

# System G3i

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## 1 Height preserving admissibility of weakening

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_0 \vdash \top} \top_R$$

- Case(s) rule  $\rightarrow_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3 \vdash F_4} \text{ax}}{h_1 : \Delta_2, F_0, F_3 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_2, F_0 \vdash F_3 \rightarrow F_4} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_2 \vdash F_3} \text{ax}}{h_1 : \Delta_2, F_0 \vdash F_3} \text{IH} \quad \frac{\frac{}{h_1 : \Delta_2 \vdash F_4} \text{ax}}{h_1 : \Delta_2, F_0 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_2, F_0 \vdash F_3 \wedge F_4} \wedge_R$$

- Case(s) rule  $\vee_1$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1 \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_2 \vdash F_3} \text{ax}}{h_1 : \Delta_2, F_0 \vdash F_3} \text{IH}}{\bullet h_1 : \Delta_2, F_0 \vdash F_3 \vee F_4} \vee_1$$

- Case(s) rule  $\vee_2$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2 \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_2 \vdash F_4} \text{ax}}{h_1 : \Delta_2, F_0 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_2, F_0 \vdash F_3 \vee F_4} \vee_2$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2} \text{ax}}{h_1 : \Delta_5, F_0, F_2 \rightarrow F_3 \vdash F_2} \text{IH} \quad \frac{\frac{}{h_1 : \Delta_5, F_3 \vdash F_4} \text{ax}}{h_1 : \Delta_5, F_0, F_3 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_5, F_0, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4} \wedge_L \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_5, F_2, F_3 \vdash F_4} \text{ax}}{h_1 : \Delta_5, F_0, F_2, F_3 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_5, F_0, F_2 \wedge F_3 \vdash F_4} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vdash F_4} \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \vee_L \quad \rightarrow \quad \frac{\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{h_1 : \Delta_5, F_0, F_2 \vdash F_4} \text{IH} \quad \frac{h_1 : \Delta_5, F_3 \vdash F_4}{h_1 : \Delta_5, F_0, F_3 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_5, F_0, F_2 \vee F_3 \vdash F_4} \vee_L \text{ax}}{\bullet h_1 : \Delta_5, F_0, F_2 \vee F_3 \vdash F_4} \vee_L$$

- Case(s) rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_2} \perp_L \quad \rightarrow \quad \frac{}{\bullet h_1 : \perp, \Delta_3, F_0 \vdash F_2} \perp_L$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_1 : \Delta_2, P_3 \vdash P_3} I \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_2, F_0, P_3 \vdash P_3} I$$

- Case(s) rule  $\top_L$

$$\frac{h_1 : \Delta_3 \vdash F_2}{\bullet h_1 : \top, \Delta_3 \vdash F_2} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_3 \vdash F_2}{h_1 : \Delta_3, F_0 \vdash F_2} \text{IH}}{\bullet h_1 : \top, \Delta_3, F_0 \vdash F_2} \top_L \text{ax}$$

## 2 Measure of derivations

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top} \top_R$$

- Case(s) rule  $\rightarrow_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3 \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH} \quad \frac{}{h_1 : \Delta_2 \vdash F_4} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \text{IH} \wedge_R$$

- Case(s) rule  $\vee_1$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1 \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1$$

- Case(s) rule  $\vee_2$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2 \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2} \text{ax}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2} \text{IH} \quad \frac{}{h_1 : \Delta_5, F_3 \vdash F_4} \text{ax}}{\bullet \bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \text{IH} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4} \wedge_L \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_2, F_3 \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\frac{h_1 : \Delta_5, F_2 \vdash F_4 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \vee_L \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_2 \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_5, F_2 \vdash F_4} \text{IH} \quad \frac{}{h_1 : \Delta_5, F_3 \vdash F_4} \text{ax}}{\bullet \bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \text{IH} \vee_L$$

- Case(s) rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_2} \perp_L \rightarrow \frac{}{\bullet \bullet h_1 : \perp, \Delta_3 \vdash F_2} \perp_L$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_1 : \Delta_2, p_3 \vdash p_3} I \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2, p_3 \vdash p_3} I$$

- Case(s) rule  $\top_L$

$$\frac{h_1 : \Delta_3 \vdash F_2}{\bullet h_1 : \top, \Delta_3 \vdash F_2} \top_L \rightarrow \frac{\frac{}{h_1 : \Delta_3 \vdash F_2} \text{ax}}{\bullet h_1 : \Delta_3 \vdash F_2} \text{IH}}{\bullet \bullet h_1 : \top, \Delta_3 \vdash F_2} \top_L$$

### 3 Invertibility of Rules

#### 3.1 Status of $\top_R$ : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \rightarrow \text{trivial}$$

- Case rule  $\rightarrow_R$

- Case rule  $\wedge_R$

- Case rule  $\vee_1$

- Case rule  $\vee_2$

- Case rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_4, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_4, F_3 \vdash \top}{\bullet h_1 : \Delta_4, F_2 \rightarrow F_3 \vdash \top} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule  $\wedge_L$

$$\frac{h_1 : \Delta_4, F_2, F_3 \vdash \top}{\bullet h_1 : \Delta_4, F_2 \wedge F_3 \vdash \top} \wedge_L \rightarrow \text{trivial}$$

- Case rule  $\vee_L$

$$\frac{h_1 : \Delta_4, F_2 \vdash \top \quad h_1 : \Delta_4, F_3 \vdash \top}{\bullet h_1 : \Delta_4, F_2 \vee F_3 \vdash \top} \vee_L \rightarrow \text{trivial}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \top} \perp_L \rightarrow \text{trivial}$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \top, \Delta_2 \vdash \top} \top_L \rightarrow \text{trivial}$$

### 3.2 Status of $\rightarrow_R$ : : Invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_4}^{\text{ax}}}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{H}$$

- Case rule  $\wedge_R$
- Case rule  $\vee_1$
- Case rule  $\vee_2$
- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \Delta_6, F_5 \vdash F_1 \rightarrow F_2}{\bullet h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_1 \rightarrow F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4}^{\text{ax}}}{h_3 : \Delta_6, F_1, F_4 \rightarrow F_5 \vdash F_4} W \frac{\overline{h_3 : \Delta_6, F_1, F_5 \vdash F_2}^{\text{ax/ind}}}{\bullet h_3 : \Delta_6, F_1, F_4 \rightarrow F_5 \vdash F_2} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \rightarrow F_2}{\bullet h_3 : \Delta_6, F_4 \wedge F_5 \vdash F_1 \rightarrow F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4, F_5 \vdash F_2}^{\text{ax/ind}}}{\bullet h_3 : \Delta_6, F_1, F_4 \wedge F_5 \vdash F_2} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \rightarrow F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \rightarrow F_2}{\bullet h_3 : \Delta_6, F_4 \vee F_5 \vdash F_1 \rightarrow F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4 \vdash F_2}^{\text{ax/ind}} \quad \overline{h_3 : \Delta_6, F_1, F_5 \vdash F_2}^{\text{ax/ind}}}{\bullet h_3 : \Delta_6, F_1, F_4 \vee F_5 \vdash F_2} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \rightarrow F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4, F_1 \vdash F_2} \perp_L$$

- Case rule  $I$
- Case rule  $\top_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \rightarrow F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \rightarrow F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash F_2}^{\text{ax/ind}}}{\bullet h_3 : \top, \Delta_4, F_1 \vdash F_2} \top_L$$

### 3.3 Status of $\wedge_R$ : (Left Premise): Invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$
- Case rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash F_3}^{\text{ax}}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{H}$$

- Case rule  $\vee_1$
- Case rule  $\vee_2$
- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \Delta_6, F_5 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_1 \wedge F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4}^{\text{ax}} \quad \overline{h_3 : \Delta_6, F_5 \vdash F_1}^{\text{ax/ind}}}{\bullet h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_1} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_6, F_4 \wedge F_5 \vdash F_1 \wedge F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, F_5 \vdash F_1}^{\text{ax/ind}}}{\bullet h_3 : \Delta_6, F_4 \wedge F_5 \vdash F_1} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \wedge F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_6, F_4 \vee F_5 \vdash F_1 \wedge F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \vdash F_1}^{\text{ax/ind}} \quad \overline{h_3 : \Delta_6, F_5 \vdash F_1}^{\text{ax/ind}}}{\bullet h_3 : \Delta_6, F_4 \vee F_5 \vdash F_1} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \wedge F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1} \perp_L$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \wedge F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1}^{\text{ax/ind}}}{\bullet h_3 : \top, \Delta_4 \vdash F_1} \top_L$$

### 3.4 Status of $\wedge_R$ (Right Premise): : Invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$
- Case rule  $\wedge_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4}}{\wedge_R} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{ ax}}{H}$$

- Case rule  $\vee_1$
- Case rule  $\vee_2$
- Case rule  $\rightarrow_L$

$$\frac{\frac{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \Delta_6, F_5 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_1 \wedge F_2}}{\rightarrow_L} \rightarrow \frac{\frac{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4}{\bullet h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_2} \text{ ax} \quad \frac{h_3 : \Delta_6, F_5 \vdash F_2}{\rightarrow_L} \text{ ax/ind}}{\rightarrow_L}$$

- Case rule  $\wedge_L$

$$\frac{\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_6, F_4 \wedge F_5 \vdash F_1 \wedge F_2}}{\wedge_L} \rightarrow \frac{\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_2}{\bullet h_3 : \Delta_6, F_4 \wedge F_5 \vdash F_2} \text{ ax/ind}}{\wedge_L}$$

- Case rule  $\vee_L$

$$\frac{\frac{h_3 : \Delta_6, F_4 \vdash F_1 \wedge F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_6, F_4 \vee F_5 \vdash F_1 \wedge F_2}}{\vee_L} \rightarrow \frac{\frac{h_3 : \Delta_6, F_4 \vdash F_2}{\bullet h_3 : \Delta_6, F_4 \vee F_5 \vdash F_2} \text{ ax/ind} \quad \frac{h_3 : \Delta_6, F_5 \vdash F_2}{\vee_L} \text{ ax/ind}}{\vee_L}$$

- Case rule  $\perp_L$

$$\frac{\frac{h_3 : \perp, \Delta_4 \vdash F_1 \wedge F_2}{\bullet h_3 : \perp, \Delta_4 \vdash F_2}}{\perp_L} \rightarrow \frac{\frac{h_3 : \perp, \Delta_4 \vdash F_2}{\bullet h_3 : \perp, \Delta_4 \vdash F_2}}{\perp_L}$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \wedge F_2}}{\top_L} \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_2} \text{ ax/ind}}{\top_L}$$



### 3.5 Status of $\vee_1$ : : Non invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$
- Case rule  $\wedge_R$
- Case rule  $\vee_1$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1 \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash F_3}}{\bullet h_1 : \Delta_2 \vdash F_3} \begin{matrix} \text{ax} \\ \text{H} \end{matrix}$$

- Case rule  $\vee_2$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2 \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_3} \text{fail}$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \Delta_6, F_5 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_1 \vee F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4} \text{ ax} \quad \overline{h_3 : \Delta_6, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_1} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_6, F_4 \wedge F_5 \vdash F_1 \vee F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \wedge F_5 \vdash F_1} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \vee F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_6, F_4 \vee F_5 \vdash F_1 \vee F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_6, F_4 \vdash F_1} \text{ ax/ind} \quad \overline{h_3 : \Delta_6, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_4 \vee F_5 \vdash F_1} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \vee F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1} \perp_L$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \vee F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \vee F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_4 \vdash F_1} \top_L$$

### 3.6 Status of $\vee_2$ : : Non invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$
- Case rule  $\wedge_R$
- Case rule  $\vee_1$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1 \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_4} \text{fail}$$

- Case rule  $\vee_2$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2 \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_4} \frac{\text{ax}}{H}$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \Delta_6, F_5 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_1 \vee F_2} \rightarrow_L \rightarrow \frac{}{h_3 : \Delta_6, F_4 \rightarrow F_5 \vdash F_4} \frac{\text{ax}}{H} \frac{}{h_3 : \Delta_6, F_5 \vdash F_2} \frac{\text{ax/ind}}{\rightarrow_L}$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_6, F_4, F_5 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_6, F_4 \wedge F_5 \vdash F_1 \vee F_2} \wedge_L \rightarrow \frac{}{h_3 : \Delta_6, F_4, F_5 \vdash F_2} \frac{\text{ax/ind}}{\wedge_L} \frac{}{\bullet h_3 : \Delta_6, F_4 \wedge F_5 \vdash F_2} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_6, F_4 \vdash F_1 \vee F_2 \quad h_3 : \Delta_6, F_5 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_6, F_4 \vee F_5 \vdash F_1 \vee F_2} \vee_L \rightarrow \frac{}{h_3 : \Delta_6, F_4 \vdash F_2} \frac{\text{ax/ind}}{\vee_L} \frac{}{h_3 : \Delta_6, F_5 \vdash F_2} \frac{\text{ax/ind}}{\vee_L}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \vee F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_2} \perp_L$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \vee F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \vee F_2} \top_L \rightarrow \frac{}{h_3 : \Delta_4 \vdash F_2} \frac{\text{ax/ind}}{\top_L} \frac{}{\bullet h_3 : \top, \Delta_4 \vdash F_2} \top_L$$

### 3.7 Status of $\rightarrow_L$ : (Left Premise): Non invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \rightarrow F_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \rightarrow F_2 \vdash F_1} \text{fail}$$

- Case rule  $\rightarrow_R$

$$\frac{h_3 : \Delta_6, F_4, F_1 \rightarrow F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_1} \text{fail}$$

- Case rule  $\wedge_R$

$$\frac{h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \wedge F_5} \wedge_R \rightarrow \frac{h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_1}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_1} \frac{\text{ax/ind}}{H}$$

- Case rule  $\vee_1$

$$\frac{h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \vee F_5} \vee_1 \rightarrow \frac{h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_1}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_1} \frac{\text{ax/ind}}{H}$$

- Case rule  $\vee_2$

$$\frac{h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \vee F_5} \vee_2 \rightarrow \frac{h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_1}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_1} \frac{\text{ax/ind}}{H}$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L \rightarrow \frac{h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \rightarrow F_5 \vdash F_1}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \rightarrow F_5 \vdash F_1} \frac{\text{ax/ind}}{H}$$

$$\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L \rightarrow \frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2} \frac{\text{ax/ind}}{H}$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L \rightarrow \frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash F_1}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \wedge F_5 \vdash F_1} \frac{\text{ax/ind}}{\wedge_L}$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash F_6} \vee_L \rightarrow \frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash F_1}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \vee F_5 \vdash F_1} \frac{\text{ax/ind}}{\vee_L} \frac{h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_1}{\vee_L}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash F_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash F_1} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_5, \Delta_4, F_1 \rightarrow F_2 \vdash p_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, p_5, F_1 \rightarrow F_2 \vdash F_1} \text{fail}$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash F_4} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash F_1} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash F_1} \top_L$$

### 3.8 Status of $\rightarrow_L$ (Right Premise): : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \rightarrow F_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_2 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{h_3 : \Delta_6, F_4, F_1 \rightarrow F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_2, F_4 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{\frac{}{h_3 : \Delta_6, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \wedge F_5} \wedge_R \quad \frac{\frac{}{h_3 : \Delta_6, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \wedge F_5} \wedge_R}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \wedge F_5} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \vee F_5} \vee_1 \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \vee F_5} \vee_1$$

- Case rule  $\vee_2$

$$\frac{h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \rightarrow F_2 \vdash F_4 \vee F_5} \vee_2 \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \vee F_5} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L \rightarrow \frac{\frac{\frac{}{h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_6} \text{ax/ind} \quad \frac{\frac{}{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$

$$\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_3 \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_5, F_3 \vdash F_4} H$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L \rightarrow \frac{\frac{\frac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash F_6} \wedge_L}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash F_6} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash F_6} \vee_L}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash F_6} \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash F_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vdash F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash F_6} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash F_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash F_4} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_5, \Delta_4, F_1 \rightarrow F_2 \vdash p_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_2, p_5 \vdash p_5} I$$

- Case rule  $\top_L$

$$\frac{\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash F_4} \top_L}{\bullet h_3 : \top, \Delta_5, F_2 \vdash F_4} \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash F_4} \top_L$$

### 3.9 Status of $\wedge_L$ : : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \wedge F_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1, F_2 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{h_3 : \Delta_6, F_4, F_1 \wedge F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \wedge F_2 \vdash F_4 \rightarrow F_5} \rightarrow_R}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \rightarrow F_5} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \rightarrow F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_3 : \Delta_6, F_1 \wedge F_2 \vdash F_4 \quad h_3 : \Delta_6, F_1 \wedge F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \wedge F_2 \vdash F_4 \wedge F_5} \wedge_R}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \wedge F_5} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \wedge F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \wedge F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \wedge F_5} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{\frac{h_3 : \Delta_6, F_1 \wedge F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \wedge F_2 \vdash F_4 \vee F_5} \vee_1}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \vee F_5} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \vee F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \vee F_5} \vee_1$$

- Case rule  $\vee_2$

$$\frac{\frac{h_3 : \Delta_6, F_1 \wedge F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \wedge F_2 \vdash F_4 \vee F_5} \vee_2}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \vee F_5} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \vee F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1, F_2 \vdash F_4 \vee F_5} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{h_3 : \Delta_7, F_4 \rightarrow F_5, F_1 \wedge F_2 \vdash F_4}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \rightarrow F_5 \vdash F_6}}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash F_4}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash F_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{\frac{h_3 : \Delta_7, F_4, F_5, F_1 \wedge F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \wedge F_5 \vdash F_6}}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash F_6} \wedge_L$$

$$\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_5, F_2, F_3 \vdash F_4} \text{ H}$$

- Case rule  $\vee_L$

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \wedge F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash F_6} \quad \frac{h_3 : \Delta_7, F_5, F_1 \wedge F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash F_6}}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash F_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash F_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash F_6} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \wedge F_2 \vdash F_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash F_4} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_5, \Delta_4, F_1 \wedge F_2 \vdash p_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1, F_2, p_5 \vdash p_5} I$$

- Case rule  $\top_L$

$$\frac{\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \wedge F_2 \vdash F_4}}{\bullet h_3 : \top, \Delta_5, F_1 \wedge F_2 \vdash F_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash F_4} \top_L$$

### 3.10 Status of $\vee_L$ : (Left Premise): Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \vee F_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{h_3 : \Delta_6, F_4, F_1 \vee F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \rightarrow F_5}}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_6, F_1, F_4 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \rightarrow F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \wedge F_5} \quad \frac{h_3 : \Delta_6, F_1 \vee F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \wedge F_5}}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_6, F_1 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \wedge F_5} \text{ ax/ind} \quad \frac{h_3 : \Delta_6, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \wedge F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \wedge F_5} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \vee F_5} \vee_1 \rightarrow \frac{\frac{h_3 : \Delta_6, F_1 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \vee F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \vee F_5} \vee_1$$

- Case rule  $\vee_2$

$$\frac{h_3 : \Delta_6, F_1 \vee F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \vee F_5} \vee_2 \rightarrow \frac{\frac{h_3 : \Delta_6, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \vee F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_6, F_1 \vdash F_4 \vee F_5} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_4 \rightarrow F_5, F_1 \vee F_2 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash F_4}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash F_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash F_6} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash F_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash F_6} \text{ ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash F_6} \vee_L$$

$$\frac{h_1 : \Delta_5, F_2 \vdash F_4 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_5, F_2 \vdash F_4} \text{ H}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash F_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash F_4} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_5, \Delta_4, F_1 \vee F_2 \vdash p_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1, p_5 \vdash p_5} I$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash F_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_1 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash F_4} \top_L$$

### 3.11 Status of $\vee_L$ (Right Premise): : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_4, F_1 \vee F_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_2 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{h_3 : \Delta_6, F_4, F_1 \vee F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_2, F_4 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \wedge F_5} \wedge_R \quad \frac{h_3 : \Delta_6, F_1 \vee F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \wedge F_5} \wedge_R}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_2 \vdash F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_6, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \wedge F_5} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \vee F_5} \vee_1 \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \vee F_5} \vee_1$$

- Case rule  $\vee_2$

$$\frac{h_3 : \Delta_6, F_1 \vee F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1 \vee F_2 \vdash F_4 \vee F_5} \vee_2 \rightarrow \frac{\frac{}{h_3 : \Delta_6, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_6, F_2 \vdash F_4 \vee F_5} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_4 \rightarrow F_5, F_1 \vee F_2 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash F_6} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash F_6} \vee_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash F_6} \vee_L$$

$$\frac{h_1 : \Delta_5, F_2 \vdash F_4 \quad h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \vee_L \rightarrow \frac{\frac{}{h_1 : \Delta_5, F_3 \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_5, F_3 \vdash F_4} \text{H}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash F_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash F_4} \perp_L$$



- Case rule  $I$

$$\frac{}{\bullet h_3 : p_5, \Delta_4, F_1 \vee F_2 \vdash p_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_2, p_5 \vdash p_5} I$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash F_4} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash F_4} \top_L$$

### 3.12 Status of $\perp_L$ : : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \top} \top_R \rightarrow \text{trivial}$$

- Case rule  $\rightarrow_R$

$$\frac{h_1 : \perp, \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \perp, \Delta_4 \vdash F_2 \rightarrow F_3} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule  $\wedge_R$

$$\frac{h_1 : \perp, \Delta_4 \vdash F_2 \quad h_1 : \perp, \Delta_4 \vdash F_3}{\bullet h_1 : \perp, \Delta_4 \vdash F_2 \wedge F_3} \wedge_R \rightarrow \text{trivial}$$

- Case rule  $\vee_1$

$$\frac{h_1 : \perp, \Delta_4 \vdash F_2}{\bullet h_1 : \perp, \Delta_4 \vdash F_2 \vee F_3} \vee_1 \rightarrow \text{trivial}$$

