System G3C

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Contents

1 Height preserving admissibility of weakening on the left

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5} \to_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5} \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \bot, \Delta_3} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \top, \Delta_3} \top_R$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4, \mathtt{F}_W \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_W \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_5}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4, \mathtt{F}_W \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3} \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2, \mathtt{F}_W \vdash \Delta_3} \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{f}_W, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{f}_W \vdash \Delta_3}{\bullet \mathtt{h}_1 : \top, \Delta_2, \mathtt{f}_W \vdash \Delta_3}\ \top_L}\ \mathtt{IH}$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5}\ {\to_R} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W, \mathtt{F}_4 \to \mathtt{F}_5}\ {\to_R}$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\wedge_R} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\wedge_R}$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\vee_R} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\vee_R}$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\bot_R} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \mathtt{F}_W}\ {\bot_R}$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\top_R} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3, \mathtt{F}_W}\ {\top_R}$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}\ {\to_L} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W}\ {\to_L}$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ {\wedge_L} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W}\ {\wedge_L}$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ {\vee_L} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W}\ {\vee_L}$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3}\ {\bot_L} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\ {\bot_L}$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ {I} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{F}_W, \mathtt{p}_3}\ {I}$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ {\top_L} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\ {\top_L}$$

3 Measure of derivations

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5}\ {\to_R} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5}\ {\to_R}$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\wedge_R} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\wedge_R}$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\vee_R} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\vee_R}$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\bot_R} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\bot_R}$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\top_R} \quad \rightsquigarrow \quad \frac{}{\bullet \bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\top_R}$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}\ {\to_L} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}\ {\to_L}$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ {\wedge_L} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ {\wedge_L}$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ {\vee_L} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ {\vee_L}$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3}\ {\bot_L} \quad \rightsquigarrow \quad \frac{}{\bullet \bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3}\ {\bot_L}$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ {I} \quad \rightsquigarrow \quad \frac{}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ {I}$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ {\top_L} \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ {\top_L}$$

4 Invertibility of Rules

4.1 Status of $\rightarrow_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ ^{\mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ height$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2} \bot_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_2}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \bot, \Delta_5, \mathtt{F}_2} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}\ \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \top, \Delta_5, \mathtt{F}_2}\ \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_6, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3} \rightarrow_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2 \vdash \Delta_1, \mathtt{F}_3, \mathtt{F}_6}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3} \wedge_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3} \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_3}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3} \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3}\ \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_1, \mathtt{F}_3}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_4 \vdash \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{p}_5 \vdash \Delta_6, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3}\ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_1, \mathtt{F}_3}\ \top_L$$

4.2 Status of $\wedge_R$: (Left Premise): `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \to \mathtt{F}_6}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ height$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1}\ \top_R$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \to_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \to_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \vee_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}^{\ ax/ind} \quad \overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_4 \vdash \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{p}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \top_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2}\ \top_L$$

4.3 Status of $\wedge_R$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}^{\ ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\ height$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \bot_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_2}\ \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \rightarrow_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_3, \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_3}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_4 \vdash \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{p}_5 \vdash \Delta_6, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_3}\ \top_L$$

4.4 Status of $\vee_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ height$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2} \bot_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2} \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_6, \mathtt{F}_2 \vee \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3} \rightarrow_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3} \wedge_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \quad \mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3} \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3} \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3} \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_4 \vdash \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2} I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{p}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_5} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3} \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3} \top_L$$

4.5 Status of $\bot_R$: : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \bot, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \to \mathtt{F}_4} \to_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \to \mathtt{F}_4} \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3} \ height$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \bot, \Delta_3} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_1, \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \bot, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \bot, \Delta_1} \to_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \Delta_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_1} \to_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, \mathtt{F}_5 \vdash \bot, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \bot, \Delta_1} \wedge_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_1} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \bot, \Delta_1 \quad \mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \bot, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \bot, \Delta_1} \vee_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_1}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \Delta_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_1} \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_3 \vdash \bot, \Delta_1} \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_3 \vdash \Delta_1} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_2 \vdash \mathtt{p}_3, \bot, \Delta_4}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_1}{\bullet \mathtt{h}_2 : \top, \Delta_3 \vdash \bot, \Delta_1}\ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \top, \Delta_3 \vdash \Delta_1}\ \top_L$$

4.6 Status of $\top_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \top, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \top, \Delta_3}\ \bot_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_1, \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \top, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \top, \Delta_1}\ \rightarrow_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, \mathtt{F}_5 \vdash \top, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \top, \Delta_1}\ \wedge_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \top, \Delta_1 \quad \mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \top, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \top, \Delta_1}\ \vee_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_3 \vdash \top, \Delta_1}\ \bot_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_2 \vdash \mathtt{p}_3, \top, \Delta_4}\ I \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_1}{\bullet \mathtt{h}_2 : \top, \Delta_3 \vdash \top, \Delta_1}\ \top_L \quad \rightsquigarrow \quad \texttt{trivial}$$

4.7 Status of $\rightarrow_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_7}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6}^{\ \texttt{ax/ind}} \quad \overline{\mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_7}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6, \mathtt{F}_7}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \bot, \Delta_5}\ \bot_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \bot, \Delta_5, \mathtt{F}_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \top, \Delta_5, \mathtt{F}_2}\ \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6}\ \to_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}^{\ \texttt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ \to_L$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}\ \to_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\ \texttt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ \mathit{height}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ {\scriptstyle \wedge_L} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ {\scriptstyle \wedge_L}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ {\scriptstyle \vee_L} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}\ {\scriptstyle \vee_L}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ {\scriptstyle \bot_L} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ {\scriptstyle \bot_L}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5}\ {\scriptstyle I} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{p}_4}\ {\scriptstyle I}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ {\scriptstyle \top_L} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ {\scriptstyle \top_L}$$

4.8 Status of $\to_L$ (Right Premise): : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_6, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7}\ {\scriptstyle \to_R} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7}\ {\scriptstyle \to_R}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ {\scriptstyle \wedge_R} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ {\scriptstyle \wedge_R}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ {\scriptstyle \vee_R} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ {\scriptstyle \vee_R}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \bot, \Delta_5}\ {\scriptstyle \bot_R} \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \bot, \Delta_5}\ {\scriptstyle \bot_R}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6}\ \to_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6}\ \to_L$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}\ \to_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \text{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ height$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_4 \vdash \Delta_5, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \text{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.9 Status of $\wedge_L$: : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_6, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_7}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}\ \text{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \bot, \Delta_5}\ \bot_R \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \bot, \Delta_5}\ \bot_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R \quad \rightsquigarrow \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ \wedge_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\ ^{\mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\ height$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ ^{\mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

- Case rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightsquigarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\dfrac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5}\ I \quad \rightsquigarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_4 \vdash \Delta_5, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.10 Status of $\vee_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_7}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6}^{\ ax/ind} \quad \overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_7}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \bot, \Delta_5} \bot_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \bot, \Delta_5} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \top, \Delta_5} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \top, \Delta_5} \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}^{\ ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\ height$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4} \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vdash \Delta_4} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{p}_4 \vdash \Delta_5, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \top_L$$

4.11 Status of $\vee_L$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_6, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_7}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \bot, \Delta_5}\ \bot_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \bot, \Delta_5}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ height$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4} \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5} I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_4 \vdash \Delta_5, \mathtt{p}_4} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4} \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4} \top_L$$

4.12 Status of $\bot_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \bot, \Delta_3} \bot_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \top, \Delta_3} \top_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4 \quad \mathtt{h}_1 : \bot, \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3} \bot_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_2, \bot, \Delta_4 \vdash \mathtt{p}_2, \Delta_3} I \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \bot, \Delta_3 \vdash \Delta_2} \top_L \quad \rightsquigarrow \quad \texttt{trivial}$$

4.13 Status of I: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_4, \mathtt{p}_2 \vdash \Delta_6, \mathtt{F}_5, \mathtt{p}_2}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash (\Delta_6, \mathtt{p}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{p}_2 \quad \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \Delta_6, \mathtt{F}_5, \mathtt{p}_2}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash (\Delta_6, \mathtt{p}_2), \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{p}_2}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash (\Delta_6, \mathtt{p}_2), \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \Delta_4, \mathtt{p}_2}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \bot, \Delta_4, \mathtt{p}_2} \bot_R \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \top, \Delta_4, \mathtt{p}_2}\ \top_R \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{p}_1 \vdash \Delta_2, \mathtt{F}_4, \mathtt{p}_1 \quad \mathtt{h}_3 : \Delta_6, \mathtt{F}_5, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1}\ \rightarrow_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1}\ \wedge_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1 \quad \mathtt{h}_3 : \Delta_6, \mathtt{F}_5, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1}\ \vee_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}\ \bot_L \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_2 : \mathtt{p}_3, \Delta_5, \mathtt{p}_1 \vdash \mathtt{p}_3, \Delta_4, \mathtt{p}_1}\ I \quad \rightsquigarrow \quad \texttt{trivial}$$

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_2 \vdash \mathtt{p}_3, \Delta_4}\ I \quad \rightsquigarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet \mathtt{h}_3 : \top, \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}\ \top_L \quad \rightsquigarrow \quad \texttt{trivial}$$

4.14 Status of $\top_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \top, \Delta_1, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4}\ \texttt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_5}\ \texttt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R}{}\quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \bot, \Delta_3}\ \bot_R}{}\quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \bot, \Delta_3}\ \bot_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \top, \Delta_3}\ \top_R \quad \rightsquigarrow \quad \dfrac{}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \top, \Delta_3}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\mathtt{h}_1 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L}{}\quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4 \quad \mathtt{h}_1 : \top, \Delta_5, \mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

- Case rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightsquigarrow \quad \dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L$$

- Case rule I

$$\dfrac{}{\bullet \mathtt{h}_1 : \mathtt{p}_2, \top, \Delta_4 \vdash \mathtt{p}_2, \Delta_3}\ I \quad \rightsquigarrow \quad \dfrac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_2 \vdash \Delta_3, \mathtt{p}_2}\ I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ \top_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ height$$

5 Height preserving admissibility of contraction on the left

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_4, \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6} \to_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_4, \mathtt{F}_6}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_4, \mathtt{F}_6}\,\mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6} \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_6}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_6}\,\mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_6}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_6}\,\mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \Delta_4}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \Delta_4}\,\mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \bot, \Delta_4} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \vdash \top, \Delta_4} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2 \vdash \top, \Delta_4} \top_R$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \to \mathtt{F}_4, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_3}\,\mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{IH\text{-}Mutual} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L$$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \to \mathtt{F}_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_5} \to_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \wedge \mathtt{F}_4, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L$$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_5} \wedge_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5 \quad \mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vee \mathtt{F}_4, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_5}\,\mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L$$

$$\frac{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \vdash \Delta_5 \quad \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_5} \vee_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_3 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \vdash \Delta_5}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_1, \bot, \bot \vdash \Delta_3}\ \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3}\ \bot_L$$

$$\frac{}{\bullet \mathtt{h}_2 : (\bot, \Delta_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_3}\ \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_4, \mathtt{F}_1 \vdash \Delta_3}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{p}_3, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_1, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I$$

$$\frac{}{\bullet \mathtt{h}_2 : (\Delta_5, \mathtt{p}_3), \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_4, \mathtt{p}_3}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \Delta_1, \top, \top \vdash \Delta_3}\ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3}\ \texttt{inv-th/ax}}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3}\ \top_L$$

$$\frac{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : (\top, \Delta_4), \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_3}\ \top_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, \mathtt{F}_1 \vdash \Delta_3}\ \texttt{ax}}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1 \vdash \Delta_3}\ \texttt{IH}}{\bullet \mathtt{h}_2 : \top, \Delta_4, \mathtt{F}_1 \vdash \Delta_3}\ \top_L$$

6 Height preserving admissibility of contraction on the Right

- Case(s) rule $\to_R$

$$\dfrac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_4 \to \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \to \mathtt{F}_5, \mathtt{F}_4 \to \mathtt{F}_5}\ {\scriptstyle \to_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_5}\ {\scriptstyle \text{inv-th/ax}}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_5}\ {\scriptstyle \text{IH-Mutual}}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_1, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \to \mathtt{F}_5}\ {\scriptstyle \to_R}$$

$$\dfrac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \to \mathtt{F}_5), \mathtt{F}_1, \mathtt{F}_1}\ {\scriptstyle \to_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_5}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \to \mathtt{F}_5}\ {\scriptstyle \to_R}$$

- Case(s) rule $\wedge_R$

$$\dfrac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_4 \wedge \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_4 \wedge \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \wedge \mathtt{F}_5, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\scriptstyle \wedge_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_4}\ {\scriptstyle \text{inv-th/ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4}\ {\scriptstyle \text{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_5}\ {\scriptstyle \text{inv-th/ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\scriptstyle \wedge_R}$$

$$\dfrac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5), \mathtt{F}_1, \mathtt{F}_1}\ {\scriptstyle \wedge_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}\ {\scriptstyle \text{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_5}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\scriptstyle \wedge_R}$$

- Case(s) rule $\vee_R$

$$\dfrac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_4 \vee \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \vee \mathtt{F}_5, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\scriptstyle \vee_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_5}\ {\scriptstyle \text{inv-th/ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\scriptstyle \vee_R}$$

$$\dfrac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5), \mathtt{F}_1, \mathtt{F}_1}\ {\scriptstyle \vee_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\scriptstyle \vee_R}$$

- Case(s) rule $\bot_R$

$$\dfrac{\mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_1}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \bot, \bot}\ {\scriptstyle \bot_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1}\ {\scriptstyle \text{inv-th/ax}}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_1}\ {\scriptstyle \bot_R}$$

$$\dfrac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_1}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\bot, \Delta_4), \mathtt{F}_1, \mathtt{F}_1}\ {\scriptstyle \bot_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_1}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1}\ {\scriptstyle \text{IH}}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_4, \mathtt{F}_1}\ {\scriptstyle \bot_R}$$

