Dyadic system for Linear Logic

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Contents

1 Weakening on the classical context

- Case(s) rule **1**

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathbf{1}}\ \mathbf{1} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \mathbf{1}}\ \mathbf{1}$$

- Case(s) rule !

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} :!\mathtt{F}_3}\ ! \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}\ \mathtt{ax}}{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} :!\mathtt{F}_3}\ !}\ \mathtt{IH}$$

- Case(s) rule ?

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_2\} : \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_4, ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_3\} : \Delta_4}\ \mathtt{ax}}{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_3, \mathtt{F}_W\} : \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_4, ?\mathtt{F}_3}\ ?}\ \mathtt{IH}$$

- Case(s) rule $

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{ax}}{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$}\ \mathtt{IH}$$

- Case(s) rule &

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \&$$

- Case(s) rule $\oplus_B$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B$$

- Case(s) rule $\oplus_A$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A$$

- Case(s) rule $\bot$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \Delta_3}\ \bot \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_3, \bot}\ \bot$$

- Case(s) rule $\top$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \Delta_3}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_3, \top}\ \top$$

- Case(s) rule I_3

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I_3 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I_3$$

- Case(s) rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_6, \mathtt{F}_4}\ \mathtt{IH}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_W\} : \Delta_5, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes$$

- Case(s) rule I_1

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, \mathtt{F}_W, p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1$$

- Case(s) rule I_2

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2)}\ I_2 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, \mathtt{F}_W, \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2)}\ I_2$$

- Case(s) rule $?_C$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \mathtt{F}_3, \Delta_2}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \Delta_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_W\} : \Delta_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_W\} : \Delta_2}\ ?_C$$

2 Contraction on the classical context

- Case(s) rule **1**

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_3, \mathtt{F}_2, \mathtt{F}_2\} : \mathbf{1}}\ \mathbf{1} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_3, \mathtt{F}_2\} : \mathbf{1}}\ \mathbf{1}$$

- Case(s) rule !

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \mathtt{F}_3, \Delta_4\} : \mathtt{F}_2}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_3\} : !\mathtt{F}_2}\ ! \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_3\} : \mathtt{F}_2}\ \mathtt{ax}}{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \mathtt{F}_2}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : !\mathtt{F}_2}\ !}\ \mathtt{IH}$$

- Case(s) rule ?

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_4, \Delta_5\} : \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_4, \mathtt{F}_4\} : \Delta_3, ?\mathtt{F}_2}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_4\} : \Delta_3}\ \mathtt{ax}}{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_2, \mathtt{F}_4\} : \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_4\} : \Delta_3, ?\mathtt{F}_2}\ ?}\ \mathtt{IH}$$

- Case(s) rule $

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \mathtt{F}_5, \Delta_6\} : \mathtt{F}_2, \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \$ \mathtt{F}_3}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax}}{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \$ \mathtt{F}_3}\ \$}\ \mathtt{IH}$$

- Case(s) rule &

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \mathtt{F}_5, \Delta_6\} : \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 :\vdash \{\mathtt{F}_5, \mathtt{F}_5, \Delta_6\} : \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \& \mathtt{F}_3}\ \& \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \& \mathtt{F}_3}\ \&$$

- Case(s) rule $\oplus_B$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \mathtt{F}_5, \Delta_6\} : \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_B$$

- Case(s) rule $\oplus_A$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \mathtt{F}_5, \Delta_6\} : \mathtt{F}_2, \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2}\ \mathtt{IH}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_A$$

- Case(s) rule $\bot$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \mathtt{F}_3, \Delta_4\} : \Delta_2}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_3\} : \bot, \Delta_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_3\} : \Delta_2}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \bot}\ \bot$$

- Case(s) rule $\top$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_3\} : \top, \Delta_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \top}\ \top$$

- Case(s) rule I_3

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_3\} : p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I_3 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I_3$$

- Case(s) rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_6, \mathtt{F}_6, \Delta_7\} : \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 :\vdash \{\mathtt{F}_6, \mathtt{F}_6, \Delta_7\} : \mathtt{F}_3, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6, \mathtt{F}_6\} : \Delta_4, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6, \mathtt{F}_6\} : \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_4, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6, \mathtt{F}_6\} : \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_5, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_4, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes$$

- Case(s) rule I_1

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, p(\mathtt{n}_2), p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1$$

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \{(\Delta_4, p(\mathtt{n}_2)), \mathtt{F}_3, \mathtt{F}_3\} : \hat{}(\mathtt{n}_2)}\ I_1 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1$$

- Case(s) rule I_2

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, \hat{}(\mathtt{n}_2), \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2)}\ I_2 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2)}\ I_2$$

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \{(\Delta_4, \hat{}(\mathtt{n}_2)), \mathtt{F}_3, \mathtt{F}_3\} : p(\mathtt{n}_2)}\ I_2 \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2)}\ I_2$$

- Case(s) rule $?_C$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \mathtt{F}_3, \Delta_4\} : \mathtt{F}_3, \Delta_2}{\bullet\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_3\} : \Delta_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2}\ ?_C$$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_4, \Delta_5\} : \mathtt{F}_3, \Delta_2}{\bullet\mathtt{h}_1 :\vdash \{(\mathtt{F}_3, \Delta_5), \mathtt{F}_4, \mathtt{F}_4\} : \Delta_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_4\} : \Delta_2, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_3, \mathtt{F}_4\} : \Delta_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_3, \mathtt{F}_4\} : \Delta_2}\ ?_C$$

3 Measure of derivations

- Case(s) rule $\mathbf{1}$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{1}}\ \mathtt{1} \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{1}}\ \mathtt{1}$$

- Case(s) rule !

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : !\mathtt{F}_3}\ !\quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : !\mathtt{F}_3}\ !$$

- Case(s) rule ?

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_2\} : \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_4, ?\mathtt{F}_3}\ ?\quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_3\} : \Delta_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_3\} : \Delta_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_4, ?\mathtt{F}_3}\ ?$$

- Case(s) rule \$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$\quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$$

- Case(s) rule &

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \&\quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \&$$

- Case(s) rule $\oplus_B$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B$$

- Case(s) rule $\oplus_A$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A$$

- Case(s) rule $\bot$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \Delta_3}\ \bot \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3, \bot}\ \bot$$

- Case(s) rule $\top$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \Delta_3}\ \top \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3, \top}\ \top$$

- Case(s) rule I_3

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I_3 \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I_3$$

- Case(s) rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_6}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes$$

- Case(s) rule I_1

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_3, p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1 \quad \rightarrow \quad \frac{}{\bullet \bullet \mathtt{h}_1 :\vdash \{\Delta_3, p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1$$

- Case(s) rule I_2

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_3, \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2)}\ I_2 \quad \rightarrow \quad \frac{}{\bullet \bullet \mathtt{h}_1 :\vdash \{\Delta_3, \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2)}\ I_2$$

- Case(s) rule $?_C$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \mathtt{F}_3, \Delta_2}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \Delta_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2}\ ?_C$$

4 Invertibility of Rules

4.1 Status of 1: : `Invertible`

- Case rule **1**

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathbf{1}}\ \mathbf{1} \quad \rightarrow \quad \texttt{trivial}$$

- Case rule !
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_2, \Delta_3\} : \mathbf{1}, \mathtt{F}_2}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_2, \Delta_3\} : \mathbf{1}}\ ?_C \quad \rightarrow \quad \texttt{trivial}$$

4.2 Status of !: : `Invertible`

- Case rule **1**
- Case rule !

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} :!\mathtt{F}_3}\ ! \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}\ ^{\mathtt{ax}}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}\ ^{\mathtt{H}}$$

- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{\mathtt{h}_2 :\vdash \{\mathtt{F}_3, \Delta_4\} : \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash \{\mathtt{F}_3, \Delta_4\} :!\mathtt{F}_{\mathbf{1}}}\ ?_C \quad \rightarrow \quad \frac{\frac{\overline{\mathtt{h}_2 :\vdash \{\Delta_4, \mathtt{F}_3\} : \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}}\ ^{\mathtt{ax}}}{\bullet \mathtt{h}_2 :\vdash \{\Delta_4, \mathtt{F}_3\} :!\mathtt{F}_{\mathbf{1}}}\ ?_C}{\bullet \mathtt{h}_2 :\vdash \{\Delta_4, \mathtt{F}_3\} : \mathtt{F}_{\mathbf{1}}}\ W B$$

4.3 Status of ?: : `Invertible`

- Case rule **1**
- Case rule !

- Case rule ?

$$\frac{\mathtt{h}_2 :\vdash \{\mathtt{F}_4, \Delta_3\} : \Delta_5, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3\} : (\Delta_5, ?\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_4}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4\} : \Delta_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_5, ?\mathtt{F}_4}\ ?$$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_2\} : \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_4, ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_3\} : \Delta_4}\ \text{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_3\} : \Delta_4}\ \mathtt{H}$$

- Case rule $

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \mathtt{F}_5, \Delta_6, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3\} : (\Delta_6, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_4\$\mathtt{F}_5}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_4\$\mathtt{F}_5}\ \$$

- Case rule &

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \Delta_6, ?\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_6, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3\} : (\Delta_6, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_4\&\mathtt{F}_5}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_4\&\mathtt{F}_5}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_6, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3\} : (\Delta_6, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \Delta_6, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3\} : (\Delta_6, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_4}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_4, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3\} : \bot, \Delta_4, ?\mathtt{F}_{\mathbf{1}}}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_4}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_4, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3\} : \top, \Delta_4, ?\mathtt{F}_{\mathbf{1}}}\ \top \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_4, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \Delta_6, ?\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_7}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3\} : (\Delta_6, ?\mathtt{F}_{\mathbf{1}}), \Delta_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_7, \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_7, \mathtt{F}_5}\ W}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \Delta_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_7, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, (\Delta_7, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_4}\ \text{ax}}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \mathtt{F}_4}\ W \quad \dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_7, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_{\mathbf{1}}\} : \Delta_6, \Delta_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{\mathtt{h}_2 :\vdash \{\mathtt{F}_4, \Delta_5\} : \mathtt{F}_4, \Delta_3, ?\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash \{\mathtt{F}_4, \Delta_5\} : \Delta_3, ?\mathtt{F}_{\mathbf{1}}}\ ?_C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4\} : \Delta_3, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 :\vdash \{\Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4\} : \Delta_3}\ ?_C$$

4.4 Status of \$: : `Invertible`

- Case rule **1**
- Case rule !
- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_6, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, ?\mathtt{F}_5}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{H}$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \& \mathtt{F}_6}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \$ \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \Delta_5, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \top, \Delta_5, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_8}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2), \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_6}\ \text{ax}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_8, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, (\Delta_8, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5}\ \text{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_6}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_5, \Delta_4, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2}\ ?_C$$

4.5 Status of &: (Left Premise): `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_6, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \mathtt{F}_{\mathbf{1}}}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_5}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5\$\mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5, \mathtt{F}_6}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5\$\mathtt{F}_6}\ \$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5\&\mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5\&\mathtt{F}_6}\ \&$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3\&\mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \text{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \text{H}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \top, \Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_8}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_6}\ \text{ax}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_8, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, (\Delta_8, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5}\ \text{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_5, \Delta_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_{\mathbf{1}}}\ ?_C$$

4.6 Status of & (Right Premise): : `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_6, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_2, ?\mathtt{F}_5}\ ?$$

- Case rule $

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5\$\mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5\$\mathtt{F}_6}\ \$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5\&\mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5\&\mathtt{F}_6}\ \&$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3\&\mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \mathtt{H}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_2, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \top, \Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_2, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_8}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_6}\ ^{\mathtt{ax}}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_8, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, (\Delta_8, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5}\ ^{\mathtt{ax}} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_5, \Delta_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_5}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2}\ ?_C$$

4.7 Status of $\oplus_B$: : `Non invertible`

- Case rule $\mathbf{1}$

- Case rule !

- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \mathtt{F}_2}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_2, ?\mathtt{F}_5}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5\$\mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5\$\mathtt{F}_6}\ \$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5\&\mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5\&\mathtt{F}_6}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ ^{\mathtt{ax}}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ ^{\mathtt{H}}$$

- Case rule $\oplus_A$

$$\frac{\vdash h_3 : \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2}{\bullet \vdash h_3 : \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \oplus F_6}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\vdash h_3 : \{\Delta_4\} : \Delta_7, F_2, F_5}\ \text{ax/ind}}{\bullet \vdash h_3 : \{\Delta_4\} : \Delta_7, F_2, F_5 \oplus F_6}\ \oplus_A$$

$$\frac{\vdash h_1 : \{\Delta_2\} : F_3, \Delta_5}{\bullet \vdash h_1 : \{\Delta_2\} : \Delta_5, F_3 \oplus F_4}\ \oplus_A \quad \rightarrow \quad \frac{}{\bullet \vdash h_1 : \{\Delta_2\} : F_4, \Delta_5}\ \text{fail}$$

- Case rule $\bot$

$$\frac{\vdash h_3 : \{\Delta_4\} : \Delta_5, F_1 \oplus F_2}{\bullet \vdash h_3 : \{\Delta_4\} : \bot, \Delta_5, F_1 \oplus F_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\vdash h_3 : \{\Delta_4\} : \Delta_5, F_2}\ \text{ax/ind}}{\bullet \vdash h_3 : \{\Delta_4\} : \Delta_5, F_2, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet \vdash h_3 : \{\Delta_4\} : \top, \Delta_5, F_1 \oplus F_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \vdash h_3 : \{\Delta_4\} : \Delta_5, F_2, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\vdash h_3 : \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2 \quad \vdash h_3 : \{\Delta_4\} : F_6, \Delta_8}{\bullet \vdash h_3 : \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), \Delta_8, F_5 \otimes F_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\vdash h_3 : \{\Delta_4\} : \Delta_7, F_2, F_5}\ \text{ax/ind} \quad \dfrac{}{\vdash h_3 : \{\Delta_4\} : \Delta_8, F_6}\ \text{ax}}{\bullet \vdash h_3 : \{\Delta_4\} : \Delta_7, \Delta_8, F_2, F_5 \otimes F_6}\ \otimes$$

$$\frac{\vdash h_3 : \{\Delta_4\} : F_5, \Delta_7 \quad \vdash h_3 : \{\Delta_4\} : F_6, \Delta_8, F_1 \oplus F_2}{\bullet \vdash h_3 : \{\Delta_4\} : \Delta_7, (\Delta_8, F_1 \oplus F_2), F_5 \otimes F_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\vdash h_3 : \{\Delta_4\} : \Delta_7, F_5}\ \text{ax} \quad \dfrac{}{\vdash h_3 : \{\Delta_4\} : \Delta_8, F_2, F_6}\ \text{ax/ind}}{\bullet \vdash h_3 : \{\Delta_4\} : \Delta_7, \Delta_8, F_2, F_5 \otimes F_6}\ \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\vdash h_3 : \{F_5, \Delta_6\} : F_5, \Delta_4, F_1 \oplus F_2}{\bullet \vdash h_3 : \{F_5, \Delta_6\} : \Delta_4, F_1 \oplus F_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{}{\vdash h_3 : \{\Delta_6, F_5\} : \Delta_4, F_2, F_5}\ \text{ax/ind}}{\bullet \vdash h_3 : \{\Delta_6, F_5\} : \Delta_4, F_2}\ ?_C$$

4.8 Status of $\oplus_A$: : `Non invertible`

- Case rule $\mathbf{1}$

- Case rule !

