System G3C

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1 Height preserving admissibility of weakening on the left

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R}$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\scriptstyle \wedge_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\scriptstyle \wedge_R}$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\scriptstyle \vee_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\scriptstyle \vee_R}$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\scriptstyle \bot_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \bot, \Delta_3}\ {\scriptstyle \bot_R}$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\scriptstyle \top_R} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \top, \Delta_3}\ {\scriptstyle \top_R}$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \rightarrow_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_W \vdash \Delta_4, \mathtt{F}_2}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \rightarrow_L}$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \wedge_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \wedge_L}$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \vee_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_W \vdash \Delta_4}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_W, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \vee_L}$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ {\scriptstyle \bot_L} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3, \mathtt{F}_W \vdash \Delta_2}\ {\scriptstyle \bot_L}$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ {\scriptstyle I} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_W, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ {\scriptstyle I}$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ {\scriptstyle \top_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_W \vdash \Delta_2}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \top, \Delta_3, \mathtt{F}_W \vdash \Delta_2}\ {\scriptstyle \top_L}$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \to \mathtt{F}_4} \to_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_W}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_W, \mathtt{F}_3 \to \mathtt{F}_4} \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_W}^{\,\mathtt{IH}} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_W}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_W, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_W}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_W, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \mathtt{F}_W} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3, \mathtt{F}_W} \top_R$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4} \to_L \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_W}^{\,\mathtt{IH}} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W} \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W} \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_W}^{\,\mathtt{IH}} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W} \vee_L$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2} \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2, \mathtt{F}_W} \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3} I \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{F}_W, \mathtt{p}_3} I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2} \top_L \quad \to \quad \frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}^{\,\mathtt{ax}}}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2, \mathtt{F}_W}^{\,\mathtt{IH}}}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2, \mathtt{F}_W} \top_L$$

3 Measure of derivations

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R}$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\scriptstyle \wedge_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ {\scriptstyle \wedge_R}$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\scriptstyle \vee_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ {\scriptstyle \vee_R}$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\scriptstyle \bot_R} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ {\scriptstyle \mathtt{IH}}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\scriptstyle \bot_R}$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\scriptstyle \top_R} \quad \rightarrow \quad \frac{}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\scriptstyle \top_R}$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \rightarrow_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \rightarrow_L}$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \wedge_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \wedge_L}$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \vee_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \mathtt{IH}}}{\bullet \bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ {\scriptstyle \vee_L}$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ {\scriptstyle \bot_L} \quad \rightarrow \quad \frac{}{\bullet \bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ {\scriptstyle \bot_L}$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ {\scriptstyle I} \quad \rightarrow \quad \frac{}{\bullet \bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ {\scriptstyle I}$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ {\scriptstyle \top_L} \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ {\scriptstyle \mathtt{IH}}}{\bullet \bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ {\scriptstyle \top_L}$$

4 Invertibility of Rules

4.1 Status of $\rightarrow_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ {\scriptstyle \rightarrow_R}$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ {\scriptstyle height}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ {\scriptstyle \wedge_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ {\scriptstyle \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ {\scriptstyle \wedge_R}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ {\scriptstyle \vee_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ {\scriptstyle \vee_R}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \bot_R} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \bot, \Delta_5, \mathtt{F}_2}\ {\scriptstyle \bot_R}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \top_R} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \top, \Delta_5, \mathtt{F}_2}\ {\scriptstyle \top_R}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \rightarrow_L} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\scriptstyle \rightarrow_L}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \wedge_L} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\scriptstyle \wedge_L}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ {\scriptstyle \vee_L} \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ {\scriptstyle \vee_L}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}\ \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vdash \Delta_4, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}\ I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \to \mathtt{F}_2}\ \top_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4, \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vdash \Delta_4, \mathtt{F}_2}\ \top_L$$

4.2 Status of $\wedge_R$: (Left Premise): `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ \text{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ height$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1}\ \top_R$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \to_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \top_L$$

4.3 Status of $\wedge_R$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}^{\ ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ height$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_2}\ \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_2}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_2}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \top_L$$

4.4 Status of $\vee_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4} \; height$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2} \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2} \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2} \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6 \vdash \mathtt{p}_5, \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2} \; I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_5} \; I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1 \vee \mathtt{F}_2} \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2} \top_L$$

4.5 Status of $\bot_R$: : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \bot, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \to \mathtt{F}_4} \to_R \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \to \mathtt{F}_4} \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3} \; height$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \bot, \Delta_3} \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \bot, \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \bot, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \bot, \Delta_4} \to_L \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4} \to_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \bot, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \bot, \Delta_4} \wedge_L \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \bot, \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \bot, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \bot, \Delta_4} \vee_L \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \bot, \Delta_2} \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_4 \vdash \mathtt{p}_3, \bot, \Delta_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \bot, \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \bot, \Delta_2}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L$$

4.6 Status of $\top_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \top, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \top, \Delta_3}\ \bot_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \top, \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \top, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \top, \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \top, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \top, \Delta_4}\ \wedge_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \top, \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \top, \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \top, \Delta_4}\ \vee_L \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \top, \Delta_2}\ \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_4 \vdash \mathtt{p}_3, \top, \Delta_2}\ I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \top, \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \top, \Delta_2}\ \top_L \quad \rightarrow \quad \texttt{trivial}$$

4.7 Status of $\rightarrow_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}^{\ \texttt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_5 \vdash \bot, \Delta_4, \mathtt{F}_1}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5 \vdash \top, \Delta_4, \mathtt{F}_1}\ \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}^{\ \texttt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \rightarrow_L$$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\ \texttt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathit{height}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1}^{\,ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1} \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4} \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4} I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_1, \mathtt{p}_5} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4} \top_L \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1} \top_L$$

4.8 Status of $\to_L$ (Right Premise): : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \to \mathtt{F}_5} \to_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \to \mathtt{F}_5} \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\,ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}^{\,ax/ind}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6}\ \to_L$$

$$\frac{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \text{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ height$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.9 Status of $\wedge_L$: : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \to \mathtt{F}_5}\ \to_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \to \mathtt{F}_5}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \rightarrow \quad \dfrac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R \quad \rightarrow \quad \dfrac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightarrow \quad \dfrac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightarrow \quad \dfrac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

$$\dfrac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}^{\ ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ height$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \rightarrow \quad \dfrac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

- Case rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\dfrac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \dfrac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.10 Status of $\vee_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_5}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \bot, \Delta_4} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \top, \Delta_4} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \top, \Delta_4} \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}^{\ ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ height$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vdash \Delta_4} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \top_L$$

4.11 Status of $\vee_L$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ height$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_5, \Delta_4} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4} \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4} \top_L$$

4.12 Status of $\bot_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \rightarrow \mathtt{F}_3} \rightarrow_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3} \wedge_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3} \vee_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \bot, \Delta_2} \bot_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \top, \Delta_2} \top_R \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4 \quad \mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2} \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \bot, \Delta_4 \vdash \mathtt{p}_3, \Delta_2} I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \bot, \Delta_3 \vdash \Delta_2} \top_L \quad \rightarrow \quad \texttt{trivial}$$

4.13 Status of I: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_3, \mathtt{p}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash (\Delta_4, \mathtt{p}_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3} \rightarrow_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{p}_5 \quad \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_3, \mathtt{p}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash (\Delta_4, \mathtt{p}_5), \mathtt{F}_2 \wedge \mathtt{F}_3} \wedge_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3, \mathtt{p}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash (\Delta_4, \mathtt{p}_5), \mathtt{F}_2 \vee \mathtt{F}_3} \vee_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \bot, \Delta_2, \mathtt{p}_3} \bot_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \top, \Delta_2, \mathtt{p}_3}\ \top_R \quad \to \quad \texttt{trivial}$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{p}_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{p}_5 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_3, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : (\Delta_6, \mathtt{p}_5), \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4, \mathtt{p}_5}\ \to_L \quad \to \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2, \mathtt{F}_3, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : (\Delta_6, \mathtt{p}_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \mathtt{p}_5}\ \wedge_L \quad \to \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_3, \mathtt{p}_5 \vdash \Delta_4, \mathtt{p}_5}{\bullet \mathtt{h}_1 : (\Delta_6, \mathtt{p}_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \mathtt{p}_5}\ \vee_L \quad \to \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ \bot_L \quad \to \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_4, \Delta_5, \mathtt{p}_3 \vdash \mathtt{p}_4, \Delta_2, \mathtt{p}_3}\ I \quad \to \quad \texttt{trivial}$$

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_4 \vdash \mathtt{p}_3, \Delta_2}\ I \quad \to \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}{\bullet \mathtt{h}_1 : \top, \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ \top_L \quad \to \quad \texttt{trivial}$$

4.14 Status of $\top_L$: : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \to \mathtt{F}_3}\ \to_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_3}\ \texttt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \to \mathtt{F}_3}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \texttt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_3}\ \texttt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \bot, \Delta_2}\ \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \bot, \Delta_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \top, \Delta_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \top, \Delta_2}\ \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4 \quad \mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \top, \Delta_4 \vdash \mathtt{p}_3, \Delta_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ height$$