- Case(s) rule $\top_R$

$$\dfrac{}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \top, \top}\ {\scriptstyle \top_R} \quad \rightsquigarrow \quad \dfrac{}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_1}\ {\scriptstyle \top_R}$$

$$\dfrac{}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\top, \Delta_4), \mathtt{F}_1, \mathtt{F}_1}\ {\scriptstyle \top_R} \quad \rightsquigarrow \quad \dfrac{}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_4, \mathtt{F}_1}\ {\scriptstyle \top_R}$$

- Case(s) rule $\to_L$

$$\dfrac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}{\bullet\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ {\scriptstyle \to_L} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_5}\ {\scriptstyle \text{ax}}}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_5}\ {\scriptstyle \text{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ {\scriptstyle \text{ax}}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ {\scriptstyle \text{IH}}}{\bullet\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ {\scriptstyle \to_L}$$

- Case(s) rule $\wedge_L$

$$\dfrac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ \wedge_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\dfrac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ \vee_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \vee_L$$

- Case(s) rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ \bot_L \quad \rightsquigarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \Delta_1, \mathtt{F}_2}\ \bot_L$$

- Case(s) rule I

$$\dfrac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash \Delta_1, \mathtt{p}_4, \mathtt{p}_4}\ I \quad \rightsquigarrow \quad \dfrac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash \Delta_1, \mathtt{p}_4}\ I$$

$$\dfrac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash (\Delta_5, \mathtt{p}_4), \mathtt{F}_1, \mathtt{F}_1}\ I \quad \rightsquigarrow \quad \dfrac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{p}_4}\ I$$

- Case(s) rule $\top_L$

$$\dfrac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ \top_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \Delta_1, \mathtt{F}_2}\ \top_L$$

7 Identity-Expansion

$$\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,\vee_L}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0 \vee \mathtt{F}_1}\,\vee_R$$

$$\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_R}{- : \mathtt{F}_0 \wedge \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \qquad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_1}\,\to_L}{- : \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_0 \to \mathtt{F}_1}\,\to_R$$

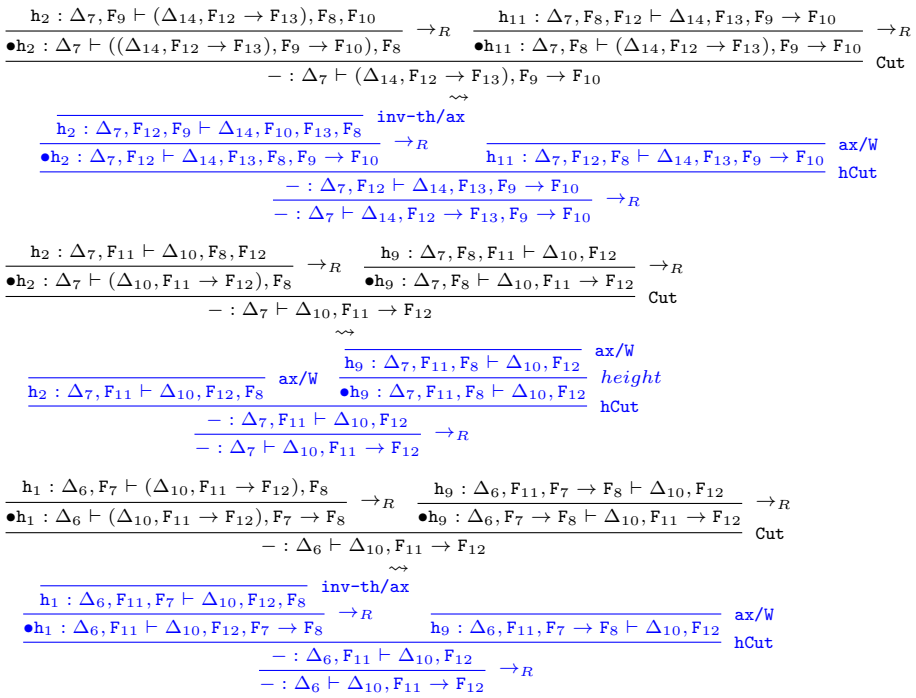
$$\dfrac{}{- : \top \vdash \top}\,\top_R$$

$$\dfrac{}{- : \bot \vdash \bot}\,\bot_L$$

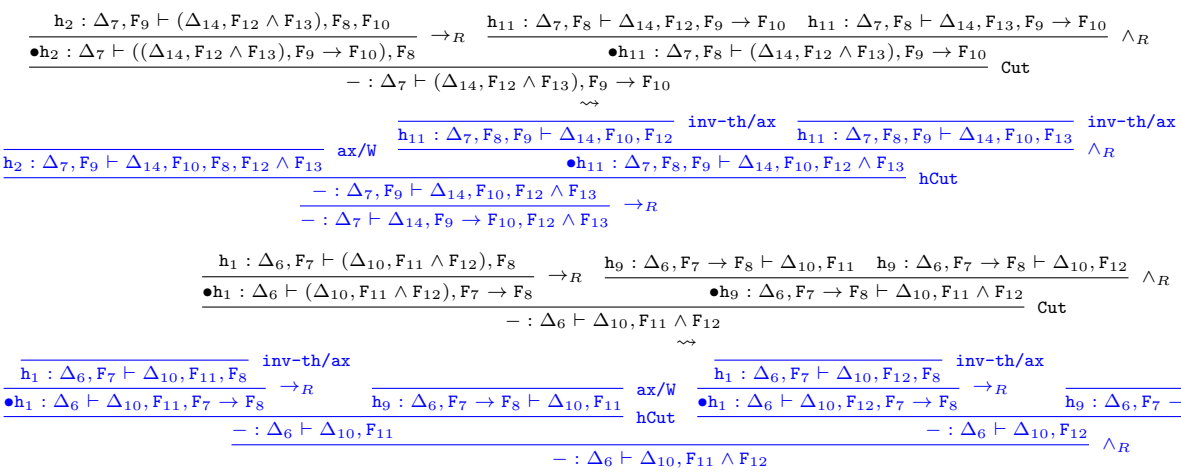
8 Cut-Elimination

8.1 Status of $\to_R$: OK

- Case rule $\to_R$



- Case rule $\wedge_R$



- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7, \mathtt{F}_9 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Delta_7 \vdash ((\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_8}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\vee_R}}{- : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_8, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,{\to_R} \quad \dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,{\vee_R}\,\mathtt{hCut}}{- : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,{\to_R}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,{\vee_R}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\,\mathtt{ax/W}\,\mathtt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\,{\vee_R}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7, \mathtt{F}_9 \vdash (\bot, \Delta_{12}), \mathtt{F}_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Delta_7 \vdash ((\bot, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_8}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\bot, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\bot_R}}{- : \Delta_7 \vdash (\bot, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{ax/W}\,\mathtt{hCut}}{- : \Delta_7 \vdash \bot, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vdash (\bot, \Delta_{10}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,{\bot_R}}{- : \Delta_6 \vdash \bot, \Delta_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{ax/W}\,\mathtt{hCut}}{- : \Delta_6 \vdash \bot, \Delta_{10}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7, \mathtt{F}_9 \vdash (\top, \Delta_{12}), \mathtt{F}_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Delta_7 \vdash ((\top, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_8}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\top_R}}{- : \Delta_7 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\top_R}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vdash (\top, \Delta_{10}), \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \top, \Delta_{10}}\,{\top_R}}{- : \Delta_6 \vdash \top, \Delta_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

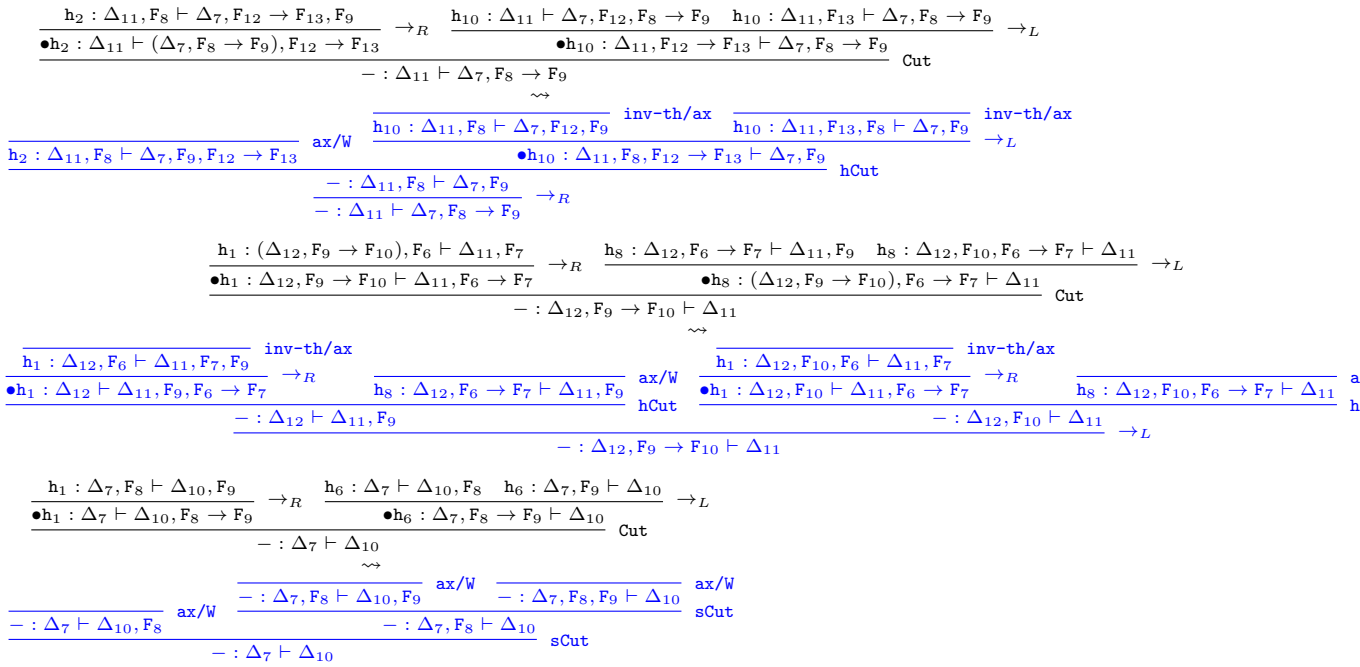
$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}}\,{\top_R}$$

- Case rule $\to_L$

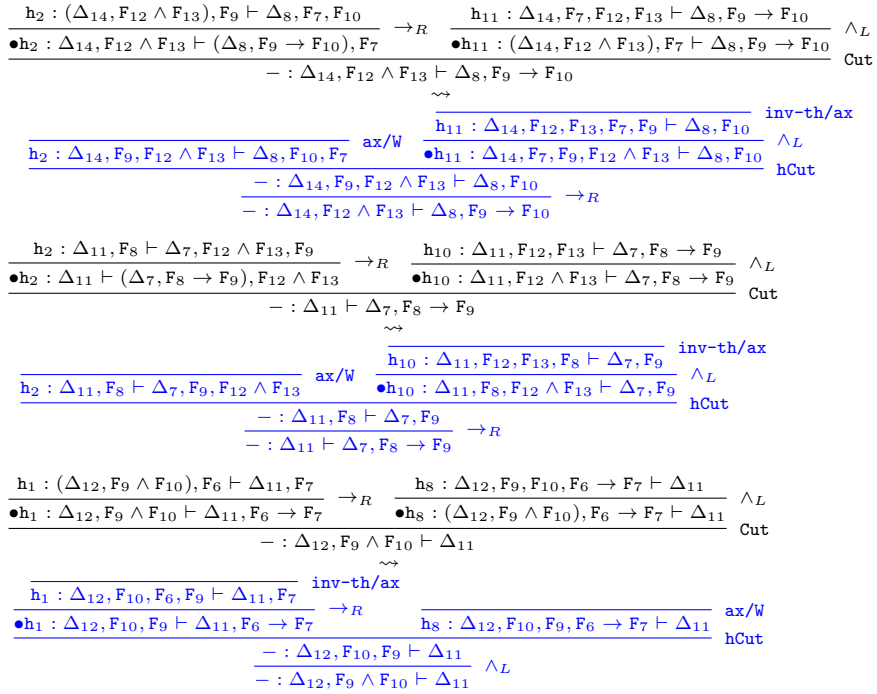
$$\dfrac{\dfrac{\mathtt{h}_2 : (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \quad \mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_L}\,\mathtt{Cut}}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}$$

$$\rightsquigarrow$$

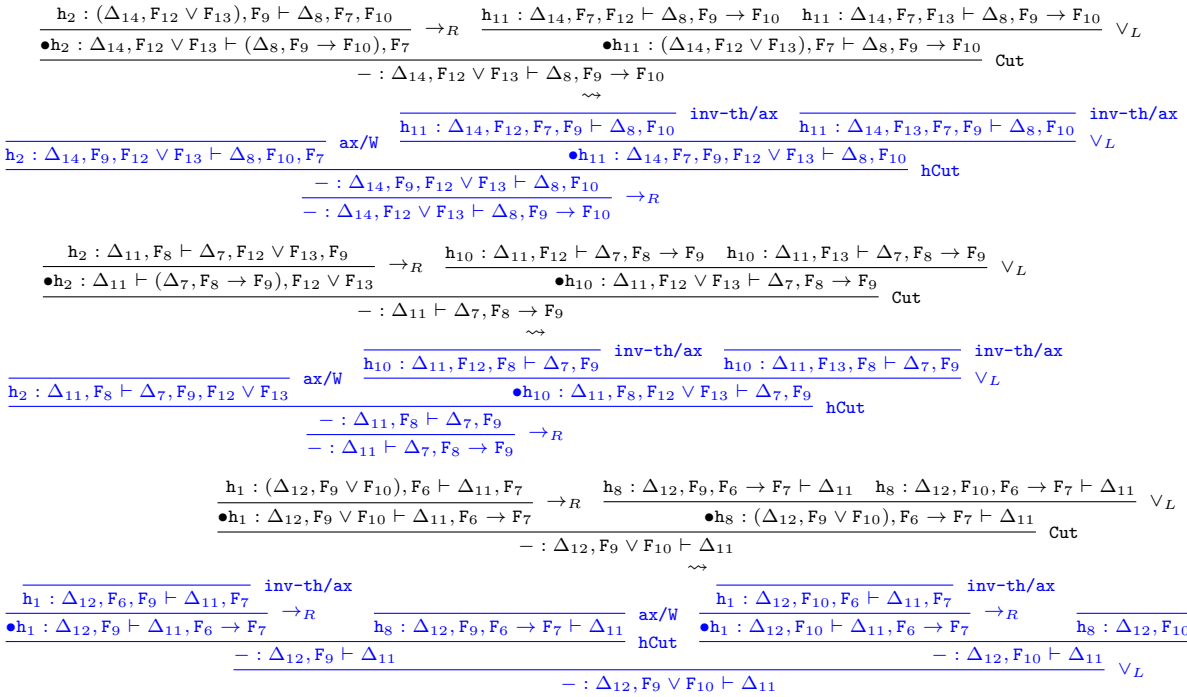
$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{inv\text{-}th/ax}\,{\to_L}}{\bullet\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}\,{\to_R}}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}$$



