System mLJ

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Contents

1 Height preserving admissibility of weakening on the left

- Case(s) rule $\to_R$

$$
\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5} \to_R
\quad\rightsquigarrow\quad
\frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4, \mathtt{F}_x \vdash \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5} \to_R
$$

- Case(s) rule $\wedge_R$

$$
\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R
\quad\rightsquigarrow\quad
\frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \Delta_3, \mathtt{F}_4}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \Delta_3, \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R
$$

- Case(s) rule $\vee_R$

$$
\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R
\quad\rightsquigarrow\quad
\frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R
$$

- Case(s) rule $\bot_R$

$$
\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R
\quad\rightsquigarrow\quad
\frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \Delta_3}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \bot, \Delta_3} \bot_R
$$

- Case(s) rule $\top_R$

$$
\frac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R
\quad\rightsquigarrow\quad
\frac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \top, \Delta_3} \top_R
$$

- Case(s) rule $\to_L$

$$
\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L
\quad\rightsquigarrow\quad
\frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_3}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_x, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_3}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4, \mathtt{F}_x \vdash \Delta_5}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_x, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L
$$

- Case(s) rule $\wedge_L$

$$
\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L
\quad\rightsquigarrow\quad
\frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_x \vdash \Delta_5}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_x, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L
$$

- Case(s) rule $\vee_L$

$$
\frac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L
\quad\rightsquigarrow\quad
\frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_x \vdash \Delta_5}\ \text{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4, \mathtt{F}_x \vdash \Delta_5}\ \text{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_x, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L
$$

- Case(s) rule $\bot_L$

$$
\frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3} \bot_L
\quad\rightsquigarrow\quad
\frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_2, \mathtt{F}_x \vdash \Delta_3} \bot_L
$$

- Case(s) rule I

$$
\frac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3} I
\quad\rightsquigarrow\quad
\frac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_x, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3} I
$$

- Case(s) rule $\top_L$

$$
\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet\mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3} \top_L
\quad\rightsquigarrow\quad
\frac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \text{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_x \vdash \Delta_3}\ \text{IH}}{\bullet\mathtt{h}_1 : \top, \Delta_2, \mathtt{F}_x \vdash \Delta_3} \top_L
$$

2   Height preserving admissibility of weakening on the right

- Case(s) rule $\to_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5} \to_R \quad \rightsquigarrow \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \mathtt{F}_5}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_x, \mathtt{F}_4 \to \mathtt{F}_5} \to_R$$

- Case(s) rule $\wedge_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_x}\ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5, \mathtt{F}_x}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_x, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case(s) rule $\vee_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_x}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_x, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case(s) rule $\bot_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_x}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \mathtt{F}_x} \bot_R$$

- Case(s) rule $\top_R$

$$\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \rightsquigarrow \quad \dfrac{}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3, \mathtt{F}_x} \top_R$$

- Case(s) rule $\to_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_3 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_x}\ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_x}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_x} \to_L$$

- Case(s) rule $\wedge_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_x}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_x} \wedge_L$$

- Case(s) rule $\vee_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5 \quad \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_x}\ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_x}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_x} \vee_L$$

- Case(s) rule $\bot_L$

$$\dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3} \bot_L \quad \rightsquigarrow \quad \dfrac{}{\bullet\mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3, \mathtt{F}_x} \bot_L$$

- Case(s) rule I

$$\dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{P}_3 \vdash \Delta_4, \mathtt{P}_3} I \quad \rightsquigarrow \quad \dfrac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{P}_3 \vdash \Delta_4, \mathtt{F}_x, \mathtt{P}_3} I$$

- Case(s) rule $\top_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet\mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \quad \rightsquigarrow \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_x}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3, \mathtt{F}_x} \top_L$$

3 Measure of derivations

- Case(s) rule $\to_R$

$$\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \mathtt{F_5}}{\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4} \to \mathtt{F_5}} \; \to_R \qquad \rightsquigarrow \qquad \dfrac{\dfrac{\overline{\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \mathtt{F_5}}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \mathtt{F_5}} \; \mathtt{IH}}{\bullet\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4} \to \mathtt{F_5}} \; \to_R$$

- Case(s) rule $\wedge_R$

$$\dfrac{\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4} \quad \mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_5}}{\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4} \wedge \mathtt{F_5}} \; \wedge_R \qquad \rightsquigarrow \qquad \dfrac{\dfrac{\overline{\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4}} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4}} \; \mathtt{IH} \quad \dfrac{\overline{\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_5}} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_5}} \; \mathtt{IH}}{\bullet\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4} \wedge \mathtt{F_5}} \; \wedge_R$$

- Case(s) rule $\vee_R$

$$\dfrac{\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4}, \mathtt{F_5}}{\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4} \vee \mathtt{F_5}} \; \vee_R \qquad \rightsquigarrow \qquad \dfrac{\dfrac{\overline{\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4}, \mathtt{F_5}} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4}, \mathtt{F_5}} \; \mathtt{IH}}{\bullet\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4} \vee \mathtt{F_5}} \; \vee_R$$

- Case(s) rule $\perp_R$

$$\dfrac{\mathtt{h_1} : \Delta_2 \vdash \Delta_3}{\bullet\mathtt{h_1} : \Delta_2 \vdash \perp, \Delta_3} \; \perp_R \qquad \rightsquigarrow \qquad \dfrac{\dfrac{\overline{\mathtt{h_1} : \Delta_2 \vdash \Delta_3} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3} \; \mathtt{IH}}{\bullet\bullet\mathtt{h_1} : \Delta_2 \vdash \perp, \Delta_3} \; \perp_R$$

- Case(s) rule $\top_R$

$$\dfrac{}{\bullet\mathtt{h_1} : \Delta_2 \vdash \top, \Delta_3} \; \top_R \qquad \rightsquigarrow \qquad \dfrac{}{\bullet\bullet\mathtt{h_1} : \Delta_2 \vdash \top, \Delta_3} \; \top_R$$

- Case(s) rule $\to_L$

$$\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3} \to \mathtt{F_4} \vdash \Delta_5, \mathtt{F_3} \quad \mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \Delta_5}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \to \mathtt{F_4} \vdash \Delta_5} \; \to_L \qquad \rightsquigarrow \qquad \dfrac{\dfrac{\overline{\mathtt{h_1} : \Delta_2, \mathtt{F_3} \to \mathtt{F_4} \vdash \Delta_5, \mathtt{F_3}} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \to \mathtt{F_4} \vdash \Delta_5, \mathtt{F_3}} \; \mathtt{IH} \quad \dfrac{\overline{\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \Delta_5} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \Delta_5} \; \mathtt{IH}}{\bullet\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \to \mathtt{F_4} \vdash \Delta_5} \; \to_L$$

- Case(s) rule $\wedge_L$

$$\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \Delta_5}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \Delta_5} \; \wedge_L \qquad \rightsquigarrow \qquad \dfrac{\dfrac{\overline{\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \Delta_5} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3}, \mathtt{F_4} \vdash \Delta_5} \; \mathtt{IH}}{\bullet\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \wedge \mathtt{F_4} \vdash \Delta_5} \; \wedge_L$$

- Case(s) rule $\vee_L$

$$\dfrac{\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vdash \Delta_5 \quad \mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \Delta_5}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vee \mathtt{F_4} \vdash \Delta_5} \; \vee_L \qquad \rightsquigarrow \qquad \dfrac{\dfrac{\overline{\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vdash \Delta_5} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vdash \Delta_5} \; \mathtt{IH} \quad \dfrac{\overline{\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \Delta_5} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \Delta_5} \; \mathtt{IH}}{\bullet\bullet\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vee \mathtt{F_4} \vdash \Delta_5} \; \vee_L$$

- Case(s) rule $\perp_L$

$$\dfrac{}{\bullet\mathtt{h_1} : \perp, \Delta_2 \vdash \Delta_3} \; \perp_L \qquad \rightsquigarrow \qquad \dfrac{}{\bullet\bullet\mathtt{h_1} : \perp, \Delta_2 \vdash \Delta_3} \; \perp_L$$

- Case(s) rule I

$$\dfrac{}{\bullet\mathtt{h_1} : \Delta_2, \mathtt{p_3} \vdash \Delta_4, \mathtt{p_3}} \; I \qquad \rightsquigarrow \qquad \dfrac{}{\bullet\bullet\mathtt{h_1} : \Delta_2, \mathtt{p_3} \vdash \Delta_4, \mathtt{p_3}} \; I$$

- Case(s) rule $\top_L$

$$\dfrac{\mathtt{h_1} : \Delta_2 \vdash \Delta_3}{\bullet\mathtt{h_1} : \top, \Delta_2 \vdash \Delta_3} \; \top_L \qquad \rightsquigarrow \qquad \dfrac{\dfrac{\overline{\mathtt{h_1} : \Delta_2 \vdash \Delta_3} \; \mathtt{ax}}{\bullet\mathtt{h_1} : \Delta_2 \vdash \Delta_3} \; \mathtt{IH}}{\bullet\bullet\mathtt{h_1} : \top, \Delta_2 \vdash \Delta_3} \; \top_L$$

4 Invertibility of Rules

4.1 Status of $\to_R$: : Non invertible

- Case rule $\to_R$

$$\frac{\mathtt{h_3} : \Delta_4, \mathtt{F_5} \vdash \mathtt{F_6}}{\bullet \mathtt{h_3} : \Delta_4 \vdash (\Delta_7, \mathtt{F_1} \to \mathtt{F_2}), \mathtt{F_5} \to \mathtt{F_6}} \to_R \quad \rightsquigarrow \quad \overline{\bullet \mathtt{h_3} : \Delta_4, \mathtt{F_1} \vdash \mathtt{F_2}} \; \mathtt{fail}$$

$$\frac{\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \mathtt{F_5}}{\bullet \mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4} \to \mathtt{F_5}} \to_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \mathtt{F_5}} \; \mathtt{ax}}{\bullet \mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \mathtt{F_5}} \; \mathtt{H}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h_3} : \Delta_4 \vdash \Delta_7, \mathtt{F_5}, \mathtt{F_1} \to \mathtt{F_2} \quad \mathtt{h_3} : \Delta_4 \vdash \Delta_7, \mathtt{F_6}, \mathtt{F_1} \to \mathtt{F_2}}{\bullet \mathtt{h_3} : \Delta_4 \vdash (\Delta_7, \mathtt{F_1} \to \mathtt{F_2}), \mathtt{F_5} \wedge \mathtt{F_6}} \wedge_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} : \Delta_4, \mathtt{F_1} \vdash \mathtt{F_2}} \; \mathtt{ax/ind}}{\bullet \mathtt{h_3} : \Delta_4, \mathtt{F_1} \vdash \mathtt{F_2}} \; \mathtt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h_3} : \Delta_4 \vdash \Delta_7, \mathtt{F_5}, \mathtt{F_6}, \mathtt{F_1} \to \mathtt{F_2}}{\bullet \mathtt{h_3} : \Delta_4 \vdash (\Delta_7, \mathtt{F_1} \to \mathtt{F_2}), \mathtt{F_5} \vee \mathtt{F_6}} \vee_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} : \Delta_4, \mathtt{F_1} \vdash \mathtt{F_2}} \; \mathtt{ax/ind}}{\bullet \mathtt{h_3} : \Delta_4, \mathtt{F_1} \vdash \mathtt{F_2}} \; \mathtt{H}$$

- Case rule $\perp_R$

$$\frac{\mathtt{h_3} : \Delta_4 \vdash \Delta_5, \mathtt{F_1} \to \mathtt{F_2}}{\bullet \mathtt{h_3} : \Delta_4 \vdash \perp, \Delta_5, \mathtt{F_1} \to \mathtt{F_2}} \perp_R \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_3} : \Delta_4, \mathtt{F_1} \vdash \mathtt{F_2}} \; \mathtt{ax/ind}}{\bullet \mathtt{h_3} : \Delta_4, \mathtt{F_1} \vdash \mathtt{F_2}} \; \mathtt{H}$$

- Case rule $\top_R$

$$\overline{\bullet \mathtt{h_3} : \Delta_4 \vdash \top, \Delta_5, \mathtt{F_1} \to \mathtt{F_2}} \; \top_R \quad \rightsquigarrow \quad \overline{\bullet \mathtt{h_3} : \Delta_4, \mathtt{F_1} \vdash \mathtt{F_2}} \; \mathtt{fail}$$

- Case rule $\to_L$

$$\frac{\mathtt{h_4} : \Delta_5, \mathtt{F_6} \to \mathtt{F_7} \vdash \Delta_1, \mathtt{F_6}, \mathtt{F_2} \to \mathtt{F_3} \quad \mathtt{h_4} : \Delta_5, \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \to \mathtt{F_3}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_6} \to \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \to \mathtt{F_3}} \to_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \Delta_5, \mathtt{F_2}, \mathtt{F_6} \to \mathtt{F_7} \vdash \mathtt{F_3}} \; \mathtt{ax/ind}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_2}, \mathtt{F_6} \to \mathtt{F_7} \vdash \mathtt{F_3}} \; \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h_4} : \Delta_5, \mathtt{F_6}, \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \to \mathtt{F_3}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_6} \wedge \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \to \mathtt{F_3}} \wedge_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \Delta_5, \mathtt{F_2}, \mathtt{F_6}, \mathtt{F_7} \vdash \mathtt{F_3}} \; \mathtt{ax/ind}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_2}, \mathtt{F_6} \wedge \mathtt{F_7} \vdash \mathtt{F_3}} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h_4} : \Delta_5, \mathtt{F_6} \vdash \Delta_1, \mathtt{F_2} \to \mathtt{F_3} \quad \mathtt{h_4} : \Delta_5, \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \to \mathtt{F_3}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_6} \vee \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \to \mathtt{F_3}} \vee_L \quad \rightsquigarrow \quad \frac{\overline{\mathtt{h_4} : \Delta_5, \mathtt{F_2}, \mathtt{F_6} \vdash \mathtt{F_3}} \; \mathtt{ax/ind} \quad \overline{\mathtt{h_4} : \Delta_5, \mathtt{F_2}, \mathtt{F_7} \vdash \mathtt{F_3}} \; \mathtt{ax/ind}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_2}, \mathtt{F_6} \vee \mathtt{F_7} \vdash \mathtt{F_3}} \vee_L$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \to F_3} \perp_L \quad \rightsquigarrow \quad \dfrac{}{\bullet h_4 : \perp, \Delta_5, F_2 \vdash F_3} \perp_L$$

- Case rule I

$$\dfrac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \to F_2} I \quad \rightsquigarrow \quad {\color{red}\dfrac{}{\bullet h_3 : \Delta_4, F_1, p_5 \vdash F_2}} {\color{red}\texttt{fail}}$$

- Case rule $\top_L$

$$\dfrac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \to F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \to F_3} \top_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{h_4 : \Delta_5, F_2 \vdash F_3} \texttt{ax/ind}}{\bullet h_4 : \top, \Delta_5, F_2 \vdash F_3} \top_L$$

4.2 Status of $\wedge_R$: (Left Premise): `Invertible`

- Case rule $\to_R$

$$\dfrac{h_3 : \Delta_4, F_5 \vdash F_6}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \to F_6} \to_R \quad \rightsquigarrow \quad \dfrac{\dfrac{}{h_3 : \Delta_4, F_5 \vdash F_6} \texttt{ax}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \to F_6} \to_R$$

- Case rule $\wedge_R$

$$\dfrac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \quad \rightsquigarrow \quad \dfrac{\dfrac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5} \texttt{ax/ind} \quad \dfrac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_6} \texttt{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \wedge_R$$

$$\dfrac{h_1 : \Delta_2 \vdash \Delta_3, F_4 \quad h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \quad \rightsquigarrow \quad \dfrac{\dfrac{}{h_1 : \Delta_2 \vdash \Delta_3, F_4} \texttt{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4} H$$

- Case rule $\vee_R$

$$\dfrac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \quad \rightsquigarrow \quad \dfrac{\dfrac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5, F_6} \texttt{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6} \vee_R$$

- Case rule $\perp_R$

$$\dfrac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \quad \rightsquigarrow \quad \dfrac{\dfrac{}{h_3 : \Delta_4 \vdash \Delta_5, F_1} \texttt{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1} \perp_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \quad \rightsquigarrow \quad \dfrac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1} \top_R$$

- Case rule $\to_L$

$$\dfrac{h_4 : \Delta_5, F_6 \to F_7 \vdash \Delta_1, F_6, F_2 \wedge F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \to F_7 \vdash \Delta_1, F_2 \wedge F_3} \to_L \quad \rightsquigarrow \quad \dfrac{\dfrac{}{h_4 : \Delta_5, F_6 \to F_7 \vdash \Delta_1, F_2, F_6} \texttt{ax/ind} \quad \dfrac{}{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2} \texttt{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \to F_7 \vdash \Delta_1, F_2} \to_L$$

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- Case rule $\wedge_L$

$$\dfrac{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3} \wedge_L \quad \rightsquigarrow \quad \dfrac{\overline{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2}\;\; ^{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2} \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2 \wedge F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3} \vee_L \quad \rightsquigarrow \quad \dfrac{\overline{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2}\;\; ^{ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2}\;\; ^{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2} \vee_L$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \perp_L \quad \rightsquigarrow \quad \dfrac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2} \perp_L$$

- Case rule I

$$\dfrac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \wedge F_2} I \quad \rightsquigarrow \quad \dfrac{}{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_1, p_5} I$$

- Case rule $\top_L$

$$\dfrac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \top_L \quad \rightsquigarrow \quad \dfrac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2}\;\; ^{ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2} \top_L$$

4.3 Status of $\wedge_R$ (Right Premise): : Invertible

- Case rule $\rightarrow_R$

$$\dfrac{h_3 : \Delta_4, F_5 \vdash F_6}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \quad \rightsquigarrow \quad \dfrac{\overline{h_3 : \Delta_4, F_5 \vdash F_6}\;\; ^{ax}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \quad \rightsquigarrow \quad \dfrac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5}\;\; ^{ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_6}\;\; ^{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

$$\dfrac{h_1 : \Delta_2 \vdash \Delta_3, F_4 \quad h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \quad \rightsquigarrow \quad \dfrac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_5}\;\; ^{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5} H$$

- Case rule $\vee_R$

$$\dfrac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \quad \rightsquigarrow \quad \dfrac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5, F_6}\;\; ^{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule $\perp_R$

$$\dfrac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \quad \rightsquigarrow \quad \dfrac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_2}\;\; ^{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_2} \perp_R$$

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- Case rule $\top_R$

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \ \top_R \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_2} \ \top_R$$

- Case rule $\rightarrow_L$

$$\frac{h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_6, F_2 \wedge F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \wedge F_3} \ \rightarrow_L \quad \rightsquigarrow \quad \frac{\dfrac{}{h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_3, F_6} \ \text{ax/ind} \quad \dfrac{}{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3} \ \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_3} \ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3} \ \wedge_L \quad \rightsquigarrow \quad \frac{\dfrac{}{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_3} \ \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_3} \ \wedge_L$$

- Case rule $\vee_L$

$$\frac{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2 \wedge F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3} \ \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_3} \ \text{ax/ind} \quad \dfrac{}{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3} \ \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_3} \ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet h_4 : \bot, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \ \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet h_4 : \bot, \Delta_5 \vdash \Delta_1, F_3} \ \bot_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \wedge F_2} \ I \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_2, p_5} \ I$$

- Case rule $\top_L$

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{h_4 : \Delta_5 \vdash \Delta_1, F_3} \ \text{ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_3} \ \top_L$$

4.4 Status of $\vee_R$: : Invertible

- Case rule $\rightarrow_R$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_6}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \rightarrow F_6} \ \rightarrow_R \quad \rightsquigarrow \quad \frac{\dfrac{}{h_3 : \Delta_4, F_5 \vdash F_6} \ \text{ax}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \vee F_2 \quad h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \wedge F_6} \ \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5} \ \text{ax/ind} \quad \dfrac{}{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \ \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\mathtt{h_3} : \Delta_4 \vdash \Delta_7, \mathtt{F_5}, \mathtt{F_6}, \mathtt{F_1} \vee \mathtt{F_2}}{\bullet \mathtt{h_3} : \Delta_4 \vdash (\Delta_7, \mathtt{F_1} \vee \mathtt{F_2}), \mathtt{F_5} \vee \mathtt{F_6}} \ \vee_R \qquad \leadsto \qquad \dfrac{\overline{\mathtt{h_3} : \Delta_4 \vdash \Delta_7, \mathtt{F_1}, \mathtt{F_2}, \mathtt{F_5}, \mathtt{F_6}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_3} : \Delta_4 \vdash \Delta_7, \mathtt{F_1}, \mathtt{F_2}, \mathtt{F_5} \vee \mathtt{F_6}} \ \vee_R$$

$$\dfrac{\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4}, \mathtt{F_5}}{\bullet \mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4} \vee \mathtt{F_5}} \ \vee_R \qquad \leadsto \qquad \dfrac{\overline{\mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4}, \mathtt{F_5}} \ \mathtt{ax}}{\bullet \mathtt{h_1} : \Delta_2 \vdash \Delta_3, \mathtt{F_4}, \mathtt{F_5}} \ \mathtt{H}$$

- Case rule $\bot_R$

$$\dfrac{\mathtt{h_3} : \Delta_4 \vdash \Delta_5, \mathtt{F_1} \vee \mathtt{F_2}}{\bullet \mathtt{h_3} : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F_1} \vee \mathtt{F_2}} \ \bot_R \qquad \leadsto \qquad \dfrac{\overline{\mathtt{h_3} : \Delta_4 \vdash \Delta_5, \mathtt{F_1}, \mathtt{F_2}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_3} : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F_1}, \mathtt{F_2}} \ \bot_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet \mathtt{h_3} : \Delta_4 \vdash \top, \Delta_5, \mathtt{F_1} \vee \mathtt{F_2}} \ \top_R \qquad \leadsto \qquad \dfrac{}{\bullet \mathtt{h_3} : \Delta_4 \vdash \top, \Delta_5, \mathtt{F_1}, \mathtt{F_2}} \ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\mathtt{h_4} : \Delta_5, \mathtt{F_6} \rightarrow \mathtt{F_7} \vdash \Delta_1, \mathtt{F_6}, \mathtt{F_2} \vee \mathtt{F_3} \quad \mathtt{h_4} : \Delta_5, \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_6} \rightarrow \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3}} \ \rightarrow_L \qquad \leadsto \qquad \dfrac{\overline{\mathtt{h_4} : \Delta_5, \mathtt{F_6} \rightarrow \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}, \mathtt{F_6}} \ \mathtt{ax/ind} \quad \overline{\mathtt{h_4} : \Delta_5, \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_6} \rightarrow \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\mathtt{h_4} : \Delta_5, \mathtt{F_6}, \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_6} \wedge \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3}} \ \wedge_L \qquad \leadsto \qquad \dfrac{\overline{\mathtt{h_4} : \Delta_5, \mathtt{F_6}, \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_6} \wedge \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\mathtt{h_4} : \Delta_5, \mathtt{F_6} \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3} \quad \mathtt{h_4} : \Delta_5, \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_6} \vee \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3}} \ \vee_L \qquad \leadsto \qquad \dfrac{\overline{\mathtt{h_4} : \Delta_5, \mathtt{F_6} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \mathtt{ax/ind} \quad \overline{\mathtt{h_4} : \Delta_5, \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_4} : \Delta_5, \mathtt{F_6} \vee \mathtt{F_7} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \vee_L$$

- Case rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h_4} : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3}} \ \bot_L \qquad \leadsto \qquad \dfrac{}{\bullet \mathtt{h_4} : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \bot_L$$

- Case rule I

$$\dfrac{}{\bullet \mathtt{h_3} : \mathtt{p_5}, \Delta_4 \vdash \mathtt{p_5}, \Delta_6, \mathtt{F_1} \vee \mathtt{F_2}} \ I \qquad \leadsto \qquad \dfrac{}{\bullet \mathtt{h_3} : \Delta_4, \mathtt{p_5} \vdash \Delta_6, \mathtt{F_1}, \mathtt{F_2}, \mathtt{p_5}} \ I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h_4} : \Delta_5 \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3}}{\bullet \mathtt{h_4} : \top, \Delta_5 \vdash \Delta_1, \mathtt{F_2} \vee \mathtt{F_3}} \ \top_L \qquad \leadsto \qquad \dfrac{\overline{\mathtt{h_4} : \Delta_5 \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \mathtt{ax/ind}}{\bullet \mathtt{h_4} : \top, \Delta_5 \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_3}} \ \top_L$$

4.5 Status of $\perp_R$: : Invertible

- Case rule $\to_R$

$$\frac{\texttt{h}_1 : \Delta_2, \texttt{F}_3 \vdash \texttt{F}_4}{\bullet\texttt{h}_1 : \Delta_2 \vdash (\perp, \Delta_5), \texttt{F}_3 \to \texttt{F}_4} \to_R \qquad \rightsquigarrow \qquad \frac{\overline{\texttt{h}_1 : \Delta_2, \texttt{F}_3 \vdash \texttt{F}_4}\ ^{\text{ax}}}{\bullet\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3 \to \texttt{F}_4} \to_R$$

- Case rule $\wedge_R$

$$\frac{\texttt{h}_1 : \Delta_2 \vdash \perp, \Delta_5, \texttt{F}_3 \quad \texttt{h}_1 : \Delta_2 \vdash \perp, \Delta_5, \texttt{F}_4}{\bullet\texttt{h}_1 : \Delta_2 \vdash (\perp, \Delta_5), \texttt{F}_3 \wedge \texttt{F}_4} \wedge_R \qquad \rightsquigarrow \qquad \frac{\overline{\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3}\ ^{\text{ax/ind}} \quad \overline{\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_4}\ ^{\text{ax/ind}}}{\bullet\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3 \wedge \texttt{F}_4} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\texttt{h}_1 : \Delta_2 \vdash \perp, \Delta_5, \texttt{F}_3, \texttt{F}_4}{\bullet\texttt{h}_1 : \Delta_2 \vdash (\perp, \Delta_5), \texttt{F}_3 \vee \texttt{F}_4} \vee_R \qquad \rightsquigarrow \qquad \frac{\overline{\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3, \texttt{F}_4}\ ^{\text{ax/ind}}}{\bullet\texttt{h}_1 : \Delta_2 \vdash \Delta_5, \texttt{F}_3 \vee \texttt{F}_4} \vee_R$$

- Case rule $\perp_R$

$$\frac{\texttt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet\texttt{h}_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \qquad \rightsquigarrow \qquad \frac{\overline{\texttt{h}_1 : \Delta_2 \vdash \Delta_3}\ ^{\text{ax}}}{\bullet\texttt{h}_1 : \Delta_2 \vdash \Delta_3} \text{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet\texttt{h}_1 : \Delta_2 \vdash \top, \perp, \Delta_3} \top_R \qquad \rightsquigarrow \qquad \frac{}{\bullet\texttt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case rule $\to_L$

$$\frac{\texttt{h}_2 : \Delta_3, \texttt{F}_4 \to \texttt{F}_5 \vdash \perp, \Delta_1, \texttt{F}_4 \quad \texttt{h}_2 : \Delta_3, \texttt{F}_5 \vdash \perp, \Delta_1}{\bullet\texttt{h}_2 : \Delta_3, \texttt{F}_4 \to \texttt{F}_5 \vdash \perp, \Delta_1} \to_L \qquad \rightsquigarrow \qquad \frac{\overline{\texttt{h}_2 : \Delta_3, \texttt{F}_4 \to \texttt{F}_5 \vdash \Delta_1, \texttt{F}_4}\ ^{\text{ax/ind}} \quad \overline{\texttt{h}_2 : \Delta_3, \texttt{F}_5 \vdash \Delta_1}\ ^{\text{ax/ind}}}{\bullet\texttt{h}_2 : \Delta_3, \texttt{F}_4 \to \texttt{F}_5 \vdash \Delta_1} \to_L$$

- Case rule $\wedge_L$

$$\frac{\texttt{h}_2 : \Delta_3, \texttt{F}_4, \texttt{F}_5 \vdash \perp, \Delta_1}{\bullet\texttt{h}_2 : \Delta_3, \texttt{F}_4 \wedge \texttt{F}_5 \vdash \perp, \Delta_1} \wedge_L \qquad \rightsquigarrow \qquad \frac{\overline{\texttt{h}_2 : \Delta_3, \texttt{F}_4, \texttt{F}_5 \vdash \Delta_1}\ ^{\text{ax/ind}}}{\bullet\texttt{h}_2 : \Delta_3, \texttt{F}_4 \wedge \texttt{F}_5 \vdash \Delta_1} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\texttt{h}_2 : \Delta_3, \texttt{F}_4 \vdash \perp, \Delta_1 \quad \texttt{h}_2 : \Delta_3, \texttt{F}_5 \vdash \perp, \Delta_1}{\bullet\texttt{h}_2 : \Delta_3, \texttt{F}_4 \vee \texttt{F}_5 \vdash \perp, \Delta_1} \vee_L \qquad \rightsquigarrow \qquad \frac{\overline{\texttt{h}_2 : \Delta_3, \texttt{F}_4 \vdash \Delta_1}\ ^{\text{ax/ind}} \quad \overline{\texttt{h}_2 : \Delta_3, \texttt{F}_5 \vdash \Delta_1}\ ^{\text{ax/ind}}}{\bullet\texttt{h}_2 : \Delta_3, \texttt{F}_4 \vee \texttt{F}_5 \vdash \Delta_1} \vee_L$$

- Case rule $\perp_L$

$$\frac{}{\bullet\texttt{h}_2 : \perp, \Delta_3 \vdash \perp, \Delta_1} \perp_L \qquad \rightsquigarrow \qquad \frac{}{\bullet\texttt{h}_2 : \perp, \Delta_3 \vdash \Delta_1} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3, \bot, \Delta_4} \ I \qquad \rightsquigarrow \qquad \frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} \ I$$

- Case rule $\top_L$

$$\frac{h_2 : \Delta_3 \vdash \bot, \Delta_1}{\bullet h_2 : \top, \Delta_3 \vdash \bot, \Delta_1} \ \top_L \qquad \rightsquigarrow \qquad \frac{\dfrac{}{h_2 : \Delta_3 \vdash \Delta_1} \ \text{ax/ind}}{\bullet h_2 : \top, \Delta_3 \vdash \Delta_1} \ \top_L$$

4.6 Status of $\top_R$: : Invertible

- Case rule $\rightarrow_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \rightarrow F_4} \ \rightarrow_R \qquad \rightsquigarrow \qquad \text{trivial}$$

- Case rule $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_5, F_3 \quad h_1 : \Delta_2 \vdash \top, \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \wedge F_4} \ \wedge_R \qquad \rightsquigarrow \qquad \text{trivial}$$

- Case rule $\vee_R$

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \vee F_4} \ \vee_R \qquad \rightsquigarrow \qquad \text{trivial}$$

- Case rule $\bot_R$

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \bot, \top, \Delta_3} \ \bot_R \qquad \rightsquigarrow \qquad \text{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \ \top_R \qquad \rightsquigarrow \qquad \text{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \top, \Delta_1, F_4 \quad h_2 : \Delta_3, F_5 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \top, \Delta_1} \ \rightarrow_L \qquad \rightsquigarrow \qquad \text{trivial}$$

- Case rule $\wedge_L$

$$\frac{h_2 : \Delta_3, F_4, F_5 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \top, \Delta_1} \ \wedge_L \qquad \rightsquigarrow \qquad \text{trivial}$$

- Case rule $\vee_L$

$$\frac{h_2 : \Delta_3, F_4 \vdash \top, \Delta_1 \quad h_2 : \Delta_3, F_5 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \top, \Delta_1} \ \vee_L \qquad \rightsquigarrow \qquad \text{trivial}$$