- Case rule ?

$$\frac{\vdash h_3 : \{F_5, \Delta_4\} : \Delta_6, F_1 \oplus F_2}{\bullet \vdash h_3 : \{\Delta_4\} : (\Delta_6, F_1 \oplus F_2), ?F_5}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\vdash h_3 : \{\Delta_4, F_5\} : \Delta_6, F_1}\ \text{ax/ind}}{\bullet \vdash h_3 : \{\Delta_4\} : \Delta_6, F_1, ?F_5}\ ?$$

- Case rule $

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \& \mathtt{F}_6}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5}\ \text{fail}$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \text{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \top, \Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_8}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_6}\ \text{ax}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_8, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, (\Delta_8, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5}\ \text{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_5, \Delta_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_{\mathbf{1}}}\ ?_C$$

4.9 Status of $\bot$: : `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_2\} : \bot, \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : (\bot, \Delta_4), ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2, \mathtt{F}_3\} : \Delta_4}\ \text{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_4, ?\mathtt{F}_3}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : (\bot, \Delta_5), \mathtt{F}_3 \$ \mathtt{F}_4}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$$$

- Case rule &

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : (\bot, \Delta_5), \mathtt{F}_3 \& \mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : (\bot, \Delta_5), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \mathtt{F}_3, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : (\bot, \Delta_5), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \text{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \Delta_3}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3}\ \text{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3}\ \mathtt{H}$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \bot, \Delta_3}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_3, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : (\bot, \Delta_5), \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \mathtt{F}_4, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, (\bot, \Delta_6), \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \bot, \mathtt{F}_3, \Delta_2}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \bot, \Delta_2}\ ?_C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2}\ ?_C$$

4.10 Status of $\top$: : `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_2\} : \top, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : (\top, \Delta_4), ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule \$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : (\top, \Delta_5), \mathtt{F}_3 \$ \mathtt{F}_4}\ \$ \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule &

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : (\top, \Delta_5), \mathtt{F}_3 \& \mathtt{F}_4}\ \& \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : (\top, \Delta_5), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \mathtt{F}_3, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : (\top, \Delta_5), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \bot, \top, \Delta_3}\ \bot \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \Delta_3}\ \top \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : (\top, \Delta_5), \Delta_6, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \texttt{trivial}$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \top, \mathtt{F}_4, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_2\} : \Delta_5, (\top, \Delta_6), \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \top, \mathtt{F}_3, \Delta_2}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \top, \Delta_2}\ ?_C \quad \rightarrow \quad \texttt{trivial}$$

4.11 Status of I_3: : `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?

- Case rule \$

- Case rule &

- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I_3 \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{\mathtt{h}_2 :\vdash \{\mathtt{F}_3, \Delta_4\} : \mathtt{F}_3, p(\mathtt{n}_1), \hat{}(\mathtt{n}_1)}{\bullet \mathtt{h}_2 :\vdash \{\mathtt{F}_3, \Delta_4\} : p(\mathtt{n}_1), \hat{}(\mathtt{n}_1)}\ ?_C \quad \rightarrow \quad \mathtt{trivial}$$

4.12 Status of $\otimes$: (Left Premise): `Non invertible`

- Case rule $\mathbf{1}$
- Case rule !
- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_5}\ ?$$

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_{\mathbf{1}}, \Delta_6}\ \mathtt{fail}$$

- Case rule \$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_{\mathbf{1}}}\ \mathtt{H}$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1, \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \& \mathtt{F}_6}\ \&$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_1, \bot}\ \bot$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_1}\ \mathtt{H}$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \top, \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \mathtt{F}_1, \top}\ \top$$

$$\frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \top, \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_1, \Delta_5}\ \text{fail}$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_9, \Delta_{10}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), (\Delta_9, \Delta_{10}), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_1, \Delta_7, \Delta_9, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \text{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_9, \Delta_{10}, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_1, \Delta_7, \Delta_9, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \text{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_9, \Delta_{10}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), (\Delta_9, \Delta_{10}), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_1, \Delta_7, \Delta_9}\ \text{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_9, \Delta_{10}, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_1, \Delta_7, \Delta_9}\ \text{fail}$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_7, \Delta_8}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5, \Delta_7}\ \text{fail}$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_6, \Delta_7\} : \mathtt{F}_6, \Delta_4, \Delta_5, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\mathtt{F}_6, \Delta_7\} : \Delta_4, \Delta_5, \mathtt{F}_1 \otimes \mathtt{F}_2}\ ?_C \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_3 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_4, \mathtt{F}_1, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_4, \mathtt{F}_1}\ ?_C$$

4.13 Status of $\otimes$ (Right Premise): : `Non invertible`

- Case rule **1**
- Case rule !
- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \Delta_7, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_6, \Delta_7, \mathtt{F}_1 \otimes \mathtt{F}_2), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_2, \Delta_7}\ \text{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_4\} : \Delta_6, \Delta_7, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_6, \Delta_7, \mathtt{F}_1 \otimes \mathtt{F}_2), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_3 :\vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_7, \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_2, ?\mathtt{F}_5}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2}\ \mathtt{H}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2}\ \mathtt{H}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind} \quad \frac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \& \mathtt{F}_6}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2}\ \mathtt{H}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2}\ \mathtt{H}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_2}\ \mathtt{H}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_2, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \top, \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_2, \Delta_6}\ \text{fail}$$

$$\frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \top, \Delta_5, \Delta_6, \mathtt{F}_1 \otimes \mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_6, \mathtt{F}_2, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_9, \Delta_{10}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), (\Delta_9, \Delta_{10}), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_2, \Delta_8, \Delta_{10}}\ \text{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_9, \Delta_{10}, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_2, \Delta_8, \Delta_{10}}\ \text{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_9, \Delta_{10}}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8, \mathtt{F}_1 \otimes \mathtt{F}_2), (\Delta_9, \Delta_{10}), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_2, \Delta_8, \Delta_{10}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \text{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7, \Delta_8 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_9, \Delta_{10}, \mathtt{F}_1 \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, \mathtt{F}_1 \otimes \mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_2, \Delta_8, \Delta_{10}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \text{fail}$$

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3, \Delta_5, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_7, \Delta_8}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_4, \Delta_6, \Delta_8}\ \text{fail}$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{\mathtt{h}_3 :\vdash \{\mathtt{F}_6, \Delta_7\} : \mathtt{F}_6, \Delta_4, \Delta_5, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \{\mathtt{F}_6, \Delta_7\} : \Delta_4, \Delta_5, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}\ ?_C \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_5, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_5, \mathtt{F}_2}\ ?_C$$

4.14 Status of I_1: : `Invertible`

- Case rule $\mathbf{1}$
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1 \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule I_2

- Case rule $?_C$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3, p(\mathtt{n_1})\} : p(\mathtt{n_1}), \hat{}(\mathtt{n_1})}{\bullet\mathtt{h}_2 :\vdash \{p(\mathtt{n_1}), \Delta_3\} : \hat{}(\mathtt{n_1})}\ ?_C \quad \rightarrow \quad \texttt{trivial}$$

$$\frac{\mathtt{h}_2 :\vdash \{\mathtt{F}_3, \Delta_4, p(\mathtt{n_1})\} : \mathtt{F}_3, \hat{}(\mathtt{n_1})}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_3, \Delta_4, p(\mathtt{n_1})\} : \hat{}(\mathtt{n_1})}\ ?_C \quad \rightarrow \quad \texttt{trivial}$$

4.15 Status of I_2: : `Invertible`

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_3, \hat{}(\mathtt{n_2})\} : p(\mathtt{n_2})}\ I_2 \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $?_C$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3, \hat{}(\mathtt{n_1})\} : p(\mathtt{n_1}), \hat{}(\mathtt{n_1})}{\bullet\mathtt{h}_2 :\vdash \{\hat{}(\mathtt{n_1}), \Delta_3\} : p(\mathtt{n_1})}\ ?_C \quad \rightarrow \quad \texttt{trivial}$$

$$\frac{\mathtt{h}_2 :\vdash \{\mathtt{F}_3, \Delta_4, \hat{}(\mathtt{n_1})\} : \mathtt{F}_3, p(\mathtt{n_1})}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_3, \Delta_4, \hat{}(\mathtt{n_1})\} : p(\mathtt{n_1})}\ ?_C \quad \rightarrow \quad \texttt{trivial}$$

4.16 Status of $?_C$: : Non invertible

- Case rule **1**

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_2, \Delta_3\} : \mathbf{1}}\ \mathbf{1} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_2, \Delta_3\} : \mathbf{1}, \mathtt{F}_2}\ \mathtt{fail}$$

- Case rule !

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \mathtt{F}_2}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : !\mathtt{F}_2}\ ! \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \mathtt{F}_3, !\mathtt{F}_2}\ \mathtt{fail}$$

- Case rule ?

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_2, \mathtt{F}_4, \Delta_5\} : \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_4, \Delta_5\} : \Delta_3, ?\mathtt{F}_2}\ ? \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_2, \mathtt{F}_4\} : \Delta_3, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_4\} : \Delta_3, \mathtt{F}_4, ?\mathtt{F}_2}\ ?$$

- Case rule $

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_2, \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_2 \$ \mathtt{F}_3}\ \$ \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_5, \mathtt{F}_2 \$ \mathtt{F}_3}\ \$$

- Case rule &

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_2 \& \mathtt{F}_3}\ \& \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind} \quad \frac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_5, \mathtt{F}_2 \& \mathtt{F}_3}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_3, \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_B \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_5, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathtt{F}_2, \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_A \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_5, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \Delta_2}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \bot, \Delta_2}\ \bot \quad \rightarrow \quad \frac{\frac{}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \top, \Delta_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3, \top}\ \top$$

- Case rule I_3

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I_3 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \mathtt{F}_3, p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ \mathtt{fail}$$

- Case rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_6, \Delta_7\} : \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 :\vdash \{\mathtt{F}_6, \Delta_7\} : \mathtt{F}_3, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_6, \Delta_7\} : \Delta_4, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_4, \mathtt{F}_2}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_5, \mathtt{F}_3, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_4, \Delta_5, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes$$

- Case rule I_1

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_3, p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_3, p(\mathtt{n}_2)\} : p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I_3$$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{(\mathtt{F}_3, \Delta_4), p(\mathtt{n}_2)\} : \hat{}(\mathtt{n}_2)}\ I_1 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4, p(\mathtt{n}_2)\} : \mathtt{F}_3, \hat{}(\mathtt{n}_2)}\ \mathtt{fail}$$

- Case rule I_2

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_3, \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2)}\ I_2 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_3, \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I_3$$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \{(\mathtt{F}_3, \Delta_4), \hat{}(\mathtt{n}_2)\} : p(\mathtt{n}_2)}\ I_2 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4, \hat{}(\mathtt{n}_2)\} : \mathtt{F}_3, p(\mathtt{n}_2)}\ \mathtt{fail}$$

- Case rule $?_C$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \mathtt{F}_3, \Delta_2}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_4\} : \Delta_2}\ ?_C \quad \rightarrow \quad \frac{}{\mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_3\} : \Delta_2, \mathtt{F}_3}\ ?_C$$

$$\frac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \mathtt{F}_4, \Delta_5\} : \mathtt{F}_3, \Delta_2}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_3, \mathtt{F}_4, \Delta_5\} : \Delta_2}\ ?_C \quad \rightarrow \quad \frac{}{\mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_3, \mathtt{F}_4\} : \Delta_2, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_3, \mathtt{F}_4\} : \Delta_2, \mathtt{F}_4}\ ?_C$$

5 Weakening on bang: $\vdash \Gamma, !F$ implies Γ, F.

- Case(s) rule **1**
- Case(s) rule !

$$\frac{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} :!\mathtt{F}_3, *}\ ! \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}\ ax}{\bullet \mathtt{h}_1 :\vdash \{\Delta_2\} : \mathtt{F}_3}\ H$$

- Case(s) rule ?

$$\frac{\mathtt{h}_2 :\vdash \{\mathtt{F}_4, \Delta_3\} : \Delta_5, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} :!\mathtt{F}_{\mathbf{1}}, \Delta_5, ?\mathtt{F}_4}\ ? \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_4\} : \Delta_5, !\mathtt{F}_{\mathbf{1}}}\ ax}{\mathtt{h}_2 :\vdash \{\Delta_3, \mathtt{F}_4\} : \Delta_5, \mathtt{F}_{\mathbf{1}}}\ IH}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_5, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_4}\ ?$$

- Case(s) rule $

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \mathtt{F}_5, \Delta_6, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} :!\mathtt{F}_{\mathbf{1}}, \Delta_6, \mathtt{F}_4 \$ \mathtt{F}_5}\ \$ \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}}}\ ax}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4, \mathtt{F}_5}\ IH}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \$ \mathtt{F}_5}\ \$$

- Case(s) rule &

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \Delta_6, !\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_6, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} :!\mathtt{F}_{\mathbf{1}}, \Delta_6, \mathtt{F}_4 \& \mathtt{F}_5}\ \& \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}\ ax}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ IH \quad \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}}}\ ax}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ IH}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \& \mathtt{F}_5}\ \&$$

- Case(s) rule $\oplus_B$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_6, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} :!\mathtt{F}_{\mathbf{1}}, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}}}\ ax}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ IH}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B$$

- Case(s) rule $\oplus_A$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \Delta_6, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} :!\mathtt{F}_{\mathbf{1}}, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}\ ax}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ IH}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A$$

- Case(s) rule $\bot$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_4, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} :!\mathtt{F}_{\mathbf{1}}, \bot, \Delta_4}\ \bot \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_4, !\mathtt{F}_{\mathbf{1}}}\ ax}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_4, \mathtt{F}_{\mathbf{1}}}\ IH}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_4, \mathtt{F}_{\mathbf{1}}, \bot}\ \bot$$

- Case(s) rule $\top$

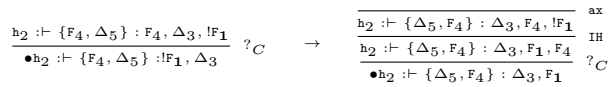
$$\frac{}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} :!\mathtt{F}_{\mathbf{1}}, \top, \Delta_4}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_4, \mathtt{F}_{\mathbf{1}}, \top}\ \top$$

- Case(s) rule I_3
- Case(s) rule $\otimes$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \Delta_6, !\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_7}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} :!\mathtt{F}_{\mathbf{1}}, \Delta_6, \Delta_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}\ ax}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ IH \quad \dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_7, \mathtt{F}_5}\ ax}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\frac{\mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_2 :\vdash \{\Delta_3\} : \mathtt{F}_5, \Delta_7, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} :!\mathtt{F}_{\mathbf{1}}, \Delta_6, \Delta_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \mathtt{F}_4}\ ax \quad \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_7, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}}}\ ax}{\mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ IH}{\bullet \mathtt{h}_2 :\vdash \{\Delta_3\} : \Delta_6, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

- Case(s) rule I_1
- Case(s) rule I_2
- Case(s) rule $?_C$



6 Identity-Expansion

$$\frac{}{- :\vdash \{*\} : \mathbf{0}, \top}\ \top$$

$$\frac{\dfrac{}{- :\vdash \{*\} : \mathbf{1}}\ \mathbf{1}}{- :\vdash \{*\} : \mathbf{1}, \bot}\ \bot$$

$$\frac{\dfrac{\dfrac{}{- :\vdash \{*\} : \mathtt{F_0}, \mathit{dual}(\mathtt{F_0})}\ \mathtt{IH}}{- :\vdash \{*\} : \mathit{dual}(\mathtt{F_0}), \mathtt{F_0} \oplus \mathtt{F_1}}\ \oplus_A \quad \dfrac{\dfrac{}{- :\vdash \{*\} : \mathtt{F_1}, \mathit{dual}(\mathtt{F_1})}\ \mathtt{IH}}{- :\vdash \{*\} : \mathit{dual}(\mathtt{F_1}), \mathtt{F_0} \oplus \mathtt{F_1}}\ \oplus_B}{- :\vdash \{*\} : \mathit{dual}(\mathtt{F_0}) \& \mathit{dual}(\mathtt{F_1}), \mathtt{F_0} \oplus \mathtt{F_1}}\ \&$$

$$\frac{\dfrac{\dfrac{}{- :\vdash \{*\} : \mathtt{F_0}, \mathit{dual}(\mathtt{F_0})}\ \mathtt{IH} \quad \dfrac{}{- :\vdash \{*\} : \mathtt{F_1}, \mathit{dual}(\mathtt{F_1})}\ \mathtt{IH}}{- :\vdash \{*\} : \mathit{dual}(\mathtt{F_0}), \mathit{dual}(\mathtt{F_1}), \mathtt{F_0} \otimes \mathtt{F_1}}\ \otimes}{- :\vdash \{*\} : \mathit{dual}(\mathtt{F_0}) \$ \mathit{dual}(\mathtt{F_1}), \mathtt{F_0} \otimes \mathtt{F_1}}\ \$$

$$\frac{\dfrac{\dfrac{\dfrac{}{- :\vdash \{*\} : \mathtt{F_0}, \mathit{dual}(\mathtt{F_0})}\ \mathtt{IH}}{- :\vdash \{*\} : \mathtt{F_0}, \mathit{dual}(\mathtt{F_0}) \oplus \mathit{dual}(\mathtt{F_1})}\ \oplus_A \quad \dfrac{\dfrac{}{- :\vdash \{*\} : \mathtt{F_1}, \mathit{dual}(\mathtt{F_1})}\ \mathtt{IH}}{- :\vdash \{*\} : \mathtt{F_1}, \mathit{dual}(\mathtt{F_0}) \oplus \mathit{dual}(\mathtt{F_1})}\ \oplus_B}{- :\vdash \{*\} : \mathtt{F_0} \& \mathtt{F_1}, \mathit{dual}(\mathtt{F_0}) \oplus \mathit{dual}(\mathtt{F_1})}\ \&$$

$$\frac{\dfrac{\dfrac{\dfrac{}{- :\vdash \{*\} : \mathtt{F_0}, \mathit{dual}(\mathtt{F_0})}\ \mathtt{IH} \quad \dfrac{}{- :\vdash \{*\} : \mathtt{F_1}, \mathit{dual}(\mathtt{F_1})}\ \mathtt{IH}}{- :\vdash \{*\} : \mathtt{F_0}, \mathtt{F_1}, \mathit{dual}(\mathtt{F_0}) \otimes \mathit{dual}(\mathtt{F_1})}\ \otimes}{- :\vdash \{*\} : \mathtt{F_0} \$ \mathtt{F_1}, \mathit{dual}(\mathtt{F_0}) \otimes \mathit{dual}(\mathtt{F_1})}\ \$$

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{- :\vdash \{*\} : \mathtt{F_0}, \mathit{dual}(\mathtt{F_0})}\ \mathtt{IH}}{- :\vdash \{\mathtt{F_0}\} : \mathtt{F_0}, \mathit{dual}(\mathtt{F_0})}\ W}{- :\vdash \{\mathtt{F_0}\} : \mathit{dual}(\mathtt{F_0})}\ ?_C}{- :\vdash \{\mathtt{F_0}\} :!\mathit{dual}(\mathtt{F_0})}\ !}{- :\vdash \{*\} :!\mathit{dual}(\mathtt{F_0}), ?\mathtt{F_0}}\ ?$$

$$\frac{\dfrac{}{- :\vdash \{*\} : \mathbf{1}}\ \mathbf{1}}{- :\vdash \{*\} : \mathbf{1}, \bot}\ \bot$$

$$\frac{}{- :\vdash \{*\} : \mathbf{0}, \top}\ \top$$

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{- :\vdash \{*\} : \mathtt{F_0}, \mathit{dual}(\mathtt{F_0})}\ \mathtt{IH}}{- :\vdash \{\mathit{dual}(\mathtt{F_0})\} : \mathtt{F_0}, \mathit{dual}(\mathtt{F_0})}\ W}{- :\vdash \{\mathit{dual}(\mathtt{F_0})\} : \mathtt{F_0}}\ ?_C}{- :\vdash \{\mathit{dual}(\mathtt{F_0})\} :!\mathtt{F_0}}\ !}{- :\vdash \{*\} :!\mathtt{F_0}, ?\mathit{dual}(\mathtt{F_0})}\ ?$$

7 Cut-Elimination

7.1 Status of 1: OK

- Case rule **1**

- Case rule !