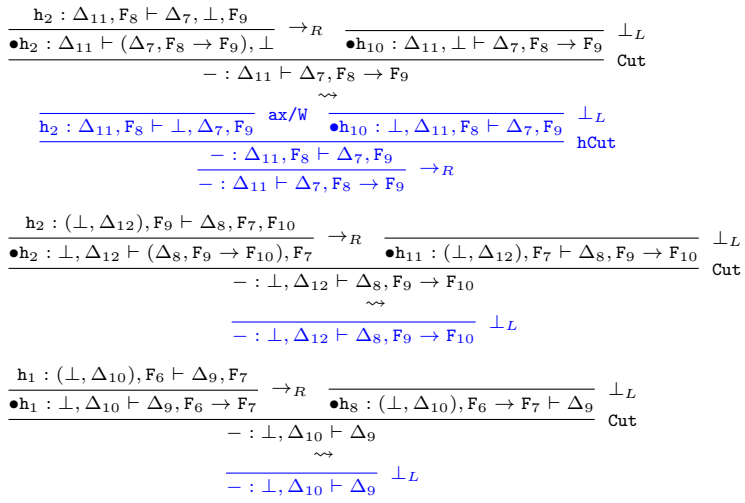
- Case rule $\wedge_L$



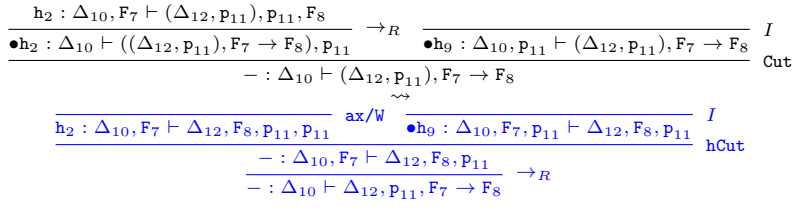
- Case rule $\vee_L$



- Case rule $\bot_L$



- Case rule I



$$\dfrac{\dfrac{\mathtt{h}_2 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\bullet \mathtt{h}_{10} : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9}\,I}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_8 \to \mathtt{F}_9}\,I$$

$$\dfrac{\dfrac{\mathtt{h}_1 : (\Delta_{11}, \mathtt{p}_9), \mathtt{F}_6 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_9), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}\,I}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \top, \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \top}\,{\to_R} \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}{\bullet \mathtt{h}_{10} : \Delta_{11}, \top \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,{\top_L}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : (\top, \Delta_{12}), \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\top, \Delta_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\top_L}}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \top, \Delta_{12}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : (\top, \Delta_{10}), \mathtt{F}_6 \vdash \Delta_9, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}\,{\top_L}}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{hCut}$$

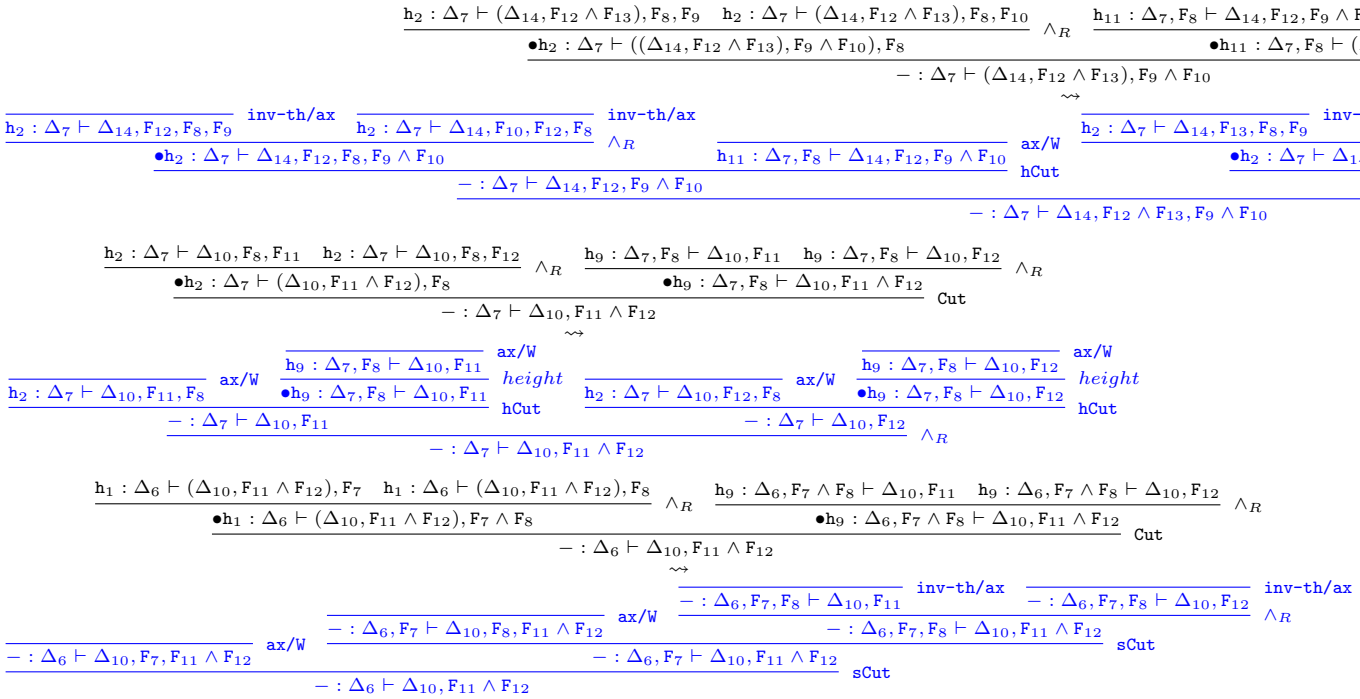
8.2 Status of $\wedge_R$: OK

- Case rule $\to_R$

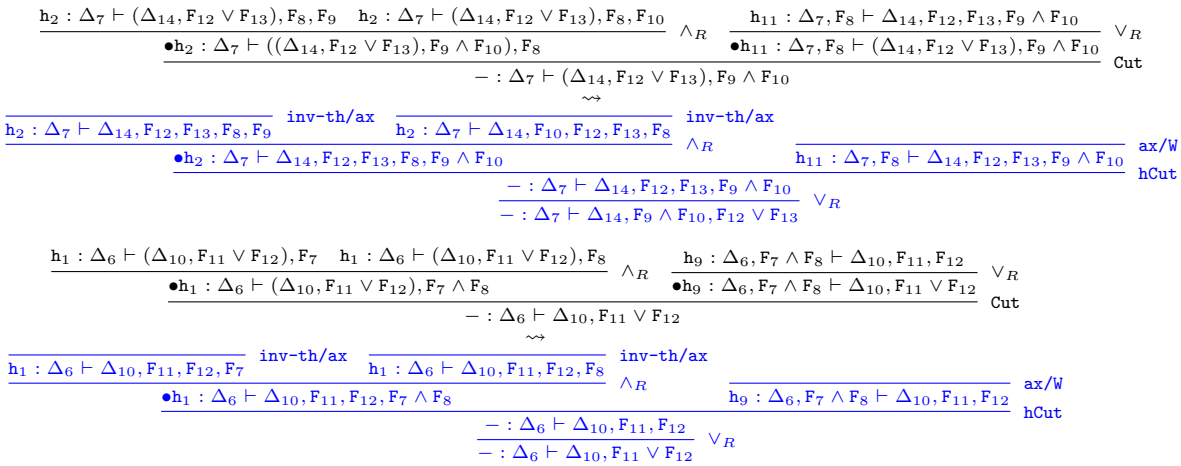
$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_8, \mathtt{F}_9 \quad \mathtt{h}_2 : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \Delta_7 \vdash ((\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_8}\,{\wedge_R} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\to_R}}{- : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_2 : \Delta_7, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_2 : \Delta_7, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\wedge_R} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{hCut}$$
$$\dfrac{}{- : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\to_R}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,{\wedge_R} \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\,{\to_R}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,{\wedge_R} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{hCut}$$
$$\dfrac{}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\,{\to_R}$$

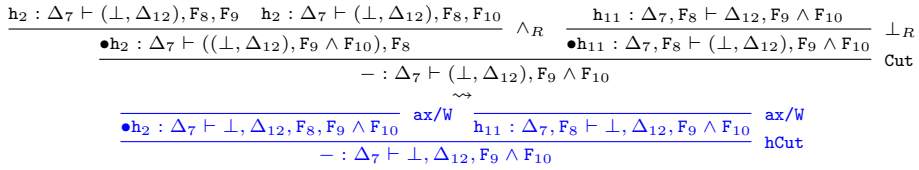
- Case rule $\wedge_R$



- Case rule $\vee_R$



- Case rule $\bot_R$



$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_{10}), \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\wedge_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\bot_R}{- : \Delta_6 \vdash \bot, \Delta_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \bot, \Delta_{10}}{}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_{12}), \mathtt{F}_8, \mathtt{F}_9 \quad \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_{12}), \mathtt{F}_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \Delta_7 \vdash ((\top, \Delta_{12}), \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_8}\,\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\top_R}{- : \Delta_7 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\top_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_{10}), \mathtt{F}_7 \quad \mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_{10}), \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_{10}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \top, \Delta_{10}}\,\top_R}{- : \Delta_6 \vdash \top, \Delta_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}}\,\top_R$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \quad \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7}\,\wedge_R \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{12}, \mathtt{F}_9 \wedge}{\bullet \mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12}}}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{12}, \mathtt{F}_9}{}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}{}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9}\,\to_L}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}{}}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_8 \quad \mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{12} \to \mathtt{F}_{13}}\,\wedge_R \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \quad \mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet \mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\to_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{- : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_{13}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_{13}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\wedge_R \quad \dfrac{}{- : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{sCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{sCut}$$

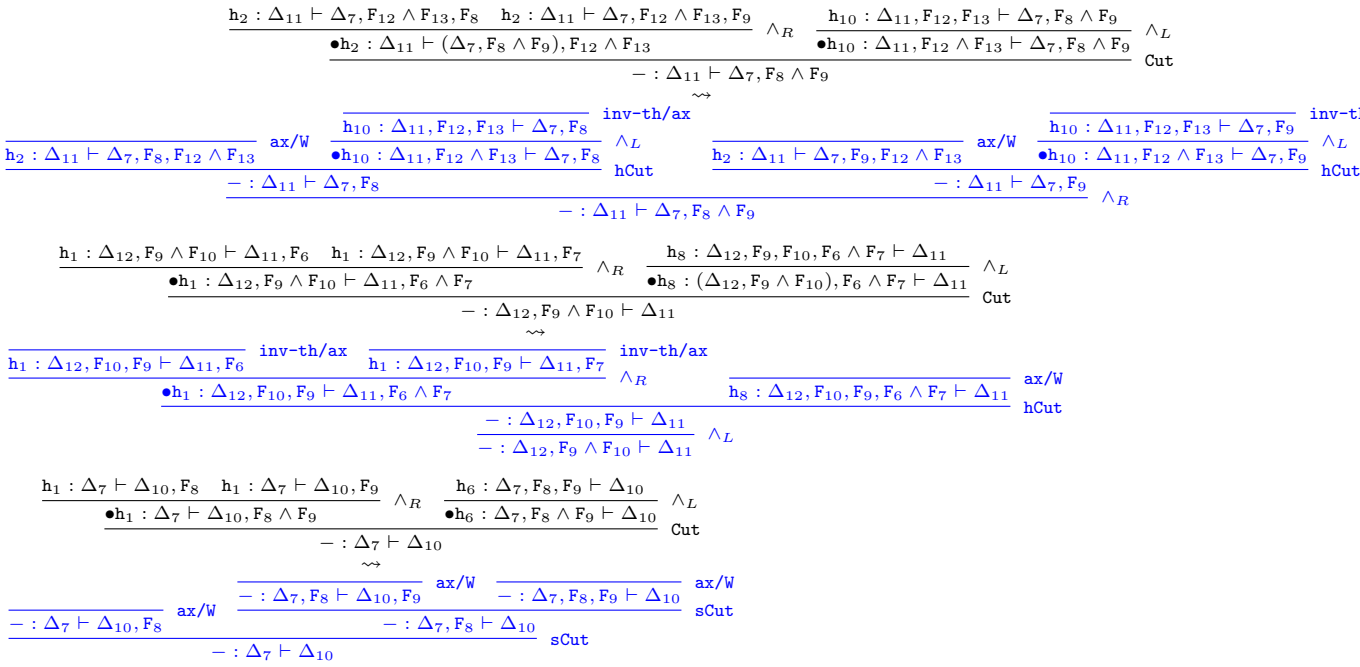
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}}\,\to_L}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{inv\text{-}th/}}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\to_L}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{sCut}}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{sCut}$$

