Delta_4}\ \bot}{- :\vdash \Delta_2, \bot, \Delta_4}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, \bot}\ ax \qquad \cfrac{}{h_3 :\vdash \mathbf{1}, \Delta_4}\ ax}{\cfrac{- :\vdash \Delta_2, \Delta_4}{- :\vdash \Delta_2, \Delta_4, \bot}\ \bot}\ hCut$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2, F_4}{\bullet h_1 :\vdash F_4, \bot, \Delta_2}\ \bot \qquad \cfrac{h_5 :\vdash \Delta_6, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \bot, \Delta_6}\ \bot}{- :\vdash (\bot, \Delta_2), \bot, \Delta_6}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 :\vdash \Delta_2, F_4}\ ax \qquad \cfrac{}{\bullet h_5 :\vdash \Delta_6, \bot, dual(F_4)}\ ax}{\cfrac{- :\vdash \Delta_2, \Delta_6, \bot}{- :\vdash \Delta_2, \Delta_6, \bot, \bot}\ \bot}\ hCut$$

- Case rule $I$


- Case rule $\otimes$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \qquad \cfrac{h_3 :\vdash \mathbf{1}, \Delta_6, F_4 \quad h_3 :\vdash \Delta_7, F_5}{\bullet h_3 :\vdash dual(\bot), \Delta_6, \Delta_7, F_4 \otimes F_5}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_7, F_4 \otimes F_5}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, \bot}\ ax \quad \cfrac{}{h_3 :\vdash \mathbf{1}, \Delta_6, F_4}\ ax}{- :\vdash \Delta_2, \Delta_6, F_4}\ hCut \qquad \cfrac{}{- :\vdash \Delta_7, F_5}\ ax}{- :\vdash \Delta_2, \Delta_6, \Delta_7, F_4 \otimes F_5}\ \otimes$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2}{\bullet h_1 :\vdash \bot, \Delta_2}\ \bot \qquad \cfrac{h_3 :\vdash \Delta_6, F_4 \quad h_3 :\vdash \mathbf{1}, \Delta_7, F_5}{\bullet h_3 :\vdash dual(\bot), \Delta_6, \Delta_7, F_4 \otimes F_5}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_7, F_4 \otimes F_5}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{- :\vdash \Delta_6, F_4}\ ax \qquad \cfrac{\cfrac{}{\bullet h_1 :\vdash \Delta_2, \bot}\ ax \quad \cfrac{}{h_3 :\vdash \mathbf{1}, \Delta_7, F_5}\ ax}{- :\vdash \Delta_2, \Delta_7, F_5}\ hCut}{- :\vdash \Delta_2, \Delta_6, \Delta_7, F_4 \otimes F_5}\ \otimes$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2, F_4}{\bullet h_1 :\vdash F_4, \bot, \Delta_2}\ \bot \qquad \cfrac{h_5 :\vdash \Delta_8, F_6, dual(F_4) \quad h_5 :\vdash \Delta_9, F_7}{\bullet h_5 :\vdash dual(F_4), \Delta_8, \Delta_9, F_6 \otimes F_7}\ \otimes}{- :\vdash (\bot, \Delta_2), \Delta_8, \Delta_9, F_6 \otimes F_7}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 :\vdash \Delta_2, F_4}\ ax \qquad \cfrac{}{\bullet h_5 :\vdash \Delta_8, \Delta_9, dual(F_4), F_6 \otimes F_7}\ ax}{\cfrac{- :\vdash \Delta_2, \Delta_8, \Delta_9, F_6 \otimes F_7}{- :\vdash \Delta_2, \Delta_8, \Delta_9, \bot, F_6 \otimes F_7}\ \bot}\ hCut$$

$$\cfrac{\cfrac{h_1 :\vdash \Delta_2, F_4}{\bullet h_1 :\vdash F_4, \bot, \Delta_2}\ \bot \qquad \cfrac{h_5 :\vdash \Delta_8, F_6 \quad h_5 :\vdash \Delta_9, F_7, dual(F_4)}{\bullet h_5 :\vdash dual(F_4), \Delta_8, \Delta_9, F_6 \otimes F_7}\ \otimes}{- :\vdash (\bot, \Delta_2), \Delta_8, \Delta_9, F_6 \otimes F_7}\ Cut$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{h_1 :\vdash \Delta_2, F_4}\ ax \qquad \cfrac{}{\bullet h_5 :\vdash \Delta_8, \Delta_9, dual(F_4), F_6 \otimes F_7}\ ax}{\cfrac{- :\vdash \Delta_2, \Delta_8, \Delta_9, F_6 \otimes F_7}{- :\vdash \Delta_2, \Delta_8, \Delta_9, \bot, F_6 \otimes F_7}\ \bot}\ hCut$$

## 5.8  Status of $I$:  `OK`

- Case rule **1**


- Case rule **!**


- Case rule ?$W$

$$\cfrac{\cfrac{\phantom{x}}{\bullet\mathtt{h}_1 :\vdash p(\mathbf{n}_3), \hat{}(\mathbf{n}_3)}\ I \quad \cfrac{\mathtt{h}_4 :\vdash \Delta_6, \hat{}(\mathbf{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(p(\mathbf{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?W}{- :\vdash \hat{}(\mathbf{n}_3), \Delta_6, ?\mathtt{F}_5}\ \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\cfrac{\phantom{x}}{- :\vdash \Delta_6, \hat{}(\mathbf{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, \hat{}(\mathbf{n}_3)}\ W}{}$$

$$\cfrac{\cfrac{\phantom{x}}{\bullet\mathtt{h}_1 :\vdash \hat{}(\mathbf{n}_3), p(\mathbf{n}_3)}\ I \quad \cfrac{\mathtt{h}_4 :\vdash \Delta_6, p(\mathbf{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(\hat{}(\mathbf{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?W}{- :\vdash p(\mathbf{n}_3), \Delta_6, ?\mathtt{F}_5}\ \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\phantom{x}}{- :\vdash \Delta_6, p(\mathbf{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, p(\mathbf{n}_3)}\ ?W$$

- Case rule ?$C$

$$\cfrac{\cfrac{\phantom{x}}{\bullet\mathtt{h}_1 :\vdash p(\mathbf{n}_3), \hat{}(\mathbf{n}_3)}\ I \quad \cfrac{\mathtt{h}_4 :\vdash \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, \hat{}(\mathbf{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(p(\mathbf{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?C}{- :\vdash \hat{}(\mathbf{n}_3), \Delta_6, ?\mathtt{F}_5}\ \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\cfrac{\phantom{x}}{- :\vdash \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, \hat{}(\mathbf{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, \hat{}(\mathbf{n}_3)}\ ?C}{}$$

$$\cfrac{\cfrac{\phantom{x}}{\bullet\mathtt{h}_1 :\vdash \hat{}(\mathbf{n}_3), p(\mathbf{n}_3)}\ I \quad \cfrac{\mathtt{h}_4 :\vdash \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, p(\mathbf{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(\hat{}(\mathbf{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?C}{- :\vdash p(\mathbf{n}_3), \Delta_6, ?\mathtt{F}_5}\ \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\phantom{x}}{- :\vdash \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, p(\mathbf{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, p(\mathbf{n}_3)}\ ?C$$

- Case rule ?

$$\cfrac{\cfrac{\phantom{x}}{\bullet\mathtt{h}_1 :\vdash p(\mathbf{n}_3), \hat{}(\mathbf{n}_3)}\ I \quad \cfrac{\mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_5, \hat{}(\mathbf{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(p(\mathbf{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?}{- :\vdash \hat{}(\mathbf{n}_3), \Delta_6, ?\mathtt{F}_5}\ \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\cfrac{\phantom{x}}{- :\vdash \Delta_6, \mathtt{F}_5, \hat{}(\mathbf{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, \hat{}(\mathbf{n}_3)}\ ?}{}$$

$$\cfrac{\cfrac{\phantom{x}}{\bullet\mathtt{h}_1 :\vdash \hat{}(\mathbf{n}_3), p(\mathbf{n}_3)}\ I \quad \cfrac{\mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_5, p(\mathbf{n}_3)}{\bullet\mathtt{h}_4 :\vdash dual(\hat{}(\mathbf{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?}{- :\vdash p(\mathbf{n}_3), \Delta_6, ?\mathtt{F}_5}\ \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\phantom{x}}{- :\vdash \Delta_6, \mathtt{F}_5, p(\mathbf{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, p(\mathbf{n}_3)}\ ?$$

- Case rule **$**

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash p(n_3),\,\hat{}(n_3)}\ I
\qquad
\cfrac{h_4 :\vdash \Delta_7, F_5, F_6, \hat{}(n_3)}{\bullet h_4 :\vdash dual(p(n_3)), \Delta_7, F_5\$F_6}\ \$
}{- :\vdash \hat{}(n_3), \Delta_7, F_5\$F_6}\ Cut
$$

$$
\rightarrow
$$

$$
\cfrac{\cfrac{}{- :\vdash \Delta_7, F_5, F_6, \hat{}(n_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, \hat{}(n_3), F_5\$F_6}\ \$
$$

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash \hat{}(n_3),\,p(n_3)}\ I
\qquad
\cfrac{h_4 :\vdash \Delta_7, F_5, F_6, p(n_3)}{\bullet h_4 :\vdash dual(\hat{}(n_3)), \Delta_7, F_5\$F_6}\ \$
}{- :\vdash p(n_3), \Delta_7, F_5\$F_6}\ Cut
$$

$$
\rightarrow
$$

$$
\cfrac{\cfrac{}{- :\vdash \Delta_7, F_5, F_6, p(n_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, p(n_3), F_5\$F_6}\ \$
$$

- Case rule $\perp$

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash p(n_3),\,\hat{}(n_3)}\ I
\qquad
\cfrac{h_4 :\vdash \Delta_5, \hat{}(n_3)}{\bullet h_4 :\vdash dual(p(n_3)), \perp, \Delta_5}\ \perp
}{- :\vdash \hat{}(n_3), \perp, \Delta_5}\ Cut
$$

$$
\rightarrow
$$

$$
\cfrac{\cfrac{}{- :\vdash \Delta_5, \hat{}(n_3)}\ \mathtt{ax}}{- :\vdash \Delta_5, \perp, \hat{}(n_3)}\ \perp
$$