- Case rule ?

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}, *}\,\mathbf{1} \quad \dfrac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_4\} : \bot, \Delta_6}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_6, ?\mathtt{F}_5}\,?}{- :\vdash \{\Delta_4\} : *, \Delta_6, ?\mathtt{F}_5}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4, \mathtt{F}_5\} : \mathbf{1}}\,\mathbf{1} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6, \bot}\,\mathtt{ax}}{- :\vdash \{\Delta_4, \mathtt{F}_5\} : \Delta_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_4\} : \Delta_6, ?\mathtt{F}_5}\,?$$

- Case rule $

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}, *}\,\mathbf{1} \quad \dfrac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \mathtt{F}_5, \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\,\$}{- :\vdash \{\Delta_4\} : *, \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \bot}\,\mathtt{ax}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\,\$$

- Case rule &

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}, *}\,\mathbf{1} \quad \dfrac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\,\&}{- :\vdash \{\Delta_4\} : *, \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5, \bot}\,\mathtt{ax}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_6, \bot}\,\mathtt{ax}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\,\&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}, *}\,\mathbf{1} \quad \dfrac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \mathtt{F}_6, \Delta_7}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_B}{- :\vdash \{\Delta_4\} : *, \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_6, \bot}\,\mathtt{ax}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}, *}\,\mathbf{1} \quad \dfrac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \mathtt{F}_5, \Delta_7}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_A}{- :\vdash \{\Delta_4\} : *, \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5, \bot}\,\mathtt{ax}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_5}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_5}\ \bot}{\dfrac{- :\vdash \{\Delta_4\} : *, \Delta_5}{- :\vdash \{\Delta_4\} : \Delta_5}\ \mathtt{ax}}\ \mathtt{Cut}$$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : dual(\mathbf{1}), \top, \Delta_5}\ \top}{\dfrac{- :\vdash \{\Delta_4\} : *, \top, \Delta_5}{- :\vdash \{\Delta_4\} : \Delta_5, \top}\ \top}\ \mathtt{Cut}$$

- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_6, \Delta_8}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes}{\dfrac{- :\vdash \{\Delta_4\} : *, \Delta_7, \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5, \bot}\ \mathtt{ax}}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_6}\ \mathtt{ax}}{- :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_3 :\vdash \{\Delta_4\} : \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_3 :\vdash \{\Delta_4\} : \bot, \mathtt{F}_6, \Delta_8}{\bullet \mathtt{h}_3 :\vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes}{\dfrac{- :\vdash \{\Delta_4\} : *, \Delta_7, \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}{\dfrac{\dfrac{}{- :\vdash \{\Delta_4\} : \Delta_7, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_4\} : \mathbf{1}}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_6, \bot}\ \mathtt{ax}}{- :\vdash \{\Delta_4\} : \Delta_8, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \{\Delta_4\} : \Delta_7, \Delta_8, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes}\ \mathtt{Cut}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_6\} : \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : \bot, \mathtt{F}_5, \Delta_4}{\bullet \mathtt{h}_3 :\vdash \{\mathtt{F}_5, \Delta_6\} : dual(\mathbf{1}), \Delta_4}\ ?_C}{\dfrac{- :\vdash \{\mathtt{F}_5, \Delta_6\} : *, \Delta_4}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_5\} : \mathbf{1}}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_5, \bot}\ \mathtt{ax}}{- :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4, \mathtt{F}_5}\ \mathtt{hCut}}{- :\vdash \{\Delta_6, \mathtt{F}_5\} : \Delta_4}\ ?_C}\ \mathtt{Cut}$$

7.2 Status of !: OK

- Case rule $\mathbf{1}$

- Case rule !

- Case rule ?

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\,! \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_6\} : \Delta_8, ?dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \Delta_8, ?\mathtt{F}_7}\,?}{- :\vdash \{\Delta_6\} : *, \Delta_8, ?\mathtt{F}_7}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\,\mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6, \mathtt{F}_7\} :!\mathtt{F}_4}\,W \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6, \mathtt{F}_7\} : \Delta_8, ?dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \{\Delta_6, \mathtt{F}_7\} : \Delta_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_6\} : \Delta_8, ?\mathtt{F}_7}\,?$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\,! \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6, dual(\mathtt{F}_4)\} : \Delta_7}{\bullet\mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \Delta_7}\,?}{- :\vdash \{\Delta_6\} : *, \Delta_7}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6, dual(\mathtt{F}_4)\} : \Delta_7}\,\mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_7}\,\mathtt{mCut}$$

- Case rule $

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\,! \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \mathtt{F}_8, \Delta_9, ?dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\,\$}{- :\vdash \{\Delta_6\} : *, \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, \mathtt{F}_8, ?dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\,\$$

- Case rule &

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\,! \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9, ?dual(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_9, ?dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\,\&}{- :\vdash \{\Delta_6\} : *, \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, ?dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\,\&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\,! \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_9, ?dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_B}{- :\vdash \{\Delta_6\} : *, \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A}{- :\vdash \{\Delta_6\} : *, \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A}$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_7, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \bot, \Delta_7}\ \bot}{- :\vdash \{\Delta_6\} : *, \bot, \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_7, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_7}\ \mathtt{hCut}}{- :\vdash \{\Delta_6\} : \Delta_7, \bot}\ \bot}$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\ !\quad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \top, \Delta_7}\ \top}{- :\vdash \{\Delta_6\} : *, \top, \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_7, \top}\ \top}$$

- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9, ?dual(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_{10}}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \{\Delta_6\} : *, \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_6\} : \Delta_{10}, \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_{10}, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4, *}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_{10}, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(!\mathtt{F}_4), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \{\Delta_6\} : *, \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :!\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_{10}, \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \{\Delta_6\} : \Delta_{10}, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_7, \Delta_8\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_7, \Delta_8\} :!\mathtt{F}_4, *}\,! \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_8\} : \mathtt{F}_7, \Delta_6, ?dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_8\} : dual(!\mathtt{F}_4), \Delta_6}\,?_C}{- :\vdash \{\mathtt{F}_7, \Delta_8\} : *, \Delta_6}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_8, \mathtt{F}_7\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_8, \mathtt{F}_7\} : \Delta_6, \mathtt{F}_7, ?dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \{\Delta_8, \mathtt{F}_7\} : \Delta_6, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \{\Delta_8, \mathtt{F}_7\} : \Delta_6}\,?_C$$

7.3 Status of ?: OK

- Case rule **1**

- Case rule !

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :?\mathtt{F}_5, \Delta_4}\,? \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(?\mathtt{F}_5), *}\,!}{- :\vdash \{\Delta_7\} : \Delta_4, *}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_5\} : \Delta_4}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5)}\,\mathtt{ax}}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} :!dual(\mathtt{F}_5)}\,!}{- :\vdash \{\Delta_7\} : \Delta_4}\,\mathtt{mCut}}$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :?\mathtt{F}_5, \Delta_4}\,? \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_7\} : \Delta_9, !dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(?\mathtt{F}_5), \Delta_9, ?\mathtt{F}_8}\,?}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5}\,\mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_8\} : \Delta_4, ?\mathtt{F}_5}\,W \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7, \mathtt{F}_8\} : \Delta_9, !dual(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \{\Delta_7, \mathtt{F}_8\} : \Delta_4, \Delta_9}\,\mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_9, ?\mathtt{F}_8}\,?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_8\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_8\} : \mathtt{F}_6, \Delta_5, ?\mathtt{F}_3}\,? \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_9, \Delta_8\} : \Delta_{10}, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \{\Delta_8\} : dual(\mathtt{F}_6), \Delta_{10}, ?\mathtt{F}_9}\,?}{- :\vdash \{\Delta_8\} : (\Delta_5, ?\mathtt{F}_3), \Delta_{10}, ?\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_8, \mathtt{F}_3\} : \Delta_5, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_7 :\vdash \{\Delta_8\} : \Delta_{10}, ?\mathtt{F}_9, dual(\mathtt{F}_6)}\,\mathtt{ax}}{\bullet\mathtt{h}_7 :\vdash \{\Delta_8, \mathtt{F}_3\} : \Delta_{10}, ?\mathtt{F}_9, dual(\mathtt{F}_6)}\,W}{- :\vdash \{\Delta_8, \mathtt{F}_3\} : \Delta_{10}, \Delta_5, ?\mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \{\Delta_8\} : \Delta_{10}, \Delta_5, ?\mathtt{F}_3, ?\mathtt{F}_9}\,?$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :?\mathtt{F}_5, \Delta_4}\,? \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}, !dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(?\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, !dual(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8\$\mathtt{F}_9}\,\$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_8\} : \mathtt{F}_6, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_8\} : \mathtt{F}_6, \Delta_5, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_7 :\vdash \{\Delta_8\} : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}, dual(\mathtt{F}_6)}{\bullet \mathtt{h}_7 :\vdash \{\Delta_8\} : dual(\mathtt{F}_6), \Delta_{11}, \mathtt{F}_9 \$ \mathtt{F}_{10}}\ \$}{- :\vdash \{\Delta_8\} : (\Delta_5, ?\mathtt{F}_3), \Delta_{11}, \mathtt{F}_9 \$ \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_8\} : \Delta_5, \mathtt{F}_6, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \{\Delta_8\} : \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, \mathtt{F}_{10}, \mathtt{F}_9, ?\mathtt{F}_3}\ \mathtt{hCut}$$

$$\dfrac{- :\vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, \mathtt{F}_{10}, \mathtt{F}_9, ?\mathtt{F}_3}{- :\vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?\mathtt{F}_3, \mathtt{F}_9 \$ \mathtt{F}_{10}}\ \$$$

- Case rule &

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : ?\mathtt{F}_5, \Delta_4}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10}, !dual(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{10}, !dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(?\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8 \& \mathtt{F}_9}\ \&}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \mathtt{F}_8 \& \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_8, !dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_9, !dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8 \& \mathtt{F}_9}\ \&$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_8\} : \mathtt{F}_6, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_8\} : \mathtt{F}_6, \Delta_5, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_7 :\vdash \{\Delta_8\} : \mathtt{F}_9, \Delta_{11}, dual(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \{\Delta_8\} : \mathtt{F}_{10}, \Delta_{11}, dual(\mathtt{F}_6)}{\bullet \mathtt{h}_7 :\vdash \{\Delta_8\} : dual(\mathtt{F}_6), \Delta_{11}, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \&}{- :\vdash \{\Delta_8\} : (\Delta_5, ?\mathtt{F}_3), \Delta_{11}, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_8, \mathtt{F}_3\} : \Delta_5, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_7 :\vdash \{\Delta_8\} : \Delta_{11}, dual(\mathtt{F}_6), \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{ax}}{\bullet \mathtt{h}_7 :\vdash \{\Delta_8, \mathtt{F}_3\} : \Delta_{11}, dual(\mathtt{F}_6), \mathtt{F}_9 \& \mathtt{F}_{10}}\ W}{- :\vdash \{\Delta_8, \mathtt{F}_3\} : \Delta_{11}, \Delta_5, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{hCut}$$

$$\dfrac{- :\vdash \{\Delta_8, \mathtt{F}_3\} : \Delta_{11}, \Delta_5, \mathtt{F}_9 \& \mathtt{F}_{10}}{- :\vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?\mathtt{F}_3, \mathtt{F}_9 \& \mathtt{F}_{10}}\ ?$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : ?\mathtt{F}_5, \Delta_4}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{10}, !dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(?\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_B}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_9, !dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_9}\ \mathtt{hCut}$$

$$\dfrac{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_9}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_B$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_8\} : \mathtt{F}_6, \Delta_5}{\bullet \mathtt{h}_1 :\vdash \{\Delta_8\} : \mathtt{F}_6, \Delta_5, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_7 :\vdash \{\Delta_8\} : \mathtt{F}_{10}, \Delta_{11}, dual(\mathtt{F}_6)}{\bullet \mathtt{h}_7 :\vdash \{\Delta_8\} : dual(\mathtt{F}_6), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\ \oplus_B}{- :\vdash \{\Delta_8\} : (\Delta_5, ?\mathtt{F}_3), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_8\} : \Delta_5, \mathtt{F}_6, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \{\Delta_8\} : \Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, \mathtt{F}_{10}, ?\mathtt{F}_3}\ \mathtt{hCut}$$

$$\dfrac{- :\vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, \mathtt{F}_{10}, ?\mathtt{F}_3}{- :\vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?\mathtt{F}_3, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : ?\mathtt{F}_5, \Delta_4}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10}, !dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(?\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_A}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_8, !dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8}\ \mathtt{hCut}$$

$$\dfrac{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_A$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_8\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_8\} : \mathtt{F}_6, \Delta_5, ?\mathtt{F}_3} \; ? \; \dfrac{\mathtt{h}_7 :\vdash \{\Delta_8\} : \mathtt{F}_9, \Delta_{11}, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \{\Delta_8\} : \mathit{dual}(\mathtt{F}_6), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}} \oplus_A}{- :\vdash \{\Delta_8\} : (\Delta_5, ?\mathtt{F}_3), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}} \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_8\} : \Delta_5, \mathtt{F}_6, ?\mathtt{F}_3} \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \{\Delta_8\} : \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)} \mathtt{ax}}{- :\vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, \mathtt{F}_9, ?\mathtt{F}_3} \mathtt{hCut}}{- :\vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?\mathtt{F}_3, \mathtt{F}_9 \oplus \mathtt{F}_{10}} \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :?\mathtt{F}_5, \Delta_4} \; ? \; \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_8, !\mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathit{dual}(?\mathtt{F}_5), \bot, \Delta_8} \bot}{- :\vdash \{\Delta_7\} : \Delta_4, \bot, \Delta_8} \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5} \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_8, !\mathit{dual}(\mathtt{F}_5)} \mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_8} \mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_8, \bot} \bot$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_8\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_8\} : \mathtt{F}_6, \Delta_5, ?\mathtt{F}_3} \; ? \; \dfrac{\mathtt{h}_7 :\vdash \{\Delta_8\} : \Delta_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \{\Delta_8\} : \mathit{dual}(\mathtt{F}_6), \bot, \Delta_9} \bot}{- :\vdash \{\Delta_8\} : (\Delta_5, ?\mathtt{F}_3), \bot, \Delta_9} \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_8\} : \Delta_5, \mathtt{F}_6, ?\mathtt{F}_3} \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \{\Delta_8\} : \Delta_9, \mathit{dual}(\mathtt{F}_6)} \mathtt{ax}}{- :\vdash \{\Delta_8\} : \Delta_5, \Delta_9, ?\mathtt{F}_3} \mathtt{hCut}}{- :\vdash \{\Delta_8\} : \Delta_5, \Delta_9, \bot, ?\mathtt{F}_3} \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :?\mathtt{F}_5, \Delta_4} \; ? \; \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathit{dual}(?\mathtt{F}_5), \top, \Delta_8} \top}{- :\vdash \{\Delta_7\} : \Delta_4, \top, \Delta_8} \mathtt{Cut}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_8, \top} \top$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_3, \Delta_8\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_8\} : \mathtt{F}_6, \Delta_5, ?\mathtt{F}_3} \; ? \; \dfrac{}{\bullet\mathtt{h}_7 :\vdash \{\Delta_8\} : \mathit{dual}(\mathtt{F}_6), \top, \Delta_9} \top}{- :\vdash \{\Delta_8\} : (\Delta_5, ?\mathtt{F}_3), \top, \Delta_9} \mathtt{Cut}$$

$$\dfrac{}{- :\vdash \{\Delta_8\} : \Delta_5, \Delta_9, \top, ?\mathtt{F}_3} \top$$

- Case rule I_3

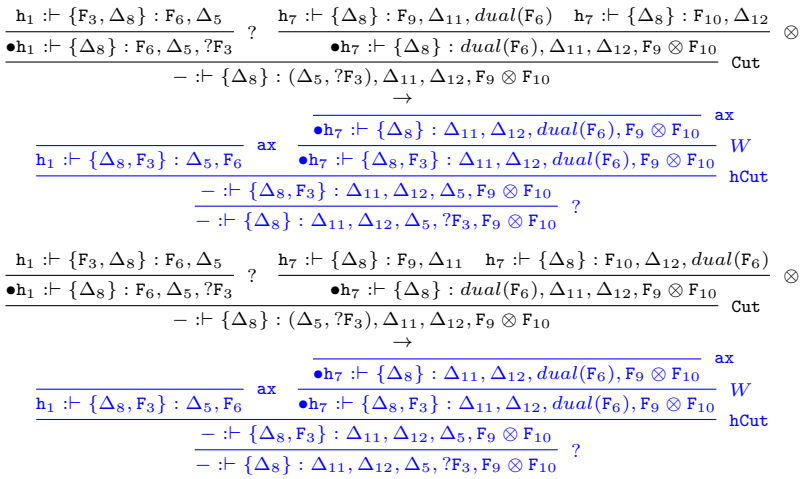
- Case rule $\otimes$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :?\mathtt{F}_5, \Delta_4} \; ? \; \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10}, !\mathit{dual}(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{11}}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathit{dual}(?\mathtt{F}_5), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9} \otimes}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9} \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5} \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_8, !\mathit{dual}(\mathtt{F}_5)} \mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8} \mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_7\} : \Delta_{11}, \mathtt{F}_9} \mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, \mathtt{F}_8 \otimes \mathtt{F}_9} \otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_5, \Delta_7\} : \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :?\mathtt{F}_5, \Delta_4} \; ? \; \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{11}, !\mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathit{dual}(?\mathtt{F}_5), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9} \otimes}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9} \mathtt{Cut}$$

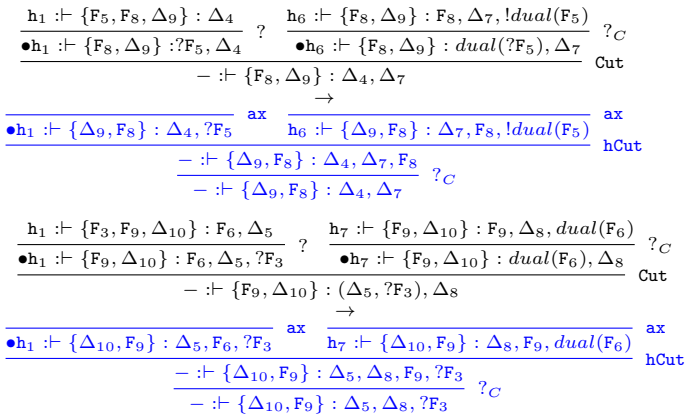
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_8} \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, ?\mathtt{F}_5} \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{11}, \mathtt{F}_9, !\mathit{dual}(\mathtt{F}_5)} \mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \mathtt{F}_9} \mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, \mathtt{F}_8 \otimes \mathtt{F}_9} \otimes$$



- Case rule I_1

- Case rule I_2

- Case rule $?_C$



7.4 Status of \$: OK

- Case rule **1**

- Case rule !