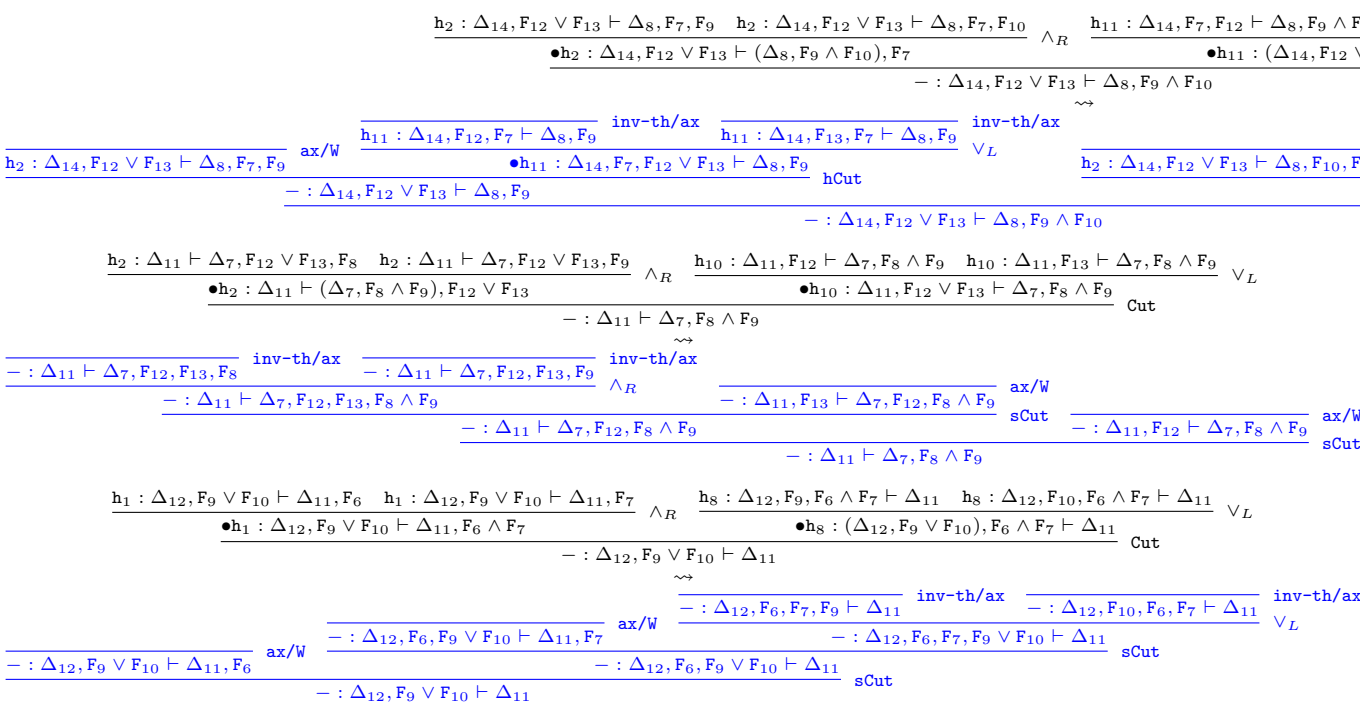
- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \quad \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7}\,\wedge_R \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\wedge_L}{- : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9}{}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7}{}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\wedge_R \quad \dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\wedge_L$$



- Case rule $\vee_L$



- Case rule $\bot_L$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \bot, \mathtt{F}_8 \quad \mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \bot, \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \bot}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_{10} : \Delta_{11}, \bot \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\bot_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11} \vdash \bot, \Delta_7, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_{10} : \bot, \Delta_{11} \vdash \Delta_7, \mathtt{F}_8}\bot_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11} \vdash \bot, \Delta_7, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_{10} : \bot, \Delta_{11} \vdash \Delta_7, \mathtt{F}_9}\bot_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\wedge_R$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \quad \mathtt{h}_2 : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \bot, \Delta_{12} \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_{11} : (\bot, \Delta_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\bot_L}{- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\bot_L$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \quad \mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}, \mathtt{F}_7 \quad \mathtt{h}_2 : \Delta_{10} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}, \mathtt{F}_8}{\bullet \mathtt{h}_2 : \Delta_{10} \vdash ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_{11}}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{10}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,I}{- : \Delta_{10} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{10}, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{p}_{11}}\,I}{- : \Delta_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{p}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10} \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_{10}, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}}\,I}{- : \Delta_{10} \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{10} \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_8 \quad \mathtt{h}_2 : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_{10} : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \wedge \mathtt{F}_9}\,I}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,I$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6 \quad \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_9), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}\,I}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \top, \mathtt{F}_8 \quad \mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \top, \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \top}\wedge_R \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet \mathtt{h}_{10} : \Delta_{11}, \top \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\top_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \quad \mathtt{h}_2 : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_{11} : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\top, \Delta_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\top_L}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \top, \Delta_{12}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \quad \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \wedge \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{hCut}$$

8.3 Status of $\vee_R$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \rightarrow \mathtt{F}_{13}), \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Delta_7 \vdash ((\Delta_{14}, \mathtt{F}_{12} \rightarrow \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \rightarrow \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\rightarrow_R}{- : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \rightarrow \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_2 : \Delta_7, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_R \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_{12} \rightarrow \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\rightarrow_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_7, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\rightarrow_R}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Delta_7 \vdash ((\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \quad \mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\wedge_R}{- : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12}, \mathtt{F}_9}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}\wedge_R}{- : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\,\mathtt{hCut}}{- : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_7, \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Delta_7 \vdash ((\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_R}{- : \Delta_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_2 : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_R \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_7 \vdash \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_{11}, \mathtt{F}_{12}}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}{\bullet \mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_R}{- : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\,\mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\,\mathit{height}}{- : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_7, \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_R}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_{12}), \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \Delta_7 \vdash ((\bot, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\bot, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\bot_R}{- : \Delta_7 \vdash (\bot, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_{12}, \mathtt{F}_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_{10}), \mathtt{F}_7, \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}}\bot_R}{- : \Delta_6 \vdash \bot, \Delta_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_{12}), \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \Delta_7 \vdash ((\top, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_8}\vee_R \quad \dfrac{}{\bullet \mathtt{h}_{11} : \Delta_7, \mathtt{F}_8 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\top_R}{- : \Delta_7 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\top_R$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_{10}), \mathtt{F}_7, \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \top, \Delta_{10}}\top_R}{- : \Delta_6 \vdash \top, \Delta_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}}\top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \quad \mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\to_L}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{12}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\to_L}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9}{\bullet\mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{12} \to \mathtt{F}_{13}} \vee_R \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \quad \mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9} \to_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathrm{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13}}{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13}}\,\mathrm{ax/W} \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9}{\mathtt{h}_{10} : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9}\,\mathrm{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\mathrm{inv\text{-}th/ax}\ \to_L}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\mathrm{hCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9} \vee_R}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}} \to_L}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathrm{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathrm{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9}\,\mathrm{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\,\mathrm{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathrm{hCut}}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}} \to_L$$

$\mathrm{inv\text{-}th/ax}$ $\mathrm{inv\text{-}th/ax}$ $\mathrm{inv\text{-}th/ax}$ $\mathrm{inv\text{-}th/ax}$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}} \wedge_L}{- : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathrm{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9}{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9}\,\mathrm{ax/W} \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}{\bullet\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathrm{inv\text{-}th/ax}\ \wedge_L}{\bullet\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathrm{hCut}}{- : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9} \vee_R}{- : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12} \wedge \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9}{\bullet\mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{12} \wedge \mathtt{F}_{13}} \vee_R \quad \dfrac{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathrm{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\,\mathrm{ax/W} \quad \dfrac{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\mathrm{inv\text{-}th/ax}\ \wedge_L}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\mathrm{hCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9} \vee_R}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathrm{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}\,\mathrm{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathrm{hCut}}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}} \wedge_L$$

$\mathrm{inv\text{-}th/ax}$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \quad \mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}} \vee_L}{- : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathrm{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9}{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9}\,\mathrm{ax/W} \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}{\bullet\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathrm{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathrm{inv\text{-}th/ax}\ \vee_L}{\bullet\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\,\mathrm{hCut}}{- : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9} \vee_R}{- : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12} \vee \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9}{\bullet\mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{12} \vee \mathtt{F}_{13}} \vee_R \quad \dfrac{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \quad \mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9} \vee_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{12} \vee \mathtt{F}_{13}}{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \vee_L}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9} \vee_R}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{ax/W}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{11}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{ax/W}\,\mathtt{hCut} \vee_L}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9} \vee_R \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{10} \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{10}} \vee_L}{- : \Delta_7 \vdash \Delta_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{sCut} \quad \dfrac{}{- : \Delta_7, \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_7 \vdash \Delta_{10}}\,\mathtt{sCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \bot, \mathtt{F}_8, \mathtt{F}_9}{\bullet\mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \bot} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_{11}, \bot \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\,\bot_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11} \vdash \bot, \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\bot_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \bot, \Delta_{12} \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\bot, \Delta_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\bot_L}{- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\bot_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6, \mathtt{F}_7}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\,\bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}, \mathtt{F}_7, \mathtt{F}_8}{\bullet\mathtt{h}_2 : \Delta_{10} \vdash ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{11}} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{10}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,I}{- : \Delta_{10} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{10}, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}\,I}{- : \Delta_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_{10} \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8} \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9}{\bullet\mathtt{h}_2 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7} \vee_R \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9}\,I}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9}\,I$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9} I}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9} I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \Delta_7, \top, \mathtt{F}_8, \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \top}\vee_R \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet \mathtt{h}_{10} : \Delta_{11}, \top \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\top_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_{11} : \Delta_{12}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\top, \Delta_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\top_L}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \top, \Delta_{12}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6, \mathtt{F}_7}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{hCut}$$

8.4 Status of $\bot_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \to \mathtt{F}_8), \bot}\bot_R \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4, \mathtt{F}_7 \vdash \Delta_6, \mathtt{F}_8}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}\to_R}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_5 \vdash (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_6}{\bullet \mathtt{h}_2 : \Delta_5 \vdash (\bot, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_6}\bot_R \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, \mathtt{F}_8 \vdash \bot, \Delta_{10}, \mathtt{F}_9}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\to_R}{- : \Delta_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_5 \vdash \bot, \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \bot}\bot_R \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \quad \mathtt{h}_5 : \bot, \Delta_4 \vdash \Delta_6, \mathtt{F}_8}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\wedge_R}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_5 \vdash (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_6}{\bullet \mathtt{h}_2 : \Delta_5 \vdash (\bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_6}\bot_R \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_{10}, \mathtt{F}_9}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\wedge_R}{- : \Delta_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_5 \vdash \bot, \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4 \vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_8}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_5 \vdash (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_6}{\bullet \mathtt{h}_2 : \Delta_5 \vdash (\bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_6}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \vee_R}{- : \Delta_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_5 \vdash \bot, \Delta_{10}, \mathtt{F}_6, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{hCut}}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\bot, \Delta_6), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4 \vdash \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \bot, \Delta_6}\ \bot_R}{- : \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_5 \vdash \Delta_8, \mathtt{F}_6}{\bullet \mathtt{h}_2 : \Delta_5 \vdash (\bot, \Delta_8), \mathtt{F}_6}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_8}\ \bot_R}{- : \Delta_5 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_5 \vdash \bot, \Delta_8, \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_5 \vdash \bot, \Delta_8}\ \mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \top, \Delta_6}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\top, \Delta_6), \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \top, \Delta_6}\ \top_R}{- : \Delta_4 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_4 \vdash \top, \Delta_6}\ \top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_5 \vdash (\top, \Delta_8), \mathtt{F}_6}{\bullet \mathtt{h}_2 : \Delta_5 \vdash (\bot, \top, \Delta_8), \mathtt{F}_6}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \bot, \top, \Delta_8}\ \top_R}{- : \Delta_5 \vdash \bot, \top, \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_5 \vdash \bot, \top, \Delta_8}\ \top_R}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \quad \mathtt{h}_4 : \bot, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \to \mathtt{F}_6), \bot \vdash \Delta_7}\ \to_L}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash (\bot, \Delta_6), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5 \vdash \bot, \Delta_6, \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_9 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \vdash \bot, \Delta_6}\ \to_L}{- : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_6, \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \Delta_5, \mathtt{F}_8 \to \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \mathtt{F}_8 \to \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \bot, \Delta_5, \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \bot, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_5}\ \to_L}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_5, \mathtt{F}_8 \to \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{hCut}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \bot}\,\bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \bot \vdash \Delta_7}\,\wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_6, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash (\bot, \Delta_6), \mathtt{F}_5}\,\bot_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8, \mathtt{F}_9 \vdash \bot, \Delta_6}{\bullet\mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \vdash \bot, \Delta_6}\,\wedge_L}{- : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_6}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_6, \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_6}\,\mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_6}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \Delta_5, \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet\mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \mathtt{F}_8 \wedge \mathtt{F}_9}\,\bot_R \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \bot, \Delta_5}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_5}\,\wedge_L}{- : \Delta_7 \vdash \bot, \Delta_5}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_5, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_5}\,\mathtt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5}\,\mathtt{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \bot}\,\bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8, \mathtt{F}_5 \vdash \Delta_7 \quad \mathtt{h}_4 : \bot, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}{\bullet\mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \bot \vdash \Delta_7}\,\vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_6, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\bot, \Delta_6), \mathtt{F}_5}\,\bot_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \bot, \Delta_6 \quad \mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_9 \vdash \bot, \Delta_6}{\bullet\mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \vdash \bot, \Delta_6}\,\vee_L}{- : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_6}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_6, \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_6}\,\mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_6}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \Delta_5, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \mathtt{F}_8 \vee \mathtt{F}_9}\,\bot_R \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_5 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \bot, \Delta_5}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_5}\,\vee_L}{- : \Delta_7 \vdash \bot, \Delta_5}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_5, \mathtt{F}_8 \vee \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_5}\,\mathtt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5}\,\mathtt{hCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \Delta_6}{\bullet\mathtt{h}_1 : \Delta_5 \vdash \Delta_6, \bot}\,\bot_R \quad \dfrac{}{\bullet\mathtt{h}_4 : \Delta_5, \bot \vdash \Delta_6}\,\bot_L}{- : \Delta_5 \vdash \Delta_6}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_5 \vdash \Delta_6}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \Delta_5, \bot}{\bullet\mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \bot}\,\bot_R \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \bot \vdash \bot, \Delta_5}\,\bot_L}{- : \Delta_7 \vdash \bot, \Delta_5}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_7 \vdash \bot, \Delta_5}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \bot, \Delta_8 \vdash \Delta_6, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \bot, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_5}\,\bot_R \quad \dfrac{}{\bullet\mathtt{h}_7 : (\bot, \Delta_8), \mathtt{F}_5 \vdash \bot, \Delta_6}\,\bot_L}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}{\bullet\mathtt{h}_1 : \Delta_7, \mathtt{p}_5 \vdash (\Delta_6, \mathtt{p}_5), \bot}\,\bot_R \quad \dfrac{}{\bullet\mathtt{h}_4 : (\Delta_7, \mathtt{p}_5), \bot \vdash \Delta_6, \mathtt{p}_5}\,I}{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}\,\mathtt{Cut}}{\rightsquigarrow \quad \dfrac{}{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}\,I}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_6 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_7}{\bullet\mathtt{h}_2 : \Delta_6 \vdash (\bot, \Delta_8, \mathtt{p}_7), \mathtt{p}_7}\,\bot_R \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{p}_7 \vdash \bot, \Delta_8, \mathtt{p}_7}\,I}{- : \Delta_6 \vdash \bot, \Delta_8, \mathtt{p}_7}\,\mathtt{Cut}}{\rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6 \vdash \bot, \Delta_8, \mathtt{p}_7, \mathtt{p}_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{p}_7 \vdash \bot, \Delta_8, \mathtt{p}_7}\,I}{- : \Delta_6 \vdash \bot, \Delta_8, \mathtt{p}_7}\,\mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_9, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_9, \mathtt{p}_7 \vdash (\bot, \Delta_8, \mathtt{p}_7), \mathtt{F}_5}\,\bot_R \quad \dfrac{}{\bullet\mathtt{h}_6 : (\Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \bot, \Delta_8, \mathtt{p}_7}\,I}{- : \Delta_9, \mathtt{p}_7 \vdash \bot, \Delta_8, \mathtt{p}_7}\,\mathtt{Cut}}{\rightsquigarrow \quad \dfrac{}{- : \Delta_9, \mathtt{p}_7 \vdash \bot, \Delta_8, \mathtt{p}_7}\,I}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5, \bot}\,\bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_6 \vdash \Delta_5}{\bullet\mathtt{h}_4 : (\top, \Delta_6), \bot \vdash \Delta_5}\,\top_L}{- : \top, \Delta_6 \vdash \Delta_5}\,\mathtt{Cut}}{\rightsquigarrow \quad \dfrac{}{- : \top, \Delta_6 \vdash \Delta_5}\,\mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \Delta_5, \top}{\bullet\mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \top}\,\bot_R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \bot, \Delta_5}{\bullet\mathtt{h}_6 : \Delta_7, \top \vdash \bot, \Delta_5}\,\top_L}{- : \Delta_7 \vdash \bot, \Delta_5}\,\mathtt{Cut}}{\rightsquigarrow \quad \dfrac{}{- : \Delta_7 \vdash \bot, \Delta_5}\,\mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \top, \Delta_8 \vdash \Delta_6, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \top, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_5}\,\bot_R \quad \dfrac{\mathtt{h}_7 : \Delta_8, \mathtt{F}_5 \vdash \bot, \Delta_6}{\bullet\mathtt{h}_7 : (\top, \Delta_8), \mathtt{F}_5 \vdash \bot, \Delta_6}\,\top_L}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\,\mathtt{Cut}}{\rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \top, \Delta_8 \vdash \bot, \Delta_6, \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \top, \Delta_8, \mathtt{F}_5 \vdash \bot, \Delta_6}\,\mathtt{ax/W}}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\,\mathtt{hCut}}$$