5 Height preserving admissibility of contraction on the left

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2 \to \mathtt{F}_3}\ \to_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2 \to \mathtt{F}_3}\ \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \Delta_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \bot, \Delta_2}\ \bot_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_4 \vdash \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_3, \mathtt{F}_4 \vdash \bot, \Delta_2}\ \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \top, \Delta_2}\ \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_3, \mathtt{F}_4 \vdash \top, \Delta_2}\ \top_R$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4}\ \to_L \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_2}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{IH\text{-}Mutual} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4}\ \to_L$$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\Delta_5, \mathtt{F}_2 \to \mathtt{F}_3), \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}\ \to_L \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_6 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_6 \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4}\ \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \to \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{IH}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3), \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}\ \wedge_L \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_6 \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{inv\text{-}th/ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

$$\frac{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4 \quad \mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3), \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}\ \vee_L \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \vdash \Delta_4}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_6, \mathtt{F}_6 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3, \mathtt{F}_6 \vdash \Delta_4}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_3, \bot, \bot \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L$$

$$\frac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_3), \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3, \mathtt{F}_4 \vdash \Delta_2}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

$$\frac{}{\bullet \mathtt{h}_1 : (\Delta_4, \mathtt{p}_3), \mathtt{F}_5, \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_5, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \Delta_3, \top, \top \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_3 \vdash \Delta_2}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \top, \Delta_3 \vdash \Delta_2}\ \top_L$$

$$\frac{\mathtt{h}_1 : \Delta_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_2}{\bullet \mathtt{h}_1 : (\top, \Delta_3), \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_2}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_2}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_3, \mathtt{F}_4 \vdash \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \top, \Delta_3, \mathtt{F}_4 \vdash \Delta_2}\ \top_L$$

6 Height preserving admissibility of contraction on the Right

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_3 \to \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \to \mathtt{F}_4, \mathtt{F}_3 \to \mathtt{F}_4} \to_R \quad \to \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_4}\,\text{inv-th/ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_4}\,\text{IH-Mutual}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\,\text{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \to \mathtt{F}_4} \to_R$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\Delta_5, \mathtt{F}_3 \to \mathtt{F}_4), \mathtt{F}_6, \mathtt{F}_6} \to_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_6}\,\text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6}\,\text{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \to \mathtt{F}_4} \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_3 \wedge \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_3 \wedge \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_3}\,\text{inv-th/ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\,\text{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_4}\,\text{inv-th/ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\,\text{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_6, \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4), \mathtt{F}_6, \mathtt{F}_6} \wedge_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_6, \mathtt{F}_6}\,\text{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_6}\,\text{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_6}\,\text{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6}\,\text{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_3 \vee \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \to \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_4}\,\text{inv-th/ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4}\,\text{IH}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\,\text{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4), \mathtt{F}_6, \mathtt{F}_6} \vee_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6, \mathtt{F}_6}\,\text{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6}\,\text{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \bot, \bot} \bot_R \quad \to \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\,\text{inv-th/ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_3), \mathtt{F}_4, \mathtt{F}_4} \bot_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_4}\,\text{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\,\text{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \mathtt{F}_4} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \top, \top} \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_3), \mathtt{F}_4, \mathtt{F}_4} \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3, \mathtt{F}_4} \top_R$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5} \to_L \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_5}\,\text{ax}}{\mathtt{h}_1 : \Delta_6 \vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_5}\,\text{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5}\,\text{ax}}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5}\,\text{IH}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5} \to_L$$

- Case(s) rule $\wedge_L$

$$\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5}\ \wedge_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5 \quad \mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5}\ \vee_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_5}\ \vee_L$$

- Case(s) rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_3}\ \bot_L \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_4 \vdash \Delta_2, \mathtt{F}_3}\ \bot_L$$

- Case(s) rule I

$$\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3, \mathtt{p}_3}\ I \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3}\ I$$

$$\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_4 \vdash (\Delta_2, \mathtt{p}_4), \mathtt{F}_3, \mathtt{F}_3}\ I \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_4 \vdash \Delta_2, \mathtt{F}_3, \mathtt{p}_4}\ I$$

- Case(s) rule $\top_L$

$$\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_3}{\bullet \mathtt{h}_1 : \top, \Delta_4 \vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_3}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_4 \vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_4 \vdash \Delta_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \top, \Delta_4 \vdash \Delta_2, \mathtt{F}_3}\ \top_L$$

7 Identity-Expansion

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,\vee_L}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0 \vee \mathtt{F}_1}\,\vee_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_R}{- : \mathtt{F}_0 \wedge \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_1}\,\to_L}{- : \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_0 \to \mathtt{F}_1}\,\to_R$$

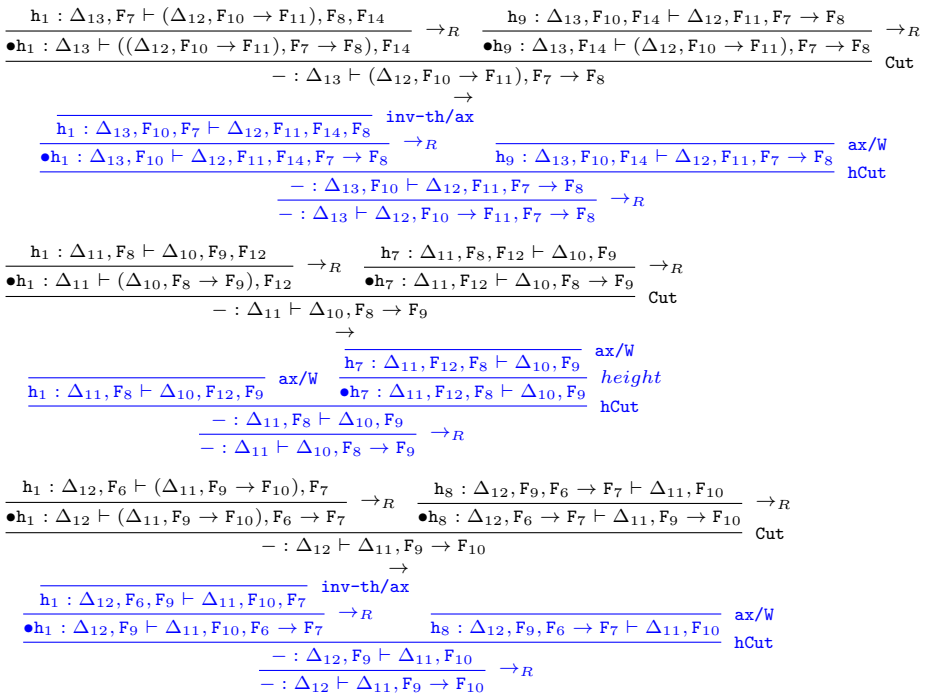
$$\dfrac{}{- : \top \vdash \top}\,\top_R$$

$$\dfrac{}{- : \bot \vdash \bot}\,\bot_L$$

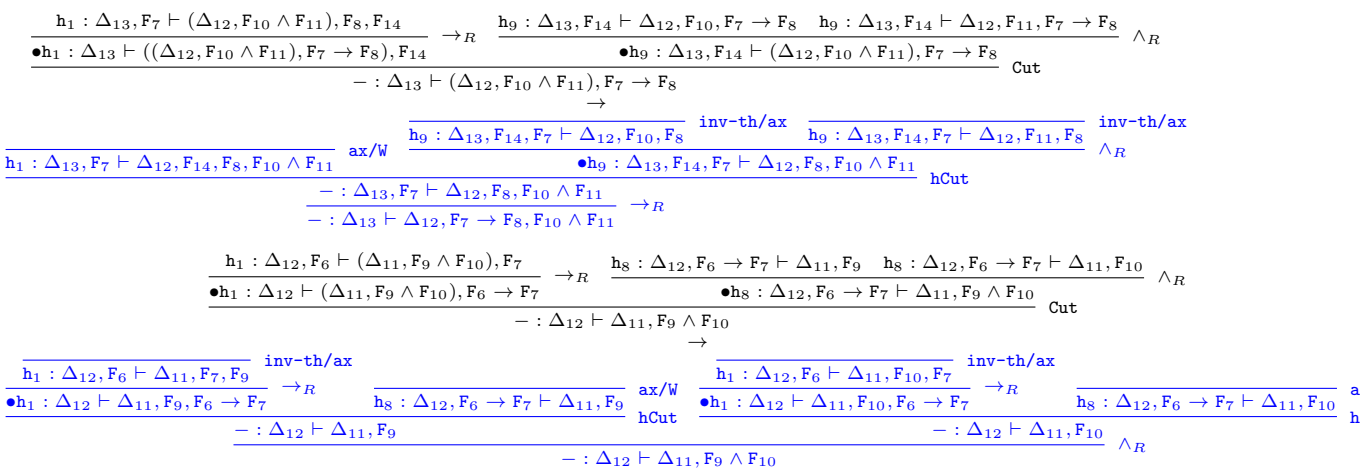
8 Cut-Elimination

8.1 Status of $\to_R$: OK

- Case rule $\to_R$



- Case rule $\wedge_R$



- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{14}}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\vee_R}}{- : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,{\vee_R}}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,{\to_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6 \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_R}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,{\vee_R}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash (\bot, \Delta_{10}), \mathtt{F}_8, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash ((\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12}}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\bot_R}}{- : \Delta_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{hCut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vdash (\bot, \Delta_9), \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_9}\,{\bot_R}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{hCut}}{\to}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash (\top, \Delta_{10}), \mathtt{F}_8, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash ((\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12}}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\top_R}}{- : \Delta_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_6 \vdash (\top, \Delta_9), \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \top, \Delta_9}\,{\top_R}}{- : \Delta_{10} \vdash \top, \Delta_9}\,\mathtt{Cut}}{\to}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{14}}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_L}}{- : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,{\to_L}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \to_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \to_L}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8} \to_R}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}} \to_L}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{a}\ \mathtt{h}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}} \to_L}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9, \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7 \to \mathtt{F}_8} \to_R \quad \dfrac{\mathtt{h}_6 : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8 \vdash \Delta_9}{\bullet \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9} \to_L}{- : \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{sCut}}{- : \Delta_{10} \vdash \Delta_9}\,\mathtt{sCut}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{14}} \to_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \wedge_L}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \wedge_L}{\mathtt{hCut}}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \to_R}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \wedge_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \wedge_L}{\mathtt{hCut}}}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8} \to_R}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \to_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut} \wedge_L}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{14}} \to_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \vee_L}{- : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}\,\vee_L}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8} \to_R}{- : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11}} \to_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8} \vee_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}\,\vee_L}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8} \to_R}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{a}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{h} \vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \bot}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \bot} \to_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{11}, \bot \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash \bot, \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8} \bot_L}{\mathtt{hCut}}}{- : \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8} \to_R}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \bot, \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12}} \to_R \quad \dfrac{}{\bullet\mathtt{h}_9 : (\bot, \Delta_{11}), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \bot_L}{- : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8} \bot_L}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\bot, \Delta_{10}), \mathtt{F}_6 \vdash \Delta_9, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9} \bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9} \bot_L}{\to}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_8, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{11}} \to_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,I}{- : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}}\,I}{\mathtt{hCut}}}{- : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}} \to_R}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_8, \mathtt{F}_{13}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{11} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13}}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_{13} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,I}{- : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\,I}\,{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9, \mathtt{p}_{10}}\,I}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\,I}\,{\to}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \top}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \top}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{11}, \top \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,{\top_L}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W}}\,{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \top, \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12}}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\top, \Delta_{11}), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,{\top_L}}{- : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{hCut}}\,{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\top, \Delta_{10}), \mathtt{F}_6 \vdash \Delta_9, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}\,{\top_L}}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{hCut}}\,{\to}$$