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- Case rule $\perp_L$

$$\frac{}{\bullet h_2 : \perp, \Delta_3 \vdash \top, \Delta_1} \perp_L \qquad \leadsto \qquad \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3, \top, \Delta_4} I \qquad \leadsto \qquad \text{trivial}$$

- Case rule $\top_L$

$$\frac{h_2 : \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \top, \Delta_3 \vdash \top, \Delta_1} \top_L \qquad \leadsto \qquad \text{trivial}$$

4.7 Status of $\to_L$: (Left Premise): `Invertible`

- Case rule $\to_R$

$$\frac{h_4 : \Delta_1, F_6, F_2 \to F_3 \vdash F_7}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \to F_7} \to_R \qquad \leadsto \qquad \frac{\dfrac{}{h_4 : \Delta_1, F_6, F_2 \to F_3 \vdash F_7} \text{ ax}}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_2, F_6 \to F_7} \to_R$$

- Case rule $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \quad h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \qquad \leadsto \qquad \frac{\dfrac{}{h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_2, F_6} \text{ ax/ind} \quad \dfrac{}{h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_2, F_7} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_2, F_6 \wedge F_7} \wedge_R$$

- Case rule $\vee_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \qquad \leadsto \qquad \frac{\dfrac{}{h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_2, F_6, F_7} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_2, F_6 \vee F_7} \vee_R$$

- Case rule $\perp_R$

$$\frac{h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \perp, \Delta_5} \perp_R \qquad \leadsto \qquad \frac{\dfrac{}{h_4 : \Delta_1, F_2 \to F_3 \vdash \Delta_5, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \top, \Delta_5} \top_R \qquad \leadsto \qquad \frac{}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule $\to_L$

$$\frac{h_3 : \Delta_7, F_1 \to F_2, F_4 \to F_5 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \to F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \to F_2), F_4 \to F_5 \vdash \Delta_6} \to_L \qquad \leadsto \qquad \frac{\dfrac{}{h_3 : \Delta_7, F_1 \to F_2, F_4 \to F_5 \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \dfrac{}{h_3 : \Delta_7, F_5, F_1 \to F_2 \vdash \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1 \to F_2, F_4 \to F_5 \vdash \Delta_6, F_1} \to_L$$

$$\frac{h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5} \to_L \qquad \leadsto \qquad \frac{\dfrac{}{h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5, F_3} \text{ ax}}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5, F_3} \text{H}$$

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- Case rule $\wedge_L$

$$\frac{\mathtt{h_3} : \Delta_7, \mathtt{F_4}, \mathtt{F_5}, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_6}{\bullet\mathtt{h_3} : (\Delta_7, \mathtt{F_1} \to \mathtt{F_2}), \mathtt{F_4} \wedge \mathtt{F_5} \vdash \Delta_6} \wedge_L \qquad \rightsquigarrow \qquad \frac{\dfrac{}{\mathtt{h_3} : \Delta_7, \mathtt{F_4}, \mathtt{F_5}, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_6, \mathtt{F_1}} \text{ ax/ind}}{\bullet\mathtt{h_3} : \Delta_7, \mathtt{F_1} \to \mathtt{F_2}, \mathtt{F_4} \wedge \mathtt{F_5} \vdash \Delta_6, \mathtt{F_1}} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h_3} : \Delta_7, \mathtt{F_4}, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_6 \quad \mathtt{h_3} : \Delta_7, \mathtt{F_5}, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_6}{\bullet\mathtt{h_3} : (\Delta_7, \mathtt{F_1} \to \mathtt{F_2}), \mathtt{F_4} \vee \mathtt{F_5} \vdash \Delta_6} \vee_L \qquad \rightsquigarrow \qquad \frac{\dfrac{}{\mathtt{h_3} : \Delta_7, \mathtt{F_4}, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_6, \mathtt{F_1}} \text{ ax/ind} \quad \dfrac{}{\mathtt{h_3} : \Delta_7, \mathtt{F_5}, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_6, \mathtt{F_1}} \text{ ax/ind}}{\bullet\mathtt{h_3} : \Delta_7, \mathtt{F_1} \to \mathtt{F_2}, \mathtt{F_4} \vee \mathtt{F_5} \vdash \Delta_6, \mathtt{F_1}} \vee_L$$

- Case rule $\perp_L$

$$\frac{}{\bullet\mathtt{h_3} : \perp, \Delta_5, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_4} \perp_L \qquad \rightsquigarrow \qquad \frac{}{\bullet\mathtt{h_3} : \perp, \Delta_5, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_4, \mathtt{F_1}} \perp_L$$

- Case rule I

$$\frac{}{\bullet\mathtt{h_3} : \mathtt{p_4}, \Delta_6, \mathtt{F_1} \to \mathtt{F_2} \vdash \mathtt{p_4}, \Delta_5} I \qquad \rightsquigarrow \qquad \frac{}{\bullet\mathtt{h_3} : \Delta_6, \mathtt{p_4}, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_5, \mathtt{F_1}, \mathtt{p_4}} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h_3} : \Delta_5, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_4}{\bullet\mathtt{h_3} : \top, \Delta_5, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_4} \top_L \qquad \rightsquigarrow \qquad \frac{\dfrac{}{\mathtt{h_3} : \Delta_5, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_4, \mathtt{F_1}} \text{ ax/ind}}{\bullet\mathtt{h_3} : \top, \Delta_5, \mathtt{F_1} \to \mathtt{F_2} \vdash \Delta_4, \mathtt{F_1}} \top_L$$

4.8 Status of $\to_L$ (Right Premise): : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h_4} : \Delta_1, \mathtt{F_6}, \mathtt{F_2} \to \mathtt{F_3} \vdash \mathtt{F_7}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \to \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \to \mathtt{F_7}} \to_R \qquad \rightsquigarrow \qquad \frac{\dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_3}, \mathtt{F_6} \vdash \mathtt{F_7}} \text{ ax/ind}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \to \mathtt{F_7}} \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \to \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \quad \mathtt{h_4} : \Delta_1, \mathtt{F_2} \to \mathtt{F_3} \vdash \Delta_5, \mathtt{F_7}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \to \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \wedge \mathtt{F_7}} \wedge_R \qquad \rightsquigarrow \qquad \frac{\dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6}} \text{ ax/ind} \quad \dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_3} \vdash \Delta_5, \mathtt{F_7}} \text{ ax/ind}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \wedge \mathtt{F_7}} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \to \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6}, \mathtt{F_7}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \to \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \vee \mathtt{F_7}} \vee_R \qquad \rightsquigarrow \qquad \frac{\dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6}, \mathtt{F_7}} \text{ ax/ind}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_3} \vdash \Delta_5, \mathtt{F_6} \vee \mathtt{F_7}} \vee_R$$

- Case rule $\perp_R$

$$\frac{\mathtt{h_4} : \Delta_1, \mathtt{F_2} \to \mathtt{F_3} \vdash \Delta_5}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_2} \to \mathtt{F_3} \vdash \perp, \Delta_5} \perp_R \qquad \rightsquigarrow \qquad \frac{\dfrac{}{\mathtt{h_4} : \Delta_1, \mathtt{F_3} \vdash \Delta_5} \text{ ax/ind}}{\bullet\mathtt{h_4} : \Delta_1, \mathtt{F_3} \vdash \perp, \Delta_5} \perp_R$$

- Case rule $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \to F_3 \vdash \top, \Delta_5} \ \top_R \quad \rightsquigarrow \quad \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top, \Delta_5} \ \top_R$$

- Case rule $\to_L$

$$\frac{h_3 : \Delta_7, F_1 \to F_2, F_4 \to F_5 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \to F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \to F_2), F_4 \to F_5 \vdash \Delta_6} \ \to_L \quad \rightsquigarrow \quad \frac{\dfrac{}{h_3 : \Delta_7, F_2, F_4 \to F_5 \vdash \Delta_6, F_4} \ \text{ax/ind} \quad \dfrac{}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \to F_5 \vdash \Delta_6} \ \to_L$$

$$\frac{h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \to F_4 \vdash \Delta_5} \ \to_L \quad \rightsquigarrow \quad \frac{\dfrac{}{h_1 : \Delta_2, F_4 \vdash \Delta_5} \ \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \ \text{H}$$

- Case rule $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \to F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \to F_2), F_4 \wedge F_5 \vdash \Delta_6} \ \wedge_L \quad \rightsquigarrow \quad \frac{\dfrac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \ \wedge_L$$

- Case rule $\vee_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \to F_2 \vdash \Delta_6 \quad h_3 : \Delta_7, F_5, F_1 \to F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \to F_2), F_4 \vee F_5 \vdash \Delta_6} \ \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6} \ \text{ax/ind} \quad \dfrac{}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \ \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \ \vee_L$$

- Case rule $\bot_L$

$$\frac{}{\bullet h_3 : \bot, \Delta_5, F_1 \to F_2 \vdash \Delta_4} \ \bot_L \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \bot, \Delta_5, F_2 \vdash \Delta_4} \ \bot_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \to F_2 \vdash p_4, \Delta_5} \ I \quad \rightsquigarrow \quad \frac{}{\bullet h_3 : \Delta_6, F_2, p_4 \vdash \Delta_5, p_4} \ I$$

- Case rule $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \to F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \to F_2 \vdash \Delta_4} \ \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{h_3 : \Delta_5, F_2 \vdash \Delta_4} \ \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \ \top_L$$

4.9 Status of $\wedge_L$: : Invertible

- Case rule $\to_R$

$$\frac{h_4 : \Delta_1, F_6, F_2 \wedge F_3 \vdash F_7}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \to F_7} \ \to_R \quad \rightsquigarrow \quad \frac{\dfrac{}{h_4 : \Delta_1, F_2, F_3, F_6 \vdash F_7} \ \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \to F_7} \ \to_R$$

- Case rule $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \quad h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \wedge F_7} \ \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6} \ \text{ax/ind} \quad \dfrac{}{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_7} \ \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \wedge F_7} \ \wedge_R$$

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- Case rule $\vee_R$

$$\dfrac{\mathrm{h}_4 : \Delta_1, \mathrm{F}_2 \wedge \mathrm{F}_3 \vdash \Delta_5, \mathrm{F}_6, \mathrm{F}_7}{\bullet\mathrm{h}_4 : \Delta_1, \mathrm{F}_2 \wedge \mathrm{F}_3 \vdash \Delta_5, \mathrm{F}_6 \vee \mathrm{F}_7} \ \vee_R \qquad \leadsto \qquad \dfrac{\overline{\phantom{\mathrm{h}_4 : \Delta_1, \mathrm{F}_2, \mathrm{F}_3 \vdash \Delta_5, \mathrm{F}_6, \mathrm{F}_7}}^{\ \mathrm{ax/ind}}\ \ \mathrm{h}_4 : \Delta_1, \mathrm{F}_2, \mathrm{F}_3 \vdash \Delta_5, \mathrm{F}_6, \mathrm{F}_7}{\bullet\mathrm{h}_4 : \Delta_1, \mathrm{F}_2, \mathrm{F}_3 \vdash \Delta_5, \mathrm{F}_6 \vee \mathrm{F}_7} \ \vee_R$$

- Case rule $\perp_R$

$$\dfrac{\mathrm{h}_4 : \Delta_1, \mathrm{F}_2 \wedge \mathrm{F}_3 \vdash \Delta_5}{\bullet\mathrm{h}_4 : \Delta_1, \mathrm{F}_2 \wedge \mathrm{F}_3 \vdash \perp, \Delta_5} \ \perp_R \qquad \leadsto \qquad \dfrac{\overline{\mathrm{h}_4 : \Delta_1, \mathrm{F}_2, \mathrm{F}_3 \vdash \Delta_5}}^{\ \mathrm{ax/ind}}{\bullet\mathrm{h}_4 : \Delta_1, \mathrm{F}_2, \mathrm{F}_3 \vdash \perp, \Delta_5} \ \perp_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet\mathrm{h}_4 : \Delta_1, \mathrm{F}_2 \wedge \mathrm{F}_3 \vdash \top, \Delta_5} \ \top_R \qquad \leadsto \qquad \dfrac{}{\bullet\mathrm{h}_4 : \Delta_1, \mathrm{F}_2, \mathrm{F}_3 \vdash \top, \Delta_5} \ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\mathrm{h}_3 : \Delta_7, \mathrm{F}_4 \rightarrow \mathrm{F}_5, \mathrm{F}_1 \wedge \mathrm{F}_2 \vdash \Delta_6, \mathrm{F}_4 \quad \mathrm{h}_3 : \Delta_7, \mathrm{F}_5, \mathrm{F}_1 \wedge \mathrm{F}_2 \vdash \Delta_6}{\bullet\mathrm{h}_3 : (\Delta_7, \mathrm{F}_1 \wedge \mathrm{F}_2), \mathrm{F}_4 \rightarrow \mathrm{F}_5 \vdash \Delta_6} \ \rightarrow_L \qquad \leadsto \qquad \dfrac{\overline{\mathrm{h}_3 : \Delta_7, \mathrm{F}_1, \mathrm{F}_2, \mathrm{F}_4 \rightarrow \mathrm{F}_5 \vdash \Delta_6, \mathrm{F}_4}}^{\ \mathrm{ax/ind}} \quad \overline{\mathrm{h}_3 : \Delta_7, \mathrm{F}_1, \mathrm{F}_2, \mathrm{F}_5 \vdash \Delta_6}^{\ \mathrm{ax/ind}}{\bullet\mathrm{h}_3 : \Delta_7, \mathrm{F}_1, \mathrm{F}_2, \mathrm{F}_4 \rightarrow \mathrm{F}_5 \vdash \Delta_6} \ \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\mathrm{h}_3 : \Delta_7, \mathrm{F}_4, \mathrm{F}_5, \mathrm{F}_1 \wedge \mathrm{F}_2 \vdash \Delta_6}{\bullet\mathrm{h}_3 : (\Delta_7, \mathrm{F}_1 \wedge \mathrm{F}_2), \mathrm{F}_4 \wedge \mathrm{F}_5 \vdash \Delta_6} \ \wedge_L \qquad \leadsto \qquad \dfrac{\overline{\mathrm{h}_3 : \Delta_7, \mathrm{F}_1, \mathrm{F}_2, \mathrm{F}_4, \mathrm{F}_5 \vdash \Delta_6}}^{\ \mathrm{ax/ind}}{\bullet\mathrm{h}_3 : \Delta_7, \mathrm{F}_1, \mathrm{F}_2, \mathrm{F}_4 \wedge \mathrm{F}_5 \vdash \Delta_6} \ \wedge_L$$

$$\dfrac{\mathrm{h}_1 : \Delta_2, \mathrm{F}_3, \mathrm{F}_4 \vdash \Delta_5}{\bullet\mathrm{h}_1 : \Delta_2, \mathrm{F}_3 \wedge \mathrm{F}_4 \vdash \Delta_5} \ \wedge_L \qquad \leadsto \qquad \dfrac{\overline{\mathrm{h}_1 : \Delta_2, \mathrm{F}_3, \mathrm{F}_4 \vdash \Delta_5}}^{\ \mathrm{ax}}{\bullet\mathrm{h}_1 : \Delta_2, \mathrm{F}_3, \mathrm{F}_4 \vdash \Delta_5} \ \mathrm{H}$$

- Case rule $\vee_L$

$$\dfrac{\mathrm{h}_3 : \Delta_7, \mathrm{F}_4, \mathrm{F}_1 \wedge \mathrm{F}_2 \vdash \Delta_6 \quad \mathrm{h}_3 : \Delta_7, \mathrm{F}_5, \mathrm{F}_1 \wedge \mathrm{F}_2 \vdash \Delta_6}{\bullet\mathrm{h}_3 : (\Delta_7, \mathrm{F}_1 \wedge \mathrm{F}_2), \mathrm{F}_4 \vee \mathrm{F}_5 \vdash \Delta_6} \ \vee_L \qquad \leadsto \qquad \dfrac{\overline{\mathrm{h}_3 : \Delta_7, \mathrm{F}_1, \mathrm{F}_2, \mathrm{F}_4 \vdash \Delta_6}}^{\ \mathrm{ax/ind}} \quad \overline{\mathrm{h}_3 : \Delta_7, \mathrm{F}_1, \mathrm{F}_2, \mathrm{F}_5 \vdash \Delta_6}^{\ \mathrm{ax/ind}}{\bullet\mathrm{h}_3 : \Delta_7, \mathrm{F}_1, \mathrm{F}_2, \mathrm{F}_4 \vee \mathrm{F}_5 \vdash \Delta_6} \ \vee_L$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet\mathrm{h}_3 : \perp, \Delta_5, \mathrm{F}_1 \wedge \mathrm{F}_2 \vdash \Delta_4} \ \perp_L \qquad \leadsto \qquad \dfrac{}{\bullet\mathrm{h}_3 : \perp, \Delta_5, \mathrm{F}_1, \mathrm{F}_2 \vdash \Delta_4} \ \perp_L$$

- Case rule I

$$\dfrac{}{\bullet\mathrm{h}_3 : \mathrm{p}_4, \Delta_6, \mathrm{F}_1 \wedge \mathrm{F}_2 \vdash \mathrm{p}_4, \Delta_5} \ I \qquad \leadsto \qquad \dfrac{}{\bullet\mathrm{h}_3 : \Delta_6, \mathrm{F}_1, \mathrm{F}_2, \mathrm{p}_4 \vdash \Delta_5, \mathrm{p}_4} \ I$$

- Case rule $\top_L$

$$\dfrac{\mathrm{h}_3 : \Delta_5, \mathrm{F}_1 \wedge \mathrm{F}_2 \vdash \Delta_4}{\bullet\mathrm{h}_3 : \top, \Delta_5, \mathrm{F}_1 \wedge \mathrm{F}_2 \vdash \Delta_4} \ \top_L \qquad \leadsto \qquad \dfrac{\overline{\mathrm{h}_3 : \Delta_5, \mathrm{F}_1, \mathrm{F}_2 \vdash \Delta_4}}^{\ \mathrm{ax/ind}}{\bullet\mathrm{h}_3 : \top, \Delta_5, \mathrm{F}_1, \mathrm{F}_2 \vdash \Delta_4} \ \top_L$$

4.10   Status of $\vee_L$:   (Left Premise): `Invertible`

- Case rule $\to_R$

$$\frac{\text{h}_4 : \Delta_1, \text{F}_6, \text{F}_2 \vee \text{F}_3 \vdash \text{F}_7}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vee \text{F}_3 \vdash \Delta_5, \text{F}_6 \to \text{F}_7} \to_R \qquad \rightsquigarrow \qquad \frac{\overline{\text{h}_4 : \Delta_1, \text{F}_2, \text{F}_6 \vdash \text{F}_7} \;\; \text{ax/ind}}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vdash \Delta_5, \text{F}_6 \to \text{F}_7} \to_R$$

- Case rule $\wedge_R$

$$\frac{\text{h}_4 : \Delta_1, \text{F}_2 \vee \text{F}_3 \vdash \Delta_5, \text{F}_6 \quad \text{h}_4 : \Delta_1, \text{F}_2 \vee \text{F}_3 \vdash \Delta_5, \text{F}_7}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vee \text{F}_3 \vdash \Delta_5, \text{F}_6 \wedge \text{F}_7} \wedge_R \qquad \rightsquigarrow \qquad \frac{\overline{\text{h}_4 : \Delta_1, \text{F}_2 \vdash \Delta_5, \text{F}_6} \; \text{ax/ind} \quad \overline{\text{h}_4 : \Delta_1, \text{F}_2 \vdash \Delta_5, \text{F}_7} \; \text{ax/ind}}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vdash \Delta_5, \text{F}_6 \wedge \text{F}_7} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\text{h}_4 : \Delta_1, \text{F}_2 \vee \text{F}_3 \vdash \Delta_5, \text{F}_6, \text{F}_7}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vee \text{F}_3 \vdash \Delta_5, \text{F}_6 \vee \text{F}_7} \vee_R \qquad \rightsquigarrow \qquad \frac{\overline{\text{h}_4 : \Delta_1, \text{F}_2 \vdash \Delta_5, \text{F}_6, \text{F}_7} \; \text{ax/ind}}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vdash \Delta_5, \text{F}_6 \vee \text{F}_7} \vee_R$$

- Case rule $\perp_R$

$$\frac{\text{h}_4 : \Delta_1, \text{F}_2 \vee \text{F}_3 \vdash \Delta_5}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vee \text{F}_3 \vdash \perp, \Delta_5} \perp_R \qquad \rightsquigarrow \qquad \frac{\overline{\text{h}_4 : \Delta_1, \text{F}_2 \vdash \Delta_5} \; \text{ax/ind}}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vdash \perp, \Delta_5} \perp_R$$

- Case rule $\top_R$

$$\frac{}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vee \text{F}_3 \vdash \top, \Delta_5} \top_R \qquad \rightsquigarrow \qquad \frac{}{\bullet\text{h}_4 : \Delta_1, \text{F}_2 \vdash \top, \Delta_5} \top_R$$

- Case rule $\to_L$

$$\frac{\text{h}_3 : \Delta_7, \text{F}_4 \to \text{F}_5, \text{F}_1 \vee \text{F}_2 \vdash \Delta_6, \text{F}_4 \quad \text{h}_3 : \Delta_7, \text{F}_5, \text{F}_1 \vee \text{F}_2 \vdash \Delta_6}{\bullet\text{h}_3 : (\Delta_7, \text{F}_1 \vee \text{F}_2), \text{F}_4 \to \text{F}_5 \vdash \Delta_6} \to_L \qquad \rightsquigarrow \qquad \frac{\overline{\text{h}_3 : \Delta_7, \text{F}_1, \text{F}_4 \to \text{F}_5 \vdash \Delta_6, \text{F}_4} \; \text{ax/ind} \quad \overline{\text{h}_3 : \Delta_7, \text{F}_1, \text{F}_5 \vdash \Delta_6} \; \text{ax/ind}}{\bullet\text{h}_3 : \Delta_7, \text{F}_1, \text{F}_4 \to \text{F}_5 \vdash \Delta_6} \to_L$$

- Case rule $\wedge_L$

$$\frac{\text{h}_3 : \Delta_7, \text{F}_4, \text{F}_5, \text{F}_1 \vee \text{F}_2 \vdash \Delta_6}{\bullet\text{h}_3 : (\Delta_7, \text{F}_1 \vee \text{F}_2), \text{F}_4 \wedge \text{F}_5 \vdash \Delta_6} \wedge_L \qquad \rightsquigarrow \qquad \frac{\overline{\text{h}_3 : \Delta_7, \text{F}_1, \text{F}_4, \text{F}_5 \vdash \Delta_6} \; \text{ax/ind}}{\bullet\text{h}_3 : \Delta_7, \text{F}_1, \text{F}_4 \wedge \text{F}_5 \vdash \Delta_6} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\text{h}_3 : \Delta_7, \text{F}_4, \text{F}_1 \vee \text{F}_2 \vdash \Delta_6 \quad \text{h}_3 : \Delta_7, \text{F}_5, \text{F}_1 \vee \text{F}_2 \vdash \Delta_6}{\bullet\text{h}_3 : (\Delta_7, \text{F}_1 \vee \text{F}_2), \text{F}_4 \vee \text{F}_5 \vdash \Delta_6} \vee_L \qquad \rightsquigarrow \qquad \frac{\overline{\text{h}_3 : \Delta_7, \text{F}_1, \text{F}_4 \vdash \Delta_6} \; \text{ax/ind} \quad \overline{\text{h}_3 : \Delta_7, \text{F}_1, \text{F}_5 \vdash \Delta_6} \; \text{ax/ind}}{\bullet\text{h}_3 : \Delta_7, \text{F}_1, \text{F}_4 \vee \text{F}_5 \vdash \Delta_6} \vee_L$$

$$\frac{\text{h}_1 : \Delta_2, \text{F}_3 \vdash \Delta_5 \quad \text{h}_1 : \Delta_2, \text{F}_4 \vdash \Delta_5}{\bullet\text{h}_1 : \Delta_2, \text{F}_3 \vee \text{F}_4 \vdash \Delta_5} \vee_L \qquad \rightsquigarrow \qquad \frac{\overline{\text{h}_1 : \Delta_2, \text{F}_3 \vdash \Delta_5} \; \text{ax}}{\bullet\text{h}_1 : \Delta_2, \text{F}_3 \vdash \Delta_5} \text{H}$$

- Case rule $\perp_L$

$$\frac{}{\bullet\text{h}_3 : \perp, \Delta_5, \text{F}_1 \vee \text{F}_2 \vdash \Delta_4} \perp_L \qquad \rightsquigarrow \qquad \frac{}{\bullet\text{h}_3 : \perp, \Delta_5, \text{F}_1 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet \texttt{h}_3 : \texttt{p}_4, \Delta_6, \texttt{F}_1 \vee \texttt{F}_2 \vdash \texttt{p}_4, \Delta_5} \; I \quad \rightsquigarrow \quad \frac{}{\bullet \texttt{h}_3 : \Delta_6, \texttt{F}_1, \texttt{p}_4 \vdash \Delta_5, \texttt{p}_4} \; I$$

- Case rule $\top_L$

$$\frac{\texttt{h}_3 : \Delta_5, \texttt{F}_1 \vee \texttt{F}_2 \vdash \Delta_4}{\bullet \texttt{h}_3 : \top, \Delta_5, \texttt{F}_1 \vee \texttt{F}_2 \vdash \Delta_4} \; \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\texttt{h}_3 : \Delta_5, \texttt{F}_1 \vdash \Delta_4} \; \texttt{ax/ind}}{\bullet \texttt{h}_3 : \top, \Delta_5, \texttt{F}_1 \vdash \Delta_4} \; \top_L$$

4.11   Status of $\vee_L$ (Right Premise): :   Invertible

- Case rule $\rightarrow_R$

$$\frac{\texttt{h}_4 : \Delta_1, \texttt{F}_6, \texttt{F}_2 \vee \texttt{F}_3 \vdash \texttt{F}_7}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6 \rightarrow \texttt{F}_7} \; \rightarrow_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\texttt{h}_4 : \Delta_1, \texttt{F}_3, \texttt{F}_6 \vdash \texttt{F}_7} \; \texttt{ax/ind}}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6 \rightarrow \texttt{F}_7} \; \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6 \quad \texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \Delta_5, \texttt{F}_7}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6 \wedge \texttt{F}_7} \; \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\texttt{h}_4 : \Delta_1, \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6} \; \texttt{ax/ind} \quad \dfrac{}{\texttt{h}_4 : \Delta_1, \texttt{F}_3 \vdash \Delta_5, \texttt{F}_7} \; \texttt{ax/ind}}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6 \wedge \texttt{F}_7} \; \wedge_R$$

- Case rule $\vee_R$

$$\frac{\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6, \texttt{F}_7}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6 \vee \texttt{F}_7} \; \vee_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\texttt{h}_4 : \Delta_1, \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6, \texttt{F}_7} \; \texttt{ax/ind}}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_3 \vdash \Delta_5, \texttt{F}_6 \vee \texttt{F}_7} \; \vee_R$$

- Case rule $\perp_R$

$$\frac{\texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \Delta_5}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \perp, \Delta_5} \; \perp_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\texttt{h}_4 : \Delta_1, \texttt{F}_3 \vdash \Delta_5} \; \texttt{ax/ind}}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_3 \vdash \perp, \Delta_5} \; \perp_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_2 \vee \texttt{F}_3 \vdash \top, \Delta_5} \; \top_R \quad \rightsquigarrow \quad \frac{}{\bullet \texttt{h}_4 : \Delta_1, \texttt{F}_3 \vdash \top, \Delta_5} \; \top_R$$

- Case rule $\rightarrow_L$

$$\frac{\texttt{h}_3 : \Delta_7, \texttt{F}_4 \rightarrow \texttt{F}_5, \texttt{F}_1 \vee \texttt{F}_2 \vdash \Delta_6, \texttt{F}_4 \quad \texttt{h}_3 : \Delta_7, \texttt{F}_5, \texttt{F}_1 \vee \texttt{F}_2 \vdash \Delta_6}{\bullet \texttt{h}_3 : (\Delta_7, \texttt{F}_1 \vee \texttt{F}_2), \texttt{F}_4 \rightarrow \texttt{F}_5 \vdash \Delta_6} \; \rightarrow_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\texttt{h}_3 : \Delta_7, \texttt{F}_2, \texttt{F}_4 \rightarrow \texttt{F}_5 \vdash \Delta_6, \texttt{F}_4} \; \texttt{ax/ind} \quad \dfrac{}{\texttt{h}_3 : \Delta_7, \texttt{F}_2, \texttt{F}_5 \vdash \Delta_6} \; \texttt{ax/ind}}{\bullet \texttt{h}_3 : \Delta_7, \texttt{F}_2, \texttt{F}_4 \rightarrow \texttt{F}_5 \vdash \Delta_6} \; \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\texttt{h}_3 : \Delta_7, \texttt{F}_4, \texttt{F}_5, \texttt{F}_1 \vee \texttt{F}_2 \vdash \Delta_6}{\bullet \texttt{h}_3 : (\Delta_7, \texttt{F}_1 \vee \texttt{F}_2), \texttt{F}_4 \wedge \texttt{F}_5 \vdash \Delta_6} \; \wedge_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\texttt{h}_3 : \Delta_7, \texttt{F}_2, \texttt{F}_4, \texttt{F}_5 \vdash \Delta_6} \; \texttt{ax/ind}}{\bullet \texttt{h}_3 : \Delta_7, \texttt{F}_2, \texttt{F}_4 \wedge \texttt{F}_5 \vdash \Delta_6} \; \wedge_L$$

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- Case rule $\vee_L$

$$\frac{\mathtt{h_3} : \Delta_7, \mathtt{F_4}, \mathtt{F_1} \vee \mathtt{F_2} \vdash \Delta_6 \quad \mathtt{h_3} : \Delta_7, \mathtt{F_5}, \mathtt{F_1} \vee \mathtt{F_2} \vdash \Delta_6}{\bullet \mathtt{h_3} : (\Delta_7, \mathtt{F_1} \vee \mathtt{F_2}), \mathtt{F_4} \vee \mathtt{F_5} \vdash \Delta_6} \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h_3} : \Delta_7, \mathtt{F_2}, \mathtt{F_4} \vdash \Delta_6} \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h_3} : \Delta_7, \mathtt{F_2}, \mathtt{F_5} \vdash \Delta_6} \mathtt{ax/ind}}{\bullet \mathtt{h_3} : \Delta_7, \mathtt{F_2}, \mathtt{F_4} \vee \mathtt{F_5} \vdash \Delta_6} \vee_L$$

$$\frac{\mathtt{h_1} : \Delta_2, \mathtt{F_3} \vdash \Delta_5 \quad \mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \Delta_5}{\bullet \mathtt{h_1} : \Delta_2, \mathtt{F_3} \vee \mathtt{F_4} \vdash \Delta_5} \vee_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \Delta_5} \mathtt{ax}}{\bullet \mathtt{h_1} : \Delta_2, \mathtt{F_4} \vdash \Delta_5} \mathtt{H}$$