8.5 Status of $\top_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \to \mathtt{F}_8), \top}\,\top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_7 \vdash \Delta_6, \mathtt{F}_8}{\bullet\mathtt{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}\,\to_R}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{Cut}}{\rightsquigarrow \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_4, \mathtt{F}_7 \vdash \top, \Delta_6, \mathtt{F}_8}\,\top_R \quad \dfrac{}{\mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_7 \vdash \Delta_6, \mathtt{F}_8}\,\mathtt{ax/W}}{- : \Delta_4, \mathtt{F}_7 \vdash \Delta_6, \mathtt{F}_8}\,\mathtt{hCut}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}\,\to_R$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_5 \vdash (\top, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_6}\,\top_R \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6, \mathtt{F}_8 \vdash \top, \Delta_{10}, \mathtt{F}_9}{\bullet\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\to_R}{- : \Delta_5 \vdash \top, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\rightsquigarrow \quad \dfrac{}{- : \Delta_5 \vdash \top, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,\top_R}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \top}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \quad \mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathtt{F}_8}{\bullet \mathtt{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathtt{F}_7}\ \top_R \quad \dfrac{}{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathtt{F}_8}\ \top_R \quad \dfrac{}{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_5 \vdash (\top, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_6}\ \top_R \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top, \Delta_{10}, \mathtt{F}_9}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \wedge_R}{- : \Delta_5 \vdash \top, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_5 \vdash \top, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \top_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8), \top}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_8}{\bullet \mathtt{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathtt{F}_7, \mathtt{F}_8}\ \top_R \quad \dfrac{}{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_8}\ \mathtt{hCut}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_5 \vdash (\top, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_6}\ \top_R \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top, \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \vee_R}{- : \Delta_5 \vdash \top, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_5 \vdash \top, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \top_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\bot, \Delta_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \top \vdash \bot, \Delta_6}\ \bot_R}{- : \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash \bot, \top, \Delta_6}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \top, \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_5 \vdash (\top, \bot, \Delta_8), \mathtt{F}_6}\ \top_R \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top, \Delta_8}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top, \bot, \Delta_8}\ \bot_R}{- : \Delta_5 \vdash \top, \bot, \Delta_8}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_5 \vdash \bot, \top, \Delta_8}\ \top_R$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\top, \Delta_6), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_4, \top \vdash \top, \Delta_6}\ \top_R}{- : \Delta_4 \vdash \top, \Delta_6}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \top, \Delta_6}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_5 \vdash (\top, \Delta_8), \mathtt{F}_6}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \top, \Delta_8}\ \top_R}{- : \Delta_5 \vdash \top, \Delta_8}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_5 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7, \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5 \quad \mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \to \mathtt{F}_6), \top \vdash \Delta_7}\,\to_L}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\,\mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\,\to_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash (\top, \Delta_6), \mathtt{F}_5}\,\top_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5 \vdash \top, \Delta_6, \mathtt{F}_8 \quad \mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_9 \vdash \top, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \vdash \top, \Delta_6}\,\to_L}{- : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \top, \Delta_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \top, \Delta_6}\,\top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \mathtt{F}_8 \to \mathtt{F}_9}\,\top_R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \top, \Delta_5, \mathtt{F}_8 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \top, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \top, \Delta_5}\,\to_L}{- : \Delta_7 \vdash \top, \Delta_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\,\top_R$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \top \vdash \Delta_7}\,\wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \top, \Delta_7}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\,\wedge_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash (\top, \Delta_6), \mathtt{F}_5}\,\top_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8, \mathtt{F}_9 \vdash \top, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \vdash \top, \Delta_6}\,\wedge_L}{- : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \top, \Delta_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \top, \Delta_6}\,\top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \mathtt{F}_8 \wedge \mathtt{F}_9}\,\top_R \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \top, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \top, \Delta_5}\,\wedge_L}{- : \Delta_7 \vdash \top, \Delta_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\,\top_R$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \top}\,\top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7 \quad \mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \top \vdash \Delta_7}\,\vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vdash \top, \Delta_7}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\,\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\,\top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\,\mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\,\vee_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\top, \Delta_6), \mathtt{F}_5}\,\top_R \quad \dfrac{\mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \top, \Delta_6 \quad \mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_9 \vdash \top, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \vdash \top, \Delta_6}\,\vee_L}{- : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \top, \Delta_6}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \top, \Delta_6}\,\top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \mathtt{F}_8 \vee \mathtt{F}_9}\,\top_R \quad \dfrac{\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vdash \top, \Delta_5 \quad \mathtt{h}_6 : \Delta_7, \mathtt{F}_9 \vdash \top, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \top, \Delta_5}\,\vee_L}{- : \Delta_7 \vdash \top, \Delta_5}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\,\top_R$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_6 \vdash \Delta_5, \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\bot, \Delta_6), \top \vdash \Delta_5}\ \bot_L}{- : \bot, \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \Delta_5}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \bot}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \bot \vdash \top, \Delta_5}\ \bot_L}{- : \Delta_7 \vdash \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \bot, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_8), \mathtt{F}_5 \vdash \top, \Delta_6}\ \bot_L}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_5 \vdash (\Delta_6, \mathtt{p}_5), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\Delta_7, \mathtt{p}_5), \top \vdash \Delta_6, \mathtt{p}_5}\ I}{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\top, \Delta_8, \mathtt{p}_7), \mathtt{p}_7}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_6, \mathtt{p}_7 \vdash \top, \Delta_8, \mathtt{p}_7}\ I}{- : \Delta_6 \vdash \top, \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_8, \mathtt{p}_7}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_9, \mathtt{p}_7 \vdash (\top, \Delta_8, \mathtt{p}_7), \mathtt{F}_5}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \top, \Delta_8, \mathtt{p}_7}\ I}{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_8, \mathtt{p}_7}\ \top_R$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_6, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_6}{\bullet \mathtt{h}_4 : \Delta_5, \top \vdash \Delta_6}\ \top_L}{- : \Delta_5 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_5 \vdash \Delta_6}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \top, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \top \vdash \top, \Delta_5}\ \top_L}{- : \Delta_7 \vdash \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \top, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_7 : \Delta_8, \mathtt{F}_5 \vdash \top, \Delta_6}{\bullet \mathtt{h}_7 : (\top, \Delta_8), \mathtt{F}_5 \vdash \top, \Delta_6}\ \top_L}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

8.6 Status of $\rightarrow_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash (\Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}), \mathtt{F}_8, \mathtt{F}_9 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}), \mathtt{F}_8}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}), \mathtt{F}_8}\,{\to_L} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_{13}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet \mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\,{\to_R}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_{13}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8}\,{\to_L} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_{13}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\,{\to_R}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash (\Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}), \mathtt{F}_8, \mathtt{F}_9 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}), \mathtt{F}_8}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}), \mathtt{F}_8}\,{\to_L} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12},}{\bullet \mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 -}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_8}\,{\to_L} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}}\,\mathtt{hCut} \quad \dfrac{\mathtt{h}_3 : \Delta_7 \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8, \mathtt{F}_9}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 -}\,\mathtt{inv}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_8, \mathtt{F}_9 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_8}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_8}\,{\to_L} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}{\bullet \mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,{\vee_R}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_8}\,{\to_L} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,{\vee_R}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash (\bot, \Delta_{12}), \mathtt{F}_8, \mathtt{F}_9 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash (\bot, \Delta_{12}), \mathtt{F}_8}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\bot, \Delta_{12}), \mathtt{F}_8}\,{\to_L} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}}{\bullet \mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_8 \vdash \bot, \Delta_{12}}\,{\bot_R}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \bot, \Delta_{12}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash (\top, \Delta_{12}), \mathtt{F}_8, \mathtt{F}_9 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash (\top, \Delta_{12}), \mathtt{F}_8}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\top, \Delta_{12}), \mathtt{F}_8}\,{\to_L} \quad \dfrac{}{\bullet \mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_8 \vdash \top, \Delta_{12}}\,{\top_R}}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \top, \Delta_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

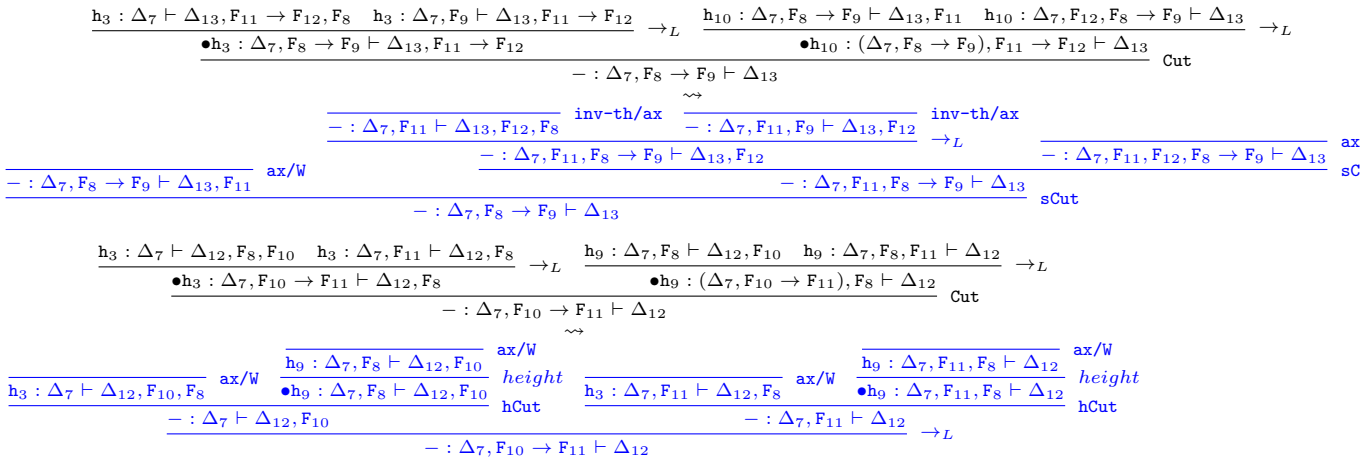
$$\dfrac{}{- : \Delta_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \top, \Delta_{12}}\,{\top_R}$$