- Case rule  $\vee_2$

$$\frac{h_1 : \perp, \Delta_4 \vdash F_3}{\bullet h_1 : \perp, \Delta_4 \vdash F_2 \vee F_3} \vee_2 \rightarrow \text{trivial}$$

- Case rule  $\rightarrow_L$

$$\frac{h_1 : \perp, \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \perp, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule  $\wedge_L$

$$\frac{h_1 : \perp, \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \wedge F_3 \vdash F_4} \wedge_L \rightarrow \text{trivial}$$

- Case rule  $\vee_L$

$$\frac{h_1 : \perp, \Delta_5, F_2 \vdash F_4 \quad h_1 : \perp, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \vee F_3 \vdash F_4} \vee_L \rightarrow \text{trivial}$$

- Case rule  $\perp_L$

$$\frac{\bullet h_1 : \perp, \Delta_3 \vdash F_2}{\bullet h_1 : \perp, \Delta_3 \vdash F_2} \perp_L \rightarrow \text{trivial}$$

- Case rule  $I$

$$\frac{\bullet h_1 : p_3, \perp, \Delta_2 \vdash p_3}{\bullet h_1 : p_3, \perp, \Delta_2 \vdash p_3} I \rightarrow \text{trivial}$$

- Case rule  $\top_L$

$$\frac{h_1 : \perp, \Delta_3 \vdash F_2}{\bullet h_1 : \top, \perp, \Delta_3 \vdash F_2} \top_L \rightarrow \text{trivial}$$

### 3.13 Status of $I$ : : Invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$
- Case rule  $\wedge_R$
- Case rule  $\vee_1$
- Case rule  $\vee_2$
- Case rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_4, p_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_4, F_3, p_5 \vdash p_5}{\bullet h_1 : (\Delta_4, p_5), F_2 \rightarrow F_3 \vdash p_5} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule  $\wedge_L$

$$\frac{h_1 : \Delta_4, F_2, F_3, p_5 \vdash p_5}{\bullet h_1 : (\Delta_4, p_5), F_2 \wedge F_3 \vdash p_5} \wedge_L \rightarrow \text{trivial}$$

- Case rule  $\vee_L$

$$\frac{h_1 : \Delta_4, F_2, p_5 \vdash p_5 \quad h_1 : \Delta_4, F_3, p_5 \vdash p_5}{\bullet h_1 : (\Delta_4, p_5), F_2 \vee F_3 \vdash p_5} \vee_L \rightarrow \text{trivial}$$

- Case rule  $\perp_L$

$$\frac{\bullet h_1 : \perp, \Delta_2, p_3 \vdash p_3}{\bullet h_1 : \perp, \Delta_2, p_3 \vdash p_3} \perp_L \rightarrow \text{trivial}$$

- Case rule  $I$

$$\frac{}{\bullet h_1 : p_2, \Delta_3 \vdash p_2} I \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\top_L$

$$\frac{h_1 : \Delta_2, p_3 \vdash p_3}{\bullet h_1 : \top, \Delta_2, p_3 \vdash p_3} \top_L \quad \rightarrow \quad \text{trivial}$$

### 3.14 Status of $\top_L$ : : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_1 : \top, \Delta_2 \vdash \top} \top_R \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{h_1 : \top, \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \top, \Delta_4 \vdash F_2 \rightarrow F_3} \rightarrow_R \quad \rightarrow \quad \frac{\frac{}{h_1 : \Delta_4, F_2 \vdash F_3} \text{ax/ind}}{\bullet h_1 : \Delta_4 \vdash F_2 \rightarrow F_3} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_1 : \top, \Delta_4 \vdash F_2 \quad h_1 : \top, \Delta_4 \vdash F_3}{\bullet h_1 : \top, \Delta_4 \vdash F_2 \wedge F_3} \wedge_R \quad \rightarrow \quad \frac{\frac{}{h_1 : \Delta_4 \vdash F_2} \text{ax/ind} \quad \frac{}{h_1 : \Delta_4 \vdash F_3} \text{ax/ind}}{\bullet h_1 : \Delta_4 \vdash F_2 \wedge F_3} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{h_1 : \top, \Delta_4 \vdash F_2}{\bullet h_1 : \top, \Delta_4 \vdash F_2 \vee F_3} \vee_1 \quad \rightarrow \quad \frac{\frac{}{h_1 : \Delta_4 \vdash F_2} \text{ax/ind}}{\bullet h_1 : \Delta_4 \vdash F_2 \vee F_3} \vee_1$$

- Case rule  $\vee_2$

$$\frac{h_1 : \top, \Delta_4 \vdash F_3}{\bullet h_1 : \top, \Delta_4 \vdash F_2 \vee F_3} \vee_2 \quad \rightarrow \quad \frac{\frac{}{h_1 : \Delta_4 \vdash F_3} \text{ax/ind}}{\bullet h_1 : \Delta_4 \vdash F_2 \vee F_3} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{h_1 : \top, \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \top, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\top, \Delta_5), F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L \quad \rightarrow \quad \frac{\frac{}{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2} \text{ax/ind} \quad \frac{}{h_1 : \Delta_5, F_3 \vdash F_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_1 : \top, \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : (\top, \Delta_5), F_2 \wedge F_3 \vdash F_4} \wedge_L \quad \rightarrow \quad \frac{\frac{}{h_1 : \Delta_5, F_2, F_3 \vdash F_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{\frac{h_1 : \top, \Delta_5, F_2 \vdash F_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash F_4} \vee_L \quad \frac{h_1 : \top, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash F_4} \vee_L}{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \text{ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \vee_L \rightarrow$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \top, \Delta_3 \vdash F_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_2} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_1 : p_3, \top, \Delta_2 \vdash p_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_2, p_3 \vdash p_3} I$$

- Case rule  $\top_L$

$$\frac{\frac{h_1 : \Delta_3 \vdash F_2}{\bullet h_1 : \top, \Delta_3 \vdash F_2} \top_L}{\bullet h_1 : \Delta_3 \vdash F_2} \text{ax} \rightarrow \frac{}{\bullet h_1 : \Delta_3 \vdash F_2} \text{H}$$

## 4 Height preserving admissibility of contraction

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2, F_3, F_3 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_3 \vdash \top} \top_R$$

- Case(s) rule  $\rightarrow_R$

$$\frac{h_1 : \Delta_4, F_2, F_5, F_5 \vdash F_3}{\bullet h_1 : \Delta_4, F_5, F_5 \vdash F_2 \rightarrow F_3} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_4, F_2, F_5, F_5 \vdash F_3}{h_1 : \Delta_4, F_2, F_5 \vdash F_3} \text{IH}}{\bullet h_1 : \Delta_4, F_5 \vdash F_2 \rightarrow F_3} \text{ax} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_1 : \Delta_4, F_5, F_5 \vdash F_2 \quad h_1 : \Delta_4, F_5, F_5 \vdash F_3}{\bullet h_1 : \Delta_4, F_5, F_5 \vdash F_2 \wedge F_3} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_4, F_5, F_5 \vdash F_2}{h_1 : \Delta_4, F_5 \vdash F_2} \text{IH} \quad \frac{h_1 : \Delta_4, F_5, F_5 \vdash F_3}{h_1 : \Delta_4, F_5 \vdash F_3} \text{IH}}{\bullet h_1 : \Delta_4, F_5 \vdash F_2 \wedge F_3} \text{ax} \wedge_R$$

- Case(s) rule  $\vee_1$

$$\frac{h_1 : \Delta_4, F_5, F_5 \vdash F_2}{\bullet h_1 : \Delta_4, F_5, F_5 \vdash F_2 \vee F_3} \vee_1 \rightarrow \frac{\frac{h_1 : \Delta_4, F_5, F_5 \vdash F_2}{h_1 : \Delta_4, F_5 \vdash F_2} \text{IH}}{\bullet h_1 : \Delta_4, F_5 \vdash F_2 \vee F_3} \text{ax} \vee_1$$

- Case(s) rule  $\vee_2$

$$\frac{h_1 : \Delta_4, F_5, F_5 \vdash F_3}{\bullet h_1 : \Delta_4, F_5, F_5 \vdash F_2 \vee F_3} \vee_2 \rightarrow \frac{\frac{h_1 : \Delta_4, F_5, F_5 \vdash F_3}{h_1 : \Delta_4, F_5 \vdash F_3} \text{IH}}{\bullet h_1 : \Delta_4, F_5 \vdash F_2 \vee F_3} \text{ax} \vee_2$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_5, F_2 \rightarrow F_3, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3, F_2 \rightarrow F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \rightarrow F_3, F_2 \rightarrow F_3 \vdash F_2}{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2} \text{IH} \quad \frac{h_1 : \Delta_5, F_3, F_3 \vdash F_4}{h_1 : \Delta_5, F_3 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \text{inv-th/ax} \rightarrow_L$$

$$\frac{h_1 : \Delta_5, F_6, F_6, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3, F_6, F_6 \vdash F_4}{\bullet h_1 : (\Delta_5, F_2 \rightarrow F_3), F_6, F_6 \vdash F_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_6, F_6, F_2 \rightarrow F_3 \vdash F_2}{h_1 : \Delta_5, F_6, F_2 \rightarrow F_3 \vdash F_2} \text{IH} \quad \frac{h_1 : \Delta_5, F_3, F_6, F_6 \vdash F_4}{h_1 : \Delta_5, F_3, F_6 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_5, F_6, F_2 \rightarrow F_3 \vdash F_4} \text{ax} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_1 : \Delta_5, F_2, F_3, F_2 \wedge F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3, F_2 \wedge F_3 \vdash F_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_2, F_3, F_3 \vdash F_4}{h_1 : \Delta_5, F_2, F_2, F_3 \vdash F_4} \text{IH} \quad \frac{h_1 : \Delta_5, F_3, F_3 \vdash F_4}{h_1 : \Delta_5, F_3 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4} \text{inv-th/ax} \wedge_L$$

$$\frac{h_1 : \Delta_5, F_2, F_3, F_6, F_6 \vdash F_4}{\bullet h_1 : (\Delta_5, F_2 \wedge F_3), F_6, F_6 \vdash F_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3, F_6, F_6 \vdash F_4}{h_1 : \Delta_5, F_2, F_3, F_6 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_5, F_6, F_2 \wedge F_3 \vdash F_4} \text{ax} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\frac{h_1 : \Delta_5, F_2, F_2 \vee F_3 \vdash F_4 \quad h_1 : \Delta_5, F_3, F_2 \vee F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3, F_2 \vee F_3 \vdash F_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_2 \vdash F_4}{h_1 : \Delta_5, F_2 \vdash F_4} \text{IH} \quad \frac{h_1 : \Delta_5, F_3, F_3 \vdash F_4}{h_1 : \Delta_5, F_3 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \text{inv-th/ax} \vee_L$$

$$\frac{h_1 : \Delta_5, F_2, F_6, F_6 \vdash F_4 \quad h_1 : \Delta_5, F_3, F_6, F_6 \vdash F_4}{\bullet h_1 : (\Delta_5, F_2 \vee F_3), F_6, F_6 \vdash F_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_6, F_6 \vdash F_4}{h_1 : \Delta_5, F_2, F_6 \vdash F_4} \text{IH} \quad \frac{h_1 : \Delta_5, F_3, F_6, F_6 \vdash F_4}{h_1 : \Delta_5, F_3, F_6 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_5, F_6, F_2 \vee F_3 \vdash F_4} \text{ax} \vee_L$$

- Case(s) rule  $\perp_L$

$$\frac{}{\bullet h_1 : \Delta_3, \perp, \perp \vdash F_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_2} \perp_L$$

$$\frac{}{\bullet h_1 : (\perp, \Delta_3), F_4, F_4 \vdash F_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3, F_4 \vdash F_2} \perp_L$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_1 : \Delta_2, p_3, p_3 \vdash p_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_2, p_3 \vdash p_3} I$$

$$\frac{}{\bullet h_1 : (\Delta_2, p_4), F_3, F_3 \vdash p_4} I \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_3, p_4 \vdash p_4} I$$

- Case(s) rule  $\top_L$

$$\frac{h_1 : \top, \Delta_3 \vdash F_2}{\bullet h_1 : \Delta_3, \top, \top \vdash F_2} \top_L \rightarrow \frac{\frac{}{h_1 : \Delta_3 \vdash F_2} \text{inv-th/ax}}{\bullet h_1 : \top, \Delta_3 \vdash F_2} \top_L$$

$$\frac{h_1 : \Delta_3, F_4, F_4 \vdash F_2}{\bullet h_1 : (\top, \Delta_3), F_4, F_4 \vdash F_2} \top_L \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_3, F_4, F_4 \vdash F_2} \text{ax}}{h_1 : \Delta_3, F_4 \vdash F_2} \text{IH}}{\bullet h_1 : \top, \Delta_3, F_4 \vdash F_2} \top_L$$

## 5 Identity-Expansion

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0 \vee F_1} \vee_1 \quad \frac{\frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0 \vee F_1} \vee_2}{- : F_0 \vee F_1 \vdash F_0 \vee F_1} \vee_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_1 \vdash F_0} W \quad \frac{\frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{- : F_0, F_1 \vdash F_0 \wedge F_1} \wedge_R}{- : F_0 \wedge F_1 \vdash F_0 \wedge F_1} \wedge_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_0 \rightarrow F_1 \vdash F_0} W \quad \frac{\frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{- : F_0, F_0 \rightarrow F_1 \vdash F_1} \rightarrow_L}{- : F_0 \rightarrow F_1 \vdash F_0 \rightarrow F_1} \rightarrow_R$$

$$\frac{}{- : \top \vdash \top} \top_R$$

$$\frac{}{- : \perp \vdash \perp} \perp_L$$

## 6 Cut-Elimination

### 6.1 Status of $\top_R$ : OK

- Case rule  $\top_R$

$$\frac{\frac{\bullet h_1 : \Delta_4 \vdash \top}{\top_R} \quad \frac{\bullet h_3 : \Delta_4, \top \vdash \top}{\top_R}}{- : \Delta_4 \vdash \top} \text{Cut}$$

$$\xrightarrow{\quad} \frac{}{- : \Delta_4 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{\bullet h_1 : \Delta_6 \vdash \top}{\top_R} \quad \frac{\frac{h_3 : \top, \Delta_6, F_4 \vdash F_5}{\rightarrow_R} \quad \bullet h_3 : \Delta_6, \top \vdash F_4 \rightarrow F_5}{\text{Cut}}}{- : \Delta_6 \vdash F_4 \rightarrow F_5} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{\bullet h_1 : \Delta_6, F_4 \vdash \top}{\top_R} \quad \frac{h_3 : \top, \Delta_6, F_4 \vdash F_5}{\text{ax/W}}}{- : \Delta_6, F_4 \vdash F_5} \text{hCut}$$

$$\xrightarrow{\quad} \frac{}{- : \Delta_6 \vdash F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{\bullet h_1 : \Delta_6 \vdash \top}{\top_R} \quad \frac{\frac{h_3 : \top, \Delta_6 \vdash F_4 \quad h_3 : \top, \Delta_6 \vdash F_5}{\wedge_R} \quad \bullet h_3 : \Delta_6, \top \vdash F_4 \wedge F_5}{\text{Cut}}}{- : \Delta_6 \vdash F_4 \wedge F_5} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{\frac{\bullet h_1 : \Delta_6 \vdash \top}{\text{ax/W}} \quad \frac{h_3 : \top, \Delta_6 \vdash F_4}{\text{hCut}} \quad \frac{\frac{\bullet h_1 : \Delta_6 \vdash \top}{\text{ax/W}} \quad \frac{h_3 : \top, \Delta_6 \vdash F_5}{\text{hCut}}}{- : \Delta_6 \vdash F_4 \wedge F_5} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{\frac{\bullet h_1 : \Delta_6 \vdash \top}{\top_R} \quad \frac{\frac{h_3 : \top, \Delta_6 \vdash F_4}{\vee_1} \quad \bullet h_3 : \Delta_6, \top \vdash F_4 \vee F_5}{\text{Cut}}}{- : \Delta_6 \vdash F_4 \vee F_5} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{\bullet h_1 : \Delta_6 \vdash \top}{\top_R} \quad \frac{h_3 : \top, \Delta_6 \vdash F_4}{\text{ax/W}}}{- : \Delta_6 \vdash F_4} \text{hCut}$$

$$\xrightarrow{\quad} \frac{}{- : \Delta_6 \vdash F_4 \vee F_5} \vee_1$$

- Case rule  $\vee_2$

$$\frac{\frac{\bullet h_1 : \Delta_6 \vdash \top}{\top_R} \quad \frac{\frac{h_3 : \top, \Delta_6 \vdash F_5}{\vee_2} \quad \bullet h_3 : \Delta_6, \top \vdash F_4 \vee F_5}{\text{Cut}}}{- : \Delta_6 \vdash F_4 \vee F_5} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{\bullet h_1 : \Delta_6 \vdash \top}{\top_R} \quad \frac{h_3 : \top, \Delta_6 \vdash F_5}{\text{ax/W}}}{- : \Delta_6 \vdash F_5} \text{hCut}$$

$$\xrightarrow{\quad} \frac{}{- : \Delta_6 \vdash F_4 \vee F_5} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{\bullet h_1 : \Delta_7, F_4 \rightarrow F_5 \vdash \top}{\top_R} \quad \frac{\frac{h_3 : \top, \Delta_7, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\rightarrow_L} \quad \bullet h_3 : (\Delta_7, F_4 \rightarrow F_5), \top \vdash F_6}{\text{Cut}}}{- : \Delta_7, F_4 \rightarrow F_5 \vdash F_6} \text{Cut}$$

$$\xrightarrow{\quad} \frac{\frac{\frac{\bullet h_1 : \Delta_7, F_4 \rightarrow F_5 \vdash \top}{\top_R} \quad \frac{h_3 : \top, \Delta_7, F_4 \rightarrow F_5 \vdash F_4}{\text{ax/W}}}{- : \Delta_7, F_4 \rightarrow F_5 \vdash F_4} \text{hCut} \quad \frac{\frac{\bullet h_1 : \Delta_7, F_5 \vdash \top}{\top_R} \quad \frac{h_3 : \top, \Delta_7, F_5 \vdash F_6}{\text{ax/W}}}{- : \Delta_7, F_5 \vdash F_6} \text{hCut}}{- : \Delta_7, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$



- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_7, F_4 \wedge F_5 \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \wedge F_5), \top \vdash F_6} \wedge_L}{- : \Delta_7, F_4 \wedge F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_7, F_4, F_5 \vdash \top}{- : \Delta_7, F_4, F_5 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{- : \Delta_7, F_4 \wedge F_5 \vdash F_6} \text{ax/W}}{- : \Delta_7, F_4 \wedge F_5 \vdash F_6} \text{hCut} \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_7, F_4 \vee F_5 \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \vee F_5), \top \vdash F_6} \vee_L}{- : \Delta_7, F_4 \vee F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_7, F_4 \vdash \top}{- : \Delta_7, F_4 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6}{- : \Delta_7, F_4 \vee F_5 \vdash F_6} \text{ax/W} \quad \frac{\frac{\bullet h_1 : \Delta_7, F_5 \vdash \top}{- : \Delta_7, F_5 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_7, F_5 \vdash F_6}{- : \Delta_7, F_5 \vdash F_6} \text{ax/W}}{- : \Delta_7, F_5 \vdash F_6} \text{hCut}}{- : \Delta_7, F_4 \vee F_5 \vdash F_6} \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_5 \vdash \top} \top_R \quad \frac{}{\bullet h_3 : (\perp, \Delta_5), \top \vdash F_4} \perp_L}{- : \perp, \Delta_5 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_5 \vdash F_4} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_4, p_5 \vdash \top} \top_R \quad \frac{}{\bullet h_3 : (\Delta_4, p_5), \top \vdash p_5} I}{- : \Delta_4, p_5 \vdash p_5} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_4, p_5 \vdash p_5} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_5 \vdash \top} \top_R \quad \frac{h_3 : \Delta_5 \vdash F_4}{\bullet h_3 : \Delta_5, \top \vdash F_4} \top_L}{- : \Delta_5 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_5 \vdash F_4} \text{ax/W}
\end{array}$$