- Case rule $\perp_L$

$$\frac{}{\bullet \mathtt{h_3} : \perp, \Delta_5, \mathtt{F_1} \vee \mathtt{F_2} \vdash \Delta_4} \perp_L \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h_3} : \perp, \Delta_5, \mathtt{F_2} \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h_3} : \mathtt{p_4}, \Delta_6, \mathtt{F_1} \vee \mathtt{F_2} \vdash \mathtt{p_4}, \Delta_5} I \quad \rightsquigarrow \quad \frac{}{\bullet \mathtt{h_3} : \Delta_6, \mathtt{F_2}, \mathtt{p_4} \vdash \Delta_5, \mathtt{p_4}} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h_3} : \Delta_5, \mathtt{F_1} \vee \mathtt{F_2} \vdash \Delta_4}{\bullet \mathtt{h_3} : \top, \Delta_5, \mathtt{F_1} \vee \mathtt{F_2} \vdash \Delta_4} \top_L \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h_3} : \Delta_5, \mathtt{F_2} \vdash \Delta_4} \mathtt{ax/ind}}{\bullet \mathtt{h_3} : \top, \Delta_5, \mathtt{F_2} \vdash \Delta_4} \top_L$$

4.12   Status of $\perp_L$: :   Invertible

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h_2} : \perp, \Delta_1, \mathtt{F_4} \vdash \mathtt{F_5}}{\bullet \mathtt{h_2} : \perp, \Delta_1 \vdash \Delta_3, \mathtt{F_4} \rightarrow \mathtt{F_5}} \rightarrow_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h_2} : \perp, \Delta_1 \vdash \Delta_3, \mathtt{F_4} \quad \mathtt{h_2} : \perp, \Delta_1 \vdash \Delta_3, \mathtt{F_5}}{\bullet \mathtt{h_2} : \perp, \Delta_1 \vdash \Delta_3, \mathtt{F_4} \wedge \mathtt{F_5}} \wedge_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h_2} : \perp, \Delta_1 \vdash \Delta_3, \mathtt{F_4}, \mathtt{F_5}}{\bullet \mathtt{h_2} : \perp, \Delta_1 \vdash \Delta_3, \mathtt{F_4} \vee \mathtt{F_5}} \vee_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\perp_R$

$$\frac{\mathtt{h_2} : \perp, \Delta_1 \vdash \Delta_3}{\bullet \mathtt{h_2} : \perp, \Delta_1 \vdash \perp, \Delta_3} \perp_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h_2} : \perp, \Delta_1 \vdash \top, \Delta_3} \top_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

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- Case rule $\rightarrow_L$

$$\frac{\mathtt{h_1} : \bot, \Delta_5, \mathtt{F_2} \rightarrow \mathtt{F_3} \vdash \Delta_4, \mathtt{F_2} \quad \mathtt{h_1} : \bot, \Delta_5, \mathtt{F_3} \vdash \Delta_4}{\bullet \mathtt{h_1} : (\bot, \Delta_5), \mathtt{F_2} \rightarrow \mathtt{F_3} \vdash \Delta_4} \rightarrow_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h_1} : \bot, \Delta_5, \mathtt{F_2}, \mathtt{F_3} \vdash \Delta_4}{\bullet \mathtt{h_1} : (\bot, \Delta_5), \mathtt{F_2} \wedge \mathtt{F_3} \vdash \Delta_4} \wedge_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h_1} : \bot, \Delta_5, \mathtt{F_2} \vdash \Delta_4 \quad \mathtt{h_1} : \bot, \Delta_5, \mathtt{F_3} \vdash \Delta_4}{\bullet \mathtt{h_1} : (\bot, \Delta_5), \mathtt{F_2} \vee \mathtt{F_3} \vdash \Delta_4} \vee_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h_1} : \bot, \Delta_2 \vdash \Delta_3} \bot_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h_1} : \mathtt{p_2}, \bot, \Delta_4 \vdash \mathtt{p_2}, \Delta_3} I \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h_1} : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h_1} : \top, \bot, \Delta_3 \vdash \Delta_2} \top_L \quad \rightsquigarrow \quad \mathtt{trivial}$$

4.13   Status of I: :   Invertible

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h_3} : \Delta_1, \mathtt{F_4}, \mathtt{p_2} \vdash \mathtt{F_5}}{\bullet \mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash (\Delta_6, \mathtt{p_2}), \mathtt{F_4} \rightarrow \mathtt{F_5}} \rightarrow_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash \Delta_6, \mathtt{F_4}, \mathtt{p_2} \quad \mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash \Delta_6, \mathtt{F_5}, \mathtt{p_2}}{\bullet \mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash (\Delta_6, \mathtt{p_2}), \mathtt{F_4} \wedge \mathtt{F_5}} \wedge_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash \Delta_6, \mathtt{F_4}, \mathtt{F_5}, \mathtt{p_2}}{\bullet \mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash (\Delta_6, \mathtt{p_2}), \mathtt{F_4} \vee \mathtt{F_5}} \vee_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash \Delta_4, \mathtt{p_2}}{\bullet \mathtt{h_3} : \Delta_1, \mathtt{p_2} \vdash \bot, \Delta_4, \mathtt{p_2}} \bot_R \quad \rightsquigarrow \quad \mathtt{trivial}$$

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- Case rule $\top_R$

$$\frac{}{\bullet\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \top, \Delta_4, \mathtt{p}_2} \ \top_R \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{p}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_2, \mathtt{F}_4, \mathtt{p}_1 \quad \mathtt{h}_3 : \Delta_6, \mathtt{F}_5, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1} \ \rightarrow_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, \mathtt{F}_5, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1} \ \wedge_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1 \quad \mathtt{h}_3 : \Delta_6, \mathtt{F}_5, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1} \ \vee_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule $\perp_L$

$$\frac{}{\bullet\mathtt{h}_3 : \perp, \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1} \ \perp_L \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_2 : \mathtt{p}_3, \Delta_5, \mathtt{p}_1 \vdash \mathtt{p}_3, \Delta_4, \mathtt{p}_1} \ I \quad \rightsquigarrow \quad \text{trivial}$$

$$\frac{}{\bullet\mathtt{h}_1 : \mathtt{p}_3, \Delta_2 \vdash \mathtt{p}_3, \Delta_4} \ I \quad \rightsquigarrow \quad \text{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet\mathtt{h}_3 : \top, \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1} \ \top_L \quad \rightsquigarrow \quad \text{trivial}$$

4.14 Status of $\top_L$: : Invertible

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \top, \Delta_1, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \ \rightarrow_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4 \vdash \mathtt{F}_5} \ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \quad \mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \ \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4} \ \text{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_5} \ \text{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \ \wedge_R$$

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- Case rule $\vee_R$

$$\dfrac{\texttt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \texttt{F}_4, \texttt{F}_5}{\bullet\texttt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \texttt{F}_4 \vee \texttt{F}_5} \ \vee_R \qquad \rightsquigarrow \qquad \dfrac{\overline{\texttt{h}_2 : \Delta_1 \vdash \Delta_3, \texttt{F}_4, \texttt{F}_5} \ \text{ax/ind}}{\bullet\texttt{h}_2 : \Delta_1 \vdash \Delta_3, \texttt{F}_4 \vee \texttt{F}_5} \ \vee_R$$

- Case rule $\perp_R$

$$\dfrac{\texttt{h}_2 : \top, \Delta_1 \vdash \Delta_3}{\bullet\texttt{h}_2 : \top, \Delta_1 \vdash \perp, \Delta_3} \ \perp_R \qquad \rightsquigarrow \qquad \dfrac{\overline{\texttt{h}_2 : \Delta_1 \vdash \Delta_3} \ \text{ax/ind}}{\bullet\texttt{h}_2 : \Delta_1 \vdash \perp, \Delta_3} \ \perp_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet\texttt{h}_2 : \top, \Delta_1 \vdash \top, \Delta_3} \ \top_R \qquad \rightsquigarrow \qquad \dfrac{}{\bullet\texttt{h}_2 : \Delta_1 \vdash \top, \Delta_3} \ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\texttt{h}_1 : \top, \Delta_5, \texttt{F}_2 \rightarrow \texttt{F}_3 \vdash \Delta_4, \texttt{F}_2 \quad \texttt{h}_1 : \top, \Delta_5, \texttt{F}_3 \vdash \Delta_4}{\bullet\texttt{h}_1 : (\top, \Delta_5), \texttt{F}_2 \rightarrow \texttt{F}_3 \vdash \Delta_4} \ \rightarrow_L \qquad \rightsquigarrow \qquad \dfrac{\overline{\texttt{h}_1 : \Delta_5, \texttt{F}_2 \rightarrow \texttt{F}_3 \vdash \Delta_4, \texttt{F}_2} \ \text{ax/ind} \quad \overline{\texttt{h}_1 : \Delta_5, \texttt{F}_3 \vdash \Delta_4} \ \text{ax/ind}}{\bullet\texttt{h}_1 : \Delta_5, \texttt{F}_2 \rightarrow \texttt{F}_3 \vdash \Delta_4} \ \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\texttt{h}_1 : \top, \Delta_5, \texttt{F}_2, \texttt{F}_3 \vdash \Delta_4}{\bullet\texttt{h}_1 : (\top, \Delta_5), \texttt{F}_2 \wedge \texttt{F}_3 \vdash \Delta_4} \ \wedge_L \qquad \rightsquigarrow \qquad \dfrac{\overline{\texttt{h}_1 : \Delta_5, \texttt{F}_2, \texttt{F}_3 \vdash \Delta_4} \ \text{ax/ind}}{\bullet\texttt{h}_1 : \Delta_5, \texttt{F}_2 \wedge \texttt{F}_3 \vdash \Delta_4} \ \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\texttt{h}_1 : \top, \Delta_5, \texttt{F}_2 \vdash \Delta_4 \quad \texttt{h}_1 : \top, \Delta_5, \texttt{F}_3 \vdash \Delta_4}{\bullet\texttt{h}_1 : (\top, \Delta_5), \texttt{F}_2 \vee \texttt{F}_3 \vdash \Delta_4} \ \vee_L \qquad \rightsquigarrow \qquad \dfrac{\overline{\texttt{h}_1 : \Delta_5, \texttt{F}_2 \vdash \Delta_4} \ \text{ax/ind} \quad \overline{\texttt{h}_1 : \Delta_5, \texttt{F}_3 \vdash \Delta_4} \ \text{ax/ind}}{\bullet\texttt{h}_1 : \Delta_5, \texttt{F}_2 \vee \texttt{F}_3 \vdash \Delta_4} \ \vee_L$$

- Case rule $\perp_L$

$$\dfrac{}{\bullet\texttt{h}_1 : \perp, \top, \Delta_3 \vdash \Delta_2} \ \perp_L \qquad \rightsquigarrow \qquad \dfrac{}{\bullet\texttt{h}_1 : \perp, \Delta_3 \vdash \Delta_2} \ \perp_L$$

- Case rule I

$$\dfrac{}{\bullet\texttt{h}_1 : \texttt{p}_2, \top, \Delta_4 \vdash \texttt{p}_2, \Delta_3} \ I \qquad \rightsquigarrow \qquad \dfrac{}{\bullet\texttt{h}_1 : \Delta_4, \texttt{p}_2 \vdash \Delta_3, \texttt{p}_2} \ I$$

- Case rule $\top_L$

$$\dfrac{\texttt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet\texttt{h}_1 : \top, \Delta_2 \vdash \Delta_3} \ \top_L \qquad \rightsquigarrow \qquad \dfrac{\overline{\texttt{h}_1 : \Delta_2 \vdash \Delta_3} \ \text{ax}}{\bullet\texttt{h}_1 : \Delta_2 \vdash \Delta_3} \ \text{H}$$

5 Height preserving admissibility of contraction on the left

- Case(s) rule $\to_R$

$$\dfrac{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2, \texttt{F}_5 \vdash \texttt{F}_6}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5 \to \texttt{F}_6} \to_R \qquad \leadsto \qquad \dfrac{\dfrac{\overline{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2, \texttt{F}_5 \vdash \texttt{F}_6}\ \text{ax}}{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_5 \vdash \texttt{F}_6}\ \text{IH}}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5 \to \texttt{F}_6} \to_R$$

- Case(s) rule $\wedge_R$

$$\dfrac{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5 \quad \texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_6}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5 \wedge \texttt{F}_6} \wedge_R \qquad \leadsto \qquad \dfrac{\dfrac{\overline{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5}\ \text{ax}}{\texttt{h}_3 : \Delta_1, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5}\ \text{IH} \quad \dfrac{\overline{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_6}\ \text{ax}}{\texttt{h}_3 : \Delta_1, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_6}\ \text{IH}}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5 \wedge \texttt{F}_6} \wedge_R$$

- Case(s) rule $\vee_R$

$$\dfrac{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5, \texttt{F}_6}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5 \vee \texttt{F}_6} \vee_R \qquad \leadsto \qquad \dfrac{\dfrac{\overline{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5, \texttt{F}_6}\ \text{ax}}{\texttt{h}_3 : \Delta_1, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5, \texttt{F}_6}\ \text{IH}}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2 \vdash \Delta_4, \texttt{F}_5 \vee \texttt{F}_6} \vee_R$$

- Case(s) rule $\perp_R$

$$\dfrac{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \perp, \Delta_4} \perp_R \qquad \leadsto \qquad \dfrac{\dfrac{\overline{\texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \Delta_4}\ \text{ax}}{\texttt{h}_3 : \Delta_1, \texttt{F}_2 \vdash \Delta_4}\ \text{IH}}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2 \vdash \perp, \Delta_4} \perp_R$$

- Case(s) rule $\top_R$

$$\dfrac{}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2, \texttt{F}_2 \vdash \top, \Delta_4} \top_R \qquad \leadsto \qquad \dfrac{}{\bullet \texttt{h}_3 : \Delta_1, \texttt{F}_2 \vdash \top, \Delta_4} \top_R$$

- Case(s) rule $\to_L$

$$\dfrac{\texttt{h}_2 : \Delta_1, \texttt{F}_3 \to \texttt{F}_4, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5, \texttt{F}_3 \quad \texttt{h}_2 : \Delta_1, \texttt{F}_4, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5}{\bullet \texttt{h}_2 : \Delta_1, \texttt{F}_3 \to \texttt{F}_4, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5} \to_L \leadsto \dfrac{\dfrac{\overline{\texttt{h}_2 : \Delta_1, \texttt{F}_3 \to \texttt{F}_4, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5, \texttt{F}_3}\ \text{ax}}{\texttt{h}_2 : \Delta_1, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5, \texttt{F}_3}\ \text{IH} \quad \dfrac{\overline{\texttt{h}_2 : \Delta_1, \texttt{F}_4, \texttt{F}_4 \vdash \Delta_5}\ \text{inv-th/ax}}{\texttt{h}_2 : \Delta_1, \texttt{F}_4 \vdash \Delta_5}\ \text{IH}}{\bullet \texttt{h}_2 : \Delta_1, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5} \to_L$$

$$\dfrac{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5, \texttt{F}_3 \quad \texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_4 \vdash \Delta_5}{\bullet \texttt{h}_2 : (\Delta_6, \texttt{F}_3 \to \texttt{F}_4), \texttt{F}_1, \texttt{F}_1 \vdash \Delta_5} \to_L \leadsto \dfrac{\dfrac{\overline{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5, \texttt{F}_3}\ \text{ax}}{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5, \texttt{F}_3}\ \text{IH} \quad \dfrac{\overline{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_4 \vdash \Delta_5}\ \text{ax}}{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_4 \vdash \Delta_5}\ \text{IH}}{\bullet \texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_3 \to \texttt{F}_4 \vdash \Delta_5} \to_L$$

- Case(s) rule $\wedge_L$

$$\dfrac{\texttt{h}_2 : \Delta_1, \texttt{F}_3, \texttt{F}_4, \texttt{F}_3 \wedge \texttt{F}_4 \vdash \Delta_5}{\bullet \texttt{h}_2 : \Delta_1, \texttt{F}_3 \wedge \texttt{F}_4, \texttt{F}_3 \wedge \texttt{F}_4 \vdash \Delta_5} \wedge_L \qquad \leadsto \qquad \dfrac{\dfrac{\dfrac{\overline{\texttt{h}_2 : \Delta_1, \texttt{F}_3, \texttt{F}_3, \texttt{F}_4, \texttt{F}_4 \vdash \Delta_5}\ \text{inv-th/ax}}{\texttt{h}_2 : \Delta_1, \texttt{F}_3, \texttt{F}_3, \texttt{F}_4 \vdash \Delta_5}\ \text{IH}}{\texttt{h}_2 : \Delta_1, \texttt{F}_3, \texttt{F}_4 \vdash \Delta_5}\ \text{IH}}{\bullet \texttt{h}_2 : \Delta_1, \texttt{F}_3 \wedge \texttt{F}_4 \vdash \Delta_5} \wedge_L$$

$$\dfrac{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_3, \texttt{F}_4 \vdash \Delta_5}{\bullet \texttt{h}_2 : (\Delta_6, \texttt{F}_3 \wedge \texttt{F}_4), \texttt{F}_1, \texttt{F}_1 \vdash \Delta_5} \wedge_L \qquad \leadsto \qquad \dfrac{\dfrac{\overline{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_3, \texttt{F}_4 \vdash \Delta_5}\ \text{ax}}{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_3, \texttt{F}_4 \vdash \Delta_5}\ \text{IH}}{\bullet \texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_3 \wedge \texttt{F}_4 \vdash \Delta_5} \wedge_L$$

- Case(s) rule $\vee_L$

$$\dfrac{\texttt{h}_2 : \Delta_1, \texttt{F}_3, \texttt{F}_3 \vee \texttt{F}_4 \vdash \Delta_5 \quad \texttt{h}_2 : \Delta_1, \texttt{F}_4, \texttt{F}_3 \vee \texttt{F}_4 \vdash \Delta_5}{\bullet \texttt{h}_2 : \Delta_1, \texttt{F}_3 \vee \texttt{F}_4, \texttt{F}_3 \vee \texttt{F}_4 \vdash \Delta_5} \vee_L \quad \leadsto \quad \dfrac{\dfrac{\overline{\texttt{h}_2 : \Delta_1, \texttt{F}_3, \texttt{F}_3 \vdash \Delta_5}\ \text{inv-th/ax}}{\texttt{h}_2 : \Delta_1, \texttt{F}_3 \vdash \Delta_5}\ \text{IH} \quad \dfrac{\overline{\texttt{h}_2 : \Delta_1, \texttt{F}_4, \texttt{F}_4 \vdash \Delta_5}\ \text{inv-th/ax}}{\texttt{h}_2 : \Delta_1, \texttt{F}_4 \vdash \Delta_5}\ \text{IH}}{\bullet \texttt{h}_2 : \Delta_1, \texttt{F}_3 \vee \texttt{F}_4 \vdash \Delta_5} \vee_L$$

$$\dfrac{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_3 \vdash \Delta_5 \quad \texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_4 \vdash \Delta_5}{\bullet \texttt{h}_2 : (\Delta_6, \texttt{F}_3 \vee \texttt{F}_4), \texttt{F}_1, \texttt{F}_1 \vdash \Delta_5} \vee_L \quad \leadsto \quad \dfrac{\dfrac{\overline{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_3 \vdash \Delta_5}\ \text{ax}}{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_3 \vdash \Delta_5}\ \text{IH} \quad \dfrac{\overline{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_1, \texttt{F}_4 \vdash \Delta_5}\ \text{ax}}{\texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_4 \vdash \Delta_5}\ \text{IH}}{\bullet \texttt{h}_2 : \Delta_6, \texttt{F}_1, \texttt{F}_3 \vee \texttt{F}_4 \vdash \Delta_5} \vee_L$$

- Case(s) rule $\perp_L$

$$\frac{}{\bullet h_2 : \Delta_1, \perp, \perp \vdash \Delta_3} \perp_L \qquad \leadsto \qquad \frac{}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3} \perp_L$$

$$\frac{}{\bullet h_2 : (\perp, \Delta_4), F_1, F_1 \vdash \Delta_3} \perp_L \qquad \leadsto \qquad \frac{}{\bullet h_2 : \perp, \Delta_4, F_1 \vdash \Delta_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_2 : \Delta_1, p_3, p_3 \vdash \Delta_4, p_3} I \qquad \leadsto \qquad \frac{}{\bullet h_2 : \Delta_1, p_3 \vdash \Delta_4, p_3} I$$

$$\frac{}{\bullet h_2 : (\Delta_5, p_3), F_1, F_1 \vdash \Delta_4, p_3} I \qquad \leadsto \qquad \frac{}{\bullet h_2 : \Delta_5, F_1, p_3 \vdash \Delta_4, p_3} I$$

- Case(s) rule $\top_L$

$$\frac{h_2 : \top, \Delta_1 \vdash \Delta_3}{\bullet h_2 : \Delta_1, \top, \top \vdash \Delta_3} \top_L \qquad \leadsto \qquad \frac{\dfrac{}{h_2 : \Delta_1 \vdash \Delta_3} \text{inv-th/ax}}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3} \top_L$$

$$\frac{h_2 : \Delta_4, F_1, F_1 \vdash \Delta_3}{\bullet h_2 : (\top, \Delta_4), F_1, F_1 \vdash \Delta_3} \top_L \qquad \leadsto \qquad \frac{\dfrac{\dfrac{}{h_2 : \Delta_4, F_1, F_1 \vdash \Delta_3} \text{ax}}{h_2 : \Delta_4, F_1 \vdash \Delta_3} \text{IH}}{\bullet h_2 : \top, \Delta_4, F_1 \vdash \Delta_3} \top_L$$

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6 Height preserving admissibility of contraction on the Right

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \to \mathtt{F}_5, \mathtt{F}_4 \to \mathtt{F}_5} \to_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \mathtt{F}_5} \; \mathtt{ax}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \to \mathtt{F}_5} \to_R$$

$$\frac{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \to \mathtt{F}_5), \mathtt{F}_1, \mathtt{F}_1} \to_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \mathtt{F}_5} \; \mathtt{ax}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \to \mathtt{F}_5} \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_4 \wedge \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_4 \wedge \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \wedge \mathtt{F}_5, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_4} \; \mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4} \; \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_5, \mathtt{F}_5} \; \mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_5} \; \mathtt{IH}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4 \quad \mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5), \mathtt{F}_1, \mathtt{F}_1} \wedge_R \quad \rightsquigarrow \quad \frac{\dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4} \; \mathtt{ax}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4} \; \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_5} \; \mathtt{ax}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_5} \; \mathtt{IH}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_4 \vee \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \vee \mathtt{F}_5, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \qquad \rightsquigarrow \qquad \frac{\dfrac{\dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_5} \; \mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_4, \mathtt{F}_5} \; \mathtt{IH}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4, \mathtt{F}_5} \; \mathtt{IH}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5), \mathtt{F}_1, \mathtt{F}_1} \vee_R \qquad \rightsquigarrow \qquad \frac{\dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5} \; \mathtt{ax}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5} \; \mathtt{IH}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case(s) rule $\perp_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \perp, \Delta_1}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \perp, \perp} \perp_R \qquad \rightsquigarrow \qquad \frac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1} \; \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \perp, \Delta_1} \perp_R$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_1}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\perp, \Delta_4), \mathtt{F}_1, \mathtt{F}_1} \perp_R \qquad \rightsquigarrow \qquad \frac{\dfrac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1, \mathtt{F}_1} \; \mathtt{ax}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_4, \mathtt{F}_1} \; \mathtt{IH}}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \perp, \Delta_4, \mathtt{F}_1} \perp_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \top, \top} \top_R \qquad \rightsquigarrow \qquad \frac{}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_1} \top_R$$

$$\frac{}{\bullet\mathtt{h}_2 : \Delta_3 \vdash (\top, \Delta_4), \mathtt{F}_1, \mathtt{F}_1} \top_R \qquad \rightsquigarrow \qquad \frac{}{\bullet\mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_4, \mathtt{F}_1} \top_R$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_5 \quad \mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}{\bullet\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2} \to_L \quad \rightsquigarrow \quad \frac{\dfrac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2, \mathtt{F}_5} \; \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_5} \; \mathtt{IH} \quad \dfrac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2} \; \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2} \; \mathtt{IH}}{\bullet\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2} \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2}{\bullet\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2} \wedge_L \qquad \rightsquigarrow \qquad \frac{\dfrac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_2} \; \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2} \; \mathtt{IH}}{\bullet\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2} \wedge_L$$

- Case(s) rule $\vee_L$

$$
\dfrac{\mathtt{h_3} : \Delta_4, \mathtt{F_5} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_2} \quad \mathtt{h_3} : \Delta_4, \mathtt{F_6} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_2}}{\bullet\mathtt{h_3} : \Delta_4, \mathtt{F_5} \vee \mathtt{F_6} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_2}} \ \vee_L
\qquad \rightsquigarrow \qquad
\dfrac{\dfrac{\overline{\mathtt{h_3} : \Delta_4, \mathtt{F_5} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_2}} \ \mathtt{ax}}{\mathtt{h_3} : \Delta_4, \mathtt{F_5} \vdash \Delta_1, \mathtt{F_2}} \ \mathtt{IH} \quad \dfrac{\overline{\mathtt{h_3} : \Delta_4, \mathtt{F_6} \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_2}} \ \mathtt{ax}}{\mathtt{h_3} : \Delta_4, \mathtt{F_6} \vdash \Delta_1, \mathtt{F_2}} \ \mathtt{IH}}{\bullet\mathtt{h_3} : \Delta_4, \mathtt{F_5} \vee \mathtt{F_6} \vdash \Delta_1, \mathtt{F_2}} \ \vee_L
$$

- Case(s) rule $\bot_L$

$$
\dfrac{}{\bullet\mathtt{h_3} : \bot, \Delta_4 \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_2}} \ \bot_L
\qquad \rightsquigarrow \qquad
\dfrac{}{\bullet\mathtt{h_3} : \bot, \Delta_4 \vdash \Delta_1, \mathtt{F_2}} \ \bot_L
$$

- Case(s) rule I

$$
\dfrac{}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{p_4} \vdash \Delta_1, \mathtt{p_4}, \mathtt{p_4}} \ I
\qquad \rightsquigarrow \qquad
\dfrac{}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{p_4} \vdash \Delta_1, \mathtt{p_4}} \ I
$$

$$
\dfrac{}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{p_4} \vdash (\Delta_5, \mathtt{p_4}), \mathtt{F_1}, \mathtt{F_1}} \ I
\qquad \rightsquigarrow \qquad
\dfrac{}{\bullet\mathtt{h_2} : \Delta_3, \mathtt{p_4} \vdash \Delta_5, \mathtt{F_1}, \mathtt{p_4}} \ I
$$

- Case(s) rule $\top_L$

$$
\dfrac{\mathtt{h_3} : \Delta_4 \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_2}}{\bullet\mathtt{h_3} : \top, \Delta_4 \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_2}} \ \top_L
\qquad \rightsquigarrow \qquad
\dfrac{\dfrac{\overline{\mathtt{h_3} : \Delta_4 \vdash \Delta_1, \mathtt{F_2}, \mathtt{F_2}} \ \mathtt{ax}}{\mathtt{h_3} : \Delta_4 \vdash \Delta_1, \mathtt{F_2}} \ \mathtt{IH}}{\bullet\mathtt{h_3} : \top, \Delta_4 \vdash \Delta_1, \mathtt{F_2}} \ \top_L
$$

7 Identity-Expansion

$$
\cfrac{
  \cfrac{
    \cfrac{\overline{-:\mathbf{F_0}\vdash\mathbf{F_0}}\ \mathtt{IH}}{-:\mathbf{F_0}\vdash\mathbf{F_0},\mathbf{F_1}}\ W
    \qquad
    \cfrac{\overline{-:\mathbf{F_1}\vdash\mathbf{F_1}}\ \mathtt{IH}}{-:\mathbf{F_1}\vdash\mathbf{F_0},\mathbf{F_1}}\ W
  }{-:\mathbf{F_0}\vee\mathbf{F_1}\vdash\mathbf{F_0},\mathbf{F_1}}\ \vee_L
}{-:\mathbf{F_0}\vee\mathbf{F_1}\vdash\mathbf{F_0}\vee\mathbf{F_1}}\ \vee_R
$$

$$
\cfrac{
  \cfrac{
    \cfrac{\overline{-:\mathbf{F_0}\vdash\mathbf{F_0}}\ \mathtt{IH}}{-:\mathbf{F_0},\mathbf{F_1}\vdash\mathbf{F_0}}\ W
    \qquad
    \cfrac{\overline{-:\mathbf{F_1}\vdash\mathbf{F_1}}\ \mathtt{IH}}{-:\mathbf{F_0},\mathbf{F_1}\vdash\mathbf{F_1}}\ W
  }{-:\mathbf{F_0},\mathbf{F_1}\vdash\mathbf{F_0}\wedge\mathbf{F_1}}\ \wedge_R
}{-:\mathbf{F_0}\wedge\mathbf{F_1}\vdash\mathbf{F_0}\wedge\mathbf{F_1}}\ \wedge_L
$$

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{\overline{-:\mathbf{F_0}\vdash\mathbf{F_0}}\ \mathtt{IH}}{-:\mathbf{F_0},\mathbf{F_0}\to\mathbf{F_1}\vdash\mathbf{F_0}}\ W
    }{-:\mathbf{F_0},\mathbf{F_0}\to\mathbf{F_1}\vdash\mathbf{F_0},\mathbf{F_1}}\ W
    \qquad
    \cfrac{\overline{-:\mathbf{F_1}\vdash\mathbf{F_1}}\ \mathtt{IH}}{-:\mathbf{F_0},\mathbf{F_1}\vdash\mathbf{F_1}}\ W
  }{-:\mathbf{F_0},\mathbf{F_0}\to\mathbf{F_1}\vdash\mathbf{F_1}}\ \to_L
}{-:\mathbf{F_0}\to\mathbf{F_1}\vdash\mathbf{F_0}\to\mathbf{F_1}}\ \to_R
$$

$$
\cfrac{}{-:\top\vdash\top}\ \top_R
$$

$$
\cfrac{}{-:\bot\vdash\bot}\ \bot_L
$$

8 Cut-Elimination

8.1 Status of $\to_R$: OK

- Case rule $\to_R$

$$
\dfrac{
\dfrac{h_2 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \to F_{13}), F_9 \to F_{10}), F_8} \to_R
\qquad
\dfrac{h_{11} : \Delta_7, F_8, F_{12} \vdash F_{13}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_9 \to F_{10}} \to_R
}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_9 \to F_{10}} \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{\dfrac{}{- : \Delta_7, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{14}, F_{12} \to F_{13}, F_9 \to F_{10}} \to_R
$$

$$
\dfrac{
\dfrac{h_2 : \Delta_7, F_{11} \vdash F_{12}}{\bullet h_2 : \Delta_7 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_8} \to_R
\qquad
\dfrac{h_9 : \Delta_7, F_8, F_{11} \vdash F_{12}}{\bullet h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R
}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \to F_{12}} \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{\dfrac{}{- : \Delta_7, F_{11} \vdash F_{12}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R
$$

$$
\dfrac{
\dfrac{h_1 : \Delta_6, F_7 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7 \to F_8} \to_R
\qquad
\dfrac{h_9 : \Delta_6, F_{11}, F_7 \to F_8 \vdash F_{12}}{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R
}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}} \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{\dfrac{}{h_1 : \Delta_6, F_{11}, F_7 \vdash F_8} \text{ax/W}}{\bullet h_1 : \Delta_6, F_{11} \vdash F_{12}, F_7 \to F_8} \to_R
\qquad
\dfrac{}{h_9 : \Delta_6, F_{11}, F_7 \to F_8 \vdash F_{12}} \text{ax/W}
}{
\dfrac{- : \Delta_6, F_{11} \vdash F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R
} \text{hCut}
$$