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash \hat{}(n_3),\,p(n_3)}\ I
\qquad
\cfrac{h_4 :\vdash \Delta_5, p(n_3)}{\bullet h_4 :\vdash dual(\hat{}(n_3)), \perp, \Delta_5}\ \perp
}{- :\vdash p(n_3), \perp, \Delta_5}\ Cut
$$

$$
\rightarrow
$$

$$
\cfrac{\cfrac{}{- :\vdash \Delta_5, p(n_3)}\ \mathtt{ax}}{- :\vdash \Delta_5, \perp, p(n_3)}\ \perp
$$

- Case rule $I$

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash p(n_4),\,\hat{}(n_4)}\ I
\qquad
\cfrac{}{\bullet h_3 :\vdash dual(p(n_4)), p(n_4)}\ I
}{- :\vdash \hat{}(n_4), p(n_4)}\ Cut
$$

$$
\rightarrow
$$

$$
\cfrac{}{- :\vdash p(n_4), \hat{}(n_4)}\ \mathtt{ax}
$$

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash \hat{}(n_4),\,p(n_4)}\ I
\qquad
\cfrac{}{\bullet h_3 :\vdash dual(\hat{}(n_4)), \hat{}(n_4)}\ I
}{- :\vdash p(n_4), \hat{}(n_4)}\ Cut
$$

$$
\rightarrow
$$

$$
\cfrac{}{- :\vdash p(n_4), \hat{}(n_4)}\ I
$$

- Case rule $\otimes$

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash p(n_3),\,\hat{}(n_3)}\ I
\qquad
\cfrac{h_4 :\vdash \Delta_7, F_5, \hat{}(n_3) \quad h_4 :\vdash \Delta_8, F_6}{\bullet h_4 :\vdash dual(p(n_3)), \Delta_7, \Delta_8, F_5 \otimes F_6}\ \otimes
}{- :\vdash \hat{}(n_3), \Delta_7, \Delta_8, F_5 \otimes F_6}\ Cut
$$

$$
\rightarrow
$$

$$
\cfrac{
\cfrac{}{- :\vdash \Delta_7, F_5, \hat{}(n_3)}\ \mathtt{ax}
\qquad
\cfrac{}{- :\vdash \Delta_8, F_6}\ \mathtt{ax}
}{- :\vdash \Delta_7, \Delta_8, \hat{}(n_3), F_5 \otimes F_6}\ \otimes
$$

$$
\cfrac{
\cfrac{}{\bullet h_1 :\vdash p(n_3),\,\hat{}(n_3)}\ I
\qquad
\cfrac{h_4 :\vdash \Delta_7, F_5 \quad h_4 :\vdash \Delta_8, F_6, \hat{}(n_3)}{\bullet h_4 :\vdash dual(p(n_3)), \Delta_7, \Delta_8, F_5 \otimes F_6}\ \otimes
}{- :\vdash \hat{}(n_3), \Delta_7, \Delta_8, F_5 \otimes F_6}\ Cut
$$

$$
\rightarrow
$$

$$
\cfrac{
\cfrac{}{- :\vdash \Delta_7, F_5}\ \mathtt{ax}
\qquad
\cfrac{}{- :\vdash \Delta_8, F_6, \hat{}(n_3)}\ \mathtt{ax}
}{- :\vdash \Delta_7, \Delta_8, \hat{}(n_3), F_5 \otimes F_6}\ \otimes
$$

$$
\cfrac{}{\bullet\mathtt{h_1} :\vdash\, \hat{}\,(\mathtt{n_3}), p(\mathtt{n_3})}\ I
\qquad
\cfrac{\mathtt{h_4} :\vdash \Delta_7, \mathtt{F_5}, p(\mathtt{n_3}) \quad \mathtt{h_4} :\vdash \Delta_8, \mathtt{F_6}}{\bullet\mathtt{h_4} :\vdash dual(\hat{}\,(\mathtt{n_3})), \Delta_7, \Delta_8, \mathtt{F_5}\otimes\mathtt{F_6}}\ \otimes
$$
$$
\cfrac{}{- :\vdash p(\mathtt{n_3}), \Delta_7, \Delta_8, \mathtt{F_5}\otimes\mathtt{F_6}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{\cfrac{}{- :\vdash \Delta_7, \mathtt{F_5}, p(\mathtt{n_3})}\ \mathtt{ax} \quad \cfrac{}{- :\vdash \Delta_8, \mathtt{F_6}}\ \mathtt{ax}}{- :\vdash \Delta_7, \Delta_8, p(\mathtt{n_3}), \mathtt{F_5}\otimes\mathtt{F_6}}\ \otimes
$$

$$
\cfrac{}{\bullet\mathtt{h_1} :\vdash\, \hat{}\,(\mathtt{n_3}), p(\mathtt{n_3})}\ I
\qquad
\cfrac{\mathtt{h_4} :\vdash \Delta_7, \mathtt{F_5} \quad \mathtt{h_4} :\vdash \Delta_8, \mathtt{F_6}, p(\mathtt{n_3})}{\bullet\mathtt{h_4} :\vdash dual(\hat{}\,(\mathtt{n_3})), \Delta_7, \Delta_8, \mathtt{F_5}\otimes\mathtt{F_6}}\ \otimes
$$
$$
\cfrac{}{- :\vdash p(\mathtt{n_3}), \Delta_7, \Delta_8, \mathtt{F_5}\otimes\mathtt{F_6}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{\cfrac{}{- :\vdash \Delta_7, \mathtt{F_5}}\ \mathtt{ax} \quad \cfrac{}{- :\vdash \Delta_8, \mathtt{F_6}, p(\mathtt{n_3})}\ \mathtt{ax}}{- :\vdash \Delta_7, \Delta_8, p(\mathtt{n_3}), \mathtt{F_5}\otimes\mathtt{F_6}}\ \otimes
$$

## 5.9  Status of ⊗:  OK

- Case rule **1**

- Case rule !

- Case rule $?W$

$$
\cfrac{\cfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_7} \quad \mathtt{h_1} :\vdash \Delta_6, \mathtt{F_3}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_4, \Delta_6, \mathtt{F_2}\otimes\mathtt{F_3}}\ \otimes \quad \cfrac{\mathtt{h_8} :\vdash \Delta_{10}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{10}, ?\mathtt{F_9}}\ ?W}{- :\vdash (\Delta_4, \Delta_6, \mathtt{F_2}\otimes\mathtt{F_3}), \Delta_{10}, ?\mathtt{F_9}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_6, \mathtt{F_7}, \mathtt{F_2}\otimes\mathtt{F_3}}\ \mathtt{ax} \quad \cfrac{}{\mathtt{h_8} :\vdash \Delta_{10}, dual(\mathtt{F_7})}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_6, \mathtt{F_2}\otimes\mathtt{F_3}}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_6, ?\mathtt{F_9}, \mathtt{F_2}\otimes\mathtt{F_3}}\ W
$$

$$
\cfrac{\cfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2} \quad \mathtt{h_1} :\vdash \Delta_5, \mathtt{F_3}, \mathtt{F_7}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_4, \Delta_5, \mathtt{F_2}\otimes\mathtt{F_3}}\ \otimes \quad \cfrac{\mathtt{h_8} :\vdash \Delta_{10}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{10}, ?\mathtt{F_9}}\ ?W}{- :\vdash (\Delta_4, \Delta_5, \mathtt{F_2}\otimes\mathtt{F_3}), \Delta_{10}, ?\mathtt{F_9}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_5, \mathtt{F_7}, \mathtt{F_2}\otimes\mathtt{F_3}}\ \mathtt{ax} \quad \cfrac{}{\mathtt{h_8} :\vdash \Delta_{10}, dual(\mathtt{F_7})}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, \mathtt{F_2}\otimes\mathtt{F_3}}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathtt{F_9}, \mathtt{F_2}\otimes\mathtt{F_3}}\ W
$$

$$
\cfrac{\cfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_6} \quad \mathtt{h_1} :\vdash \Delta_5, \mathtt{F_7}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_6}\otimes\mathtt{F_7}, \Delta_4, \Delta_5}\ \otimes \quad \cfrac{\mathtt{h_8} :\vdash \Delta_{10}, dual(\mathtt{F_6})\$ dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_6}\otimes\mathtt{F_7}), \Delta_{10}, ?\mathtt{F_9}}\ ?W}{- :\vdash (\Delta_4, \Delta_5), \Delta_{10}, ?\mathtt{F_9}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_5, \mathtt{F_6}\otimes\mathtt{F_7}}\ \mathtt{ax} \quad \cfrac{}{\mathtt{h_8} :\vdash \Delta_{10}, dual(\mathtt{F_6})\$ dual(\mathtt{F_7})}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathtt{F_9}}\ W
$$