- Case rule $\to_L$

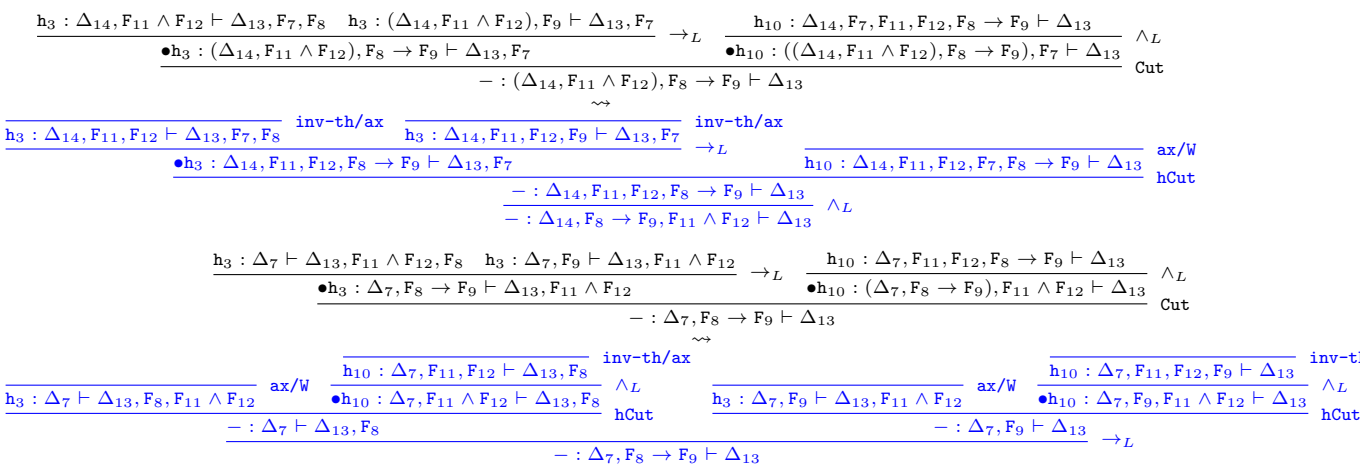
$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12} \vdash \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8 \quad \mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}{\bullet \mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\,{\to_L} \quad \dfrac{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{1}}{\bullet \mathtt{h}_{10} : ((\Delta_{14}, \mathtt{F}_{11}}}{- : (\Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}}$$

$$\rightsquigarrow$$

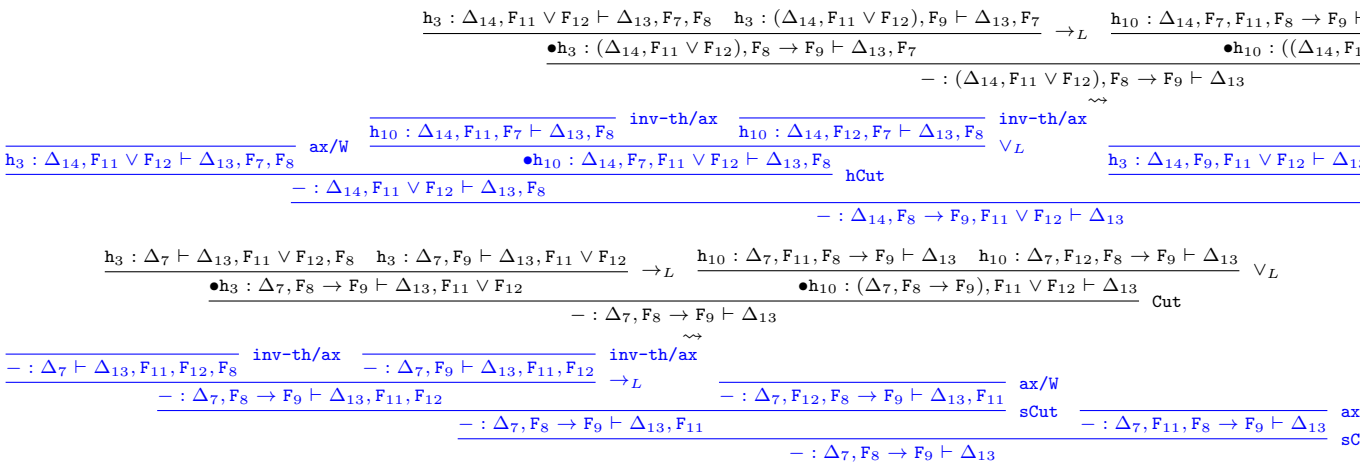
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{14} \vdash \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_3 : \Delta_{14}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7}\,{\to_L} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{12} \vdash \Delta_{13}, \mathtt{F}_7, \mathtt{F}_8}{\bullet \mathtt{h}_3 : \Delta_{14}, \mathtt{F}_1}\,\mathtt{i}}{- : \Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}}$$



- Case rule $\wedge_L$



- Case rule $\vee_L$



- Case rule $\bot_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash \Delta_{11}, \bot, \mathtt{F}_8 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{11}, \bot}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \bot}\,{\to_L} \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \bot \vdash \Delta_{11}}\,{\bot_L}}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \bot, \Delta_{11}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_7 \vdash \Delta_{11}, \mathtt{F}_8}\,{\bot_L}}{- : \Delta_7 \vdash \Delta_{11}, \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \bot, \Delta_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_7, \mathtt{F}_9 \vdash \Delta_{11}}\,{\bot_L}}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,{\to_L}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \bot, \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \quad \mathtt{h}_3 : (\bot, \Delta_{12}), \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\bot, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,{\to_L} \quad \dfrac{}{\bullet\mathtt{h}_{10} : ((\bot, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}}\,{\bot_L}}{- : (\bot, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \bot, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,{\bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}, \mathtt{F}_8 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}}\,{\to_L} \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}}\,I}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}}\,I}{- : \Delta_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{p}_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_9, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}}\,I}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\,{\to_L}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7, \mathtt{F}_8 \quad \mathtt{h}_3 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7}\,{\to_L} \quad \dfrac{}{\bullet\mathtt{h}_{10} : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{p}_{11}}\,I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash \Delta_{11}, \top, \mathtt{F}_8 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{11}, \top}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \top}\,{\to_L} \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \top \vdash \Delta_{11}}\,{\top_L}}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_8 \quad \mathtt{h}_3 : (\top, \Delta_{12}), \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\top, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,{\to_L} \quad \dfrac{\mathtt{h}_{10} : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}{\bullet\mathtt{h}_{10} : ((\top, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}}\,{\top_L}}{- : (\top, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \top, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \top, \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut}$$

8.7 Status of $\wedge_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9, \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}), \mathtt{F}_8}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}), \mathtt{F}_8}\,{\wedge_L} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\,{\to_R}}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_8}\,{\wedge_L} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_{13}, \mathtt{F}_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\,{\to_R}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9, \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}), \mathtt{F}_8}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}), \mathtt{F}_8}\,\wedge_L \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \quad \mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}{\bullet\mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,\wedge_R}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{11} : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,\wedge_R}{- : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,\wedge_L$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9, \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_8}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_8}\,\wedge_L \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}{\bullet\mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,\vee_R}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_8}\,\wedge_L \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,\vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9, \mathtt{F}_{10} \vdash (\bot, \Delta_{12}), \mathtt{F}_8}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\bot, \Delta_{12}), \mathtt{F}_8}\,\wedge_L \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}}{\bullet\mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_8 \vdash \bot, \Delta_{12}}\,\bot_R}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \bot, \Delta_{12}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9, \mathtt{F}_{10} \vdash (\top, \Delta_{12}), \mathtt{F}_8}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\top, \Delta_{12}), \mathtt{F}_8}\,\wedge_L \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_8 \vdash \top, \Delta_{12}}\,\top_R}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \top, \Delta_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \top, \Delta_{12}}\,\top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\,\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \quad \mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : ((\Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{13}}\,\rightarrow_L}{- : (\Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}}\,\rightarrow_L}{- : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\,\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \quad \mathtt{h}_{10} : \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}}\,\rightarrow_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}}\,\rightarrow_L}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\wedge_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\,\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : ((\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{13}}\,\wedge_L}{- : (\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\,\wedge_L \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11} \wedge \mathtt{F}_{12} \vdash \Delta_{13}}\,\wedge_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \wedge \mathtt{F}_{12} \vdash \Delta_{13}}\,\wedge_L}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\wedge_L \quad \dfrac{\mathtt{h}_9 : \Delta_7, \mathtt{F}_8, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}}{\bullet\mathtt{h}_9 : (\Delta_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_8 \vdash \Delta_{12}}\,\wedge_L}{- : \Delta_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{12}}\,\mathit{height}}{- : \Delta_7, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\,\wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\,\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13} \quad \mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : ((\Delta_{14}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{13}}\,\vee_L}{- : (\Delta_{14}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\,\vee_L}{\bullet\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\,\wedge_L}{- : \Delta_{14}, \mathtt{F}_8 \wedge \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\,\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13} \quad \mathtt{h}_{10} : \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\,\vee_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\,\vee_L}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\wedge_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\wedge_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{11}, \bot}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}, \bot}\,\wedge_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \bot \vdash \Delta_{11}}\,\bot_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

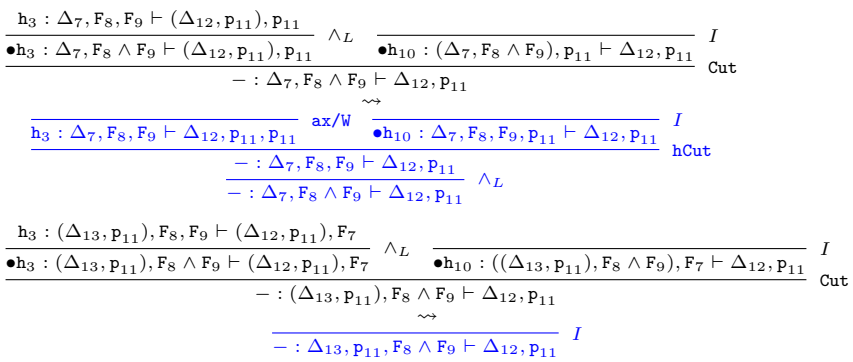
$$\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \bot, \Delta_{11}}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{11}}\,\bot_L}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\,\wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_3 : (\bot, \Delta_{12}), \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\bot, \Delta_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,\wedge_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : ((\bot, \Delta_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}}\,\bot_L}{- : (\bot, \Delta_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}$$

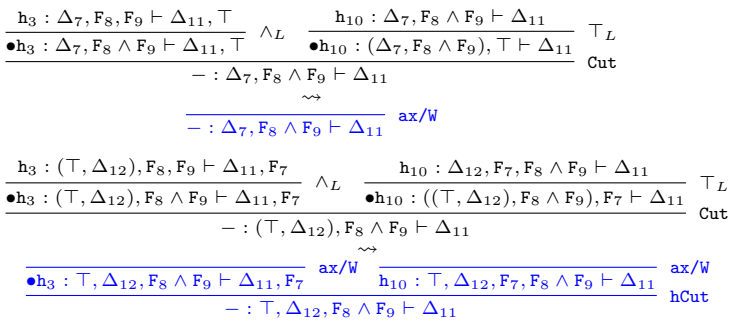
$$\rightsquigarrow$$

$$\dfrac{}{- : \bot, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\,\bot_L$$

- Case rule I

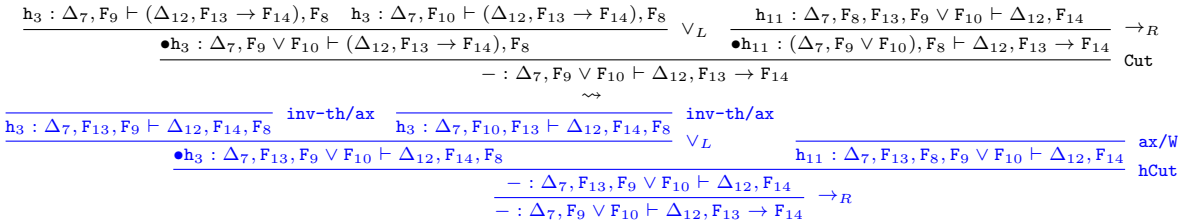


- Case rule $\top_L$

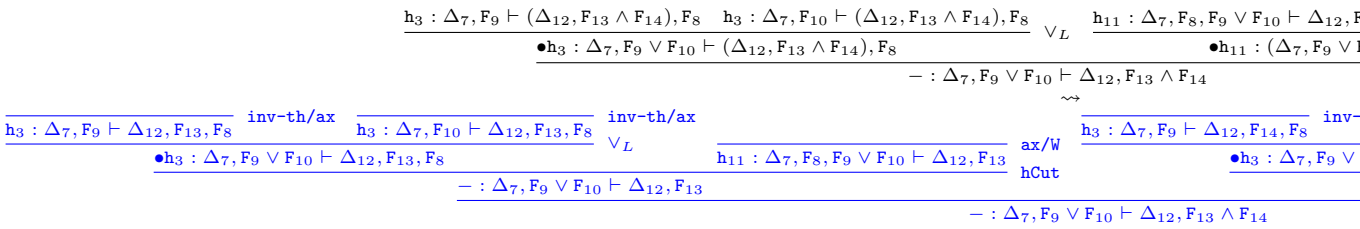


8.8 Status of $\vee_L$: OK

- Case rule $\to_R$



- Case rule $\wedge_R$



- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_8 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_8}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_8}\vee_L \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}{\bullet\mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\vee_R}{- : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_8}\vee_L \quad \dfrac{}{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash (\bot, \Delta_{12}), \mathtt{F}_8 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash (\bot, \Delta_{12}), \mathtt{F}_8}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\bot, \Delta_{12}), \mathtt{F}_8}\vee_L \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}}{\bullet\mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_8 \vdash \bot, \Delta_{12}}\bot_R}{- : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \bot, \Delta_{12}, \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\mathtt{h}_{11} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \bot, \Delta_{12}}{}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash (\top, \Delta_{12}), \mathtt{F}_8 \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_{10} \vdash (\top, \Delta_{12}), \mathtt{F}_8}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\top, \Delta_{12}), \mathtt{F}_8}\vee_L \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_8 \vdash \top, \Delta_{12}}\top_R}{- : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \top, \Delta_{12}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \top, \Delta_{12}}\top_R$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_8 \vdash \Delta_{13}, \mathtt{F}_7 \quad \mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : ((\Delta_{14}, \mathtt{F}_{11}}}{- : (\Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_8 \vdash \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7}\vee_L \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_8 \vdash \Delta_{13}, \mathtt{F}_7}\,\mathtt{in}$$

$$\bullet\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{1}$$

$$- : \Delta_{14}, \mathtt{F}_{11} \to \mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{13}, \mathtt{F}_{11} \to \mathtt{F}_{12} \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \to \mathtt{F}_{12}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \quad \mathtt{h}_{10} : \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{11} \to \mathtt{F}_{12} \vdash \Delta_{13}}\to_L}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}}{\rightsquigarrow}$$

