# System for Linear Logic

July 6, 2022

## Contents

# 1 Measure of derivations

- Case(s) rule **1**

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \rightarrow \quad \frac{\dfrac{}{\bullet\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1}}{\bullet\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathtt{IH}$$

- Case(s) rule !

$$\frac{\mathtt{h}_1 :\vdash \mathtt{F}_3, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_3}\ ! \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash ?\Upsilon 2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_3}\ !$$

- Case(s) rule $?W$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?W \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?W$$

- Case(s) rule $?C$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?C \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?C$$

- Case(s) rule $?$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?$$

- Case(s) rule $\$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$ \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$$

- Case(s) rule $\&$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4}\ \&$$

- Case(s) rule $\oplus_B$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B$$

- Case(s) rule $\oplus_A$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A$$

- Case(s) rule $\bot$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet \mathtt{h}_1 :\vdash \bot, \Delta_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 :\vdash \Delta_2, \bot}\ \bot$$

- Case(s) rule $\top$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \rightarrow \quad \frac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \top}{\bullet \bullet \mathtt{h}_1 :\vdash \Delta_2, \top}$$

- Case(s) rule $I$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I \quad \rightarrow \quad \frac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I}{\bullet \bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}$$

- Case(s) rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_5}\ \mathtt{IH}}{\bullet \bullet \mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

## 2 Weakening on bang: $\vdash \Gamma, !F$ implies $\vdash \Gamma, F$.

- Case(s) rule **1**
- Case(s) rule !

$$\frac{\mathtt{h}_1 :\vdash \mathtt{F}_3, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_3, ?\Upsilon 2}\ ! \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash ?\Upsilon 2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, \mathtt{F}_3}\ \mathtt{H}$$

- Case(s) rule $?W$

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ ?W$$

- Case(s) rule $?C$

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, !\mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \Delta_4, ?\mathtt{F}_3}\ ?C \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, !\mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ ?C$$

- Case(s) rule ?

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \Delta_4, ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ ?$$

- Case(s) rule \$

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$ \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$$

- Case(s) rule &

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \& \mathtt{F}_4}\ \&$$

- Case(s) rule $\oplus_B$

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \mathtt{IH}}{\bullet \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B$$

- Case(s) rule $\oplus_A$

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{IH}}{\bullet \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A$$

- Case(s) rule $\bot$

$$\frac{\mathtt{h}_2 :\vdash \Delta_3, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \bot, \Delta_3}\ \bot \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_{\mathbf{1}}}\ \mathtt{IH}}{\bullet \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_{\mathbf{1}}, \bot}\ \bot$$

- Case(s) rule $\top$

$$\frac{}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \top, \Delta_3}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_{\mathbf{1}}, \top}\ \top$$

- Case(s) rule $I$
- Case(s) rule $\otimes$

$$\frac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_4, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \mathtt{IH} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\frac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_{\mathbf{1}}, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_5, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \mathtt{IH}}{\bullet \mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

# 3 Invertibility of Rules

## 3.1 Status of 1: : `Invertible`

- Case rule **1**

$$\frac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$

### 3.2 Status of !: : `Invertible`

- Case rule **1**
- Case rule !

$$\frac{\mathtt{h}_1 :\vdash \mathtt{F}_3, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_3}\ ! \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash ?\Upsilon 2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $?W$

$$\frac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash (?\Upsilon 4, !\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_3}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash ?\Upsilon 4, \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ ?W$$

- Case rule $?C$

$$\frac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet \mathtt{h}_2 :\vdash (?\Upsilon 4, !\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_3}\ ?C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash ?\Upsilon 4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ ?C$$

- Case rule ?

$$\frac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash (?\Upsilon 4, !\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash ?\Upsilon 4, \mathtt{F}_3, !\mathtt{F}_{\mathbf{1}}}\ \mathtt{ax}}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ ?}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ W!$$

- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$

### 3.3 Status of $?W$: : `Non invertible`

- Case rule **1**

- Case rule !

$$\frac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash (?\Upsilon 4, ?\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3}\ ! \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash ?\Upsilon 4, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_3}\ !$$

- Case rule $?W$

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash (\Delta_4, ?\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_3}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_3}\ ?W$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{H}$$

- Case rule $?C$

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash (\Delta_4, ?\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_3}\ ?C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_3}\ ?C$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?C \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{fail}$$

- Case rule ?

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash (\Delta_4, ?\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_3}\ ?$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{fail}$$

- Case rule \$

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash (\Delta_5, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \$ \mathtt{F}_4}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$$

- Case rule &

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, ?\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash (\Delta_5, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \& \mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash (\Delta_5, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, ?\mathtt{F}_\mathbf{1}}{\bullet\mathtt{h}_2 :\vdash (\Delta_5, ?\mathtt{F}_\mathbf{1}), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_2 :\vdash \Delta_3, ?\mathtt{F}_\mathbf{1}}{\bullet\mathtt{h}_2 :\vdash \bot, \Delta_3, ?\mathtt{F}_\mathbf{1}}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_2 :\vdash \top, \Delta_3, ?\mathtt{F}_\mathbf{1}}\ \top \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_4, ?\mathtt{F}_\mathbf{1} \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash (\Delta_6, ?\mathtt{F}_\mathbf{1}), \Delta_3, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}\ \text{ax}}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\frac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_5, ?\mathtt{F}_\mathbf{1}}{\bullet\mathtt{h}_2 :\vdash \Delta_3, (\Delta_6, ?\mathtt{F}_\mathbf{1}), \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}\ \text{ax} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

## 3.4 Status of $?C$: : `Non invertible`

- Case rule $\mathbf{1}$

- Case rule !

$$\frac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, ?\mathtt{F}_\mathbf{1}}{\bullet\mathtt{h}_2 :\vdash (?\Upsilon 4, ?\mathtt{F}_\mathbf{1}), !\mathtt{F}_3}\ ! \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash ?\Upsilon 4, \mathtt{F}_3, ?\mathtt{F}_\mathbf{1}, ?\mathtt{F}_\mathbf{1}}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_3, ?\mathtt{F}_\mathbf{1}, ?\mathtt{F}_\mathbf{1}}\ !$$

- Case rule $?W$

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_\mathbf{1}}{\bullet\mathtt{h}_2 :\vdash (\Delta_4, ?\mathtt{F}_\mathbf{1}), ?\mathtt{F}_3}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_\mathbf{1}, ?\mathtt{F}_\mathbf{1}}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_\mathbf{1}, ?\mathtt{F}_\mathbf{1}, ?\mathtt{F}_3}\ ?W$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?W \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \texttt{fail}$$

- Case rule $?C$

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_\mathbf{1}, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash (\Delta_4, ?\mathtt{F}_\mathbf{1}), ?\mathtt{F}_3}\ ?C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_\mathbf{1}, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \text{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_\mathbf{1}, ?\mathtt{F}_\mathbf{1}, ?\mathtt{F}_3}\ ?C$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \text{ax/ind}}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3}\ ?C$$

- Case rule ?

$$\frac{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F}_3, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash (\Delta_4, ?\mathtt{F_1}), ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_4, \mathtt{F}_3, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \text{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_4, ?\mathtt{F_1}, ?\mathtt{F_1}, ?\mathtt{F}_3}\ ?$$

$$\frac{\mathtt{h_1} :\vdash \Delta_2, \mathtt{F}_3}{\bullet\mathtt{h_1} :\vdash \Delta_2, ?\mathtt{F}_3}\ ? \quad \rightarrow \quad \frac{}{\bullet\mathtt{h_1} :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \text{fail}$$

- Case rule $

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash (\Delta_5, ?\mathtt{F_1}), \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \text{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_5, ?\mathtt{F_1}, ?\mathtt{F_1}, \mathtt{F}_3\$\mathtt{F}_4}\ \$$

- Case rule &

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_3, ?\mathtt{F_1} \quad \mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_4, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash (\Delta_5, ?\mathtt{F_1}), \mathtt{F}_3\&\mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_3, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_4, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \text{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_5, ?\mathtt{F_1}, ?\mathtt{F_1}, \mathtt{F}_3\&\mathtt{F}_4}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_4, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash (\Delta_5, ?\mathtt{F_1}), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_4, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \text{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_5, ?\mathtt{F_1}, ?\mathtt{F_1}, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_3, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash (\Delta_5, ?\mathtt{F_1}), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_5, \mathtt{F}_3, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \text{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_5, ?\mathtt{F_1}, ?\mathtt{F_1}, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h_2} :\vdash \Delta_3, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash \bot, \Delta_3, ?\mathtt{F_1}}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_3, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \text{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_3, \bot, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h_2} :\vdash \top, \Delta_3, ?\mathtt{F_1}}\ \top \quad \rightarrow \quad \frac{}{\bullet\mathtt{h_2} :\vdash \Delta_3, \top, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F}_4, ?\mathtt{F_1} \quad \mathtt{h_2} :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h_2} :\vdash (\Delta_6, ?\mathtt{F_1}), \Delta_3, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F}_4, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h_2} :\vdash \Delta_3, \mathtt{F}_5}\ \text{ax}}{\bullet\mathtt{h_2} :\vdash \Delta_3, \Delta_6, ?\mathtt{F_1}, ?\mathtt{F_1}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\frac{\mathtt{h_2} :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h_2} :\vdash \Delta_6, \mathtt{F}_5, ?\mathtt{F_1}}{\bullet\mathtt{h_2} :\vdash \Delta_3, (\Delta_6, ?\mathtt{F_1}), \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h_2} :\vdash \Delta_3, \mathtt{F}_4}\ \text{ax} \quad \dfrac{}{\mathtt{h_2} :\vdash \Delta_6, \mathtt{F}_5, ?\mathtt{F_1}, ?\mathtt{F_1}}\ \text{ax/ind}}{\bullet\mathtt{h_2} :\vdash \Delta_3, \Delta_6, ?\mathtt{F_1}, ?\mathtt{F_1}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

### 3.5 Status of ?: : Non invertible

- Case rule **1**

- Case rule !

$$\frac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, ?\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash (?\Upsilon 4, ?\mathtt{F}_{\mathbf{1}}), !\mathtt{F}_3}\ !\quad\rightarrow\quad \frac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_{\mathbf{1}}, ?\Upsilon 4, !\mathtt{F}_3}\ \mathtt{fail}$$

- Case rule $?W$

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash (\Delta_4, ?\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_3}\ ?W\quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ ?W$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?W\quad\rightarrow\quad \frac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{fail}$$

- Case rule $?C$

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, ?\mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet \mathtt{h}_2 :\vdash (\Delta_4, ?\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_3}\ ?C\quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ ?C$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?C\quad\rightarrow\quad \frac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{fail}$$

- Case rule ?

$$\frac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, ?\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash (\Delta_4, ?\mathtt{F}_{\mathbf{1}}), ?\mathtt{F}_3}\ ?\quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_3}\ ?$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_3}\ ?\quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{H}$$

- Case rule \$

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, ?\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash (\Delta_5, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \$ \mathtt{F}_4}\ \$\quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$$

- Case rule &

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, ?\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, ?\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash (\Delta_5, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \& \mathtt{F}_4}\ \&\quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \& \mathtt{F}_4}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, ?\mathtt{F}_{\mathbf{1}}}{\bullet \mathtt{h}_2 :\vdash (\Delta_5, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B\quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash (\Delta_5, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_2 :\vdash \Delta_3, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash \bot, \Delta_3, ?\mathtt{F}_{\mathbf{1}}}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_{\mathbf{1}}, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_2 :\vdash \top, \Delta_3, ?\mathtt{F}_{\mathbf{1}}}\ \top \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_{\mathbf{1}}, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_4, ?\mathtt{F}_{\mathbf{1}} \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash (\Delta_6, ?\mathtt{F}_{\mathbf{1}}), \Delta_3, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\frac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_5, ?\mathtt{F}_{\mathbf{1}}}{\bullet\mathtt{h}_2 :\vdash \Delta_3, (\Delta_6, ?\mathtt{F}_{\mathbf{1}}), \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

### 3.6 Status of \$: : `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?$W$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2), ?\mathtt{F}_4}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, ?\mathtt{F}_4}\ ?W$$

- Case rule ?$C$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2), ?\mathtt{F}_4}\ ?C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, ?\mathtt{F}_4, ?\mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, ?\mathtt{F}_4}\ ?C$$

- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}}\$\mathtt{F}_2), ?\mathtt{F}_4}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_2, ?\mathtt{F}_4}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1\,\$\,\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1\,\$\,\mathtt{F}_2), \mathtt{F}_4\,\$\,\mathtt{F}_5}\ \$ \quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4\,\$\,\mathtt{F}_5}\ \$$$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3\,\$\,\mathtt{F}_4}\ \$ \quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_4}\ \text{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{H}$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1\,\$\,\mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1\,\$\,\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1\,\$\,\mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5}\ \& \quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \& \mathtt{F}_5}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1\,\$\,\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1\,\$\,\mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B \quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1\,\$\,\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1\,\$\,\mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A \quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_1\,\$\,\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \bot, \Delta_4, \mathtt{F}_1\,\$\,\mathtt{F}_2}\ \bot \quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_3 :\vdash \top, \Delta_4, \mathtt{F}_1\,\$\,\mathtt{F}_2}\ \top \quad\rightarrow\quad \frac{}{\bullet\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_2, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1\,\$\,\mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}{\bullet\mathtt{h}_3 :\vdash (\Delta_7, \mathtt{F}_1\,\$\,\mathtt{F}_2), \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}\ \text{ax}}{\bullet\mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5 \quad \mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1\,\$\,\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \Delta_4, (\Delta_7, \mathtt{F}_1\,\$\,\mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad\rightarrow\quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5}\ \text{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}\ \text{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

### 3.7 Status of &: (Left Premise): `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?$W$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), ?\mathtt{F}_4}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_4}\ ?W$$

- Case rule ?$C$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), ?\mathtt{F}_4}\ ?C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_4, ?\mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_4}\ ?C$$

- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), ?\mathtt{F}_4}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_4}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_4\$\mathtt{F}_5}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4\$\mathtt{F}_5}\ \$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_4\&\mathtt{F}_5}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4\&\mathtt{F}_5}\ \&$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3\&\mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}{\bullet\mathtt{h}_3 :\vdash \bot, \Delta_4, \mathtt{F}_{\mathbf{1}}\&\mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_3 :\vdash \top, \Delta_4, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}{\bullet \mathtt{h}_3 :\vdash (\Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ {\scriptstyle ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}\ {\scriptstyle ax}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5 \quad \mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \Delta_4, (\Delta_7, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5}\ {\scriptstyle ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6}\ {\scriptstyle ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

## 3.8 Status of & (Right Premise): : `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?$W$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), ?\mathtt{F}_4}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2}\ {\scriptstyle ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, ?\mathtt{F}_4}\ ?W$$

- Case rule ?$C$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), ?\mathtt{F}_4}\ ?C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, ?\mathtt{F}_4, ?\mathtt{F}_4}\ {\scriptstyle ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, ?\mathtt{F}_4}\ ?C$$

- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), ?\mathtt{F}_4}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_4}\ {\scriptstyle ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, ?\mathtt{F}_4}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), \mathtt{F}_4 \$ \mathtt{F}_5}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \$ \mathtt{F}_5}\ \$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}} \& \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}\ {\scriptstyle ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_5}\ {\scriptstyle ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \& \mathtt{F}_5}\ \&$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \& \mathtt{F}_4}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ {\scriptstyle ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ {\scriptstyle \mathtt{H}}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \oplus_B \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_5}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5} \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5} \oplus_A \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5} \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \bot, \Delta_4, \mathtt{F}_1 \& \mathtt{F}_2} \bot \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_2}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_2, \bot} \bot$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_3 :\vdash \top, \Delta_4, \mathtt{F}_1 \& \mathtt{F}_2} \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_2, \top} \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \& \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}{\bullet \mathtt{h}_3 :\vdash (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6} \otimes \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6} \otimes$$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5 \quad \mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \& \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \Delta_4, (\Delta_7, \mathtt{F}_1 \& \mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6} \otimes \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5}^{\,\mathtt{ax}} \quad \overline{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6} \otimes$$

## 3.9 Status of $\oplus_B$: : `Non invertible`

- Case rule **1**

- Case rule !