- Case rule ?

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\ \$ \qquad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_9\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\$\mathtt{F}_7), \Delta_{11}, ?\mathtt{F}_{10}}\ ?}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{11}, ?\mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\ W \qquad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, \Delta_5}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_5, ?\mathtt{F}_{10}}\ ?$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \qquad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_9\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{11}, ?\mathtt{F}_{10}}\ ?}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{11}, ?\mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}\ \mathtt{ax} \qquad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{11}, ?\mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathtt{F}_3, \mathtt{F}_4, ?\mathtt{F}_{10}}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?\mathtt{F}_{10}, \mathtt{F}_3\$\mathtt{F}_4}\ \$$$

- Case rule \$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\ \$ \qquad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\$\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\ \$}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{ax} \qquad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}, \mathtt{F}_{11}}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}\$\mathtt{F}_{11}}\ \$$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \qquad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\ \$}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\$\mathtt{F}_4}\ \mathtt{ax} \qquad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_3\$\mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}\$\mathtt{F}_{11}, \mathtt{F}_3\$\mathtt{F}_4}\ \$$$

- Case rule &

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\ \$ \qquad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\$\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\ \&}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{ax} \qquad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\ \mathtt{hCut} \qquad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{ax} \qquad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dua}}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{11}}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}\&\mathtt{F}_{11}}\ \&$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \qquad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\ \&}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}\ \mathtt{ax} \qquad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathit{dual}(\mathtt{F}_7), \mathtt{F}_{10}\&\mathtt{F}_{11}}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_{10}\&\mathtt{F}_{11}}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_{10}\&\mathtt{F}_{11}}\ \$$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\ \$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6\$\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \oplus_B}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{11}}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \oplus_B}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, dual(\mathtt{F}_7), \mathtt{F}_{10} \oplus \mathtt{F}_{11}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, dual(\mathtt{F}_7), \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\ \$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6\$\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \oplus_A}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \oplus_A$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \oplus_A}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, dual(\mathtt{F}_7), \mathtt{F}_{10} \oplus \mathtt{F}_{11}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, dual(\mathtt{F}_7), \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\ \$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\ \$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6\$\mathtt{F}_7), \bot, \Delta_{10}}\ \bot}{- :\vdash \{\Delta_9\} : \Delta_5, \bot, \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \bot}\ \bot$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \bot, \Delta_{10}}\ \bot}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \bot, \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \bot, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \bot, dual(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \bot}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \bot, \mathtt{F}_3\$\mathtt{F}_4}\ \$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\ \$ \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6\$\mathtt{F}_7), \top, \Delta_{10}}\ \top}{- :\vdash \{\Delta_9\} : \Delta_5, \top, \Delta_{10}}\ \mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \top}\ \top}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\,\$ \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \top, \Delta_{10}}\,\top}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \top, \Delta_{10}}\,\mathtt{Cut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, \mathtt{F}_3\$\mathtt{F}_4}\,\top \quad (\rightarrow)$$

- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\,\$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6\$\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}\,\otimes}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,(\rightarrow)$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\,\$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6\$\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}\,\otimes}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,(\rightarrow)$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}, dual(\mathtt{F}_6) \otimes dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{13}, \Delta_5, \mathtt{F}_{11}}\,\mathtt{hCut}\,\otimes}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\,\$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, dual(\mathtt{F}_6) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{11}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6\$\mathtt{F}_7), \Delta_{10}, \Delta_{11}}\,\mathtt{Cut}\,\otimes}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{10}, \Delta_{11}}\,(\rightarrow)$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{11}, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_5, \mathtt{F}_6}\,\mathtt{sCut} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_{11}, \Delta_5}\,\mathtt{sCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\,\$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, dual(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}\,\otimes}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,(\rightarrow)$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, dual(\mathtt{F}_7), \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\,\$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}\,\otimes}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,(\rightarrow)$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, dual(\mathtt{F}_7), \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_6\$\mathtt{F}_7, \Delta_5}\ \$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, \Delta_9, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathit{dual}(\mathtt{F}_6\$\mathtt{F}_7), \Delta_9}\ ?_C}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \Delta_5, \Delta_9}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6) \otimes \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \Delta_9, \mathtt{F}_{10}}\ \mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \Delta_9}\ ?_C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, \Delta_9, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathit{dual}(\mathtt{F}_7), \Delta_9}\ ?_C}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : (\Delta_6, \mathtt{F}_3\$\mathtt{F}_4), \Delta_9}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \Delta_9, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \Delta_9, \mathtt{F}_3\$\mathtt{F}_4}\ \$$$

7.5 Status of &: OK

- Case rule **1**

- Case rule !

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\ \& \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_9\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_{11}, ?\mathtt{F}_{10}}\ ?}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{11}, ?\mathtt{F}_{10}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\ W \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, \Delta_5}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_5, ?\mathtt{F}_{10}}\ ?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\ \& \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_9\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{11}, ?\mathtt{F}_{10}}\ ?}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{11}, ?\mathtt{F}_{10}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\ W \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?\mathtt{F}_{10}, \mathtt{F}_3\&\mathtt{F}_4}\ ?$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\ \& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\ \$}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}, \mathtt{F}_{11}}\ \mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}\$\mathtt{F}_{11}}\ \$$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\,\$}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}\$\mathtt{F}_{11}, \mathtt{F}_3\&\mathtt{F}_4}\,\$$$

- Case rule &

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\&}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{du}}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{11}}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\&$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\&}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{11}, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}\&\mathtt{F}_{11}, \mathtt{F}_3\&\mathtt{F}_4}\,\&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\oplus_B}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\oplus_B$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_{10}}\,\oplus_B}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5}\,\mathtt{sCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\oplus_B}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{11}, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\oplus_A}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\oplus_A$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_{10}}\,\oplus_A}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5}\,\mathtt{sCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\oplus_A}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\oplus_A}$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \bot, \Delta_{10}}\,\bot}{- :\vdash \{\Delta_9\} : \Delta_5, \bot, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \bot}\,\bot$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \bot, \Delta_{10}}\,\bot}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \bot, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \bot, \mathtt{F}_3\&\mathtt{F}_4}\,\bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\& \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \top, \Delta_{10}}\,\top}{- :\vdash \{\Delta_9\} : \Delta_5, \top, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \top}\,\top}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\& \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \top, \Delta_{10}}\,\top}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \top, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, \mathtt{F}_3\&\mathtt{F}_4}\,\top}$$

- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\& \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\&\quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{13}, \Delta_5, \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\&\quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\&\quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{13}, \Delta_6, \mathtt{F}_{11}, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\otimes$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_6\&\mathtt{F}_7, \Delta_5}\,\&\quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, \Delta_9, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathit{dual}(\mathtt{F}_6\&\mathtt{F}_7), \Delta_9}\,\mathtt{Cut}}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \Delta_5, \Delta_9}\,?_C}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \mathtt{F}_6\&\mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6) \oplus \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \Delta_9, \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \Delta_9}\,?_C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3\&\mathtt{F}_4}\,\&\quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, \Delta_9, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathit{dual}(\mathtt{F}_7), \Delta_9}\,?_C}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : (\Delta_6, \mathtt{F}_3\&\mathtt{F}_4), \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \Delta_9, \mathtt{F}_{10}, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \Delta_9, \mathtt{F}_3\&\mathtt{F}_4}\,?_C$$

7.6 Status of $\oplus_B$: OK

- Case rule **1**
- Case rule !
- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_9\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{11}, ?\mathtt{F}_{10}}?}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{11}, ?\mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}W \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, \Delta_5}\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_5, ?\mathtt{F}_{10}}?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_9\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{11}, ?\mathtt{F}_{10}}?}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{11}, ?\mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{11}, ?\mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathtt{F}_4, ?\mathtt{F}_{10}}\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?\mathtt{F}_{10}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B$$

- Case rule \$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\$}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}, \mathtt{F}_{11}}\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}\$\mathtt{F}_{11}}\$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\$}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3 \oplus \mathtt{F}_4}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}\$\mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\&}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6)\&\mathit{du}}\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{11}}\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}\&\mathtt{F}_{11}}\&$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{10}}\&}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5}\mathtt{sCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\&}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathit{dual}(\mathtt{F}_7), \mathtt{F}_{10}\&\mathtt{F}_{11}}\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_4, \mathtt{F}_{10}\&\mathtt{F}_{11}}\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}\&\mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_B}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_B}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_B}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10} \oplus \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B}$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_A}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_A}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_A}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, dual(\mathtt{F}_7), \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_4, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10} \oplus \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B}$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6 \oplus \mathtt{F}_7), \bot, \Delta_{10}}\bot}{- :\vdash \{\Delta_9\} : \Delta_5, \bot, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \bot}\bot}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \bot, \Delta_{10}}\bot}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \bot, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \bot, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathtt{F}_4, \bot}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \bot, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B}$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6 \oplus \mathtt{F}_7), \top, \Delta_{10}}\top}{- :\vdash \{\Delta_9\} : \Delta_5, \top, \Delta_{10}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \top}\top$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \top, \Delta_{10}}\top}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \top, \Delta_{10}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, \mathtt{F}_3 \oplus \mathtt{F}_4}\top$$

- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{13}, \Delta_5, \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, dual(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, dual(\mathtt{F}_7), \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_4, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_{10} \otimes \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, dual(\mathtt{F}_7), \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_4, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_{10} \otimes \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_7, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, \Delta_9, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : dual(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_9}?_C}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \Delta_5, \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_{10}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \Delta_9, \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \Delta_9}\,?_C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_4, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, \Delta_9, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : dual(\mathtt{F}_7), \Delta_9}?_C}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \mathtt{F}_4, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \Delta_9, \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \Delta_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B$$

7.7 Status of $\oplus_A$: OK

- Case rule **1**
- Case rule !
- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{11}, ?\mathtt{F}_{10}}?}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{11}, ?\mathtt{F}_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,W \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_{10}\} : \Delta_{11}, \Delta_5}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_5, ?\mathtt{F}_{10}}\,?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_9\} : \Delta_{11}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{11}, ?\mathtt{F}_{10}}?}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{11}, ?\mathtt{F}_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{11}, ?\mathtt{F}_{10}, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, \mathtt{F}_3, ?\mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?\mathtt{F}_{10}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\,\$}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, dual(\mathtt{F}_6)\&dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}, \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}\$\mathtt{F}_{11}}\,\$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : dual(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\,\$}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10}\$\mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}\$\mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\&}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6)\&\mathit{du}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\&$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{10}}\&}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5}\,\mathtt{sCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\&}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathit{dual}(\mathtt{F}_7), \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_3, \mathtt{F}_{10}\&\mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}\&\mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_B}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_B}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10} \oplus \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_A}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_A$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\oplus_A}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \mathtt{F}_{10} \oplus \mathtt{F}_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_7, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, \mathtt{F}_{10} \oplus \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \bot, \Delta_{10}}\bot}{- :\vdash \{\Delta_9\} : \Delta_5, \bot, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \bot}\,\bot}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \bot, \Delta_{10}}\bot}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \bot, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{10}, \bot, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \mathtt{F}_3, \bot}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \bot, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \top, \Delta_{10}}\top}{- :\vdash \{\Delta_9\} : \Delta_5, \top, \Delta_{10}}\,\mathtt{Cut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_5, \top}\,\top$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \top, \Delta_{10}}\top}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \top, \Delta_{10}}\,\mathtt{Cut}}{- :\vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\top$$

- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_5, \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash \{\Delta_9\} : \Delta_5, \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{13}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{13}, \Delta_5, \mathtt{F}_{11}}\,\mathtt{hCut}}{\dfrac{}{- :\vdash \{\Delta_9\} : \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax}}\otimes}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_5, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12}, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \mathit{dual}(\mathtt{F}_7), \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_3, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_{10} \otimes \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{10}, \Delta_{12} \quad \mathtt{h}_8 :\vdash \{\Delta_9\} : \mathtt{F}_{11}, \Delta_{13}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \mathit{dual}(\mathtt{F}_7), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash \{\Delta_9\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{12}, \Delta_{13}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \mathit{dual}(\mathtt{F}_7), \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_3, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_{12}, \Delta_{13}, \Delta_6, \mathtt{F}_{10} \otimes \mathtt{F}_{11}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_6 \oplus \mathtt{F}_7, \Delta_5}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, \Delta_9, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathit{dual}(\mathtt{F}_6 \oplus \mathtt{F}_7), \Delta_9}?_C}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \Delta_5, \Delta_9}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \mathtt{F}_6 \oplus \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)\&\mathit{dual}(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \Delta_9, \mathtt{F}_{10}}\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_5, \Delta_9}?_C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_3, \mathtt{F}_7, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_7, \Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, \Delta_9, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathit{dual}(\mathtt{F}_7), \Delta_9}?_C}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : (\Delta_6, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_9}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathit{dual}(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \Delta_9, \mathtt{F}_3}\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_6, \Delta_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A}$$

7.8 Status of $\bot$: OK

- Case rule **1**

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\bot \quad \dfrac{}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathit{dual}(\bot), *}\mathbf{1}}{- :\vdash \{\Delta_5\} : \Delta_3, *}\mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_5\} : \Delta_3}\mathtt{ax}}$$

- Case rule !

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\bot \quad \dfrac{\mathtt{h}_4 :\vdash \{\mathtt{F}_6, \Delta_5\} : \mathbf{1}, \Delta_7}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathit{dual}(\bot), \Delta_7, ?\mathtt{F}_6}?}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?\mathtt{F}_6}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot}\mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_6\} : \Delta_3, \bot}W \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5, \mathtt{F}_6\} : \mathbf{1}, \Delta_7}\mathtt{ax}}{- :\vdash \{\Delta_5, \mathtt{F}_6\} : \Delta_3, \Delta_7}\mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?\mathtt{F}_6}?}$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \qquad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_7\} : \Delta_9, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathit{dual}(\mathtt{F}_5), \Delta_9, ?\mathtt{F}_8}\,?}{- :\vdash \{\Delta_7\} : (\bot, \Delta_4), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, \mathtt{F}_5}\,\mathtt{ax} \qquad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_9, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_9, ?\mathtt{F}_8}\,\mathtt{hCut}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_9, \bot, ?\mathtt{F}_8}\,\bot$$

- Case rule $

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\,\bot \qquad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathit{dual}(\bot), \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\$}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{hCut}$$

$$\dfrac{}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\$$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \qquad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathit{dual}(\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash \{\Delta_7\} : (\bot, \Delta_4), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, \mathtt{F}_5, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8, \mathtt{F}_9, \bot}\,\mathtt{hCut}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \bot, \mathtt{F}_8\$\mathtt{F}_9}\,\$$$