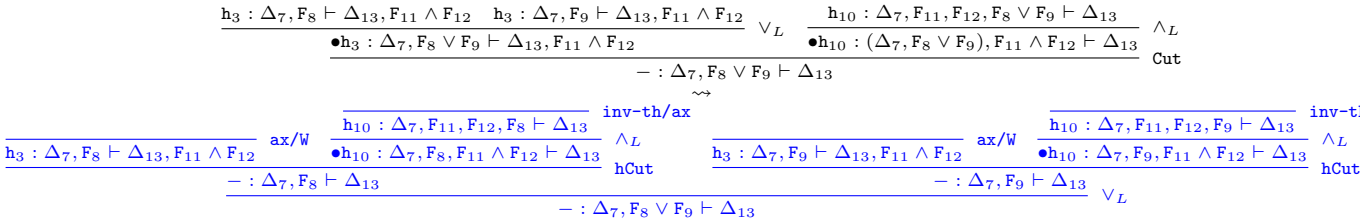
$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{13}, \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{12}}\vee_L \quad \dfrac{}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{sCut} \quad \mathtt{sCut}$$

$$\dfrac{\dfrac{}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}$$

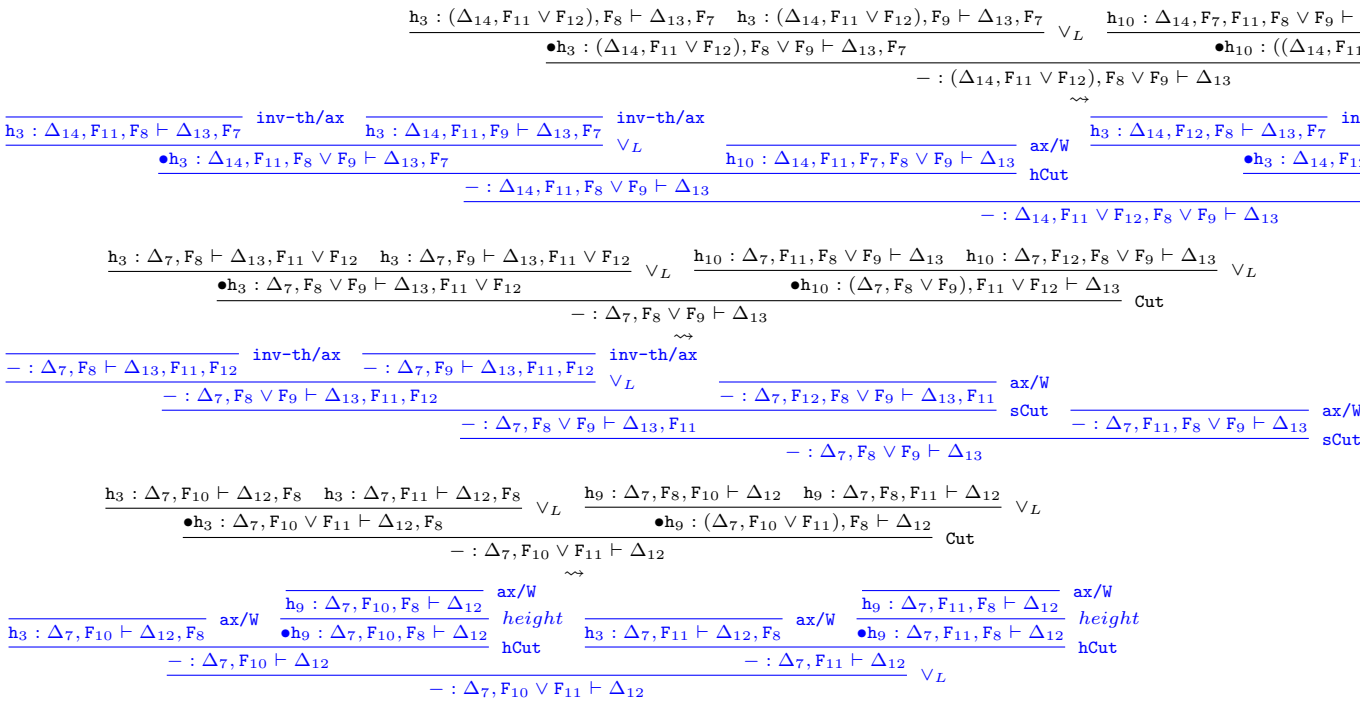
- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \vdash \Delta_{13}, \mathtt{F}_7 \quad \mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : ((\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{13}}\wedge_L}{- : (\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}}{\rightsquigarrow}$$

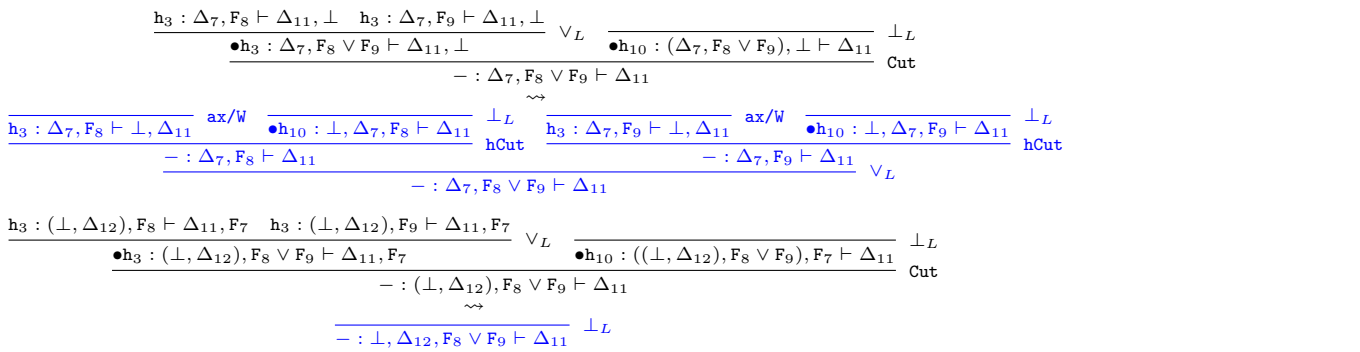
$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vdash \Delta_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\vee_L \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\wedge_L$$



- Case rule $\vee_L$



- Case rule $\bot_L$



- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11} \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}}\vee_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}}\, I}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{p}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}}\, I}{- : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{p}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{p}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_9, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}}\, I}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \quad \mathtt{h}_3 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7}\vee_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{p}_{11}}\, I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{11}, \top \quad \mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{11}, \top}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}, \top}\vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \top \vdash \Delta_{11}}\,\top_L}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : (\top, \Delta_{12}), \mathtt{F}_8 \vdash \Delta_{11}, \mathtt{F}_7 \quad \mathtt{h}_3 : (\top, \Delta_{12}), \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : (\top, \Delta_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}{\bullet\mathtt{h}_{10} : ((\top, \Delta_{12}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}}\,\top_L}{- : (\top, \Delta_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \top, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \top, \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut}$$

8.9 Status of $\bot_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6}\,\bot_L \quad \dfrac{\mathtt{h}_7 : \bot, \Delta_5, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\bot, \Delta_5), \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\to_R}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\bot_L$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6}\,\bot_L \quad \dfrac{\mathtt{h}_7 : \bot, \Delta_5, \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 : \bot, \Delta_5, \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\bot, \Delta_5), \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\wedge_R}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\bot_L$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6}\,\bot_L \quad \dfrac{\mathtt{h}_7 : \bot, \Delta_5, \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 : (\bot, \Delta_5), \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\vee_R}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\,\bot_L$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash (\bot, \Delta_8), \mathtt{F}_6}\ \bot_L \quad \dfrac{\mathtt{h}_7 : \bot, \Delta_5, \mathtt{F}_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : (\bot, \Delta_5), \mathtt{F}_6 \vdash \bot, \Delta_8}\ \bot_R}{- : \bot, \Delta_5 \vdash \bot, \Delta_8}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \bot, \Delta_8}\ \bot_L$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash (\top, \Delta_8), \mathtt{F}_6}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_5), \mathtt{F}_6 \vdash \top, \Delta_8}\ \top_R}{- : \bot, \Delta_5 \vdash \top, \Delta_8}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $\to_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5 \vdash \Delta_9, \mathtt{F}_7 \quad \mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\ \to_L}{- : \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}\ \bot_L$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \to \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \quad \mathtt{h}_6 : \bot, \Delta_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}\ \to_L}{- : \bot, \Delta_5 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_9}\ \bot_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\ \wedge_L}{- : \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \bot_L$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_5, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \wedge_L}{- : \bot, \Delta_5 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_9}\ \bot_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vdash \Delta_9 \quad \mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\ \vee_L}{- : \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\ \bot_L$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_5, \mathtt{F}_7 \vdash \Delta_9 \quad \mathtt{h}_6 : \bot, \Delta_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\ \vee_L}{- : \bot, \Delta_5 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_9}\ \bot_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_8, \mathtt{F}_6}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_5), \mathtt{F}_6 \vdash \Delta_8}\ \bot_L}{- : \bot, \Delta_5 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_8}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_7}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ I}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{p}_7}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_9, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_5}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_6 : (\bot, \Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \Delta_8, \mathtt{p}_7}\ I}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ \bot_L$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_7, \top}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_5 \vdash \Delta_7}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \top \vdash \Delta_7}\ \top_L}{- : \bot, \Delta_5 \vdash \Delta_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_7}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_8, \mathtt{F}_5 \vdash \Delta_7}{\bullet \mathtt{h}_6 : (\bot, \top, \Delta_8), \mathtt{F}_5 \vdash \Delta_7}\ \top_L}{- : \bot, \top, \Delta_8 \vdash \Delta_7}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \top, \Delta_8 \vdash \Delta_7}\ \bot_L$$