- Case rule ?$W$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_1 \oplus \mathtt{F}_2), ?\mathtt{F}_4} ?W \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, ?\mathtt{F}_4} ?W$$

- Case rule ?$C$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_1 \oplus \mathtt{F}_2), ?\mathtt{F}_4} ?C \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, ?\mathtt{F}_4, ?\mathtt{F}_4}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, ?\mathtt{F}_4} ?C$$

- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_1 \oplus \mathtt{F}_2), ?\mathtt{F}_4}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_2, ?\mathtt{F}_4}\ ?$$

- Case rule $

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_4 \$ \mathtt{F}_5}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \$ \mathtt{F}_5}\ \$$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \oplus \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \& \mathtt{F}_5}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \text{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \text{H}$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_2, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \text{fail}$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \bot, \Delta_4, \mathtt{F}_1 \oplus \mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_2}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_2, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_3 :\vdash \top, \Delta_4, \mathtt{F}_1 \oplus \mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_2, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_1 \oplus \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}{\bullet \mathtt{h}_3 :\vdash (\Delta_7, \mathtt{F}_1 \oplus \mathtt{F}_2), \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}\ \text{ax}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5 \quad \mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_1 \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \Delta_4, (\Delta_7, \mathtt{F}_1 \oplus \mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5}\ \text{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

### 3.10 Status of $\oplus_A$: : `Non invertible`

- Case rule **1**

- Case rule !

- Case rule ?$W$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), ?\mathtt{F}_4}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_4}\ ?W$$

- Case rule ?$C$

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), ?\mathtt{F}_4}\ ?C \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_4, ?\mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_4}\ ?C$$

- Case rule ?

$$\frac{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), ?\mathtt{F}_4}\ ? \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_5, \mathtt{F}_{\mathbf{1}}, ?\mathtt{F}_4}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_4 \$ \mathtt{F}_5}\ \$ \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \$ \mathtt{F}_5}\ \$$$

- Case rule &

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_4 \& \mathtt{F}_5}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \& \mathtt{F}_5}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \texttt{fail}$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4}\ \text{ax/ind}}{\bullet \mathtt{h}_3 :\vdash \Delta_6, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \text{ax}}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\bot$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \bot, \Delta_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \bot \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_3 :\vdash \top, \Delta_4, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_{\mathbf{1}}, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}{\bullet \mathtt{h}_3 :\vdash (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \Delta_4, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5}\ ^{\mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}\ ^{\mathtt{ax}}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\frac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5 \quad \mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_6, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash \Delta_4, (\Delta_7, \mathtt{F}_{\mathbf{1}} \oplus \mathtt{F}_2), \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5}\ ^{\mathtt{ax}} \quad \overline{\mathtt{h}_3 :\vdash \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_6}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 :\vdash \Delta_4, \Delta_7, \mathtt{F}_{\mathbf{1}}, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

## 3.11 Status of $\bot$: : `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?$W$

$$\frac{\mathtt{h}_1 :\vdash \bot, \Delta_3}{\bullet \mathtt{h}_1 :\vdash (\bot, \Delta_3), ?\mathtt{F}_2}\ ?W \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 :\vdash \Delta_3}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2}\ ?W$$

- Case rule ?$C$

$$\frac{\mathtt{h}_1 :\vdash \bot, \Delta_3, ?\mathtt{F}_2, ?\mathtt{F}_2}{\bullet \mathtt{h}_1 :\vdash (\bot, \Delta_3), ?\mathtt{F}_2}\ ?C \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2, ?\mathtt{F}_2}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2}\ ?C$$

- Case rule ?

$$\frac{\mathtt{h}_1 :\vdash \bot, \Delta_3, \mathtt{F}_2}{\bullet \mathtt{h}_1 :\vdash (\bot, \Delta_3), ?\mathtt{F}_2}\ ? \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_2}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 :\vdash \Delta_3, ?\mathtt{F}_2}\ ?$$

- Case rule \$

$$\frac{\mathtt{h}_1 :\vdash \bot, \Delta_4, \mathtt{F}_2, \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash (\bot, \Delta_4), \mathtt{F}_2 \$ \mathtt{F}_3}\ \$ \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2, \mathtt{F}_3}\ ^{\mathtt{ax/ind}}}{\bullet \mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2 \$ \mathtt{F}_3}\ \$$

- Case rule &

$$\frac{\mathtt{h}_1 :\vdash \bot, \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 :\vdash \bot, \Delta_4, \mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash (\bot, \Delta_4), \mathtt{F}_2 \& \mathtt{F}_3}\ \& \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2 \& \mathtt{F}_3}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_1 :\vdash \bot, \Delta_4, \mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash (\bot, \Delta_4), \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_B \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_1 :\vdash \bot, \Delta_4, \mathtt{F}_2}{\bullet\mathtt{h}_1 :\vdash (\bot, \Delta_4), \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_A \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_1 :\vdash \Delta_4, \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_2}\ \mathtt{H}$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \top, \bot, \Delta_2}\ \top \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \bot, \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash (\bot, \Delta_5), \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_5, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}\ \mathtt{ax}}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \Delta_5, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \quad \mathtt{h}_1 :\vdash \bot, \Delta_5, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \Delta_2, (\bot, \Delta_5), \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_1 :\vdash \Delta_5, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \Delta_5, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes$$

## 3.12 Status of $\top$: : `Invertible`

- Case rule **1**

- Case rule !

- Case rule ?$W$

$$\frac{\mathtt{h}_1 :\vdash \top, \Delta_3}{\bullet\mathtt{h}_1 :\vdash (\top, \Delta_3), ?\mathtt{F}_2}\ ?W \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $?C$

$$\frac{\mathtt{h}_1 :\vdash \top, \Delta_3, ?\mathtt{F}_2, ?\mathtt{F}_2}{\bullet\mathtt{h}_1 :\vdash (\top, \Delta_3), ?\mathtt{F}_2}\ ?C \quad \rightarrow \quad \texttt{trivial}$$

- Case rule ?

$$\frac{\mathtt{h}_1 :\vdash \top, \Delta_3, \mathtt{F}_2}{\bullet\mathtt{h}_1 :\vdash (\top, \Delta_3), ?\mathtt{F}_2}\ ? \quad \rightarrow \quad \texttt{trivial}$$

- Case rule \$

$$\frac{\mathtt{h}_1 :\vdash \top, \Delta_4, \mathtt{F}_2, \mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash (\top, \Delta_4), \mathtt{F}_2 \$ \mathtt{F}_3}\ \$ \quad \rightarrow \quad \texttt{trivial}$$

- Case rule &

$$\frac{\mathtt{h}_1 :\vdash \top, \Delta_4, \mathtt{F}_2 \quad \mathtt{h}_1 :\vdash \top, \Delta_4, \mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash (\top, \Delta_4), \mathtt{F}_2 \& \mathtt{F}_3}\ \& \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_1 :\vdash \top, \Delta_4, \mathtt{F}_3}{\bullet\mathtt{h}_1 :\vdash (\top, \Delta_4), \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_B \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_1 :\vdash \top, \Delta_4, \mathtt{F}_2}{\bullet\mathtt{h}_1 :\vdash (\top, \Delta_4), \mathtt{F}_2 \oplus \mathtt{F}_3}\ \oplus_A \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot$

$$\frac{\mathtt{h}_1 :\vdash \top, \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \top, \Delta_2}\ \bot \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top$

$$\frac{}{\bullet\mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_1 :\vdash \top, \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash (\top, \Delta_5), \Delta_2, \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \texttt{trivial}$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_3 \quad \mathtt{h}_1 :\vdash \top, \Delta_5, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \Delta_2, (\top, \Delta_5), \mathtt{F}_3 \otimes \mathtt{F}_4}\ \otimes \quad \rightarrow \quad \texttt{trivial}$$

### 3.13 Status of $I$: : `Invertible`

- Case rule **1**
- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$

$$\frac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_2), \hat{}(\mathtt{n}_2)}\ I \quad \rightarrow \quad \mathtt{trivial}$$

- Case rule $\otimes$

### 3.14 Status of $\otimes$: (Left Premise): `Non invertible`

- Case rule **1**
- Case rule !
- Case rule ?$W$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_2, ?\mathtt{F}_5}\ ?W$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ?W \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $?C$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ?C \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_2, ?\mathtt{F}_5}\ ?W$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ?C \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{H}$$

- Case rule ?

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_2, ?\mathtt{F}_5}\ ?W$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{H}$$

- Case rule \$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{H}$$

- Case rule &

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3 \quad \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}^{\ \mathrm{ax/ind}} \quad \overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \& \mathtt{F}_6}\ \&$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3 \quad \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

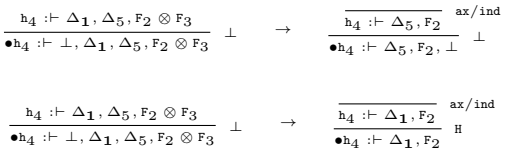
$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\oplus_A$

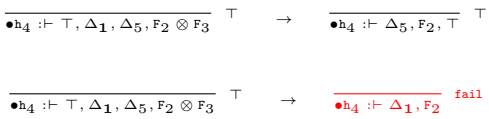
$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_2}\ \mathtt{H}$$

- Case rule $\bot$

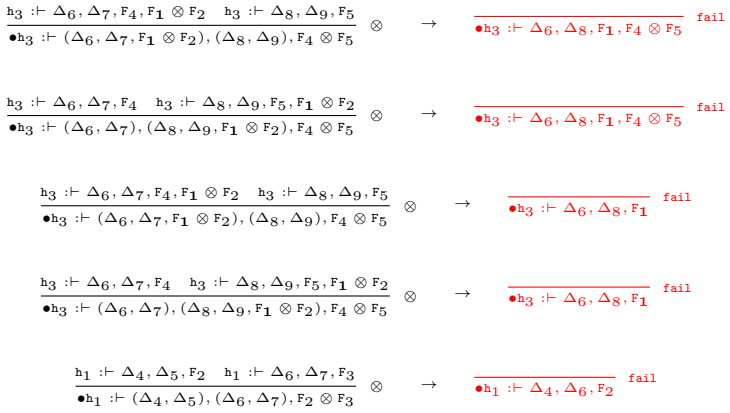


- Case rule $\top$



- Case rule $I$

- Case rule $\otimes$

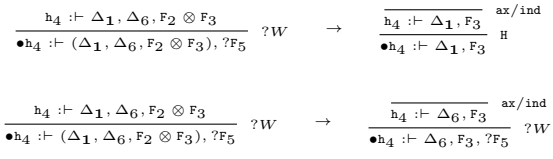


## 3.15 Status of $\otimes$ (Right Premise): : `Non invertible`

- Case rule $\mathbf{1}$

- Case rule !

- Case rule $?W$



- Case rule $?C$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ?C \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ?C \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_3}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_3, ?\mathtt{F}_5}\ ?W$$

- Case rule ?

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_6, \mathtt{F}_2 \otimes \mathtt{F}_3), ?\mathtt{F}_5}\ ? \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_3}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_3, ?\mathtt{F}_5}\ ?W$$

- Case rule \$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$ \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_3, \mathtt{F}_5, \mathtt{F}_6}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_3, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$$

- Case rule &

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3 \quad \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3 \quad \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \& \mathtt{F}_6}\ \& \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_3, \mathtt{F}_5}^{\ \mathrm{ax/ind}} \quad \overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_3, \mathtt{F}_6}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_3, \mathtt{F}_5 \& \mathtt{F}_6}\ \&$$

- Case rule $\oplus_B$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_6, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_3, \mathtt{F}_6}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_3, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

- Case rule $\oplus_A$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash (\Delta_{\mathbf{1}}, \Delta_7, \mathtt{F}_2 \otimes \mathtt{F}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_3, \mathtt{F}_5}^{\ \mathrm{ax/ind}}}{\bullet \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_3, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

- Case rule $\bot$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash \bot, \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}{\bullet \mathtt{h}_4 :\vdash \bot, \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \bot \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 :\vdash \Delta_5, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 :\vdash \Delta_5, \mathtt{F}_3, \bot}\ \bot$$

- Case rule $\top$

$$\frac{}{\bullet \mathtt{h}_4 :\vdash \top, \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 :\vdash \Delta_{\mathbf{1}}, \mathtt{F}_3}\ \mathtt{fail}$$

$$\frac{}{\bullet \mathtt{h}_4 :\vdash \top, \Delta_{\mathbf{1}}, \Delta_5, \mathtt{F}_2 \otimes \mathtt{F}_3}\ \top \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 :\vdash \Delta_5, \mathtt{F}_3, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \Delta_7, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_8, \Delta_9, \mathtt{F}_5}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), (\Delta_8, \Delta_9), \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \Delta_7, \Delta_9, \mathtt{F}_2}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \Delta_7, \mathtt{F}_4 \quad \mathtt{h}_3 :\vdash \Delta_8, \Delta_9, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \Delta_7, \Delta_9, \mathtt{F}_2}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \Delta_7, \mathtt{F}_4, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2 \quad \mathtt{h}_3 :\vdash \Delta_8, \Delta_9, \mathtt{F}_5}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \Delta_7, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), (\Delta_8, \Delta_9), \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \Delta_7, \Delta_9, \mathtt{F}_2, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_3 :\vdash \Delta_6, \Delta_7, \mathtt{F}_4 \quad \mathtt{h}_3 :\vdash \Delta_8, \Delta_9, \mathtt{F}_5, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2}{\bullet \mathtt{h}_3 :\vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, \mathtt{F}_{\mathbf{1}} \otimes \mathtt{F}_2), \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 :\vdash \Delta_7, \Delta_9, \mathtt{F}_2, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_1 :\vdash \Delta_4, \Delta_5, \mathtt{F}_2 \quad \mathtt{h}_1 :\vdash \Delta_6, \Delta_7, \mathtt{F}_3}{\bullet \mathtt{h}_1 :\vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), \mathtt{F}_2 \otimes \mathtt{F}_3}\ \otimes \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 :\vdash \Delta_5, \Delta_7, \mathtt{F}_3}\ \mathtt{fail}$$

# 4 Identity-Expansion

$$\frac{}{- :\vdash \mathbf{0}, \top}\ \top$$

$$\frac{\dfrac{}{- :\vdash \mathbf{1}}\ \mathbf{1}}{- :\vdash \mathbf{1}, \bot}\ \bot$$

$$\frac{\dfrac{\dfrac{}{- :\vdash \mathtt{F_0}, dual(\mathtt{F_0})}\ \mathtt{IH}}{- :\vdash dual(\mathtt{F_0}), \mathtt{F_0} \oplus \mathtt{F_1}}\ \oplus_A \quad \dfrac{\dfrac{}{- :\vdash \mathtt{F_1}, dual(\mathtt{F_1})}\ \mathtt{IH}}{- :\vdash dual(\mathtt{F_1}), \mathtt{F_0} \oplus \mathtt{F_1}}\ \oplus_B}{- :\vdash dual(\mathtt{F_0}) \& dual(\mathtt{F_1}), \mathtt{F_0} \oplus \mathtt{F_1}}\ \&$$

$$\frac{\dfrac{\dfrac{}{- :\vdash \mathtt{F_0}, dual(\mathtt{F_0})}\ \mathtt{IH} \quad \dfrac{}{- :\vdash \mathtt{F_1}, dual(\mathtt{F_1})}\ \mathtt{IH}}{- :\vdash dual(\mathtt{F_0}), dual(\mathtt{F_1}), \mathtt{F_0} \otimes \mathtt{F_1}}\ \otimes}{- :\vdash dual(\mathtt{F_0}) \$ dual(\mathtt{F_1}), \mathtt{F_0} \otimes \mathtt{F_1}}\ \$$

$$\frac{\dfrac{\dfrac{}{- :\vdash \mathtt{F_0}, dual(\mathtt{F_0})}\ \mathtt{IH}}{- :\vdash \mathtt{F_0}, dual(\mathtt{F_0}) \oplus dual(\mathtt{F_1})}\ \oplus_A \quad \dfrac{\dfrac{}{- :\vdash \mathtt{F_1}, dual(\mathtt{F_1})}\ \mathtt{IH}}{- :\vdash \mathtt{F_1}, dual(\mathtt{F_0}) \oplus dual(\mathtt{F_1})}\ \oplus_B}{- :\vdash \mathtt{F_0} \& \mathtt{F_1}, dual(\mathtt{F_0}) \oplus dual(\mathtt{F_1})}\ \&$$

$$\frac{\dfrac{\dfrac{}{- :\vdash \mathtt{F_0}, dual(\mathtt{F_0})}\ \mathtt{IH} \quad \dfrac{}{- :\vdash \mathtt{F_1}, dual(\mathtt{F_1})}\ \mathtt{IH}}{- :\vdash \mathtt{F_0}, \mathtt{F_1}, dual(\mathtt{F_0}) \otimes dual(\mathtt{F_1})}\ \otimes}{- :\vdash \mathtt{F_0} \$ \mathtt{F_1}, dual(\mathtt{F_0}) \otimes dual(\mathtt{F_1})}\ \$$

$$\frac{\dfrac{\dfrac{}{- :\vdash \mathtt{F_0}, dual(\mathtt{F_0})}\ \mathtt{IH}}{- :\vdash ?\mathtt{F_0}, dual(\mathtt{F_0})}\ ?}{- :\vdash !dual(\mathtt{F_0}), ?\mathtt{F_0}}\ !$$

$$\frac{\dfrac{}{- :\vdash \mathbf{1}}\ \mathbf{1}}{- :\vdash \mathbf{1}, \bot}\ \bot$$

$$\frac{}{- :\vdash \mathbf{0}, \top}\ \top$$

$$\frac{\dfrac{\dfrac{}{- :\vdash \mathtt{F_0}, dual(\mathtt{F_0})}\ \mathtt{IH}}{- :\vdash \mathtt{F_0}, ?dual(\mathtt{F_0})}\ ?}{- :\vdash !\mathtt{F_0}, ?dual(\mathtt{F_0})}\ !$$

# 5 Cut-Elimination

## 5.1 Status of 1: OK

- Case rule $\mathbf{1}$

- Case rule !