## 6.2 Status of $\rightarrow_R$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \vdash F_6}{\bullet h_1 : \Delta_8 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{}{\bullet h_7 : \Delta_8, F_5 \rightarrow F_6 \vdash \top} \top_R}{- : \Delta_8 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_5 \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \rightarrow F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10}, F_8 \vdash F_5 \rightarrow F_6}{- : \Delta_{10}, F_8 \vdash F_5 \rightarrow F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \rightarrow F_6 \vdash F_9}{- : \Delta_{10}, F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \rightarrow_R \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_5 \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_{10} \vdash F_8 \wedge F_9 \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \rightarrow F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8}{- : \Delta_{10} \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \quad \text{hCut} \\
\hline
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \rightarrow F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_9}{- : \Delta_{10} \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \wedge F_9 \quad \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_5 \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \vee F_9} \vee_1 \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \rightarrow F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8}{- : \Delta_{10} \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_5 \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \vee F_9} \vee_2 \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \rightarrow F_6}{- : \Delta_{10} \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_9}{- : \Delta_{10} \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_5 \rightarrow F_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \rightarrow F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \rightarrow F_6}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \rightarrow F_6, F_8 \rightarrow F_9 \vdash F_8}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8 \quad \text{hCut} \\
\hline
\frac{h_1 : \Delta_{11}, F_5, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \rightarrow F_6} \text{inv-th/ax} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, F_9 \vdash F_{10} \quad \rightarrow_L \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}
\end{array}$$
  

$$\begin{array}{c}
\frac{h_1 : \Delta_9, F_6 \vdash F_7}{\bullet h_1 : \Delta_9 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \rightarrow_L \\
\hline
- : \Delta_9 \vdash F_8 \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_9 \vdash F_6 \rightarrow F_7}{- : \Delta_9 \vdash F_6} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash F_6}{- : \Delta_9 \vdash F_6} \text{ax/W} \\
\hline
- : \Delta_9 \vdash F_6 \quad \text{hCut} \\
\hline
\frac{- : \Delta_9, F_6 \vdash F_7}{- : \Delta_9, F_6 \vdash F_8} \text{ax/W} \quad \frac{- : \Delta_9, F_6, F_7 \vdash F_8}{- : \Delta_9, F_6 \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_9, F_6 \vdash F_8 \quad \text{sCut} \\
\hline
- : \Delta_9 \vdash F_8
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vdash F_6 \quad \bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \rightarrow F_6 \quad \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \rightarrow F_6 \vdash F_{10}} \quad \wedge_L}{- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_5, F_8, F_9 \vdash F_6 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \rightarrow F_6} \quad \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{- : \Delta_{11}, F_8, F_9 \vdash F_{10}} \quad \text{ax/W}}{- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \quad \text{hCut} \quad \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, F_8 \vee F_9), F_5 \vdash F_6 \quad \bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \rightarrow F_6 \quad \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \rightarrow F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \rightarrow F_6 \vdash F_{10}} \quad \vee_L}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_5, F_8 \vdash F_6 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \rightarrow F_6} \quad \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \rightarrow F_6 \vdash F_{10}}{- : \Delta_{11}, F_8 \vdash F_{10}} \quad \text{ax/W}}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \quad \text{hCut} \quad \vee_L \\
\frac{\frac{h_1 : \Delta_{11}, F_5, F_9 \vdash F_6 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \rightarrow F_6} \quad \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{- : \Delta_{11}, F_9 \vdash F_{10}} \quad \text{ax/W}}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \quad \text{hCut} \quad \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : (\perp, \Delta_9), F_5 \vdash F_6 \quad \bullet h_1 : \perp, \Delta_9 \vdash F_5 \rightarrow F_6 \quad \rightarrow_R \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \rightarrow F_6 \vdash F_8} \quad \perp_L}{- : \perp, \Delta_9 \vdash F_8} \quad \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_9 \vdash F_8 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : (\Delta_8, p_9), F_5 \vdash F_6 \quad \bullet h_1 : \Delta_8, p_9 \vdash F_5 \rightarrow F_6 \quad \rightarrow_R \quad \frac{}{\bullet h_7 : (\Delta_8, p_9), F_5 \rightarrow F_6 \vdash p_9} \quad I}{- : \Delta_8, p_9 \vdash p_9} \quad \text{Cut} \\
\rightarrow \\
- : \Delta_8, p_9 \vdash p_9 \quad I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : (\top, \Delta_9), F_5 \vdash F_6 \quad \bullet h_1 : \top, \Delta_9 \vdash F_5 \rightarrow F_6 \quad \rightarrow_R \quad \frac{h_7 : \Delta_9, F_5 \rightarrow F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \rightarrow F_6 \vdash F_8} \quad \top_L}{- : \top, \Delta_9 \vdash F_8} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \top, \Delta_9 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad h_7 : \top, \Delta_9, F_5 \rightarrow F_6 \vdash F_8 \quad \text{ax/W}}{- : \top, \Delta_9 \vdash F_8} \quad \text{hCut}
\end{array}$$

### 6.3 Status of $\wedge_R$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash F_5 \quad h_1 : \Delta_8 \vdash F_6 \quad \wedge_R \quad \frac{}{\bullet h_7 : \Delta_8, F_5 \wedge F_6 \vdash \top} \quad \top_R}{- : \Delta_8 \vdash \top} \quad \text{Cut} \\
\rightarrow \\
- : \Delta_8 \vdash \top \quad \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_5 \quad h_1 : \Delta_{10} \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \wedge F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \\
\hline
\frac{\bullet h_1 : \Delta_{10}, F_8 \vdash F_5 \wedge F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \wedge F_6 \vdash F_9}{- : \Delta_{10} \vdash F_8 \rightarrow F_9} \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_5 \quad h_1 : \Delta_{10} \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_{10} \vdash F_8 \wedge F_9 \\
\hline
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \wedge F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8}{- : \Delta_{10} \vdash F_8} \text{hCut} \quad \frac{\bullet h_1 : \Delta_{10} \vdash F_5 \wedge F_6}{- : \Delta_{10} \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_9}{- : \Delta_{10} \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \wedge F_9 \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_5 \quad h_1 : \Delta_{10} \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \vee F_9} \vee_1 \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \\
\hline
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \wedge F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8}{- : \Delta_{10} \vdash F_8} \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_5 \quad h_1 : \Delta_{10} \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \vee F_9} \vee_2 \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \\
\hline
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \wedge F_6}{- : \Delta_{10} \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_9}{- : \Delta_{10} \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \wedge F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \wedge F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \\
\hline
\frac{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \wedge F_6}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \wedge F_6 \vdash F_8}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{hCut} \quad \frac{h_1 : \Delta_{11}, F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6} \text{inv-th/ax} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{inv-th/ax} \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \wedge F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \\
\hline
\frac{h_1 : \Delta_{11}, F_8, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \wedge F_6} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{11}, F_8, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \wedge F_6} \text{inv-th/ax} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}{- : \Delta_{11}, F_8, F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_9 \vdash F_6 \quad h_1 : \Delta_9 \vdash F_7}{\bullet h_1 : \Delta_9 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_5 : \Delta_9, F_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash F_8} \wedge_L \\
\hline
- : \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{}{- : \Delta_9 \vdash F_6} \text{ax/W} \quad \frac{}{- : \Delta_9, F_6 \vdash F_7} \text{ax/W} \quad \frac{}{- : \Delta_9, F_6, F_7 \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_9 \vdash F_8 \quad \text{sCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \wedge F_6 \vdash F_{10}} \vee_L \\
\hline
- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \\
\rightarrow \\
\frac{h_1 : \Delta_{11}, F_8 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \wedge F_6} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{11}, F_8 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \wedge F_6} \text{inv-th/ax} \\
\hline
- : \Delta_{11}, F_8 \vdash F_{10} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6} \text{ax/W} \quad \frac{h_1 : \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{11}, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6} \text{inv-th/ax} \\
\hline
- : \Delta_{11}, F_8 \vdash F_{10} \quad \text{hCut} \quad - : \Delta_{11}, F_9 \vdash F_{10} \\
\hline
- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_9 \vdash F_5 \quad h_1 : \perp, \Delta_9 \vdash F_6}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \wedge F_6 \vdash F_8} \perp_L \\
\hline
- : \perp, \Delta_9 \vdash F_8 \\
\rightarrow \\
- : \perp, \Delta_9 \vdash F_8 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_8, p_9 \vdash F_5 \quad h_1 : \Delta_8, p_9 \vdash F_6}{\bullet h_1 : \Delta_8, p_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{}{\bullet h_7 : (\Delta_8, p_9), F_5 \wedge F_6 \vdash p_9} I \\
\hline
- : \Delta_8, p_9 \vdash p_9 \\
\rightarrow \\
- : \Delta_8, p_9 \vdash p_9 \quad I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_9 \vdash F_5 \quad h_1 : \top, \Delta_9 \vdash F_6}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_9, F_5 \wedge F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \wedge F_6 \vdash F_8} \top_L \\
\hline
- : \top, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : \top, \Delta_9 \vdash F_5 \wedge F_6}{- : \top, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_5 \wedge F_6 \vdash F_8}{- : \top, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 6.4 Status of $\vee_1$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_8 \vdash F_5}{\bullet h_1 : \Delta_8 \vdash F_5 \vee F_6} \vee_1 \quad \frac{}{\bullet h_7 : \Delta_8, F_5 \vee F_6 \vdash \top} \top_R \\
\hline
- : \Delta_8 \vdash \top \\
\rightarrow \\
- : \Delta_8 \vdash \top \quad \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_5}{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10}, F_8 \vdash F_5 \vee F_6}{- : \Delta_{10}, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10}, F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_5}{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_{10} \vdash F_8 \wedge F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{- : \Delta_{10} \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \quad \text{hCut} \\
\hline
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6}{- : \Delta_{10} \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10} \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \wedge F_9 \quad \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_5}{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \vee F_9} \vee_1 \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{- : \Delta_{10} \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_5}{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \vee F_9} \vee_2 \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6}{- : \Delta_{10} \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10} \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \vee F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \vee F_6}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8 \quad \text{hCut} \\
\hline
\frac{h_1 : \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{- : \Delta_{11}, F_9 \vdash F_{10}} \vee_1 \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \quad \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{11}, F_8, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{- : \Delta_{11}, F_8, F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, F_8, F_9 \vdash F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{\frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_{11}, F_8 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \vee F_6} \text{inv-th/ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{- : \Delta_{11}, F_8 \vdash F_{10}} \text{ax/w}}{- : \Delta_{11}, F_8 \vdash F_{10}} \vee_1 \quad \frac{\frac{\frac{h_1 : \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{ax/w}}{- : \Delta_{11}, F_9 \vdash F_{10}} \vee_1}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{hCut} \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9 \vdash F_6}{\bullet h_1 : \Delta_9 \vdash F_6 \vee F_7} \vee_1 \quad \frac{\frac{h_5 : \Delta_9, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash F_8} \vee_L}{- : \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_9 \vdash F_6}{- : \Delta_9 \vdash F_6} \text{ax/w} \quad \frac{- : \Delta_9, F_6 \vdash F_8}{- : \Delta_9 \vdash F_8} \text{ax/w}}{- : \Delta_9 \vdash F_8} \text{sCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \perp, \Delta_9 \vdash F_5}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8}{- : \perp, \Delta_9 \vdash F_8} \perp_L}{- : \perp, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{- : \perp, \Delta_9 \vdash F_8}{- : \perp, \Delta_9 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8, p_9 \vdash F_5}{\bullet h_1 : \Delta_8, p_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{\bullet h_7 : (\Delta_8, p_9), F_5 \vee F_6 \vdash p_9}{- : \Delta_8, p_9 \vdash p_9} I}{- : \Delta_8, p_9 \vdash p_9} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_8, p_9 \vdash p_9}{- : \Delta_8, p_9 \vdash p_9} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \top, \Delta_9 \vdash F_5}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{\frac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L}{- : \top, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6}{- : \top, \Delta_9 \vdash F_8} \text{ax/w} \quad \frac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{- : \top, \Delta_9 \vdash F_8} \text{ax/w}}{- : \top, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 6.5 Status of $\vee_2$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8 \vdash F_6}{\bullet h_1 : \Delta_8 \vdash F_5 \vee F_6} \vee_2 \quad \frac{\bullet h_7 : \Delta_8, F_5 \vee F_6 \vdash \top}{- : \Delta_8 \vdash \top} \top_R}{- : \Delta_8 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_8 \vdash \top}{- : \Delta_8 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10}, F_8 \vdash F_5 \vee F_6}{- : \Delta_{10}, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10}, F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_{10} \vdash F_8 \wedge F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{- : \Delta_{10} \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \quad \text{hCut} \\
\hline
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6}{- : \Delta_{10} \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10} \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \wedge F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \wedge F_9 \quad \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \vee F_9} \vee_1 \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6}{- : \Delta_{10} \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{- : \Delta_{10} \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_8 \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10} \vdash F_6}{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \vee F_9} \vee_2 \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{10} \vdash F_5 \vee F_6}{- : \Delta_{10} \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10} \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10} \vdash F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10} \vdash F_8 \vee F_9 \quad \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \vee F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \vee F_6}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8 \quad \text{hCut} \\
\hline
\frac{h_1 : \Delta_{11}, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_9 \vdash F_6), F_5 \vee F_6 \vdash F_{10}} \vee_2 \\
\hline
- : \Delta_{11}, F_9 \vdash F_6 \quad \text{hCut} \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \quad \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{11}, F_8, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8, F_9 \vdash F_6), F_5 \vee F_6 \vdash F_{10}} \vee_2 \\
\hline
- : \Delta_{11}, F_8, F_9 \vdash F_6 \quad \text{hCut} \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \wedge_L
\end{array}$$



- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_{11}, F_8 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \vee F_6} \text{inv-th/ax}}{- : \Delta_{11}, F_8 \vdash F_{10}} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_8 \vdash F_{10}} \text{ax/w}}{- : \Delta_{11}, F_8 \vdash F_{10}} \text{hCut} \\
\frac{\frac{\frac{h_1 : \Delta_{11}, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax}}{- : \Delta_{11}, F_9 \vdash F_{10}} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_9 \vdash F_{10}} \text{ax/w}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{hCut} \\
- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_9 \vdash F_7}{\bullet h_1 : \Delta_9 \vdash F_6 \vee F_7} \vee_2 \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash F_8} \vee_L}{- : \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_9 \vdash F_7}{- : \Delta_9 \vdash F_8} \text{ax/w} \quad \frac{- : \Delta_9, F_7 \vdash F_8}{- : \Delta_9 \vdash F_8} \text{ax/w}}{- : \Delta_9 \vdash F_8} \text{sCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_9 \vdash F_6}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8} \perp_L}{- : \perp, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_9 \vdash F_8 \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_8, p_9 \vdash F_6}{\bullet h_1 : \Delta_8, p_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{}{\bullet h_7 : (\Delta_8, p_9), F_5 \vee F_6 \vdash p_9} I}{- : \Delta_8, p_9 \vdash p_9} \text{Cut} \\
\rightarrow \\
- : \Delta_8, p_9 \vdash p_9 I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_9 \vdash F_6}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L}{- : \top, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6}{- : \top, \Delta_9 \vdash F_8} \text{ax/w} \quad \frac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{- : \top, \Delta_9 \vdash F_8} \text{ax/w}}{- : \top, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 6.6 Status of $\rightarrow_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_9, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_9, F_7 \vdash F_{10}}{\bullet h_1 : \Delta_9, F_6 \rightarrow F_7 \vdash F_{10}} \rightarrow_L \quad \frac{}{\bullet h_8 : (\Delta_9, F_6 \rightarrow F_7), F_{10} \vdash \top} \top_R}{- : \Delta_9, F_6 \rightarrow F_7 \vdash \top} \text{Cut} \\
\rightarrow \\
- : \Delta_9, F_6 \rightarrow F_7 \vdash \top \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_{11}, F_7 \vdash F_{12} \rightarrow_L \quad \frac{h_8 : \Delta_{11}, F_9, F_{12}, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \rightarrow F_7), F_{12} \vdash F_9 \rightarrow F_{10}} \rightarrow_R}{\frac{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9 \rightarrow F_{10}}{\bullet h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_{12}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_{11}, F_9, F_6 \rightarrow F_7 \vdash F_{12} \quad \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_{12}, F_9, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \rightarrow F_7), F_{12} \vdash F_9 \rightarrow F_{10}} \text{ax/W}}{- : \Delta_{11}, F_9, F_6 \rightarrow F_7 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_{11}, F_7 \vdash F_{12} \rightarrow_L \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \rightarrow F_7 \vdash F_9 \quad h_8 : \Delta_{11}, F_{12}, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \rightarrow F_7), F_{12} \vdash F_9 \wedge F_{10}} \wedge_R}{\frac{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9 \wedge F_{10}}{\bullet h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_{12}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_{12} \quad \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \rightarrow F_7 \vdash F_9}{\bullet h_8 : (\Delta_{11}, F_6 \rightarrow F_7), F_{12} \vdash F_9 \wedge F_{10}} \text{ax/W}}{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9} \text{hCut}}{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9 \wedge F_{10}} \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_{11}, F_7 \vdash F_{12} \rightarrow_L \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \rightarrow F_7 \vdash F_9}{\bullet h_8 : (\Delta_{11}, F_6 \rightarrow F_7), F_{12} \vdash F_9 \vee F_{10}} \vee_1}{\frac{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}}{\bullet h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_{12}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_{12} \quad \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \rightarrow F_7 \vdash F_9}{\bullet h_8 : (\Delta_{11}, F_6 \rightarrow F_7), F_{12} \vdash F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9} \text{hCut}}{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}} \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_{11}, F_7 \vdash F_{12} \rightarrow_L \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \rightarrow F_7), F_{12} \vdash F_9 \vee F_{10}} \vee_2}{\frac{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}}{\bullet h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_{12}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_{12} \quad \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \rightarrow F_7), F_{12} \vdash F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}} \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_7 \vdash F_{13} \rightarrow_L \quad \frac{h_8 : \Delta_{12}, F_{13}, F_6 \rightarrow F_7, F_9 \rightarrow F_{10} \vdash F_9 \quad h_8 : \Delta_{12}, F_{10}, F_{13}, F_6 \rightarrow F_7 \vdash F_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7), F_{13} \vdash F_{11}} \text{Cut}}{\frac{- : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7 \vdash F_{11}}{\bullet h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7 \vdash F_{13}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{- : \Delta_{12}, F_6 \rightarrow F_7, F_9 \rightarrow F_{10} \vdash F_6 \quad \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_7, F_9 \rightarrow F_{10} \vdash F_{13} \quad \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{13}, F_7, F_9 \rightarrow F_{10} \vdash F_9 \quad \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_{13}, F_7 \vdash F_{11}}{\bullet h_8 : \Delta_{12}, F_{13}, F_7, F_9 \rightarrow F_{10} \vdash F_{11}} \text{hCut}}{- : \Delta_{12}, F_7, F_9 \rightarrow F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, F_6 \rightarrow F_7, F_9 \rightarrow F_{10} \vdash F_{11}} \rightarrow_L \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_{12}, F_7 \vdash F_9 \rightarrow F_{10} \rightarrow_L \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7, F_9 \rightarrow F_{10} \vdash F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_6 \rightarrow F_7), F_9 \rightarrow F_{10} \vdash F_{11}} \text{Cut}}{\frac{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{11}}{\bullet h_1 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9 \rightarrow F_{10}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_6 \quad \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_7 \vdash F_9 \rightarrow F_{10} \quad \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_7, F_9 \rightarrow F_{10} \vdash F_9 \quad \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_7 \vdash F_{11}}{\bullet h_8 : \Delta_{12}, F_7, F_9 \rightarrow F_{10} \vdash F_{11}} \text{hCut}}{- : \Delta_{12}, F_7 \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{11}} \rightarrow_L \\
\rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : \Delta_{10}, F_8 \vdash F_{11} \rightarrow_L \quad \frac{h_6 : \Delta_{10}, F_{11}, F_7 \rightarrow F_8 \vdash F_7 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9 \rightarrow_L}{\bullet h_6 : (\Delta_{10}, F_7 \rightarrow F_8), F_{11} \vdash F_9} \text{Cut}}{\bullet h_1 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_{11}} \rightarrow_L \\
\frac{}{- : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_9} \rightarrow \\
\frac{}{- : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_7} \text{ax/W} \quad \frac{h_1 : \Delta_{10}, F_8 \vdash F_{11} \text{ ax/W} \quad \frac{h_6 : \Delta_{10}, F_{11}, F_8 \vdash F_9 \text{ ax/W} \quad \bullet h_6 : \Delta_{10}, F_{11}, F_8 \vdash F_9}{H} \text{hCut}}{- : \Delta_{10}, F_8 \vdash F_9} \rightarrow_L \\
\frac{}{- : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_9} \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_7 \vdash F_{13} \rightarrow_L \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_{13}, F_6 \rightarrow F_7 \vdash F_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7), F_{13} \vdash F_{11}} \wedge_L}{\bullet h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash F_{13}} \text{Cut} \\
\frac{}{- : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash F_{11}} \rightarrow \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7, F_9 \wedge F_{10} \vdash F_6} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_7, F_9 \wedge F_{10} \vdash F_{13} \text{ ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_{13}, F_7, F_9 \vdash F_{11} \text{ inv-th/ax} \quad \bullet h_8 : \Delta_{12}, F_{13}, F_7, F_9 \wedge F_{10} \vdash F_{11}}{\wedge_L} \text{hCut}}{- : \Delta_{12}, F_7, F_9 \wedge F_{10} \vdash F_{11}} \rightarrow_L \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7, F_9 \wedge F_{10} \vdash F_{11}} \rightarrow_L \\
\frac{h_1 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_{12}, F_7 \vdash F_9 \wedge F_{10} \rightarrow_L \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \rightarrow F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_6 \rightarrow F_7), F_9 \wedge F_{10} \vdash F_{11}} \wedge_L}{\bullet h_1 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9 \wedge F_{10}} \text{Cut} \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{11}} \rightarrow \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_6} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_7 \vdash F_9 \wedge F_{10} \text{ ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_7, F_9 \vdash F_{11} \text{ inv-th/ax} \quad \bullet h_8 : \Delta_{12}, F_7, F_9 \wedge F_{10} \vdash F_{11}}{\wedge_L} \text{hCut}}{- : \Delta_{12}, F_7 \vdash F_{11}} \rightarrow_L \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{11}} \rightarrow_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : (\Delta_{12}, F_9 \vee F_{10}), F_7 \vdash F_{13} \rightarrow_L \quad \frac{h_8 : \Delta_{12}, F_9, F_{13}, F_6 \rightarrow F_7 \vdash F_{11} \quad h_8 : \Delta_{12}, F_{10}, F_{13}, F_6 \rightarrow F_7 \vdash F_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7), F_{13} \vdash F_{11}} \vee_L}{\bullet h_1 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash F_{13}} \text{Cut} \\
\frac{}{- : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash F_{11}} \rightarrow \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7, F_9 \vee F_{10} \vdash F_6} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_7, F_9 \vee F_{10} \vdash F_{13} \text{ ax/W} \quad \frac{h_8 : \Delta_{12}, F_{13}, F_7, F_9 \vdash F_{11} \text{ inv-th/ax} \quad \bullet h_8 : \Delta_{12}, F_{13}, F_7, F_9 \vee F_{10} \vdash F_{11}}{\vee_L} \text{hCut}}{- : \Delta_{12}, F_7, F_9 \vee F_{10} \vdash F_{11}} \rightarrow_L \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7, F_9 \vee F_{10} \vdash F_{11}} \rightarrow_L \\
\frac{h_1 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_{12}, F_7 \vdash F_9 \vee F_{10} \rightarrow_L \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash F_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_6 \rightarrow F_7), F_9 \vee F_{10} \vdash F_{11}} \vee_L}{\bullet h_1 : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}} \text{Cut} \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{11}} \rightarrow \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_6} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_7 \vdash F_9 \vee F_{10} \text{ ax/W} \quad \frac{h_8 : \Delta_{12}, F_7, F_9 \vdash F_{11} \text{ inv-th/ax} \quad \bullet h_8 : \Delta_{12}, F_7, F_9 \vee F_{10} \vdash F_{11}}{\vee_L} \text{hCut}}{- : \Delta_{12}, F_7 \vdash F_{11}} \rightarrow_L \\
\frac{}{- : \Delta_{12}, F_6 \rightarrow F_7 \vdash F_{11}} \rightarrow_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_{10}, F_7 \vdash \perp \rightarrow_L \quad \frac{}{\bullet h_8 : (\Delta_{10}, F_6 \rightarrow F_7), \perp \vdash F_9} \perp_L}{\bullet h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \perp} \text{Cut} \\
\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9} \rightarrow \\
\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_6} \text{ax/W} \quad \frac{h_1 : \Delta_{10}, F_7 \vdash \perp \text{ ax/W} \quad \bullet h_8 : \perp, \Delta_{10}, F_7 \vdash F_9 \text{ inv-th/ax}}{- : \Delta_{10}, F_7 \vdash F_9} \text{hCut} \\
\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9} \rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : (\perp, \Delta_{10}), F_7 \vdash F_{11}}{\bullet h_1 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash F_{11}} \rightarrow_L \quad \frac{\bullet h_8 : ((\perp, \Delta_{10}), F_6 \rightarrow F_7), F_{11} \vdash F_9}{\perp_L} \text{Cut} \\
\hline
\frac{}{- : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash F_9} \rightarrow \\
\hline
\frac{}{- : \perp, \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : (\Delta_9, p_{11}), F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : (\Delta_9, p_{11}), F_7 \vdash F_{10}}{\bullet h_1 : (\Delta_9, p_{11}), F_6 \rightarrow F_7 \vdash F_{10}} \rightarrow_L \quad \frac{\bullet h_8 : ((\Delta_9, p_{11}), F_6 \rightarrow F_7), F_{10} \vdash p_{11}}{- : (\Delta_9, p_{11}), F_6 \rightarrow F_7 \vdash p_{11}} I \text{Cut} \\
\hline
\frac{}{- : \Delta_9, p_{11}, F_6 \rightarrow F_7 \vdash p_{11}} \rightarrow \\
\hline
\frac{}{- : \Delta_9, p_{11}, F_6 \rightarrow F_7 \vdash p_{11}} I
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_9, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_9, F_7 \vdash p_{10}}{\bullet h_1 : \Delta_9, F_6 \rightarrow F_7 \vdash p_{10}} \rightarrow_L \quad \frac{\bullet h_8 : (\Delta_9, F_6 \rightarrow F_7), p_{10} \vdash p_{10}}{- : \Delta_9, F_6 \rightarrow F_7 \vdash p_{10}} I \text{Cut} \\
\hline
\frac{}{- : \Delta_9, F_6 \rightarrow F_7 \vdash p_{10}} \rightarrow \\
\hline
\frac{}{- : \Delta_9, F_6 \rightarrow F_7 \vdash p_{10}} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : \Delta_{10}, F_7 \vdash \top}{\bullet h_1 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top} \rightarrow_L \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9}{\bullet h_8 : (\Delta_{10}, F_6 \rightarrow F_7), \top \vdash F_9} \top_L \text{Cut} \\
\hline
\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9} \rightarrow \\
\hline
\frac{}{- : \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\top, \Delta_{10}), F_6 \rightarrow F_7 \vdash F_6 \quad h_1 : (\top, \Delta_{10}), F_7 \vdash F_{11}}{\bullet h_1 : (\top, \Delta_{10}), F_6 \rightarrow F_7 \vdash F_{11}} \rightarrow_L \quad \frac{h_8 : \Delta_{10}, F_{11}, F_6 \rightarrow F_7 \vdash F_9}{\bullet h_8 : ((\top, \Delta_{10}), F_6 \rightarrow F_7), F_{11} \vdash F_9} \top_L \text{Cut} \\
\hline
\frac{}{- : (\top, \Delta_{10}), F_6 \rightarrow F_7 \vdash F_9} \rightarrow \\
\hline
\frac{\bullet h_1 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash F_{11} \quad \text{ax/W} \quad h_8 : \top, \Delta_{10}, F_{11}, F_6 \rightarrow F_7 \vdash F_9 \quad \text{ax/W}}{- : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash F_9} \text{hCut}
\end{array}$$