- Case rule &

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\,\bot \qquad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathit{dual}(\bot), \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\,\mathtt{Cut}\,\&}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, \mathtt{F}_6}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6}\,\mathtt{hCut} \qquad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\,\&$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \qquad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10}, \mathit{dual}(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{10}, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathit{dual}(\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{Cut}\,\&}{- :\vdash \{\Delta_7\} : (\bot, \Delta_4), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, \mathtt{F}_5}\,\mathtt{ax} \qquad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_5), \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{hCut}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \bot, \mathtt{F}_8\&\mathtt{F}_9}\,\bot$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\,\bot \qquad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathit{dual}(\bot), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_B}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_7}\,\mathtt{hCut}$$

$$\dfrac{}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_B$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \qquad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{10}, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B}{- :\vdash \{\Delta_7\} : (\bot, \Delta_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, \mathtt{F}_5, \bot}\,\mathtt{ax} \qquad \overset{\rightarrow}{\dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_5)}\,\mathtt{ax}}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_9, \bot}\,\mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \bot, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\,\bot \qquad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \mathtt{F}_6, \Delta_8}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\bot), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_A}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot}\,\mathtt{ax} \qquad \overset{\rightarrow}{\dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, \mathtt{F}_6}\,\mathtt{ax}}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_A$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \qquad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10}, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}{- :\vdash \{\Delta_7\} : (\bot, \Delta_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, \mathtt{F}_5, \bot}\,\mathtt{ax} \qquad \overset{\rightarrow}{\dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5)}\,\mathtt{ax}}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \mathtt{F}_8, \bot}\,\mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \bot, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\,\bot \qquad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \Delta_6}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\bot), \bot, \Delta_6}\,\bot}{- :\vdash \{\Delta_5\} : \Delta_3, \bot, \Delta_6}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot}\,\mathtt{ax} \qquad \overset{\rightarrow}{\dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \Delta_6}\,\mathtt{ax}}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_6, \bot}\,\bot$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \qquad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_8, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \bot, \Delta_8}\,\bot}{- :\vdash \{\Delta_7\} : (\bot, \Delta_4), \bot, \Delta_8}\,\mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, \mathtt{F}_5}\,\mathtt{ax} \qquad \overset{\rightarrow}{\dfrac{}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_8, \bot, dual(\mathtt{F}_5)}\,\mathtt{ax}}}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_8, \bot}\,\mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_8, \bot, \bot}\,\bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\,\bot \qquad \dfrac{}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\bot), \top, \Delta_6}\,\top}{- :\vdash \{\Delta_5\} : \Delta_3, \top, \Delta_6}\,\mathtt{Cut}$$

$$\dfrac{}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_6, \top}\,\top$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \qquad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \top, \Delta_8}\,\top}{- :\vdash \{\Delta_7\} : (\bot, \Delta_4), \top, \Delta_8}\,\mathtt{Cut}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_8, \bot, \top}\,\top$$

- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\,\bot \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_4 :\vdash \{\Delta_5\} : \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\bot), \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot}{}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, \mathtt{F}_6}{}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_5\} : \Delta_9, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \bot, \Delta_3}\,\bot \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\bot), \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_5\} : \Delta_8, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{1}, \Delta_9, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_9, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10}, dual(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{11}}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\otimes}{- :\vdash \{\Delta_7\} : (\bot, \Delta_4), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, dual(\mathtt{F}_5), \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, \bot, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\bot$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{11}, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\otimes}{- :\vdash \{\Delta_7\} : (\bot, \Delta_4), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_7\} : \Delta_4, \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, dual(\mathtt{F}_5), \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, \bot, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\bot$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_6, \Delta_7\} : \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_6, \Delta_7\} : \bot, \Delta_3}\,\bot \quad \dfrac{\mathtt{h}_4 :\vdash \{\mathtt{F}_6, \Delta_7\} : \mathbf{1}, \mathtt{F}_6, \Delta_5}{\bullet\mathtt{h}_4 :\vdash \{\mathtt{F}_6, \Delta_7\} : dual(\bot), \Delta_5}\,?_C}{- :\vdash \{\mathtt{F}_6, \Delta_7\} : \Delta_3, \Delta_5}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_3, \bot}{}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_7, \mathtt{F}_6\} : \mathbf{1}, \Delta_5, \mathtt{F}_6}{}\,\mathtt{ax}}{- :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_3, \Delta_5, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_3, \Delta_5}\,?_C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_5, \Delta_4}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_5, \bot, \Delta_4}\,\bot \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_8, \Delta_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_9\} : dual(\mathtt{F}_5), \Delta_7}\,?_C}{- :\vdash \{\mathtt{F}_8, \Delta_9\} : (\bot, \Delta_4), \Delta_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_4, \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_7, dual(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_4, \Delta_7}\,\mathtt{hCut}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_4, \Delta_7, \bot}\,\bot$$

7.9 Status of $\top$: OK

- Case rule **1**

- Case rule !

- Case rule ?

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_4 :\vdash \{\mathtt{F}_6, \Delta_5\} : \mathbf{0}, \Delta_7}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\top), \Delta_7, ?\mathtt{F}_6}\ ?}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5, \mathtt{F}_6\} : \Delta_3, \top}\ \top \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5, \mathtt{F}_6\} : \mathbf{0}, \Delta_7}\ \mathtt{ax}}{- :\vdash \{\Delta_5, \mathtt{F}_6\} : \Delta_3, \Delta_7}\ \mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?\mathtt{F}_6}\ ?$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \top, \Delta_4}\ \top \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_7\} : \Delta_9, dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_9, ?\mathtt{F}_8}\ ?}{- :\vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_9, \top, ?\mathtt{F}_8}\ \top$$

- Case rule $

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\top), \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\ \$}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \top}\ \top \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\ \mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\ \$$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \top, \Delta_4}\ \top \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}, dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8 \$ \mathtt{F}_9}\ \$}{- :\vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, \mathtt{F}_8 \$ \mathtt{F}_9}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, \mathtt{F}_8 \$ \mathtt{F}_9}\ \top$$

- Case rule &

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\top), \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\ \&}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \top}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, \mathtt{F}_6}\ \mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \top}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, \mathtt{F}_7}\ \mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\ \&$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \top, \Delta_4}\ \top \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10}, dual(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{10}, dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8 \& \mathtt{F}_9}\ \&}{- :\vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, \mathtt{F}_8 \& \mathtt{F}_9}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, \mathtt{F}_8 \& \mathtt{F}_9}\ \top$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \mathtt{F}_7, \Delta_8}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\top), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \top}\ \top \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, \mathtt{F}_7}\ \mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \top, \Delta_4}\ \top \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{10}, dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_B}{- :\vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{Cut}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \top$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \mathtt{F}_6, \Delta_8}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\top), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \top}\ \top \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, \mathtt{F}_6}\ \mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \top, \Delta_4}\ \top \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10}, dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_A}{- :\vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{Cut}}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \top$$

- Case rule $\bot$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \Delta_6}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\top), \bot, \Delta_6}\ \bot}{- :\vdash \{\Delta_5\} : \Delta_3, \bot, \Delta_6}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \bot, \top}\ \top \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \Delta_6}\ \mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_6, \bot}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \top, \Delta_4}\ \top \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \Delta_8, dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \bot, \Delta_8}\ \bot}{- :\vdash \{\Delta_7\} : (\top, \Delta_4), \bot, \Delta_8}\ \mathtt{Cut}}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_8, \bot, \top}\ \top$$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \top, \Delta_3}\ \top \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\top), \top, \Delta_6}\ \top}{- :\vdash \{\Delta_5\} : \Delta_3, \top, \Delta_6}\ \mathtt{Cut}$$

$$\dfrac{}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_6, \top}\ \top$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \top, \Delta_4}\ \top \quad \dfrac{}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \top, \Delta_8}\ \top}{- :\vdash \{\Delta_7\} : (\top, \Delta_4), \top, \Delta_8}\ \mathtt{Cut}}{- :\vdash \{\Delta_7\} : \Delta_4, \Delta_8, \top, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \top, \Delta_3}\,\top \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_4 :\vdash \{\Delta_5\} : \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\top), \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \top}\,\top \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, \mathtt{F}_6}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \mathtt{F}_6}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_5\} : \Delta_9, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \top, \Delta_3}\,\top \quad \dfrac{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\top), \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_5\} : \Delta_8, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \Delta_3, \top}\,\top \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_5\} : \mathbf{0}, \Delta_9, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_9, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \{\Delta_5\} : \Delta_3, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \top, \Delta_4}\,\top \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10}, dual(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{11}}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\otimes}{- :\vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, \top, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\top$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_5, \top, \Delta_4}\,\top \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_6 :\vdash \{\Delta_7\} : \mathtt{F}_9, \Delta_{11}, dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\Delta_7\} : dual(\mathtt{F}_5), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\otimes}{- :\vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_{10}, \Delta_{11}, \Delta_4, \top, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\top$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_6, \Delta_7\} : \top, \Delta_3}\,\top \quad \dfrac{\mathtt{h}_4 :\vdash \{\mathtt{F}_6, \Delta_7\} : \mathbf{0}, \mathtt{F}_6, \Delta_5}{\bullet \mathtt{h}_4 :\vdash \{\mathtt{F}_6, \Delta_7\} : dual(\top), \Delta_5}\,?_C}{- :\vdash \{\mathtt{F}_6, \Delta_7\} : \Delta_3, \Delta_5}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_3, \top}\,\top \quad \dfrac{}{\mathtt{h}_4 :\vdash \{\Delta_7, \mathtt{F}_6\} : \mathbf{0}, \Delta_5, \mathtt{F}_6}\,\mathtt{ax}}{- :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_3, \Delta_5, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_7, \mathtt{F}_6\} : \Delta_3, \Delta_5}\,?_C$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_5, \top, \Delta_4}\,\top \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_8, \Delta_7, dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_9\} : dual(\mathtt{F}_5), \Delta_7}\,?_C}{- :\vdash \{\mathtt{F}_8, \Delta_9\} : (\top, \Delta_4), \Delta_7}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_4, \Delta_7, \top}\,\top$$

7.10 Status of I_3: OK

- Case rule **1**

- Case rule !

- Case rule ?

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_6\} : \Delta_8, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(\hat{}(\mathtt{n}_4)), \Delta_8, ?\mathtt{F}_7}\ ?}{- :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6, \mathtt{F}_7\} : \Delta_8, p(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_8, ?\mathtt{F}_7, p(\mathtt{n}_4)}\ ?$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_6\} : \Delta_8, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(p(\mathtt{n}_4)), \Delta_8, ?\mathtt{F}_7}\ ?}{- :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6, \mathtt{F}_7\} : \Delta_8, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_8, ?\mathtt{F}_7, \hat{}(\mathtt{n}_4)}\ ?$$

- Case rule $

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \mathtt{F}_8, \Delta_9, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(\hat{}(\mathtt{n}_4)), \Delta_9, \mathtt{F}_7 \$ \mathtt{F}_8}\ \$}{- :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \Delta_9, \mathtt{F}_7 \$ \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, \mathtt{F}_8, p(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, p(\mathtt{n}_4), \mathtt{F}_7 \$ \mathtt{F}_8}\ \$$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \mathtt{F}_8, \Delta_9, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(p(\mathtt{n}_4)), \Delta_9, \mathtt{F}_7 \$ \mathtt{F}_8}\ \$}{- :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), \Delta_9, \mathtt{F}_7 \$ \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, \mathtt{F}_8, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \hat{}(\mathtt{n}_4), \mathtt{F}_7 \$ \mathtt{F}_8}\ \$$$

- Case rule &

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9, p(\mathtt{n}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_9, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(\hat{}(\mathtt{n}_4)), \Delta_9, \mathtt{F}_7 \& \mathtt{F}_8}\ \&}{- :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \Delta_9, \mathtt{F}_7 \& \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, p(\mathtt{n}_4)}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_8, p(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, p(\mathtt{n}_4), \mathtt{F}_7 \& \mathtt{F}_8}\ \&$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9, \hat{}(\mathtt{n}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_9, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(p(\mathtt{n}_4)), \Delta_9, \mathtt{F}_7 \& \mathtt{F}_8}\ \&}{- :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), \Delta_9, \mathtt{F}_7 \& \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, \hat{}(\mathtt{n}_4)}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_8, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \hat{}(\mathtt{n}_4), \mathtt{F}_7 \& \mathtt{F}_8}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_9, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(\hat{}(\mathtt{n}_4)), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B}{- :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_8, p(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, p(\mathtt{n}_4), \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_9, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(p(\mathtt{n}_4)), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B}{- :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_8, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \hat{}(\mathtt{n}_4), \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(\hat{}(\mathtt{n}_4)), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A}{- :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, p(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, p(\mathtt{n}_4), \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(p(\mathtt{n}_4)), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A}{- :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_9, \hat{}(\mathtt{n}_4), \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_7, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(\hat{}(\mathtt{n}_4)), \bot, \Delta_7}\ \bot}{- :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \bot, \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_7, p(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_7, \bot, p(\mathtt{n}_4)}\ \bot$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \Delta_7, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(p(\mathtt{n}_4)), \bot, \Delta_7}\ \bot}{- :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), \bot, \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_7, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_7, \bot, \hat{}(\mathtt{n}_4)}\ \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\ I_3 \quad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(\hat{}(\mathtt{n}_4)), \top, \Delta_7}\ \top}{- :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \top, \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_7, \top, p(\mathtt{n}_4)}\ \top$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I_3 \quad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(p(\mathtt{n}_4)), \top, \Delta_7}\ \top}{- :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), \top, \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_7, \top, \hat{}(\mathtt{n}_4)}\ \top$$

- Case rule I_3

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : \hat{}(\mathtt{n}_6), p(\mathtt{n}_6)}\ I_3 \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(\hat{}(\mathtt{n}_6)), \hat{}(\mathtt{n}_6)}\ I_3}{- :\vdash \{\Delta_5\} : p(\mathtt{n}_6), \hat{}(\mathtt{n}_6)}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \{\Delta_5\} : p(\mathtt{n}_6), \hat{}(\mathtt{n}_6)}\ I_3$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_5\} : p(\mathtt{n}_6), \hat{}(\mathtt{n}_6)}\, I_3 \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash \{\Delta_5\} : dual(p(\mathtt{n}_6)), p(\mathtt{n}_6)}\, I_3}{- :\vdash \{\Delta_5\} : \hat{}(\mathtt{n}_6), p(\mathtt{n}_6)}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \{\Delta_5\} : p(\mathtt{n}_6), \hat{}(\mathtt{n}_6)}\, I_3$$

- Case rule $\otimes$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\, I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9, p(\mathtt{n}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_{10}}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(\hat{}(\mathtt{n}_4)), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes}{- :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, p(\mathtt{n}_4)}\, \mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_6\} : \Delta_{10}, \mathtt{F}_8}\, \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_{10}, \Delta_9, p(\mathtt{n}_4), \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\, I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_{10}, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(\hat{}(\mathtt{n}_4)), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes}{- :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7}\, \mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_6\} : \Delta_{10}, \mathtt{F}_8, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_{10}, \Delta_9, p(\mathtt{n}_4), \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\, I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9, \hat{}(\mathtt{n}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_{10}}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(p(\mathtt{n}_4)), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes}{- :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7, \hat{}(\mathtt{n}_4)}\, \mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_6\} : \Delta_{10}, \mathtt{F}_8}\, \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_{10}, \Delta_9, \hat{}(\mathtt{n}_4), \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\, I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_5 :\vdash \{\Delta_6\} : \mathtt{F}_8, \Delta_{10}, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_6\} : dual(p(\mathtt{n}_4)), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes}{- :\vdash \{\Delta_6\} : \hat{}(\mathtt{n}_4), \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6\} : \Delta_9, \mathtt{F}_7}\, \mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_6\} : \Delta_{10}, \mathtt{F}_8, \hat{}(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_6\} : \Delta_{10}, \Delta_9, \hat{}(\mathtt{n}_4), \mathtt{F}_7 \otimes \mathtt{F}_8}\, \otimes$$

- Case rule I_1

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, p(\mathtt{n}_5)\} : p(\mathtt{n}_5), \hat{}(\mathtt{n}_5)}\, I_3 \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash \{\Delta_6, p(\mathtt{n}_5)\} : dual(p(\mathtt{n}_5)), *}\, I_1}{- :\vdash \{\Delta_6, p(\mathtt{n}_5)\} : \hat{}(\mathtt{n}_5), *}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \{\Delta_6, p(\mathtt{n}_5)\} : \hat{}(\mathtt{n}_5)}\, I_1$$

- Case rule I_2

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \hat{}(\mathtt{n}_5)\} : \hat{}(\mathtt{n}_5), p(\mathtt{n}_5)}\, I_3 \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash \{\Delta_6, \hat{}(\mathtt{n}_5)\} : dual(\hat{}(\mathtt{n}_5)), *}\, I_2}{- :\vdash \{\Delta_6, \hat{}(\mathtt{n}_5)\} : p(\mathtt{n}_5), *}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \{\Delta_6, \hat{}(\mathtt{n}_5)\} : p(\mathtt{n}_5)}\, I_2$$

- Case rule $?_C$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_7, \Delta_8\} : \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\, I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_8\} : \mathtt{F}_7, \Delta_6, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_8\} : dual(\hat{}(\mathtt{n}_4)), \Delta_6}\, ?_C}{- :\vdash \{\mathtt{F}_7, \Delta_8\} : p(\mathtt{n}_4), \Delta_6}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \{\Delta_8, \mathtt{F}_7\} : \Delta_6, \mathtt{F}_7, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_8, \mathtt{F}_7\} : \Delta_6, p(\mathtt{n}_4)}\, ?_C$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_7, \Delta_8\} : p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I_3 \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_8\} : \mathtt{F}_7, \Delta_6, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_8\} : dual(p(\mathtt{n}_4)), \Delta_6}\ ?_C}{- :\vdash \{\mathtt{F}_7, \Delta_8\} : \hat{}(\mathtt{n}_4), \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_8, \mathtt{F}_7\} : \Delta_6, \mathtt{F}_7, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_8, \mathtt{F}_7\} : \Delta_6, \hat{}(\mathtt{n}_4)}\ ?_C$$

7.11 Status of $\otimes$: OK

- Case rule **1**
- Case rule !
- Case rule ?