- Case rule $?W$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \bot, \Delta_4}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_4, ?\mathtt{F}_3}\ ?W}{- :\vdash *, \Delta_4, ?\mathtt{F}_3}\ \mathtt{Cut} \qquad \begin{array}{c}\to\\ \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \bot}\ \mathtt{ax}}{- :\vdash \Delta_4}\ \mathtt{hCut}\\ \dfrac{- :\vdash \Delta_4}{- :\vdash \Delta_4, ?\mathtt{F}_3}\ ?W\end{array}$$

- Case rule $?C$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \bot, \Delta_4, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_4, ?\mathtt{F}_3}\ ?C}{- :\vdash *, \Delta_4, ?\mathtt{F}_3}\ \mathtt{Cut} \qquad \begin{array}{c}\to\\ \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \bot, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{ax}}{- :\vdash \Delta_4, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{hCut}\\ \dfrac{- :\vdash \Delta_4, ?\mathtt{F}_3, ?\mathtt{F}_3}{- :\vdash \Delta_4, ?\mathtt{F}_3}\ ?C\end{array}$$

- Case rule $?$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \bot, \Delta_4, \mathtt{F}_3}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_4, ?\mathtt{F}_3}\ ?}{- :\vdash *, \Delta_4, ?\mathtt{F}_3}\ \mathtt{Cut} \qquad \begin{array}{c}\to\\ \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \bot}\ \mathtt{ax}}{- :\vdash \Delta_4, \mathtt{F}_3}\ \mathtt{hCut}\\ \dfrac{- :\vdash \Delta_4, \mathtt{F}_3}{- :\vdash \Delta_4, ?\mathtt{F}_3}\ ?\end{array}$$

- Case rule $\$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \bot, \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$}{- :\vdash *, \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \mathtt{Cut} \qquad \begin{array}{c}\to\\ \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \bot}\ \mathtt{ax}}{- :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{hCut}\\ \dfrac{- :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}{- :\vdash \Delta_5, \mathtt{F}_3 \$ \mathtt{F}_4}\ \$\end{array}$$

- Case rule $\&$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \bot, \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \bot, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \&}{- :\vdash *, \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \bot}\ \mathtt{ax}}{- :\vdash \Delta_5, \mathtt{F}_3}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \bot}\ \mathtt{ax}}{- :\vdash \Delta_5, \mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \bot, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B}{- :\vdash *, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \bot}\ \mathtt{ax}}{- :\vdash \Delta_5, \mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \bot, \Delta_5, \mathtt{F}_3}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A}{- :\vdash *, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \bot}\ \mathtt{ax}}{- :\vdash \Delta_5, \mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \Delta_3}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_3}\ \bot}{- :\vdash *, \Delta_3}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \Delta_3}\ \mathtt{ax}$$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \top, \Delta_3}\ \top}{- :\vdash *, \top, \Delta_3}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \Delta_3, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \bot, \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes}{- :\vdash *, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathbf{1} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_4, \bot}\ \mathtt{ax}}{- :\vdash \Delta_6, \mathtt{F}_4}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{- :\vdash \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}, *}\ \mathbf{1} \quad \dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \bot, \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \mathit{dual}(\mathbf{1}), \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes}{- :\vdash *, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \mathbf{1}}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_5, \bot}\ \mathtt{ax}}{- :\vdash \Delta_6, \mathtt{F}_5}\ \mathtt{hCut}}{- :\vdash \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

## 5.2 Status of !: OK

- Case rule **1**

- Case rule !

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \mathtt{F}_6, ?\Upsilon 7, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), ?\Upsilon 7, !\mathtt{F}_6}\ !}{- :\vdash ?\Upsilon 2, ?\Upsilon 7, !\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_5 :\vdash ?\Upsilon 7, \mathtt{F}_6, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, ?\Upsilon 7, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, ?\Upsilon 7, !\mathtt{F}_6}\ !$$

- Case rule $?W$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, dual(5)}{\bullet \mathtt{h}_6 :\vdash dual(5), \Delta_8, ?\mathtt{F}_7}\ ?W}{- :\vdash (?\Upsilon 4, !\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_8, dual(5)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_8, !\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_8, !\mathtt{F}_3, ?\mathtt{F}_7}\ W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?W}{- :\vdash ?\Upsilon 2, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_7}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_7, ?\mathtt{F}_6}\ ?W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_6}\ ?W}{- :\vdash ?\Upsilon 2, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- :\vdash \Delta_6}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_6}\ W$$

- Case rule $?C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, ?\mathtt{F}_7, dual(5)}{\bullet \mathtt{h}_6 :\vdash dual(5), \Delta_8, ?\mathtt{F}_7}\ ?C}{- :\vdash (?\Upsilon 4, !\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, ?\mathtt{F}_7, dual(5)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_8, !\mathtt{F}_3, ?\mathtt{F}_7, ?\mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_8, !\mathtt{F}_3, ?\mathtt{F}_7}\ ?C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, ?\mathtt{F}_6, ?\mathtt{F}_6, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?C}{- :\vdash ?\Upsilon 2, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, ?\mathtt{F}_6, ?\mathtt{F}_6, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_7, ?\mathtt{F}_6, ?\mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_7, ?\mathtt{F}_6}\ ?C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\,!\quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, ?dual(\mathtt{F}_4), ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_6}\,?C}{- :\vdash ?\Upsilon 2, \Delta_6}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_6, ?dual(\mathtt{F}_4), ?dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_6}\,\mathtt{mCut}}$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\,!\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, \mathtt{F}_7, dual(5)}{\bullet \mathtt{h}_6 :\vdash dual(5), \Delta_8, ?\mathtt{F}_7}\,?}{- :\vdash (?\Upsilon 4, !\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_8, \mathtt{F}_7, dual(5)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_8, \mathtt{F}_7, !\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_8, !\mathtt{F}_3, ?\mathtt{F}_7}\,?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\,!\quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\,?}{- :\vdash ?\Upsilon 2, \Delta_7, ?\mathtt{F}_6}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, ?dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_7, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_7, ?\mathtt{F}_6}\,?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\,!\quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_6}\,?}{- :\vdash ?\Upsilon 2, \Delta_6}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash ?\Upsilon 2, \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_6, dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_6}\,\mathtt{sCut}}$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\,!\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8, dual(5)}{\bullet \mathtt{h}_6 :\vdash dual(5), \Delta_9, \mathtt{F}_7 \$ \mathtt{F}_8}\,\$}{- :\vdash (?\Upsilon 4, !\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \$ \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8, dual(5)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_9, \mathtt{F}_7, \mathtt{F}_8, !\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_9, !\mathtt{F}_3, \mathtt{F}_7 \$ \mathtt{F}_8}\,\$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\,!\quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\,\$}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, ?dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\,\$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\,!\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, dual(5) \quad \mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, dual(5)}{\bullet \mathtt{h}_6 :\vdash dual(5), \Delta_9, \mathtt{F}_7 \& \mathtt{F}_8}\,\&}{- :\vdash (?\Upsilon 4, !\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \& \mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, dual(5)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_9, \mathtt{F}_7, !\mathtt{F}_3}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, dual(5)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_9, \mathtt{F}_8, !\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_9, !\mathtt{F}_3, \mathtt{F}_7 \& \mathtt{F}_8}\,\&$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ ! \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, ?dual(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\ \&}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ ! \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, dual(5)}{\bullet \mathtt{h}_6 :\vdash dual(5), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B}{- :\vdash (?\Upsilon 4, !\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, dual(5)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_9, \mathtt{F}_8, !\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_9, !\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ ! \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ ! \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, dual(5)}{\bullet \mathtt{h}_6 :\vdash dual(5), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A}{- :\vdash (?\Upsilon 4, !\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, dual(5)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_9, \mathtt{F}_7, !\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_9, !\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ ! \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ ! \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, dual(5)}{\bullet \mathtt{h}_6 :\vdash dual(5), \bot, \Delta_7}\ \bot}{- :\vdash (?\Upsilon 4, !\mathtt{F}_3), \bot, \Delta_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_7, dual(5)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_7, !\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_7, \bot, !\mathtt{F}_3}\ \bot$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ ! \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, ?dual(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash dual(!\mathtt{F}_4), \bot, \Delta_6}\ \bot}{- :\vdash ?\Upsilon 2, \bot, \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_6, ?dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_6}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_6, \bot}\ \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ !\quad \dfrac{}{\bullet \mathtt{h}_6 :\vdash \mathit{dual}(5), \top, \Delta_7}\ \top}{-:\vdash (?\Upsilon 4, !\mathtt{F}_3), \top, \Delta_7}\ \mathtt{Cut}}{\dfrac{\rightarrow}{-:\vdash ?\Upsilon 4, \Delta_7, \top, !\mathtt{F}_3}\ \top}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(!\mathtt{F}_4), \top, \Delta_6}\ \top}{-:\vdash ?\Upsilon 2, \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{\rightarrow}{-:\vdash ?\Upsilon 2, \Delta_6, \top}\ \top}$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(5)\quad \mathtt{h}_6 :\vdash \Delta_7, \mathtt{F}_9}{\bullet \mathtt{h}_6 :\vdash \mathit{dual}(5), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{-:\vdash (?\Upsilon 4, !\mathtt{F}_3), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(5)}\ \mathtt{ax}}{-:\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, !\mathtt{F}_3}\ \mathtt{hCut}\quad \dfrac{}{-:\vdash \Delta_7, \mathtt{F}_9}\ \mathtt{ax}}{-:\vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, 5}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, \mathtt{F}_8\quad \mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(5)}{\bullet \mathtt{h}_6 :\vdash \mathit{dual}(5), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{-:\vdash (?\Upsilon 4, !\mathtt{F}_3), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{-:\vdash \Delta_7, \mathtt{F}_8}\ \mathtt{ax}\quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash 5, ?\Upsilon 4, !\mathtt{F}_3}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(5)}\ \mathtt{ax}}{-:\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_9, !\mathtt{F}_3}\ \mathtt{hCut}}{-:\vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, ?\mathit{dual}(\mathtt{F}_4)\quad \mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_8}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(!\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{-:\vdash ?\Upsilon 2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, ?\mathit{dual}(\mathtt{F}_4)}\ \mathtt{ax}}{-:\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7}\ \mathtt{hCut}\quad \dfrac{}{-:\vdash \Delta_6, \mathtt{F}_8}\ \mathtt{ax}}{-:\vdash ?\Upsilon 2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_7\quad \mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, ?\mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(!\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{-:\vdash ?\Upsilon 2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{-:\vdash \Delta_6, \mathtt{F}_7}\ \mathtt{ax}\quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, ?\mathit{dual}(\mathtt{F}_4)}\ \mathtt{ax}}{-:\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_8}\ \mathtt{hCut}}{-:\vdash ?\Upsilon 2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}$$

## 5.3 Status of $?W$: OK

- Case rule **1**

- Case rule !

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash ?\Upsilon 6, \mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(?\mathtt{F}_4), ?\Upsilon 6}\ !}{- :\vdash \Delta_2, ?\Upsilon 6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_2}\ \mathtt{ax}}{- :\vdash ?\Upsilon 6, \Delta_2}\ W$$

- Case rule $?W$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \mathit{dual}(\mathtt{F}_5), \Delta_8, ?\mathtt{F}_7}\ ?W}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, \mathit{dual}(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7}\ W$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, !\mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(?\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?W}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, !\mathit{dual}(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ W$$

- Case rule $?C$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, ?\mathtt{F}_7, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \mathit{dual}(\mathtt{F}_5), \Delta_8, ?\mathtt{F}_7}\ ?C}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, \mathit{dual}(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7}\ ?W$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, !\mathit{dual}(\mathtt{F}_4), ?\mathtt{F}_6, ?\mathtt{F}_6}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(?\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?C}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, !\mathit{dual}(\mathtt{F}_4), ?\mathtt{F}_6, ?\mathtt{F}_6}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6, ?\mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ ?C$$

- Case rule ?

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \mathit{dual}(\mathtt{F}_5), \Delta_8, ?\mathtt{F}_7}\ ?}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, \mathit{dual}(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7}\ W$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, !\mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(?\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, !\mathit{dual}(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ ?$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\ \$}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_9, dual(\mathtt{F}_5), \mathtt{F}_7\$\mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7\$\mathtt{F}_8}\ W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\ \$}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\ \$$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, dual(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\ \&}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_9, dual(\mathtt{F}_5), \mathtt{F}_7\&\mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8}\ W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\ \&}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_9, dual(\mathtt{F}_5), \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\ ?W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_9, dual(\mathtt{F}_5), \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\ ?W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \bot, \Delta_7}\ \bot}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \bot, \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_7, \bot, dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_7, \bot}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_7, \bot, ?\mathtt{F}_3}\ W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \bot, \Delta_6}\ \bot}{- :\vdash \Delta_2, \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_6, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_6, \bot}\ \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \top, \Delta_7}\ \top}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \top, \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- :\vdash \Delta_4, \Delta_7, \top, ?\mathtt{F}_3}\ \top}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \top, \Delta_6}\ \top}{- :\vdash \Delta_2, \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- :\vdash \Delta_2, \Delta_6, \top}\ \top}$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \Delta_7, \mathtt{F}_9}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_{10}, \Delta_7, dual(\mathtt{F}_5), \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ ?W$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, \mathtt{F}_8 \quad \mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_{10}, \Delta_7, dual(\mathtt{F}_5), \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ ?W$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, !dual(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_8}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_6, \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?W \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_7 \quad \mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_6, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes$$

## 5.4 Status of $?C$: OK

- Case rule **1**

- Case rule !