## 6.7 Status of $\wedge_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_9, F_6, F_7 \vdash F_{10}}{\bullet h_1 : \Delta_9, F_6 \wedge F_7 \vdash F_{10}} \wedge_L \quad \frac{\bullet h_8 : (\Delta_9, F_6 \wedge F_7), F_{10} \vdash \top}{\top_R} \text{Cut} \\
\hline
\frac{}{- : \Delta_9, F_6 \wedge F_7 \vdash \top} \rightarrow \\
\hline
\frac{}{- : \Delta_9, F_6 \wedge F_7 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6, F_7 \vdash F_{12}}{\bullet h_1 : \Delta_{11}, F_6 \wedge F_7 \vdash F_{12}} \wedge_L \quad \frac{h_8 : \Delta_{11}, F_9, F_{12}, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \wedge F_7), F_{12} \vdash F_9 \rightarrow F_{10}} \rightarrow_R \text{Cut} \\
\hline
\frac{}{- : \Delta_{11}, F_6 \wedge F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow \\
\hline
\frac{\bullet h_1 : \Delta_{11}, F_9, F_6 \wedge F_7 \vdash F_{12} \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_{12}, F_9, F_6 \wedge F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, F_9, F_6 \wedge F_7 \vdash F_{10}} \text{hCut} \\
\hline
\frac{}{- : \Delta_{11}, F_6 \wedge F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6, F_7 \vdash F_{12}}{\bullet h_1 : \Delta_{11}, F_6 \wedge F_7 \vdash F_{12}} \wedge_L \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \wedge F_7 \vdash F_9 \quad h_8 : \Delta_{11}, F_{12}, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \wedge F_7), F_{12} \vdash F_9 \wedge F_{10}} \wedge_R \\
\hline
- : \Delta_{11}, F_6 \wedge F_7 \vdash F_9 \wedge F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{11}, F_6, F_7 \vdash F_{12}}{\bullet h_1 : \Delta_{11}, F_6 \wedge F_7 \vdash F_{12}} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6, F_7 \vdash F_9 \quad \text{inv-th/ax} \quad h_8 : \Delta_{11}, F_{12}, F_6, F_7 \vdash F_{10} \quad \text{inv-th/ax}}{\bullet h_8 : \Delta_{11}, F_{12}, F_6, F_7 \vdash F_9 \wedge F_{10}} \wedge_R \\
\hline
- : \Delta_{11}, F_6, F_7 \vdash F_9 \wedge F_{10} \quad \text{hCut} \\
\hline
- : \Delta_{11}, F_6 \wedge F_7 \vdash F_9 \wedge F_{10} \quad \wedge_L
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6, F_7 \vdash F_{12}}{\bullet h_1 : \Delta_{11}, F_6 \wedge F_7 \vdash F_{12}} \wedge_L \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \wedge F_7 \vdash F_9}{\bullet h_8 : (\Delta_{11}, F_6 \wedge F_7), F_{12} \vdash F_9 \vee F_{10}} \vee_1 \\
\hline
- : \Delta_{11}, F_6 \wedge F_7 \vdash F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_6 \wedge F_7 \vdash F_{12} \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_{12}, F_6 \wedge F_7 \vdash F_9 \quad \text{ax/W}}{- : \Delta_{11}, F_6 \wedge F_7 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{11}, F_6 \wedge F_7 \vdash F_9 \vee F_{10} \quad \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6, F_7 \vdash F_{12}}{\bullet h_1 : \Delta_{11}, F_6 \wedge F_7 \vdash F_{12}} \wedge_L \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \wedge F_7), F_{12} \vdash F_9 \vee F_{10}} \vee_2 \\
\hline
- : \Delta_{11}, F_6 \wedge F_7 \vdash F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_6 \wedge F_7 \vdash F_{12} \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_{12}, F_6 \wedge F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, F_6 \wedge F_7 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, F_6 \wedge F_7 \vdash F_9 \vee F_{10} \quad \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6, F_7 \vdash F_{13}}{\bullet h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash F_{13}} \wedge_L \quad \frac{h_8 : \Delta_{12}, F_{13}, F_9 \rightarrow F_{10}, F_6 \wedge F_7 \vdash F_9 \quad h_8 : \Delta_{12}, F_{10}, F_{13}, F_6 \wedge F_7 \vdash F_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7), F_{13} \vdash F_{11}} \rightarrow_L \\
\hline
- : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash F_{13} \quad \text{ax/W} \quad h_8 : \Delta_{12}, F_{13}, F_6, F_7, F_9 \rightarrow F_{10} \vdash F_9 \quad \text{inv-th/ax} \quad h_8 : \Delta_{12}, F_{10}, F_{13}, F_6, F_7 \vdash F_{11} \quad \text{inv-th/ax}}{\bullet h_8 : \Delta_{12}, F_{13}, F_6, F_7, F_9 \rightarrow F_{10} \vdash F_{11}} \rightarrow_L \\
\hline
- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash F_{11} \quad \text{hCut} \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \wedge F_7 \vdash F_{11} \quad \wedge_L \\
\hline
\frac{h_1 : \Delta_{12}, F_6, F_7 \vdash F_9 \rightarrow F_{10}}{\bullet h_1 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9 \rightarrow F_{10}} \wedge_L \quad \frac{h_8 : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \wedge F_7 \vdash F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \wedge F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_6 \wedge F_7), F_9 \rightarrow F_{10} \vdash F_{11}} \rightarrow_L \\
\hline
- : \Delta_{12}, F_6 \wedge F_7 \vdash F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6, F_7 \vdash F_9 \rightarrow F_{10} \quad \text{ax/W} \quad h_8 : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash F_9 \quad \text{inv-th/ax} \quad h_8 : \Delta_{12}, F_{10}, F_6, F_7 \vdash F_{11} \quad \text{inv-th/ax}}{\bullet h_8 : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash F_{11}} \rightarrow_L \\
\hline
- : \Delta_{12}, F_6, F_7 \vdash F_{11} \quad \text{hCut} \\
\hline
- : \Delta_{12}, F_6 \wedge F_7 \vdash F_{11} \quad \wedge_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_6, F_7 \vdash F_{13}}{\bullet h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash F_{13}} \wedge_L \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_{13}, F_6 \wedge F_7 \vdash F_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7), F_{13} \vdash F_{11}} \wedge_L \\
\hline
- : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash F_{11} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_6, F_7, F_9 \wedge F_{10} \vdash F_{13} \quad \text{ax/W} \quad h_8 : \Delta_{12}, F_{10}, F_{13}, F_6, F_7, F_9 \vdash F_{11} \quad \text{inv-th/ax}}{\bullet h_8 : \Delta_{12}, F_{13}, F_6, F_7, F_9 \wedge F_{10} \vdash F_{11}} \wedge_L \\
\hline
- : \Delta_{12}, F_6, F_7, F_9 \wedge F_{10} \vdash F_{11} \quad \text{hCut} \\
\hline
- : \Delta_{12}, F_6 \wedge F_7, F_9 \wedge F_{10} \vdash F_{11} \quad \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_6, F_7 \vdash F_9 \wedge F_{10}}{\bullet h_1 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9 \wedge F_{10}} \wedge_L \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \wedge F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_6 \wedge F_7), F_9 \wedge F_{10} \vdash F_{11}} \wedge_L \\
\hline
- : \Delta_{12}, F_6 \wedge F_7 \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6, F_7 \vdash F_9 \wedge F_{10}}{\bullet h_1 : \Delta_{12}, F_6, F_7 \vdash F_9 \wedge F_{10}} \text{ ax/w} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6, F_7, F_9 \vdash F_{11}}{\bullet h_8 : \Delta_{12}, F_6, F_7, F_9 \wedge F_{10} \vdash F_{11}} \text{ inv-th/ax} \wedge_L}{- : \Delta_{12}, F_6, F_7 \vdash F_{11}} \text{ hCut} \\
\rightarrow \\
- : \Delta_{12}, F_6 \wedge F_7 \vdash F_{11} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_7, F_8 \vdash F_{11}}{\bullet h_1 : \Delta_{10}, F_7 \wedge F_8 \vdash F_{11}} \wedge_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \wedge F_8), F_{11} \vdash F_9} \wedge_L \\
\hline
- : \Delta_{10}, F_7 \wedge F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, F_7, F_8 \vdash F_{11}}{\bullet h_1 : \Delta_{10}, F_7, F_8 \vdash F_{11}} \text{ ax/w} \quad \frac{h_6 : \Delta_{10}, F_{11}, F_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_{11}, F_7, F_8 \vdash F_9} \text{ ax/w} \quad H}{- : \Delta_{10}, F_7, F_8 \vdash F_9} \text{ hCut} \\
\rightarrow \\
- : \Delta_{10}, F_7 \wedge F_8 \vdash F_9 \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{12}, F_9 \vee F_{10}), F_6, F_7 \vdash F_{13}}{\bullet h_1 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash F_{13}} \wedge_L \quad \frac{h_8 : \Delta_{12}, F_9, F_{13}, F_6 \wedge F_7 \vdash F_{11} \quad h_8 : \Delta_{12}, F_{10}, F_{13}, F_6 \wedge F_7 \vdash F_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7), F_{13} \vdash F_{11}} \vee_L \\
\hline
- : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash F_{13}}{\bullet h_1 : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash F_{13}} \text{ ax/w} \quad \frac{h_8 : \Delta_{12}, F_{13}, F_6, F_7, F_9 \vdash F_{11}}{\bullet h_8 : \Delta_{12}, F_{13}, F_6, F_7, F_9 \vee F_{10} \vdash F_{11}} \text{ inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_{13}, F_6, F_7 \vdash F_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_{13}, F_6, F_7 \vdash F_{11}} \text{ inv-th/ax} \vee_L}{- : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash F_{11}} \text{ hCut} \\
\rightarrow \\
- : \Delta_{12}, F_6 \wedge F_7, F_9 \vee F_{10} \vdash F_{11} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{12}, F_6, F_7 \vdash F_9 \vee F_{10}}{\bullet h_1 : \Delta_{12}, F_6 \wedge F_7 \vdash F_9 \vee F_{10}} \wedge_L \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \wedge F_7 \vdash F_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \wedge F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_6 \wedge F_7), F_9 \vee F_{10} \vdash F_{11}} \vee_L \\
\hline
- : \Delta_{12}, F_6 \wedge F_7 \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6, F_7 \vdash F_9 \vee F_{10}}{\bullet h_1 : \Delta_{12}, F_6, F_7 \vdash F_9 \vee F_{10}} \text{ ax/w} \quad \frac{h_8 : \Delta_{12}, F_6, F_7, F_9 \vdash F_{11}}{\bullet h_8 : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash F_{11}} \text{ inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6, F_7 \vdash F_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_6, F_7 \vdash F_{11}} \text{ inv-th/ax} \vee_L}{- : \Delta_{12}, F_6, F_7 \vdash F_{11}} \text{ hCut} \\
\rightarrow \\
- : \Delta_{12}, F_6 \wedge F_7 \vdash F_{11} \wedge_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_6, F_7 \vdash \perp}{\bullet h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash \perp} \wedge_L \quad \frac{}{\bullet h_8 : (\Delta_{10}, F_6 \wedge F_7), \perp \vdash F_9} \perp_L \\
\hline
- : \Delta_{10}, F_6 \wedge F_7 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{10}, F_6, F_7 \vdash \perp}{\bullet h_1 : \Delta_{10}, F_6, F_7 \vdash \perp} \text{ ax/w} \quad \frac{}{\bullet h_8 : \perp, \Delta_{10}, F_6, F_7 \vdash F_9} \perp_L}{- : \Delta_{10}, F_6, F_7 \vdash F_9} \text{ hCut} \\
\rightarrow \\
- : \Delta_{10}, F_6 \wedge F_7 \vdash F_9 \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\perp, \Delta_{10}), F_6, F_7 \vdash F_{11}}{\bullet h_1 : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash F_{11}} \wedge_L \quad \frac{}{\bullet h_8 : ((\perp, \Delta_{10}), F_6 \wedge F_7), F_{11} \vdash F_9} \perp_L \\
\hline
- : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash F_9 \\
\rightarrow \\
- : \perp, \Delta_{10}, F_6 \wedge F_7 \vdash F_9 \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : (\Delta_9, p_{11}), F_6, F_7 \vdash F_{10}}{\bullet h_1 : (\Delta_9, p_{11}), F_6 \wedge F_7 \vdash F_{10}} \wedge_L \quad \frac{}{\bullet h_8 : ((\Delta_9, p_{11}), F_6 \wedge F_7), F_{10} \vdash p_{11}} I \\
\hline
- : (\Delta_9, p_{11}), F_6 \wedge F_7 \vdash p_{11} \\
\rightarrow \\
- : \Delta_9, p_{11}, F_6 \wedge F_7 \vdash p_{11} I
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_9, F_6, F_7 \vdash P_{10}}{\bullet h_1 : \Delta_9, F_6 \wedge F_7 \vdash P_{10}} \wedge_L \quad \frac{h_8 : (\Delta_9, F_6 \wedge F_7), P_{10} \vdash P_{10}}{\bullet h_8 : (\Delta_9, F_6 \wedge F_7), P_{10} \vdash P_{10}} I \\
\hline
- : \Delta_9, F_6 \wedge F_7 \vdash P_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
- : \Delta_9, F_6 \wedge F_7 \vdash P_{10} \quad \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{10}, F_6, F_7 \vdash \top}{\bullet h_1 : \Delta_{10}, F_6 \wedge F_7 \vdash \top} \wedge_L \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash F_9}{\bullet h_8 : (\Delta_{10}, F_6 \wedge F_7), \top \vdash F_9} \top_L \\
\hline
- : \Delta_{10}, F_6 \wedge F_7 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
- : \Delta_{10}, F_6 \wedge F_7 \vdash F_9 \quad \text{ax/W} \\
\\
\frac{h_1 : (\top, \Delta_{10}), F_6, F_7 \vdash F_{11}}{\bullet h_1 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash F_{11}} \wedge_L \quad \frac{h_8 : \Delta_{10}, F_{11}, F_6 \wedge F_7 \vdash F_9}{\bullet h_8 : ((\top, \Delta_{10}), F_6 \wedge F_7), F_{11} \vdash F_9} \top_L \\
\hline
- : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
\frac{\bullet h_1 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash F_{11} \quad \text{ax/W} \quad h_8 : \top, \Delta_{10}, F_{11}, F_6 \wedge F_7 \vdash F_9 \quad \text{ax/W}}{- : \top, \Delta_{10}, F_6 \wedge F_7 \vdash F_9} \text{hCut}
\end{array}$$

