Modal Logic K+T+4+5

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Abstract

This system does not have cut-elimination.

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1 Height preserving admissibility of weakening on the left

• Case(s) rule \rightarrow_R

• Case(s) rule \wedge_R

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3 \land \mathbf{F}_4} \quad \wedge_R \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3}{\mathbf{h}_1: \Delta_2, \mathbf{F}_W \vdash \Delta_5, \mathbf{F}_3} \quad \mathbf{IH}}{\bullet \mathbf{h}_1: \Delta_2, \mathbf{F}_W \vdash \Delta_5, \mathbf{F}_3 \land \mathbf{F}_4} \quad \mathbf{IH}} \quad \frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_4}{\mathbf{h}_1: \Delta_2, \mathbf{F}_W \vdash \Delta_5, \mathbf{F}_4} \quad \mathbf{IH}} \quad \wedge_R \quad \wedge_$$

• Case(s) rule \vee_R

• Case(s) rule \perp_R

• Case(s) rule \top_R

$$\frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3} \ ^\top R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1 : \Delta_2, \mathbf{f}_W \vdash \top, \Delta_3} \ ^\top R$$

• Case(s) rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_4) \vdash \mathbf{f}_2}{\bullet \mathbf{h}_1: \Box \Gamma_4, \Delta_5 \vdash \Delta_3, []\mathbf{f}_2} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: unbox(\Box \Gamma_4) \vdash \mathbf{f}_2}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{f}_W, \Box \Gamma_4 \vdash \Delta_3, []\mathbf{f}_2} \quad K$$

• Case(s) rule A45

$$\frac{\mathbf{h}_1: \Box \mathbf{r}_5 \vdash \Box \mathbf{r}_3, \mathbf{f}_2}{\bullet \mathbf{h}_1: \Box \mathbf{r}_5, \Delta_6 \vdash \Box \mathbf{r}_3, \Delta_4, []\mathbf{f}_2} \quad A45 \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Box \mathbf{r}_5 \vdash \mathbf{f}_2, \Box \mathbf{r}_3} \quad ^{\mathrm{ax}}}{\bullet \mathbf{h}_1: \Delta_6, \mathbf{f}_W, \Box \mathbf{r}_5 \vdash \Delta_4, \Box \mathbf{r}_3, []\mathbf{f}_2} \quad A45$$

• Case(s) rule \rightarrow_L

$$\frac{\mathbf{h}_1:\Delta_5\vdash \mathbf{F}_2,\Delta_4\quad \mathbf{h}_1:\mathbf{F}_3,\Delta_5\vdash \Delta_4}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\to \mathbf{F}_3\vdash \Delta_4} \to_L \qquad \to \qquad \frac{\frac{\mathbf{h}_1:\Delta_5\vdash \Delta_4,\mathbf{F}_2}{\mathbf{h}_1:\Delta_5,\mathbf{F}_W\vdash \Delta_4,\mathbf{F}_2} \text{ If } \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash \Delta_4}{\mathbf{h}_1:\Delta_5,\mathbf{F}_3,\mathbf{F}_W\vdash \Delta_4} \overset{\text{ax}}{\to} L}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_W,\mathbf{F}_2\to \mathbf{F}_3\vdash \Delta_4} \to_L$$

• Case(s) rule \wedge_L

$$\frac{\underset{\bullet}{\mathbf{h}_1: \mathsf{F}_2, \mathsf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet}}{\underset{\bullet}{\mathbf{h}_1: \Delta_5, \mathsf{F}_2 \land \mathsf{F}_3 \vdash \Delta_4}{\bullet}} \wedge_L \qquad \rightarrow \qquad \frac{\overbrace{\underset{\bullet}{\mathbf{h}_1: \Delta_5, \mathsf{F}_2, \mathsf{F}_3 \vdash \Delta_4}^{\bullet}}^{\underbrace{\mathsf{h}_1: \Delta_5, \mathsf{F}_2, \mathsf{F}_3 \vdash \Delta_4}^{\bullet \mathsf{xx}}} \underset{\bullet}{\overset{\mathsf{nx}}{\mathsf{H}}} \underset{\mathsf{H}_1: \Delta_5, \mathsf{F}_2, \mathsf{F}_3 \vdash \Delta_4}{\bullet} \underset{\mathsf{h}_1: \Delta_5, \mathsf{F}_2, \mathsf{F}_3 \vdash \Delta_4}^{\bullet \mathsf{xx}}} \wedge_L$$

• Case(s) rule \vee_L

$$\frac{\mathbf{h}_1: \mathbf{f}_2, \Delta_5 \vdash \Delta_4 \quad \mathbf{h}_1: \mathbf{f}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{f}_2 \vee \mathbf{f}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \underbrace{\frac{\overset{\cdot}{\mathbf{h}_1}: \Delta_5, \mathbf{f}_2 \vdash \Delta_4}{\mathbf{h}_1: \Delta_5, \mathbf{f}_2, \mathbf{f}_W \vdash \Delta_4}}_{\bullet \mathbf{h}_1: \Delta_5, \mathbf{f}_W, \mathbf{f}_2 \vee \mathbf{f}_3 \vdash \Delta_4}^{\bullet \mathbf{h}_1: \Delta_5, \mathbf{f}_3, \mathbf{f}_W \vdash \Delta_4} \overset{\mathrm{int}}{}_{\vee_L}$$

 \bullet Case(s) rule AT

• Case(s) rule \perp_L

• Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3} \quad I \qquad \rightarrow \qquad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{f}_W, \mathtt{p}_3 \vdash \Delta_2, \mathtt{p}_3} \quad I$$

• Case(s) rule \top_L

2 Height preserving admissibility of weakening on the right

• Case(s) rule \rightarrow_R

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2 \vdash \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3 \to \mathbf{F}_4} \ \rightarrow_{R} \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_4}{\mathbf{h}_1: \Delta_2, \mathbf{F}_3 \vdash \Delta_5, \mathbf{F}_4, \mathbf{F}_W}}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_W, \mathbf{F}_3 \to \mathbf{F}_4} \xrightarrow{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_W, \mathbf{F}_3 \to \mathbf{F}_4} \rightarrow_{R}$$

• Case(s) rule \wedge_R

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3 \land \mathbf{F}_4} \quad \wedge_R \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3}{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_W} \quad \mathbf{IH} \quad \frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_4}{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_4, \mathbf{F}_W} \quad \mathbf{IH} \quad \frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_4}{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_4, \mathbf{F}_W} \quad \mathbf{IH} \quad \wedge_R \quad \wedge_R$$

• Case(s) rule \vee_R

• Case(s) rule \perp_R

• Case(s) rule \top_R

$$\frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3} \ ^\top R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3, \mathbf{f}_W} \ ^\top R$$

• Case(s) rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_4) \vdash \mathbf{f}_2}{\bullet \mathbf{h}_1: \Box \Gamma_4, \Delta_5 \vdash \Delta_3, []\mathbf{f}_2} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: unbox(\Box \Gamma_4) \vdash \mathbf{f}_2}}{\bullet \mathbf{h}_1: \Delta_5, \Box \Gamma_4 \vdash \Delta_3, \mathbf{f}_W, []\mathbf{f}_2} \quad K$$

• Case(s) rule A45

$$\frac{\mathbf{h}_1: \Box \Gamma_5 \vdash \Box \Gamma_3, \mathbf{f}_2}{\bullet \mathbf{h}_1: \Box \Gamma_5, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, []\mathbf{f}_2} \quad A45 \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Box \Gamma_5 \vdash \mathbf{f}_2, \Box \Gamma_3}}{\bullet \mathbf{h}_1: \Delta_6, \Box \Gamma_5 \vdash \Delta_4, \mathbf{f}_W, \Box \Gamma_3, []\mathbf{f}_2} \quad A45$$

• Case(s) rule \rightarrow_L

$$\frac{\mathbf{h}_1:\Delta_5\vdash \mathbf{F}_2,\Delta_4\quad \mathbf{h}_1:\mathbf{F}_3,\Delta_5\vdash \Delta_4}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\to \mathbf{F}_3\vdash \Delta_4} \to_L \qquad \to \qquad \frac{\frac{\overline{\mathbf{h}}_1:\Delta_5\vdash \Delta_4,\mathbf{F}_2}{\bullet \mathbf{h}_1:\Delta_5\vdash \Delta_4,\mathbf{F}_2}\quad \mathbf{m}}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\to \mathbf{F}_3\vdash \Delta_4,\mathbf{F}_W}\quad \mathbf{m}}_{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\to \mathbf{F}_3\vdash \Delta_4,\mathbf{F}_W}\quad \mathbf{m}} \xrightarrow{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash \Delta_4,\mathbf{F}_W}\quad \mathbf{m}}_{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash \Delta_4,\mathbf{F}_W}\quad \mathbf{m}}_{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\to\mathbf{F}_3\vdash \Delta_4,\mathbf{F}_W}\quad \mathbf{m}}$$

• Case(s) rule \wedge_L

• Case(s) rule \vee_L

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathbf{h}_1: \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \lor \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \underbrace{\frac{\overline{\mathbf{h}}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4}{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4, \mathbf{F}_W}}_{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \lor \mathbf{F}_3 \vdash \Delta_4, \mathbf{F}_W}^{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_3 \vdash \Delta_4, \mathbf{F}_W} \stackrel{\mathrm{IH}}{}_{\vee_L}$$

 \bullet Case(s) rule AT

$$\frac{\mathbf{h}_1: \mathbf{f}_2, \Delta_4, []\mathbf{f}_2 \vdash \Delta_3}{\bullet \mathbf{h}_1: \Delta_4, []\mathbf{f}_2 \vdash \Delta_3} \quad AT \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1: \Delta_4, \mathbf{f}_2, []\mathbf{f}_2 \vdash \Delta_3}{\mathbf{h}_1: \Delta_4, \mathbf{f}_2, []\mathbf{f}_2 \vdash \Delta_3, \mathbf{f}_W}}{\bullet \mathbf{h}_1: \Delta_4, []\mathbf{f}_2 \vdash \Delta_3, \mathbf{f}_W} \quad \overset{\text{int}}{}_{AT}$$

• Case(s) rule \perp_L

$$\frac{}{\bullet \mathbf{h}_1:\bot,\Delta_3\vdash \Delta_2} \ ^\bot L \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1:\bot,\Delta_3\vdash \Delta_2,\mathbf{f}_W} \ ^\bot L$$

 \bullet Case(s) rule I

• Case(s) rule \top_L

3 Measure of derivations

• Case(s) rule \rightarrow_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_3, \, \Delta_2 \vdash \mathbf{F}_4, \, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3 \rightarrow \mathbf{F}_4} \\ & \rightarrow R \end{array} \rightarrow R \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1: \, \Delta_2, \, \mathbf{F}_3 \vdash \Delta_5, \, \mathbf{F}_4}{\bullet \, \mathbf{h}_1: \, \Delta_2, \, \mathbf{F}_3 \vdash \Delta_5, \, \mathbf{F}_4} \\ & \bullet \, \mathbf{h}_1: \, \Delta_2 \vdash \Delta_5, \, \mathbf{F}_3 \rightarrow \mathbf{F}_4} \\ & \rightarrow R \end{array}$$

• Case(s) rule \wedge_R

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3 \land \mathbf{F}_4} \quad \wedge_R \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3} \quad \mathbf{IH}}{\bullet \bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3 \land \mathbf{F}_4} \quad \mathbf{IH} \quad \frac{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_4}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_4} \quad \mathbf{IH} \quad \wedge_R \quad \wedge_R$$

• Case(s) rule \vee_R

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \mathbf{F}_3, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3 \lor \mathbf{F}_4} \ \lor_R \qquad \rightarrow \qquad \frac{\overbrace{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4}^{\mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3, \mathbf{F}_4} \ _{\mathbf{1H}}^{\mathbf{H}} }{\bullet \bullet \mathbf{h}_1: \Delta_2 \vdash \Delta_5, \mathbf{F}_3 \lor \mathbf{F}_4} \ \lor_R$$

• Case(s) rule \perp_R

• Case(s) rule \top_R

• Case(s) rule K

• Case(s) rule A45

$$\frac{ \begin{smallmatrix} \mathbf{h}_1 : \Box \Gamma_5 \vdash \Box \Gamma_3, F_2 \\ \bullet \mathbf{h}_1 : \Box \Gamma_5, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, []F_2 \end{smallmatrix}}{\bullet \mathbf{h}_1 : \Box \Gamma_5, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, []F_2} \begin{tabular}{l} A45 \\ \hline \bullet \bullet \mathbf{h}_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_4, \Box \Gamma_3, []F_2 \\ \hline \bullet \bullet \mathbf{h}_1 : \Delta_6, \Box \Gamma_5 \vdash \Delta_4, \Box \Gamma_3, []F_2 \\ \hline \end{smallmatrix}$$

• Case(s) rule \rightarrow_L

$$\frac{\mathbf{h}_1:\Delta_5\vdash \mathbf{F}_2,\Delta_4\quad \mathbf{h}_1:\mathbf{F}_3,\Delta_5\vdash \Delta_4}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\to \mathbf{F}_3\vdash \Delta_4} \ \to L \qquad \to \qquad \frac{\frac{\mathbf{h}_1:\Delta_5\vdash \Delta_4,\mathbf{F}_2}{\bullet \mathbf{h}_1:\Delta_5\vdash \Delta_4,\mathbf{F}_2} \ \ \mathbf{IH}}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash \Delta_4} \ \ \frac{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash \Delta_4}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash \Delta_4} \ \ \mathbf{IH}}{\bullet L}$$

• Case(s) rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_2, \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \Delta_4} \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \Delta_4 \end{array} \wedge_L \qquad \rightarrow \qquad \begin{array}{c} \frac{\mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_4} \\ \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \Delta_4 \end{array} \wedge_L$$

• Case(s) rule \vee_L

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathbf{h}_1: \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \underbrace{\frac{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4}}_{\bullet \bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \overset{\mathrm{ax}}{\mathsf{h}} \qquad \underbrace{\frac{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4}}_{\bullet \mathsf{h}} \quad \overset{\mathrm{ax}}{\mathsf{h}} \qquad \underbrace{\mathbf{h}_1: \Delta_5, \mathbf{F}_3 \vdash \Delta_4}_{\bullet \mathsf{h}} \quad \overset{\mathrm{ax}}{\mathsf{h}} \qquad \underbrace{\mathbf{h}_1: \Delta_5, \mathbf{F}_3 \vdash \Delta_4}_{\bullet \mathsf{h}_1: \Delta_5, \mathbf{F}_3 \vdash \Delta_4}$$

 \bullet Case(s) rule AT

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \Delta_4, []\mathbf{F}_2 \vdash \Delta_3}{\bullet \mathbf{h}_1: \Delta_4, []\mathbf{F}_2 \vdash \Delta_3} \quad AT \qquad \rightarrow \qquad \frac{\overbrace{\mathbf{h}_1: \Delta_4, \mathbf{F}_2, []\mathbf{F}_2 \vdash \Delta_3}^{\mathbf{h}_1: \Delta_4, \mathbf{F}_2, []\mathbf{F}_2 \vdash \Delta_3}^{\mathbf{n}_1} \quad \mathbf{H}}{\bullet \bullet \mathbf{h}_1: \Delta_4, []\mathbf{F}_2 \vdash \Delta_3} \quad \mathbf{H}} AT$$

• Case(s) rule \perp_L

 \bullet Case(s) rule I

• Case(s) rule \top_L

$$\begin{array}{c} \mathbf{h}_1: \Delta_3 \vdash \Delta_2 \\ \hline \bullet \mathbf{h}_1: \top, \Delta_3 \vdash \Delta_2 \end{array} \ \top_L \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_1: \Delta_3 \vdash \Delta_2} \\ \hline \bullet \mathbf{h}_1: \Delta_3 \vdash \Delta_2 \end{array} \ \mathbf{H} \\ \hline \bullet \bullet \mathbf{h}_1: \top, \Delta_3 \vdash \Delta_2 \end{array} \ \top_L$$

4 Invertibility of Rules

4.1 Status of \rightarrow_R : : Invertible

• Case rule \rightarrow_R

• Case rule \wedge_R

• Case rule \vee_R

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5,\mathbf{F}_6,\Delta_7,\mathbf{F}_1\to \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4\vdash (\Delta_7,\mathbf{F}_1\to \mathbf{F}_2),\mathbf{F}_5\vee \mathbf{F}_6} \ \vee_R \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5,\mathbf{F}_6}}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5\vee \mathbf{F}_6} \ \vee_R$$

• Case rule \perp_R

$$\frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{F}_1\rightarrow\mathbf{F}_2}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{F}_1\rightarrow\mathbf{F}_2}\ \perp_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash\Delta_5,\mathbf{F}_2}}{\bullet\mathbf{h}_3:\Delta_4,\mathbf{F}_1\vdash\bot,\Delta_5,\mathbf{F}_2}\ \stackrel{\mathrm{ax/ind}}{\perp_R}$$

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{f}_1 \rightarrow \mathbf{f}_2} \ \ ^\top R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4,\mathbf{f}_1 \vdash \top,\Delta_5,\mathbf{f}_2} \ \ ^\top R$$

 \bullet Case rule K

$$\frac{\mathbf{h}_3: unbox(\Box \Gamma_6) \vdash \mathbf{f}_4}{\bullet \mathbf{h}_3: \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, \mathbf{f}_1 \rightarrow \mathbf{f}_2), []\mathbf{f}_4} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: unbox(\Box \Gamma_6) \vdash \mathbf{f}_4}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_1, \Box \Gamma_6 \vdash \Delta_5, \mathbf{f}_2, []\mathbf{f}_4} \quad K \leftarrow \mathbf{f}_3: \Delta_7, \mathbf{f}_1, \Delta_7 \vdash \Delta_7, \Delta$$

 \bullet Case rule A45

$$\frac{\mathtt{h}_3: \Box \Gamma_7 \vdash \Box \Gamma_5, \mathtt{f}_4}{\bullet \mathtt{h}_3: \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, (\Delta_6, \mathtt{f}_1 \to \mathtt{f}_2), []\mathtt{f}_4} \quad \text{A45} \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3: \Box \Gamma_7 \vdash \mathtt{f}_4, \Box \Gamma_5}}{\bullet \mathtt{h}_3: \Delta_8, \mathtt{f}_1, \Box \Gamma_7 \vdash \Delta_6, \mathtt{f}_2, \Box \Gamma_5, []\mathtt{f}_4} \quad \text{A45}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7\vdash \mathbf{F}_4,\Delta_6,\mathbf{F}_1\rightarrow \mathbf{F}_2\quad \mathbf{h}_3:\mathbf{F}_5,\Delta_7\vdash \Delta_6,\mathbf{F}_1\rightarrow \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_4\rightarrow \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_1\rightarrow \mathbf{F}_2} \quad \rightarrow_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\vdash \Delta_6,\mathbf{F}_2,\mathbf{F}_4}\quad \overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_5\vdash \Delta_6,\mathbf{F}_2}\quad \overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\rightarrow \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_2}\quad \xrightarrow{\mathbf{ax/ind}}\quad \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\rightarrow \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\rightarrow \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_2}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \mathbf{F}_5, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \rightarrow \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \land \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \rightarrow \mathbf{F}_2} \quad \land_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4, \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \land \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \stackrel{\mathrm{ax/ind}}{\wedge}_L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \rightarrow \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \rightarrow \mathbf{F}_2} \quad \vee_L \qquad \rightarrow \qquad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \lor \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \frac{\mathsf{ax/ind}}{\mathsf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2 \lor \Delta_7, \mathbf{F}_1, \mathbf{F$$

 \bullet Case rule AT

$$\begin{array}{c} \mathbf{h}_3: \mathbf{F}_4, \Delta_6, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \\ \bullet \mathbf{h}_3: \Delta_6, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \end{array} \quad AT \qquad \rightarrow \qquad \\ \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_4, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{AT} \end{array}$$

• Case rule \perp_L

 \bullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_5, \Delta_6 \vdash \mathsf{p}_5, \Delta_4, \mathsf{f}_1 \to \mathsf{f}_2} \quad I \qquad \to \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_6, \mathsf{f}_1, \mathsf{p}_5 \vdash \Delta_4, \mathsf{f}_2, \mathsf{p}_5} \quad I$$

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5\vdash\Delta_4,\mathbf{F}_1\to\mathbf{F}_2}{\bullet\mathbf{h}_3:\top,\Delta_5\vdash\Delta_4,\mathbf{F}_1\to\mathbf{F}_2}\ \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vdash\Delta_4,\mathbf{F}_2}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\vdash\Delta_4,\mathbf{F}_2}\ ^{\mathrm{ax/ind}}$$

4.2 Status of \wedge_R : (Left Premise): Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_3: \mathbf{F}_5, \Delta_4 \vdash \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \to \mathbf{F}_6} \ \rightarrow_{R} \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_4, \mathbf{F}_5 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \to \mathbf{F}_6} \overset{\mathrm{ax/ind}}{\to}_{R}$$

• Case rule \wedge_R

$$\begin{array}{c} \frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_5,\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \land \mathbf{F}_6} & \wedge_R \\ \hline \bullet \mathbf{h}_3:\Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \land \mathbf{F}_6 & \wedge_R \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3,\Delta_5}{\bullet \mathbf{h}_1:\Delta_2 \vdash \Delta_5, \mathbf{F}_3 \land \mathbf{F}_4} & \wedge_R \\ \hline \end{array} \rightarrow \begin{array}{c} \frac{\mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5}{\bullet \mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5} & \mathbf{ax/ind} \\ \hline \bullet \mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_6 & \wedge_R \\ \hline \\ \frac{\mathbf{h}_1:\Delta_2 \vdash \mathbf{F}_3,\Delta_5}{\bullet \mathbf{h}_1:\Delta_2 \vdash \Delta_5, \mathbf{F}_3 \land \mathbf{F}_4} & \wedge_R \\ \hline \end{array} \rightarrow \begin{array}{c} \frac{\mathbf{h}_1:\Delta_2 \vdash \Delta_5, \mathbf{F}_3}{\bullet \mathbf{h}_1:\Delta_2 \vdash \Delta_5, \mathbf{F}_3} & \mathbf{h} \\ \hline \\ \bullet \mathbf{h}_1:\Delta_2 \vdash \Delta_5, \mathbf{F}_3 & \mathbf{h} \\ \hline \end{array}$$

• Case rule \vee_R

$$\frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_5, \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \lor \mathbf{F}_6} \ \lor_R \qquad \rightarrow \qquad \frac{\mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5, \mathbf{F}_6}{\bullet \mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_5 \lor \mathbf{F}_6} \ \lor_R$$

• Case rule \perp_R

$$\frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{f}_1\land\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{f}_1\land\mathbf{f}_2}\ \bot_R \qquad\rightarrow\qquad \frac{\frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{f}_1}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{f}_1}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{f}_1}\ \bot_R$$

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{f}_1 \wedge \mathbf{f}_2} \ \top_R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{f}_1} \ \top_R$$

 \bullet Case rule K

$$\frac{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3: \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}}{\bullet \mathtt{h}_3: \Delta_7, \Box \Gamma_6 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_4} \quad K$$

• Case rule A45

$$\frac{h_3: \Box \Gamma_7 \vdash \Box \Gamma_5, F_4}{\bullet h_3: \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, (\Delta_6, F_1 \land F_2), []F_4} \quad \text{A45} \qquad \rightarrow \qquad \frac{\overline{h_3: \Box \Gamma_7 \vdash F_4, \Box \Gamma_5}}{\bullet h_3: \Delta_8, \Box \Gamma_7 \vdash \Delta_6, F_1, \Box \Gamma_5, []F_4} \quad \text{A45}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7\vdash \mathbf{F}_4,\Delta_6,\mathbf{F}_1\land \mathbf{F}_2\quad \mathbf{h}_3:\mathbf{F}_5,\Delta_7\vdash \Delta_6,\mathbf{F}_1\land \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_4\to \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_1\land \mathbf{F}_2} \ \to_L \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_7\vdash \Delta_6,\mathbf{F}_1,\mathbf{F}_4} \quad \overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_4\to \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_1}}{\bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_4\to \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_1} \ \xrightarrow{\bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_4\to \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_1} \quad \xrightarrow{\bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_4\to \mathbf{h}_5} \quad \xrightarrow{\bullet \mathbf{h}_3:\Delta_7,\mathbf{h}_4\to \mathbf{h}_5} \quad \xrightarrow{\bullet \mathbf{h}_4} \quad \xrightarrow{\bullet \mathbf{h}_4}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \mathbf{F}_5, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2} \ \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1} \overset{\text{ax/ind}}{\wedge}_L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_1}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1} \quad \overset{\mathrm{ax/ind}}{\vee_L} \quad \vee_L \wedge \mathbf{F}_4 \wedge \mathbf{F}_5 \wedge$$

 \bullet Case rule AT

$$\begin{array}{c} \mathbf{h}_3: \mathbf{F}_4, \Delta_6, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1 \wedge \mathbf{F}_2 \\ \bullet \mathbf{h}_3: \Delta_6, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1 \wedge \mathbf{F}_2 \end{array} \quad AT \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3: \Delta_6, \mathbf{F}_4, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1} \\ \bullet \mathbf{h}_3: \Delta_6, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1 \end{array} \quad \stackrel{\mathrm{ax/ind}}{\longrightarrow} AT \end{array}$$

• Case rule \perp_L

ullet Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_3: \Delta_5 \vdash \Delta_4, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3: \top, \Delta_5 \vdash \Delta_4, \mathbf{F}_1 \land \mathbf{F}_2} \ \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_5 \vdash \Delta_4, \mathbf{F}_1}}{\bullet \mathbf{h}_3: \top, \Delta_5 \vdash \Delta_4, \mathbf{F}_1} \overset{\mathrm{ax/ind}}{\top_L}$$

4.3 Status of \wedge_R (Right Premise): : Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_3: \mathbf{F}_5, \Delta_4 \vdash \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \to \mathbf{F}_6} \to_R \quad \to \quad \frac{\frac{\mathbf{h}_3: \Delta_4, \mathbf{F}_5 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_6}{\bullet \mathbf{h}_3: \Delta_4 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \to \mathbf{F}_6}} \xrightarrow{ax/ind} \xrightarrow{\bullet \mathbf{h}_3: \Delta_4 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \to \mathbf{F}_6} \xrightarrow{ax/ind} \xrightarrow{\bullet \mathbf{h}_3: \Delta_4 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \to \mathbf{F}_6}$$

• Case rule \wedge_R

$$\begin{array}{c} \underline{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5,\Delta_7,\mathbf{F}_1\land \mathbf{F}_2\quad \mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6,\Delta_7,\mathbf{F}_1\land \mathbf{F}_2}} \\ \bullet \mathbf{h}_3:\Delta_4\vdash (\Delta_7,\mathbf{F}_1\land \mathbf{F}_2),\mathbf{F}_5\land \mathbf{F}_6 \end{array} \quad \wedge_R \\ \end{array} \rightarrow \begin{array}{c} \underline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_6} \\ \bullet \mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5\land \mathbf{F}_6 \end{array}} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_6} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_6} \\ \bullet \mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5\land \mathbf{F}_6 \end{array}} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_6} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_6} \\ \bullet \mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5\land \mathbf{F}_6 \end{array}} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_5\land \mathbf{F}_6} \quad \underline{\mathbf{ax/ind}} \quad \underline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_2,\mathbf{F}_6} \\ \underline{\mathbf{h}_1:\Delta_2\vdash \mathbf{F}_3,\Delta_5\vdash \mathbf{h}_1:\Delta_2\vdash \mathbf{F}_4,\Delta_5} \quad } \quad \underline{\mathbf{h}_1:\Delta_2\vdash \Delta_5,\mathbf{F}_4} \quad \underline{\mathbf{h}_1:\Delta_2\vdash \Delta_5,\mathbf{h}_4} \quad \underline{\mathbf{h}_1:\Delta_2\vdash \Delta$$

• Case rule \vee_R

$$\frac{\mathbf{h}_3:\Delta_4 \vdash \mathbf{F}_5, \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \land \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \land \mathbf{F}_2), \mathbf{F}_5 \lor \mathbf{F}_6} \quad \vee_R \qquad \rightarrow \qquad \frac{\mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5, \mathbf{F}_6}{\bullet \mathbf{h}_3:\Delta_4 \vdash \Delta_7, \mathbf{F}_2, \mathbf{F}_5 \lor \mathbf{F}_6} \quad \overset{\mathsf{av/ind}}{\vee}_R$$

• Case rule \perp_R

$$\frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{f}_1\land\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{f}_1\land\mathbf{f}_2}\ \bot_R \qquad\rightarrow\qquad \frac{\mathbf{h}_3:\Delta_4\vdash\Delta_5,\mathbf{f}_2}{\bullet\mathbf{h}_3:\Delta_4\vdash\bot,\Delta_5,\mathbf{f}_2}\overset{\mathrm{ax/ind}}{\bot_R}$$

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{f}_1 \land \mathbf{f}_2} \ \top_R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{f}_2} \ \top_R$$

 \bullet Case rule K

$$\frac{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3: \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}}{\bullet \mathtt{h}_3: \Delta_7, \Box \Gamma_6 \vdash \Delta_5, \mathtt{F}_2, []\mathtt{F}_4} \quad K$$

 \bullet Case rule A45

$$\frac{h_3: \Box \Gamma_7 \vdash \Box \Gamma_5, F_4}{\bullet h_3: \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, (\Delta_6, F_1 \land F_2), []F_4} \quad {}_{A45} \qquad \rightarrow \qquad \frac{\overline{h_3: \Box \Gamma_7 \vdash F_4, \Box \Gamma_5} \quad ax}{\bullet h_3: \Delta_8, \Box \Gamma_7 \vdash \Delta_6, F_2, \Box \Gamma_5, []F_4} \quad {}_{A45}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7\vdash \mathbf{F}_4,\Delta_6,\mathbf{F}_1\land \mathbf{F}_2\quad \mathbf{h}_3:\mathbf{F}_5,\Delta_7\vdash \Delta_6,\mathbf{F}_1\land \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_4\to \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_1\land \mathbf{F}_2} \ \to_L \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_7\vdash \Delta_6,\mathbf{F}_2,\mathbf{F}_4} \quad \text{ax/ind} \quad \overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_5\vdash \Delta_6,\mathbf{F}_2} \quad \xrightarrow{\mathbf{ax/ind}} \quad \to_L \quad \to L \quad \to L$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \mathbf{F}_5, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2} \ \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \overset{\text{ax/ind}}{\wedge}_L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\vee_L} \quad \vee_L \quad \wedge_L \quad \wedge_$$

 \bullet Case rule AT

$$\begin{array}{lll} \frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_6, \left(\left[\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1 \wedge \mathbf{F}_2 \right. \right. }{\left. \bullet \mathbf{h}_3: \Delta_6, \left(\left[\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1 \wedge \mathbf{F}_2 \right. \right. \right. } \right. & AT \end{array} \\ & \rightarrow & \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_4, \left(\left[\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_2 \right. \right. }{\left. \bullet \mathbf{h}_3: \Delta_6, \left(\left[\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_2 \right. \right. \right. } \right. & AT \end{array} \\ & \rightarrow & \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_4, \left(\left[\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_2 \right. \right. \right. }{\left. \bullet \mathbf{h}_3: \Delta_6, \left(\left[\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_2 \right. \right. \right. \right. } \right. \\ & AT \end{array}$$

• Case rule \perp_L

$$\frac{}{\bullet \mathbf{h}_3: \bot, \Delta_5 \vdash \Delta_4, \mathbf{F}_1 \wedge \mathbf{F}_2} \ ^\bot L \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3: \bot, \Delta_5 \vdash \Delta_4, \mathbf{F}_2} \ ^\bot L$$

 \bullet Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5\vdash\Delta_4,\mathbf{f}_1\wedge\mathbf{f}_2}{\bullet\mathbf{h}_3:\top,\Delta_5\vdash\Delta_4,\mathbf{f}_1\wedge\mathbf{f}_2}\ \top_L \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5\vdash\Delta_4,\mathbf{f}_2}}{\bullet\mathbf{h}_3:\top,\Delta_5\vdash\Delta_4,\mathbf{f}_2} \overset{\mathrm{ax/ind}}{}{}^{}$$

4.4 Status of \vee_R : Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_3: \mathbf{F}_5, \Delta_4 \vdash \mathbf{F}_6, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4 \vdash (\Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2), \mathbf{F}_5 \to \mathbf{F}_6} \ \rightarrow_{R} \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_4, \mathbf{F}_5 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_6}}{\bullet \mathbf{h}_3: \Delta_4 \vdash \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \to \mathbf{F}_6} \xrightarrow{\mathrm{ax/ind}} \rightarrow_{R}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_3:\Delta_4\vdash \mathbf{F}_5,\Delta_7,\mathbf{F}_1\vee \mathbf{F}_2\quad \mathbf{h}_3:\Delta_4\vdash \mathbf{F}_6,\Delta_7,\mathbf{F}_1\vee \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_4\vdash (\Delta_7,\mathbf{F}_1\vee \mathbf{F}_2),\mathbf{F}_5\wedge \mathbf{F}_6} \quad \wedge_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_5}\quad ^{\mathrm{ax/ind}}\quad \overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_6}\quad ^{\mathrm{ax/ind}}\quad \overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_6}\quad \wedge_R \quad \wedge_R \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_5}\quad ^{\mathrm{ax/ind}}\quad \overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_6}\quad }^{\mathrm{ax/ind}}\quad \overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_6}\quad \overline{\mathbf{h}_3:\Delta_4\vdash \Delta_7,\mathbf{F}$$

• Case rule \vee_R

• Case rule \perp_R

$$\frac{\mathbf{h}_3: \Delta_4 \vdash \Delta_5, \mathbf{F}_1 \vee \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_4 \vdash \bot, \Delta_5, \mathbf{F}_1 \vee \mathbf{F}_2} \ \bot_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_4 \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_4 \vdash \bot, \Delta_5, \mathbf{F}_1, \mathbf{F}_2} \ \underline{}^{\mathrm{ax/ind}}$$

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{f}_1 \vee \mathbf{f}_2} \quad \top_R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_4 \vdash \top,\Delta_5,\mathbf{f}_1,\mathbf{f}_2} \quad \top_R$$

 \bullet Case rule K

$$\frac{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3: \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4} \quad \mathtt{ax}}{\bullet \mathtt{h}_3: \Delta_7, \Box \Gamma_6 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2, []\mathtt{F}_4} \quad K$$

• Case rule A45

$$\frac{h_3: \Box \Gamma_7 \vdash \Box \Gamma_5, F_4}{\bullet h_3: \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, (\Delta_6, F_1 \vee F_2), []F_4} \quad \text{$A45$} \qquad \rightarrow \qquad \frac{\overline{h_3: \Box \Gamma_7 \vdash F_4, \Box \Gamma_5}}{\bullet h_3: \Delta_8, \Box \Gamma_7 \vdash \Delta_6, F_1, F_2, \Box \Gamma_5, []F_4} \quad \text{$A45$}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7\vdash \mathbf{F}_4,\Delta_6,\mathbf{F}_1\lor \mathbf{F}_2\quad \mathbf{h}_3:\mathbf{F}_5,\Delta_7\vdash \Delta_6,\mathbf{F}_1\lor \mathbf{F}_2}{\bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_4\to \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_1\lor \mathbf{F}_2} \ \to_L \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_7\vdash \Delta_6,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_4\to \mathbf{F}_5\vdash \Delta_6,\mathbf{F}_1,\mathbf{F}_2} \quad \frac{\mathbf{ax/ind}}{\to_L} \quad \to_L$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \mathbf{F}_5, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2} \ \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4, \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \ \stackrel{\mathsf{ax/ind}}{\wedge}_L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7 \vdash \Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\vee_L} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \overset{\mathrm{ax/ind}}{\vee_L} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2}} \quad \vee_L \quad \rightarrow \quad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_2$$

 \bullet Case rule AT

$$\begin{array}{c} \mathbf{h}_3: \mathbf{F}_4, \Delta_6, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1 \vee \mathbf{F}_2 \\ \bullet \mathbf{h}_3: \Delta_6, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1 \vee \mathbf{F}_2 \end{array} \ AT \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3: \Delta_6, \mathbf{F}_4, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2} \\ \bullet \mathbf{h}_3: \Delta_6, [] \mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1, \mathbf{F}_2 \end{array} \ \begin{array}{c} \mathbf{ax/ind} \\ AT \end{array}$$

• Case rule \perp_L

ullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_5, \Delta_6 \vdash \mathsf{p}_5, \Delta_4, \mathsf{F}_1 \vee \mathsf{F}_2} \quad I \qquad \rightarrow \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_6, \mathsf{p}_5 \vdash \Delta_4, \mathsf{F}_1, \mathsf{F}_2, \mathsf{p}_5} \quad I$$

• Case rule \top_L

$$\frac{\mathbf{h}_3: \Delta_5 \vdash \Delta_4, \mathbf{f}_1 \vee \mathbf{f}_2}{\bullet \mathbf{h}_3: \top, \Delta_5 \vdash \Delta_4, \mathbf{f}_1 \vee \mathbf{f}_2} \ \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_5 \vdash \Delta_4, \mathbf{f}_1, \mathbf{f}_2}}{\bullet \mathbf{h}_3: \top, \Delta_5 \vdash \Delta_4, \mathbf{f}_1, \mathbf{f}_2} \overset{\mathrm{ax/ind}}{\top_L}$$

4.5 Status of \perp_R : : Invertible

• Case rule \rightarrow_R

$$\frac{\mathtt{h}_1:\mathtt{F}_3,\Delta_2\vdash\bot,\mathtt{F}_4,\Delta_5}{\bullet\mathtt{h}_1:\Delta_2\vdash(\bot,\Delta_5),\mathtt{F}_3\to\mathtt{F}_4}\to_R \qquad\to\qquad \frac{\frac{\mathtt{h}_1:\Delta_2,\mathtt{F}_3\vdash\Delta_5,\mathtt{F}_4}{\bullet\mathtt{h}_1:\Delta_2\vdash\Delta_5,\mathtt{F}_3\to\mathtt{F}_4}\xrightarrow{\mathtt{ax/ind}}\to_R$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_1:\Delta_2\vdash\bot,\mathbf{F}_3,\Delta_5\quad\mathbf{h}_1:\Delta_2\vdash\bot,\mathbf{F}_4,\Delta_5}{\bullet\mathbf{h}_1:\Delta_2\vdash(\bot,\Delta_5),\mathbf{F}_3\land\mathbf{F}_4}\quad\land_R\quad\quad\rightarrow\quad\quad\frac{\overline{\mathbf{h}_1:\Delta_2\vdash\Delta_5,\mathbf{F}_3}\quad\text{ax/ind}}{\bullet\mathbf{h}_1:\Delta_2\vdash\Delta_5,\mathbf{F}_3\land\mathbf{F}_4}\quad\frac{\mathbf{ax/ind}}{\land_R}\quad\quad\uparrow_R$$

• Case rule \vee_R

$$\frac{\mathbf{h}_1:\Delta_2\vdash\bot,\mathbf{F}_3,\mathbf{F}_4,\Delta_5}{\bullet\mathbf{h}_1:\Delta_2\vdash(\bot,\Delta_5),\mathbf{F}_3\vee\mathbf{F}_4}\ \vee_R \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_1:\Delta_2\vdash\Delta_5,\mathbf{F}_3,\mathbf{F}_4}}{\bullet\mathbf{h}_1:\Delta_2\vdash\Delta_5,\mathbf{F}_3\vee\mathbf{F}_4} \overset{\mathrm{ax/ind}}{\vee_R}$$

• Case rule \perp_R

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \bot, \Delta_3} \ ^\top R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3} \ ^\top R$$

ullet Case rule K

$$\frac{h_1: unbox(\Box \Gamma_4) \vdash F_2}{\bullet h_1: \Box \Gamma_4, \Delta_5 \vdash (\bot, \Delta_3), []F_2} \quad K \qquad \rightarrow \qquad \frac{\overline{h_1: unbox(\Box \Gamma_4) \vdash F_2}}{\bullet h_1: \Delta_5, \Box \Gamma_4 \vdash \Delta_3, []F_2} \quad K$$

• Case rule A45

$$\frac{\mathtt{h}_1: \Box \Gamma_5 \vdash \Box \Gamma_3, \mathtt{f}_2}{\bullet \mathtt{h}_1: \Box \Gamma_5, \Delta_6 \vdash \Box \Gamma_3, (\bot, \Delta_4), []\mathtt{f}_2} \quad {}_{A45} \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_1: \Box \Gamma_5 \vdash \mathtt{f}_2, \Box \Gamma_3}}{\bullet \mathtt{h}_1: \Delta_6, \Box \Gamma_5 \vdash \Delta_4, \Box \Gamma_3, []\mathtt{f}_2} \quad {}_{A45}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_1: \Delta_5 \vdash \bot, \mathbf{F}_2, \Delta_4 \quad \mathbf{h}_1: \mathbf{F}_3, \Delta_5 \vdash \bot, \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \bot, \Delta_4} \ \rightarrow_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5 \vdash \Delta_4, \mathbf{F}_2} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{ax/ind}} \quad \xrightarrow{\Delta_L \cap \mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{F}_2 \rightarrow \mathbf{h}_3} \quad \xrightarrow{\mathbf{h}_1: \Delta_S \cap \mathbf{h}_1: \Delta_S \cap \mathbf{h}_2: \Delta_S \cap \mathbf{$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \mathbf{F}_3, \Delta_5 \vdash \bot, \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \bot, \Delta_4} \ \land_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2, \mathbf{F}_3 \vdash \Delta_4}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \Delta_4} \overset{\mathsf{ax/ind}}{\land} L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \Delta_5 \vdash \bot, \Delta_4 \quad \mathbf{h}_1: \mathbf{F}_3, \Delta_5 \vdash \bot, \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \lor \mathbf{F}_3 \vdash \bot, \Delta_4} \quad \lor_L \qquad \to \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \lor \mathbf{F}_3 \vdash \Delta_4} \quad \overset{\mathsf{ax/ind}}{\lor_L}$$

 \bullet Case rule AT

$$\begin{array}{c} \underline{\mathbf{h}_1: \mathbf{F}_2, \Delta_4, [] \mathbf{F}_2 \vdash \bot, \Delta_3} \\ \bullet \mathbf{h}_1: \Delta_4, [] \mathbf{F}_2 \vdash \bot, \Delta_3 \end{array} \quad AT \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_1: \Delta_4, \mathbf{F}_2, [] \mathbf{F}_2 \vdash \Delta_3} \\ \bullet \mathbf{h}_1: \Delta_4, [] \mathbf{F}_2 \vdash \Delta_3 \end{array} \quad \overset{\mathrm{ax/ind}}{AT} \end{array}$$

• Case rule \perp_L

$$\frac{}{\bullet \mathbf{h}_1:\bot,\Delta_3\vdash\bot,\Delta_2} \ ^\bot L \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1:\bot,\Delta_3\vdash\Delta_2} \ ^\bot L$$

ullet Case rule I

$$\frac{}{\bullet \mathbf{h}_1: \mathbf{p}_3, \, \Delta_4 \vdash \mathbf{p}_3, \, \bot, \, \Delta_2} \quad I \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1: \, \Delta_4, \, \mathbf{p}_3 \vdash \Delta_2, \, \mathbf{p}_3} \quad I$$

• Case rule \top_L

$$\frac{\mathbf{h}_1:\Delta_3\vdash\bot,\Delta_2}{\bullet\mathbf{h}_1:\top,\Delta_3\vdash\bot,\Delta_2}\ \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1:\Delta_3\vdash\Delta_2}}{\bullet\mathbf{h}_1:\top,\Delta_3\vdash\Delta_2} \overset{\mathrm{ax/ind}}{\top_L}$$

4.6 Status of \top_R : : Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_1: \mathbf{F}_3, \Delta_2 \vdash \top, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1: \Delta_2 \vdash (\top, \Delta_5), \mathbf{F}_3 \rightarrow \mathbf{F}_4} \ \rightarrow_{R} \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \wedge_R

$$\begin{array}{ccc} \frac{\mathbf{h}_1:\Delta_2 \vdash \top, \mathbf{F}_3, \Delta_5 & \mathbf{h}_1:\Delta_2 \vdash \top, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1:\Delta_2 \vdash (\top, \Delta_5), \mathbf{F}_3 \land \mathbf{F}_4} & \wedge_R & \rightarrow & \text{trivial} \end{array}$$

• Case rule \vee_R

$$\frac{\mathbf{h}_1:\Delta_2 \vdash \top, \mathbf{F}_3, \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_1:\Delta_2 \vdash (\top, \Delta_5), \mathbf{F}_3 \vee \mathbf{F}_4} \ \lor_R \qquad \to \qquad \mathsf{trivial}$$

• Case rule \perp_R

$$\frac{\mathbf{h}_1: \Delta_2 \vdash \top, \Delta_3}{\bullet \mathbf{h}_1: \Delta_2 \vdash \bot, \top, \Delta_3} \ \bot_R \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_1 : \Delta_2 \vdash \top, \Delta_3} \ \top_R \qquad \rightarrow \qquad \mathsf{trivial}$$

 \bullet Case rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_4) \vdash \mathbf{F}_2}{\bullet \mathbf{h}_1: \Box \Gamma_4, \Delta_5 \vdash (\top, \Delta_3), []\mathbf{F}_2} \quad K \qquad \rightarrow \qquad \mathsf{trivial}$$

 \bullet Case rule A45

$$\frac{\mathtt{h}_1: \Box \Gamma_5 \vdash \Box \Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1: \Box \Gamma_5, \Delta_6 \vdash \Box \Gamma_3, (\top, \Delta_4), []\mathtt{F}_2} \quad \textit{A45} \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_1:\Delta_5 \vdash \top, \mathbf{F}_2, \Delta_4 \quad \mathbf{h}_1: \mathbf{F}_3, \Delta_5 \vdash \top, \Delta_4}{\bullet \mathbf{h}_1:\Delta_5, \mathbf{F}_2 \to \mathbf{F}_3 \vdash \top, \Delta_4} \ \to_L \qquad \to \qquad \text{trivial}$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_2, \mathbf{F}_3, \Delta_5 \vdash \top, \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \land \mathbf{F}_3 \vdash \top, \Delta_4} \ \land_L & \rightarrow & \text{trivial} \end{array}$$

• Case rule \vee_L

 \bullet Case rule AT

$$\begin{array}{ll} \mathbf{h}_1: \mathbf{F}_2, \Delta_4, []\mathbf{F}_2 \vdash \top, \Delta_3 \\ & \bullet \mathbf{h}_1: \Delta_4, []\mathbf{F}_2 \vdash \top, \Delta_3 \end{array} \ AT \qquad \to \qquad \mathrm{trivial}$$

• Case rule \perp_L

$$\frac{}{\bullet \mathbf{h}_1 : \bot, \Delta_3 \vdash \top, \Delta_2} \ ^\bot L \qquad \rightarrow \qquad \mathbf{trivial}$$

 \bullet Case rule I

$$\overline{ \bullet \mathbf{h}_1 : \mathbf{p}_3, \Delta_4 \vdash \mathbf{p}_3, \top, \Delta_2 } \quad I \qquad \rightarrow \qquad \mathbf{trivial}$$

• Case rule \top_L

$$\frac{\mathbf{h}_1:\Delta_3 \vdash \top, \Delta_2}{\bullet \mathbf{h}_1:\top, \Delta_3 \vdash \top, \Delta_2} \ \top_L \qquad \rightarrow \qquad \mathtt{trivial}$$

4.7 Status of K: Non invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_2: \Box \Gamma_6, \mathbf{F}_3, \Delta_7 \vdash \mathbf{F}_4, \Delta_5, []\mathbf{F}_1}{\bullet \mathbf{h}_2: \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathbf{F}_1), \mathbf{F}_3 \rightarrow \mathbf{F}_4} \ \rightarrow_{R} \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: unbox}(\Box \Gamma_6) \vdash \mathbf{F}_1}{\bullet \mathbf{h}_2: unbox}(\Box \Gamma_6) \vdash \mathbf{F}_1} \ \underset{\mathbb{H}}{\text{ax/ind}}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_2: \Box \Gamma_6, \Delta_7 \vdash \mathbf{F}_3, \Delta_5, []\mathbf{F}_1 \quad \mathbf{h}_2: \Box \Gamma_6, \Delta_7 \vdash \mathbf{F}_4, \Delta_5, []\mathbf{F}_1}{\bullet \mathbf{h}_2: \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathbf{F}_1), \mathbf{F}_3 \land \mathbf{F}_4} \quad \wedge_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: unbox(\Box \Gamma_6) \vdash \mathbf{F}_1}}{\bullet \mathbf{h}_2: unbox(\Box \Gamma_6) \vdash \mathbf{F}_1} \quad \mathbf{H}_1 \vdash \mathbf{H}_2 \vdash \mathbf{H}_2 \vdash \mathbf{H}_3 \vdash \mathbf{H}_3 \vdash \mathbf{H}_4 \vdash \mathbf{H}_3 \vdash \mathbf{H}_4 \vdash \mathbf{H}_3 \vdash \mathbf{H}_4 \vdash \mathbf{H}$$

• Case rule \vee_R

$$\frac{\mathbf{h}_2: \Box \Gamma_6, \Delta_7 \vdash \mathbf{F}_3, \mathbf{F}_4, \Delta_5, []\mathbf{F}_1}{\bullet \mathbf{h}_2: \Box \Gamma_6, \Delta_7 \vdash (\Delta_5, []\mathbf{F}_1), \mathbf{F}_3 \vee \mathbf{F}_4} \quad \vee_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: unbox(\Box \Gamma_6) \vdash \mathbf{F}_1}}{\bullet \mathbf{h}_2: unbox(\Box \Gamma_6) \vdash \mathbf{F}_1} \stackrel{\mathsf{ax/ind}}{\vdash}$$

• Case rule \perp_R

• Case rule \top_R

 \bullet Case rule K

 \bullet Case rule A45

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_2: \Box \Gamma_6, \Delta_7 \vdash \mathbf{F}_3, \Delta_5, []\mathbf{F}_1 \quad \mathbf{h}_2: \Box \Gamma_6, \mathbf{F}_4, \Delta_7 \vdash \Delta_5, []\mathbf{F}_1}{\bullet \mathbf{h}_2: (\Box \Gamma_6, \Delta_7), \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5, []\mathbf{F}_1} \quad \rightarrow_L \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_2: unbox(\Box \Gamma_6) \vdash \mathbf{F}_1}{\bullet \mathbf{h}_2: unbox(\Box \Gamma_6) \vdash \mathbf{F}_1}}{\bullet \mathbf{h}_2: unbox(\Box \Gamma_6) \vdash \mathbf{F}_1} \quad \underset{\mathbf{H}}{\text{ax/ind}}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_2: \Box \Gamma_6, \mathbf{F}_3, \mathbf{F}_4, \Delta_7 \vdash \Delta_5, []\mathbf{F}_1}{\bullet \mathbf{h}_2: (\Box \Gamma_6, \Delta_7), \mathbf{F}_3 \land \mathbf{F}_4 \vdash \Delta_5, []\mathbf{F}_1} \quad \land_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: unbox(\Box \Gamma_6) \vdash \mathbf{F}_1}}{\bullet \mathbf{h}_2: unbox(\Box \Gamma_6) \vdash \mathbf{F}_1} \quad \overset{\mathrm{ax/ind}}{\mathbf{H}}$$

• Case rule \vee_L

$$\frac{\mathbf{h}_2: \Box \Gamma_6, \mathbf{F}_3, \Delta_7 \vdash \Delta_5, []\mathbf{F}_1 \quad \mathbf{h}_2: \Box \Gamma_6, \mathbf{F}_4, \Delta_7 \vdash \Delta_5, []\mathbf{F}_1}{\bullet \mathbf{h}_2: (\Box \Gamma_6, \Delta_7), \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5, []\mathbf{F}_1} \quad \vee_L \qquad \rightarrow \qquad \frac{\overleftarrow{\mathbf{h}_2: unbox}(\Box \Gamma_6) \vdash \mathbf{F}_1}{\bullet \mathbf{h}_2: unbox} \stackrel{\mathrm{ax/ind}}{=} \mathbf{H}_1 + \mathbf{H}_2 + \mathbf{H}_2 + \mathbf{H}_3 + \mathbf{H}_4 + \mathbf{H}_4$$

 \bullet Case rule AT

$$\begin{array}{lll} \frac{\mathbf{h}_2: \Box \Gamma_5, \mathbf{F}_3, \Delta_6, []\mathbf{F}_3 \vdash \Delta_4, []\mathbf{F}_1}{\bullet \mathbf{h}_2: (\Box \Gamma_5, \Delta_6), []\mathbf{F}_3 \vdash \Delta_4, []\mathbf{F}_1} & AT & \rightarrow & & & & & \frac{\mathbf{h}_2: \mathbf{F}_3, unbox(\Box \Gamma_5) \vdash \mathbf{F}_1}{\bullet \mathbf{h}_2: \mathbf{F}_3, unbox(\Box \Gamma_5) \vdash \mathbf{F}_1} & \\ \end{array} \\ \xrightarrow{\bullet \mathbf{h}_2: \mathbf{F}_3, unbox(\Box \Gamma_5) \vdash \mathbf{F}_1} & \mathbf{H} & & & & & & & \\ \end{array}$$

$$\frac{\mathbf{h}_2: \Box \Gamma_5, \mathbf{F}_3, \Delta_6, []\mathbf{F}_3 \vdash \Delta_4, []\mathbf{F}_1}{\bullet \mathbf{h}_2: (\Box \Gamma_5, \Delta_6), []\mathbf{F}_3 \vdash \Delta_4, []\mathbf{F}_1} \quad AT \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: unbox(\Box \Gamma_5) \vdash \mathbf{F}_1}}{\bullet \mathbf{h}_2: unbox(\Box \Gamma_5) \vdash \mathbf{F}_1} \quad \mathbf{H}} \quad \mathbf{h}_1 : \mathbf{h}_2 : \mathbf{h}_3 : \mathbf{h}_4 : \mathbf{h}_4 : \mathbf{h}_4 : \mathbf{h}_5 : \mathbf{h$$

• Case rule \perp_L

$$\frac{}{\bullet \mathbf{h}_2:\bot,\Box\Gamma_4,\Delta_5\vdash\Delta_3,[]\mathbf{F}_1} \ ^\perp L \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_2:unbox(\Box\Gamma_4)\vdash\mathbf{F}_1} \ ^\mathrm{fail}$$

 \bullet Case rule I

$$\frac{}{\bullet \mathtt{h}_2 : \mathtt{p}_4, \, \Box \Gamma_5, \, \Delta_6 \vdash \mathtt{p}_4, \, \Delta_3, \, []\mathtt{F}_1} \quad I \qquad \rightarrow \qquad \frac{}{\bullet \mathtt{h}_2 : \mathit{unbox}(\Box \Gamma_5) \vdash \mathtt{F}_1} \quad \mathtt{fail}$$

• Case rule \top_L

$$\frac{\mathbf{h}_2: \Box \Gamma_4, \Delta_5 \vdash \Delta_3, []\mathbf{F}_1}{\bullet \mathbf{h}_2: \top, \Box \Gamma_4, \Delta_5 \vdash \Delta_3, []\mathbf{F}_1} \ \ \, \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: unbox(\Box \Gamma_4) \vdash \mathbf{F}_1}}{\bullet \mathbf{h}_2: unbox(\Box \Gamma_4) \vdash \mathbf{F}_1} \ \, \frac{\mathbf{ax/ind}}{\mathbf{H}}$$

4.8 Status of A45: Non invertible

• Case rule \rightarrow_R

$$\frac{\mathtt{h}_2: \Box \Gamma_7, \mathtt{F}_3, \Delta_8 \vdash \Box \Gamma_5, \mathtt{F}_4, \Delta_6, []\mathtt{F}_1}{\bullet \mathtt{h}_2: \Box \Gamma_7, \Delta_8 \vdash (\Box \Gamma_5, \Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \to \mathtt{F}_4} \ \to R \qquad \to \qquad \frac{\overline{\mathtt{h}_2: \Box \Gamma_7 \vdash \mathtt{F}_1, \Box \Gamma_5}}{\bullet \mathtt{h}_2: \Box \Gamma_7 \vdash \mathtt{F}_1, \Box \Gamma_5} \ _{\mathtt{H}}^{\mathtt{ax/ind}}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_2: \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, \mathbf{F}_3, \Delta_6, [[\mathbf{F}_1 \quad \mathbf{h}_2: \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, \mathbf{F}_4, \Delta_6, []\mathbf{F}_1}{\bullet \mathbf{h}_2: \Box \Gamma_7, \Delta_8 \vdash (\Box \Gamma_5, \Delta_6, [[\mathbf{F}_1]), \mathbf{F}_3 \wedge \mathbf{F}_4} \quad \wedge_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: \Box \Gamma_7 \vdash \mathbf{F}_1, \Box \Gamma_5}}{\bullet \mathbf{h}_2: \Box \Gamma_7 \vdash \mathbf{F}_1, \Box \Gamma_5} \quad \mathbf{H}_1 = \mathbf{h}_2 = \mathbf{h}_2 = \mathbf{h}_3 =$$

• Case rule \vee_R

$$\frac{\mathbf{h}_2: \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, \mathbf{F}_3, \mathbf{F}_4, \Delta_6, []\mathbf{F}_1}{\bullet \mathbf{h}_2: \Box \Gamma_7, \Delta_8 \vdash (\Box \Gamma_5, \Delta_6, []\mathbf{F}_1), \mathbf{F}_3 \vee \mathbf{F}_4} \quad \forall_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: \Box \Gamma_7 \vdash \mathbf{F}_1, \Box \Gamma_5}}{\bullet \mathbf{h}_2: \Box \Gamma_7 \vdash \mathbf{F}_1, \Box \Gamma_5} \quad \mathbf{H}$$

• Case rule \perp_R

$$\frac{\mathbf{h}_2:\Box\Gamma_5,\Delta_6\vdash\Box\Gamma_3,\Delta_4,[]\mathbf{F}_1}{\bullet\mathbf{h}_2:\Box\Gamma_5,\Delta_6\vdash\bot,\Box\Gamma_3,\Delta_4,[]\mathbf{F}_1}\ \bot_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2:\Box\Gamma_5\vdash\mathbf{F}_1,\Box\Gamma_3}}{\bullet\mathbf{h}_2:\Box\Gamma_5\vdash\mathbf{F}_1,\Box\Gamma_3}\ ^{\mathrm{ax/ind}}_{\mathrm{H}}$$

• Case rule \top_R

$$\frac{}{\bullet \mathsf{h}_2: \Box \Gamma_5, \Delta_6 \vdash \top, \Box \Gamma_3, \Delta_4, []\mathsf{F}_1} \ ^\top R \quad \rightarrow \quad \frac{}{\bullet \mathsf{h}_2: \Box \Gamma_5 \vdash \Box \Gamma_3, \mathsf{F}_1} \ ^\mathsf{fail}$$

ullet Case rule K

$$\begin{array}{c} & \underset{\bullet}{\operatorname{h}_2:unbox}(\square\Gamma_6),unbox(\square\Gamma_7) \vdash \operatorname{F}_3 \\ & \underset{\bullet}{\operatorname{h}_2:(\square\Gamma_6,\square\Gamma_7),\square\Gamma_8,\Delta_9 \vdash (\square\Gamma_4,\Delta_5,[]\operatorname{F}_1),[]\operatorname{F}_3} \\ & \\ & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & \\ & & \\$$

• Case rule A45

$$\begin{array}{c} h_2: \square_{\Gamma_8}, \square_{\Gamma_9} \vdash \square_{\Gamma_4}, \square_{\Gamma_5}, F_3, [F_1] \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, [F_1), (\square_{\Gamma_6}, \Delta_7), [F_3] \\ \hline \\ h_2: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3) \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3) \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3, [F_1], (\square_{\Gamma_6}, \Delta_7), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3, [F_1], (\square_{\Gamma_6}, \Delta_7), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3) \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}, F_3) \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1), [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_3] \\ \hline \\ \bullet_{h_2}: (\square_{\Gamma_8}, \square_{\Gamma_9}), \square_{\Gamma_{10}}, \Delta_{11} \vdash (\square_{\Gamma_4}, \square_{\Gamma_5}), (\square_{\Gamma_6}, \Delta_7, [F_1], [F_1], [F_$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_2: \Box \Gamma_7, \Delta_8 \vdash \Box \Gamma_5, \mathbf{F}_3, \Delta_6, []\mathbf{F}_1 \quad \mathbf{h}_2: \Box \Gamma_7, \mathbf{F}_4, \Delta_8 \vdash \Box \Gamma_5, \Delta_6, []\mathbf{F}_1}{\bullet \mathbf{h}_2: (\Box \Gamma_7, \Delta_8), \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Box \Gamma_5, \Delta_6, []\mathbf{F}_1} \quad \rightarrow_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: \Box \Gamma_7 \vdash \mathbf{F}_1, \Box \Gamma_5}}{\bullet \mathbf{h}_2: \Box \Gamma_7 \vdash \mathbf{F}_1, \Box \Gamma_5} \quad \mathbf{H}_1 = \mathbf{h}_2 = \mathbf{h}_3 = \mathbf$$

• Case rule \wedge_L

$$\frac{\mathtt{h}_2: \Box \Gamma_7, \mathtt{F}_3, \mathtt{F}_4, \Delta_8 \vdash \Box \Gamma_5, \Delta_6, []\mathtt{F}_1}{\bullet \mathtt{h}_2: (\Box \Gamma_7, \Delta_8), \mathtt{F}_3 \land \mathtt{F}_4 \vdash \Box \Gamma_5, \Delta_6, []\mathtt{F}_1} \ \land_L \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_2: \Box \Gamma_7 \vdash \mathtt{F}_1, \Box \Gamma_5}}{\bullet \mathtt{h}_2: \Box \Gamma_7 \vdash \mathtt{F}_1, \Box \Gamma_5} \ _{\mathtt{H}}^{\mathtt{ax/ind}}$$

• Case rule \vee_L

 \bullet Case rule AT

$$\begin{array}{lll} \frac{\mathbf{h}_2: \Box \Gamma_6, \mathbf{F}_3, \Delta_7, ([\mathbf{F}_3 \vdash \Box \Gamma_4, \Delta_5, ([\mathbf{F}_1}{\bullet \mathbf{h}_2: (\Box \Gamma_6, \Delta_7), ([\mathbf{F}_3 \vdash \Box \Gamma_4, \Delta_5, ([\mathbf{F}_1}{\bullet \mathbf{h}_2: (\Box \Gamma_6, ([\mathbf{F}_3 \vdash \mathbf{F}_1, \Box \Gamma_4}{\bullet \mathbf{h}_2: (\Box \Gamma_6, ([\mathbf{F}_3 \vdash \mathbf{F}_1, \Box \Gamma_4), ([\mathbf{F}_3 \vdash \mathbf{F}_1,$$

$$\begin{array}{lll} \underbrace{\mathbf{h}_2: \Box \Gamma_6, \mathbf{F}_3, \Delta_7, []\mathbf{F}_3 \vdash \Box \Gamma_4, \Delta_5, []\mathbf{F}_1}_{\bullet \mathbf{h}_2: (\Box \Gamma_6, \Delta_7), []\mathbf{F}_3 \vdash \Box \Gamma_4, \Delta_5, []\mathbf{F}_1} & \mathit{AT} & \rightarrow & & & & \underbrace{\overline{\mathbf{h}_2: \Box \Gamma_6 \vdash \mathbf{F}_1, \Box \Gamma_4}}_{\bullet \mathbf{h}_2: \Box \Gamma_6 \vdash \mathbf{F}_1, \Box \Gamma_4} & \mathsf{H} \\ \end{array}$$

• Case rule \perp_L

$$\frac{}{\bullet^{\mathsf{h}_2}:\bot,\Box\Gamma_5,\Delta_6\vdash\Box\Gamma_3,\Delta_4,[]\mathsf{F}_1}}\ ^{\bot}L\qquad\rightarrow\qquad \overline{\bullet^{\mathsf{h}_2}:\Box\Gamma_5\vdash\Box\Gamma_3,\mathsf{F}_1}\ ^{\mathsf{fail}}$$

 \bullet Case rule I

$$\frac{}{\bullet_{\text{h}_2:\,\text{p}_5},\,\Box\Gamma_6,\,\Delta_7\vdash\text{p}_5,\,\Box\Gamma_3,\,\Delta_4,\,[]\text{F}_1}\quad I\qquad \rightarrow\qquad \frac{}{\bullet_{\text{h}_2:\,\Box\Gamma_6\vdash\Box\Gamma_3,\,\Gamma_1}}\quad \text{fail}$$

• Case rule \top_L

$$\frac{\mathbf{h}_2: \Box \Gamma_5, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, []\mathbf{F}_1}{\bullet \mathbf{h}_2: \top, \Box \Gamma_5, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, []\mathbf{F}_1} \ \, \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: \Box \Gamma_5 \vdash \mathbf{F}_1, \Box \Gamma_3}}{\bullet \mathbf{h}_2: \Box \Gamma_5 \vdash \mathbf{F}_1, \Box \Gamma_3} \ \, \overset{\mathrm{ax/ind}}{\mathsf{H}}$$

4.9 Status of \rightarrow_L : (Left Premise): Invertible

• Case rule \rightarrow_R

$$\frac{\mathsf{h}_3:\mathsf{F}_4,\Delta_7,\mathsf{F}_1\to\mathsf{F}_2\vdash\mathsf{F}_5,\Delta_6}{\bullet\mathsf{h}_3:\Delta_7,\mathsf{F}_1\to\mathsf{F}_2\vdash\Delta_6,\mathsf{F}_4\to\mathsf{F}_5}\to_R \qquad\to\qquad \frac{\mathsf{h}_3:\Delta_7,\mathsf{F}_4\vdash\Delta_6,\mathsf{F}_1,\mathsf{F}_5}{\bullet\mathsf{h}_3:\Delta_7\vdash\Delta_6,\mathsf{F}_1,\mathsf{F}_4\to\mathsf{F}_5}\xrightarrow{\mathsf{ax/ind}}_{\mathsf{P}}$$

• Case rule \wedge_R

$$\begin{array}{c} \underline{\mathbf{h}_3:\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \mathbf{F}_4, \Delta_6 \quad \mathbf{h}_3:\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \mathbf{F}_5, \Delta_6} \\ \bullet \mathbf{h}_3:\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_6, \mathbf{F}_4 \land \mathbf{F}_5 \end{array} \quad \wedge_R \qquad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3:\Delta_7 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3:\Delta_7 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5} \quad \overline{\mathbf{ax/ind}} \\ \bullet \mathbf{h}_3:\Delta_7 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \land \mathbf{F}_5 \end{array} \quad \wedge_R \end{array} \quad \rightarrow \qquad \begin{array}{c} \overline{\mathbf{h}_3:\Delta_7 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4} \quad \overline{\mathbf{ax/ind}} \quad \overline{\mathbf{h}_3:\Delta_7 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_5} \quad \overline{\mathbf{ax/ind}} \\ \bullet \mathbf{h}_3:\Delta_7 \vdash \Delta_6, \mathbf{F}_1, \mathbf{F}_4 \land \mathbf{F}_5 \end{array} \quad \wedge_R \end{array}$$

• Case rule \vee_R

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{f}_1\rightarrow\mathbf{f}_2\vdash\mathbf{f}_4,\mathbf{f}_5,\Delta_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_1\rightarrow\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4\vee\mathbf{f}_5}\ \vee_R \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_7\vdash\Delta_6,\mathbf{f}_1,\mathbf{f}_4,\mathbf{f}_5}}{\bullet\mathbf{h}_3:\Delta_7\vdash\Delta_6,\mathbf{f}_1,\mathbf{f}_4\vee\mathbf{f}_5} \stackrel{\mathrm{ax/ind}}{\vee_R}$$

• Case rule \perp_R

• Case rule \top_R

 \bullet Case rule K

$$\frac{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3: \Box \Gamma_6, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4} \quad K \qquad \to \qquad \frac{\overline{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}}{\bullet \mathtt{h}_3: \Delta_7, \Box \Gamma_6 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_4} \quad K$$

• Case rule A45

$$\frac{h_3:\Box\Gamma_7\vdash\Box\Gamma_5,F_4}{\bullet h_3:\Box\Gamma_7,\Delta_8,F_1\to F_2\vdash\Box\Gamma_5,\Delta_6,[]F_4} \ \ {}_{A45} \qquad \to \qquad \frac{\overline{h_3:\Box\Gamma_7\vdash F_4,\Box\Gamma_5}}{\bullet h_3:\Delta_8,\Box\Gamma_7\vdash\Delta_6,F_1,\Box\Gamma_5,[]F_4} \ \ {}_{A45}$$

• Case rule \rightarrow_L

• Case rule \wedge_L

$$\frac{\mathbf{h}_3:\mathbf{F}_4,\mathbf{F}_5,\Delta_7,\mathbf{F}_1\rightarrow\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\rightarrow\mathbf{F}_2),\mathbf{F}_4\wedge\mathbf{F}_5\vdash\Delta_6} \ \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_4,\mathbf{F}_5\vdash\Delta_6,\mathbf{F}_1}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_4\wedge\mathbf{F}_5\vdash\Delta_6,\mathbf{F}_1} \overset{\mathrm{ax/ind}}{\wedge_L}$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_6 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2), \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \rightarrow \qquad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_1}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1} \quad \frac{\mathsf{ax/ind}}{\mathsf{bx}_3: \Delta_7, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6, \mathbf{F}_1} \quad \vee_L \quad \rightarrow \quad \frac{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vdash \Delta_6, \mathsf{bx}_1}{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vee \mathsf{bx}_5 \vdash \Delta_6, \mathsf{bx}_1} \quad \times_L \quad \rightarrow \quad \frac{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vdash \Delta_6, \mathsf{bx}_1}{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vee \mathsf{bx}_5 \vdash \Delta_6, \mathsf{bx}_1} \quad \times_L \quad \rightarrow \quad \frac{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vdash \Delta_6, \mathsf{bx}_1}{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vee \mathsf{bx}_5 \vdash \Delta_6, \mathsf{bx}_1} \quad \times_L \quad \rightarrow \quad \frac{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vdash \Delta_6, \mathsf{bx}_1}{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vee \mathsf{bx}_5 \vdash \Delta_6, \mathsf{bx}_1} \quad \times_L \quad \rightarrow \quad \frac{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vdash \Delta_6, \mathsf{bx}_1}{\mathsf{bx}_3: \Delta_7, \mathsf{bx}_4 \vee \mathsf{bx}_5 \vdash \Delta_6, \mathsf{bx}_1} \quad \times_L \quad \times_L$$

 \bullet Case rule AT

$$\begin{array}{c} \underline{\mathbf{h}_3: \mathbf{F}_4, \Delta_6, ([\mathbf{F}_4, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_5} \\ \bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_1 \rightarrow \mathbf{F}_2), ([\mathbf{F}_4 \vdash \Delta_5] \end{array} \ AT \end{array} \quad \rightarrow \quad \begin{array}{c} \overline{\mathbf{h}_3: \Delta_6, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1]} \\ \bullet \mathbf{h}_3: \Delta_6, ([\mathbf{F}_4 \vdash \Delta_5, \mathbf{F}_1] \end{array} \end{array} \begin{array}{c} \mathbf{ax/ind} \\ AT \end{array}$$

• Case rule \perp_L

ullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_5, \Delta_6, \mathsf{F}_1 \to \mathsf{F}_2 \vdash \mathsf{p}_5, \Delta_4} \quad I \qquad \to \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_6, \mathsf{p}_5 \vdash \Delta_4, \mathsf{F}_1, \mathsf{p}_5} \quad I$$

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{f}_1\rightarrow\mathbf{f}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{f}_1\rightarrow\mathbf{f}_2\vdash\Delta_4}\ \top_L \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5\vdash\Delta_4,\mathbf{f}_1}}{\bullet\mathbf{h}_3:\top,\Delta_5\vdash\Delta_4,\mathbf{f}_1}\overset{\mathrm{ax/ind}}{\top_L}$$

4.10 Status of \rightarrow_L (Right Premise): : Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \mathbf{F}_5, \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_6, \mathbf{F}_4 \rightarrow \mathbf{F}_5} \rightarrow_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_5}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2 \vdash \Delta_6, \mathbf{F}_4 \rightarrow \mathbf{F}_5} \xrightarrow{\mathrm{ax/ind}}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6\quad\mathbf{h}_3:\Delta_7,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_5,\Delta_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad \wedge_R \qquad \to \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}\quad \text{ax/ind}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad \frac{\mathbf{ax/ind}}{\wedge_R}$$

• Case rule \vee_R

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{f}_1\rightarrow\mathbf{f}_2\vdash\mathbf{f}_4,\mathbf{f}_5,\Delta_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_1\rightarrow\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4\vee\mathbf{f}_5} \ \vee_R \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_3:\Delta_7,\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4,\mathbf{f}_5}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4\vee\mathbf{f}_5}} \overset{\mathrm{ax/ind}}{\vee_R}$$

• Case rule \perp_R

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_3:\Delta_5,\mathbf{f}_1\to\mathbf{f}_2\vdash \top,\Delta_4} \ \ ^\top R \qquad \to \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_5,\mathbf{f}_2\vdash \top,\Delta_4} \ \ ^\top R$$

 \bullet Case rule K

$$\frac{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3: \Box \Gamma_6, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}}{\bullet \mathtt{h}_3: \Delta_7, \mathtt{F}_2, \Box \Gamma_6 \vdash \Delta_5, []\mathtt{F}_4} \quad K$$

• Case rule A45

$$\frac{h_3: \square\Gamma_7 \vdash \square\Gamma_5, F_4}{\bullet h_3: \square\Gamma_7, \Delta_8, F_1 \rightarrow F_2 \vdash \square\Gamma_5, \Delta_6, []F_4} \quad \text{$A45$} \qquad \rightarrow \qquad \frac{\overline{h_3: \square\Gamma_7 \vdash F_4, \square\Gamma_5} \quad ax}{\bullet h_3: \Delta_8, F_2, \square\Gamma_7 \vdash \Delta_6, \square\Gamma_5, []F_4} \quad \text{$A45$}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\to\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6\quad\mathbf{h}_3:\mathbf{F}_5,\Delta_7,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\to\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6}\to L \qquad \to \qquad \frac{\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \xrightarrow{\bullet\mathbf{x}/\mathrm{ind}} \to L$$

$$\frac{\mathbf{h}_1:\Delta_5\vdash \mathbf{F}_2,\Delta_4\quad \mathbf{h}_1:\mathbf{F}_3,\Delta_5\vdash \Delta_4}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_2\to \mathbf{F}_3\vdash \Delta_4}\ \to L \qquad \to \qquad \frac{\overline{\mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash \Delta_4}}{\bullet \mathbf{h}_1:\Delta_5,\mathbf{F}_3\vdash \Delta_4}\ ^{\mathrm{ax}}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3: \mathbf{f}_4, \mathbf{f}_5, \Delta_7, \mathbf{f}_1 \rightarrow \mathbf{f}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{f}_1 \rightarrow \mathbf{f}_2), \mathbf{f}_4 \wedge \mathbf{f}_5 \vdash \Delta_6} \ \land_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4, \mathbf{f}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \wedge \mathbf{f}_5 \vdash \Delta_6} \ \land_L$$

• Case rule \vee_L

 \bullet Case rule AT

$$\begin{array}{c} \mathbf{h}_3: \mathbf{F}_4, \Delta_6, [] \mathbf{F}_4, \mathbf{F}_1 \rightarrow \mathbf{F}_2 \vdash \Delta_5 \\ \bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_1 \rightarrow \mathbf{F}_2), [] \mathbf{F}_4 \vdash \Delta_5 \end{array} \quad AT \qquad \rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_2, \mathbf{F}_4, [] \mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_2, [] \mathbf{F}_4 \vdash \Delta_5} \quad \frac{\mathbf{ax/ind}}{AT} \end{array}$$

• Case rule \perp_L

$$\frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_4} \ ^\bot L \qquad \to \qquad \frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_2\vdash\Delta_4} \ ^\bot L$$

ullet Case rule I

$$\frac{}{\bullet \mathsf{h}_3: \mathsf{p}_5, \Delta_6, \mathsf{F}_1 \to \mathsf{F}_2 \vdash \mathsf{p}_5, \Delta_4} \quad I \qquad \to \qquad \frac{}{\bullet \mathsf{h}_3: \Delta_6, \mathsf{F}_2, \mathsf{p}_5 \vdash \Delta_4, \mathsf{p}_5} \quad I$$

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\to\mathbf{F}_2\vdash\Delta_4}\ \top_L \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_2\vdash\Delta_4}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_2\vdash\Delta_4}\ \top_L$$