- Case rule $?C$

$$
\cfrac{\cfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_7} \quad \mathtt{h_1} :\vdash \Delta_6, \mathtt{F_3}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_4, \Delta_6, \mathtt{F_2}\otimes\mathtt{F_3}}\ \otimes \quad \cfrac{\mathtt{h_8} :\vdash \Delta_{10}, ?\mathtt{F_9}, ?\mathtt{F_9}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{10}, ?\mathtt{F_9}}\ ?C}{- :\vdash (\Delta_4, \Delta_6, \mathtt{F_2}\otimes\mathtt{F_3}), \Delta_{10}, ?\mathtt{F_9}}\ \mathtt{Cut}
$$
$$
\rightarrow
$$
$$
\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_6, \mathtt{F_7}, \mathtt{F_2}\otimes\mathtt{F_3}}\ \mathtt{ax} \quad \cfrac{}{\mathtt{h_8} :\vdash \Delta_{10}, ?\mathtt{F_9}, ?\mathtt{F_9}, dual(\mathtt{F_7})}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_6, ?\mathtt{F_9}, ?\mathtt{F_9}, \mathtt{F_2}\otimes\mathtt{F_3}}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_6, ?\mathtt{F_9}, \mathtt{F_2}\otimes\mathtt{F_3}}\ ?C
$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2} \quad \mathtt{h_1} :\vdash \Delta_5, \mathtt{F_3}, \mathtt{F_7}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_4, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \Delta_{10}, ?\mathtt{F_9}, ?\mathtt{F_9}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{10}, ?\mathtt{F_9}} \, ?C}{- :\vdash (\Delta_4, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}), \Delta_{10}, ?\mathtt{F_9}} \, \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_5, \mathtt{F_7}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{10}, ?\mathtt{F_9}, ?\mathtt{F_9}, dual(\mathtt{F_7})} \, \text{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathtt{F_9}, ?\mathtt{F_9}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathtt{F_9}, \mathtt{F_2} \otimes \mathtt{F_3}} \, ?C$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_6} \quad \mathtt{h_1} :\vdash \Delta_5, \mathtt{F_7}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_6} \otimes \mathtt{F_7}, \Delta_4, \Delta_5} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \Delta_{10}, ?\mathtt{F_9}, ?\mathtt{F_9}, dual(\mathtt{F_6})\$dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_6} \otimes \mathtt{F_7}), \Delta_{10}, ?\mathtt{F_9}} \, ?C}{- :\vdash (\Delta_4, \Delta_5), \Delta_{10}, ?\mathtt{F_9}} \, \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_5, \mathtt{F_6} \otimes \mathtt{F_7}} \, \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{10}, ?\mathtt{F_9}, ?\mathtt{F_9}, dual(\mathtt{F_6})\$dual(\mathtt{F_7})} \, \text{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathtt{F_9}, ?\mathtt{F_9}} \, \text{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathtt{F_9}} \, ?C$$

- Case rule ?

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_7} \quad \mathtt{h_1} :\vdash \Delta_6, \mathtt{F_3}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_4, \Delta_6, \mathtt{F_2} \otimes \mathtt{F_3}} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \Delta_{10}, \mathtt{F_9}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{10}, ?\mathtt{F_9}} \, ?}{- :\vdash (\Delta_4, \Delta_6, \mathtt{F_2} \otimes \mathtt{F_3}), \Delta_{10}, ?\mathtt{F_9}} \, \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_6, \mathtt{F_7}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{10}, \mathtt{F_9}, dual(\mathtt{F_7})} \, \text{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_6, \mathtt{F_9}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_6, ?\mathtt{F_9}, \mathtt{F_2} \otimes \mathtt{F_3}} \, ?$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2} \quad \mathtt{h_1} :\vdash \Delta_5, \mathtt{F_3}, \mathtt{F_7}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_4, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \Delta_{10}, \mathtt{F_9}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{10}, ?\mathtt{F_9}} \, ?}{- :\vdash (\Delta_4, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}), \Delta_{10}, ?\mathtt{F_9}} \, \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_5, \mathtt{F_7}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{10}, \mathtt{F_9}, dual(\mathtt{F_7})} \, \text{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, \mathtt{F_9}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathtt{F_9}, \mathtt{F_2} \otimes \mathtt{F_3}} \, ?$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_6} \quad \mathtt{h_1} :\vdash \Delta_5, \mathtt{F_7}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_6} \otimes \mathtt{F_7}, \Delta_4, \Delta_5} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \Delta_{10}, \mathtt{F_9}, dual(\mathtt{F_6})\$dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_6} \otimes \mathtt{F_7}), \Delta_{10}, ?\mathtt{F_9}} \, ?}{- :\vdash (\Delta_4, \Delta_5), \Delta_{10}, ?\mathtt{F_9}} \, \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_5, \mathtt{F_6} \otimes \mathtt{F_7}} \, \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{10}, \mathtt{F_9}, dual(\mathtt{F_6})\$dual(\mathtt{F_7})} \, \text{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, \mathtt{F_9}} \, \text{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_5, ?\mathtt{F_9}} \, ?$$

- Case rule $

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2}, \mathtt{F_7} \quad \mathtt{h_1} :\vdash \Delta_6, \mathtt{F_3}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_4, \Delta_6, \mathtt{F_2} \otimes \mathtt{F_3}} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \Delta_{11}, \mathtt{F_9}, \mathtt{F_{10}}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}} \, \$}{- :\vdash (\Delta_4, \Delta_6, \mathtt{F_2} \otimes \mathtt{F_3}), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}} \, \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_6, \mathtt{F_7}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{11}, \mathtt{F_{10}}, \mathtt{F_9}, dual(\mathtt{F_7})} \, \text{ax}}{- :\vdash \Delta_{11}, \Delta_4, \Delta_6, \mathtt{F_{10}}, \mathtt{F_9}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{hCut}}{- :\vdash \Delta_{11}, \Delta_4, \Delta_6, \mathtt{F_9}\$\mathtt{F_{10}}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \$$$

$$\dfrac{\dfrac{\mathtt{h_1} :\vdash \Delta_4, \mathtt{F_2} \quad \mathtt{h_1} :\vdash \Delta_5, \mathtt{F_3}, \mathtt{F_7}}{\bullet\mathtt{h_1} :\vdash \mathtt{F_7}, \Delta_4, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}} \otimes \quad \dfrac{\mathtt{h_8} :\vdash \Delta_{11}, \mathtt{F_9}, \mathtt{F_{10}}, dual(\mathtt{F_7})}{\bullet\mathtt{h_8} :\vdash dual(\mathtt{F_7}), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}} \, \$}{- :\vdash (\Delta_4, \Delta_5, \mathtt{F_2} \otimes \mathtt{F_3}), \Delta_{11}, \mathtt{F_9}\$\mathtt{F_{10}}} \, \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1} :\vdash \Delta_4, \Delta_5, \mathtt{F_7}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{ax} \quad \dfrac{}{\mathtt{h_8} :\vdash \Delta_{11}, \mathtt{F_{10}}, \mathtt{F_9}, dual(\mathtt{F_7})} \, \text{ax}}{- :\vdash \Delta_{11}, \Delta_4, \Delta_5, \mathtt{F_{10}}, \mathtt{F_9}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \text{hCut}}{- :\vdash \Delta_{11}, \Delta_4, \Delta_5, \mathtt{F_9}\$\mathtt{F_{10}}, \mathtt{F_2} \otimes \mathtt{F_3}} \, \$$$

$$
\cfrac{
  \cfrac{h_1 :\vdash \Delta_4, F_6 \quad h_1 :\vdash \Delta_5, F_7}{\bullet h_1 :\vdash F_6 \otimes F_7, \Delta_4, \Delta_5}\ \otimes
  \quad
  \cfrac{h_8 :\vdash \Delta_{11}, F_9, F_{10}, dual(F_6)\$dual(F_7)}{\bullet h_8 :\vdash dual(F_6 \otimes F_7), \Delta_{11}, F_9\$F_{10}}\ \$
}{- :\vdash (\Delta_4, \Delta_5), \Delta_{11}, F_9\$F_{10}}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet h_1 :\vdash \Delta_4, \Delta_5, F_6 \otimes F_7}\ \text{ax}
    \quad
    \cfrac{}{h_8 :\vdash \Delta_{11}, F_{10}, F_9, dual(F_6)\$dual(F_7)}\ \text{ax}
  }{- :\vdash \Delta_{11}, \Delta_4, \Delta_5, F_{10}, F_9}\ \text{hCut}
}{- :\vdash \Delta_{11}, \Delta_4, \Delta_5, F_9\$F_{10}}\ \$
$$

$$
\cfrac{
  \cfrac{h_1 :\vdash \Delta_4, F_6 \quad h_1 :\vdash \Delta_5, F_7}{\bullet h_1 :\vdash F_6 \otimes F_7, \Delta_4, \Delta_5}\ \otimes
  \quad
  \cfrac{h_8 :\vdash \Delta_9, dual(F_6), dual(F_7)}{\bullet h_8 :\vdash dual(F_6 \otimes F_7), \Delta_9}\ \$
}{- :\vdash (\Delta_4, \Delta_5), \Delta_9}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{}{- :\vdash \Delta_4, F_6}\ \text{ax}
  \quad
  \cfrac{
    \cfrac{}{- :\vdash \Delta_5, F_7}\ \text{ax}
    \quad
    \cfrac{}{- :\vdash \Delta_9, dual(F_6), dual(F_7)}\ \text{ax}
  }{- :\vdash \Delta_5, \Delta_9, dual(F_6)}\ \text{sCut}
}{- :\vdash \Delta_4, \Delta_5, \Delta_9}\ \text{sCut}
$$

- Case rule $\perp$

$$
\cfrac{
  \cfrac{h_1 :\vdash \Delta_4, F_2, F_7 \quad h_1 :\vdash \Delta_6, F_3}{\bullet h_1 :\vdash F_7, \Delta_4, \Delta_6, F_2 \otimes F_3}\ \otimes
  \quad
  \cfrac{h_8 :\vdash \Delta_9, dual(F_7)}{\bullet h_8 :\vdash dual(F_7), \perp, \Delta_9}\ \perp
}{- :\vdash (\Delta_4, \Delta_6, F_2 \otimes F_3), \perp, \Delta_9}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet h_1 :\vdash \Delta_4, \Delta_6, F_7, F_2 \otimes F_3}\ \text{ax}
    \quad
    \cfrac{}{h_8 :\vdash \Delta_9, dual(F_7)}\ \text{ax}
  }{- :\vdash \Delta_4, \Delta_6, \Delta_9, F_2 \otimes F_3}\ \text{hCut}
}{- :\vdash \Delta_4, \Delta_6, \Delta_9, \perp, F_2 \otimes F_3}\ \perp
$$

$$
\cfrac{
  \cfrac{h_1 :\vdash \Delta_4, F_2 \quad h_1 :\vdash \Delta_5, F_3, F_7}{\bullet h_1 :\vdash F_7, \Delta_4, \Delta_5, F_2 \otimes F_3}\ \otimes
  \quad
  \cfrac{h_8 :\vdash \Delta_9, dual(F_7)}{\bullet h_8 :\vdash dual(F_7), \perp, \Delta_9}\ \perp
}{- :\vdash (\Delta_4, \Delta_5, F_2 \otimes F_3), \perp, \Delta_9}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet h_1 :\vdash \Delta_4, \Delta_5, F_7, F_2 \otimes F_3}\ \text{ax}
    \quad
    \cfrac{}{h_8 :\vdash \Delta_9, dual(F_7)}\ \text{ax}
  }{- :\vdash \Delta_4, \Delta_5, \Delta_9, F_2 \otimes F_3}\ \text{hCut}
}{- :\vdash \Delta_4, \Delta_5, \Delta_9, \perp, F_2 \otimes F_3}\ \perp
$$

$$
\cfrac{
  \cfrac{h_1 :\vdash \Delta_4, F_6 \quad h_1 :\vdash \Delta_5, F_7}{\bullet h_1 :\vdash F_6 \otimes F_7, \Delta_4, \Delta_5}\ \otimes
  \quad
  \cfrac{h_8 :\vdash \Delta_9, dual(F_6)\$dual(F_7)}{\bullet h_8 :\vdash dual(F_6 \otimes F_7), \perp, \Delta_9}\ \perp
}{- :\vdash (\Delta_4, \Delta_5), \perp, \Delta_9}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{\bullet h_1 :\vdash \Delta_4, \Delta_5, F_6 \otimes F_7}\ \text{ax}
    \quad
    \cfrac{}{h_8 :\vdash \Delta_9, dual(F_6)\$dual(F_7)}\ \text{ax}
  }{- :\vdash \Delta_4, \Delta_5, \Delta_9}\ \text{hCut}
}{- :\vdash \Delta_4, \Delta_5, \Delta_9, \perp}\ \perp
$$