## 6.8 Status of $\vee_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_9, F_6 \vdash F_{10} \quad h_1 : \Delta_9, F_7 \vdash F_{10}}{\bullet h_1 : \Delta_9, F_6 \vee F_7 \vdash F_{10}} \vee_L \quad \frac{h_8 : (\Delta_9, F_6 \vee F_7), F_{10} \vdash \top}{\bullet h_8 : (\Delta_9, F_6 \vee F_7), F_{10} \vdash \top} \top_R \\
\hline
- : \Delta_9, F_6 \vee F_7 \vdash \top \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
- : \Delta_9, F_6 \vee F_7 \vdash \top \quad \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6 \vdash F_{12} \quad h_1 : \Delta_{11}, F_7 \vdash F_{12}}{\bullet h_1 : \Delta_{11}, F_6 \vee F_7 \vdash F_{12}} \vee_L \quad \frac{h_8 : \Delta_{11}, F_9, F_{12}, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \vee F_7), F_{12} \vdash F_9 \rightarrow F_{10}} \rightarrow_R \\
\hline
- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \rightarrow F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
\frac{\bullet h_1 : \Delta_{11}, F_9, F_6 \vee F_7 \vdash F_{12} \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_{12}, F_9, F_6 \vee F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, F_9, F_6 \vee F_7 \vdash F_{10} \rightarrow F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \rightarrow F_{10} \quad \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6 \vdash F_{12} \quad h_1 : \Delta_{11}, F_7 \vdash F_{12}}{\bullet h_1 : \Delta_{11}, F_6 \vee F_7 \vdash F_{12}} \vee_L \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \vee F_7 \vdash F_9 \quad h_8 : \Delta_{11}, F_{12}, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \vee F_7), F_{12} \vdash F_9 \wedge F_{10}} \wedge_R \\
\hline
- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \wedge F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
\frac{\bullet h_1 : \Delta_{11}, F_6 \vee F_7 \vdash F_{12} \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_{12}, F_6 \vee F_7 \vdash F_9 \quad \text{ax/W} \quad \bullet h_1 : \Delta_{11}, F_6 \vee F_7 \vdash F_{12} \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_{12}, F_6 \vee F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \wedge F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \wedge F_{10} \quad \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6 \vdash F_{12} \quad h_1 : \Delta_{11}, F_7 \vdash F_{12}}{\bullet h_1 : \Delta_{11}, F_6 \vee F_7 \vdash F_{12}} \vee_L \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \vee F_7 \vdash F_9}{\bullet h_8 : (\Delta_{11}, F_6 \vee F_7), F_{12} \vdash F_9 \vee F_{10}} \vee_1 \\
\hline
- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \vee F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\hline
\frac{\bullet h_1 : \Delta_{11}, F_6 \vee F_7 \vdash F_{12} \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_{12}, F_6 \vee F_7 \vdash F_9 \quad \text{ax/W}}{- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \vee F_{10} \quad \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_6 \vdash F_{12} \quad h_1 : \Delta_{11}, F_7 \vdash F_{12}}{\bullet h_1 : \Delta_{11}, F_6 \vee F_7 \vdash F_{12}} \vee_L \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, F_6 \vee F_7), F_{12} \vdash F_9 \vee F_{10}} \vee_2 \\
\frac{\quad}{- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_{11}, F_6 \vee F_7 \vdash F_{12}}{- : \Delta_{11}, F_6 \vee F_7 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_{12}, F_6 \vee F_7 \vdash F_{10}}{- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_{11}, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \text{hCut} \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vdash F_{13} \quad h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_7 \vdash F_{13}}{\bullet h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7 \vdash F_{13}} \vee_L \quad \frac{h_8 : \Delta_{12}, F_{13}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash F_9 \quad h_8 : \Delta_{12}, F_{10}, F_{13}}{\bullet h_8 : ((\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7), F_{13} \vdash F_{11}} \vee_2 \\
\frac{\quad}{- : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7 \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash F_{13}}{- : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{13}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash F_9}{- : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{hCut} \\
\frac{\quad}{- : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{inv-th/ax} \quad \frac{\quad}{- : \Delta_{12}, F_{10}, F_7 \vdash F_{13}} \text{inv-th/ax} \quad \frac{\quad}{- : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash F_{11}} \vee_L \\
\frac{\quad}{- : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{inv-th/ax} \quad \frac{\quad}{- : \Delta_{12}, F_6 \vee F_7 \vdash F_9} \text{inv-th/ax} \quad \frac{\quad}{- : \Delta_{12}, F_6 \vee F_7 \vdash F_{11}} \vee_L \\
\frac{\quad}{- : \Delta_{12}, F_6 \vee F_7 \vdash F_{11}} \text{ax/W} \quad \frac{\quad}{- : \Delta_{12}, F_6 \vee F_7 \vdash F_{11}} \text{ax/W} \quad \frac{\quad}{- : \Delta_{12}, F_6 \vee F_7 \vdash F_{11}} \text{ax/W} \\
\frac{\quad}{- : \Delta_{12}, F_6 \vee F_7 \vdash F_{11}} \text{hCut} \quad \frac{\quad}{- : \Delta_{12}, F_6 \vee F_7 \vdash F_{11}} \text{hCut} \quad \frac{\quad}{- : \Delta_{12}, F_6 \vee F_7 \vdash F_{11}} \text{hCut}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vdash F_{13} \quad h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_7 \vdash F_{13}}{\bullet h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash F_{13}} \vee_L \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_{13}, F_6 \vee F_7 \vdash F_{11}}{\bullet h_8 : ((\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7), F_{13} \vdash F_{11}} \wedge_L \\
\frac{\quad}{- : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_{10}, F_6, F_9 \vdash F_{13}}{\bullet h_1 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash F_{13}} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_7, F_9 \vdash F_{13}}{\bullet h_1 : \Delta_{12}, F_{10}, F_7, F_9 \vdash F_{13}} \text{inv-th/ax}}{\bullet h_1 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash F_{13}} \vee_L \quad \frac{h_8 : \Delta_{12}, F_{10}, F_{13}, F_9, F_6 \vee F_7 \vdash F_{11}}{- : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash F_{11}} \text{ax/W} \\
\frac{\quad}{- : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash F_{11}} \text{hCut} \quad \frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \wedge_L \\
\frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{inv-th/ax} \quad \frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{inv-th/ax} \quad \frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \vee_L \\
\frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{ax/W} \quad \frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{ax/W} \quad \frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{ax/W} \\
\frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{hCut} \quad \frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{hCut} \quad \frac{\quad}{- : \Delta_{12}, F_9 \wedge F_{10}, F_6 \vee F_7 \vdash F_{11}} \text{hCut}
\end{array}$$

- Case rule  $\vee_L$





$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{10}, F_6 \vdash \top \quad h_1 : \Delta_{10}, F_7 \vdash \top}{\bullet h_1 : \Delta_{10}, F_6 \vee F_7 \vdash \top} \vee_L \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash F_9}{\bullet h_8 : (\Delta_{10}, F_6 \vee F_7), \top \vdash F_9} \top_L}{\frac{- : \Delta_{10}, F_6 \vee F_7 \vdash F_9}{\rightarrow} \text{Cut}} \\
\frac{}{- : \Delta_{10}, F_6 \vee F_7 \vdash F_9} \text{ax/W} \\
\frac{\frac{h_1 : (\top, \Delta_{10}), F_6 \vdash F_{11} \quad h_1 : (\top, \Delta_{10}), F_7 \vdash F_{11}}{\bullet h_1 : (\top, \Delta_{10}), F_6 \vee F_7 \vdash F_{11}} \vee_L \quad \frac{h_8 : \Delta_{10}, F_{11}, F_6 \vee F_7 \vdash F_9}{\bullet h_8 : ((\top, \Delta_{10}), F_6 \vee F_7), F_{11} \vdash F_9} \top_L}{\frac{- : (\top, \Delta_{10}), F_6 \vee F_7 \vdash F_9}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \top, \Delta_{10}, F_6 \vee F_7 \vdash F_{11}}{\rightarrow} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_{11}, F_6 \vee F_7 \vdash F_9}{\rightarrow} \text{ax/W}}{- : \top, \Delta_{10}, F_6 \vee F_7 \vdash F_9} \text{hCut}
\end{array}$$

## 6.9 Status of $\perp_L$ : OK

- Case rule  $\top_R$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_5 \vdash F_6} \perp_L \quad \frac{}{\bullet h_4 : (\perp, \Delta_5), F_6 \vdash \top} \top_R}{\frac{- : \perp, \Delta_5 \vdash \top}{\rightarrow} \text{Cut}} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_7 \vdash F_8} \perp_L \quad \frac{h_4 : \perp, \Delta_7, F_5, F_8 \vdash F_6}{\bullet h_4 : (\perp, \Delta_7), F_8 \vdash F_5 \rightarrow F_6} \rightarrow_R}{\frac{- : \perp, \Delta_7 \vdash F_5 \rightarrow F_6}{\rightarrow} \text{Cut}} \perp_L$$

- Case rule  $\wedge_R$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_7 \vdash F_8} \perp_L \quad \frac{h_4 : \perp, \Delta_7, F_8 \vdash F_5 \quad h_4 : \perp, \Delta_7, F_8 \vdash F_6}{\bullet h_4 : (\perp, \Delta_7), F_8 \vdash F_5 \wedge F_6} \wedge_R}{\frac{- : \perp, \Delta_7 \vdash F_5 \wedge F_6}{\rightarrow} \text{Cut}} \perp_L$$

- Case rule  $\vee_1$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_7 \vdash F_8} \perp_L \quad \frac{h_4 : \perp, \Delta_7, F_8 \vdash F_5}{\bullet h_4 : (\perp, \Delta_7), F_8 \vdash F_5 \vee F_6} \vee_1}{\frac{- : \perp, \Delta_7 \vdash F_5 \vee F_6}{\rightarrow} \text{Cut}} \perp_L$$

- Case rule  $\vee_2$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_7 \vdash F_8} \perp_L \quad \frac{h_4 : \perp, \Delta_7, F_8 \vdash F_6}{\bullet h_4 : (\perp, \Delta_7), F_8 \vdash F_5 \vee F_6} \vee_2}{\frac{- : \perp, \Delta_7 \vdash F_5 \vee F_6}{\rightarrow} \text{Cut}} \perp_L$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_8, F_5 \rightarrow F_6 \vdash F_9}{\vdash : \perp, \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \perp_L \quad \frac{\frac{h_4 : \perp, \Delta_8, F_9, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \perp, \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\perp, \Delta_8, F_5 \rightarrow F_6), F_9 \vdash F_7} \rightarrow_L}{\vdash : \perp, \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \perp_L \\
\\
\frac{\frac{\bullet h_1 : \perp, \Delta_8 \vdash F_5 \rightarrow F_6}{\vdash : \perp, \Delta_8 \vdash F_7} \perp_L \quad \frac{\frac{h_4 : \perp, \Delta_8, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \perp, \Delta_8, F_6 \vdash F_7}{\bullet h_4 : (\perp, \Delta_8), F_5 \rightarrow F_6 \vdash F_7} \rightarrow_L}{\vdash : \perp, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_8 \vdash F_7} \perp_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_8, F_5 \wedge F_6 \vdash F_9}{\vdash : \perp, \Delta_8, F_5 \wedge F_6 \vdash F_7} \perp_L \quad \frac{\frac{h_4 : \perp, \Delta_8, F_5, F_6, F_9 \vdash F_7}{\bullet h_4 : (\perp, \Delta_8, F_5 \wedge F_6), F_9 \vdash F_7} \wedge_L}{\vdash : \perp, \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_8, F_5 \wedge F_6 \vdash F_7} \perp_L \\
\\
\frac{\frac{\bullet h_1 : \perp, \Delta_8 \vdash F_5 \wedge F_6}{\vdash : \perp, \Delta_8 \vdash F_7} \perp_L \quad \frac{\frac{h_4 : \perp, \Delta_8, F_5, F_6 \vdash F_7}{\bullet h_4 : (\perp, \Delta_8), F_5 \wedge F_6 \vdash F_7} \wedge_L}{\vdash : \perp, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_8 \vdash F_7} \perp_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_8, F_5 \vee F_6 \vdash F_9}{\vdash : \perp, \Delta_8, F_5 \vee F_6 \vdash F_7} \perp_L \quad \frac{\frac{h_4 : \perp, \Delta_8, F_5, F_9 \vdash F_7 \quad h_4 : \perp, \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\perp, \Delta_8, F_5 \vee F_6), F_9 \vdash F_7} \vee_L}{\vdash : \perp, \Delta_8, F_5 \vee F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_8, F_5 \vee F_6 \vdash F_7} \perp_L \\
\\
\frac{\frac{\bullet h_1 : \perp, \Delta_8 \vdash F_5 \vee F_6}{\vdash : \perp, \Delta_8 \vdash F_7} \perp_L \quad \frac{\frac{h_4 : \perp, \Delta_8, F_5 \vdash F_7 \quad h_4 : \perp, \Delta_8, F_6 \vdash F_7}{\bullet h_4 : (\perp, \Delta_8), F_5 \vee F_6 \vdash F_7} \vee_L}{\vdash : \perp, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_8 \vdash F_7} \perp_L
\end{array}$$

- Case rule  $\perp_L$

$$\frac{\frac{\bullet h_1 : \perp, \Delta_6 \vdash F_7}{\vdash : \perp, \Delta_6 \vdash F_5} \perp_L \quad \frac{\bullet h_4 : (\perp, \Delta_6), F_7 \vdash F_5}{\vdash : \perp, \Delta_6 \vdash F_5} \perp_L}{\vdash : \perp, \Delta_6 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_6 \vdash F_5} \perp_L$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_5, p_7 \vdash F_6}{\vdash : \perp, \Delta_5, p_7 \vdash p_7} \perp_L \quad \frac{\bullet h_4 : (\perp, \Delta_5, p_7), F_6 \vdash p_7}{\vdash : \perp, \Delta_5, p_7 \vdash p_7} I}{\vdash : \perp, \Delta_5, p_7 \vdash p_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_5, p_7 \vdash p_7} \perp_L \\
\\
\frac{\frac{\bullet h_1 : \perp, \Delta_5 \vdash p_6}{\vdash : \perp, \Delta_5 \vdash p_6} \perp_L \quad \frac{\bullet h_4 : (\perp, \Delta_5), p_6 \vdash p_6}{\vdash : \perp, \Delta_5 \vdash p_6} I}{\vdash : \perp, \Delta_5 \vdash p_6} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \Delta_5 \vdash p_6} \perp_L
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_6 \vdash \top} \perp_L \quad \frac{h_4 : \perp, \Delta_6 \vdash F_5}{\bullet h_4 : (\perp, \Delta_6), \top \vdash F_5} \top_L}{\frac{}{- : \perp, \Delta_6 \vdash F_5} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_6 \vdash F_5} \perp_L \\
\\
\frac{\frac{}{\bullet h_1 : \perp, \top, \Delta_6 \vdash F_7} \perp_L \quad \frac{h_4 : \perp, \Delta_6, F_7 \vdash F_5}{\bullet h_4 : (\perp, \top, \Delta_6), F_7 \vdash F_5} \top_L}{\frac{}{- : \perp, \top, \Delta_6 \vdash F_5} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \top, \Delta_6 \vdash F_5} \perp_L
\end{array}$$

## 6.10 Status of $I$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_5, p_6 \vdash p_6} I \quad \frac{}{\bullet h_4 : (\Delta_5, p_6), p_6 \vdash \top} \top_R}{\frac{}{- : \Delta_5, p_6 \vdash \top} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_5, p_6 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_7, p_8 \vdash p_8} I \quad \frac{h_4 : \Delta_7, F_5, p_8, p_8 \vdash F_6}{\bullet h_4 : (\Delta_7, p_8), p_8 \vdash F_5 \rightarrow F_6} \rightarrow_R}{\frac{}{- : \Delta_7, p_8 \vdash F_5 \rightarrow F_6} \text{Cut}} \rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_7, F_5, p_8 \vdash p_8} \text{ax/W} \quad \frac{h_4 : \Delta_7, F_5, p_8, p_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_5, p_8 \vdash F_5 \rightarrow F_6} \text{ax/W}}{\frac{}{- : \Delta_7, F_5, p_8 \vdash F_5 \rightarrow F_6} \text{hCut}} \rightarrow_R \\
\frac{}{- : \Delta_7, p_8 \vdash F_5 \rightarrow F_6} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_7, p_8 \vdash p_8} I \quad \frac{h_4 : \Delta_7, p_8, p_8 \vdash F_5 \quad h_4 : \Delta_7, p_8, p_8 \vdash F_6}{\bullet h_4 : (\Delta_7, p_8), p_8 \vdash F_5 \wedge F_6} \wedge_R}{\frac{}{- : \Delta_7, p_8 \vdash F_5 \wedge F_6} \text{Cut}} \rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_7, p_8 \vdash p_8} \text{ax/W} \quad \frac{h_4 : \Delta_7, p_8, p_8 \vdash F_5}{\bullet h_4 : \Delta_7, p_8 \vdash F_5} \text{ax/W}}{\frac{}{- : \Delta_7, p_8 \vdash F_5} \text{hCut}} \frac{\frac{}{\bullet h_1 : \Delta_7, p_8 \vdash p_8} \text{ax/W} \quad \frac{h_4 : \Delta_7, p_8, p_8 \vdash F_6}{\bullet h_4 : \Delta_7, p_8 \vdash F_6} \text{ax/W}}{\frac{}{- : \Delta_7, p_8 \vdash F_5 \wedge F_6} \text{hCut}} \wedge_R \\
\frac{}{- : \Delta_7, p_8 \vdash F_5 \wedge F_6} \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_7, p_8 \vdash p_8} I \quad \frac{h_4 : \Delta_7, p_8, p_8 \vdash F_5}{\bullet h_4 : (\Delta_7, p_8), p_8 \vdash F_5 \vee F_6} \vee_1}{\frac{}{- : \Delta_7, p_8 \vdash F_5 \vee F_6} \text{Cut}} \rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_7, p_8 \vdash p_8} I \quad \frac{h_4 : \Delta_7, p_8, p_8 \vdash F_5}{\bullet h_4 : \Delta_7, p_8 \vdash F_5} \text{ax/W}}{\frac{}{- : \Delta_7, p_8 \vdash F_5} \text{hCut}} \vee_1 \\
\frac{}{- : \Delta_7, p_8 \vdash F_5 \vee F_6} \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_7, p_8 \vdash p_8} I \quad \frac{h_4 : \Delta_7, p_8, p_8 \vdash F_6}{\bullet h_4 : (\Delta_7, p_8), p_8 \vdash F_5 \vee F_6} \vee_2}{- : \Delta_7, p_8 \vdash F_5 \vee F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_7, p_8 \vdash p_8}{} I \quad \frac{h_4 : \Delta_7, p_8, p_8 \vdash F_6}{\bullet h_4 : (\Delta_7, p_8), p_8 \vdash F_5 \vee F_6} \text{ax/W}}{- : \Delta_7, p_8 \vdash F_6} \text{hCut} \\
\vee_2 \\
- : \Delta_7, p_8 \vdash F_5 \vee F_6
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : (\Delta_8, F_5 \rightarrow F_6), p_9 \vdash p_9} I \quad \frac{h_4 : \Delta_8, p_9, p_9, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \Delta_8, F_6, p_9, p_9 \vdash F_7}{\bullet h_4 : ((\Delta_8, F_5 \rightarrow F_6), p_9), p_9 \vdash F_7} \rightarrow_L}{- : (\Delta_8, F_5 \rightarrow F_6), p_9 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, p_9, F_5 \rightarrow F_6 \vdash p_9}{- : \Delta_8, p_9, F_5 \rightarrow F_6 \vdash F_5} \text{ax/W} \quad \frac{h_4 : \Delta_8, p_9, p_9, F_5 \rightarrow F_6 \vdash F_5}{- : \Delta_8, p_9, p_9, F_5 \rightarrow F_6 \vdash F_5} \text{ax/W}}{- : \Delta_8, p_9, F_5 \rightarrow F_6 \vdash F_5} \text{hCut} \\
\frac{\frac{\bullet h_1 : \Delta_8, F_6, p_9 \vdash p_9}{- : \Delta_8, F_6, p_9 \vdash F_7} I \quad \frac{h_4 : \Delta_8, F_6, p_9, p_9 \vdash F_7}{- : \Delta_8, F_6, p_9, p_9 \vdash F_7} \text{ax/W}}{- : \Delta_8, F_6, p_9 \vdash F_7} \text{hCut} \\
\rightarrow_L \\
- : \Delta_8, p_9, F_5 \rightarrow F_6 \vdash F_7
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : (\Delta_8, F_5 \wedge F_6), p_9 \vdash p_9} I \quad \frac{h_4 : \Delta_8, F_5, F_6, p_9, p_9 \vdash F_7}{\bullet h_4 : ((\Delta_8, F_5 \wedge F_6), p_9), p_9 \vdash F_7} \wedge_L}{- : (\Delta_8, F_5 \wedge F_6), p_9 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5, F_6, p_9 \vdash p_9}{- : \Delta_8, F_5, F_6, p_9 \vdash F_7} I \quad \frac{h_4 : \Delta_8, F_5, F_6, p_9, p_9 \vdash F_7}{- : \Delta_8, F_5, F_6, p_9, p_9 \vdash F_7} \text{ax/W}}{- : \Delta_8, F_5, F_6, p_9 \vdash F_7} \text{hCut} \\
\wedge_L \\
- : \Delta_8, p_9, F_5 \wedge F_6 \vdash F_7
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : (\Delta_8, F_5 \vee F_6), p_9 \vdash p_9} I \quad \frac{h_4 : \Delta_8, F_5, p_9, p_9 \vdash F_7 \quad h_4 : \Delta_8, F_6, p_9, p_9 \vdash F_7}{\bullet h_4 : ((\Delta_8, F_5 \vee F_6), p_9), p_9 \vdash F_7} \vee_L}{- : (\Delta_8, F_5 \vee F_6), p_9 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_8, F_5, p_9 \vdash p_9}{- : \Delta_8, F_5, p_9 \vdash F_7} I \quad \frac{h_4 : \Delta_8, F_5, p_9, p_9 \vdash F_7}{- : \Delta_8, F_5, p_9, p_9 \vdash F_7} \text{ax/W}}{- : \Delta_8, F_5, p_9 \vdash F_7} \text{hCut} \\
\frac{\frac{\bullet h_1 : \Delta_8, F_6, p_9 \vdash p_9}{- : \Delta_8, F_6, p_9 \vdash F_7} I \quad \frac{h_4 : \Delta_8, F_6, p_9, p_9 \vdash F_7}{- : \Delta_8, F_6, p_9, p_9 \vdash F_7} \text{ax/W}}{- : \Delta_8, F_6, p_9 \vdash F_7} \text{hCut} \\
\vee_L \\
- : \Delta_8, p_9, F_5 \vee F_6 \vdash F_7
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : (\perp, \Delta_6), p_7 \vdash p_7} I \quad \frac{}{\bullet h_4 : ((\perp, \Delta_6), p_7), p_7 \vdash F_5} \perp_L}{- : (\perp, \Delta_6), p_7 \vdash F_5} \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_6, p_7 \vdash F_5 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_5, p_6 \vdash p_6} I \quad \frac{}{\bullet h_4 : (\Delta_5, p_6), p_6 \vdash p_6} I}{- : \Delta_5, p_6 \vdash p_6} \text{Cut} \\
\rightarrow \\
- : \Delta_5, p_6 \vdash p_6 \quad I \\
\frac{\frac{}{\bullet h_1 : (\Delta_5, p_7), p_6 \vdash p_6} I \quad \frac{}{\bullet h_4 : ((\Delta_5, p_7), p_6), p_6 \vdash p_7} I}{- : (\Delta_5, p_7), p_6 \vdash p_7} \text{Cut} \\
\rightarrow \\
- : \Delta_5, p_6, p_7 \vdash p_7 \quad I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : (\top, \Delta_6), p_7 \vdash p_7} I \quad \frac{h_4 : \Delta_6, p_7, p_7 \vdash F_5}{\bullet h_4 : ((\top, \Delta_6), p_7), p_7 \vdash F_5} \top_L}{\frac{}{- : (\top, \Delta_6), p_7 \vdash F_5} \text{Cut}} \\
\frac{}{\bullet h_1 : \top, \Delta_6, p_7 \vdash p_7} I \quad \frac{h_4 : \top, \Delta_6, p_7, p_7 \vdash F_5}{\bullet h_4 : \top, \Delta_6, p_7 \vdash F_5} \text{ax/W} \\
\frac{}{- : \top, \Delta_6, p_7 \vdash F_5} \text{hCut}
\end{array}$$