4.11 Status of \wedge_L : Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_3: \mathbf{f}_4, \Delta_7, \mathbf{f}_1 \wedge \mathbf{f}_2 \vdash \mathbf{f}_5, \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_1 \wedge \mathbf{f}_2 \vdash \Delta_6, \mathbf{f}_4 \to \mathbf{f}_5} \ \rightarrow_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6, \mathbf{f}_5}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_1, \mathbf{f}_2 \vdash \Delta_6, \mathbf{f}_4 \to \mathbf{f}_5} \ \rightarrow_R$$

• Case rule \wedge_R

• Case rule \vee_R

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{f}_1\wedge\mathbf{f}_2\vdash\mathbf{f}_4,\mathbf{f}_5,\Delta_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_1\wedge\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4\vee\mathbf{f}_5} \ \vee_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{f}_1,\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4,\mathbf{f}_5}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_1,\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4\vee\mathbf{f}_5} \ \frac{\mathsf{ax/ind}}{\vee_R}$$

• Case rule \perp_R

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\bot,\Delta_4}\ \bot_R \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_4}\ \ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\bot,\Delta_4}\ \bot_R$$

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_3:\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash \top,\Delta_4} \ \top_R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash \top,\Delta_4} \ \top_R$$

ullet Case rule K

$$\frac{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3: \Box \Gamma_6, \Delta_7, \mathtt{F}_1 \land \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}}{\bullet \mathtt{h}_3: \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \Box \Gamma_6 \vdash \Delta_5, []\mathtt{F}_4} \quad K$$

 \bullet Case rule A45

$$\frac{\mathtt{h}_3: \Box \Gamma_7 \vdash \Box \Gamma_5, \mathtt{f}_4}{\bullet \mathtt{h}_3: \Box \Gamma_7, \Delta_8, \mathtt{f}_1 \land \mathtt{f}_2 \vdash \Box \Gamma_5, \Delta_6, []\mathtt{f}_4} \quad A45 \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3: \Box \Gamma_7 \vdash \mathtt{f}_4, \Box \Gamma_5}}{\bullet \mathtt{h}_3: \Delta_8, \mathtt{f}_1, \mathtt{f}_2, \Box \Gamma_7 \vdash \Delta_6, \Box \Gamma_5, []\mathtt{f}_4} \quad A45$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6-\mathbf{h}_3:\mathbf{F}_5,\Delta_7,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\wedge\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \to_L \longrightarrow \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \xrightarrow{\mathrm{ax/ind}} \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \xrightarrow{\mathrm{ax/ind}} \frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7, \mathbf{F}_1 \wedge \mathbf{F}_2 \vdash \Delta_6 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \wedge \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \wedge \mathbf{F}_2), \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \rightarrow \qquad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vee \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vee \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vee \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \mathbf{F}_5 \vee \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4 \vee \Delta_6} \quad \overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F$$

 \bullet Case rule AT

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_6, ([\mathbf{F}_4, \mathbf{F}_1 \wedge \mathbf{F}_2 \vdash \Delta_5]}{\bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_1 \wedge \mathbf{F}_2), ([\mathbf{F}_4 \vdash \Delta_5]} \quad AT \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5]}}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5]} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_4, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_2, ([\mathbf{F}_4 \vdash \Delta_5])} \quad AT \qquad \Rightarrow \qquad \frac{\mathbf{h}_$$

• Case rule \perp_L

$$\frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\Delta_4} \ \bot_L \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_4} \ \bot_L$$

 \bullet Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\wedge\mathbf{F}_2\vdash\Delta_4}\ \top_L \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_4}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1,\mathbf{F}_2\vdash\Delta_4}\ \top_L$$

4.12 Status of \vee_L : (Left Premise): Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_3: \mathbf{f}_4, \Delta_7, \mathbf{f}_1 \vee \mathbf{f}_2 \vdash \mathbf{f}_5, \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_1 \vee \mathbf{f}_2 \vdash \Delta_6, \mathbf{f}_4 \to \mathbf{f}_5} \ \to_R \qquad \to \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_1, \mathbf{f}_4 \vdash \Delta_6, \mathbf{f}_5}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_1 \vdash \Delta_6, \mathbf{f}_4 \to \mathbf{f}_5} \overset{\mathrm{ax/ind}}{\to}_R$$

• Case rule \wedge_R

$$\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6\quad\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_5,\Delta_6}{\bullet\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R\quad\quad\rightarrow\quad\frac{\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4}\quad\text{ax/ind}\quad\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_5}\\\bullet\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R\quad\quad\rightarrow\quad\frac{\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4}\quad\text{ax/ind}\quad\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_5}\\\bullet^2\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R\quad\quad\rightarrow\quad\frac{\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4}\quad\text{ax/ind}\quad\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_5}\\\bullet^2\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\quad\rightarrow\quad\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4}\quad\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_5}\\\bullet^2\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\quad\rightarrow\quad\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4}\quad\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_5}\\\bullet^2\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\quad\rightarrow\quad\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4}\quad\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_5}\\\bullet^2\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\quad\rightarrow\quad\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4}\\\bullet^2\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\quad\rightarrow\quad\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4}\\\bullet^2\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\quad\rightarrow\quad\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\\\bullet^2\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\quad\rightarrow\quad\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\\\bullet^2\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5$$

• Case rule \vee_R

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{f}_1\vee\mathbf{f}_2\vdash\mathbf{f}_4,\mathbf{f}_5,\Delta_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_1\vee\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4\vee\mathbf{f}_5}\ \vee_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{f}_1\vdash\Delta_6,\mathbf{f}_4,\mathbf{f}_5}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_1\vdash\Delta_6,\mathbf{f}_4\vee\mathbf{f}_5} \ \frac{\mathbf{ax/ind}}{\vee_R}$$

• Case rule \perp_R

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\bot,\Delta_4}\ \bot_R \qquad\rightarrow\qquad \frac{\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vdash\Delta_4}{\bullet\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vdash\bot,\Delta_4}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vdash\bot,\Delta_4}$$

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_3:\Delta_5,\mathbf{f}_1\vee\mathbf{f}_2\vdash \top,\Delta_4} \ \top_R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\Delta_5,\mathbf{f}_1\vdash \top,\Delta_4} \ \top_R$$

 \bullet Case rule K

$$\frac{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}{\bullet \mathtt{h}_3: \Box \Gamma_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_4} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_3: unbox(\Box \Gamma_6) \vdash \mathtt{F}_4}}{\bullet \mathtt{h}_3: \Delta_7, \mathtt{F}_1, \Box \Gamma_6 \vdash \Delta_5, []\mathtt{F}_4} \quad K$$

• Case rule A45

$$\frac{h_3: \Box \Gamma_7 \vdash \Box \Gamma_5, F_4}{\bullet h_3: \Box \Gamma_7, \Delta_8, F_1 \vee F_2 \vdash \Box \Gamma_5, \Delta_6, []F_4} \xrightarrow{A45} \rightarrow \frac{\overline{h_3: \Box \Gamma_7 \vdash F_4, \Box \Gamma_5}}{\bullet h_3: \Delta_8, F_1, \Box \Gamma_7 \vdash \Delta_6, \Box \Gamma_5, []F_4} \xrightarrow{A45}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6\quad\mathbf{h}_3:\mathbf{F}_5,\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \to_L \\ \bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_1\vdash\Delta_6,\mathbf{F}_4 \\ \bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6 \\ \bullet \mathbf{h}_3:\Delta_7,\mathbf{F}_1,\mathbf{F}_2\to\mathbf{h}_3 \\ \bullet \mathbf{h}_3:\Delta_7,\mathbf{h}_3\to\mathbf{h}_3 \\ \bullet \mathbf{h}_3\to\mathbf{h}_3 \\ \bullet \mathbf{h}_3:\Delta_7,\mathbf{h}_3\to\mathbf{h}_3 \\ \bullet \mathbf{h}_3\to\mathbf{h}_3 \\ \bullet \mathbf{h}_3\to\mathbf{h}_3 \\ \bullet \mathbf{h}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2), \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6} \ \, \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4, \mathbf{F}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \wedge \mathbf{F}_5 \vdash \Delta_6} \ \, \wedge_L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_6 \quad \mathbf{h}_3: \mathbf{F}_5, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2), \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vdash \Delta_6} \quad \frac{\mathsf{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1, \mathbf{F}_4 \vee \mathbf{F}_5 \vdash \Delta_6} \quad \vee_L \quad \wedge_L \quad$$

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathbf{h}_1: \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \lor \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4} \quad \mathbf{H}$$

 \bullet Case rule AT

$$\begin{array}{lll} \frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_6, []\mathbf{F}_4, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_5}{\bullet \mathbf{h}_3: (\Delta_6, \mathbf{F}_1 \vee \mathbf{F}_2), []\mathbf{F}_4 \vdash \Delta_5} & AT & \rightarrow & & & & \frac{\mathbf{h}_3: \Delta_6, \mathbf{F}_1, \mathbf{F}_4, []\mathbf{F}_4 \vdash \Delta_5}{\bullet \mathbf{h}_3: \Delta_6, \mathbf{F}_1, []\mathbf{F}_4 \vdash \Delta_5} & AT & & AT & & \\ \end{array}$$

• Case rule \perp_L

$$\frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{f}_1\vee\mathbf{f}_2\vdash\Delta_4}^{}\bot_L \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_3:\bot,\Delta_5,\mathbf{f}_1\vdash\Delta_4}^{}\bot_L$$

ullet Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}\ \top_L \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vdash\Delta_4}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\vdash\Delta_4}\ ^{\mathrm{ax/ind}}\ \top_L$$

4.13 Status of \vee_L (Right Premise): : Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_3: \mathbf{F}_4, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \mathbf{F}_5, \Delta_6}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_6, \mathbf{F}_4 \to \mathbf{F}_5} \ \rightarrow_R \qquad \rightarrow \qquad \frac{\mathbf{h}_3: \Delta_7, \mathbf{F}_2, \mathbf{F}_4 \vdash \Delta_6, \mathbf{F}_5}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2 \vdash \Delta_6, \mathbf{F}_4 \to \mathbf{F}_5} \xrightarrow{\mathsf{ax/ind}} \rightarrow_R$$

• Case rule \wedge_R

$$\frac{\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6\quad\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_5,\Delta_6}{\bullet\mathbf{a}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R\quad\quad\rightarrow\quad\frac{\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}\quad\text{ax/ind}\quad\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_5}\\\bullet\mathbf{a}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4\wedge\mathbf{F}_5}\quad\wedge_R\quad\quad\rightarrow\quad\frac{\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}\quad\text{ax/ind}}{\bullet\mathbf{a}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_5}\quad\xrightarrow{\mathbf{a}_8/\mathbf{a}_8/\mathbf{a}_8/\mathbf{a}_8/\mathbf{a}_8/\mathbf{a}_8}\quad\wedge_R\quad\rightarrow\quad\frac{\overline{\mathbf{a}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4}\quad\mathbf{a}_8/\mathbf{a}_8/\mathbf{a}_8/\mathbf{a}_8}{\bullet\mathbf{a}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_5}\quad\xrightarrow{\mathbf{a}_8/\mathbf{a}_$$

• Case rule \vee_R

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{f}_1\vee\mathbf{f}_2\vdash\mathbf{f}_4,\mathbf{f}_5,\Delta_6}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_1\vee\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4\vee\mathbf{f}_5} \ \vee_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3:\Delta_7,\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4,\mathbf{f}_5}}{\bullet\mathbf{h}_3:\Delta_7,\mathbf{f}_2\vdash\Delta_6,\mathbf{f}_4\vee\mathbf{f}_5} \overset{\mathrm{av/ind}}{\vee_R}$$

• Case rule \perp_R

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\bot,\Delta_4}\ \bot_R \qquad\rightarrow\qquad \frac{\overline{\mathbf{h}_3:\Delta_5,\mathbf{F}_2\vdash\Delta_4}}{\bullet\mathbf{h}_3:\Delta_5,\mathbf{F}_2\vdash\bot,\Delta_4}\ \bot_R$$

• Case rule \top_R

ullet Case rule K

$$\frac{\mathbf{h}_3: unbox(\Box \Gamma_6) \vdash \mathbf{F}_4}{\bullet \mathbf{h}_3: \Box \Gamma_6, \Delta_7, \mathbf{F}_1 \vee \mathbf{F}_2 \vdash \Delta_5, []\mathbf{F}_4} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: unbox(\Box \Gamma_6) \vdash \mathbf{F}_4}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{F}_2, \Box \Gamma_6 \vdash \Delta_5, []\mathbf{F}_4} \quad K$$

• Case rule A45

$$\frac{h_3:\Box\Gamma_7\vdash\Box\Gamma_5,F_4}{\bullet h_3:\Box\Gamma_7,\Delta_8,F_1\vee F_2\vdash\Box\Gamma_5,\Delta_6,[]F_4} \quad {}_{A45} \qquad \rightarrow \qquad \frac{\overline{h_3:\Box\Gamma_7\vdash F_4,\Box\Gamma_5} \quad ax}{\bullet h_3:\Delta_8,F_2,\Box\Gamma_7\vdash\Delta_6,\Box\Gamma_5,[]F_4} \quad {}_{A45}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_3:\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\mathbf{F}_4,\Delta_6\quad\mathbf{h}_3:\mathbf{F}_5,\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_6}{\bullet\mathbf{h}_3:(\Delta_7,\mathbf{F}_1\vee\mathbf{F}_2),\mathbf{F}_4\to\mathbf{F}_5\vdash\Delta_6} \to_L \\ \bullet \\ \mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4 \\ \bullet \\ \mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6,\mathbf{F}_4 \\ \bullet \\ \mathbf{h}_3:\Delta_7,\mathbf{F}_2\vdash\Delta_6 \\ \bullet \\ \mathbf{h}_3:\Delta_7,\mathbf{h}_3\vdash\Delta_6 \\ \bullet \\ \mathbf{h}_3:\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_7,\mathbf{h}_3\vdash\Delta_$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_3: \mathbf{f}_4, \mathbf{f}_5, \Delta_7, \mathbf{f}_1 \vee \mathbf{f}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{f}_1 \vee \mathbf{f}_2), \mathbf{f}_4 \wedge \mathbf{f}_5 \vdash \Delta_6} \ \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4, \mathbf{f}_5 \vdash \Delta_6}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \wedge \mathbf{f}_5 \vdash \Delta_6} \ \wedge_L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_3: \mathbf{f}_4, \Delta_7, \mathbf{f}_1 \vee \mathbf{f}_2 \vdash \Delta_6 \quad \mathbf{h}_3: \mathbf{f}_5, \Delta_7, \mathbf{f}_1 \vee \mathbf{f}_2 \vdash \Delta_6}{\bullet \mathbf{h}_3: (\Delta_7, \mathbf{f}_1 \vee \mathbf{f}_2), \mathbf{f}_4 \vee \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \text{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vee \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \text{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vee \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \text{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \text{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \text{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \text{ax/ind}}{\bullet \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_5 \vdash \Delta_6} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_2, \mathbf{f}_4 \vdash \Delta_6} \quad \mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{f}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{h}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{h}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{h}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{h}_4 \vdash \Delta_6} \quad \nabla_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_3: \Delta_7, \mathbf{$$

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathbf{h}_1: \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_3 \vdash \Delta_4}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_3 \vdash \Delta_4} \quad \mathbf{H}$$

 \bullet Case rule AT

• Case rule \perp_L

 \bullet Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_1\vee\mathbf{F}_2\vdash\Delta_4}\ \top_L \qquad\rightarrow\qquad \frac{\frac{\mathbf{h}_3:\Delta_5,\mathbf{F}_2\vdash\Delta_4}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_2\vdash\Delta_4}\ ^{\mathrm{ax/ind}}}{\bullet\mathbf{h}_3:\top,\Delta_5,\mathbf{F}_2\vdash\Delta_4}\ \top_L$$

4.14 Status of AT: : Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_2: \mathbf{F}_3, \Delta_6, []\mathbf{F}_1 \vdash \mathbf{F}_4, \Delta_5}{\bullet \mathbf{h}_2: \Delta_6, []\mathbf{F}_1 \vdash \Delta_5, \mathbf{F}_3 \rightarrow \mathbf{F}_4} \ \rightarrow_{R} \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, \mathbf{F}_3, []\mathbf{F}_1 \vdash \Delta_5, \mathbf{F}_4}}{\bullet \mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1 \vdash \Delta_5, \mathbf{F}_3 \rightarrow \mathbf{F}_4} \xrightarrow{\mathbf{ax/ind}}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_2:\Delta_6, [| \mathbf{f}_1| \vdash \mathbf{f}_3, \Delta_5 \quad \mathbf{h}_2:\Delta_6, [| \mathbf{f}_1| \vdash \mathbf{f}_4, \Delta_5}{\bullet \mathbf{h}_2:\Delta_6, [| \mathbf{f}_1| \vdash \Delta_5, \mathbf{f}_3 \wedge \mathbf{f}_4} \quad \wedge_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2:\Delta_6, \mathbf{f}_1, [| \mathbf{f}_1| \vdash \Delta_5, \mathbf{f}_3} \quad \text{ax/ind}}{\bullet \mathbf{h}_2:\Delta_6, \mathbf{f}_1, [| \mathbf{f}_1| \vdash \Delta_5, \mathbf{f}_3 \wedge \mathbf{f}_4} \quad \frac{\text{ax/ind}}{\wedge_R} \quad \wedge_R \quad$$

• Case rule \vee_R

$$\frac{\mathbf{h}_2:\Delta_6,[]\mathbf{F}_1\vdash\mathbf{F}_3,\mathbf{F}_4,\Delta_5}{\bullet\mathbf{h}_2:\Delta_6,[]\mathbf{F}_1\vdash\Delta_5,\mathbf{F}_3\vee\mathbf{F}_4}\ \vee_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2:\Delta_6,\mathbf{F}_1,[]\mathbf{F}_1\vdash\Delta_5,\mathbf{F}_3,\mathbf{F}_4}}{\bullet\mathbf{h}_2:\Delta_6,\mathbf{F}_1,[]\mathbf{F}_1\vdash\Delta_5,\mathbf{F}_3\vee\mathbf{F}_4}\ \vee_R}^{\mathsf{ax/ind}} \vee_R$$

• Case rule \perp_R

$$\frac{\mathbf{h}_2:\Delta_4, []\mathbf{F}_1 \vdash \Delta_3}{\bullet \mathbf{h}_2:\Delta_4, []\mathbf{F}_1 \vdash \bot, \Delta_3} \ \bot_R \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2:\Delta_4, \mathbf{F}_1, []\mathbf{F}_1 \vdash \Delta_3}}{\bullet \mathbf{h}_2:\Delta_4, \mathbf{F}_1, []\mathbf{F}_1 \vdash \bot, \Delta_3} \ \bot_R$$

• Case rule \top_R

$$\frac{}{\bullet \mathbf{h}_2 : \Delta_4, \, []\mathbf{F}_1 \vdash \top, \Delta_3} \ \ \, \top_R \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_2 : \Delta_4, \mathbf{F}_1, \, []\mathbf{F}_1 \vdash \top, \Delta_3} \ \ \, \top_R}$$

 \bullet Case rule K

$$\frac{\mathbf{h}_2: \mathbf{F}_1, unbox(\Box \Gamma_5) \vdash \mathbf{F}_3}{\bullet \mathbf{h}_2: (\Box \Gamma_5, []\mathbf{F}_1), \Delta_6 \vdash \Delta_4, []\mathbf{F}_3} \quad K \qquad \rightarrow \qquad \frac{\frac{\mathbf{h}_2: \mathbf{F}_1, unbox(\Box \Gamma_5) \vdash \mathbf{F}_3}{\bullet \mathbf{h}_2: \Delta_6, \mathbf{F}_1, \Box \Gamma_5, []\mathbf{F}_1 \vdash \Delta_4, []\mathbf{F}_3} \quad K \leftarrow \mathbf{h}_2: \Delta_6, \mathbf{h}_1, \Delta_6 \vdash \Delta_4, \Delta_6 \vdash \Delta_6, \Delta_$$

$$\frac{\mathtt{h}_2: unbox(\Box \Gamma_5) \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2: \Box \Gamma_5, \Delta_6, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_3} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_2: unbox(\Box \Gamma_5) \vdash \mathtt{F}_3}}{\bullet \mathtt{h}_2: \Delta_6, \mathtt{F}_1, \Box \Gamma_5, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_3} \quad K$$

 \bullet Case rule A45

$$\frac{\mathtt{h}_2:\Box\Gamma_6, []\mathtt{F}_1 \vdash \Box\Gamma_4, \mathtt{F}_3}{\bullet\mathtt{h}_2:(\Box\Gamma_6, []\mathtt{F}_1), \Delta_7 \vdash \Box\Gamma_4, \Delta_5, []\mathtt{F}_3} \quad A45 \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_2:\Box\Gamma_6, []\mathtt{F}_1 \vdash \mathtt{F}_3, \Box\Gamma_4}}{\bullet\mathtt{h}_2:\Delta_7, \mathtt{F}_1, \Box\Gamma_6, []\mathtt{F}_1 \vdash \Delta_5, \Box\Gamma_4, []\mathtt{F}_3} \quad A45$$

$$\frac{\mathtt{h}_2: \Box \Gamma_6 \vdash \Box \Gamma_4, \mathtt{F}_3}{\bullet \mathtt{h}_2: \Box \Gamma_6, \Delta_7, []\mathtt{F}_1 \vdash \Box \Gamma_4, \Delta_5, []\mathtt{F}_3} \quad {}_{A45} \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_2: \Box \Gamma_6 \vdash \mathtt{F}_3, \Box \Gamma_4} \quad ^{\mathsf{ax}}}{\bullet \mathtt{h}_2: \Delta_7, \mathtt{F}_1, \Box \Gamma_6, []\mathtt{F}_1 \vdash \Delta_5, \Box \Gamma_4, []\mathtt{F}_3} \quad {}_{A45}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_2:\Delta_6, (|\mathbf{F}_1| \vdash \mathbf{F}_3, \Delta_5 \quad \mathbf{h}_2: \mathbf{F}_4, \Delta_6, (|\mathbf{F}_1| \vdash \Delta_5)}{\bullet \mathbf{h}_2: (\Delta_6, (|\mathbf{F}_1|), \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \rightarrow_L \qquad \rightarrow \qquad \frac{\mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1| \vdash \Delta_5, \mathbf{F}_3)}{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{F}_1, (|\mathbf{F}_1|, \mathbf{F}_3 \rightarrow \mathbf{F}_4 \vdash \Delta_5)} \xrightarrow{\bullet \mathbf{h}_2:\Delta_6, \mathbf{h}_2, \mathbf$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_2: \mathbf{F}_3, \mathbf{F}_4, \Delta_6, []\mathbf{F}_1 \vdash \Delta_5}{\bullet \mathbf{h}_2: (\Delta_6, []\mathbf{F}_1), \mathbf{F}_3 \land \mathbf{F}_4 \vdash \Delta_5} \ \land_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, \mathbf{F}_3, \mathbf{F}_4, []\mathbf{F}_1 \vdash \Delta_5}}{\bullet \mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \land \mathbf{F}_4 \vdash \Delta_5} \ \land_L} \\$$

• Case rule \vee_L

$$\frac{\mathbf{h}_2: \mathbf{F}_3, \Delta_6, []\mathbf{F}_1 \vdash \Delta_5 \quad \mathbf{h}_2: \mathbf{F}_4, \Delta_6, []\mathbf{F}_1 \vdash \Delta_5}{\bullet \mathbf{h}_2: (\Delta_6, []\mathbf{F}_1), \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, \mathbf{F}_3, []\mathbf{F}_1 \vdash \Delta_5} \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\overline{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, \mathbf{F}_3, []\mathbf{F}_1 \vdash \Delta_5} \quad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_3 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{F}_1, \mathbf{F}_2 \vee \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{h}_2, \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{h}_2, \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{h}_2, \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{h}_2, \mathbf{F}_4 \vdash \Delta_5} \quad \vee_L \quad \rightarrow \qquad \frac{\mathbf{ax/ind}}{\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{h}_2: \Delta_6, \mathbf{F}_1, []\mathbf{h$$

 \bullet Case rule AT

$$\frac{\mathbf{h}_2: \mathbf{F}_3, \Delta_5, ([\mathbf{F}_1, ([\mathbf{F}_3 \vdash \Delta_4] \\ \bullet \mathbf{h}_2: (\Delta_5, ([\mathbf{F}_1), ([\mathbf{F}_3 \vdash \Delta_4] \\ \bullet \mathbf{h}_2: \Delta_5, \mathbf{F}_1, ([\mathbf{F}_1, ([\mathbf{F}_3 \vdash \Delta_4] \\ \bullet \mathbf{h}_2: \Delta_5, \mathbf{F}_1, ([\mathbf{F}_1, ([\mathbf{F}_3 \vdash \Delta_4] \\ \bullet \mathbf{h}_2: \Delta_5, \mathbf{F}_1, ([\mathbf{F}_1, ([\mathbf{F}_3 \vdash \Delta_4] \\ \bullet \mathbf{h}_2: \Delta_5, \mathbf{F}_1, ([\mathbf{F}_1, ([\mathbf{F}_3 \vdash \Delta_4] \\ \bullet \mathbf{h}_2: \Delta_5, ([\mathbf{F}_1,$$

$$\begin{array}{cccc} \underline{\mathbf{h}_1: \mathbf{F}_2, \Delta_4, []\mathbf{F}_2 \vdash \Delta_3} \\ \bullet \underline{\mathbf{h}_1: \Delta_4, []\mathbf{F}_2 \vdash \Delta_3} \end{array} \ AT & \rightarrow & & \overline{\underline{\mathbf{h}_1: \Delta_4, \mathbf{F}_2, \mathbf{F}_2, []\mathbf{F}_2 \vdash \Delta_3}} \end{array} \stackrel{\mathrm{ax/ind}}{} AT \\ \end{array}$$

• Case rule \perp_L

$$\frac{}{\bullet^{\mathrm{h}_2}:\bot,\Delta_4,\,[]\mathsf{F}_1\vdash\Delta_3}\ ^{\bot}L\qquad\rightarrow\qquad \frac{}{\bullet^{\mathrm{h}_2}:\bot,\Delta_4,\,\mathsf{F}_1,\,[]\mathsf{F}_1\vdash\Delta_3}\ ^{\bot}L$$

ullet Case rule I

$$\overline{\bullet \mathtt{h}_2 : \mathtt{p}_4, \Delta_5, []\mathtt{F}_1 \vdash \mathtt{p}_4, \Delta_3} \quad I \qquad \rightarrow \qquad \overline{\bullet \mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{p}_4, []\mathtt{F}_1 \vdash \Delta_3, \mathtt{p}_4} \quad I$$

• Case rule \top_L

$$\frac{\mathbf{h}_2:\Delta_4, []\mathbf{F}_1 \vdash \Delta_3}{\bullet \mathbf{h}_2:\top, \Delta_4, []\mathbf{F}_1 \vdash \Delta_3} \ \, \top_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_2:\Delta_4, \mathbf{F}_1, []\mathbf{F}_1 \vdash \Delta_3}}{\bullet \mathbf{h}_2:\top, \Delta_4, \mathbf{F}_1, []\mathbf{F}_1 \vdash \Delta_3} \ \, \top_L$$

4.15 Status of \perp_L : Invertible

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \bot, \mathbf{F}_2, \Delta_5 \vdash \mathbf{F}_3, \Delta_4}{\bullet \mathbf{h}_1: \bot, \Delta_5 \vdash \Delta_4, \mathbf{F}_2 \to \mathbf{F}_3} \end{array} \to_R \qquad \to \qquad \text{trivial}$$

• Case rule \wedge_R

$$\begin{array}{cccc} \mathbf{h}_1:\bot,\Delta_5 \vdash \mathbf{F}_2,\Delta_4 & \mathbf{h}_1:\bot,\Delta_5 \vdash \mathbf{F}_3,\Delta_4 \\ & \bullet \mathbf{h}_1:\bot,\Delta_5 \vdash \Delta_4, \mathbf{F}_2 \wedge \mathbf{F}_3 \end{array} \ \wedge_R & \rightarrow & \text{trivial} \end{array}$$

• Case rule \vee_R

$$\frac{\mathbf{h}_1:\bot,\Delta_5\vdash\mathbf{F}_2,\mathbf{F}_3,\Delta_4}{\bullet\mathbf{h}_1:\bot,\Delta_5\vdash\Delta_4,\mathbf{F}_2\vee\mathbf{F}_3}\ \vee_{R} \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \perp_R

$$\frac{\mathbf{h}_1:\bot,\Delta_3\vdash\Delta_2}{\bullet\mathbf{h}_1:\bot,\Delta_3\vdash\bot,\Delta_2}\ \bot_R \qquad \to \qquad \mathtt{trivial}$$

• Case rule \top_R

$$\frac{}{\bullet^{\text{h}_1}:\bot,\Delta_3\vdash\top,\Delta_2} \ \top_R \qquad \to \qquad \text{trivial}$$

 \bullet Case rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_4) \vdash \mathbf{F}_2}{\bullet \mathbf{h}_1: \Box \Gamma_4, \bot, \Delta_5 \vdash \Delta_3, []\mathbf{F}_2} \quad K \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule A45

$$\frac{\mathtt{h}_1: \Box \Gamma_5 \vdash \Box \Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1: \Box \Gamma_5, \bot, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, []\mathtt{F}_2} \quad {}_{A45} \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_1: \bot, \Delta_5 \vdash \mathbf{F}_2, \Delta_4 \quad \mathbf{h}_1: \bot, \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: (\bot, \Delta_5), \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \ \rightarrow_L \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_1: \bot, \mathbf{F}_2, \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: (\bot, \Delta_5), \mathbf{F}_2 \land \mathbf{F}_3 \vdash \Delta_4} \ \land_L \qquad \rightarrow \qquad \mathtt{trivial}$$

$$\frac{\mathbf{h}_1:\bot,\mathbf{F}_2,\Delta_5\vdash\Delta_4\quad \mathbf{h}_1:\bot,\mathbf{F}_3,\Delta_5\vdash\Delta_4}{\bullet\mathbf{h}_1:(\bot,\Delta_5),\mathbf{F}_2\vee\mathbf{F}_3\vdash\Delta_4}\ \vee_L \qquad \rightarrow \qquad \text{trivial}$$

 \bullet Case rule AT

$$\begin{array}{ll} \frac{\mathbf{h}_1:\bot,\mathbf{F}_2,\Delta_4,[]\mathbf{F}_2\vdash\Delta_3}{\bullet\mathbf{h}_1:(\bot,\Delta_4),[]\mathbf{F}_2\vdash\Delta_3} & AT & \rightarrow & \text{trivial} \end{array}$$

• Case rule \perp_L

ullet Case rule I

• Case rule \top_L

$$\frac{\mathbf{h}_1:\bot,\Delta_3\vdash\Delta_2}{\bullet\mathbf{h}_1:\top,\bot,\Delta_3\vdash\Delta_2}\ \top_L \qquad \to \qquad \mathsf{trivial}$$

4.16 Status of *I*:: Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \Delta_6, \mathbf{p}_5 \vdash \mathbf{F}_3, \Delta_4, \mathbf{p}_5}{\bullet \mathbf{h}_1: \Delta_6, \mathbf{p}_5 \vdash (\Delta_4, \mathbf{p}_5), \mathbf{F}_2 \to \mathbf{F}_3} \ \to_R \qquad \to \qquad \mathsf{trivial}$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_1:\Delta_6,\mathbf{p}_5\vdash\mathbf{F}_2,\Delta_4,\mathbf{p}_5\quad\mathbf{h}_1:\Delta_6,\mathbf{p}_5\vdash\mathbf{F}_3,\Delta_4,\mathbf{p}_5}{\bullet\mathbf{h}_1:\Delta_6,\mathbf{p}_5\vdash(\Delta_4,\mathbf{p}_5),\mathbf{F}_2\wedge\mathbf{F}_3} \quad \wedge_R \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule \vee_R

$$\frac{\mathbf{h}_1:\Delta_6,\mathbf{p}_5\vdash\mathbf{F}_2,\mathbf{F}_3,\Delta_4,\mathbf{p}_5}{\bullet\mathbf{h}_1:\Delta_6,\mathbf{p}_5\vdash(\Delta_4,\mathbf{p}_5),\mathbf{F}_2\vee\mathbf{F}_3}\ \vee_R \qquad \rightarrow \qquad \mathtt{trivial}$$

• Case rule \perp_R

$$\frac{\mathbf{h}_1:\Delta_4,\mathbf{p}_3\vdash\Delta_2,\mathbf{p}_3}{\bullet\mathbf{h}_1:\Delta_4,\mathbf{p}_3\vdash\bot,\Delta_2,\mathbf{p}_3}\ \bot_R \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule \top_R

 \bullet Case rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_5) \vdash \mathbf{F}_2}{\bullet \mathbf{h}_1: \Box \Gamma_5, \Delta_6, \mathbf{p}_4 \vdash (\Delta_3, \mathbf{p}_4), []\mathbf{F}_2} \quad K \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule A45

$$\frac{\mathtt{h}_1: \Box \Gamma_6 \vdash \Box \Gamma_3, \mathtt{F}_2}{\bullet \mathtt{h}_1: \Box \Gamma_6, \Delta_7, \mathtt{p}_5 \vdash \Box \Gamma_3, (\Delta_4, \mathtt{p}_5), []\mathtt{F}_2} \quad A45 \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_1:\Delta_6,\mathbf{p}_5\vdash\mathbf{F}_2,\Delta_4,\mathbf{p}_5\quad\mathbf{h}_1:\mathbf{F}_3,\Delta_6,\mathbf{p}_5\vdash\Delta_4,\mathbf{p}_5}{\bullet\mathbf{h}_1:(\Delta_6,\mathbf{p}_5),\mathbf{F}_2\rightarrow\mathbf{F}_3\vdash\Delta_4,\mathbf{p}_5}\ \rightarrow_L \qquad \rightarrow \qquad \text{trivial}$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \mathbf{F}_3, \Delta_6, \mathbf{p}_5 \vdash \Delta_4, \mathbf{p}_5}{\bullet \mathbf{h}_1: (\Delta_6, \mathbf{p}_5), \mathbf{F}_2 \land \mathbf{F}_3 \vdash \Delta_4, \mathbf{p}_5} \ \land_L \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule \vee_L

$$\frac{\mathbf{h}_1: \mathbf{F}_2, \Delta_6, \mathbf{p}_5 \vdash \Delta_4, \mathbf{p}_5 \quad \mathbf{h}_1: \mathbf{F}_3, \Delta_6, \mathbf{p}_5 \vdash \Delta_4, \mathbf{p}_5}{\bullet \mathbf{h}_1: (\Delta_6, \mathbf{p}_5), \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4, \mathbf{p}_5} \ \vee_L \qquad \rightarrow \qquad \text{trivial}$$

 \bullet Case rule AT

$$\begin{array}{ll} \frac{\mathbf{h}_1: \mathbf{F}_2, \Delta_5, \mathbf{p}_4, []\mathbf{F}_2 \vdash \Delta_3, \mathbf{p}_4}{\bullet \mathbf{h}_1: (\Delta_5, \mathbf{p}_4), []\mathbf{F}_2 \vdash \Delta_3, \mathbf{p}_4} & AT & \rightarrow & \text{trivial} \end{array}$$

• Case rule \perp_L

 \bullet Case rule I

$$\frac{}{\bullet \mathbf{h}_1: \mathbf{p}_4, \Delta_5, \mathbf{p}_3 \vdash \mathbf{p}_4, \Delta_2, \mathbf{p}_3} \quad I \qquad \rightarrow \qquad \mathsf{trivial}$$

$$\overline{\bullet \mathbf{h}_1: \mathbf{p}_3, \Delta_4 \vdash \mathbf{p}_3, \Delta_2} \quad I \qquad \rightarrow \qquad \mathsf{trivial}$$

• Case rule \top_L

$$\frac{\mathbf{h}_1:\Delta_4,\mathbf{p}_3\vdash\Delta_2,\mathbf{p}_3}{\bullet\mathbf{h}_1:\top,\Delta_4,\mathbf{p}_3\vdash\Delta_2,\mathbf{p}_3}\ \top_L \qquad \rightarrow \qquad \mathsf{trivial}$$

4.17 Status of \top_L : Invertible

• Case rule \rightarrow_R

$$\frac{\mathbf{h}_1: \top, \mathbf{F}_2, \Delta_5 \vdash \mathbf{F}_3, \Delta_4}{\bullet \mathbf{h}_1: \top, \Delta_5 \vdash \Delta_4, \mathbf{F}_2 \to \mathbf{F}_3} \ \rightarrow_{R} \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4, \mathbf{F}_3}}{\bullet \mathbf{h}_1: \Delta_5 \vdash \Delta_4, \mathbf{F}_2 \to \mathbf{F}_3} \overset{\mathsf{ax/ind}}{\to}_{R}$$

• Case rule \wedge_R

• Case rule \vee_R

$$\frac{\mathbf{h}_1: \top, \Delta_5 \vdash \mathbf{F}_2, \mathbf{F}_3, \Delta_4}{\bullet \mathbf{h}_1: \top, \Delta_5 \vdash \Delta_4, \mathbf{F}_2 \vee \mathbf{F}_3} \quad \vee_R \qquad \rightarrow \qquad \frac{\mathbf{h}_1: \Delta_5 \vdash \Delta_4, \mathbf{F}_2, \mathbf{F}_3}{\bullet \mathbf{h}_1: \Delta_5 \vdash \Delta_4, \mathbf{F}_2 \vee \mathbf{F}_3} \stackrel{\mathsf{ax/ind}}{\vee}_R$$

• Case rule \perp_R

• Case rule \top_R

$$\overline{\bullet_{\mathtt{h}_1}: \top, \Delta_3 \vdash \top, \Delta_2} \ ^{\top}R \qquad \rightarrow \qquad \overline{\bullet_{\mathtt{h}_1}: \Delta_3 \vdash \top, \Delta_2} \ ^{\top}R$$

 \bullet Case rule K

$$\frac{\mathbf{h}_1: unbox(\Box \Gamma_4) \vdash \mathbf{F}_2}{\bullet \mathbf{h}_1: \Box \Gamma_4, \top, \Delta_5 \vdash \Delta_3, []\mathbf{F}_2} \quad K \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: unbox(\Box \Gamma_4) \vdash \mathbf{F}_2}}{\bullet \mathbf{h}_1: \Delta_5, \Box \Gamma_4 \vdash \Delta_3, []\mathbf{F}_2} \quad K$$

• Case rule A45

$$\frac{\mathtt{h}_1: \Box \Gamma_5 \vdash \Box \Gamma_3, \mathtt{f}_2}{\bullet \mathtt{h}_1: \Box \Gamma_5, \top, \Delta_6 \vdash \Box \Gamma_3, \Delta_4, []\mathtt{f}_2} \quad \textit{A45} \qquad \rightarrow \qquad \frac{\overline{\mathtt{h}_1: \Box \Gamma_5 \vdash \mathtt{f}_2, \Box \Gamma_3}}{\bullet \mathtt{h}_1: \Delta_6, \Box \Gamma_5 \vdash \Delta_4, \Box \Gamma_3, []\mathtt{f}_2} \quad \textit{A45}$$

• Case rule \rightarrow_L

$$\frac{\mathbf{h}_1: \top, \Delta_5 \vdash \mathbf{F}_2, \Delta_4 \quad \mathbf{h}_1: \top, \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: (\top, \Delta_5), \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \ \rightarrow_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5 \vdash \Delta_4, \mathbf{F}_2} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \rightarrow \mathbf{F}_3 \vdash \Delta_4} \quad \xrightarrow{\mathbf{ax/ind}} \quad \xrightarrow{\Delta_1: \Delta_2 \vdash \Delta_2} \quad \rightarrow_L \quad \rightarrow$$

• Case rule \wedge_L

$$\frac{\mathbf{h}_1: \top, \mathbf{f}_2, \mathbf{f}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: (\top, \Delta_5), \mathbf{f}_2 \wedge \mathbf{f}_3 \vdash \Delta_4} \ \, \wedge_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{f}_2, \mathbf{f}_3 \vdash \Delta_4}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{f}_2 \wedge \mathbf{f}_3 \vdash \Delta_4} \overset{\mathrm{ax/ind}}{\wedge}_L$$

• Case rule \vee_L

$$\frac{\mathbf{h}_1: \top, \mathbf{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathbf{h}_1: \top, \mathbf{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathbf{h}_1: (\top, \Delta_5), \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{F}_2 \vee \mathbf{F}_3 \vdash \Delta_4} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vee \mathbf{h}_3} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vee \mathbf{h}_3} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vee \mathbf{h}_3} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vee \mathbf{h}_3} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \text{ax/ind}}{\bullet \mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vee \mathbf{h}_3} \quad \vee_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{h}_2 \vdash \Delta_4} \quad \nabla_L \qquad \rightarrow \qquad \frac{\overline{\mathbf{h}_1: \Delta_5, \mathbf{$$

 \bullet Case rule AT

$$\begin{array}{c} \mathbf{h}_1: \top, \mathbf{F}_2, \Delta_4, ([\mathbf{F}_2 \vdash \Delta_3 \\ \bullet \mathbf{h}_1: (\top, \Delta_4), ([\mathbf{F}_2 \vdash \Delta_3 \\ \end{array}) \quad AT \qquad \rightarrow \qquad \overline{\begin{array}{c} \mathbf{h}_1: \Delta_4, \mathbf{F}_2, ([\mathbf{F}_2 \vdash \Delta_3 \\ \bullet \mathbf{h}_1: \Delta_4, ([\mathbf{F}_2 \vdash \Delta_3 \\ \end{array}) \\ AT \end{array}} \quad \overset{\mathrm{ax/ind}}{\rightarrow} \\ AT \end{array}$$

• Case rule \perp_L

$$\frac{}{\bullet \mathbf{h}_1:\bot,\top,\Delta_3\vdash \Delta_2} \ ^\bot L \qquad \rightarrow \qquad \frac{}{\bullet \mathbf{h}_1:\bot,\Delta_3\vdash \Delta_2} \ ^\bot L$$

ullet Case rule I

$$\frac{}{\bullet \mathtt{h}_1: \mathtt{p}_3\,, \top, \Delta_4 \vdash \mathtt{p}_3, \Delta_2} \quad I \qquad \rightarrow \qquad \frac{}{\bullet \mathtt{h}_1: \Delta_4\,, \mathtt{p}_3 \vdash \Delta_2\,, \mathtt{p}_3} \quad I$$

• Case rule \top_L

5 Identity-Expansion

$$\frac{\frac{-: F_0 \vdash F_0}{-: []F_0 \vdash []F_0} \ IH}{\frac{-: F_0 \vdash F_0}{-: F_0 \vdash F_0} \ W} \frac{\frac{-: F_1 \vdash F_1}{-: F_1 \vdash F_0, F_1} \ W}{\frac{-: F_0 \vdash F_0, F_1}{-: F_0 \lor F_1 \vdash F_0, F_1} \lor_L} \bigvee_{L} \frac{\frac{-: F_0 \vdash F_0}{-: F_0 \lor F_1 \vdash F_0, F_1} \ V_L}{\frac{-: F_0 \vdash F_0}{-: F_0, F_1 \vdash F_0} \ W} \frac{\frac{-: F_1 \vdash F_1}{-: F_0, F_1 \vdash F_1} \ W}{\frac{-: F_0, F_1 \vdash F_0 \land F_1}{-: F_0 \land F_1 \vdash F_0 \land F_1} \land_L} \bigvee_{A_R} \frac{\frac{-: F_0 \vdash F_0}{-: F_0 \vdash F_0, F_1} \ W}{\frac{-: F_0 \vdash F_0, F_1}{-: F_0 \vdash F_0, F_1 \vdash F_1} \to_R} \frac{IH}{\frac{-: F_0 \vdash F_0}{-: F_0 \to F_1 \vdash F_0} \to_R} \xrightarrow{L} \frac{-: F_0 \vdash F_0}{-: T \vdash T} \vdash_R} \frac{1}{-: T \vdash T} \bigvee_{A_R} \frac{-: T \vdash T}{-: T \vdash T} \stackrel{T_R}{}$$

6 Cut-Elimination

6.1 Status of \rightarrow_R : OK

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{h_1: F_6, \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \to F_{10}}{\bullet h_1: \Delta_{12} \vdash (\Delta_{11}, F_9 \to F_{10}), F_6 \to F_7} & \frac{h_8: F_9, \Delta_{12}, F_6 \to F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8: \Delta_{12}, F_6 \to F_7 \vdash \Delta_{11}, F_9 \to F_{10}} & \xrightarrow{-: \Delta_{12} \vdash \Delta_{11}, F_9 \to F_{10}} \\ \hline -: \Delta_{12} \vdash \Delta_{11}, F_9 \to F_{10} & \xrightarrow{-: \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_{10}, F_7} & \text{inv-th/ax} \\ \hline \frac{h_1: \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_{10}, F_7 \to F_8}{\bullet h_1: \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_7 \to F_8} & \xrightarrow{h_8: \Delta_{12}, F_9, F_6 \to F_7 \vdash \Delta_{11}, F_{10}} \\ \hline -: \Delta_{12} \vdash \Delta_{11}, F_{10} \to F_{10} & \xrightarrow{-: \Delta_{12} \vdash \Delta_{11}, F_{10}} & \to R \\ \hline \frac{h_1: F_7, \Delta_{14} \vdash F_8, F_{13}, \Delta_{12}, F_{10} \to F_{11}}{-: \Delta_{12} \vdash \Delta_{11}, F_7 \to F_8} & \xrightarrow{h_9: F_{10}, F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_7 \to F_8} \\ \hline -: \Delta_{14} \vdash ((\Delta_{12}, F_{10} \to F_{11}), F_7 \to F_8), F_{13} & \xrightarrow{-: \Delta_{14} \vdash (\Delta_{12}, F_{10} \to F_{11}), F_7 \to F_8} & \xrightarrow{h_9: \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \to F_{11}), F_7 \to F_8} \\ \hline -: \Delta_{14} \vdash ((\Delta_{12}, F_{10} \to F_{11}), F_7 \to F_8) & \xrightarrow{-: \Delta_{14} \vdash (\Delta_{12}, F_{11}, F_{13}, F_8} & \xrightarrow{-: \text{inv-th/ax}} \\ \hline -: \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_{13}, F_8 & \xrightarrow{-: \text{inv-th/ax}} & \xrightarrow{-: \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \to F_8} & \xrightarrow{-: \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \to F_8} \\ \hline -: \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \to F_8 & \xrightarrow{-: \Delta_{14}, F_{10} \vdash \Delta_{12}, F_{11}, F_7 \to F_8} \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \to F_9 & \xrightarrow{-: \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} & \xrightarrow{-: \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \to F_9 & \xrightarrow{-: \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} & \xrightarrow{-: \Delta_{12}, F_8 \vdash \Delta_{10}, F_9} & \xrightarrow{-: \Delta_{12}, F_8 \vdash \Delta_{10}, F_9} \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \to F_9 & \xrightarrow{-: \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \to F_9} & \xrightarrow{-: \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} & \xrightarrow{-: \Delta_{12}, F_8 \vdash \Delta_{10}, F_9} \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \to F_9} & \xrightarrow{-: \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} & \xrightarrow{-: \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \to F_9} & \xrightarrow{-: \Delta_{12}, F_{11}, F_8 \vdash \Delta_{10}, F_9} & \xrightarrow{-: \Delta_{12}, F_8 \vdash \Delta_{10}, F_8 \to F_9} \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \to F_9} & \xrightarrow{-: \Delta_{12}, F_$$

• Case rule \wedge_R

$$\frac{ \begin{array}{c} \mathbf{h}_{1} : F_{6}, \Delta_{12} \vdash F_{7}, \Delta_{11}, F_{9} \land F_{10} \\ \bullet \mathbf{h}_{1} : \Delta_{12} \vdash (\Delta_{11}, F_{9} \land F_{10}), F_{6} \rightarrow F_{7} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12} \vdash (\Delta_{11}, F_{9} \land F_{10}), F_{6} \rightarrow F_{7} \\ \hline \\ -: \Delta_{12} \vdash \Delta_{11}, F_{9} \land F_{10} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{12}, F_{6} \vdash \Delta_{11}, F_{7}, F_{9} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12}, F_{6} \vdash \Delta_{11}, F_{7}, F_{9} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12}, F_{6} \vdash \Delta_{11}, F_{7}, F_{9} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12}, F_{6} \vdash \Delta_{11}, F_{7}, F_{9} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12}, F_{6} \vdash \Delta_{11}, F_{10}, F_{7} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, F_{9} \land F_{7} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, F_{9} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, F_{9} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, F_{9} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, F_{9} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, F_{9} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, F_{10} \\ \hline \bullet \mathbf{h}_{1} : F_{7}, \Delta_{14} \vdash F_{8}, F_{13}, \Delta_{12}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \land F_{11}), F_{7} \rightarrow F_{8}), F_{13} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14} \vdash ((\Delta_{12}, F_{10} \land F_{11}), F_{7} \rightarrow F_{8}), F_{13} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14} \vdash (\Delta_{12}, F_{10} \land F_{11}), F_{7} \rightarrow F_{8} \\ \hline \bullet \mathbf{h}_{9} : \Delta_{14} \vdash (\Delta_{12}, F_{10} \land F_{11}), F_{7} \rightarrow F_{8} \\ \hline \bullet \mathbf{h}_{9} : \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{9} : \Delta_{14}, F_{13}, F_{7} \vdash \Delta_{12}, F_{10}, F_{8} \\ \hline \bullet \mathbf{h}_{9} : \Delta_{14}, F_{13}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{7} \vdash \Delta_{12}, F_{8}, F_{10} \land F_{11} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{14}, F_{13} \vdash \Delta_{14}, F_{13},$$

• Case rule \vee_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathsf{F}_6, \Delta_{12} \vdash \mathsf{F}_7, \Delta_{11}, \mathsf{F}_9 \vee \mathsf{F}_{10}}{\bullet \mathbf{h}_1: \Delta_{12} \vdash (\Delta_{11}, \mathsf{F}_9 \vee \mathsf{F}_{10}), \mathsf{F}_6 \to \mathsf{F}_7} \to_R & \frac{\mathbf{h}_8: \Delta_{12}, \mathsf{F}_6 \to \mathsf{F}_7 \vdash \mathsf{F}_9, \mathsf{F}_{10}, \Delta_{11}}{\bullet \mathbf{h}_8: \Delta_{12}, \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_{11}, \mathsf{F}_9 \vee \mathsf{F}_{10}} & \vee_R \\ \hline & -: \Delta_{12} \vdash \Delta_{11}, \mathsf{F}_9 \vee \mathsf{F}_{10} \\ \hline \frac{\mathbf{h}_1: \Delta_{12}, \mathsf{F}_6 \vdash \Delta_{11}, \mathsf{F}_{10}, \mathsf{F}_7, \mathsf{F}_9}{\bullet \mathbf{h}_1: \Delta_{12} \vdash \Delta_{11}, \mathsf{F}_{10}, \mathsf{F}_9, \mathsf{F}_6 \to \mathsf{F}_7} & \mathbf{h}_8: \Delta_{12}, \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_{11}, \mathsf{F}_{10}, \mathsf{F}_9 \\ \hline \frac{-: \Delta_{12} \vdash \Delta_{11}, \mathsf{F}_{10}, \mathsf{F}_9}{\bullet \cdot \mathsf{h}_1: \Delta_{12} \vdash \Delta_{11}, \mathsf{F}_{10}, \mathsf{F}_9 \vee \mathsf{F}_{10}} & \vee_R \\ \hline \frac{-: \Delta_{12} \vdash \Delta_{11}, \mathsf{F}_{10}, \mathsf{F}_9 \vee \mathsf{F}_{10}}{\bullet \cdot \mathsf{h}_1: \Delta_{12} \vdash \Delta_{11}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_{10}, \mathsf{F}_9} & \mathsf{h}_{\mathsf{Cut}} \\ \hline \frac{-: \Delta_{12} \vdash \Delta_{11}, \mathsf{F}_{10} \vee \mathsf{F}_{11}}{\bullet \cdot \mathsf{h}_1: \Delta_{14} \vdash ((\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{F}_7 \to \mathsf{F}_8), \mathsf{F}_{13}} & \to_R & \frac{\mathbf{h}_9: \mathsf{F}_{13}, \Delta_{14} \vdash \mathsf{F}_{10}, \mathsf{F}_{11}, \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8}{\bullet \mathsf{h}_9: \Delta_{14}, \mathsf{F}_{13} \vdash (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{F}_7 \to \mathsf{F}_8} & \vee_R \\ \mathsf{Cut} \\ \hline -: \Delta_{14} \vdash (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{F}_7 \to \mathsf{F}_8 \\ \hline \frac{\mathbf{h}_9: \Delta_{14}, \mathsf{F}_{13}, \mathsf{F}_7 \vdash \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_8}{\bullet \mathsf{h}_9: \Delta_{14}, \mathsf{F}_{13}, \mathsf{F}_7 \vdash \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_{11}} & \mathsf{h}_{\mathsf{Cut}} \\ \hline -: \Delta_{14}, \mathsf{F}_7 \vdash \Delta_{12}, \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} & \to_R \\ \hline -: \Delta_{14}, \mathsf{F}_7 \vdash \Delta_{12}, \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \hline -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \hline -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \hline -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \hline -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \hline -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \hline -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \hline -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \hline -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \hline -: \Delta_{14} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8, \mathsf{F$$

• Case rule \perp_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_6, \Delta_{10} \vdash \mathbf{F}_7, \bot, \Delta_9}{\bullet \mathbf{h}_1: \Delta_{10} \vdash (\bot, \Delta_9), \mathbf{F}_6 \to \mathbf{F}_7} \xrightarrow{} \mathcal{F}_R & \frac{\mathbf{h}_8: \Delta_{10}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \Delta_9}{\bullet \mathbf{h}_8: \Delta_{10}, \mathbf{F}_6 \to \mathbf{F}_7 \vdash \bot, \Delta_9} & \mathcal{L}_R \\ \hline -: \Delta_{10} \vdash \bot, \Delta_9 & & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_1: \Delta_{10} \vdash \bot, \Delta_9, \mathbf{F}_6 \to \mathbf{F}_7 & \mathbf{ax/W} \\ \hline -: \Delta_{10} \vdash \bot, \Delta_9 & & \mathbf{hCut} \\ \hline \\ \bullet \mathbf{h}_1: \mathbf{F}_7, \Delta_{12} \vdash \mathbf{F}_8, \mathbf{F}_{11}, \bot, \Delta_{10} & & \mathbf{h}_9: \mathbf{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \\ \hline \bullet \mathbf{h}_1: \Delta_{12} \vdash ((\bot, \Delta_{10}), \mathbf{F}_7 \to \mathbf{F}_8), \mathbf{F}_{11} & & \mathbf{h}_9: \mathbf{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \\ \hline -: \Delta_{12} \vdash (\bot, \Delta_{10}), \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_1: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_{11}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 & & \mathbf{ax/W} \\ \hline -: \Delta_{12} \vdash \bot, \Delta_{10},$$

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathsf{F}_6, \Delta_{10} \vdash \mathsf{F}_7, \top, \Delta_9}{\bullet \mathbf{h}_1: \Delta_{10} \vdash (\top, \Delta_9), \mathsf{F}_6 \to \mathsf{F}_7} \to_R & \frac{}{\bullet \mathbf{h}_8: \Delta_{10}, \mathsf{F}_6 \to \mathsf{F}_7 \vdash \top, \Delta_9} & \mathsf{T}_R \\ & -: \Delta_{10} \vdash \top, \Delta_9 \\ & \frac{}{-: \Delta_{10} \vdash \top, \Delta_9} & \top_R \\ \\ \frac{\mathbf{h}_1: \mathsf{F}_7, \Delta_{12} \vdash \mathsf{F}_8, \mathsf{F}_{11}, \top, \Delta_{10}}{\bullet \mathbf{h}_1: \Delta_{12} \vdash ((\top, \Delta_{10}), \mathsf{F}_7 \to \mathsf{F}_8), \mathsf{F}_{11}} & \to_R & \frac{}{\bullet \mathbf{h}_9: \Delta_{12}, \mathsf{F}_{11} \vdash (\top, \Delta_{10}), \mathsf{F}_7 \to \mathsf{F}_8} \\ & -: \Delta_{12} \vdash (\top, \Delta_{10}), \mathsf{F}_7 \to \mathsf{F}_8 \\ & \frac{}{-: \Delta_{12} \vdash \top, \Delta_{10}, \mathsf{F}_7 \to \mathsf{F}_8} & \top_R \end{array}$$

\bullet Case rule K

$$\frac{\mathbf{h}_1: \mathbf{F}_6, \Box \Gamma_{11}, \Delta_{12} \vdash \mathbf{F}_7, \Delta_{10}, []\mathbf{F}_9}{\underbrace{\bullet \mathbf{h}_1: \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []\mathbf{F}_9), \mathbf{F}_6 \to \mathbf{F}_7}}_{\bullet \mathbf{h}_8: (\Box \Gamma_{11}, \Delta_{12}), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \Delta_{10}, []\mathbf{F}_9}} \xrightarrow{\bullet \mathbf{h}_8: (\Box \Gamma_{11}, \Delta_{12}), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \Delta_{10}, []\mathbf{F}_9}}_{\bullet \mathbf{h}_8: (\Box \Gamma_{11}, \Delta_{12}), \mathbf{F}_6 \to \mathbf{F}_7 \vdash \Delta_{10}, []\mathbf{F}_9}} \overset{K}{\text{cut}}$$

• Case rule A45

$$\begin{array}{c} \frac{h_1 : F_6, \Box \Gamma_{12}, \Delta_{13} \vdash F_7, \Box \Gamma_{10}, \Delta_{11}, []F_9)}{\bullet h_1 : \Box \Gamma_{12}, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, []F_9), F_6 \to F_7} & \frac{h_8 : \Box \Gamma_{12} \vdash \Box \Gamma_{10}, F_9}{\bullet h_8 : (\Box \Gamma_{12}, \Delta_{13}), F_6 \to F_7 \vdash \Box \Gamma_{10}, \Delta_{11}, []F_9} & A45 \\ \hline -: \Box \Gamma_{12}, \Delta_{13} \vdash \Box \Gamma_{10}, \Delta_{11}, []F_9 & \rightarrow \\ \hline -: \Box \Gamma_{12} \vdash F_9, \Box \Gamma_{10} & ax/W \\ \hline -: \Delta_{13}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box \Gamma_{10}, []F_9 & A45 \\ \hline \\ \frac{h_1 : F_7, \Box \Gamma_{14}, \Delta_{15} \vdash F_8, \Box F_{13}, \Box \Gamma_{11}, \Delta_{12}, []F_{10} & \rightarrow_R & h_9 : \Box \Gamma_{14}, \Box F_{13} \vdash \Box \Gamma_{11}, F_{10} \\ \hline \bullet h_1 : \Box \Gamma_{14}, \Delta_{15} \vdash ((\Box \Gamma_{11}, \Delta_{12}, []F_{10}), F_7 \to F_8), \Box F_{13} & \rightarrow_R & h_9 : \Box \Gamma_{14}, \Box \Gamma_{11}, \Delta_{12}, []F_{10}), F_7 \to F_8 \\ \hline -: \Box \Gamma_{14}, \Delta_{15} \vdash ((\Box \Gamma_{11}, \Delta_{12}, []F_{10}), F_7 \to F_8, \Box F_{13}, \Box \Gamma_{14} \vdash F_{10}, \Box \Gamma_{11} & ax/W \\ \hline -: \Box \Gamma_{14}, \Delta_{15} \vdash ((\Box \Gamma_{11}, \Delta_{12}, []F_{10}), F_7 \to F_8 & \rightarrow_R \\ \hline -: \Delta_{15}, F_7, \Box \Gamma_{14} \vdash \Delta_{12}, F_8, \Box \Gamma_{11}, []F_{10} & ax/W & h_9 : \Box \Gamma_{13}, \Delta_{15}, F_7, \Box \Gamma_{14} \vdash \Delta_{12}, F_8, \Box \Gamma_{11}, []F_{10} \\ \hline -: \Delta_{15}, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{10} & \rightarrow_R \\ \hline -: \Delta_{15}, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash F_8, F_{14}, \Box \Gamma_{11}, \Delta_{12}, []F_{10} & \rightarrow_R \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}), F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -: \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}, F_7 \to F_8 \\ \hline -$$

• Case rule \rightarrow_L

$$\frac{h_1:F_6,\Delta_{12},F_9\to F_{10}\vdash F_7,\Delta_{11}}{\bullet h_1:\Delta_{12},F_9\to F_{10}\vdash \Delta_{11},F_6\to F_7}\to R \quad \frac{h_8:\Delta_{12},F_6\to F_7\vdash F_9,\Delta_{11}}{\bullet h_8:(\Delta_{12},F_9\to F_{10}),F_6\to F_7\vdash \Delta_{11}}}{-:\Delta_{12},F_9\to F_{10}\vdash \Delta_{11}}\to Cut}\to L$$

$$\frac{h_1:\Delta_{12},F_6\vdash \Delta_{11},F_7,F_9}{\bullet h_1:\Delta_{12},F_6\vdash \Delta_{11},F_7,F_9} \quad \text{inv-th/ax}}{h_8:\Delta_{12},F_6\to F_7\vdash \Delta_{11},F_9} \quad \frac{h_8:\Delta_{12},F_6\to F_7\vdash \Delta_{11},F_9}{\bullet h_1:\Delta_{12},F_{10}\vdash A_{11},F_9\to F_7}\to R \quad h_8:\Delta_{12},F_6\to F_7\vdash \Delta_{11},F_9} \quad \frac{h_1:\Delta_{12},F_{10},F_6\vdash \Delta_{11},F_7}{\bullet h_1:\Delta_{12},F_{10}\vdash A_{11},F_6\to F_7}\to R \quad h_8:\Delta_{12},F_{10},F_6\to F_7\vdash \Delta_{11}} \\ -:\Delta_{12}\vdash \Delta_{11},F_9 \\ \hline -:\Delta_{12}\vdash \Delta_{11},F_9 \\ \hline -:\Delta_{12}\vdash A_{11},F_9 \\ \hline \bullet h_1:\Delta_{10}\vdash \Delta_9,F_7\to F_8 \\ \hline \bullet h_2:\Delta_{10}\vdash A_9 \\ \hline -:\Delta_{10}\vdash A_9 \\ \hline -:\Delta_{10},F_7\vdash A_$$

 $-:\Delta_{10}\vdash\Delta_{9}$

$$\frac{ \begin{array}{c} \frac{h_1: F_7, \Delta_{13} \vdash F_8, F_{10} \to F_{11}, \Delta_{12}}{\bullet h_1: \Delta_{13} \vdash (\Delta_{12}, F_7 \to F_8), F_{10} \to F_{11}} \\ \to R \end{array} \begin{array}{c} \frac{h_9: \Delta_{13} \vdash F_{10}, \Delta_{12}, F_7 \to F_8 \quad h_9: F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \to F_8}{\bullet h_9: \Delta_{13}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \to F_8} \end{array} } \to_L \\ & \xrightarrow{\bullet h_9: \Delta_{13}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \to F_8} Cut} \\ & \xrightarrow{\bullet h_9: \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} Cut \\ & \xrightarrow{\bullet h_9: \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} Cut \\ & \xrightarrow{\bullet h_9: \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} Cut \\ & \xrightarrow{\bullet h_9: \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} \to R \end{array} \begin{array}{c} \text{inv-th/ax} \\ \xrightarrow{\bullet h_9: \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} \\ \xrightarrow{\bullet h_9: \Delta_{13}, F_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_8} \end{array} \rightarrow_L \\ & \xrightarrow{\bullet h_9: \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} Cut \\ & \xrightarrow{\bullet h_9: \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \to F_8} Cut \\ & \xrightarrow{\bullet h_1: F_7, \Delta_{14}, F_{10} \to F_{11} \vdash (\Delta_{12}, F_7 \to F_8), F_{13}} \rightarrow_R \begin{array}{c} h_9: F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_7 \to F_8 & h_9: F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \to F_8 \\ & \xrightarrow{\bullet h_9: (\Delta_{14}, F_{10} \to F_{11}), F_{13} \vdash \Delta_{12}, F_7 \to F_8} Cut \\ & \xrightarrow{\bullet h_1: \Delta_{14}, F_{10} \to F_{11} \vdash (\Delta_{12}, F_7 \to F_8)} \xrightarrow{\bullet h_9: (\Delta_{14}, F_{10} \to F_{11}), F_{13} \vdash \Delta_{12}, F_7 \to F_8} Cut \\ & \xrightarrow{\bullet h_9: \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \to F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{11}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{13}, F_7 \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{13}, F_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{13}, F_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{13}, F_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{13}, F_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \to F_8} \xrightarrow{\bullet h_9: \Delta_{14}, F_{13}, F_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \to F_8} \xrightarrow{\bullet h_9$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \mathsf{F}_6, \Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10} \vdash \mathsf{F}_7, \Delta_{11}}{\bullet \mathbf{h}_1 : \Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10} \vdash \Delta_{11}, \mathsf{F}_6 \to \mathsf{F}_7} \to_R & \frac{\mathbf{h}_8 : \mathsf{F}_9, \mathsf{F}_{10}, \Delta_{12}, \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_{11}}{\bullet \mathbf{h}_8 : (\Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10}), \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_{11}} & \wedge_L \\ & - : \Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10} \vdash \Delta_{11} \\ & \to \\ \frac{\mathbf{h}_1 : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_6, \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_7}{\bullet \mathbf{h}_1 : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_6 \to \mathsf{F}_7} & \frac{\mathbf{h}_8 : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_9, \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_{11}}{\mathsf{h}_8 : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_9, \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_{11}} & \mathsf{ax/W} \\ & \frac{- : \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_9 \vdash \Delta_{11}}{- : \Delta_{12}, \mathsf{F}_9 \wedge \mathsf{F}_{10} \vdash \Delta_{11}} & \wedge_L \\ \\ \frac{\mathbf{h}_1 : \mathsf{F}_7, \Delta_{13} \vdash \mathsf{F}_8, \mathsf{F}_{10} \wedge \mathsf{F}_{11}, \Delta_{12}}{- : \Delta_{13} \vdash (\Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8), \mathsf{F}_{10} \wedge \mathsf{F}_{11}} & \to_R & \frac{\mathbf{h}_9 : \mathsf{F}_{10}, \mathsf{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8}{\bullet \mathbf{h}_9 : \Delta_{13}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8} & \wedge_L \\ \\ \frac{\mathbf{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8), \mathsf{F}_{10} \wedge \mathsf{F}_{11}}{- : \Delta_{13} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8} & \to_R \\ \\ \frac{\mathbf{h}_1 : \Delta_{13}, \mathsf{F}_7 \vdash \Delta_{12}, \mathsf{F}_8, \mathsf{F}_{10} \wedge \mathsf{F}_{11}}{- : \Delta_{13} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8} & \to_R \\ \\ \frac{\mathbf{h}_1 : \Delta_{14}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \mathsf{F}_8, \mathsf{F}_{13}, \Delta_{12}}{- : \Delta_{13} \vdash \Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8} & \to_R \\ \\ \frac{\mathbf{h}_1 : \mathsf{F}_7, \Delta_{14}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \mathsf{F}_8, \mathsf{F}_{13}, \Delta_{12}}{- : \Delta_{14}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \Delta_{12}, \mathsf{F}_8} & \to_R \\ \\ \frac{\mathbf{h}_1 : \Delta_{14}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash (\Delta_{12}, \mathsf{F}_7 \to \mathsf{F}_8), \mathsf{F}_{13}}{- : \Delta_{14}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \Delta_{12}, \mathsf{F}_8} & \to_R \\ \\ \frac{\mathbf{h}_1 : \Delta_{14}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \Delta_{12}, \mathsf{F}_8, \mathsf{F}_{13}}{- : \Delta_{14}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \Delta_{12}, \mathsf{F}_8} & \to_R \\ \\ \frac{\mathbf{h}_1 : \Delta_{14}, \mathsf{F}_7, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \Delta_{12}, \mathsf{F}_8, \mathsf{F}_{13}}{- : \Delta_{14}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \Delta_{12}, \mathsf{F}_8} & \to_R \\ \\ \frac{\mathbf{h}_1 : \Delta_{14}, \mathsf{F}_7, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \Delta_{12}, \mathsf{F}_8, \mathsf{F}_{13}}{- : \Delta_{14}, \mathsf{F}_{10} \wedge \mathsf{F}_{11} \vdash \Delta_{12}, \mathsf{F}_8} & \to_R \\ \\ \frac{\mathbf{h}_1 : \Delta_{14}, \mathsf{F}_$$

• Case rule \vee_L

$$\frac{\frac{\mathbf{h}_{1}: F_{6}, \Delta_{12}, F_{9} \vee F_{10} \vdash F_{7}, \Delta_{11}}{\bullet \mathbf{h}_{1}: \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11}, F_{6} \to F_{7}}}{\bullet \mathbf{h}_{1}: \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11}, F_{6} \to F_{7}}} \xrightarrow{A_{R}} \frac{\mathbf{h}_{8}: F_{9}, \Delta_{12}, F_{6} \to F_{7} \vdash \Delta_{11}}{\bullet \mathbf{h}_{8}: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \to F_{7} \vdash \Delta_{11}}}{\bullet \mathbf{h}_{8}: (\Delta_{12}, F_{9} \vee F_{10}), F_{6} \to F_{7} \vdash \Delta_{11}}} \xrightarrow{\mathbf{cut}} \times \mathbf{cut}$$

$$\frac{\mathbf{h}_{1}: \Delta_{12}, F_{6}, F_{9} \vdash \Delta_{11}, F_{7}}{\bullet \mathbf{h}_{1}: \Delta_{12}, F_{9} \vdash \Delta_{11}, F_{6} \to F_{7}} \xrightarrow{\mathbf{h}_{R}: \Delta_{12}, F_{9}, F_{6} \to F_{7} \vdash \Delta_{11}}} \times \mathbf{h}_{1} \times \mathbf{h}_{1} \times \mathbf{h}_{1} \times \mathbf{h}_{2} \times \mathbf{h$$

$$\frac{ \frac{h_1: F_7, \Delta_{13} \vdash F_8, F_{10} \lor F_{11}, \Delta_{12}}{\bullet h_1: \Delta_{13} \vdash (\Delta_{12}, F_7 \to F_8), F_{10} \lor F_{11}} \to_R \frac{h_9: F_{10}, \Delta_{13} \vdash \Delta_{12}, F_7 \to F_8}{\bullet h_9: \Delta_{13}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \to F_8} \underbrace{Cut} \\ -: \Delta_{13} \vdash \Delta_{12}, F_7 \to F_8} \\ \xrightarrow{h_9: \Delta_{13}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8} \underbrace{cut} \\ \xrightarrow{h_9: \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_8} \underbrace{cut} \\ \xrightarrow{h_9: \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_8} \underbrace{h_9: f_{11}, \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8} \\ \xrightarrow{h_9: \Delta_{13}, F_{10}, F_7 \vdash \Delta_{12}, F_8} \to_R \underbrace{h_0: \Delta_{13}, F_{11}, F_7 \vdash \Delta_{12}, F_8} \\ \xrightarrow{h_0: \Delta_{13}, F_7 \vdash \Delta_{12}, F_8} \to_R \underbrace{h_0: F_{11} \vdash \Delta_{12}, F_8} \underbrace{h_0: F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \to F_8} \\ \xrightarrow{h_1: F_7, \Delta_{14}, F_{10} \lor F_{11} \vdash F_8, F_{13}, \Delta_{12}} \to_R \underbrace{\frac{h_9: F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \to F_8}{\bullet h_9: (\Delta_{14}, F_{10} \lor F_{11}), F_{13} \vdash \Delta_{12}, F_7 \to F_8} \underbrace{cut} \\ \xrightarrow{h_1: \Delta_{14}, F_{10} \lor F_{11} \vdash (\Delta_{12}, F_7 \to F_8), F_{13}} \underbrace{\frac{h_9: F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \to F_8}{\bullet h_9: (\Delta_{14}, F_{10} \lor F_{11}), F_{13} \vdash \Delta_{12}, F_7 \to F_8}} \underbrace{cut} \\ \xrightarrow{h_1: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \to F_8}} \underbrace{\frac{h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}}} \underbrace{\frac{h_0: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}} \underbrace{\frac{h_0: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}}} \underbrace{\frac{h_0: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}} \underbrace{\frac{h_0: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}}} \underbrace{\frac{h_0: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}}} \underbrace{\frac{h_0: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}}} \underbrace{\frac{h_0: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}}} \underbrace{\frac{h_0: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}}} \underbrace{\frac{h_0: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}{\bullet h_9: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_8}}}$$

\bullet Case rule AT

$$\frac{ \frac{h_1 : F_6, \Delta_{11}, []F_9 \vdash F_7, \Delta_{10}}{\bullet h_1 : \Delta_{11}, []F_9 \vdash \Delta_{10}, F_6 \to F_7} \to_R \frac{h_8 : F_9, \Delta_{11}, []F_9, F_6 \to F_7 \vdash \Delta_{10}}{\bullet h_8 : (\Delta_{11}, []F_9), F_6 \to F_7 \vdash \Delta_{10}} Cut \\ - : \Delta_{11}, []F_9 \vdash \Delta_{10} \\ \hline \bullet h_1 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}, F_6 \to F_7 \\ \hline \bullet h_1 : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10}, F_6 \to F_7 \\ \hline - : \Delta_{11}, F_9, []F_9 \vdash \Delta_{10} \\ \hline - : \Delta_{11}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{12} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{12}, F_7 \vdash \Delta_{11}, F_8 \\ \hline - : \Delta_{12}, F_7 \vdash \Delta_{11}, F_8 \\ \hline - : \Delta_{12}, F_7 \vdash \Delta_{11}, F_8 \\ \hline - : \Delta_{12}, F_7 \vdash \Delta_{11}, F_8 \\ \hline - : \Delta_{12}, F_7 \vdash \Delta_{11}, F_8 \\ \hline - : \Delta_{12}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, []F_{10} \vdash \Delta_{11}, F_7 \to F_8 \\ \hline - : \Delta_{13}, [$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathsf{F}_6, \bot, \Delta_{10} \vdash \mathsf{F}_7, \Delta_9}{\bullet \mathbf{h}_1: \bot, \Delta_{10} \vdash \Delta_9, \mathsf{F}_6 \to \mathsf{F}_7} \xrightarrow{} \mathcal{F}_R \xrightarrow{\bullet \mathbf{h}_8: (\bot, \Delta_{10}), \mathsf{F}_6 \to \mathsf{F}_7 \vdash \Delta_9} \\ & \xrightarrow{-: \bot, \Delta_{10} \vdash \Delta_9} \xrightarrow{} \bot_L \\ \\ \frac{\mathbf{h}_1: \mathsf{F}_7, \Delta_{11} \vdash \mathsf{F}_8, \bot, \Delta_{10}}{-: \bot, \Delta_{10} \vdash \Delta_9} \xrightarrow{} \bot_L \\ \\ \frac{\bullet \mathbf{h}_1: \Delta_{11} \vdash (\Delta_{10}, \mathsf{F}_7 \to \mathsf{F}_8), \bot}{\bullet \mathbf{h}_9: \Delta_{11}, \bot \vdash \Delta_{10}, \mathsf{F}_7 \to \mathsf{F}_8} \xrightarrow{} \bot_L \\ & \xrightarrow{-: \Delta_{11} \vdash \Delta_{10}, \mathsf{F}_7 \to \mathsf{F}_8} \xrightarrow{\bullet \mathbf{h}_9: \bot, \Delta_{11}, \mathsf{F}_7 \vdash \Delta_{10}, \mathsf{F}_8} \xrightarrow{\bot_L} \\ \\ \frac{-: \Delta_{11}, \mathsf{F}_7 \vdash \bot, \Delta_{10}, \mathsf{F}_8}{\bullet \mathbf{h}_9: \bot, \Delta_{11}, \mathsf{F}_7 \vdash \Delta_{10}, \mathsf{F}_8} \xrightarrow{\bot_L} \xrightarrow{\mathsf{hCut}} \\ & \xrightarrow{-: \Delta_{11} \vdash \Delta_{10}, \mathsf{F}_7 \to \mathsf{F}_8} \xrightarrow{\to_R} \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \mathbf{F}_7, \bot, \Delta_{12} \vdash \mathbf{F}_8, \mathbf{F}_{11}, \Delta_{10} \\ \bullet \mathbf{h}_1: \bot, \Delta_{12} \vdash (\Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8), \mathbf{F}_{11} \end{array}}{ \begin{array}{c} \bullet \mathbf{h}_9: (\bot, \Delta_{12}), \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \\ \hline \\ -: \bot, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \\ \hline \\ -: \bot, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_7 \to \mathbf{F}_8 \end{array}} \begin{array}{c} \bot_L \\ \text{Cut} \end{array}$$

• Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_{1}: \mathsf{F}_{6}, \Delta_{11}, \mathsf{p}_{10} \vdash \mathsf{F}_{7}, \Delta_{9}, \mathsf{p}_{10}}{\bullet \mathbf{h}_{1}: \Delta_{11}, \mathsf{p}_{10} \vdash (\Delta_{9}, \mathsf{p}_{10}), \mathsf{F}_{6} \to \mathsf{F}_{7}} \to_{R} & \frac{\bullet \mathbf{h}_{8}: (\Delta_{11}, \mathsf{p}_{10}), \mathsf{F}_{6} \to \mathsf{F}_{7} \vdash \Delta_{9}, \mathsf{p}_{10}}{\bullet \mathbf{h}_{8}: (\Delta_{11}, \mathsf{p}_{10}), \mathsf{F}_{6} \to \mathsf{F}_{7} \vdash \Delta_{9}, \mathsf{p}_{10}} & I \\ & & -: \Delta_{11}, \mathsf{p}_{10} \vdash \Delta_{9}, \mathsf{p}_{10} \\ & \frac{-: \Delta_{11}, \mathsf{p}_{10} \vdash \Delta_{9}, \mathsf{p}_{10}}{\bullet \mathbf{h}_{1}: \mathsf{F}_{7}, \Delta_{12} \vdash \mathsf{F}_{8}, \mathsf{p}_{11}, \Delta_{10}, \mathsf{p}_{11}} \to_{R} & \frac{\bullet \mathbf{h}_{9}: \Delta_{12}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8}}{\bullet \mathbf{h}_{1}: \Delta_{12} \vdash ((\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8}), \mathsf{p}_{11}} & \mathcal{I} \\ & -: \Delta_{12} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{12}, \mathsf{F}_{7} \vdash \Delta_{10}, \mathsf{F}_{8}, \mathsf{p}_{11}, \mathsf{p}_{11} & \mathcal{A}_{10}, \mathsf{F}_{8}, \mathsf{p}_{11} \\ & -: \Delta_{12}, \mathsf{F}_{7} \vdash \Delta_{10}, \mathsf{F}_{8}, \mathsf{p}_{11} \\ \hline -: \Delta_{12} \vdash \Delta_{10}, \mathsf{p}_{11}, \mathsf{F}_{7} \to \mathsf{F}_{8} & \mathcal{A}_{11} \\ \hline \bullet \mathbf{h}_{1}: \mathsf{F}_{7}, \Delta_{13}, \mathsf{p}_{11} \vdash \mathsf{F}_{8}, \mathsf{F}_{12}, \Delta_{10}, \mathsf{p}_{11} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{13}, \mathsf{p}_{11} \vdash ((\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8}), \mathsf{F}_{12} & \mathcal{A}_{10} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{13}, \mathsf{p}_{11} \vdash ((\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8}), \mathsf{F}_{12} & \mathcal{A}_{10}, \mathsf{p}_{11} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} & \mathcal{A}_{11} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} & \mathcal{A}_{11} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} & \mathcal{A}_{11} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} & \mathcal{A}_{11} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} & \mathcal{A}_{11} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} & \mathcal{A}_{11} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{F}_{7} \to \mathsf{F}_{8} \\ \hline -: \Delta_{13}, \mathsf{p}_{11} \vdash \Delta_{10}, \mathsf{P}_{11}, \mathsf{F}_{7} \to \mathsf{F}_{8} \\ \hline -: \Delta_{13}, \mathsf{P}_{11} \vdash \Delta_{10}, \mathsf{P}_{11}, \mathsf{F}_{7} \to \mathsf{F}_{8} \\ \hline$$

• Case rule \top_L

$$\begin{array}{c} \frac{h_1:F_6,\top,\Delta_{10}\vdash F_7,\Delta_9}{\bullet h_1:\top,\Delta_{10}\vdash \Delta_9,F_6\to F_7}\to_R & \frac{h_8:\Delta_{10},F_6\to F_7\vdash \Delta_9}{\bullet h_8:(\top,\Delta_{10}),F_6\to F_7\vdash \Delta_9} & \top_L \\ \hline -:\top,\Delta_{10}\vdash \Delta_9 \\ \hline \bullet h_1:\top,\Delta_{10}\vdash \Delta_9,F_6\to F_7 & \text{ax/W} & h_8:\top,\Delta_{10},F_6\to F_7\vdash \Delta_9 \\ \hline -:\top,\Delta_{10}\vdash \Delta_9 & \text{hx}:\top,\Delta_{10}\vdash \Delta_9,F_6\to F_7 & \text{ax/W} \\ \hline -:\top,\Delta_{10}\vdash \Delta_9 & \text{hx}:\top,\Delta_{10},F_6\to F_7\vdash \Delta_9 & \text{ax/W} \\ \hline -:\top,\Delta_{10}\vdash \Delta_9 & \text{hx}:\top,\Delta_{10}\vdash \Delta_9 & \text{hx}:\top,\Delta_{10},F_7\to F_8 \\ \hline \bullet h_1:F_7,\Delta_{11}\vdash F_8,\top,\Delta_{10} & \rightarrow_R & \frac{h_9:\Delta_{11}\vdash \Delta_{10},F_7\to F_8}{\bullet h_9:\Delta_{11},\top\vdash \Delta_{10},F_7\to F_8} & \top_L \\ \hline -:\Delta_{11}\vdash \Delta_{10},F_7\to F_8 & \text{ax/W} \\ \hline -:\Delta_{11}\vdash \Delta_{10},F_7\to F_8 & \text{ax/W} \\ \hline \bullet h_1:F_7,\top,\Delta_{12}\vdash F_8,F_{11},\Delta_{10} & \rightarrow_R & \frac{h_9:F_{11},\Delta_{12}\vdash \Delta_{10},F_7\to F_8}{\bullet h_9:(\top,\Delta_{12}),F_{11}\vdash \Delta_{10},F_7\to F_8} & \top_L \\ \hline \bullet h_1:T,\Delta_{12}\vdash \Delta_{10},F_7\to F_8 & \text{ax/W} \\ \hline \bullet h_1:T,\Delta_{12}\vdash \Delta_{10},F_{11},F_7\to F_8 & \text{ax/W} \\ \hline \bullet h_1:T,\Delta_{12}\vdash \Delta_{10},F_{11},F_7\to F_8 & \text{ax/W} \\ \hline -:T,\Delta_{12}\vdash \Delta_{10},F_7\to F_8 & \text{ax/W} \\ \hline \bullet h_1:T,\Delta_{12}\vdash \Delta_{10},F_{11},F_7\to F_8 & \text{ax/W} \\ \hline \bullet h_1:T,\Delta_{12}\vdash \Delta_{10},F_{11},F_7\to F_8 & \text{ax/W} \\ \hline -:T,\Delta_{12}\vdash \Delta_{10},F_7\to F_8 & \text{ax/W} \\ \hline \bullet h_1:T,\Delta_{12}\vdash \Delta_{10},F_{11},F_7\to F_8 & \text{ax/W} \\ \hline \bullet h_1:T,\Delta_{12}\vdash \Delta_{10},F_7\to F_8 & \text{ax/W} \\ \hline \bullet h_1:T,\Delta_{12}\vdash \Delta_{10},F_$$

6.2 Status of \wedge_R : OK

• Case rule \rightarrow_R

$$\frac{\frac{h_1: \Delta_{12} \vdash F_6, \Delta_{11}, F_9 \to F_{10} \quad h_1: \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \to F_{10}}{\bullet h_1: \Delta_{12} \vdash (\Delta_{11}, F_9 \to F_{10}), F_6 \land F_7} \land_R \frac{h_8: F_9, \Delta_{12}, F_6 \land F_7 \vdash F_{10}, \Delta_{11}}{\bullet h_8: \Delta_{12}, F_6 \land F_7 \vdash \Delta_{11}, F_9 \to F_{10}} \xrightarrow{-: \Delta_{12} \vdash \Delta_{11}, F_9 \to F_{10}} \text{Cut}} \xrightarrow{-: \Delta_{12} \vdash \Delta_{11}, F_9 \to F_{10}} \frac{-: \Delta_{12} \vdash \Delta_{11}, F_{10}, F_7}{\land_R} \xrightarrow{h_8: \Delta_{12}, F_9 \vdash \Delta_{11}, F_{10}, F_1}} \xrightarrow{h_8: \Delta_{12}, F_9, F_6 \land F_7 \vdash \Delta_{11}, F_{10}} \xrightarrow{\text{ax/W}} \xrightarrow{\text{hCut}} \xrightarrow{-: \Delta_{12} \vdash \Delta_{11}, F_{10}} \to_R} \xrightarrow{-: \Delta_{12} \vdash \Delta_{11}, F_{10}} \to_R}$$

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{14}\vdash \mathbf{F}_{7},\mathbf{F}_{13},\Delta_{12},\mathbf{F}_{10}\to \mathbf{F}_{11}\quad \mathbf{h}_{1}:\Delta_{14}\vdash \mathbf{F}_{8},\mathbf{F}_{13},\Delta_{12},\mathbf{F}_{10}\to \mathbf{F}_{11}}{\bullet \mathbf{h}_{1}:\Delta_{14}\vdash ((\Delta_{12},\mathbf{F}_{10}\to \mathbf{F}_{11}),\mathbf{F}_{7}\wedge \mathbf{F}_{8}),\mathbf{F}_{13}} \quad \wedge_{R} \quad \frac{\mathbf{h}_{9}:\mathbf{F}_{10},\mathbf{F}_{13},\Delta_{14}\vdash \mathbf{F}_{11},\Delta_{12},\mathbf{F}_{7}\wedge \mathbf{F}_{8}}{\bullet \mathbf{h}_{9}:\Delta_{14},\mathbf{F}_{13}\vdash (\Delta_{12},\mathbf{F}_{10}\to \mathbf{F}_{11}),\mathbf{F}_{7}\wedge \mathbf{F}_{8}} \quad \xrightarrow{\bullet}_{R} \quad \text{Cut}} \\ \frac{-:\Delta_{14}\vdash (\Delta_{12},\mathbf{F}_{10}\to \mathbf{F}_{11}),\mathbf{F}_{7}\wedge \mathbf{F}_{8}}{\bullet \mathbf{h}_{1}:\Delta_{14},\mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{11},\mathbf{F}_{13},\mathbf{F}_{8}} \quad \text{inv-th/ax}}{\bullet \mathbf{h}_{1}:\Delta_{14},\mathbf{F}_{10}\vdash \Delta_{12},\mathbf{F}_{11},\mathbf{F}_{13},\mathbf{F}_{8}} \quad \xrightarrow{\bullet}_{R} \quad$$

• Case rule \wedge_R

$$\frac{\mathbf{h}_{1}: \Delta_{12} \vdash \mathbf{F}_{6}, \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10} \quad \mathbf{h}_{1}: \Delta_{12} \vdash \mathbf{F}_{7}, \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}}{\bullet \mathbf{h}_{1}: \Delta_{12} \vdash (\Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}), \mathbf{F}_{6} \land \mathbf{F}_{7}} \wedge \mathbf{h}_{8} : \Delta_{12}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \mathbf{F}_{10}, \Delta_{11}} \wedge \mathbf{h}_{8} : \Delta_{12}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \mathbf{F}_{10}, \Delta_{11}} \wedge \mathbf{h}_{8} : \Delta_{12}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \mathbf{F}_{10}, \Delta_{11}} \wedge \mathbf{h}_{8} : \Delta_{12}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \mathbf{F}_{10}, \Delta_{11}} \wedge \mathbf{h}_{8} : \Delta_{12}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \mathbf{F}_{10}, \Delta_{11}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{12}, \mathbf{F}_{10} \land \mathbf{F}_{11}, \mathbf{h}_{1} : \Delta_{12} \vdash \mathbf{F}_{9}, \mathbf{F}_{11}, \Delta_{11}, \mathbf{F}_{9} \land \mathbf{F}_{10}} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{12}, \mathbf{F}_{10} \land \mathbf{F}_{11}, \mathbf{h}_{1} : \Delta_{12} \vdash \mathbf{F}_{9}, \mathbf{F}_{10}, \Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \mathbf{F}_{9}, \mathbf{F}_{10}, \Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11} \wedge \mathbf{h}_{1} : \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{13} \vdash \mathbf{F}_{11}, \Delta_{14} \vdash \mathbf{F}_{11}, \Delta_{12}, \mathbf{F}_{10} \wedge \mathbf{F}_{11}, \mathbf{F}_{11}, \mathbf{F}_{12}, \mathbf{F}_{11}, \mathbf{F}_{13}, \mathbf{F}_{12} \wedge \mathbf{F}_{11}, \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{13} \vdash \mathbf{F}_{11}, \Delta_{14} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{13}, \mathbf{F}_{13} \wedge \mathbf{F}_{11} \wedge \mathbf{F}_{11}, \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{11}, \mathbf{F}_{11}, \mathbf{F}_{12}, \mathbf{F}_{11}, \mathbf{F}_{13}, \mathbf{F}_{12} \wedge \mathbf{F}_{11}, \Delta_{11} \wedge \mathbf{F}_{11}, \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{11} \wedge \mathbf{F}_{11}, \mathbf{F}_{11}, \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{11} \wedge \mathbf{F}_{11}, \Delta_{11} \wedge \mathbf{F}_{11}, \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{11}, \mathbf{F}_{11}, \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{11} \wedge \mathbf{F}_{11}, \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{11} \wedge \mathbf{F}_{11}, \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{11}, \mathbf{F}_{11} \wedge \mathbf{F}_{$$

• Case rule \vee_R

$$\frac{ \begin{array}{c} \frac{h_1: \Delta_{12} \vdash F_6, \Delta_{11}, F_9 \lor F_{10} \quad h_1: \Delta_{12} \vdash F_7, \Delta_{11}, F_9 \lor F_{10}}{\bullet h_1: \Delta_{12} \vdash (\Delta_{11}, F_9 \lor F_{10}), F_6 \land F_7} \\ \\ \hline & \begin{array}{c} \bullet h_1: \Delta_{12} \vdash (\Delta_{11}, F_9 \lor F_{10}), F_6 \land F_7 \\ \hline \\ \hline & -: \Delta_{12} \vdash \Delta_{11}, F_9 \lor F_{10} \\ \hline \\ \hline & & \\ \hline & & \\ \hline & \begin{array}{c} -: \Delta_{12} \vdash \Delta_{11}, F_9 \lor F_{10} \\ \hline \\ \hline & & \\ \hline & \\ \hline & \\ \hline & & \\$$

 $-:\Delta_{12}\vdash\Delta_{10},\mathtt{F}_8\land\mathtt{F}_9$

• Case rule \perp_R

$$\frac{\mathbf{h}_1:\Delta_{10} \vdash F_6, \bot, \Delta_9 \quad \mathbf{h}_1:\Delta_{10} \vdash F_7, \bot, \Delta_9}{\bullet \mathbf{h}_1:\Delta_{10} \vdash (\bot, \Delta_9), F_6 \land F_7} \quad \wedge_R \quad \frac{\mathbf{h}_8:\Delta_{10}, F_6 \land F_7 \vdash \Delta_9}{\bullet \mathbf{h}_8:\Delta_{10}, F_6 \land F_7 \vdash \bot, \Delta_9} \quad \bot_R \\ \hline -:\Delta_{10} \vdash \bot, \Delta_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10} \vdash \bot, \Delta_9, F_6 \land F_7} \quad \mathbf{ax/W} \quad \frac{\mathbf{h}_8:\Delta_{10}, F_6 \land F_7 \vdash \bot, \Delta_9}{\mathbf{h}_8:\Delta_{10}, F_6 \land F_7 \vdash \bot, \Delta_9} \quad \mathbf{ax/W} \\ \hline -:\Delta_{10} \vdash \bot, \Delta_9 \quad \mathbf{hCut} \\ \hline \bullet \mathbf{h}_1:\Delta_{12} \vdash F_7, F_{11}, \bot, \Delta_{10} \quad \mathbf{h}_1:\Delta_{12} \vdash F_8, F_{11}, \bot, \Delta_{10} \quad \wedge_R \quad \frac{\mathbf{h}_9:F_{11},\Delta_{12} \vdash \Delta_{10}, F_7 \land F_8}{\bullet \mathbf{h}_9:\Delta_{12}, F_{11} \vdash (\bot,\Delta_{10}), F_7 \land F_8} \quad \Delta_R \\ \hline -:\Delta_{12} \vdash (\bot,\Delta_{10}), F_7 \land F_8 \quad \mathbf{ax/W} \quad \mathbf{h}_9:\Delta_{12}, F_{11} \vdash \bot, \Delta_{10}, F_7 \land F_8} \quad \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_1:\Delta_{12} \vdash \bot, \Delta_{10}, F_{11}, F_7 \land F_8 \quad \mathbf{ax/W} \quad \mathbf{h}_9:\Delta_{12}, F_{11} \vdash \bot, \Delta_{10}, F_7 \land F_8} \quad \mathbf{ax/W} \\ \hline -:\Delta_{12} \vdash \bot, \Delta_{10}, F_7 \land F_8 \quad \mathbf{ax/W} \quad \mathbf{h}_{Cut} \\ \hline \end{array}$$

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_6, \top, \Delta_9 \quad \mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_7, \top, \Delta_9}{\bullet \mathbf{h}_1:\Delta_{10} \vdash (\top, \Delta_9), \mathbf{F}_6 \land \mathbf{F}_7} \quad \wedge_R \quad \frac{\bullet \mathbf{h}_8:\Delta_{10}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \top, \Delta_9}{\bullet \mathbf{h}_8:\Delta_{10}, \mathbf{F}_6 \land \mathbf{F}_7 \vdash \top, \Delta_9} \quad \mathsf{Cut} \\ & \xrightarrow{-:\Delta_{10} \vdash \top, \Delta_9} \quad \top_R \\ \\ \frac{\mathbf{h}_1:\Delta_{12} \vdash \mathbf{F}_7, \mathbf{F}_{11}, \top, \Delta_{10} \quad \mathbf{h}_1:\Delta_1 \vdash \mathbf{F}_8, \mathbf{F}_{11}, \top, \Delta_{10}}{-:\Delta_{12} \vdash ((\top, \Delta_{10}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{11}} \quad \wedge_R \quad \frac{\bullet \mathbf{h}_9:\Delta_{12}, \mathbf{F}_{11} \vdash (\top, \Delta_{10}), \mathbf{F}_7 \land \mathbf{F}_8}{\bullet \mathbf{h}_9:\Delta_{12}, \mathbf{F}_{11} \vdash (\top, \Delta_{10}), \mathbf{F}_7 \land \mathbf{F}_8} \quad \top_R \\ & \xrightarrow{-:\Delta_{12} \vdash (\top, \Delta_{10}), \mathbf{F}_7 \land \mathbf{F}_8} \quad \top_R \end{array}$$

\bullet Case rule K

$$\begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_1: \Box \Gamma_{11}, \Delta_{12} \vdash F_6, \Delta_{10}, []F_9 \quad h_1: \Box \Gamma_{11}, \Delta_{12} \vdash F_7, \Delta_{10}, []F_9}_{\bullet h_1: \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []F_9), F_6 \land F_7} \\ \bullet h_1: \Box \Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, []F_9), F_6 \land F_7 \\ \hline \\ \bullet h_2: (\Box \Gamma_{11}, \Delta_{12}), F_6 \land F_7 \vdash \Delta_{10}, []F_9 \\ \hline \\ -: unbox(\Box \Gamma_{11}) \vdash F_9 \\ \hline \\ -: unbox(\Box \Gamma_{11}) \vdash F_9 \\ \hline \\ -: unbox(\Box \Gamma_{11}) \vdash F_9 \\ \hline \\ \bullet h_1: \Box \Gamma_{13}, \Delta_{14} \vdash F_7, \Box F_{12}, \Delta_{11}, []F_{10} \quad h_1: \Box \Gamma_{13}, \Delta_{14} \vdash F_8, \Box F_{12}, \Delta_{11}, []F_{10} \\ \hline \\ \bullet h_1: \Box \Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []F_{10}), F_7 \land F_8), \Box F_{12} \\ \hline \\ \bullet h_1: \Box \Gamma_{13}, \Delta_{14} \vdash ((\Delta_{11}, []F_{10}), F_7 \land F_8), \Box F_{12} \\ \hline \\ \bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{13}), unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{13}), unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_2: unbox(\Box F_{13}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_1: \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []F_{10}, F_7 \land F_8), F_{13} \\ \hline \\ \bullet h_1: \Box \Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, []F_{10}, F_7 \land F_8), F_{13} \\ \hline \\ \bullet h_2: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_3: unbox(\Box F_{12}) \vdash F_{10} \\ \hline \\ \bullet h_1: \Box F_{12}, \Delta_{14} \vdash ((\Delta_{11}, []F_{10}, F_7 \land F_8), F_{13} \\ \hline \\ \bullet h_1: \Box F_{12}, \Delta_{14} \vdash ((\Delta_{11}, []F_{10}, F_7 \land F_8), F_{13} \\ \hline \\ \bullet h_1: \Box F_{12}, \Delta_{14} \vdash ((\Delta_{11}, []F_{10}, F_7 \land F_8), F_{13} \\ \hline \\ \bullet h_1: \Box F_{12}, \Delta_{14} \vdash ((\Delta_{11}, []F_{10}, F_7 \land F_8), F_{13} \\ \hline \\ \bullet h_$$

• Case rule A45

$$\frac{\mathbf{h}_{1}: \Box\Gamma_{12}, \Delta_{13} \vdash \mathbf{F}_{6}, \Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9} \quad \mathbf{h}_{1}: \Box\Gamma_{12}, \Delta_{13} \vdash \mathbf{F}_{7}, \Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}}{\bullet \mathbf{h}_{1}: \Box\Gamma_{12}, \Delta_{13} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), \mathbf{F}_{6} \land \mathbf{F}_{7}} \wedge_{R} \quad \frac{\mathbf{h}_{8}: \Box\Gamma_{12} \vdash \Box\Gamma_{10}, \mathbf{F}_{9}}{\bullet \mathbf{h}_{8}: (\Box\Gamma_{12}, \Delta_{13}), \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}} \\ -: \Box\Gamma_{12}, \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9} \\ -: \Box\Gamma_{12} \vdash \mathbf{F}_{9}, \Box\Gamma_{10} \quad \mathbf{ax/W} \\ -: \Box\Gamma_{12} \vdash \mathbf{F}_{9}, \Box\Gamma_{10}, []\mathbf{F}_{9} \quad A45 \\ \end{pmatrix}} A45$$

 $\frac{127 - 10}{-: \Delta_{14}, \Box \Gamma_{12} \vdash \Delta_{11}, []\mathsf{F}_{10}, \mathsf{F}_7 \wedge \mathsf{F}_8} \ K$

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\frac{h_1: \Box\Gamma_{14}, \Delta_{15} \vdash F_7, \Box F_{13}, \Box\Gamma_{11}, \Delta_{12}, [F_{10} \quad h_1: \Box\Gamma_{14}, \Delta_{15} \vdash F_8, \Box F_{13}, \Box\Gamma_{11}, \Delta_{12}, [F_{10} \quad h_9: \Box\Gamma_{14}, \Box F_{13}, \Box\Gamma_{14}, \Box F_{13}, \Box\Gamma_{14}, \Delta_{15}, \Box F_{13}, \Box\Gamma_{14}, \Delta_{15}, \Box F_{13}, \Box\Gamma_{14}, \Delta_{15}, \Box F_{13}, \Box\Gamma_{14}, \Delta_{15}, \Box F_{13}, \Delta_{15}, \Box F_{13}, \Box\Gamma_{14}, \Delta_{15}, \Box F_{13}, \Delta_{15}, \Box F_{14}, \Delta_{15}
```

• Case rule \rightarrow_L

$$\frac{ \underbrace{ \begin{array}{c} \underbrace{ h_1 : \Delta_{12}, F_9 \to F_{10} \vdash F_6, \Delta_{11} \quad h_1 : \Delta_{12}, F_9 \to F_{10} \vdash F_7, \Delta_{11}}_{\bullet h_1 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6 \land F_7} \\ \bullet h_1 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6 \land F_7} \\ \bullet h_2 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6 \land F_7} \\ \bullet h_3 : (\Delta_{12}, F_9 \to F_{10}), F_6 \land F_7 \vdash \Delta_{11}}_{\bullet h_3} \\ \bullet h_2 : \Delta_{12}, F_6, F_7 \to F_{10}), F_6 \land F_7 \vdash \Delta_{11}}_{\bullet h_3} \\ \bullet h_2 : \Delta_{12}, F_6, F_7 \to F_{10}), F_6 \land F_7 \vdash \Delta_{11}}_{\bullet h_3} \\ \bullet h_2 : \Delta_{12}, F_6, F_9 \to F_{10} \vdash \Delta_{11}, F_7} \\ \bullet h_2 : \Delta_{12}, F_6, F_9 \to F_{10} \vdash \Delta_{11}, F_7} \\ \bullet h_2 : \Delta_{12}, F_6, F_9 \to F_{10} \vdash \Delta_{11}}_{\bullet h_2} \\ \bullet h_1 : \Delta_{13} \vdash F_7, F_{10} \to F_{11}, \Delta_{12} \quad h_1 : \Delta_{13} \vdash F_8, F_{10} \to F_{11}, \Delta_{12} \\ \bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \land F_8), F_{10} \to F_{11} \\ \bullet h_2 : \Delta_{13} \vdash (\Delta_{12}, F_7 \land F_8), F_{10} \to F_{11}, \Delta_{12} \\ \bullet h_3 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{13} \vdash (\Delta_{12}, F_7 \land F_8), F_{10} \to F_{11} \\ \bullet h_2 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_3 : \Delta_{13}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_3 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_3 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_3 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash F_8, F_{13}, \Delta_{12} \\ \bullet h_2 : \Delta_{13}, F_{10} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash F_8, F_{13}, \Delta_{12} \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land F_8 \\ \bullet h_1 : \Delta_{14}, F_{10} \to F_{11} \vdash \Delta_{12}, F_7 \land$$

 $-:\Delta_{14},\mathtt{F}_{10}\to\mathtt{F}_{11}\vdash\Delta_{12},\mathtt{F}_{7}\land\mathtt{F}_{8}$

• Case rule \wedge_L

$$\frac{\begin{array}{c} \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \mathbf{F}_{6}, \Delta_{11} \quad \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \mathbf{F}_{7}, \Delta_{11} \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{6} \wedge \mathbf{F}_{7} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{9} \wedge \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ \hline \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11}, \mathbf{F}_{6} \wedge \mathbf{F}_{7} \\ \hline \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{9} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{10} \vdash \Delta_{11} \\ \hline \\ & -: \Delta_{12}, \mathbf{$$

$$\frac{ \frac{\mathbf{h}_{1} : \Delta_{10} \vdash \mathbf{F}_{7}, \Delta_{9} \quad \mathbf{h}_{1} : \Delta_{10} \vdash \mathbf{F}_{8}, \Delta_{9}}{\bullet \mathbf{h}_{1} : \Delta_{10} \vdash \Delta_{9}, \mathbf{F}_{7} \land \mathbf{F}_{8}} \quad \wedge_{R} \quad \frac{\mathbf{h}_{6} : \mathbf{F}_{7}, \mathbf{F}_{8}, \Delta_{10} \vdash \Delta_{9}}{\bullet \mathbf{h}_{6} : \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \vdash \Delta_{9}} \quad \wedge_{L} \\ - : \Delta_{10} \vdash \Delta_{9} \\ - : \Delta_{10} \vdash \mathbf{F}_{4}, \Delta_{9} \quad \mathbf{F}_{8} \quad \mathbf{F}_{8} \quad \mathbf{F}_{10} \land \mathbf{F}_{11}, \Delta_{12} \quad \mathbf{h}_{1} : \Delta_{13} \vdash \mathbf{F}_{8}, \mathbf{F}_{10} \land \mathbf{F}_{11}, \Delta_{12} \quad \mathbf{h}_{1} : \Delta_{13} \vdash \mathbf{F}_{10}, \mathbf{F}_{11} \vdash \Delta_{12}, \mathbf{F}_{7} \quad \mathbf{h}_{10} \quad \mathbf{h}_{11} : \Delta_{13} \vdash \mathbf{F}_{10}, \mathbf{F}_{11} \vdash \Delta_{12}, \mathbf{F}_{7} \quad \mathbf{h}_{11} \quad \mathbf{h}_{11} : \Delta_{13} \vdash \mathbf{h}_{12}, \mathbf{F}_{10}, \mathbf{F}_{11} \quad \mathbf{h}_{12}, \mathbf{F}_{10} \quad \mathbf{h}_{11} : \Delta_{13} \vdash \mathbf{h}_{12}, \mathbf{F}_{7} \land \mathbf{F}_{8} \quad \mathbf{h}_{11} : \Delta_{13} \vdash \mathbf{h}_{12}, \mathbf{F}_{10}, \mathbf{F}_{11} \vdash \Delta_{12}, \mathbf{F}_{7} \quad \mathbf{h}_{11} \quad \mathbf{h}_{11} : \Delta_{13} \vdash \mathbf{h}_{12}, \mathbf{F}_{7} \land \mathbf{F}_{8} \quad \mathbf{h}_{11} : \Delta_{13} \vdash \mathbf{h}_{12}, \mathbf{F}_{10}, \mathbf{F}_{11} \vdash \Delta_{12}, \mathbf{F}_{7} \land \mathbf{F}_{8} \quad \mathbf{h}_{11} : \Delta_{13} \vdash \mathbf{h}_{12}, \mathbf{F}_{7} \land \mathbf{F}_{8} \quad \mathbf{h}_{11} : \Delta_{14}, \mathbf{F}_{10} \land \mathbf{F}_{11} \vdash \Delta_{12}, \mathbf{F}_{7} \land \mathbf{F}_{8} \quad \mathbf{h}_{11} : \Delta_{14}, \mathbf{F}_{10} \land \mathbf{F}_{11} \vdash \Delta_{12}, \mathbf{F}_{7} \land \mathbf{F}_{8} \quad \mathbf{h}_{11} : \Delta_{14}, \mathbf{F}_{10} \land \mathbf{F}_{11} \vdash \Delta_{12}, \mathbf{F}_{13}, \mathbf{F}_{8} \quad \mathbf{h}_{11} : \Delta_{14}, \mathbf{F}_{10}$$

• Case rule \vee_L

$$\frac{ \underbrace{ \begin{array}{c} \frac{h_{1} : \Delta_{12}, F_{9} \vee F_{10} \vdash F_{6}, \Delta_{11} \quad h_{1} : \Delta_{12}, F_{9} \vee F_{10} \vdash F_{7}, \Delta_{11} \\ \bullet h_{1} : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11}, F_{6} \wedge F_{7} \end{array}}{\bullet h_{1} : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11}, F_{6} \wedge F_{7}} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{12}, F_{9} \vee F_{10} \vdash \Delta_{11} \\ - : \Delta_{13}, F_{10} \vee F_{11}, \Delta_{12} \quad h_{1} : \Delta_{13} \vdash F_{8}, F_{10} \vee F_{11}, \Delta_{12} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{13} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{14}, F_{10} \vee F_{11} \vdash F_{7}, F_{13}, \Delta_{12} \quad h_{1} : \Delta_{14}, F_{10} \vee F_{11} \vdash F_{8}, F_{13}, \Delta_{12} \\ - : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7} \wedge F_{8} \\ - : \Delta_{14}, F_{10} \vee$$

 $-: \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \wedge F_8$

\bullet Case rule AT

$$\frac{\mathbf{h}_{1} : \Delta_{11}, [] F_{9} \vdash F_{6}, \Delta_{10} \quad h_{1} : \Delta_{11}, [] F_{9} \vdash F_{7}, \Delta_{10}}{\bullet \mathbf{h}_{1} : \Delta_{11}, [] F_{9} \vdash F_{6}, \Delta_{10}} \quad \wedge_{R} \quad \frac{\mathbf{h}_{8} : F_{9}, \Delta_{11}, [] F_{9}, F_{6} \wedge F_{7} \vdash \Delta_{10}}{\bullet \mathbf{h}_{8} : (\Delta_{11}, [] F_{9}), F_{6} \wedge F_{7} \vdash \Delta_{10}} \quad AT \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline \bullet \mathbf{h}_{1} : \Delta_{11}, F_{9}, [] F_{9} \vdash \Delta_{10}, F_{6} \wedge F_{7} \quad ax/W \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash \Delta_{10} \\ \hline - : \Delta_{11}, [] F_{9} \vdash A_{10} \\ \hline - : \Delta_{11}, F_{7}, [] F_{10} \vdash \Delta_{11} \\ \hline - : \Delta_{11}, [] F_{9} \vdash A_{10} \\ \hline - : \Delta_{12} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{12} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{12} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land F_{8} \\ \hline - : \Delta_{13}, [] F_{10} \vdash \Delta_{11}, F_{7} \land$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\bot,\Delta_{10}\vdash\mathsf{F}_{6},\Delta_{9}\quad \mathbf{h}_{1}:\bot,\Delta_{10}\vdash\mathsf{F}_{7},\Delta_{9}}{\bullet \mathbf{h}_{1}:\bot,\Delta_{10}\vdash\Delta_{9},\mathsf{F}_{6}\land\mathsf{F}_{7}} & \wedge_{R} & \frac{\bullet_{\mathbf{h}_{8}}:(\bot,\Delta_{10}),\mathsf{F}_{6}\land\mathsf{F}_{7}\vdash\Delta_{9}}{\bullet \mathbf{h}_{8}:(\bot,\Delta_{10}),\mathsf{F}_{6}\land\mathsf{F}_{7}\vdash\Delta_{9}} & \bot_{L} \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & \\ & & \\ &$$

\bullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{p}_{10}\vdash \mathbf{F}_{6},\Delta_{9},\mathbf{p}_{10} \quad \mathbf{h}_{1}:\Delta_{11},\mathbf{p}_{10}\vdash \mathbf{F}_{7},\Delta_{9},\mathbf{p}_{10}}{\bullet \mathbf{h}_{1}:\Delta_{11},\mathbf{p}_{10}\vdash (\Delta_{9},\mathbf{p}_{10}),\mathbf{F}_{6}\wedge \mathbf{F}_{7}} & \wedge_{R} & \frac{\bullet \mathbf{h}_{8}:(\Delta_{11},\mathbf{p}_{10}),\mathbf{F}_{6}\wedge \mathbf{F}_{7}\vdash \Delta_{9},\mathbf{p}_{10}}{\bullet \mathbf{h}_{8}:(\Delta_{11},\mathbf{p}_{10}),\mathbf{F}_{6}\wedge \mathbf{F}_{7}\vdash \Delta_{9},\mathbf{p}_{10}} & \mathbf{Cut} \\ & -:\Delta_{11},\mathbf{p}_{10}\vdash \Delta_{9},\mathbf{p}_{10} \\ & -:\Delta_{11},\mathbf{p}_{10}\vdash \Delta_{9},\mathbf{p}_{10} & I \\ & \frac{\bullet \mathbf{h}_{1}:\Delta_{12}\vdash \mathbf{F}_{7},\mathbf{p}_{11},\Delta_{10},\mathbf{p}_{11} \quad \mathbf{h}_{1}:\Delta_{12}\vdash \mathbf{F}_{8},\mathbf{p}_{11},\Delta_{10},\mathbf{p}_{11}}{\bullet \mathbf{h}_{1}:\Delta_{12}\vdash ((\Delta_{10},\mathbf{p}_{11}),\mathbf{F}_{7}\wedge \mathbf{F}_{8}),\mathbf{p}_{11}} & \wedge_{R} & \frac{\bullet \mathbf{h}_{9}:\Delta_{12},\mathbf{p}_{11}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathbf{F}_{7}\wedge \mathbf{F}_{8}}{\bullet \mathbf{h}_{9}:\Delta_{12},\mathbf{p}_{11}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathbf{F}_{7}\wedge \mathbf{F}_{8}} & \mathbf{Cut} \\ & -:\Delta_{12}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathbf{F}_{7}\wedge \mathbf{F}_{8} & \rightarrow & \bullet \\ & \frac{\bullet \mathbf{h}_{1}:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11}}{\bullet \mathbf{h}_{21}} & \mathbf{ax/W} & \bullet \mathbf{h}_{9}:\Delta_{12},\mathbf{p}_{11}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11}} & I \\ & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11} & \wedge_{R} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11} & \wedge_{R} \\ & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11} & \wedge_{R} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11} & \wedge_{R} \\ & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11} & \wedge_{R} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11} & \wedge_{R} \\ & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11} & \wedge_{R} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11} & \wedge_{R} \\ & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11} & \wedge_{R} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11} & \wedge_{R} \\ & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11} & \wedge_{R} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11} & \wedge_{R} \\ & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11} & \wedge_{R} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11} & \wedge_{R} \\ & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11} & \wedge_{R} \\ & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{7},\mathbf{p}_{11} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_{11} & -:\Delta_{12}\vdash \Delta_{10},\mathbf{F}_{8},\mathbf{p}_$$

$$\frac{\mathbf{h}_{1}:\Delta_{13},\mathbf{p}_{11}\vdash \mathbf{F}_{7},\mathbf{F}_{12},\Delta_{10},\mathbf{p}_{11}\quad \mathbf{h}_{1}:\Delta_{13},\mathbf{p}_{11}\vdash \mathbf{F}_{8},\mathbf{F}_{12},\Delta_{10},\mathbf{p}_{11}}{\bullet \mathbf{h}_{1}:\Delta_{13},\mathbf{p}_{11}\vdash ((\Delta_{10},\mathbf{p}_{11}),\mathbf{F}_{7}\land \mathbf{F}_{8}),\mathbf{F}_{12}} \land_{R} \underbrace{\frac{\bullet \mathbf{h}_{9}:(\Delta_{13},\mathbf{p}_{11}),\mathbf{F}_{12}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathbf{F}_{7}\land \mathbf{F}_{8}}{-:\Delta_{13},\mathbf{p}_{11}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathbf{F}_{7}\land \mathbf{F}_{8}}} \underbrace{I}_{Cut}$$

• Case rule \top_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}: \top, \Delta_{10} \vdash \mathbf{F}_{6}, \Delta_{9} \quad \mathbf{h}_{1}: \top, \Delta_{10} \vdash \mathbf{F}_{7}, \Delta_{9}}{\bullet \mathbf{h}_{1}: \top, \Delta_{10} \vdash \Delta_{9}, \mathbf{F}_{6} \land \mathbf{F}_{7}} & \wedge_{R} \quad \frac{\mathbf{h}_{8}: \Delta_{10}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \Delta_{9}}{\bullet \mathbf{h}_{8}: (\top, \Delta_{10}), \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \Delta_{9}} \quad \top_{L} \\ \hline & -: \top, \Delta_{10} \vdash \Delta_{9} \\ \hline \bullet \mathbf{h}_{1}: \top, \Delta_{10} \vdash \Delta_{9}, \mathbf{F}_{6} \land \mathbf{F}_{7} \quad \mathbf{ax/W} \quad \frac{}{\mathbf{h}_{8}: \top, \Delta_{10}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \Delta_{9}} \quad \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{1}: \top, \Delta_{10} \vdash \Delta_{9}, \mathbf{F}_{6} \land \mathbf{F}_{7} \quad \mathbf{ax/W} \quad \frac{}{\mathbf{h}_{8}: \top, \Delta_{10}, \mathbf{F}_{6} \land \mathbf{F}_{7} \vdash \Delta_{9}} \quad \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{11} \vdash \mathbf{F}_{7}, \top, \Delta_{10} \quad \mathbf{h}_{1}: \Delta_{11} \vdash \mathbf{F}_{8}, \top, \Delta_{10} \quad \wedge_{R} \quad \frac{\mathbf{h}_{9}: \Delta_{11} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8}}{\bullet \mathbf{h}_{9}: \Delta_{11}, \top \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8}} \quad \top_{L} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{11} \vdash (\Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8}), \top \quad \wedge_{R} \quad \frac{\mathbf{h}_{9}: \Delta_{11} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8}}{\bullet \mathbf{h}_{9}: \Delta_{11}, \top \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8}} \quad \nabla_{L} \\ \hline \quad -: \Delta_{11} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \quad \mathbf{ax/W} \\ \hline \quad -: \Delta_{11} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \bullet \mathbf{h}_{1}: \top, \Delta_{12} \vdash \mathbf{F}_{7}, \mathbf{F}_{11}, \Delta_{10} \quad \mathbf{h}_{1}: \top, \Delta_{12} \vdash \mathbf{F}_{8}, \mathbf{F}_{11}, \Delta_{10} \quad \wedge_{R} \quad \frac{\mathbf{h}_{9}: \mathbf{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8}}{\bullet \mathbf{h}_{9}: (\top, \Delta_{12}), \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8}} \quad \top_{L} \\ \hline \quad -: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \bullet \mathbf{h}_{1}: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{11}, \mathbf{F}_{7} \land \mathbf{F}_{8} \quad \mathbf{ax/W} \\ \hline \quad -: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \mathbf{h}_{9}: \top, \Delta_{12}, \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \bullet \mathbf{h}_{1}: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{11}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad -: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \mathbf{h}_{9}: \top, \Delta_{12}, \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \mathbf{h}_{1}: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad -: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \mathbf{h}_{1}: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \mathbf{h}_{1}: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \mathbf{h}_{1}: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}_{8} \\ \hline \quad \mathbf{h}_{2}: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{7} \land \mathbf{F}$$

6.3 Status of \vee_R : OK

• Case rule \rightarrow_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_{12} \vdash \mathsf{F}_6, \mathsf{F}_7, \Delta_{11}, \mathsf{F}_9 \to \mathsf{F}_{10}}{\bullet \mathbf{h}_1: \Delta_{12} \vdash (\Delta_{11}, \mathsf{F}_9 \to \mathsf{F}_{10}), \mathsf{F}_6 \vee \mathsf{F}_7} \vee_R & \frac{\mathbf{h}_8: \mathsf{F}_9, \Delta_{12}, \mathsf{F}_6 \vee \mathsf{F}_7 \vdash \mathsf{F}_{10}, \Delta_{11}}{\bullet \mathbf{h}_8: \Delta_{12}, \mathsf{F}_6 \vee \mathsf{F}_7 \vdash \Delta_{11}, \mathsf{F}_9 \to \mathsf{F}_{10}} & \rightarrow_R \\ & -: \Delta_{12} \vdash \Delta_{11}, \mathsf{F}_9 \to \mathsf{F}_{10} & \text{Cut} \\ \hline \frac{\lambda_1: \Delta_{12}, \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_{10}, \mathsf{F}_6, \mathsf{F}_7}{\bullet \mathbf{h}_1: \Delta_{12}, \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_{10}, \mathsf{F}_6 \vee \mathsf{F}_7} \vee_R & \frac{\lambda_8: \Delta_{12}, \mathsf{F}_9, \mathsf{F}_6 \vee \mathsf{F}_7 \vdash \Delta_{11}, \mathsf{F}_{10}}{\bullet \mathbf{h}_8: \Delta_{12}, \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_{10}} & \text{ax/W} \\ \hline \frac{-: \Delta_{12}, \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_{10}}{-: \Delta_{12}, \mathsf{F}_9 \vdash \Delta_{11}, \mathsf{F}_{10}} \to_R \\ \hline \frac{\mathbf{h}_1: \Delta_{14} \vdash \mathsf{F}_7, \mathsf{F}_8, \mathsf{F}_{13}, \Delta_{12}, \mathsf{F}_{10} \to \mathsf{F}_{11}}{-: \Delta_{14} \vdash (\Delta_{12}, \mathsf{F}_{10} \to \mathsf{F}_{11}), \mathsf{F}_7 \vee \mathsf{F}_8} & \frac{\mathbf{h}_9: \mathsf{F}_{10}, \mathsf{F}_{13}, \Delta_{14} \vdash \mathsf{F}_{11}, \Delta_{12}, \mathsf{F}_7 \vee \mathsf{F}_8}{\bullet \mathbf{h}_9: \Delta_{14}, \mathsf{F}_{13} \vdash (\Delta_{12}, \mathsf{F}_{10} \to \mathsf{F}_{11}), \mathsf{F}_7 \vee \mathsf{F}_8} & \rightarrow_R \\ \hline \frac{\mathbf{h}_1: \Delta_{14} \vdash (\Delta_{12}, \mathsf{F}_{10} \to \mathsf{F}_{11}), \mathsf{F}_7 \vee \mathsf{F}_8}{\bullet \mathbf{h}_1: \Delta_{14}, \mathsf{F}_{10} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{13}, \mathsf{F}_7 \vee \mathsf{F}_8} & \mathsf{inv-th/ax} \\ \hline \bullet \mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{13}, \mathsf{F}_7 \vee \mathsf{F}_8} & \mathsf{h}_9: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{13} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_7 \vee \mathsf{F}_8} \\ \hline \bullet \mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{13}, \mathsf{F}_7 \vee \mathsf{F}_8} & \mathsf{h}_9: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{13} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_7 \vee \mathsf{F}_8} \\ \hline \bullet \mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{13}, \mathsf{F}_7 \vee \mathsf{F}_8} & \mathsf{h}_9: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{13} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_7 \vee \mathsf{F}_8} \\ \hline \bullet \mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{13}, \mathsf{F}_7 \vee \mathsf{F}_8} & \mathsf{h}_9: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{13} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_7 \vee \mathsf{F}_8} \\ \hline \bullet \mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{13}, \mathsf{F}_7 \vee \mathsf{F}_8} & \mathsf{h}_9: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{13} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_7 \vee \mathsf{F}_8} \\ \hline \bullet \mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10} \vdash \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_7 \vee \mathsf{F}_8} & \mathsf{h}_9: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{13}, \mathsf{F}_7 \vee \mathsf$$

• Case rule \wedge_R

$$\frac{\frac{h_{1}:\Delta_{12}\vdash F_{6},F_{7},\Delta_{11},F_{9}\wedge F_{10}}{\bullet h_{1}:\Delta_{12}\vdash (\Delta_{11},F_{9}\wedge F_{10}),F_{6}\vee F_{7}}}{\vee_{R}} \xrightarrow{h_{8}:\Delta_{12},F_{6}\vee F_{7}\vdash F_{9},\Delta_{11}} \xrightarrow{h_{8}:\Delta_{12},F_{6}\vee F_{7}\vdash F_{10},\Delta_{11}} \wedge_{R}} \xrightarrow{\bullet h_{8}:\Delta_{12},F_{6}\vee F_{7}\vdash \Delta_{11},F_{9}\wedge F_{10}} \text{Cut}} \wedge_{R} \xrightarrow{\bullet h_{1}:\Delta_{12}\vdash \Delta_{11},F_{9}\wedge F_{7}} \xrightarrow{\bullet h_{1}:\Delta_{12}\vdash \Delta_{11},F_{9},F_{7}} \vee_{R}} \xrightarrow{inv-th/ax} \xrightarrow{\bullet h_{1}:\Delta_{12}\vdash \Delta_{11},F_{9},F_{6}\vee F_{7}} \vee_{R} \xrightarrow{h_{1}:\Delta_{12}\vdash \Delta_{11},F_{10},F_{6}\vee F_{7}} \vee_{R}} \xrightarrow{h_{1}:\Delta_{12}\vdash \Delta_{11},F_{10},F_{6}\vee F_{7}} \vee_{R} \xrightarrow{h_{1}:\Delta_{12}\vdash \Delta_{11},F_{10}} \wedge_{R}} \xrightarrow{\bullet h_{1}:\Delta_{12}\vdash \Delta_{11},F_{9}\wedge F_{10}} \wedge_{R} \xrightarrow{\bullet h_{1}:\Delta_{12}\vdash \Delta_{11},F_{10}} \wedge_{R}$$

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: \Delta_{14} \vdash F_{7}, F_{8}, F_{13}, \Delta_{12}, F_{10} \land F_{11} \\ \bullet \mathbf{h}_{1}: \Delta_{14} \vdash ((\Delta_{12}, F_{10} \land F_{11}), F_{7} \lor F_{8}), F_{13} \end{array}}{\bullet \mathbf{h}_{9}: F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{12}, F_{7} \lor F_{8} \quad \mathbf{h}_{9}: F_{13}, \Delta_{14} \vdash F_{11}, \Delta_{12}, F_{7} \lor F_{8} \\ \bullet \mathbf{h}_{9}: \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \land F_{11}), F_{7} \lor F_{8} \\ & -: \Delta_{14} \vdash (\Delta_{12}, F_{10} \land F_{11}), F_{7} \lor F_{8} \\ & \rightarrow \\ \bullet \mathbf{h}_{9}: \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{10}, F_{7}, F_{8} \quad \mathbf{inv-th/ax} \quad \frac{\mathbf{h}_{9}: \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{11}, F_{7}, F_{8}}{\bullet \mathbf{h}_{9}: \Delta_{14}, F_{13} \vdash \Delta_{12}, F_{7}, F_{8}, F_{10} \land F_{11}} \quad \mathbf{hCut} \\ & -: \Delta_{14} \vdash \Delta_{12}, F_{7}, F_{8}, F_{10} \land F_{11} \\ & -: \Delta_{14} \vdash \Delta_{12}, F_{7}, F_{8}, F_{10} \land F_{11}, F_{7} \lor F_{8} \end{array}} \quad \forall_{R}$$

• Case rule \vee_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_{12} \vdash F_6, F_7, \Delta_{11}, F_9 \vee F_{10}}{\bullet \mathbf{h}_1: \Delta_{12} \vdash (\Delta_{11}, F_9 \vee F_{10}), F_6 \vee F_7} \quad \vee_R \quad \frac{\mathbf{h}_8: \Delta_{12}, F_6 \vee F_7 \vdash F_9, F_{10}, \Delta_{11}}{\bullet \mathbf{h}_8: \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \vee F_{10}} \\ \hline -: \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \\ \hline \bullet \mathbf{h}_1: \Delta_{12} \vdash \Delta_{11}, F_{10}, F_6, F_7, F_9 \\ \bullet \mathbf{h}_1: \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9, F_6 \vee F_7 \\ \hline -: \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{11}, F_{10}, F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \\ \hline \end{array} \quad \begin{array}{c} \mathbf{ax}/\mathbb{W} \\ \mathbf{h}_8: \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}, F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \\ \hline \end{array} \quad \begin{array}{c} \mathbf{ax}/\mathbb{W} \\ \mathbf{h}_8: \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}, F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \\ \hline \end{array} \quad \begin{array}{c} \mathbf{ax}/\mathbb{W} \\ \mathbf{h}_8: \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_{10}, F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{11}, F_9 \vee F_{10} \\ \hline \end{array} \quad \begin{array}{c} \mathbf{ax}/\mathbb{W} \\ \mathbf{h}_9: \Delta_{14}, F_{13} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash (\Delta_{12}, F_{10} \vee F_{11}), F_7 \vee F_8 \\ \hline \bullet \mathbf{h}_1: \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7, F_8 \\ \hline \bullet \mathbf{h}_1: \Delta_{14} \vdash \Delta_{12}, F_{10}, F_{11}, F_{13}, F_7, F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{12}, F_{10} \vee F_{11}, F_7 \vee F_8 \\ \hline -: \Delta_{14} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline -: \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9$$

• Case rule \perp_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_{10} \vdash \mathsf{F}_6, \mathsf{F}_7, \bot, \Delta_9}{\bullet \mathbf{h}_1:\Delta_{10} \vdash (\bot, \Delta_9), \mathsf{F}_6 \vee \mathsf{F}_7} \vee_R & \frac{\mathbf{h}_8:\Delta_{10}, \mathsf{F}_6 \vee \mathsf{F}_7 \vdash \Delta_9}{\bullet \mathbf{h}_8:\Delta_{10}, \mathsf{F}_6 \vee \mathsf{F}_7 \vdash \bot, \Delta_9} & \bot_R \\ \hline -:\Delta_{10} \vdash \bot, \Delta_9 & \\ \hline \bullet \mathbf{h}_1:\Delta_{10} \vdash \bot, \Delta_9, \mathsf{F}_6 \vee \mathsf{F}_7 & \mathsf{ax/W} & h_8:\Delta_{10}, \mathsf{F}_6 \vee \mathsf{F}_7 \vdash \bot, \Delta_9} \\ \hline \bullet \mathbf{h}_1:\Delta_{10} \vdash \bot, \Delta_9, \mathsf{F}_6 \vee \mathsf{F}_7 & \mathsf{ax/W} & h_8:\Delta_{10}, \mathsf{F}_6 \vee \mathsf{F}_7 \vdash \bot, \Delta_9 \\ \hline -:\Delta_{10} \vdash \bot, \Delta_9 & \mathsf{h}_8:\Delta_{10}, \mathsf{F}_6 \vee \mathsf{F}_7 \vdash \bot, \Delta_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{12} \vdash \mathsf{F}_7, \mathsf{F}_8, \mathsf{F}_{11}, \bot, \Delta_{10} & \mathsf{h}_9:\mathsf{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathsf{F}_7 \vee \mathsf{F}_8 \\ \hline \bullet \mathbf{h}_1:\Delta_{12} \vdash ((\bot,\Delta_{10}), \mathsf{F}_7 \vee \mathsf{F}_8), \mathsf{F}_{11} & \vee_R & \mathsf{h}_9:\Delta_{12}, \mathsf{F}_{11} \vdash (\bot,\Delta_{10}), \mathsf{F}_7 \vee \mathsf{F}_8 \\ \hline -:\Delta_{12} \vdash (\bot,\Delta_{10}), \mathsf{F}_7 \vee \mathsf{F}_8 & \mathsf{ax/W} & \mathsf{h}_9:\Delta_{12}, \mathsf{F}_{11} \vdash \bot, \Delta_{10}, \mathsf{F}_7 \vee \mathsf{F}_8 \\ \hline \bullet \mathsf{h}_1:\Delta_{12} \vdash \bot, \Delta_{10}, \mathsf{F}_{11}, \mathsf{F}_7 \vee \mathsf{F}_8 & \mathsf{ax/W} \\ -:\Delta_{12} \vdash \bot, \Delta_{10}, \mathsf{F}_7 \vee \mathsf{F}_8 & \mathsf{h}_{Cut} \\ \hline \end{array}$$

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_{10} \vdash \mathbf{F}_6, \mathbf{F}_7, \top, \Delta_9}{\bullet \mathbf{h}_1:\Delta_{10} \vdash (\top, \Delta_9), \mathbf{F}_6 \vee \mathbf{F}_7} \ \vee_R & \frac{}{\bullet \mathbf{h}_8:\Delta_{10}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \top, \Delta_9} \\ & \xrightarrow{-:\Delta_{10} \vdash \top, \Delta_9} \\ & \xrightarrow{-:\Delta_{10} \vdash \top, \Delta_9} \ \top_R \\ \\ \frac{\mathbf{h}_1:\Delta_{12} \vdash \mathbf{F}_7, \mathbf{F}_8, \mathbf{F}_{11}, \top, \Delta_{10}}{\bullet \mathbf{h}_1:\Delta_{12} \vdash ((\top, \Delta_{10}), \mathbf{F}_7 \vee \mathbf{F}_8), \mathbf{F}_{11}} \ \vee_R & \frac{}{\bullet \mathbf{h}_9:\Delta_{12}, \mathbf{F}_{11} \vdash (\top, \Delta_{10}), \mathbf{F}_7 \vee \mathbf{F}_8} \\ & \xrightarrow{-:\Delta_{12} \vdash (\top, \Delta_{10}), \mathbf{F}_7 \vee \mathbf{F}_8} \ \top_R \\ \\ \frac{}{-:\Delta_{12} \vdash \top, \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8} \end{array} \begin{array}{c} \top_R \\ \text{Cut} \end{array}$$

\bullet Case rule K

\bullet Case rule A45

• Case rule \rightarrow_L

• Case rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_{12}, F_9 \wedge F_{10} \vdash F_6, F_7, \Delta_{11}}{\bullet \mathbf{h}_1:\Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R & \frac{\mathbf{h}_8: F_9, F_{10}, \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet \mathbf{h}_8: (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} & \wedge_L \\ \hline \\ -:\Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11} \\ \hline \rightarrow & \\ \hline \mathbf{h}_1:\Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6, F_7} & \text{inv-th/ax} \\ \hline \bullet \mathbf{h}_1:\Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} & \sqrt{R} & \mathbf{h}_8:\Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}} \\ \hline \bullet \mathbf{h}_1:\Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} & \mathbf{h}_8:\Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}} \\ \hline -:\Delta_{12}, F_{10}, F_9 \vdash \Delta_{11} \\ \hline -:\Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} & \wedge_L \\ \hline \hline \bullet \mathbf{h}_1:\Delta_{13} \vdash F_7, F_8, F_{10} \wedge F_{11}, \Delta_{12} \\ \hline \bullet \mathbf{h}_1:\Delta_{13} \vdash (\Delta_{12}, F_7 \vee F_8), F_{10} \wedge F_{11}} & \vee_R & \frac{\mathbf{h}_9:F_{10}, F_{11}, \Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8}{\bullet \mathbf{h}_9:\Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7 \vee F_8} & \wedge_L \\ \hline -:\Delta_{13} \vdash \Delta_{12}, F_7 \vee F_8 \\ \hline \bullet \mathbf{h}_9:\Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7, F_8} & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \wedge F_{11} & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_9:\Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7, F_8} & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{13} \vdash \Delta_{12}, F_7, F_8, F_{10} \wedge F_{11} & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_9:\Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash F_7, F_8, F_{13}, \Delta_{12} & \vee_R \\ \hline \bullet \mathbf{h}_9:\Delta_{13}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8} & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash (\Delta_{12}, F_7 \vee F_8), F_{13} & \vee_R \\ \hline \bullet \mathbf{h}_9:\Delta_{14}, F_{10} \wedge F_{11}, F_{13}, \Delta_{14} \vdash \Delta_{12}, F_7 \vee F_8 \\ \hline \bullet \mathbf{h}_9:\Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8 & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8 & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8 & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8 & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8 & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8 & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8 & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7, F_8 & \wedge_L \\ \hline \bullet \mathbf{h}_1:\Delta_{14}, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_7$$

• Case rule \vee_L