- Case rule $I$

- Case rule $\otimes$

$$
\cfrac{
  \cfrac{h_1 :\vdash \Delta_4, F_2, F_7 \quad h_1 :\vdash \Delta_6, F_3}{\bullet h_1 :\vdash F_7, \Delta_4, \Delta_6, F_2 \otimes F_3}\ \otimes
  \quad
  \cfrac{h_8 :\vdash \Delta_{11}, F_9, dual(F_7) \quad h_8 :\vdash \Delta_{12}, F_{10}}{\bullet h_8 :\vdash dual(F_7), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \otimes
}{- :\vdash (\Delta_4, \Delta_6, F_2 \otimes F_3), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \text{Cut}
$$

$$\rightarrow$$

$$
\cfrac{
  \cfrac{
    \cfrac{}{h_1 :\vdash \Delta_4, F_2, F_7}\ \text{ax}
    \quad
    \cfrac{}{\bullet h_8 :\vdash \Delta_{11}, \Delta_{12}, dual(F_7), F_9 \otimes F_{10}}\ \text{ax}
  }{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_4, F_2, F_9 \otimes F_{10}}\ \text{hCut}
  \quad
  \cfrac{}{- :\vdash \Delta_6, F_3}\ \text{ax}
}{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_4, \Delta_6, F_2 \otimes F_3, F_9 \otimes F_{10}}\ \otimes
$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_2, F_7 \quad h_1 :\vdash \Delta_6, F_3}{\bullet h_1 :\vdash F_7, \Delta_4, \Delta_6, F_2 \otimes F_3}\ \otimes \quad \dfrac{h_8 :\vdash \Delta_{11}, F_9 \quad h_8 :\vdash \Delta_{12}, F_{10}, dual(F_7)}{\bullet h_8 :\vdash dual(F_7), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \otimes}{- :\vdash (\Delta_4, \Delta_6, F_2 \otimes F_3), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\ }{h_1 :\vdash \Delta_4, F_2, F_7}\ \text{ax} \quad \dfrac{\ }{\bullet h_8 :\vdash \Delta_{11}, \Delta_{12}, dual(F_7), F_9 \otimes F_{10}}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_4, F_2, F_9 \otimes F_{10}}\ \text{hCut} \quad \dfrac{\ }{- :\vdash \Delta_6, F_3}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_4, \Delta_6, F_2 \otimes F_3, F_9 \otimes F_{10}}\ \otimes$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_2 \quad h_1 :\vdash \Delta_5, F_3, F_7}{\bullet h_1 :\vdash F_7, \Delta_4, \Delta_5, F_2 \otimes F_3}\ \otimes \quad \dfrac{h_8 :\vdash \Delta_{11}, F_9, dual(F_7) \quad h_8 :\vdash \Delta_{12}, F_{10}}{\bullet h_8 :\vdash dual(F_7), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \otimes}{- :\vdash (\Delta_4, \Delta_5, F_2 \otimes F_3), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\ }{- :\vdash \Delta_4, F_2}\ \text{ax} \quad \dfrac{\dfrac{\ }{h_1 :\vdash \Delta_5, F_3, F_7}\ \text{ax} \quad \dfrac{\ }{\bullet h_8 :\vdash \Delta_{11}, \Delta_{12}, dual(F_7), F_9 \otimes F_{10}}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_5, F_3, F_9 \otimes F_{10}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_4, \Delta_5, F_2 \otimes F_3, F_9 \otimes F_{10}}\ \otimes$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_2 \quad h_1 :\vdash \Delta_5, F_3, F_7}{\bullet h_1 :\vdash F_7, \Delta_4, \Delta_5, F_2 \otimes F_3}\ \otimes \quad \dfrac{h_8 :\vdash \Delta_{11}, F_9 \quad h_8 :\vdash \Delta_{12}, F_{10}, dual(F_7)}{\bullet h_8 :\vdash dual(F_7), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \otimes}{- :\vdash (\Delta_4, \Delta_5, F_2 \otimes F_3), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\ }{- :\vdash \Delta_4, F_2}\ \text{ax} \quad \dfrac{\dfrac{\ }{h_1 :\vdash \Delta_5, F_3, F_7}\ \text{ax} \quad \dfrac{\ }{\bullet h_8 :\vdash \Delta_{11}, \Delta_{12}, dual(F_7), F_9 \otimes F_{10}}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_5, F_3, F_9 \otimes F_{10}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_4, \Delta_5, F_2 \otimes F_3, F_9 \otimes F_{10}}\ \otimes$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_6 \quad h_1 :\vdash \Delta_5, F_7}{\bullet h_1 :\vdash F_6 \otimes F_7, \Delta_4, \Delta_5}\ \otimes \quad \dfrac{h_8 :\vdash \Delta_{11}, F_9, dual(F_6)\$dual(F_7) \quad h_8 :\vdash \Delta_{12}, F_{10}}{\bullet h_8 :\vdash dual(F_6 \otimes F_7), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \otimes}{- :\vdash (\Delta_4, \Delta_5), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\ }{\bullet h_1 :\vdash \Delta_4, \Delta_5, F_6 \otimes F_7}\ \text{ax} \quad \dfrac{\ }{h_8 :\vdash \Delta_{11}, F_9, dual(F_6)\$dual(F_7)}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_4, \Delta_5, F_9}\ \text{hCut} \quad \dfrac{\ }{- :\vdash \Delta_{12}, F_{10}}\ \text{ax}}{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_4, \Delta_5, F_9 \otimes F_{10}}\ \otimes$$

$$\dfrac{\dfrac{h_1 :\vdash \Delta_4, F_6 \quad h_1 :\vdash \Delta_5, F_7}{\bullet h_1 :\vdash F_6 \otimes F_7, \Delta_4, \Delta_5}\ \otimes \quad \dfrac{h_8 :\vdash \Delta_{11}, F_9 \quad h_8 :\vdash \Delta_{12}, F_{10}, dual(F_6)\$dual(F_7)}{\bullet h_8 :\vdash dual(F_6 \otimes F_7), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \otimes}{- :\vdash (\Delta_4, \Delta_5), \Delta_{11}, \Delta_{12}, F_9 \otimes F_{10}}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\ }{- :\vdash \Delta_{11}, F_9}\ \text{ax} \quad \dfrac{\dfrac{\ }{\bullet h_1 :\vdash \Delta_4, \Delta_5, F_6 \otimes F_7}\ \text{ax} \quad \dfrac{\ }{h_8 :\vdash \Delta_{12}, F_{10}, dual(F_6)\$dual(F_7)}\ \text{ax}}{- :\vdash \Delta_{12}, \Delta_4, \Delta_5, F_{10}}\ \text{hCut}}{- :\vdash \Delta_{11}, \Delta_{12}, \Delta_4, \Delta_5, F_9 \otimes F_{10}}\ \otimes$$

# 6 Cut-Elimination

## 6.1 Status of 1: `OK`

- Case rule **1**

- Case rule !

- Case rule $?W$

- Case rule $?C$

- Case rule ?

- Case rule $

- Case rule $\perp$

- Case rule $I$

- Case rule $\otimes$

## 6.2 Status of !: `OK`

- Case rule **1**

- Case rule !

$$
\cfrac{
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3}\ !
\quad
\cfrac{\cfrac{\mathtt{h_6} :\vdash \mathtt{F_7}, ?\Gamma_8, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}{\bullet\mathtt{h_6} :\vdash contract(s\mathtt{n_5}, ?dual(\mathtt{F_4})), ?\Gamma_8, !\mathtt{F_7}}\ !}{}
}{- :\vdash ?\Gamma_3, ?\Gamma_8, !\mathtt{F_7}}\ \text{Cut}
}{}
$$
$$\rightarrow$$
$$
\cfrac{
\cfrac{
\cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}}\ \text{ax}
\quad
\cfrac{}{\mathtt{h_6} :\vdash ?\Gamma_8, \mathtt{F_7}, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}\ \text{ax}
}{- :\vdash ?\Gamma_3, ?\Gamma_8, \mathtt{F_7}}\ \text{hCut}
}{- :\vdash ?\Gamma_3, ?\Gamma_8, !\mathtt{F_7}}\ !
$$