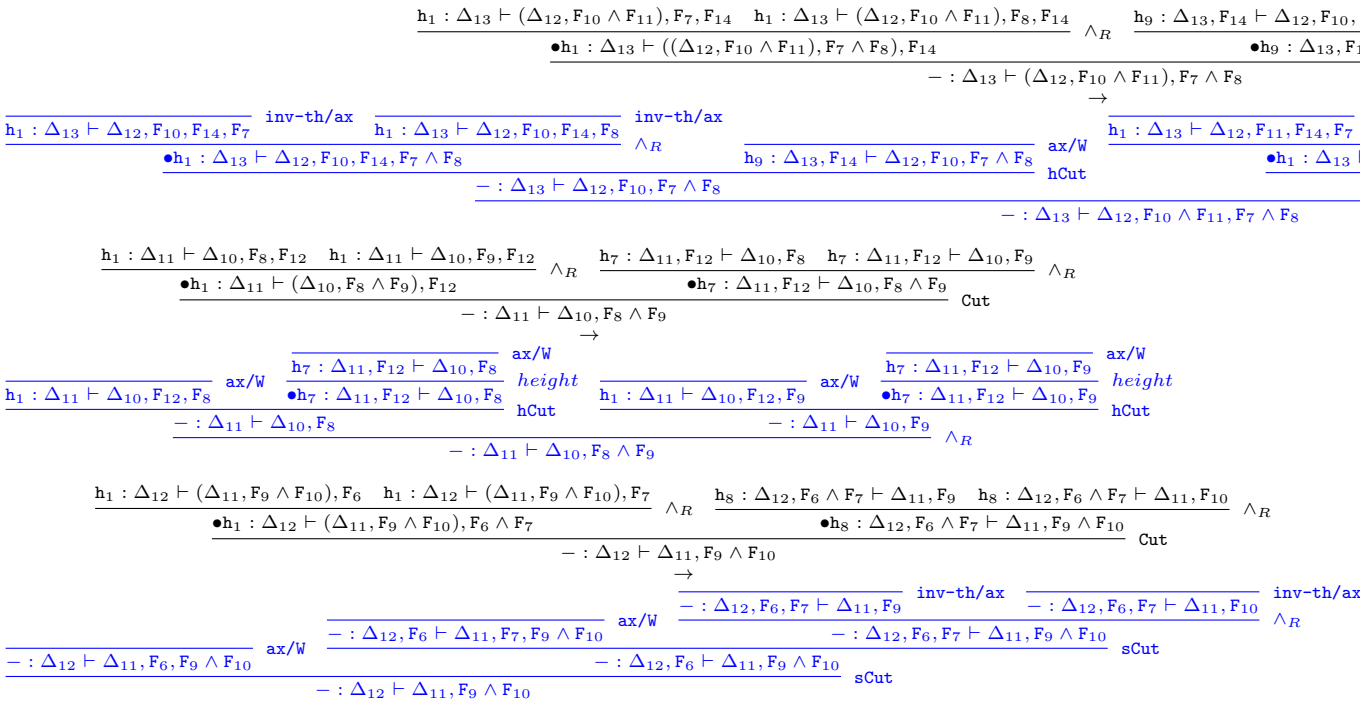
8.2 Status of $\wedge_R$: OK

- Case rule $\to_R$

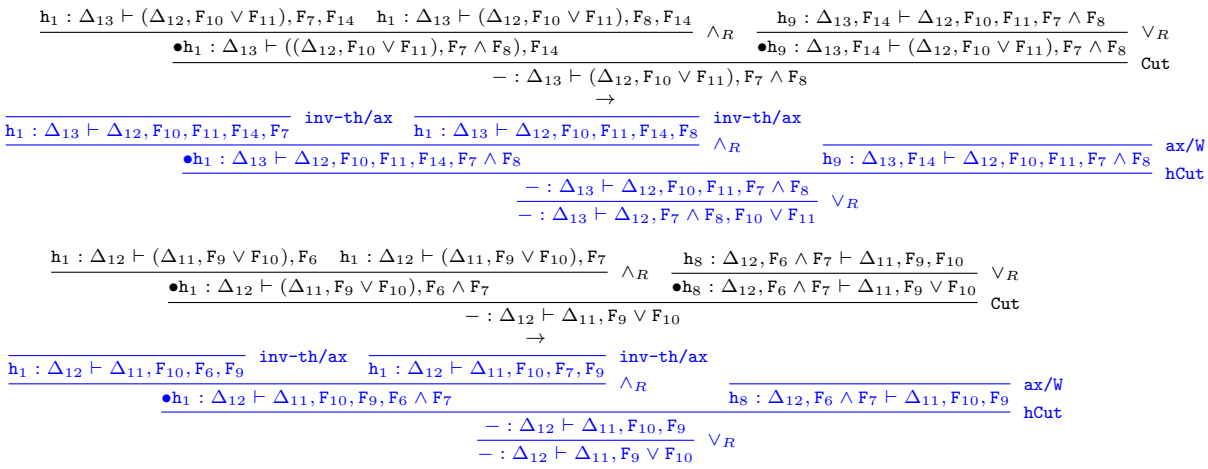
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_{14} \quad \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash ((\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14}}\,{\wedge_R} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,{\to_R}}{- : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,{\wedge_R} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,{\to_R}}\,{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7}\,{\wedge_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_R}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7}\,{\wedge_R} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_R}}\,{\to}$$

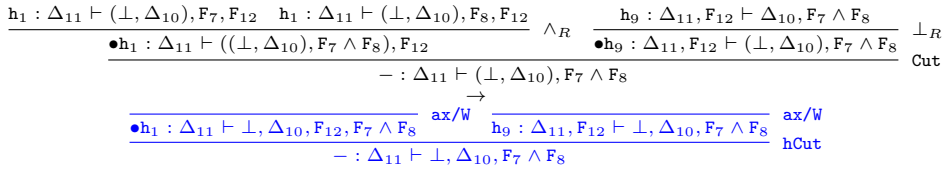
- Case rule $\wedge_R$



- Case rule $\vee_R$



- Case rule $\bot_R$



$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\,\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_9}\,\bot_R}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{hCut}}{\to}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7, \mathtt{F}_{12} \quad \mathtt{h}_1 : \Delta_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_8, \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_{11} \vdash ((\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12}}\,\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\top_R}{- : \Delta_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Delta_{11} \vdash \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\top_R}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\,\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_9}\,\top_R}{- : \Delta_{10} \vdash \top, \Delta_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_9}\,\top_R$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_{14} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14}}\,\wedge_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}}{\bullet \mathtt{h}_9 : (\Delta_{13},}}{- : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7}\,\to_L}{\mathtt{hCut}}}{- : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}}}{- : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \quad \mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\wedge_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\to_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\wedge_R \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}}{- : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{sCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}$$

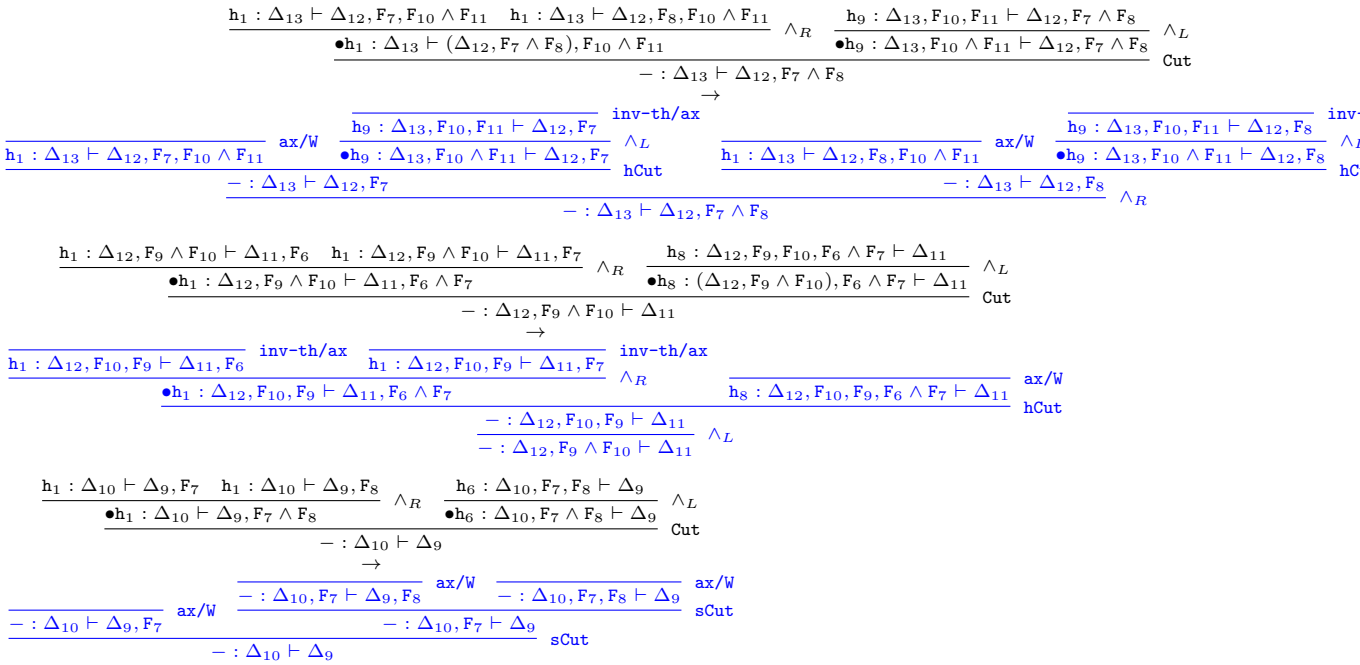
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}}\,\to_L}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{inv\text{-}th/}}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\to_L}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{sCut}}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{sCut}$$