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?C \quad \dfrac{\mathtt{h}_5 :\vdash ?\Upsilon 6, dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), ?\Upsilon 6}\ !}{- :\vdash \Delta_2, ?\Upsilon 6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_5 :\vdash ?\Upsilon 6, dual(\mathtt{F}_4)}\ \mathtt{ax}}{\bullet\mathtt{h}_5 :\vdash ?\Upsilon 6, !dual(\mathtt{F}_4)}\ !}{- :\vdash ?\Upsilon 6, \Delta_2}\ \mathtt{mCut}$$

- Case rule $?W$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_8, ?\mathtt{F}_7}\ ?W}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_8, dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7}\ W$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?W}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ ?W$$

- Case rule $?C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, ?\mathtt{F}_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_8, ?\mathtt{F}_7}\,?C}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, dual(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_3, ?\mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7}\,?C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\,?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, !dual(\mathtt{F}_4), ?\mathtt{F}_6, ?\mathtt{F}_6}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\,?C}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, !dual(\mathtt{F}_4), ?\mathtt{F}_6, ?\mathtt{F}_6}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6, ?\mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\,?C}$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, \mathtt{F}_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_8, ?\mathtt{F}_7}\,?}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, dual(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_3, ?\mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7}\,?C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\,?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\,?}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, !dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\,?$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\,\$}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_9, dual(\mathtt{F}_5), \mathtt{F}_7\$\mathtt{F}_8}\,\mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_7\$\mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7\$\mathtt{F}_8}\,?C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\,?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\$}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, !dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, dual(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\,\&}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_9, dual(\mathtt{F}_5), \mathtt{F}_7\&\mathtt{F}_8}\,\mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8}\,?C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\,?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\,\&}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4)}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !dual(\mathtt{F}_4)}\,\mathtt{ax}}{\bullet\mathtt{h}_5 :\vdash \Delta_8, !dual(\mathtt{F}_4), \mathtt{F}_6 \& \mathtt{F}_7}\,\&}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{mCut}\end{array}}$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_B}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_9, dual(\mathtt{F}_5), \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\,?C\end{array}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\,?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_B}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_B\end{array}}$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\oplus_A}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_9, dual(\mathtt{F}_5), \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\,?C\end{array}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\,?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_A}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_A\end{array}}$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \bot, \Delta_7}\,\bot}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \bot, \Delta_7}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_7, \bot, dual(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \Delta_4, \Delta_7, \bot, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash \Delta_4, \Delta_7, \bot, ?\mathtt{F}_3}\,?C\end{array}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\,?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \bot, \Delta_6}\,\bot}{- :\vdash \Delta_2, \bot, \Delta_6}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_5 :\vdash \Delta_6, !dual(\mathtt{F}_4)}\,\mathtt{ax}}{\bullet\mathtt{h}_5 :\vdash \Delta_6, \bot, !dual(\mathtt{F}_4)}\,\bot}{- :\vdash \Delta_2, \Delta_6, \bot}\,\mathtt{mCut}\end{array}}$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?C \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \top, \Delta_7}\ \top}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \top, \Delta_7}\ \mathtt{Cut}}{- :\vdash \Delta_4, \Delta_7, \top, ?\mathtt{F}_3}\ \top$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?C \quad \dfrac{}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \top, \Delta_6}\ \top}{- :\vdash \Delta_2, \top, \Delta_6}\ \mathtt{Cut}}{- :\vdash \Delta_2, \Delta_6, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \Delta_7, \mathtt{F}_9}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_{10}, \Delta_7, dual(\mathtt{F}_5), \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ ?C$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ?C \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, \mathtt{F}_8 \quad \mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_{10}, \Delta_7, dual(\mathtt{F}_5), \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ ?C$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, !dual(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_8}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, !dual(\mathtt{F}_4)}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_8}\ \mathtt{ax}}{\bullet\mathtt{h}_5 :\vdash \Delta_6, \Delta_9, !dual(\mathtt{F}_4), \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{mCut}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ?C \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_7 \quad \mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{\bullet\mathtt{h}_5 :\vdash \Delta_6, \Delta_9, !dual(\mathtt{F}_4), \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{mCut}$$

## 5.5 Status of ?: OK

- Case rule **1**

- Case rule !

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash ?\Upsilon 6, dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), ?\Upsilon 6}\ !}{- :\vdash \Delta_2, ?\Upsilon 6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_2, \mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{- :\vdash ?\Upsilon 6, dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash ?\Upsilon 6, \Delta_2}\ \mathtt{sCut}$$

- Case rule $?W$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_8, ?\mathtt{F}_7}\ ?W}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_8, dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7}\ W$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?W}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ W$$

- Case rule $?C$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, ?\mathtt{F}_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_8, ?\mathtt{F}_7}\ ?C}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, ?\mathtt{F}_7, dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7, ?\mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7}\ ?C$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, !dual(\mathtt{F}_4), ?\mathtt{F}_6, ?\mathtt{F}_6}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?C}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, !dual(\mathtt{F}_4), ?\mathtt{F}_6, ?\mathtt{F}_6}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6, ?\mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ ?C$$

- Case rule ?

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, \mathtt{F}_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_8, ?\mathtt{F}_7}\ ?}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_8, \mathtt{F}_3, ?\mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_8, ?\mathtt{F}_3, ?\mathtt{F}_7}\ ?$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_7, ?\mathtt{F}_6}\ ?$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \mathit{dual}(\mathtt{F}_5), \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\ \$}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_6 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5), \mathtt{F}_7\$\mathtt{F}_8}{\bullet\mathtt{h}_6 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5), \mathtt{F}_7\$\mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, \mathtt{F}_3, \mathtt{F}_7\$\mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7\$\mathtt{F}_8}\ ?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, !\mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\ \$}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, !\mathit{dual}(\mathtt{F}_4)}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, !\mathit{dual}(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\ \$$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \mathit{dual}(\mathtt{F}_5), \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\ \&}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7\&\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_6 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5), \mathtt{F}_7\&\mathtt{F}_8}{\bullet\mathtt{h}_6 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5), \mathtt{F}_7\&\mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, \mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7\&\mathtt{F}_8}\ ?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !\mathit{dual}(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !\mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\ \&}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !\mathit{dual}(\mathtt{F}_4)}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !\mathit{dual}(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6}\ \mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !\mathit{dual}(\mathtt{F}_4)}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !\mathit{dual}(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6\&\mathtt{F}_7}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash \mathit{dual}(\mathtt{F}_5), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_6 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5), \mathtt{F}_7 \oplus \mathtt{F}_8}{\bullet\mathtt{h}_6 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5), \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, \mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\ ?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !\mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !\mathit{dual}(\mathtt{F}_4)}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, !\mathit{dual}(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_9, dual(\mathtt{F}_5), \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_9, \mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_9, ?\mathtt{F}_3, \mathtt{F}_7 \oplus \mathtt{F}_8}\ ?}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A}$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \bot, \Delta_7}\ \bot}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \bot, \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_7, \bot, dual(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_4, \Delta_7, \mathtt{F}_3, \bot}\ \mathtt{hCut}}{- :\vdash \Delta_4, \Delta_7, \bot, ?\mathtt{F}_3}\ ?}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \bot, \Delta_6}\ \bot}{- :\vdash \Delta_2, \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_6, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_6, \bot}\ \bot}$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \top, \Delta_7}\ \top}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \top, \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- :\vdash \Delta_4, \Delta_7, \top, ?\mathtt{F}_3}\ \top}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \top, \Delta_6}\ \top}{- :\vdash \Delta_2, \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- :\vdash \Delta_2, \Delta_6, \top}\ \top}$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5) \quad \mathtt{h}_6 :\vdash \Delta_7, \mathtt{F}_9}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_{10}, \Delta_7, dual(\mathtt{F}_5), \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, \mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ ?}$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_5, \Delta_4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, \mathtt{F}_8 \quad \mathtt{h}_6 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_5)}{\bullet\mathtt{h}_6 :\vdash dual(\mathtt{F}_5), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \otimes}{- :\vdash (\Delta_4, ?\mathtt{F}_3), \Delta_7, \Delta_{10}, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_4, \mathtt{F}_3, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash \Delta_{10}, \Delta_7, dual(\mathtt{F}_5), \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, \mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_4, \Delta_7, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\ ?$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, !dual(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_8}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_6, \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_4}{\bullet\mathtt{h}_1 :\vdash ?\mathtt{F}_4, \Delta_2}\ ? \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_7 \quad \mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, !dual(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash dual(?\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_6, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, ?\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, !dual(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes$$

## 5.6 Status of \$: OK

- Case rule **1**
- Case rule !
- Case rule ?$W$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\ ?W}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3\$\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, \mathtt{F}_3\$\mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3\$\mathtt{F}_4}\ W$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\ \$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, dual(\mathtt{F}_5) \otimes dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5\$\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\ ?W}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, dual(\mathtt{F}_5) \otimes dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ ?W$$

- Case rule ?$C$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\ ?C}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3\$\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathtt{F}_3\$\mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3\$\mathtt{F}_4}\ ?C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\ \$\ \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5\$\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}\quad ?C}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ ?C$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$\ \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}\quad ?}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3\$\mathtt{F}_4}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, \mathtt{F}_8, \mathtt{F}_3\$\mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3\$\mathtt{F}_4}\ ?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\ \$\ \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5\$\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}\quad ?}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ ?$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$\ \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\ \mathtt{Cut}\quad \$}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, dual(\mathtt{F}_6), \mathtt{F}_8\$\mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8\$\mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_8\$\mathtt{F}_9}\ \$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\ \$\ \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5\$\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\ \mathtt{Cut}\quad \$}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8\$\mathtt{F}_9}\ \$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$\ \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_6)\quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\ \&}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, dual(\mathtt{F}_6), \mathtt{F}_8\&\mathtt{F}_9}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8\&\mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_8\&\mathtt{F}_9}\ \$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\ \$\ \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)\quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5\$\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\ \&}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8}\ \mathtt{hCut}\quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_5)\otimes dual(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8\&\mathtt{F}_9}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\,\$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6}{}\,\mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, dual(\mathtt{F}_6), \mathtt{F}_8 \oplus \mathtt{F}_9}{}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\$$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\,\$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_5) \otimes dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5\$\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}{}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_5) \otimes dual(\mathtt{F}_6)}{}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\,\$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6}{}\,\mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, dual(\mathtt{F}_6), \mathtt{F}_8 \oplus \mathtt{F}_9}{}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\$$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\,\$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5) \otimes dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5\$\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}{}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5) \otimes dual(\mathtt{F}_6)}{}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\,\$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \bot, \Delta_8}\,\bot}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \bot, \Delta_8}\,\mathtt{Cut} \quad \rightarrow}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6}{}\,\mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_7 :\vdash \Delta_8, \bot, dual(\mathtt{F}_6)}{}\,\mathtt{ax}}{- :\vdash \Delta_5, \Delta_8, \mathtt{F}_3, \mathtt{F}_4, \bot}\,\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_8, \bot, \mathtt{F}_3\$\mathtt{F}_4}\,\$$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\,\$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, dual(\mathtt{F}_5) \otimes dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5\$\mathtt{F}_6), \bot, \Delta_8}\,\bot}{- :\vdash \Delta_2, \bot, \Delta_8}\,\mathtt{Cut} \quad \rightarrow}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}{}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, dual(\mathtt{F}_5) \otimes dual(\mathtt{F}_6)}{}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \bot}\,\bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \top, \Delta_8}\ \top}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \top, \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \Delta_5, \Delta_8, \top, \mathtt{F}_3\$\mathtt{F}_4}\ \top$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\ \$ \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\$\mathtt{F}_6), \top, \Delta_8}\ \top}{- :\vdash \Delta_2, \top, \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \Delta_2, \Delta_8, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \otimes}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{11}, \Delta_8, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\$\mathtt{F}_4}\ \$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \otimes}{- :\vdash (\Delta_5, \mathtt{F}_3\$\mathtt{F}_4), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{11}, \Delta_8, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_3\$\mathtt{F}_4, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\ \$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5) \otimes \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\$\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5) \otimes \mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F}_9}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\ \$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_5) \otimes \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\$\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_8, \mathtt{F}_9}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\$\mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_5) \otimes \mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F}_{10}}\ \mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\$\mathtt{F}_6, \Delta_2}\ \$ \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5) \quad \mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\$\mathtt{F}_6), \Delta_8, \Delta_9}\ \otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{- :\vdash \Delta_2, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, \mathtt{F}_5}\ \mathtt{sCut} \quad \dfrac{}{- :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_8, \Delta_9}\ \mathtt{sCut}$$

## 5.7 Status of &: OK

- Case rule **1**

- Case rule !

- Case rule $?W$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\&\mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?W}{- :\vdash (\Delta_5, \mathtt{F}_3\&\mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4}\,?W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\&\mathtt{F}_6, \Delta_2}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\&\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?W}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\&\mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_9}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,?W$$

- Case rule $?C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\&\mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?C}{- :\vdash (\Delta_5, \mathtt{F}_3\&\mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4}\,?C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\&\mathtt{F}_6, \Delta_2}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\&\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?C}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\&\mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,?C$$

- Case rule $?$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\&\mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?}{- :\vdash (\Delta_5, \mathtt{F}_3\&\mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, \mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3\&\mathtt{F}_4}\,?}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\&\mathtt{F}_6, \Delta_2}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\&\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5\&\mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,?$$

- Case rule \$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\&\mathtt{F}_4}\,\&}{- :\vdash (\Delta_5, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut} \qquad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_8\$\mathtt{F}_9, \mathtt{F}_3\&\mathtt{F}_4}\,\$$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\&\mathtt{F}_6, \Delta_2}\,\&}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut} \qquad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\&\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8\$\mathtt{F}_9}\,\$$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\&\mathtt{F}_4}\,\&}{- :\vdash (\Delta_5, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{Cut} \qquad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\&}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_8\&\mathtt{F}_9}\,\&$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\&\mathtt{F}_6, \Delta_2}\,\&}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{Cut} \qquad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\&\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\&}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8\&\mathtt{F}_9}\,\&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3\&\mathtt{F}_4}\,\&}{- :\vdash (\Delta_5, \mathtt{F}_3\&\mathtt{F}_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut} \qquad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3\&\mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\&\mathtt{F}_6, \Delta_2}\,\&}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut} \qquad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\&\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5\&\mathtt{F}_6, \Delta_2}\,\&}{- :\vdash \Delta_2, \Delta_8}\,\mathtt{Cut} \qquad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5\&\mathtt{F}_6), \Delta_8}\,\oplus_B}{- :\vdash \Delta_2, \Delta_8}\,\mathtt{sCut}$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\,\&\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}{- :\vdash (\Delta_5, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \& \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_8, \mathtt{F}_3 \& \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \& \mathtt{F}_6, \Delta_2}\,\&\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \& \mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \& \mathtt{F}_6, \Delta_2}\,\&\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \& \mathtt{F}_6), \Delta_8}\,\oplus_A}{- :\vdash \Delta_2, \Delta_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash \Delta_2, \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8}\,\mathtt{sCut}}$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\,\&\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \bot, \Delta_8}\,\bot}{- :\vdash (\Delta_5, \mathtt{F}_3 \& \mathtt{F}_4), \bot, \Delta_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \& \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_5, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_8, \bot, \mathtt{F}_3 \& \mathtt{F}_4}\,\bot}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \& \mathtt{F}_6, \Delta_2}\,\&\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \& \mathtt{F}_6), \bot, \Delta_8}\,\bot}{- :\vdash \Delta_2, \bot, \Delta_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \bot}\,\bot}$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\,\&\quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \top, \Delta_8}\,\top}{- :\vdash (\Delta_5, \mathtt{F}_3 \& \mathtt{F}_4), \top, \Delta_8}\,\mathtt{Cut}}{- :\vdash \Delta_5, \Delta_8, \top, \mathtt{F}_3 \& \mathtt{F}_4}\,\top$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \& \mathtt{F}_6, \Delta_2}\,\&\quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \& \mathtt{F}_6), \top, \Delta_8}\,\top}{- :\vdash \Delta_2, \top, \Delta_8}\,\mathtt{Cut}}{- :\vdash \Delta_2, \Delta_8, \top}\,\top$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash (\Delta_5, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \& \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_5, \mathtt{F}_9, \mathtt{F}_3 \& \mathtt{F}_4}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \quad \mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \& \mathtt{F}_4}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash (\Delta_5, \mathtt{F}_3 \& \mathtt{F}_4), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_8, \mathtt{F}_9}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \& \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_5, \mathtt{F}_{10}, \mathtt{F}_3 \& \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_3 \& \mathtt{F}_4, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \& \mathtt{F}_6, \Delta_2}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \& \mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \quad \mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \& \mathtt{F}_6, \Delta_2}\,\& \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \& \mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_8, \mathtt{F}_9}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \& \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_5) \oplus \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes$$