- Case rule $\wedge_R$

$$
\dfrac{
\dfrac{h_2 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \to F_{10}), F_8} \to_R
\qquad
\dfrac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_9 \to F_{10} \quad h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{13}, F_9 \to F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \to F_{10}} \wedge_R
}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \to F_{10}} \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{\dfrac{}{- : \Delta_7, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{14}, F_9 \to F_{10}, F_{12} \wedge F_{13}} \to_R
$$

$$
\dfrac{
\dfrac{h_1 : \Delta_6, F_7 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \to F_8} \to_R
\qquad
\dfrac{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11} \quad h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R
}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{
\dfrac{\dfrac{}{h_1 : \Delta_6, F_7 \vdash F_8} \text{ax/W}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \to F_8} \to_R
\quad
\dfrac{}{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11}} \text{ax/W}
}{- : \Delta_6 \vdash \Delta_{10}, F_{11}} \text{hCut}
\quad
\dfrac{
\dfrac{\dfrac{}{h_1 : \Delta_6, F_7 \vdash F_8} \text{ax/W}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7 \to F_8} \to_R
\quad
\dfrac{}{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W}
}{- : \Delta_6 \vdash \Delta_{10}, F_{12}} \text{hCut}
}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R
$$

- Case rule $\vee_R$

$$
\dfrac{
\dfrac{h_2 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \to F_{10}), F_8} \to_R
\qquad
\dfrac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \to F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \to F_{10}} \vee_R
}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \to F_{10}} \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{\dfrac{}{- : \Delta_7, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{14}, F_9 \to F_{10}, F_{12} \vee F_{13}} \to_R
$$

$$\dfrac{\dfrac{h_1 : \Delta_6, F_7 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \lor F_{12}), F_7 \to F_8} \to_R \quad \dfrac{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11} \lor F_{12}} \lor_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \lor F_{12}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_1 : \Delta_6, F_7 \vdash F_8} \text{ax/W}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \to F_8} \to_R \quad \dfrac{}{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{\dfrac{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \lor F_{12}} \lor_R} \text{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{h_2 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\bot, \Delta_{12}), F_9 \to F_{10}), F_8} \to_R \quad \dfrac{h_{11} : \Delta_7, F_8 \vdash \Delta_{12}, F_9 \to F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\bot, \Delta_{12}), F_9 \to F_{10}} \bot_R}{- : \Delta_7 \vdash (\bot, \Delta_{12}), F_9 \to F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_7, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_{12}, F_9 \to F_{10}} \to_R$$

$$\dfrac{\dfrac{h_1 : \Delta_6, F_7 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash (\bot, \Delta_{10}), F_7 \to F_8} \to_R \quad \dfrac{h_9 : \Delta_6, F_7 \to F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \bot, \Delta_{10}} \bot_R}{- : \Delta_6 \vdash \bot, \Delta_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_6 \vdash \bot, \Delta_{10}, F_7 \to F_8} \text{ax/W} \quad \dfrac{}{h_9 : \Delta_6, F_7 \to F_8 \vdash \bot, \Delta_{10}} \text{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}} \text{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{h_2 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\top, \Delta_{12}), F_9 \to F_{10}), F_8} \to_R \quad \dfrac{}{\bullet h_{11} : \Delta_7, F_8 \vdash (\top, \Delta_{12}), F_9 \to F_{10}} \top_R}{- : \Delta_7 \vdash (\top, \Delta_{12}), F_9 \to F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_{12}, F_9 \to F_{10}} \top_R$$

$$\dfrac{\dfrac{h_1 : \Delta_6, F_7 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \to F_8} \to_R \quad \dfrac{}{\bullet h_9 : \Delta_6, F_7 \to F_8 \vdash \top, \Delta_{10}} \top_R}{- : \Delta_6 \vdash \top, \Delta_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}} \top_R$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_2 : (\Delta_{14}, F_{12} \to F_{13}), F_9 \vdash F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \to F_{13} \vdash (\Delta_8, F_9 \to F_{10}), F_7} \to_R \quad \dfrac{h_{11} : \Delta_{14}, F_7, F_{12} \to F_{13} \vdash \Delta_8, F_{12}, F_9 \to F_{10} \quad h_{11} : \Delta_{14}, F_7, F_{13} \vdash \Delta_8, F_9 \to F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \to F_{13}), F_7 \vdash \Delta_8, F_9 \to F_{10}} \to_L}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_9 \to F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_{14}, F_9, F_{12} \to F_{13} \vdash F_{10}} \text{ax/W}}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_9 \to F_{10}} \to_R$$

$$\dfrac{\dfrac{h_2 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), F_{12} \to F_{13}} \to_R \quad \dfrac{h_{10} : \Delta_{11}, F_{12} \to F_{13} \vdash \Delta_7, F_{12}, F_8 \to F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \to F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \to F_{13} \vdash \Delta_7, F_8 \to F_9} \to_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_{11}, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \to_R$$

$$\dfrac{\dfrac{h_1 : (\Delta_{12}, F_9 \to F_{10}), F_6 \vdash F_7}{\bullet h_1 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_{12}, F_6 \to F_7, F_9 \to F_{10} \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \to F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \to F_{10}), F_6 \to F_7 \vdash \Delta_{11}} \to_L}{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \text{ Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_9, F_6 \to F_7} \text{ ax/W} \quad \dfrac{}{h_8 : \Delta_{12}, F_6 \to F_7, F_9 \to F_{10} \vdash \Delta_{11}, F_9} \text{ ax/W}}{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_9} \text{ hCut} \quad \dfrac{\dfrac{\dfrac{}{h_1 : \Delta_{12}, F_{10}, F_6 \vdash F_7} \text{ inv-th/ax}}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \to F_7} \to_R \quad \dfrac{}{h_8 : \Delta_{12}, F_{10}, F_6 \to}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \to_L}{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}}$$

$$\dfrac{\dfrac{h_1 : \Delta_7, F_8 \vdash F_9}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \to F_9} \to_R \quad \dfrac{h_6 : \Delta_7, F_8 \to F_9 \vdash \Delta_{10}, F_8 \quad h_6 : \Delta_7, F_9 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash \Delta_{10}} \to_L}{- : \Delta_7 \vdash \Delta_{10}} \text{ Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8, F_8 \to F_9} \text{ ax/W} \quad \dfrac{}{h_6 : \Delta_7, F_8 \to F_9 \vdash \Delta_{10}, F_8} \text{ ax/W}}{- : \Delta_7 \vdash \Delta_{10}, F_8} \text{ hCut} \quad \dfrac{\dfrac{}{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9} \text{ ax/W} \quad \dfrac{}{- : \Delta_7, F_8, F_9 \vdash \Delta_{10}} \text{ ax/W}}{- : \Delta_7, F_8 \vdash \Delta_{10}} \text{ sCut}}{- : \Delta_7 \vdash \Delta_{10}} \text{ sCut}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_2 : (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vdash F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \to F_{10}), F_7} \to_R \quad \dfrac{h_{11} : \Delta_{14}, F_7, F_{12}, F_{13} \vdash \Delta_8, F_9 \to F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \to F_{10}} \wedge_L}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \to F_{10}} \text{ Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{- : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash F_{10}} \text{ ax/W}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \to F_{10}} \to_R$$

$$\dfrac{\dfrac{h_2 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), F_{12} \wedge F_{13}} \to_R \quad \dfrac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8 \to F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \to F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \text{ Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{- : \Delta_{11}, F_8 \vdash F_9} \text{ ax/W}}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \to_R$$

$$\dfrac{\dfrac{h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vdash F_7}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \to F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \to F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{ Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{h_1 : \Delta_{12}, F_{10}, F_6, F_9 \vdash F_7} \text{ inv-th/ax}}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \to F_7} \to_R \quad \dfrac{}{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \to F_7 \vdash \Delta_{11}} \text{ ax/W}}{\dfrac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L} \text{ hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_2 : (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vdash F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \to F_{10}), F_7} \to_R \quad \dfrac{h_{11} : \Delta_{14}, F_7, F_{12} \vdash \Delta_8, F_9 \to F_{10} \quad h_{11} : \Delta_{14}, F_7, F_{13} \vdash \Delta_8, F_9 \to F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \to F_{10}} \vee_L}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \to F_{10}} \text{ Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{- : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash F_{10}} \text{ ax/W}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \to F_{10}} \to_R$$

$$\dfrac{\dfrac{h_2 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), F_{12} \vee F_{13}} \to_R \quad \dfrac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \to F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \to F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \to F_9} \vee_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \text{ Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{- : \Delta_{11}, F_8 \vdash F_9} \text{ ax/W}}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \to_R$$

$$\frac{\dfrac{h_1 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vdash F_7}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \to F_7} \to_R \qquad \dfrac{h_8 : \Delta_{12}, F_9, F_6 \to F_7 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \to F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \to F_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{\dfrac{h_1 : \Delta_{12}, F_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \to F_7} \xrightarrow{\text{inv-th/ax}}_{\to_R} \quad \dfrac{}{h_8 : \Delta_{12}, F_9, F_6 \to F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_9 \vdash \Delta_{11}} \text{hCut} \qquad \dfrac{\dfrac{\dfrac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \to F_7} \xrightarrow{\text{inv-th/ax}}_{\to_R} \quad \dfrac{}{h_8 : \Delta_{12}, F_{10}, F_6 \to F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \text{hCut}}{}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L$$

- Case rule $\perp_L$

$$\frac{\dfrac{h_2 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), \perp} \to_R \quad \dfrac{}{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \to F_9} \perp_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{\dfrac{}{- : \Delta_{11}, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \to_R}{}$$

$$\frac{\dfrac{h_2 : (\perp, \Delta_{12}), F_9 \vdash F_{10}}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \to F_{10}), F_7} \to_R \quad \dfrac{}{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \to F_{10}} \perp_L}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \to F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \to F_{10}} \perp_L$$

$$\frac{\dfrac{h_1 : (\perp, \Delta_{10}), F_6 \vdash F_7}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \to F_7} \to_R \quad \dfrac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \to F_7 \vdash \Delta_9} \perp_L}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L$$

- Case rule I

$$\frac{\dfrac{h_2 : \Delta_{10}, F_7 \vdash F_8}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \to F_8), p_{11}} \to_R \quad \dfrac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \to F_8} I}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \to F_8} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{\dfrac{}{- : \Delta_{10}, F_7 \vdash F_8} \text{ax/W}}{- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \to F_8} \to_R}{}$$

$$\frac{\dfrac{h_2 : (\Delta_{13}, p_{11}), F_8 \vdash F_9}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \to F_9), F_7} \to_R \quad \dfrac{}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \to F_9} I}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \to F_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \to F_9} I$$

$$\frac{\dfrac{h_1 : (\Delta_{11}, p_9), F_6 \vdash F_7}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \to F_7} \to_R \quad \dfrac{}{\bullet h_8 : (\Delta_{11}, p_9), F_6 \to F_7 \vdash \Delta_{10}, p_9} I}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I$$

- Case rule $\top_L$

$$\frac{\dfrac{h_2 : \Delta_{11}, F_8 \vdash F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \to F_9), \top} \to_R \quad \dfrac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \to F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \to F_9} \top_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \to F_9} \text{ax/W}$$

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$$\frac{\dfrac{h_2 : (\top, \Delta_{12}), F_9 \vdash F_{10}}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \to F_{10}), F_7} \to_R \quad \dfrac{h_{11} : \Delta_{12}, F_7 \vdash \Delta_8, F_9 \to F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \to F_{10}} \top_L}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \to F_{10}} \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{}{- : \top, \Delta_{12}, F_9 \vdash F_{10}} \text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \to F_{10}} \to_R$$

$$\frac{\dfrac{h_1 : (\top, \Delta_{10}), F_6 \vdash F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \to F_7} \to_R \quad \dfrac{h_8 : \Delta_{10}, F_6 \to F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \to F_7 \vdash \Delta_9} \top_L}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \to F_7} \text{ax/W} \quad \dfrac{}{h_8 : \top, \Delta_{10}, F_6 \to F_7 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}$$

8.2 Status of $\wedge_R$: OK

- Case rule $\to_R$

$$\frac{\dfrac{h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_8, F_9 \quad h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_8, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \to F_{13}), F_9 \wedge F_{10}), F_8} \wedge_R \quad \dfrac{h_{11} : \Delta_7, F_8, F_{12} \vdash F_{13}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_9 \wedge F_{10}} \to_R}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \to F_{13}), F_9 \wedge F_{10}} \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{\dfrac{}{h_2 : \Delta_7 \vdash \Delta_{14}, F_8, F_9, F_{12} \to F_{13}} \text{ax/W} \quad \dfrac{\dfrac{}{h_{11} : \Delta_7, F_{12}, F_8 \vdash F_{13}} \text{ax/W}}{\bullet h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_9, F_{12} \to F_{13}} \to_R}{- : \Delta_7 \vdash \Delta_{14}, F_9, F_{12} \to F_{13}} \text{hCut} \quad \dfrac{\dfrac{}{h_2 : \Delta_7 \vdash \Delta_{14}, F_{10}, F_8, F_{12} \to F_{13}} \text{ax/W} \quad \dfrac{}{\bullet h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{10}, \ldots}{h_{11} : \Delta_7, F_{12}, F_8 \vdash \ldots}}{- : \Delta_7 \vdash \Delta_{14}, F_{10}, F_{12} \to F_{13}}}{- : \Delta_7 \vdash \Delta_{14}, F_{12} \to F_{13}, F_9 \wedge F_{10}} \wedge_R$$

$$\frac{\dfrac{h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7 \quad h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7 \wedge F_8} \wedge_R \quad \dfrac{h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}} \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{}{- : \Delta_6 \vdash \Delta_{10}, F_7, F_{11} \to F_{12}} \text{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_6, F_7 \vdash \Delta_{10}, F_8, F_{11} \to F_{12}} \text{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_6, F_{11}, F_7, F_8 \vdash F_{12}} \text{inv-th/ax}}{- : \Delta_6, F_7, F_8 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R}{- : \Delta_6, F_7 \vdash \Delta_{10}, F_{11} \to F_{12}} \text{sCut}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}} \text{sCut}$$

- Case rule $\wedge_R$

$$\frac{\dfrac{h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_8, F_9 \quad h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_8, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}), F_8} \wedge_R \quad \dfrac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{\ldots}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\ldots}}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}}$$

$$\leadsto$$

$$\frac{\dfrac{\dfrac{}{h_2 : \Delta_7 \vdash \Delta_{14}, F_{12}, F_8, F_9} \text{inv-th/ax}}{\bullet h_2 : \Delta_7 \vdash \Delta_{14}, F_{12}, F_8, F_9 \wedge F_{10}} \wedge_R \quad \dfrac{}{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}} \text{hCut} \qquad \dfrac{\dfrac{}{h_2 : \Delta_7 \vdash \Delta_{14}, F_{13}, F_8, F_9} \text{inv-}}{\bullet h_2 : \Delta_7 \vdash \Delta_{1\ldots}}}{- : \Delta_7 \vdash \Delta_{14}, F_{12} \wedge F_{13}, F_9 \wedge F_{10}}$$

where the middle conclusion $- : \Delta_7 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}$ uses $\dfrac{h_2 : \Delta_7 \vdash \Delta_{14}, F_{10}, F_{12}, F_8}{} \text{inv-th/ax}$

$$\frac{\dfrac{h_2 : \Delta_7 \vdash \Delta_{10}, F_8, F_{11} \quad h_2 : \Delta_7 \vdash \Delta_{10}, F_8, F_{12}}{\bullet h_2 : \Delta_7 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_8} \wedge_R \quad \dfrac{h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11} \quad h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut}$$

$$\leadsto$$

$$\frac{\dfrac{\dfrac{}{h_2 : \Delta_7 \vdash \Delta_{10}, F_{11}, F_8} \text{ax/W} \quad \dfrac{\dfrac{}{h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11}} \text{ax/W}}{\bullet h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11}} H}{- : \Delta_7 \vdash \Delta_{10}, F_{11}} \text{hCut} \quad \dfrac{\dfrac{}{h_2 : \Delta_7 \vdash \Delta_{10}, F_{12}, F_8} \text{ax/W} \quad \dfrac{\dfrac{}{h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W}}{\bullet h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{12}} H}{- : \Delta_7 \vdash \Delta_{10}, F_{12}} \text{hCut}}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R$$

$$\dfrac{\dfrac{h_1:\Delta_6\vdash(\Delta_{10},F_{11}\wedge F_{12}),F_7 \quad h_1:\Delta_6\vdash(\Delta_{10},F_{11}\wedge F_{12}),F_8}{\bullet h_1:\Delta_6\vdash(\Delta_{10},F_{11}\wedge F_{12}),F_7\wedge F_8}\wedge_R \quad \dfrac{h_9:\Delta_6,F_7\wedge F_8\vdash\Delta_{10},F_{11} \quad h_9:\Delta_6,F_7\wedge F_8\vdash\Delta_{10},F_{12}}{\bullet h_9:\Delta_6,F_7\wedge F_8\vdash\Delta_{10},F_{11}\wedge F_{12}}\wedge_R}{-:\Delta_6\vdash\Delta_{10},F_{11}\wedge F_{12}}\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{-:\Delta_6\vdash\Delta_{10},F_7,F_{11}\wedge F_{12}}\text{ax/W} \quad \dfrac{\dfrac{}{-:\Delta_6,F_7\vdash\Delta_{10},F_8,F_{11}\wedge F_{12}}\text{ax/W} \quad \dfrac{\dfrac{}{-:\Delta_6,F_7,F_8\vdash\Delta_{10},F_{11}}\text{inv-th/ax} \quad \dfrac{}{-:\Delta_6,F_7,F_8\vdash\Delta_{10},F_{12}}\text{inv-th/ax}}{\dfrac{-:\Delta_6,F_7,F_8\vdash\Delta_{10},F_{11}\wedge F_{12}}{-:\Delta_6,F_7\vdash\Delta_{10},F_{11}\wedge F_{12}}\text{sCut}}\wedge_R}{-:\Delta_6\vdash\Delta_{10},F_{11}\wedge F_{12}}\text{sCut}}{}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{h_2:\Delta_7\vdash(\Delta_{14},F_{12}\vee F_{13}),F_8,F_9 \quad h_2:\Delta_7\vdash(\Delta_{14},F_{12}\vee F_{13}),F_8,F_{10}}{\bullet h_2:\Delta_7\vdash((\Delta_{14},F_{12}\vee F_{13}),F_9\wedge F_{10}),F_8}\wedge_R \quad \dfrac{h_{11}:\Delta_7,F_8\vdash\Delta_{14},F_{12},F_{13},F_9\wedge F_{10}}{\bullet h_{11}:\Delta_7,F_8\vdash(\Delta_{14},F_{12}\vee F_{13}),F_9\wedge F_{10}}\vee_R}{-:\Delta_7\vdash(\Delta_{14},F_{12}\vee F_{13}),F_9\wedge F_{10}}\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2:\Delta_7\vdash\Delta_{14},F_{12},F_{13},F_8,F_9}\text{inv-th/ax} \quad \dfrac{}{h_2:\Delta_7\vdash\Delta_{14},F_{10},F_{12},F_{13},F_8}\text{inv-th/ax}}{\dfrac{\bullet h_2:\Delta_7\vdash\Delta_{14},F_{12},F_{13},F_8,F_9\wedge F_{10}}{\quad}\wedge_R} \quad \dfrac{}{h_{11}:\Delta_7,F_8\vdash\Delta_{14},F_{12},F_{13},F_9\wedge F_{10}}\text{ax/W}}{\dfrac{-:\Delta_7\vdash\Delta_{14},F_{12},F_{13},F_9\wedge F_{10}}{-:\Delta_7\vdash\Delta_{14},F_9\wedge F_{10},F_{12}\vee F_{13}}\vee_R}\text{hCut}}{}$$

$$\dfrac{\dfrac{h_1:\Delta_6\vdash(\Delta_{10},F_{11}\vee F_{12}),F_7 \quad h_1:\Delta_6\vdash(\Delta_{10},F_{11}\vee F_{12}),F_8}{\bullet h_1:\Delta_6\vdash(\Delta_{10},F_{11}\vee F_{12}),F_7\wedge F_8}\wedge_R \quad \dfrac{h_9:\Delta_6,F_7\wedge F_8\vdash\Delta_{10},F_{11},F_{12}}{\bullet h_9:\Delta_6,F_7\wedge F_8\vdash\Delta_{10},F_{11}\vee F_{12}}\vee_R}{-:\Delta_6\vdash\Delta_{10},F_{11}\vee F_{12}}\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_1:\Delta_6\vdash\Delta_{10},F_{11},F_{12},F_7}\text{inv-th/ax} \quad \dfrac{}{h_1:\Delta_6\vdash\Delta_{10},F_{11},F_{12},F_8}\text{inv-th/ax}}{\bullet h_1:\Delta_6\vdash\Delta_{10},F_{11},F_{12},F_7\wedge F_8}\wedge_R \quad \dfrac{}{h_9:\Delta_6,F_7\wedge F_8\vdash\Delta_{10},F_{11},F_{12}}\text{ax/W}}{\dfrac{-:\Delta_6\vdash\Delta_{10},F_{11},F_{12}}{-:\Delta_6\vdash\Delta_{10},F_{11}\vee F_{12}}\vee_R}\text{hCut}}{}$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{h_2:\Delta_7\vdash(\perp,\Delta_{12}),F_8,F_9 \quad h_2:\Delta_7\vdash(\perp,\Delta_{12}),F_8,F_{10}}{\bullet h_2:\Delta_7\vdash((\perp,\Delta_{12}),F_9\wedge F_{10}),F_8}\wedge_R \quad \dfrac{h_{11}:\Delta_7,F_8\vdash\Delta_{12},F_9\wedge F_{10}}{\bullet h_{11}:\Delta_7,F_8\vdash(\perp,\Delta_{12}),F_9\wedge F_{10}}\perp_R}{-:\Delta_7\vdash(\perp,\Delta_{12}),F_9\wedge F_{10}}\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet h_2:\Delta_7\vdash\perp,\Delta_{12},F_8,F_9\wedge F_{10}}\text{ax/W} \quad \dfrac{}{h_{11}:\Delta_7,F_8\vdash\perp,\Delta_{12},F_9\wedge F_{10}}\text{ax/W}}{-:\Delta_7\vdash\perp,\Delta_{12},F_9\wedge F_{10}}\text{hCut}$$

$$\dfrac{\dfrac{h_1:\Delta_6\vdash(\perp,\Delta_{10}),F_7 \quad h_1:\Delta_6\vdash(\perp,\Delta_{10}),F_8}{\bullet h_1:\Delta_6\vdash(\perp,\Delta_{10}),F_7\wedge F_8}\wedge_R \quad \dfrac{h_9:\Delta_6,F_7\wedge F_8\vdash\Delta_{10}}{\bullet h_9:\Delta_6,F_7\wedge F_8\vdash\perp,\Delta_{10}}\perp_R}{-:\Delta_6\vdash\perp,\Delta_{10}}\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet h_1:\Delta_6\vdash\perp,\Delta_{10},F_7\wedge F_8}\text{ax/W} \quad \dfrac{}{h_9:\Delta_6,F_7\wedge F_8\vdash\perp,\Delta_{10}}\text{ax/W}}{-:\Delta_6\vdash\perp,\Delta_{10}}\text{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{h_2:\Delta_7\vdash(\top,\Delta_{12}),F_8,F_9 \quad h_2:\Delta_7\vdash(\top,\Delta_{12}),F_8,F_{10}}{\bullet h_2:\Delta_7\vdash((\top,\Delta_{12}),F_9\wedge F_{10}),F_8}\wedge_R \quad \dfrac{}{\bullet h_{11}:\Delta_7,F_8\vdash(\top,\Delta_{12}),F_9\wedge F_{10}}\top_R}{-:\Delta_7\vdash(\top,\Delta_{12}),F_9\wedge F_{10}}\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{-:\Delta_7\vdash\top,\Delta_{12},F_9\wedge F_{10}}\top_R$$

$$\dfrac{\dfrac{h_1:\Delta_6\vdash(\top,\Delta_{10}),F_7 \quad h_1:\Delta_6\vdash(\top,\Delta_{10}),F_8}{\bullet h_1:\Delta_6\vdash(\top,\Delta_{10}),F_7\wedge F_8}\wedge_R \quad \dfrac{}{\bullet h_9:\Delta_6,F_7\wedge F_8\vdash\top,\Delta_{10}}\top_R}{-:\Delta_6\vdash\top,\Delta_{10}}\text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{-:\Delta_6\vdash\top,\Delta_{10}}\top_R$$

- Case rule $\to_L$

$$
\dfrac{
\dfrac{
\dfrac{h_2 : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_7, F_9 \quad h_2 : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \to F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{h_{11} : \Delta_{14}, F_7, F_{12} \to F_{13} \vdash \Delta_8, F_{12}, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \to F_{13}), F_7}}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}
}{\rightsquigarrow}
$$

$$
\dfrac{
\dfrac{\bullet h_2 : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_{12}, F_7, F_9 \wedge F_{10}}{} \text{ax/W} \quad \dfrac{h_{11} : \Delta_{14}, F_7, F_{12} \to F_{13} \vdash \Delta_8, F_{12}, F_9 \wedge F_{10}}{} \text{ax/W} \quad \dfrac{\dfrac{h_2 : \Delta_{14}, F_{13} \vdash \Delta_8, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F} \text{inv-th/ax} \quad \dfrac{h_2 : \Delta_{14}}{}}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_{12}, F_9 \wedge F_{10}} \text{hCut}}{- : \Delta_{14}, F_{12} \to F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}
$$

$$
\dfrac{
\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \to F_{13}, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \to F_{13}, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \to F_{13}} \wedge_R \quad \dfrac{h_{10} : \Delta_{11}, F_{12} \to F_{13} \vdash \Delta_7, F_{12}, F_8}{\bullet h_{10} : \Delta_{11}, F_{12} \to}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}
$$
$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_{12} \to F_{13}}{} \text{ax/W} \quad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12} \to F_{13} \vdash \Delta_7, F_{12}, F_8}{\bullet h_{10} : \Delta_{11}, F_{12} \to F_{13} \vdash \Delta_7, F_8} \text{inv-th/ax} \quad \dfrac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8}{} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_8} \text{hCut} \quad \rightarrow_L \quad \dfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_9, F_{12} \to F_{13}}{}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}
$$

$$
\dfrac{
\dfrac{h_1 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6 \quad h_1 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \dfrac{h_8 : \Delta_{12}, F_9 \to F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \to F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \text{Cut} \quad \rightarrow}{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}}
$$
$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}, F_6}{} \text{ax/W} \quad \dfrac{\dfrac{- : \Delta_{12}, F_6, F_9 \to F_{10} \vdash \Delta_{11}, F_7}{} \text{ax/W} \quad \dfrac{\dfrac{- : \Delta_{12}, F_6, F_7, F_9 \to F_{10} \vdash \Delta_{11}, F_9}{- : \Delta_{12}, F_6, F_7, F_9 \to F_{10} \vdash \Delta_{11}} \text{inv-th/ax} \quad \dfrac{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}}{}}{- : \Delta_{12}, F_6, F_7, F_9 \to F_{10} \vdash \Delta_{11}} \text{sCut}}{- : \Delta_{12}, F_6, F_9 \to F_{10} \vdash \Delta_{11}} \text{sCut}}{- : \Delta_{12}, F_9 \to F_{10} \vdash \Delta_{11}} \text{sCut}
$$

- Case rule $\wedge_L$

$$
\dfrac{
\dfrac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_7, F_9 \quad h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{\dfrac{h_{11} : \Delta_{14}, F_7, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L}{} \text{Cut}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}
$$
$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{\dfrac{h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \dfrac{h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_{10}, F_7}{} \text{inv-th/ax}}{\dfrac{\bullet h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F_{10}}{} \wedge_R \quad \dfrac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{} \text{ax/W}}{\dfrac{- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L} \text{hCut}
$$

$$
\dfrac{
\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \wedge F_{13}, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \wedge F_{13}, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \wedge F_{13}} \wedge_R \quad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \wedge F_9} \wedge_L}{} \text{Cut}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}
$$
$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_{12} \wedge F_{13}}{} \text{ax/W} \quad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8} \wedge_L}{} \text{inv-th/ax}}{\dfrac{- : \Delta_{11} \vdash \Delta_7, F_8}{} \text{hCut}} \quad \dfrac{\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_9, F_{12} \wedge F_{13}}{} \text{ax/W} \quad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_9} \wedge_L}{} \text{inv-th}}{\dfrac{- : \Delta_{11} \vdash \Delta_7, F_9}{} \text{hCut}}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \wedge_R
$$

$$
\dfrac{
\dfrac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \quad h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \dfrac{\dfrac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L}{} \text{Cut}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}}
$$
$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{\dfrac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6}{} \text{inv-th/ax} \quad \dfrac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7}{} \text{inv-th/ax}}{\dfrac{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \wedge F_7}{} \wedge_R \quad \dfrac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}}{} \text{ax/W}}{\dfrac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L} \text{hCut}
$$

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$$\dfrac{\dfrac{h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \quad h_1 : \Delta_7 \vdash \Delta_{10}, F_9}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R \quad \dfrac{h_6 : \Delta_7, F_8, F_9 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{10}} \wedge_L}{- : \Delta_7 \vdash \Delta_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_7 \vdash \Delta_{10}, F_8} \text{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9} \text{ax/W} \quad \dfrac{}{- : \Delta_7, F_8, F_9 \vdash \Delta_{10}} \text{ax/W}}{- : \Delta_7, F_8 \vdash \Delta_{10}} \text{sCut}}{- : \Delta_7 \vdash \Delta_{10}} \text{sCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9 \quad h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{h_{11} : \Delta_{14}, F_7, F_{12} \vdash \Delta_8, F_9 \wedge F}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9} \text{ax/W} \quad \dfrac{\dfrac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_9}{} \text{inv-th/ax} \quad \dfrac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9}{} \text{inv-th/ax}}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \vee_L \quad \dfrac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F}{}}{\dfrac{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9}{} \text{hCut}} {- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}$$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \vee F_{13}, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \vee F_{13}, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \vee F_{13}} \wedge_R \quad \dfrac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \wedge F_9} \vee_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8} \text{inv-th/ax} \quad \dfrac{}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_9} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8 \wedge F_9} \wedge_R \quad \dfrac{\dfrac{}{- : \Delta_{11}, F_{13} \vdash \Delta_7, F_{12}, F_8 \wedge F_9} \text{ax/W} \quad \dfrac{}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} \text{ax/W}}{} \text{sCut}}{\dfrac{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \wedge F_9}{} \text{sCut}} {- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}$$

$$\dfrac{\dfrac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \quad h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \dfrac{h_8 : \Delta_{12}, F_9, F_6 \wedge F_7 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6} \text{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7} \text{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, F_6, F_7, F_9 \vdash \Delta_{11}} \text{inv-th/ax} \quad \dfrac{}{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}} \text{inv-th/ax}}{- : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}} \text{sCut}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{sCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, \bot, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, \bot, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \bot} \wedge_R \quad \dfrac{}{\bullet h_{10} : \Delta_{11}, \bot \vdash \Delta_7, F_8 \wedge F_9} \bot_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_{11} \vdash \bot, \Delta_7, F_8} \text{ax/W} \quad \dfrac{}{\bullet h_{10} : \bot, \Delta_{11} \vdash \Delta_7, F_8} \bot_L}{- : \Delta_{11} \vdash \Delta_7, F_8} \text{hCut} \quad \dfrac{\dfrac{}{h_2 : \Delta_{11} \vdash \bot, \Delta_7, F_9} \text{ax/W} \quad \dfrac{}{\bullet h_{10} : \bot, \Delta_{11} \vdash \Delta_7, F_9} \bot_L}{- : \Delta_{11} \vdash \Delta_7, F_9} \text{hCut}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \wedge_R$$

$$\dfrac{\dfrac{h_2 : \bot, \Delta_{12} \vdash \Delta_8, F_7, F_9 \quad h_2 : \bot, \Delta_{12} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \bot, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{}{\bullet h_{11} : (\bot, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \bot_L}{- : \bot, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \bot_L$$

$$\dfrac{\dfrac{h_1 : \bot, \Delta_{10} \vdash \Delta_9, F_6 \quad h_1 : \bot, \Delta_{10} \vdash \Delta_9, F_7}{\bullet h_1 : \bot, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \dfrac{}{\bullet h_8 : (\bot, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9} \bot_L$$