- Case rule $?W$

$$
\cfrac{
\cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3}\ !
\quad
\cfrac{\cfrac{\mathtt{h_6} :\vdash \Delta_8, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}{\bullet\mathtt{h_6} :\vdash contract(s\mathtt{n_5}, ?dual(\mathtt{F_4})), \Delta_8, ?\mathtt{F_7}}\ ?W}{}
}{- :\vdash ?\Gamma_3, \Delta_8, ?\mathtt{F_7}}\ \text{Cut}
$$
$$\rightarrow$$
$$
\cfrac{
\cfrac{
\cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}}\ \text{ax}
\quad
\cfrac{}{\mathtt{h_6} :\vdash \Delta_8, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}\ \text{ax}
}{- :\vdash ?\Gamma_3, \Delta_8}\ \text{hCut}
}{- :\vdash ?\Gamma_3, \Delta_8, ?\mathtt{F_7}}\ ?W
$$

$$\cfrac{\cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \quad \cfrac{\mathtt{h_6} :\vdash \Delta_7, contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}{\bullet\mathtt{h_6} :\vdash contract(s\mathtt{n_5}, ?dual(\mathtt{F_4})), \Delta_7} \; ?W}{- :\vdash ?\Gamma_3, \Delta_7} \; \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \quad \cfrac{}{\mathtt{h_6} :\vdash \Delta_7, contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))} \; \text{ax}}{- :\vdash ?\Gamma_3, \Delta_7} \; \text{hCut}$$

- Case rule $?C$

$$\cfrac{\cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \quad \cfrac{\mathtt{h_6} :\vdash \Delta_8, ?\mathtt{F_7}, ?\mathtt{F_7}, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}{\bullet\mathtt{h_6} :\vdash contract(s\mathtt{n_5}, ?dual(\mathtt{F_4})), \Delta_8, ?\mathtt{F_7}} \; ?C}{- :\vdash ?\Gamma_3, \Delta_8, ?\mathtt{F_7}} \; \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \quad \cfrac{}{\mathtt{h_6} :\vdash \Delta_8, ?\mathtt{F_7}, ?\mathtt{F_7}, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))} \; \text{ax}}{- :\vdash ?\Gamma_3, \Delta_8, ?\mathtt{F_7}, ?\mathtt{F_7}} \; \text{hCut}}{- :\vdash ?\Gamma_3, \Delta_8, ?\mathtt{F_7}} \; ?C$$

$$\cfrac{\cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \quad \cfrac{\mathtt{h_6} :\vdash \Delta_7, ?dual(\mathtt{F_4}), ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}{\bullet\mathtt{h_6} :\vdash contract(s\mathtt{n_5}, ?dual(\mathtt{F_4})), \Delta_7} \; ?C}{- :\vdash ?\Gamma_3, \Delta_7} \; \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \quad \cfrac{}{\mathtt{h_6} :\vdash \Delta_7, ?dual(\mathtt{F_4}), ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))} \; \text{ax}}{- :\vdash ?\Gamma_3, \Delta_7} \; \text{hCut}$$

- Case rule $?$

$$\cfrac{\cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \quad \cfrac{\mathtt{h_6} :\vdash \Delta_8, \mathtt{F_7}, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}{\bullet\mathtt{h_6} :\vdash contract(s\mathtt{n_5}, ?dual(\mathtt{F_4})), \Delta_8, ?\mathtt{F_7}} \; ?}{- :\vdash ?\Gamma_3, \Delta_8, ?\mathtt{F_7}} \; \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \quad \cfrac{}{\mathtt{h_6} :\vdash \Delta_8, \mathtt{F_7}, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))} \; \text{ax}}{- :\vdash ?\Gamma_3, \Delta_8, \mathtt{F_7}} \; \text{hCut}}{- :\vdash ?\Gamma_3, \Delta_8, ?\mathtt{F_7}} \; ?$$

$$\cfrac{\cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \quad \cfrac{\mathtt{h_6} :\vdash \Delta_7, dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}{\bullet\mathtt{h_6} :\vdash contract(s\mathtt{n_5}, ?dual(\mathtt{F_4})), \Delta_7} \; ?}{- :\vdash ?\Gamma_3, \Delta_7} \; \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{}{- :\vdash ?\Gamma_3, \mathtt{F_4}} \; \text{ax} \quad \cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \quad \cfrac{}{\mathtt{h_6} :\vdash \Delta_7, dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))} \; \text{ax}}{- :\vdash ?\Gamma_3, \Delta_7, dual(\mathtt{F_4})} \; \text{hCut}}{- :\vdash ?\Gamma_3, \Delta_7} \; \text{mCut}$$

- Case rule $\$$

$$\cfrac{\cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \quad \cfrac{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7}, \mathtt{F_8}, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}{\bullet\mathtt{h_6} :\vdash contract(s\mathtt{n_5}, ?dual(\mathtt{F_4})), \Delta_9, \mathtt{F_7}\$\mathtt{F_8}} \; \$}{- :\vdash ?\Gamma_3, \Delta_9, \mathtt{F_7}\$\mathtt{F_8}} \; \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \quad \cfrac{}{\mathtt{h_6} :\vdash \Delta_9, \mathtt{F_7}, \mathtt{F_8}, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))} \; \text{ax}}{- :\vdash ?\Gamma_3, \Delta_9, \mathtt{F_7}, \mathtt{F_8}} \; \text{hCut}}{- :\vdash ?\Gamma_3, \Delta_9, \mathtt{F_7}\$\mathtt{F_8}} \; \$$$

- Case rule $\perp$

$$\cfrac{\cfrac{\mathtt{h_1} :\vdash \mathtt{F_4}, ?\Gamma_3}{\bullet\mathtt{h_1} :\vdash !\mathtt{F_4}, ?\Gamma_3} \; ! \quad \cfrac{\mathtt{h_6} :\vdash \Delta_7, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))}{\bullet\mathtt{h_6} :\vdash contract(s\mathtt{n_5}, ?dual(\mathtt{F_4})), \perp, \Delta_7} \; \perp}{- :\vdash ?\Gamma_3, \perp, \Delta_7} \; \text{Cut}$$
$$\rightarrow$$
$$\cfrac{\cfrac{\cfrac{}{\bullet\mathtt{h_1} :\vdash ?\Gamma_3, !\mathtt{F_4}} \; \text{ax} \quad \cfrac{}{\mathtt{h_6} :\vdash \Delta_7, ?dual(\mathtt{F_4}), contract(\mathtt{n_5}, ?dual(\mathtt{F_4}))} \; \text{ax}}{- :\vdash ?\Gamma_3, \Delta_7} \; \text{hCut}}{- :\vdash ?\Gamma_3, \Delta_7, \perp} \; \perp$$

- Case rule $I$

- Case rule $\otimes$

$$
\cfrac{
\cfrac{h_1 :\vdash F_4, ?\Gamma_3}{\bullet h_1 :\vdash !F_4, ?\Gamma_3}\ ! \quad
\cfrac{h_6 :\vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4)) \quad h_6 :\vdash \Delta_{10}, F_8}{\bullet h_6 :\vdash contract(sn_5, ?dual(F_4)), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \otimes
}{- :\vdash ?\Gamma_3, \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \text{Cut}
$$
$$\rightarrow$$
$$
\cfrac{
\cfrac{\phantom{\bullet h_1}}{\bullet h_1 :\vdash ?\Gamma_3, !F_4}\ \text{ax} \quad
\cfrac{h_6 :\vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- :\vdash ?\Gamma_3, \Delta_9, F_7}\ \text{ax} \quad
\cfrac{\phantom{xx}}{- :\vdash \Delta_{10}, F_8}\ \text{ax}
}{- :\vdash ?\Gamma_3, \Delta_{10}, \Delta_9, F_7 \otimes F_8}\ \otimes
$$
with hCut

$$
\cfrac{
\cfrac{h_1 :\vdash F_4, ?\Gamma_3}{\bullet h_1 :\vdash !F_4, ?\Gamma_3}\ ! \quad
\cfrac{h_6 :\vdash \Delta_9, F_7, contract(n_5, ?dual(F_4)) \quad h_6 :\vdash \Delta_{10}, F_8, ?dual(F_4)}{\bullet h_6 :\vdash contract(sn_5, ?dual(F_4)), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \otimes
}{- :\vdash ?\Gamma_3, \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \text{Cut}
$$
$$\rightarrow$$
$$
\cfrac{
\cfrac{
\cfrac{\bullet h_1 :\vdash ?\Gamma_3, !F_4\ \text{ax} \quad h_6 :\vdash \Delta_9, F_7, contract(n_5, ?dual(F_4))\ \text{ax}}{- :\vdash ?\Gamma_3, \Delta_9, F_7}\ \text{hCut} \quad
\cfrac{\bullet h_1 :\vdash ?\Gamma_3, !F_4\ \text{ax} \quad h_6 :\vdash \Delta_{10}, F_8, ?dual(F_4)\ \text{ax}}{- :\vdash ?\Gamma_3, \Delta_{10}, F_8}\ \text{hCut}
}{- :\vdash ?\Gamma_3, ?\Gamma_3, \Delta_{10}, \Delta_9, F_7 \otimes F_8}\ \otimes
}{- :\vdash ?\Gamma_3, \Delta_{10}, \Delta_9, F_7 \otimes F_8}\ C
$$

$$
\cfrac{
\cfrac{h_1 :\vdash F_4, ?\Gamma_3}{\bullet h_1 :\vdash !F_4, ?\Gamma_3}\ ! \quad
\cfrac{h_6 :\vdash \Delta_9, F_7, ?dual(F_4) \quad h_6 :\vdash \Delta_{10}, F_8, contract(n_5, ?dual(F_4))}{\bullet h_6 :\vdash contract(sn_5, ?dual(F_4)), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \otimes
}{- :\vdash ?\Gamma_3, \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \text{Cut}
$$
$$\rightarrow$$
$$
\cfrac{
\cfrac{
\cfrac{\bullet h_1 :\vdash ?\Gamma_3, !F_4\ \text{ax} \quad h_6 :\vdash \Delta_9, F_7, ?dual(F_4)\ \text{ax}}{- :\vdash ?\Gamma_3, \Delta_9, F_7}\ \text{hCut} \quad
\cfrac{\bullet h_1 :\vdash ?\Gamma_3, !F_4\ \text{ax} \quad h_6 :\vdash \Delta_{10}, F_8, contract(n_5, ?dual(F_4))\ \text{ax}}{- :\vdash ?\Gamma_3, \Delta_{10}, F_8}\ \text{hCut}
}{- :\vdash ?\Gamma_3, ?\Gamma_3, \Delta_{10}, \Delta_9, F_7 \otimes F_8}\ \otimes
}{- :\vdash ?\Gamma_3, \Delta_{10}, \Delta_9, F_7 \otimes F_8}\ C
$$

$$
\cfrac{
\cfrac{h_1 :\vdash F_4, ?\Gamma_3}{\bullet h_1 :\vdash !F_4, ?\Gamma_3}\ ! \quad
\cfrac{h_6 :\vdash \Delta_9, F_7 \quad h_6 :\vdash \Delta_{10}, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 :\vdash contract(sn_5, ?dual(F_4)), \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \otimes
}{- :\vdash ?\Gamma_3, \Delta_9, \Delta_{10}, F_7 \otimes F_8}\ \text{Cut}
$$
$$\rightarrow$$
$$
\cfrac{
\cfrac{\phantom{xxx}}{- :\vdash \Delta_9, F_7}\ \text{ax} \quad
\cfrac{\bullet h_1 :\vdash ?\Gamma_3, !F_4\ \text{ax} \quad h_6 :\vdash \Delta_{10}, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))\ \text{ax}}{- :\vdash ?\Gamma_3, \Delta_{10}, F_8}\ \text{hCut}
}{- :\vdash ?\Gamma_3, \Delta_{10}, \Delta_9, F_7 \otimes F_8}\ \otimes
$$