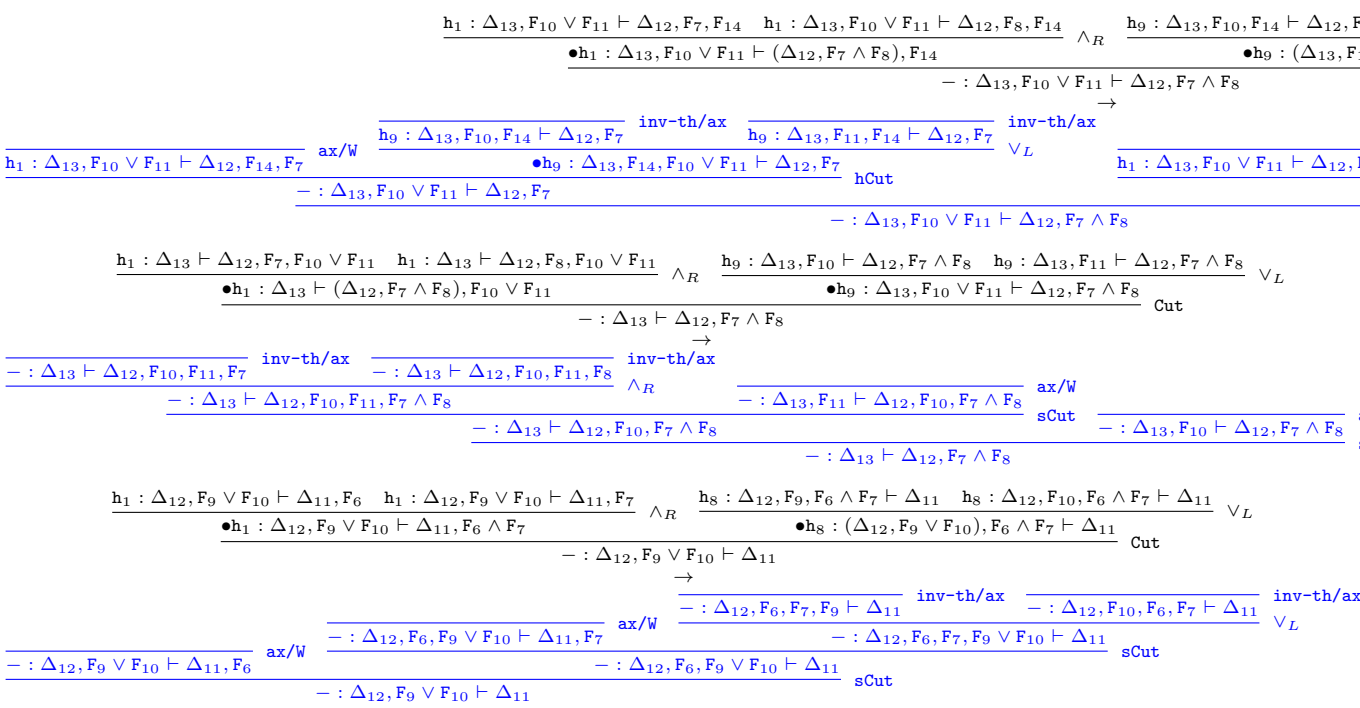
- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_{14} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14}}\,\wedge_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\wedge_L}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\wedge_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\wedge_L$$



- Case rule $\vee_L$



- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \bot \quad \mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8, \bot}{\bullet \mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \bot}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{11}, \bot \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8}\bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_{12} \quad \mathtt{h}_1 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \bot, \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12}}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : (\bot, \Delta_{11}), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\bot_L}{- : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\bot_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \quad \mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{p}_{11} \quad \mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_8, \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{12} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{11}}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}I}{- : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}}I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_{13} \quad \mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_8, \mathtt{F}_{13}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{11} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13}}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_{13} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}I}{- : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}I$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9, \mathtt{p}_{10}}I}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \top \quad \mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8, \top}{\bullet \mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \top}\wedge_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_9 : \Delta_{11}, \top \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\top_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_{12} \quad \mathtt{h}_1 : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \top, \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12}}\wedge_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_9 : (\top, \Delta_{11}), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\top_L}{- : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \quad \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{hCut}$$

8.3 Status of $\vee_R$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash ((\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14}}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\rightarrow_R}{- : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\rightarrow_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}), \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\rightarrow_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash ((\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14}}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\wedge_R}{- : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}\wedge_R}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\vee_R}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14}}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R}{- : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{12}} \vee_R \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9} \vee_R}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}{}\,\mathtt{ax/W}}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathit{height}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}} \vee_R}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}} \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash ((\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12}} \vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8} \bot_R}{- : \Delta_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\bot, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_9} \bot_R}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_9}\,\mathtt{ax/W}}{- : \Delta_{10} \vdash \bot, \Delta_9}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash ((\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12}} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8} \top_R}{- : \Delta_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{11} \vdash \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \top_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash (\top, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_9} \top_R}{- : \Delta_{10} \vdash \top, \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{10} \vdash \top, \Delta_9} \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14}} \vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8} \rightarrow_L}{- : \Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8} \rightarrow_L}{- : \Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \to \mathtt{F}_{11}} \vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8} \to_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \to_L}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8} \vee_R}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}} \to_L}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}} \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14}} \vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8} \wedge_L}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \wedge_L}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8} \vee_R}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8} \wedge_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \wedge_L}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8} \vee_R}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut} \wedge_L}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14}} \vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_L}{- : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \vee_L}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8} \vee_R}{- : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_L}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\mathtt{ax/W}} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{\mathtt{inv\text{-}th/ax}} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}{\mathtt{inv\text{-}th/ax}} \vee_L}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8} \vee_R}{- : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\mathtt{inv\text{-}th/ax}}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}} \quad \dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\mathtt{inv\text{-}th/ax}}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{ax/W}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9} \vee_L}{- : \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8}{\mathtt{ax/W}} \quad \dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_7}{\mathtt{ax/W}}\,\mathtt{sCut}}{- : \Delta_{10} \vdash \Delta_9, \mathtt{F}_7} \quad \dfrac{\mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_9}{\mathtt{ax/W}}\,\mathtt{sCut}}{- : \Delta_{10} \vdash \Delta_9}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \bot}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \bot} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{11}, \bot \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8}\,\bot_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \bot, \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12}} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : (\bot, \Delta_{11}), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\bot_L}{- : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\bot_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\,\bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{11}} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,I}{- : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}\,I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{13}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{11} \vdash ((\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13}} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_{13} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,I}{- : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\,I$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9, \mathtt{p}_{10}} I}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \top}{\bullet\mathtt{h}_1 : \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \top}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : \Delta_{11}, \top \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\top_L}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \top, \Delta_{11} \vdash (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12}}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : (\top, \Delta_{11}), \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\top_L}{- : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W}}{- : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{hCut}$$