```
 \frac{\mathbf{h}_1 : \Delta_{12}, \mathbf{F}_9 \vee \mathbf{F}_{10} \vdash \mathbf{F}_6, \mathbf{F}_7, \Delta_{11}}{\bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{F}_9 \vee \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_6 \vee \mathbf{F}_7} \quad \vee_R \quad \frac{\mathbf{h}_8 : \mathbf{F}_9, \Delta_{12}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_{11} \quad \mathbf{h}_8 : \mathbf{F}_{10}, \Delta_{12}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_{11}}{\bullet \mathbf{h}_8 : (\Delta_{12}, \mathbf{F}_9 \vee \mathbf{F}_{10}), \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_{11}} \quad \vee_L 
                                                                                                                                                                                                                                                                                                                                 -: \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}
\frac{}{h_8:\Delta_{12},F_{10},F_6\vee F_7\vdash \Delta_{11}} \ \underset{h\text{Cut}}{\text{ax/w}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{-:\Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}{-:\Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}} \vee_{L}
                                                                                                                                                                                                                                                                                                                                                                        -:\Delta_{12},\mathtt{F}_{9}\vee\mathtt{F}_{10}\vdash\Delta_{11}
                              \begin{array}{c} \mathbf{h}_1:\Delta_{10}\vdash \mathbf{F}_7,\mathbf{F}_8,\Delta_9 \\ \bullet \mathbf{h}_1:\Delta_{10}\vdash \Delta_9,\mathbf{F}_7\vee \mathbf{F}_8 \end{array} \vee_R \quad \frac{\mathbf{h}_6:\mathbf{F}_7,\Delta_{10}\vdash \Delta_9 \quad \mathbf{h}_6:\mathbf{F}_8,\Delta_{10}\vdash \Delta_9}{\bullet \mathbf{h}_6:\Delta_{10},\mathbf{F}_7\vee \mathbf{F}_8\vdash \Delta_9} \quad \mathbf{Cut} \end{array}
                          \bulleth<sub>1</sub> : \Delta<sub>10</sub> \vdash \Delta<sub>9</sub>,F<sub>7</sub> \lor F<sub>8</sub>
                                                                                                                                                                -:\Delta_{10}\vdash\Delta_{9}
    \frac{ \frac{-:\Delta_{10} \vdash \Delta_{9}, \mathsf{F}_{7}, \mathsf{F}_{8}}{-:\Delta_{10} \vdash \Delta_{9}, \mathsf{F}_{7}} \overset{\mathsf{ax/W}}{-:\Delta_{10}, \mathsf{F}_{8} \vdash \Delta_{9}, \mathsf{F}_{7}} \overset{\mathsf{ax/W}}{\mathsf{sCut}} \\ \frac{-:\Delta_{10} \vdash \Delta_{9}, \mathsf{F}_{7}}{-:\Delta_{10}, \mathsf{F}_{7} \vdash \Delta_{9}} \overset{\mathsf{ax/W}}{\mathsf{sCut}}
                                                                                                                                                                                                                                       -:\Delta_{10}\vdash\Delta_{9}
                                              \frac{\mathbf{h}_1 : \Delta_{13} \vdash \mathbf{F}_7, \mathbf{F}_8, \mathbf{F}_{10} \lor \mathbf{F}_{11}, \Delta_{12}}{\mathbf{h}_1 : \Delta_{13} \vdash (\Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8), \mathbf{F}_{10} \lor \mathbf{F}_{11}} \ \lor_R \ \frac{\mathbf{h}_9 : \mathbf{F}_{10}, \Delta_{13} \vdash \Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8 \quad \mathbf{h}_9 : \mathbf{F}_{11}, \Delta_{13} \vdash \Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8}{\bullet \mathbf{h}_9 : \Delta_{13}, \mathbf{F}_{10} \lor \mathbf{F}_{11} \vdash \Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8} \ \mathbf{Cut}
                                      \bullet h_1: \Delta_{13} \vdash (\Delta_{12}, F_7 \lor F_8), F_{\underline{10}} \lor F_{\underline{11}} \quad ...
                                                                                                                                                                                                                                                -:\Delta_{13}\vdash\Delta_{12},\mathtt{F}_{7}\vee\mathtt{F}_{8}
                                                                                                                                                                                                          \text{ax/W} \quad \frac{\text{h}_9: \Delta_{13}, \text{F}_{10} \vdash \Delta_{12}, \text{F}_7, \text{F}_8}{\text{c}} \quad \text{inv-th/ax} \quad \frac{\text{h}_9: \Delta_{13}, \text{F}_{11} \vdash \Delta_{12}, \text{F}_7, \text{F}_8}{\text{h}_9: \Delta_{13}, \text{F}_{11} \vdash \Delta_{12}, \text{F}_7, \text{F}_8} \quad \frac{\text{inv-th/ax}}{\vee_L}
                                                                                                                                                                                                                                                             -: \Delta_{13} \vdash \Delta_{12}, \mathsf{F}_7, \mathsf{F}_8 
                                                                                                                                                                                                              -:\Delta_{13}\vdash\Delta_{12},\mathtt{F}_{7}\vee\mathtt{F}_{8}
                                  \frac{\mathbf{h}_1:\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}\vdash\mathbf{F}_7,\mathbf{F}_8,\mathbf{F}_{13},\Delta_{12}}{\bullet\mathbf{h}_1:\Delta_{14},\mathbf{F}_{10}}\underbrace{\vee\mathbf{F}_{11}\vdash(\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8),\mathbf{F}_{13}}_{}\\ \vee_R \underbrace{\begin{array}{c}\mathbf{h}_9:\mathbf{F}_{10},\mathbf{F}_{13},\Delta_{14}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{14}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{14}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{14}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_8\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{14}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_9\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{14}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_9\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{14}\vdash\Delta_{12},\mathbf{F}_7\vee\mathbf{F}_9\\\bullet\mathbf{h}_9:(\Delta_{14},\mathbf{F}_{10}\vee\mathbf{F}_{11}),\mathbf{F}_{13}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14}\vdash\Delta_{14
                                                                                                                                                                                                                                                -: \Delta_{14}, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7 \vee \mathtt{F}_8
                                                                                                                                                                                                                            \mathtt{h}_1:\Delta_{14},\mathtt{F}_{10}\vee\mathtt{F}_{11}\vdash\Delta_{12},\mathtt{F}_{13},\mathtt{F}_7,\mathtt{F}_8
                                                                                                                                                                                                              \frac{1}{-:\Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7}, F_{8}} }{-:\Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_{7}, F_{8}} \vee_{R}} 
                                                                                                                                                                                                               -: \Delta_{14}, F_{10} \vee F_{11} \vdash \Delta_{12}, F_7 \vee F_8
```

\bullet Case rule AT

$$\frac{ \begin{array}{c} \mathbf{h}_{1} : \Delta_{11}, [] \mathbf{F}_{9} \vdash \mathbf{F}_{6}, \mathbf{F}_{7}, \Delta_{10} \\ \bullet \mathbf{h}_{1} : \Delta_{11}, [] \mathbf{F}_{9} \vdash \Delta_{10}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \\ \hline \\ - : \Delta_{11}, [] \mathbf{F}_{9} \vdash \Delta_{10} \\ \hline \\ - : \Delta_{11}, [] \mathbf{F}_{9} \vdash \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{11}, [] \mathbf{F}_{9}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \vdash \Delta_{10} \\ \hline \\ - : \Delta_{11}, [] \mathbf{F}_{9} \vdash \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{11}, \mathbf{F}_{9}, [] \mathbf{F}_{9} \vdash \Delta_{10}, \mathbf{F}_{6} \lor \mathbf{F}_{7} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{11}, \mathbf{F}_{9}, [] \mathbf{F}_{9} \vdash \Delta_{10} \\ \hline \\ - : \Delta_{11}, [] \mathbf{F}_{9} \vdash \Delta_{10} \\ \hline \\ - : \Delta_{11}, [] \mathbf{F}_{9} \vdash \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{12} \vdash \mathbf{F}_{7}, \mathbf{F}_{8}, [] \mathbf{F}_{10}, \Delta_{11} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{12} \vdash (\Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8}), [] \mathbf{F}_{10} \\ \hline \\ - : \Delta_{12} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{9} : \Delta_{12}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{9} : \Delta_{12}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{9} : \Delta_{12}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{9} : \Delta_{12}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{9} : \Delta_{12}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{9} : \Delta_{12}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{9} : \Delta_{12}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{9} : \Delta_{12}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{9} : \Delta_{12}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{13}, [] \mathbf{F}_{10} \vdash (\Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8}), \mathbf{F}_{12} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{13}, [] \mathbf{F}_{10} \vdash (\Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8}), \mathbf{F}_{12} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{13}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{13}, \mathbf{F}_{10}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{13}, \mathbf{F}_{10}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{13}, \mathbf{F}_{10}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{13}, \mathbf{F}_{10}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{13}, \mathbf{F}_{10}, [] \mathbf{F}_{10} \vdash \Delta_{11}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{1} : \Delta_{13}, \mathbf{F}_{10}, [] \mathbf{F}_{10}$$

• Case rule \perp_L

\bullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{11},\mathbf{p}_{10}\vdash \mathsf{F}_{6},\mathsf{F}_{7},\Delta_{9},\mathbf{p}_{10}}{\bullet \mathbf{h}_{1}:\Delta_{11},\mathbf{p}_{10}\vdash (\Delta_{9},\mathbf{p}_{10}),\mathsf{F}_{6}\vee \mathsf{F}_{7}} \vee_{R} & \frac{}{\bullet \mathbf{h}_{8}:(\Delta_{11},\mathbf{p}_{10}),\mathsf{F}_{6}\vee \mathsf{F}_{7}\vdash \Delta_{9},\mathbf{p}_{10}} & I \\ & -:\Delta_{11},\mathbf{p}_{10}\vdash \Delta_{9},\mathbf{p}_{10} & \\ & -:\Delta_{11},\mathbf{p}_{10}\vdash \Delta_{9},\mathbf{p}_{10} & I \\ \\ \hline \frac{\mathbf{h}_{1}:\Delta_{12}\vdash \mathsf{F}_{7},\mathsf{F}_{8},\mathbf{p}_{11},\Delta_{10},\mathbf{p}_{11}}{\bullet \mathbf{h}_{1}:\Delta_{12}\vdash ((\Delta_{10},\mathbf{p}_{11}),\mathsf{F}_{7}\vee \mathsf{F}_{8}),\mathbf{p}_{11}} \vee_{R} & \frac{\mathbf{h}_{9}:\Delta_{12},\mathbf{p}_{11}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathsf{F}_{7}\vee \mathsf{F}_{8}}{\bullet \mathbf{h}_{9}:\Delta_{12},\mathbf{p}_{11}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathsf{F}_{7}\vee \mathsf{F}_{8}} & I \\ \hline & -:\Delta_{12}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathsf{F}_{7}\vee \mathsf{F}_{8} & \rightarrow \\ \hline \frac{\mathbf{h}_{1}:\Delta_{12}\vdash \Delta_{10},\mathsf{F}_{7},\mathsf{F}_{8},\mathbf{p}_{11},\mathbf{p}_{11}}{\bullet \mathcal{A}_{10}\vdash \Delta_{10},\mathsf{F}_{7},\mathsf{F}_{8},\mathbf{p}_{11}} & \mathsf{hCut} \\ \hline & -:\Delta_{12}\vdash \Delta_{10},\mathbf{p}_{11},\mathsf{F}_{7}\vee \mathsf{F}_{8} & \vee_{R} \\ \hline & \frac{-:\Delta_{12}\vdash \Delta_{10},\mathbf{p}_{11}}{-:\Delta_{13},\mathbf{p}_{11}\vdash ((\Delta_{10},\mathbf{p}_{11}),\mathsf{F}_{7}\vee \mathsf{F}_{8})} & V_{R} \\ \hline & \bullet \mathbf{h}_{1}:\Delta_{13},\mathbf{p}_{11}\vdash ((\Delta_{10},\mathbf{p}_{11}),\mathsf{F}_{7}\vee \mathsf{F}_{8}),\mathsf{F}_{12} & \vee_{R} \\ \hline & \bullet \mathbf{h}_{9}:(\Delta_{13},\mathbf{p}_{11}),\mathsf{F}_{12}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathsf{F}_{7}\vee \mathsf{F}_{8} \\ \hline & -:\Delta_{13},\mathbf{p}_{11}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathsf{F}_{7}\vee \mathsf{F}_{8} \\ \hline & -:\Delta_{13},\mathbf{p}_{11}\vdash (\Delta_{10},\mathbf{p}_{11}),\mathsf{F}_{7}\vee \mathsf{F}_{8} & I \\ \hline & -:\Delta_{13},\mathbf{p}_{11}\vdash \Delta_{10},\mathbf{p}_{11},\mathsf{F}_{7}\vee \mathsf{$$

• Case rule \top_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \top, \Delta_{10} \vdash \mathbf{F}_6, \mathbf{F}_7, \Delta_9}{\bullet \mathbf{h}_1: \top, \Delta_{10} \vdash \Delta_9, \mathbf{F}_6 \vee \mathbf{F}_7} \vee_R & \frac{\mathbf{h}_8: \Delta_{10}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_9}{\bullet \mathbf{h}_8: (\top, \Delta_{10}), \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_9} & \top_L \\ \hline -: \top, \Delta_{10} \vdash \Delta_9 & \\ \hline \bullet \mathbf{h}_1: \top, \Delta_{10} \vdash \Delta_9, \mathbf{F}_6 \vee \mathbf{F}_7 & \mathbf{ax/W} & \\ \hline \bullet \mathbf{h}_1: \top, \Delta_{10} \vdash \Delta_9, \mathbf{F}_6 \vee \mathbf{F}_7 & \mathbf{ax/W} & \\ \hline -: \top, \Delta_{10} \vdash \Delta_9 & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_1: \Delta_{11} \vdash \mathbf{F}_7, \mathbf{F}_8, \top, \Delta_{10} & \vee_R & \frac{\mathbf{h}_9: \Delta_{11} \vdash \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8}{\bullet \mathbf{h}_9: \Delta_{11}, \top \vdash \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8} & \top_L \\ \hline \bullet \mathbf{h}_1: \Delta_{11} \vdash (\Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8), \top & \vee_R & \frac{\mathbf{h}_9: \Delta_{11} \vdash \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8}{\bullet \mathbf{h}_9: \Delta_{11}, \top \vdash \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8} & \top_L \\ \hline -: \Delta_{11} \vdash \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8 & \rightarrow \\ \hline -: \Delta_{11} \vdash \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8 & \mathbf{ax/W} \\ \hline \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \top, \Delta_{12} \vdash \mathbf{F}_7, \mathbf{F}_8, \mathbf{F}_{11}, \Delta_{10} \\ \bullet \mathbf{h}_1: \top, \Delta_{12} \vdash (\Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8), \mathbf{F}_{11} \end{array} \vee_R \quad \begin{array}{c} \mathbf{h}_9: \mathbf{F}_{11}, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8 \\ \bullet \mathbf{h}_9: (\top, \Delta_{12}), \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8 \end{array} \quad \begin{array}{c} \top_L \\ \mathsf{Cut} \\ \hline \\ \bullet \mathbf{h}_1: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{11}, \mathbf{F}_7 \vee \mathbf{F}_8 \end{array} \quad \begin{array}{c} \top_L \\ \mathsf{Cut} \\ \hline \\ \bullet \mathbf{h}_1: \top, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_{11}, \mathbf{F}_7 \vee \mathbf{F}_8 \end{array} \quad \begin{array}{c} \mathsf{ax/W} \\ \mathsf{h}_9: \top, \Delta_{12}, \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_7 \vee \mathbf{F}_8 \end{array} \quad \begin{array}{c} \mathsf{ax/W} \\ \mathsf{h}_{\mathsf{Cut}} \end{array}$$

6.4 Status of \perp_R : OK

• Case rule \rightarrow_R

• Case rule \wedge_R

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6 \\ \bullet \mathbf{h}_1 : \Delta_8 \vdash (\Delta_7, F_5 \wedge F_6), \bot \end{array} \\ \bot_R \\ \begin{array}{c} \mathbf{h}_4 : \bot, \Delta_8 \vdash F_5, \Delta_7 \\ \bullet \mathbf{h}_4 : \Delta_8, \bot \vdash \Delta_7, F_5 \wedge F_6 \end{array} \\ Cut \\ \\ - : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6 \\ \\ \hline - : \Delta_8 \vdash \Delta_7, F_5 \wedge F_6 \end{array} \\ \mathbf{ax/W} \\ \\ \hline \begin{array}{c} \mathbf{h}_1 : \Delta_{10} \vdash F_9, \Delta_8, F_6 \wedge F_7 \\ \bullet \mathbf{h}_1 : \Delta_{10} \vdash (\bot, \Delta_8, F_6 \wedge F_7), F_9 \end{array} \\ \bot_R \\ \hline \bullet \mathbf{h}_1 : \Delta_{10} \vdash (\bot, \Delta_8, F_6 \wedge F_7), F_9 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10} \vdash (\bot, \Delta_8, F_6 \wedge F_7) \end{array} \\ \begin{array}{c} \mathbf{h}_5 : F_9, \Delta_{10} \vdash \bot, F_6, \Delta_8 \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \end{array} \\ Cut \\ \hline \\ - : \Delta_{10} \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \hline \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \\ \bullet \mathbf{h}_5 : \Delta_{10}, F_9 \vdash \bot, \Delta_8, F_6 \wedge F_7 \\ \hline \end{array}$$

• Case rule \vee_R

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_8\vdash\Delta_7, F_5\vee F_6}{\bullet \mathbf{h}_1:\Delta_8\vdash (\Delta_7, F_5\vee F_6),\bot} & \bot_R & \frac{\mathbf{h}_4:\bot,\Delta_8\vdash F_5, F_6,\Delta_7}{\bullet \mathbf{h}_4:\Delta_8,\bot\vdash \Delta_7, F_5\vee F_6} & \lor_R \\ \hline & -:\Delta_8\vdash\Delta_7, F_5\vee F_6 & \bot_{-}:\Delta_8\vdash \Delta_7, F_5\vee F_6 \\ \hline & -:\Delta_8\vdash \Delta_7, F_5\vee F_6 & \mathbf{ax/W} \\ \hline & \frac{\mathbf{h}_1:\Delta_{10}\vdash F_9,\Delta_8, F_6\vee F_7}{\bullet \mathbf{h}_1:\Delta_{10}\vdash (\bot,\Delta_8, F_6\vee F_7), F_9} & \bot_R & \frac{\mathbf{h}_5:F_9,\Delta_{10}\vdash \bot, F_6, F_7,\Delta_8}{\bullet \mathbf{h}_5:\Delta_{10}, F_9\vdash \bot,\Delta_8, F_6\vee F_7} & \lor_R \\ \hline & -:\Delta_{10}\vdash \bot,\Delta_8, F_6\vee F_7 & \bot_{-} & \bot_{$$

• Case rule \perp_R

$$\frac{\begin{array}{c} \mathbf{h}_1:\Delta_6\vdash\bot,\Delta_5\\ \bullet\mathbf{h}_1:\Delta_6\vdash(\bot,\Delta_5),\bot\end{array} \perp_R \quad \frac{\mathbf{h}_4:\bot,\Delta_6\vdash\Delta_5}{\bullet\mathbf{h}_4:\Delta_6,\bot\vdash\bot,\Delta_5} \quad \frac{\bot_R}{\mathsf{Cut}}\\ \begin{matrix} -:\Delta_6\vdash\bot,\Delta_5\\ \hline \\ -:\Delta_6\vdash\bot,\Delta_5 \end{matrix} \quad \mathsf{ax/W} \\ \end{matrix}}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_8 \vdash \mathbf{F}_7, \Delta_6 \\ \bullet \mathbf{h}_1: \Delta_8 \vdash (\bot, \Delta_6), \mathbf{F}_7 \end{array} \perp_R \quad \begin{array}{c} \mathbf{h}_5: \mathbf{F}_7, \Delta_8 \vdash \Delta_6 \\ \bullet \mathbf{h}_5: \Delta_8, \mathbf{F}_7 \vdash \bot, \Delta_6 \end{array} }{ \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_5: \Delta_8, \mathbf{F}_7 \vdash \bot, \Delta_6 \end{array} } \quad \begin{array}{c} \bot_R \\ \mathsf{Cut} \\ \bullet \\ \bullet \mathbf{h}_5: \Delta_8, \mathbf{F}_7 \vdash \bot, \Delta_6 \end{array} } \\ \bullet \\ \bullet \mathbf{h}_5: \Delta_8, \mathbf{F}_7 \vdash \bot, \Delta_6 \\ \bullet \mathsf{h}_5: \Delta_8, \mathbf{F}_7 \vdash \bot, \Delta_6 \end{array} \quad \text{ax/W} \\ \bullet \mathsf{h}_6: \Delta_8 \vdash \bot, \Delta_6 \vdash \bot, \Delta_6 \vdash \bot, \Delta_6 \\ \bullet \mathsf{h}_7: \Delta_8 \vdash \bot, \Delta_6 \\ \bullet \mathsf{h}_7: \Delta_8 \vdash \bot, \Delta_6 \vdash \bot,$$

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_6 \vdash \top,\Delta_5}{\bullet \mathbf{h}_1:\Delta_6 \vdash (\top,\Delta_5),\bot} \ \bot_R \quad \\ \frac{\bullet}{\bullet \mathbf{h}_1:\Delta_6 \vdash (\top,\Delta_5),\bot} \ \Box_R \\ \\ -:\Delta_6 \vdash \top,\Delta_5 \\ \hline -:\Delta_6 \vdash \top,\Delta_5 \end{array} \ \top_R \\ \\ \frac{\mathbf{h}_1:\Delta_8 \vdash \mathbf{F}_7,\top,\Delta_6}{\bullet \mathbf{h}_1:\Delta_8 \vdash (\bot,\top,\Delta_6),\mathbf{F}_7} \ \bot_R \quad \\ \frac{\bullet \mathbf{h}_1:\Delta_8 \vdash (\bot,\top,\Delta_6),\mathbf{F}_7}{-:\Delta_8 \vdash \bot,\top,\Delta_6} \ \ \top_R \\ \\ \frac{-:\Delta_8 \vdash \bot,\top,\Delta_6}{-:\Delta_8 \vdash \bot,\top,\Delta_6} \ \ \top_R \end{array}$$

\bullet Case rule K

$$\begin{array}{c} \frac{h_1: \Box \Gamma_7, \Delta_8 \vdash \Delta_6, []F_5}{\bullet h_1: \Box \Gamma_7, \Delta_8 \vdash (\Delta_6, []F_5), \bot} \ \bot_R \ \frac{h_4: unbox(\Box \Gamma_7) \vdash F_5}{\bullet h_4: (\Box \Gamma_7, \Delta_8), \bot \vdash \Delta_6, []F_5} \ K \\ \hline -: \Box \Gamma_7, \Delta_8 \vdash \Delta_6, []F_5 \\ \hline -: \Delta_8, \Box \Gamma_7 \vdash \Delta_6, []F_5 \ \end{array} \\ \frac{h_5: unbox(\Box \Gamma_9), unbox(\Box F_8) \vdash F_6}{\bullet h_1: \Box \Gamma_9, \Delta_{10} \vdash (\bot, \Delta_7, []F_6), \Box F_8} \ \bot_R \ \frac{h_5: unbox(\Box \Gamma_9), unbox(\Box F_8) \vdash F_6}{\bullet h_5: (\Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \bot, \Delta_7, []F_6} \ K \\ \hline Cut \\ \hline -: \Box \Gamma_9, \Delta_{10} \vdash \bot, \Delta_7, []F_6 \ \end{array} \\ \frac{h_1: \Delta_{10}, \Box \Gamma_9 \vdash \bot, \Box F_8, \Delta_7, []F_6}{\bullet h_1: \Delta_{10}, \Box \Gamma_9 \vdash \bot, \Delta_7, []F_6} \ Ax/W \\ \hline -: \Delta_{10}, \Box \Gamma_9 \vdash \bot, \Delta_7, []F_6 \ \end{array} \\ \frac{h_1: \Delta_{10}, \Box \Gamma_9 \vdash \bot, \Box F_8, \Delta_7, []F_6}{\bullet h_1: \Box \Gamma_8, \Delta_{10} \vdash F_9, \Delta_7, []F_6} \ \bot_R \ \frac{h_5: unbox(\Box \Gamma_8) \vdash F_6}{\bullet h_5: (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \bot, \Delta_7, []F_6} \ K \\ \hline -: \Box \Gamma_8, \Delta_{10} \vdash \bot, \Delta_7, []F_6 \ \end{array} \\ \frac{h_1: \Box \Gamma_8, \Delta_{10} \vdash (\bot, \Delta_7, []F_6), F_9}{\bullet h_1: \Box \Gamma_8, \Delta_{10} \vdash (\bot, \Delta_7, []F_6), F_9} \ \bot_R \ \frac{h_5: unbox(\Box \Gamma_8) \vdash F_6}{\bullet h_5: (\Box \Gamma_8, \Delta_{10}), F_9 \vdash \bot, \Delta_7, []F_6} \ K \\ \hline -: \Box \Gamma_8, \Delta_{10} \vdash \bot, \Delta_7, []F_6 \ \to Cut \ \end{array}$$

• Case rule A45

$$\begin{array}{c} \frac{\mathbf{h}_{1}: \square\Gamma_{9}, \Delta_{11} \vdash \mathbf{F}_{10}, \square\Gamma_{7}, \Delta_{8}, []\mathbf{F}_{6}}{\bullet \mathbf{h}_{1}: \square\Gamma_{9}, \Delta_{11} \vdash (\bot, \square\Gamma_{7}, \Delta_{8}, []\mathbf{F}_{6})} \quad \bot_{R} \quad \frac{\mathbf{h}_{5}: \square\Gamma_{9} \vdash \square\Gamma_{7}, \mathbf{F}_{6}}{\bullet \mathbf{h}_{5}: (\square\Gamma_{9}, \Delta_{11}), \mathbf{F}_{10} \vdash \bot, \square\Gamma_{7}, \Delta_{8}, []\mathbf{F}_{6}} \\ \quad -: \square\Gamma_{9}, \Delta_{11} \vdash \bot, \square\Gamma_{7}, \Delta_{8}, []\mathbf{F}_{6} \\ \quad \to \\ \quad -: \square\Gamma_{9} \vdash \mathbf{F}_{6}, \square\Gamma_{7} \quad \mathbf{ax/W} \\ \hline \quad -: \square\Gamma_{9} \vdash \bot, \Delta_{8}, \square\Gamma_{7}, []\mathbf{F}_{6} \end{array} \quad A45 \end{array}$$

• Case rule \rightarrow_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_8,\mathbf{F}_5\to\mathbf{F}_6\vdash\Delta_7}{\bullet\mathbf{h}_1:\Delta_8,\mathbf{F}_5\to\mathbf{F}_6\vdash\Delta_7,\bot} \perp_R \frac{\mathbf{h}_4:\bot,\Delta_8\vdash\mathbf{F}_5,\Delta_7 \quad \mathbf{h}_4:\bot,\mathbf{F}_6,\Delta_8\vdash\Delta_7}{\bullet\mathbf{h}_4:(\Delta_8,\mathbf{F}_5\to\mathbf{F}_6),\bot\vdash\Delta_7} \quad \mathbf{Cut} \\ \hline -:\Delta_8,\mathbf{F}_5\to\mathbf{F}_6\vdash\Delta_7 \\ \hline -:\Delta_8,\mathbf{F}_5\to\mathbf{F}_6\vdash\Delta_7 \\ \hline -:\Delta_8,\mathbf{F}_5\to\mathbf{F}_6\vdash\Delta_7 \end{array} \quad \mathbf{ax/W} \\ \hline \frac{\mathbf{h}_1:\Delta_9\vdash\mathbf{F}_6\to\mathbf{F}_7,\Delta_8}{\bullet\mathbf{h}_1:\Delta_9\vdash(\bot,\Delta_8),\mathbf{F}_6\to\mathbf{F}_7} \perp_R \frac{\mathbf{h}_5:\Delta_9\vdash\bot,\mathbf{F}_6,\Delta_8 \quad \mathbf{h}_5:\mathbf{F}_7,\Delta_9\vdash\bot,\Delta_8}{\bullet\mathbf{h}_5:\Delta_9,\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8} \quad \mathbf{Cut} \\ \hline -:\Delta_9\vdash\bot,\Delta_8 \\ \hline -:\Delta_9\vdash\bot,\Delta_8 \\ \hline -:\Delta_9\vdash\bot,\Delta_8 \\ \hline \bullet\mathbf{h}_1:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\mathbf{F}_9,\Delta_8 \\ \bullet\mathbf{h}_1:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash(\bot,\Delta_8),\mathbf{F}_9} \perp_R \frac{\mathbf{h}_5:\mathbf{F}_9,\Delta_{10}\vdash\bot,\mathbf{F}_6,\Delta_8 \quad \mathbf{h}_5:\mathbf{F}_7,\mathbf{F}_9,\Delta_{10}\vdash\bot,\Delta_8}{\bullet\mathbf{h}_5:(\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7),\mathbf{F}_9\vdash\bot,\Delta_8} \quad \mathbf{Cut} \\ \hline -:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8 \\ \hline \bullet_{11}:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8,\mathbf{F}_9} \xrightarrow{\mathbf{ax/W}} \frac{\bullet}{\bullet\mathbf{h}_5:\Delta_{10},\mathbf{F}_9,\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8} \quad \mathbf{Cut} \\ \hline -:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8,\mathbf{F}_9} \xrightarrow{\mathbf{ax/W}} \xrightarrow{\bullet\mathbf{h}_5:\Delta_{10},\mathbf{F}_9,\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8} \quad \mathbf{hCut} \\ \hline \bullet_{11}:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8,\mathbf{F}_9} \xrightarrow{\mathbf{ax/W}} \xrightarrow{\bullet\mathbf{h}_5:\Delta_{10},\mathbf{F}_9,\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8} \quad \mathbf{hCut} \\ \hline \bullet_{11}:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8,\mathbf{F}_9} \xrightarrow{\mathbf{ax/W}} \xrightarrow{\bullet\mathbf{h}_5:\Delta_{10},\mathbf{F}_9,\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8} \quad \mathbf{hCut} \\ \hline \bullet_{11}:\Delta_{10},\mathbf{F}_6\to\mathbf{F}_7\vdash\bot,\Delta_8,\mathbf{F}_9} \xrightarrow{\bullet} \mathbf{h}_7\vdash\bot,\Delta_8} \xrightarrow{\bullet} \xrightarrow{\bullet} \mathbf{h}_7\vdash\bot,\Delta_8} \xrightarrow{\bullet} \mathbf{h}_$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \Delta_7}{\bullet \mathbf{h}_1:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \Delta_7, \bot} \quad \bot_R \quad \frac{\mathbf{h}_4:\bot, \mathbf{F}_5, \mathbf{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathbf{h}_4: \left(\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6\right), \bot \vdash \Delta_7} \\ \hline -:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \Delta_7 \\ \hline -:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \Delta_7 \\ \hline -:\Delta_8, \mathbf{F}_5 \wedge \mathbf{F}_6 \vdash \Delta_7 \\ \hline \bullet \mathbf{h}_1:\Delta_9 \vdash \mathbf{F}_6 \wedge \mathbf{F}_7, \Delta_8 \\ \bullet \mathbf{h}_1:\Delta_9 \vdash (\bot, \Delta_8), \mathbf{F}_6 \wedge \mathbf{F}_7 \\ \hline -:\Delta_9 \vdash \bot, \Delta_8 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \mathbf{F}_9, \Delta_8 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash (\bot, \Delta_8), \mathbf{F}_9 \\ \hline -:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash (\bot, \Delta_8), \mathbf{F}_9 \\ \hline -:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline -:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \bot, \Delta_8, \mathbf{F}_9 \\ \hline \bullet \mathbf{h}_1:\Delta_{10}, \mathbf{F}_9, \mathbf{h}_1 \wedge \mathbf{h}_1 \wedge \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1 \wedge \mathbf{h}_1 \wedge \mathbf{h}_1 \wedge \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1 \wedge \mathbf{h}_2 \wedge \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1 \wedge \mathbf{h}_2 \wedge \mathbf{h}_2 \wedge \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1 \wedge \mathbf{h}_2 \wedge \mathbf{h}_2 \wedge \mathbf{h}_2 \wedge \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1 \wedge \mathbf{h}_2 \wedge \mathbf{h}_2 \wedge \mathbf{h}_2 \wedge \mathbf{h}_2 \\ \hline \bullet \mathbf{h}_1 \wedge \mathbf{h}_2 \wedge \mathbf$$

• Case rule \vee_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \Delta_7}{\bullet \mathbf{h}_1: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \Delta_7, \bot} \ \bot_R & \frac{\mathbf{h}_4: \bot, \mathbf{F}_5, \Delta_8 \vdash \Delta_7 \quad \mathbf{h}_4: \bot, \mathbf{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathbf{h}_4: (\Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6), \bot \vdash \Delta_7} \ \mathrm{Cut} \\ & -: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \Delta_7 \\ & -: \Delta_8, \mathbf{F}_5 \vee \mathbf{F}_6 \vdash \Delta_7 \end{array} \ \mathbf{ax/W}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_9 \vdash F_6 \vee F_7, \Delta_8 \\ \hline \bullet \mathbf{h}_1: \Delta_9 \vdash (\bot, \Delta_8), F_6 \vee F_7 \end{array} \perp_{R} \begin{array}{c} \mathbf{h}_5: F_6, \Delta_9 \vdash \bot, \Delta_8 & \mathbf{h}_5: F_7, \Delta_9 \vdash \bot, \Delta_8 \\ \hline \bullet \mathbf{h}_5: \Delta_9, F_6 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{cut} \\ \hline \\ -: \Delta_9 \vdash \bot, \Delta_8 \\ \hline \bullet \mathbf{h}_5: \Delta_9, F_6 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline -: \Delta_9 \vdash \bot, \Delta_8, F_6 \vee F_7 \end{array} \\ \hline \bullet \mathbf{h}_5: \Delta_9, F_6 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline -: \Delta_9 \vdash \bot, \Delta_8 & \mathbf{hCut} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_{10}, F_6 \vee F_7 \vdash F_9, \Delta_8 & \mathbf{h}_5: F_7, F_9, \Delta_{10} \vdash \bot, \Delta_8 \\ \hline \bullet \mathbf{h}_1: \Delta_{10}, F_6 \vee F_7 \vdash (\bot, \Delta_8), F_9 & \mathbf{h}_5: (\Delta_{10}, F_6 \vee F_7), F_9 \vdash \bot, \Delta_8 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_{10}, F_6 \vee F_7 \vdash \bot, \Delta_8, F_9 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_5: \Delta_{10}, F_9, F_6 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_5: \Delta_{10}, F_9, F_6 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_5: \Delta_{10}, F_9, F_6 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_5: \Delta_{10}, F_9, F_6 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_6 \vee F_7 \vdash \bot, \Delta_8, F_9 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_6 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \\ \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \Delta_{10}, F_9 \vee F_7 \vdash \bot, \Delta_8 & \mathbf$$

\bullet Case rule AT

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_7, []\mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_1:\Delta_7, []\mathbf{F}_5 \vdash \Delta_6, \bot} \quad \bot_R \quad \frac{\mathbf{h}_4:\bot, \mathbf{F}_5, \Delta_7, []\mathbf{F}_5 \vdash \Delta_6}{\bullet \mathbf{h}_4:(\Delta_7, []\mathbf{F}_5), \bot \vdash \Delta_6} \quad AT \\ \hline -:\Delta_7, []\mathbf{F}_5 \vdash \Delta_6 \\ \hline \rightarrow \\ \hline -:\Delta_7, []\mathbf{F}_5 \vdash \Delta_6 \\ \hline \bullet \mathbf{h}_1:\Delta_8 \vdash []\mathbf{F}_6, \Delta_7 \\ \hline \bullet \mathbf{h}_1:\Delta_8 \vdash (\bot,\Delta_7), []\mathbf{F}_6 \quad \bot_R \quad \frac{\mathbf{h}_5:\mathbf{F}_6, \Delta_8, []\mathbf{F}_6 \vdash \bot, \Delta_7}{\bullet \mathbf{h}_5:\Delta_8, []\mathbf{F}_6 \vdash \bot, \Delta_7} \quad AT \\ \hline -:\Delta_8 \vdash \bot, \Delta_7 \\ \hline \hline \bullet_1:\Delta_8 \vdash \bot, \Delta_7, []\mathbf{F}_6 \quad \mathbf{ax/W} \quad \frac{\rightarrow}{\bullet \mathbf{h}_5:\Delta_8, []\mathbf{F}_6 \vdash \bot, \Delta_7} \quad \mathbf{ax/W} \\ \hline -:\Delta_8 \vdash \bot, \Delta_7 \\ \hline \bullet \mathbf{h}_1:\Delta_9, []\mathbf{F}_6 \vdash (\bot,\Delta_7), \mathbf{F}_8 \quad \Delta_8 \quad \frac{\mathbf{h}_5:\mathbf{F}_6, \mathbf{F}_8, \Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7}{\bullet \mathbf{h}_5:(\Delta_9, []\mathbf{F}_6), \mathbf{F}_8 \vdash \bot, \Delta_7} \quad \mathbf{AT} \\ \hline \bullet \mathbf{h}_1:\Delta_9, []\mathbf{F}_6 \vdash (\bot,\Delta_7), \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline -:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_1:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_5:\Delta_9, \mathbf{F}_8, []\mathbf{F}_6 \vdash \bot, \Delta_7 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_5:\Delta_9, \mathbf{F}_8, []\mathbf{F}_6 \vdash \bot, \Delta_7 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_5:\Delta_9, \mathbf{F}_8, []\mathbf{F}_6 \vdash \bot, \Delta_7 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{F}_8 \quad \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_7, \mathbf{Ax/W} \\ \hline \bullet \mathbf{h}_7:\Delta_9, []\mathbf{H}_7:\Delta_9, []\mathbf{H}_7:\Delta$$

• Case rule \perp_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_6\vdash\Delta_5}{\bullet\mathbf{h}_1:\Delta_6\vdash\Delta_5,\bot} & \bot_R & \frac{\bullet}{\bullet\mathbf{h}_4:\Delta_6,\bot\vdash\Delta_5} & \bot_L \\ & \to & \\ & \to & \\ \hline -:\Delta_6\vdash\Delta_5 & \mathbf{ax/W} \\ \\ \hline \frac{\mathbf{h}_1:\Delta_7\vdash\bot,\Delta_6}{\bullet\mathbf{h}_1:\Delta_7\vdash(\bot,\Delta_6),\bot} & \bot_R & \frac{\bullet}{\bullet\mathbf{h}_5:\Delta_7,\bot\vdash\bot,\Delta_6} & \bot_L \\ \hline -:\Delta_7\vdash\bot,\Delta_6 & \\ \hline & \to & \\ \hline -:\Delta_7\vdash\bot,\Delta_6 & \mathbf{ax/W} \\ \\ \hline \frac{\mathbf{h}_1:\bot,\Delta_8\vdash\mathsf{F}_7,\Delta_6}{\bullet\mathbf{h}_1:\bot,\Delta_8\vdash(\bot,\Delta_6),\mathsf{F}_7} & \bot_R & \frac{\bullet}{\bullet\mathbf{h}_5:(\bot,\Delta_8),\mathsf{F}_7\vdash\bot,\Delta_6} & \bot_L \\ \hline & -:\bot,\Delta_8\vdash\bot,\Delta_6 & \\ \hline & \to & \\ \hline -:\bot,\Delta_8\vdash\bot,\Delta_6 & \bot_L \\ \hline \end{array}$$

ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_7, \mathbf{p}_6 \vdash \Delta_5, \mathbf{p}_6}{\bullet \mathbf{h}_1: \Delta_7, \mathbf{p}_6 \vdash (\Delta_5, \mathbf{p}_6), \bot} \ \bot_R & \\ \frac{\bullet \mathbf{h}_4: (\Delta_7, \mathbf{p}_6), \bot \vdash \Delta_5, \mathbf{p}_6}{-: \Delta_7, \mathbf{p}_6 \vdash \Delta_5, \mathbf{p}_6} & I \\ \\ \frac{-: \Delta_7, \mathbf{p}_6 \vdash \Delta_5, \mathbf{p}_6}{-: \Delta_7, \mathbf{p}_6 \vdash \Delta_5, \mathbf{p}_6} & I \end{array}$$

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_8\vdash \mathbf{p}_7,\Delta_6,\mathbf{p}_7}{\bullet \mathbf{h}_1:\Delta_8\vdash (\bot,\Delta_6,\mathbf{p}_7),\mathbf{p}_7} \stackrel{\bot_R}{\to} \frac{\bullet \mathbf{h}_5:\Delta_8,\mathbf{p}_7\vdash \bot,\Delta_6,\mathbf{p}_7}{\bullet \mathbf{h}_5:\Delta_8,\mathbf{p}_7\vdash \bot,\Delta_6,\mathbf{p}_7} \stackrel{I}{\subset} \mathbf{cut} \\ & \xrightarrow{\bullet} \frac{\to}{\bullet \mathbf{h}_1:\Delta_8\vdash \bot,\Delta_6,\mathbf{p}_7,\mathbf{p}_7} \stackrel{\mathsf{ax/W}}{\to} \frac{\bullet \mathbf{h}_5:\Delta_8,\mathbf{p}_7\vdash \bot,\Delta_6,\mathbf{p}_7}{\bullet \mathbf{h}_5:\Delta_8,\mathbf{p}_7\vdash \bot,\Delta_6,\mathbf{p}_7} \stackrel{I}{\to} \mathbf{n}\mathbf{cut} \\ & \xrightarrow{\bullet} \frac{\mathbf{h}_1:\Delta_9,\mathbf{p}_7\vdash F_8,\Delta_6,\mathbf{p}_7}{\bullet \mathbf{h}_1:\Delta_9,\mathbf{p}_7\vdash (\bot,\Delta_6,\mathbf{p}_7),F_8} \stackrel{\bot_R}{\to} \frac{\bullet}{\bullet \mathbf{h}_5:(\Delta_9,\mathbf{p}_7),F_8\vdash \bot,\Delta_6,\mathbf{p}_7} \stackrel{I}{\subset} \mathbf{cut} \\ & \xrightarrow{\bullet} \frac{\to}{-:\Delta_9,\mathbf{p}_7\vdash \bot,\Delta_6,\mathbf{p}_7} \stackrel{I}{\to} \mathbf{cut} \\ & \xrightarrow{\bullet} \frac{\to}{-:\Delta_9,\mathbf{p}_7\vdash \bot,\Delta_6,\mathbf{p}_7} \stackrel{I}{\to} \mathbf{cut} \\ & \xrightarrow{\bullet} \frac{\to}{-:\Delta_9,\mathbf{p}_7\vdash \bot,\Delta_6,\mathbf{p}_7} \stackrel{I}{\to} \mathbf{cut} \\ \end{array}$$

• Case rule \top_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \top, \Delta_6 \vdash \Delta_5}{\bullet \mathbf{h}_1: \top, \Delta_6 \vdash \Delta_5, \bot} \perp_R & \frac{\mathbf{h}_4: \bot, \Delta_6 \vdash \Delta_5}{\bullet \mathbf{h}_4: (\top, \Delta_6), \bot \vdash \Delta_5} & \top_L \\ \hline -: \top, \Delta_6 \vdash \Delta_5 & \rightarrow & \text{Cut} \\ \hline -: \top, \Delta_6 \vdash \Delta_5 & \text{ax/W} \\ \hline \frac{\mathbf{h}_1: \Delta_7 \vdash \top, \Delta_6}{\bullet \mathbf{h}_1: \Delta_7 \vdash (\bot, \Delta_6), \top} \perp_R & \frac{\mathbf{h}_5: \Delta_7 \vdash \bot, \Delta_6}{\bullet \mathbf{h}_5: \Delta_7, \top \vdash \bot, \Delta_6} & \top_L \\ \hline -: \Delta_7 \vdash \bot, \Delta_6 & \rightarrow & \text{Cut} \\ \hline -: \Delta_7 \vdash \bot, \Delta_6 & \text{ax/W} \\ \hline \frac{\mathbf{h}_1: \top, \Delta_8 \vdash \mathbf{F}_7, \Delta_6}{\bullet \mathbf{h}_1: \top, \Delta_8 \vdash (\bot, \Delta_6), \mathbf{F}_7} \perp_R & \frac{\mathbf{h}_5: \mathbf{F}_7, \Delta_8 \vdash \bot, \Delta_6}{\bullet \mathbf{h}_5: (\top, \Delta_8), \mathbf{F}_7 \vdash \bot, \Delta_6} & \top_L \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \rightarrow & \bullet \\ \hline \hline \mathbf{h}_1: \top, \Delta_8 \vdash \bot, \Delta_6, \mathbf{F}_7 & \text{ax/W} \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \rightarrow & \bullet \\ \hline \mathbf{h}_1: \top, \Delta_8 \vdash \bot, \Delta_6, \mathbf{F}_7 & \text{ax/W} \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \rightarrow & \bullet \\ \hline \mathbf{h}_1: \top, \Delta_8 \vdash \bot, \Delta_6, \mathbf{F}_7 & \text{ax/W} \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \rightarrow & \bullet \\ \hline \mathbf{h}_1: \top, \Delta_8 \vdash \bot, \Delta_6, \mathbf{F}_7 & \text{ax/W} \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline -: \top, \Delta_8 \vdash \bot, \Delta_6 & \bullet \\ \hline \end{array}$$

6.5 Status of \top_R : OK

• Case rule \rightarrow_R

$$\begin{array}{c|c} \hline \bullet_{h_1}: \Delta_8 \vdash (\Delta_7, F_5 \to F_6), \top & \hline \bullet_{h_4}: \top, F_5, \Delta_8 \vdash F_6, \Delta_7 \\ \hline -: \Delta_8 \vdash \Delta_7, F_5 \to F_6 \\ \hline \hline \bullet_{h_1}: \Delta_8, F_5 \vdash \top, \Delta_7, F_6 & \hline \\ \bullet_{h_1}: \Delta_8, F_5 \vdash \top, \Delta_7, F_6 & \hline \\ \hline \hline \bullet_{h_1}: \Delta_8, F_5 \vdash \top, \Delta_7, F_6 & \hline \\ \hline -: \Delta_8 \vdash \Delta_7, F_6 & \hline \\ \hline -: \Delta_8 \vdash \Delta_7, F_6 & \hline \\ \hline -: \Delta_8 \vdash \Delta_7, F_6 & \hline \\ \hline -: \Delta_8 \vdash \Delta_7, F_5 \to F_6 \\ \hline \hline \hline \bullet_{h_1}: \Delta_{10} \vdash (\top, \Delta_8, F_6 \to F_7), F_9 & \hline \\ \hline \bullet_{h_2}: \Delta_{10} \vdash \top, \Delta_8, F_6 \to F_7 \\ \hline -: \Delta_{10} \vdash \top, \Delta_8, F_6 \to F_7 \\ \hline \hline -: \Delta_{10} \vdash \top, \Delta_8, F_6 \to F_7 \\ \hline \hline -: \Delta_{10} \vdash \top, \Delta_8, F_6 \to F_7 \\ \hline \hline -: \Delta_{10} \vdash \top, \Delta_8, F_6 \to F_7 \\ \hline \hline -: \Delta_{10} \vdash \top, \Delta_8, F_6 \to F_7 \\ \hline \hline \end{array}$$

• Case rule \wedge_R

$$\frac{\underbrace{\bullet \mathbf{h}_1 : \Delta_8 \vdash (\Delta_7, F_5 \land F_6), \top}_{\bullet \mathbf{h}_1 : \Delta_8 \vdash (\Delta_7, F_5 \land F_6), \top}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash F_5, \Delta_7 \quad \mathbf{h}_4 : \top, \Delta_8 \vdash F_6, \Delta_7}_{\bullet \mathbf{h}_4 : \Delta_8, \top \vdash \Delta_7, F_5 \land F_6} \land \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_5 \land F_6}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_5} \xrightarrow{\bullet}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_5}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_5} \underbrace{\bullet}_{\bullet \mathbf{h}_1 : \Delta_8 \vdash \top, \Delta_7, F_6}_{\bullet \mathbf{h}_1 : \Delta_8 \vdash \Delta_7, F_6} \xrightarrow{\bullet}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_6}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_6} \xrightarrow{\bullet}_{\bullet \mathbf{h}_1 : \Delta_8 \vdash \Delta_7, F_6}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_6} \xrightarrow{\bullet}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_6}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_6}_{\bullet \mathbf{h}_4 : \top, \Delta_8 \vdash \Delta_7, F_6}$$

$$\frac{ \bullet_{\mathbf{h}_1 : \Delta_{10} \vdash (\top, \Delta_8, \mathsf{F}_6 \land \mathsf{F}_7), \mathsf{F}_9} }{ \vdash_{\mathbf{R}} } \frac{ \vdash_{\mathbf{h}_5 : \mathsf{F}_9, \Delta_{10} \vdash \top, \mathsf{F}_6, \Delta_8} \quad \mathsf{h}_5 : \mathsf{F}_9, \Delta_{10} \vdash \top, \mathsf{F}_7, \Delta_8}{ \bullet_{\mathbf{h}_5 : \Delta_{10}, \mathsf{F}_9 \vdash \top, \Delta_8, \mathsf{F}_6 \land \mathsf{F}_7} } \quad \mathsf{Cut} \\ - : \Delta_{10} \vdash \top, \Delta_8, \mathsf{F}_6 \land \mathsf{F}_7} \\ & \xrightarrow{- : \Delta_{10} \vdash \top, \Delta_8, \mathsf{F}_6 \land \mathsf{F}_7} } \top_{\mathbf{R}}$$

• Case rule \vee_R

$$\begin{array}{c|c} \hline \bullet_{h_1}: \Delta_8 \vdash (\Delta_7, F_5 \vee F_6), \top & \frac{h_4: \top, \Delta_8 \vdash F_5, F_6, \Delta_7}{\bullet_{h_4}: \Delta_8, \top \vdash \Delta_7, F_5 \vee F_6} & \vee_R \\ \hline -: \Delta_8 \vdash \Delta_7, F_5 \vee F_6 & \rightarrow \\ \hline \bullet_{h_1}: \Delta_8 \vdash \top, \Delta_7, F_5, F_6 & \uparrow_R & \uparrow_R \\ \hline -: \Delta_8 \vdash \Delta_7, F_5, F_6 & \downarrow_R \\ \hline -: \Delta_8 \vdash \Delta_7, F_5, F_6 & \vee_R \\ \hline -: \Delta_8 \vdash \Delta_7, F_5 \vee F_6 & \vee_R \\ \hline \hline \bullet_{h_1}: \Delta_{10} \vdash (\top, \Delta_8, F_6 \vee F_7), F_9 & \top_R & \frac{h_5: F_9, \Delta_{10} \vdash \top, F_6, F_7, \Delta_8}{\bullet_{h_5}: \Delta_{10}, F_9 \vdash \top, \Delta_8, F_6 \vee F_7} & \vee_R \\ \hline -: \Delta_{10} \vdash \top, \Delta_8, F_6 \vee F_7 & \rightarrow \\ \hline -: \Delta_{10} \vdash \top, \Delta_8, F_6 \vee F_7 & \top_R \\ \hline \end{array}$$

• Case rule \perp_R

$$\begin{array}{c} \underbrace{\begin{array}{c} \bullet\mathbf{h}_1:\Delta_6\vdash(\bot,\Delta_5),\top}_{} \quad \top_R \quad \frac{\mathbf{h}_4:\top,\Delta_6\vdash\Delta_5}{\bullet\mathbf{h}_4:\Delta_6,\top\vdash\bot,\Delta_5} \ \ _{\mathbf{Cut}}^{\bot_R} \\ \hline -:\Delta_6\vdash\bot,\Delta_5 \\ \hline \\ \bullet\mathbf{h}_1:\Delta_6\vdash\bot,\top,\Delta_5 \end{array} \xrightarrow{\mathbf{ax/W}} \begin{array}{c} \bullet\mathbf{h}_4:\top,\Delta_6\vdash\bot,\Delta_5 \\ \hline \bullet\mathbf{h}_1:\Delta_6\vdash\bot,\top,\Delta_5 \end{array} \xrightarrow{\mathbf{ax/W}} \begin{array}{c} \mathbf{ax/W} \\ \bullet\mathbf{h}_2:\Delta_6\vdash\bot,\Delta_5 \end{array} \\ \hline \bullet\mathbf{h}_1:\Delta_8\vdash(\top,\bot,\Delta_6),\mathbf{F}_7 \quad T_R \quad \frac{\mathbf{h}_5:\mathbf{F}_7,\Delta_8\vdash\top,\Delta_6}{\bullet\mathbf{h}_5:\Delta_8,\mathbf{F}_7\vdash\top,\bot,\Delta_6} \end{array} \xrightarrow{\mathbf{Cut}} \begin{array}{c} \bot_R \\ \mathbf{Cut} \\ \hline -:\Delta_8\vdash\top,\bot,\Delta_6 \\ \hline -:\Delta_8\vdash\bot,\top,\Delta_6 \end{array}$$

• Case rule \top_R

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1} : \Delta_6 \vdash (\top, \Delta_5), \top & \overline{\top_R} & \hline \bullet_{\mathbf{h}_4} : \Delta_6, \top \vdash \top, \Delta_5 \\ \hline -: \Delta_6 \vdash \top, \Delta_5 & \\ \hline -: \Delta_6 \vdash \top, \Delta_5 & \\ \hline \hline -: \Delta_6 \vdash \top, \Delta_5 & \\ \hline \hline \bullet_{\mathbf{h}_1} : \Delta_8 \vdash (\top, \Delta_6), \mathbf{F}_7 & \overline{\bullet_{\mathbf{h}_5} : \Delta_8, \mathbf{F}_7 \vdash \top, \Delta_6} \\ \hline -: \Delta_8 \vdash \top, \Delta_6 & \\ \hline -: \Delta_8 \vdash \top, \Delta_6 & \\ \hline \hline -: \Delta_8 \vdash \top, \Delta_6 & \\ \hline \hline \end{array} \right.$$

 \bullet Case rule K

$$\frac{ \underbrace{ \begin{array}{c} \mathbf{h}_4 : unbox(\Box \Gamma_7) \vdash F_5 \\ \bullet \mathbf{h}_4 : (\Box \Gamma_7, \Delta_8), \top \vdash \Delta_6, []F_5 \end{array}}_{\bullet \mathbf{h}_4 : unbox(\Box \Gamma_7) \vdash F_5} \underbrace{ \begin{array}{c} K \\ \mathsf{Cut} \\ \hline -: \Box \Gamma_7, \Delta_8 \vdash \Delta_6, []F_5 \\ \hline -: unbox(\Box \Gamma_7) \vdash F_5 \\ \hline -: \Delta_8, \Box \Gamma_7 \vdash \Delta_6, []F_5 \end{array}}_{\bullet K}$$

$$\frac{ \begin{array}{c} \bullet_{h_1}: \square\Gamma_9, \Delta_{10} \vdash (\top, \Delta_7, []F_6), \square F_8 \end{array}}{-:\square\Gamma_9, \Delta_{10} \vdash (\top, \Delta_7, []F_6), \square F_8} \xrightarrow{T_R} \begin{array}{c} \frac{h_5: unbox(\square\Gamma_9), unbox(\square F_8) \vdash F_6}{\bullet_{h_5}: (\square\Gamma_9, \Delta_{10}), \square F_8 \vdash \top, \Delta_7, []F_6} \\ & \xrightarrow{-:\square\Gamma_9, \Delta_{10} \vdash \top, \Delta_7, []F_6} \xrightarrow{T_R} \\ \hline \\ \bullet_{h_1}: \square F_8, \Delta_{10} \vdash (\top, \Delta_7, []F_6), F_9 \end{array}} \xrightarrow{T_R} \begin{array}{c} \frac{h_5: unbox(\square\Gamma_8) \vdash F_6}{\bullet_{h_5}: unbox(\square\Gamma_8) \vdash F_6} \\ \bullet_{h_5}: unbox(\square\Gamma_8) \vdash F_6 \end{array}} \xrightarrow{K}$$

• Case rule A45

$$\begin{array}{c} \underbrace{\begin{array}{c} \mathbf{h}_{4} : \square \Gamma_{8} + \square \Gamma_{6}, \mathbf{F}_{5} \\ \bullet \mathbf{h}_{4} : (\square \Gamma_{8}, \Delta_{9}), \top \vdash \square \Gamma_{6}, \Delta_{7}, || \mathbf{F}_{5} \\ \bullet \mathbf{h}_{4} : (\square \Gamma_{8}, \Delta_{9}), \top \vdash \square \Gamma_{6}, \Delta_{7}, || \mathbf{F}_{5} \\ \hline \\ - : \square \Gamma_{8}, \Delta_{9} \vdash \square \Gamma_{6}, \Delta_{7}, || \mathbf{F}_{5} \\ \hline \\ - : \square \Gamma_{8} \vdash \mathbf{F}_{5}, \square \Gamma_{6} \\ \hline \\ \bullet \mathbf{h}_{1} : \square \Gamma_{10}, \Delta_{11} \vdash (\top, \square \Gamma_{7}, \Delta_{8}, || \mathbf{F}_{6}), \square \Gamma_{9} \\ \hline \\ \bullet \mathbf{h}_{1} : \square \Gamma_{10}, \Delta_{11} \vdash (\top, \square \Gamma_{7}, \Delta_{8}, || \mathbf{F}_{6}), \square \Gamma_{9} \\ \hline \\ - : \square \Gamma_{10}, \Delta_{11} \vdash \top, \square \Gamma_{7}, \Delta_{8}, || \mathbf{F}_{6} \\ \hline \\ - : \Delta_{11}, \square \Gamma_{10} \vdash \top, \Delta_{8}, \square \Gamma_{7}, || \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \Delta_{8}, || \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{F}_{6} \\ \hline \\ \bullet \mathbf{h}_{5} : \square \Gamma_{9} \vdash \square \Gamma_{7}, \mathbf{h}_{8}, || \mathbf{h}_{7} \vdash \square \Gamma_{7}, || \mathbf{h}_{7} \vdash \square \Gamma_{7$$

• Case rule \rightarrow_L

$$\frac{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top} \xrightarrow{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \top \vdash \Delta_7} \underbrace{-: \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7}_{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5} \xrightarrow{\bullet h_1 : \Delta_8, F_6 \vdash \Delta_7} \underbrace{-: \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7}_{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5} \xrightarrow{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \underbrace{-: \Delta_8, F_6 \vdash \Delta_7}_{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \xrightarrow{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \xrightarrow{\bullet h_1 : \Delta_8, F_6 \vdash \Delta_7}_{-: \Delta_8, F_6 \vdash \Delta_7} \xrightarrow{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \rightarrow F_7} \underbrace{-: \Delta_9 \vdash \top, E_6, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \top, \Delta_8}_{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \xrightarrow{\bullet h_5 : \Delta_9, F_6 \rightarrow F_7 \vdash \top, \Delta_8}_{\bullet h_5 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \top, \Delta_8} \xrightarrow{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \top, \Delta_8}_{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \top, \Delta_8} \xrightarrow{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \top, \Delta_8}_{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \top, \Delta_8} \xrightarrow{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \top, \Delta_8}_{\bullet h_5 : (\Delta_{10}, F_6 \rightarrow F_7), F_9 \vdash \top, \Delta_8}$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\bullet h_1:\Delta_8,F_5\wedge F_6\vdash \Delta_7,\top}{\bullet h_1:\Delta_8,F_5\wedge F_6\vdash \Delta_7,\top} & \frac{h_4:\top,F_5,F_6,\Delta_8\vdash \Delta_7}{\bullet h_4:(\Delta_8,F_5\wedge F_6),\top\vdash \Delta_7} & \wedge_L \\ \hline & -:\Delta_8,F_5\wedge F_6\vdash \Delta_7 & \\ \hline \bullet h_1:\Delta_8,F_5,F_6\vdash \top,\Delta_7 & \top_R & \frac{\rightarrow}{h_4:\top,\Delta_8,F_5,F_6\vdash \Delta_7} & \text{ax/W} \\ \hline & -:\Delta_8,F_5,F_6\vdash \Delta_7 & \wedge_L & \\ \hline & \frac{-:\Delta_8,F_5,F_6\vdash \Delta_7}{-:\Delta_8,F_5\wedge F_6\vdash \Delta_7} & \wedge_L \\ \hline \hline \bullet h_1:\Delta_9\vdash (\top,\Delta_8),F_6\wedge F_7 & \top_R & \frac{h_5:F_6,F_7,\Delta_9\vdash \top,\Delta_8}{\bullet h_5:\Delta_9,F_6\wedge F_7\vdash \top,\Delta_8} & \wedge_L \\ \hline & -:\Delta_9\vdash \top,\Delta_8 & \\ \hline & \frac{\rightarrow}{-:\Delta_9\vdash \top,\Delta_8} & \top_R \\ \hline \hline \hline \bullet h_1:\Delta_{10},F_6\wedge F_7\vdash (\top,\Delta_8),F_9 & \top_R & \frac{h_5:F_6,F_7,F_9,\Delta_{10}\vdash \top,\Delta_8}{\bullet h_5:(\Delta_{10},F_6\wedge F_7),F_9\vdash \top,\Delta_8} & \wedge_L \\ \hline \hline & -:\Delta_{10},F_6\wedge F_7\vdash \top,\Delta_8 & \\ \hline \end{array}$$

• Case rule \vee_L

$$\frac{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \top}{-: \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \xrightarrow{T_R} \frac{h_4 : \top, F_5, \Delta_8 \vdash \Delta_7 \quad h_4 : \top, F_6, \Delta_8 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \top \vdash \Delta_7} \vee_L \\ \xrightarrow{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7} \xrightarrow{T_R} \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7}{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7} \xrightarrow{ax/W} \xrightarrow{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \xrightarrow{T_R} \frac{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7} \xrightarrow{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7} \vee_L \\ \xrightarrow{\bullet h_1 : \Delta_9, F_5 \vdash \Delta_7} \xrightarrow{T_R} \frac{h_5 : F_6, \Delta_9 \vdash \top, \Delta_8 \quad h_5 : F_7, \Delta_9 \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \top, \Delta_8} \vee_L \\ \xrightarrow{\bullet h_1 : \Delta_9 \vdash (\top, \Delta_8), F_6 \vee F_7} \xrightarrow{T_R} \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \top, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet h_5 : \Delta_9, F_6 \vee F_7 \vdash \top, \Delta_8} \vee_L \\ \xrightarrow{\bullet h_1 : \Delta_{10}, F_6 \vee F_7 \vdash (\top, \Delta_8), F_9} \xrightarrow{T_R} \frac{h_5 : F_6, F_9, \Delta_{10} \vdash \top, \Delta_8 \quad h_5 : F_7, F_9, \Delta_{10} \vdash \top, \Delta_8}{\bullet h_5 : \Delta_{10}, F_6 \vee F_7 \vdash \top, \Delta_8} \vee_L \\ \xrightarrow{\bullet h_1 : \Delta_{10}, F_6 \vee F_7 \vdash (\top, \Delta_8), F_9} \xrightarrow{T_R} \xrightarrow{\bullet h_5 : (\Delta_{10}, F_6 \vee F_7), F_9 \vdash \top, \Delta_8} Cut$$

\bullet Case rule AT

• Case rule \perp_L

$$\begin{array}{c|c} \bullet_{\mathbf{h}_1}: \bot, \Delta_6 \vdash \Delta_5, \top & \top_R & \hline \bullet_{\mathbf{h}_4}: (\bot, \Delta_6), \top \vdash \Delta_5 & \bot_L \\ \hline & -: \bot, \Delta_6 \vdash \Delta_5 & \to \\ \hline & -: \bot, \Delta_6 \vdash \Delta_5 & \bot_L \\ \hline \hline \bullet_{\mathbf{h}_1}: \Delta_7 \vdash (\top, \Delta_6), \bot & \overline{}^T_R & \hline \bullet_{\mathbf{h}_5}: \Delta_7, \bot \vdash \top, \Delta_6 & \bot_L \\ \hline & -: \Delta_7 \vdash \top, \Delta_6 & \\ \hline & -: \Delta_7 \vdash \top, \Delta_6 & \\ \hline & -: \Delta_7 \vdash \top, \Delta_6 & \\ \hline & -: \Delta_7 \vdash \top, \Delta_6 & \\ \hline & -: \bot, \Delta_8 \vdash \top, \Delta_6 & \\ \hline & -: \bot, \Delta_8 \vdash \top, \Delta_6 & \\ \hline & -: \bot, \Delta_8 \vdash \top, \Delta_6 & \\ \hline & -: \bot, \Delta_8 \vdash \top, \Delta_6 & \\ \hline & -: \bot, \Delta_8 \vdash \top, \Delta_6 & \\ \hline \end{array}$$

ullet Case rule I

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \Delta_7, \mathbf{p}_6 \vdash (\Delta_5, \mathbf{p}_6), \top} & T_R & \hline \bullet_{\mathbf{h}_4}: (\Delta_7, \mathbf{p}_6), \top \vdash \Delta_5, \mathbf{p}_6} & I \\ \hline & -: \Delta_7, \mathbf{p}_6 \vdash \Delta_5, \mathbf{p}_6 \\ \hline & -: \Delta_7, \mathbf{p}_6 \vdash \Delta_5, \mathbf{p}_6} & I \\ \hline \hline \bullet_{\mathbf{h}_1}: \Delta_8 \vdash (\top, \Delta_6, \mathbf{p}_7), \mathbf{p}_7 & \hline T_R & \hline \bullet_{\mathbf{h}_5}: \Delta_8, \mathbf{p}_7 \vdash \top, \Delta_6, \mathbf{p}_7} & I \\ \hline & -: \Delta_8 \vdash \top, \Delta_6, \mathbf{p}_7 \\ \hline & -: \Delta_8 \vdash \top, \Delta_6, \mathbf{p}_7 & \hline T_R \\ \hline \hline \bullet_{\mathbf{h}_1}: \Delta_9, \mathbf{p}_7 \vdash (\top, \Delta_6, \mathbf{p}_7), \mathbf{F}_8 & \hline T_R & \hline \bullet_{\mathbf{h}_5}: (\Delta_9, \mathbf{p}_7), \mathbf{F}_8 \vdash \top, \Delta_6, \mathbf{p}_7} & I \\ \hline \hline & -: \Delta_9, \mathbf{p}_7 \vdash \top, \Delta_6, \mathbf{p}_7 & \hline \\ \hline & -: \Delta_9, \mathbf{p}_7 \vdash \top, \Delta_6, \mathbf{p}_7 & \hline \\ \hline & -: \Delta_9, \mathbf{p}_7 \vdash \top, \Delta_6, \mathbf{p}_7 & \hline \end{array}$$

• Case rule \top_L

$$\begin{array}{c|c} \underline{\bullet \mathbf{h}_1 : \Delta_6 \vdash \Delta_5, \top} & \top_R & \underline{\bullet \mathbf{h}_4 : \Delta_6 \vdash \Delta_5} \\ & - : \Delta_6 \vdash \Delta_5 \\ \hline & - : \Delta_6 \vdash \Delta_5 \\ \hline & - : \Delta_6 \vdash \Delta_5 \\ \hline & - : \Delta_6 \vdash \Delta_5 \end{array} & \mathbf{ax/W} \\ \\ \underline{\bullet \mathbf{h}_1 : \Delta_7 \vdash (\top, \Delta_6), \top} & \top_R & \underline{\bullet \mathbf{h}_5 : \Delta_7 \vdash \top, \Delta_6} \\ \hline & - : \Delta_7 \vdash \top, \Delta_6 \\ \hline & - : \Delta_7 \vdash \top, \Delta_6 \\ \hline & - : \Delta_7 \vdash \top, \Delta_6 \\ \hline & - : \Delta_7 \vdash \top, \Delta_6 \end{array} & \top_R \\ \\ \underline{\bullet \mathbf{h}_1 : \top, \Delta_8 \vdash (\top, \Delta_6), \mathbf{f}_7} & \top_R & \underline{\bullet \mathbf{h}_5 : \mathbf{f}_7, \Delta_8 \vdash \top, \Delta_6} \\ \hline & - : \top, \Delta_8 \vdash \top, \Delta_6 \\ \hline & - : \top, \Delta_8 \vdash \top, \Delta_6 \end{array} & \top_L \\ \mathbf{Cut} \\ \underline{\bullet \mathbf{h}_1 : \top, \Delta_8 \vdash (\top, \Delta_6), \mathbf{f}_7} & \top_R & \underline{\bullet \mathbf{h}_5 : \mathbf{f}_7, \Delta_8 \vdash \top, \Delta_6} \\ \hline & - : \top, \Delta_8 \vdash \top, \Delta_6 \\ \hline & - : \top, \Delta_8 \vdash \top, \Delta_6 \end{array}$$

6.6 Status of K: OK

• Case rule \rightarrow_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, \mathbf{F}_8 \to \mathbf{F}_9), []\mathbf{F}_6 \end{array} K \begin{array}{c} \mathbf{h}_7: \Box\Gamma_{11}, \mathbf{F}_8, \Delta_{12}, []\mathbf{F}_6 \vdash \mathbf{F}_9, \Delta_{10} \\ \hline \bullet \mathbf{h}_7: (\Box\Gamma_{11}, \Delta_{12}), []\mathbf{F}_6 \vdash \Delta_{10}, \mathbf{F}_8 \to \mathbf{F}_9 \end{array} } \begin{array}{c} \rightarrow_R \\ \mathbf{cut} \\ \hline -: \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, \mathbf{F}_8 \to \mathbf{F}_9 \\ \hline \rightarrow \mathbf{h}_1: unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_6 \end{array} \\ \hline \bullet \mathbf{h}_1: unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_6 \end{array} K \begin{array}{c} \mathbf{ax}/\mathbb{W} \\ \hline \bullet \mathbf{h}_7: \Delta_{12}, \mathbf{F}_8, \Box\Gamma_{11}, []\mathbf{F}_6 \vdash \Delta_{10}, \mathbf{F}_9 \\ \hline -: \Delta_{12}, \mathbf{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}, \mathbf{F}_9 \\ \hline -: \Delta_{12}, \mathbf{\Gamma}_{11} \vdash \Delta_{10}, \mathbf{F}_8 \to \mathbf{F}_9 \end{array} } \rightarrow_R \end{array}$$

• Case rule \wedge_R

$$\frac{h_1: unbox(\Box\Gamma_{11}) \vdash F_6}{\bullet h_1: \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \land F_9), []F_6} \quad K \quad \frac{h_7: \Box\Gamma_{11}, \Delta_{12}, []F_6 \vdash F_8, \Delta_{10} \quad h_7: \Box\Gamma_{11}, \Delta_{12}, []F_6 \vdash F_9, \Delta_{10}}{\bullet h_7: (\Box\Gamma_{11}, \Delta_{12}), []F_6 \vdash \Delta_{10}, F_8 \land F_9} \quad Cut} \\ -: \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \land F_9} \\ \hline h_1: unbox(\Box\Gamma_{11}) \vdash F_6 \quad ax/W \\ \hline \bullet h_1: \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, []F_6 \quad K \quad h_7: \Delta_{12}, \Box\Gamma_{11}, []F_6 \vdash \Delta_{10}, F_8} \quad ax/W \\ \hline -: \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8 \land F_9} \\ \hline h_1: unbox(\Box\Gamma_{12}) \vdash F_7 \quad K \quad h_8: \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, \Delta_{11}, []F_7 \quad h_8: \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{11}, []F_7} \\ \hline \bullet h_1: \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \land F_{10}), []F_7), F_{13} \quad K \quad h_8: \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_9, \Delta_{11}, []F_7 \quad h_8: \Box\Gamma_{12}, F_{13}, \Delta_{14} \vdash F_{10}, \Delta_{11}, []F_7} \\ \hline -: \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \land F_{10}), []F_7 \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: unbox(\Box\Gamma_{12}) \vdash F_7 \quad ax/W \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \land F_{10}) \quad K \quad AxW \\ \hline -: \Delta_{14}, \Box\Gamma_{14}, \Box\Gamma_{14}, \Delta\Gamma_{14}, \Delta\Gamma_$$

• Case rule \vee_R

$$\begin{array}{c} \begin{array}{c} h_1: unbox(\Box\Gamma_{11}) \vdash F_6 \\ \hline \bullet h_1: \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \vee F_9), []F_6 \\ \hline \\ \bullet h_2: \Box\Gamma_{11}, \Delta_{12} \vdash (\Delta_{10}, F_8 \vee F_9), []F_6 \\ \hline \\ & -: \Box\Gamma_{11}, \Delta_{12} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline \\ \hline \\ \bullet h_1: unbox(\Box\Gamma_{11}) \vdash F_6 \\ \hline \\ \bullet h_1: \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, F_9, []F_6 \\ \hline \\ \hline \\ \bullet h_1: \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, F_9, []F_6 \\ \hline \\ \hline \\ \bullet h_1: \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, F_9, []F_6 \\ \hline \\ \hline \\ \bullet h_1: \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, F_9 \\ \hline \\ \hline \\ -: \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8, F_9 \\ \hline \\ -: \Delta_{12}, \Box\Gamma_{11} \vdash \Delta_{10}, F_8 \vee F_9 \\ \hline \\ \bullet h_1: Unbox(\Box\Gamma_{12}) \vdash F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14} \vdash ((\Delta_{11}, F_9 \vee F_{10}), []F_7), F_{13} \\ \hline \\ \bullet h_2: \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_{11}, F_9 \vee F_{10}), []F_7 \\ \hline \\ -: Unbox(\Box\Gamma_{12}) \vdash F_7 \\ \hline \\ \hline \\ -: unbox(\Box\Gamma_{12}) \vdash F_7 \\ \hline \\ -: \Delta_{14}, \Box\Gamma_{12} \vdash \Delta_{11}, []F_7, F_9 \vee F_{10} \\ \hline \\ \hline \end{array}$$

• Case rule \perp_R

$$\frac{ \begin{array}{c} \mathbf{h}_1 : unbox(\Box \Gamma_9) \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1 : \Box \Gamma_9, \Delta_{10} \vdash (\bot, \Delta_8), [] \mathbf{F}_6 \end{array} K \quad \begin{array}{c} \mathbf{h}_7 : \Box \Gamma_9, \Delta_{10}, [] \mathbf{F}_6 \vdash \Delta_8 \\ \bullet \mathbf{h}_7 : (\Box \Gamma_9, \Delta_{10}), [] \mathbf{F}_6 \vdash \bot, \Delta_8 \end{array} }{ \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Box \Gamma_9, \Delta_{10}, \Box \Gamma_9, \Delta_{10}, [] \mathbf{F}_6 \vdash \bot, \Delta_8 \end{array} } \quad \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Box \Gamma_9, \Delta_{10}, \Box \Gamma_9, [] \mathbf{F}_6 \vdash \bot, \Delta_8 \end{array} } \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Box \Gamma_9, [] \mathbf{F}_6 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Box \Gamma_9, [] \mathbf{F}_6 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \end{array} \quad \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10}, \Delta_8 \vdash \bot, \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10} \vdash \Delta_8 \vdash \bot, \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10} \vdash \Delta_8 \vdash \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10} \vdash \Delta_8 \vdash \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10} \vdash \Delta_8 \vdash \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10} \vdash \Delta_8 \vdash \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10} \vdash \Delta_8 \vdash \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10} \vdash \Delta_8 \vdash \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10} \vdash \Delta_8 \vdash \Delta_8 \\ \bullet \mathbf{h}_7 : \Delta_{10} \vdash \Delta_8$$

$$\begin{array}{c} \mathbf{h}_1: unbox(\Box \Gamma_{10}) \vdash \mathbf{F}_7 \\ \underline{\bullet \mathbf{h}_1: \Box \Gamma_{10}, \Delta_{12} \vdash ((\bot, \Delta_9), [] \mathbf{F}_7), \mathbf{F}_{11}} \quad K \quad \frac{\mathbf{h}_8: \Box \Gamma_{10}, \mathbf{F}_{11}, \Delta_{12} \vdash \Delta_9, [] \mathbf{F}_7}{\bullet \mathbf{h}_8: (\Box \Gamma_{10}, \Delta_{12}), \mathbf{F}_{11} \vdash (\bot, \Delta_9), [] \mathbf{F}_7} \quad \underline{Cut} \\ -: \Box \Gamma_{10}, \Delta_{12} \vdash (\bot, \Delta_9), [] \mathbf{F}_7 \\ \underline{-: unbox(\Box \Gamma_{10}) \vdash \mathbf{F}_7} \quad ax/\mathsf{W} \\ \overline{-: \Delta_{12}, \Box \Gamma_{10} \vdash \bot, \Delta_9, [] \mathbf{F}_7} \quad K \end{array}$$

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1: unbox(\Box \Gamma_9) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: \Box \Gamma_9, \Delta_{10} \vdash (\top, \Delta_8), [] \mathbf{F}_6} \quad K \quad & \frac{\bullet \mathbf{h}_7: (\Box \Gamma_9, \Delta_{10}), [] \mathbf{F}_6 \vdash \top, \Delta_8}{\bullet \mathbf{h}_7: (\Box \Gamma_9, \Delta_{10}), [] \mathbf{F}_6 \vdash \top, \Delta_8} \quad \mathbf{Cut} \\ & -: \Box \Gamma_9, \Delta_{10} \vdash \top, \Delta_8 \\ & -: \Delta_{10}, \Box \Gamma_9 \vdash \top, \Delta_8 \\ \hline & \mathbf{h}_1: unbox(\Box \Gamma_{10}) \vdash \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{10}, \Delta_{12} \vdash ((\top, \Delta_9), [] \mathbf{F}_7), \mathbf{F}_{11} \quad K \quad & \bullet \mathbf{h}_8: (\Box \Gamma_{10}, \Delta_{12}), \mathbf{F}_{11} \vdash (\top, \Delta_9), [] \mathbf{F}_7 \\ \hline & -: \Box \Gamma_{10}, \Delta_{12} \vdash (\top, \Delta_9), [] \mathbf{F}_7 \\ \hline & -: \Delta_{12}, \Box \Gamma_{10} \vdash \top, \Delta_9, [] \mathbf{F}_7 \end{array} \quad \mathbf{T}_R \end{array}$$

 \bullet Case rule K

$$\frac{\mathbf{h}_{1}: unbox(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash F_{6}}{\bullet \mathbf{h}_{1}: (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_{0}, [F_{8}), [F_{6}]} K \underbrace{\bullet_{17}: ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), [F_{6} \vdash \Delta_{9}, [F_{8}]}_{\bullet \mathbf{h}_{7}: ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), [F_{6} \vdash \Delta_{9}, [F_{8}]}_{\bullet \mathbf{h}_{7}: ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), [F_{6} \vdash \Delta_{9}, [F_{8}]}_{\bullet \mathbf{h}_{7}: ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), [F_{6} \vdash \Delta_{9}, [F_{8}]}_{\bullet \mathbf{h}_{1}: \mathbf{unbox}(\Box\Gamma_{11}), \mathbf{unbox}(\Box\Gamma_{12}) \vdash F_{8}} K \underbrace{\bullet_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Delta_{9}, [F_{8}]}_{-: \mathbf{unbox}(\Box\Gamma_{10}), \mathbf{unbox}(\Box\Gamma_{11}), \mathbf{unbox}(\Box\Gamma_{11}), \mathbf{unbox}(\Box\Gamma_{11}), \mathbf{unbox}(\Box\Gamma_{11}), \mathbf{unbox}(\Box\Gamma_{11}) \vdash F_{8}}_{\bullet \mathbf{h}_{1}: \mathbf{unbox}(\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_{9}, [F_{8}], [F_{6}], K \underbrace{\bullet_{17}: ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, \Delta_{9}, [F_{8}], K \underbrace{\bullet_{17}: ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, \Delta_{15}, (\Box\Gamma_{11}, \Delta_{13}), [F_{6} \vdash \Delta_{9}, [F_{8}], K \underbrace{\bullet_{17}: ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, \Delta_{9}, [F_{8}], K \underbrace{\bullet_{17}: ((\Box\Gamma_{10}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{13}, \Box\Gamma_{12}, \Delta_{14}, \Box\Gamma_{13}, \Delta_{15}, (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}, (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Delta_{14}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Delta_{14}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Delta_{14}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Delta_{14}, \Delta_{16}, [F_{9}], F_{7}, K \underbrace{\bullet_{17}: (\Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Delta_{15}, \Gamma_{14}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Delta_{15}, \Gamma_{14}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Delta_{15}, \Gamma_{14}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{12}, \Box\Gamma_{1$$

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\frac{\mathbf{h}_{1}: unbox(\Box\Gamma_{10}, \Box\Gamma_{12}) \vdash \mathbf{F}_{8}}{\underbrace{\bullet \mathbf{h}_{1}: (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_{9}, []\mathbf{F}_{8}), \mathbf{F}_{13}}_{\bullet \mathbf{h}_{7}: ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), \mathbf{F}_{13} \vdash \Delta_{9}, []\mathbf{F}_{8}} \underbrace{K}_{\bullet \mathbf{h}_{7}: ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{14}), \mathbf{F}_{13} \vdash \Delta_{9}, []\mathbf{F}_{8}}_{-: (\Box\Gamma_{10}, \Box\Gamma_{12}), unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_{8}} \underbrace{K}_{\bullet \mathbf{h}_{7}: unbox(\Box\Gamma_{10}), unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_{8}}_{K} \underbrace{Ax/W}_{K}
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\bullet Case rule A45

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\frac{\mathbf{h}_1: unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathbf{F}_6}{\bullet \mathbf{h}_1: (\Box\Gamma_{11}, \Box\Gamma_{12}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Box\Gamma_{9}, \Delta_{10}, []\mathbf{F}_8), []\mathbf{F}_6} \quad K \quad \frac{\mathbf{h}_7: \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathbf{F}_6 \vdash \Box\Gamma_{9}, \mathbf{F}_8}{\bullet \mathbf{h}_7: ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), []\mathbf{F}_6 \vdash \Box\Gamma_{9}, \Delta_{10}, []\mathbf{F}_8} \quad \underbrace{A45}_{C...}
                                                                                                                                                                                                                                                                              -: (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Box\Gamma_{9}, \Delta_{10}, []F_{8}
                                                                                                                         \begin{array}{c|c} \hline \mathbf{h}_1: unbox(\Box \Gamma_{11}), unbox(\Box \Gamma_{13}) \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \Box \Gamma_{11}, \Box \Gamma_{12}, \Box \Gamma_{13} \vdash \mathbf{F}_8, \Box \Gamma_9, []\mathbf{F}_6 \\ \end{array} \begin{array}{c|c} \mathbf{ax/W} \\ \hline \mathbf{h}_7: \Box \Gamma_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, []\mathbf{F}_6 \vdash \mathbf{F}_8, \Box \Gamma_9 \\ \bullet \mathbf{h}_7: \Box \Gamma_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, []\mathbf{F}_6 \vdash \mathbf{F}_8, \Box \Gamma_9 \\ \bullet \mathbf{h}_7: \Box \mathbf{h}_{11}, \Box \mathbf{h}_{12}, \Box \mathbf{h}_{13}, []\mathbf{h}_7 \vdash \mathbf{h}_{13}, []\mathbf{h}_7 \vdash \mathbf{h}_8, \Box \mathbf{h}_9 \\ \bullet \mathbf{h}_7: \Box \mathbf{h}_{11}, \Box \mathbf{h}_{12}, \Box \mathbf{h}_{13}, []\mathbf{h}_7 \vdash \mathbf{h}_8, \Box \mathbf{h}_9 \\ \bullet \mathbf{h}_7: \Box \mathbf{h}_{11}, \Box \mathbf{h}_{12}, \Box \mathbf{h}_{13}, []\mathbf{h}_7 \vdash \mathbf{h}_8, \Box \mathbf{h}_9 \\ \bullet \mathbf{h}_7: \Box \mathbf{h}_{11}, \Box \mathbf{h}_{12}, \Box \mathbf{h}_{13}, []\mathbf{h}_7 \vdash \mathbf{h}_8, \Box \mathbf{h}_9 \\ \bullet \mathbf{h}_7: \Box \mathbf{h}_{11}, \Box \mathbf{h}_{12}, \Box \mathbf{h}_{13}, []\mathbf{h}_7 \vdash \mathbf{h}_8, \Box \mathbf{h}_9 \\ \bullet \mathbf{h}_7: \Box \mathbf{h}_{11}, \Box \mathbf{h}_{12}, \Box \mathbf{h}_{13}, []\mathbf{h}_7 \vdash \mathbf{h}_8, \Box \mathbf{h}_9 \\ \bullet \mathbf{h}_7: \Box \mathbf{h}_{11}, \Box \mathbf{h}_{12}, \Box \mathbf{h}_{13}, []\mathbf{h}_7 \vdash \mathbf{h}_8, \Box \mathbf{h}_9 \\ \bullet \mathbf{h}_7: \Box \mathbf{h}_{11}, \Box \mathbf{h}_{12}, \Box \mathbf{h}_{13}, []\mathbf{h}_7 \vdash \mathbf{h}_8, \Box \mathbf{h}_9 \\ \bullet \mathbf{h}_7: \Box \mathbf{h}_7
                                                                                                                                                                                                                                                                                                                                      -: \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash F_8, \Box\Gamma_9
                                                                                                                                                                                                                                                                                    \frac{}{-:\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13}\vdash\Delta_{10},\Box\Gamma_{9},[]\mathsf{F}_{8}}\ A45
                                                                                                     h_1: unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash F_6
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \mathtt{h}_7: \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Box\Gamma_{9}, \mathtt{f}_{8}
   \frac{\mathsf{h}_1: unbox(\sqcup 1:1, \sqcup 1:3) \vdash \mathsf{F}_6}{\bullet \mathsf{h}_1: (\square \Gamma_{11}, \square \Gamma_{13}), \square \Gamma_{12}, \Delta_{14} \vdash (\square \Gamma_{9}, \Delta_{10}, \llbracket \mathsf{F}_8 \rangle, \llbracket \mathsf{F}_6} \quad K \quad \frac{\mathsf{n}_7: \sqcup 1:1, \sqcup 1:2 \vdash \sqcup 1:9, \mathsf{F}_8}{\bullet \mathsf{h}_7: ((\square \Gamma_{11}, \square \Gamma_{13}), \square \Gamma_{12}, \Delta_{14}), \llbracket \mathsf{F}_6 \vdash \square \Gamma_{9}, \Delta_{10}, \llbracket \mathsf{F}_8 \rangle} \quad A45 \quad Correction (1)
                                                                                                                                                                                                                                                                              -: (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Box\Gamma_{9}, \Delta_{10}, []F_{8}
                                                                                                                                                                                                                                                                                    \frac{\longrightarrow}{-:\Box\Gamma_{11},\Box\Gamma_{12}\vdash \mathsf{F}_8,\Box\Gamma_9} \mathsf{ax/W} \\ \frac{}{-:\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13}\vdash \Delta_{10},\Box\Gamma_9,[]\mathsf{F}_8} \ A45
                                                                                                                                    \mathtt{h}_1: unbox(\Box\Gamma_{12},\Box\Gamma_{15}) \vdash \mathtt{F}_7
 \frac{\mathbf{h}_{1}: unbox(\Box\Gamma_{12}, \Box\Gamma_{15}) \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{1}: (\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash ((\Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7}), \Box\mathbf{F}_{13}} \quad K \quad \frac{\mathbf{h}_{8}: \Box\Gamma_{12}, \Box\Gamma_{14}, \Delta_{16}, \Box\Gamma_{14}, \Delta_{16}), \Box\Gamma_{14}, \Delta_{16}, \Box\Gamma_{17}, \Box
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \mathtt{h}_8: \Box\Gamma_{12}, \Box\Gamma_{14}, \Box\mathtt{F}_{13} \vdash \Box\Gamma_{10}, \mathtt{F}_9, []\mathtt{F}_7
                                                                                                                                                                                                                                                                                                                                        \frac{\rightarrow}{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{15}) \vdash F_7} \text{ ax/W} \\ \frac{-: \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_7, []F_9}{} K
                                                                                                                                      h_1: unbox(\Box\Gamma_{12}, \Box\Gamma_{15}) \vdash F_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \mathtt{h}_8: \Box\Gamma_{12}, \Box\Gamma_{14}, \Box\mathtt{F}_{13} \vdash \Box\Gamma_{10}, \mathtt{F}_9
\frac{ \begin{array}{c} \mathbf{n}_{1} : unoox(\square 1_{12}, \square 1_{15}) \sqcap \mathbf{r}_{7} \\ \bullet \mathbf{h}_{1} : (\square \Gamma_{12}, \square \Gamma_{15}), \square \Gamma_{14}, \Delta_{16} \vdash ((\square \Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7}), \square \mathbf{r}_{13} \end{array} K}{ \begin{array}{c} \mathbf{n}_{8} : \square 1_{2}, \square 1_{4}, \square \mathbf{r}_{13} \vdash \square 1_{10}, \mathbf{r}_{9} \\ \bullet \mathbf{h}_{8} : ((\square \Gamma_{12}, \square \Gamma_{15}), \square \Gamma_{14}, \Delta_{16}), \square \mathbf{r}_{14}, \Delta_{16}), \square \mathbf{r}_{13} \vdash (\square \Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7} \end{array}} \underbrace{ \begin{array}{c} A45 \\ \text{Cut} \end{array} }
                                                                                                                                                                                                                                                                                                                                           \frac{\overbrace{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{15}) \vdash F_7}^{} \text{ ax/W}}{-: \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{11}, \Box\Gamma_{10}, []F_7, []F_9} \ K
                                                                                                               \mathtt{h}_1: unbox(\Box\Gamma_{11},\Box\Gamma_{14}) \vdash \mathtt{F}_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             \mathtt{h}_7: \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\mathtt{F}_{12} \vdash \Box\Gamma_{9}, \mathtt{F}_{8}
  \begin{array}{c} \mathbf{n}_1: unoox(\square 11, \square 14) \vdash \mathsf{F8} \\ \bullet \mathbf{h}_1: (\square \Gamma_{11}, \square \Gamma_{14}), \square \Gamma_{13}, \Delta_{15} \vdash ((\square \Gamma_{9}, \Delta_{10}), []\mathsf{F_8}), \square \mathsf{F}_{12} \\ \hline \bullet \mathbf{h}_7: ((\square \Gamma_{11}, \square \Gamma_{14}), \square \Gamma_{13}, \Delta_{15}), \square \mathsf{F}_{12} \vdash (\square \Gamma_{9}, \Delta_{10}), []\mathsf{F_8} \\ \hline \bullet \mathbf{h}_7: ((\square \Gamma_{11}, \square \Gamma_{14}), \square \Gamma_{13}, \Delta_{15}), \square \mathsf{F}_{12} \vdash (\square \Gamma_{9}, \Delta_{10}), []\mathsf{F_8} \\ \hline \mathsf{Cut} \\ \end{array} 
                                                                                                                                                                                                                                                                                               -: (\Box\Gamma_{11}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{9}, \Delta_{10}), []\mathtt{F}_{8}
                                                                                                                                                                                                                                                                                                                     \frac{-: unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F_8}}{-: \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{10}, \Box\Gamma_{9}, []\mathtt{F_8}} \ K
\frac{\mathbf{h}_{1}: unbox(\Box\Gamma_{12}, \Box\Gamma_{14}) \vdash \mathbf{F}_{7}}{\underbrace{\bullet \mathbf{h}_{1}: (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash ((\Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7}), \mathbf{F}_{15}}_{\bullet \mathbf{h}_{8}: ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16}), \mathbf{F}_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7}}} \underbrace{A45}_{\bullet \mathbf{h}_{8}: ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16}), \mathbf{F}_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7}}_{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{14}) \vdash \mathbf{F}_{7}} \underbrace{\bullet \mathbf{x}/\mathbb{W}}_{-: \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathbf{F}_{7}, []\mathbf{F}_{9}}_{K}
                                                                                                                           h_1: unbox(\Box\Gamma_{12},\Box\Gamma_{14}) \vdash F_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      h_8:\Box\Gamma_{12},\Box\Gamma_{13}\vdash\Box\Gamma_{10},F_9
\frac{ \begin{array}{c} \mathbf{h}_{1} : unbox(\sqcup\Gamma_{12}, \sqcup\Gamma_{14}) \vdash \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{1} : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash ((\Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7}), \mathbf{F}_{15} \end{array} K \begin{array}{c} \mathbf{h}_{8} : \sqcup\Gamma_{12}, \sqcup\Gamma_{13} \vdash \sqcup\Gamma_{10}, \mathbf{F}_{9} \\ \hline \bullet \mathbf{h}_{8} : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16}), \mathbf{F}_{15} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7} \end{array} A45} \\ - : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash (\Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}), []\mathbf{F}_{7} \end{array} Cut
                                                                                                                                                                                                                                                                                                                                                                            \frac{}{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{14}) \vdash F_7} \text{ ax/W}
                                                                                                                                                                                                                                                                                                                            \frac{}{-:\Delta_{16},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_{11},\Box\Gamma_{10},[]\mathsf{F}_{7},[]\mathsf{F}_{9}}\ K
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$$\frac{\mathbf{h}_{1}: unbox(\Box\Gamma_{11}, \Box\Gamma_{13}) \vdash \mathbf{F}_{8}}{\bullet \mathbf{h}_{1}: (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash ((\Box\Gamma_{9}, \Delta_{10}), []\mathbf{F}_{8}), \mathbf{F}_{14}} K \xrightarrow{\bullet \mathbf{h}_{7}: ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15}), \mathbf{F}_{14} \vdash (\Box\Gamma_{9}, \Delta_{10}), []\mathbf{F}_{8}} \\ -: (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{15} \vdash (\Box\Gamma_{9}, \Delta_{10}), []\mathbf{F}_{8} \xrightarrow{\bullet} \\ -: unbox(\Box\Gamma_{11}), unbox(\Box\Gamma_{13}) \vdash \mathbf{F}_{8}} \xrightarrow{\mathbf{ax/W}} \\ -: \Delta_{15}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_{10}, \Box\Gamma_{9}, []\mathbf{F}_{8}} K$$

• Case rule \rightarrow_L

$$\frac{h_1: unbox(\Box \Gamma_{11}) \vdash F_6}{\bullet h_1: \Box \Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10}, []F_6} K \xrightarrow{h_7: \Box \Gamma_{11}, \Delta_{12}, []F_6 \vdash F_8, \Delta_{10} \quad h_7: \Box \Gamma_{11}, F_9, \Delta_{12}, []F_6 \vdash \Delta_{10}}{\bullet h_7: (\Box \Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9), []F_6 \vdash \Delta_{10}} Cut \\ -: \Box \Gamma_{11}, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{10} \\ \hline h_1: unbox(\Box \Gamma_{11}) \vdash F_6 & ax/W \\ \bullet h_1: \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8, []F_6 K \\ \hline -: \Delta_{12}, \Box \Gamma_{11}, []F_6 \vdash \Delta_{10}, F_8 \\ \hline -: \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8 \\ \hline -: \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, F_8 \\ \hline -: \Delta_{12}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10} \\ \hline -: \Delta_{12}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10} \\ \hline -: \Delta_{12}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10} \\ \hline -: \Delta_{12}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{10} \\ \hline -: \Delta_{12}, \Box \Gamma_{11}, F_8 \rightarrow F_9 \vdash \Delta_{11}, []F_7 \quad h_8: \Box \Gamma_{12}, F_9, \Box \Gamma_{11}, []F_7 \rightarrow L \\ \hline -: \Box \Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []F_7), F_9 \rightarrow F_{10} \\ \hline -: \Box \Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, []F_7 \quad h_8: \Box \Gamma_{12}, F_{10}, \Delta_{13} \vdash \Delta_{11}, []F_7 \rightarrow L \\ \hline -: unbox(\Box \Gamma_{12}) \vdash F_7 \quad ax/W \\ \hline -: \lambda_{13}, \Box \Gamma_{12} \vdash \Delta_{11}, []F_7 \quad h_8: \Box \Gamma_{12}, F_{10}, F_{13}, \Delta_{14} \vdash \Delta_{11}, []F_7 \rightarrow L \\ \hline -: \Box \Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \vdash (\Delta_{11}, []F_7), F_{13} \quad K \quad h_8: \Box \Gamma_{12}, A_{13}, F_9 \rightarrow F_{10}, F_{13}, \Delta_{14} \vdash A_{11}, []F_7 \rightarrow L \\ \hline -: \Box \Gamma_{12}, \Delta_{14}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, []F_7 \rightarrow A_8 \land A_{11}, []F_7 \rightarrow A_8 \rightarrow A_{11}, []F_7 \rightarrow A_8 \rightarrow A_{11}, []F_7 \rightarrow A_8 \rightarrow A_{11}, []F_7 \rightarrow A_1 \rightarrow A_$$

• Case rule \wedge_L

$$\begin{array}{c} h_1: unbox(\Box\Gamma_{11}) \vdash F_6 \\ \hline \bullet h_1: \Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}, []F_6 \\ \hline \\ \bullet h_1: \Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10}, []F_6 \\ \hline \\ \hline \\ -: \Box\Gamma_{11}, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{10} \\ \hline \\ \hline \\ \hline \\ \bullet h_1: unbox(\Box\Gamma_{11}) \vdash F_6 \\ \hline \\ \bullet h_1: \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, []F_6 \\ \hline \\ \hline \\ \bullet h_1: \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, []F_6 \\ \hline \\ \hline \\ \bullet h_1: \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10}, []F_6 \\ \hline \\ \bullet h_1: \Delta_{12}, F_8, F_9, \Box\Gamma_{11} \vdash \Delta_{10} \\ \hline \\ \hline \\ -: \Delta_{12}, \Gamma_{11}, F_8 \wedge F_9 \vdash \Delta_{10} \\ \hline \\ \hline \\ \bullet h_1: unbox(\Box\Gamma_{12}) \vdash F_7 \\ \hline \\ \bullet h_1: unbox(\Box\Gamma_{12}) \vdash F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{13} \vdash (\Delta_{11}, []F_7), F_9 \wedge F_{10} \\ \hline \\ \hline \\ -: Unbox(\Box\Gamma_{12}) \vdash F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash (\Delta_{11}, []F_7), F_{13} \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10} \vdash \Delta_{11}, []F_7 \\ \hline \\ \bullet h_1: \Box\Gamma_{12}, \Delta_{14}, F_9 \wedge F_{10$$

• Case rule \vee_L

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\frac{\mathbf{h}_{1}: unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}: \underline{\Box\Gamma_{11}, \Delta_{12}, \mathbf{F}_{8} \vee \mathbf{F}_{9} \vdash \Delta_{10}, []\mathbf{F}_{6}}} \ K \quad \frac{\mathbf{h}_{7}: \Box\Gamma_{11}, \mathbf{F}_{8}, \Delta_{12}, []\mathbf{F}_{6} \vdash \Delta_{10} \quad \mathbf{h}_{7}: \Box\Gamma_{11}, \mathbf{F}_{9}, \Delta_{12}, []\mathbf{F}_{6} \vdash \Delta_{10}}{\bullet \mathbf{h}_{7}: (\Box\Gamma_{11}, \Delta_{12}, \mathbf{F}_{8} \vee \mathbf{F}_{9}), []\mathbf{F}_{6} \vdash \Delta_{10}} \ \mathbf{Cut}} \quad \forall_{L} \in \mathbb{R}^{n}
                                                                                                                                                                                                                                                                                                                                                                                                    -:\Box\Gamma_{11},\Delta_{12}, F_8\vee F_9\vdash\Delta_{10}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  h_1: unbox(\Box\Gamma_{11}) \vdash F_6 ax/W
                            \boxed{ \frac{}{\mathbf{h}_1: unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_6} \quad \text{ax/W} }
 \begin{array}{c} \underline{\mathbf{n}_1 : unoox}(\Box \mathbf{1}_{11}) \vdash \mathbf{F}_6 \\ \underline{\bullet}\mathbf{h}_1 : \Delta_{12}, \mathbf{F}_8, \Box \Gamma_{11} \vdash \Delta_{10}, []\mathbf{F}_6 \\ \underline{K} \\ \underline{\mathbf{n}_7 : \Delta_{12}, \mathbf{F}_8, \Box \Gamma_{11}, []\mathbf{F}_6 \vdash \Delta_{10} \\ \underline{\mathsf{hCut}} \\ \end{array} \\ \underline{\mathbf{n}_1 : unoox}(\Box \mathbf{1}_{11}) \vdash \mathbf{F}_6 \\ \underline{\bullet}\mathbf{h}_1 : \Delta_{12}, \mathbf{F}_9, \Box \Gamma_{11} \vdash \Delta_{10}, []\mathbf{F}_6 \\ \underline{\bullet}\mathbf{h}_7 : \Delta_{12}, \mathbf{F}_9, \Box \Gamma_{11}, []\mathbf{F}_6 \vdash \Delta_{10} \\ \underline{\bullet}\mathbf{h}_7 : \Delta_{12}, \mathbf{F}_9, \Box \Gamma_{11}, []\mathbf{F}_6 \vdash \Delta_{10} \\ \underline{\bullet}\mathbf{h}_7 : \Delta_{12}, \mathbf{F}_9, \Box \Gamma_{11}, []\mathbf{F}_6 \vdash \Delta_{10} \\ \underline{\bullet}\mathbf{h}_7 : \Delta_{12}, \mathbf{F}_9, \Box \Gamma_{11}, []\mathbf{F}_6 \vdash \Delta_{10} \\ \underline{\bullet}\mathbf{h}_7 : \Delta_{12}, \mathbf{F}_9, \Box \Gamma_{11}, []\mathbf{F}_6 \vdash \Delta_{10} \\ \underline{\bullet}\mathbf{h}_7 : \Delta_{12}, \mathbf{F}_9, \Box \Gamma_{11}, []\mathbf{F}_6 \vdash \Delta_{10} \\ \underline{\bullet}\mathbf{h}_7 : \Delta_{12}, \mathbf{F}_9, \Box \Gamma_{11}, []\mathbf{F}_6 \vdash \Delta_{10} \\ \underline{\bullet}\mathbf{h}_7 : \Delta_{12}, \mathbf{F}_9, \Box \Gamma_{11}, []\mathbf{h}_7 \\ \underline{\bullet}\mathbf{h}_7 : \Delta_{12}, \mathbf{h}_7 \\ \underline{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               -: \Delta_{12}, \mathsf{F}_9, \Box \Gamma_{11} \vdash \Delta_{10} \\ --: \Delta_{12}, \mathsf{F}_9, \Box \Gamma_{11} \vdash \Delta_{10}
                                                                                                                                                                           -:\Delta_{12}, \mathtt{F}_8, \Box\Gamma_{11} \vdash \Delta_{10}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -:\Delta_{12},\Box\Gamma_{11},\mathsf{F}_8\vee\mathsf{F}_9\vdash\Delta_{10}
                                                                                                                                                                                                                                                                                           \frac{1}{(F_{10})} K \frac{\mathbf{h}_8 : \Box \Gamma_{12}, \mathbf{F}_9, \Delta_{13} \vdash \Delta_{11}, []\mathbf{F}_7 \quad \mathbf{h}_8 : \Box \Gamma_{12}, \mathbf{F}_{10}, \Delta_{13} \vdash \Delta_{11}, []\mathbf{F}_7}{\bullet \mathbf{h}_8 : (\Box \Gamma_{12}, \Delta_{13}), \mathbf{F}_9 \vee \mathbf{F}_{10} \vdash \Delta_{11}, []\mathbf{F}_7} \text{ Cut}} \vee_L
                                                              \mathtt{h}_1: unbox(\Box \Gamma_{12}) \vdash \mathtt{F}_7
 \underline{\bullet} h_1: \Box \Gamma_{12}, \underline{\Delta}_{13} \vdash (\underline{\Delta}_{11}, \overline{[]F_7), F_9 \vee F_{10}}
                                                                                                                                                                                                                                                                                         -: \Box \Gamma_{12}, \Delta_{13} \vdash \Delta_{11}, [] \mathtt{F}_7
                                                                                                                                                                                                                                                                                                                                                        \frac{}{-:unbox(\Box\Gamma_{12})\vdash F_7} \text{ ax/W}
                                                                                                                                                                                                                                                                                                                                                 -: \Delta_{13}, \square \Gamma_{12} \vdash \Delta_{11}, []F_7 \quad K
 \frac{\mathbf{h}_1 : unbox(\Box \Gamma_{12}) \vdash \mathbf{F}_7}{\underbrace{\bullet \mathbf{h}_1 : \Box \Gamma_{12}, \Delta_{14}, \mathbf{F}_9 \vee \mathbf{F}_{10} \vdash (\Delta_{11}, []\mathbf{F}_7), \mathbf{F}_{13}}_{} \quad K \quad \frac{\mathbf{h}_8 : \Box \Gamma_{12}, \mathbf{F}_9, \mathbf{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathbf{F}_7 \quad \mathbf{h}_8 : \Box \Gamma_{12}, \mathbf{F}_{10}, \mathbf{F}_{13}, \Delta_{14} \vdash \Delta_{11}, []\mathbf{F}_7}_{} \quad \mathbf{h}_8 : \Box \Gamma_{12}, \Delta_{14}, \mathbf{F}_9 \vee \mathbf{F}_{10}), \mathbf{F}_{13} \vdash \Delta_{11}, []\mathbf{F}_7}_{} \quad \mathbf{Cut}}
                                                                                                                                                                                                                                                                                                 -: \Box \Gamma_{12}, \Delta_{14}, \mathtt{F}_{9} \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_{7}
                                                                                                                                                                                                                                                                                                                                                             \frac{ \overline{-: unbox(\Box\Gamma_{12}) \vdash \mathtt{F_7}} \ ^{\mathtt{ax/W}} }{ -: \Delta_{14}, \Box\Gamma_{12}, \mathtt{F_9} \vee \mathtt{F_{10}} \vdash \Delta_{11}, []\mathtt{F_7}} \ K
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\bullet Case rule AT

$$\begin{array}{c} \frac{h_1: unbox(\Box \Gamma_{10}, []F_8) \vdash F_6}{\bullet h_1: (\Box \Gamma_{10}, []F_8), \Delta_{11} \vdash \Delta_9, []F_8} K & \frac{h_7: \Box \Gamma_{10}, F_8, \Delta_{11}, []F_6, []F_8 \vdash \Delta_9}{\bullet h_7: ((\Box \Gamma_{10}, []F_8), \Delta_{11}), []F_6 \vdash \Delta_9} & cut \\ & -: (\Box \Gamma_{10}, []F_8), \Delta_{11} \vdash \Delta_9 & ax/W \\ \hline \bullet h_1: \Delta_{11}, F_8, \Box \Gamma_{10}, []F_8 \vdash \Delta_9, AT \\ \hline -: \Delta_{11}, F_8, \Box \Gamma_{10}, []F_8 \vdash \Delta_9 & AT \\ \hline \frac{-: \Delta_{11}, F_8, \Box \Gamma_{10}, []F_8 \vdash \Delta_9}{-: \Delta_{11}, \Box \Gamma_{10}, []F_8 \vdash \Delta_9} & AT \\ \hline \bullet h_1: unbox(\Box \Gamma_{10}) \vdash F_6 & K & h_7: \Box \Gamma_{10}, F_8, \Delta_{11}, []F_6, []F_8 \vdash \Delta_9 \\ \hline \bullet h_1: \Box \Gamma_{10}, \Delta_{11}, []F_8 \vdash \Delta_9, []F_6 & M_9 \\ \hline \bullet h_1: \Delta_{11}, F_8, \Box \Gamma_{10}, []F_8 \vdash \Delta_9, []F_6 & M_9 \\ \hline \bullet h_1: \Delta_{11}, F_8, \Box \Gamma_{10}, []F_8 \vdash \Delta_9, []F_6 & M_9 \\ \hline \bullet h_1: \Delta_{11}, F_8, \Box \Gamma_{10}, []F_8 \vdash \Delta_9, []F_6 & M_9 \\ \hline \bullet h_1: \Delta_{11}, F_8, \Box \Gamma_{10}, []F_8 \vdash \Delta_9, []F_6 & M_9 \\ \hline \bullet h_1: \Delta_{11}, F_8, \Box \Gamma_{10}, []F_8 \vdash \Delta_9, []F_6 & M_9 \\ \hline \bullet h_1: \Delta_{11}, F_8, \Box \Gamma_{10}, []F_8 \vdash \Delta_9 & AT \\ \hline -: \Delta_{11}, D[T_{10}, []F_8 \vdash \Delta_9 & AT \\ \hline \bullet h_1: unbox(\Box \Gamma_9) \vdash A_8, F_7 & M_9 \\ \hline -: \Delta_{10}, \Box \Gamma_9, unbox(\Box \Gamma_9) \vdash A_8, F_7 & M_9 \\ \hline -: \Delta_{10}, \Box \Gamma_9, unbox(\Box \Gamma_9) \vdash A_8, F_7 & AT \\ \hline -: \Delta_{10}, \Box \Gamma_9 \vdash A_8, F_7 & AT \\ \hline \bullet h_1: unbox(\Box \Gamma_{11}, []F_9) \vdash F_7 & K & h_8: \Box \Gamma_{11}, F_9, F_{12}, \Delta_{13}, []F_7 \vdash \Delta_8 \\ \hline \bullet h_1: (\Box \Gamma_{11}, []F_9) \vdash A_1, F_7 & M_9 \\ \hline -: C_{10}, \Box \Gamma_9 \vdash A_8, F_7 & -: \Delta_{10}, \Box \Gamma_9 \vdash A_8 \\ \hline \bullet h_1: unbox(\Box \Gamma_{11}) \vdash F_7 & K & h_8: \Box \Gamma_{11}, F_9, F_{12}, \Delta_{13}, []F_7 \vdash \Delta_{10}, []F_7 \\ \hline \bullet h_1: unbox(\Box \Gamma_{11}) \vdash F_7 & M_8: \Box \Gamma_{11}, G_{12}, G_{12}, []F_9 \vdash \Delta_{10}, []F_7 \\ \hline \bullet h_1: unbox(\Box \Gamma_{11}) \vdash F_7 & M_8: \Box \Gamma_{11}, G_{12}, G_{12}, []F_9 \vdash \Delta_{10}, []F_7 \\ \hline \bullet h_1: unbox(\Box \Gamma_{11}) \vdash F_7 & M_8: \Box \Gamma_{11}, G_{12}, G_{12}, []F_9 \vdash \Delta_{10}, []F_7 \\ \hline \bullet h_1: unbox(\Box \Gamma_{11}) \vdash F_7 & M_8: \Box \Gamma_{11}, G_{12}, G_{12}, []F_9 \vdash \Delta_{10}, []F_7 \\ \hline \bullet h_1: Unbox(\Box \Gamma_{11}) \vdash F_7 & M_8: \Box \Gamma_{11}, G_{12}, G_{12}, []F_9 \vdash \Delta_{10}, []F_7 \\ \hline \bullet h_1: Unbox(\Box \Gamma_{11}) \vdash F_7 & M_8: \Box \Gamma_{11}, G_{12}, G_{12}, []F_9 \vdash \Delta_{10}, []F_7 \\ \hline \bullet h_1: Unbox(\Box \Gamma_{11}) \vdash F_7 & M_8: \Box \Gamma_{11}, G_{12}, G_{12}, []F_9 \vdash \Delta_{10}, []F_7$$

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_{7} \\ \\ \underline{\bullet \mathbf{h}_{1}: \Box\Gamma_{11}, \Delta_{13}, []\mathbf{F}_{9} \vdash (\Delta_{10}, []\mathbf{F}_{7}), \mathbf{F}_{12} \end{array} K \begin{array}{c} \mathbf{h}_{8}: \Box\Gamma_{11}, \mathbf{F}_{9}, \mathbf{F}_{12}, \Delta_{13}, []\mathbf{F}_{9} \vdash \Delta_{10}, []\mathbf{F}_{7} \\ \\ \underline{\bullet \mathbf{h}_{8}: (\Box\Gamma_{11}, \Delta_{13}, []\mathbf{F}_{9}), \mathbf{F}_{12} \vdash \Delta_{10}, []\mathbf{F}_{7} \\ \\ \underline{-: \Box\Gamma_{11}, \Delta_{13}, []\mathbf{F}_{9} \vdash \Delta_{10}, []\mathbf{F}_{7} \\ \\ \underline{-: unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_{7}} \end{array} \begin{array}{c} AT \\ \text{Cut} \\ \\ \underline{-: unbox(\Box\Gamma_{11}) \vdash \mathbf{F}_{7}} \end{array}$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}: unbox(\Box\Gamma_{9}) \vdash \mathbf{F}_{6}}{\bullet \mathbf{h}_{1}: \Box\Gamma_{9}, \bot, \Delta_{10} \vdash \Delta_{8}, []\mathbf{F}_{6}} \quad K \quad & \frac{\bullet \mathbf{h}_{7}: (\Box\Gamma_{9}, \bot, \Delta_{10}), []\mathbf{F}_{6} \vdash \Delta_{8}}{\bullet \mathbf{h}_{7}: (\Box\Gamma_{9}, \bot, \Delta_{10}), []\mathbf{F}_{6} \vdash \Delta_{8}} \quad \frac{\bot_{L}}{\mathsf{Cut}} \\ & \frac{-: \Box\Gamma_{9}, \bot, \Delta_{10} \vdash \Delta_{8}}{-: \bot, \Delta_{10}, \Box\Gamma_{9} \vdash \Delta_{8}} \quad \bot_{L} \\ \\ \frac{\mathbf{h}_{1}: unbox(\Box\Gamma_{10}) \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{1}: \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_{9}, []\mathbf{F}_{7}), \bot} \quad K \quad & \frac{\bullet \mathbf{h}_{8}: (\Box\Gamma_{10}, \Delta_{11}), \bot \vdash \Delta_{9}, []\mathbf{F}_{7}}{\bullet \mathbf{h}_{1}: \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_{9}, []\mathbf{F}_{7}} \quad \frac{\bot_{L}}{\mathsf{Cut}} \\ & \frac{-: \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_{9}, []\mathbf{F}_{7}}{-: \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_{9}, []\mathbf{F}_{7}} \quad K \\ \\ \frac{\bullet \mathbf{h}_{1}: unbox(\Box\Gamma_{10}) \vdash \mathbf{F}_{7}}{\bullet \mathbf{h}_{1}: \Box\Gamma_{10}, \bot, \Delta_{12} \vdash (\Delta_{9}, []\mathbf{F}_{7}), \mathbf{F}_{11}} \quad K \quad & \frac{\bullet \mathbf{h}_{8}: (\Box\Gamma_{10}, \bot, \Delta_{12}), \mathbf{F}_{11} \vdash \Delta_{9}, []\mathbf{F}_{7}}{\bullet \mathbf{h}_{1}: \Box\Gamma_{10}, \bot, \Delta_{12} \vdash (\Delta_{9}, []\mathbf{F}_{7}), \mathbf{F}_{11}} \quad C\mathbf{ut} \\ & \frac{-: \Box\Gamma_{10}, \bot, \Delta_{12} \vdash \Delta_{9}, []\mathbf{F}_{7}}{\to} \quad & \frac{\bot_{L}}{-: \Box\Gamma_{10}, \bot, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_{9}, []\mathbf{F}_{7}} \\ & \frac{-: \Box\Gamma_{10}, \bot, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_{9}, []\mathbf{F}_{7}}{\to} \quad & \frac{\bot_{L}}{-: \Box\Gamma_{10}, \bot, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_{9}, []\mathbf{F}_{7}} \end{array}$$

• Case rule I

$$\begin{array}{c} \frac{h_{1}:unbox(\Box\Gamma_{10}) \vdash F_{6}}{\bullet h_{1}:\Box\Gamma_{10},\Delta_{11},p_{9} \vdash (\Delta_{8},p_{9}), []F_{6}} \ K \\ \hline \bullet h_{7}:(\Box\Gamma_{10},\Delta_{11},p_{9}), []F_{6} \vdash \Delta_{8},p_{9} \\ \hline -:\Box\Gamma_{10},\Delta_{11},p_{9} \vdash \Delta_{8},p_{9} \\ \hline -:\Delta_{11},\Box\Gamma_{10},p_{9} \vdash \Delta_{8},p_{9} \\ \hline I \\ \hline \\ h_{1}:unbox(\Box\Gamma_{11}) \vdash F_{7} \\ \hline \bullet h_{1}:\Box\Gamma_{11},\Delta_{12} \vdash ((\Delta_{9},p_{10}), []F_{7}),p_{10} \ K \\ \hline \bullet h_{8}:(\Box\Gamma_{11},\Delta_{12}),p_{10} \vdash (\Delta_{9},p_{10}), []F_{7} \ Cut \\ \hline -:\Box\Gamma_{11},\Delta_{12} \vdash ((\Delta_{9},p_{10}), []F_{7} \\ \hline -:unbox(\Box\Gamma_{11}) \vdash F_{7} \\ \hline \bullet h_{1}:unbox(\Box\Gamma_{11}) \vdash F_{7} \\ \hline \bullet h_{1}:unbox(\Box\Gamma_{11}) \vdash F_{7} \\ \hline \bullet h_{1}:D\Gamma_{11},\Delta_{13},p_{10} \vdash ((\Delta_{9},p_{10}), []F_{7}),F_{12} \ K \\ \hline \bullet h_{8}:(\Box\Gamma_{11},\Delta_{13},p_{10}),F_{12} \vdash (\Delta_{9},p_{10}), []F_{7} \\ \hline \bullet h_{1}:\Box\Gamma_{11},\Delta_{13},p_{10} \vdash ((\Delta_{9},p_{10}), []F_{7}),F_{12} \ K \\ \hline -:\Box\Gamma_{11},\Delta_{13},p_{10} \vdash (\Delta_{9},p_{10}), []F_{7} \\ \hline -:\Box\Gamma_{11},\Delta_{13},p_{10} \vdash (\Delta_{9},p_{10}), []F_{7} \\ \hline -:\Delta_{13},\Box\Gamma_{11},p_{10} \vdash \Delta_{9},p_{10}, []F_{7} \ I \\ \hline \end{array}$$

• Case rule \top_L

$$\frac{ \begin{array}{c} \mathbf{h}_1: unbox(\Box \Gamma_9) \vdash \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_9, \top, \Delta_{10} \vdash \Delta_8, [] \mathbf{F}_6 \end{array} K \begin{array}{c} \mathbf{h}_7: \Box \Gamma_9, \Delta_{10}, [] \mathbf{F}_6 \vdash \Delta_8 \\ \hline \bullet \mathbf{h}_7: (\Box \Gamma_9, \top, \Delta_{10}), [] \mathbf{F}_6 \vdash \Delta_8 \end{array} \begin{array}{c} \top_L \\ \hline cut \\ \hline \hline \bullet \mathbf{h}_1: \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8, [] \mathbf{F}_6 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_9, [] \mathbf{F}_6 \vdash \Delta_8 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline -: \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_8 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \hline \mathbf{h}_7: \top, \Delta_{10}, \Box \Gamma_9, [] \mathbf{F}_6 \vdash \Delta_8 \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{hCut} \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_1: unbox(\Box\Gamma_{10}) \vdash \mathbf{F}_7}{\bullet \mathbf{h}_1: \Box\Gamma_{10}, \Delta_{11} \vdash (\Delta_9, []\mathbf{F}_7), \top} \quad K \quad \frac{\mathbf{h}_8: \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathbf{F}_7}{\bullet \mathbf{h}_8: (\Box\Gamma_{10}, \Delta_{11}), \top \vdash \Delta_9, []\mathbf{F}_7} \quad \top_L \\ \\ -: \Box\Gamma_{10}, \Delta_{11} \vdash \Delta_9, []\mathbf{F}_7 \\ \hline \\ -: \Delta_{11}, \Box\Gamma_{10} \vdash \Delta_9, []\mathbf{F}_7 \\ \hline \\ \bullet \mathbf{h}_1: unbox(\Box\Gamma_{10}) \vdash \mathbf{F}_7 \\ \hline \bullet \mathbf{h}_1: \Box\Gamma_{10}, \top, \Delta_{12} \vdash (\Delta_9, []\mathbf{F}_7), \mathbf{F}_{11} \quad K \quad \frac{\mathbf{h}_8: \Box\Gamma_{10}, \mathbf{F}_{11}, \Delta_{12} \vdash \Delta_9, []\mathbf{F}_7}{\bullet \mathbf{h}_8: (\Box\Gamma_{10}, \top, \Delta_{12}), \mathbf{F}_{11} \vdash \Delta_9, []\mathbf{F}_7} \quad \top_L \\ \hline \\ -: \Box\Gamma_{10}, \top, \Delta_{12} \vdash \Delta_9, []\mathbf{F}_7 \\ \hline \\ -: unbox(\Box\Gamma_{10}) \vdash \mathbf{F}_7 \quad \mathbf{ax/W} \\ \hline \\ -: unbox(\Box\Gamma_{10}) \vdash \mathbf{F}_7 \quad \mathbf{ax/W} \\ \hline \\ -: \tau, \Delta_{12}, \Box\Gamma_{10} \vdash \Delta_9, []\mathbf{F}_7 \quad K \end{array}$$

6.7 Status of A45: fail

• Case rule \rightarrow_R

• Case rule \wedge_R

$$\frac{ \begin{array}{c} h_1: \square\Gamma_{13} \vdash \square\Gamma_{11}, F_7 \\ \bullet h_1: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{11}, \Delta_{12}, F_9 \wedge F_{10}), []F_7 \end{array}}{ \begin{array}{c} \bullet h_3: \square\Gamma_{13}, \Delta_{14}, []F_7 \vdash \square\Gamma_{11}, F_9, \Delta_{12} & h_8: \square\Gamma_{13}, \Delta_{14}, []F_7 \vdash \square\Gamma_{11}, F_{10}, \Delta_{12}, F_9 \wedge F_{10}) \\ \bullet h_8: (\square\Gamma_{13}, \Delta_{14}), []F_7 \vdash \square\Gamma_{11}, \Delta_{12}, F_9 \wedge F_{10} \\ & -: \square\Gamma_{13}, \Delta_{14} \vdash \square\Gamma_{11}, \Delta_{12}, F_9 \wedge F_{10} \\ & -: \square\Gamma_{13}, \Delta_{14} \vdash \square\Gamma_{11}, \Delta_{12}, F_9 \wedge F_{10} \\ \hline \bullet h_1: \square\Gamma_{13} \vdash F_7, \square\Gamma_{11} & ax/W \\ \bullet h_1: \square\Gamma_{13} \vdash \Delta_{12}, F_9, \square\Gamma_{11}, []F_7 \end{array}} \begin{array}{c} A45 \\ h_8: \Delta_{14}, \square\Gamma_{13}, []F_7 \vdash \Delta_{12}, F_9, \square\Gamma_{11} \\ & -: \Delta_{14}, \square\Gamma_{13} \vdash \Delta_{12}, F_9, \square\Gamma_{11} \\ \hline & -: \Delta_{14}, \square\Gamma_{13} \vdash \Delta_{12}, F_9, \square\Gamma_{11} \end{array}} \begin{array}{c} ax/W \\ h_{Cut} \\ \hline & -: \Delta_{14}, \square\Gamma_{13} \vdash \Delta_{12}, F_{10}, \square\Gamma_{11}, []F_7 \end{array}} \begin{array}{c} A45 \\ h_8: \Delta_{14}, \square\Gamma_{13} \vdash \Delta_{12}, F_{10}, \square\Gamma_{11}, []F_7 \end{array}$$

• Case rule \vee_R

$$\begin{array}{c} h_1: \Box \Gamma_{13} \vdash \Box \Gamma_{11}, F_7 \\ \bullet h_1: \Box \Gamma_{13}, \Delta_{14} \vdash (\Box \Gamma_{11}, \Delta_{12}, F_9 \lor F_{10}), []F_7 \\ \bullet h_8: (\Box \Gamma_{13}, \Delta_{14}), []F_7 \vdash \Box \Gamma_{11}, F_9, F_{10}, \Delta_{12} \\ & \bullet h_8: (\Box \Gamma_{13}, \Delta_{14}), []F_7 \vdash \Box \Gamma_{11}, \Delta_{12}, F_9 \lor F_{10} \\ & -: \Box \Gamma_{13}, \Delta_{14} \vdash \Box \Gamma_{11}, \Delta_{12}, F_9 \lor F_{10} \\ & \rightarrow \\ \hline h_1: \Box \Gamma_{13} \vdash F_7, \Box \Gamma_{11} \\ \bullet h_1: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, F_{10}, F_9, \Box \Gamma_{11}, []F_7 \\ & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, F_{10}, F_9, \Box \Gamma_{11} \\ & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{12}, \Box \Gamma_{11}, F_9 \lor F_{10} \\ \hline & -: \Delta_{14}, \Box \Gamma_{14} \vdash \Box \Gamma_{12}, F_8, \Box \Gamma_{12} \\ \hline & -: \Box \Gamma_{14}, \Delta_{15} \vdash \Box \Gamma_{12}, (\Delta_{13}, F_{10} \lor F_{11}), []F_8 \\ \hline & -: \Delta_{15}, \Box \Gamma_{14} \vdash \Delta_{13}, \Box \Gamma_{12}, []F_8 \\ \hline & -: \Delta_{15}, \Box \Gamma_{14} \vdash \Delta_{13}, \Box \Gamma_{12}, []F_8, F_{10} \lor F_{11} \\ \hline & -: \Delta_{15}, \Box \Gamma_{14} \vdash \Delta_{13}, \Box \Gamma_{12}, []F_8, F_{10} \lor F_{11} \\ \hline & -: \Box \Gamma_{14}, \Delta_{16} \vdash (\Box \Gamma_{12}, (\Delta_{13}, F_{10} \lor F_{11}), []F_8), F_{15} \\ \hline & -: \Box \Gamma_{14}, \Delta_{16} \vdash \Box \Gamma_{12}, (\Delta_{13}, F_{10} \lor F_{11}), []F_8 \\ \hline & -: \Box \Gamma_{14}, \Delta_{16} \vdash \Box \Gamma_{12}, (\Delta_{13}, F_{10} \lor F_{11}), []F_8 \\ \hline & -: \Box \Gamma_{14}, \Delta_{16} \vdash \Box \Gamma_{12}, (\Delta_{13}, F_{10} \lor F_{11}), []F_8 \\ \hline & -: \Box \Gamma_{14}, \Delta_{16} \vdash \Box \Gamma_{12}, (\Delta_{13}, F_{10} \lor F_{11}), []F_8 \\ \hline & -: \Box \Gamma_{14}, \Delta_{16} \vdash \Box \Gamma_{12}, (\Delta_{13}, F_{10} \lor F_{11}), []F_8 \\ \hline & -: \Box \Gamma_{14}, \Delta_{16} \vdash \Box \Gamma_{12}, (\Delta_{13}, F_{10} \lor F_{11}), []F_8 \\ \hline & -: \Box \Gamma_{14}, \Delta_{16} \vdash \Box \Gamma_{12}, (\Delta_{13}, F_{10} \lor F_{11}), []F_8 \\ \hline & -: \Box \Gamma_{14}, \Delta_{15}, \Box \Gamma_{14} \vdash \Box \Gamma_{12},$$

• Case rule \perp_R

$$\begin{array}{c} h_1: \square\Gamma_{11} \vdash \square\Gamma_{9}, F_7 \\ \hline \bullet h_1: \square\Gamma_{11}, \Delta_{12} \vdash (\square\Gamma_{9}, \bot, \Delta_{10}), []F_7 \\ \hline \bullet h_1: \square\Gamma_{11}, \Delta_{12} \vdash (\square\Gamma_{9}, \bot, \Delta_{10}), []F_7 \\ \hline \\ \bullet h_1: \square\Gamma_{11}, \Delta_{12} \vdash (\square\Gamma_{9}, \bot, \Delta_{10}) \\ \hline \\ \hline & -: \square\Gamma_{11}, \Delta_{12} \vdash \square\Gamma_{9}, \bot, \Delta_{10} \\ \hline \\ \hline \bullet h_1: \Delta_{12}, \square\Gamma_{11} \vdash \bot, \Delta_{10}, \square\Gamma_{9}, []F_7 \\ \hline & \bullet h_3: \Delta_{12}, \square\Gamma_{11}, []F_7 \vdash \bot, \Delta_{10}, \square\Gamma_{9} \\ \hline & \bullet h_1: \Delta_{12}, \square\Gamma_{11} \vdash \bot, \Delta_{10}, \square\Gamma_{9}, []F_7 \\ \hline & \bullet h_1: \square\Gamma_{12} \vdash \square\Gamma_{10}, F_8, \square F_{14} \\ \hline & \bullet h_1: \square\Gamma_{12} \vdash \square\Gamma_{10}, F_8, \square F_{14} \\ \hline & \bullet h_1: \square\Gamma_{12}, \Delta_{13} \vdash (\square\Gamma_{10}, (\bot, \Delta_{11}), []F_8), \square F_{14} \\ \hline & \bullet h_1: \square\Gamma_{12}, \Delta_{13} \vdash (\square\Gamma_{10}, (\bot, \Delta_{11}), []F_8), \square F_{14} \\ \hline & \bullet h_1: \square\Gamma_{12}, \Delta_{13} \vdash (\square\Gamma_{10}, (\bot, \Delta_{11}), []F_8 \\ \hline & \bullet h_1: \square\Gamma_{12}, \Delta_{13} \vdash (\square\Gamma_{10}, (\bot, \Delta_{11}), []F_8 \\ \hline & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{12} \vdash \bot, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{10}, []F_8 \\ \hline \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{10}, []F_8 \\ \hline \\ \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{10}, []F_8 \\ \hline \\ \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{10}, []F_8 \\ \hline \\ \\ & \bullet h_1: \Delta_{13}, \square\Gamma_{10},$$

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 \frac{\mathbf{h}_1: \Box \Gamma_{12} \vdash \Box \Gamma_{10}, \mathbf{F}_8}{ \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Delta_{14} \vdash (\Box \Gamma_{10}, (\bot, \Delta_{11}), []\mathbf{F}_8), \mathbf{F}_{13}} A45 \quad \frac{\mathbf{h}_9: \Box \Gamma_{12}, \mathbf{F}_{13}, \Delta_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_8}{ \bullet \mathbf{h}_9: (\Box \Gamma_{12}, \Delta_{14}), \mathbf{F}_{13} \vdash \Box \Gamma_{10}, (\bot, \Delta_{11}), []\mathbf{F}_8} \quad \mathbf{Cut} \\ -: \Box \Gamma_{12}, \Delta_{14} \vdash \Box \Gamma_{10}, (\bot, \Delta_{11}), []\mathbf{F}_8 \\ \hline -: \Box \Gamma_{12} \vdash \mathbf{F}_8, \Box \Gamma_{10} \quad \mathbf{ax/W} \\ \hline -: \Delta_{14}, \Box \Gamma_{12} \vdash \bot, \Delta_{11}, \Box \Gamma_{10}, []\mathbf{F}_8 \quad A45 \\ \hline
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• Case rule \top_R

 \bullet Case rule K

$$\frac{\mathbf{h}_1: \Box \Gamma_{12}, \Box \Gamma_{14} \vdash (\Box \Gamma_{10}, [] \mathbf{F}_9), \mathbf{F}_7}{\bullet \mathbf{h}_1: (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash ((\Box \Gamma_{10}, [] \mathbf{F}_9), \Delta_{11}), [] \mathbf{F}_7} \xrightarrow{A45} \frac{\mathbf{h}_8: \mathbf{F}_7, unbox(\Box \Gamma_{12}), unbox(\Box \Gamma_{12}), unbox(\Box \Gamma_{13}) \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8: ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), [] \mathbf{F}_7 \vdash (\Box \Gamma_{10}, [] \mathbf{F}_9), \Delta_{11}} \xrightarrow{K} \xrightarrow{\mathbf{Cut}} -: (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{10}, [] \mathbf{F}_9), \Delta_{11}}$$

Axioms assumed:

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inf : C:MSFormula |-- True ; C':MSFormula
inf : False ; C:MSFormula | -- C':MSFormula
inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
                              \mathtt{h}_1: \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \Box\Gamma_{10}, \mathtt{F}_7
                                                                                                                                            h_8: F_7, unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash F_9
   \frac{\mathbf{h}_1: \sqcup \Gamma_{12}, \sqcup \Gamma_{14} \sqcap \Gamma \sqcup \Gamma_{10}, \Gamma_{7}}{\bullet \mathbf{h}_1: (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_9), []\mathbf{F}_7} \quad A45 \quad \frac{\bullet \bullet \mathbf{h}_8: ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), []\mathbf{F}_7 \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_9}{\mathsf{Cut}} \quad K \cap \mathbf{h}_8: ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), []\mathbf{F}_7 \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_9]
                                                                          -: (\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15} \vdash \Box\Gamma_{10},\Delta_{11},[]\mathtt{F}_{9}
                                                                                  = \text{ax/W} \quad \frac{ -: \mathbf{F}_7, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, unbox(\Box \Gamma_{12}), unbox(\Box \Gamma_{13}) \vdash \mathbf{F}_9, \Box \Gamma_{10} }{ -: \mathbf{F}_7, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14} \vdash \mathbf{F}_9, \Box \Gamma_{10} } \quad \text{sCut} \quad \text{$\mathbf{A}TG$} 
                                                              -:\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash F_9,\Box\Gamma_{10}
                                                           \frac{1}{-:\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_{11},\Box\Gamma_{10},[]F_{9}} A45
                         \mathtt{h}_1:\Box\Gamma_{12},\Box\Gamma_{14}\vdash(\Box\Gamma_{10},[]\mathtt{F}_9),\mathtt{F}_7
                                                                                                                                                    h_8: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash F_9
   \frac{\bullet_{\mathbf{h}_{1}}:(\Box\Gamma_{12},\Box\Gamma_{14}),(\Box\Gamma_{13},\Delta_{15}\vdash((\Box\Gamma_{10},[]\mathsf{F}_{9}),\Delta_{11}),[]\mathsf{F}_{7}}{\bullet_{\mathbf{h}_{3}}:((\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15}),[]\mathsf{F}_{7}\vdash(\Box\Gamma_{10},[]\mathsf{F}_{9}),\Delta_{11}} \underbrace{K}_{\mathsf{Cut}}
                                                                                       \frac{}{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_{9}} \ \text{ax/W}
                                                                                \frac{1}{-:\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_{11},\Box\Gamma_{10},[]\mathsf{F}_{9}}\ K
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\mathtt{h}_8: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9
                                                                             \mathtt{h}_1:\Box\Gamma_{12},\Box\Gamma_{14}\vdash\Box\Gamma_{10},\mathtt{F}_7
            \frac{1}{\bullet h_1 : (\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15} \vdash (\Box \Gamma_{10}, \Delta_{11}, []F_9), []F_7} A45 }{\bullet h_8 : ((\Box \Gamma_{12}, \Box \Gamma_{14}), \Box \Gamma_{13}, \Delta_{15}), []F_7 \vdash \Box \Gamma_{10}, \Delta_{11}, []F_9} 
                                                                                                                                                                                         -: (\Box\Gamma_{12},\Box\Gamma_{14}),\Box\Gamma_{13},\Delta_{15} \vdash \Box\Gamma_{10},\Delta_{11},[]\mathtt{F}_{9}
                                                                                                                                                                                                                 \frac{\rightarrow}{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13})} \vdash F_9 \text{ ax/W}
                                                                                                                                                                                               \frac{}{-:\Delta_{15},\Box\Gamma_{12},\Box\Gamma_{13},\Box\Gamma_{14}\vdash\Delta_{11},\Box\Gamma_{10},[]\mathsf{F}_{9}}K
                                                                 \mathtt{h}_1:\Box\Gamma_{13},\Box\Gamma_{16}\vdash(\Box\Gamma_{11},[]\mathtt{F}_{10}),\mathtt{F}_8,\Box\mathtt{F}_{15}
                                                                                                                                                                                                                                                                                                                                                                                               \mathtt{h}_9: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}), unbox(\Box\mathsf{F}_{15}) \vdash \mathsf{F}_{10}
         \overline{-: (\Box\Gamma_{13},\Box\Gamma_{16}),\Box\Gamma_{14},\Delta_{17} \vdash (\Box\Gamma_{11},[]}F_{10}),\Delta_{12},[]F_8
                                                                             \frac{\mathbf{h}_1: \Box \Gamma_{13}, \Box \Gamma_{14}, \Box \Gamma_{16} \vdash \Box F_{15}, F_8, \Box \Gamma_{11}, []F_{10}}{\mathbf{h}_1: \Box \Gamma_{13}, \Box \Gamma_{14}, \Box \Gamma_{16} \vdash \Box F_{15}, F_8, \Box \Gamma_{11}, []F_{10}} \text{ ax/W} \\ \frac{\mathbf{h}_9: unbox(\Box F_{15}), unbox(\Box \Gamma_{13}), unbox(\Box \Gamma_{14}) \vdash F_{10}}{\mathbf{h}_9: \Box F_{15}, \Box \Gamma_{13}, \Box \Gamma_{14}, \Box \Gamma_{16} \vdash F_8, \Box \Gamma_{11}, []F_{10}} \text{ hCut} \\ \frac{\mathbf{h}_9: unbox(\Box F_{15}), unbox(\Box F_{15}), unbox(\Box \Gamma_{13}), unbox(\Box \Gamma_{14}) \vdash F_{10}}{\mathbf{h}_{13}, \Box \Gamma_{14}, \Box \Gamma_{16} \vdash F_8, \Box \Gamma_{11}, []F_{10}} \text{ hCut} \\ \frac{\mathbf{h}_9: unbox(\Box F_{15}), unbox(\Box F_{15}), unbox(\Box \Gamma_{14}) \vdash F_{10}}{\mathbf{h}_{13}, \Box \Gamma_{14}, \Box \Gamma_{16} \vdash F_8, \Box \Gamma_{11}, []F_{10}} \text{ hCut} \\ \frac{\mathbf{h}_9: unbox(\Box F_{15}), unbox(\Box F_{15}), unbox(\Box \Gamma_{14}) \vdash F_{10}}{\mathbf{h}_{13}, \Box \Gamma_{14}, \Box \Gamma_{16} \vdash F_8, \Box \Gamma_{11}, []F_{10}} \text{ hCut} \\ \frac{\mathbf{h}_9: unbox(\Box F_{15}), unbox(\Box F_{15}), unbox(\Box F_{14}) \vdash F_{10}}{\mathbf{h}_{13}, \Box \Gamma_{14}, \Box \Gamma_{16} \vdash F_8, \Box \Gamma_{11}, []F_{10}} \text{ hCut} \\ \frac{\mathbf{h}_9: unbox(\Box F_{15}), unbox(\Box F_
                                                                                                                                                                                                                            -: \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{16} \vdash F_8, \Box\Gamma_{11}, []F_{10}
                                                                                                                                                                                                                 -: \Delta_{17}, \Box \Gamma_{13}, \Box \Gamma_{14}, \Box \Gamma_{16} \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{10}, []F_{8}
                                                                                                                                                                                                                                                                                                                                                                                                 h_9: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}), unbox(\Box F_{15}) \vdash F_{10}
                                                                                  \mathtt{h}_1: \Box\Gamma_{13}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \mathtt{F}_8, \Box\mathtt{F}_{15}
         \frac{\mathbf{h}_{1}: \Box\Gamma_{13}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \mathbf{F}_{8}, \Box\mathbf{F}_{15}}{\bullet\mathbf{h}_{1}: (\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{14}, \Delta_{17} \vdash (\Box\Gamma_{11}, (\Delta_{12}, []\mathbf{F}_{10}), []\mathbf{F}_{8}), \Box\mathbf{F}_{15}} \quad A45 \quad \frac{\mathbf{n}_{9} \cdot unock(\Box 1_{13}), unock(\Box 1_{14}), u
                                                                                                                                                                                                                -: (\Box \Gamma_{13}, \Box \Gamma_{16}), \Box \Gamma_{14}, \Delta_{17} \vdash \Box \Gamma_{11}, (\Delta_{12}, []\mathtt{F}_{10}), []\mathtt{F}_{8}
                                                                              \frac{\mathsf{h}_1: \Box \Gamma_{13}, \Box \Gamma_{14}, \Box \Gamma_{16}}{\mathsf{h}_1: \Box \Gamma_{13}, \Box \Gamma_{14}, \Box \Gamma_{16}} \underbrace{\vdash \Box \mathsf{F}_{15}, \mathsf{F}_8, \Box \Gamma_{11}, []\mathsf{F}_{10}}_{\mathsf{ax/W}} \underbrace{\frac{\mathsf{h}_9: unbox(\Box \mathsf{F}_{15}), unbox(\Box \Gamma_{13}), unbox(\Box \Gamma_{14}) \vdash \mathsf{F}_{10}}{\mathsf{e}^\mathsf{h}_9: \Box \mathsf{F}_{15}, \Box \Gamma_{13}, \Box \Gamma_{14}, \Box \Gamma_{16} \vdash \mathsf{F}_8, \Box \Gamma_{11}, []\mathsf{F}_{10}}_{\mathsf{hcut}}}_{\mathsf{hcut}}
                                                                                                                                                                                                                                         -: \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{16} \vdash F_8, \Box\Gamma_{11}, []F_{10}
                                                                                                                                                                                                               \frac{-:\Delta_{17},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{16}\vdash\Delta_{12},\Box\Gamma_{11},[]\mathsf{F}_{10},[]\mathsf{F}_{8}}{-:\Delta_{17},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{16}\vdash\Delta_{12},\Box\Gamma_{11},[]\mathsf{F}_{10},[]\mathsf{F}_{8}}
 \underbrace{ \begin{array}{c} \mathbf{h}_1: \Box \Gamma_{12}, \Box \Gamma_{15} \vdash \Box \Gamma_{10}, \mathbf{F}_9, \Box \mathbf{F}_{14} \\ \bullet \mathbf{h}_1: (\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16} \vdash (\Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_9), \Box \mathbf{F}_{14} \end{array} }_{\bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_9: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{10}, \Delta_{11}, \Box \mathbf{F}_{12}, \Delta_{12}, \Delta_{13}, \Delta_{16} \\ \bullet \mathbf{h}_9: \left((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16}\right), \Box \mathbf{F}_{14} \vdash \Box \mathbf{F}_{15}, \Delta_{15}, \Delta_{15}, \Delta_{16}, \Delta_{15}, \Delta_{16}, \Delta_{15}, \Delta_{15}, \Delta_{15}, \Delta_{16}, \Delta_{15}, \Delta_{15}, \Delta_{15}, \Delta_{15}, \Delta_{15}, \Delta_{16}, \Delta_{15}, \Delta_{16}, \Delta_{15}, \Delta_{15
                                                                                                                                                                -: (\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{13}, \Delta_{16} \vdash \Box \Gamma_{10}, \Delta_{11}, []_{F_9}
 Axioms assumed:
 inf : C:MSFormula |-- True ; C':MSFormula
 inf : False ; C:MSFormula | -- C':MSFormula
 inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
 suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
 suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
 suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
                                                                \mathtt{h}_1: \Box\Gamma_{13}, \Box\Gamma_{15} \vdash (\Box\Gamma_{11}, []\mathtt{F}_{10}), \mathtt{F}_8, \Box\mathtt{F}_{17}
                                                                                                                                                                                                                                                                                                                                                                                                                                    h_9: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash F_{10}
         \bullet h_1: (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash ((\Box \Gamma_{11}, []F_{10}), \Delta_{12}, []F_8), \Box F_{17} \\ \bullet h_9: ((\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16}), \Box F_{17} \vdash (\Box \Gamma_{11}, []F_{10}), \Delta_{12}, []F_8 \\ \text{Cut}
                                                                                                                                                                                                               -: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16}\vdash(\Box\Gamma_{11},[]\mathsf{F}_{10}),\Delta_{12},[]\mathsf{F}_{8}\\ \rightarrow
                                                                                                                                                                                                                           \frac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{10}}{-: \Delta_{16}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, []\mathtt{F}_{8}} \quad K
                                                                                  \mathtt{h}_1: \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \Box\Gamma_{11}, \mathtt{f}_8, \Box\mathtt{f}_{17}
                                                                                                                                                                                                                                                                                                                                                                                                                                    h_9: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash F_{10}
          \underbrace{\bullet_{h_1} : (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash (\Box \Gamma_{11}, (\Delta_{12}, []F_{10}), []F_8), \Box F_{17}}_{\bullet h_9} \quad \underbrace{A45} 
                                                                                                                                                                                                                -: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{16} \vdash \Box\Gamma_{11},(\Delta_{12},[]F_{10}),[]F_8
                                                                                                                                                                                                                                                         \frac{}{-:unbox(\Box\Gamma_{13}),unbox(\Box\Gamma_{14})\vdash F_{10}} \text{ ax/W}
                                                                                                                                                                                                                           \frac{}{-:\Delta_{16},\Box\Gamma_{13},\Box\Gamma_{14},\Box\Gamma_{15}\vdash\Delta_{12},\Box\Gamma_{11},[]\mathsf{F}_{10},[]\mathsf{F}_{8}}
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\mathtt{h}_8: \mathit{unbox}(\Box \Gamma_{12}), \mathit{unbox}(\Box \Gamma_{13}) \vdash \mathtt{F}_9
   \frac{\mathbf{h}_{1}: \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \Box\Gamma_{10}, \mathbf{F}_{9}, \Box\mathbf{F}_{16}}{\bullet\mathbf{h}_{1}: (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Box\Gamma_{10}, \underline{\Delta_{11}, []\mathbf{F}_{9}}), \Box\mathbf{F}_{16}} \quad A45 \quad \frac{\mathbf{h}_{8}: unoox(\Box\mathbf{1}_{12}), unoox(\Box\mathbf{1}_{13}), \Box\mathbf{F}_{9}}{\bullet\mathbf{h}_{8}: ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathbf{F}_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{9}} \quad K \quad \text{cut}
                                                         \mathtt{h}_1: \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \Box\Gamma_{10}, \mathtt{F}_9, \Box\mathtt{F}_{16}
                                                                                                                                                                                        -: (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_{9}
                                                                                                                                                                                                \begin{array}{c} (\Box_{112}, \Box_{114}, \Box_{113}, \Box_{113}, \Box_{113}, \Box_{114}, \Box_{
\frac{\mathbf{h}_{1}: \Box\Gamma_{13}, \Box\Gamma_{16} \vdash (\Box\Gamma_{11}, []\mathsf{F}_{10}), \mathsf{F}_{8}}{\bullet \mathbf{h}_{1}: (\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash ((\Box\Gamma_{11}, []\mathsf{F}_{10}), \Delta_{12}, []\mathsf{F}_{8}), \Box\mathsf{F}_{14}} \ A45 \ \frac{\mathbf{h}_{9}: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}), unbox(\Box\mathsf{F}_{14}) \vdash \mathsf{F}_{10}}{\bullet \mathbf{h}_{9}: ((\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17}), \Box\mathsf{F}_{14} \vdash (\Box\Gamma_{11}, []\mathsf{F}_{10}), \Delta_{12}, []\mathsf{F}_{8}} \ Cut
                                                                                                                                                                                                              \frac{-\square \Gamma_{13},\square \Gamma_{16} \vdash F_8,\square \Gamma_{11}, []F_{10}}{-: \Delta_{17},\square \Gamma_{13},\square \Gamma_{15},\square \Gamma_{16} \vdash \Delta_{12},\square \Gamma_{11}, []F_{10}, []F_8} \ \ \mathit{A45}
                                                                                         h_1: \Box \Gamma_{13}, \Box \Gamma_{16} \vdash \Box \Gamma_{11}, F_8
                                                                                                                                                                                                                                                                                                                                                                                       h_9: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{15}), unbox(\Box\mathsf{F}_{14}) \vdash \mathsf{F}_{10}
 \frac{\mathbf{h}_{1}: \Box \Gamma_{13}, \Box \Gamma_{16} \vdash \Box \Gamma_{11}, \mathbf{f}_{8}}{\bullet \mathbf{h}_{1}: (\Box \Gamma_{13}, \Box \Gamma_{16}), \underline{\Box \Gamma_{15}, \Delta_{17}} \vdash (\Box \Gamma_{11}, (\Delta_{12}, []\mathbf{f}_{10}), []\mathbf{f}_{8}), \underline{\Box F_{14}}} \quad A45 \quad \frac{\mathbf{n}_{9}: unoux(\Box \Gamma_{13}), unoux(\Box \Gamma_{15}), unoux(\Box \Gamma_{14}), \underline{\Box \Gamma_{16}}}{\bullet \mathbf{h}_{9}: ((\Box \Gamma_{13}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), \underline{\Box F_{14}} \vdash \Box \Gamma_{11}, (\Delta_{12}, []\mathbf{f}_{10}), []\mathbf{f}_{8}} \quad Cut
                                                                                                                                                                                                       \overline{-: (\Box\Gamma_{13}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash \Box\Gamma_{11}, (\Delta_{12}, []F_{10}), []F_8}
                                                                                                                                                                                                              \frac{\overbrace{-:\Box\Gamma_{13},\Box\Gamma_{16}\vdash F_8,\Box\Gamma_{11}}^{\text{/}}\text{ ax/W}}{-:\Delta_{17},\Box\Gamma_{13},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_{12},\Box\Gamma_{11},[]F_{10},[]F_8} \text{ } A45
   \frac{\mathbf{h}_1: \Box \Gamma_{12}, \Box \Gamma_{15} \vdash \Box \Gamma_{10}, \mathbf{F}_9}{\bullet \mathbf{h}_1: (\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16} \vdash (\Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_9), \Box \mathbf{F}_{13}} \quad A45 \quad \frac{\mathbf{h}_8: unbox(\Box \Gamma_{12}), unbox(\Box \Gamma_{14}), unbox(\Box \Gamma_{13}) \vdash \mathbf{F}_9}{\bullet \mathbf{h}_8: ((\Box \Gamma_{12}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{16}), \Box \mathbf{F}_{13} \vdash \Box \Gamma_{10}, \Delta_{11}, []\mathbf{F}_9} \quad K \text{ cut}
                                                                                                                                                                                        -: (\Box\Gamma_{12}, \Box\Gamma_{15}), \Box\Gamma_{14}, \Delta_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_{9}
                                                                                                                                                                                        \frac{\rightarrow}{-:\Box\Gamma_{12},\Box\Gamma_{15}\vdash F_{9},\Box\Gamma_{10}} \text{ ax/W} \\ \frac{-:\Delta_{16},\Box\Gamma_{12},\Box\Gamma_{14},\Box\Gamma_{15}\vdash \Delta_{11},\Box\Gamma_{10},[]F_{9}}{-:\Delta_{16},\Box\Gamma_{12},\Box\Gamma_{14},\Box\Gamma_{15}\vdash \Delta_{11},\Box\Gamma_{10},[]F_{9}}
                                                                  \mathtt{h}_1:\Box\Gamma_{13},\Box\Gamma_{15}\vdash(\Box\Gamma_{11},[]\mathtt{F}_{10}),\mathtt{F}_8
                                                                                                                                                                                                                                                                                                                                                                                                               \mathtt{h}_9: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{10}
 \frac{\mathbf{h}_{1} : (\Box \Gamma_{13}, \Box \Gamma_{15}) \cap (\Box \Gamma_{11}, [ F_{10}), F_{8}}{\bullet \mathbf{h}_{1} : (\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{17} \vdash ((\Box \Gamma_{11}, [ F_{10}), \Delta_{12}, [ F_{8}), F_{16}} \frac{A45}{\bullet \mathbf{h}_{9} : ((\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{17}), F_{16} \vdash (\Box \Gamma_{11}, [ F_{10}), \Delta_{12}, [ F_{8}}}{\bullet \mathbf{h}_{9} : ((\Box \Gamma_{13}, \Box \Gamma_{15}), \Box \Gamma_{14}, \Delta_{17}), F_{16} \vdash (\Box \Gamma_{11}, [ F_{10}), \Delta_{12}, [ F_{8})} \frac{K}{\mathsf{Cut}} 
                                                                                                                                                                                                         \frac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{10}}{-: \Delta_{17}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, []\mathtt{F}_{8}} \quad K
                                                                                     \mathtt{h}_1: \Box\Gamma_{13}, \Box\Gamma_{15} \vdash \Box\Gamma_{11}, \mathtt{F}_8
                                                                                                                                                                                                                                                                                                                                                                                                            h_9: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash F_{10}
 -: (\Box\Gamma_{13},\Box\Gamma_{15}),\Box\Gamma_{14},\Delta_{17} \vdash \Box\Gamma_{11},(\Delta_{12},[]\mathtt{F}_{10}),[]\mathtt{F}_{8}
                                                                                                                                                                                                          \frac{-: unbox(\Box\Gamma_{13}), unbox(\Box\Gamma_{14}) \vdash \mathtt{F}_{10}}{-: \Delta_{17}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_{10}, []\mathtt{F}_{8}} \quad K
                                                                                                                                                                                                                                                                                                                                                           \mathtt{h}_8: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathtt{F}_9
                                                                   \mathtt{h}_1: \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \Box\Gamma_{10}, \mathtt{F}_9
   \underbrace{ \begin{array}{c} \Pi_1 : \sqcup \Pi_{12}, \sqcup \Pi_{14} \vdash \sqcup \Pi_{10}, r_9 \\ \bullet h_1 : (\square \Gamma_{12}, \square \Gamma_{14}), \square \Gamma_{13}, \Delta_{16} \vdash (\square \Gamma_{10}, \Delta_{11}, [] F_9), F_{15} \end{array}}_{ \begin{array}{c} \Pi_8 : utilot X \sqcup \Pi_{12}, utilot X \sqcup \Pi_{13}) \vdash F_9 \\ \bullet h_8 : ((\square \Gamma_{12}, \square \Gamma_{14}), \square \Gamma_{13}, \Delta_{16}), F_{15} \vdash \square \Gamma_{10}, \Delta_{11}, [] F_9 \\ Cut \\ \underbrace{ \begin{array}{c} K \\ \text{Cut} \end{array}}_{ \begin{array}{c} \bullet \\ \text{Cut} \end{array}} 
                                                                                                                                                                               -: (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{16} \vdash \Box\Gamma_{10}, \Delta_{11}, []\mathtt{F}_9
                                                                                                                                                                                      \frac{ \xrightarrow{-: unbox(\Box\Gamma_{12}), unbox(\Box\Gamma_{13}) \vdash \mathsf{F}_9} \mathsf{ax/W} }{ -: \Delta_{16}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, \Box\Gamma_{10}, []\mathsf{F}_9} K
```