## 6.3   Status of $?W$:  OK

- Case rule $\mathbf{1}$

- Case rule !

$$
\cfrac{
\cfrac{h_2 :\vdash ?\Gamma_4, !F_5}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?\Gamma_3}\ ?W \quad
\cfrac{h_7 :\vdash F_8, ?\Gamma_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), ?\Gamma_9, !F_8}\ !
}{- :\vdash (?\Gamma_4, ?F_3), ?\Gamma_9, !F_8}\ \text{Cut}
$$
$$\rightarrow$$
$$
\cfrac{
\cfrac{\phantom{x}}{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3}\ \text{ax} \quad
\cfrac{h_7 :\vdash ?\Gamma_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{}\ \text{ax}
}{\cfrac{- :\vdash ?\Gamma_4, ?\Gamma_9, F_8, ?F_3}{- :\vdash ?\Gamma_4, ?\Gamma_9, !F_8, ?F_3}\ !}\ \text{hCut}
$$

- Case rule $?W$

$$\cfrac{\cfrac{h_2 :\vdash ?\Gamma_4, !F_5}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3} \; ?W \qquad \cfrac{h_7 :\vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \; ?W}{- :\vdash (?\Gamma_4, ?F_3), \Delta_9, ?F_8} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\cfrac{\overline{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3} \; ax \qquad \overline{h_7 :\vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))} \; ax}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3} \; \text{hCut}}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3, ?F_8} \; ?W}$$

$$\cfrac{\cfrac{h_2 :\vdash ?\Gamma_4, !F_5}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3} \; ?W \qquad \cfrac{h_7 :\vdash \Delta_8, contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_8} \; ?W}{- :\vdash (?\Gamma_4, ?F_3), \Delta_8} \; \text{Cut}$$

$$\to$$

$$\cfrac{\overline{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3} \; ax \qquad \overline{h_7 :\vdash \Delta_8, contract(n_6, ?dual(F_5))} \; ax}{- :\vdash ?\Gamma_4, \Delta_8, ?F_3} \; \text{hCut}$$

- Case rule $?C$

$$\cfrac{\cfrac{h_2 :\vdash ?\Gamma_4, !F_5}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3} \; ?W \qquad \cfrac{h_7 :\vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \; ?C}{- :\vdash (?\Gamma_4, ?F_3), \Delta_9, ?F_8} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3} \; ax \qquad \overline{h_7 :\vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))} \; ax}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3, ?F_8, ?F_8} \; \text{hCut}}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3, ?F_8} \; ?C$$

$$\cfrac{\cfrac{h_2 :\vdash ?\Gamma_4, !F_5}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3} \; ?W \qquad \cfrac{h_7 :\vdash \Delta_8, ?dual(F_5), ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_8} \; ?C}{- :\vdash (?\Gamma_4, ?F_3), \Delta_8} \; \text{Cut}$$

$$\to$$

$$\cfrac{\overline{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3} \; ax \qquad \overline{h_7 :\vdash \Delta_8, ?dual(F_5), ?dual(F_5), contract(n_6, ?dual(F_5))} \; ax}{- :\vdash ?\Gamma_4, \Delta_8, ?F_3} \; \text{hCut}$$

- Case rule $?$

$$\cfrac{\cfrac{h_2 :\vdash ?\Gamma_4, !F_5}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3} \; ?W \qquad \cfrac{h_7 :\vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \; ?}{- :\vdash (?\Gamma_4, ?F_3), \Delta_9, ?F_8} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3} \; ax \qquad \overline{h_7 :\vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))} \; ax}{- :\vdash ?\Gamma_4, \Delta_9, F_8, ?F_3} \; \text{hCut}}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3, ?F_8} \; ?$$

$$\cfrac{\cfrac{h_2 :\vdash ?\Gamma_4, !F_5}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3} \; ?W \qquad \cfrac{h_7 :\vdash \Delta_8, dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_8} \; ?}{- :\vdash (?\Gamma_4, ?F_3), \Delta_8} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{h_2 :\vdash ?\Gamma_4, !F_5} \; ax \qquad \cfrac{\overline{h_7 :\vdash \Delta_8, dual(F_5), contract(n_6, ?dual(F_5))} \; ax}{\bullet h_7 :\vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))} \; ?}{- :\vdash ?\Gamma_4, \Delta_8} \; \text{hCut}}{- :\vdash ?\Gamma_4, \Delta_8, ?F_3} \; ?W$$

- Case rule $\$$

$$\cfrac{\cfrac{h_2 :\vdash ?\Gamma_4, !F_5}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3} \; ?W \qquad \cfrac{h_7 :\vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8\$F_9} \; \$}{- :\vdash (?\Gamma_4, ?F_3), \Delta_{10}, F_8\$F_9} \; \text{Cut}$$

$$\to$$

$$\cfrac{\cfrac{\overline{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3} \; ax \qquad \overline{h_7 :\vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))} \; ax}{- :\vdash ?\Gamma_4, \Delta_{10}, F_8, F_9, ?F_3} \; \text{hCut}}{- :\vdash ?\Gamma_4, \Delta_{10}, ?F_3, F_8\$F_9} \; \$$

40

- Case rule $\perp$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \;?W \qquad \dfrac{\mathtt{h_7} :\vdash \Delta_8, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \perp, \Delta_8} \;\perp}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \perp, \Delta_8} \;\text{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}} \;\text{ax} \qquad \dfrac{}{\mathtt{h_7} :\vdash \Delta_8, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \;\text{ax}}{- :\vdash ?\Gamma_4, \Delta_8, ?\mathtt{F_3}} \;\text{hCut}}{- :\vdash ?\Gamma_4, \Delta_8, \perp, ?\mathtt{F_3}} \;\perp$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \;?W \qquad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5})) \quad \mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\otimes}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\text{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}} \;\text{ax} \quad \dfrac{}{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \;\text{ax}}{- :\vdash ?\Gamma_4, \Delta_{10}, \mathtt{F_8}, ?\mathtt{F_3}} \;\text{hCut} \quad \dfrac{}{- :\vdash \Delta_{11}, \mathtt{F_9}} \;\text{ax}}{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?\mathtt{F_3}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\otimes}{}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \;?W \qquad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5})) \quad \mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, ?dual(\mathtt{F_5})}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\otimes}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\text{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}} \;\text{ax} \quad \dfrac{\dfrac{}{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \;\text{ax} \quad \dfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, ?dual(\mathtt{F_5})} \;\otimes}{\bullet\mathtt{h_7} :\vdash \Delta_{10}, \Delta_{11}, ?dual(\mathtt{F_5}), \mathtt{F_8} \otimes \mathtt{F_9}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}}{\dfrac{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}}{} \;\text{hCut}}}{\dfrac{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?\mathtt{F_3}, \mathtt{F_8} \otimes \mathtt{F_9}}{} \;?W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \;?W \qquad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, ?dual(\mathtt{F_5}) \quad \mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\otimes}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\text{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}} \;\text{ax} \quad \dfrac{\dfrac{}{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, ?dual(\mathtt{F_5})} \;\text{ax} \quad \dfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \;\otimes}{\bullet\mathtt{h_7} :\vdash \Delta_{10}, \Delta_{11}, ?dual(\mathtt{F_5}), \mathtt{F_8} \otimes \mathtt{F_9}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}}{\dfrac{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}}{} \;\text{hCut}}}{\dfrac{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?\mathtt{F_3}, \mathtt{F_8} \otimes \mathtt{F_9}}{} \;?W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \;?W \qquad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8} \quad \mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\otimes}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_{10}, \Delta_{11}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\text{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_{10}, \mathtt{F_8}} \;\text{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}} \;\text{ax} \quad \dfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \;\text{ax}}{\dfrac{- :\vdash ?\Gamma_4, \Delta_{11}, \mathtt{F_9}, ?\mathtt{F_3}}{} \;\text{hCut}}}{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?\mathtt{F_3}, \mathtt{F_8} \otimes \mathtt{F_9}} \;\otimes$$

## 6.4  Status of $?C$:  OK

- Case rule $\mathbf{1}$

- Case rule $!$

$$\dfrac{\dfrac{h_2 :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3}\ {?C} \qquad \dfrac{h_7 :\vdash F_8, ?\Gamma_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), ?\Gamma_9, !F_8}\ {!}}{- :\vdash (?\Gamma_4, ?F_3), ?\Gamma_9, !F_8}\ {\text{Cut}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3}{}\ {\text{ax}} \qquad \dfrac{h_7 :\vdash ?\Gamma_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{}\ {\text{ax}}}{- :\vdash ?\Gamma_4, ?\Gamma_9, F_8, ?F_3}\ {\text{hCut}}}{- :\vdash ?\Gamma_4, ?\Gamma_9, !F_8, ?F_3}\ {!}$$