8.4 Status of $\bot_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \to \mathtt{F}_6), \bot}\bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6}{\bullet\mathtt{h}_4 : \Delta_8, \bot \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\to_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_{10}}\bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_7}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\to_R}{- : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}{\bullet\mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6), \bot}\bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \quad \mathtt{h}_4 : \bot, \Delta_8 \vdash \Delta_7, \mathtt{F}_6}{\bullet\mathtt{h}_4 : \Delta_8, \bot \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\wedge_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10}}\bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_7}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R}{- : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_8, \bot \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10}}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R}{- : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{hCut}}\ \to$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \bot, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_5), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_6, \bot \vdash \bot, \Delta_5}\ \bot_R}{- : \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \Delta_6, \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\bot, \Delta_6), \mathtt{F}_8}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_6}\ \bot_R}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_7 \vdash \bot, \Delta_6, \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{hCut}}\ \to$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \top, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_5), \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_6, \bot \vdash \top, \Delta_5}\ \top_R}{- : \Delta_6 \vdash \top, \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_6 \vdash \top, \Delta_5}\ \top_R}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\bot, \top, \Delta_6), \mathtt{F}_8}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \bot, \top, \Delta_6}\ \top_R}{- : \Delta_7 \vdash \bot, \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \bot, \top, \Delta_6}\ \top_R}\ \to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \quad \mathtt{h}_4 : \bot, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \to \mathtt{F}_6), \bot \vdash \Delta_7}\ \to_L}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash (\bot, \Delta_8), \mathtt{F}_{10}}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \bot, \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{F}_{10} \vdash \bot, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_{10} \vdash \bot, \Delta_8}\ \to_L}{- : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8, \mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{hCut}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8), \mathtt{F}_6 \to \mathtt{F}_7}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7 \vdash \bot, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \to_L}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8}\ \mathtt{hCut}}\ \to$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \bot}\,\bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \bot \vdash \Delta_7}\,\wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash (\bot, \Delta_8), \mathtt{F}_{10}}\,\bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_{10} \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10} \vdash \bot, \Delta_8}\,\wedge_L}{- : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8, \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\,\mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8), \mathtt{F}_6 \wedge \mathtt{F}_7}\,\bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7 \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\,\wedge_L}{- : \Delta_9 \vdash \bot, \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \bot, \Delta_8}\,\mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8}\,\mathtt{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \bot}\,\bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8, \mathtt{F}_5 \vdash \Delta_7 \quad \mathtt{h}_4 : \bot, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \bot \vdash \Delta_7}\,\vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash (\bot, \Delta_8), \mathtt{F}_{10}}\,\bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_{10} \vdash \bot, \Delta_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{F}_{10} \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10} \vdash \bot, \Delta_8}\,\vee_L}{- : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8, \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\,\mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_9 \vdash (\bot, \Delta_8), \mathtt{F}_6 \vee \mathtt{F}_7}\,\bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vdash \bot, \Delta_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7 \vdash \bot, \Delta_8}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\,\vee_L}{- : \Delta_9 \vdash \bot, \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_9 \vdash \bot, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \bot, \Delta_8}\,\mathtt{ax/W}}{- : \Delta_9 \vdash \bot, \Delta_8}\,\mathtt{hCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \Delta_6 \vdash \Delta_5, \bot}\,\bot_R \quad \dfrac{}{\bullet\mathtt{h}_4 : \Delta_6, \bot \vdash \Delta_5}\,\bot_L}{- : \Delta_6 \vdash \Delta_5}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_6 \vdash \Delta_5}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \Delta_6, \bot}{\bullet\mathtt{h}_1 : \Delta_7 \vdash (\bot, \Delta_6), \bot}\,\bot_R \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_7, \bot \vdash \bot, \Delta_6}\,\bot_L}{- : \Delta_7 \vdash \bot, \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_7 \vdash \bot, \Delta_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_7 \vdash \Delta_6, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \bot, \Delta_7 \vdash (\bot, \Delta_6), \mathtt{F}_8}\,\bot_R \quad \dfrac{}{\bullet\mathtt{h}_5 : (\bot, \Delta_7), \mathtt{F}_8 \vdash \bot, \Delta_6}\,\bot_L}{- : \bot, \Delta_7 \vdash \bot, \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_7 \vdash \bot, \Delta_6}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash (\Delta_5, \mathtt{p}_6), \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\Delta_7, \mathtt{p}_6), \bot \vdash \Delta_5, \mathtt{p}_6}\ I}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ I}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\bot, \Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8 \vdash \bot, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_8 \vdash \bot, \Delta_6, \mathtt{p}_7, \mathtt{p}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8 \vdash \bot, \Delta_6, \mathtt{p}_7}\ \mathtt{hCut}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{F}_9}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{p}_7 \vdash (\bot, \Delta_6, \mathtt{p}_7), \mathtt{F}_9}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_8, \mathtt{p}_7), \mathtt{F}_9 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{p}_7 \vdash \bot, \Delta_6, \mathtt{p}_7}\ I}\ \to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : (\top, \Delta_6), \bot \vdash \Delta_5}\ \top_L}{- : \top, \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \top, \Delta_6 \vdash \Delta_5}\ \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \Delta_6, \top}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\bot, \Delta_6), \top}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_7 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_7, \top \vdash \bot, \Delta_6}\ \top_L}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_7 \vdash \Delta_6, \mathtt{F}_8}{\bullet \mathtt{h}_1 : \top, \Delta_7 \vdash (\bot, \Delta_6), \mathtt{F}_8}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_5 : (\top, \Delta_7), \mathtt{F}_8 \vdash \bot, \Delta_6}\ \top_L}{- : \top, \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_7 \vdash \bot, \Delta_6, \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_5 : \top, \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \top, \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{hCut}}\ \to$$

8.5 Status of $\top_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \to \mathtt{F}_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_8, \top \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vdash \top, \Delta_7, \mathtt{F}_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_{10}}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_{10} \vdash \top, \Delta_8, \mathtt{F}_7}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R}{- : \Delta_9 \vdash \top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_9 \vdash \top, \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\ \top_R}\ \to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \quad \mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_8, \top \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10}}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \top, \Delta_8, \mathtt{F}_7}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R}{- : \Delta_9 \vdash \top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_9 \vdash \top, \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \top_R}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6}{\bullet \mathtt{h}_4 : \Delta_8, \top \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10}}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \top, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R}{- : \Delta_9 \vdash \top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_9 \vdash \top, \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \top_R}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_6, \top \vdash \bot, \Delta_5}\ \bot_R}{- : \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \bot, \top, \Delta_5}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_5}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\top, \bot, \Delta_6), \mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \top, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \top, \bot, \Delta_6}\ \bot_R}{- : \Delta_7 \vdash \top, \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \bot, \top, \Delta_6}\ \top_R}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_5), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_6, \top \vdash \top, \Delta_5}\ \top_R}{- : \Delta_6 \vdash \top, \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_6 \vdash \top, \Delta_5}\ \top_R}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \mathtt{F}_8}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \top, \Delta_6}\ \top_R}{- : \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \top, \Delta_6}\ \top_R}$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \quad \mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \to \mathtt{F}_6), \top \vdash \Delta_7}\ \to_L}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \to_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash (\top, \Delta_8), \mathtt{F}_{10}}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_{10} \vdash \top, \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{F}_{10} \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_{10} \vdash \top, \Delta_8}\ \to_L}{- : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \top, \Delta_8}\ \mathtt{Cut}}{\to}}{- : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \top, \Delta_8}\ \top_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8), \mathtt{F}_6 \to \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9 \vdash \top, \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7 \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \top, \Delta_8}\ \to_L}{- : \Delta_9 \vdash \top, \Delta_8}\ \mathtt{Cut}}{\to}}{- : \Delta_9 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \top \vdash \Delta_7}\ \wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash (\top, \Delta_8), \mathtt{F}_{10}}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_{10} \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10} \vdash \top, \Delta_8}\ \wedge_L}{- : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_8}\ \mathtt{Cut}}{\to}}{- : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_8}\ \top_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8), \mathtt{F}_6 \wedge \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7 \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \top, \Delta_8}\ \wedge_L}{- : \Delta_9 \vdash \top, \Delta_8}\ \mathtt{Cut}}{\to}}{- : \Delta_9 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7 \quad \mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \top \vdash \Delta_7}\ \vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \vee_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash (\top, \Delta_8), \mathtt{F}_{10}}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_{10} \vdash \top, \Delta_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{F}_{10} \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10} \vdash \top, \Delta_8}\ \vee_L}{- : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_8}\ \mathtt{Cut}}{\to}}{- : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_8}\ \top_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9 \vdash (\top, \Delta_8), \mathtt{F}_6 \vee \mathtt{F}_7}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vdash \top, \Delta_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7 \vdash \top, \Delta_8}{\bullet \mathtt{h}_5 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \top, \Delta_8}\ \vee_L}{- : \Delta_9 \vdash \top, \Delta_8}\ \mathtt{Cut}}{\to}}{- : \Delta_9 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_6 \vdash \Delta_5, \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\bot, \Delta_6), \top \vdash \Delta_5}\ \bot_L}{- : \bot, \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \Delta_5}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \bot}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_7, \bot \vdash \top, \Delta_6}\ \bot_L}{- : \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_7 \vdash (\top, \Delta_6), \mathtt{F}_8}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \mathtt{F}_8 \vdash \top, \Delta_6}\ \bot_L}{- : \bot, \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_6 \vdash (\Delta_5, \mathtt{p}_6), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\Delta_7, \mathtt{p}_6), \top \vdash \Delta_5, \mathtt{p}_6}\ I}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_6 \vdash \Delta_5, \mathtt{p}_6}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash (\top, \Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_8, \mathtt{p}_7 \vdash \top, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8 \vdash \top, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8 \vdash \top, \Delta_6, \mathtt{p}_7}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{p}_7 \vdash (\top, \Delta_6, \mathtt{p}_7), \mathtt{F}_9}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_8, \mathtt{p}_7), \mathtt{F}_9 \vdash \top, \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8, \mathtt{p}_7 \vdash \top, \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_8, \mathtt{p}_7 \vdash \top, \Delta_6, \mathtt{p}_7}\ \top_R$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_5, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_6, \top \vdash \Delta_5}\ \top_L}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6 \vdash \Delta_5}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_7 \vdash \top, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_7, \top \vdash \top, \Delta_6}\ \top_L}{- : \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_7 \vdash (\top, \Delta_6), \mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \top, \Delta_6}{\bullet \mathtt{h}_5 : (\top, \Delta_7), \mathtt{F}_8 \vdash \top, \Delta_6}\ \top_L}{- : \top, \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \top, \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

8.6 Status of $\rightarrow_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_{14} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{14}}\,{\to_L} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,{\to_R}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,{\to_L} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,{\to_R}\,\mathtt{hCut}}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_{14} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14}}\,{\to_L} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash}{\bullet\mathtt{h}_9 : (\Delta_{13},}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,{\to}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,{\to_L} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{13},}}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_{14} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{14}}\,{\to_L} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,{\vee_R}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\,{\to_L} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,{\vee_R}\,\mathtt{hCut}}$$