- Case rule I

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$$\dfrac{\dfrac{h_2 : \Delta_{10} \vdash (\Delta_{12}, p_{11}), p_{11}, F_7 \quad h_2 : \Delta_{10} \vdash (\Delta_{12}, p_{11}), p_{11}, F_8}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \wedge F_8), p_{11}} \wedge_R \quad \dfrac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8} I}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}, p_{11}} \text{ax/W} \quad \dfrac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_7, p_{11}} I}{- : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}} \text{hCut} \quad \dfrac{\dfrac{}{h_2 : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}, p_{11}} \text{ax/W} \quad \dfrac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_8, p_{11}} I}{- : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}} \text{hCut}}{- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \wedge F_8} \wedge_R$$

$$\dfrac{\dfrac{h_2 : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7, F_8 \quad h_2 : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7, F_9}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \wedge F_9), F_7} \wedge_R \quad \dfrac{}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9} I}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \wedge F_9} I$$

$$\dfrac{\dfrac{h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \quad h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_7}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \wedge F_7} \wedge_R \quad \dfrac{}{\bullet h_8 : (\Delta_{11}, p_9), F_6 \wedge F_7 \vdash \Delta_{10}, p_9} I}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, \top, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, \top, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \top} \wedge_R \quad \dfrac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \wedge F_9} \top_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{ax/W}$$

$$\dfrac{\dfrac{h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \quad h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \dfrac{h_{11} : \Delta_{12}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \top_L}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \text{ax/W} \quad \dfrac{}{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{hCut}$$

$$\dfrac{\dfrac{h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \quad h_1 : \top, \Delta_{10} \vdash \Delta_9, F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \dfrac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \top_L}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \text{ax/W} \quad \dfrac{}{h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}$$

8.3 Status of $\vee_R$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}), F_8} \vee_R \quad \dfrac{h_{11} : \Delta_7, F_8, F_{12} \vdash F_{13}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}} \rightarrow_R}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_7 \vdash \Delta_{14}, F_{10}, F_8, F_9, F_{12} \rightarrow F_{13}} \text{ax/W} \quad \dfrac{\dfrac{}{h_{11} : \Delta_7, F_{12}, F_8 \vdash F_{13}} \text{ax/W}}{\bullet h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \rightarrow F_{13}} \rightarrow_R}{- : \Delta_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \rightarrow F_{13}} \text{hCut}}{- : \Delta_7 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \vee F_{10}} \vee_R$$

$$\cfrac{\cfrac{h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7 \vee F_8} \vee_R \quad \cfrac{h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash F_{12}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{}{- : \Delta_6 \vdash \Delta_{10}, F_7, F_8, F_{11} \to F_{12}} \text{ax/W} \quad \cfrac{\cfrac{\cfrac{}{- : \Delta_6, F_{11}, F_8 \vdash F_{12}} \text{inv-th/ax}}{- : \Delta_6, F_8 \vdash \Delta_{10}, F_7, F_{11} \to F_{12}} \to_R}{- : \Delta_6 \vdash \Delta_{10}, F_7, F_{11} \to F_{12}} \text{sCut} \quad \cfrac{\cfrac{\cfrac{}{- : \Delta_6, F_{11}, F_7 \vdash F_{12}} \text{inv-th/ax}}{- : \Delta_6, F_7 \vdash \Delta_{10}, F_{11} \to F_{12}} \to_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}} \text{sCut}}{}$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}), F_8} \vee_R \quad \cfrac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_9 \vee F_{10} \quad h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}} \wedge_R}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_2 : \Delta_7 \vdash \Delta_{14}, F_{10}, F_8, F_9, F_{12} \wedge F_{13}} \text{ax/W} \quad \cfrac{\cfrac{}{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_9} \text{inv-th/ax} \quad \cfrac{}{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{10}, F_{13}, F_9} \text{inv-th/ax}}{\bullet h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}} \wedge_R}{\cfrac{- : \Delta_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}}{- : \Delta_7 \vdash \Delta_{14}, F_{12} \wedge F_{13}, F_9 \vee F_{10}} \vee_R}{}} \text{hCut}$$

$$\cfrac{\cfrac{h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \vee F_8} \vee_R \quad \cfrac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \quad h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7, F_8} \text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \vee F_8} \vee_R \quad \cfrac{}{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}} \text{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, F_{11}} \text{hCut} \quad \cfrac{\cfrac{\cfrac{}{h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7, F_8} \text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7 \vee F_8} \vee_R \quad \cfrac{}{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}} \text{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, F_{12}} \text{hCut}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R$$

- Case rule $\vee_R$

$$\cfrac{\cfrac{h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}), F_8} \vee_R \quad \cfrac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}} \vee_R}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{\cfrac{}{h_2 : \Delta_7 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_8, F_9} \text{inv-th/ax}}{\bullet h_2 : \Delta_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_8, F_9 \vee F_{10}} \vee_R \quad \cfrac{}{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}} \text{hCut}}{- : \Delta_7 \vdash \Delta_{14}, F_{12} \vee F_{13}, F_9 \vee F_{10}} \vee_R$$

$$\cfrac{\cfrac{h_2 : \Delta_7 \vdash \Delta_{10}, F_8, F_{11}, F_{12}}{\bullet h_2 : \Delta_7 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_8} \vee_R \quad \cfrac{h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_2 : \Delta_7 \vdash \Delta_{10}, F_{11}, F_{12}, F_8} \text{ax/W} \quad \cfrac{\cfrac{}{h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{\bullet h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11}, F_{12}} H}{- : \Delta_7 \vdash \Delta_{10}, F_{11}, F_{12}} \text{hCut}}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R$$

$$\cfrac{\cfrac{h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \vee F_8} \vee_R \quad \cfrac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{\cfrac{}{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7, F_8} \text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \vee F_8} \vee_R \quad \cfrac{}{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}} \text{hCut}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R$$

- Case rule $\bot_R$

$$\cfrac{\cfrac{h_2 : \Delta_7 \vdash (\bot, \Delta_{12}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\bot, \Delta_{12}), F_9 \vee F_{10}), F_8} \vee_R \quad \cfrac{h_{11} : \Delta_7, F_8 \vdash \Delta_{12}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\bot, \Delta_{12}), F_9 \vee F_{10}} \bot_R}{- : \Delta_7 \vdash (\bot, \Delta_{12}), F_9 \vee F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{}{\bullet h_2 : \Delta_7 \vdash \bot, \Delta_{12}, F_8, F_9 \vee F_{10}} \text{ax/W} \quad \cfrac{}{h_{11} : \Delta_7, F_8 \vdash \bot, \Delta_{12}, F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_{12}, F_9 \vee F_{10}} \text{hCut}$$

$$\cfrac{\cfrac{h_1 : \Delta_6 \vdash (\bot, \Delta_{10}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\bot, \Delta_{10}), F_7 \vee F_8} \vee_R \quad \cfrac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \bot, \Delta_{10}} \bot_R}{- : \Delta_6 \vdash \bot, \Delta_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_6 \vdash \bot, \Delta_{10}, F_7 \vee F_8} \text{ax/W} \quad \cfrac{}{h_9 : \Delta_6, F_7 \vee F_8 \vdash \bot, \Delta_{10}} \text{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}} \text{hCut}$$

- Case rule $\top_R$

$$\cfrac{\cfrac{h_2 : \Delta_7 \vdash (\top, \Delta_{12}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\top, \Delta_{12}), F_9 \vee F_{10}), F_8} \vee_R \quad \cfrac{}{\bullet h_{11} : \Delta_7, F_8 \vdash (\top, \Delta_{12}), F_9 \vee F_{10}} \top_R}{- : \Delta_7 \vdash (\top, \Delta_{12}), F_9 \vee F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- : \Delta_7 \vdash \top, \Delta_{12}, F_9 \vee F_{10}} \top_R$$

$$\cfrac{\cfrac{h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \vee F_8} \vee_R \quad \cfrac{}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \top, \Delta_{10}} \top_R}{- : \Delta_6 \vdash \top, \Delta_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}} \top_R$$

- Case rule $\rightarrow_L$

$$\cfrac{\cfrac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_9, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \cfrac{h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{12}, F_9 \vee F_{10} \quad h_{11} : \Delta_{14}, F_7, F_{13} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \rightarrow_L}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{}{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \text{ax/W} \quad \cfrac{\cfrac{h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_{12}, F_9}{} \text{inv-th/ax} \quad \cfrac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{} \text{inv-th/ax}}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_9} \rightarrow_L}{\cfrac{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \vee_R} \text{hCut}$$

$$\cfrac{\cfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \rightarrow F_{13}, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \rightarrow F_{13}} \vee_R \quad \cfrac{h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_{12}, F_8 \vee F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \vee F_9} \rightarrow_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{}{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \rightarrow F_{13}} \text{ax/W} \quad \cfrac{\cfrac{h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_{12}, F_8, F_9}{} \text{inv-th/ax} \quad \cfrac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9}{} \text{inv-th/ax}}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8, F_9} \rightarrow_L}{\cfrac{- : \Delta_{11} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R} \text{hCut}$$

$$\cfrac{\cfrac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \cfrac{h_8 : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \rightarrow_L}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{ax/W} \quad \cfrac{h_8 : \Delta_{12}, F_9 \rightarrow F_{10}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{} \text{ax/W} \quad \cfrac{\cfrac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad h_8 : \Delta_{12}, F_{10}, F_6 \vee \cdots}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} }{\cfrac{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_9}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}}} \text{hCut} \quad \rightarrow_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_7, F_9, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \ \vee_R \qquad \dfrac{h_{11} : \Delta_{14}, F_7, F_{12}, F_{13} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \ \wedge_L}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \ \text{ax/W} \qquad \dfrac{\dfrac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{\ } \ \text{inv-th/ax}}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9} \ \wedge_L}{\dfrac{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \ \vee_R} \ \text{hCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \wedge F_{13}, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \wedge F_{13}} \ \vee_R \qquad \dfrac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \vee F_9} \ \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \wedge F_{13}} \ \text{ax/W} \qquad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8, F_9}{\ } \ \text{inv-th/ax}}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8, F_9} \ \wedge_L}{\dfrac{- : \Delta_{11} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \ \vee_R} \ \text{hCut}$$

$$\dfrac{\dfrac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \ \vee_R \qquad \dfrac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \ \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6, F_7}{\ } \ \text{inv-th/ax}}{\dfrac{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \vee F_7}{\ } \ \vee_R} \qquad \dfrac{}{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}} \ \text{ax/W}}{\dfrac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \ \wedge_L} \ \text{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \ \vee_R \qquad \dfrac{h_{11} : \Delta_{14}, F_7, F_{12} \vdash \Delta_8, F_9 \vee F_{10} \quad h_{11} : \Delta_{14}, F_7, F_{13} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \ \vee_L}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \ \text{ax/W} \qquad \dfrac{\dfrac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_{10}, F_9}{\ } \ \text{inv-th/ax} \quad \dfrac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{\ } \ \text{inv-th/ax}}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9} \ \vee_L}{\dfrac{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \ \vee_R} \ \text{hCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \vee F_{13}, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \vee F_{13}} \ \vee_R \qquad \dfrac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \vee F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \vee F_9} \ \vee_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \vee F_{13}} \ \text{ax/W} \qquad \dfrac{\dfrac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8, F_9}{\ } \ \text{inv-th/ax} \quad \dfrac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9}{\ } \ \text{inv-th/ax}}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8, F_9} \ \vee_L}{\dfrac{- : \Delta_{11} \vdash \Delta_7, F_8, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \ \vee_R} \ \text{hCut}$$

$$\dfrac{\dfrac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \ \vee_R \qquad \dfrac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \ \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6, F_7}{\ } \ \text{inv-th/ax}}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \ \vee_R \quad \dfrac{}{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}} \ \text{ax/W}}{- : \Delta_{12}, F_9 \vdash \Delta_{11}} \ \text{hCut} \qquad \dfrac{\dfrac{\dfrac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\ } \ \text{inv-th/ax}}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \ \vee_R \quad \dfrac{}{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}} \ \text{ax/W}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \ \text{hCut}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \ \vee_L$$

$$\dfrac{\dfrac{h_1 : \Delta_7 \vdash \Delta_{10}, F_8, F_9}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \vee F_9} \ \vee_R \qquad \dfrac{h_6 : \Delta_7, F_8 \vdash \Delta_{10} \quad h_6 : \Delta_7, F_9 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{10}} \ \vee_L}{- : \Delta_7 \vdash \Delta_{10}} \ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_7 \vdash \Delta_{10}, F_8, F_9} \ \text{ax/W} \quad \dfrac{}{- : \Delta_7, F_9 \vdash \Delta_{10}, F_8} \ \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}, F_8} \ \text{sCut} \qquad \dfrac{}{- : \Delta_7, F_8 \vdash \Delta_{10}} \ \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}} \ \text{sCut}$$

- Case rule $\bot_L$

$$\cfrac{\cfrac{\mathtt{h_2} : \Delta_{11} \vdash \Delta_7, \bot, \mathtt{F_8}, \mathtt{F_9}}{\bullet\mathtt{h_2} : \Delta_{11} \vdash (\Delta_7, \mathtt{F_8} \vee \mathtt{F_9}), \bot} \vee_R \quad \cfrac{}{\bullet\mathtt{h_{10}} : \Delta_{11}, \bot \vdash \Delta_7, \mathtt{F_8} \vee \mathtt{F_9}} \bot_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F_8} \vee \mathtt{F_9}} \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\color{blue}\mathtt{h_2} : \Delta_{11} \vdash \bot, \Delta_7, \mathtt{F_8}, \mathtt{F_9}} \mathtt{ax/W} \quad \cfrac{}{\color{blue}\bullet\mathtt{h_{10}} : \bot, \Delta_{11} \vdash \Delta_7, \mathtt{F_8}, \mathtt{F_9}} \bot_L}{\color{blue}- : \Delta_{11} \vdash \Delta_7, \mathtt{F_8}, \mathtt{F_9}} \mathtt{hCut}}{\color{blue}- : \Delta_{11} \vdash \Delta_7, \mathtt{F_8} \vee \mathtt{F_9}} \vee_R$$

$$\cfrac{\cfrac{\mathtt{h_2} : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F_7}, \mathtt{F_9}, \mathtt{F_{10}}}{\bullet\mathtt{h_2} : \bot, \Delta_{12} \vdash (\Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}), \mathtt{F_7}} \vee_R \quad \cfrac{}{\bullet\mathtt{h_{11}} : (\bot, \Delta_{12}), \mathtt{F_7} \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}} \bot_L}{- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}} \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{\color{blue}- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}} \bot_L$$

$$\cfrac{\cfrac{\mathtt{h_1} : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F_6}, \mathtt{F_7}}{\bullet\mathtt{h_1} : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F_6} \vee \mathtt{F_7}} \vee_R \quad \cfrac{}{\bullet\mathtt{h_8} : (\bot, \Delta_{10}), \mathtt{F_6} \vee \mathtt{F_7} \vdash \Delta_9} \bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9} \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{\color{blue}- : \bot, \Delta_{10} \vdash \Delta_9} \bot_L$$

- Case rule I

$$\cfrac{\cfrac{\mathtt{h_2} : \Delta_{10} \vdash (\Delta_{12}, \mathtt{p_{11}}), \mathtt{p_{11}}, \mathtt{F_7}, \mathtt{F_8}}{\bullet\mathtt{h_2} : \Delta_{10} \vdash ((\Delta_{12}, \mathtt{p_{11}}), \mathtt{F_7} \vee \mathtt{F_8}), \mathtt{p_{11}}} \vee_R \quad \cfrac{}{\bullet\mathtt{h_9} : \Delta_{10}, \mathtt{p_{11}} \vdash (\Delta_{12}, \mathtt{p_{11}}), \mathtt{F_7} \vee \mathtt{F_8}} I}{- : \Delta_{10} \vdash (\Delta_{12}, \mathtt{p_{11}}), \mathtt{F_7} \vee \mathtt{F_8}} \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{\color{blue}\mathtt{h_2} : \Delta_{10} \vdash \Delta_{12}, \mathtt{F_7}, \mathtt{F_8}, \mathtt{p_{11}}, \mathtt{p_{11}}} \mathtt{ax/W} \quad \cfrac{}{\color{blue}\bullet\mathtt{h_9} : \Delta_{10}, \mathtt{p_{11}} \vdash \Delta_{12}, \mathtt{F_7}, \mathtt{F_8}, \mathtt{p_{11}}} I}{\color{blue}- : \Delta_{10} \vdash \Delta_{12}, \mathtt{F_7}, \mathtt{F_8}, \mathtt{p_{11}}} \mathtt{hCut}}{\color{blue}- : \Delta_{10} \vdash \Delta_{12}, \mathtt{p_{11}}, \mathtt{F_7} \vee \mathtt{F_8}} \vee_R$$

$$\cfrac{\cfrac{\mathtt{h_2} : \Delta_{13}, \mathtt{p_{11}} \vdash (\Delta_{12}, \mathtt{p_{11}}), \mathtt{F_7}, \mathtt{F_8}, \mathtt{F_9}}{\bullet\mathtt{h_2} : \Delta_{13}, \mathtt{p_{11}} \vdash ((\Delta_{12}, \mathtt{p_{11}}), \mathtt{F_8} \vee \mathtt{F_9}), \mathtt{F_7}} \vee_R \quad \cfrac{}{\bullet\mathtt{h_{10}} : (\Delta_{13}, \mathtt{p_{11}}), \mathtt{F_7} \vdash (\Delta_{12}, \mathtt{p_{11}}), \mathtt{F_8} \vee \mathtt{F_9}} I}{- : \Delta_{13}, \mathtt{p_{11}} \vdash (\Delta_{12}, \mathtt{p_{11}}), \mathtt{F_8} \vee \mathtt{F_9}} \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{\color{blue}- : \Delta_{13}, \mathtt{p_{11}} \vdash \Delta_{12}, \mathtt{p_{11}}, \mathtt{F_8} \vee \mathtt{F_9}} I$$

$$\cfrac{\cfrac{\mathtt{h_1} : \Delta_{11}, \mathtt{p_9} \vdash (\Delta_{10}, \mathtt{p_9}), \mathtt{F_6}, \mathtt{F_7}}{\bullet\mathtt{h_1} : \Delta_{11}, \mathtt{p_9} \vdash (\Delta_{10}, \mathtt{p_9}), \mathtt{F_6} \vee \mathtt{F_7}} \vee_R \quad \cfrac{}{\bullet\mathtt{h_8} : (\Delta_{11}, \mathtt{p_9}), \mathtt{F_6} \vee \mathtt{F_7} \vdash \Delta_{10}, \mathtt{p_9}} I}{- : \Delta_{11}, \mathtt{p_9} \vdash \Delta_{10}, \mathtt{p_9}} \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{\color{blue}- : \Delta_{11}, \mathtt{p_9} \vdash \Delta_{10}, \mathtt{p_9}} I$$

- Case rule $\top_L$

$$\cfrac{\cfrac{\mathtt{h_2} : \Delta_{11} \vdash \Delta_7, \top, \mathtt{F_8}, \mathtt{F_9}}{\bullet\mathtt{h_2} : \Delta_{11} \vdash (\Delta_7, \mathtt{F_8} \vee \mathtt{F_9}), \top} \vee_R \quad \cfrac{\mathtt{h_{10}} : \Delta_{11} \vdash \Delta_7, \mathtt{F_8} \vee \mathtt{F_9}}{\bullet\mathtt{h_{10}} : \Delta_{11}, \top \vdash \Delta_7, \mathtt{F_8} \vee \mathtt{F_9}} \top_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F_8} \vee \mathtt{F_9}} \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{\color{blue}- : \Delta_{11} \vdash \Delta_7, \mathtt{F_8} \vee \mathtt{F_9}} \mathtt{ax/W}$$

$$\cfrac{\cfrac{\mathtt{h_2} : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F_7}, \mathtt{F_9}, \mathtt{F_{10}}}{\bullet\mathtt{h_2} : \top, \Delta_{12} \vdash (\Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}), \mathtt{F_7}} \vee_R \quad \cfrac{\mathtt{h_{11}} : \Delta_{12}, \mathtt{F_7} \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}}{\bullet\mathtt{h_{11}} : (\top, \Delta_{12}), \mathtt{F_7} \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}} \top_L}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}} \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{}{\color{blue}\bullet\mathtt{h_2} : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F_7}, \mathtt{F_9} \vee \mathtt{F_{10}}} \mathtt{ax/W} \quad \cfrac{}{\color{blue}\mathtt{h_{11}} : \top, \Delta_{12}, \mathtt{F_7} \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}} \mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F_9} \vee \mathtt{F_{10}}} \mathtt{hCut}$$

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$$\cfrac{\cfrac{\cfrac{h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6, F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \cfrac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \top_L}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut}}{\rightsquigarrow}$$

$$\cfrac{\cfrac{}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \text{ax/W} \quad \cfrac{}{h_8 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}$$

8.4 Status of $\perp_R$: OK

- Case rule $\rightarrow_R$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \rightarrow F_8), \perp} \perp_R \quad \cfrac{h_5 : \perp, \Delta_4, F_7 \vdash F_8}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \rightarrow F_8} \rightarrow_R}{- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8} \text{Cut}}{\rightsquigarrow}$$

$$\cfrac{}{- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8} \text{ax/W}$$

$$\cfrac{\cfrac{\cfrac{h_2 : \Delta_5 \vdash (\Delta_{10}, F_8 \rightarrow F_9), F_6}{\bullet h_2 : \Delta_5 \vdash (\perp, \Delta_{10}, F_8 \rightarrow F_9), F_6} \perp_R \quad \cfrac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{Cut}}{\rightsquigarrow}$$

$$\cfrac{\cfrac{}{h_2 : \Delta_5 \vdash \perp, \Delta_{10}, F_6, F_8 \rightarrow F_9} \text{ax/W} \quad \cfrac{}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{ax/W}}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{hCut}$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \wedge F_8), \perp} \perp_R \quad \cfrac{h_5 : \perp, \Delta_4 \vdash \Delta_6, F_7 \quad h_5 : \perp, \Delta_4 \vdash \Delta_6, F_8}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \wedge F_8} \wedge_R}{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \text{Cut}}{\rightsquigarrow}$$

$$\cfrac{}{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \text{ax/W}$$

$$\cfrac{\cfrac{\cfrac{h_2 : \Delta_5 \vdash (\Delta_{10}, F_8 \wedge F_9), F_6}{\bullet h_2 : \Delta_5 \vdash (\perp, \Delta_{10}, F_8 \wedge F_9), F_6} \perp_R \quad \cfrac{h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \quad h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \wedge_R}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{Cut}}{\rightsquigarrow}$$

$$\cfrac{\cfrac{}{h_2 : \Delta_5 \vdash \perp, \Delta_{10}, F_6, F_8 \wedge F_9} \text{ax/W} \quad \cfrac{}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{ax/W}}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{hCut}$$

- Case rule $\vee_R$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \vee F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \vee F_8), \perp} \perp_R \quad \cfrac{h_5 : \perp, \Delta_4 \vdash \Delta_6, F_7, F_8}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \vee F_8} \vee_R}{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \text{Cut}}{\rightsquigarrow}$$

$$\cfrac{}{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \text{ax/W}$$

$$\cfrac{\cfrac{\cfrac{h_2 : \Delta_5 \vdash (\Delta_{10}, F_8 \vee F_9), F_6}{\bullet h_2 : \Delta_5 \vdash (\perp, \Delta_{10}, F_8 \vee F_9), F_6} \perp_R \quad \cfrac{h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8, F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \vee_R}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{Cut}}{\rightsquigarrow}$$

$$\cfrac{\cfrac{}{h_2 : \Delta_5 \vdash \perp, \Delta_{10}, F_6, F_8 \vee F_9} \text{ax/W} \quad \cfrac{}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{ax/W}}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{hCut}$$

- Case rule $\perp_R$

$$\cfrac{\cfrac{\cfrac{h_1 : \Delta_4 \vdash \perp, \Delta_6}{\bullet h_1 : \Delta_4 \vdash (\perp, \Delta_6), \perp} \perp_R \quad \cfrac{h_5 : \perp, \Delta_4 \vdash \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \perp, \Delta_6} \perp_R}{- : \Delta_4 \vdash \perp, \Delta_6} \text{Cut}}{\rightsquigarrow}$$

$$\cfrac{}{- : \Delta_4 \vdash \perp, \Delta_6} \text{ax/W}$$

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$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_5\vdash\Delta_8,\mathtt{F_6}}{\bullet\mathtt{h_2}:\Delta_5\vdash(\bot,\Delta_8),\mathtt{F_6}}\ \bot_R \quad \dfrac{\mathtt{h_7}:\Delta_5,\mathtt{F_6}\vdash\Delta_8}{\bullet\mathtt{h_7}:\Delta_5,\mathtt{F_6}\vdash\bot,\Delta_8}\ \bot_R}{-\ :\Delta_5\vdash\bot,\Delta_8}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h_2}:\Delta_5\vdash\bot,\Delta_8,\mathtt{F_6}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_7}:\Delta_5,\mathtt{F_6}\vdash\bot,\Delta_8}\ \mathtt{ax/W}}{-\ :\Delta_5\vdash\bot,\Delta_8}\ \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_4\vdash\top,\Delta_6}{\bullet\mathtt{h_1}:\Delta_4\vdash(\top,\Delta_6),\bot}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h_5}:\Delta_4,\bot\vdash\top,\Delta_6}\ \top_R}{-\ :\Delta_4\vdash\top,\Delta_6}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{-\ :\Delta_4\vdash\top,\Delta_6}\ \top_R$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_5\vdash(\top,\Delta_8),\mathtt{F_6}}{\bullet\mathtt{h_2}:\Delta_5\vdash(\bot,\top,\Delta_8),\mathtt{F_6}}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h_7}:\Delta_5,\mathtt{F_6}\vdash\bot,\top,\Delta_8}\ \top_R}{-\ :\Delta_5\vdash\bot,\top,\Delta_8}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{-\ :\Delta_5\vdash\bot,\top,\Delta_8}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_8,\mathtt{F_5}\rightarrow\mathtt{F_6}\vdash\Delta_7}{\bullet\mathtt{h_1}:\Delta_8,\mathtt{F_5}\rightarrow\mathtt{F_6}\vdash\Delta_7,\bot}\ \bot_R \quad \dfrac{\mathtt{h_4}:\bot,\Delta_8,\mathtt{F_5}\rightarrow\mathtt{F_6}\vdash\Delta_7,\mathtt{F_5}\quad \mathtt{h_4}:\bot,\Delta_8,\mathtt{F_6}\vdash\Delta_7}{\bullet\mathtt{h_4}:(\Delta_8,\mathtt{F_5}\rightarrow\mathtt{F_6}),\bot\vdash\Delta_7}\ \rightarrow_L}{-\ :\Delta_8,\mathtt{F_5}\rightarrow\mathtt{F_6}\vdash\Delta_7}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{-\ :\Delta_8,\mathtt{F_5}\rightarrow\mathtt{F_6}\vdash\Delta_7}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_{10},\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash\Delta_6,\mathtt{F_5}}{\bullet\mathtt{h_2}:\Delta_{10},\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash(\bot,\Delta_6),\mathtt{F_5}}\ \bot_R \quad \dfrac{\mathtt{h_7}:\Delta_{10},\mathtt{F_5},\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash\bot,\Delta_6,\mathtt{F_8}\quad \mathtt{h_7}:\Delta_{10},\mathtt{F_5},\mathtt{F_9}\vdash\bot,\Delta_6}{\bullet\mathtt{h_7}:(\Delta_{10},\mathtt{F_8}\rightarrow\mathtt{F_9}),\mathtt{F_5}\vdash\bot,\Delta_6}\ \rightarrow_L}{-\ :\Delta_{10},\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash\bot,\Delta_6}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h_2}:\Delta_{10},\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash\bot,\Delta_6,\mathtt{F_5}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_7}:\Delta_{10},\mathtt{F_5},\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash\bot,\Delta_6}\ \mathtt{ax/W}}{-\ :\Delta_{10},\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash\bot,\Delta_6}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_7\vdash\Delta_5,\mathtt{F_8}\rightarrow\mathtt{F_9}}{\bullet\mathtt{h_2}:\Delta_7\vdash(\bot,\Delta_5),\mathtt{F_8}\rightarrow\mathtt{F_9}}\ \bot_R \quad \dfrac{\mathtt{h_6}:\Delta_7,\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash\bot,\Delta_5,\mathtt{F_8}\quad \mathtt{h_6}:\Delta_7,\mathtt{F_9}\vdash\bot,\Delta_5}{\bullet\mathtt{h_6}:\Delta_7,\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash\bot,\Delta_5}\ \rightarrow_L}{-\ :\Delta_7\vdash\bot,\Delta_5}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h_2}:\Delta_7\vdash\bot,\Delta_5,\mathtt{F_8}\rightarrow\mathtt{F_9}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_6}:\Delta_7,\mathtt{F_8}\rightarrow\mathtt{F_9}\vdash\bot,\Delta_5}\ \mathtt{ax/W}}{-\ :\Delta_7\vdash\bot,\Delta_5}\ \mathtt{hCut}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h_1}:\Delta_8,\mathtt{F_5}\wedge\mathtt{F_6}\vdash\Delta_7}{\bullet\mathtt{h_1}:\Delta_8,\mathtt{F_5}\wedge\mathtt{F_6}\vdash\Delta_7,\bot}\ \bot_R \quad \dfrac{\mathtt{h_4}:\bot,\Delta_8,\mathtt{F_5},\mathtt{F_6}\vdash\Delta_7}{\bullet\mathtt{h_4}:(\Delta_8,\mathtt{F_5}\wedge\mathtt{F_6}),\bot\vdash\Delta_7}\ \wedge_L}{-\ :\Delta_8,\mathtt{F_5}\wedge\mathtt{F_6}\vdash\Delta_7}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{-\ :\Delta_8,\mathtt{F_5}\wedge\mathtt{F_6}\vdash\Delta_7}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}\vdash\Delta_6,\mathtt{F_5}}{\bullet\mathtt{h_2}:\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}\vdash(\bot,\Delta_6),\mathtt{F_5}}\ \bot_R \quad \dfrac{\mathtt{h_7}:\Delta_{10},\mathtt{F_5},\mathtt{F_8},\mathtt{F_9}\vdash\bot,\Delta_6}{\bullet\mathtt{h_7}:(\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}),\mathtt{F_5}\vdash\bot,\Delta_6}\ \wedge_L}{-\ :\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}\vdash\bot,\Delta_6}\ \mathtt{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\mathtt{h_2}:\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}\vdash\bot,\Delta_6,\mathtt{F_5}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h_7}:\Delta_{10},\mathtt{F_5},\mathtt{F_8}\wedge\mathtt{F_9}\vdash\bot,\Delta_6}\ \mathtt{ax/W}}{-\ :\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}\vdash\bot,\Delta_6}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_7 \vdash \Delta_5, F_8 \wedge F_9}{\bullet h_2 : \Delta_7 \vdash (\bot, \Delta_5), F_8 \wedge F_9} \; \bot_R \quad \dfrac{h_6 : \Delta_7, F_8, F_9 \vdash \bot, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \bot, \Delta_5} \; \wedge_L}{- : \Delta_7 \vdash \bot, \Delta_5} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_7 \vdash \bot, \Delta_5, F_8 \wedge F_9} \; \texttt{ax/W} \quad \dfrac{}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \bot, \Delta_5} \; \texttt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5} \; \texttt{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \bot} \; \bot_R \quad \dfrac{h_4 : \bot, \Delta_8, F_5 \vdash \Delta_7 \quad h_4 : \bot, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \bot \vdash \Delta_7} \; \vee_L}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \; \texttt{ax/W}$$

$$\dfrac{\dfrac{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash \Delta_6, F_5}{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\bot, \Delta_6), F_5} \; \bot_R \quad \dfrac{h_7 : \Delta_{10}, F_5, F_8 \vdash \bot, \Delta_6 \quad h_7 : \Delta_{10}, F_5, F_9 \vdash \bot, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \bot, \Delta_6} \; \vee_L}{- : \Delta_{10}, F_8 \vee F_9 \vdash \bot, \Delta_6} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash \bot, \Delta_6, F_5} \; \texttt{ax/W} \quad \dfrac{}{\bullet h_7 : \Delta_{10}, F_5, F_8 \vee F_9 \vdash \bot, \Delta_6} \; \texttt{ax/W}}{- : \Delta_{10}, F_8 \vee F_9 \vdash \bot, \Delta_6} \; \texttt{hCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_7 \vdash \Delta_5, F_8 \vee F_9}{\bullet h_2 : \Delta_7 \vdash (\bot, \Delta_5), F_8 \vee F_9} \; \bot_R \quad \dfrac{h_6 : \Delta_7, F_8 \vdash \bot, \Delta_5 \quad h_6 : \Delta_7, F_9 \vdash \bot, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \bot, \Delta_5} \; \vee_L}{- : \Delta_7 \vdash \bot, \Delta_5} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_7 \vdash \bot, \Delta_5, F_8 \vee F_9} \; \texttt{ax/W} \quad \dfrac{}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \bot, \Delta_5} \; \texttt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5} \; \texttt{hCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{h_1 : \Delta_5 \vdash \Delta_6}{\bullet h_1 : \Delta_5 \vdash \Delta_6, \bot} \; \bot_R \quad \dfrac{}{\bullet h_4 : \Delta_5, \bot \vdash \Delta_6} \; \bot_L}{- : \Delta_5 \vdash \Delta_6} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_5 \vdash \Delta_6} \; \texttt{ax/W}$$

$$\dfrac{\dfrac{h_2 : \Delta_7 \vdash \Delta_5, \bot}{\bullet h_2 : \Delta_7 \vdash (\bot, \Delta_5), \bot} \; \bot_R \quad \dfrac{}{\bullet h_6 : \Delta_7, \bot \vdash \bot, \Delta_5} \; \bot_L}{- : \Delta_7 \vdash \bot, \Delta_5} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7 \vdash \bot, \Delta_5} \; \texttt{ax/W}$$

$$\dfrac{\dfrac{h_2 : \bot, \Delta_8 \vdash \Delta_6, F_5}{\bullet h_2 : \bot, \Delta_8 \vdash (\bot, \Delta_6), F_5} \; \bot_R \quad \dfrac{}{\bullet h_7 : (\bot, \Delta_8), F_5 \vdash \bot, \Delta_6} \; \bot_L}{- : \bot, \Delta_8 \vdash \bot, \Delta_6} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \bot, \Delta_8 \vdash \bot, \Delta_6} \; \bot_L$$