## 6.11 Status of $\top_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \top, \Delta_5 \vdash F_6} \top_L \quad \frac{}{\bullet h_4 : (\top, \Delta_5), F_6 \vdash \top} \top_R}{\frac{}{- : \top, \Delta_5 \vdash \top} \text{Cut}} \\
\frac{}{- : \top, \Delta_5 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_7 \vdash F_8}{\bullet h_1 : \top, \Delta_7 \vdash F_8} \top_L \quad \frac{h_4 : \top, \Delta_7, F_5, F_8 \vdash F_6}{\bullet h_4 : (\top, \Delta_7), F_8 \vdash F_5 \rightarrow F_6} \rightarrow_R}{\frac{}{- : \top, \Delta_7 \vdash F_5 \rightarrow F_6} \text{Cut}} \\
\frac{}{- : \top, \Delta_7 \vdash F_5 \rightarrow F_6} \text{ax/W} \quad \frac{h_4 : \top, \Delta_7, F_8 \vdash F_5 \rightarrow F_6}{\bullet h_4 : \top, \Delta_7, F_8 \vdash F_5 \rightarrow F_6} \text{ax/W} \\
\frac{}{- : \top, \Delta_7 \vdash F_5 \rightarrow F_6} \text{hCut}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_7 \vdash F_8}{\bullet h_1 : \top, \Delta_7 \vdash F_8} \top_L \quad \frac{h_4 : \top, \Delta_7, F_8 \vdash F_5 \quad h_4 : \top, \Delta_7, F_8 \vdash F_6}{\bullet h_4 : (\top, \Delta_7), F_8 \vdash F_5 \wedge F_6} \wedge_R}{\frac{}{- : \top, \Delta_7 \vdash F_5 \wedge F_6} \text{Cut}} \\
\frac{}{- : \top, \Delta_7 \vdash F_5 \wedge F_6} \text{ax/W} \quad \frac{h_4 : \top, \Delta_7, F_8 \vdash F_5 \wedge F_6}{\bullet h_4 : \top, \Delta_7, F_8 \vdash F_5 \wedge F_6} \text{ax/W} \\
\frac{}{- : \top, \Delta_7 \vdash F_5 \wedge F_6} \text{hCut}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_7 \vdash F_8}{\bullet h_1 : \top, \Delta_7 \vdash F_8} \top_L \quad \frac{h_4 : \top, \Delta_7, F_8 \vdash F_5}{\bullet h_4 : (\top, \Delta_7), F_8 \vdash F_5 \vee F_6} \vee_1}{\frac{}{- : \top, \Delta_7 \vdash F_5 \vee F_6} \text{Cut}} \\
\frac{}{- : \top, \Delta_7 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_4 : \top, \Delta_7, F_8 \vdash F_5 \vee F_6}{\bullet h_4 : \top, \Delta_7, F_8 \vdash F_5 \vee F_6} \text{ax/W} \\
\frac{}{- : \top, \Delta_7 \vdash F_5 \vee F_6} \text{hCut}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_7 \vdash F_8}{\bullet h_1 : \top, \Delta_7 \vdash F_8} \top_L \quad \frac{h_4 : \top, \Delta_7, F_8 \vdash F_6}{\bullet h_4 : (\top, \Delta_7), F_8 \vdash F_5 \vee F_6} \vee_2}{\frac{}{- : \top, \Delta_7 \vdash F_5 \vee F_6} \text{Cut}} \\
\frac{}{- : \top, \Delta_7 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_4 : \top, \Delta_7, F_8 \vdash F_5 \vee F_6}{\bullet h_4 : \top, \Delta_7, F_8 \vdash F_5 \vee F_6} \text{ax/W} \\
\frac{}{- : \top, \Delta_7 \vdash F_5 \vee F_6} \text{hCut}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash F_9}{\bullet h_1 : \top, \Delta_8, F_5 \rightarrow F_6 \vdash F_9} \top_L \quad \frac{h_4 : \top, \Delta_8, F_9, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \top, \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\top, \Delta_8, F_5 \rightarrow F_6), F_9 \vdash F_7} \rightarrow_L}{- : \top, \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_8, F_5 \rightarrow F_6 \vdash F_9}{- : \top, \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_8, F_9, F_5 \rightarrow F_6 \vdash F_7}{\bullet h_4 : (\top, \Delta_8, F_5 \rightarrow F_6), F_9 \vdash F_7} \text{ax/W}}{- : \top, \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_8 \vdash F_5 \rightarrow F_6}{\bullet h_1 : \top, \Delta_8 \vdash F_5 \rightarrow F_6} \top_L \quad \frac{h_4 : \top, \Delta_8, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \top, \Delta_8, F_6 \vdash F_7}{\bullet h_4 : (\top, \Delta_8), F_5 \rightarrow F_6 \vdash F_7} \rightarrow_L}{- : \top, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_8 \vdash F_5 \rightarrow F_6}{- : \top, \Delta_8 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_8, F_5 \rightarrow F_6 \vdash F_7}{\bullet h_4 : (\top, \Delta_8), F_5 \rightarrow F_6 \vdash F_7} \text{ax/W}}{- : \top, \Delta_8 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8, F_5 \wedge F_6 \vdash F_9}{\bullet h_1 : \top, \Delta_8, F_5 \wedge F_6 \vdash F_9} \top_L \quad \frac{h_4 : \top, \Delta_8, F_5, F_6, F_9 \vdash F_7}{\bullet h_4 : (\top, \Delta_8, F_5 \wedge F_6), F_9 \vdash F_7} \wedge_L}{- : \top, \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_8, F_5 \wedge F_6 \vdash F_9}{- : \top, \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_8, F_9, F_5 \wedge F_6 \vdash F_7}{\bullet h_4 : (\top, \Delta_8, F_5 \wedge F_6), F_9 \vdash F_7} \text{ax/W}}{- : \top, \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_8 \vdash F_5 \wedge F_6}{\bullet h_1 : \top, \Delta_8 \vdash F_5 \wedge F_6} \top_L \quad \frac{h_4 : \top, \Delta_8, F_5, F_6 \vdash F_7}{\bullet h_4 : (\top, \Delta_8), F_5 \wedge F_6 \vdash F_7} \wedge_L}{- : \top, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_8 \vdash F_5 \wedge F_6}{- : \top, \Delta_8 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_8, F_5 \wedge F_6 \vdash F_7}{\bullet h_4 : (\top, \Delta_8), F_5 \wedge F_6 \vdash F_7} \text{ax/W}}{- : \top, \Delta_8 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8, F_5 \vee F_6 \vdash F_9}{\bullet h_1 : \top, \Delta_8, F_5 \vee F_6 \vdash F_9} \top_L \quad \frac{h_4 : \top, \Delta_8, F_5, F_9 \vdash F_7 \quad h_4 : \top, \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\top, \Delta_8, F_5 \vee F_6), F_9 \vdash F_7} \vee_L}{- : \top, \Delta_8, F_5 \vee F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_8, F_5 \vee F_6 \vdash F_9}{- : \top, \Delta_8, F_5 \vee F_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_8, F_9, F_5 \vee F_6 \vdash F_7}{\bullet h_4 : (\top, \Delta_8, F_5 \vee F_6), F_9 \vdash F_7} \text{ax/W}}{- : \top, \Delta_8, F_5 \vee F_6 \vdash F_7} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_8 \vdash F_5 \vee F_6}{\bullet h_1 : \top, \Delta_8 \vdash F_5 \vee F_6} \top_L \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash F_7 \quad h_4 : \top, \Delta_8, F_6 \vdash F_7}{\bullet h_4 : (\top, \Delta_8), F_5 \vee F_6 \vdash F_7} \vee_L}{- : \top, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_8 \vdash F_5 \vee F_6}{- : \top, \Delta_8 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_8, F_5 \vee F_6 \vdash F_7}{\bullet h_4 : (\top, \Delta_8), F_5 \vee F_6 \vdash F_7} \text{ax/W}}{- : \top, \Delta_8 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash \perp}{\bullet h_1 : \top, \Delta_6 \vdash \perp} \top_L \quad \frac{}{\bullet h_4 : (\top, \Delta_6), \perp \vdash F_5} \perp_L}{- : \top, \Delta_6 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_6 \vdash \perp}{- : \top, \Delta_6 \vdash F_5} \text{ax/W} \quad \frac{}{\bullet h_4 : \perp, \top, \Delta_6 \vdash F_5} \perp_L}{- : \top, \Delta_6 \vdash F_5} \text{hCut} \\
\\
\frac{\frac{h_1 : \perp, \Delta_6 \vdash F_7}{\bullet h_1 : \top, \perp, \Delta_6 \vdash F_7} \top_L \quad \frac{}{\bullet h_4 : (\top, \perp, \Delta_6), F_7 \vdash F_5} \perp_L}{- : \top, \perp, \Delta_6 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \top, \Delta_6 \vdash F_5} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, p_7 \vdash F_6}{\bullet h_1 : \top, \Delta_5, p_7 \vdash F_6} \top_L \quad \frac{}{\bullet h_4 : (\top, \Delta_5, p_7), F_6 \vdash p_7} I}{\frac{}{- : \top, \Delta_5, p_7 \vdash p_7} \text{Cut}} \rightarrow \\
\frac{}{- : \top, \Delta_5, p_7 \vdash p_7} I \\
\\
\frac{\frac{h_1 : \Delta_5 \vdash p_6}{\bullet h_1 : \top, \Delta_5 \vdash p_6} \top_L \quad \frac{}{\bullet h_4 : (\top, \Delta_5), p_6 \vdash p_6} I}{\frac{}{- : \top, \Delta_5 \vdash p_6} \text{Cut}} \rightarrow \\
\frac{}{- : \top, \Delta_5 \vdash p_6} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash F_7}{\bullet h_1 : \top, \Delta_6 \vdash F_7} \top_L \quad \frac{h_4 : \Delta_6, F_7 \vdash F_5}{\bullet h_4 : (\top, \Delta_6), F_7 \vdash F_5} \top_L}{\frac{}{- : \top, \Delta_6 \vdash F_5} \text{Cut}} \rightarrow \\
\frac{\frac{h_1 : \top, \Delta_6 \vdash F_7}{\bullet h_1 : \top, \Delta_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_6, F_7 \vdash F_5}{\bullet h_4 : \top, \Delta_6, F_7 \vdash F_5} \text{ax/W}}{\frac{}{- : \top, \Delta_6 \vdash F_5} \text{hCut}}
\end{array}$$



## 7 Cut-Elimination

### 7.1 Status of $\top_R$ : OK

- Case rule  $\top_R$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_4 \vdash \top} \top_R \quad \frac{\bullet h_3 : \Delta_4, \top \vdash \top}{\vdash : \Delta_2, \Delta_4 \vdash \top} \top_R}{\vdash : \Delta_2, \Delta_4 \vdash \top} \text{Cut}$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \rightarrow F_5} \top_R \quad \frac{h_3 : \top, \Delta_6, F_4 \vdash F_5}{\vdash : \Delta_2, \Delta_6, \top \vdash F_4 \rightarrow F_5} \rightarrow_R}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \rightarrow F_5} \text{Cut}$$

$$\frac{\frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_6, F_4 \vdash F_5} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_6, F_4 \vdash F_5}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \rightarrow F_5} \text{ax/W}}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \rightarrow F_5} \text{hCut}$$

- Case rule  $\wedge_R$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \wedge F_5} \top_R \quad \frac{h_3 : \top, \Delta_6 \vdash F_4 \quad h_3 : \top, \Delta_6 \vdash F_5}{\vdash : \Delta_2, \Delta_6, \top \vdash F_4 \wedge F_5} \wedge_R}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \wedge F_5} \text{Cut}$$

$$\frac{\frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_6 \vdash F_4} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_6 \vdash F_4}{\vdash : \Delta_2, \Delta_6 \vdash F_4} \text{ax/W}}{\vdash : \Delta_2, \Delta_6 \vdash F_4} \text{hCut}$$

$$\frac{\frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_6 \vdash F_5} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_6 \vdash F_5}{\vdash : \Delta_2, \Delta_6 \vdash F_5} \text{ax/W}}{\vdash : \Delta_2, \Delta_6 \vdash F_5} \text{hCut}$$

$$\frac{\vdash : \Delta_2, \Delta_6 \vdash F_4 \quad \vdash : \Delta_2, \Delta_6 \vdash F_5}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \wedge F_5} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \vee F_5} \top_R \quad \frac{h_3 : \top, \Delta_6 \vdash F_4}{\vdash : \Delta_2, \Delta_6, \top \vdash F_4 \vee F_5} \vee_1}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \vee F_5} \text{Cut}$$

$$\frac{\frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_6 \vdash F_4} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_6 \vdash F_4}{\vdash : \Delta_2, \Delta_6 \vdash F_4} \text{ax/W}}{\vdash : \Delta_2, \Delta_6 \vdash F_4} \text{hCut}$$

$$\frac{\vdash : \Delta_2, \Delta_6 \vdash F_4}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \vee F_5} \vee_1$$

- Case rule  $\vee_2$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \vee F_5} \top_R \quad \frac{h_3 : \top, \Delta_6 \vdash F_5}{\vdash : \Delta_2, \Delta_6, \top \vdash F_4 \vee F_5} \vee_2}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \vee F_5} \text{Cut}$$

$$\frac{\frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_6 \vdash F_5} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_6 \vdash F_5}{\vdash : \Delta_2, \Delta_6 \vdash F_5} \text{ax/W}}{\vdash : \Delta_2, \Delta_6 \vdash F_5} \text{hCut}$$

$$\frac{\vdash : \Delta_2, \Delta_6 \vdash F_5}{\vdash : \Delta_2, \Delta_6 \vdash F_4 \vee F_5} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\vdash : \Delta_2, \Delta_7, \top \vdash F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L}{\vdash : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_6} \text{Cut}$$

$$\frac{\frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_4} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_4}{\vdash : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_4} \text{ax/W}}{\vdash : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_4} \text{hCut}$$

$$\frac{\frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_7, F_5 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_7, F_5 \vdash F_6}{\vdash : \Delta_2, \Delta_7, F_5 \vdash F_6} \text{ax/W}}{\vdash : \Delta_2, \Delta_7, F_5 \vdash F_6} \text{hCut}$$

$$\frac{\vdash : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_4 \quad \vdash : \Delta_2, \Delta_7, F_5 \vdash F_6}{\vdash : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \wedge F_5), \top \vdash F_6} \wedge_L}{- : \Delta_2, \Delta_7, F_4 \wedge F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_7, F_4, F_5 \vdash F_6}{- : \Delta_2, \Delta_7, F_4, F_5 \vdash F_6} \text{ax/w}}{- : \Delta_2, \Delta_7, F_4 \wedge F_5 \vdash F_6} \text{hCut} \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \vee F_5), \top \vdash F_6} \vee_L}{- : \Delta_2, \Delta_7, F_4 \vee F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_7, F_4 \vdash F_6}{- : \Delta_2, \Delta_7, F_4 \vdash F_6} \text{ax/w} \quad \frac{\frac{}{\bullet h_1 : * \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_7, F_5 \vdash F_6}{- : \Delta_2, \Delta_7, F_5 \vdash F_6} \text{ax/w}}{- : \Delta_2, \Delta_7, F_4 \vee F_5 \vdash F_6} \text{hCut} \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \quad \frac{}{\bullet h_3 : (\perp, \Delta_5), \top \vdash F_4} \perp_L}{- : \Delta_2, \perp, \Delta_5 \vdash F_4} \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_2, \Delta_5 \vdash F_4 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \quad \frac{}{\bullet h_3 : (\Delta_4, p_5), \top \vdash p_5} I}{- : \Delta_2, \Delta_4, p_5 \vdash p_5} \text{Cut} \\
\rightarrow \\
- : \Delta_2, \Delta_4, p_5 \vdash p_5 \quad I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \quad \frac{h_3 : \Delta_5 \vdash F_4}{\bullet h_3 : \Delta_5, \top \vdash F_4} \top_L}{- : \Delta_2, \Delta_5 \vdash F_4} \text{Cut} \\
\rightarrow \\
- : \Delta_2, \Delta_5 \vdash F_4 \quad \text{ax/w}
\end{array}$$

## 7.2 Status of $\rightarrow_R$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{}{\bullet h_7 : \Delta_8, F_5 \rightarrow F_6 \vdash \top} \top_R}{- : \Delta_2, \Delta_8 \vdash \top} \text{Cut} \\
\rightarrow \\
- : \Delta_2, \Delta_8 \vdash \top \quad \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \rightarrow F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \rightarrow F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_8 \rightarrow F_9} \text{ ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \rightarrow F_9} \text{ hCut} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \wedge F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_8 \wedge F_9} \text{ ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \wedge F_9} \text{ hCut} \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \vee F_9} \vee_1 \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9} \text{ hCut} \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_8 \vee F_9} \vee_2 \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \rightarrow F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9} \text{ ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9} \text{ hCut} \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_5 \rightarrow F_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \rightarrow F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \rightarrow F_6, F_8 \rightarrow F_9 \vdash F_8}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{ ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10}} \text{ hCut} \rightarrow_L \\
\hline
\rightarrow \\
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \rightarrow_L \\
\hline
- : \Delta_2, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7}{- : \Delta_2, \Delta_9 \vdash F_6} \text{ ax/W} \quad \frac{h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash F_6}{- : \Delta_2, \Delta_9 \vdash F_6} \text{ ax/W} \quad \frac{- : \Delta_2, F_6 \vdash F_7}{- : \Delta_2, \Delta_9, F_6 \vdash F_8} \text{ ax/W} \quad \frac{- : \Delta_9, F_7 \vdash F_8}{- : \Delta_2, \Delta_9, F_6 \vdash F_8} \text{ ax/W}}{- : \Delta_2, \Delta_9 \vdash F_8} \text{ hCut} \text{ sCut} \\
\hline
\rightarrow \\
\frac{- : \Delta_2, \Delta_9 \vdash F_8}{- : \Delta_2, \Delta_9 \vdash F_8} C
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \rightarrow F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\text{hCut}} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \rightarrow F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \rightarrow F_6 \vdash F_{10}} \vee_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \rightarrow F_6 \vdash F_{10}}{\text{hCut}} \quad \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\text{hCut}} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}} \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \rightarrow F_6 \vdash F_8} \perp_L \\
\hline
- : \Delta_2, \perp, \Delta_9 \vdash F_8 \\
\rightarrow \\
- : \perp, \Delta_2, \Delta_9 \vdash F_8 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{}{\bullet h_7 : (\Delta_8, p_9), F_5 \rightarrow F_6 \vdash p_9} I \\
\hline
- : \Delta_2, \Delta_8, p_9 \vdash p_9 \\
\rightarrow \\
- : \Delta_2, \Delta_8, p_9 \vdash p_9 \quad I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_9, F_5 \rightarrow F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \rightarrow F_6 \vdash F_8} \top_L \\
\hline
- : \Delta_2, \top, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_5 \rightarrow F_6 \vdash F_8}{\text{hCut}} \quad \text{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash F_8}
\end{array}$$

### 7.3 Status of $\wedge_R$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{}{\bullet h_7 : \Delta_8, F_5 \wedge F_6 \vdash \top} \top_R \\
\hline
- : \Delta_2, \Delta_8 \vdash \top \\
\rightarrow \\
- : \Delta_2, \Delta_8 \vdash \top \quad \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \wedge F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \wedge F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \rightarrow F_9} \text{hCut} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \wedge F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \wedge F_9} \text{hCut} \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \vee F_9} \vee_1 \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9} \text{hCut} \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_8 \vee F_9} \vee_2 \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \wedge F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9} \text{hCut} \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \wedge F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \wedge F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \wedge F_6 \vdash F_8}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10}} \text{hCut} \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \wedge F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10}} \text{hCut} \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_5 : \Delta_9, F_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \wedge F_7 \vdash F_8} \wedge_L \\
\hline
- : \Delta_2, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{- : \Delta_2 \vdash F_7 \quad \text{ax/W} \quad - : \Delta_9, \Delta_9, F_6, F_7 \vdash F_8 \quad \text{ax/W}}{- : \Delta_2, \Delta_9, \Delta_9, F_6 \vdash F_8} \text{sCut} \\
\hline
\frac{- : \Delta_2 \vdash F_6 \quad \text{ax/W} \quad - : \Delta_2, \Delta_9, \Delta_9, F_6 \vdash F_8}{- : \Delta_2, \Delta_2, \Delta_9, \Delta_9 \vdash F_8} \text{sCut} \\
\hline
- : \Delta_2, \Delta_9 \vdash F_8 \quad C
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \wedge F_6 \vdash F_{10}} \vee_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10} \quad \text{ax/W} \quad \bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10} \quad \text{hCut} \quad - : \Delta_{11}, \Delta_2, F_9 \vdash F_{10} \quad \text{hCut}} \vee_L \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \wedge F_6 \vdash F_8} \perp_L \\
\hline
- : \Delta_2, \perp, \Delta_9 \vdash F_8 \\
\rightarrow \\
- : \perp, \Delta_2, \Delta_9 \vdash F_8 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{}{\bullet h_7 : (\Delta_8, p_9), F_5 \wedge F_6 \vdash p_9} I \\
\hline
- : \Delta_2, \Delta_8, p_9 \vdash p_9 \\
\rightarrow \\
- : \Delta_2, \Delta_8, p_9 \vdash p_9 \quad I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_9, F_5 \wedge F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \wedge F_6 \vdash F_8} \top_L \\
\hline
- : \Delta_2, \top, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6 \quad \text{ax/W} \quad h_7 : \top, \Delta_9, F_5 \wedge F_6 \vdash F_8 \quad \text{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 7.4 Status of $\vee_1$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{}{\bullet h_7 : \Delta_8, F_5 \vee F_6 \vdash \top} \top_R \\
\hline
- : \Delta_2, \Delta_8 \vdash \top \\
\rightarrow \\
- : \Delta_2, \Delta_8 \vdash \top \quad \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \rightarrow F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_8 \rightarrow F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \rightarrow F_9 \quad \text{hCut} \\
\hline
\rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \wedge F_9} \wedge_R \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \wedge F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \quad \text{hCut} \\
\hline
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \wedge F_9 \quad \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \vee F_9} \vee_1 \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_8 \quad \text{hCut} \\
\hline
\vee_1 \\
- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \vee F_9} \vee_2 \\
\hline
- : \Delta_2, \Delta_{10} \vdash F_8 \vee F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2 \vdash F_9 \quad \text{hCut} \\
\hline
\vee_2 \\
- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \vee F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8 \quad \text{hCut} \\
\hline
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10} \quad \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10} \quad \text{hCut} \\
\hline
\wedge_L \\
- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{\frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L}{- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \vee F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{hCut} \quad \frac{\frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}} \vee_L \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \quad \frac{\frac{h_5 : \Delta_9, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash F_8} \vee_L}{- : \Delta_2, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_6}{- : \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{- : \Delta_9, F_6 \vdash F_8}{- : \Delta_2, \Delta_9 \vdash F_8} \text{sCut}}{- : \Delta_2, \Delta_9 \vdash F_8}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8} \perp_L}{- : \Delta_2, \perp, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_9 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{}{\bullet h_7 : (\Delta_8, p_9), F_5 \vee F_6 \vdash p_9} I}{- : \Delta_2, \Delta_8, p_9 \vdash p_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_8, p_9 \vdash p_9} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L}{- : \Delta_2, \top, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \vee F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{- : \top, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 7.5 Status of $\vee_2$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{}{\bullet h_7 : \Delta_8, F_5 \vee F_6 \vdash \top} \top_R}{- : \Delta_2, \Delta_8 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_8 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \rightarrow F_9} \rightarrow_R}{- : \Delta_2, \Delta_{10} \vdash F_8 \rightarrow F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \vee F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_2 \vdash F_8 \rightarrow F_9} \rightarrow_R
\end{array}$$