- Case rule $\bot_R$

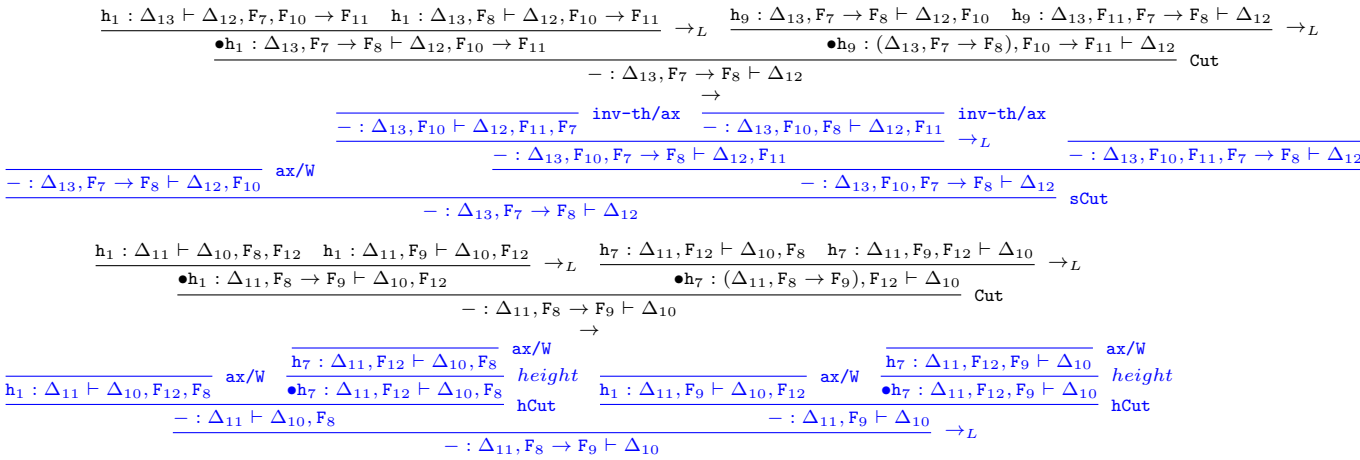
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash (\bot, \Delta_{10}), \mathtt{F}_7, \mathtt{F}_{12} \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{12}}\,{\to_L} \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12} \vdash \bot, \Delta_{10}}\,{\bot_R}}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \bot, \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{hCut}}$$

- Case rule $\top_R$

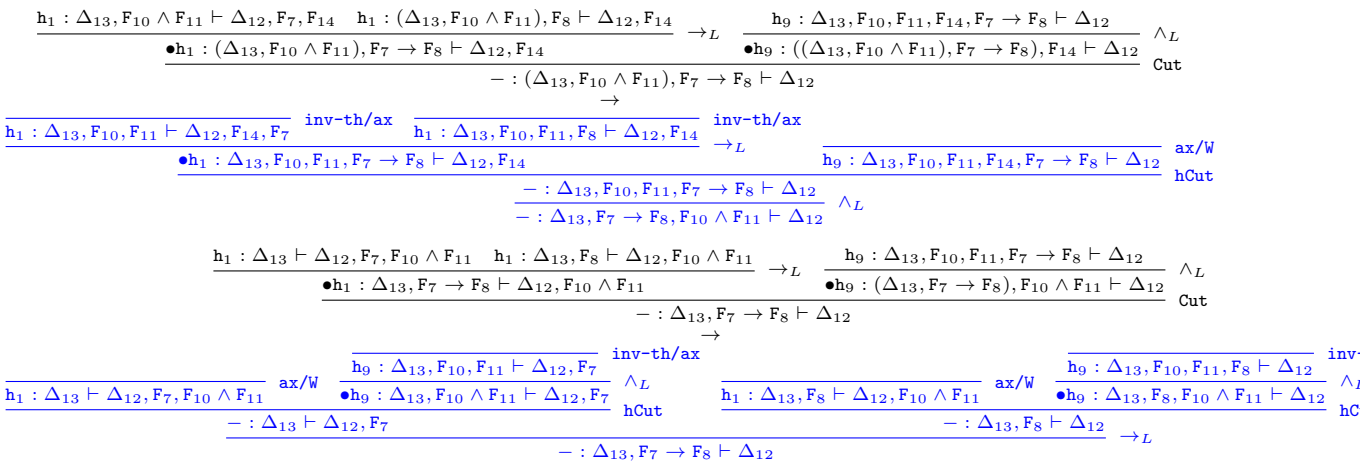
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash (\top, \Delta_{10}), \mathtt{F}_7, \mathtt{F}_{12} \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{12}}\,{\to_L} \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12} \vdash \top, \Delta_{10}}\,{\top_R}}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \top, \Delta_{10}}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \top, \Delta_{10}}\,{\top_R}}$$

- Case rule $\to_L$

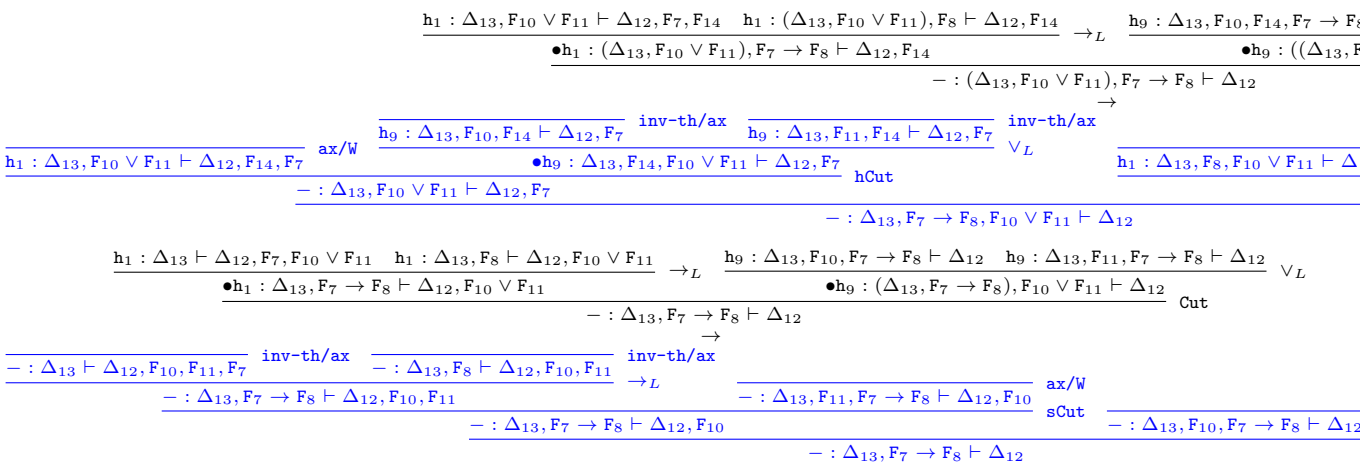
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_{14} \quad \mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}\,{\to_L} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash}{\bullet\mathtt{h}_9 : ((\Delta_{13},}}{- : (\Delta_{13}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}}\,{\to}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13} \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,{\to_L} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{13},}}$$



- Case rule $\wedge_L$



- Case rule $\vee_L$



- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \bot \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}, \bot}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \bot}\,{\to_L} \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \bot \vdash \Delta_{10}}\,{\bot_L}}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11} \vdash \bot, \Delta_{10}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\,{\bot_L}}{- : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}}\,{\bot_L}}{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,{\to_L}}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_{12} \quad \mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,{\to_L} \quad \dfrac{}{\bullet \mathtt{h}_9 : ((\bot, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}}\,{\bot_L}}{- : (\bot, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,{\bot_L}}\,\to$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{p}_{11} \quad \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}\,{\to_L} \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\,I}{- : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\,I}{- : \Delta_{12} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\,I}{- : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,{\to_L}}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{p}_{11} \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_{13} \quad \mathtt{h}_1 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{13}}{\bullet \mathtt{h}_1 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{13}}\,{\to_L} \quad \dfrac{}{\bullet \mathtt{h}_9 : ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{10}, \mathtt{p}_{11}}\,I}{- : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,I}\,\to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \top \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}, \top}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \top}\,{\to_L} \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \top \vdash \Delta_{10}}\,{\top_L}}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}}\,\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{11} \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_{12} \quad \mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,{\to_L} \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : ((\top, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}}\,{\top_L}}{- : (\top, \Delta_{11}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{hCut}}\,\to$$

8.7 Status of $\wedge_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{14}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{14}}\,{\wedge_L} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,{\to_R}}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}}\,{\wedge_L} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,{\to_R}}\,\to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\wedge_R}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{inv\text{-}th/ax}\wedge_R}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\wedge_L$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{14}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{14}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\vee_R}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}}\wedge_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{12}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12} \vdash \bot, \Delta_{10}}\bot_R}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}, \mathtt{F}_{12}}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}}{}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{12}}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12} \vdash \top, \Delta_{10}}\top_R}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \top, \Delta_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \top, \Delta_{10}}\top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet \mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : ((\Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}}\rightarrow_L}{- : (\Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax}\rightarrow_L}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}}\wedge_L}{- : \Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}}\rightarrow_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax}\rightarrow_L}{\bullet \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\wedge_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : ((\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}}\,\wedge_L}{- : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\wedge_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\,\wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\,\wedge_L}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\wedge_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{12} \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{12} \vdash \Delta_{10}}\,\wedge_L}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\,\mathit{height}}{- : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{10}}\,\wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : ((\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}}\,\vee_L}{- : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\,\vee_L}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\,\vee_L}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}}\,\vee_L}{- : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{12}}\,\wedge_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \bot}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \bot}\,\wedge_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \bot \vdash \Delta_{10}}\,\bot_L}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}}\,\bot_L}{- : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\wedge_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\bot, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}}\,\bot_L}{- : (\bot, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}\wedge_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\,I}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\,I}{- : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{13}}{\bullet\mathtt{h}_1 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{13}}\wedge_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{10}, \mathtt{p}_{11}}\,I}{- : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{- : \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \top}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \top}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \top \vdash \Delta_{10}}\,\top_L}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{- : \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : ((\top, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}}\,\top_L}{- : (\top, \Delta_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\bullet\mathtt{h}_1 : \top, \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{hCut}$$