- Case rule I

$$\dfrac{\dfrac{h_1 : \Delta_7, p_5 \vdash \Delta_6, p_5}{\bullet h_1 : \Delta_7, p_5 \vdash (\Delta_6, p_5), \bot} \; \bot_R \quad \dfrac{}{\bullet h_4 : (\Delta_7, p_5), \bot \vdash \Delta_6, p_5} \; I}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} \; I$$

$$\dfrac{\dfrac{h_2 : \Delta_6 \vdash (\Delta_8, p_7), p_7}{\bullet h_2 : \Delta_6 \vdash (\bot, \Delta_8, p_7), p_7} \; \bot_R \quad \dfrac{}{\bullet h_5 : \Delta_6, p_7 \vdash \bot, \Delta_8, p_7} \; I}{- : \Delta_6 \vdash \bot, \Delta_8, p_7} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_2 : \Delta_6 \vdash \bot, \Delta_8, p_7, p_7} \; \texttt{ax/W} \quad \dfrac{}{\bullet h_5 : \Delta_6, p_7 \vdash \bot, \Delta_8, p_7} \; I}{- : \Delta_6 \vdash \bot, \Delta_8, p_7} \; \texttt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_9,\mathtt{p_7}\vdash(\Delta_8,\mathtt{p_7}),\mathtt{F_5}}{\bullet\mathtt{h_2}:\Delta_9,\mathtt{p_7}\vdash(\bot,\Delta_8,\mathtt{p_7}),\mathtt{F_5}}\ \bot_R \qquad \dfrac{}{\bullet\mathtt{h_6}:(\Delta_9,\mathtt{p_7}),\mathtt{F_5}\vdash\bot,\Delta_8,\mathtt{p_7}}\ I}{-:\Delta_9,\mathtt{p_7}\vdash\bot,\Delta_8,\mathtt{p_7}}\ \text{Cut}$$

$$\dfrac{}{\overset{\rightsquigarrow}{\dfrac{}{-:\Delta_9,\mathtt{p_7}\vdash\bot,\Delta_8,\mathtt{p_7}}}}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h_1}:\top,\Delta_6\vdash\Delta_5}{\bullet\mathtt{h_1}:\top,\Delta_6\vdash\Delta_5,\bot}\ \bot_R \qquad \dfrac{\mathtt{h_4}:\bot,\Delta_6\vdash\Delta_5}{\bullet\mathtt{h_4}:(\top,\Delta_6),\bot\vdash\Delta_5}\ \top_L}{-:\top,\Delta_6\vdash\Delta_5}\ \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{}{-:\top,\Delta_6\vdash\Delta_5}}\ \text{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\Delta_7\vdash\Delta_5,\top}{\bullet\mathtt{h_2}:\Delta_7\vdash(\bot,\Delta_5),\top}\ \bot_R \qquad \dfrac{\mathtt{h_6}:\Delta_7\vdash\bot,\Delta_5}{\bullet\mathtt{h_6}:\Delta_7,\top\vdash\bot,\Delta_5}\ \top_L}{-:\Delta_7\vdash\bot,\Delta_5}\ \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{}{-:\Delta_7\vdash\bot,\Delta_5}}\ \text{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h_2}:\top,\Delta_8\vdash\Delta_6,\mathtt{F_5}}{\bullet\mathtt{h_2}:\top,\Delta_8\vdash(\bot,\Delta_6),\mathtt{F_5}}\ \bot_R \qquad \dfrac{\mathtt{h_7}:\Delta_8,\mathtt{F_5}\vdash\bot,\Delta_6}{\bullet\mathtt{h_7}:(\top,\Delta_8),\mathtt{F_5}\vdash\bot,\Delta_6}\ \top_L}{-:\top,\Delta_8\vdash\bot,\Delta_6}\ \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{\dfrac{}{\mathtt{h_2}:\top,\Delta_8\vdash\bot,\Delta_6,\mathtt{F_5}}\ \text{ax/W} \qquad \dfrac{}{\bullet\mathtt{h_7}:\top,\Delta_8,\mathtt{F_5}\vdash\bot,\Delta_6}\ \text{ax/W}}{-:\top,\Delta_8\vdash\bot,\Delta_6}}\ \text{hCut}$$

8.5   Status of $\top_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_1}:\Delta_4\vdash(\Delta_6,\mathtt{F_7}\to\mathtt{F_8}),\top}\ \top_R \qquad \dfrac{\mathtt{h_5}:\top,\Delta_4,\mathtt{F_7}\vdash\mathtt{F_8}}{\bullet\mathtt{h_5}:\Delta_4,\top\vdash\Delta_6,\mathtt{F_7}\to\mathtt{F_8}}\ \to_R}{-:\Delta_4\vdash\Delta_6,\mathtt{F_7}\to\mathtt{F_8}}\ \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1}:\Delta_4,\mathtt{F_7}\vdash\top,\mathtt{F_8}}\ \top_R \qquad \dfrac{}{\mathtt{h_5}:\top,\Delta_4,\mathtt{F_7}\vdash\mathtt{F_8}}\ \text{ax/W}}{-:\Delta_4,\mathtt{F_7}\vdash\mathtt{F_8}}\ \text{hCut}}{-:\Delta_4\vdash\Delta_6,\mathtt{F_7}\to\mathtt{F_8}}}\ \to_R$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_2}:\Delta_5\vdash(\top,\Delta_{10},\mathtt{F_8}\to\mathtt{F_9}),\mathtt{F_6}}\ \top_R \qquad \dfrac{\mathtt{h_7}:\Delta_5,\mathtt{F_6},\mathtt{F_8}\vdash\mathtt{F_9}}{\bullet\mathtt{h_7}:\Delta_5,\mathtt{F_6}\vdash\top,\Delta_{10},\mathtt{F_8}\to\mathtt{F_9}}\ \to_R}{-:\Delta_5\vdash\top,\Delta_{10},\mathtt{F_8}\to\mathtt{F_9}}\ \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{}{-:\Delta_5\vdash\top,\Delta_{10},\mathtt{F_8}\to\mathtt{F_9}}}\ \top_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_1}:\Delta_4\vdash(\Delta_6,\mathtt{F_7}\wedge\mathtt{F_8}),\top}\ \top_R \qquad \dfrac{\mathtt{h_5}:\top,\Delta_4\vdash\Delta_6,\mathtt{F_7}\qquad \mathtt{h_5}:\top,\Delta_4\vdash\Delta_6,\mathtt{F_8}}{\bullet\mathtt{h_5}:\Delta_4,\top\vdash\Delta_6,\mathtt{F_7}\wedge\mathtt{F_8}}\ \wedge_R}{-:\Delta_4\vdash\Delta_6,\mathtt{F_7}\wedge\mathtt{F_8}}\ \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h_1}:\Delta_4\vdash\top,\Delta_6,\mathtt{F_7}}\ \top_R \quad \dfrac{}{\mathtt{h_5}:\top,\Delta_4\vdash\Delta_6,\mathtt{F_7}}\ \text{ax/W}}{-:\Delta_4\vdash\Delta_6,\mathtt{F_7}}\ \text{hCut} \qquad \dfrac{\dfrac{}{\bullet\mathtt{h_1}:\Delta_4\vdash\top,\Delta_6,\mathtt{F_8}}\ \top_R \quad \dfrac{}{\mathtt{h_5}:\top,\Delta_4\vdash\Delta_6,\mathtt{F_8}}\ \text{ax/W}}{-:\Delta_4\vdash\Delta_6,\mathtt{F_8}}\ \text{hCut}}{-:\Delta_4\vdash\Delta_6,\mathtt{F_7}\wedge\mathtt{F_8}}}\ \wedge_R$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h_2}:\Delta_5\vdash(\top,\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}),\mathtt{F_6}}\ \top_R \qquad \dfrac{\mathtt{h_7}:\Delta_5,\mathtt{F_6}\vdash\top,\Delta_{10},\mathtt{F_8}\qquad \mathtt{h_7}:\Delta_5,\mathtt{F_6}\vdash\top,\Delta_{10},\mathtt{F_9}}{\bullet\mathtt{h_7}:\Delta_5,\mathtt{F_6}\vdash\top,\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}}\ \wedge_R}{-:\Delta_5\vdash\top,\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}}\ \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{}{-:\Delta_5\vdash\top,\Delta_{10},\mathtt{F_8}\wedge\mathtt{F_9}}}\ \top_R$$

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- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet \mathbf{h}_1 : \Delta_4 \vdash (\Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8), \top} \ \top_R \quad \dfrac{\mathbf{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_8}{\bullet \mathbf{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8} \ \vee_R}{- : \Delta_4 \vdash \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathbf{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathbf{F}_7, \mathbf{F}_8} \ \top_R \quad \dfrac{}{\mathbf{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_8}}{- : \Delta_4 \vdash \Delta_6, \mathbf{F}_7, \mathbf{F}_8} \ \substack{\text{ax/W} \\ \text{hCut}}}{- : \Delta_4 \vdash \Delta_6, \mathbf{F}_7 \vee \mathbf{F}_8} \ \vee_R$$

$$\dfrac{\dfrac{}{\bullet \mathbf{h}_2 : \Delta_5 \vdash (\top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9), \mathbf{F}_6} \ \top_R \quad \dfrac{\mathbf{h}_7 : \Delta_5, \mathbf{F}_6 \vdash \top, \Delta_{10}, \mathbf{F}_8, \mathbf{F}_9}{\bullet \mathbf{h}_7 : \Delta_5, \mathbf{F}_6 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9} \ \vee_R}{- : \Delta_5 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_5 \vdash \top, \Delta_{10}, \mathbf{F}_8 \vee \mathbf{F}_9} \ \top_R$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{}{\bullet \mathbf{h}_1 : \Delta_4 \vdash (\perp, \Delta_6), \top} \ \top_R \quad \dfrac{\mathbf{h}_5 : \top, \Delta_4 \vdash \Delta_6}{\bullet \mathbf{h}_5 : \Delta_4, \top \vdash \perp, \Delta_6} \ \perp_R}{- : \Delta_4 \vdash \perp, \Delta_6} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{\bullet \mathbf{h}_1 : \Delta_4 \vdash \perp, \top, \Delta_6} \ \text{ax/W} \quad \dfrac{}{\mathbf{h}_5 : \top, \Delta_4 \vdash \perp, \Delta_6} \ \text{ax/W}}{- : \Delta_4 \vdash \perp, \Delta_6} \ \text{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathbf{h}_2 : \Delta_5 \vdash (\top, \perp, \Delta_8), \mathbf{F}_6} \ \top_R \quad \dfrac{\mathbf{h}_7 : \Delta_5, \mathbf{F}_6 \vdash \top, \Delta_8}{\bullet \mathbf{h}_7 : \Delta_5, \mathbf{F}_6 \vdash \top, \perp, \Delta_8} \ \perp_R}{- : \Delta_5 \vdash \top, \perp, \Delta_8} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_5 \vdash \perp, \top, \Delta_8} \ \top_R$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathbf{h}_1 : \Delta_4 \vdash (\top, \Delta_6), \top} \ \top_R \quad \dfrac{}{\bullet \mathbf{h}_5 : \Delta_4, \top \vdash \top, \Delta_6} \ \top_R}{- : \Delta_4 \vdash \top, \Delta_6} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_4 \vdash \top, \Delta_6} \ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathbf{h}_2 : \Delta_5 \vdash (\top, \Delta_8), \mathbf{F}_6} \ \top_R \quad \dfrac{}{\bullet \mathbf{h}_7 : \Delta_5, \mathbf{F}_6 \vdash \top, \Delta_8} \ \top_R}{- : \Delta_5 \vdash \top, \Delta_8} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_5 \vdash \top, \Delta_8} \ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathbf{h}_1 : \Delta_8, \mathbf{F}_5 \rightarrow \mathbf{F}_6 \vdash \Delta_7, \top} \ \top_R \quad \dfrac{\mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_5 \rightarrow \mathbf{F}_6 \vdash \Delta_7, \mathbf{F}_5 \quad \mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_6 \vdash \Delta_7}{\bullet \mathbf{h}_4 : (\Delta_8, \mathbf{F}_5 \rightarrow \mathbf{F}_6), \top \vdash \Delta_7} \ \rightarrow_L}{- : \Delta_8, \mathbf{F}_5 \rightarrow \mathbf{F}_6 \vdash \Delta_7} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathbf{h}_1 : \Delta_8, \mathbf{F}_5 \rightarrow \mathbf{F}_6 \vdash \top, \Delta_7, \mathbf{F}_5} \ \text{ax/W} \quad \dfrac{}{\mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_5 \rightarrow \mathbf{F}_6 \vdash \Delta_7, \mathbf{F}_5}}{- : \Delta_8, \mathbf{F}_5 \rightarrow \mathbf{F}_6 \vdash \Delta_7, \mathbf{F}_5} \ \text{hCut} \quad \dfrac{\dfrac{}{\bullet \mathbf{h}_1 : \Delta_8, \mathbf{F}_6 \vdash \top, \Delta_7} \ \top_R \quad \dfrac{}{\mathbf{h}_4 : \top, \Delta_8, \mathbf{F}_6 \vdash \Delta_7} \ \text{ax/W}}{- : \Delta_8, \mathbf{F}_6 \vdash \Delta_7} \ \text{hCut}}{- : \Delta_8, \mathbf{F}_5 \rightarrow \mathbf{F}_6 \vdash \Delta_7} \ \rightarrow_L$$

$$\dfrac{\dfrac{}{\bullet \mathbf{h}_2 : \Delta_{10}, \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash (\top, \Delta_6), \mathbf{F}_5} \ \top_R \quad \dfrac{\mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5, \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash \top, \Delta_6, \mathbf{F}_8 \quad \mathbf{h}_7 : \Delta_{10}, \mathbf{F}_5, \mathbf{F}_9 \vdash \top, \Delta_6}{\bullet \mathbf{h}_7 : (\Delta_{10}, \mathbf{F}_8 \rightarrow \mathbf{F}_9), \mathbf{F}_5 \vdash \top, \Delta_6} \ \rightarrow_L}{- : \Delta_{10}, \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash \top, \Delta_6} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_{10}, \mathbf{F}_8 \rightarrow \mathbf{F}_9 \vdash \top, \Delta_6} \ \top_R$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \to F_9} \ \top_R \quad \dfrac{h_6 : \Delta_7, F_8 \to F_9 \vdash \top, \Delta_5, F_8 \quad h_6 : \Delta_7, F_9 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \to F_9 \vdash \top, \Delta_5} \ \to_L}{- : \Delta_7 \vdash \top, \Delta_5} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5} \ \top_R$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \top} \ \top_R \quad \dfrac{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \top \vdash \Delta_7} \ \wedge_L}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_5, F_6 \vdash \top, \Delta_7} \ \top_R \quad \dfrac{}{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7} \ \text{ax/W}}{- : \Delta_8, F_5, F_6 \vdash \Delta_7} \ \text{hCut}}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \ \wedge_L$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash (\top, \Delta_6), F_5} \ \top_R \quad \dfrac{h_7 : \Delta_{10}, F_5, F_8, F_9 \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \wedge F_9), F_5 \vdash \top, \Delta_6} \ \wedge_L}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6} \ \top_R$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \wedge F_9} \ \top_R \quad \dfrac{h_6 : \Delta_7, F_8, F_9 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \top, \Delta_5} \ \wedge_L}{- : \Delta_7 \vdash \top, \Delta_5} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5} \ \top_R$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \top} \ \top_R \quad \dfrac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7 \quad h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \top \vdash \Delta_7} \ \vee_L}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7} \ \top_R \quad \dfrac{}{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7} \ \text{ax/W}}{- : \Delta_8, F_5 \vdash \Delta_7} \ \text{hCut} \quad \dfrac{\dfrac{}{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \ \top_R \quad \dfrac{}{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7} \ \text{ax/W}}{- : \Delta_8, F_6 \vdash \Delta_7} \ \text{hCut}}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \ \vee_L$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\top, \Delta_6), F_5} \ \top_R \quad \dfrac{h_7 : \Delta_{10}, F_5, F_8 \vdash \top, \Delta_6 \quad h_7 : \Delta_{10}, F_5, F_9 \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \top, \Delta_6} \ \vee_L}{- : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6} \ \top_R$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \vee F_9} \ \top_R \quad \dfrac{h_6 : \Delta_7, F_8 \vdash \top, \Delta_5 \quad h_6 : \Delta_7, F_9 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \top, \Delta_5} \ \vee_L}{- : \Delta_7 \vdash \top, \Delta_5} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5} \ \top_R$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \perp, \Delta_6 \vdash \Delta_5, \top} \ \top_R \quad \dfrac{}{\bullet h_4 : (\perp, \Delta_6), \top \vdash \Delta_5} \ \perp_L}{- : \perp, \Delta_6 \vdash \Delta_5} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \perp, \Delta_6 \vdash \Delta_5} \ \perp_L$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), \perp} \ \top_R \quad \dfrac{}{\bullet h_6 : \Delta_7, \perp \vdash \top, \Delta_5} \ \perp_L}{- : \Delta_7 \vdash \top, \Delta_5} \ \text{Cut}$$

$$\leadsto$$

$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5} \ \top_R$$

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$$\dfrac{\dfrac{}{\bullet h_2 : \bot, \Delta_8 \vdash (\top, \Delta_6), F_5} \ \top_R \quad \dfrac{}{\bullet h_7 : (\bot, \Delta_8), F_5 \vdash \top, \Delta_6} \ \bot_L}{- : \bot, \Delta_8 \vdash \top, \Delta_6} \ \text{Cut}$$
$$\leadsto$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \top, \Delta_6} \ \top_R$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_7, p_5 \vdash (\Delta_6, p_5), \top} \ \top_R \quad \dfrac{}{\bullet h_4 : (\Delta_7, p_5), \top \vdash \Delta_6, p_5} \ I}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} \ \text{Cut}$$
$$\leadsto$$
$$\dfrac{}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} \ I$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_8, p_7), p_7} \ \top_R \quad \dfrac{}{\bullet h_5 : \Delta_6, p_7 \vdash \top, \Delta_8, p_7} \ I}{- : \Delta_6 \vdash \top, \Delta_8, p_7} \ \text{Cut}$$
$$\leadsto$$
$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_8, p_7} \ \top_R$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_9, p_7 \vdash (\top, \Delta_8, p_7), F_5} \ \top_R \quad \dfrac{}{\bullet h_6 : (\Delta_9, p_7), F_5 \vdash \top, \Delta_8, p_7} \ I}{- : \Delta_9, p_7 \vdash \top, \Delta_8, p_7} \ \text{Cut}$$
$$\leadsto$$
$$\dfrac{}{- : \Delta_9, p_7 \vdash \top, \Delta_8, p_7} \ \top_R$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_5 \vdash \Delta_6, \top} \ \top_R \quad \dfrac{h_4 : \Delta_5 \vdash \Delta_6}{\bullet h_4 : \Delta_5, \top \vdash \Delta_6} \ \top_L}{- : \Delta_5 \vdash \Delta_6} \ \text{Cut}$$
$$\leadsto$$
$$\dfrac{}{- : \Delta_5 \vdash \Delta_6} \ \text{ax/W}$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), \top} \ \top_R \quad \dfrac{h_6 : \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, \top \vdash \top, \Delta_5} \ \top_L}{- : \Delta_7 \vdash \top, \Delta_5} \ \text{Cut}$$
$$\leadsto$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5} \ \top_R$$

$$\dfrac{\dfrac{}{\bullet h_2 : \top, \Delta_8 \vdash (\top, \Delta_6), F_5} \ \top_R \quad \dfrac{h_7 : \Delta_8, F_5 \vdash \top, \Delta_6}{\bullet h_7 : (\top, \Delta_8), F_5 \vdash \top, \Delta_6} \ \top_L}{- : \top, \Delta_8 \vdash \top, \Delta_6} \ \text{Cut}$$
$$\leadsto$$
$$\dfrac{}{- : \top, \Delta_8 \vdash \top, \Delta_6} \ \top_R$$

8.6 Status of $\to_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_9 \to F_{10} \vdash (\Delta_{12}, F_{13} \to F_{14}), F_8, F_9 \quad h_3 : \Delta_7, F_{10} \vdash (\Delta_{12}, F_{13} \to F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \to F_{10} \vdash (\Delta_{12}, F_{13} \to F_{14}), F_8} \ \to_L \quad \dfrac{h_{11} : \Delta_7, F_8, F_{13}, F_9 \to F_{10} \vdash F_1}{\bullet h_{11} : (\Delta_7, F_9 \to F_{10}), F_8 \vdash \Delta_{12}, F_{13}}}{- : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_{13} \to F_{14}}$$
$$\leadsto$$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_8, F_9, F_{13} \to F_{14}}{h_3 : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_8, F_9, F_{13} \to F_{14}} \ \text{ax/W} \quad \dfrac{}{\bullet h_{11} : \Delta_7, F_8, F_9 \to F_{10} \vdash \Delta_{12}, F_9, F_{13} \to F_{14}} \ \text{ax/W}}{- : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_9, F_{13} \to F_{14}} \ \text{hCut} \quad \dfrac{\dfrac{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_8, F_{13} \to F_{14}}{\ } \ \text{ax/W} \quad \dfrac{h}{\bullet h_{11}}}{- : \Delta_7, F_{10} \vdash \Delta_{12}, F_{13}}$$
$$- : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_{13} \to F_{14}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_9 \to F_{10} \vdash (\Delta_{12}, F_{13} \land F_{14}), F_8, F_9 \quad h_3 : \Delta_7, F_{10} \vdash (\Delta_{12}, F_{13} \land F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \to F_{10} \vdash (\Delta_{12}, F_{13} \land F_{14}), F_8} \to_L \quad \dfrac{h_{11} : \Delta_7, F_8, F_9 \to F_{10} \vdash \Delta_{12}, F_{13}}{\bullet h_{11} : (\Delta_7, F_9 \to F_{10})}}{- : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_{13} \land F_{14}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_3 : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_8, F_9, F_{13} \land F_{14}} \text{ax/W} \quad \dfrac{}{\bullet h_{11} : \Delta_7, F_8, F_9 \to F_{10} \vdash \Delta_{12}, F_9, F_{13} \land F_{14}} \text{ax/W}}{- : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_9, F_{13} \land F_{14}} \text{hCut} \quad \dfrac{}{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_8, F_{13} \land F_{14}} \text{ax/W} \quad \dfrac{}{h_{11} : \Delta_7}$$
$$- : \Delta_7, F_{10} \vdash \Delta_1$$
$$- : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_{13} \land F_{14}$$

- Case rule $\lor_R$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_9 \to F_{10} \vdash (\Delta_{12}, F_{13} \lor F_{14}), F_8, F_9 \quad h_3 : \Delta_7, F_{10} \vdash (\Delta_{12}, F_{13} \lor F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \to F_{10} \vdash (\Delta_{12}, F_{13} \lor F_{14}), F_8} \to_L \quad \dfrac{h_{11} : \Delta_7, F_8, F_9 \to F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \to F_{10}), F_8 \vdash \Delta_{12}, F_{13} \lor F_{14}} \lor_R}{- : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_{13} \lor F_{14}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8, F_9} \text{inv-th/ax} \quad \dfrac{}{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8} \text{inv-th/ax}}{\bullet h_3 : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8} \to_L \quad \dfrac{}{h_{11} : \Delta_7, F_8, F_9 \to F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W}}{\dfrac{- : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{- : \Delta_7, F_9 \to F_{10} \vdash \Delta_{12}, F_{13} \lor F_{14}} \lor_R} \text{hCut}$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_9 \to F_{10} \vdash (\perp, \Delta_{12}), F_8, F_9 \quad h_3 : \Delta_7, F_{10} \vdash (\perp, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \to F_{10} \vdash (\perp, \Delta_{12}), F_8} \to_L \quad \dfrac{h_{11} : \Delta_7, F_8, F_9 \to F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_7, F_9 \to F_{10}), F_8 \vdash \perp, \Delta_{12}} \perp_R}{- : \Delta_7, F_9 \to F_{10} \vdash \perp, \Delta_{12}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet h_3 : \Delta_7, F_9 \to F_{10} \vdash \perp, \Delta_{12}, F_8} \text{ax/W} \quad \dfrac{}{h_{11} : \Delta_7, F_8, F_9 \to F_{10} \vdash \perp, \Delta_{12}} \text{ax/W}}{- : \Delta_7, F_9 \to F_{10} \vdash \perp, \Delta_{12}} \text{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_9 \to F_{10} \vdash (\top, \Delta_{12}), F_8, F_9 \quad h_3 : \Delta_7, F_{10} \vdash (\top, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \to F_{10} \vdash (\top, \Delta_{12}), F_8} \to_L \quad \dfrac{}{\bullet h_{11} : (\Delta_7, F_9 \to F_{10}), F_8 \vdash \top, \Delta_{12}} \top_R}{- : \Delta_7, F_9 \to F_{10} \vdash \top, \Delta_{12}} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7, F_9 \to F_{10} \vdash \top, \Delta_{12}} \top_R$$

- Case rule $\to_L$

$$\dfrac{\dfrac{h_3 : (\Delta_{14}, F_{11} \to F_{12}), F_8 \to F_9 \vdash \Delta_{13}, F_7, F_8 \quad h_3 : (\Delta_{14}, F_{11} \to F_{12}), F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \to F_{12}), F_8 \to F_9 \vdash \Delta_{13}, F_7} \to_L \quad \dfrac{h_{10} : \Delta_{14}, F_7, F_8 \to F_9, F_{11} \to F}{\bullet h_{10} : ((\Delta_{14},}}{- : (\Delta_{14}, F_{11} \to F_{12}), F_8 \to F_9 \vdash \Delta_{13}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_3 : \Delta_{14}, F_{11} \to F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_{11}, F_7} \text{ax/W} \quad \dfrac{}{h_{10} : \Delta_{14}, F_7, F_{11} \to F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_{11}} \text{ax/W}}{- : \Delta_{14}, F_{11} \to F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_{11}} \text{hCut} \quad \dfrac{\dfrac{}{h_3 : \Delta_{14}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_7, F_8} \text{inv-th/}}{\bullet h_3 : \Delta_{14}, F_{12}, F_8 \to F_9}}{- : \Delta_{14}, F_{11} \to F_{12}, F_8 \to F_9 \vdash \Delta_{13}}$$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \to F_{12}, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \to F_{12}}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \to F_{12}} \to_L \quad \dfrac{h_{10} : \Delta_7, F_8 \to F_9, F_{11} \to F_{12} \vdash \Delta_{13}, F_{11} \quad h_{10}}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), F_{11} \to}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_8, F_{11} \to F_{12}} \text{ax/W} \quad \dfrac{}{\bullet h_{10} : \Delta_7, F_{11} \to F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_8} \text{ax/W}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_8} \text{hCut} \quad \dfrac{}{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \to F_{12}} \text{ax/W} \quad \dfrac{}{h_{10} : \Delta_7, F_9, F_{11} \to}}{- : \Delta_7, F_9 \vdash \Delta_{13}}$$
$$- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}$$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_8, F_{10} \quad h_3 : \Delta_7, F_{11} \vdash \Delta_{12}, F_8}{\bullet h_3 : \Delta_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_8} \to_L \quad \cfrac{h_9 : \Delta_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12}, F_{10} \quad h_9 : \Delta_7, F_8, F_{11} \vdash \Delta_{12}}{\bullet h_9 : (\Delta_7, F_{10} \to F_{11}), F_8 \vdash \Delta_{12}} \to_L}{- : \Delta_7, F_{10} \to F_{11} \vdash \Delta_{12}} \text{Cut}}{\rightsquigarrow}$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_{10}, F_8} \text{ax/W} \quad \cfrac{}{\bullet h_9 : \Delta_7, F_8, F_{10} \to F_{11} \vdash \Delta_{12}, F_{10}} \text{ax/W}}{- : \Delta_7, F_{10} \to F_{11} \vdash \Delta_{12}, F_{10}} \text{hCut} \quad \cfrac{\cfrac{}{h_3 : \Delta_7, F_{11} \vdash \Delta_{12}, F_8} \text{ax/W} \quad \cfrac{\cfrac{h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{12}} \text{ax/W} }{} H}{- : \Delta_7, F_{11} \vdash \Delta_{12}} \text{hCut}}{- : \Delta_7, F_{10} \to F_{11} \vdash \Delta_{12}} \to_L$$