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\ \otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\mathtt{F}_{11}, \Delta_{10}\} : \Delta_{12}, dual(\mathtt{F}_7)\$dual(\mathtt{F}_8)}{\bullet \mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_7 \otimes \mathtt{F}_8), \Delta_{12}, ?\mathtt{F}_{11}}\ ?}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{12}, ?\mathtt{F}_{11}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}, \mathtt{F}_{11}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\ W \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}, \mathtt{F}_{11}\} : \Delta_{12}, dual(\mathtt{F}_7)\$dual(\mathtt{F}_8)}\ \mathtt{ax}}{- :\vdash \{\Delta_{10}, \mathtt{F}_{11}\} : \Delta_{12}, \Delta_5, \Delta_6}\ \mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{12}, \Delta_5, \Delta_6, ?\mathtt{F}_{11}}\ ?$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \Delta_7}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\mathtt{F}_{11}, \Delta_{10}\} : \Delta_{12}, dual(\mathtt{F}_8)}{\bullet \mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_8), \Delta_{12}, ?\mathtt{F}_{11}}\ ?}{- :\vdash \{\Delta_{10}\} : (\Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{12}, ?\mathtt{F}_{11}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_6, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}, \mathtt{F}_{11}\} : \Delta_6, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\ W \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}, \mathtt{F}_{11}\} : \Delta_{12}, dual(\mathtt{F}_8)}\ \mathtt{ax}}{- :\vdash \{\Delta_{10}, \mathtt{F}_{11}\} : \Delta_{12}, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{12}, \Delta_6, \Delta_7, ?\mathtt{F}_{11}, \mathtt{F}_3 \otimes \mathtt{F}_4}\ ?$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \mathtt{F}_8, \Delta_7}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\mathtt{F}_{11}, \Delta_{10}\} : \Delta_{12}, dual(\mathtt{F}_8)}{\bullet \mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_8), \Delta_{12}, ?\mathtt{F}_{11}}\ ?}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{12}, ?\mathtt{F}_{11}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}, \mathtt{F}_{11}\} : \Delta_5, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\ W \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}, \mathtt{F}_{11}\} : \Delta_{12}, dual(\mathtt{F}_8)}\ \mathtt{ax}}{- :\vdash \{\Delta_{10}, \mathtt{F}_{11}\} : \Delta_{12}, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{12}, \Delta_5, \Delta_7, ?\mathtt{F}_{11}, \mathtt{F}_3 \otimes \mathtt{F}_4}\ ?$$

- Case rule \$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\ \otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{13}, dual(\mathtt{F}_7)\$dual(\mathtt{F}_8)}{\bullet \mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_7 \otimes \mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11}\$\mathtt{F}_{12}}\ \$}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{13}, \mathtt{F}_{11}\$\mathtt{F}_{12}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{12}, dual(\mathtt{F}_7)\$dual(\mathtt{F}_8)}\ \mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{11}, \mathtt{F}_{12}}\ \mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{11}\$\mathtt{F}_{12}}\ \$$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{11}, dual(\mathtt{F}_7), dual(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_7 \otimes \mathtt{F}_8), \Delta_{11}}\,\$}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_5, \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{\dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_6, \mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, dual(\mathtt{F}_7), dual(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_6, dual(\mathtt{F}_7)}\,\mathtt{sCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_6}\,\mathtt{sCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{13}, dual(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11}\$\mathtt{F}_{12}}\,\$}{- :\vdash \{\Delta_{10}\} : (\Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{13}, \mathtt{F}_{11}\$\mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_6, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{12}, dual(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_6, \Delta_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_6, \Delta_7, \mathtt{F}_{11}\$\mathtt{F}_{12}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\$$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \mathtt{F}_8, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{13}, dual(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11}\$\mathtt{F}_{12}}\,\$}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{13}, \mathtt{F}_{11}\$\mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{12}, dual(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_7, \mathtt{F}_{11}\$\mathtt{F}_{12}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\$$$

- Case rule &

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \Delta_{13}, dual(\mathtt{F}_7)\$dual(\mathtt{F}_8) \quad \mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{12}, \Delta_{13}, dual(\mathtt{F}_7)\$dual($}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_7 \otimes \mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11}\&\mathtt{F}_{12}}\,\mathtt{Cut}}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{13}, \mathtt{F}_{11}\&\mathtt{F}_{12}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}, dual(\mathtt{F}_7)\$dual(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{12}, d}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{12}}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{11}\&\mathtt{F}_{12}}\,\&$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \Delta_{13}, dual(\mathtt{F}_8) \quad \mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{12}, \Delta_{13}, dual(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11}\&\mathtt{F}_{12}}\,\mathtt{Cut}\,\&}{- :\vdash \{\Delta_{10}\} : (\Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{13}, \mathtt{F}_{11}\&\mathtt{F}_{12}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_6, \mathtt{F}_3, \mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, dual(\mathtt{F}_8), \mathtt{F}_{11}\&\mathtt{F}_{12}}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_6, \mathtt{F}_3, \mathtt{F}_{11}\&\mathtt{F}_{12}}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_7, \mathtt{F}_4}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_6, \Delta_7, \mathtt{F}_{11}\&\mathtt{F}_{12}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \mathtt{F}_8, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \Delta_{13}, dual(\mathtt{F}_8) \quad \mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{12}, \Delta_{13}, dual(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11}\&\mathtt{F}_{12}}\,\mathtt{Cut}\,\&}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{13}, \mathtt{F}_{11}\&\mathtt{F}_{12}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_5, \mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_7, \mathtt{F}_4, \mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, dual(\mathtt{F}_8), \mathtt{F}_{11}\&\mathtt{F}_{12}}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_7, \mathtt{F}_4, \mathtt{F}_{11}\&\mathtt{F}_{12}}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_7, \mathtt{F}_{11}\&\mathtt{F}_{12}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\otimes$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{12}, \Delta_{13}, dual(\mathtt{F}_7)\$dual(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : dual(\mathtt{F}_7 \otimes \mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\,\oplus_B}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{12}, dual(\mathtt{F}_7)\$dual(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{12}}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\,\oplus_B$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{12}, \Delta_{13}, \mathit{dual}(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_B}{- :\vdash \{\Delta_{10}\} : (\Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_6, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{12}, \mathit{dual}(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_6, \Delta_7, \mathtt{F}_{12}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_B$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \mathtt{F}_8, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{12}, \Delta_{13}, \mathit{dual}(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_B}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{12}, \mathit{dual}(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_7, \mathtt{F}_{12}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \Delta_{13}, \mathit{dual}(\mathtt{F}_7)\$\mathit{dual}(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_7 \otimes \mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_A}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)\$\mathit{dual}(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{11}}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_A$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \Delta_{13}, \mathit{dual}(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_A}{- :\vdash \{\Delta_{10}\} : (\Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_6, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_6, \Delta_7, \mathtt{F}_{11}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_A$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \mathtt{F}_8, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \Delta_{13}, \mathit{dual}(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_8), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_A}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{13}, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_7, \mathtt{F}_{11}, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4, \mathtt{F}_{11} \oplus \mathtt{F}_{12}}\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_7)\$\mathit{dual}(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_7 \otimes \mathtt{F}_8), \bot, \Delta_{11}}\bot}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \bot, \Delta_{11}}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_7)\$\mathit{dual}(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_6, \bot}\bot$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_8), \bot, \Delta_{11}}\bot}{- :\vdash \{\Delta_{10}\} : (\Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \bot, \Delta_{11}}\,\mathtt{Cut}$$

$$\xrightarrow{\qquad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_6, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_8)}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_6, \Delta_7, \bot, \mathtt{F}_3 \otimes \mathtt{F}_4}\bot$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \mathtt{F}_8, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_8), \bot, \Delta_{11}}\bot}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \bot, \Delta_{11}}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{11}, \mathit{dual}(\mathtt{F}_8)}\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_7, \bot, \mathtt{F}_3 \otimes \mathtt{F}_4}\bot}{}\to$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\otimes \quad \dfrac{}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_7 \otimes \mathtt{F}_8), \top, \Delta_{11}}\top}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \top, \Delta_{11}}\mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_6, \top}\top}{}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_8), \top, \Delta_{11}}\top}{- :\vdash \{\Delta_{10}\} : (\Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \top, \Delta_{11}}\mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_6, \Delta_7, \top, \mathtt{F}_3 \otimes \mathtt{F}_4}\top}{}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \mathtt{F}_8, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_8), \top, \Delta_{11}}\top}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \top, \Delta_{11}}\mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_5, \Delta_7, \top, \mathtt{F}_3 \otimes \mathtt{F}_4}\top}{}\to$$

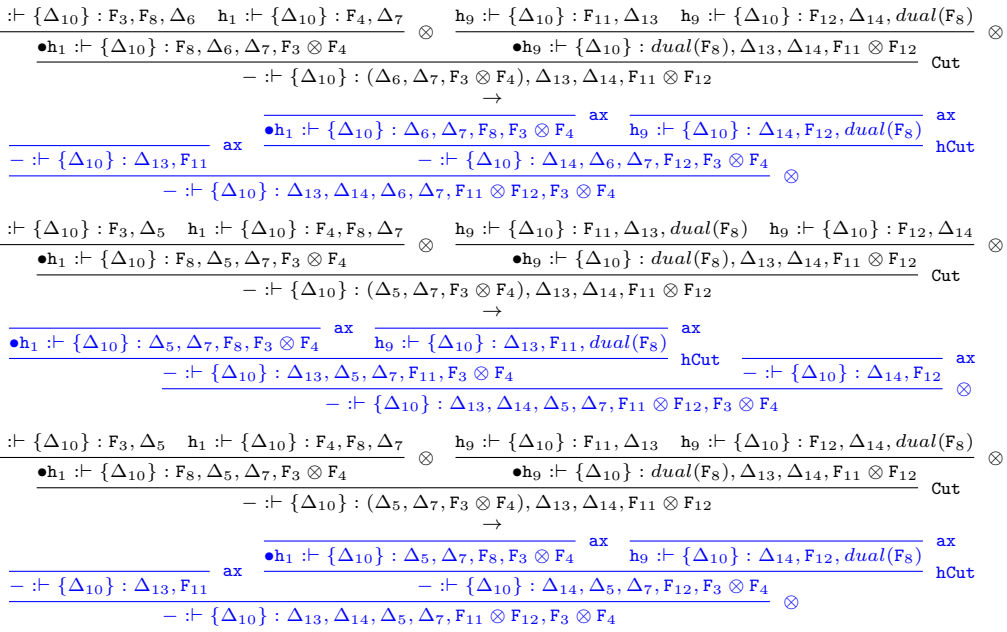
- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \Delta_{13}, \mathit{dual}(\mathtt{F}_7)\$\mathit{dual}(\mathtt{F}_8) \quad \mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{12}, \Delta_{14}}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_7 \otimes \mathtt{F}_8), \Delta_{13}, \Delta_{14}, \mathtt{F}_{11} \otimes \mathtt{F}_{12}}\otimes}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{13}, \Delta_{14}, \mathtt{F}_{11} \otimes \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)\$\mathit{dual}(\mathtt{F}_8)}\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_5, \Delta_6, \mathtt{F}_{11}}\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_{14}, \mathtt{F}_{12}}\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_{14}, \Delta_5, \Delta_6, \mathtt{F}_{11} \otimes \mathtt{F}_{12}}\otimes}{}\to$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7, \Delta_5 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_7 \otimes \mathtt{F}_8, \Delta_5, \Delta_6}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \Delta_{13} \quad \mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{12}, \Delta_{14}, \mathit{dual}(\mathtt{F}_7)\$\mathit{dual}(\mathtt{F}_8)}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_7 \otimes \mathtt{F}_8), \Delta_{13}, \Delta_{14}, \mathtt{F}_{11} \otimes \mathtt{F}_{12}}\otimes}{- :\vdash \{\Delta_{10}\} : (\Delta_5, \Delta_6), \Delta_{13}, \Delta_{14}, \mathtt{F}_{11} \otimes \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}}\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_5, \Delta_6, \mathtt{F}_7 \otimes \mathtt{F}_8}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{14}, \mathtt{F}_{12}, \mathit{dual}(\mathtt{F}_7)\$\mathit{dual}(\mathtt{F}_8)}\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_5, \Delta_6, \mathtt{F}_{12}}\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_{14}, \Delta_5, \Delta_6, \mathtt{F}_{11} \otimes \mathtt{F}_{12}}\otimes}{}\to$$

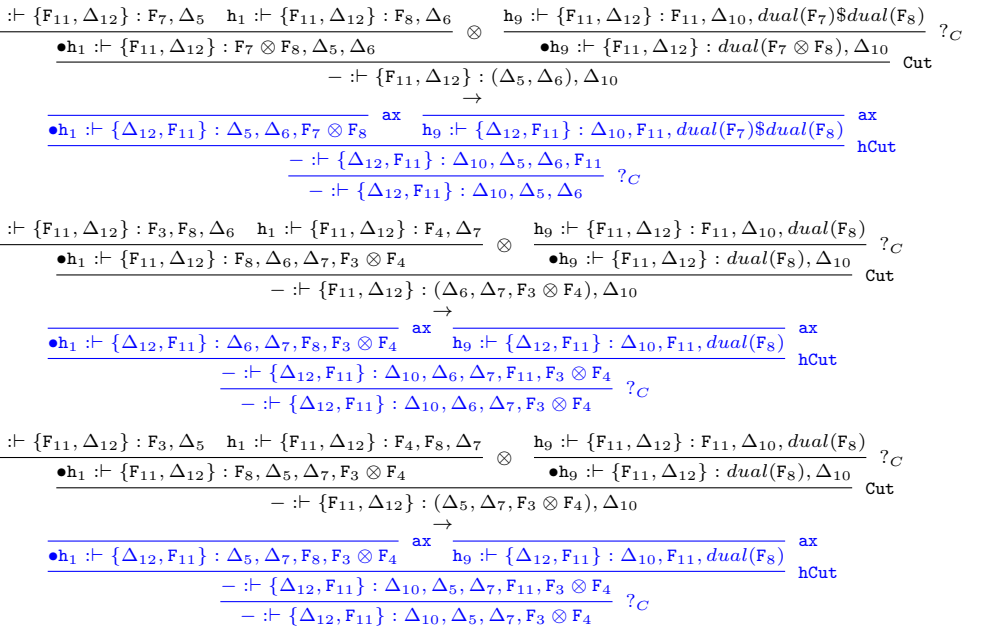
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_3, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4, \Delta_7}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_8, \Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes \quad \dfrac{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{11}, \Delta_{13}, \mathit{dual}(\mathtt{F}_8) \quad \mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathtt{F}_{12}, \Delta_{14}}{\bullet\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \mathit{dual}(\mathtt{F}_8), \Delta_{13}, \Delta_{14}, \mathtt{F}_{11} \otimes \mathtt{F}_{12}}\otimes}{- :\vdash \{\Delta_{10}\} : (\Delta_6, \Delta_7, \mathtt{F}_3 \otimes \mathtt{F}_4), \Delta_{13}, \Delta_{14}, \mathtt{F}_{11} \otimes \mathtt{F}_{12}}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \Delta_6, \Delta_7, \mathtt{F}_8, \mathtt{F}_3 \otimes \mathtt{F}_4}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_9 :\vdash \{\Delta_{10}\} : \Delta_{13}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_8)}\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_6, \Delta_7, \mathtt{F}_{11}, \mathtt{F}_3 \otimes \mathtt{F}_4}\mathtt{hCut} \quad \dfrac{}{- :\vdash \{\Delta_{10}\} : \Delta_{14}, \mathtt{F}_{12}}\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_{14}, \Delta_6, \Delta_7, \mathtt{F}_{11} \otimes \mathtt{F}_{12}, \mathtt{F}_3 \otimes \mathtt{F}_4}\otimes}{}\to$$



- Case rule I_1

- Case rule I_2

- Case rule $?_C$



7.12 Status of I_1: OK

- Case rule **1**

- Case rule !

- Case rule ?