8.10 Status of I: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_8 \vdash ((\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{p}_8), \mathtt{F}_7}\ I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{F}_{10}, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{p}_8}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_8), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{p}_8}\ \to_R}{- : \Delta_6, \mathtt{p}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{p}_8}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_6, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{p}_8, \mathtt{F}_{10} \to \mathtt{F}_{11}}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{p}_6}\ I \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_9, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \to_R}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_9, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{p}_6}\ I \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_9, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_5, \mathtt{F}_9, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{hCut}$$
$$\dfrac{}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_8 \vdash ((\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_8), \mathtt{F}_7}\ I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{p}_8 \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{p}_8}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_8), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_8}\ \wedge_R}{- : \Delta_6, \mathtt{p}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_8}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_6, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{p}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{p}_6}\ I \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \wedge_R}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9, \mathtt{p}_6}\ I \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{p}_6}\ I \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_8 \vdash ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_8), \mathtt{F}_7}\, I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{p}_8}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_8), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_8}\, \vee_R}{- : \Delta_6, \mathtt{p}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_6, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{p}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{p}_6}\, I \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\, \vee_R}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{p}_6}\, I \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\, \mathtt{hCut}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\, \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_8 \vdash ((\bot, \Delta_{10}), \mathtt{p}_8), \mathtt{F}_7}\, I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{p}_8 \vdash \Delta_{10}, \mathtt{p}_8}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_8), \mathtt{F}_7 \vdash (\bot, \Delta_{10}), \mathtt{p}_8}\, \bot_R}{- : \Delta_6, \mathtt{p}_8 \vdash (\bot, \Delta_{10}), \mathtt{p}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_6, \mathtt{p}_8 \vdash \bot, \Delta_{10}, \mathtt{p}_8}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\bot, \Delta_8), \mathtt{p}_6}\, I \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \bot, \Delta_8}\, \bot_R}{- : \Delta_5, \mathtt{p}_6 \vdash \bot, \Delta_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \bot, \Delta_8, \mathtt{p}_6}\, I \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \bot, \Delta_8}\, \mathtt{ax/W}}{- : \Delta_5, \mathtt{p}_6 \vdash \bot, \Delta_8}\, \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_8 \vdash ((\top, \Delta_{10}), \mathtt{p}_8), \mathtt{F}_7}\, I \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_8), \mathtt{F}_7 \vdash (\top, \Delta_{10}), \mathtt{p}_8}\, \top_R}{- : \Delta_6, \mathtt{p}_8 \vdash (\top, \Delta_{10}), \mathtt{p}_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_6, \mathtt{p}_8 \vdash \top, \Delta_{10}, \mathtt{p}_8}\, \top_R$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\top, \Delta_8), \mathtt{p}_6}\, I \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \top, \Delta_8}\, \top_R}{- : \Delta_5, \mathtt{p}_6 \vdash \top, \Delta_8}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_5, \mathtt{p}_6 \vdash \top, \Delta_8}\, \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{p}_7 \quad \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_{11}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : ((\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7}\, \to_L}{- : (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_{10} \to \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{p}_7 \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, \to_L}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9, \mathtt{F}_7 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : ((\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_9}\, \to_L}{- : (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_5 \vdash \Delta_9, \mathtt{F}_7, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9, \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{p}_5 \vdash \Delta_9, \mathtt{F}_7}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{10}, \mathtt{p}_5, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}\, \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : ((\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7}\, \wedge_L}{- : (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, \wedge_L}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : ((\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_9}\, \wedge_L}{- : (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{10}, \mathtt{p}_5, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\, \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_9 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_{10}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7 \quad \mathtt{h}_9 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_{11}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7}\, \vee_L}{- : (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{12}, \mathtt{p}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_{10} \vee \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{10}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7 \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, \vee_L}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9 \quad \mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : ((\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_9}\, \vee_L}{- : (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7, \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{10}, \mathtt{p}_5, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\, \vee_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \bot}\, I \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \bot \vdash \Delta_8, \mathtt{p}_7}\, \bot_L}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_2 : (\bot, \Delta_{10}), \mathrm{p}_7 \vdash (\Delta_8, \mathrm{p}_7), \mathrm{F}_6}\, I \quad \dfrac{}{\bullet \mathrm{h}_9 : ((\bot, \Delta_{10}), \mathrm{p}_7), \mathrm{F}_6 \vdash \Delta_8, \mathrm{p}_7}\, \bot_L}{- : (\bot, \Delta_{10}), \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_1 : (\bot, \Delta_8), \mathrm{p}_5 \vdash \Delta_7, \mathrm{p}_5}\, I \quad \dfrac{}{\bullet \mathrm{h}_6 : ((\bot, \Delta_8), \mathrm{p}_5), \mathrm{p}_5 \vdash \Delta_7}\, \bot_L}{- : (\bot, \Delta_8), \mathrm{p}_5 \vdash \Delta_7}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_8, \mathrm{p}_5 \vdash \Delta_7}\, \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_2 : \Delta_6, \mathrm{p}_7 \vdash ((\Delta_{10}, \mathrm{p}_9), \mathrm{p}_7), \mathrm{p}_9}\, I \quad \dfrac{}{\bullet \mathrm{h}_8 : (\Delta_6, \mathrm{p}_7), \mathrm{p}_9 \vdash (\Delta_{10}, \mathrm{p}_9), \mathrm{p}_7}\, I}{- : \Delta_6, \mathrm{p}_7 \vdash (\Delta_{10}, \mathrm{p}_9), \mathrm{p}_7}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_6, \mathrm{p}_7 \vdash \Delta_{10}, \mathrm{p}_7, \mathrm{p}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_2 : (\Delta_{11}, \mathrm{p}_9), \mathrm{p}_7 \vdash ((\Delta_{10}, \mathrm{p}_9), \mathrm{p}_7), \mathrm{F}_6}\, I \quad \dfrac{}{\bullet \mathrm{h}_8 : ((\Delta_{11}, \mathrm{p}_9), \mathrm{p}_7), \mathrm{F}_6 \vdash (\Delta_{10}, \mathrm{p}_9), \mathrm{p}_7}\, I}{- : (\Delta_{11}, \mathrm{p}_9), \mathrm{p}_7 \vdash (\Delta_{10}, \mathrm{p}_9), \mathrm{p}_7}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_{11}, \mathrm{p}_7, \mathrm{p}_9 \vdash \Delta_{10}, \mathrm{p}_7, \mathrm{p}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_2 : \Delta_6, \mathrm{p}_9 \vdash (\Delta_{10}, \mathrm{p}_9), \mathrm{F}_7}\, I \quad \dfrac{}{\bullet \mathrm{h}_8 : (\Delta_6, \mathrm{p}_9), \mathrm{F}_7 \vdash \Delta_{10}, \mathrm{p}_9}\, I}{- : \Delta_6, \mathrm{p}_9 \vdash \Delta_{10}, \mathrm{p}_9}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_6, \mathrm{p}_9 \vdash \Delta_{10}, \mathrm{p}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_1 : \Delta_5, \mathrm{p}_7 \vdash (\Delta_8, \mathrm{p}_7), \mathrm{p}_7}\, I \quad \dfrac{}{\bullet \mathrm{h}_6 : (\Delta_5, \mathrm{p}_7), \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, I}{- : \Delta_5, \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_5, \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_1 : (\Delta_9, \mathrm{p}_7), \mathrm{p}_5 \vdash (\Delta_8, \mathrm{p}_7), \mathrm{p}_5}\, I \quad \dfrac{}{\bullet \mathrm{h}_6 : ((\Delta_9, \mathrm{p}_7), \mathrm{p}_5), \mathrm{p}_5 \vdash \Delta_8, \mathrm{p}_7}\, I}{- : (\Delta_9, \mathrm{p}_7), \mathrm{p}_5 \vdash \Delta_8, \mathrm{p}_7}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_9, \mathrm{p}_5, \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_2 : \Delta_6, \mathrm{p}_7 \vdash (\Delta_8, \mathrm{p}_7), \top}\, I \quad \dfrac{\mathrm{h}_9 : \Delta_6, \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}{\bullet \mathrm{h}_9 : (\Delta_6, \mathrm{p}_7), \top \vdash \Delta_8, \mathrm{p}_7}\, \top_L}{- : \Delta_6, \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \Delta_6, \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_2 : (\top, \Delta_{10}), \mathrm{p}_7 \vdash (\Delta_8, \mathrm{p}_7), \mathrm{F}_6}\, I \quad \dfrac{\mathrm{h}_9 : \Delta_{10}, \mathrm{F}_6, \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}{\bullet \mathrm{h}_9 : ((\top, \Delta_{10}), \mathrm{p}_7), \mathrm{F}_6 \vdash \Delta_8, \mathrm{p}_7}\, \top_L}{- : (\top, \Delta_{10}), \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \top, \Delta_{10}, \mathrm{p}_7 \vdash \Delta_8, \mathrm{p}_7}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathrm{h}_1 : (\top, \Delta_8), \mathrm{p}_5 \vdash \Delta_7, \mathrm{p}_5}\, I \quad \dfrac{\mathrm{h}_6 : \Delta_8, \mathrm{p}_5, \mathrm{p}_5 \vdash \Delta_7}{\bullet \mathrm{h}_6 : ((\top, \Delta_8), \mathrm{p}_5), \mathrm{p}_5 \vdash \Delta_7}\, \top_L}{- : (\top, \Delta_8), \mathrm{p}_5 \vdash \Delta_7}\, \mathtt{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{\dfrac{}{\bullet \mathrm{h}_1 : \top, \Delta_8, \mathrm{p}_5 \vdash \Delta_7, \mathrm{p}_5}\, I \quad \dfrac{}{\mathrm{h}_6 : \top, \Delta_8, \mathrm{p}_5, \mathrm{p}_5 \vdash \Delta_7}\, \mathtt{ax/W}}{- : \top, \Delta_8, \mathrm{p}_5 \vdash \Delta_7}\, \mathtt{hCut}$$

8.11 Status of $\top_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6}\ \top_L \quad \dfrac{\mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\top, \Delta_5), \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \to_R}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \to \mathtt{F}_{10}}{}\ \mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}{}\ \mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6}\ \top_L \quad \dfrac{\mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\top, \Delta_5), \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \wedge_R}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{}\ \mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{}\ \mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6}\ \top_L \quad \dfrac{\mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9, \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\top, \Delta_5), \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_9 \vee \mathtt{F}_{10}}{}\ \mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}{}\ \mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash (\bot, \Delta_8), \mathtt{F}_6}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash (\bot, \Delta_8), \mathtt{F}_6}\ \top_L \quad \dfrac{\mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : (\top, \Delta_5), \mathtt{F}_6 \vdash \bot, \Delta_8}\ \bot_R}{- : \top, \Delta_5 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \bot, \Delta_8, \mathtt{F}_6}{}\ \mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6 \vdash \bot, \Delta_8}{}\ \mathtt{ax/W}}{- : \top, \Delta_5 \vdash \bot, \Delta_8}\ \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash (\top, \Delta_8), \mathtt{F}_6}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash (\top, \Delta_8), \mathtt{F}_6}\ \top_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\top, \Delta_5), \mathtt{F}_6 \vdash \top, \Delta_8}\ \top_R}{- : \top, \Delta_5 \vdash \top, \Delta_8}\ \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{}{- : \top, \Delta_5 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}{\bullet \mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5 \vdash \Delta_9, \mathtt{F}_7 \quad \mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\top, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\ \to_L}{- : \top, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}{}\ \mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}{}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \to \mathtt{F}_8}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \to \mathtt{F}_8}\ \top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \quad \mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\top, \Delta_5), \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}\ \to_L}{- : \top, \Delta_5 \vdash \Delta_9}\ \mathtt{Cut}}{\rightsquigarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \to \mathtt{F}_8}{}\ \mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}{}\ \mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_9}\ \mathtt{hCut}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}{\bullet\mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\,\wedge_L}{- : \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}{}\,\mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8}\,\top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\top, \Delta_5), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\,\wedge_L}{- : \top, \Delta_5 \vdash \Delta_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}{}\,\mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_9}\,\mathtt{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}{\bullet\mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vdash \Delta_9 \quad \mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\,\vee_L}{- : \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}{}\,\mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8}\,\top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_7 \vdash \Delta_9 \quad \mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\top, \Delta_5), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\,\vee_L}{- : \top, \Delta_5 \vdash \Delta_9}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}{}\,\mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_9}\,\mathtt{hCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_7, \bot}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_7, \bot}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_6 : (\top, \Delta_5), \bot \vdash \Delta_7}\,\bot_L}{- : \top, \Delta_5 \vdash \Delta_7}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_5 \vdash \bot, \Delta_7}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \bot, \top, \Delta_5 \vdash \Delta_7}\,\bot_L}{- : \top, \Delta_5 \vdash \Delta_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \bot, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}{\bullet\mathtt{h}_3 : \top, \bot, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_6 : (\top, \bot, \Delta_8), \mathtt{F}_5 \vdash \Delta_7}\,\bot_L}{- : \top, \bot, \Delta_8 \vdash \Delta_7}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \bot, \top, \Delta_8 \vdash \Delta_7}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_7}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_7}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_6 : (\top, \Delta_5), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\,I}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{p}_7}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_8, \mathtt{p}_7, \mathtt{p}_7}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_5, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{}\,I}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{p}_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_9, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_5}{\bullet\mathtt{h}_3 : \top, \Delta_9, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_5}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_6 : (\top, \Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \Delta_8, \mathtt{p}_7}\,I}{- : \top, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\,\mathtt{Cut}}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \top, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \Delta_8, \mathtt{F}_6}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_6}\,\top_L \qquad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\top, \Delta_5), \mathtt{F}_6 \vdash \Delta_8}\,\top_L}{- : \top, \Delta_5 \vdash \Delta_8}\,\mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_8, \mathtt{F}_6}\,\mathtt{ax/W} \qquad \dfrac{}{\bullet\mathtt{h}_7 : \top, \Delta_5, \mathtt{F}_6 \vdash \Delta_8}\,\mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_8}\,\mathtt{hCut}$$

9 Admissibility of $A \to A \wedge A$

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \to \mathtt{F}_5), \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\to_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \to \mathtt{F}_5}\ {\to_R}$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_1 \quad \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5), \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\wedge_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\wedge_R}$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_4}\ {\wedge_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ {\scriptstyle height}$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5), \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\vee_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\vee_R}$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_1}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\bot, \Delta_4), \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\bot_R} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_4, \mathtt{F}_1}\ {\bot_R}$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\top, \Delta_4), \mathtt{F}_1 \wedge \mathtt{F}_1}\ {\top_R} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_4, \mathtt{F}_1}\ {\top_R}$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_2 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ {\to_L} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_2 \wedge \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_5}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ {\to_L}$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ {\wedge_L} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ {\wedge_L}$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ {\vee_L} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \mathtt{F}_2}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ {\vee_L}$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ {\bot_L} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \Delta_1, \mathtt{F}_2}\ {\bot_L}$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash (\Delta_5, \mathtt{p}_4), \mathtt{F}_1 \wedge \mathtt{F}_1}\ {I} \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{p}_4}\ {I}$$

- Case(s) rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \top_L}{\vdots} \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \Delta_1, \mathtt{F}_2}\ \top_L$$

10 Admissibility of $A \wedge A \to A$

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \to \mathtt{F}_5), \mathtt{F}_1} \to_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_4 \to \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1} \to_R$$

$$\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5} \to_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5} \to_R \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5} \to_R}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, (\mathtt{F}_4 \to \mathtt{F}_5) \wedge (\mathtt{F}_4 \to \mathtt{F}_5)} \wedge_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5), \mathtt{F}_1} \wedge_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_1}\ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax} \quad \overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax} \quad \overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, (\mathtt{F}_4 \wedge \mathtt{F}_5) \wedge (\mathtt{F}_4 \wedge \mathtt{F}_5)} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5), \mathtt{F}_1} \vee_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \wedge \mathtt{F}_1}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, (\mathtt{F}_4 \vee \mathtt{F}_5) \wedge (\mathtt{F}_4 \vee \mathtt{F}_5)} \wedge_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \bot} \bot_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \bot \wedge \bot} \wedge_R$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\bot, \Delta_4), \mathtt{F}_1} \bot_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1}\ \mathtt{ax}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_1}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_1} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \top} \top_R \quad \rightsquigarrow \quad \dfrac{\overline{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R \quad \overline{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \top \wedge \top} \wedge_R$$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\top, \Delta_4), \mathtt{F}_1} \top_R \quad \rightsquigarrow \quad \frac{}{\bullet\bullet \mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_4, \mathtt{F}_1 \wedge \mathtt{F}_1} \top_R$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2} \to_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2} \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \wedge_L \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \vee_L$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \Delta_1, \mathtt{F}_2}\ \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet \bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash (\Delta_5, \mathtt{p}_4), \mathtt{F}_1}\ I \quad \rightsquigarrow \quad \frac{}{\bullet \bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash \Delta_5, \mathtt{p}_4, \mathtt{F}_1 \wedge \mathtt{F}_1}\ I$$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I \quad \rightsquigarrow \quad \frac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I \quad \dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I}{\bullet \bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3 \wedge \mathtt{p}_3}\ \wedge_R$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \Delta_1, \mathtt{F}_2}\ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_2}\ \top_L$$

Note: This rule does not preserve the height of the derivation but it produces proofs of at most $s(n)$ steps. See the specification in `prop-and-A.maude`.