- Case rule $\wedge_L$

$$\cfrac{\cfrac{h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \to F_9 \vdash \Delta_{13}, F_7, F_8 \quad h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \to F_9 \vdash \Delta_{13}, F_7} \to_L \quad \cfrac{h_{10} : \Delta_{14}, F_7, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \to F_9), F_7 \vdash \Delta_{13}} \wedge_L}{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \to F_9 \vdash \Delta_{13}} \text{Cut}$$
$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_7, F_8} \text{inv-th/ax} \quad \cfrac{}{h_3 : \Delta_{14}, F_{11}, F_{12}, F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax}}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_7} \to_L \quad \cfrac{}{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \to F_9 \vdash \Delta_{13}} \text{ax/W}}{\cfrac{- : \Delta_{14}, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_8 \to F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L} \text{hCut}$$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \to_L \quad \cfrac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \to F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}} \text{Cut}$$
$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_8, F_{11} \wedge F_{12}} \text{ax/W} \quad \cfrac{}{\bullet h_{10} : \Delta_7, F_8 \to F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}, F_8} \text{ax/W}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_8} \text{hCut} \quad \cfrac{\cfrac{}{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \text{ax/W} \quad \cfrac{\cfrac{h_{10} : \Delta_7, F_{11}, F_{12}, F_9 \dots}{\bullet h_{10} : \Delta_7, F_9, F_{11} \wedge F \dots}}{} }{- : \Delta_7, F_9 \vdash \Delta_{13}}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}} \to_L$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \to F_9 \vdash \Delta_{13}, F_7, F_8 \quad h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \to F_9 \vdash \Delta_{13}, F_7} \to_L \quad \cfrac{h_{10} : \Delta_{14}, F_7, F_{11}, F_8 \to F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee F \dots}}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \to F_9 \vdash \Delta_{13}}$$
$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_{14}, F_8 \to F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7, F_8} \text{ax/W} \quad \cfrac{}{\bullet h_{10} : \Delta_{14}, F_7, F_8 \to F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \text{ax/W}}{- : \Delta_{14}, F_8 \to F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \text{hCut} \quad \cfrac{\cfrac{}{h_3 : \Delta_{14}, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7} \text{ax/W} \quad \cfrac{\cfrac{h_{10} : \Delta_1 \dots}{}}{}}{- : \Delta_{14}, F_9, F_{11} \vee \dots}}{- : \Delta_{14}, F_8 \to F_9, F_{11} \vee F_{12} \vdash \Delta_{13}}$$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \to_L \quad \cfrac{h_{10} : \Delta_7, F_{11}, F_8 \to F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_7, F_{12}, F_8 \to F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \vee_L}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}} \text{Cut}$$
$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{\cfrac{}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11}, F_{12}, F_8} \text{inv-th/ax} \quad \cfrac{}{- : \Delta_7, F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \text{inv-th/ax}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \to_L \quad \cfrac{\cfrac{}{- : \Delta_7, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_{11}} \text{ax/W} \quad \cfrac{}{- : \Delta_7, F_{11}, F_8 \to F_9 \vdash \dots}}{- : \Delta_7, F_{12}, F_8 \to F_9 \vdash \Delta_{13}, F_{11}} \text{sCut}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}, F_{11}}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{13}}$$

- Case rule $\perp_L$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}, \perp, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{11}, \perp}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}, \perp} \to_L \quad \cfrac{}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), \perp \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}} \text{Cut}$$
$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_8 \to F_9 \vdash \perp, \Delta_{11}, F_8} \text{ax/W} \quad \cfrac{}{\bullet h_{10} : \perp, \Delta_7, F_8 \to F_9 \vdash \Delta_{11}, F_8} \text{ax/W}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}, F_8} \text{hCut} \quad \cfrac{\cfrac{}{h_3 : \Delta_7, F_9 \vdash \perp, \Delta_{11}} \text{ax/W} \quad \cfrac{}{\bullet h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_9 \vdash \Delta_{11}} \text{hCut}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}} \to_L$$

$$\cfrac{\cfrac{h_3 : (\bot, \Delta_{12}), F_8 \to F_9 \vdash \Delta_{11}, F_7, F_8 \quad h_3 : (\bot, \Delta_{12}), F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\bot, \Delta_{12}), F_8 \to F_9 \vdash \Delta_{11}, F_7} \to_L \quad \cfrac{}{\bullet h_{10} : ((\bot, \Delta_{12}), F_8 \to F_9), F_7 \vdash \Delta_{11}} \bot_L}{- : (\bot, \Delta_{12}), F_8 \to F_9 \vdash \Delta_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- : \bot, \Delta_{12}, F_8 \to F_9 \vdash \Delta_{11}} \bot_L$$

- Case rule I

$$\cfrac{\cfrac{h_3 : \Delta_7, F_8 \to F_9 \vdash (\Delta_{12}, p_{11}), p_{11}, F_8 \quad h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, p_{11}), p_{11}}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash (\Delta_{12}, p_{11}), p_{11}} \to_L \quad \cfrac{}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), p_{11} \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{12}, p_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{12}, F_8, p_{11}, p_{11}} \text{ax/W} \quad \cfrac{}{\bullet h_{10} : \Delta_7, p_{11}, F_8 \to F_9 \vdash \Delta_{12}, F_8, p_{11}} \text{ax/W}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{12}, F_8, p_{11}} \text{hCut} \quad \cfrac{\cfrac{}{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}, p_{11}} \text{ax/W} \quad \cfrac{}{\bullet h_{10} : \Delta_7, F_9, p_{11} \vdash \Delta_{12}, p}{- : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}}}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{12}, p_{11}} \to_L$$

$$\cfrac{\cfrac{h_3 : (\Delta_{13}, p_{11}), F_8 \to F_9 \vdash (\Delta_{12}, p_{11}), F_7, F_8 \quad h_3 : (\Delta_{13}, p_{11}), F_9 \vdash (\Delta_{12}, p_{11}), F_7}{\bullet h_3 : (\Delta_{13}, p_{11}), F_8 \to F_9 \vdash (\Delta_{12}, p_{11}), F_7} \to_L \quad \cfrac{}{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \to F_9), F_7 \vdash \Delta_{12}, p_{11}} I}{- : (\Delta_{13}, p_{11}), F_8 \to F_9 \vdash \Delta_{12}, p_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- : \Delta_{13}, p_{11}, F_8 \to F_9 \vdash \Delta_{12}, p_{11}} I$$

- Case rule $\top_L$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}, \top, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{11}, \top}{\bullet h_3 : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}, \top} \to_L \quad \cfrac{h_{10} : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \to F_9), \top \vdash \Delta_{11}} \top_L}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- : \Delta_7, F_8 \to F_9 \vdash \Delta_{11}} \text{ax/W}$$

$$\cfrac{\cfrac{h_3 : (\top, \Delta_{12}), F_8 \to F_9 \vdash \Delta_{11}, F_7, F_8 \quad h_3 : (\top, \Delta_{12}), F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\top, \Delta_{12}), F_8 \to F_9 \vdash \Delta_{11}, F_7} \to_L \quad \cfrac{h_{10} : \Delta_{12}, F_7, F_8 \to F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \to F_9), F_7 \vdash \Delta_{11}} \top_L}{- : (\top, \Delta_{12}), F_8 \to F_9 \vdash \Delta_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{}{\bullet h_3 : \top, \Delta_{12}, F_8 \to F_9 \vdash \Delta_{11}, F_7} \text{ax/W} \quad \cfrac{}{h_{10} : \top, \Delta_{12}, F_7, F_8 \to F_9 \vdash \Delta_{11}} \text{ax/W}}{- : \top, \Delta_{12}, F_8 \to F_9 \vdash \Delta_{11}} \text{hCut}$$

8.7 Status of $\wedge_L$: OK

- Case rule $\to_R$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_9, F_{10} \vdash (\Delta_{12}, F_{13} \to F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \to F_{14}), F_8} \wedge_L \quad \cfrac{h_{11} : \Delta_7, F_8, F_{13}, F_9 \wedge F_{10} \vdash F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \Delta_{12}, F_{13} \to F_{14}} \to_R}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \to F_{14}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_8, F_{13} \to F_{14}} \text{ax/W} \quad \cfrac{\cfrac{}{h_{11} : \Delta_7, F_{10}, F_{13}, F_8, F_9 \vdash F_{14}} \text{inv-th/ax}}{\bullet h_{11} : \Delta_7, F_{10}, F_8, F_9 \vdash \Delta_{12}, F_{13} \to F_{14}} \to_R}{- : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_{13} \to F_{14}} \text{hCut}}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \to F_{14}} \wedge_L$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_9, F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8} \wedge_L \quad \cfrac{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \quad h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_8, F_{13} \wedge F_{14}} \text{ax/W} \quad \cfrac{\cfrac{}{h_{11} : \Delta_7, F_{10}, F_8, F_9 \vdash \Delta_{12}, F_{13}} \text{inv-th/ax} \quad \cfrac{}{h_{11} : \Delta_7, F_{10}, F_8, F_9 \vdash \Delta_{12}, F_{14}} \text{inv-th/ax}}{\bullet h_{11} : \Delta_7, F_{10}, F_8, F_9 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R}{- : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{hCut}}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_L$$

- Case rule $\vee_R$

$$
\dfrac{
\dfrac{h_3 : \Delta_7, F_9, F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8} \ \wedge_L \quad
\dfrac{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \Delta_{12}, F_{13} \vee F_{14}} \ \vee_R
}{
- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}
} \ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{
\dfrac{h_3 : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8} \ \wedge_L \quad
\dfrac{}{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \ \text{ax/W}
}{
- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}
} \ \text{hCut} \ \ \text{inv-th/ax}
}{
- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}
} \ \vee_R
$$

- Case rule $\perp_R$

$$
\dfrac{
\dfrac{h_3 : \Delta_7, F_9, F_{10} \vdash (\perp, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\perp, \Delta_{12}), F_8} \ \wedge_L \quad
\dfrac{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \perp, \Delta_{12}} \ \perp_R
}{
- : \Delta_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}
} \ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}, F_8} \ \text{ax/W} \quad
\dfrac{}{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}} \ \text{ax/W}
}{
- : \Delta_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}
} \ \text{hCut}
$$

- Case rule $\top_R$

$$
\dfrac{
\dfrac{h_3 : \Delta_7, F_9, F_{10} \vdash (\top, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\top, \Delta_{12}), F_8} \ \wedge_L \quad
\dfrac{}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \top, \Delta_{12}} \ \top_R
}{
- : \Delta_7, F_9 \wedge F_{10} \vdash \top, \Delta_{12}
} \ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{}{- : \Delta_7, F_9 \wedge F_{10} \vdash \top, \Delta_{12}} \ \top_R
$$

- Case rule $\rightarrow_L$

$$
\dfrac{
\dfrac{h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \ \wedge_L \quad
\dfrac{h_{10} : \Delta_{14}, F_7, F_{11} \rightarrow F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : \Delta_{14}, F_7, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \ \rightarrow_L
}{
- : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}
} \ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{
\dfrac{}{h_3 : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_7} \ \text{ax/W} \quad
\dfrac{\dfrac{h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_{11}}{} \ \text{inv-th/ax} \quad \dfrac{h_{10} : \Delta_{14}, F_{12}, F_7, F_8, F_9 \vdash \Delta_{13}}{} \ \text{inv-th/ax}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \ \rightarrow_L
}{
- : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}
} \ \text{hCut}
}{
- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}
} \ \wedge_L
$$

$$
\dfrac{
\dfrac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \ \wedge_L \quad
\dfrac{h_{10} : \Delta_7, F_{11} \rightarrow F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : \Delta_7, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \ \rightarrow_L
}{
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}
} \ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{
\dfrac{}{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \ \text{ax/W} \quad
\dfrac{\dfrac{h_{10} : \Delta_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_{11}}{} \ \text{inv-th/ax} \quad \dfrac{h_{10} : \Delta_7, F_{12}, F_8, F_9 \vdash \Delta_{13}}{} \ \text{inv-th/ax}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \ \rightarrow_L
}{
- : \Delta_7, F_8, F_9 \vdash \Delta_{13}
} \ \text{hCut}
}{
- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}
} \ \wedge_L
$$

- Case rule $\wedge_L$

$$
\dfrac{
\dfrac{h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \ \wedge_L \quad
\dfrac{h_{10} : \Delta_{14}, F_7, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \ \wedge_L
}{
- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}
} \ \text{Cut}
$$

$$\rightsquigarrow$$

$$
\dfrac{
\dfrac{
\dfrac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_8, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \ \wedge_L \ \ \text{inv-th/ax} \quad
\dfrac{}{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{13}} \ \text{ax/W}
}{
- : \Delta_{14}, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}
} \ \text{hCut}
}{
- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}
} \ \wedge_L
$$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}\ \wedge_L \qquad \cfrac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}}\ \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}\ \text{ax/W} \qquad \cfrac{\cfrac{}{h_{10} : \Delta_7, F_{11}, F_{12}, F_8, F_9 \vdash \Delta_{13}}\ \text{inv-th/ax}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}}\ \wedge_L}{- : \Delta_7, F_8, F_9 \vdash \Delta_{13}}\ \text{hCut}}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}\ \wedge_L$$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_{10}, F_{11} \vdash \Delta_{12}, F_8}{\bullet h_3 : \Delta_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8}\ \wedge_L \qquad \cfrac{h_9 : \Delta_7, F_8, F_{10}, F_{11} \vdash \Delta_{12}}{\bullet h_9 : (\Delta_7, F_{10} \wedge F_{11}), F_8 \vdash \Delta_{12}}\ \wedge_L}{- : \Delta_7, F_{10} \wedge F_{11} \vdash \Delta_{12}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_{10}, F_{11} \vdash \Delta_{12}, F_8}\ \text{ax/W} \qquad \cfrac{\cfrac{}{h_9 : \Delta_7, F_{10}, F_{11}, F_8 \vdash \Delta_{12}}\ \text{ax/W}}{\bullet h_9 : \Delta_7, F_{10}, F_{11}, F_8 \vdash \Delta_{12}}\ H}{- : \Delta_7, F_{10}, F_{11} \vdash \Delta_{12}}\ \text{hCut}}{- : \Delta_7, F_{10} \wedge F_{11} \vdash \Delta_{12}}\ \wedge_L$$

- Case rule $\vee_L$

$$\cfrac{\cfrac{h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7}\ \wedge_L \qquad \cfrac{h_{10} : \Delta_{14}, F_7, F_{11}, F_8 \wedge F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_{14}, F_7, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}}\ \vee_L}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7}\ \text{ax/W} \qquad \cfrac{\cfrac{}{h_{10} : \Delta_{14}, F_{11}, F_7, F_8, F_9 \vdash \Delta_{13}}\ \text{inv-th/ax} \quad \cfrac{}{h_{10} : \Delta_{14}, F_{12}, F_7, F_8, F_9 \vdash \Delta_{13}}\ \text{inv-th/ax}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}}\ \vee_L}{- : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}}\ \text{hCut}}{- : \Delta_{14}, F_8 \wedge F_9, F_{11} \vee F_{12} \vdash \Delta_{13}}\ \wedge_L$$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}\ \wedge_L \qquad \cfrac{h_{10} : \Delta_7, F_{11}, F_8 \wedge F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_7, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \vee F_{12} \vdash \Delta_{13}}\ \vee_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}\ \text{ax/W} \qquad \cfrac{\cfrac{}{h_{10} : \Delta_7, F_{11}, F_8, F_9 \vdash \Delta_{13}}\ \text{inv-th/ax} \quad \cfrac{}{h_{10} : \Delta_7, F_{12}, F_8, F_9 \vdash \Delta_{13}}\ \text{inv-th/ax}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}}\ \vee_L}{- : \Delta_7, F_8, F_9 \vdash \Delta_{13}}\ \text{hCut}}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}\ \wedge_L$$

- Case rule $\bot_L$

$$\cfrac{\cfrac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{11}, \bot}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \bot}\ \wedge_L \qquad \cfrac{}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \bot \vdash \Delta_{11}}\ \bot_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{}{h_3 : \Delta_7, F_8, F_9 \vdash \bot, \Delta_{11}}\ \text{ax/W} \qquad \cfrac{}{\bullet h_{10} : \bot, \Delta_7, F_8, F_9 \vdash \Delta_{11}}\ \bot_L}{- : \Delta_7, F_8, F_9 \vdash \Delta_{11}}\ \text{hCut}}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}}\ \wedge_L$$

$$\cfrac{\cfrac{h_3 : (\bot, \Delta_{12}), F_8, F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\bot, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}, F_7}\ \wedge_L \qquad \cfrac{}{\bullet h_{10} : ((\bot, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}}\ \bot_L}{- : (\bot, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\cfrac{}{- : \bot, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}}\ \bot_L$$

- Case rule I

$$\frac{\dfrac{h_3 : \Delta_7, F_8, F_9 \vdash (\Delta_{12}, p_{11}), p_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash (\Delta_{12}, p_{11}), p_{11}} \wedge_L \quad \dfrac{}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), p_{11} \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{}{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{12}, p_{11}, p_{11}} \text{ax/W} \quad \dfrac{}{\bullet h_{10} : \Delta_7, F_8, F_9, p_{11} \vdash \Delta_{12}, p_{11}} I}{\dfrac{- : \Delta_7, F_8, F_9 \vdash \Delta_{12}, p_{11}}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \wedge_L} \text{hCut}$$

$$\frac{\dfrac{h_3 : (\Delta_{13}, p_{11}), F_8, F_9 \vdash (\Delta_{12}, p_{11}), F_7}{\bullet h_3 : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash (\Delta_{12}, p_{11}), F_7} \wedge_L \quad \dfrac{}{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, p_{11}} I}{- : (\Delta_{13}, p_{11}), F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{}{- : \Delta_{13}, p_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, p_{11}} I$$

- Case rule $\top_L$

$$\frac{\dfrac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{11}, \top}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \top} \wedge_L \quad \dfrac{h_{10} : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \top \vdash \Delta_{11}} \top_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{ax/W}$$

$$\frac{\dfrac{h_3 : (\top, \Delta_{12}), F_8, F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \wedge_L \quad \dfrac{h_{10} : \Delta_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}} \top_L}{- : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{}{\bullet h_3 : \top, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \text{ax/W} \quad \dfrac{}{h_{10} : \top, \Delta_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{ax/W}}{- : \top, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}} \text{hCut}$$

8.8 Status of $\vee_L$: OK

- Case rule $\rightarrow_R$

$$\frac{\dfrac{h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_8 \quad h_3 : \Delta_7, F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_8} \vee_L \quad \dfrac{h_{11} : \Delta_7, F_8, F_{13}, F_9 \vee F_{10} \vdash F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \text{Cut}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{}{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, F_8, F_{13} \rightarrow F_{14}} \text{ax/W} \quad \dfrac{\dfrac{h_{11} : \Delta_7, F_{13}, F_8, F_9 \vdash F_{14}}{\bullet h_{11} : \Delta_7, F_8, F_9 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R}{} \text{inv-th/ax}}{\dfrac{- : \Delta_7, F_9 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}}{}} \text{hCut} \quad \dfrac{\dfrac{}{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_8, F_{13} \rightarrow F_{14}} \text{ax/W} \quad \dfrac{h_{11} : \Delta_7, F_{10}, F_{13}, F_8}{\bullet h_{11} : \Delta_7, F_{10}, F_8 \vdash \Delta_{12},}}{- : \Delta_7, F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}}$$

$$- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14} \quad \vee_L$$

- Case rule $\wedge_R$

$$\frac{\dfrac{h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8 \quad h_3 : \Delta_7, F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8} \vee_L \quad \dfrac{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \Delta_{12},}{\bullet h_{11} : (\Delta_7, F_9 \vee}{}}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}}$$

$$\rightsquigarrow$$

$$\frac{\dfrac{\dfrac{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, F_{13}, F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_8} \text{inv-th/ax} \quad \dfrac{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_{13}, F_8}{} \text{inv-th/ax}}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_8} \vee_L \quad \dfrac{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}}{}}{\dfrac{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}}{}} \text{ax/W} \quad \dfrac{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, F_{14}, F_8 \quad \text{inv-}}{\bullet h_3 : \Delta_7, F_9 \vee}}{} \text{hCut}$$

$$- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8 \quad h_3 : \Delta_7, F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8}\ \vee_L \quad \dfrac{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \Delta_{12}, F_{13} \vee F_{14}}\ \vee_R}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_8}\ \texttt{inv-th/ax} \quad \dfrac{}{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8}\ \texttt{inv-th/ax}}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8}\ \vee_L \quad \dfrac{}{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}\ \texttt{ax/W}}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}\ \text{hCut}}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}}\ \vee_R$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_9 \vdash (\perp, \Delta_{12}), F_8 \quad h_3 : \Delta_7, F_{10} \vdash (\perp, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\perp, \Delta_{12}), F_8}\ \vee_L \quad \dfrac{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \perp, \Delta_{12}}\ \perp_R}{- : \Delta_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}, F_8}\ \texttt{ax/W} \quad \dfrac{}{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12}}\ \texttt{ax/W}}{- : \Delta_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}}\ \text{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_9 \vdash (\top, \Delta_{12}), F_8 \quad h_3 : \Delta_7, F_{10} \vdash (\top, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\top, \Delta_{12}), F_8}\ \vee_L \quad \dfrac{}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \top, \Delta_{12}}\ \top_R}{- : \Delta_7, F_9 \vee F_{10} \vdash \top, \Delta_{12}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \top, \Delta_{12}}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vdash \Delta_{13}, F_7 \quad h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7}\ \vee_L \quad \dfrac{h_{10} : \Delta_{14}, F_7, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F}{\bullet h_{10} : ((\Delta_{14}, F_{11} \rightarrow F_{12})}}{- : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vee F_9 \vdash \Delta_{13}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet h_3 : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_7}\ \texttt{ax/W} \quad \dfrac{}{h_{10} : \Delta_{14}, F_7, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}}\ \texttt{ax/W}}{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}}\ \text{hCut} \quad \dfrac{\dfrac{}{h_3 : \Delta_{14}, F_{12}, F_8 \vdash \Delta_{13}, F_7}\ \texttt{inv-th/ax} \quad h_3 : \Delta_1}{\bullet h_3 : \Delta_{14}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13},}}{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}$$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \rightarrow F_{12} \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}\ \vee_L \quad \dfrac{h_{10} : \Delta_7, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_1}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}\ \texttt{ax/W} \quad \dfrac{\dfrac{}{h_{10} : \Delta_7, F_8, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_{11}}\ \texttt{inv-th/ax} \quad \dfrac{}{h_{10} : \Delta_7, F_{12}, F_8 \vdash \Delta_{13}}\ \texttt{inv-th/ax}}{\bullet h_{10} : \Delta_7, F_8, F_{11} \rightarrow F_{12} \vdash \Delta_{13}}\ \rightarrow_L}{- : \Delta_7, F_8 \vdash \Delta_{13}}\ \text{hCut} \quad \dfrac{\rightsquigarrow}{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vdash \Delta_{13}, F_7 \quad h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7}\ \vee_L \quad \dfrac{h_{10} : \Delta_{14}, F_7, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{13}}\ \wedge_L}{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9 \vdash \Delta_{13}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vdash \Delta_{13}, F_7}\ \texttt{inv-th/ax} \quad \dfrac{}{h_3 : \Delta_{14}, F_{11}, F_{12}, F_9 \vdash \Delta_{13}, F_7}\ \texttt{inv-th/ax}}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7}\ \vee_L \quad \dfrac{}{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{13}}\ \texttt{ax/W}}{- : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}\ \text{hCut}}{- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}\ \wedge_L$$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \wedge F_{12} \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}\, \vee_L \quad \dfrac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}}\, \wedge_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \wedge F_{12}}\, \text{ax/W} \quad \dfrac{\dfrac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_{11} \wedge F_{12} \vdash \Delta_{13}}\, \wedge_L}{}\,\text{inv-th/ax}}{- : \Delta_7, F_8 \vdash \Delta_{13}}\,\text{hCut} \qquad \dfrac{\dfrac{}{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}\, \text{ax/W} \quad \dfrac{\dfrac{h_{10} : \Delta_7, F_{11}, F_{12}, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}}\, \wedge_L}{}\,\text{inv-th}}{- : \Delta_7, F_9 \vdash \Delta_{13}}\,\text{hCut}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}\ \vee_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vdash \Delta_{13}, F_7 \quad h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7}\, \vee_L \quad \dfrac{h_{10} : \Delta_{14}, F_7, F_{11}, F_8 \vee F_9 \vdash}{\bullet h_{10} : ((\Delta_{14}, F_{11})}}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_{14}, F_{11}, F_8 \vdash \Delta_{13}, F_7}\,\text{inv-th/ax} \quad \dfrac{}{h_3 : \Delta_{14}, F_{11}, F_9 \vdash \Delta_{13}, F_7}\,\text{inv-th/ax}}{\bullet h_3 : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}, F_7}\,\vee_L \quad \dfrac{}{h_{10} : \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}}\,\text{ax/W}}{- : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}}\,\text{hCut} \qquad \dfrac{\dfrac{}{h_3 : \Delta_{14}, F_{12}, F_8 \vdash \Delta_{13}, F_7}\,\text{in}}{\bullet h_3 : \Delta_{14}, F_{1}}$$
$$- : \Delta_{14}, F_{11} \vee F_{12}, F_8 \vee F_9 \vdash \Delta_{13}$$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \vee F_{12} \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}\,\vee_L \quad \dfrac{h_{10} : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_7, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \vee F_{12} \vdash \Delta_{13}}\,\vee_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_7, F_8 \vdash \Delta_{13}, F_{11}, F_{12}}\,\text{inv-th/ax} \quad \dfrac{}{- : \Delta_7, F_9 \vdash \Delta_{13}, F_{11}, F_{12}}\,\text{inv-th/ax}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_{12}}\,\vee_L \quad \dfrac{}{- : \Delta_7, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}}\,\text{ax/W}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}}\,\text{sCut} \quad \dfrac{}{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}}\,\text{ax/W}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}\,\text{sCut}$$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_8 \quad h_3 : \Delta_7, F_{11} \vdash \Delta_{12}, F_8}{\bullet h_3 : \Delta_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8}\,\vee_L \quad \dfrac{h_9 : \Delta_7, F_8, F_{10} \vdash \Delta_{12} \quad h_9 : \Delta_7, F_8, F_{11} \vdash \Delta_{12}}{\bullet h_9 : (\Delta_7, F_{10} \vee F_{11}), F_8 \vdash \Delta_{12}}\,\vee_L}{- : \Delta_7, F_{10} \vee F_{11} \vdash \Delta_{12}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_8}\,\text{ax/W} \quad \dfrac{\dfrac{h_9 : \Delta_7, F_{10}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_7, F_{10}, F_8 \vdash \Delta_{12}}\,\text{ax/W}}{}\,H}{- : \Delta_7, F_{10} \vdash \Delta_{12}}\,\text{hCut} \qquad \dfrac{\dfrac{}{h_3 : \Delta_7, F_{11} \vdash \Delta_{12}, F_8}\,\text{ax/W} \quad \dfrac{\dfrac{h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{12}}\,\text{ax/W}}{}\,H}{- : \Delta_7, F_{11} \vdash \Delta_{12}}\,\text{hCut}}{- : \Delta_7, F_{10} \vee F_{11} \vdash \Delta_{12}}\ \vee_L$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_8 \vdash \Delta_{11}, \perp \quad h_3 : \Delta_7, F_9 \vdash \Delta_{11}, \perp}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}, \perp}\,\vee_L \quad \dfrac{}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), \perp \vdash \Delta_{11}}\,\perp_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_7, F_8 \vdash \perp, \Delta_{11}}\,\text{ax/W} \quad \dfrac{}{\bullet h_{10} : \perp, \Delta_7, F_8 \vdash \Delta_{11}}\,\perp_L}{- : \Delta_7, F_8 \vdash \Delta_{11}}\,\text{hCut} \qquad \dfrac{\dfrac{}{h_3 : \Delta_7, F_9 \vdash \perp, \Delta_{11}}\,\text{ax/W} \quad \dfrac{}{\bullet h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}}\,\perp_L}{- : \Delta_7, F_9 \vdash \Delta_{11}}\,\text{hCut}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}\ \vee_L$$

$$\dfrac{\dfrac{h_3 : (\perp, \Delta_{12}), F_8 \vdash \Delta_{11}, F_7 \quad h_3 : (\perp, \Delta_{12}), F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}, F_7}\,\vee_L \quad \dfrac{}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}}\,\perp_L}{- : (\perp, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}}\ \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \perp, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}}\ \perp_L$$

- Case rule I

$$\dfrac{\dfrac{h_3 : \Delta_7, F_8 \vdash (\Delta_{12}, p_{11}), p_{11} \quad h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, p_{11}), p_{11}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash (\Delta_{12}, p_{11}), p_{11}} \vee_L \quad \dfrac{}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), p_{11} \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, p_{11}} \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{\dfrac{\dfrac{}{h_3 : \Delta_7, F_8 \vdash \Delta_{12}, p_{11}, p_{11}} \text{ax/W} \quad \dfrac{}{\bullet h_{10} : \Delta_7, F_8, p_{11} \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_8 \vdash \Delta_{12}, p_{11}} \text{hCut} \quad \dfrac{\dfrac{}{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}, p_{11}} \text{ax/W} \quad \dfrac{}{\bullet h_{10} : \Delta_7, F_9, p_{11} \vdash \Delta_{12}, p_{11}} I}{- : \Delta_7, F_9 \vdash \Delta_{12}, p_{11}} \text{hCut}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, p_{11}}} \vee_L$$

$$\dfrac{\dfrac{h_3 : (\Delta_{13}, p_{11}), F_8 \vdash (\Delta_{12}, p_{11}), F_7 \quad h_3 : (\Delta_{13}, p_{11}), F_9 \vdash (\Delta_{12}, p_{11}), F_7}{\bullet h_3 : (\Delta_{13}, p_{11}), F_8 \vee F_9 \vdash (\Delta_{12}, p_{11}), F_7} \vee_L \quad \dfrac{}{\bullet h_{10} : ((\Delta_{13}, p_{11}), F_8 \vee F_9), F_7 \vdash \Delta_{12}, p_{11}} I}{- : (\Delta_{13}, p_{11}), F_8 \vee F_9 \vdash \Delta_{12}, p_{11}} \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{}{- : \Delta_{13}, p_{11}, F_8 \vee F_9 \vdash \Delta_{12}, p_{11}} I}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{h_3 : \Delta_7, F_8 \vdash \Delta_{11}, \top \quad h_3 : \Delta_7, F_9 \vdash \Delta_{11}, \top}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}, \top} \vee_L \quad \dfrac{h_{10} : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), \top \vdash \Delta_{11}} \top_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \text{ax/W}}$$

$$\dfrac{\dfrac{h_3 : (\top, \Delta_{12}), F_8 \vdash \Delta_{11}, F_7 \quad h_3 : (\top, \Delta_{12}), F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}, F_7} \vee_L \quad \dfrac{h_{10} : \Delta_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}} \top_L}{- : (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}} \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{\dfrac{}{\bullet h_3 : \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}, F_7} \text{ax/W} \quad \dfrac{}{h_{10} : \top, \Delta_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{11}} \text{ax/W}}{- : \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}} \text{hCut}}$$