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_8, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_6, \Delta_8, p(\mathtt{n}_4)\} : \Delta_7, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_8, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \Delta_7, ?\mathtt{F}_6}\, ?}{- :\vdash \{\Delta_8, p(\mathtt{n}_4)\} : *, \Delta_7, ?\mathtt{F}_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_8, \mathtt{F}_6, p(\mathtt{n}_4)\} : \Delta_7, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_8, \mathtt{F}_6, p(\mathtt{n}_4)\} : \Delta_7}\, ?_C}{- :\vdash \{\Delta_8, p(\mathtt{n}_4)\} : \Delta_7, ?\mathtt{F}_6}\, ?$$

- Case rule $

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_8, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\, \$}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : *, \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, \mathtt{F}_7, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\, ?_C}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\, \$$$

- Case rule &

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \mathtt{F}_6, \Delta_8, p(\mathtt{n}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_8, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\, \&}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : *, \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, p(\mathtt{n}_4)}\, \mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_7, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, p(\mathtt{n}_4), \mathtt{F}_6 \& \mathtt{F}_7}\, \&}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\, ?_C$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_8, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\, \oplus_B}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : *, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_7, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_7}\, ?_C}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\, \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \mathtt{F}_6, \Delta_8, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\, \oplus_A}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : *, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6}\, ?_C}{- :\vdash \{\Delta_9, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\, \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \Delta_6, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \bot, \Delta_6}\, \bot}{\dfrac{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : *, \bot, \Delta_6}{\dfrac{\dfrac{}{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \Delta_6, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \Delta_6}\, ?_C}{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \Delta_6, \bot}\, \bot}\, \mathtt{Cut}$$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \top, \Delta_6}\, \top}{\dfrac{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : *, \top, \Delta_6}{\dfrac{}{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \Delta_6, \top}\, \top}}\, \mathtt{Cut}$$

- Case rule I_3

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, p(\mathtt{n}_5)\} : \hat{}(\mathtt{n}_5), *}\, I_1 \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash \{\Delta_6, p(\mathtt{n}_5)\} : dual(\hat{}(\mathtt{n}_5)), \hat{}(\mathtt{n}_5)}\, I_3}{\dfrac{- :\vdash \{\Delta_6, p(\mathtt{n}_5)\} : *, \hat{}(\mathtt{n}_5)}{\dfrac{}{- :\vdash \{\Delta_6, p(\mathtt{n}_5)\} : \hat{}(\mathtt{n}_5)}\, I_1}}\, \mathtt{Cut}$$

- Case rule $\otimes$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \mathtt{F}_6, \Delta_8, p(\mathtt{n}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_5 :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\, \otimes}{\dfrac{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : *, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}{\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6}\, ?_C \quad \dfrac{}{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \Delta_9, \mathtt{F}_7}\, \mathtt{ax}}{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\, \otimes}}\, \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_9, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\, \otimes}{\dfrac{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : *, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}{\dfrac{\dfrac{}{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{\dfrac{}{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \Delta_9, \mathtt{F}_7, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \Delta_9, \mathtt{F}_7}\, ?_C}{- :\vdash \{\Delta_{10}, p(\mathtt{n}_4)\} : \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\, \otimes}}\, \mathtt{Cut}$$

- Case rule I_1

- Case rule I_2

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{(\Delta_6, \hat{}(\mathtt{n}_5)), p(\mathtt{n}_5)\} : \hat{}(\mathtt{n}_5), *}\, I_1 \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash \{(\Delta_6, \hat{}(\mathtt{n}_5)), p(\mathtt{n}_5)\} : dual(\hat{}(\mathtt{n}_5)), *}\, I_2}{\dfrac{- :\vdash \{(\Delta_6, \hat{}(\mathtt{n}_5)), p(\mathtt{n}_5)\} : *, *}{\dfrac{\dfrac{}{- :\vdash \{\Delta_6, p(\mathtt{n}_5), \hat{}(\mathtt{n}_5)\} : p(\mathtt{n}_5)}\, I_2}{- :\vdash \{\Delta_6, p(\mathtt{n}_5), \hat{}(\mathtt{n}_5)\} : *}\, ?_C}}\, \mathtt{Cut}$$

- Case rule $?_C$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \Delta_6, p(\mathtt{n}_4), p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \Delta_6}\, ?_C}{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : *, \Delta_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \Delta_6, p(\mathtt{n}_4), p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \Delta_6, p(\mathtt{n}_4)}\, ?_C}{- :\vdash \{\Delta_7, p(\mathtt{n}_4)\} : \Delta_6}\, ?_C$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{(\mathtt{F}_7, \Delta_8), p(\mathtt{n}_4)\} : \hat{}(\mathtt{n}_4), *}\, I_1 \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_8, p(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_6, p(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{(\mathtt{F}_7, \Delta_8), p(\mathtt{n}_4)\} : dual(\hat{}(\mathtt{n}_4)), \Delta_6}\, ?_C}{- :\vdash \{(\mathtt{F}_7, \Delta_8), p(\mathtt{n}_4)\} : *, \Delta_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_8, \mathtt{F}_7, p(\mathtt{n}_4)\} : \Delta_6, \mathtt{F}_7, p(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_8, \mathtt{F}_7, p(\mathtt{n}_4)\} : \Delta_6, \mathtt{F}_7}\, ?_C}{- :\vdash \{\Delta_8, \mathtt{F}_7, p(\mathtt{n}_4)\} : \Delta_6}\, ?_C$$

7.13 Status of I_2: OK

- Case rule **1**
- Case rule !
- Case rule ?

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_8, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\, I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_6, \Delta_8, \hat{}(\mathtt{n}_4)\} : \Delta_7, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_8, \hat{}(\mathtt{n}_4)\} : dual(p(\mathtt{n}_4)), \Delta_7, ?\mathtt{F}_6}\, ?}{- :\vdash \{\Delta_8, \hat{}(\mathtt{n}_4)\} : *, \Delta_7, ?\mathtt{F}_6}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_8, \mathtt{F}_6, \hat{}(\mathtt{n}_4)\} : \Delta_7, \hat{}(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_8, \mathtt{F}_6, \hat{}(\mathtt{n}_4)\} : \Delta_7}\, ?_C}{- :\vdash \{\Delta_8, \hat{}(\mathtt{n}_4)\} : \Delta_7, ?\mathtt{F}_6}\, ?$$

- Case rule $

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\, I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_8, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : dual(p(\mathtt{n}_4)), \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\, \$}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : *, \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, \mathtt{F}_7, \hat{}(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\, ?_C}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\, \$$$

- Case rule &

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\, I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_6, \Delta_8, \hat{}(\mathtt{n}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_8, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : dual(p(\mathtt{n}_4)), \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\, \&}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : *, \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\, \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, \hat{}(\mathtt{n}_4)}\, \mathtt{ax} \quad \dfrac{}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_7, \hat{}(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \hat{}(\mathtt{n}_4), \mathtt{F}_6 \& \mathtt{F}_7}\, \&}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\, ?_C$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\ I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_8, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : dual(p(\mathtt{n}_4)), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : *, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_7, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_7}\ ?_C}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\ I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_6, \Delta_8, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : dual(p(\mathtt{n}_4)), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : *, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6}\ ?_C}{- :\vdash \{\Delta_9, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\ I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \Delta_6, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : dual(p(\mathtt{n}_4)), \bot, \Delta_6}\ \bot}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : *, \bot, \Delta_6}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \Delta_6, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \Delta_6}\ ?_C}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \Delta_6, \bot}\ \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\ I_2 \quad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : dual(p(\mathtt{n}_4)), \top, \Delta_6}\ \top}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : *, \top, \Delta_6}\ \mathtt{Cut}$$

$$\dfrac{}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \Delta_6, \top}\ \top$$

- Case rule I_3

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6, \hat{}(\mathtt{n}_5)\} : p(\mathtt{n}_5), *}\ I_2 \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash \{\Delta_6, \hat{}(\mathtt{n}_5)\} : dual(p(\mathtt{n}_5)), p(\mathtt{n}_5)}\ I_3}{- :\vdash \{\Delta_6, \hat{}(\mathtt{n}_5)\} : *, p(\mathtt{n}_5)}\ \mathtt{Cut}$$

$$\dfrac{}{- :\vdash \{\Delta_6, \hat{}(\mathtt{n}_5)\} : p(\mathtt{n}_5)}\ I_2$$

- Case rule $\otimes$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\ I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_6, \Delta_8, \hat{}(\mathtt{n}_4) \quad \mathtt{h}_5 :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_5 :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : dual(p(\mathtt{n}_4)), \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : *, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6}\ ?_C \quad \dfrac{}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \Delta_9, \mathtt{F}_7}\ \mathtt{ax}}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\ I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_9, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : dual(p(\mathtt{n}_4)), \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : *, \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \Delta_8, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{\dfrac{}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \Delta_9, \mathtt{F}_7, \hat{}(\mathtt{n}_4)}\ \mathtt{ax}}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \Delta_9, \mathtt{F}_7}\ ?_C}{- :\vdash \{\Delta_{10}, \hat{}(\mathtt{n}_4)\} : \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes$$

- Case rule I_1

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{(\Delta_6, p(\mathtt{n}_5)), \hat{}(\mathtt{n}_5)\} : p(\mathtt{n}_5), *}\, I_2 \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash \{(\Delta_6, p(\mathtt{n}_5)), \hat{}(\mathtt{n}_5)\} : \mathit{dual}(p(\mathtt{n}_5)), *}\, I_1}{- :\vdash \{(\Delta_6, p(\mathtt{n}_5)), \hat{}(\mathtt{n}_5)\} : *, *}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \{\Delta_6, p(\mathtt{n}_5), \hat{}(\mathtt{n}_5)\} : p(\mathtt{n}_5)}\, I_2}{- :\vdash \{\Delta_6, p(\mathtt{n}_5), \hat{}(\mathtt{n}_5)\} : *}\, ?_C$$

- Case rule I_2

- Case rule $?_C$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\, I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \Delta_6, \hat{}(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \mathit{dual}(p(\mathtt{n}_4)), \Delta_6}\, ?_C}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : *, \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \Delta_6, \hat{}(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \Delta_6, \hat{}(\mathtt{n}_4)}\, ?_C}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_4)\} : \Delta_6}\, ?_C$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{(\mathtt{F}_7, \Delta_8), \hat{}(\mathtt{n}_4)\} : p(\mathtt{n}_4), *}\, I_2 \quad \dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_7, \Delta_8, \hat{}(\mathtt{n}_4)\} : \mathtt{F}_7, \Delta_6, \hat{}(\mathtt{n}_4)}{\bullet \mathtt{h}_5 :\vdash \{(\mathtt{F}_7, \Delta_8), \hat{}(\mathtt{n}_4)\} : \mathit{dual}(p(\mathtt{n}_4)), \Delta_6}\, ?_C}{- :\vdash \{(\mathtt{F}_7, \Delta_8), \hat{}(\mathtt{n}_4)\} : *, \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_8, \mathtt{F}_7, \hat{}(\mathtt{n}_4)\} : \Delta_6, \mathtt{F}_7, \hat{}(\mathtt{n}_4)}\, \mathtt{ax}}{- :\vdash \{\Delta_8, \mathtt{F}_7, \hat{}(\mathtt{n}_4)\} : \Delta_6, \mathtt{F}_7}\, ?_C}{- :\vdash \{\Delta_8, \mathtt{F}_7, \hat{}(\mathtt{n}_4)\} : \Delta_6}\, ?_C$$

7.14 Status of $?_C$: OK

- Case rule **1**

- Case rule !

- Case rule ?

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_6, \mathtt{F}_{10}, \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_6, \Delta_3}\, ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_8, \mathtt{F}_{10}, \Delta_{11}\} : \Delta_9, \mathit{dual}(\mathtt{F}_6)}{\bullet \mathtt{h}_7 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathit{dual}(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\, ?}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \Delta_3, \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_3, \mathtt{F}_{10}, \mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\bullet \mathtt{h}_7 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}\, \mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_3, \Delta_9, \mathtt{F}_{10}, ?\mathtt{F}_8}\, \mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_3, \Delta_9, ?\mathtt{F}_8}\, ?_C$$

- Case rule \$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_6, \mathtt{F}_{11}, \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_6, \Delta_3}\, ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}, \mathit{dual}(\mathtt{F}_6)}{\bullet \mathtt{h}_7 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\, \$}{- :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \Delta_3, \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_3, \mathtt{F}_6}\, \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}\, \mathtt{ax}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \Delta_3, \mathtt{F}_8, \mathtt{F}_9}\, \mathtt{hCut}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \Delta_3, \mathtt{F}_8\$\mathtt{F}_9}\, \$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_6, \mathtt{F}_{11}, \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_6, \Delta_3}\ ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_8, \Delta_{10}, \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_9, \Delta_{10}, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \& \mathtt{F}_9}\ \&}{- :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \Delta_3, \Delta_{10}, \mathtt{F}_8 \& \mathtt{F}_9}\ \mathtt{Cut} \quad \xrightarrow{\quad}}{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_3, \mathtt{F}_{11}, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_3, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_8 \& \mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \Delta_3, \mathtt{F}_{11}, \mathtt{F}_8 \& \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \Delta_3, \mathtt{F}_8 \& \mathtt{F}_9}\ ?_C$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_6, \mathtt{F}_{11}, \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_6, \Delta_3}\ ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_9, \Delta_{10}, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_B}{- :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \Delta_3, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{Cut} \quad \xrightarrow{\quad}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_3, \mathtt{F}_6}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}{}\ \mathtt{ax}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \Delta_3, \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \Delta_3, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_6, \mathtt{F}_{11}, \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_6, \Delta_3}\ ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_8, \Delta_{10}, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_A}{- :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \Delta_3, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{Cut} \quad \xrightarrow{\quad}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_3, \mathtt{F}_6}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}{}\ \mathtt{ax}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \Delta_3, \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \Delta_3, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathtt{F}_6, \mathtt{F}_9, \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathtt{F}_6, \Delta_3}\ ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \Delta_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathit{dual}(\mathtt{F}_6), \bot, \Delta_8}\ \bot}{- :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \Delta_3, \bot, \Delta_8}\ \mathtt{Cut} \quad \xrightarrow{\quad}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_3, \mathtt{F}_6}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_8, \mathit{dual}(\mathtt{F}_6)}{}\ \mathtt{ax}}{- :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_3, \Delta_8}\ \mathtt{hCut}}{- :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_3, \Delta_8, \bot}\ \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathtt{F}_6, \mathtt{F}_9, \Delta_3}{\bullet\mathtt{h}_1 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathtt{F}_6, \Delta_3}\ ?_C \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathit{dual}(\mathtt{F}_6), \top, \Delta_8}\ \top}{- :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \Delta_3, \top, \Delta_8}\ \mathtt{Cut} \quad \xrightarrow{\quad}}{- :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_3, \Delta_8, \top}\ \top$$

- Case rule I_3

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathtt{F}_6, \mathtt{F}_{12}, \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathtt{F}_6, \Delta_3}\ ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathtt{F}_8, \Delta_{10}, \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathtt{F}_9, \Delta_{11}}{\bullet \mathtt{h}_7 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{- :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \Delta_3, \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut} \quad \to}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{13}, \mathtt{F}_{12}\} : \Delta_3, \mathtt{F}_{12}, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\bullet \mathtt{h}_7 :\vdash \{\Delta_{13}, \mathtt{F}_{12}\} : \Delta_{10}, \Delta_{11}, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \{\Delta_{13}, \mathtt{F}_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_3, \mathtt{F}_{12}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \{\Delta_{13}, \mathtt{F}_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ ?_C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathtt{F}_6, \mathtt{F}_{12}, \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathtt{F}_6, \Delta_3}\ ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathtt{F}_9, \Delta_{11}, \mathit{dual}(\mathtt{F}_6)}{\bullet \mathtt{h}_7 :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{- :\vdash \{\mathtt{F}_{12}, \Delta_{13}\} : \Delta_3, \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut} \quad \to}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{13}, \mathtt{F}_{12}\} : \Delta_3, \mathtt{F}_{12}, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\bullet \mathtt{h}_7 :\vdash \{\Delta_{13}, \mathtt{F}_{12}\} : \Delta_{10}, \Delta_{11}, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \{\Delta_{13}, \mathtt{F}_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_3, \mathtt{F}_{12}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \{\Delta_{13}, \mathtt{F}_{12}\} : \Delta_{10}, \Delta_{11}, \Delta_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ ?_C$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathtt{F}_6, \mathtt{F}_9, \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathtt{F}_6, \Delta_3}\ ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathtt{F}_9, \Delta_8, \mathit{dual}(\mathtt{F}_6)}{\bullet \mathtt{h}_7 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathit{dual}(\mathtt{F}_6), \Delta_8}\ ?_C}{- :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \Delta_3, \Delta_8}\ \mathtt{Cut} \quad \to}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_3, \mathtt{F}_6, \mathtt{F}_9}\ \mathtt{ax} \quad \dfrac{}{\bullet \mathtt{h}_7 :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_8, \mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_3, \Delta_8, \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_3, \Delta_8}\ ?_C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \mathtt{F}_9, \Delta_{11}\} : \mathtt{F}_6, \mathtt{F}_{10}, \Delta_3}{\bullet \mathtt{h}_1 :\vdash \{\mathtt{F}_{10}, \mathtt{F}_9, \Delta_{11}\} : \mathtt{F}_6, \Delta_3}\ ?_C \quad \dfrac{\mathtt{h}_7 :\vdash \{\mathtt{F}_9, \mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_9, \Delta_8, \mathit{dual}(\mathtt{F}_6)}{\bullet \mathtt{h}_7 :\vdash \{\mathtt{F}_{10}, \mathtt{F}_9, \Delta_{11}\} : \mathit{dual}(\mathtt{F}_6), \Delta_8}\ ?_C}{- :\vdash \{\mathtt{F}_{10}, \mathtt{F}_9, \Delta_{11}\} : \Delta_3, \Delta_8}\ \mathtt{Cut} \quad \to}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \{\Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9\} : \Delta_3, \mathtt{F}_{10}, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\bullet \mathtt{h}_7 :\vdash \{\Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9\} : \Delta_8, \mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9\} : \Delta_3, \Delta_8, \mathtt{F}_{10}}\ \mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9\} : \Delta_3, \Delta_8}\ ?_C$$

8 Cut-Elimination

8.1 Status of 1: OK

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.2 Status of !: OK

- Case rule **1**

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_6\} : \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \{\Delta_6\} :! \mathtt{F}_4}\ !\quad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_6\} : \mathbf{1}}\ \mathbf{1}}{- :\vdash \{\Delta_6\} : \mathbf{1}}\ \mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_6\} : \mathbf{1}}\ \mathbf{1}}\ \to$$

- Case rule !