- $$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \wedge F_9} \wedge_R}{- : \Delta_2, \Delta_{10} \vdash F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/w} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/w} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \wedge F_9} \wedge_R
\end{array}$$

- $$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \vee F_9} \vee_1}{- : \Delta_2, \Delta_{10} \vdash F_8 \vee F_9} \text{Cut}$$
- $$\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8} \text{hCut}$$
- $$\frac{- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9} \vee_1$$

- $$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{\bullet h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_8 \vee F_9} \vee_2}{- : \Delta_2, \Delta_{10} \vdash F_8 \vee F_9} \text{Cut}$$
- $$\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{- : \Delta_{10}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5 \vee F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2 \vdash F_8 \vee F_9} \text{hCut} \vee_2$$

- $$\frac{\frac{\frac{\mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \mathbf{F}_5 \vee \mathbf{F}_6} \vee_2 \quad \frac{\frac{\mathbf{h}_7 : \Delta_{11}, \mathbf{F}_8 \rightarrow \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_8 \quad \mathbf{h}_7 : \Delta_{11}, \mathbf{F}_9, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10}}{\bullet \mathbf{h}_7 : (\Delta_{11}, \mathbf{F}_8 \rightarrow \mathbf{F}_9), \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \mathbf{F}_{10}} \rightarrow_L}{\frac{- : \Delta_2, \Delta_{11}, \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash \mathbf{F}_{10}}{- : \Delta_{11}, \Delta_2, \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash \mathbf{F}_8} \text{Cut}} \rightarrow$$

- $$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10} \quad \wedge_L
\end{array}$$

- $$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{\frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L}{- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut} \quad \rightarrow \quad \frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \vee F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{hCut} \quad \frac{\frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut} \quad \vee_L}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}} \vee_L$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_2 \quad \frac{h_5 : \Delta_9, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_7 \vdash F_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash F_8} \vee_L \\
\hline
- : \Delta_2, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \Delta_2 \vdash F_7} \text{ax/W} \quad \frac{}{- : \Delta_9, F_7 \vdash F_8} \text{ax/W} \\
\hline
- : \Delta_2, \Delta_9 \vdash F_8 \quad \text{sCut}
\end{array}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8} \perp_L \\
\hline
- : \Delta_2, \perp, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \perp, \Delta_2, \Delta_9 \vdash F_8} \perp_L
\end{array}
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{}{\bullet h_7 : (\Delta_8, p_9), F_5 \vee F_6 \vdash p_9} I \\
\hline
- : \Delta_2, \Delta_8, p_9 \vdash p_9 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \Delta_2, \Delta_8, p_9 \vdash p_9} I
\end{array}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L \\
\hline
- : \Delta_2, \top, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \top, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{hCut}
\end{array}
\end{array}$$

## 7.6 Status of $\rightarrow_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_8} \rightarrow_L \quad \frac{}{\bullet h_6 : \Delta_7, F_8 \vdash \top} \top_R \\
\hline
- : (\Delta_5, F_2 \rightarrow F_3), \Delta_7 \vdash \top \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \Delta_5, \Delta_7, F_2 \rightarrow F_3 \vdash \top} \top_R
\end{array}
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_{10}} \rightarrow_L \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \rightarrow F_8} \rightarrow_R \\
\hline
- : (\Delta_5, F_2 \rightarrow F_3), \Delta_9 \vdash F_7 \rightarrow F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{}{- : \Delta_5, \Delta_9, F_7, F_2 \rightarrow F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_5, \Delta_9, F_2 \rightarrow F_3 \vdash F_7 \rightarrow F_8 \quad \rightarrow_R
\end{array}
\end{array}$$

- Case rule  $\wedge_R$



$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_7 \wedge F_8}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_7 \wedge F_8} \rightarrow_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash F_9} \wedge_L \\
\frac{\quad}{- : (\Delta_5, F_2 \rightarrow F_3), \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\quad}{- : \Delta_{10}, \Delta_5, F_2 \rightarrow F_3 \vdash F_2} \text{ax/W} \quad \frac{h_1 : \Delta_5, F_3 \vdash F_7 \wedge F_8 \quad \bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{ax/W} \\
\frac{\quad}{- : \Delta_{10}, \Delta_5, F_2 \rightarrow F_3 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_{11}} \rightarrow_L \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \vee F_8), F_{11} \vdash F_9} \vee_L \\
\frac{\quad}{- : (\Delta_5, F_2 \rightarrow F_3), \Delta_{10}, F_7 \vee F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\quad}{- : \Delta_{10}, \Delta_5, F_2 \rightarrow F_3, F_7 \vee F_8 \vdash F_2} \text{ax/W} \quad \frac{h_1 : \Delta_5, F_3 \vdash F_{11} \quad \bullet h_6 : \Delta_{10}, F_{11}, F_7 \vee F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_3, F_7 \vee F_8 \vdash F_9} \text{ax/W} \\
\frac{\quad}{- : \Delta_{10}, \Delta_5, F_2 \rightarrow F_3, F_7 \vee F_8 \vdash F_9} \text{hCut} \\
\rightarrow_L \\
\frac{\quad}{- : \Delta_{10}, \Delta_5, F_2 \rightarrow F_3 \vdash F_9}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_7 \vee F_8}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_7 \vee F_8} \rightarrow_L \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash F_9} \vee_L \\
\frac{\quad}{- : (\Delta_5, F_2 \rightarrow F_3), \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\quad}{- : \Delta_{10}, \Delta_5, F_2 \rightarrow F_3 \vdash F_2} \text{ax/W} \quad \frac{h_1 : \Delta_5, F_3 \vdash F_7 \vee F_8 \quad \bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{ax/W} \\
\frac{\quad}{- : \Delta_{10}, \Delta_5, F_2 \rightarrow F_3 \vdash F_9} \text{hCut} \\
\rightarrow_L \\
\frac{\quad}{- : \Delta_{10}, \Delta_5, F_2 \rightarrow F_3 \vdash F_9}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash \perp}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \perp} \rightarrow_L \quad \frac{\quad}{\bullet h_6 : \Delta_8, \perp \vdash F_7} \perp_L \\
\frac{\quad}{- : (\Delta_5, F_2 \rightarrow F_3), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\quad}{- : \Delta_5, \Delta_8, F_2 \rightarrow F_3 \vdash F_2} \text{ax/W} \quad \frac{h_1 : \Delta_5, F_3 \vdash \perp \quad \bullet h_6 : \perp, \Delta_8 \vdash F_7}{- : \Delta_5, \Delta_8, F_3 \vdash F_7} \perp_L \\
\frac{\quad}{- : \Delta_5, \Delta_8, F_2 \rightarrow F_3 \vdash F_7} \text{hCut} \\
\rightarrow_L \\
\frac{\quad}{- : \Delta_5, \Delta_8, F_2 \rightarrow F_3 \vdash F_7}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_9} \rightarrow_L \quad \frac{\quad}{\bullet h_6 : (\perp, \Delta_8), F_9 \vdash F_7} \perp_L \\
\frac{\quad}{- : (\Delta_5, F_2 \rightarrow F_3), \perp, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\quad}{- : \perp, \Delta_5, \Delta_8, F_2 \rightarrow F_3 \vdash F_7} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_8} \rightarrow_L \quad \frac{\quad}{\bullet h_6 : (\Delta_7, p_9), F_8 \vdash p_9} I \\
\frac{\quad}{- : (\Delta_5, F_2 \rightarrow F_3), \Delta_7, p_9 \vdash p_9} \text{Cut} \\
\rightarrow \\
\frac{\quad}{- : \Delta_5, \Delta_7, p_9, F_2 \rightarrow F_3 \vdash p_9} I
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash p_8}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash p_8} \rightarrow_L \quad \frac{\quad}{\bullet h_6 : \Delta_7, p_8 \vdash p_8} I \\
\frac{\quad}{- : (\Delta_5, F_2 \rightarrow F_3), \Delta_7 \vdash p_8} \text{Cut} \\
\rightarrow \\
\frac{\quad}{- : \Delta_5, \Delta_7, F_2 \rightarrow F_3 \vdash p_8} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash \top}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \top} \rightarrow_L \quad \frac{h_6 : \Delta_8 \vdash F_7}{\bullet h_6 : \Delta_8, \top \vdash F_7} \top_L}{\frac{- : (\Delta_5, F_2 \rightarrow F_3), \Delta_8 \vdash F_7}{\rightarrow} \text{Cut}} \\
\frac{}{- : \Delta_5, \Delta_8, F_2 \rightarrow F_3 \vdash F_7} \text{ax/W} \\
\frac{\frac{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \Delta_5, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_9} \rightarrow_L \quad \frac{h_6 : \Delta_8, F_9 \vdash F_7}{\bullet h_6 : (\top, \Delta_8), F_9 \vdash F_7} \top_L}{\frac{- : (\Delta_5, F_2 \rightarrow F_3), \top, \Delta_8 \vdash F_7}{\rightarrow} \text{Cut}} \\
\frac{\frac{}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \top, \Delta_8, F_9 \vdash F_7}{\bullet h_6 : \top, \Delta_8, F_9 \vdash F_7} \text{ax/W}}{- : \top, \Delta_5, \Delta_8, F_2 \rightarrow F_3 \vdash F_7} \text{hCut}
\end{array}$$

## 7.7 Status of $\wedge_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_8} \wedge_L \quad \frac{}{\bullet h_6 : \Delta_7, F_8 \vdash \top} \top_R}{\frac{- : (\Delta_5, F_2 \wedge F_3), \Delta_7 \vdash \top}{\rightarrow} \text{Cut}} \\
\frac{}{- : \Delta_5, \Delta_7, F_2 \wedge F_3 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{10}} \wedge_L \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \rightarrow F_8} \rightarrow_R}{\frac{- : (\Delta_5, F_2 \wedge F_3), \Delta_9 \vdash F_7 \rightarrow F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_7, F_2 \wedge F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8}{\bullet h_6 : \Delta_9, F_{10}, F_7 \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \wedge F_3 \vdash F_7 \rightarrow F_8} \text{hCut} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{10}} \wedge_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \wedge F_8} \wedge_R}{\frac{- : (\Delta_5, F_2 \wedge F_3), \Delta_9 \vdash F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2, F_3 \vdash F_7 \wedge F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \wedge F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \wedge F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \wedge F_3 \vdash F_7 \wedge F_8} \text{hCut} \wedge_L
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{10}} \wedge_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \vee F_8} \vee_1}{\frac{- : (\Delta_5, F_2 \wedge F_3), \Delta_9 \vdash F_7 \vee F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \wedge F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \wedge F_3 \vdash F_7 \vee F_8} \text{hCut} \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{10}} \wedge_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \vee F_8} \vee_2 \\
\hline
- : (\Delta_5, F_2 \wedge F_3), \Delta_9 \vdash F_7 \vee F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \wedge F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{- : \Delta_5, \Delta_9, F_2 \wedge F_3 \vdash F_8} \text{ax/W}}{- : \Delta_5, \Delta_9, F_2 \wedge F_3 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_5, \Delta_9, F_2 \wedge F_3 \vdash F_7 \vee F_8 \quad \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{11}} \wedge_L \quad \frac{h_6 : \Delta_{10}, F_{11}, F_7 \rightarrow F_8 \vdash F_7 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \rightarrow F_8), F_{11} \vdash F_9} \rightarrow_L \\
\hline
- : (\Delta_5, F_2 \wedge F_3), \Delta_{10}, F_7 \rightarrow F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_2, F_3, F_7 \rightarrow F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \rightarrow F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_7 \rightarrow F_8, F_2 \wedge F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_7 \rightarrow F_8, F_2 \wedge F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_7 \rightarrow F_8, F_2 \wedge F_3 \vdash F_9 \quad \wedge_L \\
\hline
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \rightarrow F_8}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_7 \rightarrow F_8} \wedge_L \quad \frac{h_6 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_7 \quad h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_9} \rightarrow_L \\
\hline
- : (\Delta_5, F_2 \wedge F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \rightarrow F_8}{- : \Delta_{10}, \Delta_5, F_2, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \wedge F_3 \vdash F_9 \quad \wedge_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{11}} \wedge_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \wedge F_8), F_{11} \vdash F_9} \wedge_L \\
\hline
- : (\Delta_5, F_2 \wedge F_3), \Delta_{10}, F_7 \wedge F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_2, F_3, F_7 \wedge F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \wedge F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3, F_7 \wedge F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3, F_7 \wedge F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \wedge F_3, F_7 \wedge F_8 \vdash F_9 \quad \wedge_L \\
\hline
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \wedge F_8}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_7 \wedge F_8} \wedge_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash F_9} \wedge_L \\
\hline
- : (\Delta_5, F_2 \wedge F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \wedge F_8}{- : \Delta_{10}, \Delta_5, F_2, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \wedge F_3 \vdash F_9 \quad \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_{11}} \wedge_L \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \vee F_8), F_{11} \vdash F_9} \vee_L \\
\hline
- : (\Delta_5, F_2 \wedge F_3), \Delta_{10}, F_7 \vee F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_2, F_3, F_7 \vee F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \vee F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3, F_7 \vee F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3, F_7 \vee F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \wedge F_3, F_7 \vee F_8 \vdash F_9 \quad \wedge_L \\
\hline
\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \vee F_8}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_7 \vee F_8} \wedge_L \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash F_9} \vee_L \\
\hline
- : (\Delta_5, F_2 \wedge F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_7 \vee F_8}{- : \Delta_{10}, \Delta_5, F_2, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \wedge F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \wedge F_3 \vdash F_9 \quad \wedge_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \perp}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \perp} \wedge_L \quad \frac{}{\bullet h_6 : \Delta_8, \perp \vdash F_7} \perp_L}{\frac{}{- : (\Delta_5, F_2 \wedge F_3), \Delta_8 \vdash F_7} \text{Cut}} \rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \perp}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \perp} \text{ax/W} \quad \frac{\bullet h_6 : \perp, \Delta_8 \vdash F_7}{\bullet h_6 : \perp, \Delta_8 \vdash F_7} \perp_L}{\frac{}{- : \Delta_5, \Delta_8, F_2, F_3 \vdash F_7} \text{hCut}} \wedge_L \\
\frac{}{- : \Delta_5, \Delta_8, F_2 \wedge F_3 \vdash F_7} \rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_9} \wedge_L \quad \frac{}{\bullet h_6 : (\perp, \Delta_8), F_9 \vdash F_7} \perp_L}{\frac{}{- : (\Delta_5, F_2 \wedge F_3), \perp, \Delta_8 \vdash F_7} \text{Cut}} \rightarrow \\
\frac{}{- : \perp, \Delta_5, \Delta_8, F_2 \wedge F_3 \vdash F_7} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_8} \wedge_L \quad \frac{}{\bullet h_6 : (\Delta_7, p_9), F_8 \vdash p_9} I}{\frac{}{- : (\Delta_5, F_2 \wedge F_3), \Delta_7, p_9 \vdash p_9} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_5, \Delta_7, p_9, F_2 \wedge F_3 \vdash p_9} I \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash p_8}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash p_8} \wedge_L \quad \frac{}{\bullet h_6 : \Delta_7, p_8 \vdash p_8} I}{\frac{}{- : (\Delta_5, F_2 \wedge F_3), \Delta_7 \vdash p_8} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_5, \Delta_7, F_2 \wedge F_3 \vdash p_8} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \top}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \top} \wedge_L \quad \frac{h_6 : \Delta_8 \vdash F_7}{\bullet h_6 : \Delta_8, \top \vdash F_7} \top_L}{\frac{}{- : (\Delta_5, F_2 \wedge F_3), \Delta_8 \vdash F_7} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_5, \Delta_8, F_2 \wedge F_3 \vdash F_7} \text{ax/W} \\
\frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_9} \wedge_L \quad \frac{h_6 : \Delta_8, F_9 \vdash F_7}{\bullet h_6 : (\top, \Delta_8), F_9 \vdash F_7} \top_L}{\frac{}{- : (\Delta_5, F_2 \wedge F_3), \top, \Delta_8 \vdash F_7} \text{Cut}} \rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \top, \Delta_8, F_9 \vdash F_7}{h_6 : \top, \Delta_8, F_9 \vdash F_7} \text{ax/W}}{\frac{}{- : \top, \Delta_5, \Delta_8, F_2 \wedge F_3 \vdash F_7} \text{hCut}}
\end{array}$$

## 7.8 Status of $\vee_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_8 \quad h_1 : \Delta_5, F_3 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_8} \vee_L \quad \frac{}{\bullet h_6 : \Delta_7, F_8 \vdash \top} \top_R}{\frac{}{- : (\Delta_5, F_2 \vee F_3), \Delta_7 \vdash \top} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_5, \Delta_7, F_2 \vee F_3 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{10}} \vee_L \quad \frac{h_6 : \Delta_9, F_7, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \rightarrow F_8} \rightarrow_R \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_9 \vdash F_7 \rightarrow F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_7, F_2 \vee F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10}, F_7 \vdash F_8}{- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \rightarrow F_8} \text{ax/W} \\
\hline
- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \rightarrow F_8 \quad \text{hCut} \\
\hline
- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \rightarrow F_8 \quad \rightarrow_R
\end{array}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{10}} \vee_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7 \quad h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \wedge F_8} \wedge_R \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_9 \vdash F_7 \wedge F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \wedge F_8} \text{ax/W} \\
\hline
- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \wedge F_8 \quad \text{hCut} \\
\hline
- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \wedge F_8 \quad \wedge_R
\end{array}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{10}} \vee_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \vee F_8} \vee_1 \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_9 \vdash F_7 \vee F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_7}{- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \vee F_8} \text{ax/W} \\
\hline
- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \vee F_8 \quad \text{hCut} \\
\hline
- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \vee F_8 \quad \vee_1
\end{array}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{10} \quad h_1 : \Delta_5, F_3 \vdash F_{10}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{10}} \vee_L \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{\bullet h_6 : \Delta_9, F_{10} \vdash F_7 \vee F_8} \vee_2 \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_9 \vdash F_7 \vee F_8 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{10}}{- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_{10} \vdash F_8}{- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \vee F_8} \text{ax/W} \\
\hline
- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \vee F_8 \quad \text{hCut} \\
\hline
- : \Delta_5, \Delta_9, F_2 \vee F_3 \vdash F_7 \vee F_8 \quad \vee_2
\end{array}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{11} \quad h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{11}} \vee_L \quad \frac{h_6 : \Delta_{10}, F_{11}, F_7 \rightarrow F_8 \vdash F_7 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \rightarrow F_8), F_{11} \vdash F_9} \rightarrow_L \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_{10}, F_7 \rightarrow F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{11}}{- : \Delta_{10}, \Delta_5, F_7 \rightarrow F_8, F_2 \vee F_3 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_{11}, F_7 \rightarrow F_8 \vdash F_7}{- : \Delta_{10}, \Delta_5, F_7 \rightarrow F_8, F_2 \vee F_3 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_5, F_7 \rightarrow F_8, F_2 \vee F_3 \vdash F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_7 \rightarrow F_8, F_2 \vee F_3 \vdash F_9 \quad \rightarrow_L
\end{array}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_7 \rightarrow F_8 \quad h_1 : \Delta_5, F_3 \vdash F_7 \rightarrow F_8}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_7 \rightarrow F_8} \vee_L \quad \frac{h_6 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_7 \quad h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_9} \rightarrow_L \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_7 \rightarrow F_8}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \rightarrow F_8 \vdash F_9}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \vdash F_9 \quad \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \vdash F_9 \quad \vee_L
\end{array}
\end{array}$$