- Case rule $?W$

$$\dfrac{\dfrac{h_2 :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3}\ {?C} \qquad \dfrac{h_7 :\vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8}\ {?W}}{- :\vdash (?\Gamma_4, ?F_3), \Delta_9, ?F_8}\ {\text{Cut}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3}{}\ {\text{ax}} \qquad \dfrac{h_7 :\vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{}\ {\text{ax}}}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3}\ {\text{hCut}}}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3, ?F_8}\ {?W}$$

$$\dfrac{\dfrac{h_2 :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3}\ {?C} \qquad \dfrac{h_7 :\vdash \Delta_8, contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_8}\ {?W}}{- :\vdash (?\Gamma_4, ?F_3), \Delta_8}\ {\text{Cut}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3}{}\ {\text{ax}} \qquad \dfrac{h_7 :\vdash \Delta_8, contract(n_6, ?dual(F_5))}{}\ {\text{ax}}}{- :\vdash ?\Gamma_4, \Delta_8, ?F_3}\ {\text{hCut}}$$

- Case rule $?C$

$$\dfrac{\dfrac{h_2 :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3}\ {?C} \qquad \dfrac{h_7 :\vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8}\ {?C}}{- :\vdash (?\Gamma_4, ?F_3), \Delta_9, ?F_8}\ {\text{Cut}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3}{}\ {\text{ax}} \qquad \dfrac{h_7 :\vdash \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{}\ {\text{ax}}}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3, ?F_8, ?F_8}\ {\text{hCut}}}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3, ?F_8}\ {?C}$$

$$\dfrac{\dfrac{h_2 :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3}\ {?C} \qquad \dfrac{h_7 :\vdash \Delta_8, ?dual(F_5), ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_8}\ {?C}}{- :\vdash (?\Gamma_4, ?F_3), \Delta_8}\ {\text{Cut}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3}{}\ {\text{ax}} \qquad \dfrac{h_7 :\vdash \Delta_8, ?dual(F_5), ?dual(F_5), contract(n_6, ?dual(F_5))}{}\ {\text{ax}}}{- :\vdash ?\Gamma_4, \Delta_8, ?F_3}\ {\text{hCut}}$$

- Case rule $?$

$$\dfrac{\dfrac{h_2 :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3}\ {?C} \qquad \dfrac{h_7 :\vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8}\ {?}}{- :\vdash (?\Gamma_4, ?F_3), \Delta_9, ?F_8}\ {\text{Cut}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3}{}\ {\text{ax}} \qquad \dfrac{h_7 :\vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{}\ {\text{ax}}}{- :\vdash ?\Gamma_4, \Delta_9, F_8, ?F_3}\ {\text{hCut}}}{- :\vdash ?\Gamma_4, \Delta_9, ?F_3, ?F_8}\ {?}$$

$$\dfrac{\dfrac{h_2 :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3}\ {?C} \qquad \dfrac{h_7 :\vdash \Delta_8, dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_8}\ {?}}{- :\vdash (?\Gamma_4, ?F_3), \Delta_8}\ {\text{Cut}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{h_2 :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{}\ {\text{ax}} \qquad \dfrac{\dfrac{h_7 :\vdash \Delta_8, dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}\ {?}}{}\ {\text{ax}}}{- :\vdash ?\Gamma_4, \Delta_8, ?F_3, ?F_3}\ {\text{hCut}}}{- :\vdash ?\Gamma_4, \Delta_8, ?F_3}\ {?C}$$

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- Case rule $

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet\mathtt{h_2} :\vdash !F_5, ?\Gamma_4, ?F_3}\ ?C \quad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(\mathtt{n_6}, ?dual(F_5))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(F_5)), \Delta_{10}, F_8 \$ F_9}\ \$}{- :\vdash (?\Gamma_4, ?F_3), \Delta_{10}, F_8 \$ F_9}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3}\ \text{ax} \quad \dfrac{\dfrac{}{\mathtt{h_7} :\vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(\mathtt{n_6}, ?dual(F_5))}\ \text{ax}}{\dfrac{- :\vdash ?\Gamma_4, \Delta_{10}, F_8, F_9, ?F_3}{- :\vdash ?\Gamma_4, \Delta_{10}, ?F_3, F_8 \$ F_9}\ \$}}{\ }\ \text{hCut}$$

- Case rule $\perp$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet\mathtt{h_2} :\vdash !F_5, ?\Gamma_4, ?F_3}\ ?C \quad \dfrac{\mathtt{h_7} :\vdash \Delta_8, ?dual(F_5), contract(\mathtt{n_6}, ?dual(F_5))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(F_5)), \perp, \Delta_8}\ \perp}{- :\vdash (?\Gamma_4, ?F_3), \perp, \Delta_8}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3}\ \text{ax} \quad \dfrac{}{\mathtt{h_7} :\vdash \Delta_8, ?dual(F_5), contract(\mathtt{n_6}, ?dual(F_5))}\ \text{ax}}{\dfrac{- :\vdash ?\Gamma_4, \Delta_8, ?F_3}{- :\vdash ?\Gamma_4, \Delta_8, \perp, ?F_3}\ \perp}\ \text{hCut}$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet\mathtt{h_2} :\vdash !F_5, ?\Gamma_4, ?F_3}\ ?C \quad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, F_8, ?dual(F_5), contract(\mathtt{n_6}, ?dual(F_5)) \quad \mathtt{h_7} :\vdash \Delta_{11}, F_9}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \otimes}{- :\vdash (?\Gamma_4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3}\ \text{ax} \quad \dfrac{\dfrac{}{\mathtt{h_7} :\vdash \Delta_{10}, F_8, ?dual(F_5), contract(\mathtt{n_6}, ?dual(F_5))}\ \text{ax}}{- :\vdash ?\Gamma_4, \Delta_{10}, F_8, ?F_3}\ \text{hCut} \quad \dfrac{}{- :\vdash \Delta_{11}, F_9}\ \text{ax}}{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet\mathtt{h_2} :\vdash !F_5, ?\Gamma_4, ?F_3}\ ?C \quad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, F_8, contract(\mathtt{n_6}, ?dual(F_5)) \quad \mathtt{h_7} :\vdash \Delta_{11}, F_9, ?dual(F_5)}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \otimes}{- :\vdash (?\Gamma_4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}\ \text{ax} \quad \dfrac{\dfrac{}{\mathtt{h_7} :\vdash \Delta_{10}, F_8, contract(\mathtt{n_6}, ?dual(F_5))}\ \text{ax} \quad \dfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, F_9, ?dual(F_5)}\ \text{ax}}{\bullet\mathtt{h_7} :\vdash \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, contract(\mathtt{n_6}, ?dual(F_5))}\ \otimes}{\dfrac{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9}{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9}\ ?C}\ \text{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet\mathtt{h_2} :\vdash !F_5, ?\Gamma_4, ?F_3}\ ?C \quad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, F_8, ?dual(F_5) \quad \mathtt{h_7} :\vdash \Delta_{11}, F_9, contract(\mathtt{n_6}, ?dual(F_5))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \otimes}{- :\vdash (?\Gamma_4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}\ \text{ax} \quad \dfrac{\dfrac{}{\mathtt{h_7} :\vdash \Delta_{10}, F_8, ?dual(F_5)}\ \text{ax} \quad \dfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, F_9, contract(\mathtt{n_6}, ?dual(F_5))}\ \text{ax}}{\bullet\mathtt{h_7} :\vdash \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, contract(\mathtt{n_6}, ?dual(F_5))}\ \otimes}{\dfrac{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9}{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9}\ ?C}\ \text{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3, ?F_3}{\bullet\mathtt{h_2} :\vdash !F_5, ?\Gamma_4, ?F_3}\ ?C \quad \dfrac{\mathtt{h_7} :\vdash \Delta_{10}, F_8 \quad \mathtt{h_7} :\vdash \Delta_{11}, F_9, ?dual(F_5), contract(\mathtt{n_6}, ?dual(F_5))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \otimes}{- :\vdash (?\Gamma_4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \text{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_{10}, F_8}\ \text{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !F_5, ?F_3}\ \text{ax} \quad \dfrac{}{\mathtt{h_7} :\vdash \Delta_{11}, F_9, ?dual(F_5), contract(\mathtt{n_6}, ?dual(F_5))}\ \text{ax}}{- :\vdash ?\Gamma_4, \Delta_{11}, F_9, ?F_3}\ \text{hCut}}{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9}\ \otimes$$

## 6.5 Status of ?: OK

- Case rule **1**


- Case rule !

$$\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \ ? \quad \cfrac{\mathtt{h_7} :\vdash \mathtt{F_8}, ?\Gamma_9, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), ?\Gamma_9, !\mathtt{F_8}} \ !}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), ?\Gamma_9, !\mathtt{F_8}} \ \mathtt{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}} \ \mathtt{ax} \quad \cfrac{\cfrac{}{\mathtt{h_7} :\vdash ?\Gamma_9, \mathtt{F_8}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \ \mathtt{ax}}{\cfrac{- :\vdash ?\Gamma_4, ?\Gamma_9, \mathtt{F_8}, ?\mathtt{F_3}}{- :\vdash ?\Gamma_4, ?\Gamma_9, !\mathtt{F_8}, ?\mathtt{F_3}} \ !}}{} \ \mathtt{hCut}$$

- Case rule ?W

$$\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \ ? \quad \cfrac{\mathtt{h_7} :\vdash \Delta_9, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_9, ?\mathtt{F_8}} \ ?W}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_9, ?\mathtt{F_8}} \ \mathtt{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}} \ \mathtt{ax} \quad \cfrac{\cfrac{}{\mathtt{h_7} :\vdash \Delta_9, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \ \mathtt{ax}}{\cfrac{- :\vdash ?\Gamma_4, \Delta_9, ?\mathtt{F_3}}{- :\vdash ?\Gamma_4, \Delta_9, ?\mathtt{F_3}, ?\mathtt{F_8}} \ ?W}}{} \ \mathtt{hCut}$$

$$\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \ ? \quad \cfrac{\mathtt{h_7} :\vdash \Delta_8, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_8} \ ?W}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_8} \ \mathtt{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}} \ \mathtt{ax} \quad \cfrac{}{\mathtt{h_7} :\vdash \Delta_8, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_8, ?\mathtt{F_3}} \ \mathtt{hCut}$$