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :!\mathtt{F}_4}\,!\quad\dfrac{\mathtt{h}_5 :\vdash \{\Delta_7, dual(\mathtt{F}_4)\} : \mathtt{F}_6}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_7\} :!\mathtt{F}_6}\,!}{\mathtt{Cut}}}{- :\vdash \{\Delta_7\} :!\mathtt{F}_6}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :!\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :!\mathtt{F}_4}\,\mathtt{ax}\quad\dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_7, dual(\mathtt{F}_4)\} : \mathtt{F}_6}\,\mathtt{ax}}{\mathtt{hCut}}}{\dfrac{- :\vdash \{\Delta_7\} : \mathtt{F}_6}{- :\vdash \{\Delta_7\} :!\mathtt{F}_6}\,!}$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_8\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_8\} :!\mathtt{F}_4}\,!\quad\dfrac{\mathtt{h}_5 :\vdash \{\mathtt{F}_6, \Delta_8, dual(\mathtt{F}_4)\} : \Delta_7}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_8\} : \Delta_7, ?\mathtt{F}_6}\,?}{\mathtt{Cut}}}{- :\vdash \{\Delta_8\} : \Delta_7, ?\mathtt{F}_6}}{\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_8\} :!\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_8, \mathtt{F}_6\} :!\mathtt{F}_4}\,W\quad\dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_8, \mathtt{F}_6, dual(\mathtt{F}_4)\} : \Delta_7}\,\mathtt{ax}}{\mathtt{hCut}}}{\dfrac{- :\vdash \{\Delta_8, \mathtt{F}_6\} : \Delta_7}{- :\vdash \{\Delta_8\} : \Delta_7, ?\mathtt{F}_6}\,?}}$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}\,!\quad\dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \mathtt{F}_6, \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_9\} : \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\$}{\mathtt{Cut}}}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}\,\mathtt{ax}\quad\dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{ax}}{\mathtt{hCut}}}{\dfrac{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6, \mathtt{F}_7}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\$}$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}\,!\quad\dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \mathtt{F}_6, \Delta_8\quad\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_9\} : \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\,\&}{\mathtt{Cut}}}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}}{\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}\,\mathtt{ax}\quad\dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \Delta_8, \mathtt{F}_6}\,\mathtt{ax}}{\mathtt{hCut}}}{\dfrac{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\,\&}\quad\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}\,\mathtt{ax}\quad\dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \Delta_8, \mathtt{F}_7}\,\mathtt{ax}}{\mathtt{hCut}}}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_7}\,\&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}\,!\quad\dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_9\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_B}{\mathtt{Cut}}}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}\,\mathtt{ax}\quad\dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \Delta_8, \mathtt{F}_7}\,\mathtt{ax}}{\mathtt{hCut}}}{\dfrac{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_7}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_B}$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_9\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}\,! \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \mathtt{F}_6, \Delta_8}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_9\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_A}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_9\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_9, dual(\mathtt{F}_4)\} : \Delta_8, \mathtt{F}_6}\,\mathtt{ax}}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_9\} : \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :!\mathtt{F}_4}\,! \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_7, dual(\mathtt{F}_4)\} : \Delta_6}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_7\} : \bot, \Delta_6}\,\bot}{- :\vdash \{\Delta_7\} : \bot, \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_7, dual(\mathtt{F}_4)\} : \Delta_6}\,\mathtt{ax}}{- :\vdash \{\Delta_7\} : \Delta_6}\,\mathtt{hCut}}{- :\vdash \{\Delta_7\} : \Delta_6, \bot}\,\bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :!\mathtt{F}_4}\,! \quad \dfrac{}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_7\} : \top, \Delta_6}\,\top}{- :\vdash \{\Delta_7\} : \top, \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : \Delta_6, \top}\,\top$$

- Case rule I_3

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7\} :!\mathtt{F}_4}\,! \quad \dfrac{}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_7\} : p(\mathtt{n}_6), \hat{}(\mathtt{n}_6)}\,I_3}{- :\vdash \{\Delta_7\} : p(\mathtt{n}_6), \hat{}(\mathtt{n}_6)}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- :\vdash \{\Delta_7\} : p(\mathtt{n}_6), \hat{}(\mathtt{n}_6)}\,I_3$$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_{10}\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} :!\mathtt{F}_4}\,! \quad \dfrac{\mathtt{h}_5 :\vdash \{\Delta_{10}, dual(\mathtt{F}_4)\} : \mathtt{F}_6, \Delta_8 \quad \mathtt{h}_5 :\vdash \{\Delta_{10}, dual(\mathtt{F}_4)\} : \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_{10}\} : \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes}{- :\vdash \{\Delta_{10}\} : \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_{10}, dual(\mathtt{F}_4)\} : \Delta_8, \mathtt{F}_6}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_8, \mathtt{F}_6}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \{\Delta_{10}\} :!\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \{\Delta_{10}, dual(\mathtt{F}_4)\} : \Delta_9, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}\} : \Delta_9, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}\} : \Delta_8, \Delta_9, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\otimes$$

- Case rule I_1

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7, p(\mathtt{n}_6)\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7, p(\mathtt{n}_6)\} :!\mathtt{F}_4}\,! \quad \dfrac{}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_7, p(\mathtt{n}_6)\} : \hat{}(\mathtt{n}_6)}\,I_1}{- :\vdash \{\Delta_7, p(\mathtt{n}_6)\} : \hat{}(\mathtt{n}_6)}\,\mathtt{Cut}}{\rightarrow}$$

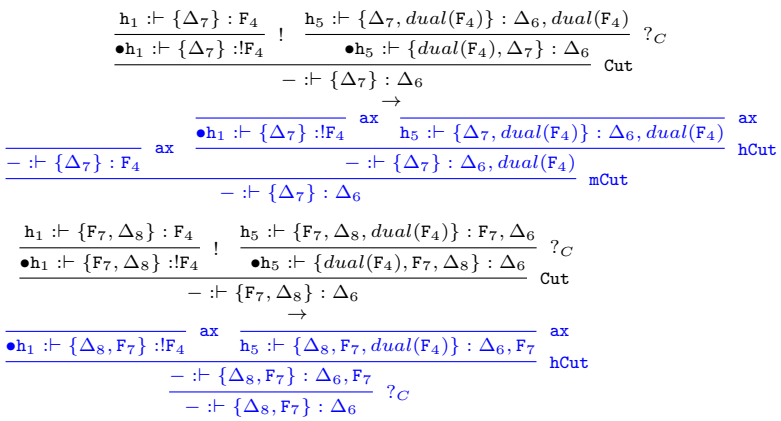
$$\dfrac{}{- :\vdash \{\Delta_7, p(\mathtt{n}_6)\} : \hat{}(\mathtt{n}_6)}\,I_1$$

- Case rule I_2

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_6)\} : \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \{\Delta_7, \hat{}(\mathtt{n}_6)\} :!\mathtt{F}_4}\,! \quad \dfrac{}{\bullet\mathtt{h}_5 :\vdash \{dual(\mathtt{F}_4), \Delta_7, \hat{}(\mathtt{n}_6)\} : p(\mathtt{n}_6)}\,I_2}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_6)\} : p(\mathtt{n}_6)}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- :\vdash \{\Delta_7, \hat{}(\mathtt{n}_6)\} : p(\mathtt{n}_6)}\,I_2$$

- Case rule $?_C$



8.3 Status of ?: OK

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2

- Case rule $?_C$

8.4 Status of $: OK

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.5 Status of &: OK

- Case rule **1**
- Case rule !
- Case rule ?

- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.6 Status of $\oplus_B$: `OK`

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$

- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.7 Status of $\oplus_A$: `OK`

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.8 Status of $\bot$: `OK`

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.9 Status of $\top$: `OK`

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule \$

- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.10 Status of I_3: `OK`

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$

- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.11 Status of $\otimes$: `OK`

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.12 Status of I_1: `OK`

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.13 Status of I_2: `OK`

- Case rule **1**
- Case rule !
- Case rule ?
- Case rule \$

- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule I_3
- Case rule $\otimes$
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.14 Status of $?_C$: OK

- Case rule **1**

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_7, \Delta_8\} : \mathtt{F}_7, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_7, \Delta_8\} :!\mathtt{F}_5}\ ?_C \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_7, \Delta_8\} : \mathbf{1}}\ \mathbf{1}}{- :\vdash \{\mathtt{F}_7, \Delta_8\} : \mathbf{1}}\ \mathtt{Cut}}{\dfrac{}{- :\vdash \{\Delta_8, \mathtt{F}_7\} : \mathbf{1}}\ \mathbf{1}}\ \rightarrow$$

- Case rule !

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_8, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} :!\mathtt{F}_5}\ ?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_9, dual(\mathtt{F}_5)\} : \mathtt{F}_7}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_8, \Delta_9\} :!\mathtt{F}_7}\ !}{- :\vdash \{\mathtt{F}_8, \Delta_9\} :!\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_9, \mathtt{F}_8\} :!\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)\} : \mathtt{F}_7}\ \mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} :!\mathtt{F}_7}\ !}\ \rightarrow$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \mathtt{F}_9, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_9, \Delta_{10}\} :!\mathtt{F}_5}\,?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_7, \mathtt{F}_9, \Delta_{10}, dual(\mathtt{F}_5)\} : \Delta_8}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_9, \Delta_{10}\} : \Delta_8, ?\mathtt{F}_7}\,?}{- :\vdash \{\mathtt{F}_9, \Delta_{10}\} : \Delta_8, ?\mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_{10}, \mathtt{F}_9\} :!\mathtt{F}_5}\,\mathtt{ax}}{\bullet\mathtt{h}_2 :\vdash \{\Delta_{10}, \mathtt{F}_7, \mathtt{F}_9\} :!\mathtt{F}_5}\,W \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_{10}, \mathtt{F}_7, \mathtt{F}_9, dual(\mathtt{F}_5)\} : \Delta_8}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}, \mathtt{F}_7, \mathtt{F}_9\} : \Delta_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}, \mathtt{F}_9\} : \Delta_8, ?\mathtt{F}_7}\,?$$

- Case rule $\$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} :!\mathtt{F}_5}\,?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_{10}, \Delta_{11}, dual(\mathtt{F}_5)\} : \mathtt{F}_7, \mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_{10}, \Delta_{11}\} : \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\,\$}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} :!\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_5)\} : \Delta_9, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\,\$$

- Case rule $\&$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} :!\mathtt{F}_5}\,?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_{10}, \Delta_{11}, dual(\mathtt{F}_5)\} : \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 :\vdash \{\mathtt{F}_{10}, \Delta_{11}, dual(\mathtt{F}_5)\} : \mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_{10}, \Delta_{11}\} : \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\,\&}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} :!\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_5)\} : \Delta_9, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} :!\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_5)\} : \Delta_9, \mathtt{F}_8}\,\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\,\&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} :!\mathtt{F}_5}\,?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_{10}, \Delta_{11}, dual(\mathtt{F}_5)\} : \mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_{10}, \Delta_{11}\} : \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_B}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} :!\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_5)\} : \Delta_9, \mathtt{F}_8}\,\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \mathtt{F}_{10}, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} :!\mathtt{F}_5}\,?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_{10}, \Delta_{11}, dual(\mathtt{F}_5)\} : \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_{10}, \Delta_{11}\} : \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_A}{- :\vdash \{\mathtt{F}_{10}, \Delta_{11}\} : \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} :!\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_5)\} : \Delta_9, \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \{\Delta_{11}, \mathtt{F}_{10}\} : \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_8, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} :!\mathtt{F}_5}\,?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_9, dual(\mathtt{F}_5)\} : \Delta_7}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_8, \Delta_9\} : \bot, \Delta_7}\,\bot}{- :\vdash \{\mathtt{F}_8, \Delta_9\} : \bot, \Delta_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \{\Delta_9, \mathtt{F}_8\} :!\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)\} : \Delta_7}\,\mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_7}\,\mathtt{hCut}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_7, \bot}\,\bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_8, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} :!\mathtt{F}_5}\ ?_C \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_8, \Delta_9\} : \top, \Delta_7}\ \top}{- :\vdash \{\mathtt{F}_8, \Delta_9\} : \top, \Delta_7}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_7, \top}\ \top$$

- Case rule I_3

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_8, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} :!\mathtt{F}_5}\ ?_C \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_8, \Delta_9\} : p(\mathtt{n}_7), \hat{}(\mathtt{n}_7)}\ I_3}{- :\vdash \{\mathtt{F}_8, \Delta_9\} : p(\mathtt{n}_7), \hat{}(\mathtt{n}_7)}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : p(\mathtt{n}_7), \hat{}(\mathtt{n}_7)}\ I_3$$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \mathtt{F}_{11}, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} :!\mathtt{F}_5}\ ?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_{11}, \Delta_{12}, dual(\mathtt{F}_5)\} : \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 :\vdash \{\mathtt{F}_{11}, \Delta_{12}, dual(\mathtt{F}_5)\} : \mathtt{F}_8, \Delta_{10}}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_{11}, \Delta_{12}\} : \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \{\mathtt{F}_{11}, \Delta_{12}\} : \Delta_9, \Delta_{10}, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} :!\mathtt{F}_5}{}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_{12}, \mathtt{F}_{11}, dual(\mathtt{F}_5)\} : \Delta_9, \mathtt{F}_7}\ \mathtt{ax}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_9, \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} :!\mathtt{F}_5}{}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_{12}, \mathtt{F}_{11}, dual(\mathtt{F}_5)\} : \Delta_{10}, \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \{\Delta_{12}, \mathtt{F}_{11}\} : \Delta_{10}, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes$$

- Case rule I_1

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{p(\mathtt{n}_7), \Delta_8\} : p(\mathtt{n}_7), !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{p(\mathtt{n}_7), \Delta_8\} :!\mathtt{F}_5}\ ?_C \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), p(\mathtt{n}_7), \Delta_8\} : \hat{}(\mathtt{n}_7)}\ I_1}{- :\vdash \{p(\mathtt{n}_7), \Delta_8\} : \hat{}(\mathtt{n}_7)}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- :\vdash \{\Delta_8, p(\mathtt{n}_7)\} : \hat{}(\mathtt{n}_7)}\ I_1$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9, p(\mathtt{n}_7)\} : \mathtt{F}_8, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9, p(\mathtt{n}_7)\} :!\mathtt{F}_5}\ ?_C \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_8, \Delta_9, p(\mathtt{n}_7)\} : \hat{}(\mathtt{n}_7)}\ I_1}{- :\vdash \{\mathtt{F}_8, \Delta_9, p(\mathtt{n}_7)\} : \hat{}(\mathtt{n}_7)}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- :\vdash \{\Delta_9, \mathtt{F}_8, p(\mathtt{n}_7)\} : \hat{}(\mathtt{n}_7)}\ I_1$$

- Case rule I_2

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\hat{}(\mathtt{n}_7), \Delta_8\} : \hat{}(\mathtt{n}_7), !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\hat{}(\mathtt{n}_7), \Delta_8\} :!\mathtt{F}_5}\ ?_C \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \hat{}(\mathtt{n}_7), \Delta_8\} : p(\mathtt{n}_7)}\ I_2}{- :\vdash \{\hat{}(\mathtt{n}_7), \Delta_8\} : p(\mathtt{n}_7)}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- :\vdash \{\Delta_8, \hat{}(\mathtt{n}_7)\} : p(\mathtt{n}_7)}\ I_2$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9, \hat{}(\mathtt{n}_7)\} : \mathtt{F}_8, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9, \hat{}(\mathtt{n}_7)\} :!\mathtt{F}_5}\ ?_C \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_8, \Delta_9, \hat{}(\mathtt{n}_7)\} : p(\mathtt{n}_7)}\ I_2}{- :\vdash \{\mathtt{F}_8, \Delta_9, \hat{}(\mathtt{n}_7)\} : p(\mathtt{n}_7)}\ \mathtt{Cut}}{\rightarrow}$$
$$\dfrac{}{- :\vdash \{\Delta_9, \mathtt{F}_8, \hat{}(\mathtt{n}_7)\} : p(\mathtt{n}_7)}\ I_2$$

- Case rule $?_C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_8, !\mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} :!\mathtt{F}_5}\,?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_9, dual(\mathtt{F}_5)\} : \Delta_7, dual(\mathtt{F}_5)}{\bullet \mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_8, \Delta_9\} : \Delta_7}\,?_C}{- :\vdash \{\mathtt{F}_8, \Delta_9\} : \Delta_7}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{- :\vdash \{\Delta_9, \mathtt{F}_8\} :!\mathtt{F}_5}\,\mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \mathtt{F}_5}\,BInv \quad \dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \{\Delta_9, \mathtt{F}_8\} :!\mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \{\Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)\} : \Delta_7, dual(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_7, dual(\mathtt{F}_5)}\,\mathtt{hCut}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_7}\,\mathtt{mCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} : \mathtt{F}_8, !\mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \{\mathtt{F}_8, \Delta_9\} :!\mathtt{F}_5}\,?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \Delta_9, dual(\mathtt{F}_5)\} : \mathtt{F}_8, \Delta_7}{\bullet \mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_8, \Delta_9\} : \Delta_7}\,?_C}{- :\vdash \{\mathtt{F}_8, \Delta_9\} : \Delta_7}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet \mathtt{h}_2 :\vdash \{\Delta_9, \mathtt{F}_8\} :!\mathtt{F}_5}{}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)\} : \Delta_7, \mathtt{F}_8}{}\,\mathtt{ax}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_7, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_9, \mathtt{F}_8\} : \Delta_7}\,?_C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \{\mathtt{F}_9, \mathtt{F}_8, \Delta_{10}\} : \mathtt{F}_9, !\mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \{\mathtt{F}_9, \mathtt{F}_8, \Delta_{10}\} :!\mathtt{F}_5}\,?_C \quad \dfrac{\mathtt{h}_6 :\vdash \{\mathtt{F}_8, \mathtt{F}_9, \Delta_{10}, dual(\mathtt{F}_5)\} : \mathtt{F}_8, \Delta_7}{\bullet \mathtt{h}_6 :\vdash \{dual(\mathtt{F}_5), \mathtt{F}_9, \mathtt{F}_8, \Delta_{10}\} : \Delta_7}\,?_C}{- :\vdash \{\mathtt{F}_9, \mathtt{F}_8, \Delta_{10}\} : \Delta_7}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet \mathtt{h}_2 :\vdash \{\Delta_{10}, \mathtt{F}_8, \mathtt{F}_9\} :!\mathtt{F}_5}{}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_6 :\vdash \{\Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, dual(\mathtt{F}_5)\} : \Delta_7, \mathtt{F}_8}{}\,\mathtt{ax}}{- :\vdash \{\Delta_{10}, \mathtt{F}_8, \mathtt{F}_9\} : \Delta_7, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \{\Delta_{10}, \mathtt{F}_8, \mathtt{F}_9\} : \Delta_7}\,?_C$$