• Case rule A45

$$\frac{ \texttt{h}_1 : (\square_{\mathsf{T}_{\mathsf{1}^4}}, \square_{\mathsf{T}_{\mathsf{1}^6}} \vdash (\square_{\mathsf{T}_{\mathsf{1}^0}}, \square_{\mathsf{T}_{\mathsf{1}^2}}, [\mathsf{F}_{\mathsf{9}}), \mathsf{F}_{\mathsf{7}} }{ \texttt{e}^{\texttt{h}_1} : (\square_{\mathsf{T}_{\mathsf{1}^4}}, \square_{\mathsf{T}_{\mathsf{1}^6}}), \square_{\mathsf{T}_{\mathsf{1}^5}}, \Delta_{\mathsf{1}^7} \vdash ((\square_{\mathsf{T}_{\mathsf{1}^0}}, \square_{\mathsf{T}_{\mathsf{2}^4}}, [\mathsf{F}_{\mathsf{9}}), \square_{\mathsf{T}_{\mathsf{1}^4}}, \Delta_{\mathsf{1}^3}), []^{\mathsf{F}}_{\mathsf{7}} } } \frac{ \texttt{A45} }{ \texttt{e}^{\texttt{h}_8} : ((\square_{\mathsf{T}_{\mathsf{1}^4}}, \square_{\mathsf{T}_{\mathsf{5}^6}}), \square_{\mathsf{T}_{\mathsf{5}^5}}, \Delta_{\mathsf{1}^7}), []^{\mathsf{F}}_{\mathsf{7}} \vdash (\square_{\mathsf{T}_{\mathsf{1}^0}}, \square_{\mathsf{T}_{\mathsf{2}^4}}, []^{\mathsf{F}}_{\mathsf{9}}), \square_{\mathsf{T}_{\mathsf{1}^4}}, \Delta_{\mathsf{1}^3} } } \\ - : ((\square_{\mathsf{T}_{\mathsf{1}^4}}, \square_{\mathsf{T}_{\mathsf{1}^6}}), \square_{\mathsf{T}_{\mathsf{2}^4}}, \square_{\mathsf{1}^4}, \Delta_{\mathsf{1}^3}) \\ - : (\square_{\mathsf{T}_{\mathsf{1}^4}}, \square_{\mathsf{T}_{\mathsf{5}^6}}), \square_{\mathsf{T}_{\mathsf{1}^4}}, \Delta_{\mathsf{1}^3} }$$