## 5.8 Status of $\oplus_B$: OK

- Case rule **1**
- Case rule !
- Case rule $?W$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?W}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\,?W$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\,\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5) \& \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?W}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5) \& \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_9}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,W$$

- Case rule $?C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?C}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_6)}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\,?C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?C}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,?C$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_6)}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, \mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\,?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\,?}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\,?$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, dual(\mathtt{F}_6)}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_8\$\mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8\$\mathtt{F}_9}\,\$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\&}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, dual(\mathtt{F}_6), \mathtt{F}_8\&\mathtt{F}_9}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, dual(\mathtt{F}_6), \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_4, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_8\&\mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\&}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8}\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_9}\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8\&\mathtt{F}_9}\&}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_8}\&}{- :\vdash \Delta_2, \Delta_8}\mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash \Delta_2, \mathtt{F}_6}\mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_6)}\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8}\mathtt{sCut}}$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_B}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6}\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_8 \oplus \mathtt{F}_9}\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_B}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_9}\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_A}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6}\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_8 \oplus \mathtt{F}_9}\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_A}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8}\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \bot, \Delta_8}\bot}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \bot, \Delta_8}\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6}\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_8, \bot, \mathit{dual}(\mathtt{F}_6)}\mathtt{ax}}{- :\vdash \Delta_5, \Delta_8, \mathtt{F}_4, \bot}\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_8, \bot, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \bot, \Delta_8}\bot}{- :\vdash \Delta_2, \bot, \Delta_8}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}{}\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{}\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8}\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \bot}\bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \top, \Delta_8}\top}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \top, \Delta_8}\mathtt{Cut}}{\dfrac{}{- :\vdash \Delta_5, \Delta_8, \top, \mathtt{F}_3 \oplus \mathtt{F}_4}\top}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \top, \Delta_8}\top}{- :\vdash \Delta_2, \top, \Delta_8}\mathtt{Cut}}{\dfrac{}{- :\vdash \Delta_2, \Delta_8, \top}\top}$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, dual(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6}{}\mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_7 :\vdash \Delta_{11}, \Delta_8, dual(\mathtt{F}_6), \mathtt{F}_9 \otimes \mathtt{F}_{10}}{}\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_4, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_4, \mathtt{F}_6}{}\mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_7 :\vdash \Delta_{11}, \Delta_8, dual(\mathtt{F}_6), \mathtt{F}_9 \otimes \mathtt{F}_{10}}{}\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_4, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}{}\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{}\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F}_9}\mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_{10}}\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_B \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash \Delta_8, \mathtt{F}_9}\mathtt{ax} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}{}\mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{}\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F}_{10}}\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes$$

## 5.9 Status of $\oplus_A$: OK

- Case rule **1**

- Case rule !

- Case rule $?W$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A\ \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\ ?W}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\ W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\ \oplus_A\ \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\ ?W}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ ?W}$$

- Case rule $?C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A\ \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\ ?C}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\ ?C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\ \oplus_A\ \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\ ?C}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ ?C}$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \oplus_A\ \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\ ?}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_9, \mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\ \mathtt{hCut}}{- :\vdash \Delta_5, \Delta_9, ?\mathtt{F}_8, \mathtt{F}_3 \oplus \mathtt{F}_4}\ ?}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\ \oplus_A\ \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_9, ?\mathtt{F}_8}\ ?}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{ax}\quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_9, \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_9, ?\mathtt{F}_8}\ ?}$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_8\$\mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\$$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8, \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8\$\mathtt{F}_9}\,\$$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\&}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_8\&\mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\&}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8\&\mathtt{F}_9}\,\&$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_8}\,\&}{- :\vdash \Delta_2, \Delta_8}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{}{- :\vdash \Delta_2, \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8}\,\mathtt{sCut}$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_B}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_9, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_B}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, dual(\mathtt{F}_6), \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\,\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}{- :\vdash \Delta_2, \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \bot, \Delta_8}\,\bot}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \bot, \Delta_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_8, \bot, dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_5, \Delta_8, \mathtt{F}_3, \bot}\,\mathtt{hCut}}{- :\vdash \Delta_5, \Delta_8, \bot, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\oplus_A}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\,\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \bot, \Delta_8}\,\bot}{- :\vdash \Delta_2, \bot, \Delta_8}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_8, dual(\mathtt{F}_5)\&dual(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_8}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_8, \bot}\,\bot}$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\oplus_A \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_6), \top, \Delta_8}\,\top}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \top, \Delta_8}\,\mathtt{Cut}}{\dfrac{}{- :\vdash \Delta_5, \Delta_8, \top, \mathtt{F}_3 \oplus \mathtt{F}_4}\,\top}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\,\oplus_A \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash dual(\mathtt{F}_5 \oplus \mathtt{F}_6), \top, \Delta_8}\,\top}{- :\vdash \Delta_2, \top, \Delta_8}\,\mathtt{Cut}}{\dfrac{}{- :\vdash \Delta_2, \Delta_8, \top}\,\top}$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{11}, \Delta_8, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_3, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_6, \Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash (\Delta_5, \mathtt{F}_3 \oplus \mathtt{F}_4), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash \Delta_{11}, \Delta_8, \mathit{dual}(\mathtt{F}_6), \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_3, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_5, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}, \mathtt{F}_3 \oplus \mathtt{F}_4}\oplus_A$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_5 \oplus \mathtt{F}_6, \Delta_2}\oplus_A \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}{\bullet\mathtt{h}_7 :\vdash \mathit{dual}(\mathtt{F}_5 \oplus \mathtt{F}_6), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash \Delta_2, \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_8, \mathtt{F}_9}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_5 \oplus \mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_5)\&\mathit{dual}(\mathtt{F}_6)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_8, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes$$

## 5.10 Status of $\bot$: OK

- Case rule **1**

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\bot \quad \dfrac{}{\bullet\mathtt{h}_3 :\vdash \mathit{dual}(\bot), *}\,\mathbf{1}}{- :\vdash \Delta_2, *}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \Delta_2}\,\mathtt{ax}$$

- Case rule !

- Case rule ?$W$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\bot \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_5}{\bullet\mathtt{h}_3 :\vdash \mathit{dual}(\bot), \Delta_5, ?\mathtt{F}_4}\,?W}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \bot}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_5}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_5}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\,W$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\bot \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, \mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\,?W}{- :\vdash (\bot, \Delta_3), \Delta_7, ?\mathtt{F}_6}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4, \bot}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, \mathit{dual}(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_3, \Delta_7, \bot}\,\mathtt{hCut}}{- :\vdash \Delta_3, \Delta_7, \bot, ?\mathtt{F}_6}\,W$$

- Case rule $?C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\,\bot \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet\mathtt{h}_3 :\vdash \mathit{dual}(\bot), \Delta_5, ?\mathtt{F}_4}\,?C}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \bot}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\,?C}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\,\bot \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, ?\mathtt{F}_6, ?\mathtt{F}_6, \mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\,?C}{- :\vdash (\bot, \Delta_3), \Delta_7, ?\mathtt{F}_6}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4, \bot}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, ?\mathtt{F}_6, ?\mathtt{F}_6, \mathit{dual}(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_3, \Delta_7, \bot, ?\mathtt{F}_6, ?\mathtt{F}_6}\,\mathtt{hCut}}{- :\vdash \Delta_3, \Delta_7, \bot, ?\mathtt{F}_6}\,?C}$$

- Case rule $?$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\,\bot \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_5, \mathtt{F}_4}{\bullet\mathtt{h}_3 :\vdash \mathit{dual}(\bot), \Delta_5, ?\mathtt{F}_4}\,?}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \bot}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_5, \mathtt{F}_4}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_5, \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\,?}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\,\bot \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, \mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\,?}{- :\vdash (\bot, \Delta_3), \Delta_7, ?\mathtt{F}_6}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4, \bot}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, \mathit{dual}(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_3, \Delta_7, \mathtt{F}_6, \bot}\,\mathtt{hCut}}{- :\vdash \Delta_3, \Delta_7, \bot, ?\mathtt{F}_6}\,?}$$

- Case rule $\$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\,\bot \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_4, \mathtt{F}_5}{\bullet\mathtt{h}_3 :\vdash \mathit{dual}(\bot), \Delta_6, \mathtt{F}_4\$\mathtt{F}_5}\,\$}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4\$\mathtt{F}_5}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \bot}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4\$\mathtt{F}_5}\,\$}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\,\bot \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\$}{- :\vdash (\bot, \Delta_3), \Delta_8, \mathtt{F}_6\$\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4, \bot}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_3, \Delta_8, \mathtt{F}_6, \mathtt{F}_7, \bot}\,\mathtt{hCut}}{- :\vdash \Delta_3, \Delta_8, \bot, \mathtt{F}_6\$\mathtt{F}_7}\,\$}$$

- Case rule $\&$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\,\bot \qquad \dfrac{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_5}{\bullet\mathtt{h}_3 :\vdash \mathit{dual}(\bot), \Delta_6, \mathtt{F}_4 \& \mathtt{F}_5}\,\&}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \& \mathtt{F}_5}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_4}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4}\,\mathtt{hCut} \qquad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_5}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \& \mathtt{F}_5}\,\&$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\,\bot \qquad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathit{dual}(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\,\&}{- :\vdash (\bot, \Delta_3), \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}\,\mathtt{ax} \qquad \dfrac{}{\bullet\mathtt{h}_5 :\vdash \Delta_8, \mathit{dual}(\mathtt{F}_4), \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{ax}}{- :\vdash \Delta_3, \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash \Delta_3, \Delta_8, \bot, \mathtt{F}_6 \& \mathtt{F}_7}\,\bot$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\,\bot \qquad \dfrac{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_5}{\bullet\mathtt{h}_3 :\vdash \mathit{dual}(\bot), \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\,\oplus_B}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_5}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\,\oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\,\bot \qquad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_B}{- :\vdash (\bot, \Delta_3), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_3, \Delta_8, \mathtt{F}_7, \bot}\,\mathtt{hCut}}{- :\vdash \Delta_3, \Delta_8, \bot, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\,\bot \qquad \dfrac{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_4}{\bullet\mathtt{h}_3 :\vdash \mathit{dual}(\bot), \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\,\oplus_A}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_6, \mathtt{F}_4}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\,\oplus_A$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\,\bot \qquad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathit{dual}(\mathtt{F}_4)}{\bullet\mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_A}{- :\vdash (\bot, \Delta_3), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathit{dual}(\mathtt{F}_4)}\,\mathtt{ax}}{- :\vdash \Delta_3, \Delta_8, \mathtt{F}_6, \bot}\,\mathtt{hCut}}{- :\vdash \Delta_3, \Delta_8, \bot, \mathtt{F}_6 \oplus \mathtt{F}_7}\,\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet\mathtt{h}_1 :\vdash \bot, \Delta_2}\,\bot \qquad \dfrac{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_4}{\bullet\mathtt{h}_3 :\vdash \mathit{dual}(\bot), \bot, \Delta_4}\,\bot}{- :\vdash \Delta_2, \bot, \Delta_4}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \bot}\,\mathtt{ax} \qquad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_4}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_4}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_4, \bot}\,\bot$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\ \bot \qquad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \bot, \Delta_6}\ \bot}{- :\vdash (\bot, \Delta_3), \bot, \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax} \qquad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \Delta_6, \bot, \mathit{dual}(\mathtt{F}_4)}\ \mathtt{ax}}{- :\vdash \Delta_3, \Delta_6, \bot}\ \mathtt{hCut}$$

$$\dfrac{- :\vdash \Delta_3, \Delta_6, \bot}{- :\vdash \Delta_3, \Delta_6, \bot, \bot}\ \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet \mathtt{h}_1 :\vdash \bot, \Delta_2}\ \bot \qquad \dfrac{}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\bot), \top, \Delta_4}\ \top}{- :\vdash \Delta_2, \top, \Delta_4}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \Delta_2, \Delta_4, \top}\ \top$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\ \bot \qquad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \top, \Delta_6}\ \top}{- :\vdash (\bot, \Delta_3), \top, \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \Delta_3, \Delta_6, \bot, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet \mathtt{h}_1 :\vdash \bot, \Delta_2}\ \bot \qquad \dfrac{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_7, \mathtt{F}_5 \quad \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\bot), \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \bot}\ \mathtt{ax} \qquad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_7, \mathtt{F}_5}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, \mathtt{F}_5}\ \mathtt{hCut} \qquad \dfrac{}{- :\vdash \Delta_4, \mathtt{F}_6}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2}{\bullet \mathtt{h}_1 :\vdash \bot, \Delta_2}\ \bot \qquad \dfrac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5 \quad \mathtt{h}_3 :\vdash \mathbf{1}, \Delta_7, \mathtt{F}_6}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\bot), \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \qquad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \bot}\ \mathtt{ax} \qquad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{1}, \Delta_7, \mathtt{F}_6}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\ \bot \qquad \dfrac{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_8}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash (\bot, \Delta_3), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax} \qquad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \Delta_6, \Delta_9, \mathit{dual}(\mathtt{F}_4), \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \bot, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \bot$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \bot, \Delta_3}\ \bot \qquad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_7 \quad \mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash (\bot, \Delta_3), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax} \qquad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \Delta_6, \Delta_9, \mathit{dual}(\mathtt{F}_4), \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{ax}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \bot, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \bot$$

## 5.11 Status of $\top$: OK

- Case rule **1**

- Case rule !

- Case rule ?$W$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_5}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \Delta_5, ?\mathtt{F}_4}\ ?W}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top, ?\mathtt{F}_4}\ \top \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_5}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?W}{- :\vdash (\top, \Delta_3), \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \Delta_3, \Delta_7, \top, ?\mathtt{F}_6}\ \top$$

- Case rule ?$C$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \Delta_5, ?\mathtt{F}_4}\ ?C}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \top \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4}\ \mathtt{ax}}{\dfrac{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4, ?\mathtt{F}_4}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\ ?C}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, ?\mathtt{F}_6, ?\mathtt{F}_6, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?C}{- :\vdash (\top, \Delta_3), \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \Delta_3, \Delta_7, \top, ?\mathtt{F}_6}\ \top$$

- Case rule ?