8.9 Status of $\perp_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{}{\bullet h_3 : \perp, \Delta_5 \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_6} \perp_L \quad \dfrac{h_7 : \perp, \Delta_5, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\perp, \Delta_5), F_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \perp_L}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet h_3 : \perp, \Delta_5 \vdash (\Delta_8, F_9 \wedge F_{10}), F_6} \perp_L \quad \dfrac{h_7 : \perp, \Delta_5, F_6 \vdash \Delta_8, F_9 \quad h_7 : \perp, \Delta_5, F_6 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\perp, \Delta_5), F_6 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \wedge F_{10}} \perp_L}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet h_3 : \perp, \Delta_5 \vdash (\Delta_8, F_9 \vee F_{10}), F_6} \perp_L \quad \dfrac{h_7 : \perp, \Delta_5, F_6 \vdash \Delta_8, F_9, F_{10}}{\bullet h_7 : (\perp, \Delta_5), F_6 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut}$$

$$\overset{\rightsquigarrow}{\dfrac{}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \vee F_{10}} \perp_L}$$

- Case rule $\perp_R$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash (\bot, \Delta_8), \mathtt{F}_6} \;\bot_L \quad \dfrac{\mathtt{h}_7 : \bot, \Delta_5, \mathtt{F}_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\bot, \Delta_5), \mathtt{F}_6 \vdash \bot, \Delta_8} \;\bot_R}{- : \bot, \Delta_5 \vdash \bot, \Delta_8} \;\text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \bot, \Delta_8} \;\bot_L$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash (\top, \Delta_8), \mathtt{F}_6} \;\bot_L \quad \dfrac{}{\bullet\mathtt{h}_7 : (\bot, \Delta_5), \mathtt{F}_6 \vdash \top, \Delta_8} \;\top_R}{- : \bot, \Delta_5 \vdash \top, \Delta_8} \;\text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \top, \Delta_8} \;\top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5} \;\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_7 \quad \mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9} \;\rightarrow_L}{- : \bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9} \;\text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9} \;\bot_L$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8} \;\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_5, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_7 \quad \mathtt{h}_6 : \bot, \Delta_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9} \;\rightarrow_L}{- : \bot, \Delta_5 \vdash \Delta_9} \;\text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_9} \;\bot_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5} \;\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9} \;\wedge_L}{- : \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9} \;\text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9} \;\bot_L$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8} \;\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_5, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9} \;\wedge_L}{- : \bot, \Delta_5 \vdash \Delta_9} \;\text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_9} \;\bot_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5} \;\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vdash \Delta_9 \quad \mathtt{h}_6 : \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9} \;\vee_L}{- : \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9} \;\text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9} \;\bot_L$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8} \;\bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_5, \mathtt{F}_7 \vdash \Delta_9 \quad \mathtt{h}_6 : \bot, \Delta_5, \mathtt{F}_8 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9} \;\vee_L}{- : \bot, \Delta_5 \vdash \Delta_9} \;\text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_9} \;\bot_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_8, \mathtt{F}_6} \;\bot_L \quad \dfrac{}{\bullet\mathtt{h}_7 : (\bot, \Delta_5), \mathtt{F}_6 \vdash \Delta_8} \;\bot_L}{- : \bot, \Delta_5 \vdash \Delta_8} \;\text{Cut}$$
$$\rightsquigarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_8} \;\bot_L$$

- Case rule I

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_7} \; {\scriptstyle \bot_L} \quad \cfrac{}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7} \; {\scriptstyle I}}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{p}_7} \; {\scriptstyle \text{Cut}}$$
$$\rightsquigarrow$$
$$\cfrac{}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{p}_7} \; {\scriptstyle \bot_L}$$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_9, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_5} \; {\scriptstyle \bot_L} \quad \cfrac{}{\bullet \mathtt{h}_6 : (\bot, \Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \Delta_8, \mathtt{p}_7} \; {\scriptstyle I}}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7} \; {\scriptstyle \text{Cut}}$$
$$\rightsquigarrow$$
$$\cfrac{}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7} \; {\scriptstyle \bot_L}$$

- Case rule $\top_L$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_7, \top} \; {\scriptstyle \bot_L} \quad \cfrac{\mathtt{h}_6 : \bot, \Delta_5 \vdash \Delta_7}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \top \vdash \Delta_7} \; {\scriptstyle \top_L}}{- : \bot, \Delta_5 \vdash \Delta_7} \; {\scriptstyle \text{Cut}}$$
$$\rightsquigarrow$$
$$\cfrac{}{- : \bot, \Delta_5 \vdash \Delta_7} \; {\scriptstyle \bot_L}$$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_3 : \bot, \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5} \; {\scriptstyle \bot_L} \quad \cfrac{\mathtt{h}_6 : \bot, \Delta_8, \mathtt{F}_5 \vdash \Delta_7}{\bullet \mathtt{h}_6 : (\bot, \top, \Delta_8), \mathtt{F}_5 \vdash \Delta_7} \; {\scriptstyle \top_L}}{- : \bot, \top, \Delta_8 \vdash \Delta_7} \; {\scriptstyle \text{Cut}}$$
$$\rightsquigarrow$$
$$\cfrac{}{- : \bot, \top, \Delta_8 \vdash \Delta_7} \; {\scriptstyle \bot_L}$$

8.10 Status of I: `OK`

- Case rule $\rightarrow_R$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_8 \vdash ((\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{p}_8), \mathtt{F}_7} \; {\scriptstyle I} \quad \cfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{F}_{10}, \mathtt{p}_8 \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_8), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{p}_8} \; {\scriptstyle \rightarrow_R}}{- : \Delta_6, \mathtt{p}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{p}_8} \; {\scriptstyle \text{Cut}}$$
$$\rightsquigarrow$$
$$\cfrac{}{- : \Delta_6, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{p}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}} \; {\scriptstyle I}$$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}), \mathtt{p}_6} \; {\scriptstyle I} \quad \cfrac{\mathtt{h}_7 : \Delta_5, \mathtt{F}_9, \mathtt{p}_6, \mathtt{p}_6 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}} \; {\scriptstyle \rightarrow_R}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}} \; {\scriptstyle \text{Cut}}$$
$$\rightsquigarrow$$
$$\cfrac{\cfrac{\cfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_9, \mathtt{p}_6 \vdash \mathtt{F}_{10}, \mathtt{p}_6} \; {\scriptstyle I} \quad \cfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_9, \mathtt{p}_6, \mathtt{p}_6 \vdash \mathtt{F}_{10}} \; {\scriptstyle \text{ax/W}}}{- : \Delta_5, \mathtt{F}_9, \mathtt{p}_6 \vdash \mathtt{F}_{10}} \; {\scriptstyle \text{hCut}}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}} \; {\scriptstyle \rightarrow_R}$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_8 \vdash ((\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_8), \mathtt{F}_7} \; {\scriptstyle I} \quad \cfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{F}_{10}, \mathtt{p}_8 \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_7, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{F}_{11}, \mathtt{p}_8}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_8), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_8} \; {\scriptstyle \wedge_R}}{- : \Delta_6, \mathtt{p}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_8} \; {\scriptstyle \text{Cut}}$$
$$\rightsquigarrow$$
$$\cfrac{}{- : \Delta_6, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{p}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}} \; {\scriptstyle I}$$

$$\cfrac{\cfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{p}_6} \; {\scriptstyle I} \quad \cfrac{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \quad \mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}} \; {\scriptstyle \wedge_R}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}} \; {\scriptstyle \text{Cut}}$$
$$\rightsquigarrow$$
$$\cfrac{\cfrac{\cfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9, \mathtt{p}_6} \; {\scriptstyle I} \quad \cfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9} \; {\scriptstyle \text{ax/W}}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9} \; {\scriptstyle \text{hCut}} \quad \cfrac{\cfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{p}_6} \; {\scriptstyle I} \quad \cfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}} \; {\scriptstyle \text{ax/W}}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}} \; {\scriptstyle \text{hCut}}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}} \; {\scriptstyle \wedge_R}$$

- Case rule $\vee_R$

$$
\dfrac{
\dfrac{}{\bullet h_2 : \Delta_6, p_8 \vdash ((\Delta_{12}, F_{10} \vee F_{11}), p_8), F_7} \ I
\qquad
\dfrac{h_9 : \Delta_6, F_7, p_8 \vdash \Delta_{12}, F_{10}, F_{11}, p_8}{\bullet h_9 : (\Delta_6, p_8), F_7 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8} \ \vee_R
}{
- : \Delta_6, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8
} \ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\dfrac{}{- : \Delta_6, p_8 \vdash \Delta_{12}, p_8, F_{10} \vee F_{11}} \ I
$$

$$
\dfrac{
\dfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \vee F_{10}), p_6} \ I
\qquad
\dfrac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_9, F_{10}}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \vee F_{10}} \ \vee_R
}{
- : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}
} \ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\dfrac{
\dfrac{
\dfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9, p_6} \ I
\qquad
\dfrac{}{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_{10}, F_9} \ \text{ax/W}
}{
- : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9
} \ \text{hCut}
}{
- : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}
} \ \vee_R
$$

- Case rule $\perp_R$

$$
\dfrac{
\dfrac{}{\bullet h_2 : \Delta_6, p_8 \vdash ((\perp, \Delta_{10}), p_8), F_7} \ I
\qquad
\dfrac{h_9 : \Delta_6, F_7, p_8 \vdash \Delta_{10}, p_8}{\bullet h_9 : (\Delta_6, p_8), F_7 \vdash (\perp, \Delta_{10}), p_8} \ \perp_R
}{
- : \Delta_6, p_8 \vdash (\perp, \Delta_{10}), p_8
} \ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\dfrac{}{- : \Delta_6, p_8 \vdash \perp, \Delta_{10}, p_8} \ I
$$

$$
\dfrac{
\dfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\perp, \Delta_8), p_6} \ I
\qquad
\dfrac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \perp, \Delta_8} \ \perp_R
}{
- : \Delta_5, p_6 \vdash \perp, \Delta_8
} \ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\dfrac{
\dfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash \perp, \Delta_8, p_6} \ \text{ax/W}
\qquad
\dfrac{}{h_7 : \Delta_5, p_6, p_6 \vdash \perp, \Delta_8} \ \text{ax/W}
}{
- : \Delta_5, p_6 \vdash \perp, \Delta_8
} \ \text{hCut}
$$

- Case rule $\top_R$

$$
\dfrac{
\dfrac{}{\bullet h_2 : \Delta_6, p_8 \vdash ((\top, \Delta_{10}), p_8), F_7} \ I
\qquad
\dfrac{}{\bullet h_9 : (\Delta_6, p_8), F_7 \vdash (\top, \Delta_{10}), p_8} \ \top_R
}{
- : \Delta_6, p_8 \vdash (\top, \Delta_{10}), p_8
} \ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\dfrac{}{- : \Delta_6, p_8 \vdash \top, \Delta_{10}, p_8} \ \top_R
$$

$$
\dfrac{
\dfrac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\top, \Delta_8), p_6} \ I
\qquad
\dfrac{}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \top, \Delta_8} \ \top_R
}{
- : \Delta_5, p_6 \vdash \top, \Delta_8
} \ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\dfrac{}{- : \Delta_5, p_6 \vdash \top, \Delta_8} \ \top_R
$$

- Case rule $\rightarrow_L$

$$
\dfrac{
\dfrac{}{\bullet h_2 : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash (\Delta_8, p_7), F_6} \ I
\qquad
\dfrac{h_9 : \Delta_{12}, F_6, p_7, F_{10} \rightarrow F_{11} \vdash \Delta_8, F_{10}, p_7 \quad h_9 : \Delta_{12}, F_6, F_{11}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \rightarrow F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \ \rightarrow_L
}{
- : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash \Delta_8, p_7
} \ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\dfrac{}{- : \Delta_{12}, p_7, F_{10} \rightarrow F_{11} \vdash \Delta_8, p_7} \ I
$$

$$
\dfrac{
\dfrac{}{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \rightarrow F_{11}} \ I
\qquad
\dfrac{h_9 : \Delta_6, p_7, F_{10} \rightarrow F_{11} \vdash \Delta_8, F_{10}, p_7 \quad h_9 : \Delta_6, F_{11}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \rightarrow F_{11} \vdash \Delta_8, p_7} \ \rightarrow_L
}{
- : \Delta_6, p_7 \vdash \Delta_8, p_7
} \ \text{Cut}
$$

$$
\rightsquigarrow
$$

$$
\dfrac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \ I
$$

$$\dfrac{\overline{\bullet h_1 : (\Delta_{10}, F_7 \to F_8), p_5 \vdash \Delta_9, p_5} \; I \quad \dfrac{h_6 : \Delta_{10}, p_5, p_5, F_7 \to F_8 \vdash \Delta_9, F_7 \quad h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \to F_8), p_5), p_5 \vdash \Delta_9} \to_L}{- : (\Delta_{10}, F_7 \to F_8), p_5 \vdash \Delta_9} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_{10}, p_5, F_7 \to F_8 \vdash \Delta_9, F_7, p_5} \; \text{ax/W} \quad \overline{h_6 : \Delta_{10}, p_5, p_5, F_7 \to F_8 \vdash \Delta_9, F_7} \; \text{ax/W}}{- : \Delta_{10}, p_5, F_7 \to F_8 \vdash \Delta_9, F_7} \; \text{hCut} \quad \dfrac{\overline{\bullet h_1 : \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5} \; I \quad \overline{h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9} \; \text{ax/W}}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9} \; \text{hCut}}{- : \Delta_{10}, p_5, F_7 \to F_8 \vdash \Delta_9} \to_L$$

- Case rule $\wedge_L$

$$\dfrac{\overline{\bullet h_2 : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash (\Delta_8, p_7), F_6} \; I \quad \dfrac{h_9 : \Delta_{12}, F_6, F_{10}, F_{11}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \wedge F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \wedge_L}{- : (\Delta_{12}, F_{10} \wedge F_{11}), p_7 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{12}, p_7, F_{10} \wedge F_{11} \vdash \Delta_8, p_7} \; I$$

$$\dfrac{\overline{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \wedge F_{11}} \; I \quad \dfrac{h_9 : \Delta_6, F_{10}, F_{11}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \wedge F_{11} \vdash \Delta_8, p_7} \wedge_L}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; I$$

$$\dfrac{\overline{\bullet h_1 : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9, p_5} \; I \quad \dfrac{h_6 : \Delta_{10}, F_7, F_8, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \wedge F_8), p_5), p_5 \vdash \Delta_9} \wedge_L}{- : (\Delta_{10}, F_7 \wedge F_8), p_5 \vdash \Delta_9} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_{10}, F_7, F_8, p_5 \vdash \Delta_9, p_5} \; I \quad \overline{h_6 : \Delta_{10}, F_7, F_8, p_5, p_5 \vdash \Delta_9} \; \text{ax/W}}{- : \Delta_{10}, F_7, F_8, p_5 \vdash \Delta_9} \; \text{hCut}}{- : \Delta_{10}, p_5, F_7 \wedge F_8 \vdash \Delta_9} \wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\overline{\bullet h_2 : (\Delta_{12}, F_{10} \vee F_{11}), p_7 \vdash (\Delta_8, p_7), F_6} \; I \quad \dfrac{h_9 : \Delta_{12}, F_6, F_{10}, p_7 \vdash \Delta_8, p_7 \quad h_9 : \Delta_{12}, F_6, F_{11}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \vee F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \vee_L}{- : (\Delta_{12}, F_{10} \vee F_{11}), p_7 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_{12}, p_7, F_{10} \vee F_{11} \vdash \Delta_8, p_7} \; I$$

$$\dfrac{\overline{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \vee F_{11}} \; I \quad \dfrac{h_9 : \Delta_6, F_{10}, p_7 \vdash \Delta_8, p_7 \quad h_9 : \Delta_6, F_{11}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \vee F_{11} \vdash \Delta_8, p_7} \vee_L}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; I$$

$$\dfrac{\overline{\bullet h_1 : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9, p_5} \; I \quad \dfrac{h_6 : \Delta_{10}, F_7, p_5, p_5 \vdash \Delta_9 \quad h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \vee F_8), p_5), p_5 \vdash \Delta_9} \vee_L}{- : (\Delta_{10}, F_7 \vee F_8), p_5 \vdash \Delta_9} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{\overline{\bullet h_1 : \Delta_{10}, F_7, p_5 \vdash \Delta_9, p_5} \; I \quad \overline{h_6 : \Delta_{10}, F_7, p_5, p_5 \vdash \Delta_9} \; \text{ax/W}}{- : \Delta_{10}, F_7, p_5 \vdash \Delta_9} \; \text{hCut} \quad \dfrac{\overline{\bullet h_1 : \Delta_{10}, F_8, p_5 \vdash \Delta_9, p_5} \; I \quad \overline{h_6 : \Delta_{10}, F_8, p_5, p_5 \vdash \Delta_9} \; \text{ax/W}}{- : \Delta_{10}, F_8, p_5 \vdash \Delta_9} \; \text{hCut}}{- : \Delta_{10}, p_5, F_7 \vee F_8 \vdash \Delta_9} \vee_L$$

- Case rule $\perp_L$

$$\dfrac{\overline{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \perp} \; I \quad \overline{\bullet h_9 : (\Delta_6, p_7), \perp \vdash \Delta_8, p_7} \; \perp_L}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; I$$

$$\dfrac{\overline{\bullet h_2 : (\bot, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6} \; I \quad \overline{\bullet h_9 : ((\bot, \Delta_{10}), p_7), F_6 \vdash \Delta_8, p_7} \; \bot_L}{- : (\bot, \Delta_{10}), p_7 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{}{- : \bot, \Delta_{10}, p_7 \vdash \Delta_8, p_7} \; \bot_L}$$

$$\dfrac{\overline{\bullet h_1 : (\bot, \Delta_8), p_5 \vdash \Delta_7, p_5} \; I \quad \overline{\bullet h_6 : ((\bot, \Delta_8), p_5), p_5 \vdash \Delta_7} \; \bot_L}{- : (\bot, \Delta_8), p_5 \vdash \Delta_7} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{}{- : \bot, \Delta_8, p_5 \vdash \Delta_7} \; \bot_L}$$

- Case rule I

$$\dfrac{\overline{\bullet h_2 : \Delta_6, p_7 \vdash ((\Delta_{10}, p_9), p_7), p_9} \; I \quad \overline{\bullet h_8 : (\Delta_6, p_7), p_9 \vdash (\Delta_{10}, p_9), p_7} \; I}{- : \Delta_6, p_7 \vdash (\Delta_{10}, p_9), p_7} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{}{- : \Delta_6, p_7 \vdash \Delta_{10}, p_7, p_9} \; I}$$

$$\dfrac{\overline{\bullet h_2 : (\Delta_{11}, p_9), p_7 \vdash ((\Delta_{10}, p_9), p_7), F_6} \; I \quad \overline{\bullet h_8 : ((\Delta_{11}, p_9), p_7), F_6 \vdash (\Delta_{10}, p_9), p_7} \; I}{- : (\Delta_{11}, p_9), p_7 \vdash (\Delta_{10}, p_9), p_7} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{}{- : \Delta_{11}, p_7, p_9 \vdash \Delta_{10}, p_7, p_9} \; I}$$

$$\dfrac{\overline{\bullet h_2 : \Delta_6, p_9 \vdash (\Delta_{10}, p_9), F_7} \; I \quad \overline{\bullet h_8 : (\Delta_6, p_9), F_7 \vdash \Delta_{10}, p_9} \; I}{- : \Delta_6, p_9 \vdash \Delta_{10}, p_9} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{}{- : \Delta_6, p_9 \vdash \Delta_{10}, p_9} \; I}$$

$$\dfrac{\overline{\bullet h_1 : \Delta_5, p_7 \vdash (\Delta_8, p_7), p_7} \; I \quad \overline{\bullet h_6 : (\Delta_5, p_7), p_7 \vdash \Delta_8, p_7} \; I}{- : \Delta_5, p_7 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{}{- : \Delta_5, p_7 \vdash \Delta_8, p_7} \; I}$$

$$\dfrac{\overline{\bullet h_1 : (\Delta_9, p_7), p_5 \vdash (\Delta_8, p_7), p_5} \; I \quad \overline{\bullet h_6 : ((\Delta_9, p_7), p_5), p_5 \vdash \Delta_8, p_7} \; I}{- : (\Delta_9, p_7), p_5 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{}{- : \Delta_9, p_5, p_7 \vdash \Delta_8, p_7} \; I}$$

- Case rule $\top_L$

$$\dfrac{\overline{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \top} \; I \quad \dfrac{h_9 : \Delta_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), \top \vdash \Delta_8, p_7} \; \top_L}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \; I}$$

$$\dfrac{\overline{\bullet h_2 : (\top, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6} \; I \quad \dfrac{h_9 : \Delta_{10}, F_6, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\top, \Delta_{10}), p_7), F_6 \vdash \Delta_8, p_7} \; \top_L}{- : (\top, \Delta_{10}), p_7 \vdash \Delta_8, p_7} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{}{- : \top, \Delta_{10}, p_7 \vdash \Delta_8, p_7} \; I}$$

$$\dfrac{\overline{\bullet h_1 : (\top, \Delta_8), p_5 \vdash \Delta_7, p_5} \; I \quad \dfrac{h_6 : \Delta_8, p_5, p_5 \vdash \Delta_7}{\bullet h_6 : ((\top, \Delta_8), p_5), p_5 \vdash \Delta_7} \; \top_L}{- : (\top, \Delta_8), p_5 \vdash \Delta_7} \; \text{Cut}$$

$$\overset{\leadsto}{\dfrac{\dfrac{}{\bullet h_1 : \top, \Delta_8, p_5 \vdash \Delta_7, p_5} \; I \quad \dfrac{}{h_6 : \top, \Delta_8, p_5, p_5 \vdash \Delta_7} \; \text{ax/W}}{- : \top, \Delta_8, p_5 \vdash \Delta_7} \; \text{hCut}}$$

8.11 Status of $\top_L$: OK

- Case rule $\to_R$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_5 \vdash (\Delta_8, F_9 \to F_{10}), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, F_9 \to F_{10}), F_6} \ \top_L \qquad \cfrac{h_7 : \top, \Delta_5, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \Delta_8, F_9 \to F_{10}} \ \to_R}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \to F_{10}} \ \text{Cut}}{\cfrac{\cfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_8, F_6, F_9 \to F_{10}} \ \text{ax/W} \quad \cfrac{}{\bullet h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9 \to F_{10}} \ \text{ax/W}}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \to F_{10}} \ \text{hCut}} \ \rightsquigarrow$$

- Case rule $\wedge_R$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_5 \vdash (\Delta_8, F_9 \wedge F_{10}), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, F_9 \wedge F_{10}), F_6} \ \top_L \qquad \cfrac{h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9 \quad h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \Delta_8, F_9 \wedge F_{10}} \ \wedge_R}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \wedge F_{10}} \ \text{Cut}}{\cfrac{\cfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_8, F_6, F_9 \wedge F_{10}} \ \text{ax/W} \quad \cfrac{}{\bullet h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9 \wedge F_{10}} \ \text{ax/W}}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \wedge F_{10}} \ \text{hCut}} \ \rightsquigarrow$$

- Case rule $\vee_R$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_5 \vdash (\Delta_8, F_9 \vee F_{10}), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, F_9 \vee F_{10}), F_6} \ \top_L \qquad \cfrac{h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9, F_{10}}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \Delta_8, F_9 \vee F_{10}} \ \vee_R}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \vee F_{10}} \ \text{Cut}}{\cfrac{\cfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_8, F_6, F_9 \vee F_{10}} \ \text{ax/W} \quad \cfrac{}{\bullet h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9 \vee F_{10}} \ \text{ax/W}}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \vee F_{10}} \ \text{hCut}} \ \rightsquigarrow$$

- Case rule $\perp_R$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_5 \vdash (\perp, \Delta_8), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\perp, \Delta_8), F_6} \ \top_L \qquad \cfrac{h_7 : \top, \Delta_5, F_6 \vdash \Delta_8}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \perp, \Delta_8} \ \perp_R}{- : \top, \Delta_5 \vdash \perp, \Delta_8} \ \text{Cut}}{\cfrac{\cfrac{}{h_3 : \top, \Delta_5 \vdash \perp, \Delta_8, F_6} \ \text{ax/W} \quad \cfrac{}{\bullet h_7 : \top, \Delta_5, F_6 \vdash \perp, \Delta_8} \ \text{ax/W}}{- : \top, \Delta_5 \vdash \perp, \Delta_8} \ \text{hCut}} \ \rightsquigarrow$$

- Case rule $\top_R$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_5 \vdash (\top, \Delta_8), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\top, \Delta_8), F_6} \ \top_L \qquad \cfrac{}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \top, \Delta_8} \ \top_R}{- : \top, \Delta_5 \vdash \top, \Delta_8} \ \text{Cut}}{\cfrac{}{- : \top, \Delta_5 \vdash \top, \Delta_8} \ \top_R} \ \rightsquigarrow$$

- Case rule $\to_L$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_{10}, F_7 \to F_8 \vdash \Delta_9, F_5}{\bullet h_3 : \top, \Delta_{10}, F_7 \to F_8 \vdash \Delta_9, F_5} \ \top_L \qquad \cfrac{h_6 : \top, \Delta_{10}, F_5, F_7 \to F_8 \vdash \Delta_9, F_7 \quad h_6 : \top, \Delta_{10}, F_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \to F_8), F_5 \vdash \Delta_9} \ \to_L}{- : \top, \Delta_{10}, F_7 \to F_8 \vdash \Delta_9} \ \text{Cut}}{\cfrac{\cfrac{}{h_3 : \top, \Delta_{10}, F_7 \to F_8 \vdash \Delta_9, F_5} \ \text{ax/W} \quad \cfrac{}{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \to F_8 \vdash \Delta_9} \ \text{ax/W}}{- : \top, \Delta_{10}, F_7 \to F_8 \vdash \Delta_9} \ \text{hCut}} \ \rightsquigarrow$$

$$\cfrac{\cfrac{\cfrac{h_3 : \Delta_5 \vdash \Delta_9, F_7 \to F_8}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \to F_8} \ \top_L \qquad \cfrac{h_6 : \top, \Delta_5, F_7 \to F_8 \vdash \Delta_9, F_7 \quad h_6 : \top, \Delta_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \to F_8 \vdash \Delta_9} \ \to_L}{- : \top, \Delta_5 \vdash \Delta_9} \ \text{Cut}}{\cfrac{\cfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \to F_8} \ \text{ax/W} \quad \cfrac{}{\bullet h_6 : \top, \Delta_5, F_7 \to F_8 \vdash \Delta_9} \ \text{ax/W}}{- : \top, \Delta_5 \vdash \Delta_9} \ \text{hCut}} \ \rightsquigarrow$$

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- Case rule $\wedge_L$

$$\dfrac{\dfrac{h_3 : \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5}{\bullet h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \top_L \quad \dfrac{h_6 : \top, \Delta_{10}, F_5, F_7, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \wedge_L}{- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \text{ax/W} \quad \dfrac{}{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \wedge F_8 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{hCut}$$

$$\dfrac{\dfrac{h_3 : \Delta_5 \vdash \Delta_9, F_7 \wedge F_8}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8} \top_L \quad \dfrac{h_6 : \top, \Delta_5, F_7, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \wedge F_8 \vdash \Delta_9} \wedge_L}{- : \top, \Delta_5 \vdash \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8} \text{ax/W} \quad \dfrac{}{\bullet h_6 : \top, \Delta_5, F_7 \wedge F_8 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_5 \vdash \Delta_9} \text{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{h_3 : \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5}{\bullet h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \top_L \quad \dfrac{h_6 : \top, \Delta_{10}, F_5, F_7 \vdash \Delta_9 \quad h_6 : \top, \Delta_{10}, F_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \vee F_8), F_5 \vdash \Delta_9} \vee_L}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \text{ax/W} \quad \dfrac{}{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \vee F_8 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{hCut}$$

$$\dfrac{\dfrac{h_3 : \Delta_5 \vdash \Delta_9, F_7 \vee F_8}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \top_L \quad \dfrac{h_6 : \top, \Delta_5, F_7 \vdash \Delta_9 \quad h_6 : \top, \Delta_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \vee F_8 \vdash \Delta_9} \vee_L}{- : \top, \Delta_5 \vdash \Delta_9} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \text{ax/W} \quad \dfrac{}{\bullet h_6 : \top, \Delta_5, F_7 \vee F_8 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_5 \vdash \Delta_9} \text{hCut}$$

- Case rule $\perp_L$

$$\dfrac{\dfrac{h_3 : \Delta_5 \vdash \Delta_7, \perp}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_7, \perp} \top_L \quad \dfrac{}{\bullet h_6 : (\top, \Delta_5), \perp \vdash \Delta_7} \perp_L}{- : \top, \Delta_5 \vdash \Delta_7} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_5 \vdash \perp, \Delta_7} \text{ax/W} \quad \dfrac{}{\bullet h_6 : \perp, \top, \Delta_5 \vdash \Delta_7} \perp_L}{- : \top, \Delta_5 \vdash \Delta_7} \text{hCut}$$

$$\dfrac{\dfrac{h_3 : \perp, \Delta_8 \vdash \Delta_7, F_5}{\bullet h_3 : \top, \perp, \Delta_8 \vdash \Delta_7, F_5} \top_L \quad \dfrac{}{\bullet h_6 : (\top, \perp, \Delta_8), F_5 \vdash \Delta_7} \perp_L}{- : \top, \perp, \Delta_8 \vdash \Delta_7} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \perp, \top, \Delta_8 \vdash \Delta_7} \perp_L$$

- Case rule I

$$\dfrac{\dfrac{h_3 : \Delta_5 \vdash (\Delta_8, p_7), p_7}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, p_7), p_7} \top_L \quad \dfrac{}{\bullet h_6 : (\top, \Delta_5), p_7 \vdash \Delta_8, p_7} I}{- : \top, \Delta_5 \vdash \Delta_8, p_7} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{\dfrac{}{h_3 : \top, \Delta_5 \vdash \Delta_8, p_7, p_7} \text{ax/W} \quad \dfrac{}{\bullet h_6 : \top, \Delta_5, p_7 \vdash \Delta_8, p_7} I}{- : \top, \Delta_5 \vdash \Delta_8, p_7} \text{hCut}$$

$$\dfrac{\dfrac{h_3 : \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5}{\bullet h_3 : \top, \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5} \top_L \quad \dfrac{}{\bullet h_6 : (\top, \Delta_9, p_7), F_5 \vdash \Delta_8, p_7} I}{- : \top, \Delta_9, p_7 \vdash \Delta_8, p_7} \text{Cut}$$

$$\rightsquigarrow$$

$$\dfrac{}{- : \top, \Delta_9, p_7 \vdash \Delta_8, p_7} I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h_3} : \Delta_5 \vdash \Delta_8, \mathtt{F_6}}{\bullet\mathtt{h_3} : \top, \Delta_5 \vdash \Delta_8, \mathtt{F_6}} \ \top_L \qquad \dfrac{\mathtt{h_7} : \Delta_5, \mathtt{F_6} \vdash \Delta_8}{\bullet\mathtt{h_7} : (\top, \Delta_5), \mathtt{F_6} \vdash \Delta_8} \ \top_L}{- : \top, \Delta_5 \vdash \Delta_8} \ \mathtt{Cut}$$

$$\leadsto$$

$$\dfrac{\dfrac{}{\mathtt{h_3} : \top, \Delta_5 \vdash \Delta_8, \mathtt{F_6}} \ \mathtt{ax/W} \qquad \dfrac{}{\bullet\mathtt{h_7} : \top, \Delta_5, \mathtt{F_6} \vdash \Delta_8} \ \mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_8} \ \mathtt{hCut}$$