Axioms assumed:

```
inf : C:MSFormula |-- True ; C':MSFormula
  inf : False ; C:MSFormula |-- C':MSFormula
  inf : P:Prop ; C:MSFormula | -- P:Prop ; C':MSFormula
  suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
  suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
  suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
                                                                                                  h_1: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), F_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \mathtt{h}_8: \Box\Gamma_{14}, \Box\Gamma_{15}, []\mathtt{F}_7 \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9
            \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : (\Box \Gamma_{14}, \Box \Gamma_{15}, \Delta_{17} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9), []\mathbf{F}_7 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{15}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \end{array} }_{\bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9 \\ \bullet \mathbf{h}_8 : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{16}, \Box \Gamma_{16}, \Box
                                                                                                                                                                                                                                            \overline{-: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}, [] F_9}
                                                                                                                                                        h_1: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash F_7, \Box\Gamma_{10}, \Box\Gamma_{12} ax/W
                                                                                                       \begin{array}{c} \bullet h_1 : \Box \Gamma_{14}, \Box \Gamma_{16}, \Box \Gamma_{16}, \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, [[F_7 \\ \bullet E_9], \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, \Box \Gamma_{1
                                                                                                                                                                                                                                                     \frac{-:\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash F_{9},\Box\Gamma_{10},\Box\Gamma_{11},\Box\Gamma_{12}}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash \Delta_{13},\Box\Gamma_{10},\Box\Gamma_{11},\Box\Gamma_{12},[]F_{9}} \ A45
                                                                                       \mathtt{h}_1:\Box\Gamma_{14},\Box\Gamma_{16}\vdash(\Box\Gamma_{10},\Box\Gamma_{12},[]\mathtt{F}_9),\mathtt{F}_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \mathtt{h}_8: \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9
            \begin{array}{c} \bullet_{h_1} : (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}, []F_9), \Box \Gamma_{11}, \Delta_{13}), []F_7 \end{array} \\ \bullet_{h_8} : ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, []F_9), \Box \Gamma_{11}, \Delta_{13} \\ & - : (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}, []F_9), \Box \Gamma_{11}, \Delta_{13} \end{array} 
                                                                                                                                                                                                                                                    \mathtt{h}_1:\Box\Gamma_{14},\Box\Gamma_{16}\vdash(\Box\Gamma_{10},\Box\Gamma_{12}),\mathtt{f}_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \mathtt{h}_8: \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9
            \underbrace{ \begin{array}{c} 1 \\ \bullet \mathbf{h}_1 : (\Box \Gamma_{14}, \Box \Gamma_{15}, \Delta_{17} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9), []\mathbf{F}_7 \end{array}}_{\boldsymbol{\Phi}\mathbf{h}_3 : ((\Box \Gamma_{14}, \Box \Gamma_{15}, \Delta_{17}), []\mathbf{F}_7 \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), \Box \Gamma_{11}, \Delta_{13}, []\mathbf{F}_9) 
                                                                                                                                                                                                                                            \overline{-: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}}, []F_9
                                                                                                                                                                                                                                                    \frac{ \overbrace{-:\Box\Gamma_{14},\Box\Gamma_{15}\vdash \mathtt{F}_{9},\Box\Gamma_{10},\Box\Gamma_{11}}^{\quad \text{ax/W}}}{-:\Delta_{17},\Box\Gamma_{14},\Box\Gamma_{15},\Box\Gamma_{16}\vdash\Delta_{13},\Box\Gamma_{10},\Box\Gamma_{11},\Box\Gamma_{12},[]\mathtt{F}_{9}} \ A45
\frac{\mathtt{h}_1: (\square\Gamma_{15}, \square\Gamma_{18} \vdash (\square\Gamma_{11}, \square\Gamma_{13}, []\mathsf{F}_{10}), \mathtt{F}_8, \square\mathsf{F}_{17}}{\bullet \mathtt{h}_1: (\square\Gamma_{15}, \square\Gamma_{18}), \square\Gamma_{16}, \Delta_{19} \vdash ((\square\Gamma_{11}, \square\Gamma_{13}, []\mathsf{F}_{10}), (\square\Gamma_{12}, \Delta_{14}), []\mathsf{F}_8), \square\mathsf{F}_{17}} \quad A45 \quad \frac{\mathtt{h}_9: (\square\Gamma_{15}, \square\Gamma_{16}, \square\Gamma_{16}, \square\Gamma_{16}, \square\Gamma_{17} \vdash \square\Gamma_{11}, \square\Gamma_{12}, \mathsf{F}_{10}, []\mathsf{F}_8}{\bullet \mathtt{h}_9: ((\square\Gamma_{15}, \square\Gamma_{18}), \square\Gamma_{16}, \Delta_{19}), \square\Gamma_{17} \vdash (\square\Gamma_{11}, \square\Gamma_{13}, []\mathsf{F}_{10}), (\square\Gamma_{12}, \Delta_{14}), []\mathsf{F}_8}
                                                                                                                                                                                                                                            : (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{16}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_{8}
 Axioms assumed:
  inf : C:MSFormula |-- True ; C':MSFormula
  inf : False ; C:MSFormula |-- C':MSFormula
  inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
  suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
  suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
  suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
                                                                                            \mathtt{h}_1: \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8, \Box\mathtt{F}_{17}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \mathtt{h}_9:\Box\Gamma_{15},\Box\Gamma_{16},\Box\mathtt{F}_{17}\vdash\Box\Gamma_{11},\Box\Gamma_{12},\mathtt{F}_{10}
           \bullet \mathbf{h}_1: (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{16}, \Delta_{19} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8), \Box F_{17} \\ \bullet \mathbf{h}_9: ((\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{16}, \Delta_{19}), \Box F_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), \Box F_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), \Box F_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), \Box F_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), \Box F_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), (\Box \Gamma_{13}, \Box \Gamma_{13}, \Box \Gamma_{13}, \Box \Gamma_{14}, \Delta_{14}), (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}), (\Box \Gamma_{12}, \Delta_{14}), (\Box \Gamma_{13}, \Box \Gamma_{13}, \Box \Gamma_{14}, \Delta_{14}), (\Box \Gamma_{14}, \Delta_{
                                                                                                                                                                                                                                                                       -: (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []F_{10}), (\Box\Gamma_{12}, \Delta_{14}), []F_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          h_9: \Box F_{17}, \Box \Gamma_{15}, \Box \Gamma_{16} \vdash F_{10}, \Box \Gamma_{11}, \Box \Gamma_{12} ax/W
                                                                                                               \frac{\mathsf{ax/W}}{\mathsf{h}_1: \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \Box\mathsf{F}_{17}, \mathsf{F}_8, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathsf{F}_{10}} \quad \mathsf{ax/W} \quad \frac{\mathsf{ax/W}}{\bullet \mathsf{h}_9: \Box\mathsf{F}_{17}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathsf{F}_8, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathsf{F}_{10}}{\bullet \mathsf{h}_9: \Box\mathsf{F}_{17}, \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathsf{F}_8, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathsf{F}_{10}}
                                                                                                                                                                                                                                                                                                    -: \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash F_8, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []F_{10}
                                                                                                                                                                                                                                                                                     \frac{-:\Delta_{19},\Box\Gamma_{15},\Box\Gamma_{16},\Box\Gamma_{18}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathsf{F}_{10},[]\mathsf{F}_{8}}{+445}
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\mathtt{h}_1: \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{f}_8, \Box\mathtt{f}_{17}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           \mathtt{h}_9: \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\mathtt{F}_{17} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8
 \bullet \mathbf{h}_1: (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{16}, \Delta_{19} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), []\mathbf{F}_8), \Box \mathbf{F}_{17}
\bullet \mathbf{h}_9: ((\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{16}, \Delta_{19}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{16}, \Delta_{19}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, \Box \Gamma_{12}, \Delta_{14}, \Box \Gamma_{14}, \Delta_{14}, \Delta_{14}, \Delta_{14}, \Delta_{14}, \Box \Gamma_{14}, \Delta_{14}, \Box \Gamma_{14}, \Delta_{14}, \Delta_{
                                                                                                                                                                                                                                                                                                                                                                                           -: (\Box\Gamma_{15}, \overline{\Box\Gamma_{18}}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []F_{10}), []F_8
                                                                                                                                                                                                                                       \overline{\mathtt{h}_1:\Box\Gamma_{15},\Box\Gamma_{18}\vdash\Box\mathtt{F}_{17},\mathtt{F}_8,\Box\Gamma_{11},\Box\Gamma_{13}}} ax/W
                                                                                                                                                          -: \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash F_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []F_8
                                                                                                                                                                                                                                                                                                                                                                                                                   -: \Delta_{19}, \overline{\square}_{\Gamma_{15}}, \overline{\square}_{\Gamma_{16}}, \overline{\square}_{\Gamma_{18}} \vdash \Delta_{14}, \overline{\square}_{\Gamma_{11}}, \overline{\square}_{\Gamma_{12}}, \overline{\square}_{\Gamma_{13}}, \overline{\parallel}_{F_{10}}, \overline{\parallel}_{F_8} 
A45
                                                                                                                                                   \mathtt{h}_1: \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{f}_8, \Box\mathtt{f}_{17}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \mathtt{h}_9:\square\Gamma_{15},\square\Gamma_{16},\square\mathtt{F}_{17}\vdash\square\Gamma_{11},\square\Gamma_{12},\mathtt{F}_{10}
  \bullet \mathbf{h}_1: (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{16}, \Delta_{19} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), []\mathbf{F}_8), \Box \mathbf{F}_{17} 
 \bullet \mathbf{h}_9: ((\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{16}, \Delta_{19}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{17} \vdash (\Box \Gamma_{11}, \Box \Gamma_{12}, \Delta_{14}, \Box \Gamma_{14}, \Delta_{14}, \Delta_{14}, \Box \Gamma_{14}, \Delta_{14}, \Delta_{
                                                                                                                                                                                                                                                                                                                                                                                              -: (\Box\Gamma_{15},\Box\Gamma_{18}),\Box\Gamma_{16},\Delta_{19} \vdash (\Box\Gamma_{11},\Box\Gamma_{13}), (\Box\Gamma_{12},\Delta_{14},[]\mathtt{F}_{10}),[]\mathtt{F}_{8}
                                                                                                                                                                                                                                      \overline{h_1:\Box\Gamma_{15},\Box\Gamma_{18}\vdash\Box F_{17},F_8,\Box\Gamma_{11},\Box\Gamma_{13}} \text{ ax/W}
                                                                                                                                                         \begin{array}{c} \mathbf{n}_1: \sqcup_{15}, \sqcup_{18} \sqcap \sqcup_{\Gamma_{17}, \Gamma_{8}}, \sqcup_{\Gamma_{11}, \sqcup_{\Gamma_{13}}} \\ \bullet \mathbf{h}_1: \square \Gamma_{15}, \square \Gamma_{16}, \square \Gamma_{18} \vdash \square \Gamma_{17}, \Gamma_{10}, \square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}, \llbracket \mathbf{f}_8 \\ \end{array} \begin{array}{c} \mathbf{a}_1 \\ \bullet_9: \square \Gamma_{17}, \square \Gamma_{15}, \square \Gamma_{16}, \square \Gamma_{18} \vdash \Gamma_{10}, \square \Gamma_{11}, \square \Gamma_{12}, \square \Gamma_{13}, \llbracket \mathbf{f}_8 \\ \bullet \\ \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                      -: \Box\Gamma_{15}, \Box\Gamma_{16}, \Box\Gamma_{18} \vdash \mathsf{F}_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, []\mathsf{F}_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                   -:\Delta_{19},\Box\Gamma_{15},\Box\Gamma_{16},\Box\Gamma_{18}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathtt{F}_{10},[]\mathtt{F}_{8}
                                                                                                                            h_1: \Box\Gamma_{14}, \Box\Gamma_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), F_9, \Box F_{16}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \mathtt{h}_8: \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\mathtt{F}_{16} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, \mathtt{F}_9
 \overline{-: (\Box\Gamma_{14}, \Box\Gamma_{17}), \Box\Gamma_{15}}, \Delta_{18} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []F_9
                                                                                                                                                       \frac{\mathsf{h}_8: \Box_{\mathsf{F}_{16}}, \Box_{\mathsf{F}_{17}} \vdash_{\mathsf{F}_{9}}, \Box_{\mathsf{F}_{10}}, \Box_{\mathsf{F}_{11}}, \Box_{\mathsf{F}_{12}}}{\mathsf{ex}^{\mathsf{W}}} \\ \frac{\mathsf{h}_8: \Box_{\mathsf{F}_{16}}, \Box_{\mathsf{F}_{14}}, \Box_{\mathsf{F}_{15}}, \Box_{\mathsf{F}_{17}} \vdash_{\mathsf{F}_{9}}, \Box_{\mathsf{F}_{10}}, \Box_{\mathsf{F}_{11}}, \Box_{\mathsf{F}_{12}}}{\mathsf{ex}^{\mathsf{W}}} \\ \frac{\mathsf{h}_8: \Box_{\mathsf{F}_{16}}, \Box_{\mathsf{F}_{14}}, \Box_{\mathsf{F}_{15}}, \Box_{\mathsf{F}_{17}} \vdash_{\mathsf{F}_{9}}, \Box_{\mathsf{F}_{10}}, \Box_{\mathsf{F}_{11}}, \Box_{\mathsf{F}_{12}}}{\mathsf{h}_{\mathsf{Cut}}} \\ \mathsf{h}_{\mathsf{Cut}} \\ \mathsf{h}_{\mathsf{
                                                                                                                                                                                                                                                                                                                                                                                                                               -: \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{17} \vdash F_9, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}
                                                                                                                                                                                                                                                                                                                                                                                            \frac{\square}{-:\Delta_{18},\square\Gamma_{14},\square\Gamma_{15},\square\Gamma_{17}\vdash\Delta_{13},\square\Gamma_{10},\square\Gamma_{11},\square\Gamma_{12},[]F_9} A45
                                                                                                                            \mathtt{h}_1:\Box\Gamma_{15},\Box\Gamma_{17}\vdash(\Box\Gamma_{11},\Box\Gamma_{13},[]\mathtt{F}_{10}),\mathtt{F}_8,\Box\mathtt{F}_{19}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     \mathtt{h}_9: \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8
  \underbrace{- \bullet h_1 : (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8), \Box F_{19} }_{\bullet h_9 : ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), (\Box \Gamma_{13}, \Box \Gamma_{14}), (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), (\Box \Gamma_{13}, \Box \Gamma_{14}), (\Box \Gamma_{15}, \Box \Gamma_{17}), (\Box \Gamma_{16}, \Delta_{18}), (\Box \Gamma_{17}, \Box \Gamma_{17}), (\Box \Gamma_{18}, \Delta_{18}), (\Box \Gamma_{19}, \Box \Gamma_{19}), (\Box \Gamma_{19}, \Delta_{18}), (\Box \Gamma_{19},
                                                                                                                                                                                                                                                                                                                                                                                            -: (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []F_{10}), (\Box\Gamma_{12}, \Delta_{14}), []F_{8} \rightarrow
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{\rightarrow}{-:\Box\Gamma_{15},\Box\Gamma_{16}\vdash \mathsf{F}_{10},\Box\Gamma_{11},\Box\Gamma_{12},[]\mathsf{F}_8}\;\mathsf{ax/W}
                                                                                                                                                                                                                                                                                                                                                                                                                      -: \Delta_{18}, \Box \Gamma_{15}, \Box \Gamma_{16}, \Box \Gamma_{17} \vdash \Delta_{14}, \Box \Gamma_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, []F_{10}, []F_8 
 A45 
                                                                                                                          \mathtt{h}_1:\Box\Gamma_{15},\Box\Gamma_{17}\vdash(\Box\Gamma_{11},\Box\Gamma_{13},[]\mathtt{F}_{10}),\mathtt{F}_8,\Box\mathtt{F}_{19}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \mathtt{h}_9: \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}
 \begin{array}{c} \bullet_{\text{h}_{1}} : (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_{8}), \Box F_{19} \\ \hline \bullet_{\text{h}_{1}} : (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_{8}), \Box F_{19} \\ \hline & \bullet_{\text{h}_{2}} : ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), \\ \hline & -: (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_{8} \\ \hline \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \frac{}{-:\Box\Gamma_{15},\Box\Gamma_{16}\vdash\mathsf{F}_{10},\Box\Gamma_{11},\Box\Gamma_{12}}\mathsf{ ax/W}
                                                                                                                                                                                                                                                                                                                                                                                                                   \frac{- \cdot \cdot \Delta_{15}, \Box_{15}, \Box_{16}, \Box_{17}, \Box_{11}, \Box_{11}, \Box_{12}}{- \cdot \cdot \Delta_{18}, \Box_{15}, \Box_{16}, \Box_{\Gamma_{17}} \vdash \Delta_{14}, \Box_{\Gamma_{11}}, \Box_{\Gamma_{12}}, \Box_{\Gamma_{13}}, []F_{10}, []F_8} \quad A45
                                                                                                                                                 \mathtt{h}_1:\Box\Gamma_{15},\Box\Gamma_{17}\vdash(\Box\Gamma_{11},\Box\Gamma_{13}),\mathtt{f}_8,\Box\mathtt{f}_{19}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \mathtt{h}_9: \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8
 \bullet h_1: (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), []F_8), \Box F_{19} \\ \bullet h_9: ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{19} \\ \bullet h_9: ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{19} \\ \bullet h_9: ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{19} \\ \bullet h_9: ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), \Box F_{19} \\ \bullet h_9: ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, \Box \Gamma_{17}), \Box F_{19} \\ \bullet h_9: ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, \Box \Gamma_{17}), \Box \Gamma_{18}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, \Box \Gamma_{17}), \Box \Gamma_{18}, \Delta_{18}), \Box F_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, \Box \Gamma_{18}), (\Box \Gamma_{11}, \Box \Gamma_{11}, \Delta_{11}, \Delta_{11}), (\Box \Gamma_{11}, \Box \Gamma_{11}, \Delta_{11}, \Delta_{11}, \Delta_{11}), (\Box \Gamma_{11}, \Delta_{11}, \Delta_{11}, \Delta_{11}, \Delta_{11}, \Delta_{11}), (\Box \Gamma_{11}, \Delta_{11}, \Delta_
                                                                                                                                                                                                                                                                                                                                                                                            \overline{-: (\Box\Gamma_{15},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{18} \vdash (\Box\Gamma_{11},\Box\Gamma_{13}), (\Box\Gamma_{12},\Delta_{14},[]F_{10}),[]F_{8}}
                                                                                                                                                                                                                                                                                                                                                                                                                   \frac{-\square \Gamma_{15},\square \Gamma_{16} \vdash \mathtt{F}_{10},\square \Gamma_{11},\square \Gamma_{12}, []\mathtt{F}_8}{-:\Delta_{18},\square \Gamma_{15},\square \Gamma_{16},\square \Gamma_{17} \vdash \Delta_{14},\square \Gamma_{11},\square \Gamma_{12},\square \Gamma_{13}, []\mathtt{F}_{10}, []\mathtt{F}_8} \ \ A45
                                                                                                                                                 \mathtt{h}_1:\Box\Gamma_{15},\Box\Gamma_{17}\vdash(\Box\Gamma_{11},\Box\Gamma_{13}),\mathtt{f}_8,\Box\mathtt{f}_{19}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \mathtt{h}_9: \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}
 \bullet \mathbf{h}_1: (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), []\mathbf{F}_8), \Box \mathbf{F}_{19}
\bullet \mathbf{h}_9: ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box \mathbf{F}_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathbf{F}_{10}), \Box \mathbf{F}_{19})
                                                                                                                                                                                                                                                                                                                                                                                              -: (\Box\Gamma_{15},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{18} \vdash (\Box\Gamma_{11},\Box\Gamma_{13}), (\Box\Gamma_{12},\Delta_{14},[]\mathtt{F}_{10}),[]\mathtt{F}_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                    \begin{array}{c} \longrightarrow \\ & \longrightarrow \\ \hline -: \square\Gamma_{15}, \square\Gamma_{16} \vdash F_{10}, \square\Gamma_{11}, \square\Gamma_{12} \end{array} \xrightarrow{\mathbf{ax/W}} \\ -: \Delta_{18}, \square\Gamma_{15}, \square\Gamma_{16}, \square\Gamma_{17} \vdash \Delta_{14}, \square\Gamma_{11}, \square\Gamma_{12}, \square\Gamma_{13}, []F_{10}, []F_{8} \end{array} A45 
                                                                                                                            \mathtt{h}_1: \Box\Gamma_{14}, \Box\Gamma_{16} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), \mathtt{F}_9, \Box\mathtt{F}_{18}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                h_8: \Box\Gamma_{14}, \Box\Gamma_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, F_9
  \bullet \mathbf{h}_1: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), []\mathbf{F}_9), \Box \mathbf{F}_{18}   \bullet \mathbf{h}_8: ((\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{17}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \mathbf{F}_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), \Box \Gamma_{11}, \Delta_{12}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}, \Box \Gamma_{12}), (\Box \Gamma_{11}
                                                                                                                                                                                                                                                                                                                                                                        -: (\Box\Gamma_{14}, \Box\Gamma_{16}), \Box\Gamma_{15}, \Delta_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), (\Box\Gamma_{11}, \Delta_{13}), []F_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        -: \Box \Gamma_{14}, \Box \Gamma_{15} \vdash \mathsf{F}_9, \Box \Gamma_{10}, \Box \Gamma_{11} \quad \mathsf{ax/W}
                                                                                                                                                                                                                                                                                                                                                                                            -: \overline{\Delta_{17}, \Box \Gamma_{14}, \Box \Gamma_{15}, \Box \Gamma_{16} \vdash \Delta_{13}, \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, []_{F_9}} \quad A45
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\mathtt{h}_1: \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8
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 \bullet \mathbf{h}_1: (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}, [] \mathbf{F}_{10}), (\Box \Gamma_{12}, \Delta_{14}), [] \mathbf{F}_{8}), \Box \mathbf{F}_{16}   \bullet \mathbf{h}_9: ((\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box \mathbf{F}_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, [] \mathbf{F}_{10}), 
                                                                                                                                                                                                                                                                                                                                                                                                                                                         -: (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []F_{10}), (\Box\Gamma_{12}, \Delta_{14}), []F_{8}
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{1}{-:\Delta_{19},\Box\Gamma_{15},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathsf{F}_{10},[]\mathsf{F}_{8}}}
                                                                                                                                                                          \mathtt{h}_1: \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []\mathtt{F}_{10}), \mathtt{F}_8
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 \underbrace{ \begin{array}{c} \bullet_{h_1} : (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8), \Box F_{16} \\ \\ \bullet \vdash_{h_2} : (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8), \Box F_{16} \\ \\ - : (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8 \\ \\ \\ \xrightarrow{\Delta} \\ \bullet \vdash_{h_2} : ((\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8 \\ \\ \xrightarrow{\Delta} \\ \bullet \vdash_{h_2} : ((\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8 \\ \\ \xrightarrow{\Delta} \\ \bullet \vdash_{h_2} : (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8 \\ \\ \xrightarrow{\Delta} \\ \bullet \vdash_{h_2} : (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8 \\ \\ \xrightarrow{\Delta} \\ \bullet \vdash_{h_2} : (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8 \\ \\ \xrightarrow{\Delta} \\ \bullet \vdash_{h_2} : (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, []F_{10}), (\Box \Gamma_{12}, \Delta_{14}), []F_8 \\ \\ \xrightarrow{\Delta} \\ \bullet \vdash_{h_2} : (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, \Box \Gamma_{18}, \Box \Gamma_{18}), \Box \Gamma_{18}, \Box \Gamma_{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{-:\Box\Gamma_{15},\Box\Gamma_{18}\vdash F_8,\Box\Gamma_{11},\Box\Gamma_{13},[]F_{10}}{-:\Delta_{19},\Box\Gamma_{15},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]F_{10},[]F_8}\ A45
                                                                                                                                                                                                    \mathtt{h}_1: \Box\Gamma_{15}, \Box\Gamma_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{f}_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           h_9:\Box\Gamma_{15},\Box\Gamma_{17},\Box F_{16}\vdash\Box\Gamma_{11},\Box\Gamma_{12},F_{10},[]F_8
\bullet h_1: (\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), []F_8), \Box F_{16} 
\bullet h_9: ((\Box \Gamma_{15}, \Box \Gamma_{18}), \Box \Gamma_{17}, \Delta_{19}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), \Box F_{16} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []F_{10}), (\Box \Gamma_{12
                                                                                                                                                                                                                                                                                                                                                                                                                                                         -: (\Box\Gamma_{15},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19} \vdash (\Box\Gamma_{11},\Box\Gamma_{13}), (\Box\Gamma_{12},\Delta_{14},[]\overline{F}_{10}),[]\overline{F}_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{-:\Box\Gamma_{15},\Box\Gamma_{18}\vdash \mathsf{F}_8,\Box\Gamma_{11},\Box\Gamma_{13}}{-:\Delta_{19},\Box\Gamma_{15},\Box\Gamma_{17},\Box\Gamma_{18}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathsf{F}_{10},[]\mathsf{F}_8}\ A45
                                                                                                                                                                                                  \mathtt{h}_1:\Box\Gamma_{15},\Box\Gamma_{18}\vdash(\Box\Gamma_{11},\Box\Gamma_{13}),\mathtt{f}_8
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\bullet_{\mathbf{h}_1}: (\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []F_{10}), []F_8), \Box F_{16}
\bullet_{\mathbf{h}_2}: ((\Box\Gamma_{15}, \Box\Gamma_{18}), \Box\Gamma_{17}, \Delta_{19}), \Box\Gamma
                                                                                                                                                                                                                                                                                                                                                                                                                                                           \overline{-: (\Box\Gamma_{15},\Box\Gamma_{18}),\Box\Gamma_{17},\Delta_{19} \vdash (\Box\Gamma_{11},\Box\Gamma_{13}), (\Box\Gamma_{12},\Delta_{14},[]F_{10}),[]F_{8}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{-:\Box\Gamma_{15},\Box\Gamma_{18} \vdash \mathsf{F}_8,\Box\Gamma_{11},\Box\Gamma_{13}}{-:\Delta_{19},\Box\Gamma_{15},\Box\Gamma_{17},\Box\Gamma_{18} \vdash \Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]\mathsf{F}_{10},[]\mathsf{F}_8} \ \ A45
                                                                                                                                                                        h_1: \Box\Gamma_{14}, \Box\Gamma_{17} \vdash (\Box\Gamma_{10}, \Box\Gamma_{12}), F_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     h_8: \Box\Gamma_{14}, \Box\Gamma_{16}, \Box F_{15} \vdash \Box\Gamma_{10}, \Box\Gamma_{11}, F_9
 \underbrace{- \bullet h_1 : (\Box \Gamma_{14}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), []F_9), \Box F_{15}}_{\bullet h_8 : ((\Box \Gamma_{14}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{18}), \Box F_{15} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), (\Box \Gamma_
                                                                                                                                                                                                                                                                                                                                                                                                                               -: (\Box\Gamma_{14},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{18} \vdash (\Box\Gamma_{10},\Box\Gamma_{12}),(\Box\Gamma_{11},\Delta_{13}), []F_9
                                                                                                                                                                                                                                                                                                                                                                                                                                                     \frac{-:\Box\Gamma_{14},\Box\Gamma_{17}\vdash F_{9},\Box\Gamma_{10},\Box\Gamma_{12}}{-:\Delta_{18},\Box\Gamma_{14},\Box\Gamma_{16},\Box\Gamma_{17}\vdash \Delta_{13},\Box\Gamma_{10},\Box\Gamma_{11},\Box\Gamma_{12},[]F_{9}} \ A45
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\bullet_{\mathbf{h}_{1}}: (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19} \vdash ((\Box\Gamma_{11}, \Box\Gamma_{13}, []F_{10}), (\Box\Gamma_{12}, \Delta_{14}), []F_{8}), F_{18}
\bullet_{\mathbf{h}_{2}}: ((\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19}), F_{18} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []F_{10}), (\Box\Gamma_{12}, \Delta_{14}), (\Box\Gamma_{13}, \Box\Gamma_{17}, \Box\Gamma
                                                                                                                                                                                                                                                                                                                                                                                                                                   -: (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}, []F_{10}), (\Box\Gamma_{12}, \Delta_{14}), []F_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                                                             \frac{ \overline{-:\Box\Gamma_{15},\Box\Gamma_{16} \vdash F_{10},\Box\Gamma_{11},\Box\Gamma_{12},[]F_8} \quad \text{ax/W} }{ \overline{-:\Delta_{19},\Box\Gamma_{15},\Box\Gamma_{16},\Box\Gamma_{17} \vdash \Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]F_{10},[]F_8}} \quad A45
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \mathtt{h}_9: \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}
-: (\Box\Gamma_{15},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{19} \vdash (\Box\Gamma_{11},\Box\Gamma_{13},[]F_{10}), (\Box\Gamma_{12},\Delta_{14}),[]F_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{-:\Box\Gamma_{15},\Box\Gamma_{16}\vdash F_{10},\Box\Gamma_{11},\Box\Gamma_{12}}{-:\Delta_{19},\Box\Gamma_{15},\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]F_{10},[]F_8} \ A45
                                                                                                                                                                                        \mathtt{h}_1: \Box\Gamma_{15}, \Box\Gamma_{17} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), \mathtt{f}_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           \mathtt{h}_9: \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, \mathtt{F}_{10}, []\mathtt{F}_8
   \begin{array}{c} \blacksquare_1 : \square_1 : 1_5, \square_1 : 1_7 : (\square_1 : 1_5, \square_1 : 1_7 : (\square_1 : 1_5, \square_1 : 1_7), \square_7 \\ \bullet_{11} : (\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19} \vdash ((\square_{\Gamma_{11}}, \square_{\Gamma_{13}}), (\square_{\Gamma_{12}}, \Delta_{14}, []_{F_{10}}), []_{F_8}), F_{18} \end{array} \right. \\ \begin{array}{c} \blacksquare_9 : ((\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19}), F_{18} \vdash (\square_{\Gamma_{11}}, \square_{\Gamma_{13}}), (\square_{\Gamma_{12}}, \Delta_{14}, []_{F_{10}}), \\ \bullet_{19} : ((\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19}), F_{18} \vdash (\square_{\Gamma_{11}}, \square_{\Gamma_{13}}), (\square_{\Gamma_{12}}, \Delta_{14}, []_{F_{10}}), \\ \bullet_{19} : ((\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19}), F_{18} \vdash (\square_{\Gamma_{11}}, \square_{\Gamma_{13}}), (\square_{\Gamma_{12}}, \Delta_{14}, []_{F_{10}}), \\ \bullet_{19} : ((\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19}), F_{18} \vdash (\square_{\Gamma_{11}}, \square_{\Gamma_{13}}), (\square_{\Gamma_{12}}, \Delta_{14}, []_{F_{10}}), \\ \bullet_{19} : ((\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19}), F_{18} \vdash (\square_{\Gamma_{11}}, \square_{\Gamma_{13}}), (\square_{\Gamma_{12}}, \Delta_{14}, []_{F_{10}}), \\ \bullet_{19} : ((\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19}), F_{18} \vdash (\square_{\Gamma_{11}}, \square_{\Gamma_{13}}), (\square_{\Gamma_{12}}, \Delta_{14}, []_{F_{10}}), \\ \bullet_{19} : ((\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19}), F_{18} \vdash (\square_{\Gamma_{11}}, \square_{\Gamma_{13}}), (\square_{\Gamma_{12}}, \Delta_{14}, []_{F_{10}}), \\ \bullet_{19} : ((\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19}), F_{18} \vdash (\square_{\Gamma_{11}}, \square_{\Gamma_{13}}), (\square_{\Gamma_{12}}, \Delta_{14}, []_{F_{10}}), \\ \bullet_{19} : (\square_{\Gamma_{15}}, \square_{\Gamma_{17}}), \square_{\Gamma_{16}}, \Delta_{19}), \\ \bullet_{19} : (\square_{\Gamma_{15}}, \square_
                                                                                                                                                                                                                                                                                                                                                                                                                                       -: (\Box\Gamma_{15}, \Box\Gamma_{17}), \Box\Gamma_{16}, \Delta_{19} \vdash (\Box\Gamma_{11}, \Box\Gamma_{13}), (\Box\Gamma_{12}, \Delta_{14}, []F_{10}), []F_{8}
                                                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{-:\Box\Gamma_{15},\Box\Gamma_{16}\vdash F_{10},\Box\Gamma_{11},\Box\Gamma_{12},[]F_{8}}{-:\Box\Gamma_{15},\Box\Gamma_{16}\vdash F_{10},\Box\Gamma_{11},\Box\Gamma_{12},[]F_{8}} \text{ ax/W}}{-:\Delta_{19},\Box\Gamma_{15},\Box\Gamma_{16},\Box\Gamma_{17}\vdash\Delta_{14},\Box\Gamma_{11},\Box\Gamma_{12},\Box\Gamma_{13},[]F_{10},[]F_{8}} \text{ } A45
                                                                                                                                                                                            \mathtt{h}_1:\Box\Gamma_{15},\Box\Gamma_{17}\vdash(\Box\Gamma_{11},\Box\Gamma_{13}),\mathtt{F}_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             h_9: \Box\Gamma_{15}, \Box\Gamma_{16} \vdash \Box\Gamma_{11}, \Box\Gamma_{12}, F_{10}
 \underbrace{- \bullet \mathsf{h}_1 : (\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{19} \vdash ((\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathsf{F}_{10}), []\mathsf{F}_8), \mathsf{F}_{18} }^{A45} }_{\bullet \mathsf{h}_9 : ((\Box \Gamma_{15}, \Box \Gamma_{17}), \Box \Gamma_{16}, \Delta_{19}), \mathsf{F}_{18} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{14}, []\mathsf{F}_{10}), (\Box \Gamma_{12}, \Delta_{13}, \Gamma_{17}), (\Box \Gamma_{17}, \Box \Gamma_{17}, \Delta_{19}), \mathsf{F}_{18} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}), (\Box \Gamma_{12}, \Delta_{13}, \Gamma_{17}, \Delta_{19}, \Gamma_{18}, \Delta_{19}), \mathsf{F}_{18} \vdash (\Box \Gamma_{11}, \Box \Gamma_{13}, \Delta_{19}, \Gamma_{18}, \Delta_{19}, \Delta_
                                                                                                                                                                                                                                                                                                                                                                                                                                       -: (\Box\Gamma_{15},\Box\Gamma_{17}),\Box\Gamma_{16},\Delta_{19} \vdash (\Box\Gamma_{11},\Box\Gamma_{13}), (\Box\Gamma_{12},\Delta_{14},[]F_{10}),[]F_8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{-:\Box\Gamma_{15},\Box\Gamma_{16}\vdash F_{10},\Box\Gamma_{11},\Box\Gamma_{12}}{-:\Box\Gamma_{15},\Box\Gamma_{16}\vdash F_{10},\Box\Gamma_{11},\Box\Gamma_{12}} \text{ ax/W}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                 -: \overline{\Delta_{19}, \Box \Gamma_{15}, \Box \Gamma_{16}, \Box \Gamma_{17} \vdash \Delta_{14}, \Box \Gamma_{11}, \Box \Gamma_{12}, \Box \Gamma_{13}, []F_{10}, []F_8} \quad A45
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 \begin{array}{c} h_1: \Box \Gamma_{14}, \Box \Gamma_{16} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), F_9 \\ \hline \bullet h_1: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{18} \vdash ((\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), []F_9), F_{17} \end{array} \\ A45 \\ \hline \begin{array}{c} h_8: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{18}, F_{17} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), []F_9 \\ \hline \\ -: (\Box \Gamma_{14}, \Box \Gamma_{16}), \Box \Gamma_{15}, \Delta_{18} \vdash (\Box \Gamma_{10}, \Box \Gamma_{12}), (\Box \Gamma_{11}, \Delta_{13}), []F_9 \\ \hline \\ -: \Box \Gamma_{14}, \Box \Gamma_{15} \vdash F_9, \Box \Gamma_{10}, \Box \Gamma_{11} \end{array} \\ \hline \\ -: \Delta_{18}, \Box \Gamma_{14}, \Box \Gamma_{15}, \Box \Gamma_{16} \vdash \Delta_{13}, \Box \Gamma_{10}, \Box \Gamma_{11}, \Box \Gamma_{12}, []F_9 \end{array} \\ A45 \\ \end{array}
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$$\frac{h_1: \Box_{\Gamma_{13}} + \Box_{\Gamma_{11}}, F_7}{\bullet h_1: \Box_{\Gamma_{13}}, \Delta_{14}, F_9 \to F_{10} + (\Box_{\Gamma_{11}}, \Delta_{12}), [F_7]}{\bullet h_1: \Box_{\Gamma_{13}}, \Delta_{14}, F_9 \to F_{10} + (\Box_{\Gamma_{11}}, \Delta_{12}), [F_7]} A45 \xrightarrow{h_8: \Box_{\Gamma_{13}}, \Delta_{14}, F_9 \to F_{10} + \Box_{\Gamma_{11}}, \Delta_{12}} \text{Cut}$$

$$-: \Box_{\Gamma_{13}}, \Delta_{14}, F_9 \to F_{10} + \Box_{\Gamma_{11}}, \Delta_{12}$$

$$-: \Box_{\Gamma_{13}}, \Delta_{14}, F_9 \to F_{10} + \Box_{\Gamma_{11}}, \Delta_{12}$$

$$\bullet h_1: \Box_{\Gamma_{13}} + F_7, \Box_{\Gamma_{11}} \text{ ax/W}$$

$$\bullet h_1: \Box_{\Gamma_{13}} + F_7, \Box_{\Gamma_{11}} \text{ ax/W}$$

$$\bullet h_1: \Delta_{14}, \Box_{\Gamma_{13}} + F_7, \Box_{\Gamma_{11}} \text{ ax/W}$$

$$\bullet h_1: \Delta_{14}, \Box_{\Gamma_{13}} + \Delta_{12}, F_9, \Box_{\Gamma_{11}} \text{ ax/W}$$

$$\bullet h_1: \Delta_{14}, \Box_{\Gamma_{13}} + \Delta_{12}, F_9, \Box_{\Gamma_{11}} \text{ ax/W}$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_8, \Box_{\Gamma_{11}} \text{ ax/W}$$

$$\bullet h_1: \Box_{\Gamma_{14}}, \Delta_{15}, F_{10} \to F_{11} + (\Box_{\Gamma_{12}}, \Delta_{13}, [F_8), D_F_1 \text{ ax/W}}$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_8, \Box_{\Gamma_{13}} \text{ ax/W}$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_8, \Box_{\Gamma_{13}} \text{ ax/W}$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_{10} \to F_{11} + (\Box_{\Gamma_{12}}, \Delta_{13}, [F_8), D_F_1 \text{ ax/W}}$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_{10} \to F_{11} + (\Box_{\Gamma_{12}}, \Delta_{13}, [F_8), D_F_1 \text{ ax/W}}$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_{10} \to F_{11} + (\Box_{\Gamma_{12}}, \Delta_{13}, [F_8), D_F_1 \text{ ax/W}}$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_{10} \to F_{11} + (\Box_{\Gamma_{12}}, \Delta_{13}, [F_8), D_F_1 \text{ ax/W}}$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_{10} \to F_{11} + (\Box_{\Gamma_{12}}, \Delta_{13}, [F_8), D_F_1 \text{ ax/W}}$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_{10} \to A_{13}, F_{10}, \Box_{\Gamma_{12}}, [F_8]$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_{10} \to A_{13}, F_{10}, \Box_{\Gamma_{12}}, [F_8]$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_{10} \to A_{13}, F_{10}, \Box_{\Gamma_{12}}, [F_8]$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, A_{13}, [F_8), F_{10}} \to F_{11} + (\Box_{\Gamma_{12}}, A_{13}, [F_8], D_{\Gamma_{12}}, [F_8]$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, A_{13}, [F_8], F_{10}} \to F_{11} + (\Box_{\Gamma_{12}}, A_{13}, [F_8], D_{\Gamma_{12}}, [F_8]$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, A_{13}, [F_8], F_{10}} \to F_{11} + (\Box_{\Gamma_{12}}, A_{13}, [F_8], D_{\Gamma_{12}}, [F_8], D_{\Gamma_{12}}, [F_8]$$

$$\bullet h_1: \Box_{\Gamma_{14}} + \Box_{\Gamma_{12}}, F_{10}} \to F_{11} + (\Box_{\Gamma_$$

• Case rule \wedge_L

$$\begin{array}{c} \mathbf{h}_{1}: \square\Gamma_{13} \vdash \square\Gamma_{11}, \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{1}: \square\Gamma_{13}, \Delta_{14}, \mathbf{F}_{9} \land \mathbf{F}_{10} \vdash (\square\Gamma_{11}, \Delta_{12}), []\mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{3}: \square\Gamma_{13}, \Delta_{14}, \mathbf{F}_{9} \land \mathbf{F}_{10} \vdash (\square\Gamma_{11}, \Delta_{12}), []\mathbf{F}_{7} \\ \hline \\ -: \square\Gamma_{13}, \Delta_{14}, \mathbf{F}_{9} \land \mathbf{F}_{10} \vdash \square\Gamma_{11}, \Delta_{12} \\ \hline \\ \hline \bullet \mathbf{h}_{1}: \square\Gamma_{13} \vdash \mathbf{F}_{7}, \square\Gamma_{11} \\ \hline \bullet \mathbf{h}_{1}: \square\Gamma_{13} \vdash \mathbf{F}_{7}, \square\Gamma_{11} \\ \hline \bullet \mathbf{h}_{1}: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{9}, \square\Gamma_{13} \vdash \Delta_{12}, \square\Gamma_{11}, []\mathbf{F}_{7} \\ \hline \\ \bullet \mathbf{h}_{1}: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{9}, \square\Gamma_{13} \vdash \Delta_{12}, \square\Gamma_{11} \\ \hline \\ -: \Delta_{14}, \Gamma_{10}, \mathbf{F}_{9}, \square\Gamma_{13} \vdash \Delta_{12}, \square\Gamma_{11} \\ \hline \\ -: \Delta_{14}, \square\Gamma_{13}, \mathbf{F}_{9} \land \mathbf{F}_{10} \vdash \Delta_{12}, \square\Gamma_{11} \\ \hline \end{array} \begin{array}{c} \mathbf{h}_{8}: \square\Gamma_{13}, \mathbf{F}_{9}, \mathbf{F}_{10}, \Delta_{14}, []\mathbf{F}_{7} \vdash \square\Gamma_{11}, \Delta_{12} \\ \hline \\ \mathbf{h}_{8}: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{9}, \square\Gamma_{13}, []\mathbf{F}_{7} \vdash \Delta_{12}, \square\Gamma_{11} \\ \hline \\ \mathbf{h}_{8}: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{9}, \square\Gamma_{13}, []\mathbf{F}_{7} \vdash \Delta_{12}, \square\Gamma_{11} \\ \hline \\ \mathbf{h}_{6} \mathbf{cut} \end{array} \begin{array}{c} \mathbf{ax/W} \\ \mathbf{h}_{6} \mathbf{cut} \\ \mathbf{h}_{7} \mathbf{cut} \\ \mathbf{h}_{8} \mathbf{cut} \\ \mathbf{h}_{7} \mathbf{cut} \\$$

• Case rule \vee_L

• Case rule AT

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 \begin{array}{c} \mathbf{h}_1: \Box \Gamma_{12}, []\mathbf{F}_9 \vdash \Box \Gamma_{10}, \mathbf{F}_7 \\ \bullet \mathbf{h}_1: (\Box \Gamma_{12}, []\mathbf{F}_9), \Delta_{\underline{13}} \vdash (\Box \underline{\Gamma}_{10}, \Delta_{11}), []\mathbf{F}_7 \end{array} \ A45 \quad \begin{array}{c} \mathbf{h}_8: \Box \Gamma_{12}, \mathbf{F}_9, \Delta_{13}, []\mathbf{F}_7, []\mathbf{F}_9 \vdash \Box \Gamma_{10}, \Delta_{11} \\ \bullet \mathbf{h}_8: ((\Box \Gamma_{12}, []\mathbf{F}_9), \Delta_{13}), []\mathbf{F}_7 \vdash \Box \Gamma_{10}, \Delta_{11} \end{array} \ AT \quad \text{Cut} 
                                                                                                                                                                                                                                     -: (\Box\Gamma_{12}, []F_9), \Delta_{13} \vdash \Box\Gamma_{10}, \Delta_{11}
                    \frac{}{\bullet h_1:\underline{\Delta_{13},F_9,\Box\Gamma_{12},[]F_9\vdash\Delta_{11},\Box\Gamma_{10},[]F_7}}\xrightarrow{ax/W}\frac{}{h_8:\Delta_{13},F_9,\Box\Gamma_{12},[]F_7,[]F_9\vdash\Delta_{11},\Box\Gamma_{10}}}\xrightarrow{ax/W}
                                                                                                                                                                                                                               -:\Delta_{13},\mathtt{F}_{9},\square\Gamma_{12},[]\mathtt{F}_{9}\vdash\Delta_{11},\square\Gamma_{10}
                                                                                                                                                                                                                                               -:\Delta_{13},\Box\Gamma_{12},[]\mathsf{F}_9\vdash\Delta_{11},\Box\Gamma_{10}
                            \frac{h_1:\square\Gamma_{12}\vdash\square\Gamma_{10},F_7}{\bullet h_1:\square\Gamma_{12},\Delta_{13},[]F_9\vdash(\square\Gamma_{10},\Delta_{11}),[]F_7} \quad A45 \quad \frac{h_8:\square\Gamma_{12},F_9,\Delta_{13},[]F_7,[]F_9\vdash\square\Gamma_{10},\Delta_{11}}{\bullet h_8:(\square\Gamma_{12},\Delta_{13},[]F_9),[]F_7\vdash\square\Gamma_{10},\Delta_{11}}
                                                                                                                                                                                                                                        -: \Box\Gamma_{12}, \Delta_{13}, []\mathtt{F}_9 \vdash \Box\Gamma_{10}, \Delta_{11}
                    \underbrace{\bullet_{\mathbf{h}_1}:\Delta_{13},\mathsf{F}_9,\Box\Gamma_{12},[]\mathsf{F}_9\vdash\Delta_{11},\Box\Gamma_{10},[]\mathsf{F}_7}_{\mathsf{h}_2} \ \ \mathsf{ax/W} \ \ \frac{\mathsf{ax/W}}{\mathsf{h}_8:\Delta_{13},\mathsf{F}_9,\Box\Gamma_{12},[]\mathsf{F}_7,[]\mathsf{F}_9\vdash\Delta_{11},\Box\Gamma_{10}}_{\mathsf{h}_2\mathsf{Cut}} \ \ \frac{\mathsf{ax/W}}{\mathsf{h}_2\mathsf{Cut}} 
                                                                                                                                                                                                                        -: \Delta_{13}, \mathsf{F}_9, \Box \Gamma_{12}, []\mathsf{F}_9 \vdash \Delta_{11}, \Box \Gamma_{10} \\ -: \Delta_{13}, \mathsf{F}_9, \Box \Gamma_{12}, []\mathsf{F}_9 \vdash \Delta_{11}, \Box \Gamma_{10} \\ -: \Delta_{11}, \Box \Gamma_{10} \\ -: \Delta_{12}, \Box \Gamma_{12}, []\mathsf{F}_9 \vdash \Delta_{11}, \Box \Gamma_{10} \\ -: \Delta_{12}, \Box \Gamma_{12}, \Box \Gamma_{12}, []\mathsf{F}_9 \vdash \Delta_{11}, \Box \Gamma_{10} \\ -: \Delta_{12}, \Box \Gamma_{12}, \Box \Gamma_{12}, \Box \Gamma_{13}, \Box \Gamma_{14}, \Box \Gamma_{15}, \Box
                                                                                                                                                                                                                                            -:\Delta_{13},\Box\Gamma_{12},[]\mathsf{F}_9\vdash\Delta_{11},\Box\Gamma_{10}
                                                                                                                                                                      \frac{\mathbf{h}_1: \Box \Gamma_{11} \vdash \Box \Gamma_9, \mathbf{f}_8}{\bullet \mathbf{h}_1: \Box \Gamma_{11}, \underline{\Delta}_{12} \vdash (\Box \Gamma_9, \underline{\Delta}_{10}), []\mathbf{f}_8} \quad A45 \quad \frac{\mathbf{h}_7: \Box \Gamma_{11}, \mathbf{f}_8, \underline{\Delta}_{12}, []\mathbf{f}_8 \vdash \Box \Gamma_9, \underline{\Delta}_{10}}{\bullet \mathbf{h}_7: (\Box \Gamma_{11}, \underline{\Delta}_{12}), []\mathbf{f}_8 \vdash \Box \Gamma_9, \underline{\Delta}_{10}} \quad \underbrace{AT}_{\mathbf{Cut}}
                                                                                                                                                                                                                                                                                                                                                               -: \Box \Gamma_{11}, \Delta_{12} \vdash \Box \Gamma_{9}, \Delta_{10}
                   \frac{-:\Delta_{12},\Box\Gamma_{11}\vdash\Delta_{10},F_8,\Box\Gamma_9}{-:\Delta_{12},\Box\Gamma_{11}\vdash\Delta_{10},F_8,\Box\Gamma_9} \text{ ax/W } \frac{\bullet h_1:\Delta_{12},F_8,\Box\Gamma_{11}\vdash\Delta_{10},\Box\Gamma_9,[]F_8}{-:\Delta_{12},F_8,\Box\Gamma_{11}\vdash\Delta_{10},\Box\Gamma_9} \frac{\text{ax/W}}{h_7:\Delta_{12},F_8,\Box\Gamma_{11},[]F_8\vdash\Delta_{10},\Box\Gamma_9} \text{ hCut}
                                                                                                                                                                                                                                                                                                                                            -:\Delta_{12},\mathsf{F}_8,\Box\Gamma_{11}\vdash\Delta_{10},\Box\Gamma_9 scut
                                                                                                                                                                                                                                                                     -:\Delta_{12},\Box\Gamma_{11}\vdash\Delta_{10},\Box\Gamma_{9}
                         \underbrace{ \begin{array}{c} \mathbf{h}_1: \Box \Gamma_{13}, [] \mathbf{F}_{10} \vdash \Box \Gamma_{11}, \mathbf{F}_8, \Box \mathbf{F}_{15} \\ \bullet \mathbf{h}_1: (\Box \Gamma_{13}, [] \mathbf{F}_{10}), \Delta_{14} \vdash (\Box \Gamma_{11}, \Delta_{12}, [] \mathbf{F}_8), \Box \mathbf{F}_{15} \end{array}}_{\phantom{a} A45} \underbrace{ \begin{array}{c} \mathbf{h}_9: \Box \Gamma_{13}, \mathbf{F}_{10}, \Delta_{14}, \Box \mathbf{F}_{15}, [] \mathbf{F}_{10} \vdash \Box \Gamma_{11}, \Delta_{12}, [] \mathbf{F}_8 \\ \bullet \mathbf{h}_9: ((\Box \Gamma_{13}, [] \mathbf{F}_{10}), \Delta_{14}), \Box \mathbf{F}_{15} \vdash \Box \Gamma_{11}, \Delta_{12}, [] \mathbf{F}_8 \end{array}}_{\phantom{a} Cut} \underbrace{ \begin{array}{c} \mathbf{A} & \mathbf{A
                                                                                                                                                                                                                                                                          -: (\Box \Gamma_{13}, []F_{10}), \Delta_{14} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{8}
                    \bullet_{h_1}:\Delta_{14},F_{10},\Box\Gamma_{13},[]F_{10}\vdash\Box F_{15},\Delta_{12},\Box\Gamma_{11},[]F_8 \qquad \mathsf{ax/W} \qquad \bullet_{h_9}:\Box F_{15},\Delta_{14},F_{10},\Box\Gamma_{13},[]F_{10}\vdash\Delta_{12},\Box\Gamma_{11},[]F_8 \qquad \mathsf{ax/W} \qquad \bullet_{h_{12}}
                                                                                                                                                                                                                                                          -: \Delta_{14}, \mathsf{F}_{10}, \Box \Gamma_{13}, []\mathsf{F}_{10} \vdash \Delta_{12}, \Box \Gamma_{11}, []\mathsf{F}_{8}
                                                                                                                                                                                                                                                                                  -: \Delta_{14}, \Box \Gamma_{13}, []F_{10} \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{8}
                              \frac{ \underbrace{ \begin{array}{c} \mathbf{h}_1: \Box \Gamma_{13} \vdash \Box \Gamma_{11}, \mathbf{F}_8, \Box \mathbf{F}_{15} \\ \hline \bullet \mathbf{h}_1: \underline{\Box \Gamma_{13}, \Delta_{14}, [] \mathbf{F}_{10} \vdash (\Box \Gamma_{11}, \Delta_{12}, [] \mathbf{F}_8), \Box \mathbf{F}_{15} \\ \end{array}} }_{\bullet \mathbf{h}_9: \underline{\Box \Gamma_{13}, \Delta_{14}, [] \mathbf{F}_{10}, \Delta_{14}, [] \mathbf{F}_{10} \vdash \Box \Gamma_{11}, \Delta_{12}, [] \mathbf{F}_8 \\ \bullet \mathbf{h}_9: \underline{\Box \Gamma_{13}, \Delta_{14}, [] \mathbf{F}_{10}, \Box \mathbf{F}_{15} \vdash \Box \Gamma_{11}, \Delta_{12}, [] \mathbf{F}_8 \\ \bullet \mathbf{h}_9: \underline{\Box \Gamma_{13}, \Delta_{14}, [] \mathbf{F}_{10}, \Box \mathbf{F}_{15} \vdash \Box \Gamma_{11}, \Delta_{12}, [] \mathbf{F}_8 \\ \bullet \mathbf{h}_9: \underline{\Box \Gamma_{13}, \Delta_{14}, [] \mathbf{F}_{10}, \Box \mathbf{F}_{15} \vdash \Box \Gamma_{11}, \Delta_{12}, [] \mathbf{F}_8 \\ \end{array}} }_{\bullet \mathbf{tt}}
                                                                                                                                                                                                                                                                               -: \Box\Gamma_{13}, \Delta_{14}, []\mathtt{F}_{10} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathtt{F}_{8}
                   \underbrace{\bullet_{h_1}:\Delta_{14},F_{\underline{10}},\square\Gamma_{13}}_{\bullet h_1},\underbrace{[]F_{10}\vdash\square F_{15},\Delta_{12},\square\Gamma_{11},[]F_8}_{} \xrightarrow{ax/W} \xrightarrow{h_9:\square F_{15},\Delta_{14},F_{10},\square\Gamma_{13},[]F_{10}\vdash\Delta_{12},\square\Gamma_{11},[]F_8}_{h_{Cut}}
                                                                                                                                                                                                                                                            \frac{-:\Delta_{14}, \mathsf{F}_{10}, \Box\Gamma_{13}, []\mathsf{F}_{10} \vdash \Delta_{12}, \Box\Gamma_{11}, []\mathsf{F}_{8}}{\Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14}, \Box\Gamma_{14}, \Box\Gamma_{15}, \Box\Gamma_{15}
                                                                                                                                                                                                                                                                                -: \Delta_{14}, \Box \Gamma_{13}, []F_{10} \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{8}
\frac{\mathbf{h}_1: \Box \Gamma_{13} \vdash \Box \Gamma_{11}, \mathbf{f}_8, []\mathbf{f}_{10}}{\bullet \mathbf{h}_1: \underline{\Box \Gamma_{13}, \Delta_{14} \vdash (\Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_8)}, []\mathbf{f}_{10}} \quad A45 \quad \frac{\mathbf{h}_9: \Box \Gamma_{13}, \mathbf{f}_{10}, \Delta_{14}, []\mathbf{f}_{10} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_8}{\bullet \mathbf{h}_9: (\Box \Gamma_{13}, \Delta_{14}), []\mathbf{f}_{10} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_8} \quad AT \\ \hline \bullet \mathbf{h}_9: (\Box \Gamma_{13}, \Delta_{14}), []\mathbf{f}_{10} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_8} \quad Cut \\ \hline
                                                                                                                                                                                             \frac{1276 \text{ G} \times 316}{-:\Box \Gamma_{13}, \Delta_{14} \vdash \Box \Gamma_{11}, \Delta_{12}, []_{F_8}}
   Axioms assumed:
   inf : C:MSFormula |-- True ; C':MSFormula
   inf : False ; C:MSFormula | -- C':MSFormula
   inf : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
   suc(hx:FNat) : C:MSFormula |-- True ; C':MSFormula
   suc(hx:FNat) : False ; C:MSFormula |-- C':MSFormula
   suc(hx:FNat) : P:Prop ; C:MSFormula |-- P:Prop ; C':MSFormula
                \frac{\mathbf{h}_{1}: \Box \Gamma_{13}, []\mathbf{F}_{10} \vdash \Box \Gamma_{11}, \mathbf{F}_{8}}{\underbrace{\bullet \mathbf{h}_{1}: (\Box \Gamma_{13}, []\mathbf{F}_{10}), \Delta_{15} \vdash (\Box \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{8}), \mathbf{F}_{14}}}_{\mathbf{4}45} \quad \frac{\mathbf{h}_{9}: \Box \Gamma_{13}, \mathbf{F}_{10}, \mathbf{F}_{14}, \Delta_{15}, []\mathbf{F}_{10} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{8}}{\bullet \mathbf{h}_{9}: ((\Box \Gamma_{13}, []\mathbf{F}_{10}), \Delta_{15}), \mathbf{F}_{14} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{8}}_{\mathbf{Cut}}} \quad \frac{AT}{\bullet \mathbf{h}_{9}: (\Box \Gamma_{13}, []\mathbf{F}_{10}), \Delta_{15}), \mathbf{F}_{14} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{8}}_{\mathbf{Cut}}}
                                                                                                                                                                                                                                                       -: (\Box \Gamma_{13}, []\mathtt{F}_{10}), \Delta_{15} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathtt{F}_{8}
                                                                                                                                                                                                                                                                                                 \overline{-:\Box\Gamma_{13},[]\mathtt{F}_{10}\vdash \mathtt{F}_{8},\Box\Gamma_{11}} ax/W
                                                                                                                                                                                                                                                            \frac{\Box}{-:\Delta_{15},\Box\Gamma_{13},[]\mathtt{F}_{10}\vdash\Delta_{12},\Box\Gamma_{11},[]\mathtt{F}_{8}}\ \ A45
```

• Case rule \perp_L

$$\begin{array}{c} \begin{array}{c} \mathbf{h}_{1}: \square\Gamma_{11} \vdash \square\Gamma_{9}, \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{1}: \square\Gamma_{11}, \bot, \Delta_{12} \vdash (\square\Gamma_{9}, \Delta_{10}), []\mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_{1}: \square\Gamma_{11}, \bot, \Delta_{12} \vdash (\square\Gamma_{9}, \Delta_{10}), []\mathbf{F}_{7} \\ \hline \\ -: \square\Gamma_{11}, \bot, \Delta_{12} \vdash \square\Gamma_{9}, \Delta_{10} \\ \hline \\ -: \bot, \Delta_{12}, \square\Gamma_{11} \vdash \Delta_{10}, \square\Gamma_{9} \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12} \vdash \square\Gamma_{10}, \mathbf{F}_{8}, \square \mathbf{F}_{14} \\ \hline \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{13} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}), \square \mathbf{F}_{14} \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{13} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}), \square \mathbf{F}_{14} \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{13} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}), \square \mathbf{F}_{14} \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{13} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{13} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{13} \vdash (\square\Gamma_{10}, \mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \Delta_{13} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}), \bot \\ \hline \\ -: \square\Gamma_{12}, \Delta_{13} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12} \vdash \square\Gamma_{10}, \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12} \vdash \square\Gamma_{10}, \mathbf{F}_{8} \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}), \mathbf{F}_{13} \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}), \mathbf{F}_{13} \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}), \mathbf{F}_{13} \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, []\mathbf{F}_{8}) \\ \hline \\ \bullet \mathbf{h}_{1}: \square\Gamma_{12}, \bot, \Delta_{14} \vdash (\square\Gamma_{10}, \Delta_{11}, [$$

\bullet Case rule I

$$\begin{array}{c} \begin{array}{c} h_1: \square\Gamma_{12} \vdash \square\Gamma_{9}, F_{7} \\ \hline \bullet h_1: \square\Gamma_{12}, \Delta_{13}, p_{11} \vdash (\square\Gamma_{9}, \Delta_{10}, p_{11}), []F_{7} \end{array} \end{array} A45 \end{array} \xrightarrow{\bullet h_8: (\square\Gamma_{12}, \Delta_{13}, p_{11}), []F_{7} \vdash \square\Gamma_{9}, \Delta_{10}, p_{11}} Cut \\ \hline \\ -: \square\Gamma_{12}, \Delta_{13}, p_{11} \vdash \square\Gamma_{9}, \Delta_{10}, p_{11} \\ \hline \\ -: \Delta_{13}, \square\Gamma_{12}, p_{11} \vdash \Delta_{10}, \square\Gamma_{9}, p_{11} \end{array} I \\ \hline \\ h_1: \square\Gamma_{13} \vdash \square\Gamma_{10}, F_{8}, \squareF_{15} \\ \hline \\ \bullet h_1: \square\Gamma_{13}, \Delta_{14}, p_{12} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}), \squareF_{15} \end{array} A45 \xrightarrow{\bullet h_9: (\square\Gamma_{13}, \Delta_{14}, p_{12}), \squareF_{15} \vdash \square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} Cut \\ \hline \\ -: \square\Gamma_{13}, \Delta_{14}, p_{12} \vdash \square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8} \xrightarrow{\bullet h_{1}: \square\Gamma_{13} \vdash \square\Gamma_{10}, F_{8}} A45 \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8})} I \\ \hline \\ \bullet h_1: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}), p_{12} \end{array} A45 \xrightarrow{\bullet h_{9}: (\square\Gamma_{13}, \Delta_{14}), p_{12} \vdash \square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} Cut \\ \hline \\ \bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}), p_{12} \xrightarrow{\bullet h_{9}: (\square\Gamma_{13}, \Delta_{14}), p_{12} \vdash \square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} Cut \\ \hline \\ -: \square\Gamma_{13}, \Delta_{14} \vdash \square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8} \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} A45 \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} Cut \\ \hline \\ -: \square\Gamma_{13}, \Delta_{14} \vdash \square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8} \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} A45 \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} A45 \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} A45 \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} A45 \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} A45 \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} A45 \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14} \vdash (\square\Gamma_{10}, (\Delta_{11}, p_{12}), []F_{8}} A45 \xrightarrow{\bullet h_{1}: \square\Gamma_{13}, \Delta_{14}, \square\Gamma_{13}, \square\Gamma$$

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: \Box \Gamma_{13} \vdash \Box \Gamma_{10}, \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_{1}: \Box \Gamma_{13}, \Delta_{15}, \mathbf{p}_{12} \vdash (\Box \Gamma_{10}, (\Delta_{11}, \mathbf{p}_{12}), [] \mathbf{F}_{8}), \mathbf{F}_{14} \end{array} }{ \begin{array}{c} \bullet \mathbf{h}_{9}: (\Box \Gamma_{13}, \Delta_{15}, \mathbf{p}_{12}), \mathbf{F}_{14} \vdash \Box \Gamma_{10}, (\Delta_{11}, \mathbf{p}_{12}), [] \mathbf{F}_{8} \\ \hline \\ -: \Box \Gamma_{13}, \Delta_{15}, \mathbf{p}_{12} \vdash \Box \Gamma_{10}, (\Delta_{11}, \mathbf{p}_{12}), [] \mathbf{F}_{8} \\ \hline \\ -: \Delta_{15}, \Box \Gamma_{13}, \mathbf{p}_{12} \vdash \Delta_{11}, \Box \Gamma_{10}, \mathbf{p}_{12}, [] \mathbf{F}_{8} \end{array} } I \end{array} } \\ \\ \mathbf{Cut}$$

$$\begin{array}{c} \mathbf{h}_1: \Box \Gamma_{11} \vdash \Box \Gamma_{9}, \mathbf{F}_{7} \\ \bullet \mathbf{h}_1: \Box \Gamma_{11}, \top, \Delta_{12} \vdash (\Box \Gamma_{9}, \Delta_{10}), [] \mathbf{F}_{7} \\ \bullet \mathbf{h}_3: \Box \Gamma_{11}, \top, \Delta_{12} \vdash (\Box \Gamma_{9}, \Delta_{10}) \\ \hline -: \Box \Gamma_{11}, \top, \Delta_{12} \vdash \Box \Gamma_{9}, \Delta_{10} \\ \hline \bullet \mathbf{h}_3: \Box \Gamma_{11}, \top, \Delta_{12} \vdash \Box \Gamma_{9}, \Delta_{10} \\ \hline -: \Box \Gamma_{11}, \top, \Delta_{12} \vdash \Box \Gamma_{9}, \Delta_{10} \\ \hline \bullet \mathbf{h}_1: \top, \Delta_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box \Gamma_{9}, [] \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Box \Gamma_{11} \vdash \Delta_{10}, \Box \Gamma_{9}, [] \mathbf{F}_{7} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12} \vdash \Box \Gamma_{10}, \mathbf{F}_{8}, \Box \mathbf{F}_{14} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \top, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \Box \mathbf{F}_{14} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \top, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \Box \mathbf{F}_{14} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \top, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \Box \mathbf{F}_{14} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \top, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \Box \mathbf{F}_{14} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \top, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \Box \mathbf{F}_{14} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Box \Gamma_{10}, \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Box \Gamma_{10}, \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Box \Gamma_{10}, \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Delta_{13}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box \Gamma_{10}, [] \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \top \\ \hline -: \Box \Gamma_{12}, \Delta_{13} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \top \\ \hline -: \Box \Gamma_{12}, \Delta_{13} \vdash \Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \top \\ \hline -: \Box \Gamma_{12}, \Delta_{13}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box \Gamma_{10}, [] \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \top \\ \hline -: \Box \Gamma_{12}, \Delta_{13}, \Box \Gamma_{12} \vdash \Delta_{11}, \Box \Gamma_{10}, [] \mathbf{F}_{8} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \Delta_{13} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \top \\ \hline -: \Box \Gamma_{12}, \nabla, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \top \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_{12}, \nabla, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \top \\ \hline -: \Box \Gamma_{12}, \nabla, \Delta_{14} \vdash (\Box \Gamma_{10}, \Delta_{11}, [] \mathbf{F}_{8}), \Box \mathbf{h}_{12}, \Box \mathbf{h}_{13}, \Box \mathbf{h}_{13},$$

6.8 Status of \rightarrow_L : OK

• Case rule \rightarrow_R

$$\frac{\frac{h_1:\Delta_{14}\vdash F_7,F_{13},\Delta_{12},F_{10}\to F_{11}}{\bullet h_1:\Delta_{14},F_7\to F_8\vdash (\Delta_{12},F_{10}\to F_{11}),F_{13}}\to L}{\frac{\bullet h_1:\Delta_{14},F_7\to F_8\vdash (\Delta_{12},F_{10}\to F_{11}),F_{13}}{\bullet h_9:(\Delta_{14},F_7\to F_8)\vdash (\Delta_{12},F_{10}\to F_{11})}\to L} \xrightarrow{\bullet h_9:F_{10},F_{13},\Delta_{14},F_7\to F_8\vdash F_{11},\Delta_{12}}\to R} Cut} \xrightarrow{-:\Delta_{14},F_{10}\vdash \Delta_{12},F_{10}\to F_{11}}\to L} \xrightarrow{h_9:\Delta_{14},F_{10}\to F_8\vdash \Delta_{12},F_{10}\to F_{11}}\to L} \xrightarrow{\bullet h_1:\Delta_{14},F_{10},F_7\to F_8\vdash \Delta_{12},F_{11},F_{13}}} \xrightarrow{inv-th/ax} \xrightarrow{h_9:\Delta_{14},F_{10},F_{13},F_7\to F_8\vdash \Delta_{12},F_{11}}\to R} \xrightarrow{\bullet h_1:\Delta_{14},F_{10},F_7\to F_8\vdash \Delta_{12},F_{11}}\to R} \xrightarrow{\bullet h_1:\Delta_{14},F_{10}\to F_{11}}\to R} \xrightarrow{\bullet h_1:\Delta_1,F_{11}\to F_{11}}\to R} \xrightarrow{\bullet h_1$$

• Case rule \wedge_R

$$\frac{\underbrace{\frac{h_1:\Delta_{14}\vdash F_7,F_{13},\Delta_{12},F_{10}\land F_{11}}{\bullet h_1:\Delta_{14},F_7\to F_8\vdash (\Delta_{12},F_{10}\land F_{11}),F_{13}}}_{\bullet h_1:\Delta_{14},F_7\to F_8\vdash (\Delta_{12},F_{10}\land F_{11}),F_{13}}\to L} \xrightarrow{\frac{h_9:F_{13},\Delta_{14},F_7\to F_8\vdash F_{12}}{\bullet h_9:(\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}\land F_{11})}}} \to L \xrightarrow{\frac{h_9:F_{13},\Delta_{14},F_7\to F_8\vdash F_{12}}{\bullet h_9:(\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}\land F_{11})}}} \xrightarrow{\frac{-:\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}}{h_9:\Delta_{14},F_{13},F_7\to F_8\vdash \Delta_{12},F_{10}}}} \xrightarrow{\frac{ax/W}{hcut}} \xrightarrow{\frac{\bullet h_1:\Delta_{14}\vdash \Delta_{12},F_{10},F_{13}}{\bullet h_1:\Delta_{14}}} \xrightarrow{\frac{-:\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}}{\bullet h_1:\Delta_{14}}} \xrightarrow{\frac{-:\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}}{\bullet h_1:\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}}} \xrightarrow{\frac{-:\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}\land F_{11}}{\bullet h_1:\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}\land F_{11}}} \xrightarrow{\frac{-:\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}\land F_{11}}{\bullet h_1:\Delta_{14},F_7\to F_8\vdash \Delta_{12},F_{10}\land F_{11}}} \xrightarrow{\frac{h_9:F_{13},\Delta_{14},F_7\to F_8\vdash F_{12},A_{14},F_7\to F_8\vdash F_{12},A_{14},F_{$$

• Case rule \vee_R

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{14}\vdash \mathbf{F}_{7},\mathbf{F}_{13},\Delta_{12},\mathbf{F}_{10}\vee \mathbf{F}_{11}}{\bullet \mathbf{h}_{1}:\mathbf{F}_{8},\Delta_{14}\vdash \mathbf{F}_{13},\Delta_{12},\mathbf{F}_{10}\vee \mathbf{F}_{11}}}_{\bullet \mathbf{h}_{1}:\Delta_{14},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash (\Delta_{12},\mathbf{F}_{10}\vee \mathbf{F}_{11}),\mathbf{F}_{13}}} \rightarrow_{L} \frac{\frac{\mathbf{h}_{9}:\mathbf{F}_{13},\Delta_{14},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \mathbf{F}_{10},\mathbf{F}_{11},\Delta_{12}}{\bullet \mathbf{h}_{9}:(\Delta_{14},\mathbf{F}_{7}\to \mathbf{F}_{8}),\mathbf{F}_{13}\vdash \Delta_{12},\mathbf{F}_{10}\vee \mathbf{F}_{11}}}_{\bullet \mathbf{tut}}}_{\bullet \mathbf{tut}} \\ -:\Delta_{14},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \Delta_{12},\mathbf{F}_{10}\vee \mathbf{F}_{11}}_{\bullet} \\ \frac{\bullet}{\mathbf{h}_{1}:\Delta_{14},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \Delta_{12},\mathbf{F}_{10},\mathbf{F}_{11},\mathbf{F}_{13}}} \xrightarrow{\mathbf{inv-th/ax}}_{\bullet \mathbf{h}_{2}:\Delta_{14},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \Delta_{12},\mathbf{F}_{10},\mathbf{F}_{11}}_{\bullet} \\ -:\Delta_{14},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \Delta_{12},\mathbf{F}_{10},\mathbf{F}_{11}}_{\bullet} \\ +\Delta_{12},\mathbf{F}_{10}\vee \mathbf{F}_{11}}_{\bullet} \\ -:\Delta_{14},\mathbf{F}_{7}\to \mathbf{F}_{8}\vdash \Delta_{12},\mathbf{F}_{10},\mathbf{F}_{11}}_{\bullet} \\ +\Delta_{12},\mathbf{F}_{10}\vee \mathbf{F}_{11}}_{\bullet} \\ +\Delta_{13},\mathbf{F}_{13}\vee \mathbf{F}_{13}\vee \mathbf{F}_{13}}_{\bullet} \\ +\Delta_{14},\mathbf{F}_{14},\mathbf{F}_{14}\vee \mathbf{F}_{14}\vee \mathbf{F}_{14$$

• Case rule \perp_R

$$\frac{\mathbf{h}_{1}:\Delta_{12} \vdash \mathsf{F}_{7}, \mathsf{F}_{11}, \bot, \Delta_{10} \quad \mathsf{h}_{1}: \mathsf{F}_{8}, \Delta_{12} \vdash \mathsf{F}_{11}, \bot, \Delta_{10}}{\bullet \mathbf{h}_{1}: \Delta_{12}, \mathsf{F}_{7} \to \mathsf{F}_{8} \vdash (\bot, \Delta_{10}), \mathsf{F}_{11}} \xrightarrow{\bullet}_{L} \frac{\mathbf{h}_{9}: \mathsf{F}_{11}, \Delta_{12}, \mathsf{F}_{7} \to \mathsf{F}_{8} \vdash \Delta_{10}}{\bullet \mathbf{h}_{9}: (\Delta_{12}, \mathsf{F}_{7} \to \mathsf{F}_{8}), \mathsf{F}_{11} \vdash \bot, \Delta_{10}} \xrightarrow{\mathsf{Cut}}_{-: \Delta_{12}, \mathsf{F}_{7} \to \mathsf{F}_{8} \vdash \bot, \Delta_{10}} \xrightarrow{\mathsf{ax/W}}_{\frac{\mathsf{h}_{9}: \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{7} \to \mathsf{F}_{8} \vdash \bot, \Delta_{10}}{\bullet \mathsf{h}_{1}: \Delta_{12}, \mathsf{F}_{7} \to \mathsf{F}_{8} \vdash \bot, \Delta_{10}} \xrightarrow{\mathsf{ax/W}}_{\mathsf{hCut}}$$

• Case rule \top_R

$$\frac{ \mathbf{h}_1 : \Delta_{12} \vdash \mathbf{F}_7, \mathbf{F}_{11}, \top, \Delta_{10} \quad \mathbf{h}_1 : \mathbf{F}_8, \Delta_{12} \vdash \mathbf{F}_{11}, \top, \Delta_{10} }{ \bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{F}_7 \rightarrow \mathbf{F}_8 \vdash (\top, \Delta_{10}), \mathbf{F}_{11} } \quad \rightarrow_L \quad \frac{ \bullet \mathbf{h}_9 : (\Delta_{12}, \mathbf{F}_7 \rightarrow \mathbf{F}_8), \mathbf{F}_{11} \vdash \top, \Delta_{10} }{ -: \Delta_{12}, \mathbf{F}_7 \rightarrow \mathbf{F}_8 \vdash \top, \Delta_{10} } \quad \mathsf{Cut} \\ \frac{ -: \Delta_{12}, \mathbf{F}_7 \rightarrow \mathbf{F}_8 \vdash \top, \Delta_{10} }{ -: \Delta_{12}, \mathbf{F}_7 \rightarrow \mathbf{F}_8 \vdash \top, \Delta_{10} } \quad \top_R$$

 \bullet Case rule K

$$\frac{\underbrace{\begin{array}{l} \mathbf{h}_1: \Box \Gamma_{13}, \Delta_{14} \vdash F_7, \Box F_{12}, \Delta_{11}, []F_{10} \quad \mathbf{h}_1: F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, []F_{10} \\ \mathbf{eh}_1: (\Box \Gamma_{13}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}), \Box F_{12} \\ \end{array}}{\underbrace{\begin{array}{l} \mathbf{eh}_1: (\Box \Gamma_{13}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}), \Box F_{12} \\ \hline \\ \mathbf{eh}_1: (\Delta_{14}, \Box \Gamma_{13} \vdash \Box F_{12}, \Delta_{11}, F_7, []F_{10} \\ \hline \\ \mathbf{eh}_2: unbox(\Box F_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10} \\ \hline \\ \mathbf{eh}_3: unbox(\Box F_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10} \\ \hline \\ \mathbf{eh}_3: unbox(\Box F_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10} \\ \hline \\ \mathbf{eh}_3: unbox(\Box F_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10} \\ \hline \\ \mathbf{eh}_3: unbox(\Box F_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10} \\ \hline \\ \mathbf{eh}_1: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, []F_{10} \\ \hline \\ \mathbf{eh}_1: \Delta_{14}, \Box \Gamma_{13} \vdash \Delta_{11}, F_7, []F_{10} \\ \hline \\ \mathbf{eh}_1: \Box \Gamma_{12}, \Delta_{14} \vdash F_7, F_{13}, \Delta_{11}, []F_{10} \quad h_1: F_8, \Box \Gamma_{12}, \Delta_{14} \vdash F_{13}, \Delta_{11}, []F_{10} \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}), F_{13} \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash (\Delta_{11}, []F_{10}) \\ \hline \\ \mathbf{eh}_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \to F_8 \vdash$$

• Case rule A45

$$\frac{\mathbf{h}_{1}: \Box \Gamma_{14}, \Delta_{15} \vdash F_{7}, \Box F_{13}, \Box \Gamma_{11}, \Delta_{12}, []F_{10} \quad \mathbf{h}_{1}: F_{8}, \Box \Gamma_{14}, \Delta_{15} \vdash \Box F_{13}, \Box \Gamma_{11}, \Delta_{12}, []F_{10}}{\bullet \mathbf{h}_{1}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}), \Box F_{13}} \xrightarrow{\bullet}_{L} \frac{\mathbf{h}_{9}: ((\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}), \Box F_{13}}{\bullet \mathbf{h}_{9}: ((\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \xrightarrow{\bullet}_{h_{9}: (\Box \Gamma_{14}, \Delta_{15}), F_{7} \rightarrow F_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10} \rightarrow \Box \Gamma_{11}, A_{12}, A_{12$$

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\frac{\mathbf{h}_{1}: \Box\Gamma_{13}, \Delta_{15} \vdash \mathbf{F}_{7}, \mathbf{F}_{14}, \Box\Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \quad \mathbf{h}_{1}: \mathbf{F}_{8}, \Box\Gamma_{13}, \Delta_{15} \vdash \mathbf{F}_{14}, \Box\Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10}}{\bullet \mathbf{h}_{1}: (\Box\Gamma_{13}, \Delta_{15}), \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash (\Box\Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10}), \mathbf{F}_{14}} \to L \quad \frac{\mathbf{h}_{9}: \Box\Gamma_{13} \vdash \Box\Gamma_{11}, \mathbf{F}_{10}}{\bullet \mathbf{h}_{9}: ((\Box\Gamma_{13}, \Delta_{15}), \mathbf{F}_{7} \to \mathbf{F}_{8}), \mathbf{F}_{14} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10}} \\ -: (\Box\Gamma_{13}, \Delta_{15}), \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Box\Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10}} \\ -: \Box\Gamma_{13} \vdash \mathbf{F}_{10}, \Box\Gamma_{11} \quad \mathbf{ax/W} \\ -: \Box\Gamma_{13} \vdash \mathbf{F}_{10}, \Box\Gamma_{11}, []\mathbf{F}_{10} \quad A45
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$$\frac{h_1: \Delta_{13} \vdash F_7, F_{10} \to F_{11}, \Delta_{12}}{\bullet h_1: \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \to F_{11}} \to L \xrightarrow{h_9: \Delta_{13}, F_7 \to F_8 \vdash F_{10}, \Delta_{12}}{\bullet h_9: (\Delta_{13}, F_7 \to F_8), F_{10} \to F_{11} \vdash \Delta_{12}} \xrightarrow{\text{Cut}} \to L \xrightarrow{\bullet h_9: (\Delta_{13}, F_7 \to F_8), F_{10} \to F_{11} \vdash \Delta_{12}} \text{Cut}} \to L \xrightarrow{-: \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{10} \to F_{11}} \xrightarrow{\bullet h_1: \Delta_{13}, F_{10} \vdash \Delta_{12}, F_{11}, F_7} \xrightarrow{\text{inv-th/ax}} \xrightarrow{-: \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}} \xrightarrow{\bullet L} \xrightarrow{-: \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}} \xrightarrow{\bullet L} \xrightarrow{-: \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}, F_{11}} \xrightarrow{\bullet L} \xrightarrow{-: \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}} \xrightarrow{\bullet L} \xrightarrow{\bullet L} \xrightarrow{-: \Delta_{13}, F_{10}, F_8 \vdash \Delta_{12}} \text{scut}$$

$$\frac{-: \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{11}}{-: \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{11}} \xrightarrow{\bullet L} \xrightarrow{\bullet L} \xrightarrow{\bullet L_1, F_{10}, F_8 \vdash \Delta_{12}, F_{11}} \xrightarrow{\bullet L_1, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{\bullet L_1, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{\bullet L_1, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{\bullet L_1, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{\bullet L_1, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{\bullet L_1, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{\bullet L_1, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}, F_{10}, F_7 \to F_8 \vdash \Delta_{12$$

• Case rule \wedge_L

$$\frac{\underbrace{\frac{h_1 : \Delta_{13} \vdash F_7, F_{10} \land F_{11}, \Delta_{12} \quad h_1 : F_8, \Delta_{13} \vdash F_{10} \land F_{11}, \Delta_{12}}_{=h_1 : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \land F_{11}}} \xrightarrow{h_2 : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_2 : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7} \xrightarrow{inv-th/ax} \xrightarrow{h_3 : \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_7} \xrightarrow{h_3 : \Delta_{13}, F_{10} \land F_{11} \vdash \Delta_{12}, F_7} \xrightarrow{h_2 : \Delta_{13}, F_3 \vdash \Delta_{12}, F_7} \xrightarrow{h_2 : \Delta_{13}, F_3 \vdash \Delta_{12}, F_7} \xrightarrow{h_2 : \Delta_{13}, F_3 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{13}, F_3 \vdash \Delta_{12}, F_7} \xrightarrow{h_2 : \Delta_{13}, F_3 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{13}, F_3 \vdash \Delta_{12}, F_3} \xrightarrow{h_3 : \Delta_{13}, F_3 \vdash \Delta_{12}, F_3} \xrightarrow{h_3 : \Delta_{13}, F_3 \vdash \Delta_{12}, F_3} \xrightarrow{h_3 : \Delta_{14}, F_{10} \land F_{11} \vdash F_7, F_{13}, \Delta_{12} \quad h_1 : F_8, \Delta_{14}, F_{10} \land F_{11} \vdash F_{13}, \Delta_{12} \quad h_2 : \Delta_{14}, F_{10} \land F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10} \land F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10} \land F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10} \land F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10} \land F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_3 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_1 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}}} \xrightarrow{h_2 : \Delta_{14}, F_{10}, F_{11}, F_7 \to F_8 \vdash \Delta_{12}} \xrightarrow{h_2 : \Delta_{14}, F_7 \to F_8, F_{10} \land F_{11} \vdash \Delta_{12}} \xrightarrow{h_2 : \Delta_{14}, F_7 \to F_8, F_{10} \land F_{11} \vdash \Delta_{12}} \xrightarrow{h_2 : \Delta_{14}, F_7 \to F_8, F_{10} \land F_{11} \vdash \Delta_{12}}} \xrightarrow{h_2 : \Delta_{14}, F_7 \to F_8, F_{10} \land F_{11} \vdash \Delta_{12}}} \xrightarrow{h_2 : \Delta_{14}, F_7 \to F_8, F_{10} \land F_{11} \vdash \Delta_{12}}} \xrightarrow{h_2 : \Delta_{14}, F_7 \to F_8, F_{10} \land F_{11} \vdash$$

• Case rule \vee_L

$$\frac{ \underbrace{ \begin{array}{c} \underbrace{ \begin{array}{c} h_1 : \Delta_{13} \vdash F_7, F_{10} \lor F_{11}, \Delta_{12} \\ \bullet h_1 : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \lor F_{11}, \Delta_{12} \\ \bullet h_1 : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \lor F_{11} \\ \end{array}}_{\bullet h_1 : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \lor F_{11}} \underbrace{ \begin{array}{c} \vdots \\ \bullet h_9 : F_{10}, \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12} \\ \bullet h_9 : (\Delta_{13}, F_7 \to F_8), F_{10} \lor F_{11} \vdash \Delta_{12} \\ \end{array}}_{Cut} \lor L$$

$$- : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{13}, F_7 \to F_8 \vdash \Delta_{12}, F_{10} \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash F_7, F_{13}, \Delta_{12} \\ \hline - : \Delta_{14}, F_{10} \lor F_{11}, F_7 \to F_8 \vdash \Delta_{12}, F_7 \\ \hline \bullet h_1 : (\Delta_{14}, F_{10} \lor F_{11}), F_7 \to F_8 \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}, F_7 \\ \hline - : \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12}$$

\bullet Case rule AT

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{12}\vdash \mathsf{F}_{7}, []\mathsf{F}_{10},\Delta_{11}\quad \mathsf{h}_{1}:\mathsf{F}_{8},\Delta_{12}\vdash []\mathsf{F}_{10},\Delta_{11}}{\bullet_{11}:\Delta_{12},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}, []\mathsf{F}_{10}}} \to_{L} \frac{\frac{\mathbf{h}_{9}:\mathsf{F}_{10},\Delta_{12}, []\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}{\bullet_{10}:(\Delta_{12},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11})}}{-:\Delta_{12},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}} \frac{AT}{\mathsf{cut}}$$

$$\frac{-:\Delta_{12},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}{\bullet_{11}:\Delta_{12}\vdash\Delta_{11},\mathsf{F}_{7}, []\mathsf{F}_{10}} \bullet_{\mathsf{h}_{9}:\Delta_{12},\mathsf{F}_{10}, []\mathsf{F}_{10}\vdash\Delta_{11},\mathsf{F}_{7}}} \frac{\mathsf{inv}\mathsf{-th/ax}}{\mathsf{A}T} \\ \bullet_{\mathsf{h}_{9}:\Delta_{12},\mathsf{F}_{10}, []\mathsf{F}_{10}\vdash\Delta_{11},\mathsf{F}_{7}}} \bullet_{\mathsf{hCut}} \underbrace{-:\Delta_{12},\mathsf{F}_{8}\vdash\Delta_{11}, []\mathsf{F}_{10}}_{\bullet_{\mathsf{h}_{9}:\Delta_{12},\mathsf{F}_{8}}, []\mathsf{F}_{10}\vdash\Delta_{11}} \bullet_{\mathsf{hCut}}}^{\bullet_{\mathsf{h}_{9}:\Delta_{12},\mathsf{F}_{10},\mathsf{F}_{8}}, []\mathsf{F}_{10}\vdash\Delta_{11}} \bullet_{\mathsf{hCut}}} \underbrace{-:\Delta_{12},\mathsf{F}_{8}\vdash\Delta_{11}, []\mathsf{F}_{10}}_{\bullet_{\mathsf{h}_{9}:\Delta_{12},\mathsf{F}_{8}}, []\mathsf{F}_{10}\vdash\Delta_{11}}^{\bullet_{\mathsf{h}_{10}}} \bullet_{\mathsf{hCut}}}^{\bullet_{\mathsf{h}_{1}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{8}}, []\mathsf{F}_{10}\vdash\Delta_{11}}} \bullet_{\mathsf{hCut}}$$

$$\frac{-:\Delta_{12},\mathsf{F}_{8}\vdash\Delta_{11}, []\mathsf{F}_{10}}{\bullet_{\mathsf{h}_{1}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11},\mathsf{F}_{12}}} \to_{L} \underbrace{\bullet_{\mathsf{h}_{1}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}^{\bullet_{\mathsf{h}_{1}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}}_{\bullet_{\mathsf{h}_{2}:\mathsf{L}_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}} \underbrace{\bullet_{\mathsf{h}_{1}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}^{\bullet_{\mathsf{h}_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}}_{\bullet_{\mathsf{h}_{1}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11},\mathsf{F}_{12}}} \underbrace{\bullet_{\mathsf{h}_{1}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11},\mathsf{F}_{12}}^{\bullet_{\mathsf{h}_{2}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}}_{\bullet_{\mathsf{h}_{2}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}} \underbrace{\bullet_{\mathsf{h}_{1}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11},\mathsf{F}_{12}}^{\bullet_{\mathsf{h}_{2}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}}_{\mathsf{h}_{\mathsf{h}_{\mathsf{L}}}} \underbrace{\bullet_{\mathsf{h}_{1}:\Delta_{13},\mathsf{F}_{10},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash\Delta_{11}}}_{\mathsf{h}_{\mathsf{h}_{\mathsf{L}}}}^{\bullet_{\mathsf{h}_{\mathsf{L}}}$$

• Case rule \perp_L

$$\begin{array}{c} \frac{h_1:\Delta_{11}\vdash F_7,\bot,\Delta_{10}\quad h_1:F_8,\Delta_{11}\vdash \bot,\Delta_{10}}{\bullet h_1:\Delta_{11},F_7\to F_8\vdash \Delta_{10},\bot} \to_L \\ \hline \bullet h_1:\Delta_{11},F_7\to F_8\vdash \Delta_{10},\bot \\ \hline \\ \frac{h_1:\Delta_{11}\vdash \bot,\Delta_{10},F_7}{\bullet h_9:\bot,\Delta_{11}\vdash \Delta_{10},F_7} \xrightarrow{\Delta_L \\ \bullet h_9:\bot,\Delta_{11}\vdash \Delta_{10},F_7} \\ \hline \\ \frac{-:\Delta_{11}\vdash \Delta_{10},F_7}{\bullet h_9:\bot,\Delta_{11}\vdash \Delta_{10},F_7} \xrightarrow{\Delta_L \\ hCut} \xrightarrow{h_1:\Delta_{11},F_8\vdash \bot,\Delta_{10}} \xrightarrow{ax/W} \xrightarrow{\bullet h_9:\bot,\Delta_{11},F_8\vdash \Delta_{10}} \\ \hline \\ \frac{-:\Delta_{11}\vdash \Delta_{10},F_7}{-:\Delta_{11}\vdash \Delta_{10},F_7} \to F_8\vdash \Delta_{10} \\ \hline \\ \frac{h_1:\bot,\Delta_{12}\vdash F_7,F_{11},\Delta_{10}\quad h_1:F_8,\bot,\Delta_{12}\vdash F_{11},\Delta_{10}}{\bullet h_9:(\bot,\Delta_{12}),F_7\to F_8),F_{11}\vdash \Delta_{10}} \xrightarrow{L_L} \\ \hline \\ \frac{\bullet h_1:\bot,\Delta_{12}\vdash F_7,F_{11},\Delta_{10}\quad h_1:F_8,\bot,\Delta_{12}\vdash F_{11},\Delta_{10}}{-:(\bot,\Delta_{12}),F_7\to F_8\vdash \Delta_{10}} \xrightarrow{L_L} \\ \hline \\ \frac{-:(\bot,\Delta_{12}),F_7\to F_8\vdash \Delta_{10}}{-:\bot,\Delta_{12},F_7\to F_8\vdash \Delta_{10}} \xrightarrow{\bot_L} \end{array}$$