- Case rule ?C

$$\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \ ? \quad \cfrac{\mathtt{h_7} :\vdash \Delta_9, ?\mathtt{F_8}, ?\mathtt{F_8}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_9, ?\mathtt{F_8}} \ ?C}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_9, ?\mathtt{F_8}} \ \mathtt{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}} \ \mathtt{ax} \quad \cfrac{\cfrac{}{\mathtt{h_7} :\vdash \Delta_9, ?\mathtt{F_8}, ?\mathtt{F_8}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \ \mathtt{ax}}{\cfrac{- :\vdash ?\Gamma_4, \Delta_9, ?\mathtt{F_3}, ?\mathtt{F_8}, ?\mathtt{F_8}}{- :\vdash ?\Gamma_4, \Delta_9, ?\mathtt{F_3}, ?\mathtt{F_8}} \ ?C}}{} \ \mathtt{hCut}$$

$$\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \ ? \quad \cfrac{\mathtt{h_7} :\vdash \Delta_8, ?dual(\mathtt{F_5}), ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_8} \ ?C}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_8} \ \mathtt{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}} \ \mathtt{ax} \quad \cfrac{}{\mathtt{h_7} :\vdash \Delta_8, ?dual(\mathtt{F_5}), ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_8, ?\mathtt{F_3}} \ \mathtt{hCut}$$

- Case rule ?

$$\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}} \ ? \quad \cfrac{\mathtt{h_7} :\vdash \Delta_9, \mathtt{F_8}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_9, ?\mathtt{F_8}} \ ?}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_9, ?\mathtt{F_8}} \ \mathtt{Cut}$$

$$\rightarrow$$

$$\cfrac{\cfrac{}{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}} \ \mathtt{ax} \quad \cfrac{\cfrac{}{\mathtt{h_7} :\vdash \Delta_9, \mathtt{F_8}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))} \ \mathtt{ax}}{\cfrac{- :\vdash ?\Gamma_4, \Delta_9, \mathtt{F_8}, ?\mathtt{F_3}}{- :\vdash ?\Gamma_4, \Delta_9, ?\mathtt{F_3}, ?\mathtt{F_8}} \ ?}}{} \ \mathtt{hCut}$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}}\ ?\quad \cfrac{\mathtt{h_7} :\vdash \Delta_8, dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_8}\ ?}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_8}\ \mathtt{Cut}}{\cfrac{\cfrac{\cfrac{- :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}}{- :\vdash ?\Gamma_4, \mathtt{F_5}, ?\mathtt{F_3}}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \mathtt{F_5}, ?\mathtt{F_3}}\ bInv\quad \cfrac{\cfrac{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}}{}\ \mathtt{ax}\quad \cfrac{\mathtt{h_7} :\vdash \Delta_8, dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_8, ?\mathtt{F_3}, dual(\mathtt{F_5})}\ \mathtt{hCut}}{- :\vdash ?\Gamma_4, \Delta_8, ?\mathtt{F_3}}\ \mathtt{mCut}}$$

- Case rule $

$$\cfrac{\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}}\ ?\quad \cfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, \mathtt{F_9}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_{10}, \mathtt{F_8}\$\mathtt{F_9}}\ \$}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_{10}, \mathtt{F_8}\$\mathtt{F_9}}\ \mathtt{Cut}}{\cfrac{\cfrac{\cfrac{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}}{}\ \mathtt{ax}\quad \cfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, \mathtt{F_9}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_{10}, \mathtt{F_8}, \mathtt{F_9}, ?\mathtt{F_3}}\ \mathtt{hCut}}{- :\vdash ?\Gamma_4, \Delta_{10}, ?\mathtt{F_3}, \mathtt{F_8}\$\mathtt{F_9}}\ \$}$$

- Case rule $\perp$

$$\cfrac{\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}}\ ?\quad \cfrac{\mathtt{h_7} :\vdash \Delta_8, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \perp, \Delta_8}\ \perp}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \perp, \Delta_8}\ \mathtt{Cut}}{\cfrac{\cfrac{\cfrac{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}}{}\ \mathtt{ax}\quad \cfrac{\mathtt{h_7} :\vdash \Delta_8, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_8, ?\mathtt{F_3}}\ \mathtt{hCut}}{- :\vdash ?\Gamma_4, \Delta_8, \perp, ?\mathtt{F_3}}\ \perp}$$

- Case rule $I$

- Case rule $\otimes$

$$\cfrac{\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}}\ ?\quad \cfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))\quad \mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_{10}, \Delta_{11}, \mathtt{F_8}\otimes\mathtt{F_9}}\ \otimes}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_{10}, \Delta_{11}, \mathtt{F_8}\otimes\mathtt{F_9}}\ \mathtt{Cut}}{\cfrac{\cfrac{\cfrac{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}}{}\ \mathtt{ax}\quad \cfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, ?dual(\mathtt{F_5}), contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_{10}, \mathtt{F_8}, ?\mathtt{F_3}}\ \mathtt{hCut}\quad \cfrac{- :\vdash \Delta_{11}, \mathtt{F_9}}{}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?\mathtt{F_3}, \mathtt{F_8}\otimes\mathtt{F_9}}\ \otimes}$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}}\ ?\quad \cfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))\quad \mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, ?dual(\mathtt{F_5})}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_{10}, \Delta_{11}, \mathtt{F_8}\otimes\mathtt{F_9}}\ \otimes}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_{10}, \Delta_{11}, \mathtt{F_8}\otimes\mathtt{F_9}}\ \mathtt{Cut}}{\cfrac{\cfrac{\cfrac{\cfrac{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}}{}\ \mathtt{ax}\ \cfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_{10}, \mathtt{F_8}, ?\mathtt{F_3}}\ \mathtt{hCut}\ \cfrac{\cfrac{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}}{}\ \mathtt{ax}\ \cfrac{\mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, ?dual(\mathtt{F_5})}{}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_{11}, \mathtt{F_9}, ?\mathtt{F_3}}\ \mathtt{hCut}}{- :\vdash ?\Gamma_4, ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?\mathtt{F_3}, ?\mathtt{F_3}, \mathtt{F_8}\otimes\mathtt{F_9}}\ \otimes}{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?\mathtt{F_3}, \mathtt{F_8}\otimes\mathtt{F_9}}\ C}$$

$$\cfrac{\cfrac{\cfrac{\mathtt{h_2} :\vdash \mathtt{F_3}, ?\Gamma_4, !\mathtt{F_5}}{\bullet\mathtt{h_2} :\vdash !\mathtt{F_5}, ?\Gamma_4, ?\mathtt{F_3}}\ ?\quad \cfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, ?dual(\mathtt{F_5})\quad \mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{\bullet\mathtt{h_7} :\vdash contract(s\mathtt{n_6}, ?dual(\mathtt{F_5})), \Delta_{10}, \Delta_{11}, \mathtt{F_8}\otimes\mathtt{F_9}}\ \otimes}{- :\vdash (?\Gamma_4, ?\mathtt{F_3}), \Delta_{10}, \Delta_{11}, \mathtt{F_8}\otimes\mathtt{F_9}}\ \mathtt{Cut}}{\cfrac{\cfrac{\cfrac{\cfrac{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}}{}\ \mathtt{ax}\ \cfrac{\mathtt{h_7} :\vdash \Delta_{10}, \mathtt{F_8}, ?dual(\mathtt{F_5})}{}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_{10}, \mathtt{F_8}, ?\mathtt{F_3}}\ \mathtt{hCut}\ \cfrac{\cfrac{\bullet\mathtt{h_2} :\vdash ?\Gamma_4, !\mathtt{F_5}, ?\mathtt{F_3}}{}\ \mathtt{ax}\ \cfrac{\mathtt{h_7} :\vdash \Delta_{11}, \mathtt{F_9}, contract(\mathtt{n_6}, ?dual(\mathtt{F_5}))}{}\ \mathtt{ax}}{- :\vdash ?\Gamma_4, \Delta_{11}, \mathtt{F_9}, ?\mathtt{F_3}}\ \mathtt{hCut}}{- :\vdash ?\Gamma_4, ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?\mathtt{F_3}, ?\mathtt{F_3}, \mathtt{F_8}\otimes\mathtt{F_9}}\ \otimes}{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?\mathtt{F_3}, \mathtt{F_8}\otimes\mathtt{F_9}}\ C}$$

$$
\cfrac{
  \cfrac{
    \cfrac{h_2 :\vdash F_3, ?\Gamma_4, !F_5}{\bullet h_2 :\vdash !F_5, ?\Gamma_4, ?F_3}\ ?
    \qquad
    \cfrac{h_7 :\vdash \Delta_{10}, F_8 \qquad h_7 :\vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 :\vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \otimes
  }{- :\vdash (?\Gamma_4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9}\ \text{Cut}
}{
  \cfrac{
    \cfrac{}{- :\vdash \Delta_{10}, F_8}\ \text{ax}
    \qquad
    \cfrac{
      \cfrac{}{\bullet h_2 :\vdash ?\Gamma_4, !F_5, ?F_3}\ \text{ax}
      \qquad
      \cfrac{}{h_7 :\vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}\ \text{ax}
    }{- :\vdash ?\Gamma_4, \Delta_{11}, F_9, ?F_3}\ \text{hCut}
  }{- :\vdash ?\Gamma_4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9}\ \otimes
}\ \rightarrow
$$

## 6.6   Status of $: OK

- Case rule **1**

- Case rule !

- Case rule ?$W$

- Case rule ?$C$

- Case rule ?

- Case rule $

- Case rule $\perp$

- Case rule $I$

- Case rule $\otimes$


## 6.7   Status of $\perp$: OK

- Case rule **1**

- Case rule !

- Case rule ?$W$

- Case rule ?$C$

- Case rule ?

- Case rule $

- Case rule $\perp$

- Case rule $I$

- Case rule $\otimes$

## 6.8  Status of $I$:  OK

- Case rule $\mathbf{1}$

- Case rule !

- Case rule $?W$

- Case rule $?C$

- Case rule ?

- Case rule \$

- Case rule $\perp$

- Case rule $I$

- Case rule $\otimes$

## 6.9  Status of $\otimes$:  OK

- Case rule $\mathbf{1}$

- Case rule !

- Case rule $?W$

- Case rule $?C$

- Case rule ?

- Case rule \$

- Case rule ⊥

- Case rule $I$

- Case rule ⊗