8.8 Status of $\vee_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{14} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{F}_{14}}\vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\to_R}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\mathtt{Cut}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}}\,\vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}}\,\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_{14}}\vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\to}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}\,\vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}\,\mathtt{hCut} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13},}$$

$$- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{14} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_{14}} \vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_R}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}} \mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}} \mathtt{inv\text{-}th/ax} \vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}} \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}} \mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash (\bot, \Delta_{10}), \mathtt{F}_{12} \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_{12}} \vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12} \vdash \bot, \Delta_{10}} \bot_R}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}, \mathtt{F}_{12}}{} \mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}}{} \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}} \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash (\top, \Delta_{10}), \mathtt{F}_{12} \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_{12}} \vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12} \vdash \top, \Delta_{10}} \top_R}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \top, \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \top, \Delta_{10}} \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{14} \quad \mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}} \vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}} \rightarrow_L}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}} \mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{14}} \mathtt{inv\text{-}th/ax} \vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}} \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}} \mathtt{hCut} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13},}}{- : \Delta_{13}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}} \vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \rightarrow \mathtt{F}_{11} \vdash \Delta_{12}} \rightarrow_L}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{11}}{} \mathtt{inv\text{-}th/ax} \quad \dfrac{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}}{} \mathtt{inv\text{-}th/ax}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{11}} \vee_L \quad \dfrac{}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{sCut}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{14} \quad \mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}} \vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : ((\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{14} \vdash \Delta_{12}} \wedge_L}{- : (\Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}} \mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}} \mathtt{inv\text{-}th/ax} \vee_L \quad \dfrac{}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}} \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}} \wedge_L}{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}} \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{12}}{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}} \wedge_L}{- : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{hC}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \vee_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{14} \quad \mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}} \vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_9 : ((\Delta_{13}, \mathtt{F}}}{- : (\Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax} \vee_L}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_{14}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{10} \vee \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \quad \mathtt{h}_1 : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}} \vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12} \quad \mathtt{h}_9 : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}} \vee_L}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{- : \Delta_{13}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{- : \Delta_{13}, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}}\,\mathtt{inv\text{-}th/ax} \vee_L}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}} \quad \dfrac{- : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{10}}{- : \Delta_{13}, \mathtt{F}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{sCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12} \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{12}} \vee_L \quad \dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_{12} \vdash \Delta_{10} \quad \mathtt{h}_7 : \Delta_{11}, \mathtt{F}_9, \mathtt{F}_{12} \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : (\Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{12} \vdash \Delta_{10}} \vee_L}{- : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{height}}{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}} \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{12}}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_9 \vdash \Delta_{10}}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_7 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{height}}{- : \Delta_{11}, \mathtt{F}_9 \vdash \Delta_{10}} \mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}} \vee_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \bot \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}, \bot}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \bot} \vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \bot \vdash \Delta_{10}} \bot_L}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash \bot, \Delta_{10}}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash \bot, \Delta_{10}}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}}\,\bot_L}{- : \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}} \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \bot, \Delta_{10}}{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : \bot, \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}}\,\bot_L}{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}} \mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}} \vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{12} \quad \mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : (\bot, \Delta_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}} \vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\bot, \Delta_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}} \bot_L}{- : (\bot, \Delta_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}} \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11} \quad \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{p}_{11}}\vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_7, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : \Delta_{12}, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{13} \quad \mathtt{h}_1 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{13}}{\bullet\mathtt{h}_1 : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{p}_{11}), \mathtt{F}_{13}}\vee_L \quad \dfrac{}{\bullet\mathtt{h}_9 : ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{13} \vdash \Delta_{10}, \mathtt{p}_{11}}\, I}{- : (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{p}_{11}}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \top \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_{10}, \top}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \top}\vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \top \vdash \Delta_{10}}\,\top_L}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{12} \quad \mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : (\top, \Delta_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\vee_L \quad \dfrac{\mathtt{h}_9 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : ((\top, \Delta_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_{12} \vdash \Delta_{10}}\,\top_L}{- : (\top, \Delta_{11}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \top, \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{hCut}$$

8.9 Status of $\bot_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{F}_{10}}\,\bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_6, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_7}{\bullet\mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\to_R}{- : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \to \mathtt{F}_7}\,\bot_L$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10}}\,\bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_7}{\bullet\mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\wedge_R}{- : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\bot_L$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10}}\,\bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\vee_R}{- : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{Cut}}{\to}$$

$$\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\bot_L$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_7 \vdash (\bot, \Delta_6), \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_7, \mathtt{F}_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \mathtt{F}_8 \vdash \bot, \Delta_6}\ \bot_R}{- : \bot, \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{Cut}$$
$$\Downarrow \rightarrow$$
$$\dfrac{}{- : \bot, \Delta_7 \vdash \bot, \Delta_6}\ \bot_L$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_7 \vdash (\top, \Delta_6), \mathtt{F}_8}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \mathtt{F}_8 \vdash \top, \Delta_6}\ \top_R}{- : \bot, \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\Downarrow \rightarrow$$
$$\dfrac{}{- : \bot, \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_7, \mathtt{F}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_{10} \vdash \Delta_8}\ \rightarrow_L}{- : \bot, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\Downarrow \rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_7 \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \rightarrow_L}{- : \bot, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\Downarrow \rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8}\ \bot_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10} \vdash \Delta_8}\ \wedge_L}{- : \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\Downarrow \rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\ \wedge_L}{- : \bot, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\Downarrow \rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8}\ \bot_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_6, \mathtt{F}_{10} \vdash \Delta_8 \quad \mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_7, \mathtt{F}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10} \vdash \Delta_8}\ \vee_L}{- : \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\Downarrow \rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_6 \vdash \Delta_8 \quad \mathtt{h}_5 : \bot, \Delta_9, \mathtt{F}_7 \vdash \Delta_8}{\bullet \mathtt{h}_5 : (\bot, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\ \vee_L}{- : \bot, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\Downarrow \rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9 \vdash \Delta_8}\ \bot_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_7 \vdash \Delta_6, \mathtt{F}_8}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \mathtt{F}_8 \vdash \Delta_6}\ \bot_L}{- : \bot, \Delta_7 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\Downarrow \rightarrow$$
$$\dfrac{}{- : \bot, \Delta_7 \vdash \Delta_6}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8), \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ I}{- : \bot, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_8, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{F}_9}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_5 : (\bot, \Delta_8, \mathtt{p}_7), \mathtt{F}_9 \vdash \Delta_6, \mathtt{p}_7}\ I}{- : \bot, \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ \bot_L$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_7 \vdash \Delta_6, \top}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_7 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\bot, \Delta_7), \top \vdash \Delta_6}\ \top_L}{- : \bot, \Delta_7 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_7 \vdash \Delta_6}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_7 \vdash \Delta_6, \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_7, \mathtt{F}_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\bot, \top, \Delta_7), \mathtt{F}_8 \vdash \Delta_6}\ \top_L}{- : \bot, \top, \Delta_7 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \top, \Delta_7 \vdash \Delta_6}\ \bot_L$$

8.10 Status of I: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{12}}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{F}_8, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_{12} \vdash (\Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{p}_{10}}\ \rightarrow_R}{- : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{p}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash (\Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{p}_{10}}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}\ \begin{matrix}\mathtt{ax/W}\\ \mathtt{hCut}\end{matrix}}{- : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{12}}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{F}_8, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_{12} \vdash (\Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}}\ \wedge_R}{- : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10}}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\ \begin{matrix}\mathtt{ax/W}\\ \mathtt{hCut}\end{matrix}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7, \mathtt{p}_{10}}\ I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}\ \begin{matrix}\mathtt{ax/W}\\ \mathtt{hCut}\end{matrix}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_7}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash ((\Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{12}}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_{12} \vdash (\Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}}\, \vee_R}{- : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\, I}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_5 : (\Delta_9, \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\, \vee_R}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\, \vee_R\, \to$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash ((\bot, \Delta_7), \mathtt{p}_8), \mathtt{F}_{10}}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{F}_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_{10} \vdash (\bot, \Delta_7), \mathtt{p}_8}\, \bot_R}{- : \Delta_9, \mathtt{p}_8 \vdash (\bot, \Delta_7), \mathtt{p}_8}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \bot, \Delta_7, \mathtt{p}_8}\, I}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_8 \vdash (\bot, \Delta_6), \mathtt{p}_8}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_8), \mathtt{p}_8 \vdash \bot, \Delta_6}\, \bot_R}{- : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_6}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_6, \mathtt{p}_8}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \bot, \Delta_6}\, \mathtt{ax/W}}{- : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_6}\, \mathtt{hCut}}\, \to$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash ((\top, \Delta_7), \mathtt{p}_8), \mathtt{F}_{10}}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_{10} \vdash (\top, \Delta_7), \mathtt{p}_8}\, \top_R}{- : \Delta_9, \mathtt{p}_8 \vdash (\top, \Delta_7), \mathtt{p}_8}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \top, \Delta_7, \mathtt{p}_8}\, \top_R}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_8 \vdash (\top, \Delta_6), \mathtt{p}_8}\, I \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_7, \mathtt{p}_8), \mathtt{p}_8 \vdash \top, \Delta_6}\, \top_R}{- : \Delta_7, \mathtt{p}_8 \vdash \top, \Delta_6}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_7, \mathtt{p}_8 \vdash \top, \Delta_6}\, \top_R}\, \to$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_{12}}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : ((\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{12} \vdash \Delta_9, \mathtt{p}_{10}}\, \to_L}{- : (\Delta_{11}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, I}\, \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, \to_L}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I}\, \to$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8}\, \to_L}{- : (\Delta_9, \mathtt{F}_6 \to \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{F}_6}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_8}\, \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_{12}}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : ((\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{12} \vdash \Delta_9, \mathtt{p}_{10}}\, \wedge_L}{- : (\Delta_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, \wedge_L}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8}\, \wedge_L}{- : (\Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\, \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_{12}}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_7, \mathtt{F}_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_{12}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : ((\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10}), \mathtt{F}_{12} \vdash \Delta_9, \mathtt{p}_{10}}\, \vee_L}{- : (\Delta_{11}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_{10} \vdash (\Delta_9, \mathtt{p}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{11}, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10} \quad \mathtt{h}_6 : \Delta_{11}, \mathtt{F}_8, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}{\bullet \mathtt{h}_6 : (\Delta_{11}, \mathtt{p}_{10}), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{p}_{10}}\, \vee_L}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_{10} \vdash \Delta_9, \mathtt{p}_{10}}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8 \quad \mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}{\bullet \mathtt{h}_5 : ((\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10}), \mathtt{p}_{10} \vdash \Delta_8}\, \vee_L}{- : (\Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_6, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8, \mathtt{p}_{10}}\, I \quad \dfrac{}{\mathtt{h}_5 : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10}, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_7, \mathtt{p}_{10} \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\, \vee_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \bot}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_8), \bot \vdash \Delta_7, \mathtt{p}_8}\, \bot_L}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, \mathtt{Cut}$$