 $\frac{-:\Delta_{13}, F_{10}, []F_{10}, F_7 \to F_8 \vdash \Delta_{11}}{-:\Delta_{13}, []F_{10}, F_7 \to F_8 \vdash \Delta_{11}} ATG$

ullet Case rule I

$$\frac{\frac{\mathbf{h}_{1}:\Delta_{12}\vdash \mathsf{F}_{7},\mathsf{p}_{11},\Delta_{10},\mathsf{p}_{11}}{\bullet} \quad \mathbf{h}_{1}:\mathsf{F}_{8},\Delta_{12}\vdash \mathsf{p}_{11},\Delta_{10},\mathsf{p}_{11}}}{\bullet} \rightarrow_{L} \quad \frac{\bullet}{\bullet} \\ \frac{\mathbf{h}_{1}:\Delta_{12},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash (\Delta_{10},\mathsf{p}_{11}),\mathsf{p}_{11}}{-:\Delta_{12},\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash (\Delta_{10},\mathsf{p}_{11}),\mathsf{p}_{11}} \quad \rightarrow_{L} \quad \frac{\bullet}{\bullet} \\ \bullet_{19}:(\Delta_{12},\mathsf{F}_{7}\to\mathsf{F}_{8}),\mathsf{p}_{11}\vdash \Delta_{10},\mathsf{p}_{11}} \quad I \\ \frac{-:\Delta_{12}\vdash \Delta_{10},\mathsf{F}_{7},\mathsf{p}_{11}}{\bullet} \quad \frac{\bullet}{\bullet} \\ \bullet_{19}:\Delta_{12},\mathsf{p}_{11}\vdash \Delta_{10},\mathsf{F}_{7},\mathsf{p}_{11}} \quad I \\ \frac{-:\Delta_{12}\vdash \Delta_{10},\mathsf{F}_{7},\mathsf{p}_{11}}{-:\Delta_{12}\vdash \Delta_{10},\mathsf{F}_{7},\mathsf{p}_{11}} \quad \Delta_{L} \\ \frac{-:\Delta_{12}\vdash \Delta_{10},\mathsf{F}_{7},\mathsf{p}_{11}}{-:\Delta_{12},\mathsf{F}_{8}\vdash \Delta_{10},\mathsf{p}_{11}} \quad \to_{L} \\ \frac{\bullet}{\bullet} \\ \bullet_{19}:(\Delta_{13},\mathsf{p}_{11}\vdash \mathsf{F}_{7},\mathsf{F}_{12},\Delta_{10},\mathsf{p}_{11} \quad \mathsf{h}_{1}:\mathsf{F}_{8},\Delta_{13},\mathsf{p}_{11}\vdash \mathsf{F}_{12},\Delta_{10},\mathsf{p}_{11}}{\bullet} \\ \frac{\bullet}{\bullet} \\ \bullet_{19}:((\Delta_{13},\mathsf{p}_{11}),\mathsf{F}_{7}\to\mathsf{F}_{8}),\mathsf{F}_{12}\vdash \Delta_{10},\mathsf{p}_{11}}{\bullet} \quad Cut \\ \frac{-:(\Delta_{13},\mathsf{p}_{11}),\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash \Delta_{10},\mathsf{p}_{11}}{-:(\Delta_{13},\mathsf{p}_{11}),\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash \Delta_{10},\mathsf{p}_{11}} \quad I \\ Cut \\ \frac{-:(\Delta_{13},\mathsf{p}_{11}),\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash \Delta_{10},\mathsf{p}_{11}}{-:(\Delta_{13},\mathsf{p}_{11}),\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash \Delta_{10},\mathsf{p}_{11}} \quad I \\ \frac{-:(\Delta_{13},\mathsf{p}_{11}),\mathsf{F}_{7}\to\mathsf{F}_{8}\vdash \Delta_{1$$

$$\begin{array}{c} \frac{\mathbf{h}_{1}:\Delta_{11} \vdash \mathbf{F}_{7}, \top, \Delta_{10} \quad \mathbf{h}_{1}: \mathbf{F}_{8}, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet \mathbf{h}_{1}: \Delta_{11}, \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10}, \top} \to_{L} \quad \frac{\mathbf{h}_{9}:\Delta_{11}, \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10}}{\bullet \mathbf{h}_{9}:(\Delta_{11}, \mathbf{F}_{7} \to \mathbf{F}_{8}), \top \vdash \Delta_{10}} \quad \top_{L} \\ & -:\Delta_{11}, \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10} \\ & \to \\ & -:\Delta_{11}, \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10} \\ \hline & \mathbf{ax/W} \\ \\ \hline \frac{\mathbf{h}_{1}: \top, \Delta_{12} \vdash \mathbf{F}_{7}, \mathbf{F}_{11}, \Delta_{10} \quad \mathbf{h}_{1}: \mathbf{F}_{8}, \top, \Delta_{12} \vdash \mathbf{F}_{11}, \Delta_{10}}{\bullet \mathbf{h}_{1}: (\top, \Delta_{12}), \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10}} \to_{L} \quad \frac{\mathbf{h}_{9}: \mathbf{F}_{11}, \Delta_{12}, \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10}}{\bullet \mathbf{h}_{9}: ((\top, \Delta_{12}), \mathbf{F}_{7} \to \mathbf{F}_{8}), \mathbf{F}_{11} \vdash \Delta_{10}} \quad \top_{L} \\ \hline & -: (\top, \Delta_{12}), \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10} \\ \hline & -: (\top, \Delta_{12}), \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10} \\ \hline & \bullet \mathbf{h}_{1}: \top, \Delta_{12}, \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{F}_{11} \quad \mathbf{ax/W} \\ \hline & -: (\top, \Delta_{12}, \mathbf{F}_{7} \to \mathbf{F}_{8} \vdash \Delta_{10}) \quad \mathbf{h}_{Cut} \\ \hline \end{array}$$

6.9 Status of \wedge_L : OK

• Case rule \rightarrow_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \mathbf{F}_7, \mathbf{F}_8, \Delta_{14} \vdash \mathbf{F}_{13}, \Delta_{12}, \mathbf{F}_{10} \to \mathbf{F}_{11} \\ \bullet \mathbf{h}_1: \Delta_{14}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash (\Delta_{12}, \mathbf{F}_{10} \to \mathbf{F}_{11}), \mathbf{F}_{13} \end{array} \wedge_L \quad \frac{\mathbf{h}_9: \mathbf{F}_{10}, \mathbf{F}_{13}, \Delta_{14}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \mathbf{F}_{11}, \Delta_{12} \\ \bullet \mathbf{h}_9: (\Delta_{14}, \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{13} \vdash \Delta_{12}, \mathbf{F}_{10} \to \mathbf{F}_{11} \\ \hline \\ -: \Delta_{14}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \Delta_{12}, \mathbf{F}_{11}, \mathbf{F}_{13} & \text{inv-th/ax} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_7, \mathbf{F}_8 \vdash \Delta_{12}, \mathbf{F}_{11}, \mathbf{F}_{13} & \text{inv-th/ax} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \Delta_{12}, \mathbf{F}_{11}, \mathbf{F}_{13} & \mathbf{h}_9: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{13}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \Delta_{12}, \mathbf{F}_{11} \\ \hline \\ -: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \Delta_{12}, \mathbf{F}_{11} & \rightarrow_R \end{array} \quad \mathbf{ax/W} \\ \quad \mathbf{hCut}$$

• Case rule \wedge_R

$$\frac{\frac{\mathbf{h}_{1}: F_{7}, F_{8}, \Delta_{14} \vdash F_{13}, \Delta_{12}, F_{10} \land F_{11}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \land F_{8} \vdash (\Delta_{12}, F_{10} \land F_{11}), F_{13}}} \land_{L} \frac{\mathbf{h}_{9}: F_{13}, \Delta_{14}, F_{7} \land F_{8} \vdash F_{10}, \Delta_{12} \quad \mathbf{h}_{9}: F_{13}, \Delta_{14}, F_{7} \land F_{8} \vdash F_{11}, \Delta_{12}}{\bullet \mathbf{h}_{9}: (\Delta_{14}, F_{7} \land F_{8}), F_{13} \vdash \Delta_{12}, F_{10} \land F_{11}} \quad \mathbf{Cut}} \\ -: \Delta_{14}, F_{7} \land F_{8} \vdash \Delta_{12}, F_{10} \land F_{11}} \rightarrow \frac{-}{\mathbf{h}_{9}: \Delta_{14}, F_{13}, F_{7}, F_{8} \vdash \Delta_{12}, F_{10}} \quad \mathbf{inv-th/ax}} \frac{\mathbf{h}_{9}: \Delta_{14}, F_{13}, F_{7}, F_{8} \vdash \Delta_{12}, F_{11}}}{\bullet \mathbf{h}_{9}: \Delta_{14}, F_{13}, F_{7}, F_{8} \vdash \Delta_{12}, F_{10} \land F_{11}}} \quad \mathbf{h}_{Cut}} \\ -: \Delta_{14}, F_{7}, F_{8} \vdash \Delta_{12}, F_{10} \land F_{11}} \\ -: \Delta_{14}, F_{7}, F_{8} \vdash \Delta_{12}, F_{10} \land F_{11}} \land_{L}}$$

• Case rule \vee_R

$$\frac{ \begin{array}{c} \mathbf{h}_{1}: \mathsf{F}_{7}, \mathsf{F}_{8}, \Delta_{14} \vdash \mathsf{F}_{13}, \Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11} \\ \bullet \mathsf{h}_{1}: \Delta_{14}, \mathsf{F}_{7} \wedge \mathsf{F}_{8} \vdash (\Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11}), \mathsf{F}_{13} \end{array} \wedge_{L} \quad \begin{array}{c} \mathbf{h}_{9}: \mathsf{F}_{13}, \Delta_{14}, \mathsf{F}_{7} \wedge \mathsf{F}_{8} \vdash \mathsf{F}_{10}, \mathsf{F}_{11}, \Delta_{12} \\ \bullet \mathsf{h}_{9}: (\Delta_{14}, \mathsf{F}_{7} \wedge \mathsf{F}_{8}), \mathsf{F}_{13} \vdash \Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11} \end{array} \quad \vee_{R} \\ \\ \leftarrow (\mathsf{Cut}) \\ \bullet \mathsf{h}_{1}: \Delta_{14}, \mathsf{F}_{7} \wedge \mathsf{F}_{8} \vdash \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_{13} \\ \bullet \mathsf{h}_{1}: \Delta_{14}, \mathsf{F}_{7} \wedge \mathsf{F}_{8} \vdash \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_{13} \end{array} \quad \overset{\mathsf{Inv-th/ax}}{\wedge_{L}} \\ \bullet \mathsf{h}_{1}: \Delta_{14}, \mathsf{F}_{7} \wedge \mathsf{F}_{8} \vdash \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_{11} \\ \bullet \mathsf{h}_{1}: \Delta_{14}, \mathsf{F}_{7} \wedge \mathsf{F}_{8} \vdash \Delta_{12}, \mathsf{F}_{10}, \mathsf{F}_{11} \\ - : \Delta_{14}, \mathsf{F}_{7} \wedge \mathsf{F}_{8} \vdash \Delta_{12}, \mathsf{F}_{10} \vee \mathsf{F}_{11} \end{array} \quad \mathsf{h}_{R} \\ \bullet \mathsf{hCut} \\ \bullet \mathsf{h$$

• Case rule \perp_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \mathbf{F}_7, \mathbf{F}_8, \Delta_{12} \vdash \mathbf{F}_{11}, \bot, \Delta_{10} \\ \bullet \mathbf{h}_1: \Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash (\bot, \Delta_{10}), \mathbf{F}_{11} \end{array} \wedge_L \quad \begin{array}{c} \mathbf{h}_9: \mathbf{F}_{11}, \Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \Delta_{10} \\ \bullet \mathbf{h}_9: (\Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \bot, \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \bot, \Delta_{10}, \mathbf{F}_{11} \end{array} \quad \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_9: (\Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{11} \vdash \bot, \Delta_{10} \\ \hline \\ \bullet \mathbf{h}_1: \Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \bot, \Delta_{10}, \mathbf{F}_{11} \end{array} \quad \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_9: (\Delta_{12}, \mathbf{F}_1, \mathbf{F}_1, \mathbf{F}_1, \mathbf{F}_1, \mathbf{F}_1, \bot, \Delta_{10}) \\ \hline \\ \bullet \mathbf{h}_1: \Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \bot, \Delta_{10} \end{array} \quad \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_9: (\Delta_{12}, \mathbf{F}_{11}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \bot, \Delta_{10}) \\ \hline \\ \bullet \mathbf{h}_1: \Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \bot, \Delta_{10} \end{array} \quad \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_1: \Delta_{12}, \mathbf{F}_1, \mathbf{h}_1: \mathbf{h}_1: \mathbf{h}_1: \Delta_{12}, \mathbf{h}_2: \mathbf{h}_1: \mathbf{h}_1: \Delta_{12}: \Delta_{12}: \mathbf{h}_1: \Delta_{12}: \Delta_{12}:$$

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_7, \mathbf{F}_8, \Delta_{12} \vdash \mathbf{F}_{11}, \top, \Delta_{10}}{\bullet \mathbf{h}_1: \Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash (\top, \Delta_{10}), \mathbf{F}_{11}} \ \land_L \\ \\ \frac{-: \Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash (\top, \Delta_{10})}{-: \Delta_{12}, \mathbf{F}_7 \land \mathbf{F}_8 \vdash \top, \Delta_{10}} \ \top_R \end{array} \quad \begin{array}{c} \top_R \\ \text{Cut} \end{array}$$

 \bullet Case rule K

 \bullet Case rule A45

$$\frac{\mathbf{h}_1: \mathbf{f}_7, \mathbf{f}_8, \Box \Gamma_{14}, \Delta_{15} \vdash \Box \mathbf{f}_{13}, \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_{10}}{\bullet \mathbf{h}_1: (\Box \Gamma_{14}, \Delta_{15}), \mathbf{f}_7 \land \mathbf{f}_8 \vdash (\Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_{10}), \Box \mathbf{f}_{13}} \land_L \frac{\mathbf{h}_9: \Box \Gamma_{14}, \Box \Gamma_{11}, \Box \Gamma_{11}, \mathbf{f}_{10}}{\bullet \mathbf{h}_9: ((\Box \Gamma_{14}, \Delta_{15}), \mathbf{f}_7 \land \mathbf{f}_8), \Box \mathbf{f}_{13} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_{10}} \land_L \\ -: (\Box \Gamma_{14}, \Delta_{15}), \mathbf{f}_7 \land \mathbf{f}_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_{10}} \rightarrow \frac{\mathbf{h}_9: \Box \mathbf{f}_{13}, \Box \Gamma_{14} \vdash \mathbf{f}_{10}, \Box \Gamma_{11}}{\bullet \mathbf{h}_9: \Box \mathbf{f}_{13}, \Box \Gamma_{14} \vdash \mathbf{f}_{10}, \Box \Gamma_{11}} \overset{\mathbf{ax/W}}{\bullet \mathbf{h}_9: \Box \mathbf{f}_{13}, \Delta_{15}, \mathbf{f}_7, \mathbf{f}_8, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, []\mathbf{f}_{10}} \land_L \\ \frac{-: \Delta_{15}, \mathbf{f}_7, \mathbf{f}_8, \Box \Gamma_{14} \vdash \Delta_{12}, \Box \Gamma_{11}, []\mathbf{f}_{10}}{-: \Delta_{15}, \Box \Gamma_{14}, \mathbf{f}_7 \land \mathbf{f}_8 \vdash \Delta_{12}, \Box \Gamma_{11}, []\mathbf{f}_{10}} \land_L \\ \end{cases} \land_L$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \mathbf{F}_7, \mathbf{F}_8, \square \Gamma_{13}, \Delta_{15} \vdash \mathbf{F}_{14}, \square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \\ \bullet \mathbf{h}_1: (\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8 \vdash (\square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10}), \mathbf{F}_{14} \end{array} \wedge_L \quad \begin{array}{c} \mathbf{h}_9: \square \Gamma_{13} \vdash \square \Gamma_{11}, \mathbf{F}_{10} \\ \bullet \mathbf{h}_9: ((\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{14} \vdash \square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \\ -: (\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8 \vdash \square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: ((\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{14} \vdash \square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: ((\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{14} \vdash \square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: ((\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{14} \vdash \square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: ((\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{14} \vdash \square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: ((\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{14} \vdash \square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: ((\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{14} \vdash \square \Gamma_{11}, \Delta_{12}, []\mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: ((\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{14} \vdash \square \Gamma_{11}, \mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: ((\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8), \mathbf{F}_{14} \vdash \square \Gamma_{11}, \mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: (\square \Gamma_{13}, \Delta_{15}), \mathbf{F}_7 \land \mathbf{F}_8 \vdash \square \Gamma_{11}, \mathbf{F}_{10} \\ -: \square \Gamma_{13} \vdash \Gamma_{13}, \mathbf{F}_{10}, \square \Gamma_{11} \end{array} \quad \begin{array}{c} A45 \\ \bullet \mathbf{h}_9: (\square \Gamma_{13} \vdash \mathbf{F}_{10}, \square \Gamma_{11}, \square \Gamma_{11}, \square \Gamma_{11}) \end{array}$$

$$\frac{ \begin{array}{c} \frac{h_1: F_7, F_8, \Delta_{13} \vdash F_{10} \to F_{11}, \Delta_{12}}{\bullet h_1: \Delta_{13}, F_7 \land F_8 \vdash \Delta_{12}, F_{10} \to F_{11}} \\ \bullet h_1: \Delta_{13}, F_7 \land F_8 \vdash \Delta_{12}, F_{10} \to F_{11} \\ \end{array}}{\bullet h_1: \Delta_{13}, F_7 \land F_8 \vdash \Delta_{12}, F_{10} \to F_{11} \\ \end{array}} \xrightarrow{\bullet h_9: (\Delta_{13}, F_7 \land F_8), F_{10} \to F_{11} \vdash \Delta_{12}} Cut \\ \hline \begin{array}{c} -: \Delta_{13}, F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_9: \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \\ \hline \end{array}} \xrightarrow{\bullet h_9: \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \\ \hline \\ \bullet h_9: \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \\ \hline \end{array}} \xrightarrow{\bullet h_9: \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \\ \hline \bullet h_9: \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \\ \hline \\ \bullet h_1: F_7, F_8, \Delta_{14}, F_{10} \to F_{11} \vdash F_{13}, \Delta_{12} \\ \hline \bullet h_1: (\Delta_{14}, F_{10} \to F_{11}), F_7 \land F_8 \vdash \Delta_{12}, F_{13} \\ \hline \\ \bullet h_1: (\Delta_{14}, F_{10} \to F_{11}), F_7 \land F_8 \vdash \Delta_{12}, F_{13} \\ \hline \\ \bullet h_1: (\Delta_{14}, F_{10} \to F_{11}), F_7 \land F_8 \vdash \Delta_{12}, F_{13} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{10} \to F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{10} \to F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{17}, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{13}, F_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{17}, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{17}, F_8, F_{10} \to F_{11} \vdash \Delta_{12} \\ \hline \\ \bullet h_9: \Delta_{14}, F_{17}, F_8, F_{10} \to F_{11} \vdash \Delta_{12}$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathsf{F}_7, \mathsf{F}_8, \Delta_{13} \vdash \mathsf{F}_{10} \land \mathsf{F}_{11}, \Delta_{12}}{\mathsf{e}_{11}: \Delta_{13}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}, \mathsf{F}_{10} \land \mathsf{F}_{11}} \ \land_L & \frac{\mathbf{h}_9: \mathsf{F}_{10}, \mathsf{F}_{11}, \Delta_{13}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}}{\mathsf{e}_{10}: (\Delta_{13}, \mathsf{F}_7 \land \mathsf{F}_8), \mathsf{F}_{10} \land \mathsf{F}_{11} \vdash \Delta_{12}} \ \land_L \\ & -: \Delta_{13}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12} \\ & \rightarrow \\ \frac{\mathbf{h}_1: \Delta_{13}, \mathsf{F}_7, \mathsf{F}_8 \vdash \Delta_{12}, \mathsf{F}_{10} \land \mathsf{F}_{11}}{\mathsf{e}_{11}: \Delta_{13}, \mathsf{F}_7, \mathsf{F}_8 \vdash \Delta_{12}, \mathsf{F}_{10} \land \mathsf{F}_{11} \vdash \Delta_{12}} \ \land_L \\ & \frac{\mathsf{h}_9: \Delta_{13}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_{7}, \mathsf{F}_8 \vdash \Delta_{12}}{\mathsf{e}_{10}: \Delta_{13}, \mathsf{F}_7, \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{-: \Delta_{13}, \mathsf{F}_7, \mathsf{F}_8 \vdash \Delta_{12}}{\mathsf{e}_{10}: \Delta_{13}, \mathsf{F}_7, \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{-: \Delta_{13}, \mathsf{F}_7, \mathsf{F}_8 \vdash \Delta_{12}}{\mathsf{e}_{11}: (\Delta_{14}, \mathsf{F}_{10} \land \mathsf{F}_{11}), \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{\mathsf{h}_1: \mathsf{F}_7, \mathsf{F}_8, \Delta_{14}, \mathsf{F}_{10} \land \mathsf{F}_{11} \vdash \mathsf{F}_{13}, \Delta_{12}}{\mathsf{e}_{11}: (\Delta_{14}, \mathsf{F}_{10} \land \mathsf{F}_{11}), \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}, \mathsf{F}_{13}} \land_L & \frac{\mathsf{h}_9: \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_{13}, \Delta_{14}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}}{\mathsf{e}_{10}: (\Delta_{14}, \mathsf{F}_{10} \land \mathsf{F}_{11}), \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{\mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10} \land \mathsf{F}_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}, \mathsf{F}_{13}}{\mathsf{h}_2} & \frac{\mathsf{h}_9: (\Delta_{14}, \mathsf{F}_{10} \land \mathsf{F}_{11}), \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}}{\mathsf{h}_9: (\Delta_{14}, \mathsf{F}_{10} \land \mathsf{F}_{11}), \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{\mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_7, \mathsf{F}_8 \vdash \Delta_{12}, \mathsf{F}_{13}}{\mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_7, \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{\mathsf{e}_{11}: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}}{\mathsf{h}_1: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{\mathsf{e}_{11}: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}}{\mathsf{h}_1: \Delta_{10}, \mathsf{F}_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{\mathsf{e}_{11}: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{11}, \Delta_{10}}{\mathsf{e}_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{\mathsf{e}_{11}: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{11}, \Delta_{10}}{\mathsf{e}_{11}: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{12}} \land_L \\ & \frac{\mathsf{e}_{11}: \Delta_{14}, \mathsf{F}_{10}, \mathsf{F}_{1$$

• Case rule \vee_L

$$\frac{ \begin{array}{c} \frac{h_1: F_7, F_8, \Delta_{13} \vdash F_{10} \lor F_{11}, \Delta_{12}}{\bullet h_1: \Delta_{13}, F_7 \land F_8 \vdash \Delta_{12}, F_{10} \lor F_{11}} \\ \bullet h_2: F_{10}, \Delta_{13}, F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_3: (\Delta_{13}, F_7 \land F_8) \vdash \Delta_{12} \\ \bullet h_3: (\Delta_{13}, F_7 \land F_8) \vdash \Delta_{12} \\ \bullet h_3: \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \lor F_{11} \\ \bullet h_2: \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \lor F_{11} \\ \bullet h_2: \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{13}, F_7, F_8 \vdash \Delta_{12}, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \\ \bullet h_2: \Delta_{13}, F_7, F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11} \vdash F_{13}, \Delta_{12} \\ \bullet h_1: (\Delta_{14}, F_{10} \lor F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11}), F_7 \land F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10}, F_{13}, F_7, F_8 \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12} \\ \bullet h_3: \Delta_{14}, F_7, F_8, F_{10} \lor F_{11} \vdash \Delta_{12$$

\bullet Case rule AT

$$\frac{ \begin{array}{c} \frac{h_1: F_7, F_8, \Delta_{12} \vdash []F_{10}, \Delta_{11}}{\bullet h_1: \Delta_{12}, F_7 \land F_8 \vdash \Delta_{11}, []F_{10}} \ \land L \\ \\ \frac{\bullet h_1: A_{12}, F_7 \land F_8 \vdash \Delta_{11}, []F_{10}}{\bullet h_9: (\Delta_{12}, F_7 \land F_8), []F_{10} \vdash \Delta_{11}} \\ \\ \frac{\to h_9: (\Delta_{12}, F_7 \land F_8), []F_{10} \vdash \Delta_{11}}{\bullet h_9: \Delta_{12}, F_7, F_8, []F_{10} \vdash \Delta_{11}} \\ \\ \frac{h_1: \Delta_{12}, F_7, F_8 \vdash \Delta_{11}, []F_{10}}{\bullet h_9: \Delta_{12}, F_7, F_8, []F_{10} \vdash \Delta_{11}} \\ \\ \frac{-: \Delta_{12}, F_7, F_8 \vdash \Delta_{11}}{\bullet h_9: \Delta_{12}, F_7 \land F_8 \vdash \Delta_{11}} \ \land L \\ \\ \frac{h_1: F_7, F_8, \Delta_{13}, []F_{10} \vdash F_{12}, \Delta_{11}}{\bullet h_1: (\Delta_{13}, []F_{10}), F_7 \land F_8 \vdash \Delta_{11}, F_{12}} \ \land L \\ \\ \frac{\bullet h_9: F_{10}, F_{12}, \Delta_{13}, []F_{10}, F_7 \land F_8 \vdash \Delta_{11}}{\bullet h_9: ((\Delta_{13}, []F_{10}), F_7 \land F_8), F_{12} \vdash \Delta_{11}} \ \land L \\ \\ \frac{\bullet h_1: \Delta_{13}, F_{10}, []F_{10}, F_7 \land F_8 \vdash \Delta_{11}}{\bullet h_9: \Delta_{13}, F_{10}, F_{12}, []F_{10}, F_7 \land F_8 \vdash \Delta_{11}} \ \land L \\ \\ \frac{\bullet h_1: \Delta_{13}, F_{10}, []F_{10}, F_7 \land F_8 \vdash \Delta_{11}}{\bullet h_9: \Delta_{13}, F_{10}, F_{12}, []F_{10}, F_7 \land F_8 \vdash \Delta_{11}} \ \land L \\ \\ \frac{-: \Delta_{13}, F_{10}, []F_{10}, F_7 \land F_8 \vdash \Delta_{11}}{\bullet h_9: \Delta_{13}, F_{10}, F_{10}, F_7 \land F_8 \vdash \Delta_{11}} \ \land L \\ \\ \frac{-: \Delta_{13}, F_{10}, []F_{10}, F_7 \land F_8 \vdash \Delta_{11}}{\bullet h_9: \Delta_{13}, F_{10}, F_{10}, F_7 \land F_8 \vdash \Delta_{11}} \ \land L \ \end{matrix} \ \land L \\ \\ \frac{-: \Delta_{13}, F_{10}, []F_{10}, F_7 \land F_8 \vdash \Delta_{11}}{\bullet h_9: \Delta_{13}, F_{10}, F_{10}, F_7 \land F_8 \vdash \Delta_{11}} \ \land L \ \end{matrix} \ \land L \ \end{matrix} \ \land L \ \land L \ \land L \ \end{matrix}$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_{1}: \mathsf{F}_{7}, \mathsf{F}_{8}, \Delta_{11} \vdash \bot, \Delta_{10}}{\bullet \mathbf{h}_{1}: \Delta_{11}, \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}, \bot} & \wedge_{L} & \frac{\bullet \mathbf{h}_{9}: (\Delta_{11}, \mathsf{F}_{7} \land \mathsf{F}_{8}), \bot \vdash \Delta_{10}}{\bullet \mathbf{h}_{9}: (\Delta_{11}, \mathsf{F}_{7} \land \mathsf{F}_{8}), \bot \vdash \Delta_{10}} & \mathsf{Cut} \\ \\ \frac{-: \Delta_{11}, \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}}{\bullet \mathbf{h}_{9}: \bot, \Delta_{11}, \mathsf{F}_{7}, \mathsf{F}_{8} \vdash \Delta_{10}} & \bot_{L} \\ \frac{-: \Delta_{11}, \mathsf{F}_{7}, \mathsf{F}_{8} \vdash \Delta_{10}}{-: \Delta_{11}, \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}} & \wedge_{L} \\ \\ \frac{\bullet \mathbf{h}_{1}: \mathsf{F}_{7}, \mathsf{F}_{8}, \bot, \Delta_{12} \vdash \mathsf{F}_{11}, \Delta_{10}}{-: \Delta_{10}, \mathsf{F}_{11}} & \wedge_{L} & \bullet_{\mathbf{h}_{9}: ((\bot, \Delta_{12}), \mathsf{F}_{7} \land \mathsf{F}_{8}), \mathsf{F}_{11} \vdash \Delta_{10}} & \bot_{L} \\ \\ \frac{\bullet \mathbf{h}_{1}: (\bot, \Delta_{12}), \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{F}_{11}}{-: (\bot, \Delta_{12}), \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}} & \bot_{L} \end{array}$$

\bullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_{1}: \mathsf{F}_{7}, \mathsf{F}_{8}, \Delta_{12} \vdash \mathsf{p}_{11}, \Delta_{10}, \mathsf{p}_{11}}{\bullet \mathsf{h}_{1}: \Delta_{12}, \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash (\Delta_{10}, \mathsf{p}_{11}), \mathsf{p}_{11}} \ \, \wedge_{L} \ \, & \frac{\bullet \mathsf{h}_{9}: (\Delta_{12}, \mathsf{F}_{7} \land \mathsf{F}_{8}), \mathsf{p}_{11} \vdash \Delta_{10}, \mathsf{p}_{11}}{\bullet \mathsf{cut}} \\ & -: \Delta_{12}, \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{p}_{11} \\ & \xrightarrow{\bullet \mathsf{h}_{9}: \Delta_{12}, \mathsf{F}_{7}, \mathsf{F}_{8}, \mathsf{p}_{11} \vdash \Delta_{10}, \mathsf{p}_{11}} \ \, & I \\ & \frac{\mathsf{h}_{1}: \Delta_{12}, \mathsf{F}_{7}, \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{p}_{11}, \mathsf{p}_{11}}{\bullet \mathsf{h}_{9}: \Delta_{12}, \mathsf{F}_{7}, \mathsf{F}_{8}, \mathsf{p}_{11} \vdash \Delta_{10}, \mathsf{p}_{11}} \ \, & \mathsf{h}_{\mathsf{Cut}} \\ & \frac{-: \Delta_{12}, \mathsf{F}_{7}, \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{p}_{11}}{-: \Delta_{12}, \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{p}_{11}} \ \, \wedge_{L} \\ & \frac{\mathsf{h}_{1}: \mathsf{F}_{7}, \mathsf{F}_{8}, \Delta_{13}, \mathsf{p}_{11} \vdash \mathsf{F}_{12}, \Delta_{10}, \mathsf{p}_{11}}{-: \Delta_{10}, \mathsf{p}_{11}, \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{p}_{11}} \ \, \wedge_{L} \\ & \frac{\bullet \mathsf{h}_{9}: ((\Delta_{13}, \mathsf{p}_{11}), \mathsf{F}_{7} \land \mathsf{F}_{8}), \mathsf{F}_{12} \vdash \Delta_{10}, \mathsf{p}_{11}}{-: (\Delta_{13}, \mathsf{p}_{11}), \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{p}_{11}} \ \, & \mathsf{Cut} \\ & \frac{-: \Delta_{13}, \mathsf{p}_{11}, \mathsf{F}_{7} \land \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{p}_{11}}{-: \Delta_{10}, \mathsf{p}_{11}} \ \, I \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \mathsf{F}_7, \mathsf{F}_8, \Delta_{11} \vdash \top, \Delta_{10} \\ \bullet \mathbf{h}_1: \Delta_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10}, \top \end{array} \wedge_L \begin{array}{c} \mathbf{h}_9: \Delta_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10} \\ \bullet \mathbf{h}_9: (\Delta_{11}, \mathsf{F}_7 \land \mathsf{F}_8), \top \vdash \Delta_{10} \end{array}}{ \begin{array}{c} -: \Delta_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10} \\ & \longrightarrow \\ \hline -: \Delta_{11}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10} \end{array} } \begin{array}{c} \mathsf{ax/W} \\ \\ \bullet \mathbf{h}_1: \mathsf{F}_7, \mathsf{F}_8, \top, \Delta_{12} \vdash \mathsf{F}_{11}, \Delta_{10} \\ \bullet \mathbf{h}_1: (\top, \Delta_{12}), \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10}, \mathsf{F}_{11} \end{array} \wedge_L \begin{array}{c} \mathbf{h}_9: \mathsf{F}_{11}, \Delta_{12}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10} \\ \bullet \mathbf{h}_9: ((\top, \Delta_{12}), \mathsf{F}_7 \land \mathsf{F}_8), \mathsf{F}_{11} \vdash \Delta_{10} \end{array} & \top_L \\ -: (\top, \Delta_{12}), \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10} \\ \hline -: (\top, \Delta_{12}), \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10} \\ \hline \bullet \mathbf{h}_1: \top, \Delta_{12}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10}, \mathsf{F}_{11} \end{array} & \mathbf{ax/W} \\ \hline \bullet \mathbf{h}_1: \top, \Delta_{12}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10}, \mathsf{F}_{11} \end{array} & \mathbf{ax/W} \\ \hline \bullet \mathsf{h}_1: \top, \Delta_{12}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10}, \mathsf{F}_{11} \end{array} & \mathbf{ax/W} \\ \hline \bullet \mathsf{h}_1: \top, \Delta_{12}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10}, \mathsf{F}_{11} \end{array} & \mathbf{ax/W} \\ \hline \bullet \mathsf{h}_1: \top, \Delta_{12}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10}, \mathsf{F}_{11} \end{array} & \mathbf{ax/W} \\ \bullet \mathsf{h}_2: \top, \Delta_{12}, \mathsf{F}_7 \land \mathsf{F}_8 \vdash \Delta_{10} \end{array} & \mathbf{ax/W} \\ \bullet \mathsf{h}_2: \mathsf{Cut} \end{array}$$

6.10 Status of \vee_L : OK

• Case rule \rightarrow_R

$$\frac{\frac{\mathbf{h}_{1}: \mathbf{F}_{7}, \Delta_{14} \vdash \mathbf{F}_{13}, \Delta_{12}, \mathbf{F}_{10} \to \mathbf{F}_{11} \quad \mathbf{h}_{1}: \mathbf{F}_{8}, \Delta_{14} \vdash \mathbf{F}_{13}, \Delta_{12}, \mathbf{F}_{10} \to \mathbf{F}_{11}}{\bullet \mathbf{h}_{1}: \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash (\Delta_{12}, \mathbf{F}_{10} \to \mathbf{F}_{11}), \mathbf{F}_{13}} \quad \vee_{L} \quad \frac{\frac{\mathbf{h}_{9}: \mathbf{F}_{10}, \mathbf{F}_{13}, \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \mathbf{F}_{11}, \Delta_{12}}{\bullet \mathbf{h}_{9}: (\Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8}), \mathbf{F}_{13} \vdash \Delta_{12}, \mathbf{F}_{10} \to \mathbf{F}_{11}} \quad \vee_{L} \quad \frac{-: \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10} \to \mathbf{F}_{11}}{\bullet \mathbf{h}_{1}: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{11}, \mathbf{F}_{13}} \quad \frac{\mathsf{inv} - \mathsf{th}/\mathsf{ax}}{\mathsf{h}_{1}: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{11}, \mathbf{F}_{13}} \quad \frac{\mathsf{inv} - \mathsf{th}/\mathsf{ax}}{\mathsf{h}_{2}: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{11}} \quad \frac{\mathsf{ax}/\mathsf{W}}{\mathsf{hCut}}} \quad \frac{-: \Delta_{14}, \mathbf{F}_{10}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{11}}{-: \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10} \to \mathbf{F}_{11}} \quad \rightarrow_{R}} \quad \frac{\mathsf{ax}/\mathsf{W}}{\mathsf{hCut}}$$

• Case rule \wedge_R

$$\frac{\underbrace{\frac{h_{1}:F_{7},\Delta_{14}\vdash F_{13},\Delta_{12},F_{10}\wedge F_{11}}{\bullet h_{1}:\Delta_{14},F_{7}\vee F_{8}\vdash (\Delta_{12},F_{10}\wedge F_{11})}_{\bullet h_{1}:\Delta_{14},F_{7}\vee F_{8}\vdash (\Delta_{12},F_{10}\wedge F_{11}),F_{13}}} \vee_{L} \underbrace{\frac{h_{9}:F_{13},\Delta_{14},F_{7}\vee F_{8}\vdash F_{10}\wedge F_{11}}{\bullet h_{9}:(\Delta_{14},F_{7}\vee F_{8}\vdash \Delta_{12},F_{10}\wedge F_{11})}_{\bullet h_{9}:(\Delta_{14},F_{7}\vee F_{8}\vdash \Delta_{12},F_{10}\wedge F_{11})}} \vee_{L} \underbrace{\frac{h_{9}:A_{14},F_{7}\vee F_{8}\vdash A_{12},F_{10}\wedge F_{11}}{\bullet h_{1}:\Delta_{14},F_{7}\vee F_{8}\vdash \Delta_{12},F_{10},F_{13}}}_{\bullet h_{1}:\Delta_{14},F_{7}\vee F_{8}\vdash \Delta_{12},F_{10},F_{13}}} \underbrace{\frac{inv-th/ax}{v_{L}}}_{h_{9}:\Delta_{14},F_{13},F_{7}\vee F_{8}\vdash \Delta_{12},F_{10}}} \underbrace{\frac{ax/w}{hCut}}_{\bullet Cut} \underbrace{\frac{h_{1}:\Delta_{14},F_{7}\vdash \Delta_{12},F_{11},F_{13}}{\bullet h_{1}:\Delta_{14},F_{7}\vee F_{8}\vdash \Delta_{12},F_{10}\wedge F_{11}}}_{\bullet Lut}$$

• Case rule \vee_R

$$\frac{\frac{\mathbf{h}_{1}: \mathbf{F}_{7}, \Delta_{14} \vdash \mathbf{F}_{13}, \Delta_{12}, \mathbf{F}_{10} \lor \mathbf{F}_{11} \quad \mathbf{h}_{1}: \mathbf{F}_{8}, \Delta_{14} \vdash \mathbf{F}_{13}, \Delta_{12}, \mathbf{F}_{10} \lor \mathbf{F}_{11}}{\bullet \mathbf{h}_{1}: \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash (\Delta_{12}, \mathbf{F}_{10} \lor \mathbf{F}_{11}), \mathbf{F}_{13}} \quad \vee_{L} \quad \frac{\mathbf{h}_{9}: \mathbf{F}_{13}, \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \mathbf{F}_{10}, \mathbf{F}_{11}, \Delta_{12}}{\bullet \mathbf{h}_{9}: (\Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8}), \mathbf{F}_{13} \vdash \Delta_{12}, \mathbf{F}_{10} \lor \mathbf{F}_{11}} \quad \nabla_{R} \quad Cut} \\ \frac{-: \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10} \lor \mathbf{F}_{11}}{\bullet} \quad \mathbf{v}_{L} \quad \mathbf{h}_{9}: \Delta_{14}, \mathbf{F}_{13}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}}}{\bullet \mathbf{h}_{1}: \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}, \mathbf{F}_{13}} \quad \mathbf{v}_{R}} \quad \mathbf{ax} / \mathbf{w}_{R} \\ \frac{-: \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}}{-: \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}} \quad \vee_{R}} \quad \mathbf{ax} / \mathbf{w}_{R} \\ \mathbf{h}_{1} : \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}} \quad \mathbf{v}_{R} \\ \mathbf{h}_{1} : \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}} \quad \mathbf{v}_{R} \\ \mathbf{h}_{1} : \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}} \quad \mathbf{v}_{R} \\ \mathbf{h}_{1} : \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}} \quad \mathbf{h}_{1} : \Delta_{14}, \mathbf{F}_{14}, \mathbf{F}_{15} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}} \\ \mathbf{h}_{2} : \Delta_{14}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{11}} \quad \mathbf{h}_{2} : \Delta_{14}, \mathbf{F}_{15} \lor \mathbf{h}_{2}, \mathbf{h}_{2} : \Delta_{14}, \mathbf{F}_{15} \lor \mathbf{h}_{2} : \Delta_$$

• Case rule \perp_R

$$\frac{ \mathbf{h}_1 : \mathbf{F}_7, \Delta_{12} \vdash \mathbf{F}_{11}, \bot, \Delta_{10} \quad \mathbf{h}_1 : \mathbf{F}_8, \Delta_{12} \vdash \mathbf{F}_{11}, \bot, \Delta_{10} }{ \bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8 \vdash (\bot, \Delta_{10}), \mathbf{F}_{11} } \quad \vee_L \quad \frac{ \mathbf{h}_9 : \mathbf{F}_{11}, \Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8 \vdash \Delta_{10} }{ \bullet \mathbf{h}_9 : (\Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8), \mathbf{F}_{11} \vdash \bot, \Delta_{10} } \quad \mathbf{Cut} \\ & \qquad \qquad - : \Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8 \vdash \bot, \Delta_{10} \\ & \qquad \qquad \longrightarrow \\ & \qquad \qquad \bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8 \vdash \bot, \Delta_{10}, \mathbf{F}_{11} \quad \mathbf{ax/W} \quad \mathbf{h}_9 : \Delta_{12}, \mathbf{F}_{11}, \mathbf{F}_7 \lor \mathbf{F}_8 \vdash \bot, \Delta_{10} \\ & \qquad \qquad - : \Delta_{12}, \mathbf{F}_7 \lor \mathbf{F}_8 \vdash \bot, \Delta_{10} \\ & \qquad \qquad \bullet \mathbf{Cut} \\ & \qquad \bullet \mathbf{Cut} \\ & \qquad \quad$$

• Case rule \top_R

$$\frac{\mathbf{h}_{1}: \mathsf{F}_{7}, \Delta_{12} \vdash \mathsf{F}_{11}, \top, \Delta_{10} \quad \mathsf{h}_{1}: \mathsf{F}_{8}, \Delta_{12} \vdash \mathsf{F}_{11}, \top, \Delta_{10}}{\underbrace{\bullet \mathsf{h}_{1}: \Delta_{12}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash (\top, \Delta_{10}), \mathsf{F}_{11}}_{\qquad -: \Delta_{12}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \top, \Delta_{10}} \underbrace{-: \Delta_{12}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \top, \Delta_{10}}_{\qquad -: \Delta_{12}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \top, \Delta_{10}} \, \quad \top_{R}}_{\qquad \mathsf{Cut}}$$

 \bullet Case rule K

$$\frac{ \begin{array}{c} \underline{h_1: F_7, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, []F_{10} \quad h_1: F_8, \Box \Gamma_{13}, \Delta_{14} \vdash \Box F_{12}, \Delta_{11}, []F_{10} \\ \underline{\bullet h_1: (\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, []F_{10}), \Box F_{12}} \\ \\ \underline{\bullet h_1: (\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, []F_{10}) \\ \underline{\bullet h_1: (\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, []F_{10}) \\ \underline{\bullet h_2: (\Box \Gamma_{13}, \Delta_{14}), F_7 \vee F_8 \vdash \Delta_{11}, []F_{10}} \\ \underline{\bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10}} \\ \underline{\bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{12}), unbox(\Box \Gamma_{13}) \vdash F_{10}} \\ \underline{\bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{13}), unbox(\Box \Gamma_{13}) \vdash F_{10}} \\ \underline{\bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{13}) \vdash F_{10}} \\ \underline{\bullet h_2: unbox(\Box F_{12}), unbox(\Box F_{13}) \vdash F_{10}} \\ \underline{\bullet h_2: unbox(\Box F_{13}), unbox(\Box F_{13}) \vdash F_{10}} \\ \underline{\bullet h_2: unbox(\Box F_{12}) \vdash F_{10}} \\ \underline{\bullet h_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, []F_{10}), F_{13}} \\ \underline{\bullet h_1: (\Box \Gamma_{12}, \Delta_{14}), F_7 \vee F_8 \vdash (\Delta_{11}, []F_{10}), F_{13}} \\ \underline{\bullet h_2: unbox(\Box F_{12}) \vdash F_{10}} \\ \underline{\bullet h_2: unbox(\Box F_{12}) \vdash F_{1$$

 \bullet Case rule A45

$$\frac{\underline{h_1: F_7, \Box \Gamma_{14}, \Delta_{15} \vdash \Box F_{13}, \Box \Gamma_{11}, \Delta_{12}, []F_{10} \quad h_1: F_8, \Box \Gamma_{14}, \Delta_{15} \vdash \Box F_{13}, \Box \Gamma_{11}, \Delta_{12}, []F_{10}}{\bullet h_1: (\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8 \vdash (\Box \Gamma_{11}, \Delta_{12}, []F_{10}), \Box F_{13}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Box \Gamma_{13}, \Phi_{19}: ((\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8)}{\bullet h_9: (\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8}{\bullet h_9: (\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8}{\bullet h_9: (\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}}{\bullet h_9: (\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}}{\bullet h_9: (\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Box \Gamma_{11}, A_{15}, F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}}{\bullet h_9: (\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}}{\bullet h_9: (\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}}{\bullet h_9: (\Box \Gamma_{14}, \Delta_{15}), F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Box \Gamma_{11}, []F_{10}}{\bullet h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}}{\bullet h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Box \Gamma_{11}, \Delta_{12}, \Box \Gamma_{11}, []F_{10}}{\bullet h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{10}}{\bullet h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{10}}{\bullet h_9: \Box \Gamma_{14}, \Delta_{15}, F_7 \vee F_8 \vdash \Delta_{12}, \Box \Gamma_{11}, []F_{10}} \vee_L \quad \frac{h_9: \Box \Gamma_{14}, \Delta_{15}, \Delta_{15}, F_7 \vee F_8 \vdash \Delta_{15}, \Delta_{15}, F_7 \vee F_8 \vdash \Delta_{15}, \Delta_{15}, F_7 \vee F_8$$

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\frac{\mathbf{h}_{1}: \mathbf{f}_{7}, \Box \Gamma_{13}, \Delta_{15} \vdash \mathbf{f}_{14}, \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_{10} \quad \mathbf{h}_{1}: \mathbf{f}_{8}, \Box \Gamma_{13}, \Delta_{15} \vdash \mathbf{f}_{14}, \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_{10}}{\bullet \mathbf{h}_{1}: (\Box \Gamma_{13}, \Delta_{15}), \mathbf{f}_{7} \lor \mathbf{f}_{8} \vdash (\Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_{10}), \mathbf{f}_{14}} \lor_{L} \quad \frac{\mathbf{h}_{9}: (\Box \Gamma_{13}, \Delta_{15}), \mathbf{f}_{7} \lor \mathbf{f}_{8}), \mathbf{f}_{14} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_{10}}{-: (\Box \Gamma_{13}, \Delta_{15}), \mathbf{f}_{7} \lor \mathbf{f}_{8} \vdash \Box \Gamma_{11}, \Delta_{12}, []\mathbf{f}_{10}} \quad A45} \quad Cut
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$$\frac{\mathbf{h}_{1}: F_{7}, \Delta_{13} \vdash F_{10} \to F_{11}, \Delta_{12}}{\bullet \mathbf{h}_{1}: \Delta_{13}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10} \to F_{11}} \vee_{L} \frac{\mathbf{h}_{9}: \Delta_{13}, F_{7} \lor F_{8} \vdash F_{10}, \Delta_{12}}{\bullet \mathbf{h}_{9}: (\Delta_{13}, F_{7} \lor F_{8}), F_{10} \to F_{11} \vdash \Delta_{12}} \vee_{L} \frac{\bullet \mathbf{h}_{9}: (\Delta_{13}, F_{7} \lor F_{8}), F_{10} \to F_{11} \vdash \Delta_{12}}{\bullet \mathbf{h}_{9}: (\Delta_{13}, F_{7} \lor F_{8}), F_{10} \to F_{11} \vdash \Delta_{12}} \wedge_{L} \frac{\bullet \mathbf{h}_{1}: \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}}{\bullet \mathbf{h}_{1}: \Delta_{13}, F_{10}, F_{11} \vdash \Delta_{12}, F_{11}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{13}, F_{10}, F_{7} \vdash \Delta_{12}, F_{11}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{13}, F_{10}, F_{7} \vdash \Delta_{12}, F_{11}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{13}, F_{10}, F_{7} \vdash A_{12}, F_{11}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{13}, F_{10}, F_{11} \vdash F_{13}, \Delta_{12}} \wedge_{L} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}, F_{13}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}, F_{13}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}, F_{13}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}, F_{13}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}, F_{13}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}, F_{13}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}, F_{13}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_{12}, F_{10}} \stackrel{\mathsf{inv} + \mathsf{th}/\mathsf{ax}}{\bullet \mathbf{h}_{1}: \Delta_{14}, F_{7} \lor F_{8} \vdash \Delta_$$

• Case rule \wedge_L

• Case rule \vee_L

$$\frac{\underbrace{\begin{array}{l} \underbrace{h_1: F_7, \Delta_{13} \vdash F_{10} \lor F_{11}, \Delta_{12} \quad h_1: F_8, \Delta_{13} \vdash F_{10} \lor F_{11}, \Delta_{12}}_{\bullet h_1: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}} \lor_L \quad \underbrace{\begin{array}{l} \underbrace{h_9: F_{10}, \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12} \quad h_9: F_{11}, \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}}_{\bullet h_9: (\Delta_{13}, F_7 \lor F_8), F_{10} \lor F_{11} \vdash \Delta_{12}} \quad \text{Cut} \\ \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10}, F_{11} \quad \text{inv-th/ax} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10}, F_{11} \quad \text{inv-th/ax} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10}, F_{11} \quad \text{ord} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \quad \text{ord} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \quad \text{ord} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \quad \text{ord} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \quad \text{ord} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12} \\ \hline \end{array}} \underbrace{\begin{array}{l} \underbrace{h_9: F_{10}, \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12} \quad h_9: F_{11}, \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}}_{\bullet u_1} \lor_L \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \\ \hline \end{array}} \underbrace{\begin{array}{l} \underbrace{h_9: F_{10}, \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12} \quad h_9: F_{11}, \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}}_{\bullet u_1} \lor_L \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10} \\ \hline -: \Delta_{13}, F_7 \lor F_8 \vdash \Delta_{12}, F_{10}, F_7 \lor F_8 \vdash \Delta_{12}, F_7 \lor F_8 \vdash \Delta_{12},$$

\bullet Case rule AT

$$\frac{ \frac{ \mathbf{h}_{1} : \mathbf{F}_{7}, \Delta_{12} \vdash [] \mathbf{F}_{10}, \Delta_{11} \quad \mathbf{h}_{1} : \mathbf{F}_{8}, \Delta_{12} \vdash [] \mathbf{F}_{10}, \Delta_{11}}{ \bullet \mathbf{h}_{1} : \Delta_{12}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{11}, [] \mathbf{F}_{10}} } \frac{ \mathbf{h}_{0} : (\Delta_{12}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{11})}{ \bullet \mathbf{h}_{9} : (\Delta_{12}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{11})} \frac{ \mathbf{h}_{0} : (\Delta_{12}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{11})}{ \bullet \mathbf{h}_{9} : \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{7}, [] \mathbf{F}_{10} \vdash \Delta_{11}} } \frac{ \mathbf{h}_{1} : \Delta_{12}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{11}}{ \bullet \mathbf{h}_{9} : \Delta_{12}, \mathbf{F}_{7}, [] \mathbf{F}_{10} \vdash \Delta_{11}} \frac{ \mathbf{h}_{1} : \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{11}, [] \mathbf{F}_{10}}{ \bullet \mathbf{h}_{9} : \Delta_{12}, \mathbf{F}_{10}, \mathbf{F}_{8}, [] \mathbf{F}_{10} \vdash \Delta_{11}} \frac{ \mathbf{h}_{1} : \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{11}, [] \mathbf{F}_{10}}{ \bullet \mathbf{h}_{9} : \Delta_{12}, \mathbf{F}_{8}, [] \mathbf{F}_{10} \vdash \Delta_{11}} \frac{ \mathbf{h}_{1} : \mathbf{h}_{1} : \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{11}, [] \mathbf{F}_{10}}{ \bullet \mathbf{h}_{9} : \Delta_{12}, \mathbf{F}_{8}, [] \mathbf{F}_{10} \vdash \Delta_{11}} \frac{ \mathbf{h}_{1} : \mathbf{h}_{1} : \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{11}, [] \mathbf{F}_{10}}{ \bullet \mathbf{h}_{9} : \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{11}} \frac{ \mathbf{h}_{1} : \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{11}, [] \mathbf{F}_{10}}{ \bullet \mathbf{h}_{9} : \Delta_{12}, \mathbf{F}_{8}, [] \mathbf{F}_{10} \vdash \Delta_{11}} \frac{ \mathbf{h}_{1} : \mathbf{h}_{1} : \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{11}, [] \mathbf{h}_{1} : \Delta_{13}, [] \mathbf{h}_{10} \vdash \mathbf{h}_{11}, \Delta_{13}, [] \mathbf{h}_{10} \vdash \mathbf{h}_{12}, \Delta_{13}, [] \mathbf{h}_{10} \vdash \mathbf{h}_{12}, \Delta_{13}, [] \mathbf{h}_{10}, \mathbf{h}_{12}, \Delta_{13}, \Delta_{1$$

• Case rule \perp_L

$$\frac{ \frac{h_1 : F_7, \Delta_{11} \vdash \bot, \Delta_{10} \quad h_1 : F_8, \Delta_{11} \vdash \bot, \Delta_{10}}{\bullet h_1 : \Delta_{11}, F_7 \lor F_8 \vdash \Delta_{10}, \bot} }{ - : \Delta_{11}, F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{\bullet h_9 : (\Delta_{11}, F_7 \lor F_8), \bot \vdash \Delta_{10}} } \frac{\bot_L}{\operatorname{Cut}}$$

$$\frac{h_1 : \Delta_{11}, F_7 \vdash \bot, \Delta_{10}}{\bullet h_9 : \bot, \Delta_{11}, F_7 \vdash \Delta_{10}} \frac{\bot_L}{\bullet \operatorname{Cut}} \frac{\bot_L}{h \operatorname{Cut}} \frac{\bot_L}{h \operatorname{Cut}} \frac{\bot_L}{h \operatorname{Cut}} \frac{\bot_L}{- : \Delta_{11}, F_8 \vdash \bot, \Delta_{10}} \frac{\operatorname{ax/W}}{\bullet h_9 : \bot, \Delta_{11}, F_8 \vdash \Delta_{10}} }{ - : \Delta_{11}, F_8 \vdash \Delta_{10}} \frac{\bot_L}{h \operatorname{Cut}} \frac{\bot_L}{h \operatorname{Cut}} \frac{\bot_L}{h \operatorname{Cut}} \frac{\bot_L}{- : \Delta_{11}, F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : \Delta_{11}, F_8 \vdash \Delta_{10}} \frac{\bot_L}{h \operatorname{Cut}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}} \frac{\bot_L}{- : (\bot, \Delta_{12}), F_7 \lor F_8 \vdash \Delta_{10}}}$$

\bullet Case rule I

$$\frac{\frac{\mathbf{h}_{1}: \mathbf{F}_{7}, \Delta_{12} \vdash \mathbf{p}_{11}, \Delta_{10}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash (\Delta_{10}, \mathbf{p}_{11}), \mathbf{p}_{11}}} }{-: \Delta_{12}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash (\Delta_{10}, \mathbf{p}_{11}), \mathbf{p}_{11}}} \underbrace{\frac{\mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash (\Delta_{10}, \mathbf{p}_{11}), \mathbf{p}_{11}}{\bullet \mathbf{h}_{9}: (\Delta_{12}, \mathbf{F}_{7} \lor \mathbf{F}_{8}), \mathbf{p}_{11} \vdash \Delta_{10}, \mathbf{p}_{11}}} }_{\bullet \mathbf{h}_{9}: \Delta_{10}, \mathbf{p}_{11}} \underbrace{\frac{\mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{7} \vdash \Delta_{10}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{9}: \Delta_{12}, \mathbf{F}_{7}, \mathbf{p}_{11} \vdash \Delta_{10}, \mathbf{p}_{11}}} }_{\bullet \mathbf{h}_{9}: \Delta_{12}, \mathbf{F}_{7}, \mathbf{p}_{11} \vdash \Delta_{10}, \mathbf{p}_{11}}} \underbrace{\frac{\mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{p}_{11}}} }_{\bullet \mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{p}_{11}} \underbrace{\frac{\mathbf{h}_{1}: \Delta_{12}, \mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{1}: \mathbf{F}_{12}, \Delta_{10}, \mathbf{p}_{11}} }_{\bullet \mathbf{h}_{1}: \mathbf{F}_{12}, \Delta_{10}, \mathbf{p}_{11}} \underbrace{\frac{\mathbf{h}_{1}: \mathbf{F}_{12}, \Delta_{10}, \mathbf{p}_{11}}{\bullet \mathbf{h}_{1}: \mathbf{F}_{8}, \Delta_{13}, \mathbf{p}_{11} \vdash \mathbf{F}_{12}, \Delta_{10}, \mathbf{p}_{11}} }_{\bullet \mathbf{h}_{9}: ((\Delta_{13}, \mathbf{p}_{11}), \mathbf{F}_{7} \lor \mathbf{F}_{8}), \mathbf{F}_{12} \vdash \Delta_{10}, \mathbf{p}_{11}} \underbrace{\frac{I}{\mathbf{h}_{1}: \mathbf{h}_{1}: \mathbf{F}_{12}, \Delta_{10}, \mathbf{p}_{11}}}_{\bullet \mathbf{h}_{1}: (\Delta_{13}, \mathbf{p}_{11}), \mathbf{F}_{7} \lor \mathbf{F}_{8} \vdash \Delta_{10}, \mathbf{p}_{11}} \underbrace{\frac{I}{\mathbf{h}_{1}: \mathbf{h}_{1}: \mathbf{h$$

$$\frac{\mathbf{h}_{1}: \mathsf{F}_{7}, \Delta_{11} \vdash \top, \Delta_{10} \quad \mathbf{h}_{1}: \mathsf{F}_{8}, \Delta_{11} \vdash \top, \Delta_{10}}{\bullet \mathbf{h}_{1}: \Delta_{11}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \Delta_{10}, \top} \lor_{L} \quad \frac{\mathbf{h}_{9}: \Delta_{11}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \Delta_{10}}{\bullet \mathbf{h}_{9}: (\Delta_{11}, \mathsf{F}_{7} \lor \mathsf{F}_{8}), \top \vdash \Delta_{10}} \, \mathsf{T}_{L} \\ -: \Delta_{11}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \Delta_{10} \\ & \xrightarrow{-: \Delta_{11}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \Delta_{10}} \, \mathsf{ax/W} \\ \\ \frac{\mathbf{h}_{1}: \mathsf{F}_{7}, \top, \Delta_{12} \vdash \mathsf{F}_{11}, \Delta_{10} \quad \mathbf{h}_{1}: \mathsf{F}_{8}, \top, \Delta_{12} \vdash \mathsf{F}_{11}, \Delta_{10}}{-: \Delta_{12}, \mathsf{F}_{11}, \Delta_{10}} \, \lor_{L} \quad \frac{\mathbf{h}_{9}: \mathsf{F}_{11}, \Delta_{12}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \Delta_{10}}{\bullet \mathbf{h}_{9}: ((\top, \Delta_{12}), \mathsf{F}_{7} \lor \mathsf{F}_{8}), \mathsf{F}_{11} \vdash \Delta_{10}} \, \, \mathsf{T}_{L} \\ & \xrightarrow{-: (\top, \Delta_{12}), \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \Delta_{10}} \\ & \xrightarrow{\bullet \mathbf{h}_{1}: \top, \Delta_{12}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{F}_{11}} \, \, \overset{\mathsf{ax/W}}{\mathsf{ax/W}} \\ & \xrightarrow{\bullet \mathbf{h}_{1}: \top, \Delta_{12}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{F}_{11}} \, \, \overset{\mathsf{ax/W}}{\mathsf{hout}} \, \, \overset{\mathsf{ax/W}}{\mathsf{hout}} \\ \\ & \xrightarrow{\bullet \mathbf{h}_{1}: \top, \Delta_{12}, \mathsf{F}_{7} \lor \mathsf{F}_{8} \vdash \Delta_{10}, \mathsf{F}_{11}} \, \, \overset{\mathsf{ax/W}}{\mathsf{hout}} \, \, \overset{\mathsf{ax/W}}{\mathsf{hout}}$$

6.11 Status of AT: OK

• Case rule \rightarrow_R

$$\frac{\frac{\mathbf{h}_{1}: \mathsf{F}_{6}, \Delta_{12}, []\mathsf{F}_{6} \vdash \mathsf{F}_{11}, \Delta_{10}, \mathsf{F}_{8} \to \mathsf{F}_{9}}{\bullet \mathsf{h}_{1}: \Delta_{12}, []\mathsf{F}_{6} \vdash (\Delta_{10}, \mathsf{F}_{8} \to \mathsf{F}_{9}), \mathsf{F}_{11}}} AT \xrightarrow{\begin{array}{c} \mathsf{h}_{7}: \mathsf{F}_{8}, \mathsf{F}_{11}, \Delta_{12}, []\mathsf{F}_{6} \vdash \mathsf{F}_{9}, \Delta_{10} \\ \bullet \mathsf{h}_{7}: (\Delta_{12}, []\mathsf{F}_{6}), \mathsf{F}_{11} \vdash \Delta_{10}, \mathsf{F}_{8} \to \mathsf{F}_{9} \\ \end{array}} \xrightarrow{-: \Delta_{12}, []\mathsf{F}_{6} \vdash \Delta_{10}, \mathsf{F}_{8} \to \mathsf{F}_{9}} \xrightarrow{\bullet \mathsf{h}_{7}: (\Delta_{12}, []\mathsf{F}_{6}), \mathsf{F}_{11} \vdash \Delta_{10}, \mathsf{F}_{8} \to \mathsf{F}_{9}} \xrightarrow{\bullet \mathsf{h}_{7}: \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{6}, []\mathsf{F}_{6} \vdash \Delta_{10}, \mathsf{F}_{8} \to \mathsf{F}_{9}} \xrightarrow{\bullet \mathsf{h}_{7}: \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{6}, []\mathsf{F}_{6} \vdash \Delta_{10}, \mathsf{F}_{8} \to \mathsf{F}_{9}} \xrightarrow{\bullet \mathsf{h}_{Cut}} \xrightarrow{\bullet \mathsf{h}_{Cut$$

• Case rule \wedge_R

$$\frac{ \frac{\mathbf{h}_1 : F_6, \Delta_{12}, []F_6 \vdash F_{11}, \Delta_{10}, F_8 \land F_9 }{\bullet \mathbf{h}_1 : \Delta_{12}, []F_6 \vdash (\Delta_{10}, F_8 \land F_9), F_{11} } }{ - : \Delta_{12}, []F_6 \vdash \Delta_{10}, F_8 \land F_9 } \underbrace{ \frac{\mathbf{h}_7 : F_{11}, \Delta_{12}, []F_6 \vdash F_8, \Delta_{10} \quad \mathbf{h}_7 : F_{11}, \Delta_{12}, []F_6 \vdash F_9, \Delta_{10} }{\bullet \mathbf{h}_7 : (\Delta_{12}, []F_6), F_{11} \vdash \Delta_{10}, F_8 \land F_9 }} \underbrace{ \begin{array}{c} \mathbf{Cut} \\ \mathbf{Cut} \\ \hline \\ \frac{\mathbf{h}_1 : \Delta_{12}, F_6, []F_6 \vdash \Delta_{10}, F_{11}, F_8 \land F_9 \\ \hline \\ - : \Delta_{12}, F_6, []F_6 \vdash \Delta_{10}, F_8 \land F_9 \\ \hline \\ - : \Delta_{12}, []F_6 \vdash \Delta_{10}, F_8 \land F_9 \end{array} } \underbrace{ \begin{array}{c} \mathbf{A}TG \\ \mathbf$$

• Case rule \vee_R

$$\frac{ \frac{\mathbf{h}_{1} : \mathbf{F}_{6}, \Delta_{12}, []\mathbf{F}_{6} \vdash \mathbf{F}_{11}, \Delta_{10}, \mathbf{F}_{8} \vee \mathbf{F}_{9}}{\bullet \mathbf{h}_{1} : \Delta_{12}, []\mathbf{F}_{6} \vdash (\Delta_{10}, \mathbf{F}_{8} \vee \mathbf{F}_{9}), \mathbf{F}_{11}} } AT \quad \frac{\mathbf{h}_{7} : \mathbf{F}_{11}, \Delta_{12}, []\mathbf{F}_{6} \vdash \mathbf{F}_{8}, \mathbf{F}_{9}, \Delta_{10}}{\bullet \mathbf{h}_{7} : (\Delta_{12}, []\mathbf{F}_{6}), \mathbf{F}_{11} \vdash \Delta_{10}, \mathbf{F}_{8} \vee \mathbf{F}_{9}} } \\ -: \Delta_{12}, []\mathbf{F}_{6} \vdash \Delta_{10}, \mathbf{F}_{8} \vee \mathbf{F}_{9} \\ \hline \frac{\mathbf{h}_{1} : \Delta_{12}, \mathbf{F}_{6}, []\mathbf{F}_{6} \vdash \Delta_{10}, \mathbf{F}_{11}, \mathbf{F}_{8} \vee \mathbf{F}_{9}}{\bullet \mathbf{h}_{7} : \Delta_{12}, \mathbf{F}_{11}, \mathbf{F}_{6}, []\mathbf{F}_{6} \vdash \Delta_{10}, \mathbf{F}_{8} \vee \mathbf{F}_{9}} } \\ \frac{-: \Delta_{12}, \mathbf{F}_{6}, []\mathbf{F}_{6} \vdash \Delta_{10}, \mathbf{F}_{8} \vee \mathbf{F}_{9}}{-: \Delta_{12}, []\mathbf{F}_{6} \vdash \Delta_{10}, \mathbf{F}_{8} \vee \mathbf{F}_{9}} ATG$$

• Case rule \perp_R

$$\frac{\mathbf{h}_1: \mathbf{F}_6, \Delta_{10}, []\mathbf{F}_6 \vdash \mathbf{F}_9, \bot, \Delta_8}{\bullet \mathbf{h}_1: \Delta_{10}, []\mathbf{F}_6 \vdash (\bot, \Delta_8), \mathbf{F}_9} \begin{array}{c} AT & \frac{\mathbf{h}_7: \mathbf{F}_9, \Delta_{10}, []\mathbf{F}_6 \vdash \Delta_8}{\bullet \mathbf{h}_7: (\Delta_{10}, []\mathbf{F}_6), \mathbf{F}_9 \vdash \bot, \Delta_8} \end{array} \begin{array}{c} \bot_R \\ \text{cut} \\ \hline \\ -: \Delta_{10}, []\mathbf{F}_6 \vdash \bot, \Delta_8 \\ \hline \\ \bullet \mathbf{h}_1: \Delta_{10}, []\mathbf{F}_6 \vdash \bot, \Delta_8, \mathbf{F}_9} \end{array} \begin{array}{c} \Delta_R \\ \bullet \mathbf{h}_7: \Delta_{10}, \mathbf{F}_9, []\mathbf{F}_6 \vdash \bot, \Delta_8 \\ \hline \\ -: \Delta_{10}, []\mathbf{F}_6 \vdash \bot, \Delta_8} \end{array} \begin{array}{c} \Delta_R \\ \text{hCut} \end{array}$$

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_6, \Delta_{10}, []\mathbf{F}_6 \vdash \mathbf{F}_9, \top, \Delta_8}{\bullet \mathbf{h}_1: \Delta_{10}, []\mathbf{F}_6 \vdash (\top, \Delta_8), \mathbf{F}_9} \quad AT \quad & \\ \hline \bullet \mathbf{h}_7: (\Delta_{10}, []\mathbf{F}_6 \vdash (\top, \Delta_8), \mathbf{F}_9 \vdash \top, \Delta_8} \\ & -: \Delta_{10}, []\mathbf{F}_6 \vdash \top, \Delta_8 \\ & \rightarrow \\ \hline -: \Delta_{10}, []\mathbf{F}_6 \vdash \top, \Delta_8} \quad \top_R \end{array} \quad \mathbf{Cut}$$

\bullet Case rule K

$$\begin{array}{c} \frac{h_1:F_6,(\square\Gamma_{11},\Delta_{12}), \|F_6\vdash\square F_{10},\Delta_9, \|F_8}{\bullet h_1:(\square\Gamma_{11},\Delta_{12}), \|F_6\vdash(\Delta_9, \|F_8), \square F_{10}} \\ AT & \frac{h_7:F_6,unbox(\square\Gamma_{11}),unbox(\square F_{10})\vdash F_8}{\bullet h_7:((\square\Gamma_{11},\Delta_{12}), \|F_6), \square F_{10}\vdash\Delta_9, \|F_8} \\ K \\ Cut \\ \hline \\ & -:(\square\Gamma_{11},\Delta_{12}), \|F_6\vdash\Delta_9, \|F_8 \\ \hline \\ & \frac{h_1:\Delta_{12},F_6,\square\Gamma_{11}, \|F_6\vdash\square F_{10},\Delta_9, \|F_8}{\bullet h_7:\square F_{10},\Delta_{12},F_6,\square\Gamma_{11}, \|F_6\vdash\Delta_9, \|F_8} \\ \hline \\ & \frac{-:\Delta_{12},F_6,\square\Gamma_{11}, \|F_6\vdash\Delta_9, \|F_8}{-:\Delta_{12},\square\Gamma_{11}, \|F_6\vdash\Delta_9, \|F_8} \\ \hline \\ & \frac{h_1:F_6,(\square\Gamma_{10},\Delta_{12}), \|F_6\vdash F_{11},\Delta_9, \|F_8}{-:\Delta_{12},\square\Gamma_{11}, \|F_6\vdash\Delta_9, \|F_8} \\ \hline \\ & \frac{-:\Delta_{12},\Gamma_{11}, \|F_6\vdash\Delta_9, \|F_8}{\bullet h_1:(\square\Gamma_{10},\Delta_{12}), \|F_6\vdash(\Delta_9, \|F_8),F_{11}} \\ \hline \\ & \frac{-:C_{12},C_{11},C_{12},C_{11},C_{12},C_{12},C_{13},C_{14}$$

• Case rule A45

$$\frac{\mathbf{h}_{1}: \mathbf{F}_{6}, (\Box \Gamma_{12}, \Delta_{13}), ([\mathbf{F}_{6} \vdash \Box \mathbf{F}_{11}, \Box \Gamma_{9}, \Delta_{10}, ([\mathbf{F}_{8}] \\ \bullet \mathbf{h}_{1}: (\Box \Gamma_{12}, \Delta_{13}), ([\mathbf{F}_{6} \vdash \Box \Gamma_{9}, \Delta_{10}, ([\mathbf{F}_{8}], \Box \mathbf{F}_{11}] \\ -: (\Box \Gamma_{12}, \Delta_{13}), ([\mathbf{F}_{6} \vdash \Box \Gamma_{9}, \Delta_{10}, ([\mathbf{F}_{8}], \Box \mathbf{F}_{11}] \\ \bullet \mathbf{h}_{7}: ((\Box \Gamma_{12}, \Delta_{13}), ([\mathbf{F}_{6}), \Box \mathbf{F}_{11} \vdash \Box \Gamma_{9}, \Delta_{10}, ([\mathbf{F}_{8}], \Box \mathbf{F}_{11}] \\ \bullet \mathbf{h}_{7}: ((\Box \Gamma_{12}, \Delta_{13}), ([\mathbf{F}_{6} \vdash \Box \Gamma_{9}, \Delta_{10}, ([\mathbf{F}_{8}], \Box \Gamma_{11}], ([\mathbf{F}_{6} \vdash \Delta_{10}, \Box \Gamma_{9}, ([\mathbf{F}_{8}], \Box \Gamma_{11}], ([\mathbf{F}_{6} \vdash \Delta_{10}, \Box \Gamma_{9}, ([\mathbf{F}_{8}], \Box \Gamma_{12}, ([\mathbf{F}_{6} \vdash \Delta_{10}, \Box \Gamma_{9}, ([\mathbf{F}_{8}], ([\mathbf{F}_{6} \vdash \Delta_{10}, \Box \Gamma_{9}, ([\mathbf{F}_{8}], ([\mathbf{F}_{6} \vdash \Delta_{10}, \Box \Gamma_{9}, ([\mathbf{F}_{8}], ([\mathbf{F}_{6} \vdash \Delta_{10}, ([\mathbf{F}_{8}], ([\mathbf{F}_{8}], ([\mathbf{F}_{8}], ([\mathbf{F}_{8}], ([\mathbf{F}_{8}], ([\mathbf{F}_{8}], ([\mathbf{F}_{8}], ([\mathbf{F}_{8}], ([$$

$$\frac{\mathbf{h}_{1}: \mathsf{F}_{6}, \Delta_{11}, []\mathsf{F}_{6} \vdash \mathsf{F}_{8} \to \mathsf{F}_{9}, \Delta_{10}}{\bullet \mathsf{h}_{1}: \Delta_{11}, []\mathsf{F}_{6} \vdash \mathsf{A}_{10}, \mathsf{F}_{8} \to \mathsf{F}_{9}]} \ AT \ \frac{\mathbf{h}_{7}: \Delta_{11}, []\mathsf{F}_{6} \vdash \mathsf{F}_{8}, \Delta_{10} \quad \mathsf{h}_{7}: \mathsf{F}_{9}, \Delta_{11}, []\mathsf{F}_{6} \vdash \Delta_{10}}{\bullet \mathsf{h}_{7}: (\Delta_{11}, []\mathsf{F}_{6}), \mathsf{F}_{8} \to \mathsf{F}_{9} \vdash \Delta_{10}} \ \mathsf{Cut} \\ \\ -: \Delta_{11}, []\mathsf{F}_{6} \vdash \Delta_{10} \\ \hline \frac{\mathsf{h}_{1}: \Delta_{11}, \mathsf{F}_{6}, []\mathsf{F}_{6} \vdash \Delta_{10}, \mathsf{F}_{8} \to \mathsf{F}_{9}]}{\bullet \mathsf{h}_{7}: \Delta_{11}, \mathsf{F}_{6}, []\mathsf{F}_{6}, \mathsf{F}_{8} \to \mathsf{F}_{9} \vdash \Delta_{10}]} \ \mathsf{ax}/\mathsf{W} \\ \mathsf{hCut} \\ \hline \frac{-: \Delta_{11}, [\mathsf{F}_{6} \vdash \Delta_{10}]}{-: \Delta_{11}, []\mathsf{F}_{6} \vdash \Delta_{10}} \ AT \\ \hline \frac{\mathsf{h}_{1}: \mathsf{F}_{6}, (\Delta_{12}, \mathsf{F}_{8} \to \mathsf{F}_{9}), []\mathsf{F}_{6} \vdash \mathsf{F}_{11}, \Delta_{10}]}{-: \Delta_{11}, []\mathsf{F}_{6} \vdash \Delta_{10}} \ \mathsf{AT} \ \frac{\mathsf{h}_{7}: \mathsf{F}_{11}, \Delta_{12}, []\mathsf{F}_{6} \vdash \mathsf{F}_{8}, \Delta_{10} \quad \mathsf{h}_{7}: \mathsf{F}_{9}, \mathsf{F}_{11}, \Delta_{12}, []\mathsf{F}_{6} \vdash \Delta_{10}}{\bullet \mathsf{h}_{7}: ((\Delta_{12}, \mathsf{F}_{8} \to \mathsf{F}_{9}), []\mathsf{F}_{6}), \mathsf{F}_{11} \vdash \Delta_{10}} \ \mathsf{Cut} \\ \hline -: (\Delta_{12}, \mathsf{F}_{8} \to \mathsf{F}_{9}), []\mathsf{F}_{6} \vdash \Delta_{10}, \mathsf{F}_{11}} \ \mathsf{ax}/\mathsf{W} \ \frac{\mathsf{h}_{7}: \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{6}, []\mathsf{F}_{6}, \mathsf{F}_{8} \to \mathsf{F}_{9} \vdash \Delta_{10}}{\bullet \mathsf{h}_{7}: \Delta_{12}, \mathsf{F}_{11}, \mathsf{F}_{6}, []\mathsf{F}_{6}, \mathsf{F}_{8} \to \mathsf{F}_{9} \vdash \Delta_{10}} \ \mathsf{hCut} \\ \hline -: \Delta_{12}, \mathsf{F}_{6}, []\mathsf{F}_{6}, \mathsf{F}_{8} \to \mathsf{F}_{9} \vdash \Delta_{10}} \ \mathsf{ATG} \ \mathsf{hCut}$$