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_5, \mathtt{F}_4}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \Delta_5, ?\mathtt{F}_4}\ ?}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_5, \mathtt{F}_4}\ \mathtt{ax}}{\dfrac{- :\vdash \Delta_2, \Delta_5, \mathtt{F}_4}{- :\vdash \Delta_2, \Delta_5, ?\mathtt{F}_4}\ ?}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_7, \mathtt{F}_6, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_7, ?\mathtt{F}_6}\ ?}{- :\vdash (\top, \Delta_3), \Delta_7, ?\mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- :\vdash \Delta_3, \Delta_7, \top, ?\mathtt{F}_6}\ \top$$

- Case rule \$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_4, \mathtt{F}_5}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \Delta_6, \mathtt{F}_4 \$ \mathtt{F}_5}\ \$}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \$ \mathtt{F}_5}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \top \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{hCut} \quad \rightarrow \quad - :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \$ \mathtt{F}_5\ \$$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\ \$}{- :\vdash (\top, \Delta_3), \Delta_8, \mathtt{F}_6 \$ \mathtt{F}_7}\ \mathtt{Cut} \quad \rightarrow \quad \dfrac{}{- :\vdash \Delta_3, \Delta_8, \top, \mathtt{F}_6 \$ \mathtt{F}_7}\ \top$$

- Case rule &

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_4 \quad \mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \Delta_6, \mathtt{F}_4 \& \mathtt{F}_5}\ \&}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \& \mathtt{F}_5}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \top \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_4}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4}\ \mathtt{hCut} \quad \rightarrow \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \top \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_5}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_5}\ \mathtt{hCut} \quad - :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \& \mathtt{F}_5\ \&$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathit{dual}(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\ \&}{- :\vdash (\top, \Delta_3), \Delta_8, \mathtt{F}_6 \& \mathtt{F}_7}\ \mathtt{Cut} \quad \rightarrow \quad \dfrac{}{- :\vdash \Delta_3, \Delta_8, \top, \mathtt{F}_6 \& \mathtt{F}_7}\ \top$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_5}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_B}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_5}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_5}\ \mathtt{hCut} \quad \rightarrow \quad - :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5\ \oplus_B$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_B}{- :\vdash (\top, \Delta_3), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut} \quad \rightarrow \quad \dfrac{}{- :\vdash \Delta_3, \Delta_8, \top, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \top$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_4}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \oplus_A}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \top \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_6, \mathtt{F}_4}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_6, \mathtt{F}_4}\ \mathtt{hCut} \quad \rightarrow \quad - :\vdash \Delta_2, \Delta_6, \mathtt{F}_4 \oplus \mathtt{F}_5\ \oplus_A$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_8, \mathtt{F}_6, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \oplus_A}{- :\vdash (\top, \Delta_3), \Delta_8, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \mathtt{Cut} \quad \rightarrow \quad \dfrac{}{- :\vdash \Delta_3, \Delta_8, \top, \mathtt{F}_6 \oplus \mathtt{F}_7}\ \top$$

- Case rule $\bot$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_4}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \bot, \Delta_4}\ \bot}{- :\vdash \Delta_2, \bot, \Delta_4}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \bot, \top}\ \top \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_4}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_4, \bot}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \bot, \Delta_6}\ \bot}{- :\vdash (\top, \Delta_3), \bot, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \Delta_3, \Delta_6, \bot, \top}\ \top$$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \top, \Delta_4}\ \top}{- :\vdash \Delta_2, \top, \Delta_4}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \Delta_2, \Delta_4, \top}\ \top$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \top, \Delta_6}\ \top}{- :\vdash (\top, \Delta_3), \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \Delta_3, \Delta_6, \top, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_7, \mathtt{F}_5 \quad \mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_6}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_7, \mathtt{F}_5}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_4, \mathtt{F}_6}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \top, \Delta_2}\ \top \quad \dfrac{\mathtt{h}_3 :\vdash \Delta_4, \mathtt{F}_5 \quad \mathtt{h}_3 :\vdash \mathbf{0}, \Delta_7, \mathtt{F}_6}{\bullet \mathtt{h}_3 :\vdash \mathit{dual}(\top), \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \top}\ \top \quad \dfrac{}{\mathtt{h}_3 :\vdash \mathbf{0}, \Delta_7, \mathtt{F}_6}\ \mathtt{ax}}{- :\vdash \Delta_2, \Delta_7, \mathtt{F}_6}\ \mathtt{hCut}}{- :\vdash \Delta_2, \Delta_4, \Delta_7, \mathtt{F}_5 \otimes \mathtt{F}_6}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_7, \mathit{dual}(\mathtt{F}_4) \quad \mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_8}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash (\top, \Delta_3), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \top, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \top$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_4, \top, \Delta_3}\ \top \quad \dfrac{\mathtt{h}_5 :\vdash \Delta_6, \mathtt{F}_7 \quad \mathtt{h}_5 :\vdash \Delta_9, \mathtt{F}_8, \mathit{dual}(\mathtt{F}_4)}{\bullet \mathtt{h}_5 :\vdash \mathit{dual}(\mathtt{F}_4), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \otimes}{- :\vdash (\top, \Delta_3), \Delta_6, \Delta_9, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \top, \mathtt{F}_7 \otimes \mathtt{F}_8}\ \top$$

## 5.12 Status of $I$: OK

- Case rule **1**

- Case rule !

- Case rule ?$W$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_6, \hat{}(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?W}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_6, ?\mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_6, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, \hat{}(\mathtt{n}_3)}\ W$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_6, p(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?W}{- :\vdash p(\mathtt{n}_3), \Delta_6, ?\mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_6, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, p(\mathtt{n}_3)}\ ?W$$

- Case rule ?$C$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, \hat{}(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?C}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_6, ?\mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, \hat{}(\mathtt{n}_3)}\ ?C$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, p(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?C}{- :\vdash p(\mathtt{n}_3), \Delta_6, ?\mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_6, ?\mathtt{F}_5, ?\mathtt{F}_5, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, p(\mathtt{n}_3)}\ ?C$$

- Case rule ?

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_5, \hat{}(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_6, ?\mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_6, \mathtt{F}_5, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, \hat{}(\mathtt{n}_3)}\ ?$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_6, \mathtt{F}_5, p(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_6, ?\mathtt{F}_5}\ ?}{- :\vdash p(\mathtt{n}_3), \Delta_6, ?\mathtt{F}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_6, \mathtt{F}_5, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_6, ?\mathtt{F}_5, p(\mathtt{n}_3)}\ ?$$

- Case rule \$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \hat{}(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, \hat{}(\mathtt{n}_3), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, p(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\ \$}{- :\vdash p(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \$ \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, \mathtt{F}_6, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, p(\mathtt{n}_3), \mathtt{F}_5 \$ \mathtt{F}_6}\ \$$$

- Case rule &

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_5, \hat{}(\mathtt{n}_3) \quad \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_6, \hat{}(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\ \&}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, \hat{}(\mathtt{n}_3)}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_7, \mathtt{F}_6, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, \hat{}(\mathtt{n}_3), \mathtt{F}_5 \& \mathtt{F}_6}\ \&$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_5, p(\mathtt{n}_3) \quad \mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_6, p(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\ \&}{- :\vdash p(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \& \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, p(\mathtt{n}_3)}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_7, \mathtt{F}_6, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, p(\mathtt{n}_3), \mathtt{F}_5 \& \mathtt{F}_6}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_6, \hat{}(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_6, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, \hat{}(\mathtt{n}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_6, p(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B}{- :\vdash p(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_6, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, p(\mathtt{n}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_5, \hat{}(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, \hat{}(\mathtt{n}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_7, \mathtt{F}_5, p(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A}{- :\vdash p(\mathtt{n}_3), \Delta_7, \mathtt{F}_5 \oplus \mathtt{F}_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_7, \mathtt{F}_5, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_7, p(\mathtt{n}_3), \mathtt{F}_5 \oplus \mathtt{F}_6}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_5, \hat{}(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \bot, \Delta_5}\ \bot}{- :\vdash \hat{}(\mathtt{n}_3), \bot, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_5, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_5, \bot, \hat{}(\mathtt{n}_3)}\ \bot$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_5, p(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \bot, \Delta_5}\ \bot}{- :\vdash p(\mathtt{n}_3), \bot, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_5, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_5, \bot, p(\mathtt{n}_3)}\ \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \top, \Delta_5}\ \top}{- :\vdash \hat{}(\mathtt{n}_3), \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \Delta_5, \top, \hat{}(\mathtt{n}_3)}\ \top$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \top, \Delta_5}\ \top}{- :\vdash p(\mathtt{n}_3), \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash \Delta_5, \top, p(\mathtt{n}_3)}\ \top$$

- Case rule $I$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I \quad \dfrac{}{\bullet \mathtt{h}_3 :\vdash dual(p(\mathtt{n}_4)), p(\mathtt{n}_4)}\ I}{- :\vdash \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ \mathtt{ax}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_4), p(\mathtt{n}_4)}\ I \quad \dfrac{}{\bullet \mathtt{h}_3 :\vdash dual(\hat{}(\mathtt{n}_4)), \hat{}(\mathtt{n}_4)}\ I}{- :\vdash p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- :\vdash p(\mathtt{n}_4), \hat{}(\mathtt{n}_4)}\ I$$

- Case rule $\otimes$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_8, \mathtt{F}_6, \hat{}(\mathtt{n}_3) \quad \mathtt{h}_4 :\vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_5, \Delta_8, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_5, \Delta_8, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_8, \mathtt{F}_6, \hat{}(\mathtt{n}_3)}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_5, \mathtt{F}_7}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_8, \hat{}(\mathtt{n}_3), \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash p(\mathtt{n}_3), \hat{}(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_5, \mathtt{F}_6 \quad \mathtt{h}_4 :\vdash \Delta_8, \mathtt{F}_7, \hat{}(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(p(\mathtt{n}_3)), \Delta_5, \Delta_8, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes}{- :\vdash \hat{}(\mathtt{n}_3), \Delta_5, \Delta_8, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- :\vdash \Delta_5, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_7, \hat{}(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_8, \hat{}(\mathtt{n}_3), \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_8, \mathtt{F}_6, p(\mathtt{n}_3) \quad \mathtt{h}_4 :\vdash \Delta_5, \mathtt{F}_7}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_5, \Delta_8, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes}{- :\vdash p(\mathtt{n}_3), \Delta_5, \Delta_8, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_8, \mathtt{F}_6, p(\mathtt{n}_3)}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_5, \mathtt{F}_7}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_8, p(\mathtt{n}_3), \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \hat{}(\mathtt{n}_3), p(\mathtt{n}_3)}\ I \quad \dfrac{\mathtt{h}_4 :\vdash \Delta_5, \mathtt{F}_6 \quad \mathtt{h}_4 :\vdash \Delta_8, \mathtt{F}_7, p(\mathtt{n}_3)}{\bullet \mathtt{h}_4 :\vdash dual(\hat{}(\mathtt{n}_3)), \Delta_5, \Delta_8, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes}{- :\vdash p(\mathtt{n}_3), \Delta_5, \Delta_8, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_5, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_7, p(\mathtt{n}_3)}\ \mathtt{ax}}{- :\vdash \Delta_5, \Delta_8, p(\mathtt{n}_3), \mathtt{F}_6 \otimes \mathtt{F}_7}\ \otimes$$

## 5.13 Status of $\otimes$: OK

- Case rule **1**
- Case rule !
- Case rule ?$W$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{10}, dual(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash dual(\mathtt{F}_7), \Delta_{10}, ?\mathtt{F}_9}\ ?W}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{10}, ?\mathtt{F}_9}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{10}, dual(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\ ?W$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{10}, dual(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash dual(\mathtt{F}_7), \Delta_{10}, ?\mathtt{F}_9}\ ?W}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{10}, ?\mathtt{F}_9}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{10}, dual(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\ ?W$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet \mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3}\ \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{10}, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash dual(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_{10}, ?\mathtt{F}_9}\ ?W}{- :\vdash (\Delta_2, \Delta_3), \Delta_{10}, ?\mathtt{F}_9}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{10}, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \Delta_3}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathtt{F}_9}\ ?W$$

- Case rule ?$C$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{10}, ?\mathtt{F}_9, ?\mathtt{F}_9, dual(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash dual(\mathtt{F}_7), \Delta_{10}, ?\mathtt{F}_9}\ ?C}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{10}, ?\mathtt{F}_9}\ \mathtt{Cut}$$

$$\xrightarrow{\quad}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{10}, ?\mathtt{F}_9, ?\mathtt{F}_9, dual(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathtt{F}_9, ?\mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\ ?C$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{10}, ?\mathtt{F}_9, ?\mathtt{F}_9, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_{10}, ?\mathtt{F}_9}\,?C}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{10}, ?\mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{10}, ?\mathtt{F}_9, ?\mathtt{F}_9, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathtt{F}_9, ?\mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,?C$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{10}, ?\mathtt{F}_9, ?\mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_{10}, ?\mathtt{F}_9}\,?C}{- :\vdash (\Delta_2, \Delta_3), \Delta_{10}, ?\mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{10}, ?\mathtt{F}_9, ?\mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathtt{F}_9, ?\mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathtt{F}_9}\,?C$$

- Case rule ?

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_{10}, ?\mathtt{F}_9}\,?}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{10}, ?\mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, \mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,?$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_{10}, ?\mathtt{F}_9}\,?}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{10}, ?\mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, \mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_3, \Delta_6, ?\mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,?$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_{10}, ?\mathtt{F}_9}\,?}{- :\vdash (\Delta_2, \Delta_3), \Delta_{10}, ?\mathtt{F}_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{10}, \Delta_2, \Delta_3, \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{10}, \Delta_2, \Delta_3, ?\mathtt{F}_9}\,?$$

- Case rule $

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_{11}, \mathtt{F}_9\$\mathtt{F}_{10}}\,\$}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{11}, \mathtt{F}_9\$\mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_9\$\mathtt{F}_{10}, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_{11}, \mathtt{F}_9\$\mathtt{F}_{10}}\,\$}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{11}, \mathtt{F}_9\$\mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_9\$\mathtt{F}_{10}, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet \mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_{11}, \mathtt{F}_9\$\mathtt{F}_{10}}\ \$}{- :\vdash (\Delta_2, \Delta_3), \Delta_{11}, \mathtt{F}_9\$\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_{10}, \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_9\$\mathtt{F}_{10}}\ \$}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet \mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6), \mathit{dual}(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_9}\ \$}{- :\vdash (\Delta_2, \Delta_3), \Delta_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash \Delta_2, \mathtt{F}_6}\ \mathtt{ax} \quad \dfrac{\dfrac{}{- :\vdash \Delta_3, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{- :\vdash \Delta_9, \mathit{dual}(\mathtt{F}_6), \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_3, \Delta_9, \mathit{dual}(\mathtt{F}_6)}\ \mathtt{sCut}}{- :\vdash \Delta_2, \Delta_3, \Delta_9}\ \mathtt{sCut}}$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_{11}, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \&}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{11}, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_4, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\bullet \mathtt{h}_8 :\vdash \Delta_{11}, \mathit{dual}(\mathtt{F}_7), \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_6, \mathtt{F}_4, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_9 \& \mathtt{F}_{10}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_{11}, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \&}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{11}, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_5, \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\bullet \mathtt{h}_8 :\vdash \Delta_{11}, \mathit{dual}(\mathtt{F}_7), \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_6, \mathtt{F}_5, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_9 \& \mathtt{F}_{10}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet \mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_{11}, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \&}{- :\vdash (\Delta_2, \Delta_3), \Delta_{11}, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_9}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_6)\$\mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_{10}}\ \mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_9 \& \mathtt{F}_{10}}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\ \oplus_B}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\ \oplus_B$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}{\bullet \mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\ \oplus_B}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}\ \mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\ \oplus_B$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash dual(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_B}{- :\vdash (\Delta_2, \Delta_3), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_{10}, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_{10}}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash dual(\mathtt{F}_7), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_A}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_A$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash dual(\mathtt{F}_7), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_A}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_A$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash dual(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_A}{- :\vdash (\Delta_2, \Delta_3), \Delta_{11}, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{11}, \mathtt{F}_9, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash \Delta_{11}, \Delta_2, \Delta_3, \mathtt{F}_9 \oplus \mathtt{F}_{10}}\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_9, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash dual(\mathtt{F}_7), \bot, \Delta_9}\bot}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \bot, \Delta_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_9, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \bot, \mathtt{F}_4 \otimes \mathtt{F}_5}\bot$$

$$\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_9, dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash dual(\mathtt{F}_7), \bot, \Delta_9}\bot}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \bot, \Delta_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_9, dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \mathtt{F}_4 \otimes \mathtt{F}_5}\,\mathtt{hCut}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \bot, \mathtt{F}_4 \otimes \mathtt{F}_5}\bot$$