$$\to$$

$$\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_9), \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \mathtt{F}_{10}}\ I \quad \dfrac{}{\bullet \mathtt{h}_6 : ((\bot, \Delta_9), \mathtt{p}_8), \mathtt{F}_{10} \vdash \Delta_7, \mathtt{p}_8}\ \bot_L}{- : (\bot, \Delta_9), \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \bot, \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_7), \mathtt{p}_8 \vdash \Delta_6, \mathtt{p}_8}\ I \quad \dfrac{}{\bullet \mathtt{h}_5 : ((\bot, \Delta_7), \mathtt{p}_8), \mathtt{p}_8 \vdash \Delta_6}\ \bot_L}{- : (\bot, \Delta_7), \mathtt{p}_8 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \bot, \Delta_7, \mathtt{p}_8 \vdash \Delta_6}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_8 \vdash ((\Delta_7, \mathtt{p}_9), \mathtt{p}_8), \mathtt{p}_9}\ I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_{10}, \mathtt{p}_8), \mathtt{p}_9 \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\ I}{- : \Delta_{10}, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8, \mathtt{p}_9}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_8 \vdash ((\Delta_7, \mathtt{p}_9), \mathtt{p}_8), \mathtt{F}_{11}}\ I \quad \dfrac{}{\bullet \mathtt{h}_6 : ((\Delta_{10}, \mathtt{p}_9), \mathtt{p}_8), \mathtt{F}_{11} \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\ I}{- : (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_9), \mathtt{p}_8}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_{10}, \mathtt{p}_8, \mathtt{p}_9 \vdash \Delta_7, \mathtt{p}_8, \mathtt{p}_9}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \mathtt{F}_{10}}\ I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_8), \mathtt{F}_{10} \vdash \Delta_7, \mathtt{p}_8}\ I}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}\ I \quad \dfrac{}{\bullet \mathtt{h}_5 : (\Delta_8, \mathtt{p}_7), \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ I}{- : \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_8, \mathtt{p}_7), \mathtt{p}_9 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_9}\ I \quad \dfrac{}{\bullet \mathtt{h}_5 : ((\Delta_8, \mathtt{p}_7), \mathtt{p}_9), \mathtt{p}_9 \vdash \Delta_6, \mathtt{p}_7}\ I}{- : (\Delta_8, \mathtt{p}_7), \mathtt{p}_9 \vdash \Delta_6, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_8, \mathtt{p}_7, \mathtt{p}_9 \vdash \Delta_6, \mathtt{p}_7}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \top}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_8), \top \vdash \Delta_7, \mathtt{p}_8}\ \top_L}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\top, \Delta_9), \mathtt{p}_8 \vdash (\Delta_7, \mathtt{p}_8), \mathtt{F}_{10}}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_9, \mathtt{F}_{10}, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}{\bullet \mathtt{h}_6 : ((\top, \Delta_9), \mathtt{p}_8), \mathtt{F}_{10} \vdash \Delta_7, \mathtt{p}_8}\ \top_L}{- : (\top, \Delta_9), \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{}{- : \top, \Delta_9, \mathtt{p}_8 \vdash \Delta_7, \mathtt{p}_8}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\top, \Delta_7), \mathtt{p}_8 \vdash \Delta_6, \mathtt{p}_8}\ I \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \Delta_6}{\bullet \mathtt{h}_5 : ((\top, \Delta_7), \mathtt{p}_8), \mathtt{p}_8 \vdash \Delta_6}\ \top_L}{- : (\top, \Delta_7), \mathtt{p}_8 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\xrightarrow{\quad}$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_7, \mathtt{p}_8 \vdash \Delta_6, \mathtt{p}_8}\ I \quad \dfrac{}{\mathtt{h}_5 : \top, \Delta_7, \mathtt{p}_8, \mathtt{p}_8 \vdash \Delta_6}\ \mathtt{ax/W}}{- : \top, \Delta_7, \mathtt{p}_8 \vdash \Delta_6}\ \mathtt{hCut}$$

8.11 Status of $\top_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_{10}}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_7}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R}{- : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10}}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_7}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R}{- : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash (\Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10}}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R}{- : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\ \mathtt{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash (\bot, \Delta_6), \mathtt{F}_8}{\bullet\mathtt{h}_1 : \top, \Delta_7 \vdash (\bot, \Delta_6), \mathtt{F}_8}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_7, \mathtt{F}_8 \vdash \Delta_6}{\bullet\mathtt{h}_5 : (\top, \Delta_7), \mathtt{F}_8 \vdash \bot, \Delta_6}\ \bot_R}{- : \top, \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_7 \vdash \bot, \Delta_6, \mathtt{F}_8}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_6}{}\ \mathtt{ax/W}}{- : \top, \Delta_7 \vdash \bot, \Delta_6}\ \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \mathtt{F}_8}{\bullet\mathtt{h}_1 : \top, \Delta_7 \vdash (\top, \Delta_6), \mathtt{F}_8}\ \top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_7), \mathtt{F}_8 \vdash \top, \Delta_6}\ \top_R}{- : \top, \Delta_7 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \top, \Delta_7 \vdash \top, \Delta_6}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10} \vdash \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_7, \mathtt{F}_{10} \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7), \mathtt{F}_{10} \vdash \Delta_8}\ \rightarrow_L}{- : \top, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10}, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}{}\ \mathtt{ax/W}}{- : \top, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \quad \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}\ \rightarrow_L}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \rightarrow \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_8}{}\ \mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8}\ \mathtt{hCut}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_{10} \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7), \mathtt{F}_{10} \vdash \Delta_8}\,\wedge_L}{- : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}{}\,\mathtt{ax/W}}{- : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}\,\wedge_L}{- : \top, \Delta_9 \vdash \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \wedge \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_8}{}\,\mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8}\,\mathtt{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6, \mathtt{F}_{10} \vdash \Delta_8 \quad \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_7, \mathtt{F}_{10} \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7), \mathtt{F}_{10} \vdash \Delta_8}\,\vee_L}{- : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}{}\,\mathtt{ax/W}}{- : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{\bullet\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \vdash \Delta_8 \quad \mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_5 : (\top, \Delta_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}\,\vee_L}{- : \top, \Delta_9 \vdash \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_9 \vdash \Delta_8, \mathtt{F}_6 \vee \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_8}{}\,\mathtt{ax/W}}{- : \top, \Delta_9 \vdash \Delta_8}\,\mathtt{hCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \Delta_6, \bot}{\bullet\mathtt{h}_1 : \top, \Delta_7 \vdash \Delta_6, \bot}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_7), \bot \vdash \Delta_6}\,\bot_L}{- : \top, \Delta_7 \vdash \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_7 \vdash \bot, \Delta_6}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \bot, \top, \Delta_7 \vdash \Delta_6}{}\,\bot_L}{- : \top, \Delta_7 \vdash \Delta_6}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_7 \vdash \Delta_6, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \top, \bot, \Delta_7 \vdash \Delta_6, \mathtt{F}_8}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \bot, \Delta_7), \mathtt{F}_8 \vdash \Delta_6}\,\bot_L}{- : \top, \bot, \Delta_7 \vdash \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \top, \Delta_7 \vdash \Delta_6}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}{\bullet\mathtt{h}_1 : \top, \Delta_8 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{p}_7}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_8), \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,I}{- : \top, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_8 \vdash \Delta_6, \mathtt{p}_7, \mathtt{p}_7}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_5 : \top, \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}{}\,I}{- : \top, \Delta_8 \vdash \Delta_6, \mathtt{p}_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{F}_9}{\bullet\mathtt{h}_1 : \top, \Delta_8, \mathtt{p}_7 \vdash (\Delta_6, \mathtt{p}_7), \mathtt{F}_9}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_5 : (\top, \Delta_8, \mathtt{p}_7), \mathtt{F}_9 \vdash \Delta_6, \mathtt{p}_7}\,I}{- : \top, \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \top, \Delta_8, \mathtt{p}_7 \vdash \Delta_6, \mathtt{p}_7}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \Delta_6, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \top, \Delta_7 \vdash \Delta_6, \mathtt{F}_8}\,\top_L \quad \dfrac{\mathtt{h}_5 : \Delta_7, \mathtt{F}_8 \vdash \Delta_6}{\bullet\mathtt{h}_5 : (\top, \Delta_7), \mathtt{F}_8 \vdash \Delta_6}\,\top_L}{- : \top, \Delta_7 \vdash \Delta_6}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 : \top, \Delta_7 \vdash \Delta_6, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \top, \Delta_7, \mathtt{F}_8 \vdash \Delta_6}\,\mathtt{ax/W}}{- : \top, \Delta_7 \vdash \Delta_6}\,\mathtt{hCut}$$