• Case rule \wedge_L

$$\frac{\frac{\mathbf{h}_1: F_6, \Delta_{11}, []F_6 \vdash F_8 \land F_9, \Delta_{10}}{\bullet \mathbf{h}_1: \Delta_{11}, []F_6 \vdash \Delta_{10}, F_8 \land F_9]} AT \quad \frac{\mathbf{h}_7: F_8, F_9, \Delta_{11}, []F_6 \vdash \Delta_{10}}{\bullet \mathbf{h}_7: (\Delta_{11}, []F_6), F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_1: \Delta_{11}, []F_6 \vdash \Delta_{10}, F_8 \land F_9]} \text{ax/W} \xrightarrow{\bullet \mathbf{h}_7: \Delta_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_1: \Delta_{11}, F_6, []F_6 \vdash \Delta_{10}, F_8 \land F_9]} \text{ax/W} \xrightarrow{\bullet \mathbf{h}_7: \Delta_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_1: F_6, (\Delta_{12}, F_8 \land F_9), []F_6 \vdash F_{11}, \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_1: (\Delta_{12}, F_8 \land F_9), []F_6 \vdash \Delta_{10}, F_{11}} \\ \xrightarrow{\bullet \mathbf{h}_7: ((\Delta_{12}, F_8 \land F_9), []F_6 \vdash \Delta_{10}, F_{11}} \\ \xrightarrow{\bullet \mathbf{h}_7: ((\Delta_{12}, F_8 \land F_9), []F_6 \vdash \Delta_{10}, F_{11}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}, F_{11}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{10}} \\ \xrightarrow{\bullet \mathbf{h}_7: \Delta_{12}, F_{11}, F_6, []F_6, F_8 \land F_9 \vdash \Delta_{1$$

• Case rule \vee_L

$$\begin{array}{c} \frac{\mathbf{h}_1 : \mathbf{f}_6, \Delta_{11}, [] \mathbf{f}_6 \vdash \mathbf{f}_8 \vee \mathbf{f}_9, \Delta_{10}}{\bullet \mathbf{h}_1 : \Delta_{11}, [] \mathbf{f}_6 \vdash \Delta_{10} \quad \mathbf{h}_7 : \mathbf{f}_8, \Delta_{11}, [] \mathbf{f}_6 \vdash \Delta_{10} \quad \mathbf{h}_7 : \mathbf{f}_9, \Delta_{11}, [] \mathbf{f}_6 \vdash \Delta_{10}}{\bullet \mathbf{h}_7 : (\Delta_{11}, [] \mathbf{f}_6), \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}} \\ & -: \Delta_{11}, [] \mathbf{f}_6 \vdash \Delta_{10} \\ & \xrightarrow{\mathbf{h}_1 : \Delta_{11}, \mathbf{f}_6, [] \mathbf{f}_6 \vdash \Delta_{10}, \mathbf{f}_8 \vee \mathbf{f}_9}} & \mathbf{ax/W} & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}} \\ & \xrightarrow{\mathbf{h}_1 : \Delta_{11}, \mathbf{f}_6, [] \mathbf{f}_6 \vdash \Delta_{10}, \mathbf{f}_8 \vee \mathbf{f}_9}} & \mathbf{ax/W} \\ & \xrightarrow{-: \Delta_{11}, [] \mathbf{f}_6 \vdash \Delta_{10}} & ATG \\ & \xrightarrow{\bullet \mathbf{h}_1 : \mathbf{f}_6, (\Delta_{12}, \mathbf{f}_8 \vee \mathbf{f}_9), [] \mathbf{f}_6 \vdash \mathbf{f}_{11}, \Delta_{10}} \\ & \xrightarrow{\bullet \mathbf{h}_1 : (\Delta_{12}, \mathbf{f}_8 \vee \mathbf{f}_9), [] \mathbf{f}_6 \vdash \Delta_{10}, \mathbf{f}_{11}}} & AT & \xrightarrow{\bullet \mathbf{h}_7 : \mathbf{f}_8, \mathbf{f}_{11}, \Delta_{12}, [] \mathbf{f}_6 \vdash \Delta_{10}, \mathbf{h}_7 : \mathbf{f}_9, \mathbf{f}_{11}, \Delta_{12}, [] \mathbf{f}_6 \vdash \Delta_{10}} \\ & \xrightarrow{\bullet \mathbf{h}_1 : (\Delta_{12}, \mathbf{f}_8 \vee \mathbf{f}_9), [] \mathbf{f}_6 \vdash \Delta_{10}, \mathbf{f}_{11}}} & AT & \xrightarrow{\bullet \mathbf{h}_7 : \mathbf{f}_8, \mathbf{f}_{11}, \Delta_{12}, [] \mathbf{f}_6 \vdash \Delta_{10}, \mathbf{h}_7 : \mathbf{f}_9, \mathbf{f}_{11}, \Delta_{12}, [] \mathbf{f}_6 \vdash \Delta_{10}} \\ & \xrightarrow{-: (\Delta_{12}, \mathbf{f}_8 \vee \mathbf{f}_9), [] \mathbf{f}_6 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_{11}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12}, \mathbf{f}_6, [] \mathbf{f}_6, \mathbf{f}_8 \vee \mathbf{f}_9 \vdash \Delta_{10}}} & \mathbf{ax/W} \\ & \xrightarrow{\bullet \mathbf{h}_7 : \Delta_{12},$$

\bullet Case rule AT

$$\frac{ \frac{\mathbf{h}_{1} : F_{6}, \Delta_{10}, ||F_{6} \vdash ||F_{8}, \Delta_{9}}{\bullet \mathbf{h}_{1} : \Delta_{10}, ||F_{6} \vdash \Delta_{9}, ||F_{8}} \ AT \ \frac{\mathbf{h}_{7} : F_{8}, \Delta_{10}, ||F_{6}, ||F_{8} \vdash \Delta_{9}}{\bullet \mathbf{h}_{7} : (\Delta_{10}, ||F_{6}), ||F_{8} \vdash \Delta_{9}} \ Cut } \\ \hline - : \Delta_{10}, ||F_{6} \vdash \Delta_{9}| \longrightarrow \\ \frac{\mathbf{h}_{1} : \Delta_{10}, F_{6}, ||F_{6} \vdash \Delta_{9}, ||F_{8}}{\bullet \mathbf{h}_{1} : \Delta_{10}, F_{6}, ||F_{6} \vdash \Delta_{9}, ||F_{8}} \ \mathbf{ax/W} \ \frac{\rightarrow \mathbf{h}_{1} : \Delta_{10}, F_{6}, ||F_{6} \vdash \Delta_{9}, ||F_{8} \vdash \Delta_{9}|}{\bullet \mathbf{h}_{1} : (\Delta_{11}, ||F_{8}), ||F_{6} \vdash F_{10}, \Delta_{9}|} \ AT \\ \hline \frac{\mathbf{h}_{1} : F_{6}, (\Delta_{11}, ||F_{8}), ||F_{6} \vdash F_{10}, \Delta_{9}|}{\bullet \mathbf{h}_{1} : (\Delta_{11}, ||F_{8}), ||F_{6} \vdash \Delta_{9}, F_{10}|} \ AT \ \frac{\mathbf{h}_{7} : F_{8}, F_{10}, \Delta_{11}, ||F_{6}, ||F_{8} \vdash \Delta_{9}|}{\bullet \mathbf{h}_{7} : ((\Delta_{11}, ||F_{8}), ||F_{6}), F_{10} \vdash \Delta_{9}|} \ AT \\ \hline - : (\Delta_{11}, ||F_{8}), ||F_{6} \vdash \Delta_{9}| \ \mathbf{ax/W} \ \frac{\rightarrow \mathbf{h}_{1} : \Delta_{11}, F_{6}, ||F_{8} \vdash \Delta_{9}, F_{10}|}{\bullet \mathbf{h}_{1} : \Delta_{11}, F_{6}, ||F_{8} \vdash \Delta_{9}, F_{10}|} \ AT \\ \hline - : \Delta_{11}, ||F_{6}, ||F_{8} \vdash \Delta_{9}| \ AT \ \frac{\rightarrow \mathbf{h}_{1} : F_{7}, \Delta_{10}, ||F_{7} \vdash F_{9}, \Delta_{8}|}{\bullet \mathbf{h}_{1} : \Delta_{10}, ||F_{7} \vdash \Delta_{8}, F_{9}|} \ AT \ \frac{\mathbf{h}_{6} : F_{7}, F_{9}, \Delta_{10}, ||F_{7} \vdash \Delta_{8}|}{\bullet \mathbf{h}_{6} : (\Delta_{10}, ||F_{7}), F_{9} \vdash \Delta_{8}|} \ AT \ \frac{\rightarrow \mathbf{h}_{1} : \Delta_{10}, F_{7}, ||F_{7} \vdash \Delta_{8}, F_{9}|}{\bullet \mathbf{h}_{6} : \Delta_{10}, F_{7}, F_{9}, ||F_{7} \vdash \Delta_{8}|} \ \mathbf{ax/W} \ \mathbf{hCut} \ \frac{- : \Delta_{10}, F_{7}, ||F_{7} \vdash \Delta_{8}|}{- : \Delta_{10}, ||F_{7} \vdash \Delta_{8}|} \ AT \ \mathbf{hCut} \ \mathbf{h$$

• Case rule \perp_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{F}_6, \Delta_9, []\mathbf{F}_6 \vdash \bot, \Delta_8}{\underbrace{\bullet \mathbf{h}_1: \Delta_9, []\mathbf{F}_6 \vdash \Delta_8, \bot}_{\bullet \mathbf{h}_7: (\Delta_9, []\mathbf{F}_6), \bot \vdash \Delta_8}} \overset{\bot_L}{\text{cut}} \\ \xrightarrow{-: \Delta_9, []\mathbf{F}_6 \vdash \Delta_8} \xrightarrow{\bullet} \overset{\bot_L}{\underbrace{\bullet \mathbf{h}_7: \bot, \Delta_9, \mathbf{F}_6, []\mathbf{F}_6 \vdash \Delta_8}_{\bullet \mathbf{h}_7: \bot, \Delta_9, \mathbf{F}_6, []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\underbrace{-: \Delta_9, []\mathbf{F}_6 \vdash \Delta_8}_{\bullet \mathbf{h}_7: \bot, \Delta_9, \mathbf{F}_6, []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\bullet \mathbf{h}_7: \bot} \\ \underbrace{\frac{-: \Delta_9, \mathbf{F}_6, []\mathbf{F}_6 \vdash \Delta_8}{-: \Delta_9, []\mathbf{F}_6 \vdash \Delta_8}} \overset{AT}{\underbrace{\bullet \mathbf{h}_7: ((\bot, \Delta_{10}), []\mathbf{F}_6), \mathbf{F}_9 \vdash \Delta_8}_{\bullet \mathbf{h}_1: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8, \mathbf{F}_9}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}} \\ \underbrace{\frac{\bot_L}{-: \bot, \Delta_{10}, []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: \bot, \Delta_{10}, []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}} \\ \underbrace{\frac{\bot_L}{-: \bot, \Delta_{10}, []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: \bot, \Delta_{10}, []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: \bot, \Delta_{10}, []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: \bot, \Delta_{10}, []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: \bot, \Delta_{10}, []\mathbf{F}_6 \vdash \Delta_8}} \overset{\bot_L}{\underbrace{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}} \\ \underbrace{\frac{\bot_L}{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}} \\ \underbrace{\frac{\bot_L}{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}} \\ \underbrace{\frac{\bot_L}{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}} \\ \underbrace{\frac{\bot_L}{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}} \\ \underbrace{\frac{\bot_L}{-: (\bot, \Delta_{10}), []\mathbf{F}_6 \vdash \Delta_8}}$$

\bullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_{1}: \mathbf{F}_{6}, \Delta_{10}, []\mathbf{F}_{6} \vdash \mathbf{p}_{9}, \Delta_{8}, \mathbf{p}_{9}]}{\bullet \mathbf{h}_{1}: \Delta_{10}, []\mathbf{F}_{6} \vdash (\Delta_{8}, \mathbf{p}_{9}), \mathbf{p}_{9}]} AT & \bullet \mathbf{h}_{7}: (\Delta_{10}, []\mathbf{F}_{6}), \mathbf{p}_{9} \vdash \Delta_{8}, \mathbf{p}_{9}] & \mathbf{Cut} \\ \hline & -: \Delta_{10}, []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9}] & \bullet \mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{6}, \mathbf{p}_{9} \vdash \Delta_{8}, \mathbf{p}_{9}] & \mathbf{I} \\ \hline & \bullet \mathbf{h}_{7}: \Delta_{10}, \mathbf{F}_{6}, []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9}] & \mathbf{h}^{\mathbf{Cut}} \\ \hline & -: \Delta_{10}, \mathbf{F}_{6}, []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9}] & AT \\ \hline & \bullet \mathbf{h}_{1}: \mathbf{F}_{6}, (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \mathbf{F}_{10}, \Delta_{8}, \mathbf{p}_{9}] & AT \\ \hline & \bullet \mathbf{h}_{1}: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash (\Delta_{8}, \mathbf{p}_{9}), \mathbf{F}_{10} & AT \\ \hline & \bullet \mathbf{h}_{1}: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash (\Delta_{8}, \mathbf{p}_{9}), \mathbf{F}_{10} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}, \mathbf{p}_{9} & AT \\ \hline & -: (\Delta_{11}, \mathbf{p}_{9}), []\mathbf{F}_{6} \vdash \Delta_{8}$$

$$\begin{array}{c} \frac{\mathbf{h}_1: \mathbf{f}_6, \Delta_9, \left[\mid \mathbf{f}_6 \vdash \top, \Delta_8 \right]}{\bullet \mathbf{h}_1: \Delta_9, \left[\mid \mathbf{f}_6 \vdash \Delta_8, \top \right]} AT & \frac{\mathbf{h}_7: \Delta_9, \left[\mid \mathbf{f}_6 \vdash \Delta_8 \right]}{\bullet \mathbf{h}_7: (\Delta_9, \left[\mid \mathbf{f}_6 \right), \top \vdash \Delta_8} & \mathbf{T}_L \\ & -: \Delta_9, \left[\mid \mathbf{f}_6 \vdash \Delta_8 \right] \\ & -: \Delta_9, \left[\mid \mathbf{f}_6 \vdash \Delta_8 \right] & \mathbf{ax/W} \\ \\ \hline \frac{\mathbf{h}_1: \mathbf{f}_6, (\top, \Delta_{10}), \left[\mid \mathbf{f}_6 \vdash \mathbf{f}_9, \Delta_8 \right]}{\bullet \mathbf{h}_1: (\top, \Delta_{10}), \left[\mid \mathbf{f}_6 \vdash \Delta_8, \mathbf{f}_9 \right]} AT & \frac{\mathbf{h}_7: \mathbf{f}_9, \Delta_{10}, \left[\mid \mathbf{f}_6 \vdash \Delta_8 \right]}{\bullet \mathbf{h}_7: ((\top, \Delta_{10}), \left[\mid \mathbf{f}_6 \mid \Delta_8 \right]} & \mathbf{T}_L \\ \hline & -: (\top, \Delta_{10}), \left[\mid \mathbf{f}_6 \vdash \Delta_8 \right] \\ \hline & -: (\top, \Delta_{10}), \left[\mid \mathbf{f}_6 \vdash \Delta_8 \right] & \mathbf{ax/W} \\ \hline & \mathbf{h}_7: \top, \Delta_{10}, \mathbf{f}_9, \left[\mid \mathbf{f}_6 \vdash \Delta_8 \right] & \mathbf{ax/W} \\ \hline & -: \top, \Delta_{10}, \left[\mid \mathbf{f}_6 \vdash \Delta_8 \right] & \mathbf{ax/W} \\ \hline & -: \top, \Delta_{10}, \left[\mid \mathbf{f}_6 \vdash \Delta_8 \right] & \mathbf{ax/W} \\ \hline \end{array}$$

6.12 Status of \perp_L : OK

• Case rule \rightarrow_R

$$\frac{\underbrace{\bullet_{h_1}: \bot, \Delta_{10} \vdash (\Delta_8, F_6 \rightarrow F_7), F_9}_{\bullet h_1} \; \bot_L \quad \frac{h_5: \bot, F_6, F_9, \Delta_{10} \vdash F_7, \Delta_8}{\bullet_{h_5}: (\bot, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \rightarrow F_7}}_{-: \bot, \Delta_{10} \vdash \Delta_8, F_6 \rightarrow F_7} \; \bot_L}_{\text{Cut}}$$

• Case rule \wedge_R

$$\frac{\bullet \mathbf{h}_1: \bot, \Delta_{10} \vdash (\Delta_8, \mathbf{F}_6 \land \mathbf{F}_7), \mathbf{F}_9}{\bot_L} \xrightarrow{\begin{array}{c} \mathbf{h}_5: \bot, \mathbf{F}_9, \Delta_{10} \vdash \mathbf{F}_6, \Delta_8 & \mathbf{h}_5: \bot, \mathbf{F}_9, \Delta_{10} \vdash \mathbf{F}_7, \Delta_8 \\ \bullet \mathbf{h}_5: (\bot, \Delta_{10}), \mathbf{F}_9 \vdash \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7 \\ \hline -: \bot, \Delta_{10} \vdash \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7 & \bot_L \\ \hline -: \bot, \Delta_{10} \vdash \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7 & \bot_L \end{array}} \land \mathbf{Cut}$$

• Case rule \vee_R

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \bot, \Delta_{10} \vdash (\Delta_8, F_6 \vee F_7), F_9 \\ - : \bot, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline \\ - : \bot, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline \\ - : \bot, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline \\ - : \bot, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline \end{array} } \begin{array}{c} \bullet_{h_5} : \bot, F_9, \Delta_{10} \vdash F_6, F_7, \Delta_8 \\ \bullet_{h_5} : (\bot, \Delta_{10}), F_9 \vdash \Delta_8, F_6 \vee F_7 \\ \hline \\ - : \bot, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline \\ - : \bot, \Delta_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline \end{array} \right. \\ \text{Cut}$$

• Case rule \perp_R

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{\mathbf{h}_1} : \bot, \Delta_8 \vdash (\bot, \Delta_6), \mathsf{F}_7 \end{array} \bot_L \quad \begin{array}{c} h_5 : \bot, \mathsf{F}_7, \Delta_8 \vdash \Delta_6 \\ \bullet_{\mathbf{h}_5} : (\bot, \Delta_8), \mathsf{F}_7 \vdash \bot, \Delta_6 \end{array}}_{-: \bot, \Delta_8 \vdash \bot, \Delta_6} \quad \begin{array}{c} \bot_R \\ \mathsf{Cut} \end{array} } \\ \underbrace{ \begin{array}{c} -: \bot, \Delta_8 \vdash \bot, \Delta_6 \\ -: \bot, \Delta_8 \vdash \bot, \Delta_6 \end{array}}_{-: \bot, \Delta_8 \vdash \bot, \Delta_6} \quad \bot_L \end{array} }_{-: \bot, \Delta_8 \vdash \bot, \Delta_6}$$

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \bot, \Delta_8 \vdash (\top, \Delta_6), \mathsf{F}_7 & \bot_L & \hline \bullet_{\mathbf{h}_5}: (\bot, \Delta_8), \mathsf{F}_7 \vdash \top, \Delta_6 \\ \hline -: \bot, \Delta_8 \vdash \top, \Delta_6 \\ \hline -: \bot, \Delta_8 \vdash \top, \Delta_6 & \top_R \\ \hline \hline -: \bot, \Delta_8 \vdash \top, \Delta_6 & \top_R \\ \end{array}$$

ullet Case rule K

$$\begin{array}{c|c} \underline{\bullet_{h_1}: \bot, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, []F_6), \Box F_8} & \bot_L & \frac{h_5: unbox(\Box \Gamma_9), unbox(\Box F_8) \vdash F_6}{\bullet h_5: (\bot, \Box \Gamma_9, \Delta_{10}), \Box F_8 \vdash \Delta_7, []F_6} & K \\ \underline{-: \bot, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, []F_6} & \bot_L \\ \hline \underline{-: \bot, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, []F_6} & \bot_L \\ \hline \underline{\bullet_{h_1}: \bot, \Box \Gamma_8, \Delta_{10} \vdash (\Delta_7, []F_6), F_9} & \bot_L & \frac{h_5: unbox(\Box \Gamma_8) \vdash F_6}{\bullet h_5: (\bot, \Box \Gamma_8, \Delta_{10}), F_9 \vdash \Delta_7, []F_6} & K \\ \underline{-: \bot, \Box \Gamma_8, \Delta_{10} \vdash \Delta_7, []F_6} & \bot_L \\ \hline \underline{-: \bot, \Delta_{10}, \Box \Gamma_8 \vdash \Delta_7, []F_6} & \bot_L \end{array}$$

 \bullet Case rule A45

$$\frac{ \begin{array}{c} \bullet_{h_1} : \bot, \Box \Gamma_{10}, \Delta_{11} \vdash (\Box \Gamma_7, \Delta_8, [] F_6), \Box F_9 \\ \hline \\ \bullet_{h_1} : \bot, \Box \Gamma_{10}, \Delta_{11} \vdash (\Box \Gamma_7, \Delta_8, [] F_6), \Box F_9 \\ \hline \\ - : \bot, \Box \Gamma_{10}, \Delta_{11} \vdash \Box \Gamma_7, \Delta_8, [] F_6 \\ \hline \\ - : \bot, \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_8, \Box \Gamma_7, [] F_6 \\ \hline \\ \bullet_{h_5} : (\bot, \Box \Gamma_{10}, \Delta_{11}), \Box F_9 \vdash \Box \Gamma_7, \Delta_8, [] F_6 \\ \hline \\ \bullet_{h_5} : (\bot, \Box \Gamma_9 \vdash \Box \Gamma_7, F_6 \\ \hline \\ \bullet_{h_5} : (\bot, \Box \Gamma_9, \Delta_{11}), F_{10} \vdash \Box \Gamma_7, \Delta_8, [] F_6 \\ \hline \\ - : \bot, \Box \Gamma_9, \Delta_{11} \vdash \Box \Gamma_7, \Delta_8, [] F_6 \\ \hline \\ - : \bot, \Delta_{11}, \Box \Gamma_9 \vdash \Delta_8, \Box \Gamma_7, [] F_6 \\ \hline \\ - : \bot, \Delta_{11}, \Box \Gamma_9 \vdash \Delta_8, \Box \Gamma_7, [] F_6 \\ \hline \end{array} \right.$$

• Case rule \rightarrow_L

• Case rule \wedge_L

$$\frac{ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : \bot, \Delta_9 \vdash \Delta_8, F_6 \land F_7 \end{array}}_{\bullet \mathbf{h}_1} \ \bot_L \ \ \frac{ \begin{array}{c} \mathbf{h}_5 : \bot, F_6, F_7, \Delta_9 \vdash \Delta_8 \\ \bullet \mathbf{h}_5 : (\bot, \Delta_9), F_6 \land F_7 \vdash \Delta_8 \end{array}}_{\bullet \mathbf{h}_5 : (\bot, \Delta_9), F_6 \land F_7 \vdash \Delta_8} \ \ \begin{array}{c} \land_L \\ \bullet \mathbf{h}_5 : \bot, \Delta_9 \vdash \Delta_8 \end{array}}_{\bullet \mathbf{h}_5 : (\bot, \Delta_9), F_6 \land F_7 \vdash \Delta_8} \ \ \mathbf{h}_5 : \bot_{\bullet} \\ \bullet \mathbf{h}_5 : \bot_{\bullet} \\ \bullet$$

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \bot, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8, F_9 \\ \hline \\ \bullet_{h_2} : \bot, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8, F_9 \end{array} \, \bot_L \quad \underbrace{ \begin{array}{c} \bullet_{h_5} : \bot, F_6, F_7, F_9, \Delta_{10} \vdash \Delta_8 \\ \hline \\ \bullet_{h_5} : (\bot, \Delta_{10}, F_6 \wedge F_7), F_9 \vdash \Delta_8 \\ \hline \\ - : \bot, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8 \end{array} \, \subset_L }_{\quad \ \ - : \bot, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_8} \ \bot_L$$

 \bullet Case rule \vee_L

ullet Case rule AT

$$\begin{array}{c|c} \underbrace{ \begin{array}{c} \bullet_{h_1} : \bot, \Delta_8 \vdash \Delta_7, [] F_6 \\ } \end{array} \begin{array}{c} \bot_L & \frac{h_5 : \bot, F_6, \Delta_8, [] F_6 \vdash \Delta_7}{\bullet h_5 : (\bot, \Delta_8), [] F_6 \vdash \Delta_7} \\ & \xrightarrow{-: \bot, \Delta_8 \vdash \Delta_7} & \bot_L \\ \\ \hline \bullet_{h_1} : \bot, \Delta_9, [] F_6 \vdash \Delta_7, F_8 & \bot_L & \frac{h_5 : \bot, F_6, F_8, \Delta_9, [] F_6 \vdash \Delta_7}{\bullet h_5 : (\bot, \Delta_9, [] F_6), F_8 \vdash \Delta_7} \\ \hline \\ \bullet_{h_1} : \bot, \Delta_9, [] F_6 \vdash \Delta_7, F_8 & \xrightarrow{-: \bot, \Delta_9, [] F_6 \vdash \Delta_7} \end{array} \begin{array}{c} AT \\ \text{Cut} \\ \hline \\ -: \bot, \Delta_9, [] F_6 \vdash \Delta_7 \end{array} \end{array}$$

• Case rule \perp_L

ullet Case rule I

• Case rule \top_L

$$\frac{ \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : \bot, \Delta_7 \vdash \Delta_6, \top}_{\bullet \mathbf{h}_1} \ \bot_L \end{array} \begin{array}{c} \mathbf{h}_5 : \bot, \Delta_7 \vdash \Delta_6 \\ \bullet \mathbf{h}_5 : (\bot, \Delta_7), \top \vdash \Delta_6 \end{array}}_{-: \bot, \Delta_7 \vdash \Delta_6} \ \mathbf{Cut} \\ \underbrace{ \begin{array}{c} -: \bot, \Delta_7 \vdash \Delta_6 \\ \hline -: \bot, \Delta_7 \vdash \Delta_6 \end{array} \end{array}}_{-: \bot, \Delta_7 \vdash \Delta_6} \ \bot_L$$

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \bot, \top, \Delta_8 \vdash \Delta_6, \mathsf{F}_7 \\ - : \bot, \top, \Delta_8 \vdash \Delta_6 \end{array}}_{ \begin{array}{c} \bullet_{h_5} : \bot, \mathsf{F}_7, \Delta_8 \vdash \Delta_6 \\ \bullet_{h_5} : (\bot, \top, \Delta_8), \mathsf{F}_7 \vdash \Delta_6 \\ - : \bot, \top, \Delta_8 \vdash \Delta_6 \\ \hline - : \bot, \top, \Delta_8 \vdash \Delta_6 \end{array}}_{ \begin{array}{c} \bot_L \end{array}} \mathsf{Cut}$$

6.13 Status of I: OK

• Case rule \rightarrow_R

$$\frac{ \bullet \mathbf{h}_1 : \Delta_9, \mathbf{p}_{10} \vdash (\Delta_8, \mathbf{F}_6 \to \mathbf{F}_7), \mathbf{p}_{10} }{I} \frac{ \mathbf{h}_5 : \mathbf{F}_6, \Delta_9, \mathbf{p}_{10}, \mathbf{p}_{10} \vdash \mathbf{F}_7, \Delta_8 }{\bullet \mathbf{h}_5 : (\Delta_9, \mathbf{p}_{10}), \mathbf{p}_{10} \vdash \Delta_8, \mathbf{F}_6 \to \mathbf{F}_7 } \xrightarrow{\rightarrow_R} \mathbf{Cut}$$

$$\frac{ \bullet \mathbf{h}_1 : \Delta_9, \mathbf{p}_{10} \vdash \Delta_8, \mathbf{F}_7, \mathbf{p}_{10} }{I} \xrightarrow{\rightarrow} \frac{1}{\mathbf{h}_5 : \Delta_9, \mathbf{F}_6, \mathbf{p}_{10}, \mathbf{p}_{10} \vdash \Delta_8, \mathbf{F}_7 } \mathbf{ncut}$$

$$\frac{ \bullet \mathbf{h}_1 : \Delta_9, \mathbf{F}_6, \mathbf{p}_{10} \vdash \Delta_8, \mathbf{F}_7, \mathbf{p}_{10} }{I} \xrightarrow{\bullet \mathbf{h}_5 : \Delta_9, \mathbf{F}_6, \mathbf{p}_{10}, \mathbf{p}_{10} \vdash \Delta_8, \mathbf{F}_7 } \mathbf{ncut}$$

$$\frac{ - : \Delta_9, \mathbf{F}_6, \mathbf{p}_{10} \vdash \Delta_8, \mathbf{F}_7 }{- : \Delta_9, \mathbf{p}_{10} \vdash \Delta_8, \mathbf{F}_6 \to \mathbf{F}_7 } \xrightarrow{\bullet} \mathbf{ncut}$$

$$\frac{ \bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{p}_{10} \vdash ((\Delta_9, \mathbf{F}_7 \to \mathbf{F}_8), \mathbf{p}_{10}), \mathbf{F}_{11} }{I} \xrightarrow{\bullet \mathbf{h}_6 : \mathbf{F}_7, \mathbf{F}_{11}, \Delta_{12}, \mathbf{p}_{10} \vdash \mathbf{F}_8, \Delta_9, \mathbf{p}_{10} } \xrightarrow{\bullet} \mathbf{ncut}$$

$$\frac{\bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{p}_{10} \vdash ((\Delta_9, \mathbf{F}_7 \to \mathbf{F}_8), \mathbf{p}_{10}), \mathbf{F}_{11} }{\bullet} \xrightarrow{\bullet} \mathbf{ncut}$$

$$\frac{\bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{p}_{10} \vdash ((\Delta_9, \mathbf{F}_7 \to \mathbf{F}_8), \mathbf{p}_{10})}{\bullet} \xrightarrow{\bullet} \mathbf{ncut}$$

$$\frac{\bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{p}_{10} \vdash ((\Delta_9, \mathbf{F}_7 \to \mathbf{F}_8), \mathbf{p}_{10})}{\bullet} \xrightarrow{\bullet} \mathbf{ncut}$$

$$\frac{\bullet \mathbf{h}_1 : \Delta_{12}, \mathbf{p}_{10} \vdash ((\Delta_9, \mathbf{F}_7 \to \mathbf{F}_8), \mathbf{p}_{10})}{\bullet} \xrightarrow{\bullet} \mathbf{ncut}$$

• Case rule \wedge_R

$$\frac{\underbrace{\frac{\bullet h_1 : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \land F_7), p_{10}}{- : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \land F_7), p_{10}}} I \xrightarrow{\begin{array}{c} h_5 : \Delta_9, p_{10}, p_{10} \vdash F_6, \Delta_8 & h_5 : \Delta_9, p_{10}, p_{10} \vdash F_7, \Delta_8 \\ \bullet h_5 : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \land F_7 \\ \hline \\ \bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, p_{10} \end{array} I \xrightarrow{\begin{array}{c} h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6 \land F_7 \\ \hline \\ \bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, p_{10} \end{array} I \xrightarrow{\begin{array}{c} h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6 \land F_7 \\ \hline \\ \bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_7, p_{10} \end{array} I \xrightarrow{\begin{array}{c} h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_7 \\ \hline \\ \bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_7, p_{10} \end{array} I \xrightarrow{\begin{array}{c} h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_7 \\ \hline \\ \bullet h_1 : \Delta_1, p_{10} \vdash \Delta_8, F_6 \land F_7 \end{array}} A_R$$

$$\frac{\bullet h_1 : \Delta_1, p_{10} \vdash \Delta_8, F_6 \land F_7}{- : \Delta_1, p_{10} \vdash (\Delta_9, F_7 \land F_8), p_{10}} \land A_R$$

$$\frac{\bullet h_1 : \Delta_1, p_{10} \vdash (\Delta_9, F_7 \land F_8), p_{10}}{- : \Delta_1, p_{10} \vdash (\Delta_9, F_7 \land F_8), p_{10}} \land A_R$$

$$\frac{\bullet h_1 : \Delta_1, p_{10} \vdash (\Delta_9, F_7 \land F_8), p_{10}}{- : \Delta_1, p_{10} \vdash \Delta_9, p_{10}, F_7 \land F_8} I$$

• Case rule \vee_R

$$\frac{ \underbrace{ \begin{array}{c} \bullet_{h_1} : \Delta_9, p_{10} \vdash (\Delta_8, F_6 \vee F_7), p_{10} \\ \bullet h_2 : (\Delta_9, p_{10}), p_{10} \vdash F_6, F_7, \Delta_8 \\ - : \Delta_9, p_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline \bullet_{h_1} : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7, p_{10} \\ \end{array} \begin{array}{c} \to \\ \bullet_{h_2} : (\Delta_9, p_{10}), p_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline \bullet_{h_1} : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7, p_{10} \\ \hline & \bullet_{h_2} : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7 \\ \hline & - : \Delta_9, p_{10} \vdash \Delta_8, F_6, F_7 \\ \hline & - : \Delta_9, p_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline & \bullet_{h_2} : \Delta_9, p_{10} \vdash \Delta_8, F_6 \vee F_7 \\ \hline & \bullet_{h_3} : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10} \\ \hline & \bullet_{h_4} : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10} \\ \hline & \bullet_{h_5} : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10} \\ \hline & - : \Delta_{12}, p_{10} \vdash (\Delta_9, F_7 \vee F_8), p_{10} \\ \hline & \bullet_{h_3} : \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}, F_7 \vee F_8 \\ \hline & \bullet_{h_4} : \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}, F_7 \vee F_8 \\ \hline & \bullet_{h_5} : \Delta_{12}, p_{10} \vdash \Delta_9, p_{10}, F_7 \vee F_8 \\ \hline \end{array} \begin{array}{c} \vee_R \\ \vee_R$$

• Case rule \perp_R

$$\begin{array}{c} \underbrace{ \begin{array}{c} \bullet \mathbf{h}_1 : \Delta_7, \mathbf{p}_8 \vdash (\bot, \Delta_6), \mathbf{p}_8 \\ \hline \bullet \mathbf{h}_2 : (\Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \Delta_6 \\ \hline - : \Delta_7, \mathbf{p}_8 \vdash \bot, \Delta_6 \\ \hline \\ \bullet \mathbf{h}_1 : \Delta_7, \mathbf{p}_8 \vdash \bot, \Delta_6, \mathbf{p}_8 \\ \hline \end{array} \begin{array}{c} \bullet \mathbf{h}_2 : (\Delta_7, \mathbf{p}_8), \mathbf{p}_8 \vdash \bot, \Delta_6 \\ \hline \bullet \mathbf{h}_1 : \Delta_7, \mathbf{p}_8 \vdash \bot, \Delta_6, \mathbf{p}_8 \\ \hline \hline \bullet \mathbf{h}_1 : \Delta_7, \mathbf{p}_8 \vdash \bot, \Delta_6, \mathbf{p}_8 \\ \hline - : \Delta_7, \mathbf{p}_8 \vdash \bot, \Delta_6 \\ \hline \end{array} \begin{array}{c} \bullet \mathbf{h}_2 : \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \bot, \Delta_6 \\ \hline \bullet \mathbf{h}_3 : \Delta_7, \mathbf{p}_8 \vdash \bot, \Delta_6 \\ \hline \bullet \mathbf{h}_4 : \Delta_7, \mathbf{p}_8 \vdash \Delta_7, \mathbf{p}_8 \\ \hline - : \Delta_{10}, \mathbf{p}_8 \vdash (\bot, \Delta_7), \mathbf{p}_8 \\ \hline - : \Delta_{10}, \mathbf{p}_8 \vdash (\bot, \Delta_7, \mathbf{p}_8) \\ \hline - : \Delta_{10}, \mathbf{p}_8 \vdash \bot, \Delta_7, \mathbf{p}_8 \\ \hline - : \Delta_{10}, \mathbf{p}_8 \vdash \bot, \Delta_7, \mathbf{p}_8 \\ \hline \end{array} \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_2 : \Delta_7, \mathbf{h}_3 \vdash \Delta_7, \mathbf{h}_4 \\ \hline - : \Delta_{10}, \mathbf{h}_3 \vdash \Delta_7, \mathbf{h}_8 \\ \hline - : \Delta_{10}, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline - : \Delta_{10}, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline \end{array} \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_4 : \mathbf{h}_7, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline - : \Delta_{10}, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline - : \Delta_{10}, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline \end{array} \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_4 : \mathbf{h}_7, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline - : \Delta_{10}, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline - : \Delta_{10}, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline \end{array} \begin{array}{c} \bot_R \\ \bullet \mathbf{h}_7 : \Delta_7, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline - : \Delta_{10}, \mathbf{h}_8 \vdash \Delta_7, \mathbf{h}_8 \\ \hline \end{array}$$

$$\begin{array}{c|c} \hline \bullet_{\mathbf{h}_1}: \Delta_7, \mathbf{p}_8 \vdash (\top, \Delta_6), \mathbf{p}_8 & I & \bullet_{\mathbf{h}_5}: (\Delta_7, \mathbf{p}_8), \mathbf{p}_8 \vdash \top, \Delta_6 \\ \hline & -: \Delta_7, \mathbf{p}_8 \vdash \top, \Delta_6 \\ \hline & -: \Delta_7, \mathbf{p}_8 \vdash \top, \Delta_6 \\ \hline & -: \Delta_7, \mathbf{p}_8 \vdash \top, \Delta_6 \\ \hline \hline \bullet_{\mathbf{h}_1}: \Delta_{10}, \mathbf{p}_8 \vdash ((\top, \Delta_7), \mathbf{p}_8), \mathbf{F}_9 & I & \bullet_{\mathbf{h}_6}: (\Delta_{10}, \mathbf{p}_8), \mathbf{F}_9 \vdash (\top, \Delta_7), \mathbf{p}_8 \\ \hline & -: \Delta_{10}, \mathbf{p}_8 \vdash (\top, \Delta_7), \mathbf{p}_8 \\ \hline & -: \Delta_{10}, \mathbf{p}_8 \vdash \top, \Delta_7, \mathbf{p}_8 \\ \hline & -: \Delta_{10}, \mathbf{p}_8 \vdash \top, \Delta_7, \mathbf{p}_8 \end{array} \right.$$

\bullet Case rule K

$$\frac{\bullet_{h_1}: (\Box\Gamma_8, \Delta_9), p_{10} \vdash (\Delta_7, []F_6), p_{10}}{\bullet_{h_2}: (\Box\Gamma_8, \Delta_9), p_{10} \vdash \Delta_7, []F_6} I \xrightarrow{\bullet_{h_3}: ((\Box\Gamma_8, \Delta_9), p_{10}), p_{10} \vdash \Delta_7, []F_6} Cut$$

$$-: (\Box\Gamma_8, \Delta_9), p_{10} \vdash \Delta_7, []F_6 \xrightarrow{\bullet} Ax/W$$

$$-: unbox(\Box\Gamma_8) \vdash F_6 ax/W$$

$$-: \Delta_9, \Box\Gamma_8, p_{10} \vdash \Delta_7, []F_6 K$$

$$\bullet_{h_1}: (\Box\Gamma_{11}, \Delta_{12}), p_9 \vdash ((\Delta_8, []F_7), p_9), \Box F_{10} I \xrightarrow{\bullet_{h_6}: ((\Box\Gamma_{11}, \Delta_{12}), p_9), \Box F_{10} \vdash (\Delta_8, []F_7), p_9} K$$

$$-: (\Box\Gamma_{11}, \Delta_{12}), p_9 \vdash ((\Delta_8, []F_7), p_9) \xrightarrow{\bullet} I$$

$$-: \Delta_{12}, \Box\Gamma_{11}, p_9 \vdash \Delta_8, p_9, []F_7 I$$

$$\bullet_{h_6}: ((\Box\Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, []F_7), p_9), F_{11} \vdash (\Delta_8, []F_7), p_9$$

$$-: \Delta_{12}, \Box\Gamma_{11}, p_9 \vdash (\Delta_8, []F_7), p_9 \xrightarrow{\bullet} Cut$$

$$-: (\Box\Gamma_{10}, \Delta_{12}), p_9 \vdash ((\Delta_8, []F_7), p_9 \vdash (\Delta_8, []F_7), p_9$$

$$-: \Delta_{12}, \Box\Gamma_{10}, p_9 \vdash (\Delta_8, []F_7), p_9$$

$$-: \Delta_{12}, \Box\Gamma_{10}, p_9 \vdash (\Delta_8, []F_7), p_9$$

$$-: \Delta_{12}, \Box\Gamma_{10}, p_9 \vdash (\Delta_8, []F_7), p_9$$

• Case rule A45

$$\frac{\bullet_{h_1}: (\Box\Gamma_9, \Delta_{10}), p_{11} \vdash (\Box\Gamma_7, \Delta_8, []F_6), p_{11}}{\bullet_{h_5}: ((\Box\Gamma_9, \Delta_{10}), p_{11}), p_{11} \vdash \Box\Gamma_7, \Delta_8, []F_6} \underbrace{A45}_{\text{Cut}} \\ -: (\Box\Gamma_9, \Delta_{10}), p_{11} \vdash \Box\Gamma_7, \Delta_8, []F_6}_{-: \Box\Gamma_9 \vdash F_6, \Box\Gamma_7} \underbrace{ax/W}_{-: \Box\Gamma_9, p_{11} \vdash \Delta_8, \Box\Gamma_7, []F_6} \underbrace{A45}_{\text{Cut}} \\ \underbrace{-: \Box\Gamma_9 \vdash F_6, \Box\Gamma_7}_{-: \Delta_{10}, \Box\Gamma_9, p_{11} \vdash \Delta_8, \Box\Gamma_7, []F_6} \underbrace{A45}_{\text{h}_6} : \Box\Gamma_{12}, \Box F_{11} \vdash \Box\Gamma_8, F_7} \\ \underbrace{\bullet_{h_1}: (\Box\Gamma_{12}, \Delta_{13}), p_{10} \vdash ((\Box\Gamma_8, \Delta_9, []F_7), p_{10}), \Box F_{11}}_{-: (\Box\Gamma_{12}, \Delta_{13}), p_{10} \vdash (\Box\Gamma_8, \Delta_9, []F_7), p_{10}} \underbrace{A45}_{\text{Cut}} \\ \underbrace{-: (\Box\Gamma_{12}, \Delta_{13}), p_{10} \vdash (\Box\Gamma_8, \Delta_9, []F_7), p_{10}}_{-: \Delta_{13}, \Box\Gamma_{12}, p_{10} \vdash \Delta_9, \Box\Gamma_8, p_{10}, []F_7} I$$

$$\frac{ \underbrace{ \bullet_{\mathbf{h}_1} : (\Box \Gamma_{11}, \Delta_{13}), \mathsf{p}_{10} \vdash ((\Box \Gamma_8, \Delta_9, []\mathsf{F}_7), \mathsf{p}_{10}), \mathsf{F}_{12} }_{ \bullet \mathsf{h}_6 : ((\Box \Gamma_{11}, \Delta_{13}), \mathsf{p}_{10}), \mathsf{F}_{12} \vdash (\Box \Gamma_8, \Delta_9, []\mathsf{F}_7), \mathsf{p}_{10} }_{ \bullet \mathsf{t}_6 : ((\Box \Gamma_{11}, \Delta_{13}), \mathsf{p}_{10}), \mathsf{F}_{12} \vdash (\Box \Gamma_8, \Delta_9, []\mathsf{F}_7), \mathsf{p}_{10} }_{ \mathsf{Cut} } \\ \underbrace{ - : (\Box \Gamma_{11}, \Delta_{13}), \mathsf{p}_{10} \vdash (\Box \Gamma_8, \Delta_9, []\mathsf{F}_7), \mathsf{p}_{10} }_{ - : \Delta_{13}, \Box \Gamma_{11}, \mathsf{p}_{10} \vdash \Delta_9, \Box \Gamma_8, \mathsf{p}_{10}, []\mathsf{F}_7 } I$$

$$\frac{\underbrace{\bullet h_1 : (\Delta_9, F_6 \to F_7), p_{10} \vdash \Delta_8, p_{10}}_{\bullet h_1 : (\Delta_9, F_6 \to F_7), p_{10} \vdash \Delta_8, p_{10}} I \xrightarrow{\bullet h_5 : (\Delta_9, P_{10}, P_{10} \vdash F_6, \Delta_8 \quad h_5 : F_7, \Delta_9, p_{10}, p_{10} \vdash \Delta_8}_{\bullet h_5 : ((\Delta_9, F_6 \to F_7), p_{10}), p_{10} \vdash \Delta_8} Cut} \xrightarrow{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6, p_{10}} I \xrightarrow{h_5 : \Delta_9, p_{10}, p_{10} \vdash \Delta_8, F_6}_{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6} \underbrace{\bullet h_1 : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10}}_{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_8, F_6} I \xrightarrow{\bullet h_1 : \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10}} I \xrightarrow{h_5 : \Delta_9, F_7, p_{10} \vdash \Delta_8}_{\bullet h_1 : \Delta_9, p_{10} \vdash \Delta_9, p_{10}} \to L$$

$$\underbrace{\bullet h_1 : \Delta_1, p_{10} \vdash \Delta_9, p_{10} \vdash \Delta_9, p_{10} \vdash \Delta_9, p_{10}}_{\bullet h_6 : (\Delta_{11}, p_{10}) \vdash F_7, \Delta_9, p_{10} \vdash h_6 : F_8, \Delta_{11}, p_{10} \vdash \Delta_9, p_{10}}_{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \to F_8 \vdash \Delta_9, p_{10}} Cut$$

$$\underbrace{\bullet h_1 : \Delta_{11}, p_{10} \vdash (\Delta_9, p_{10}), F_7 \to F_8}_{\bullet h_6 : (\Delta_{11}, p_{10}), F_7 \to F_8 \vdash \Delta_9, p_{10}}_{\bullet h_6 : ((\Delta_{12}, F_7 \to F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}} Cut$$

$$\underbrace{\bullet h_1 : (\Delta_{12}, F_7 \to F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}}_{\bullet h_6 : ((\Delta_{12}, F_7 \to F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}}_{\bullet h_6 : ((\Delta_{12}, F_7 \to F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}}} Cut$$

$$\underbrace{\bullet h_1 : (\Delta_{12}, F_7 \to F_8), p_{10} \vdash (\Delta_9, p_{10}), F_{11}}_{\bullet h_6 : ((\Delta_{12}, F_7 \to F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}}_{\bullet h_6 : ((\Delta_{12}, F_7 \to F_8), p_{10}), F_{11} \vdash \Delta_9, p_{10}}} Cut$$

• Case rule \wedge_L

• Case rule \vee_L

$$\frac{\bullet h_1: (\Delta_9, F_6 \vee F_7), p_{10} \vdash \Delta_8, p_{10}}{I} \xrightarrow{\begin{array}{c} h_5: F_6, \Delta_9, p_{10}, p_{10} \vdash \Delta_8 & h_5: F_7, \Delta_9, p_{10}, p_{10} \vdash \Delta_8 \\ \bullet h_5: ((\Delta_9, F_6 \vee F_7), p_{10}), p_{10} \vdash \Delta_8 \end{array}} \xrightarrow{\bullet h_5: ((\Delta_9, F_6 \vee F_7), p_{10}), p_{10} \vdash \Delta_8} \xrightarrow{Cut} \xrightarrow{\bullet h_1: \Delta_9, F_6, p_{10} \vdash \Delta_8, p_{10}} \xrightarrow{I} \xrightarrow{\begin{array}{c} h_5: \Delta_9, F_6, p_{10}, p_{10} \vdash \Delta_8 \\ \bullet h_1: \Delta_9, F_6, p_{10} \vdash \Delta_8, p_{10} \end{array}} \xrightarrow{As/W} \xrightarrow{\bullet h_1: \Delta_9, F_6, p_{10} \vdash \Delta_8, p_{10}} \xrightarrow{I} \xrightarrow{\begin{array}{c} h_5: \Delta_9, F_7, p_{10} \vdash \Delta_8 \\ \bullet h_1: \Delta_9, F_7, p_{10} \vdash \Delta_8, p_{10} \end{array}} \vee_L$$

\bullet Case rule AT

• Case rule \perp_L

$$\begin{array}{c|c} \hline \bullet_{h_1}: (\bot, \Delta_7), p_8 \vdash \Delta_6, p_8 & I & \hline \bullet_{h_5}: ((\bot, \Delta_7), p_8), p_8 \vdash \Delta_6 \\ \hline -: (\bot, \Delta_7), p_8 \vdash \Delta_6 & \\ \hline -: \bot, \Delta_7, p_8 \vdash \Delta_6 & \bot_L \\ \hline \hline \bullet_{h_1}: \Delta_9, p_8 \vdash (\Delta_7, p_8), \bot & \hline \bullet_{h_6}: (\Delta_9, p_8), \bot \vdash \Delta_7, p_8 \\ \hline -: \Delta_9, p_8 \vdash \Delta_7, p_8 & \\ \hline -: \Delta_9, p_8 \vdash \Delta_7, p_8 & I \\ \hline \hline \bullet_{h_1}: (\bot, \Delta_{10}), p_8 \vdash (\Delta_7, p_8), F_9 & \hline \bullet_{h_6}: ((\bot, \Delta_{10}), p_8), F_9 \vdash \Delta_7, p_8 \\ \hline -: (\bot, \Delta_{10}), p_8 \vdash \Delta_7, p_8 & \bot_L \\ \hline -: (\bot, \Delta_{10}), p_8 \vdash \Delta_7, p_8 & \bot_L \\ \hline -: \bot, \Delta_{10}, p_8 \vdash \Delta_7, p_8 & \bot_L \\ \hline \end{array}$$

$\bullet\,$ Case rule I

$$\begin{array}{c} \frac{\bullet \mathbf{h}_1 : (\top, \Delta_7), \mathbf{p}_8 \vdash \Delta_6, \mathbf{p}_8}{I} & \frac{\mathbf{h}_5 : \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \Delta_6}{\bullet \mathbf{h}_5 : ((\top, \Delta_7), \mathbf{p}_8), \mathbf{p}_8 \vdash \Delta_6} & \top_L \\ \hline -: (\top, \Delta_7), \mathbf{p}_8 \vdash \Delta_6 & \\ \hline \bullet \mathbf{h}_1 : (\top, \Delta_7, \mathbf{p}_8 \vdash \Delta_6, \mathbf{p}_8) & I & \xrightarrow{\bullet} \mathbf{h}_5 : \top, \Delta_7, \mathbf{p}_8, \mathbf{p}_8 \vdash \Delta_6} & \mathbf{ax/W} \\ \hline -: \top, \Delta_7, \mathbf{p}_8 \vdash \Delta_6 & \mathbf{hCut} \\ \hline \bullet \mathbf{h}_1 : \Delta_9, \mathbf{p}_8 \vdash (\Delta_7, \mathbf{p}_8), \top & I & \frac{\mathbf{h}_6 : \Delta_9, \mathbf{p}_8 \vdash \Delta_7, \mathbf{p}_8}{\bullet \mathbf{h}_6 : (\Delta_9, \mathbf{p}_8), \top \vdash \Delta_7, \mathbf{p}_8} & \top_L \\ \hline -: \Delta_9, \mathbf{p}_8 \vdash \Delta_7, \mathbf{p}_8 & I & \\ \hline \bullet \mathbf{h}_1 : (\top, \Delta_{10}), \mathbf{p}_8 \vdash (\Delta_7, \mathbf{p}_8), \mathbf{f}_9 & I & \frac{\mathbf{h}_6 : \mathbf{f}_9, \Delta_{10}, \mathbf{p}_8 \vdash \Delta_7, \mathbf{p}_8}{\bullet \mathbf{h}_6 : ((\top, \Delta_{10}), \mathbf{p}_8), \mathbf{f}_9 \vdash \Delta_7, \mathbf{p}_8} & \top_L \\ \hline -: (\top, \Delta_{10}), \mathbf{p}_8 \vdash \Delta_7, \mathbf{p}_8 & \to \\ \hline -: (\top, \Delta_{10}), \mathbf{p}_8 \vdash \Delta_7, \mathbf{p}_8 & \to \\ \hline -: (\top, \Delta_{10}), \mathbf{p}_8 \vdash \Delta_7, \mathbf{p}_8 & I & \\ \hline \end{array}$$

6.14 Status of \top_L : OK

• Case rule \rightarrow_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_{10} \vdash \mathbf{F}_9, \Delta_8, \mathbf{F}_6 \to \mathbf{F}_7 \\ \bullet \mathbf{h}_1: \top, \Delta_{10} \vdash (\Delta_8, \mathbf{F}_6 \to \mathbf{F}_7), \mathbf{F}_9 \end{array} \top_L \quad \begin{array}{c} \mathbf{h}_5: \top, \mathbf{F}_6, \mathbf{F}_9, \Delta_{10} \vdash \mathbf{F}_7, \Delta_8 \\ \bullet \mathbf{h}_5: (\top, \Delta_{10}), \mathbf{F}_9 \vdash \Delta_8, \mathbf{F}_6 \to \mathbf{F}_7 \end{array} \xrightarrow[]{} \begin{array}{c} \rightarrow_R \\ \mathsf{Cut} \\ \hline \\ \bullet \mathbf{h}_1: \top, \Delta_{10} \vdash \Delta_8, \mathbf{F}_9, \mathbf{F}_6 \to \mathbf{F}_7 \end{array} \xrightarrow[]{} \begin{array}{c} \mathbf{h}_5: \top, \mathbf{F}_6, \mathbf{F}_9, \Delta_{10} \vdash \mathbf{F}_7, \Delta_8 \\ \bullet \mathbf{h}_5: (\top, \Delta_{10}), \mathbf{F}_9 \vdash \Delta_8, \mathbf{F}_6 \to \mathbf{F}_7 \end{array} \xrightarrow[]{} \begin{array}{c} \rightarrow_R \\ \mathsf{Cut} \\ \hline \\ \bullet \mathbf{h}_1: \top, \Delta_{10} \vdash \Delta_8, \mathbf{F}_9, \mathbf{F}_6 \to \mathbf{F}_7 \end{array} \xrightarrow[]{} \begin{array}{c} \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_9 \vdash \Delta_8, \mathbf{F}_6 \to \mathbf{F}_7 \\ \bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_9 \vdash \Delta_8, \mathbf{F}_6 \to \mathbf{F}_7 \end{array} \xrightarrow[]{} \begin{array}{c} \mathsf{Ax/W} \\ \mathsf{hCut} \end{array}$$

• Case rule \wedge_R

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_{10} \vdash \mathbf{F}_9, \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7 \\ \bullet \mathbf{h}_1: \top, \Delta_{10} \vdash (\Delta_8, \mathbf{F}_6 \land \mathbf{F}_7), \mathbf{F}_9 \end{array} }{ \begin{array}{c} \bullet \mathbf{h}_5: \top, \mathbf{F}_9, \Delta_{10} \vdash \mathbf{F}_6, \Delta_8 & \mathbf{h}_5: \top, \mathbf{F}_9, \Delta_{10} \vdash \mathbf{F}_7, \Delta_8 \\ \bullet \mathbf{h}_5: (\top, \Delta_{10}), \mathbf{F}_9 \vdash \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7 \end{array} } \begin{array}{c} \mathbf{Cut} \\ \bullet \mathbf{h}_5: \top, \Delta_{10} \vdash \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7 \end{array} } \\ & \xrightarrow{\bullet \mathbf{h}_5: \top, \Delta_{10} \vdash \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7} \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_9 \vdash \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7 \end{array} } \begin{array}{c} \mathbf{ax/W} \\ \bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_9 \vdash \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7 \end{array} } \\ & \xrightarrow{\bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_9 \vdash \Delta_8, \mathbf{F}_6 \land \mathbf{F}_7} \begin{array}{c} \mathbf{h} \mathbf{h} \mathbf{Cut} \end{array}$$

• Case rule \vee_R

• Case rule \perp_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_8 \vdash \mathbf{F}_7, \bot, \Delta_6}{\bullet \mathbf{h}_1: \top, \Delta_8 \vdash (\bot, \Delta_6), \mathbf{F}_7} \; \top_L \quad \frac{\mathbf{h}_5: \top, \mathbf{F}_7, \Delta_8 \vdash \Delta_6}{\bullet \mathbf{h}_5: (\top, \Delta_8), \mathbf{F}_7 \vdash \bot, \Delta_6} \quad \frac{\bot_R}{\mathsf{Cut}} \\ \\ \underline{-: \top, \Delta_8 \vdash \bot, \Delta_6} \quad \longrightarrow \quad \\ \underline{\mathbf{h}_1: \top, \Delta_8 \vdash \bot, \Delta_6, \mathbf{F}_7} \quad \underbrace{\mathsf{ax/W}}_{\bullet \mathbf{h}_5: \top, \Delta_8, \mathbf{F}_7 \vdash \bot, \Delta_6} \quad \mathsf{ax/W}_{\mathsf{hCut}} \\ \\ \underline{-: \top, \Delta_8 \vdash \bot, \Delta_6} \quad \\ \end{array}$$

• Case rule \top_R

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_8 \vdash \mathbf{F}_7, \top, \Delta_6}{\bullet \mathbf{h}_1: \top, \Delta_8 \vdash (\top, \Delta_6), \mathbf{F}_7} \; \top_L \quad \\ \bullet \mathbf{h}_5: (\top, \Delta_8), \mathbf{F}_7 \vdash \top, \Delta_6}{-: \top, \Delta_8 \vdash \top, \Delta_6} \quad \\ \frac{-: \top, \Delta_8 \vdash \top, \Delta_6}{-: \top, \Delta_8 \vdash \top, \Delta_6} \; \top_R \end{array}$$

 \bullet Case rule K

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Box \Gamma_9, \Delta_{10} \vdash \Box \mathbf{F}_8, \Delta_7, [] \mathbf{F}_6 \\ \bullet \mathbf{h}_1: \top, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, [] \mathbf{F}_6), \Box \mathbf{F}_8 \end{array}}{ -: \top, \Box \Gamma_9, \Delta_{10} \vdash (\Delta_7, [] \mathbf{F}_6), \Box \mathbf{F}_8} \xrightarrow{\mathbf{A}_X/\mathbf{W}} \begin{array}{c} \mathbf{h}_5: unbox(\Box \Gamma_9), unbox(\Box \mathbf{F}_8) \vdash \mathbf{F}_6 \\ \bullet \mathbf{h}_5: (\top, \Box \Gamma_9, \Delta_{10}), \Box \mathbf{F}_8 \vdash \Delta_7, [] \mathbf{F}_6 \end{array}} \xrightarrow{\mathbf{K} \text{ Cut}} \\ -: \top, \Box \Gamma_9, \Delta_{10} \vdash \Delta_7, [] \mathbf{F}_6 \\ \hline{ \bullet \mathbf{h}_1: \top, \Delta_{10}, \Box \Gamma_9 \vdash \Box \mathbf{F}_8, \Delta_7, [] \mathbf{F}_6} \xrightarrow{\mathbf{A}_X/\mathbf{W}} \xrightarrow{\bullet \mathbf{h}_5: \top, \Box \mathbf{F}_8, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [] \mathbf{F}_6} \xrightarrow{\mathbf{A}_3/\mathbf{W}} \xrightarrow{\mathbf{h}_{01}: \top, \Delta_{10}, \Box \Gamma_9 \vdash \Delta_7, [] \mathbf{F}_6} \xrightarrow{\mathbf{h}_5: unbox(\Box \Gamma_8) \vdash \mathbf{F}_6} \xrightarrow{\mathbf{K} \text{ Cut}} \\ \hline{ \bullet \mathbf{h}_1: \Box \Gamma_8, \Delta_{10} \vdash (\Delta_7, [] \mathbf{F}_6), \mathbf{F}_9} \xrightarrow{\top_L} \xrightarrow{\bullet \mathbf{h}_5: unbox(\Box \Gamma_8) \vdash \mathbf{F}_6} \xrightarrow{\mathbf{K} \text{ Cut}} \\ \hline{ -: \top, \Box \Gamma_8, \Delta_{10} \vdash \Delta_7, [] \mathbf{F}_6} \xrightarrow{-: unbox(\Box \Gamma_8) \vdash \mathbf{F}_6} \xrightarrow{\mathbf{K}} \xrightarrow{\mathbf{K}} \\ \hline{ -: unbox(\Box \Gamma_8) \vdash \mathbf{F}_6} \xrightarrow{\mathbf{K}/\mathbf{W}} \\ \hline{ -: \tau, \Delta_{10}, \Box \Gamma_8 \vdash \Delta_7, [] \mathbf{F}_6} \xrightarrow{\mathbf{K}} \end{array}$$

 \bullet Case rule A45

$$\begin{array}{c} \frac{\mathbf{h}_1: \Box \Gamma_{10}, \Delta_{11} \vdash \Box \mathbf{F}_9, \Box \Gamma_7, \Delta_8, [] \mathbf{F}_6}{\bullet \mathbf{h}_1: \top, \Box \Gamma_{10}, \Delta_{11} \vdash (\Box \Gamma_7, \Delta_8, [] \mathbf{F}_6), \Box \mathbf{F}_9} \ \top_L & \frac{\mathbf{h}_5: \Box \Gamma_{10}, \Box \mathbf{F}_9 \vdash \Box \Gamma_7, \mathbf{F}_6}{\bullet \mathbf{h}_5: (\top, \Box \Gamma_{10}, \Delta_{11}), \Box \mathbf{F}_9 \vdash \Box \Gamma_7, \Delta_8, [] \mathbf{F}_6} \\ \hline -: \top, \Box \Gamma_{10}, \Delta_{11} \vdash \Box \Gamma_7, \Delta_8, [] \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \top, \Delta_{11}, \Box \Gamma_{10} \vdash \Box \mathbf{F}_9, \Delta_8, \Box \Gamma_7, [] \mathbf{F}_6} & \mathbf{ax/W} & \bullet \\ \hline -: \top, \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_8, \Box \Gamma_7, [] \mathbf{F}_6} & \mathbf{ax/W} \\ \hline -: \top, \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_8, \Box \Gamma_7, [] \mathbf{F}_6} & \mathbf{h}_5: \top, \Box \mathbf{F}_9, \Delta_{11}, \Box \Gamma_{10} \vdash \Delta_8, \Box \Gamma_7, [] \mathbf{F}_6} \\ \hline \bullet \mathbf{h}_1: \Box \Gamma_9, \Delta_{11} \vdash \mathbf{F}_{10}, \Box \Gamma_7, \Delta_8, [] \mathbf{F}_6} & \top_L & \mathbf{h}_5: \Box \Gamma_9 \vdash \Box \Gamma_7, \mathbf{F}_6 \\ \hline \bullet \mathbf{h}_1: \top, \Box \Gamma_9, \Delta_{11} \vdash (\Box \Gamma_7, \Delta_8, [] \mathbf{F}_6), \mathbf{F}_{10}} & \mathbf{A}_{45} \\ \hline -: \top, \Box \Gamma_9, \Delta_{11} \vdash \Box \Gamma_7, \Delta_8, [] \mathbf{F}_6} & \rightarrow \\ \hline -: \Box \Gamma_9 \vdash \mathbf{F}_6, \Box \Gamma_7} & \mathbf{ax/W} \\ \hline -: \Box \Gamma_9 \vdash \mathbf{F}_6, \Box \Gamma_7, [] \mathbf{F}_6} & \mathbf{A}_{45} \\ \hline \end{array}$$

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_9 \vdash \mathbf{F}_6 \rightarrow \mathbf{F}_7, \Delta_8}{\bullet \mathbf{h}_1: \top, \Delta_9 \vdash \Delta_8, \mathbf{F}_6 \rightarrow \mathbf{F}_7} \ \top_L \ \frac{\mathbf{h}_5: \top, \Delta_9 \vdash \mathbf{F}_6, \Delta_8 \ \mathbf{h}_5: \top, \mathbf{F}_7, \Delta_9 \vdash \Delta_8}{\bullet \mathbf{h}_5: (\top, \Delta_9), \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_8} \ \mathbf{Cut} \\ \\ -: \top, \Delta_9 \vdash \Delta_8 \\ \rightarrow \\ \frac{\mathbf{h}_1: \top, \Delta_9 \vdash \Delta_8, \mathbf{F}_6 \rightarrow \mathbf{F}_7}{\bullet \mathbf{h}_1: \top, \Delta_9 \vdash \Delta_8, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_8} \ \frac{\mathbf{ax/W}}{\bullet \mathbf{h}_5: \top, \Delta_9, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_8} \ \frac{\mathbf{ax/W}}{\mathsf{hCut}} \\ \\ \frac{\mathbf{h}_1: \Delta_{10}, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \mathbf{F}_9, \Delta_8}{\bullet \mathbf{h}_1: \top, \Delta_{10}, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9} \ \top_L \ \frac{\mathbf{h}_5: \top, \mathbf{F}_9, \Delta_{10} \vdash \mathbf{F}_6, \Delta_8 \ \mathbf{h}_5: \top, \mathbf{F}_7, \mathbf{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet \mathbf{h}_5: (\top, \Delta_{10}, \mathbf{F}_6 \rightarrow \mathbf{F}_7), \mathbf{F}_9 \vdash \Delta_8} \ \mathbf{Cut} \\ \\ -: \top, \Delta_{10}, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9 \ \frac{\mathsf{ax/W}}{\bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_9, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_8} \ \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \\ -: \top, \Delta_{10}, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_8 \\ -: \top, \Delta_{10}, \mathbf{F}_6 \rightarrow \mathbf{F}_7 \vdash \Delta_8 \end{array} \ \frac{\mathsf{ax/W}}{\bullet \mathbf{hCut}} \ \mathbf{hCut}$$

• Case rule \wedge_L

$$\begin{array}{c} \frac{\mathbf{h}_1: \Delta_9 \vdash \mathbf{F}_6 \wedge \mathbf{F}_7, \Delta_8}{\bullet \mathbf{h}_1: \top, \Delta_9 \vdash \Delta_8, \mathbf{F}_6 \wedge \mathbf{F}_7} \; \top_L \quad \frac{\mathbf{h}_5: \top, \mathbf{F}_6, \mathbf{F}_7, \Delta_9 \vdash \Delta_8}{\bullet \mathbf{h}_5: (\top, \Delta_9), \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \Delta_8} \; \bigwedge_L \\ \qquad \qquad \qquad \qquad \qquad \qquad \\ \frac{-: \top, \Delta_9 \vdash \Delta_8}{\bullet \mathbf{h}_1: \top, \Delta_9 \vdash \Delta_8, \mathbf{F}_6 \wedge \mathbf{F}_7} \; \underset{\bullet}{\mathsf{ax/W}} \; \underbrace{\bullet \mathbf{h}_5: \top, \Delta_9, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \Delta_8}_{\bullet \mathbf{h}_5: \top, \Delta_9, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \Delta_8} \; \underset{\bullet}{\mathsf{ax/W}} \\ \frac{\mathbf{h}_1: \Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \mathbf{F}_9, \Delta_8}{\bullet \mathbf{h}_1: \top, \Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9} \; \top_L \; \underbrace{\begin{array}{c} \mathbf{h}_5: \top, \mathbf{F}_6, \mathbf{F}_7, \mathbf{F}_9, \Delta_{10} \vdash \Delta_8 \\ \bullet \mathbf{h}_5: (\top, \Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7), \mathbf{F}_9 \vdash \Delta_8 \end{array}}_{\bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7), \mathbf{F}_9 \vdash \Delta_8} \; \bigwedge_L \\ \underbrace{\begin{array}{c} \mathbf{h}_1: \Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9 \\ \bullet \mathbf{h}_1: \top, \Delta_{10}, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9 \end{array}}_{\bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_9, \mathbf{F}_6 \wedge \mathbf{F}_7 \vdash \Delta_8} \; \underbrace{\begin{array}{c} \mathbf{h}_5: \nabla, \mathbf{h}_{10}, \mathbf$$

• Case rule \vee_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_9 \vdash \mathbf{F}_6 \vee \mathbf{F}_7, \Delta_8}{\bullet \mathbf{h}_1: \top, \Delta_9 \vdash \Delta_8, \mathbf{F}_6 \vee \mathbf{F}_7} & \top_L & \frac{\mathbf{h}_5: \top, \mathbf{F}_6, \Delta_9 \vdash \Delta_8 & \mathbf{h}_5: \top, \mathbf{F}_7, \Delta_9 \vdash \Delta_8}{\bullet \mathbf{h}_5: (\top, \Delta_9), \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_8} & \mathbf{Cut} \\ \hline & -: \top, \Delta_9 \vdash \Delta_8 & \\ \hline & \frac{\rightarrow}{\mathbf{h}_1: \top, \Delta_9 \vdash \Delta_8, \mathbf{F}_6 \vee \mathbf{F}_7} & \mathbf{ax/W} & \frac{\rightarrow}{\bullet \mathbf{h}_5: \top, \Delta_9, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_8} & \mathbf{ax/W} \\ & -: \top, \Delta_9 \vdash \Delta_8 & \mathbf{hCut} \\ \hline & \frac{\mathbf{h}_1: \Delta_{10}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \mathbf{F}_9, \Delta_8}{\bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_8} & \mathbf{T}_L & \frac{\mathbf{h}_5: \top, \mathbf{F}_6, \mathbf{F}_9, \Delta_{10} \vdash \Delta_8 & \mathbf{h}_5: \top, \mathbf{F}_7, \mathbf{F}_9, \Delta_{10} \vdash \Delta_8}{\bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9} & \mathbf{Cut} \\ \hline & -: \top, \Delta_{10}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9} & \mathbf{ax/W} & \frac{\rightarrow}{\bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_9, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_8} & \mathbf{ax/W} \\ \hline & -: \top, \Delta_{10}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9} & \mathbf{ax/W} & \frac{\rightarrow}{\bullet \mathbf{h}_5: \top, \Delta_{10}, \mathbf{F}_9, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_8} & \mathbf{ax/W} \\ \hline & -: \top, \Delta_{10}, \mathbf{F}_6 \vee \mathbf{F}_7 \vdash \Delta_8, \mathbf{F}_9} & \mathbf{ax/W} & \mathbf{hCut} \\ \hline \end{array}$$

 \bullet Case rule AT

$$\frac{ \begin{array}{c} \mathbf{h}_1 : \Delta_8 \vdash [] \mathbf{F}_6, \Delta_7 \\ \bullet \mathbf{h}_1 : \top, \Delta_8 \vdash \Delta_7, [] \mathbf{F}_6 \end{array} \top_L \quad \begin{array}{c} \mathbf{h}_5 : \top, \mathbf{F}_6, \Delta_8, [] \mathbf{F}_6 \vdash \Delta_7 \\ \bullet \mathbf{h}_5 : (\top, \Delta_8), [] \mathbf{F}_6 \vdash \Delta_7 \end{array} \quad \begin{array}{c} AT \\ \text{Cut} \\ \hline \\ \bullet \mathbf{h}_5 : \top, \Delta_8 \vdash \Delta_7 \end{array} \\ \\ \hline \\ \bullet \mathbf{h}_5 : \top, \Delta_8 \vdash \Delta_7 \\ \hline \\ \bullet \mathbf{h}_5 : \top, \Delta_8, [] \mathbf{F}_6 \vdash \Delta_7 \end{array} \quad \begin{array}{c} AT \\ \text{Cut} \\ \bullet \mathbf{h}_5 : \top, \Delta_8, [] \mathbf{F}_6 \vdash \Delta_7 \end{array} \quad \begin{array}{c} AT \\ \text{Cut} \\ \bullet \mathbf{h}_5 : \top, \Delta_8, [] \mathbf{F}_6 \vdash \Delta_7 \end{array}$$

$$\frac{ \begin{array}{c} \mathbf{h}_1: \Delta_9, [] \mathbf{F}_6 \vdash \mathbf{F}_8, \Delta_7 \\ \bullet \mathbf{h}_1: \top, \Delta_9, [] \mathbf{F}_6 \vdash \Delta_7, \mathbf{F}_8 \end{array} \top_L \quad \frac{\mathbf{h}_5: \top, \mathbf{F}_6, \mathbf{F}_8, \Delta_9, [] \mathbf{F}_6 \vdash \Delta_7 }{\bullet \mathbf{h}_5: (\top, \Delta_9, [] \mathbf{F}_6), \mathbf{F}_8 \vdash \Delta_7} \quad \frac{AT}{\mathsf{Cut}} \\ \\ \underline{ \begin{array}{c} -: \top, \Delta_9, [] \mathbf{F}_6 \vdash \Delta_7 \\ \hline \\ \mathbf{h}_1: \top, \Delta_9, [] \mathbf{F}_6 \vdash \Delta_7, \mathbf{F}_8 \end{array}} \quad \frac{\mathsf{ax/W}}{\bullet \mathbf{h}_5: \top, \Delta_9, \mathbf{F}_8, [] \mathbf{F}_6 \vdash \Delta_7} \quad \frac{\mathsf{ax/W}}{\mathsf{hCut}} \\ \\ \underline{ \begin{array}{c} -: \top, \Delta_9, [] \mathbf{F}_6 \vdash \Delta_7 \end{array}} \quad \mathbf{hCut} \end{array}$$

• Case rule \perp_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1:\Delta_7\vdash \bot,\Delta_6}{\bullet \mathbf{h}_1:\top,\Delta_7\vdash \Delta_6,\bot} & \top_L & \frac{}{\bullet \mathbf{h}_5:(\top,\Delta_7),\bot\vdash \Delta_6} & \bot_L \\ \hline & -:\top,\Delta_7\vdash \Delta_6 & \\ & \to \\ \hline \frac{\mathbf{h}_1:\top,\Delta_7\vdash \bot,\Delta_6}{\bullet \mathbf{h}_5:\bot,\top,\Delta_7\vdash \Delta_6} & \frac{\bot_L}{\bullet \mathbf{h}\mathbf{Cut}} \\ \hline -:\top,\Delta_7\vdash \Delta_6 & \\ \hline \frac{\mathbf{h}_1:\bot,\Delta_8\vdash \mathbf{F}_7,\Delta_6}{\bullet \mathbf{h}_1:\top,\bot,\Delta_8\vdash \Delta_6,\mathbf{F}_7} & \top_L & \frac{}{\bullet \mathbf{h}_5:(\top,\bot,\Delta_8),\mathbf{F}_7\vdash \Delta_6} & \overset{\bot_L}{\mathsf{Cut}} \\ \hline & -:\top,\bot,\Delta_8\vdash \Delta_6 & \to \\ \hline & -:\bot,\top,\Delta_8\vdash \Delta_6 & \bot_L \\ \hline \end{array}$$

ullet Case rule I

$$\begin{array}{c} \frac{\mathbf{h}_1:\Delta_8 \vdash \mathbf{p}_7,\Delta_6,\mathbf{p}_7}{\bullet \mathbf{h}_1:\top,\Delta_8 \vdash (\Delta_6,\mathbf{p}_7),\mathbf{p}_7} \; \top_L \quad \\ \hline \bullet \mathbf{h}_5:(\top,\Delta_8),\mathbf{p}_7 \vdash \Delta_6,\mathbf{p}_7}{-:\top,\Delta_8 \vdash \Delta_6,\mathbf{p}_7} \quad \mathbf{Cut} \\ \hline \\ \frac{\mathbf{h}_1:\top,\Delta_8 \vdash \Delta_6,\mathbf{p}_7,\mathbf{p}_7}{\bullet \mathbf{h}_5:\top,\Delta_8,\mathbf{p}_7 \vdash \Delta_6,\mathbf{p}_7} \quad I \\ \hline \bullet \mathbf{h}_5:\top,\Delta_8,\mathbf{p}_7 \vdash \Delta_6,\mathbf{p}_7} \quad I \\ \hline \\ \bullet \mathbf{h}_1:\Delta_9,\mathbf{p}_7 \vdash \mathbf{F}_8,\Delta_6,\mathbf{p}_7} \quad \\ \hline \bullet \mathbf{h}_1:\top,\Delta_9,\mathbf{p}_7 \vdash (\Delta_6,\mathbf{p}_7),\mathbf{F}_8} \quad \top_L \quad \hline \bullet \mathbf{h}_5:(\top,\Delta_9,\mathbf{p}_7),\mathbf{F}_8 \vdash \Delta_6,\mathbf{p}_7} \quad I \\ \hline \\ \bullet \mathbf{h}_1:\top,\Delta_9,\mathbf{p}_7 \vdash \Delta_6,\mathbf{p}_7} \quad \\ \hline \\ \bullet \mathbf{h}_7:\top,\Delta_9,\mathbf{p}_7 \vdash \Delta_6,\mathbf{p}_7} \quad I \\ \hline \end{array}$$

• Case rule \top_L

$$\begin{array}{c|c} \frac{\mathbf{h}_1: \Delta_8 \vdash \mathbf{F}_7, \Delta_6}{\bullet \mathbf{h}_1: \top, \Delta_8 \vdash \Delta_6, \mathbf{F}_7} & \top_L & \frac{\mathbf{h}_5: \mathbf{F}_7, \Delta_8 \vdash \Delta_6}{\bullet \mathbf{h}_5: (\top, \Delta_8), \mathbf{F}_7 \vdash \Delta_6} & \top_L \\ \hline & -: \top, \Delta_8 \vdash \Delta_6 \\ \hline \frac{\mathbf{h}_1: \top, \Delta_8 \vdash \Delta_6, \mathbf{F}_7}{\bullet \mathbf{h}_5: \top, \Delta_8, \mathbf{F}_7 \vdash \Delta_6} & \frac{\mathsf{ax/W}}{\mathsf{hCut}} \end{array}$$