$$\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_9, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash dual(\mathtt{F}_6 \otimes \mathtt{F}_7), \bot, \Delta_9}\bot}{- :\vdash (\Delta_2, \Delta_3), \bot, \Delta_9}\,\mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_9, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}\,\mathtt{ax}}{- :\vdash \Delta_2, \Delta_3, \Delta_9}\,\mathtt{hCut}}{- :\vdash \Delta_2, \Delta_3, \Delta_9, \bot}\bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \top, \Delta_9} \top}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \top, \Delta_9}\ \mathtt{Cut}}{\rightarrow}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \top, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \top$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \top, \Delta_9} \top}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \top, \Delta_9}\ \mathtt{Cut}}{\rightarrow}}{- :\vdash \Delta_3, \Delta_6, \Delta_9, \top, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \top$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3} \otimes \quad \dfrac{}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_6 \otimes \mathtt{F}_7), \top, \Delta_9} \top}{- :\vdash (\Delta_2, \Delta_3), \top, \Delta_9}\ \mathtt{Cut}}{\rightarrow}}{- :\vdash \Delta_2, \Delta_3, \Delta_9, \top}\ \top$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \Delta_9, \mathtt{F}_{11}}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}} \otimes}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\ \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}{}\ \mathtt{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_9, \mathtt{F}_{11}}\ \mathtt{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, \mathtt{F}_{10} \otimes \mathtt{F}_{11}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_9, \mathtt{F}_{10} \quad \mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}} \otimes}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\ \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- :\vdash \Delta_9, \mathtt{F}_{10}}\ \mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}{}\ \mathtt{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \mathtt{F}_{11}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{hCut}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, \mathtt{F}_{10} \otimes \mathtt{F}_{11}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \Delta_9, \mathtt{F}_{11}}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}} \otimes}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\ \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{10}, \mathit{dual}(\mathtt{F}_7)}{}\ \mathtt{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \mathtt{F}_{10}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_9, \mathtt{F}_{11}}\ \mathtt{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, \mathtt{F}_{10} \otimes \mathtt{F}_{11}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 :\vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash \mathtt{F}_7, \Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5} \otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_9, \mathtt{F}_{10} \quad \mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash \mathit{dual}(\mathtt{F}_7), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}} \otimes}{- :\vdash (\Delta_3, \Delta_6, \mathtt{F}_4 \otimes \mathtt{F}_5), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\ \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- :\vdash \Delta_9, \mathtt{F}_{10}}\ \mathtt{ax} \quad \dfrac{\bullet\mathtt{h}_2 :\vdash \Delta_3, \Delta_6, \mathtt{F}_7, \mathtt{F}_4 \otimes \mathtt{F}_5}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{11}, \mathit{dual}(\mathtt{F}_7)}{}\ \mathtt{ax}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \mathtt{F}_{11}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \mathtt{hCut}}{- :\vdash \Delta_{12}, \Delta_3, \Delta_6, \Delta_9, \mathtt{F}_{10} \otimes \mathtt{F}_{11}, \mathtt{F}_4 \otimes \mathtt{F}_5}\ \otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{10}, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7) \quad \mathtt{h}_8 :\vdash \Delta_9, \mathtt{F}_{11}}{\bullet\mathtt{h}_8 :\vdash dual(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{10}, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_{10}}\mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_9, \mathtt{F}_{11}}\mathtt{ax}}{- :\vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \Delta_2, \mathtt{F}_6 \quad \mathtt{h}_1 :\vdash \Delta_3, \mathtt{F}_7}{\bullet\mathtt{h}_1 :\vdash \mathtt{F}_6 \otimes \mathtt{F}_7, \Delta_2, \Delta_3}\otimes \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_9, \mathtt{F}_{10} \quad \mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{11}, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}{\bullet\mathtt{h}_8 :\vdash dual(\mathtt{F}_6 \otimes \mathtt{F}_7), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes}{- :\vdash (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- :\vdash \Delta_9, \mathtt{F}_{10}}\mathtt{ax} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 :\vdash \Delta_2, \Delta_3, \mathtt{F}_6 \otimes \mathtt{F}_7}\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_{12}, \mathtt{F}_{11}, dual(\mathtt{F}_6)\$dual(\mathtt{F}_7)}\mathtt{ax}}{- :\vdash \Delta_{12}, \Delta_2, \Delta_3, \mathtt{F}_{11}}\mathtt{hCut}}{- :\vdash \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, \mathtt{F}_{10} \otimes \mathtt{F}_{11}}\otimes$$

# 6 Cut-Elimination

## 6.1 Status of 1: OK

- Case rule **1**
- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$

## 6.2 Status of !: OK

- Case rule **1**
- Case rule !

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\,!\quad \dfrac{\mathtt{h}_6 :\vdash \mathtt{F}_7, ?\Upsilon 8, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{\bullet \mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), ?\Upsilon 8, !\mathtt{F}_7}\,!}{- :\vdash ?\Upsilon 2, ?\Upsilon 8, !\mathtt{F}_7}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash ?\Upsilon 8, \mathtt{F}_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 2, ?\Upsilon 8, \mathtt{F}_7}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 2, ?\Upsilon 8, !\mathtt{F}_7}\,!$$

- Case rule $?W$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{\bullet \mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), \Delta_8, ?\mathtt{F}_7}\ ?W}{- :\vdash ?\Upsilon 2, \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_8, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_8}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_8, ?\mathtt{F}_7}\ ?W$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_5, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_5, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_4, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet \mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_4}\ ?W}{- :\vdash ?\Upsilon 2, \Delta_4}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_4, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_4}\ \mathtt{hCut}$$

- Case rule $?C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, ?\mathtt{F}_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{\bullet \mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), \Delta_8, ?\mathtt{F}_7}\ ?C}{- :\vdash ?\Upsilon 2, \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_8, ?\mathtt{F}_7, ?\mathtt{F}_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_8, ?\mathtt{F}_7, ?\mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_8, ?\mathtt{F}_7}\ ?C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_5, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_5, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_4, ?dual(\mathtt{F}_5), ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet \mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_4}\ ?C}{- :\vdash ?\Upsilon 2, \Delta_4}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_4, ?dual(\mathtt{F}_5), ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_4}\ \mathtt{hCut}$$

- Case rule $?$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_8, \mathtt{F}_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{\bullet \mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), \Delta_8, ?\mathtt{F}_7}\ ?}{- :\vdash ?\Upsilon 2, \Delta_8, ?\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_8, \mathtt{F}_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_8, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_8, ?\mathtt{F}_7}\ ?$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_5, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_5, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_4, dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet \mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_4}\ ?}{- :\vdash ?\Upsilon 2, \Delta_4}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- :\vdash ?\Upsilon 2, \mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_5}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_4, dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_4, dual(\mathtt{F}_5)}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_4}\ \mathtt{mCut}$$

- Case rule $\$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet \mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{\bullet \mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\ \$}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}\ \mathtt{ax} \quad \dfrac{}{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, \mathtt{F}_8, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7, \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7\$\mathtt{F}_8}\ \$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet\mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{\bullet\mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), \Delta_9, \mathtt{F}_7 \& \mathtt{F}_8}\ \mathtt{Cut} \quad \mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7 \& \mathtt{F}_8}\ \&}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_8}\ \mathtt{hCu}}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7 \& \mathtt{F}_8}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet\mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{\bullet\mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_8}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet\mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{\bullet\mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_9, \mathtt{F}_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_9, \mathtt{F}_7 \oplus \mathtt{F}_8}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet\mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{\bullet\mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), \bot, \Delta_7}\ \bot}{- :\vdash ?\Upsilon 2, \bot, \Delta_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_1 :\vdash ?\Upsilon 2, !\mathtt{F}_4}{}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_6 :\vdash \Delta_7, ?dual(\mathtt{F}_4), contract(\mathtt{n}_5, ?dual(\mathtt{F}_4))}{}\ \mathtt{ax}}{- :\vdash ?\Upsilon 2, \Delta_7}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 2, \Delta_7, \bot}\ \bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 :\vdash \mathtt{F}_4, ?\Upsilon 2}{\bullet\mathtt{h}_1 :\vdash !\mathtt{F}_4, ?\Upsilon 2}\ !\quad \dfrac{}{\bullet\mathtt{h}_6 :\vdash contract(s\mathtt{n}_5, ?dual(\mathtt{F}_4)), \top, \Delta_7}\ \top}{- :\vdash ?\Upsilon 2, \top, \Delta_7}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- :\vdash ?\Upsilon 2, \Delta_7, \top}\ \top$$

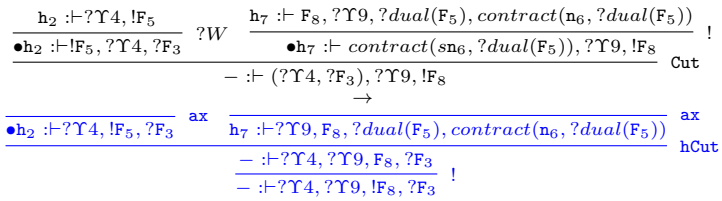
- Case rule $I$

- Case rule $\otimes$

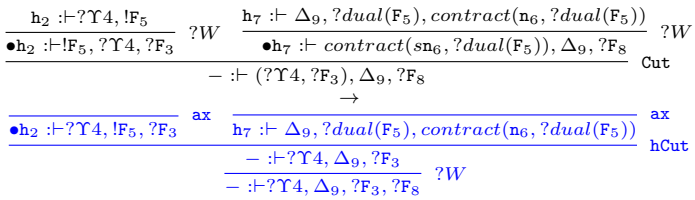
## 6.3 Status of $?W$: OK

- Case rule **1**

- Case rule !



- Case rule $?W$



$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_6, ?\Upsilon 4, ?\mathtt{F}_3}\,?W \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_5, \mathit{contract}(\mathtt{n}_7, ?\mathit{dual}(\mathtt{F}_6))}{\bullet\mathtt{h}_8 :\vdash \mathit{contract}(\mathit{sn}_7, ?\mathit{dual}(\mathtt{F}_6)), \Delta_5}\,?W}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_5}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_5, \mathit{contract}(\mathtt{n}_7, ?\mathit{dual}(\mathtt{F}_6))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_5, ?\mathtt{F}_3}\,\mathtt{hCut}$$

- Case rule $?C$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?W \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, ?\mathit{dual}(\mathtt{F}_5), \mathit{contract}(\mathtt{n}_6, ?\mathit{dual}(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash \mathit{contract}(\mathit{sn}_6, ?\mathit{dual}(\mathtt{F}_5)), \Delta_9, ?\mathtt{F}_8}\,?C}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, ?\mathit{dual}(\mathtt{F}_5), \mathit{contract}(\mathtt{n}_6, ?\mathit{dual}(\mathtt{F}_5))}\,\mathtt{ax}}{\dfrac{- :\vdash ?\Upsilon 4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_8, ?\mathtt{F}_8}{- :\vdash ?\Upsilon 4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_8}\,?C}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_6, ?\Upsilon 4, ?\mathtt{F}_3}\,?W \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_5, ?\mathit{dual}(\mathtt{F}_6), ?\mathit{dual}(\mathtt{F}_6), \mathit{contract}(\mathtt{n}_7, ?\mathit{dual}(\mathtt{F}_6))}{\bullet\mathtt{h}_8 :\vdash \mathit{contract}(\mathit{sn}_7, ?\mathit{dual}(\mathtt{F}_6)), \Delta_5}\,?C}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_5}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_5, ?\mathit{dual}(\mathtt{F}_6), ?\mathit{dual}(\mathtt{F}_6), \mathit{contract}(\mathtt{n}_7, ?\mathit{dual}(\mathtt{F}_6))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_5, ?\mathtt{F}_3}\,\mathtt{hCut}$$

- Case rule $?$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?W \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, ?\mathit{dual}(\mathtt{F}_5), \mathit{contract}(\mathtt{n}_6, ?\mathit{dual}(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash \mathit{contract}(\mathit{sn}_6, ?\mathit{dual}(\mathtt{F}_5)), \Delta_9, ?\mathtt{F}_8}\,?}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, ?\mathit{dual}(\mathtt{F}_5), \mathit{contract}(\mathtt{n}_6, ?\mathit{dual}(\mathtt{F}_5))}\,\mathtt{ax}}{\dfrac{- :\vdash ?\Upsilon 4, \Delta_9, \mathtt{F}_8, ?\mathtt{F}_3}{- :\vdash ?\Upsilon 4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_8}\,?}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_6, ?\Upsilon 4, ?\mathtt{F}_3}\,?W \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_5, \mathit{dual}(\mathtt{F}_6), \mathit{contract}(\mathtt{n}_7, ?\mathit{dual}(\mathtt{F}_6))}{\bullet\mathtt{h}_8 :\vdash \mathit{contract}(\mathit{sn}_7, ?\mathit{dual}(\mathtt{F}_6)), \Delta_5}\,?}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_5}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_8 :\vdash \Delta_5, \mathit{dual}(\mathtt{F}_6), \mathit{contract}(\mathtt{n}_7, ?\mathit{dual}(\mathtt{F}_6))}\,\mathtt{ax}}{\bullet\mathtt{h}_8 :\vdash \Delta_5, ?\mathit{dual}(\mathtt{F}_6), \mathit{contract}(\mathtt{n}_7, ?\mathit{dual}(\mathtt{F}_6))}\,?}{\dfrac{- :\vdash ?\Upsilon 4, \Delta_5}{- :\vdash ?\Upsilon 4, \Delta_5, ?\mathtt{F}_3}\,?W}\,\mathtt{hCut}$$

- Case rule \$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?W \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, ?\mathit{dual}(\mathtt{F}_5), \mathit{contract}(\mathtt{n}_6, ?\mathit{dual}(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash \mathit{contract}(\mathit{sn}_6, ?\mathit{dual}(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut} \quad \rightarrow \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, ?\mathit{dual}(\mathtt{F}_5), \mathit{contract}(\mathtt{n}_6, ?\mathit{dual}(\mathtt{F}_5))}\,\mathtt{ax}}{\dfrac{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, ?\mathtt{F}_3}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8\$\mathtt{F}_9}\,\$}\,\mathtt{hCut}$$

- Case rule $\&$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8 \& \mathtt{F}_9}\ \mathtt{Cut} \quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8 \& \mathtt{F}_9}\ \&}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, ?dual(\mathtt{F}_5), \mathtt{F}_8 \& \mathtt{F}_9, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8 \& \mathtt{F}_9}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8 \& \mathtt{F}_9}\ ?W}\ \&$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_B}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_9, ?\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_A}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, ?\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \bot, \Delta_8}\ \bot}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\mathtt{h}_7 :\vdash \Delta_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_8, ?\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_8, \bot, ?\mathtt{F}_3}\ \bot$$

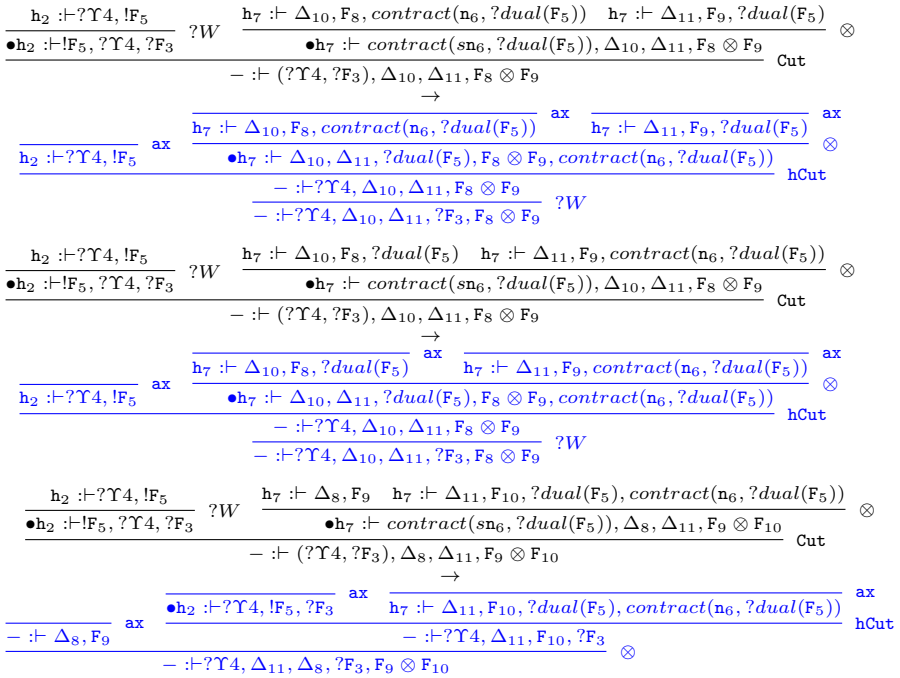
- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?W \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \top, \Delta_8}\ \top}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \top, \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- :\vdash ?\Upsilon 4, \Delta_8, \top, ?\mathtt{F}_3}\ \top}$$

- Case rule $I$

- Case rule $\otimes$

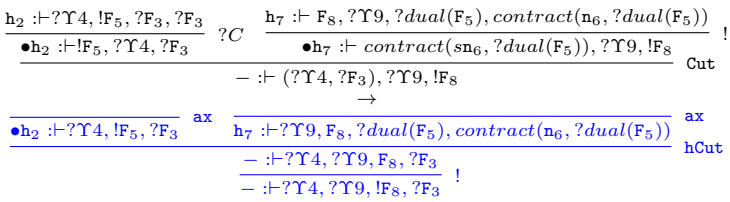
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?W \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \mathtt{Cut} \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \otimes}{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\ \mathtt{ax} \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{- :\vdash ?\Upsilon 4, \Delta_{11}, \mathtt{F}_9, ?\mathtt{F}_3}\ \mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?\mathtt{F}_3, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\ \otimes$$



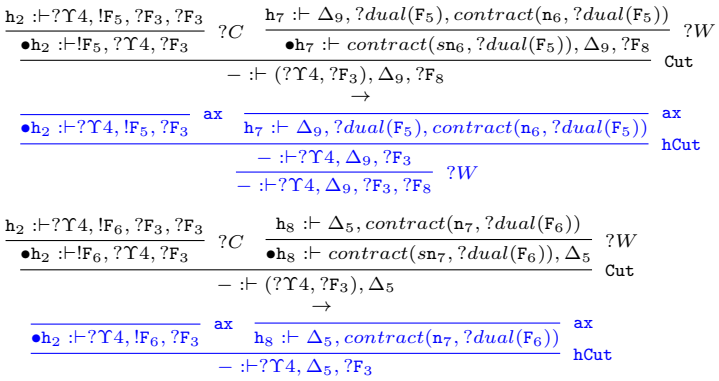
## 6.4 Status of $?C$: OK

- Case rule **1**

- Case rule !