- Case rule  $\wedge_L$



$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{11} \quad h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{11}} \vee_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \wedge F_8), F_{11} \vdash F_9} \wedge_L \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_{10}, F_7 \wedge F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{11}} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_{11}, F_7, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_{11}, F_7 \vee F_8) \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_7, F_8, F_2 \vee F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_7 \wedge F_8, F_2 \vee F_3 \vdash F_9 \quad \wedge_L \\
\hline
\frac{h_1 : \Delta_5, F_2 \vdash F_7 \wedge F_8 \quad h_1 : \Delta_5, F_3 \vdash F_7 \wedge F_8}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_7 \wedge F_8} \vee_L \quad \frac{h_6 : \Delta_{10}, F_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash F_9} \wedge_L \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \wedge F_8}{\bullet h_1 : \Delta_5, F_2 \vdash F_7 \wedge F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{hCut} \\
\hline
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_7 \wedge F_8}{\bullet h_1 : \Delta_5, F_3 \vdash F_7 \wedge F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \wedge F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \vee F_3 \vdash F_9 \quad \vee_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_{11} \quad h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{11}} \vee_L \quad \frac{h_6 : \Delta_{10}, F_7, F_{11} \vdash F_9 \quad h_6 : \Delta_{10}, F_8, F_{11} \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \vee F_8), F_{11} \vdash F_9} \vee_L \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_{10}, F_7 \vee F_8 \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \vee F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \vee F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2, F_7 \vee F_8 \vdash F_9} \text{hCut} \\
\hline
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_{11}}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \vee F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_{11}, F_7 \vee F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_3, F_7 \vee F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \vee F_3, F_7 \vee F_8 \vdash F_9 \quad \vee_L \\
\hline
\frac{h_1 : \Delta_5, F_2 \vdash F_7 \vee F_8 \quad h_1 : \Delta_5, F_3 \vdash F_7 \vee F_8}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_7 \vee F_8} \vee_L \quad \frac{h_6 : \Delta_{10}, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash F_9} \vee_L \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_{10} \vdash F_9 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_7 \vee F_8}{\bullet h_1 : \Delta_5, F_2 \vdash F_7 \vee F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_2 \vdash F_9} \text{hCut} \\
\hline
\frac{\frac{h_1 : \Delta_5, F_3 \vdash F_7 \vee F_8}{\bullet h_1 : \Delta_5, F_2 \vdash F_7 \vee F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_7 \vee F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_5, F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_5, F_2 \vee F_3 \vdash F_9 \quad \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash \perp \quad h_1 : \Delta_5, F_3 \vdash \perp}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \perp} \vee_L \quad \frac{}{\bullet h_6 : \Delta_8, \perp \vdash F_7} \perp_L \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash \perp}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \perp} \text{ax/W} \quad \frac{\bullet h_6 : \perp, \Delta_8 \vdash F_7}{\bullet h_6 : \perp, \Delta_8 \vdash F_7} \text{ax/W}}{- : \Delta_5, \Delta_8, F_2 \vdash F_7} \text{hCut} \\
\hline
\frac{\frac{h_1 : \Delta_5, F_3 \vdash \perp}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \perp} \text{ax/W} \quad \frac{\bullet h_6 : \perp, \Delta_8 \vdash F_7}{\bullet h_6 : \perp, \Delta_8 \vdash F_7} \text{ax/W}}{- : \Delta_5, \Delta_8, F_3 \vdash F_7} \text{hCut} \\
\hline
- : \Delta_5, \Delta_8, F_2 \vee F_3 \vdash F_7 \quad \vee_L \\
\hline
\frac{h_1 : \Delta_5, F_2 \vdash F_9 \quad h_1 : \Delta_5, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_9} \vee_L \quad \frac{}{\bullet h_6 : (\perp, \Delta_8), F_9 \vdash F_7} \perp_L \\
\hline
- : (\Delta_5, F_2 \vee F_3), \perp, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\hline
\rightarrow \\
- : \perp, \Delta_5, \Delta_8, F_2 \vee F_3 \vdash F_7 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_2 \vdash F_8 \quad h_1 : \Delta_5, F_3 \vdash F_8}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_8} \vee_L \quad \frac{}{\bullet h_6 : (\Delta_7, p_9), F_8 \vdash p_9} I \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_7, p_9 \vdash p_9 \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_5, \Delta_7, p_9, F_2 \vee F_3 \vdash p_9 \quad I \\
\hline
\frac{h_1 : \Delta_5, F_2 \vdash p_8 \quad h_1 : \Delta_5, F_3 \vdash p_8}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash p_8} \vee_L \quad \frac{}{\bullet h_6 : \Delta_7, p_8 \vdash p_8} I \\
\hline
- : (\Delta_5, F_2 \vee F_3), \Delta_7 \vdash p_8 \quad \text{Cut} \\
\hline
\rightarrow \\
- : \Delta_5, \Delta_7, F_2 \vee F_3 \vdash p_8 \quad \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5, F_2 \vdash \top \quad h_1 : \Delta_5, F_3 \vdash \top}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \top} \vee_L \quad \frac{h_6 : \Delta_8 \vdash F_7}{\bullet h_6 : \Delta_8, \top \vdash F_7} \top_L}{\frac{- : (\Delta_5, F_2 \vee F_3), \Delta_8 \vdash F_7}{\rightarrow} \text{Cut}} \\
\frac{}{- : \Delta_5, \Delta_8, F_2 \vee F_3 \vdash F_7} \text{ax/W} \\
\frac{\frac{h_1 : \Delta_5, F_2 \vdash F_9 \quad h_1 : \Delta_5, F_3 \vdash F_9}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_9} \vee_L \quad \frac{h_6 : \Delta_8, F_9 \vdash F_7}{\bullet h_6 : (\top, \Delta_8), F_9 \vdash F_7} \top_L}{\frac{- : (\Delta_5, F_2 \vee F_3), \top, \Delta_8 \vdash F_7}{\rightarrow} \text{Cut}} \\
\frac{\frac{}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \top, \Delta_8, F_9 \vdash F_7}{\bullet h_6 : \top, \Delta_8, F_2 \vee F_3 \vdash F_7} \text{ax/W}}{- : \top, \Delta_5, \Delta_8, F_2 \vee F_3 \vdash F_7} \text{hCut}
\end{array}$$

## 7.9 Status of $\perp_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_6} \perp_L \quad \frac{}{\bullet h_4 : \Delta_5, F_6 \vdash \top} \top_R}{\frac{- : (\perp, \Delta_3), \Delta_5 \vdash \top}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_3, \Delta_5 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_8} \perp_L \quad \frac{h_4 : \Delta_7, F_5, F_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \rightarrow F_6} \rightarrow_R}{\frac{- : (\perp, \Delta_3), \Delta_7 \vdash F_5 \rightarrow F_6}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_3, \Delta_7 \vdash F_5 \rightarrow F_6} \perp_L
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_8} \perp_L \quad \frac{h_4 : \Delta_7, F_8 \vdash F_5 \quad h_4 : \Delta_7, F_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \wedge F_6} \wedge_R}{\frac{- : (\perp, \Delta_3), \Delta_7 \vdash F_5 \wedge F_6}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_3, \Delta_7 \vdash F_5 \wedge F_6} \perp_L
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_8} \perp_L \quad \frac{h_4 : \Delta_7, F_8 \vdash F_5}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \vee F_6} \vee_1}{\frac{- : (\perp, \Delta_3), \Delta_7 \vdash F_5 \vee F_6}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_3, \Delta_7 \vdash F_5 \vee F_6} \perp_L
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_8} \perp_L \quad \frac{h_4 : \Delta_7, F_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \vee F_6} \vee_2}{\frac{- : (\perp, \Delta_3), \Delta_7 \vdash F_5 \vee F_6}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_3, \Delta_7 \vdash F_5 \vee F_6} \perp_L
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_9} \perp_L \quad \frac{\frac{h_4 : \Delta_8, F_9, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), F_9 \vdash F_7} \rightarrow_L}{- : (\perp, \Delta_3), \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_3, \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \perp_L \\
\frac{\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_5 \rightarrow F_6} \perp_L \quad \frac{\frac{h_4 : \Delta_8, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \Delta_8, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \rightarrow_L}{- : (\perp, \Delta_3), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_3, \Delta_8 \vdash F_7} \perp_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_9} \perp_L \quad \frac{\frac{h_4 : \Delta_8, F_5, F_6, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), F_9 \vdash F_7} \wedge_L}{- : (\perp, \Delta_3), \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_3, \Delta_8, F_5 \wedge F_6 \vdash F_7} \perp_L \\
\frac{\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_5 \wedge F_6} \perp_L \quad \frac{\frac{h_4 : \Delta_8, F_5, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \wedge F_6 \vdash F_7} \wedge_L}{- : (\perp, \Delta_3), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_3, \Delta_8 \vdash F_7} \perp_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_9} \perp_L \quad \frac{\frac{h_4 : \Delta_8, F_5, F_9 \vdash F_7 \quad h_4 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), F_9 \vdash F_7} \vee_L}{- : (\perp, \Delta_3), \Delta_8, F_5 \vee F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_3, \Delta_8, F_5 \vee F_6 \vdash F_7} \perp_L \\
\frac{\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_5 \vee F_6} \perp_L \quad \frac{\frac{h_4 : \Delta_8, F_5 \vdash F_7 \quad h_4 : \Delta_8, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \vee F_6 \vdash F_7} \vee_L}{- : (\perp, \Delta_3), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_3, \Delta_8 \vdash F_7} \perp_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \perp} \perp_L \quad \frac{}{\bullet h_4 : \Delta_6, \perp \vdash F_5} \perp_L}{- : (\perp, \Delta_3), \Delta_6 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_3, \Delta_6 \vdash F_5} \perp_L \\
\frac{\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_7} \perp_L \quad \frac{}{\bullet h_4 : (\perp, \Delta_6), F_7 \vdash F_5} \perp_L}{- : (\perp, \Delta_3), \perp, \Delta_6 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \perp, \Delta_3, \Delta_6 \vdash F_5} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_6} \perp_L \quad \frac{}{\bullet h_4 : (\Delta_5, p_7), F_6 \vdash p_7} I}{- : (\perp, \Delta_3), \Delta_5, p_7 \vdash p_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_3, \Delta_5, p_7 \vdash p_7} \perp_L
\end{array}$$

- Case rule  $\top_L$

- Case rule  $\top_R$

- Case rule  $\rightarrow_R$

- Case rule  $\wedge_R$

- Case rule  $\vee_1$

- Case rule  $\vee_2$

- Case rule  $\rightarrow_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_9 \vdash p_9} I \quad \frac{h_4 : \Delta_8, p_9, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \Delta_8, F_6, p_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), p_9 \vdash F_7} \rightarrow_L}{\frac{}{- : (\Delta_2, p_9), \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \text{Cut}}{\frac{}{- : \Delta_2, \Delta_8, p_9, F_5 \rightarrow F_6 \vdash F_7} \text{ax/w}}$$

- Case rule  $\wedge_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_9 \vdash p_9} I \quad \frac{h_4 : \Delta_8, F_5, F_6, p_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), p_9 \vdash F_7} \wedge_L}{\frac{}{- : (\Delta_2, p_9), \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{Cut}}{\frac{}{- : \Delta_2, \Delta_8, p_9, F_5 \wedge F_6 \vdash F_7} \text{ax/w}}$$

- Case rule  $\vee_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_9 \vdash p_9} I \quad \frac{h_4 : \Delta_8, F_5, p_9 \vdash F_7 \quad h_4 : \Delta_8, F_6, p_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), p_9 \vdash F_7} \vee_L}{\frac{}{- : (\Delta_2, p_9), \Delta_8, F_5 \vee F_6 \vdash F_7} \text{Cut}}{\frac{}{- : \Delta_2, \Delta_8, p_9, F_5 \vee F_6 \vdash F_7} \text{ax/w}}$$

- Case rule  $\perp_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_7 \vdash p_7} I \quad \frac{}{\bullet h_4 : (\perp, \Delta_6), p_7 \vdash F_5} \perp_L}{\frac{}{- : (\Delta_2, p_7), \perp, \Delta_6 \vdash F_5} \text{Cut}}{\frac{}{- : \perp, \Delta_2, \Delta_6, p_7 \vdash F_5} \perp_L}}$$

- Case rule  $I$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_6 \vdash p_6} I \quad \frac{}{\bullet h_4 : (\Delta_5, p_7), p_6 \vdash p_7} I}{\frac{}{- : (\Delta_2, p_6), \Delta_5, p_7 \vdash p_7} \text{Cut}}{\frac{}{- : \Delta_2, \Delta_5, p_6, p_7 \vdash p_7} I}$$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_6 \vdash p_6} I \quad \frac{}{\bullet h_4 : \Delta_5, p_6 \vdash p_6} I}{\frac{}{- : (\Delta_2, p_6), \Delta_5 \vdash p_6} \text{Cut}}{\frac{}{- : \Delta_2, \Delta_5, p_6 \vdash p_6} I}$$

- Case rule  $\top_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_7 \vdash p_7} I \quad \frac{h_4 : \Delta_6, p_7 \vdash F_5}{\bullet h_4 : (\top, \Delta_6), p_7 \vdash F_5} \top_L}{\frac{}{- : (\Delta_2, p_7), \top, \Delta_6 \vdash F_5} \text{Cut}}{\frac{}{- : \top, \Delta_2, \Delta_6, p_7 \vdash F_5} \text{ax/w}}$$

## 7.11 Status of $\top_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash F_6}{\bullet h_1 : \top, \Delta_3 \vdash F_6} \top_L \quad \frac{}{\bullet h_4 : \Delta_5, F_6 \vdash \top} \top_R \\
\hline
- : (\top, \Delta_3), \Delta_5 \vdash \top \\
\rightarrow \\
\frac{}{- : \top, \Delta_3, \Delta_5 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \top, \Delta_3 \vdash F_8} \top_L \quad \frac{h_4 : \Delta_7, F_5, F_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \rightarrow F_6} \rightarrow_R \\
\hline
- : (\top, \Delta_3), \Delta_7 \vdash F_5 \rightarrow F_6 \\
\rightarrow \\
\frac{h_1 : \Delta_3 \vdash F_8}{- : \top, \Delta_3, \Delta_7 \vdash F_5 \rightarrow F_6} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_7, F_8 \vdash F_5 \rightarrow F_6}{\text{hCut}} \text{ax/W}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \top, \Delta_3 \vdash F_8} \top_L \quad \frac{h_4 : \Delta_7, F_8 \vdash F_5 \quad h_4 : \Delta_7, F_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \wedge F_6} \wedge_R \\
\hline
- : (\top, \Delta_3), \Delta_7 \vdash F_5 \wedge F_6 \\
\rightarrow \\
\frac{h_1 : \Delta_3 \vdash F_8}{- : \top, \Delta_3, \Delta_7 \vdash F_5 \wedge F_6} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_7, F_8 \vdash F_5 \wedge F_6}{\text{hCut}} \text{ax/W}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \top, \Delta_3 \vdash F_8} \top_L \quad \frac{h_4 : \Delta_7, F_8 \vdash F_5}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \vee F_6} \vee_1 \\
\hline
- : (\top, \Delta_3), \Delta_7 \vdash F_5 \vee F_6 \\
\rightarrow \\
\frac{h_1 : \Delta_3 \vdash F_8}{- : \top, \Delta_3, \Delta_7 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_7, F_8 \vdash F_5 \vee F_6}{\text{hCut}} \text{ax/W}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \top, \Delta_3 \vdash F_8} \top_L \quad \frac{h_4 : \Delta_7, F_8 \vdash F_6}{\bullet h_4 : \Delta_7, F_8 \vdash F_5 \vee F_6} \vee_2 \\
\hline
- : (\top, \Delta_3), \Delta_7 \vdash F_5 \vee F_6 \\
\rightarrow \\
\frac{h_1 : \Delta_3 \vdash F_8}{- : \top, \Delta_3, \Delta_7 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_7, F_8 \vdash F_5 \vee F_6}{\text{hCut}} \text{ax/W}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \top, \Delta_3 \vdash F_9} \top_L \quad \frac{h_4 : \Delta_8, F_9, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), F_9 \vdash F_7} \rightarrow_L \\
\hline
- : (\top, \Delta_3), \Delta_8, F_5 \rightarrow F_6 \vdash F_7 \\
\rightarrow \\
\frac{h_1 : \Delta_3 \vdash F_9}{- : \top, \Delta_3, \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_8, F_9, F_5 \rightarrow F_6 \vdash F_7}{\text{hCut}} \text{ax/W}
\end{array}$$
  

$$\begin{array}{c}
\frac{h_1 : \Delta_3 \vdash F_5 \rightarrow F_6}{\bullet h_1 : \top, \Delta_3 \vdash F_5 \rightarrow F_6} \top_L \quad \frac{h_4 : \Delta_8, F_5 \rightarrow F_6 \vdash F_5 \quad h_4 : \Delta_8, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \rightarrow F_6 \vdash F_7} \rightarrow_L \\
\hline
- : (\top, \Delta_3), \Delta_8 \vdash F_7 \\
\rightarrow \\
\frac{h_1 : \Delta_3 \vdash F_5 \rightarrow F_6}{- : \top, \Delta_3, \Delta_8 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_4 : \top, \Delta_8, F_5 \rightarrow F_6 \vdash F_7}{\text{hCut}} \text{ax/W}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \top, \Delta_3 \vdash F_9} \top_L \quad \frac{h_4 : \Delta_8, F_5, F_6, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), F_9 \vdash F_7} \wedge_L}{- : (\top, \Delta_3), \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \top, \Delta_3 \vdash F_9} \text{ax/W} \quad \frac{h_4 : \top, \Delta_8, F_9, F_5 \wedge F_6 \vdash F_7}{\bullet h_4 : \top, \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{ax/W}}{- : \top, \Delta_3, \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \wedge F_6}{\bullet h_1 : \top, \Delta_3 \vdash F_5 \wedge F_6} \top_L \quad \frac{h_4 : \Delta_8, F_5, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \wedge F_6 \vdash F_7} \wedge_L}{- : (\top, \Delta_3), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \wedge F_6}{\bullet h_1 : \top, \Delta_3 \vdash F_5 \wedge F_6} \text{ax/W} \quad \frac{h_4 : \top, \Delta_8, F_5 \wedge F_6 \vdash F_7}{\bullet h_4 : \top, \Delta_8, F_5 \wedge F_6 \vdash F_7} \text{ax/W}}{- : \top, \Delta_3, \Delta_8 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \top, \Delta_3 \vdash F_9} \top_L \quad \frac{h_4 : \Delta_8, F_5, F_9 \vdash F_7 \quad h_4 : \Delta_8, F_6, F_9 \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), F_9 \vdash F_7} \vee_L}{- : (\top, \Delta_3), \Delta_8, F_5 \vee F_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \top, \Delta_3 \vdash F_9} \text{ax/W} \quad \frac{h_4 : \top, \Delta_8, F_9, F_5 \vee F_6 \vdash F_7}{\bullet h_4 : \top, \Delta_8, F_5 \vee F_6 \vdash F_7} \text{ax/W}}{- : \top, \Delta_3, \Delta_8, F_5 \vee F_6 \vdash F_7} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \vee F_6}{\bullet h_1 : \top, \Delta_3 \vdash F_5 \vee F_6} \top_L \quad \frac{h_4 : \Delta_8, F_5 \vdash F_7 \quad h_4 : \Delta_8, F_6 \vdash F_7}{\bullet h_4 : \Delta_8, F_5 \vee F_6 \vdash F_7} \vee_L}{- : (\top, \Delta_3), \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_5 \vee F_6}{\bullet h_1 : \top, \Delta_3 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_4 : \top, \Delta_8, F_5 \vee F_6 \vdash F_7}{\bullet h_4 : \top, \Delta_8, F_5 \vee F_6 \vdash F_7} \text{ax/W}}{- : \top, \Delta_3, \Delta_8 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash \perp}{\bullet h_1 : \top, \Delta_3 \vdash \perp} \top_L \quad \frac{}{\bullet h_4 : \Delta_6, \perp \vdash F_5} \perp_L}{- : (\top, \Delta_3), \Delta_6 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash \perp}{\bullet h_1 : \top, \Delta_3 \vdash \perp} \text{ax/W} \quad \frac{}{\bullet h_4 : \perp, \top, \Delta_6 \vdash F_5} \perp_L}{- : \top, \Delta_3, \Delta_6 \vdash F_5} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \top, \Delta_3 \vdash F_7} \top_L \quad \frac{}{\bullet h_4 : (\perp, \Delta_6), F_7 \vdash F_5} \perp_L}{- : (\top, \Delta_3), \perp, \Delta_6 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \top, \Delta_3, \Delta_6 \vdash F_5} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash F_6}{\bullet h_1 : \top, \Delta_3 \vdash F_6} \top_L \quad \frac{}{\bullet h_4 : (\Delta_5, p_7), F_6 \vdash p_7} I}{- : (\top, \Delta_3), \Delta_5, p_7 \vdash p_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_3, \Delta_5, p_7 \vdash p_7} I \\
\\
\frac{\frac{h_1 : \Delta_3 \vdash p_6}{\bullet h_1 : \top, \Delta_3 \vdash p_6} \top_L \quad \frac{}{\bullet h_4 : \Delta_5, p_6 \vdash p_6} I}{- : (\top, \Delta_3), \Delta_5 \vdash p_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_3, \Delta_5 \vdash p_6} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_3 \vdash \top}{\bullet h_1 : \top, \Delta_3 \vdash \top} \top_L \quad \frac{h_4 : \Delta_6 \vdash F_5}{\bullet h_4 : \Delta_6, \top \vdash F_5} \top_L}{- : (\top, \Delta_3), \Delta_6 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_3, \Delta_6 \vdash F_5} \text{ax/W} \\
\\
\frac{\frac{h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \top, \Delta_3 \vdash F_7} \top_L \quad \frac{h_4 : \Delta_6, F_7 \vdash F_5}{\bullet h_4 : (\top, \Delta_6), F_7 \vdash F_5} \top_L}{- : (\top, \Delta_3), \top, \Delta_6 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \top, \top, \Delta_3 \vdash F_7} \text{ax/W} \quad \frac{h_4 : \Delta_6, F_7 \vdash F_5}{\bullet h_4 : \top, \top, \Delta_6, F_7 \vdash F_5} \text{ax/W}}{- : \top, \top, \Delta_3, \Delta_6 \vdash F_5} \text{hCut}
\end{array}$$