- Case rule $?W$



- Case rule $?C$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_9, ?\mathtt{F}_8}\,?C}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, ?\mathtt{F}_8, ?\mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_8, ?\mathtt{F}_8}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_8}\,?C$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_6, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_5, ?dual(\mathtt{F}_6), ?dual(\mathtt{F}_6), contract(\mathtt{n}_7, ?dual(\mathtt{F}_6))}{\bullet\mathtt{h}_8 :\vdash contract(s\mathtt{n}_7, ?dual(\mathtt{F}_6)), \Delta_5}\,?C}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_5}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_8 :\vdash \Delta_5, ?dual(\mathtt{F}_6), ?dual(\mathtt{F}_6), contract(\mathtt{n}_7, ?dual(\mathtt{F}_6))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_5, ?\mathtt{F}_3}\,\mathtt{hCut}}$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_9, ?\mathtt{F}_8}\,?}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_9, ?\mathtt{F}_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_9, \mathtt{F}_8, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_8}\,?$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_6, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_8 :\vdash \Delta_5, dual(\mathtt{F}_6), contract(\mathtt{n}_7, ?dual(\mathtt{F}_6))}{\bullet\mathtt{h}_8 :\vdash contract(s\mathtt{n}_7, ?dual(\mathtt{F}_6)), \Delta_5}\,?}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_5}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_8 :\vdash \Delta_5, dual(\mathtt{F}_6), contract(\mathtt{n}_7, ?dual(\mathtt{F}_6))}\,\mathtt{ax}}{\bullet\mathtt{h}_8 :\vdash \Delta_5, ?dual(\mathtt{F}_6), contract(\mathtt{n}_7, ?dual(\mathtt{F}_6))}\,?}{- :\vdash ?\Upsilon 4, \Delta_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_5, ?\mathtt{F}_3}\,?C$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\$}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8\$\mathtt{F}_9}\,\$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5)) \quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\&}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, ?dual(\mathtt{F}_5), \mathtt{F}_8\&\mathtt{F}_9, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\&}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_8\&\mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8\&\mathtt{F}_9}\,?C$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_9, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_B$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \bot, \Delta_8}\,\bot}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \bot, \Delta_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_8, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_8, \bot, ?\mathtt{F}_3}\,\bot$$

- Case rule $\top$

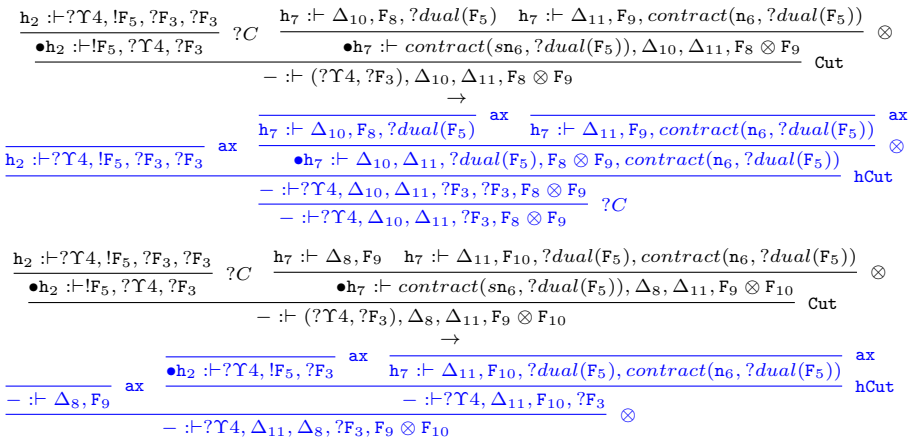
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \top, \Delta_8}\,\top}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \top, \Delta_8}\,\mathtt{Cut}}{- :\vdash ?\Upsilon 4, \Delta_8, \top, ?\mathtt{F}_3}\,\top$$

- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5)) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{11}, \mathtt{F}_9, ?\mathtt{F}_3}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?\mathtt{F}_3, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes$$

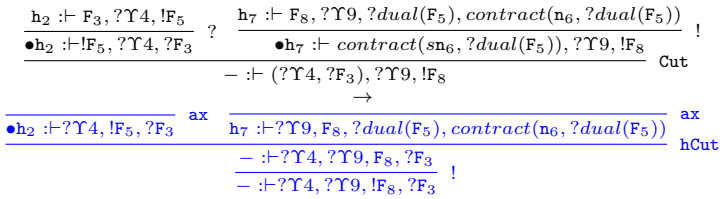
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\,?C \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5)) \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5)}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\otimes}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{\dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5)}\,\mathtt{ax}}{\bullet\mathtt{h}_7 :\vdash \Delta_{10}, \Delta_{11}, ?dual(\mathtt{F}_5), \mathtt{F}_8 \otimes \mathtt{F}_9, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\otimes}{- :\vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\,?C$$



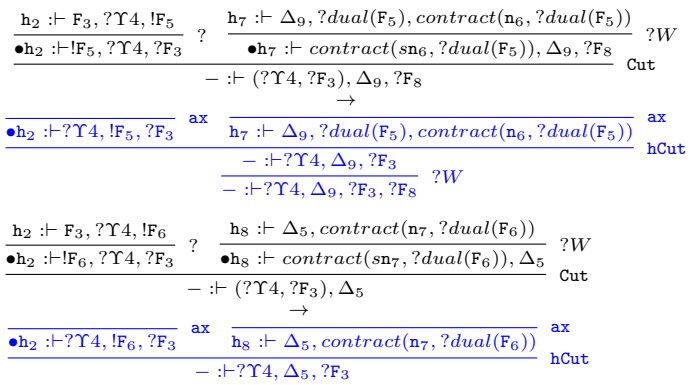
## 6.5 Status of ?: OK

- Case rule **1**

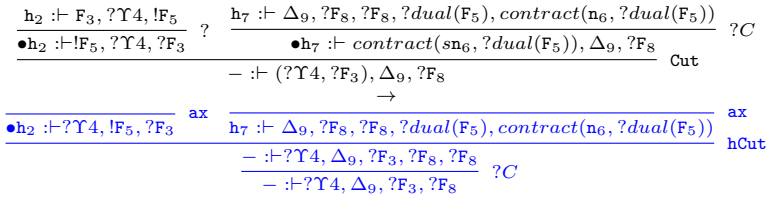
- Case rule !



- Case rule ?$W$



- Case rule ?$C$



$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_6}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_6, ?\Upsilon 4, ?\mathtt{F}_3}\ ?\quad \dfrac{\mathtt{h}_8 :\vdash \Delta_5, ?dual(\mathtt{F}_6), ?dual(\mathtt{F}_6), contract(\mathtt{n}_7, ?dual(\mathtt{F}_6))}{\bullet\mathtt{h}_8 :\vdash contract(s\mathtt{n}_7, ?dual(\mathtt{F}_6)), \Delta_5}\ ?C}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_5}\ \mathtt{Cut}}{\dfrac{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6, ?\mathtt{F}_3}{- :\vdash ?\Upsilon 4, \Delta_5, ?\mathtt{F}_3}\ \mathtt{hCut}}\ \mathtt{ax}\quad \dfrac{\mathtt{h}_8 :\vdash \Delta_5, ?dual(\mathtt{F}_6), ?dual(\mathtt{F}_6), contract(\mathtt{n}_7, ?dual(\mathtt{F}_6))}{}\ \mathtt{ax}$$

- Case rule ?

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_9, ?\mathtt{F}_8}\ ?}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_9, ?\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}{- :\vdash ?\Upsilon 4, \Delta_9, \mathtt{F}_8, ?\mathtt{F}_3}\ ?}{- :\vdash ?\Upsilon 4, \Delta_9, ?\mathtt{F}_3, ?\mathtt{F}_8}\ \mathtt{hCut}}\ \mathtt{ax}\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_9, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{}\ \mathtt{ax}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_6}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_6, ?\Upsilon 4, ?\mathtt{F}_3}\ ?\quad \dfrac{\mathtt{h}_8 :\vdash \Delta_5, dual(\mathtt{F}_6), contract(\mathtt{n}_7, ?dual(\mathtt{F}_6))}{\bullet\mathtt{h}_8 :\vdash contract(s\mathtt{n}_7, ?dual(\mathtt{F}_6)), \Delta_5}\ ?}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_5}\ \mathtt{Cut}}{\dfrac{\dfrac{- :\vdash ?\Upsilon 4, !\mathtt{F}_6, ?\mathtt{F}_3}{- :\vdash ?\Upsilon 4, \mathtt{F}_6, ?\mathtt{F}_3}\ bInv\quad \dfrac{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_6, ?\mathtt{F}_3}{}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_5, ?\mathtt{F}_3, dual(\mathtt{F}_6)}\ \mathtt{mCut}\quad \dfrac{\mathtt{h}_8 :\vdash \Delta_5, dual(\mathtt{F}_6), contract(\mathtt{n}_7, ?dual(\mathtt{F}_6))}{}\ \mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_5, ?\mathtt{F}_3}\ \mathtt{hCut}$$

- Case rule $

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\ \$}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8\$\mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, ?\mathtt{F}_3}\ \$}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8\$\mathtt{F}_9}\ \$}\ \mathtt{ax}\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{}\ \mathtt{ax}$$

- Case rule &

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))\quad \mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\ \&}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8\&\mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, ?\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8\&\mathtt{F}_9}\ \&}\ \mathtt{ax}\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{}\ \mathtt{ax}\quad \dfrac{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}{}\ \mathtt{ax}\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dua}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_9, ?\mathtt{F}_3}\ \mathtt{hCut}$$

- Case rule $\oplus_B$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet\mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ?\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet\mathtt{h}_7 :\vdash contract(s\mathtt{n}_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_B}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_9, ?\mathtt{F}_3}\ \mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8 \oplus \mathtt{F}_9}\ \oplus_B}\ \mathtt{ax}\quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{}\ \mathtt{ax}$$

- Case rule $\oplus_A$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet \mathtt{h}_7 :\vdash contract(sn_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\oplus_A}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_{10}, ?\mathtt{F}_3, \mathtt{F}_8 \oplus \mathtt{F}_9}\,\oplus_A$$

- Case rule $\bot$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet \mathtt{h}_7 :\vdash contract(sn_6, ?dual(\mathtt{F}_5)), \bot, \Delta_8}\bot}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \bot, \Delta_8}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_8, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_8, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, \Delta_8, \bot, ?\mathtt{F}_3}\,\bot$$

- Case rule $\top$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ? \quad \dfrac{}{\bullet \mathtt{h}_7 :\vdash contract(sn_6, ?dual(\mathtt{F}_5)), \top, \Delta_8}\top}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \top, \Delta_8}\,\mathtt{Cut}}{- :\vdash ?\Upsilon 4, \Delta_8, \top, ?\mathtt{F}_3}\,\top$$

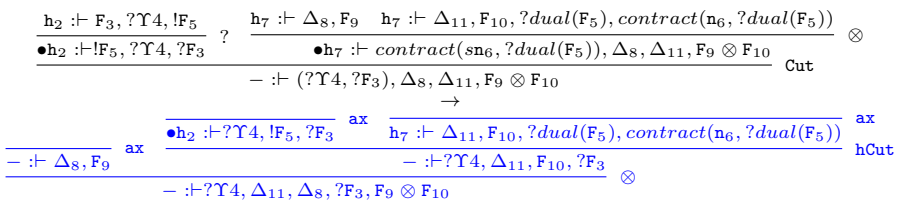
- Case rule $I$

- Case rule $\otimes$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5)) \quad \mathtt{h}_7 :\vdash \Delta_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_7 :\vdash contract(sn_6, ?dual(\mathtt{F}_5)), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\otimes}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_8, \Delta_{11}, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5), contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{11}, \mathtt{F}_9, ?\mathtt{F}_3}\,\mathtt{hCut} \quad \dfrac{}{- :\vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?\mathtt{F}_3, \mathtt{F}_9 \otimes \mathtt{F}_{10}}\,\otimes$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5)) \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5)}{\bullet \mathtt{h}_7 :\vdash contract(sn_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\otimes}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, ?\mathtt{F}_3}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, ?dual(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{11}, \mathtt{F}_9, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\otimes}{- :\vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\,C$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 :\vdash \mathtt{F}_3, ?\Upsilon 4, !\mathtt{F}_5}{\bullet \mathtt{h}_2 :\vdash !\mathtt{F}_5, ?\Upsilon 4, ?\mathtt{F}_3}\ ? \quad \dfrac{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5) \quad \mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}{\bullet \mathtt{h}_7 :\vdash contract(sn_6, ?dual(\mathtt{F}_5)), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\otimes}{- :\vdash (?\Upsilon 4, ?\mathtt{F}_3), \Delta_{10}, \Delta_{11}, \mathtt{F}_8 \otimes \mathtt{F}_9}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{10}, \mathtt{F}_8, ?dual(\mathtt{F}_5)}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{10}, \mathtt{F}_8, ?\mathtt{F}_3}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_2 :\vdash ?\Upsilon 4, !\mathtt{F}_5, ?\mathtt{F}_3}\,\mathtt{ax} \quad \dfrac{}{\mathtt{h}_7 :\vdash \Delta_{11}, \mathtt{F}_9, contract(\mathtt{n}_6, ?dual(\mathtt{F}_5))}\,\mathtt{ax}}{- :\vdash ?\Upsilon 4, \Delta_{11}, \mathtt{F}_9, ?\mathtt{F}_3}\,\mathtt{hCut}}{- :\vdash ?\Upsilon 4, ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?\mathtt{F}_3, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\otimes}{- :\vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?\mathtt{F}_3, \mathtt{F}_8 \otimes \mathtt{F}_9}\,C$$



## 6.6 Status of $: OK

- Case rule **1**
- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule $
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$

## 6.7 Status of &: OK

- Case rule **1**
- Case rule !

- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$

### 6.8 Status of $\oplus_B$: `OK`

- Case rule **1**
- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$

- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$

### 6.9 Status of $\oplus_A$: `OK`

- Case rule **1**
- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$

### 6.10 Status of $\perp$: OK

- Case rule **1**
- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\perp$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$

### 6.11 Status of $\top$: OK

- Case rule **1**
- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?

- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$

### 6.12 Status of $I$: `OK`

- Case rule **1**
- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$

- Case rule $I$
- Case rule $\otimes$

### 6.13 Status of $\otimes$: `OK`

- Case rule **1**
- Case rule !
- Case rule ?$W$
- Case rule ?$C$
- Case rule ?
- Case rule \$
- Case rule &
- Case rule $\oplus_B$
- Case rule $\oplus_A$
- Case rule $\bot$
- Case rule $\top$
- Case rule $I$
- Case rule $\otimes$